Combat Aircraft Mission Tradeoff Models

for Conceptual Design Evaluation

by

Lev A. Malakhoff

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APPROVED:

D.R. Drew

S.A. Ardekani

B.S. Blanchard

J.W. Dickey

A.G. Hobeika

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Blacksburg, Virginia

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(ABSTRACT)

A methodology is developed to address the analyses of combat aircraft attrition. The operations of an aircraft carrier task force are modeled using the systems dynamics simulation language DYNAMO. The three mission-roles include: surface attack, fighter escort, and carrier defense. The level of analysis is performed over the entire campaign, going beyond the traditional single-sortie analysis level.

These analyses are performed by determining several measures of effectiveness (MOEs) for whatever constraints are applied to the model. The derived MOEs include: Campaign Survivability (CS), Fraction of Force Lost (FFL), Exchange Ratio (ER), Relative Exchange Ratio (RER), Possible Crew Loss (PCL), and Replacement Cost (RC). RER is felt to be the most useful MOE since it considers the initial inventory levels of both friendly and enemy forces, and its magnitude is easy for the analyst to relate to (an RER greater than one is a prediction of a friendly force's victory).

The simulation model developed in this research is run for several experiments. The effects of force size on the MOEs is studied, as well as a hypothetical *multimission* aircraft deployed to perform any of the three missions (albeit at lower effectiveness than the specialized aircraft for their given roles but nonetheless with a higher availability).

Evaluation of specific technological improvements such as smaller radar cross section, higher thrust/weight, improved weapons ranges, is made using the MOEs. Also, a cost-effectiveness tradeoff methodology is developed by determining the acquisition cost ratio (ACR) for certain modified alternatives the baseline by determining the required initial inventory of modified aircraft to produce the same total effectiveness of the baseline aircraft.

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1.0 Introduction

Civil Engineering has traditionally been a discipline of public service. It serves the people through its support of the nation's *infrastructure*.

Highways, bridges, buildings, water supply, wastewater treatment, are all aspects of infrastructure. However, all of these elements rely on the unspoken premise of national security -- directly or indirectly. Considering highways, in the case of the national Interstate Highway system, this aspect of infrastructure was conceived with direct applications for national defense (specifically, to facilitate the movement of troops and materiel in times of war).

This leads to the concept of *military infrastructure*. In at least an indirect sense, all other aspects of a nation's infrastructure rely on that nation's military ability to protect them. This is not to say that there are not other processes which deter aggression, there are many: diplomacy, economic pressures, world opinion, etc. However, it is important to note how highly many countries hold military infrastructure. The unfortunate truth is that most countries spend more on their military than on any other category of infrastructure.

Keeping this in mind, the research presented in this document is concerned with a particular aspect of military infrastructure, notably, aircraft carrier operations.

1.1 The Problem

Carrier aircraft, as a rule, must be designed for more capability than their landbased counterparts. This results from the fact that there are only so many aircraft that can fit on a carrier; the option of laying down additional parking area and achieving the same effectiveness by using a greater number of individually less capable aircraft simply doesn't exist. Also, carrier aircraft tend to operate in areas away from the support of surface installations and are found in hostile airspace a greater percentage of the time.

In order to counter the projected threat in the 1990-2000 time frame and beyond, government and industry must identify, analyze, budget, and plan for cost-effective and affordable aircraft force modernization. Tactical aircraft force modernization can be accomplished in several ways:

1. By introducing new aircraft
2. By enhancing the capability of production aircraft through preplanned product improvement initiatives (P^3I)
3. By tailoring derivative aircraft configurations to meet specific mission roles
4. By retrofitting/modifying inventory aircraft

1.2 Purpose & Objectives

The objective of this research is to develop combat aircraft mission tradeoff models to be used in conjunction with mission scenario models, being researched in a companion research project, to arrive at a methodology to be used for conceptual design evaluation by determining the effect of aircraft performance and survivability enhancement features on combat measures of merit. Mission tradeoff models consider the results of all the sorties required to achieve the goal in the context of theater-wide combat.

1.3 Plan of Research

A research project is under way to develop a methodology for conceptual design evaluation by determining the effect of aircraft performance parameters and survivability enhancement features on combat survivability and effectiveness. While the project was conceived with the ATA (Advanced Tactical Aircraft) in mind, the methodology, to the extent possible, is not to be aircraft specific. Hereafter, reference to CBF is meant to describe a generic "carrier based fighter" for the Navy.

This research presents an overview of work accomplished in the development of a computer simulation model to be used (1) to microscopically evaluate different survivability-related performance concepts such as susceptibility, vulnerability, lethality and inventory in a total weapon system environment for a new or modified combat aircraft, and (2) to perform trade-off analysis between the aircraft's design/operational features and its mission requirements. This work is an application and extension of research performed under the long-term cooperative effort between the Joint Technical Coordinating Group for Aircraft Survivability (JTCG/AS) and Virginia Polytechnic Institute and State University (VPI&SU) in the area of survivability management (SURMAN).

A basic tenet of the survivability discipline is that survivability considerations must be incorporated at the earliest time in the development cycle of the aircraft in order to optimize costs/benefits. This is also true for other considerations such as mission effectiveness, force effectiveness, and affordability. Thus, in order to achieve a complete and sound development plan for future aircraft, these considerations must be studied and evaluated throughout their development cycle, from concept formulation to design and implementation.

There are five major areas of emphasis to be covered in a complete survivability study:

- Susceptibility
- Vulnerability
- Lethality
- Inventory
- Systems evaluation

Susceptibility focuses on those mechanisms that lead to the detection of the aircraft and avoidance of its being hit. Vulnerability addresses the ability of the aircraft to withstand different levels of damage. Lethality addresses the effectiveness of the aircraft as a defensive and offensive weapon system. Inventory concerns such issues as procurement and affordability because one of the objectives of the project is to provide a mechanism for performing tradeoff analyses among the overall force effectiveness. Finally, the system's evaluation part of the model synthesized the results of all the essential elements of consideration to provide a means for evaluating the effectiveness of the aircraft at the tactical level as well as at the force-on-force level.

The first three areas mentioned above are engineering oriented. Their purpose is to synthesize the engineering knowledge for making management decisions. Thus, the model will be structured in order to identify technological changes that have leverage on the overall performance of the aircraft. Additionally, the model will be able to receive outputs of offline studies as inputs.

Survivability-related modeling can be thought of as spanning several tiers as shown in Table 1.1 In this context, engineering represents a bottom-up approach starting at the lowest tier, Level 1, and survivability management can be thought of as a top-down approach. Levels 2, 3, and 4 provide the mechanism relating the engineering/design parameters (e.g., speed, signatures, weight, excess energy, etc.), and the operational parameters (e.g., payload, range, flight profile, etc.) to the aircraft survivability. In Levels 5 and 6, the thrust area is in relating

Table 1.1 Survivability Analysis Hierarchy

Level	Description	Key Parameters
1	Vulnerability Assessment Munitions Effectiveness Assessment	Aircraft Single Shot Prob. of Kill, Pssk(A/C) Threat Single Shot Prob. of Kill, Pssk(Threat)
2	Encounter Survivability Assessment Encounter Lethality Assessment	Aircraft Survivability/Encounter, Ps/e Aircraft Lethality/Encounter, Pk/e
3	Sortie Survivability Assessment Sortie Lethality Assessment	Aircraft Survivability/Sortie, Ps/s Aircraft Lethality/Sortie, Pk/s
4	Mission Attainment Survivability/Effectiveness Determination	Campaign Survivability, CS Mission Attainment Measure, MAM Measure of Mission Success, MOMS
5	Survivability Cost-Effectiveness Tradeoffs	Relative Exchange Ratio, RER Aircraft Replacement Cost, RC Possible Crew Loss, PCL
6	Resource Allocation Tradeoffs	Budget to R&D, O&M, and Procurement

the aircraft survivability to the parameters that describe the overall force effectiveness so that the changes in technology, operations and/or management policies can be evaluated.

The contents of this research address the work accomplished in Levels 4 and 5. The work on the lower-tier model -- (i.e., the mechanisms that relate the aircraft's engineering/design and operational parameters to measuring effectiveness -- Levels 1-3 in Table 1.1) is reported elsewhere [1][2]. The inputs to the ASALT models are the probabilities of single-shot kill and the outputs are $P_{S/S}$, the probability of the aircraft surviving a sortie, and $P_{K/S}$, the probability that the aircraft destroys the threat on a sortie. It should be stressed that these parameters are means to higher ends such as longer term mission-oriented measures of effectiveness including: the mission attainment measure, MAM, the campaign survivability, CS, and the measure of mission success, MOMS (see Chapter 5).

The system dynamics approach is utilized in developing this model (see Chapter 2). The strength of the methodology lies mainly in its ability to represent causal relationships and feedbacks within a system, and to incorporate and synthesize interdisciplinary inputs.

1.3.1 Relation to Companion Research

The thrust of this research is to develop a methodology to aid in the assessment process that provides sufficient understanding of the feasibility, performance, and cost-effectiveness of alternatives to help decision makers in generating modernization strategies. The methodology contains two main elements:

1. Mission Scenario Models
2. Mission Tradeoff Models

In the current cooperative research effort, the methodology is being applied to four missions:

1. Surface Attack
2. Fighter Escort
3. Carrier Defense
4. Multimission Aircraft

Multimission aircraft would integrate the first three missions on the above list.

Since the mission tradeoff models discussed in this research are dependent on the mission scenario models of the companion research, it is appropriate to discuss these very important inputs.

Combat performance of a tactical aircraft can be studied broadly under three types of analyses:

1. Single Sortie
2. Threat Sector
3. Campaign

Using these three types of analyses, we can apply them to each of the four mission scenario cases. For the surface attack, fighter escort, and carrier defense missions, the single sortie analysis level is utilized. To analyze multimission aircraft a level of analysis between the threat sector and campaign levels is utilized. The following sections contain descriptions of the mission scenarios and the levels of analyses.

1.3.1.1 Surface Attack Mission

A "carrier" is a ship whose primary purpose is to carry and operate aircraft. Without the aircraft, there is no reason for the ship's existence. As a matter of fact, without the attack aircraft (about 1/3 of the 90 odd aircraft carried), it would be hard to justify the existence of carriers.

The A-6 Intruder is the primary striking force of the carrier for both land and maritime missions. The A-6's have normally operated without fighter escort. Unfortunately, the air threat has increased and fighters for escort are now needed.

1.3.1.2 Fighter Escort Mission

Escorting is the most difficult of all fighter missions. Historically, it has proved impossible to protect friendly strike aircraft completely, even with large numbers of escort fighters.

Many types of escort are possible. The most effective is the "fighter sweep," which precedes the strike force, ranging out across the most probable line of approach of the enemy fighters, on a search-and-destroy mission. The limiting factor is the carrier capacity; to provide anywhere near adequate protection runs the risk of weakening the carrier air defense.

1.3.1.3 Carrier Defense Mission

At present, the air threat to the carrier is considered the most serious. Whether launched from ship, submarine, or an airborne platform, stopping the incoming missiles will be a function of carrier air defense. Because the bomber-launched missile has the greatest range and the highest performance, it is considered to be the primary threat.

For air defense, the first screen would be composed of four F-14's (the only aircraft that can engage a Backfire or Blackjack bomber for a sustained period) in two different combat air patrols. With the arrival of the F-14D in the 1990's, the goal would be to destroy the bombers before they launched. If they launch, the F-14's would have to ignore the bombers to go after the missiles.

1.3.1.4 *Multimission Aircraft*

The VFMX, conceived as a combined F-14/A-6 replacement, was abandoned because it was found that updated F-14's and A-6's could come very close to performance of the new aircraft at much less cost. It was thought better to wait to take advantage of technological growth.

Intended to enter service in the 1990's is Navy's Advanced Tactical Aircraft (ATA). It is not clear at this time if the ATA is to be simply a new attack aircraft to be developed from either the A-6 or the F/A-18, or a wholly new dual role fighter/attack aircraft which would possess high supersonic performance, some stealth characteristics, and a V/STOL concept. Another idea is that the ATA would be an all-weather attack aircraft with self-defense and escort capabilities.

1.3.1.5 *Single-Sortie Level of Analysis*

The single-sortie analysis provides a snapshot viewpoint and gives perhaps the greatest degree of traceability between technological input and effectiveness. For system assessment a mini-scenario is specified (e.g. four defenders vs. eight intruders). Typical measures of effectiveness include kills, losses, and the exchange ratio.

Unfortunately, extrapolating results into a larger combat context is risky and may be unrealistic, because success (victory) in one mission scenario does not insure success thoughout the campaign with its variety of scenarios. The desired degree of improvement between survivability, lethality, and inventory is difficult to investigate. Therefore, the usefulness of single-sortie results is as input to higher-order analyses -- for this research in particular as "mission tradeoff analysis."

1.3.1.6 *Threat Sector Level of Analysis*

This level integrates the cumulative effect of the many sorties that produce the total kills required to counter the threat. Threat sector analysis provides a framework to assess force effectiveness where scenario constraints of conflict duration, kill requirements, and aircraft inventory are important. Typical measures of effectiveness include: sorties generated, targets destroyed, and aircraft lost; all of which are time dependent.

Limitations include predetermined resource allocation which is not responsive to reactions by the threat. For example, the impact of carrier-attack is reduced by sortie generation; this important feedback would not be treated explicitly. Similarly, multimission aircraft are not placed dynamically into swing-role assignments.

1.3.1.7 *Campaign Level of Analysis*

Campaign analysis provides a theater-wide assessment of force-on-force effectiveness. The model accounts for interaction among various elements such as carrier deck-cycle performance and sortie generation; impact of carrier attack; attrition of assets due to both air-to-air and surface-to-air engagements; and threat removal due to defense suppression.

For this level the basic unit of aircraft may be the combat wing. However, assessment of individual technological enhancements at this level presents a challenge because of reduced resolution in input characterization and small variation in output.

2.0 Methodology

The world is complex. This ineluctable statement may well be the superlative of understatements, but the fact that the world *is* complex has contributed to a never-ending source of challenge or frustration to researchers. In order to analyze a problem within a useable framework, the analyst has had to make series of assumptions, approximations, and oftentimes intuitive "leaps" to arrive at a solution. All the processes the analyst goes through are an attempt to filter the complex world and produce a model which satisfactorily explains the relevant worldly processes.

This requires a systems approach; in other words, all elements and processes occuring in the entire *universe* should be considered, but not necessarily included, in the problem formulation. As an example, the amount of solar radiation striking Earth's surface can be neglected in a study of stock market price fluctuations, unless however, astronomers predict a supernova explosion within six months - such a prediction, if creditable, would surely have an effect on the world's economy.

In the broadest and most general sense, *systems* refer to a way of thinking. To begin with, since systems vary widely in scope and complexity, all systems can be regarded as subsystems of larger systems. To illustrate systems in order of increasing scope and complexity [3]:

1. A single machine viewed as a group of interrelated parts and performing one or more functions.
2. A group of machines working together to produce an end product.
3. All machines in a total assembly process.
4. The work of a group of assemblers acting together to produce an end product.
5. A collection of persons, processes and goals in an organizational setting, such as a company or a government agency -- the management system.
6. A collection of all the information flows -- formal and informal, manual or automated -- the management information system.

The larger systems are a part, but are not all, of the total environment. The environment of a system contains conditions, forces and components which are not defined formally as part of the system being studied but nevertheless influence that system and, in turn, this environment is influenced by the system. While systems and their environments are tangible objective entities, they are studied subjectively. Different researchers will likely have different purposes, frames of reference and interests, and therefore will inevitably conceptualize the same system differently.

To better understand the systems concept, it is helpful to establish a relationship between the system and its environment. Basically, the environment for a given system [4]:

1. Completely encloses the system,
2. Includes only those things which influence the system,
3. Provides the constraints on the system,
4. Supplies the boundary conditions,
5. Is the source of all knowledge and contains all the resources and catalysts for implementing the system, and
6. Establishes the context within which any systems engineering effort must be judged.

To address systems problems, the following section describes a methodology termed "Simulation Modeling," often referred to simply as simulation.

2.1 Simulation

In its broadest sense, simulation modeling is the process of designing a mathematical-logical model of a real system and experimenting with this model on a computer. In this way simulation encompasses a model-building process as well as the design and implementation of an appropriate experiment involving that model. These experiments, or simulations, permit inferences to be drawn about the real systems

- Without building them, if they are only propsed systems;
- Without disturbing them, if they are operating systems that are costly or unsafe to experiment with;
- Without destroying them, if the object of an experiment is to determine their limits of stress.

In this way, simulation models can be used for design, procedural analysis, and performance assessment [5].

Simulation modeling assumes that we can describe a system in terms acceptable to a computer system. In this regard, a key concept is that of a *system state description*. If a system can be characterized by a set of variables, with each combination of variable values representing a unique state or condition of the system, then manipulation of the variable values simulates movement of the system from state to state. A simulation experiment involves observing the dynamic behavior of a model by moving from state to state in accordance with well-defined operating rules designed into the model.

Changes in the state of a system can occur continuously over time or at discrete instants in time. The discrete instants can be established deterministically or stochastically depending on the nature of model inputs. Although the procedures for describing the dynamic behavior of discrete and continuous change models differ, the basic concept of simulating a system by portraying the changes in the state of the system over time remains the same.

Simulation modeling is an experimental and applied methodology which seeks to:

1. Describe the behavior of systems;
2. Construct theories or hypotheses that account for the observed behavior;
3. Use these theories to predict future behavior, that is, the effects that will be produced by changes in the system or in its method of operation.

A simulation model is "run" rather than "solved" in order to obtain results and can be thought of as a tool of analysis of the behavior of the system under any specified conditions. Simulation is not a theory; rather, it is a problem-solving methodology. That is to say, simulation models are "if-then" devices – "if" a certain input is specified, which includes any strategy, "then" the output can be determined.

The usefulness of simulation experiments is multifold [6]:

1. *Evaluation*: determining how good a proposed system design performs in an absolute sense when evaluated against specific criteria.
2. *Comparison*: comparing competitive systems designed to carry out a specified function, or comparing several proposed operating policies or procedures.
3. *Prediction*: estimating the performance of the system under some projected set of conditions.
4. *Sensitivity Analysis*: determining which of many factors are the most significant in affecting overall system performance.
5. *Optimization*: determining exactly which combination of factor levels will produce the best overall response of the system.

6. *Functional Relations*: establishing the nature of the relationship among one or more significant factors and the system's response.

Some advantages of simulation include:

- Solution to the problem,
- Insight into the behavior of the system,
- Results are easy to sell (realism),
- Provides a systematic approach to the problem,
- Control over time

Some of the disadvantages of simulation include:

- Costly (programming, computer time, etc.),
- No analytical results obtained,
- Sometimes not efficient and time consuming,
- Answers obtained from stochastic models are only estimates,
- Analysis of results may be difficult,
- Difficult to optimize,
- Usually more is read into the results than should be,
- Not so difficult to do, but skill is needed for a good job.

2.2 *Systems Dynamics*

An important aspect of any system is *feedback*, which can be either positive or negative. Feedback occurs in the modelling of a problem where the analyst determines that two or more levels are interdependent upon each other. Positive feedback can occur as either a benevolent cycle (e.g. the more money I have - level 1 - the more I pay my accountants - level 2 - who save me from paying taxes, hence I may end up with even more money), or as a vicious cycle (e.g. if the number of well-educated persons in a country - level 1 - is low, than some measure of national wealth - level 2 - is also likely to be low, thus encouraging these well-educated persons to emigrate to a more attractive country.

Both types of positive feedback cycles, benevolent and vicious, are formulated to produce ultimate levels of infinity or negative infinity, respectively. Negative feedback on the other hand, manifests itself by approaching a limit, or goal. As an example, the thermostat setting of a given home may be 70°F which directs the furnace or air conditioning systems to bring the temperature down or up as appropriate.

An ideal approach to these types of problems is the use of *systems dynamics*. Systems dynamics is a type of simulation technique developed by Professor Jay W. Forrester and applied originally to industrial production situations [7]. Lately it has been applied to a variety of local to world-wide social, economic, environmental, and military problems.

Professor Forrester's methodology provides a foundation for constructing computer models to do what the human mind cannot do -- rationally analyze the structure, interactions, and modes of behavior of complex social systems in order to provide a framework whereby strategies can be tested and tradeoffs performed while options are still open. The three basic steps are:

1. The formulation of a mental model in the form of a verbal description,
2. The expression of this verbal model in the form of a flow diagram,
3. The conversion of this flow diagram to a set of simultaneous difference equations.

These equations can be solved manually but this is not practical; a digital computer is necessary.

In converting from a verbal description of the system to be studied to a flow diagram, the structure of the system is perceived as consisting of two basic components -- levels and flows. Levels represent accumulations of flows in the system such as inventories of goods, population, amounts of different types of housing, industrial capital, arable land, etc. Rates of flow represent the activities and decision functions in the system such as the movement of goods, migration, construction and demolition of housing, generation and depreciation of capital, and the bringing of land into production through irrigation [4]. The conversion of these processes into a flow diagram useful to systems dynamics is done by a method known as *Causal Diagramming* and is described in the next section.

2.2.1 Causal Diagramming

The first step in developing the flow or causal diagram of the model is to identify the key variables which describe the problem situation and record the way the system to be analyzed works. These key variables which are deduced from the mental model or verbal description of the system, are arranged on a sheet of paper. Cause-effect relationships between pairs of variables are depicted by arrows in the second step. The third step in developing causal a causal diagram is giving each link or arrow a plus or minus directional sign usually placed near the arrow head.

Thus, to begin the causal diagramming process, the key variables that describe the system behavior are identified from the verbal description of the problem. These key variables are then connected by arrows to show the cause-effect relationship. The variable at the tail of the arrow is the dependent variable; it affects the variable at the head of the arrow. The causal relationship can also be understood as the variable at the tail of the arrow "affecting or influencing" the variable at the head of the arrow depending upon the context. One point to be borne in mind is that the pair-wise relationship is made under the assumption of "all other things being the same". The next step is to identify and mark the nature of the cause-effect relationship. If the relationship is one in which there is a direct proportionality between the two variables, then a plus sign is placed near the head of the arrow. This indicates an increase in the value of the variable at the tail of the arrow causes an increase in the value of the variable at the head of the arrow. A minus sign is placed at the head of the arrow when there is an inversely proportional relationship between the two variables, i.e., when an increase of the variable at the tail of the arrow causes the value of the variable at the head of the arrow to decrease.

There are two basic components of the structure of a system from the system dynamics perspective - flows and levels. Levels are state variables that represent the accumulation or build up of resources in the systems such as inventories of aircraft, ordnance, weapons, etc.. Rates of flow represent action in the system such as aircraft procurement and attrition. The flow can be categorized into two types: 1) physical flow and 2) information flow. Physical flows are causal relationships between a rate variable such as aircraft attrition (lost aircraft/day) and a level variable such as inventory (aircraft). Solid lines are used to indicate physical flows. The relationship between a level variable such as procurement and a rate variable such as procurement rate is an information flow. It is the information regarding the state of the inventory that influences the procurement rate. Such relationships depicting information flow are presented using dotted lines. Thus level variables will always appear at the head of solid arrows and rate variables always appear at the tail of solid arrows. Any intermediate variable on the path from a level variable or exogenous input to a rate variable is called an auxiliary variable. Supplementary variables do not form part of the system itself, but merely indicate its performance and therefore appear always at the end of a dashed arrow, and having no arrows emanating from them. Exogenous inputs, have no arrows leading to them but have dashed arrows emanating from them.

Fig. 2.1 depicts a causal diagram displaying the basic combat interactions between the opposing fighter aircraft. The numbers of Carrier Based Fighters (CBF) and Aggressor Fighters (AGG) are level variables, Aircraft Attrition Rates for CBF and AGG respectively are CBFKR and AGGKR are rate variables. Finally, Sortie Survivabilities, CBFSS and AGGSS, and Aircraft Lethalities, CBFL and AGGL, are treated as constants in this represention.

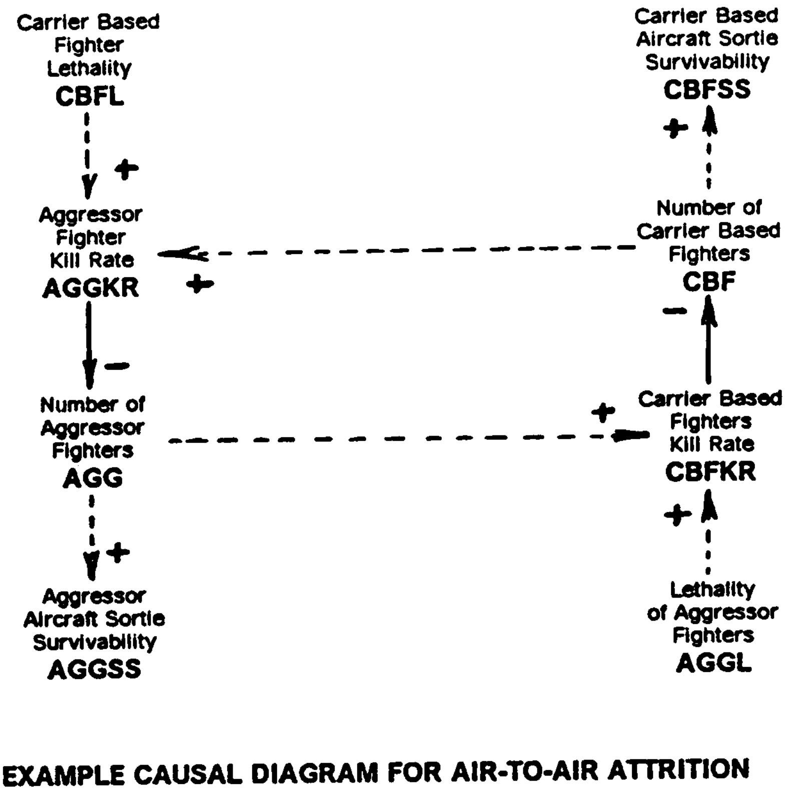


Figure 2.1 Basic Model Combat Interactions.

2.2.2 System Dynamic Equations

Integration (or accumulation) is the basis of the level and rate structure used in systems dynamics. A level variable $L(t)$ denotes the accumulation of of some physical entity at time t. RI and RO represent the rate variables, rate-in and rate-out, denoting the change in the level variable over the interval from $t-1$ to t. The relationships between the level $L(t)$ and the rates can be expressed mathematically as,

$$L(t) = L(t-1) + \int_{t-1}^{t} (RI - RO)dt \qquad \{2.1\}$$

or, using DYNAMO notation (see next section),

$$L.K = L.J + (DT)(RI.JK - RO.JK) \qquad \{2.1\}$$

In difference equation terminology, any level variable L_i is expressed as functions of rate variables R_j and the previous value of the level,

$$L_i(t) = L(t-1) + (dt)\sum_{j=1}^{n} R_j(t-1) \qquad i = 1,\dots\dots,m \qquad \{2.3\}$$

With the R_j is assumed to be constant over the interval from $t - dt$ to t. The rate variables are of the form

$$R_j(t) = f\,[L_i(t), E_k(t), A_{ij}(t), A_{kj}(t)] \qquad \{2.4\}$$

Where E_k is the set of of exogenous inputs that affect R_j directly and A_{ij} and A_{kj} are the impacts of auxiliary variables in the causal streams from the *ith* level to the *kth* exogenous input, respectively. Since the exogenous inputs are known time functions or constants, and if the initial values of the level variables are known, then all other variables can be computed from them for that time. Then the new values of the level variables for the next point in time can then be found from the "level" Eq. 2.4.

An outgrowth of systems dynamics is a unique type of FORTRAN-based computer programming language known as *DYNAMO* (for *DYNamic MOdelling*) and will be described in the next section. DYNAMO is a language which compiles and executes system dynamic models. Because the inability of a computer language to handle subscripts, DYNAMO uses a postscript notation in which *.K* stands for the present time t, *.J* stands for the past time $t-dt$, and *.L* stands for the future time $t+dt$. As in all computer programming upper case letters are used and *DT* is called the simulation interval, the time between successive computations in the simulation. Since rate variables are assumed to be constant over *DT*, the double postscript is used., *.JK* for the rates on the right side of an equation and *.KL* for the rates on the left side.

2.3 DYNAMO Simulation Language

DYNAMO makes use of series of level and rate equations relating them through simulation time. Level equations are essentially difference equations, where the level at present time is equal to the level at past time plus the interval multiplied times the difference in the rates in and out. Rates may be constant, functions of time, functions of levels or other rates, or some nonlinear combination of all the above.

There are at least six well documented versions of DYNAMO [8] and all of them support the basic functions to be described in the following paragraphs.

2.3.1 Logical Functions

DYNAMO supports logical functions similar in nature to those available in higher level languages such as FORTRAN and PASCAL. There are two types of conditional statements to compare two alternatives simultaneously. CLIP and FIFGE are used interchangeably though the former definition will be used in the models for this research.

CLIP(A,B,C,D) or FIFGE(A,B,C,D)	A if $C \geq D$ B if $C < D$

A special case of this rule is the comparison of the numerical value of a variable C with zero. This capability is achieved with either SWITCH or FIFZE functions as shown below.

SWITCH(A,B,C) or FIFZE(A,B,C)	A if $C = 0$ B if $C \neq 0$

Another important pair of functions are MAX and MIN which select the largest and smallest of two given variables A and B.

MAX(A,B)	A if $A \geq B$ B if $A < B$
MIN(A,B)	A if $A \leq B$ B if $A > B$

Furthermore, the function MAX can be used to compute the absolute value of a variable specifying the variable and its negative inside the two parenthesis positions.

MAX(A,-A)

2.3.2 Table Functions

A very important part of many simulation models is the use of tabular data, either empirically or theoretically derived. DYNAMO is capable to handle data by means of several types of table functions. TABLE, TABXT, TABHL, and TABPL are all invoked in the same manner though there are significant changes in their respective interpolating schemes. Two statements are required to describe a table function. The first one involves five ordered elements inside a parenthesis (see below) to specify the dependent variable name, the independent variable name, the lowest, highest values of the independent variable, and the interval between each independent variable. A general definition of a table is given below followed by a numerical example to help the understanding of this concept.

TABHL(Y-variable,x-variable,low x,high x,x-interval)

Y-variable=numerical values

2.3.2.1 Example 2.1

Suppose it is required to express in table form the probability of survival of a friendly aircraft given varying sortie force sizes of both friendly and enemy aircraft. For this example suppose that the friendly forces fly sortie strengths of two or four aircraft and that the enemy forces fly sortie strengths of three or five aircraft. The kill fractions for the various combinations of force strengths are shown in Table 2.1.

Table 2.1 - Kill Fractions for Example 2.1

Enemy Sortie Force Size	Friendly Sortie Force Size 2	4
3	0.25	0.10
5	0.40	0.15

The kill fractions are input into the TABLE functions using the following statements. Notice that a CLIP function is used to select the proper table (if enemy sortie force size, X, is greater than or equal to 5, kill fractions will be calculated by K$X5).

```
A  K$X.K=CLIP(K$X5.K,K$X3.K,X.K,5)
A    K$X5.K=TABLE(K$X5T,$.K,2,4,2)
A      K$X5T=0.40/0.15
A    K$X3.K=TABLE(K$X3T,$.K,2,4,2)
A      K$X3T=0.25/0.10
```

The main use of these table functions relies on the fact that every time the variable represented by the table function is invoked in the code the value of that variable will be interpolated or extrapolated from the known function values. TABLE, TABHL, and TABXT interpolate linearly between declared elements while TABPL performs cubic interpolation. TABHL upgrades TABLE by assigning the extreme point function values to the desired function if extrapolation is required. TABXT performs linear extrapolation at the desired value of the function. A last remark in the definition of the numerical values for TABPL is the fact that a zero must be added for each numerical value defined for the Y-variable.

The analyst must determine through logical relationships and statistical data analyses the values for constants and the multipliers which relate the rates to the levels. Once the model has been calibrated and validated, the DYNAMO model can be used for a variety of applications, including: forecasting and prediction, sensitivity analysis, and testing of various scenarios.

2.4 DYNAMO vs. FORTRAN

In developing a simulation model, the analyst needs to select a conceptual framework for describing the system to be modeled. The framework or perspective contains a "world view" within which the system functional relationships are perceived and described. If the modeler is employing a simulation language, the world views will normally be implicit in the language.

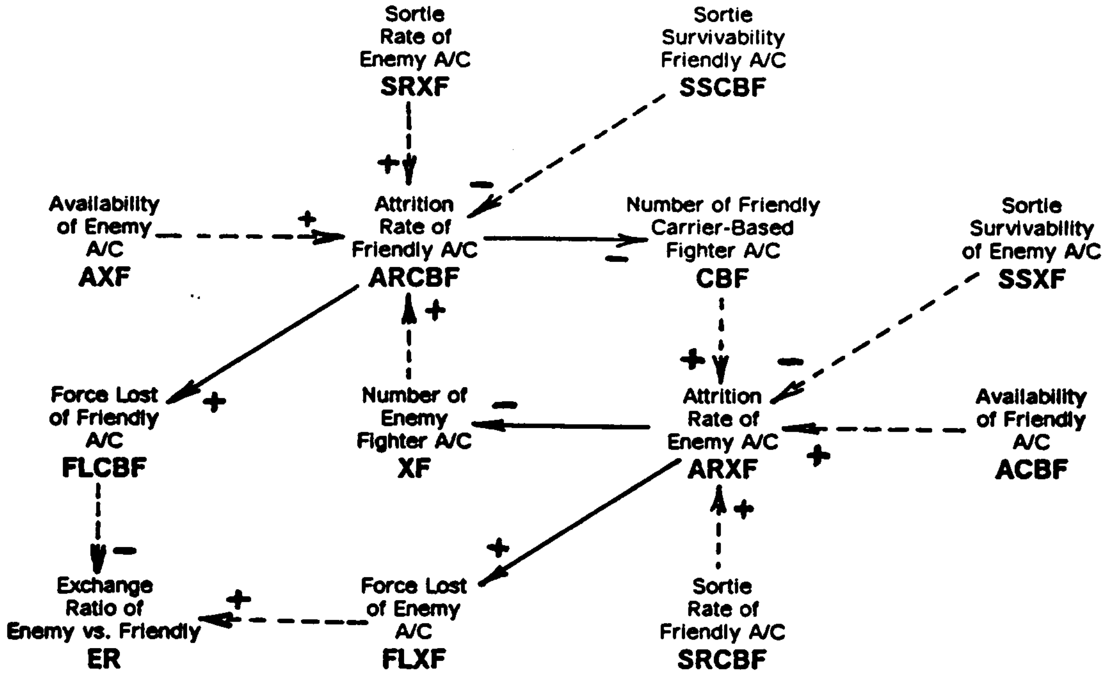
However, if the modeler elects to employ a general purpose language such as FORTRAN, then the perspective for organizing the system description is the responsibility of the modeler. In either case, the world view employed by the modeler provides a conceptual mechanism for articulating the system description.

DYNAMO utilizes a world view referred to as *process interaction* which is ideally suited to the types of problems undertaken in this research. In a process interaction world view, the modeler identifies the components (or subsystems) of a model and describes the sequence of actions of each component individually. The process interaction world view emphasizes defining system behavior by defining the action sequences of each component [6].

Fig. 2.3 contains a causal diagram for a basic air-to-air combat attrition relationship. As an example, the model equations are written both in DYNAMO and FORTRAN code (see Figs. 2.4-2.5, respectively).

Comparing the codes from Figs. 2.4-2.5, the total length including the comments is approximately the same. However, one important aspect of DYNAMO, notably the TABLE function stands out. By defining the attrition rates (ARCBF and ARXF) to be constant, the code lengths remain the same but we sacrifice the versatility to change these rates with respect to say, time. Using exactly the same length of DYNAMO code, a non-linear relationship between ARCBF and time (if, for example, weapons system effectiveness decreases with time and thus ARCBF and ARXF also would decrease) could be substituted into the TABLE functions defined for the dummy variables ARCBFT and ARXFT.

Figs. 2.6-2.7 show the listings for the DYNAMO and FORTRAN code, respectively. It may be of interest to note that the DYNAMO output is automatically formatted and can also provide useable plots.



EXAMPLE CAUSAL DIAGRAM FOR AIR-TO-AIR ATRITION MODEL

Figure 2.2 - Causal Diagram for Example 2.2

```
NOTE  CBF - FRIENDLY CARRIER-BASED FIGHTER AIRCRAFT
NOTE  CBFN - INITIAL NUMBER OF CBF
L     CBF.K=MAX(0,CBF.J+(DT)(-ARCBF.JK))
N     CBF=CBFN
C     CBFN=20
NOTE
NOTE  ARCBF - ATTRITION RATE OF CBF
NOTE  AXF - AVAILABILITY OF XF
NOTE  SRXF - SORTIE RATE OF XF
NOTE  SSCBF - SORTIE SURVIVABILITY OF CBF
NOTE
R     ARCBF.KL=AXF.K*SRXF*XF.K*(1-SSCBF)
A       AXF.K=TABLE(AXFT,TIME,0,25,5)
T         AXFT=0.85/0.85/0.85/0.85/0.85/0.85
C       SRXF=2
C       SSCBF=0.985
NOTE
NOTE  XF - ENEMY FIGHTER AIRCRAFT
NOTE  XFN - INITIAL NUMBER OF XF
NOTE
L     XF.K=MAX(0,XF.J+(DT)(-ARXF.JK))
N     XF=XFN
C     XFN=20
NOTE
NOTE  ARXF - ATTRITION RATE OF XF
NOTE  ACBF - AVAILABILITY OF CBF
NOTE  SRCBF - SORTIE RATE OF CBF
NOTE  SSXF - SORTIE SURVIVABILITY OF XF
NOTE
R     ARXF.KL=ACBF.K*SRCBF*CBF.K*(1-SSXF)
A       ACBF.K=TABLE(ACBFT,TIME,0,25,5)
T         ACBFT=0.90/0.90/0.90/0.90/0.90/0.90
C       SRCBF=2
C       SSXF=0.97
NOTE
NOTE  FLCBF - FORCE LOST OF CBF
NOTE  FLCBFN - INITIAL NUMBER OF LOST CBF (NOT = 0 FOR CALCULATIONS)
NOTE
L     FLCBF.K=FLCBF.J+(DT)(ARCBF.JK)
N     FLCBF=FLCBFN
C     FLCBFN=1E-6
NOTE
NOTE    FLXF - FORCE LOST OF XF
NOTE    FLXFN - INITIAL NUMBER OF LOST XF
NOTE
L     FLXF.K=FLXF.J+(DT)(ARXF.JK)
N     FLXF=FLXFN
C     FLXFN=0
NOTE
NOTE    ER - EXCHANGE RATIO OF XF VS. CBF
NOTE
S     ER.K=FLXF.K/FLCBF.K
NOTE
SPEC  DT=1/LENGTH=25/PRTPER=1/PLTPER=1
PRINT CBF,XF,FLCBF,FLXF,ER
PLOT  CBF,XF
PLOT  ER
RUN
QUIT
```

Figure 2.3 - DYNAMO Code for Example 2.2

```
$JOB
C
C    CBF - NUMBER OF CARRIER-BASED FIGHTERS (INITIALLY=20)
C    ACBF - AVAILABILITY OF CBF
C    SRCBF - SORTIE RATE OF CBF
C    SSCBF - SORTIE SURVIVABILITY OF CBF
C
     CBF=20
     ACBF=0.90
     SRCBF=2
     SSCBF=0.985
C
C    XF - NUMBER OF ENEMY FIGHTERS (INITIALLY=20)
C    AXF - AVAILABILITY OF XF
C    SRXF - SORTIE RATE OF XF
C    SSXF - SORTIE SURVIVABILITY OF XF
C
     XF=20
     AXF=0.85
     SRXF=2
     SSXF=0.97
C
C    FLCBF - FORCE LOST OF CBF (CLOSE TO 0 INITIALLY)
C    FLXF - FORCE LOST OF XF (INITIALLY 0)
C
     FLCBF=1E-6
     FLXF=0
C
C    IDT - TIME INCREMENT
     IDT=1
C
     WRITE(6,19)
19   FORMAT(//,2X,' TIME ',8X,'CBF',11X,'XF',9X,'FLCBF',9X,'FLXF',10X
    $,'ER')
C
     DO 10 ITIME=1,26,IDT
C    INCREMENT TIME BY IDT
     ARCBF=AXF*SRXF*XF*(1-SSCBF)
C    ARCBF - ATTRITION RATE OF CBF
     IF(ARCBF.LT.0)ARCBF=0
     IF(CBF.LE.0)ARCBF=0
C    ABOVE: NO ATTRITION IF ARCBF OR CBF ARE NEGATIVE
     ARXF=ACBF*SRCBF*CBF*(1-SSXF)
C    ATTRITION RATE OF XF
     IF(ARXF.LT.0)ARXF=0
     IF(XF.LE.0)ARXF=0
C    ABOVE: NO ATTRITION IF ARXF OR XF ARE NEGATIVE
     WRITE(6,20)ITIME,CBF,XF,FLCBF,FLXF,ER
20   FORMAT(I6,5F14.3)
     CBF=CBF-IDT*ARCBF
C    ABOVE: DECREASES CBF ACCORDING TO ARCBF
     XF=XF-IDT*ARXF
C    ABOVE: DECREASES XF ACCORDING TO ARXF
     FLCBF=FLCBF+IDT*ARCBF
C    ABOVE: INCREASES FLCBF ACCORDING TO ARCBF
     FLXF=FLXF+IDT*ARXF
C    ABOVE: INCREASES FLXF ACCORDING TO ARXF
     ER=FLXF/FLCBF
C    ER - EXCHANGE RATIO
10   CONTINUE
     STOP
     END
$ENTRY
```

Figure 2.4 - FORTRAN Code for Example 2.2

TIME=	.00	CBF=	20.00	XF=	20.00	FLCBF=	.000	FLXF=	.00	ER=	.000
TIME=	1.00	CBF=	19.49	XF=	18.92	FLCBF=	.510	FLXF=	1.08	ER=	2.118
TIME=	2.00	CBF=	19.01	XF=	17.87	FLCBF=	.992	FLXF=	2.13	ER=	2.149
TIME=	3.00	CBF=	18.55	XF=	16.84	FLCBF=	1.448	FLXF=	3.16	ER=	2.181
TIME=	4.00	CBF=	18.12	XF=	15.84	FLCBF=	1.878	FLXF=	4.16	ER=	2.216
TIME=	5.00	CBF=	17.72	XF=	14.86	FLCBF=	2.281	FLXF=	5.14	ER=	2.253
TIME=	6.00	CBF=	17.34	XF=	13.90	FLCBF=	2.660	FLXF=	6.10	ER=	2.291
TIME=	7.00	CBF=	16.99	XF=	12.97	FLCBF=	3.015	FLXF=	7.03	ER=	2.333
TIME=	8.00	CBF=	16.65	XF=	12.05	FLCBF=	3.346	FLXF=	7.95	ER=	2.376
TIME=	9.00	CBF=	16.35	XF=	11.15	FLCBF=	3.653	FLXF=	8.85	ER=	2.422
TIME=	10.00	CBF=	16.06	XF=	10.27	FLCBF=	3.937	FLXF=	9.73	ER=	2.472
TIME=	11.00	CBF=	15.80	XF=	9.40	FLCBF=	4.199	FLXF=	10.60	ER=	2.524
TIME=	12.00	CBF=	15.56	XF=	8.55	FLCBF=	4.439	FLXF=	11.45	ER=	2.580
TIME=	13.00	CBF=	15.34	XF=	7.71	FLCBF=	4.657	FLXF=	12.29	ER=	2.640
TIME=	14.00	CBF=	15.15	XF=	6.88	FLCBF=	4.853	FLXF=	13.12	ER=	2.704
TIME=	15.00	CBF=	14.97	XF=	6.06	FLCBF=	5.029	FLXF=	13.94	ER=	2.772
TIME=	16.00	CBF=	14.82	XF=	5.25	FLCBF=	5.183	FLXF=	14.75	ER=	2.845
TIME=	17.00	CBF=	14.68	XF=	4.45	FLCBF=	5.317	FLXF=	15.55	ER=	2.924
TIME=	18.00	CBF=	14.57	XF=	3.66	FLCBF=	5.431	FLXF=	16.34	ER=	3.009
TIME=	19.00	CBF=	14.48	XF=	2.87	FLCBF=	5.524	FLXF=	17.13	ER=	3.100
TIME=	20.00	CBF=	14.40	XF=	2.09	FLCBF=	5.597	FLXF=	17.91	ER=	3.200
TIME=	21.00	CBF=	14.35	XF=	1.31	FLCBF=	5.651	FLXF=	18.69	ER=	3.307
TIME=	22.00	CBF=	14.32	XF=	.54	FLCBF=	5.684	FLXF=	19.46	ER=	3.424
TIME=	23.00	CBF=	14.30	XF=	.00	FLCBF=	5.698	FLXF=	20.23	ER=	3.551
TIME=	24.00	CBF=	14.30	XF=	.00	FLCBF=	5.698	FLXF=	21.01	ER=	3.687
TIME=	25.00	CBF=	14.30	XF=	.00	FLCBF=	5.698	FLXF=	21.78	ER=	3.822

Figure 2.5 - DYNAMO Output for Example 2.2

TIME	CBF	XF	FLCBF	FLXF	ER
1	20.000	20.000	0.000	0.000	
2	19.490	18.920	0.510	1.080	2.118
3	19.008	17.868	0.992	2.132	2.149
4	18.552	16.841	1.448	3.159	2.181
5	18.122	15.839	1.878	4.161	2.216
6	17.719	14.861	2.281	5.139	2.253
7	17.340	13.904	2.660	6.096	2.291
8	16.985	12.968	3.015	7.032	2.333
9	16.654	12.050	3.346	7.950	2.376
10	16.347	11.151	3.653	8.849	2.422
11	16.063	10.268	3.937	9.732	2.472
12	15.801	9.401	4.199	10.599	2.524
13	15.561	8.548	4.439	11.452	2.580
14	15.343	7.707	4.657	12.293	2.640
15	15.147	6.879	4.853	13.121	2.704
16	14.971	6.061	5.029	13.939	2.772
17	14.817	5.252	5.183	14.747	2.845
18	14.683	4.452	5.317	15.548	2.924
19	14.569	3.660	5.431	16.340	3.009
20	14.476	2.873	5.524	17.127	3.100
21	14.403	2.091	5.597	17.909	3.200
22	14.349	1.313	5.651	18.687	3.307
23	14.316	0.539	5.684	19.461	3.424
24	14.302	-0.235	5.698	20.234	3.551
25	14.302	-0.235	5.698	20.234	3.551
26	14.302	-0.235	5.698	20.234	3.551

Figure 2.6 - FORTRAN Listing for Example 2.2

3.0 Mathematics of Survivability

Survivability can be considered to be the capability of a weapon system, such as a combat aircraft, to continue to carry out its required missions. Survivability is a function of both susceptibility, which includes a combination of factors to determine the probability of hit by a given threat, and vulnerability, which is the probability of kill of the system after having been hit by the threat propagators. This relationship can be seen symbolically by:

$$SURVIVABILITY = 1 - SUSCEPTABILITY \times VULNERABILITY \tag{3.1}$$

where all three parameters are expressed as probabilities.

The probability of survival is actually a summary measure; depending on the particular application it may be computed for various aspects of a sortie or a complete sortie. Two popular parameters include the probability of survival per sortie, $P_{S/S}$, and the probability of survival per encounter, P_{S/E_i} The probability that an aircraft will survive a single sortie during which it may have multiple engagements with the various weapons of a zone defense is given in [9][10] by

$$P_{S/S} = \prod_{i=1}^{N} e^{-N_i(D)2R_{eff_i}(1-P_{S/E_i})/A_i} \tag{3.2}$$

where

$P_{S/S}$ = probability of sortie survivability over i engagements with the zone defense weapons mixture

A_i = the area in which the weapon systems or firing units are expected to be randomly distributed

N_i = the number of the i^{th} type weapon system in area A

R_{eff_i} = the effective range of the i^{th} type weapon system

D = the distance the aircraft flies through area A without significantly changing altitude or airspeed

P_{S/E_i} = the probability of the aircraft surviving a single encounter with the i^{th} type weapon system.

All of the above parameters with the exception of P_{S/E_i} are environmentally oriented. The probability of survival per encounter can be shown to be

$$P_{S/E_i} = 1 - (P_{LOS})(P_D)(P_L)(P_G)(P_{DET})\left[1 - \prod_{i=1}^{n}(1 - P_{SSK})\right] \qquad \{3.3\}$$

where

P_{LOS} = probability of line-of-sight to the target

P_D = probability of detection, given line-of-sight

P_L = probability of launch or firing, given detection

P_G = probability of successful guidance, given launch or firing

P_{DET} = probability of warhead detonation (fuzed warheads) given successful guidance

n = number of shots fired during an encounter

P_{SSK} = single-shot kill probability

A number of essential functions must be performed in sequence by any type of threat system to be effective. First, a radar or visual line-of-sight to the target must exist. Next, the target aircraft must be detected, recognized, and identified as an enemy. Projectile or missile launch can then occur given that the target is within its range and angular-rate limitations. Successful guidance to the target is a function of the threat system type, the launched projectile or missile associated with the threat system (i.e. free flight projectile, radar or IR guided missiles, etc.), and the kinetic aspects of the target. Referring to Eq. 3.3, the five terms preceding the bracketed term represent the threat system effectiveness, TSE. Since it is the product of five probabilities, it is itself a probability, P_{TSE}.

Using the binomial distribution with parameters n (the number of shots fired) and P_{SSK} (the single shot kill probability) the following relation is derived:

$$P_{K/TSE}(x) = \sum_{x=1}^{n} \frac{n!}{x!(n-x)!} P_{SSK}^{x} (1 - P_{SSK})^{n-x} \tag{3.4}$$

where $P_{K/TSE}$ is the probability of being killed for a certain threat system effectiveness. This is one minus the probability of not being killed in n shots, or,

$$P_{K/TSE}(x) = 1 - P_{K/TSE}(0) = 1 - (1 - P_{SSK})^{n} \tag{3.5}$$

Repeating the derivation while assuming the event, a single shot kill, with probability P_{SSK}, is represented with a Poisson distribution, Eq. 3.4 becomes

$$P_{K/TSE}(x) = \sum_{x=1}^{\infty} e^{-m} \frac{m^{x}}{x!} \tag{3.6}$$

where $m = nP_{SSK}$. Instead of Eq. 3.5, one now obtains

$$P_{K/TSE}(x) = 1 - P_{K/TSE}(0) = 1 - \exp(-nP_{SSK}) \tag{3.7}$$

Substituting Eq. 3.10 into Eq. 3.5 gives

$$P_{S/E_i} = 1 - P_{TSE} + P_{TSE}e^{-nP_{SSK}} \tag{3.8}$$

Eq. 3.7 is an approximation of Eq. 3.5 just as the Poisson is an approximation of the binomial distribution. That is:

$$\prod_{i=1}^{n}(1 - P_{SSK}) = (1 - P_{SSK})^{n} = [(1 - P_{SSK})^{\frac{-1}{P_{SSK}}}]^{-nP_{SSK}} = e^{-nP_{SSK}} \tag{3.9}$$

In order to convert Eq. 3.11 to a form popularly found as

$P(survival) = 1 - P(detection)P(hit/detection)P(kill/hit)$

we can expand the last term into an exponential series, neglecting higher order terms –

$$P_{S/E_i} = 1 - P_{TSE}\ n\,P_{SSK} \tag{3.10}$$

where

$$P_{SSK} = P_{SSH}\,P_{K/H} \tag{3.11}$$

and

$$P_{S/E_i} = 1 - P_{TSE}\ n\,P_{SSH}\,P_{K/H} \tag{3.12}$$

Substitution of Eq. 3.11 into Eq. 3.2 gives the expression for sortie survivability as

$$P_{S/S} = \prod_{i=1}^{N} e^{(ZDF)P_{TSE}(n\,P_{SSH})P_{K/H}} \tag{3.13}$$

where $ZDF = N_iD(2R_{eff_i}/A_i)$ and is called the "zone defense factor" as implied by the parameter definitions given in Eq. 3.2.

The degree of sophistication employed in an analysis of defense system effectiveness depends on the intended use of the results and the confidence level placed on the inputs. Realistic results required the inclusion of many parameters, some of which, such as aircraft vulnerability areas, blast and fragmentation envelopes, weapon system accuracy, etc., are elusive. Consider the last term of Eq. 3.11: the probability of a random hit killing the aircraft is equal to the ratio of its vulnerable area A_V to its presented area A_P Substituting this into eq. 3.11 gives

$$P_{S/E_i} = 1 - P_{TSE}\, n\, P_{SSH} \frac{A_V}{A_P} \tag{3.14}$$

Now consider P_{SSH} in Eq. 3.11. the probability of hitting an aircraft in a single firing from a threat. This single shot probability determination can be approached in many ways including the following. Using the concept of CEP (the circular error probability of a group of n shots defining a circle within which half will hit) the probability of the aircraft being hit [11] will be

$$P(x) = \frac{m^x}{x!} e^{-m} = \frac{m^x}{x!}\left(1 - m + \frac{m^2}{2!} - \ldots\right) \tag{3.15}$$

where

$$m = \frac{A_P}{\pi (CEP)^2} \frac{1}{2} \tag{3.16}$$

Neglecting higher order terms in the exponential expansion,

$$P_{SSH} = \frac{A_P}{2\pi (CEP)^2} \tag{3.17}$$

and after substituting in Eq. 3.11

$$P_{S/E_i} = 1 - P_{TSE} \frac{n A_V}{2\pi (CEP)^2} \tag{3.18}$$

It is interesting to note that the development in Eqs. 3.3 to 3.18 define the look-shoot, look-shoot case. For the look-shoot-shoot-shoot case the Bernoulli form is

$$P(x) = \sum_{x=1}^{n} \frac{n!}{x!(n-x)!} (P_{TSE}P_{SSK})^{x}(1 - P_{TSE}P_{SSK})^{n-x} \tag{3.19}$$

giving a probability of survival per encounter of

$$P_{S/E_i} = (1 - P_{TSE}P_{SSK})^{n}(1 - P_{TSE}P_{SSK})^{n-x} \tag{3.20}$$

These cases are defined in the following section.

3.1 Targeting and Firing Cases

Analyzing survivability in a multiple vehicle engagement sortie (referred to as an M on N encounter) requires a number of assumptions for assigning the relationship between single-shot-kill and encounter survivability. Two possible targeting cases studied here are Coordinated Targeting and Random Targeting. The firing cases are Look-Shoot-Shoot-Shoot and Look-Shoot-Look-Shoot.

Coordinated Targeting implies that a given sortie force of say, M aircraft, would each select one of N aircraft with no overlapping in the selection (unless of course N < M, in which case the remainder of the M force would be assigned one by one to the N force). Random Targeting, on the other hand, implies that each aircraft of the M force is not aware of the selections taken by the rest of the M force, and "ganging up" may occur on some of the N force while others remain untargeted.

The Look-Shoot-Shoot-Shoot firing case assumes that once a target is acquired, the pilot fires a number of propagators at the target. If the propagators are identical in performance, they will each give identical probabilities of single-shot-kill. However, the pilot can alternatively affect his enemy's survival probability by using the Look-Shoot-Look-Shoot approach in which he waits to see the effect of each firing before spending more propagators.

The following terms will be used in the formulations of the following sections:

- $P_{S/E}$ = probability of survival given an encounter
- $\$_0$ = sortie force level of friendly forces
- X_0 = sortie force level of enemy forces
- P_{K,R_X} = probability of being killed by enemy radar guided missiles
- P_{K,I_X} = probability of being killed by enemy infrared guided missiles
- P_{XPE} = probability of enemy platform effectiveness
- P_{XRE} = probability of enemy radar guided missile effectiveness
- $P_{K/XRE}$ = probability of being killed, given radar guided missile effectiveness
- P_{XIE} = probability of enemy infrared guided missile effectiveness
- $P_{K/XIE}$ = probability of being killed, given infrared guided missile effectiveness
- N_{R_X} = number of radar guided missiles carried/fired by enemy aircraft
- N_{I_X} = number of infrared guided missiles carried/fired by enemy aircraft
- n_{R_X} = relative number of radar guided missiles carried/fired by enemy aircraft
- n_{I_X} = relative number of infrared guided missiles carried/fired by enemy aircraft
- P_{SSK,R_X} = probability of single shot kill by an enemy radar guided missile
- P_{SSK,I_X} = probability of single shot kill by an enemy infrared guided missile

3.1.1 Coordinated Targeting (Look-Shoot-Shoot-Shoot)

$$P_{S/E} = (1 - P_{K,R_X})(1 - P_{K,I_X}) \qquad \{3.21\}$$

where

$$P_{K,R_X} = P_{XPE} P_{XRE} P_{K|XRE} \tag{3.22}$$

and

$$P_{K,I_X} = P_{XPE} P_{XIE} P_{K|XIE} \tag{3.23}$$

$$P_{K|XRE} = \sum_{i=1}^{n_{R_X}} \frac{n!}{i!(n-i)!} P^i_{SSK,R_X} (1 - P_{SSK,R_X})^{n-i} = 1 - (1 - P_{SSK,R_X})^{n_{R_X}} \tag{3.24}$$

$$P_{K|IRE} = 1 - (1 - P_{SSK,I_X})^{n_{I_X}} \tag{3.25}$$

$$P_{S/E} = \{1 - P_{XPE} P_{XRE}[1 - (1 - P_{SSK,R_X})^{n_{R_X}}]\}\{1 - P_{XPE} P_{XIE}[1 - (1 - P_{SSK,I_X})^{n_{I_X}}]\} \tag{3.26}$$

Since $n_{R_X} = N_{R_X} X_0/\$_0$

$$P_{S/E} = \{1 - P_{XPE} P_{XRE}[1 - (1 - P_{SSK,R_X})^{N_{R_X} X_0/\$_0}]\} \times$$

$$\{1 - P_{XPE} P_{XIE}[1 - (1 - P_{SSK,I_X})^{N_{I_X} X_0/\$_0}]\} \tag{3.27}$$

3.1.2 Coordinated Targeting (Look-Shoot; Look-Shoot)

$$P_{S/E} = (1 - P_{K,R_X})(1 - P_{K,I_X}) \tag{3.28}$$

where

$$P_{K,R_X} = P_{XPE} P_{XRE} P_{K|XRE} \tag{3.29}$$

and

$$P_{K,I_X} = P_{XPE} P_{XIE} P_{K|XIE} \tag{3.30}$$

$$P_{XRE}\,P_{K/XRE} = \sum_{i=1}^{n_{R_X}} \frac{n!}{i!(n-i)!}(P_{XRE}\,P_{SSK,R_X})^i(1 - P_{XRE}\,P_{SSK,R_X})^{n-i}$$

$$= 1 - (1 - P_{XRE}\,P_{SSK,R_X})^{n_{R_X}} \qquad \{3.31\}$$

$$P_{XIE}\,P_{K/XIE} = 1 - (1 - P_{XIE}\,P_{SSK,I_X})^{n_{I_X}} \qquad \{3.32\}$$

$$P_{S/E} = \{1 - P_{XPE}[1 - (1 - P_{XRE}\,P_{SSK,R_X})^{N_{R_X}X_0/\$_0}]\} \times$$

$$\{1 - P_{XPE}[1 - (1 - P_{XIE}\,P_{SSK,I_X})^{N_{I_X}X_0/\$_0}]\} \qquad \{3.33\}$$

3.1.3 Random Targeting (Look-Shoot-Shoot-Shoot)

$$P_{S/E} = (1 - P_{K,R_X})(1 - P_{K,I_X}) \qquad \{3.34\}$$

where

$$P_{K,R_X} = P_{XPE}\,P_{XRE}\,P_{K/XRE} \qquad \{3.35\}$$

and

$$P_{K,I_X} = P_{XPE}\,P_{XIE}\,P_{K/XIE} \qquad \{3.36\}$$

$$P_{K/XRE} = \sum_{i=1}^{\infty} \frac{m^i}{i!} e^{-m} = 1 - e^{-m} \qquad \{3.37\}$$

where

$$m = P_{SSK,R_X}\,N_{R_X}\,X_0/\$_0 \qquad \{3.38\}$$

$$P_{K/XIE} = 1 - e^{-P_{SSK,R_X} N_{R_X} X_0/\$_0} \qquad \{3.39\}$$

$$P_{S/E} = \{1 - P_{XPE} P_{XRE}(1 - e^{-P_{SSK,R_X} N_{R_X} X_0/\$_0})\}\{1 - P_{XPE} P_{XIE}(1 - e^{-P_{SSK,I_X} N_{I_X} X_0/\$_0})\} \qquad \{3.40\}$$

3.1.4 Random Targeting (Look Shoot; Look Shoot)

$$P_{S/E} = (1 - P_{K,R_X})(1 - P_{K,I_X}) \qquad \{3.41\}$$

where

$$P_{K,R_X} = P_{XPE} P_{XRE} P_{K/XRE} \qquad \{3.42\}$$

and

$$P_{K,I_X} = P_{XPE} P_{XIE} P_{K/XIE} \qquad \{3.43\}$$

$$P_{K/XRE} = \sum_{i=1}^{\infty} \frac{m^i}{i!} e^{-m} = 1 - e^{-m} \qquad \{3.44\}$$

where

$$m = P_{XRE} P_{SSK,R_X} N_{R_X} X_0/\$_0 \qquad \{3.45\}$$

$$P_{S/E} = \{1 - P_{XPE}(1 - e^{P_{XRE} P_{SSK,R_X} N_{R_X} X_0/\$_0})\}\{1 - P_{XPE}(1 - e^{P_{XRE} P_{SSK,I_X} N_{I_X} X_0/\$_0})\} \qquad \{3.46\}$$

3.2 Cost vs. Effectiveness

The mathematical treatment of cost/effectiveness tradeoffs begins by developing the expression for the cumulative (total) number of sorties (TS) flown by an aircraft wih survivability P_S after n scheduled sorties, which is given by the following geometric series:

$$TS = 1 + P_S + P_S^2 + P_S^3 + \ldots + P_S^{n-1} \qquad \{3.47\}$$

Multiplying both sides by P_S gives

$$P_S\,TS = P_S + P_S^2 + P_S^3 + \ldots + P_S^{n-1} + P_S^n \qquad \{3.48\}$$

Subtracting Eq. 3.48 from Eq. 3.47, and solving for TS, one obtains

$$TS = \frac{1 - P_S^n}{1 - P_S} \qquad \{3.49\}$$

Taking the limit as n approaches infinity, the expected number of sorties scheduled by an aircraft in its lifetime is

$$E(n) = \frac{1}{1 - P_S} \qquad \{3.50\}$$

Frequently, measures of effectiveness are expressed in terms of an exchange ratio, ER. For this approach, the exchange ratio can be expressed as:

$$ER = E(n)\,TDPS \qquad \{3.51\}$$

where TDPS is the number of targets destroyed per sortie.

The analysis developed in this section forms the theoretical basis for two additional measures of effectiveness. These MOE's are cost for equal effectiveness (CEE) and effectiveness obtained for equal cost (EEC).

3.2.1 Cost for Equal Effectiveness

If we desire to determine the force size of aircraft with two different configurations, say, by adding an additional ECM (Electronic Counter Measures) pod at the expense of an air-to-air missile, then to maintain equal effectiveness we merely equate the total sorties flown and returning by a force of N aircraft in regard to Eq. 3.50:

$$\frac{N_B P_{S,B}}{P_{K,B}} = \frac{N_M P_{S,M}}{P_{K,M}} \tag{3.52}$$

where the subscripts B and M identify the baseline and modified configuration, respectively. Eq. 3.52 can then be used to determine the number of sorties required by the new configuration to yield the same effectiveness as the old configuration. Solving for N_M and substituting Eq. 3.50:

$$N_M = \frac{N_B P_{K,M} P_{S,B}}{P_{K,B} P_{S,M}} = N_B E(n)_B \frac{P_{k,M}}{P_{S,M}} P_{S,B} \tag{3.53}$$

3.2.2 Effectiveness for Equal Cost

Effectiveness for Equal Cost, as a measure of effectiveness, determines the number of sorties that can be flown for an equal total life cycle cost, C_T. Life cycle cost for the baseline configuration is determined and is used as the equal cost level for the analysis. In order to determine the unit cost, C_U for a given modified configuration, the life cycle operating cost, new equipment costs, and equipment operating costs are obtained. From these inputs, the number of aircraft, N_M which can be modified and operated for a given total cost, C_T is given by:

$$N_M = \frac{C_T}{C_U} \tag{3.54}$$

3.3 Time Dependency

There are two significant shortcomings of the time invariant approaches described in the previous two sections. First, knowing how fast that a given number of targets can be destroyed is as important as knowing the ultimate number to be destroyed. Second, attrition and the threat force size responsible for this attrition are not constant, but change over time. Time can be brought into the picture as follows:

$$n = SR\,t \tag{3.55}$$

where n is the number of sorties flown, SR is the sortie rate, and t is the analysis time period. The sortie rate, a complex function involving maintenance considerations, repair time, reliability, crew ratio, etc., is assumed to be constant. Using Eq. 3.55, the total number of sorties TS flown by an aircraft with survivability P_S and sortie rate SR in t days is:

$$TS = \frac{1 - P_S^{SR\,t}}{1 - P_S} \tag{3.56}$$

The total number of sorties flown by a force size of N aircraft, TS_N is obtained by multiplying Eq. 3.56 by N:

$$TS_N = N\frac{1 - P_S^{SR\,t}}{1 - P_S} \tag{3.57}$$

The total number of targets destroyed by a force of N aircraft, TTD_N, is obtained by multiplying Eq. 3.57 by the target kill potential TDPS (targets destroyed per sortie):

$$TTD_N = N\frac{1-P_S^{SRt}}{1-P_S}TDPS \qquad \{3.58\}$$

The above relations are useful in calculating several MOE's outlined in Chapter 5.

The fraction of force remaining, FFR, and fraction of force lost, FFL, can be calculated by:

$$FFR = P_S^{SRt} \qquad \{3.59\}$$

$$FFL = 1 - P_S^{SRt} \qquad \{3.60\}$$

3.4 Co-Kill Probability

Consider an aerial combat engagement integrated forwards in time to the first firing opportunity at $t = t_1$. At this moment let vehicle i fire at vehicle j with single shot kill probability $P_{K_{ij}}(t1)$ Vehicle j has a survival probability

$$P_{S_j}(\tau_1) = P_{S_j}(0) - P_{K_{ij}}(t_1) \qquad \{3.61\}$$

where τ_1 is the time at which weapon impact occurs on vehicle j and it is assumed that an active weapon requiring no assistance following firing is employed. Now suppose that at some later time, t_2, vehicle j obtains a firing opportunity on vehicle k and that its single shot kill probability is $P_{K_{jk}}(t_2)$; then the co-kill probability of this shot, accounting for the previous shot at t_1, is

$$P_{K_k}(\tau_2) = P_{S_j}(t_2)P_{K_{jk}}(t_2) \qquad \{3.62\}$$

where τ_2 is the impact time of the shot fired at t_2 Then suppose at time t_3 that vehicle k returns fire at vehicle j, then the co-kill probability of this shot is

$$P_{K_j}(\tau_3) = P_{S_k}(t_3)P_{K_{kj}}(t_3) \tag{3.63}$$

To generalize the above equations, suppose vehicle m fires at vehicle n at time t_i and that vehicles m and n have survival probabilities $P_{S_m}(t_i)$ and $P_{S_n}(\tau_i)$. The co-kill probability of this shot is

$$P_{K_n}(\tau_i) = P_{S_m}(t_i)P_{S_n}(\tau_i)P_{K_{mn}}(t_i) \tag{3.64}$$

and the survival probability of vehicle n is

$$P^{+}_{S_n}(t_i + \Delta t_i) = P_{S_n}(t_i) - P_{S_m}(t_i)P^{-}_{S_n}(t_i + \Delta t_i)P_{K_{mn}}(t_i) \tag{3.65}$$

where $\tau_i = t_i + \Delta t_i$, and $-/+$ are before/after impact. Eqs. 3.64 and 3.65 are the M on N co-kill probability equations. These equations are integrated forward in time subject to the initial condition

$$P_{S_m}(0) = P_{S_n}(0) = 1 \tag{3.66}$$

Eq. 3.64 has a straightforward physical interpretation; aircraft which have been destroyed cannot be destroyed again (P_{S_n}, and aircraft which have been destroyed cannot fire a weapon (P_{S_m}).

At the end of a sortie, force survival probabilities can be found by

$$\bar{P}_{S_\$}(T_F) = \frac{1}{M}\sum_{l=1}^{M} P_{S_l}(T_F) \tag{3.67}$$

and

$$\bar{P}_{S_X}(T_F) = \frac{1}{N}\sum_{l=M+1}^{M+N} P_{S_l}(T_F) \tag{3.68}$$

where T_F is the sortie termination time

It may be interesting to note that a draw occurs if

$$\bar{P}_{S_\$}(T_F) = \bar{P}_{S_X}(T_F) \quad \{3.69\}$$

The number of aircraft surviving on each side are:

$$N_\$ = M\,\bar{P}_{(S_\$)}(T_F)$$

$$N_X = N\,\bar{P}_{(S_X)}(T_F) \quad \{3.70\}$$

The exchange ratio can be expressed as:

$$ER = \frac{M - N_\$}{N - N_X} \quad \{3.71\}$$

4.0 Mission-Role Tradeoff Models

The Co-Kill approach to modeling survivability and lethality tradeoffs has several favorable aspects. For any given sortie, the fraction of surviving aircraft for both friendly and enemy forces can be determined using the mathematical approach described in Chapter 3. In other words, if, say, on a fighter escort mission, four friendly aircraft encounter six of the enemy's, then the Co-Kill probabilities can directly be determined. These probabilities may assert that, for example, 25 percent of the friendly forces are destroyed while 35 percent of the enemy aircraft are destroyed in this sortie (due to the greater effectiveness of the friendly aircraft despite the numerical superiority of the enemy).

The mission trade-off models described in this chapter use Co-Kill probabilities calculated through the mission scenario models of the companion research. The Co-Kill probabilities were determined for each mission type (surface attack, fighter escort, or carrier defense) along with selected combinations of friendly vs. enemy force sizes. These probabilities are reported in Tables 4.1-4.14 as "Kill Fractions" -- the fraction of the sortie force which is destroyed.

Table 4.1 - Kill Fractions: $SA by XSA

Enemy Force Size	Friendly Sortie Force Size 4	6	8
12	0.13	0.12	0.11
10	0.10	0.09	0.08
8	0.07	0.06	0.05

Table 4.2 - Kill Fractions: $SA by XFE

Enemy Force Size	Friendly Sortie Force Size 4	6	8
7	0.20	0.19	0.18
6	0.17	0.16	0.15
5	0.14	0.13	0.12

Table 4.3 - Kill Fractions: $FE by XSA

Enemy Force Size	Friendly Sortie Force Size 3	4	5
12	0.15	0.14	0.13
9	0.12	0.11	0.10
6	0.09	0.08	0.07

Table 4.4 - Kill Fractions: $FE by XCD

Enemy Force Size	Friendly Sortie Force Size 3	4	5
12	0.26	0.24	0.22
9	0.20	0.18	0.16
6	0.14	0.12	0.10

Table 4.5 - Kill Fractions: $CD by XFE

Enemy Force Size	Friendly Sortie Force Size 4	6	8
7	0.30	0.28	0.26
6	0.24	0.22	0.20
5	0.18	0.16	0.14

Table 4.6 - Kill Fractions: $CD by XCD

Enemy Force Size	Friendly Sortie Force Size 4	6	8
12	0.15	0.14	0.13
9	0.12	0.11	0.10
6	0.09	0.08	0.07

Table 4.7 - Kill Fractions: $FE by XFE

Enemy Force Size	2	Friendly Sortie Force Size 3	4	5
7	0.25	0.23	0.21	0.19
6	0.20	0.18	0.16	0.14
5	0.15	0.13	0.11	0.09
4	0.10	0.08	0.06	0.04

Table 4.8 - Kill Fractions: XFE by $FE

Friendly Force Size	Enemy Sortie Force Size 4	5	6	7
5	0.80	0.78	0.76	0.74
4	0.75	0.73	0.71	0.69
3	0.70	0.68	0.66	0.64
2	0.65	0.63	0.61	0.59

Table 4.9 - Kill Fractions: XSA by $SA

Friendly Force Size	Enemy Sortie Force Size 6	9	12
8	0.93	0.91	0.89
6	0.87	0.85	0.83
4	0.81	0.79	0.77

Table 4.10 - Kill Fractions: XSA by $FE

Friendly Force Size	Enemy Sortie Force Size 6	9	12
5	0.20	0.19	0.18
4	0.17	0.16	0.15
3	0.14	0.13	0.12

Table 4.11 - Kill Fractions: XFE by $SA

Friendly Force Size	Enemy Sortie Force Size 5	6	7
8	0.55	0.53	0.51
6	0.50	0.48	0.46
4	0.44	0.42	0.40

Table 4.12 - Kill Fractions: XFE by $CD

Friendly Force Size	Enemy Sortie Force Size 5	6	7
8	0.55	0.53	0.51
6	0.49	0.47	0.45
4	0.43	0.41	0.39

Table 4.13 - Kill Fractions: XCD by $FE

Friendly Force Size	Enemy Sortie Force Size 6	9	12
5	0.60	0.57	0.54
4	0.51	0.48	0.45
3	0.42	0.39	0.36

Table 4.14 - Kill Fractions: XCD by $CD

Friendly Force Size	Enemy Sortie Force Size 6	9	12
8	0.70	0.68	0.66
6	0.64	0.62	0.60
4	0.58	0.56	0.54

One condition difficult to predict is the enemy sortie force strength. Likewise, availability and sortie rate considerations make it difficult to provide friendly sortie force sizes as large as may be desired. Therefore, some mix of sortie strengths of both friendly and enemy forces will occur throughout the air-combat campaign. This implies that the encounters are governed by stochastic processes. For these models it is assumed that the sortie size of friendly forces is uncorrelated with that of the enemy forces. Also, the events referred to where a given mission type "sortie exists" will be modelled stochastically, since these are the only occasions where engagements ensue, and are, of course, not scheduled.

For a first approximation, the sortie strengths and sortie event times will be modelled with a uniform distribution, assigning each state (e.g., sortie force size) to a portion of the distribution. The details of this development are given in the next section.

The models therefore are hybrids. By using both the Co-Kill and stochastic approaches, the analysis can be performed both efficiently and realistically. If stochasticity were used in place of the Co-Kill probabilities for the survivability/lethality analysis of the combat engagements (e.g., assigning 1 or 0 for each aircraft surviving or killed, respectively), then a large number of simulation runs (probably at least 1000 since survivability is often expressed to four decimal places) would be required, each using a different seed for the random number generator. Using Co-Kill probabilities, analysis can be performed in a single simulation run, with results converging over a sufficient amount of simulation time.

4.1 Baseline Carrier Operations Model

The Carrier Operations Model (COM) is used to analyze the tradeoffs of different mission scenarios by attriting the various specialized aircraft types. For the purposes of analysis, three mission types have been specified:

1. Surface Attack
2. Fighter Escort
3. Carrier Defense

Each of the above roles require aircraft or installations, both friendly and enemy, to perform and interact. Of course, on a fighter escort mission, for example, many times no encounters are made; for the purpose of this model, these sorties will not be included in the survivability analysis.

It follows that there should be at least six level variables to be included in the model. These variables are:

- CB$SA - Carrier Based aircraft for Surface Attack
- CB$FE - Carrier Based aircraft for Fighter Escort
- CB$CD - Carrier Based aircraft for Carrier Defense
- AGXCD - Aggressor aircraft against Carrier Defense
- AGXFE - Aggressor aircraft for Fighter Escort
- AGXSA - Aggressor units against Surface Attack

It should be noted that these variables are written with either a "$" or "X" to precede the mission role. This proves useful in identifying to which participant that the aircraft belong, as will be shown when writing succeeding variables.

As an aside, the rationale for selecting $ to represent the friendly forces is somewhat patriocentric. Historically, the dollar symbol was originally drawn by overlaying a capital U (as in *United*) on a capital S (as in *States*) in such a manner:

By eliminating the bottom curve of the U the dollar symbol is often drawn as an S with two vertical lines, while the more modern version seen here makes use of only one: $.

Of course, it begs the question to assign an american symbol to the *friendly* forces; if the reader would feel more comfortable, he or she should feel free to let $ represent any of the NATO countries or other U.S. ally. The capital X is not meant to represent any particular enemy. It is often meant to represent an unknown, and therefore mysterious, foe; in this case, an eXogenous force.

4.1.1 Model Code

The following subsections provide examples of the mechanisms by which the model is processed. A complete listing of the model code is presented in Appendix B.

4.1.1.1 Level Variables by Aircraft Type

As mentioned above, the model contains six level variables. As an example, this section will be devoted to describing the mechanisms in determining CB$SA. The level equation has the form

```
L     CB$SA.K=MAX(0,CB$SA.J-(DT)(A$SAMSA.JK+A$SADOD.JK)
```

where,

A$SAMSA - Attrition of friendly surface attack aircraft on a surface attack mission

A$SADOD - Attrition of friendly surface attack aircraft which are "dead on deck"

The above equation uses the MAX function which prevents CB$SA from becoming negative. Notice the timescripts .K, .J, and .JK which are explained in Chapter 2.

The attrition rate, A$SAMSA, is calculated by adding the component attritions that the friendly surface attack aircraft experiences on a surface attack mission. In DYNAMO notation:

```
R     A$SAMSA.KL=A$SAXSA.K+A$SAXFE.K
```

Where A$SAXSA is the attrition of the friendly surface attack aircraft attributed to enemy surface attack aircraft and A$SAXFE is the attrition of the friendly surface attack aircraft attributed to the enemy fighters. These component attritions are calculated by:

A A$SAXSA.K = P$SAMSA.K*K$SAMSA.K*$SA.K

A A$SAXFE.K = (1-P$SAMSA.K)*K$SAMSA.K*$SA.K

Where K$SAMSA is the overall kill fraction of friendly surface attack aircraft on a mission, P$SAMSA is the proportion of the overall attrition assigned to the primary threat (i.e., the defensive weapons sites) and $SA is the number of friendly surface attack aircraft performing the mission and is calculated by equations given in a following section.

K$SAMSA is actually a composite of two other kill fractions. An attack aircraft may be attrited by both the enemy units against surface attack, XSA (i.e., the defensive weapons sites), as well as enemy fighters encountered during the surface attack mission, XFESA. This kill fraction takes on the form:

$$K_{\$1m} = \frac{1-(1-K_{\$1/X1})(1-K_{\$1/X2})}{1+\left[\dfrac{K_{X1/\$2}(1-K_{\$2/X1})+K_{X2/\$2}(1-K_{\$2/X2})}{K_{X1/\$2}+K_{X2/\$2}}\right]} \qquad \{4.1\}$$

Where $K_{\$1m}$ is the kill fraction of $1 aircraft (accompanied on the sortie by $2) through attrition by both X1 (the primary threat) and X2 (the primary threat for $2) aircraft. $K_{M/N}$ are the kill fractions of sortie force *M* by sortie force *N*, respectively, which can be found using the ASALT models [1][2]. For this example, $1 and $2 are the friendly sortie forces of surface attack and fighter escort aircraft, respectively, and X1 and X2 are the enemy forces of defensive weapon sites and fighters engaged in the sortie.

In DYNAMO notation, dummy variables are used to enhance readability and facilitate processing, as shown in the following code.

```
A     K$SAMSA.K=(1-(1-K$SAXSA.K)*(1-K$SAXFE.K))/B$SAMSA.K
A       B$SAMSA.K=(1+KXSA$FE.K*(1-K$FEXSA.K)+KXFEUFE.K*(1-K$FEYFE.K))
X        /D$SAMSA.K
A         D$SAMSA.K=KXSA$FE.K+KXFEUFE.K+1E-6
```

Where,

- K$SAXSA - Kill fraction of friendly surface attack aircraft by enemy units against surface attack
- K$SAXFE - Kill fraction of friendly surface attack aircraft by enemy fighter escort aircraft
- KXSA$FE - Kill fraction of enemy surface attack units by friendly fighter escort aircraft
- KXFEUFE - Kill fraction of enemy fighter escort aircraft by friendly fighter escort aircraft
- K$FEXSA - Kill fraction of friendly fighter escort aircraft by enemy surface attack units
- K$FEYFE - Kill fraction of friendly fighter escort aircraft by enemy fighters
- B$SAMSA & D$SAMSA - Dummy variables for intermediate calculations

The proportion of the overall sortie attrition assigned to the principal threat (e.g., P$SAMSA) has the form:

$$P_{\$1m} = \frac{K_{\$1/X1}}{K_{\$1/X1} + K_{\$1/X2}} + \left(1 - \frac{K_{\$1/X1}}{K_{\$1/X1} + K_{\$1/X2}}\right) \frac{K_{X1/\$2}}{K_{X1/\$2} + K_{X2/\$2}} \qquad \{4.2\}$$

Where $P_{\$1m}$ is the proportion of the sortie attrition of $1 assigned to the primary threat (i.e., X1). The DYNAMO code to calculate the proportion for this example (i.e., P$SAMSA) is as follows:

```
A  P$SAMSA.K=C$SAMSA.K+(1-C$SAMSA.K)*K$SAXFE.K/(K$SAXFE.K+K$FEYFE.K+1E-6)
A   C$SAMSA.K=K$SAXSA.K/(K$SAXSA.K+K$SAXFE.K+1E-6)
```

Where C$SAMSA is a dummy variable for intermediate calculations. The above kill fractions are calculated by the Co-Kill approach, and their processing is given in a following subsection.

The second attrition rate, A$SADOD (Attrition of friendly Surface Attack aircraft "Dead On Deck") is formulated by the following:

R A$SADOD.KL=FDOD.K*(CB$SA.K-$SA.K)

where FDOD is the fraction of aircraft "dead on deck." The above equation demonstrates that aircraft already flying a sortie ($SA) cannot be attrited by this mechanism. FDOD is calculated by:

A FDOD.K=RK$*K$CDMCD.K

where RK$ is a model parameter defining a "Relative Kill" of the aircraft currently parked on the carrier, and K$CDMCD is the kill fraction of friendly aircraft for carrier defense on a carrier defense mission.

4.1.1.2 Sortie Force Sizes

The number of aircraft sent to perform a given mission as well as the number of enemy encountered is a function of availability, launch rate, as well as a number of other factors. For the purposes of this analysis, the sortie force size of friendly attack aircraft ($SA) is assumed to take on the discrete values of 4, 6, or 8 according to the following code:

A $SA.K=CLIP($SA$.K,0,SASEX.K,1)

A SA.K=CLIP(4,$SA$$.K,0.30,RN4.K)

A $SA$$.K=CLIP(6,8,0.70,RN4.K) Where,

- SASEX - Is 1 if a "Surface Attack Sortie EXists," assigning SA to $SA, otherwise, $SA = 0
- SA - Intermediate Variable, assigns $SA = 4 with a probability of 30%, otherwise, $SA = $SA$$
- $SA$$ - Intermediate Variable, assigns $SA = 6 with a probability of 70-30 = 40%, otherwise, $SA = 8
- RN4 - Number randomly generated between zero and one.

4.1.1.3 Kill Fractions used for Attrition

As an example, the code used to determine the variable K$SAXSA (Kill fraction of friendly Surface Attack aircraft by enemy units against Surface Attack) will be presented here. K$SAXSA is calculated by:

```
A    K$SAXSA.K=CLIP(CLIP(K$1A.K,K$$1.K,XSA.K,12),0,SASEX.K,1)
A     K$$1.K=CLIP(K$1B.K,K$1C.K,XSA.K,9)
A     K$$1A.K=TABHL(K$1AT,$SA.K,4,8,2)
A     K$$1B.K=TABHL(K$1BT,$SA.K,4,8,2)
A     K$$1C.K=TABHL(K$1CT,$SA.K,4,8,2)
T      K$1AT=0.13/0.12/0.11
T      K$1BT=0.10/0.09/0.08
T      K$1CT=0.07/0.06/0.05
```

Where,

- K$$1 - Is used when XSA < 12 to assign the kill fraction to K$1B or K$1C
- XSA - The number of enemy surface attack targets for a given sortie

- SASEX - "Surface Attack Sortie EXists," when = 1, kill fractions are calculated (see next section)
- K$1A - Used when XSA > 12 (for this model when XSA = 12)
- K$1B - Used when XSA = 9
- K$1C - Used when XSA < 9 (here, when XSA = 6)
- K$1AT - Kill fractions of four $SA when XSA = 12, 9, and 6
- K$1BT - Kill fractions of six $SA when XSA = 12, 9, and 6
- K$1CT - Kill fractions of eight $SA when XSA = 12, 9, and 6

4.1.1.4 Sortie Scheduling Mechanism

The occurance of combat sorties where there is engagement cannot be feasibly predicted. In order to model these occurances, a random distribution is assumed. For the example of scheduling surface attack missions, a value of 1 is assigned to SASEX (Surface Attack Sortie EXists) when a random number exceeds a certain value as shown in the following code:

```
A    SASEX.K=CLIP(1,0,RN1.K,0.80)
```

Where RN1.k is a number, randomly generated, taking on values between zero and one. In this example, surface attack would occur with probability of 20% in any given time period, however, the interactions of the model would not allow more than one engagement surface attack sortie to occur in the same time period.

4.1.1.5 *Measures of Effectiveness*

Using the simulation model, it is possible to calculate the levels of the various combat systems. For an analysis purpose, these numbers are not enough to ascertain the effectiveness of the overall system or even the long-range effectiveness of the individual systems. One of the measures of effectiveness which gives a good description of a particular system's effectiveness is the Relative Exchange Ratio as introduced in Chapter 2. A more thorough derivation is given in Chapter 5; the following is the form closely followed in the model.

$$RER = \left[1 - \sqrt{1 - \frac{1 - P_{S/S}}{P_{K/S}}(X_0/\$_0)^2}\,\right]^{-1} \tag{4.3}$$

Where $P_{S/S}$ is the sortie survivability, $P_{K/S}$ is the sortie lethality (both with respect to the friendly forces), and X_0 and $\$_0$ are the initial inventory levels of enemy and friendly forces, respectively. If we introduce an intermediate variable,

$$T_{\$/X} = \frac{1 - P_{S/S}}{P_{K/S}}(X_0/\$_0) \tag{4.4}$$

then Eq. 4.3 can be written as

$$RER = [1 - \sqrt{1 - T_{\$/X}}\,]^{-1} \tag{4.5}$$

The expression shown in Eqs. 4.3 and 4.5 was formulated on the assumption that the friendly forces are victorious (i.e., $X \rightarrow 0$). However, it is expected that during the testing of the model that results will give relative exchange ratios less than one, results which Eq. 4.3 would be unable to provide (since the expression is unsolvable for negative values under the radical). Thus, for a victorious enemy, while RER is still from the friendly forces viewpoint:

$$RER = 1 - \sqrt{1 - \frac{P_{K/S}}{1 - P_{S/S}}(\$_0/X_0)^2} \tag{4.6}$$

or,

$$RER = 1 - \sqrt{1 - \frac{1}{T_{\$/X}}} \qquad \{4.7\}$$

Therefore, when the variable $T_{\$/X}$ is less than one, Eq. 4.5 should be used, otherwise, Eq. 4.7 would govern.

$P_{S/S}$ and $P_{K/S}$ are the sortie survivability and sortie lethality, respectively, from the viewpoint of the friendly forces. Since these probabilities are scenario dependant, they will be calculated throughout the simulation, as averages over time and estimates converging on the "actual" probabilities. When estimating the sortie survivability of one role-type of friendly force vs. one of the enemy's:

$$P_{S/S_\tau} = 1 - \frac{\sum_{t=0}^{\tau} A_{\$/X_t}}{\sum_{t=0}^{\tau} \$_{S_t}} \qquad t = 0,1,2,\ldots,\tau \qquad \{4.8\}$$

where τ is the time at which $P_{S/S}$ is being calculated, $A_{\$/X_t}$ is the attrition of the friendly forces by enemy forces (number of aircraft) at time t, and $\$_{S_t}$ is the number of friendly aircraft engaged in a sortie at time t.

Likewise, the friendly aircraft's sortie lethality can be found by:

$$P_{K/S_\tau} = \frac{\sum_{t=0}^{\tau} A_{X/\$_t}}{\sum_{t=0}^{\tau} X_{S_t}} \qquad t = 0,1,2,\ldots,\tau \qquad \{4.9\}$$

where $A_{X/\$_t}$ is the attrition of the enemy aircraft (or, in the case of surface attack, the number of defensive weapons sites destroyed) at time t and X_{S_t} is the number of enemy units involved in a sortie at time t

The logic is presented in DYNAMO notation in the following lines for the example of determining the relative exchange ratio for the enemy surface attack installations (XSA) vs. the friendly surface attack aircraft.

```
A  R$SAXSA.K=CLIP(R1A.K,R1B.K,1,T$SAXSA.K)
A    T$SAXSA.K=((1-PSS1.K)/(PKS1.K+1E-6))*((AGXSAN/CB$SAN)**2)
A  R1A.K=1/(1+D1.K-(1-T1A.K)**0.5)
A    D1.K=SWITCH(1,CLIP(0,1,T$SAXSA.K),W$1.K)
A    T1A.K=CLIP(T$SAXSA.K,0,1,T$SAXSA.K)
A  R1B.K=1-((1-(1/(T1B.K+1E-6)))**0.5)
A    T1B.K=CLIP(T$SAXSA.K,1,T$SAXSA.K,1)
A  PSS1.K=1-(W$1.K/T$1.K)
A  PKS1.K=WX1.K/TX1.K
L   W$1.K=W$1.J+A$SAXSA.J
L   T$1.K=T$1.J+$SA.J
N      W$1=0
N      T$1=1E-6
L   WX1.K=WX1.J+KXSA$SA.J*XSA.J
L   TX1.K=TX1.J+XSA.J
N      WX1=0
N      TX1=1E-6
```

where,

- R$SAXSA - Relative exchange ratio of $SA by XSA
- T$SAXSA - $T_{\$/X}$ (see Eq. 4.4)
- R1A - RER when T$SAXSA < 1
- D1A - Prevents division by zero in R1A when W$1 = 0
- T1A - T$SAXSA when T$SAXSA < 1

- R1B - RER when T$SAXSA > 1
- T1B - T$SAXSA when T$SAXSA > 1
- PSS1 - Sortie survivability (Case 1)
- PKS1 - Sortie Lethality (Case 1)
- W$1 - Sum of attrition of $SA by XSA
- T$1 - Sum of friendly aircraft flying sorties
- WX1 - Sum of attrition of XSA by $SA
- TX1 - Sum of defensive weapons sites involved in surface attack sorties
- Case 1 considers the primary participants of the surface attack mission

Notice that several functions include and several variables' initial value a constant -- 1E-6. Proper placement of this small number (0.000001) helps to prevent fatal errors such as division by zero or negative numbers under a radical.

4.1.2 Baseline Results

Figures 4.1-4.6 show the decline of the six levels through the simulation time of 75 hours. The initial inventory levels are an aggregation of ten carriers and their enemy counterparts. By making the analysis with such large inventories it is possible to schedule more frequent sorties than might be expected (e.g., surface attack sorties are occuring twenty percent of the time), revealing results sooner. Recalling the level variables,

- CB$SA - Carrier based aircraft for surface attack
- CB$FE - Carrier based aircraft for fighter escort
- CB$CD - Carrier based aircraft for carrier defense
- AGXSA - Aggressor units against surface attack
- AGXFE - Aggressor aircraft for fighter escort

- AGXCD - Aggressor aircraft against carrier defense

These results are better seen with the use of the relative exchange ratio (RER) as given in Eqs. 4.3 and 4.6. For the attrition directly due to a primary threat (e.g., an enemy fighter force against the friendly fighter force on, say, a carrier defense sortie where the secondary threat would be the enemy bombers) relative exchange ratios were calculated cumulatively with respect to time and can be seen in Figures 4.7-4.9.

A question may arise upon studying the plots of the level variables CB$SA and AGXSA (Figs. 4.1 and 4.4) and comparing them to the RER plot in Fig. 4.7. How can a relative exchange ratio greater than one be realized when more of the friendly force is attrited than that of the enemy? This occurs because RER considers only the attrition due to and from the *primary* threat. In other words, since the friendly surface attack aircraft are also being attrited by the enemy fighters and bombers, the level of CB$SA may decline more rapidly than AGXSAN despite the superiority of the friendly bombers.

By making use of high engagement probabilities with large initial inventory levels, the simulation model's results are generated over a calendar period of 75 hours. Of course, the same end-results could be achieved over almost any time period provided that the distribution of sortie mission types were the same. This reduces the importance of the plots of the level variables shown in Figs. 4.1-4.6 since at some point such a large number of aircraft will be lost, reducing sortie generation and sortie force sizes. However, it is reasonable to allow the simulation to run for a longer period when estimating the relative exchange ratio, provided, of course, that the level variables are not allowed to decay to a level less than that required to generate the respective sorties. The RER plots shown in the following figures are estimates of the "actual" relative exchange ratio, the more engagements which occur result in a better estimate of the real RER.

The RER's at the simulation termination time (i.e., 75 hours) are presented in Table 4.15. Note that these results are included for two experimental (i.e., Force Size Ratio and

Multimission) cases as well as the baseline scenario. The experimental cases are described in the following sections. simulation.

Table 4.15 - Relative Exchange Ratios at Simulation Termination

Model	Surface Attack	Primary Interaction Role Fighter Escort	Carrier Defense
Baseline	2.813	1.420	1.602
Force Size Ratio			
1.5 : 1	2.141	0.657	0.809
2 : 1	2.669	1.307	1.524
Multimission	2.650	1.374	1.474

4.2 *Effects of Force Size*

Mathematical representations of combat have long fascinated analysts and practitioners. In 1805 at the Battle of Trafalger, Nelson faced a combined French and Spanish force of 46 ships with his fleet of 40 ships. History records that he deployed eight of his ships to hold half of the enemy fleet downwind while his main force of 32 ships engaged the remainder of 23 ships in the rear.

Over a hundred years later Lanchester [12] tested his classical theory of combat which states that superiority is proportional to the squares of the respective numerical strengths. According to Lanchester's model, Nelson faced a fighting strength of $23^2 + 23^2 = 1058$. By dividing his force as he did, he met the enemy with a margin of superiority of $\sqrt{32^2 + 8^2} = 5.5$ ships. Eight ships were the most that could have been deployed and still have

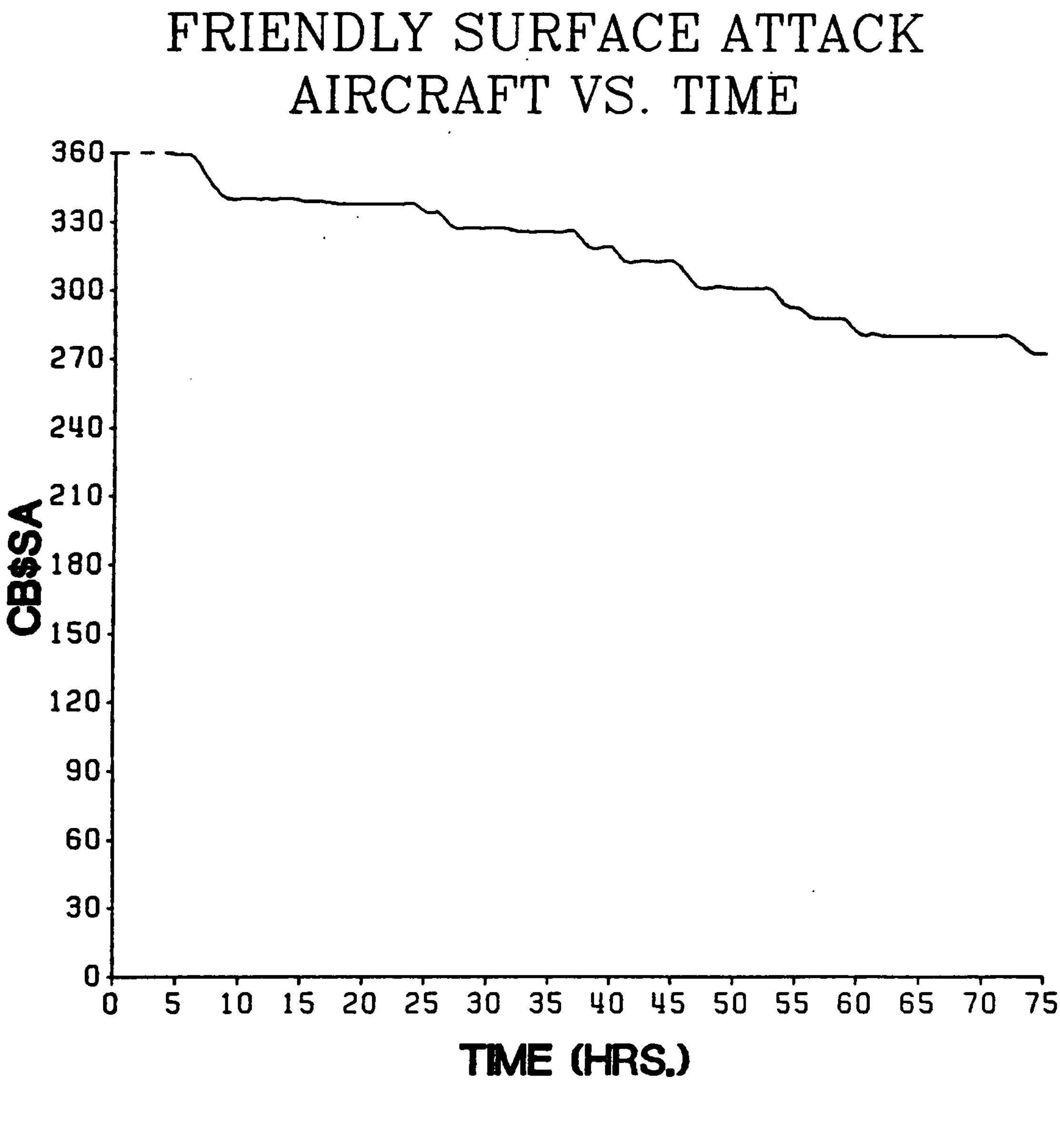


Figure 4.1 - Carrier Based Aircraft for Surface Attack vs. Time

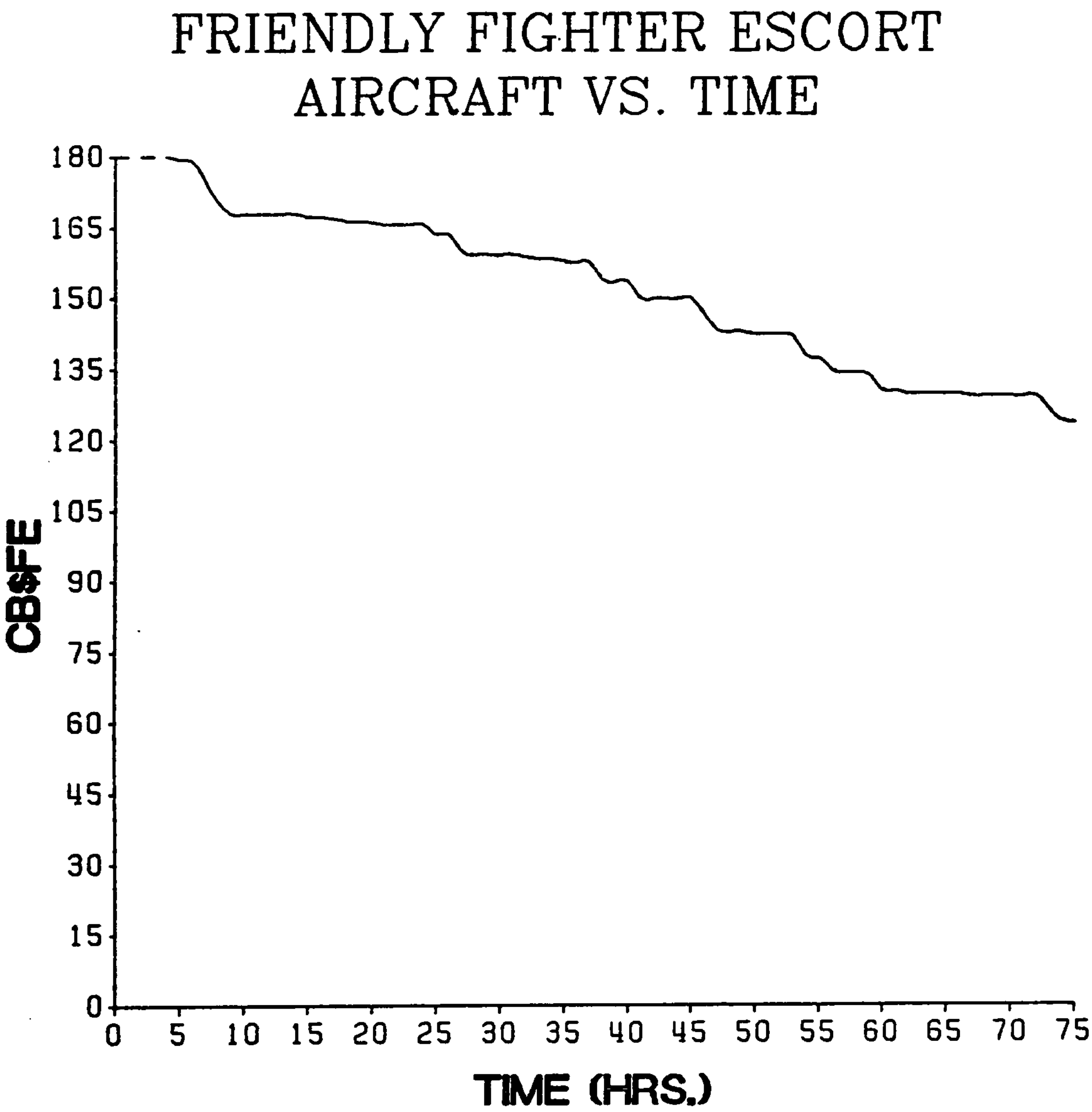


Figure 4.2 - Carrier Based Aircraft for Fighter Escort vs. Time

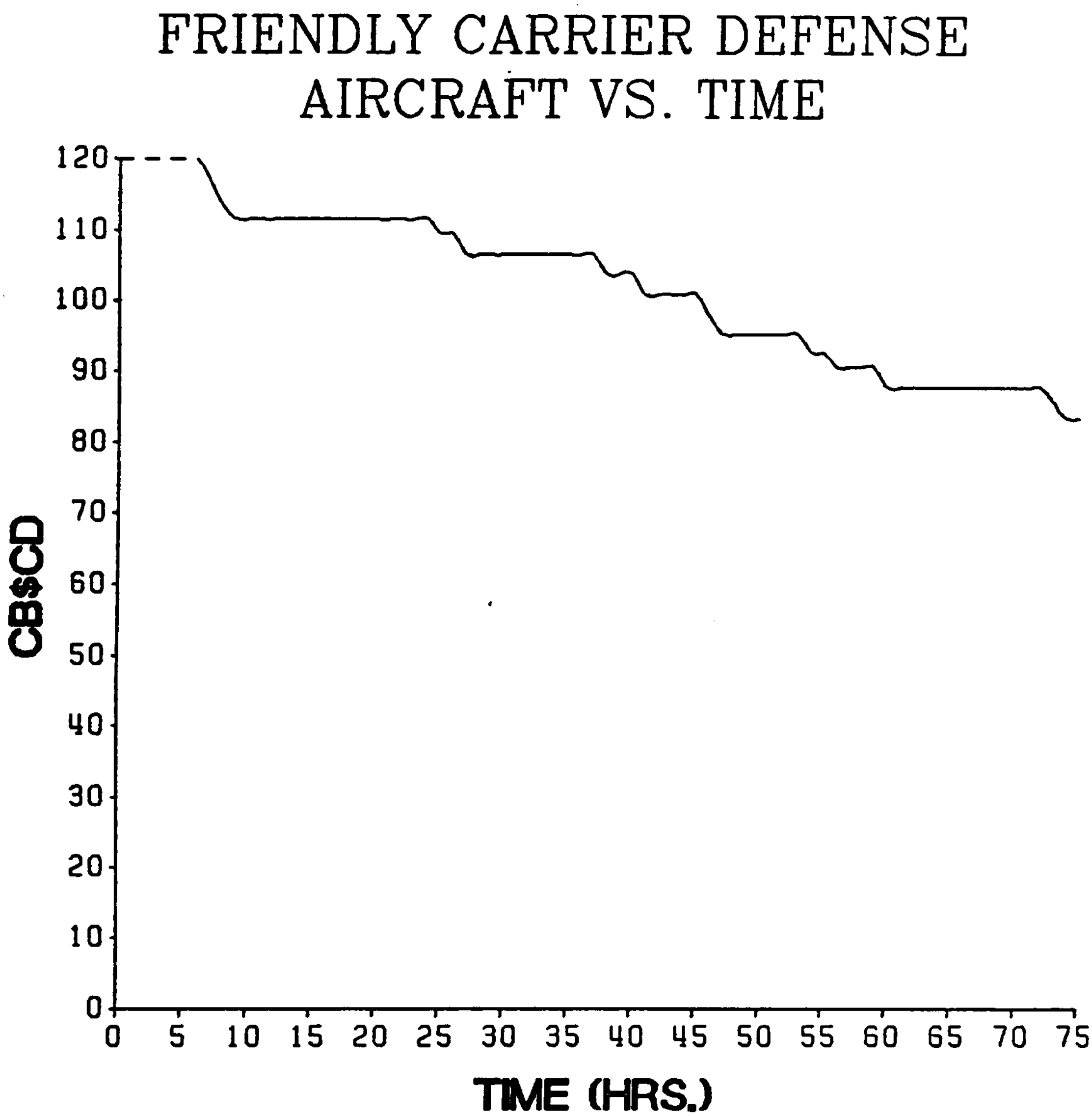


Figure 4.3 - Carrier Based Aircraft for Carrier Defense vs. Time

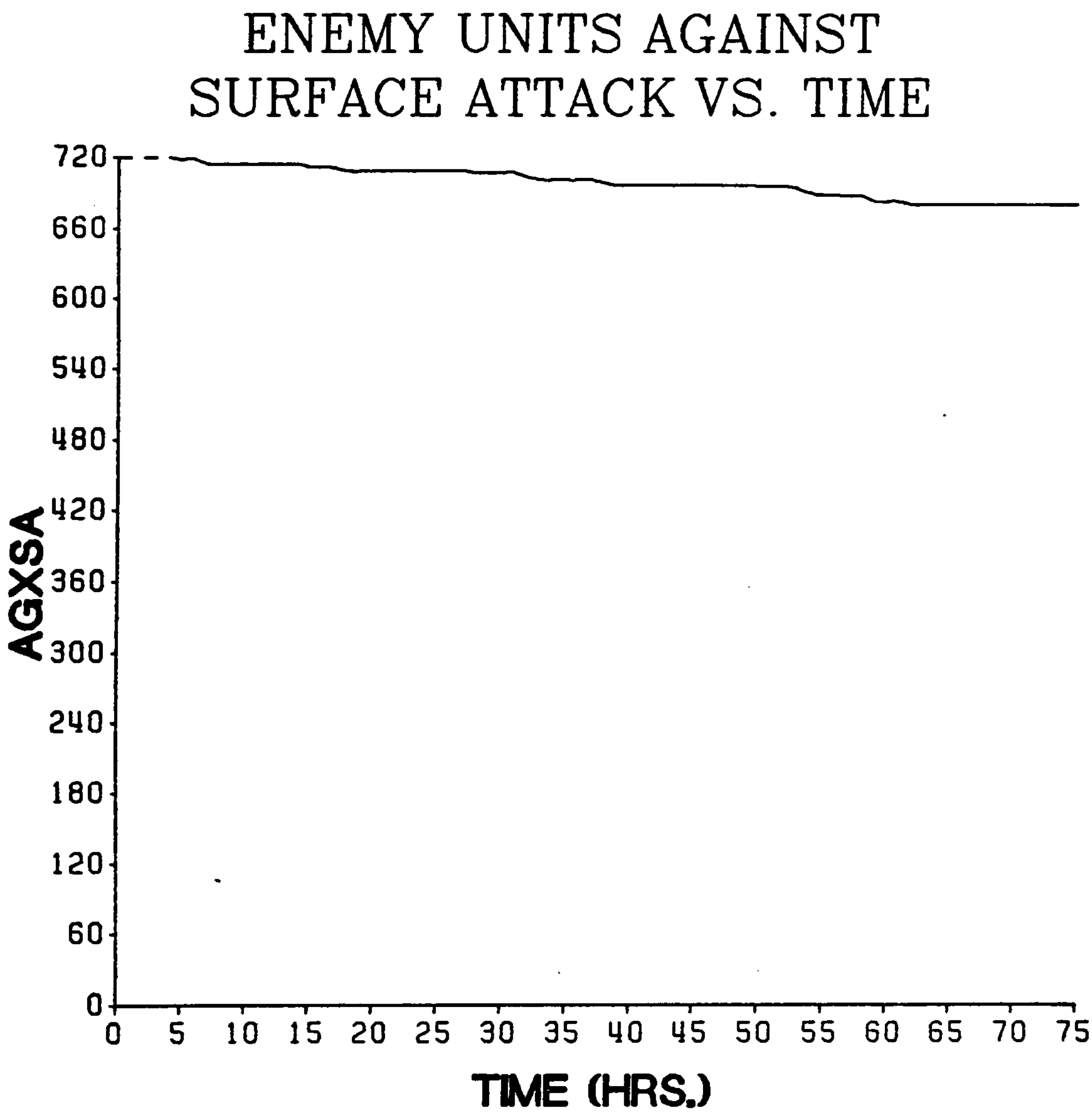


Figure 4.4 - Enemy Defensive Weapon Sites against Surface Attack vs. Time

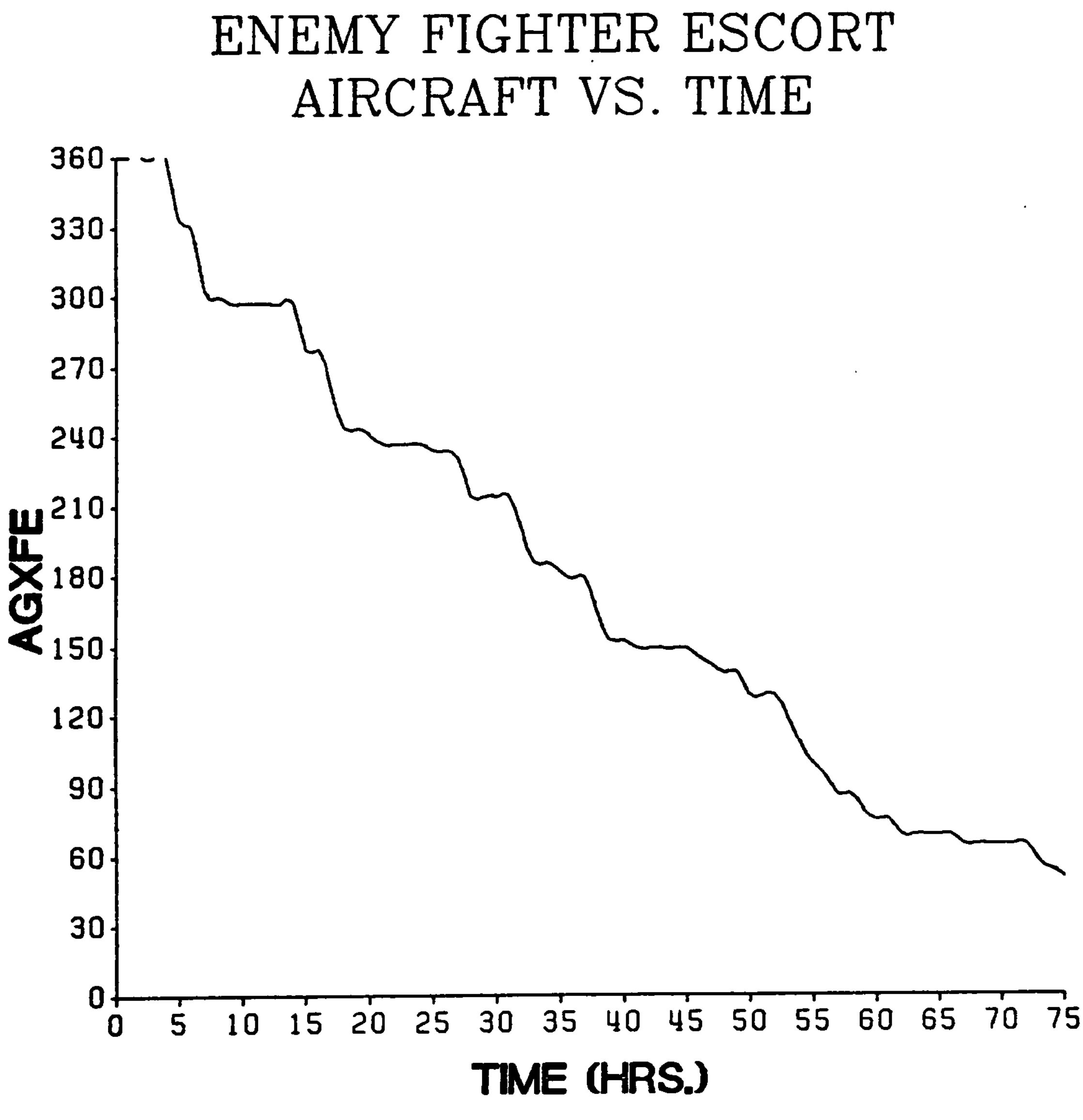


Figure 4.5 - Enemy Aircraft for Fighter Escort vs. Time

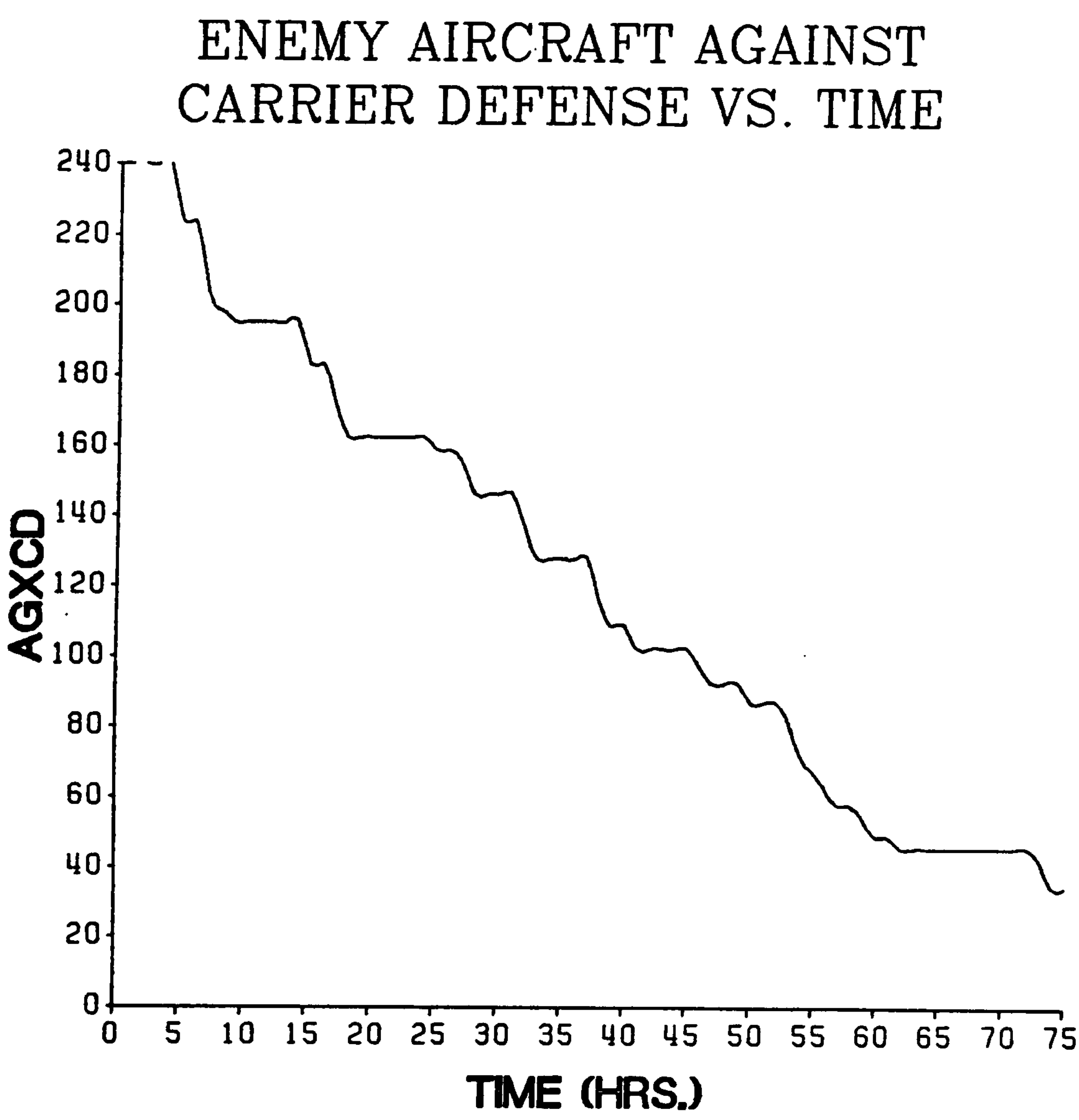


Figure 4.6 - Enemy Aircraft against Carrier Defense vs. Time

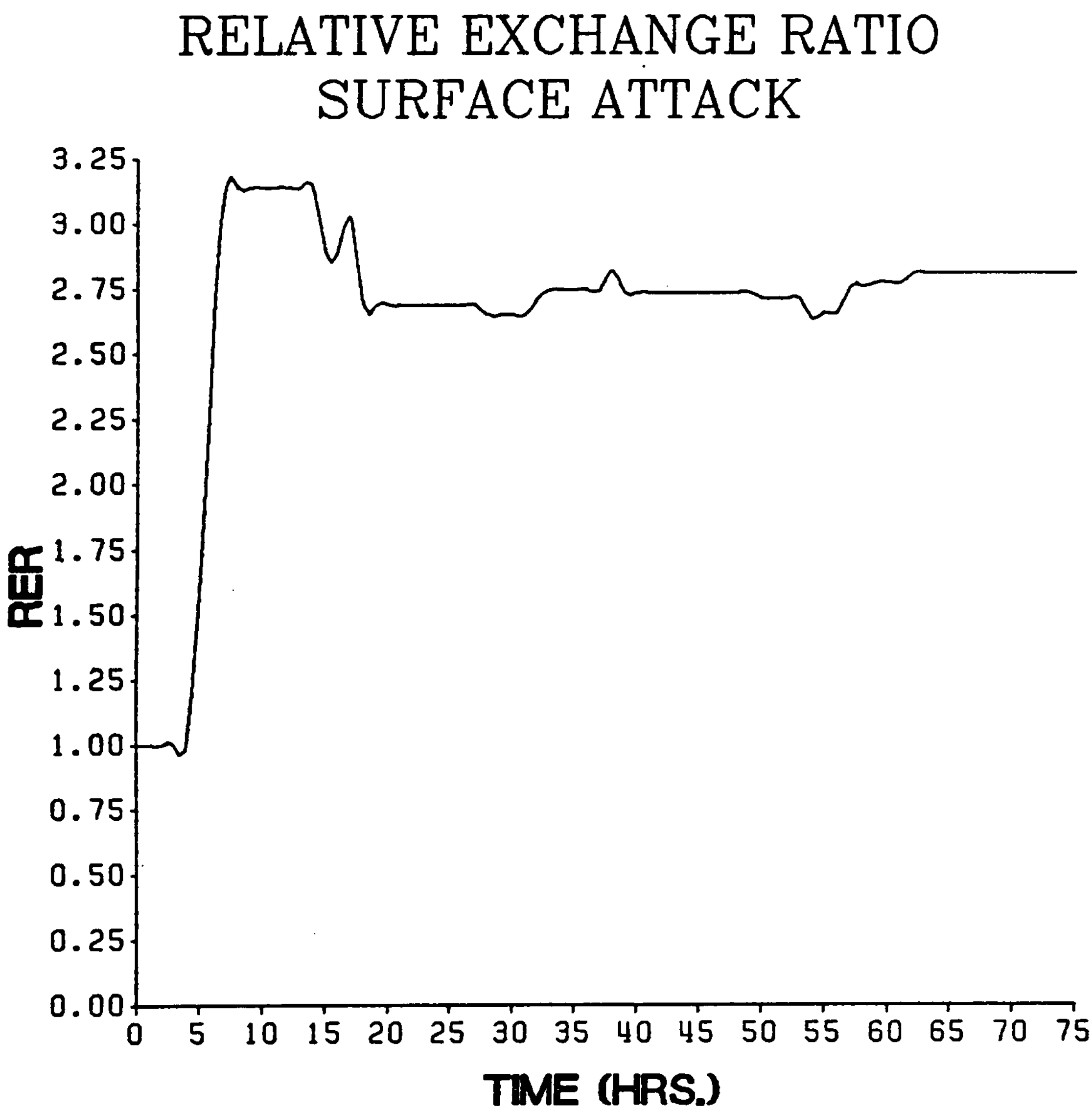


Figure 4.7 - Relative Exchange Ratio (Baseline) Surface Attack

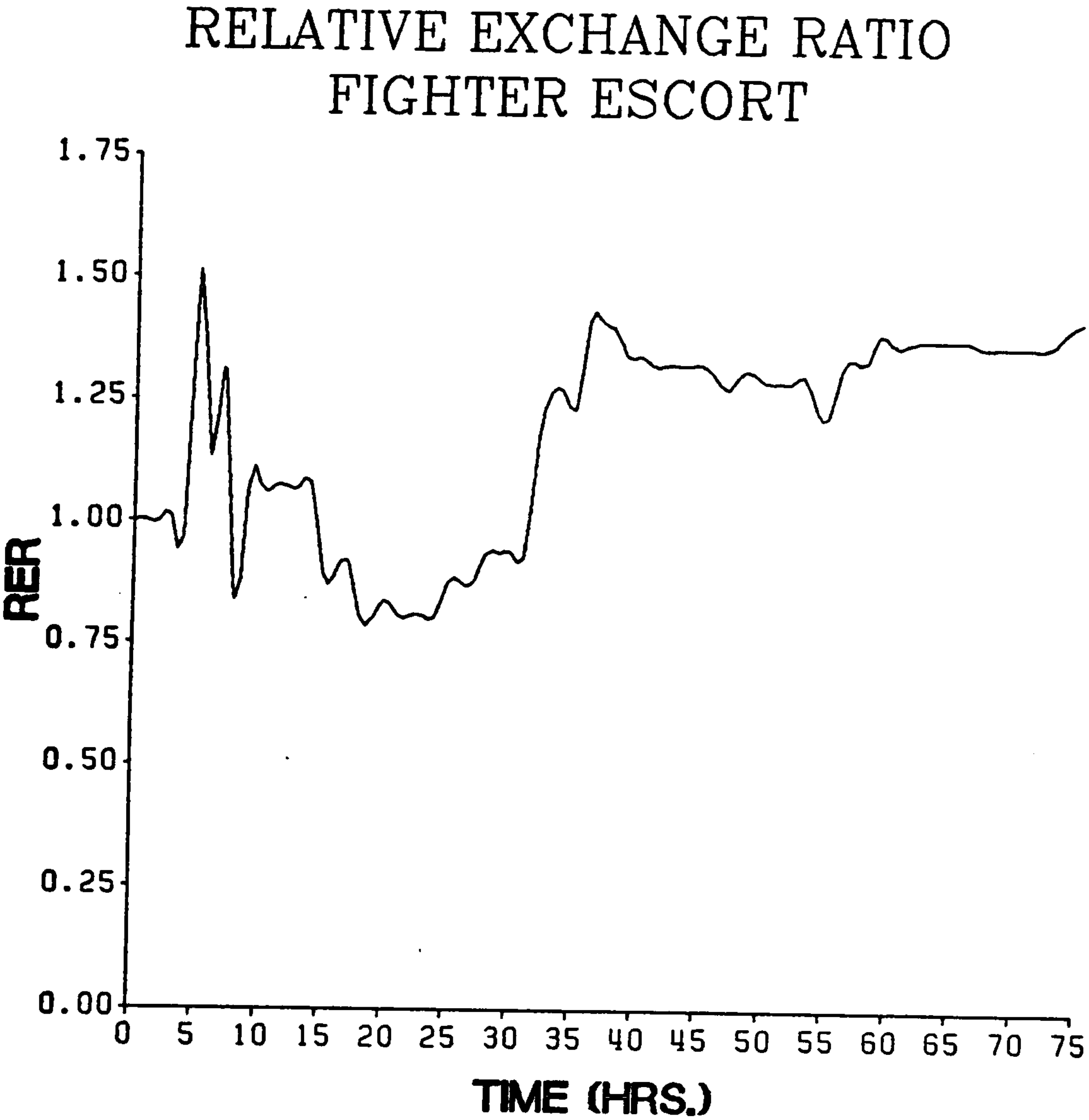


Figure 4.8 - Relative Exchange Ratio (Baseline) Fighter Escort

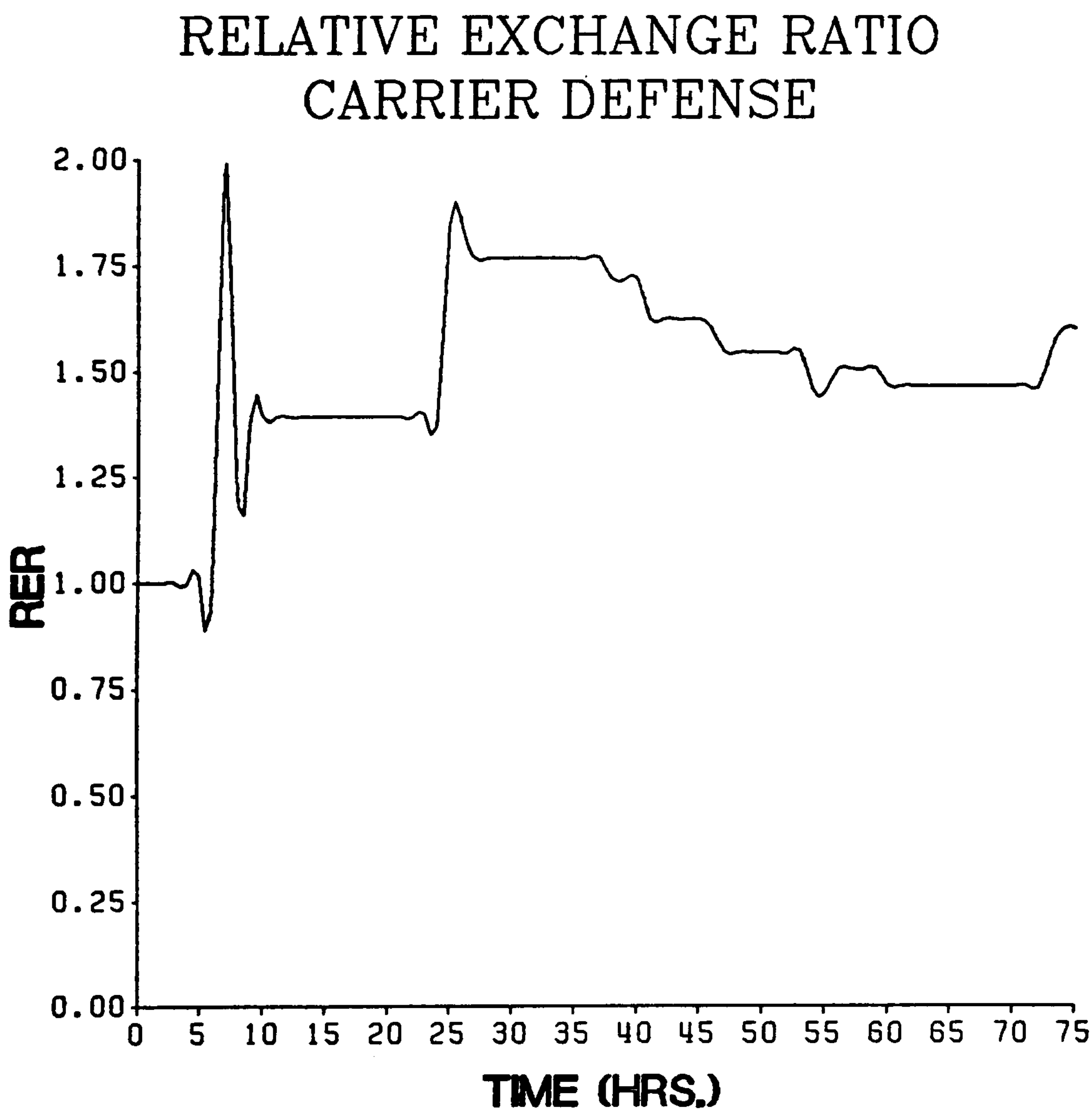


Figure 4.9 - Relative Exchange Ratio (Baseline) Carrier Defense

maintained this superiority. If less had been assigned, they might not have been able to have contained the 23 until his force of 32 had destroyed the enemy and come to their aid.

This little discourse inspires the question: how can we adapt Nelson's strategies to modern air combat? It is assumed that the enemy inventories are much larger than those of friendly forces (for modelling purposes this ratio is assumed to be 2:1). However, if, through superior logistics and higher availability the enemy sortie force sizes encountered were only 1.5 times as large as the friendly sorties, the required effectiveness would be somewhat less than that of "average" case, where

$$\frac{X_0}{\$_0} = \frac{X_S}{\$_0} = \text{ (e.g., 2)} \tag{4.10}$$

The baseline model does include enemy sortie force sizes larger than those of the friendly forces. For this example, however, the sortie sizes will be artificially fixed to an enemy/friendly ratio of 1.5:1 according to the following values:

"Average" Case (2:1)

- \$SA = 6 vs. XSA = 12
- \$FESA = 3 vs. XFESA = 6
- \$FEFE = 3 vs. XFEFE = 6
- \$FECD = 3 vs. XFECD = 6
- \$CD = 6 vs. XCD = 12

"Special" Case (1.5:1)

- \$SA = 8 vs. XSA = 12
- \$FESA = 4 vs. XFESA = 6
- \$FEFE = 4 vs. XFEFE = 6
- \$FECD = 4 vs. XFECD = 6

- \$CD = 8 vs. XCD = 12

The relative exchange ratios for the primary threats are shown in Figs. 4.10-4.4.15.

4.3 Multimission Aircraft Carrier Operations

The specialization of an aircraft to a particular role, whether it be surface attack, fighter escort, or carrier defense, necessitates a whole regime of logistic and operations considerations to cater to each aircraft type. On the other hand, if an aircraft can perform each of these missions with an acceptable loss of effectiveness in any given role, there is a possibility that an overall *increase* in the measures of effectiveness could be realized. This result would occur if, say in the baseline case, all of the surface attack aircraft were destroyed, an important component of the overall system's effectiveness would be reduced to zero. With other aircraft able to assume the role of the dedicated attack aircraft the system's lethality is protected.

This marked contrast raises the basic question: how much mission versatility should be built into a single aircraft? A *multimission* aircraft can lead to large savings in development costs, in production costs through economies of scale, and in operating and maintenance costs through standardization. The advantage of a special purpose aircraft is that since it is designed for only one task, the performance characteristics most essential to carrying out that task effectively need not be compromised in the aircraft's design.

This leads to the need to study the effectiveness of multimission aircraft and the relation of their contribution to the MOE's versus that found for the aircraft of the carrier operations model described in the above sections.

It is expected that a carrier-based multimission aircraft (CBM) would not enjoy the same effectiveness in each role as its counterparts of the carrier operations model. Determining

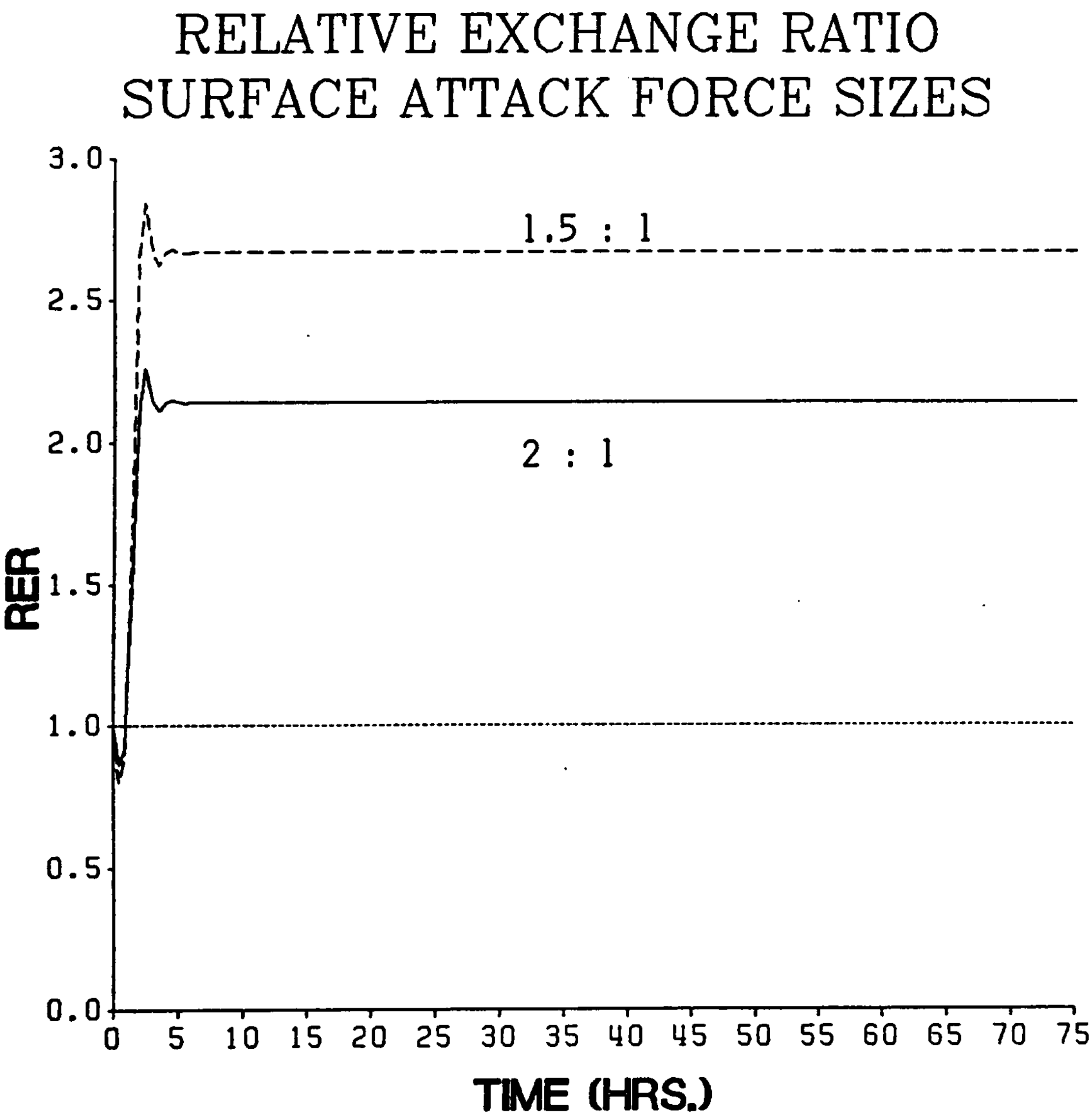


Figure 4.10 - Relative Exchange Ratio - Surface Attack - Force Sizes

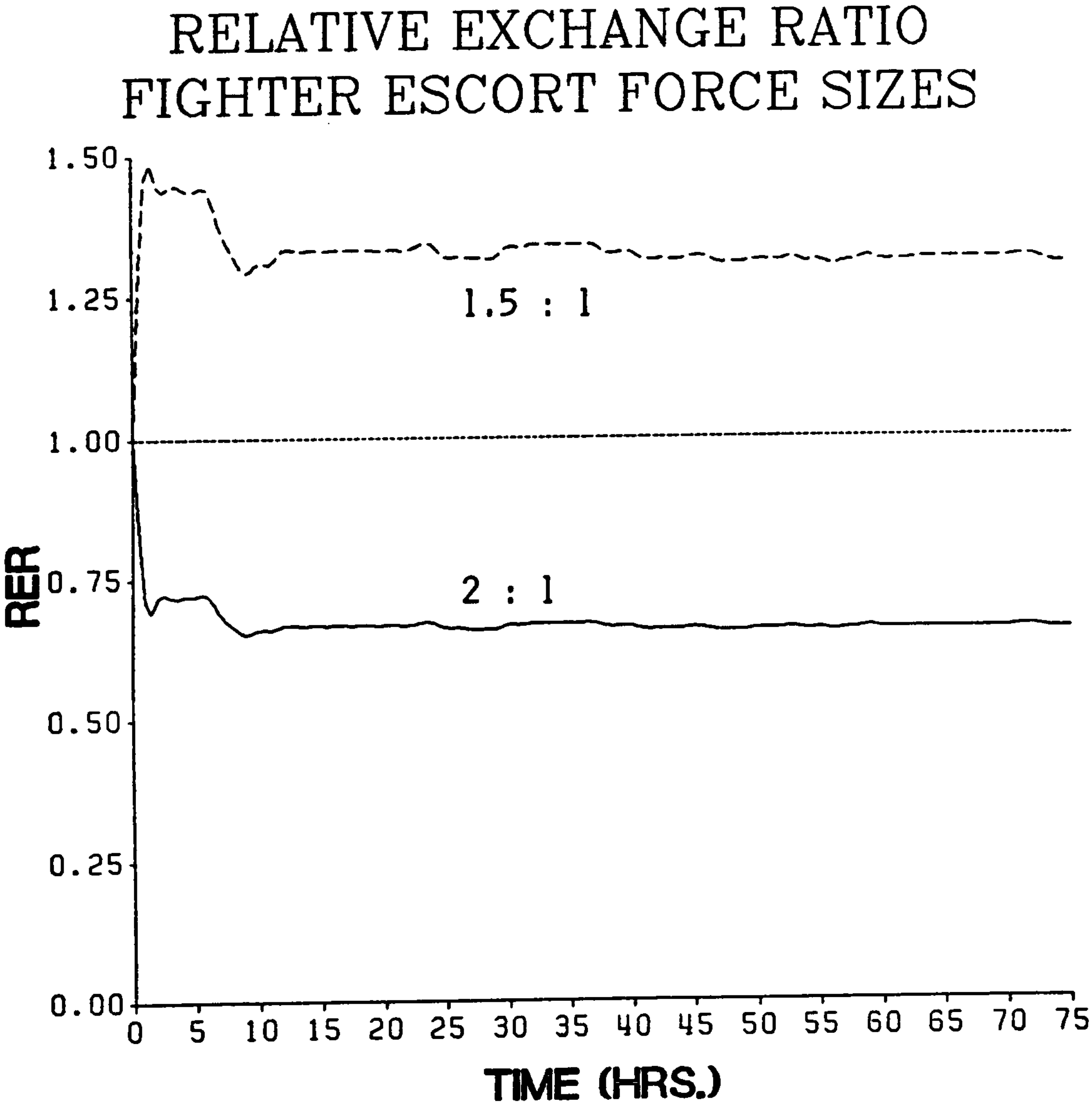


Figure 4.11 - Relative Exchange Ratio - Fighter Escort - Force Sizes

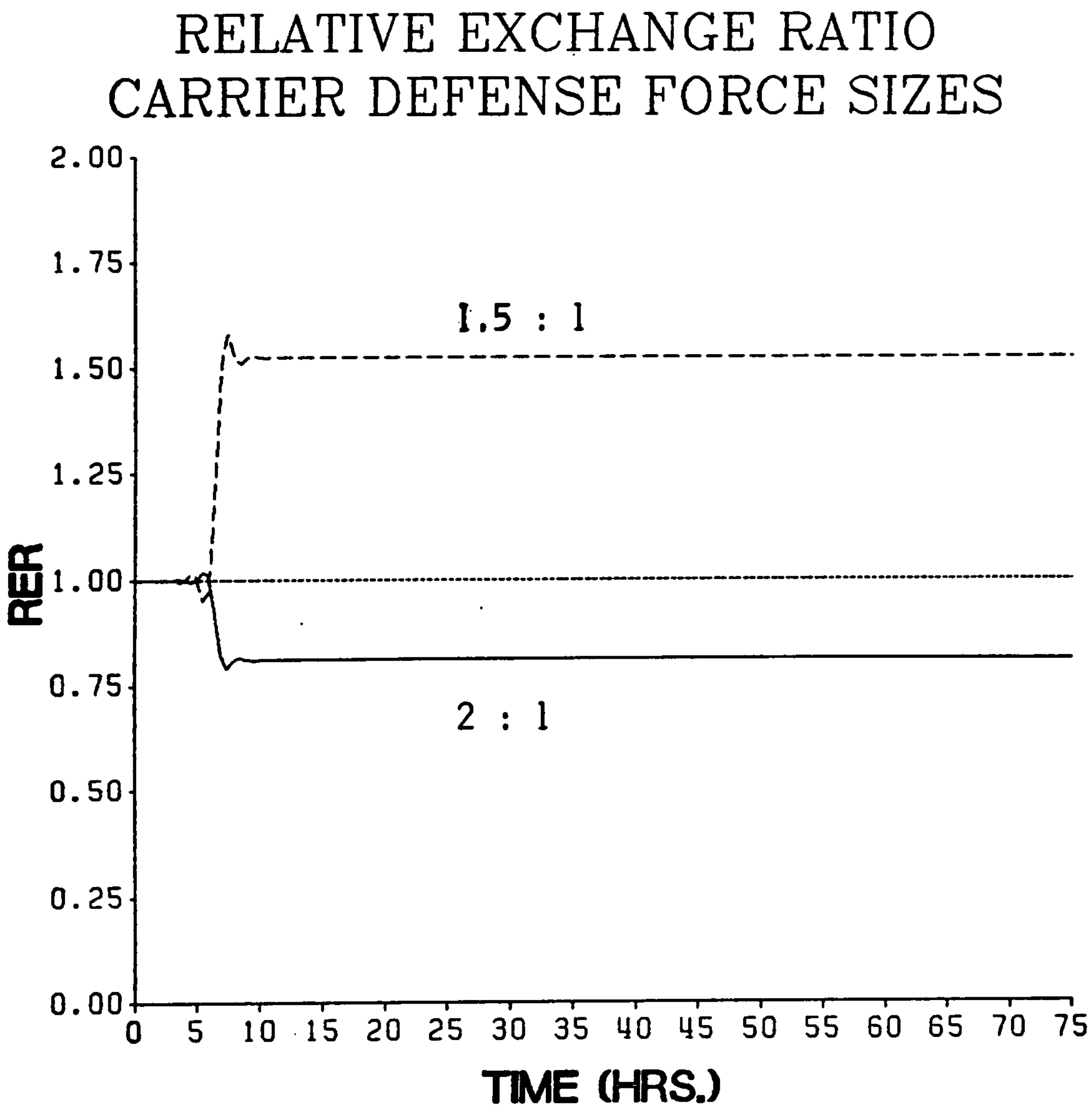


Figure 4.12 - Relative Exchange Ratio - Carrier Defense - Force Sizes

reliable values for the sortie survivability $P_{S/S}$ and sortie lethality $P_{K/S}$ would require information that is nor available at this time, but is reasonable to assume a five percent decrease in both $P_{S/S}$ and $P_{K/S}$ for the purpose of analysis.

This is more directly accomplished by decreasing the kill fraction of enemy forces and by increasing the kill fraction of friendly forces by enemy forces by dividing and multipying these probabilities by 1.05 respectively.

It is expected that the resulting increase in availability by selecting the multimission configuration would allow larger sortie forces to be flown. This is affected in the model by skewing the baseline distribution to the modified one as shown in Table 4.16. The table headings "Lower," "Mid," and "Upper" refer to the three possible sortie force sizes for a given role. For example, the friendly surface attack aircraft sorties may consist of one of three force sizes: Lower=4, Mid=6, and Upper=8. The three levels for each force type can be seen in the column and row headings in Tables 4.1-4.14. The probabilities used in the multimission modification are only to be applied to the friendly forces, no change is expected in the enemy sortie force size distribution.

Table 4.16 - Sortie Force Size Distributions

	Force Level Probability		
	Lower	Mid	Upper
Baseline	0.30	0.40	0.30
Multimission	0.15	0.35	0.50

4.3.1 Multimission Results

As can be seen in Figs. 4.13-4.15, the multimission configurations enjoy nearly the same relative exchange ratios as those found for the baseline case. These figures show nearly identical RER histories for the multimission cases, differing only in magnitude (since all other conditions of the baseline case have been unperturbed). The most favorable multimission case is that for fighter escort (see Fig. 4.14), possibly since fighters participate in each of the three missions - thus the versatility is most apparent.

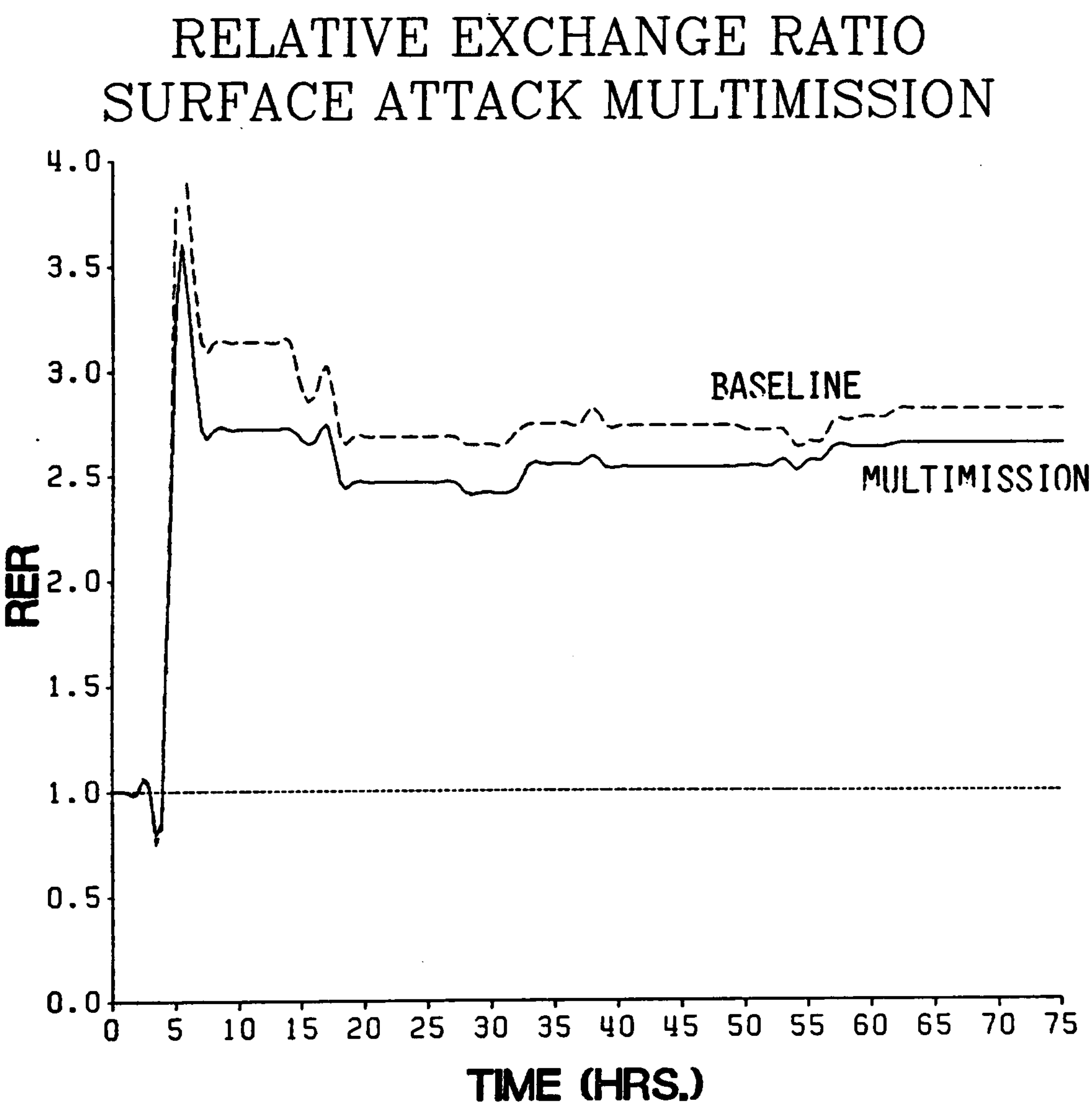


Figure 4.13 - Relative Exchange Ratio - Surface Attack - Multimission

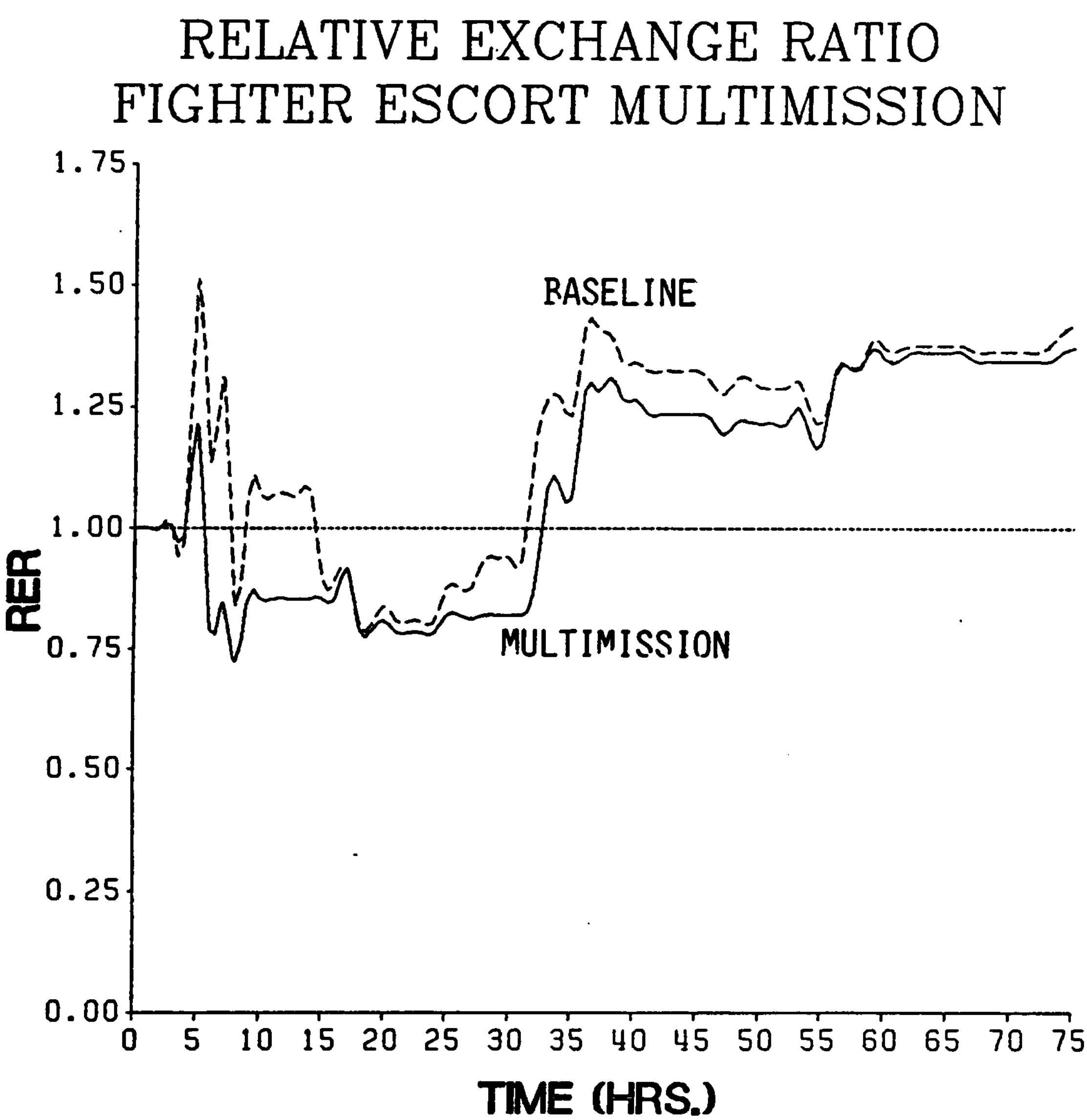


Figure 4.14 - Relative Exchange Ratio - Fighter Escort - Multimission

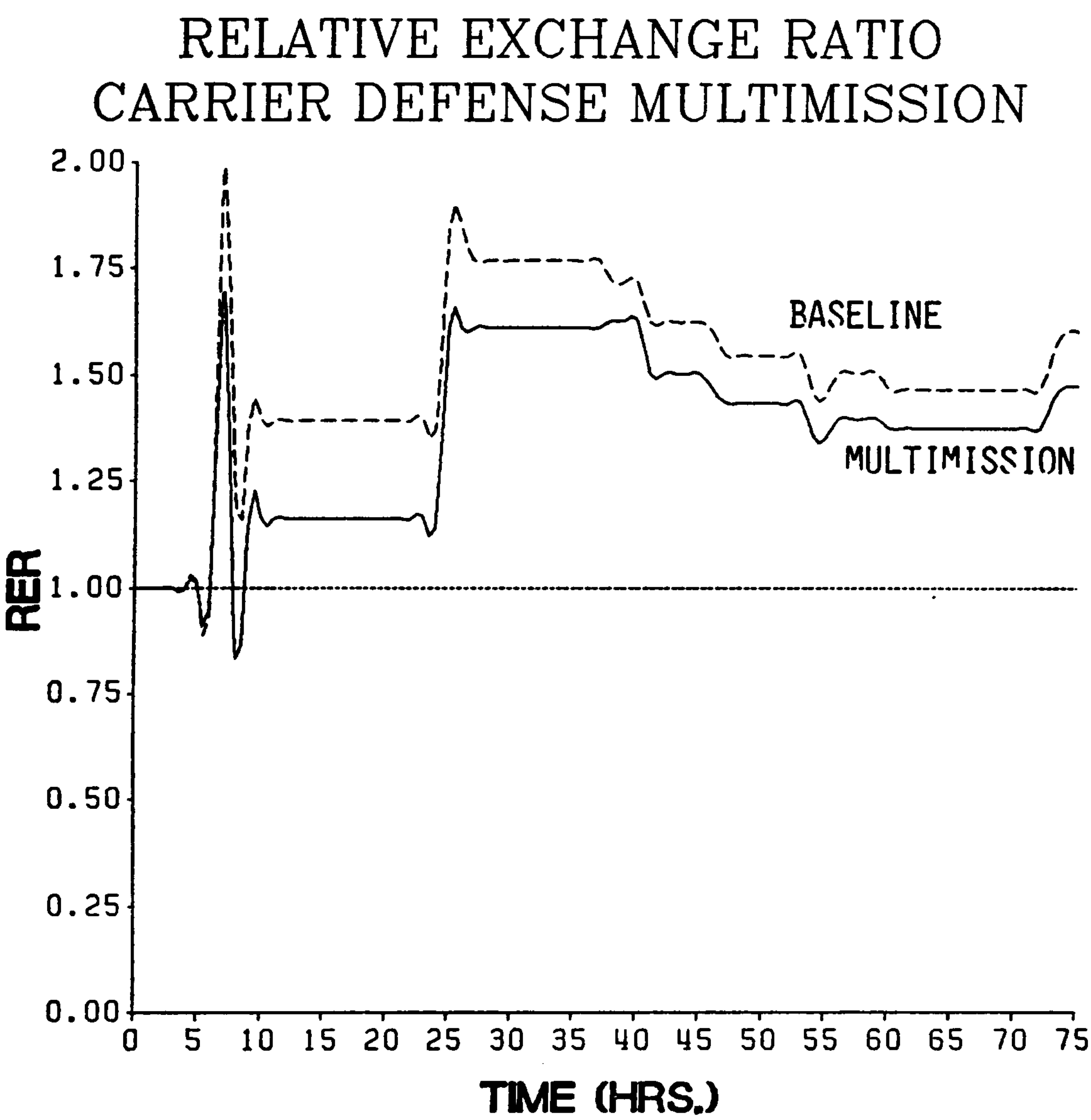


Figure 4.15 - Relative Exchange Ratio - Carrier Defense - Multimission

5.0 Performance Tradeoffs Methodology

The survivability of an aircraft can be enhanced by creative design -- one that minimizes weight, cost and performance penalties by the addition of extra elements to the design -- and by proper utilization of the aircraft. Survivability enhancement refers to any particular characteristics of the aircraft or subsystem, any design technique, or any tactic that reduces either the vulnerability or the susceptibility of the aircraft. Vulnerability reduction concepts include damage suppression (passive and active) and component shielding, location, redundancy and elimination. Susceptibility reduction is accomplished through electronic warfare features such as passive warning receivers that warn the crew of a threat, electronic countermeasures such as jammers and deceivers carried onboard or in supporting units to prevent tracking by the threat, signature reduction that makes the aircraft more difficult to detect and track, the use of expendables such as chaff and flares to frustrate radar and IR systems, threat suppression using radar-seeking missiles, and tactics. While all survivability reduction considerations are best handled during aircraft design, it is imperative that susceptibility features such as signature reduction be incorporated at the inception of conceptual design.

Since performance capabilities and handling qualities of the aircraft as manifest through such indicators as speed and altitude impact strongly on aircraft survivability and combat effectiveness, it is evident that such considerations must be implemented at the earliest opportunity in the aircraft life cycle. Aircraft performance capabilities influence combat effectiveness by limiting the mission radius, payload, speed, loiter time and maneuverability. Undesirable handling characteristics compromise effectiveness by limiting the flight envelope and complicating low level or nap-of-the-Earth flight.

The Aircraft Survivability and Lethality Tradeoffs (ASALT) Models developed in the companion research [1][2] has as its output two parameters: (1) sortie survivability $P_{S/S}$ (the fraction of aircraft surviving a sortie) and (2) sortie lethality $P_{K/S}$ (the fraction of enemy aircraft destroyed per sortie). In many cases it is difficult to determine the overall effect of varying a given design/decision parameter. For example, as aircraft speed increases, $P_{S/S}$ increases but $P_{K/S}$ decreases. similarly, as a fraction of payload devoted to ECM increases, $P_{S/S}$ increases but $P_{K/S}$ decreases. In this section a methodology is described for trading off the conflicting goals of increasing aircraft survivability and increasing combat effectiveness.

5.1 Aircraft Survivability and Combat Effectiveness

The effectiveness of a particular military aircraft depends on its characteristics, its mission, and the nature of the threat. For attack aircraft flying an interdiction mission exposed to surface-to-air threat systems, which is what is modeled here, the aircraft's effectiveness can be measured by its "lethality," $P_{K/S}$ A transport aircraft's effectiveness would be measured by its ability to deliver troops and material, a reconnaissance aircraft by its capability to locate threats and targets, etc. Survivability and effectiveness are obviously interrelated. The higher the aircraft's survivability, the more sorties that it can fly, thus increasing its effectiveness by whatever measure is used.

Ball [11] describes how the effectiveness of an aircraft in the offensive view of the mission can be quantified by the mission attainment measure, MAM, which ranges from 0 to 1. It is a relative measure of the ability of the aircraft to accomplish its objectives in the presence of the threat without the consideration of the threat effects. The threat effects are considered in the defensive view of the mission in which the effectiveness of the aircraft can be measured by the aircraft's survivability, S, the fraction of aircraft remaining after a sortie, or mission. A combined measure of effectiveness of an aircraft in a particular scenario as the measure of mission success, MOMS, which is given by the equation:

$$MOMS = (S)(MAM) \qquad \{5.1\}$$

The number of aircraft launched, L, to achieve a mission goal, G, is:

$$L = \frac{G}{MOMS} \qquad \{5.2\}$$

Now survivability S as the fraction of aircraft surviving can be written:

$$S = \frac{L - CLG}{L} \qquad \{5.3\}$$

where CLG is the cumulative losses in achieving the goal. Substituting Eq. 5.2 into Eq. 5.3 and solving for CLG give:

$$CLG = \frac{G(1-S)}{MOMS} = \frac{G(1-S)}{(MAM)(S)} \qquad \{5.4\}$$

Eq. 5.4 is the basis of the Mission Trade-Off Model (MTOM) that was specifically developed to assess the impact of survivability enhancement features on mission effectiveness and costs [13][14]. The model calculates the life cycle costs associated with a group of aircraft necessary to accomplish a prescribed mission. MTOM is a computer simulation model consisting of two submodels: the mission tradeoff and effectiveness (MTOE) submodel evaluates the number of aircraft to meet a goal and the Mission Tradeoff and Cost (MTOC) submodel calculates the life-cycle cost for the aircraft.

Table 5.1 presents a simple example tradeoff study for a carrier based fighter whose primary mission is air superiority escort. For this purpose, a carrier carries 1800 air-to-air missiles to be delivered by the CBFE. The increase in survivability for the modified aircraft over the baseline is accommodated by the substitution of an ECM pod for a missile in one of the external stores hard points, giving two ECM pods.

Table 5.1 Example Mission Tradeoff Study.

Aircraft B or M	Sortie Survivability S	Missiles Per Aircraft N	Acquisition Cost AC	Peacetime 15-Yr LCC LCC
Baseline	0.980	9	\$25.00 M	\$12,000 M
Modified	0.983	8	\$25.10 M	\$12,050 M

In the context of Eq. 5.4, the mission goal, G, is to fire 1800 missiles per carrier times 15 carriers at enemy aircraft and the mission attainment measure, MAM, is the payload. Substituting in Eq. 5.4 for the baseline and modified cases gives:

$$CLGB = \frac{(27000)(0.020)}{(9)(0.980)} = 61.22$$

and

$$CLGM = \frac{(27000)(0.017)}{(8)(0.983)} = 58.37$$

The combat replacement costs are \$1,531 million and \$1,459 million for the baseline and modified aircraft respectively. Added to the peacetime life-cycle costs gives \$13,531 million and \$13,509 million, respectively, making the modified alternative superior.

5.2 Survivability-Effectiveness-Attrition Relationships

Aircraft survivability and combat effectiveness are inextricably related to attrition. Any mission tradeoff methodology utilizes some attrition model, explicit or implied. Herein lies an irony: many practitioners of aircraft survivability would seem to prefer not to know the details of the assumptions regarding attrition. It is only when these attrition considerations are made explicit that they challenge them. The position taken here is that the attrition basis for tradeoff methodology should be explicit so that it can be challenged along with all other aspects of the methodology.

Consider the mission tradeoff methodology illustrated in Table 5.2 which is widely accepted throughout the survivability community [13][14], what do we know about the attrition principles upon which this methodology rests? To shed some light on this, let us devise an attrition model which will generate Eq. 5.4, the basis for the mission tradeoff methodology in Table 5.2.

Table 5.2 Attrition Formulation of Tradeoff Example of Table 5.1

$\$_t = \$_{t-1} - (DT)A\$TI_t$	No. of A/C at Time t
$A\$TI_t = \$_t Q$	Attrition Fighter Escort
Q = (SR)(A)(1-SS)	Attrition Factor
SR = 2	Sortie Rate
A = 0.5	Aircraft Availability
CLG_t = G/MFPL sub t	Cum. Losses in Achieving Goal
G = 27,000	Goal (27,000 missiles fired)
$MFPL_t = (N)CS_t/CL_t$	Missiles Fired Per Loss
$TS_t = TS_{t-1} + (DT)(SR)(A)(SS)\$_t$	Cum. Sorties
$TL_t = TL_{t-1} + (DT)A\$TI_t$	Cumulative losses

SS is the Sortie Survivability and N = No. of Missiles per A/C

The differential equation of the attrition formulation of Table 5.2 is formed from the difference equation for $\$_t$:

$$\lim_{DT\to 0}\frac{\$_t - \$_{t-1}}{DT} = \frac{d\$_t}{dt} = -A\$TI_t \qquad \{5.5\}$$

Substitution in the right hand side and separation of variables gives:

$$\frac{d\$_t}{\$_t} = -Qdt \qquad \{5.6\}$$

Integration and application of limits gives

$$\$_t = \$_0 e^{-Qt} \qquad \{5.7\}$$

Forming the differential equation for TS_t in the same manner gives:

$$\frac{d\,TS_t}{dt} = (SR)(A)(SS)(\$_0 e^{-Qt}) \qquad \{5.8\}$$

The solution of Eq. 5.8 is:

$$TS_t = \frac{(SR)(A)(SS)(\$_0)}{Q}(1 - e^{-Qt}) = \frac{(SS)(\$_0)}{1 - SS}(1 - e^{-Qt}) \qquad \{5.9\}$$

Repeating this procedure for TL_t yields the differential equation:

$$\frac{dTL_t}{dt} = A\$TI_t = \$_t Q = (Q)\$_0 e^{-Qt} \qquad \{5.10\}$$

with solution:

$$TL_t = \$_0(1 - e^{-Qt}) \qquad \{5.11\}$$

Now the cumulative losses in achieving the goal, CLG_t, in Table 5.2 is:

$$CLG_t = \frac{G}{PDTPL_t} = \frac{(G)(TL_t)}{(P)(TS_t)} \qquad \{5.12\}$$

or,

$$CLG_t = \frac{(G)(\$)(\$_0)(1 - e^{-Qt})}{(P)(SS)(\$)(\$_0)(1 - e^{-Qt})/(1 - SS)} = \frac{G(1 - SS)}{(P)(SS)} \qquad \{5.13\}$$

Eq. 5.13 is the form of Eq. 5.4 used in Example 5.1 in which the aircraft survivability, S, is the sortie survivability, SS, and the mission attainment measure, MAM, is the payload, P. Indeed, these were the values used to calculate CLGB and CLGM in comparing the baseline and modified aircraft in Example 5.1.

The point here is that the traditional mission tradeoff methodology is based on the naive attrition model shown in the equations of Table 5.2. The model is naive because none of the characteristics of the threat are even included. Surely a discipline that calculates aircraft survivability to four decimal places should be as concerned with accuracy as it is with precision.

5.3 General Attrition Model

The choice of attrition model profoundly affects the results of cost-effectiveness analysis, in general, and mission tradeoff analysis, in particular. One would expect, then, that the methodologies would agree on the basic form of attrition model for a given combat mission. This unfortunately is not the case. In [15], for example, surface-to-air missiles attrite attack aircraft to the so-called "linear" law, while in [16] the analysts use an approximation of the "square" law attrition equation for the same combat situation. Moreover, in spite of the dependence of air combat survivability decisions on attrition, there is a paucity of treatment of attrition. Where it is utilized, the incorporation of attrition principles is either implicit or simplistic. In this section a general attrition with relevance to mission tradeoff methodology is developed.

The approach used to model attrition dominates all facets of a combat model. Not only does it determine winners and losers and the movement of the front-line, but also gives casualty levels, equipment losses, ammunition expenditures, and resupply/reinforcement requirements.

In order that a mathematical model may be developed in a concise manner, it is necessary that now we define the fundamental concepts.

A "force" is a body or collection of organized fighting units. These "fighting units" may consist of men, ships, airplanes, tanks or any other component which is capable of contributing to the victory of its side. There are many factors which contribute to this capability. These could include weapon effectiveness, speed, maneuverability, vulnerability, etc. A fighting unit is said to be destroyed when it is no longer capable of contributing to the success of its own side.

Two adversaries, Ivan and Sam, each deploying a force are engaged in a conflict, the conflict ending when one force breaks off the encounter. The force which does not break off the conflict is said to have won the battle.

The properties of the fighting units may be summarized into two sets of constants, b and c, called "attrition coefficients." The manner in which this summarization is performed is dependent on the situation. Details will be delayed until specific models are considered.

The rate at which the size of a given force is diminished is called the "rate of attrition."

The formulation of a model for a particular situation depends on many factors. Among the most important of these are:

1. types of units involved and weapons possessed,
2. tactics employed by each side.

Having then determined the interaction of one opponent on the other, one considers the type of mathematical analysis to be attempted, and its objectives.

The most famous approach to attrition modeling was introduced by Frederick W. Lanchester in World War I and attempted to describe the effects of concentration in warfare by means of differential equations [12]. The sets have come to be known as Lanchester's linear law and Lanchester's square law. The former is a representation of combat where there is no concentration of force, but rather unconcentrated area fire where shooters do not know when a target is killed. The latter is a representation of concentration, or aimed fire; that is, shooters know when a target is killed, and concentrate their fire on the survivors.

Let \$ and X be the two opponents, $\$_0$ and X_0 their respective strengths, and $\$_t$ and X_t the strengths at time t after the start of the conflict. The rate of decrease in $\$_t$ and X_t is governed by differential equations of the type:

$$\frac{d\$_t}{dt} = -Q(\$_t) - B(X_t) - U(X_t)(\$_t) \qquad \{5.14\}$$

and

$$\frac{dX_t}{dt} = -P(X_t) - C(\$_t) - V(X_t)(\$_t) \qquad \{5.15\}$$

When the first terms in the equation correspond to attrition during target interdiction, the second terms to attrition due to aimed fire, and the third terms to attrition due to indiscriminate fire. The parameters Q and P are attrition factors for the aircraft and threat, respectively; the parameters B and C are unit effectiveness factors for the threat and the aircraft, respectively; and the parameters U and V are interaction effectiveness factors for area fire. All six parameters are functions of weapon system availability, utilization (sortie rate) and survivability and/or lethality. Four special cases have been solved and are of particular interest. These are summarized in Fig. 5.1.

FOUR SPECIAL CASES OF GENERAL ATTRITION MODEL

1. Target Interdiction Mission (B=C=U=V=0)
Transient solution:

$\$_t = \$_0 e^{-Qt}$ & $X_t = X_0 e^{-Pt}$

2. Threat Suppression Due to Aimed Fire (Q=P=U=V=0)
Transient:

$$\$_t = \$_0 \cosh \sqrt{BC}\,t - X_0\sqrt{B/C}\,\sinh \sqrt{BC}\,t$$

$$X_t = X_0 \cosh \sqrt{BC}\,t - \$_0\sqrt{C/B}\,\sinh \sqrt{BC}\,t$$

State Equation:

$$C(\$_0^2 - \$_t^2) = B(X_0^2 - X_t^2)$$

3.Threat Suppression Due to Area Fire (Q=P=B=C=0)
Transient:

$$\$_t = \frac{X_0U - \$_0V}{\left[\dfrac{X_0U}{\$_0V} e^{(X_0U - \$_0V)t} - 1\right]} V = \frac{\$_0(\$_0V - X_0U)}{X_0Ue^{(X_0U-\$_0V)t} - \$_0V}$$

State Equation:

$$V(\$_0 - \$_t) = U(X_0 - X_t)$$

4. Combined Interdiction/Suppression Mission (U=V=0)

$$\$_t = \frac{(B)(X_0)(e^{\lambda_2 t} - e^{\lambda_1 t}) + \$_0\left[(Q + \lambda_1)e^{\lambda_2 t} - (Q + \lambda_2)e^{\lambda_1 t}\right]}{\lambda_1 - \lambda_2}$$

$$X_t = \frac{-\dfrac{\$_0}{B}(Q + \lambda_1)(e^{\lambda_2 t} - e^{\lambda_1 t}) + X_0\left[(\lambda_1 + Q)e^{\lambda_1 t} - (Q + \lambda_2)e^{\lambda_2 t}\right]}{\lambda_1 - \lambda_2}$$

Where

$$\lambda_1 = \frac{-(Q + P) + \sqrt{(Q - P)^2 + 4(B)(C)}}{2}$$

$$\lambda_2 = \frac{-(Q + P) - \sqrt{(Q - P)^2 + 4(B)(C)}}{2}$$

The derivations for these cases are presented in Appendix B.

Figure 5.1 Special Cases of Attrition.

5.4 Application to Tradeoff Methodology

The simulation model developed in the companion research is based on attrition rates for both the attacking aircraft and the threat, the defense weapon sites, achieved through "aimed" fire. Indeed, most of the model deals with the susceptibility of the aircraft to detection, acquisition, and tracking by the threat. Therefore, Case 2 in Fig. 5.1 applies. Consider the State Equation for Case 2,

$$C(\$_0^2 - \$_t^2) = B(X_0^2 - X_t^2) \tag{5.16}$$

where

$$B = (SRX)(AX)(1 - P_{S/S}), \tag{5.17}$$

$$C = (SR\$)(A\$)(P_{K/S}) \tag{5.18}$$

and SR\$ is the aircraft sortie rate, A\$ is aircraft availability, $P_{S/S}$ is the sortie survivability and $P_{K/S}$ is the sortie lethality. It is important to note that $P_{S/S}$ is a composite of *all* the sorties that an aircraft flies, many of which are not marked by encounters. Since the survivability probability occuring during sorties where there are no encounters can be considered to be equal to one,

$$P_{S/S} = \frac{P_{S/e\$} + n_{e\$}}{1 + n_{e\$}} \tag{5.19}$$

where $P_{S/e\$}$ is the probability of survival in an encounter sortie and $n_{e\$}$ is the average number of sorties occuring between encounter sorties of friendly forces. Similarly,

$$P_{K/S} = 1 - \frac{P_{S/eX} + n_{eX}}{1 + n_{eX}} \tag{5.20}$$

where P_{S/s_X} is the probability of survival in an encounter sortie and n_{s_X} is the average number of sorties occuring between encounter sorties of enemy forces.

Substituting Eqs. 5.16 and 5.17 into 5.15 yields:

$$\$_t^2 = \$_0^2 - \frac{(SRX)(AX)}{(SR\$)(A\$)}\frac{1-P_{S/S}}{P_{K/S}}(X_0^2 - X_t^2) \tag{5.21}$$

For sake of convenience, an "Attrition and Sortie Rate" factor, ASR, is defined as follows:

$$ASR = \frac{(SRX)(AX)}{(SR\$)(A\$)} \tag{5.22}$$

Assuming that complete annihilation occurs for the side that loses, the battle will be complete and the number of units surviving will be positive for the victorious force, \$, and zero for the vanquished force, X. Thus,

$$\$_t = \$_0\sqrt{1 - ASR\frac{1-P_{S/S}}{P_{K/S}}(X_0/\$_0)^2} \tag{5.23}$$

Several measures of effectiveness (MOEs) can be defined. First, there is the campaign survivability, CS:

$$CS = \frac{\$_t}{\$_0} = \sqrt{1 - ASR\frac{1-P_{S/S}}{P_{K/S}}(X_0/\$_0)^2} \tag{5.24}$$

The fraction force lost, FFL, useful in calculating several MOEs, is:

$$FFL = 1 - \sqrt{1 - ASR\frac{1-P_{S/S}}{P_{K/S}}(X_0/\$_0)^2} \tag{5.25}$$

The exchange ratio, ER, is the ratio of opponents killed versus friendly forces killed, or:

$$ER = \frac{X_0 - X_t}{\$_0 - \$_t} = (X_0/\$_0)\left[1 - \sqrt{1 - ASR\frac{1-P_{S/S}}{P_{K/S}}(X_0/\$_0)^2}\right]^{-1} \tag{5.26}$$

The relative exchange ratio, RER, corrects the kill rate divided by the loss rate, for the peacetime inventory ratio, $X_0/\$_0$:

$$RER = \frac{(X_0 - X_t)}{(\$_0 - \$_t)}\frac{\$_0}{X_0} = \left[1 - \sqrt{1 - ASR\frac{1 - P_{S/S}}{P_{K/S}}(X_0/\$_0)^2}\right]^{-1} \tag{5.27}$$

The possible crew loss PCL is the number of aircraft lost which is, in equation form,

$$PCL = \$_0 - \$_t = \$_0\left[1 - \sqrt{1 - ASR\frac{1 - P_{S/S}}{P_{K/S}}(X_0/\$_0)^2}\right] \tag{5.28}$$

The aircraft replacement cost, RC, is the number of aircraft lost multiplied by the aircraft acquisition cost, AC,

$$RC = (\$_0 - \$_t)AC = \$_0(AC)\left[1 - \sqrt{1 - ASR\frac{1 - P_{S/S}}{P_{K/S}}(X_0/\$_0)^2}\right] \tag{5.29}$$

The measure of effectiveness can be thought of as the outputs of the mission tradeoff study. The inputs to the equations for the MOEs (Eqs. 5.24 - 5.29) are $P_{S/S}$, $P_{K/S}$, X_0 , and $\$_0$. $P_{S/S}$ (sortie survivability) and $P_{K/S}$ (sortie lethality) can be directly calculated from $P_{S/e_\$}$ and P_{S/e_X} (encounter sortie survivabilities of friendly and opposing forces, respectively) which are the outputs of the attack simulation model described in ASALT II. X_0, the threat inventory, is the goal -- the number of enemy aircraft to be destroyed, and $\$_0$ is the inventory of friendly aircraft at the start of the campaign.

While all six of the MOEs defined here are cost-dependent in the sense that $\$_0$ would decrease as the aircraft acquisition cost increases (assuming a fixed acquisition budget), insight can still be gained by ignoring costs temporarily. In Figs. 5.2 to 5.11, four MOEs are plotted for five aircraft design/decision variables used in the scenario simulation model of the companion research. These graphs pick up where the results of the mission scenario models leave off -- meaning that while resulting plots of $P_{S/e}$ and $P_{K/e}$ versus the various aircraft design/decision variables can be obtained, Figs. 5.2 to 5.11 show plots in which overall MOEs are used which permits us to reconcile or tradeoff conflicting changes in survivability, $P_{S/S}$, and lethality, $P_{K/S}$

5.5 Example Tradeoff Studies - Fighter Escort

For the fighter escort mission, a hypothetical aircraft, the CBFE (Carrier-Based aircraft for Fighter Escort) is used. In Figs. 5.2-5.3, four measures of effectiveness (CS, FFL, ER and RER) are determined and plotted against the range of the CBFE's long range missiles. The ranges of the CBFE weapons and the related encounter sortie survivabilities are shown in Table 5.3. In Fig. 5.2, Campaign Survivability is shown to increase for ranges greater than 104 km. The lack of data for ranges less than 104 km can be explained by Fig. 5.3, where the Relative Exchange Ratio becomes smaller than one in this region; that is to say, for unsatisfactory missile ranges, CS is essentially zero.

Similar results are found for the MOE's relating to T/W in Figs. 5.4-5.5. Greater values of T/W result in more favorable Exchange Ratios, as well as a higher Campaign Survivability. The various T/W configurations and their associated survivabilities can be seen in Table 5.4.

Table 5.5 lists the various Radar Cross Sections analyzed along with their respective encounter sortie survivability values. In Fig. 5.6, it can be seen that an increase in Radar Cross Section will give rise to a decrease in Campaign Survivability, a finding which is in agreement with intuition. It is important to point out that in Fig. 5.6, the trend shows a marked change in slope somewhere between RCS values of 1 and 5 sq. m. Fig. 5.7 shows this inflection with more resolution at RCS values at about 3 sq. m. This would imply that an CBFE with an RCS of 3 will not suffer much decrease in Exchange Ratios for increases in its RCS, but any decrease in its RCS will result in a markedly higher ER.

Table 5.3 - Weapon Ranges

Configuration	Ranges (km) LR/MR/SR/GUNS	P_{S/ε_s}	P_{S/ε_x}
Baseline	130/47/7.5/2.5	0.682	0.155
Mod. #1	182/66/10.5/3.5	0.786	0.128
Mod. #2	156/57/9.0/3.0	0.739	0.145
Mod. #3	104/38/6.0/2.0	0.633	0.150
Mod. #4	78/28/4.5/1.5	0.455	0.205
Mod. #5	52/19/3.0/1.0	0.204	0.458

Table 5.5 - Radar Cross Section Variations

Configuration	RCS m^2	P_{S/ε_s}	P_{S/ε_x}
Baseline	5	0.682	0.155
Mod. #1	50	0.667	0.159
Mod. #2	2	0.685	0.153
Mod. #3	1	0.694	0.152
Mod. #4	0.5	0.714	0.149

The baseline tactical engagement vector for the previous scenarios utilizes a 0° (head-to-head) angle. Figs. 5.8-5.11 show results for variations on the initial vectors as they affect the MOE's. Tactics (A) referred to in Figs. 5.8-5.9 make use of a 0° angle assumed by the aggressor (enemy) force with variations imposed by the CBFE force; encounter sortie survivability values for these tactics are shown in Table 5.6. Tactics (B) as shown in Figs. 5.10-5.11 and assumes a constant initial vector of 15° for the CBFE force relative to variations in the headings of the aggressor force; survivability values are shown in Table 5.7.

Table 5.6 - Tactics (A) Variations

Configuration 0° Aggressor	Tactic CBFE Angle	P_{S/ε_s}	P_{S/ε_x}
Baseline	0	0.682	0.155
Mod. #1	15	0.638	0.206
Mod. #2	30	0.691	0.204

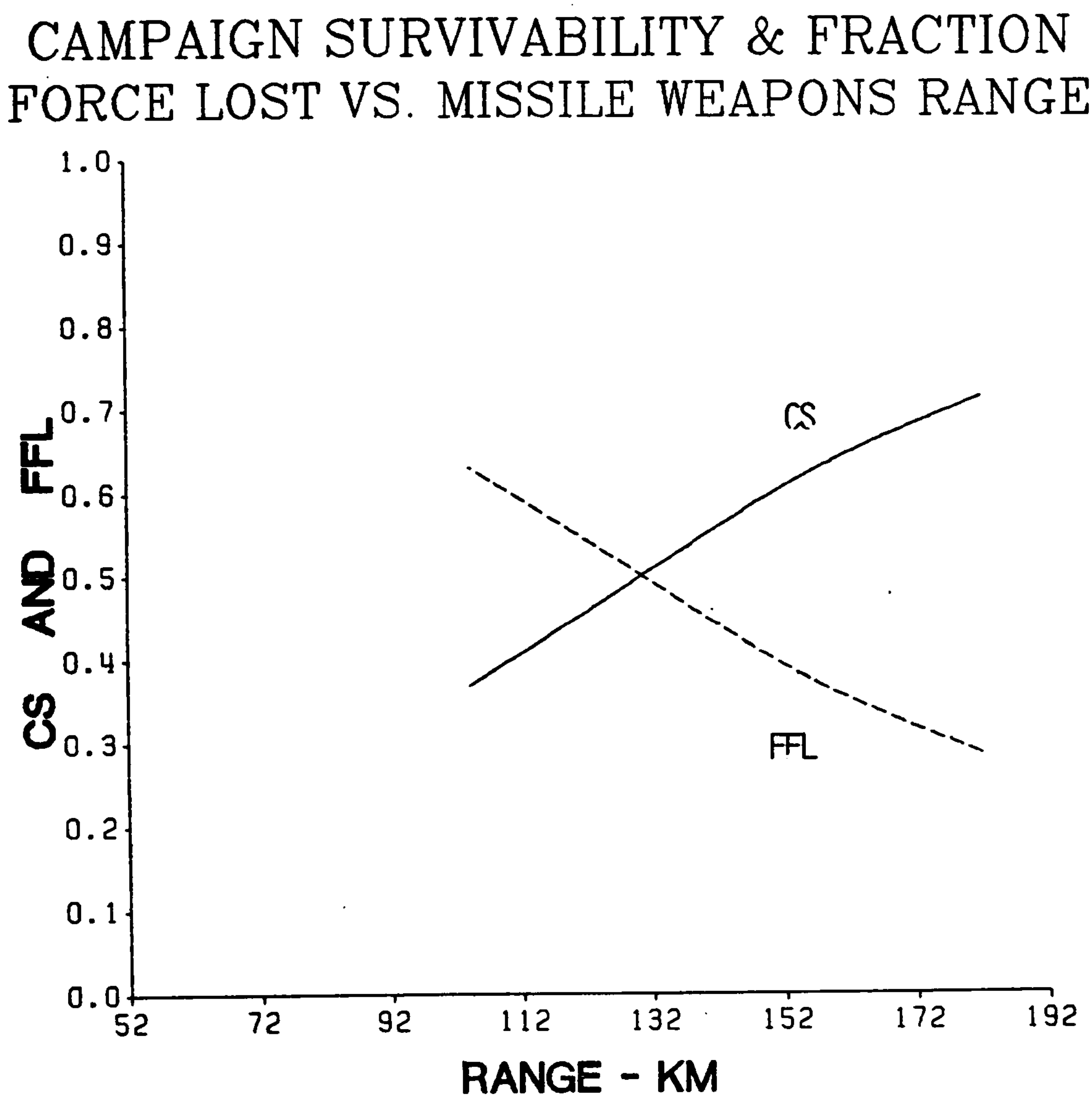


Figure 5.2 Missile Weapons Range vs. CS and FFL

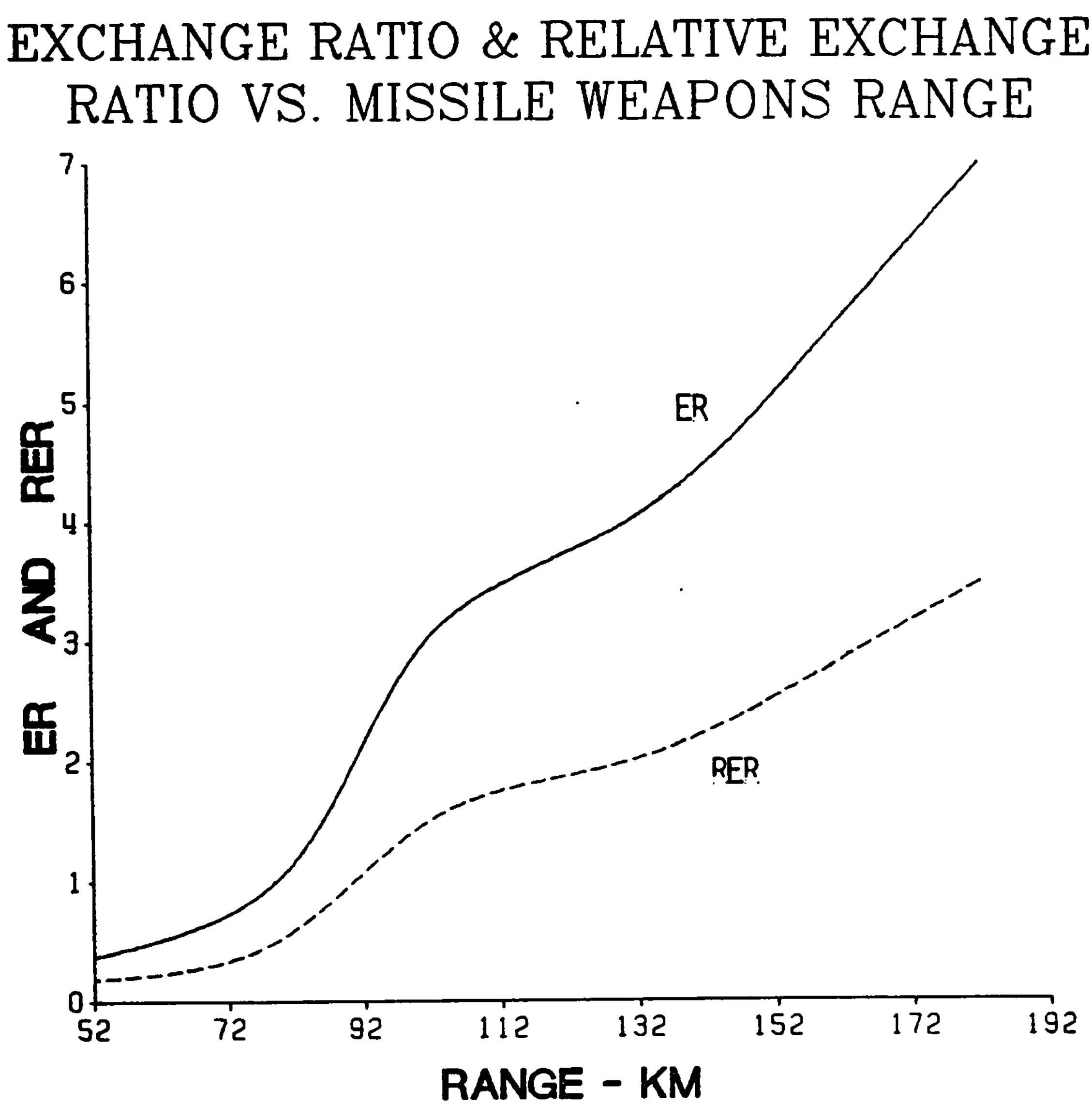


Figure 5.3 Missile Weapons Range vs. ER and RER

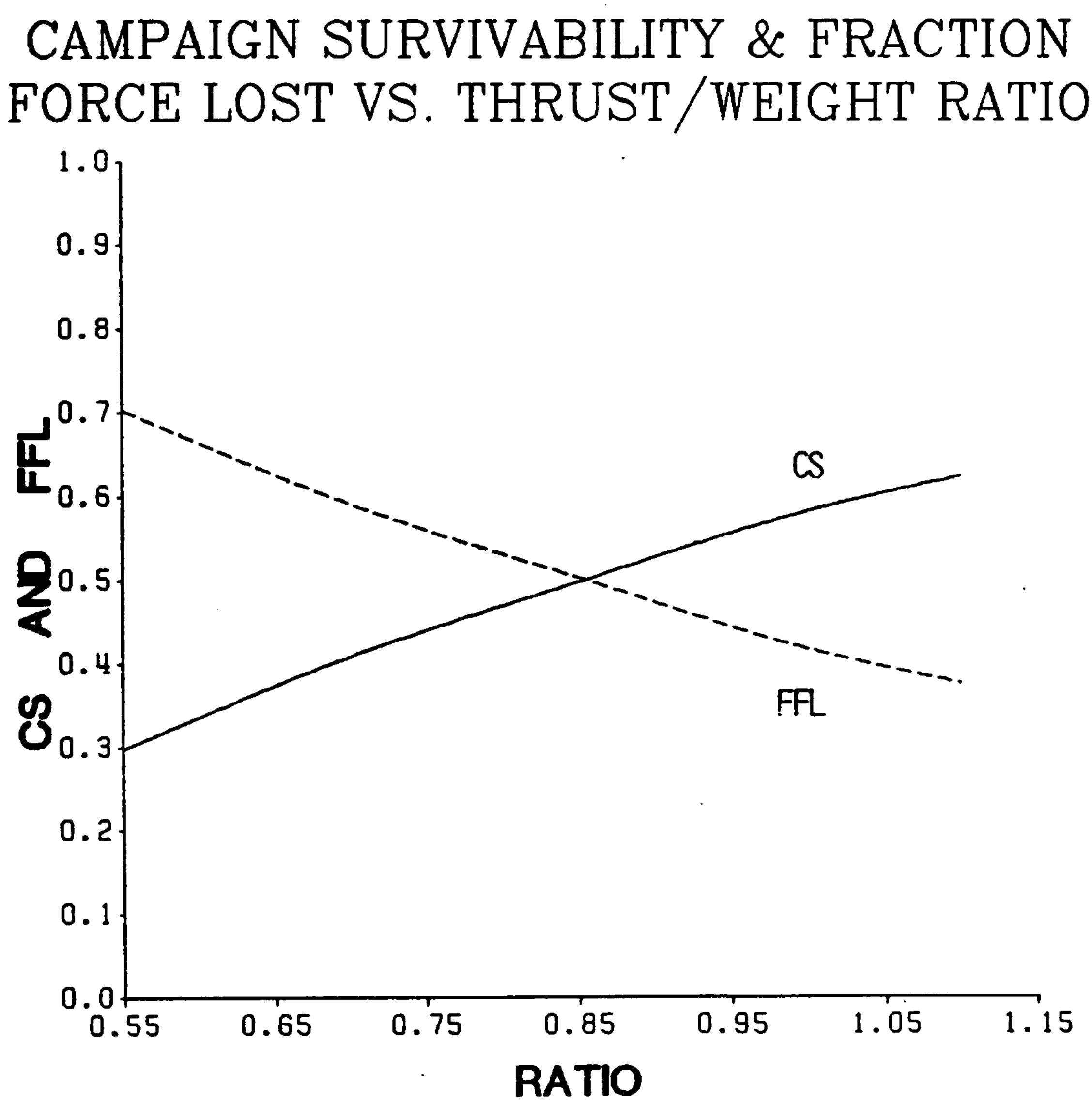


Figure 5.4 Thrust-to-Weight Ratio vs. CS and FFL

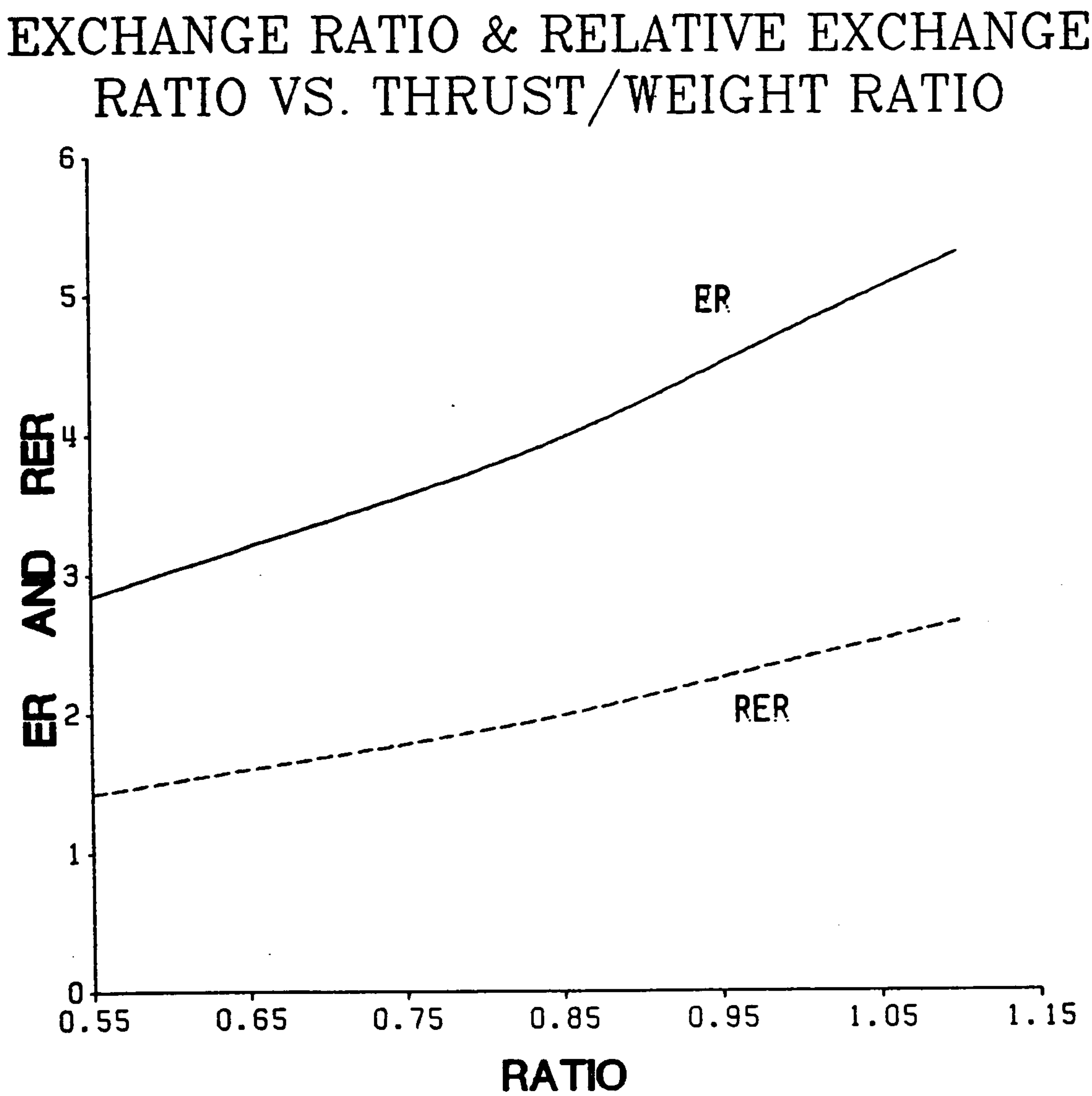


Figure 5.5 Thrust-to-Weight Ratio vs. ER and RER

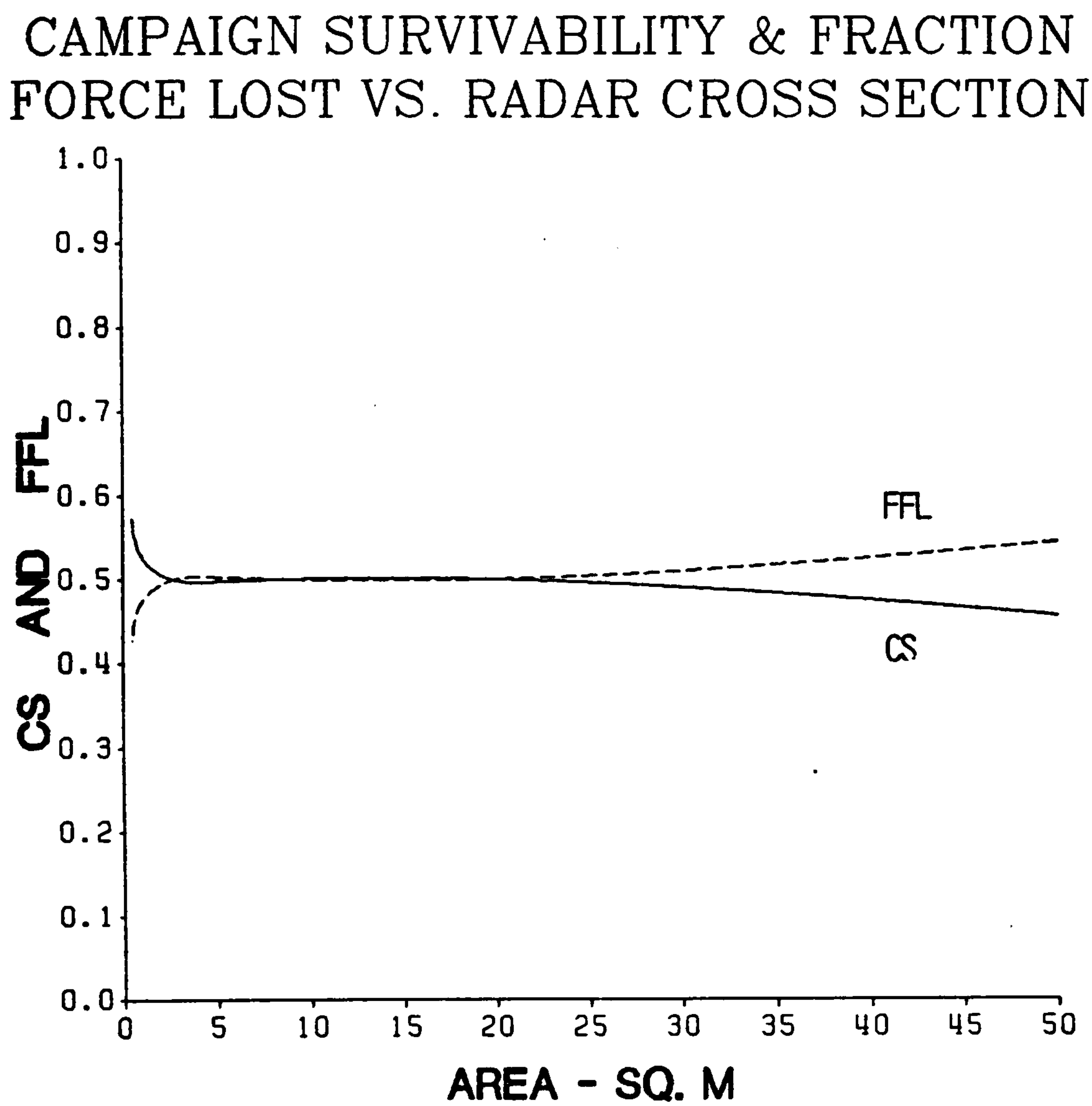


Figure 5.6 Radar Cross Section vs. CS and FFL

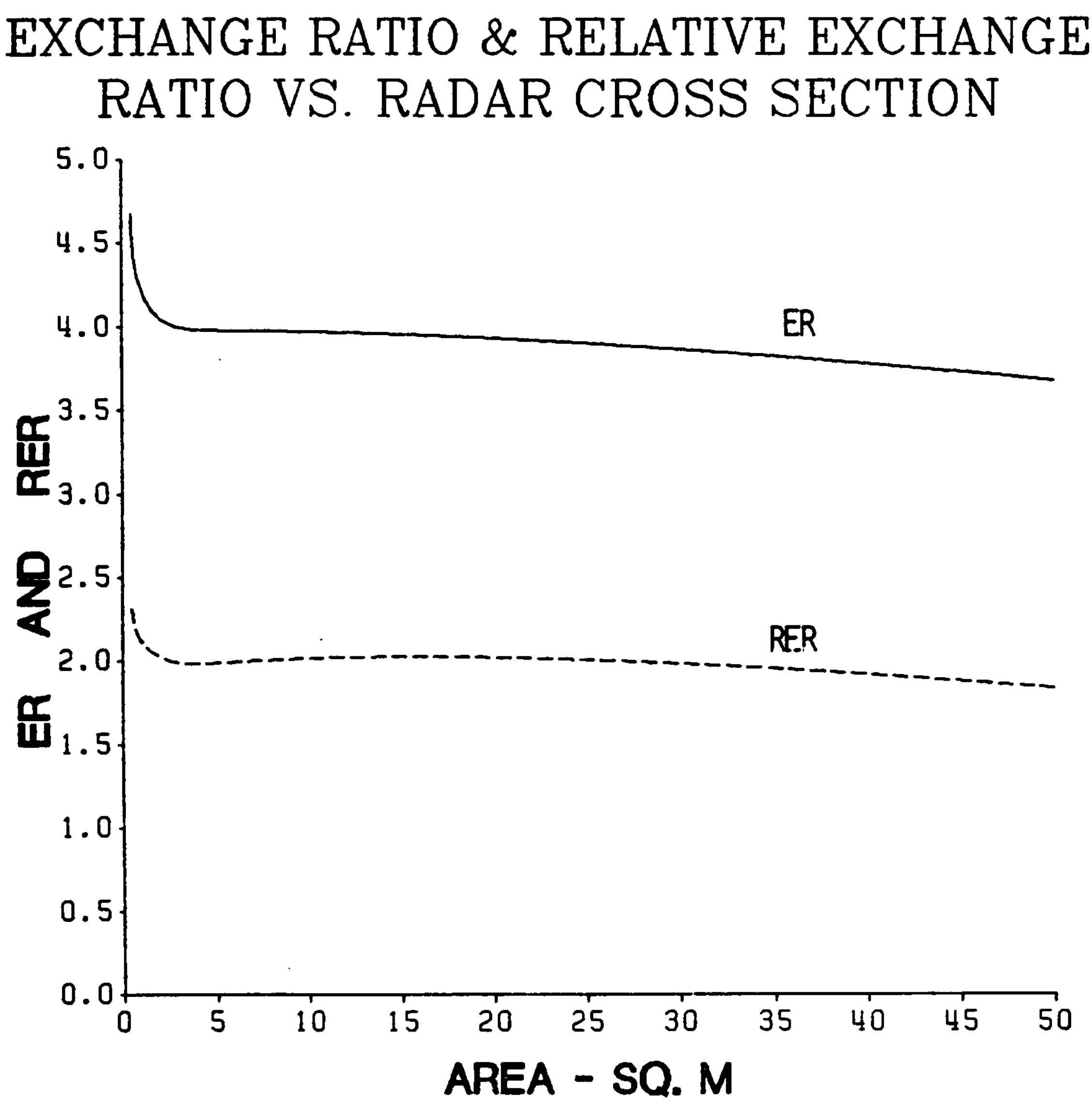


Figure 5.7 Radar Cross Section vs. ER and RER.

Table 5.4 - Thrust/Weight Variations

Configuration	T/W Ratios	P_{S/ε_s}	P_{S/ε_x}
Baseline	0.85	0.682	0.155
Mod. #1	0.55	0.618	0.162
Mod. #2	0.70	0.649	0.157
Mod. #3	1.00	0.718	0.147
Mod. #4	1.10	0.737	0.140

Table 5.7 - Tactics (B) Variations

Configuration 15^{o} CBFE	Tactic Aggressor Angle	P_{S/ε_s}	P_{S/ε_x}
Mod. #0	15	0.667	0.223
Mod. #1	30	0.653	0.218
Mod. #2	75	0.627	0.273

5.5.1 Example 5.1

Suppose that four modifications of a projected attack aircraft (i.e a baseline design) are to be investigated using the aircraft effectiveness tradeoff methodology previously described. These modifications are simply changes to the baseline aircraft powerplant in order to appreciate the variations of thrust/weight on the four measures of effectiveness (MOE) previously described. The goal here is to choose the "best" airframe-powerplant configuration that yields the minimum acquisition cost ratio (ACR), minimum possible crew loss (PCL), and the highest campaign survivability (CS) for a given mission scenario. Several assumptions have to be made in order to simplify the problem and they are as follows: 1) The main airframe geometric characteristics are kept constant except those associated with the inlet size to accommodate the various mass flow rate requirements associated with each particular powerplant installation to be studied. 2) Aerodynamic drag characteristics are varied to account for changes in inlet drag. 3) Radar cross section (RCS) characteristics are also varied

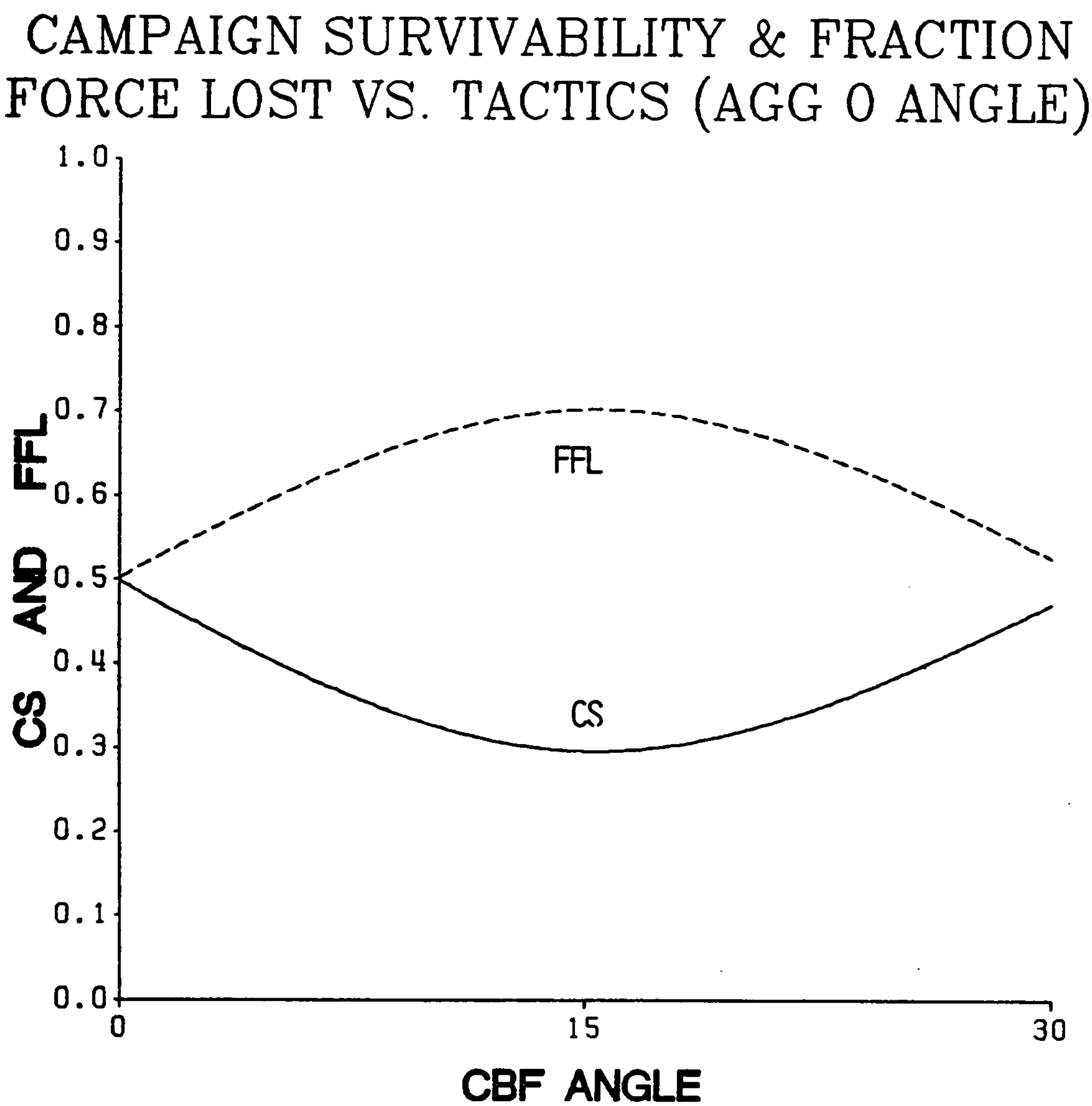


Figure 5.8 Tactics (A) vs. CS and FFL

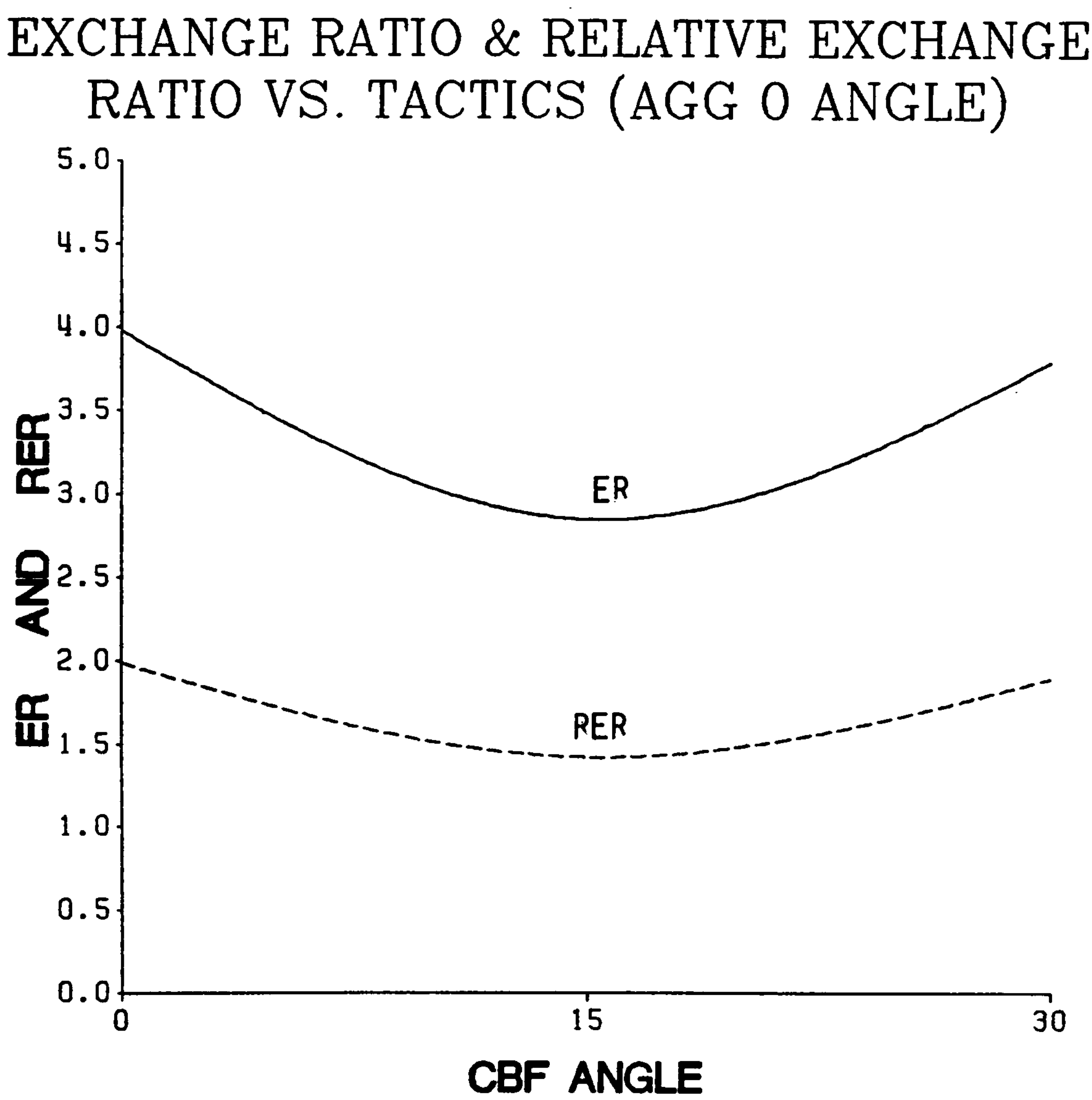


Figure 5.9 Tactics (A) vs. ER and RER.

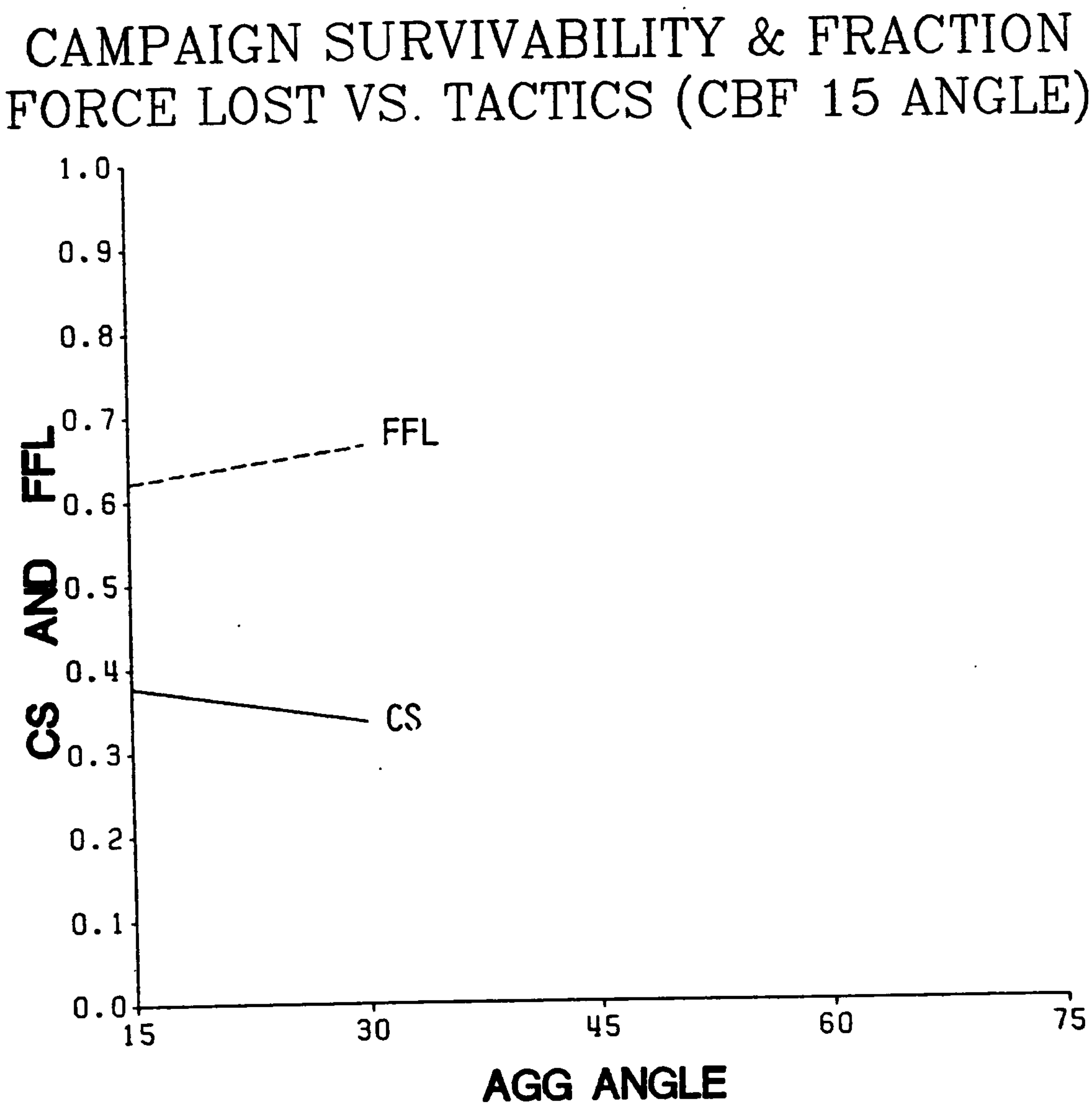


Figure 5.10 Tactics (B) vs. CS and FFL

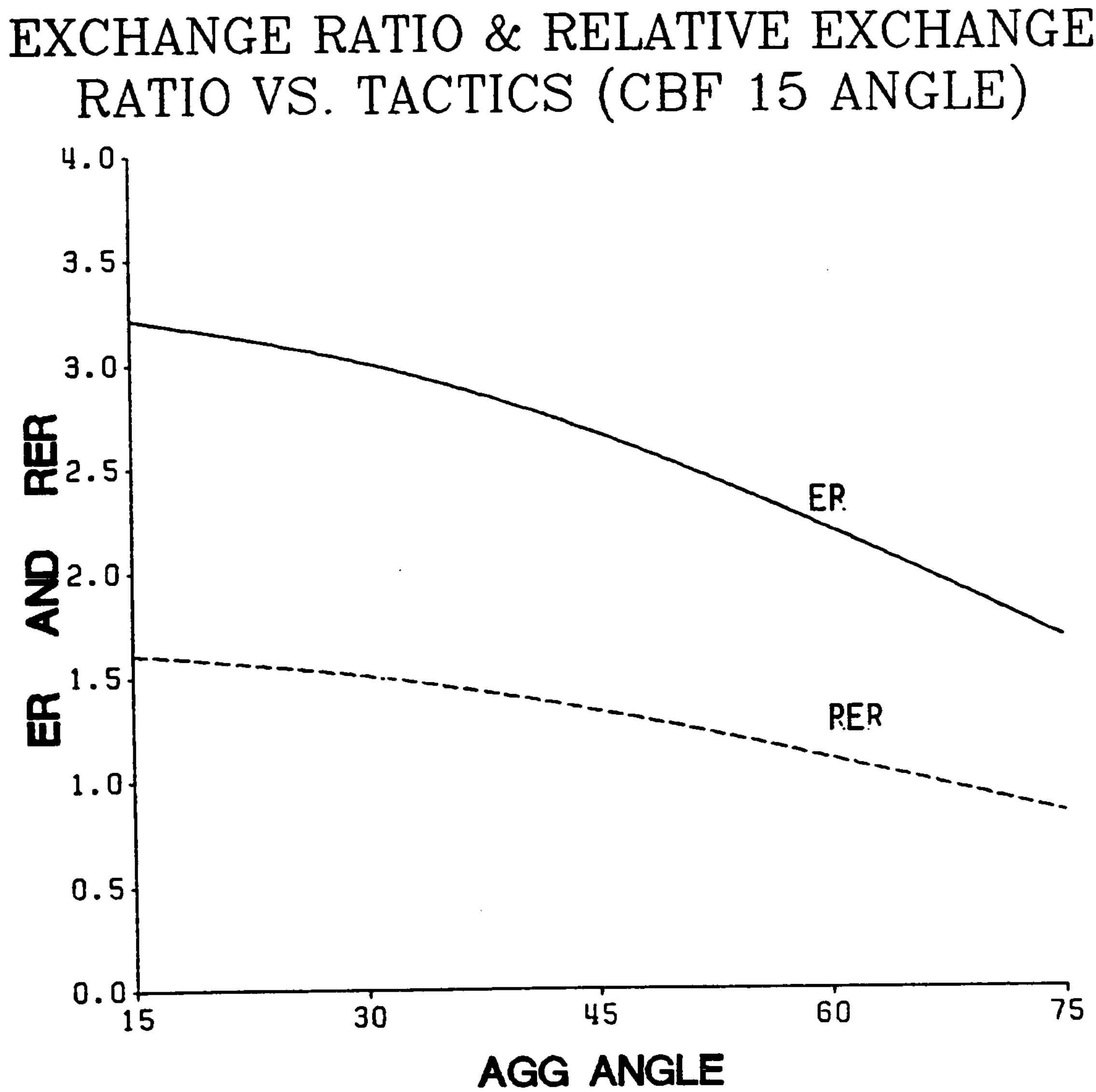


Figure 5.11 Tactics (B) vs. ER and RER.

to account for changing inlet sizes. 4) Changes in aircraft weight are solely due to the different engines installed.

The relevant characteristics of the baseline design are given in Table 5.8.

Table 5.8 Aircraft Configurations for Example 5.1

Configuration	Thrust/Weight	Static Thrust at Sea Level (kg)*
Baseline	0.85	16262
Mod. #1	0.55	10526
Mod. #2	0.70	13682
Mod. #3	1.00	18946
Mod. #4	1.10	21050

* max. afterburner

The driving parameter for each modification was the thrust-to-weight ratio for the particular mission weight. Under the assumption of a fixed mission weight (e.g. good only for short missions) it is possible to size the powerplant to be used to exactly match the desired thrust/weight parameter. Table 5.9 summarizes the selected of thrust-to-weight ratios used in this example.

The first step to be performed in this tradeoff study is to determine the new physical engine characteristics of the modified aircraft in order to later on estimate radar cross section (RCS) and drag variations from those of the baseline design.

For the purpose of preliminary engine sizing it is assumed that the modified powerplant area requirements can be estimated from the basic thrust equation for a turbojet/turbofan engine. Defining T_{mod} and T_{base} as the modified and baseline thrusts, respectively; $\dot{m}_{mod}$ and $\dot{m}_{base}$ as the corresponding mass flow rates; also, A_{mod} and A_{base} are the modified and baseline engine compressor face areas. It can be shown that, for similar operating speed and pressure conditions,

$$\frac{T_{mod}}{T_{base}} \propto \frac{\dot{m}_{mod}}{\dot{m}_{base}} \propto \frac{A_{mod}}{A_{base}} \tag{5.30}$$

Table 5.9 CBFE Aircraft Characteristics for Example 5.1

Aircraft weights	
Maximum takeoff gross weight	25000 kg
Empty weight	14000 kg
Maximum fuel weight	6000 kg
Maximum payload weight	6500 kg
Mission weight	19300 kg
Wing characteristics	
Aspect ratio	3.5
Wing area	40 sq. m
Wing efficiency factor	0.79
Engine characteristics	
Number of engines	2
Static sea level thrust	8131 kg/engine
Engine diameter	0.95 m
Engine frontal area	0.709 sq. m
Engine mass	1054 kg
Engine thrust/mass	7.71:1
Thrust variations with Mach number and altitude	
Aircraft signature	
Aircraft radar cross section	5 sq. meters
Aircraft aerodynamic characteristics	
Aircraft C_{D0} (clean)	0.011 below mach 0.8
Aircraft C_{D0} (loaded)	0.023 below mach 0.8
Aircraft C_{D0} varies with Mach number	

from where an estimate of the compressor area requirements can be derived from either the mass flow rate or the thrust. The engine diameter can be readily estimated from which, ultimately, engine masses corresponding to each modification can be assessed. A useful relationship for these calculations is:

$$D_{mod} = D_{base}\sqrt{\frac{\dot{m}_{mod}}{\dot{m}_{base}}} \tag{5.31}$$

where D_{mod} and D_{base} represent the engine diameters for the modified and the baseline aircraft, respectively.

Another important engine parameter considered in preliminary engine sizing is the length of the powerplant, L. According to Nicolai [17] the following expression has been found applicable in current fighter designs.

$$L_{mod} = L_{base}\left(\frac{\dot{m}_{mod}}{\dot{m}_{base}}\right)^{n-\frac{1}{2}} \tag{5.32}$$

where L_{mod} and L_{base} are the modified and baseline engine lengths, respectively, and n is a scaling factor usually taken as unity for most practical purposes.

Regression analysis was performed for 11 current fighter aircraft engines in the same thrust category of the modifications sought in order to estimate trends in engine mass versus engine diameter [1]. This study showed good correlation (90%) and is used to estimate engine masses for all modifications; where the derived relationship is as follows:

$$W_{engine} = k_1 + k_2 D_{engine} \tag{5.33}$$

where k_1 and k_2 are constants, and D_{engine} is the engine diameter taken as reference. It follows that for a modified vs. a baseline engine,

$$W_{mod} = W_{base}\frac{D_{mod}}{D_{base}} \tag{5.34}$$

For added structural mass similar analogies are used -- starting with the added fuselage structural volume to support the modified powerplant. Assuming an average aircraft structural density and that the volume of the modified engine is proportional to the new required structural volume to support it, then expression 5.35 can be used to estimate the mass changes associated with the new powerplant.

$$\frac{V_{mod}}{V_{base}} = \frac{D^2_{mod}L_{mod}}{D^2_{base}L_{base}} \tag{5.35}$$

and,

$$M_{mod} = M_{base}\frac{V_{mod}}{V_{base}} \tag{5.36}$$

where, V_{mod} and V_{base} are the structural volumes of the modified and baseline aircraft, respectively; M_{mod} and M_{base} are the masses of the modified and baseline aircraft, respectively.

For the present analysis the baseline fuselage mass affected in the modification process is taken as 3500 kg for computational purposes. The final values of the mass changes for each modification are shown in Table 5.10.

Table 5.10 - Dimension/Mass Variations

Configuration	D (m)	L (m)	ΔM (kg)
Baseline	0.950	4.040	0.0
Mod. #1	0.760	3.250	-1698
Mod. #2	0.867	3.705	-828
Mod. #3	1.019	4.361	843
Mod. #4	1.073	4.590	1569

Another important issue is the drag increase associated with installation of a different engine for each modification. The simplifying assumption is that only additive drag is considered to increase CD_0 at a rate of 7.5 counts per 0.1 square meter of increased engine frontal area (one count of C_{D0} is 1/10000). The reasoning behind this seemingly arbitrary assumption is the fact that additive drag (spillage drag as sometimes referred) is very sensity to the geometry of the particular inlet, information not available and beyond the scope of this simple example. The numerical results of drag changes for each configuration are in terms of the change in clean aircraft zero lift drag coefficient (CD_0).

The last parameter affected directly with the installation of a new engine is the radar cross section area (RCS) associated with it. Crispin [18] offers a simple equation to approximate cavities such as that those characteristic of aircraft inlets as shown in Eq. 5.37.

$$\sigma \propto \frac{A_c^2}{\lambda^2} \tag{5.37}$$

and for a constant λ:

$$\frac{\sigma_{\text{mod}}}{\sigma_{base}} = \frac{A_{\text{mod}}^2}{A_{base}^2} \tag{5.38}$$

where σ_{mod} and σ_{base} represent the radar cross sectional areas of the modified and baseline aircraft, respectively; A_i is the engine area and λ is the radar wavelength. Finally it should be realized that the inlets only constitute one portion of the many radar reflecting surfaces found in an aircraft and for this matter it has been assumed that 60 % of the total aircraft radar cross section (RCS) is due to the inlets. The corresponding numerical results of the computation of RCS for each modification are shown in Table 5.5.

Items 1,2,5, and 8 in Table 5.11 constitute the principal inputs to investigate the new survivability and lethality of each modified aircraft. The engine thrust is scaled down or up in the code according to the known values of engine thrust at sea level (item 1 in Table 5.11). The second input to the program is the aircraft weight. The weights shown in Table 5.11 correspond to the mission weights to be used for each run since it has been assumed that the only two sources of mass reduction/increase are different engine masses and the structural modifications to each aircraft. Values for ΔC_{D0} are also to be incorporated. Lastly, the

Table 5.11 Summary of Modifications for Baseline Aircraft.					
		ALTERNATIVE A/C			
DESCRIPTION	BASE	MOD. 1	MOD. 2	MOD. 3	MOD. 4
1) Engine Thrust SSL (kg)	8131	526	684	9473	10525
2) Aircraft Mass (kg)	19300	17602	18473	20143	20869
3) Thrust/Weight	0.85	0.55	0.70	1.00	1.10
4) ΔC_{D0} (counts)	0.0	-30.60	-14.25	12.90	23.40
5) Engine Capture Area (sq. m)	0.363	0.472	0.567	0.653	0.723
6) Engine Diameter (m)	0.95	0.76	0.87	1.02	1.07
7) Radar Cross Section (sq. m)	5.00	3.23	4.08	5.98	6.88
8) Normalized RCS	1.00	0.646	0.816	1.196	1.376

normalized RCS values shown in Table 5.11 correspond to the values of a multiplier for each modification.

Using the outputs of the ASALT model presented in the companion research (i.e. encounter sortie survivability and lethality) and the tradeoff methodology previously described one obtains Tables 5.12 and 5.13 where two important tradeoff cases are depicted variations in thrust/weight.

In the first one (i.e., Table 5.12) a constant procurement run of 1000 aircraft is assumed for all the configurations studied and the idea is to determine the six measures of effectiveness (MOE's) defined by Eqs. 5.24-5.29 (items 15-20 of Table 5.12).

Analysis of these six MOE's reveals a logical trend for an increase in survivability with respect to increases in the thrust-to-weight ratio. However, when comparing Table 5.12 to Table 5.4, it is important to recall that the $P_{S/e}$ values listed in Table 5.4 consider *only* the effects to increasing T/W, and do *not* take into account the accompanying increases in weight, radar cross section, and drag. A comparison of these encounter sortie survivability values (between Tables 5.4 and 5.12) shows that increases in T/W do not have as great an effect on survivability when the other design variables are included. For example, survivability for modification 4 found in Table 5.4 has a value of 0.737, and in Table 5.12, 0.722. Nonetheless, Table 5.12 shows monotonic improvements in all the MOE's, implying that the baseline configuration is suboptimal.

Table 5.12 Aircraft Effectiveness Tradeoff Methodology.

		Alternative A/C				
DESCRIPTION	FORMULATION	BASE	MOD. 1	MOD. 2	MOD. 3	MOD. 4
1) Threat Inventory	X_0 = Goal	2000	2000	2000	2000	2000
2) Thrust-to-Weight Ratio		0.85	0.55	0.70	1.00	1.10
3) Acquisition Cost Ratio	ACR=ACM/ACB			NOT APPLICABLE		
4) Production Run	$\$_0M = \$_0B(ACR)$	1000	1000	1000	1000	1000
5) Encounter Sortie Surv.	$P_{S/\varepsilon_\$}$	0.682	0.631	0.658	0.702	0.722
6) No. to Encounter Sorties	$n_{\varepsilon_\$}$	19	19	19	19	19
7) Sortie Survivability	$P_{S/S}$ Eq.{5.19}	0.984	0.982	0.983	0.985	0.986
8) Effectiveness of X	B Eq.{5.17}	.016	.018	.017	.015	.014
9) Encounter Sortie Surv.	P_{S/ε_X}	0.155	0.160	0.158	0.150	0.143
10) No. to Encounter Sorties	n_{ε_X}	9	9	9	9	9
11) Sortie Lethality	$P_{K/S}$ Eq.{5.20}	0.0845	0.0840	0.0842	0.0850	0.0857
12) Effectiveness of $	C Eq.{5.18}	0.0845	0.0840	0.0842	0.0850	0.0857
13) Surviving No. of $	$\$_t$ Eq.{5.23}	497	348	433	547	593
14) No. of X Destroyed	$X_0 - X_t = X_0$	2000	2000	2000	2000	2000
15) Campaign Survivability	CS Eq.{5.24}	0.497	0.348	0.433	0.547	0.593
16) Fraction Force Lost	FFL Eq.{5.25}	0.503	0.652	0.567	0.453	0.407
17) Exchange Ratio	ER Eq.{5.26}	3.979	3.070	3.528	4.412	4.910
18) Rel. Exchange Ratio	RER Eq.{5.27}	1.989	1.535	1.764	2.206	2.455
19) Possible Crew Loss	PCL Eq.{5.28}	503	652	567	453	407
20) Replacement Cost	RC Eq.{5.29}			NOT APPLICABLE		

The second tradeoff study that can be derived from this methodology is the analysis of the economic implications that modifying a baseline design can bring into tactical MOE's such as Campaign Survivability and Exchange Ratio. For the sake of illustration and following the same lines of the previous example suppose it is desired to know the procurement numbers (i.e.production run) necessary for each configuration to yield an equivalent Campaign Survivability to that of the baseline configuration.

Table 5.13 shows the results of such a study using the same values for encounter sortie survivability, initial enemy force inventory, and T/W modifications.

Since CS is constant for the four modifications, it is necessary to express variables ER, PCL, and RC (replacement cost) in terms of the known quantities CS, P_{SSS}, and $P_{K/S}$, (calculated directly from P_{S/ϵ_j} Eqs. 5.19-20) and then solve Eqs. 5.26, 5.28, and 5.29 for ER, PCL, and RC respectively.

Table 5.11 is again used as input to the ASALT program and values of $P_{S/\epsilon_\$}$ and P_{S/ϵ_X} are readily available as outputs. Table 5.7 shows the final results of this Cost Effectiveness Tradeoff Methodology. A preliminary conclusion that can be drawn from this data is that to achieve the maximum effectiveness for the minimum enemy counter-effectiveness (Modification 4 – C=0.0857 & B=0.014) that 932 modified aircraft would be required, an acquisition cost ratio of 1.073. That is to say that the Modification-4 aircraft could cost up to 1.073 times as much as the baseline aircraft and still be favored for cost-effectiveness.

5.5.2 Effects of Force Size Variations

The decision of determining an appropriate patrol size to send up on a fighter escort mission is affected by elements such as availability, turn around time, and crew availability, etc. Obviously, a larger sortie force would be an advantage, but for the sake of this study, insight can be gained by observing the relationships between patrol size, modifications, and survivability.

Table 5.13 Aircraft Cost-Effectiveness Tradeoff Methodology.

DESCRIPTION	FORMULATION	Alternative A/C BASE	MOD. 1	MOD. 2	MOD. 3	MOD. 4
1) Threat Inventory	X_0 = Goal	2000	2000	2000	2000	2000
2) Thrust-to-Weight Ratio		0.85	0.55	0.70	1.00	1.10
3) Acquisition Cost Ratio	ACR=ACM/ACB	1.000	0.937	0.965	1.033	1.073
4) Production Run	$\$_0M = \$_0B(ACR)$	1000	1067	1036	968	932
5) Encounter Sortie Surv.	$P_{S/\varepsilon_\$}$	0.682	0.631	0.658	0.702	0.722
6) No. to Encounter Sorties	$n_{\varepsilon_\$}$	19	19	19	19	19
7) Sortie Survivability	$P_{S/S}$ Eq.{5.19}	0.984	0.982	0.983	0.985	0.986
8) Effectiveness of X	B Eq.{5.17}	0.016	0.018	0.017	0.015	0.014
9) Encounter Sortie Surv.	P_{S/ε_X}	0.155	0.160	0.158	0.150	0.143
10) No. to Encounter Sorties	n_{ε_X}	9	9	9	9	9
11) Sortie Lethality	$P_{K/S}$ Eq.{5.20}	0.0845	0.0840	0.0842	0.0850	0.0857
12) Effectiveness of \$	C Eq.{5.18}	0.0845	0.0840	0.0842	0.0850	0.0857
13) Surviving No. of \$	$\$_t$ Eq.{5.23}	497	530	515	481	463
14) No. of X Destroyed	$X_0 - X_t = X_0$	2000	2000	2000	2000	2000
15) Campaign Survivability	CS Eq.{5.24}	0.497	0.497	0.497	0.497	0.497
16) Fraction Force Lost	FFL Eq.{5.25}	0.503	0.503	0.503	0.503	0.503
17) Exchange Ratio	ER Eq.{5.26}	3.979	3.726	3.838	4.108	4.266
18) Rel. Exchange Ratio	RER Eq.{5.27}	1.989	1.989	1.989	1.989	1.989
19) Possible Crew Loss	PCL Eq.{5.28}	503	537	521	487	469
20) Replacement Cost	RC Eq.{5.29}	1.00	1.00	1.00	1.00	1.00

Fig. 5.12 shows the relationship between patrol size (using 2, 3, and 4 CBFE aircraft against a constant 4 enemy aircraft), Thrust-to-Weight ratio (using the process as outlined in the previous section) and the Relative Exchange Ratio (Eq. 5.27).

A similar tradeoff trend is shown in Fig. 5.13 for Weapons Ranges. The data for the CBFE=4 case are identical to those that can be found Table 5.4.

Notice that in Figs. 5.12-13 that a horizontal line has been drawn at RER=1.0. This is a critical boundary for the friendly forces since crossing below it essentially deems the enemy victorious.. For this reason RER was chosen as the critical MOE because, unlike ER, it considers the initial inventory ratio. A question analysts should ask themselves is "What good is an Exchange Ratio of 2:1 when then enemy outnumbers us 3:1 ?"

The families of curves shown in Figs. 5.12-13 should used carefully. Interpolation between curves to say, a CBFE=3.5, is unrealistic (even if patrols are 50% 3 aircraft and 50% 4 aircraft) since force/survivability relationships are not linear. This is to say that the results are for integer values of aircraft; and it is important to remember that four aircraft are not necessarily twice as effective as two aircraft.

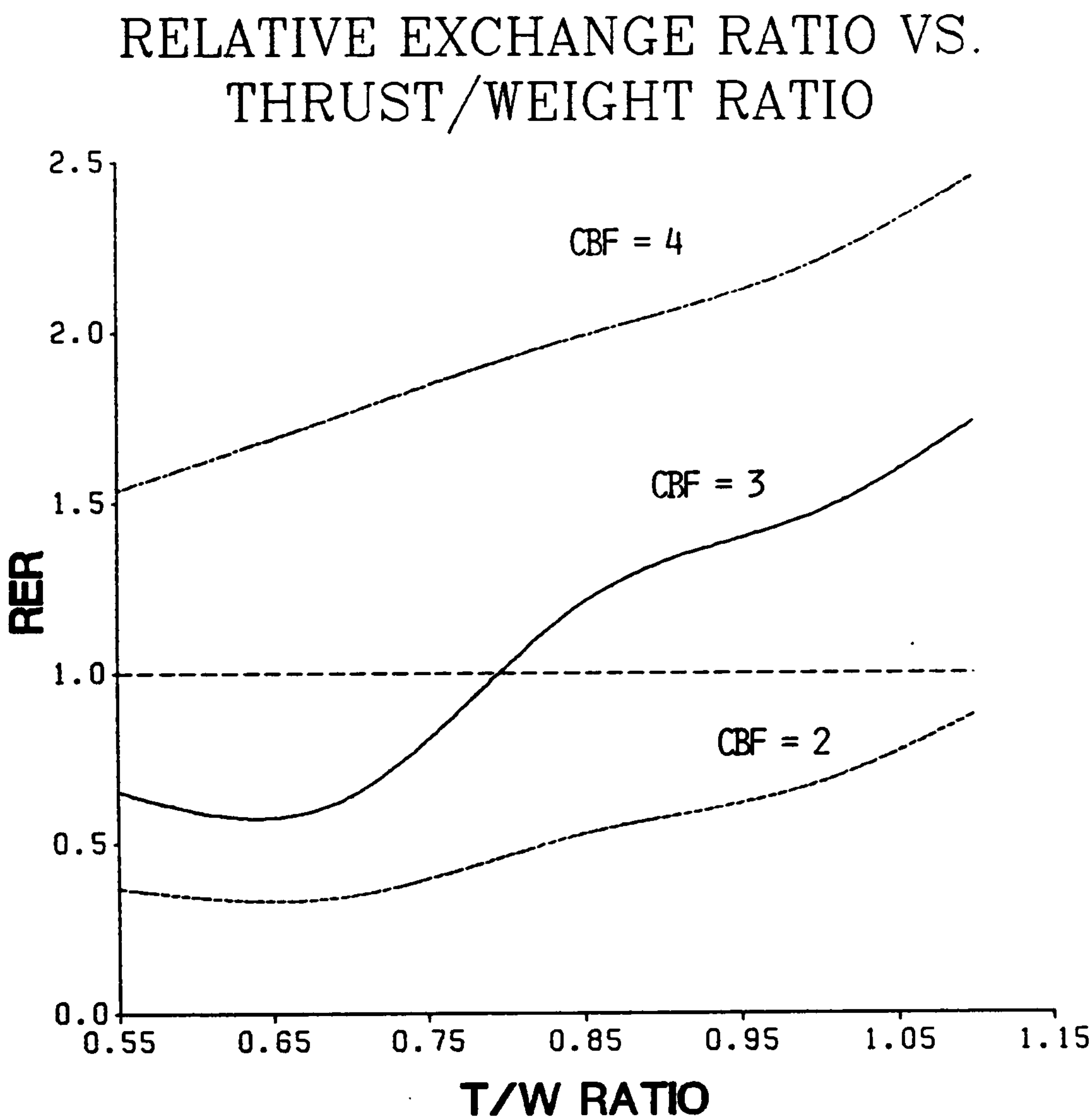


Figure 5.12 Family of Curves - T/W Ratio

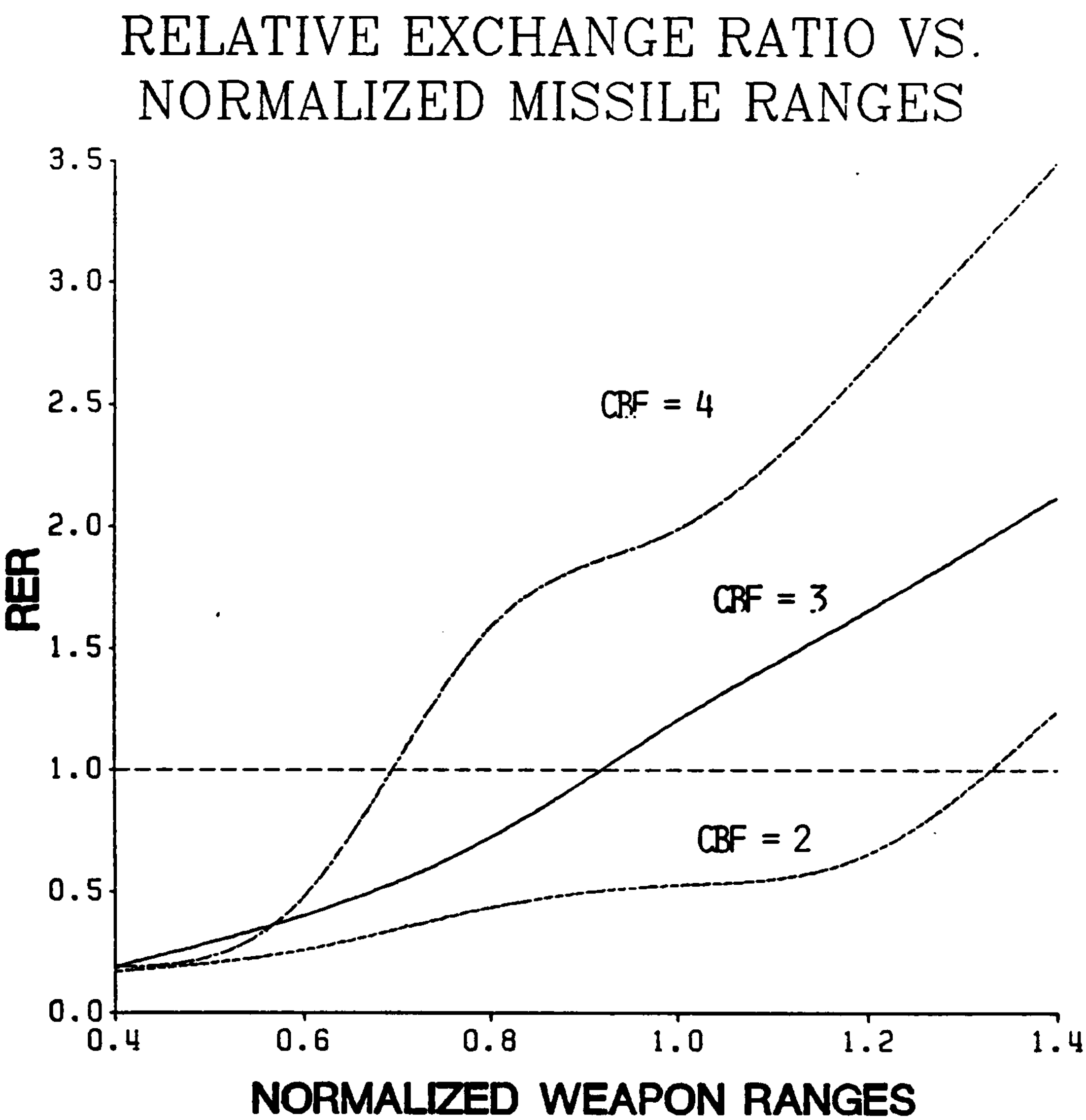


Figure 5.13 Family of Curves - Normalized Weapons Ranges

5.6 Example Tradeoff Studies - Surface Attack

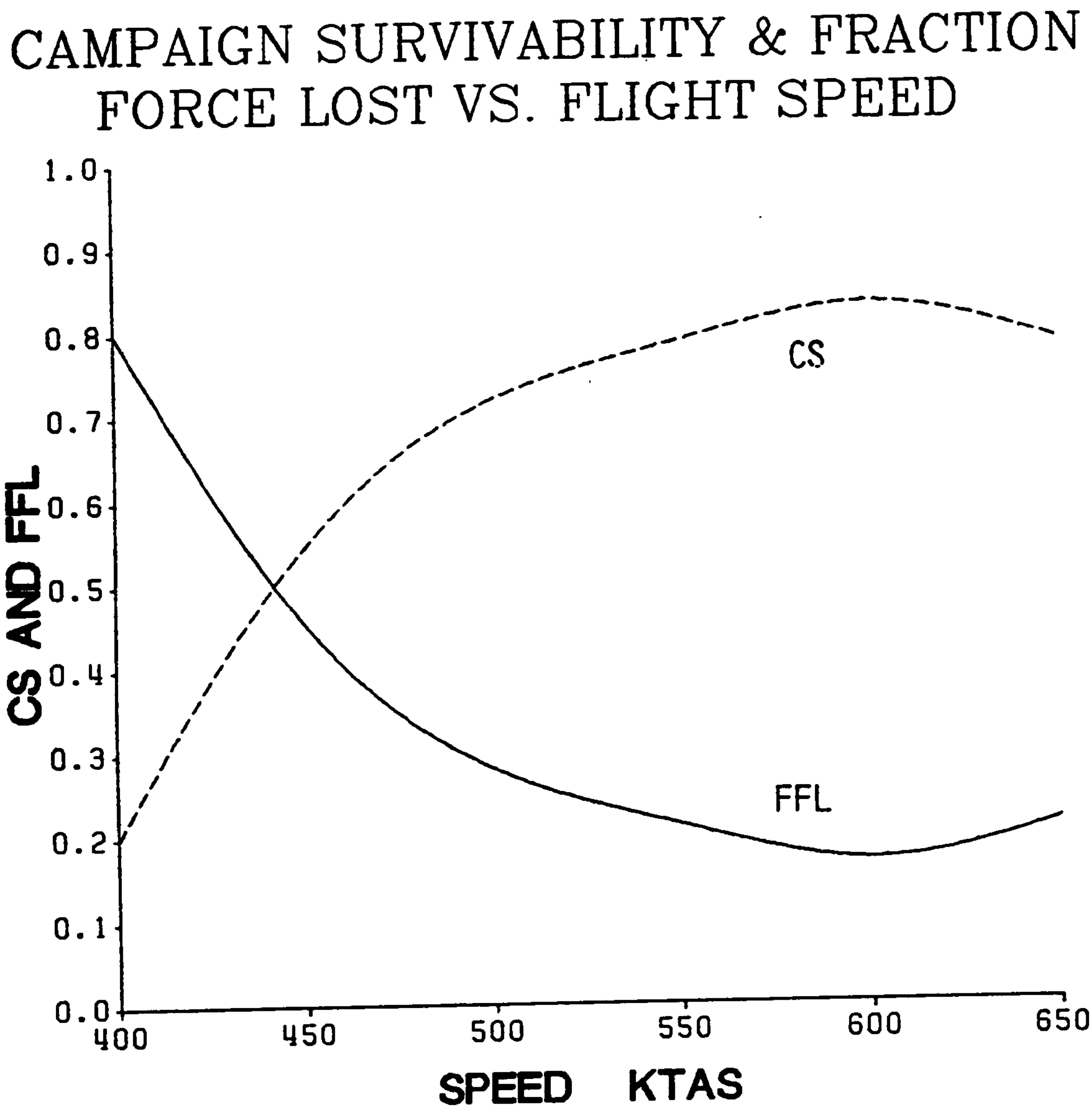
To perform the surface attack mission, a hypothetical aircraft designated CBSA (Carrier-Based aircraft for Surface Attack) is used. In Figs. 5.14-5.23, the five measures of effectiveness are determined and plotted against CBSA air speed. In Fig. 5.14, Campaign Survivability is shown to increase up until about 600 ktas, and then decreases with higher velocities. The increase is due to the increasing difficulty that the defense weapon sites may have in tracking the CBSA. The tradeoff in lethality occurs when the CBSA can no longer acquire and deliver to targets while traveling at higher speeds.

Fig. 5.15 shows a similar inflection at 600 ktas for an analysis of the Exchange Ratio. Using any of the four measures of effectiveness, each trend points toward an optimal air speed of 600 ktas for the CBSA.

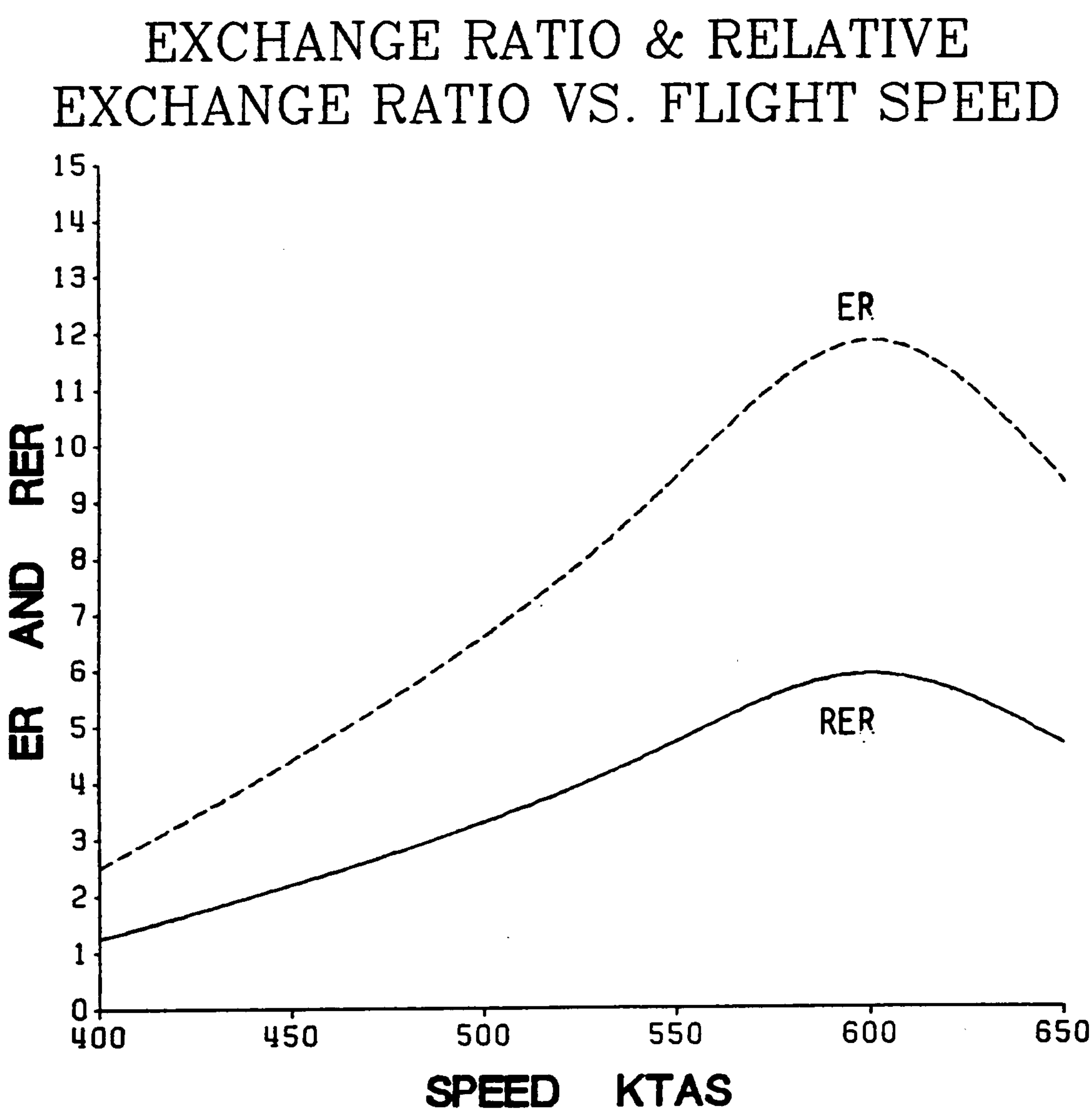
Analyses of Thrust to Weight ratio versus Campaign Survivability and Fraction of Force Lost in Fig. 5.16 show monotonic trends. In other words, any increase in T/W will result in an increase in survivability, however, the range of T/W plotted show only a reasonable range of values which would be incorporated in an economically feasible CBSA. Similar results are found for the Exchange Ratios relating to T/W in Fig. 5.17. Greater values of T/W result in more favorable Exchange Ratios.

In Fig. 5.18, it can be seen that an increase in Radar Cross Section will give rise to a decrease in Campaign Survivability, a finding which is in agreement with intuition. It is important to point out that in Fig. 5.18, the trend shows a marked change in slope somewhere between RCS values of 10 and 15 sq. m. Fig. 5.19 shows this inflection with more resolution at RCS values at about 10 sq. m. This would imply that an CBSA with an RCS of 10 will not suffer much decrease in Exchange Ratios for increases in its RCS, but any decrease in its RCS will result in substantially higher ER values.

The two measures of effectiveness plotted against altitude in Fig. 5.20 are determined for three regions. Region I shows Campaign Survivability and Fraction Force Lost for altitudes



Figures 5.14 Flight Speed vs. CS and FFL



Figures 5.15 Flight Speed vs. ER and RER

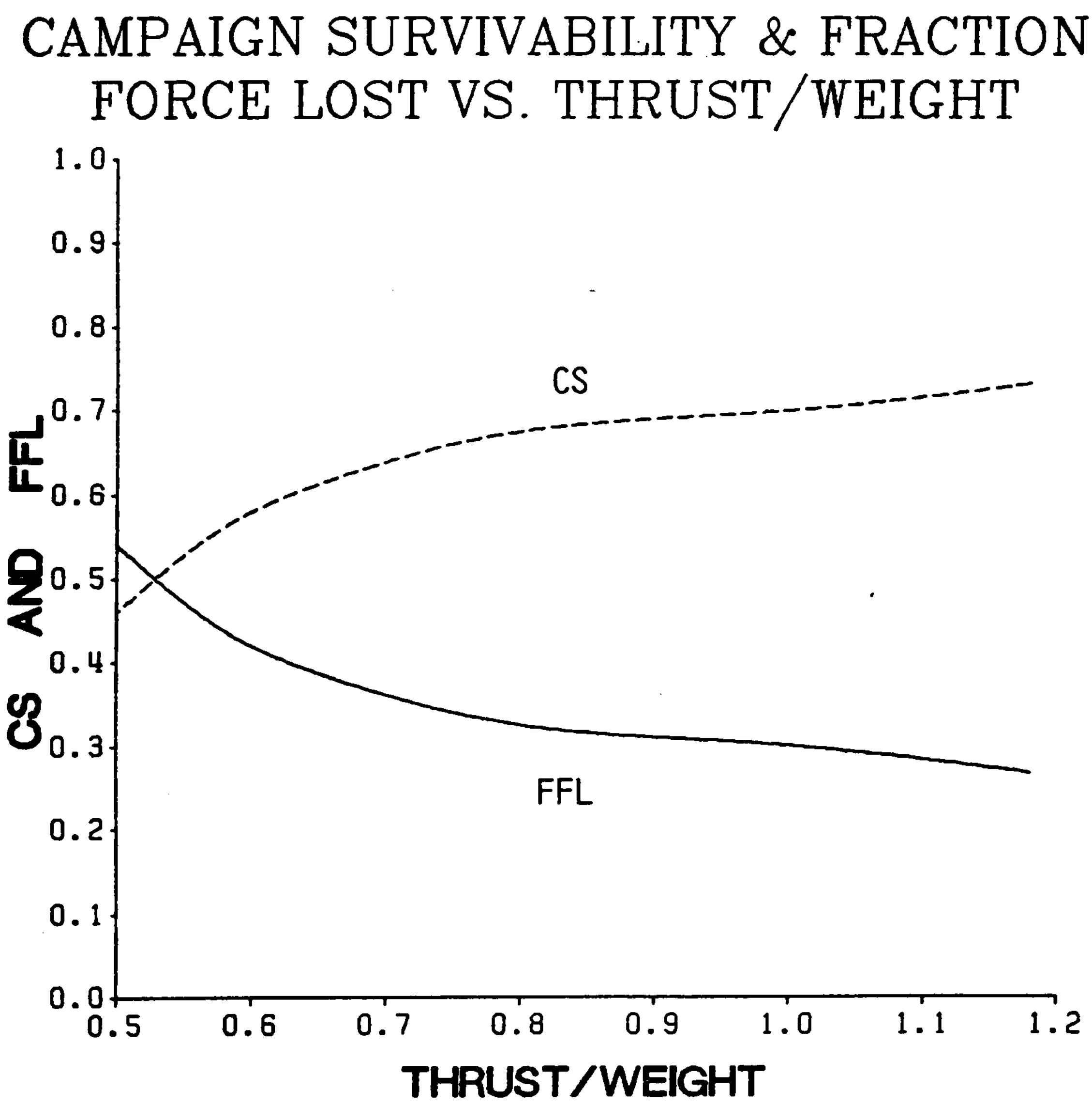


Figure 5.16 Thrust-to-Weight Ratio vs. CS and FFL

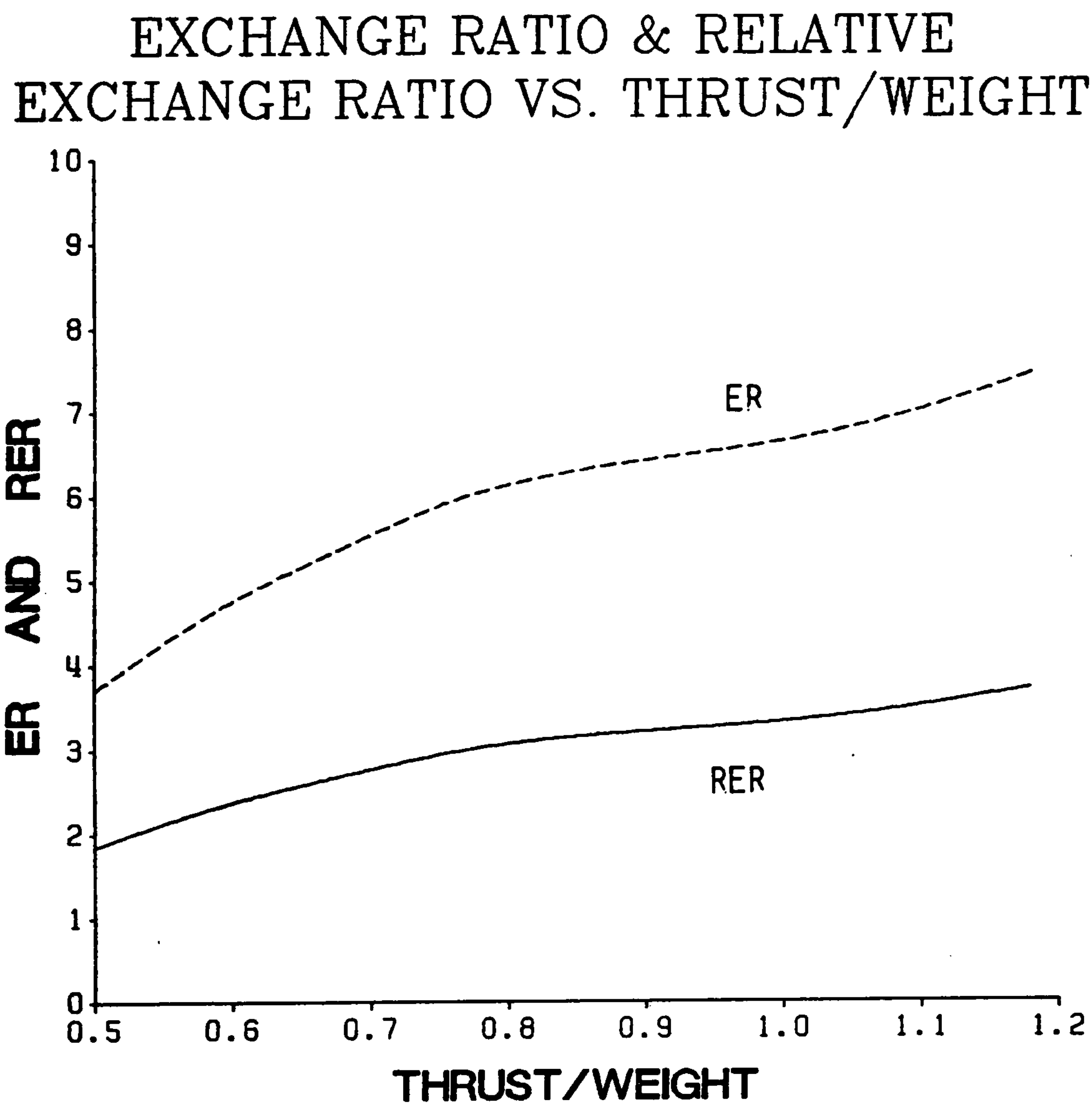


Figure 5.17 Thrust-to-Weight Ratio vs. ER and RER

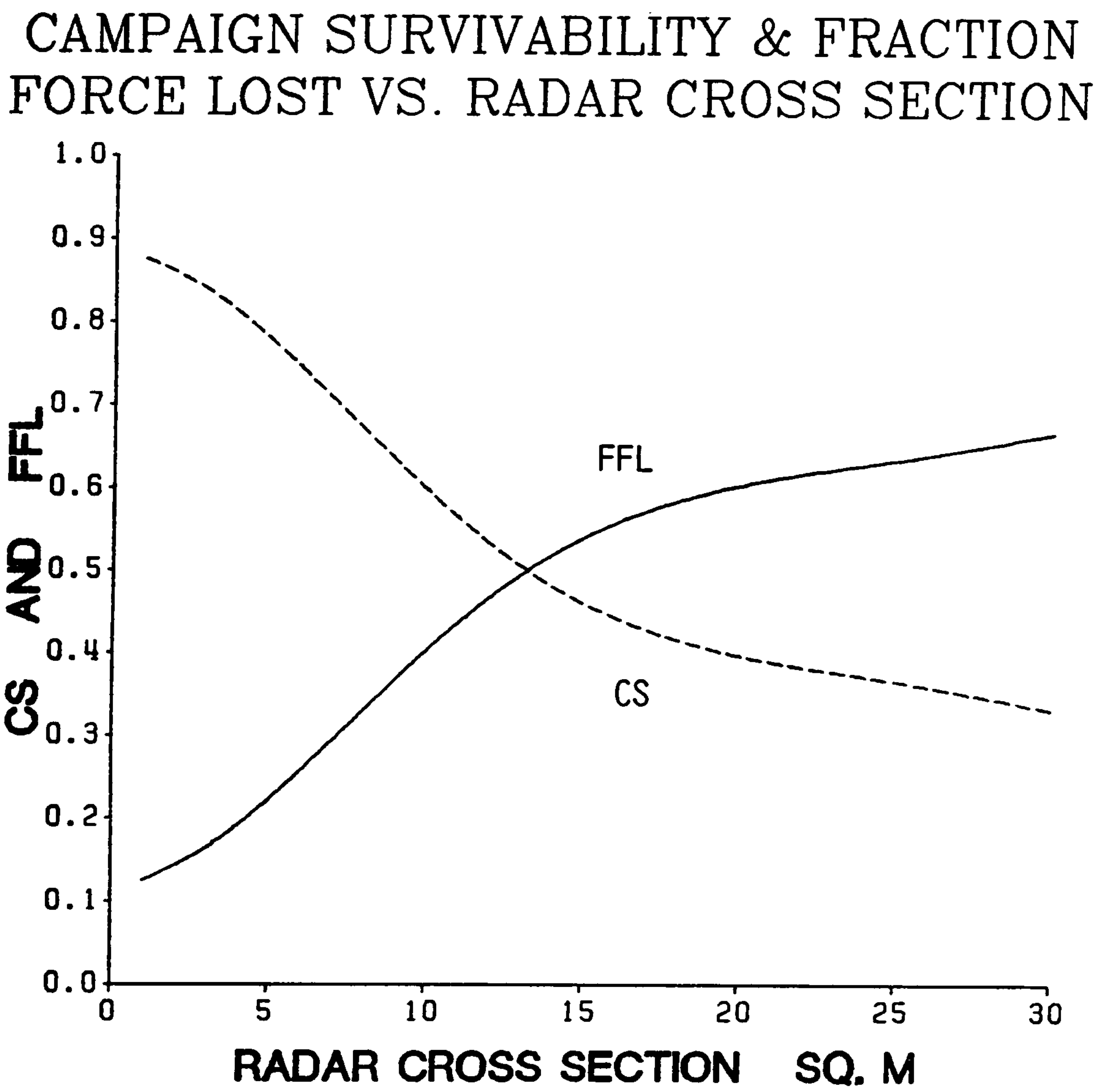


Figure 5.18 Radar Cross Section vs. CS and FFL

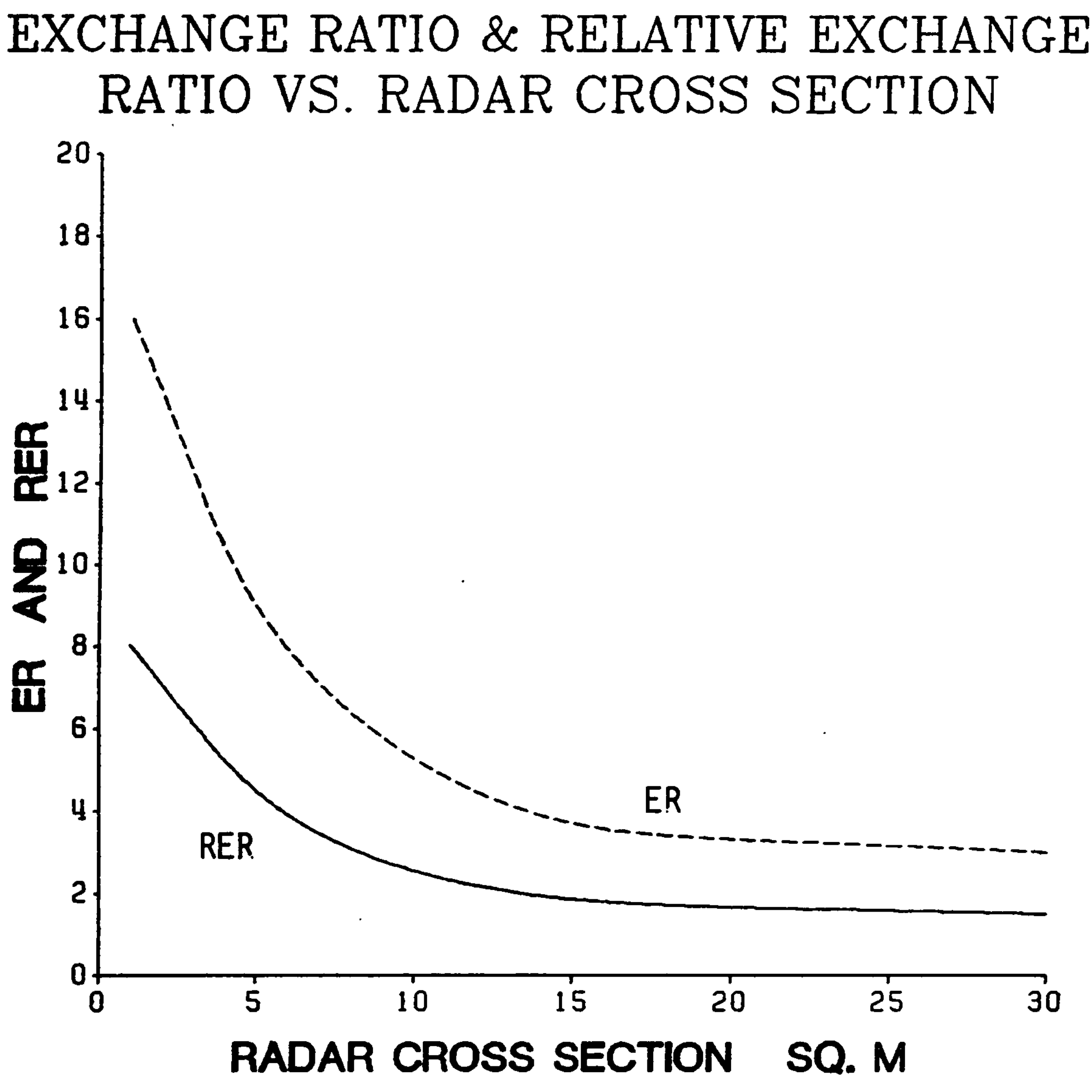


Figure 5.19 Radar Cross Section vs. ER and RER

between 50 and 2000 meters. The basis for selecting this upper limit is the highest reach of the anti-aircraft artillery. The second discontinuity occurs between Regions II and III at 6500 m which is the upper limit of the IR missiles. Region III survivability continues to decrease up to 10000 m due to the radar-guided missiles and the difficulty of the CBSA to inflict damage on defense weapon sites from such high altitudes. Fig. 5.21 reflects the same trends when considering Exchange Ratios as a function of altitude, showing an optimum ER at 2000 m.

Figs. 5.22 and 5.23 represent plots of ECM fraction carriage versus CS and FFL. There is a favored level of CS at an ECM fraction mounted externally of about 18%, with decreasing levels of CS for greater ECM fractions. This could be attributed to the loss in aircraft lethality by carrying to much ECM externally thus reducing the amount of ordnance delivered per sortie and eventually reducing CS. Fig. 5.23 shows a similar trend for Exchange Ratio.

In order to illustrate the potential use of the model as a tool to be used in conceptual design a simple example is given next.

5.6.1 Example 5.2

Suppose that four modifications of a projected attack aircraft (i.e a baseline design) are to be investigated using the aircraft effectiveness tradeoff methodology previously described. These modifications are simply changes to the baseline aircraft powerplant in order to appreciate the variations of thrust/weight on the four measures of effectiveness (MOE) previously described. The goal here is to choose the "best" airframe-powerplant configuration that yields the minimum acquisition cost ratio (ACR), minimum possible crew loss (PCL), and the highest campaign survivability (CS) for a given mission scenario. Several assumptions have to be made in order to simplify the problem and they are as follows: 1) The main airframe geometric characteristics are kept constant except those associated with the inlet size to accommodate the various mass flow rate requirements associated with each particular powerplant

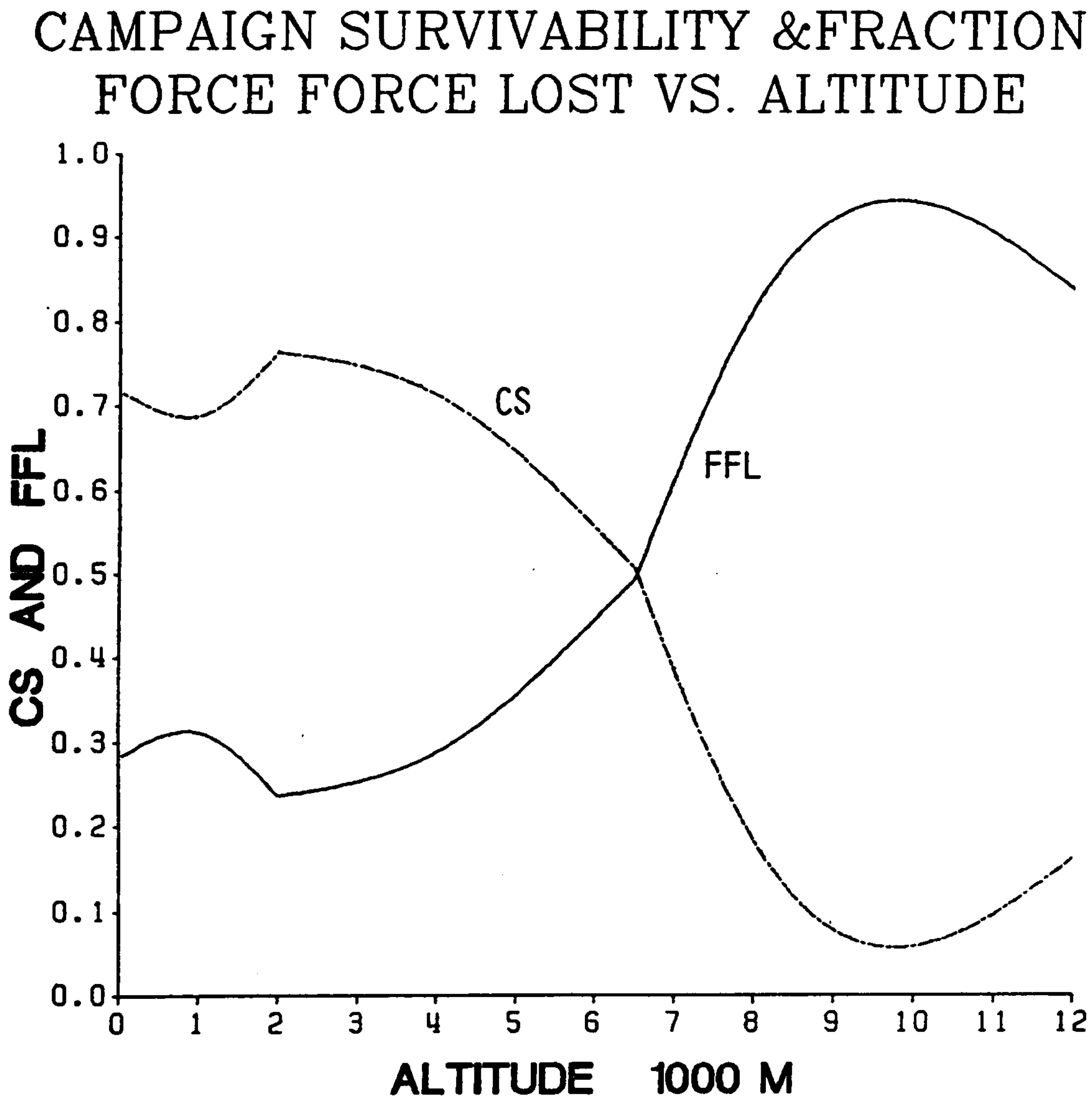


Figure 5.20 Altitude vs. CS and FFL

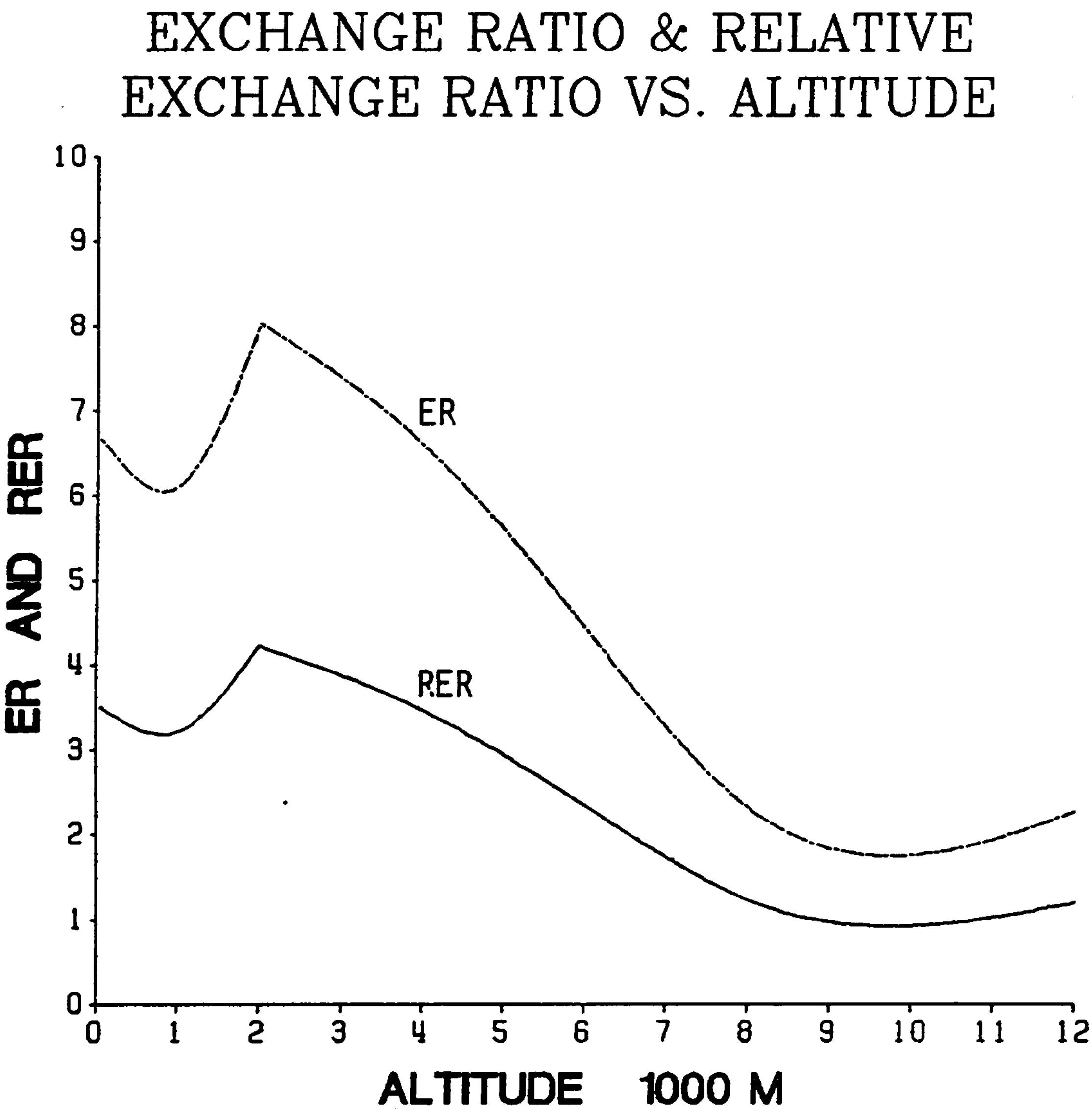


Figure 5.21 Altitude vs. ER and RER

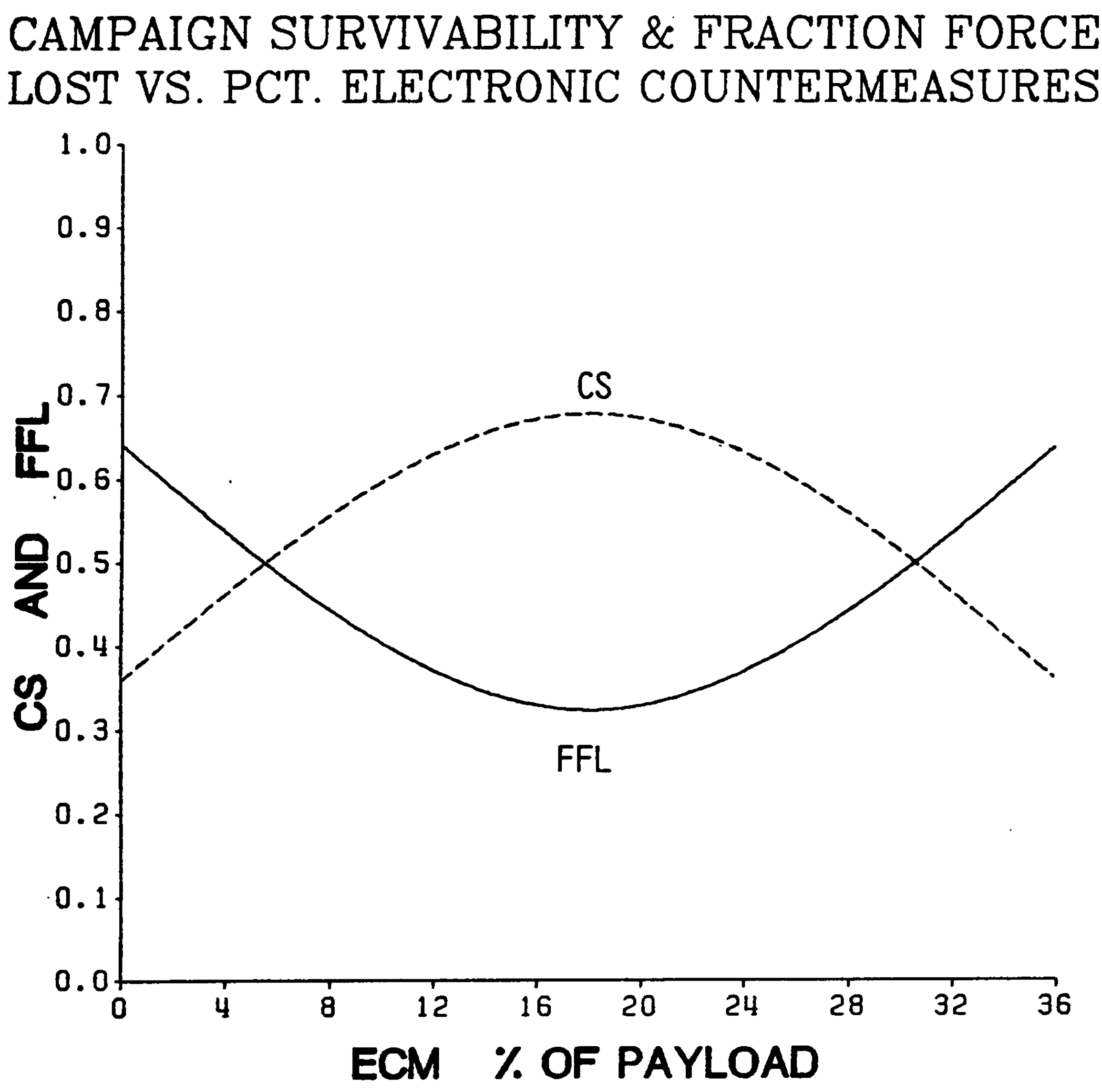


Figure 5.22 Percent ECM Fraction vs. CS and FFL

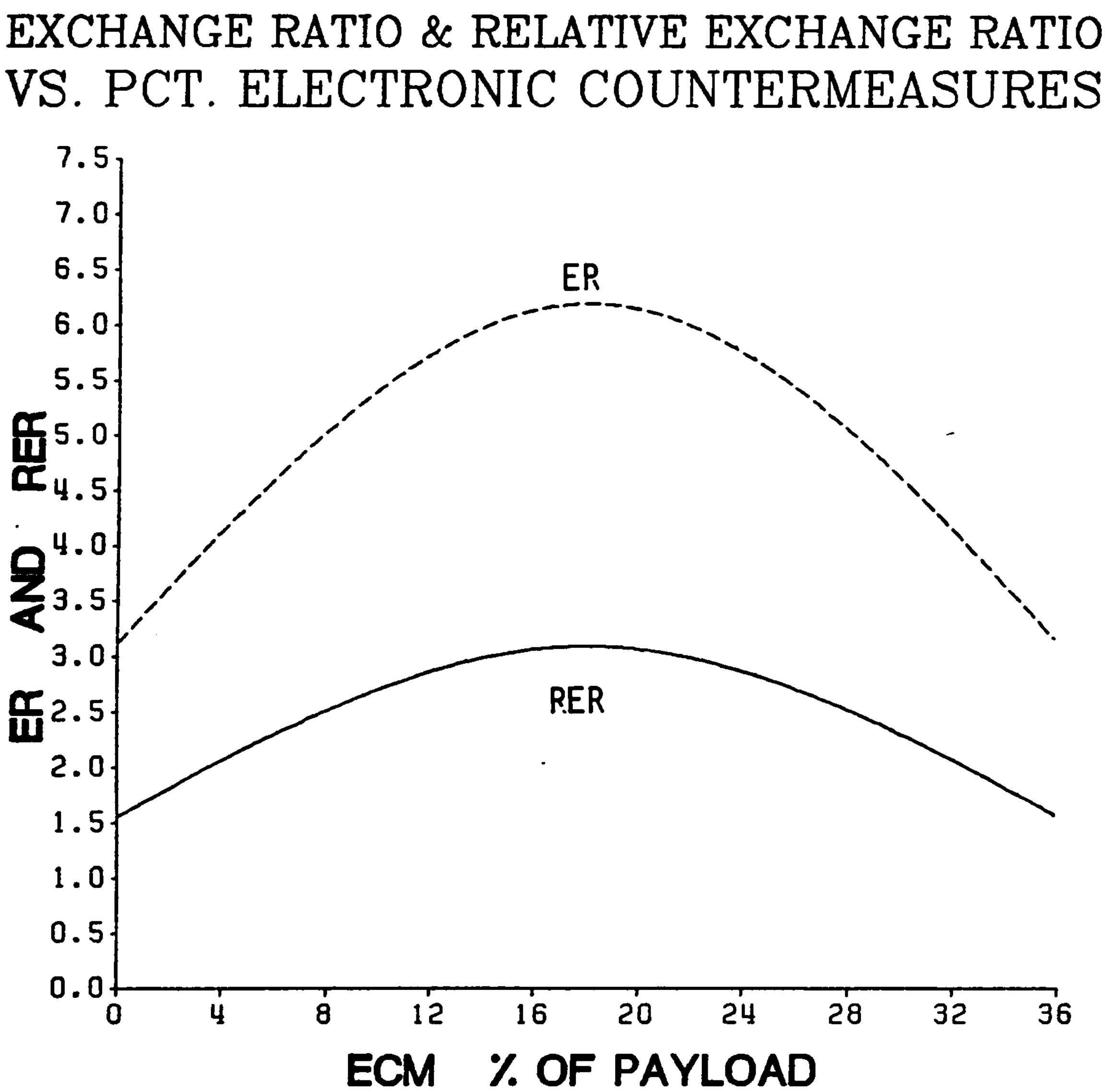


Figure 5.23 Percent ECM Fraction vs. ER and RER

installation to be studied. 2) Aerodynamic drag characteristics are varied to account for changes in inlet drag. 3) Radar cross section (RCS) characteristics are also varied to account for changing inlet sizes. 4) Changes in aircraft weight are solely due to the different engines installed. The relevant characteristics of the baseline design are given in Table 5.15.

The driving parameter for each modification was the thrust-to-weight ratio for the particular mission weight. Under the assumption of a fixed mission weight (e.g. good only for short missions) it is possible to size the powerplant to be used to exactly match the desired thrust/weight parameter. Table 5.14 summarizes the selected of thrust-to-weight ratios used in this example.

Table 5.14 Aircraft Configurations for Example 5.2

Configuration	Thrust/Weight	Static Thrust at Sea Level (lbs)
Baseline	0.84	18000
Mod. #1	0.50	10392
Mod. #2	0.70	14761
Mod. #3	1.10	23719
Mod. #4	1.20	26005

The first step to be performed in this tradeoff study is to determine the new physical engine characteristics of the modified aircraft in order to later on estimate radar cross section (RCS) and drag variations from those of the baseline design. For the purpose of simplification in preliminary engine sizing it is assumed that the modified powerplant area requirement varies with thrust to the power .85 as shown in Eq. 5.39.

$$\frac{T_1}{T_2} = \left(\frac{A_1}{A_2}\right)^{.85} \tag{5.39}$$

Where A_1 and A_2 represent the areas of the baseline and modified engines, and T_1 and T_2 are their corresponding thrust values under static sea level conditions. From Eq. 5.39 it is possible to solve for A_2 and obtain the frontal area the powerplant in question from where the engine diameter can be found. Results of this computation are shown in Table 5.16.

Table 5.15 Aircraft Characteristics for Example 5.2

Aircraft weights	
Maximum takeoff gross weight	53000 lbs.
Empty weight	21000 lbs.
Maximum fuel weight	22000 lbs.
Maximum payload weight	18000 lbs.
Mission weight	42550 lbs.
Wing characteristics	
Aspect ratio	3.9
Wing area	420
Wing efficiency factor	.83
Engine characteristics	
Number of engines	2
Static sea level thrust	18000 lbs.t./engine
Engine diameter	38.0 in.
Engine frontal area	7.87 sq. ft.
Engine weight	2320 lbs.
Engine thrust/weight	7.76:1
Thrust variations with Mach number and altitude	
Aircraft signature	
Aircraft radar cross section	10 sq. meters
Aircraft aerodynamic characteristics	
Aircraft C_{D0} (clean)	0.015 below mach .7
Aircraft C_{D0} (loaded)	0.024 below mach .7
Aircraft C_{D0} varies with Mach number	
Ordnance	
Air-to-surface missiles	6 @ 1000 lb/each
Bombs	16 @ 500 lb/each

Regression analysis was performed for 11 current fighter engines in the same thrust category of the baseline engine in order to determine current trends in engine weight versus engine diameter relationships an be able to predict the weights of the powerplants to be used in the modified aircraft. The regression equation yields:

$$W_e = 127.62\ d - 2531.25 \qquad \{5.40\}$$

Where W_e is the engine weight in pounds and d its diameter in inches. The regression correlation coefficient was .90. Numerical results for the various configurations investigated are shown in Table 5.16.

Another important issue is the drag increase associated with installation of a different engine for each modification. The simplifying assumption is that only additive drag is considered to increase CD_0 at a rate of 18 counts per square foot of increased engine frontal area (one count of C_{D0} is 1/10000). The reasoning behind this seemingly arbitrary assumption is the fact that additive drag (sometimes referred to as spillage drag) is very sensitive to the geometry of the particular inlet -- information not available and beyond the scope of this simple example. The numerical results of drag changes for each configuration are shown in Table 5.16 in terms of the change in clean aircraft zero lift drag coefficient (CD_0).

In order assess these drag increases for each configuration, a simple performance analysis gives great insight about the speed capabilities of the modified aircraft. Note that this analysis is only necessary to determine the impact of the modified CD_0 in the overall aircraft performance characteristics and thus determine the new flight envelopes for the various configurations.

For non-accelerated flight and neglecting thrust offset angles it is known that thrust (T) and drag (D) are equal. Writing explicitly the expression for drag:

$$T = D = q\ S\ C_D \qquad \{5.41\}$$

Where q is the compressable equivalent of dynamic pressure, S represents the wing area, and C_D is the drag coefficient. Further breaking down C_D in its lifting and non-lifting terms yields,

$$C_D = C_{Di} + C_{D0} \qquad \{5.42\}$$

Where C_{Di} is the induced drag coefficient (i.e., lift dependent) and C_{D0} is the zero-lift drag coefficient (i.e., a function of Mach number). Also it is known that the dynamic pressure takes the form:

$$q = \frac{1}{2}\gamma \ P_0 \ \delta \ M^2 \qquad \{5.43\}$$

Where γ is the ratio of specific heats for air, P_0 is the atmospheric pressure at sea level conditions, δ is the air pressure ratio at altitude, and M is the flight mach number. Writing C_{Di} in terms of lift coefficient C_L, wing efficiency factor e, and wing aspect ratio AR,

$$C_{Di} = \frac{C_L^2}{\pi \ AR \ e} \qquad \{5.44\}$$

For straight and level flight lift (L) equals weight ($W_{a/c}$). Therefore,

$$C_L = \frac{2 \ W_{a/c}}{\gamma \ P_0 \ \delta \ M^2 \ S} \qquad \{5.45\}$$

Substituting Eqns. 5.42-5.45 into 5.41 yields,

$$T = \frac{1}{2} \gamma \, P_0 \, \delta \, M^2 \, S \, C_{D0} + \frac{4 \, W_{a/c}^{\ 2}}{\pi \, AR \, e \, \gamma^2 \, P_0^2 \, \delta^2 \, M^4} \qquad \{5.46\}$$

Which represents a general form to express the thrust requirements of an aircraft in steady flight for a desire Mach number. Results from Eq. 5.46 are extremely useful since the ASALT model uses speed as an input to arrive to the sortie survivability. It is then necessary to know the variability of this parameter for the various configurations investigated. Maximum Mach numbers at sea level conditions are shown in Table 5.16. Notice that these values seem optimistic since no wave drag is accounted for in the transonic region.

The last parameter affected directly with the installation of a new engine is the radar cross section area (RCS) associated with it. For a more usable form of Eq. 5.37, Crispin [18] offers a simple equation to approximate cavities such as that those characteristic of aircraft inlets as follows.

$$\sigma = 4\ \pi\ \frac{A_e^2}{\lambda^2} \tag{5.47}$$

where A_e is the engine area and λ is the radar wavelength. For this analysis λ is assumed to be .7 meters. Finally, it should be realized that the inlets only constitute one portion of the many radar reflecting surfaces found in an aircraft and for this matter it has been assumed that 75 % of the total aircraft radar cross section (RCS) is due to the inlets. The corresponding numerical results of the computation of RCS for each modification are shown in Table 5.16.

Items 1,2,5, and 8 in Table 5.16 constitute the principal inputs to investigate the new survivability and lethality of each modified aircraft. The engine thrust is scaled down or up in the code according to the known values of engine thrust at sea level (Item 1 in Table 5.16). A multiplicative scaling factor is introduced in the program to match the baseline sea level thrust to the desired thrust for the new modification under the same conditions (i.e. sea level). The rest of the values are scaled proportionately. The second input to the program is the aircraft weight. The weights shown in Table 5.16 correspond to the mission weights to be used for each run since it has been assumed that the only source of weight reduction is the different engine weight installed in each modified aircraft. Values for ΔC_{D0} are also important to be incorporated into the model by adding a dummy variable to account for ΔC_{D0} . Lastly, the RCS values shown in Table 5.16 correspond to the values for each modification.

Using the outputs of the ASALT model and the tradeoff methodology previously described one obtains Tables 5.17 and 5.18 where two important tradeoff cases are depicted for a mission speed of 500 knots.

Table 5.16 Summary of Modifications for Baseline Aircraft - Example 5.2

		ALTERNATIVE A/C			
DESCRIPTION	BASE	MOD. 1	MOD. 2	MOD. 3	MOD. 4
1) Engine Thrust SSL	18000	10392	14761	23719	26005
2) Aircraft Weight (lbs.)	42550	41566	42174	43125	43555
3) Thrust/Weight	0.84	0.50	0.70	1.1	1.2
4) MM_0	1.27	1.032	1.13	1.45	1.50
5) ΔC_{D0}	0	-5.1	-2.11	3.54	5.05
6) Engine Frontal Area (sq. ft.)	7.87	5.035	6.7	9.836	10.66
7) Engine Diameter (in.)	38.0	30.38	35.05	42.50	44.20
8) Radar Cross Section (sq. mt.)	10	5.56	7.9	12.92	18.9
9) Normalized RCS	1.00	0.556	0.790	1.292.	1.890

In the first one (i.e., Table 5.17) a constant procurement run of 1000 aircraft is assumed for all the configurations studied and the idea is to determine the six measures of effectiveness (MOE's) defined by Eqns. 5.24-5.29 (Items 12-17 of Table 5.17).

Analysis of these 6 MOE's reveals the following: 1) Campaign Survivability increases as thrust-to-weight ratio increases up to T/W values around .84 with a noticeable decline for higher T/W (i.e. modifications 3 and 4). The explanation for this "unexpected" result lies in the original assumptions of the example. It is recalled that an increase in thrust-to-weight ratio has an inherent increase in the aircraft radar cross section due to the larger inlet size. Thus at high T/W the performance gain is small (since the drag divergence speeds for all four modifications are the same in the low transonic regime) compared with the large RCS penalty associated with the installation of a much powerful and bigger engine. Note a 24% decrease in Campaign Survivability when going from .84 to 1.2 for T/W at 500 knots (Table 5.17). 2) The Exchange Ratio (ER) behaves in a similar way as $P_{S/S}$ and it is seen from the same table that the largest ER value shown is that for the baseline configuration (i.e., ER=5.533), implying that 5.533 defense sites are destroyed per each aircraft destroyed. 3) The Possible Crew Loss is a direct consequence of Campaign Survivability and as should be expected is minimized for the maximum value of $P_{S/S}$.

These three conclusions seem to favor the baseline configuration for this particular mission if it were flown at 500 knots. The value of the speed parameter is extremely important

Table 5.17 Aircraft Effectiveness Tradeoff Methodology - Example 5.2

DESCRIPTION	FORMULATION	Alternative A/C BASE	MOD. 1	MOD. 2	MOD. 3	MOD. 4
1) Threat Inventory	X_0 = Goal	1900	1900	1900	1900	1900
2) Speed		500	500	500	500	500
3) Thrust-to-Weight Ratio		.84	.5	.7	1.1	1.2
4) Acquisition Cost Ratio	ACR = ACM/ACB			NOT APPLICABLE		
5) Production Run	$\$_0M = \$_0B(ACR)$	1000	1000	1000	1000	1000
6) Sortie Survivability	$P_{s/s}$	.976	.964	.975	.972	.968
7) Effectiveness of DWS	B Eq.{5.17}	.024	.036	.025	.028	.032
8) Sortie Lethality	$P_{K/S}$	.1523	.1526	.1524	.1526	.1527
9) Effectiveness of A/C	C Eq.{5.18}	.1523	.1526	.1524	.1526	.1527
10) Surviving No. of A/C	$\$_t$ Eq.{5.23}	656	385	638	581	493
11) No. of DWS Destroyed	$X_0 - X_t = X_0$	1900	1900	1900	1900	1900
12) Campaign Survivability	CS Eq.{5.24}	.6566	.3851	.6386	.5810	.4934
13) Fraction Force Lost	FFL Eq.{5.25}	.3434	.6149	.3614	.4190	.5066
14) Exchange Ratio	ER Eq.{5.26}	5.533	3.090	5.258	4.535	3.751
15) Rel. Exchange Ratio	RER Eq.{5.27}	2.910	1.626	2.767	2.387	1.974
16) Possible Crew Loss	PCL Eq.{5.28}	344	615	361	419	507
17) Replacement Cost	RC Eq.{5.29}			NOT APPLICABLE		

in this type of analysis since an increase in mission speed could favor higher thrust-to-weight ratio configurations.

A subsequent tradeoff study that can be derived from this methodology is the analysis of the economic implications that modifying a baseline design can bring into tactical MOE's such as Campaign Survivability and Exchange Ratio. For the sake of illustration and following the same lines of the previous example suppose it is desired to know the procurement numbers (i.e., production run) necessary for each configuration to yield an equivalent Campaign Survivability to that of the baseline configuration at 500 knots.

Since $P_{S/S}$ is constant for the four modifications, it is necessary to express variables ER, PCL, and RC (replacement cost) in terms of the known quantities $P_{S/S}$, $P_{K/S}$, and CS and then solve Eqs. 5.26, 5.28, and 5.29 for ER, PCL, and RC respectively. Table 5.16 is again used as input to the ASALT program and values of $P_{S/S}$and $P_{K/S}$ are readily available as outputs. Table 5.18 shows the final results of this Cost Effectiveness Tradeoff Methodology.

Several preliminary conclusions can be drawn from this data. 1) The highest Exchange Ratio (5.533) is achieved at the baseline configuration which appears to combine good performance characteristics with relatively low radar cross section area. Notice that since the program inputs are the same as those for the previous tradeoff study (i.e., the same speed is used), that the values of $P_{S/S}$ are unchanged from Table 5.17. 2) Also, it it seen from Table 5.18 that the baseline configuration has the lowest values for PCL and the initial number to be procured in order to complete the goal of destroying 1900 defense sites features that give the baseline design the best all-around performance from the configurations studied under the given mission constraints. As in the previous tradeoff study, the high thrust-to-weight configurations did not achieve improvement in the three MOE's previously mentioned due to the large increase in RCS due to the larger inlet requirements (i.e., higher engine thrust) which makes these configurations more vulnerable of being detected ultimately increasing the attacking aircraft attrition rate.

An important parameter included in Table 5.18 is the Acquisition Cost Ratio (ACR) which expresses the quotient between the baseline cost to that of the particular modification that

Table 5.18 Aircraft Cost-Effectiveness Tradeoff Methodology - Ex. 5.2

DESCRIPTION	FORMULATION	Alternative A/C BASE	MOD. 1	MOD. 2	MOD. 3	MOD. 4
1) Threat Inventory	X_0 = Goal	1900	1900	1900	1900	1900
2) Speed		500	500	500	500	500
3) Acquisition Cost Ratio	ACR=ACM/ACB	1.000	.817	.979	.926	.867
4) Production Run	$\$_0M = \$_0B(ACR)$	1000	1224	1021	1080	1154
5) Sortie Survivability	$P_{s/s}$	.976	.964	.975	972	968
6) Effectiveness of DWS	B Eq.{5.17}	.024	.036	.025	.028	.032
7) Sortie Lethality	$P_{K/S}$	.1523	.1526	.1524	.1526	.1527
8) Effectiveness of A/C	C Eq.{5.18}	.1523	.1526	.1524	.1526	.1527
9) Surviving No. of A/C	$\$_t$ Eq.{5.23}	656	803	669	708	757
10) No. of DWS Destroyed	$X_0 - X_t = X_0$	1900	1900	1900	1900	1900
11) Campaign Survivability	CS Eq.{5.24}	.6566	.6566	.6566	0.6566	.6566
12) Fraction Force Lost	FFL Eq.{5.25}	.3434	.3434	.3434	.3434	.3434
13) Exchange Ratio	ER Eq.{5.26}	5.533	4.522	5.423	5.128	4.798
14) Rel. Exchange Ratio	RER Eq.{5.27}	2.911	2.911	2.911	2.911	2.911
15) Possible Crew Loss	PCL Eq.{5.28}	344	421	351	371	396
16) Replacement Cost	RC Eq.{5.29}	1.000	1.000	1.000	1.000	1.000

will yield the same total procurement cost. For example, in Table 5.18 the ACR for modification 3 is quoted as 0.926 times the baseline ACR cost. This implies that if modification 3 is chosen the unit aircraft price should be .926 that of the baseline design in order to match a fixed procurement budget. Note that eighty more aircraft are to be procured in modification 3 at the same total expense. When one considers that the price of the engine and engine related systems is proportional to the thrust delivered by the powerplant it is not difficult to deduce that modifications with higher T/W (i.e., mods. 3 and 4) will cost significantly more thus making them economically and tactically impractical for this particular mission profile and speed. The role of the Acquisition Cost Ratio (ACR) is to establish a link between the decision making variables (e.g., ACR, RC) and tactic related variables such as CS, ER, and PCL.

5.6.2 Example 5.3

As a second example let us suppose that a baseline configuration and three modifications are examined. The major assumption here is that the aircraft do not change externally and that a reduction in RCS is achieved by some internal electronics or by promoting the use of new non-reflective materials. The idea is to realize improvements in the MOE's with changes in radar cross section.

Table 5.19 Aircraft Effectiveness Tradeoff Methodology - Example 5.3

DESCRIPTION	FORMULATION	BASE	Alternative A/C MOD. 1	MOD. 2	MOD. 3
1) Threat Inventory	X_0 = Goal	1900	1900	1900	1900
2) Radar Cross Section (m^2)		10	20	5	1
3) Acquisition Cost Ratio	ACR = ACM/ACB			NOT APPLICABLE	
4) Production Run	$\$_0 M = \$_0 B(ACR)$	1000	1000	1000	1000
5) Sortie Survivability	$P_{s/s}$	.976	.964	.981	.988
6) Effectiveness of DWS	B Eq.{5.17}	.024	.036	.019	.012
7) Sortie Lethality	$P_{K/S}$	.1523	.1520	.1520	.1529
8) Effectiveness of A/C	C Eq.{5.18}	.1523	.1520	.1520	.1529
9) Surviving No. of A/C	$\$_t$ Eq.{5.23}	656	380	740	846
10) No. of DWS Destroyed	$X_0 - X_t = X_0$	1900	1900	1900	1900
11) Campaign Survivability	CS Eq.{5.24}	.6566	.3808	.7408	0.8466
12) Fraction Force Lost	FFL Eq.{5.25}	.3434	.6192	.2592	.1534
13) Exchange Ratio	ER Eq.{5.26}	5.533	3.068	7.330	12.383
14) Rel. Exchange Ratio	RER Eq.{5.27}	2.910	1.615	3.858	6.518
15) Possible Crew Loss	PCL Eq.{5.28}	343	620	260	154
16) Replacement Cost	RC Eq.{5.29}			NOT APPLICABLE	

The values of RCS were modified from 1 to 20 squared meters and Table 5.19 illustrates the results obtained for this particular analysis. As expected, the value of $P_{S/S}$ increases from .3808 for $A = 20m^2$ to .8466 at one square meter. Similar monotonic behavior is shown for Exchange Ratio (ER) and Number of Aircraft Surviving. PCL, on the other hand, decreases from a high 343 to a low of 154 (55% decrease) thus making the low RCS option very desirable from the tactical point of view.

The last issue to be addressed here is the cost analysis. Table 5.9 depicts results for the same four configurations investigated. For a constant Campaign Survivability (e.g., .6566) the values of other three MOE's are found giving the following conditions: 1) Exchange Ratio increases with reductions in aircraft RCS. 2) Crew Losses decreased with decreased RCS though not as dramatically as in the unconstrained CS case.

Table 5.20 Aircraft Cost-Effectiveness Tradeoff Methodology - Ex. 5.3

			Alternative A/C		
DESCRIPTION	FORMULATION	BASE	MOD. 1	MOD. 2	MOD. 3
1) Threat Inventory	X_0 = Goal	1900	1900	1900	1900
2) Radar Cross Section (m^2)		10	20	5	1
3) Acquisition Cost Ratio	1.00	.816	1.124	1.416	
4) Production Run	$\$_0 M = \$_0 B(ACR)$	1000	1226	890	706
5) Sortie Survivability	$P_{s/s}$	.976	.964	.981	.988
6) Effectiveness of DWS	B Eq.{5.17}	.024	.036	.019	.012
7) Sortie Lethality	$P_{K/S}$	.1523	.1520	.1520	.1529
8) Effectiveness of A/C	C Eq.{5.18}	.1523	.1520	.1520	.1529
9) Surviving No. of A/C	$\$_t$ Eq.{5.23}	656	805	584	463
10) No. of DWS Destroyed	$X_0 - X_t = X_0$	1900	1900	1900	1900
11) Campaign Survivability	CS Eq.{5.24}	.6566	.6566	.6566	0.6566
12) Fraction Force Lost	FFL Eq.{5.25}	.3434	.3434	.3434	.3434
13) Exchange Ratio	ER Eq.{5.26}	5.533	4.513	6.212	7.840
14) Rel. Exchange Ratio	RER Eq.{5.27}	2.912	2.912	2.912	2.912
15) Possible Crew Loss	PCL Eq.{5.28}	344	421	306	243
16) Replacement Cost	RC Eq.{5.29}	344	344	344	344

6.0 Discussion and Conclusions

A basic tenet of the survivability discipline is that survivability considerations must be incorporated at the earliest time in the development cycle of the aircraft in order to optimize tradeoffs between benefits and costs. This is also true for other considerations more important to this research such as mission effectiveness, force effectiveness, and affordability. It is with these thoughts in mind that the example applications found in this research were made; for the three combat aircraft mission roles,

1. Surface Attack
2. Fighter Escort
3. Carrier Defense

it is necessary that the survivability considerations must be studied and evaluated throughout their development cycle, from concept formation to design and implementation.

The level of analysis used in evaluating survivability enhancements is critical to the usefulness of the conclusions. To study the effectiveness of a particular enhancement for a given mission scenario and make decisions based on that scenario *only* is ludicrous. The situation as it stands is that combat operations are made up of a variety of scenarios, a commander can not choose to participate in those with favorable outcomes and abstain from joining with those in which the enemy has the advantage.

Several embellishments, enhancements, and extensions are discussed and recommended in the following sections; these are only a few of the many which have suggested themselves during the course of this research, but are felt important enough to be addressed despite their omission in the actual modelling or methodology.

6.1 Modelling Approach Retrospective

The models and methodologies developed in this research are rooted in the Co-Kill approach to calculating survivability and lethality probabilities. The Co-Kill approach provides the analyst a method to determine these probabilities for given numbers of friendly and enemy aircraft engaged in combat. For any given sortie, the fraction of surviving aircraft for both friendly and enemy forces can be determined using the mathematical approach described in Chapter 3. The methodology is used, for example, when four friendly aircraft encounter six of the enemy's, Co-Kill probabilities can be directly determined. These probabilities may assert that, for example, 25 percent of the friendly forces are destroyed while the enemy loses 35 percent of its sortie force.

These probabilities are calculated using DYNAMO computer programs developed in the companion research. For each of the special cases (e.g., 2 vs. 4, 3 vs. 4, 4 vs. 4, etc.), a single simulation run of an ASALT [1][2] model can be run; the output of ASALT giving the fractions of the sortie forces (both enemy and friendly) which survive. Subtracting these fractions from one and multiplying the result by the respective sortie force size yields the attrition incurred for that given sortie for two respective opponents.

One condition difficult to predict is the enemy sortie force strength. Likewise, availability and sortie rate considerations make it difficult to provide friendly sortie force sizes as large as may be desired. Therefore, some mix of sortie strengths of both friendly and enemy forces will occur thoughout the air-combat campaign.

It is logical that, in modeling the operations of the carrier's combat aircraft, the occurance of certain *state variables* would be determined stochastically. Specifically, values of these variables determine when the state of the system may be such that:

- A surface attack sortie is occurring
- A fighter escort sortie is occurring
- A carrier defense sortie is occurring
- The number of the respective aircraft type for the above missions is allocated as they would be in combat.

Since these state variables are determined stochastically, and because direct inputs into the attrition equations are the Co-Kill probabilities, the modelling approach is a hybridization of both techniques, enjoying the benefits of each. By using both the Co-Kill and stochastic approaches, the analysis can be performed both efficiently and realistically. If stochasticity were used in place of the Co-Kill probabilities for the survivability/lethality analysis of the combat engagements, then a large number of simulation model runs would be required, each using a different random seed. This "large number" would be on the order of 10^3 since survivability is often expressed to four decimal places. However, by using the Co-Kill approach, analyses can be performed in a single simulation run, with results converging over a sufficient amount of simulation time.

6.2 *Mission-Role Tradeoff Modelling*

The specialization of an aircraft to a particular role necessitates a whole regime of logistic and operations to cater to each aircraft type. The investigations in this research reveal that an aircraft may perform each of the three missions with certain losses of effectiveness in any given role, but overall increases in aircraft availability make possible larger sortie sizes, and thus increases in overall system effectiveness. Also, savings could be enjoyed through economies of scale (three versatile aircraft may be cheaper than three specialized ones in the long run), and operating and maintenance costs through standardization. These cost considerations are addressed in a following section with other recommendations for future research.

Another topic treated in this research was to study the effectiveness of varying the ratio of the sortie force size with respect to that of the initial inventory levels. It was found that an "overwhelming" enemy, initially enjoying a 2:1 numerical superiority, does not realize victory if the sortie force ratios can be reduced to, say 1.5:1.

Some interesting results were found for the baseline case as presented in Chapter 4. Exchange ratios between primary threats were calculated over time and plotted in various combinations. The losses of friendly aircraft for surface attack were found to be greater than the number of destroyed enemy defensive weapon sites, yet the exchange ratio between these two primary threats gave the advantage to the friendly forces. This is due to the interactions of the secondary threats. Friendly surface attack aircraft are not only attrited by the surface batteries, but also by the enemy fighters flying against the friendly fighter escort and the enemy carrier attack aircraft destroying the on-board aircraft during the carrier defense mission. On the other hand, the enemy defensive weapons sites are attritied mainly by the friendly surface attack aircraft, with the accompanying fighter escort inflicting minor damage.

In evaluating the effectiveness of an aircraft designed to fly all three of the designated missions, the baseline model was modified to reflect the use of a hypothetical *multimission* aircraft. It is expected that a carrier-based multimission aircraft (CBM) would not enjoy the same effectiveness in each role as its counterparts of the baseline carrier operations model. Assumptions were made on the resulting reductions in sortie lethality and survivability. However, it is also expected that the resulting increase in availability by selecting the multimission configuration would allow larger sortie forces to be flown, and for the example given in Chapter 4, the effectiveness of the multimission configuration was close to that of the baseline.

6.3 Analyzing Performance Tradeoffs

At the sortie level, a number of vulnerability and susceptability reduction concepts were investigated for their effect on survivability enhancement. Vulnerability reduction concepts include damage suppression (both passive and active) as well as the shielding, location, redundancy or elimination of components. Susceptability reduction is accomplished through electronic countermeasures such as jammers and deceivers, signature reduction, and tactics. Susceptability considerations included in this research are: thrust/weight (T/W) ratio, radar cross section, flight altitude, electronic countermeasures as a percentage of payload, among others.

These considerations were then analyzed at the campaign level for their contributions/detractions to the measures of effectiveness (MOEs). The six MOEs derived in Chapter 5 are:

- CS - Campaign Survivability
- FFL - Fraction Force Lost
- ER - Exchange Ratio
- RER - Relative Exchange Ratio

- PCL - Possible Crew Loss
- RC - Replacement Cost

Each of the six MOEs is a function of the initial inventory levels of friendly and enemy forces at the start of the campaign. The MOEs are therefore cost-dependent in the sense that the initial inventory would be smaller for larger aircraft acquisition costs (for given survivability enhancement modifications), but, insight can still be gained by ignoring costs temporarily.

Of these six, one such measure of effectiveness, the relative exchange ratio RER, was felt to be the most descriptive since it essentially determines the victor at the end of the campaign (referring to the derivation, RER values greater than one indicate total annihilation of the enemy's force).

Investigation into the contributions of increasing the T/W ratio on the RER revealed what may be counter-intuitive results. Larger T/W means bigger engines = more weight, more drag, a bigger radar cross section; smaller T/W of course means smaller engines = less maneuverability and payload. The performance tradeoff comes when making the decision on the right engine size to realize the largest value of RER.

When analyzing the fighter escort missions, RER and other MOEs were determined and plotted against the range of the CBFE's (Carrier-Based aircraft for Fighter Escort) long-range missiles. Results for ranges less than 104 km predict an RER less than one. So, all other variables kept constant, the example of Chapter 5 requires that the friendly forces deploy long range missiles effective for more than 104 km. Of course, when costs are considered, evaluation of a 125 km missile vs. a 150 km missile may favor the lesser, since many more may be afforded to be deployed.

In the surface attack mission analyses of Chapter 5, the MOEs plotted against the aircraft's RCS (radar cross section) show marked changes in slope near RCS values of 10 m^2. This would imply that an aircraft with an RCS of 10 will not suffer much decrease in, say, RER for increases in its RCS, but any decrease in its RCS will result in substantially higher RER values.

An important parameter used in studying the tradeoffs between cost and effectiveness is the Acquisition Cost Ratio (ACR) which expresses the quotient between the baseline cost to that of the particular modification that will yield the same total procurement cost. If, for example, a modification is quoted to have an ACR of 0.9 times the baseline ACR cost, than the unit price of the modified aircraft should be 0.9 times that of the baseline design (or less) in order to match a fixed procurement budget. The role of the ACR is to establish a link between the decision-making variables (e.g., RC) and tactic related variables such as CS, RER, and PCL.

6.4 Recommendations for Future Research

The following sections are devoted to extensions and enhancement features that could be made to the models and methodology found in this research.

6.4.1 Logistics

An important consideration in the Carrier Operations Model (Chapter 4) that could be included are the effects of differing levels of damage on turn around time. By assigning these levels in a way similar to [11], a respective maintenance level could be assigned, each with its own crew and time requirement included in the model.

These considerations would provide a logical mechanism for varying both the availability and the sortie rate, and most importantly, the sortie force size. Larger sortie sizes would realize smaller kill fractions (of and for friendly forces), thus less severe maintenance levels would be assigned, thus accelerating the turn around time and increasing availability.

6.4.2 Stochasticity and Time Variance

A simpler, but harder to justify, approach to the above embellishments for modelling would be to vary sortie rate, availability, and any number of the other variables stochastically. Once data is available and relationships are established, calculations for the kill fractions may include functions of random variables to help account for such unknowns as weather, mechanical failures, and the pilot's human frailties. These calculations may have as a significant input the level of training of the pilot, since ability to respond under these varying conditions can have a great effect on the outcomes.

People experienced with carrier operations may discover that certain tasks are performed more efficiently with time (the learning curve [19]); these functions can be enhanced with time variant variables. An important time-variant concern is the ongoing procurement of new aircraft to replace those lost throughout the campaign (the procurement may be at a constant rate but the inventory level cannot be considered in the same way as the time-invariant baseline case as a "constant" initial inventory level).

6.4.3 Costs

To correctly analyze some of these tradeoffs, knowing the initial inventory levels is not satisfactory, especially if ongoing procurement is expected to occur. As an example, say with the initial inventory levels known and a RER predicted to be some moderately large number favorable to the friendly forces, that the enemy are able to procure their much cheaper aircraft at a rate extremly faster than the friendly forces, then they may do so possibly at a rate such that these enemy forces overwhelm the friendly.

The most reasonable means to analyze such a problem would be to determine the cost of these replacement aircraft, both in terms of money (possibly normalized as a function of the nation's GNP) and time (aircraft produced at a faster rate are expected to cost more money).

6.5 Relevance to Research Objectives

The models and methodology developed in this research can be used to help in the evaluation of the conceptual design of combat aircraft by determining the effect of aircraft performance parameters and survivability enhancement features on combat survivability and effectiveness.

Appendix A. Model Code

```
NOTE
NOTE _____________FRIENDLY_FORCES__________________
NOTE
NOTE ------------------------------------------------
NOTE CB$SA - CARRIER BASED A/C FOR SURFACE ATTACK
NOTE
L   CB$SA.K=MAX(0,CB$SA.J-(DT)(A$SAMSA.JK+A$SADOD.JK))
N   CB$SA=CB$SAN
C   CB$SAN=360
NOTE
NOTE A$SAMSA - ATTRITION $ SURFACE ATTACK ON MISSION SURFACE ATTACK
NOTE A$SAXSA - ATTRITION $ SURFACE ATTACK BY ENEMY SURFACE ATTACK
NOTE A$SAXFE - ATTRITION $ SURFACE ATTACK BY ENEMY FIGHTER ESCORT
NOTE K$SAMSA - KILL FRACTION $ SURFACE ATTACK ON MISSION SURFACE ATTACK
NOTE B$SAMSA - INTERMED. VARIABLE
NOTE D$SAMSA - INTERMED. VARIABLE
NOTE P$SAMSA - PROPORTION OF ATTRITION ATTRIBUTED TO PRIMARY THREAT
NOTE C$SAMSA - INTERMED. VARIABLE
NOTE
R   A$SAMSA.KL=A$SAXSA.K+A$SAXFE.K
A   A$SAXSA.K=P$SAMSA.K*K$SAMSA.K*$SA.K
A   A$SAXFE.K=(1-P$SAMSA.K)*K$SAMSA.K*$SA.K
A     K$SAMSA.K=(1-(1-K$SAXSA.K)*(1-K$SAXFE.K))/B$SAMSA.K
A       B$SAMSA.K=1+(KXSA$FE.K*(1-K$FEXSA.K)+KXFEUFE.K*(1-K$FEYFE.K))
X    /D$SAMSA.K
A         D$SAMSA.K=KXSA$FE.K+KXFEUFE.K+1E-6
A P$SAMSA.K=C$SAMSA.K+(1-C$SAMSA.K)*K$SAXFE.K/(K$SAXFE.K+K$FEYFE.K+1E-6)
A   C$SAMSA.K=K$SAXSA.K/(K$SAXSA.K+K$SAXFE.K+1E-6)
NOTE
NOTE A$SADOD - ATTRITION $ SURFACE ATTACK (DEAD ON DECK)
NOTE
R   A$SADOD.KL=FDOD.K*(CB$SA.K-$SA.K)
NOTE
NOTE $SA - $ SURFACE ATTACK SORTIE FORCE SIZE FOR SURFACE ATTACK
```

```
NOTE $SA MAY TAKE ON VALUES OF 4, 6, OR 8 ACCORDING TO FOLLOWING
NOTE
A   $SA.K=CLIP($SA$.K,0,SASEX.K,1)
A     $SA$.K=CLIP(4,$SA$$.K,0.30,RN4.K)
A     $SA$$.K=CLIP(6,8,0.70,RN4.K)
NOTE ------------------------------------------
NOTE CB$FE - CARRIER BASED A/C FOR FIGHTER ESCORT
NOTE
L   CB$FE.K=MAX(0,CB$FE.J-(DT)(A$FEMFE.JK+A$FEMSA.JK+A$FEMCD.JK
X    +A$FEDOD.JK))
N   CB$FE=CB$FEN
C   CB$FEN=180
NOTE
NOTE A$FEMFE - ATTRITION $ FIGHTER ESCORT ON MISSION FIGHTER ESCORT
NOTE
R   A$FEMFE.KL=K$FEXFE.K*$FEFE.K
NOTE
NOTE A$FEMSA - ATTRITION $ FIGHTER ESCORT ON MISSION SURFACE ATTACK
NOTE A$FEXSA - ATTRITION $ FIGHTER ESCORT BY X SURFACE ATTACK
NOTE A$FEYFE - ATTRITION $ FIGHTER ESCORT BY X FIGHTER ESCORT
NOTE K$FEMSA - KILL FRACTION $ FIGHTER ESCORT ON MISSION SURFACE ATTACK
NOTE B$FEMSA - INTERMED. VARIABLE
NOTE D$FEMSA - INTERMED. VARIABLE
NOTE P$FEMSA - PROPORTION OF ATTRITION ATTRIBUTED TO PRIMARY THREAT
NOTE C$FEMSA - INTERMED. VARIABLE
NOTE
R   A$FEMSA.KL=A$FEXSA.K+A$FEYFE.K
A   A$FEYFE.K=P$FEMSA.K*K$FEMSA.K*$FESA.K
A   A$FEXSA.K=(1-P$FEMSA.K)*K$FEMSA.K*$FESA.K
A     K$FEMSA.K=(1-(1-K$FEYFE.K)(1-K$FEXSA.K))/B$FEMSA.K
A       B$FEMSA.K=1+(KXFE$SA.K*(1-K$SAXFE.K)+KXSA$SA.K*(1-K$SAXSA.K))
X    /D$FEMSA.K
A         D$FEMSA.K=KXFE$SA.K+KXSA$SA.K+1E-6
A P$FEMSA.K=C$FEMSA.K+(1-C$FEMSA.K)*KXFE$SA.K/(KXFE$SA.K+KXSA$SA.K+1E-6)
A   C$FEMSA.K=K$FEYFE.K/(K$FEYFE.K+K$FEXSA.K+1E-6)
NOTE
NOTE A$FEMCD - ATTRITION $ FIGHTER ESCORT ON MISSION CARRIER DEFENSE
NOTE A$FEXCD - ATTRITION $ FIGHTER ESCORT BY X CARRIER DEFENSE
NOTE A$FEZFE - ATTRITION $ FIGHTER ESCORT BY X FIGHTER ESCORT
NOTE K$FEMCD - KILL FRACTION $ FIGHTER ESCORT ON MISSION CARRIER DEFENSE
NOTE B$FEMCD - INTERMED. VARIABLE
NOTE D$FEMCD - INTERMED. VARIABLE
NOTE P$FEMCD - PROPORTION OF ATTRITION ATTRIBUTED TO PRIMARY THREAT
NOTE C$FEMCD - INTERMED. VARIABLE
NOTE
R   A$FEMCD.KL=A$FEXCD.K+A$FEZFE.K
A   A$FEZFE.K=P$FEMCD.K*K$FEMCD.K*$FECD.K
A   A$FEXCD.K=(1-P$FEMCD.K)*K$FEMCD.K*$FECD.K
A     K$FEMCD.K=(1-(1-K$FEZFE.K)*(1-K$FEXCD.K))/B$FEMCD.K
A       B$FEMCD.K=1+(KXFE$CD.K*(1-K$SAXFE.K)+KXCD$CD.K*(1-K$CDXCD.K))
X    /D$FEMCD.K
A         D$FEMCD.K=KXFE$CD.K+KXCD$CD.K+1E-6
A P$FEMCD.K=C$FEMCD.K+(1-C$FEMCD.K)*KXFE$CD.K/(KXFE$CD.K+KXCD$CD.K+1E-6)
A   C$FEMCD.K=K$FEZFE.K/(K$FEZFE.K+K$FEXCD.K+1E-6)
NOTE
NOTE A$FEDOD - ATTRITION $ FIGHTER ESCORT (DEAD ON DECK)
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NOTE
R    A$FEDOD.KL=FDOD.K*(CB$FE.K-($FEFE.K+$FESA.K+$FECD.K))
NOTE
NOTE $FESA - $ FIGHTER ESCORT SORTIE FORCE SIZE FOR SURFACE ATTACK
NOTE $FESA MAY TAKE ON VALUES OF 3, 4, OR 5 ACCORDING TO FOLLOWING
NOTE
A    $FESA.K=CLIP($FESA$.K,0,SASEX.K,1)
A      $FESA$.K=CLIP(3,$FESA$$.K,0.30,RN5.K)
A      $FESA$$.K=CLIP(4,5,0.70,RN5.K)
NOTE
NOTE $FEFE - $ FIGHTER ESCORT SORTIE FORCE SIZE FOR FIGHTER ESCORT
NOTE $FEFE MAY TAKE ON VALUES OF 2, 3, OR 4 ACCORDING TO FOLLOWING
NOTE
A    $FEFE.K=CLIP($FEFE$.K,0,FESEX.K,1)
A      $FEFE$.K=CLIP(2,$FEFE$$.K,0.30,RN6.K)
A      $FEFE$$.K=CLIP(3,4,0.70,RN6.K)
NOTE
NOTE $FECD - $ FIGHTER ESCORT SORTIE FORCE SIZE FOR CARRIER DEFENSE
NOTE $FECD MAY TAKE ON VALUES OF 3, 4, OR 5 ACCORDING TO FOLLOWING
NOTE
A    $FECD.K=CLIP($FECD$.K,0,CDSEX.K,1)
A      $FECD$.K=CLIP(3,$FECD$$.K,0.30,RN7.K)
A      $FECD$$.K=CLIP(4,5,0.70,RN7.K)
NOTE ------------------------------------------------
NOTE CB$CD - CARRIER BASED AIRCRAFT FOR CARRIER DEFENSE
NOTE
L    CB$CD.K=MAX(0,CB$CD.J-(DT)(A$CDMCD.JK+A$CDDOD.JK))
N    CB$CD=CB$CDN
C    CB$CDN=120
NOTE
NOTE A$CDMCD - ATTRITION $ CARRIER DEFENSE ON MISSION CARRIER DEFENSE
NOTE A$CDXCD - ATTRITION $ CARRIER DEFENSE BY X CARRIER DEFENSE
NOTE A$CDXFE - ATTRITION $ CARRIER DEFENSE BY X FIGHTER ESCORT
NOTE K$CDMCD - KILL FRACTION $ CARRIER DEFENSE ON MISSION CARRIER DEFENSE
NOTE B$CDMCD - INTERMED. VARIABLE
NOTE D$CDMCD - INTERMED. VARIABLE
NOTE P$CDMCD - PROPORTION OF ATTRITION ATTRIBUTED TO PRIMARY THREAT
NOTE C$CDMCD - INTERMED. VARIABLE
NOTE
R    A$CDMCD.KL=A$CDXCD.K+A$CDXFE.K
A    A$CDXCD.K=P$CDMCD.K*K$CDMCD.K*$CD.K
A    A$CDXFE.K=(1-P$CDMCD.K)*K$CDMCD.K*$CD.K
A      K$CDMCD.K=(1-(1-K$CDXCD.K)*(1-K$CDXFE.K))/B$CDMCD.K
A        B$CDMCD.K=1+(KXCD$FE.K*(1-K$FEXCD.K)+KXFETFE.K*(1-K$FEZFE.K))
X     /D$CDMCD.K
A          D$CDMCD.K=KXCD$FE.K+KXFETFE.K+1E-6
A P$CDMCD.K=C$CDMCD.K+(1-C$CDMCD.K)*KXCD$FE.K/(KXCD$FE.K+KXFETFE.K+1E-6)
A    C$CDMCD.K=K$CDXCD.K/(K$CDXCD.K+K$CDXFE.K+1E-6)
NOTE
NOTE A$CDDOD - ATTRITION $ CARRIER DEFENSE (DEAD ON DECK)
NOTE FDOD - FRACTION DEAD ON DECK
NOTE RK$ - RELATIVE KILL OF FRIENDLY FORCES
NOTE
R    A$CDDOD.KL=FDOD.K*(CB$CD.K-$CD.K)
A      FDOD.K=RK$*K$CDMCD.K
C        RK$=0.10
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NOTE
NOTE $CD - $ CARRIER DEFENSE SORTIE FORCE SIZE FOR CARRIER DEFENSE
NOTE $CD MAY TAKE ON VALUES OF 4, 6, OR 8 ACCORDING TO FOLLOWING
NOTE
A   $CD.K=CLIP($CD$.K,0,CDSEX.K,1)
A     $CD$.K=CLIP(4,$CD$$.K,0.30,RN8.K)
A     $CD$$.K=CLIP(6,8,0.70,RN8.K)
NOTE
NOTE _____________ENEMY_FORCES_________________________
NOTE
NOTE ----------------------------------------
NOTE AGXCD - AGGRESSOR A/C AGAINST CARRIER DEFENSE
NOTE
L   AGXCD.K=MAX(0,AGXCD.J-(DT)(AXCDMCD.JK+AXCDDOG.JK))
N   AGXCD=AGXCDN
C   AGXCDN=240
NOTE
NOTE AXCDMCD - ATTRITION X CARRIER DEFENSE ON MISSION CARRIER DEFENSE
NOTE AXCD$CD - ATTRITION X CARRIER DEFENSE BY $ CARRIER DEFENSE
NOTE AXCD$FE - ATTRITION X CARRIER DEFENSE BY $ FIGHTER ESCORT
NOTE KXCDMCD - KILL FRACTION X CARRIER DEFENSE ON MISSION CARRIER DEFENSE
NOTE BXCDMCD - INTERMED. VARIABLE
NOTE DXCDMCD - INTERMED. VARIABLE
NOTE PXCDMCD - PROPORTION OF ATTRITION ATTRIBUTED TO PRIMARY THREAT
NOTE CXCDMCD - INTERMED. VARIABLE
NOTE
R   AXCDMCD.KL=AXCD$CD.K+AXCD$FE.K
A   AXCD$CD.K=PXCDMCD.K*KXCDMCD.K*XCD.K
A   AXCD$FE.K=(1-PXCDMCD.K)*KXCDMCD.K*XCD.K
A     KXCDMCD.K=(1-(1-KXCD$CD.K)*(1-KXCD$FE.K))/BXCDMCD.K
A       BXCDMCD.K=1+(K$CDXFE.K*(1-KXFE$CD.K)+K$FEZFE.K*(1-KXFETFE.K))
X    /DXCDMCD.K
A         DXCDMCD.K=K$CDXFE.K+K$FEZFE.K+1E-6
A PXCDMCD.K=CXCDMCD.K+(1-CXCDMCD.K)*K$CDXFE.K/(K$CDXFE.K+K$FEZFE.K+1E-6)
A   CXCDMCD.K=KXCD$CD.K/(KXCD$CD.K+KXCD$FE.K+1E-6)
NOTE
NOTE AXCDDOG - ATTRITION X CARRIER DEFENSE (DEAD ON GROUND)
NOTE
R   AXCDDOG.KL=FDOG.K*(AGXCD.K-XCD.K)
NOTE
NOTE XCD - X CARRIER DEFENSE SORTIE FORCE SIZE AGAINST CARRIER DEFENSE
NOTE XCD MAY TAKE ON VALUES OF 6, 9, OR 12 ACCORDING TO FOLLOWING
NOTE
A   XCD.K=CLIP(XCDX.K,0,CDSEX.K,1)
A     XCDX.K=CLIP(6,XCDXX.K,0.30,RN9.K)
A     XCDXX.K=CLIP(9,12,0.70,RN9.K)
NOTE ----------------------------------------
NOTE AGXFE - AGGRESSOR A/C FOR FIGHTER ESCORT
NOTE
L   AGXFE.K=MAX(0,AGXFE.J-(DT)(AXFEMFE.JK+AXFEMCD.JK+AXFEMSA.JK
X    +AXFEDOG.JK))
N   AGXFE=AGXFEN
C   AGXFEN=360
NOTE
NOTE AXFEMFE - ATTRITION X FIGHTER ESCORT ON MISSION FIGHTER ESCORT
NOTE
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R    AXFEMFE.KL=KXFE$FE.K*XFEFE.K
NOTE
NOTE AXFEMCD - ATTRITION X FIGHTER ESCORT ON MISSION CARRIER DEFENSE
NOTE AXFE$CD - ATTRITION X FIGHTER ESCORT BY $ CARRIER DEFENSE
NOTE AXFETFE - ATTRITION X FIGHTER EXCORT BY $ FIGHTER ESCORT
NOTE KXFEMCD - KILL FRACTION X FIGHTER ESCORT ON MISSION CARRIER DEFENSE
NOTE BXFEMCD - INTERMED. VARIABLE
NOTE DXFEMCD - INTERMED. VARIABLE
NOTE PXFEMCD - PROPORTION OF ATTRITION ATTRIBUTED TO PRIMARY THREAT
NOTE CXFEMCD - INTERMED. VARIABLE
NOTE
R    AXFEMCD.KL=AXFE$CD.K+AXFETFE.K
A    AXFETFE.K=PXFEMCD.K*KXFEMCD.K*XFECD.K
A    AXFE$CD.K=(1-PXFEMCD.K)*KXFEMCD.K*XFECD.K
A      KXFEMCD.K=(1-(1-KXFE$CD.K)*(1-KXFETFE.K))/BXFEMCD.K
A        BXFEMCD.K=1+(K$FEXCD.K*(1-KXCD$FE.K)+K$CDXCD.K*(1-KXCD$CD.K))
X     /DXFEMCD.K
A          DXFEMCD.K=K$FEXCD.K+K$CDXCD.K+1E-6
A PXFEMCD.K=CXFEMCD.K+(1-CXFEMCD.K)*K$FEXCD.K/(K$FEXCD.K+K$CDXCD.K+1E-6)
A    CXFEMCD.K=KXFETFE.K/(KXFETFE.K+KXFE$CD.K+1E-6)
NOTE
NOTE AXFEMSA - ATTRITION X FIGHTER ESCORT ON MISSION SURFACE ATTACK
NOTE AXFE$SA - ATTRITION X FIGHTER ESCORT BY $ SURFACE ATTACK
NOTE AXFEUFE - ATTRITION X FIGHTER ESCORT BY $ FIGHTER ESCORT
NOTE KXFEMSA - KILL FRACTION X FIGHTER ESCORT ON MISSION SURFACE ATTACK
NOTE BXFEMSA - INTERMED. VARIABLE
NOTE DXFEMSA - INTERMED. VARIABLE
NOTE PXFEMSA - PROPORTION OF ATTRITION ATTRIBUTED TO PRIMARY THREAT
NOTE CXFEMSA - INTERMED. VARIABLE
NOTE
R    AXFEMSA.KL=AXFE$SA.K*AXFEUFE.K
A    AXFEUFE.K=PXFEMSA.K*KXFEMSA.K*XFESA.K
A    AXFE$SA.K=(1-PXFEMSA.K)*KXFEMSA.K*XFESA.K
A      KXFEMSA.K=(1-(1-KXFE$SA.K)*(1-KXFEUFE.K))/BXFEMSA.K
A        BXFEMSA.K=1+(K$FEXSA.K*(1-KXSA$FE.K)+K$SAXSA.K*(1-KXSA$SA.K))
X     /DXFEMSA.K
A          DXFEMSA.K=K$FEXSA.K+K$SAXSA.K+1E-6
A PXFEMSA.K=CXFEMSA.K+(1-CXFEMSA.K)*K$FEXSA.K/(K$FEXSA.K+K$SAXSA.K+1E-6)
A    CXFEMSA.K=KXFEUFE.K/(KXFEUFE.K+KXFE$SA.K+1E-6)
NOTE
NOTE AXFEDOG - ATTRITION X FIGHTER ESCORT (DEAD ON GROUND)
NOTE
R    AXFEDOG.KL=FDOG.K*(AGXFE.K-(XFEFE.K+XFECD.K+XFESA.K))
NOTE
NOTE XFECD - X FIGHTER ESCORT SORTIE FORCE SIZE FOR CARRIER DEFENSE
NOTE XFECD MAY TAKE ON VALUES OF 5, 6, OR 7 ACCORDING TO FOLLOWING
NOTE
A    XFECD.K=CLIP(XFECDX.K,0,CDSEX.K,1)
A      XFECDX.K=CLIP(5,XFECDXX.K,0.30,RN10.K)
A      XFECDXX.K=CLIP(6,7,0.70,RN10.K)
NOTE
NOTE XFEFE - X FIGHTER ESCORT SORTIE FORCE SIZE FOR FIGHTER ESCORT
NOTE XFEFE MAY TAKE ON VALUES OF 4, 5, OR 6 ACCORDING TO FOLLOWING
NOTE
A    XFEFE.K=CLIP(XFEFEX.K,0,FESEX.K,1)
A      XFEFEX.K=CLIP(4,XFEFEXX.K,0.30,RN11.K)
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A     XFEFEXX.K=CLIP(5,6,0.70,RN11.K)
NOTE
NOTE XFESA - X FIGHTER ESCORT SORTIE FORCE SIZE FOR SURFACE ATTACK
NOTE XFESA MAY TAKE ON VALUES OF 5, 6, OR 7 ACCORDING TO FOLLOWING
NOTE
A    XFESA.K=CLIP(XFESAX.K,0,SASEX.K,1)
A     XFESAX.K=CLIP(5,XFESAXX.K,0.30,RN12.K)
A     XFESAXX.K=CLIP(6,7,0.70,RN12.K)
NOTE ------------------------------------
NOTE AGXSA - AGGRESSOR UNITS AGAINST SURFACE ATTACK
NOTE
L    AGXSA.K=MAX(0,AGXSA.J-(DT)(AXSAMSA.JK))
N    AGXSA=AGXSAN
C    AGXSAN=720
NOTE
NOTE AXSAMSA - ATTRITION X SURFACE ATTACK ON MISSION SURFACE ATTACK
NOTE AXSA$SA - ATTRITION X SURFACE ATTACK BY $ SURFACE ATTACK
NOTE AXSA$FE - ATTRITION X SURFACE ATTACK BY $ FIGHTER ESCORT
NOTE KXSAMSA - KILL FRACTION X FIGHTER ESCORT ON MISSION SURFACE ATTACK
NOTE BXSAMSA - INTERMED. VARIABLE
NOTE DXSAMSA - INTERMED. VARIABLE
NOTE PXSAMSA - PROPORTION OF ATTRITION ATTRIBUTED TO PRIMARY THREAT
NOTE CXSAMSA - INTERMED. VARIABLE
NOTE
R    AXSAMSA.KL=AXSA$SA.K*AXSA$FE.K
A    AXSA$SA.K=PXSAMSA.K*KXSAMSA.K*XSA.K
A    AXSA$FE.K=(1-PXSAMSA.K)*KXSAMSA.K*XSA.K
A     KXSAMSA.K=(1-(1-KXSA$SA.K)*(1-KXSA$FE.K))/BXSAMSA.K
A      BXSAMSA.K=1+(K$SAXFE.K*(1-KXFE$SA.K)+K$FEYFE.K*(1-KXFEUFE.K))
X    /DXSAMSA.K
A        DXSAMSA.K=K$SAXFE.K+K$FEYFE.K+1E-6
A PXSAMSA.K=CXSAMSA.K+(1-CXSAMSA.K)*K$SAXFE.K/(K$SAXFE.K+K$FEYFE.K+1E-6)
A    CXSAMSA.K=KXSA$SA.K/(KXSA$SA.K+KXSA$FE.K+1E-6)
NOTE
NOTE FDOG - FRACTION DEAD ON GROUND
NOTE RKX - RELATIVE KILL OF X TARGETS
NOTE
A    FDOG.K=RKX*KXSAMSA.K
C     RKX=0.10
NOTE
NOTE XSA - X SURFACE ATTACK TARGET FORCE SIZE FOR SURFACE ATTACK
NOTE XSA MAY TAKE ON VALUES OF 6, 9, OR 12 ACCORDING TO FOLLOWING
NOTE
A    XSA.K=CLIP(XSAX.K,0,SASEX.K,1)
A     XSAX.K=CLIP(6,XSAXX.K,0.30,RN13.K)
A     XSAXX.K=CLIP(9,12,0.70,RN13.K)
NOTE
NOTE ______KILL_FRACTION_CALCULATIONS___________________________
NOTE
NOTE K$SAXSA - KILL FRACTION $ SURFACE ATTACK BY X SURFACE ATTACK
A    K$SAXSA.K=CLIP(CLIP(K$1A.K,K$$1.K,XSA.K,12),0,SASEX.K,1)
A     K$$1.K=CLIP(K$1B.K,K$1C.K,XSA.K,9)
A     K$1A.K=TABHL(K$1AT,$SA.K,4,8,2)
A     K$1B.K=TABHL(K$1BT,$SA.K,4,8,2)
A     K$1C.K=TABHL(K$1CT,$SA.K,4,8,2)
T      K$1AT=0.13/0.12/0.11
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T       K$1BT=0.10/0.09/0.08
T       K$1CT=0.07/0.06/0.05
NOTE
NOTE K$SAXFE - KILL FRACTION $ SURFACE ATTACK BY X FIGHTER ESCORT
NOTE
A    K$SAXFE.K=CLIP(CLIP(K$2A.K,K$$2.K,XFESA.K,7),0,SASEX.K,1)
A      K$$2.K=CLIP(K$2B.K,K$2C.K,XFESA.K,6)
A      K$2A.K=TABHL(K$2AT,$SA.K,4,8,2)
A      K$2B.K=TABHL(K$2BT,$SA.K,4,8,2)
A      K$2C.K=TABHL(K$2CT,$SA.K,4,8,2)
T       K$2AT=0.20/0.19/0.18
T       K$2BT=0.17/0.16/0.15
T       K$2CT=0.14/0.13/0.12
NOTE
NOTE K$FEXFE - KILL FRACTION $ FIGHTER ESCORT BY X FIGHTER ESCORT
NOTE
A    K$FEXFE.K=CLIP(CLIP(K$3A.K,K$$3.K,XFEFE.K,6),0,FESEX.K,1)
A      K$$3.K=CLIP(K$3B.K,K$3C.K,XFEFE.K,5)
A      K$3A.K=TABHL(K$3AT,$FEFE.K,2,4,1)
A      K$3B.K=TABHL(K$3BT,$FEFE.K,2,4,1)
A      K$3C.K=TABHL(K$3CT,$FEFE.K,2,4,1)
T       K$3AT=0.20/0.18/0.16
T       K$3BT=0.15/0.13/0.11
T       K$3CT=0.10/0.08/0.06
NOTE
NOTE K$FEXSA - KILL FRACTION $ FIGHTER ESCORT BY X SURFACE ATTACK
NOTE
A    K$FEXSA.K=CLIP(CLIP(K$4A.K,K$$4.K,XSA.K,12),0,SASEX.K,1)
A      K$$4.K=CLIP(K$4B.K,K$4C.K,XSA.K,9)
A      K$4A.K=TABHL(K$4AT,$FESA.K,3,5,1)
A      K$4B.K=TABHL(K$4BT,$FESA.K,3,5,1)
A      K$4C.K=TABHL(K$4CT,$FESA.K,3,5,1)
T       K$4AT=0.15/0.14/0.13
T       K$4BT=0.12/0.11/0.10
T       K$4CT=0.09/0.08/0.07
NOTE
NOTE K$FEYFE - KILL FRACTION $ FIGHTER ESCORT BY X FIGHTER ESCORT
NOTE
A    K$FEYFE.K=CLIP(CLIP(K$5A.K,K$$5.K,XFESA.K,7),0,SASEX.K,1)
A      K$$5.K=CLIP(K$5B.K,K$5C.K,XFESA.K,6)
A      K$5A.K=TABHL(K$5AT,$FESA.K,3,5,1)
A      K$5B.K=TABHL(K$5BT,$FESA.K,3,5,1)
A      K$5C.K=TABHL(K$5CT,$FESA.K,3,5,1)
T       K$5AT=0.23/0.21/0.19
T       K$5BT=0.18/0.16/0.14
T       K$5CT=0.13/0.11/0.09
NOTE
NOTE K$FEXCD - KILL FRACTION $ FIGHTER ESCORT BY X CARRIER DEFENSE
NOTE
A    K$FEXCD.K=CLIP(CLIP(K$6A.K,K$$6.K,XCD.K,12),0,CDSEX.K,1)
A      K$$6.K=CLIP(K$6B.K,K$6C.K,XCD.K,9)
A      K$6A.K=TABHL(K$6AT,$FECD.K,3,5,1)
A      K$6B.K=TABHL(K$6BT,$FECD.K,3,5,1)
A      K$6C.K=TABHL(K$6CT,$FECD.K,3,5,1)
T       K$6AT=0.26/0.24/0.22
T       K$6BT=0.20/0.18/0.16
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T      K$6CT=0.14/0.12/0.10
NOTE
NOTE K$FEZFE - KILL FRACTION $ FIGHTER ESCORT BY X FIGHTER ESCORT
NOTE
A    K$FEZFE.K=CLIP(CLIP(K$7A.K,K$$7.K,XFECD.K,7),0,CDSEX.K,1)
A      K$$7.K=CLIP(K$7B.K,K$7C.K,XFECD.K,6)
A      K$7A.K=TABHL(K$7AT,$FECD.K,3,5,1)
A      K$7B.K=TABHL(K$7BT,$FECD.K,3,5,1)
A      K$7C.K=TABHL(K$7CT,$FECD.K,3,5,1)
T      K$7AT=0.23/0.21/0.19
T      K$7BT=0.18/0.16/0.14
T      K$7CT=0.13/0.11/0.09
NOTE
NOTE K$CDXCD - KILL FRACTION $ CARRIER DEFENSE BY X CARRIER DEFENSE
NOTE
A    K$CDXCD.K=CLIP(CLIP(K$8A.K,K$$8.K,XCD.K,12),0,CDSEX.K,1)
A      K$$8.K=CLIP(K$8B.K,K$8C.K,XCD.K,9)
A      K$8A.K=TABHL(K$8AT,$CD.K,4,8,2)
A      K$8B.K=TABHL(K$8BT,$CD.K,4,8,2)
A      K$8C.K=TABHL(K$8CT,$CD.K,4,8,2)
T      K$8AT=0.15/0.14/0.13
T      K$8BT=0.12/0.11/0.10
T      K$8CT=0.09/0.08/0.07
NOTE
NOTE K$CDXFE - KILL FRACTION $ CARRIER DEFENSE BY X FIGHTER ESCORT
NOTE
A    K$CDXFE.K=CLIP(CLIP(K$9A.K,K$$9.K,XFECD.K,7),0,CDSEX.K,1)
A      K$$9.K=CLIP(K$9B.K,K$9C.K,XFECD.K,6)
A      K$9A.K=TABHL(K$9AT,$CD.K,4,8,2)
A      K$9B.K=TABHL(K$9BT,$CD.K,4,8,2)
A      K$9C.K=TABHL(K$9CT,$CD.K,4,8,2)
T      K$9AT=0.30/0.28/0.26
T      K$9BT=0.24/0.22/0.20
T      K$9CT=0.18/0.16/0.14
NOTE
NOTE ----------------------------------------
NOTE KXCD$CD - KILL FRACTION X CARRIER DEFENSE BY $ CARRIER DEFENSE
NOTE
A    KXCD$CD.K=CLIP(CLIP(KX1A.K,KXX1.K,$CD.K,8),0,CDSEX.K,1)
A      KXX1.K=CLIP(KX1B.K,KX1C.K,$CD.K,6)
A      KX1A.K=TABHL(KX1AT,XCD.K,6,12,3)
A      KX1B.K=TABHL(KX1BT,XCD.K,6,12,3)
A      KX1C.K=TABHL(KX1CT,XCD.K,6,12,3)
T      KX1AT=0.70/0.68/0.66
T      KX1BT=0.64/0.62/0.60
T      KX1CT=0.58/0.56/0.54
NOTE
NOTE KXCD$FE - KILL FRACTION X CARRIER DEFENSE BY $ FIGHTER ESCORT
NOTE
A    KXCD$FE.K=CLIP(CLIP(KX2A.K,KXX2.K,$FECD.K,5),0,CDSEX.K,1)
A      KXX2.K=CLIP(KX2B.K,KX2C.K,$FECD.K,4)
A      KX2A.K=TABHL(KX2AT,XCD.K,6,12,3)
A      KX2B.K=TABHL(KX2BT,XCD.K,6,12,3)
A      KX2C.K=TABHL(KX2CT,XCD.K,6,12,3)
T      KX2AT=0.60/0.57/0.54
T      KX2BT=0.51/0.48/0.45
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T       KX2CT=0.42/0.39/0.36
NOTE
NOTE KXFE$FE - KILL FRACTION X FIGHTER ESCORT BY $ FIGHTER ESCORT
NOTE
A    KXFE$FE.K=CLIP(CLIP(KX3A.K,KXX3.K,$FEFE.K,4),0,FESEX.K,1)
A      KXX3.K=CLIP(KX3B.K,KX3C.K,$FEFE.K,3)
A      KX3A.K=TABHL(KX3AT,XFEFE.K,4,6,1)
A      KX3B.K=TABHL(KX3BT,XFEFE.K,4,6,1)
A      KX3C.K=TABHL(KX3CT,XFEFE.K,4,6,1)
T       KX3AT=0.75/0.73/0.71
T       KX3BT=0.70/0.68/0.66
T       KX3CT=0.65/0.63/0.61
NOTE
NOTE KXFE$CD - KILL FRACTION X FIGHTER ESCORT BY $ CARRIER DEFENSE
NOTE
A    KXFE$CD.K=CLIP(CLIP(KX4A.K,KXX4.K,$CD.K,8),0,CDSEX.K,1)
A      KXX4.K=CLIP(KX4B.K,KX4C.K,$CD.K,6)
A      KX4A.K=TABHL(KX4AT,XFECD.K,5,7,1)
A      KX4B.K=TABHL(KX4BT,XFECD.K,5,7,1)
A      KX4C.K=TABHL(KX4CT,XFECD.K,5,7,1)
T       KX4AT=0.55/0.53/0.51
T       KX4BT=0.49/0.47/0.45
T       KX4CT=0.43/0.41/0.39
NOTE
NOTE KXFETFE - KILL FRACTION X FIGHTER EXCORT BY $ FIGHTER ESCORT
NOTE
A    KXFETFE.K=CLIP(CLIP(KX5A.K,KXX5.K,$FECD.K,5),0,CDSEX.K,1)
A      KXX5.K=CLIP(KX5B.K,KX5C.K,$FECD.K,4)
A      KX5A.K=TABHL(KX5AT,XFECD.K,5,7,1)
A      KX5B.K=TABHL(KX5BT,XFECD.K,5,7,1)
A      KX5C.K=TABHL(KX5CT,XFECD.K,5,7,1)
T       KX5AT=0.78/0.76/0.74
T       KX5BT=0.73/0.71/0.69
T       KX5CT=0.68/0.66/0.64
NOTE
NOTE KXFE$SA - KILL FRACTION X FIGHTER ESCORT BY $ SURFACE ATTACK
NOTE
A    KXFE$SA.K=CLIP(CLIP(KX6A.K,KXX6.K,$SA.K,8),0,SASEX.K,1)
A      KXX6.K=CLIP(KX6B.K,KX6C.K,$SA.K,6)
A      KX6A.K=TABHL(KX6AT,XFESA.K,5,7,1)
A      KX6B.K=TABHL(KX6BT,XFESA.K,5,7,1)
A      KX6C.K=TABHL(KX6CT,XFESA.K,5,7,1)
T       KX6AT=0.55/0.53/0.52
T       KX6BT=0.50/0.48/0.46
T       KX6CT=0.44/0.42/0.40
NOTE
NOTE KXFEUFE - KILL FRACTION X FIGHTER ESCORT BY $ FIGHTER ESCORT
NOTE
A    KXFEUFE.K=CLIP(CLIP(KX7A.K,KXX7.K,$FESA.K,5),0,SASEX.K,1)
A      KXX7.K=CLIP(KX7B.K,KX7C.K,$FESA.K,4)
A      KX7A.K=TABHL(KX7AT,XFESA.K,5,7,1)
A      KX7B.K=TABHL(KX7BT,XFESA.K,5,7,1)
A      KX7C.K=TABHL(KX7CT,XFESA.K,5,7,1)
T       KX7AT=0.78/0.76/0.74
T       KX7BT=0.73/0.71/0.69
T       KX7CT=0.68/0.66/0.64
```

```
NOTE
NOTE KXSA$SA - KILL FRACTION X SURFACE ATTACK BY $ SURFACE ATTACK
NOTE
A    KXSA$SA.K=CLIP(CLIP(KX8A.K,KXX8.K,$SA.K,8),0,SASEX.K,1)
A      KXX8.K=CLIP(KX8B.K,KX8C.K,$SA.K,6)
A      KX8A.K=TABHL(KX8AT,XSA.K,6,12,3)
A      KX8B.K=TABHL(KX8BT,XSA.K,6,12,3)
A      KX8C.K=TABHL(KX8CT,XSA.K,6,12,3)
T        KX8AT=0.93/0.91/0.89
T        KX8BT=0.87/0.85/0.83
T        KX8CT=0.81/0.79/0.77
NOTE
NOTE KXSA$FE - KILL FRACTION X SURFACE ATTACK BY $ FIGHTER ESCORT
NOTE
A    KXSA$FE.K=CLIP(CLIP(KX9A.K,KXX9.K,$FESA.K,5),0,SASEX.K,1)
A      KXX9.K=CLIP(KX9B.K,KX9C.K,$FESA.K,4)
A      KX9A.K=TABHL(KX9AT,XSA.K,6,12,3)
A      KX9B.K=TABHL(KX9BT,XSA.K,6,12,3)
A      KX9C.K=TABHL(KX9CT,XSA.K,6,12,3)
T        KX9AT=0.20/0.19/0.18
T        KX9BT=0.17/0.16/0.15
T        KX9CT=0.14/0.13/0.12
NOTE
NOTE
NOTE __________SORTIE_SCHEDULING__________________________
NOTE
NOTE SASEX - SURFACE ATTACK SORTIE EXISTS
NOTE FESEX - FIGHTER ESCORT SORTIE EXISTS
NOTE CDSEX - CARRIER DEFENSE SORTIE EXISTS
NOTE
A    SASEX.K=CLIP(1,0,RN1.K,0.80)
A    FESEX.K=CLIP(1,0,RN2.K,0.90)
A    CDSEX.K=CLIP(1,0,RN3.K,0.85)
NOTE
NOTE RNI - RANDOM NUMBER I BETWEEN 0 AND 1
NOTE
A    RN1.K=NOISE()+0.5
A    RN2.K=NOISE()+0.5
A    RN3.K=NOISE()+0.5
A    RN4.K=NOISE()+0.5
A    RN5.K=NOISE()+0.5
A    RN6.K=NOISE()+0.5
A    RN7.K=NOISE()+0.5
A    RN8.K=NOISE()+0.5
A    RN9.K=NOISE()+0.5
A    RN10.K=NOISE()+0.5
A    RN11.K=NOISE()+0.5
A    RN12.K=NOISE()+0.5
A    RN13.K=NOISE()+0.5
NOTE
NOTE ___EXAMPLE_MEASURE_OF _EFFECTIVENESS______________________
NOTE
NOTE R$SAXSA - RELATIVE EXCHANGE RATIO: $SA BY XSA
NOTE T$SAXSA - INTERMED. TERM FOR $SA BY XSA
NOTE R1A - RER WHEN T$SAXSA < 1
NOTE R1B - RER WHEN T$SAXSA > 1
```

```
NOTE T1A - T$SAXSA WHEN T$SAXSA < 1 (OTHERWISE = 1)
NOTE T1B - T$SAXSA WHEN T$SAXSA > 1 (OTHERWISE = 1)
NOTE
A    R$SAXSA.K=CLIP(R1A.K,R1B.K,1,T$SAXSA.K)
A      T$SAXSA.K=((1-PSS1.K)/(PKS1.K+1E-6))*((AGXSAN/CB$SAN)**2)
A    R1A.K=1/(1+D1.K-(1-T1A.K)**0.5)
A      D1.K=SWITCH(1,CLIP(0,1,1,T$SAXSA.K),W$1.K)
A      T1A.K=CLIP(T$SAXSA.K,0,1,T$SAXSA.K)
A    R1B.K=1-((1-(1/(T1B.K+1E-6)))**0.5)
A      T1B.K=CLIP(T$SAXSA.K,1,T$SAXSA.K,1)
NOTE
NOTE PSS1 - ESTIMATED SORTIE SURVIVABILITY $SA VS. XSA
NOTE PKS1 - ESTIMATED SORTIE LETHALITY $SA VS. XSA
NOTE W$1 - SUM OF ATTRITION $SA BY XSA
NOTE WX1 - SUM OF ATTRITION XSA BY $SA
NOTE T$1 - SUM OF SORTIE SIZES $SA
NOTE TX1 - SUM OF SORTIE SIZES XSA
NOTE
A    PSS1.K=1-(W$1.K/T$1.K)
A    PKS1.K=WX1.K/TX1.K
L     W$1.K=W$1.J+A$SAXSA.J
L     T$1.K=T$1.J+$SA.J
N       W$1=0
N       T$1=1E-6
L     WX1.K=WX1.J+AXSA$SA.J
L     TX1.K=TX1.J+XSA.J
N       WX1=0
N       TX1=1E-6
NOTE
NOTE ------------------------------------------------
NOTE R$FEXFE - RELATIVE EXCHANGE RATIO: $FE BY XFE
NOTE T$FEXFE - INTERMED. TERM FOR $FE BY XFE
NOTE R2A - RER WHEN T$FEXFE < 1
NOTE R2B - RER WHEN T$FEXFE > 1
NOTE T2A - T$FEXFE WHEN T$FEXFE < 1 (OTHERWISE = 1)
NOTE T2B - T$FEXFE WHEN T$FEXFE > 1 (OTHERWISE = 1)
NOTE
A    R$FEXFE.K=CLIP(R2A.K,R2B.K,1,T$FEXFE.K)
A      T$FEXFE.K=((1-PSS2.K)/(PKS2.K+1E-6))*((AGXFEN/CB$FEN)**2)
A    R2A.K=1/(1+D2.K-(1-T2A.K)**0.5)
A      D2.K=SWITCH(1,CLIP(0,1,1,T$FEXFE.K),W$2.K)
A      T2A.K=CLIP(T$FEXFE.K,0,1,T$FEXFE.K)
A    R2B.K=1-((1-(1/(T2B.K+1E-6)))**0.5)
A      T2B.K=CLIP(T$FEXFE.K,1,T$FEXFE.K,1)
NOTE
NOTE PSS2 - ESTIMATED SORTIE SURVIVABILITY $FE VS. XFE
NOTE PKS2 - ESTIMATED SORTIE LETHALITY $FE VS. XFE
NOTE W$2 - SUM OF ATTRITION $FE BY XFE
NOTE WX2 - SUM OF ATTRITION XFE BY $FE
NOTE T$2 - SUM OF SORTIE SIZES $FE
NOTE TX2 - SUM OF SORTIE SIZES XFE
NOTE
A    PSS2.K=1-(W$2.K/T$2.K)
A    PKS2.K=WX2.K/TX2.K
L     W$2.K=W$2.J+A$FEMFE.JK+A$FEYFE.J+A$FEZFE.J
L     T$2.K=T$2.J+$FEFE.J+$FESA.J+$FECD.J
```

```
N       W$2=0
N       T$2=1E-6
L     WX2.K=WX2.J+AXFEMFE.JK+AXFETFE.J+AXFEUFE.J
L     TX2.K=TX2.J+XFEFE.J+XFECD.J+XFESA.J
N       WX2=0
N       TX2=1E-6
NOTE
NOTE ------------------------------------------------
NOTE R$CDXCD - RELATIVE EXCHANGE RATIO: $CD BY XCD
NOTE T$CDXCD - INTERMED. TERM FOR $CD BY XCD
NOTE R3A - RER WHEN T$CDXCD < 1
NOTE R3B - RER WHEN T$CDXCD > 1
NOTE T3A - T$CDXCD WHEN T$CDXCD < 1 (OTHERWISE = 1)
NOTE T3B - T$CDXCD WHEN T$CDXCD > 1 (OTHERWISE = 1)
NOTE
A    R$CDXCD.K=CLIP(R3A.K,R3B.K,1,T$CDXCD.K)
A      T$CDXCD.K=((1-PSS3.K)/(PKS3.K+1E-6))*((AGXCDN/CB$CDN)**2)
A    R3A.K=1/(1+D3.K-(1-T3A.K)**0.5)
A      D3.K=SWITCH(1,CLIP(0,1,1,T$CDXCD.K),W$3.K)
A      T3A.K=CLIP(T$CDXCD.K,0,1,T$CDXCD.K)
A    R3B.K=1-((1-(1/(T3B.K+1E-6)))**0.5)
A      T3B.K=CLIP(T$CDXCD.K,1,T$CDXCD.K,1)
NOTE
NOTE PSS3 - ESTIMATED SORTIE SURVIVABILITY $CD VS. XCD
NOTE PKS3 - ESTIMATED SORTIE LETHALITY $CD VS. XCD
NOTE W$3 - SUM OF ATTRITION $CD BY XCD
NOTE WX3 - SUM OF ATTRITION XCD BY $CD
NOTE T$3 - SUM OF SORTIE SIZES $CD
NOTE TX3 - SUM OF SORTIE SIZES XCD
NOTE
A    PSS3.K=1-(W$3.K/T$3.K)
A    PKS3.K=WX3.K/TX3.K
L     W$3.K=W$3.J+A$CDXCD.J
L     T$3.K=T$3.J+$CD.J
N       W$3=0
N       T$3=1E-6
L     WX3.K=WX3.J+AXCD$CD.J
L     TX3.K=TX3.J+XCD.J
N       WX3=0
N       TX3=1E-6
NOTE
NOTE ____________CONTROL_STATEMENTS________________________________
NOTE
SPEC DT=1/LENGTH=75/PRTPER=1
NOTE PRINT CB$SA,CB$FE,CB$CD,AGXCD,AGXFE,AGXSA
PRINT R$SAXSA,R$FEXFE,R$CDXCD
RUN
QUIT
```

Appendix B. Analytical Solution of Attrition Models

The attrition model referred to as Case 4 in Chapter 5 is represented mathematically by the following system of differential equations:

$$\frac{d\$_t}{dt} = -Q(\$_t) - B(X_t) \tag{B-1}$$

$$\frac{dX_t}{dt} = -C(\$_t) - P(X_t) \tag{B-2}$$

where the symbols are as explained earlier. In the state space form, the same can be written as:

$$\begin{matrix} \dot{\$}_t \\ \dot{X}_t \end{matrix} = \begin{bmatrix} -Q & -B \\ -C & -P \end{bmatrix} \begin{matrix} \$_t \\ X_t \end{matrix} \tag{B-3}$$

In vector notation,

$$\dot{\overline{X}} = (\overline{A})(\overline{X}) \tag{B-4}$$

where,

- $\dot{\overline{X}}$ is the vector on the left hand side of equation B-3
- $\overline{A}$ is the matrix on the right hand side of equation B-3
- $\overline{X}$ is the vector on the right hand side of equation B-3

Since the coefficients of the X vector are constants, the problem is referred to as a constant coefficient problem. The standard solution to such a problem is:

$$\overline{X} = e^{\overline{A}t}\overline{X}_0 \qquad \{B-5\}$$

The Cayley-Hamilton Theorem [??] is used to obtain e^{At} The characteristic equation of A is given by:

$$\begin{vmatrix} \lambda + Q & B \\ C & \lambda + P \end{vmatrix} = 0 \qquad \{B-6\}$$

The characteristic polynomial in λ, $EP(\lambda)$, can thus be obtained from equation B-6 as:

$$CP(\lambda) - \lambda^2 + (Q+P)\lambda + (Q)(P) - (B)(C) = 0 \qquad \{B-7\}$$

The roots of this equation are:

$$\lambda_1 = \left[-(Q+P) + \sqrt{(Q-P)^2 + 4(B)(C)} \right]/2 \qquad \{B-8\}$$

$$\lambda_2 = \left[-(Q+P) - \sqrt{(Q-P)^2 + 4(B)(C)} \right]/2 \qquad \{B-9\}$$

e^{λ} can be expressed in terms of the $CP(\lambda)$ as

$$e^{\lambda} = CP(\lambda)f(\lambda) + \text{ remainder} \qquad \{B-10\}$$

where $f(\lambda)$ is any function (to be assumed) and *remainder* is a term leftover after factorizing e^{λ} by $CP(\lambda)$ and $f(\lambda)$. Clearly the degree of the remainder has to be less than that of $CP(\lambda)$

Let,

$$remainder = U + (V)(\lambda) \qquad \{B-11\}$$

Substituting the values of la_1 and la_2 in equation B-10, and since $CP(\lambda_1) = CP(\lambda_2) = C$, we have:

$$e^{\lambda_1} = U + (V)(\lambda_1) \qquad \{B-12\}$$

and

$$e^{\lambda_2} = U + (V)(\lambda_2) \qquad \{B-13\}$$

Solving Equations B-12 and B-13 simultaneously,

$$U = \frac{\lambda_2 e^{\lambda_1} - \lambda_1 e^{\lambda_2}}{\lambda_2 - \lambda_1} \qquad \{B-14\}$$

and

$$V = \frac{e^{\lambda_2} - e^{\lambda_1}}{\lambda_2 - \lambda_1} \qquad \{B-15\}$$

Invoking the Cayley-Hamilton theorem, we can write

$$At = U(I) + V(A) \qquad \{B-16\}$$

where I is the identity matrix. Therefore,

$$e^{At} = \begin{bmatrix} -Q & -B \\ -C & -P \end{bmatrix} \left(\frac{e^{\lambda_2 t} - e^{\lambda_1 t}}{\lambda_2 - \lambda_1} \right) \left(\frac{\lambda_2 e^{\lambda_1 t} - \lambda_1 e^{\lambda_2 t}}{\lambda_2 - \lambda_1} \right) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \{B-18\}$$

Thus, from equations B-5 and B-17 we obtain:

$$\$_t = \frac{B(X_0)(e^{\lambda_2 t} - e^{\lambda_1 t}) + \$_0[(Q + \lambda_1)e^{\lambda_2 t} - (Q + P)e^{\lambda_1 t}]}{\lambda_2 - \lambda_1} \qquad \{B-18\}$$

and

$$X_t = \frac{(-\$_0/B)(Q+\lambda_1)(Q+\lambda_2)(e^{\lambda_2 t} - e^{\lambda_1 t}) + X_0[(Q+\lambda_1)e^{\lambda_1 t} - (Q+\lambda_2)e^{\lambda_2 t}]}{\lambda_2 - \lambda_1} \qquad \{B-19\}$$

Case 1 is obtained from this general case by letting B=0 and C=0 giving $\lambda_1 = -P$, $\lambda_2 = -Q$, and

$$\$_t = \frac{\$_0[(Q-P)e^{-Qt} - (Q-Q)e^{-Pt}]}{Q-P} = \$_0 e^{-Qt} \qquad \{B-20\}$$

$$X_t = \frac{X_0[(Q-P)e^{-Pt} - (Q-Q)e^{-Qt}]}{Q-P} = X_0 e^{-Pt} \qquad \{B-21\}$$

Case 2 is obtained from Eqs. B-18 and B-19 by letting Q=0 and P=0 so that

$$\$_t = \frac{B(X_0)(e^{-\sqrt{BC}\,t} - e^{\sqrt{BC}\,t}) - \$_0\sqrt{BC}\,(e^{-\sqrt{BC}\,t} + e^{\sqrt{BC}\,t})}{-2\sqrt{BC}} \qquad \{B-22\}$$

$$= \$_0 \cosh \sqrt{BC}\,t - X_0\sqrt{B/C}\, \sinh \sqrt{BC}\,t \qquad \{B-23\}$$

and

$$X_t = \frac{C(\$_0)(e^{-\sqrt{BC}\,t} - e^{\sqrt{BC}\,t}) - X_0\sqrt{BC}\,(e^{\sqrt{BC}\,t} + e^{-\sqrt{BC}\,t})}{-2\sqrt{BC}} \qquad \{B-24\}$$

$$= X_0 \cosh \sqrt{BC}\,t - \$_0\sqrt{B/C}\, \sinh \sqrt{BC}\,t \qquad \{B-25\}$$

Case 3 differs from the other cases in that the interaction between the aircraft force and the threat force is accounted for. Starting with the differential equations,

$$\frac{d\$_t}{dt} = -U(\$_t)(X_t) \qquad \{B-26\}$$

$$\frac{dX_t}{dt} = -V(\$_t)(X_t) \qquad \{B-27\}$$

one obtains:

$$\frac{d\$}{dX} = \frac{U}{V} \qquad \{B-28\}$$

Separating variables,

$$\int_{\$_0}^{\$_t} (V) d\$ = \int_{X_0}^{X_t} (U) dX \qquad \{B-29\}$$

Integrating, one obtains the equations of state for this "linear" attrition law:

$$V(\$_t - \$_0) = U(X_t - X_0) \qquad \{B-30\}$$

Solving for one of the state variables in terms of the other,

$$X_t = X_0 - (\$_0 - \$_t)V/U \qquad \{B-31\}$$

Now, to solve for the remaining state variables, substitute Eq. B-31 into Eq. B-26 yielding:

$$d\frac{\$_t}{dt} = -U(\$_t)[X_0 - (\$_0 - \$_t)V/U] = -U(\$_t)\left[\frac{X_0 U}{V} - (\$_0 - \$_t)\right]\frac{V}{U} \qquad \{B-33\}$$

$$= -V(\$_t)\left[\left(\frac{X_0 U}{V} - \$_0\right) + \$_t\right] \qquad \{B-34\}$$

Separating variables,

$$\int_{\$_0}^{\$_t} \frac{d\$_t}{\$_t\left[\left(\frac{X_0 U - \$_0 V}{V}\right) + \$_t\right]} = -V\int_0^t dt \qquad \{B-35\}$$

Converting the left-hand side to partial fractions,

$$\frac{V}{X_0U-\$_0V}\int_{\$_0}^{\$_t}\frac{d\$_t}{\$_t}-d\frac{\$_t}{\left(\frac{X_0U-\$_0V}{V}\right)+\$_t}=-V\int_0^t dt \qquad \{B-36\}$$

Integrating,

$$\ln\left(\frac{\$_t}{\frac{X_0U-\$_0V}{V}+\$_t}\right)\Bigg]_{\$_0}^{\$_t}=-(X_0U-\$_0V)t \qquad \{B-37\}$$

Applying the limits of integration and converting to the base of the natural logarithms gives:

$$\frac{\$_t}{\frac{X_0U-\$_0V}{V}+\$_t}=\frac{\$_t}{\frac{X_0U-\$_0V}{V}+\$_0}e^{-(X_0U-\$_0V)t} \qquad \{B-38\}$$

Inverting both sides,

$$\frac{X_0U-\$_0V}{V(\$_t)}+1=X_0\frac{U}{\$_0}Ve^{(X_0U-\$_0V)t} \qquad \{B-39\}$$

Solving for \$t gives:

$$\$_t=\frac{X_0U-\$_0V}{\left[\frac{X_0U}{\$_0V}e^{(X_0U-\$_0V)t}-1\right]V}=\frac{\$_0(\$_0V-X_0U)}{X_0Ue^{(X_0U-\$_0V)t}-\$_0V} \qquad \{B-40\}$$

Appendix C. Sensitivity Analyses

The following figures show the effects of varying certain kill fractions on the relative exchange ratios of the three missions. The Carrier Operations Model described in Chapter 4 and listed in Appendix A was modified to permit a sensitivity analysis.

The kill fraction calculations for the three mission types were modified by multiplying a constant variable. For example, the kill fraction of enemy surface attack targets by friendly surface attack aircraft, KXSA$SA, is now coded in DYNAMO by:

```
A    KXSA$SA.K=NSA*(CLIP(CLIP(KX8A.K,KXX8.K,$SA.K,8),0,SASEX.K,1)
```

with the only modification to the baseline code being the multiplication of the expression on the right-hand side by NSA. NSA is then varied by +/- 2.5% and +/- 5% and the simulation is run for the standard 75 hours. The curve in Fig. C.1 shows the effect of NSA on the relative exchange ratio between the primary combatants of the surface attack mission. Likewise, there are two other terms inserted into the model, NFE and NCD, for the fighter escort and carrier defense missions, respectively, which are kept equal to one while varying NSA. The results of varying NFE and NCD are presented in Figs. C.2 and C.3.

Table C.1 is a listing of the relative exchange ratios found by varying the values of the normalizer, N__. It is important to remember that these RER values are all from different simulation runs, except for the N__ values of 1.000 (since these are from the baseline where NSA=NFE=NCD).

Table C.1 - Relative Exchange Ratios for Varying Kill Fractions

N__ (Normalizer	Relative Exchange Ratio Surface Attack	Fighter Escort	Carrier Defense
0.950	2.652	1.308	1.518
0.975	2.732	1.366	1.560
1.000	2.813	1.420	1.602
1.025	2.892	1.472	1.643
1.050	2.972	1.522	1.683

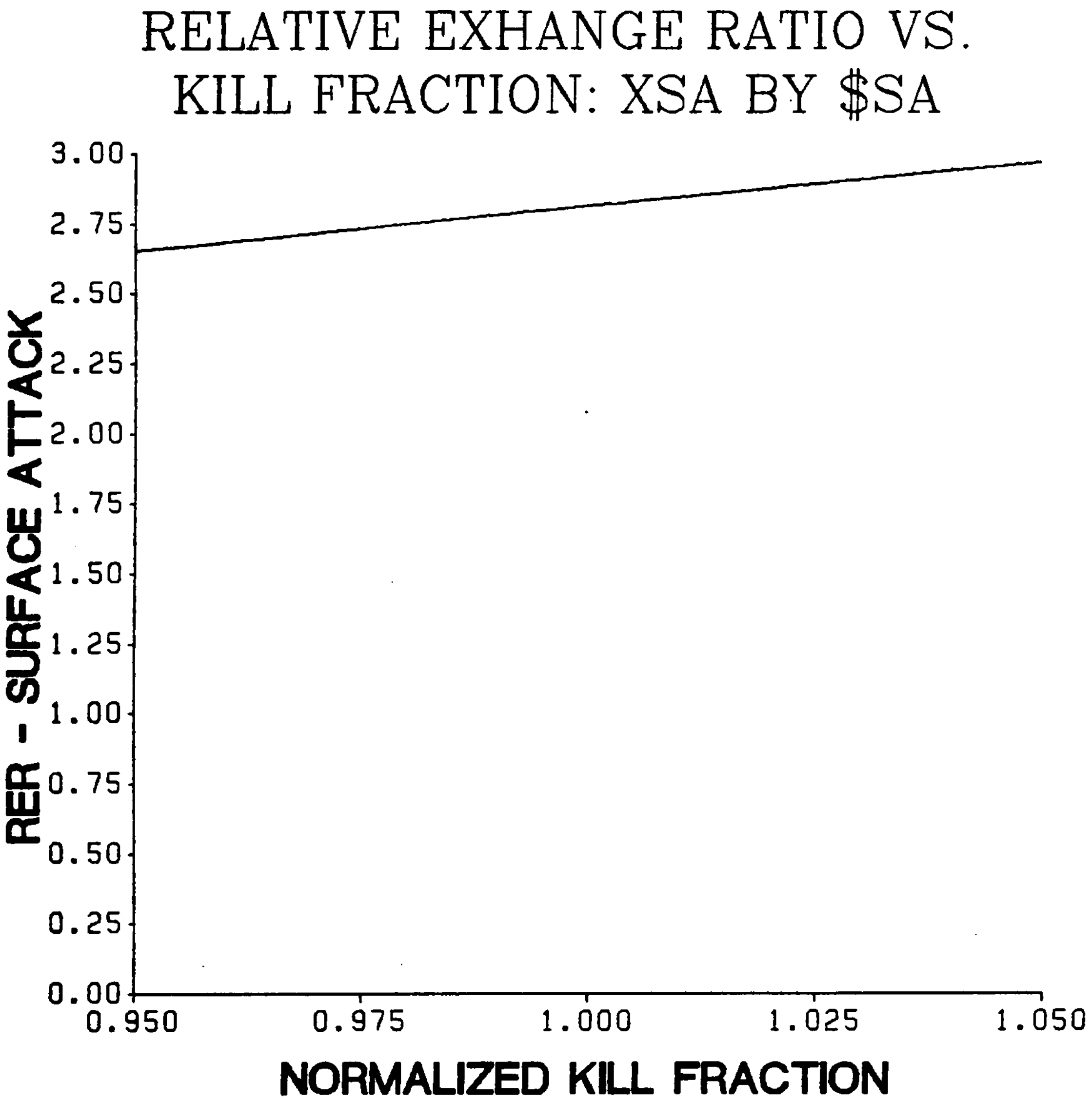

Figure C.1 - Sensitivity Analysis - Surface Attack Kill Fractions

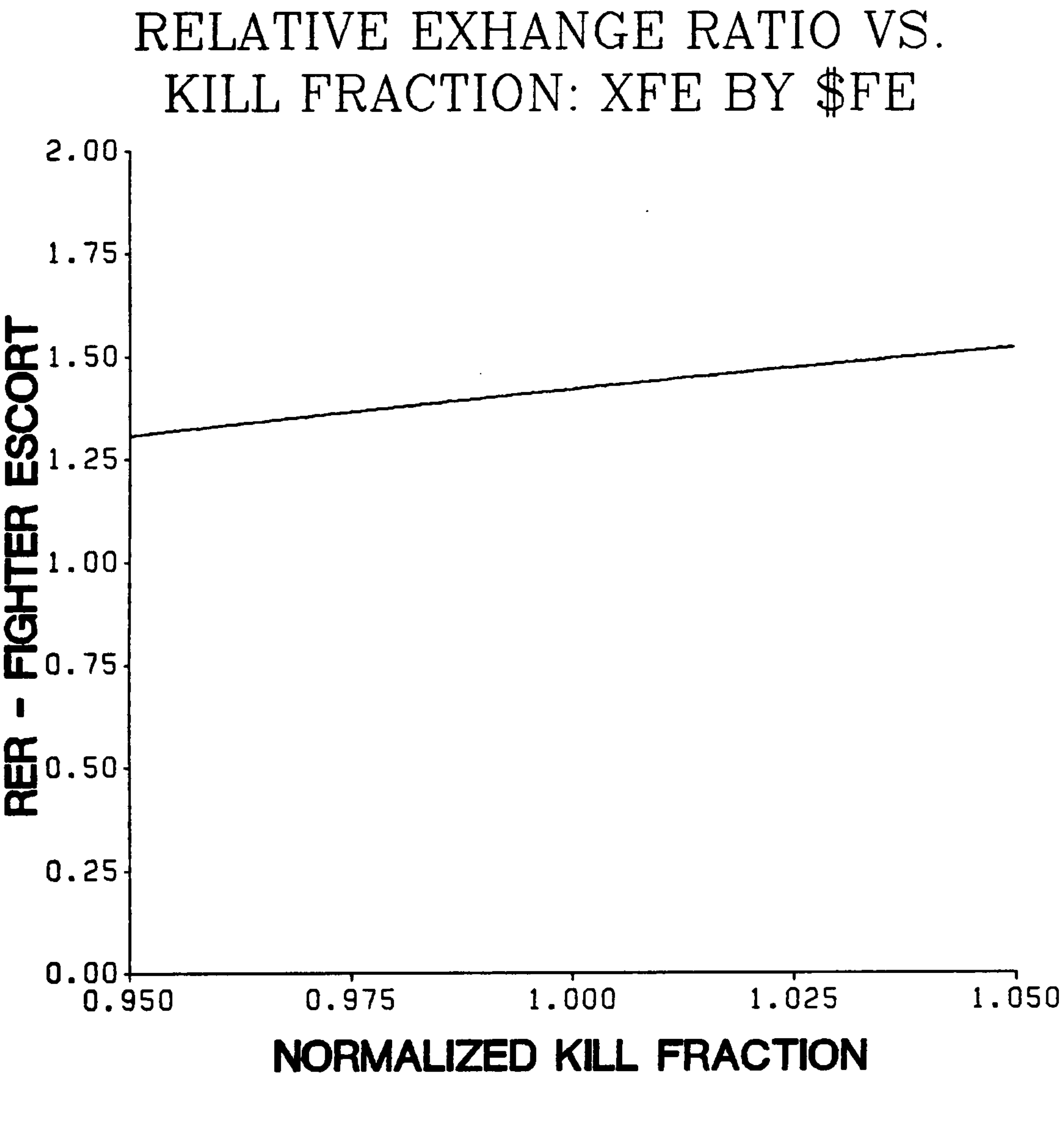

Figure C.2 - Sensitivity Analysis - Fighter Escort Kill Fractions

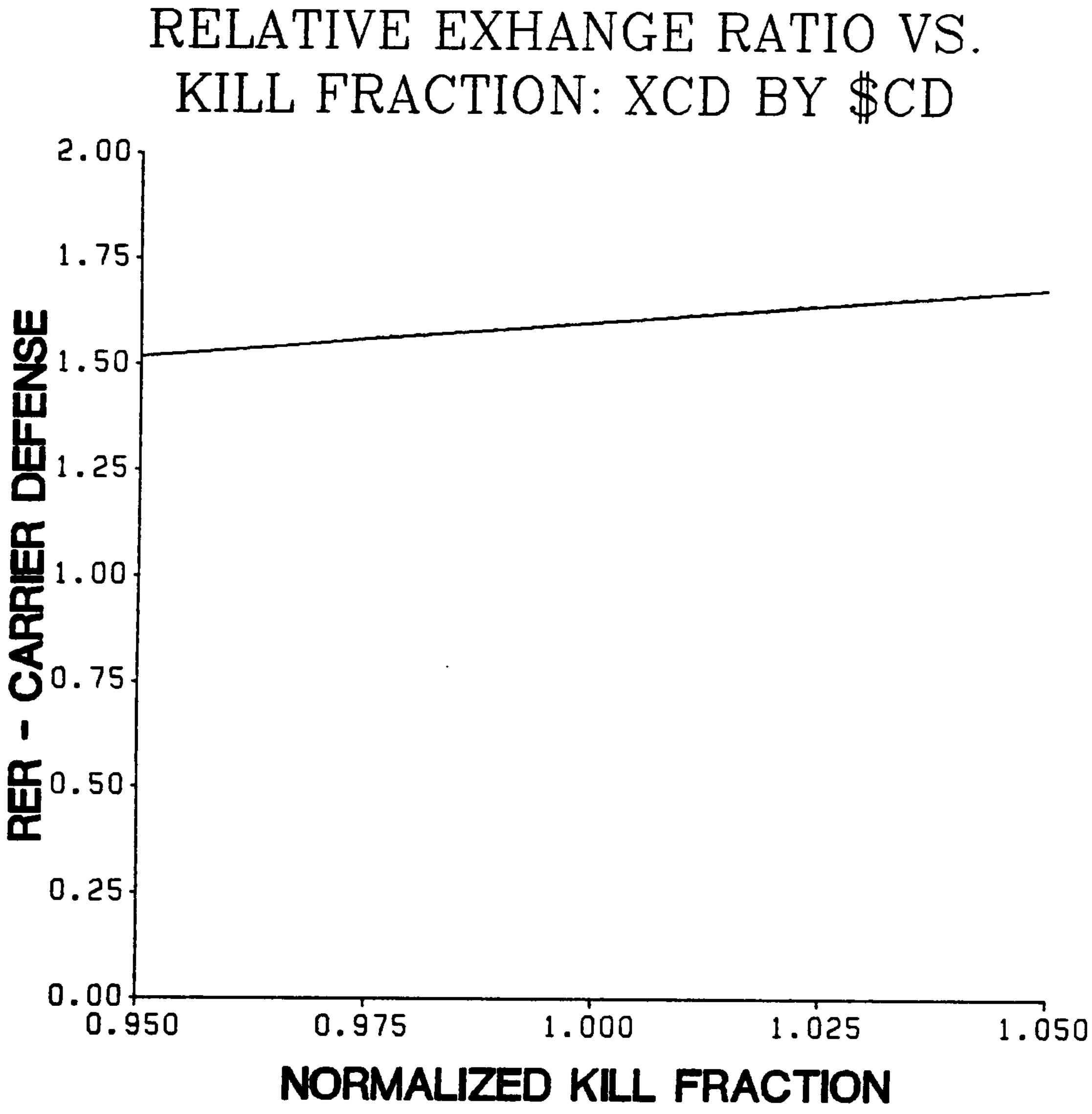

Figure C.3 - Sensitivity Analysis - Carrier Defense Kill Fractions

Appendix D. Glossery of Variables

ACR	acquistion cost ratio
AGXCD	aggressor aircraft against carrier defense
AGXFE	aggressor aircraft for fighter escort
AGXSA	aggressor units against surface attack
A$SAMSA	attrition of friendly surface attack aircraft on surface attack mission
A$SAXFE	attrition of friendly surface attack aircraft by enemy fighters
A$SAXSA	attrition of friendly surface attack aircraft by enemy defensive weapons sites
B	enemy effectiveness
C	friendly forces effectiveness
CB$SA	carrier based aircraft for surface attack
CB$FE	carrier based aircraft for fighter escort
CB$CD	carrier based aircraft for carrier defense
CLG	cumulative losses in achieving goal
CS	campaign survivability

DWS	defensive weapons sites
ECM	electronic countermeasures
ER	exchange ratio
FFL	fraction of force lost
FFR	fraction of force remaining
$K_{X1/\$1}$	kill fraction: X1 by \$1
$K_{X1/\$2}$	kill fraction: X1 by \$2
$K_{X2/\$1}$	kill fraction: X2 by \$1
$K_{X2/\$2}$	kill fraction: X2 by \$2
$K_{\$1_m}$	mission kill fraction of \$1
K$SAMSA	kill fraction of friendly surface attack aircraft on surface attack mission
K$SAXSA	kill fraction of friendly surface attack aircraft by defensive weapons sites
$K_{\$1/X1}$	kill fraction: \$1 by X1
$K_{\$1/X2}$	kill fraction: \$1 by X2
$K_{\$2/X1}$	kill fraction: \$2 by X1
$K_{\$2/X2}$	kill fraction: \$2 by X2
MAM	mission attainment measure
MOMS	measure of mission success
PCL	possible crew loss
P_K	probability of kill
P_{K,I_X}	probability of being killed by enemy infrared guided missiles
P_{K,R_X}	probability of being killed by enemy radar guided missiles
$P_{K/S}$	probability of kill given a sortie
$P_{K/TSE}$	probability of being killed given threat

	system effectiveness
$P_{K/XIE}$	probability of being killed, given infrared guided missile effectiveness
$P_{K/XRE}$	probability of being killed, given radar guided missile effectiveness
P_S	probability of survival
$P_{S/E}$	probability of survival, given an encounter
$P_{S/S}$	probability of survival, given a sortie
P_{SSH}	probability of single shot hit
P_{SSK}	probability of single shot kill
$P_{\$1_m}$	proportion of attrition of \$1 due to primary threat
P\$SAMSA	proportion of attrition of friendly surface attack aircraft due to enemy DWS
RC	replacement cost
RCS	radar cross section
RER	relative exchange ratio
SR	sortie rate
TL	total losses
TS	total number of sorties
TTD_N	total no. of targets destroyed by N aircraft
T/W	thrust to weight ratio
$T_{\$/X}$	intermediate term useful in calculating RER
X_t	enemy forces at time t
X1	primary threat in a sortie
X2	secondary (accompanying) threat in a sortie
$\$_t$	friendly forces at time t
\$1	primary friendly forces in a sortie

S2	secondary (accompanying) friendly forces in a sortie

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