Fatigue Analysis and Reconstruction of Helicopter Load Spectra

by

Abolhassan K. Khosrovaneh

Dissertation submitted to the Faculty of the

Virginia Polytechnic Institute and State University

in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

in

Engineering Mechanics

APPROVED:

N. E. Dowling, Chairman

C. W. Smith

R. A. Heller

M. P. Singh

R. H. Plaut

February 1989

Blacksburg, Virginia

Fatigue Analysis and Reconstruction of Helicopter Load Spectra

by

Abolhassan K. Khosrovaneh

N. E. Dowling, Chairman

Engineering Mechanics

(ABSTRACT)

Helicopter load histories applied to notched metal samples are taken as examples, and their fatigue lives are predicted by using a simplified version of the local strain approach. This simplified method requires an input load history in the form of the rain-flow matrix and places bounds on the fatigue life. A peak-valley reconstructed history is generated based on the standard spectrum Helix. A second history studied is a more irregular one based on actual flight data. It is used to generate three reconstructed histories based on three principles: peak-valley, to-from, and rain-flow. Emphasis is given to the rain-flow reconstruction method, and different reconstruction methods based on rain-flow cycle counting are presented. Life predictions are presented for all of the above cases, and the comparison with test data and other considerations suggest that the most promising reconstruction approach is one based on rain-flow cycle counting. Finally, a method is presented which reconstructs a history with the same rain-flow cycles and also the same distribution of relative time increments between adjacent peaks and valleys. This reconstructed history gives the same fatigue life as the original history.

Acknowledgements

I would like to express my sincere appreciation and respect to my committee chairman, Dr. N. E. Dowling, for his invaluable guidance, suggestions and assistance in conducting this investigation and throughout my graduate work. Furthermore, a sincere note of thanks is also extended to the other members of my advisory committee, Prof. C. W. Smith, Dr. R. A. Heller, Dr. M. P. Singh, and Dr. R. Plaut, for their counsel and guidance throughout this investigation.

I wish to express my gratitude to the U.S. Army Aviation Applied Technology Directorate, Ft. Eustis, Virginia, and to the U.S. Army Aerostructues Directorate at NASA Langley, Hampton, Virginia, for financing this investigation. I am also grateful to my friend, Dr. S. Thangjitham, for his valuable suggestions during the course of this investigation. Special thanks is also extended to my friends, and for their encouragement and support during my graduate work.

Special appreciation goes to my parents, I will be forever thankful for the love, encouragement, confidence, and financial support they provided throughout all phases of my education. Special thanks is also extended to my brothers, and for their love and encouragement. This dissertation is dedicated to my parents.

Table of Contents

List of Illustrations

List of Tables

1.0 Introduction

The study of lengthy irregular time histories are of interest in analysis of fatigue damage. In order to make the data manageable for future use, it is desirable to summarize this lengthy irregular load history by a concise description. This concise description is needed to provide sufficient information to estimate fatigue life and also to reconstruct a load history similar to the original one, in the form of time sequence, which can then be used in component testing. The description and reconstruction of load spectra is of interest to the helicopter industry. These are the means of interpreting flight loads data so that the life calculations and component tests can later be done which contain considerable detailed information from the actual loads data. The goal of this work is to develop a fatigue life prediction method and a reconstruction method using a concise description of the original history.

Fatigue lives are predicted by a variety of methods, including stress-life curves, local strain approach, and fracture mechanics. In this work, fatigue life is predicted based on a simplified method. This method is based on the local strain approach and has the advantage that it requires the load history in only the reduced form of range, mean, and cycles from the rain-flow cycle counting method. This reduced description is in the form of a matrix and is called a rain-flow matrix. The local strain approach predicts fatigue life based on the local strain at the notch, and the strain-life curve. Note that one area of importance in predicting fatigue life is the sequence effects that can result from occasional high loads. These introduce local residual stresses that act as mean stresses and therefore affect the fatigue life. The local strain approach [1-4] permits these local mean stress effects on life to be quantitatively evaluated and predicted.

In order to predict fatigue lives based on a cumulative damage type approach, a cycle counting method is required. Dowling [5] showed that the preferred cycle counting method for this purpose is the rain-flow method, and this method has become widely accepted. In the rain-flow cycle counting method, two pairs of adjacent ranges which are joined by a peak and valley are compared in each step. If the first range is less than or equal to the second, a cycle is counted. This procedure continues until all the ranges in the history are covered. The rain-flow method cannot be misled by any synthetic loading sequence, and it always counts the cycles in a physically realistic manner. This is due to the fact that closed stress-strain hysteresis loops are counted as cycles and these are representative of fatigue damaging events. By using the rain-flow cycle counting method, a service load history can be reduced to a convenient compact form. The compact description is in the form of a matrix giving combinations of range and mean, or peak and valley values. Although some detail is lost, such a matrix can be used with the aid of the local strain approach to place upper and lower bounds [6,7] on the life that would result from analysis of the original, unsummarized history.

Various methods of developing concise descriptions of load spectra and then regenerating equivalent load spectra are investigated. The methods considered include: peak-valley cycle counting method [7], the to-from matrix method [8-10] and the rain-flow reconstruction method. Note that the rain-flow reconstruction method consists of taking the results of the rain-flow cycles of the original history and regenerating a new history based on the above results, which gives the same rain-flow cycles as the original history.

In this work the emphasis is given to the rain-flow reconstruction method. This method has an advantage over the other methods, because it gives the same rain-flow cycle count as the original history. Note that since the rain-flow cycles are the same, therefore the life is similar.

Four methods of reconstruction based on the rain-flow cycle counting method are developed. All these methods give the same rain-flow cycle as the original history. Two of the methods are based on a two dimensional matrix (peak-valley) or (valley-peak) [11-13]. Further pursuit of the above approaches are the development of two new procedures. One of these procedures is based on a three dimensional matrix (peak-valley-peak), or (valley-peak-valley), while in the other procedure both rain-flow cycles and distribution of relative times between peaks and valleys of the original history are preserved. This task is achieved by adding another three dimensional matrix containing peak-valley-time of the original history to the rain-flow matrix.

Finally, all of the work and results on analysis and reconstruction of load histories, and the obtained experimental results, are drawn together and evaluated, and significant conclusions are reached.

2.0 Literature Review

In this work the literature is discussed in two different sections. In one section the life prediction methods for the load histories are surveyed, while in the other section the load reconstruction methods from a concise description are surveyed.

2.1 Life Prediction for the Load Histories

The problem of fatigue life prediction under cyclic loading is known in the literature as the cumulative damage problem. Fatigue life prediction under variable cyclic loading can be carried out using a variety of cumulative damage methods such as the Palmgren-Miner (P-M) rule [14,15], the double linear damage rule [16], the Hashin and Rotem rule [17], the Miller and Zachariah rule [18], the Kujawski rule [19], etc. [19]. Schutz [20] compared most of the cumulative damage methods, and concluded that the best available method is the P-M rule, and could not be improved. Dowling [5] showed that the P-M rule can be used successfully if it is used with care in three areas, namely: cycle counting, the mean stress effect, and the overstrain effect. He concluded that by considering these three areas the use of a more complex method can be avoided. The P-M rule states that fatigue failure is expected when the summation of the life fraction reaches unity. However, it was shown [5] that the summation of the life fraction is less than unity when the cycle sequence is from a high level to a low level, and greater than unity for an opposite case. Ref. [5] contains a literature survey regarding the above conditions.

Moreover, the P-M rule must be used with care in three areas [4,5]. The first area of concern requires proper handling of cycle counting. The cycle counting approach simply is a tool of breaking the load history down into individual events. Figure 2.1 describes six relatively simple methods of cycle counting that have been used [5]. A more detailed explanation of the above methods are given in [21].
These cycle counting methods may produce a major error in life. A more advanced method (rain-flow) was developed by Mitsuishi and Endo [22], experimentally verified by Dowling [5], and introduced to the Europe and U.S. The rain-flow cycle counting method is the best method available because it always count the major cycles arising from the variation of the mean level, as well as the low level cycles; hence this method is widely used as the preferred cycle counting method. This method is illustrated in Fig. 2.2. A new definition of the rain-flow method has been developed [21,23] for computer use, and this new method is used in this work. In this method , two pairs of adjacent ranges, which are joined by a peak and valley, are compared. If the first range is less than or equal to the second, a cycle is counted and the corresponding peak and valley are discarded. This procedure continues until all the peaks and valleys in the history are covered. This method is explained in detail in Chapter 3.

Recently Glinka et al. [24] also developed an algorithm for very long time histories, while Rychlik [25] developed a new definition of the rain-flow cycle counting method based on a statistical approach. Lindgren [26] also developed a new definition for a rain-flow cycle counting method. This method calculates the rain-flow cycle distribution based on a Markov approximation of local maxima and minima.

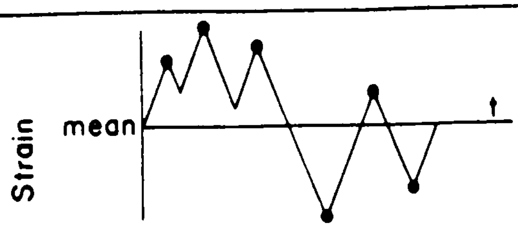
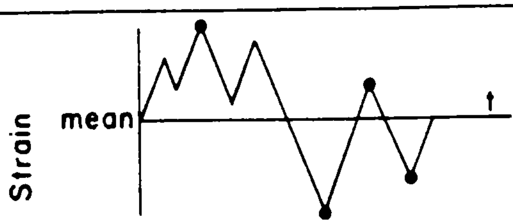
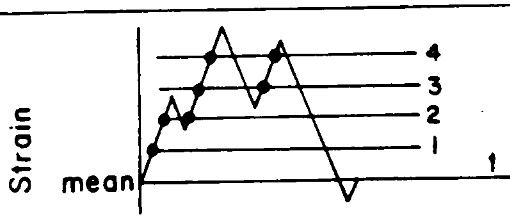
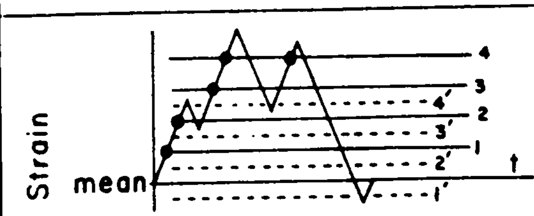
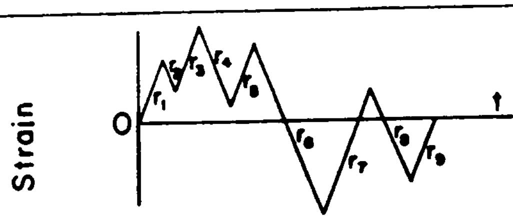
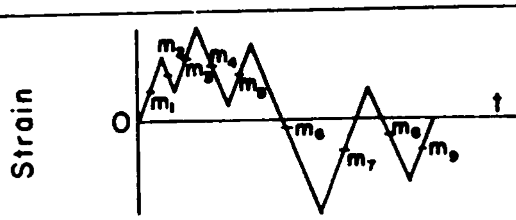
Name	Example	Description
Peak		All maximums above the mean and all minimums below the mean are counted.
Mean crossing peak		Only the largest peak between successive crossings of the mean is counted.
Level crossing		All positive slope level crossings above the mean, and negative slope level crossings below the mean, are counted.
Fatigue - meter		Similar to level crossing except that only one count is made between successive crossings of a lower level associated with each counting level.
Range		Each range, i.e. the difference between successive peak values, is counted as 1/2 cycle, the amplitude of which is half the range.
Range - mean		Ranges are counted as above and the mean value of each range is also considered.

Figure 2.1. Example of Simple Cycle Counting Methods [5].

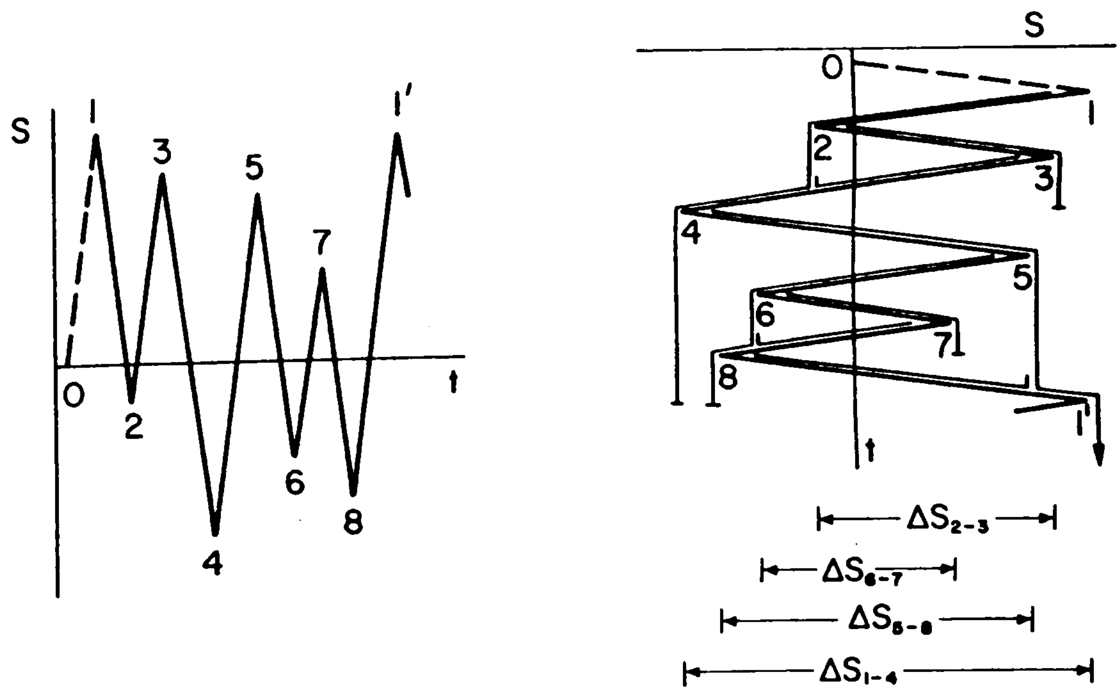


Figure 2.2. Illustration of Rain-flow Cycle Counting (Traditional Method) [4].

The second area of concern in using the P-M rule is the effect of the mean stresses. Fatigue lives are predicted by a variety of methods, including stress-life curves [4,27,28], and the local strain approach [1-4]. Stress life curves is the traditional method and is based primarily on linear-elastic stress analysis [4,27,28]. Note that in this method the effect of plasticity and mean stresses are not included; therefore there was a need for the development of a more advanced method, such as the local strain approach. Figure 2.3 shows a compressive nominal stress and a tensile local mean stress that is caused by the sequence effects. Accounting for mean stress effects based on nominal stress, S, produces a correction in the wrong direction [21].

The local strain approach [1-4,29,30] permits local mean stress effects on life to be quantitatively evaluated and predicted. This approach predicts fatigue life using the local notch stress-strain history [29,30]. The local notch stress-strain history is obtained by converting the load history using the cyclic stress-strain curve and the nominal stress-strain curve. The determination and use of the cyclic stress-strain curve and the nominal stress-strain curve are discussed in detail in [31] and [32], respectively. Note that the modeling of the material's stress-strain behavior is the key to estimating the local stress-strain histories. This modeling behavior is explained in detail in [1,33], and was confirmed experimentally in [4,29,30,34]. The local strain approach was also confirmed experimentally in [1-4,29]. A complete explanation of the local strain approach is given in [1-4,29]. This method has a disadvantage in that it requires the history in full length; therefore there is a need for a method that only requires the history in a concise form.

A new method called the simplified method [6,7], based on the local strain approach, is completed and used in this work. This simplified method has an advantage in that it only requires the load history in the reduced form of range, mean, and cycles from the rain-flow cycle counting method. By using this information, and with the aid of the local strain approach, upper and lower bounds can be placed on the life that would result from analysis of the original, unsummarized history. This method is briefly described in a paper by Conle [6]; however, no

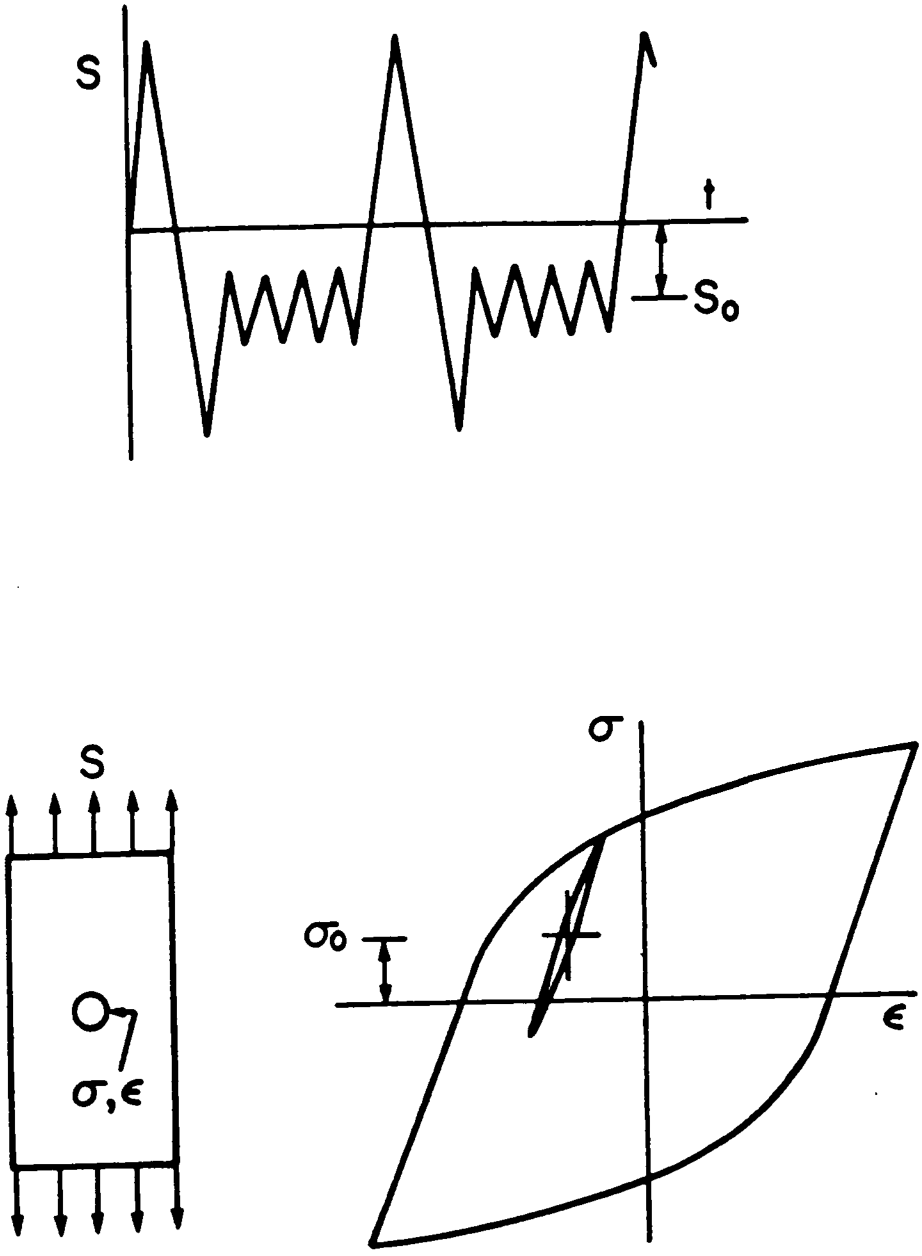


Figure 2.3. Comparison of Nominal and Local Mean Stresses: The local mean stress is strongly affected by the sequence of prior loading [27].

detailed explanation of the procedure is given. The first description in detail of this method was published by Dowling and Khosrovaneh [7].

Different methods are available of accounting for mean stresses [35,36] in the local strain approach. Comparison of the calculated lives using all these correction methods gave fair agreement with the test data [5] However,in this work the effect of the mean stresses is considered using the rule of Morrow [35].

Finally, the third area of concern in using the P-M rule is the overstrain effect. The term overstrain is defined as the application of a few very high loads among a majority of lower level loads. This effect caused by the higher stresses needs to be considered since it increases the damage done by the lower stress levels [7,37]. This effect can be included in life predictions by simply basing life predictions on data for specimens that have been prestrained [5,38]. An example of such a strain-life curve for prestrain material [39] is shown in Fig. 2.4. The literature also reveals that several investigators [39-41] showed that the application of initial overstrains reduces the fatigue life and also causes a reduction in the fatigue or endurance limit, while others [2,41,42] observed that this effect was magnified by the application of periodic overstrains.

To summarize, it has been concluded that, by considering the three areas that are explained above, the P-M rule can be used successfully, and a more complex method can be avoided.

2.2 History Reconstruction Methods

A variety of methods exist for summarizing load histories, such as power spectrum density (PSD) [43], to-from matrix [8-10] and cycle counting [5,21,41]. The goal of this work is to use

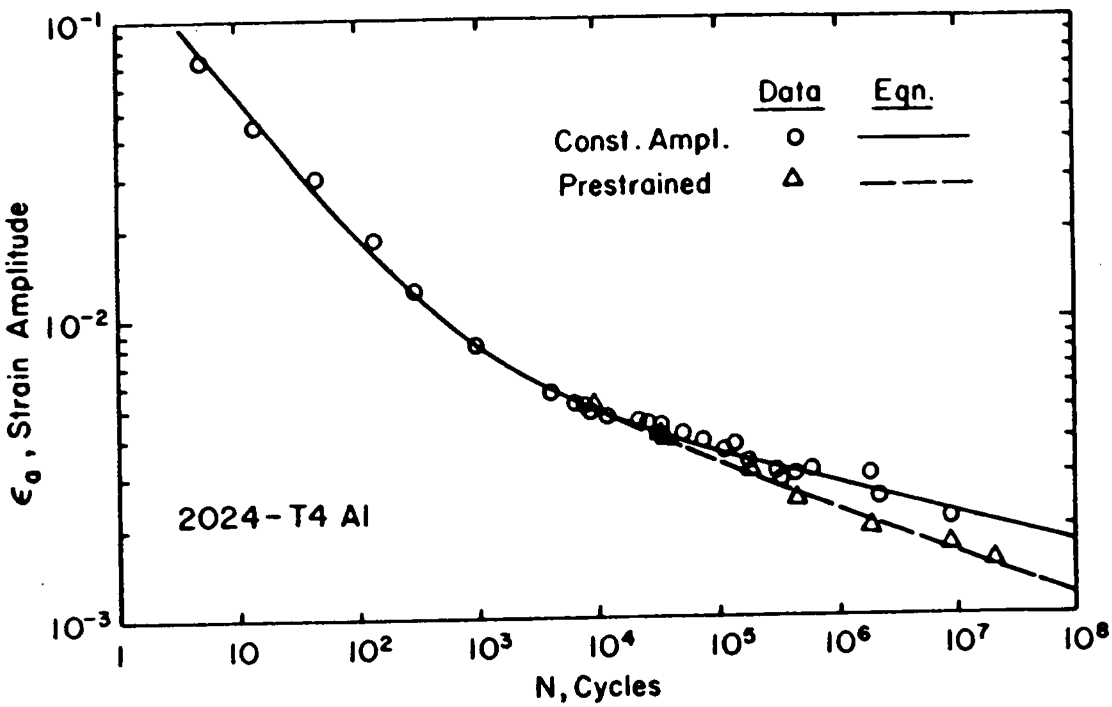


Figure 2.4. Strain Life Data and Curves for 2024-T4 Aluminum.

the load histories in the summary forms and to reconstruct histories that produce similar fatigue lives.

In the power spectrum density method, Wirsching [43] reconstructed a history based on the statistical definitions such as irregularity factor [44] and number of zero crossings [44]. As explained, this method has an advantage of having a probabilistic basis, but it has a disadvantage of handling loads with a deterministic mean variation. Note that this method may not produce the same life as the original history.

The to-from reconstruction method was developed by Haibach et al. [8]. This method [8-9] assumes that the original history is a stationary Guassian process. It is further can be approximately described as a Markovian sequence of peaks and valleys. This method [10] also has the same advantage and disadvantage as the PSD method. Also, it is difficult to set a limitation on the basic assumption for the to-from matrix of a Guassian random process [10,13].

Finally, reconstruction based on cycle counting can be done using a variety of methods, such as the level crossing histogram [41,45-47], peak-valley [7,13,41], and the rain-flow method [11-13,41]. In the past, the most popular reconstruction method, given the level crossing histogram, was the programmed step test. This method was developed by Gassner [47] and is described in detail in [41]. To run this test on a specimen or component, the cumulative occurrence or the level crossing histogram is divided into a number of levels and the corresponding cycle numbers are divided in blocks of first increasing and then decreasing amplitude. The literature reveals that the programmed step test gives lives which are longer than the original lives [41,48-50]; therefore, regardless of its popularity, this method is not an ideal reconstruction method.

Conle [41], in his literature review, concludes that the probable reason for life prolongation under a many cycle block program loading is the delay of the overstrain effect of the large cycle. A further problem can also be created by uncontrolled generation of residual mean stresses during the change from large amplitude blocks to smaller amplitude blocks. Also, the condition of large amplitude blocks as to whether they end on tensile or compressive half cycles cannot be determined by the level crossing summary; therefore, no account can be taken of that effect during reconstruction. Note that depending on whether the large block was ended on a compressive or tensile cycle, different residual stress conditions can be induced in the subsequent smaller level block. The compressive residuals extend the fatigue life, while tensile residuals shorten it [39]. Conle [41] concluded that an extensive testing program is required to determine the effect of the residual stresses and also events concerning the large blocks.

Finally, several investigations [5,45-47] have found that significant counting errors can occur when the level crossing count method is used to reconstruct a history using the program test. The errors have been well defined in Ref. [5].

Peak-valley [7,13,41,50] is perhaps the simplest possible form of reconstruction. It has been found [13] that this method produces histories which have lives that are excessively conservative. This method is explained in detail in Chapter 5. In this work emphasis is given to the rain-flow reconstruction method. Conle et al. [41,50] performed a rain-flow reconstruction in a quite sophisticated form which attempts to reproduce the local mean stresses from the original history. Conle concluded that this method is possible to achieve, but too complex for wide use. In this work a different procedure is used, and four methods of reconstruction based on the rain-flow cycle counting are developed. All these methods give the same rain-flow cycle as the original history; therefore, the lives are identical. Two of the methods are based on a two dimensional matrix (peak-valley). The literature reveals that ten-Haven [11] and Perrett [12] also did some work on the above subjects, but little detail is available on their methods. A description in detail of this method was published by Khosrovaneh and Dowling [13].

Further pursuit of the above approaches is the development of two new procedures. One of these procedures is based on a three dimensional matrix (peak-valley-peak). The literature reveals no information on this method. In the other method, both rain-flow cycles and distribution of relative time increments between peaks and valleys are preserved. An intensive literature search has been done for this method in order to determine the statistical relationship between each peak and valley and the intermediate data points. The literature reveals no information on the above subject; however, valuable information is gathered to accomplish the above task using a different direction.

Rice [44] found the statistical distribution of peaks and valleys in the general case, and it has also been shown that when a load history has a narrow frequency spectrum, the peaks and valleys are distributed according to the Rayleigh distribution [51]. Cartwright [52] found that for a narrow frequency spectrum the range is distributed according to the Rayleigh distribution. Cartwright also came to the conclusion that the statistical distribution of a range is very difficult to determine in the general case. However, Leybold [53] found that the distribution ranges could be approximated by the sum of a Rayleigh distribution and a normal distribution. He also found that the distribution of means could be approximated by a normal distribution. Though the distribution of ranges and means (peaks and valleys) in the load history can theoretically be computed from the power spectrum [54], such a computation is very difficult; however, Leybold's approximate method eases the computation. Amini and Trifunac [55] developed a method to determine the expected value of the local peak using [52]. In addition to the described technical papers, there are some excellent books that cover these subjects. Lin [51] and Newland [56] are good examples of books that provide some introductory discussion on these subjects.

3.0 Fatigue Life Prediction

3.1 Introduction

As indicated in the previous Chapter, it is desirable to predict the fatigue life using a simplified method. This simplified method is based on the local strain and has the advantages that it only requires the original history in a concise form. This concise form is in the form of a matrix that is obtained from the rain-flow cycle counting of the original history. Using this matrix with the aid of the local strain approach, upper and lower bounds can be placed on the life that would result from the analysis of the original, unsummarized history [6,7]. After calculating the lives for all the cycles, then the final step is to apply the P-M rule. This rule states that the fatigue failure occurs when the summation of life fraction reaches unity [14,15]. The successful use of this rule requires proper handling of cycle counting, local notch mean stress effects, and overstrain effects. In this paper, the P-M rule will be used, and the above three complexities will be included in life calculation. These three complexities are explained in detail below.

Figure 3.1 shows the obvious cycles at stress range ΔS_1 and ΔS_2 plus a major cycle ΔS_3. A cycle counting method is required to count all the cycles including the major ones. Otherwise, for example, in this case the third term shown in brackets (major cycle) is missing. A drastic error would result in the predicted life if the first two terms in the P-M equation are small compared to the third term. Therefore, a correct cycle counting method such as the rain-flow is needed. The rain-flow cycle counting method includes the major cycles arising from the effect of the mean variations, as well as the smaller cycles. This method is explained later in this Chapter.

The second area, mean stress effects, must correctly be determined and evaluated. The sequence effects that can result from occasional high loads are very important. These introduce local residual stresses at stress raisers that act as mean stresses and therefore affect the fatigue life. Note that in the absence of a mean applied load [27], local mean stresses of either sign may exist, and the mean load and the local stress may even be opposite in sign [7,27]. This situation is shown in Fig. 2.3. By using the local strain approach [1-4] these local effects on life will be qualitatively evaluated and predicted.

Third, and finally, is the overstrain effect. This effect caused by the higher stress levels needs to be considered since it increases the damage done by the lower stress levels [7,37,39]. Figure 2.4 illustrates this effect for 2024-T4 aluminum with data from Ref. [39]. As mentioned earlier, this effect can be included in life predictions by simply basing life predictions on data for specimens that have been prestrained [7].

In the remainder of this Chapter the rain-flow cycle counting, the local strain approach and a simplified method based on the local strain approach which results in upper and lower bounds on the predicted life will be discussed.

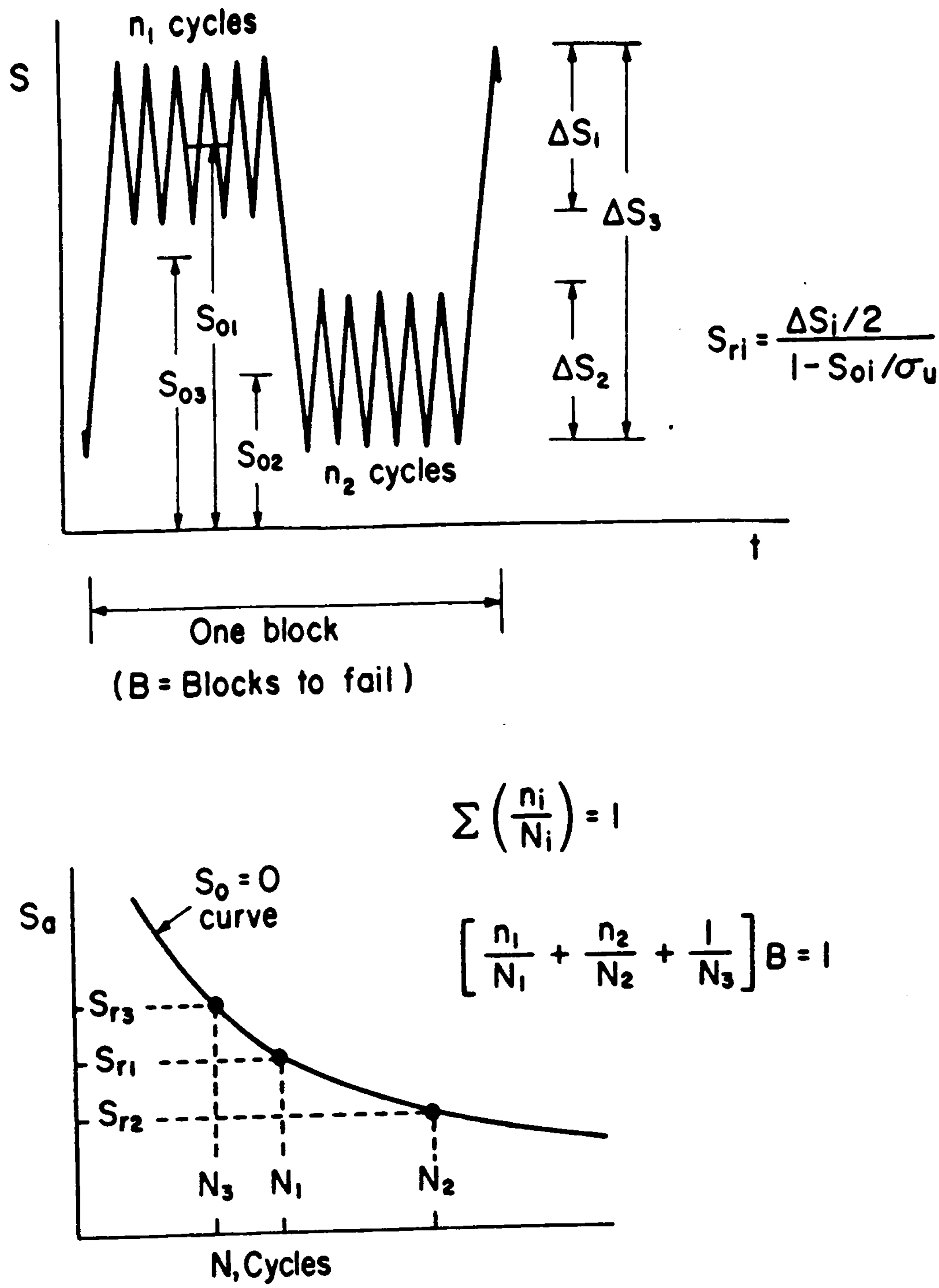


Figure 3.1. Application of the P-M Rule [27].

3.2 Rain-flow Cycle Counting

In order to predict a fatigue life based on a cumulative damage type of approach, a cycle counting method is required. The rain-flow cycle counting method has become widely accepted as the best method for analyzing an irregular time history for this purpose.

The rain-flow cycle counting method may be defined in two ways, traditional and new method. In the traditional method [5,22], the history is plotted with the time axis vertical such that the process can be considered as a sequence of roofs in which water is falling on them. Then the rain-flow paths are defined according to the following rules [5]:

1. When the path moves toward the left and down, the flow stops if it reaches opposite a reversal which is more to the right, or equal to, the one from which it started.

2. When the path moves toward the right and down, the flow stops if it reaches opposite a reversal which is more to the left, or equal to, the one from which it started.

3. The flow stops to avoid intercepting a path from a roof above.

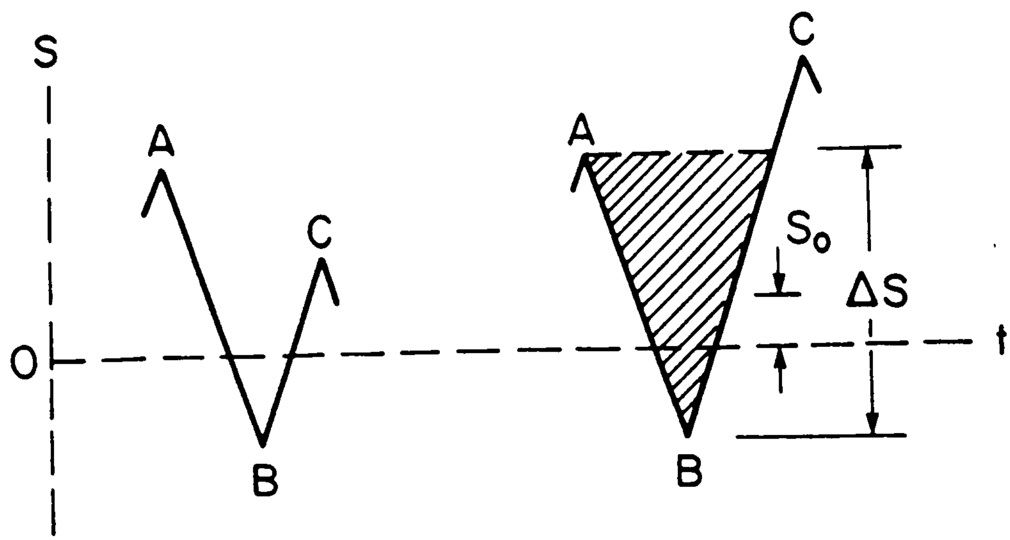
Horizontal lengths of continuous rain-flow are taken as ranges of stress cycles if paired, otherwise as ranges of half cycles. Figure 2.2(b) shows the four rain-flow cycles (2-3-2′, 6-7-6′, 5-8-5′, and 1-4-4′) that are obtained from cycle counting of the load history in Fig 2.2(a) [4].

In the new version, cycles are counted depending on the comparison of two adjacent ranges as illustrated in Fig. 3.2, which also defines range and mean of a cycle. If the first range is less than or equal to the second range, a cycle is counted. This procedure [21] is illustrated in more detail by the example in Fig. 3.3. The most extreme event in the history, that is, either the highest peak or the lowest valley, is first located. Then the history is arranged to start with the most extreme peak or valley, and all the peaks and valleys which occur prior to the maximum are moved to the end of the history. Fig 3.3(a) and 3.3(b) illustrates this.

Cycle counting then proceeds, and if a cycle is counted, this information is recorded, and its peak and valley are discarded for purposes of further cycle counting. If no cycle is counted, then one moves ahead until a count can be made. This procedure continues until all peaks and valleys in the history are covered. The results are then recorded in a matrix which gives the numbers of cycles at various combinations of range and mean or peak and valley. In order to limit the size of the matrix, rounding to discrete values of ranges and means are of course required. Table 3.1 shows the range and mean matrix for the example of Fig. 3.3.

3.3 Local Strain Approach

This approach assumes that fatigue life is controlled primarily by the notch surface strain, and considers plasticity and mean stress effects in a fairly complete manner. The various strains and stresses of interest may be either estimated or measured. In order to measure the various strains and stresses, consider Fig. 3.4(a) which shows a constant amplitude loading, S, vs. time, that is applied to a notch member until it fails. The local strain at the notch can be measured by bonding a strain gage at the notch where the strains are highest [1-4]. Fig. 3.4(c) illustrates such a case. Now the next step is to determine the local notch stress histories. Note that the local notch stress histories , as for the situation of Fig. 3.4, cannot be directly measured by any practical experimental method available today. However, if the notch member is assumed to be thin enough to approximate plane stress , then the measured strain history may be enforced upon an unnotched axial specimen of the same material, and stresses necessary to do this measured. Figure 3.4(d) shows the obtained local stress- strain response.



$\overline{BC} < \overline{AB}$

No cycle

$\overline{BC} \geq \overline{AB}$

AB = cycle

<u>For cycle A - B</u>

Peak = S_A

Valley = S_B

Range = $\Delta S = S_A - S_B$

Mean = $S_0 = (S_A + S_B)/2$

Figure 3.2. **Condition for Recording an Event During Rain-flow Cycle Counting: This cycle counting method is based on the new version.**

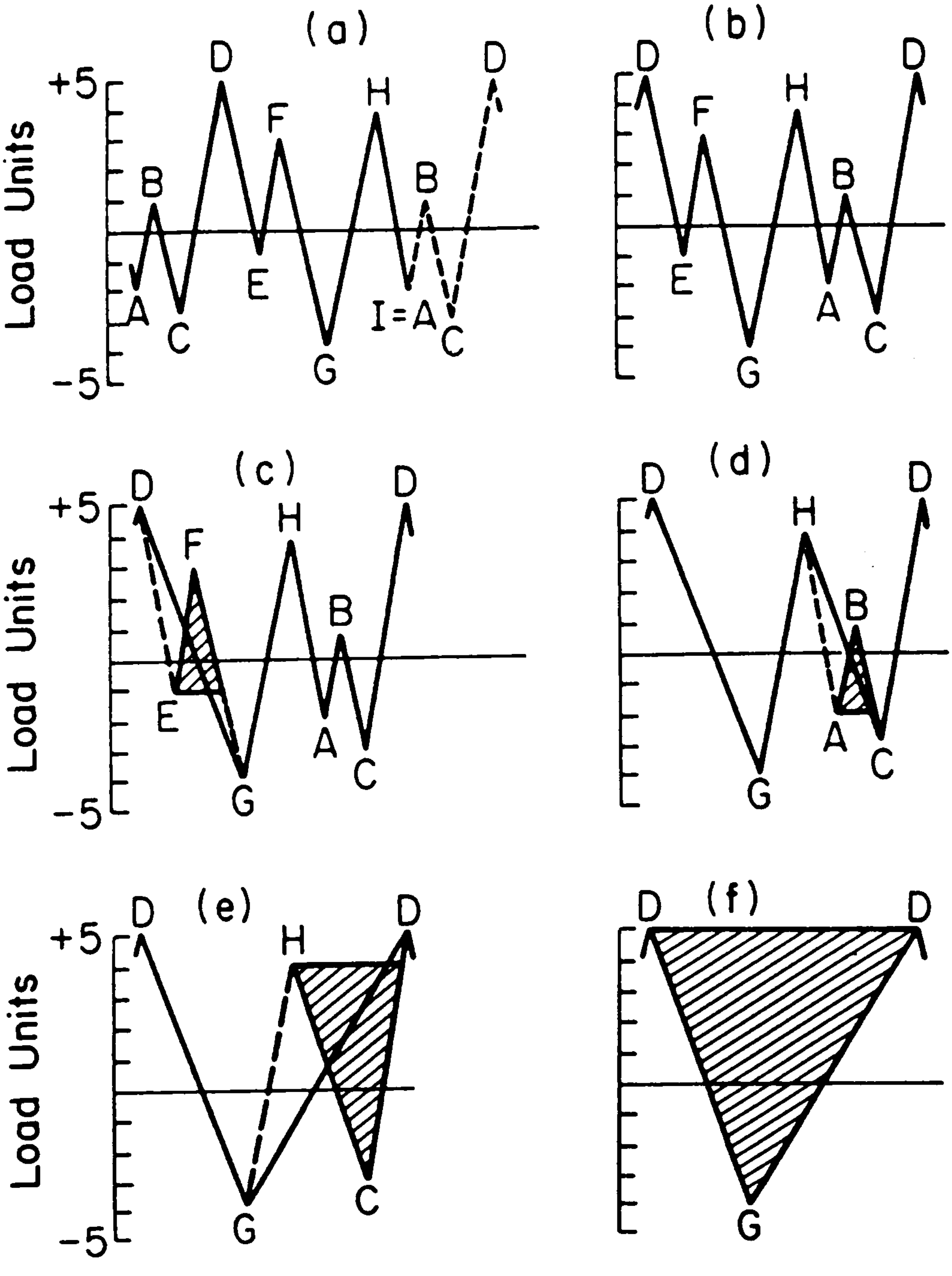


Figure 3.3. Example of Rain-flow Cycle Counting from ASTM Standards [21].

Table 3.1. Rain-Flow Matrix of the Load History in Fig. 3.3 [21]

Event	Range	Mean (units)							
	(units)	-1.5	-1.0	-0.5	0	0.5	1.0	1.5	All
--	0.5	-	-	-	-	-	-	-	-
--	1.0	-	-	-	-	-	-	-	-
--	1.5	-	-	-	-	-	-	-	-
--	2.0	-	-	-	-	-	-	-	-
--	2.5	-	-	-	-	-	-	-	-
AB	3.0	-	-	1	-	-	-	-	1
--	3.5	-	-	-	-	-	-	-	-
EF	4.0	-	-	-	-	-	1	-	1
--	4.5	-	-	-	-	-	-	-	-
--	5.0	-	-	-	-	-	-	-	-
--	5.5	-	-	-	-	-	-	-	-
--	6.0	-	-	-	-	-	-	-	-
--	6.5	-	-	-	-	-	-	-	-
HC	7.0	-	-	-	-	1	-	-	1
--	7.5	-	-	-	-	-	-	-	-
--	8.0	-	-	-	-	-	-	-	-
--	8.5	-	-	-	-	-	-	-	-
DG	9.0	-	-	-	-	1	-	-	1
--	9.5	-	-	-	-	-	-	-	-
--	10.0	-	-	-	-	-	-	-	-

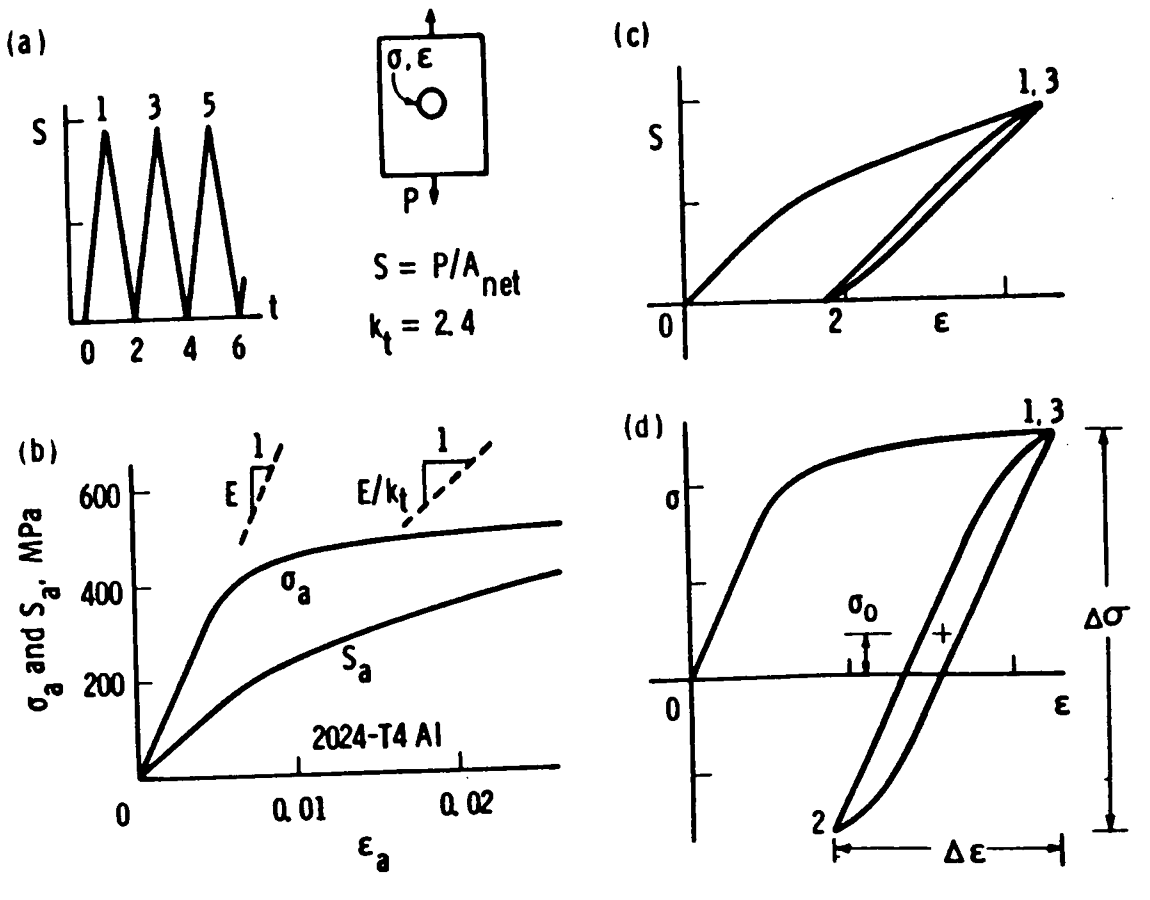


Figure 3.4. Estimation of Local Stress and Strain for Zero to Max. Loading: (a) applied load history; (b) baseline stress-strain and load-strain curves; (c) load vs. notch strain response; and (d) notch stress vs. notch strain response [4].

As explained earlier, these stresses and strains can also be estimated. Note that the local strain approach consists of replacing with analysis the experimental procedures for obtaining the various strains and stresses of interest. A cyclic stress-strain curve is needed:

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{A}\right)^{\frac{1}{s}} \tag{3.1}$$

where ε, σ are amplitudes of strain and stress, respectively, E is the elastic modulus, and A and s are material constants. Using the cyclic stress-strain curve, a curve relating nominal stress, S, and strain , ε, is also obtained. An approximate analysis based on Neuber' s rule in the form of the Eq. 3.2 is often used for this purpose:

$$\sigma\varepsilon = \frac{(k_t S)^2}{E} \tag{3.2}$$

where k_t is the elastic stress concentration factor. Using the above two curves and following the load history, a local stress-strain curve could be estimated.

3.3.1 Life Prediction by the Local Strain Approach

Life prediction by the local strain approach consists of two steps. First, the local notch stress and strain histories must be predicted, and second, the life corresponding to the local stress and strain histories must be estimated. Figure 3.5 shows the initial and most difficult step. This step is difficult because it requires specific handling of the complex nonlinearity among load, strain, and stress. In order to achieve the above task, a cyclic stress-strain curve using Eq. 3.1 is needed. Fig 3.5(b) shows this curve. Next, by employing Eq. 3.1 and with the aid of Neuber's rule in the form of Eq. 3.2, a curve relating nominal stress and strain is obtained. These curves are then used to estimate the local stress-strain response at the notch by following the load history while modeling the hysteresis loop behavior of the materials. For convenience, the history is assumed to start at the highest absolute value. The discussion of

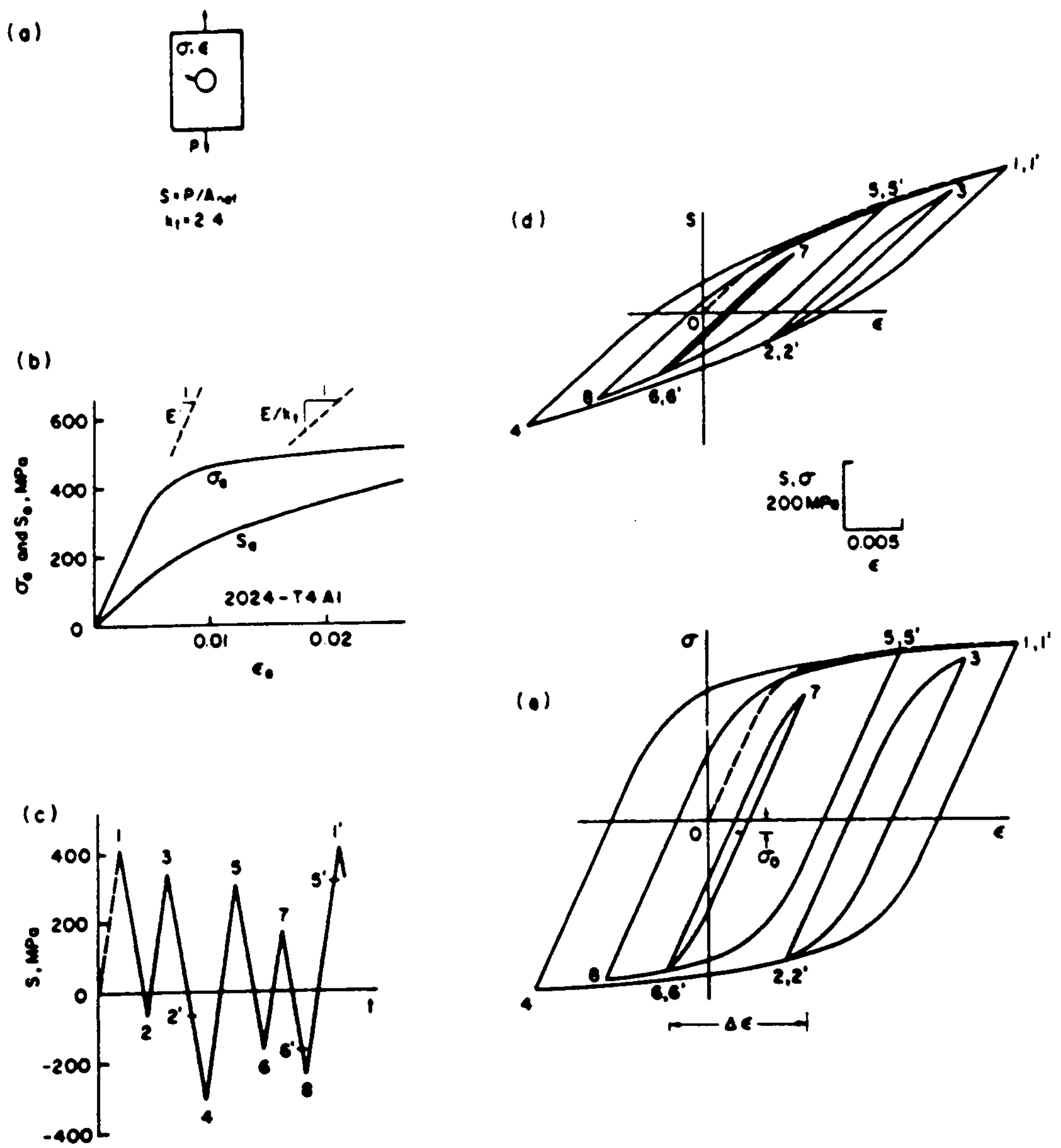


Figure 3.5. **Illustration of Local-Strain Approach for an Irregular History:** Notched member (a), having cyclic stress-strain and load-strain curves as in (b), is subjected to load history (c). The resulting load-strain response is shown in (d), and (e) is the local- notch stress strain response [2].

modeling the stress-strain behavior is explained in details in Refs [4,31]; however, here the basic idea will be discussed briefly. This modeling is based on dividing the nominal stress-strain curve and cyclic stress strain loop into a series of numbers of elements or segments (Fig. 3.6(a)), and then the response may be modeled by following a set of rules governing the use of these elements [5]. These rules are explained in Fig. 3.6.

Using the described procedure, nominal stress, S, vs. ε , and σ vs. ε could be determined as shown in 3.5(d) and (e). Also the above curves consist of a set of closed loops, such as 2-3-2', 6-7-6', 5-8-5', and 1-4-1' for the example in Fig. 3.5. Note that each closed loop is identified as a cycle, and the same cycle counting result obtained for the history in Fig. 3.5(c) by applying the rain-flow cycle counting method (see Fig. 2.2); therefore, each closed loop represents a rain-flow cycle. Each cycle now has a known strain range, $\Delta\varepsilon$, and a mean stress, σ_0 , as shown for cycle 6-7-6' in Fig 3.5(e). In order to find the life , N, a strain vs. life curve is needed in the form of Eq. 3.3:

$$\varepsilon_a = \frac{\sigma'_f}{E}(2N')^b + \varepsilon'_f(2N')^c \tag{3.3}$$

where ε_a is the strain amplitude corresponding to a closed loop, N' is the life in cycles for zero mean stress, and σ'_f ,b, ε'_f and c are additional constants. If the rule of Morrow [35] is used to account for mean stress effects, the fatigue life is given by:

$$N = N'\left(1 - \frac{\sigma_o}{\sigma'_f}\right)^{-\frac{1}{b}} \tag{3.4}$$

where σ_0 is the mean stress. The final step is then to apply the P-M rule. For a load history that is assumed to repeat till failure occurs, the rule takes the form:

$$B\left[\sum_{per\ block} \frac{n_i}{N_i}\right] = 1 \tag{3.5}$$

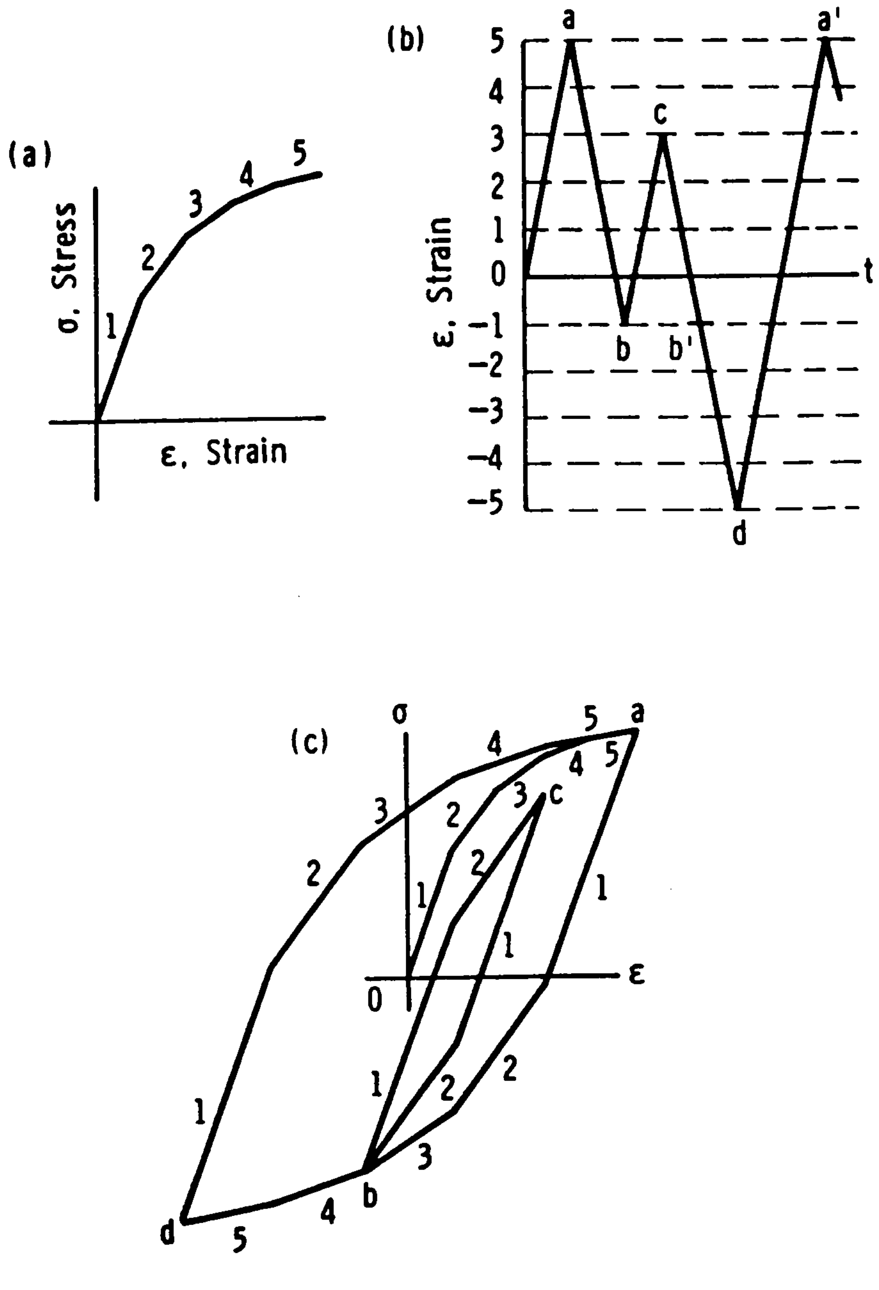


Figure 3.6. Illustration of Stress-Strain Model: (a) baseline stress-strain curve, (b) a strain vs. time history, (c) the resulting stress-strain behavior. Straight line segments, 1, 2, etc., called elements, are used as follows: (1) Initially, and after each reversal of strain direction, elements are used in order, starting with the first. (2) Each element may be used once in either direction with its original length; thereafter, the length is doubled while retaining the same shape. (3) An exception to rule (1) is that an element must be skipped if its most recent use was not in the opposite direction of its impending use [4].

where n_i is the number of occurrences per block of a cycle corresponding to life N_i, and B is the unknown number of blocks (repetitions) to failure for the irregular history.

3.3.2 Fatigue Life Calculation Neglecting Mean Stress Effects

Consider the load history in Fig. 3.3. If mean stress can be neglected, then a much simpler procedure may be used to predict the fatigue life without obtaining the stress-strain modeling. In order to achieve this task, a rain-flow cycle counting of the load history is required. The cycles are then stored in a matrix such as Table 3.1. Fatigue life can be predicted by only using the sum of each row, where each row corresponds to a discrete value of a range. The last column in Table 3.1 shows such a value. Note that in this case a simple history is used for illustration purposes; therefore, a simple rain-flow matrix is obtained. A more detailed matrix is shown in Chapter 5.

These ranges (from the rain-flow matrix), ΔS, are then used with Neuber's rule in the form of Eq. 3.2 and with the aid of the cyclic stress-strain curve in the form of Eq. 3.1 to obtain the strain amplitude, ε_a. Then the life, N, for each strain amplitude is obtained using the strain-life relationship, Eq. 3.3. An iterative solution may be obtained using a computer program to solve for ε_a. The final step is to apply the P-M rule, Eq. 3.5, to estimate the number of blocks to failure.

3.3.3 Calculation of Upper-Lower Bounds on Mean Stress Effect

The local stress as explained earlier requires a knowledge of the full sequence of the load history. However, in this approach the load history may be used in a concise form. Consider a notch member as in Fig. 3.5(a), subjected to an irregular load history , such as Fig. 3.5(c).

In order to determine the fatigue life, the first step is to determine the rain-flow matrix that contains the information on range and mean loads of the rain-flow cycles. Note that in order to predict the fatigue life neglecting mean stress effects, only the range of each cycle is required; however, it is possible to take the advantage of additional information in the rain-flow matrix, namely mean values, S_0, to place upper and lower bounds on the life as influenced by the local mean stress.

The principle behind this bounding is shown in Fig. 3.7 for one cycle, namely 6-7 from Fig. 3.5. The guiding principle is that both load-strain loop 6-7 and stress-strain loop must lie within the corresponding loop for the largest cycle in the history, namely 1-4. The load limits S_6 and S_7 are known; therefore limits can be placed on the mean strain of cycle 6-7. As shown in Fig. 3.7(a), loop 6-7 can be so far to the right that it is attached at A, or so far to the left that is attached at B. Similarly, the same line of reasoning can be applied to a load-stress loop as in Fig. 3.7(c), where loop 6-7 could be so low that it is attached at A, or so high that is attached at B. Fig. 3.7(b) shows the extreme stress-strain loops which satisfy both sets of constraints, so that these correspond to the upper and lower bounds on mean stresses σ_{0B} and σ_{0A}.

Knowing theses bounds on the mean stress for cycle 6-7 allows bounds to be placed on life, N, from Eq. 3.4. The upper bound values on life are similarly obtained for all the cycles in the history, and these are used with the P-M rule in the form of Eq. 3.5 to obtain an upper bound on life for irregular load history. The same procedure, but using the lower bound N's for each cycle, may be used to obtain a lower bound on life for the irregular history.

If there is no plastic strain during the history , the stress-strain loop for the major cycle, such as 1-4 in the example, will be reduced to a straight line, and therefore the upper and lower bounds on life are then identical. Also at high load levels, the cycle causing most of the damage may not have significant mean stresses due to the large plastic strains, and the bounds will then be close. Hence, the widest separation between the bounds is expected at the intermediate load levels. Note that if most of the damage is done by low-level cycles, then

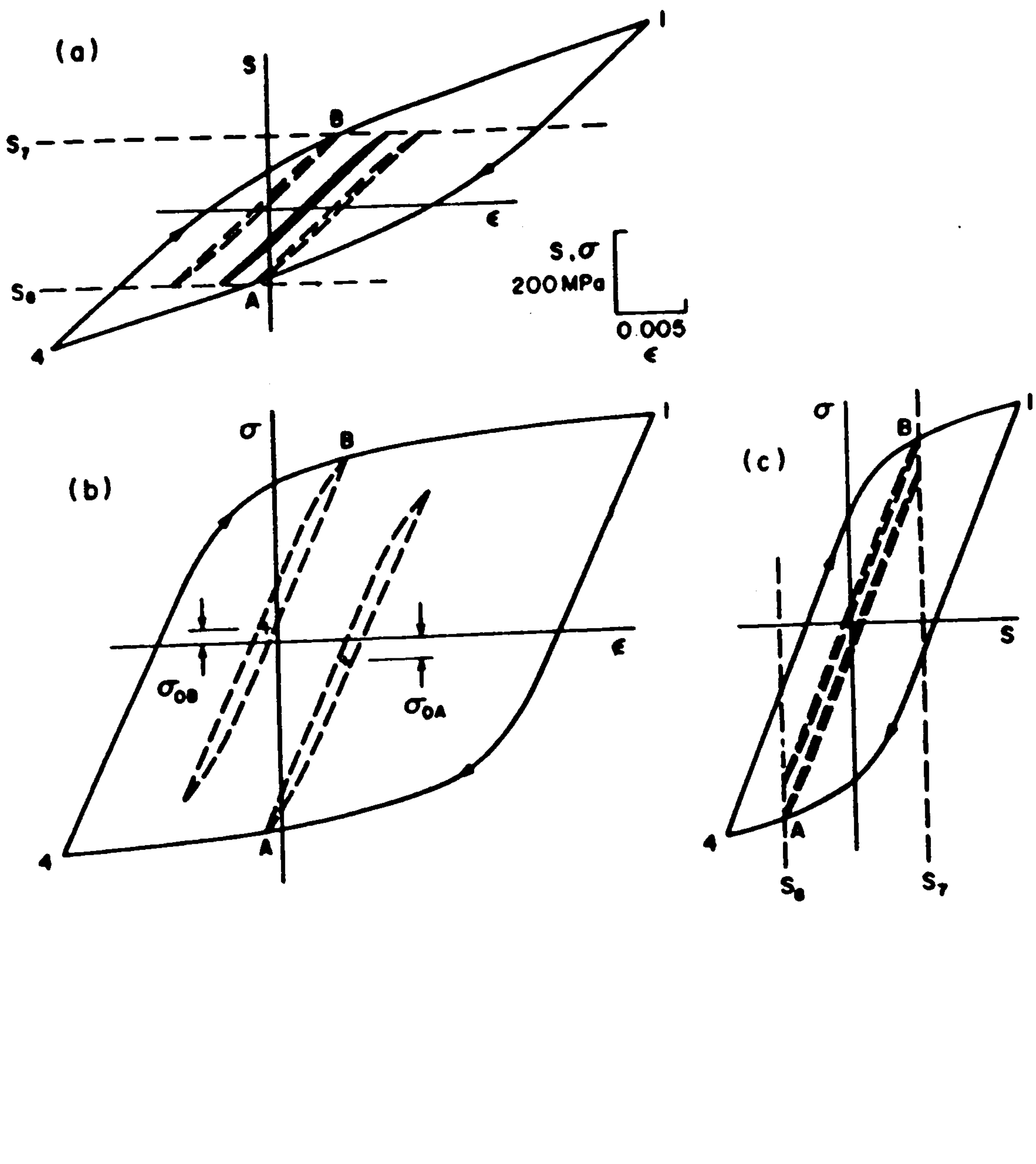


Figure 3.7. **Illustration of Placing Bounds on the Mean Stress of a Subcycle:** This illustration is based on Fig. 3.5. Note that the sequence of the applied loads is not known, and the mean stress for 6-7 must lie between σ_{0A} and σ_{0B} values shown.

the degree of separation of the bounds will be greatest. Also, if all but a negligible fraction of the damage is done by the major cycle , then the two bounds are again identical [7].

The details of calculations needed to obtain the upper and lower bounds on stresses are given in Appendix A to this dissertation. A computer program was developed and successfully used for the analysis of the load histories.

4.0 Techniques for History Reconstruction

4.1 Introduction

A variety of methods exist for summarizing load histories, such as power spectrum density (PSD) [43], the to-from matrix [8-10], and cycle counting [16]. This summarization must be complete, meaning that it must provide sufficient information for reconstructing a load history which can be used in component testing. The reconstructed histories using these methods must produce lives similar to the original histories; otherwise, these methods could not be adapted. In this work the load histories are summarized using to-from matrix and cycle counting method. The cycle counting methods used for this purpose are the peak-valley (P-V) and the rain-flow (R-F). Emphasis is given to the rain-flow reconstruction methods, and four different methods of reconstruction based on this method are developed and compared with the other reconstruction methods.

Histories are reconstructed using the above methods, and their lives are compared with the original histories and conclusions are drawn.

In the remainder of this Chapter, the peak-valley, the to-from, and the four rain-flow reconstruction methods will be discussed.

4.2 Peak-Valley Reconstruction Method

Peak-Valley reconstruction method is perhaps the simplest possible form of reconstruction. In this method the history is summarized by obtaining the distributions of peaks and valleys. In order to reconstruct a history, the highest peak and lowest valley are combined to form a cycle, and then the second highest and second lowest, and so on, are similarly combined until all events are used. Note that the sequence events of the reconstructed history are not identical to the original history, and pairings into cycles forms more severe cycles than in the original history.

Two load histories are reconstructed using the above methods, and the procedures are explained in detail in Chapter 5.

4.3 To-From Reconstruction Method

This method assumes that the original history is a stationary Guassian process. It is further assumed that the process can be approximately described as a Markovian sequence of peaks and valleys [8-10]. To apply the to-from matrix approach to a service load (or stress, etc.) history, the history is divided into discrete intervals numbered usually from 1 to 32. The transitions between adjacent peaks and valleys , or valleys and peaks are then plotted in a 32 by 32 matrix. Figure 4.2 shows a 'to-from' matrix which is obtained from the transition between adjacent peaks and valleys of Fig. 4.1. For simplicity , matrices of reduced size 7 by 7 are used in this example. The to-from matrix observes some general rules. For example, the diagonal elements in the matrix are set to zero. The reason is that the transition from one level to the same level has no physical meaning. Furthermore, due to the fact that the number of transitions into one particular level is equal to the number of transitions from that level, it follows that the sum of elements in row i is equal to the sum of elements in column i. The algorithm described in Refs [8,9] can then be used to develop a computer program to reconstruct a history from the matrix.

The sequence of events of the reconstructed and original histories using the to-from method are not identical; however, if the above assumptions apply, then the original and the reconstructed history are statistically equivalent. A load history is reconstructed using this approach and is explained in details in Chapter 5.

4.4 Rain-flow Reconstruction Method Based on a 2-D Matrix

In this method the load histories are first summarized by applying the rain-flow cycle counting method in a compact form of a 32 by 32 matrix giving combinations of peak and valley values which correspond to the rain-flow cycles. Such a matrix can be defined in two forms. In one form it can be defined so that all the values above the diagonal line are zero, which indicates that the directions of the cycles are not considered. The other form of the matrix considers the directions of the cycles and has values on both sides of the diagonal.

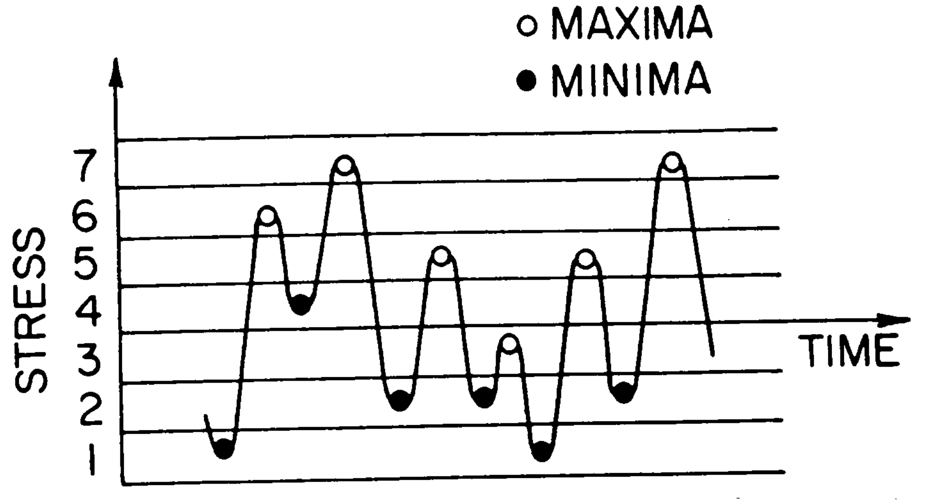


Figure 4.1. Peaks and Valleys and Transition Between Adjacent Peaks and Valleys [57].

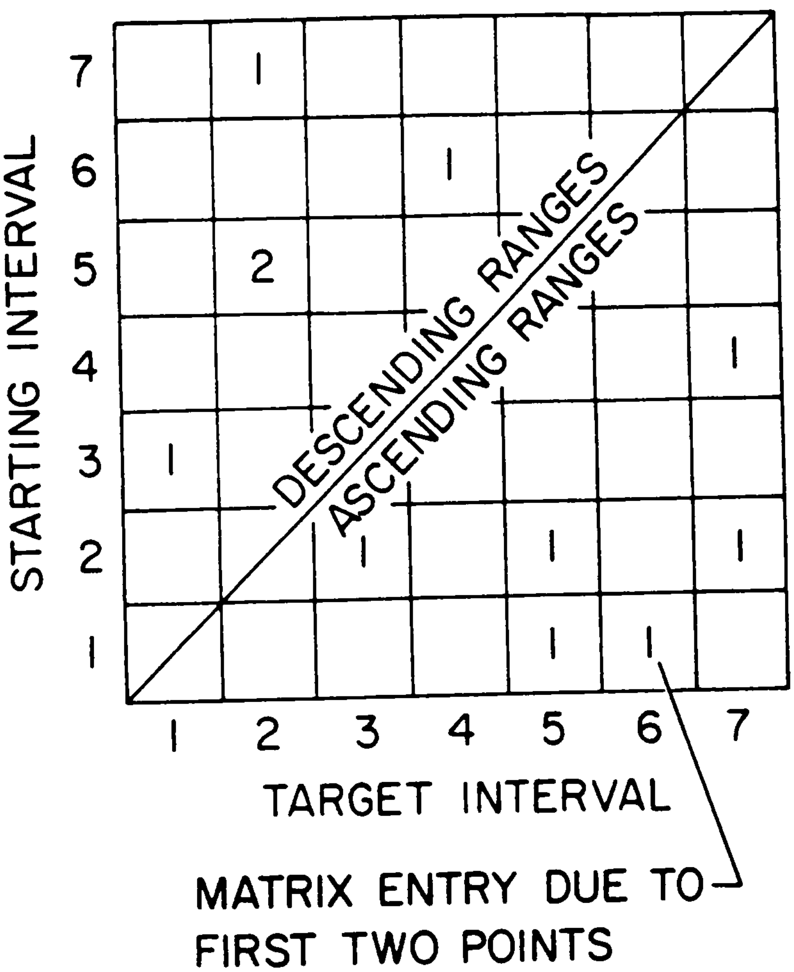


Figure 4.2. To-From Matrix for the History of Fig. 4.1 [57].

Figure 4.3(a) shows a short load history and, for simplicity, matrices of reduced size of 5 by 5 are used in this example. The corresponding rain-flow cycle counting matrices for that history are shown, both the one without consideration of the cycle directions, 4.3(b), and the one considering the cycle directions, 4.3(c). The differences between the two matrices can be seen in cycle b-c and g-h. If the directions are not considered, these cycles are identical. But if directions are considered, b-c is plotted in the matrix as 2-3, whereas g-h is plotted as 3-2.

The detailed procedures, as employed here, for reconstruction based on these two types of rain-flow matrix are explained below. Two computer programs were developed for the above methods. The computer program listing for reconstructing histories considering the directions of rain-flow cycles is presented in Appendix B.

If the first storage method is applied, that is, if the rain-flow directions are not considered, then the matrix has a triangular form. Figure 4.3(b) illustrates such a matrix. In order to reconstruct, first the largest cycle, which is in the first column and last row, is considered, and then all the columns corresponding to the same row are considered in order. Then the preceding row and corresponding columns are considered in order. This procedure continues until all the elements of the matrix are covered. The cycle can be placed within any cycle in the matrix with equal or more extreme peak and valley, that is, greater or equal row number and less or equal column number. A random location is chosen among all the possibilities, and then the partially reconstructed sequence is rearranged accordingly. The simplest reconstructed history can be formed by placing all the cycles having the same range and mean in a single location. However, if a more irregular history is desired, the number of cycles for a given peak-valley combination is divided into "n" groups, and each group is placed in a possible location randomly.

For the other type of matrix where the information retained includes the directions of the cycles, a potentially full 32 by 32 matrix is formed. The elements within the matrix are chosen in the order given by the numbers in Fig. 4.4 so that the largest cycles are employed first. In

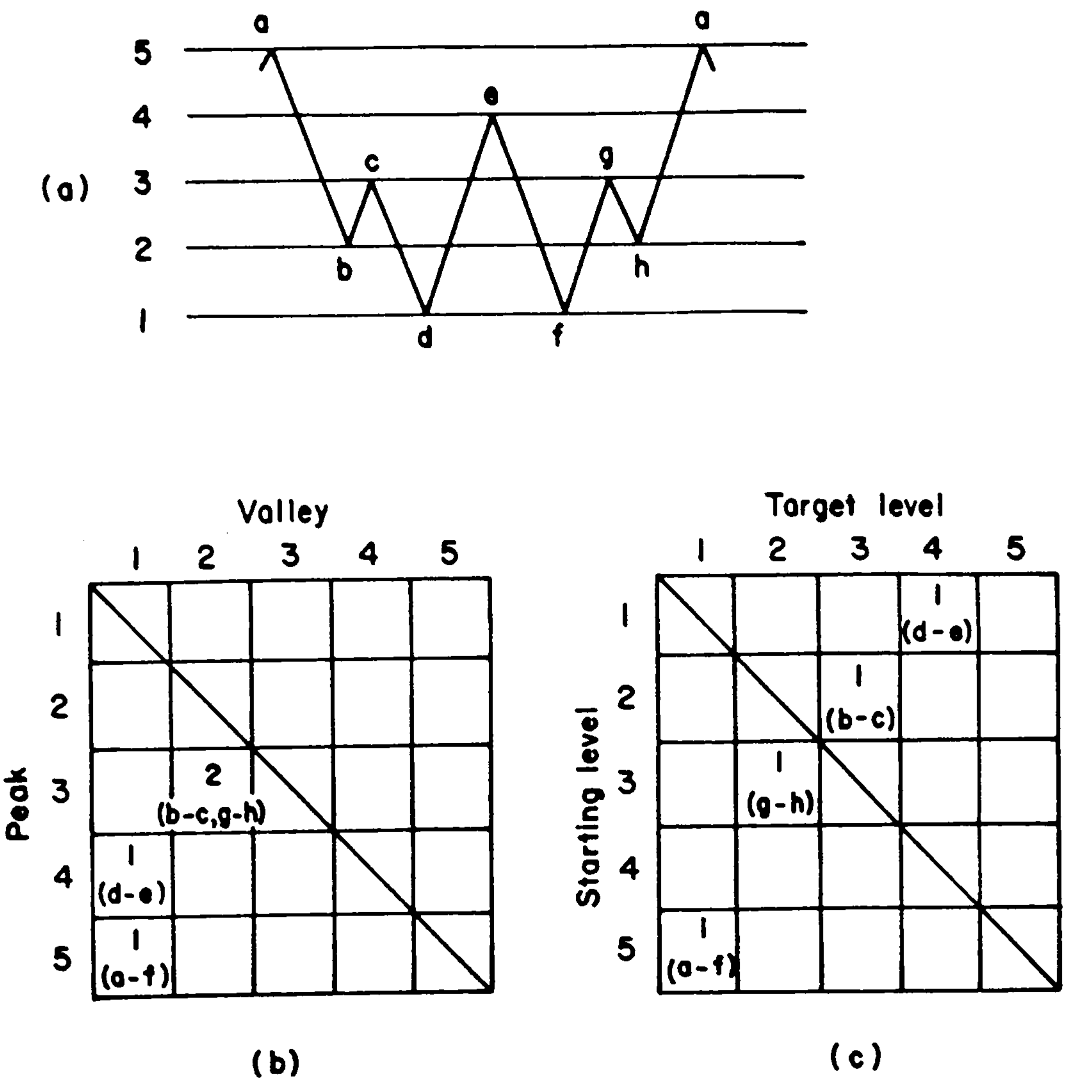


Figure 4.3. Two Forms of Matrix Def. for the Purpose of the R-F Reconstruction: (a) example history, and matrices without (b) and with (c) directions of cycles recorded.

Fig. 4.4 an 8 by 8 matrix is used for illustration purposes; the same procedure is extended for a 32 by 32 matrix. Four rules for inserting a cycle into a partially reconstructed history are illustrated in Fig. 4.5 and described below.

If the cycle that is being inserted (inserting cycle) has a greater row than column, that is, if it is ordered peak-valley, then it can be placed within any cycle (receiving cycle), provided:

1. If the receiving cycle is ordered valley-peak, that is, if it has a row less than the column, then the receiving row must be less than or equal to the inserting column, and the receiving column must be greater than or equal to the inserting row. Figure 4.5(a) illustrates this case.

2. If the receiving cycle is ordered peak-valley, that is, if it has a row greater than column, then the receiving row must be greater than or equal to the inserting row, and the receiving column must be less than or equal to the inserting column. Figure 4.5(b) illustrates this case.

On the other hand, if the inserting cycle has a greater column than row, that is, if it is ordered valley-peak, then it can be placed within any cycle, provided:

3. If the receiving cycle is ordered peak-valley, that is, if it has a row greater than the column, then the receiving row must be greater than or equal to the inserting column, and the receiving column must be less than or equal to the inserting row. Figure 4.5(c) illustrates this case.

4. If the receiving cycle is ordered valley-peak, that is, if it has a row less than column, then the receiving row must be less than or equal to the inserting row, and the receiving column must be greater than or equal to the inserting column. Figure 4.5(d) illustrates this case.

TARGET LEVEL

STARTING LEVEL	1	2	3	4	5	6	7	8
1		(43)	(31)	(21)	(17)	(10)	(5)	(2)
2	(50)		(44)	(32)	(22)	(18)	(11)	(6)
3	(37)	(51)		(45)	(33)	(23)	(19)	(12)
4	(26)	(38)	(52)		(46)	(34)	(24)	(20)
5	(13)	(37)	(39)	(53)		(47)	(35)	(25)
6	(7)	(14)	(28)	(40)	(54)		(48)	(36)
7	(3)	(8)	(15)	(29)	(41)	(55)		(49)
8	(1)	(4)	(9)	(16)	(30)	(42)	(56)	

Figure 4.4. Order of Insertion of Cycles Into the Partially Reconstructed History.

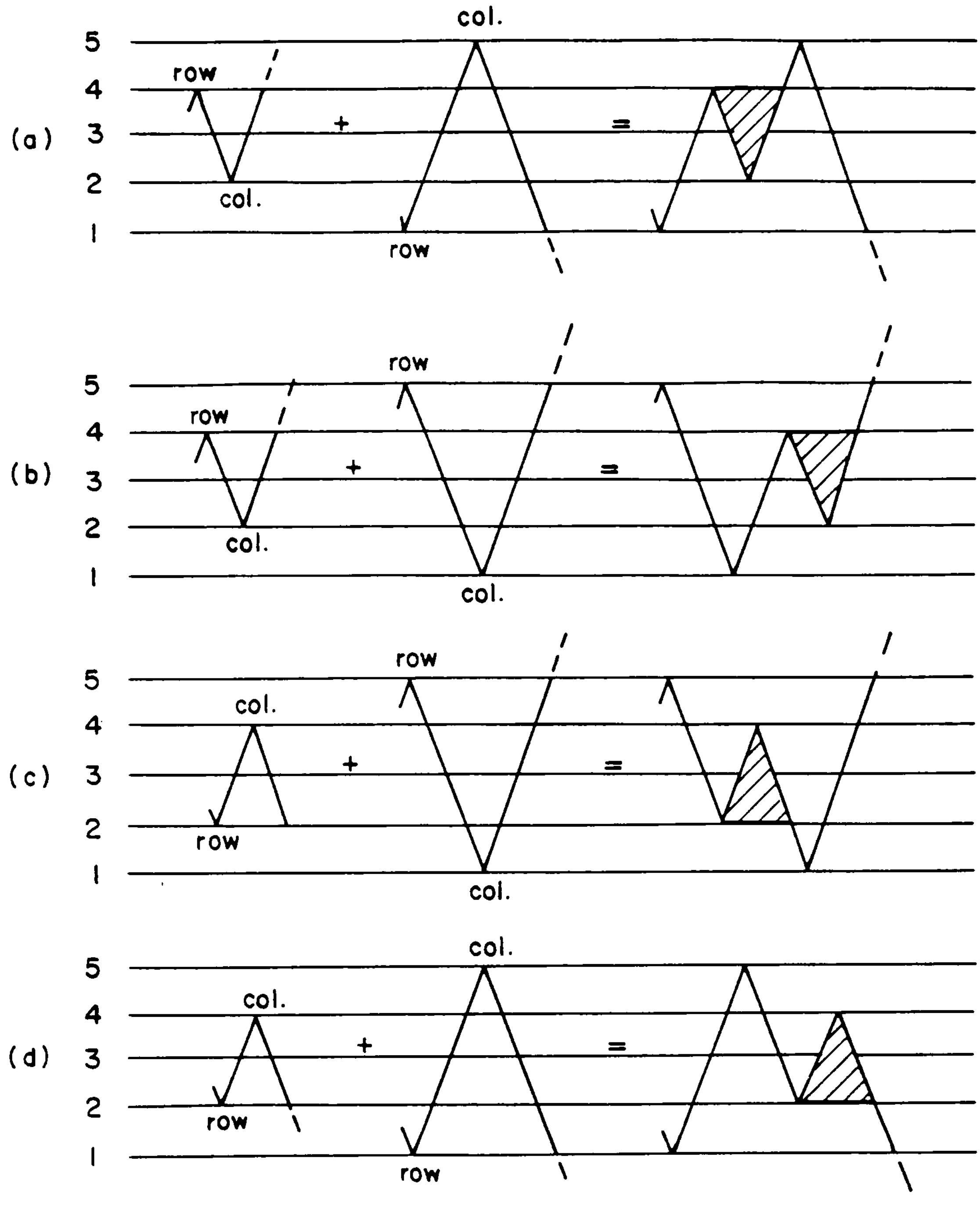


Figure 4.5. Illustration of Rules 1-4 for the Rain-flow Reconstruction: These rules are applied when the directions of the cycles are considered.

In addition, the reconstruction must alternate between peaks and valleys. This results in the insertion being made in the rising branch of the receiving cycle for cases (1) and (2), and in the falling branch for (3) and (4). Also, the major cycle could be considered to be either a peak-valley or a valley-peak cycle; the latter is arbitrarily chosen here.

These rules simply ensure that the inserting cycle is within the bounds of the receiving cycle. As an example, consider the history of Fig. 4.6(a). The inserting cycle 5-4 has a row 5 and a column 4. The partially reconstructed history has the following four rain-flow cycles: 2-6, 6-3, 7-2, and the major cycle, 1-7. Figure 4.6(b) shows these cycles. If rule number 1 is applied, then the possible receiving cycles would be 2-6 and 1-7. These cycles satisfy rule (1); therefore a random location is chosen between these two possibilities. Figure 4.6(c) shows the history if 2-6 is chosen as the receiving cycle, while Fig. 4.6(d) shows the history if 1-7 is chosen as the receiving cycle.

If rule number 2 is applied, then the possible receiving cycle would be 6-3 and 7-2. Note that both cycles satisfy rule number 2. Figures 4.6(d) and 4.6(e) show the history with the receiving cycle being 6-3 and 7-2, respectively.

Consider the history in Fig. 4.7(a). The inserting cycle is 4-5, and the partially reconstructed history is the same as for Fig. 4.6(a), having the same cycles 2-6, 6-3, 7-2 and 1-7, as already shown in Fig 4.6(b). If rule number 3 is applied, then the possible receiving cycles would be 6-3 and 7-2. Note that both cycles satisfy rule number 3. Figures 4.7(b) and (c) show the history with receiving cycles 6-3 and 7-2, respectively.

If rule number 4 is applied, then the possible receiving cycles would be 2-6 and 1-7. Both cycles satisfy rule (4). Figures 4.7(d) and (e) show the history with the receiving cycle 2-6 and 1-7, respectively.

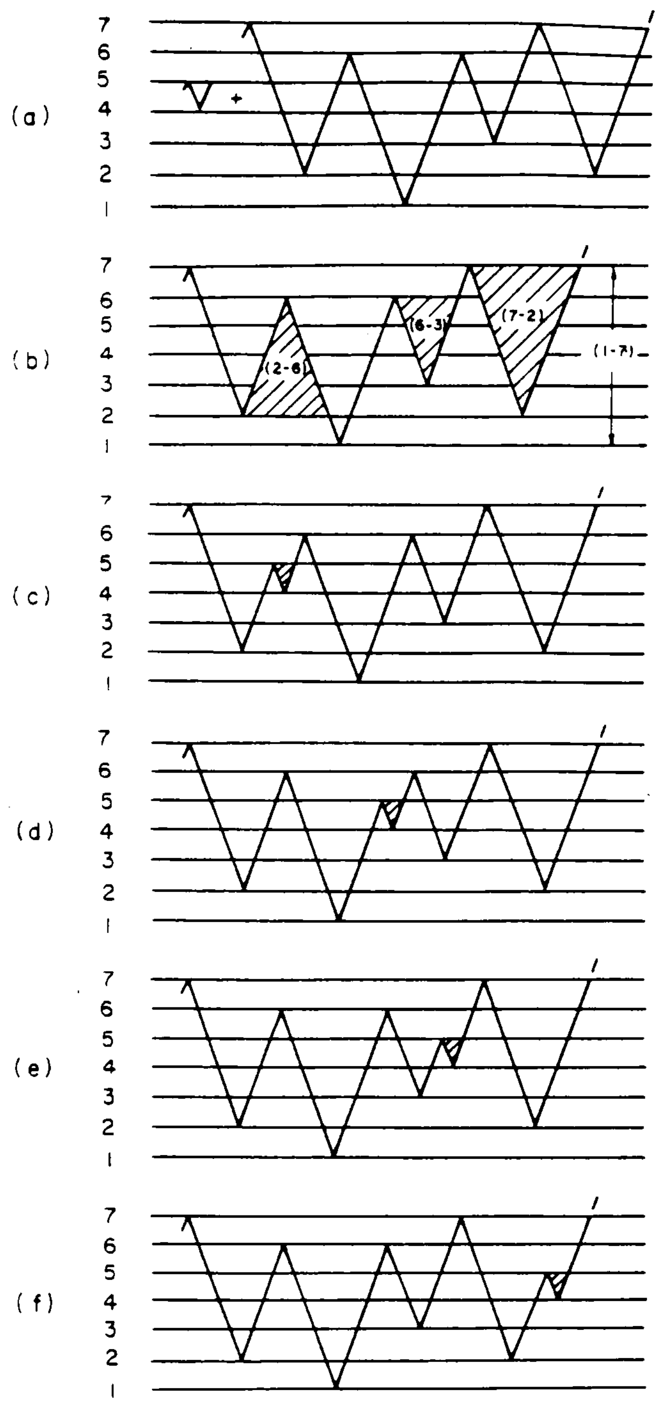


Figure 4.6. Example of R-F Reconstruction where the Inserting Cycle is Ordered P-V: The insertion of (a) can be made into any of the rain-flow cycles of (b), with the four possibilities being (c), (d), (e) and (f).

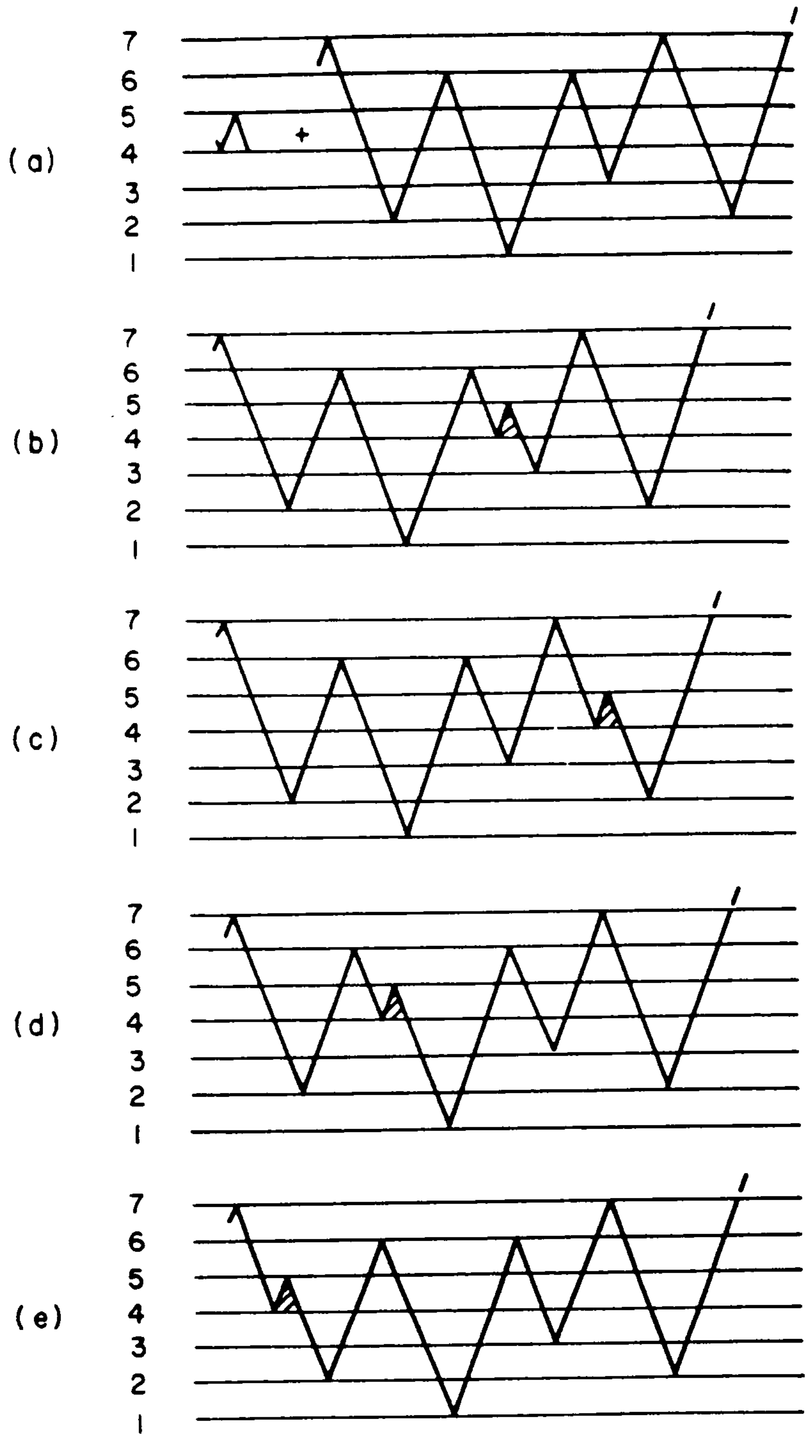


Figure 4.7. Example of R-F Reconstruction where the Inserting Cycle is Ordered V-P: The insertion of (a) can be made into any of the rain-flow cycles of Fig. 4.6(b), with four possibilities being (b), (c), (d) and (e).

Note that the location for the cycle under consideration will be chosen among all the possibilities, and as before there exist two general options for the reconstructed history, namely the simplest form and the irregular form. In the latter case, the number of cycles for a given peak-valley combination is divided into "n" groups, and each group is placed in a possible location randomly.

The reconstructed histories by either of the above described procedures will not have the same sequence as the original history. The reason is that a given minor cycle can be placed in a variety of locations, as its original location was not preserved by the peak-valley matrix of the rain-flow cycles. However, the predicted life for the reconstructed history will be identical to that for the original history as to the upper and lower bounds from the simplified local-strain approach.

4.5 Rain-flow Reconstruction Method Based on a 3-D Peak-Valley-Peak Matrix

This method is similar to the two dimensional reconstruction method (cycle directions considered), except another dimension is added to the matrix. Also, the rain-flow matrix was used with a size of 16 by 16 by 16. Greater resolution, say 32 by 32 by 32, could be used subject to limitations on computational time and cost. This method follows rules 2 and 4 (section 4.4) plus an additional rule which must also be introduced. The new rule states that if the inserting cycle is ordered peak-valley, then the receiving cycle must also be ordered peak-valley, and the receiving peak must be equal to the third dimension of the inserting cycle. However, if the inserting cycle is ordered valley-peak, then the receiving cycle must also be ordered valley-peak, and the receiving valley must be equal to the third dimension of the inserting cycle.

For example, consider Fig. 4.3(c). In the two dimensional approach, cycle g-h, has a starting level of 3 and a target level of 2, while in the three dimensional approach, this cycle is stored as g-h-a, which has a starting level of 3, an intermediate level of 2, and a target level of 5. A computer program was developed for this case. Application of this method is explained in Chapter 5.

4.6 Rain-flow Reconstruction Methods Based on a 3-D Peak-Valley-Time Matrix

The goal of these two similar methods is to preserve both rain-flow cycles of the original history and distribution of relative time increments between adjacent peaks and valleys. In the previous rain-flow reconstruction methods, the load history was already transferred into peak and valley points. However, in this method the history is used in the original form of peaks, valleys, and intermediate points between each peak and valley corresponding to the data points at equal time intervals. Two methods were developed and programmed on the computer and are explained below.

4.6.1 Method One

In this method the original history is stored in a three dimensional matrix for reconstruction purposes. This matrix contains the information on the rain-flow cycle peaks and valleys and the relative time intervals between each rain-flow peak and valley expressed as the number of equal time intervals. In this method an additional rule plus the previous four rules are required. The new rule states that the interval number of the receiving cycle must be greater than for the inserting cycle. Also, when a cycle is received, its interval number is deducted from the interval number of the receiving cycle. This rule allows the interval number of each rain-flow cycle to be preserved.

Using the interval numbers, a cosine wave in the form of Eq. 4.1 is used to fill the intermediate points between each peak and valley:

$$v = \frac{v_i + v_{i+1}}{2} + \frac{v_i - v_{i+1}}{2} \cos \frac{\pi(t - t_i)}{t_{i+1} - t_i} \tag{4.1}$$

where v_i and v_{i+1} are the peak and valley or valley and peak under consideration and t_i and t_{i+1} are their corresponding times. A computer program was developed for the above case. The reconstructed history using the above method gives the same rain-flow cycles and relative time intervals between adjacent peaks and valleys as the original history.

4.6.2 Method Two

This method is much simpler than method one, and preserves both rain-flow cycles and relative time intervals between adjacent peaks and valleys.

In order to achieve the above task, a three dimensional matrix of peak-valley-time of the original history is required. Note that the above matrix is not the rain-flow matrix, but a matrix consisting of peaks, valleys, and intermediate points between each peak and valley corresponding to data points of equal time intervals. Next, the original history is transferred into peaks and valleys, and its rain-flow matrix is obtained. The reconstruction procedure begins by reconstructing a history using the two dimensional rain-flow matrix of peak-valley. This reconstructed history is used with the aid of the three dimensional matrix to obtain the reconstructed history in the final form. Note that the difference between the reconstructed history in its final form and its intermediate form is that the interval numbers are added. Therefore, the next step is the determination of the interval numbers. Interval numbers are obtained by orderly taking the peak and valley of the reconstructed history in its intermediate form and matching them with the peak-valley-time matrix. There is a possibility that more than one interval size exists; therefore, a random selection is made among all the possibilities. After interval numbers are determined, a cosine wave in the form of Eq. 4.1 is used to fill the intermediate points between each peak and valley. A computer program was developed for the above case.

In summary, this procedure preserves both rain-flow cycles and relative time intervals between adjacent peaks and valleys. Since this method is simpler to program and less costly than method one it was adopted for further study.

5.0 Analysis and Reconstruction of Helix and Maneuver Histories

5.1 *Material Properties Used*

For this study, two materials were used, 2024-T4 aluminum and titanium 6Al-4V. 2024-T4 aluminum was only used in the preliminary study of the dissertation and titanium 6Al-4V was used for the main portions of the dissertation.

Unnotched axial specimen data for 2024-T4 aluminum from Ref. [39] were fitted to a cyclic stress-strain curve in the form of Eq. 3.1 and a strain-life curve in the form of Eq.3.3. This was done for both ordinary constant amplitude tests and for tests on previously prestrained specimens. Table 5.1 gives the resulting material constants. Figure 2.4 shows the strain vs. life data for this material and also curves corresponding to Eq. 3.1 fitted to the data.

In the case of titanium 6Al-4V, ordinary constant amplitude data were obtained from Ref. [58]. Constants fitted to these were obtained from A. Conle of Ford Motor Company and are given

Table 5.1. Cyclic Stress-Strain and Strain-Life Constants for 2024-T4 Al

Symbols, Units	2024-T4 Aluminum	
	Const. Ampl.	Prestrained
E, GPa (ksi)	73.1 (10600)	73.1 (10600)
A, MPa (ksi)	738 (107.0)	738 (107.0)
s	0.080	0.080
ε'_f	0.327	0.327
c	-0.645	-0.645
σ'_f, MPa (ksi)	900 (130.6)	1294 (187.6)
b	-0.102	-0.142

in Table 5.2. Figures 5.1 and 5.2 show strain vs. life and stress vs. strain data for this material, and also equations fitted to the data. Some constant amplitude data were also obtained at Virginia Tech on the same material. These data are also shown in Figs 5.1 and 5.2, and agree well with the data obtained from Ref. [37] and also the fitted constants. In the case of pre-strained materials, the data and constants were obtained from Ref. [59]. These data and constants are also shown in Figs 5.1 and 5.2 and are given in Table 5.2.

5.2 General Description of the Load Histories

Two load histories, Helix and Maneuver, are used in fatigue life predictions and reconstruction purposes. Helix is a standard load spectrum obtained from Ref. [60], and Maneuver history is obtained from the University of Dayton Research Institute (UDRI). These two load histories are explained below.

5.2.1 General Description of Helix

Helix [60] gives standard loading sequences for the main rotors of helicopters with articulated rotors. Helix represents a load history for a 190.5-hour (2,132,024 cycles) sequence of 140 flights. Each flight in the sequence represents one of either training, transport , antisubmarine warfare, or search and rescue. Each of these appears in the sequence in three different lengths of 0.75 hour, 2.25 hours, and 3.75 hours. There are twelve unique flights, which are applied in specific sequence and number of repetitions to obtain the total 140 flights. Figure 5.3 shows the load vs. time history for portions of a transport flight in Helix.

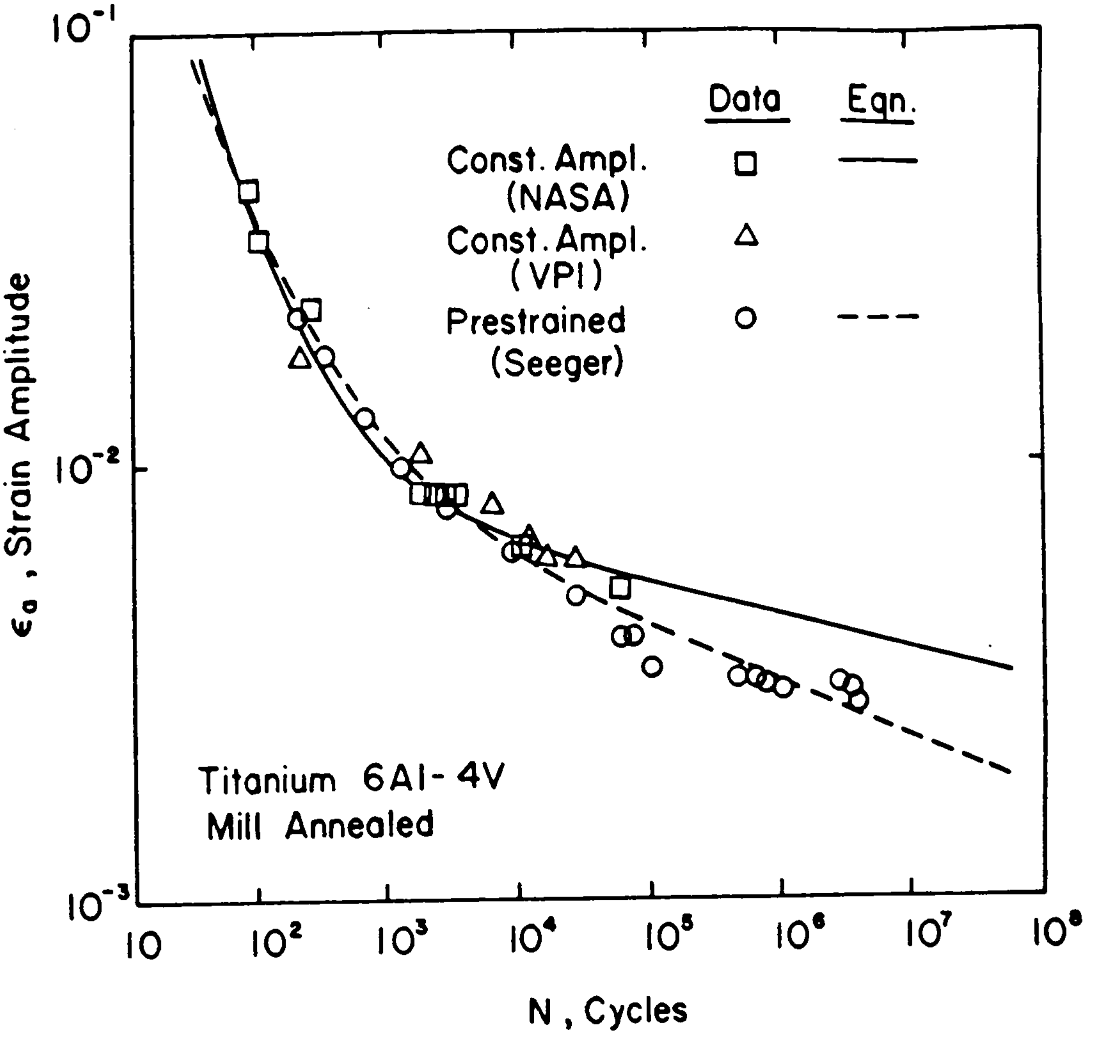


Figure 5.1. Strain vs. Life Test Data and Curves for Ti-6Al-4V.

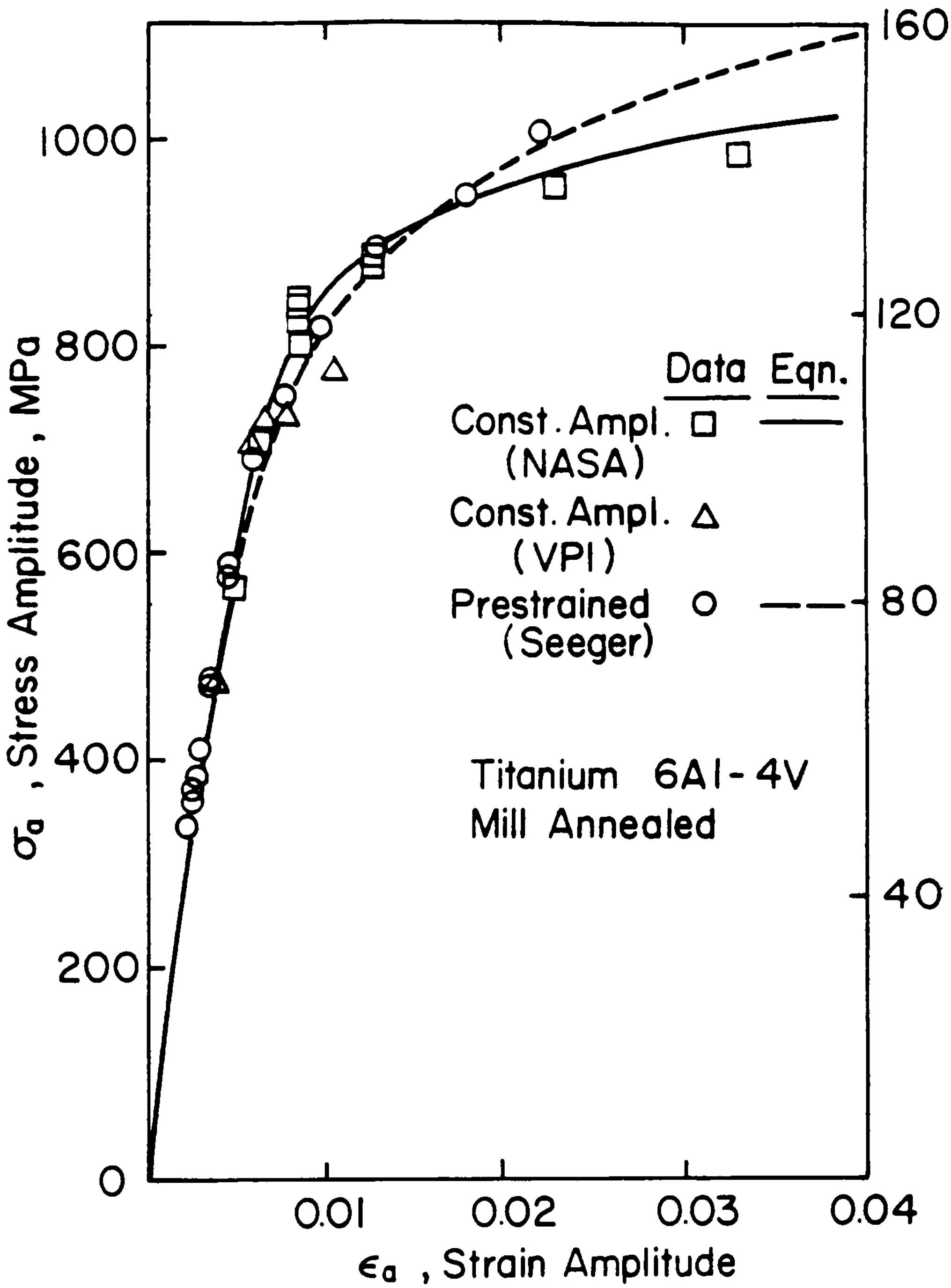


Figure 5.2. Cyclic Stress-Strain Test Data and Curves for Ti-6Al-4V.

Table 5.2. Cyclic Stress-Strain and Strain-Life Constants for Ti 6Al-4V

Symbols, Units	Titanium 6Al-4V	
	Const. Ampl.	Prestrained
E, GPa (ksi)	113.8 (16500)	113.8(16500)
A, MPa (ksi)	1327 (192.4)	1702 (246.9)
s	0.0755	0.127
ε'_f	6.22	2.802
c	-1.01	-0.860
σ'_f, MPa (ksi)	1523 (220.9)	2207 (320.0)
b	-0.0763	-0.126

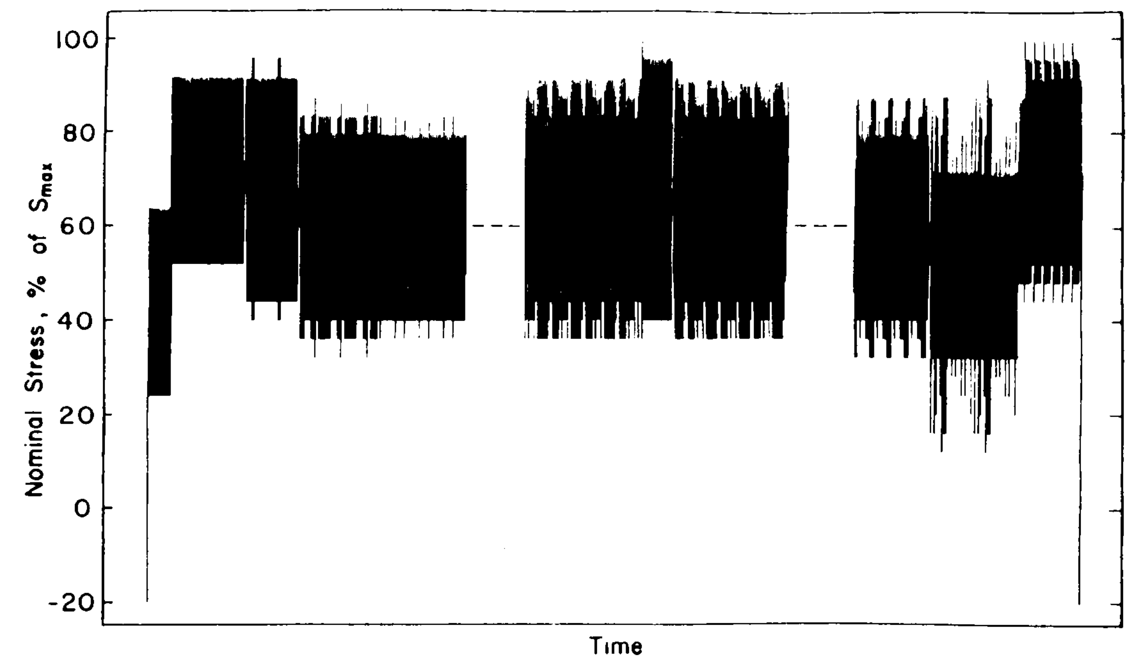


Figure 5.3. Example of the Load History for Portions of a Transport Flight in Helix [60].

Helix [7,60] is composed of 24 unique maneuvers, which are repeated in various sequences and numbers of repetitions to compose the various flights. The maneuvers such as take off, forward flight of various load levels, turns, etc. each consists of a mean level and a relatively small number of cycles. These cycles occur at one or more stress amplitudes, with the number of cycles being between 1 and 40.

Helix also has a relatively high mean level for the various maneuvers, ranging from 60 % to 68 % of the maximum nominal stress in the spectrum , S_{max} . Helix reaches 100 % of the S_{max} level at least once in each flight and returns to -20 % at the end of each flight. Hence Helix has a large ground-air-ground cycle and a large number of cycles at relatively high mean levels. Table 5.3 gives the cycles at various ranges from rain-flow cycle counting of Helix, and Table 5.4 gives the range-mean matrix for Helix from rain-flow cycle counting. The matrix entries in Table 5.4 were obtained from all of Helix by dividing each by 140, the number of flights. These range and mean values correspond to the history scaled so that S_{max}= 100 units.

5.2.2 General Description of Maneuver History

A load history for the tail rotor pitch beam of an AUH-76 helicopter was selected as representative, and load histories from each of 30 distinct severe maneuvers were assumed to occur once each in a specific sequence. This produced a load history containing 33,470 cycles, which was then modified by UDRI to eliminate minor events, shortening it to 8777 cycles, that is, 8777 peaks and 8777 valleys. This new history is called the modified history. Figure 5.4(a) shows about one third of the modified history. Table 5.5 gives the ranges of the rain-flow cycles, and Fig. 5.5 shows the complete range and mean matrix of the rain-flow cycle count. Note that the original and modified history are scaled so that the highest load is 1 unit, which results in the lowest load being -0.516; therefore, the overall range is 1.516.

Table 5.3. Cycles from R-F Counting of Helix and Its P-V Reconstruction

Stress Range for S_{max} = 100 units	Helix, Cycles Per Flight, Average⁺		Reconstruction Cycles per Flight	
	Number	Cumulative	Number	Cumulative
120	1	1	1	1
116	0	1	0	1
112	0	1	0	1
108	0	1	0	1
104	0	1	0	1
100	0	1	0	1
96	0	1	0	1
92	0	1	0	1
88	1	2	2	3
84	0	2	10	13
80	7	9	20	33
76	3	12	0	33
72	42	54	80	113
68	2	56	50	163
64	74	130	120	283
60	16	146	280	563
56	8344	8490	8030	8593
52	26	8516	190	8783
48	3252	11768	2730	11513
44	3	11771	1810	13323
40	3425	15196	100	13423
36	2	15198	1510	14933
32	0	15198	40	14973
28	2	15200	130	15103
24	0	15200	50	15153
20	1	15201	50	15203
16	0	15201	0	15203
12	2	15203	0	15203
8	5	15208	0	15203
4	21	15229	0	15203

⁺The average values are obtained by dividing the cycles counted for all Helix by 140, the number of flights.

Table 5.4. Range-Mean Matrix for Helix from Rain-flow Cycle Counting

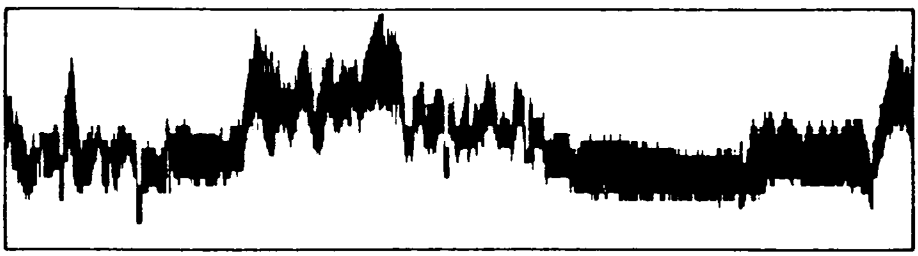
	Mean									
Range	40	44	48	52	56	60	64	68	72	ALL
4	0	0	0	1	0	2	16	2	0	21
8	0	0	0	0	1	0	4	0	0	5
12	0	0	0	0	0	0	2	0	0	2
16	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	1	0	0	0	1
24	0	0	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	2	0	0	2
32	0	0	0	0	0	0	0	0	0	0
36	0	0	0	1	0	0	0	1	0	2
40	0	12	0	43	30	1742	1348	27	223	3425
44	0	0	0	1	0	1	0	1	0	3
48	0	0	0	14	5	453	2613	155	12	3252
52	0	0	0	0	0	5	20	1	0	26
56	0	0	0	24	6	65	7785	460	4	8344
60	0	0	0	0	1	0	15	0	0	16
64	0	0	0	6	8	30	6	24	0	74
68	0	0	0	1	1	0	0	0	0	2
72	0	0	0	10	4	28	0	0	0	42
76	0	0	0	1	1	1	0	0	0	3
80	0	0	0	1	1	5	0	0	0	7
84	0	0	0	0	0	0	0	0	0	0
88	0	0	0	0	1	0	0	0	0	1
92	0	0	0	0	0	0	0	0	0	0
96	0	0	0	0	0	0	0	0	0	0
100	0	0	0	0	0	0	0	0	0	0
104	0	0	0	0	0	0	0	0	0	0
108	0	0	0	0	0	0	0	0	0	0
112	0	0	0	0	0	0	0	0	0	0
116	0	0	0	0	0	0	0	0	0	0
120	1	0	0	0	0	0	0	0	0	1
ALL	1	12	0	103	59	2333	11811	671	239	15229

Table 5.5. Rain-Flow Counted Ranges for the Maneuver History and Derivaties

Range+ (32 Levels)	Modified History	Rain-Flow Filtered History	Peak-Valley Reconstruction	To-From Reconstruction
1	1160	0	500	280
2	400	0	520	512
3	500	0	300	666
4	1055	0	510	1250
5	1684	0	490	1645
6	1677	0	230	1180
7	1044	0	570	749
8	577	0	100	480
9	309	139	310	279
10	177	177	380	168
11	95	95	30	135
12	45	45	320	62
13	19	19	290	59
14	13	13	150	38
15	4	4	170	32
16	0	0	310	32
17	5	5	110	22
18	1	1	160	20
19	3	3	210	22
20	2	2	200	12
21	0	0	20	8
22	1	1	70	29
23	0	0	0	39
24	2	2	20	9
25	1	1	40	3
26	0	0	10	0
27	2	2	60	0
28	0	0	30	1
29	0	0	0	1
30	1	1	5	1
31	0	0	0	0
32	0	0	0	0
total	8777	510	6115	7734

+For scaling so that the highest peak is 1 unit, range 1 is 0.05 units,range 32 is 1.60 units,and the increments between each range level is 0.05 units.

(a)



(b)

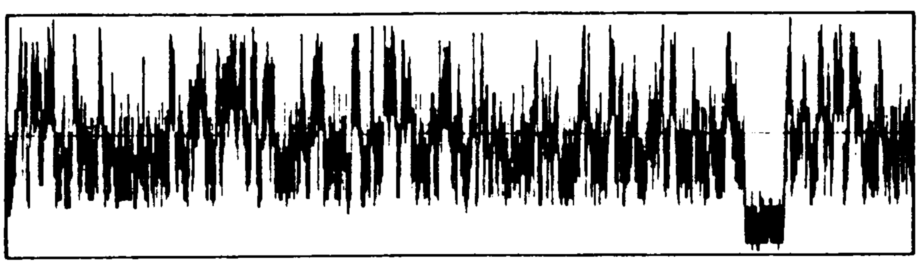


Figure 5.4. Portions of (a) Modified Maneuver and (b) The To-from Recon. History.

RANGE \ MEAN	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
1				7	17						6	271	404	268	178	7	2															
2			9	28	16							77	36	64	124	23	20	4														
3			10	12	10								21	69	106	107	70	39	26	20	6	4										
4				6	9	2	2	1	16	66	99	74	97	103	99	96	93	96	88	56	27	24	2									
5				8	17	4			6	148	150	193	225	130	167	99	110	96	88	82	80	56	20	6	1							
6				4	3				4	78	202	166	236	158	171	68	94	82	68	68	109	83	49	26	7	1						
7					1					13	140	80	146	116	77	31	44	46	33	49	74	60	71	42	17	4						
8										3	32	30	56	56	37	20	14	30	22	31	53	48	60	58	27	1						
9											3	14	22	26	14	12	7	14	11	13	29	34	39	43	26	2						
10												4	4	6	10	6	7	7	11	12	22	18	16	29	21	6						
11													2	3	7	9	8	6	5	3	11	6	6	16	14							
12													1		3	6	2	5	3	6	5	2	1	5	6							
13												1		1	1	4	1		4	1	5	1										
14															1	4	3	1	2	1		1										
15																1	2		1													
16																																
17															1		1			2	1											
18															1																	
19																	1			1	1											
20																	1				1											
21																																
22																		1														
23																																
24																	1		1													
25																			1													
26																																
27																	1	1														
28																																
29																																
30																1																
31																																
32																																

Figure 5.5. Range-Mean Matrix from R-F Cycle Counting of The Modified History.

Fatigue life calculation was done for both original and modified history for the titanium 6Al-4V. It was found that the differences are small, and the two histories appear to cause the same damage; therefore, all further testing and analysis is based on the modified history. The fatigue life predictions will be discussed later in this Chapter.

5.3 Detailed Local Stain Analysis of Helix

Life calculations were done for the plate with hole specimens of 2024-T4 Al subjected to Helix using the full local strain approach. This task is achieved by using a computer program which is developed by Wirsching [61] from an earlier version of the program developed by Brose [62]. The program was used as received except for minor revisions to make it compatible with the system at Virginia Tech. Some minor corrections were also made for roundoff type errors associated with the use of discrete strain levels in the program.

As mentioned earlier, Helix consists of twelve unique flights. Helix reaches the 100 % of S_{max} level at least once in each flight and returns to -20 % at the end of each flight. The first step is to determine the fatigue lives for each of these twelve unique flights. These flights also repeat themselves since Helix consists of 140 flights. Therefore, the life for Helix was calculated by combining the calculated life for each unique flight and also considering the number of times a unique flight is repeated. Note that this procedure works because all of the flights return to -20 % level. This gives common minimum stresses and strains locally at the notch for each flight, which results in no sequence effects from one flight affecting another.

The calculated results are compared to test data [60] and are shown in Fig. 5.6. Table 5.6 gives the calculated lives corresponding to an elastic stress concentration factor (S.C.F.) of $k_t = 2.5$.

As is seen from Fig. 5.6, the agreement is quite good for the predicted values using the pre-strained data, except for the lowest stress level, where the calculations are conservative.

5.4 Upper-Lower Bound Analysis of Helix

The fatigue lives for the plate-with-hole specimens of 2024-T4 Al subjected to Helix were calculated using the new simplified approach. As already discussed, this method is based on the local strain approach and only the rain-flow matrix of the history is required as input information for fatigue life analysis. This simplified version places bounds on the fatigue life.

In order to achieve the above task, as the previous case, each of the twelve flights must be analyzed separately. Therefore, the first step is to determine the rain-flow matrix of each unique flight. Then, by using this matrix , upper and lower bounds can be determined for each unique flight. These lives then combine to obtain overall bounds by considering the number of times each unique flight was repeated. Another option is also available for predicting the lives. In this option only one matrix will be used which is derived from all of Helix. Comparison between these two options shows no significant difference; therefore, the simpler one was adopted.

Hence, the rain-flow matrix for an average flight in the form of Table 5.4 is used for the purpose of calculating the upper and lower bounds on life. A computer program was developed using the algorithm explained in Chapter 3 and Appendix A.

The resulting bounds on life are plotted in Fig. 5.6 and are given in Table 5.6. As it is seen from Fig. 5.6, the bounds are very tight except at the higher stress levels. Also, reasonable agreement exists between the full local strain approach and the upper and lower bounds. Note that

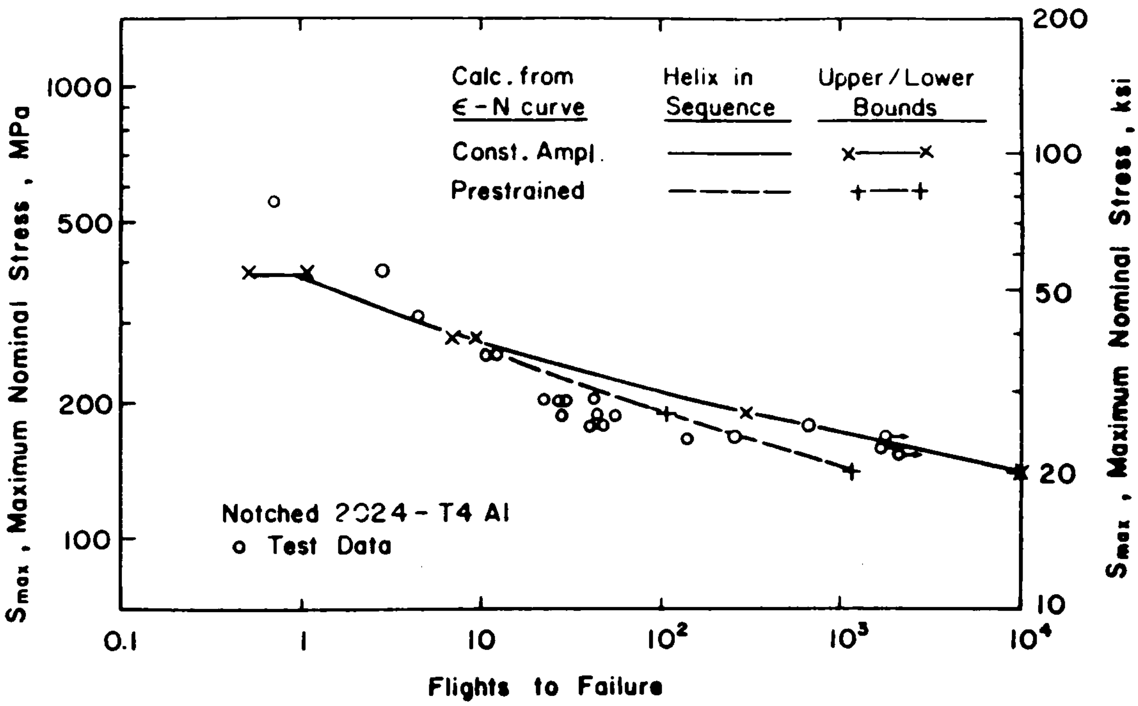


Figure 5.6. Analysis of Helix Compared to Test Data for 2024-T4 Aluminum.

Table 5.6. Calculated Fligts to Failure for Helix for Notched Specimens (S.C.F. =2.5)

S_{max}, MPa (ksi) (net area)	Helix Full Analysis	Rain-flow Matrix Lower Bd.	Upper Bd.
(a) 2024-T4 Al, Const. Ampl. Strain-Life Curve			
138 (20)	9614	10005	10006
186 (27)	295	294	296
276 (40)	7.95	6.88	8.57
379 (55)	0.95	0.50	1.07
(b) 2024-T4 Al, Prestrained Strain-Life curve			
138 (20)	1107	1149	1149
186 (27)	107.0	108.0	108.2
276 (40)	7.94	7.55	8.37
317 (46)	-----	3.10	3.86
(c) Ti 6Al-4V, Const. Ampl. Strain-Life Curve			
317 (46)	-----	26193	26213
358 (52)	-----	4415	4433
407 (59)	-----	857	876
455 (66)	-----	221	240
517 (75)	-----	46.9	62.8
(d) Ti 6Al-4V, Prestrained Strain-Life Curve			
317 (46)	-----	681	687
455 (66)	-----	32.7	36.0
517 (75)	-----	11.8	13.9
530 (77)	-----	9.59	11.5

when the bounds are tight, the two methods are in complete agreement, except for various sources of minor numerical error, such as roundoff errors.

Based on the obtained results for aluminum, it was concluded that it is unnecessary to do the more detailed full analysis for the other cases, and the calculated lives based on upper and lower bounds are as reliable as the case for more involved full local strain analysis. Note that the calculated lives obtained for all cases correspond to an elastic stress concentration factor of $k_t = 2.5$.

The upper and lower bound life calculations were also done for titanium 6Al-4V. The resulting calculated lives are plotted in Fig.5.7 and are given in Table 5.6. See the curves identified as "original Helix", with one curve based on constant amplitude strain-life data, and the other on prestrained data. Figure 5.7 plots $k_t\ S_{max}$ so that the various k_t can be shown on the same plot. This is expected to be valid based on Neuber's rule, Eq. 3.2, as long as net section yielding does not occur, in which case Eq. 3.2 is not valid.

The data obtained from RAE scatter over a broad range, and the lives tend to be longer than those obtained from UDRI. Also, from Fig. 5.7, it is evident that general agreement is obtained between data and analysis.

Finally, it is noteworthy to mention that most of the fatigue damage in Helix is concentrated at a single level. This case is illustrated by Fig. 5.8 for one stress level, for Al. 2024-T4. Also, the major cycle (ground-air-ground) does not cause any significant damage directly, as is seen from Fig. 5.8; however, by affecting the local mean stresses of the lower cycles, the fatigue damage done by these levels can be increased through the prestrain effect.

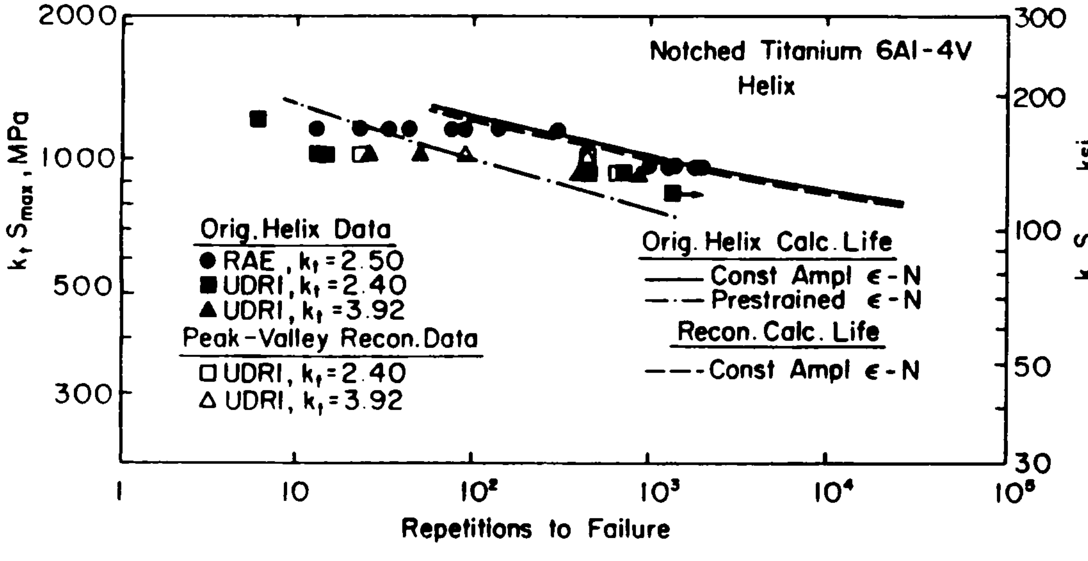


Figure 5.7. Analysis of Helix Compared to Test Data for Ti 6Al-4V: The lines shown are the middle of the bounds.

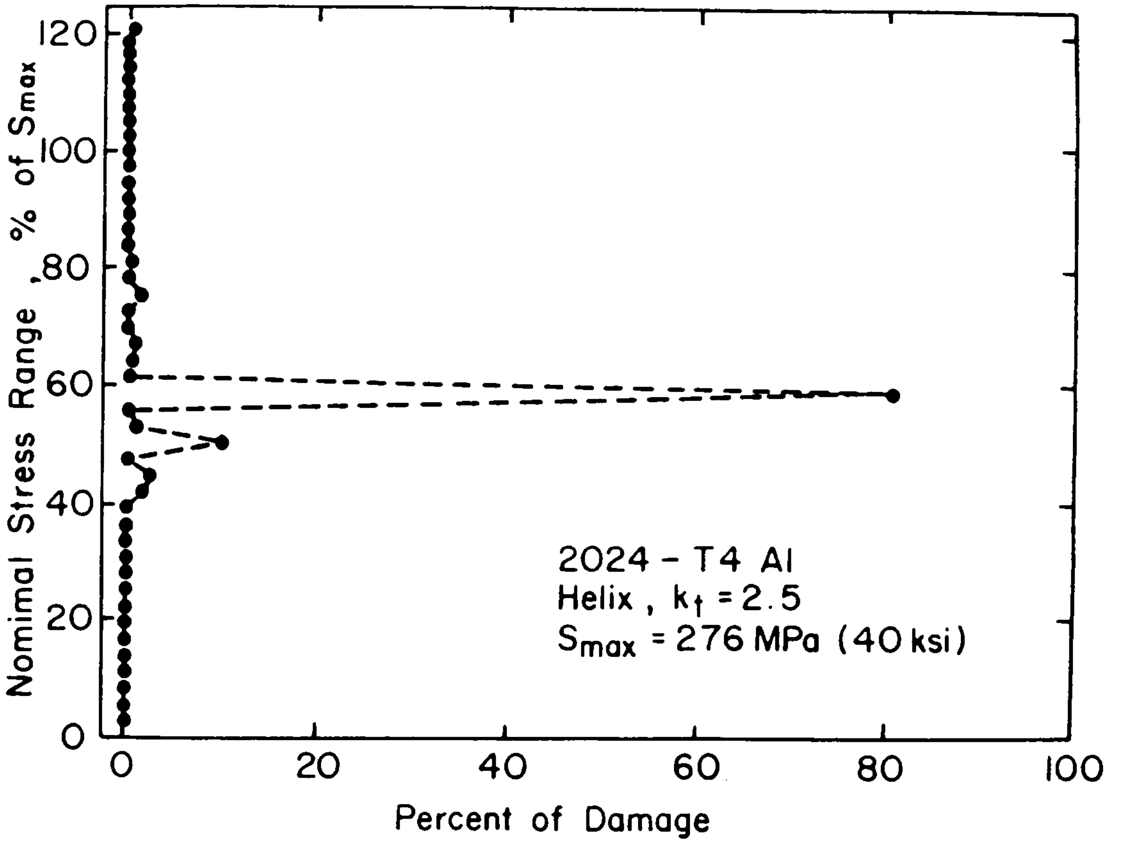


Figure 5.8. Distribution of Fatigue Damage with Stress Level for One Case of Helix.

5.5 *Upper and Lower Bound Analysis of Maneuver History*

Upper and lower bound life calculation for original and modified history for titanium 6Al-4V are given in Table in Table 5.7. As is evident, the differences are small, and the two histories appear to cause the same fatigue damage; therefore all the testing and analysis is based on the modified version.

Figure 5.9 shows the calculated results, as well as the test data. Table 5.7 gives the calculated lives for notched specimens ($k_t = 2.5$). Reasonable agreement is obtained by comparing the test data and the calculated values, especially when prestrained data is used. An exception is at the lowest stresses where these calculations are conservative.

Figure 5.10 shows the distributions of cycles vs. ranges which are also given in Table 5.5. During the determination of the fatigue lives, it was noted that most of the fatigue damage was done by the higher range levels, and none by the lower levels. In this regard, it was also noted that a potentially large saving of test time can be realized by eliminating lower level non-damaging stress cycles from the load history. Therefore, a rain-flow filtering was done on the maneuver history.

The basis of this filtering was the fatigue "damage", more properly called the usage fraction, which is obtained from the P-M rule calculation. Usage fraction is defined as:

$$\text{usage fraction} = \frac{\frac{n_i}{N_i}}{\sum_{i=1}^{l} \frac{n_i}{N_i}} \tag{5.1}$$

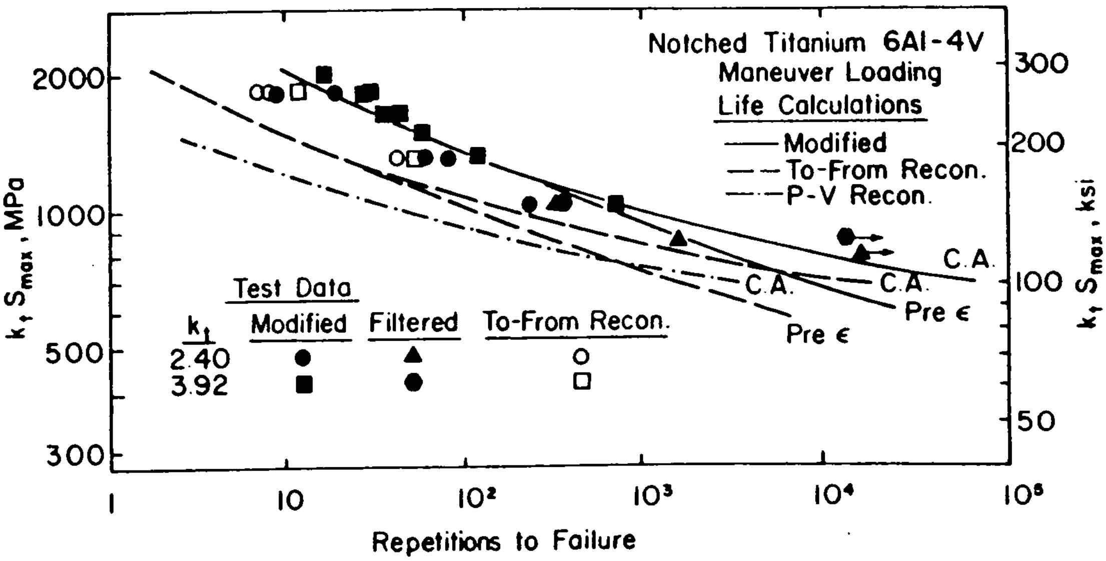


Figure 5.9. Comparison of Maneuver History and Its Reconstructions to Test Data.

Table 5.7. Calculated Repetitions to Failure for Maneuver for Notched Specimens (S.C.F. =2.5)

S_{max}, MPa (ksi) (net area)	Modified Sequence Lower Bd.	Modified Sequence Upper Bd.	Original Sequence Lower Bd.	Original Sequence Upper Bd.
(a)Ti 6Al-4V, Const. Ampl. Strain-Life Curve				
276 (40)	67700	67700	61800	61800
327 (47)	7940	7940	7320	7320
379 (55)	1410	1410	1310	1340
455 (66)	291	315	274	298
586 (85)	59.3	68.9	52.6	67.4
827 (120)	6.14	13.8	6.01	13.6
(b) Ti 6Al-4V, Prestrained Strain-Life Curve				
276 (40)	8770	8930		
379 (55)	888	934		
827 (127)	7.89	14.1		

where n_i is the number of cycle applied at a stress level corresponding to life N_i, and I is the number of different discrete stress levels. Figure 5.11 shows the usage fraction at each level vs. range for the modified history.

As is evident from Figs 5.10 and 5.11, most cycles occur at the lower ranges, whereas most damage is estimated to be done at higher ranges. Therefore the modified history was shortened by filtering all the ranges less than 0.45 units, that is, 30 % of the largest rain-flow range of 1.516. Table 5.5 gives the rain-flow ranges, and the range mean matrix is shown in Fig. 5.12. This filtered history contains 510 cycles, that is, 510 peaks and 510 valleys. Figure 5.13(a) shows the filtered history. By filtering the lowest stress levels of the modified history, the usage fraction is decreased only slightly, specifically by about 1 %.

Note that both modified and filtered histories produce similar fatigue lives, since most of the damage is done at higher levels that are not filtered out in the latter. Also, from Fig. 5.8, comparison of the test data for modified and filtered history at one stress level, where both were tested, shows quite good agreement, which indicates that the filtering was a success.

5.6 *Peak-Valley Reconstruction of Helix*

The method described in Chapter 4 is employed here to obtain a peak-valley (P-V) reconstructed history based on Helix. In order to reconstruct Helix some important factors must be mentioned. First, from Table 5.4, the majority of cycles occur at a few levels. Also, as explained before , most of the fatigue damage is concentrated at a single level (Fig. 5.8). Hence the most important features of the reconstructed Helix would be to produce similar cycles at the few levels with the highest cycles, and also at the most damaging levels.

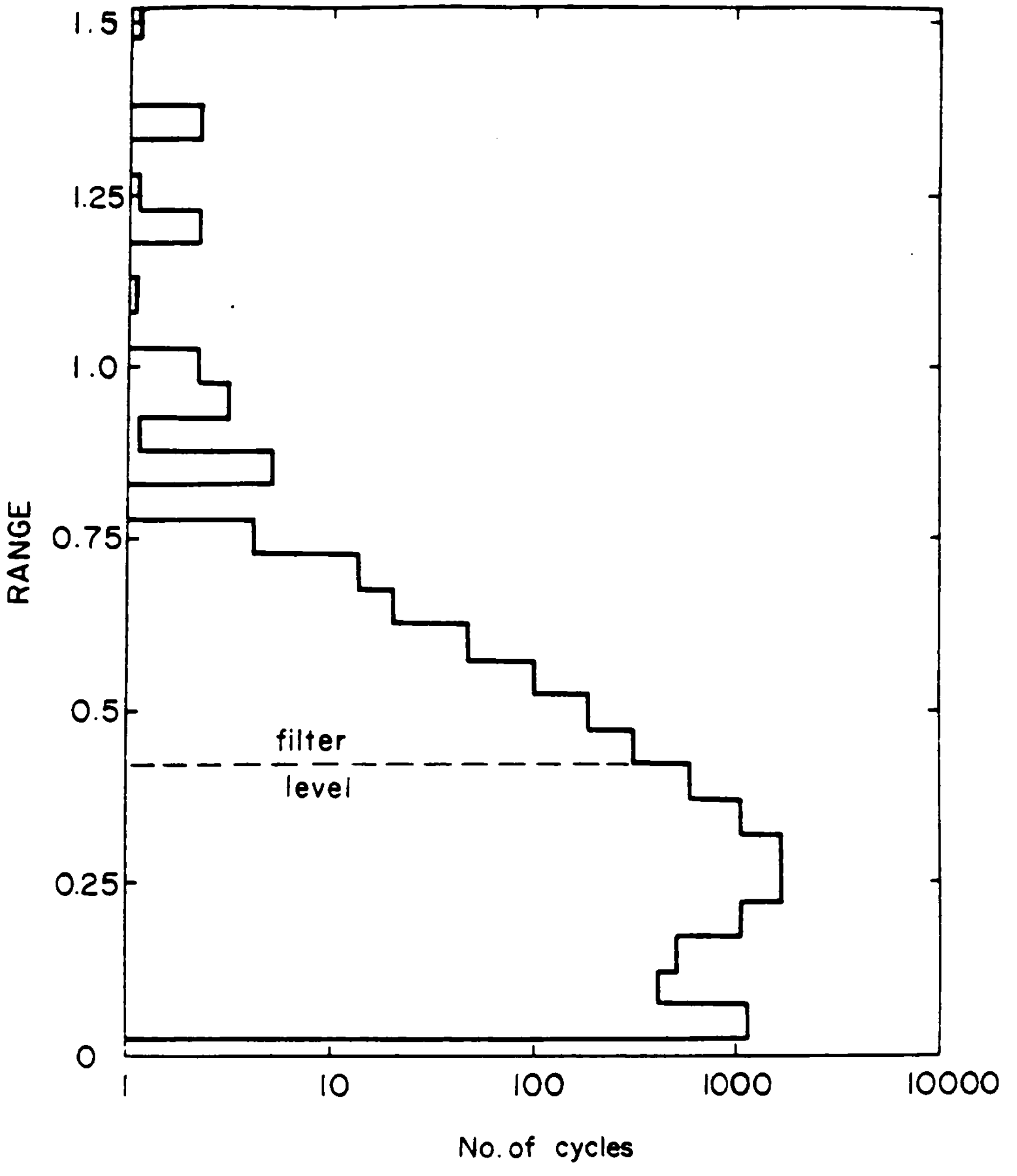


Figure 5.10. No. of Cycles vs. Range of the R-F Counted for the Modified History.

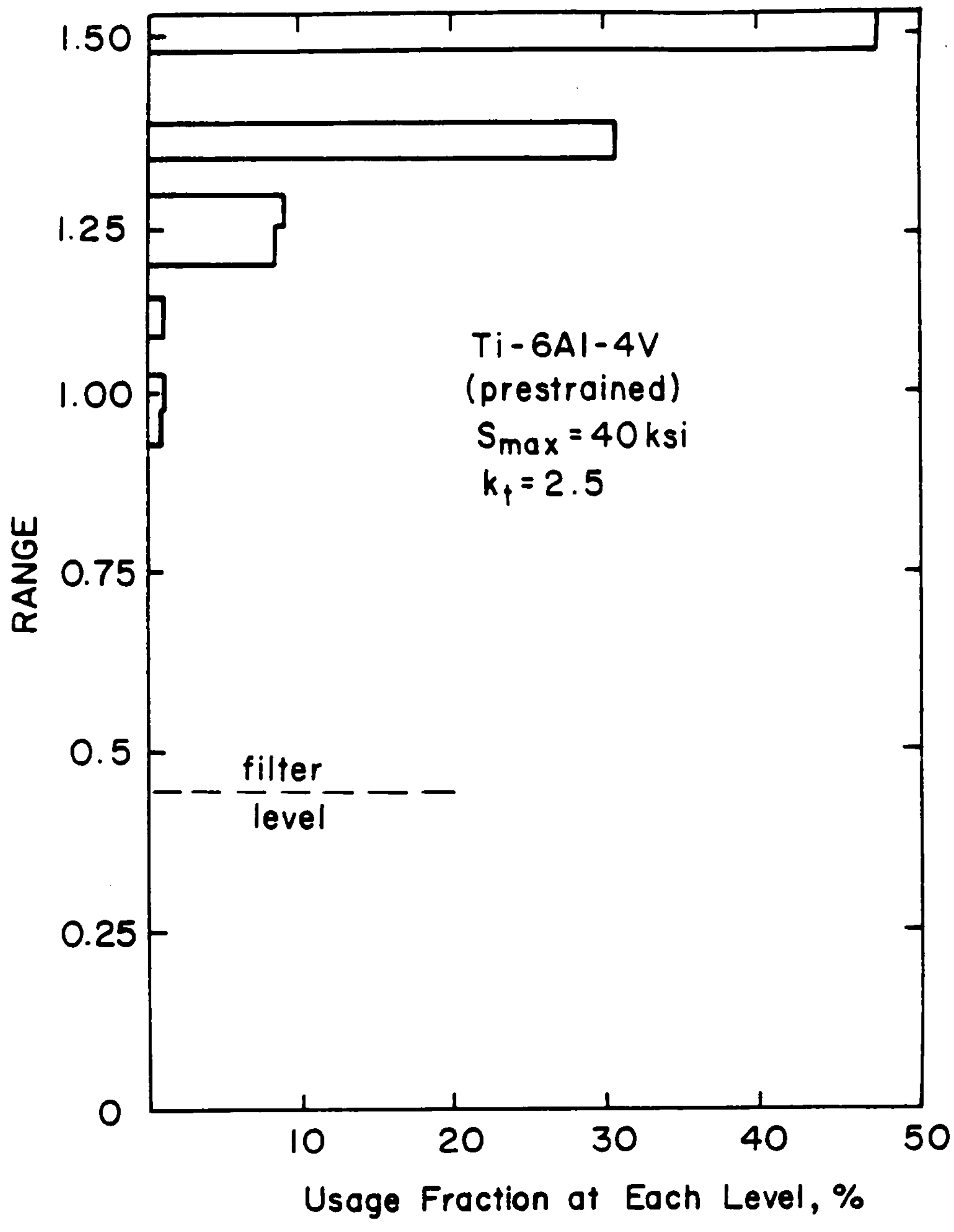


Figure 5.11. Usage Fraction vs. Range of the R-F Counted Cycle for the Modified History.

RANGE \ MEAN	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
1																																
2																																
3																																
4																																
5																																
6																																
7																																
8																																
9												6	10	9	7	7	6	9	7	5	8	14	19	19	12	2						
10												4	4	6	10	6	7	7	11	12	22	18	15	29	21	6						
11													2	3	7	9	8	6	5	3	11	6	6	16	14							
12													1		3	6	2	5	3	6	6	2	1	6	6							
13												1		1	1	4	1		4	1	5	1										
14															1	4	3	1	2	1		1										
15																1	2		1													
16																																
17															1		1			2	1											
18															1																	
19																	1			1	1											
20																	1				1											
21																																
22																		1														
23																																
24																	1		1													
25																			1													
26																																
27																	1	1														
28																																
29																																
30																1																
31																																
32																																

Figure 5.12. Range-Mean Matrix from R-F Cycle Counting for the Filtered History.

(a)

(b)

(c)

(d)

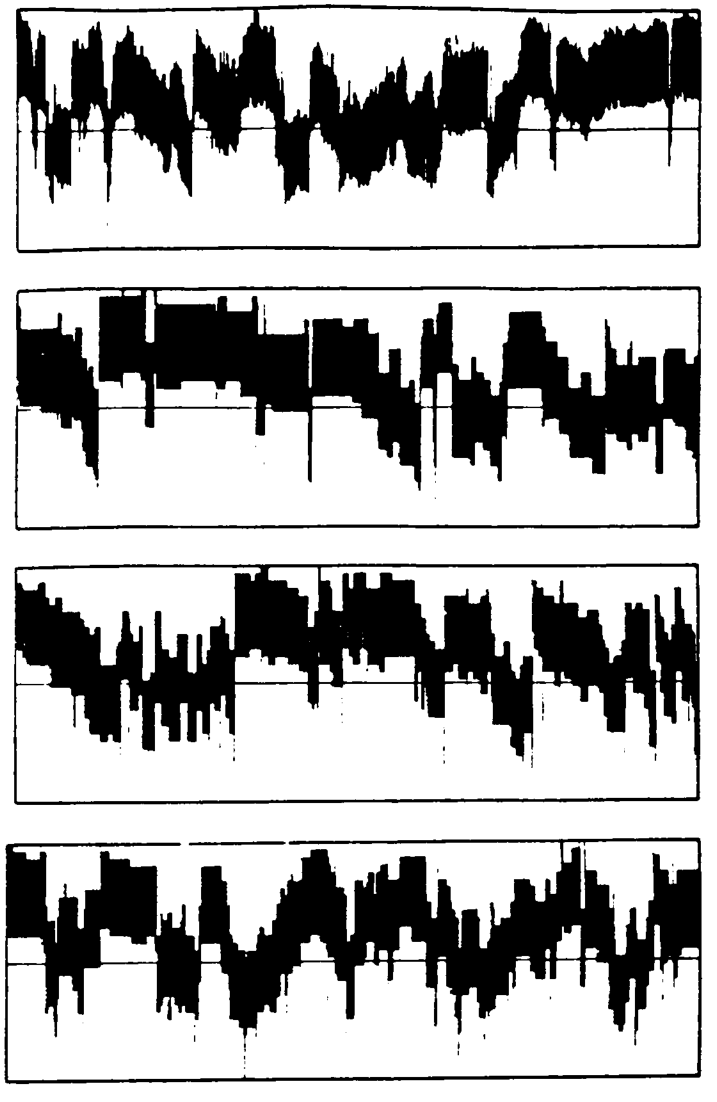


Figure 5.13. Rain-Flow Reconstruction of the Filtered Maneuver History: (a)filtered maneuver history, (b) reconstructed with all cycles of a given peak and valley in one location, where the direction of the cycle not considered, (c) reconstructed similarly but with large blocks of cycles split into three different locations, and (d) reconstructed with all cycles of a given peak and valley in one location, but where the directions of the cycles are now considered.

In order to reconstruct a history from Helix, the highest peak and lowest valley are combined to form a cycle, and the second highest and second lowest, and so on, are similarly combined. This procedure continues until all the events are covered. Helix consists of 140 flights; therefore the result was divided by 140, in order to obtain a single representative flight. The result is given in Table 5.8. As is shown in Table 5.8 the cycles are applied in a high-low-high (hi-lo-hi) sequence that repeated 5 times. An exception to the above are the three cycles at the beginning and the cycle at the end which only occur once.

Table 5.3 compares the rain-flow cycles for the reconstructed and the original history. As is seen, some cycles appear at the higher rain-flow ranges for the reconstructed history than for the original history. Recall that in this case most of the fatigue damage is done at one particular stress level. Therefore, the difference in the rain-flow matrix of the original, compared to the peak-valley reconstruction, does not affect the fatigue life significantly. The calculated results for the peak-valley reconstructed history using both materials are given in Table 5.9 and are plotted for titanium in Fig. 5.7. Comparison shows that the lives calculated are slightly shorter than those for Helix; therefore, as expected, the reconstructed history is more conservative.

5.7 Peak-Valley Reconstruction of Maneuver History

A peak-valley reconstruction was done for the modified maneuver history using the same procedure as described for Helix. The result is shown in Table 5.10. The upper and lower bounds on life were predicted for titanium 6Al-4V using only the constant amplitude strain-life curve. The results are given in Table 5.11 and are plotted on Fig. 5.9.

Table 5.8. Peak-Valley Reconstruction of Helix

(a) Blocks of Stress Cycles				
Block No.	Stresses for $S_{max} = 100$ units Minimum	Maximum	Number of Repetitions	Comment
1	-20	100	1	Do once at start
2	12	100	2	of each flight
3	16	100	1	
4	20	100	2	
5	24	96	8	
6	28	96	5	
7	32	96	12	
8	36	96	28	Repeat 10 times
9	36	92	803	(1520 Cycles each time)
10	40	92	19	
11	40	88	273	
12	40	84	181	
13	40	80	10	
14	44	80	151	
15	48	80	4	
16	52	80	13	
17	52	76	5	
18	52	72	5	
19	52	68	1	Omit as trivial
20	56	68	0	(occurs 10 times)
21	60	68	1	
22	-20	--	1	Return to level to start next flight

(b) Sequence of Blocks by No. for Comp. Flight (15,203 Cycles)	
1	Start flight
2	
3 thru 18, 18 thru 3	This is a hi-lo-hi
3 thru 18, 18 thru 3	sequence of blocks
3 thru 18, 18 thru 3	repeated 5 times
3 thru 18, 18 thru 3	
3 thru 18, 18 thru 3	
22	End flight

Note: if the flight as in (b) is repeated 140 times, this will approximate one repition of all of Helix, and it will contain 2,128,429 cycles.

Table 5.9. Calculated Flights to Failure for P-V Reconstruction of Helix (S.C.F. =2.5)

S_{max}, MPa (ksi) (net area)	Peak-Valley Reconstruction Lower Bd.	Upper Bd.
(a) 2024-T4 Al, Const. Ampl. Strain-Life Curve		
138 (20)	8562	8563
(b) 2024-T4 Al, Prestrained Strain-Life Curve		
138 (20)	1057	1057
276 (40)	7.12	7.90
317 (46)	2.95	3.66
(c) Ti 6Al-4V, Const. Ampl. Strain-Life Curve		
317 (46)	20258	20272
358 (52)	3431	3443
407 (59)	673	686
455 (66)	176	190
517 (75)	38.5	50.2

Comparison of the reconstructed and original histories in Table 5.5 shows that rain-flow cycles are introduced at relatively high levels that do not occur in the original history. As a result, the calculated lives are shortened by more than a factor of 10. This reconstructed history produces a life which is grossly conservative; therefore it was not used in testing.

5.8 To-From Reconstruction of Maneuver History

The maneuver load history is divided into discrete intervals numbered from 1 to 32. The transition between adjacent peaks and valleys, or valleys and peaks, is then plotted in a 32 by 32 matrix. Figure 5.14 shows this matrix. A computer program [63] was obtained for reconstruction purposes. Figure 5.4(b) shows a portion of the to-from reconstruction of this history.

The distribution of number of cycles with rain-flow range is given in Table 5.5 and is plotted in Fig. 5.15. The comparison of the distribution of cycles as shown in Fig. 5.15 with the original history in Fig. 5.10 shows that the to-from reconstruction shifts some cycles to relatively high ranges which occur at considerably lower range in the original history.

Figure 5.16 shows the usage factor vs. range of rain-flow counted cycles for the to-from reconstruction of the modified maneuver history. Comparison of Fig. 5.16 with 5.11 shows that the distribution of usage fraction with range for the to-from reconstruction is quite different from that for the modified history for the same stress level. In particular, the calculated most damaging level for the reconstructed history is about 75 % of the highest range, rather than 100 %. In general, the distribution of the usage fraction with range of the two histories is not similar.

Table 5.10. Peak-Valley Reconstruction of Maneuver History

(a) Blocks of Stress Cycles

Block No.	Stresses for S_{max} = 100 units Minimum	Maximum	Number of Repetitions	Comment
1	-0.517	1.000	1	Do once (5 Cycles)
2	-0.517	0.980	4	
3	-0.517	0.933	3	
4	-0.467	0.833	6	
5	-0.417	0.883	1	
6	-0.417	0.833	4	
7	-0.367	0.833	2	
8	-0.267	0.833	7	Repeat 10 times
9	-0.217	0.833	2	(611 Cycles each time)
10	-0.217	0.783	20	
11	-0.217	0.733	21	
12	-0.217	0.683	16	
13	-0.167	0.683	11	
14	-0.167	0.633	31	
15	-0.167	0.583	17	
16	-0.117	0.583	15	
17	-0.117	0.533	29	
18	-0.117	0.483	32	
19	-0.067	0.483	3	
20	-0.067	0.433	38	
21	-0.067	0.383	31	
22	-0.017	0.383	10	
23	-0.017	0.333	57	
24	-0.017	0.283	23	
25	0.033	0.283	49	
26	0.033	0.233	51	
27	0.083	0.233	30	
28	0.083	0.183	52	
29	0.133	0.183	50	
30	-0.517	----	1	Return to starting pt.

(b) Sequence of Blocks by No. for One Repetition (6115 Cycles)

1	Start flight
2	
3 thru 29, 29 thru 3	This is a hi-lo-hi
3 thru 29, 29 thru 3	sequence of blocks
3 thru 29, 29 thru 3	repeated 5 times
3 thru 29, 29 thru 3	
3 thru 29, 29 thru 3	
30	End flight

Table 5.11. Calculated Repetitions to Failure for Recons. of Maneuver History (S.C.F.= 2.5)

S_{max}, MPa (ksi) (net area)	Peak-Valley Reconstruction Lower Bd.	Peak-Valley Reconstruction Upper Bd.	To-From Reconstruction Lower Bd.	To-From Reconstruction Upper Bd.
(a) Ti 6Al-4V, Const. Ampl. Strain-Life Curve				
276 (40)	3490	3490	19700	19700
379 (55)	77.0	81.0	342	366
586 (85)	2.06	2.99	10.5	10.5
827 (120)	0.327	0.489	1.34	1.90
(b) Ti 6Al-4V, Prestrained Strain-Life Curve				
276 (40)	----	----	1660	1680
379 (55)	----	----	155	165
586 (85)	----	----	11.3	13.3
827 (120)	----	----	2.25	2.82

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
1				1	1	2																										
2				3	19	6	16	1																								
3				2	18	7	10	7								1																
4		4	2		5	12	3	4	4			1																				
5	3	28	11	1		1								1																		
6		2	9	16							2	5	2	1	3																	
7	1	9	15	4							4	30	80	39	26	8	2										1					
8		1	7	4								36	197	171	196	42	17	3	1													
9				3	1								86	132	141	101	61	29	9	3	4	4	2	1								
10														63	208	252	173	76	21	8	9	5	4	4	1							
11						2	4						11	1	70	171	179	88	33	15	6	6	4	1	1							
12			1			6	36	30					104	67	38	94	119	152	104	23	14	6	3	2	1	1						1
13				1		2	71	206	87	5	20	88		3	37	56	96	139	126	38	14	7	6	5	5	3	3					
14					1		43	171	118	72	6	62	6		39	120	129	103	90	90	61	21	19	16	3	5	1	1				
15						3	21	192	162	211	62	44	38	34		14	24	82	82	117	88	53	21	10	8	14	4	2				
16						1	11	42	93	251	175	102	53	118	13		1	9	67	80	100	81	53	27	18	16	13	2				
17							3	12	63	159	179	120	96	134	24			2	2	23	70	90	63	67	42	36	16	5	2	1		
18							1	6	26	74	79	161	147	97	74	17	1			1	18	49	79	91	89	61	32	26	9	1	1	
19								1	8	22	36	101	113	94	97	49	5				1	14	26	74	98	67	53	62	24	13	2	2
20							1	1	6	8	13	23	40	94	104	93	13	3					3	23	38	42	69	63	61	33	17	4
21									2	8	7	14	14	46	100	87	78	20						3	3	16	17	29	69	26	17	2
22									5	4	6	6	6	27	36	88	93	56	9							1	4	8	8	10	3	2
23									3	4	2	4	10	17	28	42	65	61	43	3								1	1	2	1	
24									1	4	2	2	4	11	14	30	68	99	68	20	1											
25									1	1			2	10	16	12	40	90	88	41	6											
26													6	5	9	21	34	50	70	53	12	1										
27													3	2	2	13	10	46	56	60	18	2										
28												1			2	3	9	18	46	61	41	7										
29																1	1	9	30	62	46	14	1									
30												1				1	1	3	10	27	32	7	4									
31																		1	4	19	12	5										
32																			1	7	3											

Figure 5.14. To-From Matrix for the Modified Maneuver History.

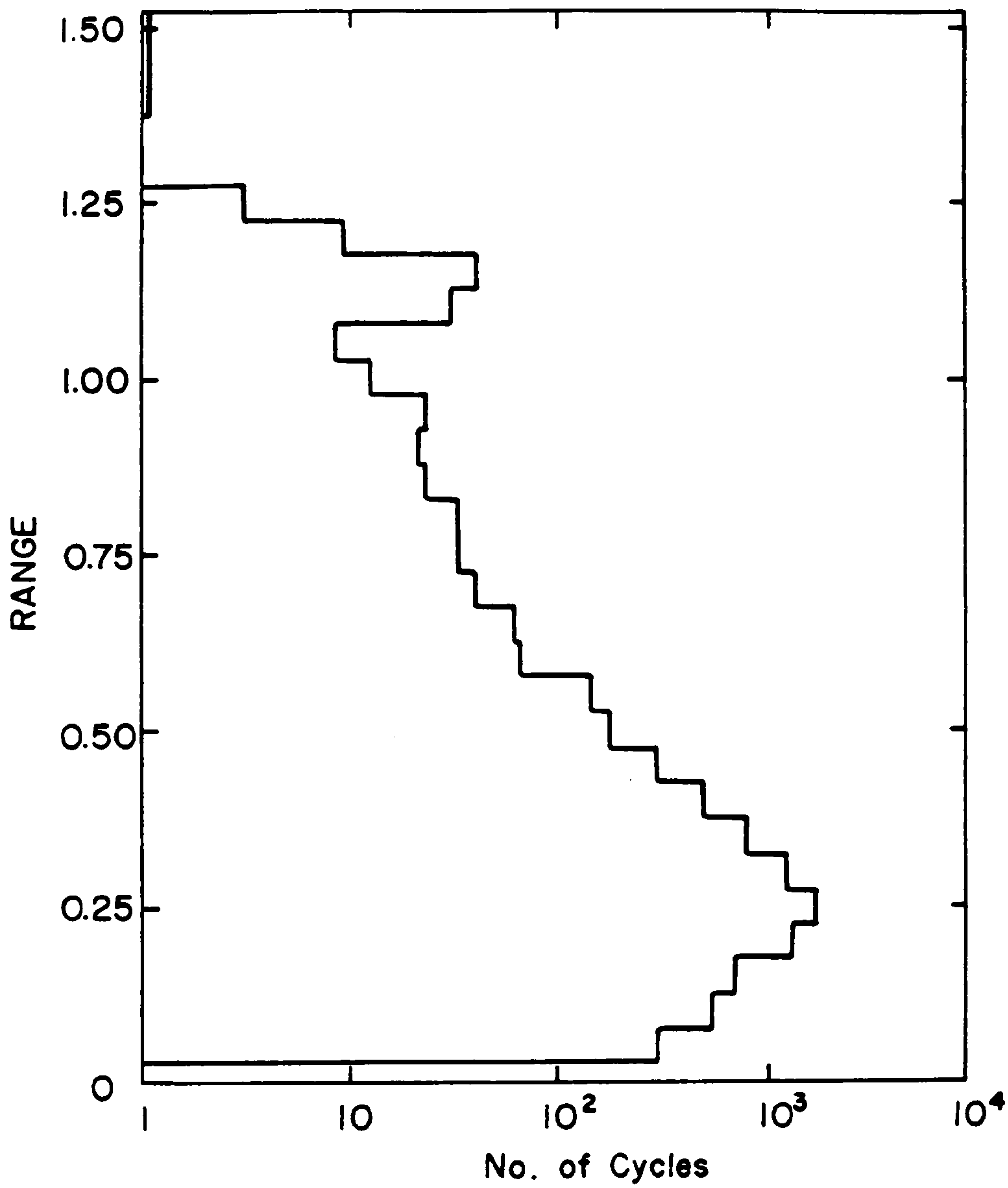


Figure 5.15. No. of Cycles vs. Range of the R-F Method for the To-From History: The to-from reconstruction is based on the maneuver history.

Life calculations for the to-from reconstruction are given in Table 5.11 and are plotted along with the test data in Fig. 5.9. Comparing these with the calculations for peak-valley reconstruction and modified history, it is evident that this reconstruction gives shorter lives than the original history. That is, this method is conservative, but not as conservative as the peak-valley reconstruction. Based on the above discussion, it is clear that neither the peak-valley nor the to-from reconstructions were very accurate for the maneuver history.

5.9 Rain-Flow Reconstruction of Maneuver History

The procedure described in Chapter 4 was used for reconstruction of the filtered maneuver history. The filtered history was first summarized using the rain-flow cycle counting into the compact form of a 32 by 32 matrix giving combinations of peak and valley values which correspond to rain-flow cycles. Recall that a history could be reconstructed in two ways depending on the definitions of the rain-flow matrix.

Figures 5.13(b) and 5.13(c) show the simplest reconstructed history and a more irregular reconstructed history for the filtered maneuver history, when the cycle directions are **not** considered. Recall that the simplest history is obtained by placing all the cycles having the same range and mean in a single location. The more irregular version can be formed by simply dividing the number of cycles for a given peak-valley combination into "n" groups, and each group is placed into a possible location randomly.

Figure 5.13(d) shows the simplest reconstruction history for the filtered history when the directions of the cycles are retained.

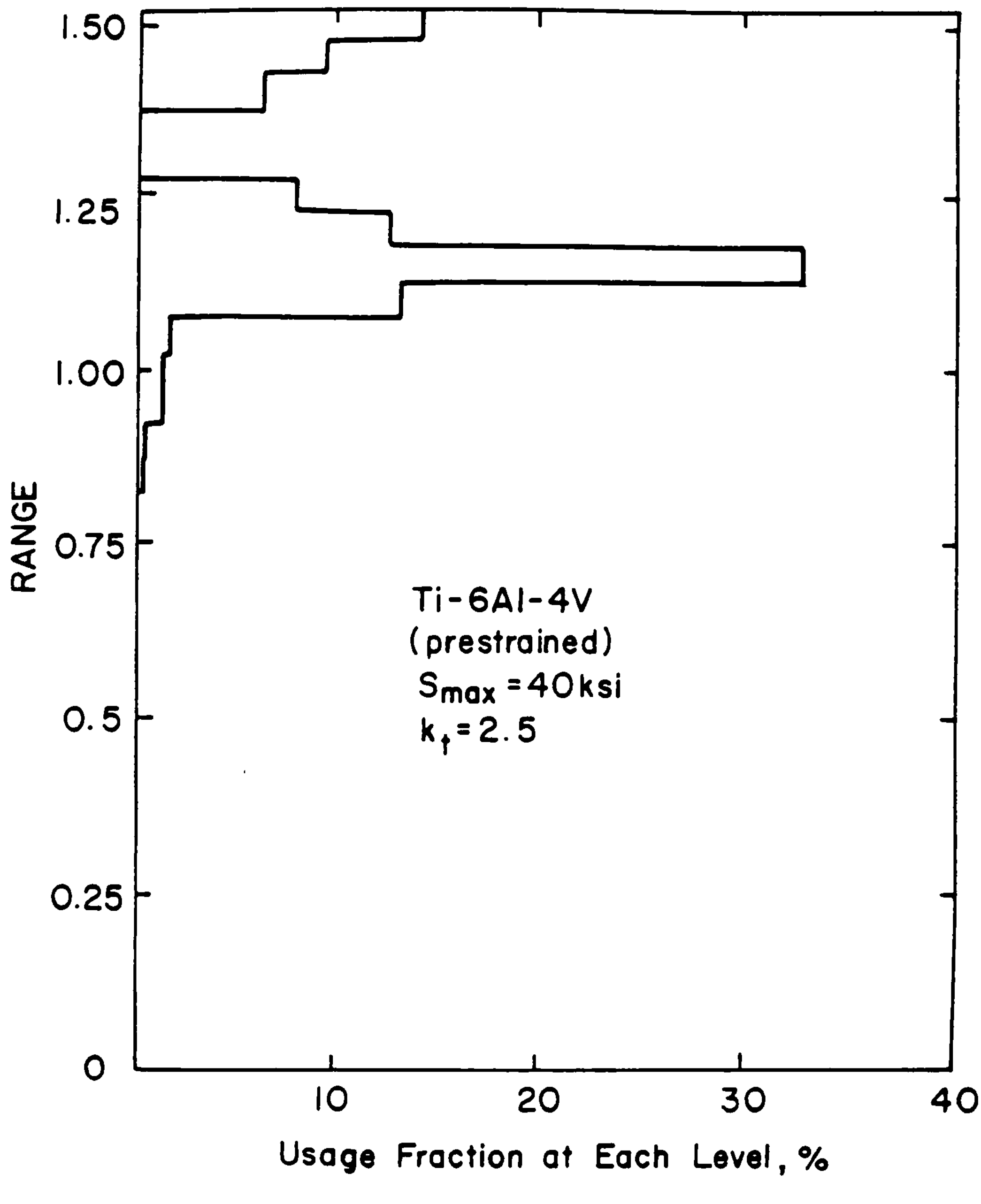


Figure 5.16. Usage Fraction vs. Range of the R-F Method for the To-From History.

The calculated lives for the reconstructed histories based on the rain-flow counting method will be similar to that for the original history, since the reconstructed histories have the same peak-valley matrix of the rain-flow cycles as the original history.

The filtered history was also summarized by using the rain-flow method in the compact form of a three dimensional matrix of 16 by 16 by 16 giving combinations of peak, valley, and peak, or valley, peak, and valley. The algorithm explained in Chapter 4 was used to reconstruct a history from the rain-flow matrix of the filtered history. Figure 5.17 shows the simplest form of the reconstruction history for the filtered history using the three dimensional matrix. Note that the fatigue life of this reconstructed history is also similar to the original filtered history.

5.10 Reconstruction of a History Not Reduced to Peaks and Valleys

In order to proceed with this method a history in its original form of peak,valley, and intermediate data points is required. Note that peaks and valleys were known for both Helix and Maneuver histories. Therefore, it was decided to generate a history for this purpose. In order to achieve the above, an arbitrary power spectrum density (PSD) is required. Using this PSD a sample of x(t) is simulated using the following equation [64]:

$$x(t) = \sum_{k=1}^{N} [2S(\omega_k)\Delta\omega_k]^{\frac{1}{2}} \cos(\omega_k t + \psi_k) \tag{5.2}$$

where $S(\omega)$ is the one sided spectral density function, $\psi_k(k=1,2,......)$ are independent and identically distributed random phases with a uniform density function $1/2\pi$ in $(0,2\pi)$.

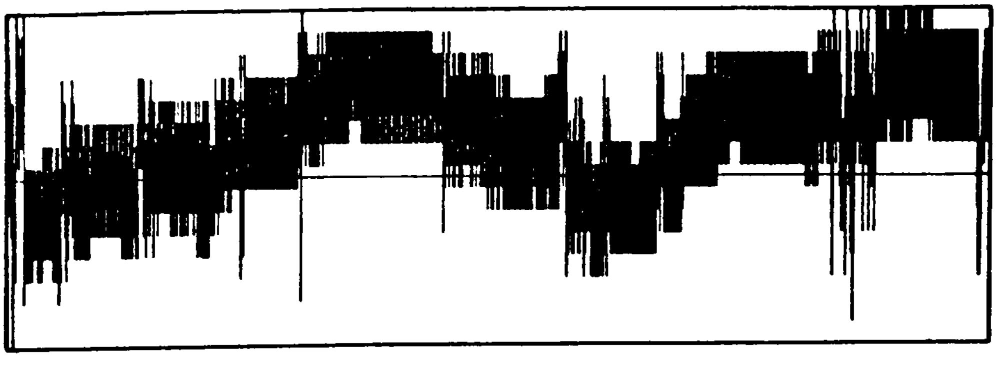


Figure 5.17. 3-D Reconstructed History from the Filtered History.

A rectangular-shaped PSD is chosen and, using the above equation, a history containing 50,000 points is generated. Then this history is transformed to a peak-valley point process containing 4231 cycles. Table 5.12 gives the cycles from rain-flow counting of the generated history. Method two explained in Chapter 4 was used to reconstruct a history from the generated history. This reconstructed history contains 49,800 points. Note that in this method both rain-flow cycles and distributions of relative time increments between adjacent peaks and valleys are preserved. Mean and standard deviation values of the original and reconstructed histories were obtained using Eqs. 5.3 and 5.4:

$$\tilde{x} = \frac{1}{N}\sum_{i=1}^{N} x_i \tag{5.3}$$

$$\sigma^2 = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \tilde{x})^2 \tag{5.4}$$

where $\tilde{x}$ is the mean process, N is the total number of peaks and valleys in the history, σ^2 is the variance, and σ is the standard deviation. It was found that the original and reconstructed histories have similar mean and standard deviation values. PSD's of these histories are not similar, as is seen in Fig.5.18, but both produce similar lives since the reconstructed history has the same rain-flow cycles as the original history. It is likely that for a larger history, say 1,000,000 points, the PSD's would be more similar.

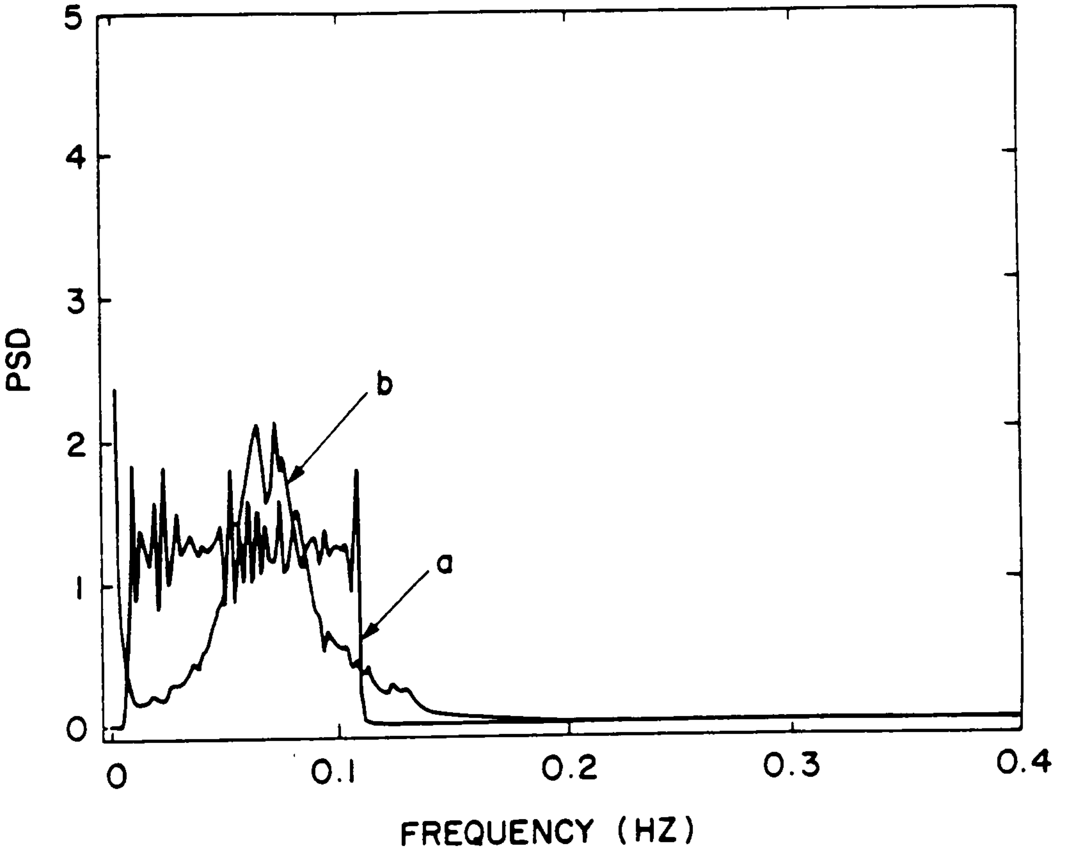


Figure 5.18. Comparison of the PSD's of the Original (a), and Reconstructed (b) Histories.

Table 5.12. Cycles from Rain-Flow Counting of the Generated History

Range (16 Levels)	No. of Cycles
1	3
2	6
3	38
4	146
5	307
6	563
7	628
8	600
9	564
10	615
11	410
12	245
13	81
14	23
15	1
16	1

6.0 Discussion

A simplified method (upper-lower bound) for calculating fatigue crack initiation life, based on the local strain approach, is introduced in detail, programmed, and demonstrated for various load histories. This method has two very distinct advantages. First, only a rain-flow matrix in the compact form of range-mean values is required as the input information. This compact form is much easier to handle and store than the full history in the form of the peaks and valleys. Recall that in the local strain approach the history in its full size is required. Secondly, the life calculations are simpler and more economical. Fatigue lives were calculated for Helix using both methods. It was found that the computer time requirements for the simplified method were less by an order of magnitude than those for the local strain approach.

In order to verify that the fatigue process has been properly modeled, fatigue lives should be calculated for different values of S_{max} values. Then these values should be compared with test data. Based on the comparison of the calculated lives and test data, conclusions can be reached. The calculated lives and the test data are shown in Figs. 5.6, 5.7, and 5.9.

Figure 5.6 shows a comparison between calculated lives using the local strain approach and the upper-lower bound. There is a reasonable agreement between the two methods and where the bounds are tight, the calculated lives are similar.

From Figs. 5.6, 5.7, and 5.9, it is seen that the calculated lives and the test data are in good agreement when the prestrained data are used. The only exceptions are in the lowest levels, where the data approach or even exceed the calculation based on the constant amplitude strain-life data. The reason for this trend appears to be that these stress levels are so low that even the highest stress level (ground-air-ground) is approaching the endurance limit. The interpretation is made that when the major cycle is sufficiently low, an endurance limit is expected for the spectrum loading. Based on the above discussion, small cycles with amplitudes below the endurance limit can cause fatigue damage if preceded by the major cycle above the endurance limit.

The calculated lives tend to be reasonably accurate at high stress levels, although there is considerable scatter. Recall that in the case of Helix (Fig. 5.8) most damage was done by the relatively low levels. This contrasts with the situation for the Maneuver history, where the most damaging levels were calculated to be those near the maximum rain-flow range in the history.

Finally, from the above discussion and comparison between test data and calculated lives, it can be concluded that the upper and lower bound provides a reliable method, and it is recommended that this method be used as the crack initiation algorithm for predicting the fatigue life.

On the subject of methods of reconstructing histories, it is seen that the peak-valley and the to-from methods can produce histories which have lives that are excessively conservative. This is the case where rain-flow cycles with relatively large ranges are introduced by large mean level variations. Also, note that one of the assumptions in reconstructing load history based on the to-from matrix is that the real history is a random Gaussian process [8]. Therefore, the question arises as to how to decide from a fatigue point of view if the random assumption is valid, and as to the consequences of various degrees of departure from this assumption [10].

Based on the life calculation comparisons for the reconstructed histories, it appears that the most promising reconstructing method is some version of a rain-flow reconstructed method. Although rain-flow reconstructed histories do not generally produce the same loading sequence as the original history, this is not generally expected to affect the life. This occurs because the rain-flow matrix will be identical to the original rain-flow matrix; therefore the same upper and lower bounds on life will be estimated from a simplified local strain approach. Since the differences in life due to any possible sequence are expected to be within the upper and lower bounds, it is of course necessary for these bounds to be reasonably tight. This is the case for the Helix and the maneuver loading, and it is also expected to be so for most irregular loading histories from actual in-service field data.

Perrett's work [12] also suggests that reconstructed histories producing identical rain-flow cycles give similar lives as original histories. This conclusion is based on test data for total life for crack initiation plus growth, and also on analysis of crack growth for standard aircraft spectra. He also suggests that, in all cases where the reconstructions process has been randomized, there is not a significant load interaction effect beyond that accounted for by rain-flow counting. The evidence to date suggests that all of the possible rain-flow reconstructions from a given rain-flow matrix cause similar lives for both crack initiation and growth.

Finally, on the subject of reconstructing histories in their original forms, it is shown that the reconstructed histories produce identical rain-flow cycles and the same distribution of relative time increments. However, the power spectral density's (PSD's) of the two histories are not identical. It should be mentioned that more work is needed in this approach, especially for determination of a statistical relationship between the intermediate data points and the corresponding peak and valley. In this case also the reconstructed history gives the same (relatively narrow) bounds on fatigue life as the original history regardless of different PSD's, since the rain-flow matrices of both histories are identical. Note that the influence of the power spectral density (PSD) as a service history fatigue life parameter has been discounted in the literature [41,64,65]. Based on the past investigations, it can be concluded that PSD does not affect the fatigue life; therefore, two histories with the same rain-flow matrix and different PSD's will have similar fatigue life.

7.0 Conclusions and Recommendations

The following conclusions and recommendations are drawn based on this work:

1. The simple P-M rule may be employed in cumulative damage calculations provided there is proper handling of cycle counting, local notch mean stress effects, and overstrain effects. By using the rain-flow cycle counting, local strain approach, and prestrained material constants, the above are considered in the life calculation.

2. The endurance limit should be used only with caution. This limit can be assumed to exist only when the major cycle is below it.

3. Life calculations based on non-overstrained constant amplitude are often non-conservative.

4. The simplified version of the local strain approach (upper and lower bounds) should be used more widely, since only a rain-flow matrix is required as input information, and since it is easy to program on a digital computer. This approach is more economical than analysis of the large amount of data involved in full time sequence load histories.

5. Both peak-valley and to-from reconstructed histories may produce lives which are excessively conservative.

6. Reconstruction based on the rain-flow method appears to be a promising approach since it produces a history with a rain-flow cycle counting matrix identical to the one for the original history. As a result, the life is expected to be similar to the life for the original history.

7. All reconstruction approaches based on the rain-flow cycle counting method generate histories which give identical calculated upper and lower bounds on life to the original history, where the local strain approach is used as the basis of life calculation for crack initiation.

8. Reconstructions of histories that preserve both rain-flow cycles and distribution of relative time intervals between adjacent peaks and valleys also appear to be a promising approach. These results should be regarded as a preliminary effort, and additional work is needed to develop and experimentally verify this method.

References

1. Landgraf, R. W., Richards, F. D., and Lapointe, N. R., "Fatigue Life Predictions for a Notched Member Under Complex Load Histories," *Fatigue Under Complex Loading: Analyses and Experiments*, Vol. AE6, The Society of Automotive Engineers, 1977, pp. 95-106.

2. Dowling, N. E., Brose, W. R. and Wilson, W.K., "Notched Member Fatigue Life Prediction by the Local Strain Approach," *Fatigue Under Complex Loading; Analyses and Experiments*, Vol. AE-6, The Society of Automotive Engineers, 1977, pp. 55-84.

3. Socie, D. F., "Fatigue Life Prediction Using Local Stress-Strain Concepts," *Experimental Mechanics*, SESA, Vol. 17, No. 2, Feb. 1977, pp. 50-56.

4. Dowling, N. E., "Fatigue Failure Prediction for Complex Load Versus Time Histories," *Section 7.4 of Pressure Vessels and piping: Design Technology–1982– Decade of Progress*, Ed. by S. Y. Zamrik and D. Dietrich, Book No. G00213, ASME, 1982. Also published in *Journal of Engineering Materials and Technology*, ASME Vol. 105, July 1983, pp.206-214, with Erratum, Oct. 1983, p. 321.

5. Dowling, N. E., "Fatigue Failure Prediction for Complicated Stress-Strain Histories," *Journal of Materials*, ASTM, Vol. 7, No. 1, March 1972. pp. 71-87.

6. Conle, A. and Landgraf, R. W., "A Fatigue Analysis Program for Ground Vehicle Components," *Proceedings of the International Conferences on Digital Techniques in Fatigue (SEECO '83)*, Society of Environmental Engineers, March 1983, London, pp. 1-28.

7. Dowling, N. E. and Khosrovaneh, A. K., "Simplified Analysis of Helicopter Load Spectra," Paper for the ASTM Symposium on Development of Fatigue Loading Spectra, April 1987, Cincinnati, OH.

8. Haibach, E., Fischer, W., and Huck, M., "A Standard Random Loading Sequence of Gaussian Type Recommended for General Application in Fatigue Testing: Its Mathematical Background and digital Generation," *Fatigue Testing and Design*, Vol 2, 1976.

9. Fash, J. W., Conle, F. A., and Minter, G.L., "Analysis of Irregular Loading Histories for SAE Biaxial Fatigue Program," to be published by *the Society of Automotive Engineers*, Warrendale, Pa.

10. Dowling, N. E. and Thangitham, S., "Concise Description and Reconstruction of Spectrum Loading," *Proceeding of the 14th ICAF Symposium*, Ottawa, Canada, June 1987.

11. ten Have, A. A.,"European Approaches in Standard Spectrum Development," NLR-MP-87007-U, National Aerospace Laboratory, Emmerloord, The Netherlands. Paper for ASTM Symposium on Development of Fatigue Loading Spectra, Cincinnati, Ohio, April 1987.

12. Perrett, B., "An Evaluation of a Method for Reconstructing Fatigue Test Loading Sequences From Load Data Acquired via Rain-flow Counting," paper for the 14th ICAF Symposium, Ottawa, Ontario, Canada, June 1987.

13. Khosrovaneh, A. K. and Dowling, N. E.,"Analysis and Reconstruction of Helicopter Load Spectra," *Paper for the American Helicopter Society National Technical Specialists Meeting on Advanced Rotorcraft Structures,* Williamsburg, VA, Oct. 1988

14. Palmgren, A., *Ball and Roller Engineering*, Translated by G. Palmgren and B. Ruley, SFK Industries, Inc., Philadelphia, 1945, pp. 82-83.

15. Miner, M. A.,"Cumulative Damage in Fatigue," *Journal of Applied Mechanics,* ASME, Vol. 12, Sept. 1945, pp. A-159-164.

16. Manson, S. S., Freche, J. C., and Ensign, C.R., "Application of a Double Linear Damage Rule to Cumulative Fatigue," NASA TN D-3839. 1967.

17. Hashin, Z. and Rotem, A., " A Cumulative Damage Theory of Fatigue Failure," *Material Science and Engineering,* Vol. 34, 1978, pp. 147-160.

18. Miller, K. J. and Zachariah, K. P., "Cumulative Damage Laws for Fatigue Initiation and Stage 1 Propagation," *Journal of Strain Analysis,* Vol. 12, 1977, pp. 262-270.

19. Kujawski, D. and Ellyin, F., "A Cumulative Damage Theory for Fatigue Crack Initiation and Propagation," *International Journal of Fatigue,* Vol. 6, No.2, April 1984.

20. Schutz, W., "Lebensdauer-Berechnung bei Bean Spruchungen mit beliebiegen Last-Zeit-Funktionen," VDI Berichte, No. 268, 1976, pp. 113-138.

21. "Standard Practice for Cycle Counting in Fatigue Analysis," *1986 Annual Book of ASTM Standards, Vol. 03.01,* Standard NO. 1949, pp. 836-848.

22. Matsuishi, M. and Endo, T., "Fatigue of Metals Subjected to Varying Stress," presented at Japan Society of Mechanical Engineers, Funkuoka, Japan, March 1968.

23. Downing, S. D. and Socie, D. F.,"Simple Rain-Flow Counting Algorithm" *International Journal of Fatigue,* Vol. 4, No. 1, Jan. 1982, pp. 31-40.

24. Glinka, G. and Kam, J. C., "Rain-Flow Counting Algorithm for Very Long Stress Histories," *International Journal of Fatigue,* Vol. 9, No. 4, Oct. 1987.

25. Rychlik, I., "A New Definition of the Rain-Flow Cycle Counting Method" *International Journal of Fatigue,* Vol. 9, No. 2, April 1987, pp. 119-121.

26. Lindgren, G. and Rychlik, I., "Rain-Flow Cycle Distribution for Fatigue Life Prediction Under Gaussian Load Processes," *Fatigue & Fracture of Engineering Material and Structure,* Vol.10, No. 3. 1987.

27. Dowling, N. E., "A Review of Fatigue Life Prediction Methods," *SAE Technical Paper Series,* Passenger Car Meeting and Exposition, Deaborn, Michigan, Oct. 19-22, 1987.

28. Graham, J. A., Millian, J. F., and Appl, F. J., Editors, *Fatigue Design Handbook.* New York, Society of Automotive Engineers, 1968.

29. Wetzel, R. M., "A Method of Fatigue Damage Analysis," Ph.D. Thesis, Dept. of Civil Engineering, University of Waterloo, Ontario, Canada, 1971. See also Technical Report No. SR 71-107, Scientific Research Staff, Ford Motor Company, Dearborn, Michigan, Aug. 1971.

30. Martin, J. F., Topper, T. H., and Sinclair, G. M., "Computer Based Simulation of Cyclic Stress-Strain Behavior with Applications to Fatigue," *Material Research and Standards,* ASTM, Vol. 11, No. 2, Feb. 1971, pp. 23-29.

31. Landgraf, R. W., Morrow, J. and Endo, T., "Determination of the Cyclic Stress-Strain Curve," *Journal of Materials,* Vol.4, No.1, March 1969, pp.176-188.

32. Neuber, H., " Theory of Stress Concentration for Shear Strained Prismatical Bodies with Arbitrary Nonlinear Stress-Strain Law," *Journal of Applied Mechanics*, Vol. 28, Dec. 1961. pp.544-550.

33. Landgraf, R. W., "The Resistance of Metals to Cyclic Deformation," *Achievement of High Fatigue Resistance in Metals and Alloys*, ASTM STP 467, American Society for Testing Materials, 1970, pp. 3-36.

34. Jhansale, H. R., "A New Parameter for the Hysteretic Stress-Strain Behavior of Metals," *Journal of Engineering Metals and Technology*, ASME, Jan. 1975, pp. 33-38.

35. Morrow, J., "Fatigue Properties of Metals," Section 3.2 of *Fatigue Design Handbook* , Society of Automotive Engineers, 1668. Section 3.2 is a summary of a paper presented at a meeting of division 4 of the SAE iron and steel Technical Committee, Nov. 4, 1964.

36. Smith, K. N., Watson P. and Topper, T. H., "A Stress Strain Function for the Fatigue of Metals," *Journal of Metals*, ASTM, Vol. 5, No. 4, Dec. 1970, pp. 767-778.

37. Conle, A. and Topper, T. H., "Overstrain Effects During Variable Amplitude Service History Testing," *International Journal of Fatigue*, Vol. 2, No. 3, July 1980, pp.130-136.

38. Watson, P. and Topper, T. H.,"The Effects of Overstrains on the Fatigue Behavior of Steels," Paper Presented at 1970 Fall Meeting of the Metallurgical Society of AIME, Cleveland, Ohio, Oct. 1970.

39. Topper, T. H. and Sandor, B. I., "Effects of Mean Stresses and Prestrain on Fatigue Damage Summation," ASTM STP 462, 1970, pp. 93- 104.

40. Watson, P. and Topper, T. H., "An Evaluation of the Fatigue Performance of Automotive Steels," *SAE MID-YEAR MEETING*, Montreal, Paper No.710597, June 1971.

41. Conle, A., "An Examination of Variable Amplitude Histories in Fatigue," Ph.D. Thesis, Dept. of Civil Engineering, University of Waterloo, Ontario, Canada, 1979.

42. Brose, W. R., Dowling, N. E. and Morrow, J., "Effect of Periodic Large Strain Cycles on the Fatigue Behavior of Steels," SAE, Report No. 740221, 1974.

43. Wirshing, P. H. and Light, M. C., "Fatigue Under Wide Band Random Stresses," *Journal of the Structural Division, ASCE,* Vol. 106, No. ST7, Proc. paper 15574, July 1980, pp. 1593-1607.

44. Rice, S .O., "Mathematical Analysis of Random Noise," *Bell System Technical Journal,* Vol. 23, 1944, pp. 232-282.

45. van Dijk, G. M., "Statistical Load Data Processing," 6th ICAF Symp., Miami, Fla., NASA SP 309, 1971.

46. Schijve, J., "The Analysis of Random Load-Time Histories with Relation to Fatigue Tests and Life Calculations," 2nd ICAF-AGARD Symp., Paris, France, May 1961.

47. Gassner, E., "Festigkeitsversuche mit wiederholter Beansprunchung im Flugzeugbau," Luftwissen, Vol. 6, 1939, Translation NACA TM-1087.

48. Jacoby, G. H., "Comparison of Fatigue Lives Under Conventional Program Loading and Digital Random Loading," ASTM STP 462, 1970.

49. Schijve, J., "Effect of Load Sequences on Crack Propagation Under Random Loading and Program Loading," Engineering Fracture Mechanics, Vol. 5, 1973, pp. 269-280.

50. Conle, A. and Topper, T. H., "Fatigue Service Histories: Technique for Data Collection and History Reconstruction," Paper No. 820093, Society of Automotive Engineers, 1983.

51. Lin, Y. K. *Probabilistic Theory of Structural Dynamics* McGraw- Hill, Book Co., Inc. New York, N. Y., 1967.

52. Cartwright, D. E. and Longuet-Higgins, M. S.,"The Statistical Distribution of Maxima of a Random Function," Proceeding of the Royal Society of London, Ser. A327, 1956, pp. 212-232.

53. Leybold, H., "Technique for Examining the Statistical and Power Spectra Properties of Random Time Histories," M.S. Thesis, V.P.I., Blacksburg, Virginia, May 1963.

54. Beer, F. P., et al, "An Approach to the Study of Crack Growth Under Random Loading," Leigh University, Instiute of Research Rept., June 1961.

55. Amini, A. and Trifunac, M., "Distribution of Peaks in Linear Earthquake Response," *Journal of the Engineering Mechanics Division,* ASCE, Vol. 107, 1981, pp. 207-226.

56. Newland, D. E., *An Introduction to Random Vibration and Spectral Analysis,* Longman, London, 1975.

57. Buxbaum, O., "Random Load Analysis as a Link Between Operational Stress Measurement and Fatigue Life Assessment," *Service Fatigue Loads Monitoring, Simulation, and Analysis,* ASTM Special Technical Publication, No. 671, 1971, pp. 5-20.

58. Natchigall, A. J., "Strain-Cyclic Fatigue Behavior of Ten Structural Metals Tested in Liquid Helium (4K), in Liquid Nitrogen (78K), and in Ambient Air (300K)," NASA TN D-7532, National Aeronautics and Space Administration, Lewis Research Center, Cleveland, OH, 1974.

59. Boller, C. and Seeger, T. *Material Data for Cyclic Loading* , Technische Hochschule Darmstadt, Darmstadt, FRG, book to be published by Elsevier.

60. Edwards, P.R., and Darts, J., "Standardized Fatigue Loading Seqeunce for Helicopter Rotors (Helix and Felix), Parts 1 and 2," Reports Nos. TR84084 and TR84085, Royal Aircraft Establishment, Ministry of Defense, Farnborough, Hants, England (also ICAF Documents No. 1442), 1984.

61. Wirsching, P. H., "Computer Program for Local Strain Analysis - User Manual," Aerospace and Mechanical Engineering Dept., University of Arizona, Tuscon, AZ. Report prepared under Grant 3-41-3 from NASA Lewis Research Center, Cleveland, OH, July 1982.

62. Brose, W. R., "Fatigue Life Predictions for a Notched Plate with Analysis of Mean Stress and Overstrain Effects," Report No.402, Dept. of Theoretical and Applied Mechanics, University of Illinois, Urbana, Illinois, Sept. 1975. See also *Fatigue Under Complex Loading: Analyses and Experiments,* Vol. AE-6, Society of Automotive Engineers, 1977, pp. 117-135.

63. Berens, A. P., Gallagher, J.P., Dowling, N. E., Khosrovaneh, A.K., and Thangjitham, S. ,"Helicopter Fatigue Methodology," DAAJO2-85-C-0047, Aviation Applied Technology Directorate, U.S. Army Resarch and Technology Activity, Ft. Eustis, VA, July 1987.

64. Yang, J. N., "Simulation of Random Envelope Processes," Journal of Sound and Vibration, Vol. 21, 1972, pp. 73-85.

65. Linsley, R. S., Hillberry, R. M., "Random Fatigue of 2024-T3 Al Under Two Spectra with Identical Peak Probability Density Function," ASTM STP 511, 1972.

66. Watson, P., Hoddinott, D. S., and Norman, J. P., "Periodic Overloads and Random Fatigue Behavior," ASTM STP 519, 1973.

Appendix A. Details of Upper/lower Bound Calculations

The detailed equations needed to accomplish the procedure described in Chapter 3 are given here. The example of cycle 6-7-6′ of Fig 3.5 is further employed as an example with the aid of Fig. 3.7.

It is convenient to write Eqs. 3.1 and 3.2 in general form without subscripts:

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{A}\right)^{\frac{1}{s}} \qquad (A-1)$$

$$\sigma\varepsilon = \frac{(k_t S)^2}{E} \qquad (A-2)$$

The values of the constants E, A, s and k_t are of course unchanged.

Combining Eqs. A-1 and A-2 gives a relationship involving only strain, ε, and nominal stress, S:

$$\varepsilon = \left(\frac{k_t S}{E}\right)^2 \frac{1}{\varepsilon} + \left[\frac{(k_t S)^2}{E\varepsilon A}\right]^{\frac{1}{s}} \qquad (A-3)$$

Considering Fig. 3.7, the goal is to determine the bounds on mean stress, such as σ_{0A} and σ_{0B}. Point 1 corresponds to the maximum load in the history, and 4 corresponds to the minimum load in the history. As a convenience, it is assumed that the largest absolute value of the load is positive. If not, then what follows will need to be modified with appropriate sign changes. Note that the load history is known, which implies that S values are known for all calculations, so that the unknowns are the σ and ε values. These calculations take advantage of the fact that various loop curves in either Fig. 3.7(a) or (b) have the same shape, which is that of the corresponding curve from Fig. 3.5(b) expanded with a scale factor of two.

To obtain the unknowns for point 1, let:

$$S = S_1 \tag{A-4a}$$

$$\varepsilon = \varepsilon_1 \tag{A-4b}$$

$$\sigma = \sigma_1 \tag{A-4c}$$

Substitute Eq. A-4 into Eqs. A-1 and A-3, and solve for ε_1 from Eq. A-3. Then use Eq. A-1 and solve for σ_1. To analyze the range of the major cycle 1-4-4′, let:

$$S = \frac{\Delta S_{1-4}}{2} \tag{A-5a}$$

$$\varepsilon = \frac{\Delta\varepsilon_{1-4}}{2} \tag{A-5b}$$

$$\sigma = \frac{\Delta\sigma_{1-4}}{2} \tag{A-5c}$$

Using a parrallel procedure to that just described, $\Delta\varepsilon_{1-4}$ and $\Delta\sigma_{1-4}$ are obtained. Then the stress and strain at point 4 are:

$$\varepsilon_4 = \varepsilon_1 - \Delta\varepsilon_{1-4} \tag{A-6a}$$

$$\sigma_4 = \sigma_1 - \Delta\sigma_{1-4} \qquad (A-6b)$$

To analyze the range of the minor cycle, such as 6-7, let:

$$S = \frac{\Delta S_{6-7}}{2} \qquad (A-7a)$$

$$\varepsilon = \frac{\Delta\varepsilon_{6-7}}{2} \qquad (A-7b)$$

$$\sigma = \frac{\Delta\sigma_{6-7}}{2} \qquad (A-7c)$$

Using the same procedure, $\Delta\varepsilon_{6-7}$ and $\Delta\sigma_{6-7}$ are determined.

Once the stress and strain at points 1 and 4 obtained, the points of attachment of loops A and B in Fig. 3.7 must be determined. In order to determine the point of attachment of loop A, let:

$$S = \frac{S_1 - S_A}{2} \qquad (A-8a)$$

$$\varepsilon = \frac{\varepsilon_1 - \varepsilon_A}{2} \qquad (A-8b)$$

$$\sigma = \frac{\sigma_1 - \sigma_A}{2} \qquad (A-8c)$$

Then substitute Eq. A-8 into Eqs. A-1 and A-3 , and obtain ε_A and σ_A . Then, to find the point of attachment of loop B, let:

$$S = \frac{S_B - S_4}{2} \qquad (A-9a)$$

$$\varepsilon = \frac{\varepsilon_B - \varepsilon_4}{2} \qquad (A-9b)$$

$$\sigma = \frac{\sigma_B - \sigma_4}{2} \qquad (A-9c)$$

Again substitute into Eqs. A-1 and A-3, and obtain ε_B and σ_B.

The bounds on mean stresses are then:

$$\sigma_{0B} = \sigma_B - \frac{\Delta\sigma_{6-7}}{2} \qquad (A-10a)$$

$$\sigma_{0A} = \sigma_A + \frac{\Delta\sigma_{6-7}}{2} \qquad (A-10b)$$

Next, into Eq. 3.3 substitute:

$$\varepsilon_a = \frac{\Delta\varepsilon_{6-7}}{2} \qquad (A-16)$$

and obtain N', the life for zero mean stress. Finally, substitute this N' and σ_{0B} into Eq. 3.4 to obtain the lower bound in life, N, for cycle 6-7-6′. Similarly, substitute N' and σ_{0A} to get the upper bound on N.

Following a similar procedure for all cycles smaller than the major one then allows the P-M rule, Eq. 3.5, to be employed once for all of the lower bound N values, and a second time with all the upper bound N values, to obtain bounds on the calculated number of blocks (repitation) to failure, B.

A computer program consistent with the above was developed and used in the analysis of Helix and Maneuver load histories.

Appendix B. Computer Program RAINF2 for Rain-flow Cycle Counting Analysis

A computer program for rain-flow cycle counting is provided. The program can take a lengthy load history and reduce it to a compact form of a matrix giving combinations of range and mean or peak and valley values. This information can be used for fatigue analysis. The input values are defined, and two examples using different options of the program are provided. The differences with an earlier version (RAINF1) are minor and are confined to two areas of the program as indicated by comment statements.

B.1 Program Logic

The following logic is used consistent with the ASTM Standard E1049 (Ref. 21 of this dissertation):

Let x denote the absolute value of the range under consideration, and y the previous absolute range adjacent to x.

Step 1: Determine the maximum absolute value in the history. (Note that this value can be either a peak or a valley.)

Step 2: Arrange the history to start with the maximum absolute value. Move all peaks and valleys which occur prior to the maximum load to the end as illustrated in Fig. B.1(b).

Step 3: Read the next value. If out of data, go to step 9.

Step 4: Three points are needed to define x and y. If there are less than three points, go back to step 3. Define x and y using the three most recent peaks and valleys that have not been discarded.

Step 5: Compare the two ranges, namely x and y. If x is less than y, go to step 3; otherwise go to step 6.

Step 6. If a rain-flow filtered history is not desired, go to step 8.

Step 7: If y is less than or equal to the filter level specified in the program input, discard the peak and valley of the range y in the array in memory, which is the original history of step 2.

Step 8: Count range y as one cycle, determine the mean value of the peak and valley of y, discard the peak and valley of y in the array set up in step 3, and go to step 4.

Step 9: Stop.

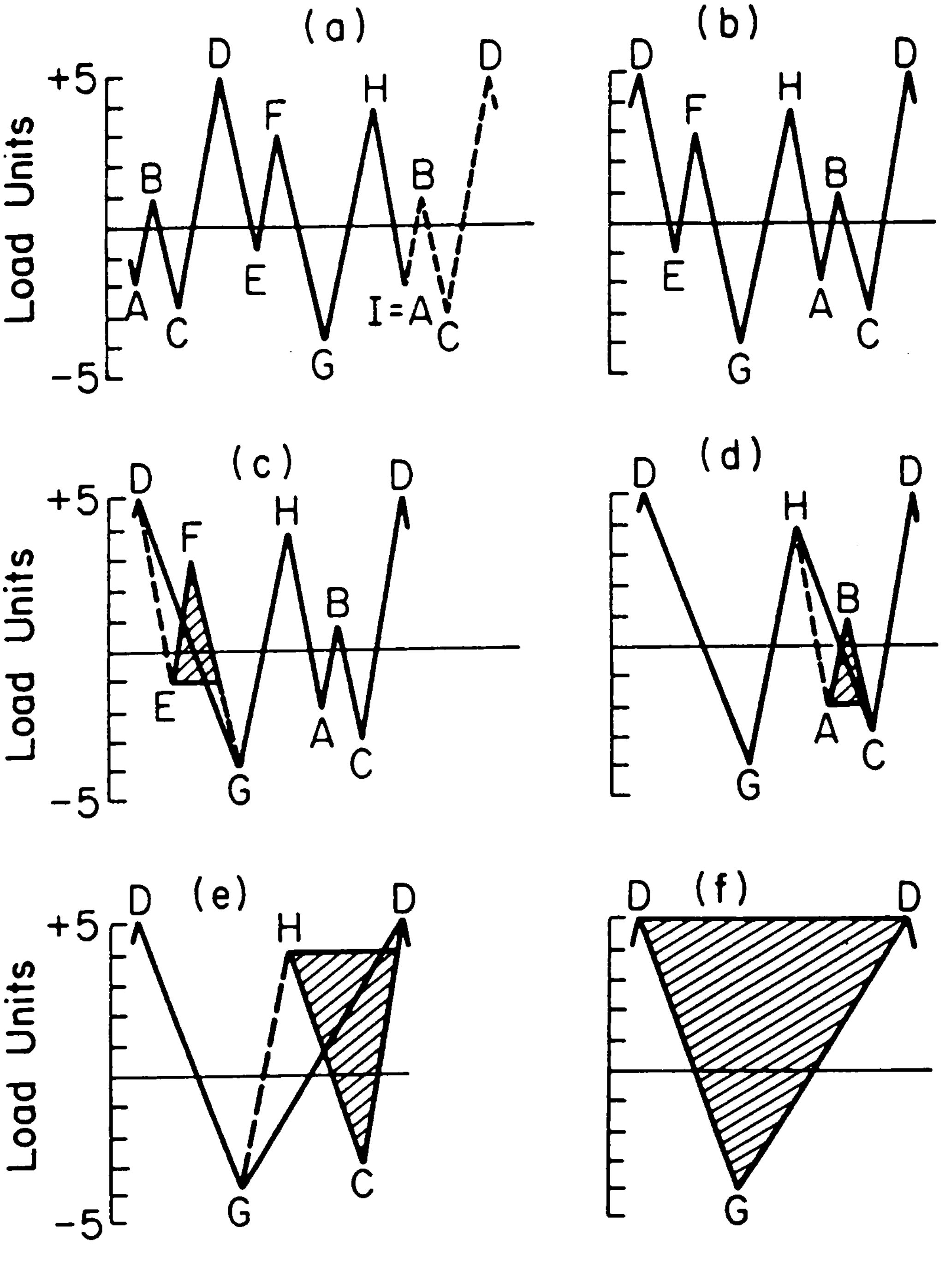


Figure B.1. Example of Rain-flow Program Logic.

B.2 Definition of Input Data

Data line 1: OPTION = 1 List filtered history as peak-valley sequence; also print range-mean matrix of rain-flow cycles for original history.

2 List range, mean, minimum, and maximum of rain-flow cycles not in matrix form.

3 Print range-mean matrix of rain-flow cycles.

4 Print starting versus target level, 32 by 32, matrix of rain-flow cycles. Note the directions of cycles are stored in this matrix. The starting level for a cycle is either a peak (max) value or a valley (min) value, whichever occurs first, and the target level is the other peak or valley that defines the cycle. Also, the history is converted to a minimum value of 1 and maximum value of 32.

Data line 2: FL = Filter level as a range value. Note that this data is required only for OPTION =1, and is otherwise omitted.

Data line 3: NN = Number of peak-valley points in the history. The history must start and end with the same value, so that this starting/ending value is counted twice in NN, and NN must be an odd number.

Data line 4: XIM = Constant increment between mean values in the range-mean matrix. Note that this data is required only if OPTION =1 or 3, and is otherwise omitted.

XIR = Constant increment between range values in the range-mean matrix. Note that this data is required only if OPTION =1 or 3, and is otherwise omitted.

Data line 5: P( ) = Input load history as peaks and valleys in sequence. Note that the history must start and end with the same value, and the direction of loading must reverse at each value.

B.3 Example 1

The history of Fig. B.1 is used for this rain-flow cycle counting example. Option 2 of the program is used; Therefore, the result is shown as a list of range, mean, minimum(valley), and maximum (peak) values. The entire program listing and program input and output for this example follow.

Program Input

```
2
9
-2.,1.,-3.,5.,-1.,3.,-4.,4.,-2.
```

PROGRAM LISTING

```
C         RAIN-FLOW COUNTING PROGRAM (RAINF2)
C   NOTE THAT THE HISTORY MUST START AND END WITH THE SAME VALUE.
C
C   INPUT
C   DATA LINE 1.  OPTION=1   LIST FILTER HISTORY AS PEAK/VALLEY
C                      SEQUENCE.ALSO PRINT RANGE/MEAN
C                      MATRIX OF RAINFLOW CYCLES FOR
C                      ORIGINAL HISTORY.
C                 =2   LIST RANGE,MEAN,MIN,AND MAX OF RAIN-
C                      FLOW CYCLES NOT IN MATRIX FORM.
C                 =3   PRINT RANGE/MEAN MATRIX OF RAINFLOW
C                      CYCLES.
C                 =4   PRINT START/TARGET MATRIX OF RAINFLOW
C                      CYCLES.
C   DATA LINE 2.       FL=FILTER VALUE AS A RANGE
C   DATA LINE 3.       NN=NUMBER OF PEAK/VALLEY POINTS IN HISTORY
C   DATA LINE 4.       XIM=CONSTANT INCREMENT BETWEEN MEAN VALUES
C                    IN THE RANGE/MEAN MATRIX.
C                XIR=CONSTANT INCREMENT BETWEEN RANGE VALUES
C                    IN THE RANGE/MEAN MATRIX.
C            NOTE THAT XIM AND XIR REQUIRED FOR OPTION=1 OR 3
C   DATA LINE 5.       P( )=INPUT LOAD HISTORY AS PEAKS AND VALLEYS
C                    IN SEQUENCE.
C
C
C
C
      REAL P(10000),PE(10000),PP(10000),PC(10000),PCC(10000),MM(64),
     *RA(64),PI(10000),MEAN(5005),R(5005)
      INTEGER M(64,64),SUM(64),SUMM,MI(32,32),OPTION
      READ(5,*)OPTION
      IF(OPTION.EQ.1)READ(5,*)FL
      READ(5,*)NN
      IF(OPTION.EQ.4)GO TO 40
      IF(OPTION.EQ.2)GO TO 40
      READ(5,*)XIR,XIM
40    READ(5,*)(P(I),I=1,NN)
      N=NN
C
C
C     DETERMINATION OF LARGEST PEAK OR VALLEY
      LCOUNT=1
      DO 100 I=1,N
      PE(I)=P(I)
100   CONTINUE
      PMAX=ABS(P(1))
      DO 200 I=2,N
      IF(PMAX.GE.PE(I)) GO TO 200
      PMAX=ABS(PE(I))
      LCOUNT=I
200   CONTINUE
      IF(OPTION.EQ.4)THEN
      SMAX=P(1)
      SMIN=P(1)
      DO 301 I=2,NN
```

```
      IF(P(I).GT.SMAX)SMAX=P(I)
      IF(P(I).LT.SMIN)SMIN=P(I)
301   CONTINUE
      CF1=SMIN
      CF2=SMAX
      CF3=SMAX-SMIN
      END IF
C
C
C     ARRANGE THE PEAK OR VALLEY
      JK=LCOUNT+1
      J=N-JK+1
      KKK=LCOUNT
      DO 300 I=1,J
      PP(I)=P(KKK)
      KKK=KKK+1
300   CONTINUE
      J=J+1
      DO 350 I=1,LCOUNT
      PP(J)=P(I)
      J=J+1
350   CONTINUE
      DO 500 I=1,NN
      PC(I)=PP(I)
500   CONTINUE
      NNN=N+1
      IF(OPTION.EQ.2)WRITE(6,210)
C
C
C     FINDING THE CYCLE
      AA=3.1422
      DO 194 I=1,32
      DO 195 J=1,32
      MI(I,J)=0
195   CONTINUE
194   CONTINUE
      I=0
      K=1
      IF(OPTION.EQ.2)WRITE(6,107)
      J=1
2     I=I+1
      IF(I.LT.3) GO TO 2
      J=J+1
      IF(I.EQ.NNN) GO TO 400
50    IF(PP(J).EQ.AA) THEN
      J=J-1
      GO TO 50
      END IF
      JM1=J-1
60    IF(PP(JM1).EQ.AA) THEN
      JM1=JM1-1
      GO TO 60
      END IF
70    IF(I.GT.NNN) GO TO 400
      X=ABS(PP(I)-PP(J))
      Y=ABS(PP(J)-PP(JM1))
```

```
      XX=(PP(J)+PP(JM1))/2.
5      IF(X.GE.Y)THEN
      IF(OPTION.NE.1)GO TO1600
      IF(Y.LE.FL) THEN
      PC(J)=AA
      PC(JM1)=AA
      END IF
1600  IF(OPTION.EQ.2)GO TO 41
      IF(OPTION.EQ.4)THEN
      PI(J)=((32.*(-CF1+PP(J)))+CF2-PP(J))/CF3
      PI(JM1)=((32.*(-CF1+PP(JM1)))+CF2-PP(JM1))/CF3
      PMAX=PI(J)
      PMIN=PI(JM1)
C
C      THE NEXT 4 LINES REPLACE 8 LINES IN RAINF1.
C
      END IF
      IF(OPTION.EQ.4)THEN
      EIJ=PMIN+.5
      EJI=PMAX+.5
      IJ=INT(EIJ)
      JI=INT(EJI)
      MI(IJ,JI)=MI(IJ,JI)+1
      END IF
      IF(OPTION.EQ.4)GO TO 42
      R(K)=Y
      MEAN(K)=XX
      K=K+1
      GO TO 42
41     PMAX=PP(J)
      PMIN=PP(JM1)
      IF(PMAX.LT.PMIN)THEN
      PMAX=PP(JM1)
      PMIN=PP(J)
      END IF
      WRITE(6,108)Y,XX,PMAX,PMIN
42     PP(J)=AA
      PP(JM1)=AA
      J=J-1
3      J=J-1
      IF(J.LT.1) GO TO 11
      IF(PP(J).EQ.AA) GO TO 3
      IF(I.EQ.4) THEN
      I=5
      J=I-1
      END IF
      JM1=J-1
      IF(JM1.LT.1)THEN
      JM1=J
      J=I
      I=I+1
      GO TO 70
      END IF
6      IF(PP(JM1).EQ.AA) GO TO 4
      X=ABS(PP(I)-PP(J))
      Y=ABS(PP(J)-PP(JM1))
```

```
      XX=(PP(J)+PP(JM1))/2.
      GO TO 5
4     JM1=JM1-1
      IF(JM1.LT.1)THEN
      JM1=J
      J=I
      I=I+1
      END IF
      GO TO 6
11    I=I+2
      J=I-1
      JM1=J-1
      GO TO 70
      ELSE
      XX=(PP(J)+PP(JM1))/2.
      J=I-1
      END IF
      GO TO 2
C
C
C
400   IF(OPTION.EQ.2) GO TO 999
      IF(OPTION.EQ.4)GO TO 998
      K=K-1
      RMAX=R(1)
      RMIN=R(1)
      RMMAX=MEAN(1)
      RMMIN=MEAN(1)
      DO 1800 I=2,K
      IF(R(I).GT.RMAX)RMAX=R(I)
      IF(R(I).LT.RMIN)RMIN=R(I)
      IF(MEAN(I).GT.RMMAX)RMMAX=MEAN(I)
      IF(MEAN(I).LT.RMMIN)RMMIN=MEAN(I)
1800  CONTINUE
      DIFR=RMAX-RMIN
      DIFM=RMMAX-RMMIN
      ER=DIFR/XIR
      EM=DIFM/XIM
      ER=ER+2
      EM=EM+2
      LEVLM=INT(EM)
      LEVLR=INT(ER)
      DO 192 L=1,LEVLR
      DO 193 LL=1,LEVLM
      M(L,LL)=0
193   CONTINUE
192   CONTINUE
      YA=RMIN-XIR
      XA=RMMIN-XIM
      WRITE(6,111)RMIN,RMAX,RMMIN,RMMAX
      XB=.50
      YB=.50
      DO 1900 I=1,K
      EI=((R(I)-YA)/XIR)+YB
      EJ=((MEAN(I)-XA)/XIM)+XB
      II=INT(EI)
```

```
      JJ=INT(EJ)
      IF(II.EQ.0)II=1
      IF(JJ.EQ.0)JJ=1
      M(II,JJ)=M(II,JJ)+1
1900  CONTINUE
C
C
C     FILTERING PROCESS
      IF(OPTION.NE.1)GO TO 1102
      KN=1
      DO 1000 II=1,NN
      IF(PC(II).EQ.AA) GO TO 1000
      PCC(KN)=PC(II)
      KN=KN+1
1000  CONTINUE
      KN=KN-1
      WRITE(6,112)
112   FORMAT('1',//15X,'FILTER HISTIRY-PEAK/VALLEY SEQUENCE')
      WRITE(6,113)FL
113   FORMAT(//15X,'FILTER LEVEL=',F7.3)
      WRITE(6,1103) KN
1103  FORMAT(//15X,'NUMBER OF POINTS IN FILTER HISTORY=',I5,//)
      WRITE(6,1001)(PCC(I),I=1,KN)
1001  FORMAT(1X,8(2X,F6.1))
C
C
C     MATRIX PREPRATION
      GO TO 1102
998   LEVLM=32
      LEVLR=32
      XIR=1
      XIM=1
      RMMIN=1
      RMIN=1
      DO 201 I=1,32
      DO 202 J=1,32
      M(I,J)=MI(I,J)
202   CONTINUE
201   CONTINUE
1102  IF(OPTION.EQ.2)GO TO 999
      MM(1)=RMMIN
      DO 900 L=2,LEVLM
      LL=L-1
      MM(L)=MM(LL)+XIM
900   CONTINUE
      RA(1)=RMIN
      DO 1100 L=2,LEVLR
      LL=L-1
1100  RA(L)=RA(LL)+XIR
      I=0
99    I=I+1
      IF(I.GT.LEVLR) GO TO 1153
      SUM(I)=0.
      DO 98 J=1,LEVLM
      SUM(I)=SUM(I)+M(I,J)
98    CONTINUE
```

```
      GO TO 99
1153  CONTINUE
999   IF(OPTION.EQ.2)GO TO 997
1151  L=1
      LB=8
1152  IF(OPTION.EQ.1)GO TO 996
      WRITE(6,116)
      GO TO 1154
996   WRITE(6,114)
      WRITE(6,115)FL
1154  IF(OPTION.EQ.4)GO TO 604
      GO TO 1150
604   WRITE(6,605)
C
C     THE NEXT STATEMENT DIFFERS FROM RAINF1
C
605   FORMAT(77X,'TOTAL',/1X,'START /****************************TARGET
     ******************************',4X,'CYCLES',/)
      GO TO 2100
1150  WRITE(6,600)
600   FORMAT(77X,'TOTAL',/1X,'RANGE  /******************************MEA
     *N****************************',3X,'CYCLES',/)
2100  WRITE(6,101)(MM(LL),LL=L,LB)
      DO 1300 I=1,LEVLR
      WRITE(6,102)RA(I),SUM(I)
      WRITE(6,103)(M(I,J),J=L,LB)
1300  CONTINUE
      IF(LB.EQ.LEVLM)GO TO 1400
      L=L+8
      LB=LB+8
      IF(LB.GT.LEVLM)LB=LEVLM
      GO TO 1152
1400  SUMM=0
      DO 1500 I=1,LEVLR
1500  SUMM=SUMM+SUM(I)
      WRITE(6,104)SUMM
101   FORMAT(12X,8(F6.1,2X))
102   FORMAT(2X,F6.1,69X,I4)
103   FORMAT('+',11X,8(I6,2X))
104   FORMAT(//,5X,'TOTAL NO OF CYCLES=',3X,I5)
997   CONTINUE
107   FORMAT(15X,'RANGE',15X,'MEAN',15X,' MAX',15X,'   MIN')
108   FORMAT(14X,F7.3,12X,F7.3,13X,F7.3,13X,F7.3)
111   FORMAT('1',//15X,'MIN RANGE=',F8.3,//15X,'MAX RANGE=',F8.3,
     *//15X,'MIN MEAN=',F8.3,//15X,'MAX MEAN=',F8.3)
116   FORMAT('1',//35X,'RAINFLOW CYCLES ')
114   FORMAT('1',//20X,'RAINFLOW CYCLES FOR ORIGINAL HISTORY')
115   FORMAT(/5X,'NO RANGE LESS THAN OR EQUAL TO FILTER LEVEL=',F7.3
     *,3X,'OCCUR IN FILTER HISTORY')
210   FORMAT('1',//10X,'RANGES COUNTED AS CYCLES BY RAINFLOW CYCLE COUNT
     *ING METHOD.')
      STOP
      END
```

PROGRAM OUTPUT:

RANGES COUNTED AS CYCLES BY RAINFLOW CYCLE COUNTING METHOD.

RANGE	MEAN	MAX	MIN
4.000	1.000	3.000	-1.000
3.000	-0.500	1.000	-2.000
7.000	0.500	4.000	-3.000
9.000	0.500	5.000	-4.000

B.4 Example 2

The history of Fig. B.1 is again used for the rain-flow cycle counting example. Option 4 is used; therefore, the results are given in the form of a compact 32 by 32 matrix containing the peak and valley values of rain-flow cycles. Also, the directions of the cycles are stored in this matrix. Note that the history is converted using linear interpolation to have a minimum value of 1 and a maximum value of 32.

This option of the cycle counting program produces the output needed for use in a separate program (RECON2) that reconstructs a loading sequence having the same rain-flow cycles as the original sequence. The reconstructed history also produces rain-flow cycles with directions the same as in the original history. RECON2 is described in Appendix C.

In scaling to 32 levels, the level number, P_m, is related to any peak or valley value from the original history, P_0, as follows:

$$P_m = \frac{31 \times (P_0 - P_{0min})}{(P_{0max} - P_{0min})} + 1 \qquad (B-1)$$

except that P_m is rounded to the nearest integer. P_{0min} and P_{0max} are the lowest valley and highest peak, respectively, in the original history. For this example, P_{0min} = -4, and P_{0max} = 5. Cycle EF, which starts at P_0 = -1, and goes to P_0 = 3, has a starting level of 11 and a target level of 25.

Program Input

```
4
9
-2.,1.,-3.,5.,-1.,3.,-4.,4.,-2.
```

PROGRAM OUTPUT:

RAINFLOW CYCLES

START /****************************TARGET****************************

	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	TOTAL CYCLES
1.0	0	0	0	0	0	0	0	0	0
2.0	0	0	0	0	0	0	0	0	0
3.0	0	0	0	0	0	0	0	0	0
4.0	0	0	0	0	0	0	0	0	0
5.0	0	0	0	0	0	0	0	0	0
6.0	0	0	0	0	0	0	0	0	0
7.0	0	0	0	0	0	0	0	0	0
8.0	0	0	0	0	0	0	0	0	1
9.0	0	0	0	0	0	0	0	0	0
10.0	0	0	0	0	0	0	0	0	0
11.0	0	0	0	0	0	0	0	0	1
12.0	0	0	0	0	0	0	0	0	0
13.0	0	0	0	0	0	0	0	0	0
14.0	0	0	0	0	0	0	0	0	0
15.0	0	0	0	0	0	0	0	0	0
16.0	0	0	0	0	0	0	0	0	0
17.0	0	0	0	0	0	0	0	0	0
18.0	0	0	0	0	0	0	0	0	0
19.0	0	0	0	0	0	0	0	0	0
20.0	0	0	0	0	0	0	0	0	0
21.0	0	0	0	0	0	0	0	0	0
22.0	0	0	0	0	0	0	0	0	0
23.0	0	0	0	0	0	0	0	0	0
24.0	0	0	0	0	0	0	0	0	0
25.0	0	0	0	0	0	0	0	0	0
26.0	0	0	0	0	0	0	0	0	0
27.0	0	0	0	0	0	0	0	0	0
28.0	0	0	0	0	0	0	0	0	0
29.0	0	0	0	1	0	0	0	0	1
30.0	0	0	0	0	0	0	0	0	0
31.0	0	0	0	0	0	0	0	0	0
32.0	1	0	0	0	0	0	0	0	1

RAINFLOW CYCLES

START /****************************TARGET**************************** TOTAL CYCLES

	9.0	10.0	11.0	12.0	13.0	14.0	15.0	16.0	TOTAL CYCLES
1.0	0	0	0	0	0	0	0	0	0
2.0	0	0	0	0	0	0	0	0	0
3.0	0	0	0	0	0	0	0	0	0
4.0	0	0	0	0	0	0	0	0	0
5.0	0	0	0	0	0	0	0	0	0
6.0	0	0	0	0	0	0	0	0	0
7.0	0	0	0	0	0	0	0	0	0
8.0	0	0	0	0	0	0	0	0	1
9.0	0	0	0	0	0	0	0	0	0
10.0	0	0	0	0	0	0	0	0	0
11.0	0	0	0	0	0	0	0	0	1
12.0	0	0	0	0	0	0	0	0	0
13.0	0	0	0	0	0	0	0	0	0
14.0	0	0	0	0	0	0	0	0	0
15.0	0	0	0	0	0	0	0	0	0
16.0	0	0	0	0	0	0	0	0	0
17.0	0	0	0	0	0	0	0	0	0
18.0	0	0	0	0	0	0	0	0	0
19.0	0	0	0	0	0	0	0	0	0
20.0	0	0	0	0	0	0	0	0	0
21.0	0	0	0	0	0	0	0	0	0
22.0	0	0	0	0	0	0	0	0	0
23.0	0	0	0	0	0	0	0	0	0
24.0	0	0	0	0	0	0	0	0	0
25.0	0	0	0	0	0	0	0	0	0
26.0	0	0	0	0	0	0	0	0	0
27.0	0	0	0	0	0	0	0	0	0
28.0	0	0	0	0	0	0	0	0	0
29.0	0	0	0	0	0	0	0	0	1
30.0	0	0	0	0	0	0	0	0	0
31.0	0	0	0	0	0	0	0	0	0
32.0	0	0	0	0	0	0	0	0	1

RAINFLOW CYCLES

START /****************************TARGET****************************

START	17.0	18.0	19.0	20.0	21.0	22.0	23.0	24.0	TOTAL CYCLES
1.0	0	0	0	0	0	0	0	0	0
2.0	0	0	0	0	0	0	0	0	0
3.0	0	0	0	0	0	0	0	0	0
4.0	0	0	0	0	0	0	0	0	0
5.0	0	0	0	0	0	0	0	0	0
6.0	0	0	0	0	0	0	0	0	0
7.0	0	0	0	0	0	0	0	0	0
8.0	0	1	0	0	0	0	0	0	1
9.0	0	0	0	0	0	0	0	0	0
10.0	0	0	0	0	0	0	0	0	0
11.0	0	0	0	0	0	0	0	0	1
12.0	0	0	0	0	0	0	0	0	0
13.0	0	0	0	0	0	0	0	0	0
14.0	0	0	0	0	0	0	0	0	0
15.0	0	0	0	0	0	0	0	0	0
16.0	0	0	0	0	0	0	0	0	0
17.0	0	0	0	0	0	0	0	0	0
18.0	0	0	0	0	0	0	0	0	0
19.0	0	0	0	0	0	0	0	0	0
20.0	0	0	0	0	0	0	0	0	0
21.0	0	0	0	0	0	0	0	0	0
22.0	0	0	0	0	0	0	0	0	0
23.0	0	0	0	0	0	0	0	0	0
24.0	0	0	0	0	0	0	0	0	0
25.0	0	0	0	0	0	0	0	0	0
26.0	0	0	0	0	0	0	0	0	0
27.0	0	0	0	0	0	0	0	0	0
28.0	0	0	0	0	0	0	0	0	0
29.0	0	0	0	0	0	0	0	0	1
30.0	0	0	0	0	0	0	0	0	0
31.0	0	0	0	0	0	0	0	0	0
32.0	0	0	0	0	0	0	0	0	1

RAINFLOW CYCLES

START /****************************TARGET**************************** TOTAL CYCLES

START	25.0	26.0	27.0	28.0	29.0	30.0	31.0	32.0	TOTAL CYCLES
1.0	0	0	0	0	0	0	0	0	0
2.0	0	0	0	0	0	0	0	0	0
3.0	0	0	0	0	0	0	0	0	0
4.0	0	0	0	0	0	0	0	0	0
5.0	0	0	0	0	0	0	0	0	0
6.0	0	0	0	0	0	0	0	0	0
7.0	0	0	0	0	0	0	0	0	0
8.0	0	0	0	0	0	0	0	0	1
9.0	0	0	0	0	0	0	0	0	0
10.0	0	0	0	0	0	0	0	0	0
11.0	1	0	0	0	0	0	0	0	1
12.0	0	0	0	0	0	0	0	0	0
13.0	0	0	0	0	0	0	0	0	0
14.0	0	0	0	0	0	0	0	0	0
15.0	0	0	0	0	0	0	0	0	0
16.0	0	0	0	0	0	0	0	0	0
17.0	0	0	0	0	0	0	0	0	0
18.0	0	0	0	0	0	0	0	0	0
19.0	0	0	0	0	0	0	0	0	0
20.0	0	0	0	0	0	0	0	0	0
21.0	0	0	0	0	0	0	0	0	0
22.0	0	0	0	0	0	0	0	0	0
23.0	0	0	0	0	0	0	0	0	0
24.0	0	0	0	0	0	0	0	0	0
25.0	0	0	0	0	0	0	0	0	0
26.0	0	0	0	0	0	0	0	0	0
27.0	0	0	0	0	0	0	0	0	0
28.0	0	0	0	0	0	0	0	0	0
29.0	0	0	0	0	0	0	0	0	1
30.0	0	0	0	0	0	0	0	0	0
31.0	0	0	0	0	0	0	0	0	0
32.0	0	0	0	0	0	0	0	0	1

TOTAL NO OF CYCLES= 4

Appendix C. Computer Program RECON2 for Two Dimensional Rain-flow Reconstruction

A computer program for reconstructing a load history from the result of rain-flow cycle counting in matrix form is provided. Note that the directions of the cycles are indicated by the position in the matrix. The numbers in the matrix , that is, the " matrix elements", are the numbers of cycles at the various peak-valley-direction combinations. The program takes the results of rain-flow cycle counting in the form of such a matrix and uses this information to reconstruct a load history. The resulting history has the same rain-flow matrix as the original history but is not generally identical as to ordering of the cycles. The program logic is explained in detail in Chapter 4 of this dissertation, with a few examples, and also in a forthcoming paper.

Note that in its simplest application, the program places all cycles having the same peak, valley and direction in a single location in the history. If a more irregular version is desired, then the number of cycles for a given peak-valley-direction combination is divided into n groups. In this program n is called NOC. This is done only if the number of cycles is greater than a specific value (NP). The group size is then rounded down to the nearest whole number, and this many cycles are placed in the first location, the same in the second location, etc., except that the number in the last location includes the rounded down number plus all of the residual from rounding. If NOC=NP=1, then cycles are placed individually, but this choice may be costly due to computer run time for lengthy histories.

C.1 . Definition of Input Data

Data line 1:	NP=	Largest matrix element which is placed in a single location. If a given row and column has this value or less, then all of these cycles are placed in one location.
Data line 2:	NOC=	The number of different locations in which cycles from a matrix element are placed. If the number of cycles for a given row and column is greater than NP, then these cycles are placed in this many randomly chosen locations.
Data line 3:	AB(,)=	The rain-flow matrix elements themselves, where directions of rain-flow cycles are considered.

Note that the rain-flow peak-valley matrix AB(,) is read into a two dimensional array, starting with the first row, moving left to right, and completing each row before going to the next row. This data can be obtained using the RAINF2 computer program (OPTION=4). The standard size of the matrix for this program is 32 by 32. However, if a different size is desired, say 16 by 16, then the LEVEL value inside the program should be changed from 32 to 16. The columns are starting levels (peaks or valleys) of cycles, and the rows are target levels.

C.2 Example 1

A history containing 9 peak-valley points is used with NP=NOC=1. Figure B.1 shows this history. The input is the rain-flow matrix of this history, with directions of rain-flow cycles considered, as obtained using the RAINF2 computer program (Appendix B, OPTION 4). Note that the reconstructed history gives the same rain-flow peak-valley matrix (with directions considered) as the original history; however, it does not have the same sequence as the original history. Figure C.1 compares the original and reconstructed histories. Note that the user must convert the history obtained (min=1, max=32) using linear interpolation to get a history compatible with the original history. The entire program listing and program input and output for this example follow.

PROGRAM INPUT

```
1
1
    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0
    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0
    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0
    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0
    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0
    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0
    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0
    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0    1    0    0    0    0    0    0    0    0    0    0    0    0
  0    0
    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0
    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0
    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0    0    0    0    0    0    0    0    1    0    0    0    0    0
  0    0
    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0
    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0
    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0
    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0
    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
```

```
0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
0  0
  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
0  0
  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
0  0
  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
0  0
  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
0  0
  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
0  0
  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
0  0
  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
0  0
  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
0  0
  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
0  0
  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
0  0
  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
0  0
  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
0  0
  0  0  0  1  0  0  0  0  0  0  0  0  0  0  0
0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
0  0
  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
0  0
  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
0  0
  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0
0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
0  0
```

PROGRAM LISTING

```
C        2-D RAIN-FLOW RECONSTRUCTION PROGRAM (RECON2)
C
C
C        THIS PROGRAM CONSTRUCT A HISTORY USING THE R-F MATRIX
C        (DIRECTIONS OF THE RAIN-FLOW CYCLES ARE CONSIDERD)
C
C         INPUT:
C
C         DATA LINE 1.  NP=A GIVEN ROW AND COLUMN HAS THIS VALUE OR
C                       LESS, THEN ALL PLACED IN ONE LOCATION.
C         DATA LINE 2.  NOC=THE NUMBER OF CYCLES FOR EACH ROW AND
C                        COLUMN, IF GREATER THAN NP, IS PLACED
C                        IN THIS MANY RANDOMLY CHOSEN LOCATIONS.
C         DATA LINE 3.  AB( , )=THE RAIN-FLOW MATRIX.
C                     (CYCLES DIRECTIONS CONSIDERED.)
C         NOTE: SIMPLEST FORM OF A RAIN-FLOW RECONSTRUCTED HISTORY
C               IS OBTAINED BY SETTING NP=100000
C
      INTEGER A(32,32),KA(15024),P(15000),PP(15000),IL(15000)
     &, PE(15000),DUM,DUM1,DUM2,JL(15000),AB(32,32)
      DIMENSION R(20000)
C
C      LEVEL IS THE NO. OF COLUMNS OR ROWS.
C
      LEVEL=32
C
      READ(5,*)NP
      READ(5,*)NOC
      DO 90 I=1,LEVEL
      READ(5,*)(AB(I,J),J=1,LEVEL)
90     CONTINUE
C
C      RANDOM NUMBER GENERATION
C
      DO 91 I=1,20000
      CALL TR(U)
      R(I)=U
91     CONTINUE
C
C      REARRANGING THE R-F MATRIX
C
      ISS=0
      IB=1
      ILA=LEVEL
      IL(1)=1
      JL(1)=LEVEL
      P(1)=1
      P(2)=LEVEL
      PP(1)=1
      PP(2)=LEVEL
      IL(2)=1
      IL(2)=LEVEL
      AB(LEVEL,1)=0
      AB(1,LEVEL)=1
```

```
      MTOTAL=2
      IS=1
      ILAST=LEVEL
      ICOUNT=0
      ISUM=3
      GO TO 3
4     J=1
      ISUM=ISUM+(2*IS)-ICOUNT
3     KI=ILAST-1
      IF(KI.LT.1)GO TO 5
      IL(ISUM)=ILAST-1
      ILAST=ILAST-1
      J=1
      JL(ISUM)=J
2     ILL=IL(ISUM)
      ILL=ILL+1
      IF(ILL.GT.LEVEL)GO TO 1
      ISUM=ISUM+1
      J=J+1
      IL(ISUM)=ILL
      JL(ISUM)=J
      GO TO 2
1     IS=IS+1
      ICOUNT=ICOUNT+1
      GO TO 4
C
C
C
5     IS=1
      ICOUNT=0
      ILAST=1
      ISUM=2
      J=LEVEL+1
      ILAST=1
      GO TO 7
6     ILAST=1
      ISUM=ISUM+(2*IS)-ICOUNT
7     JL(ISUM)=J-1
      IF(J.LT.1)GO TO 10
      J=J-1
      IL(ISUM)=ILAST
8     JLL=JL(ISUM)
      JLL=JLL+1
      IF(JLL.GT.LEVEL)GO TO 9
      ISUM=ISUM+1
      ILAST=ILAST+1
      JL(ISUM)=JLL
      IL(ISUM)=ILAST
      GO TO 8
9     IS=IS+1
      ICOUNT=ICOUNT+1
      GO TO 6
C
C      RECONSTRUCTION BEGINS
C
10     ITOTAL=LEVEL*LEVEL
```

```
      KCOUNT=1
      ISUM=2
9999   ISUM=ISUM+1
      KCOUNT=1
      IF(ISUM.GT.ITOTAL)GO TO 999
      L=0
      ILAST=IL(ISUM)
      J=JL(ISUM)
      IF(AB(ILAST,J).EQ.0)GO TO 9999
      NOC2=10000
      INUM=0
99     IF(INUM.EQ.NOC2)GO TO 9999
      IF(AB(ILAST,J).GT.NP) THEN
      INUM=INUM+1
      KNUM=AB(ILAST,J)/NOC
      A(ILAST,J)=KNUM
      NOC1=KNUM*NOC
      IF(NOC1.NE.AB(ILAST,J))THEN
      IF(INUM.EQ.NOC)A(ILAST,J)=KNUM+(AB(ILAST,J)-NOC1)
      NOC2=NOC
      ELSE
      NOC2=NOC
      END IF
      ELSE
      A(ILAST,J)=AB(ILAST,J)
       NOC2=INUM
      END IF
      IF(ILAST.LT.J)GO TO 300
      JM1=J-1
      ISU=ISUM-1
      DO 100 LI=1,ISU
      IPU=ILA-ILAST+1
      IDUM=IL(LI)
      JDUM=JL(LI)
      IF(A(IDUM,JDUM).EQ.0)GO TO 100
      IF(IL(LI).LT.JL(LI))THEN
      IF(IL(LI).LT.J.AND.JL(LI).GT.ILAST)GO TO 102
      GO TO 100
      END IF
      IF(IL(LI).GT.JL(LI))THEN
      IF(IL(LI).GT.ILAST.AND.JL(LI).LT.J)GO TO 102
      GO TO 100
      END IF
102    L=L+1
      KA(LI)=L
100    CONTINUE
710    IR=L*R(IB)
      KCOUNT=KCOUNT+1
      IB=IB+1
      NRAN=IR+1
C
C
C
      IF(KCOUNT.EQ.5)GO TO 711
776    IF(NRAN.EQ.1)THEN
```

```
711     KAM=0
98      DO 600 IA=1,MTOTAL
      IF(P(IA).EQ.1.AND.IA.EQ.1)GO TO 700
      IF(MTOTAL.LE.2)THEN
      IF(P(IA).EQ.1)GO TO 700
      END IF
      IF(P(IA).EQ.1.AND.P(IA+1).GT.LEVEL)THEN
      IF(P(IA-1).GT.LEVEL)THEN
      IF(P(IA-1).GT.116)GO TO 700
      IAX=IA-1
      IAMX=IAX
7009    IAMX=IAMX-1
      IF(IAMX.LT.1)GO TO 600
      IF(P(IAMX).EQ.P(IA-1))GO TO 700
      GO TO 7009
      END IF
      END IF
600     CONTINUE
700     IA3=IA+1
721     IF(P(IA3).GT.LEVEL)THEN
      IA3=IA3+1
      GO TO 721
      END IF
762     IF(P(IA3).LT.ILAST)THEN
      IA4=IA3-1
      IT=P(IA4)
      IA3=IA3+1
      DO 722 LE=IA3,MTOTAL
      IF(P(LE).EQ.IT)GO TO 723
722       CONTINUE
723     IA1=LE
      IA=LE
      LEE=LE+1
761     IF(LEE.GT.MTOTAL)GO TO 777
      IF(P(LEE).GT.LEVEL)THEN
      LEE=LEE+1
      GO TO 761
      END IF
      IA3=LEE
      GO TO 762
      END IF
777     IA1=IA
      ISS=ISS+A(ILAST,J)
724     KK=0
713     JI=IA
      DO 714 IM=1,JI
714     PP(IM)=P(IM)
      LLL=JI+1
      PP(LLL)=(ILAST*100)+J
      JK=JI+(A(ILAST,J)*2)+1
      JE=JI+2
      DO 701 IMM=JE,JK,2
      PP(IMM)=ILAST
      IM1=IMM+1
      PP(IM1)=J
701     CONTINUE
```

```
      MTOTAL=MTOTAL+(A(ILAST,J)*2)+2
      IM1=IM1+1
      PP(IM1)=(ILAST*100)+J
      IM1=IM1+1
      J11=JI
      DO 702 IB=IM1,MTOTAL
      J11=J11+1
      PP(IB)=P(J11)
702   CONTINUE
      DO 703 IN=1,MTOTAL
703   P(IN)=PP(IN)
      GO TO 99
      END IF
      IF(KCOUNT.EQ.5)GO TO 99
C
C
C
      K=1
      DO 200 LI=1,ISU
      IPU=ILA-ILAST+1
      IDUM=IL(LI)
      JDUM=JL(LI)
      IF(A(IDUM,JDUM).EQ.0)GO TO 200
      IF(IL(LI).LT.JL(LI))THEN
      IF(IL(LI).LT.J.AND.JL(LI).GT.ILAST)GO TO 201
      GO TO 200
      END IF
      IF(IL(LI).GT.JL(LI))THEN
      IF(IL(LI).GT.ILAST.AND.JL(LI).LT.J)GO TO 201
      GO TO 200
      END IF
201   K=K+1
      KA(I)=K
      IF(NRAN.EQ.K)GO TO 500
200   CONTINUE
500   I=IL(LI)
      II=JL(LI)
      ISS=A(ILAST,J)+ISS
      IF(I.EQ.1.AND.II.EQ.LEVEL)THEN
      NRAN=1
      GO TO 776
      END IF
      IF(I.LT.II)THEN
      IF(I.GE.J)GO TO 710
      END IF
      CALL SORT (I,II,ILAST,J,LK,KC,P,PP,MTOTAL,LEVEL,A)
      IF(I.EQ.1.AND.II.EQ.LEVEL)GO TO 98
      GO TO 99
C
C
C
300   ISU=ISUM-1
      DO 800 LB=1,ISU
      IPU=ILA-ILAST+1
      IDUM=IL(LB)
      JDUM=JL(LB)
```

```
      IF(A(IDUM,JDUM).EQ.0)GO TO 800
      IF(IL(LB).GT.JL(LB))THEN
      IF(IL(LB).GT.J.AND.JL(LB).LT.ILAST)GO TO 801
      GO TO 800
      ELSE
      IF(IL(LB).LT.ILAST.AND.JL(LB).GT.J)GO TO 801
      GO TO 800
      END IF
801    L=L+1
      KA(I)=L
800    CONTINUE
      KCOUNT=1
1711   IR=L*R(IB)
      KCOUNT=KCOUNT+1
      IB=IB+1
      NRAN=IR+1
      IF(KCOUNT.EQ.5)GO TO 1712
C
C
C
      IF(NRAN.EQ.1)THEN
1712    NAM=0
101    IC=A(ILAST,J)
      DO 1901 IA=1,MTOTAL
      IF(P(IA).EQ.1.AND.IA.EQ.1)GO TO 1916
      IF(MTOTAL.LE.2)THEN
      IF(P(IA).EQ.1)GO TO 1916
      END IF
      IF(P(IA).EQ.1.AND.P(IA+1).GT.LEVEL)THEN
      IF(P(IA-1).GT.LEVEL)THEN
      IF(P(IA-1).GT.116)GO TO 1916
      IAX=IA-1
      IAMX=IAX
7010    IAMX=IAMX-1
      IF(IAMX.LT.1)GO TO 1901
      IF(P(IAMX).EQ.P(IA-1))GO TO 1916
      GO TO 7010
      END IF
      END IF
1901    CONTINUE
1916    IF(IA.EQ.1)GO TO 2103
      IAM1=IA-1
      PIAM1=P(IAM1)
      IAM2=IA-1
2101    IAM2=IAM2-1
      IF(P(IAM2).EQ.PIAM1)GO TO 2100
      GO TO 2101
2100    IAM1=IAM2+1
2108    IF(P(IAM1).LT.ILAST)THEN
      IA1=IAM2
      IA=IA1
      IAM3=IAM2-1
2105    IF(IAM3.LE.0)GO TO 2103
      IF(P(IAM3).GT.LEVEL)GO TO 2104
      IAM3=IAM3-1
```

```
      GO TO 2105
2104    IAM4=IAM3-1
2107    IF(IAM4.LE.0)GO TO 2103
      IF(P(IAM4).EQ.P(IAM3))GO TO 2106
      IAM4=IAM4-1
       GO TO 2107
2106    IAM1=IAM4+1
      IAM2=IAM4
      GO TO 2108
      END IF
2103    IA1=IA
2102    JI=IA
      JIM1=JI-1
      IF(JI.EQ.1)GO TO 1909
      DO 1910 I=1,JIM1
1910    PP(I)=P(I)
1909    JK=JI+(IC*2)-1
      PP(JI)=(ILAST*100)+J
      JE=JI+1
      DO 1911 I=JE,JK,2
      PP(I)=ILAST
      IP=I+1
      PP(IP)=J
1911    CONTINUE
      MTOTAL=MTOTAL+(IC*2)+2
      IP=IP+1
      PP(IP)=(ILAST*100)+J
      IP=IP+1
      J11=JI-1
      DO 1912 IB=IP,MTOTAL
      J11=J11+1
      PP(IB)=P(J11)
1912    CONTINUE
      DO 1914 KB=1,MTOTAL
1914    P(KB)=PP(KB)
      GO TO 99
      END IF
      IF(KCOUNT.EQ.5)GO TO 99
C
C
C
      K=0
      DO 900 LB=1,ISU
      IPU=ILA-ILAST+1
      IDUM=IL(LB)
      JDUM=JL(LB)
      IF(A(IDUM,JDUM).EQ.0)GO TO 900
      IF(IL(LB).GT.JL(LB))THEN
      IF(IL(LB).GT.J.AND.JL(LB).LT.ILAST)GO TO 901
      GO TO 900
      ELSE
      IF(IL(LB).LT.ILAST.AND.JL(LB).GT.J)GO TO 901
      GO TO 900
      END IF
901     K=K+1
      KA(I)=K
```

```
      IF(NRAN.EQ.K)GO TO 1200
900     CONTINUE
1200    I=IL(LB)
      II=JL(LB)
      ISS=ISS+A(ILAST,J)
      CALL SORT (I,II,ILAST,J,LK,KC,P,PP,MTOTAL,LEVEL,A)
      IF(I.EQ.1.AND.II.EQ.LEVEL)GO TO 101
      GO TO 99
999     MT=1
      PE(1)=LEVEL
      DO 1001 I=1,MTOTAL
      IF(PP(I).GT.LEVEL)GO TO 1001
      MT=MT+1
      PE(MT)=PP(I)
1001    CONTINUE
      WRITE(6,*)MT
      WRITE(6,1300)(PE(I),I=1,MT)
1300    FORMAT(11(I5,1X))
      STOP
      END
C
C
      SUBROUTINE TR(X)
C
C      THIS SUBROUTINE GENERATES RANDOM NUMBER BETWEEN 0 AND 1.
C      (APPLIED TIME SERIES ANALYSIS BY OTNES.)
C
      DATA L/783637/
      L=125*L
      L=L-(L/2796203)*2796203
      X=FLOAT(L)/2796202.
      RETURN
      END
C
      SUBROUTINE SORT(I,II,ILAST,J,LK,KC,P,PP,MTOTAL,LEVEL,A)
C
C      THIS SUBROUTINE PUT THE INSERTING CYCLES INTO A PARTIALLY
C      RECONSTRUCTED HISTORY.
C
C      ILAST,J= INSERTING PEAK AND VALLEY , OR INSERTING VALLEY
C      AND PEAK.
C      I,II= RECEIVING PEAK AND VALLEY , OR RECEIVING VALLEY
C      AND PEAK.
C
      INTEGER PP(15000),A(32,32),P(15000)
      IF(I.EQ.1.AND.II.EQ.LEVEL)GO TO 200
      LEVEL=32
      IP=(I*100)+II
C
C
      IF(I.GT.II)THEN
C
      IF(ILAST.GT.J)THEN
      DO 1 L=1,MTOTAL
      LP1=L+1
1     IF(P(L).EQ.II.AND.P(LP1).EQ.IP)GO TO 2
```

```
2      LPL=LP1
500    LPL=LPL+1
      IF(P(LPL).GT.LEVEL)GO TO 500
      LPL1=LPL
501    LPL1=LPL1+1
      IF(P(LPL1).GT.LEVEL)GO TO 501
      IF(P(LPL).LT.ILAST.AND.P(LPL1).LT.J)THEN
      I=1
      II=LEVEL
      GO TO 200
      END IF
      DO 3 LK=LP1,MTOTAL
3     IF(P(LK).LE.LEVEL)GO TO 4
4     IF(P(LK).GT.ILAST)GO TO 45
      LK1=LK-1
48     DO 410 LK2=LK,MTOTAL
410    IF(P(LK2).EQ.P(LK1))GO TO 42
42     LK3=LK2
      LK3=LK2+1
      IF(P(LK3).LE.LEVEL)GO TO 43
      LK4=LK3
      LK4=LK4+1
      IF(P(LK4).GT.ILAST)GO TO 44
      LK1=LK3
      LK=LK4
      GO TO 48
43     IF(P(LK3).EQ.LEVEL)THEN
      LP1=LK3-1
      END IF
      GO TO 45
44     LP1=LK3-1
45     DO 5 LI=1,LP1
5      PP(LI)=P(LI)
      IPM1=LP1+1
      PP(IPM1)=(ILAST*100)+J
      IPM2=IPM1+1
      K=A(ILAST,J)
      KK=1
      L=IPM2
6      PP(L)=ILAST
      L=L+1
      PP(L)=J
      IF(K.EQ.KK)GO TO 8
      KK=KK+1
      L=L+1
      GO TO 6
8      L=L+1
      PP(L)=(ILAST*100)+J
      L=L+1
      L2=IPM1
      DO 9 IK=L2,MTOTAL
      PP(L)=P(IK)
      L=L+1
9      CONTINUE
      MTOTAL=L-1
```

```
      ELSE
C
C
      DO 10 JI=2,MTOTAL
      JI1=JI-1
10     IF(P(JI).EQ.I.AND.P(JI1).EQ.IP)GO TO 11
11     JIP1=JI+1
      DO 12 LE=JIP1,MTOTAL
12     IF(P(LE).EQ.P(JI1))GO TO 13
13     LE=LE-2
      IF(P(LE).GT.LEVEL)GO TO 14
      JI=LE
      GO TO 16
14     LEE=LE
140    LEE=LEE-1
      IF(P(LEE).EQ.P(LE))GO TO 150
      GO TO 140
150    LE1=LEE+1
      IF(P(LE1).GT.ILAST)THEN
      DO 17 LC=LE1,MTOTAL
      LC1=LC+1
17     IF(P(LC).EQ.P(LE))GO TO 18
18     JI=LC
      ELSE
220    LE2=LEE-1
      IF(P(LE2).LE.LEVEL)GO TO 190
      LE3=LE2
210    LE3=LE3-1
      IF(P(LE3).EQ.P(LE2))GO TO 180
      GO TO 210
180    LE4=LE3+1
      IF(P(LE4).LT.ILAST)THEN
      LEE=LE3
      GO TO 220
      ELSE
      JI=LE2
      END IF
      GO TO 16
190    IF(P(LE2).EQ.I)THEN
      JI=LE2
      END IF
      END IF
16     DO 19 IK=1,JI
19     PP(IK)=P(IK)
      ICC=A(ILAST,J)*2
      IK=0
      IC=JI
      IC=IC+1
      PP(IC)=(ILAST*100)+J
20     IC=IC+1
      IK=IK+2
      PP(IC)=ILAST
      IC=IC+1
      PP(IC)=J
      IF(IK.EQ.ICC)GO TO 21
```

```
      GO TO 20
21     MTOTAL=MTOTAL+ICC+2
      IC=IC+1
      PP(IC)=(ILAST*100)+J
      IC=IC+1
      IA=JI
      DO 22 ID=IC,MTOTAL
      IA=IA+1
22     PP(ID)=P(IA)
      END IF
      END IF
      IF(I.GT.II)GO TO 2000
C
C
      IF(ILAST.LT.J)THEN
      DO 23 KI=1,MTOTAL
      KI1=KI+1
23     IF(P(KI).EQ.II.AND.P(KI1).EQ.IP)GO TO 24
24     DO 25 KC=KI1,MTOTAL
25     IF(P(KC).LE.LEVEL)GO TO 250
250     IF(P(KC).LT.ILAST)GO TO 231
      KCM1=KC-1
      DO 233 KB=KC,MTOTAL
233     IF(P(KB).EQ.P(KCM1))GO TO 234
234      KI1=KB
      GO TO 24
231     KTRY=KI1+1
      IF(KC.EQ.KTRY)THEN
      LIP=KC-1
      ELSE
      LIP=KC-2
      END IF
      IP1=LIP+1
      DO 26 KB=1,LIP
26     PP(KB)=P(KB)
      PP(IP1)=(ILAST*100)+J
      KK=1
      K=A(ILAST,J)
      L=IP1+1
27     PP(L)=ILAST
      L=L+1
      PP(L)=J
      IF(K.EQ.KK)GO TO 28
      KK=KK+1
      L=L+1
      GO TO 27
28     L=L+1
      PP(L)=(ILAST*100)+J
      L=L+1
      L2=IP1
      DO 29 IK=L2,MTOTAL
      PP(L)=P(IK)
      L=L+1
29     CONTINUE
      MTOTAL=L-1
```

```
      ELSE
C
C
      DO 30 NI=2,MTOTAL
      NI1=NI-1
30    IF(P(NI1).EQ.IP.AND.P(NI).EQ.I)GO TO 31
31    DO 32 NC=NI,MTOTAL
      NC1=NC+1
32    IF(P(NC).EQ.II.AND.P(NC1).EQ.IP)GO TO 333
333   NC1=NC1-2
      GO TO 41
33    NC1=NC-2
41    IF(P(NC1).LE.LEVEL)GO TO 300
      NC2=NC1
320   NC2=NC2-1
      IF(P(NC2).EQ.P(NC1))GO TO 310
      GO TO 320
310   NC3=NC2+1
      IF(P(NC3).LT.ILAST)GO TO 330
      NC=NC3
      GO TO 33
300   IF(P(NC1).EQ.I)THEN
      JI=NC1
      END IF
      GO TO 380
330   JI=NC1
380   DO 37 IE=1,JI
37    PP(IE)=P(IE)
      PP(IE)=(ILAST*100)+J
      K=A(ILAST,J)
      KK=1
      L=IE+1
39    PP(L)=ILAST
      L=L+1
      PP(L)=J
      IF(K.EQ.KK)GO TO 38
      KK=KK+1
      L=L+1
      GO TO 39
38    L=L+1
      PP(L)=(ILAST*100)+J
      L=L+1
      L2=JI+1
      DO 40 IS=L2,MTOTAL
      PP(L)=P(IS)
      L=L+1
40    CONTINUE
      MTOTAL=L-1
      END IF
2000  DO 1000 K=1,MTOTAL
1000  P(K)=PP(K)
200   RETURN
      END
```

GENERATED HISTORY

32 11 25 8 18 1 29 4 32

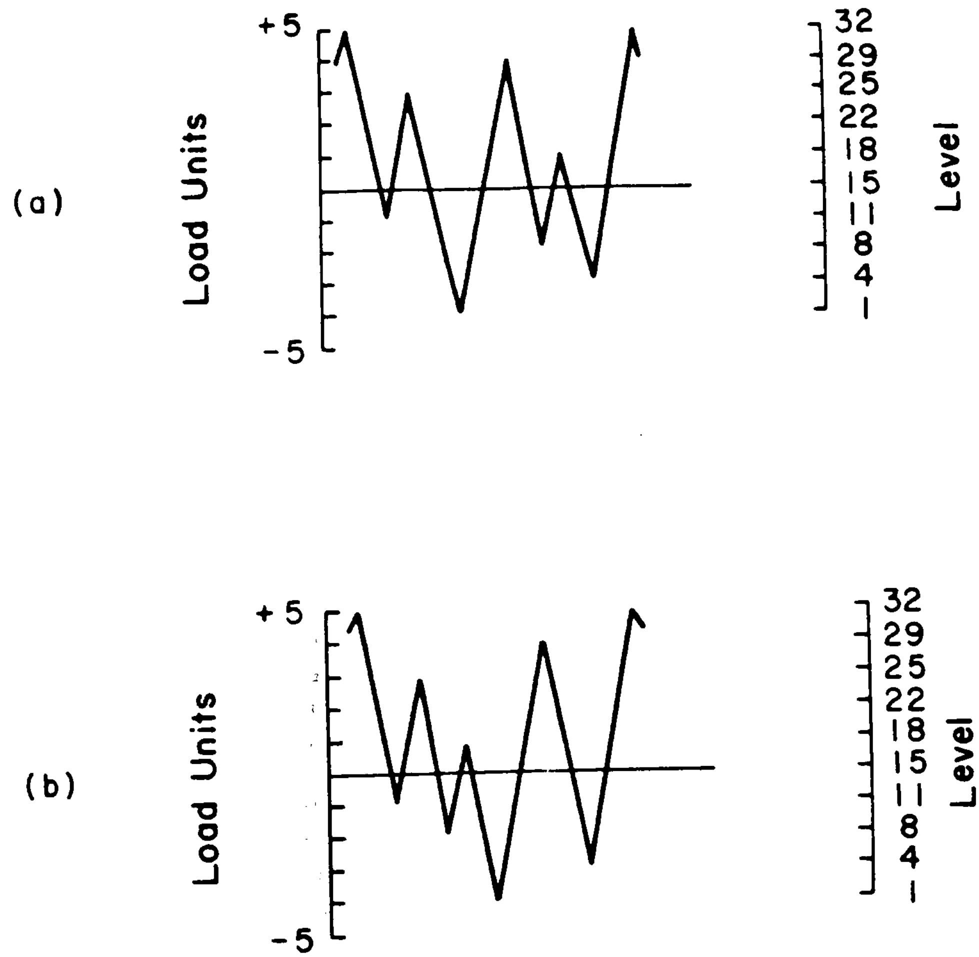


Figure C.1. Comparison of the Original (a), and Reconstructed (b) Histories.

C.3 Example 2

A history (Maneuver History) containing 1021 peak-valley points is used. This history is explained in detail in Chapter 4 of this dissertation. The input is the rain-flow matrix of this history with directions of rain-flow cycles considered, as obtained using the RAINF2 computer program (Appendix B, OPTION 4). Note that the reconstructed history gives the same rain-flow peak-valley matrix as the original history, and also the directions of the cycles are preserved. Note that as mentioned before, the user must convert the history obtained (min = 1, max = 32) using linear interpolation to get a history on a scale compatible with the original history. The program input and output are attached. Cycles from each matrix element are placed in two randomly chosen locations (NOC = 2), provided these are more than eight (NP = 8). Figure C.2 shows the original and the reconstructed histories.

Input for the Reconstruction Program

```
8
2
     0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0
     0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0
     0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0
     0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0
     0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0
     0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0    0    0    0    0    0    0    0    0    0    0    0    0    1
  0    1
     0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    2    0    1    0    0    0    0    1    0    1    0    0    1    0
  0    0
     0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    3    4    1    0    0    0    0    0    0    0    0    0    0    0
  0    0
     0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0    4    4    2    0    3    0    1    1    0    0    0    0    0
  0    0
     0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0    0    2    7    4    2    0    2    0    0    0    0    0    0
  0    0
     0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0    0    0    4    3    2    1    2    1    0    0    0    0    1
  0    0
     0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0    0    0    0    2    5    3    1    1    0    0    0    0    0
  0    0
     0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0    0    0    0    0    3    3    1    3    3    0    0    0    0
  0    0
     0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0    0    0    0    0    0    1    7    2    6    0    1    0    0
  0    0
     0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0    0    0    0    0    0    0    0    3    5    4    4    2    1    0
  0    0
     0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
```

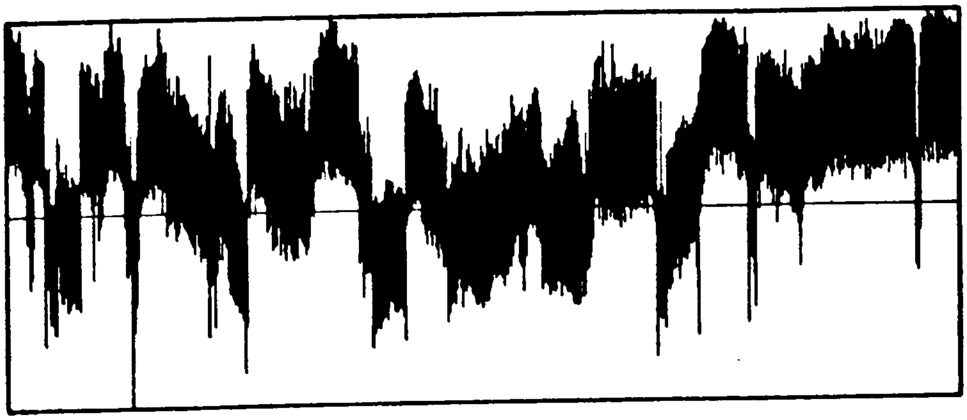
```
0   0   0   0   0   0   0   0   0   1   9   5   2   1   0
0   0
  0   0   0   0   0   0   1   2   0   0   0   0   0   0   0
0   0   0   0   0   0   0   0   0   0   3   6   3   1   0
0   0
  0   0   0   0   0   0   1   6   2   0   0   0   0   0   0
0   0   0   0   0   0   0   0   0   0   0   3   8   2   1
2   0
  0   0   0   0   0   1   1   0   2   6   0   0   0   0   0
0   0   0   0   0   0   0   0   0   0   0   0   5  12  12
2   1
  0   0   0   0   0   0   1   0   3   2   1   0   0   0   0
0   0   0   0   0   0   0   0   0   0   0   0   0   5  15
10  4
  0   0   0   0   0   0   0   0   1   5   3   4   0   0   0
0   0   0   0   0   0   0   0   0   0   0   0   0   0   4
6   0
  0   0   0   0   0   0   0   0   1   3   4   3   1   0   0
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0   0
  0   0   0   0   0   0   0   1   4   3   2   6   5   4   0
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0   0
  0   0   0   0   0   0   0   0   0   3   0   3   3   2   4
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0   0
  0   0   0   0   0   1   0   0   0   1   0   0   0   4   4
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0   0
  0   0   0   0   0   0   0   0   1   0   0   1   1   2   3
12  2   0   0   0   0   0   0   0   0   0   0   0   0   0
0   0
  0   0   0   0   0   0   0   0   0   0   0   1   2   3   2
7  10   4   0   0   0   0   0   0   0   0   0   0   0   0
0   0
  0   0   0   0   0   0   1   0   0   0   0   0   1   0   3
4   3  14   4   0   0   0   0   0   0   0   0   0   0   0
0   0
  0   0   0   0   0   0   0   0   0   0   1   1   0   0   1
0   0   6  13   5   0   0   0   0   0   0   0   0   0   0
0   0
  0   0   0   0   0   0   0   0   0   0   0   0   1   0   0
0   0   1   7  10   7   0   0   0   0   0   0   0   0   0
0   0
  0   0   0   1   0   0   1   0   0   0   1   0   0   0   0
0   0   2   4   6   6   0   0   0   0   0   0   0   0   0
0   0
  1   0   0   0   1   0   0   0   0   0   0   1   0   0   0
0   0   0   1   0   1   0   0   0   0   0   0   0   0   0
0   0
```

GENERATED HISTORY

32	21	31	21	31	21	31	21	31	21	31
21	31	20	32	20	32	20	32	20	32	19
32	20	30	20	30	20	30	20	30	20	30
20	30	20	30	20	30	6	17	7	18	7
23	10	23	10	23	10	26	16	26	16	26
16	26	16	26	16	26	16	27	15	27	17
26	17	26	17	26	15	27	12	29	14	25
14	25	12	26	15	26	15	26	15	28	17
28	17	28	17	29	15	27	15	27	15	27
15	27	11	21	12	21	12	21	12	21	12
30	20	30	20	30	20	30	20	30	20	31
20	31	20	31	20	31	20	31	20	31	20
31	19	31	19	31	19	31	19	32	20	31
20	31	20	31	20	31	20	31	21	30	21
30	21	30	21	30	20	31	20	31	20	31
20	31	20	31	20	30	20	30	20	30	20
30	20	30	20	30	20	30	19	31	19	31
19	30	19	30	19	30	19	30	19	30	19
30	20	29	20	29	20	29	20	29	20	29
19	30	19	30	19	30	19	30	19	30	19
30	18	31	18	31	17	27	17	27	17	27
17	27	17	27	17	27	16	26	16	26	16
26	16	26	16	26	16	26	16	26	16	26
16	26	15	28	15	27	18	27	18	27	18
27	18	28	16	25	10	19	10	19	7	22
11	22	11	22	11	22	11	26	12	23	12
23	12	23	12	21	12	21	12	25	15	24
15	24	15	24	9	20	10	20	10	28	13
29	18	28	18	28	18	28	18	28	18	28
18	28	18	28	18	28	18	27	18	27	18
27	16	28	16	28	8	18	8	18	8	18
8	18	7	24	6	17	8	17	8	20	11
22	13	23	12	23	12	23	12	23	12	23
12	23	12	24	14	24	14	24	12	24	12
24	12	24	10	24	10	24	10	25	10	29
18	29	18	29	18	29	18	29	18	29	19
28	19	28	19	28	19	28	19	28	18	30
15	29	14	28	14	24	14	24	14	24	14
24	14	24	14	24	14	24	9	21	10	21
10	21	10	21	10	21	10	24	13	23	13
23	13	23	11	22	11	22	10	21	10	21
10	21	10	21	10	20	10	20	10	20	10
20	10	20	10	20	10	20	9	22	9	22
9	22	7	23	9	23	9	23	9	23	9
22	12	22	12	22	12	26	11	24	11	24
10	24	10	24	10	22	10	22	9	19	9
19	9	19	9	19	7	17	7	17	1	18
9	18	9	18	8	18	8	18	8	18	8

18	8	18	8	19	8	17	8	17	8	17
7	19	6	23	11	23	11	23	9	20	9
20	8	24	13	24	13	24	13	26	17	26
17	26	16	26	16	26	16	26	16	26	16
26	16	27	16	27	16	27	16	27	16	27
16	27	16	27	16	27	14	27	14	27	14
28	19	28	19	28	19	28	19	28	18	28
18	28	18	28	18	28	18	28	18	28	18
29	19	29	19	29	19	29	19	29	19	29
19	29	15	28	16	28	16	28	16	28	16
29	19	29	19	29	19	29	19	29	19	29
19	29	19	30	21	30	21	30	21	30	21
30	21	30	21	30	21	30	20	30	20	30
20	30	20	30	20	30	19	30	19	30	19
30	19	30	19	30	19	30	19	30	18	29
20	29	20	29	20	29	20	29	20	31	21
31	21	31	21	31	21	31	21	31	21	31
18	31	18	31	4	19	9	19	9	22	9
19	10	19	10	19	10	19	10	19	10	19
10	25	7	19	9	18	9	18	9	18	9
18	6	22	10	22	10	22	10	28	14	26
14	26	14	26	14	26	14	26	14	26	11
23	7	32	21	32	19	32	19	29	19	29
19	29	19	29	19	29	19	29	12	23	13
23	13	23	13	23	13	23	13	28	15	28
15	28	15	27	17	27	17	27	17	27	17
27	17	27	17	27	17	27	17	27	17	27
17	32	19	29	19	29	19	29	19	29	19
29	19	29	17	28	17	28	17	28	13	22
13	22	13	22	11	25	12	24	5	20	8
19	7	20	9	20	9	20	9	26	15	25
15	25	15	25	15	25	15	25	12	23	14
23	14	23	14	23	14	31	18	30	18	29
18	29	11	26	13	30	16	28	18	28	18
28	18	28	18	28	18	28	18	28	18	29
16	27	16	27	16	27	16	27	16	27	13
25	13	25	13	25	13	24	12	22	12	22
12	22	12	22	12	22	11	21	11	21	11
21	11	20	11	20	11	20	11	20	9	25
7	21	9	21	11	21	11	21	11	27	13
27	15	26	15	26	15	26	15	26	13	25
15	25	15	25	15	25	15	30	17	29	13
25	14	25	14	25	14	25	14	31	13	26
13	26	13	24	15	24	15	24	15	24	15
26	14	23	11	26	14	26	14	32		

(a)



(b)

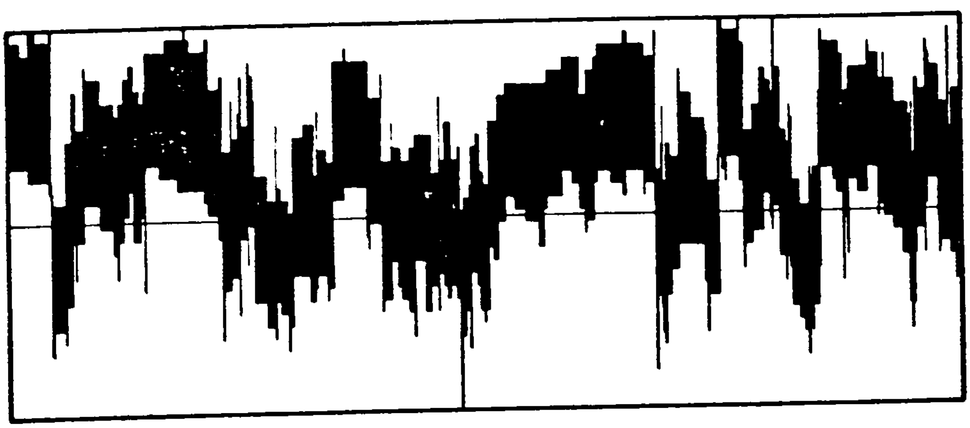


Figure C.2. Comparison of the Original (a), and Reconstructed (b) Histories.

The vita has been removed from the scanned document