Essays on Coalition Formation under Asymmetric Information

by

Daesik Lee

Dissertation submitted to the Faculty of the

Virginia Polytechnic Institute and State University

in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

in

Economics

APPROVED:

Jacques Cremer, Chairman

Herve Moulin

Yannis Ioannides

Catherine Eckel

David Salant

February, 1988

Blacksburg, Virginia

Essays on Coalition Formation under Asymmetric Information

by

Daesik Lee

Jacques Cremer Chairman

Economics

(ABSTRACT)

We consider the applicability of the Revelation Principle under the possibility of collusive behavior among players in some Bayesian framework. In doing this, since the coalition formation itself suffers information asymmetry problems, we assume that the coalition is formed if the colluding parties can successfully find some coalitional mechanism whose outcome is a set of messages in the original mechanism. Recently Cremer[1986] proposes a coalitional mechanism in the framework of the well known Vickrey-Clark-Groves mechanism. We assume that the agents successfully collude if they can find coalitional a mechanism such that (i) coalitional mechanism is incentive-compatible and (ii) the payoff of this mechanism is strictly Pareto-improving in terms of the agent's expected utility. Our analysis is undertaken in a one principal / two agent framework.

We first find that the Revelation Principle is still applicable in the pure adverse selection model. We then extend this result to a model with both adverse selection and moral hazard aspects. Finally, we consider a three-tier principal/ supervisor/ agent hierarchical organization, as in Tirole(1986). We explicitly present the coalitional mechanism as a side-contract between the supervisor and the agent. We apply the previous result of applicability of the Revelation Principle and characterize the coalition-proof mechanism. We find that the principal can design an optimal collusion free contract with some additional cost by specifying proper individual and coalitional incentive-compatibility conditions and individual rationality conditions. Moreover, we find that the results of Tirole(1986)'s paper hinge on the fact that he considers only " hard," verifiable, information.

Acknowledgements

To all those who have offered support and assistance in the completion of this dissertation I would like to extend my sincere thanks. Foremost, I thank my parents for their prayers and unending support over all these years. I also would like to thank to the members of my committee: Herve Moulin, Yannis Ionnides, David Salant and Catherine Eckel. Hans Haller, Stephen Sheppard, and Sangmoon Hahm also provided much appreciated guidence and encouragement.

This dissertation could not have been completed without the expertise and dedication of my committee chairman Jacques Cremer. His patience, kindness, and encouragement helped me achieve the level of confidence necessary to successfully complete this endeavor.

Finally, I dedicate this dissertation to my wife, , in token of my love and appreciation.

Table of Contents

CHAPTER I INTRODUCTION

The Walrasian general-equilibrium model of price mechanism is the foundation of economic theory. The two fundamental welfare theorems show the marvel of the price system, which was described as an invisible hand by Adam Smith. Hayek argued that the real power of the price system is its efficiency in communicating information " a system in which the knowledge of the relevant facts are dispersed among many people. " (Hayek [1945], p. 525) However, the market is not always as effective at transmitting information as Hayek argued. More precisely, this is because actual market does not always satisfies the heuristic description of the circumstances under which economic agents are " price takers ". In many incidence, the economic agents in a market try to use his information strategically. In a sense, this is not a matter of authenticity of the efficiency of competitive allocation; rather, it is a matter of its relevance of the relationship.

Some of the most exciting of the recent advances in microeconomic theory have been in the modeling of strategic behavior under asymmetric information. Asymmetric information in social and economic affairs comes mainly from the following two factors: First, one person can have private information which is not available to the others. Second, a person can have his private decision domain, or behavior that others cannot control or monitor. In other words, a person with private information cannot be compelled to reveal it unless he is given the proper incentive and similarly, he cannot be directed to choose any particular decision or action unless he is given some incentive to do so. Obviously, asymmetric information by itself does not necessarily cause problems.

However, as is pointed out in Laffont and Maskin [1982], asymmetric information does matter in a broad class of economic environment where one party's objectives do not coincide with those of the other party's and moreover, one party's utility depends on the other party's private information and/or behavior.[1]

In most of literature about asymmetric information problems, this economic environment has been described as a " Principal (planner, designer, government, or monopolist, depending on context) / Agent (member of society or of organizations) relationship. " The principal tries to solve this incentive problem by characterizing mechanisms (equivalently contracts or game forms) which can give the agents proper incentives to reveal their private decisions.

This design of the mechanism can, in general, be quite complicated, involving a specification of what strategies are feasible for each agent at each stage, what the agent knows at each stage, and how the final allocation depends on the whole history of signals of the agents. Fortunately, however, it is well known now that the Revelation Principle makes mechanism design drastically simple and technically feasible. In other words, using the Revelation Principle, we can characterize simply the optimal incentive mechanism by choosing a prespecified allocation rule that maximizes the principal's payoff subject to some incentive compatibility and individual rationality conditions properly specified. One major assumption in this approach, however, is that the agents cannot make binding agreements to cheat together. In other words, collusive behavior among the agents is assumed away in the analysis. However, this is quite a limiting assumption as we consider the real economic institutions. Many studies of organizations and bureaucracies have shown that collusive behavior, implicit or explicit, does exist and is often prominent. Then the natural question is how does this possible collusive behavior affect the design of incentive-compatible mechanisms? This question, which is truly of major importance, has hardly been studied at all, partly because of its analytical complexity.

Our research starts with the question, " If the agents are able to collude among themselves, is the Revelation Principle still applicable? Thus can we design a coalition-proof mechanism by characterizing prespecified allocation rules which maximize the principal's pay off subject to incentive compatibility condition for individuals and possible coalitions? " Another important limitation for this question is how the agents can successfully agree on some type of coalition where coalition formation itself suffers from the information asymmetry. Following Cremer [1986]'s concept of coalitional mechanism, we assume that the agents can successfully collude if they can find a coalitional mechanism where each colluding party is given proper incentives to tell the truth about the private information relevant to the objectives of coalition and moreover, the outcome of the coalition should be strictly Pareto-improving.

Chapter II and III are mainly devoted to this question and chapter IV was motivated by Tirole [1986]'s recent analysis about the effect of collusive behavior in multilateral hierarchies. Tirole's approach views the side contract (collusive behavior) as incorporated into the main contract. As is pointed out, this is not meant to be descriptive of real situation. In other words, in his approach, the coalition formation is only implicit. Chapter IV has two purposes: to present explicitly a coalitional mechanism which is implicit in Tirole's paper and to confirm the result of the second chapter; and to pursue his analysis with different assumptions. We consider soft (unverifiable) information, whereas Tirole confined his analysis to only hard (verifiable) information. The rest of chapter contains brief review of some ideas and concepts of incentive economics and the problem of coalition formation.

1. Models of asymmetric information

For the last two decades, the literature about the incentive and information problems has grown rapidly in various economic contexts. For example, most resource allocation problems such as public good provision, resource allocation system design, auctions and procurement, monopoly pricing and so on have been reexamined in this light, as well as implementation in social choice theory and contract design and classical moral hazard problem in insurance market, labor market etc. We cannot attempt to review such a vast literature, we will only survey some basic ideas and try to briefly explain the nature of these models.

There are several criteria to classify this literature. First, we can distinguish an incentive problem caused by pure private information from that caused by unobservability of the agent's behavior. For example, since an insurance company cannot distinguish a low risk person from high risk person, it cannot get insurers to reveal unfavorable information. This is called the problem of " adverse selection. " Also, the insurance company has difficulties getting fully insured individuals to exert enough effort for their insured loss. This is called the " moral hazard " problem. [2]

The theories of adverse selection and moral hazard share many common features, however, they have some important conceptual differences and therefore, the economic interpretation must be also different. The most important feature of a pure moral hazard model is that the agents are assumed risk averse and the outcome of agent's action is determined stochastically. Because of risk averse assumption, the agent's full residual-claimancy cannot be optimal (see Shavell [1979] for detail). Hence in this model, the agents are usually supposed to choose their decision before the state of nature is revealed. Multi-agent consideration extends to team theoretic approach (Holmstrom [1982]) and some tournament literature (Lazear and Rosen [1981] and Green and Stokey [1983]). At this moment, we will refer to McDonald's [1984] survey for further development and details.

Unlike a pure moral hazard model, most of pure adverse selection models assume that the agents are risk neutral for simplicity of analysis and that the state of nature (in this case private information of each agent) is already revealed but not observable to other players. Notice that even if the agents take actions in the model, it is basically considered as an adverse selection problem when the output is nonstochastically determined and observable to the principal (for example, see Sappington [1983] and Tirole [1986]).

However, many economic problems simultaneously involve these two factors : inability of the principal to observe the private information of the agent and to monitor his behavior. For example, in the owner-manager set up, the owner may be unable to observe the effort level of the manager and at the same time, some profitability parameters may be private knowledge to the manager. This direction of extension include the income tax model of Mirlees[1971], the literature on the new soviet incentive scheme (Weitzman [1976]) and recent papers by Baron and Holmstrom[1980], Baron[1982]. Laffont and Tirole [1985] and Picard [1985] treat the regulatory authority / regulated firm set up where the cost parameter is a private information to the firm and also a regulatory body cannot observe the effort level in production. And recently, McAfee and McMillan [1986] analyzed optimal procurement schemes under both moral hazard and adverse selection.

Second, if we consider a most simple case i.e., one principal / one agent set up, the principal tries to solve this asymmetry of information problem by designing a mechanism, a rule that specifies, in advance, his behavior (decisions on allocations, compensations and so on) on the basis of the agent's message and some future observation of the results of agents behavior.[3] Usually, the principal can design a mechanism considering the agent's possible optimal behavior and guaranteeing a minimal expected payoff to induce the agent to accept the mechanism. When he specifies a mechanism considering the agent's possible optimal behavior, he should impose proper incentives (restrictions) for the agent not to misrepresent his private information and not to shirk. This is called " incentive-compatibility condition " (or self- selection constraint) and the latter restriction is called "individual-rationality condition." Hence the principal's feasible mechanism is restricted by these two constraints. [4] However, if there is more than one agent, the optimal behavior of one agent will depend upon the other agent's behavior. Thus, with more than one agent, a mechanism induces a game among the agents and the principal optimizes his payoff subject to the agents being in equilibrium. In other words, incentive-compatibility condition in this case should be some equilibrium conditions of game where truth-telling or obedient behavior maximizes the agent's payoff.

In this one principal / multi agent set up or recently, in the multi principal / one agent set up, [5] there are several alternative hypotheses about the way in which an agent might act under this game situation. That is, an equilibrium concept is a way to resolve the agents' strategic uncertainty. Among others, dominant strategy equilibrium, Nash equilibrium and Bayesian Nash equilibrium are most frequently applied. The relevance of each equilibrium concept might differ according to the area of application. [6] Conceptually, the dominant strategy equilibrium is most appealing under asymmetric information, however, the problem is that the principal can hardly find mechanisms permitting both optimality and dominant strategy equilibrium.[7] Social choice theory has developed a vast literature using different equilibrium concepts. Implementation theory studies the design of a mechanism (game form) which always has at least one equilibrium, and whose possible outcomes in equilibrium all belong to the appropriate social choice set for the individual's true types. Muller and Satterthwaite[1985] provides a survey for the implementation in dominant strategy whereas Maskin[1985] and Myerson[1985] provides a survey for the implementation in Nash equilibrium and Bayesian Nash equilibrium respectively.

In the implementation literature, the idea of the Revelation Principle was observed by Gibbard [1977], Rosenthal [1978], Green and Laffont [1977], and proved by Myerson [1979] and fully exploited by Dasgupta, Hammond and Maskin [1979]. In the allocation literature, it was observed by Holmstrom [1977] and fully explained in extensive form game set up by Harris and Townsend [1981]. Since we will formally present this principle in the model of next two chapters, we just mention some conceptual problems of this idea.

As is pointed out by Dasgupta, Hammond and Maskin [1979], even if original mechanism has a unique equilibrium (or, alternatively, all equilibria generate the same payoff for the principal), the direct mechanism constructed from this original mechanism may have multiple equilibria, some of which give the principal a low payoff. For example, consider a revelation principle of implementation in dominant strategy. For any mechanism that implements a social choice rule in dominant strategies, there exists a direct mechanism which truthfully implements it in dominant strategies. If there exist several dominant strategy equilibria in this direct mechanism, there is nothing that guarantees that agents will choose the truth-telling dominant strategy. Furthermore, these alternative strategies need not correspond to dominant strategies in the original mechanism, which suggests that the direct mechanism given by the revelation principle may not implement the given social choice rule in dominant strategies. Repullo [1985] pointed out that the reason for this failure comes from the fact that the original mechanism may have alternative Nash or Bayesian Nash equilibria (under incomplete information) which cannot be ruled out a priori, and which may yield outcomes outside the social choice sets. Turnbell [1985] and Ma and Moore [1986]'s studies about the collusive behavior are originated from this multiplicity of equilibrium.

Myerson [1982] extends the revelation principle to the more general set up where the agents choose their private actions and have private information. Recently, McAfee and McMillan [1986] extend it to a case where it is costly for the principal to communicate with the agents. We have never observed any published or unpublished written work that studies the revelation principle where collusion is explicitly introduced.

2. Models of coalition formation

The consideration of collusive behavior in incentive theory certainly deserves some motivation. First, it has been observed by many organization theorists that the collusive behavior within organization does exist and is often prominent. For example, the work of Crozier [1963] and Dalton[1959] showed that the behavior of agents within the organization is under the strong influence of implicit collusive behavior such as reciprocation in general conduct, as well as some explicit form of collusive behavior, bribes. Recently, for example, Holmstrom and Tirole summarized the consideration of collusive behavior within the firm as followes:

> A major reason for multi-lateral contracting is that agents can enter into side-contracts with each other. On an informal basis this is commonplace in all organizations. Personal relationships and the like fall in this category. More

generally, reciprocation in the conduct of tasks represents side-contracting that cannot fully be controlled by a comprehensive contract. The most explicit form of side-transfers are bribes. They may be paid as monetary compensation for services or they may take more subtle forms, a promotion in exchange for another favor, for instance. It has been alleged that auditing firms occasionally obtain favorable contracts from their clients in exchange for good audits. Civil servants are known to have received lucrative job offers after they have quit their government jobs. The list could be extended. The point is that side-contracting in the form of bribes, personal relationships and promises of reciprocation are prevalent. How does this affect the design of incentive and tasks in a hierarchy ? (Holmstrom and Tirole [1987], pp. 93-94.)

Second and important motivation is that the recontracting the terms of existing contract can be interpreted as the formation of coalition (For this view, see Cremer and Riordan[1987]). Incentive theory studies various types of organizations which are seen as contractual, whether these contracts are implicit or explicit. Specifying a contract is viewed as a way to mitigate the incentive problems created by information asymmetry. This approach is widely used, however, as is pointed out in Cremer[1987], very little research has been conducted on the possibility of finding a general equilibrium in contracts. Moreover, some studies have been made to imbed personalized contract theory (such as principal/agent set up) into a competitive general equilibrium theory. See Hellwig[1987], Bester[1985] and Laffont and Tirole[1985], McAfee and McMillan[1986], and Riordan and Sappington[1985] for this direction of research.

The collusive behavior can be mainly represented by the manipulation of the information. There are several ways in which information may be manipulated: to conceal or ignore existing evidence; to distort the evidence or create false information. Ignoring or concealing relevant information might be considered as a implicit form of collusion. For example, sometimes information may be hard to dispose of. It may then be useful not to detain it:

Inside the firm, normal surprise was also a preventive of conflict. For example, safety and health inspectors usually telephoned in advance of visit so that they would not see unsafe practices on conditions they would feel obliged to report (Dalton[1959], p.48) [8]

Distorting the information can be considered as a more explicit form of collusive behavior. Accounting distortions are the conspicuous examples of this kind. Furthermore, Tirole[1986] emphasize the vertical structure of collusion in multilateral hierarchies like modern corporation and bureaucries. He introduces the principal / supervisor / agent three-layer hierarchy as a convenient abstraction of a hierarchical structure of a firm. We can think of many similar examples like shareholder/ manager/ worker, owner/ auditor/manager or voter/ government agency/ regulated firm etc. We will analyze this vertical collusion example in chapter IV. We briefly introduce some motivation of the study for collusion and emphasize the pervasiveness of collusive behavior, implicit or explicit, with in the organization.

This problem of collusion, however, has hardly been studied. In the rest of this section, we mention some literature about the study of coalition. Before we proceed, some remarks on the noncooperative tacit coalition from repeated game approach are in order. In a repeated game approach, the emphasis is given on the result that the agents can achieve more efficient outcome than the single period game by explicitly considering the repetition of their relationship. In other words, some collusive allocation (called trigger strategy) can be supported even under the noncooperative behavioral assumption by considering repeated time horizon and some discount rate of the future payoff. Friedman[1971],[1974] provides methodological grounds for this application of repeated game. Radner[1981] studies a tacit collusion of oligopoly under the perfect information and Green and Porter[1984] analyze the cartel formation under the price uncertainty using this repeated geme model. However, this line of research never mentions about how exactly the agents can agree on the trigger strategy at the outset of the game. McDonald[1984] provides some details of this literature.

The problem of coalition formation has been treated by a number of authors. Benett and Conn [1977] considers Groves mechanism and show that there does not exist any revelation mechanism which is immune to manipulation by coalition. Green and Laffont [1979] extends this result, attempting to find some way to ameliorate this situation. They introduce some cost of communication within the coalition, and show that for a fixed coalition, the expected gain to cheating compared to telling the truth decrease with the number of agents in the population. Then, considering the population as a random sample in a fixed distribution, they obtains the probability that the cheating coalition will exist with a per-capita gain large than any fixed number approaches to zero as the sample size grows.This gives some possibility that the coalition manipulation can be somewhat mitigated. Barber [1979] uses the same approach with Green and Laffont [1979] in a more generalized framework.

Recently, coalitions in some Bayesian context has been studied by Cremer and Riordan [1987] and Tirole [1986]. Cremer and Riordan [1987] consider the hierarchical contract which is a tree-like structure of transactions and communications. They solve for the optimal grand design and show that, by using expected externality payments, the optimal contract is immune to side-contracting which can be interpreted as coalition. The idea is that such payment forces each party to internalize the externality imposed by its decision on the other parties.

These studies, however, do not consider any problem of asymmetry of information within the coalition. In other words, they assume that coalitions face no information asymmetry when deciding upon a joint strategy. Recently, Cremer[1986] studies the coalition formation under asymmetric information in the Groves mechanism case. He introduces coalitional mechanisms in which colluding parties are given proper incentive to reveal their private information correctly and they can find strictly parto-improving payoff with coalition. Then, first, he shows same general negative result than the literature which assumes away problems of asymmetric information within coalitions; Groves mechanisms cannot at the same time be robust to coalitions of size 2 and to coalitions of size 3. However, he shows that there exist mechanisms robust to coalitions of size 2 when the coalition formation suffers from asymmetric information. Tirole[1986] studies the effect of collusive behavior within multilateral organizations. Specifically, he considers three-tier Principal/ Supervisor/ Agent hierarchy and characterizes the coalition free contract when the agent and the supervisor can collude about their reports to the principal. The approach in this study is to view the side-contracts(coalitions) as incorporated into the grand design. In other words, the principal try to design the main contract which leaves no opportunity for the agents to engage in future side-contracting opportunities. However, as is pointed out, this approach is not descriptive of the real situation.[9] In chapter IV, we consider same model with Tirole[1986] and explicitly consider the coalition formation mechanism, following Cremer[1986].

FOOTNOTES

1 Laffont and Maskin [1982] pointed out this noncoincidence of goals distinguishes incentive theory from the theory of teams (Marschak and Radner [1972]), which postulates identical objectives, but which otherwise shares many features with incentive problems. Also, the assumption that each party (sometimes as a surrogate for society itself) has well-defined objective function separates incentive theory from some social choice theory (Arrow [1951]). See Laffont and Maskin [1982], p 31.

2 Myerson [1985] modeled this incentive problem using a Bayesian game with incomplete information, as defined by Harsanyi [1967, 1968]. His Bayesian game corresponds to a model of moral hazard and Bayesian collective choice problem fits in a adverse selection model. " Bayesian incentive problem " considers these two aspects of information asymmetry simultaneously.

3 The term " mechanism " may vary depending on the area of application. Usually, mechanism applies in the allocation literature and the term " contract " is often used in specific market or firm, whereas " game form " or " voting scheme " applies in the social choice context. Sometimes, " incentive scheme " can be used for general purposes.

4 In this sense, the allocation which is feasible under these two constraints is called " second-best allocation " whereas the principal's full information allocation is first-best.

5 See Bernheim and Whinston [1985] , [1986] and Baron [1985] for this model.

[6] Interesting discussions about the problems of each solution concept in mechanism design are found in Laffont and Maskin [1982] and Groves [1982].

[7] In implementation literature, we can find several versions of impossibility theorems. Mostly, see Arrow [1963] and Gibbard- Satterwaite [1973].

[8] This is recited from Tirole[1985]. p 10.

[9] See Holmstrom and Tirole[1987] for detail.

CHAPTER II THE REVELATION PRINCIPLE AND COALITION FORMATION

1. Introduction

In economic theory, the literature about the problems of information and incentives has grown up rapidly for the last decade. The main problem in this literature is to characterize mechanisms that solve economic problems when the players have some private information and personal decision domains. This problem is mainly analyzed in the principal/agent paradigm (alternatively, the planner, or government and the members of the society depending on the context). That is, the principal tries to design an incentive-compatible mechanism which gives the agents proper incentives to reveal their private information and which influences their private decisions.

It is well understood, now, that we have a simple way to characterize an incentive-compatible mechanism by the "Revelation Principle". By this principle, we can confine our search of mechanisms to direct mechanisms and we can characterize a optimal incentive-compatible mechanism by maximizing the principal's utility (social welfare depending on the context) subject to some properly specified incentive-compatible constraints. The incentive literature is replete with both implicit and explicit applications of this Revelation Principle.

One major assumption in this approach, however, is that the agents play a noncooperative game. In other words, the possibility of collusive behavior among the agents is totally excluded. This is quite a limiting assumption as we consider real economic institutions. Recently, Tirole[1986] has emphasized the effect of collusive behavior in multilateral hierarchies like modern corporations and bureaucracies.[1] Considering the possibility of coalitions in a general incentive model is, of course, a major problem to be analyzed. A most important preliminary question is whether the Revelation Principle is still applicable or not.

In this chapter, the analysis is confined to a special case; one principal and two agents. However, the question itself is worth analyzing and the answer is not so obvious as it might appear at first blush. Moreover, we have one more complicated problem to consider. Under the presence of private information, we model this situation in a Bayesian setup. Then, coalition formation among the agents itself will suffer from problems of information asymmetry. Hence, agents should find some way to overcome this problem and successfully form a coalition. Following Cremer[1986], we assume that the agents use a coalitional mechanism where each colluding party is given proper incentives to reveal his private information correctly and the coalition is strictly Pareto-improving.[2]

Before proceeding, we want to mention some related works. Demsky and Sappington[1984] considered a model in which agents can collude and decide which equilibrium to play. No side-payments are considered in this model, because both agents prefer a different equilibrium from the principal. This possibility requires a design change, which is costly for the principal. However, Ma and Moore[1986] and Turnbell[1986] independently discuss mechanisms that can avoid this problem costlessly. They start with the same framework as Demsky and Sappington's, which is an adverse selection model with one principal and two correlated agents, and consider a game where there are multiple equilibria played by agents.

Although their setup is restrictive, they showed that the Revelation Principle still works without loss of efficiency where the agents are allowed to choose a better equilibrium through a coalition. We consider these results to support the rather general results of this chapter.

This chapter is organized as follows: in section 2, we describe a general adverse selection model and explain the concept of a mechanism. Then, we introduce the Revelation Principle and discuss why this principle is so important and how it works. In section 3, we present the coalitional mechanism following Cremer[1986] and examine whether the Revelation principle still works under the possibility of agents' coalition. Some remarks conclude this chapter.

2. Revelation Principle in an Adverse Selection Model

We describe the standard adverse selection model common to various areas of the incentive literature. Suppose there are one principal and two agents, i. e., $N=\{0,1,2\}$ where 0 represents the principal and 1,2 represent the agents. For each agent i, we let T_i denote the set of all possible states of agent i's private information, That is, T_i is the set of all possible types for agent i, where each type $t_i \in T_i$ represents a complete description of all the private information i might have about his preferences, abilities and his environment.

We use the notation $T = T_1 x T_2 \in R^2$ where $t=(t_1, t_2) \in T$ describes the state of nature. Each t_i is a random variable drawn from the same distribution. We assume that the realization of t_i is private information to agent i. P is a probability distribution on T such that P(t) is the probability, assessed by the principal, or by any agent ex ante, that $t=(t_1, t_2)$ will be the vector of types of the agents.

For the principal, we let D represent his decision domain. Each decision $d \in D$ may have various economic interpretations depending on the economic context of the model. For example, this might describe how the principal plans to allocate his resources to the agents or how he plans to pay them as a function of some future observations about output. For mathematical simplicity, we assume that D and T_i are finite and nonempty.[3] Finally, given $t \in T$, the preference of the principal is given by a Von Neumann-Morgenstern utility function U_0 : DxT → R and that of agents is defined by $U_i : DxT_i \rightarrow R$, i= 1,2. In other words, for any given (d, t_i) in DxT_i, $U_i(d, t_i)$ represents the utility payoff for agent i if the principal acts according to decision d, and if the agent's information is represented by t_i. The structure of model is completely specified by [N, D, U_0, (T_i, U_i) , P], which we assume is common knowledge among all the players. In addition, each agent i knows his actual type $t_i \in T_i$ as his private information.

A mechanism is any specification of how the economic decisions are determined as a function of individual's information. Here, we want to define more precisely the concept of a mechanism. Given the general structure of the model described above, the principal's problem (equivalently the economic decision) is to coordinate his decision (the allocation) in order to maximize his expected utility.[4]

The principal's solution to the problem would typically be a procedure in which he first asks every agent for some information about his type and then selects a decision $d \in D$, using messages sent by the agents. This solution process can be formally modeled by defining a mechanism, which in effect defines a game to be played by the agents.

Before we formally define a mechanism, we should mention some basic assumptions about the process described above. The first assumption is, of course, that the principal has complete control over all communication between the agents, that he can request any information which the agent is willing to send. Hence we can think of the principal as a Stackleberg leader in a two moves game: he moves first by choosing a mechanism (a game to be played by the agents); then the agents react to that mechanism. The second and most important assumption is that the message chosen by each agent is communicated to the principal confidentially and noncooperatively. By this assumption,we exclude the possibility of any collusive behavior among agents.

However, as we mentioned before, the principal faces a constraining factor in designing a mechanism: the agent has some private information which the principal cannot observe directly. Since he cannot force the agent to reveal his private information truthfully, he should design a mechanism by which each agent is given proper incentives to reveal his information honestly. Actually, the principal can successfully achieve this goal by designing a game to be played by agents when the Bayesian Nash equilibrium of this game can maximize his expected utility.[5]

Now, the principal designs this mechanism by choosing a message space M_i for each agent i and decision function (allocation rule) d : M $\rightarrow$ D, which associates with each vector of messages (m_1, m_2) in M = $M_1 x M_2$ a decision (an allocation) d = d(m_1, m_2) in D. For the agents, this mechanism [M, d] is a normal form game where M_i represents player i's strategy space and function d describes the payoff to each agent i which depends on the message sent to the principal. [6]

Agent i's behavior depends on his own type t_i. Thus, given t_i, we can represent his strategy as a mapping $\mu_i : T_i \rightarrow M_i$ where $\mu_i(t_i) \in M_i$. For convenience, we consider μ_i as some truthful response map depending on what the principal ask the agent such that $\mu_i(t_i)$ would be i's truthful response if he were type t_i. (Hence in a direct mechanism this truthful response map would be a identity map, $\mu_i(t_i) = t_i$)

Given the mechanism [M, d], since agent i's optimal strategy will depend on the strategy of the other agent through d, his optimal strategy will depend on how he believes others behave. In other words, the agent may face strategic uncertainty: uncertainty about the other agent's strategy. The hypotheses (drawn from game theory) about how an agent might act under strategic uncertainty are embodied in alternative equilibrium concepts.

Following most incentive literature, we use the Bayesian Nash equilibrium. In this equilibrium, it is assumed that each agent chooses his best strategy given his prior beliefs about the others' type. In other words, a vector of strategies (μ_1, μ_2) is a Bayesian equilibrium of [M, d] if, for each i and for each possible value of his type t_i, the message $m_i = \mu_i(t_i)$ maximizes i's expected utility given that he believes that j will use μ_j and given his beliefs about t_j.

Formally, the strategies (μ_1, μ_2) form a Bayesian Nash equilibrium of the game [M, d] if and only if

$$\forall i,\ i=1,2,\ \ \forall t_i \in T_i,\ \ \forall m_i \in M_i$$

$$\sum_{t_j \in T_j} p(t_j \mid t_i)\, U_i\{\, d(\mu_i(t_i), \mu_j(t_j))\ ; t_i\} \ \ \geq \sum_{t_j \in T_j} p(t_j \mid t_i)\, U_i\{\, d(m_i, \mu_j(t_j))\ ; t_i\}$$

Then, the allocation (decision) of this model is $d(\mu_1(t_1), \mu_2(t_2))$ given a realization of (t_1, t_2) The principal's problem is to design a mechanism [M,d] such that there is an equilibrium of strategies (μ_1, μ_2) which gives him the highest expected utility.

We are now in a position to define a direct mechanism and to motivate the Revelation Principle. Following Dasgupta, Hammond and Maskin [1979], a mechanism is direct if and only if each $M_i = T_i$. In other words, in a direct mechanism, the principal directly asks each agent about his type, hence the message space is the set of possible types T_i. Furthermore, the agents reveal their true type in an equilibrium. In other words, a direct mechanism where truth telling is an equilibrium is one in which each player is given an incentive (by the decision rule prespecified) not to lie, if he expects other player to tell the truth. Now, the Revelation Principle is:

> any equilibrium decision (allocation) of any mechanism can
> be achieved by a truthful, direct mechanism.

This Principle has been proven by many authors in very general models.[7] However, for the consistency of the model, we provide a proof of a two agents case.

[Proof]

Given a general mechanism [M,d] and its equilibrium decision $d(\mu_1, \mu_2)$ described earlier, we can define a direct mechanism [M′,d′] where $M_i' = T_i$ i=1,2 and $d'\colon T \rightarrow D$ with $d'(t) = d(\mu_1(t_1), \mu_2(t_2))$. Formally,

(i) The strategies (μ_i, μ_j) satisfies the following condition

$\forall\, i,\ i = 1, 2,\ \ \forall t_i \in T_i,\ \ \forall m_i \in M_i$

$$\sum_{t_j \in T_j} p(t_j \mid t_i)\, U_i\{\, d(\mu_i(t_i), \mu_j(t_j))\ ;\, t_i\} \quad \geq \sum_{t_j \in T_j} p(t_j \mid t_i)\, U_i\{\, d(m_i, \mu_j(t_j))\ ;\, t_i\} \tag{1}$$

(ii) In a direct mechanism, the principal wants to have truth-telling as a Bayesian Nash equilibrium of [M′,d′]. We define a strategy of the agent in the direct mechanism as a map $\phi : T_i \rightarrow T_i$ where truth-telling is a identity map $\phi_i(t_i) = t_i$. Then we must have:

$\forall\, i,\ i = 1, 2,\ \ \forall t_i \in T_i,\ \ \forall \tilde{t}_i \in T_i$

$$\sum_{t_j \in T_j} p(t_j \mid t_i)\, U_i\{\, d'(\, t_i, t_j)\ ;\, t_i\} \quad \geq \sum_{t_j \in T_j} p(t_j \mid t_i)\, U_i\{\, d'(\, \tilde{t}_i, t_j)\ ;\, t_i\} \tag{2}$$

(iii) Assume that (2) is not true. In other words, there exist some t_i and $\tilde{t}_i$ such that

$$\sum_{t_j \in T_j} p(t_j \mid t_i)\, U_i\{\, d'(\, \tilde{t}_i, t_j)\ ;\, t_i\} \quad > \sum_{t_j \in T_j} p(t_j \mid t_i)\, U_i\{\, d'(\, t_i, t_j)\ ;\, t_i\} \tag{3}$$

Then, since we define d′(t) = d($\mu_i(t_i)$, $\mu_j(t_j)$), there exist some t_i and $\tilde{t}_i$ such that

$$\sum_{t_j \in T_j} p(t_j \mid t_i)\, U_i\{\, d(\mu_i(\tilde{t}_i), \mu_j(t_j))\ ;\, t_i\} \quad > \sum_{t_j \in T_j} p(t_j \mid t_i)\, U_i\{\, d(\mu_i(t_i), \mu_j(t_j))\ ;\, t_i\} \tag{4}$$

This contradicts (1). Observe that these two mechanisms implement the same decision i.e., $d'(t_i, t_j) = d(\mu_i(t_i), \mu_j(t_j))$

Q. E. D.

The essential idea is that, given any equilibrium of reporting strategies in any mechanism, we can implement an equivalent incentive-compatible decision rule in a direct mechanism. Think of the allocation rule d as a computer program that uses m_i, m_j as an input and d(m_i,m_j) as an output. Suppose (μ_i, μ_j) is equilibrium of this mechanism. Then, we program the computer to calculate messages for each i using μ_i and have each player simply inputs his true type t_i. The computer could then use these messages to compute a decision using the rule d. This new program uses each agent's type as an input and calculate the same output with d. This is a direct mechanism. If any agent has any incentive to lie about his true type in a new program, then he would have an incentive to lie to himself in the original program. This contradicts the fact that original reporting strategy (μ_i, μ_j) is a Bayesian Nash equilibrium.

Before closing this section, we should mention why we pay so much attention to direct mechanisms and the Revelation Principle. The Revelation Principle enables us to limit our search for optimal mechanisms to direct mechanisms without fear of ignoring a more complicated mechanism which could produce better outcomes.

Generally, a mechanism can be quite complicated. It specifies what strategies are feasible and what each agent knows at each stage, and how the final allocation depends on the whole process of reportings and actions taken. However, by the revelation principle, we can use a direct mechanism by specifying an allocation rule which is function from the set of parameters of types to the set of feasible allocations.

Moreover, the Revelation Principle implies that we can restrict attention to direct mechanisms in which truth-telling is an equilibrium. By this, we can find an optimal mechanism under the presence of information asymmetries as a solution to a simple mathematical programming problem: maximize the principal's objective function (or social welfare function) subject to some self-selection constraints and feasibility conditions. Indeed, in the literature on incentives, information and contracts, the standard approach to characterize the optimal mechanism is through on implicit or explicit application of the Revelation Principle.

For example, consider some allocation problem (contract) with information asymmetries. Suppose that $d^*(t)$ is an allocation of *some mechanism*. By Revelation Principle, $d^*(t)$ is also the truthful equilibrium of a direct mechanism. It means that the direct mechanism has *an allocation rule* F which gives each agent an incentive to reveal his type truthfully.We have

$\forall i,\ i=1,2,\ \forall t_i \in T_i,\ \forall \tilde{t}_i \in T_i$

$$E_{t_j \mid t_i}[\,U_i\{F(t_i, t_j)\ ; t_i\}\,] \ \geq\ E_{t_j \mid t_i}[\,U_i\{F(\tilde{t}_i, t_j)\ ; t_i\}\,]$$

If everyone reports truthfully, the direct mechanism represented by allocation rule F must have same result $d^*(t)$, hence we have $F(t) \equiv d^*(t)$. Therefore, the direct mechanism whose truthful equilibrium allocation implements the original allocation d^*(observed) is simply the direct mechanism whose allocation rule F is d^*. Hence, in any general situation with information asymmetry, the search for an optimal mechanism (rule) F can be solved simply by searching an optimal allocation in the space of feasible allocations that satisfy the self-selection conditions, using standard mathematical programming techniques.

3. Coalitional Mechanism and Revelation Principle

In the model described in the previous section, we assumed that the agent acts noncooperatively as a utility maximizer who would passively accept any Nash equilibrium of the game designed by the principal. By this assumption, we exclude any possibility of collusive behavior among the agents. However, as is already pointed out in organization theory, collusive behavior among the agents, implicit or explicit, is pervasive in real economic institutions. These collusive behaviors, like bribes, personal relationships, and the promise of reciprocation, do affect the design of incentive mechanisms.

We want to relax the assumption about this noncooperative behavior and to introduce the possibility of agents' coalitions. Then, the immediate question is whether or not the Revelation principle would still hold under the possibility of the agents' coalition. If it does, we can enjoy all the advantages of the Revelation Principle even under the possible collusive behavior of the agents. In other words, we can limit our search for an optimal coalition-proof mechanism to an optimal coalition-proof direct mechanism without loss of generality. Moreover, we can find the optimal coalition-proof mechanism using mathematical programming: maximizing the principal's expected utility subject to some individual and coalitional incentive compatibility conditions and feasibility conditions.

In our Bayesian setup, however, coalition formation itself suffers from the problem of information asymmetry. Therefore, in order to collude successfully, the agents should use some "coalitional mechanism" whose outcome is a set of messages in the original mechanism. Consider the following scenario: First, the principal designs a mechanism [M,d] where the Bayesian Nash equilibrium of this mechanism maximizes his expected utility. However, before the agents actually send the messages to the principal, they can meet and agree on the message to send to the principal. But, since t_i is i's private information, j cannot observe the realized value of t_i and vice versa. Therefore, the coalitional mechanism also should be an incentive-compatible mechanism. In other words, when they meet and announce their types to decide what message to send to the principal, each agent should be given proper incentives not to deceive his true type. This leads to the following conditions for successful coalition formation under information asymmetry:

We assume that there emerges a coalition (equivalently a side contract) if [8]

(i) The coalition can find an incentive-compatible mechanism

(ii) The coalition is feasible (i.e., the coalition is strictly Pareto- improving)

Then, we can define the coalitional mechanism as a decision rule $\delta: T \rightarrow M$ within the coalition, where $\delta_i: T \rightarrow M_i$. Observe that $\delta_i(t) \in M_i$ represents agent i's report to the principal agreed in the coalition. Then, given the original mechanism [M,d] and equilibrium message (μ_i, μ_j), δ(t) is a coalitional mechanism if and only if it satisfies:

(i) Coalitional incentive compatibility conditions

$\forall\, i,\; i = 1, 2,\;\; \forall t_i \in T_i,\;\; \forall \tilde{t}_i \in T_i$

$$\sum_{t_j \in T_j} p(t_j \mid t_i)\, U_i\{\, d\,(\delta_i(t_i, t_j), \delta_j(t_i, t_j)\,)\,; t_i\} \;\;\geq\; \sum_{t_j \in T_j} p(t_j \mid t_i)\, U_i\{\, d\,(\delta_i(\tilde{t}_i, t_j), \delta_j(\tilde{t}_i, t_j)\,)\,; t_i\}$$

(ii) Individual Rationality conditions

$\forall\, i,\ i = 1, 2$

$$\sum_{t_j \in T_j} p(t_j \mid t_i)\, U_i\{ d(\delta_i(t), \delta_j(t)) ; t_i\} \quad \geq \sum_{t_j \in T_j} p(t_j \mid t_i)\, U_i\{ d(\mu_i(t_i), \mu_j(t_j)) ; t_i\}$$

with at least one strict inequality

The welfare level of agents in this Individual Rationality condition is measured interim. Observe that if this condition is true interim, it implies that this is also true ex-ante.

Then, we can define a coalition-proof mechanism as follows: A mechanism [M,d] is coalition proof if and only if there does not exist a coalitional mechanism δ(t), which satisfies both condition (i) and condition (ii). Now, the main result can be summarized in the following theorem[9]

Theorem : If there exists a coalition proof mechanism, then there exists a equivalent coalition proof direct mechanism.

[Proof]

The theorem can be restated as follows: the fact that the original mechanism is coalition proof implies that the direct mechanism is also coalition proof. We can prove this by showing that if there exists a coalitional mechanism which threatens the direct mechanism then we can build a coalitional mechanism which threatens the original mechanism. The proof is done by the following three steps.

step 1

Given the original mechanism [M,d] and its equilibrium message (μ_i, μ_j), we can build a truthful direct mechanism [M′, d′] where $M_i' = T_i$, i=1,2 , and d′(t)= $d(\mu_i(t_i), \mu_j(t_j))$. Proof is given in the previous section.

step 2

Applying the revelation principle in specifying a coalitional mechanism, we can always have a direct coalitional mechanism. Hence, assume that there exists a coalitional mechanism [M′, δ'] which threatens this direct mechanism [M′,d′] where M′=T and d′(t)= $d(\mu_i(t_i), \mu_j(t_j))$) and $\delta' : T \rightarrow T$ with $\delta_i' : T \rightarrow T_i$ such that

$\forall i,\ i=1,2,\ \forall t_i \in T_i,\ \forall \tilde{t}_i \in T_i$

$$\sum_{t_j \in T_j} p(t_j \mid t_i)\, U_i\{ d'(\delta_i(t_i, t_j), \delta_j(t_i, t_j)) ; t_i \} \geq \sum_{t_j \in T_j} p(t_j \mid t_i)\, U_i\{ d'(\delta_i(\tilde{t}_i, t_j), \delta_j(\tilde{t}_i, t_j)) ; t_i \} \quad (5)$$

and

$\forall i,\ i=1,2$

$$\sum_{t_j \in T_j} p(t_j \mid t_i)\, U_i\{ d'(\delta_i(t), \delta_j(t)) ; t_i \} \geq \sum_{t_j \in T_j} p(t_j \mid t_i)\, U_i\{ d'(t_i, t_j) ; t_i \} \quad (6)$$

step 3

The theorem is proved if we can build a coalitional mechanism which threatens the original mechanism [M,d]. To do so, we must find a mapping $\delta^* : T \rightarrow M$ such that

$\forall i,\ i=1,2,\ \forall t_i \in T_i,\ \forall \tilde{t}_i \in T_i$

$$\sum_{t_j \in T_j} p(t_j \mid t_i)\, U_i\{ d(\delta_i^*(t_i, t_j), \delta_j^*(t_i, t_j)) ; t_i \} \geq \sum_{t_j \in T_j} p(t_j \mid t_i)\, U_i\{ d(\delta_i^*(\tilde{t}_i, t_j), \delta_j^*(\tilde{t}_i, t_j)) ; t_i \} \quad (7)$$

and

$\forall i,\ i=1,2$

$$\sum_{t_j \in T_j} p(t_j \mid t_i)\, U_i\{ d(\delta_i^*(t), \delta_j^*(t)) ; t_i \} \geq \sum_{t_j \in T_j} p(t_j \mid t_i)\, U_i\{ d(\mu_i(t_i), \mu_j(t_j)) ; t_i \} \quad (8)$$

Since $d'(t) = d(\mu_i(t_i), \mu_j(t_j))$, (5) and (6) are equivalent to:

$\forall i,\ i = 1, 2,\ \ \forall t_i \in T_i,\ \ \forall \tilde{t}_i \in T_i$

$$\sum_{t_j \in T_j} p(t_j \mid t_i)\, U_i \{\, d\, [\, \mu_i(\delta_i(t_i, t_j)), \mu_j(\delta_j(t_i, t_j))\,]\ ; t_i \}$$
$$\geq \sum_{t_j \in T_j} p(t_j \mid t_i)\, U_i \{\, d\, [\, \mu_i(\delta_i(\tilde{t}_i, t_j)), \mu_j(\delta_j(\tilde{t}_i, t_j))\,]\ ; t_i \} \tag{9}$$

and

$\forall i,\ i = 1, 2$

$$\sum_{t_j \in T_j} p(t_j \mid t_i)\, U_i \{\, d\, [\mu_i(\delta_i(t)), \mu_j(\delta_j(t))] ; t_i \} \geq \sum_{t_j \in T_j} p(t_j \mid t_i)\, U_i \{\, d\, [\mu_i(t_i), \mu_j(t_j)] ; t_i \} \tag{10}$$

It is then clear that δ^*, with $\delta_i^*(\cdot) = \mu_i[\delta_i(\cdot)]$, satisfy (7) and (8). Observe that δ_i^* is a function from T to M_i, because δ_i is a function from T to T_i and μ_i is a function from T_i to M_i. Hence, by choosing δ^* as $\delta_i^*(\cdot) = \mu_i[\delta_i(\cdot)]$, we can find a mapping δ^* satisfying (7) and (8) from (9) and (10).

Q. E. D.

The idea of $\delta_i^*(\cdot)$ is the following: if the agents can find coalitional mechanism $\delta_i : T \rightarrow T_i$ that threatens the direct mechanism, then, in the original mechanism, they can find a very simple way to cheat. That is, they will play noncooperative equilibrium of the main contract (μ_i, μ_j), pretending their true type to be δ_i (t) $\in T_i$ instead of $t_i \in T_i$, i=1,2. Observe that the agents can easily cheat the principal since they appear to passively follow the equilibrium strategy.

The power of this theorem is that it enables us to limit our search for the optimal coalition proof mechanism to the optimal coalition proof direct mechanism without concerning any other complex mechanisms. For example, in Tirole[1986] and chapter IV of dissertation, this theorem is applied to characterize a coalition proof contract. That is, the decision process and timing of these models is quite complex. However, when the principal tries to design a mechanism at the outset, he does not have to worry about the complex structure of coalition formation. He can consider only a coalition proof direct mechanism, and moreover, this mechanism can be found by maximizing his expected utility subject to some individual and coalitional incentive-compatibility conditions and feasibility conditions.[10]

4. Concluding Remarks

In this chapter, we showed that we can confine our search for coalition proof mechanisms to a coalition proof direct mechanisms. By this result, we can characterize the coalition proof mechanisms in a very simple way. In other words, this result enhances the value of Revelation Principle as a tool for the study of incentive problems. Turnbell[1986] and Ma and Moore[1986] independently studied the mechanism which can avoid the effect from the possibility of coalition without any loss of efficiency. They designed mechanisms under which the principal can prevent the possible coalition without additional cost. However, this is only possible in the very restricted set up (their model is initiated by Demsky and Sappington[1984]). In chapter IV, we show that the principal should pay the additional cost to prevent the possible coalitions between the supervisor and the agent. In our model in chapter IV, imposing individual incentive constraints is not enough to prevent all the coalitional incentive problems. However, these studies can be considered as special examples to support the result of this chapter.

We can easily extend this result to a n person case. It is never clear whether there exists a coalition proof mechanism or, if it exists, how it could be characterized. This is because we must consider every possible intermediate size coalition as well as the coalition of the whole. However, if we assume there exists a coalition proof mechanism in an n players set up, then we can show that there exists a coalition proof direct mechanism. In the next chapter, we will extend this result to a more general set up where agents have a private decision domain as well as private information.

FOOTNOTES

1 Indeed, many sociological studies of hierarchical organizations show that collusive behavior, implicit or explicit, does exist and is often prominent. Moreover, Tirole[1986] pointed out that the observed collusive behavior is only a tip of iceberg. See Tirole[1986] for detail.

2 See Cremer [1986] for detail. Actually his model is developed in non-Bayesian setup. However, we can use same assumptions here.

3 It is natural for the choice of $d \in D$ to be made in a randomized fashion, if gains can be achieved there by. Then, we can consider D as a set of probability distribution over choices in some finite decision set D_0. In other words, if D_f is finite with cordinality m, then D is a simplex in R^m. Hence, D includes the possibility of randomized decision rule and our analysis can be carried out using D without loss of generality.

4 For example, in the public good provision problem, the principal would be represented by a government. The principal's expected utility would be some measure of social welfare and the decision would be whether or not to provide a specific public good and the tax scheme when they decide to provide.

5 For a given mechanism, of course, there may be no equilibrium or there may be several. The former possibility poses no great conceptual difficulties: the principal can simply confine his attention to those mechanisms that have an equilibrium. We will discuss the second problem later.

6 This 'message' could actually be a whole sequence of functions specifying at each stage what message to send as a function of the history of the game to that stage. The Revelation Principle can be also proved for the extensive from mechanism. See Harris and Townsend [1981] for detail.

7 This idea has been observed by Gibbard[1973] and Green and Laffont[1977], proved by Myerson[1979] and Harris and Townsend[1981] and fully exploited by Dasgupta, Hammond and Maskin[1979].

8 Cremer [1986] defines a coalitional mechanism [δ (t), v(t)] where δ(t) is the agreed message of agents sent the the principal and v(t) is the monetary transfer in the coalition. However, for the purpose of this paper we don't have to consider transfers explicitly.

9 We don't have to worry about the proof in the other direction. This is because if we have a coalition proof direct mechanism, it automatically implies that we have at least one coalition proof mechanism.

10 The way we set up this coalition incentive-compatibility constraint is as follows: the Principal thinks about the final allocation i.e., in his mind, he allows the agents to collude during the process of model. Then, for this final allocation, he puts the constraints such that the total allocation to the agents should be self-selected.

CHAPTER III EXTENSION TO THE GENERALIZED MODEL

1. Introduction

In the adverse selection model described in chapter 2, the only constraining factor to the principal is agents' private information. However, agents may have a private decision domain which affects the principal's utility as well as their own payoff. The unobservability of the agents' actions causes a so-called moral hazard problem. In this chapter, we consider the general class of problem which includes both adverse selection and moral hazard aspects. Many principal-agent problems involve these two problems at the same time. For instance, in the regulatory authority-regulated firms relationship, the regulatory authority may be unable to observe the effort level of each firm and each firm's cost condition may be its private information. Likewise, the owner of a firm cannot observe some productivity parameter of workers and, at the same time, he cannot directly monitor their effort level. Furthermore, we can consider insurance markets when an insurer might be unable to distinguish high risk persons from low risk persons and, simultaneously, he will not observe the level of care taken by the insured person.

This chapter is organized as follows: in section 2, we present a general model with combination of the moral hazard and adverse selection problems. This model can be considered as a nonrandomized version of Myerson[1982]'s set up. In section 3, we present the direct mechanism for this general coordination mechanism and add a proof of the Revelation Principle for expositional consistency. In section 4, we introduce a coalitional mechanism with some institutional assumptions. Finally, we show that the Revelation principle is still applicable under the class of possible coalitions we considered.

2. Model

We describe a model of this generalized principal/agent problem, closely following Myerson[1982]'s set up. Suppose there are three players N $=$ {0,1,2} where 0 represents the principal and 1,2 represent the agents. For each agent i , i=1,2, T_i denotes the set of possible types for agent i. Each type $t_i \in T_i$ describes all the private information of agent i. D_0 denotes the principal's decision set. That is, any d_0 in D_0 represents the principal's decision or action which can be contractually specified. Hence, d_0 must be a public and enforceable decision and the principal is assumed to commit himself to his preannounced mechanism. Let assume D_i represents the set of all possible private actions(decision) controlled by agent i.

We use the notation T $= T_1 x T_2 \in R^2$ where t $= (t_1, t_2) \in T$ describes the information state of the model. Similarly, we use D $= D_0 \, x \, D_1 \, x \, D_2 \in R^3$ where d $= (d_0, d_1, d_2) \in D$ denotes a decision vector or outcome in D. We assume that T and D are nonempty, finite sets for mathematical simplicity.[1] Given any vector of type t and decision d, the preference of the principal is given by a Von Neumann-Morgenstern utility function U_0 : D x T $\rightarrow$ R, and that of agent is defined by U_i : D x T_i $\rightarrow$ R. Finally, agent i's beliefs concerning the state are given by a probability measure p_i on T, where p_i (t) represents the subjective probability that the agent i would assign to the state t before he learns his type. The structure of the model is completely specified by [N, D_0, $U_0(d_i, T_i, U_i)$, p(t)], which we assume is common knowledge among all the players. Given this structure, the principal's problem is to coordinate his decision and those of the agents in order to maximize his expected utility. However, as we mentioned, the principal has two constraining factors in achieving this goal: he cannot directly observe an agent's type t_i in T_i and cannot directly control the agent's private decision d_i in D_i.

Now, we describe a typical coordination mechanism designed by the principal. Let M_i denote the set of all possible messages that agent i could use for sending reports to the principal given his own private information and R_i denote the set of all possible responses that the agent i might receive from the principal in the mechanism.[2] Since the response to one agent will depend on the messages received from the other agent, the principal would specify the response function (it can be random) $r_i : M \rightarrow R_i$ where $r_i(m) \in R_i$ and m = $(m_1, m_2) \in M = M_1 x M_2$. Finally, $d_0 : M \rightarrow D_0$ represents the principal's decision rule where $d_0(m) \in D_0$. Each option d_0 in D_0, may for example, represent a description of some future observation about output which might be determined by the revealed parameter of each agent's private information and the agent's level of effort (private decision).[3] Thus, the mechanism designed by the principal can be summarized as $[M_i, R_i, d_0, r_i]_{i=1,2}$. Again, this is a game to be played by the agents. Given the structure of this coordination mechanism, each agent chooses a reporting strategy in M_i as a function of his type and chooses his decision in D_i as a function of the response from the principal and his own type. Formally, we denote agent i's reporting strategy as a function $\mu_i : T \rightarrow M_i$ where $\mu_i(t_i) \in M_i$ and agent i's private decision strategy as a function $\delta_i : R_i x T_i \rightarrow D_i$ where $\delta_i(r_i, t_i) \in D_i$.

Given the choices of strategies $[(\mu_1, \delta_1), (\mu_2, \delta_2)]$ for the agents, the expected utility for agent i would be:

$$V_i[(\mu_1, \delta_1), (\mu_2, \delta_2)] = E_t[U_i\{d_0(\mu(t)), \delta_i(r_i[\mu(t)], t_i), \delta_j(r_j[\mu(t)], t_j), t\}]$$
$$= \sum_{t \in T} p_i(t)[U_i\{d_0(\mu(t)), \delta_i(r_i[\mu(t)], t_i), \delta_j(r_j[\mu(t)], t_j), t\}]$$
$$\text{where } \mu(t) \equiv [\mu_i(t_i), \mu_j(t_j)]$$

Since we assumed that each agent's strategy (μ_i, δ_i) is chosen independently, the principal can expect that the agents will play an equilibrium of a noncooperative game, where each agent's strategy maximizies his expected utility. Formally, the agents' choices of strategies $[(\mu_1, \delta_1), (\mu_2, \delta_2)]$ form a Bayesian Nash equilibrium, if and only if

$$\forall i, \ \forall \mu_i' : T_i \rightarrow M_i, \ \forall \delta_i' : R_i x T_i \rightarrow D_i$$

$$V_i[(\mu_i, \delta_i), (\mu_j, \delta_j)] \geq V_i[(\mu_i', \delta_i'), (\mu_j, \delta_j)]$$

Then the allocation induced by this coordination mechanism will be $d_0(\mu_1(t_1), \mu_2(t_2)) \in D_0$ and principal's response to each agent i will be $r_i(\mu_1(t_1), \mu_2(t_2)) \in R_i$

3. Direct Mechanism and Revelation Principle

We are in a position to define a direct mechanism for this general coordination mechanism. Following Dasgupta, Hammond and Maskin [1979], a mechanism is direct if and only if each M_i is T_i and each R_i is D_i. That is, in a direct mechanism, each agent i is asked to report his type $t_i \in T_i$ to the principal and, in return, the principal will directly suggest a decision $d_i \in D_i$ to each agent i.

Given any mechanism $[M_i, R_i, d_0, r_i]_{i=1,2}$ and its equilibrium strategy $(\mu_i, \delta_i)_{i=1,2}$ we can define a direct mechanism $[\overline{M}_i, \overline{R}_i, \overline{d}_0, \overline{r}_i]_{i=1,2}$ such that

$$\forall i \ \ \overline{M}_i = T_i \quad \text{and} \quad \forall i \ \ \overline{R}_i = D_i$$

$\bar{d}_0 : T \to D_0 \;\; \bar{d}_0(t) \in D_0$ with $\bar{d}_0(t) = d_o(\mu(t))$ where $\mu(t) = [\mu_i(t_i),\; \mu_j(t_j)]$

$\bar{r}_i : T \to D_i \;\; \bar{r}_i(t) \in D_i$ with $\bar{r}_i(t) = \delta_i[r_i(\mu(t)), t_i]$

For notational convenience, we shall use $d_i^r(t) \equiv \bar{r}_i(t) = \delta_i[r_i(\mu(t)), t_i]$, since $\bar{r}_i(t)$ is actually a recommended decision for the agent i. The strategy of the agent in a direct mechanism $(\bar{\mu}_i, \bar{\delta}_i)_{i=1,2}$ is defined as follows:

$\bar{\mu}_i : T_i \to T_i\,,\;\; \bar{\mu}_i(t_i) \in T_i$ is an agent's reporting strategy

$\bar{\delta}_i : D_i x T_i \to D_i\,,\;\; \bar{\delta}_i(d_i^r, t_i) \in D_i$ is an agent's private decision strategy.

Following Myerson [1982], we say that agent i is " honest and obedient " if he uses the strategy $(\bar{\mu}_i, \bar{\delta}_i)$ satisfying

$\forall t_i, \forall d_i \qquad \bar{\mu}_i(t_i) = t_i$ and $\bar{\delta}_i(d_i(t), t_i) = d_i(t)$

Note that the d_i(t) in the function $\bar{\delta}_i(d_i(t), t_i)$ always represents a recommendation from the Principal. Hence $d_i(t)$ in $\bar{\delta}$ function represents d_i^r(t).

Then, a direct mechanism is incentive-compatible if and only if the honest and obedient strategies [$(\bar{\mu}_i, \bar{\delta}_i),(\bar{\mu}_j, \bar{\delta}_j)$] form a Bayesian Nash equilibrium for the direct mechanism. In other words, the direct mechanism $[\bar{M}_i, \bar{R}_i, \bar{d}_0, \bar{\delta}_2]_{i=1,2}$ is incentive compatible if and only if it satisfies

$\forall i,\;\; \forall\, \bar{\mu}_i' : T_i \to T_i,\;\; \forall\, \bar{\delta}_i' : D_i x T_i \to D_i$

$$
\begin{aligned}
&E_t[\, U_i\{\, \bar{d}_0(t_i, t_j),\; d_i(t),\; d_j(t),\; t\,\}\,] \\
\geq\; &E_t[\, U_i\{\, \bar{d}_0(\bar{\mu}_i'(t_i), t_j),\; \bar{\delta}_i'[d_i^r(\bar{\mu}_i'(t_i), t_j),\; t_i],\, \bar{\delta}_j[d_j^r(\bar{\mu}_i'(t_i), t_j),\; t_j],\; t\,\}\,]\;.
\end{aligned}
$$

This inequality must hold for all $\bar{\mu}_i'$, and t_i. In particular, choose $\bar{\mu}_i'$ such that $\bar{\mu}_i'(\tau_i) = \tau_i'$ for some $\tau_i \in T_i$ and $\bar{\mu}_i'(t_i) = t_i$ for $t_i \neq \tau_i$. Then, we must have

$\forall i,\ \forall \tau_i \in T_i,\ \forall \tau_i' \in T_i,\ \forall \bar{\delta}_i' : D_i x T_i \rightarrow D_i$

$$\underset{t_i = \tau_i}{E_t} [U_i \{ \bar{d}_0 (t_i, t_j) ,\ d_i(t) ,\ d_j(t) ,\ t \}]$$
$$\geq \underset{t_i = \tau_i}{E_t} [U_i \{ \bar{d}_0 (\tau_i', t_j) ,\ \bar{\delta}_i' [d_i^r (\tau_i', t_j),\ t_j] , \bar{\delta}_j [d_j^r (\tau_i', t_j),\ t_j] ,\ t \}]$$

The Revelation Principle of this generalized principal-agent problem was proved by Myerson[1982] with n players in a randomized set up. However, we go through the proof in this nonrandomized set up for expositional consistency to the next step. The Revelation Principle is:

The optimal incentive-compatible direct mechanism is also optimal in the class of any coordination mechanism.

[Proof]

(i) Given a mechanism $[M_i, R_i, d_0, r_i]_{i=1,2}$ and its equilibrium $[(\mu_1, \delta_1),\ (\mu_2, \delta_2)]$, we must have:

$\forall i,\ \forall \mu_i' : T_i \rightarrow M_i,\ \forall \delta_i' : R_i x T_i \rightarrow D_i$

$$\begin{aligned} &E_t [U_i \{ d_0 (\mu(t)) ,\ \delta_i[r_i (\mu(t)), t_i] ,\ \delta_j[r_j (\mu(t)), t_j] , t \}] \\ &\geq E_t [U_i \{ d_0 (\mu_i'(t_i), \mu_j(t_j)) , \delta_i' (r_i [\mu_i'(t_i), \mu_j(t_j)] ,\ t_j) , \delta_j' (r_j [\mu_i'(t_i), \mu_j(t_j)] ,\ t_j) ,\ t \}] \end{aligned} \tag{1}$$

(ii) Consider a direct mechanism $[\bar{M}_i, \bar{R}_i, \bar{d}_0, \bar{r}_i]_{i=1,2}$ where $\bar{M}_i = T_i$, $\bar{R}_i = D_i$ and $\bar{d}_0(t) = d_0(\mu(t))$, $\bar{r}_i(t) = d_i^r(t) = \delta_i[r_i(\mu(t)), t_i]$. If it is incentive-compatible, then we must have:

$\forall i,\ \forall \tau_i \in T_i,\ \forall \tau_i' \in T_i,\ \forall \bar{\delta}_i' : D_i x T_i \rightarrow D_i$

$$\begin{aligned} &\underset{t_i = \tau_i}{E_t} [U_i \{ d_0 (t_i, t_j) ,\ d_i(t) ,\ d_j(t) ,\ t \}] \\ &\geq \underset{t_i = \tau_i}{E_t} [U_i \{ d_0 (\tau_i', t_j) ,\ \bar{\delta}_i' [d_i^r (\tau_i', t_j),\ t_j] , \bar{\delta}_j [d_j^r (\tau_i', t_j),\ t_j] ,\ t \}] \end{aligned} \tag{2}$$

(iii) Assume this were not true. Then we can find some τ_i, τ_i' and $\bar{\delta}_i'$ such that

$$\underset{t_i=\tau_i}{E_t}\ [\,U_i\{\,\bar{d}_0(\tau_i',t_j)\,,\ \bar{\delta}_i'\,[d_i^r(\tau_i',t_j),\ t_j\,]\,,\bar{\delta}_j\,[d_j^r(\tau_i',t_j),\ t_j\,]\ ,\ t\,\}\,]$$
$$> \underset{t_i=\tau_i}{E_t}\ [\,U_i\{\,\bar{d}_0(\,t_i,t_j\,)\,,\ d_i(t)\,,\ d_j(t)\,,\ t\,\}\,] \tag{3}$$

Since $\bar{d}_0(t)=d_0(\mu(t))$, $d_i(t)=d_i^r(t)=\delta_i[r_i(\mu(t)),t_i]$, where $\mu(t)=[\mu_i(t_i),\mu_j(t_j)]$, from (3) we can find some τ_i, τ_i' and $\bar{\delta}_i'$ such that

$$\underset{t_i=\tau_i}{E_t}\ [\,U_i\{\,d_0(\,\mu_i(\tau_i')\,,\ \mu_j(t_j))\,,\ \bar{\delta}_i'\,[d_i^r(\tau_i',t_j),\ t_j\,]\,,\bar{\delta}_j\,[d_j^r(\tau_i',t_j),\ t_j\,]\ ,\ t\,\}\,]$$
$$> \underset{t_i=\tau_i}{E_t}\ [\,U_i\{\,d_0(\,\mu(t))\,,\ \delta_i\,[\,r_i(\mu(t))\,,\ t_i\,]\,,\delta_j\,[\,r_j(\mu(t))\,,\ t_j\,]\,,t\,\}\,] \tag{4}$$

Observe that we can find some μ_i' such that $\mu_i'(t_i)=\mu_i(\tau_i')$ when $t_i=\tau_i$, $\mu_i'(t_i)=\mu_i(t_i)$ when $t_i\neq\tau_i$. Also, we can find some $\bar{\delta}_i'$ which produce $\bar{\delta}_i'[\,d_i^r(t_i',t_j),\ t_i\,]=d_i'(t_i',t_j)$ and this decision is equivalent to the recommendation $d_i^{r\prime}$. Then, since we defined $d_i^r(t)=\delta_i[r_i(\mu(t)),t_i]$, where $\mu(t)=[\mu_i(t_i),\mu_j(t_j)]$, we can find some δ_i' such that $d_i^{r\prime}(t_i',t_j)\ =\ \delta_i'\,[\,r_i(\mu_i(t_i'),\mu_j(t_j)),\ t_i\,]$. Observe that $\bar{\delta}_j$ is an obedient strategy. Hence, this implies $\bar{\delta}_j\,[\,d_j^r(\tau_i',t_j),\ t_j\,]\ =\ d_j^r(\tau_i',t_j)\ =\ \delta_j\,[\,r_j(\mu_i(\tau_i'),\mu_j(t_j)),\ t_j\,]$. Hence, from (4) we can find some μ_i' and δ_i' such that

$$E_t[\,U_i\{\,d_0\,(\mu_i'(t_i),\mu_j(t_j))\,,\delta_i'\,(\,r_i\,[\mu_i'(t_i),\mu_j(t_j)\,]\,,\ t_j\,)\,,\delta_j'\,(\,r_j\,[\mu_i'(t_i),\mu_j(t_j)\,]\,,\ t_j\,)\,,\ t\,\}\,]$$
$$>\quad E_t[\,U_i\{\,d_0\,(\,\mu(t))\,,\ \delta_i[\,r_i\,(\mu(t)),t_i]\,,\ \delta_j[\,r_j\,(\mu(t)),t_j]\,,t\,\}\,] \tag{5}$$

This contradicts (1).

The intuition behind this principle is similar to the case of pure adverse selection model. We can construct a direct mechanism which simulates any given mechanism through the following program built in a direct mechanism: First, the principal asks each agent i (simultaneouly and independently) to reveal his type t_i. Then the direct mechanism computes what reports (m_i) would have been sent by the agents in the original mechanism using the equilibrium strategy of the original mechanism μ_i. Second, the direct mechanism computes what actions (d_i) would have been carried out by the agents as a function of the principal's recommendation $(d_i=\delta_i(r_i,t_i))$. Finally, the principal tells each agent to take the action computed for him.

4. Coalitional mechanism and Revelation Principle

As is pointed out in the previous section, when we allow the agents to collude, the coalition formation itself has a problem of information asymmetry. We assumed that the agents must find a coalitional mechanism in order to reach agreement. However, it is never clear when they meet, how they reach agreements and whether they are allowed to renegotiate or not.

In order to resolve this uncertainty about the coalition formation itself, we should restrict our analysis to certain types of coalitions by describing institutional assumptions about the coalition formation process. We assume that each of colluding parties can check whether the other party keeps the agreement. One easy way for this is to assume that the messages $(\hat{m}_i, \hat{m}_j)$ and the decisions $(\hat{d}_i, \hat{d}_j)$ agreed in the coalition are observable by each party. Moreover, we assume that the agents can enforce this coalitional mechanism by assuming that the agents are in some form of long-run relationship. By this, the coalition can punish any agent who deviates from the agreememt of coalition at the next stage. These assumptions might be too strong. Since the spectrum of real collusive behavior is extremely varied, we can never have exact knowledge about the actual process of coalition formation. Sometimes, a colluding party might not observe the other party's real message. This may inhibit the formation of coalitions. However, we consider the case where the coalition is enforceable: not knowing the tools at the disposition of agents, the principal wants to protect himself from collusive behavior even in what is the worst possible case for him. This corresponds to a risk-minimizing behavior by the mechanism designer.

Given this assumption, we consider the simplest institutional set up: First, we assume that the agents choose a coalitional mechanism (or sign a side contract) before any private information t_i is revealed.[4] This means that the mechanism is given by a principal ex-ante. We can imagine a decision process of this model as follows:

time

P offers mechanism (M, R, d_0, r)	Agents specify coalition mechanism $(\hat{\mu}, \hat{\delta})$	private information t_i is revealed to each i	Each agent send message m_i	P responds to each i with r_i. P choose his decision	Agents choose their private decisions δ_i, δ_j	P's decision may be excuted

Second, after they choose a coalitional mechanism, the agents are supposed to commit themselves to this mechanism. For instance, they cannot meet again after they receive responses from the principal and renegotiate about their private decisions. Third, we assume that the principal does not have any hidden information, which makes the computation of responses from the principal straight forward. Finally, observe that the response from the principal is assumed to be private information throughout the whole analysis.

Following Cremer [1986], we assume that the coalition is successful under the information asymmetry, if (i) the coalition can find an incentive- compatible mechanism and (ii) the coalition is strictly Pareto-improving. Formally, we can define a coalitional mechanism as follows: Given the main mechanisms $[M_i, R_i, D_0, r_i]$ a coalitional mechanism is a set of decision rules $[\hat{\mu}_i(t), \hat{\delta}_i(t)]_{i=1,2}$ such that

$$\hat{\mu}_i : T \rightarrow M_i$$

where $\hat{\mu}_i(t)$ denotes agent i's report to the principal agreed in the coalition

$$\hat{\delta}_i : T \rightarrow D_i$$

where $\hat{\delta}_i(t) \equiv \hat{\delta}_i(r_i(\hat{\mu}(t)), t_i)$ denotes agent i's decision agreed in the coalition.

Observe that, in the previous section, the agent's private decision is $\hat{\delta}_i(r_i(\hat{\mu}(t)), t_i)$. However, with given the institutional assumptions we have made, the agreement about their private decision $\hat{\delta}_i$ directly depends upon their announced types. In other words, since they are not allowed to renegotiate and the principal has no private information, the colluding parties can calculate the responses from the principal straightforwardly. Moreover, since we assumed that the agents can observe their reports to the principal and the private decisions agreed in the coalition, actually the response from the principal cannot convey additional information. Therefore, they can consent directly on their messages and private decisions. In other words, once they successfully agree on their messages $\hat{\mu}_i(t), \hat{\mu}_j(t)$, then each agent can calculate the possible response (or recommendation) from the pincipal using the given main contract. Afterwards, they can agree on their private decisions to successfully improve their utility given the choice of messages $\hat{\mu}_i(t), \hat{\mu}_j(t)$. However, the possible choice of a agreed report to the principal ($\hat{\mu}_i, \hat{\mu}_j$) is restricted by the possible choice of agreed decisions ($\hat{\delta}_i$, $\hat{\delta}_j$) through the response function r_i, r_j specified in the main contract.

Now, given a main mechanism $[M_i, R_i, d_0, r_i]_{i=1,2}$, and its equilibrium $[(\mu_i, \delta_i),(\mu_j, \delta_j)]$, the coalitional mechanism [$\hat{\mu}_i(t), \hat{\delta}_i(t)$] is successful if and only if it satisfies:

(i) Coalitional incentive compatibility conditions

$\forall i, \ \forall \alpha_i : T_i \rightarrow T_i$

$$
\begin{aligned}
& E_t[\, U_i\{ d_0(\hat{\mu}(t)),\ \hat{\delta}_i(t),\ \hat{\delta}_j(t),\ t \}\,] \\
& \quad \geq\ E_t[\, U_i\{ d_0(\hat{\mu}(\alpha_i(t_i), t_j)\,],\ \hat{\delta}_i(\alpha_i(t_i), t_j),\ \hat{\delta}_j(\alpha_i(t_i), t_j),\ t \}\,]
\end{aligned}
$$

(ii) Individual rationality conditions

$\forall i$ i=1,2

$$
\begin{aligned}
& E_t[\, U_i\{ d_0(\hat{\mu}(t)),\ \hat{\delta}_i(t),\ \hat{\delta}_j(t),\ t \}\,] \\
& \quad \geq\ E_t[\, U_i\{ d_0(\mu(t)),\ \delta_i(r_i(\mu(t)),\ t_i),\ \delta_j(r_j(\mu(t)),\ t_j),\ t \}\,]
\end{aligned}
$$

with at least one strict inequality

Then, the mechanism $[M_i, R_i, d_0, r_i]_{i=1,2}$ is coalition proof, if and only if we can find a coalitional mechanism which satisfies both conditions (i) and conditions (ii). Our main question is whether the Revelation Principle is still applicable or not, with this coalition proof mechanism. In other words, we would like to show that the optimal coalition proof direct mechanism is still optimal in any class of coalition proof coordination mechanism.

Theorem : Revelation Principle with coalition

The optimal incentive-compatible coalition proof direct mechanism is also optimal in the class of all coalition proof coordination mechanism.

[Proof]

We can prove the theorem by showing that the existence of a coalition proof coordination mechanism implies the existence of coalition proof direct mechanism with the same outcome. We use counterpositive way, following the previous proof in chapter 2.

Step 1

Given original coordination mechanism $[M_i, R_i, d_0, r_i]_{i=1,2}$ and its equilibrium strategy $[(\mu_i, \delta_i), (\mu_j, \delta_j)]$, we can build an incentive-compatible direct mechanism $[M_i, R_i, d_0, r_i]_{i=1,2}$ where $\overline{M}_i = T_i$ and $\overline{R}_i = D_i$ and $\overline{d}_0(t) = d_0(\mu(t))$ and $\overline{r}_i = \delta_i[r_i(\mu(t)), t_i]$. The proof is given in the previous section.

Step 2

Given a direct mechanism $[T_i, D_i, \overline{d}_0, \overline{r}_i]_{i=1,2}$ and its equilibrium $\overline{\mu}_i(t_i) = t_i$ and $\overline{\delta}_i(d_i'(t), t_i) = d_i(t)$ (remember that d_i' represents $\overline{r}_i(t) = \delta_i[r_i(\mu(t)), t_i]$), let assume that we can find a coalitional mechanism $[\tilde{\mu}, \tilde{\delta}]$ which threatens the direct mechanism as follows:

$$\tilde{\mu}_i : T \rightarrow T_i, \text{ where } \tilde{\mu}_i(t) \in T_i \text{ and } \tilde{\delta}_i : T \rightarrow D_i, \text{ where } \tilde{\delta}_i(t) \in D_i \tag{6}$$

$\forall i, \ \forall \beta_i : T_i \rightarrow T_i$

$$E_t [U_i \{ \bar{d}_0 (\tilde{\mu}(t)) , \tilde{\delta}_i(t) , \tilde{\delta}_j(t) , t \}] \geq E_t [U_i \{ \bar{d}_0 [\tilde{\mu} (\beta_i(t_i), t_j)] , \tilde{\delta}_i (\beta_i(t_i), t_j) , \tilde{\delta}_j (\beta_i(t_i), t_j) , t \}] \tag{7}$$

$\forall i$ i=1,2

$$E_t [U_i \{ \bar{d}_0 (\tilde{\mu}(t)) , \tilde{\delta}_i(t) , \tilde{\delta}_j(t) , t \}] \geq E_t [U_i \{ \bar{d}_0 (t) , d_i(t) , d_j(t) , t \}] \tag{8}$$

with at least one strict inequality

Step 3

We are done if we can build a coalitional mechanism which threatens the original mechanism, given coalitional mechanism $[\tilde{\mu}, \tilde{\delta}]$ which threatens the direct mechanism $[T_i, D_i, \bar{d}_0, \bar{r}_i]_{i=1,2}$ That is, we should find some coalitional mechanism $[\mu^*, \delta^*]$ which threatens the original mechanism such that

$$\mu_i^* : T \rightarrow M_i \quad \text{and} \quad \delta_i^* : T \rightarrow D_i \tag{9}$$

$\forall i, \ \forall \alpha_i : T_i \rightarrow T_i$

$$E_t [U_i \{ d_0 (\mu^*(t)) , \delta^*_i(t) , \delta^*_j(t) , t \}] \geq E_t [U_i \{ d_0 [\mu^* (\alpha_i(t_i), t_j)] , \delta^*_i (\alpha_i(t_i), t_j) , \delta^*_j (\alpha_i(t_i), t_j) , t \}] \tag{10}$$

$\forall i$

$$E_t[\, U_i\{ d_0(\mu^*(t)),\ \delta^*_i(t),\ \delta^*_j(t),\ t\}\,]$$
$$\geq\ E_t[\, U_i\{ d_0(\mu(t)),\ \delta_i(r_i(\mu(t)),\ t_i),\ \delta_j(r_j(\mu(t)),\ t_j),\ t\}\,] \tag{11}$$

with at least one strict inequality

Remember that $\bar{d}_0(t_i, t_j) = d_0(\mu_i(t_i), \mu_j(t_j))$. Hence (10) and (11) becomes

$\forall i,\ \ \forall \beta_i : T_i \rightarrow T_i$

$$E_t[\, U_i\{ d_0[\, \mu_i(\widetilde{\mu}_i(t)),\ \mu_j(\widetilde{\mu}_j(t))\,],\ \delta tlde_i(t),\ \widetilde{\delta}_j(t),\ t\}\,]$$
$$\geq\ E_t[\, U_i\{ d_0[\, \mu_i(\widetilde{\mu}_i(\beta_i(t_i), t_j)),\ \mu_j(\widetilde{\mu}_j(\beta_i(t_i), t_j))\,],\ \widetilde{\delta}_i(\beta_i(t_i), t_j),\ \widetilde{\delta}_j(\beta_i(t_i), t_j),\ t\}\,] \tag{12}$$

$\forall i$ i=1,2

$$E_t[\, U_i\{ d_0[\, \mu_i(\widetilde{\mu}_i(t)),\ \mu_j(\widetilde{\mu}_j(t))\,],\ \widetilde{\delta}_i(t),\ \widetilde{\delta}_j(t),\ t\}\,]$$
$$\geq\ E_t[\, U_i\{ d_0(\mu_i(t_i),\ \mu_j(t_j)),\ d_i(t),\ d_j(t),\ t\}\,] \tag{13}$$

with at least one strict inequality

Observe that $d_i(t) = d_i^r(t) = \delta_i[r_i(\mu(t)),\ t_i]$. Thus, we can build [μ^*, δ^*] by simply choosing $\mu_i^*(\cdot) = \mu_i[\hat{\mu}_i(\cdot)]$ and $\delta_i^*(\cdot) = \delta_i(\cdot)$

Q. E. D.

The intuition behind this function [μ_i^*, δ_i^*] is the following: we know that the main contract is given. We assume that the agents can find a coalitional mechanism which threatens the equivalent direct mechanism to this given original coordination mechanism. Then, given this coalitional mechanism for the direct mechanism, one simple way for the agent i to cheat in original mechanism is to follow the Bayesian Nash equilibrium strategy as if his true type is $\hat{\mu}_i(t)$ instead of t_i. However, this is possible only under the institutional assumptions we made.

5. Concluding Remarks

We considered the most simple possible institutional set up. However, since the agent's private decisions are still privately controlled by the agents, the model considered in this chapter analyzes a more general problem than the model in the previous chapter.

We can easily extend this results to a richer institutional set up. For instance, we can allow the possible randomness about the response from the principal. Possibly, we can assume that the principal have some private information. In this case, it is obvious that the coalitional mechanism is some decision rules ($\hat{\mu}, \hat{\delta}$) such that $\hat{\mu}_i : T \rightarrow M_i$ and $\hat{\delta}_i : R_i x T \rightarrow D_i$. Remember that r_i is still private information and the colluding parties are not allowed to meet again and exchange information about the responses from the principal. With this set up, we can show that the Revelation Principle is applicable. Considering the explicit second stage coalition will be a much more complex problem. At this moment, we choose not to model this.

Another problem is about the timing of the coalition formation. We can consider the interim time horizon where the agents posecess their private information before the principal design a contract. In this interim set up, the analysis will be bascally same except the measure of the agents' utility. However, conceptually we need possibly more limited assumptions to find a successful coalitional mechanism. When we consider the ex-ante model, the colluding parties sign a side-contract before any private information is revealed. In this case, we expect that the contents of the side contract itself will be restricted by the future revelation of information asymmetry problem. But, they face no difficulity in agreeing on the contents of side-contract. Actually, however, when each agent has his private information before they sign a side-contract, the coalition formation process become a bargaining situation. We do not have good a theory to predict the outcome of this bargaining process. Hence, we may need another strong assumption such as this bargaining ends up with the some coalitional mechanism we described.

FOOTNOTES

1 See the note (3) of chapter II. We can have same arguememt here.

2 This communication system may not necessarily be a one-stage affair. See Harris and Townsend[1981].

3 In this model, each agent's private decision is not explicitly connected to the principal's decision. However, the principal's decision itself might be contingent on the revelation of some future parameter, which may depend on the agent's private decisions.

4 How an individual's welfare should be measured depends crucially on what information he possesses at the time. We have three stages : ex ante, before individuals have received any private information; interim, where each individual has received his private information t_i but does not know the other's information; and ex post, when the information state t is public knowledge. See Holmstrom and Myerson [1983] for the detail.

CHAPTER IV COALITIONS FORGING EVIDENCE

1. Introduction

This chapter has two purposes: First, we explicitly present the model of coalition formation implicit in Tirole(1986)'s paper. Second, we pursue his analysis with different assumptions and try to figure out the effect of collusive behavior within the hierarchical organizations.

Recently, Tirole [1986] has analyzed the effects of collusive behavior in hierarchical organizations. Organizations (firms) are often considered as networks of overlapping principal-agent relationships (See e.g., Williamson [1967b], Alchian and Demsetz [1972], Mirrlees [1976] and Calvo and Wellisz [1978]). As is well known, in principal-agent models, there is no room for collusive behavior, because two parties have strictly conflicting objectives.[1] However, many studies of the organizations and bureaucracies have shown that collusive behavior, implicit or explicit, does exist and is often prominent. [2] For example, there have been many of manager/auditor collusion.[3] Therefore, the analysis of the hierarchical organizations cannot boil down to two-tier principal/agent structures.

In his important paper, Tirole [1986] considers a three-tier one principal / one supervisor / one agent relationship as a simple abstraction for the analysis of coalition problems in hierarchical organizations. In this paper, we concentrate on collusion within the firm. [4] The principal is the owner or top manager of the firm. The principal needs to hire a supervisor for various reasons. For example, he needs the supervisor's productive activities such as coordination, organization, counseling and selecting the agent etc. Also, the principal wants to monitor the agent and sometimes has to observe the agent's private information. However, he may not have time for this kind of monitoring because either he has too many agents to monitor, or he wants to spend his time at other activities such as long term planning. Because of this, the principal needs a specialist for monitoring.

In this chapter, we assume away any productive role of the supervisor. In other words, the principal hires the supervisor only for gathering the agent's private information which affects the profit of the firm. In this context, collusive behavior manifests itself through the manipulation of relevant information by the supervisor and the agent. Sociological studies of collusive behavior have observed two types of manipulation of information [5] :

(I) ignoring or concealing relevant information.

(II) distorting the information.

Ignoring or concealing relevant information might be considered as the more implicit form of collusion. For example, supervisors usually neglect to report the employee's minor use of materials and services for personal ends, as far as it is not too serious. This is because reporting this information often leads to bad evaluation of the supervisor himself and/or processing this information is costly. Hence, the supervisor usually ignores this observation, and implicitly colludes for the benefit of both parties. Distorting the information can be considered as a more explicit form of collusion. A well known example is the coalition between top managers and accountants. For example, if the top managers' salary level depends on the total sales for a year, they may try to manipulate the sales record, for instance, by adding the sales of the first month of this year to last year's sales level. Distorting information includes both changing the level of given parameters or records and creating false information. In this paper, we consider both type of manipulation, whereas Tirole [1986] confined himself to type

(I) behavior. In other words, Tirole considered only the " hard (verifiable) " information, whereas we consider both " hard and soft (unverifiable) " information. Tirole interprets his results as showing that the supervisor naturally acts as an advocate for the agent. However, we will show that this result hinges crucially on the fact that he considers only type (I) behavior.

Section 2 describes the one principal / one supervisor / one agent model of Tirole [1986]. The principal cannot observe either the agent's effort or his private information (for example, realization of some production parameter). By hiring the supervisor, the principal can get some information. Uncertainties, information and the supervisor's reporting technology is described and some institutional assumptions about the bargaining process within the coalition are introduced .

In section 3, we investigate the mechanism (contract) which can guarantee a collusion free outcome to the principal. For this purpose, we explicitly derive the coalitional incentive compatibility conditions for the collusion under asymmetric information as well as under symmetric information. The principal can design a contract which can implement the collusion free allocation by imposing the proper constraints. We investigate the properties of coalition proof contracts and also find that it will be costly for the principal to implement this mechanism. This follows becausethe suboptimality of the agent's effort in the coalition proof mechanism is more severe than that in the overt contract case (the possibility of collusion is not allowed). The remaining part describes some implications of the model. Under our assumptions about coalitional behavior, the supervisor does not naturally act as an advocate for the agent. He has a degree of freedom to act as an advocate either for the principal or for the agent, or for neither. And even if the supervisor is infinitly risk averse, he may play some role.

2. The Model

The model presented in this section closely follows that of Tirole [1986]. We consider a firm as an example of the three tier principal/supervisor/agent relationship. The owner of a production process, the principal, wants to hire a worker, the agent, to perform some productive activity. As mentioned in the introduction, the principal also wants to hire a supervisor in order to monitor the agent. It is assumed implicitly that the principal lacks the time and/or the special knowledge to efficiently monitor the agent. We start with the standard principal-agent type model, and then place more structure on this model in order to investigate the effect of coalition between two parties.

2.1. The Players: The agent (worker) chooses a level of effort $e \geq 0$, which together with the realization of an exogenous productivity parameter θ determines the profit X.

$X = f(\theta, e)$ or simply $X = \theta + e$

The function g(e) represents the agent's disutility of effort in monetary terms, where g(•) is increasing, strictly convex and satisfies $g(0) = g'(0) = 0$. The agent has an increasing, differentiable and strictly concave Von Neuman-Morganstern utility function U(•). Hence, his expected utility is EU(W-g(e)), where W is the wage he receives from the principal. We assume that there exists ex-ante competition in the supply of agents, with reservation wage W_0 and reservation utility level $\bar{U} \equiv EU(W_0)$. This gives the agent's participation constraint (equivalently the agent's individual rationality constraint. From now on, we will represent this as AIR) :

$$(AIR) \qquad EU(W - g(e)) \geq \bar{U}$$

The supervisor has a Von Neuman-Morgenstern utility function, V(•), which is increasing, differentiable and strictly concave. Let S be the wage which the supervisor receives from the principal. In this paper, following Tirole, we suppose that the supervisor has no directly productive role. Furthermore, we assume he exerts no effort in supervising the agent. In this model, supervision involves only information gathering and reporting.[6] Again, we assume that there exists an ex-ante competitive supply of supervisors, with reservation wage S_0 and reservation utility $\bar{V} \equiv V(S_0)$. The supervisor's individual rationality condition is :[7]

$$(SIR) \qquad EV(S) \geq \bar{V}$$

Later, when we discuss coalitions, we will explain in more detail the supervisor's role and responsibilities.

The principal owns the production process, and offers contracts to the supervisor and the agent. His expected utility is E(X-S-W). This assumption implies that the principal is risk neutral. In other words, he takes all the risk and the supervisor has no risk sharing role. Finally, all players are assumed to be expected utility maximizers.

2.2. Information Structure: The uncertainty in the model stems from the randomness of productivity parameter. The principal cannot observe the realization of θ. However, the agent, after he accepts the contract, can observe θ before he chooses his effort level. The principal hires the supervisor in order to get some information about θ and asks the supervisor to report his observation. We assume, however, that the supervisor cannot always observe the productivity parameter θ which can take only two values: $\bar{\theta}$ and $\underline{\theta}$ such that $\Delta\theta \equiv \bar{\theta} - \underline{\theta}$ is strictly positive. $\underline{\theta}$ represents the low productivity state and $\bar{\theta}$ the high productivity state. By combining the two levels of productivity parameter and the cases whether the supervisor can observe θ or not, we have 4 states of nature as follows (S and A means supervisor and agent respectively):

state 1 : $\theta = \underline{\theta}$, observed by both A and S

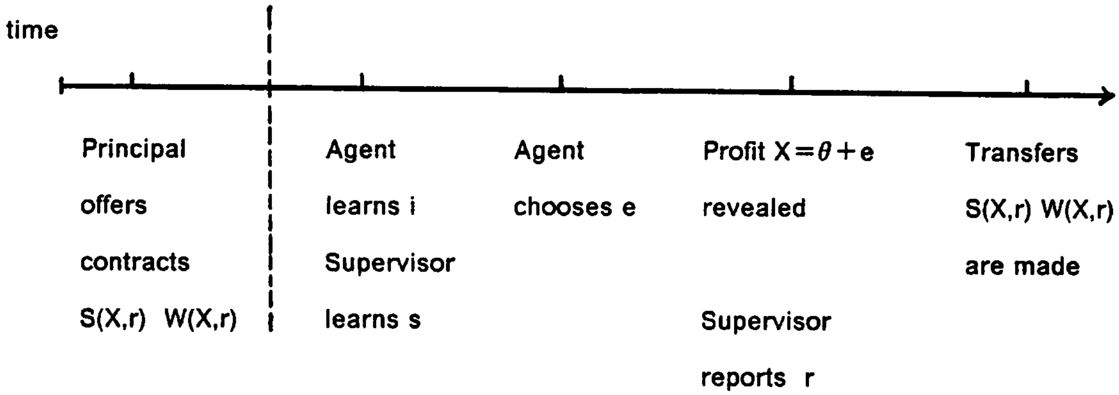
state 2 : $\theta = \underline{\theta}$, observed only by A (S observes nothing)

state 3 : $\theta = \overline{\theta}$, observed only by A (S observes nothing)

state 4 : $\theta = \overline{\theta}$, observed by both A and S

We assume that state of nature i occurs with probability $p_i\,(\sum_{i=1}^{4} p_i = 1)$.[8] Thus, in states 2 and 3, the supervisor cannot observe the realized productivity parameter. In other words, he cannot distinguish whether the agent observes $\underline{\theta}$ or $\overline{\theta}$. The agent's observation set is $a \in \{\underline{\theta}, \overline{\theta}\}$. Formally, we will will say that the signal, that the supervisor receives about the productivity parameter s is $\underline{\theta}$ in state of nature 1, ϕ in states of nature 2 and 3 and $\overline{\theta}$ in state of nature 4 i.e., $s \in \{\underline{\theta}, \overline{\theta}, \phi\}$, where ϕ means no observation. The possible reports by the supervisor to the principal will also be $r \in \{\underline{\theta}, \overline{\theta}, \phi\}$. If r is identical to s in each state of nature, then the supervisor truthfully conveys the information. Also, we assume that the agent knows whether the supervisor observes θ or not. By this, we assume the agent knows the state of nature, not only the level of productivity.

We summarize the decision process of the model as follows: the principal offers contracts to the supervisor and to the agent which are function of observable variables, i.e., the profit X and the supervisor's report r to the principal. The principal moves first, offering the contract S(X,r) and W(X,r) to the supervisor and the agent respectively. After accepting these offers, the agent knows the realized state of nature and the supervisor observes his signal s, which might be some θ or ϕ. Then the agent chooses his optimal level of effort e and then the output $X = \theta + e$ is revealed. In this model, the principal can choose the timing of the supervisor's report. Finally, the payment S and W are determined.[9]



2.3. Contract Without Coalition: We want to deal with the case without collusion as a reference point for the discussion of the coalition. For the purpose of comparison, we first consider the full information (first-best) allocation. For the time being, we ignore the information structure described in the previous section and we assume that the principal can directly observe the productivity parameter as well as the effort exerted by the agent. Hence, there is no uncertainty in this case and the supervisor has no supervisory role. Then, the agent will exert the optimal level of effort e^* where the marginal disutility of effort is equal to the marginal contribution to the profit. In this simple model, we get e^* such as $g'(e^*)=1$ for all θ. If we denote $g^* \equiv g(e^*)$ as a corresponding disutility of optimal effort, then the agent's wage in each state will be $W_0 + g^*$, which is independent of the state of nature. In this case, as is well known, the principle can enforce a first-best contract by offering the wage level $W_0 + g^*$ in all states of nature. Then, the agent receives no more than his reservation utility level in any state. Finally, since the supervisor has no supervisory function, he gets reservation wage S_0 for all states of nature.

Now, we return to the information structure described in the section 2.2. We assume that collusion between the supervisor and agent (from now on, I sometimes use collusion S/A or coalition S/A for this) is not allowed or not feasible. Then the supervisor does not have any incentive to misreport. Also, given reservation wage level S_0 in all states of nature, the supervisor obtains full insurance and is willing to participate. Therefore, the principal can have the supervisor's information by paying S_0 in all states of nature. Actually, in this case, the three tier relationship will boil down to the usual principal-agent relationship, where the principal pays constant wage S_0 and inherits the supervisor's information. Then, the principal can design an enforceable contract by making the contract based on the publicly observed variable X (output). Furthermore, he can confine his contract to the output level in each state of nature by the revelation principle.[10] In other words, the principal can offer the contract $W_i \equiv w(X_i)$ and $S_i \equiv s(X_i)$ where $X_i = \theta_i + e_i$, i = 1, 2, 3, 4 (See Harris and Townsend [1981] and Myerson [1979], [1982]). Then, we find an optimal enforceable contract for the principal as a mathematical program (CF) where the principal's expected utility is maximized subject to the agent's individual rationality constraints and the self-selection constraints:

$$(CF) \qquad \underset{\{W_i, e_i\}}{\text{Max}} \sum_{i=1}^{4} P_i(\theta_i + e_i - W_i)$$

s. t.

$$(AIR) \qquad \sum_{i=1}^{4} P_i U(W_i - g(e_i)) \geq \bar{U}$$

$$(AIC1) \qquad W_3 - g(e_3) \geq W_2 - g(e_2 - \Delta\theta)$$

$$(AIC2) \qquad W_2 - g(e_2) \geq W_3 - g(e_3 + \Delta\theta)$$

In this model, the agent has private information in the states 2 and 3. The agent's incentive compatibility constraint $W_3 - g(e_3) \geq W_2 - g(e_2 - \Delta\theta)$ means that the agent cannot claim that he is in the state 2 by producing X_2 and exerts the effort level $e_2 - \Delta\theta$ in state 3. Without (AIC1), we could have $W_3 - g(e_3) < W_2 - g(e_2 - \Delta\theta)$ and the agent would prefer to produce

X_2 by exerting only $e_2 - \Delta\theta$ in state 3. (Observe that since $X_2 = e_2 + \bar{\theta}$ by definition, which is equal to $(e_2 - \Delta\theta) + \bar{\theta}$, the workers in state 3 can produce X_2 by exerting $e_2 - \Delta\theta$) Hence, by enforcing this constraint, the agent in state 3 does not have any incentive to misreport his true state. [11] In state 2, the agent may have an incentive to claim his true state is 3 by producing X_3. However, we can easily see that this constraint is not binding at an optimum, since the agent can only cheat the principal by misrepresenting from $\bar{\theta}$ to $\underline{\theta}$.

lemma 1 : (proposition 1 of Tirole [1986])

The solution of (CF) has the following features:

a) The supervisor gets S_0 for all state of nature

b) $e_4 = e_3 = e_1 = e^* > e_2$

c) $W_3 - g(e_3) > W_4 - g(e_4) = W_1 - g(e_1) > W_2 - g(e_2)$

Hence, $W_3 > W_4 = W_1 > W_2$

d) AIC1 is binding and AIC2 is not.

We consider this case just for comparison with the discussion of the coalition case (the proof of this lemma is provided in the appendix (A)). Interpretation of these results clarifies characteristics of this model. In states of nature 1 and 4, the principal can perfectly monitor the agent as he knows the true θ and can observe the output. Therefore, the agent cannot shirk, hence $e_1 = e_4 = e^*$, and W_1 must be equal to W_4. In the state 3, by offering high wage level W_3, the principal can give the agent incentive to reveal his true information $\bar{\theta}$ and in the state 2, offering low wage level W_2, make it less atractive for the agent to shirk in the good state of productivity. Since the principal can find out the true parameter in the state 3, he can sucessfully monitor the agent's effort, thus the agent exerts e^*. Hence the principal can guarantee to himself the outcome in which the agent cannot shirk in state 3. However, low wage level in state 2 cannot gives the agent enough incentive to exert the effort level e^*. Thus, the cost of truthful revelation is some suboptimal level of effort in state 2 i.e., $e_2 < e^*$.

2.4. The Supervisor/Agent Coalition: As we have seen above, the possibility of collusion is the key reason why the analysis of organizations and/or hierarchies dose not boil down to the two tier principal-agent relationship. If we assume in the simple model introduced in the previous section, that either the main contract does not forbid any side contracts between the supervisor and the agent or that the side payment is not observable by the supervisor, we can easily see that the allocations in lemma 1 would not be sustainable. For example, the supervisor gets the same wage in states 3 and 4, however, the agent gets higher utility at state 3. This gives the agent some incentives to bribe the supervisor to conceal his information. Also in states 2, the agent is willing to pay the supervisor for reporting $\underline{\theta}$, as an excuse for bad performance. In this section, we add some more structures and assumptions in order to discuss the S/A coalition problem.

As we assumed earlier, agents collude by manipulating information. Under the S/A coalition, the supervisor may report false information, in one of two ways :

(I) by concealing relevant information.

(II) by reporting a false level of the parameter θ.
(misrepresentation of evidence)

Tirole [1986] deals only with type (I) misrepresentation , since concealing information might be more plausible behavior in terms of verifiability of the report. However, we frequently observe colluding parties trying to manipulate evidence. In other words, once they agree to collude, they sometimes become aggressive enough to produce false evidence. For example, consider the stockholder / top manager / worker hierarchy. It is well known that manager / worker coalitions may try to manipulate the level of profit and/or the level of the sales when the manager's reward depends on the level of profit and/or the level of sales respectively. Hence, we have:

if $s=\underline{\theta}$, $r \in \{\underline{\theta}, \bar{\theta}, \phi\}$

$$\text{if } s=\phi, \quad r \in \{\phi, \underline{\theta}, \overline{\theta}\}$$

$$\text{if } s=\theta, \quad r \in \{\overline{\theta}, \underline{\theta}, \phi\}$$

Whereas, considering only type (I) behavior, Tirole assumed

$$\text{if } s=\theta, \quad r \in \{\theta, \phi\}$$

$$\text{if } s=\phi, \quad r=\phi$$

In this model only the supervisor is supposed to report the productivity parameter to the principal.[12] In many cases, the agent may not be in a position to produce proper information by himself. For example, if the production process is highly sophisticated, then even though the agent knows the contents of information that the principal needs he cannot transmit this information convincingly. In other words, sometimes the agent may not be able to put the evidence in a form that the principal can understand or use. Alternatively, the agent may not have time to accumulate evidence by himself.Obviously, the agent can also convey information properly in some cases. We will discuss the supervisor's role when the agent can convey information properly by himself later. We need another institutional assumption in order to ensure that coalition formation is not too complex. We assume that the supervisor's report is public information both to the principal and to the agent. Without this assumption, the agent cannot be sure about whether the supervisor's report is what they agree to send. By this, we can assume that if anyone unilaterly deviates from the coalition, then the other can observe this and both will be sevely punished by the principal. Within this structure, we assume that the supervisor and the agent try to sign a Pareto optimal side contract and each of the colluding parties can guarantee itself a strictly greater payoff than he would receive without side-contracts.[13]

With these assumptions about the S/A coalition, the decision process of the model is follows: The principal offers contract $\{ S_i, W_i \}$ and recommendation about e_i to the supervisor and the agent respectively. Then, given this main contract, the supervisor and the agent try to sign a side contract before the uncertainties are revealed. Specifically, given main contract, they assent to do the following: if some state of nature i occurs, then we would misrepresent our real state of nature as state j by manipulating the supervisor's report r and the output X and decide the side payments σ_i according to given main contract W_i, W_j and S_i, S_j. We denote this side-contract $\sigma_i \equiv \sigma(X_i, r_i)$ as a monetary side payment from the agent to the supervisor. Then the rest of decision process of the model is same with in the previous section. Finally, the supervisor and the agent will receive $S_i + \sigma_i$ and $W_i - \sigma_i$ respectively. However, this side-contract has serious difficulity: the supervisor cannot distinguish the state of nature 2 from the state of nature 3 and vice versa. In states 2 and 3, we suffer from the problem of coalition formation under asymmetric information whereas, in state 1 and 4, both of the colluding parties share the same information. In the next section, we explicitly present the coalitional mechanism under which the supervisor and the agent manage this informational asymmetry and form a coalition successfully.

3. Derivation of Optimization Problem and Results

In this section, we try to find some optimization problem whose solution gives an optimal coalition proof contract to the principal.

The principal maximizes his expected payoff under some constraints including the usual participation constraints, the agent's incentive compatibility constraint and the constraints which can prevent the formation of S/A coalitions. Following Tirole[1986], we call this last constraint the coalition incentive compatibility constraints (From now on, we represent this constraint as CIC).

3.1. Coalition Incentive Compatibility Constraints: In the states of nature 1 and 4, the agent and the supervisor have the same information. So, they can easily form a coalition by coordinating the supervisor's report to the principal and the output level. Therefore, the principal can easily find constraints, which ensure that the supervisor and the agent have no incentive to collude. In the states of nature 1 and 4, we have following CIC's:

$$(CIC1) \qquad S_1 + W_1 - g(e_1) \geq S_2 + W_2 - g(e_2)$$

$$(CIC2) \qquad S_1 + W_1 - g(e_1) \geq S_3 + W_3 - g(e_3 + \Delta\theta)$$

$$(CIC3) \qquad S_1 + W_1 - g(e_1) \geq S_4 + W_4 - g(e_4 + \Delta\theta)$$

$$(CIC4) \qquad S_4 + W_4 - g(e_4) \geq S_3 + W_3 - g(e_3)$$

$$(CIC5) \qquad S_4 + W_4 - g(e_4) \geq S_2 + W_2 - g(e_2 - \Delta\theta)$$

$$(CIC6) \qquad S_4 + W_4 - g(e_4) \geq S_1 + W_1 - g(e_1 - \Delta\theta)$$

The meaning of these constraints is clear. If the allocation specified in the main contract does not meet (CIC1) i.e., $S_2 + W_2 - g(e_2) > S_1 + W_1 - g(e_1)$, then, the S/A coalition tries to increase their total wage bill by concealing the true information $\underline{\theta}$ thus, reporting nothing (r= ϕ) in state 1. Similary, if (CIC2) is not met, i.e., $S_3 + W_3 - g(e_3 + \Delta\theta) > S_1 + W_1 - g(e_1)$, then S/A coalition conceals the true information $\underline{\theta}$ and, furthermore, agrees to disguise the real θ by producing X_3 in state 1. Observe that if $S_3 + W_3 - g(e_3 + \Delta\theta) > S_1 + W_1 - g(e_1)$, then we know $W_3 - g(e_3 + \Delta\theta) > W_1 - g(e_1)$, and $S_3 > S_1$ since we assumed that each of colluding parties can guarantee itself a greater payoff than before. Then the agent prefers to produce X_3 by exerting the effort level $e_3 + \Delta\theta$ in order to get W_3 since $X_3 = e_3 + \bar{\theta} = e_3 + \Delta\theta + \underline{\theta}$. Similarly, without the constraint (CIC 3), we can have $S_4 + W_4 - g(e_4 + \Delta\theta) > S_1 + W_1 - g(e_1)$. This means the total wage bill of the supervisor and the agent can increase by reporting false level r= $\bar{\theta}$ instead of reporting true one r= $\underline{\theta}$. This gives them incentive to collude. Hence we need (CIC 3). The same story is true for (CIC4), (CIC5) and (CIC 6) at state 4. Tirole [1986], however, did not consider (CIC 3) and (CIC 6) as he assumed away the possibility of reporting false level of evidence.

In the states of nature 2 and 3, however, there is an informational asymmetry between the supervisor and the agent, only the agent can distinguish state 2 from state 3 whereas the supervisor cannot. Because of this asymmetry problem, we cannot directly derive the coalition incentive compatibility conditions. Thus, in a first step,we investigate all the possible cases of coalition in the states of nature 2 and 3. Then, we try to find the proper CIC's which can prevent such collusions.

We need to describe all the possible contents of side-contracts which can be signed between the supervisor and the agent given any possible allocation (contract) W_i, S_i and e_i. The actual side-contract depends on the main contract offered by the principal. At this point, however, we should consider all the possible cases of side contract given any main contracts (allocations). This is because we want to design a coalition proof contract from the principal's view point.

Since the supervisor cannot distinguish state 2 from state 3, we can easily imagine that the side-contract will specify the coordinated report and the production level in the state 2 and the state 3 simultaneously. For example, given some main contract allocation, S/A coalition agrees to report $\underline{\theta}$ whatever the real state is (i.e. both state 2 and state 3). Therefore, we have following list of all the possible contents of a side-contract between the supervisor and the agent in state 2 and state 3. (Note that the LHS of arrow shows the actual state of nature and the RHS of the arrow represents the state which the colluding parties agree to claim by manipulating the report and the output level. Here S and A represent the supervisor and the agent respectively)

I	state 2 -->	1	S	reporting $\underline{\theta}$
	state 3 -->	1	S	reporting $\underline{\theta}$
II-1	state 2 -->	1	S	reporting $\underline{\theta}$

state 3 --> 2 S reporting ϕ and A producing X_2

II-2 state 2 --> 1 S reporting $\underline{\theta}$

state 3 --> 3 truth telling

III state 2 --> 1 S reporting $\underline{\theta}$

state 3 --> 4 S reporting $\overline{\theta}$

IV-1 state 2 --> 2 truth telling

state 3 --> 1 S reporting $\underline{\theta}$

IV-2 state 2 --> 3 S reporting ϕ but A producing X_3

state 3 --> 1 S reporting $\underline{\theta}$

V-1 state 2 --> 2 truth telling

state 3 --> 2 S reporting ϕ but A producing X_2

V-2 state 2 --> 3 S reporting ϕ but A producing X_3

state 3 --> 2 S reporting ϕ but A producing X_2

V-3 state 2 --> 3 S reporting ϕ but A producing X_3

state 3 --> 3 truth telling

VI-1 state 2 --> 2 truth telling

state 3 --> 4 S reporting $\overline{\theta}$

VI-2 state 2 --> 3 S reporting ϕ but A producing X_3

state 3 --> 4 S reporting $\overline{\theta}$

VII	state 2 --> 4	S reporting $\bar{\theta}$
	state 3 --> 1	S reporting $\underline{\theta}$
VIII-1	state 2 --> 4	S reporting $\bar{\theta}$
	state 3 --> 2	S reporting ϕ but A producing X_2
VIII-2	state 2 --> 4	S reporting $\bar{\theta}$
	state 3 --> 3	truth telling
IX	state 2 --> 4	S reporting $\bar{\theta}$
	state 3 --> 4	S reporting $\bar{\theta}$

The story of each case of side-contract is as follows: Case I means that colluding parties agree to misrepresent their state of nature as state 1 by agreeing to report $\underline{\theta}$ whatever the real state of nature is. Similary, in case IX, they agree to report $\bar{\theta}$ in both states 2 and 3, hence behave as in state 4. Case III represents following agreements between the supervisor and the agent: If the real state claimed by the agent is 2, the supervisor is supposed to report $\underline{\theta}$ instead of reporting nothing and if the real state claimed by the agent is 3, he is supposed to report $\bar{\theta}$ instead of reporting nothing. Notice that the supervisor cannot actually distinguish the state 2 from 3. However, if colluding parties find that case III coalition can increase their utility level given some main contract, they can manage this problem and try to behave according to the agreement of case III. They face an informational asymmetry problem in forming coalitions. We will analyze this explicitly by investigating coalitional mechanisms later.

Consider the cases V-1, V-2 and V-3. In terms of the supervisor's report, they are identical, he tells the truth. However, since S_i and W_i depends not only on report but also on the output, which is again function of θ and e, θ and e will be different in these two states even if the report by the supervisor is the same in state 2 and state 3. Therefore,

S_2 and S_3, $W_2 - g(e_2)$ and $W_3 - g(e_3)$ and thus X_2 and X_3 might be also different. For example, if the allocation (contract) offered by the principal satisfies the following relationship; $S_2 + W_2 - g(e_2 - \Delta\theta) > S_3 + W_3 - g(e_3)$, then S and A have incentive to collude as in V-1 if they can manage the problem of informational asymmetry within the coalition (See the Coalitional Individual Rationality condition of V-1 in figure 2 at the end of appendix). In other words, with the AIC condition $W_3 - g(e_3) \geq W_2 - g(e_2 - \Delta\theta)$ the supervisor has incentive to bribe the agent to behave in state 3 as in state 2 (See $S_2 > S_3$). Contrary to this, in V-3 coalition, the supervisor wants the agent to behave in state 2 as in state 3. We can easily see the stories for the rest of cases by applying similar arguments.

Now, we describe the coalitional mechanism when the colluding parties have an informational asymmetry problem, which is implicit in Tirole [1986]. The question is under what conditions the supervisor and the agent can successfully form a coalition given informational asymmetry.

We assume that there emerge the S/A coalitions if [14]

(i) The coalition can find an incentive compatible mechanism
(The agent has no incentive to misrepresent his own stste)

(ii) The coalition is feasible (both colluding parties get more than before)

We can find conditions for coalition formation from these assumptions. First, since the agent has private information, he can misrepresent his real state of nature to the supervisor within the coalition. In order to prevent this problem, we need to put some constraints under which truth telling is optimal for the agent in the coalition. We call these "coalitional agent's incentive compatibility constraints (CAIC)". Next, in order to form a coalition, the agent's net expected utility after the side contract in state 2 and 3 should be strictly greater than his net expected utility before coalition. Similarly, the supervisor's average wage after the side contract must be strictly greater than his wage without coalition. We call this "coalitional individual rationality constraint of the agent (CIRA)" and "coalitional individual rationality constraints of the supervisor (CIRS)" respectively. We derive (CIRA), (CIRS) and (CAIC) in each and every possible case of the S/A coalition in states of nature 2 and 3. (See the appendix (B)).

Here, we choose arbitrary one case, say case II-2 (state 2 --> 1, state 3 --> 3) and discuss this case in detail. This coalition represents the following story: if the true parameter claimed by the agent is $\underline{\theta}$, then the supervisor reports $\underline{\theta}$, instead of reporting nothing (observe that the supervisor reports nothing without coalition). On the other hand, if the true parameter claimed by the agent is $\overline{\theta}$, then the colluding parties agree to ignore this information, hence the supervisor reports nothing (This is equivalent to the supervisor's telling the truth).

The individual rationality constraints of both parties to the coalition are:

$$\text{(CIRA)} \quad P'_2(W_1 - g(e_1) - \sigma_2) + P'_3(W_3 - g(e_3) - \sigma_3) > P'_2(W_2 - g(e_2)) + P'_3(W_3 - g(e_3))$$

$$\text{(CIRS)} \quad P'_2(S_1 + \sigma_2) + P'_3(S_3 + \sigma_3) > P'_2 S_2 + P'_3 S_3$$

Here, σ_2 and σ_3 represent the side payment from the agent to the supervisor in states of nature 2 and 3 respectively, which is equal to $\sigma(X_2, r_2)$ and $\sigma(X_3, r_3)$ respectively and P'_2 is equal to $P_2 / (P_2 + P_3)$ and P'_3 is equal to $P_3 / (P_2 + P_3)$, hence $P'_2 + P'_3 = 1$

From (CIRA) and (CIRS), we get:

$$S_1 + W_1 - g(e_1) > S_2 + W_2 - g(e_2)$$

We call this constraint simply coalitional individual rationality (CIR) of coalition II-2. This implies that if this constraint is not satisfied, then one or both of the parties' individual rationality constraints is not met. In other words, if (CIR) is not satisfied, a coalition of case II-2 will not be feasible.

In addition to this, since the agent can misrepresent his true information, we need CAIC in states of nature 2 and 3. Thus we have:

$$\text{(CAIC in state 2)} \quad W_1 - g(e_1) - \sigma_2 \geq W_3 - g(e_3 + \Delta\theta) - \sigma_3$$

If this holds in the state of nature 2, the agent does not have any incentive to behave as in the state of nature 3. Similarly we have:

$$\text{(CAIC in state 3)} \quad W_3 - g(e_3) - \sigma_3 \;\geq\; W_1 - g(e_1 - \Delta\theta) - \sigma_2$$

These two conditions are equivalent to

$$W_1 - g(e_1) - W_3 + g(e_3 + \Delta\theta) \;\geq\; \sigma_2 - \sigma_3$$

and

$$\sigma_2 - \sigma_3 \;\geq\; W_1 - g(e_1 - \Delta\theta) - W_3 + g(e_3)$$

They imply :

$$W_1 - g(e_1) - W_3 + g(e_3 + \Delta\theta) \geq W_1 - g(e_1 - \Delta\theta) - W_3 + g(e_3)$$

$$<==> g(e_3 + \Delta\theta) - g(e_1) \;\geq\; g(e_3) - g(e_1 - \Delta\theta)$$

$$<==> g(e_3 + \Delta\theta) - g(e_3) \;\geq\; g(e_1) - g(e_1 - \Delta\theta)$$

Since g(•) is continuous and strictly convex, we have $e_3 + \Delta\theta \;\geq\; e_1$, or equivalently $e_3 \;\geq\; e_1 - \Delta\theta$. We summarize these conditions in the following lemma.

lemma 2 : There emerges the coalition of case II-2 if and only if

both CAIC $e_3 \geq e_1 - \Delta\theta$ and CIR $S_1 + W_1 - g(e_1) > S_2 + W_2 - g(e_2)$

are true at the same time.

[Proof]

Proof of only if part : This is obvious. If the coalition II-2 arises, it must satisfy conditions (i) and (ii). We already have shown that CAIC and CIR of lemma 1 must hold from conditions (i) and (ii).

Proof of if part : We want to find σ_2 and σ_3 which satisfy the CIRA , CIRS and CAIC in state 2 and CAIC in state 3 from the given conditions $e_3 \geq e_1 - \Delta\theta$ and $S_1 + W_1 - g(e_1) > S_2 + W_2 - g(e_2)$

[1] If $e_3 \geq e_1 - \Delta\theta$, then by the strict convexity of $g(\cdot)$, we have

$$g(e_3 + \Delta\theta) - g(e_3) \;\geq\; g(e_1) - g(e_1 - \Delta\theta)$$

$$<==> \; g(e_3 + \Delta\theta) - g(e_1) \;\geq\; g(e_3) - g(e_1 - \Delta\theta)$$

Then for all W_1 and W_3, we have

$$W_1 - g(e_1) - W_3 + g(e_3 + \Delta\theta) \;\geq\; W_1 - g(e_1 - \Delta\theta) - W_3 + g(e_3)$$

Then, there exists ω such that

$$W_1 - g(e_1) - (W_3 - g(e_3 + \Delta\theta)) \geq \omega \geq W_1 - g(e_1 - \Delta\theta) - (W_3 + g(e_3)) \tag{1}$$

[2] We also have $S_1 + W_1 - g(e_1) > S_2 + W_2 - g(e_2)$

$$<==> W_1 - g(e_1) - (W_2 - g(e_2)) \;>\; S_2 - S_1$$

$$==> P_2(W_1 - g(e_1)) - P_2(W_2 - g(e_2)) \;>\; P_2(S_2 - S_1) \text{ for all } P_2 > 0$$

Then there exists v such that

$$P_2(W_1 - g(e_1)) - P_2(W_2 - g(e_2)) \;>\; v \;>\; P_2(S_2 - S_1) \tag{2}$$

[3] Choose σ_2 and σ_3 such that :

$$\omega = \sigma_2 - \sigma_3 \text{ and } \nu = P_2\sigma_2 + P_3\sigma_3 \tag{3}$$

This is possible if we pick $\sigma_2 = \dfrac{P_3\omega + \nu}{P_2 + P_3}$, $\sigma_3 = \dfrac{\nu - P_2\omega}{P_2 + P_3}$

Then, it is clear that with such a choice of σ_2 and σ_3, CIRA, CIRS, (CAIC in state 2) and (CAIC in state 3) hold. Hence, by conditions (i) and (ii), the coalition II-2 emerges. ■

We can apply the same procedure to get the CIR and CAIC of each and every case. See the appendix B for the detail and Figure 2 in the appendix B shows (CAIC) and the (CIR) of each case.

For each case of possible coalition, as we have shown in lemma 2 of case II-2, if (CIR) and (CAIC) are satisfied at the same time, then the coalition described in each case will emerge. Hence, as a corollary of this lemma we can derive the coalitional incentive compatibility constraints as following structure:

{ If (CAIC) is true, then (CIR) should not be true }

By using this structure, we can derive the following constraints:[15]

(CIC7)
$$\begin{aligned} &P'_2\{S_2 + W_2 - g(e_2)\} + P'_3\{S_3 + W_3 - g(e_3)\} \geq \\ &P'_2\{S_1 + W_1 - g(e_1)\} + P'_3\{S_1 + W_1 - g(e_1 - \Delta\theta)\} \end{aligned} \qquad [I]$$

(CIC8)
If $e_2 \geq e_1$, then
$$\begin{aligned} &P'_2\{S_2 + W_2 - g(e_2)\} + P'_3\{S_3 + W_3 - g(e_3)\} \geq \\ &P'_2\{S_1 + W_1 - g(e_1)\} + P'_3\{S_2 + W_2 - g(e_2 - \Delta\theta)\} \end{aligned} \qquad [II-1]$$

(CIC9)
If $e_3 + \Delta\theta \geq e_1$, then
$$S_2 + W_2 - g(e_2) \geq S_1 + W_1 - g(e_1) \qquad [II-2]$$

If $e_4 + \Delta\theta \geq e_1$, then

(C/C10) $$P'_2\{S_2 + W_2 - g(e_2)\} + P'_3\{S_3 + W_3 - g(e_3)\} \geq P'_2\{S_1 + W_1 - g(e_1)\} + P'_3\{S_4 + W_4 - g(e_4)\}$$ [III]

If $e_1 \geq e_2$, then

(C/C11) $$S_3 + W_3 - g(e_3) \geq S_1 + W_1 - g(e_1 - \Delta\theta)$$ [IV – 1]

If $e_4 \geq e_3$, then

(C/C12) $$P'_2\{S_2 + W_2 - g(e_2)\} + P'_3\{S_3 + W_3 - g(e_3)\} \geq P'_2\{S_3 + W_3 - g(e_3 + \Delta\theta)\} + P'_3\{S_4 + W_4 - g(e_4)\}$$ [VI – 2]

(C/C13) $$S_3 + W_3 - g(e_3) \geq S_2 + W_2 - g(e_2 - \Delta\theta)$$ [V – 1]

If $e_2 \geq e_3 + \Delta\theta$, then

(C/C14) $$P'_2\{S_2 + W_2 - g(e_2)\} + P'_3\{S_3 + W_3 - g(e_3)\} \geq P'_2\{S_3 + W_3 - g(e_3 + \Delta\theta)\} + P'_3\{S_2 + W_2 - g(e_2 - \Delta\theta)\}$$ [V – 2]

(C/C15) $$S_2 + W_2 - g(e_2) \geq S_3 + W_3 - g(e_3 + \Delta\theta)$$ [V – 3]

If $e_4 + \Delta\theta \geq e_2$, then

(C/C16) $$S_3 + W_3 - g(e_3) \geq S_4 + W_4 - g(e_4)$$ [VI – 1]

If $e_1 \geq e_3 + \Delta\theta$, then

(C/C17) $$P'_2\{S_2 + W_2 - g(e_2)\} + P'_3\{S_3 + W_3 - g(e_3)\} \geq P'_2\{S_3 + W_3 - g(e_3 + \Delta\theta)\} + P'_3\{S_1 + W_1 - g(e_1 - \Delta\theta)\}$$ [IV – 2]

If $e_1 \geq e_4 + \Delta\theta$, then

(C/C18) $$P'_2\{S_2 + W_2 - g(e_2)\} + P'_3\{S_3 + W_3 - g(e_3)\} \geq P'_2\{S_4 + W_4 - g(e_4 + \Delta\theta)\} + P'_3\{S_1 + W_1 - g(e_1 - \Delta\theta)\}$$ [VII]

If $e_2 \geq e_4 + \Delta\theta$, then

(C/C19) $$P'_2\{S_2 + W_2 - g(e_2)\} + P'_3\{S_3 + W_3 - g(e_3)\} \geq P'_2\{S_4 + W_4 - g(e_4 + \Delta\theta)\} + P'_3\{S_2 + W_2 - g(e_2 - \Delta\theta)\}$$ [VIII – 2]

If $e_3 \geq e_4$, then

(C/C20) $$S_2 + W_2 - g(e_2) \geq S_4 + W_4 - g(e_4 + \Delta\theta)$$ [VIII – 2]

$$
(CIC21) \quad \begin{aligned} &P'_2\{S_2 + W_2 - g(e_2)\} + P'_3\{S_3 + W_3 - g(e_3)\} \geq \\ &P'_2\{S_4 + W_4 - g(e_4 + \Delta\theta)\} + P'_3\{S_4 + W_4 - g(e_4)\} \end{aligned} \qquad [IX]
$$

3.2. The Optimization Problems and the Results: We can finally derive the optimization problem that can guarantee coalition proof allocation to the principal. That is, we want to solve the following program (C):

$$
(C) \qquad \underset{\{S_i,\, W_i, e_i\}}{\text{Max}} \quad \sum_{i=1}^{4} P_i\,(\theta_i + e_i - S_i - W_i)
$$

s. t. (SIR), (AIR), (AIC), (CIC1) - (CIC21)

We have a very complicated and severely restricted feasible set. However, we can relax this feasible set and study this optimization problem in a simplified form. The CICs′ we already derived are ex-ante constraints in the following sense: The principal figures at the outset the constraints under which the agent and the supervisor have no incentive to try to form any coalitions. However, we can have ex-post constraints by considering the self-selection constraints of the total wage bill net of disutility in each state of nature. In other words, the principal considers final allocations, which are the net wages to the supervisor and the agent after side payment through coalition. Then he can impose constraints such that this final total wage bill net of disutility in one state of nature must be greater than that of any other state of nature (Note that even if the principal considers the final allocation, it does not imply that the S/A coalition really emerged. This only happened in the principal's logical process in order to calculate his optimal contract). Then there is no room for any S/A coalition to get the higher total wage bill net of disutility by misrepresenting their true state of nature.

Thus, we consider the larger feasible set by imposing the following ex-post constraints

$$
(CIC1') \qquad S_1 + W_1 - g(e_1) = S_2 + W_2 - g(e_2)
$$

$$(CIC2') \qquad S_3 + W_3 - g(e_3) = S_4 + W_4 - g(e_4)$$

$$(CIC3') \qquad S_3 + W_3 - g(e_3) \geq S_2 + W_2 - g(e_2 - \Delta\theta)$$

$$(CIC4') \qquad S_4 + W_4 - g(e_4) \geq S_1 + W_1 - g(e_1 - \Delta\theta)$$

$$(CIC5') \qquad S_2 + W_2 - g(e_2) \geq S_3 + W_3 - g(e_3 + \Delta\theta)$$

$$(CIC6') \qquad S_1 + W_1 - g(e_1) \geq S_4 + W_4 - g(e_4 + \Delta\theta)$$

These inequalities are derived from the self selection constraints of the total wage bill net of disutility in each state of nature. (The details are provided in the proof of proposition 2 i.e., Appendix (E)). Observe that the equality constraints (CIC1′) and (CIC2′) come from the inequalities in both direction respectively. Hence, the simplified program (C′) is:

$$(C') \qquad \underset{\{S_i, W_i, e_i\}}{\text{Max}} \sum_{i=1}^{4} P_i\,(\theta_i + e_i - S_i - W_i)$$

s. t. (SIR), (AIR), (AIC1) and (CIC1′) - (CIC5′)

We will solve problem (C′) and show that the optimal solution is feasible for the original problem (C). The solution to this problem is described in the following proposition. (The proof of this proposition is provided in appendix C)

Proposition 1 : The solution to (C′) has the following features:[16]

a) $S_4 > S_3 \geq S_2 > S_1$

b) $e_4 = e_3 = e^* > e_1 \geq e_2$

c) $W_3 - g(e_3) > W_4 - g(e_4) > W_1 - g(e_1) > W_2 - g(e_2)$

Hence, $W_3 > W_4 > W_1 > W_2$

d) (AIC1) and (CIC4′) are binding while (CIC5′) is not.

e) $0 \leq S_3 - S_2 < W_2 - g(e_2) - \{W_3 - g(e_3 + \Delta\theta)\}$

f) $S_4 + W_4 - g(e_4) = S_3 + W_3 - g(e_3) > S_2 + W_2 - g(e_2) = S_1 + W_1 - g(e_1)$

There are two constraints that we ignore in this problem (C′): AIC 2 and CIC 6′. However we can see these constraints will indeed be automatically satisfied by the solution to problem (C′).

lemma 3 : The solution to the program (C′) described in proposition 1 is actually the solution to the original program (C).

Lemma 3 means that if the principal offers a contract which satisfies all the conditions of proposition 1, then he can guarantee himself a coalition free allocation. This lemma explicitly proves that we can use the Revelation Principle without loss of generality. In other words, when we imposed the constraints of program (C′), we considered the direct coalition proof mechanism through the finall allocation is self-selected individually and coalitionally. We prove lemma 3 by showing that the allocation described in lemma 2 satisfies all the constraints in the original problem (C). See the details in appendix (D).

By lemma 3, we know the principal indeed guarantees himself the coalition proof contract. However,when the principal figures out the optimal contract, he already considers all the possible type of S/A coalition. Therefore, even if the principal allows them to collude, the final outcome shall satisfy all the conditions of proposition 1. In other words, if the principal offers the contract described in proposition 1, there is no state of nature in which the total wage bill net of disutility of effort (after side contract) can be increased by changing the report or the effort level. Thus, we restate this in the following proposition. The proof is provided in appendix (E).

Proposition 2 : The principal can guarantee himself the final allocation $\{S_i, W_i, e_i\}$ described in proposition 1 even if the agent and the supervisor is allowed to form a coalition.

Now, we need some interpretation of the results of proposition 1. First, observe that (f) of proposition 1 shows that the total wage bill of the supervisor and agent net of disutility from effort depends only on the true level of productivity parameters. However the principal can get the optimal coalition proof allocation by assigning different wages for the supervisor and for the agent respectively. For instance, the agent in the state 3 tries to shirk as the supervisor cannot provide the evidence. Hence the agent must be paid a higher wage in state 3 than in state 4 i.e., $W_3 > W_4$. On the other hand the principal could make S_4 higher than S_3 in order to prevent the agent from bribing the supervisor to conceal this information $\bar{\theta}$. Furthermore, by making S_4 the highest, the principal tries to make the supervisor report $\bar{\theta}$ whenever he observe it. Contrary to this, when the agent has private information $\underline{\theta}$ in state 2, he wants this information transmitted to the supervisor as an excuse for his bad performance , because the supervisor cannot provide the evidence for this also. So, in this case the agent suffers the lowest level of utility as this information $\underline{\theta}$ actually is valuable only to the agent as a proof of low output level. Observe that when the agent privately observe $\underline{\theta}$, he has no room for shirking because $S_2 + W_2 - g(e_2) > S_3 + W_3 - g(e_3 + \Delta\theta) = S_4 + W_4 - g(e_4 + \Delta\theta)$ from the fact that (CIC5′) is not binding. Hence the principal sets S_2 (also S_3 and S_4) higher than S_1 in order for the agent not to bribe the supervisor to report a good excuse for his bad performance.

In this model, we do not consider the supervisor's effort in observing the productivity parameter, which means the supervisor's wage level is irrelevant to the information cost. [17] The essence of the problem in this model is the private information of the agent which is unobservable by the supervisor. Part (c) of proposition 1 shows that the agent enjoys the highest net utility when he has a private information $\bar{\theta}$ which is valuable to the principal. Contrary to this, the agent suffers the lowest net utility when he has a private information $\underline{\theta}$, which is not valuable to the principal. Finally, by comparing the contract described in proposition 1 with that of lemma 1, we can find the cost for preventing all the possible coalitions. First, under collusion, the supervisor's information is more costly to obtain. Without the possibility of collusion, the principal can pay the constant reservation wage S_0 in each state. However, under the possiblity of side contract, the principal must pay a risk premium to the supervisor, since the supervisor is risk-averse and S_i in proposition 1 is not constant. Second, another source of cost of preventing the possible collusion is some suboptimal level of effort in state of nature 1 as well as state 2. The principal needs to make W_1 and W_2 lower than W_3 and W_4 (same with the supervisor's wage) in order to make it less attractive for the agent to shirk in good state of productivity and this low wage cannot give enough incentive for the agent to exert e^* in the state 1 and 2.

Before going further, we want to examine Tirole's results (lemma 1 of Tirole [1986]). He assumes that the supervisor can only conceal the information. In other words, if $s=\theta$, $r \in \{\theta, \phi\}$ and if $s=\phi, r=\phi$. Under this assumption, the coalitional incentive compatibility constraints are reduced to (CIC1), (CIC4), (CIC13), (CIC14) and (CIC15) of the program (C). The solution described in the lemma 1 of Tirole [1986] satisfies all constraint mentioned above.[18] This explicitly proves his "equivalence principle". Tirole [1986] did not consider the coalitional mechanism in states of nature 2 and 3 explicitly. However, we provide the constraints (CIC13), (CIC14) and (CIC15) which can prevent all the possible type of coalitions under his assumptions and we have shown that the allocations described in lemma 1 of Tirole satisfies the constraints (CIC1), (CIC4), (CIC13), (CIC14) and (CIC15).

For the rest of this section, we try to investigate several special cases of model in order to clarify the intuition of the results. This will also allow us to compare the implications of this model with Tirole's. First, let us consider two extreme cases of the supervisor's preference, following Tirole [1986].

Proposition 3 : If the supervisor is risk neutral, the optimal contract is the same as the no collusion case described in lemma 1

Proof is provided in the appendix (F). The intuition behind this proposition is obvious. Since the supervisor is risk neutral, the principal can sell the ownership of the production process to the supervisor at the price of the expected profit minus supervisor's reservation wage. Then the relationship between the risk neutral supervisor and the risk averse agent becomes a typical principal-agent relationship, hence there is no room for coalition.

Proposition 4 : If the supervisor is infinitely risk averse, he receives a fixed wage S_0.

Then the principal has one of the following three types of information to monitor the agent:

$\{s_1 = s_2 = s_3 = s_4 = \overline{\theta}\}$, $\{s_1 = s_2 = s_3 = s_4 = \underline{\theta}\}$ and $\{s_1 = s_2 = s_3 = s_4 = \phi\}$

Proof is provided in appendix (F). Since the supervisor is infinitely risk averse, he only cares about the certain wage no matter what the level of wage is. As is shown in the proof, if S_i is constant then the allocations described in part b) and c) of proposition 1 change to

$$(b') \qquad W_3 - g(e_3) = W_4 - g(e_4) \; > \; W_1 - g(e_1) = W_2 - g(e_2)$$

$$(c') \qquad e_3 = e_4 = e^* \; > \; e_1 = e_2$$

In other words, given constant S_i , the principal does not try to distinguish state 1 from state 2 and state 3 from state 4 respectively. This makes the role of the supervisor trivial as the distinction of the state 1 from 2 and the state 3 from 4 comes from the existence of the supervisor. Moreover, b′) and c′) show that the principal wants to distinguish the states 1 and 2 from the states 3 and 4. However, since the supervisor cannot distinguish state 2 from state 3, his report cannot have the following structures: $\{s_1 = s_2 = \phi,\ s_3 = s_4 = \overline{\theta}\}$, $\{s_1 = s_2 = \underline{\theta},\ s_3 = s_4 = \overline{\theta}\}$ or $\{s_1 = s_2 = \underline{\theta},\ s_3 = s_4 = \phi\}$. Thus we get the information structure described in the proposition. Intuition is clear. Since the supervisor strongly prefers a constant wage, the principal expects that he always tries to report the same signal in all the states of nature. In our model, unlike Tirole's, the supervisor does not necessairly act as an advocate for the agent.

We can imagine situations where the agent inherits the supervisor's reporting technology. In other words, the agent is supposed to report to the principal in all the states of nature and in the state 1 and 4 he can convey his information in a verifiable way. Then obviously the agent always tries to report $\underline{\theta}$ since this gives not only some room for shirking when the actual θ is $\overline{\theta}$ but also gives an excuse for low output level when the actual θ is $\underline{\theta}$. This information structure (reporting $\underline{\theta}$ in all states of nature) is exactly same as the infinitely risk averse supervisor acts as an advocate for the agent. We summarize this result in the following proposition.

Proposition 5 : Even if the agent inherits the supervisor's reporting technology in states 1 and 4, we need the supervisor except when the supervisor is infinitely risk averse and he acts as an advocate for the agent.

4. Conclusion

In this paper we studied the problem of designing some optimal collusion free contract under simple three-tier Principal/ supervisor/ agent hierarchical structure. We figured out the coalitional incentive compatibility conditions by explicitly presenting the coalitional mechanism when the coalition formation suffers informational asymmetry. We showed that the principal can design an optimal collusion free contract by putting proper incentive compatibility conditions and individual rationality conditions. Of course, the principal must pay some additional cost for this mechanism. The most important feature of this optimal collusion free contract is that the allocation rule (in our model the specification of S_i, W_i, e_i) is prespecified so that the evaluation about the agent does not depend on the report by the supervisor who is so-called " simultaneously judge and party".

The analysis in this paper is very restictive, of course. Actual organizations have more complex hierarchy than one principal/ one supervisor/ one agent structure: A supervisor will monitor several agents and an agent may have more than one supervisor. Also we might have several layers of supervisors. Furthermore, in this model, we ruled out the supervisor's productive role. Considering the supervisor's effort might be an interesting future work. Finally, we did not consider the dynamic aspects of coalitions Actually, the long term relationship between players has been considered to improve the performance of it's organization. For example, as is shown in the repeated moral hazard literature, repeated relationships alleviates incentive problem (see Radner [1986]) and sometimes, help to accumulate specific assets and reduce transation costs (see Williamson [1975]). However, if we consider the effect of collusive behavior in the organization, the long term relationship, as is pointed out in Tirole[1986], may not always be a blessing since the long term relationship will also strengthen the bounds within coalitions.

FOOTNOTES

[1] We have one principal-many agents type model (e.g., Demski and Sappington [1984], Holstrom [1982] and Nalebuff and Stiglitz [1983]) and recently many principals- one agent type model (e.g., Baron [1985], Bernheim and Whinston [1985], [1986], Cremer and Riordan [1987]). However, in these models, they did not explicitly consider the collusive behavior.

[2] See part 2 of Tirole [1986]. He largely referred this point the studies of Crozier [1963] and Dalton [1959].

[3] See part 2 of Tirole [1986]. Also Antle [1982],[1894] and Williamson [1975] discuss this problem explicitly. I will discuss this problem also in part 2.

[4] However, we can find three tier structure in other context. For example, voter / congressman or senator / government, voter(people) / department of defense / defense contractor, publisher of journal / referee / paper submitter and stockholder / manager / worker and so on. Tirole [1986] provided lots of interesting examples (see p183 of Tirole[1986]).

[5] See Crozier (1963) and Dalton (1959) for the detail.

[6] This assumption is also made for the simplicity of the model. We can introduce the supervisor's effort for information gathering in two different directions. First, we can assume that if the supervisor put some effort level a* or more, he can always observe the true θ , otherwise he observes nothing. Second, we can consider the uncertainty structure such that the probability for the supervisor to observe θ depends on the supervisor's level of effort for gathering the information.

[7] We know that the supervisor's opportunity cost of gathering the information is zero. This means the principal hires the supervisor for the other productive activity. However, it will be more realistic to assume that.

[8] We assume this probability distribution is a common knowledge for every player in this model. This model differs from the models of Green and Storky [1983] and Sappington [1980], [1983], all of which consider precontract asymmetry of information.

[9] This model is different from a pure moral hazard model. This is because the agent has perfect information and the output X is determind nonstochastically in this model. Therefore, the main problem of this model is asymmetry of information. If the principal could observe θ, then the principal would have full information when he observes output X, hence he could monitor the agent's effort perfectly.

[10] The principal can choose direct mechanism without any loss of generality under which the agent is supposed to declare his private observation and then some allocation (here, W_i and recommandation about e_i) is effected following the prespecified allocation rule as a function of declaration of the agent.

[11] AIC's come from direct application of self-selection conditions discussed in Harris and Townsend [1981]. The allocation W_i and X_i must be self selected by the agent in the state 2 and 3. Hence we have :

$$W_2 - g(X_2 - \underline{\theta}) \geq W_3 - g(X_3 - \underline{\theta}) \text{ in the state 2}$$

$$W_3 - g(X_3 - \overline{\theta}) \geq W_2 - g(X_2 - \overline{\theta}) \text{ in the state 3}$$

These conditions are equivalent to AIC1 and AIC2 respectively.

[12] This institutional assumption reflects the observation that the reporting system of hierarchical organization itself is often hierarchical. Obviously, the principal can ask the agent to report either. However, our analysis of coalitions does not change even if the principal ask the agent to send a message as well, because S/A coalition can always coordinate the message to be sent to the principal.

[13] This is, of course, very weak restriction. By this, we ignore all the problems associated with bargaining within the coalition. However, in reality, the problem of allocating the surplus from coalition may hinder the formation of coalition itself.

[14] Cremer [1986] recently proposed and analyzed this coalitional mechanism under the nonbayesian context. Since we don't have established bargaining theory to make strong prediction within the coalition, we can use these two conditions, which are tractable and not overly restrictive.

[15] Note that for the constraints (CIC7), (CIC13), (CIC15), and (CIC21), agent's incentive compatibility conditions within the coalition are always met. This is because the supervisor and the agent agree to misrepresent only one state of nature regardless of the real state of nature in the cases of side-contract [I], [V-1], [V-3], and [IX] respectively.

[16] We should mention the supervisor's incentive problem without colluding with the agent. Given contract a) of proposition 1, $S_4 > S_3 \geq S_2 > S_1$, the supervisor has incentive to change his report in the following way:

(i) r = $\underline{\theta}$ --> r = ϕ (ii) r = $\underline{\theta}$ --> r = $\bar{\theta}$

(iii-1) r = ϕ (but actual $\theta = \underline{\theta}$) --> r = $\bar{\theta}$ (iii-2) r = ϕ (but actual $\theta = \bar{\theta}$) --> r = $\bar{\theta}$

First, consider the case (i). Since r is public information, the agent can observe the supervisor's misrepresentation, which reduces the agent's payoff. Hence, even if the agent is not supposed to speak, he voluntarily tells that the supervisor conceals true information intentionally. Second, consider case (ii) and (iii-1). The principal can prevent this kind of misrepresentations by making the timing of report after the output X is revealed. This is because $X_1 = \underline{\theta} + e_1$ and $X_2 = \underline{\theta} + e_2$ can never be compatible with the report $\bar{\theta}$. Of course, we assume that the supervisor is severly punished if any shirking behavior is known to the principal. Finally, since the supervisor actually cannot distinguish (iii-2) from (iii-1), even if he has some possibility to successfully increase his payoff with case (iii-2), he will not try to report $\bar{\theta}$ alone when he observes nothing.

[17] Tirole's interpretation of his equality $S_4 > S_1 > S_2 = S_3$ is misleading. This result comes mainly from his assumption on the coalitional behavior of the supervisor within the coalition. The reason for high S_1 ($S_1 > S_2 = S_3$) cannot be a information cost since the supervisor pays nothing for gathering the information. In our model, the supervisor does not act as a advocate for the agent. He act as a advocate for the agent only when it is good for himself. Tirole's this interpretation also comes from his assumption on the supervisor's reporting technology.

[18] lemma 1 of Tirole [1986] has the following relationship among the allocations.

(a) $W_3 - g(e_3) = W_2 - g(e_2)$

(b) $S_1 + W_1 - g(e_1) > S_2 + W_2 - g(e_2)$

(c) $S_4 + W_4 - g(e_4) = S_3 + W_3 - g(e_3)$

(d) $S_3 + W_3 - g(e_3) = S_2 + W_2 - g(e_2 - \Delta\theta)$

(e) $e_1 = e_3 = e_4 = e^* > e_2$

(CIC 1) is satisfied with strict inequality by (b). (CIC 4) is met with equality by (c). (CIC 15) is met with inequality by the fact that $S_2 = S_3$ and $W_2 - g(e_2) > W_3 - g(e_3 + \Delta\theta)$. (CIC13) is satisfied with equality by (d) and finally we don't have to worry about (CIC14), because (CAIC) in (CIC14) is not met by (e).

CHAPTER V. CONCLUSIONS

In incentive theory, considering the effect of collusive behavior in design of incentive-compatible mechanism is recognized as a major extension of the existing theoretical results in this area. Moreover, the issue of coalition formation under asymmetric information is also, we believe, of primary importance for a number of fields in Economics in which contract has been accepted as an important paradigm. A most important preliminary step for this problem is considered in this dissertation. The main results of chapter II and III shows that the Revelation Principle is still applicable in some classes of incentive models, even if we take into account the possibility of collusive behavior among agents. This result, even though it is not general enough, shows some directions in this complicated research topic. In this chapter, we present the conclusions drawn from the analysis in this dissertation. We also present some possible extensions and future directions of this research.

1. Conclusions

In chapter 2, we showed that the Revelation Principle is still applicable with the possibility collusive behavior among the agents in the pure adverse selection model. In other words, the principal can limit his search of coalition proof mechanisms to coalition proof direct mechanisms without loss of efficiency. This implies that the characterization of coalition proof mechanisms can be achieved by solvinga mathematical programming problem: maximizing the principal's payoff subject to individual rationality conditions and some individual and coalitional incentive compatibility condition properly specified.

In chapter III, we extend this analysis to a generalized model, where the principal cannot monitor the agent's personal behavior or control the agent's private decisions as well as the agent's private information. With quite restricted institutional assumptions about the process of coalition, we can prove that the Revelation Principle still holds in a generalized principal-agent set up. However, this positive result is vulnerable to various institutional assumption on the formation of coalition.

In chapter IV, we explicitly present the coalitional mechanism for the supervisor/ agent coalition in three-tier principal/ supervisor/ agent hierarchy. The model we considered can be interpreted as a adverse selection model with acting agents. In this special model, we applied the result of chapter II and characterize the coalition proof mechanism with the assumption of soft information. This results implies that Tirole[1986]'s theories of coalitions hinge on the fact that he considers only hard information.

2. Suggestions for Further Research

As we mentioned earlier, we can easily extend the results of chapter II and III in n player set up. In chapter III, there are several interesting questions for future research. First, we can consider the case that the response from the principal is public information. In this case, we conjecture that the Revelation Principle may not work with the possibility of collusion. Since the response from the principal "r" conveys additional information about the other party's true type, this messy information can reduce the range of successful coalition given main mechanism. Thus, even if the principal can characterize coalition-proof mechanisms, there can be some way to cheat in given original mechanism.

Second, we can explicitly model the second stage coalition. In other words, we can consider the following scenario about the coalition formation process. The colluding parties, first, meet and communicate on their private information agreeing on their report to the principal. After they receive recommendations about their private decisions, they meet again and communicate about responses from the principal and decide on their private behavior based on this communication. Modeling this situation might be a very complicated form of sequential game.

In chapter IV, we can either introduce the supervisor's productive function or consider the supervisor's cost of information gathering. In the model we considered, the supervisor is assumed to be a mere information conduit. The supervisor, however, can play an important productive roles like coordination and monitoring and consulting. Hence, considering the supervisor's action might be an interesting future work. Moreover, the model in chapter IV consider only discrete number of states of nature. Considering a continuous parameter is not a trivial extension. This is because many studies in this incentive literature shows that the results in discrete model does not automatically extend to the continuous case. Finally, as is suggested by Tirole[1986], analyzing the effect of collusive behavior in dynamic models of the organization is also an interesting future study. Some recent studies (e.g. see Radner[1981]) suggest that long-run relationship can produce a stable environment, which enhances the performance of its organization. This conclusion, however, may not always hold if we consider the effect of collusive behavior.

BIBLIOGRAPHY

Alchain, A. and H. Demsetz. [1972], " Production, Information Costs, and Economic Organization," *American Economic Review* Vol. LXII, pp 777-795.

Antle, R. [1982], " The Auditor as an Economic Agent," *Journal of Accounting Research,* Vol. 20, pp 503-527.

Antle, R. [1984], " Auditor Independence," *Journal of Accounting Research,* Vol. 22, pp 1-20.

Baron, D. P. [1982], "A Model of the Demand for Investment Banking and Advising and Distribution Services for New Issues," *Journal of Finance*, Vol. 37, pp 955-976.

Baron, D. P. [1985], "Noncooperative Regulation of a Nonlocalized Externality, " *Rand Journal of Economics*, Vol. 16, No. 4, pp 553-568.

Baron, D. P., and B. Holmstrom [1980], "The Investment Banking Contract for New Issues under Asymmetric Information: Delegation and the Incentive Problem," *Journal of Finance*, Vol. 35, pp 1115-1138.

Bernheim, B. D., B. Peleg and M. Whinston [1987], "Coalition Proof Nash Equilibria : I. Concepts," *Journal of Economic Theory*, 42(1), pp 1-12.

Bernheim, B. D. and M. D. Whinston [1985], "Common Marketing Agency as a Device for Facilitating Collusion, " *Rand Journal of Economics*, Vol. 16, No. 2, pp 269-281.

Bernheim, B. D. and M. D. Whinston [1986], "Common Agency, " *Econometrica*, Vol. 54, No. 4, pp 923-942.

Bester, H. [1985], "Screening vs Rationing in Credit Markets with Imperpect Information," *American Economic Review*, Vol. 75, pp 850-855.

Calvo, G. and s. wellisz. [1978], " Supervisor, Loss of control, and the Optimum size of the Firm, " *Journal of Political Economy,* Vol. 86, pp 943- 952.

Cremer, J. [1986], "Manipulation by Coalitions under Asymmetric Information: The Case of Groves Mechanism," VPI&SU Working Paper, December(revised).

Cremer, J., and R. P. McLean [1985], "Optimal Selling Strategies Under Uncertainty for a Discriminating Monopolist Where Demands Are Interdependent," *Econometrica*, 53(2), pp 345-361.

Cremer, J., and M. Riordan. [1987], " On Governing Multilateral Transactions with Bilateral Contracts," Forthcoming, *Rand Journal of Economics.*

Crozier, M. [1963], *The Bureaucratic Phenomenon,* Chicago: University of Chicago Press.

Dalton, M. [1959], *Men Who Manage,* New York: Wiley and Sons.

Dasgupta, P. S., P. J.Hammond and E.S.Maskin [1979], "The Implementation of Social Choice Rules : Some Results on Incentive Compatibility," *Review of Economic Studies*, 46, pp 185-216.

Demski J., and D. Sappington [1984], "Optimal Incentive Schemes With Multiple Agent," *Journal of Economic Theory*, 33(1), pp 152-171.

Gibbard, A. [1973], "Manipulation of Voting Schemes: A General Result," *Econometrica*, Vol. 41, pp 587-602.

Green, J. R., and Laffont, J. J. [1977], "Characterization of Satisfactory Mechanisms for the Revelation of Preferences for Public Goods," *Econometrica*, Vol. 45, pp 427-38.

Green E. J. and R. H. Porter [1984], "Noncooperative Collusion under Imperpect Price Information," *Econometrica*, Vol. 52, No. 1 pp 87-100.

Green, J. R., and N. L. Stokey [1983], "A Comparison of Tournaments and Contracts," *Journal of Political Economy*, Vol. 91, pp 349-364.

Grossman, S., and O. Hart. [1983], " An Analysis of the Principal- Agent Problem," *Econometrica,* Vol. 51, pp 7-46.

Harris, M., and M. Townsend [1981], "Resource Allocation under Asymmetric Information," *Econometrica*, Vol. 49, pp 231-259.

Harris, M., and R. M. Townsend [1985], "Allocation Mechanism, Asymmetric Information and The Revelation Principal," In George R. Feiwel(ed.) *Issues in Contemporary Microeconomics & Welfare*, Albany: State University of New York Press, pp 379-394.

Holmstrom, B. [1977], "On Incentives and Control in Organizations," Unpublished Ph. D. dissertation, Stanford University, Calif.

Holstrom, B. [1979], " Moral Hazard and Observability," *Bell Journal of Economics,* Vol. 10, pp 74-91.

Holmstrom, B. [1982], "Moral Hazard in Teams," *Bell Journal of Economics*, Vol. 13, pp 324-340.

Holmstrom, B., and R. B. Myerson [1983], "Efficient and Durable Decision Rules with Incomplete Information," *Econometrica*, Vol. 51, pp 1799-1819.

Laffont, J. J., and E. Maskin [1982], "The Theory of Incentives: an Overview," In Hildenbrand, W.,(ed.), *Advances in Economic Theory*, Cambridge: Cambridge University Press, pp 31-94.

Laffont, J. J. and J. Tirole [1985a], "Auctioning Incentive Contracts," Working Paper, Universite des Sciences Sociales de Toulouse n^0 8523.

Laffont, J. J. and J. Tirole [1985b], "Using Cost Observation to Regulate Firms," *Journal of Political Economy*, Vol. 94, No. 3, pp 614-641.

Lazear, E. P., and S. Rosen [1981], "Rank-Order Tournaments as Optimum Labor Contracts," *Journal of Political Economy*, Vol. 89, pp 841-64.

Ma, C., and J. Moore [1986], "Stopping Agents 'Cheat'," STICERD Discussion Paper #86/135, London School of Economics, March(revised).

Maskin, E. S. [1985], "The Theory of Implementation in Nash Equilibrium," In Leonid Hurwicz et al. (eds.), *Social Goals and Social Organizations*, Cambridge: Cambridge University Press, pp 173-204.

McAfee, R. P. and J McMillan [1986], "Bidding for Contracts: A Principal-Agent Analysis," *The Rand Journal of Economics*, Vol. 17, No. 3, pp 326-338.

McDonald, G. M. [1984], "New Directions in the Economic Theory of Agency," *Canadian Journal of Economics*, Vol. 17, NO. 3, pp 415-440.

Mirlees, J. [1971], "An Exploration in the Theory of Optimum Income Taxation," *Review of Economic Studies*, Vol. 38, pp 175-208.

Muller, E., and M. A. Satterthwaite [1985], "Strategy-Proofness: The Existence of Dominant-Strategy Mechanisms," In Leonid Hurwicz et al. (eds.), *Social Goals and Social Organizations*, Cambridge: Cambridge University Press, pp 131-172.

Myerson, R. B. [1979], "Incentive Compatibility and the Bargaining Problem," *Econometrica*, Vol. 47, pp 61-73.

Myerson, R. B. [1982], "Optimal Coordination Mechanisms in Generalized Principal-Agent Problems," *Journal of Mathematical Economics*, Vol. 10, pp 67-81.

Myerson, R. [1983], "Mechanism Design by an Informed Pricipal," *Econometrica*, Vol. 51, pp 61-73.

Myerson, R. B. [1984], "Cooperative Games with Incomplete Information," *International Journal of Game Theory*, Vol. 13, pp 69-96.

Myerson, R. B. [1985], "Bayesian Equilibrium and Incentive-compatibility: an introduction," In Leonid Hurwicz et al. (eds.), *Social Goals and Social Organizations*, Cambridge: Cambridge University Press, pp 229-260

Nalebuff, B., and J. Stiglitz. [1983], " Prizes and Incentives: Towards a General Theory of Compensation and Competition," *Bell Journal of Economics*, Vol. 14, pp 21-43.

Picard, P. [1985], "On the Design of Incentive Schemes under Moral Hazard and Adverse Selection," Working Paper, No. 8602, CEPREMAP.

Postlewaite, A. [1985], "Implementation via Nash Equilibrium in Economic Environment," In Leonid Hurwicz et al. (eds.), *Social Goals and Social Organizations*, Cambridge: Cambridge University Press, pp 205-228.

Radner, R. [1981], "Monitering Cooperative Agreements in a Repeated Pricipal-agent Relationship," *Econometrica*, Vol. 49, pp 1127-48.

Radner, R. [1986], " Optimal Equilibria in Some Repeated Partnership Games with Imperfect Information and No Discounting," *Review of Economic Studies* LIII (1), pp 43-56.

Repullo, R. [1985], "Implementation in Dominant Strategies under Complete and Incomplete Information," *Review of Economic Studies*, 52(2), pp 223-230.

Riordan, M., and D. Sappington [1985], "Franchise Monopoly," *Mimeo.*

Rosenthal, R. W. [1978], "Arbitration of Two-Party Disputes under Uncertainty," *Review of Economic Studies*, Vol. 45, pp 595-604.

Ross, S. A. [1973], "The Economic Theory of Agency: the Principal's Problem," *American Economic Review*, Vol. 63, pp 134-139.

Sappington, D. [1980], " Precontractual Information Asymmetry between Principal and Agent: The Continuous case," *Economic Letters,* Vol. 5, pp 371-375.

Sappington, D. [1983], "Limited Liability Contracts between Principal and Agent," *Journal of Economic Theory*, Vol. 29, pp 1-21.

Shavell, S [1979], "Risk Sharing and Incentives in the Principal and Agent Relationship," *Bell Journal of Economics*, Vol. 10, pp 55-73.

Tirole, J. [1986], "Hierarchies and Bureaucracies," *Journal of Law, Economics and Organizations*, Vol. 2, pp 181-214.

Turnbull, s. [1986], "The Revelation Principal and Principal's Pet in Multiple Agency Problems," Working Paper #86-1, Ohio State University, April.

Weitzman, M. [1976], "The New Soviet-Incentive Model," *Bell Journal of Economics*, Vol. 7, pp 251-257.

Williamson, O. [1967,a]. " The Economics of Defense Contracting: Incentives and Performance," in R. Mckean, ed., *Issues in Defense Economics,* New York: Columbia University Press.

Williamson, O. [1967,b]. " Hierarchical Control and Optimum Firm Size," *Journal of Political Economy,* Vol. 75, pp 123-138.

Williamson, O. [1975]. *Market and Hierarchies: Analysis and Antitrust Implications,* New York: Free Press.

Appendix A. APPENDIX

APPENDIX (A)

Proof of lemma 1 (Collusion free case)

We ignore (AIC2) for the time being and then show that at the optimum (AIC) is not binding. The Lagrangian function is:

$$L = \sum_{i=1}^{4} P_i\,(\,\theta_i + e_i - W_i\,) + \mu\,[\,\sum_{i=1}^{4} P_i\,U\,(\,W_i - g(e_i)\,) - \bar{U}\,]$$
$$+ \gamma\,(\,W_3 - g(e_3) - W_2 - g(e_2 - \Delta\theta)\,)$$

(F. O. C.)

$$\frac{dL}{dW_1} = 0 \quad ==> \quad \mu\,U'\,(\,W_1 - g(e_1)) = 1 \tag{1}$$

$$\frac{dL}{dW_2} = 0 \quad ==> \quad \mu\,U'\,(\,W_2 - g(e_2)) = 1 + \frac{\gamma}{P_2} \tag{2}$$

$$\frac{dL}{dW_3} = 0 \quad ==> \quad \mu U'(W_3 - g(e_3)) = 1 - \frac{\gamma}{P_3} \tag{3}$$

$$\frac{dL}{dW_4} = 0 \quad ==> \quad \mu U'(W_4 - g(e_4)) = 1 \tag{4}$$

$$\frac{dL}{de_1} = 0 \quad ==> \quad \mu U'(W_1 - g(e_1))\, g'(e_1) = 1 \tag{5}$$

$$\frac{dL}{de_2} = 0 \quad ==> \quad \mu U'(W_2 - g(e_2))\, g'(e_2) = 1 + \frac{\gamma}{P_2} g'(e_2 - \Delta\theta) \tag{6}$$

$$\frac{dL}{de_3} = 0 \quad ==> \quad \mu U'(W_3 - g(e_3))\, g'(e_3) = 1 - \frac{\gamma}{P_3} g'(e_3) \tag{7}$$

$$\frac{dL}{de_4} = 0 \quad ==> \quad \mu U'(W_4 - g(e_1))\, g'(e_1) = 1 \tag{8}$$

Conditions (1) and (5) give $g'(e_1) = 1$. Similary, (3) and (7) give $g'(e_3) = 1$ and (4) and (8) give $g'(e_4) = 1$. Hence We have $e_1 = e_3 = e_4 = e^*$. However, by (2) and (6) we have

$$\left(1 + \frac{\gamma}{P_2}\right) g'(e_2) = 1 + \frac{\gamma}{P_2} g'(e_2 - \Delta\theta)$$

This implies

$$(1 - g'(e_2)) = \frac{\gamma}{P_2}\left(g'(e_2) - g'(e_2 - \Delta\theta)\right) \tag{9}$$

Show that (AIC1) is binding. Assume not. Then we have $\gamma = 0$ and $W_3 - g(e_3) > W_2 - g(e_2 - \Delta\theta)$. Since $g(\cdot)$ is strictly convex and increasing, we have $W_3 - g(e_3) > W_2 - g(e_2 - \Delta\theta) > W_2 - g(e_2)$ However, from (2) and (3) we have $W_3 - g(e_3) = W_2 - g(e_2)$. This is contradiction. Hence (AIC1) is binding and $\gamma > 0$. Since $\gamma > 0$, we get $g'(e_2) < 1$ by (9). Hence, we have $e_2 < e^*$. By the strict concavity of the agent's utility function $U(\cdot)$, conditions (1), (2), (3), and (4) imply $W_3 - g(e_3) > W_4 - g(e_4) = W_1 - g(e_1) > W_2 - g(e_2)$. Also, we already have $e_4 = e_3 = e_1 = e^* > e_2$ where e^* is the level of effort such as $g'(e^*) = 1$.

APPENDIX (B)

We get (CIRA), (CIRS) and (CAIC) in every case. Note that we use P_2 and P_3 for the notational conveniency. However, actually $P_2 \equiv P_2' = \frac{P_2}{P_2 + P_3}$, $P_3 \equiv P_3' = \frac{P_3}{P_2 + P_3}$. Hence $P_2 + P_3 = 1$.

Case (I)

First, we get the individual rationality conditions within the coalition as follows:

$$CIRA \quad \begin{aligned} & P_2 (W_1 - g(e_1) - \sigma_2) + P_3 (W_1 - g(e_1 - \Delta\theta) - \sigma_3) \\ & > P_2 (W_2 - g(e_2)) + P_3 (W_3 - g(e_3)) \end{aligned} \tag{1}$$

$$CIRS \quad P_2 (S_1 + \sigma_2) + P_3 (S_1 + \sigma_3) > P_2 S_2 + P_3 S_3 \tag{2}$$

Then, (1) and (2) give the Coalitional Individual Rationality Condition as follows:

$$(CAIC) \quad \begin{aligned} & P_2 (S_1 + W_1 - g(e_1)) + P_3 (S_1 + W_1 - g(e_1 - \Delta\theta)) \\ & > P_2 (S_2 + W_2 - g(e_2)) + P_3 (S_3 + W_3 - g(e_3)) \end{aligned} \tag{3}$$

Second, in this case, since the colluding parties agree to represent the state 1 regardless of the real state of nature, the atgent has no opportunity to use his private information. Hence, CAIC is always met. Observe that this is same with case (V-1), (V-3), and (IX).

Case (II-1)

First, we get the individual rationality conditions within the coalition as follows:

$$CIRA \quad \begin{aligned} & P_2(W_1 - g(e_1) - \sigma_2) + P_3(W_2 - g(e_2 - \Delta\theta) - \sigma_3) \\ & > P_2(W_2 - g(e_2)) + P_3(W_3 - g(e_3)) \end{aligned} \tag{1}$$

$$CIRS \quad P_2(S_1 + \sigma_2) + P_3(S_2 + \sigma_3) > P_2 S_2 + P_3 S_3 \tag{2}$$

Then, (1) and (2) give the Coalitional Individual Rationality condition as follows:

$$(CIR) \quad \begin{aligned} & P_2(S_1 + W_1 - g(e_1)) + P_3(S_2 + W_2 - g(e_2 - \Delta\theta)) \\ & > P_2(S_2 + W_2 - g(e_2)) + P_3(S_3 + W_3 - g(e_3)) \end{aligned} \tag{3}$$

Second, we get the agent's incentive rationality conditions as follows:

$$\text{(CAIC in state 2)} \quad W_1 - g(e_1) - \sigma_2 \geq W_2 - g(e_2) - \sigma_3 \tag{4}$$

$$\text{(CAIC in state 3)} \quad W_2 - g(e_2 - \Delta\theta) - \sigma_3 \geq W_1 - g(e_1 - \Delta\theta) - \sigma_2 \tag{5}$$

Then (4) and (5) imply

$$g(e_2) - g(e_1) \geq g(e_2 - \Delta\theta) - g(e_1 - \Delta\theta)$$

$$<==> g(e_2) - g(e_2 - \Delta\theta) \geq g(e_1) - g(e_1 - \Delta\theta)$$

$==> g'(e_2) \geq g'(e_1) ==> e_2 \geq e_1$ by the strict convexity of g(e). Hence, we have the Coalitional Agent's Incentive Condition as follows:

$$(CAIC) \quad e_1 \geq e_2 \tag{6}$$

Case (II-2)

First, we get the individual rationality conditions within the coalition as follows:

$$CIRA \quad \begin{aligned} &P_2 (W_1 - g(e_1) - \sigma_2) + P_3 (W_3 - g(e_3) - \sigma_3) \\ &> P_2 (W_2 - g(e_2)) + P_3 (W_3 - g(e_3)) \end{aligned} \tag{1}$$

$$CIRS \quad P_2 (S_1 + \sigma_2) + P_3 (S_3 + \sigma_3) > P_2 S_2 + P_3 S_3 \tag{2}$$

(1) and (2) imply

$$\begin{aligned} &P_2 (S_1 + W_1 - g(e_1)) + P_3 (S_3 + W_3 - g(e_3)) \\ &> P_2 (S_2 + W_2 - g(e_2)) + P_3 (S_3 + W_3 - g(e_3)) \end{aligned}$$

Then we have Coalition Individual Rational condition as follows:

$$(CIR) \quad S_1 + W_1 - g(e_1) > S_2 + W_2 - g(e_2) \tag{3}$$

Second, we get the agent's incentive rationality conditions as follows:

$$(\text{CAIC in state 2}) \quad W_1 - g(e_1) - \sigma_2 \geq W_3 - g(e_3 + \Delta\theta)\_\sigma_3 \tag{4}$$

$$(\text{CAIC in state 3}) \quad W_3 - g(e_3) - \sigma_3 \geq W_1 - g(e_1 - \Delta\theta) - \sigma_2 \tag{5}$$

Then (4) and (5) implies

$$g(e_3 + \Delta\theta) + g(e_1 - \Delta\theta) \geq g(e_1) + g(e_3)$$

$$<==> g(e_3 + \Delta\theta) - g(e_3) \geq g(e_1) - g(e_1 - \Delta\theta)$$

$==> \quad g'(e_3) \geq g'(e_1 - \Delta\theta) \quad ==> \quad e_3 \geq e_1 - \Delta\theta$ by the strict convexity of g(e). Hence, we have the Coalitional Agent's Incentive Condition as follows:

$$(CAIC) \quad e_3 + \Delta\theta \geq e_1 \tag{6}$$

Case (III)

First, we get the individual rationality conditions within the coalition as follows:

$$\begin{aligned} CIRA \quad & P_2(W_1 - g(e_1) - \sigma_2) + P_3(W_4 - g(e_4) - \sigma_3) \\ & > P_2(W_2 - g(e_2)) + P_3(W_3 - g(e_3)) \end{aligned} \tag{1}$$

$$CIRS \quad P_2(S_1 + \sigma_2) + P_3(S_4 + \sigma_3) > P_2 S_2 + P_3 S_3 \tag{2}$$

Then, (1) and (2) give the Coalitional Individual Rationality condition as follows:

$$\begin{aligned} (CIR) \quad & P_2(S_1 + W_1 - g(e_1)) + P_3(S_4 + W_4 - g(e_4)) \\ & > P_2(S_2 + W_2 - g(e_2)) + P_3(S_3 + W_3 - g(e_3)) \end{aligned} \tag{3}$$

Second, we get the agent's incentive rationality conditions as follows:

$$(\text{CAIC in state 2}) \quad W_1 - g(e_1) - \sigma_2 \geq W_4 - g(e_4 + \Delta\theta) - \sigma_3 \tag{4}$$

$$(\text{CAIC in state 3}) \quad W_4 - g(e_4) - \sigma_3 \geq W_1 - g(e_1 - \Delta\theta) - \sigma_2 \tag{5}$$

Then (4) and (5) imply

$$g(e_4 + \Delta\theta) + g(e_1 - \Delta\theta) \geq g(e_1) + g(e_4)$$

$$<==> g(e_4 + \Delta\theta) - g(e_4) \geq g(e_1) - g(e_1 - \Delta\theta)$$

$==> \; g'(e_4) \geq g'(e_1 - \Delta\theta) \;\; ==> \;\; e_4 \geq e_1 - \Delta\theta$ by the strict convexity of g(e). Hence, we have the Coalitional Agent's Incentive Condition as follows:

$$(CAIC) \quad e_4 \geq e_1 - \Delta\theta \tag{6}$$

Case (IV-1)

First, we get the individual rationality conditions within the coalition as follows:

$$\begin{array}{ll} CIRA & P_2(W_2 - g(e_2) - \sigma_2) + P_3(W_1 - g(e_1 - \Delta\theta) - \sigma_3) \\ & > P_2(W_2 - g(e_2)) + P_3(W_3 - g(e_3)) \end{array} \tag{1}$$

$$CIRS \quad P_2(S_2 + \sigma_2) + P_3(S_1 + \sigma_3) > P_2 S_2 + P_3 S_3 \tag{2}$$

(1) and (2) imply

$$\begin{array}{l} P_2(S_2 + W_2 - g(e_2)) + P_3(S_1 + W_1 - g(e_1 - \Delta\theta)) \\ > P_2(S_2 + W_2 - g(e_2)) + P_3(S_3 + W_3 - g(e_3)) \end{array}$$

Then we have Coalition Individual Rational condition as follows:

$$(CIR) \quad S_1 + W_1 - g(e_1 - \Delta\theta) > S_3 + W_3 - g(e_3) \tag{3}$$

Second, we get the agent's incentive rationality conditions as follows:

$$\text{(CAIC in state 2)} \quad W_2 - g(e_2) - \sigma_2 \geq W_1 - g(e_1) - \sigma_3 \tag{4}$$

$$\text{(CAIC in state 3)} \quad W_1 - g(e_1 - \Delta\theta) - \sigma_3 \geq W_2 - g(e_2 - \Delta\theta) - \sigma_2 \tag{5}$$

Then (4) and (5) imply

$$g(e_1) + g(e_2 - \Delta\theta) \geq g(e_2) + g(e_1 - \Delta\theta)$$

$$<==> g(e_1) - g(e_1 - \Delta\theta) \geq g(e_2) - g(e_2 - \Delta\theta)$$

$==> g'(e_1) \geq g'(e_2) ==> e_1 \geq e_2$ by the strict convexity of g(e). Hence, we have the Coalitional Agent's Incentive Condition as follows:

$$(CAIC) \quad e_1 \geq e_2 \tag{6}$$

Case (IV-2)

First, we get the individual rationality conditions within the coalition as follows:

$$CIRA \quad \begin{array}{l} P_2\,(W_3 - g(e_3 + \Delta\theta) - \sigma_2) + P_3\,(W_1 - g(e_1 - \Delta\theta) - \sigma_3) \\ > P_2\,(W_2 - g(e_2)) + P_3\,(W_3 - g(e_3)) \end{array} \tag{1}$$

$$CIRS \quad P_2\,(S_3 + \sigma_2) + P_3\,(S_1 + \sigma_3) > P_2\,S_2 + P_3\,S_3 \tag{2}$$

Then, (1) and (2) give the Coalitional Individual Rationality condition as follows: leqno (CIR) reqno (3)

$$\begin{array}{l} P_2\,(S_3 + W_3 - g(e_3 + \Delta\theta)) + P_3\,(S_1 + W_1 - g(e_1 - \Delta\theta)) \\ > P_2\,(S_2 + W_2 - g(e_2)) + P_3\,(S_3 + W_3 - g(e_3)) \end{array}$$

Second, we get the agent's incentive rationality conditions as follows:

$$\text{(CAIC in state 2)} \quad W_3 - g(e_3 + \Delta\theta) - \sigma_2 \geq W_1 - g(e_1) - \sigma_3 \tag{4}$$

$$\text{(CAIC in state 3)} \quad W_1 - g(e_1 - \Delta\theta) - \sigma_3 \geq W_3 - g(e_3) - \sigma_2 \tag{5}$$

Then (4) and (5) imply

$$g(e_1) + g(e_3) \geq g(e_3 + \Delta\theta) + g(e_1 - \Delta\theta)$$

$$<==> g(e_1) - g(e_1 - \Delta\theta) \geq g(e_3 + \Delta\theta) - g(e_3)$$

$==> \; g'(e_1) \geq g'(e_3 + \Delta\theta) \; ==> \; e_1 \geq e_3 + \Delta\theta$ by the strict convexity of g(e). Hence, we have the Coalitional Agent's Incentive Condition as follows:

$$(CAIC) \quad e_1 \geq e_3 + \Delta\theta \tag{6}$$

Case (V-1)

First, we get the individual rationality conditions within the coalition as follows:

$$CIRA \quad \begin{aligned} &P_2(W_2 - g(e_2) - \sigma_2) + P_3(W_2 - g(e_2 - \Delta\theta) - \sigma_3) \\ &> P_2(W_2 - g(e_2)) + P_3(W_3 - g(e_3)) \end{aligned} \tag{1}$$

$$CIRS \quad P_2(S_2 + \sigma_2) + P_3(S_2 + \sigma_3) > P_2 S_2 + P_3 S_3 \tag{2}$$

(1) and (2) imply

$$\begin{aligned} &P_2(S_2 + W_2 - g(e_2)) + P_3(S_2 + W_2 - g(e_2 - \Delta\theta)) \\ &> P_2(S_2 + W_2 - g(e_2)) + P_3(S_3 + W_3 - g(e_3)) \end{aligned}$$

Then we have Coalition Individual Rational condition as follows:

$$(CIR) \quad S_2 + W_2 - g(e_2 - \Delta\theta) > S_3 + W_3 - g(e_3) \tag{3}$$

Second, CAIC is always satisfied as we observed in the case (I).

Case (V-2)

First, we get the individual rationality conditions within the coalition as follows:

$$CIRA \quad \begin{aligned} &P_2(W_3 - g(e_3 + \Delta\theta) - \sigma_2) + P_3(W_2 - g(e_2 - \Delta\theta) - \sigma_3) \\ &> P_2(W_2 - g(e_2)) + P_3(W_3 - g(e_3)) \end{aligned} \tag{1}$$

$$CIRS \quad P_2(S_3 + \sigma_2) + P_3(S_2 + \sigma_3) > P_2 S_2 + P_3 S_3 \tag{2}$$

Then, (1) and (2) gives the Coalitional Individual Rationality condition as follows:

$$(CIR) \quad \begin{aligned} &P_2(S_3 + W_3 - g(e_3 + \Delta\theta)) + P_3(S_2 + W_2 - g(e_2 - \Delta\theta)) \\ &> P_2(S_2 + W_2 - g(e_2)) + P_3(S_3 + W_3 - g(e_3)) \end{aligned} \tag{3}$$

Second, we get the agent's incentive rationality conditions as follows:

$$\text{(CAIC in state 2)} \quad W_3 - g(e_3 + \Delta\theta) - \sigma_2 \;\geq\; W_2 - g(e_2) - \sigma_3 \tag{4}$$

$$\text{(CAIC in state 3)} \quad W_2 - g(e_2 - \Delta\theta) - \sigma_3 \;\geq\; W_3 - g(e_3) - \sigma_2 \tag{5}$$

Then (4) and (5) imply

$$g(e_2) + g(e_3) \;\geq\; g(e_3 + \Delta\theta) + g(e_2 - \Delta\theta)$$

$$<==> \; g(e_2) - g(e_2 - \Delta\theta) \;\geq\; g(e_3 + \Delta\theta) - g(e_3)$$

$==> \; g'(e_2) \;\geq\; g'(e_3 + \Delta\theta) \quad ==> \quad e_2 \;\geq\; e_3 + \Delta\theta$ by the strict convexity of g(e). Hence, we have the Coalitional Agent's Incentive Condition as follows:

$$(CAIC) \qquad e_2 \;\geq\; e_3 + \Delta\theta \tag{6}$$

Case (V-3)

First, we get the individual rationality conditions within the coalition as follows:

$$CIRA \quad \begin{array}{l} P_2\,(\,W_3 - g(e_3 + \Delta\theta) - \sigma_2\,) \;+\; P_3\,(\,W_3 - g(e_3) - \sigma_3\,) \\ > \; P_2\,(\,W_2 - g(e_2)) \;+\; P_3\,(\,W_3 - g(e_3)) \end{array} \tag{1}$$

$$CIRS \quad P_2\,(\,S_3 + \sigma_2\,) \;+\; P_3\,(\,S_3 + \sigma_3\,) \;>\; P_2\,S_2 \;+\; P_3\,S_3 \tag{2}$$

(1) and (2) imply

$$\begin{array}{l} P_2\,(\,S_3 + W_3 - g(e_3 + \Delta\theta)\,) \;+\; P_3\,(\,S_3 + W_3 - g(e_3)\,) \\ > \; P_2\,(\,S_2 + W_2 - g(e_2)) \;+\; P_3\,(\,S_3 + W_3 - g(e_3)) \end{array}$$

Then we have Coalition Individual Rational condition as follows:

$$(CIR) \qquad S_3 + W_3 - g(e_3 + \Delta\theta) > S_2 + W_2 - g(e_2) \qquad (3)$$

Second, CAIC is always satisfied as we observed in the case (I).

Case (VI-1)

First, we get the individual rationality conditions within the coalition as follows:

$$CIRA \qquad \begin{aligned} & P_2 (W_2 - g(e_2) - \sigma_2) + P_3 (W_4 - g(e_4) - \sigma_3) \\ & > P_2 (W_2 - g(e_2)) + P_3 (W_3 - g(e_3)) \end{aligned} \qquad (1)$$

$$CIRS \qquad P_2 (S_2 + \sigma_2) + P_3 (S_4 + \sigma_3) > P_2 S_2 + P_3 S_3 \qquad (2)$$

(1) and (2) imply

$$\begin{aligned} & P_2 (S_2 + W_2 - g(e_2)) + P_3 (S_4 + W_4 - g(e_4)) \\ & > P_2 (S_2 + W_2 - g(e_2)) + P_3 (S_3 + W_3 - g(e_3)) \end{aligned}$$

Then we have Coalition Individual Rational condition as follows:

$$(CIR) \qquad S_4 + W_4 - g(e_4) > S_3 + W_3 - g(e_3) \qquad (3)$$

Second, we get the agent's incentive rationality conditions as follows:

$$(\text{CAIC in state 2}) \qquad W_2 - g(e_2) - \sigma_2 \geq W_4 - g(e_4 + \Delta\theta) - \sigma_3 \qquad (4)$$

$$(\text{CAIC in state 3}) \qquad W_4 - g(e_4) - \sigma_3 \geq W_2 - g(e_2 - \Delta\theta) - \sigma_2 \qquad (5)$$

Then (4) and (5) imply

$$g(e_4 + \Delta\theta) + g(e_2 - \Delta\theta) \geq g(e_2) + g(e_4)$$

$$<==> g(e_4 + \Delta\theta) - g(e_4) \geq g(e_2) - g(e_2 - \Delta\theta)$$

$==> \; g'(e_4 + \Delta\theta) \;\geq\; g'(e_2) \;\; ==> \;\; e_4 + \Delta\theta \;\geq\; e_2$ by the strict convexity of g(e). Hence, we have the Coalitional Agent's Incentive Condition as follows:

$$(CAIC) \qquad e_4 + \Delta\theta \;\geq\; e_2 \tag{6}$$

Case (VI-2)

First, we get the individual rationality conditions within the coalition as follows:

$$CIRA \qquad \begin{aligned} &P_2\,(\,W_3 - g(e_3 + \Delta\theta) - \sigma_2\,) \;+\; P_3\,(\,W_4 - g(e_4) - \sigma_3\,) \\ &>\; P_2\,(\,W_2 - g(e_2))\;+\;P_3\,(\,W_3 - g(e_3)) \end{aligned} \tag{1}$$

$$CIRS \qquad P_2\,(\,S_3 + \sigma_2\,) \;+\; P_3\,(\,S_4 + \sigma_3\,) \;>\; P_2\,S_2 \;+\; P_3\,S_3 \tag{2}$$

(1) and (2) imply

$$\begin{aligned} &P_2\,(\,S_3 + W_3 - g(e_3 + \Delta\theta)\,) \;+\; P_3\,(\,S_4 + W_4 - g(e_4)\,) \\ &>\; P_2\,(\,S_2 + W_2 - g(e_2))\;+\;P_3\,(\,S_3 + W_3 - g(e_3)) \end{aligned}$$

Then we have Coalition Individual Rational condition as follows:

$$(CIR) \qquad S_4 + W_4 - g(e_4) \;>\; S_3 + W_3 - g(e_3) \tag{3}$$

Second, we get the agent's incentive rationality conditions as follows:

$$(\text{CAIC in state 2}) \qquad W_3 - g(e_3 + \Delta\theta) - \sigma_2 \;\geq\; W_4 - g(e_4 + \Delta\theta) - \sigma_3 \tag{4}$$

$$(\text{CAIC in state 3}) \qquad W_4 - g(e_4) - \sigma_3 \;\geq\; W_3 - g(e_3) - \sigma_2 \tag{5}$$

Then (4) and (5) imply

$$g(e_4+\Delta\theta)+g(e_3) \;\geq\; g(e_3+\Delta\theta)+g(e_4)$$

$$<==> g(e_4+\Delta\theta)-g(e_4) \;\geq\; g(e_3+\Delta\theta)-g(e_3)$$

$==> \; g'(e_4) \;\geq\; g'(e_3) \;\; ==> \;\; e_4 \;\geq\; e_2$ by the strict convexity of g(e). Hence, we have the Coalitional Agent's Incentive Condition as follows:

$$(CAIC) \qquad e_4 \;\geq\; e_2 \tag{6}$$

Case (VII)

First, we get the individual rationality conditions within the coalition as follows:

$$CIRA \qquad \begin{aligned} &P_2\,(W_4-g(e_4+\Delta\theta)-\sigma_2)\;+\;P_3\,(W_1-g(e_1-\Delta\theta)-\sigma_3)\\ &>\;P_2\,(W_2-g(e_2))\;+\;P_3\,(W_3-g(e_3))\end{aligned} \tag{1}$$

$$CIRS \qquad P_2\,(S_4+\sigma_2)\;+\;P_3\,(S_1+\sigma_3)\;>\;P_2\,S_2\;+\;P_3\,S_3 \tag{2}$$

Then, (1) and (2) gives the Coalitional Individual Rationality condition as follows:

$$(CIR) \qquad \begin{aligned} &P_2\,(S_4+W_4-g(e_4+\Delta\theta))\;+\;P_3\,(S_1+W_1-g(e_1-\Delta\theta))\\ &>\;P_2\,(S_2+W_2-g(e_2))\;+\;P_3\,(S_3+W_3-g(e_3))\end{aligned} \tag{3}$$

Second, we get the agent's incentive rationality conditions as follows:

$$(\text{CAIC in state 2}) \qquad W_4-g(e_4+\Delta\theta)-\sigma_2 \;\geq\; W_1-g(e_1)-\sigma_3 \tag{4}$$

$$(\text{CAIC in state 3}) \qquad W_1-g(e_1-\Delta\theta)-\sigma_3 \;\geq\; W_4-g(e_4)-\sigma_2 \tag{5}$$

Then (4) and (5) imply

$$g(e_1)+g(e_4) \;\geq\; g(e_4+\Delta\theta)+g(e_1-\Delta\theta)$$

$< == > \; g(e_1) - g(e_1 - \Delta\theta) \;\geq\; g(e_4 + \Delta\theta) - g(e_3)$

$==> \; g'(e_1) \;\geq\; g'(e_4 + \Delta\theta) \;\; ==> \;\; e_1 \;\geq\; e_4 + \Delta\theta$ by the strict convexity of g(e). Hence, we have the Coalitional Agent's Incentive Condition as follows:

$$(CAIC) \qquad e_1 \;\geq\; e_4 + \Delta\theta \tag{6}$$

Case (VIII-1)

First, we get the individual rationality conditions within the coalition as follows:

$$CIRA \qquad \begin{aligned} & P_2\,(\,W_4 - g(e_4 + \Delta\theta) - \sigma_2\,) \;+\; P_3\,(\,W_2 - g(e_2 - \Delta\theta) - \sigma_3\,) \\ & > \; P_2\,(\,W_2 - g(e_2)) \;+\; P_3\,(\,W_3 - g(e_3)) \end{aligned} \tag{1}$$

$$CIRS \qquad P_2\,(\,S_4 + \sigma_2\,) \;+\; P_3\,(\,S_2 + \sigma_3\,) \;>\; P_2\,S_2 \;+\; P_3\,S_3 \tag{2}$$

Then, (1) and (2) gives the Coalitional Individual Rationality condition as follows:

$$(CIR) \qquad \begin{aligned} & P_2\,(\,S_4 + W_4 - g(e_4 + \Delta\theta)\,) \;+\; P_3\,(\,S_2 + W_2 - g(e_2 - \Delta\theta)\,) \\ & > \; P_2\,(\,S_2 + W_2 - g(e_2)) \;+\; P_3\,(\,S_3 + W_3 - g(e_3)) \end{aligned} \tag{3}$$

Second, we get the agent's incentive rationality conditions as follows:

$$(\text{CAIC in state 2}) \qquad W_4 - g(e_4 + \Delta\theta) - \sigma_2 \;\geq\; W_2 - g(e_2) - \sigma_3 \tag{4}$$

$$(\text{CAIC in state 3}) \qquad W_2 - g(e_2 - \Delta\theta) - \sigma_3 \;\geq\; W_4 - g(e_4) - \sigma_2 \tag{5}$$

Then (4) and (5) imply

$$g(e_2) + g(e_4) \;\geq\; g(e_4 + \Delta\theta) + g(e_2 - \Delta\theta)$$

$< == > \; g(e_2) - g(e_2 - \Delta\theta) \;\geq\; g(e_4 + \Delta\theta) - g(e_4)$

$==> \; g'(e_2) \; \geq \; g'(e_2 + \Delta\theta) \; ==> \; e_2 \; \geq \; e_2 + \Delta\theta$ by the strict convexity of g(e). Hence, we have the Coalitional Agent's Incentive Condition as follows:

$$(CAIC) \qquad e_2 \; \geq \; e_4 + \Delta\theta \tag{6}$$

Case (VIII-2)

First, we get the individual rationality conditions within the coalition as follows:

$$CIRA \qquad \begin{aligned} & P_2 \, (W_4 - g(e_4 + \Delta\theta) - \sigma_2) \; + \; P_3 \, (W_3 - g(e_3) - \sigma_3) \\ & > \; P_2 \, (W_2 - g(e_2)) \; + \; P_3 \, (W_3 - g(e_3)) \end{aligned} \tag{1}$$

$$CIRS \qquad P_2 \, (S_4 + \sigma_2) \; + \; P_3 \, (S_3 + \sigma_3) \; > \; P_2 \, S_2 \; + \; P_3 \, S_3 \tag{2}$$

(1) and (2) imply

$$\begin{aligned} & P_2 \, (S_4 + W_4 - g(e_4 + \Delta\theta)) \; + \; P_3 \, (S_3 + W_3 - g(e_3)) \\ & > \; P_2 \, (S_2 + W_2 - g(e_2)) \; + \; P_3 \, (S_3 + W_3 - g(e_3)) \end{aligned}$$

Then we have Coalition Individual Rational condition as follows:

$$(CIR) \qquad S_4 + W_4 - g(e_4 + \Delta\theta) \; > \; S_2 + W_2 - g(e_2) \tag{3}$$

Second, we get the agent's incentive rationality conditions as follows:

$$(\text{CAIC in state 2}) \qquad W_4 - g(e_4 + \Delta\theta) - \sigma_2 \; \geq \; W_3 - g(e_3 + \Delta\theta) - \sigma_3 \tag{4}$$

$$(\text{CAIC in state 3}) \qquad W_3 - g(e_3) - \sigma_3 \; \geq \; W_4 - g(e_4) - \sigma_2 \tag{5}$$

Then (4) and (5) imply

$$g(e_3 + \Delta\theta) + g(e_4) \ \geq \ g(e_4 + \Delta\theta) + g(e_3)$$

$$<==> \ g(e_3 + \Delta\theta) - g(e_3) \ \geq \ g(e_4 + \Delta\theta) - g(e_4)$$

$==> \ g'(e_3) \ \geq \ g'(e_4) \ ==> \ e_3 \ \geq \ e_4$ by the strict convexity of g(e). Hence, we have the Coalitional Agent's Incentive Condition as follows:

$$(CAIC) \qquad e_3 \ \geq \ e_4 \tag{6}$$

Case (IX)

First, we get the individual rationality conditions within the coalition as follows:

$$CIRA \qquad \begin{array}{l} P_2\,(\,W_4 - g(e_4 + \Delta\theta) - \sigma_2\,) \ + \ P_3\,(\,W_4 - g(e_4) - \sigma_3\,) \\ > \ P_2\,(\,W_2 - g(e_2)) \ + \ P_3\,(\,W_3 - g(e_3)) \end{array} \tag{1}$$

$$CIRS \qquad P_2\,(\,S_4 + \sigma_2\,) \ + \ P_3\,(\,S_4 + \sigma_3\,) \ > \ P_2\,S_2 \ + \ P_3\,S_3 \tag{2}$$

Then, (1) and (2) give the Coalitional Individual Rationality Condition as follows:

$$\begin{array}{l} P_2\,(\,S_4 + W_4 - g(e_4 + \Delta\theta)\,) \ + \ P_3\,(\,S_4 + W_4 - g(e_4)\,) \\ > \ P_2\,(\,S_2 + W_2 - g(e_2)) \ + \ P_3\,(\,S_3 + W_3 - g(e_3)) \end{array}$$

Second, CAIC is always satisfied as we observed in the case (I).

We summarize all (CIR)'s and (CAIC)'s in the following Figure.

APPENDIX (C)

Proof of proposotion 1 (coalition case)

Since we have two equality constraints, we put

$$S_2 + W_2 - g(e_2) = S_1 + W_1 - g(e_1) = \phi_1$$
$$S_4 + W_4 - g(e_4) = S_3 + W_3 - g(e_3) = \phi_4$$

Then, we can rewrite the program (C′) as follows:

$$\max_{\{S_i,\, e_i,\, \phi_1,\, \phi_4\}} \quad P_1(\theta_1 + e_1 - g(e_1) - \phi_1) + P_2(\theta_2 + e_2 - g(e_2) - \phi_1) + P_3(\theta_3 + e_3 - g(e_3) - \phi_4) + P_4(\theta_4 + e_4 - g(e_4) - \phi_4)$$

s. t.

$$(SIR) \qquad \sum_{i=1}^{4} P_i V(S_i) \geq \bar{V}$$

$$(AIR) \qquad P_1 U(\phi_1 - S_1) + P_2 U(\phi_1 - S_2) + P_3 U(\phi_4 - S_3) + P_4 U(\phi_4 - S_4) \geq \bar{U}$$

$$(AIC) \qquad \phi_4 - S_3 - \{ \phi_1 - S_2 + g(e_2) - g(e_2 - \Delta\theta) \} \geq 0$$

$$(CIC3') \qquad \phi_4 - \{ \phi_1 + g(e_2) - g(e_2 - \Delta\theta) \} \geq 0$$

$$(CIC4') \qquad \phi_4 - \{ \phi_1 + g(e_1) - g(e_1 - \Delta\theta) \} \geq 0$$

$$(CIC5') \qquad \phi_1 - \{ \phi_4 + g(e_3) - g(e_3 + \Delta\theta) \} \geq 0$$

Lagrangian function is:

$$
\begin{aligned}
L^{c'} &= P_1(\theta_1 + e_1 - g(e_1) - \phi_1) + P_2(\theta_2 + e_2 - g(e_2) - \phi_1) \\
&+ P_3(\theta_3 + e_3 - g(e_3) - \phi_4) + P_4(\theta_4 + e_4 - g(e_4) - \phi_4) + \rho(\sum_{i=1}^{4} P_i V(S_i) - \overline{V}) \\
&+ \mu\{P_1U(\phi_1 - S_1) + P_2U(\phi_1 - S_2) + P_3U(\phi_4 - S_3) + P_4U(\phi_4 - S_4) - \overline{U}\} \\
&+ \gamma\{\phi_4 - S_3 - \phi_1 + S_2 - g(e_2) + g(e_2 - \Delta\theta)\} + \pi\{\phi_4 - \phi_1 - g(e_2) + g(e_2 - \Delta\theta)\} \\
&+ \alpha\{\phi_4 - \phi_1 - g(e_1) + g(e_1 - \Delta\theta)\} + \beta\{\phi_1 - \phi_4 - g(e_3) + g(e_3 + \Delta\theta)\}
\end{aligned}
$$

(F. O. C.)

$$\frac{dL}{dS_1} = 0 \quad ==> \quad \rho V'(S_1) = \mu U'(\phi_1 - S_1) \tag{1}$$

$$\frac{dL}{dS_2} = 0 \quad ==> \quad \rho V'(S_2) = \mu U'(\phi_1 - S_2) - \frac{\gamma}{P_2} \tag{2}$$

$$\frac{dL}{dS_3} = 0 \quad ==> \quad \rho V'(S_3) = \mu U'(\phi_4 - S_3) + \frac{\gamma}{P_3} \tag{3}$$

$$\frac{dL}{dS_4} = 0 \quad ==> \quad \rho V'(S_4) = \mu U'(\phi_4 - S_4) \tag{4}$$

From (1) and (4), we have

$$\frac{V'(S_1)}{V'(S_4)} = \frac{U'(\phi_1 - S_1)}{U'(\phi_4 - S_4)} \quad ==> \quad \{S_1 \geq S_4 \quad \text{iff} \quad \phi_1 - S_1 \geq \phi_4 - S_4\} \tag{5}$$

$$\frac{dL}{d\phi_1} = 0 ==> -P_1 - P_2 + P_1\mu U'(\phi_1 - S_1) + P_2\mu U'(\phi_1 - S_2) - \gamma - \pi - \alpha + \beta = 0$$

From (1) and (2), we have

$$\rho(P_1V'(S_1) + P_2V'(S_2)) = P_1 + P_2 + \pi + \alpha - \beta \tag{6}$$

Similarly, $\frac{dL}{d\phi_4} = 0$ gives

$$\rho\,(P_3V'(S_3)+P_4V'(S_4))=P_3+P_4-\pi-\alpha+\beta \tag{7}$$

$$\frac{dL}{de_1}=0 \quad ==> \quad P_1(1-g'(e_1))=\alpha\,(g'(e_1)-g'(e_1-\Delta\theta))$$

By the strict convexity of g(•), $g'(e_1)-g'(e_1-\Delta\theta)$ is always positive. Hence, we have

$$\begin{aligned} e_1 &= e^* \quad \text{if} \quad \alpha=0 \\ e_1 &< e^* \quad \text{if} \quad \alpha>0 \end{aligned} \tag{8}$$

$$\frac{dL}{de_2}=0 \quad ==> \quad P_2(1-g'(e_2))=(\gamma+\pi)\,(g'(e_2)-g'(e_2-\Delta\theta))$$

By the strict convexity of g(•), $g'(e_2)-g'(e_2-\Delta\theta)$ is always positive. Hence, we have

$$\begin{aligned} e_2 &= e^* \quad \text{if} \quad \gamma+\pi=0 \\ e_1 &< e^* \quad \text{if} \quad \gamma+\pi>0 \end{aligned} \tag{9}$$

$$\frac{dL}{de_3}=0 \quad ==> \quad P_3(1-g'(e_3))=\beta\,(g'(e_3)-g'(e_3+\Delta\theta))$$

By the strict convexity of g(•), $g'(e_3)-g'(e_3+\Delta\theta)$ is always negative. Hence, we have

$$\begin{aligned} e_3 &= e^* \quad \text{if} \quad \beta=0 \\ e_3 &> e^* \quad \text{if} \quad \beta>0 \end{aligned} \tag{10}$$

$$\frac{dL}{de_4}=0 \quad ==> \quad g'(e_4)=1$$

This gives

$$e_4 = e^* \tag{11}$$

I. Show that (AIC) constraint is binding

Assume that (AIC) is not binding. Then we have $\gamma = 0$ and

$$\phi_4 - S_3 - \{ \phi_1 - S_2 + g(e_2) - g(e_2 - \Delta\theta) \} > 0 \tag{12}$$

Since $\gamma = 0$, (2) and (3) imply that Borch's rule holds between the states 2 and 3:

$$\frac{V'(S_2)}{V'(S_3)} = \frac{U'(\phi_1 - S_2)}{U'(\phi_4 - S_3)} \tag{13}$$

From (12), since $g(e_2) - g(e_2 - \Delta\theta) > 0$, We know that $\phi_4 - S_3 > \phi_1 - S_2$. Then, from (13) we have

$$S_3 > S_2 \tag{14}$$

From (12) and (14), we have

$$\phi_4 - \{ \phi_1 + g(e_2) - g(e_2 - \Delta\theta) \} > 0 \tag{15}$$

This implies that (CIC3') is not binding, which implies $\phi = 0$. Since $\gamma = \phi = 0$, $e_2 = e^*$ by (9). Then, we can say $e_2 \geq e_1$. From (15), we have $\phi_4 - \phi_1 > g(e_2) - g(e_2 - \Delta\theta)$. And $e_2 \geq e_1$ implies $g(e_2) - g(e_2 - \Delta\theta) > g(e_1) - g(e_1 - \Delta\theta)$ by the strict convexity of g(•). These two relationship gives

$$\phi_4 - \phi_1 > g(e_1) - g(e_1 - \Delta\theta) \tag{16}$$

This implies (CIC4') is not binding, which implies $\alpha = 0$.

However, we can show that $\gamma = \phi = \alpha = 0$ is impossible. If $\gamma = \phi = \alpha = 0$, (8) and (9) gives $e_1 = e_2 = e^*$ and (1)and (2) give $\frac{V'(S_1)}{V'(S_2)} = \frac{U'(\phi_1 - S_1)}{U'(\phi_1 - S_2)}$. Thus we have $S_1 = S_2$. Similary, (3) and (4) give $S_4 = S_3$. Since $S_1 = S_2$ and $S_3 = S_4$, we have following relationship by (6) and (7):

$$\rho V'(S_1) = 1 - \frac{\beta}{P_1 + P_2} < \rho V'(S_4) = 1 + \frac{\beta}{P_3 + P_4} \tag{17}$$

By the concavity of V(•), (17) implies $S_1 > S_4$. From (5), we have $\phi_1 - S_1 > \phi_4 - S_4$, which implies $\phi_1 - \phi_4 > S_1 - S_4 > 0$. Hence, we get $\phi_1 > \phi_4$. This contradicts with $\phi_4 - \phi_1 > g(e_1) - g(e_1 - \Delta\theta) = g(e_2) - g(e_2 - \Delta\theta) > 0$. Hence, (AIC) is binding.

Furthermore, we can easily observe that γ cannot be 0. Assume $\gamma = 0$. Then, as we have shown eariler, we get equation (13). Even if (AIC) is binding, (thus $(\phi_4 - S_3) - (\phi_1 - S_2) = g(e_2) - g(e_2 - \Delta\theta)$) we still have $\phi_4 - S_3 > \phi_1 - S_2$, which gives $S_3 > S_2$. Then we can apply the same argument to derive the contradiction.

II Show that constraint (CIC4′) is binding

Claim 1 : we cannot have $\gamma > 0$ and $\pi = \alpha = 0$

(Proof)

Assume that we have $\gamma > 0$ and $\pi = \alpha = 0$. Since (AIC) is binding, we have $\phi_4 - \phi_1 - (g(e_2) - g(e_2 - \Delta\theta)) = S_3 - S_2$. However we know $\phi_4 - \phi_1 - (g(e_2) - g(e_2 - \Delta\theta)) \geq 0$ by (CIC3′). Hence we have $S_3 \geq S_2$. Observe that $\gamma > o$ implies $S_2 > S_1$ and $S_4 > S_3$ from (1), (2), (3), and (4). Hence, we have $S_4 > S_3 \geq S_2 > S_1$, which implies $V'(S_4) < V'(S_3) \leq V'(S_2) < V'(S_1)$ by the strict concavity of V(•). Then we have

$$P_1V'(S_1) + \frac{P_2V'(S_2)}{P_3V'(S_3) + P_4V'(S_4)} > \frac{(P_1 + P_2)V'(S_2)}{(P_3 + P_4)V'(S_3)} > \frac{P_1 + P_2}{P_3 + P_4} \tag{18}$$

However, since $\pi = \alpha = 0$, (6) and (7) give

$$P_1V'(S_1) + \frac{P_2V'(S_2)}{P_3V'(S_3) + P_4V'(S_4)} = \frac{P_1 + P_2 - \beta}{P_3 + P_4 + \beta} \leq \frac{P_1 + P_2}{P_3 + P_4}$$

This contradicts with (17).

Claim 2 : if $\gamma > 0$ and (CIC3′) is binding, then (CIC4′) must be binding.

(Proof)

Assume (CIC4′) is not binding. Then we have

$$\alpha = 0 \text{ and } \phi_4 - \phi_1 > g(e_1) - g(e_1 - \Delta\theta) \tag{19}$$

Observe that $\gamma > 0$ and $\alpha = 0$ imply $e_1 = e^* > e_2$ by (8) and (9). This gives $g(e_1) - g(e_1 - \Delta\theta) > g(e_2) - g(e_2 - \Delta\theta)$ by the strict convexity of g(•). However, since (CIC3′) is binding, we have $g(e_2) - g(e_2 - \Delta\theta) = \phi_4 - \phi_1$. By these two relationships, we get $g(e_1) - g(e_1 - \Delta\theta) > \phi_4 - \phi_1$, which contradicts with (19).

Finally, by claim 1 and 2, we can claim that (CIC4′) is always binding.

(Proof)

Assume (CIC4′) is not binding. If (CIC4′) is not binding, then we have $\alpha = 0$. However, by claim 1, we cannot have $\gamma > 0$ and $\pi = \alpha = 0$, thus π must be positive. Hence, (CIC3′) must be binding. However, if (CIC3′) is binding, (CIC4′) must not be binding by claim 2. This is contradiction.

III Show that constraint (CIC5′) is not binding

Assume that (CIC5′) is binding. Then we have $\phi_4 - \phi_1 = g(e_3 + \Delta\theta) - g(e_3)$. However, since (CIC4′) is binding, we have

$$g(e_3 + \Delta\theta) - g(e_3) = g(e_1) - g(e_1 - \Delta\theta) \tag{20}$$

However, since $\alpha \geq 0$ and $\beta \geq 0$, we have $e_3 \geq e_1$ by (8) and (10). Then by the strict convexity of g(•), we have

$$g(e_3 + \Delta\theta) - g(e_3) > g(e_3) - g(e_3 - \Delta\theta) \geq g(e_1) - g(e_1 - \Delta\theta)$$

This contradicts with (20).

We can show the rest of proposition 1 as follows: Since $\gamma > 0$, we have $S_4 > S_3$ and $S_2 > S_1$ from (1), (2), (3), and (4). Since (AIC) is binding, we have $W_3 - g(e_3) = W_2 - g(e_2 - \Delta\theta)$. Then, by (CIC3′) we have $S_3 \geq S_2$ (Note that we cannot determine whether (CIC3′) is binding or not). Hence we have $S_4 > S_3 \geq S_2 > S_1$.

Observe that $S_4 > S_3$ implies $W_3 - g(e_3) > W_4 - g(e_4)$ and $S_2 > S_1$ implies $W_1 - g(e_1) > W_2 - g(e_2)$ respectively. Now, since $S_4 > S_1$ by (5), we have $\phi_4 - S_4 > \phi_1 - S_1$, which is equvalent to $W_4 - g(e_4) - W_1 - g(e_1)$. Hence, we have $W_3 - g(e_3) > W_4 - g(e_4) > W_1 - g(e_1) > W_2 - g(e_2)$.

Since (CIC5′) is not binding, we have $\beta = 0$, which means $e_3 = e^*$. Since (CIC4′) is binding, we have $\phi_4 - \phi_1 = g(e_1) - g(e_1 - \Delta\theta)$. However, by (CIC3′), we have $\phi_4 - \phi_1 \geq g(e_2) - g(e_2 - \Delta\theta)$. These two relationships give $g(e_1) - g(e_1 - \Delta\theta) \geq g(e_2) - g(e_2 - \Delta\theta)$, which implies $e_1 \geq e_2$ by the strict convexity of g(•). Hence, we have $e_4 = e_3 = e^* > e_1 \geq e_2$.

Finally, the fact that (CIC5′) is not binding, (AIC) is binding, and constraint (CIC3 ′) can produce some limitation on $S_3 - S_2$, which is (e) of proposition 1.

APPENDIX (D)

Proof of lemma 3

The solution of (C′) gives following relationships among optimal allocations.

(*a*) AIC is binding $==>$ $W_3 - g(e_3) = W_2 - g(e_2 - \Delta\theta)$

(*b*) $(CIC1')$ and $(CIC2')$ $==>$ $\begin{array}{l} S_1 + W_1 - g(e_1) = S_2 + W_2 - g(e_2) \\ S_4 + W_4 - g(e_4) = S_3 + W_3 - g(e_3) \end{array}$

(*c*) $(CIC3')$ $==>$ $S_3 + W_3 - g(e_3) \geq S_2 + W_2 - g(e_2 - \Delta\theta)$

(*d*) $(CIC4')$ is binding $==>$ $S_4 + W_4 - g(e_4) = S_1 + W_1 - g(e_1 - \Delta\theta)$

(*e*) $(CIC5')$ is not binding $==>$ $S_2 + W_2 - g(e_2) \geq S_3 + W_3 - g(e_3 + \Delta\theta)$

(*f*) $e_4 = e_3 = e^* > e_1 \geq e_2$

We can show that the allocations which meet conditions (a) to (f) satisfy all the constraints of the original program (C).

First, (AIR) and (SIR) are met by assumption. By (a), (AIC) is met with equality and the other (AIC) is met with strict inequality. Since $W_3 - g(e_3) = W_2 - g(e_2 - \Delta\theta)$ and $e_3 > e_2$, we have $W_3 - g(e_3 + \Delta\theta) < W_2 - g(e_2)$ by the strict convexity of g(•).

Second, we don't have to worry about (CIC12), (CIC14), (CIC18), and (CIC19), since the " if " part of these constraints are not met by (f).

Next, (CIC1), (CIC4), (CIC9), and (CIC16) are met with equality by (b). Since $e_3 = e_4$ and (b) is true, (CIC2), (CIC3), (CIC15), and (CIC20) are met with strict inequality by the property (e). Given property (d), (CIC6) and (CIC8) are met with equality and (CIC5) is met since (CIC6) is true and $e_1 \geq e_2$. Observe that (CIC9) is equivalent to (c).

Finally, we can easily see the rest of constrints (CIC7), (CIC8), (CIC10) , (CIC17), and (CIC21) are met by using the other constraints that we already proved to be true.

APPENDIX (E)

Proof of proposition 2

By lemma 3, the solution of original program (C) has the same features (a) to (f) described in appendix (D). Let assume the allocation { S_i, W_i, e_i } is a final (after side contract) allocation. Then we are through if we can show the allocation with features of (a) to (f) satiesfies following twelve inequalities. This is because if the principal chooses the main contract { S_i, W_i, e_i } satisfying all the twelve inequalities, then since we assumed that each colluding party can guarantee itself the payoff before the coalition, S/A coalition cannot find any incentive to misrepresent their true state of nature.

$$S_1 + W_1 - g(e_1) \geq S_2 + W_2 - g(e_2) \tag{1}$$

$$S_1 + W_1 - g(e_1) \geq S_3 + W_3 - g(e_3 + \Delta\theta) \tag{2}$$

$$S_1 + W_1 - g(e_1) \geq S_4 + W_4 - g(e_4 + \Delta\theta) \tag{3}$$

$$S_2 + W_2 - g(e_2) \geq S_1 + W_1 - g(e_1) \tag{4}$$

$$S_2 + W_2 - g(e_2) \geq S_3 + W_3 - g(e_3 + \Delta\theta) \tag{5}$$

$$S_2 + W_2 - g(e_2) \geq S_4 + W_4 - g(e_4 + \Delta\theta) \tag{6}$$

$$S_3 + W_3 - g(e_3) \geq S_1 + W_1 - g(e_1 - \Delta\theta) \tag{7}$$

$$S_3 + W_3 - g(e_3) \geq S_2 + W_2 - g(e_2 - \Delta\theta) \tag{8}$$

$$S_3 + W_3 - g(e_3) \geq S_4 + W_4 - g(e_4) \tag{9}$$

$$S_4 + W_4 - g(e_4) \geq S_1 + W_1 - g(e_1 - \Delta\theta) \tag{10}$$

$$S_4 + W_4 - g(e_4) \geq S_2 + W_2 - g(e_2 - \Delta\theta) \tag{11}$$

$$S_4 + W_4 - g(e_4) \;\geq\; S_3 + W_3 - g(e_3) \tag{12}$$

Observe that (1), (2), and (3) are equivalent to (4), (5), and (6) respectively and (7), (8), and (9) are equivalent to (10), (11), and (12) respectively by property (b) of lemma 3. Thus we are through if we can show that the allocation with properties (a) to (f) in appendix (D) satisfies inequalities (1), (2), (3), (7), (8), and (9).

First, (1) and (9) are met with equality by (b) of lemma 3. Second, (2) is satisfied with strict inequality by (e) of lemma 3. Also by (f), $e_3 = e_4$ and by (b), $S_3 + W_3 - g(e_3) = S_4 + W_4 - g(e_4)$. These two imply $S_3 + W_3 - g(e_3 + \Delta\theta) = S_4 + W_4 - g(e_4 + \Delta\theta)$, which shows that (3) is equivalent to (2). Next, (8) is equivalent to (c) of lemma 3 i.e., (CIC3′). Finally, (7) is met with equality by (d) of lemma 3.

APPENDIX (F)

Proof of proposition 3

Since the supervisor is risk neutral, $V'(S)$ is constant. By choosing ρ appropriately, the first order conditions of program (C') become equivalent to that of (CF). First, all the (CIC)'s are not binding, hence $\pi = \alpha = \beta = 0$. Second, we choose ρ such that $\rho V'(S) = 1$, then we have:

$$\mu U'(W_1 - g(e_1)) = 1 \qquad \text{from (1) of } (C')$$

$$\mu U'(W_2 - g(e_2)) = 1 + \frac{\gamma}{P_2} \qquad \text{from (2) of } (C')$$

$$\mu U'(W_3 - g(e_3)) = 1 - \frac{\gamma}{P_3} \qquad \text{from (3) of } (C')$$

$$\mu U'(W_4 - g(e_4)) = 1 \qquad \text{from (4) of } (C')$$

$$P_2(1 - g'(e_2)) = \gamma(g'(e_2) - g'(e_2 - \Delta\theta)) \qquad \text{from (9) of } (C')$$

These (F. O. C.)'s are exactly that of program (CF).

Proof of proposition 4

Since the supervisor is infinitely risk averse, S_i in each state must be same. Otherwise, we should have unbounded level of wages for the agent. This is impossible. Hence, S_i is constant. Since S_i is constant, we have $W_3 = W_4$ and $e_3 = e_4$. Moreover, $S_2 = S_3$ implies that CIC3' is binding since we already know that AIC is binding. Then we have $S_3 + W_3 - g(e_3) = S_2 + W_2 - g(e_2 - \Delta\theta)$ and $S_4 + W_4 - g(e_4) = S_1 + W_1 - g(e_1 - \Delta\theta)$. This gives $S_2 + W_2 - g(e_2 - \Delta\theta) = S_1 + W_1 - g(e_1 - \Delta\theta)$ by (CIC2'). Since we have $S_2 + W_2 - g(e_2) = S_1 + W_1 - g(e_1)$ by (CIC1'), these two relationships give $g(e_1) - g(e_1 - \Delta\theta) = g(e_2) - g(e_2 - \Delta\theta)$.

Thus we get $e_1 = e_2$ by the strict convexity of g(•). This also gives $W_1 = W_2$ since $S_1 = S_2$ and $S_1 + W_1 - g(e_1) = S_2 + W_2 - g(e_2)$. Same is true between the state 3 and the state 4. In otherwords, since $e_3 = e_4$, $S_3 = S_4$ and $S_3 + W_3 - g(e_3) = S_4 + W_4 - g(e_4)$, we can have $W_3 = W_4$. All these results imply that the principal does not try to distinguish state 2 from state 1 and state 4 from state 3 respectively. Observe that $W_3 - g(e_3) = W_2 - g(e_2 - \Delta\theta) > W_2 - g(e_2)$ and $W_4 - g(e_4) = W_1 - g(e_1 - \Delta\theta) > W_1 - g(e_1)$. This means that the principal wants to distinguish state 1 from state 4 and state 2 from state 3 respectively. However, since the supervisor cannot distinguish state 2 from state 3, his report will be confined to the following trivial structures: $\{ s_1 = s_2 = s_3 = s_4 = \underline{\theta} \}$, $\{ s_1 = s_2 = s_3 = s_4 = \overline{\theta} \}$ and $\{ s_1 = s_2 = s_3 = s_4 = \phi \}$.

The vita has been removed from the scanned document