Computer Aided Design of a Switched Reluctance Motor

by

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(ABSTRACT)

In recent years, the product life cycle has decreased and demands for new products have emerged due to competition, modern industrial needs and rapidly changing technology. This has necessitated changes in design, development and manufacturing processes to improve the quality and efficiency and to reduce the cost. Computer Aided Design (CAD) helps to meet this challenge both in the design evaluation and product design stages.

This thesis contains the development of an interactive software for the optimal design of a motor intended for variable speed applications. The use of finite element analysis methods is proposed as an indispensable part of the CAD system for electrical machine design. An illustration of the method is presented for the design of a switched reluctance motor.

Acknowledgements

I express my deep sense of thanks to my loving mother and father for encouraging and helping me throughout my educational career and other pursuits.

I would also like to express my gratitude to my advisor, Dr. Krishnan Ramu, for guiding me throughout the course of my graduate studies without which this work would have been impossible. I would also like to thank Dr. W.A. Blackwell and Dr. C.E. Nunnally for consenting to be in my committee amidst their busy schedule. My thanks are also due to A.O. Smith Corporation for funding this work. Last but not the least, I also appreciate the role played by my collegues   and

in the machine design and finite element analysis verification which enabled me to complete this work.

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1.0   Introduction

1.1   Overview

In recent years, the trend in industry is to reduce cost and to improve the efficiency and quality. Computer Aided Design or "CAD" as it is commonly known, is one of the methods currently used to achieve this objective.

The main advantages of CAD is that it provides the means for the designer to create, test, prove, modify and demonstrate models of possible future products. In theory, there is no constraint on the size of the product or the complexity of the device or system to be modeled.

In some areas, there has been a dramatic increase in efficiency in the design office due to the use of CAD. In other areas, where, apparently the same conditions exist and the same software and hardware facilities are available for adoption, the new

techniques have not been applied. The electrical machine design constitutes one such major area where CAD would make a difference in the technology.

The design of electrical machines consists of solutions to a number of related problems in at least two areas of expertise. They are:

1. Electromagnetic design.
2. Mechanical design.
3. Thermal design.

The overall process of design of any rotating machine from the specifications to the determination of all the other dimensions and other items of information required for the manufacture of the machine may be integrated as a single engineering problem. CAD, due to its capability of handling such a complex problem is gaining ground in this application.

1.2   Need for Computer aided design of a switched reluctance motor.

The Switched Reluctance Motor (SRM) is doubly salient, i.e., both the rotor and stator poles are salient. It consists of no windings on the rotor and very simple windings on the stator. Currents in the stator circuits are switched on and off in accordance with the rotor position and torque is produced by the attraction of rotor poles into alignment with the excited stator poles.

Though the operation of the machine is simple, the design process remains complex. This is due to the fact that the machine operates under saturation most of the time. Due to the nonlinear nature of the magnetic circuit, the force produced in the machine varies nonlinearly as the poles move in and out of the alignment. The prediction of torque, current, flux, voltage and hence the performance of the machine is obtained through the iterative process. This has necessitated the use of CAD for obtaining an accurate and optimal design of the SRM.

The motor has drawn attention due to its inherent advantages. The main advantage of a switched reluctance motor is its extreme low cost compared with a dc machine because of its simple construction. Other advantages are its high reliability, negligible maintenance, suitability for hazardous environment, high-speed capability and excellent thermal characteristics.

All of these factors make it potentially attractive for a wide range of applications in the manufacturing and transportation sectors and in domestic appliances. The ease of adaptability of switched reluctance machines for retrofitting is a significant advantage too. A significant portion of the cost of the SRM drive is in its power and control circuitry. Hence with the falling cost of the power and control circuits, the SRM drive systems are expected to become cost effective.

1.3 Organization of the thesis

The main tasks of the thesis are identified as follows. The main objective is to develop a highly interactive modular software to perform the design and performance evaluation of any given SRM. A finite element package is used to verify the validity of the results obtained through the design package. It serves to enhance the solution accuracy of the existing method. The modified algorithm has been incorporated to produce accurate and reliable results. Since the performance of the SRM is very much dependent on the BH characteristics of the material used, a very efficient curve fitting technique has to be devised and incorporated in the package.

The organization of the chapters in the thesis will be discussed in the next few paragraphs. Chapter 2 outlines broadly the field of Computer Aided Design. The various ingredients required in the development of a good engineering CAD package are presented. Also the research and development work done in the field of electrical machine design using a CAD package is discussed in detail.

Chapter 3 discusses the typical Computer Aided Design Package of electrical machines for variable speed applications. The evolution of new machines, such as the Switched Reluctance Motors (SRM) and its design process is discussed briefly. The CAD procedure in a modular form is described in detail. The various design aspects are also discussed in this chapter.

Chapter 4 gives the illustration of the CAD package developed for a switched reluctance motor which forms the main part of the thesis. The principle and operation of the SRM is discussed for getting an insight into the complexity of the problem. The

various factors involved in the design methodology are discussed and relevant mathematical equations are derived. Since a two dimensional finite element method package is used for the verification of the CAD package, the method adopted in the finite element calculations is also discussed. The system requirements and the features incorporated in the CAD package are described in detail.

Chapter 5 describes the various results obtained with the CAD package. The validity of these results are verified by comparing it with the finite element analysis results. The curve fitting and the design module accuracy are also discussed. Then the various menus available in the system are presented. Chapter 6 discusses the conclusions and recommendations for future study. The original contributions of the thesis have been mentioned in this chapter. Since this is the first work in the area, various improvements to this package are also discussed in detail. This would further enhance the validity of the package.

2.0   Computer Aided Design

2.1   Introduction

This chapter contains detailed description about the field of Computer Aided Design (CAD). It begins with a discussion about th scope of CAD. Then the various requirements of an ideal CAD packages are presented. This is followed by a detailed discussion about the past, present and future status of CAD and its applications. The chapter concludes with a comprehensive literature survey on the work that has been performed in the field of CAD in the design of electric motors.

2.2   Scope of Computer Aided Design

Computer Aided Design can be defined as the process of creating a product or system model in graphic or data form. It can also be thought of as a grouping of those

activities concerned with decision making, planning and creative design and the provision of the means by which they can be attained.

With the aid of CAD the performance of a product can be analyzed through interactive features and computer analysis. This reduces the dependency on the traditional prototype testing, thus saving the time and resources. Due to the fact that it reduces the design time and improves the productivity and design quality, the CAD has grown significantly from its modest beginning in the 1960's. Three important developments which assisted CAD in its growth are [9] :

1. Decentralized computer processing with inexpensive and reliable minicomputers.
2. Development of 3-D 'wireframe' technology.
3. Raster graphics display.

The CAD systems mainly fall into three categories [9]. They are,

1. **Workstation based systems:** The advantages of the personal computers and more powerful engineering workstations are integrated to make it very powerful.
2. **Minicomputer and Microcomputer based systems:** These systems support more users than workstation based systems and may handle anywhere between 4 - 12 users at a time.
3. **Mainframe based system:** Such systems handle 12 to 50 users at the same time. It is more sophisticated but costlier than the other systems.

Other types of CAD systems existing for electrical engineering applications are mainly in-house software. Such programs, are of course unmaintained and usually either specialized to a particular application or subject to the peculiar requirements

of an industrial or educational setting. Most of these are developed by university groups and workers at publicly supported research laboratories.

The main advantage of computerization for electrical engineering applications is that it allows the integration of all the design phases in product development in an industry. This can be seen from Figure 1 on page 9. It can be noted that, with a powerful personal computer or a network of personal computers the different stages in engineering design and the data bases needed can be very well integrated leading to automation, the dream of the industry.

Another advantage of CAD is, an alternate design's validity can be checked in a very short time by performing the analysis again just by plugging in the new values in place of the old ones. The design can be performed iteratively till the product specifications are met. This not only saves time, but also eliminates errors which are made even by an experienced and careful designer.

2.3   Requirements of a CAD system

The main requirements of any CAD package for engineering applications is that it should be highly interactive and capable of accurate and reliable results. Also, every time a particular design is performed a permanent data bank should be created. This helps in the reduction of the design time if the same model is to be used in the future.

After the design process is completed, an analysis has to be performed on the system model. The next step is to test the model with all the practical constraints such

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                        End product
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Figure 1.   Engineering design - Integration of design phases

as, thermal constraints, mode of vibration, component availability, etc. If the model is developed for commercial purposes, the cost factor has to be incorporated in the design. In such cases, the cost optimization routine becomes an important part of the CAD task.

The final step is to build the prototype using increasingly precise design information and test its performance. In all these stages the package should be highly interactive for making changes in the model parameters. An optimum design is released for manufacture. The series of procedures involved in a product development using CAD is shown in Figure 2 on page 11.

But the world's most perfect software is of no use if there is insufficient documentation to inform the user of the facilities built in the system and how to use them. Three levels of documentation are required for a CAD system to make it powerful [9]. They are:

1. **Aides memoire to the user:** A short manual, often known as the "reference card" is useful to jog the experienced users' memory on how to do things. It is often better to have the reference card to be built in the system than having it as an external documentation. This would help in the reduction of time for reference.

2. **Full user documentation:** This defines all the facilities built in the system along with the response to each of the command. An ideal form of this document is tough to realize since no software can satisfy all the requirements to all kinds of users. But with the passage of time, a CAD system can mature enough to be acceptable and so would be its manual.
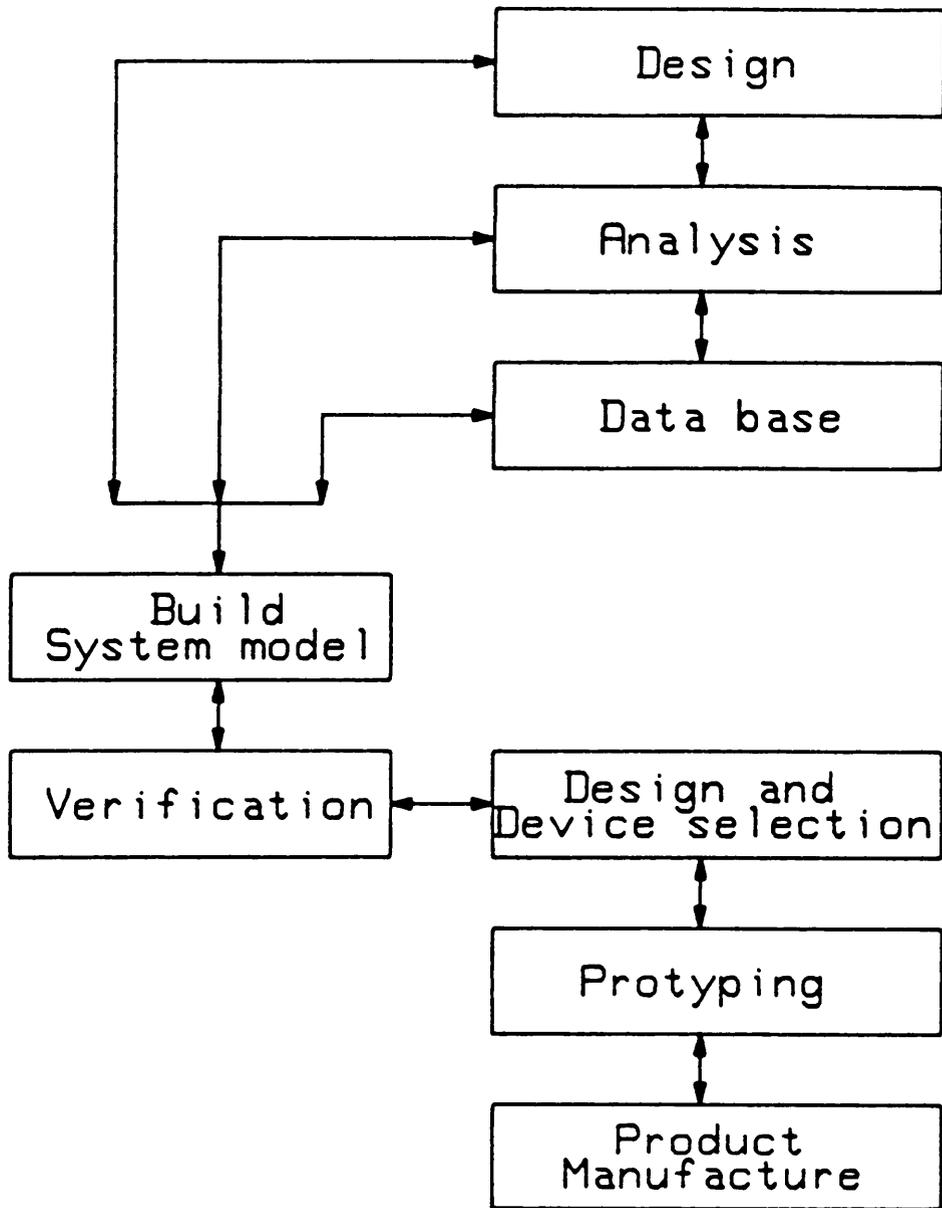
Figure 2. Product development using CAD

3. **Maintenance documentation:** The design principles should be included in an appropriate document. Subroutines calling graphs, details of common block structures and inter-relationships should be made clear.

In industry, the main advantage of the CAD process is that with a central data base established, design time can be very much reduced. This data base is accessed by both the product design and manufacturing organizations and would help in assessing the manufacturability of a product. Further it can help to set up optimum manufacturing schedules, optimize the use of components and keep the manufacturing costs as low as possible.

The main requirement of such integrated CAD systems is the interactive graphics feature. Computer graphics offer two important features:

1. It enables the analysis to be presented in a form with which an engineer is familiar and can quickly assimilate.
2. It enables simple and straightforward interaction, which is vital in making a computer system acceptable to non computer personnel.

The performance of the product can be analyzed and this enables the user to optimize the design in less time since the graphic display gives him an idea about the trend in the machine performance.

Another requirement for any CAD software is that it should be highly user friendly i.e. even a novice in the field should be able to operate it easily. The package should lead the designer through all the processes and help him in case of trouble. This feature becomes important in an industrial environment since the design of a product may

be done even by an incipient engineer. In such cases a tutorial to go through the design process is highly desired. This would help a new user to get adapted to the system without any external help.

2.4 CAD past, present and future

In the late 60's and the early 70's, the name CAD was used to describe the application of computers to design. But this had been going on right from the arrival of computers. Since then, the development of the CAD art from computer aided analysis to the present form as computer aided engineering, is moving it steadily closer to the professional practice of engineering. This can be attributed to the benefits accrued from CAD. Calculations which took many weeks to perform were computed in just a few seconds by the computer. Also complex problems which took days to compute were simplified by CAD. Even if correct results were not given, it was at least possible to know the trend of the results. Applications such as finite element analysis, multi-component distillation and most tasks requiring numerical methods of solution would not have made the present progress but for CAD.

Software for Computer Aided Design, which in the seventies amounted to straight-forward scientific application program, has increasingly begun to take system orientation associated with the business operating systems in the past. With the advent of personal computers in the 80's, computer aided design took a giant step in indus-trial applications. These computers very much reduced the cost of computation compared with the mainframe computers. But the only problem with such technology

was the time associated with each design. This disadvantage was reduced to a certain extent by networking. By making available the results of a design to more number of users and storing it in a permanent data bank, repetition of design was eliminated. But even all these developments could not eliminate the problem of time completely. This is due to the fact that a computer has to be completely tied up to a problem for long periods of time for each design.

But due to the current developments like multitasking even such disadvantages are being eliminated. The multitasking feature allows the user to carry on his normal work with the computer, while the computer performs complex design calculations, such as finite element analysis,etc., which might even take two to three days to arrive at a solution.

There are many constraints at present which impede the growth of the CAD such as cost and the gap which exists between the engineers who need some software and the system analysts who are assigned to develop it. Another major barrier is the fear that use of the CAD will inhibit the learning experience of junior staff. The major hindrance to introducing the CAD system in companies where complex engineering products are designed is the lack of ability to handle large amounts of engineering data. Database technology has yet to develop to the required level of proficiency and various database management systems currently available becomes hopeless when storing huge chunks of data for certain engineering applications.

In the future, these barriers are bound to be overcome. To reduce cost and time, CAD software might be built into dedicated hardware contained in the back cover of the manual describing its use. The hardware could be inserted into a small, inexpensive keyboard which is also linked to the main computer system which carries

large all-purpose programs and allows access to the project data base and general data bank.

2.5 Applications of CAD

A partial list of the current applications gives an indication of the extent to which the CAD is able to work in an interdisciplinary manner across the whole field of engineering. The major fields of application are mechanical, civil, chemical, industrial and electrical engineering. The areas of electrical engineering where CAD is currently used are [8] :

1.   Integrated circuit analysis and design

2.   Printed circuit board layout

3.   Component simulation

4.   Fault level interactive analysis

5.   Electric motor design

6.   Electron lens design

2.6 Status of CAD in the design of electric motors

Many forms of the CAD packages have been developed for design of electric motors and a number of them are used commercially at the present time. Some of the work

done in the various fields of electric motor design is discussed in the following paragraphs.

Even as early as in 1954, when CAD was just in its infancy, researchers were working on the design of electrical machines using a digital computer. Punched card techniques were used effectively for the solutions of problems involving [10],

1. Field distribution.
2. Optimum design for a synchro.
3. Optimum cage design for an induction motor.

Due to the advantage in solving problems involving iterative techniques, digital computers were used for the solution of distributed parameter problems in electrical machines [11]. This analysis technique was helpful in making intelligent decisions in the machine design, based on the computer output. A similar approach was used to obtain shaft deflection curves and critical speeds for a wide variety of design involving both ac and dc machines [2,11]. This resulted in a better mechanical design as well as better electrical and magnetic design.

In the area of dc machines, the CAD procedures were in vogue for the design of small dc machines [14]. A computer based solution of the equations arising from the application of optimal control theory, to a wide speed range dc drive has been developed [12]. This optimal analysis helped in response improvements in industrial dc drives. Also a piecewise linear computer model has been used to calculate the control characteristics of a typical dc series motor [13]. The computer program is used to obtain a normalized per-unit torque model with the relative propagation of series

to diverted turns as a parameter, and to generate the characteristics of 'field strength' with the variation of mark/period ratio.

A CAD package has been reported for overhead equipment designs for electric railways [15]. The derivation of the complete specifications of an overhead electrification system to satisfy a customer's specification is a complex process with many interacting parameters. Use of the CAD reduced lead time considerably and included additional design features which are not possible otherwise.

In the early past of 1959, an effort was made for the design of a synchronous machine using a digital computer [16]. The design method chosen was not a novel one and many assumptions were made. But, a feasible design of the synchronous machine was performed with the aid of the computer. Also, there had been in actual use a lot of computer programs for rotating machine design [17,18]. All these packages were based on the modern principles of CAD. One such package was a combination of three programs for the design of synchronous motor which had many of the features of a complete synthesis program [17]. Later a detailed design procedure for the design of salient pole synchronous machine was also presented [19].

A majority of work performed in electrical machinery is in the design of the induction motor. This is natural as the majority of the machines used are induction motors. Various methods have been developed and used for the design of a single phase induction motor [20,21,22]. Work had been reported on the design of the polyphase induction motor [23]. A program was developed for the calculation of the performance of a squirrel cage induction motor [21]. As the computer technology advanced, methods were developed for the optimal design of machines [24]. These packages

utilized the capabilities of the modern, high speed computers to meet all the performance specifications at a minimum cost.

Another area where the CAD has played a vital role is in the design of permanent magnet (PM) synchronous machine. PM AC machines are becoming increasingly significant with robotics and machine tools market. Software has been written which can be used in the assessment of the relative outputs for high-field configurations [25]. A CAD tool has also been developed for a disc-geometry homopolar synchronous machine with field excitation on the primary side [26]. This was developed with the aid of a three dimensional magnetostatic computer program 'GFUN' which accurately predicted the flux densities prevailing in the machine.

Recently an extremely efficient steady state Pulse Width Modulated Variable Speed Drive (PWM VSD) modeling package has been used for transient performance investigation of a wide range of steady state operational modes and is in regular use [27]. Various other packages based on Finite Element Analysis (FEA) [9,28] have been developed and are being used for an accurate analysis of electrical machines. But they have the inherent disadvantage in that the solutions are time intensive and the designer cannot interpret the design analytically.

Apart from the above CAD packages, various in-house software have been developed for the design of induction, synchronous and other electrical machines. These are in use in large research centers and universities. But all these packages deal with specific and peculiar requirements of the user. Hence there are only a limited number of CAD packages available commercially which can handle a problem in its most general sense. One such CAD package for the design of an electric motor for variable speed application is discussed in the next chapter.

3.0 Computer Aided Design of electrical machines for variable speed applications

3.1 Introduction

The rapid development of electronics has made the variable speed drives cost effective. The interaction of the power converter with the electric machine has added some problems in the form of additional losses due to harmonics and suitable control techniques for optimized yet stable operation. Matching of the converter to the machine requires both mechanical and electromagnetic modifications to the machine. Hence, a sizeable amount of redesign of existing machines is presently done in the industry. And added to that is the design of new machines such as permanent magnet synchronous and switched reluctance machines for both high and low performance variable speed applications [29], respectively.

Altogether, these events have placed a strong emphasis on the machine design. The classical design procedures, while applicable to conventional machines, are not suitable for new machines. A research trend has emerged combining the classical and modern analysis and design procedures [31]. Such a step necessitates the need for computer aided design of electrical machines.

This chapter describes the computer aided design of electrical machines in a schematic and modular form. While this procedure is commonly applicable to the design of any electrical machine, an illustration for the design of a switched reluctance motor will be given in the next chapter. The present procedure has been successfully applied to the design of the axial-field permanent magnet synchronous machine, induction machine and switched reluctance motor. A number of prototypes have been tested based on the computer aided design procedure.

3.2   Computer Aided Design Procedure

The design of an electrical machine consists of two basic functions. They are:

1. Given a set of specifications, finding the relevant machine dimensions.

2. Evaluation of the machine performance given its dimensions. This is required to check whether the designed machine performs as specified.

This is shown in Figure 3 on page 21.
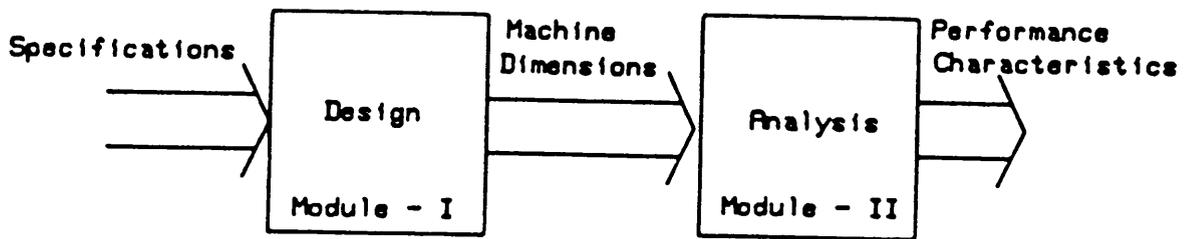
Figure 3.   Basic functional modules of machine design software

The module - I in some respects is the inverse of module- II so far as performance is concerned. The material characteristics go into module - I. The system equations governing the modules I and II are very different and highly coupled. In some details, the modules I and II are considered next to demonstrate their complexity.

3.2.1 Module - I

The specifications to the design module are in general,

1. Machine output characteristics

2. Power supply characteristics

3. Magnetic and electric loadings

4. Material characteristics

5. Type of motor for design

These submodules of the specification module are common to any motor design and their individual descriptions may have any number of options, shown in Figure 4 on page 23. Apart from the above specifications, there may be special considerations such as the type of winding and overall cost. These specifications are related to an electric machine by an output equation, given as,

$$S = KBA_{sp}D^2L\omega \qquad (2.1)$$

where,

        S is Volt ampere rating of the machine.

        K is constant dependent on machine type.
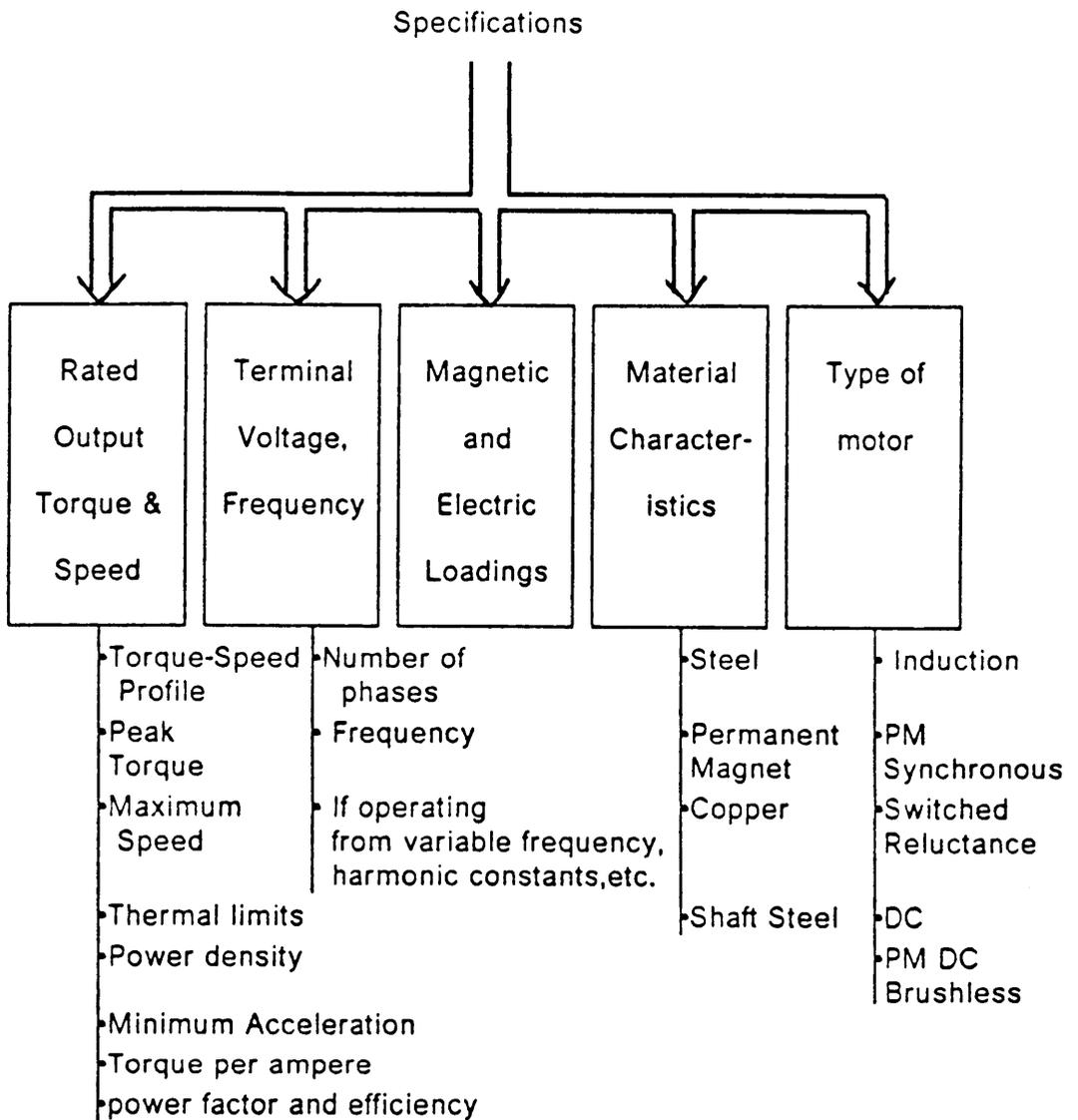
Figure 4. Input Specifications

A_{sp} is Electric loading.

B is Magnetic loading.

D is bore diameter.

L is effective core length.

ω is angular speed of the machine.

These constants are different for different machines and accordingly considered at the appropriate stages of software development.

3.2.2 Design Process in Module - I

The output equation generates the bore diameter and iron length of the machine. Based on these parameters, the following variables are calculated.

1. Number of rotor and stator slots, their shape and dimensions

2. Number of turns per phase and their arrangement

3. Airgap

4. In case of salient pole machines, the pole arcs

5. In case of dc machines, the commutator design

The input specifications number around 20 and the output of the design module - I has about 10 variables.

3.2.3 Module - II

The machine dimensions are used to determine the parameters of the electrical equivalent circuit of the machine. There are two methods to do that and they are,

1. Classical method

2. Finite element method

A considerable number of approximations are made in the former while the latter can use very little approximations, depending on the finite element method used. With the aid of the estimated machine parameters, the performance of the machine can be computed. This is shown in the Figure 5 on page 26. If the difference between the predicted performances by these two methods is not within the accepted norm, then the classical method can be fine tuned to narrow down the error. This eventually helps to eliminate the finite element analysis for all future designs of a particular motor, saving a considerable amount of time and effort.

At the output of the module - II, the following are calculated:

1. The output torque

2. Speed

3. Current

4. Losses

5. Thermal rise

6. Torque per unit current

7. Power density
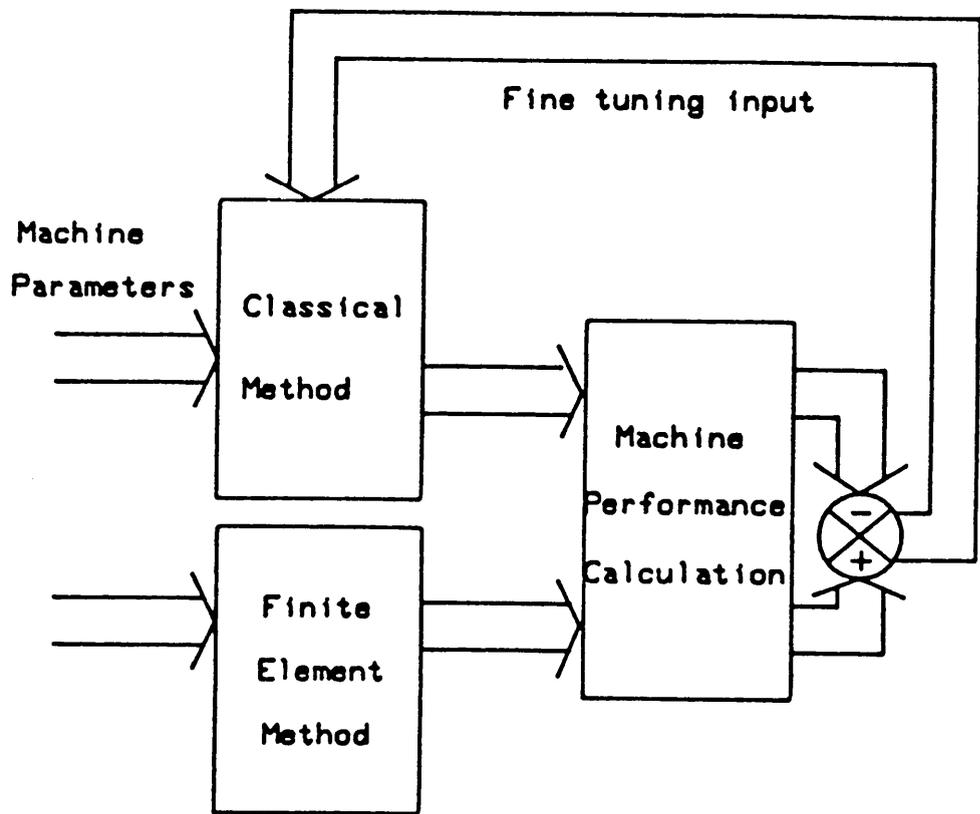
Figure 5.   Module - II in details

8. Cost per kW

9. Acceleration, etc.

These may or may not satisfy the specifications. A further procedure is required and is named module - III and is considered next.

3.2.4 Module - III

The module - III comes into design process when there is a discrepancy between the specifications and output performance of the machine. The computational process of module - III is shown in Figure 6 on page 28. In this module, the output performance obtained is compared with that of the input specifications and checked for accuracy. If there is an error in the prediction, then the machine dimensions are varied accordingly. Usually the value of the external diameter or the length is fixed. Hence the optimization is performed to obtain these values. After recalculating the machine dimensions, the performance is again predicted with the aid of Module II till the desired design is obtained. The overall flow chart is shown in Figure 7 on page 29. An illustration of the method in the design of a complex machine such as a switched reluctance motor is discussed in the next chapter.
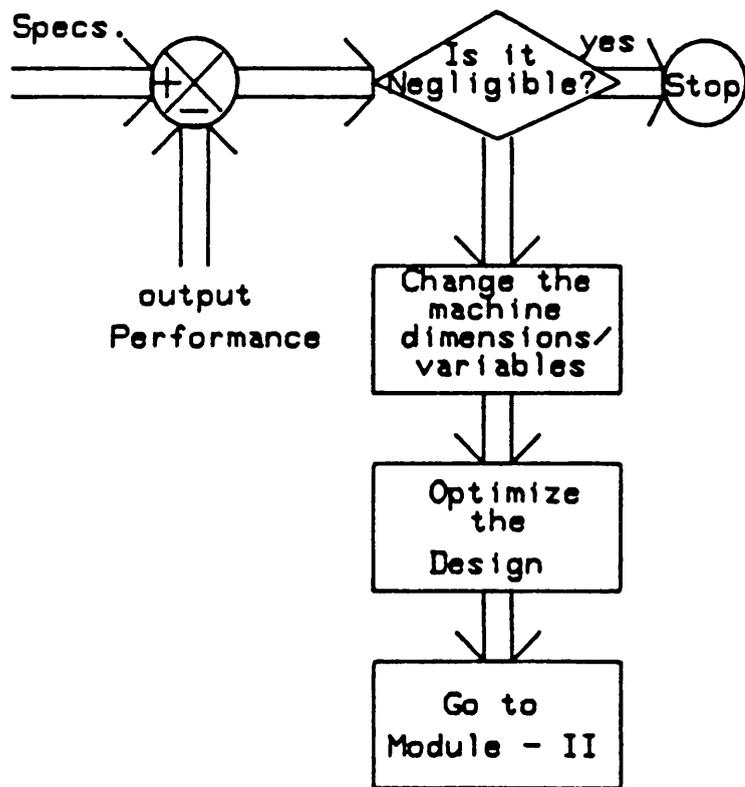
Figure 6.   Module III, the feedback process in machine design

Figure 7. Overall flowchart for machine design

4.0 Computer Aided Design of a Switched Reluctance Motor (SRM)

4.1 *Introduction*

This chapter contains the main crux of the thesis topic. To begin with, the objective of the thesis is presented. This is followed by a brief discussion on the principle and operation of the SRM. This would help in getting an insight into the problem and understanding its complexity. Then the approach used in the design and analysis is presented. It is followed by a discussion about the various requirements of this particular package and how they were implemented. The chapter concludes with a sketch of the main features which have been incorporated in the system implemented.

4.2 Definition of the problem

The design of electric machines is a highly iterative process. It involves the derivation of the functional dependence between the main dimensions, electric and magnetic loadings, frequency, number of poles, the allowable temperature and costs. The main objective of this thesis is to develop a user friendly interactive system using a personal computer for the optimal design of a variable speed motor in general, and a switched reluctance motor in particular. The design scheme is based on a systematic approach proposed earlier [30,31].

The design of the switched reluctance motor involves the following two aspects:

1. To translate the desired performance specifications into optimal dimensions of the machine.
2. From the dimensions of the machine, to determine the performance of the motor.

To handle all these design variables using classical methods would not only result in a loss of accuracy but also expensive in engineering effort. On the other hand, using a workstation based CAD system, the designer can conveniently handle any number of design variables and manipulate them using software commands. By making the CAD system highly interactive and in conjunction with a powerful graphics capability, the designer (not necessarily experienced) can arrive at an optimal design with minimal effort and minimal cost. A simple implementation of the switched reluctance motor design is shown in Figure 8 on page 32.
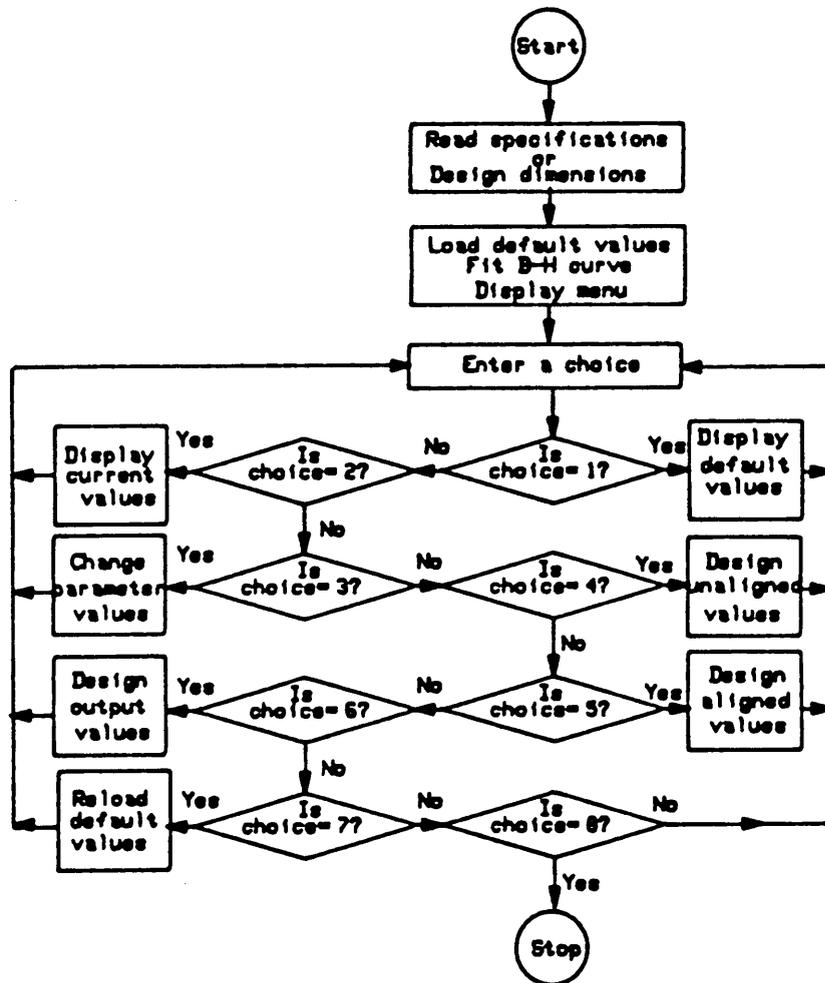
Figure 8.   Design implementation for a switched reluctance motor

4.3   Principle and operation of the switched reluctance motor

Switched reluctance motors are inherently variable speed motors. They might be treated as brushless machines having parallels with either the ac or dc machines without field excitation. The motor has salient stator and rotor poles. The stator windings are concentric and similar to the dc field winding and the rotor has no windings at all. Because of the simple stator windings, the manufacturing cost is very much reduced.

The basic principle of working of an SRM can be described as follows. The current in the stator circuit is switched on and off with respect to the rotor position. The torque is produced by the attraction of the rotor poles into alignment with the set of stator poles carrying current.

Figure 9 on page 34 illustrates the a configuration of a SRM with salient pole rotor and salient pole stator. It is a 6-4 arrangement. The number of poles in the stator and rotor are different and there are various such combinations such as 4-2, 8-6, etc., which are of practical importance. The windings on the stator poles are of simple concentrated form with diametrically opposite coils connected in series to form a single phase. Thus the nature of the stator pole is physically equivalent to a dc stator without interpoles. But, electromagnetically it is equivalent to the armature winding in the dc motor. The number of phases for operating the SRM is decided by various factors. This in turn determines the stator and rotor pole combination in the machine.
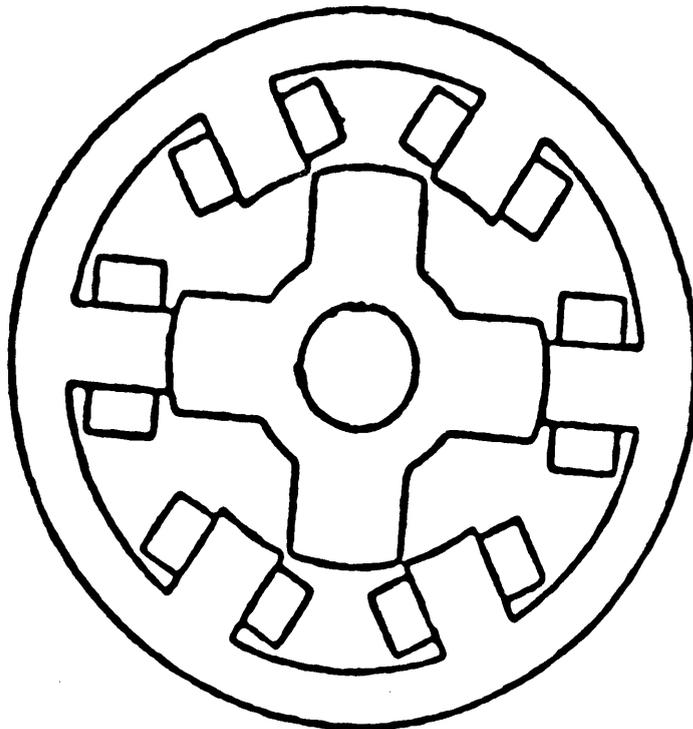
Figure 9. Configuration of a typical 6-4 switched reluctance motor

Computer Aided Design of a Switched Reluctance Motor (SRM)

As noted earlier, torque generation is by the tendency of the magnetic circuit to adopt a minimum reluctance configuration i.e., for the rotors to align with the stator poles and hence maximize the inductance of the excited coils. Since the torque developed is independent of the direction of the current flow, unidirectional currents can be used thus simplifying the electronic driving circuits. The instantaneous gross torque, $T(\theta, i)$ transmitted to the rotor for a given rotor position θ is given by,

$$T(\theta, i) = \frac{dW'(\theta, i)}{d\theta} \tag{4.1}$$

where $W'(\theta, i)$ is the winding co-energy for a given rotor position and is defined as,

$$W'(\theta, i) = \int_0^i iL(\theta, i)di \tag{4.2}$$

From equations (4.1) and (4.2), it is seen that for the given dimensions and physical parameters of the motor, performance prediction of the motor requires an accurate determination of the inductance profile as a function of the rotor position. The preliminary design to obtain an initial estimate of the dimensions is obtained using some empirical approximations. From these values, a solution for the inductance profile can be obtained using classical methods. The torque is dependent on current and position which in turn is a function of the magnetic characteristics of the core material used in the machine and excitation, thus making it a nonlinear function. Since linear approximations is valid only for certain small regions of operation, numerical solutions are necessary.

For the purpose of understanding, the generation of torque in a SRM is considered on a qualitative basis and linearity is assumed. By neglecting magnetic nonlinearity, equaticn(4.1) simplifies to

$$T(\theta, i) = \frac{i^2}{2} \frac{dL}{d\theta} \tag{4.3}$$

where L is the self inductance of the circuit for any value of θ. Figure 10(a) illustrates the variation of inductance of one stator circuit as a function of rotor position when a pair of rotor poles moves with respect to a pair of stator poles [32]. The relative angle is zero when the poles begin to overlap. Now the operation of the motor in different regions will be discussed.

R_{ab} : From θ_a to θ_b the leading edge of the rotor poles starts moving from the edges of the stator pole to full alignment. This can be noted from the fact that there is a linear increase in the value of the inductance till it reaches the maximum at L_{max}.

R_{bc} : This period from θ_b to θ_c is called the 'dead zone'. In this region, both the poles are in full alignment and hence the inductance is maintained constant at L_{max}.

R_{cd} : From θ_c to θ_d the inductance decreases linearly from L_{max} to L_{min} indicating that the poles are moving out of alignment.

R_{de} : This region θ_d to θ_e has stator and rotor poles which are not overlapped and hence the inductance remains constant at L_{min}.

The associated variation of torque for a constant current is illustrated in Figure 10(b). The torque is positive (i.e. motor action) as poles come into alignment and is negative (i.e. generator action) if current continues to flow as the poles move out of alignment. But in practice, because of the saturation and fringing, the characteristics become complex due to nonlinearities.
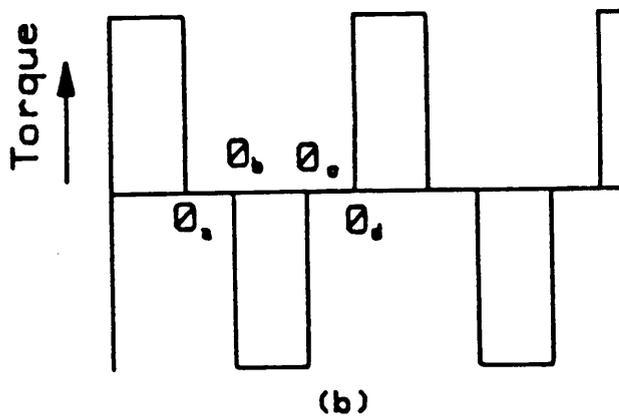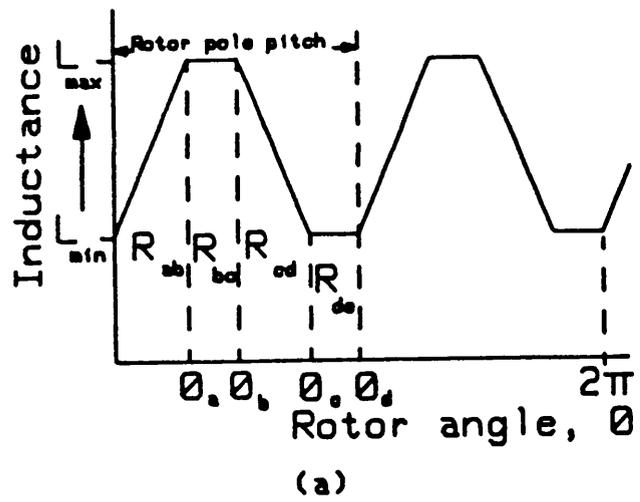
Figure 10. Simplified waveforms for a switched reluctance motor

4.4 Method adopted for design

The design of an electrical machine proceeds in three steps. First, a preliminary design is arrived at with the aid of a set of empirical assumptions. Secondly, the dimensions are used to determine the flux distribution and consequently the performance of the machine. The third step involves the modification of dimensions and assumed parameters to meet the desired specifications and constraints. While developing the CAD package for the design of a Switched Reluctance Motor (SRM), the approach adopted is very much similar to the conventional method.

The performance prediction from the dimensions is based on a method proposed earlier [30]. The relavent equations corresponding to the performance prediction are discussed in Appendix C. The minimum and maximum inductances corresponding to fully unaligned and fully aligned positions, respectively, are very important parameters in determining the behavior of the machine. After these values have been estimated, the power, torque and other parameters are calculated from the following equations. From Figure 11 on page 39,

$$\text{Energy/stroke} = W_{\text{aligned}} - W_{\text{unaligned}} = \text{Co-energy} \tag{4.4}$$

where,

$$W_{\text{unaligned}} = \frac{1}{2}\psi_u \times i_p \tag{4.5}$$

$$W_{\text{aligned}} = \Delta i[\psi_1 + \psi_2 + ... + \frac{1}{2}\psi_n] \tag{4.6}$$
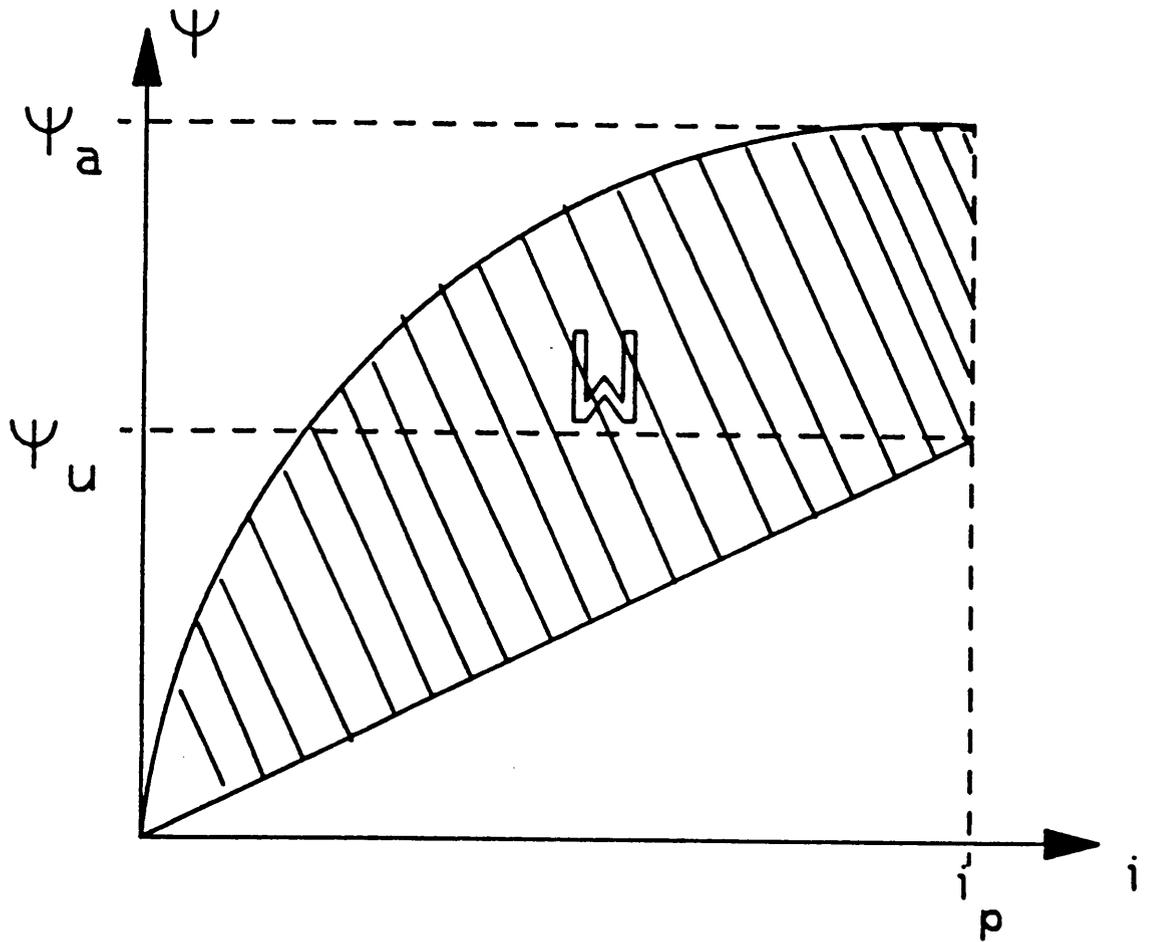
Figure 11. Flux linkage characteristics illustration for energy calculation.

Equation 4.6 calculates the area under the curve using the trapezoidal method. The accuracy is affected by the number of points considered in the curve. A more refined method, such as the Simpson's method, can overcome this error. But, since many points in the curve are known, the simpler method was adopted. The average torque is calculated as,

$$T_{av} = \frac{\text{Total work done/revolution}}{2\pi}$$
(4.7)

And,

$$
\begin{aligned}
\text{Total work done/revolution} &= W \times \text{No. of strokes/revolution} \\
&= W \times q.N_r \\
&= W \times \frac{N_s}{2}.N_r
\end{aligned}
$$
(4.8)

where,

W is the Co-energy.

N_s is the number of stator poles.

N_r is the number of rotor poles.

q is the number of phases.

Hence,

$$T_{av} = \frac{W.N_s.N_r}{4\pi}$$
(4.9)

If ω is the rotor angular speed and n is the rotor speed in rpm, then

$$P_d = T \times \omega$$

$$= q\frac{W.N_r}{2\pi}.\omega$$

$$= \frac{qWN_r}{2\pi}.\frac{2\pi n}{60} \qquad (4.10)$$

$$= \frac{qWN_r n}{60}$$

As,

$$q = \frac{N_s}{2} \qquad (4.11)$$

The output power is,

$$P_d = \frac{WN_r N_s n}{120} \qquad (4.12)$$

4.5   Determination of magnetic loading

The performance of an electric machine is critically dependent on the magnetic loading which is defined as the average rms value of the radial component of flux density in the entire air gap. The flux density is determined from the equation,

$$B_{av} = \frac{k\varphi_m}{\pi DL} \qquad (4.13)$$

where,

φ_m is the maximum flux.

B_{av} is the average flux density.

k is a constant dependent on the conduction angle per phase.

Hence, from the dimensions and the flux distribution of the machine, the flux density can be determined. The magnetic flux distribution is verified with a two dimensional finite element method (FEM). In the initial development stage, the FEM is used to improve the accuracy of the analytical methods.

The inductances can be calculated by two methods. They are:

1. Flux linkage method.

2. Stored energy method.

3. Maxwell's stress tensor method.

The Maxwell's method needs one field solution while the other two methods demand a minimum of two. Though the Maxwell stress approach has the advantage of being computationally fast and inexpensive, it is not used for the following reason. The torque is evaluated using several contours and the final result is averaged. This is a very complicated process and the accuracy level achieved depends on the number of contours taken. Hence, for the purpose of comparison with the analytic method the first two methods were performed in the finite element analysis. The average torque in each case is then calculated and compared with the analytic method. The procedure followed in the FEM is described as follows.

4.5.1 The energy method

$$L = \frac{2W_f}{i_p^2} \tag{4.14}$$

For the two dimensional FEA method and assuming anisotropy, the magnetic vector potential exists only in the z-direction so that,

$$W_f = \int_c \int_0^B \vec{H} \cdot d\vec{B} \, dl = Z \cdot \int_0^B \vec{H} \cdot d\vec{B} \tag{4.15}$$

and,

$$L = \frac{2Z}{i_p^2} \int_0^B \vec{H} \cdot d\vec{B} \tag{4.16}$$

Z = Length of a coil side measured along the axis of the core (z - direction). The vector integral and other arithmetic is easily performed by the postprocessor of the FEA CAD package.

4.5.2 Flux Linkage Method

$$L = \frac{1}{i_p^2} \int_U \vec{A} \cdot \vec{J} \, dU = \frac{Z}{i_p^2} \int_S A \cdot J dS \tag{4.17}$$

for the two dimensional FEA method. These operations were also performed using the postprocessor provided.

4.6 Calculation of torque from FEM results

Using the postprocessor, the flux linkages were calculated from the following equation.

$$\psi = \frac{Z}{i} \int_s A J dS \qquad (4.18)$$

Knowing the co-energy in the analytical case, the co-energy in the FEA case is calculated from,

$$W_c(FEA) = W_c(ANLT) - \delta W_1 + \delta W_2 \qquad (4.19)$$

δW_1 and δW_2 are the absolute differences between the energy values obtained using the analytic and FEM in the unaligned and aligned positions respectively. They are evaluated by numerical approximation. The average torque produced by the motor is given by,

$$\frac{\text{Work / Stroke} \times \text{Number of strokes / revolution}}{2\pi} = T_{av} \qquad (4.20)$$

$$\text{Number of strokes / revolution} = \frac{N_s}{2}.N_r \qquad (4.21)$$

So,

$$T_{av} = \frac{W_c N_s N_r}{4\pi} \qquad (4.22)$$

where,

N_s = Number of stator poles.

N_r = Number of rotor poles.

For the 6/4 pole motor studied,

$$N_s = 6 \tag{4.23}$$

$$N_r = 4 \tag{4.24}$$

The torque is then compared with that obtained with the analytic method. Then the error is reduced by making suitable modifications in the inductance calculation. This is performed with the aid of the flux plot obtained from the finite element method.

The FEM is used as a supporting tool for checking the accuracy of the analytical solution. In the FEM, the performance is predicted from the flux distribution of the machine. This analysis takes into account the nonlinear nature of the magnetic circuit. Thus a better accuracy is achieved when the machine operates in deep saturation compared to the approximate analytic methods. The FEM can be eliminated once the corrections are made in the analytical method. This has the advantage in that an accurate solution is obtained in a short time. Also the designer can relate the machine with a set of equations which is not possible in the finite element method.

4.7 Determination of machine dimensions and parameters

For designing the dimensions from given performance characteristics, a systematic design procedure has been adopted [31]. In this method, the output equation of the motor is developed in a manner very similar to that used for conventional rotating machines. This helps in using the designer's experience in developing the design for SRM. From the output equation and the performance specifications of an electrical machine, a minimum of eight main variables are identified. The approach used to obtain these variables is described as follows. The output equation of the switched reluctance motor is represented similar to that of conventional machines as,

$$P_{rated} = k_d \times \eta \times k_1 \times B_{av} \times A_{sp} \times k_2 \times k \times n_{rated} \times D^3 \qquad (4.25)$$

where,

k_d = duty cycle.

η = efficiency.

$k_1 = \dfrac{\pi^2}{120}$

B_{av} = Average flux density over the core.

A_{sp} = Specific Electric loading in Amp.cond/m.

D = Bore diameter.

k = Bore diameter to stack length ratio.

n_{rated} = rated speed of the motor.

$k_2 = 1 - \dfrac{L_u}{L_a^s}$

L_u = Unaligned inductance per phase.

L_a^s = Aligned saturated inductance per phase.

With empirical approximation and assuming constant speed [31], equation (4.25) can be reduced as,

$$P_{rated} = K.D^3 \qquad (4.26)$$

where,

K is a constant.

From equation (4.26), the bore diameter is calculated. Using this value of the bore diameter, the stack length and turns per phase are calculated. Then the stator pole arc and rotor pole arc are calculated for the given pole configuration. The stator pole height is calculated by computing the winding area and clearance. At this stage, the wire size to be used is also determined. Then the cross-section and thickness of the back iron is computed using the B-H characteristic of the material. The other values such as, stator outer diameter, interpolar air gap, rotor pole height, etc., are then derived from the above values.

The determination of dimensions depends on the performance index selected. The most common performance index is thermal consideration. Others which may have to be simultaneously satisfied include the per unit magnetizing current (in induction motors), the motor reactances (motors for special applications), volume, shape and cost. The design procedure adopted which constitutes the module 1 (discussed earlier) is illustrated in Figure 12 on page 48 and Figure 13 on page 49. The corresponding equations and the algorithm adopted are given in Appendix B.

```
┌─────────────────────┐                           ( 1 )
│  Enter given data   │                            │
│  and constraints    │              ┌─────────────▼────────────┐
└──────────┬──────────┘              │   Modify initial         │
           │                         │ estimates as desired     │
           ▼                         └──────────────────────────┘
┌─────────────────────┐
│ Read estimates of   │
│ B_max, A_sp, K_2, k, N_r, N_s │
└──────────┬──────────┘        ┌──────────────────────────┐
           │                   │ Compute stator and       │
           └──────────────────▶│   rotor pole arcs        │
┌─────────────────────┐        └─────────────┬────────────┘
│ Compute peak phase  │        ┌─────────────▼────────────┐
│ current; Select SWG.│◀───────│  Determine B_av          │
└──────────┬──────────┘        │ from B_max and B_s       │
           │                   └──────────────────────────┘
           ▼
┌─────────────────────┐
│   Compute bore      │
│   diameter, D       │
└──────────┬──────────┘
           │
           ▼
┌─────────────────────┐        ┌──────────────────────────┐
│ Determine L from    │───────▶│ Compute turns/phase      │
│     L = kD          │        │ T_ph = A_sp πD/2m.i_p    │
└─────────────────────┘        └─────────────┬────────────┘
                                             │
┌─────────────────────┐        ┌─────────────▼────────────┐
│ Calculate thickness │◀───────│ Compute stator and       │
│   of back iron      │        │  rotor pole height       │
└──────────┬──────────┘        └──────────────────────────┘
           │
           ▼
         ( 2 )
```

Figure 12. Flowchart for design module 1
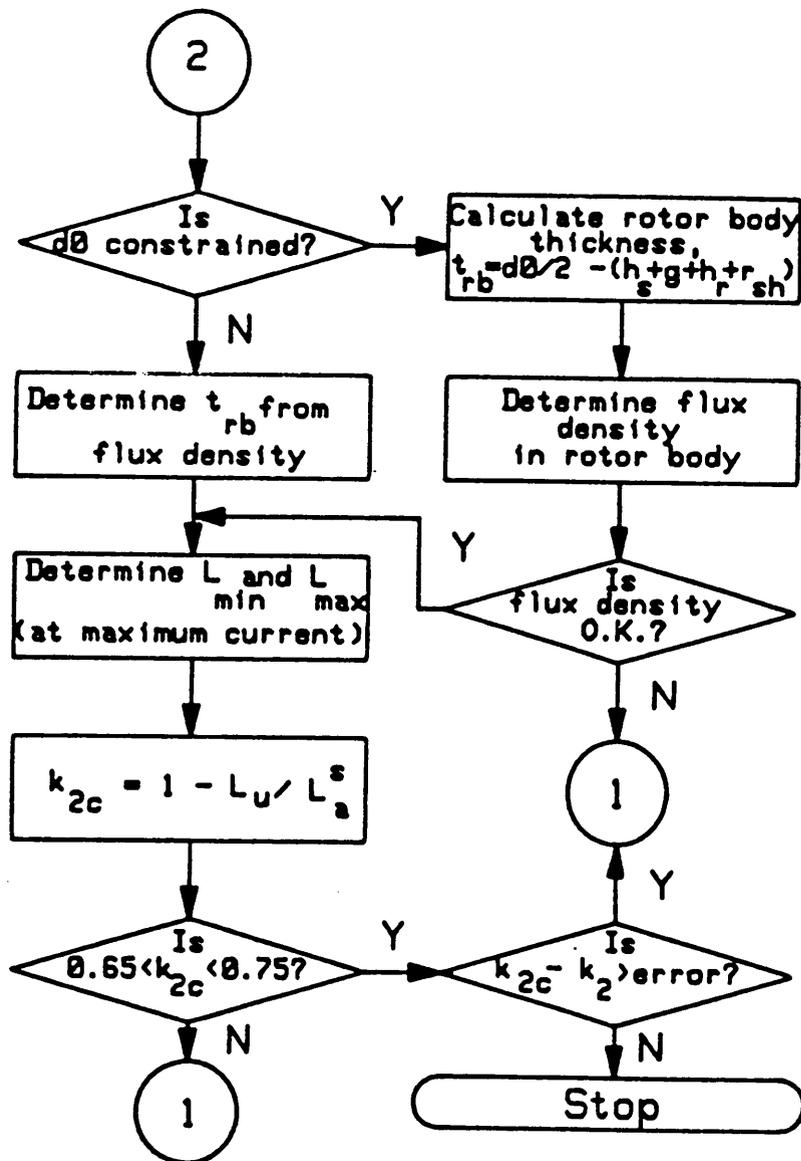
Figure 13. Flowchart for design module 1 (contd.)

4.8 System requirements

The determination of the magnetic loading is very crucial to the success of the design and it is complicated due to the nonlinear nature of the magnetic materials. In classical design methods, the complexity of design is reduced by applying the approximation that the machine operates on the linear part of the characteristic and therefore treated as a linear magnetic circuit. This approximation can produce significant errors in the prediction of performance. Hence the main requirement of the system developed is not just to realize the electromagnetic function. Instead, in doing so, it also optimizes shape, size and weight, cost, noise, electromagnetic interference, flexibility of manufacturing and fabrication, efficiency, etc. In special applications additional factors like minimization of harmonic effects, etc. may become important requirements of the system.

Another requirement of the system is that it should be able to output the required characteristics interactively, both in graphical and tabular form, if and when necessary. This would facilitate elegant analysis of the results achieved and optimization of the design. Optimization is on the basis of cost, feasibility of manufacture, special design constraints and other factors the designer may be given with. Another requirement for any such system is the B-H curve fitting of the material. This will be discussed in detail in the following section.

4.9 B-H curve fitting

Irrespective of the given specifications, the magnetic characteristics of the material have to be available to the system in order to establish the machine dimensions and performance characteristics. Hence, the B-H curve of the material used has to be accurately fit. Various methods for B-H curve fitting have been proposed. These are conveniently categorized into:

1. Least squares method [38].

2. Five term method [36].

3. Brauer's three point method [37].

4. Spline method [38].

For modeling the B-H curve, only the mean curve is considered. Anisotropy and hysteresis are neglected. Except in methods 3 and 4, the Raleigh region of the curve is neglected. The method described in 3 has also the advantage in that it requires only three points with reasonable accuracy. Hence this method was considered. The algorithm is as follows.

Three strategically chosen points, one just above the origin, one below the knee and the other above the knee are chosen. This is represented using the equation,

$$H = (k_1 e^{k_2 B^2} + k_3)B \qquad (4.27)$$

where k_1, k_2 and k_3 are material dependent constants. This gives a third order nonlinear system of equations. This is solved using the Newton Raphson method. This

method involves the use of the first derivative. Since the slope of the B-H curve varies widely (0 to 10^4), the initial guess is very critical. This leads to convergence problems. Hence the method to solve it is to eliminate k_1 and k_2 by arithmetic manipulation. The value of k_2 is obtained iteratively. The values of k_1 and k_3 are determined by back substitution.

In practical cases, the B-H curve becomes linearly increasing after a certain period of saturation. Since method 3 assumes an asymptotic B-H curve, the error in this region is $\geq$ 200 percent. Hence the algorithm is modified as follows. Equation (4.27) is used till the saturation region for the exponential part of the B-H curve which amounts to an error of $\leq$ 4 percent. Beyond that, the curve is modeled as a straight line and solved as a first order equation which also produces an error of $\leq$ 4 percent. This error is justified since the data sheet itself might have errors.

For the system designed, since it was desired to operate with a special kind of material, E-140 steel, which does not follow the normal characteristics of a steel [39], the spline technique is adopted. In this method, the curve is split piecewise and separate equations are computed for two consecutive points. The main disadvantage of this method is that the range of curve fitting is limited by the extremal points given. This is overcome by extrapolating the curve beyond the end points by approximating it as a straight line. The error due to this approximation is acceptable since the BH curve is almost linear at high saturation regions. The accuracy of this technique is proportional to the number of points chosen. So, even though it has this disadvantage compared to Brauer's method, this technique was chosen due to the peculiarity of the material used.

4.10 Interactive feature requirements

Since the user of the system may be an inexperienced engineer, it is essential that the system is built user friendly and interactive. The major factors to be taken into consideration are as follows [9] :

1. **Control of processes:** altering the contents (editing) of data, initiating processes of steps (eg. solution of equations).

2. **File management:** creating, deleting, merging, cataloguing of data files; attaching and detaching files.

3. **Display management:** specifying what data should be displayed and in what form.

4. **System management:** identifying passwords, listing available files, signing off.

The user should be prompted for design specifications. If the output performance is specified, then the system should calculate the machine dimensions and then proceed to calculate the inductances and performance. But, on the other hand, if the machine dimensions are specified, the system should calculate the inductance directly and then the performance. When the user is required to choose the input parameters, it is desired that the system interactively guides the user through the process giving the various possible options. After the calculation of the inductances, their values should be compared with that obtained using FEM and then adjusted using a correction factor. With the corrected value, the output performance is then evaluated. The system should also be able to display the output results either in a graphical or a tabular form.

In the present system, a hierarchical command structure has been used. The program boots up with a set of tested default machine values and the performance is evaluated accurately. This feature accomplishes three purposes:

1. As a tutorial for new users.

2. As a tool to check the computational accuracy of a design.

3. As a means to verify the status of the system.

4.11   Main features of the system

The salient features incorporated in the CAD package are:

*   The design for desired output performance or predicting its performance characteristics from the given geometric dimensions, magnetic characteristics and electric loading.

*   Interactive verification of the points entered for B-H curve fitting.

*   B-H curve fitting for a set of strategically chosen points on the curve.

*   Design with default parameters in case repeated design of the same machine has to be performed.

*   Display of current and/or default parameters at any stage of the design.

*   Interactive alteration of parameter values at any design stage.

*   Built-in mechanism for checking computational accuracy and status of the system.

*   Built-in graphics capability for interactive plotting of desired characteristics.

- Prediction of inductance for any desired rotor position.

- Tabulation of output results.

- Provision for conversion factors of M.K.S. and F.P.S. units.

The results are presented and discussed in the next chapter.

5.0   Results and discussions

5.1   Introduction

This chapter contains the results obtained using a default model which has been verified earlier [31]. These results are discussed in detail and their verification is also presented. The results of the curve fitting techniques used are presented in the beginning of this chapter. It is followed by results obtained for the design and analysis of the default model used. The various assumptions made in the algorithm are also sketched briefly. The chapter concludes with a discussion on the various results presented. It includes a detailed analysis about the various sources of errors in the results.

5.2   Curve fitting results

Figure 14 on page 62 illustrates the default machine model being used in the CAD package.

Figure 15 on page 63 through Figure 18 on page 66 illustrate the spline fit and the Brauer fit for typical M 19 and E140 steels. For the case of M-19 steel, the curve fit produced by the Brauer method or the spline technique is not significantly different. But, in the case of E-140 steel, there is a marked deviation in the case of Brauer method even below the knee region of the curve as illustrated in Figure 17 on page 65. Using spline fit as shown in Figure 18 on page 66, it can be noted that there is a very close correspondence of the results. This justifies the reason for using spline technique for curve fitting.

5.3   Analysis and design results

Figures 18, 19 and 20 illustrate the normalized inductance, torque and the flux linkage characteristics in the linear region for the default machine. The following assumptions were made while evaluating the performance characteristics.

1. **Unaligned position.**

    a. Two dimensional field.

    b. End effects negligible.

    c. Permeability of iron infinity.

2. Aligned position.

   a.   No flux leakages.

   b.   Flux linked with all turns.

   c.   Uniform flux distribution.

The parabolic nature of the torque in the linear region can be explained as follows. The torque is proportional to the square of the current and inversely proportional to the rate of change of inductance, i.e.,

$$T = \frac{i^2}{2} \frac{dL}{d\theta} \qquad (5.1)$$

But from Figure 18, it can be noted that the inductance is constant over the linear region. This implies that the rate of change of inductance is also constant. Hence, equation (5.1) reduces to,

$$T = k \times i^2 \qquad (5.2)$$

where, k is a constant. This in turn is the equation of parabola. The normalizing factor for torque is defined as 18.89 N.m. for this model. The normalizing factor for flux linkages is 0.673 while for inductance is 37.74 mH. for this model. The results obtained are in close correspondence with the work done earlier [31].

Three strategically chosen points, namely 0.05 p.u, 0.5 p.u and 1.0 p.u stator current are used for computations for finite element analysis. The reason for choosing only three points is due to the amount of time taken for the solutions. Also by choosing three points, one in the low excitation level, one in the medium excitation level and one in the deeply saturated region, the profile can be determined with reasonable accuracy. For comparison with analytical method, a fair judgement can be passed

by using these three points. Figure 22 on page 70, Figure 23 on page 71 and Figure 24 on page 72 gives the comparison with the finite element results. It can be noted that the results are in close agreement even in the nonlinear region. Figure 25 on page 73 gives the variation of induction as function of rotor position for low excitation currents. Work is currently being done to refine this method which has been formulated very recently. It is not needed for torque prediction which requires the knowledge of the maximum and minimum inductances only. But this module has been incorporated in the package only for research purposes.

5.4 Discussion

The following observations can be made from the characteristics presented. For low excitation levels, results obtained by the analytical method are of the same order of accuracy as those obtained by finite element method. In the unaligned position, the analytical method gives an error of 51.4 percent in the value of phase inductance calculated using the method proposed earlier [30]. But with slight modifications to that method, the technique implemented has resulted in a reduction of error less than 13 percent. Also the error in the calculated torque is less than 5 percent of that calculated using the finite element method. But, it should be noted that the error in the calculation of unaligned inductance and torque might be higher for other motor geometries.

Comparing the flux linkage method and the energy method for inductance calculation using finite element analysis, the error was less than 1 percent for the three cases

studied. Hence, for comparison with the analytical method, any one of those methods can be used without much loss of accuracy. The accuracy of the results using a two dimensional finite element analysis package is less than 5 percent for this particular model.

The possible sources of error can be discussed as follows:

1. **Unaligned position.**
   a. The flux lines in path 1, 3, 4 and 5 as explained in Appendix C are not con-centric as assumed in the derivation of the expressions used in the analytical method.
   b. End effects have been neglected.
2. **Aligned position.**
   a. At high excitations, the effective cross-section of the airgap is larger than the geometrical gap due to fringing effects. The magnitude of this effect depends on the level of excitation (phase current). The Carter's coefficient used in deriving the analytical expressions does not take into account this depend-ence on excitation level.
   b. The increased error at 10A and 20A can be attributed to the fact that the three assumptions are violated at medium and high excitations.
   c. Fringe flux at the ends of the core has been neglected. The L/D ratio for the model studied is 0.4154 which is very small. This might result in substantial end effects.

The effects of the change in the rotor geometry on the performance of the SRM are beyond the scope of this thesis. However, it can be noted that the results are in close agreement for the geometries studied. For the region where the machine is designed

to operate there is very close correspondence between the two results. For this particular model, it is desired to operate at 20A with a rotor pole arc of 36 degrees and the pole arc ratio greater than or equal to 1. The torque predicted using the CAD package has an error of less than 5 percent. But for a pole arc ratio less than 1, the accuracy of the CAD package deteriorates. This is due to the fact that the assumed flux paths are no longer accurate. However, it should be noted that the results obtained using the modified algorithm are more accurate than those obtained using the method proposed earlier [30]. The results of various rotor positions are summarized in Figure 26 on page 74. Figure 27 on page 75 through Figure 30 on page 78 gives the various menus which are displayed in the CAD package. Figure 28 on page 76 displays the machine parameters for which the characteristics have been presented. With the values used in design module illustrated in Figure 30 on page 78, the machine is shown in Figure 29 on page 77. The close correspondence between these values and the default machine values are noted. These values were obtained with the stator outside diameter constrained. A machine based on this design is currently being built. The experimental verification will be performed when it is available. This would help to further enhance the validity of the design package.

```
           ┌─────────────┐
           │    START    │
           └──────┬──────┘
                  │
    ┌─────────────┴──────────────┐
    │ Ns = 6                     │
    │ Nr = 4                     │
    │  s = 23.91 degrees.        │
    │  r = 35.92 degrees.        │
    │  g = 0.025 mm.             │
    │ Gi = 27.0 mm.              │
    │  d = 122.06 mm.            │
    │ d0 = 194.0 mm.             │
    │  c = 12.50 mm.             │
    │  l = 50.76 mm.             │
    │                            │
    │  N = 536 turns             │
    │ Ip = 20.0 Amperes.         │
    │ u0 = 0.125e-6 H/m          │
    └─────────────┬──────────────┘
                  │
           ┌──────┴──────┐
           │   RETURN    │
           └─────────────┘
```
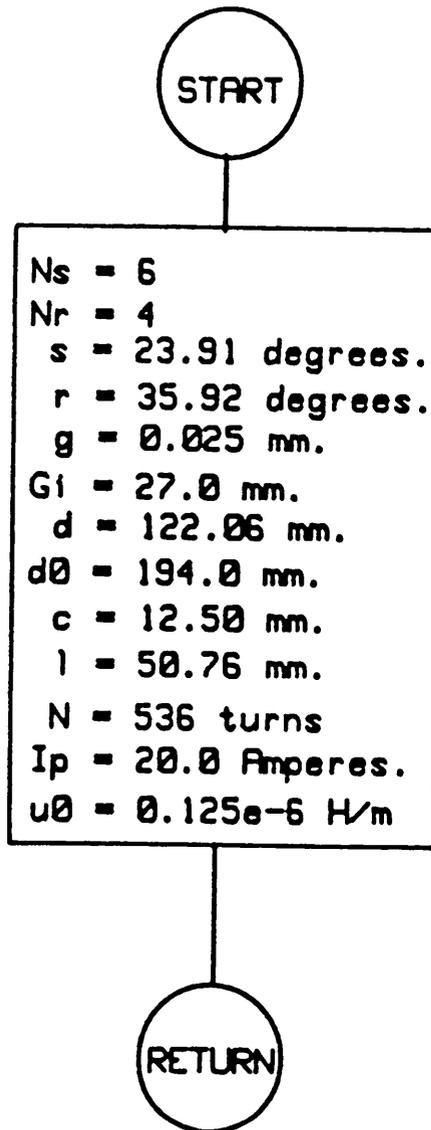
Figure 14.   Default machine parameters used in the system
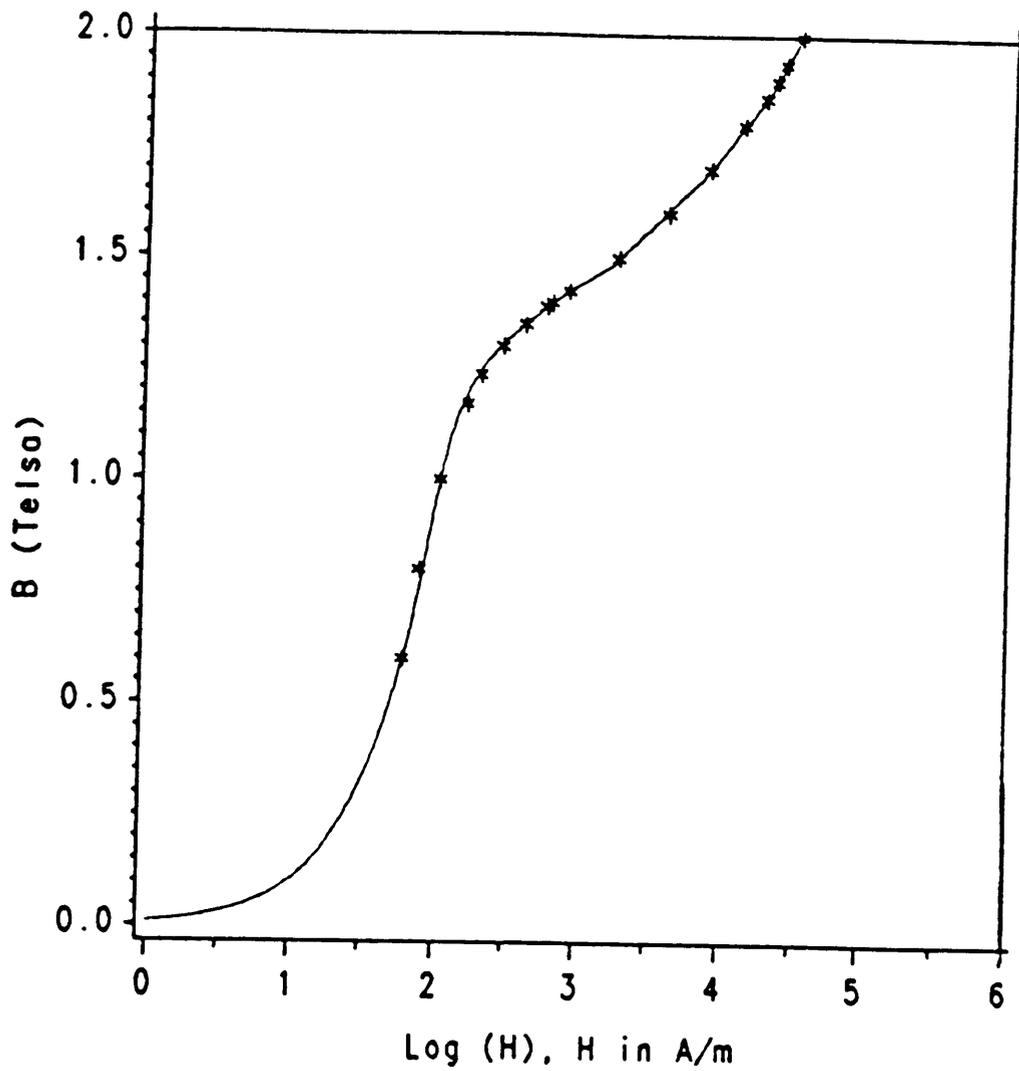
Figure 15. Brauer fit of M-19 steel BH characteristics
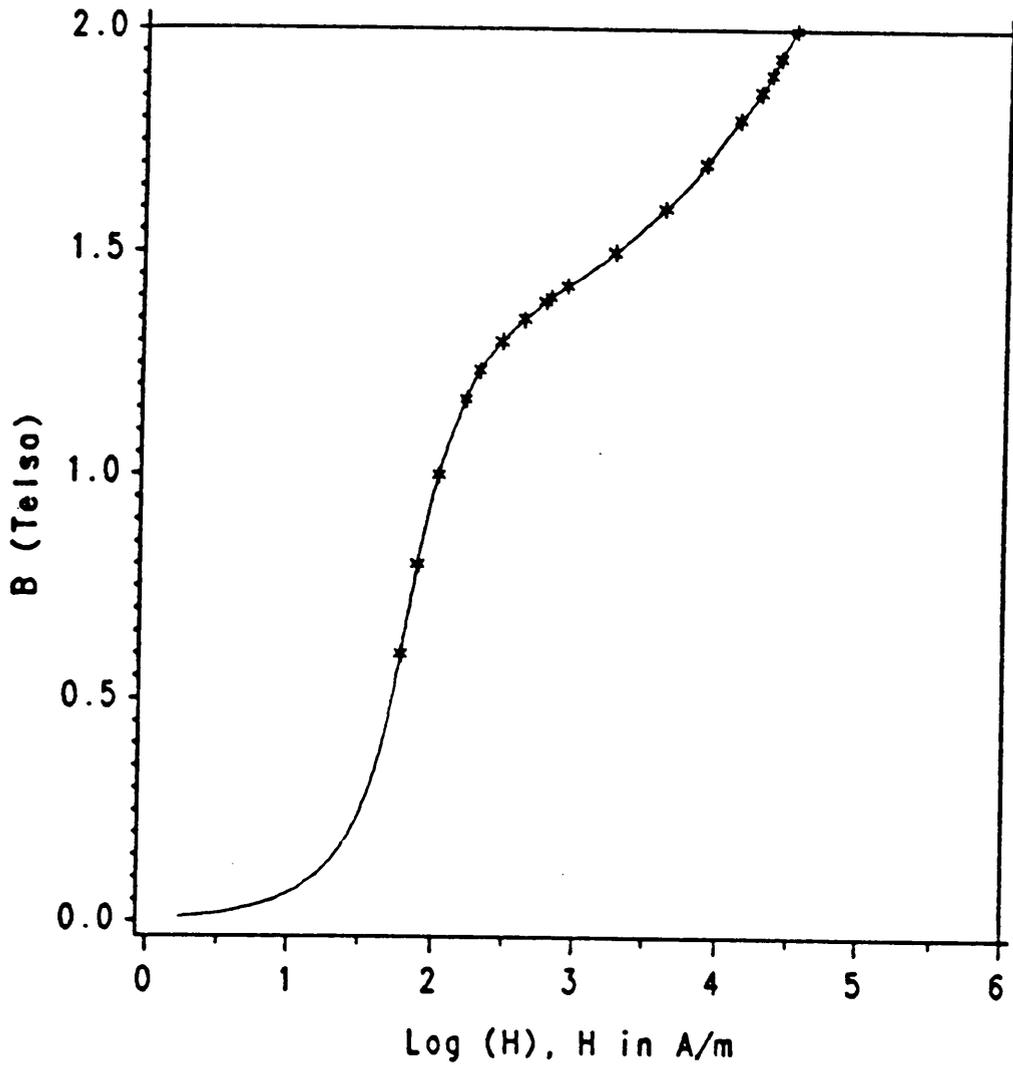
Results and discussions

Figure 16.   Spline fit of M-19 steel BH characteristics
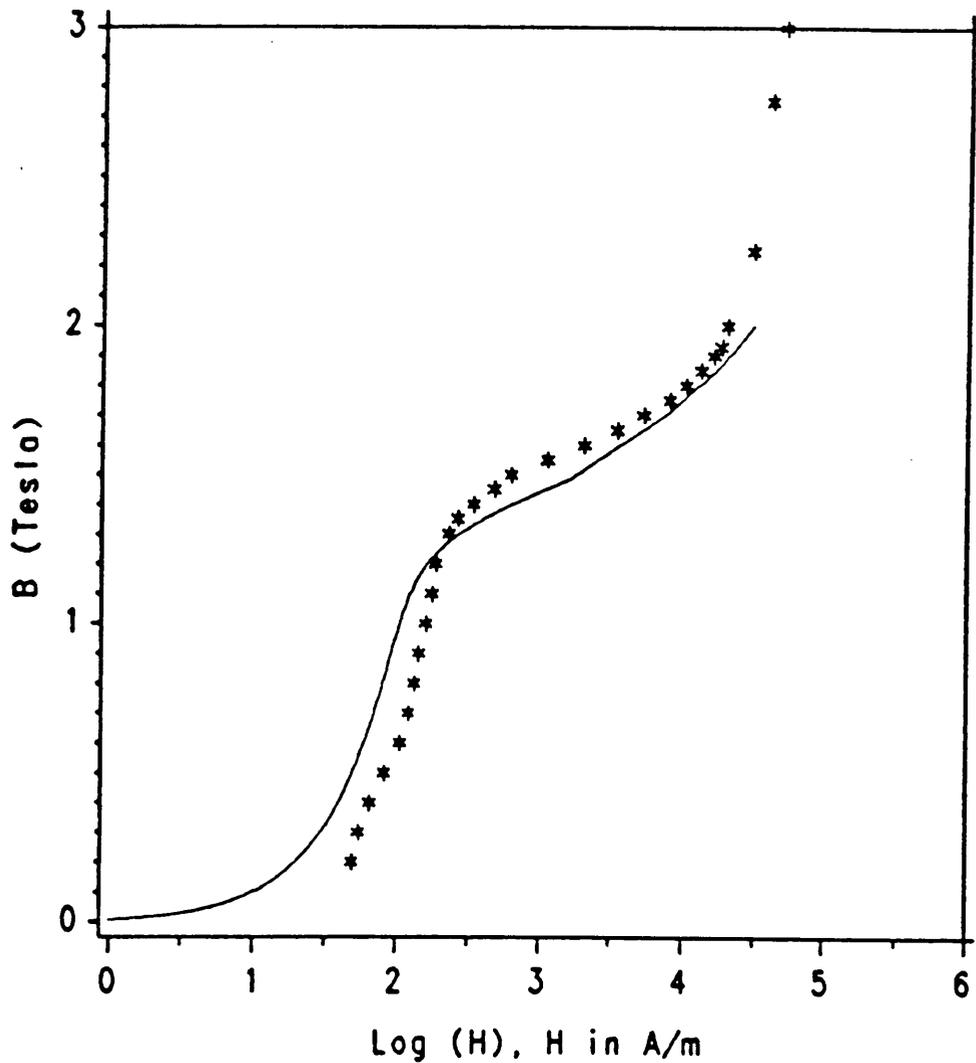
Figure 17. Brauer fit of E140 steel BH characteristics
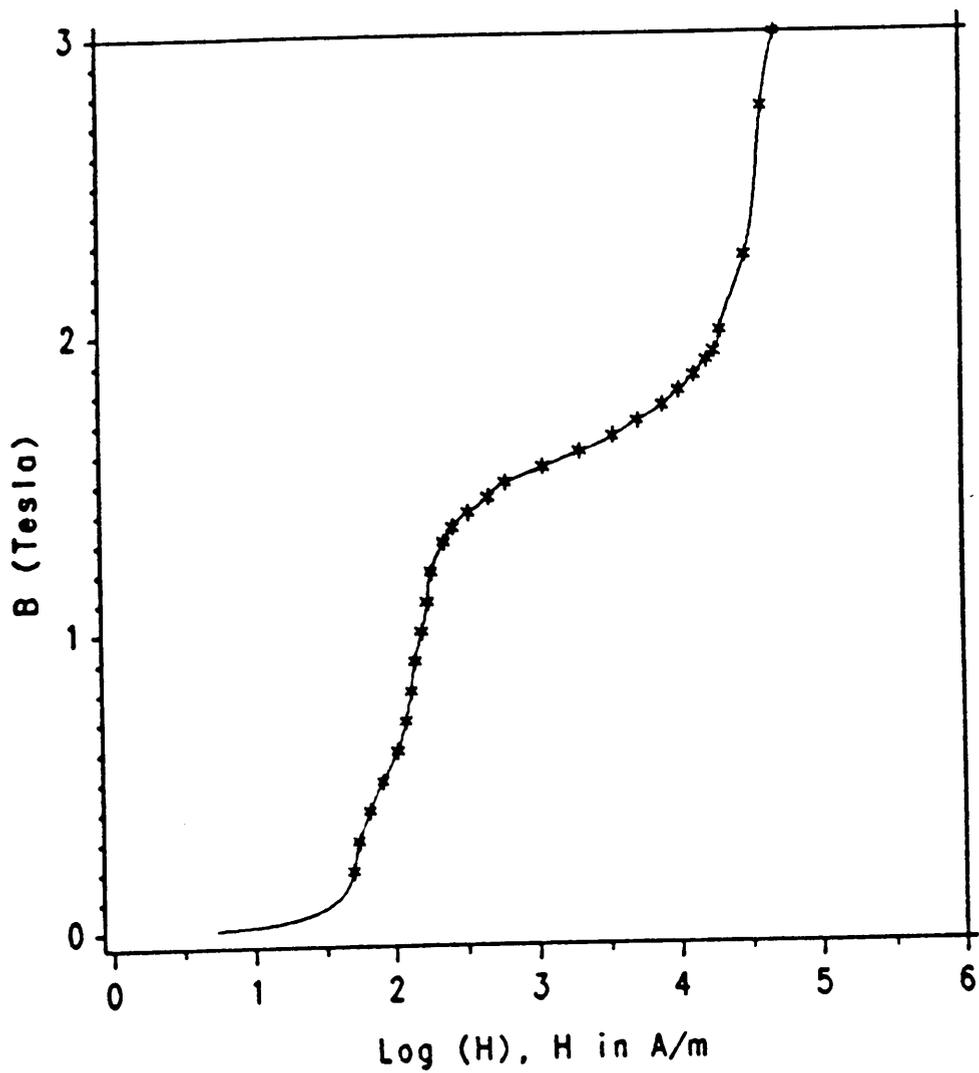
Results and discussions

Figure 18.   Spline fit of E140 steel BH characteristics

Results and discussions
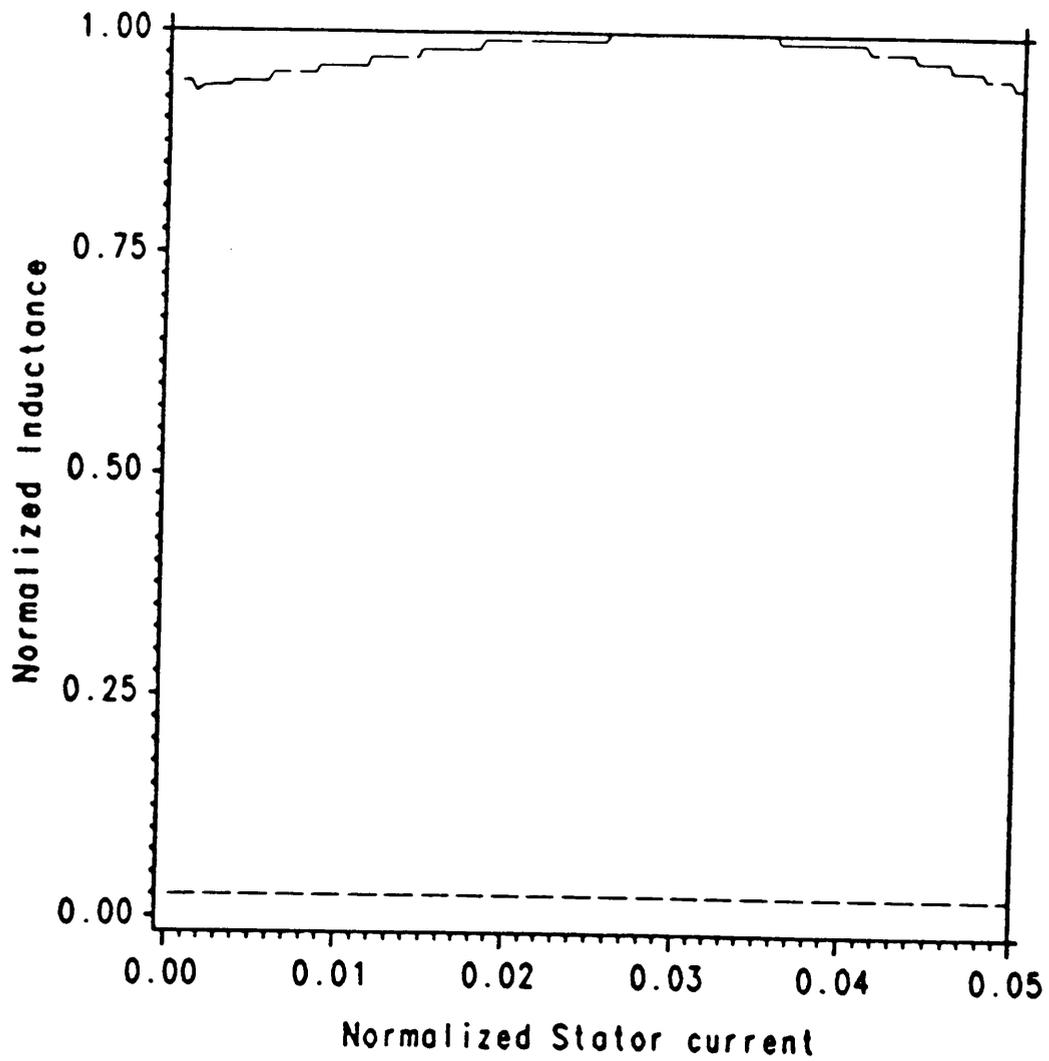
Figure 19. Normalized Inductance profile for linear region

Results and discussions
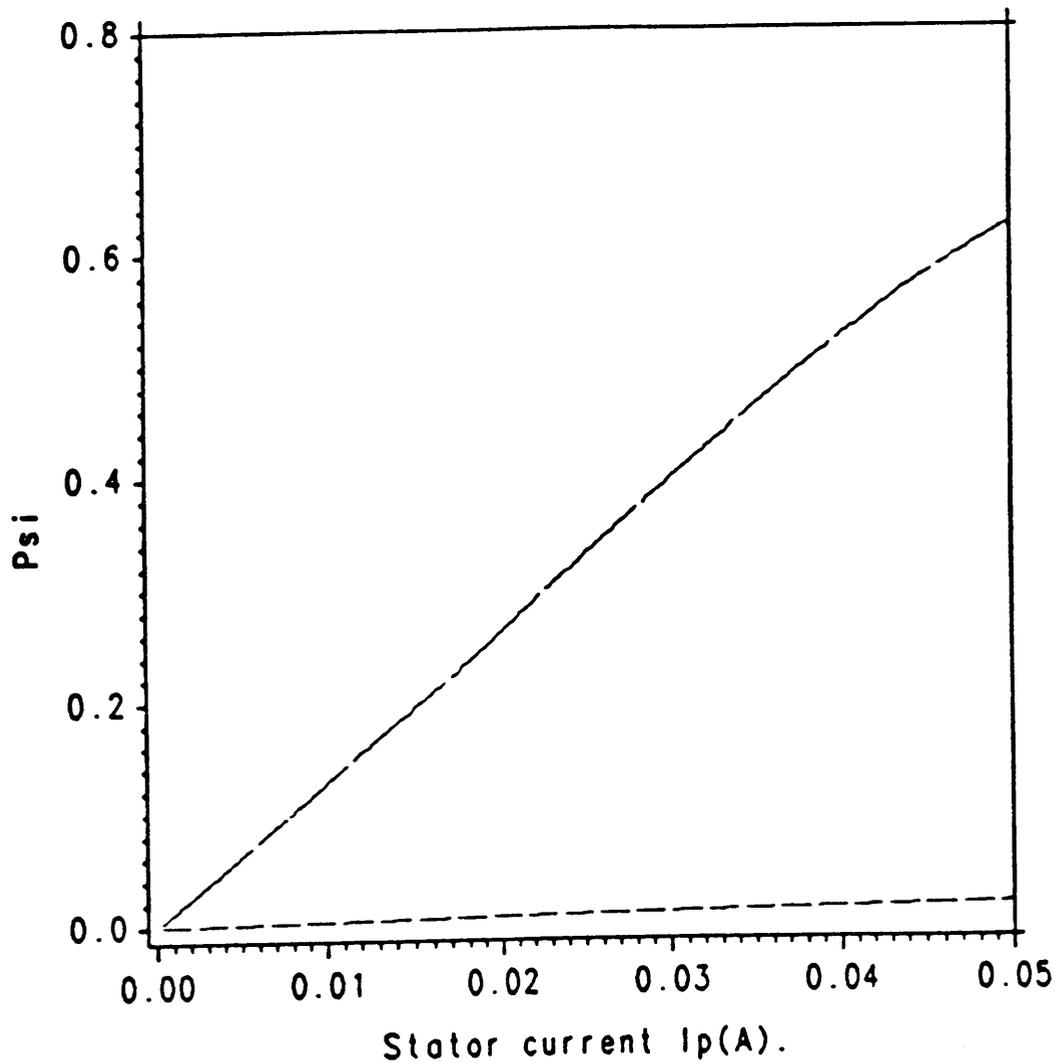
Figure 20. Normalized flux linkage characteristics for linear region
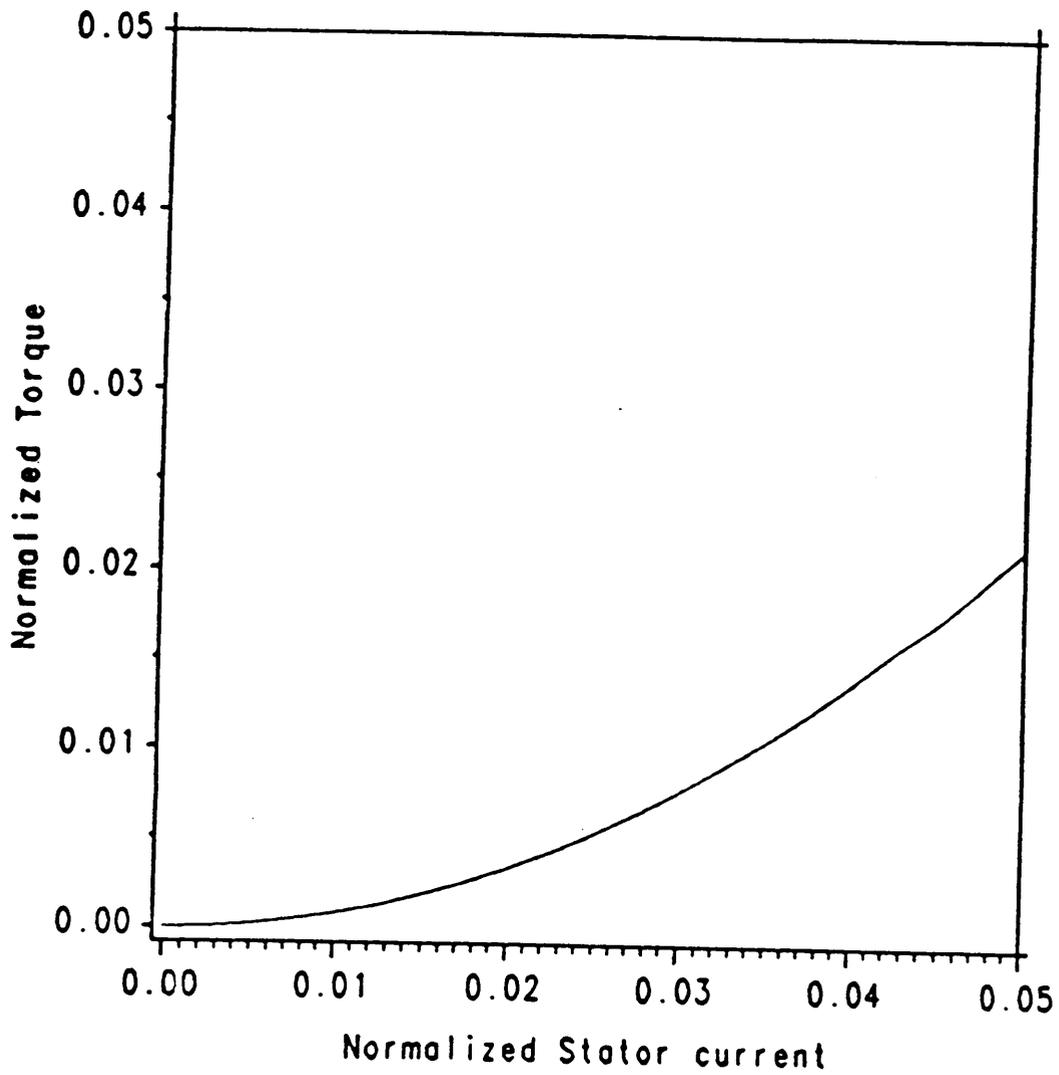
Figure 21.   Normalized torque characteristics for linear region

Results and discussions
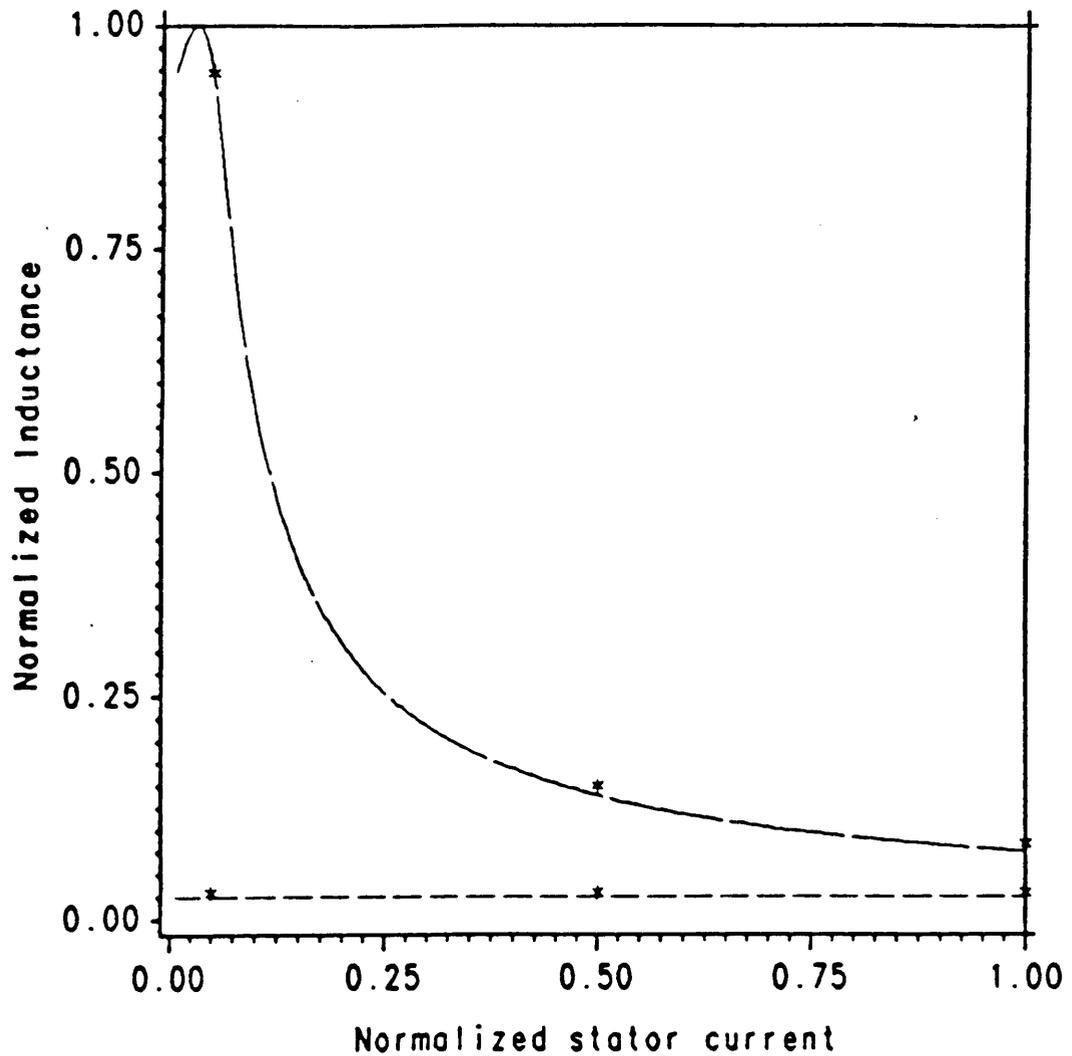
Figure 22. Comparison of normalized inductance profile

Results and discussions
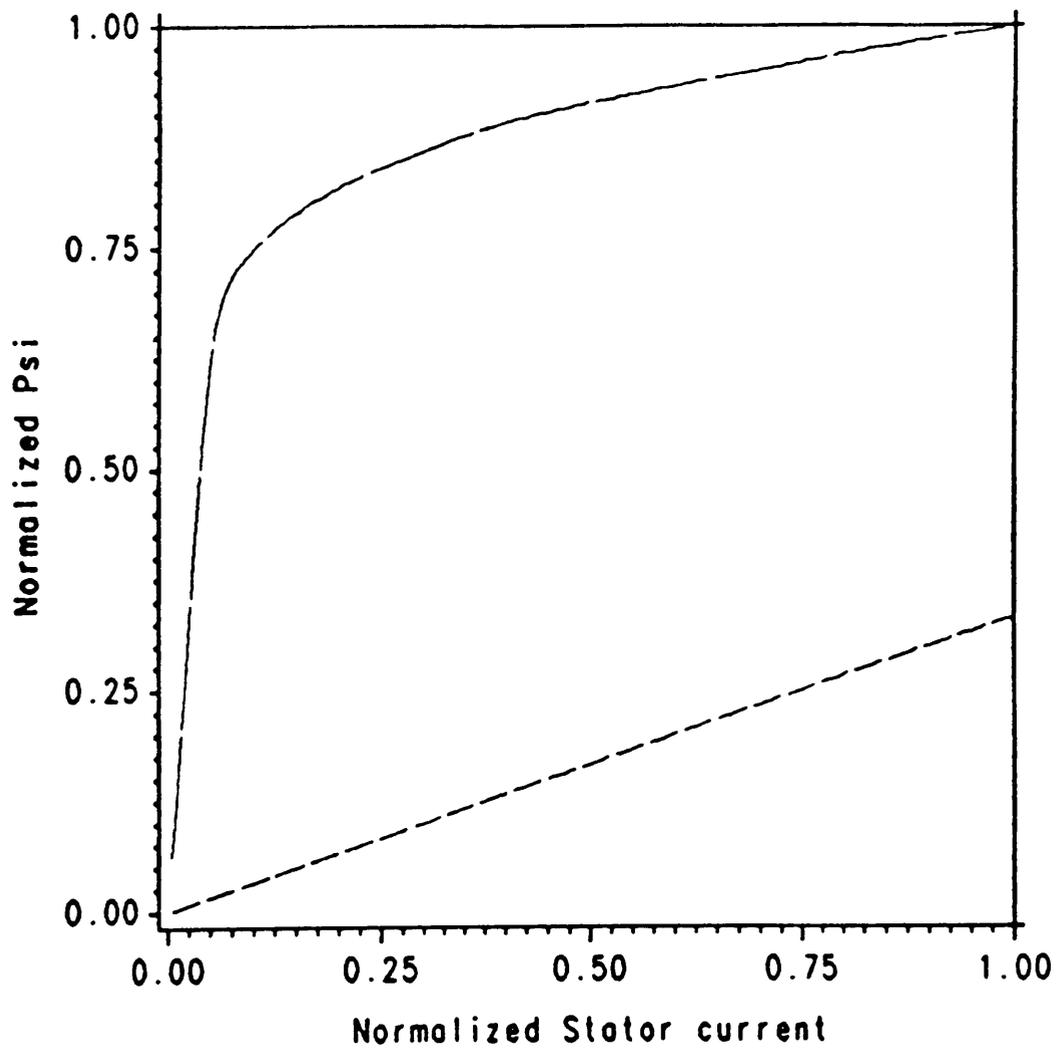
Figure 23. Normalized flux linkage characteristics
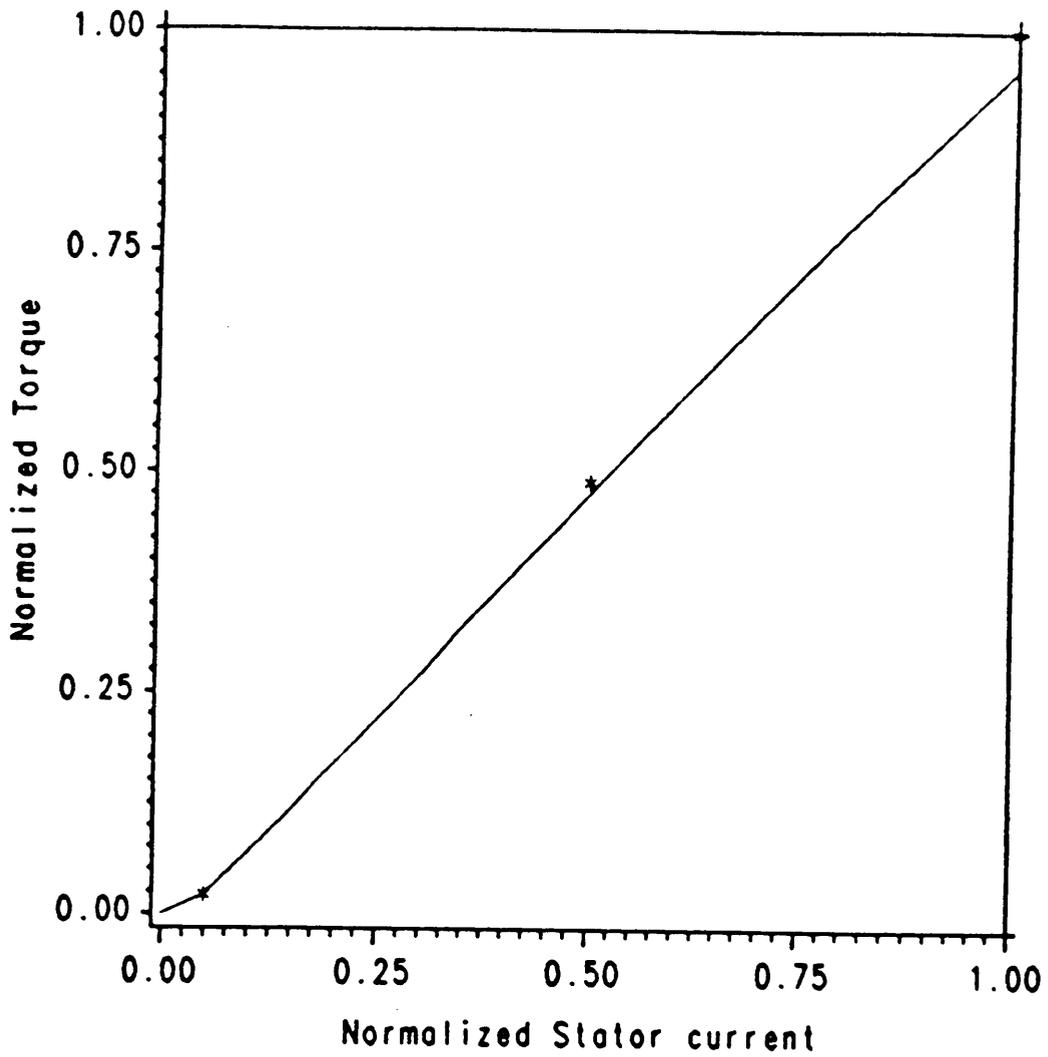
Figure 24. Comparison of normalized torque characteristics
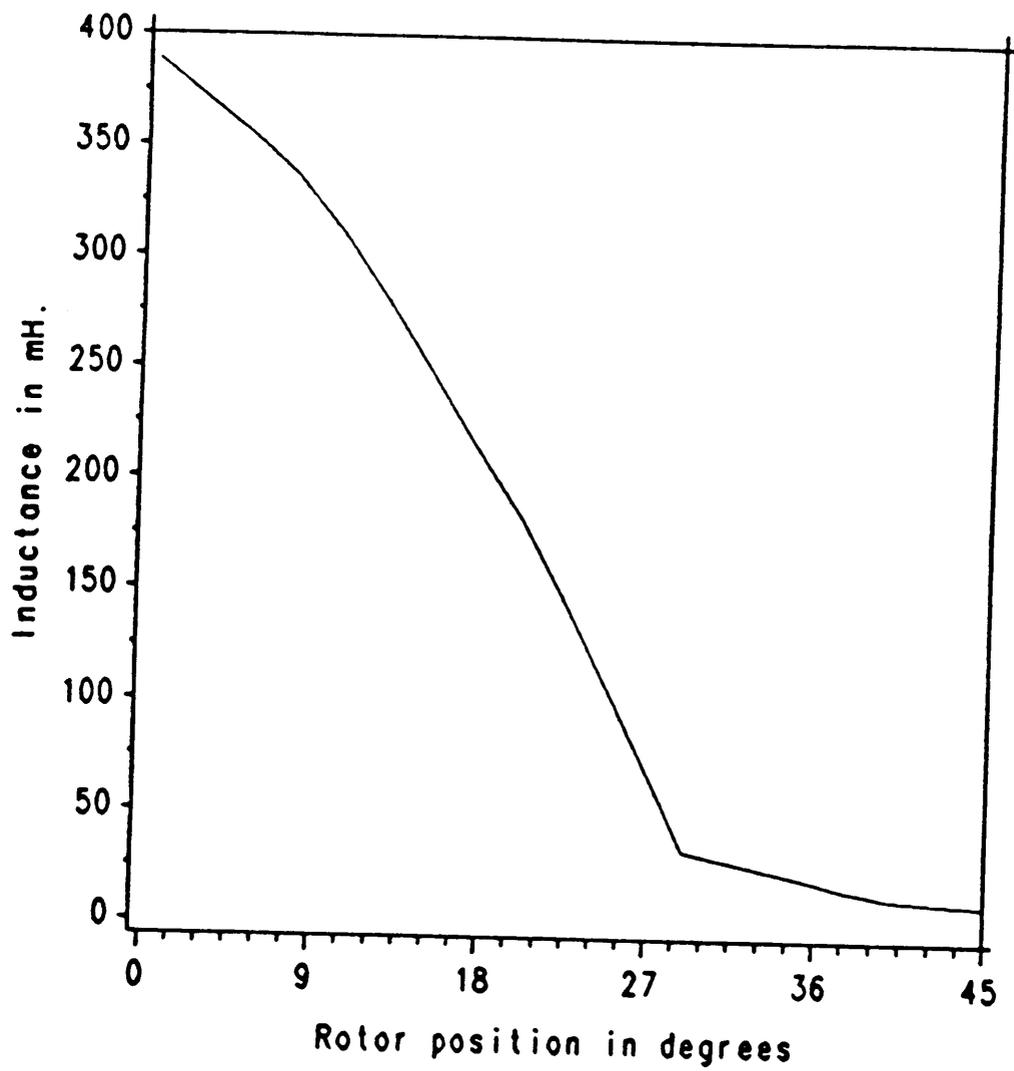
Figure 25. Inductance as a function of rotor position for linear region

| B_s | B_r | FINITE ELEMENT ANALYSIS RESULTS | | | | | ANALYTICAL RESULTS | | | |
		L_{min}	L_{max} i=10	L_{max} i=20	Torque i=10	Torque i=20	L_{min}	L_{max}	Torque i=10	Torque i=20
18	13.5	10.33	37.29		4.72		7.41	42.2	6.03	
	18.0	10.51	46.48		6.68		7.45	54.4	8.25	
	22.5	10.62	47.70		6.78		7.52	54.4	8.05	
	27.0	10.97	52.03		7.41		7.63	54.6	8.05	
24	18.0	11.29	55.22	32.71	7.71	15.93	9.94	61.5	9.05	18.4
	24.0	11.53	54.01	35.31	9.16	18.50	10.20	61.8	9.09	18.5
	30.0	12.07	55.06	36.64	9.08	18.76	10.60	61.5	8.98	18.2
	36.0	12.88	56.48	37.75	9.23	18.89	11.30	61.6	8.92	17.9
32	24.0	12.65	61.01		8.74		14.40	63.0	8.73	
	32.0	13.75	63.04		9.18		15.70	63.0	8.99	
	40.0	15.84	68.60		9.31		18.20	63.0	8.73	
	48.0	19.61	70.05		9.16		24.20	61.5	8.15	

Figure 26.   Tabulated summary of results

Results and discussions

```
*****************************************************************
                              MENU
     1. DESIGN MACHINE DIMENSIONS     2. DISPLAY DEFAULT VALUES

     3. DISPLAY CURRENT VALUES        4. CHANGE PARAMETER VALUES

     5. DESIGN UNALIGNED VALUES       6. ALIGNED POSITION VALUES

     7. DESIGN FOR REGION I           8. DESIGN FOR REGION II

     9. DESIGN FOR REGION III        10. DESIGN FOR REGION IV

    11. DESIGN FOR REGION V          12. OUTPUT PARAMETER PLOTS

    13. RELOAD DEFAULT VALUES        14. QUIT

*****************************************************************

MAKE A SELECTION: (Choose a number)
```

Figure 27.   Main menu of the system

```
***************************************************************
                    DEFAULT VALUES
 1. Number of Stator Poles,               Ns =    6
 2. Number of Rotor Poles,                Nr =    4
 3. Stator Pole arc,                       s =   23.91  deg.
 4. Rotor Pole arc,                        r =   35.92  deg.
 5. Air Gap length,                        g =    0.25  mm.
 6. Rotor interpolar space airgap length,Gi =   27.0    mm.
 7. Rotor Diameter,                        d =  122.06  mm.
 8. Stator outside diameter,              d0 =  194.0   mm.
 9. Back iron width,                       c =   12.50  mm.
10. Core length,                           l =   50.76  mm.
11. Number of turns per phase,             N =  536
12. Peak Stator Current,                  Ip =   20.0    A.
13. Permittivity of free space,           u0 =  4PI*E-7 H/m
***************************************************************

Pause.
Please press <return> to continue.
```

Figure 28. Menu for default machine parameters

```
*****************************************************************
                     CURRENT VALUES
 1. Number of Stator Poles,              Ns =    6
 2. Number of Rotor Poles,               Nr =    4
 3. Stator Pole arc,                      s =   24.000 deg.
 4. Rotor Pole arc,                       r =   36.000 deg.
 5. Air Gap length,                       g =    0.250 mm.
 6. Rotor interpolar space airgap length,Gi =   26.852 mm.
 7. Rotor Diameter,                       d =  122.104 mm.
 8. Stator outside diameter,             d0 =  194.602 mm.
 9. Back iron width,                      c =   12.551 mm.
10. Core length,                          l =   50.722 mm.
11. Number of turns per phase,            N =  536
12. Peak Stator Current,                 Ip =   20.000 A.
13. Permittivity of free space,          u0 =    .126E-05H/m
*****************************************************************

Pause.
Please press <return> to continue.
```

Figure 29.   Menu describing the machine model in use

```
****************************************************************
                  CURRENT MACHINE DIMENSIONS
 1. Number of Stator Poles,               Ns =  6
 2. Number of Rotor Poles,                Nr =  4
 3. Average flux density,                 Bn =   .240    Tesla
 4. Stator vs. Rotor pole ratio,          k2 =   .667
 5. Number of phases,                      m = 3.000
 6. Air Gap length,                        g =   .250     mm.
 7. Current Density,                       J =   .800E+01A/mm2
 8. Rated speed,                           n =   .180E+04rpm.
 9. Rated Power,                           P = 2880.000Watts
10. Average flux density of stator yoke,Bb =   .245   Tesla
11. Efficiency,                          Eff = 85.000   %
12. Supply voltage,                        V = 150.000 V.
13. Duty Cycle,                           kd =  1.000
14. Electrical loading,                  Asp =   .1686E+06
15. Length to Diameter ratio,            L/D =   .415E+00
16. Packing factor,                       FF =   .950E+00
17. Clearance for winding height design,Cl =   .100E+01mm.
****************************************************************
```

Pause.
Please press <return> to continue.

Figure 30. Menu for design module 1

6.0   Conclusions

6.1   Introduction

A very general method for the computer aided design of electrical machines has been presented. The modular form of various transfer blocks facilitates the design development of any electrical machine. Also it lends the facility to highly sophisticate the algorithms and their implementation individually. An illustration of the method in the design of a complex machine such as a switched reluctance motor is given. The work done in this field has been very minimal.

This chapter discusses the various aspects of the CAD package implemented. It concludes with a list of future recommendations which could be performed on this package.

6.2 Contributions

The main accomplishments of the thesis are listed below:

- Development of a PC based user-friendly and interactive CAD package for the design and analysis of the switched reluctance motor (SRM).

- The validity of the CAD package results have been verified with finite element analysis results.

- As the operation of the SRM is highly dependent on the BH characteristics of the material used, a very accurate curve fitting technique valid for any material has been formulated and incorporated.

6.3 Recommendations

The CAD package developed is the first step in the design of electrical machines for variable speed applications. This leaves a wide scope for future development. One important feature which can be included in this package is optimization. This plays an important role in the industry. A robust optimization routine is quite detailed. The basic factors which can be included are cost, size, thermal considerations, etc. Another feature which can be built is expanding the present system to design any existing electrical machine. This can be performed by suitably modifying certain modules in the existing package to characterize the peculiarities of the machines.

The next enhancement is linking the FEM program with the CAD package. This would help in reduction of time whenever a new product has to be developed. The accuracy of the analytical design can be simultaneously verified with the FEM results. With multitasking and other technological innovations this would be made possible in the near future.

Since the present CAD package is still in its development stage and not tested in an industrial environment, the user-friendly features may not be quite sufficient. By linking the package with the product development and manufacturing sections, any model can be produced in one step from the designer's table to the workshop. This feature will help in automation.

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Appendix A. LIST OF SYMBOLS

The symbols which are used in the equations and their definitions are explained in this section. This is useful just as a reference card since most of the symbols are defined with the equations where it appears.

a_b	Cross sectional area of the rotor body
a_c	Conductor cross-sectional area
a_g	Cross sectional area of the air gap
a_r	Cross sectional area of the rotor pole
a_s	Cross sectional area of the stator pole
a_y	Cross sectional area of the stator yoke
A_{sp}	Specific electric loading
B_{av}	Average Flux density
B_b	Average Flux density of stator yoke
c	Thickness of back-iron
Cl	Clearance for winding height design
d_w	Diameter of the wire

D	Bore diameter
D_0	Stator outer diameter
eff	Efficiency
ff	Packing factor
g	Gap length
g_i	Interpolar air-gap length
h_w	Winding height
h_r	Height of rotor pole
h_s	Height of stator pole
i_p	Peak current in the stator
J	Current density
k	Ratio of stator pole length to bore diameter (L/D)
k_1	Mathematical constant
k_2	Operating point constant
k_d	Duty cycle
l	Core length
l_b	Length of the rotor body
l_g	Length of the air gap
l_r	Length of the rotor pole
l_s	Length of the stator pole
l_y	Length of the stator yoke
L	Axial length of the stator
L_a	Aligned inductance per phase
L_u	Unaligned inductance per phase
m	Number of phases
n_{rated}	Rated speed

N_r	Number of rotor poles
N_s	Number of stator poles
pf	Power factor (at converter terminals)
P_d	Output power
P_{rated}	Rated output power
R	Winding resistance per phase
t_r	Rotor pole arc-length
t_s	Stator pole arc-length
T	Torque
T_{ph}	Number of turns per phase
V	Applied Voltage
w	Winding width
β_r	Rotor pole arc-angle
β_s	Stator pole arc-angle
λ_r	Rotor pole pitch
λ_s	Stator pole pitch
ψ	Flux linkage
μ_0	Permittivity of the free space

Appendix B. DESIGN EXPRESSIONS

The design algorithm adopted is given in detail in this section. The geometric dimensions used are explained in Figure 31 on page 90. Before proceeding with the design, the system is supplied with the following specifications by the designer.

1. Number of stator poles, N_s.

2. Number of rotor poles, N_r.

3. Average flux density, B_{av}.

4. Stator vs. rotor pole arc ratio, k_2.

5. Number of phases, m.

6. Air gap length, g.

7. Current density, J.

8. Rated speed, n_{rated}.

9. Rated power, P_{rated}.

10. Average flux density of stator yoke, B_b.

11. Efficiency, eff.

12. Supply voltage, V.
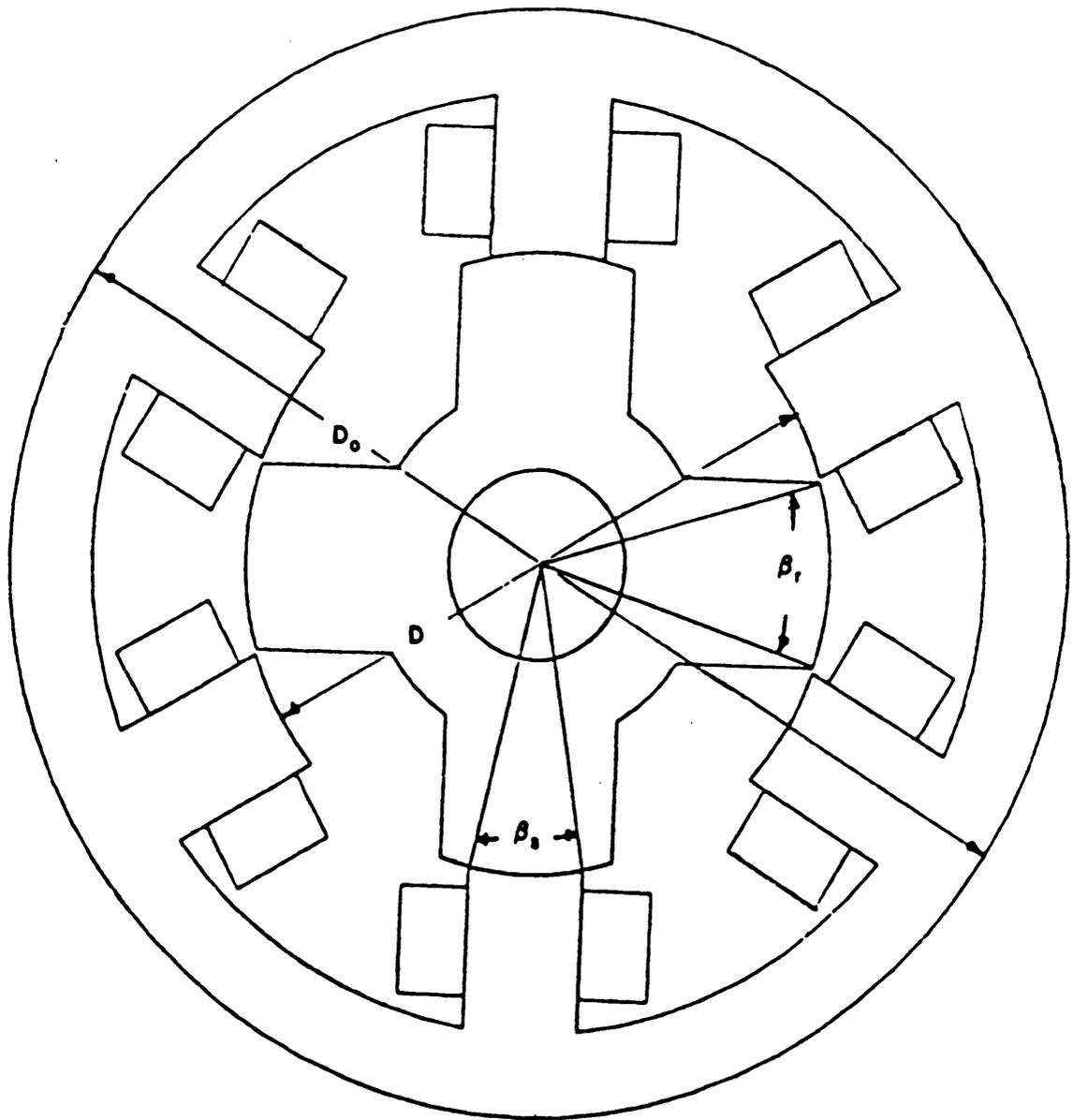
Figure 31. Dimensional description of a 6-4 SRM

13. Duty cycle, k_d.

14. Specific ectrical loading, A_{sp}.

15. Length to Diameter ratio, k.

16. Packing factor, ff.

17. Clearance for winding height design, Cl.

Also the B-H curve of the material used in the machine is fit prior to the design. The peak current in the stator, i_p, is calculated using the following equation,

$$i_p = \frac{P_{rated}}{(V \times eff \times k_d)} \tag{B.1}$$

The stator pole arc, β_s and the rotor pole arc, β_r are calculated using the following equations,

$$\beta_s = 0.8 \frac{\pi}{N_s} \tag{B.2}$$

$$\beta_r = \frac{2 \times \pi}{N_s} - \beta_s \tag{B.3}$$

The bore diameter, D, is obtained from the following equation as,

$$P_{rated} = k_d \times eff \times k_2 \times k_1 \times B_{av} \times A_{sp} \times D^3 \times k \times n_{rated} \tag{B.4}$$

where,

$$k_1 = \frac{\pi^2}{120} \tag{B.5}$$

Then, the number of turns per phase, T_{ph} is given as,

$$T_{ph} = A_{sp} \times \frac{\pi \times D}{2 \times m \times i_p} \tag{B.6}$$

The axial length of the stator, L, is given as,

$$L = k \times D \tag{B.7}$$

Using the above values, the stator pole arc length, t_s, the rotor pole arc length, t_r and the stator pole pitch, λ_s are calculated from the following equations.

$$t_s = \beta_s \left(\frac{D}{2} + g \right) \tag{B.8}$$

$$t_r = \beta_r \left(\frac{D}{2} \right) \tag{B.9}$$

$$\lambda_s = \frac{t_s}{0.4} \tag{B.10}$$

The wire gauge (AWG) is selected by calculating the conductor cross-sectional area, a_c and the diameter of the wire, d_w given as,

$$a_c = \frac{i_p}{(\sqrt{m} \times J)} \tag{B.11}$$

$$d_w = \sqrt{4a_c/\pi} \tag{B.12}$$

The winding area is calculated using which the winding height, h_w and the winding width, W_t, are calculated as follows.

$$Z = \lambda_s - t_s \tag{B.13}$$

$$W_t = \frac{(Z - Cl)}{2} \tag{B.14}$$

$$N_{layer} = \frac{W_t \times ff}{d_w} \tag{B.15}$$

$$N_{tlayer} = \frac{T_{ph}}{(2 \times N_{layer})} + 1 \tag{B.16}$$

$$h_w = N_{tlayer} \times d_w/ff \tag{B.17}$$

where N_{layer} and N_{tlayer} have to be integers. The stator pole height, h_s, is given as,

$$h_s = h_w + (2 \times W_t \tan(\frac{\beta_s}{2})) \tag{B.18}$$

The next parameters which have to be evaluated are the back iron thickness, c_b and cross-section. The flux density of the back iron, $B_{back\text{-}iron}$, is estimated from the B-H curve fit. Then assuming,

$$B_{gap} = B_{av} \tag{B.19}$$

The back iron thickness is given as,

$$c = \frac{B_{gap} \times a_{gap}}{B_{back\text{-}iron} \times L} \tag{B.20}$$

If the value of the stator outer diameter, D_0, is specified, then

$$c = \frac{D_0}{2} - (\frac{D}{2} + h_s + g) \tag{B.21}$$

Before proceeding with the above calculations, the following condition should be satisfied.

$$B_{\text{back-iron}} < B_{\text{av}} \qquad (B.22)$$

The stator outer diameter, D_0, if not specified, can be calculated as,

$$D_0 = D + 2g + 2h_s + 2c \qquad (B.23)$$

The interpolar air gap, g_i is given by the equation,

$$g_i = 1.05 \times \frac{\beta_s D}{2} \qquad (B.24)$$

Finally the rotor pole height, h_r, is calculated as,

$$h_r = g_i - g \qquad (B.25)$$

From these dimensions obtained, the unaligned and the aligned inductances are calculated using the algorithm given in Appendix C. They are then compared with the finite element analysis results and are validated if the error is within the tolerance limit specified by the designer.

Appendix C. ANALYTIC EXPRESSIONS

The algorithm used for the prediction of performance is detailed in this section. The flux paths used in the calculation of minimum inductance, i.e. in the fully unaligned position, is illustrated in Figure 32 on page 96. The assumptions made in this simplified map inside the machine are as follows:

    i.   Field lines of the path 1 consist of concentric circular arcs with the center at the point C_1.

    ii.   Field lines of the path 3 does not exist.

    iii.   The field lines of the path 4 are as given below.

    iv.   Field lines of the path 5 consists of concentric circular arcs with the center at the point, C_5, and

    v.   Field lines of the path 6 consist of parallel straight line segments.

The equations used in predicting the minimum inductance, L_{min} are as follows. All the geometrical dimensions are explained in Figure 33 on page 97.
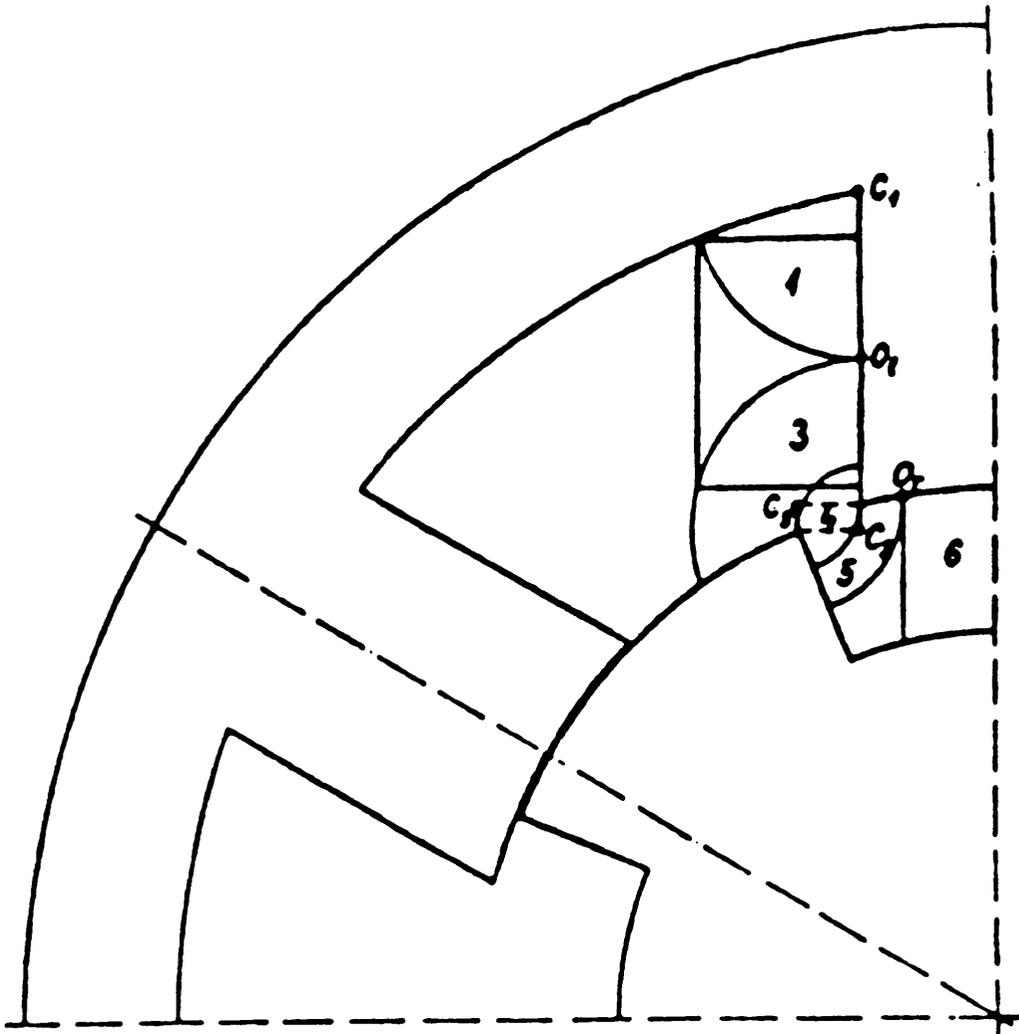
Figure 32. Approximated flux paths in unaligned position

Appendix C. ANALYSIS EQUATIONS
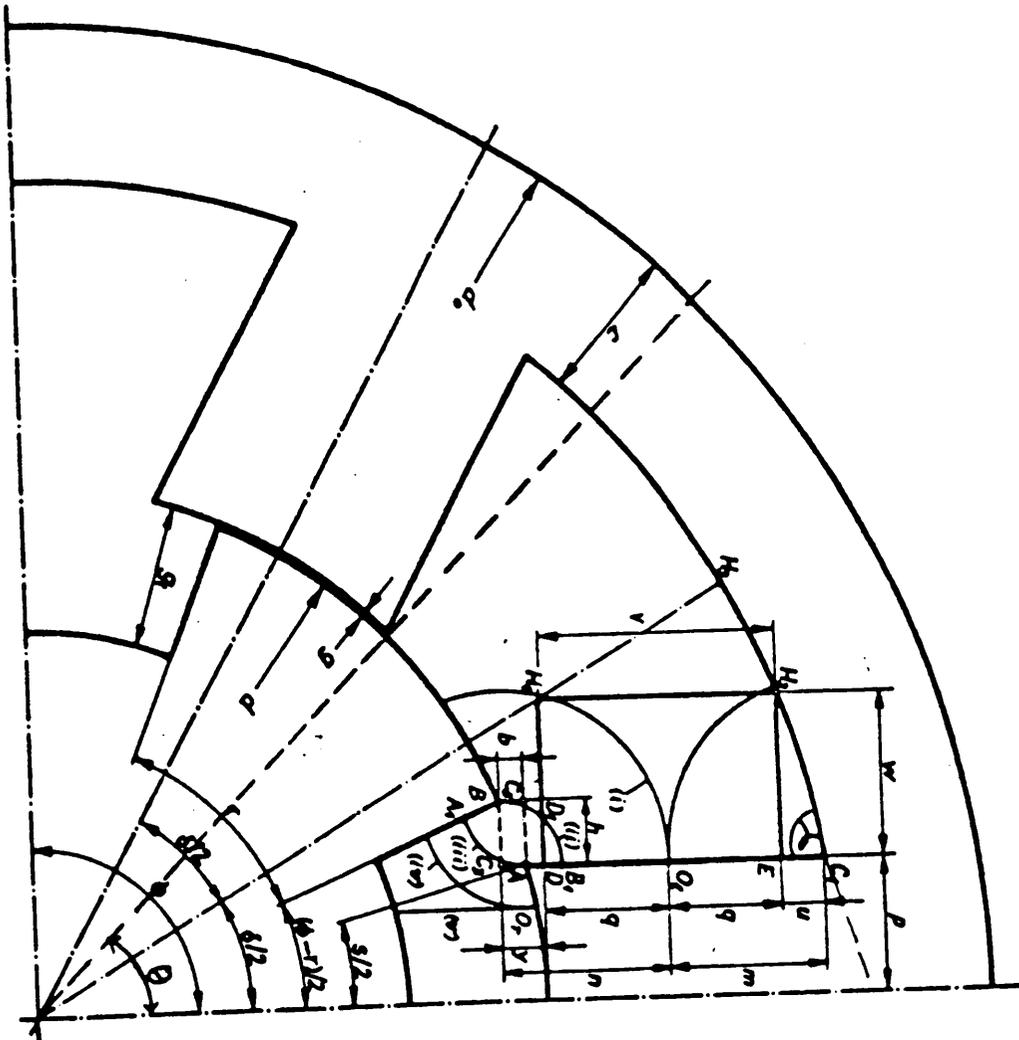
Figure 33. Dimensional description of magnetic configuration

C.1 Calculation of minimum inductance

The magnetic configuration is described by the following set of parameter:

1. Number of stator poles, N_s.

2. Number of rotor poles, N_r.

3. Stator pole arc, s.

4. Rotor pole arc, r.

5. Airgap length, g.

6. Airgap length of rotor interpolar space, g_i.

7. Stator outside diameter, d_0.

8. Back iron width, c.

9. Core length, l.

10. Number of turns per phase, N.

11. Permittivity of free space, μ_0.

The upper case letters represent the normalized dimensional parameters, i.e.,

$$G = \frac{g}{d_0} \tag{C.1}$$

$$g_i = \frac{g_i}{d_0} \tag{C.2}$$

$$D = \frac{d}{d_0} \tag{C.3}$$

$$C = \frac{c}{d_0} \tag{C.4}$$

$$L = \frac{l}{d_0} \tag{C.5}$$

The other angles and dimensions represented in Figure 33 on page 97 are as explained below.

$$\varphi = \frac{2\pi}{N_r} \tag{C.6}$$

$$\delta = \frac{2\pi}{N_s} \tag{C.7}$$

$$\gamma = \text{ang } H_2 C_1 E \cong \frac{\pi}{2} - \frac{\delta}{2} \tag{C.8}$$

$$P = \frac{p}{d_0} = \left(\frac{D}{2} + G\right) \sin \frac{s}{2} \tag{C.9}$$

$$W = \frac{w}{d_0} = \left(\frac{D}{2} + G\right) \tan \frac{\delta}{2} - P \tag{C.10}$$

$$V = \frac{v}{d_0} = \left(\frac{1}{2} - C - \frac{\frac{D}{2} + G}{\cos \frac{\delta}{2}}\right) \frac{1}{\cos \frac{\delta}{2}} \tag{C.11}$$

$$Y = \frac{y}{d_0} = \frac{D}{2} + G - \frac{D}{2} \cos(\frac{\varphi - r}{2}) \tag{C.12}$$

$$H = \frac{h}{d_0} = \frac{D}{2} \sin(\frac{\varphi - r}{2}) - P \tag{C.13}$$

$$B = \frac{b}{d_0} = \left(\frac{D}{2} + G\right)\cos\frac{s}{2} - \frac{D}{2}\cos(\frac{\varphi - r}{2}) \qquad (C.14)$$

$$Q = \frac{q}{d_0} = \frac{V}{2} \qquad (C.15)$$

$$U = \frac{u}{d_0} = \frac{W}{\tan\gamma} \qquad (C.16)$$

$$M = \frac{m}{d_0} = U + Q \qquad (C.17)$$

$$N = \frac{n}{d_0} = Y + Q \qquad (C.18)$$

$$Perm = \mu_0 / \left(\frac{N}{2}\right)^2 \qquad (C.19)$$

The inductances over the various paths are given by the following equations.

$$L_1 = Perm \times \frac{\gamma(0.6 \times m)^4}{4w^2(2v + u)^2} \qquad (C.20)$$

$$L_3 = \frac{Perm \times (\frac{\pi}{2} - \alpha - \beta) \times H^2}{2} \qquad (C.21)$$

where,

$$\alpha = \sin^{-1}(\frac{p}{g_i}) \qquad (C.22)$$

and,

$$\beta = \tan^{-1} \frac{g_i}{\tan \frac{s}{2} \times \left(\frac{D}{2} + G - g_i\right)} \qquad (C.23)$$

$$L_4 = 0 \qquad (C.24)$$

$$L_5 = Perm \times \left\{ \frac{2}{\pi - (\varphi - r)} \ln \frac{2G_i}{H[\pi - (\varphi - r)]} \right\} \qquad (C.25)$$

$$L_6 = Perm \times \left\{ \frac{P + H}{G_i} - \frac{2}{\pi - (\varphi - r)} \right\} \qquad (C.26)$$

Now, the minimum inductance value can be calculated from the above values as,

$$L_{min} = L_1 + L_3 + L_4 + L_5 + L_6 \qquad (C.27)$$

C.2 Calculation of maximum inductance

The assumptions made for calculating the maximum inductance are as follows:

i.   When a phase winding is excited, the magnetic circuit is treated as a simple '2-pole' pattern as illustrated in Figure 34 on page 103.

ii. There is no flux leakage, i.e., all flux passes from the stator and back.

iii. The flux is linked with all the turns.

iv. The flux is uniformly distributed in the cross-section normal to the field lines.

The cross-sectional area of the two stator poles, a_s and their lengths, l_s are calculated as,

$$a_s = \frac{d}{2}(\sin\frac{s}{2})l \tag{C.28}$$

$$l_s = 2\left(\frac{d_0}{2} - c - \frac{d}{2} - g\right) \tag{C.29}$$

The airgap cross-sectional area, a_g and the airgap length, l_g are obtained from the following equations.

$$a_g = \left[\frac{1}{2}\left(\frac{d}{2} + g\right)s + (1 - \sigma)i\right]l \tag{C.30}$$

where,

$$i = \frac{1}{2}\frac{d}{2}(r - s) \tag{C.31}$$

$$\sigma = \frac{2}{\pi}\left[\tan^{-1}(\frac{i}{g}) - \frac{g}{2i}\ln\left(l + (\frac{i}{g})^2\right)\right] \tag{C.32}$$

$$l_g = 2 \times g \tag{C.33}$$

The cross-sectional area of the two rotor poles, a_r and their lengths, l_r are given as,

$$a_r = a_g \tag{C.34}$$

$$l_r = 2 \times g_i \tag{C.35}$$
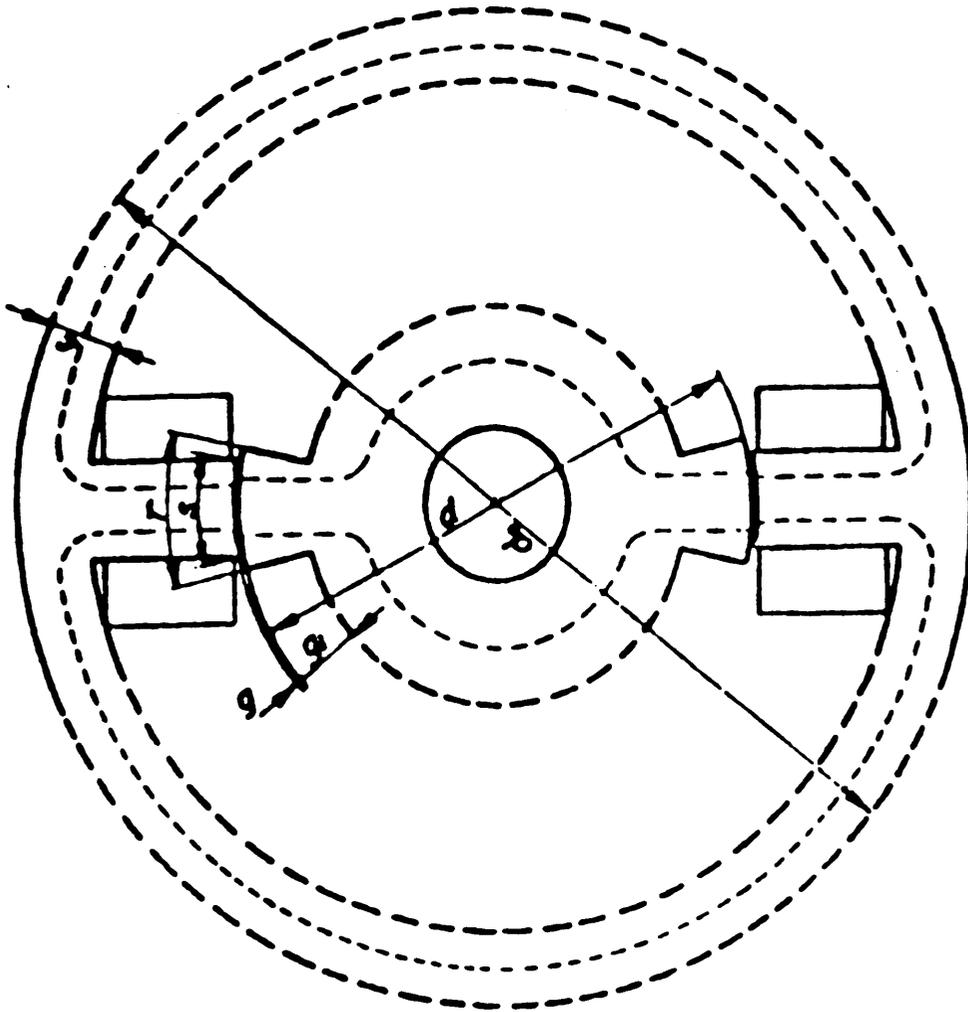
Figure 34.   Simple '2-pole' pattern for aligned position

The rotor body cross-sectional area, a_b and its length, l_b are given as,

$$a_b = \left(\frac{d}{2} - g_i\right)l \tag{C.36}$$

$$l_b = \frac{1}{2}\left(\frac{d}{2} - g_i\right)\pi \tag{C.37}$$

The stator yoke cross-sectional area, a_y and its length, l_y are given as,

$$a_y = c \times l \tag{C.38}$$

$$l_y = \frac{1}{2}(d_0 - c)\pi \tag{C.38}$$

The mmf equation is given as,

$$Ni = \frac{B_g}{\mu_0}l_g + H_s l_s + H_r l_r + H_b l_b + H_y l_y \tag{C.40}$$

The subscripts, g, s, r, b and y represent the airgap, stator pole, rotor pole, rotor body and the stator yoke respectively. B is the flux density and H is the magnetic field intensity and ψ is the flux linkage. But,

$$B = \frac{\psi}{Na} \tag{C.41}$$

Hence,

$$B_g = \frac{\psi}{N} \times \frac{1}{a_g} \tag{C.42}$$

$$B_s = \frac{\psi}{N} \times \frac{1}{a_s} = B_g \times \frac{a_g}{a_s} \tag{C.43}$$

Similarly,

$$B_r = B_g \times \frac{a_g}{a_r} \qquad (C.44)$$

$$B_b = B_g \times \frac{a_g}{a_b} \qquad (C.45)$$

$$B_y = B_g \times \frac{a_g}{a_y} \qquad (C.46)$$

The iterative procedure to solve for equation(C.40) is as follows:

1. Assume N and i.

2. Assume ψ and calculate $B_{g, Bs,} B_r, B_b$ and B_y.

3. From B_g to B_y, using B-H curve, calculate H_r, H_s, H_b and H_y .

4. Substitute the values in equation (C.40) and check if it is equal to the assumed value.

5. If not, adjust the value of ψ (till equation (C.40) is satisfied). Go to 2. Repeat the steps till the mmf equation is satisfied.

From the value of ψ calculated, the maximum inductance is calculated using the equation,

$$L_{max} = \frac{\psi}{i} \qquad (C.47)$$

C.3 Prediction of performance

With the minimum and the maximum values of the inductances and the corresponding flux linkage characteristics obtained, the performance characteristics are obtained in the following manner. The energy per stroke, W, is given as,

$$W = W_{aligned} - W_{unaligned} = \text{Co-energy} \qquad (C.48)$$

where,

$$W_{unaligned} = \frac{1}{2}\psi_u \times i_p \qquad (C.49)$$

$$W_{aligned} = \Delta i[\psi_1 + \psi_2 + ... + \frac{1}{2}\psi n] \qquad (C.50)$$

The average torque, T_{av}, is calculated as,

$$T_{av} = \frac{\text{Total work done/revolution}}{2\pi} \qquad (C.51)$$

And,

$$\begin{aligned}
\text{Total work done/revolution} &= W \times \text{No. of strokes/revolution} \\
&= W \times q.N_r \\
&= W \times \frac{N_s}{2}.N_r
\end{aligned} \qquad (C.52)$$

where,

W is the Co-energy.

N_s is the number of stator poles.

N_r is the number of rotor poles.

q is the number of phases.

Hence,

$$T_{av} = \frac{W.N_s.N_r}{4\pi} \qquad (C.53)$$

If ω is the rotor angular speed and n is the rotor speed in rpm, then

$$
\begin{aligned}
P_d &= T \times \omega \\
&= q\frac{W.N_r}{2\pi}.\omega \\
&= \frac{qWN_r}{2\pi}.\frac{2\pi n}{60} \\
&= \frac{qWN_r n}{60}
\end{aligned}
\qquad (C.54)
$$

As,

$$q = \frac{N_s}{2} \qquad (C.55)$$

The output power is,

$$P_d = \frac{WN_r N_s n}{120} \qquad (C.56)$$

If the performance had been given to the designer to start with, then the performance predicted can be compared with it. If the prediction exceeds a certain tolerance level, then suitable modifications can be made in the dimensions calculation and the whole process repeated again.