AN ANALYTICAL STUDY AND COMPUTER ANALYSIS OF THREE-DIMENSIONAL, STEADY-STATE VIBRATION OF MULTISHAFT GEARED-ROTOR SYSTEMS

by

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TABLE OF CONTENTS

TABLE OF CONTENTS (Cont'd)

TABLE OF CONTENTS (Cont'd)

LIST OF FIGURES

LIST OF FIGURES (Cont'd)

LIST OF TABLES

NOMENCLATURE

The nature of the current work demands the following extensive nomenclature list which includes terms of initial description and final form regarding derivations. It does not include terms which are defined in the text during intermediate steps in derivations and which are not referred to again.

a Variable in the Mathieu Equation

$\underline{\underline{B}}_{\alpha\beta}$ Block element submatrices containing bearing translational stiffnesses in α,β = x, y, z system coordinates used in the bearing transfer matrix, N/m (lb-in)

$\underline{\underline{B}}^{*}_{\alpha\beta}$ Block element submatrices containing bearing rotational stiffnesses in α,β = x, y, z system coordinates used in the bearing transfer matrix, N-m/rad (lb-in/rad)

b Tooth face width, m (in)

C^{HC}_{ij} Tooth-pair compliance tensor in line-of-action coordinates representing Hertzian contact deformation, m/N (in/lb)

C^{L}_{ij} Tooth-pair compliance tensor in line-of-action coordinates representing bending, shear, and compression; m/N (in/lb)

$C_{\alpha\beta}$ Bearing translational damping characteristics in system global coordinates, N-s/m (lb-s/in)

NOMENCLATURE (Cont'd)

$C^{*}_{\alpha\beta}$ Bearing rotational damping characteristics in system global coordinates, N-m-s/rad (lb-in-s/rad)

E Young's modulus, N/m^2 (lb/in^2)

e Eccentricity of disk center of gravity from rotational center, m (in)

f Mesh force per unit face width acting along the line-of-action, N/m (lb/in)

f_c Component of f normal to the tooth centerline, N/m (lb/in)

f_s Component of f parallel to the tooth centerline, N/m (lb/in)

F Mesh force acting along the line-of-action, N (lb)

F_c Component of the line-of-action mesh force acting normal to the tooth centerline, N (lb)

F_s Component of the line-of-action mesh force acting parallel to the tooth centerline, N (lb)

$F(t)$ Forcing function involving transcendental functions, N (lb) or N-m (lb-in)

NOMENCLATURE (Cont'd)

$F^{\sigma}(t)$ Time-varying gear-tooth machining error occurring at tooth-passing frequency, σ, and integer multiples; m (in)

F^{β} Coefficients of transcendental functions at frequency β making up $F^{\sigma}(t)$; β ending in c or s implies cosine or sine, respectively; m (in)

$F^{\sigma}_{\alpha i}(t)$ Mesh forcing function applicable to gear i in the α direction global (x_s,y_s,z_s) coordinates due to gear-tooth machining errors appearing at tooth-passing frequency and integer multiples; N (lb), $\alpha = x_s$ or y_s; N-m (lb-in), $\alpha = z_s$

$F^{\beta}_{\alpha i}$ Transfer matrix extension column forcing function associated with shaft i, global coordinate direction α, due to gear-tooth machining errors appearing at frequency β; β ending in c or s implies cosine or sine, respectively; N (lb)

$F^{\sigma}_{\alpha\beta}$ Gear-tooth error due to machining errors at tooth-passing frequency and multiples in the α direction of the β coordinate system.

β = g: gear (x_g,y_g); m (in)

β = q: global 2-D (x_s,y_s); m (in)

β = s: global 3-D (x_s,y_s,z_s); N (lb), $\alpha = x_s$ or y_s
N-m (lb-in), $\alpha = z_s$

NOMENCLATURE (Cont'd)

$\underline{\underline{F}}$ Transfer matrix representing an externally applied force

$G^{\gamma}(t)$ Time-varying gear-tooth machining error occurring at frequencies, γ, other than tooth passing and multiples, m (in)

G^{β} Gear-tooth machining error coefficient at frequency β making up $G^{\gamma}(t)$; β ending in c or s implies cosine or sine, respectively; m (in)

$G^{\gamma}_{\alpha\beta}(t)$ Mesh forcing function in direction α of coordinate system β due to gear-tooth machining errors at (frequencies, γ, other than tooth passing and multiples)

β = g: gear (x_g, y_g); m (in)

β = q: global 2-D (x_s, y_s); m (in)

β = s: global 3-D (x_s, y_s, z_s); N (lb), $\alpha = x_s$ or y_s

N-m (lb-in), $\alpha = z_s$

$G^{\gamma}_{\alpha\beta}$ Coefficients of transcendental terms comprising $G^{\gamma}_{\alpha\beta}(t)$ where γ is the particular frequency involved; γ ending in c or s implies cosine or sine, respectively

$\underline{G}^{*}_{Z}$ Extension column subvector of forcing in the transfer matrix for the visco-elastic bearing, N-m (lb-in)

NOMENCLATURE (Cont'd)

g Universal gravitational constant, 9.8 m/s^2 (386.088 in/s^2)

h Thickness of tapered cantilever beam representation of the gear tooth; variable over length; m (in)

h_o Thickness of tapered cantilever beam representation of the gear tooth at the base, m (in)

h_1 Thickness of tapered cantilever beam representation of the gear tooth at the point of contact, m (in)

I Area moment of inertia, m^4 (in^4)

I_d Disk diametral mass moment of inertia, $N\text{-}m\text{-}s^2$ ($lb\text{-}in\text{-}s^2$)

I_p Disk polar mass moment of inertia, $N\text{-}m\text{-}s^2$ ($lb\text{-}in\text{-}s^2$)

$\underline{\underline{I}}$ Identity matrix

J Polar mass moment of inertia, $N\text{-}m\text{-}s^2$ ($lb\text{-}in\text{-}s^2$)

$\underline{\underline{K}}^{\ell}$ Mesh-stiffness tensor in the line-of-action coordinates, N/m (lb/in)

$k_{\alpha\beta}^{\ell}$ Elements of the (2x2) $\underline{\underline{K}}$ mesh-stiffness tensor in line-of-action coordinates where α and β take on the values 1,2; for x,y directions, respectively; N/m (lb/in)

NOMENCLATURE (Cont'd)

$\underline{\underline{K}}^g$ Mesh-stiffness tensor in gear coordinates, N/m (lb/in)

$k^g_{\alpha\beta}$ Elements of the (2x2) $\underline{\underline{K}}^g$ mesh-stiffness tensor in gear coordinates where α and β take on the values 1,2; for x,y directions, respectively; N/m (lb/in)

$\underline{\underline{K}}^q$ Mesh-stiffness tensor in system global (x_{s1}, y_{si}) coordinates, N/m (lb/in)

$k^q_{\alpha\beta}$ Elements of the (2x2) mesh-stiffness tensor in system global (x_{s1}, y_{s1}) coordinates where α and β each take on values 1,2 for the x_{s1}, y_{s1} directions, respectively; N/m (lb/in)

$\underline{\underline{K}}^s$ Mesh-stiffness tensor in system global (x_{si}, y_{si}, z_{si}) coordinates; units depend on particular element, see $k^s_{\alpha\beta}$

$k^s_{\alpha\beta}$ Elements of the $\underline{\underline{K}}^s$ mesh-stiffness tensor. 144 elements are involved where α and β each take on w_{xi}, w_{yi}, w_{zi}, θ_{xi}, θ_{yi}, θ_{zi}; i = gears 1,2. The current work deals with only 36 elements involving ω_{xi}, ω_{yi}, θ_{zi} which are abbreviated in the current work as xi, yi, zi; respectively. Example: $k^s_{\omega_{x1}\theta_{z2}}$ is abbreviated k^s_{x1z2} and has units N/rad (lb/rad)

k Stiffness, N/m (lb/in)

NOMENCLATURE (Cont'd)

$K(t)$ Time-varying torsional mesh stiffness, N-m/rad (in-lb/rad)

$k_{\alpha\beta}(t)$ Time-varying mesh stiffness relating force in the α direction due to a displacement in the β direction, all other displacements zero; units depend on α and β for rotational and translational force and response

k^{β} Torsional mesh-stiffness coefficient of transcendental term at frequency β; β ending in c or s implies cosine or sine, respectively, N-m/rad (lb-in/rad)

$\underline{\underline{K}}$ Stiffness transfer matrix

L Length of the tapered cantilever beam tooth model, m (in)

$\underline{\underline{\ell}}^{\ell q}$ Cartesian tensor direction cosines for transformation from line-of-action to gear coordinates.

$\underline{\underline{\ell}}^{gq}$ Cartesian tensor direction cosines for transformation from gear to system global coordinates.

M Moment, N-m (lb-in)

M or m Mass, N-s^2/m (lb-s^2/in)

$M_{\alpha}(t)$ Time-varying internal moment in the α direction, N-m (lb-in)

NOMENCLATURE (Cont'd)

$\underline{M}_\alpha$ Internal moment state subvector containing all M_α^β in the α direction, N-m (lb-in)

M_α^β Internal moment state vector in the α direction at frequency β; β ending in c or s implies cosine or sine, respectively; N-m (lb-in)

$\underline{\underline{M}}$ Mass transfer matrix

n Used with direction cosines for coordinate transformation; n has the value +1 or -1 for negative or positive driver rotation, respectively in system global coordinates

N State-vector component of internal force, N (lb)

0_α Coordinate axis origin of system

$\alpha = t$, tooth local (x_t, y_t)

$= \ell$, tooth line-of-action (x_ℓ, y_ℓ)

$= g_i$, gear (x_{gi}, y_{gi})

$= s_i$, rotor system global (x_{si}, y_{si}, z_{si})

i = gears 1,2

$\underline{0}$ Null vector

$\underline{\underline{0}}$ Null matrix

q Variable in the Mathieu Equation

NOMENCLATURE (Cont'd)

$\underline{q}$ State vector

R_i Pitch radius of body i, m (in)

R_{bi} Base radius of gear i, m (in)

R_{ci} Radius of curvature of body i, m (in)

U_T Total strain energy, N-m (lb-in)

U_α Strain energy due to shear (α = s), bending (α = b), and compression (α = c), N-m (lb-in)

$V_\alpha(t)$ Time-varying internal shear in the α direction, N (lb)

$\underline{V}_\alpha$ Internal shear state subvector containing all V_α^β in the α direction, N (lb)

V_α^β Internal shear state vector in the α direction at frequency β; β ending in c or s implies cosine or sine, respectively; N (lb)

$w_\alpha(t)$ Time-varying translational response in the α direction, m (in)

w_α Translational state subvector containing all ω_α^β for response in the α direction, m (in)

NOMENCLATURE (Cont'd)

w_{α}^{β} Translational state-vector response in the α direction at frequency β; β ending in c or s implies cosine or sine, respectively; m (in)

X_i Angle between the line-of-action and tooth local coordinates for tooth i, rad

$x(t)$ Translational time history of response, m (in)

x State-vector component of displacement, m (in)

Greek Characters

β Phase reference angle locating disk imbalance, rad

δ_i Hertzian contact deformation of body i, m (in)

δ_L Hertzian contact deformation along the line-of-action of a contacting tooth-pair, m (in)

γ Frequency, rad/s

μ_i Poisson ratio for body i, dimensionless

σ Tooth-passing frequency, rad/s

τ Shear stress, N/m^2 (lb/in^2)

τ Time, s

NOMENCLATURE (Cont'd)

τ Disk wobble angle defined by Daws [10] as measured positive about the system global x axis, rad

$\theta(t)$ Rotational time history of response, rad

$\theta_{\alpha}(t)$ Time-varying rotational response in the α direction, rad

$\underline{\theta}_{\alpha}$ Rotational state subvector containing all θ_{α}^{β} for response in the α direction, rad

θ_{α}^{β} Rotational state-vector response in the α direction at frequency β; β ending in c or s implies cosine or sine, respectively; rad

ω Frequency of shaft-running speed, rad/s

1. INTRODUCTION TO GEARED ROTOR VIBRATION

The field of rotor dynamics has engaged the efforts of a large sector of the engineering community for decades but increasingly in recent years. The geared system of shafting has no serious competition in high torque transmission whether the application is aircraft, automotive, industrial, or marine in nature. World economy and competition in the 1980's have created industry-wide emphasis on cost-effectiveness with explicit emphasis on cost. Implicitly, however, hardware must be reliable. In few areas is the engineering challenge stiffer than in gearbox design. Due to the many and diverse analytical problems yet unsolved, rotor dynamics is still regarded as a very young and developing technology.

Significant studies of rotor systems address diverse concerns including bearing instability and nonlinearity, nonlinear dynamic terms arising from inertial elements, inaccuracies in machining and assembly, non-constant stiffness and damping characteristics of gear teeth, and three-dimensional effects to name but a few.

1.1 Goals of the Current Work

The goal of the current work is the development of a family of computer programs automating model development and solution for three-dimensional linear dynamic response in geared rotors. The Geared Rotor Developer (GRD) computer program described in Appendix A is the interactive front end for the Gear-Mesh Stiffness (GMS) and Geared Rotor Solver (GRS) computer programs of Appendices B and C, respectively. The contributions of the current work reside in two primary developmental areas, described below, culminating in the GMS and GRS programs.

Evaluation of Gear-Mesh Stiffness

Particular attention is paid to the involute spur-gear mesh, the inescapable mode of coupling in power transmission. Succinctly, the gear-mesh element displays stiffness varying throughout engagement by as much as a factor of two, with significant dynamic force generating capabilities.

To illustrate the task at hand, consider the gear pair of Figure 1.1. The calculated rotational stiffness values at discrete increments throughout a tooth passing appear as plus (+) signs in Figure 1.2. These values reflect bending, tension, shear, and local contact modes of stiffness. Recognizing that successive teeth of a gear are similar, the stiffness characterizing a mesh at constant speed is periodic at tooth-passing frequency.

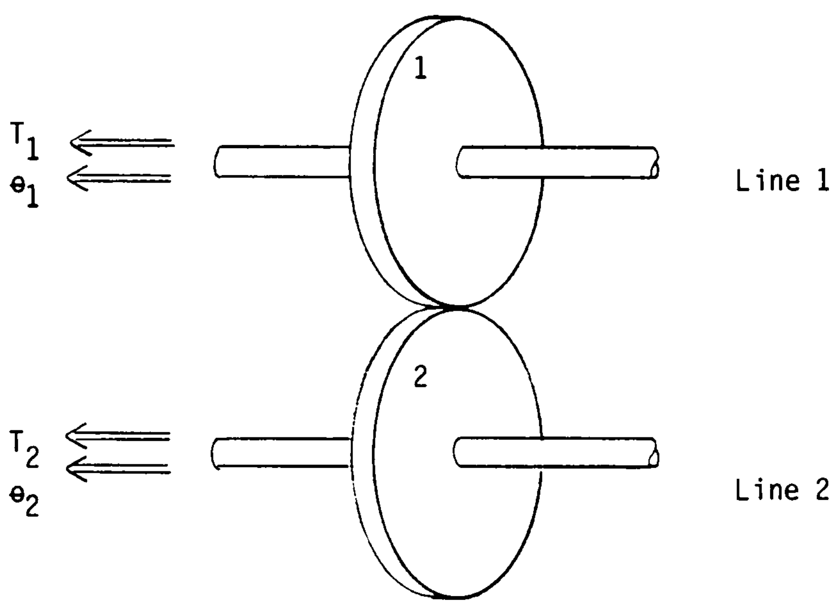


Figure 1.1 Meshing Spur Gears; example case, 30 teeth each, 20° pressure angle, 4 pitch, 0.0254 m (1") face width, 271 N-m (2400 in-lb) torque.

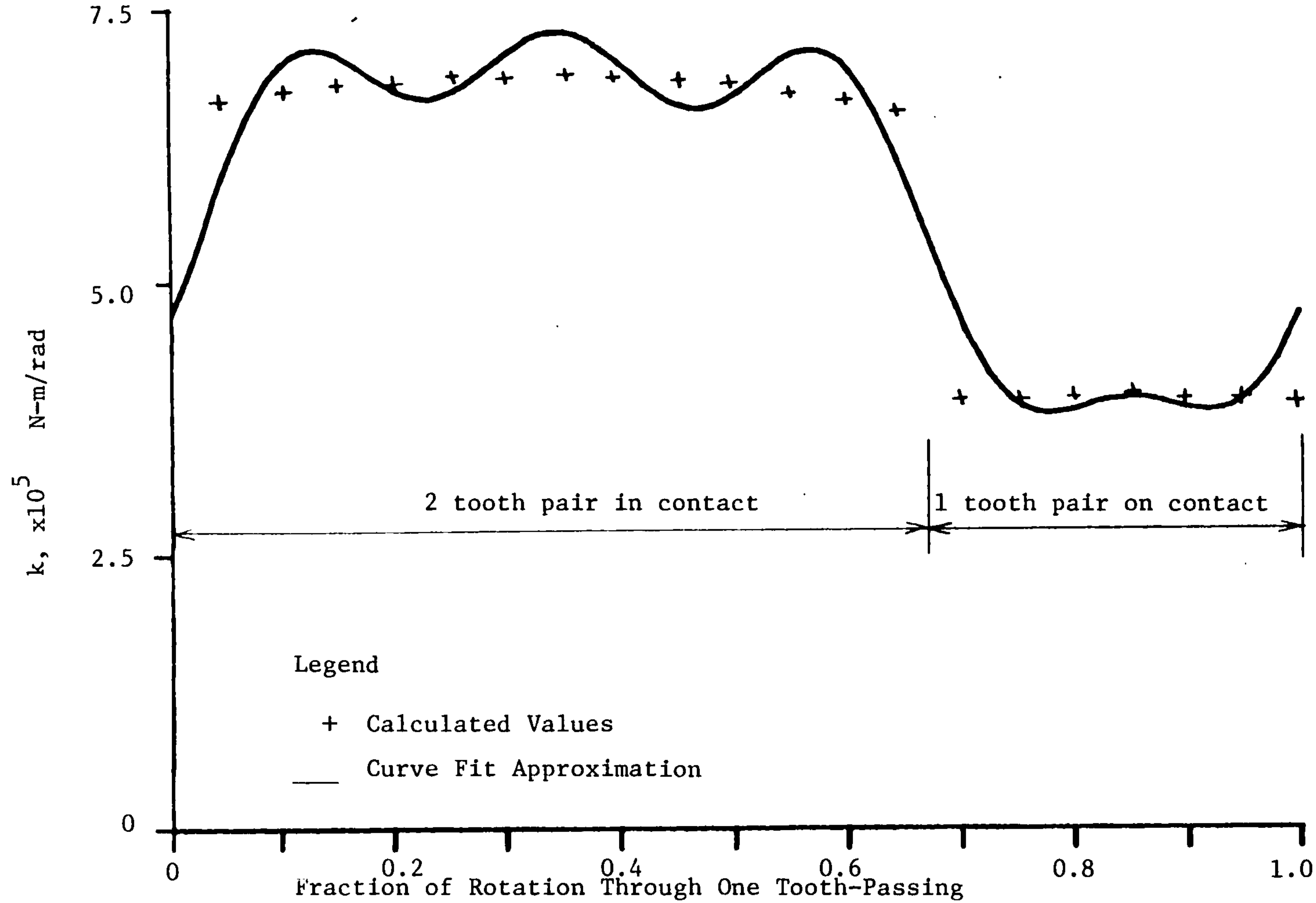


Figure 1.2 Rotational Stiffness of Figure 1.1 Gear Mesh

If a Fourier series truncated with the fourth harmonic according to

$$K(t) = k^{o} + \sum_{n=1}^{4} (k^{n\sigma c} \cos n\sigma t + k^{n\sigma s} \sin n\sigma t), \qquad (1.1)$$

is used to approximate these values, a least squares fit of calculated values yields the coefficients k^{o}, $k^{n\sigma c}$, and $k^{n\sigma s}$.

The harmonic terms $k^{n\sigma c}$ and $k^{n\sigma s}$ are significant relative to the mean stiffness k^{o}. Section 2.2 will illustrate the role of these harmonic stiffness terms in "parametric excitation" or "frequency branching". The example problem of Section 5.2 will demonstrate the significance of these forces in a dynamic response analysis.

Chapter 3 develops the equations for the representation of time-varying mesh stiffness in three dimensions which becomes the foundation for the GMS computer program described in Appendix B. In a full three-dimensional analysis the single equation (1.1) expands to 144 of the form

$$k_{\alpha\beta}(t) = k^{o}_{\alpha\beta} + \sum_{n=1}^{4} (k^{n\sigma c}_{\alpha\beta} \cos n\sigma t + k^{n\sigma s}_{\alpha\beta} \sin n\sigma t) \qquad (1.2)$$

where α and β each take on the twelve values among x, y, and z translational and rotational motions of gears 1 and 2 (3x2x2 = 12).

The current work deals with the 36 of these 144 equations for spur gearing involving only the shaft rotational and two transverse directions for the two shafts. Future work expanding treatment to helical and other gearing will generate the remaining 108 equations which additionally involve shaft axial and transverse rotational directions.

Gear-tooth machining errors appearing at tooth-passing frequency and integer multiples will be accounted for by

$$F^{\sigma}(t) = \sum_{n} (F^{n\sigma c} \cos n\sigma t + F^{n\sigma s} \sin n\sigma t) \tag{1.3}$$

and errors appearing at other frequencies will be accounted for by

$$G^{\gamma}(t) = \sum_{j} (G^{\gamma_j c} \cos \gamma_j t + G^{\gamma_j s} \sin \gamma_j t) \tag{1.4}$$

Dynamic Response of Geared-Rotor Systems

The second area of concentration in the current work is the development of a generalized computer program for dynamic analysis of geared systems. Chapter 2 develops extensive modifications to the classical transfer matrix method for this purpose.

The Geared Rotor Solver (GRS) computer program of Appendix C utilizes this development to generate the particular solution to a set of linear, periodic coefficient differential equations. This solution includes the full three-dimensional six degrees-of-freedom of internal force and displacemental response. The state vector carries all frequencies (necessitated by the parametric mesh excitation) and all shafts simultaneously.

One final introductory note is in order with regard to the periodic stiffness, geared-rotor problem. The most fundamental scalar equivalent mathematically is the well known Mathieu problem whose solution has regions of stability and instability. The current work does not deal directly with this instability, assuming all gearbox dynamic response to be linear and stable. Appendix D addresses the matter of Mathieu instability as it relates to the geared-rotor problem, concluding, probablistically, that the physical parameters involved render real systems free of this instability.

1.2 Literature Review

General computer programs for three-dimensional dynamic response of multishaft geared systems are largely unreported in the literature.

In 1977 Lund [1]* revealed his development of a general technique involving influence coefficients placed at mesh locations. This reportedly uncoupled the system for transfer matrix solution of each rotor independently. The technique is not three-dimensional nor multifrequencied. Hence, physical and frequency branching phenomena, discussed in the next chapter, are not included. Mesh stiffness is treated as constant by Lund, whereas the current work will demonstrate that the effect of mesh-stiffness variations on response is significant.

Sciarra, et al [2, 3] in 1978 and Hartman [4, 5] reported the development of elaborate computer codes aimed at the prediction of dynamics of gearboxes of heavy-lift helicopters due to problems experienced by the U.S. Army. The analysis was torsional only and hence physical branching, i.e., torsional/lateral/axial dynamic interaction is not included.

*Numbers in brackets designate references at the end of the dissertation.

Benton and Seireg [6, 7, 8, 9] recognized the significant effect of non-constant gear-mesh stiffness on the dynamics of rotor systems. Digital phase/plane analyses were performed on torsional gear/pinion systems represented by the scalar Mathieu equation. Comparison with experimental test rig data demonstrated the effect of periodic stiffness on response for very simple geared rotors and did not lead to a general solution technique.

Recently Daws [10] and Mitchell [11] explored and developed the physical and frequency branching techniques which have led to the modified transfer matrices employed in the current effort. The ability of the method to perform a three-dimensional, multifrequency dynamic analysis on a multishafted compressor drive train with periodic mesh stiffness was demonstrated. Of significant value also is Daws' development of the general form of the modified transfer matrix for the three-dimensional inertial rigid disk including unbalance, wobble, and gyroscopic effects.

Among recommendations for further work, Daws [10] cited the gear-mesh stiffness representation. Researchers exploring this topic, notably Rebbechi [12] and Crisp [13], developed closed-form empirical relations for tooth deformation, and hence, stiffness which include bending, shear, and compressive modes of deflection.

1.3 The Transfer Matrix Method

Pestel and Leckie [14] provide an excellent theoretical and applied mechanics reference in the transfer matrix method. The classical method establishes a transfer matrix relating the force-response states, or state vectors, on both sides of an element. This is done for each element and the matrices are multiplied to yield a global system matrix. Inherently the solution proceeds unidirectionally, and hence the technique is also called the line solution method.

Observe the simple oscillator of Figure (1.3). The state vector involves the deflection, x(t), the internal force, N(t), and an extension location to accomodate any externally applied forces. This yields the state-vector definition

$$\underline{q} = \begin{Bmatrix} x \\ N \\ \underline{1} \end{Bmatrix} \qquad (1.5)$$

For the stiffness element, 1, the equations

$$x_1^R = x_1^L + N_1^L \frac{1}{k} \qquad (1.6)$$

$$N_1^R = N_1^L \qquad (1.7)$$

lead to the extended stiffness transfer matrix

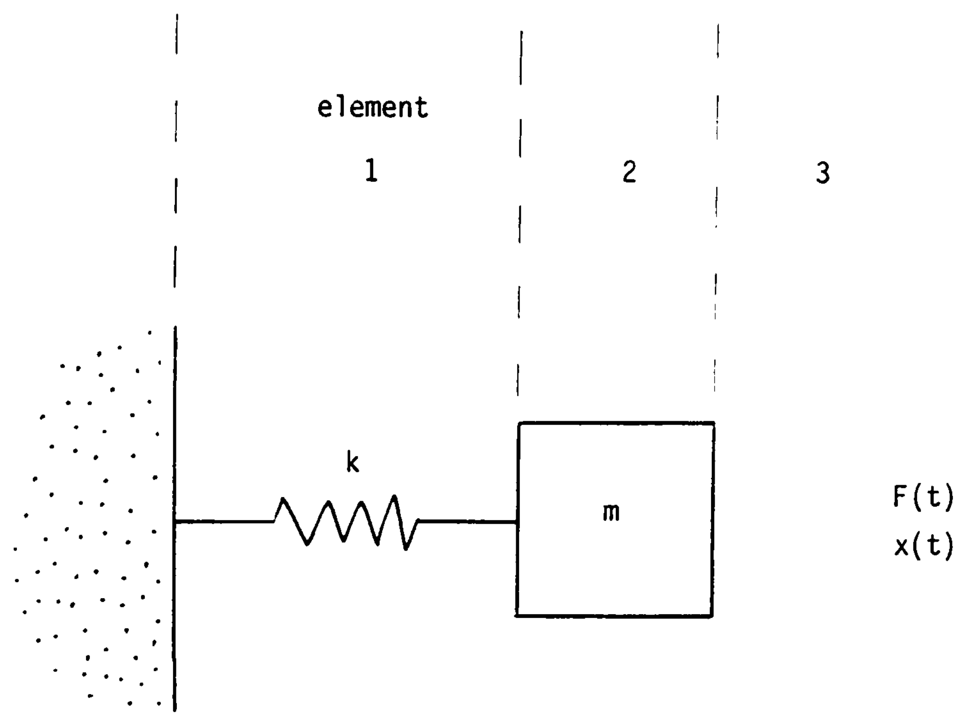


Figure 1.3 Simple Oscillator

$$\underline{q}_1^R = \left[\begin{array}{cc|c} 1 & 1/k & 0 \\ 0 & 1 & 0 \\ \hline 0 & 0 & 1 \end{array}\right] \underline{q}_1^L = \underline{K}\,\underline{q}_1^L \tag{1.8}$$

By assuming a harmonic response at frequency ω, a force balance done on element 2 yields

$$x_2^R = x_2^L \tag{1.9a}$$

$$N_2^R = N_2^L - m\omega^2 x_2^L \tag{1.9b}$$

In transfer matrix form Equations (1.9) become

$$\underline{q}_2^R = \left[\begin{array}{cc|c} 1 & 0 & 0 \\ -m\omega^2 & 1 & 0 \\ \hline 0 & 0 & 1 \end{array}\right] \underline{q}_2^L = \underline{M}\,\underline{q}_2^L \tag{1.10}$$

If an external force, $F(t) = F^{\omega} \cos \omega t$, applied to the mass, is added and denoted element 3, then its transfer matrix is

$$\underline{q}_3^R = \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & -F^{\omega} \\ \hline 0 & 0 & 1 \end{array}\right] \underline{q}_3^L = \underline{F}\,\underline{q}_3^L \tag{1.11}$$

By observing that $\underline{q}_2^L = \underline{q}_1^R$ and $\underline{q}_3^L = \underline{q}_2^R$, Equations (1.8), (1.10), and (1.11) may be combined to yield the system global transfer matrix equation

$$\underline{q}_3^R = \underline{F}\,\underline{M}\,\underline{K}\,\underline{q}_1^L = \left[\begin{array}{cc|c} 1 & 1/k & 0 \\ -m\omega^2 & 1 - m\omega^2/k & -F^{\omega} \\ \hline 0 & 0 & 1 \end{array}\right] \underline{q}_1^L \tag{1.12}$$

By applying boundary conditions

$$x_1^L = 0$$
$$N_3^R = 0 \tag{1.13}$$

Equation (1.12) may be solved for the unknown left side state-vector quantity

$$N_1^L = F^\omega (1 - m\omega^2/k)^{-1} \tag{1.14}$$

Successive substitution of the now known left-end state vector, $\underline{q}_1^L$, into Equations (1.8), (1.10), and (1.11) yield the states throughout the system

$$\begin{aligned}
\underline{q}_1^R &= \left\{ F^\omega (1 - m\omega^2/k)^{-1} \quad F^\omega (1 - m\omega^2/k)^{-1} \;\middle|\; 1 \right\}^T \\
\underline{q}_2^R &= \left\{ F^\omega (k - m\omega^2)^{-1} \quad F^\omega \;\middle|\; 1 \right\}^T \\
\underline{q}_3^R &= \left\{ F^\omega (k - m\omega^2)^{-1} \quad 0 \;\middle|\; 1 \right\}^T
\end{aligned} \tag{1.15}$$

The concepts for obtaining state-vector form and transfer matrix content remain the same for higher order systems. In three-dimensional analyses the six degress of freedom of translation and rotation replace the x_i of the above example. Six internal forces in these directions replace N_i. In geared systems, many shafts and many frequencies are carried simultaneously in the state vector in the modified transfer matrix method.

2. THE MODIFIED TRANSFER MATRIX METHOD

2.1 The Concept of Physical Branching

The example of Section 1.3 illustrates the transfer matrix method of building a system global matrix by multiplication in a "line" fashion. Historically, this has caused difficulty in applying the method to branched systems, such as the geared-rotor problem.

Recent work of Hibner [15], however, in analysis of multispooled gas turbine engines is gratefully acknowledged for its contribution to the state-of-the-art in transfer matrix analysis of branched systems. Daws [10] initially addressed the task of adapting the technique to geared shafting and Mitchell [11] was finally successful at establishing an elegant technique. To illustrate this method, observe Figure 2.1. For simplicity, consider shaft rotation as the response variable and let k represent the translational mesh stiffness along the line-of-action. By noting the sign conventions of Figure 2.1, the right side states are established as

$$\theta_1^R = \theta_1^L$$

$$\theta_2^R = \theta_2^L$$

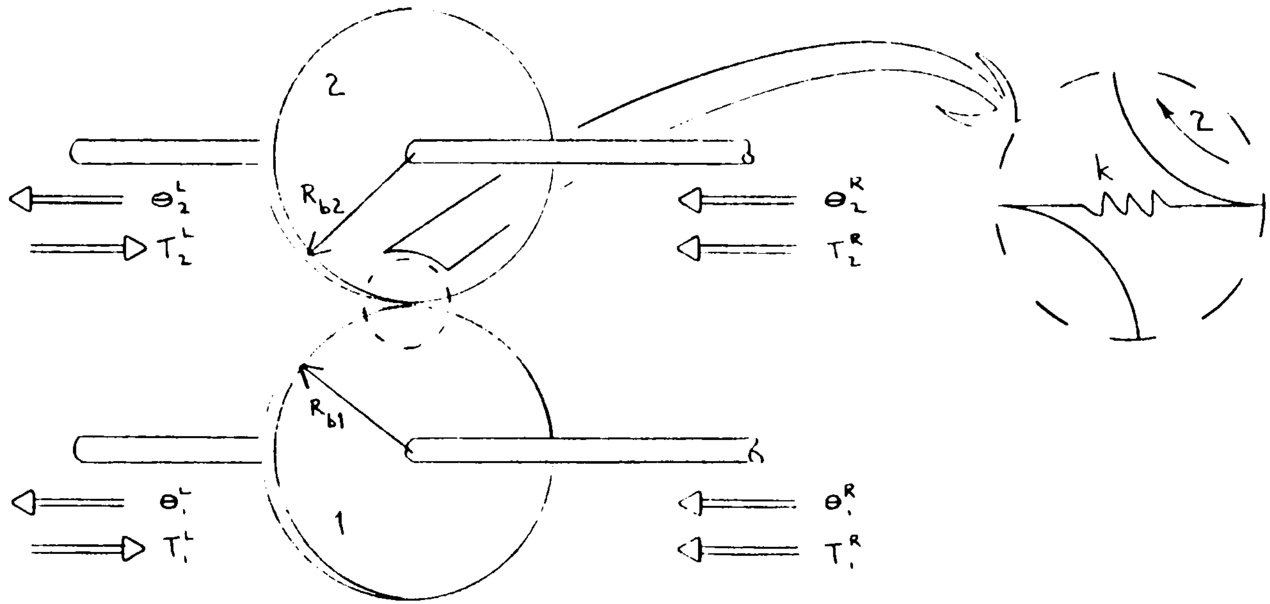


Figure 2.1 Physical Branching Conceptual Model For Axial Rotational Motion

$$T_1^R = T_1^L + k\theta_1^L R_{b1}^2 + k\theta_2^L R_{b1} R_{b2}$$

$$T_2^R = T_2^L + k\theta_2^L R_{b2}^2 + k\theta_1^L R_{b1} R_{b2}$$

These equations may be expressed in the double-sized transfer matrix form

$$\underline{q}^R = \begin{Bmatrix} \theta_1 \\ T_1 \\ \theta_2 \\ T_2 \\ \hline 1 \end{Bmatrix}^R = \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ kR_{b1}^2 & 1 & kR_{b1}R_{b2} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ kR_{b1}R_{b2} & 0 & kR_{b2}^2 & 1 & 0 \\ \hline 0 & 0 & 0 & 0 & 1 \end{array}\right] \underline{q}^L \qquad (2.1)$$

The significance of the Hibner Branching Method of simultaneously carrying multishaft state-vector information is the elimination of cumbersome equivalencing techniques. A more rigorous development of the physical branching method, including techniques in analytical solution methods is provided by Mitchell [11].

Computer storage problems associated with larger systems of equations is severe in this class of problems and is made worse due to inherent ill-conditioning and non-bandedness of the matrices. A suitable matrix solver could not be identified which circumvented these problems so one was developed in the current work, see Appendix C.

2.2 The Concept of Frequency Branching

Recall from Chapter 1 that the mesh stiffness is time-varying at tooth-passing frequency. This section will demonstrate that periodic stiffness produces forces and hence, responses at an infinite number of harmonics of the frequency of that periodicity as well as sideband frequencies.

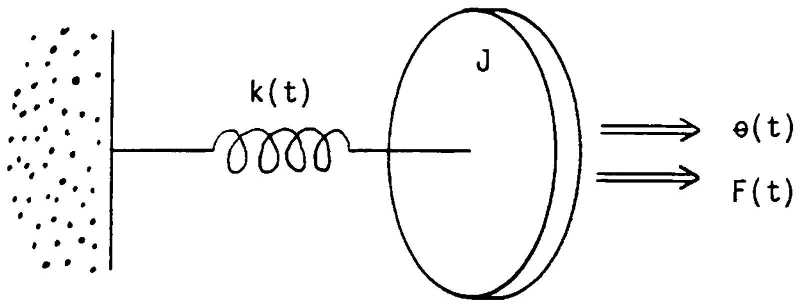
Consider the oscillator of Figure (2.2) as a single degree-of-freedom model of a simple geared system. The system stiffness reflects mesh stiffness which is periodic at tooth-passing frequency, σ; J represents system inertia; and excitation is D.C. and at shaft-running speed, ω.

The differential equation for this system is

$$J\ddot{\theta}(t) + (k^{O} + k^{\sigma} \cos \sigma t)\theta(t) = F^{O} + F^{\omega} \cos \omega t \tag{2.2}$$

whose solution can be shown to be

$$\theta(t) = \theta^{O} + \theta^{\omega} \cos \omega t + \sum_{m=1}^{\infty} (\theta^{m\sigma} \cos m\sigma t + \theta^{m\sigma+\omega} \cos (m\sigma+\omega)t + \theta^{m\sigma-\omega} \cos (m\sigma-\omega)t) \tag{2.3}$$



$$k(t) = k^{o} + k^{\sigma} \cos \sigma t$$

$$F(t) = F^{o} + F^{\omega} \cos \omega t$$

Figure 2.2 Rotational Oscillator with Time-Varying Stiffness

To verify the assumed particular solution, $\theta(t)$ is substituted into the differential equation. Differentiating $\theta(t)$ twice

$$\ddot{\theta}(t) = -\omega^2 \theta^{\omega} \cos \omega t - \sum_{m=1}^{\infty} [(m\sigma)^2 \, \theta^{m\sigma} \cos m\sigma t \tag{2.4}$$

$$+ (m\sigma+\omega)^2 \, \theta^{m\sigma+\omega} \cos (m\sigma+\omega)t + (m\sigma-\omega)^2 \, \theta^{m\sigma-\omega} \cos (m\sigma-\omega)t]$$

By multiplying $k(t)$ and $\theta(t)$ the second term of Equation (2.2) is

$$k(t)\,\theta(t) = k^{o}\theta(t) + k^{\sigma}\theta^{\omega} \cos \sigma t \cos \omega t$$

$$+ k^{\sigma} \sum_{m=1}^{\infty} [\theta^{m\sigma} \cos \sigma t \cos m\sigma t$$

$$+ \theta^{m\sigma+\omega} \cos \sigma t \cos (m\sigma+\omega)t$$

$$+ \theta^{m\sigma-\omega} \cos \sigma t \cos (m\sigma-\omega)t] \tag{2.5}$$

By using the trigonometric relation

$$\cos x \cos y = 1/2 \cos(x+y) + 1/2 \cos(x-y)$$

Equation (2.5) becomes

$$k(t)\theta(t) = k^{o}\theta(t) + 0.5\, k^{\sigma}\theta^{\omega} (\cos (\sigma+\omega)t + \cos (\sigma-\omega)t)$$

$$+ k^{\sigma} \sum_{m=1}^{\infty} [0.5\, \theta^{m\sigma} (\cos (m+1)\sigma t + \cos (m-1)\sigma t) \tag{2.6}$$

$$+ 0.5\theta^{m\sigma+\omega} (\cos ((m+1)\sigma+\omega)t + \cos ((m-1)\sigma-\omega)t]$$

$$+ 0.5\theta^{m\sigma-\omega} (\cos ((m+1)\sigma-\omega)t + \cos ((m-1)\sigma+\omega)t]$$

Thus Equation (2.2) is seen to involve forcing terms at all the frequencies of the assumed solution; D.C., ω, $m\sigma$, $m\sigma+\omega$, $m\sigma-\omega$, $m=1,2,\ldots,\infty$; and none additional.

Practically, of course, a tractable finite number of these frequencies must be identified for solution. Selection of frequencies of interest is discussed in the next section, see also Mitchell and Lynch [16].

The concept of "frequency branching", or "parametric excitation" as it is also called, is evident in Equation (2.6). Since a response at one frequency leads to forces and hence responses at other frequencies, all frequencies must be carried simultaneously in the state vector.

2.3 The Assumed Form of the State Vector

Truncation of the particular solution, Equation (2.3), necessary to effect a real-world numerical solution, occurs with respect to the harmonics of tooth-passing frequency, σ. In the current work frequencies involving tooth passing are the fundamental, second harmonic, and shaft-running speed and gear error* frequency sidebands. Where

n_ω = number of distinct shaft speeds
n_ϵ = number of distinct "error" frequencies
n_σ = number of distinct tooth-passing frequencies

then, for example, one of the degrees-of-freedom for shaft i can be expressed

$$
\begin{aligned}
w_{xi}(t) = {} & w_{xi}^{o} + \sum_{m=1}^{n_\omega} \left(w_{xi}^{\omega_m c} \cos \omega_m t + w_{xi}^{\omega_m s} \sin \omega_m t \right) \\
& + \sum_{p=1}^{n_\epsilon} \left(w_{xi}^{\epsilon_p c} \cos \epsilon_p t + w_{xi}^{\epsilon_p s} \sin \epsilon_p t \right) \\
& + \sum_{n=1}^{2} \sum_{q=1}^{n_\sigma} \left(w_{xi}^{n\sigma_q c} \cos n\sigma_q t + w_{xi}^{n\sigma_q s} \sin n\sigma_q t \right) \qquad (2.7) \\
& + \sum_{q=1}^{n_\sigma} \sum_{m=1}^{n_\omega} \left(w_{xi}^{\sigma_q \pm \omega_m c} \cos (\sigma_q \pm \omega_m) t + w_{xi}^{\sigma_q \pm \omega_m s} \sin (\sigma_q \pm \omega_m)\, t \right) \\
& + \sum_{q=1}^{n_\sigma} \sum_{p=1}^{n_\epsilon} \left(w_{xi}^{\sigma_q \pm \epsilon_p c} \cos (\sigma_q \pm \epsilon_p) t + w_{xi}^{\sigma_q \pm \epsilon_p s} \sin (\sigma_q \pm \epsilon_p)\, t \right)
\end{aligned}
$$

* "Error" frequencies are introduced in Section 3.3. Succinctly, they enable the modified transfer matrix method to deal with machining errors in gear teeth.

All terms of the state vector; w, θ, M and V; for the global coordinate x, y, and z directions are of the form of Equation (2.7). The state vector then takes the form

$$\underline{q} = \begin{Bmatrix} \underline{w}_{x1} \\ \underline{w}_{y1} \\ \underline{w}_{z1} \\ \underline{\theta}_{x1} \\ \underline{\theta}_{y1} \\ \underline{\theta}_{z1} \\ \underline{M}_{x1} \\ \underline{M}_{y1} \\ \underline{M}_{z1} \\ \underline{V}_{x1} \\ \underline{V}_{y1} \\ \underline{V}_{z1} \\ \underline{w}_{x2} \\ \underline{w}_{y2} \\ \underline{w}_{z2} \\ \underline{\theta}_{x2} \\ \vdots \\ \underline{V}_{z2} \\ \underline{w}_{x3} \\ \vdots \\ \underline{V}_{z3} \\ \vdots \\ \underline{w}_{xr} \\ \vdots \\ \underline{V}_{zr} \\ \hline 1 \end{Bmatrix} \qquad (2.8)$$

where, for example,

$$\underline{w}_{x1} = \left\{ \begin{matrix} w_{xi}^{o} \\ w_{x1}^{\omega_1 c} \\ w_{x1}^{\omega_1 s} \\ w_{x1}^{\omega_2 c} \\ \vdots \\ w_{x1}^{\omega_{n_\omega} s} \\ w_{x1}^{\epsilon_1 c} \\ \vdots \\ w_{x1}^{\epsilon_{n_\epsilon} s} \\ w_{x1}^{\sigma_1 c} \\ \vdots \\ w_{x1}^{2\sigma_{n_\sigma} s} \\ w_{x1}^{\sigma_1+\omega_1 c} \\ \vdots \\ w_{x1}^{\sigma_{n_\sigma}-\omega_{n_\omega} s} \\ w_{x1}^{\sigma_1+\epsilon_1 c} \\ \vdots \\ w_{x1}^{\sigma_{n_\omega}-\epsilon_{n_\epsilon} s} \end{matrix} \right\} \tag{2.9}$$

Where there are 'm' meshes and 'n' shafts, the frequencies involved in Equation (2.9) are

n_ω shaft-running speed frequencies, $n_\omega \leq n$*

n_ε error frequencies; $n_\varepsilon \leq \sum_{i=1}^{m} n_{\varepsilon i}$, where $n_{\varepsilon i}$ is the number of error frequencies associated with mesh i

n_σ tooth-passing frequencies, $n_\sigma \leq m$

$n_{\sigma \pm \omega}$ shaft-speed sidebands of tooth-passing frequencies; $n_{\sigma \pm \omega} \leq 4m$, $4m \Rightarrow 2$ sidebands for 2 shafts for each of m meshes

$n_{2\sigma}$ twice tooth-passing frequencies, $n_{2\sigma} \leq m$

$n_{\sigma \pm \varepsilon}$ error-frequency sidebands of tooth-passing frequencies; $n_{\sigma \pm \varepsilon} \leq 2 \sum_{i=1}^{m} n_{\varepsilon i}$, where $n_{\varepsilon i}$ is the number of error frequencies associated with mesh i

Thus, the vector Equation (2.9) contains $(2n_f+1)$ elements where

$$n_f = n_\omega + n_\varepsilon + n_\sigma + n_{\sigma \pm \omega} + n_{2\sigma} + n_{\sigma \pm \varepsilon}$$

---

* The inequality ($\leq$) exists since there otherwise may be duplication of frequencies; i.e., two shafts at the same speed.

The state-vector Equation (2.8) size then is

$$n_{sv} = 12\ (2n_f+1)n + 1$$

For example, a system of 4 shafts, 3 meshes, and one error frequency per mesh results in $n_f = 31$ non-zero frequencies and a state-vector size of 3025 with no duplication of frequencies.

3. THE GEAR MESH

The involute spur gear tooth is at the heart of the current periodic coefficient problem. Observe from Figure 1.2 that mesh stiffness is strongly a function of the number of instantaneously contacting tooth pairs and weakly a function of other factors such as load position. The analysis of this element and the development of the stiffness tensor

$$k_{\alpha\beta}(t) = k^{o}_{\alpha\beta} + \sum_{n=1}^{4} (k^{n\sigma c}_{\alpha\beta} \cos n\sigma t + k^{n\sigma s}_{\alpha\beta} \sin n\sigma t) \tag{1.2}$$

and error tensors

$$F^{\sigma}(t) = \sum_{n} (F^{n\sigma c} \cos n\sigma t + F^{n\sigma s} \sin n\sigma t) \tag{1.3}$$

$$G^{\gamma}(t) = \sum_{j} (G^{\gamma_j c} \cos \gamma_j t + G^{\gamma_j s} \sin \gamma_j t) \tag{1.4}$$

are discussed in this chapter.

3.1 Analysis of the Time-Varying Mesh-Stiffness Tensor

Figure 3.1 illustrates an involute gear-tooth profile with three straight-sided tapered cantilever approximations superimposed. Caldwell [17] proposed an approximation defined by a root thickness equal to that of the actual tooth at its point of maximum stress and a tangent to the tooth fillet at that point. Based on a comparison of analytics and measurement, Furrow [18] concluded that the pitch and addendum points should define the cantilever. Daws [10] allowed the addendum and base circle intersections with the involute to define the tooth model, citing better agreement with experimental work of Chabert [19] than the other two tooth models. The model used in the current work is that of Daws. To complete the definition, the model extends from the dedendum to the addendum of the actual tooth as shown in Figure 3.1.

It is necessary to establish the stiffness characteristics of the tooth. The mesh contact force, F of Figure 3.2, is resolved into components F_s and F_c in the x- and y- coordinate directions, respectively.

The area moment of inertia is,

$$I = I_{zz} = bh^3/12 \tag{3.1}$$

$$h = h_1 \; (1 + kx)$$

$$k = (h_o - h_1)/h_1L$$

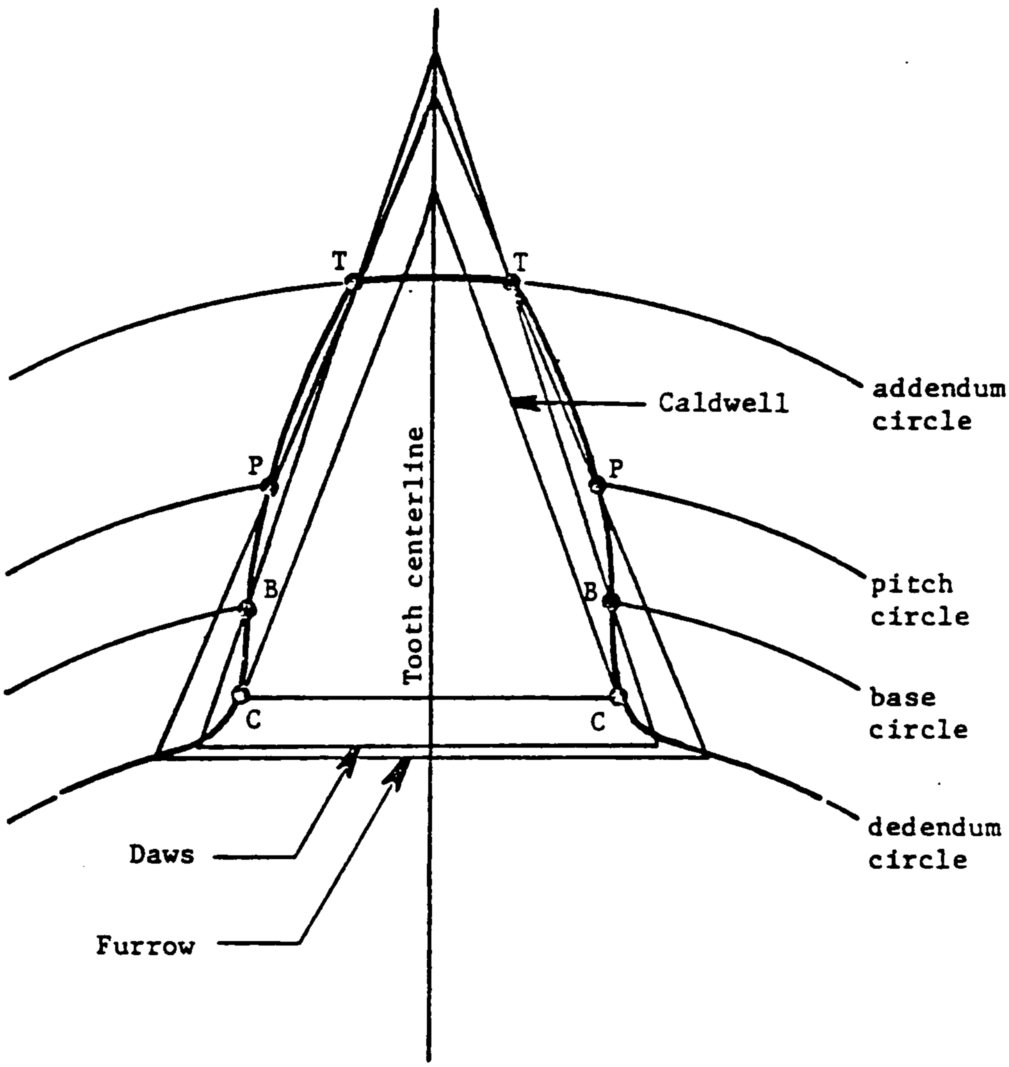


Figure 3.1 Gear Tooth Showing Various Tapered Cantilever Beam Representations. After Daws [10]

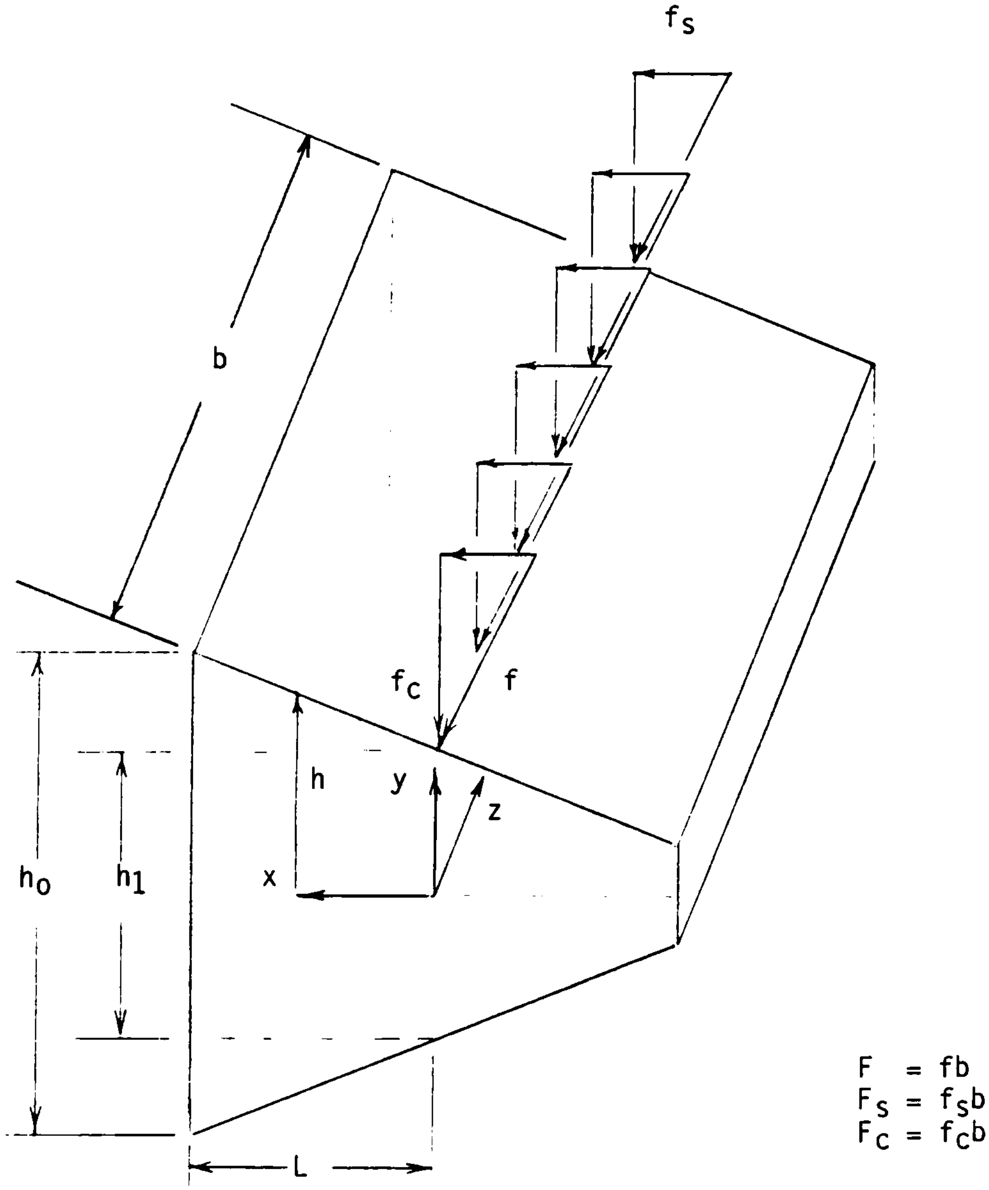


Figure 3.2 Tapered Cantilever Model of the Involute Spur Gear Tooth.

Four modes of deformation are considered in evaluating the flexibility of the model. Components due to bending, compression, and shear are evaluated by applying Castigliano's Theorem. Local contact mode, which is nonlinear with load, is linearized about the mean operating torque loading condition.

The moment caused by F is

$$M = M_z = F_s h_1/2 - F_c x \tag{3.2}$$

From Juvinall [20] the shear energy, dU_s, in a volume b(dx)dy is

$$dU_s = (\tau^2\, b/2G)\, dxdy \tag{3.3}$$

where, from Timoshenko [21]

$$\tau = \frac{3M}{bh^2} \frac{dh}{dx} + \frac{6}{b}\left(\frac{h^2}{4} - y^2\right) \frac{d(M/h^3)}{dx} \tag{3.4}$$

where h is defined in Equation (3.1). Noting that

$$\frac{M}{h^3} = \frac{F_s h_1\ /2 - F_c x}{h_1^3\ (1 + kx)^3}$$

$\bar{h}$ and $\bar{M}$ are defined as

$$\bar{h} = dh/dx = h_1 k$$

$$\overline{M} = \frac{d\,(M/h^3)}{dx} = \frac{3F_c kx}{h_1^3\,(1+kx)^4} - \frac{F_c}{h_1^3\,(1+kx)^3} - \frac{3F_s k}{h_1^2\,(1+kx)^4} \tag{3.6}$$

The shear stress Equation (3.4) becomes

$$\tau = Py^2 + Q$$

where

$$P = -6\overline{M}/b$$

$$Q = \frac{3M\overline{h}}{bh^2} + \frac{3h^2\overline{M}}{2b} \tag{3.7}$$

and

$$\tau^2 = P^2y^4 + Q^2 + 2PQy^2 \tag{3.8}$$

The shear strain energy becomes

$$U_s = \frac{b}{G}\int_0^L\int_0^{h/2} \tau^2\, dydx = \frac{b}{G}\int_0^L \left[\left(\frac{A^2y^5}{5} + \frac{2ABy^3}{3} + B^2y\right)\Big|_0^{h/2}\right]dx \tag{3.9}$$

By defining

$$A' = \frac{bP^2h^5}{160G} = \frac{9h^5\overline{M}}{2b}$$

$$B' = \frac{PQbh^3}{12G} = \frac{-3h\overline{h}M\overline{M}}{2bG} - \frac{3h^5\overline{M}^2}{4bG} \tag{3.10}$$

$$C' = \frac{Q^2bh}{2G} = \frac{9M^2\overline{h}^2}{2bGh^3} + \frac{9h^5\overline{M}^2}{8bG} + \frac{9h\overline{h}M\overline{M}}{2bG}$$

the shear strain energy becomes

$$U_s = \int_0^L (A' + B' + C')\, dx. \tag{3.11}$$

Expanding terms from Equations (3.10) yields

$$\begin{aligned}\overline{M}^2 = {} & 9F_c^2\, k^2\, h_1^{-6}\, x^2\, (1 + kx)^{-8} + F_c^2\, h_1^{-6}\, (1 + kx)^{-6} \\ & + 2.25\, F_s^{\,2}\, k^2\, h_1^{-4} (1 + kx)^{-8} - 6F_c^2\, kh_1^{-6}\, x\, (1 + kx)^{-7} \\ & - 9F_cF_s\, k^2 h_1^{-5}\, x\, (1 + kx)^{-8} + 3\, F_cF_s\, kh_1^{-5}\, (1 + kx)^{-7}\end{aligned} \tag{3.12}$$

$$\begin{aligned}h^5\overline{M}^2 = {} & 9F_c k^2\, h_1^{-1}\, x^2\, (1 + kx)^{-3} + F_c^2\, h_1^{-1}\, (1 + kx)^{-1} \\ & + 2.25\, F_s^2\, k^2\, h_1 (1 + kx)^{-3} - 6\, F_c^2\, kh_1^{-1}\, x\, (1 + kx)^{-2} \\ & - 9F_cF_s\, k^2\, x\, (1 + kx)^{-3} + 3F_cF_s k\, (1 + kx)^{-2}\end{aligned} \tag{3.13}$$

$$\begin{aligned}M^2 h^{-3}\overline{h}^{\,2} = {} & F_c^2\, k^2 h_1^{-1}\, x^2\, (1 + kx)^{-3} - F_cF_s k^2\, x\, (1 + kx)^{-3} \\ & + 0.25\, F_s^2\, k^2 h_1\, (1 + kx)^{-3}\end{aligned} \tag{3.14}$$

$$\begin{aligned}h\overline{h}M\overline{M} = {} & 1.5\, F_cF_s\, k^2 x\, (1 + kx)^{-3} - 0.5F_cF_s k\, (1 + kx)^{-2} \\ & -0.75\, F_s^2\, k^2\, h_1\, (1 + kx)^{-3} - 3F_c^2\, k^2 h_1^{-1}\, x^2\, (1 + kx)^{-3} \\ & -F_c^2\, kh_1^{-1}\, x\, (1 + kx)^{-2} + 1.5F_cF_s k^2 x\, (1 + kx)^{-3}\end{aligned} \tag{3.15}$$

Equations (3.11) through (3.15) lead to

$$A' + B' + C' = \frac{3F_c^2}{5bGh_1}\ (1 + kx)^{-1} + \frac{3F_cF_sk}{10bG}\ (1 + kx)^{-2}$$

$$- \frac{3F_c^2k\ x\ (1 + kx)^{-2}}{5bGh_1} + \frac{9F_s^2\ k^2h_1(1 + kx)^{-3}}{40bG} \tag{3.16}$$

$$- \frac{9F_cF_sk^2\ x\ (1 + kx)^{-3}}{10bG} + \frac{9F_c^2k\ x^2\ (1 + kx)^{-3}}{10bGh_1}$$

Combining Equations (3.11) and (3.16) and evaluating yields the shear strain energy

$$U_s = a_s\ F_c^{\ 2} + b_s\ F_cF_s + c_sF_s^{\ 2} \tag{3.17}$$

$$a_s = \frac{3}{5bGh_1k}\left(\frac{3\ln\ (1+kL)}{2} - \frac{3}{4\ (1+kL)^2} + \frac{2}{(1+kL)} - \frac{5}{4}\right)$$

$$b_s = \frac{3}{10bG}\left(\frac{-3}{2\ (1+kL)^2} + \frac{2}{(1+kL)} - \frac{1}{2}\right)$$

$$c_s = \frac{9h_1k}{80bG}\left(1 - \frac{1}{(1+kL)^2}\right)$$

The bending strain energy is

$$U_b = \int_o^L (M^2/2EI)\ dx \tag{3.18}$$

$$M^2 = F_c^2\ x^2 - F_cF_sh_1x + F_s^2\ h_1^2\ /4$$

Combining Equation (3.18) with Equation (3.1) and evaluating yields

$$U_b = a_b\, F_c^{\,2} + b_b\, F_c F_s + c_b\, F_s^{\,2} \tag{3.19}$$

$$a_b = \frac{6}{Eb(h_1 k)^3} \left(\ln(1+kL) - \frac{1}{2(1+kL)^2} + \frac{2}{(1+kL)} - \frac{3}{2} \right)$$

$$b_b = \frac{-6}{Eb(h_1 k)^2} \left(\frac{1}{2(1+kL)^2} - \frac{1}{(1+kL)} + \frac{1}{2} \right)$$

$$c_b = \frac{3}{4Ebh_1 k} \left(1 - (1+kL)^{-2} \right)$$

The compressive strain energy is

$$U_c = \int_0^L \frac{F_s^{\,2}}{2EA}\, dx \tag{3.20}$$

Evaluated, this expression becomes

$$U_c = c_c\, F_s^{\,2} \tag{3.21}$$

$$c_c = \frac{\ln(1+kL)}{2Ebh_1 k}$$

The total strain energy in the tooth caused by the mesh contact force, F, due to shear, bending, and compression is

$$U_T = U_S + U_B + U_C = AF_c^2 + BF_c F_s + CF_s^2 \tag{3.22}$$

$A = a_s + a_b$

$B = b_s + b_b$

$C = c_s + c_b + c_c$

The kinematic gear pair supports a load only along the line of action with any components tangent to the teeth resulting in slip. Hence, tooth stiffness lies only on the line-of-action. By applying Castigliano's Theorem the line-of-action deflection is

$$\delta_L = \partial U_T / \partial F \tag{3.23}$$

Introducing

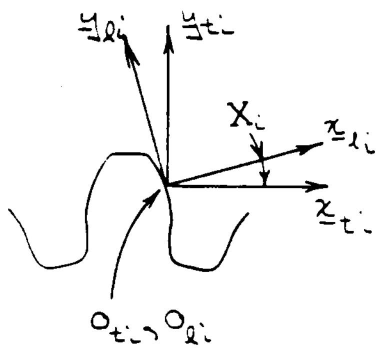
$$U_T = F^2(A \cos^2 X_i + B \cos X_i \sin X_i + C \sin^2 X_i) \tag{3.24}$$

where X_i locates the tooth coordinates in relation to the line-of-action coordinates as shown in Figure 3.3(a), Equation (3.23) becomes

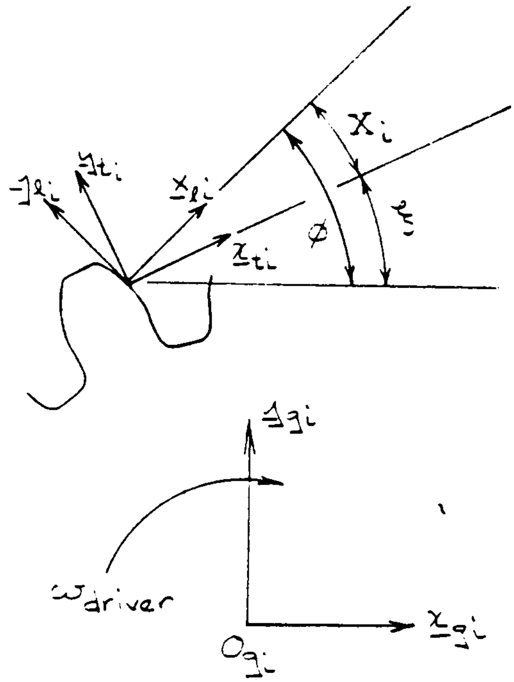
$$\delta_L = 2F\ (A \cos^2 X_i + B \cos X_i \sin X_i + C \sin^2 X_i) \tag{3.25}$$

Thus, tooth i compliance of gear j due to bending, shear, and tension may be represented by a plane truss (bar) member along the line-of-action of magnitude

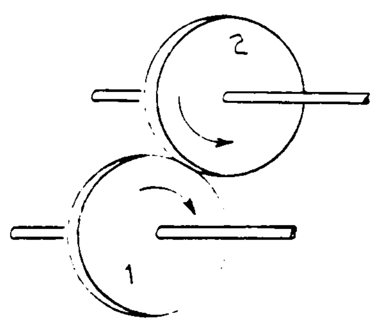
$$C^L_{ij} = \delta_L / F \Big|_{\substack{\text{tooth } i \\ \text{gear } j}} = 2\ A \cos^2 X_i + 2\ B \cos X_i \sin X_i + 2\ C \sin^2 X_i \tag{3.26}$$

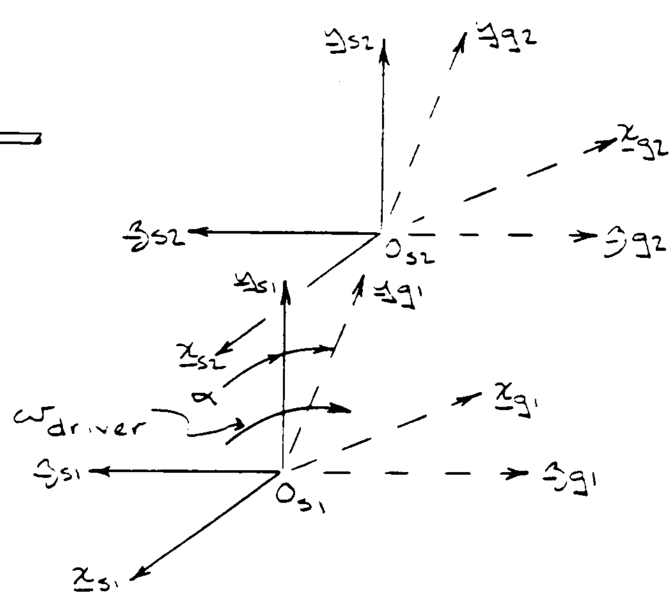


(a) Tooth (x_{ti}, y_{ti}) Coordinates and Line-of-Action ($x_{\ell i}$, $y_{\ell i}$) Coordinates



(b) Gear Coordinates (x_{gi}, y_{gi})





(c) System Global Coordinates (x_{si}, y_{si}, z_{si})

Figure 3.3 Coordinate Systems Used in the Analysis of the Gear Mesh

Contact mode tooth compliance may also be represented by a truss element along the line-of-action but is nonlinear with load. From Roark and Young [22] for two cylinders of contact radii R_{c1} and R_{c2}, axes parallel, pressed together with force, f, per unit length, their center distance is reduced by

$$\delta = (fH/\pi)\ [2/3 + \ln(4R_{c1}/b) + \ln(4R_{c2}/b)] \tag{3.27}$$

$$b \equiv 1.6\ (fK_d H)^{1/2}$$

$$K_d \equiv 2\ R_{c1}R_{c2}/(R_{c1} + R_{c2})$$

$$H \equiv (1-\mu_1^{\ 2})/E_1 + (1-\mu_2^{\ 2})/E_2$$

Furrow [18] investigated two assumptions regarding individual tooth contact deflection; 1) deflection is equally divided between the teeth for all points of contact, and 2) each tooth deflects inversely with its contact radius. The second assumption, originally suggested by Walker [23], is the more intuitively pleasing. However, Furrow discounted this assumption, citing Weber [24], contending that the contact area is not planar. Moreover, the first assumption displayed better agreement with experimental data, and therefore is used in the current work. Hence, each body, j, deflects according to

$$\delta_j = (fH/2\pi)\ [2/3 + \ln(4R_{c1}/b) + \ln(4R_{c2}/b)] \tag{3.28}$$

Contact mode compliance for tooth i of gear j linearized about the mean-operating deflection is evaluated according to

$$C_{ij}^{HC} = \partial\delta_j/\partial f\Big|_{\text{tooth } i} = (H/2\pi)\,[2/3 + \ln(4R_{c1}/b) + \ln(4R_{c2}/b)]$$

$$+ (FH/2\pi)\,(\partial/\partial f)\,[\ln(4R_{c1}/b) + \ln(4R_{c2}/b)] \qquad (3.29)$$

Employing

$$d(\ln(u(x))/dx = u(x)^{-1}\,\partial u(x)/\partial x$$

$$(\partial/\partial f)\,\ln(4R_{c1}/b) = -(2F)^{-1} \qquad (3.30)$$

then

$$C_{ij}^{HC} = (H/2\pi)\,(\ln(4R_{c1}/b) + \ln(4R_{c2}/b) - 1/3) \qquad (3.31)$$

The complete mesh-stiffness tensor, involving all contacting teeth, in line-of-action coordinates (x_ℓ, y_ℓ) of Figure 3.3(a) is desired of the form

$$\underline{\underline{K}} = \begin{bmatrix} k_{11}^{\ell} & k_{12}^{\ell} \\ k_{21}^{\ell} & k_{22}^{\ell} \end{bmatrix} \qquad (3.32)$$

The elements of this stiffness tensor may be evaluated by combining Equations (3.26) and (3.31) according to

$$k_{11}^{\ell} = \sum_{i}^{\substack{\text{Number of} \\ \text{Teeth}}} \; \sum_{j=1}^{2 \text{ Gears}} \left(C_{ij}^{L} + C_{ij}^{HC} \right)^{-1} \qquad (3.32a)$$

$$k_{12}^{\ell} = k_{21}^{\ell} = k_{22}^{\ell} = 0 \qquad (3.32b)$$

The direction cosines

$$\underline{\ell}^{\ell g} = \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix} \qquad (3.33)$$

are used to transform stiffness along the line-of-action coordinates to gear coordinates according to

$$k^{g}_{mn} = \sum_{p=1}^{2} \sum_{r=1}^{2} \ell^{\ell g}_{mp} \; \ell^{\ell g}_{nr} \; k^{\ell}_{pr} \, , \qquad \begin{matrix} m = 1,2 \\ n = 1,2 \end{matrix} \qquad (3.34)$$

This yields gear-tooth stiffness in gear coordinates of Figure 3.3(b) in the form

$$\underline{K}^{g} = \begin{bmatrix} k^{g}_{11} & k^{g}_{12} \\ k^{g}_{21} & k^{g}_{22} \end{bmatrix} \qquad (3.35)$$

The system global coordinate axes of Figure 3.3(c); ($\underline{x}_{si}$, $\underline{y}_{si}$, $\underline{z}_{si}$), i = 1,2; are associated with the driving (i=1) and driven (i=2) shafts of the geared system. The $\underline{z}_{si}$ axis is defined as the i^{th} shaft axial direction, and $\underline{x}_{si}$ normal to $\underline{z}_{si}$, both lying in the horizontal plane. The $\underline{y}_{si}$ axis is vertical such that the system obeys the right-hand rule. In the illustration of Figure 3.3(c) the global 0_{s1} system is that of the driving gear with its rotation in the positive $\underline{z}_{s1}$ direction. The angle α is defined as the angle between the positive $\underline{y}_{s1}$ axis and the line of centers $0_{s1}0_{s2}$ measured in the positive $\underline{z}_{s1}$ direction. The direction cosines for transforming the stiffness tensor in gear coordinates, $\underline{\underline{K}}^{g}$, to system global coordinates, $\underline{\underline{K}}^{q}$, are

$$\underline{\underline{\ell}}^{gp} = \begin{bmatrix} \ell_{11}^{gq} & \ell_{12}^{gq} \\ \ell_{21}^{gq} & \ell_{22}^{gq} \end{bmatrix} = \begin{bmatrix} n\cos\alpha & -\sin\alpha \\ n\sin\alpha & \cos\alpha \end{bmatrix} \qquad (3.36)$$

where n is +1 or -1 according to negative or positive driver rotation, respectively. The elements of $\underline{\underline{K}}^{qp}$ become

$$k_{ij}^{qp} = \sum_{m=1}^{2} \sum_{n=1}^{2} \ell_{im}^{gq} \ell_{jn}^{gq} k_{mn}^{g}, \quad \begin{aligned} i &= 1,2 \\ j &= 1,2 \\ p &= 1,2 \text{ contacting tooth pairs} \end{aligned} \qquad (3.37)$$

$\underline{\underline{K}}^{qp}$ is the translational stiffness tensor associated with the coordinate $(\underline{x}_{s1}, \underline{y}_{s1})$ directions. A full three-dimensional stiffness tensor includes the coordinate $\underline{z}_{si}$ direction and rotational as well as translational stiffnesses. The resulting 36 element (6x6) tensor for arbitrary spur gearing is the goal of the current three-dimensional geared-rotor analysis. Spur gear-stiffness elements exist only in the translational $\underline{x}_{si}, \underline{y}_{si}$ and rotational $\underline{z}_{si}$ directions. Though the rotational $\underline{x}_{si}, \underline{y}_{si}$, and translational $\underline{z}_{si}$ mesh stiffnesses are absent in the current analysis, the geared-rotor solution carries all six degrees of freedom allowing their inclusion as a result of future work. The stiffnesses involving rotational $\underline{z}_{si}$ motion are a function of the elements of the translational stiffness tensor, $\underline{\underline{K}}^{qp}$, and the mesh contact position.

Let $(x_{s1}^{qi}, y_{s1}^{qi})$ and $(x_{s2}^{qi}, y_{s2}^{qi})$ denote the mesh contact location in the $\underline{O}_{s1}$ and $\underline{O}_{s2}$ systems respectively. The superscript, i, ranges from 1 to m for the m contact locations in multiple tooth-pair portions of engagement. Allow $k_{\alpha\beta}^{s}$ to denote the 36 elements of the mesh-stiffness tensor $\underline{\underline{K}}^{s}$ where α and β take on the values xi, yi translational and zi rotational directions; i = gears 1,2.

For example, k_{x2z1}^{s} denotes the force horizontally through gear center 2 required to maintain unit positive rotation about gear center 1, all other displacements constrained to zero.

The equations relating the elements of $\underline{\underline{K}}^{s}$ to $\underline{\underline{K}}^{qp}$ and the mesh locations are

$$k_{x1x1}^{s} = -k_{x2x1}^{s} = -k_{x1x2}^{s} = k_{x2x2}^{s} = \sum_{i=1}^{m} k_{11}^{qi} \qquad (3.38\ a)$$

$$k_{x1y1}^{s} = -k_{x1y2}^{s} = -k_{x2y1}^{s} = k_{x2y2}^{s} = \sum_{i=1}^{m} k_{12}^{qi} \qquad (3.38\ b)$$

$$k_{y1x1}^{s} = -k_{y1x2}^{s} = -k_{y2x1}^{s} = k_{y2x2}^{s} = \sum_{i=1}^{m} k_{21}^{qi} \qquad (3.38\ c)$$

$$k_{y1y1}^{s} = -k_{y1y2}^{s} = -k_{y2y1}^{s} = k_{y2y2}^{s} = \sum_{i=1}^{m} k_{22}^{qi} \qquad (3.38\ d)$$

$$k_{z1x2}^{s} = -k_{z1x2}^{s} = \sum_{i=1}^{m} (-k_{11}^{qi} y_{s1}^{qi} + k_{21}^{qi}\ x_{s1}^{qi}\ ) \qquad (3.38\ e)$$

$$k^{s}_{z1y1} = -k^{s}_{z1y2} = \sum_{i=1}^{m} (k^{qi}_{22} x^{qi}_{s1} - k^{qi}_{12}\, y^{qi}_{s1}) \tag{3.38 f}$$

$$k^{s}_{z2x1} = -k^{s}_{z2x2} = \sum_{i=1}^{m} (k^{qi}_{11} y^{qi}_{s2} - k^{qi}_{21}\, x^{qi}_{s2}) \tag{3.38 g}$$

$$k^{s}_{z2y1} = -k^{s}_{z2y2} = \sum_{i=1}^{m} (-k^{qi}_{22} x^{qi}_{s2} + k^{qi}_{12}\, y^{qi}_{s2}) \tag{3.38 h}$$

$$k^{s}_{x1z1} = -k^{s}_{x2z1} = \sum_{i=1}^{m} (-k^{qi}_{11} y^{qi}_{s1} + k^{qi}_{12}\, x^{qi}_{s1}) \tag{3.38 i}$$

$$k^{s}_{x1z2} = -k^{s}_{x2z2} = \sum_{i=1}^{m} (k^{qi}_{11} y^{qi}_{s2} - k^{qi}_{12}\, x^{qi}_{s2}) \tag{3.38 j}$$

$$k^{s}_{y1z1} = -k^{s}_{y2z1} = \sum_{i=1}^{m} (k^{qi}_{22} x^{qi}_{s1} - k^{qi}_{21}\, y^{qi}_{s1}) \tag{3.38 k}$$

$$k^{s}_{y1z2} = -k^{s}_{y2z2} = \sum_{i=1}^{m} (-k^{qi}_{22} x^{qi}_{s2} + k^{qi}_{21}\, y^{qi}_{s2}) \tag{3.38 l}$$

$$\begin{aligned} k^{s}_{z1z1} = \sum_{i=1}^{m} (&k^{qi}_{11}\, y^{qi}_{s1}\, y^{qi}_{s1} - k^{qi}_{12}\, x^{qi}_{s1}\, y^{qi}_{s1} \\ &- k^{qi}_{21}\, y^{qi}_{s1}\, x^{qi}_{s1} + k^{qi}_{22}\, x^{qi}_{s1}\, x^{qi}_{s1}) \end{aligned} \tag{3.38 m}$$

$$\begin{aligned} k^{s}_{z1z2} = \sum_{i=1}^{m} (&-k^{qi}_{11}\, y^{qi}_{s2}\, y^{qi}_{s1} + k^{qi}_{12}\, x^{qi}_{s2}\, y^{qi}_{s1} \\ &+ k^{qi}_{21}\, y^{qi}_{s2}\, x^{qi}_{s1} - k^{qi}_{22}\, x^{qi}_{s2}\, x^{qi}_{s1}) \end{aligned} \tag{3.38 n}$$

$$\begin{aligned} k^{s}_{z2z1} = \sum_{i=1}^{m} (&-k^{qi}_{11}\, y^{qi}_{s2}\, y^{qi}_{s1} + k^{qi}_{12}\, x^{qi}_{s1}\, y^{qi}_{s2} \\ &+ k^{qi}_{21}\, y^{qi}_{s1}\, x^{qi}_{s2} - k^{qi}_{22}\, x^{qi}_{s2}\, x^{qi}_{s1}) \end{aligned} \tag{3.38 o}$$

$$k^{s}_{z2z2} = \sum_{i=1}^{m} (k^{qi}_{11}\, y^{qi}_{s2}\, y^{qi}_{s2} \;-\; k^{qi}_{12}\, x^{qi}_{s2}\, y^{qi}_{s2}$$

$$- k^{qi}_{21}\, y^{qi}_{s2}\, x^{qi}_{s2} \;+\; k^{qi}_{22}\, x^{qi}_{s2}\, x^{qi}_{s2}\,) \tag{3.38 p}$$

Since tooth stiffness is periodic at tooth-passing frequency, then evaluating Equations (3.38) at several discrete increments during one tooth passing characterizes mesh stiffness for all time. The $\underline{\underline{K}}^{s}$ tensor, for those discrete increments may be fit by a Fourier Series to yield stiffness expressed in the transcendental form of Equation (1.2) required by the dynamic response model.

3.2 The Gear Mesh Transfer Matrix

The mesh stiffness characterized in the last section is sufficient to establish the general form of the associated transfer matrix.

Consider an arbitrary pair of gears, i and j, of a geared system, and the general form of the state vector

$$\underline{q} = \left\{ \begin{array}{c} \underline{w}_{x1} \\ \underline{w}_{y1} \\ \underline{w}_{z1} \\ \underline{\theta}_{x1} \\ \underline{\theta}_{y1} \\ \underline{\theta}_{z1} \\ \underline{M}_{x1} \\ \underline{M}_{y1} \\ \underline{M}_{z1} \\ \underline{V}_{x1} \\ \underline{V}_{y1} \\ \underline{V}_{z1} \\ \underline{w}_{x2} \\ \underline{w}_{y2} \\ \vdots \\ \underline{V}_{z2} \\ \underline{w}_{x3} \\ \vdots \\ \underline{V}_{z3} \\ \vdots \\ \underline{w}_{xn} \\ \vdots \\ \underline{V}_{zn} \\ \hline 1 \end{array} \right\} \tag{2.8}$$

where, for example

$$\underline{w}_{x1} = \begin{Bmatrix} w_{xi}^{o} \\ w_{x1}^{\omega_1 c} \\ w_{x1}^{\omega_1 s} \\ w_{x1}^{\omega_2 c} \\ \vdots \\ w_{x1}^{\omega_r s} \\ w_{x1}^{\varepsilon_1 c} \\ \vdots \\ w_{x1}^{\varepsilon_s s} \\ w_{x1}^{\sigma_1 c} \\ \vdots \\ w_{x1}^{2\sigma_t s} \\ w_{x1}^{\sigma_1+\omega_1 c} \\ \vdots \\ w_{x1}^{\sigma_t-\omega_r s} \\ w_{x1}^{\sigma_1+\varepsilon_1 c} \\ \vdots \\ w_{x1}^{\sigma_t-\varepsilon_s s} \end{Bmatrix} \tag{2.9}$$

The general form of the extended transfer matrix equation for the gear mesh is given by Equation (3.39).

$$\underline{g}^R = \left\{ \begin{array}{c} \underline{W}_{x1} \\ \vdots \\ \underline{V}_{z\,i-1} \\ \underline{W}_{xi} \\ \underline{W}_{yi} \\ \underline{W}_{zi} \\ \underline{\Theta}_{xi} \\ \underline{\Theta}_{yi} \\ \underline{\Theta}_{zi} \\ \underline{M}_{xi} \\ \underline{M}_{yi} \\ \underline{M}_{zi} \\ \underline{V}_{xi} \\ \underline{V}_{yi} \\ \underline{V}_{zi} \\ \underline{W}_{x\,i+1} \\ \vdots \\ \underline{V}_{z\,j-1} \\ \underline{W}_{xj} \\ \underline{W}_{yj} \\ \underline{W}_{zj} \\ \underline{\Theta}_{xj} \\ \underline{\Theta}_{yj} \\ \underline{\Theta}_{zj} \\ \underline{M}_{xj} \\ \underline{M}_{yj} \\ \underline{M}_{zj} \\ \underline{V}_{xj} \\ \underline{V}_{yj} \\ \underline{V}_{zj} \\ \underline{W}_{x\,j+1} \\ \vdots \\ \underline{V}_{zn} \\ 1 \end{array} \right\} = \left[\begin{array}{cccccccc|c} I & & & & & & & & \\ & \ddots & & & & & & & \\ & & I & & & & & & \\ A_{z_ix_i}\,A_{z_iy_i} & A_{z_iz_i} & & I & & A_{z_ix_j}\,A_{z_iy_j} & A_{z_iz_j} & & F_{z_i} \\ A_{x_ix_i}\,A_{x_iy_i} & A_{x_iz_i} & & & I & A_{x_ix_j}\,A_{x_iy_j} & A_{x_iz_j} & & F_{x_i} \\ A_{y_ix_i}\,A_{y_iy_i} & A_{y_iz_i} & & & & A_{y_ix_j}\,A_{y_iy_j} & A_{y_iz_j} & & F_{y_i} \\ & & & & \ddots & & & & \\ A_{z_jx_i}\,A_{z_jy_i} & A_{z_jz_i} & & & & A_{z_jx_j}\,A_{z_jy_j} & A_{z_jz_j} & I & F_{z_j} \\ A_{x_jx_i}\,A_{x_jy_i} & A_{x_jz_i} & & & & A_{x_jx_j}\,A_{x_jy_j} & A_{x_jz_j} & I & F_{x_j} \\ A_{y_jx_i}\,A_{y_jy_i} & A_{y_jz_i} & & & & A_{y_jx_j}\,A_{y_jy_j} & A_{y_jz_j} & I & F_{y_j} \\ & & & & & & & \ddots\; I & \\ \hline & & & & & & & & 1 \end{array} \right] \cdot \underline{g}^L \qquad (3.39)$$

The thirty-six (n_f x n_f) block stiffness submatrices, $A_{\alpha\beta}$, and the six (n_f x 1) extension column force vectors, F_α, contain the only non-zero, non-unity elements in Equation (3.39). The vectors F_α are described in the next section. The form of the submatrices $A_{\alpha\beta}$ are identical to one another. They are established by carrying out the multiplication of stiffness and assumed response yielding the desired mesh force as done in the elementary examples of Sections 1.3 and 2.2. The general form of $A_{\alpha\beta}$ is given in Equation (3.41) using the submatrix A_{zixj} for illustration where line "j" is the driver. Consistent with Equation (1.2), the mesh stiffness relating line i, shaft rotational (z_i) and line j, x translational (x_j) motion is

$$k_{z2x1}\,(t) = k^{o}_{z2x1} + \sum_{n=1}^{4} (k^{n\sigma c}_{z2x1} \cos n\sigma_{ij}\, t + k^{n\sigma s}_{z2x1} \sin n\sigma_{ij}\, t) \quad (3.40)$$

where non-zero elements of Equation (3.41) are defined according to

$$a^{o} = k^{o}_{z2x1}$$

$$a^{c} = (1/2)\; k^{\sigma c}_{z2x1}$$

$$a^{s} = (1/2)\; k^{\sigma s}_{z2x1}$$

$$
A_{j;x_j} = \begin{array}{l}
M^{0}_{j;i} \\ \vdots \\ M^{\omega_i;c} \\ M^{\omega_i;s} \\ \vdots \\ M^{\omega_j;c} \\ M^{\omega_j;s} \\ \vdots \\ M^{\sigma_n;c} \\ M^{\sigma_n;s} \\ \vdots \\ M^{\epsilon p_i;c} \\ M^{\epsilon p_i;s} \\ \vdots \\ M^{\epsilon p_j;c} \\ M^{\epsilon p_j;s} \\ \vdots \\ M^{(\sigma+\omega)_i;c} \\ M^{(\sigma+\omega)_i;s} \\ M^{(\sigma-\omega)_i;c} \\ M^{(\sigma-\omega)_i;s} \\ \vdots \\ M^{(\sigma+\omega)_j;c} \\ M^{(\sigma+\omega)_j;s} \\ M^{(\sigma-\omega)_j;c} \\ M^{(\sigma-\omega)_j;s} \\ \vdots \\ M^{2\sigma_n;c} \\ M^{2\sigma_n;s} \\ \vdots \\ M^{(\sigma+\epsilon)p_i;c} \\ M^{(\sigma+\epsilon)p_i;s} \\ M^{(\sigma-\epsilon)p_i;c} \\ M^{(\sigma-\epsilon)p_i;s} \\ \vdots \\ M^{(\sigma+\epsilon)p_j;c} \\ M^{(\sigma+\epsilon)p_j;s} \\ M^{(\sigma-\epsilon)p_j;c} \\ M^{(\sigma-\epsilon)p_j;s}
\end{array}
\left[\begin{array}{ccccccccccc}
a^0 \cdots & & & a^c\ a^s & & & & & & & \\
& a^0\ a^0 \cdots & & & & & \begin{smallmatrix} a^c & a^s & a^c & a^s \\ -a^s & a^c & a^s & -a^c \end{smallmatrix} & & & & \\
& & a^0\ a^0 \cdots & & & & & \begin{smallmatrix} a^c & a^s & a^c & a^s \\ -a^s & a^c & a^s & -a^c \end{smallmatrix} & & & \\
\begin{smallmatrix} 2a^c \\ 2a^s \end{smallmatrix} & & & a^0\ a^0 \cdots & & & & & \begin{smallmatrix} a^c & a^s \\ -a^s & a^c \end{smallmatrix} & & \\
& & & & a^0\ a^0 \cdots & & & & & \begin{smallmatrix} a^c & a^s & a^c & a^s \\ -a^s & a^c & a^s & -a^c \end{smallmatrix} & \\
& & & & & a^0\ a^0 \cdots & & & & & \begin{smallmatrix} a^c & a^s & a^c & a^s \\ -a^s & a^c & a^s & -a^c \end{smallmatrix} \\
& \begin{smallmatrix} a^c & -a^s \\ a^s & a^c \\ a^c & a^s \\ a^s & -a^c \end{smallmatrix} & & & & & a^0\ a^0\ a^0\ a^0 \cdots & & & & \\
& & \begin{smallmatrix} a^c & -a^s \\ a^s & a^c \\ a^c & a^s \\ a^s & -a^c \end{smallmatrix} & & & & & a^0\ a^0\ a^0\ a^0 \cdots & & & \\
& & & \begin{smallmatrix} a^c & -a^s \\ a^s & a^c \end{smallmatrix} & & & & & a^0\ a^0 \cdots & & \\
& & & & \begin{smallmatrix} a^c & -a^s \\ a^s & a^c \\ a^c & a^s \\ a^s & -a^c \end{smallmatrix} & & & & & a^0\ a^0\ a^0\ a^0 \cdots & \\
& & & & & \begin{smallmatrix} a^c & -a^s \\ a^s & a^c \\ a^c & a^s \\ a^s & -a^c \end{smallmatrix} & & & & & a^0\ a^0\ a^0\ a^0
\end{array}\right] \qquad (3.41)
$$

3.3 Machining Errors in Gear Teeth

Machining errors in gear teeth are often lumped into three categories; runout, pitch, and profile. Runout is due to variations in the distance from the gear center of rotation to the teeth. In meshing gears this error manifests itself in angular velocity variations at the running-speeds of the gears. Pitch error, due to variations in tooth spacing, shows up at integer multiples of the running speeds. Profile error is any irregularity of the tooth surface compared to the ideal involute shape. Commonly repeated on successive teeth of a gear, this error is evident at tooth-passing frequency and its integer multiples.

Rotative speed variations in lightly contacting gears may be seen at frequencies other than running-speed, tooth passing and their multiples. Examples of this are errors mapped into the gear being cut by either the indexing or cutting gears in the fellows or hobbing processes, respectively. The four types of machining errors recognized in the current work are summarized in Table 3.1.

Gear errors may be crudely estimated from American Gear Manufacturers Association (AGMA) gear quality classification but preferably, are evaluated experimentally. This section discusses the general procedure for experimentally determining gear errors but a rigorous treatment is beyond the current scope, see Robinson [25].

Gear Error Type	Frequencies of Speed Variations in Lightly Contacting Gears
Run-Out	Gear-Running Speeds
Pitch	Gear-Running Speeds and Integer Multiples
Profile	Tooth-Passing Frequency and Integer Multiples
Indexing	Integer Multiples of Running Speed, Tooth-Passing Frequency Associated with the Cutting Wheel, and Others

Table 3.1 Types of Gear Errors and Frequencies at Which They Occur

Consider the gear pair of Figure 3.4 engaged with centers fixed at operating center distance, lightly loaded to maintain contact, and rotated at any convenient speed. An optical encoder transducer mounted on each shaft emits N_s pulses for each revolution (Figure 3.4(a)). Signal conditioning is then necessary to yield a pulse train of constant width and height pulses (Figure 3.4 (b)). As the gears speed up and slow down due to the various types of machining errors involved, the pulse train becomes more and less dense, respectively. Low-pass filtering these signals produces voltage signals (Figure 3.4 (c)) which are proportional to the gear rotational speeds. Subtracting these signals yields a voltage signal (Figure 3.4 (d)) proportional to the relative angular velocity of the gear pair. For perfect gears this signal is constant. Variations in this signal reflect gear errors which may be resolved into frequency components via an FFT analysis. This approach evaluates the actual error magnitudes and frequencies for the particular gear pair to be used.

Suppose calibration is done such that all speed variations are reflected in $\omega_1(t)$ ($\omega_2(t)$ = constant). Represented in Figure 3.4(d) are speed fluctuations expressible as

$$\begin{aligned} \dot{\theta}_1\,(t) = & \sum_n (\dot{\theta}^{n\sigma c} \cos n\sigma t + \dot{\theta}^{n\sigma s} \sin n\sigma t) \\ & + \sum_j (\dot{\theta}^{\gamma_j c} \cos \gamma_j t + \dot{\theta}^{\gamma_j s} \sin \gamma_j t\,) \end{aligned} \tag{3.42}$$

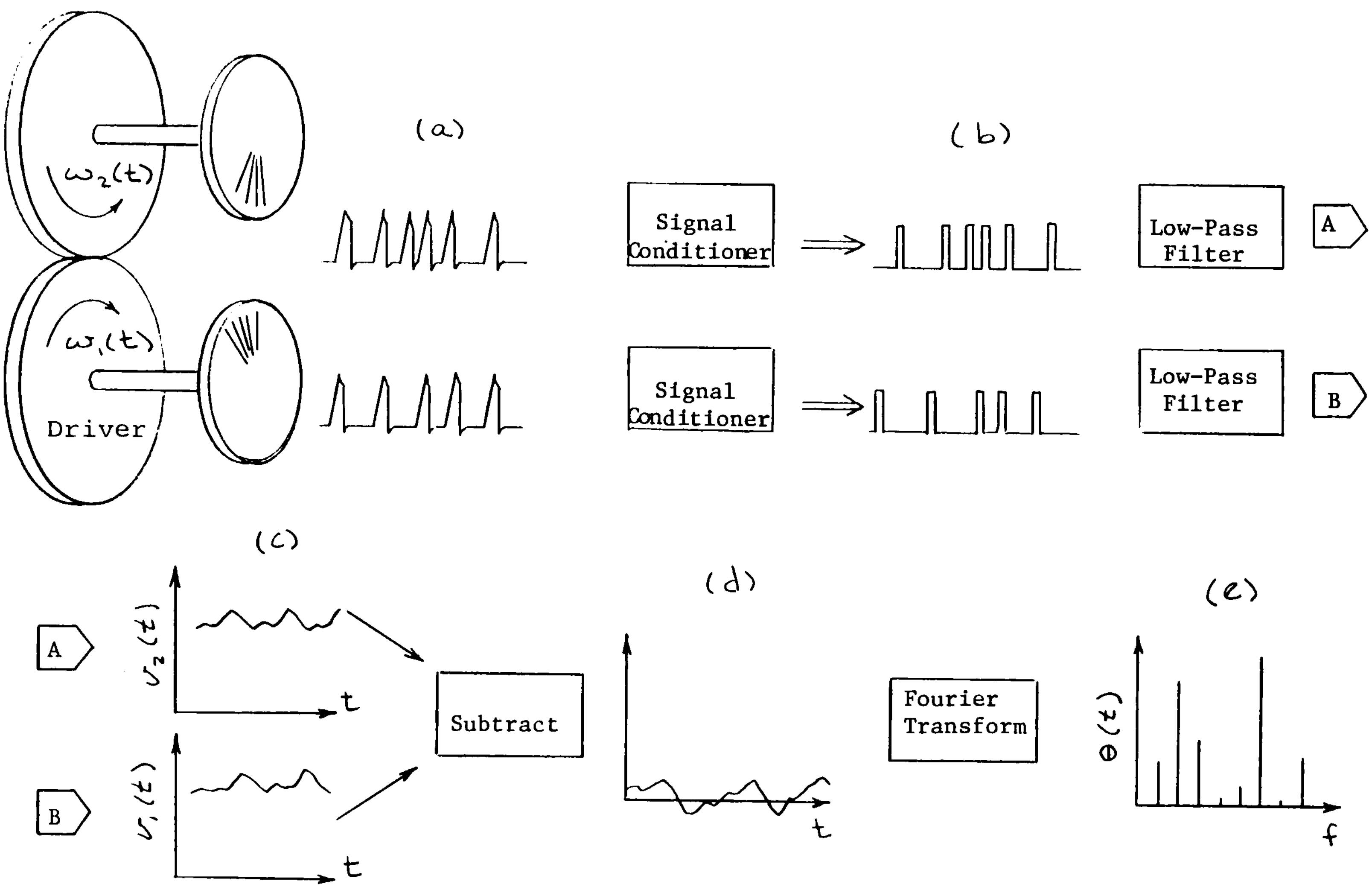


Figure 3.4 Experimental Technique For Determining Gear Machining Errors

Equation (3.42) contains speed variations at tooth-passing frequency and multiples, and various error frequencies, including shaft speeds and multiples.

For ease in mathematical treatment the terms of Equation (3.42) are translated into variations in tooth surface compared to geometrically perfect gears for errors at tooth-passing frequency and integer multiples according to

$$F^{\sigma}(t) = \sum_{n} (F^{n\sigma c} \cos n\sigma t + F^{n\sigma s} \sin n\sigma t) \tag{1.3}$$

and for all other error frequencies including shaft speed and multiples according to

$$G^{\gamma}(t) = \sum_{j} (G^{\gamma_j c} \cos \gamma_j t + G^{\gamma_j s} \sin \gamma_j t) \tag{1.4}$$

Equations (1.3) and (1.4) have units of length and represent material (+/-) superimposed on the ideal involute measured normal to that ideal tooth surface along the ideal operating line-of-action which would cause the same rotational speeding up and slowing down indicated in Equation (3.42).

Recognizing that

$$\dot{\theta}_1(t) = (1/R_{b1})\ (dF^{\sigma}(t)/dt + dG^{\gamma}(t)/dt) \tag{3.43}$$

and expanding yields

$$\dot{\theta}_1(t) = (1/R_{b1}) \sum_n (-n\sigma F^{n\sigma c} \sin n\sigma t + n\sigma F^{n\sigma s} \cos n\sigma t)$$

$$+ (1/R_{b1}) \sum_j (-\gamma_j (G^{\gamma_j c} \sin \gamma_j t + G^{\gamma_j s} \cos \gamma_j t)) \qquad (3.44)$$

Thus, the error coefficients of Equations (1.3) and (1.4) may be expressed in terms of the experimentally determined coefficients of Equation (3.42) as

$$F^{n\sigma c} = -R_{b1} \dot{\theta}^{n\sigma s} / n\sigma \qquad (3.45\ a)$$

$$F^{n\sigma s} = R_{b1} \dot{\theta}^{n\sigma c} / n\sigma \qquad (3.45\ b)$$

$$G^{\gamma_j c} = -R_{b1} \dot{\theta}^{\gamma_j s} / \gamma_j \qquad (3.45\ c)$$

$$G^{\gamma_j s} = R_{b1} \dot{\theta}^{\gamma_j c} / \gamma_j \qquad (3.45\ d)$$

It remains only to develop the form of the subvectors; $\underline{F}_{\alpha i}$, α = x, y, z; of the extension column of the mesh transfer matrix Equation (3.39) in terms of Equations (1.3) and (1.4).

To illustrate, by example using the line 1 translational x elements, the extension column subvector $\underline{F}_{x1}$ and the corresponding state variable subvector $\underline{V}_{x1}$ are written side-by-side in Equation (3.46).

$$\underline{V}_{x_1} = \begin{Bmatrix} V_{x_1}^{0} \\ V_{x_1}^{\omega,c} \\ V^{\omega,s} \\ \vdots \\ V^{\omega_r,s} \\ V^{\sigma,c} \\ V^{\sigma,s} \\ V^{\sigma_2 c} \\ \vdots \\ V^{\varepsilon,c} \\ V^{\varepsilon,s} \\ \vdots \\ V^{\varepsilon_t s} \\ V^{(\sigma+\omega),c} \\ V^{(\sigma+\omega),s} \\ V^{(\sigma-\omega),c} \\ V^{(\sigma-\omega),s} \\ V^{(\sigma+\omega)_2 c} \\ \vdots \\ V^{(\sigma-\omega)_n s} \\ V^{(2\sigma),c} \\ V^{(2\sigma),s} \\ V^{(2\sigma)_2 c} \\ \vdots \\ V^{(2\sigma)_r s} \\ V^{(\sigma+\varepsilon)c} \\ V^{(\sigma+\varepsilon)s} \\ V^{(\sigma-\varepsilon)c} \\ V^{(\sigma-\varepsilon)s} \\ V_{x_1}^{(\sigma+\varepsilon)_2 c} \\ \vdots \\ V_{x_1}^{(\sigma-\varepsilon)_w s} \end{Bmatrix} \qquad (3.46\,a)$$

$$\underline{F}_{x_1} = \begin{Bmatrix} F_{x_1}^{0} \\ F_{x_1}^{\omega,c} \\ F_{x_1}^{\omega,s} \\ \vdots \\ 0 \\ F^{\sigma,c} \\ F^{\sigma,s} \\ 0 \\ \vdots \\ F^{\varepsilon,c} \\ F^{\varepsilon,s} \\ \vdots \\ 0 \\ F^{(\sigma+\omega),c} \\ F^{(\sigma+\omega),s} \\ F^{(\sigma-\omega),c} \\ F^{(\sigma-\omega),s} \\ 0 \\ \vdots \\ 0 \\ F^{(2\sigma),c} \\ F^{(2\sigma),s} \\ 0 \\ \vdots \\ 0 \\ F^{(\sigma+\varepsilon),c} \\ F^{(\sigma+\varepsilon),s} \\ F^{(\sigma-\varepsilon),c} \\ F_{x_1}^{(\sigma-\varepsilon),s} \\ 0 \\ \vdots \\ 0 \end{Bmatrix} \qquad (3.46\,b)$$

Evaluation of F^{o}_{x1}, $F^{\sigma c}_{x1}$, $F^{\sigma s}_{x1}$, $F^{2\sigma c}_{x1}$, $F^{2\sigma s}_{x1}$, of Equation (3.46)

The errors at tooth-passing frequency and multiples, $F^{\sigma}(t)$ of Equation (1.3), take on the values $F^{\sigma}(t_j)$, $j = 1,2,\ldots,m$; at m discrete increments throughout an engagement. These may be resolved into gear 1 (x_{g1}, y_{g1}) coordinates in accordance with Figure 3.3(b) using the relations

$$\left. F^{\sigma}_{xg} \right|_{t = t_j} = F^{\sigma}(t_j) \cos \phi$$

$$\left. F^{\sigma}_{yg} \right|_{t = t_j} = F^{\sigma}(t_j) \sin \phi \tag{3.47}$$

These quantities may be transformed to system global coordinates via the Equation (3.36) direction cosines according to

$$F^{\sigma}_{xq} = \ell^{gq}_{11} \; F^{\sigma}_{xg} + \ell^{gq}_{12} \; F^{\sigma}_{yg}$$

$$F^{\sigma}_{yq} = \ell^{gq}_{21} \; F^{\sigma}_{xg} + \ell^{gq}_{22} \; F^{\sigma}_{yg} \tag{3.48}$$

Recall the stiffness elements $k^{qp}_{\alpha\beta}$ of Equation (3.37) in system 1 coordinates (x_{s1}, y_{s1}), where p varies according to the number of tooth pairs instantaneously contacting. The mesh forces due to gear errors of the form of Equation (1.3) may be evaluated for gear i and the $j^{\underline{th}}$ increment according to

$$F^{\sigma}_{xs_i} = -F^{\sigma}_{xs_2} = \sum_p (F^{\sigma}_{xq}\, k^{qp}_{11}\Big|_j - F^{\sigma}_{yq}\, k^{qp}_{12}\Big|_j)$$

$$F^{\sigma}_{ys_i} = -F^{\sigma}_{ys_2} = \sum_p (F^{\sigma}_{xq}\, k^{qp}_{21}\Big|_j - F^{\sigma}_{yq}\, k^{qp}_{22}\Big|_j) \tag{3.49}$$

$$F^{\sigma}_{zs_1} = -F^{\sigma}_{xs_1}\, y_{s_1}\Big|_j + F^{\sigma}_{ys_1}\, x_{s_1}\Big|_j$$

$$F^{\sigma}_{zs_2} = -F^{\sigma}_{xs_2}\, y_{s_2}\Big|_j + F^{\sigma}_{ys_2}\, x_{s_2}\Big|_j$$

where $k^{qp}_{\alpha\beta}\Big|_j$, $x_{si}\Big|_j$, and $y_{si}\Big|_j$ are the values of $k^{qp}_{\alpha\beta}$, x_{si}, and y_{si} evaluated at increment j. The discrete $F^{\sigma}_{\alpha si}$; $\alpha = x, y, z$; i = gears 1,2; for all increments, j, throughout engagement are least squares fit to the coefficients of the equations

$$F^{\sigma}_{\alpha i}(t) = F^{o}_{\alpha i} + F^{\sigma c}_{\alpha i} \cos \sigma t + F^{\sigma s}_{\alpha i} \sin \sigma t$$
$$+ F^{2\sigma c}_{\alpha i} \cos 2\sigma t + F^{2\sigma s}_{\alpha i} \sin 2\sigma t \tag{3.50}$$

These coefficients are the elements F^{o}_{x1}, $F^{\sigma c}_{x1}$, $F^{\sigma s}_{x1}$, $F^{2\sigma c}_{x1}$, $F^{2\sigma s}_{x1}$, for $\alpha = x$ in this example, of Equation (3.46).

Evaluation of $F_{x1}^{\gamma c}$, $F_{x1}^{\gamma s}$, $F_{x1}^{(\sigma \pm \gamma)c}$, $F_{x1}^{(\sigma \pm \gamma)s}$; $\gamma = \omega$, ϵ of Equation (3.46)

Gear errors, $G^{\gamma}(t)$ of the form of Equation (1.4), may be expressed in gear 1 coordinates (x_{g1}, y_{g1}) according to

$$
\begin{aligned}
G_{xg}^{\gamma}(t) &= \sum_j (G^{\gamma_j c} \cos \phi \cos \gamma_j t + G^{\gamma_j s} \cos \phi \sin \gamma_j t) \\
&= \sum_j (G_{xg}^{\gamma_j c} \cos \gamma_j t + G_{xg}^{\gamma_j s} \sin \gamma_j t)
\end{aligned}
\tag{3.51}
$$

$$
\begin{aligned}
G_{yg}^{\gamma}(t) &= \sum_j (G^{\gamma_j c} \sin \phi \cos \gamma_j t + G^{\gamma_j s} \sin \phi \sin \gamma_j t) \\
&= \sum_j (G_{yg}^{\gamma_j c} \cos \gamma_j t + G_{yg}^{\gamma_j s} \sin \gamma_j t)
\end{aligned}
$$

The expression of these errors in system 1 global coordinates is desired in the form

$$
\begin{aligned}
G_{xq}^{\gamma}(t) &= \sum_j (G_{xq}^{\gamma_j c} \cos \gamma_j t + G_{xq}^{\gamma_j s} \sin \gamma_j t) \\
G_{yq}^{\gamma}(t) &= \sum_j (G_{yq}^{\gamma_j c} \cos \gamma_j t + G_{yq}^{\gamma_j s} \sin \gamma_j t)
\end{aligned}
\tag{3.52}
$$

The direction cosines of Equation (3.36) perform this transformation according to

$$
\begin{aligned}
G_{xq}^{\gamma_j c} &= \ell_{11}^{gq} \; G_{xg}^{\gamma_j c} + \ell_{12}^{gq} \; G_{yg}^{\gamma_j c} \\
G_{xq}^{\gamma_j s} &= \ell_{11}^{gq} \; G_{xg}^{\gamma_j s} + \ell_{12}^{gq} \; G_{yg}^{\gamma_j s}
\end{aligned}
\tag{3.53}
$$

$$G_{yq}^{\gamma_j c} = \ell_{21}^{gq} \; G_{xg}^{\gamma_j c} + \ell_{22}^{gq} \; G_{yg}^{\gamma_j c}$$

$$G_{yq}^{\gamma_j s} = \ell_{21}^{gq} \; G_{xg}^{\gamma_j s} + \ell_{22}^{gq} \; G_{yg}^{\gamma_j s}$$

Recall the fundamental time-varying mesh stiffness equation

$$k_{\alpha\beta}(t) = k_{\alpha\beta}^{o} + \sum_{n=1}^{4} (k_{\alpha\beta}^{n\sigma c} \cos n\sigma t + k_{\alpha\beta}^{n\sigma s} \sin n\sigma t) \qquad (1.2)$$

where α, β take on the values x_i (translational), y_i (translational), and z_i (rotational).

The resulting mesh forces in system global coordinates ($\alpha = x_{si}, y_{si}, z_{si}$; i = gear 1,2) may be expressed

$$\begin{aligned} G_{\alpha s}^{\gamma}(t) = \sum_j (F_\alpha^{\gamma_j c} \cos \gamma_j t &+ F_\alpha^{\gamma_j s} \sin \gamma_j t \\ + F_\alpha^{(\sigma+\gamma_j)c} \cos (\sigma+\gamma_j)t &+ F_\alpha^{(\sigma+\gamma_j)s} \sin (\sigma+\gamma_j)t \qquad (3.53) \\ + F_\alpha^{(\sigma-\gamma_j)c} \cos (\sigma-\gamma_j)t &+ F_\alpha^{(\sigma-\gamma_j)s} \sin (\sigma-\gamma_j)t \end{aligned}$$

Alternatively, in terms of Equations (3.52) and (1.2), Equation (3.53) may be expressed

$$G_{x_1 s}^{\gamma}(t) = -G_{x_2 s}^{\gamma}(t) = k_{x1x1}(t) G^{\gamma}{}_{xq}(t) - k_{x1y1}(t) G_{yq}^{\gamma}(t)$$

$$G_{y_1 s}^{\gamma}(t) = -G_{y_2 s}^{\gamma}(t) = k_{y1x1}(t) G^{\gamma}{}_{xq}(t) - k_{y1y1}(t) G_{yq}^{\gamma}(t) \qquad (3.54)$$

$$G^{\gamma}_{z_1 s}(t) = -G^{\gamma}_{x_1 s}(t) y_{s1} + G^{\gamma}_{y_1 s}(t)\ x_{s1}$$

$$G^{\gamma}_{z_2 s}(t) = -G^{\gamma}_{x_2 s}(t) y_{s2} + G^{\gamma}_{y_2 s}(t)\ x_{s2}$$

where (x_{si}, y_{si}) is the operating pitch point location in system i coordinates. Substituting Equations (1.2) and (3.52) into (3.54), expanding, and discarding terms at frequencies not included in the assumed state vector yields, for example

$$\begin{aligned} G^{\gamma}_{x_1 s}(t) = \sum_j \Big[& (\overset{o}{k}_{x1x1} G^{\gamma_j s}_{xq} - \overset{o}{k}_{x1y1} G^{\gamma_j s}_{xq})\ \sin \gamma_j t \\ & + (\overset{o}{k}_{x1x1} G^{\gamma_j c}_{xq} - \overset{o}{k}_{x1y1} G^{\gamma_j c}_{xq})\ \cos \gamma_j t \\ & + 0.5(k^{\sigma c}_{x1x1} G^{\gamma_j s}_{xq} + k^{\sigma s}_{x1x1} G^{\gamma_j c}_{xq} - k^{\sigma c}_{x1y1} G^{\gamma_j s}_{xq} - k^{\sigma s}_{x1y1} G^{\gamma_j c}_{xq})\ \sin(\sigma+\gamma_j)t \\ & + 0.5(k^{\sigma c}_{x1x1} G^{\gamma_j c}_{xq} - k^{\sigma s}_{x1x1} G^{\gamma_j s}_{xq} - k^{\sigma c}_{x1y1} G^{\gamma_j c}_{xq} + k^{\sigma s}_{x1y1} G^{\gamma_j s}_{xq})\ \cos(\sigma+\gamma_j)t \\ & + 0.5(k^{\sigma s}_{x1x1} G^{\gamma_j c}_{xq} - k^{\sigma c}_{x1x1} G^{\gamma_j s}_{xq} - k^{\sigma s}_{x1y1} G^{\gamma_j c}_{xq} + k^{\sigma c}_{x1y1} G^{\gamma_j s}_{xq})\ \sin(\sigma-\gamma_j)t \\ & + 0.5(k^{\sigma c}_{x1x1} G^{\gamma_j c}_{xq} + k^{\sigma s}_{x1x1} G^{\gamma_j s}_{xq} - k^{\sigma c}_{x1y1} G^{\gamma_j c}_{xq} - k^{\sigma s}_{x1y1} G^{\gamma_j s}_{xq})\ \cos(\sigma-\gamma_j)t \Big] \end{aligned} \tag{3.55}$$

Equation (3.55) comprises the coefficients $F^{\gamma_j c}_{x1}$, $F^{\gamma_j s}_{x1}$, $F^{(\sigma+\gamma_j)c}_{x1}$, $F^{(\sigma+\gamma_j)s}_{x1}$, $F^{(\sigma-\gamma_j)c}_{x1}$, $F^{(\sigma-\gamma_j)s}_{x1}$ of the subvector Equation (3.46) and Equation (3.53). Similar substitutions yield the terms for the extension column subvectors corresponding to translational y and rotational z forces as well.

4. EXTENDED TRANSFER MATRIX ELEMENTS

The development of mesh stiffness, gear-tooth error effects, physical branching techniques, and the extended transfer matrix for the spur gear mesh presented in the last chapter form a major cornerstone for the current work contribution. Additional transfer matrix elements must be considered in order that a complete dynamic analysis of a rotor system be performed.

The current work utilizes the contributions of Daws [10] who developed the form of the rotor bearing and inertial disc extended transfer matrix elements. The extended transfer matrix for the massless elastic shaft is developed in the current work.

A word of caution is in order when using transfer matrices from various sources, as well as future development, in regard to the coordinate system. It may be noted that the signs of some terms in the current work are different from other sources, Pestle and Leckie [14] for example. The reason for this is that the current work, as does Daws [10], uses a coordinate system with a left-facing positive face; i.e., while the system obeys the right-hand rule, the outward directed axial z displacement occurs on the left face.

4.1 The Rotor Bearing Element

Daws [10] developed the general form of the extended transfer matrix for the rotor bearing element which is presented in this section for completeness. Equation (4.1) relates the state vectors on the left-hand and right-hand sides of a bearing.

The only bearing force not a function of dynamics is friction damping associated with shaft i D.C. rotation at ω_i. This is accounted for in the extension column subvector indicated in Equation (4.1) as $\underline{G}_z^*$ which contains

$$\underline{G}_Z^* = (-c_{zz}^* \omega_i \ o \ o \ . \ . \ . \ . \ . \ o)^T \tag{4.2}$$

The submatrices $\underline{B}_{\alpha\beta}^*$ and $\underline{B}_{\alpha\beta}$ for rotational and translational state-vector quantities, respectively, are identical in form. The expanded form is illustrated in Equation (4.3) for the case $\underline{B}_{yx}^*$.

$$\underline{q}^R = \begin{Bmatrix} W_{x1} \\ \vdots \\ V_{z\,i-1} \\ W_{xi} \\ W_{yi} \\ W_{zi} \\ \Theta_{xi} \\ \Theta_{yi} \\ \Theta_{zi} \\ M_{xi} \\ M_{yi} \\ M_{zi} \\ V_{xi} \\ V_{yi} \\ V_{zi} \\ W_{x\,i+1} \\ \vdots \\ V_{zn} \\ \hline 1 \end{Bmatrix} = \left[\begin{array}{ccccccccccccccccc|c} I & & & & & & & & & & & & & & & & & \\ & I & & & & & & & & & & & & & & & & \\ & & I & & & & & & & & & & & & & & & \\ & & & I & & & & & & & & & & & & & & \\ & & & & I & & & & & & & & & & & & & \\ & & & & & I & & & & & & & & & & & & \\ & & & & & & I & & & & & & & & & & & \\ & & & & & & & I & & & & & & & & & & \\ & & & & B^*_{xx} & B^*_{xy} & & & I & & & & & & & & & \\ & & & & B^*_{yx} & B^*_{yy} & & & & I & & & & & & & & G_z^* \\ & & & & & & B^*_{zz} & & & & I & & & & & & & \\ & & B_{xx} & B_{xy} & & & & & & & & I & & & & & & \\ & & B_{yx} & B_{yy} & & & & & & & & & I & & & & & \\ & & & & B_{zz} & & & & & & & & & I & & & & \\ & & & & & & & & & & & & & & I & & & \\ & & & & & & & & & & & & & & & I & & \\ & & & & & & & & & & & & & & & & I & \\ \hline & & & & & & & & & & & & & & & & & 1 \end{array}\right] \cdot \underline{q}^L \qquad (4.1)$$

$$
\underline{\underline{B}}^{*}_{yx} = \begin{array}{c|c}
 & \begin{array}{ccccccccccccccccc} \Phi^{0}_{xi} & \Phi^{\omega_r c} & \Phi^{\omega_r s} & \Phi^{\sigma_s c} & \Phi^{\sigma_s s} & \Phi^{\varepsilon_t c} & \Phi^{\varepsilon_t s} & \Phi^{(\sigma+\omega)_u c} & \Phi^{(\sigma+\omega)_u s} & \Phi^{(\sigma-\omega)_u c} & \Phi^{(\sigma-\omega)_u s} & \Phi^{(2\sigma)_v c} & \Phi^{(2\sigma)_v s} & \Phi^{(\sigma+\varepsilon)_w c} & \Phi^{(\sigma+\varepsilon)_w s} & \Phi^{(\sigma-\varepsilon)_w c} & \Phi^{(\sigma-\varepsilon)_w s} \end{array} \\
\begin{array}{c} M^{0}_{yi} \\ M^{\omega_r c} \\ M^{\omega_r s} \\ M^{\sigma_s c} \\ M^{\sigma_s s} \\ M^{\varepsilon_t c} \\ M^{\varepsilon_t s} \\ M^{(\sigma+\omega)_u c} \\ M^{(\sigma+\omega)_u s} \\ M^{(\sigma-\omega)_u c} \\ M^{(\sigma-\omega)_u s} \\ M^{(2\sigma)_r c} \\ M^{(2\sigma)_r s} \\ M^{(\sigma+\varepsilon)_w c} \\ M^{(\sigma+\varepsilon)_w s} \\ M^{(\sigma-\varepsilon)_w c} \\ M^{(\sigma-\varepsilon)_w s}_{yi} \end{array} &
\left[\begin{array}{ccccccccccccccccc}
b^{k*}_{yx} &&&&&&&&&&&&&&&& \\
& b^{k*}_{yx} & b^{c*}_{yx} &&&&&&&&&&&&&& \\
& b^{c*}_{yx} & b^{k*}_{yx} &&&&&&&&&&&&&& \\
&&& b^{k*}_{yx} & b^{c*}_{yx} &&&&&&&&&&&& \\
&&& b^{c*}_{yx} & b^{k*}_{yx} &&&&&&&&&&&& \\
&&&&& b^{k*}_{yx} & b^{c*}_{yx} &&&&&&&&&& \\
&&&&& b^{c*}_{yx} & b^{k*}_{yx} &&&&&&&&&& \\
&&&&&&& b^{k*}_{yx} & b^{c*}_{yx} &&&&&&&& \\
&&&&&&& b^{c*}_{yx} & b^{k*}_{yx} &&&&&&&& \\
&&&&&&&&& b^{k*}_{yx} & b^{c*}_{yx} &&&&&& \\
&&&&&&&&& b^{c*}_{yx} & b^{k*}_{yx} &&&&&& \\
&&&&&&&&&&& b^{k*}_{yx} & b^{c*}_{yx} &&&& \\
&&&&&&&&&&& b^{c*}_{yx} & b^{k*}_{yx} &&&& \\
&&&&&&&&&&&&& b^{k*}_{yx} & b^{c*}_{yx} && \\
&&&&&&&&&&&&& b^{c*}_{yx} & b^{k*}_{yx} && \\
&&&&&&&&&&&&&&& b^{k*}_{yx} & b^{c*}_{yx} \\
&&&&&&&&&&&&&&& b^{c*}_{yx} & b^{k*}_{yx}
\end{array}\right]
\end{array} \qquad (4.3)
$$

The two terms involved in Equation (4.3) are b_{yx}^{k*} and b_{yx}^{c*}, defined

$$b_{yx}^{k*} = -k_{yx}^{*} \tag{4.4}$$

$$b_{yx}^{c*} = -\gamma c_{yx}^{*} \tag{4.5}$$

where γ is the circular frequency of the particular state-vector quantity involved. The $k_{\alpha\beta}^{*}$ and $k_{\alpha\beta}$ are user provided three-dimensional bearing stiffness characteristics rotationally and translationally, respectively, similarly $c_{\alpha\beta}^{*}$ and $c_{\alpha\beta}$ for damping.

4.2 The Inertial Disk Element

A significant contribution arising from the work of Daws [10] is the development of the general form of the extended transfer matrix for the rigid inertial disk in three-dimensional space. The resultant matrix appears in Equation (4.6), next page. The submatrices $\underline{\underline{A}}$, $\underline{\underline{B}}$, and $\underline{\underline{C}}$ are diagonal submatrices with main diagonal elements a, b, and c; respectively; where

$$a = - I_d \gamma^2 \tag{4.7 a}$$

$$b = - M\gamma^2 \tag{4.7 b}$$

$$c = - I_p \gamma^2 \tag{4.7 c}$$

The quantities M, I_d, and I_p are the disk mass, and diametral and polar mass moments of inertia, respectively, and γ is the circular frequency of the involved state-vector quantity.

The form of the submatrix, $\underline{\underline{D}}$, is given in Equation (4.8), second following page, where

$$d = I_p \gamma \omega \tag{4.7 d}$$

and ω is the shaft-running speed.

$$\underline{q}^R = \left\{ \begin{matrix} w_{x1} \\ \vdots \\ v_{zi-1} \\ w_{xi} \\ w_{yi} \\ w_{zi} \\ \theta_{xi} \\ \theta_{yi} \\ \theta_{zi} \\ M_{xi} \\ M_{yi} \\ M_{zi} \\ V_{xi} \\ V_{yi} \\ V_{zi} \\ w_{xi+1} \\ \vdots \\ V_{zn} \\ 1 \end{matrix} \right\} = \begin{array}{c} \begin{array}{ccccccccccccccccccc} w_{x1} & \cdots & v_{zi-1} & w_{xi} & w_{yi} & w_{zi} & \theta_{xi} & \theta_{yi} & \theta_{zi} & M_{xi} & M_{yi} & M_{zi} & V_{xi} & V_{yi} & V_{zi} & w_{xi+1} & \cdots & V_{zn} & 1 \end{array} \\ \left[\begin{array}{cccccccccccccccccc|c} I&&&&&&&&&&&&&&&&&& \\ &\ddots&&&&&&&&&&&&&&&&& \\ &&I&&&&&&&&&&&&&&&& \\ &&&I&&&&&&&&&&&&&&& \\ &&&&I&&&&&&&&&&&&&& \\ &&&&&I&&&&&&&&&&&&& \\ &&&&&&I&&&&&&&&&&&& \\ &&&&&&&I&&&&&&&&&&& \\ &&&&&&&&I&&&&&&&&&& \\ &&&&&&A&-D&&I&&&&&&&&&E \\ &&&&&&D&A&&&I&&&&&&&&F \\ &&&&&&&&C&&&I&&&&&&&G \\ &&&B&&&&&&&&&I&&&&&&H \\ &&&&B&&&&&&&&&I&&&&&J \\ &&&&&B&&&&&&&&&I&&&& \\ &&&&&&&&&&&&&&&I&&& \\ &&&&&&&&&&&&&&&&\ddots&& \\ &&&&&&&&&&&&&&&&&I& \\ \hline &&&&&&&&&&&&&&&&&&1 \end{array} \right] \end{array} \cdot \underline{q}^L \qquad (4.6)$$

$$
\underline{\underline{D}} = \begin{array}{cc}
 & \begin{array}{ccccccccccccccccc} \Theta^{0}_{xi} & \cdots\Theta^{\omega_r c} & \Theta^{\omega_r s} & \cdots\Theta^{\sigma_s c} & \Theta^{\sigma_s s} & \cdots\Theta^{\varepsilon_t c} & \Theta^{\varepsilon_t s} & \cdots\Theta^{(\sigma+\omega)_u c} & \Theta^{(\sigma+\omega)_u s} & \Theta^{(\sigma-\omega)_u c} & \Theta^{(\sigma-\omega)_u s} & \cdots\Theta^{(2\sigma)_v c} & \Theta^{(2\sigma)_v s} & \cdots\Theta^{(\sigma+\varepsilon)_w c} & \Theta^{(\sigma+\varepsilon)_w s} & \Theta^{(\sigma-\varepsilon)_w c} & \Theta^{(\sigma-\varepsilon)_w s} \end{array} \\
\begin{array}{c} M^{0}_{yi} \\ \vdots M^{\omega_r c} \\ M^{\omega_r s} \\ \vdots M^{\sigma_s c} \\ M^{\sigma_s s} \\ \vdots M^{\varepsilon_t c} \\ M^{\varepsilon_t s} \\ \vdots M^{(\sigma+\omega)_u c} \\ M^{(\sigma+\omega)_u s} \\ M^{(\sigma-\omega)_u c} \\ M^{(\sigma-\omega)_u s} \\ \vdots M^{(2\sigma)_v c} \\ M^{(2\sigma)_v s} \\ \vdots M^{(\sigma+\varepsilon)_w c} \\ M^{(\sigma+\varepsilon)_w s} \\ M^{(\sigma-\varepsilon)_w c} \\ M^{(\sigma-\varepsilon)_w s}_{yi} \end{array} &
\left[\begin{array}{ccccccccccccccccc}
 & & & & & & & & & & & & & & & & \\
 & & -d & & & & & & & & & & & & & & \\
 & d & & & & & & & & & & & & & & & \\
 & & & & -d & & & & & & & & & & & & \\
 & & & d & & & & & & & & & & & & & \\
 & & & & & & -d & & & & & & & & & & \\
 & & & & & d & & & & & & & & & & & \\
 & & & & & & & & -d & & & & & & & & \\
 & & & & & & & d & & & & & & & & & \\
 & & & & & & & & & & -d & & & & & & \\
 & & & & & & & & & d & & & & & & & \\
 & & & & & & & & & & & & -d & & & & \\
 & & & & & & & & & & & d & & & & & \\
 & & & & & & & & & & & & & & -d & & \\
 & & & & & & & & & & & & & d & & & \\
 & & & & & & & & & & & & & & & & -d \\
 & & & & & & & & & & & & & & & d &
\end{array}\right]
\end{array} \qquad (4.8)
$$

The vector, $\underline{E}$, of Equation (4.6) is null except for the $M_{xi}^{\omega_i c}$ row, or

$$\underline{E} = (o \ldots o \ e \ o \ldots o)^T \tag{4.9 a}$$

where

$$e = \tau \ (I_d - I_p) \ \omega_i^2 \tag{4.9 b}$$

and τ is the disk wobble angle shown in Figure 4.1. The vector $\underline{F}$ of Equation (4.6) is null except for the $M_{yi}^{\omega_i s}$ row, or

$$\underline{F} = (o \ldots \ -e \ o \ldots o)^T \tag{4.10}$$

with e defined in Equation (4.9 b). The vector, $\underline{G}$, is null except for the $M_{zi}^{\omega_i c}$ and $M_{zi}^{\omega_i s}$ rows, or

$$\underline{G} = (o \ldots o \ g_s \ g_c \ o \ldots o)^T \tag{4.11 a}$$

where

$$g_s = mg \ e \sin \beta \tag{4.11 b}$$

$$g_c = -mg \ e \cos \beta \tag{4.11 c}$$

and where the quantities e and β locate the disk mass center from the disk center of rotation as shown in Figure 4.1.

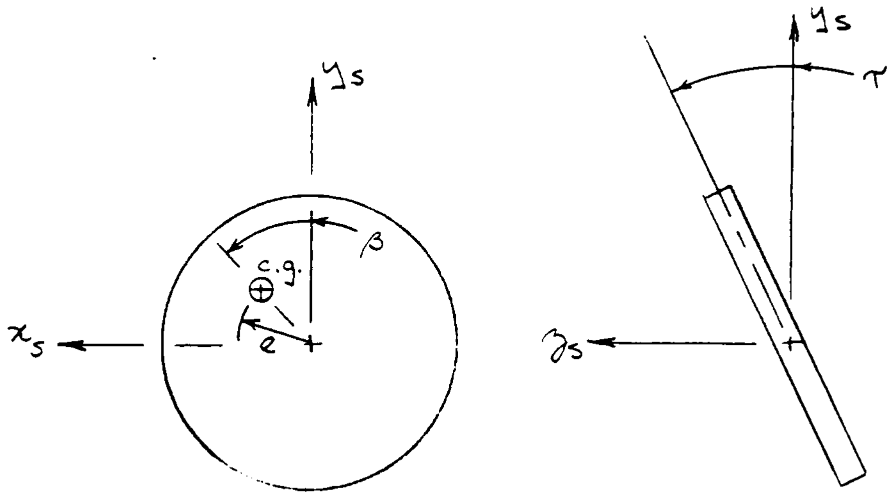


Figure 4.1 Rigid Inertial Disk

The vector, $\underline{H}$, of Equation (4.6) is null except for the $V_{xi}^{\omega_i c}$ and $V_{xi}^{\omega_i s}$ rows, or

$$\underline{H} = (o \ldots h_c \; h_s \; o \ldots o)^T \tag{4.12 a}$$

where

$$h_c = -me\omega_1^2 \sin \beta \tag{4.12 b}$$

$$h_s = -me\omega_1^2 \cos \beta \tag{4.12 c}$$

The vector, $\underline{J}$, is null except for the $V_{yi}^{\omega_i c}$ and $V_{yi}^{\omega_i s}$ rows, or

$$\underline{J} = (J^\sigma \; o \ldots -h_s \; h_c \; o \ldots o)^T \tag{4.13 a}$$

where

$$J^\sigma = -mg \tag{4.13 b}$$

and where h_c and h_s are defined in Equation (4.12).

4.3 The Elastic Shaft Element

The extended transfer matrix for the massless elastic rotor element is developed by applying simple beam deflection theory and recognizing the sign conventions of Figure 4.2.

The right-face (see Figure 4.2) state-vector conditions in terms of left-face conditions and element properties begins with the equality of shear on the left and right sides, or

$$V^R_\alpha = V^L_\alpha \quad , \quad \alpha = x, y, z \tag{4.14}$$

Summing and setting to zero, moments about the right-hand end yields

$$\Sigma M_x = 0 = M^R_x - M^L_x + V^L_y L$$

$$\Sigma M_y = 0 = M^R_y - M^L_y - V^L_x L$$

$$\Sigma M_z = 0 = M^R_z - M^L_z$$

or, rewritten

$$M^R_x = - LV^L_y + M^L_x \tag{4.15 a}$$

$$M^R_y = LV^L_x + M^L_y \tag{4.15 b}$$

$$M^R_z = M^L_z \tag{4.15 c}$$

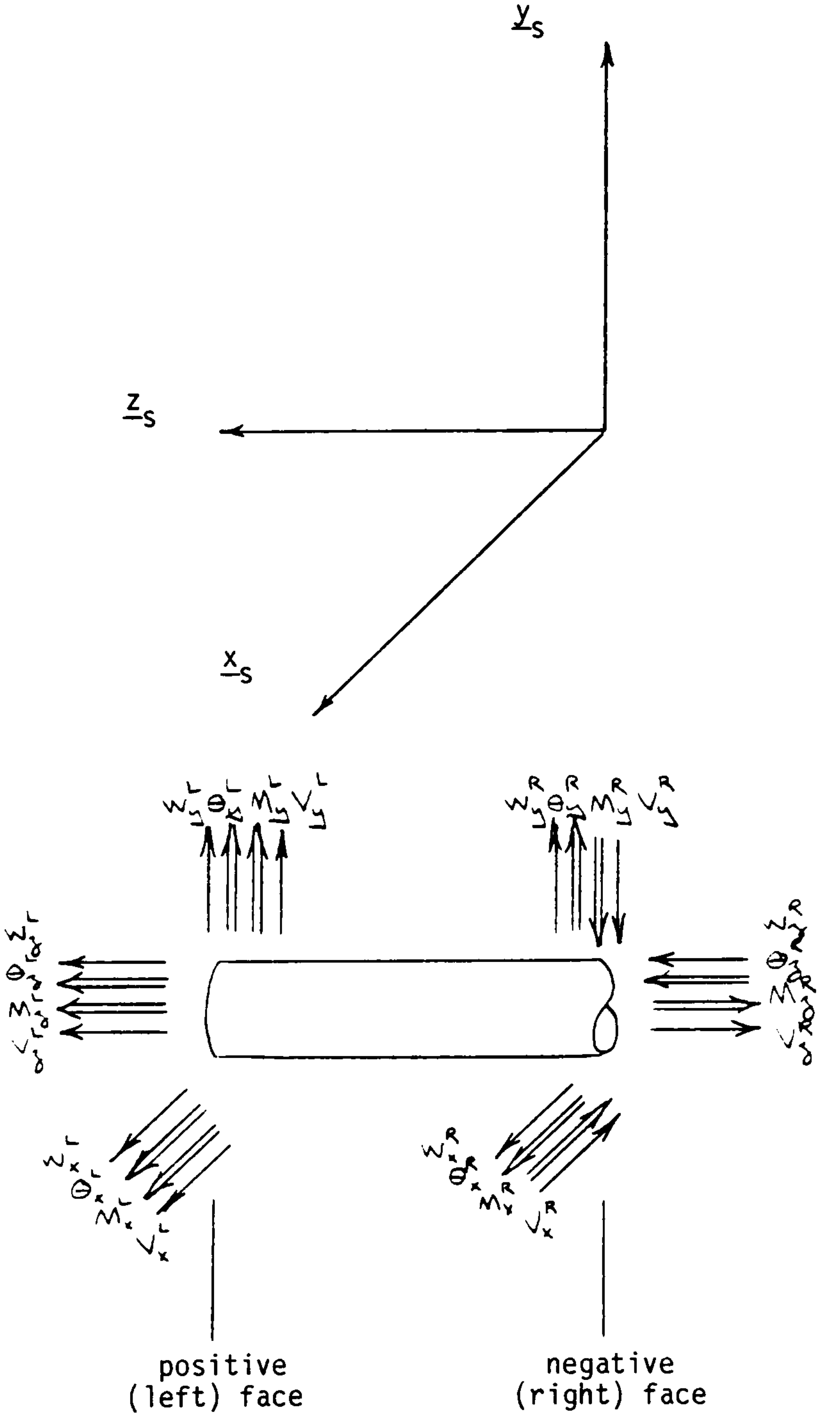


Figure 4.2 Global Coordinate System and Modified Transfer Matrix Sign Convention

From simple beam deflection relations involving shear and moment, the right end slopes are

$$\theta_x^R = \theta_x^L - V_y^R L^2/2EI - M_x^R L/EI$$

$$\theta_y^R = \theta_y^L + V_x^R L^2/2EI - M_y^R L/EI$$

$$\theta_z^R = \theta_z^L - M_z^R L/JG$$

Combining Equations (4.14) and (4.15) with these relations yields

$$\theta_x^R = V_y^L L^2/2EI - M_x^L L/EI + \theta_x^L \qquad (4.16\ a)$$

$$\theta_y^R = -V_x^L L^2/2EI - M_y^L L/EI + \theta_y^L \qquad (4.16\ b)$$

$$\theta_z^R = -M_z^L L/JG + \theta_z^L \qquad (4.16\ c)$$

Right-end deflections are

$$w_x^R = w_x^L - L\theta_y^L + M_y^R L^2/2EI - V_x^R L^3/3EI$$

$$w_y^R = w_y^L + L\theta_x^L - M_x^R L^2/2EI - V_y^R L^3/3EI$$

$$w_z^R = w_z^L - V_z^R L/EA$$

Combining Equations (4.14), (4.15), and (4.16) with these relations yields

$$w_x^R = (L^3/6EI)V_x^L + (L^2/2EI)M_y^L - L\theta_y^L + w_x^L \qquad (4.17\ a)$$

$$w_y^R = (L^3/6EI)\ V_y^L - (L^2/2EI)\ M_x^L - L\theta_x^L + w_y^L \qquad (4.17\ b)$$

$$w_t^R = -\ (L/EA)\ V_z^L + w_z^L \qquad (4.17\ c)$$

The form of the transfer matrix for the massless elastic rotor element appears in Equation (4.18), next page, for the i^{th} shaft. The submatrices A, B, C, D, E and F are diagonal with non-zero elements; a, b, c, d, e, f, respectively; defined

$$a = L/GJ \qquad (4.19\ a)$$

$$b = L/EA \qquad (4.19\ b)$$

$$c = L/EI \qquad (4.19\ c)$$

$$d = L^2/2EI \qquad (4.19\ d)$$

$$e = L^3/6EI \qquad (4.19\ e)$$

$$f = L \qquad (4.19\ f)$$

Equation (4.18) containing diagonal submatrices with the elements of Equation (4.19) thus complete the description of the form of the elastic rotor element transfer matrix.

The foregoing sections describe the form of the extended transfer matrix for the rotor bearing, inertial disk, and elastic shaft elements of the geared-rotor system. These and the mesh transfer matrix described in the last chapter comprise all rotor elements considered in the current work and in the family of computer programs developed in the current work.

$$
g_b^R = \begin{array}{cc}
 & \begin{array}{ccccccccccccccccccc} W_{x1} & \cdots & V_{z,i-1} & W_{xi} & W_{yi} & W_{zi} & \Theta_{xi} & \Theta_{yi} & \Theta_{zi} & M_{xi} & M_{yi} & M_{zi} & V_{xi} & V_{yi} & V_{zi} & W_{x,i+1} & \cdots & V_{zn} & 1 \end{array} \\
\begin{array}{c} W_{x1} \\ \vdots \\ V_{z,i-1} \\ W_{xi} \\ W_{yi} \\ W_{zi} \\ \Theta_{xi} \\ \Theta_{yi} \\ \Theta_{zi} \\ M_{xi} \\ M_{yi} \\ M_{zi} \\ V_{xi} \\ V_{yi} \\ V_{zi} \\ W_{x,i+1} \\ \vdots \\ V_{zn} \\ 1 \end{array} &
\left[\begin{array}{cccccccccccccccccc|c}
I & & & & & & & & & & & & & & & & & & \\
 & \ddots & & & & & & & & & & & & & & & & & \\
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 & & & I & & & & -F & & & D & & E & & & & & & \\
 & & & & I & & F & & & -D & & & & E & & & & & \\
 & & & & & I & & & & & & & & & -B & & & & \\
 & & & & & & I & & & -C & & & & D & & & & & \\
 & & & & & & & I & & & -C & & -D & & & & & & \\
 & & & & & & & & I & & & -A & & & & & & & \\
 & & & & & & & & & I & & & & -F & & & & & \\
 & & & & & & & & & & I & & F & & & & & & \\
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\end{array}\right]
\end{array} \cdot g_b^L \qquad (4.18)
$$

Chapter 5 discusses the interactive front end Geared Rotor Developer, GRD; the Gear Mesh Stiffness program, GMS; the Geared Rotor Solver program, GRS; and their application to a test case gearbox.

5. APPLICATION OF THE TECHNOLOGY ON THE DIGITAL COMPUTER

The Gear Mesh Stiffness (GMS) computer program 1) reflects bending, shear, compression, and local contact deformation in the mesh; 2) places the mesh force along the line of action; and 3) calculates stiffness incrementally throughout engagement recognizing both one and two tooth pair contact. The incremental values are fit to a Fourier series including mean and four harmonics of tooth passing. Other methods in the literature typically 1) reflect only bending and shear; 2) place the mesh force at the pitch point normal to the tooth centerline; and 3) calculate only mean stiffness for contact at the pitch point. Section 5.1 compares the current approach to classical methods indicating the significance of individual terms in the evaluation of stiffness.

The Geared Rotor Solver (GRS) computer program of the current work 1) is three-dimensional with six degrees of freedom; 2) employs physical branching to carry all shafting simultaneously; 3) employs frequency branching to carry multiple frequencies simultaneously; and 4) utilizes a newly developed mesh transfer matrix which involves gear errors. Section 5.2 presents a sample problem illustrating many of these analytical capabilities.

5.1 The Significance of Terms in the Evaluation of Gear-Tooth Stiffness

The derivations of Chapter 3 result in gear-tooth stiffness along the line-of-action expressed as

$$K_{11}^{L} = \sum_{i=1}^{\text{Number of Tooth Pairs}} \sum_{j=1}^{2} (C_{ij}^{L} + C_{ij}^{HC})^{-1} \qquad (3.32\text{ a})$$

for n instantaneously contacting tooth pairs.

Consider identical gears in mesh at the pitch point; 4P, 28T, 20°ϕ, 0.019m (0.75") face width, transmitting 271 N-m (2400 in-lb) torque. At this instant only one tooth pair is in contact. For this special case, Equation (3.32 a) reduces to

$$K_{11}^{L} = (2k_s^{-1} + 2k_b^{-1} + 2k_c^{-1} + 2k_h^{-1})^{-1} \qquad (5.1)$$

where k_s, k_b, k_c, and k_h represent individual tooth stiffness components due to shear, bending, compression, and local contact; respectively. Equation (5.1) with Equations (3.25) and (3.26) yield

$$k_s = (2a_s \cos^2 X + 2b_s \cos X \sin X + 2\, c_s \sin^2 X)^{-1} \qquad (5.2\text{ a})$$

$$k_b = (2a_b \cos^2 X + 2b_b \cos X \sin X + 2\, c_b \sin^2 X)^{-1} \qquad (5.2\text{ b})$$

$$k_c = (2c_c \sin^2 X)^{-1} \qquad (5.2\text{ c})$$

Equation (5.1) with Equation (3.31) yields

$$k_h = [(H/2\pi)(\ln(4R_{c1}/b) + \ln(4R_{c2}/b) - 1/3)]^{-1}$$

Table 5.1 applies to this example providing a breakdown of terms comprising mesh stiffness. Notice that the compression term, k_c, is an order of magnitude higher than the other three, so its effect on overall mesh stiffness at this contact location is negligible.

Many authors have presented expressions for stiffness due to bending and shear considering only the component of the line of action tooth load which is normal to the tooth centerline. This produces a stiffness which is erroneous in both magnitude and in physical orientation. In evaluating the significance of these errors consider the tapered cantilever tooth model shown in Figure (3.2) subject to only the force component, F_c, normal to the centerline. Mabie [26] solved exactly this problem developing, in the nomenclature of this work, expressions for deflection due to bending

$$\delta_b = \frac{12\ L^3 F \cos\phi}{Ebh_1^{\ 3}} \left\{ (k-1)^{-3} \quad (\ln k + \frac{2}{k} - \frac{1}{2k^2} - \frac{3}{2}) \right\}$$

$$k = h_0/h_1$$

and due to shear

$$\delta_s = \frac{91\ F \cos\phi}{5Gbh_1} \left(\frac{\ln k}{K-1} + \frac{1}{2k^2} - \frac{5}{6k} \right)$$

I. CURRENT WORK

	Stiffness*	Compliance**
(a) c_c		2.03 (1.67)
(b) k_c	296. (359)	
(c) a_b		3.29 (2.71)
(d) b_b		-5.89 (-4.85)
(e) c_b		3.98 (3.28)
(f) k_b	29.1 (35.3)	
(g) a_s		5.56 (4.58)
(h) b_s		0.69 (0.57)
(i) c_s		0.57 (0.47)
(j) k_s	9.39 (11.4)	
(k) k_h	5.68 (6.9)	
(l) k_{11}^{2}	1.6 (1.9)	

II. COMPUTED FROM MABIE [26]

	Stiffness*	Compliance**
(a) $\delta_b/F_c \cos\phi$		3.29 (2.71)
(b) $k_b' = (2\delta_b/F_c \cos\phi)^{-1}$	15.2 (18.5)	
(c) $\delta_s/F_c \cos\phi$		5.56 (4.58)
(d) $k_s' = (2\delta_s/F_c \cos\phi)^{-1}$	8.98 (10.9)	
(e) $k' = k_s' k_b' / (k_s' + k_b')$	5.68 (6.9)	

* Stiffness, 10^{8} N/m (10^{6} lb/in)

** Compliance, 10^{-10} m/N (10^{-8} in/lb)

Table 5.1 Terms Comprising Gear-Mesh Stiffness of Mating Identical 4P, 28T, 20°ϕ, 0.019m (0.75 in) Face Width Gears Transmitting 271 N-m (2400 in-lb) Torque for Instantaneous Contact at the Pitch Point

The evaluation of these expressions for the current example appears in Table 5.1 (II).

With regard to bending, the component of compliance due to the normal component, F_c, is the same for the equations of the current work (I-c) as for the Mabie equation (II-a). The effect of the tangential component, F_s, as a restoring moment opposing F_c in bending, is significant however. This leads to approximately a factor of two in calculated bending stiffness; 29.1 x 10^8 N/m (I-f) vs 15.2 x 10^8 N/m (II-b). However, this difference becomes washed out as shear compliance, and to a greater degree, contact mode compliance, are much higher.

The shear term from the Mabie equation (II-d) is about the same as that from the current work (I-j). This is not surprising since only the F_c component contributes to shear.

The most significant upshot of this example, however, is the predominance of the contact mode term. Hertzian contact mode compliance, in this example, is actually slightly higher than the combined compliance of shear, bending, and compression.

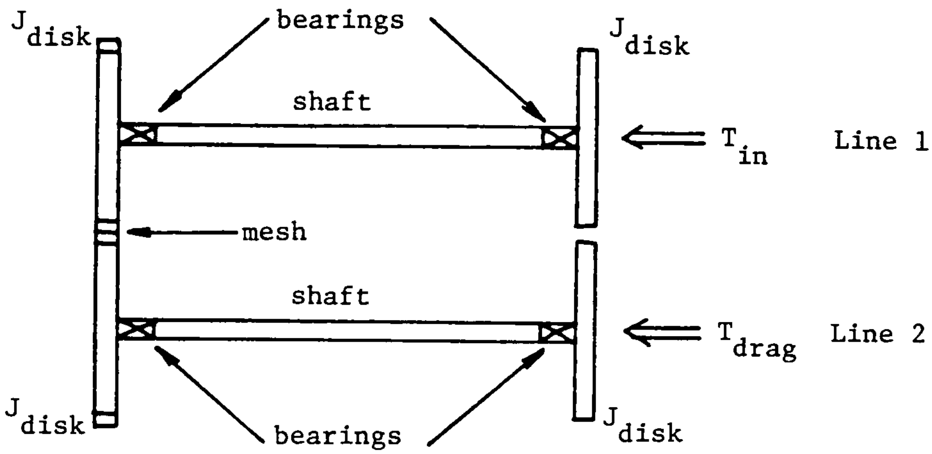
5.2 Dynamic Response of a Gear Coupled Two Rotor System

This section illustrates a sample problem applying the GMS and GRS computer programs in a complete geared-rotor analysis from model development to dynamic response solution. More specifically, this problem shows the significance of time-varying stiffness as an excitation source.

Consider the two identical rotors of Figure 5.1 which are gear coupled. For the purpose of analysis, the system of Figure 5.1 is sectioned into elemental components according to Figure 5.2. The interactive data entry to the front-end program, GRD, is shown in Appendix A.

Torque input to the system occurs at the right-hand side of line 1 which thus becomes the driving rotor with rotation positive according to the global sign convention. Line 2 is the driven shaft with negative axial rotation. The torque drag on the system is positive at the right-hand side of line 2.

Due to the symmetry of the two shafts, there exists a torsional natural frequency whose mode shape is characterized by the shafts flexing exactly in phase such that each shaft responds as though the center of the mesh were fixed inertially. The problem is formulated such that the shafting is very much more compliant than



Bearings: 4 identical

$k_{xx} = k_{yy} = 8.24 \times 10^{10}$ N/m (1.0×10^9 lb/in)

$k_{xx} = k_{yy} = 1.13 \times 10^8$ N-m/rad (1.0×10^9 in-lb/rad)

$c_{zz} = 0.0113$ N-m-s (0.1 in-lb-s)

Gears: 2 identical - 4P, 28T, 20°ϕ, 4.76×10^{-3}m (3/16") FW

Disks: 4 identical - 8.37 N (4 lb), 0.1524 m (6") diameter

Shafts: 2 identical - 0.3048 m (12") x 6.35 m (0.25") diameter

$k_{shaft} = JG/L = 41.52$ N-m/rad (367.5 in-lb/rad)

$J_{disk} = md^2/8 = 5.265 \times 10^{-3}$ N-m-s^2 (0.0466 in-lb-s^2)

$k_{gear} = 1.13 \times 10^5$ N-m/rad (1.0×10^6 in-lb/rad) >> k_{shaft}

$\omega_{n_{\substack{torsional\\fundamental}}} = (k_{shaft}/J_{disk})^{1/2} = 88.8$ rad/s

$c_{c_{\substack{torsional\\fundamental}}} = 2(k_{shaft}J_{disk})^{1/2} = 0.9355$ N-m-s (8.28 in-lb-s)

$\zeta = c_{zz}/c_c = 0.0113/0.9355 = 1.2$

Figure 5.1 Sample Problem Two Shaft Geared System

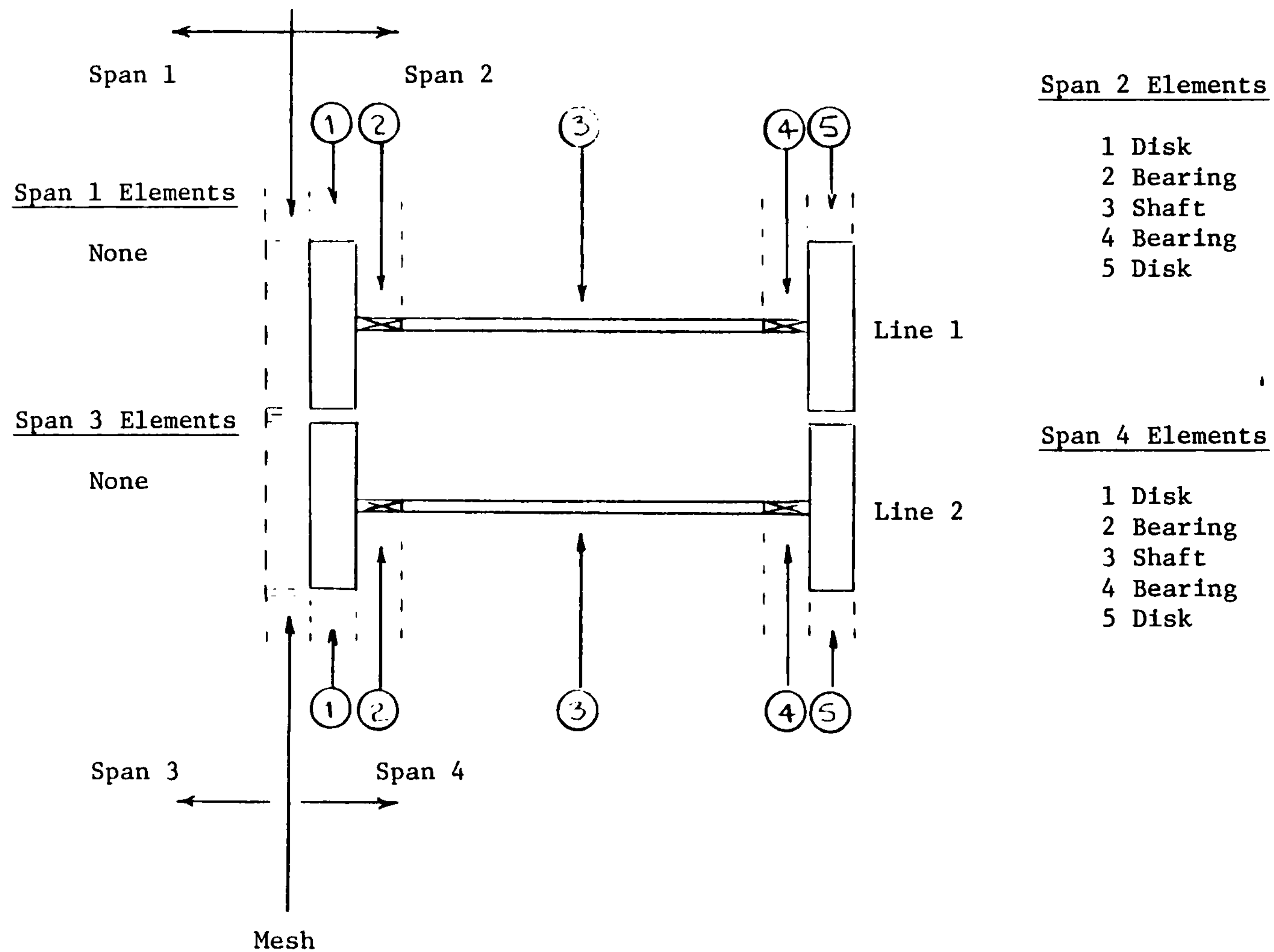


Figure 5.2 Sample Problem of Figure 5.1 Broken Down Into Model Elements

the gear mesh. Therefore, this natural frequency is approximately

$$\omega_n \Big|_{\substack{\text{fundamental} \\ \text{torsional}}} = \sqrt{k_{shaft}/J_{disk}}$$

For the numerical data of Figure 5.1 this resonance occurs at 88.8 rad/s which corresponds to tooth-passing frequency for 30.3 rpm operation. Since disk imbalance, misalignment, and gear errors are zero, it is apparent that the only source of dynamic excitation is parametric due to the time-varying tooth stiffness. Table 5.2 is the computer print-out of GRS for this simulation; see also Appendix A. Analyzing this output, many observations can be made.

Initial Left-Hand Side State Vector (Table 5.2(a))

The left-hand side state vector of Table 5.2(a) shows the translational and rotational response state solved for by GRS for both lines. The corresponding force state is not printed because it is identically zero. All responses are essentially zero except the mean shaft rotation. The GRS algorithm places a very weak (relative to rotor system stiffness) torsional spring to ground at the right-hand end of line 1 to stabilize the torsional solution, which is otherwise unconstrained. Hence, the left-hand torsional response is due to the 1.695 N-m (15 lb-in) input torque acting on the 41.52 N-m/rad (367.5 lb-in/rad) shaft stiffness. This calculates to 0.0408 rad negative rotation of line 1 left-hand rotation which is

INITIAL STATE VECTOR SOLUTION ITERATED 0 TIMES. IT IS :

LINE 1 RESPONSES,

W, RPS	ID	WXC	WXS	WYC	WYS	WZC	WZS	θXC	θXS	θYC	θYS	θZC
0	DC	-0.428D-07		-0.244D-07		0.000D 00		-0.374D-13		-0.103D-12		-0.408D-01
3	RS	-0.153D-16	-0.230D-16	0.557D-17	0.836D-17	0.000D 00	0.000D 00	-0.135D-22	-0.202D-22	-0.370D-22	-0.557D-22	0.577D-08
89	TPF	-0.159D-08	-0.798D-09	0.580D-09	0.290D-09	0.000D 00	0.000D 00	-0.139D-14	-0.696D-15	-0.382D-14	-0.191D-14	0.233D-06
86	T-R	0.115D-13	0.209D-13	-0.420D-14	-0.762D-14	0.000D 00	0.000D 00	0.101D-19	0.183D-19	0.277D-19	0.502D-19	-0.780D-09
92	T+R	0.229D-13	-0.298D-14	-0.833D-14	0.109D-14	0.000D 00	0.000D 00	0.200D-19	-0.260D-20	0.548D-19	-0.714D-20	0.159D-08
178	2*TP	0.877D-13	0.158D-12	-0.319D-13	-0.575D-13	0.000D 00	0.000D 00	0.765D-19	0.138D-18	0.210D-18	0.378D-18	-0.313D-07

LINE 2 RESPONSES,

W, RPS	ID	WXC	WXS	WYC	WYS	WZC	WZS	θXC	θXS	θYC	θYS	θZC
0	DC	0.428D-07		-0.556D-07		0.000D 00		0.374D-13		0.103D-12		0.408D-01
3	RS	0.153D-16	0.230D-16	-0.557D-17	-0.836D-17	0.000D 00	0.000D 00	0.135D-22	0.192D-22	0.342D-22	0.595D-22	0.577D-08
89	TPF	0.159D-08	0.798D-09	-0.580D-09	-0.290D-09	0.000D 00	0.000D 00	0.139D-14	0.696D-15	0.382D-14	0.191D-14	0.232D-06
86	T-R	-0.115D-13	-0.209D-13	0.420D-14	0.762D-14	0.000D 00	0.000D 00	-0.101D-19	-0.183D-19	-0.277D-19	-0.502D-19	-0.780D-09
92	T+R	-0.229D-13	0.298D-14	0.833D-14	-0.109D-14	0.000D 00	0.000D 00	-0.200D-19	0.260D-20	-0.548D-19	0.714D-20	0.159D-08
178	2*TP	-0.877D-13	-0.158D-12	0.319D-13	0.575D-13	0.000D 00	0.000D 00	-0.773D-19	-0.139D-18	-0.209D-18	-0.375D-18	-0.313D-07

Table 5.2(a) GRS Output For The Section 5.2 Sample Problem - Initial State Vector

AFTER MESH 1 STATE VECTOR IS,

LINE 1 RESPONSES,

H, RPS	ID	MXC	MXS	MYC	MYS	MZC	MZS	θXC	θXS	θYC	θYS	θZC
0	DC	-0.428D-07		-0.244D-07		0.000D 00		-0.374D-13		-0.103D-12		-0.408D-01
3	RS	-0.153D-16	-0.230D-16	0.557D-17	0.836D-17	0.000D 00	0.000D 00	-0.135D-22	-0.202D-22	-0.370D-22	-0.557D-22	0.577D-08
89	TPF	-0.159D-08	-0.798D-09	0.580D-09	0.290D-09	0.000D 00	0.000D 00	-0.139D-14	-0.696D-15	-0.382D-14	-0.191D-14	0.233D-06
86	T-R	0.115D-13	0.209D-13	-0.420D-14	-0.762D-14	0.000D 00	0.000D 00	0.101D-19	0.183D-19	0.277D-19	0.502D-19	-0.780D-09
92	T+R	0.229D-13	-0.298D-14	-0.833D-14	0.109D-14	0.000D 00	0.000D 00	0.200D-19	-0.260D-20	0.548D-19	-0.714D-20	0.159D-08
178	2*TP	0.877D-13	0.158D-12	-0.319D-13	-0.575D-13	0.000D 00	0.000D 00	0.765D-19	0.138D-18	0.210D-18	0.378D-18	-0.313D-07

LINE 2 RESPONSES,

H, RPS	ID	MXC	MXS	MYC	MYS	MZC	MZS	θXC	θXS	θYC	θYS	θZC
0	DC	0.428D-07		-0.556D-07		0.000D 00		0.374D-13		0.103D-12		0.408D-01
3	RS	0.153D-16	0.230D-16	-0.557D-17	-0.836D-17	0.000D 00	0.000D 00	0.135D-22	0.192D-22	0.342D-22	0.595D-22	0.577D-08
89	TPF	0.159D-08	0.798D-09	-0.580D-09	-0.290D-09	0.000D 00	0.000D 00	0.139D-14	0.696D-15	0.382D-14	0.191D-14	0.232D-06
86	T-R	-0.115D-13	-0.209D-13	0.420D-14	0.762D-14	0.000D 00	0.000D 00	-0.101D-19	-0.183D-19	-0.277D-19	-0.502D-19	-0.780D-09
92	T+R	-0.229D-13	0.298D-14	0.833D-14	-0.109D-14	0.000D 00	0.000D 00	-0.200D-19	0.260D-20	-0.548D-19	0.714D-20	0.159D-08
178	2*TP	-0.877D-13	-0.158D-12	0.319D-13	0.575D-13	0.000D 00	0.000D 00	-0.773D-19	-0.139D-18	-0.209D-18	-0.375D-18	-0.313D-07

LINE 1 FORCES,

H, RPS	ID	MXC	MXS	MYC	MYS	MZC	MZS	VXC	VXS	VYC	VYS	VZC
0	DC	0.000D 00		0.000D 00		-0.150D 02		-0.428D 01		0.156D 01		0.000D 00
3	RS	0.000D 00	0.000D 00	0.000D 00	0.000D 00	-0.536D-08	-0.804D-08	-0.153D-08	-0.230D-08	0.557D-09	0.836D-09	0.000D 00
89	TPF	0.000D 00	0.000D 00	0.000D 00	0.000D 00	-0.558D 00	-0.279D 00	-0.159D 00	-0.798D-01	0.580D-01	0.290D-01	0.000D 00
86	T-R	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.404D-05	0.733D-05	0.115D-05	0.209D-05	-0.420D-06	-0.762D-06	0.000D 00
92	T+R	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.801D-05	-0.104D-05	0.229D-05	-0.298D-06	-0.833D-06	0.109D-06	0.000D 00
178	2*TP	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.307D-04	0.552D-04	0.877D-05	0.158D-04	-0.319D-05	-0.575D-05	0.000D 00

LINE 2 FORCES,

H, RPS	ID	MXC	MXS	MYC	MYS	MZC	MZS	VXC	VXS	VYC	VYS	VZC
0	DC	0.000D 00		0.000D 00		-0.150D 02		0.428D 01		-0.156D 01		0.000D 00
3	RS	0.000D 00	0.000D 00	0.000D 00	0.000D 00	-0.536D-08	-0.804D-08	0.153D-08	0.230D-08	-0.557D-09	-0.836D-09	0.000D 00
89	TPF	0.000D 00	0.000D 00	0.000D 00	0.000D 00	-0.558D 00	-0.279D 00	0.159D 00	0.798D-01	-0.580D-01	-0.290D-01	0.000D 00
86	T-R	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.404D-05	0.733D-05	-0.115D-05	-0.209D-05	0.420D-06	0.762D-06	0.000D 00
92	T+R	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.801D-05	-0.104D-05	-0.229D-05	0.298D-06	0.833D-06	-0.109D-06	0.000D 00
178	2*TP	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.307D-04	0.552D-04	-0.877D-05	-0.158D-04	0.319D-05	0.575D-05	0.000D 00

Table 5.2(b) GRS Output For The Section 5.2 Sample Problem - State Vector On The Right Hand Side Of The Gear Mesh

AFTER ELEMENT 1 STATE VECTOR IS,

LINE 1 RESPONSES,

H, RPS	ID	HXC	HXS	HYC	HYS	HZC	HZS	ØXC	ØXS	ØYC	ØYS	ØZC
0	DC	-0.428D-07		-0.244D-07		0.000D 00		-0.374D-13		-0.103D-12		-0.408D-01
3	RS	-0.153D-16	-0.230D-16	0.557D-17	0.836D-17	0.000D 00	0.000D 00	-0.135D-22	-0.202D-22	-0.370D-22	-0.557D-22	0.577D-08
89	TPF	-0.159D-08	-0.798D-09	0.580D-09	0.290D-09	0.000D 00	0.000D 00	-0.139D-14	-0.696D-15	-0.382D-14	-0.191D-14	0.233D-06
86	T-R	0.115D-13	0.209D-13	-0.420D-14	-0.762D-14	0.000D 00	0.000D 00	0.101D-19	0.183D-19	0.277D-19	0.502D-19	-0.780D-09
92	T+R	0.229D-13	-0.298D-14	-0.833D-14	0.109D-14	0.000D 00	0.000D 00	0.200D-19	-0.260D-20	0.548D-19	-0.714D-20	0.159D-08
178	2*TP	0.877D-13	0.158D-12	-0.319D-13	-0.575D-13	0.000D 00	0.000D 00	0.765D-19	0.138D-18	0.210D-18	0.378D-18	-0.313D-07

LINE 2 RESPONSES,

H, RPS	ID	HXC	HXS	HYC	HYS	HZC	HZS	ØXC	ØXS	ØYC	ØYS	ØZC
0	DC	0.428D-07		-0.556D-07		0.000D 00		0.374D-13		0.103D-12		0.408D-01
3	RS	0.153D-16	0.230D-16	-0.557D-17	-0.836D-17	0.000D 00	0.000D 00	0.135D-22	0.192D-22	0.342D-22	0.595D-22	0.577D-08
89	TPF	0.159D-08	0.798D-09	-0.580D-09	-0.290D-09	0.000D 00	0.000D 00	0.139D-14	0.696D-15	0.382D-14	0.191D-14	0.232D-06
86	T-R	-0.115D-13	-0.209D-13	0.420D-14	0.762D-14	0.000D 00	0.000D 00	-0.101D-19	-0.183D-19	-0.277D-19	-0.502D-19	-0.780D-09
92	T+R	-0.229D-13	0.298D-14	0.833D-14	-0.109D-14	0.000D 00	0.000D 00	-0.200D-19	0.260D-20	-0.548D-19	0.714D-20	0.159D-08
178	2*TP	-0.877D-13	-0.158D-12	0.319D-13	0.575D-13	0.000D 00	0.000D 00	-0.773D-19	-0.139D-18	-0.209D-18	-0.375D-18	-0.313D-07

LINE 1 FORCES,

H, RPS	ID	MXC	MXS	MYC	MYS	MZC	MZS	VXC	VXS	VYC	VYS	VZC
0	DC	0.000D 00		0.000D 00		-0.150D 02		-0.428D 01		-0.244D 01		0.000D 00
3	RS	0.228D-22	-0.223D-22	0.185D-22	0.720D-23	-0.265D-08	-0.404D-08	-0.153D-08	-0.230D-08	0.557D-09	0.836D-09	0.000D 00
89	TPF	-0.240D-12	-0.183D-12	0.737D-12	0.346D-12	-0.558D 00	-0.279D 00	-0.159D 00	-0.798D-01	0.580D-01	0.290D-01	0.000D 00
86	T-R	0.115D-17	0.359D-17	-0.514D-17	-0.877D-17	0.378D-05	0.685D-05	0.115D-05	0.209D-05	-0.420D-06	-0.762D-06	0.000D 00
92	T+R	0.418D-17	0.214D-18	-0.112D-16	0.173D-17	0.863D-05	-0.112D-05	0.229D-05	-0.298D-06	-0.833D-06	0.109D-06	0.000D 00
178	2*TP	0.484D-16	0.110D-15	-0.164D-15	-0.286D-15	-0.154D-04	-0.276D-04	0.877D-05	0.158D-04	-0.319D-05	-0.575D-05	0.000D 00

LINE 2 FORCES,

H, RPS	ID	MXC	MXS	MYC	MYS	MZC	MZS	VXC	VXS	VYC	VYS	VZC
0	DC	0.000D 00		0.000D 00		-0.150D 02		0.428D 01		-0.556D 01		0.000D 00
3	RS	-0.246D-22	0.207D-22	-0.173D-22	-0.814D-23	-0.265D-08	-0.404D-08	0.153D-08	0.230D-08	-0.557D-09	-0.836D-09	0.000D 00
89	TPF	0.240D-12	0.183D-12	-0.737D-12	-0.346D-12	-0.558D 00	-0.279D 00	0.159D 00	0.798D-01	-0.580D-01	-0.290D-01	0.000D 00
86	T-R	-0.115D-17	-0.359D-17	0.514D-17	0.877D-17	0.378D-05	0.685D-05	-0.115D-05	-0.209D-05	0.420D-06	0.762D-06	0.000D 00
92	T+R	-0.418D-17	-0.214D-18	0.112D-16	-0.173D-17	0.863D-05	-0.112D-05	-0.229D-05	0.298D-06	0.833D-06	-0.109D-06	0.000D 00
178	2*TP	-0.491D-16	-0.111D-15	0.163D-15	0.284D-15	-0.154D-04	-0.276D-04	-0.877D-05	-0.158D-04	0.319D-05	0.575D-05	0.000D 00

Table 5.2(c) GRS Output For The Section 5.2 Sample Problem - State Vector On The Right Hand Side Of The First Disk

AFTER ELEMENT 2 STATE VECTOR IS,

LINE 1 RESPONSES,

H, RPS	ID	HXC	HXS	HYC	HYS	HZC	HZS	ΘXC	ΘXS	ΘYC	ΘYS	ΘZC
0	DC	-0.428D-07		-0.244D-07		0.000D 00		-0.374D-13		-0.103D-12		-0.408D-01
3	RS	-0.153D-16	-0.230D-16	0.557D-17	0.836D-17	0.000D 00	0.000D 00	-0.135D-22	-0.202D-22	-0.370D-22	-0.557D-22	0.577D-08
89	TPF	-0.159D-08	-0.798D-09	0.580D-09	0.290D-09	0.000D 00	0.000D 00	-0.139D-14	-0.696D-15	-0.382D-14	-0.191D-14	0.233D-06
86	T-R	0.115D-13	0.209D-13	-0.420D-14	-0.762D-14	0.000D 00	0.000D 00	0.101D-19	0.183D-19	0.277D-19	0.502D-19	-0.780D-09
92	T+R	0.229D-13	-0.298D-14	-0.833D-14	0.109D-14	0.000D 00	0.000D 00	0.200D-19	-0.260D-20	0.548D-19	-0.714D-20	0.159D-08
178	2*TP	0.877D-13	0.158D-12	-0.319D-13	-0.575D-13	0.000D 00	0.000D 00	0.765D-19	0.138D-18	0.210D-18	0.378D-18	-0.313D-07

LINE 2 RESPONSES,

H, RPS	ID	HXC	HXS	HYC	HYS	HZC	HZS	ΘXC	ΘXS	ΘYC	ΘYS	ΘZC
0	DC	0.428D-07		-0.556D-07		0.000D 00		0.374D-13		0.103D-12		0.408D-01
3	RS	0.153D-16	0.230D-16	-0.557D-17	-0.836D-17	0.000D 00	0.000D 00	0.135D-22	0.192D-22	0.342D-22	0.595D-22	0.577D-08
89	TPF	0.159D-08	0.798D-09	-0.580D-09	-0.290D-09	0.000D 00	0.000D 00	0.139D-14	0.696D-15	0.382D-14	0.191D-14	0.232D-06
86	T-R	-0.115D-13	-0.209D-13	0.420D-14	0.762D-14	0.000D 00	0.000D 00	-0.101D-19	-0.183D-19	-0.277D-19	-0.502D-19	-0.780D-09
92	T+R	-0.229D-13	0.298D-14	0.833D-14	-0.109D-14	0.000D 00	0.000D 00	-0.200D-19	0.260D-20	-0.548D-19	0.714D-20	0.159D-08
178	2*TP	-0.877D-13	-0.158D-12	0.319D-13	0.575D-13	0.000D 00	0.000D 00	-0.773D-19	-0.139D-18	-0.209D-18	-0.375D-18	-0.313D-07

LINE 1 FORCES,

H, RPS	ID	MXC	MXS	MYC	MYS	MZC	MZS	VXC	VXS	VYC	VYS	VZC
0	DC	0.374D-05		0.103D-04		-0.150D 02		-0.171D-05		0.623D-06		0.000D 00
3	RS	0.135D-14	0.202D-14	0.370D-14	0.557D-14	-0.268D-08	-0.402D-08	-0.613D-15	-0.925D-15	0.224D-15	0.335D-15	0.000D 00
89	TPF	0.139D-06	0.696D-07	0.382D-06	0.191D-06	-0.558D 00	-0.279D 00	-0.637D-07	-0.319D-07	0.232D-07	0.116D-07	0.000D 00
86	T-R	-0.101D-11	-0.183D-11	-0.277D-11	-0.502D-11	0.378D-05	0.685D-05	0.461D-12	0.837D-12	-0.168D-12	-0.304D-12	0.000D 00
92	T+R	-0.200D-11	0.260D-12	-0.548D-11	0.714D-12	0.863D-05	-0.112D-05	0.914D-12	-0.119D-12	-0.333D-12	0.433D-13	0.000D 00
178	2*TP	-0.765D-11	-0.138D-10	-0.210D-10	-0.378D-10	-0.154D-04	-0.276D-04	0.351D-11	0.631D-11	-0.128D-11	-0.229D-11	0.000D 00

LINE 2 FORCES,

H, RPS	ID	MXC	MXS	MYC	MYS	MZC	MZS	VXC	VXS	VYC	VYS	VZC
0	DC	-0.374D-05		-0.103D-04		-0.150D 02		0.171D-05		-0.623D-06		0.000D 00
3	RS	-0.135D-14	-0.192D-14	-0.342D-14	-0.595D-14	-0.268D-08	-0.402D-08	0.566D-15	0.102D-14	-0.225D-15	-0.318D-15	0.000D 00
89	TPF	-0.139D-06	-0.696D-07	-0.382D-06	-0.191D-06	-0.558D 00	-0.279D 00	0.637D-07	0.319D-07	-0.232D-07	-0.116D-07	0.000D 00
86	T-R	0.101D-11	0.183D-11	0.277D-11	0.502D-11	0.378D-05	0.685D-05	-0.461D-12	-0.837D-12	0.168D-12	0.304D-12	0.000D 00
92	T+R	0.200D-11	-0.260D-12	0.548D-11	-0.714D-12	0.863D-05	-0.112D-05	-0.914D-12	0.119D-12	0.333D-12	-0.433D-13	0.000D 00
178	2*TP	0.773D-11	0.139D-10	0.209D-10	0.375D-10	-0.154D-04	-0.276D-04	-0.348D-11	-0.624D-11	0.130D-11	0.232D-11	0.000D 00

Table 5.2(d) GRS Output For The Section 5.2 Sample Problem - State Vector On The Right Hand Side Of The First Bearing

AFTER ELEMENT 3 STATE VECTOR IS,

LINE 1 RESPONSES,

H, RPS	ID	HXC	HXS	HYC	HYS	HZC	HZS	ØXC	ØXS	ØYC	ØYS	ØZC
0	DC	-0.507D-11		-0.400D-07		0.000D 00		0.216D-12		0.116D-11		0.132D-06
3	RS	0.294D-18	0.442D-18	-0.121D-18	-0.193D-18	0.000D 00	0.000D 00	-0.138D-19	-0.295D-19	-0.440D-19	-0.442D-19	0.578D-08
89	TPF	0.413D-12	-0.158D-11	-0.134D-12	0.376D-12	0.000D 00	0.000D 00	-0.288D-13	0.933D-13	-0.107D-12	0.393D-12	0.152D-02
86	T-R	-0.621D-19	-0.110D-18	0.323D-20	-0.105D-19	0.000D 00	0.000D 00	0.632D-20	0.143D-19	0.219D-19	0.484D-19	-0.111D-07
92	T+R	0.955D-19	-0.218D-19	0.147D-19	-0.418D-20	0.000D 00	0.000D 00	0.231D-19	-0.252D-20	0.553D-19	-0.764D-20	-0.219D-07
178	2*TP	0.488D-15	0.309D-15	-0.840D-16	0.382D-16	0.000D 00	0.000D 00	-0.209D-16	0.812D-17	-0.122D-15	-0.773D-16	0.104D-07

LINE 2 RESPONSES,

H, RPS	ID	HXC	HXS	HYC	HYS	HZC	HZS	ØXC	ØXS	ØYC	ØYS	ØZC
0	DC	0.525D-11		-0.400D-07		0.000D 00		-0.121D-12		-0.110D-11		0.816D-01
3	RS	0.796D-18	-0.368D-18	0.438D-19	-0.190D-18	0.000D 00	0.000D 00	-0.224D-20	0.354D-19	0.599D-19	-0.370D-18	0.578D-08
89	TPF	-0.218D-12	0.608D-12	0.603D-13	-0.409D-12	0.000D 00	0.000D 00	-0.209D-14	-0.766D-13	0.127D-12	-0.119D-12	0.152D-02
86	T-R	0.317D-19	0.859D-19	-0.420D-19	-0.536D-19	0.000D 00	0.000D 00	-0.130D-19	-0.269D-19	-0.404D-19	-0.664D-19	-0.111D-07
92	T+R	-0.132D-18	0.218D-19	0.374D-19	-0.786D-20	0.000D 00	0.000D 00	-0.103D-19	-0.177D-21	-0.412D-19	0.561D-20	-0.219D-07
178	2*TP	-0.552D-15	-0.143D-14	0.117D-15	-0.108D-15	0.000D 00	0.000D 00	0.114D-15	0.762D-16	0.145D-16	-0.190D-16	0.104D-07

LINE 1 FORCES,

H, RPS	ID	MXC	MXS	MYC	MYS	MZC	MZS	VXC	VXS	VYC	VYS	VZC
0	DC	-0.374D-05		-0.103D-04		-0.150D 02		-0.171D-05		0.623D-06		0.000D 00
3	RS	-0.134D-14	-0.199D-14	-0.366D-14	-0.553D-14	-0.268D-08	-0.402D-08	-0.613D-15	-0.925D-15	0.224D-15	0.335D-15	0.000D 00
89	TPF	-0.139D-06	-0.697D-07	-0.382D-06	-0.192D-06	-0.558D 00	-0.279D 00	-0.637D-07	-0.319D-07	0.232D-07	0.116D-07	0.000D 00
86	T-R	0.101D-11	0.183D-11	0.277D-11	0.502D-11	0.378D-05	0.685D-05	0.461D-12	0.837D-12	-0.168D-12	-0.304D-12	0.000D 00
92	T+R	0.200D-11	-0.260D-12	0.548D-11	-0.714D-12	0.863D-05	-0.112D-05	0.914D-12	-0.119D-12	0.333D-12	0.433D-13	0.000D 00
178	2*TP	0.767D-11	0.138D-10	0.211D-10	0.379D-10	-0.154D-04	-0.276D-04	0.351D-11	0.631D-11	-0.128D-11	-0.229D-11	0.000D 00

LINE 2 FORCES,

H, RPS	ID	MXC	MXS	MYC	MYS	MZC	MZS	VXC	VXS	VYC	VYS	VZC
0	DC	0.374D-05		0.103D-04		-0.150D 02		0.171D-05		-0.623D-06		0.000D 00
3	RS	0.135D-14	0.189D-14	0.337D-14	0.630D-14	-0.268D-08	-0.402D-08	0.566D-15	0.102D-14	-0.225D-15	-0.318D-15	0.000D 00
89	TPF	0.139D-06	0.697D-07	0.382D-06	0.191D-06	-0.558D 00	-0.279D 00	0.637D-07	0.319D-07	-0.232D-07	-0.116D-07	0.000D 00
86	T-R	-0.101D-11	-0.183D-11	-0.277D-11	-0.502D-11	0.378D-05	0.685D-05	-0.461D-12	-0.837D-12	0.168D-12	0.304D-12	0.000D 00
92	T+R	-0.200D-11	0.260D-12	-0.548D-11	0.714D-12	0.863D-05	-0.112D-05	-0.914D-12	0.119D-12	0.333D-12	-0.433D-13	0.000D 00
178	2*TP	-0.784D-11	-0.139D-10	-0.209D-10	-0.375D-10	-0.154D-04	-0.276D-04	-0.348D-11	-0.624D-11	0.130D-11	0.232D-11	0.000D 00

Table 5.2(e) GRS Output For The Section 5.2 Sample Problem - State Vector On The Right Hand Side Of The Shaft

AFTER ELEMENT 4 STATE VECTOR IS,

LINE 1 RESPONSES,

H, RPS	ID	HXC	HXS	HYC	HYS	HZC	HZS	θXC	θXS	θYC	θYS	θZC
0	DC	-0.507D-11		-0.400D-07		0.000D 00		0.216D-12		0.116D-11		0.132D-06
3	RS	0.294D-18	0.442D-18	-0.121D-18	-0.193D-18	0.000D 00	0.000D 00	-0.138D-19	-0.295D-19	-0.440D-19	-0.442D-19	0.578D-08
89	TPF	0.413D-12	-0.158D-11	-0.134D-12	0.376D-12	0.000D 00	0.000D 00	-0.288D-13	0.933D-13	-0.107D-12	0.393D-12	0.152D-02
86	T-R	-0.621D-19	-0.110D-18	0.323D-20	-0.105D-19	0.000D 00	0.000D 00	0.632D-20	0.143D-19	0.219D-19	0.484D-19	-0.111D-07
92	T+R	0.955D-19	-0.218D-19	0.147D-19	-0.418D-20	0.000D 00	0.000D 00	0.231D-19	-0.252D-20	0.553D-19	-0.764D-20	-0.219D-07
178	2*TP	0.488D-15	0.309D-15	-0.840D-16	0.382D-16	0.000D 00	0.000D 00	-0.209D-16	0.812D-17	-0.122D-15	-0.773D-16	0.104D-07

LINE 2 RESPONSES,

H, RPS	ID	HXC	HXS	HYC	HYS	HZC	HZS	θXC	θXS	θYC	θYS	θZC
0	DC	0.525D-11		-0.400D-07		0.000D 00		-0.121D-12		-0.110D-11		0.816D-01
3	RS	0.796D-18	-0.368D-18	0.438D-19	-0.190D-18	0.000D 00	0.000D 00	-0.224D-20	0.354D-19	0.599D-19	-0.370D-18	0.578D-08
89	TPF	-0.218D-12	0.608D-12	0.603D-13	-0.409D-12	0.000D 00	0.000D 00	-0.209D-14	-0.766D-13	0.127D-12	-0.119D-12	0.152D-02
86	T-R	0.317D-19	0.859D-19	-0.420D-19	-0.536D-19	0.000D 00	0.000D 00	-0.130D-19	-0.269D-19	-0.404D-19	-0.664D-19	-0.111D-07
92	T+R	-0.132D-18	0.218D-19	0.374D-19	-0.786D-20	0.000D 00	0.000D 00	-0.103D-19	-0.177D-21	-0.412D-19	0.561D-20	-0.219D-07
178	2*TP	-0.552D-15	-0.143D-14	0.117D-15	-0.108D-15	0.000D 00	0.000D 00	0.114D-15	0.762D-16	0.145D-16	-0.190D-16	0.104D-07

LINE 1 FORCES,

H, RPS	ID	MXC	MXS	MYC	MYS	MZC	MZS	VXC	VXS	VYC	VYS	VZC
0	DC	-0.253D-04		-0.126D-03		-0.150D 02		0.506D-03		0.400D 01		0.000D 00
3	RS	0.138D-11	0.295D-11	0.440D-11	0.442D-11	-0.271D-08	-0.400D-08	-0.294D-10	-0.442D-10	0.121D-10	0.193D-10	0.000D 00
89	TPF	0.274D-05	-0.940D-05	0.103D-04	-0.395D-04	-0.558D 00	-0.279D 00	-0.414D-04	0.158D-03	0.134D-04	-0.376D-04	0.000D 00
86	T-R	0.375D-12	0.398D-12	0.579D-12	0.180D-12	0.378D-05	0.685D-05	0.667D-11	0.119D-10	-0.491D-12	0.742D-12	0.000D 00
92	T+R	-0.314D-12	-0.834D-14	-0.450D-13	0.494D-13	0.863D-05	-0.112D-05	-0.864D-11	0.206D-11	-0.181D-11	0.461D-12	0.000D 00
178	2*TP	0.210D-08	-0.798D-09	0.122D-07	0.777D-08	-0.154D-04	-0.276D-04	-0.488D-07	-0.309D-07	0.840D-08	-0.382D-08	0.000D 00

LINE 2 FORCES,

H, RPS	ID	MXC	MXS	MYC	MYS	MZC	MZS	VXC	VXS	VYC	VYS	VZC
0	DC	0.158D-04		0.120D-03		-0.150D 02		-0.523D-03		0.400D 01		0.000D 00
3	RS	0.225D-12	-0.354D-11	-0.598D-11	0.370D-10	-0.271D-08	-0.400D-08	-0.796D-10	0.368D-10	-0.438D-11	0.190D-10	0.000D 00
89	TPF	0.348D-06	0.773D-05	-0.123D-04	0.121D-04	-0.558D 00	-0.279D 00	0.218D-04	-0.608D-04	-0.605D-05	0.409D-04	0.000D 00
86	T-R	0.294D-12	0.868D-12	0.128D-11	0.162D-11	0.378D-05	0.685D-05	-0.363D-11	-0.942D-11	0.436D-11	0.566D-11	0.000D 00
92	T+R	-0.962D-12	0.278D-12	-0.136D-11	0.154D-12	0.863D-05	-0.112D-05	0.123D-10	-0.206D-11	-0.340D-11	0.743D-12	0.000D 00
178	2*TP	-0.114D-07	-0.763D-08	-0.147D-08	0.187D-08	-0.154D-04	-0.276D-04	0.552D-07	0.143D-06	-0.117D-07	0.108D-07	0.000D 00

Table 5.2(f) GRS Output For The Section 5.2 Sample Problem
State Vector On The Right Hand Side Of The Second Bearing

AFTER ELEMENT 5 STATE VECTOR IS,

LINE 1 RESPONSES,

W, RPS	ID	MXC	MXS	MYC	MYS	MZC	MZS	θXC	θXS	θYC	θYS	θZC
0	DC	-0.507D-11		-0.400D-07		0.000D 00		0.216D-12		0.116D-11		0.132D-06
3	RS	0.294D-18	0.442D-18	-0.121D-18	-0.193D-18	0.000D 00	0.000D 00	-0.138D-19	-0.295D-19	-0.440D-19	-0.442D-19	0.578D-08
89	TPF	0.413D-12	-0.158D-11	-0.134D-12	0.376D-12	0.000D 00	0.000D 00	-0.288D-13	0.933D-13	-0.107D-12	0.393D-12	0.152D-02
86	T-R	-0.621D-19	-0.110D-18	0.323D-20	-0.105D-19	0.000D 00	0.000D 00	0.632D-20	0.143D-19	0.219D-19	0.484D-19	-0.111D-07
92	T+R	0.955D-19	-0.218D-19	0.147D-19	-0.418D-20	0.000D 00	0.000D 00	0.231D-19	-0.252D-20	0.553D-19	-0.764D-20	-0.219D-07
178	2*TP	0.488D-15	0.309D-15	-0.840D-16	0.382D-16	0.000D 00	0.000D 00	-0.209D-16	0.812D-17	-0.122D-15	-0.773D-16	0.104D-07

LINE 2 RESPONSES,

W, RPS	ID	MXC	MXS	MYC	MYS	MZC	MZS	θXC	θXS	θYC	θYS	θZC
0	DC	0.525D-11		-0.400D-07		0.000D 00		-0.121D-12		-0.110D-11		0.816D-01
3	RS	0.796D-18	-0.368D-18	0.438D-19	-0.190D-18	0.000D 00	0.000D 00	-0.224D-20	0.354D-19	0.599D-19	-0.370D-18	0.578D-08
89	TPF	-0.218D-12	0.608D-12	0.603D-13	-0.409D-12	0.000D 00	0.000D 00	-0.209D-14	-0.766D-13	0.127D-12	-0.119D-12	0.152D-02
86	T-R	0.317D-19	0.859D-19	-0.420D-19	-0.536D-19	0.000D 00	0.000D 00	-0.130D-19	-0.269D-19	-0.404D-19	-0.664D-19	-0.111D-07
92	T+R	-0.132D-18	0.218D-19	0.374D-19	-0.786D-20	0.000D 00	0.000D 00	-0.103D-19	-0.177D-21	-0.412D-19	0.561D-20	-0.219D-07
178	2*TP	-0.552D-15	-0.143D-14	0.117D-15	-0.108D-15	0.000D 00	0.000D 00	0.114D-15	0.762D-16	0.145D-16	-0.190D-16	0.104D-07

LINE 1 FORCES,

W, RPS	ID	MXC	MXS	MYC	MYS	MZC	MZS	VXC	VXS	VYC	VYS	VZC
0	DC	-0.253D-04		-0.126D-03		-0.150D 02		0.506D-03		-0.964D-04		0.000D 00
3	RS	0.138D-11	0.295D-11	0.440D-11	0.442D-11	0.578D-12	0.854D-12	-0.294D-10	-0.442D-10	0.121D-10	0.193D-10	0.000D 00
89	TPF	0.274D-05	-0.940D-05	0.103D-04	-0.395D-04	0.152D-06	0.760D-07	-0.414D-04	0.158D-03	0.134D-04	-0.376D-04	0.000D 00
86	T-R	0.375D-12	0.398D-12	0.579D-12	0.180D-12	-0.111D-11	-0.200D-11	0.667D-11	0.119D-10	-0.491D-12	0.742D-12	0.000D 00
92	T+R	-0.314D-12	-0.834D-14	-0.450D-13	0.494D-13	-0.219D-11	0.285D-12	-0.864D-11	0.206D-11	-0.181D-11	0.461D-12	0.000D 00
178	2*TP	0.210D-08	-0.798D-09	0.122D-07	0.777D-08	0.829D-12	0.177D-11	-0.488D-07	-0.309D-07	0.840D-08	-0.382D-08	0.000D 00

LINE 2 FORCES,

W, RPS	ID	MXC	MXS	MYC	MYS	MZC	MZS	VXC	VXS	VYC	VYS	VZC
0	DC	0.158D-04		0.120D-03		-0.150D 02		-0.523D-03		0.711D-04		0.000D 00
3	RS	0.225D-12	-0.354D-11	-0.598D-11	0.370D-10	0.139D-16	-0.319D-16	-0.796D-10	0.368D-10	-0.438D-11	0.190D-10	0.000D 00
89	TPF	0.348D-06	0.773D-05	-0.123D-04	0.121D-04	-0.277D-13	-0.283D-14	0.218D-04	-0.608D-04	-0.605D-05	0.409D-04	0.000D 00
86	T-R	0.294D-12	0.868D-12	0.128D-11	0.162D-11	-0.589D-18	-0.104D-17	-0.363D-11	-0.942D-11	0.436D-11	0.566D-11	0.000D 00
92	T+R	-0.962D-12	0.278D-12	-0.136D-11	0.154D-12	-0.496D-19	-0.244D-18	0.123D-10	-0.206D-11	-0.340D-11	0.743D-12	0.000D 00
178	2*TP	-0.114D-07	-0.763D-08	-0.147D-08	0.187D-08	-0.155D-12	-0.344D-13	0.552D-07	0.143D-06	-0.117D-07	0.108D-07	0.000D 00

Table 5.2(g) GRS Output For The Section 5.2 Sample Problem
State Vector On The Right Hand Side Of The Second Disk

FINAL STATE VECTOR IS,

LINE 1 RESPONSES,

W, RPS	ID	WXC	WXS	WYC	WYS	WZC	WZS	ΘXC	ΘXS	ΘYC	ΘYS	ΘZC
0	DC	-0.507D-11		-0.400D-07		0.000D 00		0.216D-12		0.116D-11		0.132D-06
3	RS	0.294D-18	0.442D-18	-0.121D-18	-0.193D-18	0.000D 00	0.000D 00	-0.138D-19	-0.295D-19	-0.440D-19	-0.442D-19	0.578D-08
89	TPF	0.413D-12	-0.158D-11	-0.134D-12	0.376D-12	0.000D 00	0.000D 00	-0.288D-13	0.933D-13	-0.107D-12	0.393D-12	0.152D-02
86	T-R	-0.621D-19	-0.110D-18	0.323D-20	-0.105D-19	0.000D 00	0.000D 00	0.632D-20	0.143D-19	0.219D-19	0.484D-19	-0.111D-07
92	T+R	0.955D-19	-0.218D-19	0.147D-19	-0.418D-20	0.000D 00	0.000D 00	0.231D-19	-0.252D-20	0.553D-19	-0.764D-20	-0.219D-07
178	2*TP	0.488D-15	0.309D-15	-0.840D-16	0.382D-16	0.000D 00	0.000D 00	-0.209D-16	0.812D-17	-0.122D-15	-0.773D-16	0.104D-07

LINE 2 RESPONSES,

W, RPS	ID	WXC	WXS	WYC	WYS	WZC	WZS	ΘXC	ΘXS	ΘYC	ΘYS	ΘZC
0	DC	0.525D-11		-0.400D-07		0.000D 00		-0.121D-12		-0.110D-11		0.816D-01
3	RS	0.796D-18	-0.368D-18	0.438D-19	-0.190D-18	0.000D 00	0.000D 00	-0.224D-20	0.354D-19	0.599D-19	-0.370D-18	0.578D-08
89	TPF	-0.218D-12	0.608D-12	0.603D-13	-0.409D-12	0.000D 00	0.000D 00	-0.209D-14	-0.766D-13	0.127D-12	-0.119D-12	0.152D-02
86	T-R	0.317D-19	0.859D-19	-0.420D-19	-0.536D-19	0.000D 00	0.000D 00	-0.130D-19	-0.269D-19	-0.404D-19	-0.664D-19	-0.111D-07
92	T+R	-0.132D-18	0.218D-19	0.374D-19	-0.786D-20	0.000D 00	0.000D 00	-0.103D-19	-0.177D-21	-0.412D-19	0.561D-20	-0.219D-07
178	2*TP	-0.552D-15	-0.143D-14	0.117D-15	-0.108D-15	0.000D 00	0.000D 00	0.114D-15	0.762D-16	0.145D-16	-0.190D-16	0.104D-07

LINE 1 FORCES,

W, RPS	ID	MXC	MXS	MYC	MYS	MZC	MZS	VXC	VXS	VYC	VYS	VZC
0	DC	-0.253D-04		-0.126D-03		-0.177D-09		0.506D-03		-0.964D-04		0.000D 00
3	RS	0.138D-11	0.295D-11	0.440D-11	0.442D-11	-0.160D-16	0.586D-16	-0.294D-10	-0.442D-10	0.121D-10	0.193D-10	0.000D 00
89	TPF	0.274D-05	-0.940D-05	0.103D-04	-0.395D-04	-0.397D-13	-0.951D-11	-0.414D-04	0.158D-03	0.134D-04	-0.376D-04	0.000D 00
86	T-R	0.375D-12	0.398D-12	0.579D-12	0.180D-12	-0.613D-18	-0.119D-17	0.667D-11	0.119D-10	-0.491D-12	0.742D-12	0.000D 00
92	T+R	-0.314D-12	-0.834D-14	-0.450D-13	0.494D-13	0.125D-18	0.677D-20	-0.864D-11	0.206D-11	-0.181D-11	0.461D-12	0.000D 00
178	2*TP	0.210D-08	-0.798D-09	0.122D-07	0.777D-08	-0.216D-12	-0.108D-12	-0.488D-07	-0.309D-07	0.840D-08	-0.382D-08	0.000D 00

LINE 2 FORCES,

W, RPS	ID	MXC	MXS	MYC	MYS	MZC	MZS	VXC	VXS	VYC	VYS	VZC
0	DC	0.158D-04		0.120D-03		-0.120D-09		-0.523D-03		0.711D-04		0.000D 00
3	RS	0.225D-12	-0.354D-11	-0.598D-11	0.370D-10	0.139D-16	-0.319D-16	-0.796D-10	0.368D-10	-0.438D-11	0.190D-10	0.000D 00
89	TPF	0.348D-06	0.773D-05	-0.123D-04	0.121D-04	-0.277D-13	-0.283D-14	0.218D-04	-0.608D-04	-0.605D-05	0.409D-04	0.000D 00
86	T-R	0.294D-12	0.868D-12	0.128D-11	0.162D-11	-0.589D-18	-0.104D-17	-0.363D-11	-0.942D-11	0.436D-11	0.566D-11	0.000D 00
92	T+R	-0.962D-12	0.278D-12	-0.136D-11	0.154D-12	-0.496D-19	-0.244D-18	0.123D-10	-0.206D-11	-0.340D-11	0.743D-12	0.000D 00
178	2*TP	-0.114D-07	-0.763D-08	-0.147D-08	0.187D-08	-0.155D-12	-0.344D-13	0.552D-07	0.143D-06	-0.117D-07	0.108D-07	0.000D 00

Table 5.2(h) GRS Output For The Section 5.2 Sample Problem
Final Right-Hand Side State Vector

verified in Table 5.2(a). Again, since the mesh is very much stiffer than the shaft, line 2 left-hand mean rotation is the same magnitude as for line 1, only positive, as seen in Table 5.2(a).

State Vector to the Right of the Mesh (Table 5.2(b))

The state vector to the right of the mesh is shown in Table 5.2(b). The following observations are made

- The x and y internal moments and the z internal shear remain zero as we know the spur gear treatment of the current work does not affect these terms.

- The mean torque transmission across the mesh is handled properly with the internal z moment shown as 1.69 N-m (15 lb-in) with negative sign for both lines.

- The mean mesh force along the line-of-action calculates to 20.29N (4.56 lb) which is resolved into 19.06N (4.28 lb) in the x direction and 6.94N (1.56 lb) in the y direction. These magnitudes are verified in Table 5.2(b) and their signs are in accordance with the physical problem and the global sign convention.

- Dynamic internal forces and moments are very low with the exception of those at tooth-passing frequency. This is expected since the example is designed for mesh parametric excitation at this frequency.

- All responses are identical to those before the mesh which is expected for a "point" transfer matrix which the mesh, disk, and bearing all are. Changes in response occur only across a "field" transfer matrix; the elastic shaft in rotor analysis.

State Vector to the Right of the First Disk (Table 5.2 (c))

The state vector to the right of the first disk is shown in Table 5.2(c). The following observations are made.

- The disk is a "point" transfer matrix and again no change in response is seen.

- The dynamic responses, however small, are seen to produce shears and moments, as expected, due to the disk inertial characteristics. Taking one of these terms at random to check magnitude and sign, consider the line 2 z moment sine term at tooth-passing frequency. The value is -0.032 N-m (-0.279 lb-in) according to Table 5.2(c). The corresponding response is -0.343×10^{-6} rad which produces a differential moment across the disk equal to $\theta_z^{\sigma s} I_p \omega^2$ which calculates out to -0.032 N-m (-0.279 lb-in).

The sign is consistent with the system sign convention and hence checks.

- Checking the mean x and y shear terms notice there is no appreciable change in the x term. The y shear terms change by -17.78N (-4 lb) which is the disk weight.

State Vector to the Right of the First Bearing (Table 5.2(d))

The state vector to the right of the first bearing is shown in Table 5.2(d). The following observations are made.

- The bearing is a "point" transfer matrix and no changes in response occur.

- Changes in the force state occur in relation to deflections, velocities, and user provided bearing stiffness and damping. Taking one term at random to check magnitude and sign consider line 1 cosine at ω in the y direction. The response is $+1.415 \times 10^{-19}$m ($+0.557 \times 10^{-17}$ in) and the stiffness is 1.75×10^{10} N/m (10^{8} lb/in). This should result in a change in the corresponding shear term of -2.48×10^{-9}N (-5.57×10^{10}). The value of $V_{y1}^{\omega s}$ from Table 5.2(c) is 2.48×10^{-9}N (0.557×10^{-9} lb) and from Table 5.2(d) is 9.96×10^{-16}N (0.224×10^{-15} lb), essentially zero; thus verifying this check.

State Vector to the Right of the Shaft (Table 5.2(e))

The state vector to the right of the shaft is shown in Table 5.2(e). The following observations are made.

- Recalling the equations from section 4.3 for the elastic shaft. No changes in shear terms are expected and comparing Tables 5.2(d) and (e) in fact none occur.

- Changes in translational and rotational responses and internal moments will all occur in relation to one another. Taking one term at random to check magnitude and sign consider line 1 cosine at 2σ in the x direction. Recalling

$$M_x^R = - LV_y^L + M_x^L \qquad (4.15\ a)$$

where from Table 5.2(d)

$$M_{x1}^{2\sigma c}\Big|^L = -8.643 \times 10^{-13}\ \text{N-m}\ (-0.765 \times 10^{-11}\ \text{lb-in})$$

$$V_{y1}^{2\sigma c}\Big|^L = -5.69 \times 10^{-12}\ \text{N}\ (-0.128 \times 10^{-11}\ \text{lb})$$

then, where L = 0.3048m (12 in)

$$M_{x1}^{2\sigma c}\Big|^{R} = -0.3048\ (-5.69 \times 10^{-12}) + (-8.643 \times 10^{-13})$$

$$= 0.87 \times 10^{-12}\ \text{N-m}\ (0.77 \times 10^{-11}\ \text{lb-in})$$

This checks with $M_{x1}^{2\sigma c}$ of Table 5.2(e).

The transfer matrix steps across the second bearing (Table 5.2(f)) and second disk (Table 5.2(g)) exhibit the same behavior as the first bearing and disk and therefore will not be explored in detail here. Discussion continues with the last transfer matrix.

Final State Vector on the Right-Hand Side of the System (Table 5.2 (h))

The final right-hand side state vector is shown in Table 5.2 (h). The only differences between the last transfer matrix and the identity matrix are as follows.

- A ficticious spring is attached to the right-hand side of line 1 to stabilize the solution relative to the shaft torsional response; which otherwise is unconstrained. This stiffness is chosen to be extremely weak relative to system stiffness elements. This provides a reference for mean torsional response and otherwise does not effect dynamics.

- The user specified right-hand side torques are applied in the last transfer matrix. Notice from Table 5.2(g) that the mean torques in both lines equal in magnitude but opposite in sign to the applied torques. Thus, the application of the last transfer matrix is seen in Table 5.2(h) to produce the (essentially) zero force state associated with the free-free boundary conditions.

Resonance Excitation in a Speed Run-up Analysis

The current example problem was run for varying shaft speeds to show the levels of torsional response when the 88.8 Hz resonance corresponds to $\sigma+\omega$, σ, $\sigma-\omega$, and ω. The simplified results of this speed run-up analysis showing only torsional oscillation of the line 1 right-hand end is shown in Table 5.3 for the various frequencies. Tabulated with response is the corresponding shear stress (τ) on the shaft surface and the von Mises stress (σ_{vm}) calculated according to

$$\sigma_{vm} = \sqrt{3\tau^2} \tag{5.3}$$

based only on the uniaxial stress state at the particular frequency.

Notice that for this example problem the fatique stresses due to gear mesh parametric excitation at resonance are not significant (except at σ), though one could certainly devise a problem in which they are. This example shows the mesh-excitation energy to produce

S.I. (θ, x10^6 rad; τ, x10^3 N/m^2; σ_{vm}, x10^3 N/m^2)

N,	ω			$\sigma+\omega$			σ			$\sigma+\omega$		
rpm	θ	τ	σ_{vm}	θ	τ	σ_{vm}	θ	τ	σ_{vm}	θ	τ	σ_{vm}
28.7	0.01	0.	0.	0.01	0.	0.	4.0	1.6	2.6	0.4	0.03	0.03
29.2	0.01	0.	0.	0.01	0.	0.	6.1	2.3	4.2	6.59	2.56	4.44
29.7	0.01	0.	0.	0.02	0.	0.	10.8	4.2	7.1	0.05	0.03	0.03
30.3	0.01	0.	0.	0.02	0.	0.	1700.	660.3	1143.	0.02	0.	0.
30.8	0.01	0.	0.	0.04	0.03	0.03	11.9	4.5	8.1	0.01	0.	0.
31.4	0.01	0.	0.	6.59	2.56	4.44	5.4	2.3	3.6	0.01	0.	0.
32.0	0.01	0.	0.	0.04	0.03	0.03	3.5	1.3	2.3	0.01	0.	0.
840.0	0.63	0.27	0.42	0.	0.	0.	0.	0.	0.	0.	0.	0.
847.8	47.5	18.5	31.9	0.	0.	0.	0.	0.	0.	0.	0.	0.
855.0	0.31	0.13	0.19	0.	0.	0.	0.	0.	0.	0.	0.	0.

English (θ, x10^6 rad; τ, lb/in^2; σ_{vm}, lb/in^2)

N,	ω			$\sigma+\omega$			σ			$\sigma+\omega$		
rpm	θ	τ	σ_{vm}	θ	τ	σ_{vm}	θ	τ	σ_{vm}	θ	τ	σ_{vm}
28.7	0.01	0.	0.	0.01	0.	0.	4.0	0.5	0.8	0.4	0.01	0.01
29.2	0.01	0.	0.	0.01	0.	0.	6.1	0.7	1.3	6.59	0.79	1.37
29.7	0.01	0.	0.	0.02	0.	0.	10.8	1.3	2.2	0.05	0.01	0.01
30.3	0.01	0.	0.	0.02	0.	0.	1700.	203.6	352.6	0.02	0.	0.
30.8	0.01	0.	0.	0.04	0.01	0.01	11.9	1.4	2.5	0.01	0.	0.
31.4	0.01	0.	C	6.59	0.79	1.37	5.4	0.7	1.1	0.01	0.	0.
32.0	0.01	0.	0.	0.04	0.01	0.01	3.5	0.4	0.7	0.01	0.	0.
840.0	0.63	0.08	0.13	0.	0.	0.	0.	0.	0.	0.	0.	0.
847.8	47.5	5.69	9.85	0.	0.	0.	0.	0.	0.	0.	0.	0.
855.0	0.31	0.04	0.06	0.	0.	0.	0.	0.	0.	0.	0.	0.

Table 5.3 Sample Problem Results for Shaft Torsional Response of GMS/GRS Speed Run-up Through the Fundamental Torsional Resonance - Shaft Stress Levels

significantly higher resonance response at tooth-passing frequency than at the sidebands (258 times more). The resonance response at running speed is also (7.2 times) higher than at the sideband frequencies. Both resonance and nonresonance response is approximately equal at the two sideband frequencies. Moreover, classical gearbox dynamic analyses which do not include time-varying mesh stiffness are blind to this excitation and response phenomenon.

6. CONCLUSIONS AND RECOMMENDATIONS

Conclusions

The primary conclusions regarding the current work consist of the following.

- Analysis of the spur gear mesh stiffness has been established which

  - places the tooth load properly along the line-of-action

  - calculates stiffness due to Hertzian contact (linearized about the mean static torque), bending, shear, and compression modes of deformation

  - calculates stiffness incrementally throughout engagement reflecting one and two tooth pair contact regions

  - calculates stiffness involving shaft rotational and two transverse directions in three-dimensional system global coordinates incorporating the true contact locations in space for each increment

  + calculates gear-error forcing functions arising from gear tooth machining errors at tooth-passing frequency and multiples

  + presents stiffnesses and tooth-passing frequency gear error forces in the form necessary to be incorporated in the current modified transfer matrix method

- A modified transfer matrix method has been established which

  + is fully three-dimensional with six degrees-of-freedom at each location

  + carries all shafting simultaneously

  + carries all frequencies simultaneously; necessitated by the periodic coefficient mesh stiffness

  + incorporates the modified transfer matrices developed by Daws [10] for the inertial disk and visco-elastic bearing

  + incorporates the modifed transfer matrices developed in the current work for the massless elastic shaft and the gear mesh

  + utilizes a sparse matrix solver developed in the current work which literally stores only non-zero elements

  + is automated by an integrated system for interactive/batch computer processing for geared systems of arbitrary construction

- Gear mesh stiffness has been shown to be strongly a function of the number of teeth in contact and weakly a function of load position

- Individual tooth stiffness has been shown to be most highly a function of contact mode deflection followed by shear and bending with compression being nearly negligible

- While the Mathieu problem and attendent instability is well known mathematically, it has not been shown in the literature to be a measurable phenomenon. Moreover, Appendix D suggests that for real gear coupled rotor systems, such instablities are probably not physically realizable

- Time-varying stiffness, however, does provide a source of excitation in geared system of sufficient magnitude to be of concern in dynamics

- All transfer matrix elements have been checked for their proper functioning and accuracy. Specifically, the mesh, disk, and bearing element function as "point" transfer matrices in that responses do not change across the elements. Moreover, the transfer of shears and torques occur properly across the mesh; the relationships among forces and responses traversing the disk and bearing elements have been spot-checked for accuracy. The relationships among deflections, slopes, shears, and moments across the elastic shaft have also been spot-checked for accuracy.

Recommendations

Many prime areas for further development exist for which the need is clear. Among them are the following.

1. Frequency response analysis capability is a current need which could be developed with the current computer model. Analytical studies of dynamics problems in general begin by assessing the system frequency response characteristics. Analysis proceeds by comparing resonances with excitation frequencies in the "as designed" system. Since system modeling parameters can never be determined "exactly", forced response analysis is done assuming coincidence of excitation frequencies and neighboring natural frequencies to obtain worst case estimates of dynamics. If these are excessive redesign takes the form of shifting excitation and resonance frequencies well away from one another. The entire design, analysis, and redesign process points up the importance of obtaining system frequency response.

2. The current six degree-of-freedom, multi-frequency, multi-shaft model is a prime application for which graphical output is needed. The tabular output is sufficiently voluminous that it is too easy for the engineer to overlook important system dynamics without

graphics. Post processing and graphics capabilities of current microcomputer is particularly timely in this regard.

3. Experimental work continues to be a need in geared-rotor dynamics research. This serves a two-fold purpose; 1) to verify the accuracy of a computer model, and equally importantly 2) to determine which identifiable mechanisms have a significant effect on dynamics vs. those which have scientific merit but whose effects on dynamics are largely immeasureable. Examples of one or the other are so numerous as to be limited only by one's imagination. However, the majority are nonlinear terms.

4. Analytical studies of deformation and stiffness of gear teeth have been extensive over the years [10,12,13,17, 18,19,23,24,27]. However, many fundamental aspects of the gear mesh model merit further work.

   - Helical, worm, and other types of gearing clearly displaying stiffness involving translational z and rotational x and y directions is not assessed by the current model. Since these types of gearing are very much the rule rather than the exception in gearboxes this is a prime need area.

- The gear mesh is a well known source of system damping which is not developed in the current model and, therefore, is an area of need.

- The effect of static deflections in three-dimensions of the gear hub has an effect on mesh stiffness and damping characteristics which is largely linear. Daws [10] investigated this concluding its effect to be potentially significant.

- Section 5.1 demonstrates the significance of contact deformation on overall mesh stiffness. The current work bases contact deformation on equivalent cylinders of radii equal to the tooth contact radii. In fact, however, gear teeth are crowned, hence, are more accurately modeled as spheroids with two curvature radii for each body.

REFERENCES

1. Lund, J. N., "Rotor-Bearing Dynamics Design Technology - Part V: Computer Program Manual for Rotor Response and Stability", Air Force Aero Propulsion Laboratory Technical Report AFAPL-TR-65-45, Part V, May 1965, pp. 16-19.

2. Sciarra, J. J., et al, "Helicopter Transmission Vibration and Noise Reduction Program - Vol. I - Technical Report", March 1978, Report Number USARTL-TR-78-2A, U.S. Army Research Technology Laboratory (AVRADCOM), Ft. Eustis, VA 23604.

3. Sciarra, J. J., et al, "Helicopter Transmission Vibration and Noise Reduction Program - Vol. II - User's Manual", Report Number USARTL-TR-78-2B, U.S. Army Research Technology Laboratory, Ft. Eustis, VA 23604.

4. Hartman, R. M., "A Dynamic Approach to Helicopter Noise Reduction and Improved Reliability", Preprint No. 772, presented at the 29th Annual National Forum of the American Helicopter Society, May 1973, Washington, DC.

5. Hartman, R. M., et al, "Model 301 HLH/ATC (Heavy Lift Helicopter/Advanced Technology Component) Transmission Noise Reduction Program", available NTIS, Springfield, VA 22151, Doc. No. AD-784132.

6. Benton, M. and A. Seireg, "Simulation of Resonances and Instability Conditions in Pinion-Gear Systems", ASME, Journal of Mechanical Design, Paper No. 77-DET-50, Design Technical Conference, Chicago, IL, September 26-30, 1977.

7. Benton, M. and A. Seireg, "Normal Mode Uncoupling of Systems with Time Varying Stiffness", ASME, Journal of Mechanical Design, Vol. 102, April 1980, pp. 379-383.

8. Benton, M. and A. Seireg, "The Application of the Ritz Averaging Method to Determine the Response of Systems with Time Varying Stiffness to Harmonic Excitation", ASME, Journal of Mechanical Design, Vol. 102, April 1980, pp. 384-390.

9. Benton, M. and A. Seireg, "Factors Influencing Instability and Resonances in Geared Systems", ASME, Journal of Mechanical Design, Paper No. 80-C2/DET-8, Century 2 International Power Transmissions and Gearing Conference, San Francisco, CA, August 18-21, 1980.

10. Daws, J. W., "An Analytical Investigation of Three-Dimensional Vibration in Gear-Coupled Rotor Systems", Dissertation, Virginia Polytechnic Institute and State University, Blacksburg, VA 24061, May 1979.

11. Mitchell, L. D., "A New Branching Technique for the Static and Dynamic Analysis of Geared Systems", I. Mech. E, 1980.

12. Rebbechi, B. E., "The Kinematics and Kinetics of Spur Gears", Thesis, Monash University, Victoria, Australia, October 1975.

13. Crisp, J. D. C., and B. E. Rebbechi, "On the Elastokinematics of the Involute Gear Pair", ASME, Journal of Engineering for Industry, Paper No. 76-DET-34, Design Engineering Technical Conference, Montreal, Canada, September 26-29, 1976.

14. Pestel, E. C. and F. A. Leckie, Matrix Methods in Elastomechanics, McGraw-Hill Book Co., 1963, pp. 51-94.

15. Hibner, D. H., "Dynamic Response of Viscous Damped Multi-Shaft Jet Engines", Journal of Aircraft, Vol. 12, No. 4, April 1975, pp. 305-312.

16. Mitchell, L. D. and Lynch, G. A., "Origins of Noise", Machine Design, Vol. 41, No. 10, May 1, 1969, pp. 174-178.

17. Caldwell, S. M., "Analysis of Spur Gear Teeth for Static Deflections", Thesis, Virginia Polytechnic Institute, Blacksburg, VA 24061, 1966.

18. Furrow, R. W., "The Measurements of Static Deflections in Spur Gear Teeth", Thesis, Virginia Polytechnic Institute, Blacksburg, VA 24061, 1968.

19. Chabert, G., "An Evaluation of Stresses and Deflections of Spur Gear Teeth Under Strain", ASME Paper No. 72-PTG-27, Mechanisms Conference and International Symposium on Gearing and Transmissions, San Francisco, CA, October 8-12, 1972.

20. Juvinal, R. C., Engineering Consideration of Stress, Strain, and Strength, McGraw-Hill Book Co., 1967, pp. 147-149.

21. Timoshenho, S., Strength of Materials, Part II, Advanced Theory and Problems, 3rd Edition, D. Van Nostrand Company, 1956, pp. 62-64.

22. Roark, R. J. and W. C. Young, Formulas for Stress and Strain, 5th Edition, McGraw-Hill Book Co., 1975, pp. 513-522.

23. Walker, H., "Gear Tooth Deflection and Profile Modification", The Engineer, Vol. 166, 1938, pp. 409-412, 434-436.

24. Weber, C., "The Deformation of Loaded Gears and the Effect on Their Load Carrying Capacity, Part I", Department of Scientific and Industrial Research, Sponsored Research Report No. 3, London, England, 1939.

25. Robinson, C. H., "Measurement of Gear Pair Transmission Error Using Optical Encoders", Thesis, Virginia Polytechnic Institute and State University, Blacksburg, VA 24061, July 1981.

26. Mabie, H. H., "Deflections of Tapered Cantilever Beams", Sandia Corporation Technical Memorandum SCTM-342-61(13), November 1961.

27. Timoshenko, S. and R. V. Baud, "Strength of Gear Teeth is Greatly Affected by Fillet Radius", Automotive Industries, Vol. 55, No. 4, July 22, 1926, pp. 138-142.

28. Eisinger, K. and H. C. Merchant, "Amplitude Modulation of a Forced System by Parametric Variation", ASME, Journal of Applied Mechanics, Paper No. 79-APM-8, Joint ASME-CSME Applied Mechanis, Fluids Engineering, and Bioengineering Conference, Niagara Falls, NY, June 18-20, 1979.

29. Forsythe, G. and C. B. Moler, Computer Solution of Linear Algebraic Systems, Prentice-Hall, Englewood Cliffs, NJ, p. 27.

APPENDIX A

INPUT/OUTPUT FOR THE GMS/GRS ALGORITHMS VIA THE GEARED-ROTOR DEVELOPER (GRD)

A complete analysis of three-dimensional vibration of a geared-rotor system is initiated by executing the Geared Rotor Developer (GRD) computer program. This interactive front end queries the user for rotor system description data, assembles input data files, and executes the Gear Mesh Stiffness (GMS, Appendix B) and Geared Rotor Solver (GRS, Appendix C) algorithms.

Table A.1 illustrates an interactive session with GRD in which the user is analyzing the two rotor test problem of Chapter 5.

All three programs; GRD, GMS, and GRS; were developed on an IBM 3033 and run under FORTRAN H-extended and WATFIV compilers.

A FORTRAN source code listing of GRD immediately follows the interactive session.

LINE LEFT DC TORQUES

LINE NO.	TQ(IN-LB)
1	0.0
2	0.0

LINE RIGHT DC TORQUES

LINE NO.	TQ(IN-LB)
1	15.00
2	15.00

LINE SPEEDS AND SPAN ID'S

LINE NO.	SPEED(RPM)	S1	S2	S3	S4
1	30.28	1	2	0	0
2	-30.28	3	4	0	0

SPAN TYPES (SHAFT = 1, DISK = 2, & BEARING = 3) AND ELEMENTAL ID'S

SPAN	1		2		3		4		5		6		7		8		9		0		1		2		3		4		5		6		7		8	
NO.	T	E	T	E	T	E	T	E	T	E	T	E	T	E	T	E	T	E	T	E	T	E	T	E	T	E	T	E	T	E	T	E	T	E	T	E
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	2	1	3	1	1	1	3	1	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	2	1	3	1	1	1	3	1	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

SHAFT PARAMETERS

SHAFT NO.	L (IN)	D (IN)	MOD EL (LB/SQ IN)	MOD RIG (LB/SQ IN)
1	12.0000	0.2500	0.3000D+08	0.1150D+08

DISK PARAMETERS

DISK NO.	WEIGHT (LB)	D (IN)	L (IN)	ECC (IN)	WOB (DEG)	B (DEG)
1	4.0000	6.0000	1.0000	0.0	0.0	0.0

Table A.1 GRD Interactive Input For The Section 5.2 Sample Problem

```
BEARING STIFFNESSES (LB/IN) OR (IN LB/RAD)
 BRG    KXX        KXY    KYX    KYY        KZZ        KTXX       KTXY   KTYX   KTYY       KTZZ
   1 0.10D+09      0         0 0.10D+09 0.10D+09 0.10D+09          0         0 0.10D+09    0.0
BEARING DAMPING RATES (LB S/IN) OR (IN LB S/RAD)
 BRG    CXX        CXY    CYX    CYY        CZZ        CTXX       CTXY   CTYX   CTYY       CTZZ
   1 0.0           0         0 0.0        0.0        0.0           0         0 0.0        0.1
MESH LINE AND GEAR CONFIGURATIONS AND TORQUES
MESH NO.  HIERARCHY  L(DR)  L(DN)  G(DR)  G(DN)  ALF(DEG) TORQUE (IN LB)
   1          1        1      2      1      1      180.00        15.00
GEAR DATA
GEAR NO.  FACE(IN)  DIA PITCH  PHI(DEG)  NO. TEETH  MOD ELAS   MOD RIG
   1      0.1875      4.00       20.00         28.  0.3000D+08  0.1150D+08
GEAR ERRORS AT IET(K) * TOOTH PASSING FREQUENCY
MESH NO.   K    IET(K)    ERR(SINE)      ERR(COS)
   1       1     0       0.0            0.0
GEAR ERRORS NOT AT TOOTH PASSING FREQUENCY
MESH NO.  GEAR (1/2)  FREQ(RPM)  ERR(SINE)  ERR(COS)
   1          1           0.0    0.0          0.0
   1          2           0.0    0.0          0.0
```

Table A.1 (Continued) GRD Interactive Input For The Section 5.2 Sample Problem

```
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 LLT(5),LRT(5),MAL,MTQ
      INTEGER SPI,SPT,SPE
      DIMENSION OME(5),SPI(5,4),SPT(15,18),SPE(15,18),SHD(8),SHE(8),
     +SHG(8),SHL(8),DIB(8),DID(8),DIE(8),DIL(8),DIM(8),DIT(8),GNT(10),
     +IBC(4,2,2,2),IBK(4,2,2,2),IBKZ(4,2),IBCZ(4,2),
     +BC(4,2,2,2),BK(4,2,2,2),BKZ(4,2),BCZ(4,2),GFW(10),GDP(10),GPA(10),
     +GYM(10),GMR(10),ETS(5,3),ETC(5,3),EEF(5,2),EES(5,2),EEC(5,2),
     +INE(5),ID(5),IEE(5,2),IET(5,3),MHR(5),ML(2,5),MG(2,5),MAL(5),
     +MTQ(5)
      DATA PI/3.141592653589793D0/
      DATA IBG,IDI,IGR,IL,IM,ISH,ISP,IWR/0,0,0,0,0,0,0,4/
      DO 30 I = 1,15
      DO 30 J = 1,18
      SPT(I,J) = 0
      SPE(I,J) = 0
30    CONTINUE
      DO 40 I = 1,5
      LLT(I) = 0.0D0
      LRT(I) = 0.0D0
      ID(I) = 1
      INE(I) = 0
      DO 40 J = 1,2
      IEE(I,J) = 0
      EEF(I,J) = 0.0D0
      EES(I,J) = 0.0D0
      EEC(I,J) = 0.0D0
      MG(J,I) = 0
      ML(J,I) = 0
40    CONTINUE
      DO 50 I = 1,5
      DO 50 J = 1,3
      IET(I,J) = 0
      ETS(I,J) = 0.0D0
      ETC(I,J) = 0.0D0
      SPI(I,J) = 0
      SPI(I,J+1) = 0
50    CONTINUE
85    WRITE (4,1)
100   CONTINUE
      READ (2,*,END=110) I,LLT(I)
      GO TO 100
110   REWIND 2
      WRITE (4,2)
120   CONTINUE
      READ (2,*,END=130) I,LRT(I)
      GO TO 120
130   REWIND 2
      WRITE (4,3)
140   CONTINUE
      READ (2,*,END=150) I,OME(I),(SPI(I,J),J=1,4)
      IL = MAX0(IL,I)
      GO TO 140
150   REWIND 2
      WRITE (4,4)
160   CONTINUE
      READ (2,*,END=170) I,(SPT(I,J),SPE(I,J),J=1,18)
      ISP = MAX0(ISP,I)
      GO TO 160
170   REWIND 2
```

```
      WRITE (4,5)
180   CONTINUE
      READ (2,*,END=190) I,SHL(I),SHD(I),SHE(I),SHG(I)
      ISH = MAXO(ISH,I)
      GO TO 180
190   REWIND 2
      WRITE (4,6)
200   CONTINUE
      READ (2,*,END=210) I,DIM(I),DID(I),DIL(I),DIE(I),DIT(I),DIB(I)
      IDI = MAXO(IDI,I)
      GO TO 200
210   REWIND 2
      WRITE (4,7)
220   CONTINUE
      READ (2,*,END=230) I,(((BK(I,J,L,M),L=1,2),M=1,2),BKZ(I,J),
     +J=1,2)
      IBG = MAXO(IBG,I)
      GO TO 220
230   REWIND 2
      WRITE (4,8)
240   CONTINUE
      READ (2,*,END=250) I,(((BC(I,J,L,M),L=1,2),M=1,2),BCZ(I,J),
     +J=1,2)
      GO TO 240
250   DO 258 I = 1,IBG
      DO 258 J = 1,2
      IBKZ(I,J) = (BKZ(I,J) + 5.0D-1)
      IBCZ(I,J) = (BCZ(I,J) + 5.0D-1)
      DO 258 K = 1,2
      DO 258 L = 1,2
      IBK(I,J,K,L) = (BK(I,J,K,L) + 5.0D-1)
258   IBC(I,J,K,L) = (BC(I,J,K,L) + 5.0D-1)
      REWIND 2
      WRITE (4,9)
260   CONTINUE
      READ (2,*,END=270) I,MHR(I),ML(1,I),ML(2,I),MG(1,I),MG(2,I),
     +MAL(I),MTQ(I)
      IM = MAXO(IM,I)
      GO TO 260
270   IF (IM.EQ.0) GO TO 330
      REWIND 2
      WRITE (4,10)
280   CONTINUE
      READ (2,*,END=290) I,GFW(I),GDP(I),GPA(I),GNT(I),GYM(I),GMR(I)
      IGR = MAXO(IGR,I)
      GO TO 280
290   REWIND 2
      WRITE (4,11)
300   CONTINUE
      READ (2,*,END=310) I,K,IET(I,K),ETS(I,K),ETC(I,K)
      INE(I) = MAXO(INE(I),K)
      GO TO 300
310   REWIND 2
      WRITE (4,12)
320   CONTINUE
      READ (2,*,END=330) I,J,EEF(I,J),EES(I,J),EEC(I,J)
      IEE(I,J) = 1
      GO TO 320
330   REWIND 2
      WRITE (4,13)
```

```
      READ (2,*) I
      GO TO (1000,340,85), I
1000  IWR = 8
340   WRITE (IWR,1)
      WRITE (IWR,14) (I,LLT(I),I=1,IL)
      WRITE (IWR,2)
      WRITE (IWR,14) (I,LRT(I),I=1,IL)
      WRITE (IWR,3)
      WRITE (IWR,15) (I,OME(I),(SPI(I,J),J=1,4),I=1,IL)
      WRITE (IWR,4)
      WRITE (IWR,16) (I,(SPT(I,J),SPE(I,J),J=1,18),I=1,ISP)
      WRITE (IWR,5)
      WRITE (IWR,17) (I,SHL(I),SHD(I),SHE(I),SHG(I),I=1,ISH)
      WRITE (IWR,6)
      WRITE (IWR,18) (I,DIM(I),DID(I),DIL(I),DIE(I),DIT(I),DIB(I),
     +I=1,IDI)
      WRITE (IWR,7)
      WRITE (IWR,19) (I,BK(I,1,1,1),IBK(I,1,1,2),IBK(I,1,2,1),
     +BK(I,1,2,2),BKZ(I,1),BK(I,2,1,1),IBK(I,2,1,2),IBK(I,2,2,1),
     +BK(I,2,2,2),IBKZ(I,2),I=1,IBG)
      WRITE (IWR,8)
      WRITE (IWR,19) (I,BC(I,1,1,1),IBC(I,1,1,2),IBC(I,1,2,1),
     +BC(I,1,2,2),BCZ(I,1),BC(I,2,1,1),IBC(I,2,1,2),IBC(I,2,2,1),
     +BC(I,2,2,2),IBCZ(I,2),I=1,IBG)
      IF (IM.EQ.0) GO TO 350
      WRITE (IWR,9)
      WRITE (IWR,20) (I,MHR(I),ML(1,I),ML(2,I),MG(1,I),MG(2,I),MAL(I),
     +MTQ(I),I=1,IM)
      WRITE (IWR,10)
      WRITE (IWR,21) (I,GFW(I),GDP(I),GPA(I),GNT(I),GYM(I),GMR(I),
     +I=1,IGR)
      WRITE (IWR,11)
      WRITE (IWR,22) (I,ID(I),IET(I,ID(I)),ETS(I,ID(I)),
     +ETC(I,ID(I)),I=1,IM)
      WRITE (IWR,12)
      WRITE (IWR,23) ((I,J,EEF(I,J),EES(I,J),EEC(I,J),J=1,2),I=1,IM)
350   IF (IWR.EQ.4) GO TO 330
      IF (IDI.EQ.0) GO TO 80
      DO 90 I = 1,IDI
      DIM(I) = DIM(I)/3.86088D2
      DIT(I) = DIT(I)*PI/1.8D2
      DIB(I) = DIB(I)*PI/1.8D2
90    CONTINUE
80    WRITE (1,26) IM,IL,IGR
      WRITE (1,25) (OME(I),I=1,IL)
      IF (IM.EQ.0) GO TO 70
      WRITE (1,25) (GFW(I),GDP(I),GPA(I),GYM(I),GMR(I),GNT(I),I=1,IGR)
      WRITE (1,26) ((MG(I,J),ML(I,J),I=1,2),J=1,IM)
      WRITE (1,25) (MTQ(J),MAL(J),J=1,IM)
      WRITE (1,26) (INE(I),I=1,IM)
      DO 60 I = 1,IM
      K = INE(I)
      IF (K.EQ.0) GO TO 60
      WRITE (1,25) (ETS(I,J),ETC(I,J),J=1,K)
      WRITE (1,26) (IET(I,J),J=1,K)
60    CONTINUE
70    WRITE (3,26) IM,IL,ISP,ISH,IDI,IBG,IGR
      WRITE (3,25) (LLT(I),LRT(I),OME(I),I=1,IL)
      WRITE (3,26) ((SPI(I,J),I=1,IL),J=1,4)
      WRITE (3,26) ((SPT(I,J),SPE(I,J),I=1,ISP),J=1,18)
```

```
      WRITE (3,25) (SHL(I),SHD(I),SHE(I),SHG(I),I=1,ISH)
      WRITE (3,25) (DIM(I),DID(I),DIL(I),DIE(I),DIT(I),DIB(I),I=1,IDI)
      WRITE (3,25) ((((BK(I,J,L,M),L=1,2),M=1,2),BKZ(I,J),J=1,2),
     +I=1,IBG)
      WRITE (3,25) ((((BC(I,J,L,M),L=1,2),M=1,2),BCZ(I,J),J=1,2),
     +I=1,IBG)
      IF (IM.EQ.0) GO TO 360
      WRITE (3,26) (MHR(I),(IEE(I,J),ML(J,I),MG(J,I),J=1,2),I=1,IM)
      WRITE (3,25) (GNT(I),I=1,IGR)
      WRITE (3,25) ((EEF(I,J),EES(J,I),EEC(J,I),I=1,IM),J=1,2)
1     FORMAT (/'LINE LEFT DC TORQUES'//'LINE NO.   TQ(IN-LB)'/)
2     FORMAT (/'LINE RIGHT DC TORQUES'//'LINE NO.   TQ(IN-LB)'/)
3     FORMAT (/'LINE SPEEDS AND SPAN ID''S'//'LINE NO.  SPEED(RPM)',
     +'  S1 S2 S3 S4'/)
4     FORMAT (/'SPAN TYPES (SHAFT = 1, DISK = 2, & BEARING = 3)',
     +' AND ELEMENTAL ID''S'//'SPAN   1   2',
     +3X,'3   4   5   6   7   8   9   0   1   2   3   4   5   6   7   8'
     +/' NO. T E T E T E T E T E T E T E T E T E T E T E T E T E T E T E ',
     +'T E T E T E T E'/)
5     FORMAT (/'SHAFT PARAMETERS'//'SHAFT NO.    L (IN)        D',
     +' (IN)      MOD EL (LB/SQ IN)   MOD RIG (LB/SQ IN)'/)
6     FORMAT (/'DISK PARAMETERS'//'DISK NO.  WEIGHT (LB)  D',
     +' (IN)      L (IN)      ECC (IN)    WOB (DEG)     B (DEG)'/)
7     FORMAT (/'BEARING STIFFNESSES (LB/IN) OR (IN LB/RAD)'//' BRG    ',
     +'KXX        KXY    KYX    KYY        KZZ         KTXX       KTXY  KTYX   ',
     +'KTYY       KTZZ'/)
8     FORMAT (/'BEARING DAMPING RATES (LB S/IN) OR (IN LB S/RAD)'//
     +' BRG    CXX         CXY    CYX     CYY         CZZ        CTXX        CTXY  ',
     +'CTYX   CTYY      CTZZ'/)
9     FORMAT (/'MESH LINE AND GEAR CONFIGURATIONS AND TORQUES'//
     +'MESH NO.  HIERARCHY  L(DR)  L(DN)  G(DR)  G(DN)  ALF(DEG) ',
     +'TORQUE (IN LB)'/)
10    FORMAT (/'GEAR DATA'//'GEAR NO.  FACE(IN)  DIA PITCH  PHI',
     +'(DEG)  NO. TEETH  MOD ELAS   MOD RIG'/)
11    FORMAT (/'GEAR ERRORS AT IET(K) * TOOTH PASSING FREQUENCY'//
     +'MESH NO.   K     IET(K)      ERR(SINE)       ERR(COS)'/)
12    FORMAT (/'GEAR ERRORS NOT AT TOOTH PASSING FREQUENCY'//
     +'MESH NO.  GEAR (1/2)  FREQ(RPM)  ERR(SINE)  ERR(COS)'/)
13    FORMAT (/'INDICATE BY NO. (1/2/3)'//5X,'(1)  INPUT DATA IS',
     +' FINE - EXIT ROUTINE,'/5X,'(2)  LOOK AT INPUT DATA, OR'/5X,
     +'(3)  CHANGE INPUT')
14    FORMAT (I4,F15.2)
15    FORMAT (I4,F16.2,I5,3I3)
16    FORMAT (2I3,35I2)
17    FORMAT (I4,2F14.4,2D20.4)
18    FORMAT (I5,6F12.4)
19    FORMAT (I4,D9.2,2I6,3D9.2,2I6,D9.2,I6)
20    FORMAT (I4,I11,I9,3I7,2F12.2)
21    FORMAT (I4,F12.4,F10.2,F11.2,F11.0,2D12.4)
22    FORMAT (I4,I8,I7,D16.4,D14.4)
23    FORMAT (I4,I11,F15.2,2D11.3)
25    FORMAT (4D20.12)
26    FORMAT (25I3)
360   STOP
      END
```

APPENDIX B

THE GEAR MESH STIFFNESS (GMS) COMPUTER PROGRAM

The gear mesh stiffness (GMS) computer program calculates spur-gear-tooth stiffness incrementally throughout engagement which reflects bending, shear, tension, and local contact modes of deflection. Table B.1 illustrates typical output of this instantaneous stiffness in 'P' coordinates, or gear local (x_g, y_g) coordinates described in Section 3.1.

Stiffness is transformed to system global 'Q' coordinates (x_s, y_s) via the direction cosines Equation (3.36), see Table B.2. These instantaneous stiffness values are related to system global 'S' coordinates (x_s, y_s, z_s) according to Equations (3.38), see Table B.3.

A Fourier series of incremental stiffness according to Equation (1.2) yields the representation of tooth stiffness coefficients of Table B.4 required by the mesh transfer matrix in GRS.

A FORTRAN source code listing of GMS immediately follows Table B.4.

INCREMENTATION NUMBER 16. 'P' VARIABLES, GEAR COORDINATE (X,Y) DIRECTIONS

	PRESSURE ANGLE (RAD)	CENTER DIST (IN)		ANGULAR VEL- OCITY (1/S)		CONTACT LOC (IN)	CONTACT AREA (SQ-IN)	CONTACT FORCE (LBF)	SUMMED ERROR (IN)	STIFFNESS TENSOR (LB/IN) X	Y
LH:	0.0000D 00	0.0000D 00	G1:	0.0000D 00	X:	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00
	0.3491D 00	0.7000D 01		0.3171D 01		-0.1318D-01	0.1972D-03	0.4286D 01	0.0000D 00	0.3525D 06	0.1283D 06
					Y:	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00
						0.3495D 01	0.5417D-03	0.1560D 01	0.0000D 00	0.1283D 06	0.4670D 05
			G2:	0.0000D 00	X:	0.0000D 00					
				0.3171D 01		-0.1318D-01					
					Y:	0.0000D 00					
						-0.3505D 01					
RH:	0.0000D 00	0.0000D 00	G1:	0.0000D 00	X:	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00
	0.0000D 00	0.0000D 00		0.0000D 00		0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00
					Y:	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00
						0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00
			G2:	0.0000D 00	X:	0.0000D 00					
				0.0000D 00		0.0000D 00					
					Y:	0.0000D 00					
						0.0000D 00					

Table B.1 GMS Output In Gear Coordinates For The Section 5.2 Sample Problem

'Q' VARIABLES, SYSTEM (GLOBAL) (X,Y)

	CONTACT LOCATION (IN)	CONTACT AREA (SQ-IN)	CONTACT FORCE (LBF)	SUMMED ERROR (IN)	STIFFNESS TENSOR (LBF/IN) X	Y
LH G1 X:	0.000000D 00	0.000000D 00	0.000000D 00	0.000000D 00	0.000000D 00	0.000000D 00
	-0.131784D-01	0.197181D-03	0.428571D 01	0.000000D 00	0.352501D 06	-0.128300D 06
Y:	0.000000D 00	0.000000D 00	0.000000D 00	0.000000D 00	0.000000D 00	0.000000D 00
	-0.349520D 01	0.541750D-03	0.155987D 01	0.000000D 00	-0.128300D 06	0.466974D 05
G2 X:	0.000000D 00					
	-0.131784D-01					
Y:	0.000000D 00					
	0.350480D 01					
RH G1 X:	0.000000D 00	0.000000D 00	0.000000D 00	0.000000D 00	0.000000D 00	0.000000D 00
	0.000000D 00	0.000000D 00	0.000000D 00	0.000000D 00	0.000000D 00	0.000000D 00
Y:	0.000000D 00	0.000000D 00	0.000000D 00	0.000000D 00	0.000000D 00	0.000000D 00
	0.000000D 00	0.000000D 00	0.000000D 00	0.000000D 00	0.000000D 00	0.000000D 00
G2 X:	0.000000D 00					
	0.000000D 00					
Y:	0.000000D 00					
	0.000000D 00					

Table B.2 GMS Output In System Global (x,y) Coordinates For The Section 5.2 Sample Problem

'S' VARIABLES, SYSTEM (GLOBAL) COORDINATE (X,Y,Z) DIRECTIONS

	BLOCK STIFFNESS TERMS G1 X	G1 Y	G1 Z	BLOCK STIFFNESS TERMS G2 X	G2 Y	G2 Z	EXT COL ERR TERMS	EXT COL FRICT TERMS
G1 X:	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00
	0.3525D 06	-0.1283D 06	0.1234D 07	-0.3525D 06	0.1283D 06	0.1234D 07	0.0000D 00	-0.1170D 00
Y:	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00
	-0.1283D 06	0.4670D 05	-0.4490D 06	0.1283D 06	-0.4670D 05	-0.4490D 06	0.0000D 00	-0.3214D 00
Z:	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00
	0.1234D 07	-0.4490D 06	0.4318D 07	-0.1234D 07	0.4490D 06	0.4318D 07	0.0000D 00	-0.4047D 00
G2 X:	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00
	-0.3525D 06	0.1283D 06	-0.1234D 07	0.3525D 06	-0.1283D 06	-0.1234D 07	0.0000D 00	0.1170D 00
Y:	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00
	0.1283D 06	-0.4670D 05	0.4490D 06	-0.1283D 06	0.4670D 05	0.4490D 06	0.0000D 00	0.3214D 00
Z:	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00
	0.1234D 07	-0.4490D 06	0.4318D 07	-0.1234D 07	0.4490D 06	0.4318D 07	0.0000D 00	-0.4143D 00

Table B.3 GMS Output In System Global (x,y,z) Coordinates For The Section 5.2 Sample Problem

'T' VARIABLES, SYSTEM (GLOBAL) COORDINATE (X,Y,Z) DIRECTIONS. HARMONIC FUNCTIONS AT N X TPF

		BLOCK STIFFNESS TERMS			BLOCK STIFFNESS TERMS			EXTENSION
		G1 X	G1 Y	G1 Z	G2 X	G2 Y	G2 Z	ERR AT N*TPF
G1 X	DC:	0.5016D 06	-0.1826D 06	0.1756D 07	-0.5016D 06	0.1826D 06	0.1756D 07	-0.0000D 00
	1S:	0.1395D 06	-0.5078D 05	0.4883D 06	-0.1395D 06	0.5078D 05	0.4883D 06	-0.0000D 00
	1C:	-0.7064D 05	0.2571D 05	-0.2472D 06	0.7064D 05	-0.2571D 05	-0.2472D 06	-0.0000D 00
	2S:	0.3360D 05	-0.1223D 05	0.1176D 06	-0.3360D 05	0.1223D 05	0.1176D 06	-0.0000D 00
	2C:	0.2399D 05	-0.8733D 04	0.8398D 05	-0.2399D 05	0.8733D 04	0.8398D 05	0.0000D 00
	3S:	0.5419D 04	-0.1972D 04	0.1897D 05	-0.5419D 04	0.1972D 04	0.1897D 05	
	3C:	-0.3264D 05	0.1188D 05	-0.1142D 06	0.3264D 05	-0.1188D 05	-0.1142D 06	
	4S:	0.3655D 05	-0.1330D 05	0.1279D 06	-0.3655D 05	0.1330D 05	0.1279D 06	
	4C:	-0.1194D 05	0.4347D 04	-0.4180D 05	0.1194D 05	-0.4347D 04	-0.4180D 05	
G1 Y	DC:	-0.1826D 06	0.6646D 05	-0.6390D 06	0.1826D 06	-0.6646D 05	-0.6390D 06	-0.0000D 00
	1S:	-0.5078D 05	0.1848D 05	-0.1777D 06	0.5078D 05	-0.1848D 05	-0.1777D 06	-0.0000D 00
	1C:	0.2571D 05	-0.9358D 04	0.8998D 05	-0.2571D 05	0.9358D 04	0.8998D 05	-0.0000D 00
	2S:	-0.1223D 05	0.4451D 04	-0.4281D 05	0.1223D 05	-0.4451D 04	-0.4281D 05	-0.0000D 00
	2C:	-0.8733D 04	0.3179D 04	-0.3057D 05	0.8733D 04	-0.3179D 04	-0.3057D 05	0.0000D 00
	3S:	-0.1972D 04	0.7179D 03	-0.6904D 04	0.1972D 04	-0.7179D 03	-0.6904D 04	
	3C:	0.1188D 05	-0.4324D 04	0.4158D 05	-0.1188D 05	0.4324D 04	0.4158D 05	
	4S:	-0.1330D 05	0.4842D 04	-0.4656D 05	0.1330D 05	-0.4842D 04	-0.4656D 05	
	4C:	0.4347D 04	-0.1582D 04	0.1522D 05	-0.4347D 04	0.1582D 04	0.1522D 05	
G1 Z	DC:	0.1756D 07	-0.6390D 06	0.6145D 07	-0.1756D 07	0.6390D 06	0.6145D 07	-0.0000D 00
	1S:	0.4883D 06	-0.1777D 06	0.1709D 07	-0.4883D 06	0.1777D 06	0.1709D 07	-0.0000D 00
	1C:	-0.2472D 06	0.8998D 05	-0.8653D 06	0.2472D 06	-0.8998D 05	-0.8653D 06	-0.0000D 00
	2S:	0.1176D 06	-0.4281D 05	0.4116D 06	-0.1176D 06	0.4281D 05	0.4116D 06	-0.0000D 00
	2C:	0.8398D 05	-0.3057D 05	0.2939D 06	-0.8398D 05	0.3057D 05	0.2939D 06	0.0000D 00
	3S:	0.1897D 05	-0.6904D 04	0.6639D 05	-0.1897D 05	0.6904D 04	0.6639D 05	
	3C:	-0.1142D 06	0.4158D 05	-0.3998D 06	0.1142D 06	-0.4158D 05	-0.3998D 06	
	4S:	0.1279D 06	-0.4656D 05	0.4477D 06	-0.1279D 06	0.4656D 05	0.4477D 06	
	4C:	-0.4180D 05	0.1522D 05	-0.1463D 06	0.4180D 05	-0.1522D 05	-0.1463D 06	

Table B.4 GMS Output Stiffness Values For GRS For The Section 5.2 Sample Problem

'T' VARIABLES, SYSTEM (GLOBAL) COORDINATE (X,Y,Z) DIRECTIONS. HARMONIC FUNCTIONS AT N X TPF

		BLOCK STIFFNESS TERMS			BLOCK STIFFNESS TERMS			EXTENSION
		G1 X	G1 Y	G1 Z	G2 X	G2 Y	G2 Z	ERR AT N*TPF
G2 X	DC:	-0.5016D 06	0.1826D 06	-0.1756D 07	0.5016D 06	-0.1826D 06	-0.1756D 07	-0.0000D 00
	1S:	-0.1395D 06	0.5078D 05	-0.4883D 06	0.1395D 06	-0.5078D 05	-0.4883D 06	-0.0000D 00
	1C:	0.7064D 05	-0.2571D 05	0.2472D 06	-0.7064D 05	0.2571D 05	0.2472D 06	-0.0000D 00
	2S:	-0.3360D 05	0.1223D 05	-0.1176D 06	0.3360D 05	-0.1223D 05	-0.1176D 06	-0.0000D 00
	2C:	-0.2399D 05	0.8733D 04	-0.8398D 05	0.2399D 05	-0.8733D 04	-0.8398D 05	0.0000D 00
	3S:	-0.5419D 04	0.1972D 04	-0.1897D 05	0.5419D 04	-0.1972D 04	-0.1897D 05	
	3C:	0.3264D 05	-0.1188D 05	0.1142D 06	-0.3264D 05	0.1188D 05	0.1142D 06	
	4S:	-0.3655D 05	0.1330D 05	-0.1279D 06	0.3655D 05	-0.1330D 05	-0.1279D 06	
	4C:	0.1194D 05	-0.4347D 04	0.4180D 05	-0.1194D 05	0.4347D 04	0.4180D 05	
G2 Y	DC:	0.1826D 06	-0.6646D 05	0.6390D 06	-0.1826D 06	0.6646D 05	0.6390D 06	-0.0000D 00
	1S:	0.5078D 05	-0.1848D 05	0.1777D 06	-0.5078D 05	0.1848D 05	0.1777D 06	-0.0000D 00
	1C:	-0.2571D 05	0.9358D 04	-0.8998D 05	0.2571D 05	-0.9358D 04	-0.8998D 05	-0.0000D 00
	2S:	0.1223D 05	-0.4451D 04	0.4281D 05	-0.1223D 05	0.4451D 04	0.4281D 05	-0.0000D 00
	2C:	0.8733D 04	-0.3179D 04	0.3057D 05	-0.8733D 04	0.3179D 04	0.3057D 05	0.0000D 00
	3S:	0.1972D 04	-0.7179D 03	0.6904D 04	-0.1972D 04	0.7179D 03	0.6904D 04	
	3C:	-0.1188D 05	0.4324D 04	-0.4158D 05	0.1188D 05	-0.4324D 04	-0.4158D 05	
	4S:	0.1330D 05	-0.4842D 04	0.4656D 05	-0.1330D 05	0.4842D 04	0.4656D 05	
	4C:	-0.4347D 04	0.1582D 04	-0.1522D 05	0.4347D 04	-0.1582D 04	-0.1522D 05	
G2 Z	DC:	0.1756D 07	-0.6390D 06	0.6145D 07	-0.1756D 07	0.6390D 06	0.6145D 07	-0.0000D 00
	1S:	0.4883D 06	-0.1777D 06	0.1709D 07	-0.4883D 06	0.1777D 06	0.1709D 07	-0.0000D 00
	1C:	-0.2472D 06	0.8998D 05	-0.8653D 06	0.2472D 06	-0.8998D 05	-0.8653D 06	-0.0000D 00
	2S:	0.1176D 06	-0.4281D 05	0.4116D 06	-0.1176D 06	0.4281D 05	0.4116D 06	-0.0000D 00
	2C:	0.8398D 05	-0.3057D 05	0.2939D 06	-0.8398D 05	0.3057D 05	0.2939D 06	0.0000D 00
	3S:	0.1897D 05	-0.6904D 04	0.6639D 05	-0.1897D 05	0.6904D 04	0.6639D 05	
	3C:	-0.1142D 06	0.4158D 05	-0.3998D 06	0.1142D 06	-0.4158D 05	-0.3998D 06	
	4S:	0.1279D 06	-0.4656D 05	0.4477D 06	-0.1279D 06	0.4656D 05	0.4477D 06	
	4C:	-0.4180D 05	0.1522D 05	-0.1463D 06	0.4180D 05	-0.1522D 05	-0.1463D 06	

Table B.4 (Continued) GMS Output Stiffness Values For GRS For The Section 5.2 Sample Problem

```
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 INV,KC,KI,KLA,KT,KXY,MP,MU,N,NCT
      INTEGER GMG,GML
      DIMENSION GFW(10),GDP(10),GPA(10),GYM(10),GMR(10),GNT(10),GTQ(5),
     +GOME(5),GMG(2,5),GAL(5),GML(2,5),INE(5),ETS(5,3),ETC(5,3),
     +IET(5,3)
      COMMON /MESS/ AA(50,9),A9(9),A99(9,9),AG(2),ALF,ALP,AX(2),AY(2),
     +B9(9),BET(2),BP,C1(2,2),CA(2),CHTH(2),CI(2,2,2),CK(2,2,2,2),
     +CLA(2,2,2,2),CP,COF,FID(2),CRN,TH(4),CT(2,2),DC2,DCS,DS2,
     +CTR,CTT(2,2),CXY(2,2,2,2),XX(2),DC(3),DS(3),DP,DTH(2),E(2),
     +ETA(2),FF(2),FJB(2),FLA,FXY(2,2),FW(2),G(2),GAM(2),H1(2),H2(2),
     +FU(2),H1A(2),H1B(2),H1C(2),
     +KC(2,2,2,2),KI(2,2,2),KLA(2,2,2,2),KT(2,2),KXY(2,2,2,2),
     +MP,MU(2),N(2),NCT(2,2),OME(2),OTP,PHE(2),PHG(2),PHI,PI,POT,
     +PSI(2),R(2,2),RB(2),RC(2,2),RD(2),RITER,RETA(2),RK,RN(2,2),RO(2),
     +RP(2),TPA,TB(2),THA(2,2),THE(2,4),TLA(2,2),TQ(2),TRQ,TXY(2,2),
     +V(2,2),XA(2),XI,XT(2,2),XTU(2),YA(2),YT(2,2),YTU(2),Z,ZET,
     +XQ(2),YQ(2),I12(50),I21,ID(3),ITER,NOE,KD1
      COMMON /PQST/ PAR(2,2,2,50),PC(2,2,50),PER(2,2,2,50),
     +PFN(2,2,2,50),PK(2,2,2,2,50),PL(2,2,2,2,50),PPA(2,2,50),
     +PT(2,50),PV(2,2,2,50),PW(2,2,2,50),
     +QAR(2,2,2,50),QER(2,2,2,50),QFN(2,2,2,50),QK(2,2,2,2,50),
     +QL(2,2,2,2,50),QV(2,2,2,50),
     +SFC(2,2,3,50),SFK(2,2,3,50),SK(2,2,2,3,3,50),SFE(2,2,3,2,50),
     +TFC(2,2,3,5),TFK(2,2,3,5),TK(2,2,2,3,3,9),TFE(2,2,3,2,3)
     +,UK(2,2,2,3,3,50),UFC(2,2,3,50),UFK(2,2,3,50),UFE(2,2,3,2,50),
     +VK(2,2,2,3,3),VFC(2,2,3),VFK(2,2,3),VFE(2,2,3,2)
      COMMON /LET/ A,A2,B,B2,C,C2,D,D2,F,F2,H(2)
      DASIN(A) = DATAN(A/DSQRT(1.0D0-A*A))
      DACOS(A) = DATAN(DSQRT(1.0D0-A*A)/A)
      INV(A) = DTAN(A) - A
      GA(PHI) = 1.0D0/(DCOS(PHI)*DCOS(PHI))
      GB(RP,PHI) = 2.0D0*RP*DTAN(PHI)
      GC(RP,RO) = RP*RP-RO*RO
      GG(A,B,C,R) = DASIN((DSQRT(B*B-4.0D0*A*C)-B)/(2.0D0*A*R))
      ITER = 20
      PI = 3.141592653589793D0
      COF = 7.5D-2
      POT = PI/2.0D0
C
C  *  READ DATA FROM GRD
C
      READ (5,*) IM,IL,IGR
      READ (5,*) (GOME(I),I=1,IL)
      IF (IM.EQ.0) STOP
      READ (5,*) (GFW(I),GDP(I),GPA(I),GYM(I),GMR(I),GNT(I),I=1,IGR)
      READ (5,*) ((GMG(I,J),GML(I,J),I=1,2),J=1,IM)
      READ (5,*) (GTQ(J),GAL(J),J=1,IM)
      READ (5,*) (INE(I),I=1,IM)
      DO 20 I = 1,IM
      K = INE(I)
      IF (K.EQ.0) GO TO 20
      READ (5,*) (ETS(I,J),ETC(I,J),J=1,K)
      READ (5,*) (IET(I,J),J=1,K)
20    CONTINUE
C
C  * INITIALIZE VALUES
C
      DO 240 ING = 1,IM
```

```
      GTQ(ING) = DABS(GTQ(ING))
      DP = GDP(ING)
      DO 30 I = 1,2
      FW(I) = GFW(GMG(I,ING))
      E(I) = GYM(GMG(I,ING))
      G(I) = GMR(GMG(I,ING))
      N(I) = GNT(GMG(I,ING))
      OME(I) = GOME(GML(I,ING))
30    CONTINUE
      PHI = GPA(GMG(1,ING))
      TRQ = GTQ(ING)
      ALF = GAL(ING)
      NOE = INE(ING)
      IF (NOE.EQ.0) GO TO 50
      DO 40 I = 1,NOE
      DS(I) = ETS(ING,I)
      DC(I) = ETC(ING,I)
      ID(I) = IET(ING,I)
40    CONTINUE
C
C  *  GEAR AND INVOLUTE QUANTITIES  *
C
50    OME(1) = OME(1)*PI/3.0D1
      ALF = ALF*PI/1.8D2
      PHI = PHI*PI/1.8D2
      DC2 = DCOS(PHI)
      DS2 = DSIN(PHI)
      DCS = DC2*DS2
      DC2 = DC2*DC2
      DS2 = DS2*DS2
      CRN = N(2)/N(1)
      OME(2) = -OME(1)/CRN
      OTP = DABS(OME(1)*N(1))
      ADD = 1.0D0/DP
      DED = 1.25D0/DP
      IF (PHI.LT.3.0D-1) DED = 1.157D0/DP
      DO 60 I = 1,2
      MU(I) = 5.0D-1*E(I)/G(I) - 1.0D0
      RP(I) = 5.0D-1*N(I)/DP
      RO(I) = RP(I) + ADD
      RD(I) = RP(I) - DED
60    RB(I) = RP(I)*DCOS(PHI)
      CTR = RP(1) + RP(2)
      FLA = TRQ/RB(2)
      CP = PI/DP
      BP = CP*DCOS(PHI)
C
C          INSERT APPROPRIATE STATEMENTS IF NON-STANDARD,
C
C               (1)  PITCH LINE TOOTH THICKNESS ( TPT(I) )
C               (2)  ADDENDA ( ADD,RO(I) ),
C               (3)  DEDENDA ( DED,RD(I) ), AND/OR
C               (4)  OPERATING CENTER DISTANCE ( CTR ).
C
C          BE ESPECIALLY CAREFUL WITH 20 DEGREE PRESSURE ANGLE GEARS !!!
C          PRIOR TO 1968 THE STANDARD WAS EVEN DIFFERENT !!!
C
      Z = DSQRT(RO(1)*RO(1)-RB(1)*RB(1)) + DSQRT(RO(2)*RO(2)-RB(2)*RB(2)
     +) - CTR*DSIN(PHI)
      MP = Z/BP
```

```
      IF (MP.GT.2.0D0) MP = DSQRT(-1.0D0)
C
C  *  MODEL AND INCREMENTATION PREPARATION  *
C
      DO 90 I = 1,2
      E(I) = E(I)*FW(I)
      G(I) = G(I)*FW(I)
      J = 1
      IF (I.EQ.1) J = 2
      H(I) = (1.0D0-MU(I)*MU(I))/E(I)
      TPA = PI/(4.0D0*DP)
      PHO = DACOS(RP(I)*DCOS(PHI)/RO(I))
      TAA = RO(I)*(TPA/RP(I)+INV(PHI)-INV(PHO))
      TA = RO(I)*DSIN(TAA/RO(I))
      DA = RO(I)*DCOS(TAA/RO(I))
      IF (RB(I).GT.RD(I)) GO TO 70
      PHD = DACOS(RP(I)*DCOS(PHI)/RD(I))
      TDA = RD(I)*(TPA/RP(I)+INV(PHI)-INV(PHD))
      TD = RD(I)*DSIN(TDA/RD(I))
      DD = RD(I)*DCOS(TDA/RD(I))
      BET(I) = DATAN((TD-TA)/(DA-DD))
      H2(I) = 2.0D0*TD
      GO TO 80
70    TBA = RB(I)*(TPA/RP(I)+INV(PHI))
      PSI(I) = TBA/RB(I)
      TB(I) = RB(I)*DSIN(TBA/RB(I))
      DB = RB(I)*DCOS(TBA/RB(I))
      BET(I) = DATAN((TB(I)-TA)/(DA-DB))
      ES = 1.0D0 + 1.0D0/((DTAN(BET(I)))**2)
      TE = -2.0D0*(DB+TB(I)/DTAN(BET(I)))/DTAN(BET(I))
      YU = (DB+TB(I)/DTAN(BET(I)))**2-RD(I)**2
      DD = DB + TB(I)/DTAN(BET(I)) + (TE+DSQRT(TE*TE-4.0D0*ES*YU))/
     +(2.0D0*ES*DTAN(BET(I)))
      H2(I) = -(TE+DSQRT(TE*TE-4.0D0*ES*YU))/ES
80    H1A(I) = 1.0D0+ 1.0D0/((DTAN(BET(I)))**2)
      H1B(I) = -(2.0D0*DD + H2(I)/DTAN(BET(I)))/DTAN(BET(I))
      H1C(I) = (DD + H2(I)/(2.0D0*DTAN(BET(I))))**2
      BETA = BET(I)*1.8D2/PI
      A = GA(PHI)
      B = GB(RP(I),PHI)
      C = GC(RP(I),RO(I))
      GAM(I) = GG(A,B,C,RO(I))
      ETA(J) = DATAN(RO(I)*DSIN(GAM(I))/(CTR-RO(I)*DCOS(GAM(I))))
      RETA(J) = (CTR-RO(I)*DCOS(GAM(I)))/DCOS(ETA(J))
      PHE(J) = PHI - ETA(J)
      PHG(I) = PHI + GAM(I)
      THE(J,1) = DTAN(PHE(J))
      THE(J,3) = THE(J,1) + Z/(MP*RB(J))
      THE(I,4) = DTAN(PHG(I))
      THE(I,2) = THE(I,4) - Z/(MP*RB(I))
      RITER = ITER
      DTH(I) = Z/(RITER*MP*RB(I))
      EYE = I
90    CONTINUE
C
C  *  INCREMENT THROUGH THE MESH  *
C
      DO 230 K = 1,ITER
      RK = K
      THA(1,1) = THE(1,1) + RK*DTH(1)
```

```
      THA(1,2) = THE(1,3) + RK*DTH(1)
      THA(2,1) = THE(2,4) - RK*DTH(2)
      THA(2,2) = THE(2,2) - RK*DTH(2)
      I12(K) = 1
      IF (THA(1,2).LT.THE(1,4)) I12(K) = 2
      I21 = I12(K)
C  *
C  *  SKIP DEFLECTION ANALYSIS - FUTURE WORK
C  *
      IDEFL = 1
C  *
      DO 110 KD1 = 1,IDEFL
      DO 100 I = 1,2
      DO 100 J =1,I21
      R(I,J) = RB(I)*DSQRT(1.0D0+THA(I,J)*THA(I,J))
      IF (KD1.EQ.1) RC(I,J) = DSQRT(R(I,J)*R(I,J)-RB(I)*RB(I))
100   CALL STIFF (I,J)
      IF (KD1.EQ.2) GO TO 110
      CALL P(K,1)
      CALL QS (K,1)
110   CONTINUE
C  *
C  *  SKIP DEFLECTION ANALYSIS - FUTURE WORK
C  *
      IF (IDEFL.EQ.1) GO TO 220
120   IF (I21.EQ.2) GO TO 130
      TQ(1) = TRQ
      CALL DEFLCT(1)
      GO TO 210
130   DO 140 L = 1,2
      DO 140 M = 1,2
140   KT(L,M) = KI(1,L,M) + KI(2,L,M)
      CALL INVERT (KT(1,1),KT(1,2),KT(2,2),CT(1,1),CT(1,2),CT(2,1),CT(2,
     +2))
      TQ(1) = KI(1,1,1)*TRQ/(KI(1,1,1) + KI(2,1,1))
      TQ(2) = KI(2,1,1)*TRQ/(KI(1,1,1) + KI(2,1,1))
      LLL = 0
      DO 150 L = 1,2
      DO 150 M = 1,2
150   V(M,L) = R(M,L)
160   LLL = LLL + 1
170   DO 180 L = 1,2
      CALL DEFLCT(L)
      TH2 = TH(2)
      TH(2) = THA(2,L) - PSI(2) - PHI
      CHTH(L) = TH2 - TH(2)
180   CHK = DABS((CHTH(1) - CHTH(2))*2.0D2/(CHTH(1) + CHTH(2)))
      IF (LLL.GT.3.OR.CHK.LT.1.0D-3) GO TO 200
      KD1 = 2
      DO 190 L = 1,2
      DO 190 M = 1,2
      R(M,L) = V(M,L)
190   CALL STIFF (L,M)
      TQ(1) = TQ(1)*CHTH(2)/CHTH(1)
      RATIO = TQ(1)/TQ(2)
      TQ(2) = TRQ/(1.0D0+RATIO)
      TQ(1) = TRQ - TQ(2)
      GO TO 160
200   IF (CHK.GT.5.0D-1) WRITE (6,2) CHK
210   CONTINUE
```

```
C
C  *  REDUCE DATA - DEFLECTED TEETH
C
220   CALL P(K,2)
      CALL QS (K,2)
230   CONTINUE
      CALL TUV
      IF (ING.EQ.1) REWIND 9
      DO 240 I = 1,3
      DO 240 J = 1,2
      WRITE (9,1) TFK(1,J,I,1),TFK(1,J,I,3),TFK(1,J,I,2),
     +TFK(1,J,I,5),TFK(1,J,I,4)
      DO 240 M = 1,2
      WRITE (9,1) TFE(1,J,I,M,1),TFE(1,J,I,M,3),TFE(1,J,I,M,2)
      DO 240 L = 1,3
240   WRITE (9,1) TK(1,J,M,I,L,1),TK(1,J,M,I,L,3),TK(1,J,M,I,L,2)
1     FORMAT (D42.32)
2     FORMAT (//' *** WARNING :  THETA TWO''S ONLY CLOSE TO ',D12.4,
     +' PERCENT OF EACH OTHER !!! ***'//)
      STOP
      END

      SUBROUTINE INVERT (A,B,C,D,E,F,G)
      DOUBLE PRECISION A,B,C,D,E,F,G,H
      H=A*C-B*B
      D=C/H
      E=-B/H
      F=E
      G=A/H
      RETURN
      END
C
      SUBROUTINE STIFF (I,J)
C
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 INV,KC,KI,KLA,KT,KXY,MP,MU,N,NCT
      COMMON /MESS/ XX(50,9),A9(9),A99(9,9),AG(2),ALF,ALP,AX(2),AY(2),
     +B9(9),BET(2),BP,C1(2,2),CA(2),CHTH(2),CI(2,2,2),CK(2,2,2,2),
     +CLA(2,2,2,2),CP,COF,FID(2),CRN,TH(4),CT(2,2),DC2,DCS,DS2,
     +CTR,CTT(2,2),CXY(2,2,2,2),DD(2),DC(3),DS(3),DP,DTH(2),E(2),
     +ETA(2),FF(2),FJB(2),FLA,FXY(2,2),FW(2),G(2),GAM(2),H1(2),H2(2),
     +FU(2),H1A(2),H1B(2),H1C(2),
     +KC(2,2,2,2),KI(2,2,2),KLA(2,2,2,2),KT(2,2),KXY(2,2,2,2),
     +MP,MU(2),N(2),NCT(2,2),OME(2),OTP,PHE(2),PHG(2),PHI,PI,POT,
     +PSI(2),R(2,2),RB(2),RC(2,2),RD(2),RITER,RETA(2),RK,RN(2,2),RO(2),
     +RP(2),TPA,TB(2),THA(2,2),THE(2,4),TLA(2,2),TQ(2),TRQ,TXY(2,2),
     +V(2,2),XA(2),XI,XT(2,2),XTU(2),YA(2),YT(2,2),YTU(2),Z,ZET,
     +XQ(2),YQ(2),I12(50),I21,ID(3),ITER,NOE,KD1
      COMMON /PQST/ PAR(2,2,2,50),PC(2,2,50),PER(2,2,2,50),
     +PFN(2,2,2,50),PK(2,2,2,2,50),PL(2,2,2,2,50),PPA(2,2,50),
     +PT(2,50),PV(2,2,2,50),PW(2,2,2,50),
     +QAR(2,2,2,50),QER(2,2,2,50),QFN(2,2,2,50),QK(2,2,2,2,50),
     +QL(2,2,2,2,50),QV(2,2,2,50),
     +SFC(2,2,3,50),SFK(2,2,3,50),SK(2,2,2,3,3,50),SFE(2,2,3,2,50),
     +TFC(2,2,3,5),TFK(2,2,3,5),TK(2,2,2,3,3,9),TFE(2,2,3,2,3)
     +,UK(2,2,2,3,3,50),UFC(2,2,3,50),UFK(2,2,3,50),UFE(2,2,3,2,50),
     +VK(2,2,2,3,3),VFC(2,2,3),VFK(2,2,3),VFE(2,2,3,2)
      COMMON /LET/ A,A2,B,B2,C,C2,D,D2,F,F2,H(2)
      H1(I) = -(H1B(I)+DSQRT(H1B(I)*H1B(I)-4.0D0*H1A(I)*
```

```
     +(H1C(I)-R(I,J)*R(I,J)))))/H1A(I)
      A = H2(I)/H1(I)
      A2 = A*A
      B = DLOG(A)
      C = 2.0D0*DTAN(BET(I))
      C2 = C*C
      C3 = C*C2
      C1(1,I) = 6.0D0/(E(I)*C2*H1(I))*(1.0D0+1.0D0/A2-2.0D0/A)
      C1(2,I) = 3.0D0/(E(I)*C*H1(I))*(1.0D0/A2-1.0D0)
      IF (KD1.NE.3) GO TO 10
      M = I + 2
      TH(M) = C1(1,I)*FJB(J)*DCOS(XI) + C1(2,I)*FJB(J)*DSIN(XI)
10    AUB = 6.0D0/(E(I)*C3)*(B+2.0D0/A-5.0D-1/A2-1.5D0)
      AUS = 3.0D0/(5.0D0*G(I)*C)*(1.5D0*B-7.5D-1/A2+2.0D0/A-5.0D0/4.0D0)
      BUB = 6.0D0/(E(I)*C2)*(1.0D0/A-5.0D-1/A2-5.0D-1)
      BUS = 3.0D0/(1.0D1*G(I))*(-1.5D0/A2+2.0D0/A-5.0D-1)
      CUB = 6.0D0/(E(I)*C)*(-1.0D0/(8.0D0*A2)+1.0D0/8.0D0)
      CUS = 9.0D0*C/(8.0D1*G(I))*(1.0D0-1.0D0/A2)
      CUC = B/(2.0D0*E(I)*C)
      DO 20 K = 1,2
      DO 20 L = 1,2
      CLA(I,J,K,L) = 0.0D0
20    CONTINUE
      WRITE (6,1)
      XI = THA(I,J) - PSI(I)
      C2 = DCOS(XI)*DCOS(XI)
      S2 = DSIN(XI)*DSIN(XI)
      SC = DSIN(XI)*DCOS(XI)
      CS = 2.0D0*(C2*AUS+SC*BUS+S2*CUS)
      CB = 2.0D0*(C2*AUB+SC*BUB+S2*CUB)
      CC = 2.0D0*S2*CUC
      CLA(I,J,1,1) = CS + CB + CC
      WRITE (6,2) I,J,CS,CB,CC
      IF (I.EQ.1.OR.J.LT.I21) GO TO 99
      FJB(1) = FLA
      WRITE (6,3)
      IF (J.EQ.1) GO TO 30
      CLEFT = CLA(1,1,1,1) + CLA(2,1,1,1)
      CRIGHT = CLA(1,2,1,1) + CLA(2,2,1,1)
      FJB(1) = FLA*CRIGHT/(CLEFT+CRIGHT)
      FJB(2) = FLA - FJB(1)
30    DO 40 K = 1,I21
      A = RC(1,K)
      B = RC(2,K)
      DK = 2.0D0*A*B/(A+B)
      HH = H(1) + H(2)
      B = 1.6D0*DSQRT(FJB(K)*DK*HH)
      DO 40 L = 1,2
      CH = (HH/(2.0D0*PI))*(DLOG(4.D0*RC(1,K)/B) + DLOG(4.D0*RC(2,K)/B)
     + - 1.0D0/3.0D0)
      WRITE (6,2) L,K,CH
      CLA(L,K,1,1) = CLA(L,K,1,1) + CH
40    CONTINUE
1     FORMAT (/' COMPLIANCES'/' GEAR TOOTH',
     +'   SHEAR   BENDING     COMPRESSION'/)
2     FORMAT (2I5,3D15.7)
3     FORMAT (/' GEAR TOOTH     HERTZIAN CONT'/)
99    RETURN
      END
C
```

```
      SUBROUTINE DEFLCT (ITH)
C
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 MAA,MAB,MAC,MAD,MAQ,MLA,MQT
      REAL*8 INV,KC,KI,KLA,KT,KXY,MP,MU,N,NCT
      COMMON /MESS/ AA(50,9),A9(9),A99(9,9),AG(2),ALF,ALP,AX(2),AY(2),
     +B9(9),BET(2),BP,C1(2,2),CA(2),CHTH(2),CI(2,2,2),CK(2,2,2,2),
     +CLA(2,2,2,2),CP,COF,FID(2),CRN,TH(4),CT(2,2),DC2,DCS,DS2,
     +CTR,CTT(2,2),CXY(2,2,2,2),DD(2),DC(3),DS(3),DP,DTH(2),E(2),
     +ETA(2),FF(2),FJB(2),FLA,FXY(2,2),FW(2),G(2),GAM(2),H1(2),H2(2),
     +FU(2),H1A(2),H1B(2),H1C(2),
     +KC(2,2,2,2),KI(2,2,2),KLA(2,2,2,2),KT(2,2),KXY(2,2,2,2),
     +MP,MU(2),N(2),NCT(2,2),OME(2),OTP,PHE(2),PHG(2),PHI,PI,POT,
     +PSI(2),R(2,2),RB(2),RC(2,2),RD(2),RITER,RETA(2),RK,RN(2,2),RO(2),
     +RP(2),TPA,TB(2),THA(2,2),THE(2,4),TLA(2,2),TQ(2),TRQ,TXY(2,2),
     +V(2,2),XA(2),XI,XT(2,2),XTU(2),YA(2),YT(2,2),YTU(2),Z,ZET,
     +XQ(2),YQ(2),I12(50),I21,ID(3),ITER,NOE,KD1
      COMMON /PQST/ PAR(2,2,2,50),PC(2,2,50),PER(2,2,2,50),
     +PFN(2,2,2,50),PK(2,2,2,2,50),PL(2,2,2,2,50),PPA(2,2,50),
     +PT(2,50),PV(2,2,2,50),PW(2,2,2,50),
     +QAR(2,2,2,50),QER(2,2,2,50),QFN(2,2,2,50),QK(2,2,2,2,50),
     +QL(2,2,2,2,50),QV(2,2,2,50),
     +SFC(2,2,3,50),SFK(2,2,3,50),SK(2,2,2,3,3,50),SFE(2,2,3,2,50),
     +TFC(2,2,3,5),TFK(2,2,3,5),TK(2,2,2,3,3,9),TFE(2,2,3,2,3)
     +,UK(2,2,2,3,3,50),UFC(2,2,3,50),UFK(2,2,3,50),UFE(2,2,3,2,50),
     +VK(2,2,2,3,3),VFC(2,2,3),VFK(2,2,3),VFE(2,2,3,2)
      COMMON /LET/ A,A2,B,B2,C,C2,D,D2,F,F2,H(2)
      INV(A) = DTAN(A) - A
      DASIN(A) = DATAN(A/DSQRT(1.0D0-A*A))
      DACOS(A) = DATAN(DSQRT(1.0D0-A*A)/A)
C
C  *  INITIALIZE
C
      KD1 = 3
      FID(ITH) = PHI
      FJB(ITH) = TQ(ITH)/RB(2)
      DO 9 I = 1,2
9     TH(I) = THA(I,ITH) - PSI(I) - PHI
      K = 0
      L = 0
10    FF(1) = FJB(ITH)*DCOS(FID(ITH))
      FF(2) = FJB(ITH)*DSIN(FID(ITH))
      DO 30 I = 1,2
      FU(I) = DACOS(RB(I)/R(I,ITH))
C
C  *  LOCATE UNDEFLECTED POINTS 'T' & 'Q'
C
      VETU = TH(I) + PSI(I) - INV(FU(I))
      XTU(I) = R(I,ITH)*DSIN(VETU)
      YTU(I) = R(I,ITH)*DCOS(VETU)
      TP = R(I,ITH)*(TPA/RP(I) + INV(PHI) - INV(FU(I)))
      QT = R(I,ITH)*DASIN(TP/R(I,ITH))
      XQU = XTU(I) - QT*DCOS(TH(I))
      YQU = YTU(I) + QT*DSIN(TH(I))
C
C  *  LOCATE DEFLECTED CONTACT POINTS 'T' & 'Q'
C
      CALL STIFF(I,ITH)
      DO 20 J = 1,2
20    DD(J) = FF(1)*CXY(I,ITH,J,1) + FF(2)*CXY(I,ITH,J,2)
```

```
      XT(I,ITH) = XTU(I) - DD(1)
      YT(I,ITH) = YTU(I) - DD(2)
      MQT = DTAN(TH(I+2) - TH(I))
      XQ(I) = XT(I,ITH) - QT*DCOS(TH(I)-TH(I+2))
      YQ(I) = YT(I,ITH) + QT*DSIN(TH(I)-TH(I+2))
C
C  *  LOCATE THE POINTS 'A'
C
      EQU = DSQRT(XQU*XQU + YQU*YQU)
      MAQ = -1.0D0/MQT
      VAQ = DATAN(1.0D0/MAQ)
      XA(I) = XQ(I) - EQU*DSIN(VAQ)
      YA(I) = YQ(I) - EQU*DCOS(VAQ)
30    CONTINUE
C
C  *  LOCATE NEW PITCH POINT, 'P', & THE POINTS 'D' & 'B', GEAR 1
C
      EX = XA(1) + XA(2)
      WY = CTR - YA(1) - YA(2)
      CA(ITH) = DSQRT(EX*EX + WY*WY)
      FIP = DACOS(CTR*DCOS(PHI)/CA(ITH))
      FID(ITH) = FIP + DATAN(EX/WY)
      RP1 = N(1)*CA(ITH)/(N(1)+N(2))
      MAA = -WY/EX
      VAP = DATAN(1.0D0/MAA)
      XP = XA(1) + RP1*DSIN(VAP)
      YP = YA(1) + RP1*DCOS(VAP)
      MLA = DTAN(FID(ITH))
      BLA = YP - MLA*XP
      MAD = -1.0D0/MLA
      DO 40 I = 1,2
      BAD = YA(I) - MAD*XA(I)
      XD = (BAD - BLA)/(MLA - MAD)
      YD = MAD*XD + BAD
      IF (I.EQ.2) GO TO 40
      Y1 = YD - YA(1)
      Y2 = YQ(1) - YA(I)
      X1 = XD - XA(1)
      X2 = XQ(1) - XA(I)
40    CONTINUE
      R1 = DSQRT(X1*X1+Y1*Y1)
      R2 = DSQRT(X2*X2+Y2*Y2)
      DAQ = DACOS((Y1*Y2+X1*X2)/(R1*R2))
      ALPHA = DATAN(PSI(1)+DAQ)
      MAB = 1.0D0/DTAN(DATAN(1.0D0/MAD)+ALPHA)
      BAB = YA(1) - MAB*XA(1)
      XB = (BAB - BLA)/(MLA - MAB)
      YB = MAB*XB + BAB
      R(1,ITH) = DSQRT((XB-XA(1))**2 + (YB-YA(1))**2)
      RC(1,ITH) = DSQRT(R(1,ITH)*R(1,ITH) - RB(1)*RB(1))
C
C  *  NOW GO TO THE X2 Y2 SYSTEM AND LOCATE 'B2' A DIST 'HCD' FROM 'B'
C
      XB = -XB
      YB = CTR - YB
      XB2 = XB
      YB2 = YB
      AP = DATAN(MLA)
      DO 60 II = 1,2
      I = 3-II
```

```
      A = RC(1,ITH)
      B = RC(2,ITH)
      DK = 2.0D0*A*B/(A+B)
      B = 1.6D0*DSQRT(FJB(ITH)*DK*(H(1)+H(2)))
      HCD = (H(ITH)/PI)*(1.0D0/6.0D0+DLOG(4.0*RC(I,ITH)/B))
      XB2 = XB2 + HCD*DCOS(AP)
      YB2 = YB2 + HCD*DSIN(AP)
      IF (II.EQ.2) GO TO 50
      XT(I,ITH) = -XB2
      YT(I,ITH) = CTR - YB2
      GO TO 60
50    R(2,ITH) = DSQRT((XB2-XA(2))**2 + (YB2-YA(2))**2)
      RC(2,ITH) = DSQRT(R(2,ITH)*R(2,ITH)-RB(2)*RB(2))
60    CONTINUE
C
C  *  LOCATE THE POINT 'C' ON GEAR 2 A DIST R(2) FROM 'A2'
C
      QAC = PSI(2) - INV(DACOS(RB(2)/R(2,ITH)))
      VAC = QAC + DATAN(1.0D0/MAQ)
      XC = XA(2) + R(2,ITH)*DSIN(VAC)
      YC = YA(2) + R(2,ITH)*DCOS(VAC)
C
C  *  FIND 'DEL' ROTATION OF GEAR 2
C
      Y1 = YC  - YA(2)
      Y2 = YB2 - YA(2)
      X1 = XC  - XA(2)
      X2 = XB2 - XA(2)
      R1 = DSQRT(X1*X1 + Y1*Y1)
      R2 = DSQRT(X2*X2 + Y2*Y2)
      ARG = (Y1*Y2+X1*X2)/(R1*R2)
      ARG = DMIN1(1.0D0,ARG)
      ARG = DMAX1(-1.0D0,ARG)
      DEL = DACOS(ARG)
      IF (XC.GT.XB2) DEL = -DEL
      TH(2) = TH(2) + DEL
      XN = BLA/(MAD-MLA)
      YN = MAD*XN
      RN(2,ITH) = DSQRT(XN*XN+YN*YN)
      FJB(ITH) = TQ(ITH)/RN(2,ITH)
      DEL = DABS(DEL)
      IF (K.NE.0) GO TO 65
      DELP = DEL
      DELC = DEL*1.0D-6
      GO TO 70
65    IF (DEL.LT.DELP) L = 0
      IF (DEL.GT.DELP) L = L + 1
      DELP = DEL
      IF (L.LE.5) GO TO 70
      STOP
70    K = K + 1
      IF (K.LT.10.AND.DEL.GT.DELC) GO TO 10
      KD1 = 4
      DO 80 I = 1,2
80    CALL STIFF (I,ITH)
      RETURN
      END
C
      SUBROUTINE P (K,M)
```

```
C
C
C STMT    VAR   DIMENSIONING
C NOS     NAME  A B C D E F  COORD    COMMENTS
C
C - 99     PK   2   2 2 2 50  GEAR    K, STIFFNESS, 2ND ORDER TENSOR
C 100-     PL   2 2 2 2   50  GEAR    CONTACT LOCATION
C 200-     PT   2         50   N/A    TIME AT LOCATION, 0 < T < (2*PI/TPF)
C 300-     PPA  2   2     50  GEAR    PRESSURE ANGLE, ABSCISSA-CCW-COM NORM
C 400-     PC   2   2     50   N/A    CENTER DISTANCE (FICTITIOUS GEARS)
C 500-     PV   2   2 2   50  GEAR    CONT PT REL VEL (V2-V1, 1 = DRIVER)
C 600-     PW   2 2 2     50   N/A    ANGULAR VELOCITY
C 700-     PFN  2   2 2   50  GEAR    CONTACT FORCE
C 800-     PAR  2   2 2   50  GEAR    CONTACT AREA
C 900-     PER  2   2 2   50  GEAR    ERROR DUE TO (SUMMED) TERMS AT N*TPF
C
C DIMENSIONING     A = 1,2  UNDEFLECTED, DEFLECTED, RESP
C                  B = 1,2  GEAR 1 (DR), GEAR 2 (DN)
C                  C = 1,2  TOOTH CONT 1 (LH), 2 (RH), IN MULTI-CONT ZONE
C                  D = 1,2  X,Y GEAR LOCAL COORDINATE DIRECTIONS
C                  E = 1,2  X,Y GEAR LOCAL COORDINATE DIRECTIONS
C                  F = 1,2,...,10  INCREM THROUGH ONE MESH ENGAGEMENT
C
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 INV,KC,KI,KLA,KT,KXY,MP,MU,N,NCT
      COMMON /MESS/ AA(50,9),A9(9),A99(9,9),AG(2),ALF,ALP,AX(2),AY(2),
     +B9(9),BET(2),BP,C1(2,2),CA(2),CHTH(2),CI(2,2,2),CK(2,2,2,2),
     +CLA(2,2,2,2),CP,COF,FID(2),CRN,TH(4),CT(2,2),DC2,DCS,DS2,
     +CTR,CTT(2,2),CXY(2,2,2,2),DD(2),DC(3),DS(3),DP,DTH(2),E(2),
     +ETA(2),FF(2),FJB(2),FLA,FXY(2,2),FW(2),G(2),GAM(2),H1(2),H2(2),
     +FU(2),H1A(2),H1B(2),H1C(2),
     +KC(2,2,2,2),KI(2,2,2),KLA(2,2,2,2),KT(2,2),KXY(2,2,2,2),
     +MP,MU(2),N(2),NCT(2,2),OME(2),OTP,PHE(2),PHG(2),PHI,PI,POT,
     +PSI(2),R(2,2),RB(2),RC(2,2),RD(2),RITER,RETA(2),RK,RN(2,2),RO(2),
     +RP(2),TPA,TB(2),THA(2,2),THE(2,4),TLA(2,2),TQ(2),TRQ,TXY(2,2),
     +V(2,2),XA(2),XI,XT(2,2),XTU(2),YA(2),YT(2,2),YTU(2),Z,ZET,
     +XQ(2),YQ(2),I12(50),I21,ID(3),ITER,NOE,KD1
      COMMON /PQST/ PAR(2,2,2,50),PC(2,2,50),PER(2,2,2,50),
     +PFN(2,2,2,50),PK(2,2,2,2,50),PL(2,2,2,2,50),PPA(2,2,50),
     +PT(2,50),PV(2,2,2,50),PW(2,2,2,50),
     +QAR(2,2,2,50),QER(2,2,2,50),QFN(2,2,2,50),QK(2,2,2,2,50),
     +QL(2,2,2,2,50),QV(2,2,2,50),
     +SFC(2,2,3,50),SFK(2,2,3,50),SK(2,2,2,3,3,50),SFE(2,2,3,2,50),
     +TFC(2,2,3,5),TFK(2,2,3,5),TK(2,2,2,3,3,9),TFE(2,2,3,2,3)
     +,UK(2,2,2,3,3,50),UFC(2,2,3,50),UFK(2,2,3,50),UFE(2,2,3,2,50),
     +VK(2,2,2,3,3),VFC(2,2,3),VFK(2,2,3),VFE(2,2,3,2)
      COMMON /LET/ A,A2,B,B2,C,C2,D,D2,F,F2,H(2)
      DACOS(A) = DATAN(DSQRT(1.0D0-A*A)/A)
C
C  *  INITIALIZE  *
C
      PT(M,K) = 0.0D0
      DO 11 I = 1,2
      PPA(M,I,K) = 0.0D0
      PC(M,I,K) = 0.0D0
      DO 11 J = 1,2
      PV(M,I,J,K) = 0.0D0
      PW(M,I,J,K) = 0.0D0
      PFN(M,I,J,K) = 0.0D0
      PAR(M,I,J,K) = 0.0D0
```

```
      PER(M,I,J,K) = 0.0D0
      DO 11 L = 1,2
      PK(M,I,J,L,K) = 0.0D0
11    PL(M,I,J,L,K) = 0.0D0
C  *
C  *  SKIP DEFLECTION ANALYSIS - FUTURE WORK
C  *
      IF (M.EQ.2) GO TO 925
C  *
      DO 310 I =1,I21
      PPA(M,I,K) = PHI
      IF (M.EQ.2) PPA(M,I,K) = FID(I)
      TXY(1,1) =  DCOS(PPA(M,I,K))
      TXY(1,2) = -DSIN(PPA(M,I,K))
      TXY(2,1) =  DSIN(PPA(M,I,K))
      TXY(2,2) =  DCOS(PPA(M,I,K))
      DKLA = 1.0D0/(CLA(1,I,1,1) + CLA(2,I,1,1))
      DO 10 J = 1,2
      DO 10 L = 1,2
      PK(M,I,J,L,K) = TXY(J,1)*TXY(L,1)*DKLA
10    CONTINUE
310   CONTINUE
100   GO TO (150), M
      DO 710 I =1,I21
      PL(2,1,I,1,K) = XT(1,I)
      PL(2,1,I,2,K) = YT(1,I)
      PL(2,2,I,1,K) = -XT(1,I)
      PL(2,2,I,2,K) = -YT(1,I)
400   PC(2,I,K) = CA(I)
700   PFN(2,I,1,K) = FJB(I)*DCOS(PPA(M,I,K))
710   PFN(2,I,2,K) = FJB(I)*DSIN(PPA(M,I,K))
200   A = (CHTH(1)+CHTH(2))/2.0D0
      IF (I21.EQ.1) A = CHTH(1)
      PT(2,K) = RK*PI/(RITER*OTP) + (RK*DTH(2)-A)/(2.0D0*DABS(OME(2)))
      GO TO 900
150   C = DCOS(POT-PHI)
      C2 = C*C
      A = RP(1)
      A2 = A*A
      DO 760 I =1,I21
      D = R(1,I)
      D2 = D*D
      B = A*C - DSQRT(D2+A2*(C2-1.0D0))
      PL(1,1,I,1,K) = -B*DSIN(POT-PHI)
      PL(1,1,I,2,K) =  A - B*C
      PL(1,2,I,1,K) = PL(1,1,I,1,K)
      PL(1,2,I,2,K) = PL(1,1,I,2,K) - CTR
450   PC(1,I,K) = CTR
      DO 870 J = 1,2
870   RC(J,I) = DSQRT(R(J,I)*R(J,I)-RB(J)*RB(J))
750   S = TRQ/RB(2)
      PFN(1,I,1,K) = S*DCOS(PHI)
      PFN(1,I,2,K) = S*DSIN(PHI)
      GO TO (760), I21
      S = PK(M,I,1,1,K)/(PK(M,I,1,1,K)+PK(M,I,2,1,K))
      PFN(1,I,1,K) = S*PFN(1,I,1,K)
      PFN(1,I,2,K) = S*PFN(1,I,2,K)
760   CONTINUE
250   PT(1,K) = (RK/RITER)*2.0D0*PI/OTP
900   A = 0.0D0
```

```
      IF (NOE.EQ.0) GO TO 500
      DO 910 J = 1,NOE
      ARG = ID(J)*OTP*PT(M,K)
910   A = A + DS(J)*DSIN(ARG) + DC(J)*DCOS(ARG)
      A1 = A
500   DO 920 I =1,I21
      A = PL(M,1,I,1,K)
      B = PL(M,1,I,2,K)
      R1 = DSQRT(A*A + B*B)
      R2 = DSQRT(A*A + (CTR-B)*(CTR-B))
      T1 = DATAN(A/B)
      T2 = DATAN(-A/(CTR-B))
      V1 = DABS(OME(1))*R1
      ARG1 = T1 + PPA(M,I,K)
      ARG2 = T2 + PPA(M,I,K)
      VN = V1*DCOS(ARG1)
      V1R = V1*DSIN(ARG1)
      V2 = VN/DCOS(ARG2)
      OME2P = V2/R2
      A = DABS(OME(1)/OME2P)
      B = 5.0D-1*(RB(1)*DABS(OME(1))+RB(2)*DABS(OME(2)))
      OME2PP = 2.0D0*B/(RB(1)*A+RB(2))
      OME1PP = OME2PP*A
      V1 = OME1PP*V1
      VN = V1*DCOS(ARG1)
      V1R = V1*DSIN(ARG1)
      V2 = VN/DCOS(ARG2)
      V2R = V2*DSIN(ARG2)
      PV(M,I,1,K) = (V1R-V2R)*DCOS(PPA(M,I,K))
      PV(M,I,2,K) = (V2R-V1R)*DSIN(PPA(M,I,K))
600   PW(M,1,I,K) = OME1PP
      PW(M,2,I,K) = OME2PP
      F = DSQRT(PFN(M,I,1,K)**2+PFN(M,I,2,K)**2)
      A = 2.0D0/RC(I,1) + 2.0D0/RC(I,2)
      B = DSQRT(F*(H(1)+H(2))/(POT*A))
      PAR(M,I,1,K) = B*DSIN(PPA(M,I,K))
800   PAR(M,I,2,K) = B*DCOS(PPA(M,I,K))
      IF (NOE.EQ.0) GO TO 920
      PER(M,I,1,K) = A1*DCOS(PPA(M,I,K))
      PER(M,I,2,K) = A1*DSIN(PPA(M,I,K))
920   CONTINUE
      IF (M.EQ.1) GO TO 930
925   WRITE (6,1) K
      WRITE (6,2) PPA(2,1,K),PC(2,1,K),PW(2,1,1,K),PL(2,1,1,1,K),
     +PAR(2,1,1,K),PFN(2,1,1,K),PER(2,1,1,K),
     +PK(2,1,1,1,K),PK(2,1,1,2,K)
      WRITE (6,3) PPA(1,1,K),PC(1,1,K),PW(1,1,1,K),PL(1,1,1,1,K),
     +PAR(1,1,1,K),PFN(1,1,1,K),PER(1,1,1,K),
     +PK(1,1,1,1,K),PK(1,1,1,2,K)
      WRITE (6,5) PL(2,1,1,2,K),PAR(2,1,2,K),PFN(2,1,2,K),
     +PER(2,1,2,K),PK(2,1,2,1,K),PK(2,1,2,2,K)
      WRITE (6,6) PL(1,1,1,2,K),PAR(1,1,2,K),PFN(1,1,2,K),
     +PER(1,1,2,K),PK(1,1,2,1,K),PK(1,1,2,2,K)
      WRITE (6,7) PW(2,2,1,K),PL(2,2,1,1,K)
      WRITE (6,8) PW(1,2,1,K),PL(1,2,1,1,K)
      WRITE (6,5) PL(2,2,1,2,K)
      WRITE (6,6) PL(1,2,1,2,K)
      WRITE (6,4) PPA(2,2,K),PC(2,2,K),PW(2,1,2,K),PL(2,1,2,1,K),
     +PAR(2,2,1,K),PFN(2,2,1,K),PER(2,2,1,K),
     +PK(2,2,1,1,K),PK(2,2,1,2,K)
```

```
      WRITE (6,3) PPA(1,2,K),PC(1,2,K),PW(1,1,2,K),PL(1,1,2,1,K),
     +PAR(1,2,1,K),PFN(1,2,1,K),PER(1,2,1,K),
     +PK(1,2,1,1,K),PK(1,2,1,2,K)
      WRITE (6,5) PL(2,1,2,2,K),PAR(2,2,2,K),PFN(2,2,2,K),
     +PER(2,2,2,K),PK(2,2,2,1,K),PK(2,2,2,2,K)
      WRITE (6,6) PL(1,1,2,2,K),PAR(1,2,2,K),PFN(1,2,2,K),
     +PER(1,2,2,K),PK(1,2,2,1,K),PK(1,2,2,2,K)
      WRITE (6,7) PW(2,2,2,K),PL(2,2,2,1,K)
      WRITE (6,8) PW(1,2,2,K),PL(1,2,2,1,K)
      WRITE (6,5) PL(2,2,2,2,K)
      WRITE (6,6) PL(1,2,2,2,K)
1     FORMAT (1H1,'INCREMENTATION NUMBER',I3,
     +'. ''P'' VARIABLES, GEAR ',
     +'COORDINATE (X,Y) DIRECTIONS'//6X,' PRESSURE     CENTER',
     +7X,'ANGULAR VEL-     CONTACT      CONTACT      ',
     +'CONTACT    SUMMED    STIFFNESS TENSOR (LB/IN)'/5X,
     +'ANGLE (RAD)  DIST (IN)      OCITY  (1/S)      LOC (IN) ',
     +' AREA (SQ-IN) FORCE (LBF) ERROR (IN)',6X,'X',11X,'Y'/)
2     FORMAT (' LH:',2D12.4,' G1:',D12.4,' X:',7D12.4)
3     FORMAT (4X,2D12.4,4X,D12.4,3X,7D12.4)
4     FORMAT (' RH:',2D12.4,' G1:',D12.4,' X:',7D12.4)
5     FORMAT (45X,'Y:',7D12.4)
6     FORMAT (47X,7D12.4)
7     FORMAT (29X,'G2:',D12.4,' X:',D12.4)
8     FORMAT (29X,2D15.4)
930   RETURN
      END
C
      SUBROUTINE QS (K,M)
C
C  VAR  DIMENSIONING     GLOBAL
C NAME  A B C D E F G     COORD   COMMENTS
C
C  QK   2     2 2 2 50     X/Y    K, STIFFNESS, 2ND ORDER TENSOR
C  QL   2 2   2 2   50     X/Y    CONTACT LOCATION
C  QV   2     2 2   50     X/Y    CONT PT REL VEL (V2-V1), 1 = DRIVER)
C  QFN  2     2 2   50     X/Y    CONTACT FORCE
C  QAR  2     2 2   50     X/Y    CONTACT AREA
C  QER  2     2 2   50     X/Y    ERROR DUE TO (SUMMED) TERMS AT N*TPF
C
C  SK   2 2 2   3 3 50    X/Y/Z   K, BLOCK STIFFNESS TERMS
C  SFK  2 2     3   50    X/Y/Z   EXT COL - ERRORS (N*TPF) * STIFFNESSES
C  SFC  2 2     3   50    X/Y/Z   EXT COL - SLIDING FRICTION IN RUNNING
C  SFE  2 2     3 2 50    X/Y/Z   EXT COL - SFE * ERR ( NOT AT N*TPF )
C
C  DIMENSIONING     A = 1,2  UNDEFL, DEFLECTED, RESP
C                   B = 1,2  GEAR 1 (DR), GEAR 2 (DN)
C                   C = 1,2  GEAR 1 (DR), GEAR 2 (DN)
C                   D = 1,2  TOOTH CONT 1 (LH), 2 (RH), IN MULTI-CONT ZONE
C                   E = 1,2(,3)  X,Y(,Z) SYSTEM GLOBAL COORD DIRECTIONS
C                   F = 1,2(,3)  X,Y(,Z) SYSTEM GLOBAL COORD DIRECTIONS
C                   G = 1,2,...,50  INCREM THROUGH ONE MESH ENGAGEMENT
C
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 INV,KC,KI,KLA,KT,KXY,MP,MU,N,NCT
      COMMON /MESS/ AA(50,9),A9(9),A99(9,9),AG(2),ALF,ALP,AX(2),AY(2),
     +B9(9),BET(2),BP,C1(2,2),CA(2),CHTH(2),CI(2,2,2),CK(2,2,2,2),
     +CLA(2,2,2,2),CP,COF,FID(2),CRN,TH(4),CT(2,2),DC2,DCS,DS2,
     +CTR,CTT(2,2),CXY(2,2,2,2),DD(2),DC(3),DS(3),DP,DTH(2),E(2),
```

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     +ETA(2),FF(2),FJB(2),FLA,FXY(2,2),FW(2),G(2),GAM(2),H1(2),H2(2),
     +FU(2),H1A(2),H1B(2),H1C(2),
     +KC(2,2,2,2),KI(2,2,2),KLA(2,2,2,2),KT(2,2),KXY(2,2,2,2),
     +MP,MU(2),N(2),NCT(2,2),OME(2),OTP,PHE(2),PHG(2),PHI,PI,POT,
     +PSI(2),R(2,2),RB(2),RC(2,2),RD(2),RITER,RETA(2),RK,RN(2,2),RO(2),
     +RP(2),TPA,TB(2),THA(2,2),THE(2,4),TLA(2,2),TQ(2),TRQ,TXY(2,2),
     +V(2,2),XA(2),XI,XT(2,2),XTU(2),YA(2),YT(2,2),YTU(2),Z,ZET,
     +XQ(2),YQ(2),I12(50),I21,ID(3),ITER,NOE,KD1
      COMMON /PQST/ PAR(2,2,2,50),PC(2,2,50),PER(2,2,2,50),
     +PFN(2,2,2,50),PK(2,2,2,2,50),PL(2,2,2,2,50),PPA(2,2,50),
     +PT(2,50),PV(2,2,2,50),PW(2,2,2,50),
     +QAR(2,2,2,50),QER(2,2,2,50),QFN(2,2,2,50),QK(2,2,2,2,50),
     +QL(2,2,2,2,50),QV(2,2,2,50),
     +SFC(2,2,3,50),SFK(2,2,3,50),SK(2,2,2,3,3,50),SFE(2,2,3,2,50),
     +TFC(2,2,3,5),TFK(2,2,3,5),TK(2,2,2,3,3,9),TFE(2,2,3,2,3)
     +,UK(2,2,2,3,3,50),UFC(2,2,3,50),UFK(2,2,3,50),UFE(2,2,3,2,50),
     +VK(2,2,2,3,3),VFC(2,2,3),VFK(2,2,3),VFE(2,2,3,2)
      COMMON /LET/ A,A2,B,B2,C,C2,D,D2,F,F2,H(2)
C
C  *  INITIALIZE  *
C
      DO 25 I = 1,2
      DO 25 J = 1,3
      IF (J.EQ.3) GO TO 15
      QV(M,I,J,K) = 0.0D0
      QFN(M,I,J,K) = 0.0D0
      QAR(M,I,J,K) = 0.0D0
      QER(M,I,J,K) = 0.0D0
15    SFK(M,I,J,K) = 0.0D0
      SFC(M,I,J,K) = 0.0D0
      DO 25 L = 1,2
      SFE(M,I,J,L,K) = 0.0D0
      IF (J.EQ.3) GO TO 20
      QK(M,I,J,L,K) = 0.0D0
      QL(M,I,J,L,K) = 0.0D0
20    DO 25 II = 1,3
25    SK(M,I,L,J,II,K) = 0.0D0
C  *
C  *  SKIP DEFLECTION ANALYSIS - FUTURE WORK
C  *
      IF (M.EQ.2) GO TO 95
C  *
      EN = 1.0D0
      IF (OME(1).GT.0.0D0) EN = -1.0D0
      CTT(1,1) = EN*DCOS(ALF)
      CTT(1,2) =   -DSIN(ALF)
      CTT(2,1) = EN*DSIN(ALF)
      CTT(2,2) =    DCOS(ALF)
      ALP = DABS(ALF)
      IF (ALP.GT.POT) ALP = PI - ALP
      NCT(1,1) = DCOS(ALP)
      NCT(1,2) = DSIN(ALP)
      NCT(2,1) = NCT(1,2)
      NCT(2,2) =  NCT(1,1)
C
C                            ST
C  *  TRANSFORMATION OF 1-- ORDER CARTESIAN TENSORS
C
      DO 30 I =1,I21
      DO 30 J = 1,2
```

```
      DO 30 L = 1,2
      QV(M,I,J,K) = QV(M,I,J,K) + CTT(J,L)*PV(M,I,L,K)
      DO 30 LL = 1,2
30    QL(M,J,I,L,K) = QL(M,J,I,L,K) + CTT(L,LL)*PL(M,J,I,LL,K)
C
C                               ST
C  *  TRANSFORMATION OF 1-- ORDER NON-TENSOR QUANTITIES  *
C
      DO 40 I =1,I21
      DO 40 J = 1,2
      DO 40 JJ = 1,2
      QFN(M,I,J,K) = QFN(M,I,J,K) + NCT(J,JJ)*PFN(M,I,JJ,K)
      QAR(M,I,J,K) = QAR(M,I,J,K) + NCT(J,JJ)*PAR(M,I,JJ,K)
40    QER(M,I,J,K) = QER(M,I,J,K) + NCT(J,JJ)*PER(M,I,JJ,K)
C
C                               ND
C  *  TRANSFORMATION OF 2-- ORDER STIFFNESS TENSOR  *
C
      DO 50 I =1,I21
      DO 50 J = 1,2
      DO 50 L = 1,2
      DO 50 JJ = 1,2
      DO 50 LL = 1,2
50    QK(M,I,J,L,K) = QK(M,I,J,L,K)+ CTT(J,JJ)*CTT(L,LL)*PK(M,I,JJ,LL,K)
C
C  *  READY BLOCK AND EXT COL TERMS FOR LEAST SQUARES ANAL  *
C
C  *  BLOCK STIFFNESS TERMS  *
C
      DO 60 L =1,I21
      DO 60 I = 1,3
      DO 60 J = 1,3
      DO 60 II = 1,2
      DO 60 JJ = 1,2
      EN = -1.0D0
      IF (II.EQ.JJ) EN = 1.0D0
      A = EN*(QL(M,II,L,2,K)*QL(M,JJ,L,2,K)*QK(M,L,1,1,K) -
     + QL(M,II,L,2,K)*QL(M,JJ,L,1,K)*QK(M,L,1,2,K) -
     + QL(M,II,L,1,K)*QL(M,JJ,L,2,K)*QK(M,L,2,1,K) +
     + QL(M,II,L,1,K)*QL(M,JJ,L,1,K)*QK(M,L,2,2,K))
      IF (J.NE.3) A = EN*(QL(M,II,L,1,K)*QK(M,L,2,J,K) -
     + QL(M,II,L,2,K)*QK(M,L,1,J,K))
      IF (I.NE.3) A = EN*(QL(M,JJ,L,1,K)*QK(M,L,I,2,K) -
     + QL(M,JJ,L,2,K)*QK(M,L,I,1,K))
      IF (I.NE.3.AND.J.NE.3) A = EN*QK(M,L,I,J,K)
60    SK(M,II,JJ,I,J,K) = SK(M,II,JJ,I,J,K) + A
C
C  *  EXT COL STIFFNESS TERMS DUE TO ERRORS (N*TPF) ACTING ON STIFF'S  *
C
      DO 80 I = 1,3
      DO 80 J = 1,2
      MM = 1
      IF (J.EQ.2) MM = -1
      DO 80 L =1,I21
      B = QK(M,L,1,1,K)*QL(M,J,L,2,K) - QK(M,L,2,1,K)*QL(M,J,L,1,K)
      C = QK(M,L,1,2,K)*QL(M,J,L,2,K) - QK(M,L,2,2,K)*QL(M,J,L,1,K)
      IF (I.EQ.3) GO TO 70
      B = MM*QK(M,L,I,1,K)
      C = MM*QK(M,L,I,2,K)
70    A = QER(M,L,1,K)*B + QER(M,L,2,K)*C
```

```
      SFK(M,J,I,K) = SFK(M,J,I,K) + A
      SFE(M,J,I,1,K) = SFE(M,J,I,1,K) + B
      SFE(M,J,I,2,K) = SFE(M,J,I,2,K) + C
80    CONTINUE
C
C  *  EXT COL DAMPING TERMS DUE TO SLIDING FRICTION IN NORMAL RUNNING  *
C
      DO 90 L =1,I21
      DO 90 J = 1,2
      MX = 1
      MY = 1
      IF (QV(M,L,1,K).LT.0.0D0) MX = -1
      IF (QV(M,L,2,K).LT.0.0D0) MY = -1
      IF (J.EQ.1) GO TO 85
      MX = -MX
      MY = -MY
85    A = MX*QFN(M,L,2,K)*COF
      B = MY*QFN(M,L,1,K)*COF
      SFC(M,J,1,K) = SFC(M,J,1,K) + A
      SFC(M,J,2,K) = SFC(M,J,2,K) + B
90    SFC(M,J,3,K) = SFC(M,J,3,K) - A*QL(M,J,L,2,K) + B*QL(M,J,L,1,K)
      IF (M.EQ.1) GO TO 999
95    WRITE (6,1)
      WRITE (6,2) QL(2,1,1,1,K),
     +QAR(2,1,1,K),QFN(2,1,1,K),QER(2,1,1,K),
     +QK(2,1,1,1,K),QK(2,1,1,2,K)
      WRITE (6,3) QL(1,1,1,1,K),
     +QAR(1,1,1,K),QFN(1,1,1,K),QER(1,1,1,K),
     +QK(1,1,1,1,K),QK(1,1,1,2,K)
      WRITE (6,4) QL(2,1,1,2,K),QAR(2,1,2,K),QFN(2,1,2,K),
     +QER(2,1,2,K),QK(2,1,2,1,K),QK(2,1,2,2,K)
      WRITE (6,3) QL(1,1,1,2,K),QAR(1,1,2,K),QFN(1,1,2,K),
     +QER(1,1,2,K),QK(1,1,2,1,K),QK(1,1,2,2,K)
      WRITE (6,5) QL(2,2,1,1,K)
      WRITE (6,3) QL(1,2,1,1,K)
      WRITE (6,4) QL(2,2,1,2,K)
      WRITE (6,3) QL(1,2,1,2,K)
      WRITE (6,6) QL(2,1,2,1,K),
     +QAR(2,2,1,K),QFN(2,2,1,K),QER(2,2,1,K),
     +QK(2,2,1,1,K),QK(2,2,1,2,K)
      WRITE (6,3) QL(1,1,2,1,K),
     +QAR(1,2,1,K),QFN(1,2,1,K),QER(1,2,1,K),
     +QK(1,2,1,1,K),QK(1,2,1,2,K)
      WRITE (6,4) QL(2,1,2,2,K),QAR(2,2,2,K),QFN(2,2,2,K),
     +QER(2,2,2,K),QK(2,2,2,1,K),QK(2,2,2,2,K)
      WRITE (6,3) QL(1,1,2,2,K),QAR(1,2,2,K),QFN(1,2,2,K),
     +QER(1,2,2,K),QK(1,2,2,1,K),QK(1,2,2,2,K)
      WRITE (6,5) QL(2,2,2,1,K)
      WRITE (6,3) QL(1,2,2,1,K)
      WRITE (6,4) QL(2,2,2,2,K)
      WRITE (6,3) QL(1,2,2,2,K)
      WRITE (6,7)
      WRITE (6,8) SK(2,1,1,1,1,K),SK(2,1,1,1,2,K),SK(2,1,1,1,3,K),
     +SK(2,1,2,1,1,K),SK(2,1,2,1,2,K),SK(2,1,2,1,3,K),SFK(2,1,1,K),
     +SFC(2,1,1,K),SFE(2,1,1,1,K),SFE(2,1,1,2,K)
      WRITE (6,11) SK(1,1,1,1,1,K),SK(1,1,1,1,2,K),SK(1,1,1,1,3,K),
     +SK(1,1,2,1,1,K),SK(1,1,2,1,2,K),SK(1,1,2,1,3,K),SFK(1,1,1,K),
     +SFC(1,1,1,K),SFE(1,1,1,1,K),SFE(1,1,1,2,K)
      WRITE (6,12) SK(2,1,1,2,1,K),SK(2,1,1,2,2,K),SK(2,1,1,2,3,K),
     +SK(2,1,2,2,1,K),SK(2,1,2,2,2,K),SK(2,1,2,2,3,K),SFK(2,1,2,K),
```

```
     +SFC(2,1,2,K),SFE(2,1,2,1,K),SFE(2,1,2,2,K)
      WRITE (6,11) SK(1,1,1,2,1,K),SK(1,1,1,2,2,K),SK(1,1,1,2,3,K),
     +SK(1,1,2,2,1,K),SK(1,1,2,2,2,K),SK(1,1,2,2,3,K),SFK(1,1,2,K),
     +SFC(1,1,2,K),SFE(1,1,2,1,K),SFE(1,1,2,2,K)
      WRITE (6,13) SK(2,1,1,3,1,K),SK(2,1,1,3,2,K),SK(2,1,1,3,3,K),
     +SK(2,1,2,3,1,K),SK(2,1,2,3,2,K),SK(2,1,2,3,3,K),SFK(2,1,3,K),
     +SFC(2,1,3,K),SFE(2,1,3,1,K),SFE(2,1,3,2,K)
      WRITE (6,11) SK(1,1,1,3,1,K),SK(1,1,1,3,2,K),SK(1,1,1,3,3,K),
     +SK(1,1,2,3,1,K),SK(1,1,2,3,2,K),SK(1,1,2,3,3,K),SFK(1,1,3,K),
     +SFC(1,1,3,K),SFE(1,1,3,1,K),SFE(1,1,3,2,K)
      WRITE (6,14) SK(2,2,1,1,1,K),SK(2,2,1,1,2,K),SK(2,2,1,1,3,K),
     +SK(2,2,2,1,1,K),SK(2,2,2,1,2,K),SK(2,2,2,1,3,K),SFK(2,2,1,K),
     +SFC(2,2,1,K),SFE(2,2,1,1,K),SFE(2,2,1,2,K)
      WRITE (6,11) SK(1,2,1,1,1,K),SK(1,2,1,1,2,K),SK(1,2,1,1,3,K),
     +SK(1,2,2,1,1,K),SK(1,2,2,1,2,K),SK(1,2,2,1,3,K),SFK(1,2,1,K),
     +SFC(1,2,1,K),SFE(1,2,1,1,K),SFE(1,2,1,2,K)
      WRITE (6,12) SK(2,2,1,2,1,K),SK(2,2,1,2,2,K),SK(2,2,1,2,3,K),
     +SK(2,2,2,2,1,K),SK(2,2,2,2,2,K),SK(2,2,2,2,3,K),SFK(2,2,2,K),
     +SFC(2,2,2,K),SFE(2,2,2,1,K),SFE(2,2,2,2,K)
      WRITE (6,11) SK(1,2,1,2,1,K),SK(1,2,1,2,2,K),SK(1,2,1,2,3,K),
     +SK(1,2,2,2,1,K),SK(1,2,2,2,2,K),SK(1,2,2,2,3,K),SFK(1,2,2,K),
     +SFC(1,2,2,K),SFE(1,2,2,1,K),SFE(1,2,2,2,K)
      WRITE (6,13) SK(2,2,1,3,1,K),SK(2,2,1,3,2,K),SK(2,2,1,3,3,K),
     +SK(2,2,2,3,1,K),SK(2,2,2,3,2,K),SK(2,2,2,3,3,K),SFK(2,2,3,K),
     +SFC(2,2,3,K),SFE(2,2,3,1,K),SFE(2,2,3,2,K)
      WRITE (6,11) SK(1,2,1,3,1,K),SK(1,2,1,3,2,K),SK(1,2,1,3,3,K),
     +SK(1,2,2,3,1,K),SK(1,2,2,3,2,K),SK(1,2,2,3,3,K),SFK(1,2,3,K),
     +SFC(1,2,3,K),SFE(1,2,3,1,K),SFE(1,2,3,2,K)
1     FORMAT (' ''Q'' VARIABLES, SYSTEM (GLOBAL) (X,Y)',
     +//13X,'CONTACT          CONTACT          ',
     +' CONTACT',8X,'SUMMED      STIFFNESS TENSOR (LBF/IN)'/10X,
     +'LOCATION (IN)  AREA (SQ-IN)   FORCE (LBF)     ER',
     +'ROR (IN)',9X,'X',13X,'Y'/)
2     FORMAT (' LH G1 X:',7D14.6)
3     FORMAT (9X,7D14.6)
4     FORMAT (7X,'Y:',7D14.6)
5     FORMAT (4X,'G2 X:',7D14.6)
6     FORMAT (' RH G1 X:',7D14.6)
7     FORMAT (/' ''S'' VARIABLES, SYSTEM (GLOBAL) COORDINATE (X,Y,Z)',
     +' DIRECTIONS'//14X,'BLOCK STIFFNESS TERMS',15X,'BLOCK STIFFNES',
     +'S TERMS',10X,'EXT COL    EXT COL    EXT COL ERR'
     +,'''S NOT AT N*TPF',/11X,'G1 X',8X,'G1 Y',
     +8X,'G1 Z',8X,'G2 X',8X,'G2 Y',8X,'G2 Z',5X,'ERR TERMS  ',
     +'FRICT TERMS',6X,'X',11X,'Y'/)
8     FORMAT (' G1 X:',10D12.4)
11    FORMAT (6X,10D12.4)
12    FORMAT (4X,'Y:',10D12.4)
13    FORMAT (4X,'Z:',10D12.4)
14    FORMAT (' G2 X:',10D12.4)
999   RETURN
      END
C
      SUBROUTINE TUV
C
C  VAR  DIMENSIONING       GLOBAL
C NAME  A B C D E F  G     COORDS   COMMENTS
C
C  TK   2 2 2 3 3 9        X/Y/Z/F  K, BLOCK STIFFNESSES
C  TFK  2 2   3   5        X/Y/Z/F  EXT COL - ERRORS (N*TPF) * STIFFNESSES
```

```
C   TFC   2 2    3   5      X/Y/Z/F  EXT COL - SLIDING FRICTION IN RUNNING
C   TFE   2 2    3 2 3      X/Y/Z/F  EXT COL - TFE * ERR (NOT AT TPF)
C
C   UK    2 2 2 3 3    50   X/Y/Z/F  * 'U' VARIABLES CALCULATED FROM
C   UFK   2 2    3     50   X/Y/Z/F  * 'T' VAR'S AND IDEALLY SHOULD
C   UFC   2 2    3     50   X/Y/Z/F  * EQUAL 'S' VAR'S ('DATA'). SEEK
C   UFE   2 2    3 2   50   X/Y/Z/F  * STATS ON HOW CLOSE 'S' & 'U' ARE.
C
C   VK    2 2 2 3 3                  * 'V' VAR'S ARE RMS RESIDUAL (%)
C   VFK   2 2    3                   * OF 'U' COMPARED TO 'S'. THIS IS A
C   VFC   2 2    3                   * MEASURE OF HOW WELL 'T' VAR'S
C   VFE   2 2    3 2                 * APPROXIMATE THE MESH QUANTITIES.
C
C   DIMENSIONING    A = 1,2  UNDEFL, DEFLECTED, RESP
C                   B = 1,2  GEAR 1 (DR), GEAR 2 (DN)
C                   C = 1,2  GEAR 1 (DR), GEAR 2 (DN)
C                   D = 1,2,3  X,Y,Z SYSTEM GLOBAL COORD DIRECTIONS
C                   E = 1,2,3  X,Y,Z SYSTEM GLOBAL COORD DIRECTIONS
C                   F = 1,2,3(,...,9)  DC AND TPF (& HARM UP TO 3RD) SINE
C                                      AND COSINE COEFFICIENTS
C                   G = 1,2,...,50 INCREM'S THROUGH AN ENGAGEMENT
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION L1(2),L2(3),L3(2)
      REAL*8 INV,KC,KI,KLA,KT,KXY,MP,MU,N,NCT
      COMMON /MESS/ AA(50,9),A9(9),A99(9,9),AG(2),ALF,ALP,AX(2),AY(2),
     +B9(9),BET(2),BP,C1(2,2),CA(2),CHTH(2),CI(2,2,2),CK(2,2,2,2),
     +CLA(2,2,2,2),CP,COF,FID(2),CRN,TH(4),CT(2,2),DC2,DCS,DS2,
     +CTR,CTT(2,2),CXY(2,2,2,2),DD(2),DC(3),DS(3),DP,DTH(2),E(2),
     +ETA(2),FF(2),FJB(2),FLA,FXY(2,2),FW(2),G(2),GAM(2),H1(2),H2(2),
     +FU(2),H1A(2),H1B(2),H1C(2),
     +KC(2,2,2,2),KI(2,2,2),KLA(2,2,2,2),KT(2,2),KXY(2,2,2,2),
     +MP,MU(2),N(2),NCT(2,2),OME(2),OTP,PHE(2),PHG(2),PHI,PI,POT,
     +PSI(2),R(2,2),RB(2),RC(2,2),RD(2),RITER,RETA(2),RK,RN(2,2),RO(2),
     +RP(2),TPA,TB(2),THA(2,2),THE(2,4),TLA(2,2),TQ(2),TRQ,TXY(2,2),
     +V(2,2),XA(2),XI,XT(2,2),XTU(2),YA(2),YT(2,2),YTU(2),Z,ZET,
     +XQ(2),YQ(2),I12(50),I21,ID(3),ITER,NOE,KD1
      COMMON /PQST/ PAR(2,2,2,50),PC(2,2,50),PER(2,2,2,50),
     +PFN(2,2,2,50),PK(2,2,2,2,50),PL(2,2,2,2,50),PPA(2,2,50),
     +PT(2,50),PV(2,2,2,50),PW(2,2,2,50),
     +QAR(2,2,2,50),QER(2,2,2,50),QFN(2,2,2,50),QK(2,2,2,2,50),
     +QL(2,2,2,2,50),QV(2,2,2,50),
     +SFC(2,2,3,50),SFK(2,2,3,50),SK(2,2,2,3,3,50),SFE(2,2,3,2,50),
     +TFC(2,2,3,5),TFK(2,2,3,5),TK(2,2,2,3,3,9),TFE(2,2,3,2,3)
     +,UK(2,2,2,3,3,50),UFC(2,2,3,50),UFK(2,2,3,50),UFE(2,2,3,2,50),
     +VK(2,2,2,3,3),VFC(2,2,3),VFK(2,2,3),VFE(2,2,3,2)
      COMMON /LET/ A,A2,B,B2,C,C2,D,D2,F,F2,H(2)
      DATA L1,L2,L3/2HG1,2HG2,2H X,2H Y,2H Z,2HS:,2HC:/
C
      NOK = 9
      NOF = 5
      NFE = 3
      N1 = MAX0(NOK,NOF,NFE)
      N2 = N1/2
C
C  *  INITIALIZE  *
C
      DO 16 I1 = 1,2
      DO 16 I2 = 1,2
      DO 16 I3 = 1,3
```

```
      VFK(I1,I2,I3) = 0.0D0
      VFC(I1,I2,I3) = 0.0D0
      DO 16 I4 = 1,5
      TFK(I1,I2,I3,I4) = 0.0D0
      TFC(I1,I2,I3,I4) = 0.0D0
      IF (I4.GE.3) GO TO 16
      VFE(I1,I2,I3,I4) = 0.0D0
      DO 15 I5 = 1,ITER
      UFE(I1,I2,I3,I4,I5) = 0.0D0
      IF (I5.LE.3) TFE(I1,I2,I3,I4,I5) = 0.0D0
      IF (I4.GT.1) GO TO 15
      UFK(I1,I2,I3,I5) = 0.0D0
      UFC(I1,I2,I3,I5) = 0.0D0
15    CONTINUE
16    CONTINUE
C   *
C   * SKIP DEFLECTION ANALYSIS - FUTURE WORK
C   *
      IDEFL= 1
C   *
      DO 68 I = 1,IDEFL
      DO 10 J = 1,ITER
      AA(J,1) = 1.0D0
      DO 10 K = 1,N2
      IC1 = 2*K
      IC2 = IC1 + 1
      DK = K
      ARG = DK*OTP*PT(I,J)
      AA(J,IC1) = DSIN(ARG)
10    AA(J,IC2) = DCOS(ARG)
      DO 20 J = 1,N1
      DO 20 K = 1,N1
      A99(J,K) = 0.0D0
      DO 20 L = 1,ITER
20    A99(J,K) = A99(J,K) + AA(L,K)*AA(L,J)
      DO 40 JJ = 1,2
      DO 40 J = 1,3
      DO 40 K = 1,2
      DO 30 L = 1,NOF
      A9(L) = 0.0D0
      DO 30 M = 1,ITER
      C = SFK(I,K,J,M)
      IF (JJ.EQ.2) C = SFC(I,K,J,M)
30    A9(L) = A9(L) + AA(M,L)*C
      CALL LUD (NOF,A99,A9,B9)
      DO 40 II = 1,NOF
      IF (JJ.EQ.1) TFK(I,K,J,II) = B9(II)
40    IF (JJ.EQ.2) TFC(I,K,J,II) = B9(II)
      IF (NOE.EQ.0) GO TO 135
      DO 46 J = 1,3
      DO 46 K = 1,2
      VFK(I,K,J) = 0.0D0
      VFC(I,K,J) = 0.0D0
      DO 44 M = 1,ITER
      UFK(I,K,J,M) = 0.0D0
      UFC(I,K,J,M) = 0.0D0
      DO 42 L = 1,NOF
      UFK(I,K,J,M) = UFK(I,K,J,M) + AA(M,L)*TFK(I,K,J,L)
      UFC(I,K,J,M) = UFC(I,K,J,M) + AA(M,L)*TFC(I,K,J,L)
42    CONTINUE
```

```
      A1 = SFK(I,K,J,M)
      A2 = UFK(I,K,J,M)
      VFK(I,K,J) = VFK(I,K,J) + (A2-A1)*(A2-A1)
      A1 = SFC(I,K,J,M)
      A2 = UFC(I,K,J,M)
      VFC(I,K,J) = VFC(I,K,J) + (A2-A1)*(A2-A1)
44    CONTINUE
      VFK(I,K,J) = DSQRT(VFK(I,K,J)/RITER)*1.0D2/TFK(I,K,J,1)
      VFC(I,K,J) = DSQRT(VFC(I,K,J)/RITER)*1.0D2/TFC(I,K,J,1)
46    CONTINUE
135   DO 130 KK = 1,2
      DO 100 J = 1,3
      DO 100 K = 1,2
      DO 90 L = 1,NFE
      A9(L) = 0.0D0
      DO 90 M = 1,ITER
      C = SFE(I,K,J,KK,M)
90    A9(L) = A9(L) + C*AA(M,L)
      CALL LUD(NFE,A99,A9,B9)
      DO 100 II = 1,NFE
100   TFE(I,K,J,KK,II) = B9(II)
      DO 130 J = 1,3
      DO 130 K = 1,2
      VFE(I,K,J,KK) = 0.0D0
      DO 120 M = 1,ITER
      UFE(I,K,J,KK,M) = 0.0D0
      DO 110 L = 1,NFE
      UFE(I,K,J,KK,M) = UFE(I,K,J,KK,M) + AA(M,L)*TFE(I,K,J,KK,L)
110   CONTINUE
      A1 = SFE(I,K,J,KK,M)
      A2 = UFE(I,K,J,KK,M)
      VFE(I,K,J,KK) = VFE(I,K,J,KK) + (A2-A1)*(A2-A1)
120   CONTINUE
      VFE(I,K,J,KK) = DSQRT(VFE(I,K,J,KK)/RITER)*1.0D2/TFE(I,K,J,KK,1)
130   CONTINUE
      DO 60 J = 1,3
      DO 60 K = 1,2
      DO 60 JJ = 1,3
      DO 60 KK = 1,2
      DO 50 L = 1,NOK
      A9(L) = 0.0D0
      DO 50 M = 1,ITER
50    A9(L) = A9(L) + AA(M,L)*SK(I,K,KK,J,JJ,M)
      CALL LUD (NOK,A99,A9,B9)
      DO 60 II = 1,NOK
60    TK(I,K,KK,J,JJ,II) = B9(II)
      DO 68 J = 1,3
      DO 68 K = 1,2
      DO 68 JJ = 1,3
      DO 68 KK = 1,2
      VK(I,K,KK,J,JJ) = 0.0D0
      DO 66 M = 1,ITER
      UK(I,K,KK,J,JJ,M) = 0.0D0
      DO 64 L = 1,NOK
      UK(I,K,KK,J,JJ,M) = UK(I,K,KK,J,JJ,M) + AA(M,L)*TK(I,K,KK,J,JJ,L)
64    CONTINUE
      A1 = SK(I,K,KK,J,JJ,M)
      A2 = UK(I,K,KK,J,JJ,M)
      VK(I,K,KK,J,JJ) = VK(I,K,KK,J,JJ) + (A2-A1)*(A2-A1)
66    CONTINUE
```

```
      VK(I,K,KK,J,JJ) = DSQRT(VK(I,K,KK,J,JJ)/RITER)*1.0D2/
     +TK(I,K,KK,J,JJ,1)
68    CONTINUE
      DO 80 I = 1,2
      WRITE (6,1)
      DO 80 J = 1,3
      WRITE (6,2) L1(I),L2(J),((TK(1,I,L,J,K,1),K=1,3),L=1,2),
     +TFK(1,I,J,1),TFC(1,I,J,1),TFE(1,I,J,1,1),TFE(1,I,J,2,1)
      DO 80 K = 1,4
      DO 80 L = 1,2
      K1 = 2*K - 1 + L
      GO TO (72,74,70,70), K
72    WRITE (6,4) K,L3(L),((TK(1,I,M1,J,M2,K1),M2=1,3),M1=1,2),
     +TFK(1,I,J,K1),TFC(1,I,J,K1),TFE(1,I,J,1,K1),TFE(1,I,J,2,K1)
      GO TO 80
74    WRITE (6,4) K,L3(L),((TK(1,I,M1,J,M2,K1),M2=1,3),M1=1,2),
     +TFK(1,I,J,K1),TFC(1,I,J,K1)
      GO TO 80
70    WRITE (6,4) K,L3(L),((TK(1,I,M1,J,M2,K1),M2=1,3),M1=1,2)
80    CONTINUE
      WRITE (6,7)
      WRITE (6,8) VK(2,1,1,1,1),VK(2,1,1,1,2),VK(2,1,1,1,3),
     +VK(2,1,2,1,1),VK(2,1,2,1,2),VK(2,1,2,1,3),VFK(2,1,1),
     +VFC(2,1,1),VFE(2,1,1,1),VFE(2,1,1,2)
      WRITE (6,11) VK(1,1,1,1,1),VK(1,1,1,1,2),VK(1,1,1,1,3),
     +VK(1,1,2,1,1),VK(1,1,2,1,2),VK(1,1,2,1,3),VFK(1,1,1),
     +VFC(1,1,1),VFE(1,1,1,1),VFE(1,1,1,2)
      WRITE (6,12) VK(2,1,1,2,1),VK(2,1,1,2,2),VK(2,1,1,2,3),
     +VK(2,1,2,2,1),VK(2,1,2,2,2),VK(2,1,2,2,3),VFK(2,1,2),
     +VFC(2,1,2),VFE(2,1,2,1),VFE(2,1,2,2)
      WRITE (6,11) VK(1,1,1,2,1),VK(1,1,1,2,2),VK(1,1,1,2,3),
     +VK(1,1,2,2,1),VK(1,1,2,2,2),VK(1,1,2,2,3),VFK(1,1,2),
     +VFC(1,1,2),VFE(1,1,2,1),VFE(1,1,2,2)
      WRITE (6,13) VK(2,1,1,3,1),VK(2,1,1,3,2),VK(2,1,1,3,3),
     +VK(2,1,2,3,1),VK(2,1,2,3,2),VK(2,1,2,3,3),VFK(2,1,3),
     +VFC(2,1,3),VFE(2,1,3,1),VFE(2,1,3,2)
      WRITE (6,11) VK(1,1,1,3,1),VK(1,1,1,3,2),VK(1,1,1,3,3),
     +VK(1,1,2,3,1),VK(1,1,2,3,2),VK(1,1,2,3,3),VFK(1,1,3),
     +VFC(1,1,3),VFE(1,1,3,1),VFE(1,1,3,2)
      WRITE (6,14) VK(2,2,1,1,1),VK(2,2,1,1,2),VK(2,2,1,1,3),
     +VK(2,2,2,1,1),VK(2,2,2,1,2),VK(2,2,2,1,3),VFK(2,2,1),
     +VFC(2,2,1),VFE(2,2,1,1),VFE(2,2,1,2)
      WRITE (6,11) VK(1,2,1,1,1),VK(1,2,1,1,2),VK(1,2,1,1,3),
     +VK(1,2,2,1,1),VK(1,2,2,1,2),VK(1,2,2,1,3),VFK(1,2,1),
     +VFC(1,2,1),VFE(1,2,1,1),VFE(1,2,1,2)
      WRITE (6,12) VK(2,2,1,2,1),VK(2,2,1,2,2),VK(2,2,1,2,3),
     +VK(2,2,2,2,1),VK(2,2,2,2,2),VK(2,2,2,2,3),VFK(2,2,2),
     +VFC(2,2,2),VFE(2,2,2,1),VFE(2,2,2,2)
      WRITE (6,11) VK(1,2,1,2,1),VK(1,2,1,2,2),VK(1,2,1,2,3),
     +VK(1,2,2,2,1),VK(1,2,2,2,2),VK(1,2,2,2,3),VFK(1,2,2),
     +VFC(1,2,2),VFE(1,2,2,1),VFE(1,2,2,2)
      WRITE (6,13) VK(2,2,1,3,1),VK(2,2,1,3,2),VK(2,2,1,3,3),
     +VK(2,2,2,3,1),VK(2,2,2,3,2),VK(2,2,2,3,3),VFK(2,2,3),
     +VFC(2,2,3),VFE(2,2,3,1),VFE(2,2,3,2)
      WRITE (6,11) VK(1,2,1,3,1),VK(1,2,1,3,2),VK(1,2,1,3,3),
     +VK(1,2,2,3,1),VK(1,2,2,3,2),VK(1,2,2,3,3),VFK(1,2,3),
     +VFC(1,2,3),VFE(1,2,3,1),VFE(1,2,3,2)
1     FORMAT (1H1,'''T'' VARIABLES, SYSTEM (GLOBAL) COORDINATE (X,Y,Z)',
     +' DIRECTIONS. COEFFICIENTS OF HARMONIC FUNCTIONS AT N X TPF',
     +//17X,'BLOCK STIFFNESS TERMS',15X,'BLOCK STIFFNESS',
```

```
     +' TERMS',10X,'EXTENSION COL TERMS  EXT COL ERR NOT AT N*TPF'/
     +14X,'G1 X',8X,'G1 Y',8X,'G1 Z',8X,'G2 X',8X,'G2 Y',8X,'G2 Z',
     +3X,'ERR AT N*TPF  FRICTION',8X,'X',11X,'Y'/)
2     FORMAT (1X,2A2,' DC:',10D12.4)
3     FORMAT (9X,10D12.4)
4     FORMAT (6X,I1,A2,10D12.4)
7     FORMAT (1H1,'''V'' VARIABLES, RMS ERROR IN SOLUTION, % OF MEAN',
     +' VALUE'//14X,'BLOCK STIFFNESS TERMS',15X,'BLOCK STIFFNES',
     +'S TERMS',10X,'EXTENSION COLUMN     EXT COL ERR'
     +,'''S NOT AT N*TPF',/11X,'G1 X',8X,'G1 Y',
     +8X,'G1 Z',8X,'G2 X',8X,'G2 Y',8X,'G2 Z',5X,'ERR TERMS  ',
     +'FRICT TERMS',6X,'X',11X,'Y'/)
8     FORMAT (' G1 X:',10D12.4)
11    FORMAT (6X,10D12.4)
12    FORMAT (4X,'Y:',10D12.4)
13    FORMAT (4X,'Z:',10D12.4)
14    FORMAT (' G2 X:',10D12.4)
      RETURN
      END
C

      SUBROUTINE LUD (ISZ,AA,BA,XS)
C
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 LD(9,9)
      REAL*8 AA(9,9),BA(9),BB(9),FI(2,4),XS(9),XX(9,2),YY(9)
      DMS(A,B) = (2.0D0*(A-B)/(A+B))**2
C
C  *  S = (L + I) * (U + D)  *
C
      LD(1,1) = AA(1,1)
      DO 130 I = 2,ISZ
      DO 130 J = 1,I
      IF (I.EQ.J) GO TO 110
      LD(I,J) = AA(I,J)/LD(J,J)
      IF (J.EQ.1) GO TO 110
      I1 = J - 1
      DO 100 K = 1,I1
100   LD(I,J) = LD(I,J) - LD(I,K)*LD(K,J)/LD(J,J)
110   LD(J,I) = AA(J,I)
      IF (J.EQ.1) GO TO 130
      I1 = J - 1
      DO 120 K = 1,I1
120   LD(J,I) = LD(J,I) - LD(J,K)*LD(K,I)
130   CONTINUE
C
C  *  SOLVE FOR Y ;  (L + I) * Y = B  *
C
      YY(1) = BA(1)
      DO 140 I = 2,ISZ
      YY(I) = BA(I)
      I1 = I - 1
      DO 140 J = 1,I1
140   YY(I) = YY(I) - YY(J)*LD(I,J)
C
C  *  SOLVE FOR X ; (U + D) * X = Y  *
C
      XX(ISZ,1) = YY(ISZ)/LD(ISZ,ISZ)
      DO 150 II = 2,ISZ
```

```
      I = ISZ + 1 - II
      XX(I,1) = YY(I)/LD(I,I)
      I1 = II - 1
      DO 150 JJ = 1,I1
      J = ISZ + 1 - JJ
150   XX(I,1) = XX(I,1) - LD(I,J)*XX(J,1)/LD(I,I)
      DO 155 I = 1,ISZ
155   XS(I) = XX(I,1)
C
C  *  REGENERATE 'B' VECTOR  *
C
      IF (J.GT.-1000) GO TO 999
      DO 160 I = 1,ISZ
160   YY(I) = XX(I,1)
      DO 290 LQ = 1,2
      KQ = 1
      IF (LQ.EQ.1) GO TO 165
      DO 162 I = 1,ISZ
162   XX(I,1) = YY(I)
165   DO 170 I = 1,4
170   FI(1,I) = 0.0D0
      IQ = 2
      JQ = 1
      I1 = -1
180   I1 = I1 + 1
      IQ = JQ
      JQ = 1
      IF (IQ.EQ.1) JQ = 2
      DO 210 I = 1,ISZ
      BB(I) = 0.0D0
      DO 190 J = 1,ISZ
190   BB(I) = BB(I) + XX(J,IQ)*AA(I,J)
      FI(1,1) = FI(1,1) + (BB(I)-BA(I))**2
      A1 = DABS(BB(I) + BA(I))
      IF (A1.LE.1.0D-20) GO TO 210
      FI(1,2) = FI(1,2) + DMS(BB(I),BA(I))
210   CONTINUE
      A1 = ISZ
      DO 220 I = 1,4
220   FI(1,I) = DSQRT(FI(1,I)/A1)
      K = 1
      L = 0
      DO 225 I = 1,4
      A1 = DABS(FI(1,I))
      IF (A1.GT.1.0D10) L = 1
      IF (I1.EQ.0) GO TO 225
      IF (FI(1,I).LE.FI(2,I)) K = 0
225   FI(2,I) = FI(1,I)
      IF (L.EQ.1) GO TO 230
      IF (I1.EQ.0.OR.I1.EQ.10) GO TO 230
C
C  *  NEXT STMT EXPER - FORCES TEN ITERATIONS  *
C
      IF (K.EQ.1) GO TO 240
      IF (K.EQ.0) GO TO 240
230   IF (KQ.EQ.2) GO TO 280
      KQ = 2
C
C  *  ITERATIVE REFINEMENT  *
C
```

```
240   DO 250 I = 1,4
250   FI(1,I) = 0.0D0
      DO 270 I = 1,ISZ
      XX(I,JQ) = BA(I)/AA(I,I)
      DO 260 J = 1,ISZ
      IF (I.EQ.J) GO TO 260
      IF (I.GT.J.AND.LQ.EQ.2) GO TO 255
      XX(I,JQ) = XX(I,JQ) - XX(J,IQ)*AA(I,J)/AA(I,I)
      GO TO 260
255   XX(I,JQ) = XX(I,JQ) - XX(J,JQ)*AA(I,J)/AA(I,I)
260   CONTINUE
      FI(1,3) = FI(1,3) + (XX(I,IQ)-XX(I,JQ))**2
      A1 = DABS(XX(I,IQ)+XX(I,JQ))
      IF (A1.LE.1.0D-20) GO TO 270
      FI(1,4) = FI(1,4) + DMS(XX(I,IQ),XX(I,JQ))
270   CONTINUE
      GO TO 180
280   CONTINUE
290   CONTINUE
999   RETURN
      END
```

APPENDIX C

THE GEARED ROTOR SOLVER (GRS) COMPUTER PROGRAM

The Geared Rotor Solver (GRS) performs the three-dimensional steady-state response analysis of arbitrary geared systems using the extended transfer matrix method. It is designed to accept input from the GRD interactive front end and the GMS algorithm for gear-tooth stiffness calculation.

GRS Program Structure

The FORTRAN source listing of GRS, immediately following this discussion, consists of a main program and eight subroutines as follows:

- MAIN Program -
  - Reads data from GRD and GMS.
  - Establishes analysis frequencies and components of the state vector.
  - Applies "left-hand" boundary conditions, builds the matrix from left to right, and applies "right-hand" boundary conditions.
  - Solves the matrix.
  - Rebuilds the system from the left-hand state vector solution and prints the state vector after each element

- o SHAFT, DISK, BRG, MESH subroutines are called by MAIN during the "build" and "rebuild" steps in the analysis. These subroutines generate the extended transfer matrix elements for the massless elastic shaft, rigid inertial disk, visco-elastic rotor bearing, and elastic gear mesh respectively.

- o TLAST subroutine is called by MAIN to establish the extended transfer matrix element which imposes the boundary conditions at the "right-hand" end of the system.

- o MULT subroutine is called by MAIN for successive multiplication by each successive elemental extended transfer matrix during the "build" and "rebuild" steps in the analysis.

- o LUD subroutine is the "lower-upper-decomposition" algorithm which solves the global transfer matrix for the left-hand side state vector.

- o REBLD subroutine is called by MAIN to print out the state vector after each elemental increment during the "rebuild" step in the analysis.

LUD Decomposition and Solution of Large Sparse Matrices

The extended transfer matrix method applied to geared rotors creates matrices which, as with other structural analysis programs, are often of order greater than 2000. NASTRAN for example, a highly developed workhorse finite element program, tailors its solution to the sparsity, symmetry, and bandedness which are characteristic of that method. Transfer matrices are not symmetric and the branching techniques of the current work render them highly non-banded. The matrix solution algorithm developed and described here applies generally to large sparse matrices with non-zeroes on the main diagonal.

It is well known [28] that the matrix equation, $\underline{A}\ \underline{X} = \underline{B}$, with $\underline{A}$, nxn; and $\underline{b}$, nx1; may be decomposed into lower and upper triangular matrices such that

$$\underline{A}\ \underline{X} = \underline{L}\ \underline{U}\ \underline{X} = \underline{B} \tag{C.1}$$

where the elements of $\underline{L}$ and $\underline{U}$ are evaluated explicitly from the recursive relations,

$$u_{ij} = a_{ij} - \sum_{k=1}^{i-1} \ell_{jk}\, u_{ki} \tag{C.2}$$

$$\ell_{ji} = u_{ij} / u_{ii} \tag{C.3}$$

Equation (C.1) is solved for X explicitly in two steps. Defining the intermediate vector $\underline{Y}$ according to

$$\underline{U}\ \underline{X} = \underline{Y} \tag{C.4}$$

Equations (C.1) and (C.4) may be combined to form

$$\underline{L}\ \underline{Y} = \underline{B} \tag{C.5}$$

The vector $\underline{Y}$ is obtained by a foreward pass of Equation (C.5) which allows the explicit solution of $\underline{X}$ by a backward pass of Equation (C.4). The RMS residual error, given by

$$\epsilon_{RMS} = [(\underline{B} - \underline{A}\ \underline{X})^T(\underline{B} - \underline{A}\ \underline{X})/n]^{1/2} \tag{C.6}$$

is monitored during a standard Gauss-Seidel iterative refinement as a measure of the quality of the solution.

The FORTRAN coding of the above "lower-upper - decomposition" (LUD) algorithm with no space economizing is a very simple programming exercise. As on example, this technique applied to a 400 order matrix required 3 megabytes of storage and 40 minutes execution time. The uniqueness of subroutine LUD of in the program listing for GMS is that only the non-zero matrix values and their row and column locations are stored. While this results in complex programming logic, the method is extremely efficient with regard to storage space, input/output time, and computation time. The same example problem was run with less than 1 megabyte and took 90 seconds.

```
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION BB(2000),DE(2000),S(50000,2),SD(50000,2),XX(2000),
     +YY(2000)
      REAL*8 LLT,LRT,LOM,OME,KO,KC,KS,KPO,KPC,KPS
      INTEGER*2 ILC,ILR,IUC,IUR(25000,2),ISC,ISR,ITC,ITR,IN(2000)
      INTEGER*2 MG,MHR,ML,NSP,SPE,SPI,SPT,TYP,MPR,MTP
      INTEGER*2 IRS,ITP,IE,IER,IPL,IML,ITT,IPE,IME,IX(2000),IBR(2000)
      INTEGER STM,SV,SVH
      INTEGER DS,DF,OID,US,UF,UST
      COMMON /Z1/ KO(5,3,2,3,2),KC(5,3,2,3,2),KS(5,3,2,3,2),BC(4,2,2,2),
     +BK(4,2,2,2),FPO(5,3,2),FPC1(5,3,2),FPC2(5,3,2),FPS1(5,3,2),FPS2(5,
     +3,2),KPO(5,3,2,2),KPC(5,3,2,2),KPS(5,3,2,2),BCZ(4,2),BKZ(4,2),
     +EF(2,5),EC(5,2),ES(5,2),DIB(8),DID(8),DIE(8),DIL(8),DIM(8),DIT(8),
     +GNT(4),LOM(5),LLT(5),LRT(5),OME(25),SHD(8),SHE(8),SHG(8),SHL(8),
     +CONV,IL,IM,ISZ,JF,LHM,NFQ,PI,ILL,STM,SV,SVH,LM,LN,IQ,JQ,IE(2,5),
     +IER(2,5),IME(2,5),IML(2,5),IOME(25),IPE(2,5),IPL(2,5)
     +,MG(2,5),ML(2,5),MTP(2,25),NSP(5,5),SPE(15,18),SPI(5,4),SPT(15
     +,18),IRS(5),ITP(5),ITT(5),OID(25),MHR(5),MPR
      COMMON /Z2/ YU(25000),EL(25000),YUP(25000),ILC(25000),ILR(25000),
     +IUC(25000,2),T(8008),ISC(50000,2),ISR(50000,2),ITC(8008),ITR(8008)
      EQUIVALENCE (YU(1),S(1,1),SD(1,1)),(T(9),BB(1)),(T(2009),YY(1),
     +XX(1)),(ISR(1,1),IUR(1,1)),(ITC(2009),IN(1)),
     +(ITC(4009),IX(1)),(ITC(6009),IBR(1)),(ITR(9),DE(1))
      DATA JDC,JRS,JTP,JER,JPR,JMR,JTT,JPE,JME/4H  DC,4H  RS,
     +4H TPF,4H  ER,4H T+R,4H T-R,4H2*TP,4H T+E,4H T-E/
      DIF(A,B) = DABS(DABS(A)-DABS(B))
      DMS(AA1,AA2) = (2.0D0*(AA1-AA2)/(AA1+AA2))**2
      PI = 3.141592653589793D0
      CONV = PI/3.0D1
      MHR(1) = 0
      REWIND 9
C
C  *  READ DATA
C
      READ (5,*) IM,IL,ISP,ISH,IDI,IBG,IGR
      READ (5,*) (LLT(I),LRT(I),LOM(I),I=1,IL)
      READ (5,*) ((SPI(I,J),I=1,IL),J=1,4)
      READ (5,*) ((SPT(I,J),SPE(I,J),I=1,ISP),J=1,18)
      IF (ISH.GT.0) READ (5,*) (SHL(I),SHD(I),SHE(I),SHG(I),I=1,ISH)
      READ (5,*) (DIM(I),DID(I),DIL(I),DIE(I),DIT(I),DIB(I),I=1,IDI)
      READ (5,*) ((((BK(I,J,L,M),L=1,2),M=1,2),BKZ(I,J),J=1,2),
     +I=1,IBG)
      READ (5,*) ((((BC(I,J,L,M),L=1,2),M=1,2),BCZ(I,J),J=1,2),
     +I=1,IBG)
      IF (IM.EQ.0) GO TO 100
      READ (5,*) (MHR(I),(IE(J,I),ML(J,I),MG(J,I),J=1,2),I=1,IM)
      READ (5,*) (GNT(I),I=1,IGR)
      READ (5,*) ((EF(J,I),ES(I,J),EC(I,J),I=1,IM),J=1,2)
100   IB1 = 1
      ILL = IL
      IF (IM.EQ.0) GO TO 120
      DO 110 K = 1,IM
      DO 110 I = 1,3
      DO 110 J = 1,2
      READ (9,*) FPO(K,I,J),FPC1(K,I,J),FPS1(K,I,J),FPC2(K,I,J),
     +FPS2(K,I,J)
      DO 110 M = 1,2
      READ (9,*) KPO(K,I,J,M),KPC(K,I,J,M),KPS(K,I,J,M)
      DO 110 L = 1,3
```

```
110   READ (9,*) K0(K,I,J,L,M),KC(K,I,J,L,M),KS(K,I,J,L,M)
      REWIND 9
120   LHM = MHR(1)
      IF (IM.LE.1) GO TO 140
      DO 130 I = 2,IM
      I1 = MHR(I)
130   LHM = MAX0(LHM,I1)
140   K = 1
      IN(1) = 1
      IF (IM.EQ.0) GO TO 190
      DO 150 J = 1,IL
      IN(J) = 1
      DO 150 I = 1,LHM
150   NSP(I,J) = 0
160   DO 180 I = 1,IM
      IF (MHR(I).NE.K) GO TO 180
      DO 170 J = 1,2
      NSP(K,ML(J,I)) = SPI(ML(J,I),IN(ML(J,I)))
170   IN(ML(J,I)) = IN(ML(J,I)) + 1
180   CONTINUE
      K = K + 1
      IF (K.LE.LHM) GO TO 160
190   DO 200 I = 1,IL
200   NSP(K,I) = SPI(I,IN(I))
C
C  *  BEGIN SET FREQUENCIES  *
C
      OME(1) = 0.0D0
      OID(1) = JDC
      N = 1
C
C  *  LINE RUN SPEEDS  *
C
      DO 230 I=1,IL
      RS = DABS(LOM(I)*CONV)
      M = 0
      DO 210 J = 1,N
210   IF (DIF(RS,OME(J)).LT.1.0D-2) M=J
      IF (M.EQ.0) GO TO 220
      IRS(I) = M
      GO TO 230
220   N = N + 1
      IRS(I) = N
      OME(N) = RS
      OID(N) = JRS
230   CONTINUE
C
C  *  TOOTH PASSING  *
C
      IF (IM.EQ.0) GO TO 450
      DO 270 I = 1,IM
      TPF = 0.0D0
      DO 240 J = 1,2
240   TPF = TPF + DABS(LOM(ML(J,I))*GNT(MG(J,I))*CONV/2.0D0)
      M = 0
      DO 250 J = 1,N
250   IF (DIF(TPF,OME(J)).LT.1.0D-2) M=J
      IF (M.EQ.0) GO TO 260
      ITP(I) = M
      GO TO 270
```

```
260   N = N+1
      ITP(I) = N
      OME(N) = TPF
      OID(N) = JTP
270   CONTINUE
C
C  *  ERROR FREQUENCIES  *
C
      DO 310 L = 1,2
      DO 310 I = 1,IM
      EF(L,I) = DABS(EF(L,I)*CONV)
      IF (IE(L,I).EQ.1) GO TO 280
      IER(L,I) = ITP(I)
      GO TO 310
280   M = 0
      DO 290 K = 1,N
290   IF (DIF(EF(L,I),OME(K)).LT.1.0D-2) M = K
      IF (M.EQ.0) GO TO 300
      IER(L,I) = M
      GO TO 310
300   N = N + 1
      IER(L,I) = N
      OME(N) = EF(L,I)
      OID(N) = JER
310   CONTINUE
C
C  *  TOOTH PASSING PLUS/MINUS RUN SPEED  *
C
      MPR = N + 1
      DO 350 L = 1,2
      DO 350 I = 1,IM
      DO 350 KK = 1,3,2
      K = KK - 2
      FREQ = OME(ITP(I)) + K*OME(IRS(ML(L,I)))
      M = 0
      DO 320 J = 1,N
320   IF (DIF(FREQ,OME(J)).LT.1.0D-2) M = J
      IF (M.GT.0) GO TO 340
      N = N + 1
      OME(N) = FREQ
      IF (K.EQ.-1) GO TO 330
      IPL(L,I) = N
      OID(N) = JPR
      GO TO 350
330   IML(L,I) = N
      OID(N) = JMR
      GO TO 350
340   IF (K.EQ.-1) IML(L,I) = M
      IF (K.EQ.1)  IPL(L,I) = M
350   CONTINUE
C
C  *  TWICE TOOTH PASSING FREQUENCIES  *
C
      DO 380 I = 1,IM
      FREQ = 2.0D0*OME(ITP(I))
      M = 0
      DO 360 J = 1,N
360   IF (DIF(FREQ,OME(J)).LT.1.0D-2) M=J
      IF (M.EQ.0) GO TO 370
      ITT(I) = M
```

```
      GO TO 380
370   N = N + 1
      ITT(I) = N
      OME(N) = FREQ
      OID(N) = JTT
380   CONTINUE
C
C  *  TOOTH PASSING PLUS/MINUS ERROR  *
C
      DO 440 L = 1,2
      DO 440 I = 1,IM
      IF (IE(L,I).EQ.1) GO TO 390
      IPE(L,I) = ITT(I)
      IME(L,I) = 1
      GO TO 440
390   DO 430 KK = 1,3,2
      K = KK - 2
      FREQ = DABS(OME(ITP(I))+K*EF(L,I))
      M = 0
      DO 400 J = 1,N
400   IF (DIF(FREQ,OME(J)).LT.1.0D-2) M = J
      IF (M.GT.0) GO TO 420
      N = N + 1
      OME(N) = FREQ
      IF (K.EQ.-1) GO TO 410
      IPE(L,I) = N
      OID(N) = JPE
      GO TO 430
410   IME(L,I) = N
      OID(N) = JME
      GO TO 430
420   IF (K.EQ.-1) IME(L,I) = M
      IF (K.EQ.1)  IPE(L,I) = M
430   CONTINUE
440   CONTINUE
450   NFQ = N
      JF = (2*NFQ) - 1
C
C  *  END SET FREQUENCIES  *
C
      WRITE (6,30) (I,OME(I),OID(I),I=1,NFQ)
      WRITE (6,31) (I,IRS(I),I=1,IL)
      IF (IM.EQ.0) GO TO 455
      WRITE (6,32) (I,ITP(I),I=1,IM)
      WRITE (6,33) ((I,J,IER(I,J),I=1,2),J=1,IM)
      WRITE (6,34) ((I,J,IML(I,J),IPL(I,J),I=1,2),J=1,IM)
      WRITE (6,35) (I,ITT(I),I=1,IM)
      WRITE (6,36) ((I,J,IME(I,J),IPE(I,J),I=1,2),J=1,IM)
455   DO 460 I = 1,NFQ
460   IOME(I) = (OME(I)+5.0D-1)
C
C  *  SET MESH TYPES  *
C
      IF (IM.EQ.0) GO TO 490
      DO 470 I = 1,IM
      DO 470 J = 2,NFQ
470   MTP(I,J) = 0
      DO 480 I = 1,IM
      MTP(I,ITP(I)) = -ITT(I)
      MTP(I,ITT(I)) = ITP(I)
```

```
      DO 480 J = 1,2
      MTP(I,IPL(J,I)) = IRS(ML(J,I))
      MTP(I,IML(J,I)) = -IRS(ML(J,I))
      MTP(I,IRS(ML(J,I))) = IPL(J,I)
      IF (IE(J,I).EQ.0) GO TO 480
      MTP(I,IER(J,I)) = IPE(J,I)
      MTP(I,IPE(J,I)) = IER(J,I)
      MTP(I,IME(J,I)) = -IER(J,I)
480   CONTINUE
C
C  *  DEVELOP FIRST S MATRIX  *
C
490   SVH = 6*JF
      SV = 2*SVH
      STM = IL*SV+1
      ISZ = IL*SVH
      LN = 0
      DO 500 I = 1,IL
      DO 500 J = 1,SVH
      LN = LN + 1
      S(LN,1) = 1.0D0
      ISC(LN,1) = (I-1)*SV+J
      ISR(LN,1) = ISC(LN,1)
500   CONTINUE
      DO 510 I = 1,IL
      IF (LLT(I).LT.1.0D-2) GO TO 510
      LN = LN + 1
      S(LN,1) = LLT(I)
      ISR(LN,1) = (I-1)*SV+8*JF+1
      ISC(LN,1) = STM
510   CONTINUE
      LN = LN + 1
      S(LN,1) = 1.0D0
      ISC(LN,1) = STM
      ISR(LN,1) = STM
C
C  *  BUILD  *
C
520   IQ = 2
      JQ = 1
      NHM = 1
      NEL = 1
540   JQ = IQ
      IQ = 1
      IF (JQ.EQ.1) IQ = 2
550   IST = 1
      LM = 1
      T(LM) = 0.0D0
      DO 560 I = 1,IL
      I1 = NSP(NHM,I)
      IF (I1.EQ.0) GO TO 560
      I2 = SPE(I1,NEL)
      TYP = SPT(I1,NEL)
      IF (TYP.EQ.0) GO TO 560
      IST = 0
      IF (TYP.EQ.1) CALL SHAFT(I,I2)
      IF (TYP.EQ.2) CALL DISK(I,I2)
      IF (TYP.EQ.3) CALL BRG(I,I2)
560   CONTINUE
      IF (IST.EQ.1) GO TO 570
```

```
      IF (IB1.EQ.0) CALL REBLD (NEL,NHM,1)
      IF (IB1.EQ.1) CALL MULT
      NEL = NEL + 1
      GO TO 540
570   IF (NHM.GT.LHM) GO TO 590
      DO 580 I = 1,IM
      IF (MHR(I).NE.NHM) GO TO 580
      CALL MESH(I)
      IF (IB1.EQ.0) CALL REBLD (NEL,NHM,2)
      IF (IB1.EQ.1) CALL MULT
      JQ = IQ
      IQ = 1
      IF (JQ.EQ.1) IQ = 2
580   CONTINUE
      NHM = NHM + 1
      NEL = 1
      GO TO 550
C
C  *  LAST T MATRIX  *
C
590   LM = 1
      T(LM) = 0.0D0
      CALL TLAST
      IF (IB1.EQ.1) GO TO 600
      CALL REBLD (NEL,NHM,3)
      GO TO 999
600   CALL MULT
C
C  *  REDUCE GLOBAL TO QUARTER SIZE  *
C
      JQ = IQ
      IQ = 1
      IF (JQ.EQ.1) IQ = 2
      IB = -1
      L = 0
      M = 1
      IN(1) = 1
      DO 640 I = 2,LN
      J = (ISC(I,IQ)-1)/SVH
      K = (ISR(I,IQ)-1)/SVH
      IF (MOD(K,2).NE.1) GO TO 640
      L = L + 1
      S(L,JQ) = S(I,IQ)
      ISC(L,JQ) = ISC(I,IQ) - (J/2)*SVH
      ISR(L,JQ) = ISR(I,IQ) - ((K+1)/2)*SVH
      IF (IB.EQ.-1) GO TO 630
      IB = IB + 1
      BB(IB) = -S(L,JQ)
      IBR(IB) = ISR(L,JQ)
      GO TO 640
630   IF (ISC(I+1,IQ).NE.STM) GO TO 640
      IB = 0
      LL = L
640   CONTINUE
      I1 = 1
650   LN = LL
      LB = IB
      JQ = IQ
      IQ = 1
      IF (JQ.EQ.1) IQ = 2
```

```
C
C  *  REORDER EQUATIONS FROM M,V,M,V,... TO V,M,V,M,...  *
C
      IF (I1.EQ.2) GO TO 710
      I1 = 2
      DO 660 I = 1,LB
      XX(I) = BB(I)
660   IX(I) = IBR(I)
      LL = 0
      I = 1
      DO 680 K = 1,ISZ
      DO 680 KK =1,ILL
      IST = I - 1
      DO 680 L = 1,2
      I = IST
670   I = I + 1
      IF (I.GT.LN) GO TO 680
      IF (ISC(I,IQ).NE.K) GO TO 680
      I2 = 1 + (ISR(I,IQ)-1)/(SVH/2)
      I3 = (I2+1)/2 - KK
      IF (I3.NE.0) GO TO 680
      I2 = L - MOD(I2,2)
      IF (I2.NE.1) GO TO 670
      LL = LL + 1
      S(LL,JQ) = S(I,IQ)
      ISC(LL,JQ) = K
      ISR(LL,JQ) = ISR(I,IQ) + (2*L-3)*3*JF
      GO TO 670
680   CONTINUE
      IB = 0
      I =1
      DO 700 KK = 1,ILL
      IST = I - 1
      DO 700 L = 1,2
      I = IST
690   I = I + 1
      IF (I.GT.LB) GO TO 700
      I2 = 1 + (IX(I)-1)/(SVH/2)
      I3 = (I2+1)/2 - KK
      IF (I3.NE.0) GO TO 700
      I2 = L - MOD(I2,2)
      IF (I2.NE.1) GO TO 690
      IB = IB + 1
      BB(IB) = XX(I)
      IBR(IB) = IX(I) + (2*L-3)*3*JF
      GO TO 690
700   CONTINUE
      GO TO 650
C
C  *  S = SL + SUD  *
C
710   IN(1) = 1
      DO 720 I = 2,ISZ
720   IN(I) = 0
      DO 730 I = 2,LN
730   IF (ISC(I,IQ).NE.ISC(I-1,IQ)) IN(ISC(I,IQ)) = I
      DO 740 I = 1,ISZ
740   IX(I) = 0
      MJ = 0
      DO 790 I = 1,ISZ
```

```
      K = 0
      MI = IN(I)
750   IF (ISC(MI,IQ).EQ.I) GO TO 760
      IN(I) = 0
      GO TO 790
760   IF (ISR(MI,IQ).GT.ISC(MI,IQ)) GO TO 780
      MJ = MJ + 1
      IF (K.EQ.1) GO TO 770
      IX(I) = MJ
      K = 1
770   SD(MJ,JQ) =S(MI,IQ)
      ISC(MJ,JQ) = ISC(MI,IQ)
      ISR(MJ,JQ) = ISR(MI,IQ)
      MI = MI + 1
      IF (MI.GT.LN) GO TO 790
      GO TO 750
780   IN(I) = MI
790   CONTINUE
      LST = MJ + 1
      DO 800 I = 2,ISZ
      I1 = I - 1
      DO 800 J = 1,I1
      IF (IN(J).EQ.0) GO TO 800
      IF (ISR(IN(J),IQ).GT.I) GO TO 800
      MJ = MJ + 1
      SD(MJ,JQ) = S(IN(J),IQ)
      ISC(MJ,JQ) = ISC(IN(J),IQ)
      ISR(MJ,JQ) = ISR(IN(J),IQ)
      IN(J) = IN(J) + 1
      IF (ISC(IN(J),IQ).NE.J) IN(J) = 0
800   CONTINUE
      WRITE (9,9) (I,BB(I),IBR(I),I=1,LB)
      WRITE (9,2) (IX(I),I=1,ISZ)
      WRITE (9,5) (I,SD(I,JQ),ISR(I,JQ),ISC(I,JQ),I=1,LN)
      IF (JQ.EQ.1) GO TO 820
      DO 810 I = 1,LN
810   SD(I,1) = SD(I,2)
      GO TO 840
820   DO 830 I = 1,LN
      ISC(I,2) = ISC(I,1)
830   ISR(I,2) = ISR(I,1)
C
C  *  L-U-D DECOMPOSITION   *
C
840   CALL LUD (LB,LST,JITER)
C
C  *  PRINT THE SOLUTION
C
      I1 = 2*NFQ - 1
      I2 = 6*I1
      DO 860  I = 1,ILL
      I3 = I2*(I-1) + 1
      I4 = I3 + 5*I1
      IF (I.EQ.1) WRITE (6,6) JITER
      WRITE (6,24) I,IOME(1),OID(1),(XX(K),K=I3,I4,I1)
      DO 860  J = 2,NFQ
      I4 = I3 + 2*J - 3
      I5 = I4 + 5*I1
860   WRITE (6,25) IOME(J),OID(J),(XX(K),XX(K+1),K = I4,I5,I1)
C
```

```
C  *  REBUILD  *
C
      IB1 = 0
      IL = ILL
      DO 870  I = 1,ISZ
      J = (I-1)/SVH
      K1 = I + J*SVH
      K2 = K1 + SVH
      S(K1,1) = XX(I)
870   S(K2,1) = 0.0D0
      S(STM,1) = 1.0D0
      DO 880  I = 1,ILL
      J = (I-1)*SV + 8*JF + 1
880   S(J,1) = S(J,1) + LLT(I)
      GO TO 520
C
C  *  FINI  *
C
2     FORMAT (8I10)
5     FORMAT (I6,D42.32,2I4)
6     FORMAT (1H1,' INITIAL STATE VECTOR SOLUTION ITERATED',I2,
     +' TIMES. IT IS :'//)
9     FORMAT (I6,D42.32,I4)
24    FORMAT (/' LINE',I2,' RESPONSES,',//' W, RPS ID',4X,
     +'WXC         WXS         WYC         WYS         WZC',
     +7X,'WZS         OXC         OXS         OYC',7X,
     +'OYS         OZC         OZS'/1H+,73X,'-',9X,'-',9X,'-',9X,'-',9X,'-',
     +9X,'-'//I5,1X,A4,D11.3,5D20.3)
25    FORMAT (I5,1X,A4,1X,12D10.3)
30    FORMAT (/' FREQUENCIES'//5X,'I',6X,'OME(I)',4X,'OID(I)'/,
     +25(I6,D14.4,3X,A4/))
31    FORMAT (/' RUNNING SPEEDS'//5X,'I',6X,'IRS(I)',/
     +25(I6,I10/))
32    FORMAT (/' TOOTH PASSING FREQUENCIES'//5X,'I',6X,'ITP(I)'/,
     +25(I6,I10/))
33    FORMAT (/' ERROR FREQUENCIES'//5X,'I',9X,'J',5X,'IER(I,J)'/,
     +25(I6,2I10/))
34    FORMAT (/' TOOTH PASSING PLUS/MINUS RUN SPEED'//5X,'I',9X,'J',
     +5X,'IML(I,J)',2X,'IPL(I,J)'/,25(I6,3I10/))
35    FORMAT (/' TWICE TOOTH PASSING'//5X,'I',6X,'ITT(I)'/,
     +25(I6,I10/))
36    FORMAT (/' TOOTH PASSING PLUS/MINUS ERRORS'//5X,'I',9X,'J',
     +5X,'IME(I,J)  IPE(I,J)'/,25(I6,3I10/))
999   STOP
      END
C

      SUBROUTINE SHAFT (IL,J)
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION BB(2000),DE(2000),S(50000,2),SD(50000,2),XX(2000),
     +YY(2000)
      REAL*8 LLT,LRT,LOM,OME,K0,KC,KS,KP0,KPC,KPS
      INTEGER*2 ILC,ILR,IUC,IUR(25000,2),ISC,ISR,ITC,ITR,IN(2000)
      INTEGER*2 MG,MHR,ML,NSP,SPE,SPI,SPT,TYP,MPR,MTP
      INTEGER*2 IRS,ITP,IE,IER,IPL,IML,ITT,IPE,IME,IX(2000),IBR(2000)
      INTEGER STM,SV,SVH
      INTEGER DS,DF,OID,US,UF,UST
      COMMON /Z1/ K0(5,3,2,3,2),KC(5,3,2,3,2),KS(5,3,2,3,2),BC(4,2,2,2),
     +BK(4,2,2,2),FP0(5,3,2),FPC1(5,3,2),FPC2(5,3,2),FPS1(5,3,2),FPS2(5,
```

```
     +3,2),KPO(5,3,2,2),KPC(5,3,2,2),KPS(5,3,2,2),BCZ(4,2),BKZ(4,2),
     +EF(2,5),EC(5,2),ES(5,2),DIB(8),DID(8),DIE(8),DIL(8),DIM(8),DIT(8),
     +GNT(4),LOM(5),LLT(5),LRT(5),OME(25),SHD(8),SHE(8),SHG(8),SHL(8),
     +CONV,IZ,IM,ISZ,JF,LHM,NFQ,PI,ILL,STM,SV,SVH,LM,LN,IQ,JQ,IE(2,5),
     +IER(2,5),IME(2,5),IML(2,5),IOME(25),IPE(2,5),IPL(2,5)
     +,MG(2,5),ML(2,5),MTP(2,25),NSP(5,5),SPE(15,18),SPI(5,4),SPT(15
     +,18),IRS(5),ITP(5),ITT(5),OID(25),MHR(5),MPR
      COMMON /Z2/ YU(25000),EL(25000),YUP(25000),ILC(25000),ILR(25000),
     +IUC(25000,2),T(8008),ISC(50000,2),ISR(50000,2),ITC(8008),ITR(8008)
      EQUIVALENCE (YU(1),S(1,1),SD(1,1)),(T(9),BB(1)),(T(2009),YY(1),
     +XX(1)),(ISR(1,1),IUR(1,1)),(ITC(2009),IN(1)),
     +(ITC(4009),IX(1)),(ITC(6009),IBR(1)),(ITR(9),DE(1))
      IS = (IL-1)*SV
      AR = PI*SHD(J)*SHD(J)/4.0D0
      PJ = AR*SHD(J)*SHD(J)/8.0D0
      DJ = PJ/2.0D0
      A = SHL(J)/(SHG(J)*PJ)
      B = SHL(J)/(SHE(J)*AR)
      C = SHL(J)/(SHE(J)*DJ)
      D = C*SHL(J)/2.0D0
      E = D*SHL(J)/3.0D0
      DO 10 I = 1,JF
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = -SHL(J)
      ITC(LM) = IS + 4*JF + I
      ITR(LM) = IS + I
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = D
      ITC(LM) = IS + 7*JF + I
      ITR(LM) = IS + I
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = E
      ITC(LM) = IS + 9*JF + I
      ITR(LM) = IS + I
10    CONTINUE
      DO 20 I = 1,JF
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = SHL(J)
      ITC(LM) = IS + 3*JF + I
      ITR(LM) = IS + JF + I
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = -D
      ITC(LM) = IS + 6* JF + I
      ITR(LM) = IS + JF + I
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = E
      ITC(LM) = IS + 10*JF + I
      ITR(LM) = IS + JF + I
20    CONTINUE
      DO 30 I = 1,JF
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = -B
      ITC(LM) = IS + 11*JF + I
      ITR(LM) = IS + 2*JF + I
30    CONTINUE
      DO 40 I = 1,JF
      ROW = IS + 3*JF + I
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = -C
      ITC(LM) = IS + 6*JF + I
```

```
      ITR(LM) = ROW
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = D
      ITC(LM) = IS + 10*JF + I
      ITR(LM) = ROW
40    CONTINUE
      DO 50 I = 1,JF
      ROW = IS + 4*JF + I
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = -C
      ITC(LM) = IS + 7*JF + I
      ITR(LM) = ROW
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = -D
      ITC(LM) = IS + 9*JF + I
      ITR(LM) = ROW
50    CONTINUE
      DO 60 I = 1,JF
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = -A
      ITC(LM) = IS + 8*JF + I
      ITR(LM) = IS + 5*JF + I
60    CONTINUE
      DO 70 I = 1,JF
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = -SHL(J)
      ITC(LM) = IS + 10*JF + I
      ITR(LM) = IS +  6*JF + I
70    CONTINUE
      DO 80 I = 1,JF
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = SHL(J)
      ITC(LM) = IS + 9*JF + I
      ITR(LM) = IS + 7*JF + I
80    CONTINUE
C
      RETURN
      END
C
      SUBROUTINE DISK(IL,J)
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION BB(2000),DE(2000),S(50000,2),SD(50000,2),XX(2000),
     +YY(2000)
      REAL*8 LLT,LRT,LOM,OME,K0,KC,KS,KP0,KPC,KPS
      INTEGER*2 ILC,ILR,IUC,IUR(25000,2),ISC,ISR,ITC,ITR,IN(2000)
      INTEGER*2 MG,MHR,ML,NSP,SPE,SPI,SPT,TYP,MPR,MTP
      INTEGER*2 IRS,ITP,IE,IER,IPL,IML,ITT,IPE,IME,IX(2000),IBR(2000)
      INTEGER STM,SV,SVH
      INTEGER DS,DF,OID,US,UF,UST
      COMMON /Z1/ K0(5,3,2,3,2),KC(5,3,2,3,2),KS(5,3,2,3,2),BC(4,2,2,2),
     +BK(4,2,2,2),FP0(5,3,2),FPC1(5,3,2),FPC2(5,3,2),FPS1(5,3,2),FPS2(5,
     +3,2),KP0(5,3,2,2),KPC(5,3,2,2),KPS(5,3,2,2),BCZ(4,2),BKZ(4,2),
     +EF(2,5),EC(5,2),ES(5,2),DIB(8),DID(8),DIE(8),DIL(8),DIM(8),DIT(8),
     +GNT(4),LOM(5),LLT(5),LRT(5),OME(25),SHD(8),SHE(8),SHG(8),SHL(8),
     +CONV,IZ,IM,ISZ,JF,LHM,NFQ,PI,ILL,STM,SV,SVH,LM,LN,IQ,JQ,IE(2,5),
     +IER(2,5),IME(2,5),IML(2,5),IOME(25),IPE(2,5),IPL(2,5)
     +,MG(2,5),ML(2,5),MTP(2,25),NSP(5,5),SPE(15,18),SPI(5,4),SPT(15
     +,18),IRS(5),ITP(5),ITT(5),OID(25),MHR(5),MPR
```

```
      COMMON /Z2/ YU(25000),EL(25000),YUP(25000),ILC(25000),ILR(25000),
     +IUC(25000,2),T(8008),ISC(50000,2),ISR(50000,2),ITC(8008),ITR(8008)
      EQUIVALENCE (YU(1),S(1,1),SD(1,1)),(T(9),BB(1)),(T(2009),YY(1),
     +XX(1)),(ISR(1,1),IUR(1,1)),(ITC(2009),IN(1)),
     +(ITC(4009),IX(1)),(ITC(6009),IBR(1)),(ITR(9),DE(1))
      IS = (IL-1)*SV
      RID = DIM(J)*(3.0D0*DID(J)*DID(J)+4.0D0*DIL(J)*DIL(J))/48.0D0
      RIP = DIM(J)*DID(J)*DID(J)/8.0D0
      DO 20 I = 2,NFQ
      A = RID*OME(I)*OME(I)
      D = -RIP*OME(I)*OME(IRS(IL))
      IC1 = IS + 3*JF + 2*(I-1)
      IC2 = IC1 + 1
      IC3 = IC1 + JF
      IC4 = IC3 + 1
      IR1 = IS + 6*JF + 2*(I-1)
      IR2 = IR1 + 1
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = A
      ITC(LM) = IC1
      ITR(LM) = IR1
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = D
      ITC(LM) = IC4
      ITR(LM) = IR1
      IF (OME(I).NE.OME(IRS(IL)).OR.DIT(J).EQ.0.0D0) GO TO 10
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = DIT(J)*(RID-RIP)*OME(I)*OME(I)
      ITC(LM) = STM
      ITR(LM) = IR1
10    IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = A
      ITC(LM) = IC2
      ITR(LM) = IR2
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = -D
      ITC(LM) = IC3
      ITR(LM) = IR2
20    CONTINUE
      DO 30 I = 2,NFQ
      A = -RID*OME(I)*OME(I)
      D = RIP*OME(I)*OME(IRS(IL))
      IC1 = IS + 3*JF + 2*(I-1)
      IC2 = IC1 + 1
      IC3 = IC1 + JF
      IC4 = IC3 + 1
      IR1 = IS + 7*JF + 2*(I-1)
      IR2 = IR1 + 1
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = -D
      ITC(LM) = IC2
      ITR(LM) = IR1
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = A
      ITC(LM) = IC3
      ITR(LM) = IR1
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = D
      ITC(LM) = IC1
      ITR(LM) = IR2
```

```
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = A
      ITC(LM) = IC4
      ITR(LM) = IR2
      IF (OME(I).NE.OME(IRS(IL)).OR.DIT(J).EQ.0.0D0) GO TO 30
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = -DIT(J)*(RID-RIP)*OME(I)*OME(I)
      ITC(LM) = STM
      ITR(LM) = IR2
30    CONTINUE
      DO 40 I = 2,NFQ
      C = RIP*OME(I)*OME(I)
      IC = IS + 5*JF + 2*(I-1) - 1
      IR = IS + 8*JF + 2*(I-1) - 1
      DO 40 K = 1,2
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = C
      ITC(LM) = IC + K
      ITR(LM) = IR + K
      IF (I.NE.IRS(IL)) GO TO 40
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = -DIM(J)*3.86088D2*DIE(J)*DCOS(DIB(J))
      IF (K.EQ.1) T(LM) = -T(LM)*DTAN(DIB(J))
      ITC(LM) = STM
      ITR(LM) = IR + K
      IF (T(LM).EQ.0.0) LM = LM - 1
40    CONTINUE
      K = 1
50    IC = IS + (K-1)*JF
      IR = IS + (K+8)*JF
      IF (K.NE.2) GO TO 60
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = -DIM(J)*3.86088D2
      ITC(LM) = STM
      ITR(LM) = IS + 10*JF + 1
60    DO 70 I = 2,NFQ
      B = DIM(J)*OME(I)*OME(I)
      F = -DIM(J)*DIE(J)*OME(I)*OME(I)*DCOS(DIB(J))
      H = F*DTAN(DIB(J))
      DO 70 L = 1,2
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = B
      ITC(LM) = IC + 2*I + L - 3
      ITR(LM) = IR + 2*I + L - 3
      IF (K.EQ.3.OR.I.NE.IRS(IL)) GO TO 70
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = H
      IF (K.EQ.2.AND.L.EQ.1) T(LM) = F
      IF (K.EQ.1.AND.L.EQ.2) T(LM) = -F
      ITC(LM) = STM
      ITR(LM) = ITR(LM-1)
70    IF (T(LM).EQ.0.0D0) LM = LM - 1
      IF (K.EQ.3) GO TO 80
      IF (K.EQ.2) K = 3
      IF (K.EQ.1) K = 2
      GO TO 50
80    CONTINUE
C
      RETURN
      END
```

```
C
      SUBROUTINE BRG(IL,J)
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION BB(2000),DE(2000),S(50000,2),SD(50000,2),XX(2000),
     +YY(2000)
      REAL*8 LLT,LRT,LOM,OME,K0,KC,KS,KP0,KPC,KPS
      INTEGER*2 ILC,ILR,IUC,IUR(25000,2),ISC,ISR,ITC,ITR,IN(2000)
      INTEGER*2 MG,MHR,ML,NSP,SPE,SPI,SPT,TYP,MPR,MTP
      INTEGER*2 IRS,ITP,IE,IER,IPL,IML,ITT,IPE,IME,IX(2000),IBR(2000)
      INTEGER STM,SV,SVH
      INTEGER DS,DF,OID,US,UF,UST
      COMMON /Z1/ K0(5,3,2,3,2),KC(5,3,2,3,2),KS(5,3,2,3,2),BC(4,2,2,2),
     +BK(4,2,2,2),FP0(5,3,2),FPC1(5,3,2),FPC2(5,3,2),FPS1(5,3,2),FPS2(5,
     +3,2),KP0(5,3,2,2),KPC(5,3,2,2),KPS(5,3,2,2),BCZ(4,2),BKZ(4,2),
     +EF(2,5),EC(5,2),ES(5,2),DIB(8),DID(8),DIE(8),DIL(8),DIM(8),DIT(8),
     +GNT(4),LOM(5),LLT(5),LRT(5),OME(25),SHD(8),SHE(8),SHG(8),SHL(8),
     +CONV,IZ,IM,ISZ,JF,LHM,NFQ,PI,ILL,STM,SV,SVH,LM,LN,IQ,JQ,IE(2,5),
     +IER(2,5),IME(2,5),IML(2,5),IOME(25),IPE(2,5),IPL(2,5)
     +,MG(2,5),ML(2,5),MTP(2,25),NSP(5,5),SPE(15,18),SPI(5,4),SPT(15
     +,18),IRS(5),ITP(5),ITT(5),OID(25),MHR(5),MPR
      COMMON /Z2/ YU(25000),EL(25000),YUP(25000),ILC(25000),ILR(25000),
     +IUC(25000,2),T(8008),ISC(50000,2),ISR(50000,2),ITC(8008),ITR(8008)
      EQUIVALENCE (YU(1),S(1,1),SD(1,1)),(T(9),BB(1)),(T(2009),YY(1),
     +XX(1)),(ISR(1,1),IUR(1,1)),(ITC(2009),IN(1)),
     +(ITC(4009),IX(1)),(ITC(6009),IBR(1)),(ITR(9),DE(1))
      IS = (IL-1)*SV
      L = 2
10    K = 1
20    IR = IS + (11-3*L+K)*JF + 1
      IC = IS + 3*(L-1)*JF + 1
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = -BK(J,L,K,1)
      ITC(LM) = IC
      ITR(LM) = IR
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = -BK(J,L,K,2)
      ITC(LM) = IC + JF
      ITR(LM) = IR
      DO 30 I = 2,NFQ
      IC1 = IC + 2*I - 3
      IC2 = IC1 + 1
      IC3 = IC1 + JF
      IC4 = IC3 + 1
      IR1 = IR + 2*I - 3
      IR2 = IR1 + 1
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = -BK(J,L,K,1)
      ITC(LM) = IC1
      ITR(LM) = IR1
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = -BC(J,L,K,1)*OME(I)
      ITC(LM) = IC2
      ITR(LM) = IR1
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = -BK(J,L,K,2)
      ITC(LM) = IC3
      ITR(LM) = IR1
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
```

```
      T(LM) = -BC(J,L,K,2)*OME(I)
      ITC(LM) = IC4
      ITR(LM) = IR1
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = BC(J,L,K,1)*OME(I)
      ITC(LM) = IC1
      ITR(LM) = IR2
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = -BK(J,L,K,1)
      ITC(LM) = IC2
      ITR(LM) = IR2
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = BC(J,L,K,2)*OME(I)
      ITC(LM) = IC3
      ITR(LM) = IR2
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = -BK(J,L,K,2)
      ITC(LM) = IC4
      ITR(LM) = IR2
30    CONTINUE
      IF (K.EQ.2) GO TO 40
      K = 2
      GO TO 20
40    CHK = DMAX1(BKZ(J,L),BCZ(J,L))
      IF (CHK.LE.1.D-3) GO TO 100
      IR = IS + (14-3*L)*JF + 1
      IF (BKZ(J,L).LE.1.D-3) GO TO 50
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = -BKZ(J,L)
      ITC(LM) = IS + (3*L-1)*JF + 1
      ITR(LM) = IR
50    IF (L.EQ.1.OR.BCZ(J,L) .LE.1.D-3) GO TO 60
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = -BCZ(J,L)*OME(IRS(IL))
      ITC(LM) = STM
      ITR(LM) = IR
60    DO 90 I = 2,NFQ
      IC1 = IS + (3*L-1)*JF + 2*(I-1)
      IC2 = IC1 + 1
      IR1 = IS + (14-3*L)*JF + 2*(I-1)
      IR2 = IR1 + 1
      IF (BKZ(J,L).LE.1.D-3) GO TO 70
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = -BKZ(J,L)
      ITC(LM) = IC1
      ITR(LM) = IR1
70    IF (BCZ(J,L).LE.1.D-3) GO TO 80
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = -BCZ(J,L)*OME(I)
      ITC(LM) = IC2
      ITR(LM) = IR1
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = BCZ(J,L)*OME(I)
      ITC(LM) = IC1
      ITR(LM) = IR2
80    IF (BKZ(J,L).LE.1.D-3) GO TO 90
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = -BKZ(J,L)
      ITC(LM) = IC2
      ITR(LM) = IR2
```

```
90    CONTINUE
100   IF (L.EQ.1) GO TO 999
      L = 1
      GO TO 10
999   RETURN
      END
C
      SUBROUTINE MESH(IM)
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION BB(2000),DE(2000),S(50000,2),SD(50000,2),XX(2000),
     +YY(2000)
      REAL*8 LLT,LRT,LOM,OME,KO,KC,KS,KPO,KPC,KPS
      INTEGER*2 IC(6),IR(6)
      INTEGER*2 ILC,ILR,IUC,IUR(25000,2),ISC,ISR,ITC,ITR,IN(2000)
      INTEGER*2 MG,MHR,ML,NSP,SPE,SPI,SPT,TYP,MPR,MTP
      INTEGER*2 IRS,ITP,IE,IER,IPL,IML,ITT,IPE,IME,IX(2000),IBR(2000)
      INTEGER STM,SV,SVH
      INTEGER DS,DF,OID,US,UF,UST
      COMMON /Z1/ KO(5,3,2,3,2),KC(5,3,2,3,2),KS(5,3,2,3,2),BC(4,2,2,2),
     +BK(4,2,2,2),FPO(5,3,2),FPC1(5,3,2),FPC2(5,3,2),FPS1(5,3,2),FPS2(5,
     +3,2),KPO(5,3,2,2),KPC(5,3,2,2),KPS(5,3,2,2),BCZ(4,2),BKZ(4,2),
     +EF(2,5),EC(5,2),ES(5,2),DIB(8),DID(8),DIE(8),DIL(8),DIM(8),DIT(8),
     +GNT(4),LOM(5),LLT(5),LRT(5),OME(25),SHD(8),SHE(8),SHG(8),SHL(8),
     +CONV,IL,IZ,ISZ,JF,LHM,NFQ,PI,ILL,STM,SV,SVH,LM,LN,IQ,JQ,IE(2,5),
     +IER(2,5),IME(2,5),IML(2,5),IOME(25),IPE(2,5),IPL(2,5)
     +,MG(2,5),ML(2,5),MTP(2,25),NSP(5,5),SPE(15,18),SPI(5,4),SPT(15
     +,18),IRS(5),ITP(5),ITT(5),OID(25),MHR(5),MPR
      COMMON /Z2/ YU(25000),EL(25000),YUP(25000),ILC(25000),ILR(25000),
     +IUC(25000,2),T(8008),ISC(50000,2),ISR(50000,2),ITC(8008),ITR(8008)
      EQUIVALENCE (YU(1),S(1,1),SD(1,1)),(T(9),BB(1)),(T(2009),YY(1),
     +XX(1)),(ISR(1,1),IUR(1,1)),(ITC(2009),IN(1)),
     +(ITC(4009),IX(1)),(ITC(6009),IBR(1)),(ITR(9),DE(1))
      LM = 1
      T(1) = 0.0D0
      I1 = ML(1,IM)
      I2 = ML(2,IM)
      MIN = MIN0(I1,I2)
      MAX = MAX0(I1,I2)
      IC(1) = (MIN-1)*SV
      IC(2) = IC(1) + JF
      IC(3) = IC(1) + 5*JF
      IC(4) = (MAX-1)*SV
      IC(5) = IC(4) + JF
      IC(6) = IC(4) + 5*JF
      IR(1) = IC(1) + 8*JF
      IR(2) = IR(1) + JF
      IR(3) = IR(2) + JF
      IR(4) = IC(4) + 8*JF
      IR(5) = IR(4) + JF
      IR(6) = IR(5) + JF
      DO 190 I = 1,6
      I1 = 1
      IF (I.EQ.3.OR.I.EQ.6) I1 = 2
      IF (I.EQ.1.OR.I.EQ.4) I1 = 3
      I2 = 1
      IF (I.LE.3.AND.ML(2,IM).LT.ML(1,IM)) I2 = 2
      IF (I.GT.3.AND.ML(2,IM).GT.ML(1,IM)) I2 = 2
      DO 190 J = 1,JF
```

```
      NF = (J/2) + 1
      DO 90 K = 1,6
      I3 = K
      IF (K.EQ.4) I3 = 1
      IF (K.EQ.5) I3 = 2
      IF (K.EQ.6) I3 = 3
      I4 = 1
      IF (K.LE.3.AND.ML(2,IM).LT.ML(1,IM)) I4 = 2
      IF (K.GT.3.AND.ML(2,IM).GT.ML(1,IM)) I4 = 2
      IF (J.NE.1) GO TO 10
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = K0(IM,I1,I2,I3,I4)
      ITC(LM) = IC(K) + 1
      ITR(LM) = IR(I) + 1
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = KC(IM,I1,I2,I3,I4)/2.0D0
      ITC(LM) = ITC(LM-1) + 2*(ITP(IM)-1) - 1
      ITR(LM) = ITR(LM-1)
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = KS(IM,I1,I2,I3,I4)/2.0D0
      ITC(LM) = ITC(LM-1) + 1
      ITR(LM) = ITR(LM-1)
      GO TO 90
10    IF (MTP(IM,NF)) 30,20,60
20    IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = K0(IM,I1,I2,I3,I4)
      ITC(LM) = IC(K) + J
      ITR(LM) = IR(I) + J
      IF (MTP(IM,NF).GE.MPR) GO TO 80
      GO TO 90
30    IF (MPR+MTP(IM,NF)) 40,40,50
40    IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = KC(IM,I1,I2,I3,I4)
      IF (MOD(J,2).EQ.1) T(LM) = KS(IM,I1,I2,I3,I4)
      ITC(LM) = IC(K) + 1
      ITR(LM) = IR(I) + J
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = K0(IM,I1,I2,I3,I4)
      ITC(LM) = IC(K) + J
      ITR(LM) = ITR(LM-1)
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = KC(IM,I1,I2,I3,I4)/2.0D0
      IF (MOD(J,2).EQ.1) T(LM) = -KS(IM,I1,I2,I3,I4)/2.0D0
      ITC(LM) = IC(K) - 2*(MTP(IM,NF)+1)
      ITR(LM) = ITR(LM-1)
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = KS(IM,I1,I2,I3,I4)/2.0D0
      IF (MOD(J,2).EQ.1) T(LM) = KC(IM,I1,I2,I3,I4)/2.0D0
      ITC(LM) = ITC(LM-1) + 1
      ITR(LM) = ITR(LM-1)
      GO TO 90
50    IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = KC(IM,I1,I2,I3,I4)/2.0D0
      IF (MOD(J,2).EQ.1) T(LM) = KS(IM,I1,I2,I3,I4)/2.0D0
      ITC(LM) = IC(K) - 2*(MTP(IM,NF)+1)
      ITR(LM) = IR(I) + J
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = KS(IM,I1,I2,I3,I4)/2.0D0
      IF (MOD(J,2).EQ.1) T(LM) = -KC(IM,I1,I2,I3,I4)/2.0D0
      ITC(LM) = ITC(LM-1) + 1
```

```
      ITR(LM) = ITR(LM-1)
      GO TO 20
60    IF (MPR-MTP(IM,NF)) 20,20,70
70    IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = KC(IM,I1,I2,I3,I4)/2.0D0
      IF (MOD(J,2).EQ.1) T(LM) = KS(IM,I1,I2,I3,I4)/2.0D0
      ITC(LM) = IC(K) + 2*(MTP(IM,NF)-1)
      ITR(LM) = IR(I) + J
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = -KS(IM,I1,I2,I3,I4)/2.0D0
      IF (MOD(J,2).EQ.1) T(LM) = KC(IM,I1,I2,I3,I4)/2.0D0
      ITC(LM) = ITC(LM-1) + 1
      ITR(LM) = ITR(LM-1)
      GO TO 20
80    IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = KC(IM,I1,I2,I3,I4)/2.0D0
      IF (MOD(J,2).EQ.1) T(LM) = -KS(IM,I1,I2,I3,I4)/2.0D0
      ITC(LM) = IC(K) + 2*(MTP(IM,NF)-1)
      ITR(LM) = IR(I) + J
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = KS(IM,I1,I2,I3,I4)/2.0D0
      IF (MOD(J,2).EQ.1) T(LM) = KC(IM,I1,I2,I3,I4)/2.0D0
      ITC(LM) = ITC(LM-1) + 1
      ITR(LM) = IR(I) + J
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = KC(IM,I1,I2,I3,I4)/2.0D0
      IF (MOD(J,2).EQ.1) T(LM) = KS(IM,I1,I2,I3,I4)/2.0D0
      ITC(LM) = ITC(LM-1) + 1
      ITR(LM) = ITR(LM-1)
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = KS(IM,I1,I2,I3,I4)/2.0D0
      IF (MOD(J,2).EQ.1) T(LM) = -KC(IM,I1,I2,I3,I4)/2.0D0
      ITC(LM) = ITC(LM-1) + 1
      ITR(LM) = ITR(LM-1)
90    CONTINUE
      IF (J.NE.1) GO TO 100
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = FP0(IM,I1,I2)
      GO TO 180
100   IF (NF.NE.ITP(IM)) GO TO 110
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = FPC1(IM,I1,I2)
      IF (MOD(J,2).EQ.1) T(LM) = FPS1(IM,I1,I2)
      GO TO 180
110   IF (NF.NE.ITT(IM)) GO TO 120
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = FPC2(IM,I1,I2)
      IF (MOD(J,2).EQ.1) T(LM) = FPS2(IM,I1,I2)
      GO TO 180
120   IF (IE(1,IM).EQ.0.AND.IE(2,IM).EQ.0) GO TO 190
      IF (NF.NE.IER(1,IM).AND.NF.NE.IER(2,IM)) GO TO 140
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = 0.0D0
      DO 130 L = 1,2
      IF (NF.NE.IER(L,IM)) GO TO 130
      IF (MOD(J,2).EQ.0) T(LM) = T(LM) + EC(IM,L)*KP0(IM,I1,I2,1)
      IF (MOD(J,2).EQ.1) T(LM) = T(LM) + ES(IM,L)*KP0(IM,I1,I2,2)
130   CONTINUE
      GO TO 180
140   IF (NF.NE.IPE(1,IM).AND.NF.NE.IPE(2,IM)) GO TO 160
```

```
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = 0.0D0
      DO 150 L = 1,2
      IF (NF.NE.IPE(L,IM)) GO TO 150
      IF (MOD(J,2).EQ.0) T(LM) = T(LM) + (EC(IM,L)*KPC(IM,I1,I2,1)-
     +ES(IM,L)*KPS(IM,I1,I2,2))/2.0D0
      IF (MOD(J,2).EQ.1) T(LM) = T(LM) + (EC(IM,L)*KPS(IM,I1,I2,1)+
     +ES(IM,L)*KPC(IM,I1,I2,2))/2.0D0
150   CONTINUE
      GO TO 180
160   IF (NF.NE.IME(1,IM).AND.NF.NE.IME(2,IM)) GO TO 190
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = 0.0D0
      DO 170 L = 1,2
      IF (NF.NE.IPE(L,IM)) GO TO 170
      IF (MOD(J,2).EQ.0) T(LM) = T(LM) + (ES(IM,L)*KPS(IM,I1,I2,2)+
     +EC(IM,L)*KPC(IM,I1,I2,1))/2.0D0
      IF (MOD(J,2).EQ.1) T(LM) = T(LM) + (EC(IM,L)*KPS(IM,I1,I2,1)+
     +ES(IM,L)*KPC(IM,I1,I2,2))/2.0D0
170   CONTINUE
180   ITC(LM) = STM
      ITR(LM) = ITR(LM-1)
190   IF (T(LM).EQ.0.0D0) LM = LM - 1
      DO 200 I = 1,LM
200   IF (ITC(I).NE.STM) T(I) = -T(I)
      RETURN
      END
C
      SUBROUTINE TLAST
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION BB(2000),DE(2000),S(50000,2),SD(50000,2),XX(2000),
     +YY(2000)
      REAL*8 LLT,LRT,LOM,OME,K0,KC,KS,KP0,KPC,KPS
      INTEGER*2 ILC,ILR,IUC,IUR(25000,2),ISC,ISR,ITC,ITR,IN(2000)
      INTEGER*2 MG,MHR,ML,NSP,SPE,SPI,SPT,TYP,MPR,MTP
      INTEGER*2 IRS,ITP,IE,IER,IPL,IML,ITT,IPE,IME,IX(2000),IBR(2000)
      INTEGER STM,SV,SVH,TRIG
      INTEGER DS,DF,OID,US,UF,UST
      COMMON /Z1/ K0(5,3,2,3,2),KC(5,3,2,3,2),KS(5,3,2,3,2),BC(4,2,2,2),
     +BK(4,2,2,2),FP0(5,3,2),FPC1(5,3,2),FPC2(5,3,2),FPS1(5,3,2),FPS2(5,
     +3,2),KP0(5,3,2,2),KPC(5,3,2,2),KPS(5,3,2,2),BCZ(4,2),BKZ(4,2),
     +EF(2,5),EC(5,2),ES(5,2),DIB(8),DID(8),DIE(8),DIL(8),DIM(8),DIT(8),
     +GNT(4),LOM(5),LLT(5),LRT(5),OME(25),SHD(8),SHE(8),SHG(8),SHL(8),
     +CONV,IL,IM,ISZ,JF,LHM,NFQ,PI,ILL,STM,SV,SVH,LM,LN,IQ,JQ,IE(2,5),
     +IER(2,5),IME(2,5),IML(2,5),IOME(25),IPE(2,5),IPL(2,5)
     +,MG(2,5),ML(2,5),MTP(2,25),NSP(5,5),SPE(15,18),SPI(5,4),SPT(15
     +,18),IRS(5),ITP(5),ITT(5),OID(25),MHR(5),MPR
      COMMON /Z2/ YU(25000),EL(25000),YUP(25000),ILC(25000),ILR(25000),
     +IUC(25000,2),T(8008),ISC(50000,2),ISR(50000,2),ITC(8008),ITR(8008)
      EQUIVALENCE (YU(1),S(1,1),SD(1,1)),(T(9),BB(1)),(T(2009),YY(1),
     +XX(1)),(ISR(1,1),IUR(1,1)),(ITC(2009),IN(1)),
     +(ITC(4009),IX(1)),(ITC(6009),IBR(1)),(ITR(9),DE(1))
      AK = 1.0D0
      DO 10 I = 1,IL
      TQ1 = DABS(LLT(I))
      TQ2 = DABS(LRT(I))
 10   AK = DMAX1(AK,TQ1,TQ2)
      AK = -1.0D-4
```

```
      T(1) = AK
      ITC(1) = 5*JF + 1
      ITR(1) = 8*JF + 1
      T(2) = LRT(1)
      ITC(2) = STM
      ITR(2) = ITR(1)
      LM = 2
      IF (LRT(1).EQ.0.0D0) LM = 1
      DO 20 I = 2,NFQ
      DO 20 J = 1,2
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = AK
      ITC(LM) = 5*JF + 2*I + J - 3
      ITR(LM) = 8*JF + 2*I + J - 3
 20   CONTINUE
      IF (IL.EQ.1) GO TO 99
      DO 30 I = 2,IL
      IF (T(LM).NE.0.0D0.OR.LM.EQ.0) LM = LM + 1
      T(LM) = LRT(I)
      ITC(LM) = STM
      ITR(LM) = SV*(I-1) + 8*JF + 1
 30   IF (T(LM).EQ.0.0) LM = LM - 1
 99   RETURN
      END
C
      SUBROUTINE MULT
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION BB(2000),DE(2000),S(50000,2),SD(50000,2),XX(2000),
     +YY(2000)
      REAL*8 LLT,LRT,LOM,OME,K0,KC,KS,KP0,KPC,KPS
      INTEGER*2 ILC,ILR,IUC,IUR(25000,2),ISC,ISR,ITC,ITR,IN(2000)
      INTEGER*2 MG,MHR,ML,NSP,SPE,SPI,SPT,TYP,MPR,MTP
      INTEGER*2 IRS,ITP,IE,IER,IPL,IML,ITT,IPE,IME,IX(2000),IBR(2000)
      INTEGER STM,SV,SVH,TRIG
      INTEGER DS,DF,OID,US,UF,UST
      COMMON /Z1/ K0(5,3,2,3,2),KC(5,3,2,3,2),KS(5,3,2,3,2),BC(4,2,2,2),
     +BK(4,2,2,2),FP0(5,3,2),FPC1(5,3,2),FPC2(5,3,2),FPS1(5,3,2),FPS2(5,
     +3,2),KP0(5,3,2,2),KPC(5,3,2,2),KPS(5,3,2,2),BCZ(4,2),BKZ(4,2),
     +EF(2,5),EC(5,2),ES(5,2),DIB(8),DID(8),DIE(8),DIL(8),DIM(8),DIT(8),
     +GNT(4),LOM(5),LLT(5),LRT(5),OME(25),SHD(8),SHE(8),SHG(8),SHL(8),
     +CONV,IL,IM,ISZ,JF,LHM,NFQ,PI,ILL,STM,SV,SVH,LM,LN,IQ,JQ,IE(2,5),
     +IER(2,5),IME(2,5),IML(2,5),IOME(25),IPE(2,5),IPL(2,5)
     +,MG(2,5),ML(2,5),MTP(2,25),NSP(5,5),SPE(15,18),SPI(5,4),SPT(15
     +,18),IRS(5),ITP(5),ITT(5),OID(25),MHR(5),MPR
      COMMON /Z2/ YU(25000),EL(25000),YUP(25000),ILC(25000),ILR(25000),
     +IUC(25000,2),T(8008),ISC(50000,2),ISR(50000,2),ITC(8008),ITR(8008)
      EQUIVALENCE (YU(1),S(1,1),SD(1,1)),(T(9),BB(1)),(T(2009),YY(1),
     +XX(1)),(ISR(1,1),IUR(1,1)),(ITC(2009),IN(1)),
     +(ITC(4009),IX(1)),(ITC(6009),IBR(1)),(ITR(9),DE(1))
      L = 0
      M = 1
      N = 1
      NST = 1
      TRIG = 1
10    IR = ITR(M)
      IC = ISC(N,IQ)
20    IF (ISC(TRIG,IQ).GT.IC) GO TO 30
      IF (ISC(TRIG,IQ).EQ.IC.AND.ISR(TRIG,IQ).GE.IR) GO TO 30
```

```
      L = L + 1
      S(L,JQ) = S(TRIG,IQ)
      ISC(L,JQ) = ISC(TRIG,IQ)
      ISR(L,JQ) = ISR(TRIG,IQ)
      TRIG = TRIG + 1
      IF (TRIG.GT.LN) GO TO 180
      GO TO 20
30    L = L + 1
      S(L,JQ) = 0.0E0
      I1 = 0
      IF (ISC(TRIG,IQ).NE.IC.OR.ISR(TRIG,IQ).NE.IR) GO TO 40
      I1 = 1
      S(L,JQ) = S(TRIG,IQ)
      TRIG = TRIG + 1
40    IF (ISR(N,IQ)-ITC(M)) 60,50,70
50    I1 = 1
      S(L,JQ) = S(L,JQ) + S(N,IQ)*T(M)
      M = M + 1
60    N = N + 1
      GO TO 80
70    M = M + 1
80    IF (M.GT.LM.OR.N.GT.LN) GO TO 90
      IF (ISC(N,IQ).EQ.IC.AND.ITR(M).EQ.IR) GO TO 40
90    ISC(L,JQ) = IC
      ISR(L,JQ) = IR
      IF (I1.EQ.0) L = L - 1
      IF (M.LE.LM.OR.N.LE.LN) GO TO 120
100   IF (TRIG.GT.LN) GO TO 180
      DO 110 I = TRIG,LN
      L = L + 1
      S(L,JQ) = S(I,IQ)
      ISC(L,JQ) = ISC(I,IQ)
110   ISR(L,JQ) = ISR(I,IQ)
      GO TO 180
120   IF (N.GT.LN) GO TO 130
      IF (M.GT.LM) GO TO 150
      IF (ITR(M)-IR) 160,160,140
130   IF (M.GT.LM) GO TO 100
      IF (ITR(M).GT.IR) GO TO 140
      M = M + 1
      GO TO 130
140   N = NST
      GO TO 10
150   IF (N.GT.LN) GO TO 100
      IF (ISC(N,IQ).NE.IC) GO TO 170
      N = N + 1
      GO TO 150
160   IF (M.GT.LM) GO TO 170
      IF (ITR(M).NE.IR) GO TO 140
      M = M + 1
      GO TO 160
170   NST = N
      M = 1
      GO TO 10
180   LN = L
      RETURN
      END
C

      SUBROUTINE REBLD (NEL,NHM,IRT)
```

```
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION BB(2000),DE(2000),S(50000,2),SD(50000,2),XX(2000),
     +YY(2000)
      REAL*8 LLT,LRT,LOM,OME,KO,KC,KS,KPO,KPC,KPS
      INTEGER*2 ILC,ILR,IUC,IUR(25000,2),ISC,ISR,ITC,ITR,IN(2000)
      INTEGER*2 MG,MHR,ML,NSP,SPE,SPI,SPT,TYP,MPR,MTP
      INTEGER*2 IRS,ITP,IE,IER,IPL,IML,ITT,IPE,IME,IX(2000),IBR(2000)
      INTEGER STM,SV,SVH
      INTEGER DS,DF,OID,US,UF,UST
      COMMON /Z1/ KO(5,3,2,3,2),KC(5,3,2,3,2),KS(5,3,2,3,2),BC(4,2,2,2),
     +BK(4,2,2,2),FPO(5,3,2),FPC1(5,3,2),FPC2(5,3,2),FPS1(5,3,2),FPS2(5,
     +3,2),KPO(5,3,2,2),KPC(5,3,2,2),KPS(5,3,2,2),BCZ(4,2),BKZ(4,2),
     +EF(2,5),EC(5,2),ES(5,2),DIB(8),DID(8),DIE(8),DIL(8),DIM(8),DIT(8),
     +GNT(4),LOM(5),LLT(5),LRT(5),OME(25),SHD(8),SHE(8),SHG(8),SHL(8),
     +CONV,IL,IM,ISZ,JF,LHM,NFQ,PI,ILL,STM,SV,SVH,LM,LN,IQ,JQ,IE(2,5),
     +IER(2,5),IME(2,5),IML(2,5),IOME(25),IPE(2,5),IPL(2,5)
     +,MG(2,5),ML(2,5),MTP(2,25),NSP(5,5),SPE(15,18),SPI(5,4),SPT(15
     +,18),IRS(5),ITP(5),ITT(5),OID(25),MHR(5),MPR
      COMMON /Z2/ YU(25000),EL(25000),YUP(25000),ILC(25000),ILR(25000),
     +IUC(25000,2),T(8008),ISC(50000,2),ISR(50000,2),ITC(8008),ITR(8008)
      EQUIVALENCE (YU(1),S(1,1),SD(1,1)),(T(9),BB(1)),(T(2009),YY(1),
     +XX(1)),(ISR(1,1),IUR(1,1)),(ITC(2009),IN(1)),
     +(ITC(4009),IX(1)),(ITC(6009),IBR(1)),(ITR(9),DE(1))
      DO 10 I = 1,STM
10    S(I,JQ) = 0.0D0
      DO 20 I = 1,LM
      A = S(ITC(I),IQ)
      B = T(I)
      IF (A.EQ.0.0D0.OR.B.EQ.0.0D0) GO TO 20
      C = DLOG10(DABS(A)) + DLOG10(DABS(B))
      IF (C.GT.7.5D1.OR.C.LT.-7.5D1) GO TO 20
      S(ITR(I),JQ) = S(ITR(I),JQ) + A*B
20    CONTINUE
      DO 25 I = 1,STM
25    S(I,JQ) = S(I,JQ) + S(I,IQ)
      IF (IRT.EQ.1) WRITE (6,1) NEL
      IF (IRT.EQ.2) WRITE (6,2) NHM
      IF (IRT.EQ.3) WRITE (6,3)
      I1 = 2*NFQ - 1
      I2 = 6*I1
      DO 30 K = 1,2
      DO 30 I = 1,ILL
      I3 = 1 + 2*I2*(I-1) + I2*(K-1)
      I4 = I3 + 5*I1
      IF (K.EQ.1) WRITE (6,4) I,IOME(1),OID(1),(S(L,JQ),L=I3,I4,I1)
      IF (K.EQ.2) WRITE (6,5) I,IOME(1),OID(1),(S(L,JQ),L=I3,I4,I1)
      DO 30 J = 2,NFQ
      I4 = I3 + 2*J - 3
      I5 = I4 + 5*I1
30    WRITE (6,7) IOME(J),OID(J),(S(L,JQ),S(L+1,JQ),L=I4,I5,I1)
1     FORMAT (1H1,' AFTER ELEMENT',I3,' STATE VECTOR IS,')
2     FORMAT (1H1,' AFTER MESH',I3,' STATE VECTOR IS,')
3     FORMAT (1H1,' FINAL STATE VECTOR IS,')
4     FORMAT (/' LINE',I2,' RESPONSES,',//' W, RPS ID',4X,
     +'WXC       WXS       WYC       WYS       WZC',
     +7X,'WZS       OXC       OXS       OYC',7X,
     +'OYS       OZC       OZS'/1H+,73X,'-',9X,'-',9X,'-',9X,'-',9X,'-',
     +9X,'-'//I5,1X,A4,D11.3,5D20.3)
5     FORMAT (/' LINE',I2,' FORCES,',//' W, RPS ID',4X,
```

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     +'MXC         MXS          MYC          MYS          MZC',
     +7X,'MZS          VXC          VXS          VYC',7X,
     +'VYS         VZC          VZS'//I5,1X,A4,D11.3,5D20.3)
7     FORMAT (I5,1X,A4,1X,12D10.3)
      RETURN
      END
C
      SUBROUTINE LUD (LB,LST,JITER)
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION BB(2000),DE(2000),S(50000,2),SD(50000,2),XX(2000),
     +YY(2000)
      REAL*8 LLT,LRT,LOM,OME,K0,KC,KS,KP0,KPC,KPS
      INTEGER*2 ILC,ILR,IUC,IUR(25000,2),ISC,ISR,ITC,ITR,IN(2000)
      INTEGER*2 MG,MHR,ML,NSP,SPE,SPI,SPT,TYP,MPR,MTP
      INTEGER*2 IRS,ITP,IE,IER,IPL,IML,ITT,IPE,IME,IX(2000),IBR(2000)
      INTEGER STM,SV,SVH
      INTEGER DS,DF,OID,US,UF,UST
      COMMON /Z1/ K0(5,3,2,3,2),KC(5,3,2,3,2),KS(5,3,2,3,2),BC(4,2,2,2),
     +BK(4,2,2,2),FP0(5,3,2),FPC1(5,3,2),FPC2(5,3,2),FPS1(5,3,2),FPS2(5,
     +3,2),KP0(5,3,2,2),KPC(5,3,2,2),KPS(5,3,2,2),BCZ(4,2),BKZ(4,2),
     +EF(2,5),EC(5,2),ES(5,2),DIB(8),DID(8),DIE(8),DIL(8),DIM(8),DIT(8),
     +GNT(4),LOM(5),LLT(5),LRT(5),OME(25),SHD(8),SHE(8),SHG(8),SHL(8),
     +CONV,IL,IM,ISZ,JF,LHM,NFQ,PI,ILL,STM,SV,SVH,LM,LN,IQ,JQ,IE(2,5),
     +IER(2,5),IME(2,5),IML(2,5),IOME(25),IPE(2,5),IPL(2,5)
     +,MG(2,5),ML(2,5),MTP(2,25),NSP(5,5),SPE(15,18),SPI(5,4),SPT(15
     +,18),IRS(5),ITP(5),ITT(5),OID(25),MHR(5),MPR
      COMMON /Z2/ YU(25000),EL(25000),YUP(25000),ILC(25000),ILR(25000),
     +IUC(25000,2),T(8008),ISC(50000,2),ISR(50000,2),ITC(8008),ITR(8008)
      EQUIVALENCE (YU(1),S(1,1),SD(1,1)),(T(9),BB(1)),(T(2009),YY(1),
     +XX(1)),(ISR(1,1),IUR(1,1)),(ITC(2009),IN(1)),
     +(ITC(4009),IX(1)),(ITC(6009),IBR(1)),(ITR(9),DE(1))
C
C  *  L-U-D DECOMPOSITION  *
C
      LL1 = 1
      LU1 = 1
      DE(1) = SD(1,1)
      ISL = LST
      ISUD = 2
      LL = 0
      LU = 0
      DO 430 I = 2,ISZ
      IU = 1
      II = 1
100   IF (ISL.GT.LN) GO TO 250
      IF (ISR(ISL,2)-I) 110,120,250
110   ISL = ISL + 1
      GO TO 100
120   LL = LL + 1
      EL(LL) = SD(ISL,1)/DE(ISC(ISL,2))
      ILC(LL) = ISC(ISL,2)
      ILR(LL) = I
      IF (LU.EQ.0) GO TO 110
      ILS = LL
      IL = LL
130   ISL = ISL + 1
      IB = ILC(LL) + 1
      IF (ISL.LE.LN) GO TO 140
```

```
      II = 0
      IEND = I - 1
      GO TO 150
140   IF (ISR(ISL,2).NE.I) II = 0
      IEND = ISC(ISL,2)
      IF (II.EQ.0) IEND = I - 1
150   IF (IB.GT.IEND) GO TO 250
      KK = 0
      IF (IU.GT.LU) GO TO 230
      DO 220 J = IB,IEND
      KK = 0
      IL = ILS
      IF (IU.GT.LU) GO TO 220
160   IF (IUC(IU,2)-J)  210,170,220
170   IF (ILC(IL)-IUR(IU,2)) 200,180,210
180   IF (KK.EQ.1) GO TO 190
      KK = 1
      LL = LL + 1
      EL(LL) = 0.0D0
      ILC(LL) = J
      ILR(LL) = I
190   EL(LL) = EL(LL) - EL(IL)*YUP(IU)/DE(J)
      IL = IL + 1
      IU = IU + 1
      IF (IL.GT.LL.OR.IU.GT.LU) GO TO 220
      GO TO 160
200   IL = IL + 1
      IF (IL.GT.LL) GO TO 220
      GO TO 170
210   IU = IU + 1
      IF (IU.LE.LU) GO TO 160
220   CONTINUE
230   IF (II.EQ.0) GO TO 250
      IF (KK.EQ.1) GO TO 240
      LL = LL + 1
      EL(LL) = 0.0D0
      ILC(LL) = IEND
      ILR(LL) = I
240   EL(LL) = EL(LL) + SD(ISL,1)/DE(IEND)
      GO TO 130
250   IN(I) = 0
      II = 1
260   IF (ISC(ISUD,2)-I)  270,280,420
270   ISUD = ISUD + 1
      IF (ISUD.GE.LST) GO TO 420
      GO TO 260
280   LU = LU + 1
      YUP(LU) = SD(ISUD,1)
      IUC(LU,2) = I
      IUR(LU,2) = ISR(ISUD,2)
      UST = LU
      IF (ISR(ISUD,2).EQ.ISC(ISUD,2)) GO TO 400
      IF (II.EQ.1) IN(I) = LU
      IF (LL.EQ.0) II = 0
      IF (II.EQ.0) GO TO 270
      IL = 1
290   IB = ISR(ISUD,2) + 1
      IEND = ISR(ISUD+1,2)
      IF (ISC(ISUD+1,2).NE.I) II = 0
      IF (II.EQ.0) IEND = I
```

```
      IF (IL.GT.LL) GO TO 380
      DO 370 J = IB,IEND
      JJ = 0
300   IF (IL.GT.LL) GO TO 370
      IF (ILR(IL)-J) 310,320,370
310   IL = IL + 1
      GO TO 300
320   IU = UST
330   IF (ILC(IL)-IUR(IU,2)) 310,340,360
340   IF (JJ.EQ.1) GO TO 350
      JJ = 1
      LU = LU + 1
      YUP(LU) = 0.0D0
      IUC(LU,2) = I
      IUR(LU,2) = J
350   YUP(LU) = YUP(LU) - EL(IL)*YUP(IU)
      IL = IL + 1
      IF (IL.GT.LL) GO TO 370
      IF (ILR(IL).GT.J) GO TO 370
360   IU = IU + 1
      IF (IU.GT.LU) GO TO 370
      GO TO 330
370   CONTINUE
      IF (JJ.EQ.1) GO TO 390
380   LU = LU + 1
      YUP(LU) = 0.0D0
      IUC(LU,2) = I
      IUR(LU,2) = IEND
390   IF (II.NE.0) YUP(LU) = YUP(LU) + SD(ISUD+1,1)
      IF (IEND.NE.I) GO TO 410
400   DE(ISC(ISUD,2)) = YUP(LU)
      LU = LU - 1
      GO TO 420
410   ISUD = ISUD + 1
      GO TO 290
420   CONTINUE
430   CONTINUE
      LGU = LU
C
C  *  TO SOLVE, RENUMBER U FROM COLUMNAR TO ROW-WISE; (L+I)(U+D)=S  *
C
      LU = 0
      DO 450 I = 2,ISZ
      J = I - 1
      DO 450 L = I,ISZ
      IF (IN(L).EQ.0) GO TO 450
      IF (IUR(IN(L),2).NE.J) GO TO 450
      LU = LU + 1
      YU(LU) = YUP(IN(L))
      IUC(LU,1) = IUC(IN(L),2)
      IUR(LU,1) = IUR(IN(L),2)
      IN(L) = IN(L) + 1
      IF (IN(L).LE.LGU) GO TO 440
      IN(L) = 0
      GO TO 450
440   IF (IUC(IN(L),2).NE.L)  IN(L) = 0
450   CONTINUE
      IF (LGU.EQ.LU) GO TO 460
C
C  *  SOLVE ; FIRST FIND Y OF (L + I) Y = B  *
```

```
C
460   REWIND 9
      READ (9,4) (J,BB(I),IBR(I),I=1,LB)
      LC = 1
      I1 = IBR(1)
      YY(I1) = BB(1)
      IN(I1) = 1
      IB = 2
      IR = I1 + 1
      IF (I1.EQ.1) GO TO 480
      DO 470 I = 2,I1
470   IN(I-1) = 0
480   DO 520 I = IR,ISZ
      IN(I) = 0
      YY(I) = 0.0D0
      IF (IB.GT.LB) GO TO 490
      IF (IBR(IB).GT.I) GO TO 490
      IN(I) = 1
      YY(I) = BB(IB)
      IB = IB + 1
490   IF (LC.GT.LL) GO TO 520
      IF (ILR(LC) - I) 500,510,520
500   LC = LC + 1
      IF (LC.GT.LL) GO TO 520
      GO TO 490
510   IF (IN(ILC(LC)).EQ.0) GO TO 500
      IN (I) = 1
      YY(I) = YY(I) - YY(ILC(LC))*EL(LC)
      GO TO 500
520   CONTINUE
C
C  *  NOW FIND X OF (U + D) X = Y
C
      BB(ISZ) = YY(ISZ)/DE(ISZ)
      IX(ISZ) = IN(ISZ)
      IU = LGU
      DO 560 J = 2,ISZ
      I = ISZ + 1 - J
      IX(I) = IN(I)
      BB(I) = 0.0D0
      IF (IN(I).EQ.0) GO TO 530
      BB(I) = YY(I)
530   IF (IU.EQ.0) GO TO 550
      IF (IUR(IU,1).LT.I) GO TO 550
      IF (IX(IUC(IU,1)).EQ.0) GO TO 540
      IX(I) = 1
      BB(I) = BB(I) - BB(IUC(IU,1))*YU(IU)
540   IU = IU - 1
      GO TO 530
550   IF (IX(I).EQ.0) GO TO 560
      BB(I) = BB(I)/DE(I)
560   CONTINUE
      DO 570 I = 1,ISZ
570   XX(I) = BB(I)
      REWIND 9
      READ (9,4) (J,BB(I),IBR(I),I=1,LB)
      READ (9,1) (IN(I),I=1,ISZ)
      READ (9,2) (J,SD(I,1),ISR(I,2),ISC(I,2),I=1,LN)
      REWIND 9
C
```

```
C  *  RENUMBER UD FROM COLUMNAR TO ROW-WISE AND SPLIT INTO U + D  *
C              *  S = (L) + (UD) = (L) + (U) + (D)  *
C
      LU = 0
      US = 1
      DO 580 I = 2,ISZ
      J = I - 1
      DO 580 L = I,ISZ
      IF (IN(L).EQ.0) GO TO 580
      IF (ISR(IN(L),2).NE.J) GO TO 580
      LU = LU + 1
      SD(LU,2) = SD(IN(L),1)
      ISC(LU,1) = ISC(IN(L),2)
      ISR(LU,1) = ISR(IN(L),2)
      IN(L) = IN(L) + 1
      IF (ISC(IN(L),2).NE.L) IN(L) = 0
580   CONTINUE
      UF = LU
      DS = LU + 1
      DO 590 I = 1,ISZ
      IF (IN(I).EQ.0) GO TO 590
      LU = LU + 1
      SD(LU,2) = SD(IN(I),1)
590   CONTINUE
      DF = LU
      I = LU + 1
      IQ = 1
      JITER = -1
      IF (I.EQ.LST) GO TO 600
C
C  *  REGENERATE B FROM FIRST X, S X = B  *
C
600   JITER = JITER + 1
      IB = 1
      LD = LST - 1
      IL = LST
      ID = DS
      IU = US
      FI = 0.0D0
      DO 640  I = 1,ISZ
      DE(I) = 0.0D0
      AA1 = 0.0D0
      IF (IB.GT.LB) GO TO 610
      IF (IBR(IB).NE.I) GO TO 610
      AA1 = BB(IB)
      IB = IB + 1
610   IF (IL.GT.LN) GO TO 620
      IF (ISR(IL,2).NE.I) GO TO 620
      DE(I) = DE(I) + XX(ISC(IL,2))*SD(IL,1)
      IL = IL + 1
      GO TO 610
620   IF (IU.GT.UF) GO TO 630
      IF (ISR(IU,1).NE.I) GO TO 630
      DE(I) = DE(I) + XX(ISC(IU,1))*SD(IU,2)
      IU = IU + 1
      GO TO 620
630   DE(I) = DE(I) + XX(I)*SD(ID,2)
      ID = ID + 1
      DE(I) = DE(I) - AA1
640   FI = FI + DE(I)*DE(I)
```

```
      AA1 = ISZ
      FI = DSQRT(FI/AA1)
      WRITE (6,5) JITER,FI
      REWIND 9
      IF (JITER.EQ.0) GO TO 650
      IF (FI.LT.FJ) GO TO 650
      READ (9,3) (XX(I),I=1,ISZ)
      JITER = JITER - 1
      GO TO 999
650   WRITE (9,3) (XX(I),I=1,ISZ)
      FJ = FI
C
C  *  GAUSS-SEIDEL ITERATIVE REFINEMENT  *
C
      FI = 0.0D0
      DO 655 I = 1,ISZ
      DE(I) = 0.0D0
655   CONTINUE
      DO 710 I = 1,ISZ
      AA1 = 0.0D0
      DO 660 J = 1,ISZ
      AA2 = DABS(XX(J))
      IF (AA2.LT.AA1.OR.DE(J).GT.1.0D0) GO TO 660
      AA1 = AA2
      JROW = J
660   CONTINUE
      DE(JROW) = 2.0D0
      AA1 = 0.0D0
      IB = 0
      IU = US - 1
      IL = LST - 1
665   IL = IL + 1
      IF (IL.GT.LN) GO TO 680
      IF (ISR(IL,2)-JROW) 665,670,680
670   IF (ISC(IL,2).EQ.JROW) GO TO 665
      AA1 = AA1 - SD(IL,1)*XX(ISC(IL,2))
      GO TO 665
680   IU = IU + 1
      IF (IU.GT.UF) GO TO 700
      IF (ISR(IU,1)-JROW) 680,690,700
690   IF (ISC(IU,1).EQ.JROW) GO TO 680
      AA1 = AA1 - SD(IU,2)*XX(ISC(IU,1))
      GO TO 680
700   IB = IB + 1
      IF (IB.GT.LB) GO TO 710
      IF (IBR(IB)-JROW) 700,705,710
705   AA1 = AA1 + BB(IB)
710   XX(JROW) = AA1/SD(DS-1+JROW,2)
      GO TO 600
1     FORMAT (8I10)
2     FORMAT (I6,D42.32,2I4)
3     FORMAT (D42.32)
4     FORMAT (I6,D42.32,I4)
5     FORMAT (/' ITERATION',I3,' - RMS RESIDUAL =',D26.18/)
999   RETURN
      END
```

APPENDIX D

MATHIEU STABILITY

The existence of periodic coefficients describing gear mesh stiffness creates the potential for Mathieu instability. The well known standard form of the scalar Matthieu equation is

$$\ddot{y} + (a - 2q \cos 2t)\, y = 0 \tag{D.1}$$

Consider Figure (2.2) as a simplified model of a rotor system with one gear mesh where J represents the system inertia and k(t) the system flexibility. If the mesh is much more flexible than the shafting then Equation (1.3) approximates the system stiffness. Governing the homogeneous solution of rotational response, $\theta(t)$, then is

$$J\ddot{\theta}(t) + (k^{o} + \sum_{n=1}^{4} k^{n\sigma c} \cos n\sigma t + k^{n\sigma s} \sin n\sigma t))\, \theta(t) = 0 \tag{D.2}$$

For the purpose of illustration allow the series representation of stiffness to be truncated further, yielding

$$J\ddot{\theta}(t) + (k^{o} + k^{\sigma c} \cos \sigma t + k^{\sigma s} \sin \sigma t)\, \theta(t) = 0 \tag{D.3}$$

A shift in phase and in the time variable allows Equation (D.3) to be expressed in the standard form on the Mathieu Equation (D.1), where

$y = y\ (\tau) = \theta(t)$

$\tau = \sigma t/2$

$a = 4k^{o}/\ (J\sigma)^2$

$q = -2\ \sqrt{k^{\sigma c^2} + k^{\sigma s^2}}\ /(J\sigma^2)$

The linear, periodic coefficient Mathieu Equation does not possess a strictly periodic solution; nor is its solution expressible in closed form. The Mathieu stability graph of Figure D.1 depicts regions of stable response as well as narrow regions of instability where the response to an arbitrary force or perturbation grows in time without bound. For the gear pair of Figure 1.1, the ratio of stiffnesses (mean/time-varying) is about 3/1. The corresponding ratio of a/q is 6. Hence it is immediately recognized that concern with Mathieu stability in geared systems is limited to the first quadrant of Figure D.1, and more specifically to a narrow sector between the ordinate and a line of slope ~6. In actuality shafts and bearings contribute greatly to overall system flexibility, hence the region of interest regarding Mathieu stability is even smaller than the area to the left of the a/q = 6 line.

Hence, it is not surprising that a review of literature has not revealed reports of Mathieu instability in rotor systems. While Mathieu instability is a recognized mathematical and physical phenomenon, it is believed that it is improbable that rotors will exhibit this unstable behavior.

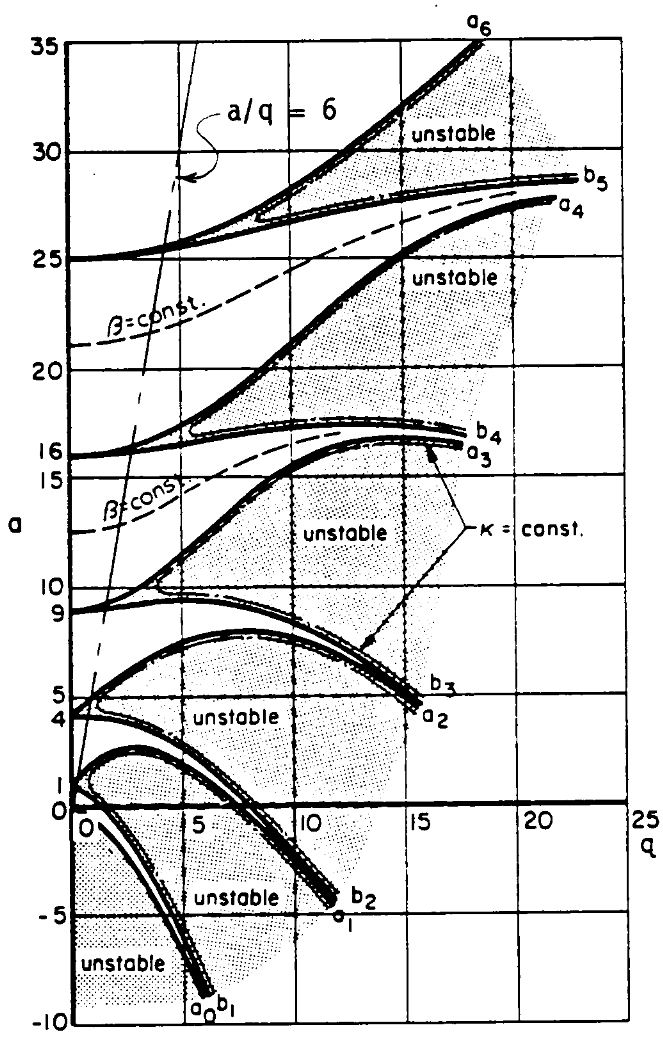

Figure D.1 Mathieu Stability Graph; Stability of Solutions to Equation (D.1). After Eisinger [29], Modified.

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AN ANALYTICAL STUDY AND COMPUTER ANALYSIS OF THREE-DIMENSIONAL, STEADY-STATE VIBRATION OF MULTISHAFT GEARED-ROTOR SYSTEMS

by

James Michael Blanding

(ABSTRACT)

A unique multifrequencied transfer matrix method performs three-dimensional harmonic, steady-state response calculations on geared-rotor systems. The full six degrees-of-freedom method includes physical branching to accommodate multiple shafting and frequency branching to simultaneously accommodate multiple frequencies and their interdependence resulting from time-varying mesh stiffness.

Areas of emphasis include development of a modified transfer matrix to handle multiple frequencies and shafting; description of the time-varying stiffness tensor representing the involute spur gear mesh based on bending, shear, compression, and local contact deformation; development of the mesh transfer matrix; development of an automatic system solver to allow the engineer to analyze systems of arbitrary construction; and the development of a matrix solver to efficiently handle large systems.

A computer analysis demonstrates the significance of terms included in the stiffness evaluation as compared with less rigorous treatment in the literature. An analytical example problem

illustrates the automated model generation through complete rotor system dynamic response analysis produced by the current work with special attention to the significance of parametric excitation due to the gear mesh.