A STUDY OF THE LINK-CHAIN LIFO CONTROVERSY

by

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(ABSTRACT)

The accounting literature contains no evidence on the reliability of the link-chain variant of dollar-value LIFO as a method of inventory accounting as compared to the double-extension variant. The research produced the first evidence on the topic.

Process analyses of the two methods found both to be flawed, with the link-chain method seriously flawed. The link-chain method inappropriately incorporates the price-levels of periods when there is no annual layer to be restated. The resulting, and all subsequent, inventory valuations are misstated. The link-chain and double-extension methods can both produce misstated valuations in periods with layer erosion. The study identifies procedures to correct these errors.

Two quantitative experiments were conducted to evaluate the relationship of the inventory valuations produced by the two methods. The first experiment used a small amount real data; the second experiment used a large amount of synthesized data. The experimental results indicated the relationship of the valuations to be circumstantial. Based on the process analyses and the quantitative experiments, the link-chain LIFO method was determined to be an unreliable method of inventory accounting.

The quantitative experiments were also used to investigate related issues. The first experiment concluded that a dollar-value LIFO method based on Fisher's "ideal" index methodology was practicable. The second experiment concluded that adopting the method used by the Bureau of Labor Statistics to assign base-date costs to new or changed items that enter the CPI market basket of goods and services to

the double-extension LIFO method was practicable and would not diminish tax revenues. The study recommended that consideration be given to replacing the current double-extension and link-chain methods with a double-extension LIFO with the BLS method.

The second experiment also concluded that the IRS's inventory "turnover" test is probably based on the IRS's perception of what constitutes taxpayer practicality. The definition of practicality, however, is questioned and alternative definitions are suggested.

The study indicated that further research on the relationship of inventory valuations to the income taxation process is needed.

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Introduction

Inventory Valuations and Income Taxation

Taxpayers were first made aware of the income taxation implications of inventory valuations in 1918.[1] From that time, all taxpayers who generate income from the "production, purchase, or sale of merchandise" have been required to value their inventories (using specified methods) at the beginning and end of each taxable year.[2]

The taxpayer's income tax liability is affected because the inventory valuation is part of the taxpayer's cost-of-goods-sold figure computation, as follows:

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1 Revenue Act of 1918.

2 Regs. §1.471-1.

	Beginning Inventory
Plus :	Purchases & Inventory-Related Costs
Equals:	Cost of Goods Available For Sale
Less :	**Ending Inventory**
Equals:	Cost of Goods Sold

For taxpayers engaged in manufacturing, mining or merchandising businesses, the cost-of-goods-sold figure is part of the taxpayers' gross income from operations computation, as follows:[3]

	Total Sales
Less :	**Cost of Goods Sold**
Equals:	Gross Income From Operations

For purposes of income taxation, gross income is defined as "all income from whatever source derived, including **gross income** (emphasis added) from business activities."[4] From the preceding computations, it is seen that the ending inventory valuation is directly related to both the taxpayer's gross income and income tax liability. Thus, a valuation method that increases the cost-of-goods-sold figure also lowers both the ending inventory valuation and the income tax liability.

Conversely, a method that decreases the cost-of-goods-sold figure also increases both the ending inventory valuation and the income tax liability. The impounding of increased replacement costs in the ending inventory led to an illusory inventory

3 Regs. §1.61-3(a).

4 Internal Revenue Code §61.

profit, resulting in the mismatching of revenues and expenses. Many taxpayers were concerned by the income tax liability arising from inventory profits, and adopted a "normal" method of inventory valuation.

The normal stock concept asserted that businesses required a "base stock" of inventory for smooth and efficient operations. The taxpayer assigned a subjectively determined cost to the base stock. By holding the assigned cost constant, the taxpayer could effectively change the computation of the cost-of-goods-sold figure to:

Purchases and Inventory-Related Costs = Cost of Goods Sold

The increased costs of inventory replacement passed directly to the income statement, and taxpayers' inventory profit concerns were successfully addressed. The normal concept was widely adopted by taxpayers, despite a lack of legitimizing legislation.[5] The IRS officially discredited the normal concept in 1930 by prevailing against a base-stock method taxpayer.[6]

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5 Revenue Act of 1918, §203.

6 See *Lucas vs. Kansas City Structural Steel Co.*, U.S. 264, 74 L. Ed. 848 (1930). The Court did not discuss the logic of the normal concept. The method failed because of the arbitrariness of cost assignment to the "normal" stock.

LIFO Inventory Valuation

The demise of the normal concept forced many taxpayers to lobby Congress for remedial legislation that addressed the inventory profit issue. In 1938, taxpayers in fungible-goods industries were authorized to use a restricted version of the Last-In-First-Out (LIFO) inventory valuation method.[7] In 1939, the authorization was extended to all taxpayers with inventories.[8]

The LIFO method presumes that inventory replacement costs will generally be increasing, and views an inventory pool as containing two categories of items. The first category is made up of the items "in the opening inventory of the taxable year, in the order of acquisition and to the extent thereof."[9] LIFO acknowledges the normal concept by retaining the prior-period costs of the first-category items.

The second category is made up of the items "acquired during the taxable year."[10] These items are assigned costs during the current-period valuation process and are appended to the first-category items. Thus, a LIFO inventory pool consists of a base-date layer and a series of periodic increments, or annual layers.

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7 Revenue Act of 1938, §22(d). The LIFO option was extended only to fungible goods industries, principally the raw materials inventories of non-ferrous metal and tanning industries.

8 Revenue Act of 1939, §22(d).

9 Regs. §1.472-1(a).

10 Ibid.

LIFO Liquidations

Annual layers occur only when the total quantities in the ending inventory pool exceeds the total quantities in the beginning inventory pool. If there are equal quantities in the beginning and ending inventory pools, the opening inventory pool valuation is carried forward unchanged. If the quantities in the ending inventory pool fall short of the quantities in the beginning inventory pool, a LIFO layer liquidation has occurred.[11]

The taxpayer recognizes this erosion of the first-category items by reducing successive inventory layers, most recent layer first. The layers are reduced until the shortfall has been completely removed. When replacement prices have been generally increasing, the layer liquidation causes an inventory profit recognition. Moreover, the taxpayer cannot restore liquidated LIFO layers: subsequent annual layers must impound the higher price levels of the subsequent periods, not the price levels associated with the liquidated layers.

Specific-Goods LIFO

The LIFO authorized by the Congress in 1938 (the specific-goods LIFO) is tied to the physical flow of the items in the LIFO inventory pool. As originally contemplated, specific-goods LIFO would be used only by taxpayers in fungible-goods industries: the LIFO inventory pool would contain only items with

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11 Regs. §1.472-8(e)(2)(d)

virtually identical characteristics.[12] Thus, the measurement of quantities in an inventory pool would be expressed in terms of such common physical units as bushels, pounds, etc.

The fungibility requirement in the LIFO inventory pool precluded specific-goods LIFO from addressing the inventory profit concerns of taxpayers not in fungible-goods industries. The taxpayer was forced to create a new LIFO inventory pool when adding an item that was not "virtually identical" to items in an existing inventory pool. If the new item replaced another item, the decrease in the quantity of the replaced item would cause a liquidation in the older inventory pool. An inventory profit recognition, therefore, would occur.

Dollar-Value LIFO

McAnly suggested another method of measuring inventory pool quantities that addressed the concerns of taxpayers not in fungible-goods industries. The measurements of the quantities in a specific-goods LIFO inventory pool were fungible because the items in the inventory pool were fungible. McAnly suggested that the use of a fungible measuring unit would also produce fungible measurements of inventory pool quantities, even when the items in the inventory pool were not

12 Regs. §§1.472-1(d), 1.472-3(a) and 1.472-8(b)(3)(ii).

fungible.[13]

The fungible measuring unit was a stable-dollar value that represented the purchasing power applicable to the taxpayer's LIFO base-date inventory pool. The taxpayer could convert the current-period costs of annual layers to stable-dollar values by using a price-level index, and vice versa.[14] The expansion of the inventory pool directly addressed the inventory profit concern because one item's quantity increase could offset another item's quantity decrease.

The IRS opposed the dollar-value LIFO concept because of a perception that tax revenues would be inappropriately diminished.[15] In the 1947 *Hutzler Bros. Co.* case, the Service argued that LIFO method taxpayers were required to identify the specific articles in an inventory.[16] The Tax Court, stating that the IRS's position did not reflect Congressional intent, ruled for the taxpayer. In the 1948 *Basse* case, the

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13 See Kenneth G. Humer, William G. Galliher, Jr. and George A. Stewart, 69-4th T.M., *LIFO:Fundamentals, Pooling, and Computations*, at A-39; Herbert T. McAnly, "The Case for LIFO:It Realistically States Income and Is Applicable to Any Industry," *The Journal of Accountancy*, June 1953, pp. 693-97; and Herbert T. McAnly, "The Current Status of LIFO,"*The Journal of Accountancy*, May 1958, pp. 56-59.

14 For more complete information on dollar-value LIFO's evolution and development please see: Carman G. Blough, Samuel J. Broad and Robert M. Trueblood, "Pooling of LIFO Inventories By Use of Dollar-Value Method," memo to Fred C. Scribner, Jr., Under Secretary of the Treasury, dated February 23, 1960, pp. 2-4; Humer, Galliher and Stewart, at A-34 through A-39: McCanly, (1953), pp. 693-97, (1958), pp. 56-59; and Alan Silver, "LIFO: Summing Up," *Industrial Distributor*, March 1979, pp. 57-62.

15 Humer, Galliher, and Stewart at A-37.

16 *Hutzler Bros. Co. v. Commissioner*, 8 T.C. 14(1947).

Service's argument that LIFO method taxpayers were required to match the items in their ending inventory against the items in the beginning inventory was rejected, and the taxpayer again prevailed.[17]

After the decisions in *Hutzler* and *Basse*, the IRS authorized retailers to use a special dollar-value LIFO variant. The 1949 extension of the dollar-value LIFO option to all taxpayers did not permit taxpayers to use the expanded dollar-value LIFO inventory pools. Not until 1960 were manufacturers and processors able to gain authorization to use dollar-value LIFO with greatly expanded pools.[18]

Double-Extension Method

The double-extension variant of dollar-value LIFO is defined as:[19]

> a technique used in applying dollar value LIFO in which the current and base year costs of each item in inventory are extended, or multiplied, by the units on hand at the current year valuation date.

The IRS prefers this dollar-value LIFO method, which must generally be used by a taxpayer.[20] The need for accounting records from both the base-date and current periods may be inconvenient, but the method's major weakness is the process used

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17 *Basse v. Commissioner*, 10 T.C. 328(1948), acq. 1950-1 C.B. 1.

18 Regs. §1.472-8(e).

19 AICPA Task Force on LIFO Inventory Problems, *Identification and Discussion of Certain Financial Accounting and Reporting Issues Concerning LIFO Inventories - Issues Paper (ISUPAP No 42)*, (AICPA: November 30, 1984), Appendix VII.

20 Regs. §1.472-8(e)(2)(i).

to enter new items into the inventory pool.[21] A double-extension method taxpayer is required to ascertain a base-period cost for an item that enters the inventory pool for the first time, subsequent to the LIFO base date.[22]

Unless the taxpayer can satisfactorily establish or reconstruct either the base-period cost or some earlier-period cost, the current-period cost must be used as the base-period cost. For items in existence on the LIFO base date, the taxpayer may use catalogs, price lists or other documentation to establish a cost. The prospects for a successful base-period cost reconstruction, however, diminish as the LIFO base date becomes more distant. It seems unlikely that any taxpayer could satisfactorily reconstruct a 1960 base-period cost for a personal computer.

When the current-period cost is used as base-date cost, the new item's cost is assumed to have remained stable from the base period. This view of cost behavior is inconsistent with LIFO's basic assumption of generally-rising inventory prices and with common knowledge. The view of cost behavior, however, is consistent with the specific-goods LIFO requirement for creation of a new inventory pool when a dissimilar item enters the taxpayer's inventory. In both cases, an inventory profit recognition may arise.

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21 See AICPA Emerging Issues Task Force, *Issue Summary: LIFO Issues*, October 8, 1984, at Section IV-D; AICPA Task Force on LIFO Inventory Problems, Accounting Standards Division, *Identification and Discussion of Certain Financial Accounting and Reporting Issues Concerning LIFO Inventories - Issue Paper (ISUPAP 42)*. n.p., AICPA, November 30, 1984, Pars. 4-14 to 4-27; Willard J. De Fillips, "LIFO for Small Businesses," *The Practical Accountant*, May 1984, pp. 32-33; Humer, Galliher and Stewart, at A-11; Parker, p. 15; Silver, pp. 58, 62; Securities and Exchange Commission, *Certification of Financial Reporting Policies*, Financial Reporting Releases, 17 CFR Part 211, April 15, 1982, at Section 205.02.a; and Rasoul H. Tondkar et al., "How Small Businesses Can Benefit From LIFO," *The Ohio CPA Journal*, Spring 1982, p. 68.

22 Regs. §1.472-8(e)(2)(d)(iii)

Link-Chain Method

The link-chain variant of dollar-value LIFO is defined as:[23]

> a technique used in applying dollar value LIFO in which the base year cost of ending inventory is determined by applying a cumulative index to the dollar value of the ending inventory. The cumulative index is the relationship of the current year's prices to those of the prior year (based on either double extension or internal index) multiplied by the prior year's cumulative index, causing each year's index to be characterized as a link in a chain of indexes back to the base year.

Taxpayers are attracted to the link-chain method because of the ease in which new item introductions are processed. The establishment or ascertainment of a base-period cost for a new item is greatly simplified over the double-extension method. For computational purposes, the link-chain method considers the beginning of the current year to be the LIFO base date. Therefore, a new item's cost need be established or ascertained only as of the beginning of the current year. Unlike a double-extension taxpayer, the link-chain taxpayer is not plagued by a lack of data for reconstructing base-period costs.[24]

The IRS, however, severely restricts taxpayer access to this dollar-value LIFO variant. To use the link-chain method, a taxpayer must convince the district director that the use of either the double-extension or index methods is impractical or unsuitable in view of the taxpayer's particular LIFO inventory pool.[25] It is

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23 AICPA Task Force, Appendix VII.

24 See C. Richard Cox and Carl L. Glassberg, "LIFO: "The Deflator"- A Current Review and Analysis," *The Tax Adviser*, January 1971, pp. 30-31; Humer, Galliher and Stewart, at A-11; Taylor, p. 52; and Tondkar, et al., p. 68.

25 Regulations §1.472-8(e)(1).

understood that the so-called inventory "turnover" test, the only known IRS criterion for link-chain authorization, is based on the IRS's perception of taxpayer practicality.[26]

The taxpayer must provide the IRS with inventory-activity data covering at least a five-year period. From the submitted data, the IRS computes ratios of the items added to and dropped from the taxpayer's inventory pool. The taxpayer "passes" the test by having ratios that meet or exceed a five-year cumulative "turnover" rate that has varied from 95% to 75%.[27]

The Problem

Some taxpayers perceive the labyrinthine and restrictive LIFO regulations and policies as representing an IRS opposition to dollar-value LIFO. The IRS's opposition is purportedly based on a belief that more taxpayers would adopt dollar-value LIFO if the link-chain method could be used.[28] The literature contains no evidence to support such a supposition, however, and such a motive would violate the principle of taxpayer equity.

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26 See R. R. Collins, "Dollar-value LIFO: IRS criteria for changes to link-chain method," *The Tax Adviser*, July 1981, p. 420; De Fillips, p. 32; Taylor, p. 53; Schneider, pp. 14-38 to 14-40; and Schwartz, p. 43.

27 Collins, p. 421.

28 See De Fillips, p. 41; and Clayton T. Rumble, "So You Still Have Not Adopted LIFO," *Management Accounting*, October 1983, p. 64.

The IRS, however, apparently believes that the link-chain method is typically not an acceptable method of inventory accounting. This view of the link-chain method may reflect the IRS's belief that link-chain inventory valuations are numerically lower than the comparable double-extension valuations. The view may also result from a belief that the link-chain valuations may have a consistency that is lower than the consistency of the comparable double-extension valuations. The IRS has been mute on the basis for denying access to the link-chain method. Taxpayer consternation with the lack of IRS justification is typified by Humer, Galliher and Stewart, who state:[29]

> It is difficult to comprehend the reasons for the Commissioner's preference for methods other than the link-chain technique, in view of the many advantages this procedure has from an administrative viewpoint.

The proponents of increased link-chain access, however, have failed to shoulder the taxpayer's traditional burden of proof. No evidence that indicates that the link-chain method is an acceptable method of inventory accounting has been presented. A review of the accounting and income tax literatures produced only assertions and assumptions, not evidence, that a taxpayer who uses the link-chain method has an income tax advantage over a taxpayer who uses the double-extension method.[30] The problem, therefore, is the lack of evidence on the link-chain method's acceptability.

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29 Humer, Galliher and Stewart, at A-11.

30 See Bill N. Schwartz, "Dollar Value LIFO - Evaluating One Experience," *The CPA Journal*, April 1982, p. 44; and Silver, p. 58.

Importance of the Problem

The lack of easy taxpayer access to link-chain LIFO has been called "probably the most significant and controversial LIFO-related issue."[31] Humer, Galliher and Stewart agree with the seriousness of the problem by stating:[32]

> Indeed the endurance of LIFO as a fundamentally sound method of inventory accounting may well depend upon the adaptation of its computational procedures in a flexible and practical manner to ever-changing operating and technological business circumstances.

The requests of link-chain advocates for increased access can be legitimized only if accompanied by evidence that the link-chain method produces inventory valuations that are not lower than the double-extension method valuations, and that do not vary circumstantially. This study is important because it will produce evidence on the consistency and valuation-comparability issues.

The Research Approach

The research consists of comparing certain characteristics (attributes) of the double-extension method with the same characteristics of the link-chain method, and drawing conclusions from the results of the comparisons. There are two

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31 De Fillips, p. 32.

32 Humer, Galliher, and Stewart, at A-11.

standard methods of making such pair-wise comparisons. The first method involves an indirect comparison: a third entity is used as a common referent (*benchmark*) against which the two entities of concern are independently compared. The second method involves the direct comparison of the two entities of interest against each other: one of the two entities is assumed to be of benchmark quality.

The research will assume that the double-extension method is of benchmark quality, as implied by the IRS's preference for the method. The accounting literature, however, provides neither evidence nor IRS statements to support an assumption that the double-extension method produces fair inventory valuations. The research, therefore, must appropriately offer an assessment of the merits of the double-extension method and the valuations it yields.

The two issues of concern to the research are the consistency issue and the valuation-comparability (quantity) issue. The quality of an inventory valuation method is measured by the consistency of it's valuations. Quality is lost when the valuation process is flawed and misstated valuations are produced: when valuations of the same inventory pool vary circumstantially.

The valuation-comparability issue, actually a quantity issue, is concerned with the relationship between the process's numeric output. The study uses the output from the double-extension method as the benchmark against which the link-chain method is compared. The finding of the issue will depend on the relationship of the link-chain valuations to the comparable double-extension valuations. If the link-chain valuations are less than the double-extension valuations, the study will find against the link-chain method.

The consistency and valuation-comparability (quantity) issues form a classical analysis of the risk of replacing an older process. For example, consider a manufacturing operation that has two manufacturing processes suitable for outputting a certain good. Both processes are in use throughout the manufacturing operation, although management has a preference for one of the processes and limits the use of the other to what it considers to be hardship cases.

One unit of the manufacturing process is currently using the preferred process, but wishes to change to the other process for reasons of operating efficiency. Because the administrative infrastructure for both processes already exist, setup costs are irrelevant. Management needs the answers to only two questions to assess the risk of changing from the preferred process to the non-preferred process. They are: (1) "Will the quantity of output decrease?", and (2) "Will the consistency of the output decrease?" If the answers to both questions are, "No", management will lose nothing by permitting the change.

To the IRS, the inventory valuation process is a component of the larger process of collecting income tax revenues. The study assumes the IRS's view on the valuation-comparability (quantity) issue to be that replacing the double-extension method with the link-chain method will decrease the inventory-valuation process's output. Likewise, the study assumes the IRS's view on the consistency issue to be that replacing the double-extension method with the link-chain method will lower the inventory-valuation process's output.

This study, therefore, is a risk analysis that involves the two dollar-value LIFO variants. A difference between this study and a generic risk analysis is that the costs associated with a changeover are irrelevant because they will be absorbed by the taxpayer. The study will recognize the taxpayer's burden of proof in matters before the IRS by finding the link-chain method to be an acceptable method of inventory valuation only if both analytic findings are favorable to the link-chain method.

Outline of the Study

The analysis of the consistency issue is contained in Chapter 2. Because the processes are more easily analyzed in their symbolic form, a symbology is developed and the two valuation methods are described by sets of equations. The processes' elements are analyzed and two general areas are identified as having the potential to decrease the consistency of the processes' valuations. Authoritative research on price-level indexes is introduced and used to analyze the price-level index type area.

In Chapter 3, the analysis of the valuation-comparability (quantity) issue begins by developing an experimental methodology. The sets of equations describing the two valuation processes are converted to a computer program to facilitate the numerous computations involved with the dollar-value LIFO processes. Two quantitative experiments are designed to produce data to be used in tests of the relationship between the double-extension and link-chain valuation, hypotheses are developed, and statistical tests are conducted.

Two topics that grew out of the analysis of the consistency issue are integrated into the two quantitative experiments. In the first experiment, a dollar-value LIFO valuation method based on Fisher's "ideal" index is introduced. Small amounts of real data are used to validate the computer model's computations and to test the practicability of the "ideal" dollar-value LIFO method.

In the second experiment, the methodology used by the Bureau of Labor Statistics to update the market basket of goods and services indexed by the Consumer Price Index (CPI). Large amounts of synthesized data are used to test for differences between the double-extension and link-chain valuations, for the impact of using the BLS methodology, and for a tax-revenue related rationale for the IRS's so-called inventory "turnover" test.

The analysis of the valuation-comparability issue ends in Chapter 4 with discussions of the hypotheses tested in the preceding chapter. Also reported are the results of sensitivity analyses performed on the hypotheses tests of the second quantitative experiment. In Chapter 5, the findings of the study are summarized, conclusions are reached, the limitations on the generalizability of the research results are discussed, and possible extensions of the research are identified.

The Consistency Issue

The consistency issue is concerned with the ability of the valuation processes to produce inventory pool valuations that do not vary circumstantially. This chapter contains two main sections. The first main section is devoted to price-level index concerns, and is divided into price-level index use and price-level index type subsections. Fisher's research on price-level indexes is integrated into the analysis, and the CPI and link-chain LIFO are compared and contrasted. The second main section contains an analysis of the quality implications of the methods' processes for new item introduction. The price-level literature and the BLS method for new item introductions are examined.

Dollar-Value LIFO Introduction

The double-extension and link-chain dollar-value LIFO processes were converted to systems of equations prior to beginning the data analyses.[33] A standard pq notation was used: the p representing the unit-cost attribute and the q representing the quantity attribute. An individual item's dollar value is represented as pq and an inventory pool's dollar-value can be represented by $\sum pq$.

It was necessary to use subscripts to distinguish between time periods. For example, the current-period (*n*) ending inventory's dollar value at base-period costs is $\sum p_0 q_n$; the dollar value at beginning-of-the-current-year costs is $\sum p_{n-1} q_n$. The double-extension LIFO equations are found in Figure 1 on page 20; the link-chain LIFO equations are contained in Figure 2 on page 21.

33 For previous discussion of the double-extension method; see above, pp. 8-9. For previous discussion of the link-chain method; see above, pp. 9-11.

Base-Period Cost Of The Ending Inventory, EI_0

$$EI_0 = \sum p_0 q_n$$

where:

p_0 is the base-period cost.

q_n is the current-period quantity.

Base-Period Cost Of The Annual Layer, AL_0

$$AL_0 = EI_0 - BI_0$$

where:

BI_0 is the beginning inventory's base-period cost.

If amount is negative, a LIFO liquidation has occurred.
If amount is zero, the valuation is unchanged and the process stops.

Double-Extension Price-Level Index, IX_{DEP}

$$IX_{DEP} = \frac{\sum p_n q_n}{EI_0}$$

where:

p_n is the current-period cost.

LIFO Valuation Of The Annual Layer

$$AL_0 \times IX_{DEP}$$

Figure 1. Equations for Dollar-Value LIFO, Double-Extension Process

Annual Price-Level Index, ANN_n

$$ANN_n = \frac{\sum p_n q_n}{\sum p_{n-1} q_n}$$

where:

p_{n-1} is the prior-period cost.

Cumulative Price-Level Index, CUM_n

$$CUM_n = CUM_{n-1} \times ANN_n$$

where:

CUM_{n-1} is the prior-period CUM.

Base-Period Cost Of The Ending Inventory, EI_0

$$EI_0 = \frac{\sum p_n q_n}{CUM_n}$$

Base-Period Cost Of The Annual Layer, AL_0

$$AL_0 = EI_0 - BI_0$$

If amount is negative, a LIFO liquidation has occurred.
If amount is zero, the valuation is unchanged and the process stops.

LIFO Valuation Of The Annual Layer

$$AL_0 \times CUM_n$$

Figure 2. Equations for the Dollar-Value LIFO, Link-Chain Process

The analyses of this study require an expanded understanding of the dollar-value LIFO process.[34] A dollar-value LIFO inventory pool can be viewed as a series of costs and the dollar-value LIFO price-level indexes associated with those costs. The two costs in each layer of the series are denominated in dollars. One cost (base-period cost of the annual layer) is expressed in terms of the costs applicable to that inventory pool on the LIFO base date. The second cost (LIFO valuation of the annual layer) is expressed in terms of the costs of the period in which the annual layer occurred.

The relationship between the base-period cost of an annual layer and that layer's LIFO valuation is determined by the dollar-value LIFO price-level index. Algebraically, the relationship is as follows:

$$LAYERBD_n \times INDEX_n = LAYERLIFO_n$$

where:

LAYERBD is a layer's base-date cost.

INDEX is a dollar-value LIFO price-level index.

n denotes the associated measurement period.

LAYERLIFO is a layer's LIFO valuation.

The relationship can also be stated as follows:

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34 For discussion of dollar-value LIFO; see above, pp. 6-8.

$$LAYERBD_n = \frac{LAYERLIFO_n}{INDEX_n}$$

Thus, a dollar-value LIFO inventory pool for *n* periods can be represented by the following series of annual layers' base-date costs and price-level indexes:

$LAYERBD_0$	100%
$LAYERBD_1$	$INDEX_1$
$LAYERBD_2$	$INDEX_2$
...	...
$LAYERBD_n$	$INDEX_n$

This representation of a dollar-value LIFO inventory pool shows that the LIFO-layer liquidation process is a process of replacing a pre-liquidation series with a new, post-liquidation series.[35] The price-level index's role in the dollar-value LIFO valuation process is clearly seen. Each price-level index captures the relationship between the price-level of the base period and the price-level of a subsequent period. The function of the price-level index is to convert an annual layer's base-period cost into terms of the price levels of the period of the annual layer.

As seen in the above tabular representation, there is a one-to-one relationship between an annual layer (represented by its base-period cost) and its price-level index. Without an associated annual layer, there is no place for a price-level index.

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35 For discussion of the LIFO layer-liquidation process; see above pp. 4-5.

Price-Level Index Concerns

The dollar-value LIFO price-level index is an integral element of both the double-extension and link-chain valuation methods. Therefore, the analysis of the consistency issue began by examining the role of the price-level index in the double-extension and link-chain processes. That examination was eased by splitting the price-level index issues into two separate areas. The two areas, separately discussed below, deal with the methods' uses of price-level indexes and the type of price-level indexes.

Price-Level Index Use

Dollar-value LIFO inventory pools expand through the process of appending annual layers (second-category items) to beginning inventories (first-category items), producing the beginning inventory of the following period.[36] The conversion of the base-date cost of the annual layers to LIFO values is accomplished through the use of the valuation method's price-level index. If the inventory pool does not expand (there is no annual layer), the creation and use of a dollar-value LIFO price-level index seems inappropriate.

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36 For discussion of dollar-value LIFO; see above, pp. 6-8.

The double-extension method's price-level index is derived only when there is an annual layer; thus, the one-to-one relationship between annual layers and price-level indexes is maintained. The link-chain method, however, derives and uses its annual price-level index regardless of the existence of an annual layer. The link-chain method, therefore, violates the one-to-one relationship between annual layers and price-level indexes. The reason given for this seemingly inappropriate use of a dollar-value LIFO price-level index is that the annual price-level index must be used in determining the base-period cost of the link-chain LIFO inventory pool.[37]

Is it necessary to use the link-chain method's annual price-level index to determine the ending inventory's base-period cost? The formula used by the link-chain method to determine the base-period cost of the ending inventory is as follows:[38]

$$\sum p_0 q_n = \frac{\sum p_n q_n}{CUM_n}$$

where:

$\sum p_0 q_n$ is the base-period price of the ending inventory.

$\sum p_n q_n$ is the current-period price of the ending inventory.

CUM_n is the current-period cumulative price-level index.

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37 For link-chain method equations; see above, Fig. 2, on p.21.

38 Ibid.

It is possible to decompose the CUM_n as follows:[39]

$$CUM_n = CUM_{n-1} \times ANN_n$$

where:

CUM_{n-1} is the prior-period cumulative price-level index.

ANN_n is the current-period annual price-level index.

The ANN_n is itself decomposable, as follows:[40]

$$ANN_n = \frac{\sum p_n q_n}{\sum p_{n-1} q_n}$$

where:

$\sum p_{n-1} q_n$ is the ending inventory's prior-period cost extension.

Substituting into the formula for the base-period cost of the ending inventory produces the following:

39 Ibid.

40 Ibid.

$$\sum p_0 q_n = \frac{\sum p_n q_n}{CUM_n}$$

$$= \frac{\sum p_n q_n}{CUM_{n-1} \times ANN_n}$$

$$= \frac{\sum p_n q_n}{CUM_{n-1}} \div \frac{\sum p_n q_n}{\sum p_{n-1} q_n}$$

$$= \frac{\sum p_n q_n}{CUM_{n-1}} \times \frac{\sum p_{n-1} q_n}{\sum p_n q_n}$$

$$= \frac{\sum p_{n-1} q_n}{CUM_{n-1}}$$

The link-chain method, therefore, does not require the current-period price-level index to compute the base-period cost of the ending inventory. Does the incorporation of the ANN_n lower the quality (consistency) of the inventory valuation? Assume that a taxpayer's inventory pool is valued by using the link-chain method. The taxpayer's LIFO inventory pool has not expanded in the current period. Thus, the ANN_n is incorporated into the CUM_n.[41]

The taxpayer again values the inventory pool at the end of the next measurement period. The base-period cost is derived as follows:

$$\sum p_0 q_{n+1} = \frac{\sum p_n q_{n+1}}{CUM_n}$$

41 Ibid.

In the following numerical example, a "zero" layer occurred in Period 1 and the previous inventory pool valuation was carried forward unchanged. There was, therefore, no annual layer and no need to update the CUM_n by incorporating into it the ANN_n. Has the inappropriate incorporation of the annual price-level index diminished the consistency of the link-chain method's output?

Period	0 (n-1)		1 n		2 (n+1)	
Item	P	Q	P	Q	P	Q
A	5	6	6	10	5	12
B	5	6	6	4	5	12
C	1	40	2	30	1	80

	0	1	2
Annual Index	100.00%	144.00%	65.79%
Cumulative Index	100.00%	144.00%	94.74%
Base-Date Cost	\$100.00	\$100.00	\$211.11
Layer	\$100.00	\$0.00	\$111.11
Cum. LIFO Valuation	\$100.00	\$100.00	\$205.27

The ending inventory pool of Period 2 holds exactly twice the quantities of the base-period inventory pool. Thus, the appropriate base-period cost of the ending inventory of Period 2 is seen to be \$200.00. From the above numerical example, the ending inventory pool of Period 2 was assigned a base-period cost of \$211.11. This indicates that the inappropriate incorporation of the ANN_n has damaged the ability of the link-chain method to produce consistent inventory valuations.

If the base-period cost of the ending inventory of Period 2 is to be correctly computed, Period 1 should be ignored. Therefore, the correct formula is:

$$\sum p_0 q_{n+1} = \frac{\sum p_{n-1} q_{n+1}}{CUM_{n-1}}$$

A numerical example demonstrating the validity of ignoring Period 1 is presented below.

Period	0		2	
Item	P	Q	P	Q
A	5	6	5	12
B	5	6	5	12
C	1	40	1	80

	0	2
Annual Index	100.00%	100.00%
Cumulative Index	100.00%	100.00%
Base-Date Cost	$100.00	$200.00
Layer	$100.00	$100.00
Cum. LIFO Valuation	$100.00	$200.00

By ignoring Period 1 and not incorporating the ANN_1, the base-period cost of $200.00 was correctly computed. Therefore, the ANN_n need not be captured for periods in which there is no annual layer. The algebraic formula for the magnitude of the misstatement caused by the ANN_n can be derived as follows.

The correct formula for computing the base-period cost of the ending inventory of Period 2 was:

$$\sum p_0 q_{n+1} = \frac{\sum p_{n-1} q_{n+1}}{CUM_{n-1}}$$

where:

$$\sum p_0 q_{n+1} = \sum p_0 q_2$$

$$\sum p_{n-1} q_{n+1} = \sum p_0 q_2$$

$$CUM_{n-1} = CUM_0$$

The incorrect formula for computing the base-period cost of the ending inventory of Period 2 was:

$$\sum p_0 q_{n+1} = \frac{\sum p_n q_{n+1}}{CUM_n}$$

where:

$$\sum p_0 q_{n+1} = \sum p_0 q_2$$

$$\sum p_n q_{n+1} = \sum p_1 q_2$$

$$CUM_n = CUM_1$$

The formula for the magnitude of the misstatement of the base-period cost of the ending inventory of Period 2, therefore, can be determined by subtracting the correct formula from the incorrect formula, as follows:

$$\frac{\sum p_n q_{n+1}}{CUM_n} - \frac{\sum p_{n-1} q_{n+1}}{CUM_{n-1}}$$

It is possible to isolate the effect of the ANN_n by remembering that:

$$CUM_n = CUM_{n-1} \times ANN_n$$

After substituting for the CUM_n, and then factoring out the CUM_{n-1} term, the formula for the misstatement is:

$$\frac{1}{CUM_{n-1}} \left(\frac{\sum p_n q_{n+1}}{ANN_n} - \sum p_{n-1} q_{n+1} \right)$$

Examining the data from the two preceding numerical examples shows the following:

$n = 1$

$CUM_{n-1} = CUM_0 = 1.00$

$\sum p_n q_{n+1} = \sum p_1 q_2 = \$304$

$ANN_n = ANN_1 = 1.44$

$\sum p_{n-1} q_{n+1} = \sum p_0 q_2 = \$200.$

This data can be used to validate the correctness of the formula that represents the magnitude of the misstatement. Substituting the numerical data into the formula, the misstatement of the base-period cost of the ending inventory of Period 2 is correctly stated as:

$$\frac{1}{1}\left(\frac{\$304}{1.44}-\$200\right)=\$11.11$$

In the preceding example, a "zero" layer led to the inappropriate incorporation of the ANN_n, and the false inflation of the base-period cost of the next-period's ending inventory. Three additional numerical examples are given below for other possible inappropriate incorporations of the ANN_n.

In the following example, the second of the four numerical examples, a "zero" layer causes the inappropriate incorporation of the ANN_n. The result is that the base-period cost of the ending inventory of the next period is falsely deflated, causing an unwarranted LIFO-layer liquidation.

Period	0		1		2	
Item	P	Q	P	Q	P	Q
A	5	6	4	10	5	6
B	5	6	5	4	5	6
C	1	40	1	30	1	40

	0	1	2
Annual Index	100.00%	90.00%	113.64%
Cumulative Index	100.00%	90.00%	102.27%
Base-Date Cost	$100.00	$100.00	$97.78
Layer	$100.00	$0.00	($2.22)
Cum. LIFO Valuation	$100.00	$100.00	$97.78

Period	0		2	
Item	P	Q	P	Q
A	5	6	5	6
B	5	6	5	6
C	1	40	1	40

Annual Index	100.00%	100.00%
Cumulative Index	100.00%	100.00%
Base-Date Cost	$100.00	$100.00
Layer	$100.00	$0.00
Cum. LIFO Valuation	$100.00	$100.00

LIFO-layer liquidations can also cause the ANN_n to be inappropriately impounded. Below, in the third of the four numerical examples, the LIFO-layer liquidation causes the base-date cost of the next period's ending inventory to be falsely inflated.

Period	0		1		2	
Item	P	Q	P	Q	P	Q
A	5	6	6	5	5	6
B	5	6	6	3	5	6
C	1	40	2	20	1	40

Annual Index	100.00%	146.67%	65.79%
Cumulative Index	100.00%	146.67%	96.49%
Base-Date Cost	$100.00	$60.00	$103.64
Layer	$100.00	($40.00)	$43.64
Cum. LIFO Valuation	$100.00	$60.00	$102.11

Period	0		2	
Item	P	Q	P	Q
A	5	5	5	6
B	5	3	5	6
C	1	20	1	40

Annual Index	100.00%	100.00%
Cumulative Index	100.00%	100.00%
Base-Date Cost	$60.00	$100.00
Layer	$60.00	$40.00
Cum. LIFO Valuation	$60.00	$100.00

In the fourth numerical example, a LIFO-layer liquidation leads to the inappropriate incorporation of the ANN_n. The result is a false deflation of the base-period cost of the next-period's ending inventory that precipitates an unwarranted liquidation.

Period	0		1		2	
Item	P	Q	P	Q	P	Q
A	5	6	1	2	5	6
B	5	6	6	6	5	6
C	1	40	1	40	1	40

	0	1	2
Annual Index	100.00%	97.50%	121.95%
Cumulative Index	100.00%	97.50%	118.90%
Base-Date Cost	$100.00	$80.00	$84.10
Layer	$100.00	($20.00)	$4.10
Cum. LIFO Valuation	$100.00	$80.00	$84.88

Period	0		2	
Item	P	Q	P	Q
A	5	6	5	6
B	5	6	5	6
C	1	40	1	40

	0	2
Annual Index	100.00%	100.00%
Cumulative Index	100.00%	100.00%
Base-Date Cost	$100.00	$100.00
Layer	$100.00	$0.00
Cum. LIFO Valuation	$100.00	$100.00

The misstatement caused by the inappropriate-incorporation error is propagated to all subsequent inventory valuations through the multiplicative mechanism of the link-chain cumulative price-level index. As seen above in the four numerical examples, the misstatement may be either an overstatement or an understatement.

The study makes no comment on the materiality of the errors caused by the inappropriate incorporation of the ANN_n. However, misstatements will be found in the accounting records of link-chain method taxpayers whose inventory pools have undergone either a "zero" layer or a LIFO-layer liquidation subsequent to the LIFO base date. The existence of this procedural flaw means that the link-chain method cannot be relied on to produce consistent valuations. Because this error is not found in the double-extension method, the analytic finding is that the consistency of the link-chain method's valuations is lower than the consistency of the comparable double-extension valuations.

The possibility of the *inappropriate-incorporation* type of index-use error can be eliminated by updating the CUM_n only when an annual layer exists. This means that the base-period cost of the ending inventory must be computed without use of the ANN_n as developed in this study.[42] A link-chain process so modified was developed, and the equations describing the modified link-chain process are found in Figure 3 on page 36.

[42] For development of base-period cost computation; see above, pp. 25-28.

Base-Period Cost Of The Ending Inventory, EI_0

$$EI_0 = \frac{\sum p_{n-1} q_n}{CUM_{n-1}}$$

where:

p_{n-1} is the prior-period cost.

CUM_{n-1} is the prior-period CUM.

Base-Period Cost Of The Annual Layer, AL_0

$$AL_0 = EI_0 - BI_0$$

where:

BI_0 is the beginning inventory's base-period cost.

If amount is negative, a LIFO liquidation has occurred.
If amount is zero, the valuation is unchanged and the process stops.

Annual Price-Level Index, ANN_n

$$ANN_n = \frac{\sum p_n q_n}{\sum p_{n-1} q_n}$$

Cumulative Price-Level Index, CUM_n

$$CUM_n = CUM_{n-1} \times ANN_n$$

LIFO Valuation Of The Annual Layer

$$AL_0 \times CUM_n$$

Figure 3. Equations for the Modified Link-Chain Process

LIFO-Layer Liquidation Errors

A LIFO-layer liquidation may lead to the deterioration of the quality of the valuations produced by both the double-extension and link-chain methods. The liquidation process replaces the pre-liquidation series of layers and associated price-level indexes with a new, post-liquidation series of layers and associated indexes.[43] The LIFO-layer liquidation error occurs when a pre-liquidation index is inappropriately used to assign a LIFO valuation to a post-liquidation layer.

This error has different effects on the double-extension and link-chain methods because the link-chain computation of the base-period cost of the post-liquidation layer may be imprecise. The LIFO-layer liquidation error, therefore, is separately discussed below for the double-extension and link-chain methods.

Double-Extension Error

Period	0		1		2	
Item	P	Q	P	Q	P	Q
A	5	6	7	11	5	7
B	5	6	8	9	5	7
C	1	40	2	50	1	40

	0	1	2
Price-Level Index	100.00%	166.00%	N/A
Base-Date Cost	$100.00	$150.00	$110.00
Layer	$100.00	$50.00	($40.00)
Cum. LIFO Valuation	$100.00	$183.00	$116.60

Period	0		2	
Item	P	Q	P	Q

43 For discussion of LIFO-layer liquidations; see above, pp. 4-5, 23.

A	5	6	5	7
B	5	6	5	7
C	1	40	1	40

Price-Level Index	100.00%	100.00%
Base-Date Cost	$100.00	$110.00
Layer	$100.00	$10.00
Cum. LIFO Valuation	$100.00	$110.00

By referring to the preceding numerical data, it is seen that the LIFO-layer liquidation in Period 2 caused the partial liquidation of the Period 1 annual layer. The base-period cost of the ending layer has been precisely stated because the double-extension method determines the cost by a direct extension at base-period costs. There is a misstatement in the inventory's LIFO valuation in the amount of $6.60, ($116.60 - 110.00).

The misstatement was caused by multiplying the post-liquidation layer by a pre-liquidation price-level index that was not appropriate to the product mix in the ending inventory of Period 2. There is no misstatement if the product mix from which the pre-liquidation price-level index was derived is the same as the product mix in the current-period's ending inventory. Otherwise, the magnitude of the misstatement can be determined as below.

The correct formula for the LIFO valuation is:

$$Layer^{POST} \times Index^{POST}$$

where:

$Layer^{POST}$ is the layer's post-liquidation, base-period cost.

$Index^{POST}$ is the appropriate index for the layer's product mix.

The incorrect formula for the LIFO valuation is:

$$Layer^{POST} x\, Index^{PRE}$$

where:

$Index^{PRE}$ is the pre-liquidation, price-level index.

The formula for the amount of the misstatement is obtained by subtracting the correct formula for the LIFO valuation from the incorrect formula for the LIFO valuation, as follows:

$$\left(Layer^{POST} \times Index^{PRE}\right) - \left(Layer^{POST} \times Index^{POST}\right)$$

This equation can be simplified by factoring out the $Layer^{POST}$ term, and results in:

$$Layer^{POST} \times \left(Index^{PRE} - Index^{POST}\right)$$

The data from the immediately preceding numerical example can be used to validate the formula for the magnitude of the misstatement. Extracting from the example, it is seen that:

$$Layer^{POST} = \$100.00$$

$$Index^{PRE} = 166\%$$

$Index^{POST} = 100\%$

Substituting the data into the formula for the misstatement of the double-extension valuation caused by the LIFO-layer liquidation error results in the following correct computation:

$$\$100.00 \times (166\% - 100\%) = \$10.00 \times 66\% = \$6.60$$

The relationship between the pre-liquidation and product-mix appropriate price-level indexes determines whether the misstatement is an understatement or overstatement. The possibility of the LIFO-layer liquidation error can be eliminated by requiring the taxpayer to recompute the double-extension price-level index for the most recent annual layer, after the recognition of a LIFO-layer liquidation.

Link-Chain Error

Period	0		1		2	
Item	P	Q	P	Q	P	Q
A	5	6	7	11	5	7
B	5	6	8	9	5	7
C	1	40	2	50	1	40

	0	1	2
Annual Index	100.00%	166.00%	59.46%
Cumulative Index	100.00%	166.00%	98.70%
Base-Date Cost	$100.00	$150.00	$111.45
Layer	$100.00	$50.00	($38.55)
Cum. LIFO Valuation	$100.00	$148.75	$119.01

Period	0		2	
Item	P	Q	P	Q
A	5	6	5	7
B	5	6	5	7
C	1	40	1	40
Annual Index	100.00%		100.00%	
Cumulative Index	100.00%		100.00%	
Base-Date Cost	$100.00		$110.00	
Layer	$100.00		$10.00	
Cum. LIFO Valuation	$100.00		$110.00	

Referring to the immediately preceding numerical data, the LIFO-layer liquidation in Period 2 caused the partial liquidation of the Period 1 annual layer. Unlike the double-extension method, however, the link-chain method does not compute the base-period cost by directly extending the ending inventory quantities at base-period costs. Thus, the base-period cost of the post-liquidation layer is overstated in the amount of $1.45, ($11.45 - $10.00).

This misstatement was not caused by an inappropriate incorporation of the link-chain method's annual price-level index. The corrective action, however, is the same: the valuation of Period 1 is ignored. The formula previously developed for inappropriate-incorporation misstatements can be used to determine the amount of this misstatement of the base-period cost; the formula is:[44]

$$\frac{1}{CUM_{n-1}} \left(\frac{\sum p_n q_{n+1}}{ANN_n} - \sum p_{n-1} q_{n+1} \right)$$

44 For discussion of inappropriate-incorporation error and formula; see above, pp. 27-35.

By examining the preceding numerical data, it is seen that:

n, the period of concern, is equal to 1

$$CUM_{n-1} = CUM_0 = 100\%$$

$$\sum p_n q_{n+1} = \sum p_1 q_2 = \$185$$

$$ANN_n = ANN_1 = 166\%$$

$$\sum p_{n-1} q_{n+1} = \sum p_0 q_2 = \$110$$

The amount of the misstatement in this case is determined by substituting the numerical data into the formula. The correct computation of $1.45 is arrived at as follows:

$$\frac{1}{1}\left(\frac{\$185}{1.66} - \$110\right) = \$1.45$$

The inventory's LIFO valuation has also been misstated; the overstatement is in the amount of $9.01 ($119.01 - $110.00). The formula previously developed for misstatements caused by multiplying a post-liquidation layer by an inappropriate pre-liquidation index can be used to examine this misstatement; the formula is:[45]

$$Layer^{POST} \times \left(Index^{PRE} - Index^{POST}\right)$$

From the data of this numerical example, it is seen that:

$$Layer^{POST} = \$11.45$$

45 For discussion of using inappropriate, pre-liquidation index; see above, pp. 37-40.

$$Index^{PRE} = 166\%$$

$$Index^{POST} = 100\%$$

Entering the data into the formula produces the following computation:

$$\$11.45 \times (166\% - 100\%) = \$11.45 \times 66\% = \$7.56$$

There are two LIFO-layer liquidation errors in the link-chain method. The base-period cost of the post-liquidation layer has been overstated in the amount of $1.45, and the use of an inappropriate price-level index caused an overstatement of $7.56. Added together, the individual errors ($1.45 and $7.56) sum to the amount ($9.01) of the overstatement of the LIFO value assigned to the post-liquidation layer.

The possibility of link-chain LIFO-layer liquidation errors can be eliminated. The first step in the elimination process is to require the taxpayer to recompute the base-period cost of the ending inventory in the manner developed in this study.[46] The second step is to require the taxpayer to recompute the annual and cumulative price-level indexes appropriate to the most recent, post-liquidation annual layer.

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46 For discussion of the method of recomputing the base-period cost of the ending inventory; see above, pp. 29-32.

Summary

Both the link-chain and double-extension methods are flawed by the potential of producing misstated LIFO valuations after the recognition of a LIFO-layer liquidation. Because both methods have this flaw, the link-chain method's consistency is not diminished more than is the double-extension method's consistency. The link-chain method, however also has the potential of misstating the base-period cost of the post-liquidation layer. Thus, the analytic finding is that the consistency of the link-chain measurement is lower than the comparable double-extension measurement.

This study makes no attempt to assess the materiality of the misstatements caused by the LIFO-layer liquidation errors. However, the accounting records of both double-extension and link-chain LIFO taxpayers whose inventories have undergone LIFO-layer liquidations are misstated to the extent defined in this section.

Price-Level Index Type

Irving Fisher performed the authoritative research on the types of index numbers, and their relative ranking.[47] His research objective was to determine the construction of index number that was best able to represent changes to the value of collections of items over time. Fisher used price and quantity data for 36

47 Irving Fisher, *The Making of Index Numbers*, (Houghton Mifflin Company, Boston, 1927.) Although Fisher investigated more than 180 types, many formulae were reduced to duplicates. As a result, the final ratings compared 119 unique index-number types.

commodities marketed in the United States during WWI to test more than 180 types of price-level and quantity indexes. A copy of Fisher's price data is in Table 18 on page 158; a copy of Fisher's quantity marketed data is in Table 19 on page 159.

Index numbers are yardsticks (often stated as percentages) used to measure changes in groups of data over time. In this study's notation, $\sum p_0 q_0$, and $\sum p_n q_n$ represent the valuations of a taxpayer's inventory pool at times *0* and *n*. The taxpayer can express the relationship (stated in percentages) between the two valuations by dividing the base-period valuation into the current valuation, and multiplying by 100, as below:

$$I_v = \frac{\sum p_n q_n}{\sum p_0 q_0} \times 100$$

The resulting value index, (I_v), is a single, verifiable number that describes the change in the value of the taxpayer's inventory pool. The value of an inventory pool, however, is a function of the price and quantity attributes of the items in the inventory pool. Thus, a value index is seen to be the product of two other indexes, as below:

$$I_v = I_p \times I_q$$

where:

I_p is a price level index.

I_q is a quantity index.

Derivation Methods

Fisher's research also encompassed the two alternatives for deriving any type of index number: the fixed-base system and the chain system.[48] Under the fixed-base system alternative, index numbers are computed relatively to the same base period. Under the chain system alternative, a two-step process is used to compute index numbers. The first step of the chain system process is to compute an index number for the current period by treating the end of the prior year as the base period. In the second step, the first step's annual index is multiplied by all previous annual links, forming a "chain" to the actual base period.

After investigating the two derivation alternatives, Fisher concluded that, "The chain system is of little or no real use"[49] because the fixed-base system:[50]

1. is simpler to conceive and to calculate, and means something clear and definite to everybody;
2. has no cumulative error as does the chain system;
3. is graphically indistinguishable from the chain system.

The point must be made that the fixed-base and chain system methods of index number derivation are not synonyms for the double-extension and link-chain variants of dollar-value LIFO.

48 Ibid., pp. 15-22.

49 Ibid., p. 308.

50 Ibid., p. 312.

Fisher's Ratings

Fisher began his research by valuing the collections of items represented by the 36 commodities for each of the years 1913-18. Fisher derived the value indexes for the annual collections of commodities. The price-level and quantity indexes produced by the various index number type constructions were rated on their ability to duplicate the value indexes.

One common type of index number researched and ranked by Fisher was the Paasche index, that takes the following form:

$$\text{Paasche Index} = \frac{\sum p_n q_n}{\sum p_0 q_n}$$

The distinguishing characteristic of Paasche price-level indexes is the use of current-period quantity weighting, as indicated by the q_n term in both the denominator and numerator. The Paasche index is biased by the current-period weighting because the actual purchases of the base period must be restated to reflect the purchases of the current period. This restatement ignores the tendency of consumers to purchase more of items that experience small price increases over time. Thus, the Paasche price-level index understates the effects of price-level changes.

The price-level index used by the double-extension LIFO method is:

$$\frac{\sum p_n q_n}{\sum p_0 q_n}$$

The link-chain method, for computational purposes, assumes that the base-period is the beginning of the current year. The annual price-level index used by the link-chain LIFO method is:

$$\frac{\sum p_n q_n}{\sum p_{n-1} q_n}$$

where:

p_{n-1} is the prior-period cost.

The q_n term is present in both the denominator and numerator of the price-level indexes used by both the double-extension and link-chain LIFO methods. Therefore, both indexes are Paasche price-level indexes. It must be noted, however, that the link-chain method's cumulative price-level index is not a true Paasche index, but the product of a number of Paasche indexes.

Fisher tested the Paasche index number and rated the index only 38 of the 119 types tested because of index's current-period weighting. The inferior rating of the Paasche index appears to question the ability of valuation methods using the index to produce fair inventory valuations. It is not the validity of the mechanical aspects of the procedure (double-extension or link-chain) that is being questioned: the questionable aspect is the use of the Paasche index.

Another common type of index number researched by Fisher was the Laspeyres index, that takes the following form:

$$\text{Laspeyres Index} = \frac{\sum p_n q_0}{\sum p_0 q_0}$$

The distinguishing characteristic of Laspeyres price-level indexes is the use of base-period weighting, as indicated by the q_0 term in both the denominator and numerator. The Laspeyres index is biased by the base-period weighting because the purchases of the current-period must be restated to reflect the purchases of the base-period.

The Laspeyres' restatement, like the Paasche's, ignores the tendency of consumers to purchase more of items that experience small price increases over time. Unlike the Paasche index, however, the Laspeyres index overstates the effects of price-level changes. Fisher determined that the Laspeyres index was superior to the Paasche index, but its base-period bias resulted in a rating of 36 of 119.

The Laspeyres price-level index is used most prominently in the Consumer Price Index (CPI). The CPI monitors and reports on changes to the cost of a quality-of-life market basket of goods and services. Prepared by the U.S. Department of Labor's Bureau of Labor Statistics (BLS) since 1919, the CPI is:[51]

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51 Julius Shiskin, *The Consumer Price Index: Concepts and Content Over the Years*, Report 517, U.S. Department of Labor, Bureau of Statistics, May 1978(revised).

> The country's principal measure of price change, the index is used as an indicator of inflation to evaluate economic policy, as a deflator to adjust other economic indicators, such as the Gross National Product, and as a monitor of how well income payments keep up with the cost of living.

Cox and Glassberg have attempted to justify link-chain LIFO by citing the BLS's use of a chain methodology in deriving the CPI.[52] It is true that both indexes are derived using a chain methodology, but Cox and Glassberg either chose to ignore significant differences between the indexes and the processes, or were unaware of such differences.

The BLS must maintain a constant quantity and quality of goods and services in the CPI's market basket of goods and services to ensure the validity of inter-period comparisons that use the CPI. Thus, the base-period weighting of the CPI's Laspeyres index produces annual links of a constant dimensionality. There is no assurance of product-mix constancy in the link-chain LIFO procedure. The current-quantity weighting of the Paasche index used in the link-chain LIFO method means that the dimensionality of its annual links will vary with the product mix in the LIFO inventory pool.

To summarize, Cox and Glassberg's analogy fails for two reasons. One reason is that the Laspeyres index used in the CPI is superior to the Paasche index used in both the double-extension and link-chain variants of dollar-value LIFO. The other reason is that the product mix is kept constant in the chain system used in the CPI

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52 See C. Richard Cox and Carl L. Glassberg, "LIFO: "The Deflator"- A Current Review and Analysis," *The Tax Adviser*, January 1971, pp. 30-31.

process, while there is no assurance of the same constancy of product mix in the link-chain LIFO procedure. Cox and Glassberg's analogy, therefore can be characterized as incomplete and misleading.

Fisher's research concluded with the development of an "ideal" index number. The "ideal" index number capitalized on the offsetting biases possessed by the Paasche (current-period) and Laspeyres (base-period) indexes. The biases were neutralized by taking the geometric mean (square root of the product) of the two indexes, and the resulting index number exhibited the smallest variance from Fisher's computed value indexes. The "ideal" index is computed as:

$$\text{Ideal Index} = \sqrt{\text{Laspeyres Index} \times \text{Paasche Index}}$$

OR

$$\text{Ideal Index} = \sqrt{\frac{\sum p_n q_0}{\sum p_0 q_0} \times \frac{\sum p_n q_n}{\sum p_0 q_n}}$$

Index Type Summary

Both the double-extension and link-chain methods use Paasche price-level indexes. The price-level index used in the double-extension method is derived using a fixed-base methodology; the price-level index used in the link-chain method is derived using a chain methodology. Although Fisher cited a cumulative error in chain-system indexes as a reason for preferring the fixed-base derivation, the error is not documented.[53] Thus, there is no evidence that the Paasche index reduces the consistency of the link-chain valuation more than it reduces the consistency of the double-extension valuation.

New Item Introduction

Under a fixed-base derivation, computing a price-level index requires both base-period and current-period cost data for the items in the collection that is being indexed. Fisher indicates the controversy of this requirement by reporting the complaints of 1920-era chain-system advocates about the fixed-base alternative's problems with[54]

> complications arising out of the *withdrawal* of any commodity from the index number, or the *entry* of a new commodity, or both at once, *i.e.* the *substitution* of a new for an old.

The complaints of today's taxpayers directed at the double-extension LIFO method's cost ascertainment procedure for new items echo the complaints cited by Fisher.[55] The price-level index literature, however, reveals that the 1920-era complaints were successfully addressed by Fisher and by the BLS.

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53 Fisher, p. 312.

54 Fisher, p. 310.

55 For complaints about current practice of new item introduction; see above, pp. 9, 13.

Fisher's approach to the problem of introducing new items after the base date was eminently practical. Fisher suggested that, similar to patching a tire, a new replacement item literally take the place of the older item: the base-period cost of the replaced item became a surrogate for the replacement item's base-period cost.[56] Fisher differed the procedure for new items that entered, but not as replacement items. The price-level index would be computed for the collection of items as if the new, non-replacement items were not present. Then, the new, non-replacement items would be assigned base-period costs equal to their current-period cost divided by the computed price-level index.

To maintain constancy, the BLS must occasionally modify the contents of the CPI market basket. Modifications are required when changes are made to the specifications of market basket items, or when the patterns of consumer tastes and consumption change over time. To maintain the precision of the CPI, the BLS has expanded Fisher's two categories of new item introductions into four categories, separately discussed below.[57]

Directly Comparable Change

The new item is a substitute for another item with substantially identical characteristics. The BLS uses Fisher's "splicing" methodology, and the new item assumes the replaced item's base-period cost:

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56 Fisher, pp. 310-12.

57 This section is derived from material contained in U.S. Department of Labor, Bureau of Labor

$$p_0^{NEW} = p_0^{OLD}$$

where:

p_0^{NEW} is the new item's base-period cost.

p_0^{OLD} is the old item's base-period cost.

Direct Quality Adjustment

The new item is a substitute for a similar item, but the new item has a *Quality Difference*--one or more component parts are different. Quality differences (QA) occur either when previously optional items have become standard, or when previously existing features have been replaced or modified.

A dollar value (*Quality Adjustment*) is assigned to the quality difference. The quality adjustment (QA) for the first type of quality difference is based on a weighted average of the former-option price and the producer cost; the QA for the second type is based on the difference between the total production costs for the new and the old features. A previous-period cost for the new item is then computed as:

$$p_{n-1}^{NEW} = p_{n-1}^{OLD} + QA$$

where:

Statistics, *BLS Handbook of Methods: Volume II-The Consumer Price Index*, Bulletin 2134-2, April 1984, pp. 17-19.

p_{n-1}^{OLD} is the old item's previous-period price.

Now, the base-period cost can be computed as:

$$p_0^{NEW} = \frac{p_0^{OLD} \times p_{n-1}^{NEW}}{p_{n-1}^{OLD}}$$

Linking With Overlap Pricing

The new item is a substitute for a dissimilar item, but current-period prices are available for both items. The new item's base-period cost is:

$$p_0^{NEW} = p_n^{NEW} \times \frac{p_0^{OLD}}{p_n^{OLD}}$$

where:

p_n^{NEW} is the new item's current-period price.

p_n^{OLD} is the old item's current-period price.

Linking Without Overlap Pricing

The new item either enters by itself (not as a substitute), or as a substitute for a dissimilar item, but current-period prices are not available for both items. The assumption is made that the new-item's prior-period price has changed at a rate equal to the average change rate of the new item's *item stratum*.

An item stratum is an element of the third tier of the BLS's four-tier hierarchical arrangement of CPI market-basket items, and holds groupings of items judged to provide similar utility to the consumer.[58] The BLS approach differs from Fisher's approach for entering non-replacement items by basing the determination of base-period cost an index of items having similar economic utilities, not on the index applicable to the entire collection of items. The new item's base-period price is:

$$P_0^{NEW} = \frac{P_n^{NEW}}{I_{0\to(n-1)}^{STRATA} \times I_n^{STRATA}}$$

where:

$I_{0\to(n-1)}^{STRATA}$ is the prior-period CUM for the item's strata.

I_n^{STRATA} is the current-period index for the item's strata.

Analysis

Neither the IRS nor taxpayers can reasonably expect that any index number will exhibit absolute precision. Some loss of precision must be tolerated to gain the ability to simply represent the amalgamations of changes that are being indexed. Fisher was able to define what could reasonably be expected of an index number, by stating:[59]

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58 Ibid., p. 10.

59 Fisher, p. 10.

> The fundamental purpose of an index number is that it shall *fairly represent*, so far as one figure can, the general trend of the many diverging ratios from which it is calculated. It should be the "just compromise" among conflicting elements, the "fair average," the "golden mean." Without some kind of fair splitting of the differences involved, an index number is apt to be unsatisfactory, if not absurd.

There can be no "fair splitting of the differences" when current-period cost is used for the base-period cost of a new item that enters the inventory pool subsequent to the LIFO base date. The resulting index number will almost certainly be "unsatisfactory, if not absurd." In addition to not defending the current practice of new item introduction, the IRS has provided no evidence that dollar-value LIFO, even without the introduction of new items, can produce a fair inventory valuation when the Paasche index is used.

The BLS's methodology, entering new items under the assumption that they have experienced the same price changes as other items, produces a price-level index that cannot be considered to be precise. The procedure, however, does acknowledge the folly of attempting to recreate history in an attempt to create a "precise" index number. Supporting the BLS methodology is its continued use in maintaining the CPI.

In the double-extension method, the effect of the current practice of new item introduction travels directly from the current period to the base period. In the link-chain method, however, the effect travels indirectly from the current period to the base period. The annual price-level index bears the full impact of the new item introduction. The impact is abated ($CUM_{n-1} < 1$), unchanged ($CUM_{n-1} = 1$), or exacerbated ($CUM_{n-1} > 1$) when the cumulative price-level index is computed.[60] Thus, the impact of the current practice of new item introduction on the link-chain method is circumstantial and uncertain.

In both valuation methods, it is a Paasche price-level index that is influenced by the new item introduction. There is no evidence that the Paasche index used in the link-chain method is either more or less influenced by the current practice of new item introduction than is the Paasche index used in the double-extension method. The double-extension method, however, has a flaw in its new item introduction process that is not shared by the link-chain method. The flaw is the LIFO regulation that gives taxpayers a possible choice of base-period costs.[61] Humer, Galliher, and Stewart highlighted the taxpayer's flexibility by stating:[62]

> In this connection, Tax Management believes that the regulations permit, but do not require, the use of a lower unit cost as of an earlier date (subsequent to the base date) if the taxpayer is able to reconstruct or otherwise establish such lower unit cost.

A taxpayer who is able to determine several prior-period unit costs for a new item is also able to select the one cost that best suits the taxpayer's purpose. Although the flaw exists, it must be noted that a taxpayer will typically want the earliest (lowest) cost in order to minimize the deterioration of the inventory pool's price-level index. The effect of this flaw on the consistency of the double-extension valuations cannot be determined, but the potential does exist.

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60 For link-chain method equations; see above, Fig. 2 on p. 21.

61 Regs. §1.472-8(e)(2)(iii).

62 Humer, Galliher and Stewart, at A-19.

The analytic finding, therefore, is that the current practice of new item introduction does not decrease the consistency of the link-chain method's valuations more than it decreases the consistency of the double-extension method's valuations.

Chapter Summary

Two major topics relevant to the consistency issue have been analyzed and discussed: (1) price-level index concerns, and (2) new item introduction concerns. Two topics introduced and developed in this chapter are carried forward to the next chapter. The topics are (1) the possible use of a better price-level index methodology in dollar-value LIFO and (2) the BLS method of new item introduction.

The findings of this chapter are summarized in Table 1 on page 61, and the conclusions that can be drawn from these findings are enumerated in Chapter 5. The analysis of the valuation-comparability issue begins in the following chapter.

Table 1

Summary of Chapter 2 Findings

1. The consistency of the valuations produced by the link-chain LIFO method will be more variable than the consistency of valuations produced by the double-extension method in periods when the inventory pool has either a "zero" layer or a LIFO-layer liquidation.

2. The consistency of the valuations produced by both the link-chain and double-extension LIFO methods will be diminished when the inventory pool undergoes a LIFO-layer liquidation and an inappropriate index is used to assign a LIFO valuation to a post-liquidation layer. The consistency of the link-chain valuation, however, suffers the greatest diminution.

3. The double-extension variant of dollar-value LIFO can produce misstatements when a taxpayer is allowed to establish multiple prior-period costs for a new item entering the inventory pool subsequent to the LIFO base date.

4. The current-practice of new item introduction used in the double-extension and link-chain LIFO methods is without theoretical support or justification in the price-level index or accounting literatures.

The Valuation-Comparability Issue

The concern of the valuation-comparability issue is whether a taxpayer gains an income tax advantage by using the link-chain method to value an inventory pool. The advantage results from obtaining an inventory valuation that is lower than the comparable double-extension method valuation. A review of the accounting literature discloses no prior research on the valuation-comparability issue. The experiments of this chapter are performed in order to provide evidence on the issue.

The chapter contains three main sections. The experimental approach and a generic, experimental data structure are developed in the first main section. The second main section contains the design of this study's first experiment. The experimental data requirements are defined and the experimental data structure is specified, the experimental hypotheses are stated and discussed, and statistical procedures to analyze the experimental output are specified. Also, a dollar-value LIFO valuation method based on Fisher's "ideal" index is introduced.

The study's second experiment is designed in the third section. The experimental data requirements are defined, the data structure is specified, experimental hypotheses are stated and discussed, and statistical procedures to analyze the experimental output are specified. This experimental design also accommodates tests to determine the effect of the rate at which inventory pool items are "turned over", and to determine whether the use of the BLS's methodology of reconstructing base-period costs for new items would decrease tax revenues.

Experimental Approach

Experiments that use small samples of real data typically lack internal validity: the variables of interest are ill-defined and uncontrollable, and treatment effects cannot be randomized. Also, such experiments usually lack external validity because the characteristics of the sample data are not representative of the characteristics of the population proper. In order to be relevant to this study, the data must detail multi-period changes to the cost and quantity attributes of individual items in taxpayers' inventory pools.

Several attempts were made to obtain significant amounts of real data concerning inventory-pool item quantity and price attributes. Trade associations of glass and paper manufacturers were approached for assistance. Both trade associations responded with statements that member businesses did not disclose unaggregated inventory data, even to the trade associations.

The Bureau of Labor Statistics (BLS) was contacted. It is well known that the BLS receives quantity and cost data from individual manufacturers and processors for use in computing various indexes. The BLS responded by stating that such data could be obtained only by assuring the suppliers complete anonymity, and by destroying the unaggregated data immediately after the BLS's data processing was complete.

The researcher then contacted numerous individual manufacturers and processors to no avail. The reluctance of companies to disclose inventory data is understandable: there is a shared perception that there is nothing to gain from such disclosures. Competitors may gain valuable insights through inventory analyses, the research may result in changes to current processes with accompanying compliance costs, and income tax liabilities may be increased.

The lack of appropriate data led to the decision to use models to analyze the valuation-comparability issue. The decision was also influenced by the prior production of mathematical models of the double-extension and link-chain valuation methods.[63] There are two general solution approaches that are applicable to the mathematical models of the valuation processes. The analytical approach yields a mathematical-logical solution that exhibits perfect external validity and generalizability because the truth of the solution is independent of data characteristics.

The simulation approach produces a solution whose generalizability is limited by the characteristics of the data used in the simulation. Properly designed simulations are marked by a high degree of internal validity: variables can be precisely defined, controlled and randomized. The simulation solution is highly generalizable to the portion of the population proper whose characteristics are similar to the characteristics of the data used in the simulation. The simulation approach was chosen for this study because of the lack of prior research on the research topic and the probability that an analytical solution would be difficult to understand.

A Monte Carlo simulation was selected because the distribution of changes to the cost and quantity attributes of inventory pool items can be probabilistically described. The study assumes that the cost and quantity attributes have equal probabilities of being changed in a given measurement period; a review of the

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63 For double-extension equations; see above, Fig. 1, p. 20. For link-chain equations; see above, Fig. 2, p. 21.

accounting literature yielded no evidence to the contrary. Monte Carlo simulations are widely used in income tax research and dissertations[64] and are prominent in accounting and finance research.[65]

Numerical simulations, such as Monte Carlo simulations, are based on the Law of Large Numbers. Synthesized data are manipulated through a mathematical model of the system or process being studied to produce numerical outcomes that are treated as though they came from the actual system or process, not the model. Distributions of numerical outcomes are produced by repetitions of the manipulation (simulation) process. The Law of Large Numbers states that (when sufficient repetitions of the manipulation process are performed) the value of the mean of the distribution of numerical outcomes will approximate the mean result produced by the real-life system.

Computer Program

The dollar-value LIFO valuation processes are computationally intensive in nature.

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64 Brighton, Gerald D. and Robert H. Michaelsen, "Profile of Tax Dissertations in Accounting: 1967-1984," *The Journal of the American Tax Association*, Spring 1985, p. 80.

65 See Benjamin S. Blanchard and Wolter J. Fabrycky, *Systems Engineering And Analysis*, (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1981), pp. 87, 159, 164, 173-176; Lynn E. Bussey, *The Economic Analysis of Industrial Projects*, (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1978), pp. 392-97; David B. Hertz, "Risk Analysis in Capital Investment," *Harvard Business Review*, January-February, 1964, pp. 95-106; David B. Hertz, "Investment Policies That Pay Off," *Harvard Business Review*, January-February, 1968, pp. 96-108; Sidney W. Hess and Harry A. Quigley, "Analysis of Risk in Investments Using Monte Carlo Techniques," *Chemical Engineering Progress Symposium Series 42: Statistics and Numerical Methods in Chemical Engineering*, (New York: American Institute of Chemical Engineering, 1963; and Wayne E. Leininger, *Quantitative Methods in Accounting*, (New York: D. Van Nostrand Company, 1980), pp. 325, 328.

In order to perform the required computations efficaciously, it was necessary to write a computer program that would process input inventory data to produce the needed inventory valuations. Because inventory data is often presented in array or matrix form, the computer language chosen for the valuation program was APL.[66]

The mathematical models of the valuation processes were converted to an APL program.[67] The purpose of the APL program was to output two strings of numerical inventory valuations for each group of input inventory pool data. One numerical string contains the valuation resulting from the application of the double-extension method; the other string contains the valuations produced by the link-chain method. An integral component of the program was the development of the experimental data structure, described below.

Data Structure Development

The data-structure development began by identifying the inventory-pool item attributes relevant to the double-extension method, and to the link-chain method. Those attributes identified are as follows:

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66 Extensive examples of APL's applications and discussions of APL's suitability for accounting are contained in Yuji Ijiri, *Accounting Structured in APL*, Accounting Education Series, No. 6, (Sarasota, Florida: American Accounting Association, 1984); Theodore J. Mock and Miklos A. Vasarhelyi, *APL for Management*, (Boston: John Wiley and Sons, 1972); and Miklos A. Vasarhelyi, *TREAT: Terminal-Related Educational Audit Tools*, (New York: Touche Ross Foundation, 1980).

A structured introduction to using APL is in James Martin, *Application Development Without Programmers*, (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1982), Chapter 13, pp. 189-205.

67 For double-extension equations; see above, Fig. 1, p. 20. For link-chain equations; see above, Fig. 2, p. 21.

DOUBLE-EXTENSION	LINK-CHAIN
Quantity	Quantity
Current-Period Cost	Current-Period Cost
LIFO Base-Date Cost	Beginning-of-Current-Year Cost

The development continued by isolating the inventory-pool item attributes relevant to either the double-extension method or the link-chain method. The isolation was accomplished by eliminating the redundant elements in the two preceding lists of attributes. The resulting list of inventory-pool item attributes relevant to either the double-extension or link-chain methods is as follows:

<u>RELEVANT ATTRIBUTES</u>
Quantity
Current-Period Cost
LIFO-Base Date Cost
Beginning-of-Current-Year Cost

An inventory pool is a collection of items. For purposes of the double-extension and link-chain valuation methods, the collection of items can be described by a collection of lists of the relevant attributes. It is convenient to organize a collection of related lists into tabular form. Figure 4 on page 71 contains a layout of a generic table that is suitable for holding the relevant attributes (as defined above) of the items in an inventory pool.

The generic, inventory-item attribute table has one column for each item type found in the inventory pool that is being described. The rows contain the numerical data describing the relevant attributes. One table is required for the LIFO base-date valuation, and one additional table is required for each subsequent annual inventory valuation.

Both the double-extension and link-chain valuation methods require cumulative data that is attached to the inventory pool, not to the individual items in the inventory pool. The needed cumulative data could be reconstructed from the prior periods' inventory-item attribute tables, but such processing is time consuming. The models of the valuation processes were analyzed to identify the cumulative data needed to avoid the reconstruction process, and those data were as follows:

CUMULATIVE DATA
Double-Extension Index
Double-Extension Increment
Link-Chain Cumulative Index
Link-Chain Increment

Figure 5 on page 71 contains a layout of a generic table that is suitable for holding the needed cumulative data. Within the table, there is one column for each measurement period: one column for the LIFO base-date valuation, and one additional column for each subsequent measurement period. The rows contain the cumulative, numerical data. One cumulative-data table is used for each taxpayer's inventory pool.

This tabular data structure can accommodate any number of valuations for any number of inventory pools of any number of item types. Changes to the number of item types in an individual inventory pool are recognized by changing the number of columns in that inventory pool's inventory-item attribute table. Subsequent annual valuations are recognized by increasing the number of columns in that inventory pool's cumulative-date table. Additional inventory pools require the use of additional inventory-item attribute and cumulative-data tables.

Item	1	2	3	...	*N*
Quantity					
Current-Period Cost					
LIFO Base Date Cost					
Beginning-of-Current-Year Cost					

Figure 4

Data Structure: Inventory-Item Attribute Table

Period	0	1	2	...	*Y*
Double-Extension Index					
Double-Extension Increment					
Link-Chain Cumulative Index					
Link-Chain Increment					

Figure 5

Data Structure: Cumulative-Data Table

The First Experiment

The first experiment was designed to satisfy several purposes. One purpose of the experiment was to validate the APL program by using Fisher's data as the experimental data.[68] Fisher's research had required the extension of the 36 commodities at various price levels; those computations were available for matching against the computer-generated inventory valuations.[69]

Another purpose of the experiment was the introduction of a dollar-value LIFO method based on Fisher's "ideal" index methodology. The introduction was suggested by the poor rating that Fisher assigned to the Paasche price-level index used in the double-extension and link-chain LIFO methods.[70] An additional purpose was to compare the valuations produced of an inventory pool made up of a relatively small number of item types by the double-extension and link-chain dollar-value LIFO methods.

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68 For discussion of APL program; see above, pp. 66-67.

69 For discussion of Fisher's research; see above, pp. 44-49, 51-53.

70 For discussion of "ideal" index; see above, p. 51.

Experimental Data & Data Structure

The experimental data are the data chosen by Fisher to put his research to a severe test, describing the prices and quantities marketed of 36 commodities in the 1913-18 period.[71] The data capture the economic fluctuations in the United States that occurred immediately prior to the beginning of World War I, and a portion of the period of that war. Copies of Fisher's data and the extensions relevant to this experiment are contained in Appendix A, Tables 18-24 on pages 158-64.

Table 2 on page 75 contains descriptive statistics on Fisher's data that indicate the extraordinary price and quantity fluctuations during the 1913-18 period. Both the average price-level and quantity marketed attributes of the 36 commodities were generally increasing over the period. The quantity marketed increased at the simple average rate of 4.81% per year; the price-level increased at the simple average rate of 15.25% per year.

The small sample size and the non-representative fluctuations in the experimental data limit the generalizability of this experiment's results, especially in light of the IRS standards for inventory measurement. The standards, promulgated to support an increased reliance on inferential statistics, include a sample size of 500 and a .05 α-level.[72] The relatively small number (36) of item types in the sample data means

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71 Fisher, p. 14.

72 William L. Felix, Jr. and Robert S. Roussey, "Statistical Inference and the IRS", *Journal of Accountancy*, June 1985, p. 38.

that changes to an individual commodity may materially effect the difference between the valuations obtained by the double-extension and link-chain LIFO methods.

The configuration of this experiment's data structure can now be specified.[73] Using 1913 as the LIFO base date leaves the five years in the 1914-18 period as subsequent annual measurement dates. Thus, six (one for the 1913 base date, and five for the years in the 1914-18 period) inventory-item attribute tables are needed. Because the inventory pool contains 36 commodities, the inventory-item attribute tables must have 36 (one for each commodity) columns. One cumulative-data table is needed because only one taxpayer's inventory pool is being valued; the cumulative-data table must contain six (one for the 1913 base date, and five for the years in the 1914-18 period) columns.

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[73] For discussion of the general data structure; see above, pp. 67-70. For data-structure table layouts; see above, Figs. 4&5, p. 71.

Table 2

Fisher's 36-Commodity Data
Selected Descriptive Statistics

	Dollar-Value Change		
	Minimum	Mean	Maximum
FIVE-YEAR DATA			
Price-Level	-31.98%	86.70%	181.17%
Quantity	-89.55%	25.65%	203.54%
ANNUAL DATA			
Price-Level	-6.31%	15.25%	33.42%
Quantity	-26.94%	4.81%	27.59%

"Ideal" Valuation Method

The belief that a dollar-value LIFO method based on a price-level index superior to the Paasche index should produce a fairer inventory valuation than currently obtained grew out of Fisher's findings. Fisher rated the Paasche index only 38 of the 119 index number types he tested. The Paasche index is used in both the double-extension and link-chain LIFO methods, although its derivation differs between the two dollar-value LIFO variants.[74]

Fisher's top-ranked index number type, the "ideal" index, uses both the Paasche and Laspeyres index numbers.[75] The offsetting biases of the Paasche and Laspeyres indexes are neutralized to produce the "ideal" index.[76] Logically, a dollar-value LIFO valuation method based on the "ideal" index methodology should produce the fairest inventory valuation. The double-extension and link-chain dollar-value LIFO methods both produce Paasche-based valuations. Therefore, only a Laspeyres-based inventory valuation is needed for an inventory valuation method based on Fisher's "ideal" index methodology.[77]

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74 For discussion of Paasche index; see above, pp. 47-48.

75 For discussion of Laspeyres index; see above, p. 49.

76 For discussion of "ideal" index; see above, p. 51.

77 Ibid.

The double-extension method was chosen for conversion to use of a Laspeyres index number because Fisher cited the presence of a cumulative error in chain-system index numbers.[78] In defense of the link-chain method, Fisher documented neither the source of the chain-system index number error nor the significance (if any) of the error. The "ideal" dollar-value LIFO inventory valuation will be computed as the geometric mean of the Paasche-based and Laspeyres-based double-extension inventory valuations, similar to the manner in which Fisher computed his "ideal" index.

The Laspeyres double-extension method was easily adapted to use of a Laspeyres index; the equations for the Laspeyres, double-extension method are found in Figure 6 on page 79. The current-period weighting of the Laspeyres index greatly simplified the computation of the annual price-level indexes. In this respect, the double-extension method with the Laspeyres index is computationally simpler than the dollar-value LIFO double-extension method with the Paasche index: the Paasche requires that the base-period cost of the base-date inventory be restated to reflect the ending inventory quantities of the current period.

The Laspeyres price-level indexes for Fisher's data were computed, and were compared with Fisher's *pq* extensions to verify their correctness. The indexes were then used to produce the Laspeyres, double-extension valuations. Apart from the index computation, only one procedural difference was encountered. It was

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78 Fisher, p. 312.

necessary to divide the Laspeyres price-level index into the current-period cost extension of the ending inventory in order to determine the ending inventory's base-period cost.

Price-Level Index, IX_{DEL}

$$IX_{DEL} = \frac{\sum p_n q_0}{\sum p_0 q_0}$$

where:

q_0 is the base-period quantity.

Base-Period Cost Of The Ending Inventory, EI_0

$$EI_0 = \frac{\sum p_n q_n}{IX_{DEL}}$$

Base-Period Cost Of The Annual Layer, AL_0

$$AL_0 = EI_0 - BI_0$$

If amount is negative, a LIFO liquidation has occurred.
If amount is zero, the valuation is unchanged and the process stops.

LIFO Valuation Of The Annual Layer

$$AL_0 \times IX_{DEL}$$

Figure 6. Equations for the Laspeyres, Double-Extension Process.

The Experimental Hypotheses

The research hypotheses for the first experiment are:

H1: The means of the distribution of inventory valuations resulting from the link-chain method will be less than or equal to the means of the distribution of inventory valuations produced by the double-extension method.

H2: The means of the distributions of inventory valuations resulting from the double-extension method will not differ from the means of the distribution resulting from use of an "ideal" valuation method.

H3: The means of the distributions of inventory valuations resulting from the link-chain method will not differ from the means of the distribution resulting from use of an "ideal" valuation method.

The first hypothesis test is concerned with the relationship between the valuations produced by the IRS-preferred double-extension method and the taxpayer-preferred link-chain method. The second and third hypotheses require the previously developed Laspeyres, double-extension inventory valuation in order to compute the "ideal" dollar-value LIFO inventory valuation.[79] The second hypothesis is concerned with whether the valuation produced by the double-extension method is a

[79] For discussion of Laspeyres, double-extension valuations; see above, pp. 76-77.

reasonable approximation of the "ideal" valuation.[80] The third hypothesis is concerned with the relationship of the valuations produced by the link-chain method and the "ideal" method.

Statistical Considerations

Because the same inventory pools are valued with the double-extension method and link-chain methods, the corresponding valuation measurements are both paired and dependent. The differences between the measurements are solely attributable to the treatment (valuation method) differences. The appropriate statistical test for hypotheses concerning treatment differences is the paired t-test.[81] The IRS statistical requirement of a .05 α-level is used in all of this study's statistical procedures.[82]

It seems reasonable to believe that the IRS perceives that maintaining the *status quo* minimizes its risk of diminishing tax revenues. The null hypotheses used in this study recognize the IRS's perception, and the accompanying belief that the IRS resists suggested changes to the dollar-value LIFO methods. The null hypothesis of

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80 For discussion of "ideal" valuation method; see above, pp. 76-78.

81 Dennis E. Hinkle, William Wiersma and Stephen G. Jurs, *Applied Statistics for the Behavioral Sciences*, (Boston: Houghton Mifflin Co., 1979), pp. 211-15; Geoffrey Keppel, *Design and Analysis: A Researcher's Handbook*, (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1982), pp. 85-87; J. H. Pollard, *A Handbook of Numerical and Statistical Techniques*, Cambridge, Great Britain:Cambridge University Press, 1977; reprint Ed., Cambridge, Great Britain: Cambridge University Press, 1981), pp. 178, 185; and, Russell Langley, *Practical Statistics Simply Explained*, (New York: Dover Publications, Inc., 1970), p. 158.

82 Felix, Jr. and Roussey, p. 38.

this experiment's first hypothesis is that the means of the distribution of inventory valuations produced from the link-chain method (suggested alternative) are less than or equal to the means of the distribution of inventory valuations produced by the IRS-preferred (*status quo*) double-extension method.

A rejection of the null hypotheses indicates a lack of support for the IRS's *status quo* preference; a failure to reject the null hypotheses supports the IRS's position. A failure to reject the null hypothesis, however, does not necessarily mean that the link-chain method (alternative treatment) imparts a tax advantage relative to the double-extension (*status quo*) method. The null hypothesis subsumes two possible comparisons: the mean of the alternative treatment may either be less than or equal to the mean of the IRS-preferred treatment.

From the IRS's viewpoint, distinguishing between the two possibilities is unimportant: the *status quo* treatment is acceptable in both instances. If the mean of the alternative treatment is less than the mean of the *status quo* treatment, a change would cause tax revenues to be lost. If the means are equal, there is no reason for the IRS to change from current practice.

From the taxpayer's viewpoint, however, distinguishing between the two possibilities is important. If the means are equal, the valuation is the same for income tax purposes whether the taxpayer uses the double-extension method or the link-chain method. The IRS could not deny the taxpayer the use of the link-chain method (alternative treatment) on the grounds that tax revenues would be diminished.

There are three possible results of the paired t-test of the null hypothesis that the means of the distribution of inventory valuations produced from the link-chain (alternative treatment) is less than or equal to the means of the distribution of inventory valuations produced by the IRS-preferred (*status quo*) double-extension method. The possibilities are as follows:

TEST RESULT	IMPLICATION
$t_{obs} < -1.645$	Reject the null hypothesis; the evidence does not support the IRS's *status quo*.
$-1.645 < t_{obs} < +1.645$	Fail to reject the null hypothesis and conclude that there is no difference between the tested means. The evidence does support the IRS's *status quo*.
$t_{obs} > +1.645$	Fail to reject the null hypothesis and conclude that the use of the alternative treatment does yield a tax advantage relative to the IRS's *status quo*.

The directional (one-tail) paired t-test is used to test the first hypothesis. If the null hypothesis is rejected, the evidence will not support the IRS's *status quo* position. If there is a failure to reject the null hypothesis, further analysis of the results to determine whether the alternative treatment produces an income tax advantage is indicated.

The tests of the second and third hypothesis are concerned whether the tested valuation method (double-extension or link-chain) produces a valuation that is a reasonable approximation of the "ideal" dollar-value LIFO valuation. Because the analysis is concerned only with the existence, not the directional nature, of the difference, a standard two-tail, paired t-test is used. The second hypothesis is concerned with the relationship of the double-extension valuation to the "ideal" valuation; the third hypothesis is concerned with the relationship of the link-chain valuation and the "ideal" valuation.

The lack of a significant difference in the second hypothesis test will indicate that the double-extension method produced a reasonable approximation to the "ideal" valuation. The lack of a significant difference in the third hypothesis test will indicate that the link-chain method produced a reasonable approximation to the "ideal" valuation. The results of the statistical tests of this experiment's hypotheses are reported in the next chapter.

The Second Experiment

The second experiment's primary purpose was to produce evidence on the valuation-comparability issue: to test the nature of the relationship of the valuations produced by the double-extension and link-chain LIFO methods. Another purpose was to examine the impact of using the BLS's methodology for determining the base-period cost of new items entering the inventory pool subsequent to the LIFO base-date.[83] The methodology, suggested by Fisher, was refined by the BLS into the

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83 For discussion of the BLS method of entering new items; see above, pp. 53-56.

Linking Without Overlap Pricing procedure.[84] Yet another purpose of the second experiment was to test for a revenue-related rationale for the IRS's inventory "turnover" test.[85]

Experimental Data & Data Structure

The need for synthesized data in this experiment created three areas of concern related to experimental data. The three areas of concern were (1) providing LIFO base-date inventory pools, (2) changing the LIFO inventory pools over time, and (3) introducing new items into the inventory pools subsequent to the LIFO base date. The three areas are separately discussed below, and are followed by a discussion of the development of the experimental data structure.

LIFO Base-Date Inventory Pools

A review of the accounting and inventory literatures failed to produce parameters describing either typical or standard inventory configurations. Therefore, the base-date cost and quantity attributes were randomly generated. From the layout of the inventory-item attribute table contained in Figure 4 on page 71, it is seen that only the current-period cost and quantity attributes must be provided on the LIFO base date. The current-period cost, on the LIFO base date, is also the LIFO base-date cost and the beginning-of-current-year cost.

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[84] Prior discussion of *Linking W/O Overlap Pricing*; see above, pp. 55-56.

[85] Previous discussion of "turnover" test; see above, pp. 10-11.

It was determined that a base-date inventory pool would contain 500 item types, in acknowledgment of the IRS's standard sample size requirement.[86] The attribute row, current-period cost or quantity, was filled with a string of 500 randomly selected numbers, ranging from unity to 100,000. Once the current-period cost and quantity attribute rows were both filled, the current-period costs were copied into the LIFO base-date cost and beginning-of-current-year cost attribute rows.

Next, the cumulative-data table was updated.[87] The base-date quantities were extended at base-date costs, and the valuation was placed into both the double-extension and link-chain increment rows under the Period 0 (base date) column in the cumulative-data table. Both the double-extension and link-chain cumulative indexes were assigned a base-date value of 100%.

Annual Change Data

The change in an inventory pool's dollar value is the aggregate of the changes in the inventory-pool items' dollar values. Therefore, the inventory pool's dollar value can be updated by changing the current-period cost and quantity attributes. The need to specify the extent of the possible changes to the current-period cost and quantity attributes highlights the great weakness of the simulation approach: the impossibility of considering all possible data parameters in a numeric simulation.

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86 Felix, Jr. and Roussey, p. 38.

87 For layout of cumulative-data table; see above, Fig. 5, p. 71.

Thus, simulation results can be validly generalized only to those subsets of the population proper that have parameters similar to those of the data used in the simulation.

The current-period cost and quantity attribute rows in the inventory-item attribute tables can, during a measurement period, have a mean change in the ±9% range. The number of measurement periods was set at the LIFO base-date plus ten subsequent periods on the assumption that this length of time (10 years) would approximate a typical business cycle. The mean change applicable to the attribute row (current-period cost and quantity) was randomly selected from a discrete uniform distribution.

For example, assume that the mean of the current-period cost row in an inventory pool's inventory-item attribute table is $10,000. The integer 5 is selected from a uniform distribution that ranges from -9% to +9%. This means that the expected mean of the updated current-period cost row is 105% of the row's previous mean of $10,000, or $10,500.

The changes that are applied to the individual items' attributes in the inventory-item attribute row are dependent on the mean change of the entire inventory-item attribute row. The change applied to the individual items is in the range of the attribute row mean change ±30%. The range of ±30% was chosen to prevent the assigning of a negative dollar value to an individual item.

Continuing the example of the preceding paragraph, assume that an individual item in the applicable inventory pool has $30,000 as its current-period cost attribute. Because 5% was selected as the mean change for the attribute row, the change percentage for the individual item is randomly selected from a uniform distribution that ranges from -25% (+5% - 30%) to +35% (+5% + 30%). Assume that the integer 10 is selected from the preceding range. The updated current-cost attribute for the individual item is 110% of the item's previous current-period cost of $30,000, or $33,000.

This process of assigning change parameters is repeated for the current-period cost and quantity attribute rows in each of this experiment's 500 base-date inventory-item attribute tables.[88] The minimum value of the updated mean of the inventory-item attribute row (current-period cost or quantity) is 91% (100% - 9%) of the previous mean. The maximum value of the updated mean of the inventory-item attribute row is seen to be 109% (100% + 9%) of the previous mean. Therefore, the maximum decrease in dollar value that can be experienced by an inventory pool during one measurement period is 17.19%.

This percentage (17.19%) is obtained by multiplying the minimum value (91% of the previous mean) of the updated current-period cost attribute mean by the minimum value (91% of the previous mean) of the updated quantity attribute mean, resulting in a dollar-value that is 82.81 % (91% x 91%) of the previous dollar value.

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88 For discussion of inventory-item attribute table; see above, pp. 68-69, Fig. 4 on p. 71.

The percentage of decrease is derived by subtracting the old dollar value (100.00%) from the new dollar value (82.81%). The expected means and limits of the changes to inventory-pools' dollar values caused by eight combinations of inventory-item attribute row mean changes are shown in Table 3 on page 90.

Table 3

Second Experiment
Inventory-Pool Dollar-Value Change Limits

	Dollar-Value Change		
	Minimum	Expected Mean	Maximum
INVENTORY POOL			
Experiment-Wide	-17.19%	0.00%	18.81%
Price-Level Increasing	-9.00%	4.50%	18.81%
Quantity Increasing	0.00%	9.20%	18.81%
Quantity Decreasing	-9.00%	0.00%	9.00%
Price-Level Decreasing	-17.19%	-4.50%	9.00%
Quantity Increasing	-9.00%	0.00%	9.00%
Quantity Decreasing	-17.19%	-8.80%	0.00%
Price-Level Constant	-9.00%	0.00%	9.00%
Quantity Constant	-9.00%	0.00%	9.00%
INDIVIDUAL ITEM			
Experiment-Wide	-62.79%	0.00%	93.21%
Price-Level Increasing	-39.00%	4.50%	93.21%
Quantity Increasing	0.00%	9.20%	93.21%
Quantity Decreasing	-39.00%	-0.20%	39.00%
Price-Level Decreasing	-62.79%	-4.50%	39.00%
Quantity Increasing	-39.00%	-0.20%	39.00%
Quantity Decreasing	-62.79%	-8.80%	0.00%
Price-Level Constant	-9.00%	0.00%	9.00%
Quantity Constant	-9.00%	0.00%	9.00%

New Item Entry

The first step in introducing new items into this experiment's inventory pools subsequent to the LIFO base date is to determine how many items to introduce. The limit established by the IRS's so-called inventory "turnover" test was used to establish the range of the number of items that could be replaced during a measurement period. The IRS started with a required five-year turnover rate of 95%, a 19% annual average. For reasons known only to the IRS, the required five-year "turnover" rate was lowered to 75%, a 15% annual average.

The 15% annual average equates to 75 (15% x 500) item types in an experimental inventory pool. Because the 15% is the floor for link-chain use, the maximum number of items that could be introduced into an experimental inventory pool was set at 74, one item below the link-chain threshold. The number of items to be introduced into an inventory pool in this experiment was determined by randomly selecting an integer in the 0 to 74 range.

The next step was to identify the particular items in the inventory pool that were to be replaced. This was accomplished by identifying the positions the items occupied in the 500-item inventory-pool array. For example, 13 identified the item whose inventory-item attribute table column was the thirteenth from the left side of the table. The positions were identified by selecting integers from a range of 1 to 500, with the number of integers selected being equal to the number of items to be replaced. Then, the replacement items were assigned current-period costs and quantities in the inventory-item attribute tables as though they were base-date items: the numbers were randomly selected from a range of 1 to 100,000.

New Item Introduction Alternatives

There are two methods of assigning base-period costs to the replacement items of this experiment, the current-practice and the BLS method.[89] The current-practice method used in dollar-value LIFO requires the use of the current-period cost as base-period cost unless a base-period cost can otherwise be successfully ascertained by the taxpayer.[90] The double-extension method requires the ascertainment of the cost as of the LIFO base date; the link-chain method requires the ascertainment of the cost only as of the beginning of the current period.

The BLS method is based on Fisher's approach to entering new, non-replacement items into a collection of items.[91] First, the price-level index for the inventory pool is computed as though the new, non-replacement replacement items are not in the inventory pool. Then, base-period costs for the new, non-replacement, items are calculated by dividing the computed price-level index into the items' current-period costs.

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89 Previous discussion of new item introduction alternatives; see above, pp. 8-9, 53-59.

90 Regs. §1.472-8(e)(2)(d)(iii).

91 For discussion of Fisher's approach to entering new items; see above, p. 53.

The BLS method is similar to an established procedure in the regulations.[92] The procedure is used to determine a price-level index for a dollar-value LIFO inventory pool that contains items for which no purchases have been made during the current period and for which a reconstruction of current-period cost is not possible. These items are excluded from the index-computation process, but are included in the inventory-pool's overall LIFO valuation.

Because the items must be included in the LIFO valuation, the prior-period cost is used as the current-period cost. During the LIFO valuation, however, the assumed costs are multiplied by the price-level index as though the items had been included during the index-computation process. Thus, the IRS has implicitly assigned a base-period cost to the omitted items equal to the assumed current-period cost divided by the inventory pool's average price-level increase.

This IRS procedure differs from the BLS method only by using an assumed current-period cost; the BLS method uses the actual current-period cost. In both procedures, items are omitted from the index-computation procedure, and base-period costs for the omitted items are set equal to current-period cost (assumed or real) divided by the inventory pool's price-level index. The BLS method is seen to be quite similar to an established IRS procedure. Thus, the IRS should not object to the use of the BLS method if if tax revenues are not decreased.

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92 Revenue Ruling 79-103, 1979-1 C.B. 192.

Generalizability of Results

As stated previously, the results of simulations cannot be universally generalized.[93] The results are validly generalized only to the subsets of the population proper whose data characteristics are similar to the characteristics of the data used in the simulation. The specifications of the simulation's experimental data are now complete, and the data characteristics that must be exhibited by inventory pools in order that the simulation results can be validly generalized to the inventory pool can now be specified. The characteristics are found in Table 4 on page 95.

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93 For discussion of generalizability of simulation results; see above, pp. 65, 86-87.

Table 4

Second Experiment
Inventory-Pool Characteristics for Generalizability

1. There are 500 or more types of items in the inventory pool

2. Base-date quantities are in the 1-100,000 unit range

3. Base-date costs are in the $1-$100,000 per-unit cost range

4. Replacement items enter in quantities and current-period costs that are in the 1-100,000 unit range

5. Annual dollar-value changes at the inventory-pool level range between -17.79% to +18.81%

6. Annual dollar-value changes at the individual-item level range between -62.72% to +93.21% range

7. Annual mean dollar-value changes at the individual-item-attribute level range between -9% to +9%

8. Annual inventory "turnover" rates are in the 0% to 14.8% range

Data Structure

The two experiments were designed to isolate the effects of the double-extension and link-chain methods on inventory valuations by valuing the same inventory pools with both methods. The design for the second experiment also allowed the effects of the two methods of new item introduction on inventory valuations to be isolated.[94]

The isolation of the effects of the new item introduction treatments was accomplished by using parallel data structures, configured as follows:

CURRENT PRACTICE	BLS METHOD
Double-Extension	Double-Extension
Link-Chain	Link-Chain

In the current-practice data structure, new items were entered at base-period costs equal to their current-period costs. In the BLS-method data structure, new items were entered at base-period costs equal to their current-period costs divided by the price-level index computed as though the replacement items were not in the inventory pool.

Within a data structure, the difference between the double-extension and link-chain methods' measurements was solely attributable to the effect of the valuation method. Between data structures, the difference between the corresponding valuation methods' measurements was attributable solely to the effect of the new item introduction alternative.

94 For discussion of new item introduction alternatives; see above, pp. 8-9, 53-59, 92-93.

It is now possible to specify the configuration of the experimental data structure.[95] Because the inventory pools contain 500 item types, the inventory-item attribute tables must have 500 (one for each item type) columns. The use of ten periods subsequent to the LIFO base date indicates the need for 11 (one for the LIFO base date, and 10 for the subsequent periods) inventory-item attribute tables for each inventory pool within each data structure. There are 500 inventory pools being valued in the experiment. Thus, 5,500 (500 inventory pools times the eleven tables needed per inventory pool) inventory-item attribute tables are needed per data structure. Because there are two data structures in the experiment, a total of 11,000 (two data structures times 5,500 tables per data structure) inventory-item attribute tables are needed for the experiment.

One cumulative-data table is needed for each inventory pool being valued. Thus, 500 (one for each of the 500 inventory pools) cumulative-data tables are needed per data structure. There are two data structures in the experiment. Thus, 1,000 (two data structures times 500 tables per structure) are needed for the experiment. Because measurements are made for the LIFO base date and ten subsequent periods, 11 (one for the LIFO base date and one for each subsequent period) columns are required in the cumulative-data tables.

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95 For discussion of general data structure; see above, pp. 68-70. For layout of data structure tables, see Figs. 4&5, p. 71.

The Experimental Hypotheses

The research hypotheses of the second experiment are:[96]

H1: In the current-practice data structure, the means of the distribution of inventory valuations produced by the link-chain method will be less than or equal to the means of the distribution of inventory valuations produced by the double-extension method.

H2: In the BLS-method data structure, the means of the distribution of inventory valuations produced by the link-chain method will be less than or equal to the means of the distribution of inventory valuations produced by the double-extension method.

H3: The means of the distribution of inventory valuations produced by the double-extension method in the BLS-method data structure will be less than or equal to the means of the distribution of inventory valuation produced by the double-extension method in the current-practice data structure.

H4: The means of the distribution of inventory valuations produced by the link-chain method in the BLS-method data structure will be less than or equal to the means of the distribution of inventory valuation produced by the link-chain method in the current-practice data structure.

H5: The means of the differences between the double-extension and link-chain method valuations under the current practice of new item introduction will not become smaller as the item "turnover" rate becomes larger.

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96 For discussion of the new item introduction alternatives; see above, pp. 8-9, 53-59, 92-93. For discussion of the parallel current-practice and BLS method data structures; see above, pp. 96-97.

Statistical Considerations

The statistical design of the tests of the first four hypotheses of this experiment was the design of the test of the first hypothesis of the first experiment of this study.[97] The null hypotheses of the first four hypotheses recognize the IRS's preference for the *status quo*: the double-extension method is preferred to the link-chain method, and the current practice of new item introduction is preferred to the BLS method of new item introduction.

The question being asked is whether the taxpayer will gain an advantage (lowered inventory valuation) by using a treatment other than the *status quo* treatment. The experimental data structure maintains the paired and dependent nature of the measurements being compared in the first four hypotheses.[98] Thus, the one-tail, paired t-test is the appropriate test for the first four hypotheses.

The concern of the first hypothesis is whether (using the current practice of new item introduction) the taxpayer gains an advantage by using the link-chain method to value an inventory pool. The rejection of the null hypothesis will mean that there is no evidence to support the IRS's preference for the double-extension method when the current practice of new item introduction is used. The failure to reject the

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97 For discussion of first hypothesis of first experiment; see above, pp. 80-83.

98 For discussion of second experiment's data structure; see above, pp. 96-97.

null hypothesis indicates the need for additional analysis to determine if the taxpayer would gain an advantage by using the link-chain method with the current practice of new item introduction.[99]

The concern of the second hypothesis is whether (using the BLS method of new item introduction) the taxpayer gains an advantage by using the link-chain method to value an inventory pool. The rejection of the null hypothesis will mean that there is no quantitative evidence to support the IRS's preference for the double-extension method if the BLS method of new item introduction is used. The failure to reject the null hypothesis indicates the need for additional analysis to determine if the taxpayer would gain an advantage by using the link-chain method with the BLS method of new item introduction.

The concern of the third hypothesis is whether the adoption of the BLS method of new item introduction would produce an income tax advantage for the double-extension method taxpayer. The rejection of the null hypothesis will mean that there is no quantitative evidence to support the IRS's preference for the current practice of new item introduction, instead of the BLS method of new item introduction, in conjunction with the double-extension method. The failure to reject the null hypothesis indicates the need for additional analysis to determine if the taxpayer would gain an advantage by using the BLS method of new item introduction in conjunction with the double-extension method.

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99 Previous discussion of need for additional analysis; see above, pp. 82-83.

The concern of the fourth hypothesis is whether the adoption of the BLS method of new item introduction would produce an income tax advantage for the link-chain method taxpayer. The rejection of the null hypothesis will mean that there is no quantitative evidence to support the IRS's preference for the current practice of new item introduction, instead of the BLS method of new item introduction, in conjunction with the link-chain method. The failure to reject the null hypothesis indicates the need for additional analysis to determine if the taxpayer would gain an advantage by using the BLS method of new item introduction in conjunction with the link-chain method.

The fifth hypothesis is concerned with the rationale for the IRS's inventory "turnover" test.[100] While it is understood that the test is grounded in practicality, this assumption is not supported in the literature. It is also possible that the "turnover" test is based on tax revenue considerations: the IRS may believe that the difference between the double-extension and link-chain valuations becomes less as the "turnover" rate approaches the 15% link-chain threshold.

If the "turnover" test is grounded in tax-revenue concerns, the "turnover" rate will be negatively correlated to the difference between the double-extension and link-chain methods' valuations. As the "turnover" rate approaches the link-chain threshold, the magnitude of the difference between the valuations will diminish. Testing for a negative correlation is not a straight-forward process.

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100 For discussion of "turnover" test; see above, pp. 10-11.

The first step is computing the Pearson-*r* statistic (correlation coefficient) for the "turnover" rate and the difference between the double-extension and link-chain valuations produced under the current practice of new item introduction.[101] The Pearson-*r* is the index number that describes the strength of the linear relationship between the two variables.

Before interpreting the meaning of the correlation coefficient, however, it must be established that the relationship between the "turnover" rate and the difference between the double-extension and link-chain valuations is linear in nature. If the relationship is non-linear, the Pearson-r statistic will understate the strength of the relationship between the two variables. The eta, η, coefficient is a measure of correlation that does not depend on the linearity of the regression.[102] It is necessary to compute two eta coefficients because the coefficient depends on which of the two variables is categorized during the computation.[103]

The "turnover" rate variable is divided into 15 categories, each containing five consecutive integers. In this way, the 0-74 range of the number of items replaced in the experimental inventory pools is encompassed. The eta coefficient produced from this categorization is called the *row-eta* in this experiment. The differences between the double-extension and link-chain valuations are categorized into 10

101 Hinkle, Wiersma, and Jurs, pp.75-79.

102 Ibid., pp. 112-15.

103 Ibid.

contiguous groups, with each group having a width equal to one-tenth of the range of valuation differences. The eta coefficient produced from this categorization is called the *column-eta* in this experiment.

The next step is to compute two zeta, ζ, coefficients of nonlinearity, one for each of the η coefficients. The ζ coefficients is computed as follows:

$$\zeta = \eta^2 - r^2$$

where:

r is the Pearson-r statistic.

If the larger of the two ζ exceeds 10% (0.10), non-linear regression will be used to determine whether the Pearson-*r* statistic has understated the degree of correlation, and to determine the largest coefficient of correlation. In any case, the largest coefficient of correlation produced will be used for the remainder of the testing.

Regardless of the magnitude of the correlation coefficient, there may or may not be any correlation between the two variables: the coefficient may have resulted from chance. Mason and Hinkle, Wiersma and Jurs suggest the use of the *z*-statistic to test the null hypothesis of no correlation between the variables.[104] In this case, the

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[104] Ibid., pp. 181-83; and, Mason, pp. 546-48.

alternative hypothesis is one-tailed in nature, being that a negative correlation exists between the two variables, with the critical value for rejection of the null hypothesis being -1.645.

A failure to reject the null hypothesis means that there is no evidence of a negative correlation; the rejection of the null hypothesis indicates the existence of a negative correlation. If a negative correlation exists, the strength of the correlation must be assessed for significance. Hinkle, Wiersma and Jurs state that a correlation coefficient below 0.30 (30%) indicates "little if any correlation" between the two variables under investigation.[105]

A coefficient of correlation greater than 0.30, therefore, will indicate a significant, negative correlation between the "turnover" rate and the difference between the methods' valuations. The significant, negative correlation will indicate that the "turnover" test is based on tax-revenue considerations. The lack of a significant, negative correlation makes it probable that the "turnover" test is based on the IRS's perception of taxpayer practicality.

Sensitivity Analyses

The high level of internal validity in well designed simulations may either eliminate or mask significant differences during statistical testing. It is necessary to perform sensitivity analyses to reduce the probability that significant differences have been

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105 See Hinkle, Wiersma and Jurs, p. 85, Table 5.5.

"randomized out" and overlooked during statistical testing. In this experiment, the sensitivity analyses will consist of replicating the experiment's hypotheses tests on subgroups of the simulated distribution.

Two sets of subgroups were established for the sensitivity analyses. One set consists of six subgroups that represent various combinations of price-level and quantity change directions. In a period, the current-period cost and quantity attribute rows of the inventory-item attribute tables can either increase, be unchanged, or decrease.[106] The set of price-level and quantity change direction groups is defined in Figure 7 on page 108.

The second set of subgroups was formed from the ±9% range for the mean changes to the current-period cost attribute rows.[107] These subgroups refer only to the price-level changes, not to both quantity and price-level changes because the price-level index is the sole index used in dollar-value LIFO. In addition, the first set of subgroups already address the combinations of price-level and quantity changes. These eight data groups are hierarchically arranged with two two major subgroups, the Generally-Increasing and Generally-Decreasing categories, that are themselves each divided into three subgroups. The set of price-level change subgroups is depicted in Figure 8 on page 108.

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106 For discussion of inventory-item attribute table; see above, pp. 68-69, Fig. 4, p. 71.

107 Previous discussion of ±9% range; see above, pp. 86-89.

The sensitivity analyses of the tests of the first four hypotheses are performed as follows. The paired t-test appropriate to the hypothesis is repeated on the 14 distributional data subgroups. Then, a One-Way Analysis of Variance (ANOVA) procedure is used to analyze the subcategories within the Generally Decreasing and Generally Increasing categories.[108] The results of an ANOVA may indicate that there is a significant difference between the means of the Small, Medium and Large subgroups. If a significant difference is indicated, the Tukey procedure is used to perform pair-wise comparisons to identify the homogeneous subgroups, and to maintain the experimental .05-α-level.[109]

The sensitivity analysis of the test of the fifth hypothesis consists of repeating the correlation significance process on the 14 distributional data subgroups.[110] The results of the statistical tests of the five hypotheses of this experiment are reported in the next chapter, as are the results of the sensitivity analyses of the results of the five hypotheses tests.

Chapter Summary

Eight hypotheses were developed and discussed in this chapter: three hypotheses in

108 Ibid., pp. 258-60.

109 Ibid., pp. 271-72.

110 Previous discussion of correlation analysis procedure; see above, pp. 101-04.

the first experiment and five hypotheses in the second experiment. A major difference in the experiments is the nature of the experimental data. The first experiment used a limited amount of real data, while the second experiment used a large amount of synthesized data. The results of the statistical analyses of the eight hypotheses are reported and discussed in the following chapter.

• Price-Level increasing, Quantity increasing	P > 1.0 & Q > 1.0
• Price-Level increasing, Quantity decreasing	P > 1.0 & Q < 1.0
• Price-Level decreasing, Quantity increasing	P < 1.0 & Q > 1.0
• Price-Level decreasing, Quantity decreasing	P < 1.0 & Q < 1.0
• Price-Level constant	P = 1.0
• Quantity constant	Q = 1.0

Figure 7

Second Experiment
Sensitivity Analyses - Price-Level & Quantity Subgroups

Generally Decreasing			Generally Increasing		
-9%	-6%	-3%	+3%	+6%	+9%
Large	Moderate	Small	Small	Moderate	Large

Figure 8

Second Experiment
Sensitivity Analyses - Price-Level Change Subgroups

Experimental Results

This chapter contains the results of the tests of the eight hypotheses developed in the preceding chapter. For the first experiment from the preceding chapter, the results of the tests are reported and discussed. For the second experiment from the preceding chapter, the results of the tests and the associated sensitivity analyses are reported and discussed. The chapter is divided into two main sections: one section for each of the experiments designed in the preceding chapter.

First Experiment

The first experiment was performed, and statistical tests were made of the three hypotheses. The hypotheses were concerned with the differences between the valuations of Fisher's data produced by the link-chain and double-extension methods, and differences between the valuations produced by a proposed "ideal" valuation method, and the double-extension and link-chain valuation methods. Descriptive statistics and summary data for the first experiment are contained in Table 5 on page 113.

The first hypothesis test was concerned with the relationship between the double-extension and link-chain valuations produced from Fisher's data. The result of the paired t-test, from Table 6 on page 114, indicated the failure to reject (at the .05 level) the null hypothesis that the mean of the distribution of valuations produced by the link-chain method was less than or equal to the mean of the distribution of valuations produced by the double-extension method.

Because the null hypothesis could not be rejected, additional analysis of the test results became necessary.[111] The failure to reject the null hypothesis meant either that there was no difference between the means of the two distributions or that the mean of the alternative treatment (link-chain method) was less than the mean of the IRS's *status quo* preference (double-extension method). In this case, the evidence (observed t-statistic of 2.9438) indicated that the link-chain method produced lower inventory valuations than does the double-extension method. Thus, the IRS's *status quo* preference was upheld.

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111 For discussion of need for additional analysis; see above, pp. 82-83.

The second and third hypotheses were concerned with the valuation that was produced by an "ideal" dollar-value LIFO method.[112] The second hypothesis was concerned with the relationship of the inventory valuations produced by the "ideal" and double-extension methods. The result of the paired t-test, from Table 6 on page 114, indicated that the null hypothesis of no difference between the means of the distributions of valuations produced by the "ideal" and double-extension methods could not be rejected at the .05 level. Because this test did not use a directional null hypothesis, no additional analysis was necessary. The evidence indicates that the double-extension valuations of Fisher's data is a fair approximation of the "ideal" valuations of that data.

The concern of the third hypothesis was with the relationship between the valuations produced by the "ideal" and link-chain methods. From Table 6 on page 114, the result of the paired t-test indicates that the null hypothesis of no difference between the means of the distributions of valuations produced by the "ideal" and link-chain methods was rejected at the .05 level. Thus, the evidence indicated that the link-chain valuations of Fisher's data did not fairly approximate the "ideal" valuations of that data.

Using Fisher's data, the IRS's concern with revenue diminution was shown to be justified because the link-chain method produced lower inventory valuations than the double-extension method. The link-chain method valuations were also

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112 For discussion of discussion of "ideal" valuation method; see above, pp. 76-78.

significantly different from the "ideal" valuations. The results of the three hypotheses tests of the first experiment are summarized in Table 17 on page 140; the conclusions that can be drawn from these results are detailed in the next chapter.

Table 5

First Experiment
Descriptive Data & Summary Statistics

LIFO Valuations (in $Millions)

	1914	1915	1916	1917	1918
Double-Extension	12,992	14,268	15,744	15,880	17,312
Link-Chain	12,992	14,238	15,683	15,773	17,217
Double-Extension, Laspeyres	13,041	14,323	15,767	15,711	17,248
Double-Extension, "Ideal"	13,017	14,296	15,756	15,795	17,280

Price-Level Indexes

	1914	1915	1916	1917	1918
Double-Extension	100.72%	100.10%	114.35%	161.05%	177.43%
Link-Chain	100.32%	100.33%	114.83%	162.02%	178.43%
Double-Extension, Laspeyres	99.94%	99.68%	114.09%	162.07%	177.88%

Summary Statistics (in $Millions)

	Double-Extension			Link-Chain
		Laspeyres	"Ideal"	
Mean	15,239	15,218	15,229	15,181
Median	15,744	15,711	15,756	15,683
Standard Deviation	1,655	1,598	1,625	1,615
Standard Error - Mean	740	714	727	722
Minimum	12,992	13,041	13,017	12,992
Maximum	17,312	17,248	17,280	17,217
Range	4,320	4,207	4,263	4,225

Table 6

First Experiment - Statistical Tests

FIRST HYPOTHESIS

Paired t-Test:	Degrees of Freedom	T Value	$\underline{\alpha}$[a]
	4	2.9438	0.9789

SECOND HYPOTHESIS

Paired t-Test:	Degrees of Freedom	T Value	$\underline{\alpha}$
	4	-0.4833	0.6541

THIRD HYPOTHESIS

Paired t-Test:	Degrees of Freedom	T Value	$\underline{\alpha}$
	4	4.6436	0.0097[s]

[a] $\underline{\alpha}$ is the lowest α-level leading to rejection of the null hypothesis. In this study, levels that are less than 0.0001 are reported as 0.0000.

[s] indicates significance at the .05 level.

Second Experiment

The second experiment was performed, and statistical tests were made of the experiment's five hypotheses. The hypotheses were concerned with the differences between the valuations produced by the double-extension and link-chain methods, and the effect on those valuations of the BLS method of new item introduction.[113] The fifth hypothesis was concerned with the nature of the correlation of an increase in the so-called "turnover" rate with the difference between the valuations produced by the double-extension and link-chain valuation methods under the current practice of new item introduction.

First Hypothesis

The concern of this hypothesis was with the relationship between the valuations of the synthesized data produced by the double-extension and link-chain methods, under the current practice of new item introduction. The current practice of new item introduction assumes that the current-period cost of an item that enters the inventory pool subsequent to the LIFO base date is used as the item's base-period cost.

113 For discussion of new item introduction alternatives; see above, pp. 56-61.

The result of the paired t-test, from Table 7 on page 120, indicated that the null hypothesis that the means of the distribution of valuations produced by the link-chain method are less than or equal to the means of the distribution of valuations produced by the double-extension method was rejected at the .05 level. The evidence indicated that, under the current practice of new item introduction, the link-chain valuations of the synthesized data are greater than the comparable double-extension valuations. Thus, under the current practice of new item introduction, the link-chain method did not produce lower inventory valuations than did the double-extension method, and tax revenues were not diminished.

The results of the sensitivity analysis of the test of first hypothesis are contained in Table 8 on page 121. The sensitivity analyses of the second experiment replicated the hypothesis tests on 14 subgroups of the simulated distribution.[114] The sensitivity analysis indicated that the result of the hypothesis test was valid across the simulated distribution.

Some degree of distributional sensitivity, however, was indicated by the One-Way ANOVA tests of the Generally-Increasing and Generally-Decreasing price-level change subgroups. The Tukey test of the data groups in the Generally-Decreasing category indicated a direct relationship between the price-level changes and the

[114] For discussion of sensitivity analyses; see above, pp. 104-6. For composition of 14 data subgroups; see above, Figs. 7&8, p. 108.

excess of the link-chain valuation over the double-extension valuation: as the rate of price-level change decreased, the excess of the link-chain valuation over the double-extension valuation also decreased.

The Tukey test of the data groups in the Generally-Increasing category similarly indicated a direct relationship between the price-level changes and the excess of the link-chain valuations over the double-extension valuations: as the rate of price-level change increased, the excess of the link-chain valuation over the double-extension valuation also increased.

One explanation for the distributional sensitivity is the multiplicative nature of the link-chain cumulative price-level index. Assume that there is a 9% price-level increase for four consecutive periods. The effective increase to a double-extension taxpayer is 36%; the effective price-level increase to a link-chain taxpayer is 41.16%. The difference occurs because of the compounding effect of the link-chain method. Similarly, assume that the price level falls by 9% for four years: the net decrease to the double-extension taxpayer is 36%; to the link-chain taxpayer, the net decrease is only 31.43%.

Another source of the distributional sensitivity is the difference between the manner in which new items are introduced into the double-extension and link-chain inventory pools subsequent to the LIFO base date. For this hypothesis, the new items were assigned base-period costs equal to their current-period costs under both the double-extension and link-chain methods. However, the impact of assigning the current-period cost as the base-period cost for the new items differs between the double-extension and link-chain methods.

Both the double-extension and link-chain methods' price-level indexes are ratios. Mathematically, adding the same number to both the denominator and numerator of a ratios causes the ratio to approach unity. Those ratios that were greater than unity will decrease; those ratios less than unity will be increased.

The double-extension method's index absorbs the full impact of the new item introduction: when the price-level is increasing, the price-level index will be decreased; when the price-level is decreasing, the price-level index will be increased. In the link-chain method, it is the annual price-level index that bears the brunt of the new item introduction. The multiplicative nature of the cumulative price-level index serves to buffer the impact of the new item introduction.

For example, consider the case where two taxpayers (one using the double-extension method, one using the link-chain method) have identical inventory pools. In the same period, both taxpayers identically replace their entire inventory pools; neither can ascertain any base-period costs for the new items. Also assume that both taxpayers have identical, positive annual layers.

The taxpayer using the double-extension method computes a price-level index of 100%; the LIFO valuation of the annual layer is equal to its current-period cost.

The taxpayer using the link-chain method computes an annual price-level index to be 100%, meaning that the prior period's cumulative price-level index will be used to assign a LIFO value to the annual layer. In periods where the price levels have been generally increasing, the prior-period cumulative index will likely exceed unity: the link-chain method will produce a higher LIFO valuation than does the double-extension method. In periods where the price levels have been generally decreasing, the opposite will be true; the link-chain method will produce a lower LIFO valuation then does the double-extension method.

Table 7

Second Experiment
First Hypothesis - Statistical Test

Summary Statistics ($):

	Double-Extension	Link-Chain	Difference
Mean	4,916,332	4,958,969	-42,637
Median	4,909,103	4,962,789	-12,029
Standard Deviation	456,739	468,172	136,240
Standard Error - Mean	6,549	6,620	1,926
Maximum	7,029,284	7,045,844	560,347
Minimum	3,362,330	3,320,895	-749,750
Range	3,366,953	3,724,949	1,310,098

Paired t-Test:

Degrees of Freedom	T Value	$\underline{\alpha}$
4,999	-22.1292	.0000[S]

[S] indicates significance at the .05 level.

Table 8

Second Experiment
First Hypothesis - Sensitivity Analysis

Price-Level & Quantity Combinations:

Paired t-Tests:		df	T Value	α
Price-Level Increasing:				
	Quantity Increasing	1,142	-13.8683	0.0000^S
	Quantity Decreasing	1,102	-13.4242	0.0000^S
Price-Level Decreasing:				
	Quantity Increasing	1,065	-6.1654	0.0000^S
	Quantity Decreasing	1,170	-8.3720	0.0000^S
Price-Level Stable		259	-6.3429	0.0000^S
Quantity Stable		266	-5.6315	0.0000^S

Generally-Increasing Price Level:

Paired t-Tests:	String	df	T Value	α
	Combined	2,370	-19.9221	0.0000^S
	Small	524	-7.4612	0.0000^S
	Medium	780	-10.4800	0.0000^S
	Large	1,064	-15.5499	0.0000^S

95% Tukey HSD Test:		Average	Homogeneous Groups
	Large	-67,476.95	*
	Medium	-52,773.02	**
	Small	-45,930.08	*

Generally-Decreasing Price Level:

Paired t-Tests:	String	df	T Value	α
	Combined	2,368	-10.7257	0.0000^S
	Small	540	-7.5600	0.0000^S
	Medium	788	-5.9277	0.0000^S
	Large	1,038	-5.3929	0.0000^S

95% Tukey HSD Test:		Average	Homogeneous Groups
	Small	-45,766.61	*
	Medium	-27,208.91	*
	Large	-20,507.37	*

^S indicates significance at the .05 level.

Second Hypothesis

The concern of this hypothesis was with the relationship between the valuations of the synthesized data produced by the double-extension and link-chain methods, under the BLS method of new item introduction. The BLS method of new item introduction assumes that the base-period cost of a new item that enters the inventory pool subsequent to the LIFO base date is equal to the item's current-period cost divided by a price-level index computed by ignoring the presence of the new items in the inventory pool.

The result of the paired t-test, from Table 9 on page 124, indicated that the null hypothesis that the means of the distribution of valuations produced by the link-chain method are less than or equal to the means of the distribution of valuations produced by the double-extension method could not be rejected at the .05 level. The failure to reject the null hypothesis indicated that further analysis was needed.[115]

The observed T-statistic of 0.1207 fell short of the critical value of 1.645 that would be needed to reject a null hypothesis that there is no difference between the means.

[115] For discussion on need for additional analysis; see above, pp. 82-83.

There was no evidence to reject a null hypothesis of no difference between the means at the .05 level. Therefore, the evidence indicated that both valuation methods produced the same valuations; tax revenue was not diminished.

The results of the sensitivity analysis of the second hypothesis are contained in Table 10 on page 125. The sensitivity analysis indicated that the result of the hypothesis test was valid across the simulated distribution. The paired t-tests of the sensitivity analysis indicated failures to reject the null hypothesis at the .05 level, and indicated no difference between the valuations produced by the double-extension and link-chain methods.

The discussion of the first hypothesis of this experiment indicated that the double-extension method should be more responsive than the link-chain method to the removal of the impact of new item introductions.[116] The comparison of summary valuation data presented in Table 11 on page 126 justifies this expectation: the double-extension method is seen to have been more sensitive to the new item introduction alternative. Although the mean valuation produced by both valuation methods increases, the rate at which the double-extension mean increases is seen to be triple that of the rate of increase to the link-chain mean. The effect of the accelerated increase in the double-extension valuations is to decrease the difference between the link-chain and double-extension method's valuations.

116 For discussion of first hypothesis of second experiment; see above, pp. 115-21.

Table 9

Second Experiment
Second Hypothesis - Statistical Test

Summary Statistics ($):

	Double-Extension	Link-Chain	Difference
Mean	4,978,035	4,977,924	111
Median	4,978,245	4,979,150	905
Standard Deviation	471,696	473,254	65,211
Standard Error - Mean	6,670	6,692	922
Maximum	6,984,617	7,032,790	307,245
Minimum	3,270,156	3,310,264	-329,815
Range	3,714,460	3,722,526	637,060

Paired t-Test:

Degrees of Freedom	T Value	$\underline{\alpha}$
4,999	0.1207	0.2373

Table 10

Second Experiment
Second Hypothesis - Sensitivity Analysis

Price-Level & Quantity Combinations:

Paired t-Tests:	df	T Value	α
Price-Level Increasing:			
Quantity Increasing	1,142	0.2680	0.6056
Quantity Decreasing	1,102	0.6034	0.7268
Price-Level Decreasing:			
Quantity Increasing	1,065	-0.2531	0.4001
Quantity Decreasing	1,170	0.4282	0.6657
Price-Level Stable	259	-0.6615	0.2542
Quantity Stable	266	-1.1831	0.1189

Generally-Increasing Price Level:

Paired t-Tests:

String	df	T Value	α
Combined	2,370	0.7686	0.7789
Small	524	1.1994	0.8845
Medium	780	0.3823	0.6488
Large	1,064	-0.0482	0.4808

Generally-Decreasing Price Level:

Paired t-Tests:

String	df	T Value	α
Combined	2,368	-0.4049	0.3428
Small	540	-0.5107	0.3049
Medium	788	0.1929	0.5765
Large	1,038	-0.4053	0.3427

Table 11

Second Experiment
Double-Extension & Link-Chain Valuation Methods' Sensitivity to New Item Introduction Alternative

	Double-Extension			Link-Chain		
	C-P	BLS	%	C-P	BLS	%
Mean	4,916,332	4,978,035	1.26	4,958,969	4,977,924	0.38
Standard Deviation	456,739	471,696	3.27	468,172	473,254	1.09
Standard Error - Mean	6,549	6,670	1.81	6,620	6,692	1.09
Range	3,366,953	3,714,460	9.36	3,724,949	3,722,526	-0.07

Key to Table Column Headings

C-P	Current-Practice method of new item introduction
BLS	BLS method of new item introduction
%	Change in measurement under current-practice method to measurement under BLS method, expressed as a percentage of the current-practice measurement

Third Hypothesis

The concern of this hypothesis was with the effect of the BLS method of new item introduction on the valuations produced by the double-extension method. The result of the paired t-test, reported in Table 12 on page 129, indicated that the null hypothesis that the means of the distribution of inventory valuations produced under the BLS method of new item introduction by the double-extension method are less than or equal to the mean of the distribution of valuations produced under the current practice of new item introductions by the double-extension method was rejected at the .05 level.

The evidence, therefore, indicated that the use of the BLS method of new item introduction led to higher double-extension method valuations of the synthesized data than produced by the double-extension method using the current practice of new item introductions. The introduction of the BLS method of new item introduction did not lead to decreased double-extension method valuations, and tax revenues were not diminished.

The results of the sensitivity analysis of this hypothesis test are reported in Table 13 on page 130. The sensitivity analysis produced no evidence contradictory to the results of this hypothesis test. The paired t-tests failed to disclose any distributional sensitivity, and the ANOVA procedures indicated the homogeneity of the data groups in both the Generally-Increasing and Generally-Decreasing price-level change categories.

The results of this hypothesis test are consistent with the findings of this experiment's second hypothesis test which showed that the use of the BLS method of new item introduction would not decrease tax revenues.[117] Those results and the data in Table 11 on page 126 showed that the mean of the double-extension method's valuations increased when the impact of the current-practice of new item introduction was removed. Thus, the results of the test of this hypothesis were somewhat predictable.

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117 For discussion of second hypothesis of second experiment; see above, pp. 122-26.

Table 12

Second Experiment
Third Hypothesis - Statistical Test

Summary Statistics ($):

	Current-Practice	BLS method	Difference
Mean	4,916,332	4,978,035	-61,703
Median	4,909,103	4,978,245	-27,083
Standard Deviation	456,739	471,696	159,125
Standard Error - Mean	6,549	6,670	2,250
Maximum	7,029,284	6,984,617	635,887
Minimum	3,362,330	3,270,156	-954,5333
Range	3,366,953	3,7144,460	1,590,420

Paired t-Test:

Degrees of Freedom	T Value	α
4,999	-27.4194	0.0000[S]

[S] indicates significance at the .05 level.

Table 13

Second Experiment
Third Hypothesis Sensitivity Analysis

Price-Level & Quantity Combinations:

Paired t-Tests:	df	T Value	α
Price-Level Increasing:			
Quantity Increasing	1,142	-13.2339	0.0000s
Quantity Decreasing	1,102	-13.0264	0.0000s
Price-Level Decreasing:			
Quantity Increasing	1,065	-11.4897	0.0000s
Quantity Decreasing	1,170	-14.8451	0.0000s
Price-Level Stable	259	-4.9462	0.0000s
Quantity Stable	266	-6.2599	0.0000s

Generally-Increasing Price Level:

Paired t-Tests: String	df	T Value	α
Combined	2,370	-19.2426	0.0000s
Small	524	-8.7931	0.0000s
Medium	780	-10.8871	0.0000s
Large	1,064	-13.2003	0.0000s

Generally-Decreasing Price Level:

Paired t-Tests: String	df	T Value	α
Combined	2,368	-18.9714	0.0000s
Small	540	-9.5231	0.0000s
Medium	788	-10.7794	0.0000s
Large	1,038	-12.3940	0.0000s

s indicates significance at the .05 level.

Fourth Hypothesis

The concern of this hypothesis was with the effect of the BLS method of new item introduction on the valuations produced by the link-chain method. The result of the paired t-test, reported in Table 14 on page 134, indicated that the null hypothesis that the means of the distribution of inventory valuations produced under the BLS method of new item introduction by the link-chain method are less than or equal to the mean of the distribution of valuations produced under the current practice of new item introduction by the link-chain method was rejected at the .05 level.

The evidence indicated that the use of the BLS method of new item introduction led to higher link-chain method valuations of the synthesized data than produced by the link-chain method using the current practice of new item introductions. Thus, the introduction of the BLS method of new item introduction did not cause a drop in the link-chain method valuations, and tax revenues were not diminished.

The results of the sensitivity analysis of this hypothesis test are reported in Table 15 on page 135. With one exception, the results support the results of the hypothesis test. The lone exception was the Large subgroup (change level > 6%) of the Generally-Increasing category. The observed t-value (-0.9691) meant that a null hypothesis of equality of means could not be rejected at the .05 level. Thus, the evidence indicated that the means of the valuations were equal, and tax revenues were not decreased.

The One-Way ANOVA tests indicated significant differences in the means of the data groups in both the Generally-Decreasing and Generally-Increasing categories. The Tukey tests of both categories indicated an inverse relation between the price-level change rates and the excess of the BLS-method valuation over the current-practice valuation.

The inverse relationship results from the interaction of the chain-system methodology and the difference between the two alternative methods of new item introduction. Under the current practice of new item introduction, new items are entered at a beginning of the year cost that is equal to the items' current-period costs. Thus, the current practice of new item introduction can influence the link-chain price-level indexes. Under the BLS method of new item introduction, the link-chain price-level indexes are unaffected by the new items.

For example, assume that two taxpayers have identical link-chain LIFO inventory pools. One taxpayer uses the current practice of entering new items and the other taxpayer uses the BLS method of entering new items. Assume that the identical prior-period cumulative indexes are greater than unity and that identical items enter both inventory pools during the current period.

For the taxpayer using the current practice, the introduction of new items decreases the price-level index toward unity. For the taxpayer using the BLS method, the introduction of the new items has no effect on the price-level index. It is this difference that causes the inverse relationship. As the rate of price-level change increases, the price level will also increase. Under the BLS method, the link-chain index will increase; under the current practice, the price-level index will decrease. Thus, the BLS-method, link-chain valuation under the BLS method will decrease toward the current practice, link-chain valuation when the price-level increases.

Further comment on this trend appears to be appropriate. From Table 15 on page 135, it is seen that there is a dramatic decrease in the average of the three Tukey HSD Test groups in the Generally-Increasing category. The trend may indicate that (at rates of price-level increase higher than used in this experiment) the BLS-method, link-chain valuation may be lower than the current-practice, link-chain method.

Table 14

Second Experiment
Fourth Hypothesis Statistical Test

Summary Statistics ($):

	Current-Practice	BLS method	Difference
Mean	4,958,969	4,977,924	-18,955
Median	4,962,789	4,979,150	-16,361
Standard Deviation	468,172	473,254	-5,082
Standard Error - Mean	6,620	6,692	-72
Maximum	7,045,844	7,032,790	13,054
Minimum	3,320,895	3,310,264	10,631
Range	3,724,949	3,722,526	2,423

Paired t-Test:

Degrees of Freedom	T Value	$\underline{\alpha}$
4,999	-23.3299	0.0000[s]

[s] indicates significance at the .05 level.

Table 15

Sensitivity Analysis
Second Experiment - Fourth Hypothesis

Price-Level & Quantity Combinations:

Paired t-Tests:	df	T Value	α
Price-Level Increasing:			
Quantity Increasing	1,142	-4.8320	0.0000[S]
Quantity Decreasing	1,102	-5.3370	0.0000[S]
Price-Level Decreasing:			
Quantity Increasing	1,065	-23.2692	0.0000[S]
Quantity Decreasing		-26.0206	0.0000[S]
Price-Level Stable	259	-7.0598	0.0000[S]
Quantity Stable	266	-7.2275	0.0000[S]

Generally-Increasing Price Level:

Paired t-Tests:

String	df	T Value	α
Combined	2,370	-7.4310	0.0000[S]
Small	524	-7.2247	0.0000[S]
Medium	780	-6.2114	0.0000[S]
Large	1,064	-0.9691	0.1664

95% Tukey HSD Test:

	Average	Homogeneous Groups
Small	-13,669.40	*
Medium	-9,945.88	*
Large	-1,368.29	*

Generally-Decreasing Price Level:

Paired t-Tests:

String	df	T Value	α
Combined	2,368	-35.9994	0.0000[S]
Small	540	-13.5256	0.0000[S]
Medium	788	-19.5931	0.0000[S]
Large	1,038	-27.8689	0.0000[S]

95% Tukey HSD Test:

	Average	Homogeneous Groups
Large	-37,066.40	*
Medium	-27,819.70	*
Small	-24,751.37	*

[S] indicates significance at the .05 level.

Fifth Hypothesis

The concern of this hypothesis was whether the IRS's "turnover" test was grounded in the IRS's perception of taxpayer practicality. The results of the Z-test, reported in Table 16 on page 139, indicated the failure to reject, at the .05 level, the null hypothesis of no negative correlation between the inventory "turnover" rate and the difference between the double-extension and link-chain method valuations under the current practice of new item introduction. Thus, the evidence indicates that the "turnover" rate is based on practicality concerns.

The results of the sensitivity analysis of this hypothesis test are contained in Table 16 on page 139. The results indicated that the null hypothesis was rejected at the .05 level for three distributional subgroups: the price-level stable group of the Price-Level & Quantity Combination groupings, and the medium and large data groups in the Generally Increasing category. For these three data groups, the evidence indicated the existence of a negative correlation between the "turnover" rate and the difference between the double-extension and link-chain valuations.

The correlation analysis procedure was applied, and it was determined that the correlation coefficients for the three groups were all less than the 30% required to indicate significant strength.[118] Thus, the results of the sensitivity analysis

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118 For discussion of correlation analysis procedure; see above, pp. 101-4.

supported the result of the hypothesis test. Throughout the simulated distribution, the evidence does not indicate the existence of a negative correlation between the inventory "turnover" rate and the difference between the double-extension and link-chain valuations produced under the current practice of new item introduction.

Although the evidence indicates that practicality is at the core of the "turnover" test, it is appropriate to examine the IRS's operational definition of practicality. Consider two taxpayers with equal sales volume: one with 80 item types in an inventory pool; the other with 5,000 item types. To pass the "turnover" test and gain access to the link-chain method, the first taxpayer has to "turnover" an average of 12 item types per year; the second taxpayer must have an average "turnover" of 750 item types per year.

Assume that the second taxpayer has an actual average "turnover" of "only" 600 item types per year, and the first taxpayer has an actual "turnover" of 12 item types per year. Under double-extension LIFO, the second taxpayer must reconstruct or ascertain 588 (600 - 12) more base-date costs than must the first taxpayer. Because the taxpayers have equal sales volumes, the second taxpayer incurs a much larger compliance cost, both absolute and sales-volume relative, under double-extension LIFO than the first taxpayer.

The first taxpayer, however, will be eligible to use the link-chain method because of the 15% "turnover" rate. The second taxpayer will not be eligible because of the

12% "turnover" rate; therefore, the double-extension method must be used. A definition of taxpayer practicality in terms of compliance cost seems more appropriate than a definition in terms of "turnover" rate.

One alternative is to substitute an absolute number of items "turned over" for the current, relative "turnover" rate. For example, any taxpayer who "turned over" an average of 20 item types would pass the "turnover" test; thus, all taxpayers would be treated equally. Another alternative is to express the "turnover" rate in a sliding scale that is relative to sales volume. The larger the taxpayer's sales volume, the more item types needed to be "turned over" to pass the "turnover" test.

Chapter Conclusion

The results of the eight hypotheses tests designed in the preceding chapter were reported and discussed. Those results are summarized in Table 17 on page 140. The conclusions that can be drawn from the results are summarized in the next chapter.

Table 16

Second Experiment - Fifth Hypothesis
Statistical Tests & Sensitivity Analysis

Statistical Tests:

	Row-Eta	Column-Eta	Pearson-r	ζ[a]	n	Z-score
	0.2112	0.0799	0.0060	4.44%	5,000	0.4242

Sensitivity Analysis:

	Row Eta	Column Eta	Pearson-r	ζ	n	Z-score
Price-Level & Quantity Combinations:						
Price-Level Increasing						
Quantity Increasing	0.2560	0.1993	0.0933	5.685%	1,143	3.1512
Quantity Decreasing	0.3251	0.2696	-0.2267	5.434%	1,103	-7.5220[S]
Price-Level Decreasing						
Quantity Increasing	0.2191	0.1748	-0.0219	4.751%	1,066	-0.7131
Quantity Decreasing	0.2099	0.2113	-0.0187	4.429%	1,171	-0.6386
Price-Level Stable	0.2770	0.2529	0.1207	6.219%	260	1.9423
Quantity Stable	0.3633	0.2771	-0.1068	13.131%	267	-1.7414[S]
Generally-Increasing Price Level:						
Combined	0.2530	0.1688	-0.0056	6.400%	2,371	-0.2702
Small	0.2187	0.1668	-0.0561	4.467%	525	1.2833
Medium	0.2317	0.3131	-0.0608	9.431%	781	-1.6981[S]
Large	0.3316	0.2250	-0.0714	10.548%	1,065	-2.3293[S]
Generally-Decreasing Price Level:						
Combined	0.2348	0.1853	0.0450	5.311%	2,369	2.1903
Small	0.2902	0.2598	0.5071	8.166%	541	1.1784
Medium	0.3114	0.2419	0.0327	9.588%	789	0.9165
Large	0.2696	0.2579	0.0937	6.391%	1,039	3.0175

[a] ζ is zeta, the coefficient of nonlinearity.

[S] indicates significance at the .05 level.

Table 17

Summary of Experimental Findings

FIRST EXPERIMENT

	Hypothesis	Finding
H1:	Mean of the link-chain method distribution is less than the mean of the double-extension method distribution.	Not rejected
H2:	Means of the distributions are the same for the "ideal" and double-extension methods.	Not rejected
H3:	Means of the distributions are the same for the "ideal" and link-chain methods.	Rejected at .05 level

SECOND EXPERIMENT

	Hypothesis	Finding
H1:	Under the current practice of new item introduction, the mean of the link-chain distribution is less than the mean of the double-extension distribution.	Rejected at .05 level
H2:	Under the BLS method of new item introduction, the mean of the link-chain distribution is less than the mean of the double-extension distribution.	Not rejected
H3:	Mean of the double-extension distribution is less under the BLS method of new item introduction than under the current practice of new item introduction.	Rejected at .05 level
H4:	Mean of the link-chain distribution is less under the BLS method of new item introduction than under the current practice of new item introduction.	Rejected at .05 level*
H5:	Under the current practice of new item introduction, there is no negative correlation between the inventory "turnover" rate and the difference between the double-extension and link-chain valuations.	Not rejected

* Not rejected for the Large subcategory of the Generally-Increasing category.

Conclusions, Limitations, and Extensions

The research was conducted in order to reach a conclusion on the acceptability of link-chain LIFO as an inventory valuation method. To reach that conclusion, it was necessary to produce the first evidence on the nature of the relationship between the valuations produced by the double-extension and link-chain variants of dollar-value LIFO. Prior to this study, there existed only speculation and assumptions that taxpayers could reduce their tax liabilities by using the link-chain LIFO method.

Two research issues were separately analyzed in this study. The two issues were: (1) the consistency issue that inquired whether the link-chain LIFO method produced consistent inventory valuations; and (2) the valuation-comparability issue that inquired whether the valuations produced by the link-chain method were numerically lower than those produced by the double-extension method.

A process-analysis approach was used to analyze the consistency issue. In order to use the double-extension LIFO method as a benchmark, it was necessary to model both the double-extension and link-chain LIFO processes. The mathematical models were analyzed to identify procedural flaws that might lead to the production of inconsistent inventory valuations. The identified procedural elements were rigorously analyzed to determine the impact of the flaw on the production of consistent valuations.

The valuation-comparability issue was analyzed using a simulation approach owing to the lack of access to large amounts of real data and to the perceived inappropriateness of an analytical solution. The simulation approach encompassed the design of two experiments to produce quantitative, inventory-valuation data for hypothesis testing. In the first experiment, a small amount of real data was processed to produce double-extension and link-chain LIFO valuations, and valuations from an "ideal" dollar-value LIFO method based on Fisher's "ideal" index methodology. Statistical tests were made of hypotheses that were concerned with the relationship between the double-extension and link-chain valuations, and the relationship between each of those valuations and the "ideal" valuation.

The second experiment processed a large amount of synthesized data to produce double-extension and link-chain LIFO valuations under two methods of new item introduction. The two methods of new item introduction were (1) the current practice where new items may be entered at a base-period cost that is equal to their current-period cost, and (2) the BLS method where new items are, in effect, entered on the assumptions that their base-period cost has varied at a rate equal to the price-level index for all non-new items in the inventory pool.

Statistical tests were made of hypotheses that were concerned with the relationships between the double-extension and link-chain under both methods of new item introduction, the effect of the BLS method of new item introduction on those valuations, and the relationship between inventory "turnover" and the difference between the link-chain and double-extension valuations. The hypotheses tests of the second experiment were subjected to sensitivity analyses to minimize the possibility that significant differences had been "randomized out."

The remainder of this chapter contains four major sections: (1) a summary of the results of this study; (2) a discussion of the limitations of this study; (3) a statement of the conclusions that can be reached; and (4) possible extensions of the study.

Summary of Results

This section contains separate summaries of the analyses of the consistency issue and the valuation-comparability issue. The valuation-comparability summary contains separate discussion of the two experiments used to analyze the issue.

The Consistency Issue

The major flaw of the link-chain method is the inappropriate incorporation of the annual price-level index in periods having no annual layer. The cumulative price-level index is distorted, and all future inventory valuations are misstated. The study discussed and developed a method of eliminating the inappropriate-incorporation error.[119] The study identified the circumstances that cause the error, but made no comment on the materiality (if any) of the misstatements that may currently exist in the accounting records of link-chain, dollar-value LIFO taxpayers.

The link-chain and double-extension methods disproportionately share a flaw that occurs after the recognition of a LIFO-layer liquidation. The flaw is the use of a

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119 For discussion of inappropriate-incorporation modification; see above, p. 35. For equations describing modified link-chain process; see above, Fig. 3, p. 36.

pre-liquidation price-level index to assign a LIFO valuation to a post-liquidation layer. The LIFO-layer liquidation may also cause the link-chain method to improperly compute the base-period cost of the ending inventory.

In the double-extension method, the inventory's LIFO valuations will be misstated for the current and all subsequent periods. In the link-chain method, both the inventory's base-period cost and LIFO valuations will be misstated for the current and all subsequent periods. The study discussed methods of eliminating the LIFO-liquidation error from the double-extension method and from the link-chain method.[120] The study identified the circumstances that produce the LIFO-layer liquidation errors, but made no comment on the materiality (if any) of the misstatements that may currently exist in the accounting records of either double-extension or link-chain taxpayers.

The double-extension method is flawed in that a taxpayer is allowed to satisfactorily reconstruct multiple prior-period costs for a new item entering an inventory pool subsequent to the LIFO base date. The taxpayer can "pick and choose" that prior-period cost deemed most advantageous. The seriousness of this flaw is questionable because a taxpayer most often will choose the lowest (earliest) cost. The flaw, however, exists and cannot be corrected with a simple, procedural modification.

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120 For discussion of double-extension, LIFO-layer liquidation error elimination; see above, p. 41. For discussion of link-chain, LIFO-layer liquidation error elimination; see above, p. 43.

The price-level indexes used by the double-extension and link-chain methods were examined for structural flaws having the potential to cause either of the methods to produce inconsistent valuations. The authoritative research performed by Fisher was reviewed, and it was discovered that both the double-extension and link-chain methods use Paasche indexes. Also discovered was that Fisher had rated the Paasche price-level index only 38 of the 119 index types researched. The inferiority is is not indigenous to the dollar-value LIFO methods, but to the Paasche index itself.

The ability of either the double-extension or link-chain method to produce fair inventory valuations with the Paasche index is suspect. In response to the Paasche's inferiority, a dollar-value LIFO method based on Fisher's "ideal" index methodology was proposed and developed. The "ideal" index uses both Paasche and Laspeyres indexes. Therefore, an examination of the Laspeyres index was necessary. From that examination, it was seen that attempting to justify the use of link-chain LIFO by citing the BLS's use of a chain-method in the CPI is inappropriate, and that the Bureau of Labor Statistics has a comprehensive procedure for assigning base-period costs to new or changed items entering the CPI's market basket of goods and services.

The Valuation-Comparability Issue

The first experiment was designed to accomplish two main tasks. The two tasks were (1) the validation of the computer model by comparing its output valuation against Fisher's earlier computations, and (2) the introduction and testing of a valuation method based on Fisher's "ideal" index methodology.[121] The second experiment was designed to produce generalizable results on the relationship of double-extension valuations to link-chain valuations. The experiment also analyzed the effect of the BLS method of new item introduction, and the nature of the relationship between the inventory "turnover" rate and the difference between the double-extension and link-chain methods' valuations.[122]

First Experiment

The conclusion of the first hypothesis test was that the link-chain method had produced lower inventory valuations than the double-extension method. The conclusion demonstrated that a link-chain method taxpayer can, circumstantially, gain a tax advantage over a double-extension method taxpayer. The results of the second and third hypotheses tests indicated that the "ideal" method produced a valuation that was consistent with the double-extension valuation, but not with the link-chain valuation.

The APL model was validated against Fisher's earlier computations of the extensions needed to derive the Paasche price-level indexes used by both the double-extension and link-chain methods. The computer-generated output was

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121 For discussion of "ideal"index; see above, p. 51. For discussion of "ideal" valuation method; see above, pp. 76-78.

122 For discussion of "turnover" test; see above, pp. 10-11, 101-02, 137-38.

recursively matched against Fisher's computations until the two sets of figures matched. The experiment also demonstrated the practicability of computing and using a dollar-value LIFO method based on Fisher's "ideal" index methodology.

Second Experiment

The results of the first hypothesis test showed that link-chain valuations are not necessarily always lower than the comparable double-extension valuations. The results of the tests of the first four hypotheses indicated that the IRS could not expect tax revenues to decrease if (1) taxpayers were authorized to use link-chain LIFO under the current-practice of base-cost reconstruction, or if (2) double-extension LIFO taxpayers were authorized to use the BLS method of base-cost reconstruction, or (3) both. There was some indication, however, that a link-chain taxpayer using the BLS method of new item introduction might gain a tax advantage over a link-chain taxpayer using the current practice of new item introduction at higher rates of price-level change than those used in the experiment. [123]

The result of the fifth hypothesis test indicated that the IRS's inventory "turnover" test is grounded in taxpayer practicality. The study, however, questioned the appropriateness of the current "turnover" ratio, and suggested alternative quantitative "practicality" criteria.

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123 For discussion of trend at higher rates of price-level change; see above, p. 133.

Limitations of the Study

The generalizability of the findings on the two issues differ. Therefore, the limitations applicable to the issues are separately discussed below.

Consistency Issue

The analysis of the consistency issue was applied to the current practices of the double-extension and link-chain methods. The two valuation processes are abstract entities that are independent from environmental considerations. Therefore, the analytic findings are generalizable to the valuation processes without limit. The assessment of the materiality of the flaws identified during the analysis is a separate issues. The study did not offer an opinion on the matter of materiality.

Valuation-Comparability Issue

The statistical tests in this study used a .05 α-level in recognition of the IRS standard.[124] A *post hoc* inspection of the observed test statistics indicated that a lower α-level could have been used without producing contradictory results. There is, however, always a risk in inferential statistics that $\alpha\%$ of the test results may be significant by chance alone. The point being made is that the experimental results can appropriately be generalized only to a class of inventories, and not to individual inventories.

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124 Felix, Jr. and Roussey, p. 38.

First Experiment

The statistical findings of the first experiment, at best, have extremely limited generalizability. The experiment used a small sample of data possessing cost and quantity attributes of extreme volatility that cannot be considered to be representative of the universe of taxpayers' inventories. Two non-statistical findings were produced that do not suffer from limited generalizability. Those findings are (1) that it is possible for link-chain LIFO valuations to be lower than the comparable double-extension method valuations, and (2) that the development and use of a dollar-value LIFO method based on a superior index methodology is practicable.

Second Experiment

The use of a simulation approach means that the research results have limited generalizability.[125] The research results are validly generalized only to inventory pools that have the same characteristics as the synthesized data used in the simulation. These characteristics were defined, and are contained in Table 4 on page 95.

Sensitivity analyses were performed on the tests of hypotheses. The analyses indicated that the results of the statistical tests of the hypotheses were consistent in the subgroups of the simulated distribution. Therefore, the results of the hypotheses tests can be generalized to the data subgroups. The IRS requirement for a sample

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125 For discussion of generalizability of simulation results; see above, pp. 65, 86-87, 94.

size of 500 in inventory measurements means that the results may not be validly generalizable to the price-level stable and quantity-stable subgroups of the price-level and quantity categories.[126] As shown in Figure 7 on page 108, both data groups contain fewer than 500 measurements.

The generalizability of the experimental results were also limited by three experimental design decisions. The experimental design made no provision for: (1) bargain purchases, (2) involuntary liquidations, or (3) negative inventory account balances. The results, therefore, are not validly generalizable to inventory pools that exhibit any of the three preceding characteristics.

Conclusions

This section contains the conclusions that can be drawn from the findings of the study. There are two subsections. The first sub-section contains the study's conclusion on the acceptability of the link-chain LIFO method; the second contains the conclusions on issues other than the research question.

Research Question

The study was undertaken to answer the research question, "Compared to the double-extension LIFO method, is the link-chain LIFO method an acceptable method of inventory valuation?" Two research issues, the consistency issue and the

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126 Ibid.

valuation-comparability issue. were examined in order to answer the research question. The link-chain method was to be found acceptable only if the findings of both research issues were favorable to the link-chain method.

The consistency issue asked, "Does the link-chain method produce valuation measurements that are equal in quality (consistency) to the valuation measurements produced by the double-extension method?" The analysis produced the finding that link-chain measurements were of a lower consistency than double-extension measurements. Two factors contributing to the diminished consistency were (1) the inappropriate impounding of the annual price-level index in periods without an annual layer, and (2) the potential to compound errors because of the multiplicative nature of the cumulative price-level index.

Therefore, the answer to the consistency issue's question is "No." The analytic finding on the consistency issue is against the link-chain method.

The valuation-comparability issue asked, "Does the link-chain method produce valuation measurements that are numerically lower than the comparable double-extension method valuation measurements?" The analysis produced the finding that the answer varied circumstantially. The first experiment used a small amount of real data, and concluded that the link-chain valuations were less than the comparable double-extension valuations. The second experiment used a large amount of synthesized data, and concluded that the link-chain valuations were not less than the comparable double-extension valuations.

Therefore, the answer to the valuation-comparability issue's question is a qualified "No." The analytic finding on the valuation-comparability issue is against the link-chain method.

To be found an acceptable method of inventory valuation, the link-chain method needed favorable findings from both analyses. Neither analytic finding was favorable to the link-chain method. Thus, the study concludes that the link-chain method is not an acceptable method of inventory valuation.

Secondary Conclusions

The study determined that the *inappropriate-incorporation* and *LIFO-layer liquidation* errors, as identified in this study, could be eliminated.[127] The procedural modifications necessary to eliminate the errors were identified and discussed.[128] It is recommended that these modifications be implemented as soon as possible. If the resulting misstatements are material, taxpayers' financial statements will be distorted and misleading.

The study cannot, however, conclude that the link-chain method will become an acceptable method of inventory valuation after correction of these errors. The

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127 For discussion of *inappropriate-incorporation* errors; see above, pp. 25-35. For discussion of double-extension, *LIFO-layer liquidation* errors; see above, pp. 37-41. For discussion of link-chain, *LIFO-layer liquidation* error; see above, pp. 41-44.

128 For discussion of modification relevant to the *inappropriate-incorporation* error; see above, p. 35 aand Fig.5, p. 36. For discussion of modification relevant to the double-extension, *LIFO-layer liquidation* error; see above, p. 41. For discussion of modification relevant to the link-chain, *LIFO-layer liquidation* error; see above, p. 43.

multiplicative nature of the link-chain price-level index lends itself to the compounding of measurement and computational errors. Although the CPI is prepared using a link-chain methodology, it cannot be expected that a taxpayer will devote comparable resources or care to link-chain LIFO computations.

Although the study concluded that the IRS's inventory "turnover" test is based on "practicality" concerns, the fairness of the current criterion was questioned. The "turnover" ratio currently used seems less equitable than a criterion based either on absolute compliance cost or on compliance cost relative to sales volume. If the link-chain method is continued, consideration should be given to modifying the criterion for adoption.

The IRS prefers that taxpayers not use the link-chain variant of dollar-value LIFO. This study is supportive of the IRS's bias against link-chain LIFO because of the method's potential for promulgating errors. The study recommends that consideration be given to replacing the current double-extension and link-chain dollar-value LIFO variants with the double-extension method using the BLS method for entering new items into the inventory pool. The BLS method of estimating base-period costs is well established and supported in the price-level index literature.

This study established that, for inventory pools describable by the study's experimental parameters, double-extension LIFO with the BLS method can be used without causing tax revenues to decrease.[129] Double-extension LIFO with the BLS method eliminates the link-chain method's propensity for error compounding, and also eliminates the ability of double-extension LIFO taxpayers to "pick and choose" between alternative base-period costs.[130] In view of these benefits and the uniformity of application offered, the IRS should not object to the implementation of double-extension LIFO with the BLS method.

The major complaint of taxpayers is with the problem that the current practice of double-extension LIFO has with new items entering the inventory pool after the LIFO base date. Double-extension LIFO with the BLS method eliminates this complaint. The AICPA's Emerging Issues Task Force has recommended that taxpayers estimate a base-date cost "if it is not otherwise objectively determinable."[131] The BLS method provides a structured process for estimating base-period costs. In view of these benefits, the accounting community should not object to the implementation of double-extension LIFO with the BLS method.

Extensions to the Study

Currently, there exists no data base of inventory-pool item price and quantity attributes. Such a data base would be of value to accounting researchers, to

129 For generalizable, inventory-pool characteristics; see above, Table 4, p. 95. Previous discussion of research to identify subsets; see above, pp. 104-6.

130 For discussion of this potential; see above, p. 58.

131 AICPA Emerging Issues Task Force, §4-D.

governmental economists and planners, to individual taxpayers and industry groups, and to the IRS. As previously mentioned, taxpayers believe they have nothing to gain by disclosing their inventory data. Therefore, the establishment of the data base might best be accomplished under the aegis of a branch of the U.S. Government.

Another possible extension consists of replications of part of the second experiment to qualify or disqualify additional classes of taxpayer inventory pools to use double-extension LIFO with the BLS method of new item introduction. Because link-chain related computations are unnecessary, only the third hypothesis of the second experiment need be replicated.[132]

Alternatively, the repeated simulations can be eliminated by deriving an analytical solution. Mathematical models of the valuation processes were developed and can be used in the analytical solution. The IRS's statistical procedures presume a normal distribution indicating the necessity of integrating the equation for the Gaussian distribution into the solution.[133] The analytical solution would permit the use of simple descriptive statistics to identify acceptable classes of inventory pools.

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132 For discussion of the third hypothesis of the second experiment; see above, p. 100. For discussion of the test of the third hypothesis of the second experiment; see above, pp. 127-28.

133 For discussion of model development; see above, p. 19. For dollar-value LIFO, double-extension method equations; see above, Fig. 1, p. 20. For dollar-value LIFO, link-chain method equations; see above, Fig. 2, p. 21.

Fisher's evaluation of the Paasche index suggests research into the fairness of dollar-value LIFO valuations.[134] The practicability of a valuation method based on Fisher's "ideal" index methodology with the double-extension LIFO method was demonstrated in the study.[135] Additional detailed research, performed as modified replications of Fisher's research, is indicated. The research's objective, however, would be to determine the fairest valuation method, not the fairest index number type.

There is a need for research into the presence and materiality of the inventory-valuation misstatements that may have been produced by the procedural weaknesses identified in this study.[136] If such misstatements have occurred, they must be corrected. Else, the users of financial statements may be misled, and dollar-value LIFO's credibility will suffer.

This study has produced evidence on the link-chain LIFO controversy, and has demonstrated the need for more research on the role of dollar-value LIFO inventory valuations in the income taxation process.

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134 For discussion of Paasche index findings; see above, pp. 47-48.

135 For discussion of "ideal" valuation method; see above, pp. 76-78.

136 For discussion of *inappropriate-incorporation* errors; see above, pp. 25-35. For discussion of *LIFO-layer liquidation* errors; see above, pp. 37-43.

Appendix A

Data of the First Experiment

Table 18

Prices of the 36 Commodities, 1913-18 ($)

No.	Commodity	1913	1914	1915	1916	1917	1918
1	Bacon	0.1236	0.1295	0.1129	0.1462	0.2382	0.2612
2	Barley	0.6263	0.6204	0.7103	0.8750	1.3232	1.4611
3	Beef	0.1295	0.1364	0.1289	0.1382	0.1672	0.2213
4	Butter	0.2969	0.2731	0.2743	0.3179	0.4034	0.4857
5	Cattle	12.0396	11.9208	12.1354	12.4375	15.6354	18.8646
6	Cement	1.5800	1.5800	1.4525	1.6888	2.0942	2.6465
7	Coal, anth.	5.0636	5.0592	5.0464	5.2906	5.6218	6.5089
8	Coal, bit.	1.2700	1.1700	1.0400	2.0700	3.5800	2.4000
9	Coffee	0.1113	0.0816	0.0745	0.0924	0.0929	0.0935
10	Coke	3.0300	2.3200	2.4200	4.7800	10.6600	7.0000
11	Copper	0.1533	0.1318	0.1676	0.2651	0.2764	0.2468
12	Cotton	0.1279	0.1121	0.1015	0.1447	0.2350	0.3178
13	Eggs	0.2468	0.2660	0.2597	0.2945	0.4015	0.4827
14	Hay	11.2500	12.3182	11.6250	10.0625	17.6042	21.8958
15	Hides	0.1727	0.1842	0.2076	0.2391	0.2828	0.2144
16	Hogs	8.3654	8.3608	7.1313	9.6459	15.7047	17.5995
17	Iron Bars	1.5100	1.2000	1.3700	2.5700	4.0600	3.5000
18	Iron, pig	14.9025	13.3900	13.5758	18.6708	38.8082	36.5340
19	Lead (white)	0.0676	0.0675	0.0698	0.0927	0.1121	0.1271
20	Lead	0.0437	0.0386	0.0467	0.0686	0.0879	0.0741
21	Lumber	90.3974	90.9904	90.5000	91.9000	105.0400	121.0455
22	Mutton	0.1025	0.1010	0.1073	0.1250	0.1664	0.1982
23	Petroleum	0.1233	0.1200	0.1208	0.1217	0.1242	0.1695
24	Pork	0.1486	0.1543	0.1429	0.1618	0.2435	0.2495
25	Rubber	0.8071	0.6158	0.5573	0.6694	0.6477	0.5490
26	Silk	3.9083	4.0573	3.6365	5.4458	5.9957	6.9770
27	Silver	0.5980	0.5481	0.4969	0.6566	0.8142	0.9676
28	Skins	2.5833	2.6250	2.7188	4.1729	5.5208	5.5625
29	Steel rails	28.0000	28.0000	28.0000	31.3333	38.0000	54.0000
30	Tin, pig	44.3200	35.7000	38.6600	43.4800	61.6500	87.1042
31	Tin plate	3.5583	3.3688	3.2417	5.1250	9.1250	7.7300
32	Wheat	0.9131	1.0412	1.3443	1.4165	2.3211	2.2352
33	Wool	0.5883	0.5975	0.7375	0.7900	1.2841	1.6600
34	Lime	1.2500	1.2500	1.2396	1.4050	1.7604	2.3000
35	Lard	0.1101	0.1037	0.0940	0.1347	0.2170	0.2603
36	Oats	0.3758	0.4191	0.4958	0.4552	0.6372	0.7747

SOURCE: Irving Fisher, *The Making of Index Numbers* (Boston: Houghton Mifflin Co., 1927):489, Table 63.

Table 19

Quantities Marketed of the 36 Commodities, 1913-18 (Millions of units)

No.	Commodity	1913	1914	1915	1916	1917	1918
1	Bacon, lb.	1,077.00	1,069.00	1,869.00	1,481.00	1,187.00	1,498.00
2	Barley, bu.	178.20	195.00	228.90	182.30	209.00	256.40
3	Beef, lb.	6,589.00	6,522.00	6,820.00	7,134.00	8,417.00	10,244.00
4	Butter, lb.	1,757.00	1,780.00	1,800.00	1,820.00	1,842.00	1,916.00
5	Cattle, cwt.	69.80	67.60	71.50	83.10	103.50	118.30
6	Cement, bbl.	85.80	84.40	84.40	92.00	88.10	69.40
7	Coal, anth., ton	6.90	6.86	6.78	6.75	7.83	7.69
8	Coal, bit., ton	477.00	424.00	443.00	502.00	552.00	583.00
9	Coffee, lb.	863.00	1,002.00	1,119.00	1,201.00	1,320.00	1,144.00
10	Coke, short ton	46.30	34.60	41.60	54.50	56.70	55.00
11	Copper, lb.	812.30	620.50	1,043.50	1,429.80	1,316.50	1,648.30
12	Cotton, lb.	2,785.00	2,820.00	2,838.00	3,235.00	3,423.00	3,298.00
13	Eggs, doz.	1,772.00	1,759.00	1,791.00	1,828.00	1,882.00	1,908.00
14	Hay, ton	79.20	83.00	103.00	111.00	94.90	89.80
15	Hides, lb.	672.00	924.00	1,227.00	1,212.00	1,113.00	663.00
16	Hogs, cwt.	68.40	65.10	76.80	86.20	67.80	82.40
17	Iron Bar, cwt.	79.20	50.40	82.60	132.40	133.00	132.00
18	Iron, pig, ton	31.00	23.30	29.90	39.40	38.70	38.10
19	Lead (white), lb.	286.00	318.00	312.00	258.00	230.00	216.00
20	Lead, lb.	823.70	1,025.60	1,014.10	1,104.50	1,099.80	1,083.00
21	Lumber, M bd. ft.	21.80	20.70	20.50	22.30	21.20	19.20
22	Mutton, lb.	732.00	734.00	629.00	618.00	474.00	513.00
23	Petroleum, gal.	10,400.00	11,200.00	11,840.00	12,640.00	14,880.00	15,680.00
24	Pork, lb.	9,211.00	8,871.00	9,912.00	10,524.00	8,427.00	11,426.00
25	Rubber, lb.	115.80	136.60	231.40	258.80	375.90	351.50
26	Silk, lb.	19.10	19.10	20.00	24.40	29.40	27.10
27	Silver, oz.	146.10	144.00	173.40	139.30	133.60	140.70
28	Skins, skin	6.70	5.90	4.30	5.60	2.70	0.70
29	Steel rails, ton	3.50	1.95	2.20	2.86	2.94	2.37
30	Tin, pig, cwt.	1.04	0.95	1.16	1.43	1.56	1.59
31	Tin plate, cwt.	15.30	17.30	19.70	22.80	29.50	28.00
32	Wheat, bu.	555.00	654.00	588.00	642.00	605.00	562.00
33	Wool, lb.	448.00	550.00	699.00	737.00	707.00	752.00
34	Lime, bbl., 300 lb.	23.30	22.50	25.00	27.10	24.00	20.20
35	Lard, lb.	1,100.00	955.00	1,050.00	1,141.00	927.00	1,107.00
36	Oats, bu.	1,122.00	1,240.00	1,360.00	1,480.00	1,587.00	1,538.00

SOURCE: Irving Fisher, *The Making of Index Numbers* (Boston: Houghton Mifflin Co., 1927):490, Table 64.

Table 20

Dollar Values (in $Millions) for 36 Commodities, 1913-18

$p_0 q_n$

No.	Commodity	1913	1914	1915	1916	1917	1918
1	Bacon	133.12	132.13	231.01	183.05	146.71	185.15
2	Barley	111.61	122.13	143.36	114.17	130.90	160.58
3	Beef	853.28	844.60	883.19	923.85	1,090.00	1,326.60
4	Butter	521.65	528.48	534.42	540.36	546.89	568.86
5	Cattle	840.36	813.88	860.83	1,000.49	1,246.10	1,424.28
6	Cement	135.56	133.35	133.35	145.36	139.20	109.65
7	Coal, anth.	34.94	34.74	34.33	34.18	39.65	38.94
8	Coal, bit.	605.79	538.48	562.61	637.54	701.04	740.41
9	Coffee	96.05	111.52	124.54	133.67	146.92	127.33
10	Coke	140.29	104.84	126.05	165.14	171.80	166.65
11	Copper	124.53	95.12	159.97	219.19	201.82	252.68
12	Cotton	356.20	360.68	362.98	413.76	437.80	421.81
13	Eggs	437.33	434.12	442.02	451.15	464.48	470.89
14	Hay	891.00	933.75	1,158.75	1,248.75	1,067.63	1,010.25
15	Hides	116.05	159.57	211.90	209.31	192.22	114.50
16	Hogs	572.19	544.59	642.46	721.10	567.17	689.31
17	Iron Bars	119.59	76.10	124.73	199.92	200.83	199.32
18	Iron, pig	461.98	347.23	445.58	587.16	576.73	567.79
19	Lead (white)	19.33	21.50	21.09	17.44	15.55	14.60
20	Lead	36.00	44.82	44.32	48.27	48.06	47.33
21	Lumber	1,970.66	1,871.23	1,853.15	2,015.86	1,916.42	1,735.63
22	Mutton	75.03	75.24	64.47	63.35	48.58	52.58
23	Petroleum	1,282.32	1,380.96	1,459.87	1,558.51	1,834.70	1,933.34
24	Pork	1,368.75	1,318.23	1,472.92	1,563.87	1,252.25	1,697.90
25	Rubber	93.46	110.25	186.76	208.88	303.39	283.70
26	Silk	74.65	74.65	78.17	95.36	114.90	105.91
27	Silver	87.37	86.11	103.69	83.30	79.89	84.14
28	Skins	17.31	15.24	11.11	14.47	6.97	1.81
29	Steel rails	98.00	54.60	61.60	80.08	82.32	66.36
30	Tin, pig	46.09	42.10	51.41	63.38	69.14	70.47
31	Tin plate	54.44	61.56	70.10	81.13	104.97	99.63
32	Wheat	506.77	597.17	536.90	586.21	552.43	513.16
33	Wool	263.56	323.57	411.22	433.58	415.93	442.40
34	Lime	29.13	28.13	31.25	33.88	30.00	25.25
35	Lard	121.11	105.15	115.61	125.62	102.06	121.88
36	Oats	421.65	465.99	511.09	556.18	596.39	577.98

Table 21

Dollar Values (in $Millions) for 36 Commodities, 1913-18

$p_{n-1} q_n$

No.	Commodity	1913	1914	1915	1916	1917	1918
1	Bacon	133.12	132.13	242.04	167.20	173.54	356.82
2	Barley	111.61	122.13	142.01	129.49	182.88	339.27
3	Beef	853.28	844.60	930.25	919.57	1,163.23	1,712.80
4	Butter	521.65	528.48	491.58	499.23	585.57	772.91
5	Cattle	840.36	813.88	852.34	1,008.45	1,287.28	1,849.67
6	Cement	135.56	133.35	133.35	133.63	148.78	145.34
7	Coal, anth.	34.94	34.74	34.30	34.06	41.43	43.23
8	Coal, bit.	605.79	538.48	518.31	522.08	1,142.64	2,087.14
9	Coffee	96.05	111.52	91.31	89.47	121.97	106.28
10	Coke	140.29	104.84	96.51	131.89	271.03	586.30
11	Copper	124.53	95.12	137.53	239.63	349.00	455.59
12	Cotton	356.20	360.68	318.14	328.35	495.31	775.03
13	Eggs	437.33	434.12	476.41	474.73	554.25	766.06
14	Hay	891.00	933.75	1,268.77	1,290.38	954.93	1,580.86
15	Hides	116.05	159.57	226.01	251.61	266.12	187.50
16	Hogs	572.19	544.59	642.11	614.72	653.99	1,294.07
17	Iron Bars	119.59	76.10	99.12	181.39	341.81	535.92
18	Iron, pig	461.98	347.23	400.36	534.89	722.56	1,478.59
19	Lead (white)	19.33	21.50	21.06	18.01	21.32	24.21
20	Lead	36.00	44.82	39.14	51.58	75.45	95.20
21	Lumber	1,970.66	1,871.23	1,865.30	2,018.15	1,948.28	2,016.77
22	Mutton	75.03	75.24	63.53	66.31	59.25	85.36
23	Petroleum	1,282.32	1,380.96	1,420.80	1,526.91	1,810.90	1,947.46
24	Pork	1,368.75	1,318.23	1,529.42	1,503.88	1,363.49	2,782.23
25	Rubber	93.46	110.25	142.50	144.23	251.63	227.67
26	Silk	74.65	74.65	81.15	88.73	160.11	162.48
27	Silver	87.37	86.11	95.04	69.22	87.72	114.56
28	Skins	17.31	15.24	11.29	15.23	11.27	3.86
29	Steel rails	98.00	54.60	61.60	80.08	92.12	90.06
30	Tin, pig	46.09	42.10	41.41	55.28	67.83	98.02
31	Tin plate	54.44	61.56	66.37	73.91	151.19	255.50
32	Wheat	506.77	597.17	612.23	863.04	856.98	1,304.46
33	Wool	263.56	323.57	417.65	543.54	558.53	965.64
34	Lime	29.13	28.13	31.25	33.59	33.72	35.56
35	Lard	121.11	105.15	108.89	107.25	124.87	240.22
36	Oats	421.65	465.99	569.98	733.78	722.40	980.01

Table 22

Dollar Values (in $Millions) for 36 Commodities, 1913-18

$p_n q_0$

No.	Commodity	1913	1914	1915	1916	1917	1918
1	Bacon	133.12	139.47	121.59	157.46	256.54	281.31
2	Barley	111.61	110.56	126.58	155.93	235.79	260.37
3	Beef	853.28	898.74	849.32	910.60	1,101.68	1,458.15
4	Butter	521.65	479.84	481.95	558.55	708.77	853.37
5	Cattle	840.36	832.07	847.05	868.14	1,091.35	1,316.75
6	Cement	135.56	135.56	124.62	144.90	179.68	227.07
7	Coal, anth.	34.94	34.91	34.82	36.51	38.79	44.91
8	Coal, bit.	605.79	558.09	496.08	987.39	1,707.66	1,144.80
9	Coffee	96.05	70.42	64.29	79.74	80.17	80.69
10	Coke	140.29	107.42	112.05	221.31	493.56	324.10
11	Copper	124.53	107.06	136.14	215.34	224.52	200.48
12	Cotton	356.20	312.20	282.68	402.99	654.47	885.07
13	Eggs	437.33	471.35	460.19	521.85	711.46	855.34
14	Hay	891.00	975.60	920.70	796.95	1,394.25	1,734.15
15	Hides	116.05	123.78	139.51	160.68	190.04	144.08
16	Hogs	572.19	571.88	487.78	659.78	1,074.20	1,203.81
17	Iron Bars	119.59	95.04	108.50	203.54	321.55	277.20
18	Iron, pig	461.98	415.09	420.85	578.79	1,203.05	1,132.55
19	Lead (white)	19.33	19.31	19.96	26.51	32.06	36.35
20	Lead	36.00	31.79	38.47	56.51	72.40	61.04
21	Lumber	1,970.66	1,983.59	1,972.90	2,003.42	2,289.87	2,638.79
22	Mutton	75.03	73.93	78.54	91.50	121.80	145.08
23	Petroleum	1,282.32	1,248.00	1,256.32	1,265.68	1,291.68	1,762.80
24	Pork	1,368.75	1,421.26	1,316.25	1,490.34	2,242.88	2,298.14
25	Rubber	93.46	71.31	64.54	77.52	75.00	63.57
26	Silk	74.65	77.49	69.46	104.01	114.52	133.26
27	Silver	87.37	80.08	72.60	95.93	118.95	141.37
28	Skins	17.31	17.59	18.22	27.96	36.99	37.27
29	Steel rails	98.00	98.00	98.00	109.67	133.00	189.00
30	Tin, pig	46.09	37.13	40.21	45.22	64.12	90.59
31	Tin plate	54.44	51.54	49.60	78.41	139.61	118.27
32	Wheat	506.77	577.87	746.09	786.16	1,288.21	1,240.54
33	Wool	263.56	267.68	330.40	353.92	575.28	743.68
34	Lime	29.13	29.13	28.88	32.74	41.02	53.59
35	Lard	121.11	114.07	103.40	148.17	238.70	286.33
36	Oats	421.65	470.23	556.29	510.73	714.94	869.21

Table 23

Dollar Values (in $Millions) for 36 Commodities, 1913-18

$p_n q_{n-1}$

No.	Commodity	1913	1914	1915	1916	1917	1918
1	Bacon	133.12	139.47	120.69	273.25	352.77	310.04
2	Barley	111.61	110.56	138.51	200.29	241.22	305.37
3	Beef	853.28	898.74	840.69	942.52	1,192.80	1,862.68
4	Butter	521.65	479.84	488.25	572.22	734.19	894.66
5	Cattle	840.36	832.07	820.35	889.28	1,299.30	1,952.49
6	Cement	135.56	135.56	122.59	142.53	192.67	233.16
7	Coal, anth.	34.94	34.91	34.62	35.87	37.95	50.96
8	Coal, bit.	605.79	558.09	440.96	917.01	1,797.16	1,324.80
9	Coffee	96.05	70.42	74.65	103.40	111.57	123.42
10	Coke	140.29	107.42	83.73	198.85	580.97	396.90
11	Copper	124.53	107.06	104.00	276.63	395.20	324.91
12	Cotton	356.20	312.20	286.23	410.66	760.22	1,087.83
13	Eggs	437.33	471.35	456.81	527.45	733.94	908.44
14	Hay	891.00	975.60	964.88	1,036.44	1,954.07	2,077.91
15	Hides	116.05	123.78	191.82	293.38	342.75	238.63
16	Hogs	572.19	571.88	464.25	740.81	1,353.75	1,193.25
17	Iron Bars	119.59	95.04	69.05	212.28	537.54	465.50
18	Iron, pig	461.98	415.09	316.32	558.26	1,529.04	1,413.87
19	Lead (white)	19.33	19.31	22.20	28.92	28.92	29.23
20	Lead	36.00	31.79	47.90	69.57	97.09	81.50
21	Lumber	1,970.66	1,983.59	1,873.35	1,883.95	2,342.39	2,566.16
22	Mutton	75.03	73.93	78.76	78.63	102.84	93.95
23	Petroleum	1,282.32	1,248.00	1,352.96	1,440.93	1,569.89	2,522.16
24	Pork	1,368.75	1,421.26	1,267.67	1,603.76	2,562.59	2,102.54
25	Rubber	93.46	71.31	76.13	154.90	167.62	206.37
26	Silk	74.65	77.49	69.46	108.92	146.30	205.12
27	Silver	87.37	80.08	71.55	113.85	113.42	129.27
28	Skins	17.31	17.59	16.04	17.94	30.92	15.02
29	Steel rails	98.00	98.00	54.60	68.93	108.68	158.76
30	Tin, pig	46.09	37.13	36.73	50.44	88.16	135.88
31	Tin plate	54.44	51.54	56.08	100.96	208.05	228.04
32	Wheat	506.77	577.87	879.17	832.90	1,490.15	1,352.30
33	Wool	263.56	267.68	405.63	552.21	946.38	1,173.62
34	Lime	29.13	29.13	27.89	35.13	47.71	55.20
35	Lard	121.11	114.07	89.77	141.44	247.60	241.30
36	Oats	421.65	470.23	614.79	619.07	943.06	1,229.45

Table 24

Dollar Values (in $Millions) for 36 Commodities, 1913-18

$p_n q_n$

No.	Commodity	1913	1914	1915	1916	1917	1918
1	Bacon	133.12	138.44	211.01	216.52	282.74	391.28
2	Barley	111.61	120.98	162.59	159.51	276.55	374.63
3	Beef	853.28	889.60	879.10	985.92	1,407.32	2,267.00
4	Butter	521.65	486.12	493.74	578.58	743.06	930.60
5	Cattle	840.36	805.85	867.68	1,033.56	1,618.26	2,231.68
6	Cement	135.56	133.35	122.59	155.37	184.50	183.67
7	Coal, anth.	34.94	34.71	34.21	35.71	44.02	50.05
8	Coal, bit.	605.79	496.08	460.72	1,039.14	1,976.16	1,399.20
9	Coffee	96.05	81.76	83.37	110.97	122.63	106.96
10	Coke	140.29	80.27	100.67	260.51	604.42	385.00
11	Copper	124.53	81.78	174.89	379.04	363.88	406.80
12	Cotton	356.20	316.12	288.06	468.10	804.41	1,048.10
13	Eggs	437.33	467.89	465.12	538.35	755.62	920.99
14	Hay	891.00	1,022.41	1,197.38	1,116.94	1,670.64	1,966.24
15	Hides	116.05	170.20	254.73	289.79	314.76	142.15
16	Hogs	572.19	544.29	547.68	831.48	1,064.78	1,450.20
17	Iron Bars	119.59	60.48	113.16	340.27	539.98	462.00
18	Iron, pig	461.98	311.99	405.92	735.63	1,501.88	1,391.95
19	Lead (white)	19.33	21.47	21.78	23.92	25.78	27.45
20	Lead	36.00	39.59	47.36	75.77	96.67	80.25
21	Lumber	1,970.66	1,883.50	1,855.25	2,049.37	2,226.85	2,324.07
22	Mutton	75.03	74.13	67.49	77.25	78.87	101.68
23	Petroleum	1,282.32	1,344.00	1,430.27	1,538.29	1,848.10	2,657.76
24	Pork	1,368.75	1,368.80	1,416.42	1,702.78	2,051.97	2,850.79
25	Rubber	93.46	84.12	128.96	173.24	243.47	192.97
26	Silk	74.65	77.49	72.73	132.88	176.27	189.08
27	Silver	87.37	78.93	86.16	91.46	108.78	136.14
28	Skins	17.31	15.49	11.69	23.37	14.91	3.89
29	Steel rails	98.00	54.60	61.60	89.61	111.72	127.98
30	Tin, pig	46.09	33.92	44.85	62.18	96.17	138.50
31	Tin plate	54.44	58.28	63.86	116.85	269.19	216.44
32	Wheat	506.77	680.94	790.45	909.39	1,404.27	1,256.18
33	Wool	263.56	328.63	515.51	582.23	907.86	1,248.32
34	Lime	29.13	28.13	30.99	38.08	42.25	46.46
35	Lard	121.11	99.03	98.70	153.69	201.16	288.15
36	Oats	421.65	519.68	674.29	673.70	1,011.24	1,191.49

Bibliography

AICPA Emerging Issues Task Force. *Issue Summary: LIFO Issues.* n.p.: AICPA, October 8, 1984.

AICPA Task Force on LIFO Inventory Problems Accounting Standards Division. *Identification and Discussion of Certain Financial Accounting and Reporting Issues Concerning LIFO Inventories - Issue Paper (ISUPAP 42).* n.p.: AICPA, November 30, 1984.

Blanchard, Benjamin S. and Wolter J. Fabrycky. *Systems Engineering And Analysis.* Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1981.

Blough, Carman G., Samuel J. Broad, and Robert M. Trueblood. "Pooling of LIFO Inventories By Use of Dollar-Value Method," memorandum to Mr. Fred C. Scribner, Jr., Under-Secretary of the Treasury, dated February 23, 1960, that was sent by Mr. John L. Carey, Executive Director, American Institute of Certified Public Accountants.

Brighton, Gerald D. and Robert H. Michaelsen. "Profile of Tax Dissertations in Accounting: 1967-1984." *The Journal of the American Tax Association*, Spring 1985 p. 76-91.

Bussey, Lynn E. *The Economic Analysis of Industrial Projects*. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1978.

Collins, R. R., Jr. "Dollar-value LIFO: IRS criteria for change to link-chain method." *The Tax Adviser*, July 1981, p. 420.

Cook, Thomas M. and Robert A. Russell. *Introduction to Management Science*. 3rd. Ed., Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1985.

Cox, C. Richard and Carl L. Glassberg. "LIFO: The Deflator - a Current Review and Analysis." *The Tax Adviser*, January 1971, pp. 28-32.

De Fillips, Willard J. "LIFO for Small Businesses." *Practical Accountant*, May 1984, pp. 26-36.

Felix, William L., Jr. and Robert S. Roussey, "Statistical Inference and the IRS," *Journal of Accountancy*, June 1985, pp. 38-45.

Hertz, David B. "Risk Analysis in Capital Investment." *Harvard Business Review*, January-February, 1964, pp. 95-106.

------------. "Investment Policies That Pay Off." *Harvard Business Review*, January-February, 1968, pp. 96-108.

Hess, Sidney W. and Harry A. Quigley. "Analysis of Risk in Investments Using Monte Carlo Techniques." *Chemical Engineering Progress Symposium Series 42: Statistics and Numerical Methods in Chemical Engineering*. New York: American Institute of Chemical Engineering, 1963.

Hinkle, Dennis E., William Wiersma, and Stephen G. Jurs. *Applied Statistics for the Behavioral Sciences*. Boston: Houghton Mifflin Company, 1979.

Humer, Kenneth G., William G. Galliher, Jr., and George A. Stewart. *LIFO: Fundamentals, Pooling, and Computations*, 69-4th T. M. Tax Management, Inc., 1983.

Ijiri, Yuji. *Accounting Structured in APL*, Accounting Education Series, Volume No. 6. Sarasota, Florida: American Accounting Association, 1984.

Keppel, Geoffrey. Design and Analysis: *A Researcher's Handbook*, 2nd Ed. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1982.

Langley, Russell. *Practical Statistics Simply Explained*. New York: Dover Publications, Inc., 1970.

Lee, Sang M. and Laurence J. Moore. *Introduction to Decision Science*. New York: Petrocelli/Charter, 1975.

Leininger, Wayne E. *Quantitative Methods in Accounting*. New York: D. Van Nostrand Company, 1980.

Luker, Paul A. "Computer Simulation: A Developer's Perspective." *Academic Computing*, Spring 1987, pp. 18-21, 67-68.

Martin, James. *Applications Development Without Programmers*. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1982.

McAnly, Herbert T. "The Current Status of LIFO." *The Journal of Accountancy*. May 1958, pp. 55-62.

------------. "The Case For LIFO: It Realistically States Income and is Applicable to Any Industry." *The Journal of Accountancy*, June 1953, pp. 691-700.

Mock, Theodore J. and Miklos A. Vasarhelyi. *APL for Management*. New York: Wiley, Becker and Hayes, 1972.

Parker, James E. "LIFO Accounting Made Simpler for Small Business." *Journal of Small Business Management*. April 1983, pp. 13-21.

Pollard, J. H. *A Handbook of Numerical and Statistical Techniques*. Cambridge, Great Britain: Cambridge University Press, 1977; reprint Ed., Cambridge, Great Britain: Cambridge University Press, 1981.

Reeve, James M. and Keith G. Stanga. "The LIFO Pooling Decision: Some Empirical Results From Accounting Practice." *Accounting Horizons*, June 1987, pp. 25-33.

Rumble, Clayton T. "So You Still Have Not Adopted LIFO." *Management Accounting*, October 1983, pp. 59-67.

Schneider, Leslie B. *Federal Income Taxation of Inventories*. New York: Matthew Bender and Co., 1980.

Schwartz, Bill N. "Dollar Value LIFO - Evaluating One Experience." *CPA Journal*, April 1982, pp. 42-46.

Securities and Exchange Commission. *Codification of Financial Reporting Policies - Financial Reporting Releases*. 17 CFR Part 211, April 15, 1982.

Silver, Alan. "LIFO:Summing Up." *Industrial Distribution*, March 1979, pp. 57-62.

Taylor, John L. "LIFO: A Different Approach." *Journal of Accountancy*, April 1981, pp. 50-60.

Tondkar, Rasoul H., Edward N. Coffman, Robert F. Mizell, and Jackie G. Williams. "How Small Businesses Can Benefit From LIFO." *Ohio CPA Journal*, Spring 1982, pp. 67-72.

Vasarhelyi, Miklos A. *TREAT: Terminal-Related Educational Audit Tools*. New York: Touche-Ross Foundation, 1980.

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