COMPARISON OF A TWO-HINGED AND A THREE-HINGED SPANDREL-BRACED ARCH HIGHWAY BRIDGE

by

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J. A. C.

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INTRODUCTION

Exposition

The essential characteristic of an arch is that it develops convergent reactions when loaded. This characteristic distinguishes the arch from structures which develop divergent reactions, such as suspension bridges, and from the large group of structures which develop parallel reactions under normal loading. The arch may be segmental or curved and built of wood, stone, brick, concrete, aluminum, or steel. The arch, however, is not defined by geometric proportions or material but by its structural action.

Being simple to build and pleasing to the eye, the arch is one of the oldest forms of construction. Archaeologists believe some examples of masonry arches to be more than 5000 years old. Many centuries passed before the beginning of scientific analysis, consequently, the earlier builders usually depended upon great mass and weight for strength and rigidity. The old types of masonry arches were statically indeterminate to the third degree. During the last century, the use of concrete and steel with greatly increased knowledge of structural action has produced many changes in the types of arches. The insertion of hinges has reduced their degree of indeterminacy. As a result, the arch has become light, graceful, and consequently more desirable structurally and architecturally. It is stronger and more efficient, and arch bridge spans have increased from 300 to 1650 feet. While acquiring new shape and form, the arch has been adapted to a variety of applications. Welded steel-arch roof frames, for example, bear little resemblance to the classic stone arch bridge. Nevertheless, the same structural action occurs and the identical structural theory applies.

The first major steel-arch bridge was the Eads Bridge at St. Louis, completed in 1874. Since that time, hingless, two-hinged, and three-hinged steel arches of many varieties have achieved prominence in the bridge field. The three-hinged arch has been the most popular type because of its ease of analysis and erection, but the notable long span steel arches today are of the two-hinged type. This is exemplified by the world's two longest arch spans, the Sidney Harbor Bridge in Australia and the Bayonne Bridge over the Kill van Kull at New York City.

The spandrel-braced arch is an important type of steel arch for bridges. Perhaps from an aesthetic viewpoint it does not rank with other arch types, but in all cases it is a more handsome structure than ordinary truss bridges. For bridging deep gorges where natural abutments permit sufficient rise and are capable of resisting large thrusts, the spandrel-braced arch is particularly adapted. A deep gorge, however, is not a necessary condition. This type of arch can be used to bridge streams with gently sloping banks if base rock is near the surface and approach spans are added.

Whether spanning steep canyons, swift channels, or sites of water-traffic activity where a bridge cannot be erected by falsework, the spandrel-braced arch presents the advantage of easier erection which can be performed by the cantilever method. This is done by treating each half as a cantilever held by temporary tie-backs until the center connections are made.

Compared with other types of steel arches, the spandrel-braced arch is the stiffest and most economical for bridges of intermediate spans. They are constructed with either two or three hinges and have been used extensively for both railway and highway bridges. (See Appendix A for a table of notable spandrel-braced arch bridges.)

Need for the Study

The relative merits of two-hinged and three-hinged arches have been a subject of much controversy among structural engineers. The main point of disagreement has concerned the relative economy of the two types of structures.

Hudson[1] made a comparative study of the weights of a 200-foot two-hinged and three-hinged spandrel-braced arch designed for highway loading. Considering only the weight of the main members, the two-hinged type showed a saving of 3.9%.

Jacoby[2] conducted an investigation of the relative weights of a 500-foot spandrel-braced railway arch designed as a two-hinged and three-hinged structure. He reported a saving of steel in the main truss members of 0.84% for the three-hinged arch.

Waddell[3], in a treatise on steel arch bridges, included a comparison of weights of main material for a 500-foot span two-hinged and three-hinged arch of the braced-rib type. The two-hinged arch showed a saving of 8% designed for highway loading and a saving of 5% for railway loading.

Howe[4] reported that for a 416-foot steel arch bridge the saving in weight for a two-hinged over a three-hinged type was 6.9%.

Kunz[5] considered that, in general, the weight of metal in a three-hinged arch bridge and its lateral bracing was about 15% less than that for a comparable two-hinged arch.

The variations in results of these studies indicate that no general rule can be established concerning the relative economy of the two-hinged and three-hinged arches.

Another point in considering the two types of structures has been their relative stiffness. It is an accepted fact that for arch structures of the same over-all dimensions, the two-hinged is stiffer than the three-hinged arch—but is the difference in stiffness of any major importance?

A further point of importance and a classical argument against indeterminate structures, in which category lies the two-hinged arch, has been the effects of displacement of supports upon the stresses in the main structure. The importance of such stresses will vary with arches of different proportions.

It appears, then, that when an arch structure is considered for a particular location and use, an intelligent choice between a two-hinged and a three-hinged arch can be based only upon comparative designs of the two types made under identical specifications.

The Problem

The problem considered in this thesis was the analysis and design of a two-hinged and a three-hinged spandrel braced arch highway bridge and a comparison of the two structures. The comparison was made on the basis of weight of the main trusses, their relative stiffness, and the influence of lateral displacement upon the two structures.

The outline and general dimensions of the spandrel-braced parabolic arch, designed with two hinges (at L0 and L10) and with three hinges (at L0, L5, and L10), are shown in Fig. 1. For the bridge structure, the arch trusses were assumed vertical and spaced 28 feet center to center with a 26 foot roadway. This particular arch structure was chosen because the writer had made a partial analysis for the two-hinged type in C.E. 534.*

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* Virginia Polytechnic Institute. Catalog, 1950.p.165.

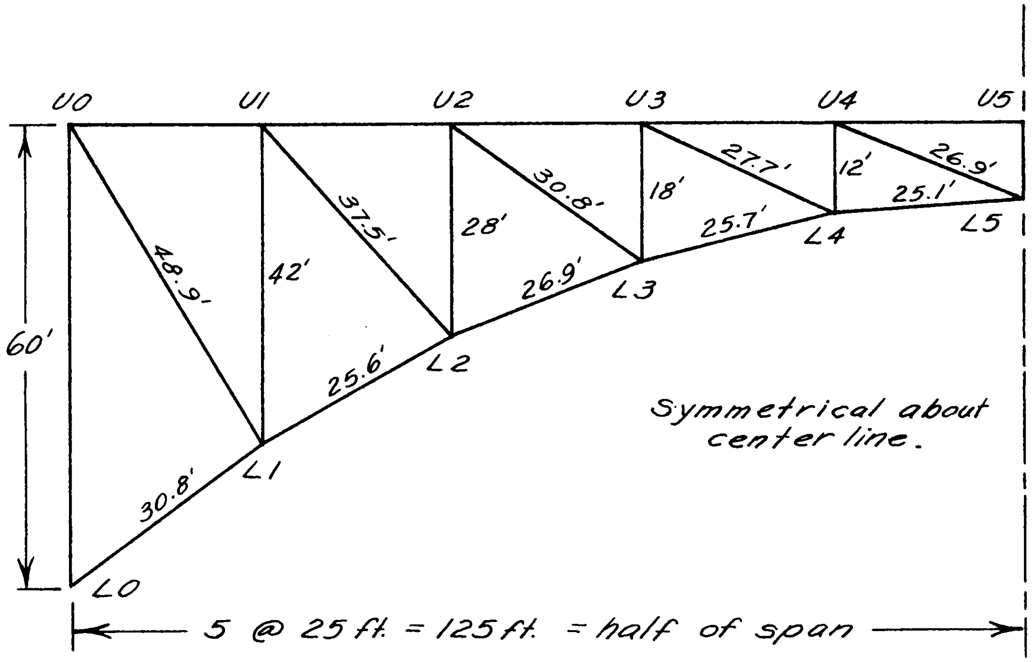


Fig. 1. Spandrel-braced Parabolic Arch

Notation

In all tables, stresses were indicated by + for tension and - for compression. All other symbols were defined as they were used.

II

ANALYSIS AND DESIGN OF THE THREE-HINGED SPANDREL-BRACED ARCH

Loads

1. Dead Load

Based on an investigation of designs for similar structures, the following weights were assumed:

Floor system	100 lb per sq ft
Arch trusses	300 lb per lin ft of truss
Details and laterals	50% wt of arch trusses

It was also assumed that the weight of the arch trusses, details, and laterals was distributed to the upper and lower panel points by the ratio of one to two respectively. The resulting panel point loads were determined as follows:

Upper Chord

Floor system	100 x 25 x 29 x 0.5	= 36,200 lb
Arch truss	300 x 25 x 1/3	= 2,500 lb
Details and laterals	0.5 x 2500	= 1,250 lb
	Total	39,950 lb
		say 40 kips

Lower Chord

Arch truss	300 x 25 x 2/3	= 5,000 lb
Details and laterals	0.5 x 5000	= 2,500 lb
	Total	7,500 lb
		or 7.5 kips

It is evident, for a structure of this type, that the weight per foot varies as the depth of the spandrel. Since the panel point weights of the trusses, details, and laterals were small in comparison with the weight of the floor system, which was uniform across the entire span, the above calculated panel dead loads were considered constant.

2. Live Load

The live loading used was the H20-S16-44 equivalent lane loading specified by the A.A.S.H.O.[6] For a maximum live loading, the two lane loadings were assumed shifted to one side of the roadway (Fig. 2).

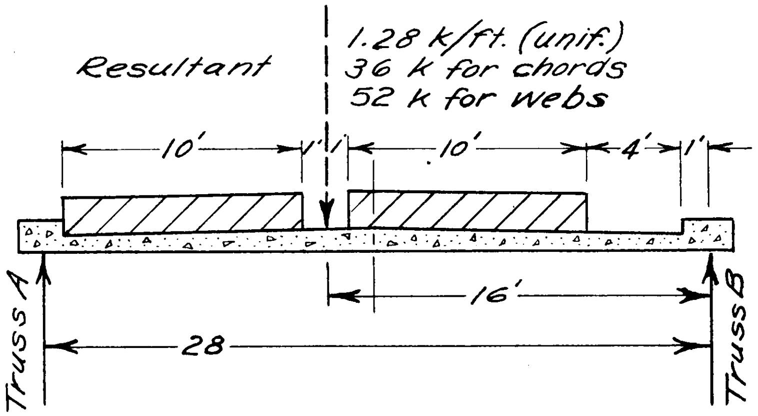


Fig. 2. Cross Section of Roadway

Taking moments about truss B, the maximum panel live loads on truss A were obtained as follows:

Uniform load .... 1.28 x 25 x 16/28 = 18.3 kips

Concentrated load
for chord members .... 36 x 16/28 = 20.6 kips

Concentrated load
for web members .... 52 x 16/28 = 29.7 kips

3. Wind Load

The main members of the arch trusses are stressed as a result of wind stresses in the lateral bracing. An exact analysis for wind would have been extremely tedious and certainly needless, since the actual wind forces and the wind stress distribution are uncertain. A lateral system in the plane of the upper chord could not be made continuous from end to end of the structure since it would prohibit freedom of the crown hinge movement. Consequently, a continuous lateral system in the planes of the lower chord and transverse bracing in the planes of the verticals were considered to resist the lateral wind forces. It was assumed that both systems were composed of tension diagonals and compression struts.

The wind forces used were those specified by the A.A.S.H.O[7] stated in part in Appendix B. Also included in Appendix B are the calculations of the vertical and horizontal loads on the leeward truss as shown in Fig. 3. Only half of the truss was considered because of symmetry about the crown hinge.

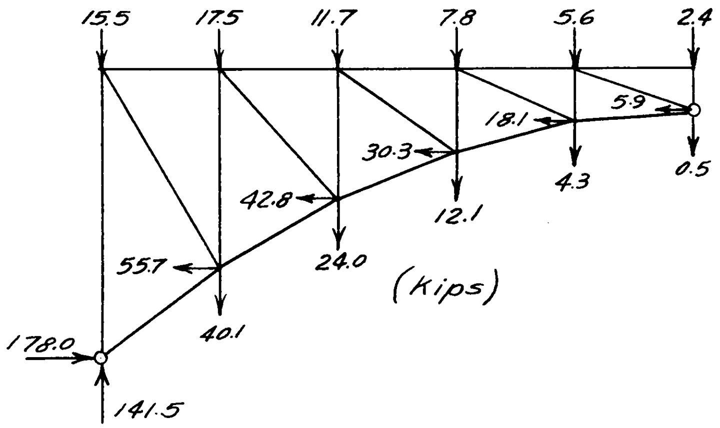


Fig. 3. Wind Loads Resisted by Leeward Truss

The resulting wind loads on the windward truss would be numerically equal but opposite in direction to those on the leeward truss, and the lower chord loads would be applied one joint nearer the abutment in each case. It was assumed that the resulting stresses in the windward truss members were equal in magnitude but opposite in sign, a conservative assumption but sufficient for the accuracy this analysis required.

Analysis

The reactions and stresses for the three-hinged spandrel-braced arch are statically determinate. For any loading the vertical reactions are determined the same as for a simple beam. The horizontal reactions may be determined by taking moments about L_8 (Fig. 4) since for equilibrium the moments about the crown hinge must equal zero. After finding the reactions for any given loading, the stresses in the members were found by statics.

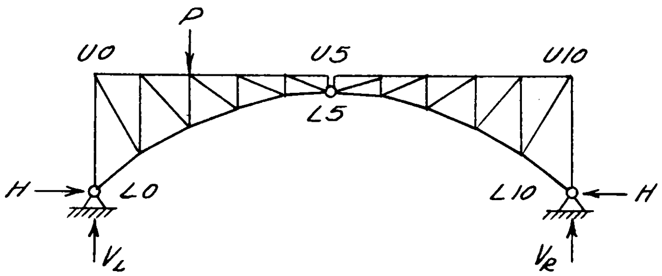


Fig. 4. Three-hinged Spandrel-braced Arch

Since it is not apparent from inspection what positions the live load must occupy to produce maximum stresses in the various members, a structure of this type is best analyzed by the method of influence lines. Instead of actually constructing influence lines, it was considered more practical to list the influence ordinates in tabular form.

In preparing the influence table for stress in each member, (Table 1) only half of the arch needed consideration because of symmetry about the crown hinge. Nevertheless, it was necessary to move the unit load across the entire structure.

With Table 2 (Summary of Influence Table) and the loads and stresses previously calculated, the design stresses were readily determined. Special attention was given specifications[8] 3.2.12, 3.6.5, and 3.4.1 concerning impact, stress reversal, and stress combinations respectively. In reference to the latter, instead of increasing the allowable unit stresses by 25% for members subjected to wind stress (W) in combination with dead (D), live (L), and impact (I) stresses, combined stresses were decreased by 20%. The design stresses were then taken as the larger of either D + L + I or 0.8 (D + L + I + W).

Design

The main truss members were designed in strict accordance with the current A.A.S.H.O. specifications[9]. Since L/r (L is unsupported length; r is least radius of gyration) was the controlling factor in the design of members U0-L0, U0-L1 and U1-L1, their unsupported lengths were reduced by including a brace from joint L2 horizontally to member U0-L0. By doing this, smaller channels were used for verticals and diagonals throughout the structure which resulted in a considerable saving in steel.

TABLE 1

INFLUENCE TABLE FOR STRESS IN EACH MEMBER
(Three-hinged Spandrel-braced Arch)

Member	Stress when unit load is at panel point										
	U0	UI	U2	U3	U4	U5	U6	U7	U8	U9	U10
U0-UI	0.000	-0.429	-0.262	-0.095	+0.071	+0.238	+0.191	+0.143	+0.095	+0.048	0.000
UI-U2	0.000	-0.429	-0.857	-0.393	+0.071	+0.536	+0.429	+0.322	+0.214	+0.107	0.000
U2-U3	0.000	-0.389	-0.778	-1.166	-0.167	+0.833	+0.667	+0.500	+0.333	+0.167	0.000
U3-U4	0.000	-0.250	-0.500	-0.750	-1.000	+0.833	+0.667	+0.500	+0.333	+0.167	0.000
U4-U5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
L0-LI	0.000	-0.308	-0.616	-0.924	-1.232	-1.540	-1.232	-0.924	-0.616	-0.308	0.000
LI-L2	0.000	+0.204	-0.272	-0.750	-1.225	-1.702	-1.362	-1.021	-0.681	-0.341	0.000
L2-L3	0.000	+0.192	+0.384	-0.384	-1.153	-1.921	-1.538	-1.153	-0.769	-0.384	0.000
L3-L4	0.000	+0.143	+0.286	+0.428	-0.856	-2.140	-1.712	-1.284	-0.856	-0.428	0.000
L4-L5	0.000	0.000	0.000	0.000	0.000	-2.090	-1.672	-1.254	-0.836	-0.418	0.000
U0-LI	0.000	+0.839	+0.512	+0.186	-0.140	-0.466	-0.372	-0.280	-0.186	-0.093	0.000
UI-L2	0.000	0.000	+0.893	+0.447	0.000	-0.447	-0.358	-0.268	-0.178	-0.089	0.000
U2-L3	0.000	-0.049	-0.097	+0.954	+0.294	-0.366	-0.294	-0.221	-0.147	-0.074	0.000
U3-L4	0.000	-0.154	-0.308	-0.462	+0.924	0.000	0.000	0.000	0.000	0.000	0.000
U4-L5	0.000	-0.269	-0.538	-0.807	-1.076	+0.896	+0.718	+0.538	+0.358	+0.180	0.000
U0-L0	-1.000	-0.720	-0.440	-0.160	+0.120	+0.400	+0.320	+0.240	+0.160	+0.080	0.000
UI-LI	0.000	-1.000	-0.666	-0.334	0.000	+0.333	+0.267	+0.200	+0.133	+0.067	0.000
U2-L2	0.000	+0.029	-0.943	-0.557	-0.172	+0.214	+0.172	+0.129	+0.086	+0.043	0.000
U3-L3	0.000	+0.067	+0.133	-0.800	-0.400	0.000	0.000	0.000	0.000	0.000	0.000
U4-L4	0.000	+0.100	+0.200	+0.300	-0.600	-0.333	-0.267	-0.200	-0.133	-0.067	0.000
U5-L5	0.000	0.000	0.000	0.000	0.000	-1.000	0.000	0.000	0.000	0.000	0.000

TABLE 2

SUMMARY OF INFLUENCE TABLE
(Three-hinged Spandrel-braced Arch)

Member	Sum of Ordinates		Maximum Ordinate		Loaded Length	
	+	-	+	-	+	-
U0-UI	0.786	0.786	0.238	0.095	162.5	87.5
UI-U2	1.679	1.679	0.536	0.393	162.5	87.5
U2-U3	2.500	2.500	0.833	0.389	137.5	112.5
U3-U4	2.500	2.500	0.833	0.250	137.5	112.5
U4-U5	0.000	0.000	0.000	0.000	0.0	0.0
L0-LI	0.000	7.700	0.000	1.540	0.0	250.0
LI-L2	0.204	7.354	0.204	1.702	37.5	212.5
L2-L3	0.576	7.302	0.384	1.921	62.5	187.5
L3-L4	0.857	7.276	0.428	2.140	87.5	162.5
L4-L5	0.000	6.270	0.000	2.090	0.0	150.0
U0-LI	1.537	1.537	0.839	0.466	87.5	162.5
UI-L2	1.340	1.340	0.893	0.447	75.0	150.0
U2-L3	1.248	1.248	0.954	0.366	50.0	200.0
U3-L4	0.923	0.923	0.924	0.462	37.5	87.5
U4-L5	2.690	2.690	0.896	1.076	137.5	112.5
U0-L0	1.320	2.320	0.400	1.000	162.5	87.5
UI-LI	1.000	2.000	0.334	1.000	150.0	100.0
U2-L2	0.673	1.673	0.214	0.943	175.0	75.0
U3-L3	0.200	1.200	0.133	0.800	62.5	62.5
U4-L4	0.600	1.600	0.300	0.600	87.5	162.5
U5-L5	0.000	1.000	0.000	1.000	0.0	25.0

TABLE 3

STRESS TABLE

(Three-hinged Spandrel-braced Arch)

Dead Panel Loads — Upper Chord .. 40.0 kips — Lower Chord .. 7.5 kips

Live Panel Loads — Uniform .. 18.3 kips — Concentrated: for chords .. 20.6 kips; for webs .... 29.7 kips

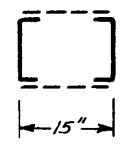
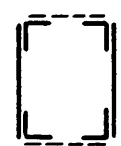
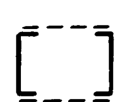
Member	Dead Stress	Live Stress Unif.	Live Stress Conc.	Live Stress Total	Impact Factor	Impact Stress	L + I	D+L+I *	Wind Stress	D+L+I +W	0.8(D+L +I +W)	Design Stress
U0-UI		+ 14.4 - 14.4	+ 4.9 - 2.0	+ 19.3 - 16.4	0.174 0.236	+ 3.4 - 3.9	+ 22.7 - 20.3	+ 32.9 - 30.5	+ 1.3 - 1.3	+ 34.2 - 31.8	+ 27 - 25	+ 33 - 31
UI-U2		+ 30.7 - 30.7	+11.0 - 8.1	+ 41.7 - 38.8	0.174 0.236	+ 7.3 - 9.2	+ 49.0 - 48.0	+ 73.0 - 72.0	+ 2.0 - 2.0	+ 75.0 - 74.0	+ 60 - 59	+ 73 - 72
U2-U3		+ 45.8 - 45.8	+17.2 - 8.0	+ 63.0 - 53.8	0.191 0.210	+12.0 -11.3	+ 75.0 - 65.1	+107.6 - 97.7	+ 1.8 - 1.8	+109.4 - 99.5	+ 88 - 80	+108 - 98
U3-U4		+ 45.8 - 45.8	+17.2 - 5.2	+ 63.0 - 51.0	0.191 0.210	+12.0 -10.7	+ 75.0 - 61.7	+105.8 - 92.5	+ 0.8 - 0.8	+106.6 - 93.3	+ 85 - 75	+106 - 93
U4-U5												
L0-LI	-366	-141.0	-31.8	-172.8	0.133	-23.0	-195.8	-561.8	-220.0	-781.8	-625	-625
LI-L2	-340	+ 3.7 -134.7	+ 4.2 -35.1	+ 7.9 -169.8	0.300 0.148	+ 2.4 -25.1	+ 10.3 -194.9	-227.7 -534.9	+141.3 -141.3	- 86.4 -676.2	-541	-541
L2-L3	-320	+ 10.5 -133.6	+ 7.9 -39.6	+ 18.4 -173.2	0.266 0.160	+ 4.9 -27.8	+ 23.3 -201.0	-200.7 -521.0	+ 87.6 - 87.6	-113.1 -608.6	-486	-521
L3-L4	-305	+ 15.7 -133.2	+ 8.8 -44.1	+ 24.5 -177.3	0.236 0.174	+ 5.8 -30.9	+ 30.3 -208.2	-183.2 -513.2	+ 52.5 - 52.5	-130.7 -565.7	-453	-513
L4-L5	-298	-115.0	-43.0	-158.0	0.182	-28.8	-186.8	-484.8	- 32.1	-516.9	-414	-485
U0-LI		+ 28.1 - 28.1	+24.9 -13.8	+ 53.0 - 41.9	0.236 0.174	+12.5 - 7.3	+ 65.5 - 49.2	+ 90.1 - 73.8	+ 2.5 - 2.5	+ 92.6 - 76.3	+ 74 - 61	+ 90 - 74
UI-L2		+ 24.5 - 24.5	+26.5 -13.3	+ 51.0 - 37.8	0.250 0.182	+12.8 - 6.9	+ 63.8 - 44.7	+ 86.2 - 67.1	+ 1.1 - 1.1	+ 87.3 - 68.2	+ 70 - 55	+ 86 - 67
U2-L3		+ 22.8 - 22.8	+28.3 -10.9	+ 51.1 - 33.7	0.286 0.154	+14.6 - 5.2	+ 65.7 - 38.9	+ 85.1 - 58.5	+ 0.2 - 0.2	+ 85.3 - 58.5	+ 68 - 47	+ 85 - 58
U3-L4		+ 16.9 - 16.9	+27.4 -13.7	+ 44.3 - 30.6	0.300 0.236	+13.3 - 7.2	+ 57.6 - 37.8	+ 76.5 - 56.7	+ 1.2 - 1.2	+ 77.7 - 57.9	+ 62 - 46	+ 77 - 57
U4-L5		+ 49.2 - 49.2	+26.6 -31.9	+ 75.8 - 81.1	0.191 0.210	+14.5 -17.0	+ 90.3 - 98.1	+135.4 -143.2	+ 0.8 - 0.8	+136.2 -144.0	+109 -115	+135 -143
U0-L0	- 20	+ 24.2 - 33.4	+11.9 -29.7	+ 36.1 - 63.1	0.174 0.236	+ 6.3 -14.9	+ 42.4 - 78.0	+ 42.6 -112.2	+ 13.3 - 13.3	+ 55.9 -125.5	+ 45 -100	-113
UI-LI	- 40	+ 18.3 - 36.6	+ 9.9 -29.7	+ 28.2 - 66.3	0.182 0.222	+ 5.1 -14.7	+ 33.3 - 81.0	+ 7.9 -123.6	+ 16.7 - 16.7	+ 24.6 -140.3	+ 20 -112	-124
U2-L2	- 40	+ 12.3 - 30.6	+ 6.4 -28.0	+ 18.7 - 58.6	0.167 0.250	+ 3.1 -14.7	+ 21.8 - 73.3	- 6.2 -113.3	+ 11.8 - 11.8	+ 5.6 -125.1	+ 5 -100	-114
U3-L3	- 40	+ 3.7 - 22.0	+ 3.4 -23.8	+ 7.1 - 45.8	0.266 0.266	+ 1.9 -12.2	+ 9.0 - 58.0	- 19.0 - 98.0	+ 8.3 - 8.3	- 10.7 -106.3	- 85	- 98
U4-L4	- 40	+ 11.0 - 29.2	+ 8.9 -17.8	+ 19.9 - 47.0	0.236 0.174	+ 4.7 - 8.2	+ 24.6 - 55.2	- 3.4 - 95.3	+ 5.9 - 5.9	+ 2.5 -101.2	+ 2 - 81	- 96
U5-L5	- 20	- 9.2	-14.8	- 24.0	0.300	- 7.2	- 31.2	- 51.2	- 2.4	- 53.6	- 43	- 51

Note: Stresses expressed in kips.
* Corrected for reversal.

TABLE 4

DESIGN SUMMARY

(Three-hinged Spandrel-braced Arch)

Member	Length (ft)	Design Stress (kips)	Allow. Stress (k/in²)	Area Req'd (in²)	Section	Assembly	Least r (in.)	Area Prv'd (in²)	Weight	
									lb/ft	lb
U0-U1	25.0	+ 33 - 31	13.2	2.4	2 9 ⊔ 13.4		3.49	7.9	26.8	670
U1-U2	25.0	+ 73 - 72	13.2	5.5	2 9 ⊔ 13.4		3.49	7.9	26.8	670
U2-U3	25.0	+108 - 98	13.2	7.4	2 9 ⊔ 13.4		3.49	7.9	26.8	670
U3-U4	25.0	+106 - 93	13.2	7.0	2 9 ⊔ 13.4		3.49	7.9	26.8	670
U4-U5	25.0				2 9 ⊔ 13.4		3.49	7.9	26.8	670
L0-L1	30.8	-625	14.2	44.0	4 Ls. 4 x 4 x 5/8 2 Pls. 18 x 3/4		6.47	45.4	154.6	4760
L1-L2	28.6	-541	14.3	37.9	4 Ls. 4 x 4 x 1/2 2 Pls. 18 x 11/16		6.40	39.8	135.4	3880
L2-L3	26.9	-521	14.4	36.2	4 Ls. 4 x 4 x 1/2 2 Pls. 18 x 5/8		6.47	37.5	127.8	3440
L3-L4	25.7	-513	14.4	35.6	4 Ls. 4 x 4 x 1/2 2 Pls. 18 x 5/8		6.47	37.5	127.8	3280
L4-L5	25.1	-485	14.5	33.4	4 Ls. 4 x 4 x 1/2 2 Pls. 18 x 9/16		6.54	35.3	120.0	3010
U0-L1	48.9	+ 90 - 74	11.8	6.3	2 9 ⊔ 13.4		3.49	7.8	26.8	1310
U1-L2	37.5	+ 86 - 67	10.8	6.2	2 9 ⊔ 13.4		3.49	7.8	26.8	1000
U2-L3	30.8	+ 85 - 58	12.2	4.8	2 9 ⊔ 13.4		3.49	7.8	26.8	830
U3-L4	27.7	+ 77 - 57	12.7	4.5	2 9 ⊔ 13.4		3.49	7.8	26.8	740
U4-L5	26.9	+135 -143	12.5	11.4	2 9 ⊔ 20		3.22	11.7	40.0	1080
U0-L0	60.0	-113	12.3	9.2	2 9 ⊔ 20		3.22	11.7	40.0	2400
U1-L1	42.0	-124	12.3	10.1	2 9 ⊔ 20		3.22	11.7	40.0	1680
U2-L2	28.0	-114	12.3	9.3	2 9 ⊔ 20		3.22	11.7	40.0	1120
U3-L3	18.0	- 98	14.0	7.0	2 9 ⊔ 13.4		3.49	7.8	26.8	480
U4-L4	12.0	- 96	14.6	6.6	2 9 ⊔ 13.4		3.49	7.8	26.8	320
U5-L5	10.0	- 51	14.7	3.5	2 9 ⊔ 13.4		3.49	7.8	26.8	270

III

ANALYSIS AND DESIGN OF THE TWO-HINGED SPANDREL-BRACED ARCH

Loads

1. Dead Load

The dead weights were assumed to be the same as for the three-hinged arch, with the exception that the weight of the trusses, details, and laterals was assumed distributed equally between the upper and lower panel points. The resulting dead panel loads are shown in Table 7.

2. Live Load

The live load was, in all respects, the same as previously described for the three-hinged arch.

3. Wind Load

The lateral wind forces were assumed to be the same as for the three-hinged arch, however, a slightly different lateral system was considered to resist these forces. Continuous lateral trusses in the planes of the upper and lower chords were assumed as tension diagonals and compression struts. Also, tension diagonals were considered in the vertical end planes of the structure for taking the wind force from the upper lateral system to the abutments. The resulting forces acting in the plane of the leeward arch were calculated in a manner similar to that shown in Appendix B for the three-hinged arch.

<u>Analysis</u>

The reactions and stresses for the two-hinged spandrel-braced arch are statically indeterminate. A structure of this type is readily solved by applying Castigliano's theorem.

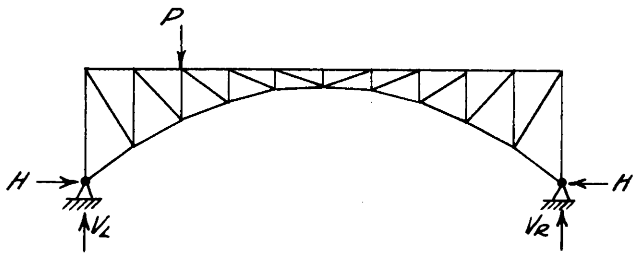
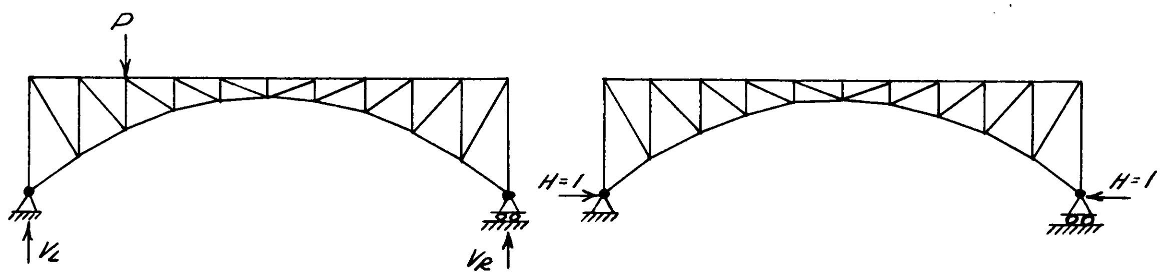


Fig. 5. Two-hinged Spandrel-braced Arch

For any loading P (Fig. 5), the load and vertical reactions and the horizontal reactions were considered in two parts as shown in Fig. 6.



(a) Condition H = 0 (b) Condition H = 1

Fig. 6

As a result, the stress in any member (Fig. 5) was equal to

$$F = S + Hu$$

in which S = stress in any member of Fig. 6a.

u = stress in any member of Fig. 6b.

H = horizontal thrust

The total internal work

$$W = \sum \frac{F^2 L}{2AE} = \sum \frac{(S + Hu)^2 L}{2AE}$$

where A = cross section area of any member.

E = modulus of elasticity.

From Castigliano's theorem,

$$\frac{\partial W}{\partial H} = \sum \frac{(Su + Hu^2) L}{AE} = 0$$

from which

$$H = -\frac{\sum \frac{SuL}{AE}}{\sum \frac{u^2 L}{AE}}$$

At this point is was necessary to make as assumption regarding the unknown areas of the various members. By assuming that both A and E were constant for all members, the above equation was reduced to

$$H = -\frac{\sum SuL}{\sum u^2 L}$$

The accuracy of this assumption was checked after the cross section areas were obtained for the various members. A comparison of the values of H follows:

Unit Load at	Constant Areas	Variable Areas	% Variation
U1	0.245	0.247	0.82
U2	0.479	0.472	1.46
U3	0.690	0.681	1.30
U4	0.855	0.846	1.05
U5	0.921	0.918	0.33

Like the three-hinged arch, this structure was analysed by an influence table, however, the process by which the table was obtained was more laborious. A unit load was considered to move across the entire span (Fig. 6a), and for each of its panel point positions, the stress (S), in each member for half of the truss was calculated. Also, the stress (u), in each member was calculated for the condition H = 1 (Fig. 6b). From these values of S and u, H was calculated for the unit load at each upper panel point. Once these values were obtained, the calculation of the influence ordinates for stress in each member followed.

With the aid of Table 6 (Summary of Influence Table) and the dead and live loads previously determined, the dead and live stresses shown in Table 7 were calculated.

For the wind loading on the leeward truss, a new value for H was obtained with which the wind stress in each member was calculated. As shown in Table 7, these stresses were assumed to be equal in magnitude but opposite in sign for the windward truss.

Also, stresses were calculated for a uniform temperature variation of $\pm$ 60° Fahrenheit. The temperature stress in any member was equal to

$$T = Hu$$

where

$$H = \frac{etL'}{\sum \frac{u^2 L}{AE}}$$

in which e = coefficient of linear expansion (.0000065 per degree)

t = change in temperature (±60°)

L′ = length of span (250 ft)

u = stress in any member due to H = unity

A = area (assumed constant at 25 in.^{2})

E = modulus of elasticity (29,000 K/in.^{2})

Temperature stresses thus determined appear in Table 7. After the cross-sectional areas were determined, new temperature stresses were calculated for those members whose areas differed greatly from 25 sq. in. However, no changes in the sections were required.

The design stresses were determined in the same manner as for the three-hinged arch except that here it was necessary to include the temperature stresses.

Design

Design of the main members was made in accordance with the current A.A.S.H.O. specifications[10]. The unsupported lengths of members U0-L0, U0-L1, and U1-L1 were reduced by placing a stiffening brace from joint L1 horizontally to member U0-L0 in the same manner as in the three-hinged arch.

TABLE 5

INFLUENCE TABLE FOR STRESS IN EACH MEMBER
(Two-hinged Spandrel-braced Arch)

Member	Stress when unit load is at panel point										
	U0	U1	U2	U3	U4	U5	U6	U7	U8	U9	U10
U0-U1	+0.003	-0.429	-0.273	-0.124	+0.006	+0.096	+0.124	+0.114	+0.083	+0.046	+0.003
U1-U2	+0.009	-0.433	-0.890	-0.472	-0.105	+0.155	+0.250	+0.241	+0.181	+0.103	+0.009
U2-U3	+0.018	-0.397	-0.846	-1.332	-0.528	+0.058	+0.308	+0.339	+0.268	+0.160	+0.018
U3-U4	+0.031	-0.262	-0.612	-1.026	-1.616	-0.493	+0.052	+0.225	+0.222	+0.155	+0.031
U4-U5	+0.039	-0.015	-0.140	-0.350	-0.770	-1.660	-0.770	-0.350	-0.140	-0.015	+0.039
L0-L1	-0.010	-0.304	-0.582	-0.840	-1.044	-1.131	-1.044	-0.840	-0.582	-0.304	-0.010
L1-L2	-0.013	+0.209	-0.228	-0.638	-0.976	-1.161	-1.113	-0.911	-0.638	-0.337	-0.013
L2-L3	-0.018	+0.198	+0.448	-0.224	-0.799	-1.160	-1.184	-0.995	-0.706	-0.379	-0.018
L3-L4	-0.026	+0.153	+0.380	+0.661	-0.331	-1.010	-1.180	-1.055	-0.764	-0.419	-0.026
L4-L5	-0.039	+0.014	+0.138	+0.342	+0.766	-0.430	-0.906	-0.912	-0.698	-0.404	-0.039
U0-L1	-0.007	+0.837	+0.536	+0.244	-0.009	-0.186	-0.242	-0.221	-0.162	-0.092	-0.007
U1-L2	-0.008	+0.006	+0.925	+0.521	+0.166	-0.087	-0.189	-0.192	-0.147	-0.085	-0.008
U2-L3	-0.011	-0.044	-0.056	+1.056	+0.520	+0.116	-0.072	-0.123	-0.108	-0.070	-0.011
U3-L4	-0.014	-0.149	-0.256	-0.336	+1.210	+0.618	+0.288	+0.129	+0.054	+0.006	-0.014
U4-L5	-0.008	-0.268	-0.508	-0.732	-0.910	+1.268	+0.886	+0.615	+0.390	+0.181	-0.008
U0-L0	-0.994	-0.722	-0.460	-0.210	+0.010	+0.160	+0.210	+0.190	+0.140	+0.078	+0.006
U1-L1	+0.006	-1.005	-0.692	-0.392	-0.127	+0.063	+0.139	+0.141	+0.108	+0.063	+0.006
U2-L2	+0.007	+0.026	-0.967	-0.617	-0.305	-0.068	+0.041	+0.071	+0.063	+0.041	+0.007
U3-L3	+0.006	+0.065	+0.111	-0.854	-0.524	-0.266	-0.124	-0.055	-0.023	-0.002	+0.006
U4-L4	+0.003	+0.099	+0.189	+0.272	-0.661	-0.468	-0.329	-0.229	-0.145	-0.068	+0.003
U5-L5	0.000	0.000	0.000	0.000	0.000	-1.000	0.000	0.000	0.000	0.000	0.000

TABLE 6

SUMMARY OF INFLUENCE TABLE
(Two-hinged Spandrel-braced Arch)

Member	Sum of Ordinates		Maximum Ordinate		Loaded Length	
	+	-	+	-	+	-
UO-UI	0.475	0.826	0.124	0.429	175	75
UI-U2	0.948	1.900	0.250	0.890	150	100
U2-U3	1.169	3.103	0.339	1.332	150	100
U3-U4	0.716	4.009	0.225	1.616	125	125
U4-U5	0.078	4.210	0.039	1.660	25	225
LO-LI	0.000	6.691	0.000	1.131	0	250
LI-L2	0.209	6.028	0.209	1.161	25	225
L2-L3	0.646	5.501	0.448	1.184	50	200
L3-L4	1.194	4.820	0.661	1.189	75	175
L4-L5	1.260	3.428	0.766	0.912	100	150
UO-LI	1.617	0.926	0.837	0.242	75	175
UI-L2	1.618	0.716	0.925	0.192	100	150
U2-L3	1.692	0.495	1.056	0.123	75	175
U3-L4	2.305	0.769	1.210	0.336	150	100
U4-U5	3.340	2.434	1.268	0.910	125	125
UO-LO	0.794	2.386	0.210	0.994	162	88
UI-LI	0.526	2.216	0.141	1.005	150	100
U2-L2	0.256	1.957	0.071	0.967	150	100
U3-L3	0.188	1.848	0.111	0.854	75	175
U4-L4	0.566	1.900	0.272	0.661	100	150
U5-L5	0.000	1.000	0.000	1.000	0	50

TABLE 7

STRESS TABLE

(Two-hinged Spandrel-braced Arch)

Dead Panel Loads   Upper chord .. 41.8 kips   Lower chord .. 5.7 kips

Live Panel Loads   Uniform .. 18.3 kips   Concentrated: for chords .. 20.6 kips
for webs .... 29.7 kips

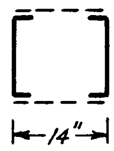
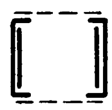
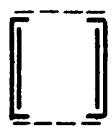
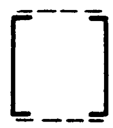
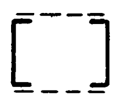
Member	Dead Stress	Live Stress			Impact Factor	Impact Stress	L + I	D+L+I *	Temp. Stress	Wind Stress	D+L+I +T+W	0.8(D+L +I+T+W)	Design Stress
		Unif.	Conc.	Total									
U0-UI	- 16.7	+ 8.7 - 15.1	+ 2.6 - 8.8	+ 11.3 - 23.9	0.167 0.250	+ 1.9 - 6.0	+ 13.2 - 29.9	+ 2.3 - 47.4	+ 5.5 - 5.5	+ 2.8 - 2.8	+ 10.6 - 55.7	+ 9 - 45	- 47
UI-U2	- 45.2	+ 17.2 - 34.8	+ 5.2 -18.3	+ 22.4 - 53.1	0.182 0.222	+ 4.1 -11.8	+ 26.5 - 64.9	- 5.1 -110.1	+14.6 -14.6	+ 30.8 - 30.8	+ 40.3 -155.5	+ 32 - 125	-125
U2-U3	- 92.0	+ 21.4 - 56.7	+ 7.0 -27.7	+ 28.4 - 84.4	0.182 0.222	+ 5.2 -18.7	+ 33.6 -103.1	- 30.8 -195.1	+29.8 -29.8	+ 48.1 - 48.1	+ 47.1 -273.0	+ 38 - 218	-218
U3-U4	-156.3	+ 13.1 - 73.2	+ 4.7 -33.6	+ 17.8 -106.8	0.200 0.200	+ 3.6 -21.4	+ 21.4 -128.2	- 88.0 -284.5	+51.1 -51.1	+ 50.4 - 50.4	+ 13.5 -388.6	+ 11 - 312	-312
U4-U5	-196.0	+ 1.4 - 77.0	+ 0.8 -34.2	+ 2.2 -111.2	0.300 0.143	+ 0.7 -15.9	+ 2.9 -127.1	-134.3 -323.1	+64.0 -64.0	+ 51.0 - 51.0	- 19.0 -438.1	- 350	-350
L0-LI	-318.0	-122.4	-23.3	-145.7	0.133	-19.4	-165.1	-318.0 -483.1	+15.8 -15.8	+ 40.8 - 40.8	-261.4 -539.7	- 431	-483
LI-L2	-276.0	+ 3.8 -110.3	+ 4.3 -23.9	+ 8.1 -134.2	0.300 0.143	+ 2.4 -19.2	+ 10.5 -153.4	-182.7 -429.4	+21.0 -21.0	+ 17.4 - 17.4	-140.1 -467.8	- 374	-429
L2-L3	-231.0	+ 11.8 -100.8	+ 9.2 -24.4	+ 21.0 -125.2	0.286 0.154	+ 6.0 -19.3	+ 27.0 -144.5	-134.7 -375.5	+29.6 -29.6	+ 3.5 - 3.5	-101.6 -408.6	- 327	-376
L3-L4	-172.0	+ 21.8 - 88.2	+13.6 -24.5	+ 35.4 -112.7	0.250 0.167	+ 8.8 -18.8	+ 44.2 -131.5	- 76.4 -303.5	+43.9 -43.9	+ 25.5 - 25.5	- 7.0 -372.9	- 298	-304
L4-L5	-103.0	+ 23.1 - 62.7	+15.8 -18.8	+ 38.9 - 81.5	0.222 0.182	+ 8.6 -14.8	+ 47.5 - 96.3	- 24.6 -199.3	+64.3 -64.3	+ 50.0 - 50.0	+ 89.7 -313.6	+ 72 - 251	-251
U0-LI	+ 32.8	+ 29.6 - 16.9	+24.6 - 7.2	+ 54.2 - 24.1	0.250 0.167	+13.6 - 4.0	+ 67.8 - 28.1	+103.2 - 7.7	+10.7 -10.7	+ 5.6 - 5.6	+119.5 - 24.0	+ 96 - 19	+103
UI-L2	+ 42.8	+ 29.6 - 13.1	+27.4 - 5.7	+ 57.0 - 18.8	0.222 0.182	+12.7 - 3.4	+ 69.7 - 22.2	+112.5 + 7.8	+13.7 -13.7	+ 10.4 - 10.4	+136.6 - 16.3	+ 109 - 13	+113
U2-L3	+ 56.8	+ 31.0 - 9.1	+31.4 - 3.7	+ 62.4 - 12.8	0.250 0.167	+15.6 - 2.1	+ 78.0 - 14.9	+134.8 + 24.8	+18.7 -18.7	+ 16.5 - 16.5	+170.0 - 10.4	+ 136 - 8	+136
U3-L4	+ 72.9	+ 42.2 - 14.1	+36.0 -10.0	+ 78.2 - 24.1	0.182 0.222	+14.2 - 5.4	+ 92.4 - 29.5	+165.3 + 21.5	+23.6 -23.6	+ 22.0 - 22.0	+210.9 - 24.1	+ 169 - 19	+169
U4-L5	+ 43.0	+ 61.0 - 44.5	+37.6 -27.0	+ 98.6 - 71.5	0.200 0.200	+19.7 -14.3	+118.3 - 85.8	+189.1 - 83.5	+13.8 -13.8	+ 13.2 - 13.2	+216.1 -110.5	+ 173 - 88	+189
U0-L0	- 49.2	+ 14.5 - 34.6	+ 6.2 -29.5	+ 20.7 - 64.1	0.174 0.235	+ 3.6 -15.1	+ 24.3 - 79.2	- 10.1 -128.4	+ 9.2 - 9.2	0.0 -117.4	- 0.9 -255.0	- 204	-204
UI-LI	- 74.6	+ 9.6 - 40.5	+ 4.2 -29.8	+ 13.8 - 70.3	0.182 0.222	+ 2.5 -15.6	+ 16.3 - 85.9	- 35.9 -160.5	+10.2 -10.2	+ 9.2 - 9.2	- 16.5 -179.9	- 144	-161
U2-L2	- 75.2	+ 4.7 - 35.8	+ 2.1 -28.7	+ 6.8 - 64.5	0.182 0.222	+ 1.2 -14.3	+ 8.0 - 78.8	- 44.6 -154.0	+10.9 -10.9	+ 11.0 - 11.0	- 22.7 -175.9	- 141	-154
U3-L3	- 73.2	+ 3.4 - 33.8	+ 3.3 -25.4	+ 6.7 - 59.2	0.250 0.167	+ 1.7 - 9.9	+ 8.4 - 69.1	- 42.8 -142.3	+10.2 -10.2	+ 10.8 - 10.8	- 21.8 -163.3	- 131	-142
U4-L4	- 57.7	+ 10.4 - 34.8	+ 8.1 -19.6	+ 18.5 - 54.4	0.222 0.182	+ 4.1 - 9.9	+ 22.6 - 64.3	- 17.8 -122.0	+ 5.1 - 5.1	+ 6.3 - 6.3	- 5.4 -133.4	- 107	-122
U5-L5	- 41.8	- 18.3	-29.7	- 48.0	0.286	-13.7	- 61.7	- 41.8 -103.5	- 0.0 - 0.0	+ 1.4 - 1.4	- 40.4 -104.9	- 84	-104

Note: Stresses expressed in kips.
* Corrected for reversal.

TABLE 8

DESIGN SUMMARY

(Two-hinged Spandrel-braced Arch)

Member	Length (ft)	Design Stress (kips)	Allow. Stress (k/in²)	Area Req'd (in²)	Section	Assembly	Least r (in.)	Area Prv'd (in²)	Weight	
									lb/ft	lb
U0-UI	25.0	- 47	13.9	3.4	2 12 ⊔ 20.7		4.61	12.1	41.4	1040
UI-U2	25.0	-125	13.9	9.0	2 12 ⊔ 20.7		4.61	12.1	41.4	1040
U2-U3	25.0	-218	13.8	15.8	2 12 ⊔ 30		4.28	17.6	60.0	1500
U3-U4	25.0	-312	13.6	23.0	2 12 ⊔ 30 2 Pls. 10 x 5/16		3.96	23.8	81.2	2030
U4-U5	25.0	-350	13.5	25.9	2 12 ⊔ 30 2 Pls. 10 x 7/16		3.87	26.3	89.8	2250
L0-LI	30.8	-483	13.6	35.5	2 15 ⊔ 50 2 Pls. 12 x 5/16		4.92	36.8	125.6	3870
LI-L2	28.6	-429	13.8	31.1	2 15 ⊔ 40 2 Pls. 12 x 3/8		4.98	32.4	110.6	3160
L2-L3	26.9	-376	14.0	26.8	2 15 ⊔ 50		5.24	29.3	100.0	2690
L3-L4	25.7	-304	14.2	21.4	2 15 ⊔ 40		5.44	23.4	80.0	2060
L4-L5	25.1	-251	14.3	17.5	2 15 ⊔ 33.9		5.62	19.8	67.8	1700
U0-LI	48.9	+103	18.0	5.7	2 10 ⊔ 15.3		3.87	7.2	30.6	1500
UI-L2	37.5	+113	18.0	6.3	2 10 ⊔ 15.3		3.87	7.2	30.6	1150
U2-L3	30.8	+136	18.0	7.6	2 10 ⊔ 20		3.66	10.0	40.0	1230
U3-L4	27.7	+169	18.0	9.4	2 10 ⊔ 20		3.66	10.0	40.0	1110
U4-L5	26.9	+189	18.0	10.5	2 10 ⊔ 25		3.42	12.6	50.0	1350
U0-L0	60.0	-204	11.9	17.1	2 10 ⊔ 30		3.42	17.6	60.0	3600
UI-LI	42.0	-161	12.7	12.7	2 10 ⊔ 25		3.52	14.7	50.0	2100
U2-L2	28.0	-154	12.7	12.1	2 10 ⊔ 25		3.52	14.7	50.0	1400
U3-L3	18.0	-142	14.1	10.1	2 10 ⊔ 20		3.66	11.7	40.0	720
U4-L4	12.0	-122	14.7	8.3	2 10 ⊔ 15.3		3.87	8.9	30.6	370
U5-L5	10.0	-104	14.8	7.0	2 10 ⊔ 15.3		3.87	8.9	30.6	310

IV

STIFFNESS OF THE TWO ARCH STRUCTURES

In determining the stiffness of the two structures, their deflections under live load plus impact was considered. For both arches, the deflections of two points were determined, (1) U5 with the live load across the entire span, and (2) U3 with the live load across nearly half of the span. The live loading used was that previously determined (see Fig. 2) with the concentrated load acting at the points for which the deflection was being determined.

For the uniform live load across the entire span, the impact factor was $\frac{50}{125 + 250} = 0.133$ and the resulting live plus impact panel loads were as follows:

Uniform .... 18.3 x 1.133 = 20.7 kips

Concentrated: for chords ..... 20.6 x 1.133 = 23.3 kips

for webs .... 29.7 x 1.133 = 33.6 kips

For the uniform live load from L0 to halfway between U4 and U5 the impact factor was $\frac{50}{125 + 112} = 0.211$ and the resulting live plus impact panel loads were as follows:

Uniform .... 18.3 x 1.211 = 22.2 kips

Concentrated: for chords ..... 20.6 x 1.211 = 25.0 kips

for webs .... 29.7 x 1.211 = 36.0 kips

By virtual work, all deflections were determined as follows:

$$\Delta = \sum \frac{SuL}{AE}$$

where Δ = deflection of any point (in.).

S = stress in any member due to applied live plus impact loads (kips).

u = stress in any member due to a unit load at any point where the deflection is required.

The calculations for the deflections shown in Table 9 are included in Appendix C.

TABLE 9

COMPARISON OF DEFLECTIONS

	Full Loading $\Delta U5$ (in.)	Partial Loading $\Delta U3$ (in.)
Two-hinged arch	0.90	0.79
Three-hinged arch	1.28	1.01

V

EFFECTS OF LATERAL DISPLACEMENT OF SUPPORTS UPON THE TWO STRUCTURES

Lateral yielding of abutments of a three-hinged arch produce no stresses of any major importance so long as the over-all geometry of the structure does not change appreciably. On the other hand, for a two-hinged arch, small horizontal displacements of supports produce stresses which bear investigation.

The stress in any member of the two-hinged arch due to lateral yielding of supports equals Hu where

$$H = \frac{E\delta}{\sum \frac{u^2 L}{A}}$$

in which δ = increase in span due to a relative lateral movement of supports. (Fig. 7a)

u = stress in any member due to H = unity (Fig. 7b).

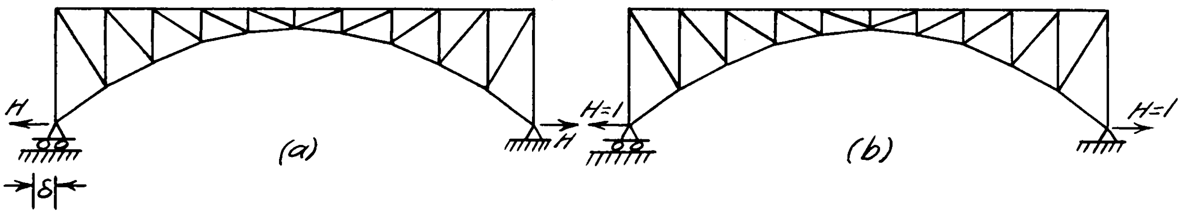


Fig. 7

For a relative lateral displacement of 0.25 in.

$$H = \frac{29,000 \times 0.25}{2 \times 1626} = 2.23 \text{ kips}$$

The resulting stresses are shown in Table 9.

TABLE 10

CALCULATION OF STRESSES CAUSED BY A RELATIVE LATERAL YIELDING OF SUPPORTS OF 0.25 INCHES

(Two-hinged Spandrel-braced Arch)

Member	$\frac{L}{A}$	u	$\frac{u^2L}{A}$	Hu (kips)
Uo-U1	24.9	-0.429	+ 4.6	- 0.97
U1-U2	24.9	-1.142	+ 32.6	- 2.55
U2-U3	17.1	-2.330	+ 92.9	- 5.20
U3-U4	12.6	-4.000	+202.0	- 8.92
U4-U5	11.4	-5.000	+285.0	-11.15
L0-L1	10.0	1.232	+ 15.2	+ 2.75
L1-L2	10.6	1.639	+ 28.5	+ 3.65
L2-L3	11.0	2.310	+ 58.7	+ 5.15
L3-L4	13.2	3.430	+155.1	+ 7.65
L4-L5	15.2	5.020	+384.0	+11.20
U0-L1	65.6	0.835	+ 45.8	+ 1.86
U1-L2	50.4	1.070	+ 57.6	+ 2.39
U2-L3	31.4	1.466	+ 67.6	+ 3.27
U3-L4	28.3	1.846	+ 96.6	+ 4.11
U4-L5	22.0	1.076	+ 25.5	+ 2.40
U0-L0	40.9	-0.720	+ 21.2	- 1.61
U1-L1	34.4	-0.798	+ 21.9	- 1.78
U2-L2	22.9	-0.857	+ 16.8	- 1.91
U3-L3	18.4	-0.800	+ 11.8	- 1.79
U4-L4	16.1	-0.400	+ 2.6	- 0.89
U5-L5	13.4	0	0	0
$\sum\frac{u^2L}{A}$ for half of span = +1626.0				

VI

SUMMARY AND CONCLUSIONS

Summary

The weights of the main members of the two structures are as follows:

Two-hinged arch .... 72,000 lb per truss

Three-hinged arch .... 65,900 lb per truss

Considering the weights of details and laterals to vary in the same ratio as the main members of the respective arch trusses, it was found that the three-hinged arch was lighter than the two-hinged arch by 8.5%.

The maximum deflection for each arch structure was found at the center with the live load across the entire span. They compare as follows:

Two-hinged arch .... 0.90 in.

Three-hinged arch .... 1.28 in.

An inspection of Tables 8 and 10 indicates that for the two-hinged arch, a relative lateral yielding of supports of small magnitude (0.25 in.) produces no critical stresses, whereas, a lateral displacement of larger magnitude would over-stress upper chord and diagonal members.

Conclusions

1. From the standpoint of weight of the main members, the three-hinged arch is more suitable.

2. On the basis of stiffness, the two-hinged arch is the stiffer of the two arches. However, their maximum deflections are about the same order of magnitude and the slight difference is of no major importance.

3. Neither the two-hinged nor the three-hinged arch is stressed critically by a slight lateral yielding of abutments.

4. The three-hinged spandrel-braced arch highway bridge is shown to be more satisfactory for the configuration investigated.

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APPENDIX A

TABLE 1-A

NOTABLE SPANDREL-BRACED ARCH BRIDGES

Location	Span (ft)	Type of Traffic	No. of Hinges	Year Completed	Reference *
Marble Canyon, Ariz.	616	Hy.	2	1928	1
St. John, N.B., Canada	560	Hy.	2	1916	2
Niagara, G.T. Ry.	550	Ry.	2	1897	3
Zambesi River, Rhodesia, Africa	500	Ry.	2	1905	4
Lake Street, Minneapolis, Minn.	456	Hy.	3	1888	5
Frazer River, Canada	425	Ry.	3	1915	6
Driving Park, Rochester, N.Y.	416	Hy.	3	1890	7
Ft. Snelling, Minn.	364	Hy.	3	1896	8
Szegedin, Hungary	361	Hy.	3	1884	9
Panther Hollow, Pittsburgh, Pa.	360	Hy.	2	1896	7
St. Croix, Minn.	350	Hy.	3	1910	9
Crooked River, Ore.	340	Hy.	2	1913	10
Bonn, Germany	307	Hy.	2	1899	11
Surprise Creek, Canada	291	Ry.	3	1898	7
Cambridge, New Zealand	290	Hy.	3	1905	9
Salmon River, B.C., Canada	270	Ry.	3	1893	12
Spokane, Washington	255	Ry.	3	1904	9
Iron Mountain, Mich.	207	Ry.	3	1903	13
Fairmount Park, Philadelphia, Pa.	200	Hy.& Ry.	3	1898	14
St. Guistiana, Austria	197	Hy.	2	1889	9
Woodstock, Vt.	188	Ry.	3	1911	9

* References are listed on the following page.

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APPENDIX B

CALCULATIONS OF WIND LOADS ON LEEWARD TRUSS

Lateral Loads

The lateral wind forces specified by the A.A.S.H.O. are stated in part as follows:

(1) 30 pounds per square foot on $1\frac{1}{2}$ times the projected area of the structure as seen in elevation.

(2) 200 pounds per linear foot on live road to be applied 6 feet above the roadway.

(3) The minimum lateral force to be 300 pounds per linear foot on the loaded chord and 150 pounds per linear foot on the unloaded chord.

For estimating lateral wind loads, the following assumptions were made:

(1) Depth of chords .... 18 in.

(2) Depth of webs .... 12 in.

(3) Depth of roadway (curb and stringer) .... 18 in.

(4) Depth of hand rail .... 6 in.

(5) Distance from center of floor beam to crown of roadway .... 24 in.

The lateral wind concentration at any lower panel point from each member intersecting at the point was

$$\frac{L}{2} \times \frac{w}{12} \times 30 \times 1.5 = 1.875\ Lw$$

where: L = length of member (ft); w = width of member (in.).

Denoting chord members by (c), vertical members by (v), and diagonal members by (d), the lower panel point concentrations were determined as follows:

L0	(c)	1.875 x 30.8 x 18	=	1040
	(v)	1.875 x 60 x 12	=	1350
				2390 lb
L1	(c)	1.875 (28.7 + 30.8) 18	=	2010
	(v)	1.875 x 42 x 12	=	950
	(d)	1.875 x 48.8 x 12	=	1100
				4060 lb
L2	(c)	1.875 (28.7 + 26.9) 18	=	1880
	(v)	1.875 x 28 x 12	=	630
	(d)	1.875 x 27.5 x 12	=	840
				3350 lb
L3	(c)	1.875 (26.9 + 25.7) 18	=	1770
	(v)	1.875 x 18 x 12	=	410
	(d)	1.875 x 30.8 x 12	=	690
				2870 Lb
L4	(c)	1.875 (25.7 + 25.1) 18	=	1720
	(v)	1.875 x 12 x 12	=	270
	(d)	1.875 x 27.7 x 12	=	620
				2560 lb
L5	(c)	1.875 x 25.1 x 18	=	850
	(v)	1.875 x 10 x 12	=	230
	(d)	1.875 x 26.9 x 12	=	610
				1690 lb

Total for half of truss = 16,920 lb

Lateral force per ft of span on unloaded chord = $\frac{16,920}{125}$ = 135 lb; therefore, the minimum force of 150 lb per ft was used, making the loads for interior panel points 150 x 25 = 3,750 lb, and for exterior panel points 150 x 12.5 = 1,875 lb.

For lateral wind concentrations on the upper panel points, the wind force was included on:

live load	200 x 25	=	5,000 lb
road way	1.875 x 50 x 18	=	1,690 lb
hand rail	1.875 x 50 x 6	=	560 lb
chord	1.875 x 50 x 18	=	1,690 lb
verticals } varies with joints			
diagonals }			
			8,940 lb +

The total for each of the upper panel points was calculated as follows:

U0				4,470
	(v)	1.875 x 60 x 12	=	1,350
	(d)	1.875 x 48.9 x 12	=	1,100
				6,920 lb
U1				8,940
	(v)	1.875 x 42 x 12	=	940
	(d)	1.875 x 37.5 x 12	=	840
				10,730 lb
U2				8,940
	(v)	1.875 x 28 x 12	=	630
	(d)	1.875 x 30.8 x 12	=	690
				10,250 lb
U3				8,940
	(v)	1.875 x 18 x 12	=	410
	(d)	1.875 x 27.7 x 12	=	620
				9,970 lb
U4				8,940
	(v)	1.875 x 12 x 12	=	270
	(d)	1.875 x 26.9 x 12	=	610
				9,820 lb
U5				4,470
	(v)	1.875 x 10 x 12	=	230
				4,700 lb

Total force on half of truss = 52,400 lb

Lateral force per foot of span = $\frac{52,400}{125}$ = 420 lb

Since this was greater than the 300-lb minimum, the above calculated loads were used.

Vertical Loads at Upper Panel Points

The overturning effect of the wind on the live load was resolved into a couple acting on the upper panel points as shown in Fig. 1-B.

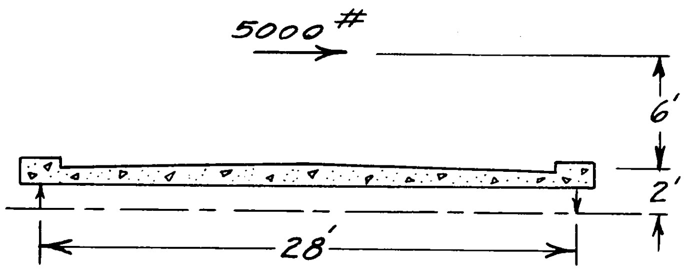

Fig. 1-B

Wind on Live Load

Vertically downward loads on the leeward truss at U2, U3, and U4 were 500 x 8/28 = 1,430 lb and at U1 and U5, 715 lb.

Lateral wind concentrations (w), at the upper panel points, acting through the transverse bracing, have an effect on the leeward truss equivalent to that of vertically downward forces applied to the upper panel points (see Fig. 2-B).

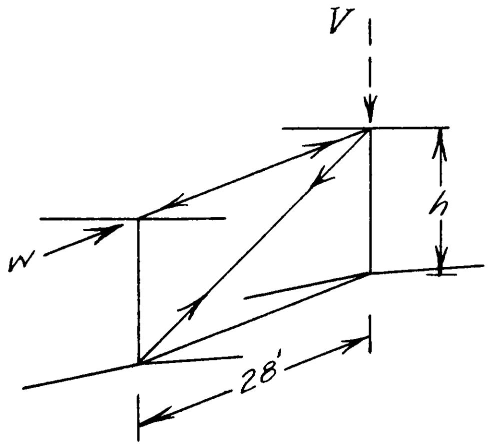

Fig. 2-B

A Typical Transverse Bracing Section

These equivalent vertical forces (V), equal in magnitude to the vertical stress components of the transverse tension diagonals, were determined by:

$$V = \frac{wh}{28}$$

where: h = depth of truss at any panel point

28 = distance between trusses

For the various panel points, V was determined as follows:

U0	6920 x 60/28	=	14,800 lb
U1	10,730 x 42/28	=	16,100 lb
U2	10,260 x 28/28	=	10,260 lb
U3	8,970 x 18/28	=	6,400 lb
U4	9,820 x 12/28	=	4,210 lb
U5	4,700 x 10/28	=	1,680 lb

The sum of the above two vertical effects is expressed in kips and shown in Fig. 3, Page 10.

Vertical and Horizontal Loads at Lower Panel Points

The loads on the lower lateral system were the sum of the upper and lower panel point wind concentrations. (Fig. 3-B)

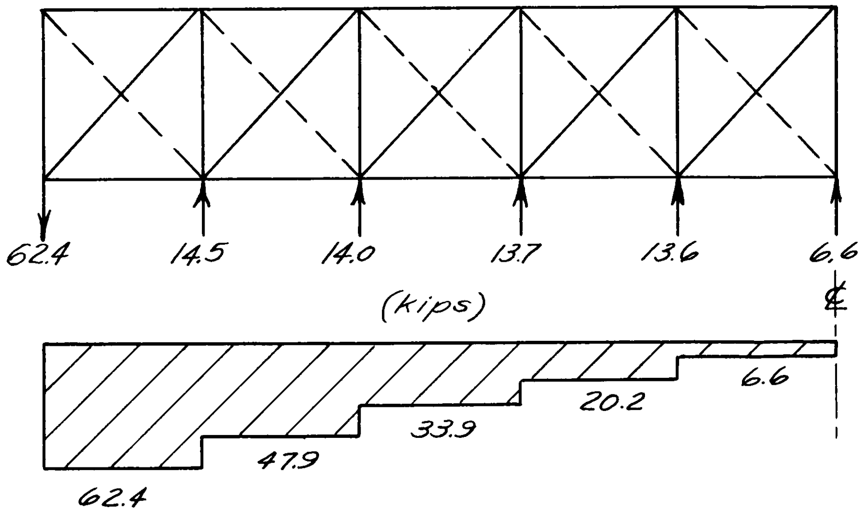

Fig. 3-B

Loading and Shear Diagrams for Lower Lateral System

The shear in the lower laterals was taken by the tension diagonals. Since these diagonals were in inclined planes, they had vertical and horizontal stress components which acted upon the arch trusses as vertical and horizontal loads.

The vertical component at any panel point was

$$\text{panel shear} \times \frac{\text{vertical projection of diagonal length}}{\text{truss spacing}}$$

and for the various points was determined as follows:

L1	62.4 x 18/28 =	40.1 kips
L2	47.9 x 14/28 =	24.0 kips
L3	33.9 x 10/28 =	12.1 kips
L4	20.2 x 6/28 =	4.3 kips
L5	6.6 x 2/28 =	0.5 kips

The horizontal component at any panel point was

$$\text{panel shear} \times \frac{\text{panel length}}{\text{truss spacing}}$$

and for the various points was determined as follows:

L1 62.4 x 25/28 = 55.7 kips

L2 47.9 x 25/28 = 42.8 kips

L3 33.9 x 25/28 = 30.3 kips

L4 20.2 x 25/28 = 18.1 kips

L5 6.6 x 25/28 = 5.9 kips

The above calculated loads are shown on the leeward truss in Fig. 3, Page 10.

APPENDIX C

CALCULATIONS OF DEFLECTIONS

TABLE 1-C

DEFLECTION OF U5 BY VIRTUAL WORK
(Three-hinged Spandrel-braced Arch)

Member	S			u	$\frac{L}{A}$	$\frac{SuL}{A}$
	Unif.	Conc.	Total			
U0-U1	0	+ 5.6	+ 5.6	+0.238	38.8	+ 52
U1-U2	0	+12.5	+ 12.5	+0.536	38.8	+ 260
U2-U3	0	+19.4	+ 19.4	+0.833	38.8	+ 626
U3-U4	0	+19.4	+ 19.4	+0.833	38.8	+ 626
U4-U5	0	0	0	0	38.8	0
L0-L1	-159.4	-35.9	-195.3	-1.540	8.2	+2450
L1-L2	-148.0	-39.7	-187.7	-1.702	8.6	+2750
L2-L3	-139.1	-44.9	-184.0	-1.921	8.6	+3040
L3-L4	-133.0	-50.0	-183.0	-2.140	8.6	+3380
L4-L5	-130.0	-48.7	-178.7	-2.090	8.5	+3180
U0-L1	0	-15.8	- 15.8	-0.466	75.5	+ 556
U1-L2	0	-15.0	- 15.0	-0.447	57.8	+ 387
U2-L3	0	-12.3	- 12.3	-0.366	47.5	+ 214
U3-L4	0	0	0	0	42.7	0
U4-L5	0	+30.1	+ 30.1	+0.896	27.7	+ 746
U0-L0	- 10.4	+13.4	+ 3.0	+0.400	61.4	+ 74
U1-L1	- 20.7	+11.2	- 9.5	+0.333	43.0	- 136
U2-L2	- 20.7	+ 7.2	- 13.5	+0.214	27.6	- 80
U3-L3	- 20.7	0	- 20.7	0	27.8	0
U4-L4	- 20.7	-11.2	- 31.9	-0.333	18.5	+ 196
U5-L5	- 10.4	-16.8	- 27.2	-0.500	15.4	+ 210

$$\sum \frac{SuL}{A} \text{ for half of span } = +18{,}531$$

$$\Delta U5 = \frac{2 \times 18{,}531}{29{,}000} = 1.28 \text{ in.}$$

TABLE 2-C

DEFLECTION OF U3 BY VIRTUAL WORK
(Three-hinged Spandrel-braced Arch)

Member	S			u	$\frac{L}{A}$	$\frac{SuL}{A}$
	Unif.	Conc.	Total			
U0-U1	-15.9	- 2.4	- 18.3	-0.095	38.8	+ 67
U1-U2	-35.6	- 9.8	- 45.4	-0.393	38.8	+ 693
U2-U3	-55.5	-29.1	- 84.6	-1.166	38.8	+3820
U3-U4	-55.5	-18.7	- 75.2	-0.750	38.8	+2190
U4-U5	0	0	0	0	38.8	0
L0-L1	-68.4	-23.1	- 91.5	-0.924	8.2	+ 693
L1-L2	-45.3	-18.7	- 74.0	-0.750	8.6	+ 477
L2-L3	-21.3	- 9.6	- 30.9	-0.384	8.6	+ 102
L3-L4	0	+10.7	+ 10.7	+0.428	8.6	+ 40
L4-L5	0	0	0	0	8.5	0
U0-L1	+31.0	+ 6.7	+ 37.7	+0.186	75.5	+ 530
U1-L1	+29.8	+16.1	+ 45.9	+0.447	57.8	+1187
U2-L3	+24.5	+34.4	+ 58.9	+0.954	47.5	+2670
U3-L4	0	-16.6	- 16.6	-0.462	42.7	+ 328
U4-L5	-59.7	-29.1	- 88.8	-0.807	27.5	+1970
U0-L0	-48.8	- 5.8	- 54.6	-0.160	61.4	+ 536
U1-L1	-44.4	-12.0	- 56.4	-0.334	43.0	+ 810
U2-L2	-36.4	-20.1	- 56.5	-0.557	27.6	+ 869
U3-L3	-22.2	-28.8	- 51.0	-0.800	27.8	+1133
U4-L4	0	+10.8	+ 10.8	+0.300	18.5	+ 60
U5-L5	0	0	0	0	15.4	0
U6-L6	-14.8	- 7.2	- 32.0	-0.200	18.5	+ 118
U7-L7	0	0	0	0	27.8	0
U8-L8	+ 9.5	+ 4.6	+ 14.1	+0.129	27.6	+ 50
U9-L9	+14.8	+ 7.2	+ 22.0	+0.200	43.0	+ 189
U10-L10	+17.8	+ 8.6	+ 26.4	+0.240	61.4	+ 389
U6-L5	+39.8	+19.4	+ 59.2	+0.538	27.5	+ 876
U7-L6	0	0	0	0	42.7	0
U8-L7	-16.3	- 8.0	- 24.3	-0.221	27.5	+ 255
U9-L8	-19.8	- 9.6	- 29.4	-0.268	57.8	+ 455
U10-L9	-20.7	-10.1	- 30.8	-0.280	75.5	+ 650
L5-L6	-92.6	-31.4	-124.0	-1.254	8.53	+1328
L6-L7	-95.0	-32.1	-127.1	-1.284	8.64	+1410
L7-L8	-85.2	-28.8	-114.0	-1.153	8.60	+1310
L8-L9	-75.5	-25.6	-101.1	-1.021	8.62	+ 890
L9-L10	-68.4	-23.1	- 91.5	-0.924	8.15	+ 689
U5-U6	0	0	0	0	38.8	0
U6-U7	+37.0	+12.5	+ 49.5	+0.500	38.8	+ 960
U7-U8	+37.0	+12.5	+ 49.5	+0.500	38.8	+ 960
U8-U9	+23.8	+ 8.0	+ 31.8	+0.322	38.8	+ 497
U9-U10	+10.6	+ 3.6	+ 24.2	+0.143	38.8	+ 134

$$\sum \frac{SuL}{A} = +\ 29{,}335$$

$$\Delta U3 = \frac{29{,}335}{29{,}000} = 1.01 \text{ in.}$$

TABLE 3-C

DEFLECTION OF U5 BY VIRTUAL WORK
(Two-hinged Spandrel-braced Arch)

Member	S			u	$\frac{L}{A}$	$\frac{SuL}{A}$
	Unif.	Conc.	Total			
U0-U1	- 7.3	+ 2.2	- 5.1	+0.096	24.9	- 12
U1-U2	- 19.7	+ 3.6	- 16.1	+0.155	24.9	- 62
U2-U3	- 40.0	+ 1.4	- 38.6	+0.058	17.1	- 38
U3-U4	- 68.1	-11.5	- 79.6	-0.493	12.6	+ 495
U4-U5	- 85.5	-38.7	-124.2	-1.660	11.4	+2350
L0-L1	-138.5	-26.4	-164.9	-1.131	10.0	+1865
L1-L2	-120.5	-27.0	-147.5	-1.161	10.6	+1818
L2-L3	-100.8	-27.0	-127.8	-1.160	11.0	+1631
L3-L4	- 75.2	-23.5	- 98.7	-1.010	13.2	+1316
L4-L5	- 45.0	-10.0	- 55.0	-0.430	15.2	+ 360
U0-L1	+ 14.3	- 6.2	+ 8.1	-0.186	65.6	- 99
U1-L2	+ 18.7	- 3.0	+ 15.7	-0.087	50.4	- 69
U2-L3	+ 24.8	+ 3.9	+ 28.7	+0.116	31.5	+ 105
U3-L4	+ 31.8	+20.8	+ 52.6	+0.618	28.3	+ 920
U4-L5	+ 18.8	+42.5	+ 61.3	+1.268	22.0	+1710
U0-L0	- 22.9	+ 5.4	- 17.5	+0.160	40.9	- 114
U1-L1	- 35.0	+ 2.1	- 32.9	+0.063	34.4	- 71
U2-L2	- 35.2	- 2.3	- 37.5	-0.068	22.9	+ 58
U3-L3	- 34.6	- 8.9	- 43.5	-0.266	18.4	+ 213
U4-L4	- 27.6	-15.7	- 43.3	-0.468	16.1	+ 326
U5-L5	- 20.7	-33.6	- 54.3	-0.500	13.4	+ 364

$$\sum \frac{SuL}{A} \text{ for half of span} = +13{,}066$$

$$\Delta U5 = \frac{2 \times 13{,}066}{29{,}000} = 0.90 \text{ in.}$$

TABLE 4-C

DEFLECTION OF U3 BY VIRTUAL WORK
(Two-hinged Spandrel-braced Arch)

Member	S			u	L/A	SuL/A
	Unif.	Conc.	Total			
U0-U1	-18.1	- 3.1	- 21.2	-0.124	24.9	+ 65
U1-U2	-42.0	-11.8	- 53.8	-0.472	24.9	+ 633
U2-U3	-68.5	-33.3	-101.8	-1.332	17.1	+2320
U3-U4	-77.3	-25.6	-102.9	-1.026	12.6	+1330
U4-U5	-27.4	- 8.8	- 36.2	-0.350	11.4	+ 144
L0-L1	-61.7	-21.0	- 82.7	-0.840	10.0	+ 695
L1-L2	-36.5	-15.9	- 52.4	-0.638	10.6	+ 354
L2-L3	- 8.8	- 5.6	- 14.4	-0.224	11.0	+ 35
L3-L4	+18.6	+16.5	+ 35.1	+0.661	13.2	+ 306
L4-L5	+27.1	+ 8.6	+ 35.7	+0.342	15.2	+ 185
U0-L1	+35.6	+ 8.8	+ 44.4	+0.244	65.5	+ 711
U1-L2	+35.8	+18.8	+ 54.6	+0.521	50.4	+1433
U2-L3	-32.5	+38.0	+ 70.5	+1.056	31.5	+2340
U3-L4	+10.1	-12.1	- 2.0	-0.336	28.3	+ 19
U4-L5	-53.9	-26.3	- 80.2	-0.732	22.0	+1293
U0-L0	-52.9	- 7.6	- 60.5	-0.210	40.9	+ 520
U1-L1	-49.0	-14.1	- 63.1	-0.392	34.4	+ 850
U2-L2	-41.4	-22.2	- 63.6	-0.617	22.9	+ 900
U3-L3	-26.5	-30.7	- 57.2	-0.854	18.4	+ 898
U4-L4	- 2.2	+ 9.8	+ 7.6	+0.272	16.1	+ 33
U5-L5	0	0	0	0	13.4	0
U6-L6	-17.0	- 8.3	- 25.3	-0.229	16.1	+ 93
U7-L7	- 4.4	- 2.0	- 6.4	-0.055	18.4	+ 6
U8-L8	+ 5.0	+ 2.6	+ 7.6	+0.071	22.9	+ 12
U9-L9	+10.1	+ 5.1	+ 15.2	+0.141	34.4	+ 74
U10-L10	+13.8	+ 6.8	+ 20.6	+0.190	40.9	+ 162
U6-L5	+45.8	+22.2	+ 68.0	+0.615	22.0	+ 919
U7-L6	+10.3	+ 4.6	+ 14.9	+0.129	28.3	+ 54
U8-L7	- 8.5	- 4.5	- 13.0	-0.123	31.5	+ 50
U9-L8	-13.8	- 6.9	- 20.7	-0.192	50.4	+ 200
U10-L9	-17.1	- 8.0	-24.00	-0.221	65.6	+ 348
L5-L6	-65.7	-22.8	- 88.5	-0.912	15.2	+1228
L6-L7	-76.5	-26.4	-102.9	-1.055	13.2	+1430
L7-L8	-72.8	-24.9	- 97.7	-0.995	11.0	+1070
L8-L9	-66.8	-22.8	- 89.6	-0.911	10.6	+ 866
L9-L10	-61.7	-21.0	- 82.7	-0.840	10.0	+ 695
U5-U6	-27.4	- 8.8	- 36.2	-0.350	11.4	+ 144
U6-U7	+15.2	+ 5.6	+ 20.8	+0.225	12.6	+ 59
U7-U8	+24.2	+ 8.5	+ 32.7	+0.339	17.1	+ 190
U8-U9	+17.4	+ 6.0	+ 23.1	+0.241	24.9	+ 141
U9-U10	+ 8.2	+ 2.8	+ 11.0	+0.114	24.9	+ 31

$$\sum \frac{SuL}{A} = + 22,836$$

$$\Delta U3 = \frac{22,836}{29,000} = 0.79 \text{ in.}$$

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