

STABILITY OF A CONDENSATE FILM  
FLOWING DOWN A VERTICAL PLANE

by

Mazhar Unsal

Dissertation submitted to the Graduate Faculty of the  
Virginia Polytechnic Institute and State University  
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

in

Mechanical Engineering

APPROVED:

---

W. C. Thomas, Chairman

---

J. R. Mahan

---

R. D. Riess

---

F. J. Pierce

---

H. L. Moses

May, 1975

Blacksburg, Virginia

## ACKNOWLEDGMENTS

I am indebted to my major professor, Dr. William C. Thomas, for his guidance and constructive attitude during the research and preparation of this thesis.

## TABLE OF CONTENTS

	<u>Page</u>
ACKNOWLEDGMENTS . . . . .	ii
LIST OF FIGURES . . . . .	v
LIST OF TABLES . . . . .	vi
 CHAPTER	
I. INTRODUCTION . . . . .	1
II. REVIEW OF LITERATURE . . . . .	5
1. PHYSICAL DESCRIPTION OF CONDENSATION AND INTERFACIAL PHENOMENA . . . . .	5
2. LAMINAR FILM CONDENSATION ON VERTICAL FLAT PLATES . . . . .	7
A. ANALYTICAL STUDIES . . . . .	7
B. SMALL PRANDTL NUMBER LIQUID CONDENSATION- DISCREPANCY BETWEEN EXPERIMENT AND THEORY . . . . .	9
C. EXPERIMENTS WITH LIQUIDS OF PRANDTL NUMBER ON THE ORDER OF UNITY OR LARGER . . . . .	11
3. HYDRODYNAMIC STABILITY THEORY . . . . .	14
A. STABILITY OF LIQUID FILMS FLOWING DOWN A FLAT SURFACE . . . . .	16
B. STABILITY OF LAMINAR FILM CONDENSATION ON A VERTICAL WALL . . . . .	19
III. LINEARIZED STABILITY ANALYSIS . . . . .	21
1. ASYMPTOTIC SOLUTION WITH NUSSELT'S BASE FLOW . . . . .	21
2. ASYMPTOTIC SOLUTION WITH ACCELERATION AND CONVECTION EFFECTS . . . . .	29

## TABLE OF CONTENTS - continued

CHAPTER	<u>Page</u>
IV. RESULTS AND DISCUSSION . . . . .	33
V. CONCLUSIONS . . . . .	46
REFERENCES . . . . .	47
APPENDIX I. PROBLEM FORMULATION . . . . .	53
APPENDIX II. SOLUTION OF THE STEADY STATE EQUATIONS-- THE BASE FLOW . . . . .	63
APPENDIX III. DERIVATION OF THE LINEARIZED STABILITY PROBLEM . . . . .	70
VITA . . . . .	77
ABSTRACT	

## LIST OF FIGURES

<u>Figure</u>	<u>Page</u>
1. Neutral Stability Curve when $\mathfrak{S}=0.001$ , $Pr=1.76$ , $f^*=2.08 \times 10^6$ . . . . .	34
2. Neutral Stability Curve when $\mathfrak{S}=0.01$ , $Pr=1.84$ , $f^*=4.36 \times 10^5$ . . . . .	35
3. Neutral Stability Curve when $\mathfrak{S}=0.05$ , $Pr=2.17$ , $f^*=1.35 \times 10^5$ . . . . .	36
4. Nondimensional Critical Distance vs Heat Capacity Parameter. . . . .	40
A1.1 Geometry and Coordinates of Flow . . . . .	54
A1.2 The Interfacial Control Volume . . . . .	56
A2.1 Comparison of the First-order Solution with the Exact Solution . . . . .	69

LIST OF TABLES

<u>Table</u>		<u>Page</u>
I	Critical Distances in Various Experiments . . . . .	42

NOMENCLATURE

$c_p$	specific heat
$C$	$C_r + iC_i$ , defined in equation (A.3.28)
$g$	gravitational acceleration
$h_{fg}$	latent heat of vaporization
$\bar{h}$	Nusselt's mean coefficient of heat transfer
$k$	thermal conductivity
$\ell$	$\lambda/2\pi$ , length scale of disturbance
$L$	length of the wall
$Pr$	$\rho \nu c_p / k$ , Prandtl number
$\tilde{p}, \bar{p}, p$	pressure, dimensionless base flow pressure, dimensionless disturbance pressure
$Re$	$\frac{\tilde{u}_0 \tilde{\eta}}{\nu} = \frac{\tilde{\eta} \bar{\eta}^3}{2 Pr}$ , Reynolds number
$Re_L$	$Re(\tilde{x} = L)$
$\tilde{t}, t$	time coordinate, dimensionless time coordinate
$\tilde{u}_0$	$\tilde{x}$ -component of the free surface velocity
$\tilde{u}, \bar{u}$	$\tilde{x}$ -component velocity, $x$ -component dimensionless velocity of the base flow
$\tilde{v}, \bar{v}$	$\tilde{y}$ -component velocity, $y$ -component dimensionless velocity of the base flow
$\tilde{x}, x$	parallel coordinate, dimensionless parallel coordinate
$\tilde{y}, y$	normal coordinate, dimensionless normal coordinate
$\alpha$	$\left[ k \nu \Delta \tau / (1 - \gamma) \rho g h_{fg} \right]^{1/3}$

$\delta$	$\rho_v/\rho$
$\Gamma$	$4\pi^2\sigma/\rho g\lambda^2$
$\Gamma^*$	$\sigma/\rho g\alpha^2$ , surface tension parameter
$\epsilon$	$\alpha/\ell$ , dimensionless wavenumber
$\mathcal{J}$	$c_p \Delta\tau/h_{fg}$ , heat-capacity parameter
$\eta_0$	defined in equation (A3.27)
$\tilde{\eta}, \bar{\eta}, \eta$	film thickness, dimensionless base flow film thickness, dimensionless disturbance film thickness
$\bar{\theta}, \theta$	dimensionless base flow temperature, dimensionless disturbance temperature
$\lambda$	wavelength of disturbance
$\nu$	kinematic viscosity
$\rho_v, \rho$	vapor density, liquid density
$\sigma$	surface tension
$\tilde{\tau}, \tau_w, \tau_s$	temperature, wall temperature, saturation temperature
$\Delta\tau$	$\tau_s - \tau_w$
$\phi$	defined in equation (A3.25)
$\varphi$	defined in equation (A3.24)
$\tilde{\psi}, \bar{\psi}, \psi$	stream function, dimensionless base flow stream function, dimensionless disturbance stream function

## I. INTRODUCTION

When a vapor comes in contact with a body below the saturation temperature of the vapor, it is observed that the vapor in the vicinity of the body experiences a phase change. This process is called condensation and the liquid phase resulting from the process is called the condensate. If a liquid condensate completely "wets" the surface of the body, it appears as a film of liquid on the body. This is called film condensation.

Heat transfer by film condensation is of major importance in many industrial applications. Consequently, the problem has been subject to experimental and theoretical studies by many investigators. While these studies have been carried out for film condensation on various different body geometries, the vertical flat plate has been studied most extensively. The previous experimental and theoretical work on the subject has helped to identify many important features of the mechanisms of heat transfer, but there still exists a discrepancy between experiment and theory. This results in a considerable uncertainty range when predictions are to be made. In a review of the literature, Saberksy [1] noted that the role of surface waves and film stability characteristics on condensation heat transfer is still not well understood and suggested that further study must be in the transient behavior of the condensate film. Recently, Gregorig et al. [2] presented an experimental correlation of film condensation based on the "most carefully" conducted experiments. They recognized the importance of the Weber number in film condensation. This

suggests that surface tension and hence surface waves are likely to contribute to the discrepancy between theory and experiment.

Because of the complexity of the mathematical problem governing the motion and temperature of the condensate film (Appendix I), the previous theoretical studies of the problem have been based on various simplifying approximations. One such approximation is the well known Prandtl's boundary layer simplifications. Sparrow and Gregg [3] further simplified the problem by making the steady-state assumption. Under these simplifications they obtained a solution to the so-called "boundary layer equations of laminar film condensation" by employing a similarity transformation and using a high speed digital computer. The available theoretical analyses based on the steady-state assumption cannot take into account the effects of the surface waves.

The discrepancy between the existing theoretical analysis of laminar film condensation based on the assumption of steady state condensate flow and the carefully conducted experimental measurements compounded by the observation of waves on the condensate film suggests that the theoretical models must be modified to take into account the waves on the condensate. The first step in the development of such a transient theory is the investigation of the linearized stability of condensate flow. Linearized stability analyses of laminar film condensation have only very recently appeared in the literature [4-6]. The present analytical study has been undertaken to contribute improvements and extensions to the linearized stability analysis of laminar film condensation.

The purpose of the present analytical study is threefold. First, it is observed that the interfacial conditions used in [4-6] are different from the general formulation of these conditions as presented by Slattery [7] and Hsieh [8]. In particular, the kinematic surface condition used in [4-6], while valid for an isothermal liquid film, does not describe the physical situation for the condensation problem. During the condensation process, the phase change which occurs at the interface results in a net mass transfer across the interface. When the problem formulation is carried out by properly accounting for the condensation mass transfer across the interface (Appendix I), the surface kinematic condition derived for an interface with mass transfer differs from that used in [4-6]. In fact, the kinematic condition used in these earlier investigations violates the law of conservation of energy across the interface. It is also observed that the effect of the condensation mass transfer on the stability of the condensate film has been heuristically accounted for by introducing a vapor velocity term into the normal stress interfacial condition in [4-6]. It is shown in Appendix I that the proper form of this interfacial condition is radically different from that used by these investigators. In view of these shortcomings in the previous analyses of the linearized stability of condensate flow, reinvestigation of the same problem using the proper form of the interfacial conditions is in order. Secondly, the perturbation solution to the linearized stability problem is carried out to second-order with respect to the dimensionless wave number. The determination of the second-order solution is important and it will provide an

estimate on the region of validity of the first-order solution. The comparison of the first-order solution and the second-order solution will be employed when judgement is made on the validity of the results. Finally, the linearized stability analysis is extended to account for the effects of the acceleration and convection in the base flow.

The literature relevant to the problem is discussed in Chapter II. The derivation of the linearized stability problem is presented in the appendices. The equations and boundary conditions governing the motion and the temperature of the condensate film are formulated in Appendix I. In Appendix II, the problem is solved by a perturbation method under the steady-state assumption and the boundary-layer approximations. The solution found in Appendix II is essentially a perturbation solution of the same problem which was solved by Sparrow and Gregg [3]. This solution is used as base flow when analyzing the stability problem in Chapter III. The linearized stability problem is derived in Appendix III. Descriptions of the geometry of the flow are given in the Appendices. The linearized stability problem, as formulated in Appendix III, is solved by employing a perturbation method in Chapter III. The solutions from the linearized stability analysis are discussed in Chapter IV. In analyzing the results, emphasis is given to the determination of the neutral stability curves which identifies the conditions under which a condensate film is stable or unstable, and to the predictions of critical distances from the leading edge of the vertical wall up to which the condensate film is stable. Results are discussed with comparison to the previous stability analyses and available experimental data.

## II. REVIEW OF LITERATURE

### 1. PHYSICAL DESCRIPTION OF CONDENSATION AND INTERFACIAL PHENOMENA

The start for condensation involves nuclei, i.e., some particles on which liquid begins to form. The origin of condensation is accordingly called nucleation. Nucleation can be subdivided into two categories depending on the type of nuclei: bulk nucleation and surface nucleation.

Bulk nucleation occurs within the bulk of the vapor away from boundaries such as when a vapor experiences cooling by an adiabatic expansion. Bulk nucleation can be subdivided into two categories depending on whether foreign particles are absent or present in the vapor: homogeneous nucleation and heterogeneous nucleation. In homogeneous nucleation, sudden expansion results in spontaneous formation of nuclei whereas in heterogeneous nucleation condensation can start on the foreign particles already present in the system. Processes like condensation by expansion in a nozzle, and formation of clouds in the center of a low-pressure region are examples of bulk nucleation.

Film condensation is an example of surface nucleation. Surface condensation can be subdivided into homogeneous and heterogeneous surface condensation depending upon whether the condensation takes place on the surface of the same substance (its liquid or solid phase) or on a foreign surface. Under this classification, the initial stage

of film condensation is heterogeneous surface condensation in many applications where the nuclei for condensation are located on a foreign solid surface. After the condensate forms a liquid film on the solid surface, the process is then homogeneous surface condensation where the condensing surface is the liquid phase of the vapor. The "interfacial zone" separating the liquid from the vapor is called the liquid/vapor interface or simply the interface. The formation of an interface results in a surface stress (called surface tension) which acts in a manner to contract the surface. It is also well known that the normal component of fluid stress is discontinuous at a curved interface [9]. The physical reason for the existence of surface tension at an interface has been disputed. According to an argument given by Davies and Rideal [10], the formation of a free surface causes the molecules at the surface of the liquid to be attracted to the interior of the liquid depleting the molecules at the surface. This increases the intermolecular spacing at the surface, resulting in added attractive forces between molecules in the surface. The added attractive force at the interface is said to be responsible for the surface tension phenomena.

An important question often raised regarding the nature of the interface is its thickness. This question has been answered upon strong experimental evidence that the change in density from vapor to liquid is very abrupt, the interfacial region being only 1-2 molecules

thick [11]. This observation is consistent with the assumption in the theoretical studies of laminar film condensation that the interface is a surface of infinitesimal thickness from the macroscopic viewpoint. The possibility of a temperature discontinuity at the interface and its effects on the heat transfer through the condensate film has been studied in the past. This is discussed in the next section.

## 2. LAMINAR FILM CONDENSATION ON VERTICAL FLAT PLATES

### A. Analytical Studies

Heat transfer with film condensation on a vertical wall was first analyzed by Nusselt [12] assuming that the condensate film is laminar and neglecting the effects of acceleration, convection and time dependence. Following Nusselt's analysis, one can find the following expression for the mean heat transfer coefficient for the case of a constant wall temperature.

$$\bar{h} = \frac{4}{3} \left[ \frac{(1-\delta) \rho g h_{fg} k^3}{4L\nu\Delta T} \right]^{1/4} \quad (1)$$

Later Rohsenow [13] extended Nusselt's analysis to take into account the heat convection in the condensate film. Subsequently Sparrow and Gregg [3] solved the boundary layer equations of laminar film condensation to evaluate the effects of acceleration in the condensation film on the heat transfer. Their results showed that acceleration and convection are important mechanisms contributing to heat transfer when  $\zeta$ , the heat-capacity parameter, is sufficiently large.

The results reduced to Nusselt's simple analysis when  $\mathcal{J}$  approached zero. Chen [14], showed using perturbation methods to solve the integrated boundary layer equations, that the effect of vapor drag from a stagnant vapor is negligible for condensate films of Prandtl number order unity or larger. For small Prandtl number liquids, the previous results of Sparrow and Gregg were modified slightly as a result of vapor drag. Yang [15] used perturbation methods to investigate the effect of a nonisothermal wall on the heat transfer through the condensate film. His results are only qualitative and his method of solution requires a numerical procedure for calculating the heat flux. Denny and Mills [16] used a finite difference method to solve the boundary layer equations of film condensation taking into account variations in fluid properties, interfacial drag, and variable wall temperature. They found that when  $\mathcal{J} \approx 0.02$  and  $Pr \approx 2$ , the solution of the full boundary layer equations for a nonisothermal wall shows good agreement with the solution from a Nusselt type analysis. They did not, however, present results for large  $\mathcal{J}$  for which the nonlinear terms in the governing equations are important. In order to account for the effects of acceleration and convection in a condensate film adjacent to a nonisothermal wall, Nagendra and Tirunarayanan [17] and, later, Lienhard and Dhir [18] analyzed the nonisothermal wall problem using the similarity solution techniques. The results in [17] showed that nonisothermal boundary conditions have an important effect on the heat transfer. (See also the discussion in [19]). The solutions of the boundary layer equations

subject to a nonisothermal wall condition as presented in [18] did not converge to the solution from a Nusselt type analysis in the limit as  $\mathfrak{J} \rightarrow 0$  in contrast to the previous numerical results in [16]. This discrepancy between the finite difference results and similarity solution results were resolved in [20] where it was shown that the application of the similarity method leads to erroneous results for the nonisothermal wall problem. Subsequently, the nonisothermal wall problem was reinvestigated by using perturbation methods [21] where it was shown that the effect of the nonisothermal wall on the heat transfer through the condensate film disappears as  $\mathfrak{J} \rightarrow 0$  irrespective of the particular wall temperature distributions considered if equation (1) is evaluated at the mean value of the wall temperature distribution.

#### B. Small Prandtl Number Liquid Condensation-- Discrepancy Between Experiment and Theory

The results from the theoretical studies [3,12-21] show a wide discrepancy with experiments conducted with liquids of  $Pr \ll 1$  such as liquid metals. The measurements of Misra and Bonilla [22] for the condensation of sodium and mercury vapors are 5-15 percent of the values predicted from the theoretical studies. Sukhatme and Rohsenow [23] argued that the previously published experimental data contained errors due to the presence of noncondensable gases in the vapor, however, their measurements were still an order of magnitude lower the theoretical predictions. In order to explain the discrepancy between theory and experiment, they used the kinetic theory, originally developed by Schrage [24], to derive an expression for the interfacial

resistance to heat transfer. Their modified theory and experimental results showed that the interfacial resistance to heat transfer is important for liquid metal condensation and that significant interfacial temperature drop is present during the condensation of metallic vapors. In discussing their work, however, Bonilla [25] pointed out that their theoretical model was grossly inadequate. Wilhelm [26] reviewed the literature on the condensation of metallic vapors and argued that the kinetic theory of condensation is an inadequate description of the interfacial transport processes. He concluded that the theoretical model in its present form provides only a qualitative idea of the relative importance of the interfacial resistance to heat transfer. Fedorovich and Rohsenow [27] accounted for the effect of possible vapor subcooling on the interfacial resistance to heat transfer in an attempt to improve the kinetic theory of interfacial heat transfer. Their results showed better agreement with the experimental data, the data being up to an order of magnitude lower than the theoretical results. Later, Rohsenow [28] rejected the theoretical study of reference [27] and mentioned that vapor subcooling does not occur. Based on an error analysis of the experimental literature, he concluded that the deviation between data and the kinetic theory of condensation results from uncertainty and error in experimental measurements. Merte [11], on the other hand, suggested that the discrepancy between experiment and theory results from the variation of the interfacial resistance to heat transfer with vapor pressure. The present status of the discrepancy suggests further experimental

work with metallic vapor condensation and theoretical work directed to improve the kinetic theory model for the interfacial resistance to heat transfer.

C. Experiments With Liquids of Prandtl Number on the Order of Unity or Larger

In contrast with the situation with liquids of small Prandtl numbers, the experimental data for film condensation with liquids of Prandtl number on the order of unity or larger show much better agreement with the theoretical results in [3,12-21]. Experimental data from different investigations generally give greater heat transfer coefficients than the analytical predictions [29]. The data of Hebbard and Badger [30], Shea and Krase [31], Meisenburg, et. al. [32] and Baker, et. al. [33], for the condensation of steam give 20-50 percent higher heat transfer coefficients than the analytical results. The data of [34] and [35] give heat transfer coefficients which are 0-50 percent lower than the Nusselt theory when  $Re > 10$ . According to Rohsenow [28], the higher heat transfer coefficients observed in the experiments are a result of ripples on the liquid film and vapor velocity at the interface which tends to make the condensate film thinner. He also explained that the lower heat transfer coefficients observed in other experiments were a result of the presence of noncondensable gases such as air in the experimental apparatus. In view of the past experimental literature, he concluded that when proper account of these various effects is taken, the theoretical studies which neglect the interfacial resistance to heat transfer

agree well with data obtained for condensation of non-metallic vapors. Rohsenow's assertion has been previously verified by the experimental measurements of Mills and Seban [36] where it was found that the interfacial resistance to heat transfer is negligible in industrial applications of filmwise condensation. Also, their measurements of heat transfer coefficients showed excellent comparison with the simple Nusselt theory of film condensation. Slegers and Seban [37] conducted an experiment with the condensation of n-butyl alcohol in an attempt to identify any possible interfacial resistance to heat transfer. They also found that the measured heat transfer coefficients showed excellent agreement with the simple Nusselt theory with the maximum difference being only 2 percent. Their results therefore also confirm that interfacial resistance to heat transfer is indeed negligible during the condensation of ordinary vapors. Sukhatme and Rohsenow [23] evaluated the magnitude of a possible temperature drop across the interface for the condensation of different vapors. While the estimated magnitude of the temperature jump was significant for liquid metals, it was only a fraction of a Fahrenheit degree for water vapor condensation. The results from these investigations suggest that the interfacial temperature is well approximated by the saturation temperature of the vapor for most applications of filmwise condensation except for metallic vapors.

Walt and Kröger [38] argued that most of the existing experimental data for film condensation is not reliable. They suggested that the excellent agreement between the Nusselt theory and the experimental

results in [36,37] was because the condensate Reynolds number was less than 5 in these investigations, a situation in which the effect of surface waves must be negligible considering the stability criteria presented by Benjamin [39] and Yih [40]. They presented an experimental study where possible errors from the presence of noncondensable gases, vapor velocity, and mode of condensation were eliminated. Their results showed excellent agreement with the Nusselt theory in the limit as the condensate Reynolds number approached zero. The measured heat transfer coefficients showed a gradual increase over the theoretical prediction of Nusselt with increasing Reynolds number, the difference between theory and experiment being as large as 16 percent for  $Re_L \approx 90$ . They concluded that the deviation from the Nusselt analysis is determined by the state of the film surface, an increase in the degree of "film instability" resulting in a larger heat transfer coefficient than predicted by Nusselt's theory.

Gregorig et al. [2] discussed the experimental literature for the condensation of non-metallic vapors. Upon elimination of much of the earlier literature with regard to accuracy in measurement techniques, visual control of the type of condensation, and the existence of non-condensable gases, they concluded that only the data of Ritani and Shekrihadse [41], and Kirschbaum [42] can be considered reliable. Their measurements together with those of [41,42] showed excellent comparison when a correlation was made taking into account the variation of the Nusselt number with Reynolds number and Weber number. These results when compared with Nusselt's analysis show that agreement

is excellent as Reynolds number approaches zero in accordance with the measurements of Walt and Kröger [38] even though these studies have been conducted independently and with different experimental setups. The experimental correlation in [2] gives gradually increasing heat transfer coefficients from the Nusselt solution with increasing Reynolds number, the deviation being about 20 percent for  $Re_L \approx 90$  (from Fig. 6 of [2]). Gregorig et al. also conjectured the most probable reason for the deviation between theory and experiment for larger Reynolds numbers to be the smooth condensate film assumption of Nusselt, the slightest wave formation being responsible for increased heat transfer rates.

### 3. HYDRODYNAMIC STABILITY THEORY

The problem of finding solutions to the Navier-Stokes equations which govern the motion of the liquid film presents insuperable mathematical difficulties [43]. It has therefore been customary to make physically justifiable simplifying assumptions leading to mathematically tractable problems. Time independent motion is one such simplifying assumption. Solutions based on this assumption, however, have been found to yield poor descriptions of flows at high Reynolds numbers where turbulent rather than laminar motion occurs. This raises a question on the validity of the assumption on time independence in situations where experiment and solutions of the steady state equations show disagreement. This question when cast into a slightly different form: Is the steady state flow stable with respect to disturbances

imposed on it?, constitutes the problem of hydrodynamic stability.

The stability of flows with fixed boundaries and those with one or more of the boundaries specified at infinity (such as boundary-layer type flows) has been extensively studied in the literature [44-49]. The general conclusion from the previous literature is that even finding solutions to the linearized stability problem is a difficult task. In the case of flows with fixed boundaries and the Blasius boundary layer on a flat plate, accurate solution of the linearized stability problem has been only recently demonstrated by use of electronic computers or expansions in terms of some particular orthogonal functions showing rapid convergence behavior [50-53].

The problem of laminar film condensation differs from those problems with fixed boundaries or that of the Blasius boundary layer on a flat plate in that one of the boundaries is located at a flexible free surface (or the interface) whose location is not known a priori and must be determined simultaneously with the solution of the problem. Additionally, the boundary conditions specified on the interface are of mixed type which further complicates the problem. The laminar film condensation problem is therefore an example of the so-called "free-boundary problems". Fluid flows with free boundaries constitute a class of problems for which there are very few exact mathematical solutions, and these problems are generally studied by means of numerical or perturbation methods [54]. The stability problem associated with isothermal liquid films flowing down a flat surface is closely related to the condensate film stability problem and is discussed in

the following.

A. Stability of Liquid Films Flowing  
Down a Flat Surface

The motion of liquids with a free surface has been observed and studied by Rayleigh [55] and Lamb [56]. Taylor [57] presented an analytical study of the stability of an initially quiescent liquid interface subjected to an acceleration such as gravity. The stability of an initially quiescent liquid interface when subjected to an acceleration has been alternately called Taylor instability [58], Lamb-Taylor instability [59], and Rayleigh-Taylor instability [60]. This type of instability is closely related to the stability of a liquid film flowing down a flat surface in which the liquid adjacent to the free surface is not quiescent initially but has a steady state velocity profile.

Although the problem of the stability of condensate flow has only very recently been investigated, the literature on a very similar problem, stability of isothermal liquid film flow, is extensive. Because the governing equations and the boundary conditions of the condensate flow stability problem are similar in nature to those of the isothermal film stability problem, the solution methods so far developed for the latter can be utilized to investigate the former.

Benjamin [39] formulated the linearized stability problem of laminar liquid film flow down a plane surface and presented an approximate solution based on a finite power series approximation to the disturbance stream function. Yih [40] demonstrated the application

of the parameter perturbation method to the same problem. His results were in agreement with Benjamin's, yet his method of solution was less laborious. Krantz and Goren [61] investigated the same problem by assuming a finite power series approximation to the disturbance stream function and using a momentum integral approach. Anshus [62] studied the asymptotic behaviour of the solution of the linearized stability problem for large and small values of the Reynolds number and a surface tension parameter. Boyadjiev and Krylov [63] assumed a parabolic profile for the disturbance velocity and used the method of moments (see reference [64]) to solve the linearized stability problem. They also presented a review of the previous theoretical and experimental work on the subject. The results showed that the different approximate solutions of the problem give reasonable predictions for the wave velocity and amplification rate for the data of reference [61] taken for Reynolds number on the order of unity. Portalski and Clegg [65] obtained experimental data for  $Re \approx 18$  and found disagreement with the theoretical predictions of Benjamin [39]. It should be noted, however, that the approximate solutions based on a power series approximation to the stream function break down at high Reynolds numbers because the stream function exhibits boundary layer behaviour as  $Re \rightarrow \infty$  [62]. Finding a uniformly valid approximate solution for large Reynolds numbers, on the other hand, is a difficult task because of the sharp changes in the disturbance stream function near the free surface. In order to find a solution valid for large Reynolds numbers, Anshus and Goren [66] simplified the Orr-Sommerfeld equation into an

equation with constant coefficients by replacing the base flow  $x$ -component velocity with its value at the free surface. Their results based on the solution of the simplified equation showed fair agreement with an unpublished numerical solution of the Orr-Sommerfeld equation by C. V. Sternling and G. D. Towell. Their results, however, contain an error of an uncertain magnitude because of the simplification made on the governing equation. The solution methods for the linearized stability problem so far available in the literature employ different approximation schemes and a reliable exact solution valid for all Reynolds numbers is still not available.

While the linearized stability theory is capable of determining the behaviour of infinitesimal disturbances on the base flow, it is apt to break down as the disturbances become finite in amplitude. It is also incapable of determining the existence of an equilibrium amplitude for the waves on the liquid film. In order to study the behaviour of finite amplitude waves and to investigate the possibility of an equilibrium amplitude, Lin [67-70] formulated the non-linear stability problem and presented an approximate solution based on the non-linear stability theory of Stuart [71] and Watson [72]. Lin's non-linear stability analysis leading to the concept of the so-called "Landau's second coefficient" showed that the linearly unstable waves do not grow indefinitely but reach an equilibrium finite amplitude. Penev et al. [73] also predicted equilibrium finite amplitude waves by using the method of moments. The predictions of these investigators showed excellent comparison with the experimental measurements of the wave amplitudes on thin liquid films. Byatt-Smith [74], and Krantz

and Goren [75] also studied the non-linear stability problem and qualitatively established the existence of an equilibrium amplitude, however, they did not present quantitative results for the magnitude of the equilibrium amplitude. The non-linear stability analyses so far studied in the literature have been based on various approximate solution procedures because finding exact solutions to the non-linear stability problem is an even more difficult task than finding exact solutions to the linearized stability problem.

#### B. Stability of Laminar Film Condensation on a Vertical Wall

The linearized stability problem of laminar film condensation on a vertical wall has been only very recently investigated in [4-6]. Marshall and Lee [4,5] used the parameter perturbation method originally developed by Yih [40] to find a first order solution with respect to the wave number. Lee and Marschall [6] used numerical methods and found a solution to the problem valid for any wave number. The results from these investigations show that the laminar condensate film flowing down a vertical wall is unstable with respect to infinitesimal disturbances starting after a negligible distance from the leading edge of the vertical wall. In these studies the Nusselt solution has been taken as the base flow because it has a simple algebraic representation which simplifies the stability analysis considerably. Lee and Marschall [6] noted that the effects of acceleration and convection in the base flow can be accounted for by finding algebraic representations to the numerical solutions of the boundary layer equations of laminar film

condensation given by Sparrow and Gregg [3]. One of the purposes of the present investigation is to present a stability analysis which evaluates these effects. Rather than trying to represent the numerical results of Sparrow and Gregg in algebraic form, however, the base flow is taken as the first-order perturbation solution of the boundary layer equations of film condensation. The first-order perturbation solution which accounts for the effects of acceleration and convection in the base flow is presented in Appendix II.

### III. LINEARIZED STABILITY ANALYSIS

In this chapter, the linearized stability problem (derived in Appendix III) is analyzed by using perturbation methods. The solution procedure used is similar to that of Yih [40] except for minor modifications. (For instance the complex characteristic value,  $C$ , is not expanded in a power series with respect to the wave number but is determined by a new procedure). In the first part, a second-order solution is found with respect to the wave number using the Nusselt solution as base flow. The first-order solution with the Nusselt base flow leads to a closed form expression for the neutral stability curve whereas the second-order solution leads to a nonlinear algebraic equation for the neutral stability curve which can be solved numerically. In the second part a first-order solution is found with respect to the wave number using the base flow given in Appendix II. By means of using the latter form for the base flow, the effects of convection and acceleration in the base flow can be identified through comparison with the solution given in the first part.

#### 1. THE ASYMPTOTIC SOLUTION WITH NUSSELT'S BASE FLOW

The mathematical problem governing the behaviour of infinitesimal disturbances is given by (from Appendix III):

$$\phi'''' - 2\epsilon^2\phi'' + \epsilon^4\phi = i\epsilon\frac{\mathcal{J}}{Pr}((\bar{u}-c)(\phi'' - \epsilon^2\phi) - \bar{u}_{yy}\phi) \quad (A3.29)$$

$$\psi'' - \epsilon^2\psi = i\epsilon\mathcal{J}((\bar{u}-c)\psi - \bar{\theta}_y\phi) \quad (A3.30)$$

$$\phi = \phi' = \psi = 0 \quad \text{at } \gamma = 0 \quad (A3.31)$$

$$\psi = -\eta_0\bar{\theta}_y \quad \text{at } \gamma = \bar{\eta} \quad (A3.32)$$

$$\phi'' + \epsilon^2\phi + \eta_0\bar{u}_{yy} = 0 \quad \text{at } \gamma = \bar{\eta} \quad (A3.33)$$

$$\phi'''' + (i\epsilon\frac{\mathcal{J}}{Pr}(c-\bar{u}) - \epsilon^2(3-2\gamma))\phi' - i\eta_0\epsilon\Gamma + i\epsilon\frac{\mathcal{J}}{Pr}\bar{u}_y\phi = 0 \quad \text{at } \gamma = \bar{\eta} \quad (A3.34)$$

$$\psi' + \eta_0\bar{\theta}_{yy} = i\epsilon\phi + i\epsilon\eta_0(\bar{u}-c)$$

$$\text{at } \gamma = \bar{\eta} \quad (A3.35)$$

Equation (A3.29), which governs the disturbance stream function amplitude,  $\phi$ , is the well known Orr-Sommerfeld equation. Equation (A3.30) governs the disturbance temperature amplitude,  $\psi$ . Here,  $\epsilon$  is the dimensionless wave number given by

$$\epsilon = \alpha/l$$

where  $l = \frac{\lambda}{2\pi}$  is the length scale of the disturbance and  $C = C_r + iC_i$ ,  $C_r$  being the dimensionless wave velocity and  $\epsilon C_i$  being the rate of amplification or decay. For a given base flow, equations (A3.29) and (A3.30) when solved subject to the six boundary conditions (A3.31-A3.34) lead to the determination of  $\psi/\eta_0$  and  $\phi/\eta_0$  as functions of  $C$ . These solutions, when substituted into (A3.35), yield an equation for the determination of  $C$ . Infinitesimal disturbances will grow or decay with time depending on whether  $C_i > 0$  or  $C_i < 0$ . The condition for neutral stability is  $C_i = 0$ .

A perturbation solution of the above problem valid for long waves is found by assuming expansions of the form

$$\begin{aligned}\phi &= \phi_0 + \epsilon \phi_1 + \epsilon^2 \phi_2 + O(\epsilon^3) \\ \psi &= \psi_0 + \epsilon \psi_1 + \epsilon^2 \psi_2 + O(\epsilon^3)\end{aligned}\tag{2}$$

Substituting (2) into (A3.29-A3.34) and equating like powers of  $\epsilon$  yields the zeroth-order problem

$$\begin{aligned}\phi_0'''' &= 0 \\ \psi_0'' &= 0\end{aligned}$$

$$\begin{aligned}
\phi_0 &= \phi_0' = \psi_0 = 0 & \text{at } y=0 \\
\psi_0 + \eta_0 \bar{\theta}_y &= 0 & \text{at } y=\bar{\eta} \\
\phi_0'' + \eta_0 \bar{u}_{yy} &= 0 & \text{at } y=\bar{\eta} \\
\phi_0''' &= 0 & \text{at } y=\bar{\eta}
\end{aligned} \tag{3}$$

the first-order problem

$$\begin{aligned}
\phi_1'''' &= i \frac{\mathcal{J}}{\bar{r}} ((\bar{u}-c) \phi_0'' - \bar{u}_{yy} \phi_0) \\
\psi_1'' &= i \mathcal{J} ((\bar{u}-c) \psi_0 - \bar{\theta}_y \phi_0) \\
\phi_1 &= \phi_1' = \psi_1 = 0 & \text{at } y=0 \\
\psi_1 &= \phi_1'' = 0 & \text{at } y=\bar{\eta} \\
\phi_1''' + i \frac{\mathcal{J}}{\bar{r}} (c-\bar{u}) \phi_0' - i \eta_0 \Gamma + i \frac{\mathcal{J}}{\bar{r}} \bar{u}_y \phi_0 &= 0 & \text{at } y=\bar{\eta}
\end{aligned} \tag{4}$$

and the second-order problem

$$\begin{aligned}
\phi_2'''' &= 2 \phi_0'' + i \frac{\mathcal{J}}{\bar{r}} ((\bar{u}-c) \phi_1'' - \bar{u}_{yy} \phi_1) \\
\psi_2'' &= \psi_0'' + i \mathcal{J} ((\bar{u}-c) \psi_1 - \bar{\theta}_y \phi_1) \\
\phi_2 &= \phi_2' = \psi_2 = 0 & \text{at } y=0 \\
\psi_2 &= 0 & \text{at } y=\bar{\eta} \\
\phi_2'' + \phi_0 &= 0 & \text{at } y=\bar{\eta} \\
\phi_2''' + i \frac{\mathcal{J}}{\bar{r}} (c-\bar{u}) \phi_1' - (3-2\mathcal{J}) \phi_0' &= 0 & \text{at } y=\bar{\eta}
\end{aligned} \tag{5}$$

If  $\bar{u}$  and  $\bar{\theta}$  are given, the above problems can be solved in succession for the determination of  $\phi_i$  and  $\psi_i$ . If the heat-capacity parameter,  $\gamma$ , is sufficiently small, then the Nusselt solution may be used as base flow to solve the stability problem. For moderate values of  $\gamma$ , however, the solution given in Appendix II better describes the solution of the boundary-layer equations and may be used as the base flow to evaluate the effects of acceleration and convection in the base flow on the stability of the condensate film. For example, if  $Pr > 1$  then Fig. A2.1 suggests that the Nusselt solution is an appropriate description of the base flow if  $\gamma < 0.2$ . The base flow given by Nusselt's solution is found by solving the base flow problem given by equations (A2.7-A2.11) and neglecting the nonlinear terms in equations (A2.8) and (A2.9). It is therefore given by the zeroth-order solution of the boundary-layer problem in Appendix II, i.e.,

$$\begin{aligned}\bar{u} &= \bar{\eta} \gamma - \frac{1}{2} \gamma^2 \\ \bar{\theta} &= \frac{1}{\bar{\eta}} \\ \bar{\eta} &= \sqrt{2} x^{1/4}\end{aligned}\tag{6}$$

Substituting (6) into equations (3) and (4), and solving these problems in succession yields

$$\begin{aligned}\phi_0 &= a_0 \gamma^2, \quad \phi_1 = i(a_1 \gamma^2 + a_2 \gamma^3 + a_3 \gamma^4 + a_4 \gamma^5) \\ \psi_0 &= b_0 \gamma, \quad \psi_1 = i(b_1 \gamma + b_2 \gamma^3 + b_3 \gamma^4 + b_4 \gamma^5)\end{aligned}\tag{7}$$

where

$$\begin{aligned} a_0 &= \frac{\eta_0}{2}, \quad a_1 = \frac{\mathfrak{J}}{4R} \eta_0 c \bar{\eta}^2 - \frac{\mathfrak{J}}{12R} \eta_0 \bar{\eta}^4 - \frac{1}{2} \eta_0 \bar{\eta} \Gamma, \quad a_2 = \frac{1}{6} \eta_0 \Gamma, \\ a_3 &= -\frac{\mathfrak{J}}{24R} \eta_0 c, \quad a_4 = \frac{\mathfrak{J}}{120R} \eta_0 \bar{\eta}, \quad b_0 = -\frac{\eta_0}{\bar{\eta}^2}, \quad b_1 = \mathfrak{J} \eta_0 \left( \frac{\bar{\eta}^2}{10} - \frac{c}{6} \right) \\ b_2 &= \frac{\mathfrak{J} \eta_0 c}{6 \bar{\eta}^2}, \quad b_3 = -\frac{\mathfrak{J} \eta_0}{8 \bar{\eta}}, \quad b_4 = \frac{\mathfrak{J} \eta_0}{40 \bar{\eta}^2} \end{aligned}$$

Substituting the first-order solution given by (2) and (7) into the interfacial condition (A3.35) yields the following expression for the eigenvalue of the problem,  $C$ ,

$$C = \frac{B_1 + i B_2}{B_3 + i B_4} \quad (8)$$

where

$$\begin{aligned} B_1 &= \frac{1}{\bar{\eta}_0^2} + \frac{3\mathfrak{J}}{40R} \epsilon^2 \bar{\eta}_0^6 + \frac{1}{3} \Gamma^* \bar{\eta}_0^3 \epsilon^4 \\ B_2 &= \epsilon \bar{\eta}_0^2 \left( 1 + \frac{11}{40} \mathfrak{J} \right) \\ B_3 &= \frac{5\mathfrak{J}}{24R} \epsilon^2 \bar{\eta}_0^4 \\ B_4 &= \epsilon \left( 1 + \frac{\mathfrak{J}}{3} \right) \end{aligned}$$

The condition for neutral stability separating the stable from the unstable disturbances requires that the imaginary part of  $C$  must vanish. Thus for neutral stability we have

$$B_1 B_4 - B_2 B_3 = 0$$

This equation when solved for  $\epsilon$  yields

$$\epsilon = \left[ \frac{-D_2 \pm \sqrt{D_2^2 - 4D_1D_3}}{2D_1} \right]^{1/2} \quad (9)$$

where

$$D_1 = \frac{1}{3} \Gamma^* \left(1 + \frac{\mathcal{J}}{3}\right) \bar{\eta}_0^3 \equiv D_{11} \Gamma^* \bar{\eta}_0^3$$

$$D_2 = \frac{\mathcal{J}}{Pr} \left( \frac{3}{40} \left(1 + \frac{\mathcal{J}}{3}\right) - \frac{5}{24} \left(1 + \frac{11}{40} \mathcal{J}\right) \right) \bar{\eta}_0^6 \equiv D_{22} \bar{\eta}_0^6$$

$$D_3 = \left(1 + \frac{\mathcal{J}}{3}\right) \frac{1}{\bar{\eta}_0^2} \equiv D_{33} \frac{1}{\bar{\eta}_0^2}$$

The equation for the neutral stability curve (9) which is based on the first-order solution of the problem is only an approximation. The regions of validity of this equation may be determined by means of finding an equation for the neutral stability curve based on the second-order solution of the problem.

Substituting (7) into (5) and solving the resulting problem, one finds.

$$\begin{aligned} \phi_2 &= \alpha_5 \gamma^2 + \alpha_6 \gamma^3 + \alpha_7 \gamma^4 + \alpha_8 \gamma^5 + \alpha_9 \gamma^6 + \alpha_{10} \gamma^7 + \alpha_{11} \gamma^8 + \alpha_{12} \gamma^9 \\ \psi_2 &= b_5 \gamma + b_6 \gamma^3 + b_7 \gamma^4 + b_8 \gamma^5 + b_9 \gamma^6 + b_{10} \gamma^7 + b_{11} \gamma^8 + b_{12} \gamma^9 \end{aligned} \quad (10)$$

where

$$\begin{aligned} \alpha_5 &= \left(2\mathcal{J} - \frac{5}{2}\right) a_0 \bar{\eta}^2 + \frac{\mathcal{J}}{2Pr} \left(-a_1 c \bar{\eta}^2 + \frac{1}{3} a_1 \bar{\eta}^4 - a_2 c \bar{\eta}^3 + \frac{2}{5} a_2 \bar{\eta}^5 - a_3 c \bar{\eta}^4 + \frac{13}{30} a_3 \bar{\eta}^6 \right. \\ &\quad \left. - a_4 c \bar{\eta}^5 + \frac{19}{42} a_4 \bar{\eta}^7\right), \quad \alpha_6 = \left(\frac{1}{3} - \frac{2\mathcal{J}}{3}\right) a_0 \bar{\eta}, \quad \alpha_7 = \frac{a_2}{6} + \frac{\mathcal{J}}{12Pr} a_1 c, \\ \alpha_8 &= \frac{\mathcal{J}}{20Pr} a_2 c - \frac{\mathcal{J}}{60Pr} a_1 \bar{\eta}, \quad \alpha_9 = \frac{\mathcal{J}}{30Pr} a_3 c - \frac{\mathcal{J}}{60Pr} a_2 \bar{\eta}, \quad \alpha_{10} = \frac{\mathcal{J}}{420Pr} a_2 \\ &\quad + \frac{\mathcal{J}}{42Pr} a_4 c - \frac{\mathcal{J}}{70Pr} a_3 \bar{\eta}, \quad \alpha_{11} = \frac{\mathcal{J}}{336Pr} a_3 - \frac{\mathcal{J}}{84Pr} a_4 \bar{\eta}, \quad \alpha_{12} = \frac{\mathcal{J}}{336Pr} a_4 \end{aligned}$$

$$\begin{aligned}
b_5 &= -\mathcal{J} \left( \frac{b_1 c \bar{\eta}^2}{6} + \frac{\bar{\eta}^3}{12} \left( \frac{a_1}{\bar{\eta}} - \bar{\eta} b_1 \right) + \frac{\bar{\eta}^4}{20} \left( c b_2 + \frac{b_1}{2} + \frac{a_2}{\bar{\eta}} \right) + \frac{\bar{\eta}^5}{30} \left( c b_3 - \bar{\eta} b_2 \right. \right. \\
&+ \left. \left. \frac{a_3}{\bar{\eta}} \right) + \frac{\bar{\eta}^6}{42} \left( c b_4 - \bar{\eta} b_3 + \frac{b_2}{2} + \frac{a_4}{\bar{\eta}} \right) + \frac{\bar{\eta}^7}{56} \left( \frac{b_3}{2} - \bar{\eta} b_4 \right) + \frac{\bar{\eta}^8}{144} b_4 \right), \\
b_6 &= \frac{\mathcal{J}}{6} c b_1, \quad b_7 = \frac{\mathcal{J}}{12} \left( \frac{a_1}{\bar{\eta}} - \bar{\eta} b_1 \right), \quad b_8 = \frac{\mathcal{J}}{20} \left( c b_2 + \frac{b_1}{2} + \frac{a_2}{\bar{\eta}} \right), \quad b_9 = \frac{\mathcal{J}}{30} \left( c b_3 - \right. \\
&\left. \bar{\eta} b_2 + \frac{a_3}{\bar{\eta}} \right), \quad b_{10} = \frac{\mathcal{J}}{42} \left( c b_4 - \bar{\eta} b_3 + \frac{b_2}{2} + \frac{a_4}{\bar{\eta}} \right), \quad b_{11} = \frac{\mathcal{J}}{56} \left( \frac{b_3}{2} - \bar{\eta} b_4 \right), \quad b_{12} = \frac{\mathcal{J}}{144} b_4.
\end{aligned}$$

Substituting the second-order solution given by (2), (7) and (10), into (A3.35) one obtains

$$(X_r + iX_i)c^2 + (Y_r + iY_i)c + Z_r + iZ_i = 0 \quad (11)$$

where

$$X_r = -\frac{1}{45} \mathcal{J}^2 \epsilon^2 \bar{\eta}^2, \quad X_i = \frac{61 \mathcal{J}^2}{720 P_r^2} \epsilon^3 \bar{\eta}^6$$

$$Y_r = \left( \frac{127}{5040} \mathcal{J}^2 + \frac{\mathcal{J}^2}{18 P_r} + \frac{5 \mathcal{J}}{24 P_r} \right) \epsilon^2 \bar{\eta}^4$$

$$Y_i = \left( 1 + \frac{\mathcal{J}}{3} \right) \epsilon - \frac{2 \mathcal{J} \Gamma^*}{15 P_r} \epsilon^5 \bar{\eta}^5 - \frac{2407 \mathcal{J}^2}{40320 P_r^2} \epsilon^3 \bar{\eta}^8$$

$$\begin{aligned}
Z_r &= -\frac{1}{\bar{\eta}^2} - \left( \frac{1}{3} + \frac{11}{120} \mathcal{J} \right) \Gamma^* \epsilon^4 \bar{\eta}^3 - \left( \frac{1349}{201600} \mathcal{J}^2 + \frac{11 \mathcal{J}^2}{560 P_r} \right. \\
&+ \left. \frac{3 \mathcal{J}}{40 P_r} \right) \epsilon^2 \bar{\eta}^6
\end{aligned}$$

$$\begin{aligned}
Z_i &= -\left( 1 + \frac{11}{40} \mathcal{J} \right) \epsilon \bar{\eta}^2 + \left( 1 - \frac{2 \mathcal{J}}{3} \right) \epsilon^3 \bar{\eta}^4 + \frac{37 \mathcal{J} \Gamma^*}{840 P_r} \epsilon^5 \bar{\eta}^7 \\
&+ \frac{431 \mathcal{J}^2}{40320 P_r^2} \epsilon^3 \bar{\eta}^{10}
\end{aligned}$$

Hence for neutral stability, the following condition must be satisfied

$$Y_r X_i - X_i \sqrt{Y_r^2 - 4X_r Z_r} - Y_i X_r + X_r \sqrt{Y_i^2 - 4X_i Z_i} = 0 \quad (12)$$

## 2. ASYMPTOTIC SOLUTION WITH ACCELERATION AND CONVECTION EFFECTS

In order to evaluate the effects of acceleration and convection in the base flow, we solve the first-order stability problem posed by (3) and (4) using for  $\bar{U}$  and  $\bar{\Theta}$  the equations (A2.12), (A2.15), (A2.16), and (A2.24). Following the same procedure as in part 1, we find

$$\begin{aligned} \phi_0 &= \eta_0 \left( \frac{1}{2} - \frac{\mathcal{J}}{4R} \bar{\eta}_x \bar{\eta}^3 \right) y^2 \\ \phi_1 &= i \left( -\frac{1}{2} \eta_0 \bar{\eta} \Gamma + \eta_0 \frac{\mathcal{J}}{R} \left( 1 - \frac{\mathcal{J}}{2R} \bar{\eta}_x \bar{\eta}^3 \right) \left( \frac{1}{4} c \bar{\eta}^2 + \frac{5\mathcal{J}}{288R} \bar{\eta}_x \bar{\eta}^7 \right. \right. \\ &\quad \left. \left. - \frac{1}{12} \bar{\eta}^4 \right) \right) y^2 + \frac{1}{6} i \eta_0 \Gamma y^3 + i \eta_0 \frac{\mathcal{J}}{R} \left( 1 - \frac{\mathcal{J}}{2R} \bar{\eta}_x \bar{\eta}^3 \right) \left( \frac{1}{120} (\bar{\eta} - \right. \\ &\quad \left. \frac{\mathcal{J}}{6R} \bar{\eta}_x \bar{\eta}^4) y^5 - \frac{\mathcal{J}}{8064R} \bar{\eta}_x \bar{\eta} y^8 - \frac{c}{24} y^4 \right) \\ \psi_0 &= -\eta_0 \left( \frac{1}{\bar{\eta}^2} - \frac{11\mathcal{J}}{40} \bar{\eta}_x \bar{\eta} \right) y \quad (13) \\ \psi_1 &= - \left( d_1 \bar{\eta}^2 + d_2 \bar{\eta}^3 + d_3 \bar{\eta}^4 + d_4 \bar{\eta}^6 + d_5 \bar{\eta}^7 \right) y + d_1 y^3 \\ &\quad + d_2 y^4 + d_3 y^5 + d_4 y^7 + d_5 y^8 \end{aligned}$$

where

$$\begin{aligned}
 d_1 &= \frac{1}{6} i \mathcal{J} \eta_0 c \left( \frac{1}{\eta^2} - \frac{11}{40} \mathcal{J} \bar{\eta}_x \bar{\eta} \right), \quad d_2 = -\frac{1}{12} i \mathcal{J} \eta_0 \left( \left( \frac{1}{\eta^2} - \frac{11}{40} \mathcal{J} \bar{\eta}_x \bar{\eta} \right) \right. \\
 &\quad \left. \bar{\eta} - \frac{\mathcal{J}}{6R} \bar{\eta}_x \bar{\eta}^4 \right) + \left( \frac{1}{2} - \frac{\mathcal{J}}{4R} \bar{\eta}_x \bar{\eta}^3 \right) \left( \frac{1}{\eta} + \frac{\mathcal{J}}{10} \bar{\eta}_x \bar{\eta}^2 \right), \quad d_3 = \frac{1}{40} i \mathcal{J} \eta_0 \left( \frac{1}{\eta^2} - \frac{11}{40} \mathcal{J} \bar{\eta}_x \bar{\eta} \right), \\
 &\quad d_4 = \frac{1}{168} i \mathcal{J} \eta_0 \bar{\eta}_x \left( \mathcal{J} \left( \frac{1}{\eta} - \frac{\mathcal{J}}{2R} \bar{\eta}_x \bar{\eta}^2 \right) - \frac{\mathcal{J}}{6R} \left( \frac{1}{\eta} - \frac{11}{40} \mathcal{J} \bar{\eta}_x \bar{\eta}^2 \right) \right), \\
 &\quad d_5 = -\frac{1}{896} i \mathcal{J}^2 \eta_0 \bar{\eta}_x \left( \frac{1}{\eta^2} - \frac{\mathcal{J}}{2R} \bar{\eta}_x \bar{\eta} \right)
 \end{aligned}$$

Substituting the first-order solution given by (2) and (13) into (A3.35) one obtains

$$c = \frac{\bar{B}_1 + i \bar{B}_2}{\bar{B}_3 + i \bar{B}_4} \quad (14)$$

where

$$\begin{aligned}
 \bar{B}_1 &= \left( \frac{1}{A^2} + \frac{29}{40} \mathcal{J} A^2 \right) \frac{1}{\eta_0^2} - \epsilon^2 \frac{\mathcal{J}}{R} \left( 1 - \frac{\mathcal{J}}{2R} A^4 \right) \left( \frac{71}{4480R} A^{10} - \frac{3}{40} A^6 \right) \bar{\eta}_0^6 + \frac{1}{3} \epsilon^4 \pi^* A^3 \bar{\eta}_0^3 \\
 \bar{B}_2 &= \epsilon \left( \left( 1 + \frac{11}{40} \mathcal{J} \right) A^2 - \left( \frac{1269}{22400} \mathcal{J}^2 + \frac{11 \mathcal{J}^2}{112R} + \frac{3 \mathcal{J}}{8R} \right) A^6 + \frac{157 \mathcal{J}^3}{8960R} A^{10} \right) \bar{\eta}_0^2 \\
 \bar{B}_3 &= \frac{5 \mathcal{J}}{24R} \epsilon^2 \left( 1 - \frac{\mathcal{J}}{2R} A^4 \right) A^4 \bar{\eta}_0^4 \\
 \bar{B}_4 &= \epsilon \left( 1 + \frac{\mathcal{J}}{3} - \frac{11 \mathcal{J}^2}{120} A^4 \right) \\
 A &= 1 + \mathcal{J} \left( \frac{9}{160R} - \frac{11}{160} \right)
 \end{aligned}$$

The condition for neutral stability is then given by

$$\bar{B}_1 \bar{B}_4 - \bar{B}_2 \bar{B}_3 = 0$$

Hence, the equation for the neutral stability curve is found to be

$$\epsilon = \left[ \frac{-\bar{D}_2 \pm \sqrt{\bar{D}_2^2 - 4\bar{D}_1\bar{D}_3}}{2\bar{D}_1} \right]^{1/2} \quad (15)$$

where

$$\bar{D}_1 = \frac{1}{3} \Gamma^* A^3 \left( 1 + \frac{\mathcal{J}}{3} - \frac{11}{120} \mathcal{J}^2 A^4 \right) \bar{\eta}_0^3 \equiv \bar{D}_{11} \Gamma^* \bar{\eta}_0^3$$

$$\bar{D}_2 = \frac{\mathcal{J}}{R} \left( 1 - \frac{\mathcal{J}}{2R} A^4 \right) A^6 \left( \left( \frac{3}{40} - \frac{71\mathcal{J}}{4480R} A^4 \right) \left( 1 + \frac{\mathcal{J}}{3} - \frac{11\mathcal{J}^2}{120} A^4 \right) \right.$$

$$\left. - \frac{5}{24} \left( 1 + \frac{11}{40} \mathcal{J} - \left( \frac{1269}{22400} \mathcal{J}^2 + \frac{11\mathcal{J}^2}{112R} + \frac{3\mathcal{J}}{40R} \right) A^4 \right. \right.$$

$$\left. + \frac{157\mathcal{J}^3}{8960R} A^8 \right) \bar{\eta}_0^6 \equiv \bar{D}_{22} \bar{\eta}_0^6$$

$$\bar{D}_3 = \left( \frac{1}{A^2} + \frac{29}{40} \mathcal{J} A^2 \right) \left( 1 + \frac{\mathcal{J}}{3} - \frac{11}{120} \mathcal{J}^2 A^4 \right) \frac{1}{\bar{\eta}_0^2} \equiv$$

$$\bar{D}_{33} \frac{1}{\bar{\eta}_0^2}$$

Having obtained the zeroth-order and the first-order solutions, it is possible to solve this problem to second-order by the same procedure as in the previous section. This procedure, however, requires a vast amount of algebra while the benefits of the enormous undertaking are believed to be small. It will be observed in the next section that the first-order and the second-order solutions with Nusselt's base flow give essentially the same critical Reynolds number for common engineering applications. This suggests that the same situation is likely to prevail when equations (A2.12), (A2.15), (A2.16) and (A2.24) are used as base flow in the stability analysis. This is because the two problems are the same except for the small order (with respect to the Nusselt solution) convection and acceleration terms in the base flow.

#### IV. RESULTS AND DISCUSSION

Given the parameters  $\zeta$ ,  $Pr$ , and  $f^*$ , the neutral stability curve for the Nusselt base flow separating stable from unstable disturbances is determined from equation (9) or more accurately from equation (12). The neutral stability curves are presented in Figs. 1, 2, and 3 for three sets of values for  $\zeta$ ,  $Pr$ , and  $f^*$ . The values of the parameters so chosen correspond to the condensation of saturated steam at 100 C with  $\Delta\tau$  being equal to 0.54, 5.4, and 27 centigrade degrees respectively for each curve. The parameters  $Pr$  and  $f^*$  were determined by evaluating the variable properties at the reference temperature  $\tau_w + 0.33\Delta\tau$  as suggested in [16]. The Reynolds number  $Re$  is defined by  $Re = \frac{\tilde{U}_0 \tilde{\eta}}{\nu} = \frac{\zeta \tilde{\eta}^3}{2Pr}$  where  $\tilde{U}_0$  is the x-component velocity at the free surface,  $\tilde{\eta}$  is the film thickness, and  $\nu$  is the kinematic viscosity. In these figures, the dashed curve corresponds to equation (9). For fixed values of  $\epsilon$ , equation (12) was solved numerically by the Newton-Raphson method. The solid curves in the figures correspond to equation (12). The critical Reynolds number,  $Re_c$ , is defined as the largest Reynolds number for which disturbances of all wavelengths decay with time.

It is observed from these figures that the difference between the first-order and the second-order neutral stability curves is small over most of the  $\epsilon-Re$  plane. Generally, the first-order solution is valid in those regions of the  $\epsilon-Re$  plane where the

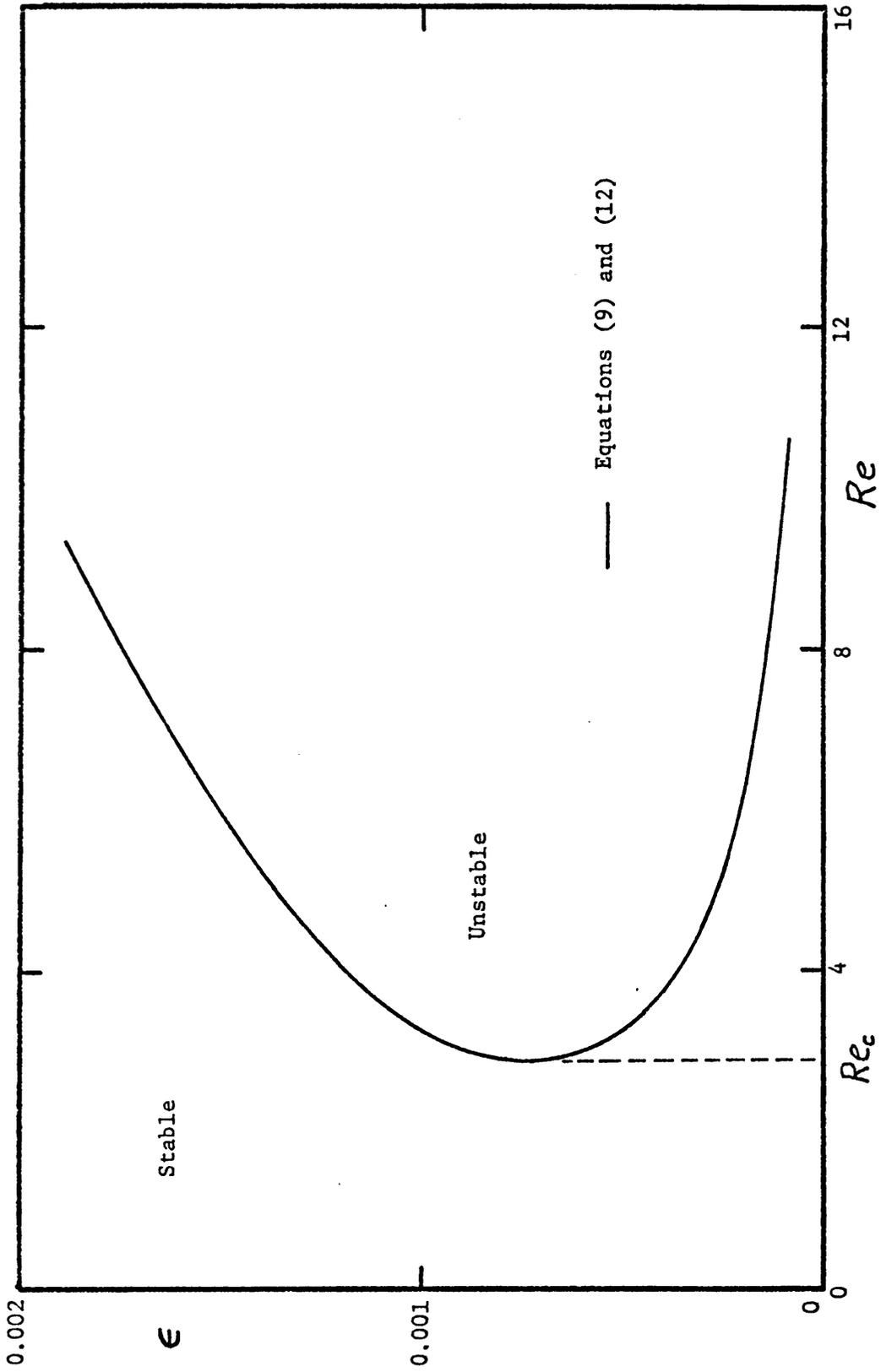


Figure 1. Neutral stability curve when  $S = 0.001$ ,  $Pr = 1.76$ ,  $\Gamma^* = 2.08 \times 10^6$

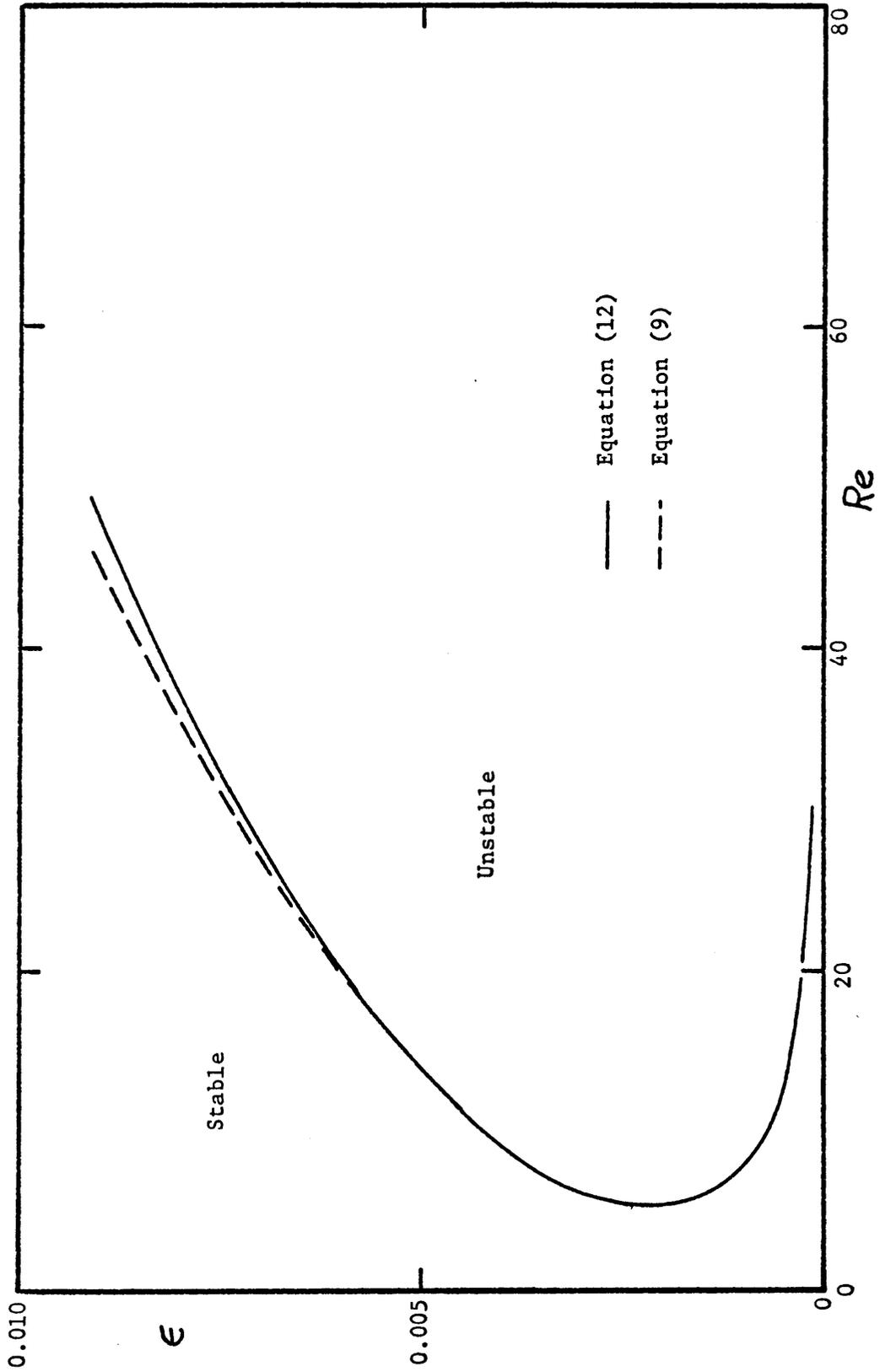


Figure 2. Neutral stability curve when  $\zeta = 0.01$ ,  $Pr = 1.84$ ,  $\Gamma^* = 4.36 \times 10^5$ .

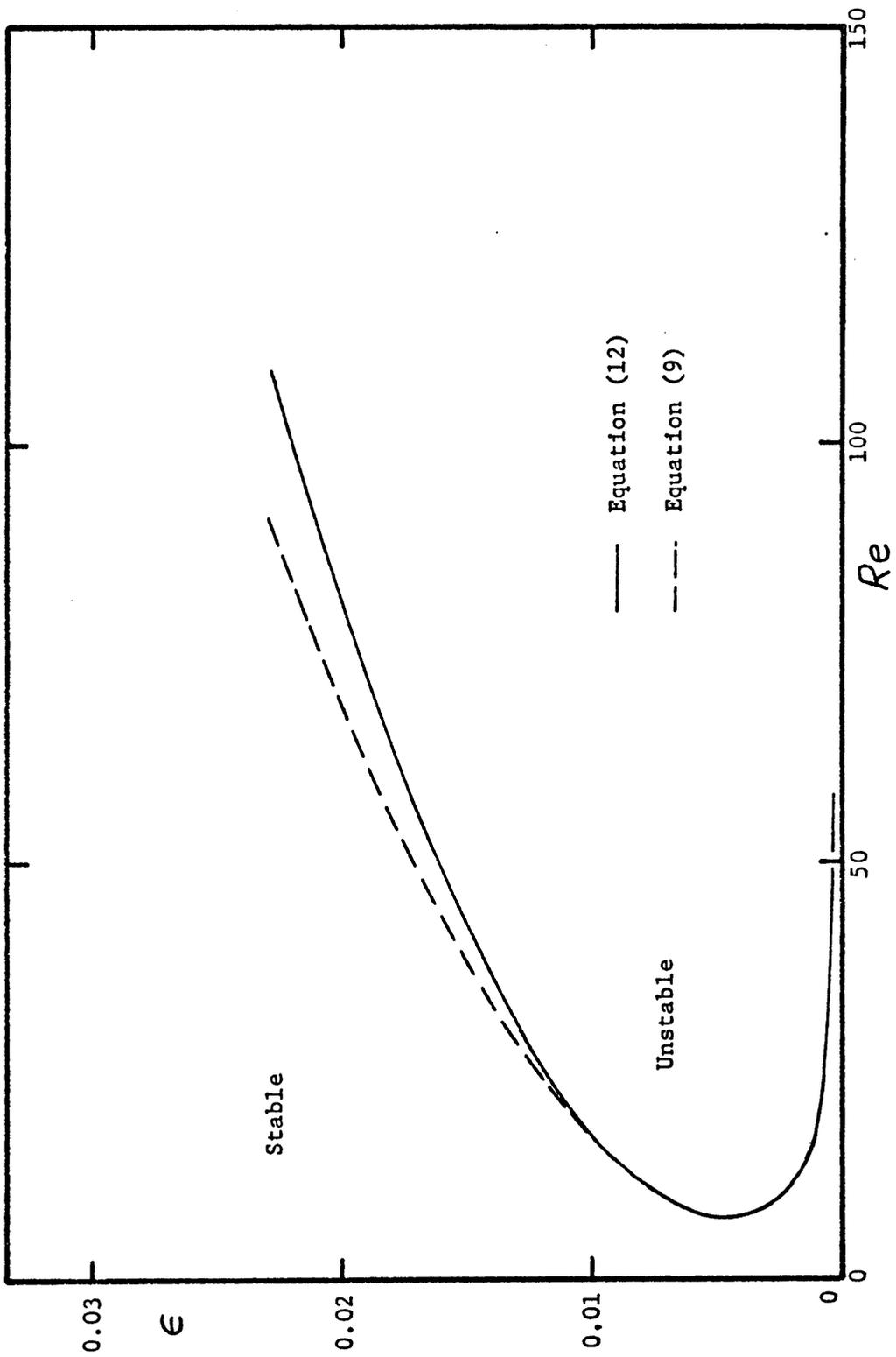


Figure 3. Neutral stability curve when  $\delta = 0.05$ ,  $Pr = 2.17$ ,  $\Gamma^* = 1.35 \times 10^5$ .

product  $\epsilon Re$  is small. In particular, equation (9) adequately specifies the critical Reynolds number. The critical Reynolds number in these figures are 2.9, 5.2, and 7.4. The corresponding critical Reynolds numbers from the Marshall and Lee analysis [4] are 0.017, 0.17, and 0.70. These are smaller than the present predictions by at least an order of magnitude. The reason for this disagreement is the difference in the kinematic surface conditions used in each analysis. The wave number,  $\epsilon$ , corresponding to  $Re_c$  are 0.00074, 0.0022, 0.0047 respectively for each curve. These wave numbers correspond to wave lengths which tend to amplify as  $Re$  increases past  $Re_c$ . Noting that for saturated steam at 100 C and with  $\Delta T$  corresponding to 0.54, 5.4, and 27 C  $\alpha$  is  $1.7 \times 10^{-4}$ ,  $3.8 \times 10^{-4}$  and  $6.7 \times 10^{-4}$  centimeters respectively, the wave lengths can be computed from  $\lambda = \frac{2\pi\alpha}{\epsilon}$ . The wave lengths for each curve are found to be 1.4, 1.1, and 0.90 centimeters. These are in qualitative agreement with the experimental observations of Tailby and Portalski (see [66]) with isothermal water film flow where the reported wave length is close to one centimeter.

When the Reynolds number is larger than  $Re_c$ , there is a band of unstable wave lengths as is observed from the neutral stability curves. Given any Reynolds number, there exists a band of very long waves and a band of very short waves that are stable. As the surface tension parameter,  $\sigma^*$ , increases, the upper branch of the neutral stability curve is lowered. This implies that surface tension stabilizes the short waves. On the other hand,

the condensation mass transfer stabilizes the long waves because if the condensation mass transfer is neglected in the analysis then the lower branch of the neutral stability curve would have been the  $\epsilon=0$  axis.

There are two notable differences in the neutral stability curves from the present analysis and the results in [4,6]. One is that the critical Reynolds numbers from the present study while being small are not negligibly small as was found in [4-6]. The other is that there is always a band of very long waves that are stable in the present results. Marshall and Lee analysis, on the other hand, has the  $\epsilon=0$  axis for the lower branch of the neutral stability curve.

The analyses of laminar film condensation based on the steady state assumption is strictly valid only in the region from the leading edge of the vertical plate to the critical distance up to which all infinitesimal disturbances will decay with time. This critical distance may be determined by considering only the first-order solution since the second-order solution does not contribute any noticeable correction. When the base flow is taken to be the Nusselt's solution then the critical distance is found by letting the discriminant in equation (9) vanish. Denoting the nondimensional critical distance by  $X_c$ , one finds

$$\frac{X_c}{\Gamma^{*4/11}} = \frac{1}{4} \left[ \frac{4 D_{11} D_{33}}{D_{22}^2} \right]^{4/11} \quad (16)$$

Similarly, with the more general form for the base flow the critical distance is found by letting the discriminant in equation (15) vanish. One finds

$$\frac{\chi_c}{\Gamma^*{}^{4/11}} = \frac{1}{4} \left[ \frac{4\bar{D}_{11}\bar{D}_{33}}{\bar{D}_{22}^2} \right]^{4/11} \quad (17)$$

The right hand sides of equations (16) and (17) depend only on the parameters  $\zeta$ , and  $Pr$ . Hence these equations generate a family of curves (Fig. 4) as the Prandtl number assumes different values. Inspection of Fig. 4 reveals that when  $\zeta$  is fixed, increasing  $Pr$  or  $\Gamma^*$  increases the critical distance and hence has a stabilizing effect on the condensate film. When the Prandtl number is fixed, then decreasing  $\zeta$  (or  $\Delta\tau$ ) increases  $\chi_c$  and hence has a stabilizing effect. This observation is in agreement with the experimental measurements where the Nusselt solution has been found to agree with measurements as  $\Delta\tau \rightarrow 0$ . It is also observed that for the Prandtl numbers corresponding to the curves in Fig. 4, the more accurate representation for the base flow has a slight stabilizing effect on the condensate film when  $\zeta$  is large. This correction which is a result of taking into account the acceleration and convection effects in the base flow, however, is small.

The Prandtl numbers in Fig. 4 correspond to the experimental conditions of Ritani and Shekriladze [41], Walt and Kröger [38], Mills and Seban [36], and Slegers and Seban [37]. In these experimental investigations, agreement with the Nusselt solution was

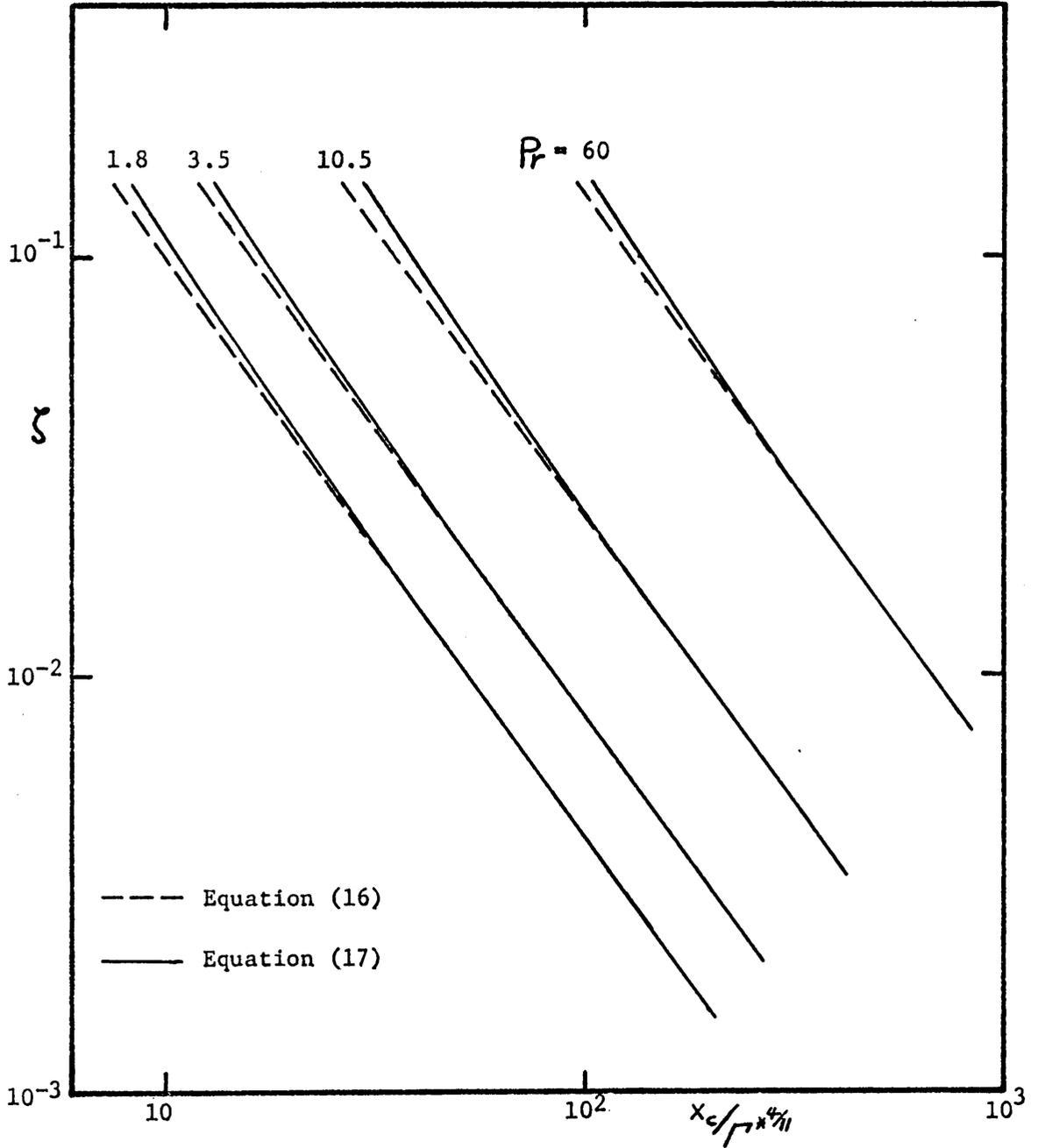


Figure 4. Nondimensional critical distance vs heat capacity parameter.

observed to be within 1-2 percent when  $\Delta T$  was 1, 1, 4, 30 centigrade degrees, respectively, in each experimental study. Table 1 summarizes the experimental conditions, the relevant parameters and the critical distance estimation for each of these experiments. Given the parameters  $\xi$  and  $Pr$ ,  $x_c/\Gamma^{*4/11}$  is determined from Fig. 4. Then the critical distance is computed from  $\tilde{x}_c = \alpha x_c$ . It is observed from Table I that, according to Fig. 4, only in the experiments of Mills and Seban is the condensate film stable over the major part of the test section. Thus the present theory supports the comparison of the measurements with the Nusselt theory. For the other experiments, however the film is found to be unstable over the major portion of the condensate film. Since good comparison with the experimental measurements and the Nusselt theory is also observed in these other experiments, this situation appears to be not self-explanatory and needs to be explained. One possible explanation for this is that while the film is unstable over the major portion of the test section, the equilibrium amplitude of the waves on the condensate film can be so small there that the waves may not contribute any noticeable effect on the heat transfer through the film. This assertion cannot be tested within the framework of the present linearized stability analysis. In order to test it, one needs to predict the amplitudes of the waves on the condensate film and then determine the effects of the waves on the heat transfer through the film. The prediction of an equilibrium amplitude, however, is only possible through a nonlinear stability analysis. This situation,

TABLE I. CRITICAL DISTANCES IN VARIOUS EXPERIMENTS

Experiment	Liquid $\Delta T$ (C°)	Length of test section (cm)	Relevant Parameters	$x_c / \Gamma^* 4/11$ (from Fig. 4)	Calculated critical distance, $\tilde{x}_c$ (cm)
Ritani and Shekriladze	Water $\Delta T = 1$	21.7	$\zeta = 0.00186$ $Pr = 1.8$ $\Gamma^* = 1.37 \times 10^6$ $\alpha = 2.13 \times 10^{-4}$ cm	176	6.4
Walt and Kröger	Freon 12 $\Delta T = 1$	5.7	$\zeta = 0.00786$ $Pr = 3.5$ $\Gamma^* = 1.78 \times 10^5$ $\alpha = 2.04 \times 10^{-4}$ cm	102	1.7
Mills and Seban	Water $\Delta T = 4$	12.7	$\zeta = 0.00686$ $Pr = 10.5$ $\Gamma^* = 2.95 \times 10^5$ $\alpha = 5.06 \times 10^{-4}$ cm	250	12.3
Slegers and Seban	n-Butyl alcohol $\Delta T = 30$	12.7	$\zeta = 0.152$ $Pr = 60$ $\Gamma^* = 1.0 \times 10^4$ $\alpha = 1.66 \times 10^{-3}$ cm	103	4.9

therefore, cannot be explained any further at this point.

If the heat capacity parameter,  $\zeta$ , is sufficiently small, then we can approximate  $D_2$  in equation (9) by

$$D_2 \approx -\frac{4}{30} \frac{\zeta}{Pr} \bar{\eta}_0^6$$

With this value for  $D_2$  and with equation (16), one finds the following equation for the critical distance up to which the film is completely stable

$$\tilde{x}_c = \frac{1}{4} \left[ (75\sigma)^4 \left( \frac{h_{fg}}{k\Delta\tau} \right)^7 \frac{\rho^3 \nu^9}{g^5} \right]^{1/11} \quad (18)$$

This equation exhibits the effects of the various properties on the stability of the condensate film. For instance, increasing the surface tension or viscosity stabilizes the film. An important observation is that given any film condensation situation, the condensate film can be stabilized completely by making  $\Delta\tau \rightarrow 0$ . This observation is in agreement with the experimental results in [37] and [38] where gradual better agreement between experimental results and the Nusselt theory is observed as  $\Delta\tau$  is decreased. It is noted that equation (18) and the critical distance prediction of Marschall and Lee [4] are very different. The predicted critical distance in reference [4] is proportional to  $(\Delta\tau)^{1/4}$  whereas the

present  $\tilde{\chi}_c$  is proportional to  $(\Delta\tau)^{-7/11}$ . This difference in the expressions for  $\tilde{\chi}_c$  results from the fact that the kinematic condition used in [4] does not take into account the condensation mass transfer across the liquid/vapor interface. It is noted that the critical distance prediction in [4] has the opposite trend that as  $\Delta\tau \rightarrow 0$  the condensate film is predicted to become more unstable. Also the critical distance predictions from [4] are practically negligible whereas the present predictions are small but not negligible.

Unfortunately the available experimental investigations from the literature have only concentrated on the heat transfer measurements and not on the study of the wavy nature of the condensate film. The present work identifies the important parameters and the conditions when the condensate film is stable or unstable, however, a quantitative comparison of the present results with experimental data is not possible. To this end, more experimental work is needed to establish the validity of the assumptions and the solution techniques used in the present work. The linearized stability analysis determines the region of stability and instability in the  $\epsilon-Re$  plane. It cannot, however, predict equilibrium amplitudes and hence it is incapable of predicting quantitative heat transfer results through a wavy condensate film. The prediction of heat transfer through a wavy condensate film is only possible by means of predicting equilibrium amplitudes. To this end, nonlinear stability analysis remains to be investigated. A nonlinear

stability analysis for this problem can be pursued by choosing either of the approximate solution procedures developed in [67], [73], or [76].

The validity of the hypothesis on the negligence of the non-parallelism in the base flow must be checked through developing a stability analysis which accounts for the effects of nonparallelism. Such an analysis is in principle possible by using either of the methods developed in [77] or [78].

## V. CONCLUSIONS

1. The first-order solution with respect to the nondimensional wavenumber,  $\epsilon$ , provides quantitative results for determining the neutral stability curve in the linearized stability analysis of laminar film condensation in those regions of the  $\epsilon - Re$  plane where the product  $\epsilon Re$  is sufficiently small. In particular the critical Reynolds number can be determined from the first-order solution for most practical situations.
2. The effects of acceleration and convection in the base flow on the stability characteristics of the condensate film is small and can be neglected in situations of usual engineering importance. The Nusselt solution may be used as base flow in stability analyses of laminar film condensation without noticeable error in common engineering situations.
3. The linearized stability characteristics of the condensate film are determined by three parameters;  $\zeta$ ,  $Pr$ , and  $\Gamma^*$ . Decreasing  $\zeta$ , increasing  $Pr$ , or increasing  $\Gamma^*$  has a stabilizing effect on the condensate film. Surface tension stabilizes the short waves whereas condensation mass transfer stabilizes the long waves.
4. The interfacial conditions used in the previous analyses of the stability of condensate flow do not describe the physical situation satisfactorily. The future stability analyses must be based on the present formulation or equivalently on those of [7] or [8].

## REFERENCES

1. R. H. Sabersky, Heat transfer in the seventies, Int. J. Heat Mass Transfer 14, 1927-1949 (1971).
2. R. Gregorig, J. Kern, and K. Turek, Improved correlation of film condensation data based on a more rigorous application of similarity parameters, Wärme-und Stoffübertragung 7, 1-13 (1974).
3. E. M. Sparrow and J. L. Gregg, A boundary-layer treatment of laminar-film condensation, J. Heat Transfer 81, 13-18 (1959).
4. E. Marschall and C. Y. Lee, Stability of condensate flow down a vertical wall, Int. J. Heat Mass Transfer 16, 41-48 (1973).
5. E. Marschall and C. Y. Lee, Stability characteristics of condensate films, Wärme-und Stoffübertragung 1, 32-37 (1973).
6. C. Y. Lee and E. Marschall, Laminar stability analysis of condensate film flow, Wärme-und Stoffübertragung 7, 14-21 (1974).
7. J. G. Slattery, General balance equation for a phase interface, I/EC Fundamentals 6, 108-115 (1967).
8. D. Y. Hsieh, Effects of heat and mass transfer on Rayleigh-Taylor instability, J. Basic Engng. 94, 156-162 (1972).
9. L. E. Scriven, Dynamics of a fluid interface, Chem. Engng. Sci. 12, 98-108 (1960).
10. J. T. Davies and E. K. Rideal, Interfacial Phenomena, Academic Press, New York (1963).
11. H. Merte, Condensation heat transfer, in Advances in Heat Transfer (ed. T. F. Irvine and J. P. Hartnett), Academic Press, New York, 181-272 (1973).
12. M. Jacob, Heat Transfer, John Wiley, New York, 663-667, (1967).
13. W. M. Rohsenow, Heat transfer and temperature distribution in laminar-film condensation, Trans. Am. Soc. Mech. Engrs. 78, 1645-1648 (1956).
14. M. M. Chen, An analytical study of laminar film condensation: part 1 - flat plates, J. Heat Transfer 83, 48-54 (1961).

15. K-T. Yang, Laminar film condensation on a vertical nonisothermal plate, J. Appl. Mech. 88, 203-205 (1966).
16. V. E. Denny and A. F. Mills, Nonsimilar solutions for laminar film condensation on a vertical surface, Int. J. Heat Mass Transfer 12, 965-979 (1969).
17. H. R. Nagendra and M. A. Tirunarayanan, Laminar film condensation from nonisothermal vertical plates, Chem. Engng. Sci. 25, 1073-1079 (1970).
18. J. H. Lienhard and V. K. Dhir, Laminar film condensation on non-isothermal and arbitrary-heat-flux surfaces, and on fins, J. Heat Transfer 96, 197-203 (1974).
19. E. R. G. Eckert, E. M. Sparrow, R. J. Goldstein, C. J. Scott, W. E. Ibele, and E. Pfender, Heat Transfer - A review of 1970 Literature, Int. J. Heat Mass Transfer 14, 1883-1926 (1971).
20. M. Ünsal and W. C. Thomas, Discussion, J. Heat Transfer, 97, 159 (1975).
21. M. Ünsal and W. C. Thomas, Perturbation solutions of laminar film condensation on nonisothermal walls, to be published.
22. B. Misra and C. F. Bonilla, Heat Transfer in the condensation of metal vapors: mercury and sodium up to atmospheric pressure, Chem. Engng. Prog. Symposium Series 52 No. 18, 7-21 (1956).
23. S. P. Sukhatme and W. M. Rohsenow, Heat transfer during film condensation of a liquid metal vapor, J. Heat Transfer 88, 19-27 (1966).
24. R. W. Schrage, A theoretical study of interphase mass transfer, Columbia University Press, New York (1953).
25. C. F. Bonilla, Discussion, J. Heat Transfer 88, 28 (1966).
26. D. J. Wilhelm, Condensation of metal vapors: mercury and the kinetic theory of condensation, Argonne National Laboratory report ANL-6948 (1964).
27. E. D. Fedorovich and W. M. Rohsenow, The effect of vapor subcooling on film condensation of metals, Int. J. Heat Mass Transfer 12, 1525-1529 (1969).
28. W. M. Rohsenow, Status of and problems in boiling and condensation heat transfer, Prog. Heat Mass Transfer 6, 1-44 (1972).

29. W. H. McAdams, Heat Transmission, McGraw Hill, New York (1954).
30. G. M. Hebbard and W. L. Badger, Steam film heat transfer coefficients for vertical tubes, Trans. Amer. Inst. Chem. Eng. 30, 194-214 (1934).
31. F. L. Shea and N. W. Krase, Drop-wise and film condensation of steam, Trans. Amer. Inst. Chem. Eng. 36, 463-487 (1940).
32. S. J. Meisenburg, R. M. Boarts, and W. L. Badger, The influence of small concentrations of air in steam on the steam film coefficients of heat transfer, Trans. Amer. Inst. Chem. Eng. 31, 622-638 (1935).
33. E. M. Baker, E. W. Kazmark, and S. W. Stroebe, Steam film heat transfer coefficients for vertical tubes, Trans. Amer. Inst. Chem. Eng. 35, 127-135 (1938).
34. D. L. Spencer and W. E. Ibele, Laminar film condensation of a saturated and superheated vapor on a surface with a controlled temperature distribution, Proc. Int. Heat Transfer Conf., 3rd, 337-347 (1966).
35. H-C. Moy, On laminar film condensation of vapors on vertical flat plate, Ph.D. Thesis, The University of Iowa, Iowa City, Iowa, (1969).
36. A. F. Mills and R. A. Seban, The condensation coefficient of water, Int. J. Heat Mass Transfer 10, 1815-1826 (1967).
37. L. Slegers and R. A. Seban, Nusselt condensation of n-butyl alcohol, Int. J. Heat Mass Transfer 12, 237-239 (1969).
38. J. V. D. Walt and D. G. Kröger, Heat transfer during film condensation of saturated and superheated freon-12, Prog. Heat Mass Transfer 6, 75-98 (1972).
39. T. B. Benjamin, Wave formation in laminar flow down an inclined plane, J. Fluid Mech. 2, 554-574 (1957).
40. C-S. Yih, Stability of liquid flow down an inclined plane, Phys. Fluids 6, 321-334 (1963).
41. G. V. Ritani and T. G. Shekriladze, An experimental study of the heat exchange process on transition from laminar to turbulent flow of the film, Thermal Engineering II, 101-103 (1964).
42. E. Kirschbaum, Neues zum wärmeübergang mit und ohne änderung des aggregatzustandes, Chem. Ing. Techn. 24, 393-400 (1952).

43. H. Schlichting, Boundary-Layer Theory, McGraw Hill, New York (1968).
44. J. T. Stuart, Hydrodynamic stability, in Laminar Boundary Layers (ed. L. Rosenhead), Clarendon Press, Oxford, 492-628 (1963).
45. S. F. Shen, Stability of laminar flows, in Theory of Laminar Flows (ed. F. K. Moore), Princeton Univ. Press, New Jersey, 719-853 (1964).
46. W. H. Reid, The stability of parallel flows, in Basic Developments in Fluid Dynamics (ed. M. Holt), Academic Press, New York, 249-308 (1965).
47. J. T. Stuart, Hydrodynamic stability, Appl. Mech. Rev. 18, 523-531 (1965).
48. C. C. Lin, The theory of hydrodynamic stability, Cambridge (1967).
49. R. Betchov and W. O. Criminale, Stability of parallel flows, Academic Press (1967).
50. R. Jordinson, The flat plate boundary layer. Part 1. Numerical integration of the Orr-Sommerfeld equation, J. Fluid Mech. 43, 801-811 (1970).
51. S. A. Orszag, Accurate solution of the Orr-Sommerfeld stability equation, J. Fluid Mech. 50, 689-703 (1971).
52. D. T. Mook, Stability of parallel flows of second-order liquids, Phys. Fluids, 15, 219-223 (1972).
53. B. D. Hassard, T. S. Chang, and G. S. S. Ludford, Expansion functions with boundary-layer behavior and the Orr-Sommerfeld equation, J. Math. Analysis and Appl. 46, 463-477 (1974).
54. A. B. Tayler, Singularities at flow separation points, Quart. Journ. Mech. and Applied Math. 26, 153-172 (1973).
55. J. W. S. Rayleigh, The Theory of Sound, Vol. 2, Dover, New York, 343-346 (1945).
56. H. Lamb, Hydrodynamics, Dover, New York, 363-375 (1945).
57. G. I. Taylor, The instability of liquid surfaces when accelerated in a direction perpendicular to their planes I, Proc. Roy. Soc. A 201, 192-196 (1950).

58. R. Bellman and R. H. Pennington, Effects of surface tension and viscosity on Taylor instability, Quart. J. Appl. Math. 12, 151-162 (1954).
59. A. H. Nayfeh, On the non-linear Lamb-Taylor instability, J. Fluid Mech. 38, 619-631 (1969).
60. B. J. Daly, Numerical study of the effect of surface tension on interface instability, Phys. Fluid. 12, 1340-1354 (1969).
61. W. B. Krantz and S. L. Goren, Stability of thin liquid films flowing down a plane, I/EC Fundamentals 10, 91-101 (1971).
62. B. E. Anshus, On the asymptotic solution to the falling film stability problem, I/EC Fundamentals 11, 502-508 (1972).
63. Ch. Boyadjiev and V. S. Krylov, Analysis of stability of laminar liquid film flow, Chem. Engng. Journal 6, 225-232 (1973).
64. B. A. Finlayson, The Method of Weighted Residuals and Variational Principles, Academic Press, New York (1972).
65. S. Portalski and A. S. Clegg, An experimental study of wave inception on falling liquid films, Chem. Engng. Sci. 27, 1257-1265 (1972).
66. B. E. Anshus and S. L. Goren, A method of getting approximate solutions to the Orr-Sommerfeld equation for flow on a vertical wall, A. I. Ch. E. J. 12, 1004-1008 (1966).
67. S. P. Lin, Finite-amplitude stability of a parallel flow with a free surface, J. Fluid Mech. 36, 113-126 (1969).
68. S. P. Lin, Roles of surface tension and Reynolds stresses on the finite amplitude stability of a parallel flow with a free surface, J. Fluid Mech. 40, 307-314 (1970)
69. S. P. Lin, Profile and speed of finite amplitude waves in a falling liquid layer, Phys. Fluids 14, 263-268 (1971).
70. S. P. Lin, Finite amplitude stability of a contaminated liquid film, Progr. Heat Mass Transfer 6, 263-275 (1972).
71. J. T. Stuart, On the non-linear mechanics of wave disturbances in stable and unstable parallel flows, J. Fluid Mech. 9, 353-370 (1960).
72. J. Watson, On the non-linear mechanics of wave disturbances in stable and unstable parallel flows, J. Fluid Mech. 9, 371-389 (1960).

73. V. Penev, V. S. Krylov, Ch. Boyadjiev and V. P. Vorotilin, Wavy flow of thin liquid films, Int. J. Heat Mass Transfer 15, 1395-1406 (1972).
74. J. G. B. Byatt-Smith, Waves on a thin film of viscous liquid, A. I. Ch. E. J. 17, 557-561 (1971).
75. W. B. Krantz and S. L. Goren, Finite-amplitude long waves on liquid films flowing down a plane, I/EC Fundamentals 9, 107-113 (1970).
76. A. H. Nayfeh and N. S. Saric, Nonlinear stability of a liquid film adjacent to a supersonic stream, J. Fluid Mech. 58, 39-51 (1973).
77. C-H. Ling and W. C. Reynolds, Non-parallel flow corrections for the stability of shear flows, J. Fluid Mech. 59, 571-591 (1973).
78. A. H. Nayfeh, W. S. Saric, and D. T. Mook, Stability of non-parallel flows, Archives of Mechanics 26, 409-414 (1974).
79. H. S-J. Kao, Stability analysis of transition from laminar to turbulent flow during two-phase film boiling on a vertical surface, Ph.D. Thesis, Virginia Polytechnic Institute and State University, Blacksburg, Virginia (1969).
80. R. P. Dring and B. Gebhart, A theoretical investigation of disturbance amplification in external laminar natural convection, J. Fluid Mech. 34, 551-564 (1968).
81. B. Gebhart, Natural convection flow, instability and transition, J. Heat Transfer 91, 293-309 (1969).
82. B. Gebhart, Instability, transition, and turbulence in buoyancy-induced flows, Ann. Rev. Fluid Mech. 5, 213-246 (1973).

## APPENDIX I

### PROBLEM FORMULATION

The liquid motion and temperature in a two dimensional condensate film are governed by the following equations (see Fig. A1.1):

continuity:

$$\tilde{U}_{\tilde{x}} + \tilde{V}_{\tilde{y}} = 0 \quad (A1.1)$$

$\tilde{x}$ -direction momentum:

$$\tilde{U}_{\tilde{t}} + \tilde{U}\tilde{U}_{\tilde{x}} + \tilde{V}\tilde{U}_{\tilde{y}} + \tilde{\rho}^{-1}\tilde{P}_{\tilde{x}} = g + \nu(\tilde{U}_{\tilde{x}\tilde{x}} + \tilde{U}_{\tilde{y}\tilde{y}}) \quad (A1.2)$$

$\tilde{y}$ -direction momentum:

$$\tilde{V}_{\tilde{t}} + \tilde{U}\tilde{V}_{\tilde{x}} + \tilde{V}\tilde{V}_{\tilde{y}} + \tilde{\rho}^{-1}\tilde{P}_{\tilde{y}} = \nu(\tilde{V}_{\tilde{x}\tilde{x}} + \tilde{V}_{\tilde{y}\tilde{y}}) \quad (A1.3)$$

energy:

$$\tilde{c}_{\tilde{t}} + \tilde{U}\tilde{c}_{\tilde{x}} + \tilde{V}\tilde{c}_{\tilde{y}} = k\tilde{\rho}^{-1}\tilde{c}_p^{-1}(\tilde{c}_{\tilde{x}\tilde{x}} + \tilde{c}_{\tilde{y}\tilde{y}}) \quad (A1.4)$$

In order to complete the formulation we need to specify, in addition to equations (A1.1)-(A1.4), the boundary conditions. At the solid/liquid interface, the liquid velocity components  $\tilde{u}$  and  $\tilde{v}$  vanish and the liquid temperature is equal to the wall temperature; that is

$$\tilde{u} = \tilde{v} = 0, \quad \tilde{c} = \tau_w \quad \text{at } \tilde{y} = 0 \quad (A1.5)$$

The vapor phase is assumed to be saturated and

$$\tilde{c} = \tau_s \quad \text{at } \tilde{y} = \tilde{\eta} \quad (A1.6)$$

Conditions to be satisfied at the liquid/vapor interface are the conservation of mass, momentum, and energy across the interface.

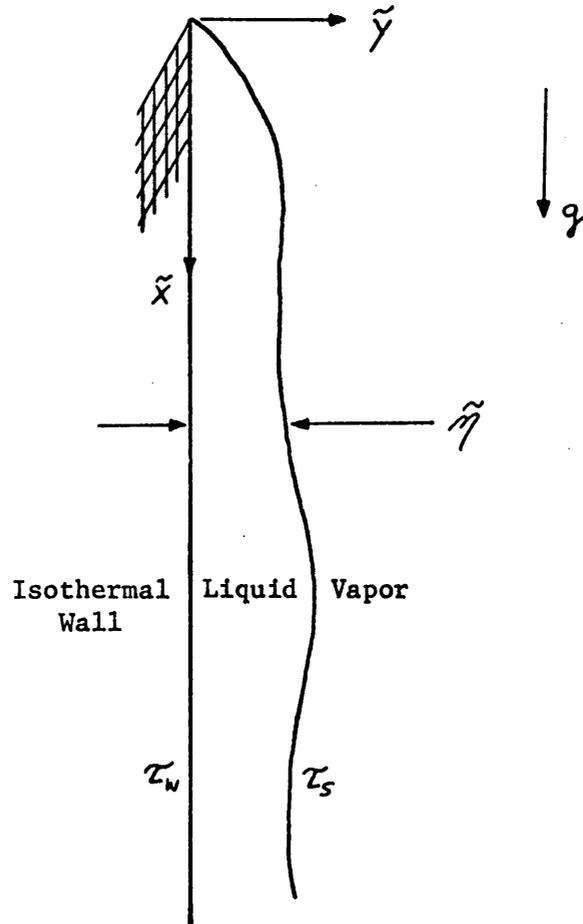


Figure A1.1. Geometry and coordinates of flow.

In addition to these three conditions we have the condition of no-slip between the vapor and the liquid at the interface and the definition of the velocity of the interface. In order to derive these conditions, we use the control volume approach and take a control volume of infinitesimal thickness which contains the liquid/vapor interface (Fig. A1.2). In this figure, the unit normal and tangential vectors to the interface are given by:

$$\vec{n} = (1 + \tilde{\eta}_x^2)^{-1/2} (-\tilde{\eta}_x \vec{i} + \vec{j}) \quad (A1.7)$$

$$\vec{t} = (1 + \tilde{\eta}_x^2)^{-1/2} (\vec{i} + \tilde{\eta}_x \vec{j}) \quad (A1.8)$$

The velocities of the liquid, interface, and vapor are defined by:

$$\vec{V} = \tilde{U} \vec{i} + \tilde{V} \vec{j} \quad (A1.9)$$

$$\vec{V}_i = \tilde{U}_i \vec{i} + \tilde{V}_i \vec{j} \quad (A1.10)$$

$$\vec{V}_v = \tilde{U}_v \vec{i} + \tilde{V}_v \vec{j} \quad (A1.11)$$

The liquid is taken to be viscous, but the viscosity of the vapor is neglected. Thus, the liquid stress tensor and the vapor stress tensor are given by:

$$T = \begin{bmatrix} -\tilde{p} + 2\rho\nu\tilde{U}_x & \rho\nu(\tilde{V}_x + \tilde{U}_y) \\ \rho\nu(\tilde{V}_x + \tilde{U}_y) & -\tilde{p} + 2\rho\nu\tilde{V}_y \end{bmatrix} \quad (A1.12)$$

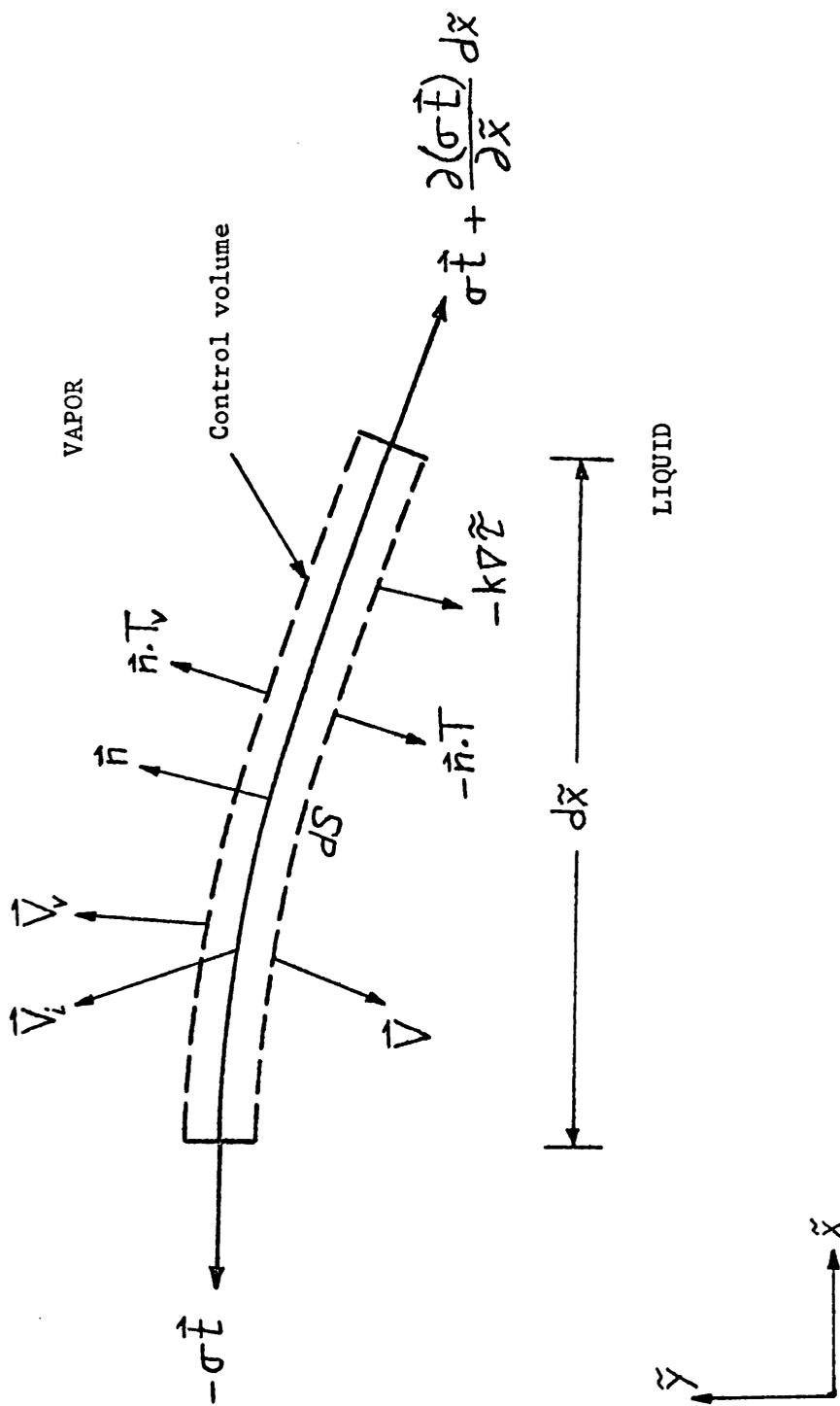


Figure A1.2. The interfacial control volume.

$$\underline{T}_v = \begin{bmatrix} -\tilde{p}_v & 0 \\ 0 & -\hat{p}_v \end{bmatrix} \quad (A1.13)$$

Because of the phase change at the interface, mass is transferred across the interface. Since the control volume is taken to be of infinitesimal thickness, the condition of conservation of mass is obtained by letting the net mass flux into the control volume vanish:

$$\rho_v (\vec{V}_v - \vec{V}_i) \cdot \vec{n} + \rho (\vec{V} - \vec{V}_i) \cdot (-\vec{n}) = 0 \quad \text{at } \tilde{y} = \tilde{\eta} \quad (A1.14)$$

To obtain the condition representing the conservation of momentum, we note that the inertia and body forces which are proportional to the volume are negligible for the control volume of infinitesimal thickness. Hence this condition is obtained by letting the sum of the forces on the control volume balance the momentum flux into the control volume:

$$\begin{aligned} \vec{n} \cdot \underline{T}_v dS + (-\vec{n} \cdot \underline{T}) dS + \sigma \frac{\partial \vec{t}}{\partial \tilde{x}} d\tilde{x} &= \rho_v (\vec{V}_v - \vec{V}_i) \cdot (-\vec{n}) \vec{V}_v dS \\ + \rho (\vec{V} - \vec{V}_i) \cdot \vec{n} \vec{V} dS &\quad \text{at } \tilde{y} = \tilde{\eta} \quad (A1.15) \end{aligned}$$

The vapor phase is assumed to be isothermal and the temperature of the liquid/vapor interface is taken to be equal to the vapor temperature. Under this assumption, no energy transfer by conduction takes place into the control volume from the vapor. The condition of conservation of energy is obtained by balancing the heat flux of the liquid phase at the interface by the latent heat released as a result of phase change:

$$(-k \nabla \tilde{x}) \cdot (-\vec{n}) = h_{fg} \rho (\vec{V} - \vec{V}_i) \cdot (-\vec{n}) \quad \text{at } \tilde{y} = \tilde{\eta} \quad (\text{A1.16})$$

Continuity of tangential velocities at the liquid/vapor interface yields:

$$(\vec{V} - \vec{V}_v) \cdot \vec{t} = 0 \quad \text{at } \tilde{y} = \tilde{\eta} \quad (\text{A1.17})$$

The liquid/vapor interface moves at its velocity:

$$\tilde{v}_i = \frac{d\tilde{\eta}}{d\tilde{t}} = \tilde{\eta}_{\tilde{t}} + \tilde{U}_i \tilde{\eta}_{\tilde{x}} \quad (\text{A1.18})$$

The interfacial conditions (A1.14-A1.17) derived here from a control volume approach can alternately be obtained from reference [7] where they have been derived from basic principles of continuum mechanics or from reference [8] where they have been obtained by integrating the conservation equations valid in either phase over an infinitesimal volume astride of the interface.

When the proper expressions are substituted into equations

(A1.14-A1.16), one finds the following conditions:

Conservation of mass across the liquid/vapor interface:

$$\delta(\tilde{U}_v - \tilde{U}_i) \tilde{\eta}_{\tilde{x}} - \delta(\tilde{V}_v - \tilde{V}_i) - (\tilde{U} - \tilde{U}_i) \tilde{\eta}_{\tilde{x}} + \tilde{V} - \tilde{V}_i = 0 \quad \text{at } \tilde{y} = \tilde{\eta} \quad (\text{A1.19})$$

Conservation of tangential momentum across the liquid/vapor interface:

$$\begin{aligned} & \tilde{U}_{\tilde{y}} + \tilde{V}_{\tilde{x}} + 4 \frac{\tilde{U}_{\tilde{x}} \tilde{\eta}_{\tilde{x}}}{\tilde{\eta}_{\tilde{x}}^2 - 1} + \frac{((\tilde{U} - \tilde{U}_i) \tilde{\eta}_{\tilde{x}} - (\tilde{V} - \tilde{V}_i))(\tilde{U} + \tilde{V} \tilde{\eta}_{\tilde{x}})}{\nu (\tilde{\eta}_{\tilde{x}}^2 - 1)} \\ & - \frac{\delta((\tilde{U}_v - \tilde{U}_i) \tilde{\eta}_{\tilde{x}} - (\tilde{V}_v - \tilde{V}_i))(\tilde{U}_v + \tilde{V}_v \tilde{\eta}_{\tilde{x}})}{\nu (\tilde{\eta}_{\tilde{x}}^2 - 1)} = 0 \\ & \text{at } \tilde{y} = \tilde{\eta} \end{aligned} \quad (\text{A1.20})$$

Conservation of normal momentum across the liquid/vapor interface

$$\begin{aligned} & \tilde{p} + 2\rho\nu\tilde{U}_{\tilde{x}} \frac{1 + \tilde{\eta}_{\tilde{x}}^2}{1 - \tilde{\eta}_{\tilde{x}}^2} + \sigma \frac{\tilde{\eta}_{\tilde{x}\tilde{x}}}{(1 + \tilde{\eta}_{\tilde{x}}^2)^{3/2}} \\ & - \frac{\rho((\tilde{U} - \tilde{U}_i) \tilde{\eta}_{\tilde{x}} - (\tilde{V} - \tilde{V}_i))(\tilde{U} \tilde{\eta}_{\tilde{x}} + \tilde{V})}{\tilde{\eta}_{\tilde{x}}^2 - 1} \\ & + \frac{\rho\delta((\tilde{U}_v - \tilde{U}_i) \tilde{\eta}_{\tilde{x}} - (\tilde{V}_v - \tilde{V}_i))(\tilde{U}_v \tilde{\eta}_{\tilde{x}} + \tilde{V}_v)}{\tilde{\eta}_{\tilde{x}}^2 - 1} = \tilde{p}_v \quad \text{at } \tilde{y} = \tilde{\eta} \end{aligned} \quad (\text{A1.21})$$

Conservation of energy across the liquid/vapor interface:

$$\tilde{z}_{\tilde{y}} - \tilde{z}_{\tilde{x}} \tilde{\eta}_{\tilde{x}} + \frac{\rho h_{fg}}{k} (\tilde{v} - \tilde{v}_i - (\tilde{u} - \tilde{u}_i) \tilde{\eta}_{\tilde{x}}) = 0 \quad \text{at } \tilde{y} = \tilde{\eta} \quad (\text{A1.22})$$

Continuity of tangential velocities at the liquid/vapor interface:

$$\tilde{u} + \tilde{v} \tilde{\eta}_{\tilde{x}} - \tilde{u}_v - \tilde{v}_v \tilde{\eta}_{\tilde{x}} = 0 \quad \text{at } \tilde{y} = \tilde{\eta} \quad (\text{A1.23})$$

The proper interfacial conditions representing the conservation of tangential momentum, normal momentum and energy across the interface are obtained by eliminating the velocities of the vapor and of the interface from (A1.20-A1.22). Substituting (A1.18), (A1.19), and (A1.23) into (A1.20-A1.22) one obtains

$$\tilde{u}_{\tilde{y}} + \tilde{v}_{\tilde{x}} + 4 \frac{\tilde{u}_{\tilde{x}} \tilde{\eta}_{\tilde{x}}}{\tilde{\eta}_{\tilde{x}}^2 - 1} = 0 \quad \text{at } \tilde{y} = \tilde{\eta} \quad (\text{A1.24})$$

$$\tilde{p} + 2\rho\nu \tilde{u}_{\tilde{x}} \frac{1 + \tilde{\eta}_{\tilde{x}}^2}{1 - \tilde{\eta}_{\tilde{x}}^2} + \sigma \frac{\tilde{\eta}_{\tilde{x}\tilde{x}}}{(1 + \tilde{\eta}_{\tilde{x}}^2)^{3/2}} + \frac{\rho(1-\gamma)(\tilde{v} - \tilde{u} \tilde{\eta}_{\tilde{x}} - \tilde{\eta}_{\tilde{x}})^2}{\gamma(1 + \tilde{\eta}_{\tilde{x}}^2)} = \tilde{p}_v \quad \text{at } \tilde{y} = \tilde{\eta} \quad (\text{A1.25})$$

$$\tilde{z}_{\tilde{y}} - \tilde{z}_{\tilde{x}} \tilde{\eta}_{\tilde{x}} + \frac{\rho h_{fg}}{k} (\tilde{v} - \tilde{u} \tilde{\eta}_{\tilde{x}} - \tilde{\eta}_{\tilde{x}}) = 0 \quad \text{at } \tilde{y} = \tilde{\eta} \quad (\text{A1.26})$$

Equation (A1.26) when written in the form

$$\frac{1}{\rho h_{fg}} q^* + \tilde{\eta}_{\tilde{t}} + \frac{\partial}{\partial \tilde{x}} \int_0^{\tilde{\eta}} \tilde{u} d\tilde{y} = 0 \quad \text{at } \tilde{y} = \tilde{\eta}, \quad (\text{A1.27})$$

$$q^* = -k (\tilde{t}_{\tilde{y}} - \tilde{t}_{\tilde{x}} \tilde{\eta}_{\tilde{x}})$$

represents the macroscopic mass balance in the liquid film. Hence this condition can also be called the "macroscopic mass balance equation", or "the kinematic condition of phase changing liquid films". It is observed from equation (A1.26) that the kinematic surface condition used in references [2-4]

$$\tilde{v} - \tilde{u} \tilde{\eta}_{\tilde{x}} - \tilde{\eta}_{\tilde{t}} = 0 \quad \text{at } \tilde{y} = \tilde{\eta} \quad (\text{A1.28})$$

holds only if the component of the heat flux in the liquid film directed normal to the interface vanishes. This, however, is not possible in the condensation problem because the latent heat released by phase change must be removed by conduction. It is noted that if  $\tilde{t} = \text{constant}$ , equations (A1.24-A1.26) reduce to the correct interfacial conditions for an isothermal liquid film as given in reference [73]. Under the boundary layer approximations and steady state assumption, equation (A1.26) reduces to

$$\tilde{t}_{\tilde{y}} + \frac{\rho h_{fg}}{k} (\tilde{v} - \tilde{u} \tilde{\eta}_{\tilde{x}}) = 0 \quad \text{at } \tilde{y} = \tilde{\eta} \quad (\text{A1.29})$$

which is just the differential form of the over-all energy balance equation given in [3].

The governing equations (A1.1-A1.4) and the boundary conditions (A1.5), (A1.6), and (A1.24-A1.26) describe the flow of a condensate film flowing down a vertical plane and undergoing heat and mass transfer across the liquid/vapor interface. The last expression on the left hand side of equation (A1.25) represents the normal stress resulting from the phase change. This term differs from that given in [2-4]. We note, however, that the formulation presented by these authors is heuristic and that in order to account for the normal stress resulting from phase change, they have inserted a vapor velocity term into the normal stress interfacial condition valid for an isothermal liquid film without any justification.

APPENDIX II

SOLUTION OF THE STEADY STATE

EQUATIONS--THE BASE FLOW

Under the boundary layer approximations and steady state assumption, equations (A1.1-A1.4) and conditions (A1.5), (A1.6), (A1.24-A1.26) reduce to

$$\tilde{U}_{\tilde{x}} + \tilde{V}_{\tilde{y}} = 0 \quad (A2.1)$$

$$\tilde{U}\tilde{U}_{\tilde{x}} + \tilde{V}\tilde{U}_{\tilde{y}} + \tilde{\rho}^{-1}\tilde{P}_{\tilde{x}} = g + \nu\tilde{U}_{\tilde{y}\tilde{y}} \quad (A2.2)$$

$$\tilde{U}\tilde{t}_{\tilde{x}} + \tilde{V}\tilde{t}_{\tilde{y}} = k(\rho c_p)^{-1}\tilde{t}_{\tilde{y}\tilde{y}} \quad (A2.3)$$

$$\tilde{U} = \tilde{V} = 0, \quad \tilde{t} = \tau_w \quad \text{at } \tilde{y} = 0 \quad (A2.4)$$

$$\tilde{t} = \tau_s, \quad \tilde{U}_{\tilde{y}} = 0, \quad \tilde{P} = \tilde{P}_v,$$

$$\tilde{t}_{\tilde{y}} + \rho h_{fg} k^{-1}(\tilde{v} - \tilde{U}\tilde{\eta}_{\tilde{x}}) = 0 \quad \text{at } \tilde{y} = \tilde{\eta} \quad (A2.5)$$

In this problem, the vapor pressure is taken to be given by the hydrostatic pressure distribution, that is

$$\tilde{P}_v = \rho_v g \tilde{x} + P_0$$

Also, the variation of the liquid pressure in the  $\tilde{y}$ -direction is neglected. This problem is made dimensionless by introducing the following transformations

$$\begin{aligned} \tilde{x} &= \alpha x, \quad \tilde{y} = \alpha y, \quad \tilde{\eta} = \alpha \bar{\eta} \\ \tilde{U} &= \frac{k \Delta \tau}{\alpha \rho h_{fg}} \bar{U}, \quad \tilde{V} = \frac{k \Delta \tau}{\alpha \rho h_{fg}} \bar{V}, \quad \tilde{t} = \tau_w + \Delta \tau \bar{\theta} \end{aligned} \quad (A2.6)$$

Substituting (A2.6) into (A2.1-A2.5) one finds

$$\bar{u}_x + \bar{v}_y = 0 \quad (\text{A2.7})$$

$$\zeta \left( \frac{1}{Pr} \bar{u} \bar{u}_x + \frac{1}{Pr} \bar{v} \bar{u}_y \right) = 1 + \bar{u}_{yy} \quad (\text{A2.8})$$

$$\zeta (\bar{u} \bar{\theta}_x + \bar{v} \bar{\theta}_y) = \bar{\theta}_{yy} \quad (\text{A2.9})$$

subject to

$$\bar{u} = \bar{v} = \bar{\theta} = 0 \quad \text{at } y = 0 \quad (\text{A2.10})$$

$$\bar{\theta} = 1, \bar{u}_y = 0, \bar{\theta}_y - \bar{u} \bar{\eta}_x + \bar{v} = 0 \quad \text{at } y = \bar{\eta} \quad (\text{A2.11})$$

Here,  $\zeta$  is the so-called heat-capacity parameter. A first order perturbation solution of the problem posed by equations (A2.7-A2.11) is found by assuming the following expansions

$$\bar{u} = \bar{u}_0 + \zeta \bar{u}_1 + O(\zeta^2)$$

$$\bar{v} = \bar{v}_0 + \zeta \bar{v}_1 + O(\zeta^2) \quad (\text{A2.12})$$

$$\bar{\theta} = \bar{\theta}_0 + \zeta \bar{\theta}_1 + O(\zeta^2)$$

Substituting equations (A2.12) into (A2.7-A2.11) and equating like powers of  $\zeta$  yields the zeroth-order problem

$$\bar{u}_{0x} + \bar{v}_{0y} = 0$$

$$\bar{u}_{0yy} + 1 = 0$$

$$\bar{\theta}_{0yy} = 0 \quad (A2.13)$$

$$\bar{u}_0 = \bar{v}_0 = \bar{\theta}_0 = 0 \quad \text{at } y=0$$

$$\bar{u}_{0y} = 0, \bar{\theta}_0 = 1 \quad \text{at } y=\bar{\eta}$$

and the first-order problem

$$\bar{u}_{1x} + \bar{v}_{1y} = 0$$

$$\bar{u}_{1yy} = \frac{1}{Pr} \bar{u}_0 \bar{u}_{0x} + \frac{1}{Pr} \bar{v}_0 \bar{u}_{0y}$$

$$\bar{\theta}_{1yy} = \bar{u}_0 \bar{\theta}_{0x} + \bar{v}_0 \bar{\theta}_{0y} \quad (A2.14)$$

$$\bar{u}_1 = \bar{v}_1 = \bar{\theta}_1 = 0 \quad \text{at } y=0$$

$$\bar{u}_{1y} = \bar{\theta}_1 = 0 \quad \text{at } y=\bar{\eta}$$

The solution of (A2.13) is

$$\bar{u}_0 = \bar{\eta}y - \frac{1}{2}y^2$$

$$\bar{v}_0 = -\frac{1}{2}\bar{\eta}_x y^2 \quad (A2.15)$$

$$\bar{\theta}_0 = \frac{y}{\bar{\eta}}$$

Substituting (A2.15) into (A2.14) and solving the resulting problem gives

$$\begin{aligned}\bar{U}_1 &= \frac{1}{24Pr} \bar{\eta} \bar{\eta}_x y^4 - \frac{1}{6Pr} \bar{\eta}^4 \bar{\eta}_x y \\ \bar{V}_1 &= -\frac{1}{120Pr} (\bar{\eta}_x^2 + \bar{\eta} \bar{\eta}_{xx}) y^5 \\ &\quad + \frac{1}{12Pr} (4\bar{\eta}^3 \bar{\eta}_x^2 + \bar{\eta}^4 \bar{\eta}_{xx}) y^2 \\ \bar{\theta}_1 &= \frac{1}{40} \frac{\bar{\eta}_x}{\bar{\eta}^2} y^5 - \frac{1}{8} \frac{\bar{\eta}_x}{\bar{\eta}} y^4 + \frac{1}{10} \bar{\eta}_x \bar{\eta}^2 y\end{aligned}\tag{A1.16}$$

Substituting (A2.12) into the last condition of (A2.11) now leads to the following ordinary differential equation for the dimensionless film thickness,  $\bar{\eta}$

$$1 - \bar{\eta}^3 \bar{\eta}_x + \mathfrak{J} \left( -\frac{11}{40} \bar{\eta}^3 \bar{\eta}_x + \frac{9}{20Pr} \bar{\eta}^6 \bar{\eta}_x^2 + \frac{3}{40Pr} \bar{\eta}^7 \bar{\eta}_{xx} \right) + O(\mathfrak{J}^2) = 0 \tag{A2.17}$$

A perturbation solution of equation (A2.17) subject to the condition

$$\bar{\eta}(0) = 0 \tag{A2.18}$$

is found by assuming

$$\bar{\eta} = \bar{\eta}_0 + \mathfrak{J} \bar{\eta}_1 + O(\mathfrak{J}^2) \tag{A2.19}$$

Substituting (A2.19) into (A2.17) and (A2.18), and equating like powers of  $\mathfrak{J}$  gives the zeroth-order problem

$$\bar{\eta}_0^3 \bar{\eta}_{0x} = 1 \quad (\text{A2.20})$$

$$\bar{\eta}_0 = 0 \quad \text{at } x=0$$

and the first-order problem

$$\bar{\eta}_{1x} + \frac{3\bar{\eta}_{0x}}{\bar{\eta}_0} \bar{\eta}_1 = -\frac{11}{40} \bar{\eta}_{0x} + \frac{9}{20Pr} \bar{\eta}_0^3 \bar{\eta}_{0x}^2 + \frac{3}{40Pr} \bar{\eta}_0^4 \bar{\eta}_{0xx} \quad (\text{A2.21})$$

$$\bar{\eta}_1 = 0 \quad \text{at } x=0$$

The solution of (A2.20) is

$$\bar{\eta}_0 = \sqrt{2} x^{1/4} \quad (\text{A2.22})$$

Substituting (A2.22) into (A2.21) and solving the resulting problem, one finds

$$\bar{\eta}_1 = \sqrt{2} \left( \frac{9}{160Pr} - \frac{11}{160} \right) x^{1/4} \quad (\text{A2.23})$$

Thus  $\bar{\eta}$  is given by

$$\bar{\eta} = \sqrt{2} \left( 1 + \mathcal{O} \left( \frac{9}{160Pr} - \frac{11}{160} \right) \right) x^{1/4} + \mathcal{O}(\mathcal{O}^2) \quad (\text{A2.24})$$

In order to find the effects of acceleration and convection in the condensate film on the heat transfer across the film, one can now evaluate the Nusselt number defined by

$$Nu = -\frac{\tilde{x}}{\tau_s - \tau_w} \frac{\partial \tilde{t}}{\partial \tilde{y}} \quad \text{at } \tilde{y} = 0 \quad (A2.25)$$

If  $Nu_0$  is to represent the Nusselt number when  $\mathfrak{J} = 0$  then the ratio  $Nu/Nu_0$  is found to be:

$$\frac{Nu}{Nu_0} = \frac{1 + \frac{1}{10} \mathfrak{J}}{1 + \frac{9\mathfrak{J}}{160Pr} - \frac{11}{160} \mathfrak{J}} + O(\mathfrak{J}^2) \quad (A2.26)$$

Equation (A2.26) is compared with the results from the exact numerical solution of the same mathematical problem in Fig. A2.1. It is observed that the first order solution is a valid approximation to the exact solution of the problem, particularly if  $\mathfrak{J}$  is small. Therefore the first order solution given by equations (A2.12), (A2.15), (A2.16) and (A2.24) can be used as base flow in the stability analysis to identify the effects of acceleration and convection on the stability characteristics of condensate flow.

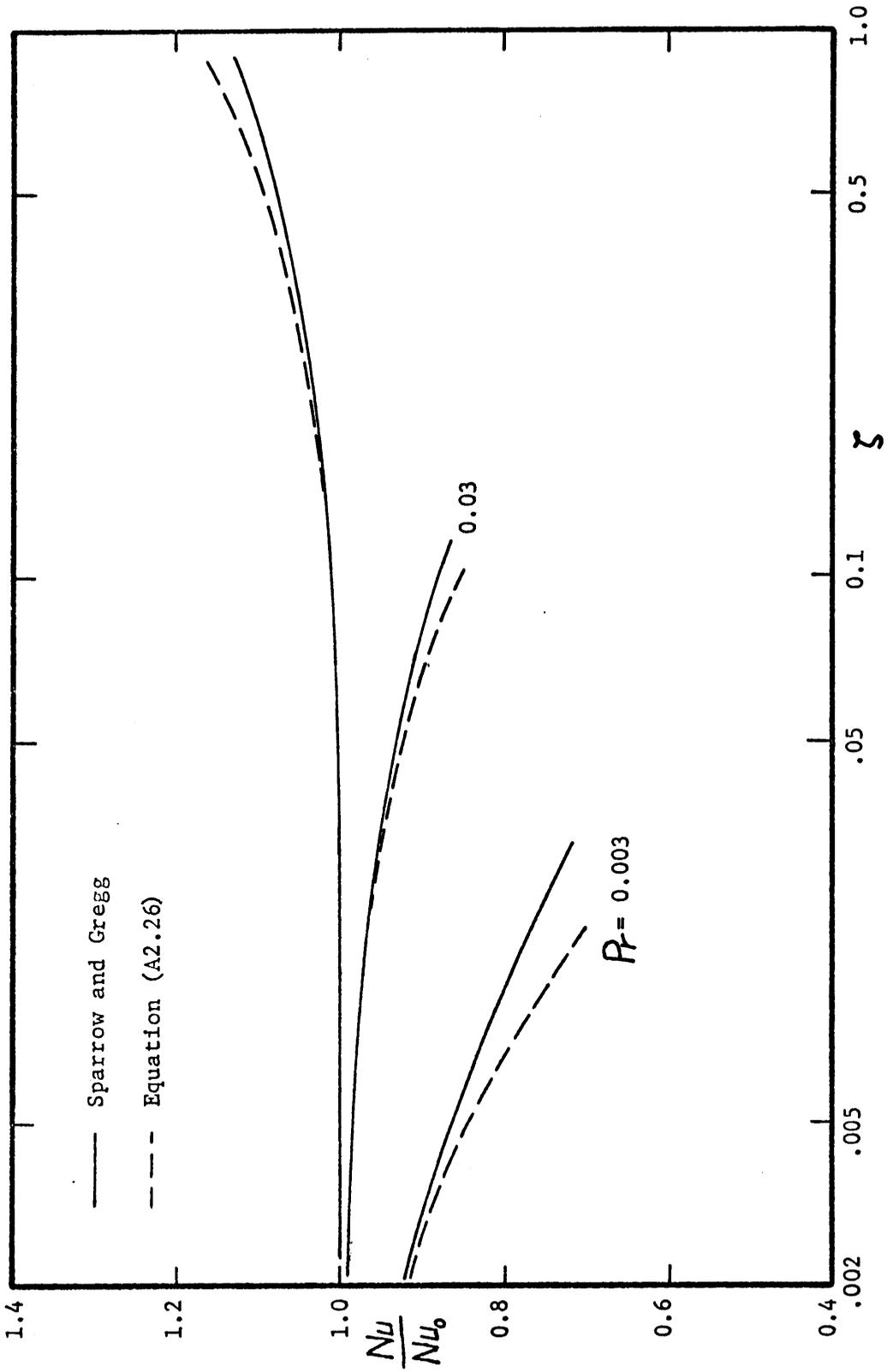


Figure A2.1. Comparison of the first-order solution with the exact solution.

### APPENDIX III

#### DERIVATION OF THE LINEARIZED STABILITY PROBLEM

In this section we will derive the linearized stability problem starting from the general formulation given in Appendix I. The derivation given here is essentially the same of that given by Yih [40] for the case of an isothermal liquid film flow except for an additional approximation that needs to be made in order to render the stability problem tractable. This additional approximation is the so-called "parallel flow assumption". The necessity for this approximation arises from the fact that while the base flow of an isothermal liquid film is a "parallel flow" (the field quantities do not change in the x-direction), the base flow of laminar film condensation is a boundary layer type "nearly parallel flow" (the field quantities vary slowly in the x-direction). This approximation has been made in different nearly parallel flows. Kao [79] made the approximation when studying the stability of two-phase film boiling. The same approximation is standard in the studies of the stability of natural convection flows [80-82]. The Blasius boundary layer, the two dimensional jet, and the two-dimensional flat-plate wake are also examples of nearly parallel flows where this approximation is common. Ling and Leynolds [77] investigated the validity of the parallel flow assumption on the stability analysis. Their results showed that this assumption is particularly valid for boundary layer type flows, the non-parallel flow corrections

being important in other flows such as jets and wakes. Recently, Nayfeh et al. [78] presented an analysis based on the method of multiple scales for the investigation of the stability of non-parallel flows. Their results showed that the parallel flow assumption is the correct first approximation in the studies of stability of nearly parallel flows. In the present investigation, the parallel flow approximation will be utilized. It is noted, however, that the effects of non-parallelism in the base flow can be accounted for by the methods presented in [77] or [78].

In the derivation of the linearized stability problem, it is convenient to introduce the stream function,  $\tilde{\psi}$ , defined by

$$\tilde{u} = \tilde{\psi}_y, \quad \tilde{v} = -\tilde{\psi}_x \quad (A3.1)$$

Introducing (A3.1) into (A1.1-A1.6) and (A1.24-A1.26) yields

$$\nu \nabla^4 \tilde{\psi} = \tilde{\psi}_y \nabla^2 \tilde{\psi}_x - \tilde{\psi}_x \nabla^2 \tilde{\psi}_y + \nabla^2 \tilde{\psi}_z \quad (A3.2)$$

$$\bar{p}' \bar{p}_x = g + \nu \nabla^2 \tilde{\psi}_y - \tilde{\psi}_y \tilde{\psi}_{xy} + \tilde{\psi}_x \tilde{\psi}_{yy} - \tilde{\psi}_y \tilde{z} \quad (A3.3)$$

$$\tilde{z}_z + \tilde{\psi}_y \tilde{z}_x + \tilde{\psi}_x \tilde{z}_y = k(\rho c_p)^{-1} \nabla^2 \tilde{z} \quad (A3.4)$$

$$\tilde{\psi}_y = \tilde{\psi}_x = 0, \quad \tilde{z} = z_w \quad \text{at } \tilde{y} = 0 \quad (A3.5)$$

$$\tilde{z} = z_s, \quad \tilde{\psi}_{\tilde{y}\tilde{y}} - \tilde{\psi}_{\tilde{x}\tilde{x}} + 4 \tilde{\psi}_{\tilde{x}\tilde{y}} \tilde{\eta}_{\tilde{x}} (\tilde{\eta}_{\tilde{x}}^2 - 1)^{-1} = 0 \text{ at } \tilde{y} = \tilde{\eta} \quad (\text{A3.6})$$

$$\tilde{p} + 2 \rho \nu \tilde{\psi}_{\tilde{x}\tilde{y}} (1 + \tilde{\eta}_{\tilde{x}}^2) (1 - \tilde{\eta}_{\tilde{x}}^2)^{-1} + \sigma \tilde{\eta}_{\tilde{x}\tilde{x}} (1 + \tilde{\eta}_{\tilde{x}}^2)^{-3/2}$$

$$+ \rho (1 - \gamma) \gamma^{-1} (\tilde{\psi}_{\tilde{x}} + \tilde{\psi}_{\tilde{y}} \tilde{\eta}_{\tilde{x}} + \tilde{\eta}_{\tilde{z}})^2 (1 + \tilde{\eta}_{\tilde{x}}^2)^{-1} = \tilde{p}_v$$

$$\text{at } \tilde{y} = \tilde{\eta} \quad (\text{A3.7})$$

$$\tilde{z}_{\tilde{y}} - \tilde{z}_{\tilde{x}} \tilde{\eta}_{\tilde{x}} - k \rho h_{fg} (\tilde{\psi}_{\tilde{x}} + \tilde{\psi}_{\tilde{y}} \tilde{\eta}_{\tilde{x}} + \tilde{\eta}_{\tilde{z}}) = 0 \text{ at } \tilde{y} = \tilde{\eta} \quad (\text{A3.8})$$

In accordance with the linearized stability theory [48], we introduce the following transformations

$$\frac{\tilde{z} - z_w}{\Delta \tau} = \bar{\theta} + \theta \quad (\text{A3.9})$$

$$\frac{\rho h_{fg} \tilde{\psi}}{k \Delta \tau} = \bar{\psi} + \psi \quad (\text{A3.10})$$

$$\frac{\tilde{p}}{\alpha \rho g} = \bar{p} + p \quad (\text{A3.11})$$

$$\frac{\tilde{\eta}}{\alpha} = \bar{\eta} + \eta \quad (\text{A3.12})$$

and nondimensionalize the coordinates by

$$\tilde{x} = \alpha x, \quad \tilde{y} = \alpha y, \quad \tilde{z} = \frac{\alpha^2 \rho h_{fg}}{k \Delta \tau} z \quad (\text{A3.13})$$

In this transformation,  $\bar{\theta}$ ,  $\bar{\psi}$ ,  $\bar{p}$ , and  $\bar{\eta}$  are the base flow quantities, whereas  $\theta$ ,  $\psi$ ,  $p$ , and  $\eta$  represent the disturbance quantities.  $\bar{\psi}$  is related to the base flow velocity components  $\bar{u}$  and  $\bar{v}$  through

$$\bar{u} = \bar{\psi}_y, \quad \bar{v} = -\bar{\psi}_x \quad (A3.14)$$

and  $\bar{p}$  is given by

$$\bar{p} = \frac{\bar{P}}{\alpha \rho g} \quad (A3.15)$$

When (A3.9-A3.13) are substituted into (A3.2-A3.8) and the following action is taken:

- (i)  $\bar{u}, \bar{v}, \bar{\theta}, \bar{\eta}$  satisfies the base flow equations (A2.7-A2.11).
- (ii) The x-derivatives with respect to the base flow quantities  $\bar{\psi}, \bar{\theta}, \bar{\eta}$ , are neglected from the equations (parallel flow approximation)
- (iii) Nonlinear terms with respect to the disturbance quantities,  $\theta, \psi, \eta$  are neglected from the equations (linearization)

one obtains

$$\begin{aligned} \psi_{yyyy} + 2\psi_{xxyy} + \psi_{xxxx} &= \frac{J}{Pr} (\psi_{tyy} + \psi_{txx} + \bar{u}\psi_{xxx} \\ &+ \bar{u}\psi_{xyy} - \bar{u}_y\psi_x) \end{aligned} \quad (A3.16)$$

$$p_x = \psi_{xxy} + \psi_{yyy} - \frac{J}{Pr} (\psi_{yt} + \bar{u}\psi_{xy} - \bar{u}_y\psi_x) \quad (A3.17)$$

$$\theta_{yy} + \theta_{xx} = \mathcal{J}(\theta_t + \bar{u}\theta_x - \bar{\theta}_y\psi_x) \quad (A3.18)$$

$$\psi = \psi_y = \theta = 0 \quad \text{at } y = 0 \quad (A3.19)$$

$$\theta = -\bar{\theta}_y\eta \quad \text{at } y = \bar{y} \quad (A3.20)$$

$$\psi_{yy} - \psi_{xx} + \bar{u}_{yy}\eta = 0 \quad \text{at } y = \bar{y} \quad (A3.21)$$

$$p + 2(1-\delta)\psi_{xy} + \sigma(\rho g \alpha^2)^{-1} \eta_{xx} = 0 \quad \text{at } y = \bar{y} \quad (A3.22)$$

$$\theta_y + \bar{\theta}_{yy}\eta = \psi_x + \bar{u}\eta_x + \eta_t \quad \text{at } y = \bar{y} \quad (A3.23)$$

Here, the interfacial conditions (A3.20-A3.23) have been transformed from  $y = \bar{y} + \eta$  to  $y = \bar{y}$  by means of a Taylor series expansion for small  $\eta$ . The mathematical problem posed by (A3.16-A3.23) is linear with respect to the disturbance quantities and hence admits traveling wave solutions of the form

$$\theta = \varphi(y) \exp(i\beta) + CC \quad (A3.24)$$

$$\psi = \phi(y) \exp(i\beta) + CC \quad (A3.25)$$

$$p = P(y) \exp(i\beta) + CC \quad (A3.26)$$

$$\eta = \eta_0 \exp(i\beta) + CC \quad (A3.27)$$

Here CC denotes the complex conjugate and  $\beta$  is given by

$$\beta = \epsilon(x - ct) \quad (A3.28)$$

$\epsilon$  is the wavenumber and  $c$  is the complex characteristic value of the following "linearized stability problem" obtained by substituting (A3.24-A3.27) into (A3.16-A3.23):

$$\phi'''' - 2\epsilon^2\phi'' + \epsilon^4\phi = i\epsilon\frac{\mathcal{J}}{\rho r}((\bar{u}-c)(\phi'' - \epsilon^2\phi) - \bar{u}_{yy}\phi) \quad (\text{A3.29})$$

$$\psi'' - \epsilon^2\psi = i\epsilon\mathcal{J}((\bar{u}-c)\psi - \bar{\theta}_y\phi) \quad (\text{A3.30})$$

$$\phi = \phi' = \psi = 0 \quad \text{at } y = 0 \quad (\text{A3.31})$$

$$\psi = -\eta_0\bar{\theta}_y \quad \text{at } y = \bar{y} \quad (\text{A3.32})$$

$$\phi'' + \epsilon^2\phi + \eta_0\bar{u}_{yy} = 0 \quad \text{at } y = \bar{y} \quad (\text{A3.33})$$

$$\begin{aligned} \phi''' + (i\epsilon\frac{\mathcal{J}}{\rho r}(c-\bar{u}) - \epsilon^2(3-2\gamma))\phi' - i\eta_0\epsilon\Gamma \\ + i\epsilon\frac{\mathcal{J}}{\rho r}\bar{u}_y\phi = 0 \quad \text{at } y = \bar{y} \end{aligned} \quad (\text{A3.34})$$

$$\psi' + \eta_0\bar{\theta}_{yy} = i\epsilon\phi + i\epsilon\eta_0(\bar{u}-c) \quad \text{at } y = \bar{y} \quad (\text{A3.35})$$

The formulation of the linearized stability problem as presented above is in accordance with the formulation procedure suggested by Lin [48]. We note that under the parallel flow approximation, the base flow quantities  $\bar{u}$ ,  $\bar{\theta}$ ,  $\bar{y}$  are treated as invariant with respect to the  $x$ -coordinate when analyzing the stability problem. In this regard, the solutions to the stability problems of nearly parallel flows have sometimes been called "local solutions". In view of the agreement between local solutions and experimental

measurements, and an order of magnitude analysis of the errors introduced by the parallel flow assumption, this approximation is generally accepted [48]. The linearized stability problem posed by (A3.29-A3.35) is "homogeneous" with respect to the unknowns  $\phi$ ,  $\psi$ , and  $\eta_0$ . This property compels the problem into a characteristic-value problem. A necessary condition for the existence of a nontrivial solution for this problem is a relation among the parameters of the form

$$C = C(\mathcal{J}, Pr, \epsilon, \Gamma, \gamma, \bar{M}) \quad (A3.36)$$

This condition determines the regions of stability and instability of the disturbances induced on the steady condensate flow. The criteria separating stability from instability is discussed in Chapter III.

**The vita has been removed from  
the scanned document**

STABILITY OF A CONDENSATE FILM  
FLOWING DOWN A VERTICAL PLANE

by  
Mazhar Ünsal

(ABSTRACT)

The mathematical problem governing the motion and temperature in a two dimensional condensate film are formulated. The linearized stability problem is derived under the parallel flow assumption. A second-order perturbation solution of the stability problem is presented using the Nusselt solution as base flow. The boundary layer equations of laminar film condensation are solved to first-order by using perturbation methods. A first-order perturbation solution of the stability problem is determined using the first-order solution of the boundary layer equations as base flow. The first-order solutions of the stability problem lead to approximate closed form expressions for the neutral stability curve. A nonlinear algebraic equation for the neutral stability curve is found from the second-order solution of the stability problem which is solved numerically.

It is found that decreasing the temperature drop across the condensate film has a stabilizing effect. This is observed to be in qualitative agreement with the previous experimental studies. It is also found that increasing the Prandtl number or the surface tension

has a stabilizing effect on the condensate film. Critical distances up to which the condensate film is completely stable are predicted. The predictions are discussed with reference to the previous experimental studies. The critical distance and neutral stability predictions disagree with the results from the previous stability analyses. The reasons for this disagreement are discussed. The results indicate that the effects of acceleration and convection in the base flow are negligible in the linearized stability problem for almost all practical situations.