A MARKOV PROCESS METHODOLOGY FOR MODELING MACHINE INTERACTIONS IN TIMBER HARVESTING SYSTEMS

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ABSTRACT

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Recent advancements in timber harvesting systems analysis have been almost exclusively simulation based. A similar degree of effort in developing analytic models has been conspicuously absent.

That part of timber harvesting analysis where simulation plays its most vital role is the study of machine interactions. The importance of machine interactions lies in determining the proportions of delay, idle and productive time for the interacting machines. This in turn, is important for balancing productivity so that no single component of the interaction is accumulating excessive amounts of delay or idle time.

The objective of this study was to determine the feasibility of applying Markov process theory to the analysis of timber harvesting systems and components. Through modeling the interaction between a fixed location slasher and a grapple skidder, it is shown how a Markov model can be used to obtain proportions of delay, idle and productive time. Unlike the statistical solutions derived

from simulation models, the Markov model improves upon this by providing an analytic solution. The Markov model also avoids the problems of correlated output data from simulations by explicitly recognizing that any possible future state is dependent only on the current state of the system and is conditionally independent of the past history of the system.

The methodology for building a Markov model requires dealing with only two probability distributions, the Erlang and mixed Erlang, for modeling time based activities (such as cycle times) of the interacting machines. These probability distributions in turn, provide the necessary data for developing a system of algebraic equations for solving the Markov process model.

While this is the first step in applying stochastic process theory to timber harvesting analysis, the results of this study indicate that the technique has considerable potential for application in timber harvesting system modeling.

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1. INTRODUCTION

The last 15-20 years have witnessed a surge of interest and research into production and economic analyses of timber harvesting machines and systems. Studies have been conducted on a wide range of systems and harvesting methods, including: highly mechanized sytems, labor intensive systems, single and multi-product systems, clearcutting, thinning, residue harvests and others.

The advent of mechanized logging helped transform the relatively stable universe of simple logging methods to one in which confusion and complexity are the rule (26). Similarly, the evolution of techniques used in estimating productivity of men, machines, and systems has followed a path from the simplified to the complex.

The primary tool for developing more sophisticated timber harvesting models has been simulation. Simulation modeling offered a unique opportunity to investigate problems that were considered unusually complex for other methods. The basic tools of the simulation modeler; a computer and proficiency with a higher-level computer language permitted access to solutions of complex problems by a much larger group of researchers. As a result, the proliferation of simulation models has been one of significant proportions.

The rapid movement to simulation has tended to overlook possible analytic formulations of timber harvesting problems. Because many of the analytic models are more mathematically oriented, there seems to have been a tendency to avoid them. This study represents the first step in exploring the possibility of adapting analytic models to the more complex aspects of timber harvesting problems and for enhancing certain aspects of modeling that simulation does not handle well.

2. REVIEW OF THE LITERATURE

Timber Harvesting System Analysis--An Extrapolation From Traditional Industrial Methods?

The movements of men and machines and their modeling is an integral part of harvesting productivity and inevitably, economic studies. Unlike production studies in other manufacturing industries, timber harvesting productivity is unique and out of this particular uniqueness arises a certain degree of complexity. Traditional manufacturing industries usually operate on a homogeneous raw material (or product), in a restricted environment with closely controlled processing guidelines.

Unfortunately, the timber harvesting industry lacks the constancy of a traditional industry. The logging industry must continually deal with a raw material that comes in many different shapes, sizes, and species and exists in an environment that varies from site to site, as well as within each site. Furthermore, loggers rarely have the opportunity to follow a predetermined production sequence, as men and machines must maneuver as conditions dictate. As Donnelly (11) stated, "The range of differences and combinations is large and so are the effects".

With well developed techniques for analyzing productivity in traditional industries, we see a great deal of this methodology extrapolated to the non-traditional timber harvesting environment. The constancy of traditional industrial productivity has led to standardization in a very deterministic fashion (i.e., usually via regression estimates). Managers are most concerned with developing normal performance ratings that provide benchmarks for evaluating employee performance.

Unlike the logging industry, the traditional industries usually deal only with the variability arising out of differences in employees, such as: inherent knowledge, physical capacity, health, physical dexterity and training.

Also, traditional industrial productivity is assumed to approximate a normal distribution (31). Normally distributed production times are taken as indicative of consistent work habits and normal work pacing. Steffy and Darby (31) cite non-symmetric production distributions as evidence of some abnormality. Positively skewed distributions are deemed to be indicative of restricted work output, due possibly to machine cycle time restrictions or other dependent operations. Alternatively, negatively skewed productivity distributions may indicate a worker is motivated to work slowly, but sets a limit on maximum cycle time. Applying general statements of this type to timber harvesting would be improper, since the variation in operating conditions will inevitably cause departures from symmetry in performance distributions.

One of the more questionable extensions from traditional industries is the application of least squares regression techniques for describing timber harvesting productivity. That is, does the use of regression equations to model stochastic systems adequately capture the probabilistic behavior of the system and provide reasonable performance estimates?

Further, it is not evident that timber harvesting cycle times should be symmetrically distributed. While typical industrial operator work characteristics remain, the added variation of timber and tract characteristics confound a straightforward application of the normal performance concept.

Traditional Techniques for Analyzing Harvesting Systems

Various methods have been utilized to study timber harvesting systems, including: case studies, regression analysis, and simulation. Case studies generally focus upon developing simple summary statistics from observations on a system operating in a restricted setting (i.e., quite often a single tract of timber) (2,4,19,21). There is generally no attempt to build a model to analyze the system and test possible modifications in system components.

Regression Analysis

Regression analyses take a somewhat broader view of timber harvesting. Data is usually collected on pertinent logging system components, over a range of conditions. From this data, regression equations are formulated to describe productivity. The level of detail ranges from several equations describing a harvesting function (e.g., one equation for each element of a skidder cycle) to a single function-level equation (e.g., one equation describing the volume produced per unit time by the skidder). The individual equations describing the components of the system are then aggregated and estimates of machine interactions are made, usually via delay or utilization factors, in order to develop system productivities and costs (1,8,13,17,18,23,24).

Several potential problems can arise using regression analysis. First, it is incumbent on the modeler to conform to the underlying least squares assumptions. Failure in this regard could lead to models and performance estimates of questionable value. Second, a review of 17 harvesting papers, which modeled productivity via regression equations, showed that the range in explained variation (i.e., r-squared) was 8.3 to 99.0 percent. For a random process like timber harvesting, one must question the ability of these models to capture the probabilistic behavior of the system and its components for decisionmaking purposes.

Cottell et al. (9) used regression equations to model skidder productivity and found that one-fourth to two-thirds of the variation in components of aggregate productivity was unexplained by the regression models. Much of this unexplained variation was attributed to worker motivation. Cottell et al. (9) note that greater detail in time studies or in measuring environmental and operating factors cannot make up for lack of measurement on worker motivation and physiological effort.

While modeling a system stochastically would not specifically identify the degree of worker motivation, such an approach would inherently account for this variation.

Harvesting Simulation

The third, and currently most advanced technique for analyzing timber harvesting systems is simulation (3,6,10,22,32,33,34). The general intent of simulation models is to "duplicate the essence of a system without attaining the reality of that system" (14). This is generally accomplished through the development of computer code to model each harvesting activity. The models usually allow for input flexibility (e.g., time study data that can range in form from point estimates, to regression equations, to empirical and theoretical probability distributions).

Men and Machine Interactions

The most important feature of simulation, that is at best only roughly approximated by other techniques, is the handling of complex interactions between men and machines (33). Interactions occur when a system component is dependent upon the initiation or completion of an activity of another component in order to proceed with its activities. Simulation models generally rely on a discrete event approach to modeling interactions. That is, the inputs for each component of the interaction provide a means for maintaining a simulation clock. Events are initiated, continued or completed as the clock is continuously updated. When an activity is forced to wait for initiation or completion of the dependent activity, an interaction occurs. Two broad categories of men/machine interactions can be identified.

Controllable Interactions

Controllable interactions are those which can be manipulated by management to the benefit of the firm. For example, consider the potential interaction between a feller-buncher and a grapple skidder. By keeping the feller-buncher far enough ahead of the skidder, the logging manager can, through proper means, maintain a productive buffer between these functions. In this way, the skidder will always have material available, so that any skidder delays resulting from insufficient material are avoided. The important consideration here is how far in advance should the felling phase begin before the skidding phase, so as to maintain a productive buffer? The necessary information for successfully implementing such a strategy is simply an estimate of the productivity of each machine. Then, using this information, the logger can start and maintain the feller-buncher far enough ahead of the skidder(s) so that no interaction ever occurs.

Although important, this type of interaction does not pose a major obstacle in studying system productivity. When a system is built upon controllable interactions, system performance can generally be determined by analyzing the productivity of each non-interacting component and determining the minimum of these as representative of system productivity.

Uncontrollable Interactions

Uncontrollable interactions are those that management cannot avoid, without affecting the productivity of another component or incurring an unreasonable cost. For example, a fixed location processor (i.e., slasher, whole-tree chipper, Hahn harvester, etc.) and some type of in-woods transporter (i.e., cable skidders, grapple skidders, forwarders, etc.) that supplies material to the processor would constitute an uncontrollable interaction. The processor's stationary position requires continuous processing of the material being supplied by the in-woods transporter. Since the processor has limited reach capabilities (as defined by the maximum boom reach of its loader), only a finite number of stems can be within reach of the loader at any given time. This limited area for inventorying stems, hereafter referred to as a production buffer, is a key element in modeling the interaction of the processor and in-woods transporter. If the production buffer is full when the in-woods transporter arrives at the processor, it must accumulate delay time waiting for the processor to deplete the buffer to a level at which it can unload. Similarly, if the processor returns to the production buffer to find it empty, it must accumulate idle time until the in-woods transporter arrives with the next load.

Thus, the productive buffer plays a key role in determining delay and idle times for the in-woods transporter and processor, respectively. The proportions of idle and delay time, in turn, are important in system balancing. That is, if a very expensive piece of machinery is accumulating large amounts of delay time due to the interaction with a relatively inexpensive piece of machinery, the economic success of the operation may be in jeopardy due to the imbalance in system components. By making modifications in the system, the logging manager might improve system balance, thereby increasing productivity or reducing costs, which presumably, should improve system economics.

Simulation as a Tool

To gain some perspective about simulation as a modeling tool, one must recall that simulation should be used in cases where it becomes too complex to model a problem mathematically, or even if such models could be constructed, available techniques may not be conducive to problem solution. As Garner (14) states, "The reduction of the system to its essential elements is both the strength and weakness of simulation. Oversimplification often leads to unwieldy models whose costs of construction and use may outweigh any benefits from its increased accuracy." From a historic perspective, as the wave of interest quickly passed over possible analytic formulations, enthusiasm for simulation witnessed a rapid progression to more complex models as researchers attempted to model finer and finer details of harvesting systems.

As a consequence, we find that the introduction of operations research techniques, of which simulation is a part, into woodlands applications "resulted in the development of models geared to demonstrating techniques rather than solving manager's problems" (14). This ultimately led to considerable, but presumably unnecessary misunderstanding, which fostered a lack of confidence in the techniques (14).

One might argue that through this demonstration of techniques there has been a tendency to lure those using simulation models into a belief that modeling a system is no more difficult than simply inputting data into the model. Such a casual approach can lead to potentially serious errors. For example, an important consideration is the recognition and handling of possible dependencies between random variables. Mitchell et al. (30) found that for a tandem queue in a maintenance shop (where the first station inspected and marked incoming items for repairs and the second station carried out the repairs) the time in the first station was positively correlated with time in the second. Their analysis showed that ignoring this correlation in modeling the system could lead to serious inaccuracies in the results.

Further, the internal assumptions of any given model may not be fully exposed to the user and therefore the model may not be treating the system as intended. As Goulet et al. (15) noted concerning five state-of-the-art computer simulation models, all models needed a skilled programmer to install and maintain the computer codes. Likewise, any changes that might be needed would require reprogramming by a skilled programmer. Thus, it is imperative, albeit difficult, for the users of these models to become familiar with the logic, intricacies and special assumptions of a particular model before using it as a decisionmaking tool.

Analysis of Simulation Output

While there has been an intense effort to develop harvesting simulation models, there has been little effort expended on developing adequate guidelines for generating statistically sound output data. Law and Kelton (25) provide support for this contention by observing that, 'in many simulation studies a large amount of time and money is spent on model development and programming but little effort is made to analyze the simulation output data in an appropriate manner. As a matter of fact, the most common mode of operation is to make a single simulation run of somewhat arbitrary length and then treat the resulting simulation estimates as being true answers for the model. Since these estimates are random variables which may have large variances, they could, in a particular simulation run, differ greatly from the corresponding true answers. The net effect is, of course, that there may be significant probability of making erroneous inferences about the system under study."

Results from studies of the type just described are further complicated by the fact that the simulation output data are always correlated (25). Thus, classical statistical analysis, based on independent and identically distributed (IID) observations is not applicable. For example, consider the inventory of stems for a whole-tree chipper, where the inventory is changing as a result of processing by the chipper and additions by a skidder(s). The inventory level is important in determining idle time for the chipper. Suppose a simulation model was designed to collect data on the number of trees in the inventory just before the chipper makes a removal from the inventory. Each observation is at least correlated with the previous observation. That is, the size of the buffer now, is dependent on its size at the last sampling time. Thus, classical statistics, requiring IID observations is inappropriate for determining anything but a simple average of trees in the buffer.

Law and Kelton (25) discuss procedures for constructing approximate IID observations from single run, steady-state simulations, in order to apply classical statistical techniques to the estimation of confidence intervals on performance variables. However, one must be extremely careful in applying these procedures, since they are only approximations to IID observations and require specific criteria to be met in order to achieve a sound statistical basis.

For instance, one of the methods discussed by Law and Kelton (25) requires subdividing a steady-state simulation run into n batches of equal length. To develop confidence intervals on performance variables, the batch length must be chosen large enough so that the individual observations are covariance-stationary (i.e., the mean and variance are stationary over time and the covariance between any two sequential observations depends only on the length of the interval separating them and not on the actual times at which they occur), the batch means are approximately normally distributed and are uncorrelated. If the batch means are correlated, variance estimates will be severely biased. The key question then, revolves around searching for an appropriate batch length in order satisfy these conditions.

Another method for constructing IID observations is to identify, from the simulation, random times at which the process probabilistically starts over and to use these regeneration periods as independent observations of the process. However, the way in which regeneration points can be consistently identified is not expanded upon. Further, in real-life applications, processes may not even contain regeneration points, or if they do exist, are of such length that only a very few cycles can be simulated.

For either of the above techniques, the selection of overall simulation length (i.e., total sample size) is extremely important. A total sample size that is too small can lead to confidence intervals that do not provide the desired coverage. Further, total sample size is extremely model dependent so that no specific guidelines for sample size selection are possible (25).

With respect to the timber harvesting literature, no evidence of any attempts to construct IID observations in order to develop confidence intervals on performance variables is evident (3,5,12,22,27,32,33,34).

Enhancements to Simulation and Harvesting Analysis

Goulet et al. (16) in summarizing the state-of-the-art in computer simulation of timber harvesting conclude that "continued development and refinement in timber harvesting computer simulation is needed to effectively analyze existing and proposed timber harvesting strategies". Among their recommendations are recognition of variability in performance variables, maintenance of a uniform level of detail across all functions, use of element within a function as the lowest level of detail and the development of models that faithfully reproduce the harvesting operations being modeled.

While these recommendations are very reasonable, with some being necessary, others should not be accepted without further investigation, since they may not contribute to the overall advancement of timber harvesting productivity analysis. For example, what basis is there for modeling individual elements if an aggregate measure, such as cycle time, produces similar results? Cottell et al. (9) showed that when modeling skidder productivity using regression equations, shift-level estimates were as adequate as estimates using much more detailed data. While this conclusion may not be applicable to a strictly stochastic analysis, it does serve to illustrate the point that recommendations such as those provided by Goulet et al. (16) should be investigated before implementation. It also implies that the level of detail needed in a particular analysis should depend on the objectives of the analysis and not on arbitrary guidelines.

Meng (28,29) describes analytical solutions to harvesting machine interactions treated as special queuing situations, where the input device may, for example, be a skidder and the processing device a chipper or slasher. In particular, Meng (28) develops the analytic solutions where these queues are limited to one or no skidders waiting and the input times are normally distributed and the processing times are either Poisson or normally distributed. The major limitations of this approach lie in the recognition that harvesting times cannot be normally distributed, since times must be positive and there is a positive probability of negative times using the normal distribution, and that the queues may not be limited to length 0 or 1.

Other studies purporting to expand the frontiers of harvesting simulation may or may not do so in a manner that is any less complex or conceptually preferable. For example, a recent study by Hines et al. (20) has extended harvesting system analysis to 2 phases. The first phase uses a Monte Carlo technique in conjunction with an experimental design procedure to statistically determine the minimum level of detail for input variables (i.e., point estimates, regression, or distributions) that will provide satisfactory estimates of output parameters. Phase 2 represents the actual harvesting system simulation, in the traditional sense, from which the information necessary for decisionmaking is gleaned. One question that arises is, why consider modeling a stochastic system in some alternative form by introducing a sophisticated technique (that appears conceptually and computationally more difficult) to determine when a random variable can be summarized into a deterministic form?

Some attempts at streamlining harvesting simulations have been made. Bradley et al. (7) provide an analytic technique for locating trees in random spatial patterns for tree harvest simulators. All that is required to locate trees in random stands is harvesting strip width and stand density. The authors also provide a means of determining optimum strip width, which is designed to minimize travel distance between trees.

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3. OBJECTIVES

The uncontrollable interaction offers a unique and complex problem for effectively analyzing timber harvesting systems. For any new technique to become generally accepted, it must be able to handle the analysis of uncontrollable interactions. To become competitive with simulation modeling, a new technique must not only handle the uncontrollable interaction, but improve upon the performance of simulation in these situations.

The objective of this research was to explore some alternatives for modeling uncontrollable interactions, that have the potential for improving upon state-of-the-art models. The specific objectives of this study are:

1. Adapt the theoretical aspects of a Markov process model to the analysis of uncontrollable interactions in timber harvesting systems. As improvements on simulation, the model to be developed should provide analytic solutions, avoid the problems with correlated output data and reduce or even eliminate the need for advanced programming skills in order to use and modify the model.

2. Provide a means of analyzing data that is compatible with the assumptions of the Markov model.

3. Formalize and solve the Markov process model for a particular type of uncontrollable interaction.

4. Illustrate the flexibility of the model by considering various modifications to the uncontrollable interaction.

An Uncontrollable Interaction for Illustration

For purposes of illustrating a model, a fixed location slasher (i.e., Siiro Model S-110-VM or the equivalent, with a purchase price of $45-50k) and a grapple skidder (i.e., Caterpillar 518 or the equivalent, with a purchase price of $85-90k) will be used. This interaction is assumed to be contained within a system that mechanically fells trees using a limited area, track-type feller-buncher (e.g., Drott 40). The feller-buncher places sheared trees in bunches, where they are limbed and topped using a chainsaw. The treelength bunches are then transported to the slasher by the grapple skidder. Figure 1 contains a flow diagram illustrating the sequence of activities for this harvesting system.

The treelength stems delivered to the slasher are processed into fixed or variable length logs and bolts. Figure 2 illustrates the key components of the interaction. Production buffer size is defined by the boom reach of the slasher's loader, so that only a finite number of skidder loads can be inventoried in the buffer at a given time. The fluctuations in the production buffer, which will determine idle and delay times, will be tied directly to:

1. Slasher cycle time--the time it takes the slasher to process a skidder load of treelength stems.

2. Skidder cycle time--the time it takes the skidder, upon leaving the buffer area, to return with its next load.

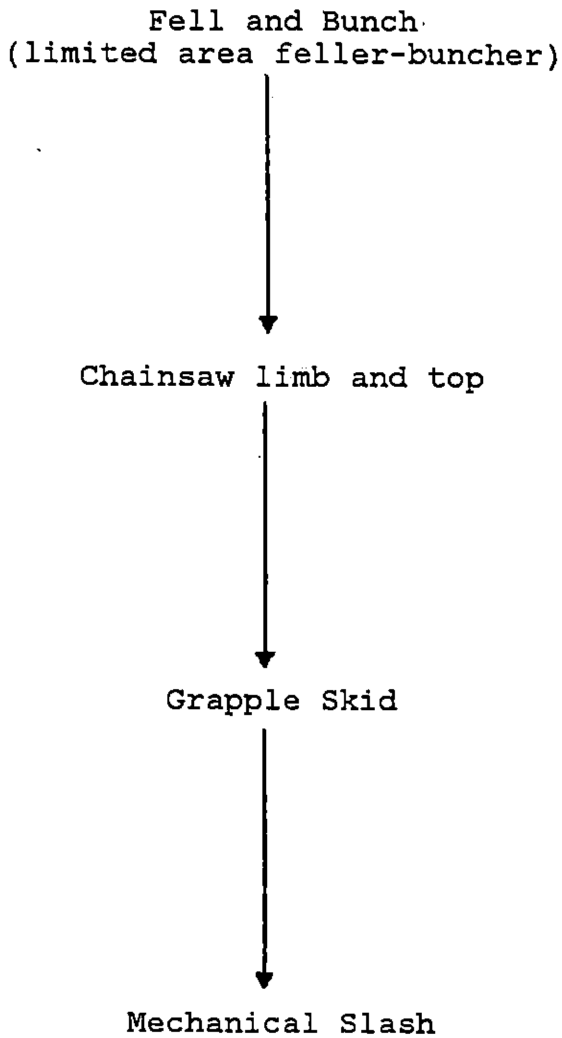

Figure 1. Flow diagram for the mechanical fell, chainsaw limb and top, grapple skid and mechanical slash harvesting system.

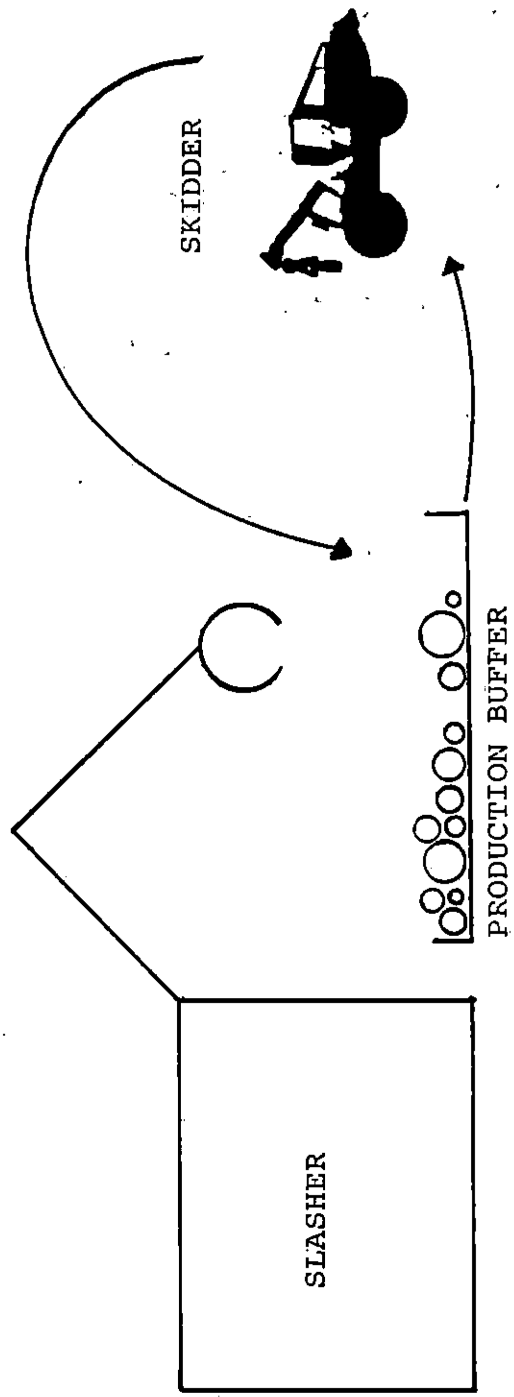

Figure 2. Components of the one slasher/one skidder interaction.

4. METHODS

4.1 RANDOM PROCESS THEORY

By observing the evolution of natural or man-made phenomenon, over time, it becomes increasingly apparent that events or activities occur in ways that are difficult to predict. The task of modeling these systems, or components thereof, with the objective of adequately capturing their dynamic probabilistic behavior, can be difficult at best. The theory of random or stochastic processes provides a rich class of tools for analytically modeling these real-life phenomenon.

Random Processes

Consider a logging operation at work. By observing the system, over a period of time, one can begin to distinguish certain recurring events or activities. Some of the activities may appear to occur at random for random periods of time, while others may recur in a particular sequence, but where each component of this sequence still occurs for a random length of time. It is not difficult, then, to begin thinking of these events as random variables, where the random variable provides a rule for assigning numbers to sample points. For instance, consider the time it takes a grapple skidder to acquire (or grapple) a load of trees at an arbitrary inwoods location. The sample point would be the bunch to be acquired and the number assigned to the sample point by the random variable would be the time to grapple the load.

A logical extension from a single activity (or random variable) would be the ordering of activities of a machine or individual. That is, the time it takes a skidder to travel to its loading point may be denoted by a random variable X, the time to load as random variable Y and the time to travel back to the landing as random variable Z. What emerges from this ordering is the notion of a random process, as opposed to a collection of arbitrary events evolving over time.

In a more formal sense a random process has been defined as "an indexed family of random variables" (4). Or, let $\{X_t: t \in T\}$ denote a random process, where for every t (or index) in the set T, X_t is a random variable. For the purposes of this study, the index t will denote time and X_t will be the state of the system at time t.

Classifying Random Processes

It is useful to categorize a random process according to (4,11):

1. The index set T of the process, and
2. The state space of the process.

The indexing set T is referred to as the parameter set of the random process and may be either discrete or continuous. For a discrete parameter random process, T is a countable set such as the non-negative integers. For example, consider the inventory of logs at a roadside landing. If one were to observe the inventory at the end of each workday, the random inventory level would have a discrete parameter.

In a continuous parameter process, T is usually some subset of the real numbers (eg., $0 \leq t \leq \infty$). For the roadside inventory example, one could choose to observe the inventory at any arbitrary time during the workday, in which case the random inventory level would have a continuous parameter.

The state space of a random process is defined as the set of all possible values that the random variable can take. Like the parameter space, the state space can be either discrete or continuous. A state space is discrete if it contains a finite or countably infinite number of states. For instance, if the random process of interest was the number of logs or bolts in the roadside inventory, the number of logs in inventory would represent a discrete state process, defined by the non-negative integers. Those processes that do not have discrete state spaces are termed continuous.

Markov Processes

Having defined the general class of random processes, how one might go about completely specifying the probabilistic behavior of a random process? Ordinarily, a large number of probabilities must be identified in order to accomplish this. Take the random process associated with the activities of the skidder discussed earlier. In this case one would need to know the probability distribution for each random variable (X,Y,Z) in the sequence. If X, Y and Z were not independent random variables, the joint distributions of all possible combinations of the three would be required. For practical applications, one has no hope of satisfying this requirement, for large numbers of random variables, unless some simplifying structure can be introduced.

Fortunately, there are cases where only a small subset of probabilities is necessary to describe the probabilistic behavior of the system. A class of random processes that provides a reasonable structure with which to work and is flexible enough to handle real-life applications is referred to as Markov processes. The distinguishing feature of these processes is that they satisfy the Markov property. Clarke and Disney (4) define the Markov property for discrete state random processes as:

$$\Pr[X_{t_{n+1}}=k_{n+1} \mid X_{t_n}=k_n, \ldots, X_{t_1}=k_1] = \Pr[X_{t_{n+1}}=k_{n+1} \mid X_{t_n}=k_n] \quad (1)$$

for every $k_1, \ldots, k_{n+1}$ in the state space, $n=0,1,2,\ldots,$ and $t_1 \le t_2 \le \ldots \le t_{n+1}$ in T. For a physical system like timber harvesting, this says that the probability the system will be in a given state at time t_{n+1} may be determined from the knowledge of its state at t_n and is conditionally independent of the history of the system before t_n (4).

Using the classification scheme described earlier, Markov processes may be in any of four basic forms: (discrete state, discrete parameter), (discrete state, continuous parameter), (continuous state, discrete parameter) or (continuous state, continuous parameter).

Discrete state, discrete parameter Markov processes have seen limited use in forestry applications. Bruner and Moser (3), Peden et al. (14), Seth and Shulka (15) and Suzuki (18) used this type of process to model future diameter distributions, number of surviving trees, number of mortality trees and number of harvested trees in uneven-aged stands of timber.

Because timber harvesting systems are configured with finite numbers of machines and personnel, operating on finite numbers of trees or stems, discrete state processes are of interest here, eliminating the need to consider continuous state processes.

A Markov Process For Timber Harvesting Analysis

For application to logging systems analysis the Markov process of interest is the discrete state, continuous parameter process. Because logging operations, and interactions in particular, are dynamic, where activities and production levels change continuously, the continuous parameter Markov process will permit an analysis at any arbitrary time t. Using either of the discrete parameter Markov processes would restrict the analysis to fixed points in time. Such a process would not adequately describe the probabilistic behavior of timber harvesting systems, from a decisionmaking standpoint.

For the discrete state, continuous parameter Markov process one can draw upon the well developed theory describing these processes. This theory can then be combined with certain assumptions about timber harvesting systems in order to build a flexible analytic model for describing the probabilistic behavior of these systems.

Traditional analyses (especially simulation models) of timber harvesting systems have focused almost exclusively on the long-run or steady-state behavior of a system. Adopting this assumption here is reasonable in at least two ways. First, the discrete state, continuous parameter Markov model is amenable to solution of the steady state probability behavior of the harvesting system under study. Solutions that include the transient or start up period of the process require manipulation of a system of differential equations. Except for the simplest cases, a solution is retrievable only through methods that are beyond the scope of this study. Second, for a rational decisionmaker, modifications in system and component configurations and decisions as to where the system can operate efficiently should be based on steady-state rather than transient behavior of the system.

Continuous Parameter Markov Theory

The objective of this study is to show how Markov process theory can be adapted to the modeling of uncontrollable interactions in timber harvesting systems. Therefore, a comprehensive theoretical development of these processes is not a primary objective. However, it is necessary to discuss some important theoretical results that will aid in understanding model development.

The basis of the discrete state Markov model lies in developing a state space that describes the operation of the system to a degree that will provide the necessary information to meet the objectives of the analysis. Once these states have been defined it is necessary to determine how the process moves, probabilistically, from one state to the next. The movement from state to state can be thought of as being repeated over and over again many times when studying the steady-state behavior of the system. The ultimate objective is to determine the steady-state probability of the system being in a given state. Referring back to the skidder example, we might be interested in developing a Markov model to determine the steady-state probability that the skidder is in any of its three states: travel to woods, acquire a load, travel back to a landing.

The probabilistic behavior of a Markov process is governed by two quantities. One is the vector of probabilities for the initial states of the system. Since steady-state behavior is of interest here, the system is assumed to have been in operation sufficiently long that any effect of these initial conditions is largely irrelevant.

The other governing component of the probabilistic behavior is the matrix of transition probabilities, referred to as P(t). Each element of P(t) denotes the probability of moving from state i to some future state j, over an interval of time of length t. The elements are denoted by $p_{ij}(t)$. The basic idea is that one chooses a time interval, t, and asks for the probability that the state of the system is j at time (s+t), given that the state was i at time s. For each (i,j) then, one has a function of t and s, and these functions represent the elements of the transition matrix P(t).

In most applications, including the one here, one assumes that the Markov process is time homogeneous. That is, the elements of the transition matrix depend only on the length of the time interval (s,s+t) and not on its initial point, s. Thus, the P(t) matrix has the same entries regardless of where one takes the interval of length t. Of course, changing the interval length will change the elements of P(t).

Also of considerable importance in continuous parameter Markov process theory, as used here, is the requirement that the process spend a negative exponential distributed amount of time in state i before making a transition into a different state j (4). The implication is that the time the process spends in state i is independent of the next state visited. If this dependency existed, then information about the current state and the time the process had been in the current state would be relevant to the prediction of the next state. This violates the Markov property. The key being that the forgetfulness property of the exponential is what guarantees that the process is Markovian.

Unfortunately, the negative exponential assumption necessary to maintain the Markovian property is not compatible with what is known about the probability distributions of most harvesting system components (1,12). Most probability distributions of time, in harvesting system applications, are unimodal and range from positively skewed distributions to those approaching symmetry (1,12).

Alternative Probability Distributions

In order to use continuous parameter Markov process theory to model timber harvesting systems, a device is needed that will permit a degree of flexibility in modeling time-based random variables, yet maintains the integrity of the Markov model. This requirement is met by a class of distributions referred to as phase type distributions (13). The first of these distributions is known as the Erlang family. The Erlang density function can be expressed as:

$$f(t) = [\lambda(\lambda t)^{k-1} \exp(-\lambda t)]/(k-1)! \qquad (3)$$

where k is a positive integer and $\lambda, t > 0$. Parameters k and λ are often referred to as shape and scale parameters, respectively. The expected value of the Erlang distribution is:

$$E(t) = k/\lambda \qquad (4)$$

and the variance is:

$$Var(t) = k/\lambda^2 \qquad (5)$$

The potential shapes that can be modeled with the Erlang family range from the purely random exponential to a variety of positively skewed distributions (13).

What makes these Erlang distributions important from a Markov process standpoint? Recall the basic probability result that the distribution of the sum of k independent, identically distributed exponential random variables is distributed as an Erlang distribution with shape parameter k. Now, consider the physical construct presented in Figure 1. Call the rectangular box a holding device and each cell within the box a stage. Suppose that an entity, perhaps a skidder, must, upon entering the holding device go through the first stage, followed by stage 2, and so on until the final or k^{th} stage is completed, at which time the skidder moves out of the holding device altogether. For instance, recall the skidder process which was a sequence of travel to the woods, acquire a load, and travel back to the landing. This sequence may be thought of as containing 3 stages and could conceivably, for illustration purposes, be modeled as an Erlang distribution with shape parameter 3. The skidder must enter stage 1, travel to the woods, followed by stage 2, acquire a bunch, and finally stage 3, travel back to the landing, which completes the cycle. Only when the skidder has exited stage 3 can it or another skidder enter the holding device.

In general, suppose at each of the k-stages the time spent in the j^{th} stage is exponentially distributed as follows:

$$f(t) = \lambda \exp(-\lambda t) \qquad (6)$$

where, $\lambda, t>0$. For modeling purposes this construction provides the following desirable properties:

1. The time between entities exiting the holding device is Erlang with parameter k, where the times to traverse each stage are identically distributed negative exponentials, with scale parameter λ. Although the time it takes to traverse the holding device is not exponential, the combination of k-exponential stages (or phases) can be considered the statistical equivalent for the purposes of the Markov model (13).

2. By permitting only 1 entity in the holding device at a time guarantees that the sequence of times between consecutive departures is one of independent random variables.

3. The time until an entity departs the holding device depends only on what cell it is in, because of the forgetfulness property of the exponential distribution.

4. The concept of stages allows one to describe the movements of an entity in discrete terms. That is, the skidder can only be in one of three possible states, denoted by the three Erlang stages.

While the Erlang family is reasonably flexible in modeling different distributions, it is not flexible enough to describe the wide variety of distributional shapes encountered in harvesting applications. A second family of distributions, known as the mixed Erlang, provides a greater degree of flexibility (12). Several forms of the mixed Erlang family exist, but the following form will be used for illustration purposes:

$$f(t) = [p^{K+1}(K\lambda_K)^K t^{K-1} \exp(-pK\lambda_K t)]/(K-1)! + [q^{L+1}(L\lambda_L)^L t^{L-1} \exp(-qL\lambda_L t)]/(L-1)! \qquad (7)$$

where, $p+q=1$, K and L are positive integers and λ_K, λ_L, $t>0$. The expected value of the mixed Erlang distribution is:

$$E(t) = 1/\lambda_K + 1/\lambda_L \qquad (8)$$

and the variance is:

$$Var(t) = (K+1)/K\lambda^2 p + (L+1)/L\lambda^2 q - [1/\lambda_K + 1/\lambda_L]^2 \qquad (9)$$

The physical construction for the mixed Erlang distribution is similar to that for the Erlang family. Figure 2 shows that the mixed Erlang distribution can be conceptualized as two holding devices, with K and L stages, respectively. The probability of an entity entering holding device one is p, while the probability of entering holding device two is q (or 1-p). Again, only one entity can be traversing the holding devices at any time. That is, if an entity is in holding device one, no entity can simultaneously be in holding device two. Similarly, once an entity enters a holding device, it must enter stage one then proceed through either K or L stages, depending on the holding device, until it exits the holding device.

For the mixed Erlang, the probability distribution of time in the j^{th} stage is exponential, with density function:

$$f(t) = pK\lambda_K \exp(-pK\lambda_K t) \qquad (10)$$

for holding device one, and

$$f(t) = qL\lambda_L \exp(-Lq\lambda_L t) \qquad (11)$$

for holding device two. Just as in the simple Erlang case, the time to traverse a holding device is distributed as an Erlang K for the first holding device and Erlang L for the second holding device. Again, all the desirable properties of the simple Erlang model are retained by the mixed Erlang construction. The key to this physical construction is that a cycle time, for instance, with a peculiar distribution can be modeled using the mixed Erlang and still retain the Markovian property.

Solution Of The Markov Process Model

Clark and Disney (4) show how the P(t) matrix, discussed earlier, can be used to develop a system of differential equations for describing the entire time path for every state j and every initial state i. Unfortunately, for practical purposes there are few explicit solutions to these differential equations for large state spaces.

Further, for even moderately sized problems the numerical solution of these equations would tax the capabilities of many large computers. However, a steady-state solution will require dealing with a less complicated system of equations.

In the theory of Markov processes, as considered here, it can be shown that a steady-state solution can be determined from the following system of algebraic equations (4):

$$\Pi\Lambda = 0 \qquad (12)$$

where, Π is a vector of steady-state probabilities ($\Pi = \pi_1$, π_2, ... ,π_n) and Λ is referred to as the rate matrix or generator of the Markov process.

The elements of the rate matrix (λ_{ij}'s) are the derivatives of the $p_{ij}(t)$ evaluated at t=0. Thus, knowing λ_{ij} for every (i,j) is "equivalent" to knowing $p_{ij}(t)$ for every i, j, and $t \geq 0$. For practical applications, the λ_{ij}'s are what the analyst seeks and are referred to as transition rates (or instantaneous rates) of the Markov process.

Two very important theoretical results for discrete state, continuous parameter Markov processes can be applied to the determination of a unique probability solution. These properties are referred to as a finite state space and irreducibility.

Because timber harvesting systems are configured with finite numbers of men and machines, operating on finite numbers of logs, trees, bunches, etc., the realization of finite state spaces does not pose a problem.

Irreducibility is obtained when there exists a positive probability of eventually entering any state j from any state i (4). In practice this property is not difficult to obtain and will be evident from the final structure of the rate matrix.

With irreducibility and finite state space the following theorems can be applied to the steady-state solution of the model:

1. For an irreducible Markov process, the π_j's exist and always have the same value, irrespective of the initial probability distribution of the process.

2. If the Markov process is irreducible and has a finite state space, then there exists exactly one solution for the π_j's, where $0 \leq \pi_j \leq 1$ and $\Sigma_j(\pi_j) = 1$.

The steady-state probabilities of the process can be obtained by utilizing matrix methods for solving the system of algebraic equations presented in equation (12). Although these systems will generally be too large for hand calculation, they can be handled with a computer and one of the many software packages or routines available for solving these types of problems.

Determining the Transition Rates

The basic problem now, is how to compute the λ_{ij}'s from data. Since the λ_{ij}'s are the building blocks of all Markov process theory, their determination is crucial to the development of a workable Markov process model.

Consider skidder cycle time modeled as a mixed Erlang distribution. Upon beginning a cycle of productive work (i.e., depart the landing for woods), the skidder chooses with probability p to enter a series of K-stages and with probability q (i.e., 1-p) to enter a series of L-stages. If it enters the K-stage series it must traverse K-stages before returning to the landing. Each stage holds the skidder for a random time that is chosen from an exponential distribution of the form $[pK\lambda_K \exp(-pK\lambda_K t)]$. Thus, to complete a tour of useful work, the skidder must traverse all K-stages. The total cycle time is a random variable that is the sum of K independent, identically distributed exponential random variables.

To compute the requisite λ_{ij} the argument is as follows. We want the derivative of $p_{ij}(t)$ at t=0. Using the homogeneity assumption, choose the interval $(0,\Delta t)$ to determine the rates. Suppose the skidder is now in stage i of holding device K. At time Δt where can it be? It could be anywhere depending on how much time it is given to change. Then, for example, it could be in stage j, assuming $i,j<K$.

Now, compute λ_{ij} for $j=i+1$ to start. Suppose the skidder only made one move, from i to i+1 in Δt. At what rate $\lambda_{i,i+1}$ does it make this transition? Since the holding times are exponentially distributed:

$$p_{i,i+1}(\Delta t) = \int_0^{\Delta t} pK\lambda_K \exp[-pK\lambda_K s]\exp[-pK\lambda_K(\Delta t-s)]ds \qquad (13)$$

That is, the skidder moved to stage i+1 at some time s (with probability $pK\lambda_K \exp[-pK\lambda_K s]ds$) and did not move again in the remaining time (with probability $\exp[-pK\lambda_K(\Delta t-s)]$). Since this move could have occurred anywhere in $(0,\Delta t)$, the integral gives the total probability of the transition. Integrating equation (13) yields:

$$p_{ij}(\Delta t) = pK\lambda_K \exp(-pK\lambda_K \Delta t) \int_0^{\Delta t} ds \qquad (14)$$

$$= pK\lambda_K \Delta t \exp(-pK\lambda_K \Delta t) \qquad (15)$$

Expansion of this in a Taylor series about zero yields:

$$p_{ij}(\Delta t) = pK\lambda_K \Delta t[1 - pK\lambda_K \Delta t + (pK\lambda_K \Delta t)^2/2! - \ldots] \qquad (16)$$

$$= pK\lambda_K \Delta t + o(\Delta t) \qquad (17)$$

where, $o(\Delta t)$ is a function that has the property $[o(\Delta t)/\Delta t]\to 0$ for $\Delta t\to 0$. Typically, $o(\Delta t)$ is of the form $[a_{ij}(\Delta t)^2 + b_{ij}(\Delta t)^3 + \ldots]$. Upon differentiation then, $\lambda_{ij} = pK\lambda_K$ for $j=i+1$ when the skidder makes only one move.

Now, compute $p_{i,i+2}(\Delta t)$ for $i, i+2<K$, where the skidder moves two stages from its start in Δt. As before, it will make some move in $(0,s_1)$ to stage i+1, then make some move to stage i+2 after s_1, say at s_2, and it will stay in i+2 thereafter until at least Δt. The probabilities here are given by:

$$p_{i,i+2}(\Delta t) = \int_0^{\Delta t}\int_0^{s_2} pK\lambda_K \exp[-pK\lambda_K s_1]\ pK\lambda_K \exp[-pK\lambda_K(s_2-s_1)]$$

$$\exp[-pK\lambda_K(\Delta t-s_2)]\ ds_1\ ds_2 \qquad (18)$$

Again performing the indicated integration, and expanding the result in a Taylor series one finds:

$$p_{i,i+2}(\Delta t) = (pK\lambda_K\Delta t)^2 + o(\Delta t) \qquad (19)$$

The crucial observation is that $p_{i,i+2}$, the probability of moving two stages in Δt, does not have a Δt term. Its first term is in Δt^2. This means that the rate of this change is a second order phenomenon. By choosing Δt small enough, (remember λ_{ij} is the derivative of $p_{ij}(t)$, so Δt will eventually go to zero) $p_{i,i+2}$ can be made arbitrarily small, so in the limit it is zero. Notice that this does not say that it is impossible to make the transition, but rather, the rate at which it happens is zero.

Observe, now, that the rate of making exactly 2 transitions among the stages is zero. It seems perfectly reasonable and can be proven precisely (as was done in the

2-case) that to make 3 transitions (from i to i+3), the rate is also zero and in fact to make any more than 1 transition, the rate is zero. This is critical, for it means that when computing the rate matrix, Λ, one must only be concerned with the skidder moving one stage and any other possible move occurs with rate zero. Furthermore, observe that this rate is only the parameter of the exponential distribution representing the holding times in each stage. So, not only are there relatively few λ_{ij} to compute, they can be determined by inspection of the cycle time model (i.e., Erlang or mixed Erlang). In this case, they are all $pK\lambda_K$ for the holding device with K-stages and $qL\lambda_L$ for the holding device with L-stages. For the simple Erlang model the rate is just λ.

There are two large holes in the above argument:

1. What happens if the skidder does not move?

2. What happens when the skidder and some other piece of equipment being studied moves in Δt?

Using precisely the same logic as for the skidder move, it can be shown that the rate of change of probability if the skidder does not move is $-pK\lambda_K$. Thus, for the skidder alone, the rate of change of probability (λ_{ij}) is zero unless $j=i+1$ (in which case the rate is $pK\lambda_K$) or $j = i$ (in which case the rate is $-pK\lambda_K$).

Regarding the second question, one can show as before, that the rate of change of probability is zero if both the skidder and another piece of equipment try to change states simultaneously. Thus, either the skidder changes with its rate or the other machine changes with its rate or neither changes (which is the negative of the sum of these two rates). Any other change has zero rate of change in its probability.

The foregoing development is the logic used in constructing the Λ matrix. Three items that can serve as a check on this construction are:

1. For any rate matrix the off-diagonal terms are positive and since they are not probabilities but rates of change of probability, they may be larger or smaller than 1.

2. The diagonal terms are all negative.

3. The diagonal terms are the negative of the sum of the positive terms on the same row (i.e., the row sums of the rate matrix are 0).

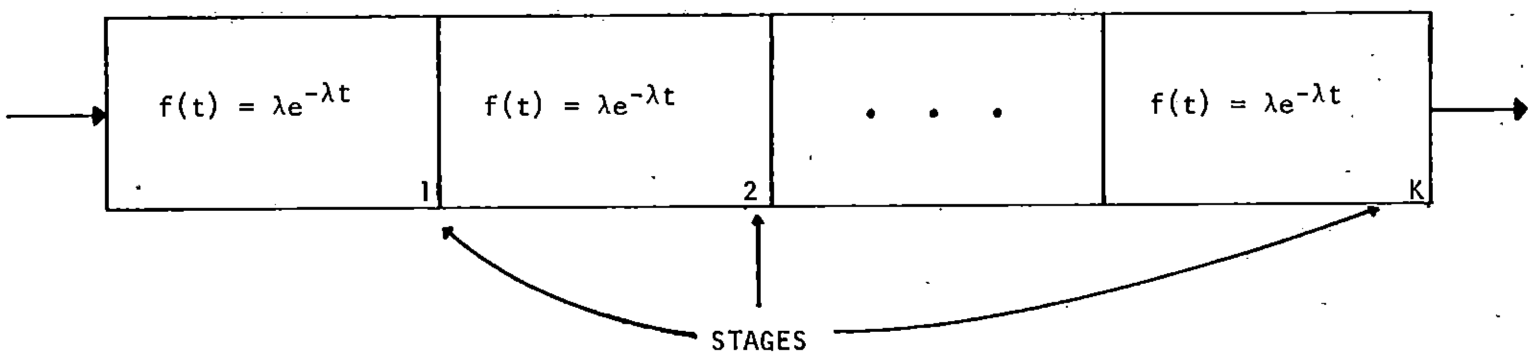

Figure 1. Physical construct for the simple Erlang cycle time model.

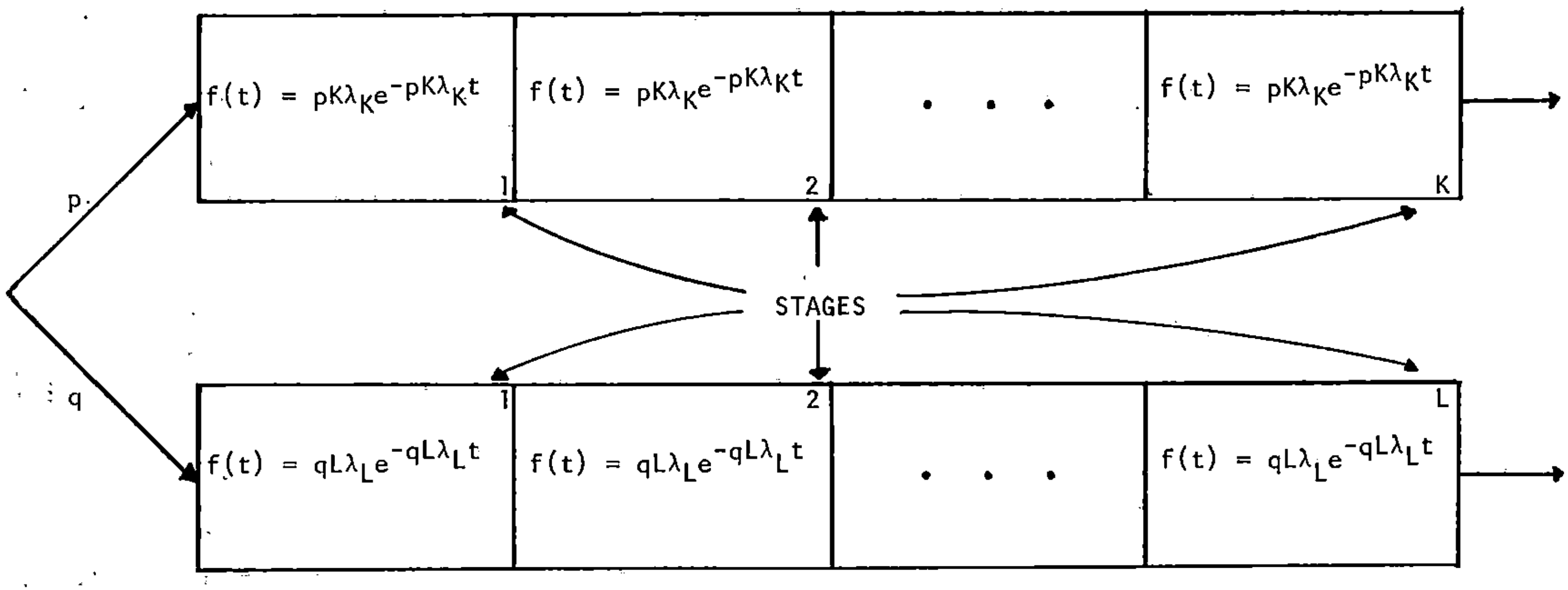

Figure 2. Physical construct for the mixed Erlang cycle time model.

4.2 DATA ANALYSIS

Data analysis is a crucial component of any model building effort. The manner in which the data is to be analyzed and the choice of potential models should ideally be considered simultaneously. If not, the analyst runs the risk of selecting a model that is not appropriate for the data. Or, the results of an arbitrary data analysis may lead the analyst to choose an inappropriate or ineffective model.

A thorough data analysis provides several important advantages:

1. It can help provide a clearer understanding of the phenomenon being studied. This, in turn, can be beneficial in defining the type of model that will be most effective in solving the problem at hand.

2. It can provide invaluable guidance for supplemental data collection, as well as, for data collection in future studies.

3. An intense data analysis will help identify those components of a problem that are most influential in affecting the outcome and how these components can be altered or redesigned to produce a desired result.

As is so often the case, the data for this study was collected prior to setting formal objectives. Data collection consisted of time studies on individual components of slasher and skidder cycle times. Since the Markov model requires cycle time data the following analysis serves to illustrate the problems encountered when using data that was not collected for a specific model. It also illustrates some of the alternatives the analyst can draw upon when faced with data in a form that is not directly compatible with the model of interest.

Interaction Components To Be Modeled

In order to build a model to analyze the slasher-skidder interaction, statistical models must be developed for the following components:

1. The time it takes the slasher to process a skidder load of treelength stems (i.e., slasher cycle time).

2. The time it takes a skidder to pick up a load and deposit it at the slasher for processing (i.e., skidder cycle time).

The previous section provided a discussion of the statistical models that could be used in conjuntion with the Markov process model. They were the Erlang and mixed Erlang probability distributions.

The objective of this chapter is to develop cycle time probability distributions, for the slasher and grapple skidder, using the Erlang and mixed Erlang distributions.

Modeling Skidder Cycle Time Components

Data for modeling skidder cycle times consisted of four components:

1. The time to travel from the landing to the in-woods loading location (travel-out).

2. The time to maneuver into a position to grapple a load.

3. The time to grapple a load.

4. The time to travel from the loading point to the landing (travel-in).

5. The time to drop the load at the production buffer.

For modeling purposes, the maneuver and loading activities were combined into a single component. Also, the unloading time is assumed to occur instantaneously (i.e., take zero time) as the skidder makes its transition from travel-in to travel-out.

Skidder Travel Times And Distances

The key aspect in modeling skidder cycle times lies in the relation between travel times and travel distances. Accompanying each observation of travel time (out or in) was a distance estimate. For travel-out there were 117 observations of time and distance and for travel-in there were 188 times and distances.

Of these observations, 41 cycles had all four times and distances from which a Pearson product moment correlation analysis could be performed (17). All pairwise correlations were found to be highly significant (i.e., at a significance level < 0.0001). A partial correlation for the times, holding both distance variables constant, was still signficant at a 0.05 significance level.

The indication is that a joint distribution between the four random variables (travel times and distances) is necessary to model this phenomenon. Unfortunately, the amount of data available is not sufficient for fitting a joint distribution of four variables. Since time is the variable of interest, any simplifying assumptions must be made on the travel distances.

One solution is to fix distance, thereby making time conditional on distance. This is not altogether unreasonable. Developing a joint distribution of even two variables, for a particular set of field conditions, would require large amounts of data. Attempting to obtain a general joint density over varying field conditions would require huge amounts of data. From a practical standpoint, the best solution is to focus on studying a particular system under a specific set of conditions. In this case, the specific set of conditions would be the distance distributions and the terrain and site factors under which the data was collected.

From a solutions standpoint, the idea of modeling only specific conditions avoids the potential for drawing imprecise conclusions or making inaccurate inferences. Such a problem arises when data collected under different conditions is pooled in an effort to obtain a general set of results for the system under study. The conclusions drawn from such an analysis are necessarily general in nature and provide little basis for inferring system performance under more specific sets of conditions. Ideally, a model should be used to test different sets of conditions individually, providing several sets of results which can then be aggregated to draw inferences.

The set of distance distributions applicable to this study are presented in Figures 1 and 2 for travel-out and travel-in distances, respectively. With distance assumed fixed, the fact that travel times remain correlated still poses a problem. From a modeling standpoint, the key question is whether the correlation will have an effect on the type of answers desired. One way to explore this is to examine the extremes of the correlation. That is, assume that the times are perfectly correlated in the first case and independent in the second. Then, by developing a cycle time distribution for each situation and using these as input to the Markov model, the impact of the correlation can be judged by comparing the respective results generated by the model.

Perfectly Correlated Travel Times

The travel-out and travel-in data sets were combined, since they are assumed to be perfectly correlated. Initially the data was fit to a general gamma probability distribution using maximum likelihood methods (7,10). The only difference between the general gamma and the Erlang distribution presented in Chapter 4, is that the shape parameter, k, of the gamma may take any value greater than zero. The maximum likelihood estimates for the general gamma merely serve as a basis for choosing an Erlang shape parameter that is a positive integer. The maximum likelihood estimate of k is (7,10).

$$\log_e(k) - \psi(k) = \log_e(X/X_G) \qquad (1)$$

where,

X = the arithmetic mean of the data,

X_G = the geometric mean of the data,

ψ = the psi-function or logarithmic derivative of the gamma function.

The maximum likelihood estimate of λ is (7,10):

$$\lambda = k/X \qquad (2)$$

The maximum likelihood estimates of the general gamma for the combined travel time data were $k=2.318$ and $\lambda=4.580$. Figure 3 shows the empirical distribution of these travel times superimposed on the fitted gamma distribution.

Visually, the theoretical curve does not fit the data very well. A more formal assessment was performed using a Kolmogorov-Smirnov (K-S) goodness-of-fit test of the empirical versus the fitted distribution (5).

The K-S test statistic is a measure of the maximum absolute vertical distance between the empirical and fitted distributions. When the parameters of the distribution are estimated from the data, Crutcher (6) states that the K-S test is conservative with respect to the Type I error. That is, if the test statistic is obtained where the parameters are estimated from the data, the hypothesis that the two distribution functions are equal is rejected with considerable confidence. Crutcher (6) provides alternate tables of critical values for exponential, gamma, normal and extreme value distributions. Using these tables, the empirical and fitted distributions were found to be statistically different at a 0.01 level.

From Figure 3, a hump or dogleg is evident in the middle range of the empirical distribution. Others have attributed this to two or more simpler distributions being combined into a single complicated distribution, occurring when 2 distinct phenomenon are inadvertently sampled as one (8,16). In this case, for example, the travel times could conceivably be coming from 2 distinct distance distributions, one for short distances and one for long distances.

The standard theoretical distributions do not perform well where doglegs, reverse curvature or S-curves are evident (8,16). One alternative is to fit a mixed distribution, which effectively decomposes the complicated distribution into more simpler distributions. Thus, not only is the mixed Erlang an important distribution for obtaining compatibility with the Markov process model assumptions, but it can be used here to model travel times based on a physical phenomenon.

In line with the assumptions of the mixed Erlang, that K and L be positive integers and $0 \leq p \leq 1$, candidate curves were fit by trial and error. That is, K, L and p were fixed and moment estimators for λ_k and λ_l were computed. The mixed Erlang chosen for fitting this data is presented in Figure 4. Visually, the fit is much better than the gamma fit, with a K-S value of 0.056. Unfortunately, the author is not aware of any formal K-S test for mixed distributions. Thus, the goodness-of-fit is based solely on the visual assessment and the relative improvement in the K-S value.

It is important to note at this point, that while the K-S test is a popular and commonly used goodness-of-fit procedure, it is insensitive in the tails of distributions (7). Because the tails of a distribution (and their relative size) exert a significant influence on the solution of a stochastic model, it is not always prudent to place a major emphasis on the results of K-S goodness-of-fit tests alone. A visual assessment of the fit can often be more informative. From a practical standpoint, for instance, it may be more important to explore the sensitivity of different fitted distributions, that appear statistically similar, but perform differently in the tails of the distribution. Because of this, statistical goodness-of-fit will remain important, but will not serve as the sole criteria in selecting statistical models.

Independent Travel Times

Figures 5 and 6 show the empirical distributions of travel-out and travel-in, respectively, superimposed on the maximum likelihood fits of the general gamma distribution. Again, the presence of doglegs in the mid-range of these distributions causes a poor fit of the gamma. Using the mixed Erlang distribution, the fits were improved substantially as illustrated in Figures 7 and 8. Based on the visual fit and the relative improvement in the K-S statistic, the mixed Erlangs were chosen to model travel-out and travel-in times.

In-woods Maneuvering and Loading

The data consisted of 129 observations with a mean time of 0.40 minutes and a standard deviation of 0.15 minutes. In order to illustrate some of the flexibility available to an analyst in developing statistical models from component distributions, a different distribution is used here. In the course of fitting the in-woods maneuver and loading data to various distributions, the best fit was obtained using a 3-parameter gamma distribution. The 3-parameter gamma density function is as follows (7,10):

$$f(t) = [\lambda^{k}(t-\gamma)^{k-1} \exp(-\lambda(t-\gamma))]/(k-1)! \qquad (3)$$

where, λ, k are as before, and γ is the lower bound of the distribution. Using a lower bound of 0.15, the maximum likelihood estimates of λ and k were 11.91 and 3.019, respectively. The K-S statistic was 0.0488 and was not significant at a 0.20 significance level. Figure 9 shows the empirical distribution superimposed on the fitted curve.

Developing a Cycle Time Distribution

Because cycle time is composed of travel times and loading times, a method is needed to combine these components in order to get a distribution of cycle times. The most desirable method would be to convolve the component distributions in order to analytically determine the cycle time distribution. However, the convolution of mixed distributions with other mixed distributions and/or standard distributions is not generally an easy task. More importantly, the resulting distribution, if tractable, is not guaranteed to be in an Erlang or mixed Erlang form.

Therefore, random travel and maneuver/loading times were generated from the fitted distributions and then summed to obtain cycle time estimates. This approach does have some advantages. If any component distributions can not be reasonably approximated with the Erlang or mixed Erlang distributions, other distributions can be used for estimation (as with the maneuver/load data here).

Finally, it is assumed that travel times and maneuver/loading times are independent. No physical reason could be offered for assuming otherwise.

Perfectly Correlated Travel Times

Using the mixed Erlang distribution for perfectly correlated travel times and the 3-parameter gamma for maneuver/loading times, 1000 random times were generated for each and summed to give cycle time estimates.

Figure 10 shows a 2-parameter gamma, fit using maximum likelihood estimates, superimposed on the estimated distribution. The fit is poor both from a visual and statistical standpoint, with the K-S test showing the two to be different at a 0.01 significance level. Using a mixed Erlang distribution a better fit of the cycle time distribution had parameters: K=3, L=6, p=0.15, λ_K=2.612, and λ_L=1.013. Figure 11 shows the fitted distribution superimposed on the empirical distribution. The K-S value for this fit was 0.056. Again, the selection of this model was based on a visual assessment and the relative improvement in the K-S statistic.

Independent Travel Times

The mixed Erlang distributions for travel-out and travel-in and the 3-parameter gamma for maneuver/load time were each randomly sampled 1000 times, and each component summed to estimate cycle times. Figure 12 shows the estimated cycle time distribution superimposed on an Erlang distribution with parameter 7. While the visual fit is reasonable, with a K-S statistic of 0.031, the K-S test still rejected the hypothesis that they are equal at a 0.01 significance level. Although, they are statistically different, no mixed Erlang could be found that fit the data any better.

Modeling Slasher Cycle Time Components

Even though available slasher data was collected on individual elements, the time for the slasher to process one of its own grapple loads was collected in continuous time, so that the time to process these loads was easily recoverable. That is, the time it takes the slasher to remove a load of stems (using its self-contained loader) from the productive buffer, until it returns to the buffer for another grapple load. In order to get cycle time to process a skidder load, this data must be coupled with data on the number of stems grappled by the slasher's loader and skidder load sizes.

A total of 252 slasher loads, with times and numbers of stems, were available. Mean time for the slasher to process a load was 2.515 minutes and the standard deviation was 1.276 minutes.

Fitting the empirical distribution to a 2-parameter gamma indicated the candidate shape parameters to be k=3 and k=4, which yielded scale parameters of 1.193 and 1.591, respectively. Figures 13 and 14 show the fitted distributions superimposed on the empirical distributions. Visually, both curves appear to fit the data reasonably well.

Using the K-S test, the hypothesis that the empirical and fitted distributions are the same could be first rejected at a significance level of $0.15 \leq \alpha \leq 0.20$. Therefore, the combined results of the visual and statistical tests indicate that either distribution provides a reasonable model. For the sake of simplicity, the Erlang with shape parameter 3 was chosen for modeling purposes.

Slasher Load Sizes

The average number of stems grappled per cycle was 2.159 with a standard deviation of 1.040. Three truncated, discrete probability densities were considered for fitting this data: binomial, negative binomial and Poisson. Truncation occurs for the zero class, since the slasher will always pick up at least one stem.

The negative binomial was deemed inappropriate since the sample variance is less than the sample mean (9). Of the remaining two, the truncated binomial provided the best fit in terms of the differences between the empirical and fitted distributions.

The truncated binomial density is (9):

$$Pr[X=k] = \{\{N!/[k!(N-k)!]\}[(p^k)(q^{N-k})]/[1-q^N]\} \qquad (4)$$

where,

N = the number of independent trials, or stems grappled,

p = the probability of a given outcome,

q = 1-p.

The maximum likelihood estimator of p satisfies the following equation:

$$x = Np'(1-q'^N)^{-1}) \qquad (5)$$

where, p' and q' are estimators of p and q. Using the data to test different values of N, and solving for p, the best fit was for N=6 and p=0.326 (i.e., the slasher can pick up any number of stems between 1 and 6). The maximum absolute difference between the empirical and fitted functions was 0.017, which was deemed adequate for modeling purposes.

Skidder Load Sizes

No data was available on skidder load sizes. Therefore, an estimate of these load sizes was extracted from the literature, for illustration purposes. Bradley et al. (2) show that the number of stems cut by a track-type feller buncher with a rotatable boom, at each stopping point, follows a truncated Poisson distribution. This result was applicable to stands exhibiting a random spatial pattern of trees. Since the number of trees felled at each stop will, in large measure, define the bunch size, this should provide a reasonable estimate of load sizes.

The truncated Poisson density is (9):

$$Pr[x=n] = \exp[-\theta][1-\exp(-\theta)]/n! \qquad (6)$$

where,

θ = the Poisson parameter,

n = the number of stems.

Bradley et al. (2) provide the following relationship for θ:

$$\theta = 1.7861 + 0.0134T \qquad (7)$$

where, T = the stand density in trees per acre. Thus, for any stand density the probability of a load of size n can be determined directly from equation (6). For illustration purposes, stand density is assumed to be 300 trees per acre.

Slasher Cycle Times

In order to estimate cycle times the following procedure was implemented:

1. Randomly generate a skidder load size from its truncated Poisson distribution.

2. Randomly generate a slasher load size from its negative binomial distribution.

3. Randomly generate a time to process the slasher load from the Erlang-3 distribution.

4. Repeat steps 2 and 3 until the sum of slasher load sizes is at least equal to the random skidder load size generated in step 1 and at the same time sum the individual times to process a slasher load.

This procedure was repeated 1000 times and the resulting distribution of cycle times was fit to an Erlang distribution. After fitting a 2-parameter gamma, the best Erlang fits were again with shape parameters 3 and 4, scale parameters 0.340 and 0.455, and K-S statistics 0.031 and 0.033, respectively (Figures 15 and 16). Although these goodness-of-fit statistics indicate that the empirical and fitted distributions are significantly different, the fit was sufficient for addressing the objectives of the study and the quality of fit was comparable to the mixed Erlang fit for skidder cycle times. For illustrating the Markov model, the Erlang with shape parameter 3 will be used.

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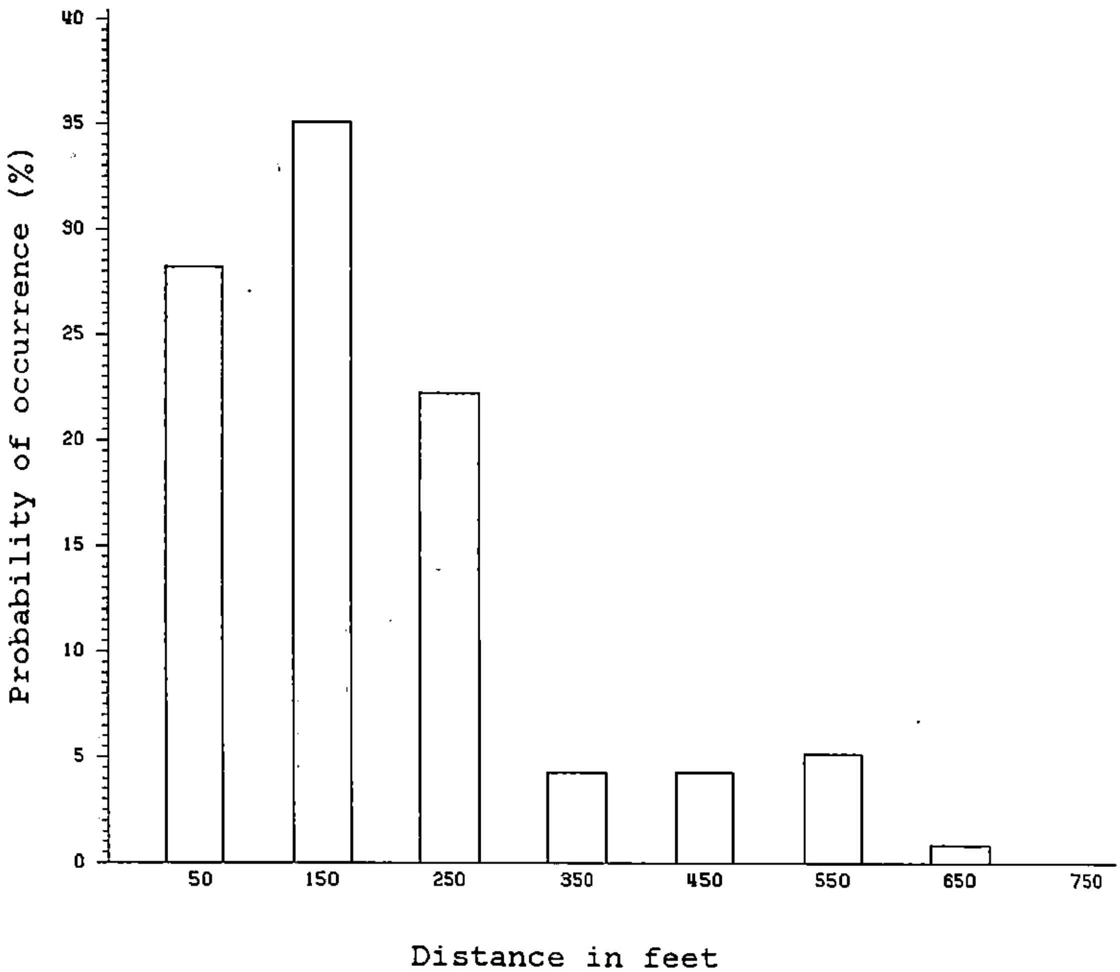

Figure 1. A histogram of grapple skidding travel-out distances.

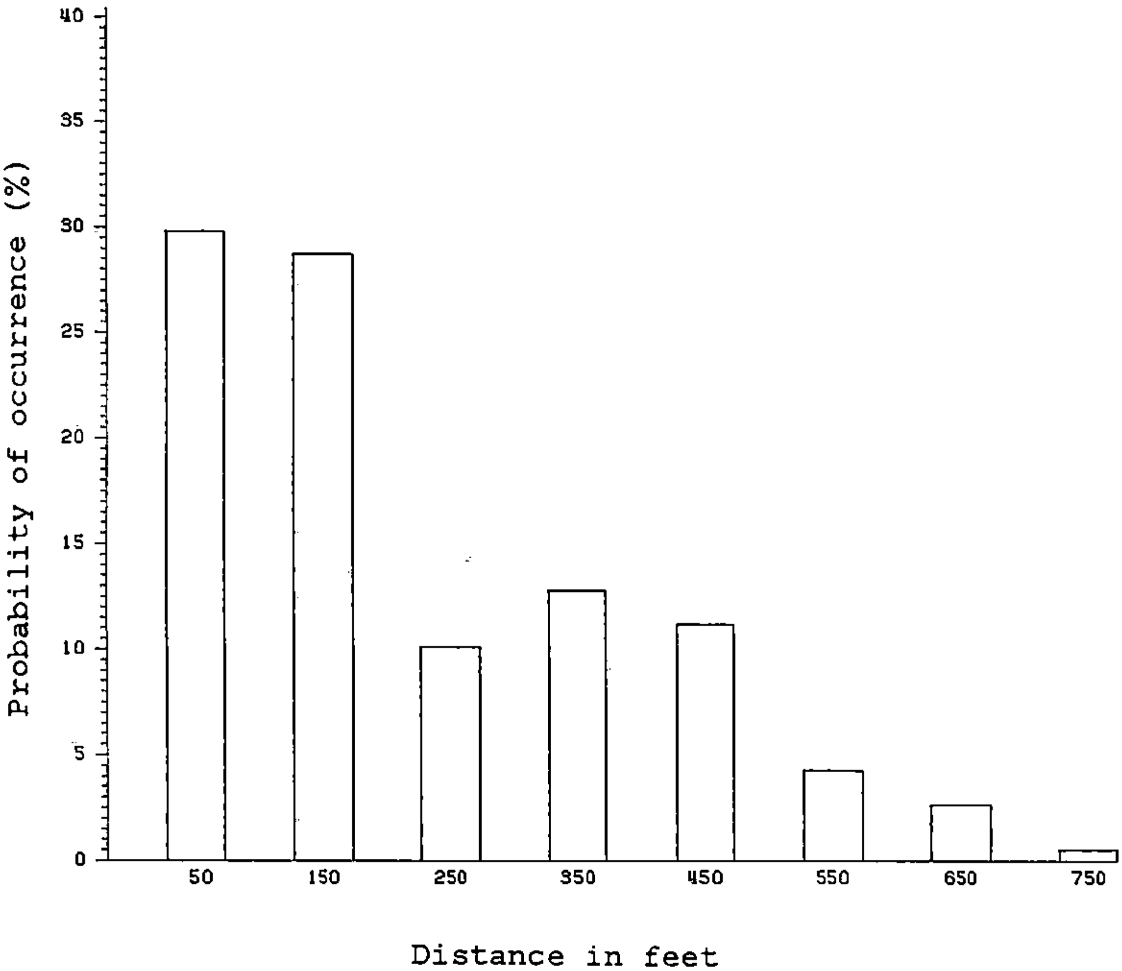

Figure 2. A histogram of grapple skidding travel-in distances.

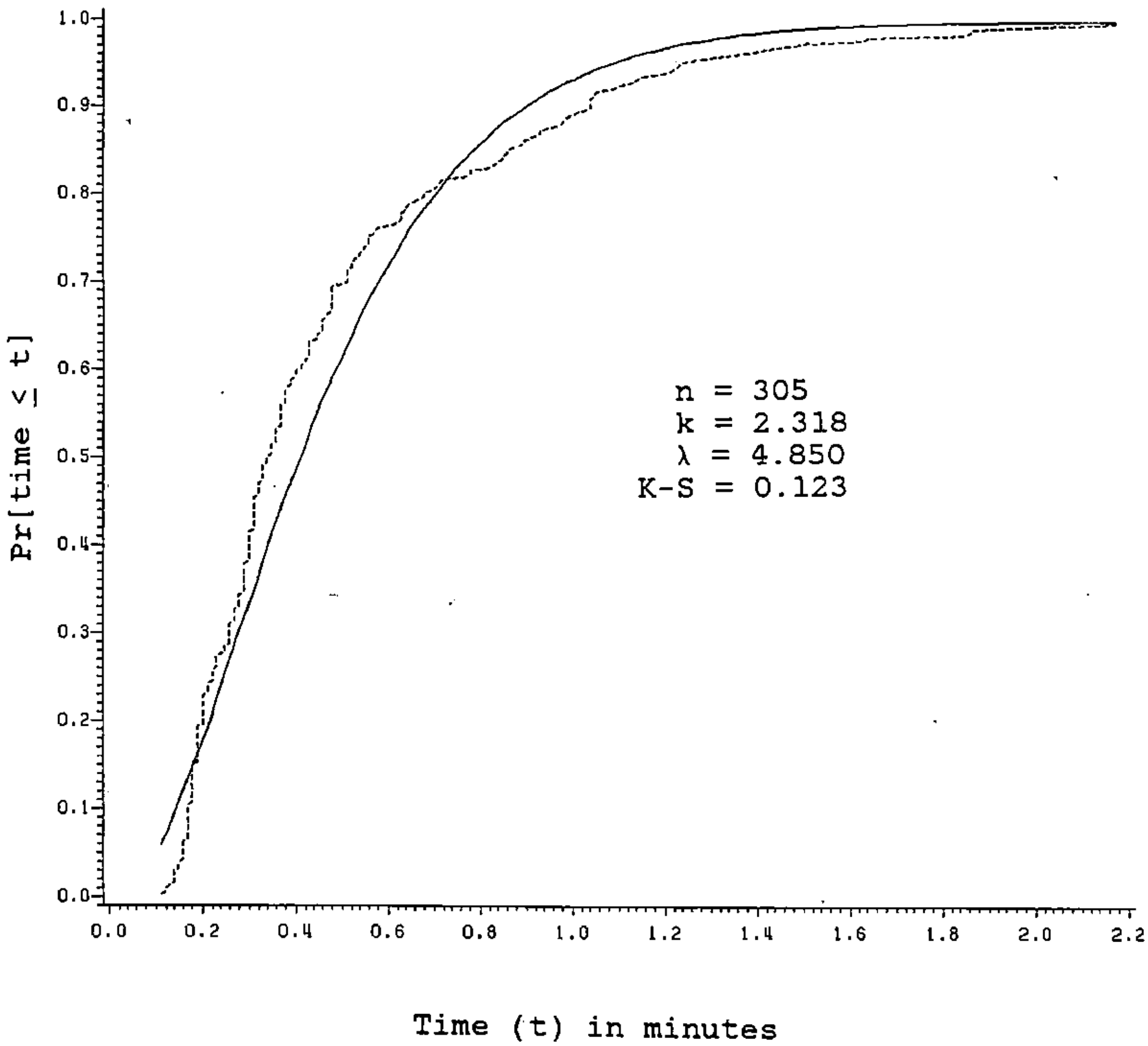

Figure 3. Two parameter gamma distribution (solid line) versus empirical distribution (dashed line) of combined travel-out and travel-in grapple skidding travel times.

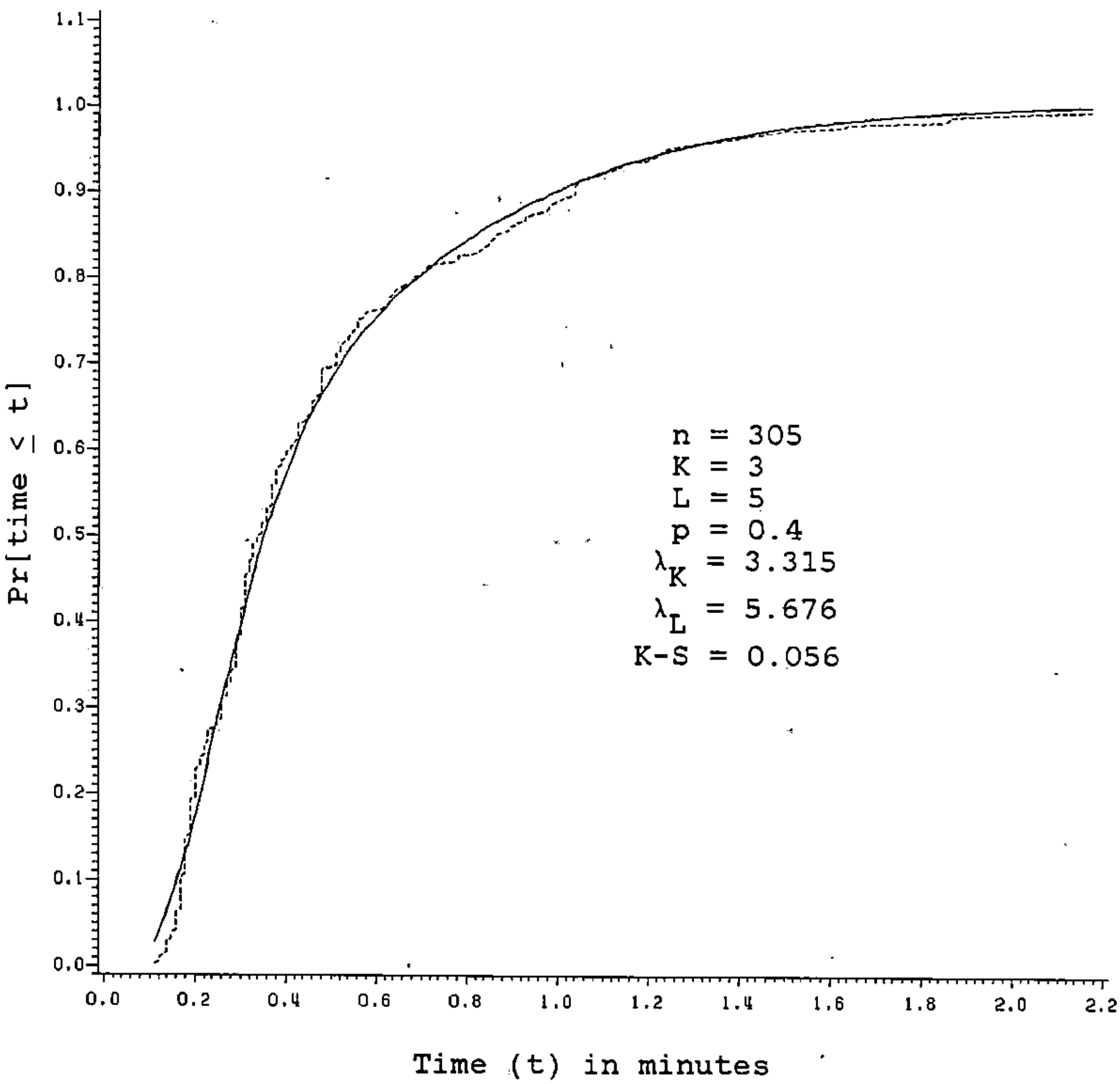

Figure 4. Mixed Erlang distribution (solid line) versus empirical distribution (dashed line) of combined travel-out and travel-in grapple skidding travel times.

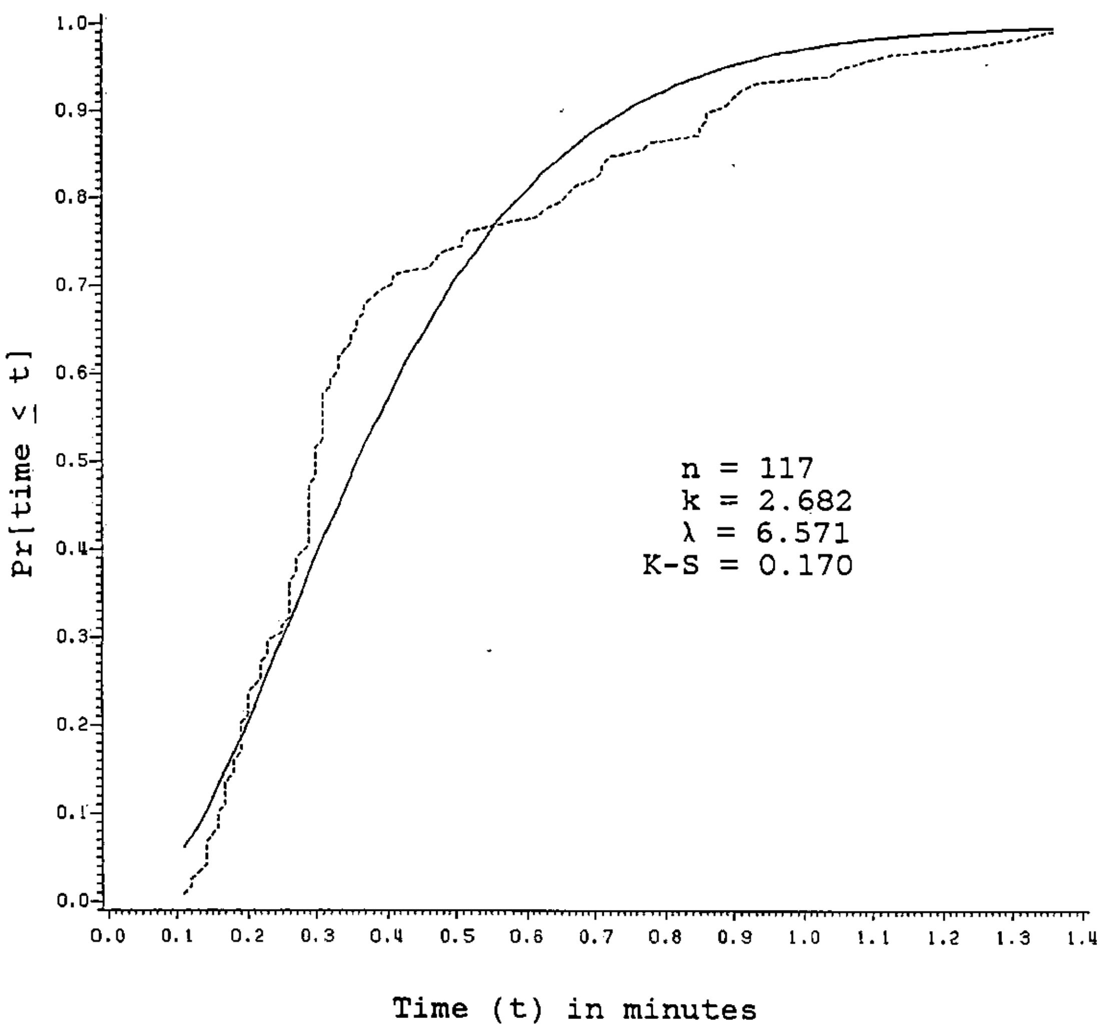

Figure 5. Two parameter gamma distribution (solid line) versus empirical distribution (dashed line of travel-out times for grapple skidding.

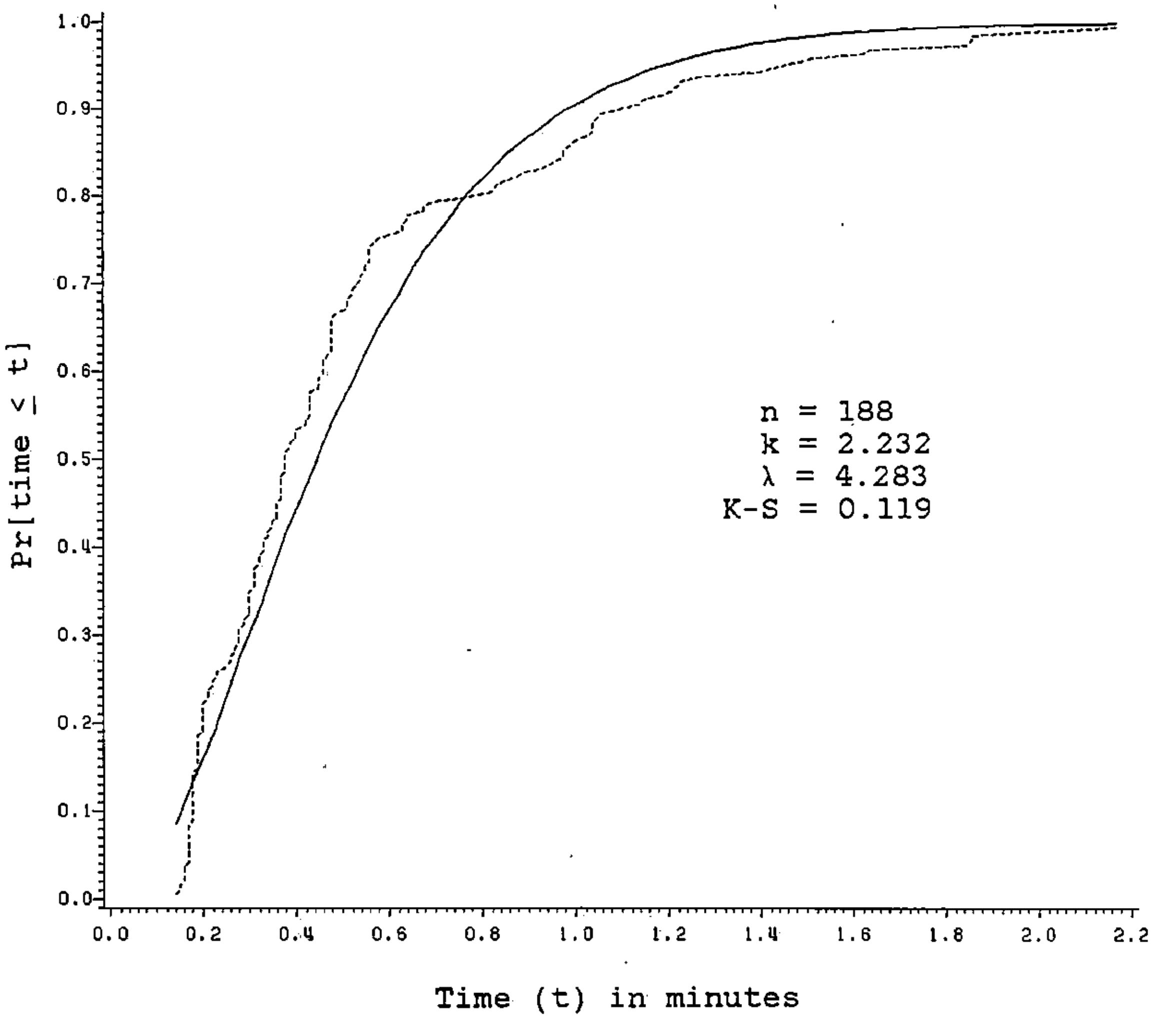

Figure 6. Two parameter gamma distribution (solid line) versus empirical distribution (dashed line) of travel-in times for grapple skidding.

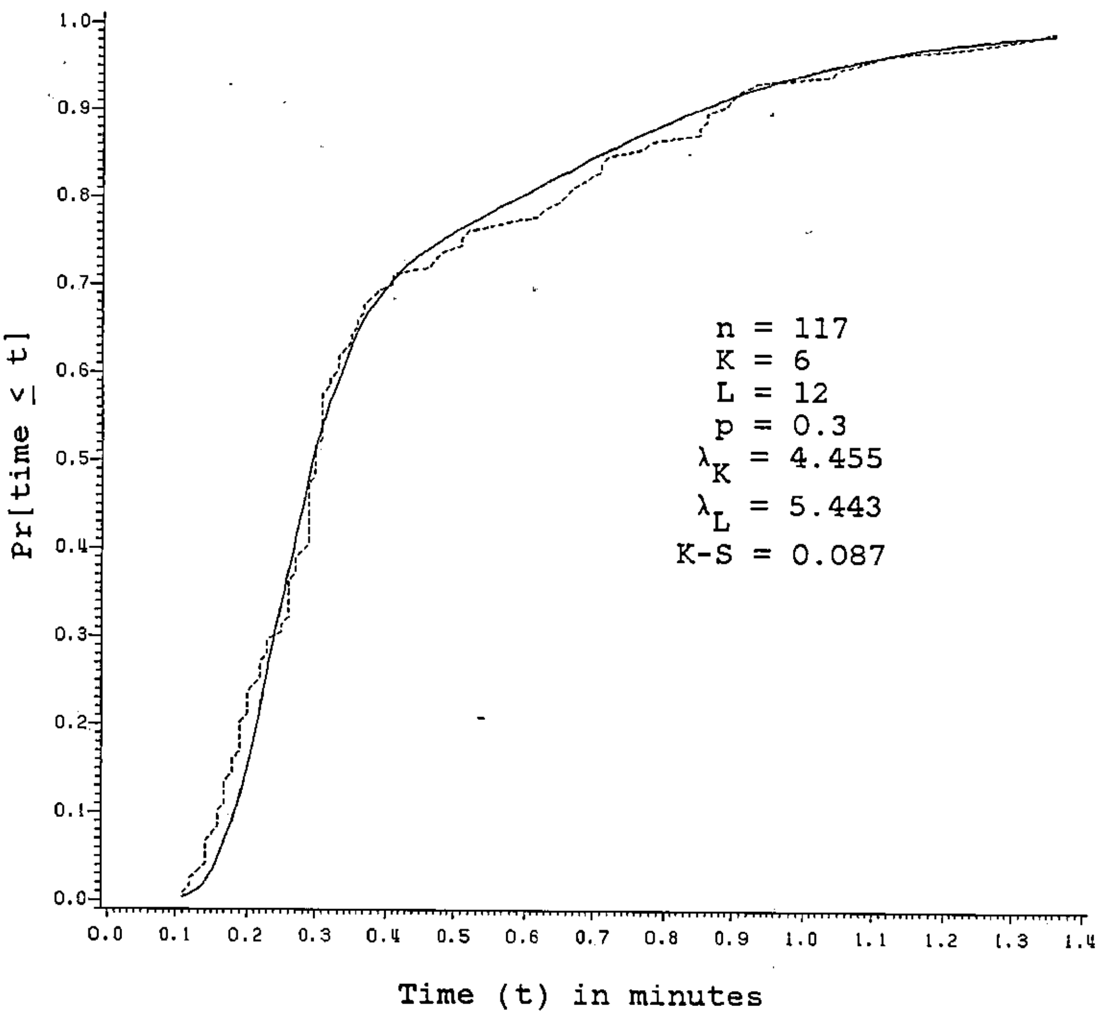

Figure 7. Mixed Erlang distribution (solid line) versus empirical distribution (dashed line) of travel-out times for grapple skidding.

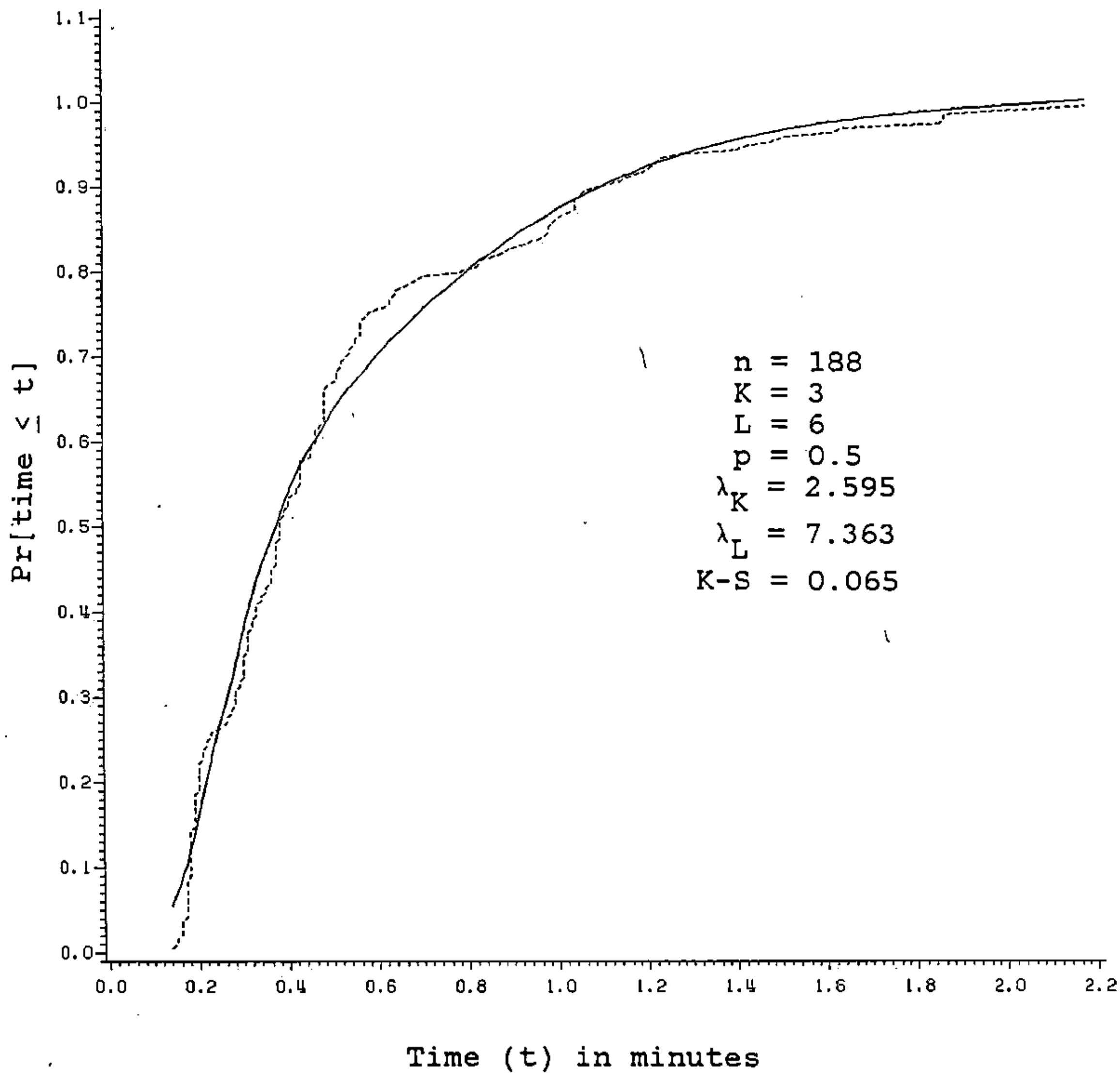

Figure 8. Mixed Erlang distribution (solid line) versus empirical distribution (dashed line) of travel-in times for grapple skidding.

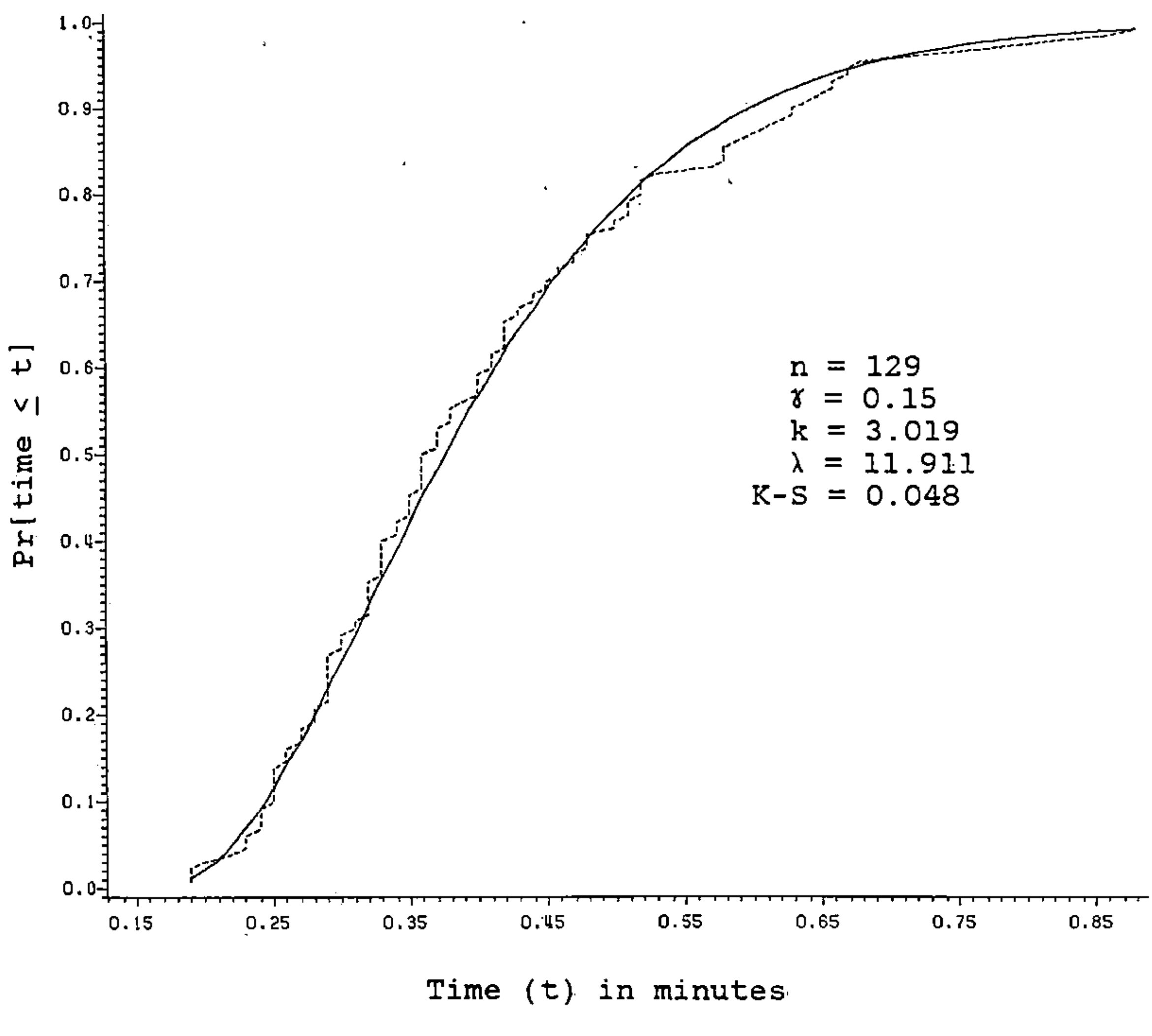

Figure 9. Three parameter gamma distribuiton (solid line) versus empirical distribution (dashed line) of maneuver/loading times for grapple skidding.

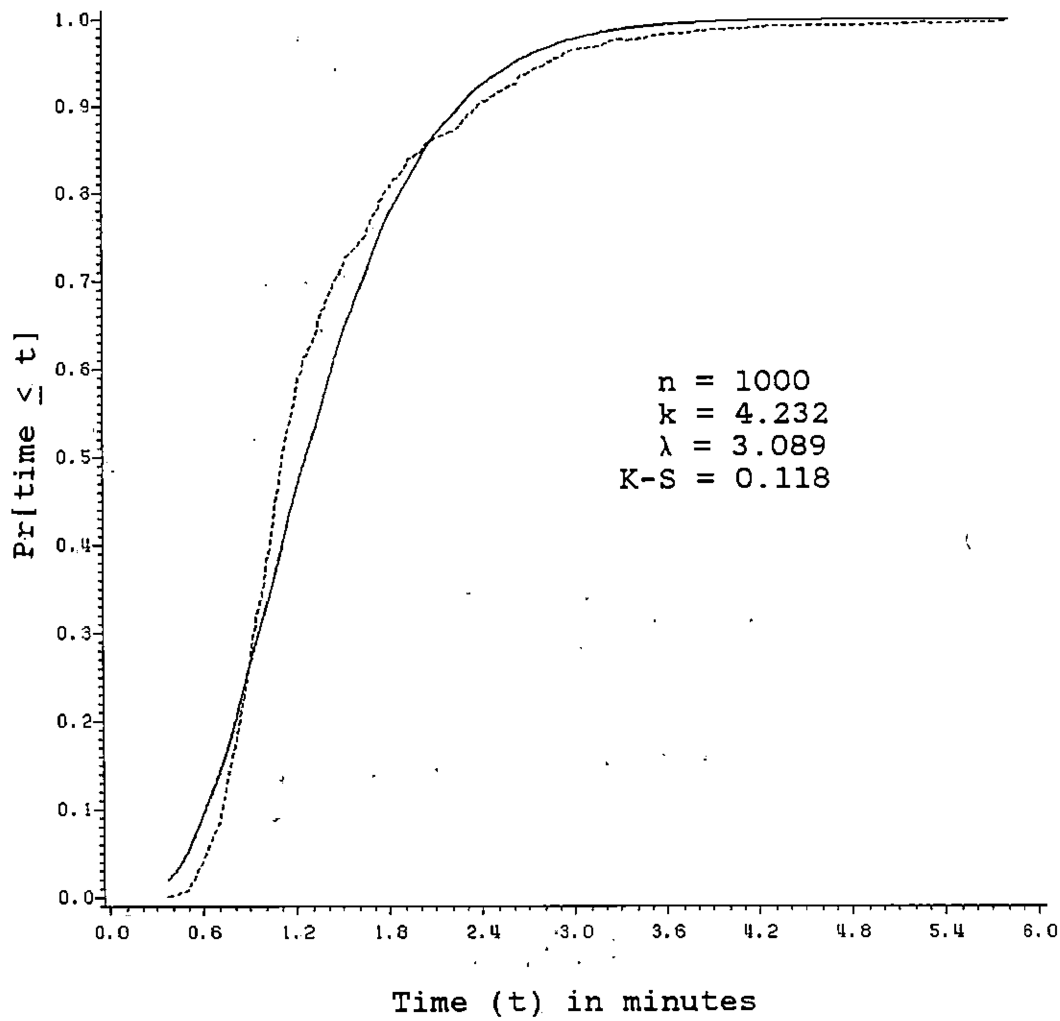

Figure 10. Two-parameter gamma distribution (solid line) versus estimated cycle time distribution (dashed line) for grapple skidding, assuming perfectly correlated travel times.

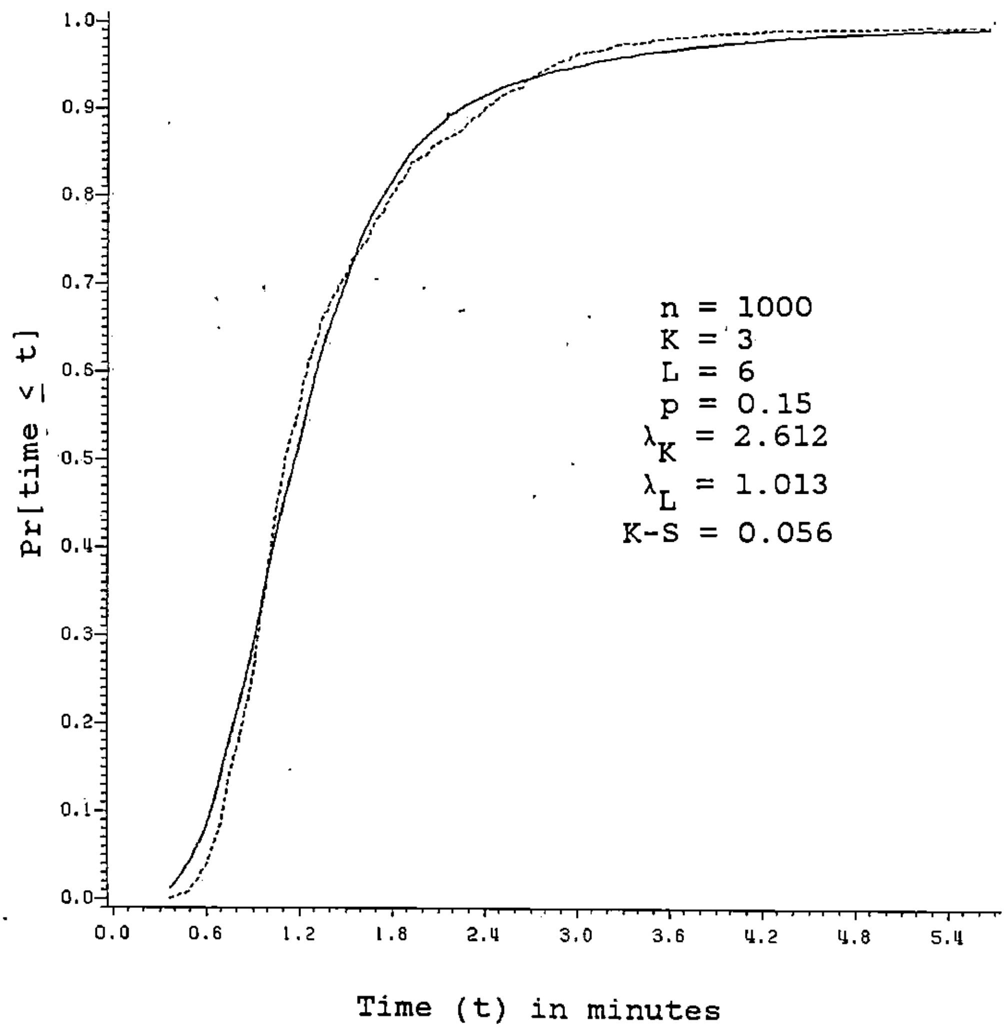

Figure 11. Mixed Erlang distribution (solid line) versus estimated cycle time distribution (dashed line) for grapple skidding, assuming perfectly correlated travel times.

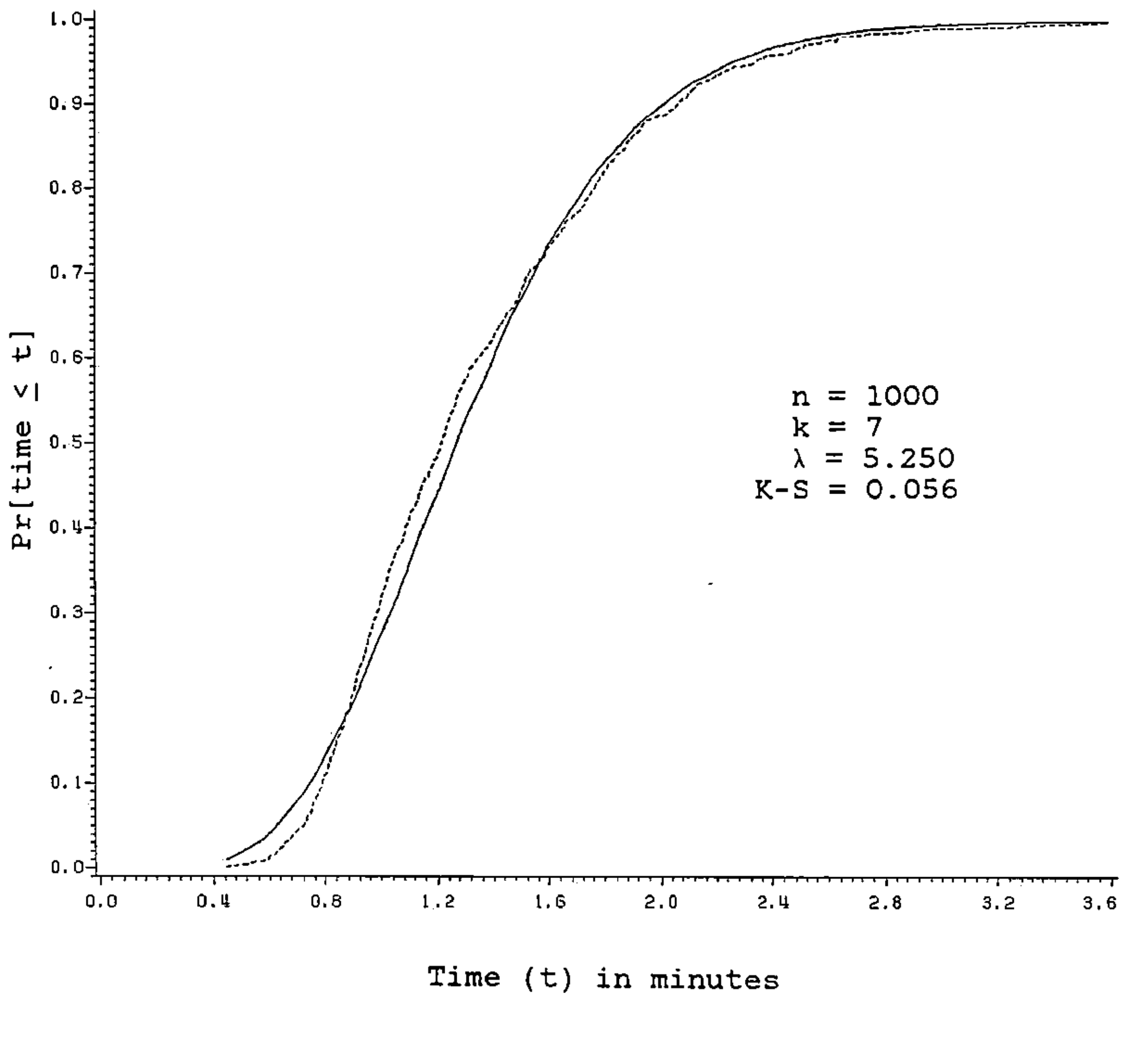

Figure 12. Erlang distribution (solid line) versus estimated cycle time distribution (dashed line) for grapple skidding, assuming independent travel times.

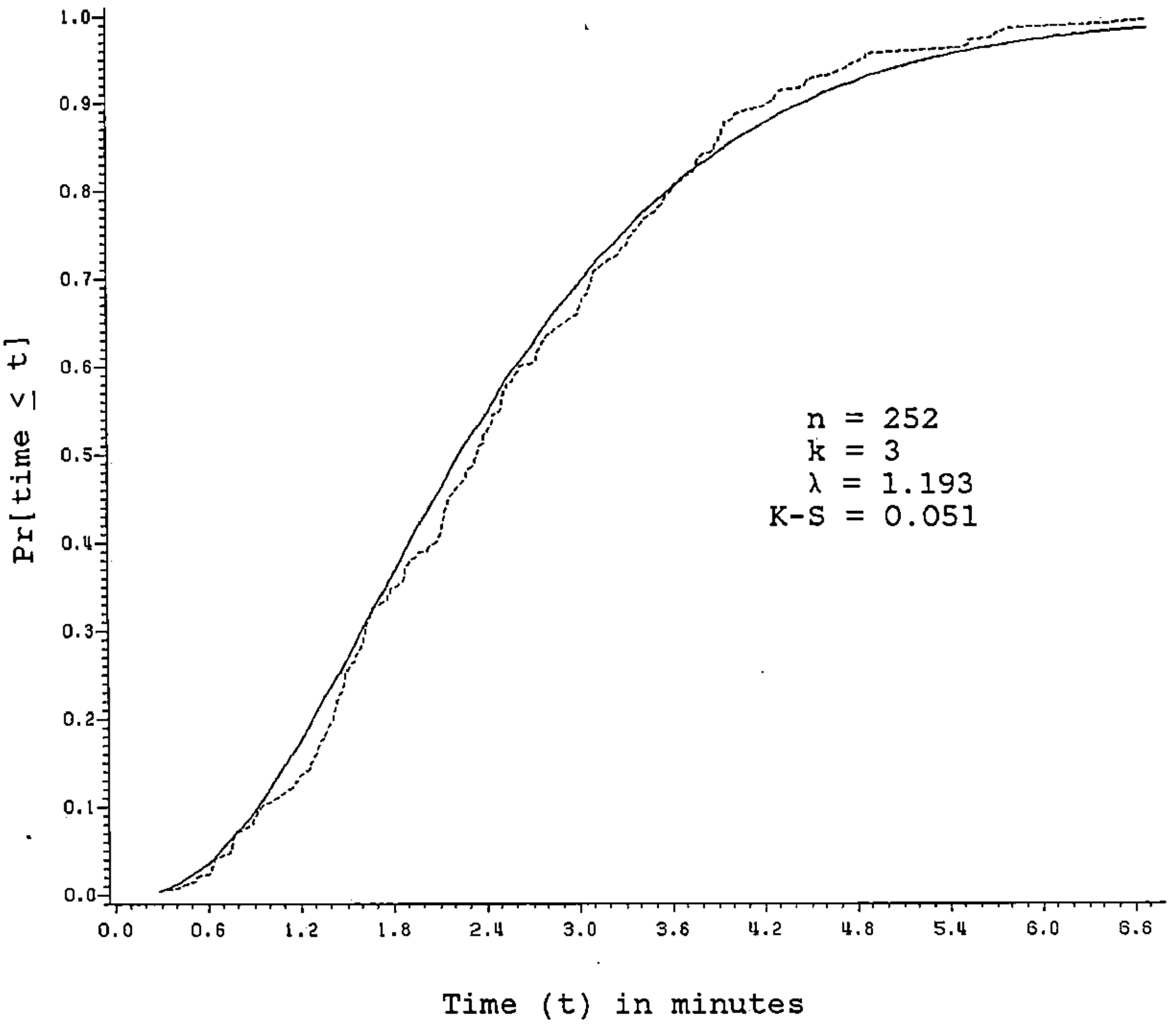

Figure 13. Erlang distribution (solid line) versus empirical distribution (dashed line) for the time it takes the slasher to process one of its own grapple loads.

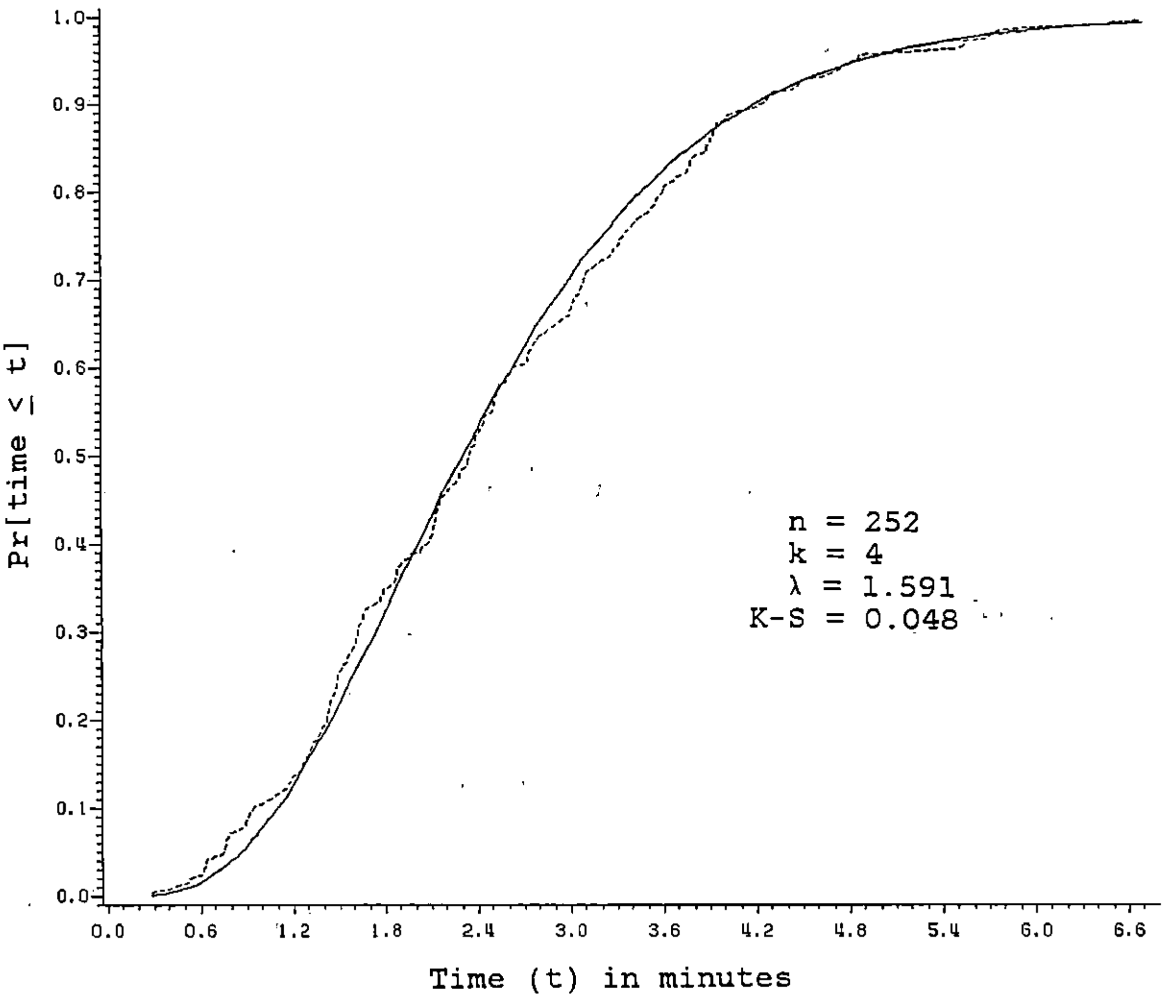

Figure 14. Erlang distribution (solid line) versus empirical distribution (dashed line) for the time it takes the slasher to process one of its own grapple loads.

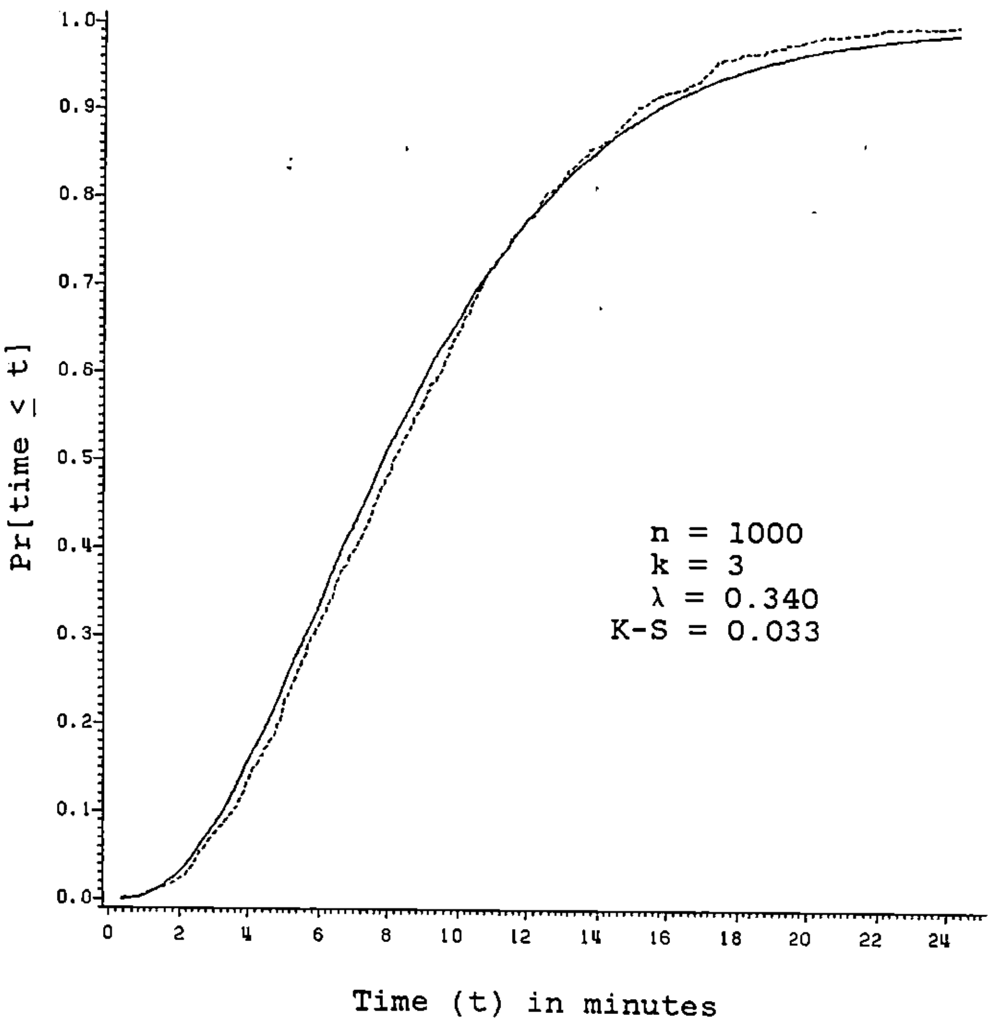

Figure 15. Erlang distribution (solid line) versus estimated slasher cycle time distribution (dashed line) assuming a shape parameter of 3.

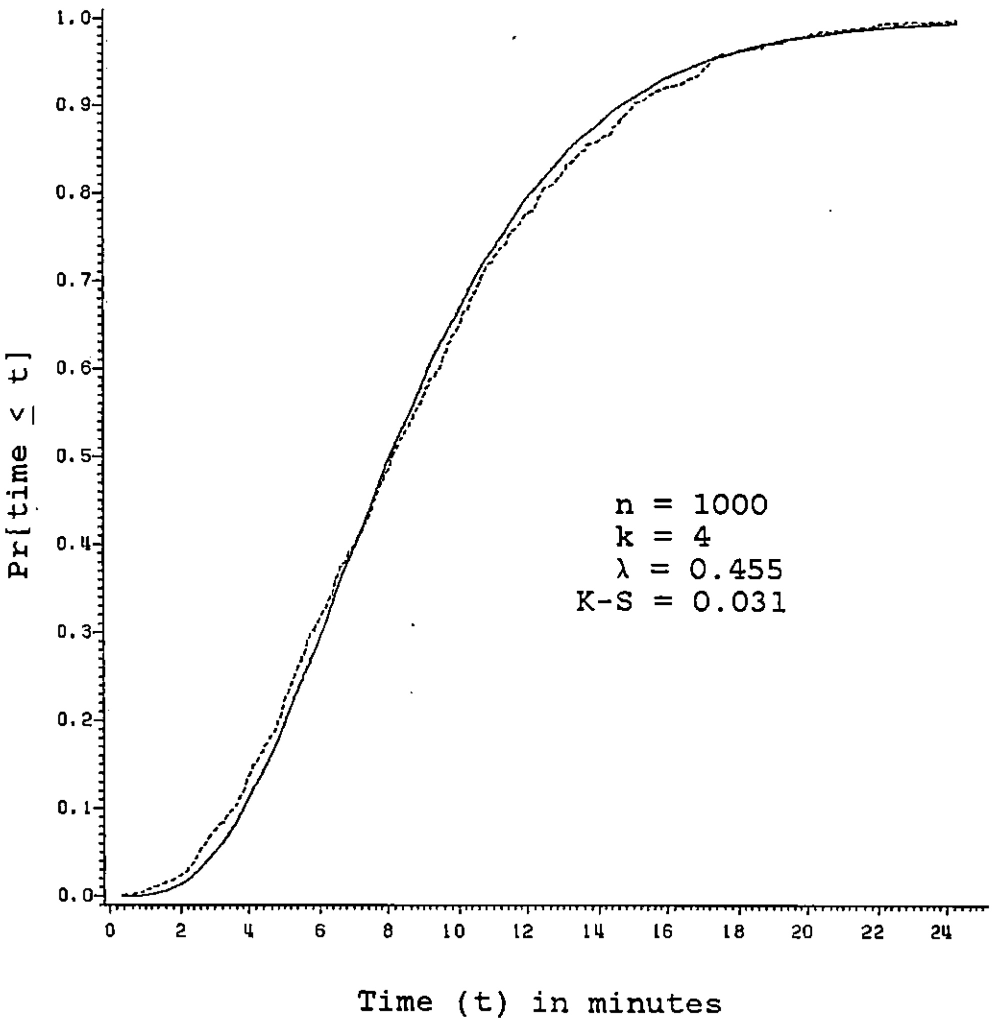

Figure 16. Erlang distribution (solid line) versus estimated slasher cycle time distribution (dashed line) assuming a shape parameter of 4.

5. DISCUSSION

5.1 A MARKOV MODEL OF THE SLASHER/SKIDDER INTERACTION

The theory and statistical components necessary to model the slasher/skidder interaction as a Markov process have been developed in previous sections. It remains to combine the components into a formal model in order to determine a steady-state solution to the problem. This section will focus on:

1. Formalizing and solving the Markov process model for the steady state probabilities of delay and idle time for the slasher and skidder. This will be performed for the cases where skidder travel times are either perfectly correlated or they are independent, in order to assess the importance of the correlation.

2. Determining the probability distribution of slasher and/or skidder waiting times, under the condition that the slasher and/or skidder must wait. This will be illustrated for both perfectly correlated and independent skidder travel times.

A Markov Process Model for Perfectly Correlated Skidder Travel Times

The three components of this process are the skidder cycle times, the slasher cycle times and the level of the productive buffer. Because the states of the skidder require a 2-part specification (i.e., which holding device and which stage within a holding device), the productive states of the process can be described using a 4-dimensional specification as follows:

State descriptor = (i,j,k,m)

where,

i = 1 or 2 and indicates the mixed Erlang cycle time or holding device in which the skidder is traveling,

j = 1,2,3 or 1,2,3,4,5,6 and indicates which stage of the mixed Erlang holding device (i.e., holding device 1 or 2, respectively) the skidder is traversing,

k = 1,2,3 and indicates which stage of its Erlang cycle time the slasher is in,

m = 0,1,2,3 and indicates the number of skidder loads left to be processed in the productive buffer and the capacity of the production buffer is 3 skidder loads.

With the 4-dimensional specification it is now possible to describe the productive states of the process. For example, (1,2,2,1) represents the productive state where the skidder is currently in holding device one, stage 2 of its mixed Erlang cycle time, the slasher is in the second stage of its Erlang cycle time, and there is one skidder load left in the productive buffer. In all, there are 108 productive states (i.e., for i=1; j=3 times k=3 times m=4 equals 36 states plus, for i=2; j=6 times k=3 times m=4 equals 72 states or 108 total states).

Describing Possible State Changes

It is important to understand how the process moves from one state to another. In moving from a current state to possible future states, recall that Markov process theory allows only one transition between states, since the transition rate for any other move is zero. That is, if the slasher is in its first stage, the next transition for the slasher can only be to stage 2 and not stage 3.

To this point no mention has been made as to when the skidder may unload into the productive buffer or when the slasher depletes the buffer by one skidder load. For convenience, it is assumed that as the slasher exits the third stage of its holding device (i.e., completes a cycle) it instantaneously depletes the productive buffer by one skidder load, assuming it found at least one skidder load in the buffer. Similarly, as the skidder exits the third stage of its first holding device or the sixth stage of its second holding device, it instantaneously increases the productive buffer by one load, assuming it finds the buffer at less than full capacity.

Some examples of possible state changes are:

1. Assume the system is in (1,1,1,2). The skidder can move into the next stage of holding device 1, giving the transition to (1,2,1,2); the slasher can move into the second stage of its holding device which is state (1,1,2,2); or the process can stay in its current state (1,1,1,2).

2. The system is in (1,2,3,3). The possible transitions are (1,3,3,3), (1,2,1,2) or stay in the current state (1,2,3,3). Note, that the slasher, in completing a cycle, depleted the buffer by one skidder load (i.e, (1,2,3,3) to (1,2,1,2)).

3. The system is in (1,3,3,1). Possible transitions include (1,1,3,2), (2,1,3,2), since the skidder can choose to enter holding device 1 with probability p and to enter holding device 2 with probability q, upon completion of the current cycle; (1,3,1,0) where the slasher completes a cycle and begins processing the next load, thereby depleting the buffer by one skidder load; and to stay in the current state (1,3,3,1). Note, that the skidder, in completing a cycle through the first holding device, increased the buffer by one load.

Delay and Idle States

In order to completely specify the states of the system, a number of additional states are needed. These states will indicate when the slasher is idle or the skidder is delayed. One additional state specification will be required for the slasher (in addition to its 3 Erlang stages) and will come into play when the slasher finishes processing the current skidder load and returns to the productive buffer and finds it empty. At that point the slasher would enter the idle state. This can occur for any of the possible productive states the skidder can occupy in its mixed Erlang model, while the productive buffer is empty. The possible number of idle states for the slasher is 9 (i.e., (1,1,4,0), (1,2,4,0), (1,3,4,0), (2,1,4,0), (2,2,4,0), (2,3,4,0), (2,4,4,0), (2,5,4,0) and (2,6,4,0), where the idle state is denoted as k=4).

For example, the slasher can find the productive buffer empty when the skidder is in stage 2 of holding device 1 (i.e. (1,2,3,0)), as well as, stage 4 of holding device 2 (i.e., (2,4,3,0)). In these example cases, the slasher would enter the idle states (1,2,4,0) and (2,4,4,0), respectively.

For the skidder two additional state specifications must be included. These states will be entered from the respective mixed Erlang holding devices when the skidder arrives at the slasher, thereby completing a cycle of productive work, to find the productive buffer full. This can occur for any of 6 possible slasher states (i.e., (1,4,1,3), (1,4,2,3), (1,4,3,3), (2,7,1,3), (2,7,2,3), and (2,7,3,3)) when the buffer is full, which gives a total of 6 possible delay states for the skidder. The delay states are denoted as j=4 and j=7 for holding device 1 and holding device 2, respectively. For example, the skidder would enter delay states (1,4,1,3) and (2,7,2,3), from productive states (1,3,1,3) and (2,6,2,3), respectively.

The transitions to and from delay and idle states are illustrated below:

1. Suppose the system was at (2,6,2,3). The possible transitions are (2,7,2,3) since the buffer is full when the skidder finishes the current cycle and must wait; (2,6,3,2) since the slasher can still continue on its cycle or (2,6,2,3) where the process stays in the current state.

2. If the system was in skidder delay state (2,7,3,3), the possible transitions are (1,3,1,3) with probability p since the skidder simultaneously unloaded as the slasher depleted the buffer by one skidder load, (2,1,1,3) with probability q for the same reason, or the system stayed at its current state (2,7,3,3).

3. If the buffer is empty and the system is in (1,2,3,0), the possible transitions are to slasher idle state (1,2,4,0) since the slasher has no material to remove from the buffer, (1,3,3,0) or stay in (1,2,3,0).

4. If the slasher is idle and the process is in state (1,3,4,0), the skidder could complete a cycle, at which time it unloads and the slasher instantaneously depletes a load from the buffer. Since the skidder can enter holding device 1 or 2 at this point the possible transitions are (1,1,1,0), (2,1,1,0) or stay in (1,3,4,0).

All possible states of the system, productive and idle states, can now be described using a 4-dimensional specification. The total number of states is 123.

The Rate Matrix

It was shown earlier how the Markov model could be solved to obtain the steady-state probabilistic behavior of the process. The rate matrix was a key component in deriving a solution. One can visualize the rate matrix by considering the 123 possible states of the slasher/skidder interaction process. Let the rate matrix have 123 rows, where each row is designated as a current state of the process. Now, let the rate matrix also have 123 columns, which are designated as future states of the process, yielding a 123 X 123 element matrix. The values that are entered in the cells of this matrix are the rates of change of probability for moving from a current state (rows) to possible future states of the process (columns). Since the possible transitions from a current state to a future state are limited, as illustrated with the transition examples, the number of nonzero elements in the rate matrix is relatively small in comparison to the overall size of the matrix.

The rates are taken from the parameter of the exponential distribution for holding times in the stages of each holding device and are:

1. For state changes involving the slasher, the rate is the parameter λ from the exponential holding time in a stage of the simple Erlang holding device and is 0.34.

2. For state changes involving the first holding device of the skidder, the rate is the parameter $pK\lambda_K$ from the exponential holding time in a stage and is 1.18..

3. For state changes involving the second holding device of the skidder, the rate is the parameter $qL\lambda_L$ from the exponenetial holding time in a stage and is 5.17.

4. In changing skidder holding devices, p = 0.15 for entering holding device 1 and q = 0.85 for entering holding device 2 for the purpose of determining the rate. For example, if the skidder exits the third stage of the first holding device and immediately begins another productive cycle, the transition rate for reentering the first holding device is 0.15 times 1.18 or 0.177 and for entering the second holding device is 0.85 times 1.18 or 1.003. Similarly, when exiting its second holding device the skidder can begin another cycle immediately by entering holding device one with rate 0.15 times 5.17 or 0.776 or reenter holding device two with rate 0.85 times 5.17 or 4.394.

The structure of the rate matrix can most easily be illustrated by presenting a series of submatrices, of the rate matrix, then showing how these submatrices can be combined to give the final structure of the rate matrix. Figures 1-6 show the submatrix components of the rate matrix, indexed by current and future states, and Figure 7 shows how these submatrices are ordered in the overall rate matrix.

The rate matrix is then used in the matrix equation (2):

$$\Pi\Lambda = 0 \qquad (1)$$

where, Π is the vector of steady-state probabilities ($\Pi = \pi_1, \pi_2, \ldots, \pi_n$), Λ is the rate matrix and the 0 on the right hand side of the equation is a vector containing all zeroes. The solution of this system of equations was obtained by using the PROC MATRIX procedure in SAS (1). Although the rate matrix is theoretically singular (which was verified using SAS) it is possible to solve this system of equations by recalling that the steady-state probabilities must sum to one. This requires setting one row (or column) of the rate matrix to ones and setting the appropriate right hand side element of the zero vector to a one.

Upon solution of the system of equations, it was found that the skidder was delayed 84.5% of the time, the slasher was never idle, and the slasher and skidder were simultaneously performing useful work 15.5% of the time. These probabilities were determined by summing over the individual steady-state probabilities. That is, the 15.5% of the time when both were performing useful work is the sum of the steady-state probabilities for the 108 productive states. Similarly, the 84.5% skidder delay time is the sum of the steady-state probabilities for the 6 skidder delay states. The sum of steady-state probabilities for the 9 slasher idle states was zero to 5 decimal places.

A Markov Process Model for Independent Skidder Travel Times

While the three components, skidder, slasher and productive buffer, do not change for this case, the dimensionality of the problem does. Since skidder cycle times were modeled as a simple Erlang with 7 stages, the process can be described using a 3-dimensional specification as follows:

$$\text{State descriptor} = (i,j,k)$$

where,

$i = 1,2,3,4,5,6,7$ and indicates the stage of the simple Erlang cycle time that the skidder is traversing,

$j = 1,2,3$ and indicates which stage of its Erlang cycle time the slasher is in;

$k = 0,1,2,3$ and indicates the number of skidder loads left to be processed in the productive buffer.

In this case, there are 63 productive states of the process (i.e., i=7 times j=3 times k=3 equals 63). Once again, to fully describe the states of the system idle and delay states for the slasher and skidder, respectively, must be included. As before, the slasher will enter the idle state when it finishes processing the current skidder load and returns to the buffer to find it empty. Since this can occur for any of the possible skidder states (i.e., 1 thru 7) when the buffer is empty, there are a total of 7 slasher idle states: (1,4,0), (2,4,0), (3,4,0), (4,4,0), (5,4,0), (6,4,0) and (7,4,0). Similarly, the skidder will be delayed when it ends its Erlang cycle time to find the buffer full. This can occur for any of the 3 possible slasher states when the buffer is full, yielding 3 skidder delay states: (8,1,0), (8,2,0) and (8,3,0). A total of 73 states, then, are required to model the independent skidder travel time case.

The assumptions concerning when the slasher and skidder can deplete or increase the buffer are the same as before. The only difference being that the skidder has only one possible cycle time and can increase the buffer only when exiting the seventh stage of the Erlang holding device. State transitions are the same as before, except the skidder now does not have a choice of holding devices to enter. Upon completing a cycle the skidder merely reenters the Erlang holding device. Thus, the possible transitions and determination of rates is conceptually easier for this model.

The Rate Matrix (Independent Travel Times)

The rate matrix here is 73 X 73 with rates:

1. For states changes involving the slasher, the rate is the parameter λ from the exponential holding time in a stage of the slasher's Erlang cycle time and is once again 0.34.

2. For state changes involving the skidder, the rate is the parameter λ from the exponential holding time in a stage of the skidder's Erlang cycle time and is 5.25.

Figures 8-12 provide the submatrix components of the rate matrix, while Figure 13 illustrates the aggregation of these submatrices into the full rate matrix. Again using the PROC MATRIX procedure in SAS (1) to solve this system of equations, the results show that the skidder was delayed 85.3% of the time, the slasher was never idle and both slasher and skidder were productive 14.7% of the time.

Effect of Correlated Versus Independent Skidder Travel Times

Comparing the results for perfectly correlated and independent skidder travel times indicates a negligible effect on the steady-state probabilities. Thus, the analyst may choose either model for further study of the slasher/skidder interaction, where proportions of delay, idle and productive time are the factors of interest.

Distribution of Delay Times for the Skidder (Perfectly Correlated Travel Times)

Since the slasher is never idle in this model, the skidder delay time will be used to illustrate the determination of the probability distribution of skidder delay times. The question of interest is: Given that the skidder arrives at the productive buffer and finds it full, how long does the skidder have to wait before unloading?

The components necessary to derive this distribution are as follows:

1. The steady-state probability that the skidder is delayed in one of its delay states. For the perfectly correlated travel time case, the total number of delay states for the skidder is 6: (1,4,1,3), (1,4,2,3), (1,4,3,3), (2,7,1,3), (2,7,2,3), and (2,7,3,3). In solving the Markov model, the steady-state probability of being in any one of these states was directly retrievable and were 0.023, 0.039, 0.046, 0.193, 0.264, and 0.280, respectively.

2. The length of time a skidder must wait before the slasher depletes the buffer by one, thereby allowing the skidder to unload. For example, if the skidder is in delay state (1,4,1,3) or (2,7,1,3), how long will it remain there, or more specifically how long until the slasher completes its current cycle. Because of the way in which the Markov model was constructed, the remaining time in any stage is always exponential. Thus, if the slasher is in stage one of

its Erlang cycle time, the time until it exits the first stage is exponential, and the time until it completes the cycle is the sum of 3 independent and identical exponential random variables. This sum is, by definition, an Erlang probability distribution with shape parameter 3. The probability density is:

$$\begin{aligned} f(t) &= [\lambda(\lambda t)^{k-1} \exp(-\lambda t)]/(k-1)! \\ &= [0.34(0.34t)^{3-1} \exp(-0.34t)]/(3-1)! \\ &= [0.34(0.34t)^{2} \exp(-0.34t)]/2 \qquad (2) \end{aligned}$$

Similarly, if the skidder is in state (1,4,2,3) or (2,7,2,3) the length of time it must wait before unloading is distributed as an Erlang with shape parameter 2. The probability density is:

$$f(t) = [0.34(0.34t) \exp(-0.34t)] \qquad (3)$$

Finally, if the skidder is in state (1,4,3,3) or (2,7,2,3) the length of time it must wait before unloading is exponentially distributed (i.e., Erlang with shape parameter 1). The density is:

$$f(t) = 0.34 \exp(-0.34t) \qquad (4)$$

For convenience, denote equations 2, 3 and 4 as E_1, E_2 and E_3, respectively.

The probability statement of interest in solving the waiting time problem is as follows:

$$\Pr[\text{waiting time} = t, D_j \mid \text{skidder is idle}] \qquad (5)$$

That is, what is the probability that the delay or waiting time of the skidder is t minutes and given that the skidder is delayed, it is in one of the D_j delay states (where, D_j, and j=1,2,3,4,5,6 denotes any of the six delay states of the skidder)?

This joint probability can be decomposed, using basic probability theory into:

$$\begin{aligned}&\Pr[\text{waiting time} = t, D_j \mid \text{skidder is idle}] = \\ &\quad \Pr[\text{waiting time} = t \mid D_j \text{ and skidder is idle}] \times \\ &\quad \Pr[D_j \mid \text{skidder is idle}] \qquad (6)\end{aligned}$$

Decomposing the joint probability into two parts has effectively simplified the determination of the probability distribution of waiting time. Consider the first part of the decomposition. This is the conditional distribution of the time the skidder waits, given that it is idle and in state D_j. However, this is just an Erlang distribution with shape parameter 1, 2 or 3, depending on D_j (i.e., which stage the slasher was in when the skidder began its delay). The second conditional distribution of the decomposition is just the probability of being in state D_j given that the skidder is idle. The idle time of the skidder was determined earlier to be 84.5%. For example, the probability of being in (1,4,1,3) given that the skidder is idle is 0.023/0.845 = 0.027.

The probability distribution for any of the D_j can now be determined as a weighted Erlang distribution. In order to determine the total probability of delay times, it is necessary to sum over all D_j's. The probability density for the total delay time distribution is:

$$f(t) = (0.023/0.845)E_3 + (0.039/0.845)E_2 + (0.046/0.845)E_1 + (0.193/0.845)E_3 + (0.264/0.845)E_2 + (0.280/0.845)E_1$$

$$= 0.255E_3 + 0.358E_2 + 0.387E_1 \qquad (7)$$

Expanding and combining terms yields:

$$f(t) = \exp(-0.34t)[0.0050t^2 + 0.0414t + 0.1316] \qquad (8)$$

Figure 15 shows a plot of this density. The expected value and variance of delay time are respectively:

$$E(t) = \int_0^\infty \exp(-0.34t)[0.005t^3 + 0.0414t^2 + 0.1316t]dt$$

$$E(t) = 5.490 \text{ minutes}$$

$$E(t^2) = \int_0^\infty \exp(-0.34t)[0.005t^4 + 0.0414t^3 + 0.1316t^2]dt$$

$$E(t^2) = 50.696$$

$$Var(t) = E(t^2) - [E(t)]^2$$

$$= 21.556$$

Distribution of Delay Times for the Skidder (Independent Skidder Travel Times)

The determination of the delay time distribution for this model is somewhat easier than in the perfectly correlated case. There are only 3 delay states for the skidder: (1,8,1,3), (1,8,2,3) and (1,8,3,3) with steady-state probabilities 0.214, 0.306, and 0.329, respectively. Using the same idea as in the perfectly correlated travel time case, it can be shown that the distribution of delay times for the skidder is:

$$f(t) = (0.214/0.849)E_3 + (0.306/0.849)E_2 + (0.387/0.849)E_1$$

$$= \exp(-0.34t)\ [0.005t^2 + 0.0416t + 0.1319] \qquad (9)$$

The expected value and variance of delay time are respectively:

$$E(t) = \int_0^\infty \exp(-0.34t)[0.005t^3 + 0.0416t^2 + 0.1319t]dt$$

$$E(t) = 5.503 \text{ minutes}$$

$$E(t^2) = \int_0^\infty \exp(-0.34t)[0.005t^4 + 0.0416t^3 + 0.1319t^2]dt$$

$$E(t^2) = 51.801$$

$$Var(t) = E(t^2) - [E(t)]^2$$

$$= 21.518$$

Figure 16 shows a plot of the delay time distribution for independent skidder travel times.

Based on the above results and the plots in Figures 15 and 16, the delay time distributions are negligibly different. Therefore, for further analysis it will result in very little impact to the final solution by using either perfectly correlated or independent travel times.

Slasher Idle Time Distribution

The approach to developing a waiting or idle time distribution for the slasher, given that it must wait, is much the same as with the skidder. The necessary components are:

1. The steady-state probabilities for slasher delay states.

2. The length of time the slasher must wait before the skidder arrives with another load.

The difference now is that the times until the skidder arrives come from two different distributions, one with an Erlang-3 cycle time and one with an Erlang-6 cycle time. Thus, it is important to keep sight of which skidder holding device goes with each slasher delay state. With this data, it is simply a matter of developing the distributions of idle time for each possible slasher idle state, then summing over all slasher idle states in order to get the total distribution of idle time.

Future States

Current States

	(i,j,1,0)	(i,j,1,1)	(i,j,1,2)	(i,j,1,3)	(i,j,2,0)	(i,j,2,1)	(i,j,2,2)	(i,j,2,3)	(i,j,3,0)	(i,j,3,1)	(i,j,3,2)	(i,j,3,3)	(i,j+1,1,0)	(i,j+1,1,1)	(i,j+1,1,2)	(i,j+1,1,3)	(i,j+1,2,0)	(i,j+1,2,1)	(i,j+1,2,2)	(i,j+1,2,3)	(i,j+1,3,0)	(i,j+1,3,1)	(i,j+1,3,2)	(i,j+1,3,3)
(i,j,1,0)	A				B								C											
(i,j,1,1)		A				B								C										
(i,j,1,2)			A				B								C									
(i,j,1,3)				A				B								C								
(i,j,2,0)					A				B								C							
(i,j,2,1)						A				B								C						
(i,j,2,2)							A				B								C					
(i,j,2,3)								A				B								C				
(i,j,3,0)									A												C			
(i,j,3,1)	B									A												C		
(i,j,3,2)		B									A												C	
(i,j,3,3)			B									A												C

I (columns (i,j,1,0)–(i,j,3,3))   II (columns (i,j+1,1,0)–(i,j+1,3,3))

Figure 1. Submatrices of the rate matrix for the slasher/skidder interaction, where skidder travel times are perfectly correlated and; I is submatrix A1 when A=-1.52, B=0.34, and i=1; I is submatrix A2 when A=-5.51, B=0.34 and i=2; II is submatrix A3 when c=1.18 and i=1: and II is submatrix A4 when c=5.17 and i=2.

Future States

Current States	(1,1,1,0)	(1,1,1,1)	(1,1,1,2)	(1,1,1,3)	(1,1,2,0)	(1,1,2,1)	(1,1,2,2)	(1,1,2,3)	(1,1,3,0)	(1,1,3,1)	(1,1,3,2)	(1,1,3,3)	(2,1,1,0)	(2,1,1,1)	(2,1,1,2)	(2,1,1,3)	(2,1,2,0)	(2,1,2,1)	(2,1,2,2)	(2,1,2,3)	(2,1,3,0)	(2,1,3,1)	(2,1,3,2)	(2,1,3,3)
(i,3,1,0)		A												B										
(i,3,1,1)			A												B									
(i,3,1,2)				A												B								
(i,3,1,3)																								
(i,3,2,0)						A												B						
(i,3,2,1)							A												B					
(i,3,2,2)								A												B				
(i,3,2,3)																								
(i,3,3,0)										A												B		
(i,3,3,1)											A												B	
(i,3,3,2)												A												B
(i,3,3,3)																								
						I												II						

Figure 2. Submatrices of the rate matrix for the slasher/ skidder interaction, where skidder travel times are perfectly correlated and; I is submatrix A5 when A=0.177 and i=1; I is submatrix A6 when A=0.776 and i=2; II is submatrix A7 when B=1.003 and i=1; and II is submatrix A8 when B=4.394 and i=2.

Future States

Current States	(1,1,4,0)	(1,2,4,0)	(1,3,4,0)	(2,1,4,0)	(2,2,4,0)	(2,3,4,0)	(2,4,4,0)	(2,5,4,0)	(2,6,4,0)	(1,4,1,3)	(1,4,2,3)	(1,4,3,3)	(2,7,1,3)	(2,7,2,3)	(2,7,3,3)	
(1,1,1,0)																
(1,1,1,1)																
(1,1,1,2)																
(1,1,1,3)																
(1,1,2,0)																
(1,1,2,1)																A9
(1,1,2,2)																
(1,1,2,3)																
(1,1,3,0)	A															
(1,1,3,1)																
(1,1,3,2)																
(1,1,3,3)																
(1,2,1,0)																
(1,2,1,1)																
(1,2,1,2)																
(1,2,1,3)																
(1,2,2,0)																
(1,2,2,1)																A10
(1,2,2,2)																
(1,2,2,3)																
(1,2,3,0)		A														
(1,2,3,1)																
(1,2,3,2)																
(1,2,3,3)																
(1,3,1,0)																
(1,3,1,1)																
(1,3,1,2)																
(1,3,1,3)										B						
(1,3,2,0)																
(1,3,2,1)																A11
(1,3,2,2)																
(1,3,2,3)											B					
(1,3,3,0)			A													
(1,3,3,1)																
(1,3,3,2)																
(1,3,3,3)												B				

Figure 3. Submatrices A9, A10 and A11 of the rate matrix for the slasher/skidder interaction, where skidder travel times are perfectly correlated and A=0.34 and B=1.18.

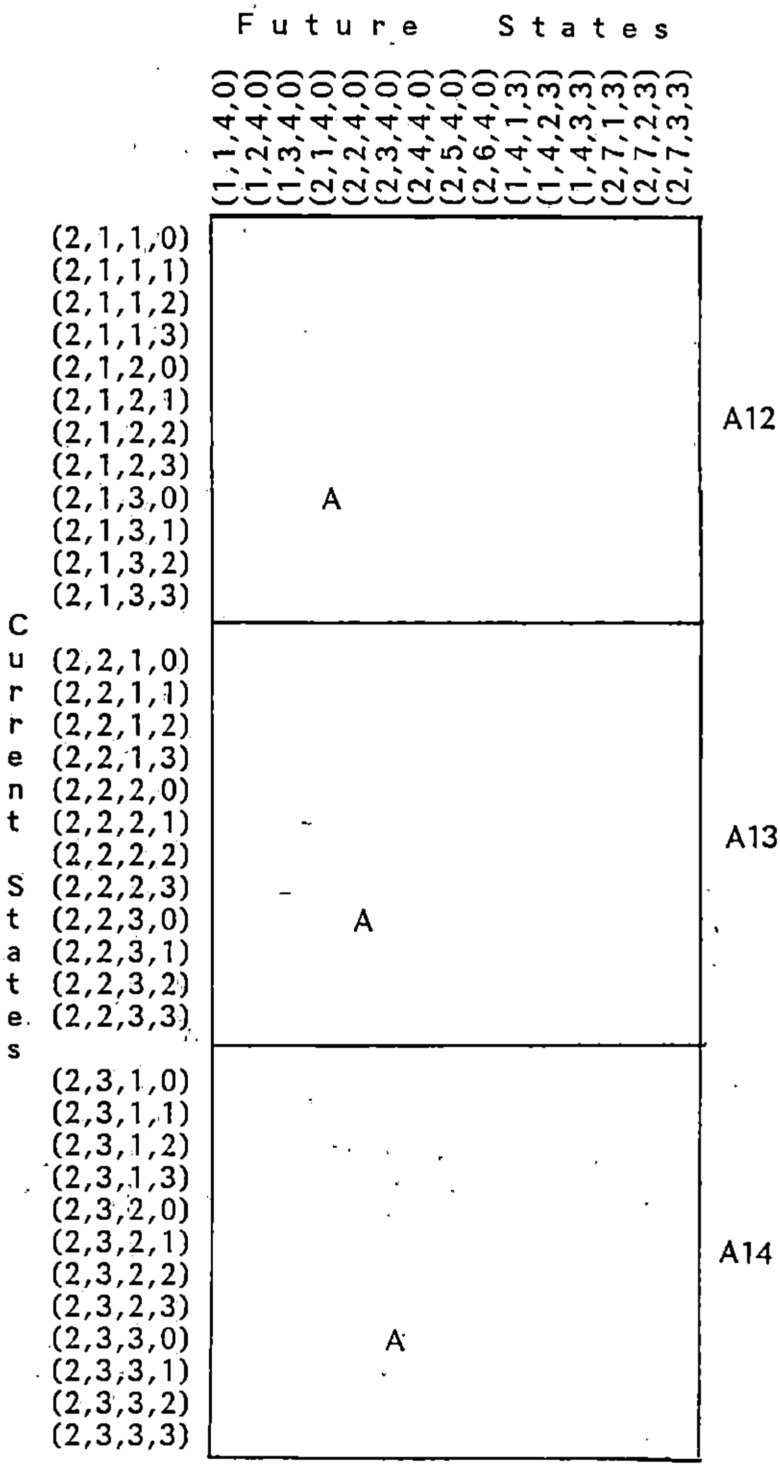

Figure 4. Submatrices A12, A13 and A14 of the rate matrix for the slasher/skidder interaction, where skidder travel times are perfectly correlated and A=0.34.

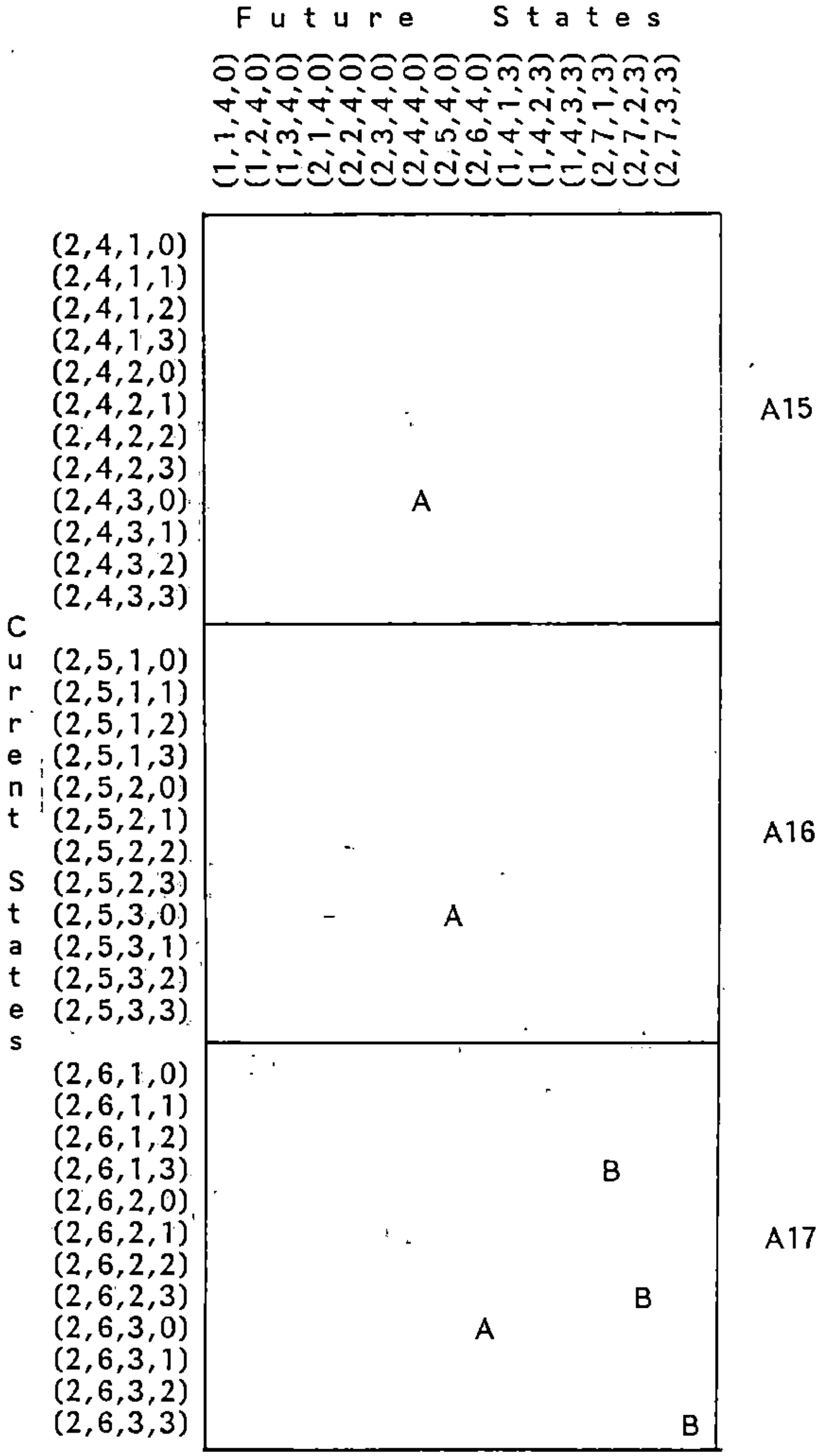

Figure 5. Submatrices A15, A16 and A17 of the rate matrix for the slasher/skidder interaction, where skidder travel times are perfectly correlated and A=0.34 and B=5.17.

Future States

A18

Current States	(1,1,1,0)	(1,1,1,1)	(1,1,1,2)	(1,1,1,3)	(1,1,2,0)	(1,1,2,1)	(1,1,2,2)	(1,1,2,3)	(1,1,3,0)	(1,1,3,1)	(1,1,3,2)	(1,1,3,3)
(1,1,4,0)												
(1,2,4,0)												
(1,3,4,0)	A											
(2,1,4,0)												
(2,2,4,0)												
(2,3,4,0)												
(2,4,4,0)												
(2,5,4,0)												
(2,6,4,0)	B											
(1,4,1,3)												
(1,4,2,3)												
(1,4,3,3)				E								
(2,7,1,3)												
(2,7,2,3)												
(2,7,3,3)				E								

A19

Current States	(2,1,1,0)	(2,1,1,1)	(2,1,1,2)	(2,1,1,3)	(2,1,2,0)	(2,1,2,1)	(2,1,2,2)	(2,1,2,3)	(2,1,3,0)	(2,1,3,1)	(2,1,3,2)	(2,1,3,3)
(1,1,4,0)												
(1,2,4,0)												
(1,3,4,0)	C											
(2,1,4,0)												
(2,2,4,0)												
(2,3,4,0)												
(2,4,4,0)												
(2,5,4,0)												
(2,6,4,0)	D											
(1,4,1,3)												
(1,4,2,3)												
(1,4,3,3)				F								
(2,7,1,3)												
(2,7,2,3)												
(2,7,3,3)				F								

A20

Current States	(1,1,4,0)	(1,2,4,0)	(1,3,4,0)	(2,1,4,0)	(2,2,4,0)	(2,3,4,0)	(2,4,4,0)	(2,5,4,0)	(2,6,4,0)	(1,4,1,3)	(1,4,2,3)	(1,4,3,3)	(2,7,1,3)	(2,7,2,3)	(2,7,3,3)
(1,1,4,0)	G	H													
(1,2,4,0)		G	H												
(1,3,4,0)			G												
(2,1,4,0)				I	J										
(2,2,4,0)					I	J									
(2,3,4,0)						I	J								
(2,4,4,0)							I	J							
(2,5,4,0)								I	J						
(2,6,4,0)									I						
(1,4,1,3)										K	L				
(1,4,2,3)											K	L			
(1,4,3,3)												K			
(2,7,1,3)													K	L	
(2,7,2,3)														K	L
(2,7,3,3)															K

Figure 6. Submatrices A19, A20 and A21 of the rate matrix for the slasher/skidder interaction, where skidder travel times are perfectly correlated and A=0.177, B=0.776, C=1.003, D=4.394, E=0.051, F=0.289, G=-1.18, H=1.18, I=-5.17, J=5.17, K=-0.34 and L=0.34.

Future States

Current States

A1	A3								A9
	A1	A3							A10
A5		A1	A7						A11
			A2	A4					A12
				A2	A4				A13
					A2	A4			A14
						A2	A4		A15
							A2	A4	A16
A6		A8						A2	A17
A18		A19							A20

Figure 7. Structure of the rate matrix for the slasher/ skidder interaction, where skidder travel times are perfectly correlated and the structure of submatrices A1-A20 is presented in Figures 1-6.

Future States

	(i,1,0)	(i,1,1)	(i,1,2)	(i,1,3)	(i,2,0)	(i,2,1)	(i,2,2)	(i,2,3)	(i,3,0)	(i,3,1)	(i,3,2)	(i,3,3)	(i+1,1,0)	(i+1,1,1)	(i+1,1,2)	(i+1,1,3)	(i+1,2,0)	(i+1,2,1)	(i+1,2,2)	(i+1,2,3)	(i+1,3,0)	(i+1,3,1)	(i+1,3,2)	(i+1,3,3)
(i,1,0)	A				B								C											
(i,1,1)		A				B								C										
(i,1,2)			A				B								C									
(i,1,3)				A				B								C								
(i,2,0)					A				B								C							
(i,2,1)						A				B								C						
(i,2,2)							A				B								C					
(i,2,3)								A				B								C				
(i,3,0)									A												C			
(i,3,1)	B									A												C		
(i,3,2)		B									A												C	
(i,3,3)			B									A												C

B1 B2

Figure 8. Submatrices B1 and B2 of the rate matrix for the slasher/skidder interaction, where skidder travel times are independent and A=-5.59, B=0.34, C=5.25, and i=1,2,3,4,5,6,7.

Future States

Current States	(1,1,0)	(1,1,1)	(1,1,2)	(1,1,3)	(1,2,0)	(1,2,1)	(1,2,2)	(1,2,3)	(1,3,0)	(1,3,1)	(1,3,2)	(1,3,3)	(1,4,0)	(2,4,0)	(3,4,0)	(4,4,0)	(5,4,0)	(6,4,0)	(7,4,0)	(8,1,3)	(8,2,3)	(8,3,3)
(7,1,0)		A																				
(7,1,1)			A																			
(7,1,2)				A																		
(7,1,3)																				A		
(7,2,0)						A																
(7,2,1)							A															
(7,2,2)								A														
(7,2,3)																					A	
(7,3,0)										A									B			
(7,3,1)											A											
(7,3,2)												A										
(7,3,3)																						A

B3 (columns (1,1,0)–(1,3,3)) B10 (columns (1,4,0)–(8,3,3))

Figure 9. Submatrices B3 and B10 of the rate matrix for the slasher/skidder interaction, where skidder travel times are independent and A=5.25 and B=0.34.

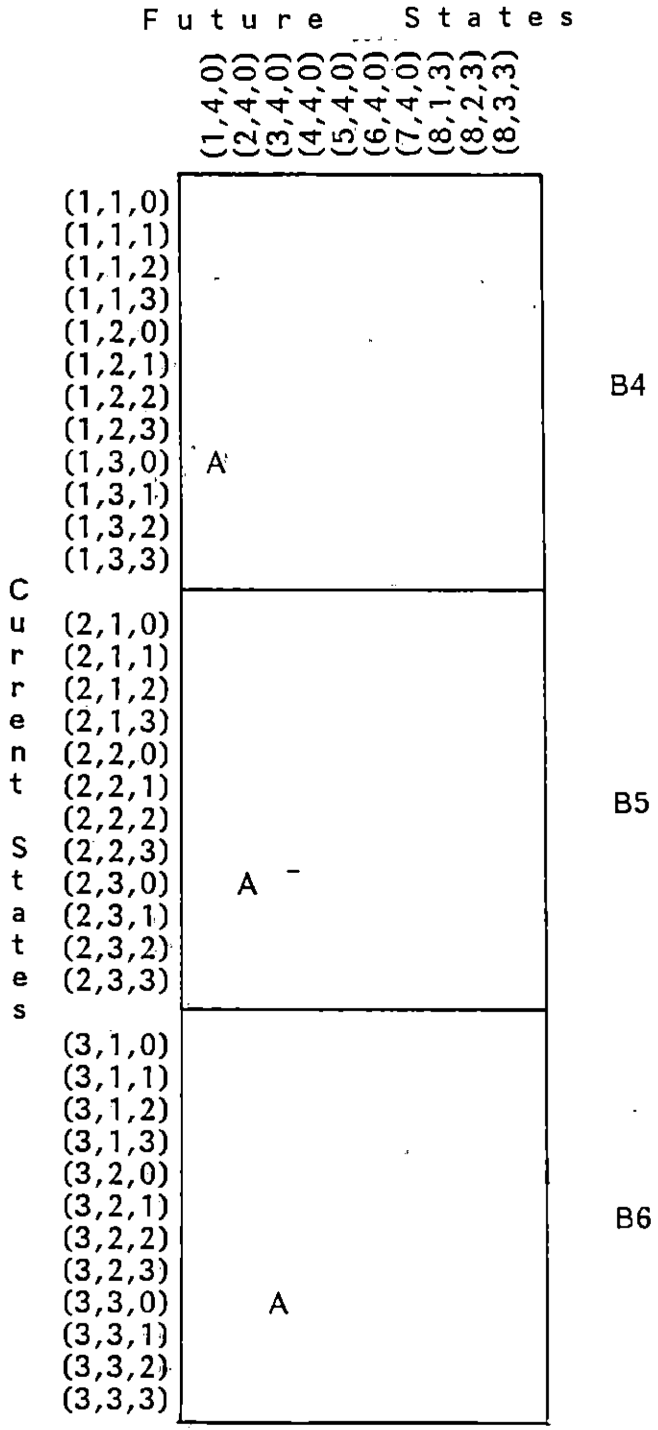

Figure 10. Submatrices B4, B5 and B6 of the rate matrix for the slasher/skidder interaction, where skidder travel times are independent and A=0.34.

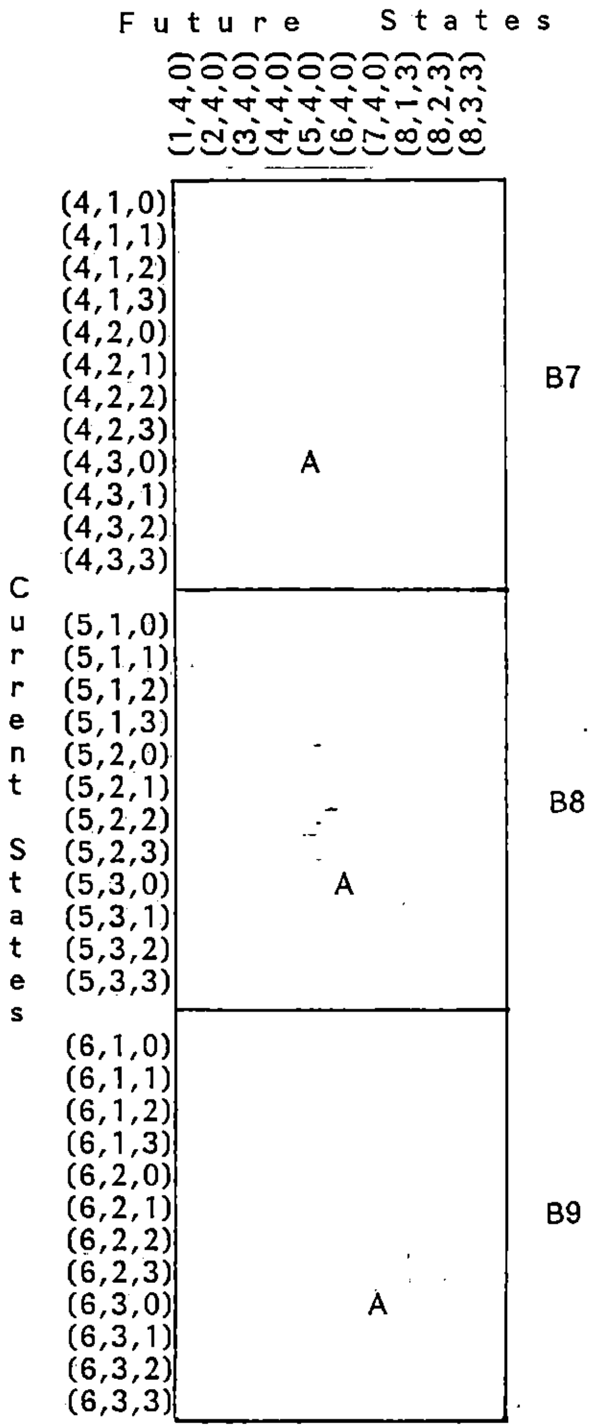

Figure 11. Submatrices B7, B8 and B9 of the rate matrix for the slasher/skidder interaction, where skidder travel times are independent and A=0.34.

Future States

Current States

B11

	(1,1,0)	(1,1,1)	(1,1,2)	(1,1,3)	(1,2,0)	(1,2,1)	(1,2,2)	(1,2,3)	(1,3,0)	(1,3,1)	(1,3,2)	(1,3,3)
(1,4,0)												
(2,4,0)												
(3,4,0)												
(4,4,0)												
(5,4,0)												
(6,4,0)												
(7,4,0)	A											
(8,1,3)												
(8,2,3)												
(8,3,3)				B								

B12

	(1,4,0)	(2,4,0)	(3,4,0)	(4,4,0)	(5,4,0)	(6,4,0)	(7,4,0)	(8,1,3)	(8,2,3)	(8,3,3)
(1,4,0)	C	A								
(2,4,0)		C	A							
(3,4,0)			C	A						
(4,4,0)				C	A					
(5,4,0)					C	A				
(6,4,0)						C	A			
(7,4,0)							C			
(8,1,3)								D	B	
(8,2,3)									D	B
(8,3,3)										D

Figure 12. Submatrices B11 and B12 of the rate matrix for the slasher/skidder interaction, where skidder travel times are independent and A=5.25, B=0.34, C=-5.25 and D=-0.34.

Future States

Current States

B1	B2						B4
	B1	B2					B5
		B1	B2				B6
			B1	B2			B7
				B1	B2		B8
					B1	B2	B9
B3						B1	B10
B11							B12

Figure 13. Structure of the rate matrix for the slasher/skidder interaction, where skidder travel times are independent and the structure of submatrices B1-B12 is presented in Figures 8-12.

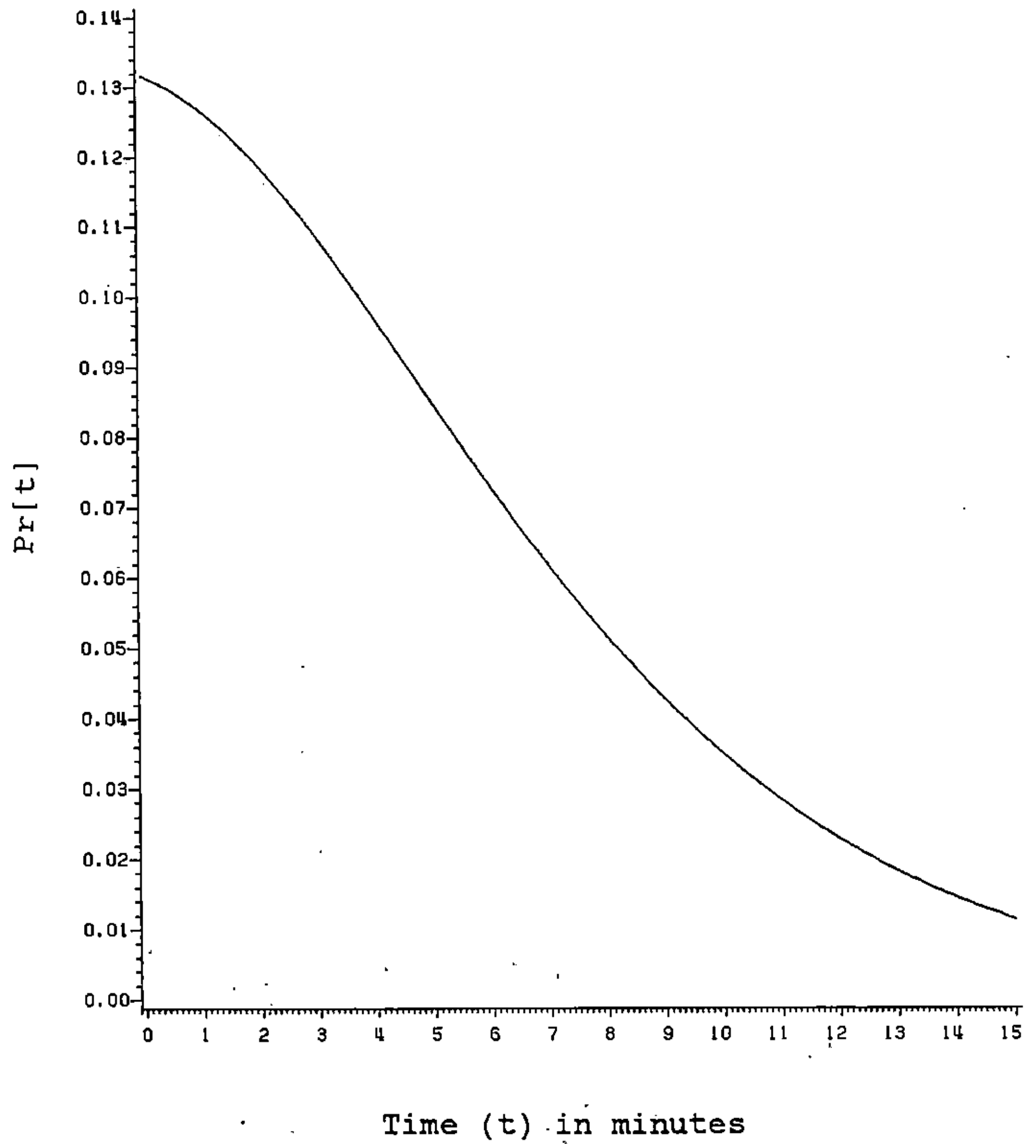

Figure 14. Probability density of the time a skidder must wait before unloading, given that the skidder is forced to wait, assuming perfectly correlated skidder travel times.

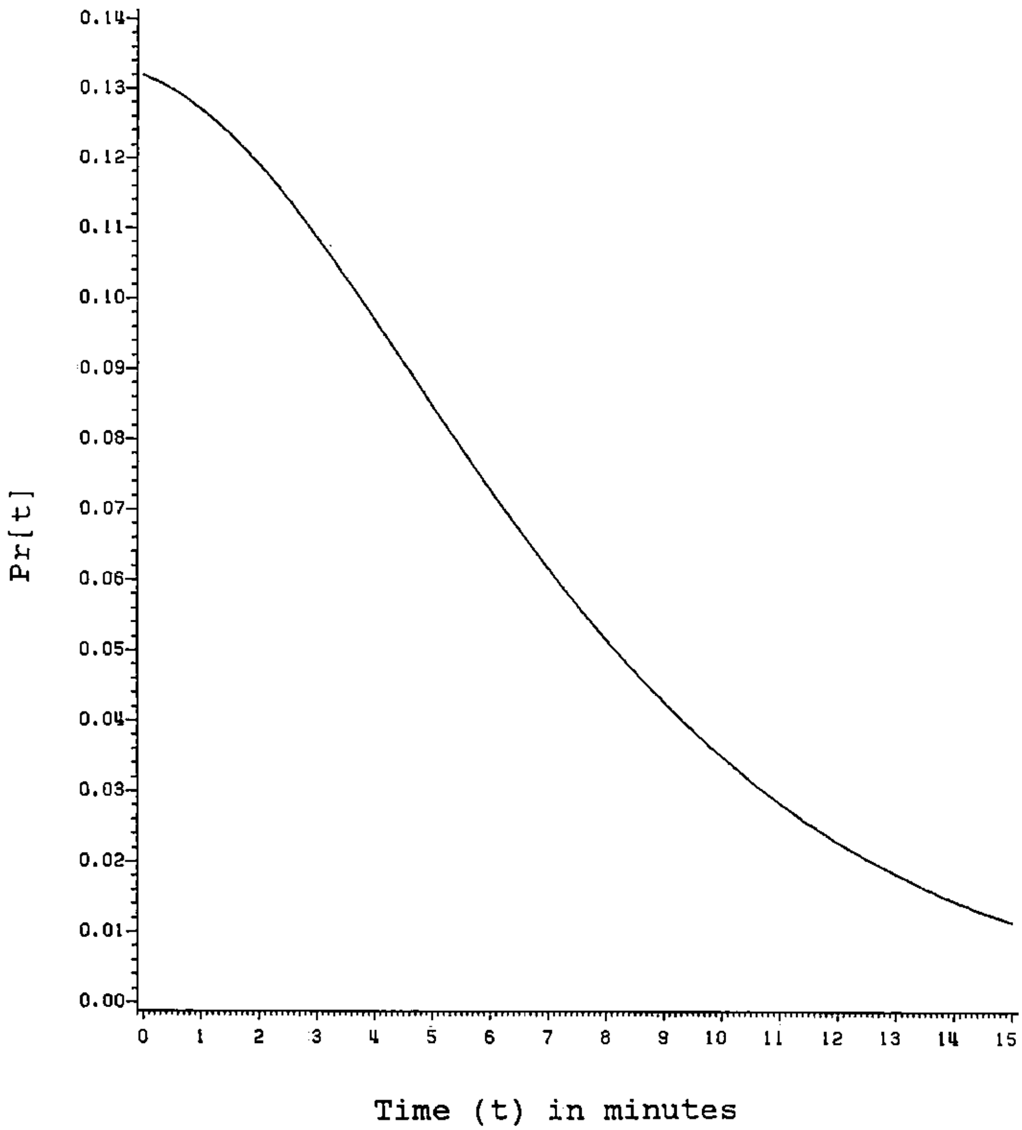

Figure 15. Probability density of the time a skidder must wait before unloading, given that the skidder is forced to wait, assuming independent skidder travel times.

5.2 BALANCING THE SLASHER/SKIDDER INTERACTION

The purpose of this section is to show how an analyst might use the Markov process model to make a more complete problem analysis. It was evident that a significant imbalance existed between the slasher and skidder. From a logging managers point of view this is not a desirable situation. The $90k skidder spent most of the time delayed, while the $50k slasher was always busy. The fixed cost of having the skidder idle represents a significant drain on the firms resources. How then, might a manager seek to alleviate this imbalance in the logging system?

Possible Modifications in the Slasher/Skidder Interaction

Two broad areas of potential modifications are available to the logging manager. First, are those alterations that can be made to the current system. These might include, among others:

1. Speed up the cycle time of the slasher. By focusing more closely on the activities of the slasher, the manager might identify areas where efficiency may be increased, through modifying some operational aspect(s) of the slasher.

2. Slow the skidder so that cycle times become longer. The distances the skidder was traveling, as illustrated earlier, were uncommonly short. In fact, one might have expected the skidder to be at a productive advantage over

the slasher, based solely on this observation. One possible modification is to increase skid distances, thereby increasing skidder cycle time, which in turn can reduce the number of landings required or decrease the number of roads to be built.

3. Increasing the size of the productive buffer. This may be accomplished through better placement of loads by the skidder or perhaps extending boom reach of the slasher's loader.

The other category of possible system modifications encompasses alternatives that require redesigning the system. Some of the possible redesigns might include:

1. Add another slasher, as a means of speeding up processing cycle times.

2. Replace the grapple skidder with one or more cable skidders, in an effort to slow the rate at which material is supplied to the slasher.

There are many ways for a logger to address the problem of balancing the slasher/skidder interaction, using the modifications suggested above. Different approaches may be dictated by the prevailing logging conditions or the financial status of the firm, among others. Since the primary purpose of this study was to introduce and illustrate a Markov model for analyzing machine interactions, only one approach to balancing the skidder/slasher interaction will be discussed.

The modifications to be tried will begin with:

1. Increasing skidder cycle time, as an indirect means of increasing skid distances, with the arbitrary constraint that mean skidder cycle time cannot be increased by more than a factor of 3.

2. If increasing skidder cycle time does not provide a reasonable balance, explore the opportunity of adding another slasher.

3. Determine if changing the productive buffer capacity has a significant influence on balancing the interaction.

Also, since the effect due to the correlation in skidder travel times was negligible, all analyses in this chapter will assume that the travel times are perfectly correlated.

Increasing Skidder Cycle Times

Because of the uncommonly short skid distances used to model skidder cycle times, the obvious system modification is to slow the skidder down by increasing skid distances. The approach will be to alter mean skidder cycle time as a means of emulating increased skid distance. Unfortunately, doubling mean skidder cycle time, for instance, does not directly imply a doubling of skid distances. However, for lack of a firm relationship between the two, it is assumed that a change in mean cycle time roughly approximates the same change in skid distance, at least for exploring the possible modifications in the interaction.

As illustrated earlier, the mean of the mixed Erlang distribution was a function of λ_K and λ_L. Thus, the choice of λ_K and λ_L, for the purpose of doubling, tripling, etc. of the mean value, may be an arbitrary one. That is, many different combinations of λ_K and λ_L will provide the desired change in the mean value.

However, the choice of λ_K and λ_L will also have an effect on the variance of the cycle time distribution. Recall, that the variance of the mixed Erlang is a function of K, L, p, λ_K, and λ_L. Therefore, care must be taken in selecting values of λ_K and λ_L that result in a reasonable modification in the variance of the cycle time distribution.

With longer cycle times, and the associated implication of increased skid distances, one might reasonably expect increased variation in cycle times. However, the exact physical relationship between increasing mean cycle times and the resulting change in variance is unknown. The relationship could be linear, quadratic, exponential, etc. If the relationship were known, alterations in the mean cycle time could be made so that the change in the variance would be of the appropriate magnitude. Mathematically, this could be accomplished by fixing K and L and then selecting values of λ_K, λ_L and p in the equations for the mean and variance that satisfy the mean-variance relationship.

Lacking a mean-variance relationship, an assumption must be made that provides a method of selecting λ_K and λ_L. First, it is assumed that the variance will increase with increased cycle time. Second, for convenience, p will remain fixed. Then, the values of λ_K and λ_l that minimize the increase in variance will be the ones chosen for lengthening mean cycle time.

Mean cycle time for the original model was 1.37 minutes and the variance was 0.7637. Two modifications in mean cycle time will be made. One will be to double the mean cycle time to 2.74 minutes and the other to triple it to 4.11 minutes. The resulting minimum variances are 1.4044 and 3.1599, respectively. Choice of λ_K and λ_L yielded 2.73 and 0.42, respectively, for doubling mean cycle time and 1.82 and 0.28, respectively, for tripling mean cycle time.

The same model, used to solve the slasher/skidder interaction earlier, can be used to determine the results due to modifying mean skidder cycle time. All that is required is to change all elements of the rate matrix corresponding to the skidder as follows:

1. For holding device one the new skidder rates are 1.23 (i.e., rate = $pK\lambda_K$ where, $p = 0.15$, $K = 3$ and λ_K = 2.73) where the mean cycle time is doubled and 0.82 (i.e., rate = $pK\lambda_K$ where, $p = 0.15$, $K = 3$ and $\lambda_K = 1.82$) where the mean is tripled.

2. For holding device two the new rates are 2.15 (i.e., rate = $qL\lambda_L$ where, $q = 0.85$, $L = 6$ and $\lambda_L = 0.42$) where the mean cycle time is doubled and 1.43 (i.e., rate = $qL\lambda_L$ where, $q = 0.85$, $L = 6$ and $\lambda_L = 0.28$) where the mean is tripled.

Solving the 123 X 123 system of equations for these modifications, the steady-state probabilities are:

1. For a doubling of mean skidder cycle time:

   Pr[skidder is delayed] = 0.690

   Pr[slasher is idle] = 0.0

   Pr[both are active] = 0.310

2. For a tripling of mean cycle time:

   Pr[skidder is delayed] = 0.534

   Pr[slasher is idle] = 0.0

   Pr[both are active] = 0.466

From a logging manager's standpoint the skidder is still delayed too large a proportion of time. If for some reason, mean cycle time can not be increased further by lengthening skid distance (perhaps tract size is limiting the maximum possible skid distance) what is the next possible alteration that might be tried?

Adding Another Slasher to the System

If the logger can afford it, the next possible modification might be a redesign of the system by adding a slasher. The additional slasher is assumed to be identical to the first slasher in terms of its productivity, so that they both have the same cycle time distribution. At least two possible designs can be explored with the addition of the second slasher. Figure 1 illustrates the first design. Here the two slashers are positioned so as to work from a common productive buffer. The operation of the skidder does not change since it is still servicing a single production buffer.

Figure 2 illustrates the second design. In this case, the two slashers have their own separate buffers. The skidder now has the choice of unloading at buffer 1 or buffer 2.

Modeling the Two Slasher/Common Buffer Design

The two slasher/common buffer model differs from the single slasher model in several ways. First, there are now five components to the problem, which will require a 5-dimensional specification in order to describe the states of the system:

$$\text{State Descriptor} = (i,j,k,m,n)$$

where,

i = 1 or 2 and indicates the mixed Erlang cycle time or holding device in which the skidder is traveling,

j = 1,2,3 or 1,2,3,4,5,6 and indicates which stage of the mixed Erlang holding device (i.e., holding device 1 or 2, respectively) the skidder is traversing,

k = 1,2,3 and indicates which stage of its Erlang cycle time slasher 1 is in,

m = 1,2,3 and indicates which stage of its Erlang cycle time slasher 2 is in,

n = 0,1,2,3 and indicates the number of skidder loads left to be processed in the productive buffer.

For example, a possible productive state of the system is (2,4,2,3,2). The skidder is in the fourth stage of its second holding device, the first slasher is in stage 2 of its Erlang cycle time, slasher 2 is in stage 3 of its Erlang cycle time and the productive buffer has 2 skidder loads left to process. The total number of productive states in this model is 324 (i.e., for i=1; j=3 times k=3 times m=3 times n=4 equals 108 plus, for i=2; j=6 times k=3 times m=3 times n=4 equals 216 or 324 total).

To fully describe the states of the system, idle and delay states must be included. Either slasher will enter the idle state when it finishes processing its current skidder load and returns to the buffer to find it empty. This can occur for any of the possible skidder states, when the buffer is empty, and any of the possible states of the other slasher. For example, if the skidder is in the first stage of its first holding device and the buffer is empty, the 3 idle states for slasher 2 are (1,1,1,4,0), (1,1,2,4,0) and (1,1,3,4,0); the 3 idle states for slasher 1 are (1,1,4,1,0), (1,1,4,2,0) and (1,1,4,3,0); and (1,1,4,4,0) denotes the state where both slashers are idle. Note, that as a matter of convenience a 4 is used to indicate that a slasher is in the idle state. Since there are 9 possible skidder states when the buffer is full, there are 9 times 7 or 63 possible idle states for the slasher.

Similarly, the skidder will be delayed when it ends the current cycle to find the productive buffer full. This can occur for any of the possible active states of the slashers when the buffer is full. For example, having just completed a cycle through the first holding device and finding the productive buffer full, the 9 possible delay states that the skidder could enter are: (1,4,1,1,3), (1,4,1,2,3), (1,4,1,3,3,), (1,4,2,1,3), (1,4,2,2,3), (1,4,2,3,3), (1,4,3,1,3), (1,4,3,2,3) or (1,4,3,3,3), where 4 indicates the skidder delay state. The same idea applies when the skidder exits the second holding device and finds the buffer full. Thus, there are 9 + 9 or 18 total delay states for the skidder.

The total number of states for describing the two slasher/one skidder interaction with a common buffer is 405. Rates for the slasher are the same as before (i.e. 0.34), while the skidder rate is the same as in the earlier model where the mean cycle time was tripled. Solving the 405 X 405 system of equations showed the steady-state probabilities to be:

Pr[skidder is delayed] = 0.100

Pr[only slasher 1 is idle] = 0.028

Pr[only slasher 2 is idle] = 0.028

Pr[both slashers are idle] = 0.007

Pr[skidder and both slashers are active] = 0.837

From these results, it is evident that the additional slasher proved significant in reducing delay time for the skidder. However, the tradeoff is an increased proportion of idle time for the slashers. Any attempt at balancing a system by changing the productivity (i.e., cycle time) of a machine(s) will introduce certain tradeoffs in the delay and idle times of the component machines. The key is to reduce the idle and delay times for the component machines to a uniformly acceptable level. In redesigning the system by adding another slasher, the balance of delay and idle time, depending on the logging manager in charge, is probably within reasonable limits.

Modeling the Two Slasher/Two Buffer Design

The physical separation of the two slashers in order to allow each one to have its own buffer is not as costly a redesign as making the increment to two slashers. Nor is it a modification that attempts to alter the cycle times of the equipment. Therefore, allowing two buffers differs from the earlier changes by attempting to affect delay and idle times through rearranging the components of the system in a physical sense. However, some additional cost may be incurred if the physical separation of the two slashers forces a logger to increase landing size, for instance.

From an analyst's point of view, the addition of a second buffer does cause a moderate change in the modeling aspects of the problem. The model is now composed of 6 elements, requiring a 6-dimensional state specification as follows:

State Descriptor = (i,j,k,m,n,o)

where,

i = 1 or 2 and indicates the mixed Erlang holding device in which the skidder is traveling,

j = 1,2,3 or 1,2,3,4,5,6 and indicates which stage of the mixed Erlang holding device (i.e., holding device 1 or 2, respectively) the skidder is traversing,

k = 1,2,3 and indicates which stage of its Erlang cycle time slasher 1 is in,

m = 0,1,2,3 and indicates the number of skidder loads left to be processed in slasher 1's productive buffer,

n = 1,2,3 and indicates which stage of its Erlang cycle time slasher 2 is in,

o = 0,1,2,3 and indicates the number of skidder loads left to be processed in slasher 2's productive buffer.

An additional assumption must be added here in order to specify state transitions. The problem arises since the skidder now has the choice of unloading at either slasher. First, it is assumed that the skidder always unloads at the slasher having the fewest number of loads remaining to be processed. If the respective buffers have an equal number of loads remaining to be processed, it is assumed that the skidder will always unload at slasher 1. Thus, it can be expected that slasher 1 should be idle a smaller proportion of time than slasher 2.

The number of active states for this problem is 1296 (i.e., for i=1; j=3 times k=3 times m=4 times n=3 times o=4 equals 432 plus, for i=2; j=6 times k=3 time m=4 times n=3 times o=4 equals 864 or 1296 total). The idle states of each slasher total 12 for each possible skidder state when the buffer is empty (i.e., for slasher 2 the 12 states are (1,1,1,0,4,0), (1,1,1,1,4,0), (1,1,1,2,4,0), (1,1,1,3,4,0), (1,1,2,0,4,0,), (1,1,2,1,4,0), (1,1,2,2,4,0), (1,1,2,3,4,0), (1,1,3,0,4,0), (1,1,3,1,4,0), (1,1,3,2,4,0) and (1,1,3,3,4,0)), plus one idle state denoting that both slashers are idle (i.e., (1,1,4,0,4,0)). For each possible skidder state where the buffer is empty, then, there are a 12 + 12 + 1 or 25 idle states. Since there are 9 possible skidder states, the total number of idle states for the slashers is 9 times 25 or 225.

The possible skidder delay states once again total 18 (i.e., for the skidder exiting holding device one to find the buffer full, the possible delay states are (1,4,1,3,1,3), (1,4,1,3,2,3), (1,4,1,3,3,3), (1,4,2,3,1,3), (1,4,2,3,2,3), (1,4,2,3,3,3), (1,4,3,3,1,3), (1,4,3,3,2,3), (1,4,3,3,3,3)). The total number of states to describe the system is 1539. Rates for the process are identical to those used in the 2 slasher/1 skidder/1 buffer model. Solving the 1539 X 1539 system of equations yielded the following steady-state probabilities:

Pr[skidder is delayed] = 0.081

Pr[only slasher 1 is idle] = 0.005

Pr[only slasher 2 is idle] = 0.015

Pr[both slasher are idle] = 0.001

Pr[skidder and both slashers are active] = 0.8987

Although the improvement in idle and delay times is not very substantial, it is interesting to note that simply rearranging the configuration to include individual buffers did not produce a tradeoff in delay and idle times. That is, the change actually reduced both delay time for the skidder and idle time for the slashers. So it is important to recognize that certain changes in a system will produce tradeoffs, while others do not. However, the desirability of this change, as mentioned earlier, must be viewed in light of any additional cost that might be incurred.

Increasing the Size of the Productive Buffer

All models to this point have assumed a maximum buffer size of 3 skidder loads. The assumption here will be that the maximum capacity of the buffer is 4 skidder loads. The effect of increased buffer size will be determined using the 2 slasher/common buffer model. The model remains the same in that the states of the system will again require a 5-dimensional specification indicating the holding device and stage for the skidder, which stage slasher 1 is in, which stage slasher 2 is in and the number of skidder loads remaining to be processed in the buffer. The only difference now is that the level of the buffer can be 0,1,2,3, or 4. The number of productive states of the process therefore increases to 405, from 324, while the number of delay and idle states remains unchanged. The total number of states is 486.

The transitions and rates are determined the same as before, so that this modification introduces nothing new new in terms of setting up the model. Solving the 486 X 486 system of equations showed that increasing the buffer size did have an effect on the steady-state probabilities:

Pr[skidder is delayed] = 0.090

Pr[slasher 1 is idle] = 0.019

Pr[slasher 2 is idle] = 0.019

Pr[both slashers are idle] = 0.005

Pr[skidder and both slashers are active] = 0.867

It appears that increasing buffer size, especially if it is of minimal cost, can provide some improvement in idle and delay times. In this case, skidder delay was reduced by 10% and slasher idle time was improved about 32%.

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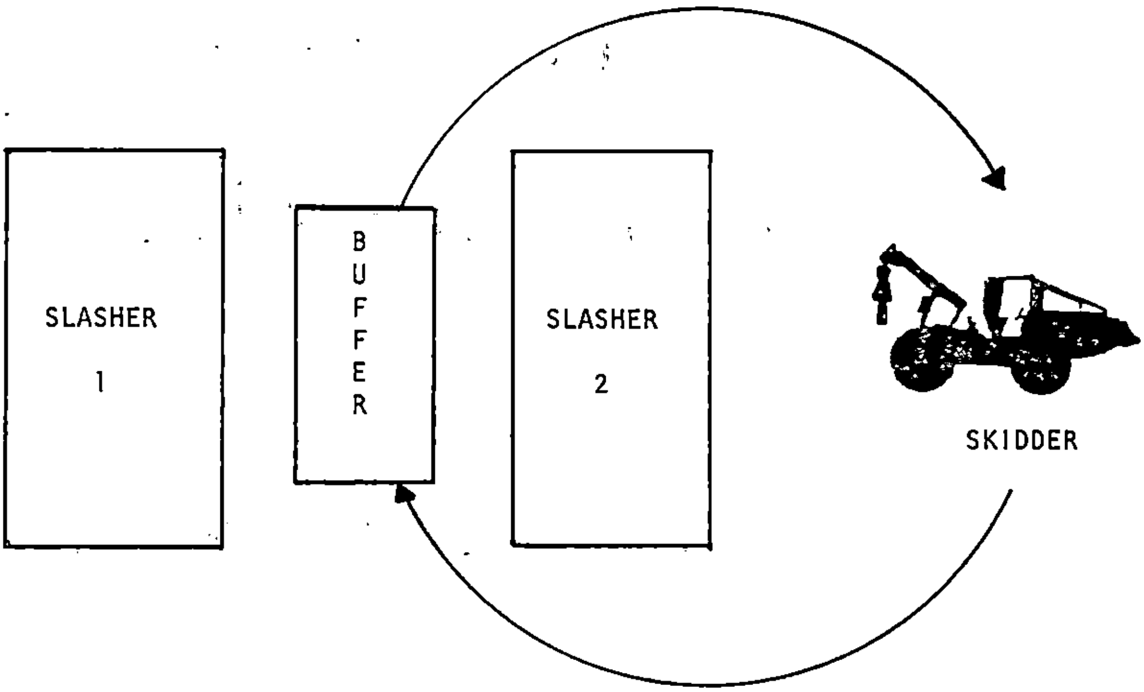

Figure 1. Components of the two slasher/one skidder interaction with a common productive buffer.

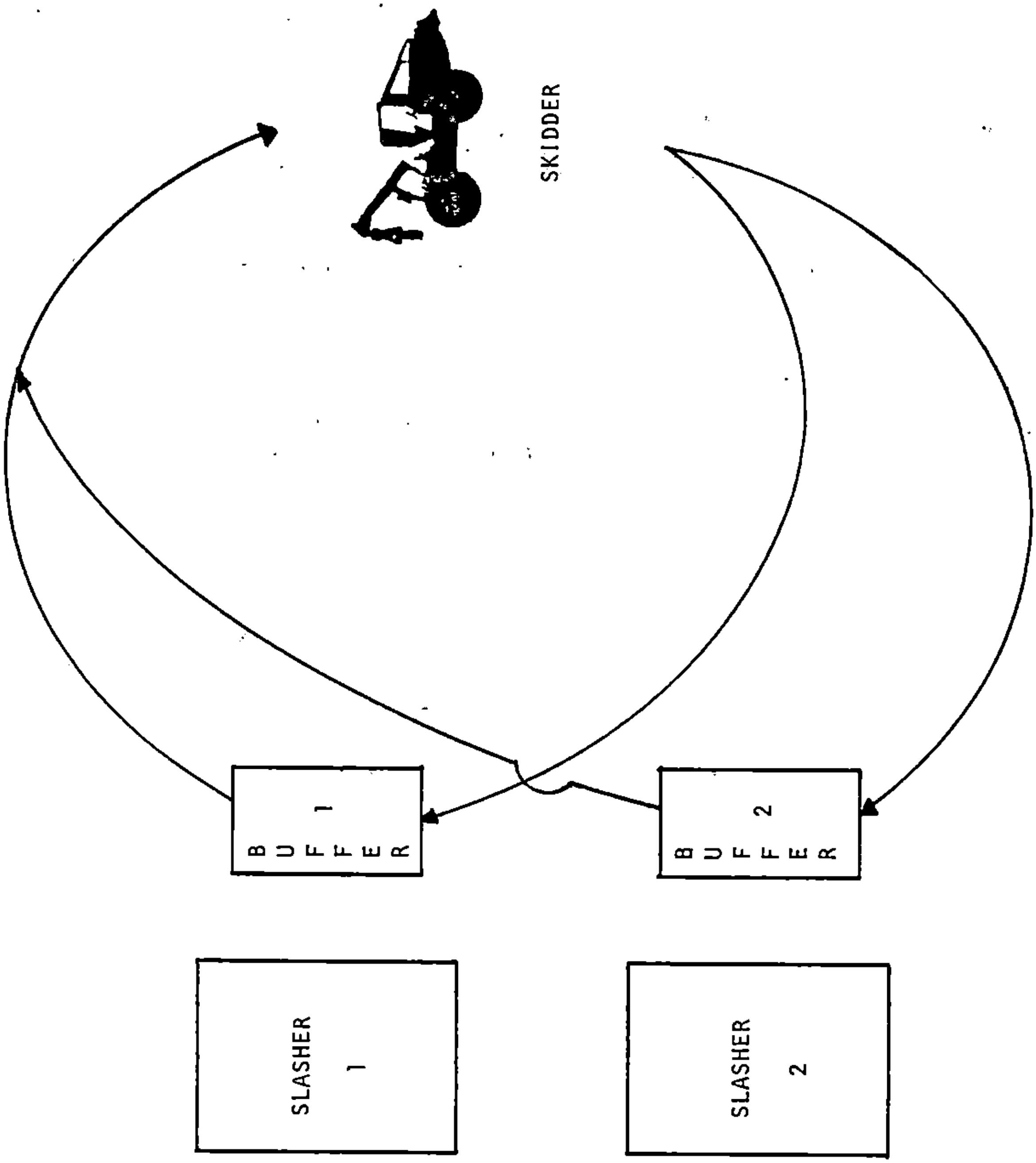

Figure 2. Components of the two slasher/one skidder interaction with separate productive buffers for each slasher.

6. SUMMARY

This study represents the first application of Markov process theory to the analysis of timber harvesting systems. By focusing on the most difficult aspect of modeling timber harvesting systems, machine interactions, it was shown how to adapt and successfully apply Markov process theory to the modeling of this problem.

As a package for analyzing complex interactions, the methodology developed here provides considerable flexibility both in the types of interactions that can be modeled and in modeling the time-based probability distributions required by the Markov process model. Any number of interacting machines can theoretically be modeled. For statistically modeling the variety of time-based distributions one can expect to encounter in timber harvesting applications, the Erlang and mixed Erlang distributions provide the necessary flexibility. In addition, and more importantly, the Erlang and mixed Erlang distributions provide the needed compatability with the assumptions of the Markov model, establishing the key link between data analysis and formulation of the model.

Actual solution of the Markov process model, for steady-state probabilities, is a straightforward application of matrix algebra techniques on a system of algebraic equations. The major limitation at this point is the dimensionality of problems with many machines and the capabilities of current computer facilities in handling problems of this size.

Flexibility of the model was illustrated by applying several modifications to the one slasher/one skidder system. The ideas of increasing skidder cycle time, adding another slasher and increasing productive buffer size were all handled easily by the model. A reasonable balance between interacting components was achieved by tripling mean skidder cycle time and adding another slasher.

Some of the key advantages that the Markov process model offers over simulation based studies are:

1. An analytic solution to proportions of delay, idle and productive time and to waiting time distributions is possible.

2. The Markov process model is explicit in recognizing the correlation between current states of the system and future states of the system, in developing a solution to the model. On the other hand, the existence of correlated outputs in a simulation offers theoretical obstacles in developing anything more than an expected value solution to performance of the system.

3. The Markov process model can offer certain efficiencies in actually obtaining a solution. Once the model is specified and the rate matrix developed, the

investigation of alterations in the system simply requires changing the rates of the process. Whereas, an effective simulation requires not only changing the parameters of the model, but determining how many times the model should be run in order to get a sufficiently large sample to perform statistical tests and finally making the statistical analysis of the output data.

4. The Markov process model does not require that a user be competent in a higher level computer language in order to develop and/or make changes to a model.

5. No allowance is made for inputting non-random components into a system that is non-random by nature.

6. The model presented here requires familiarity and understanding of only two types of probability distributions: the Erlang and mixed Erlang. With these 2 distributions a wide range of potential shapes is available for modeling the components of the random process.

Although the adaptation of stochastic process theory, and Markov processes in particular, to timber harvesting analysis is in its infancy, the results of this study indicate that that the approach has significant potential and deserves additional attention.

7. AREAS FOR ADDITIONAL RESEARCH

1. Maintaining each individual active state in the model is not necessary when delay and idle times are of interest. Therefore, the notion of lumping the active states into a single state could be of considerable importance in reducing problem size and solution time.

2. Incorporating random non-productive components into the model, including breakdowns, operator delays, etc.

3. Using the results of the machine interaction model to determine system productivity.

4. Using the Markov model to analyze the activities of a single machine as a means for discovering inefficiencies in the current operational scheme and identifying possible changes or redesigns of the machine.

5. Develop optimum fitting procedures for the mixed Erlang distribution, as well as , goodness-of-fit statistics.

6. Determine waiting time distributions where the time a machine has to wait is dependent on the minimum amount of time it takes one of two or more machines to complete its current activity. For example, in the 2 slasher models, the time the skidder must wait when arriving to find the common buffer or two individual buffers full will be the minimum time of the two random variables indicating when the slashers will complete the current cycle.

7. Investigate the techniques for solving large sparse matrices as a means of handling large sized problems.

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