A DETERMINATION OF THE ACCELERATION DUE TO GRAVITY AT BLACKSBURG, VIRGINIA

by

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Blacksburg, Virginia

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INTRODUCTION

The purpose of the project herein described was to determine the acceleration due to gravity for the locale of Blacksburg, Virginia, to five significant figures. As far as the author has been able to determine, there has never been a formal determination of "g" made for this area.

Supporting this prime purpose was the fact that the equipment required for this experiment, i.e. a secondary standard of length, a recording system, and several portable photocell relay circuits, and the vacuum tank, would be useful to the department in the future.

Plan of the project

The project was located in Room 106, Davidson Hall, on the campus of the Virginia Polytechnic Institute. The elevation of the knife edge was 2037.39 ft.. A Coast and Geodetic Survey bench mark is located on the first step on the east side of Patton Hall at an elevation of 2057.13 ft.. It was from this mark that the Civil Engineering Department established as a secondary reference the first floor elevation of Davidson Hall which was used in determining the elevation of the knife edge. The geographic location of a point between the Lutheran church and the V.P.I. barracks is North latitude 37° 13' 54.456" and West longitude 80° 25' 10.961". The exact location of this point is not known. The monument was destroyed during recent construction in that area.

It is generally accepted that an experiment employing a Kater's type pendulum is one of the better methods for making an accurate determination of "g". For this reason such a pendulum was used in this experiment; however, in designing the equipment the following considerations were kept in mind:

1. The pendulum should have a small coefficient of expansion.
2. Friction at the point of support should be kept at a minimum.
3. The pendulum should vibrate in a vacuum in order to reduce the damping effect of the air on the pendulum.
4. The data to be used in the determination of the period of the pendulum should be recorded automatically.
5. The amplitude of vibration should be known.

On the other hand, corrections for such phenomena as the motion of the support, the elongation of the pendulum due to its own weight, flexure of the pendulum, and the general dissipation of the energy of the pendulum while in motion in the vacuum tank have been disregarded. A review of the literature indicated that these are second-order effects and sufficiently small as to be negligible in this determination.

THE THEORY OF KATER'S REVERSIBLE PENDULUM

Any rigid object which is hinged about a horizontal axis not passing through its center of gravity, and acted upon by its own weight as its restoring force when displaced through an angle θ from its equilibrium position, is defined as a compound pendulum. However, the equation of motion for such a pendulum requires knowledge of its moment of inertia about its axis of suspension.

In 1818 Captain Henry Kater (9) eliminated this difficulty by devising a pendulum with two axes of suspension from which it could be suspended alternately. He showed that when these axes were adjusted so that the period of vibration of the pendulum was the same for both, the distance between the axes was equal to the length of an equivalent simple pendulum.

A simple pendulum is defined as a heavy particle suspended by a weightless thread. If such a pendulum vibrates under the action of gravity alone, its equation of motion is

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\sin\theta \qquad (1)$$

where $\frac{d^2\theta}{dt^2}$ is the angular acceleration

g is the acceleration due to gravity
l is the length of the pendulum
θ is the angular displacement of the pendulum from its equilibrium position

Integration of this equation gives for the time, t, of a complete vibration of the pendulum,

$$* \; t = 2\pi\sqrt{\frac{l}{g}}\left(1 + \frac{1}{4}\sin^2\frac{\theta}{2} + \frac{9}{64}\sin^4\frac{\theta}{2} + \ldots\right) \qquad (2)$$

For the ideal case of amplitude, $\theta = 0$:

$$t = 2\pi\sqrt{\frac{l}{g}} \qquad (2a)$$

In Figure 1, let h_1 and h_2 be the distances from the points of suspensions, s_1 and s_2 respectively, to the center of gravity, C of G. Then, when suspended from s_1 and pulled to one side through an angle θ, the torque which will cause it to swing toward its position of stable equilibrium, i.e. the restoring torque, will be

$$\mathcal{T} = I\frac{d^2\theta}{dt^2} = -Wh_1 \sin\theta \qquad (3)$$

where $\underline{I}$ is its moment of inertia about $\underline{s_1}$ as its axis, and $\underline{W}$ as its weight.

However, LaGrange's equation for the moment of inertia of a body about any axis parallel to an axis through the center of gravity is

$$I_A = I_G + Md^2 \qquad (4)$$

where I_A = Moment of Inertia about the new axis
I_G = Moment of Inertia about the center of gravity
M = Mass of the body
d = Distance between new axis and the center of gravity

* The derivation of this equation can be found in <u>Higher Mathematics for Engineers and Physicists</u>, I.S. and E.S. Sokolnikoff. McGraw Hill Book Company, New York, N.Y. 1933.

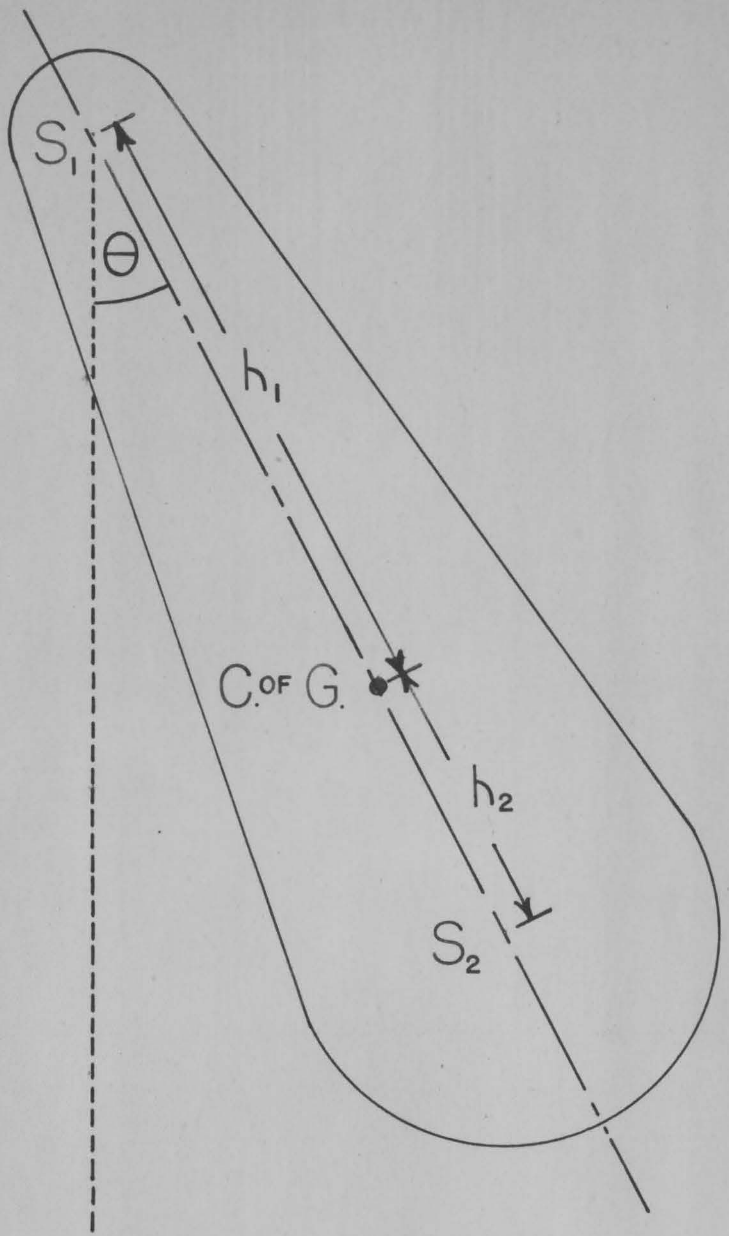

FIG. I SCHEMATIC DIAGRAM OF A KATER'S PENDULUM

But $I_G = MR^2$ where R is the radius of gyration about the center of gravity. Therefore, equation (3) becomes

$$M\,(R^2 + h_1^{\,2})\,\frac{d^2\theta}{dt^2} = -Wh_1 \sin\theta \tag{5}$$

$$\frac{d^2\theta}{dt^2} = -\,\frac{Wh_1 \sin\theta}{M\,(R^2 + h_1^{\,2})} \tag{6}$$

In absolute units, $W = mg$, and hence equation (6) becomes

$$\frac{d^2\theta}{dt^2} = -\,\frac{g}{(h_1 + \frac{R^2}{h_1})}\,\sin\theta \tag{7}$$

By comparing the corresponding equation for a simple pendulum,

$$\frac{d^2\theta}{dt^2} = -\,\frac{g}{l}\,\sin\theta \tag{8}$$

we note that if $\underline{l}$ is equal to $h_1 + \frac{R^2}{h_1}$ then the two pendulums will have the same period of vibration.

By the same process, we can arrive at a similar statement for the equation of motion for the compound pendulum about its other axis of suspension S_2. Hence

$$\frac{d^2\theta}{dt^2} = -\,\frac{g}{(h_2 + \frac{R^2}{h_2})}\,\sin\theta \tag{9}$$

where again if $(h_2 + \frac{R^2}{h_2})$ is equal to the length of the equivalent simple pendulum, their periods will be equal.

Consequently, we have the two equalities

$$l = h_1 + \frac{R^2}{h_1} \tag{10}$$

and

$$l = h_2 + \frac{R^2}{h_2} \tag{11}$$

By eliminating R we can arrive at

$$l = h_1 + h_2 \tag{12}$$

Consequently we have demonstrated that the distance between the two axes of suspension, $h_1 + h_2$, is equal to the length of an equivalent simple pendulum. Therefore, we can write equation (1) as

$$t_1 = 2\pi \sqrt{\frac{h_1^2 + R^2}{h_1\, g}} \tag{13}$$

and also

$$t_2 = 2\pi \sqrt{\frac{h_2^2 + R^2}{h_2\, g}} \tag{14}$$

provided that amplitude of vibration for the pendulum is small.

Squaring and subtracting equations (13) and (14), we have

$$h_2 g t_2^2 - h_1 g t_1^2 = 4\pi^2(h_2^2 + R^2 - h_1^2 - R^2) \tag{15}$$

or

$$\frac{4\pi^2}{g} = \frac{h_2 t_2^2 - h_1 t_1^2}{h_2^2 - h_1^2} \tag{16}$$

Upon expanding the right-hand side of this equation into partial fractions we arrive at our final equation

$$\frac{4\pi^2}{g} = \frac{t_1^2 + t_2^2}{2(h_1 + h_2)} + \frac{t_1^2 - t_2^2}{2(h_1 - h_2)} \qquad (17)$$

If t_1 were to equal t_2 then this term would become zero and our equation would revert to the form of (2a)

$$t = 2\pi\sqrt{\frac{l}{g}}$$

which is the equationfor the period of a simple pendulum.

However, if the apparatus is adjusted so that t_1 and t_2 are very nearly equal, and yet a considerable difference is maintained between h_1 and h_2, then the second term becomes a correction term of relatively small magnitude.

As a consequence of this second term becoming such a correctinn term, the distances h_1 and h_2 need not be determined beyond the accuracy of three significant places. Instead, the accuracy of the experiment is determined by the length, $l = h_1 + h_2$, of the pendulum and the periods, t_1 and t_2.

APPARATUS

The Pendulum

The pendulum used was a metal bar, practically straight, with an axis of support at each of its two conjugate points.

The bar of the pendulum was made of Gamma metal, an alloy manufactured by the Midvale Steel Corporation, whose coefficient of linear expansion was taken as 1.0×10^{-6} per °C. The bar itself was 131.5 cm. long and had a cross-sectional area of 1.9 cm. x 0.6 cm..

Plate glass planes were attached to the pendulum by means of brackets located approximately at the conjugate points of the pendulum. These planes supported the pendulum as it vibrated back and forth on a knife edge. They were made of two 2-inch sections of 2" x 1/4" angle iron which were slotted and bolted back to back. The result was a T-shaped bracket containing an opening into which the stem of the pendulum could be press-fit. See Figure 2a.

These two brackets were fitted to the bar of the pendulum facing one another. To their larger faces, and on either side of the stem, were cemented the glass planes. This was accomplished in such a manner, to be described later, that the planes on each bracket were approximately perpendicular to the center line of the pendulum and in a plane with each other. See Figure 2b.

Also attached to the stem of the pendulum were two solid cylindrically shaped cast iron bobs. One of these bobs was 4.3 cm. in diameter and 5.4

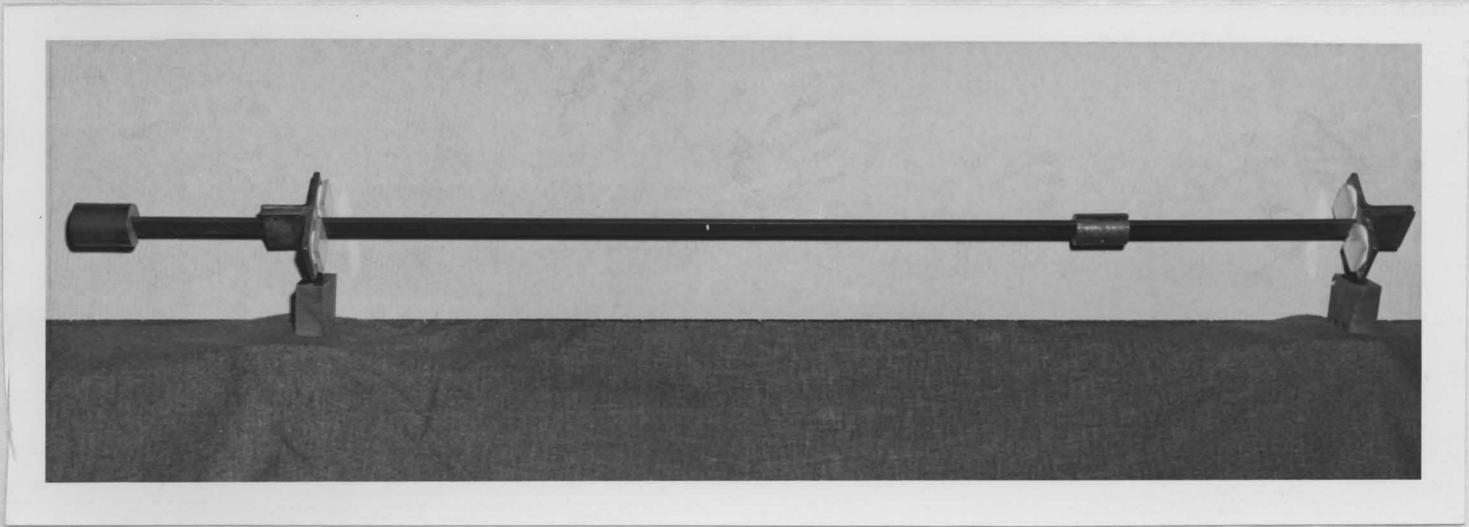

Fig. 2a Photograph of pendulum

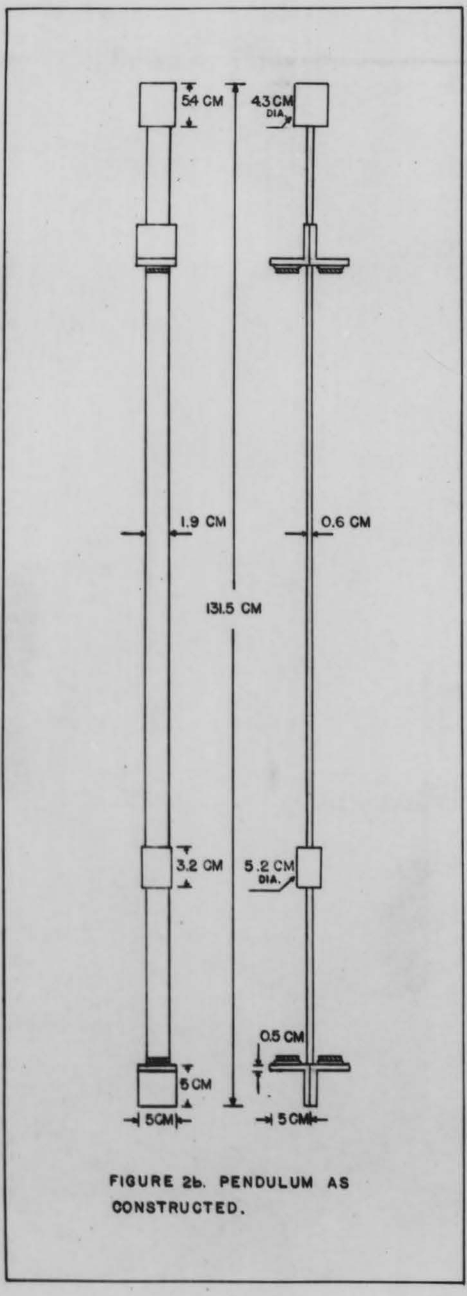

Fig. 2b Pendulum as constructed

cm. long, and was placed at one end of the pendulum. Its purpose was to destroy the symmetry of the straight bar, i.e. to make h_1 not equal to h_2. The second bob, smaller than the first, was 5.2 cm. in diameter and 3.2 cm. long, and it was positioned between the brackets. This bob served as an adjustment for bringing t_1 and t_2 to near equality.

In order to attach the plate glass planes to each bracket with the previously mentioned restrictions, a wooden rack was built. This supported a flat steel disc 16 inches in diameter and one inch thick. Near the center of this disc a hole was cut of the size and shape shown in Figure 3. This opening permitted the pendulum to be passed through it freely. The disc rested upon three bolts in the rack, which served as leveling screws. On the upper surface of this disc, which had been machined flat, were placed the glass plates, one on either side of the slotted portion of the hole in the disc. A sufficient quantity of thickened plaster of Paris was then sandwiched between the faces of the brackets and these glass plates and the pendulum was allowed to settle in a vertical position. A system of guides and a plumb line were used for this vertical adjustment. See Figure 4.

The Vacuum Tank and Supporting Apparatus

For simplicity, the vacuum tank was designed as a single unit consisting of the three main parts, the lower portion, the supporting shelf, and the cap. The assembled tank and its associated equipment is shown in block-diagram form in Figure 5.

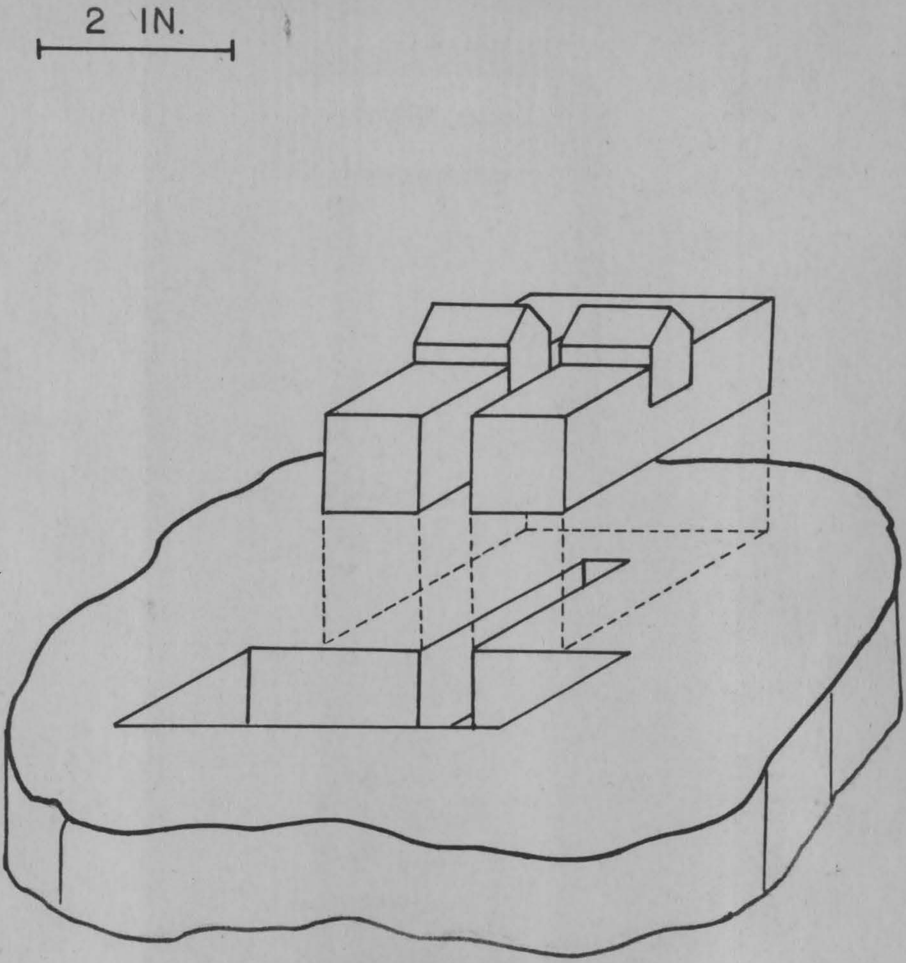

FIGURE 3 KNIFE EDGE ASSEMBLY SHOWING POSITION OVER HOLE IN SUPPORTING PLATE

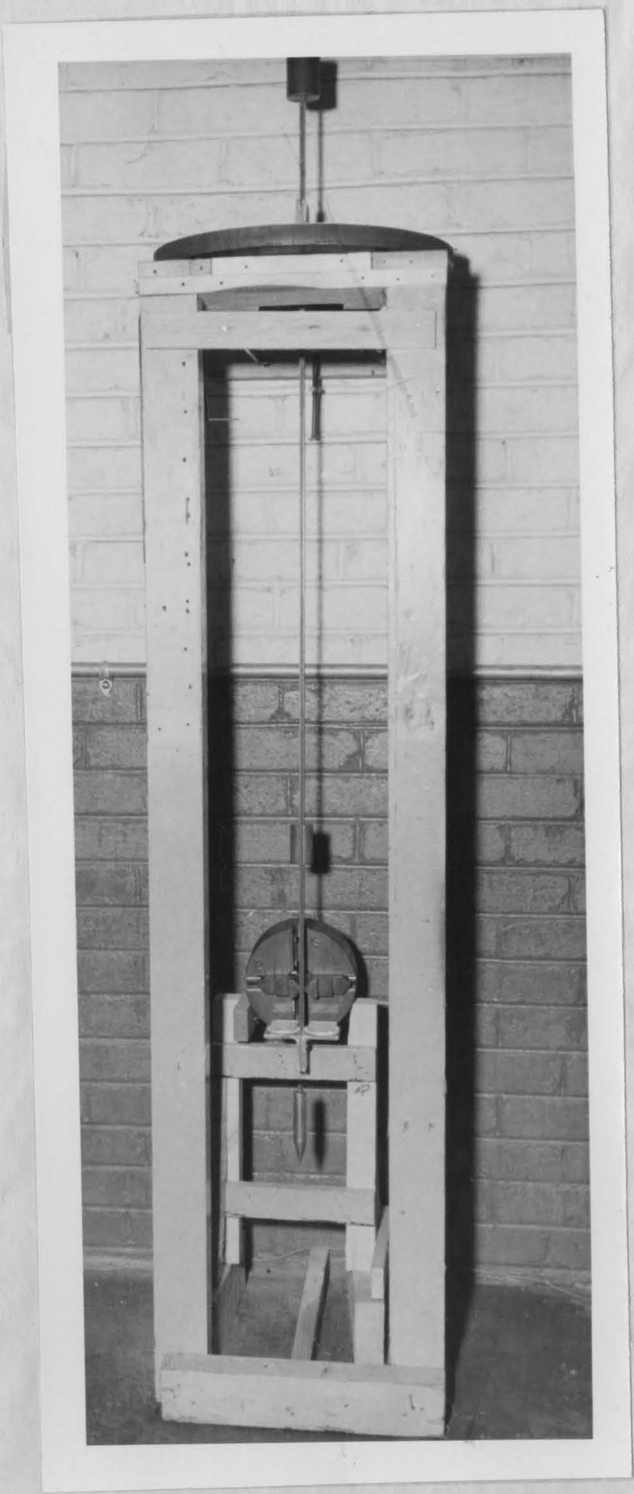

Fig. 4 Photograph of auxiliary assembly rack

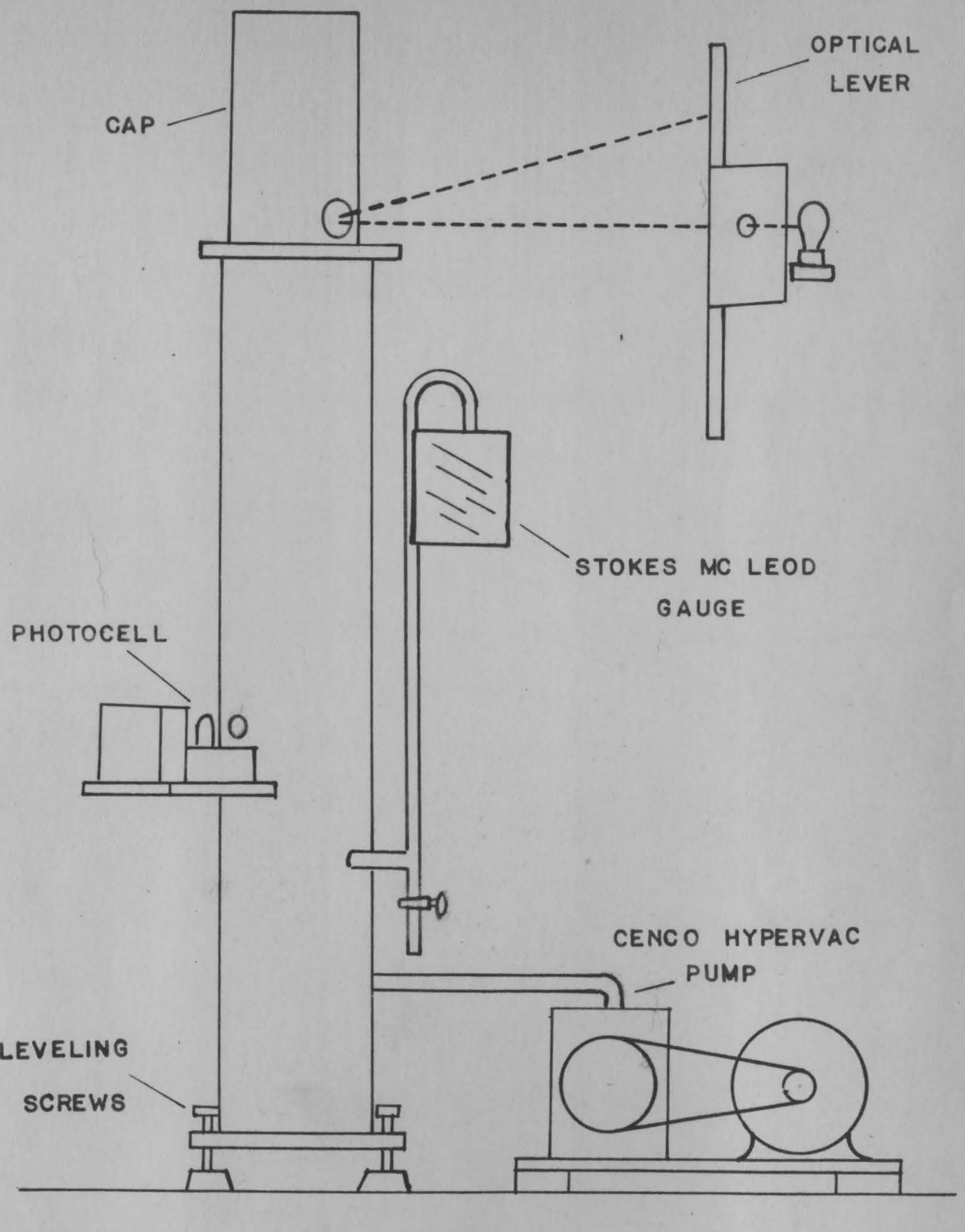

FIGURE 5 BLOCK DIAGRAM OF VACUUM TANK AND ASSOCIATED EQUIPMENT

The lower portion of the tank was a five-foot section of 10" I.D. wrought-iron steam pipe. The upper end of this was machined to a square shoulder, and the lower end was closed by welding on a 14" square of 3/4" boiler plate. This was fitted with a 3/8" cap screw in each corner. These screws served as leveling devices for the entire tank.

To complete the chamber, a cap was made from a 16" length of 8" I.D. steam pipe. The bottom of this cap rested on the shelf arrangement, and the top was sealed off with a 10" square aluminum plate waxed in place.

The bottom section rested on 2" thick wooden blocks on the concrete ground floor and positioned under the cap screws, and was made rigid to the wall at its upper end. The cap was small enough to be removable. This was necessary in order to reverse the position of the pendulum, and also to have access to the inside of the tank.

All dimensions were dictated somewhat by the availability of the material, but mainly by the necessary clearances for the pendulum as it vibrated within the tank.

The seals between the removable portions of the tank were accomplished with neoprene gaskets, and only the force exerted by the atmosphere was required to make them vacuum tight.

Two nipples were welded into the side of the bottom section of the tank. One was used for the rubber vacuum hose connection to the Cenco Hypervac pump, and it was positioned about a foot up from the floor of the tank. In this position the air stream as the pump evacuated the tank was generally directed down along the pendulum.

The other nipple served three purposes. First, it was used for the vacuum hose connection to the Stokes type McCleod gauge. Secondly, it was used as the inlet valve to the vacuum tank. And thirdly, because it was intentionally positioned about 100 cm. down the tank from the level of the knife edge, it was also used to build up the initial amplitude of the pendulum by admitting puffs of air into the tank, which impinged upon the side of the lower bracket.

The Knife Edge

In the literature are recorded many experiments which were made to determine the best type of knife edge to be used in pendulum work of this nature. The results indicate that in order for the edge to withstand the successive sidewise thrust of the pendulum as it vibrates from side to side, the material must be hard and tough but not brittle, and the angle between the faces forming the edge should be about 120° to insure sufficient strength.

The knife edge used in this experiment, Figure 3, was made of tool steel which had been shaped, hardened, and then hand ground to an angle of 136°. However, because the pendulum carried the glass planes with it, and they in turn supported the pendulum on the "knife edge", it was required that the knife edge used actually be made in two segments so that it could support the pendulum on both sides of the stem. Therefore, the two segments, after they had been shaped and hardened, were press-fit into a U-shaped block of cast iron. For grinding, then, this entire assembly was turned over so that one of the edges of the block and the knife edge itself rested upon a piece of plate glass wetted with an aqueous solution of levitated alumina. Therefore, not only did the cast iron block serve as a holder for the two segments of the knife edge, but it also served as a convenient jig for grinding the final edge.

The combination of glass and tool steel used to reduce the friction between the bearing surfaces appeared to be satisfactory. Examination of the knife edges before and after the project with a high-powered microscope showed the edge to be apparently free of any defects. However, examination of the glass planes did reveal surface scratches; but these did not appear to be arranged in any regular fashion. This indicated that they were not necessarily breakdowns of the surface of the glass caused by the pendulum oscillating back and forth. Instead, it was supposed that they were caused by such things as the initial abrupt setting of the pendulum down on the edge, foreign particles between the knife edge and the glass, and the like. Improvement over this condition might have been attained if polished quartz planes could have been substituted for the plate glass.

Measurement of Length

For a standard of length a bar of Gamma metal similar to the stem of the pendulum was sent to the National Bureau of Standards for calibration. On this bar there were three fine parallel lines approximately 99, 100, and 101 centimeters respectively away from a similar line which served as the zero line. This secondary standard, with its certified calibration, is on file in the Physics Department office. A reprint of this certification is made in Figure 6.

United States Department of Commerce
Washington

C
O
P
Y

NATIONAL BUREAU OF STANDARDS

Certificate
for
One-Meter Bar

Maker: Virginia Polytechnic Institute
Maker's No. 105

Virginia Polytechnic Institute,
Department of Physics,
Blacksburg, Virginia.

This meter bar has been compared with the standards of the United States, and a calibration has been made of the two one-centimeter sub-intervals. The results obtained are as follows:

Length of the interval from the zero to the one-meter graduation (labeled 0 to 2 on the bar) at 25° centigrade: 99.9381 centimeter.

The observations were made at a mean temperature of 19.75° centigrade, and in reducing to 25° centigrade, the coefficient of linear expansion of 0.000001 per degree centigrade was used. This value for the coefficient is an average value for gamma steel.

Length of the sub-intervals at 25° centigrade:

Interval (cm)	Length (cm)
1 to 2	1.0018
2 to 3	1.0004

Measurements were made approximately 0.05 centimeter from the ends of the graduations farthest from the engraved numbers.

It is estimated that the values for the lengths given above are not in error by more than 0.0005 centimeter.

For the Director,

Lewis V. Judson,
Chief, Length Section
Optics and Metrology Division

Test No. 2.4/G-7780
Test completed: January 18, 1952.

Figure 6

Calibration of Secondary Standard Length

Travelling microscopes* were positioned above the zero and 101 centimeter lines on the standard bar, and their relative positions were found on the scales of the comparators. Replacing the standard bar with the pendulum, the distances between the edges of the faces of the plate glass planes could then be determined by observing the changes in the positions of the comparators. These comparators could be read to a thousandth of a centimeter and estimated to a ten-thousandth of a centimeter.

Measurement of Time

In Room 305, Davidson Hall, there was a Seth Thomas clock having approximately a seconds pendulum. Using this pendulum as a shutter to interrupt a beam of light directed toward a photocell, an electrical pulse was sent down to the recording apparatus in Room 106. This pulse actuated a pen relay on the chronograph, and the trace thus made on the drum of the chronograph served as the time calibration for the determination of the period.

The rate of the clock was determined by frequent comparison with the Station W.W.V.** time signals. By knowing the rate of the clock, it was possible to correct the value of the elapsed time of each determination.

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* Manufactured by Gaertner Scientific Corporation, Chicago, Ill.

** Station W.W.V. of the National Bureau of Standards broadcasts continuously, day and night, on standard radio frequencies of 2.5, 5, 10, 15, 20, 25, 30 and 35 Mc. Time intervals as received are normally accurate within $\pm$ (2 parts in 10^8 + 1 millisecond). Transient conditions in the ionosphere at times cause received pulses to scatter by several milliseconds. (13)

If S is the daily rate of the clock, and T the recorded elapsed time, the corrected period is

$$T' = T\left(1 - \frac{S}{86,400}\right)$$

The Chronograph

The recording instrument was a chronograph, Figure 7, driven by an 1800-rpm synchronous motor. Drum speeds of 1.33, 4, and 12 rpm were possible through the use of a gear box. All runs for the project were made at the slowest speed. This allowed the recording and subsequent determinatinn of the two periods of the pendulum to five significant figures on a single record.

In operation, a carriage having two pen relays moved parallel to the length of the drum once for every 66 revolutions of the drum. As was previously mentioned, one of these pens was actuated by the time pulse originating at the clock. The second relay was actuated by a similar photocell located on the outside of the vacuum tank. Here the beam of light came from a source on the opposite side of the tank, and was directed through the tank so that the Kater's pendulum bar itself would act as a shutter. Two windows were installed in the tank approximately 80 cm. down from the knife edge and diametrically opposite one another for this purpose.

Determination of the period of vibration was made by taking the ratio of the corrected elapsed time to the number of vibrations as recorded on the chronograph.

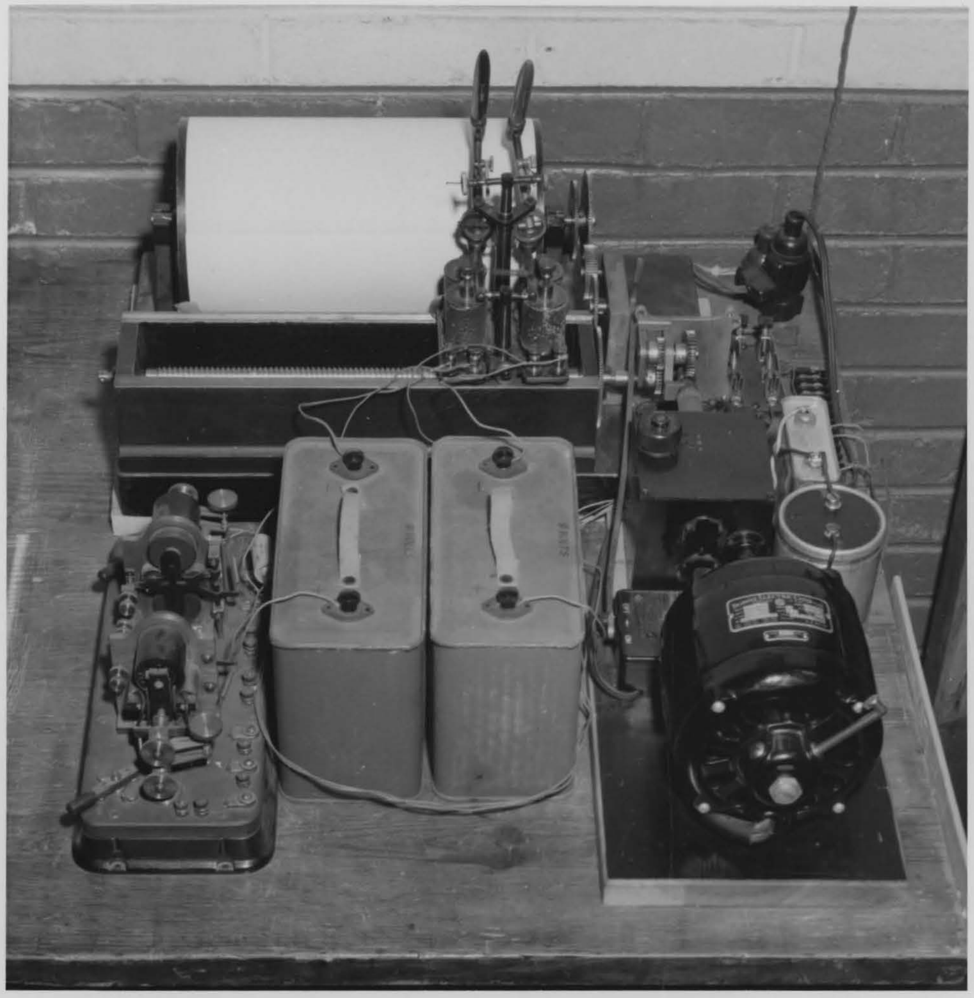


Fig. 7 Photograph of Chronograph

The amplitude of the pendulum was determined through the use of an optical lever. The mirror for this system was located on the side of the bracket which was supporting the pendulum on the knife edge. Theoretically, this mirror should be at the knife edge itself, but the correctinn for this small displacement was found to be negligible when the amplitude is of the order of a few degrees. The scale, toward which the optical lever was directed, was about 300 centimeters away from the tank, and an oval window in the cap of the tank permitted the light beam to penetrate into the tank.

METHOD OF CALCULATION

As was mentioned, the electrical impulses originating at the clock and pendulum are each recorded by individual traces on the record of the chronograph. Because two pens filled with different colored inks were used, it was necessary to adjust the distance between the pens such that they would interweave as the record progressed.

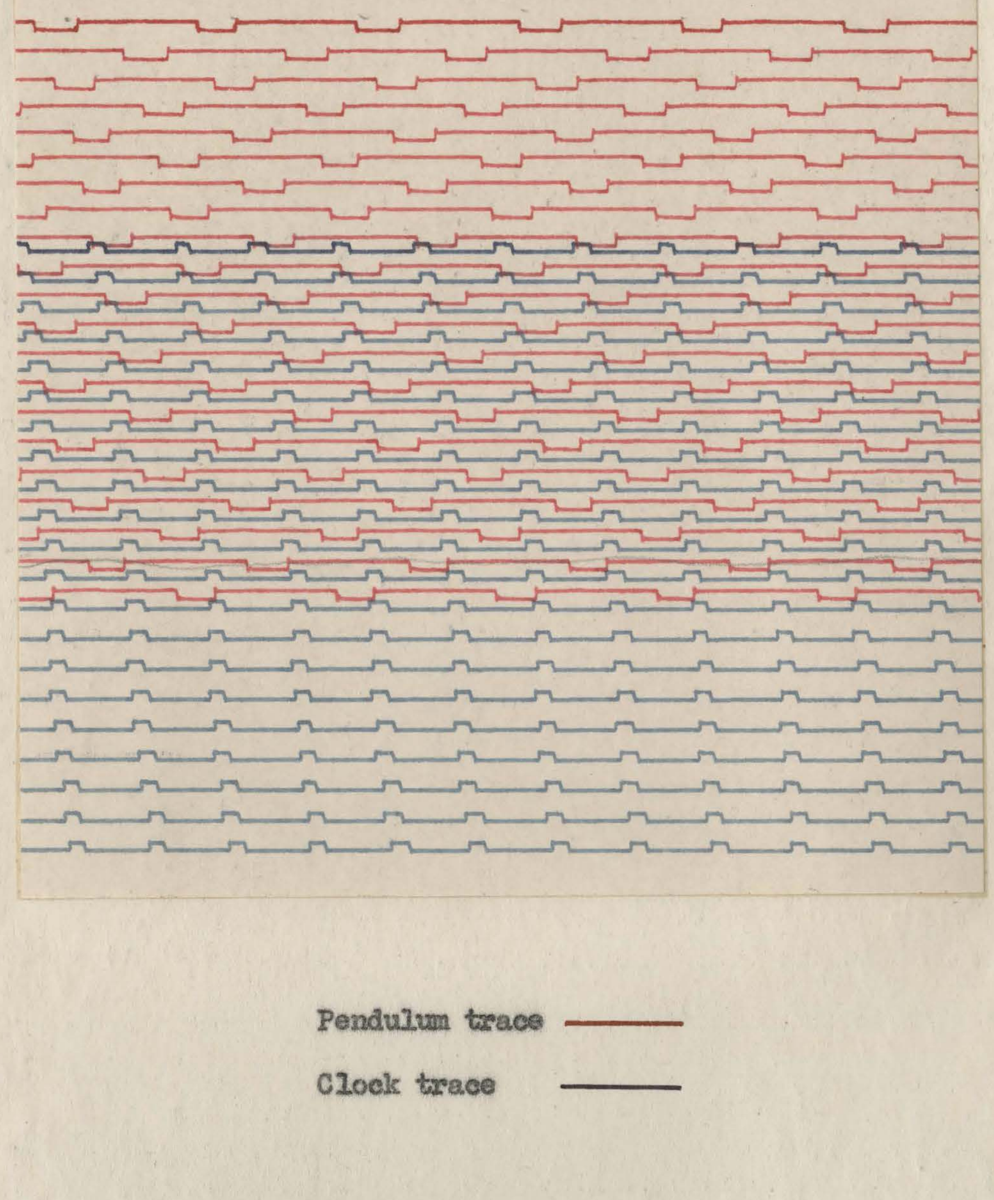
In order to determine the period as recorded, it was necessary to remove the record from the drum of the chronograph, count the total number of vibrations - whole and parts - and obtain the ratio between the elapsed time and the number of vibrations of the pendulum.

The total elapsed time for each determination was about nine hundred seconds, and the number of vibrations of the pendulum was about four hundred and fifty. Therefore, the period could be determined to five significant figures considering the parts of the first and last vibrations as well as the total number in between. A sample record can be found in Figure 8.

Corrections

(a) Clock Rate

The Seth Thomas clock used for this project was compared frequently with the time signal broadcast continually by the National Bureau of Standards over Station W.W.V.. A graph of the daily rates of the clock during the experiment is shown in Figure 9. After making any necessary adjustments on the clock, care was taken that enough time had elapsed before data was again taken in order to allow the clock to take up its new rate.



Pendulum trace

Clock trace

Fig. 8 Sample Portion of Record

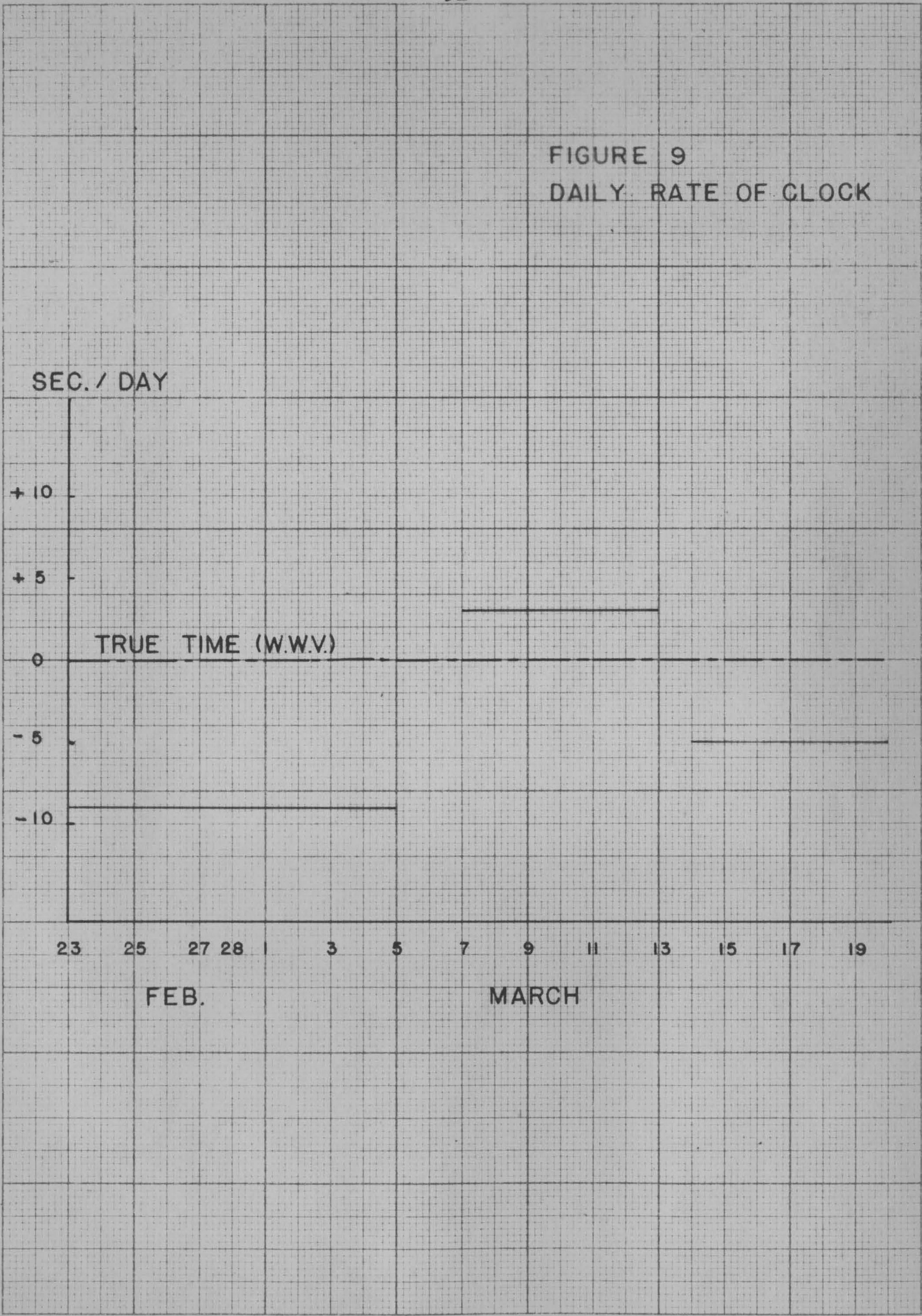


FIGURE 9
DAILY RATE OF CLOCK
SEC. / DAY
+ 10
+ 5
0
- 5
-10
TRUE TIME (W.W.V.)
23
25
27 28
1
3
5
7
9
11
13
15
17
19
FEB.
MARCH

It was possible to correct for the overall rate of the clock. However, slight diurnal variations of the rate, due to effects of changing temperature and humidity upon the clock, could not be eliminated. Even so, correcting the elapsed time of the determinations for the clock rate did tend to group the final values of the periods closer together. But upon examining Figure 10 it is apparent that there is a greater spread of values of the periods in morning than in the afternoon. A statistical F test was made which indicated that the difference in the variances is just significant at the five percent level.*

An explanation of this difference could be that the steady state conditions for the equipment had been reached by afternoon. As was mentioned before, it was not possible to account for or control such factors as temperature, humidity, or the A.C. line frequency which might affect the clock or the chronograph.

(b) Horizontal Adjustments of the Knife Edge

If the knife edge is not absolutely horizontal then the effective length, L', of the pendulum will be

$$L' = L \cos \theta$$

where θ is the angle the knife edge makes with the horizontal.

For this project the knife edge was leveled with a Gurley bubble

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* Significance at the five percent level means that if there were no real difference present the probability of obtaining an observed difference as large or larger than the one observed is less than five percent.

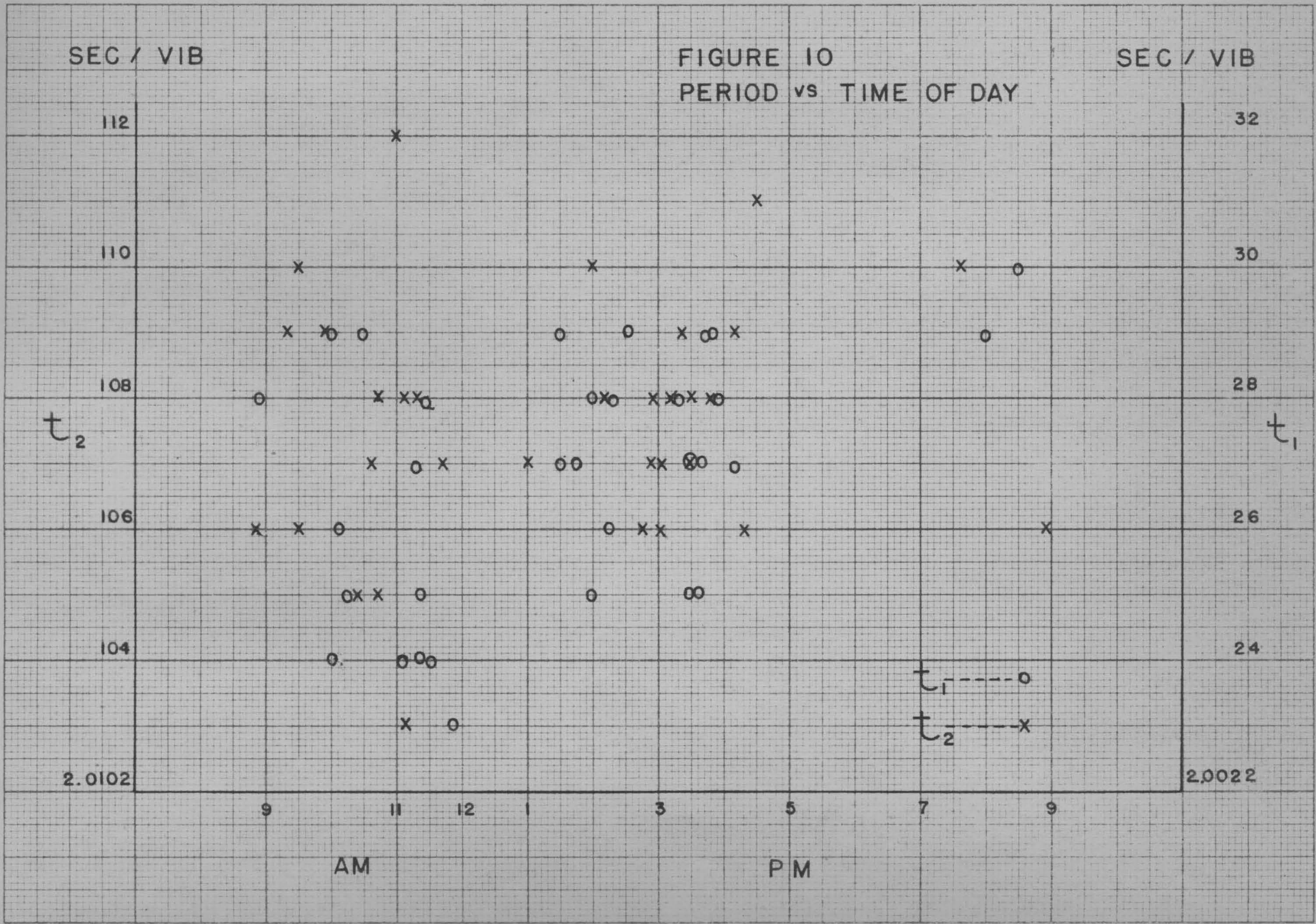


FIGURE 10
PERIOD vs TIME OF DAY
SEC / VIB
SEC / VIB
112
110
108
106
104
2.0102
32
30
28
26
24
2.0022
t_2
t_1
9
11
12
1
3
5
7
9
AM
PM
t_1 ----- o
t_2 ----- x

level whose sensitivity in minutes of arc for the first three divisions to either side of the center are given in Figure 11.

Div.	Side 1	Side 2
1	0.1'	0.1'
2	0.8'	0.6'
3	1.6'	1.3'

Fig. 11

It is readily seen that even if the knife edge were as much as 3' off-level, the cosine is still equal to 1.00000, and, therefore, the effective length of the pendulum to five significant places is still equal to the real length.

(c) Adjustment of Glass Planes

The measurement of the distances between the corresponding edges for the two sets of glass planes indicated that they were not parallel. Theoretically, this would alter the effective length of the pendulum in the same manner as the adjustment of the knife edge. However, several determinations of the periods were made with the pendulum reversed on the knife edge, i.e. rotated about a vertical axis through 180° from its normal position. The fact that the periods determined in this manner were well within the limits of the original data eliminated the necessity for correcting for the non-parallelism of the planes.

(d) Temperature

The temperature of the pendulum was assumed to be the same as the temperature of the room. Because this temperature at the time of the runs never varied more than two degrees from 25° C., no attempt was made to correct for the expansion of the pendulum. As given by the National Bureau of Standards, the coefficient of linear expansion for the pendulum bar was 1.0×10^{-6} per °C. which would have meant a total maximum expansion of 5.2×10^{-4} cm. Because the comparators could only be read to a thousandth, and estimated to a ten-thousandth, of a centimeter, the correction for this expansion was negligible.

(e) Damping

In an experiment of this type, the two sources of damping are (1) the residual air, and (2) the friction at the knife edge. The effect of friction cannot be predicted, whereas the damping caused by the residual air will cause the amplitude of the pendulum to decrease exponentially.

Figure 12 is a curve which demonstrates the damping of this pendulum for a forty-eight hour run.

The period of a damped vibration (8, p.825), where the damping effect is assumed proportional to the velocity, is

$$T = \frac{2\pi}{\sqrt{\frac{g}{l} - b^2}} = 2\pi\sqrt{\frac{l}{g}}\left\{1 + \frac{b^2 l}{2g} + ---\right\} \qquad (18)$$

where $b = \frac{1}{nT} \log_e \left(\frac{\alpha_o}{\alpha_n}\right)$

n = total number of swings

α_o = initial amplitude

α_n = final amplitude

If typical values from the data of this project are inserted, the magnitude of this correction becomes 10^{-11}, which in our case is a negligible amount.

It will also be noticed from Figure 13 that there was no over-all trend in the data between the amplitude of vibration and its period.

(f) Motion of the Support

Theoretically, the support of a pendulum executes forced vibrations caused by the motion of the pendulum as it swings through its appointed arc. This motion will alter the period of the pendulum and can be corrected for, if necessary.

With this in mind, a simple test was made to determine if the supporting plate for the pendulum was in motion. A dish containing a pool of mercury was placed on the supporting plate, and while the pendulum was in motion it was observed that there were no apparent surface disturbances, which indicated that the support was comparatively at rest.

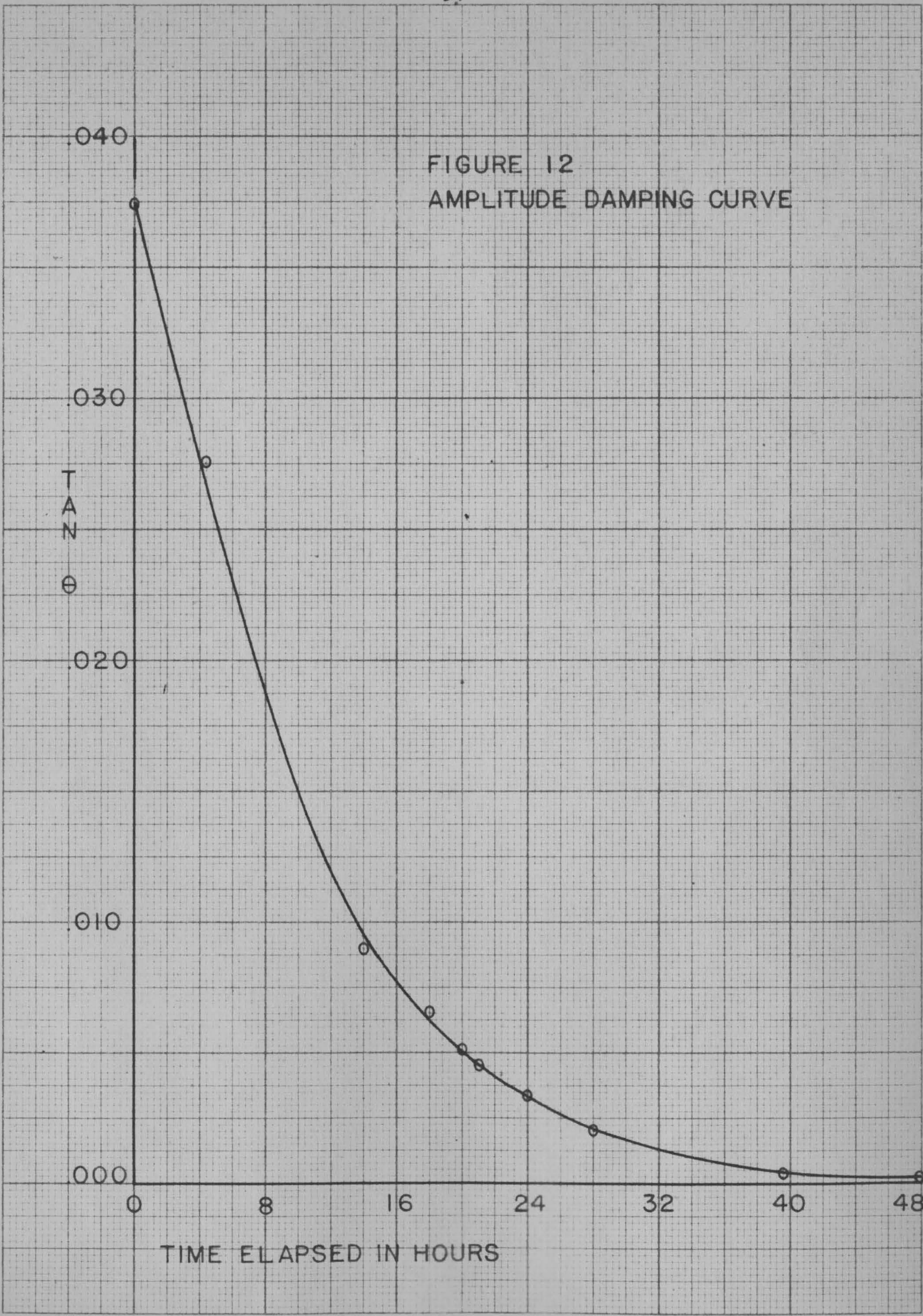


FIGURE 12
AMPLITUDE DAMPING CURVE
.040
.030
.020
.010
.000
TAN θ
0
8
16
24
32
40
48
TIME ELAPSED IN HOURS

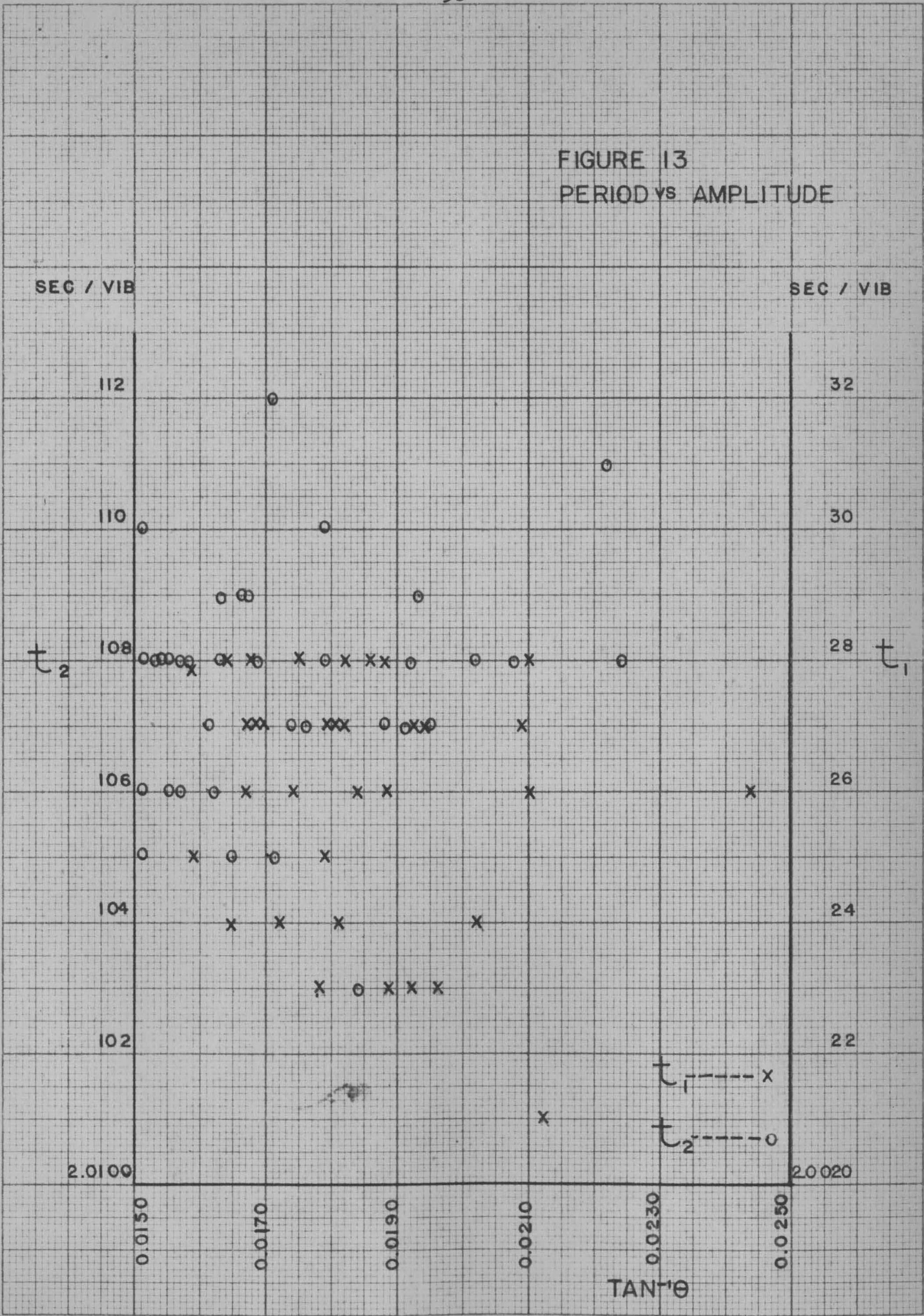


FIGURE 13
PERIOD VS AMPLITUDE

(g) Other Factors

Scatter diagrams, Figures 14 through 17, of the data were made with the thought that they might assist in uncovering additional interactions between combinations of "g", pressure, amplitude and period. Examination of these diagrams did not reveal the existence of any such interactions insofar as they could be detected with the equipment employed.

A plot of the day-by-day variation of "g", Figure 18, was examined and it appeared at first that there might have been a gradual increasing trend in the values of "g" as determined, and an analysis of variance was made to test the hypothesis that the slope of this "trend" was zero. This hypothesis, however, proved to be most tenable, the evidence of trend being very insignificant.

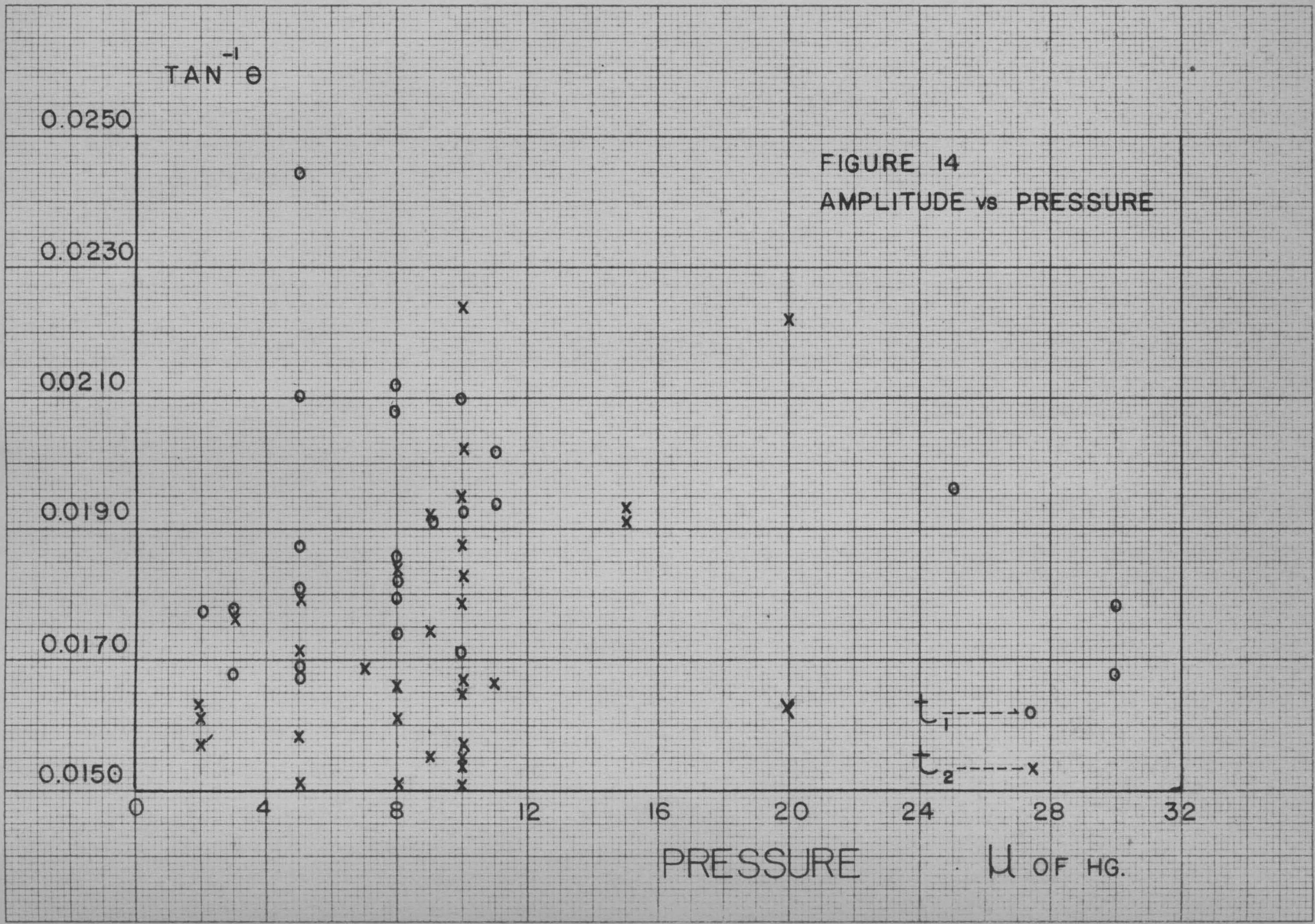
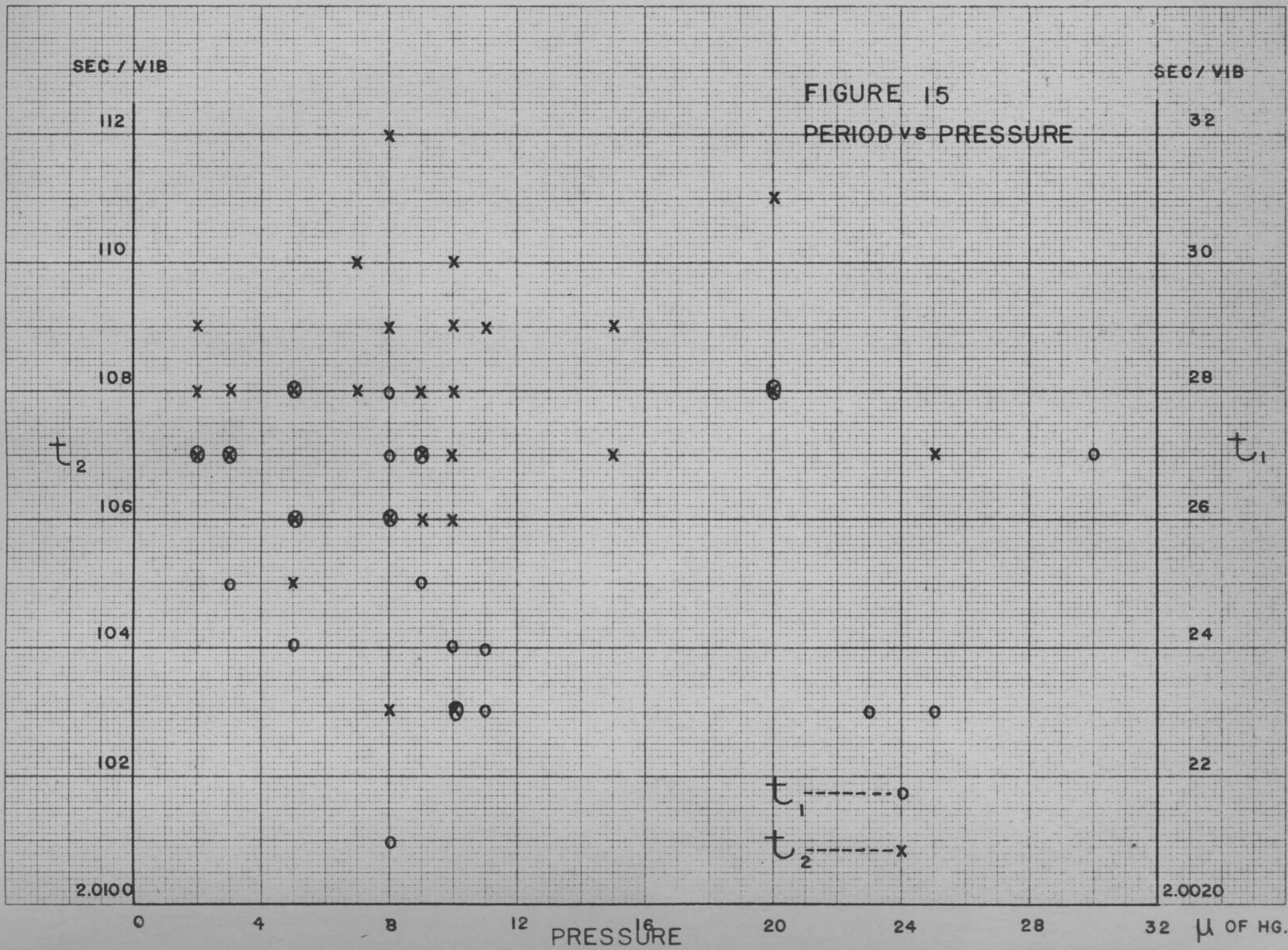


FIGURE 14
AMPLITUDE vs PRESSURE
TAN^{-1} θ
0.0250
0.0230
0.0210
0.0190
0.0170
0.0150
0 4 8 12 16 20 24 28 32
PRESSURE μ OF HG.
t_1 -----→ o
t_2 ----- x



SEC / VIB
SEC / VIB
FIGURE 15
PERIOD vs PRESSURE
112
110
108
106
104
102
2.0100
32
30
28
26
24
22
2.0020
0
4
8
12
16
20
24
28
32
μ OF HG.
PRESSURE
t2
t1
t1 --------o
t2 --------x

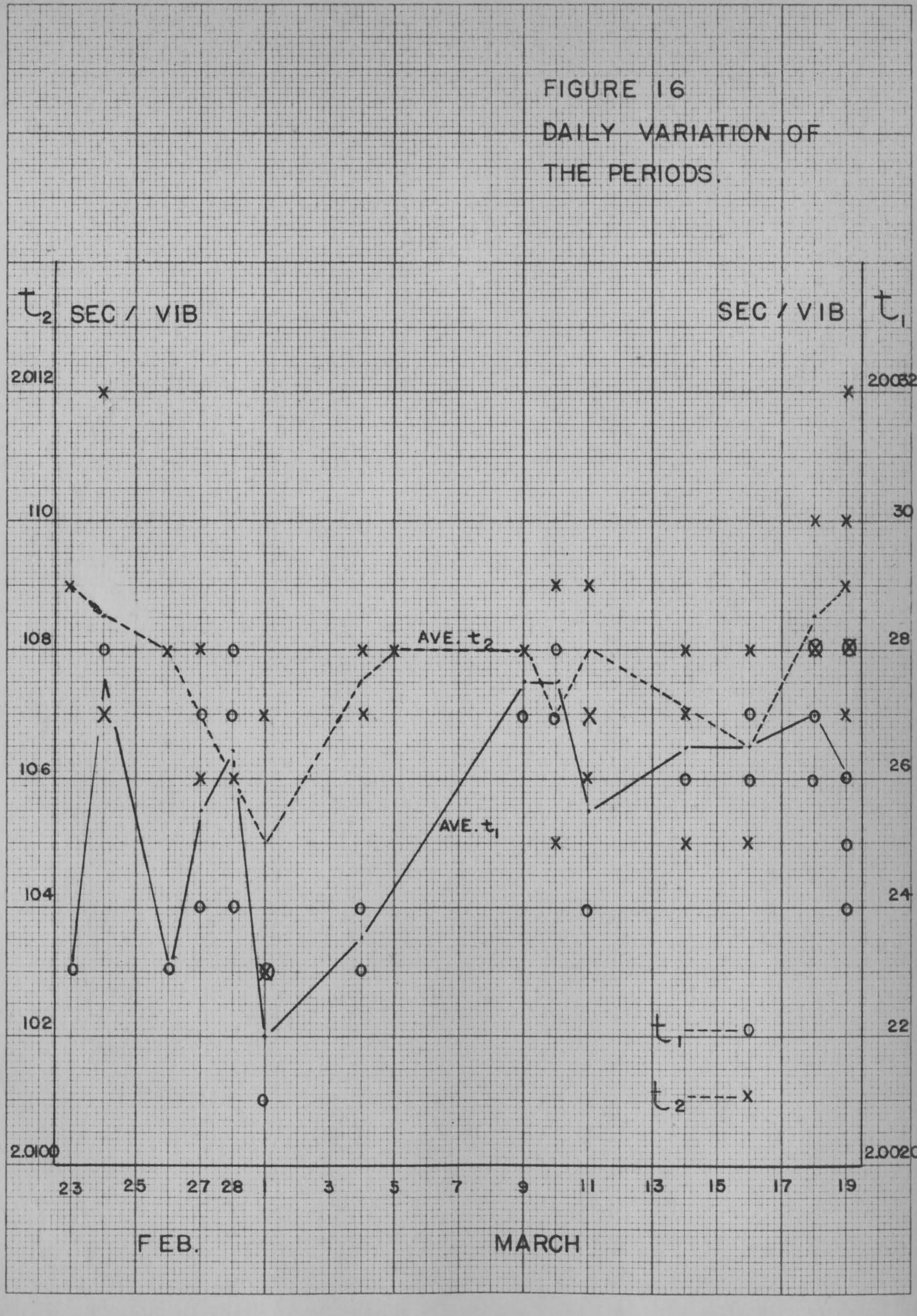


FIGURE 16
DAILY VARIATION OF
THE PERIODS.
t_2 SEC / VIB
SEC / VIB t_1
2.0112
110
108
106
104
102
2.0100
2.0032
30
28
26
24
22
2.0020
AVE. t_2
AVE. t_1
t_1 ---- o
t_2 ---- x
23
25
27
28
1
3
5
7
9
11
13
15
17
19
FEB.
MARCH
20 X 20 PER INCH

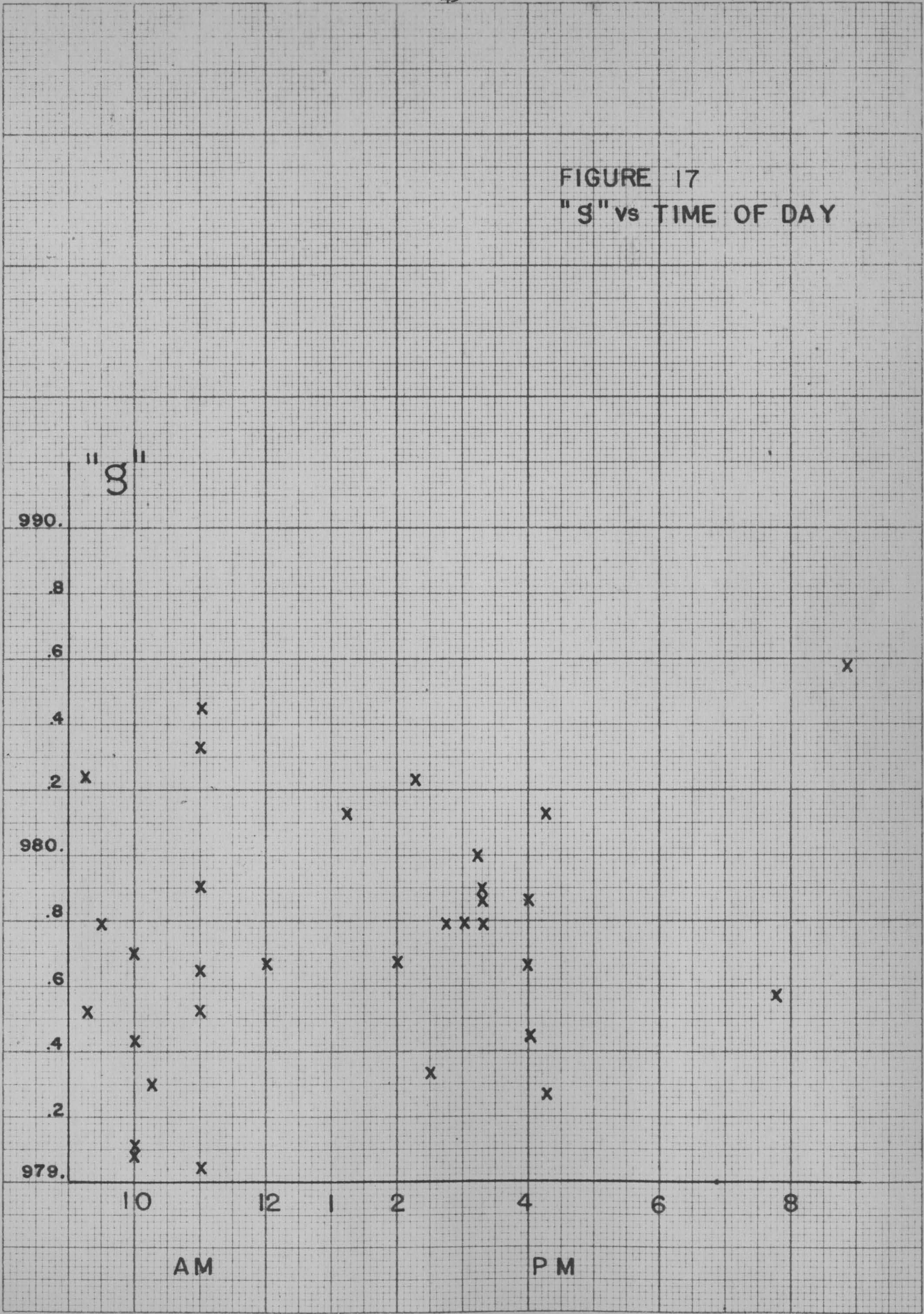


FIGURE 17
"g" vs TIME OF DAY
"g"
990.
.8
.6
.4
.2
980.
.8
.6
.4
.2
979.
10
12
1
2
4
6
8
AM
PM

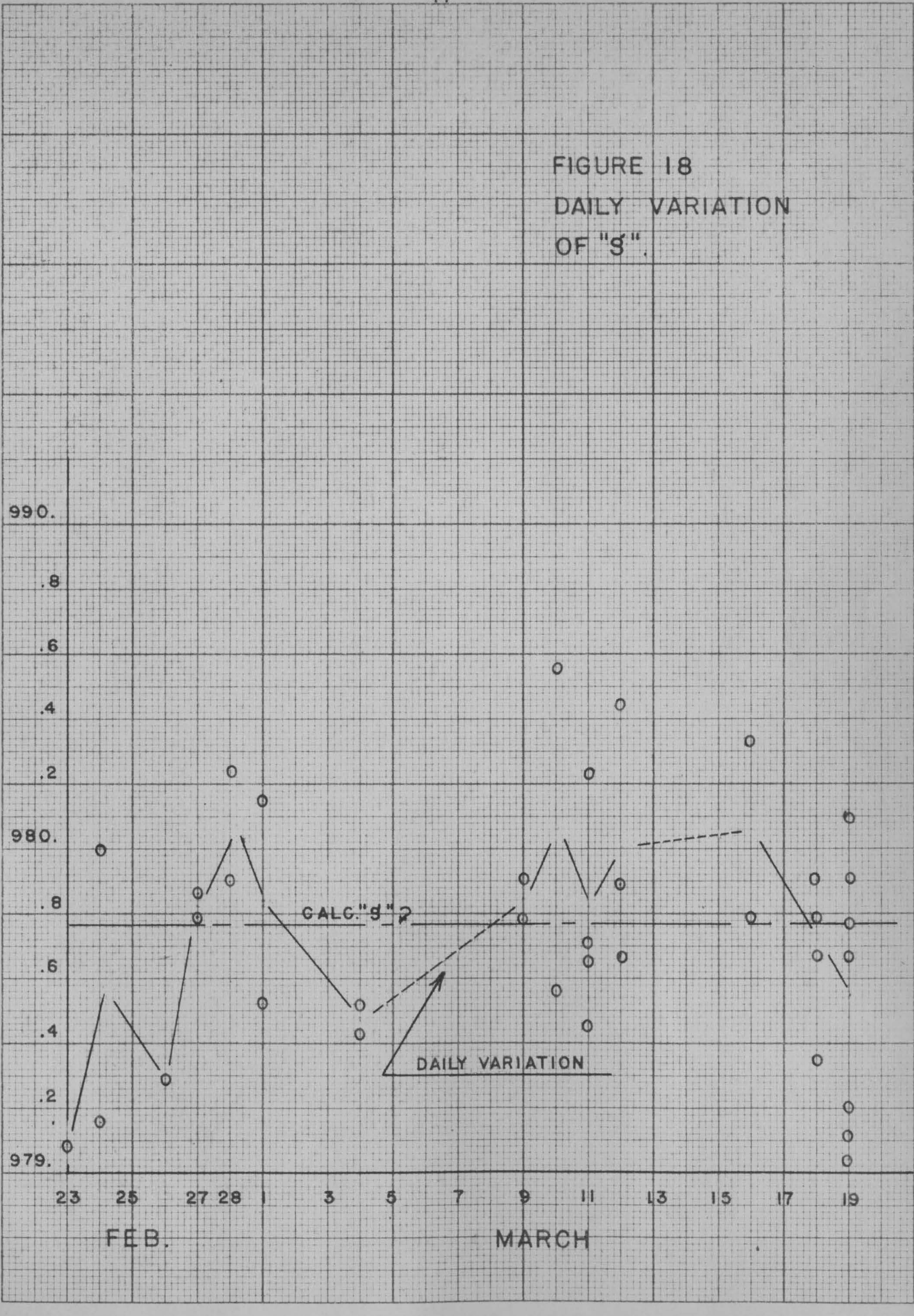


FIGURE 18
DAILY VARIATION
OF "g".
990.
.8
.6
.4
.2
980.
.8
.6
.4
.2
979.
CALC."g"?
DAILY VARIATION
23
25
27 28
1
3
5
7
9
11
13
15
17
19
FEB.
MARCH

DATA AND RESULTS

On pages 47 and 49 are recorded in tabular form the data as obtained in this determination.

Table 1 presents the data used to determine the length of the pendulum. The equation of the method used is

$$\text{Length of Pendulum} = \left[101.9385 + (R_2 - R_1)_{Pend} - (R_2 - R_1)_{Stand.}\right] \text{ cm.}$$

As an example, to determine the length of the pendulum for position A using data from February 16.

$$\text{Length of Pendulum at A} = 101.9385 + (1.4860) - (1.0496) = 101.3749 \text{ cm.}$$

By this method, the following lengths for the pendulum at its four positions were obtained

A (cm)	B (cm)	C (cm)	D (cm)
101.3730	101.3720	101.3674	101.3672
101.3732	101.3667	101.3661	101.3669
101.3749	101.3727		
	101.3768		

By averaging the values for A and D, and B and C, the distance between the center lines of corresponding planes at opposite ends of the pendulum were obtained as 101.3704 cm and 101.3703 cm. Each of these values is based on at least fifty individual determinations and, therefore, we can accept the length of the pendulum as 101.37 cm.

The values of h_1 and h_2 were obtained as 64.9 cm and 36.5 cm respectively by balancing the pendulum bar on a knife edge and measuring these distances with a meter stick. Greater accuracy was not warranted as the difference between these distances appears only in the denominator of the correction term.

The data used for the determination of the periods of the pendulum is given as obtained in Table 2. It constitutes the last seventy-six runs, i.e. thirty-eight for each of the two positions of the pendulum, of several hundred runs. These last runs were used for they were made after the glass planes had been reset on the pendulum. They were originally found to be far from parallel with each other, which, of course, was not desirable, and therefore they were reset.

The frequencies of the quantity, $\frac{4\pi^2}{g}$, and of the periods, t_1 and t_2, are plotted in Figures 19 and 20. Each one of these appears sufficiently close to a normal distribution to indicate the absence of important assignable causes for variation.

Date	Standard Length R_1 cm.	Standard Length R_2 cm.	Pend. Position A R_1 cm.	Pend. Position A R_2 cm.	Pend. Position B R_1 cm.	Pend. Position B R_2 cm.	Pend. Position C R_1 cm.	Pend. Position C R_2 cm.	Pend. Position D R_1 cm.	Pend. Position D R_2 cm.
2-13	2.8496	3.9010	2.7345	4.2202	2.7278	4.2120	1.5792	3.0592	1.4720	2.9520
	91	05	40	00	73	23	91	92	20	18
	94	05	47	03	75	23	90	90	20	20
	94	05	48	00	78	23	91	93	23	18
	96	05	49	01	78	23	88	92	22	18
	95	03	48	02	78	21	90	90	25	18
	98	00	50	01	82	22	95	92	22	18
	95	05	45	01	78	21	95	93	20	18
	98	06	49	00	79	22	92	89	21	20
	95	03			77	21	92	91	25	18
	2.8495	3.9005	2.7346	4.2201	2.7277	4.2122	1.5792	3.0591	1.4722	2.9519
			2.7330	4.2191	2.6967	4.1755				
			35	85	65	60				
			30	88	70	58				
			30	90	68	60				
			32	90	69	62				
			30	90	70	60				
			32	90	67	62				
			32	90	68	60				
			32	90	69	62				
			34	90	69	61				
			2.7332	4.2189	2.6968	4.1760				
					0.8270	2.3127				
					73	30				
					73	26				
					74	25				
					75	25				
					75	25				
					72	22				
					75	30				
					75	25				
					75	25				
					0.8274	2.3126				

First page of
Table 1

	Standard Length		Pend. Position A		Pend. Position B		Pend. Position C		Pend. Position D	
Date	R_1 cm.	R_2 cm.	R_1 cm.	R_2 cm.	R_1 cm.	R_2 cm.	R_1 cm.	R_2 cm.	R_1 cm.	R_2 cm.
2-16	1.1110	2.1608	1.4227	2.9091	1.3780	2.8660	1.1895	2.6666	1.0917	2.5682
	10	05	24	89	81	60	91	68	19	75
	08	05	27	88	80	60	95	70	20	82
	08	05	30	85	83	62	98	67	19	80
	10	05	29	88	82	61	98	63	21	81
	10	04	25	86	8;	60	93	63	19	78
	08	05	29	87	83	58	93	67	20	80
	10	05	28	88	81	60	96	66	19	80
	08	03	23	85	83	60	97	68	19	78
	09	04	30	85	79	60	94	65	18	82
	1.1109	2.1605	1.4227	2.9087	1.3781	2.8660	1.1895	2.6667	1.0919	2.5679

TABLE 1

Data for Determination of the Length of the Pendulum

Run No.[1]	Time	Press.[2]	Temp. °C	Amplitude Before	Amplitude After	No.vib.	Elapsed Time (sec) Uncorr.	Elapsed Time (sec) Corr.	Periods (sec) Up	Periods (sec) Down	$\frac{4\pi^2}{g}$	"g" cm/sec^2
2-23-1	9:50 a.m.	15	24	1°9'	1°6'	425.30	855.14	855.22		2.0109		
2	10:20 a.m.	10	24	1°9'	1°6'	449.38	899.71	899.79	2.0023		0.0403217	979.086
2-24-1	3:00 p.m.	15	24	1°6'	1°5'	447.67	900.07	900.15		2.0107		
2	3:25 p.m.	30	24	0°59'	0°57'	449.51	900.15	900.23	2.0027		0.0402837	980.009
3	3:40 p.m.	20	25	0°40'	0°39'	449.52	900.23	900.31	2.0028			
4	4:30 p.m.	20	25	1°17'	1°16'	447.56	900.00	900.08		2.0111	0.0403145	979.260
2-26-1	10:05 a.m.	30	23	1°2'	1°1'	449.52	900.00	900.08	2.0023			
2	10:40 a.m.	20	24	0°59'	0°56'	447.52	899.79	899.87		2.0108	0.0403127	979.304
2-27-1	2:25 p.m.	10	23.5	1°10'	1°9'	447.59	899.94	900.02		2.0108		
2	3:20 p.m.	10	25	0°58'	0°57'	449.30	899.74	899.82	2.0027		0.0402927	979.790
3	3:35 p.m.	5	25	0°57'	0°56'	449.48	899.98	900.06	2.0024			
4	4:20 p.m.	8	24.5	0°52'	0°51'	447.85	900.37	900.45		2.0106	0.0402897	979.863
2-28-1	1:00 p.m.	10	23.5	1°8'	1°7'	447.67	900.06	900.14		2.0107		
2	1:30 p.m.	8	23.5	1°6'	1°3'	449.66	900.49	900.57	2.0028		0.0402787	980.131
3	2:10 p.m.	10	24	1°6'	1°3'	449.63	900.38	900.46	2.0027			
4	2:40 p.m.	10	24	0°55'	0°54'	447.78	900.21	900.29		2.0106	0.0402747	980.228
5	3:00 p.m.	9	23	0°54'	0°53'	447.89	900.44	900.52		2.0106		
6	3:30 p.m.	10	23	1°0'	0°59'	449.41	899.82	899.90	2.0024		0.0402897	979.863
7	3:50 p.m.	3	23	0°59'	0°58'	449.33	899.81	899.89	2.0027			
8	4:20 p.m.	5	24	0°52'	0°52'	447.63	899.91	899.99		2.0106	0.0402747	980.228
3-1-1	9:00 a.m.	11	25	1°7'	1°5'	449.48	900.02	899.99	2.0023			
2	9:30 a.m.	10	24	1°5'	1°4'	447.66	900.15	900.12		2.0107	0.0403037	979.523
3	10:05 a.m.	8	25	1°4'	1°3'	447.37	899.39	899.36		2.0103		
4	10:50 a.m.	8	25	1°15'	1°14'	449.37	899.72	899.69	2.0021		0.0402778	980.153
3-4-1	9:45 a.m.	9	25	1°7'	1°6'	447.81	900.50	900.47		2.0108		
2	10:15 a.m.	11	25	1°11'	1°9'	449.48	900.08	900.05	2.0024		0.0403077	979.426
3	10:30 a.m.	25	25	1°9'	1°7'	449.67	900.42	900.39	2.0023			
4	11:40 a.m.	25	25	0°54'	-	425.07	854.67	854.69		2.0107	0.0403037	979.523
3-5-1	2:55 p.m.	10	24.5	1°18'	1°17'	447.22	899.31	899.28		2.0108		
3-9-1	3:05 p.m.	10	23.5	0°54'	0°53'	425.30	855.19	855.21		2.0108	0.0402927	979.790
2	3:30 p.m.	9	25	1°7'	1°5'	449.37	900.00	899.97	2.0027			
3	3:45 p.m.	5	25	0°59'	0°58'	426.96	855.15	855.13	2.0028		0.0402877	979.912
3-10-1	7:35 p.m.	11	24	0°58'	0°57'	447.39	899.67	899.64		2.0109		
2	8:00 p.m.	11	24	1°8'	1°6'	449.15	899.53	899.50	2.0027		0.0403017	979.572
3	8:30 p.m.	8	25	1°5'	1°4'	449.26	899.80	899.77	2.0028			
4	8:50 p.m.	10	25	0°53'	0°52'	447.52	899.78	899.75		2.0105	0.0402607	980.569
3-11-1	8:50 a.m.	8	25	0°30'?	0°29'?	447.78	900.33	900.30		2.0106		
2	9:18 a.m.	8	26	0°58'	0°57'	447.86	900.61	900.58		2.0109	0.0402747	980.228
3	9:45 a.m.	10	24.5	1°7'	1°6'	449.30	899.86	899.83	2.0027			
4	10:00 a.m.	5	25	1°6'	1°5'	449.33	899.97	899.94	2.0028		0.0402966	979.696
5	10:35 a.m.	9	26	1°1'	1°0'	447.74	900.29	900.26		2.0107		
6	11:20 a.m.	5	26.5	1°34'	-	449.67	900.46	900.43	2.0024		0.0402987	979.645
7	3:40 p.m.	5	26	1°25'	1°24'	449.37	899.92	899.89	2.0020			
8	4:10 p.m.	10	26	1°8'	1°7'	447.70	900.29	900.26		2.109	0.0403066	979.452

First Page of Table 2

Run No.[1]	Time	Press.[2]	Temp. °C	Amplitude Before	Amplitude After	No.vib.	Elapsed Time (sec) Uncorr.	Elapsed Time (sec) Corr.	Periods (sec) Up	Periods (sec) Down	$\frac{4\pi^2}{g}$	"g" cm/sec^2
3-14-1	10:40 a.m.	5	26	1°0'	0°59'	447.85	900.36	900.41		2.0105		
2	11:20 a.m.	8	26	1°15'	1°12'	449.59	900.35	900.40	2.0027		0.0402657	980.447
3	1:30 p.m.	5	25	1°14'	1°12'	449.04	899.19	899.24	2.0026			
4	2:15 p.m.	10	25	1°3'	1°2'	447.60	900.00	900.05		2.0108	0.0402976	979.671
5	2:45 p.m.	3	26	1°2'	1°0'	447.60	899.92	899.97		2.0107		
6	3:30 p.m.	8	25.5	1°0'	1°0'	449.37	899.85	899.89	2.0026		0.0402886	979.890
3-16-1	8:55 a.m.	8	25	1°4'	1°1'	898.67	1799.69	1799.79	2.0027			
2	9:45 a.m.	7	25	0°59'	0°58'	895.30	1800.14	1800.24		2.0108	0.0402927	979.790
3	10:25 a.m.	10	25	0°58'	0°57'	895.34	1799.96	1800.06		2.0105		
4	11:15 a.m.	5	25	0°58'	0°57'	899.22	1800.64	1800.74	2.0026		0.0402707	980.326
3-18-1	10:00 a.m.	10	25	1°15'	1°12'	449.52	900.23	900.28	2.0028			
2	11:05 a.m.	10	25	0°54'	0°53'	447.59	899.97	900.02		2.0108	0.0402877	979.912
3	2:00 p.m.	5	25	1°2'	1°1'	447.63	900.13	900.18		2.0110		
4	2:25 p.m.	10	25	1°4'	1°2'	426.93	854.98	855.02	2.0027		0.0403106	979.355
5	2:45 p.m.	2	25	1°2'	1°1'	449.04	899.25	899.30	2.0027			
6	3:10 p.m.	10	25	0°54'	0°53'	425.41	855.36	855.41		2.0108	0.0402927	979.790
7	3:30 p.m.	3	25	0°53'	0°52'	425.03	854.59	854.64		2.0108		
8	4:10 p.m.	10	25	0°58'	0°57'	449.74	900.59	900.64	2.0026		0.0402976	979.671
3-19-1	9:30 a.m.	10	25.5	0°55'	0°54'	447.63	900.15	900.20		2.0110		
2	10:05 a.m.	9	26	0°56'	0°55'	426.96	854.95	854.99	2.0025		0.0403206	979.112
3	10:25 a.m.	8	26	0°55'	0°54'	449.29	899.78	899.83	2.0028			
4	11:00 a.m.	8	26	0°56'	0°55'	447.66	900.30	900.35		2.0112	0.0403235	979.042
5	11:15 a.m.	5	25	0°55'	0°54'	447.77	900.32	900.37		2.0108		
6	1:30 p.m.	10	25.5	1°6'	1°5'	449.33	899.78	899.83	2.0026		0.0402976	979.671
7	1:45 p.m.	8	25	1°5'	1°3'	449.63	900.36	900.41	2.0026			
8	2:00 p.m.	5	25	1°3'	1°2'	449.41	899.85	899.90	2.0024		0.0402798	980.104 (a)
9	2:15 p.m.	3	25	1°2'	1°1'	449.37	899.81	899.86	2.0025			
10	2:30 p.m.	5	24.5	1°1'	1°0'	449.03	899.25	899.30	2.0028		0.0403166	979.210 (b)
11	3:00 p.m.	10	24	0°58'	0°57'	447.59	899.87	899.92		2.0106		
12	3:15 p.m.	2	25	0°57'	0°56'	447.78	900.37	900.42		2.0109	0.0402937	979.766 (c)
13	3:30 p.m.	2	25	0°56'	0°55'	447.56	899.87	899.92		2.0107		
14	3:45 p.m.	2	25	0°55'	0°54'	447.41	899.59	899.64		2.0108	0.0402877	979.912 (d)

[1] First two figures signify month and day of run

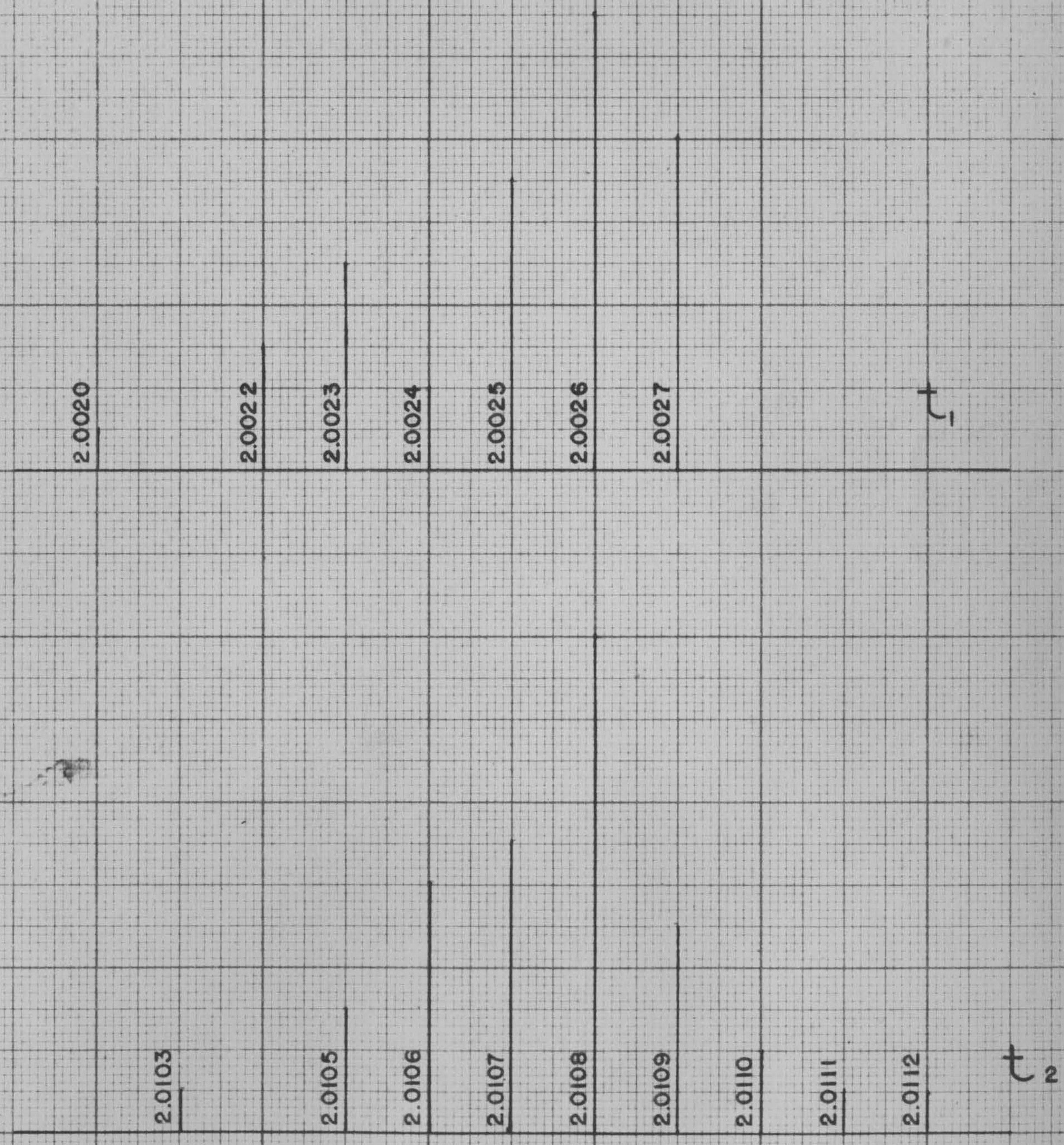
[2] Pressure as expressed by microns of Hg.

(a) Computed from 3-19-7 and 3-19-11
(b) Computed from 3-19-8 and 3-19-12
(c) Computed from 3-19-9 and 3-19-13
(d) Computed from 3-19-10 and 3-19-14

TABLE 2

Data for determination of "g"

FIGURE 19
PERIOD DISTRIBUTIONS



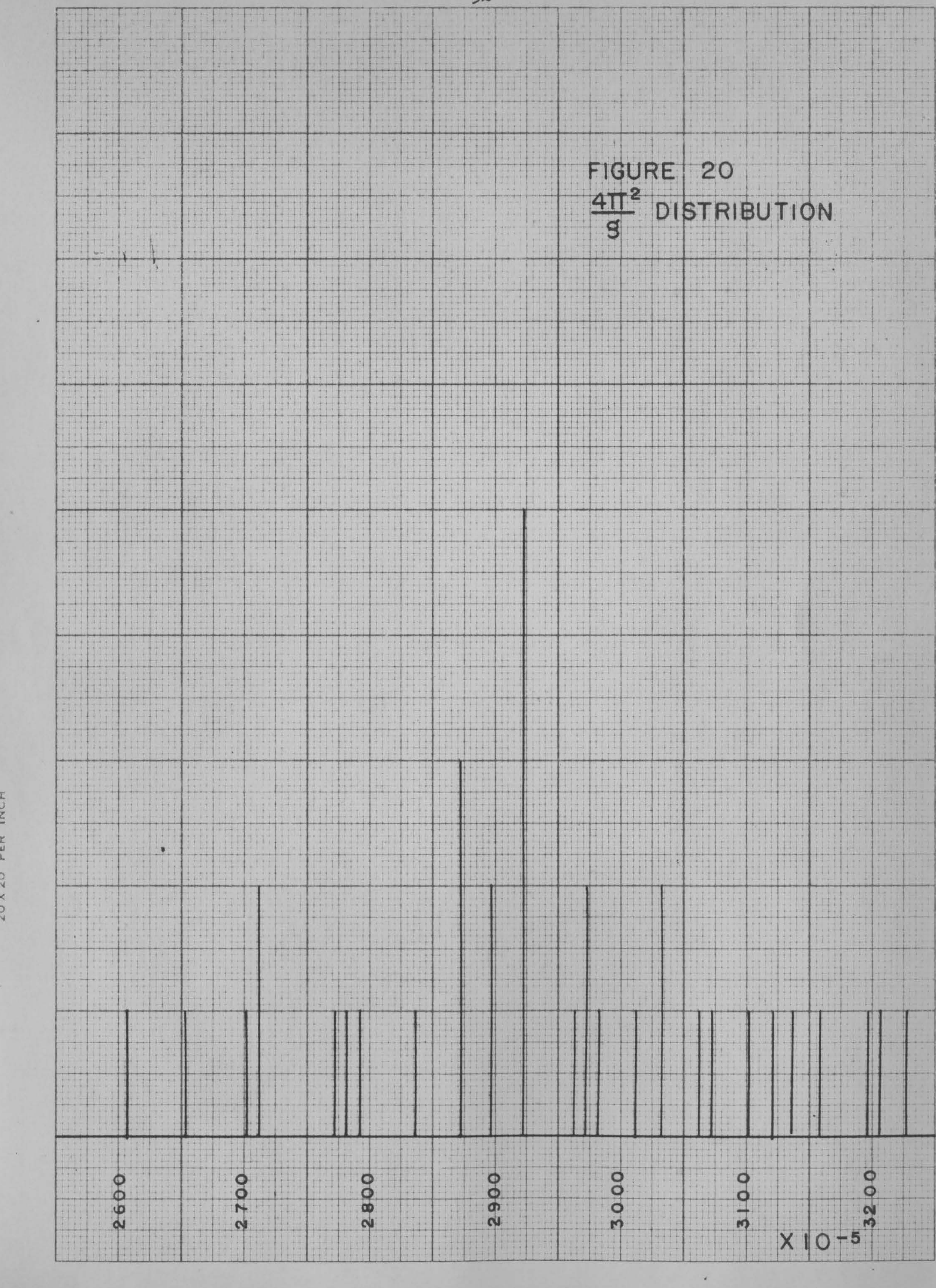


FIGURE 20
$\frac{4\pi^2}{g}$ DISTRIBUTION
2600
2700
2800
2900
3000
3100
3200
X 10^-5
20 X 20 PER INCH

EVALUATION OF DATA

If we consider the equation

$$\frac{4\pi^2}{g} = \frac{t_1^{\,2} + t_2^{\,2}}{2(h_1 + h_2)} + \frac{t_1^{\,2} - t_2^{\,2}}{2(h_1 - h_2)}$$

used to calculate the value of "g", we find that the quantity $\frac{4\pi^2}{g}$ is a linear function of $t_1^{\,2}$ and $t_2^{\,2}$. Therefore, it is convenient to determine the variance of these quantities and from this determine the confidence limits on the final value of "g".

This was done giving the following 95 percent confidence limits for the true value of "g" as obtained from data of this project:

$$\text{"g"} = 979.7625 \pm 0.1276 \frac{\text{cm}}{\text{sec}^2}$$

Wider 99 percent confidence limits are:

$$\text{"g"} = 979.7619 \pm 0.1690 \frac{\text{cm}}{\text{sec}^2}$$

On the other hand, the International Gravity Formula, as adopted by the International Association of Geodesy in 1930 (6, pg. 4) reads

$$\gamma_0 = 979.049\ (1 + 0.0052884 \sin^2 \phi - 0.0000059 \sin^2 2\phi)$$

where γ_0 is the theoretical value of gravity at sea level for latitude ϕ.

The correction for the elevation of the station is given by the formula

$$\text{Elevation Correction} = -0.00030855h - 0.00000022h \cos 2\phi + 0.000072 \left(\frac{h}{1000}\right)^2$$

where h is the elevation of the station in meters.

Solving for the theoretical value of gravity as determined by these two equations, one obtains

$$\gamma = 979.7442 \frac{cm}{sec^2}$$

On the basis of the close agreement between the predicted and experimental values the final value of acceleration due to gravity for the locale of Blacksburg, Virginia, should be placed at

$$"g" = 979.76 \pm 0.128 \frac{cm}{sec^2}$$

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BIBLIOGRAPHY

1. Bowie, William. Investigations of Gravity and Isostasy, Special Publication No. 40, Coast and Geodetic Survey, U.S. Department of Commerce, Washington, D.C., 1917.

2. Clements, G.R., and Wilson, L.T. Analytical and Applied Mechanics, pp. 317-319, pp. 358-361, McGraw-Hill Book Co., Inc., New York, N.Y., 1935.

3. Cochran, W.G., and Cox, G.M. Experimental Designs, John Wiley & Sons, Inc., New York, N.Y., 1950.

4. Den Hartog, J.P. Mechanics, pp. 234-237, pp. 264-265, p. 292, McGraw-Hill Book Co., Inc., New York, N.Y., 1948.

5. Dryden, H.L. A Reexamination of the Potsdam Absolute Determination of Gravity. Journal of Research of the National Bureau of Standards, Vol. 29, U.S. Department of Commerce, Washington, D.C., Nov. 1942.

6. Duerksen, J.A. Pendulum Gravity Data in the United States, Special Publication No. 244, Coast and Geodetic Survey, U.S. Department of Commerce, Washington, D.C., 1949.

7. Dunwoody, H. Laboratory Exercises, Notes, and Problems in Physics, pp. 75-89, John Wiley and Sons, Inc., New York, N.Y., 1917.

8. Heyl, P.R. and Cook, G.S. The Value of Gravity at Washington, Research Paper RP 946, Journal of Research of the National Bureau of Standards, Vol. 17, U.S. Department of Commerce, Washington, D.C., Dec. 1936.

9. Kater, Henry. An Account of Experiments for Determining the Length of the Pendulum Vibrating Seconds in the Latitude of London, Philosophical Transactions of the Royal Society of London, pp. 38-102, Vol. 108, 1818.

10. Kater, Henry. A Treatise on Mechanics, pp. 145-159, Longman, Rees, Orme, Brown, and Green; and John Taylor, London, 1830.

11. Kaye, G.W.C., and Laby, T.H. Physical and Chemical Constants and Some Mathematical Functions, pp. 11-15, Longmans, Green and Co., Ltd., London, 1928.

12. Moss, S.A. The Exact Values of "g" Through the United States, General Electric Review, Vol. 21, pp. 344-352, May 1918.

BIBLIOGRAPHY (Continued)

13. National Bureau of Standards Central Radio Propagation Laboratory. Standard Frequencies and Time Signals, W.W.V. and W.W.V.H., Letter Circular LC 974, National Bureau of Standards, U.S. Department of Commerce, Washington, D.C., Jan. 12, 1950.

14. Robeson, Frank L. Physics, pp. 174-182, The Macmillan Company, New York, N.Y., 1947.

15. Routh, E.J. Elementary Rigid Dynamics, pp. 64-80, Macmillan & Co., London, 1930.

16. ----------- Advanced Rigid Dynamics, pp. 255-257, Macmillan & Co., London, 1930.

17. Shedd, J.C., and Birchby, J.A. The Reversible Pendulum, The Physical Review, pp. 274-293, Vol. 25, Oct. 1907.

18. -----------, ----------------, and Birchby, W.N. Reversible Pendulum II, The Physical Review, pp. 110-124, Vol. 34, Feb. 1912.

19. Shortt, W.H. The Dissipation of Energy by a Pendulum Oscillating in Air at Low Pressures. Proceedings of the Physical Society, Section B, Vol. 36, Part 6, June 1950.

20. Swick, C.H. Determination of "g" by Means of the Free Swinging Pendulum, Bulletin 78, pp. 151-166, The National Research Council, The National Academy of Sciences, Washington, D.C., Feb. 1931.

21. ----------- Pendulum Gravity Measurements and Isostatic Reductions, Special Publication No. 232, Coast and Geodetic Survey, U.S. Department of Commerce, Washington, D.C., 1942.

22. von Karman, T., and Biot, M.A. Mathematical Methods in Engineering, pp. 115-130, McGraw-Hill Book Company, New York, N.Y., 1940.

23. Watson, W. The Pendulum-Measurement of "g" and Rating a Chronometer, A Textbook of Practical Physics, pp. 118-138, Longmans, Green & Co., London, 1908.

24. Webster, A.G. The Dynamics of Particles and of Rigid, Elastic, and Fluid Bodies, p. 48, Hafner Publishing Co., Inc., New York, N.Y., 1949.

BIBLIOGRAPHY (Continued)

25. Weld, L.R. Theory of Errors and Least Squares, "Reduction of Pendulum to Zero Arc" pp. 107-110, The Macmillan Company, New York, N.Y., 1916.

26. Young, Thomas. Remarks on the Probabilities of Error in Physical Observations and on the Density of the Earth, Considered, Especially with Regard to the Reductinn of Experiments on the Pendulum, Philosophical Transactions of the Royal Society of London, Vol. 109, pp. 70-95, 1819.

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