CAPACITY, ENTRY DETERRENCE, AND HORIZONTAL MERGER

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(ABSTRACT)

This dissertation examines the free rider problem of entry deterrence, the profitability of a horizontal merger, and the effects of a horizontal merger on the outsiders' profits and industry prices, in the markets where firms' capacity costs are sunk.

We investigate the free rider problem of entry deterrence in the subgame perfect Nash equilibria of a three-stage game in which in the first stage multiple incumbent firms choose their capacities simultaneously and independently, in the second stage a potential entrant, after observing the incumbent firms' capacity vector, chooses its capacity, and in the third stage the firms engage in capacity-constrained Cournot competition. We show that the free rider problem may occur: there are situations where both entry prevention and allowing entry are equilibria but entry prevention is Pareto superior for the incumbent firms. We also show that increasing the number of incumbent firms may cause the equilibrium price to increase and thus consumer welfare to decrease. The free rider problem is still manifested in a modified model in which multiple potential entrants choose their capacities sequentially after the first stage incumbents' capacity decisions.

Several recent papers which theoretically analyze the profitability of a

horizontal merger and its effects on the outsiders' profits and industry prices, all observe that a merger never decreases industry prices, a merger to a monopoly is always profitable, and a merger never hurts the outsiders.

However, we demonstrate, in a market for a homogeneous product where firms with sunk capacities compete in quantities and there are potential entrants, that a merger can decrease industry price and a merger of incumbent firms to a monopoly may not be profitable. We also show, in a market for a homogeneous product where firms with sunk capacities engage in capacity-constrained price competition, that a merger can hurt the outsiders.

To My Parents

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CHAPTER I

INTRODUCTION

1. STRATEGIC ENTRY DETERRENCE AND SUNK CAPACITY COSTS

Consider a market situation in which there are a single established firm (or a coordinated cartel), called the incumbent, and a single potential entrant. There are two periods: preentry and postentry. The market demand is constant through the periods. In the preentry period, the incumbent chooses and produces at an output level which it will *maintain* in the postentry period. At the beginning of the postentry period, taking the incumbent's output level as given, the potential entrant decides whether or not to enter and if so how much to produce. It enters if and only if it can make a positive profit. After the decisions of the potential entrant are made, the active firm(s) in the market produce(s) at the predetermined output level(s). Their products are homogeneous and consumers can switch firms without any costs.

These are all essential assumptions of the classic model of strategic entry deterrence, called the Bain-Sylos-Modigliani (BSM) limit pricing model. The potential entrant in the model believes that the incumbent will produce, in the postentry period, at the output level committed in the preentry period regardless of the potential entrant's decisions (the Bain-Sylos postulate). Thus if the entrant enters, it becomes a Cournot follower and chooses a Cournot best response to the incumbent's output. The incumbent, on the other hand, acts as a Stackelberg leader in output. If entry deterrence is profitable when compared with accommodation, the incumbent commits to the "limit output" and prevents the potential entrant from entering the market. Otherwise, it allows the entrant to enter.

However, the assumption that the incumbent can convince the potential entrant that it will maintain the same output in the postentry period as its preentry output regardless of whether or not entry occurs, is dubious. The incumbent's optimal response to entry is usually an accommodating output reduction and given the entrant's knowledge of the fact, the entrant treats the incumbent's threat that it will continue to produce at the committed output as empty and thus enters the market.

Recognizing that the incumbent's *prior* (to the entrant's decisions) and *irreversible* investment in entry deterrence can be a credible threat (equivalent to commitment), Spence (1977) proposed a model in which the incumbent installs capacity in the preentry period and capacity costs are *sunk*. The potential entrant believes, in that model, that if it enters the incumbent's output will be equal to its installed capacity. Thus, entry prevention is always feasible like in the BSM limit pricing model and excess capacity may be observed even in the case of a linear demand.

However, the incumbent's threat that it will produce up to its capacity if the entrant enters, can be empty. For example, in the two-stage model of Dixit (1980), if the incumbent's installed capacity is greater than its output at the intersection of its variable-cost Cournot reaction function and the entrant's full-cost reaction function, then the incumbent's optimal response to entry is to choose the output at the intersection.

Dixit (1980) studies a perfect equilibrium in a model which is basically the same as Spence's and shows that in the case where each firm's marginal revenue is always decreasing in the other's output, excess capacity is never observed in a perfect equilibrium. There are situations where preventing entry is profitable in Spence's model but not feasible in Dixit's (1980).

Dixit (1980) assumes that in the second stage while the incumbent's capacity costs are sunk, those of the entrant are variable. Ware (1984) argues that if the incumbent's capacity costs are sunk, then those of the entrant are equally so and must also be committed before production takes place. He thus proposes a *three-stage model* in which the incumbent and the entrant sequentially choose their capacities and then engage in capacity-constrained Cournot competition. The incumbent has a harder time preventing entry than suggested by Dixit (1980) but it still maintains a strategic advantage over the entrant because it sinks capacity before the entrant does.

In Chapters II, III, and IV of this dissertation, we consider market situations where firms have *sunk capacity costs*, and develop our arguments based on *three-stage models*, following Ware's proposal.

2. ENTRY DETERRENCE AND THE FREE RIDER PROBLEM

Beginning with the seminal work of Bain (1956), the early literature on strategic entry deterrence has concentrated on a market situation in which a single incumbent firm (or group of colluding incumbents) confronts a single potential entrant. This includes, for example, Spence (1977), Salop (1979), Dixit (1979), (1980), Spulber (1981), Schmalensee (1981), Kreps and Wilson (1982), Milgrom and Roberts (1982), Fudenberg and Tirole (1983), Ware (1984), Bulow, Geanakoplos, and Klemperer (1985), and Allen (1986). In these works, the incumbent prevents the potential entrant from entering the market if entry deterrence is feasible and also profitable when compared with accommodation.

Recently, the literature has been dominated by market situations in which an established oligopoly of competing firms faces a potential entrant (or multiple entrants), and in which all noncooperative firms enter the market sequentially. For example, Prescott and Visscher (1977), pioneering the ideas of sequential entry and endogenous market structure, analyze a model in which firms with perfect foresight about subsequent entry and location decisions enter the market and choose locations sequentially. Bernheim (1984) develops a model of sequential entry into an industry and demonstrates that standard government policies may have the perverse effect of increasing industrial concentration. He also investigates the ability of a noncooperative oligopoly to prevent entry when member firms simultaneously commit to deterrence investments. Gilbert and Vives (1986) consider a market where several incumbents facing a potential entrant choose simultaneously and independently postentry output levels. Harrington (1987) examines the effect on entry deterrence of asymmetric information about the constant marginal cost between a noncooperative oligopoly and potential entrants. Vives (1988) analyzes a model with quantity commitments where an incumbent (or several incumbents) faces a sequence of potential entrants. Eaton and Ware (1987) study noncooperative entry deterrence in a sequential entry model where firms sink capacity costs at the time of entry and market structure is determined endogenously, and McLean and Riordan (1989) do so in a sequential entry model where upon entering, each firm irreversibly chooses a type of technology and market structure is determined endogenously.

Entry deterrence in the models characterized by multiple incumbents has the properties of a public good. If some firms prevent entry, all firms in the industry are protected from the new competitors. That is, "consumption" of entry deterrence is not exclusive. Also, one firm's consumption of entry deterrence does not reduce its amount enjoyed by other firms. Since firms in the above-mentioned studies cannot collude on investments in entry deterrence, each firm may free-ride on its rivals' provision of the public good and the free rider problem suggests that there would be underinvestment in entry deterrence.

The seminal work which discusses the free rider problem of entry deterrence is Bernheim's. In his model there are cases where both entry prevention and allowing entry are equilibria, but entry prevention is mutually more profitable than allowing entry. Thus one may say that there may be underinvestment in entry deterrence. He argues, however, that there is no such phenomenon because whenever entry prevention is jointly profitable when compared to allowing entry, sufficient deterrence investment is normally expected to be undertaken even by a noncooperative oligopoly. He actually applies to the model the solution concept of a coalition-proof Nash equilibrium introduced in Bernheim, Peleg, and Whinston (1987). Gilbert and Vives demonstrates that the free rider problem never occurs in their model. Waldman (1987a) shows that in the model of Gilbert and Vives there is no evidence of underinvestment in entry deterrence even if the incumbent firms are uncertain about the exact investment in entry deterrence needed to deter entry. He shows, on the other hand, that in Bernheim's model there is a strong tendency to underinvest in entry deterrence if incumbent firms are uncertain about the exact investment in entry deterrence needed to deter entry. Harrington finds the same situations as Berheim and states that potentially the free rider problem of entry deterrence does exist. Eaton and Ware never find underinvestment in entry deterrence in the sense that the number of firms in the equilibrium is the smallest that can deter entry. McLean and Riordan observe that the free rider problem frequently occurs. Waldman (1987b) points out that in the model of McLean and Riordan the free rider problem arises even if a group of early entrants (or incumbent firms) are assumed to move simultaneously. He also argues there that the free rider problem matters if some factor is present which smooths the return to investing in entry deterrence.

Gilbert and Vives examine the free rider problem of entry deterrence in a two-stage model where multiple incumbent firms facing a single potential entrant commit noncooperatively to postentry output levels and then the entrant's decisions follow. In Chapter II of the dissertation, we show that their observation of nonexistence of the free rider problem totally relies on the Bain-Sylos postulate which they assume. Indeed, introducing sunk capacity costs which are typically considered as credible entry deterrence investment and adopting a three-stage game *à la* Ware (1984), we demonstrate that the free rider probem can occur. We illustrate that there are situations where both entry prevention and allowing entry are subgame perfect equilibria but entry prevention is mutually more profitable than allowing entry. The reason why our observation differs from that of Gilbert and Vives is that entry prevention is not always feasible in our model while it is in theirs.

3. HORIZONTAL MERGERS AND THREE COMMON OBSERVATIONS

Defining a horizontal merger as a union of independent firms in the same market into a single entity under the control of a single decision maker, several recent papers theoretically analyze its profitability and its effects on the outsiders' profits and industry prices.

Are mergers beneficial to the merging parties (insiders)? The answers to the question differ across the studies. Szidarovszky and Yakowitz (1982), Salant, Switzer, and Reynolds (1983), Davidson and Deneckere (1984), and Perry and Porter (1985) all demonstrate that mergers may reduce the joint profits of the constituent firms (insiders) in quantity-setting games. Davidson and Deneckere (1984) and Deneckere and Davidson (1985) show that mergers are never disadvantageous in static price-setting games.

The above-mentioned studies, on the other hand, coincide in the following observations: (i) a merger, profitable or not, never decreases industry prices, (ii) a merger to a monopoly is always profitable, and (iii) a merger never hurts the outsiders.

However, drawing on the models prevalent in the industrial organization literature, we illustrate in Chapters III and IV of the dissertation that the three common observations can be reversed. In particular, Chapter III provides both an example in which a merger causes industry price to decrease, and an example in which a merger to a monopoly is not profitable. Chapter IV provides an example in which a merger hurts outsiders.

We consider, in Chapter III, a market for a homogeneous product where firms with sunk capacities compete in quantities and there are potential entrants, and in Chapter IV, a market for a homogeneous product where firms with sunk capacities engage in capacity-constrained price competition. In both markets firms cannot decrease their capacities since their capacity costs are sunk. Our arguments are made in the context of three-stage games. We treat each merged entity as a single firm with the combined capacity of insiders and then give it a strategic advantage that it can decide whether or not to increase its capacity before outsiders do. The latter assumption, which is not innocuous for the result in Chapter IV, seems to be natural because the merged firm can commit to its postmerger capacity before it announces the merger.

CHAPTER II

CAPACITY, ENTRY DETERRENCE, AND THE FREE RIDER PROBLEM

1. INTRODUCTION

Since the seminal work of Bain (1956), the literature on strategic entry deterrence has grown rapidly. The early works in this area restricted attention to a market situation in which a single incumbent firm (or a coordinated cartel) faces a single potential entrant. Works by Spence (1977), Salop (1979), Dixit (1979), (1980), Spulber (1981), Schmalensee (1981), Fudenberg and Tirole (1983), and Ware (1984) are examples. Recently, attention has moved from the one incumbent one entrant framework to models in which multiple competing incumbent firms confront a single potential entrant (e.g., Gilbert and Vives (1986), Harrington (1987), and Waldman (1987a), (1987b)) and to models in which firms sequentially enter an industry (e.g., Prescott and Visscher (1977), Bernheim (1984), Eaton and Ware (1987), and McLean and Riordan (1989)). These works focus on the free rider problem of entry deterrence and/or the effects of sequential entry on elements of market structure. Entry deterrence in them has the characteristics of a public good. If some firms prevent entry, all firms in the industry are protected from the new competitors. Since firms in these models cannot collude on investment in entry deterrence, examining the free rider problem of entry deterrence should be an interesting issue.

Gilbert and Vives examine the free rider problem of entry deterrence in a two-stage model in which in the first stage multiple incumbent firms facing a potential entrant which must pay a fixed cost to enter the industry, decide independently how much to produce and in the second stage the potential entrant decides whether to enter or not, and if it enters how much to produce. They conclude that in their model the free rider problem never occurs: there is no underinvestment in entry deterrence.[1] Waldman (1987a) shows that in the model of Gilbert and Vives even if the incumbent firms are uncertain about the cost of entry and thus uncertain about the exact investment in entry deterrence needed to deter entry, there is no evidence of an underinvestment in entry deterrence.[2]

However, the result of Gilbert and Vives critically relies on the assumption that the incumbent firms are able to convince the potential entrant that they will produce at their committed output levels regardless of the potential entrant's actions. Indeed, their model is an extension of the Bain-Sylos-Modigliani limit pricing model to the case of multiple incumbent firms.[3] Notice that in the recent literature on strategic entry deterrence, output has been typically regarded as a noncredible entry deterrence instrument.

We introduce the installation of capacity as a sunk investment in entry deterrence, which has been considered a credible entry deterrent in this area (e.g., Spence (1977), Dixit (1979), (1980), Schmalensee (1981), Spulber (1981), Ware (1984)). In particular we adopt the proposal in Ware (1984) that as long as the instrument of strategic commitment takes the form of sunk costs, which both a potential entrant and an incumbent firm must incur, a three-stage model is required. Since a potential entrant in our model must also install capacity (sink capacity costs) before production takes place, we set up a three-stage model.

We consider a market for a homogeneous product with multiple incumbent firms facing a potential entrant. In the first stage, the incumbent firms choose capacities simultaneously and independently. In the second stage, after observing the capacity combination of the incumbent firms, the potential entrant chooses a capacity. If it enters the market (chooses a positive capacity), it must pay a sunk fixed cost of entry as well as sunk costs of capacity. In the third stage the firms engage in Cournot competition, subject to the capacity constraints chosen in the first two stages. At each stage all firms have perfect foresight about actions in future stages. Hence, a relevant equilibrium concept for the game is that of a subgame perfect Nash equilibrium.

We first show that, unlike the result of Gilbert and Vives, there may be underinvestment in entry deterrence in our model. In particular, we construct an example with two symmetric equilibria, one of which is an entry-preventing and the other is an entry-allowing equilibrium. In this example the profits of the incumbent firms are higher at the entry-preventing equilibrium than at the entry-allowing one. In such a case colluding incumbent firms may well prevent entry but noncooperating incumbent firms might not. The reason why this case arises is that even if an incumbent firm builds an infinite capacity given the capacity combination of the other incumbent firms, the potential entrant ignores the portion of the capacity which will be unused after it enters with the best capacity. In our model it should be noticed that there are situations where even infinite capacities of the incumbent firms cannot prevent entry.

Second, we show that increasing the number of incumbent firms may cause the equilibrium price to increase and thus consumer welfare to decrease. It can happen when preventing entry which is not feasible with smaller number of incumbent firms becomes feasible by increasing the number of incumbent firms. A change in the number of incumbent firms may result from a horizontal merger of incumbent firms. Our result then implies that consumers may benefit from a horizontal merger of incumbent firms.

The outline for the chapter is as follows. In Section 2 we present our three-stage game on strategic entry deterrence and review some properties of the Cournot reaction functions. Section 3 defines the equilibria of two types of subgames and develops some of their properties. In Section 4 we first show that in all subgame perfect equilibria no firm has excess capacity. Second, we prove the existence of an equilibrium of our full game in a series of propositions by constructing symmetric equilibria. Finally, we illustrate in an example that there may exist a continuum of entry-allowing equilibria and the total equilibrium output may be different across these equilibria. In the example we also find that the profit of a potential entrant can be higher than that of an incumbent firm. It implies that a first mover does not always have strategic advantage. Section 5 provides an example in which incumbent firms may underinvest in entry deterrence. The comparative statics are examined in Section 6.

Section 7 modifies the original model, allowing for multiple potential entrants which choose capacities in sequence. We first show that whenever preventing entry is feasible there is an entry-preventing equilibrium. This contrasts to the result when there is only one potential entrant. We next show that even with multiple potential entrants there may be underinvestment in entry deterrence. Section 8 contains several conclusions.

2. THE MODEL

The m incumbent firms (first movers) and one potential entrant (second mover) form the set of players in a three-stage noncooperative game with complete but imperfect information and perfect recall. They seek to maximize their own profits. The potential entrant is denoted firm m+1. Let $M = \{1, \cdots, m\}$ be the set of the incumbent firms and $\overline{M} = \{1, \cdots, m+1\}$ the set of players of the game. In the first stage, the incumbent firms choose capacities simultaneously and independently. Let $k_i \geq 0$ be the capacity chosen by firm i for $i \in M$. In the second stage, after learning the capacity each incumbent firm installed, firm m+1 chooses a capacity, $k_{m+1} \geq 0$. We assume that firm m+1 enters (chooses a positive capacity) if and only if it can make positive profits. In the third stage, the firms engage in capacity-constrained Cournot competition, given $k^{m+1} = (k_1, \cdots, k_{m+1})$.

All firms have perfect foresight about actions in future stages. No firm can make empty (noncredible) threats which would not be carried out. Thus a relevant equilibrium concept for the game is that of a subgame perfect Nash equilibrium.

A single homogeneous good is produced by the firms. Denote the output of firm i, $i \in \overline{M}$, in the third stage by x_i, where x_i belongs to $[0, k_i]$. The cost function of firm i, $i \in M$, is

$$C_i(x_i, k_i) = \begin{cases} S + Vx_i + Rk_i & \text{for } x_i \leq k_i \text{ and } k_i > 0 \\ 0 & \text{for } k_i = 0 \end{cases}$$

where $S \geq 0$ is a fixed set-up cost, $V \geq 0$ is a constant unit variable cost up to k_i, and $R > 0$ is a constant unit capacity cost. We assume that once incurred, the capacity costs, S and Rk_i, are sunk. Similarly, the cost function of firm m+1 is

$$C(x_{m+1}, k_{m+1}) = \begin{cases} F + Vx_{m+1} + Rk_{m+1} & \text{for } x_{m+1} \leq k_{m+1} \text{ and } k_{m+1} > 0 \\ 0 & \text{for } k_{m+1} = 0 \end{cases}$$

where $F \geq S$ is a fixed cost of entry. Once incurred, F and the capacity costs are sunk.

Let $p = f(X)$ be the market inverse demand function with $X = \sum_{i \in \overline{M}} x_i$. We assume that for some $\hat{X} > 0$, $f(X)$ is positive on the interval $[0, \hat{X})$, on which it is twice continuously differentiable, strictly decreasing ($f'(X) < 0$), and concave ($f''(X) \leq 0$). For $X \geq \hat{X}$, $f(X) = 0$. We consider situations where the profit of firm i, $i \in M$, is positive at the full cost m-firm Cournot equilibrium. For simplicity, we set $S = 0$.

The assumption that the structure of the game is common knowledge for all firms completes its description.

It is helpful to first define two Cournot reaction functions and to review some facts about them.

Definition 1. Let $t(Z) = \underset{x \geq 0}{\text{argmax}}\ xf(Z+x) - Vx \quad$ for $Z \in [0, \infty)$.

That is, $t(Z)$ is the best response of a firm when the other firms produce a total output of Z, with no fixed costs, no capacity constraints, and a constant marginal cost V.

Definition 2. Let $r(Z) = \underset{x \geq 0}{\text{argmax}}\ xf(Z+x) - (V+R)x \quad$ for $Z \in [0, \infty)$.

Similarly, $r(\cdot)$ is the Cournot reaction function with a constant marginal cost $V+R$ when there are no fixed costs and no capacity constraints.

Lemma 1. There exist $\bar{Z} > 0$ such that $t(Z) > 0$ for $Z < \bar{Z}$ and $t(Z) = 0$ otherwise, and $\tilde{Z} > 0$ such that $r(Z) > 0$ for $Z < \tilde{Z}$ and $r(Z) = 0$ otherwise. (a) $t(Z)$ and $r(Z)$ are continuously differentiable and decreasing in $Z \in [0, \bar{Z})$ and $Z \in [0, \tilde{Z})$, respectively. (b) $-1 < t' < 0$ for $Z \in [0, \bar{Z})$ and $-1 < r' < 0$ for $Z \in [0, \tilde{Z})$, and thus $Z+t(Z)$ and $Z+r(Z)$ are increasing in Z. (c) $t(Z) \geq r(Z)$ for all Z, with strict inequality for $Z \in [0, \bar{Z})$. (d) The maximum profits of a firm, $t(Z)f(Z+t(Z)) - Vt(Z)$ and $r(Z)f(Z+r(Z)) - (V+R)r(Z)$, are decreasing in $Z \in [0, \bar{Z})$ and $Z \in [0, \tilde{Z})$, respectively.

The proof is given in Lemma 1 of Kreps and Scheinkman (1983) and in Appendix of Gilbert and Vives (1986).

3. CAPACITY-CONSTRAINED SUBGAMES

The game formalized above contains two types of subgames different from the full game itself. The first type consists of subgames starting at the beginning of the second stage, when capacities chosen by incumbent firms become common knowledge. Let $k^m = (k_1, \cdots, k_m)$ be the vector of capacities chosen by the incumbent firms in the first stage. Then this subgame is called the k^m (capacity-constrained) subgame.

The other type consists of subgames starting at the beginning of the third stage, the point where capacities chosen by all firms become common knowledge. If k^{m+1} is the capacity vector at the end of the first two stages, then the subgame is called the k^{m+1} (capacity-constrained) subgame.

We study first the latter type of subgames. In the second type of subgames, firms are only concerned with variable costs since capacity and entry costs are already sunk. That is, the m+1 firms engage in capacity-constrained Cournot competition, with a constant marginal cost V and without fixed costs.

<u>Definition 3</u>. The output vector, $\bar{x}^{m+1} = (\bar{x}_1, \cdots, \bar{x}_{m+1})$, is a Cournot-Nash equilibrium of the k^{m+1} subgame if

$$\bar{x}_i = \operatorname*{argmax}_{0 \leq x_i \leq k_i} \; x_i f(\bar{X}_{-i} + x_i) - V x_i \qquad \text{for all } i \in \overline{M}$$

where $\bar{X}_{-i} = \sum_{j \in \overline{M}_{-i}} \bar{x}_j$ with $\overline{M}_{-i} = \overline{M} \setminus \{i\}$.

Notice in Definition 3 that $\bar{x}_i = \min\{ k_i, t(\bar{X}_{-i}) \}$ for all $i \in \bar{M}$.

Lemma 2. The Cournot-Nash equilibrium, $\bar{x}^{m+1}$, of the k^{m+1} subgame is unique.

The proof is given in Proposition 1 of Eaton and Ware (1987).

In the Cournot-Nash equilibrium of the k^{m+1} subgame, three types of firms are possible.

Definition 4. If $\bar{x}_i < k_i$, for $i \in \bar{M}$, then firm i is said to be nonconstrained. If $\bar{x}_i = k_i$, for $i \in \bar{M}$, then firm i is said to be constrained. If $\bar{x}_i = k_i < t(\bar{X}_{-i})$, for $i \in \bar{M}$, then firm i is said to be strictly constrained.

Let $D(k^{m+1})$ be the set of strictly constrained firms and $L(k^{m+1})$ the set of constrained firms in the Cournot-Nash equilibrium of the k^{m+1} subgame.

By Lemma 2 we can write $\bar{x}_i$, for all $i \in \bar{M}$, as a function of k^{m+1}, $\bar{x}_i(k^{m+1})$. We derive several properties of the function $\bar{x}_i(\cdot)$ and several properties of the Cournot-Nash equilibria.

Lemma 3. (a) $\bar{x}_i(\cdot)$ is continuous in k^{m+1} and so is $\bar{X}(\cdot) = \sum_{i \varepsilon \bar{M}} \bar{x}_i(\cdot)$.

(b) If firms i and j have the same capacities in the k^{m+1} subgame, then their equilibrium outputs are the same.

(c) If the capacity of firm i is smaller than that of firm j and firm j is constrained in equilibrium, then firm i is strictly constrained in equilibrium.

(d) If firms i and j are not strictly constrained in equilibrium, then their outputs are the same.

(e) If in equilibrium firm i is strictly constrained but firm j is not, then the output of firm i is smaller than that of firm j.

(f) If the capacity of firm i is equal to or smaller than t $(\sum_{j \in \overline{M}_{-i}} k_j)$, then it is constrained in equilibrium.

The proof is trivial and is omitted.

For any k^{m+1}, let $(k_{-i}^{m+1}, \tilde{k}_i) = (k_1, \cdots, k_{i-1}, \tilde{k}_i, k_{i+1}, \cdots, k_{m+1})$: namely, $(k_{-i}^{m+1}, \tilde{k}_i)$ is the capacity vector k^{m+1} with $\tilde{k}_i$ substituted in place of k_i. This notation is utilized in the proof of part (e) in the following lemma.

<u>Lemma 4</u>. Consider the k^{m+1} and the $\tilde{k}^{m+1}$ subgames where for some i, $k_i > \tilde{k}_i$ and $k_j = \tilde{k}_j$ for all $j \in \overline{M}_{-i}$.

(a) If firm i is constrained in the k^{m+1} subgame, then it is strictly constrained in the $\tilde{k}^{m+1}$ subgame.

(b) If firm i is not strictly constrained in the $\tilde{k}^{m+1}$ subgame, then each firm produces the same quantity in the two subgames, $\bar{x}_j(k^{m+1}) = \bar{x}_j(\tilde{k}^{m+1})$ for all $j \in \overline{M}$, and thus the total outputs in the two subgames are the same.

(c) If a firm is strictly constrained [constrained] in the k^{m+1} subgame, then it is so in the $\tilde{k}^{m+1}$ subgame, i.e. $D(k^{m+1}) \subseteq D(\tilde{k}^{m+1})$ and $L(k^{m+1}) \subseteq L(\tilde{k}^{m+1})$.

(d) If firm i is strictly constrained in the $\tilde{k}^{m+1}$ subgame and $D(\tilde{k}^{m+1}) = L(k^{m+1}) = L$, then $\bar{x}_j(k^{m+1}) < \bar{x}_j(\tilde{k}^{m+1})$ for all $j \in \overline{M} \setminus L$ and the total output in the k^{m+1} subgame is larger than that in the $\tilde{k}^{m+1}$ subgame.

(e) If firm i is strictly constrained in the $\tilde{k}^{m+1}$ subgame, then the total output in the k^{m+1} subgame is larger than that in the $\tilde{k}^{m+1}$ subgame.

The proof of parts (a) and (b) is trivial and is omitted. We present the proof of parts (c), (d), and (e) in Appendix A.

Proofs of the following three lemmas are also provided in Appendix A.

<u>Lemma 5</u>. Consider the k^{m+1} and the $\tilde{k}^{m+1}$ subgames and let $k^m > \tilde{k}^m$, $k_{m+1} = r(\sum_{i \varepsilon M} k_i)$, and $\tilde{k}_{m+1} = r(\sum_{i \varepsilon M} \tilde{k}_i)$.[4] If all firms are constrained in the k^{m+1} subgame, then so are they in the $\tilde{k}^{m+1}$ subgame.

<u>Lemma 6</u>. Consider the k^{m+1} and the $\tilde{k}^{m+1}$ subgames where $k_i \geq \tilde{k}_i$ for some $i \in \overline{M}$. If firm i is constrained in the k^{m+1} subgame but nonconstrained in the $\tilde{k}^{m+1}$ subgame, then the total output in the k^{m+1} subgame is smaller than that in the $\tilde{k}^{m+1}$ subgame.

<u>Lemma 7</u>. Consider the k^{m+1} and the $\tilde{k}^{m+1}$ subgames and let $k^m > \tilde{k}^m$ and $\overline{X}(k^{m+1}) = \overline{X}(\tilde{k}^{m+1})$. If all firms are constrained in the k^{m+1} subgame and firm m+1 is constrained in the $\tilde{k}^{m+1}$ subgame, then all firms are constrained in the $\tilde{k}^{m+1}$ subgame.

We move now to study the first type of subgames. In such subgames, firm m+1 first chooses a capacity, given the capacity vector of incumbent firms, and then all the firms engage in capacity-constrained Cournot competition. When it chooses its capacity, firm m+1 exercises perfect foresight with respect to the Cournot-Nash equilibrium in the next stage. In the following we state several definitions and discuss some properties of the k^m subgame.

Definition 5. Let $E(k^m)$ be the maximum profit (or minimum loss) that firm m+1 can make if it enters, given incumbent firms' capacity vector k^m:

$$E(k^m) = \max_{k_{m+1}>0} \Pi(k^m, k_{m+1})$$

where $\Pi(k^m, k_{m+1}) = \bar{x}_{m+1}(k^{m+1})\, f(\, \bar{X}(k^{m+1})\,) - C(\, \bar{x}_{m+1}(k^{m+1}), k_{m+1}\,)$.

$E(k^m)$ is continuous and nonincreasing in k_i with $i \in M$. Once firm i, $i \in M$, is not strictly constrained, Lemma 4 (b) implies that $E(k^m)$ is constant as k_i increases.

Definition 6. Let $A(k^m) = \{k_{m+1} |\ \text{argmax}_{k_{m+1}>0} \Pi(k^m, k_{m+1})\}$ if $\text{argmax}_{k_{m+1}>0} \Pi(k^m, k_{m+1})$ exists and $A(k^m) = \{\, 0 \,\}$ otherwise.

That is, a positive element in $A(k^m)$ is the capacity of firm m+1 which yields the maximum profit to the firm among positive capacities, given k^m. Notice that $A(k^m) = \{\, 0 \,\}$ implies that $r(\, \sum_{i \in M} k_i\,) = 0$ but the converse is not true. Trivially if $E(k^m) > 0$, then $A(k^m)$ contains a positive capacity (or capacities).

Definition 7. $(\bar{k}_{m+1}, \bar{x}^{m+1})$ is a subgame perfect Nash equilibrium of the k^m subgame if $\bar{k}_{m+1} = 0$ when $E(k^m) \leq 0$, $\bar{k}_{m+1} \in A(k^m)$ when $E(k^m) > 0$, and $\bar{x}_i = \bar{x}_i(k^m, \bar{k}_{m+1})$ for all $i \in \overline{M}$.

Notice that there may exist more than one equilibria in the k^m subgame.

However, when firm m+1 stays out, the equilibrium is unique (see Lemma 2). Notice also that in the k^m subgame, $\bar{k}_{m+1}$ may not be an element of $A(k^m)$. It happens when $E(k^m) \leq 0$ but $A(k^m)$ contains a positive capacity (or capacities).

Lemma 8. In the k^m subgame, if firm m+1 chooses a capacity in $A(k^m)$, then it is constrained.

We provide the proof in Appendix A. Lemma 9 is immediate from Lemma 8. Appendix A also presents the proof of Lemma 10.

Lemma 9. In all equilibria of the k^m subgame, firm m+1 is constrained.

Lemma 10. Consider the k^m and the $\tilde{k}^m$ subgames with $k^m > \tilde{k}^m$. Let $k_{m+1} = r(\ \sum_{i\varepsilon M} k_i\ )$ and $\tilde{k}_{m+1} = r(\ \sum_{i\varepsilon M} \tilde{k}_i\ )$. If $k_{m+1} \in A(k^m)$ and $L(k^{m+1}) = \overline{M}$, then $A(\tilde{k}^m) = \{\ \tilde{k}_{m+1}\ \}$.

Definition 8. Let $Y = \min \{Z \in [0, \infty) \mid [\ \max_{x \geq 0} xf(Z+x) - (V+R)x - F\ ] = 0\}$.

That is, Y is the Bain-Sylos-Modigliani limit output. It is well defined if $F \leq \max_{x \geq 0} [\ xf(x) - (V+R)x\ ]$. Notice that Z is unique when F is positive.

Let $(Y/m)^m$ be the capacity vector of the incumbent firms where all capacities are the same and equal to Y/m. Denote by k^m_∞ the capacity vector of the incumbent firms for which each incumbent firm has an infinite capacity. For example, if all incumbent firms are nonconstrained in the $k^{m+1} = (k^m, 0)$ subgame, then the capacity vector k^m can be represented by k^m_∞.

Lemma 11. Consider the case where $E(k_\infty^m) \leq 0$. In the equilibrium of the $(Y/m)^m$ subgame, firm m+1 stays out and all firms are constrained.

We present the proof in Appendix A.

Lemma 12. Consider the k^m and the $\tilde{k}^m$ subgames where $Y = \sum_{i\varepsilon M} k_i$ and $k^m > \tilde{k}^m$. If firm m+1 stays out in the equilibrium of the k^m subgame, then it enters with the capacity of $r(\sum_{i\varepsilon M} \tilde{k}_i)$ and all firms are constrained in the equilibrium of the $\tilde{k}^m$ subgame.

Appendix A provides the proof.

Lemma 13. Consider the case where $E(k_\infty^m) \leq 0$. If $k^m < (Y/m)^m$, then in the equilibrium of the k^m subgame, firm m+1 enters with the capacity of $r(\sum_{i\varepsilon M} k_i)$ and all firms are constrained.

Proof. It is established by Lemma 11 and Lemma 12.

4. SUBGAME PERFECT EQUILIBRIA OF THE FULL GAME

In this section, we first show that an additional assumption is required for the existence of a subgame perfect equilibrium of the full game. The assumption is that firm m+1 chooses the smallest capacity whenever it is indifferent between capacities. Second, we establish that given a capacity vector of the other incumbent firms, an incumbent firm never chooses a capacity under which it will be nonconstrained in the following subgame. This result, combined with Lemma 9, implies that in all subgame perfect equilibria all firms are constrained. It also implies that incumbent firms cannot prevent entry by holding excess capacity. Third, we show that whenever there is an entry-preventing equilibrium, there always exists a unique symmetric entry-preventing equilibrium in which all incumbent firms choose the same capacities. This property simplifies the task of checking whether there is an entry-preventing equilibrium in specific models. It also simplifies the proof of the existence of an equilibrium. Fourth, in a series of propositions we prove the existence of an equilibrium by constructing symmetric equilibria. Finally, we illustrate in Example 2 that in our model there may exist a continuum of entry-allowing equilibria and the total output may be different across these equilibria. In the example, we also show that the profit of an entrant can be larger than that of an incumbent firm.

Example 1. This example illustrates that an equilibrium may not exist. Suppose that there are just one incumbent firm, called firm 1, and a potential entrant, called firm 2. The inverse demand function is linear: $p = 2-X$ for $0 \leq X \leq 2$ and $p = 0$ for $X > 2$. Let $V = .8$, $R = .2$, and $F = 0$.[5]

In Figure 2.1, the line ABC is firm 2's Cournot reaction function, $x_2 = r(x_1) = .5 - .5x_1$, computed with a constant marginal cost of $V+R = 1$. Firm 2's profit decreases as we move from left to right along ABC. The lines, $x_1 = t(x_2) = .6 - .5x_2$ and $x_2 = t(x_1) = .6 - .5x_1$, are firm 1's and firm 2's Cournot reaction functions, respectively, with a constant marginal cost of $V = .8$. Point U is their intersection: $(x_1^U, x_2^U) = (.4, .4)$. Firm 2's isoprofit curve passing through point B, computed with a marginal cost equal to 1, is tangent to the line $x_1 = t(x_2)$ at point U. Firm 2's profit on that isoprofit curve is .08.

Now firm 1 chooses k_1 in the first stage and firm 2 chooses k_2 in the second stage, both exercising perfect foresight about the actions in future stages. In the third stage both firms engage in capacity-constrained Cournot competition, given the capacities.

We first focus on firm 2's subgame perfect reaction correspondence. Indeed, for $k_1 \geq .4$, if firm 2 chooses a capacity $k_2 = .4$, then the equilibrium of the $(k_1, .4)$ subgame will be (x_1^U, x_2^U) and its profit will be .08. But for k_1 in the interval $[.4, \tilde{k}_1)$, with $\tilde{k}_1 = (5-2\sqrt{2})/5) \simeq .4343$, it can make more profit by choosing $k_2 = r(k_1)$. Then, the equilibrium of the (k_1, k_2) subgame is $\bar{x}_1(k_1, k_2) = k_1$, $\bar{x}_1(k_1, k_2) = k_2$. More generally, for any $k_1 < \tilde{k}_1$, firm 2 will choose a capacity $r(k_1)$. These best responses of firm 2 are represented by the segment [A, B) in Figure 2.1.

For $k_1 > \tilde{k}_1$, on the other hand, firm 2 is better off by choosing $k_2 = .4$ and its best responses are given by the half line (G, ∞). For $k_1 = \tilde{k}_1$, firm 2 has two best responses, $r(\tilde{k}_1) = \sqrt{2}/5$ and .4. As a consequence, firm 2's subgame perfect correspondence comprises the union of [A, B] and $[G, \infty)$.

We can now study firm 1's choice of a capacity. Clearly, it will not choose a capacity greater than $\tilde{k}_1$. This is so since such a capacity will result in (.4, .4) as the Cournot-Nash equilibrium in the third stage and thus firm 1's profit with the capacity is smaller than that with, for example,

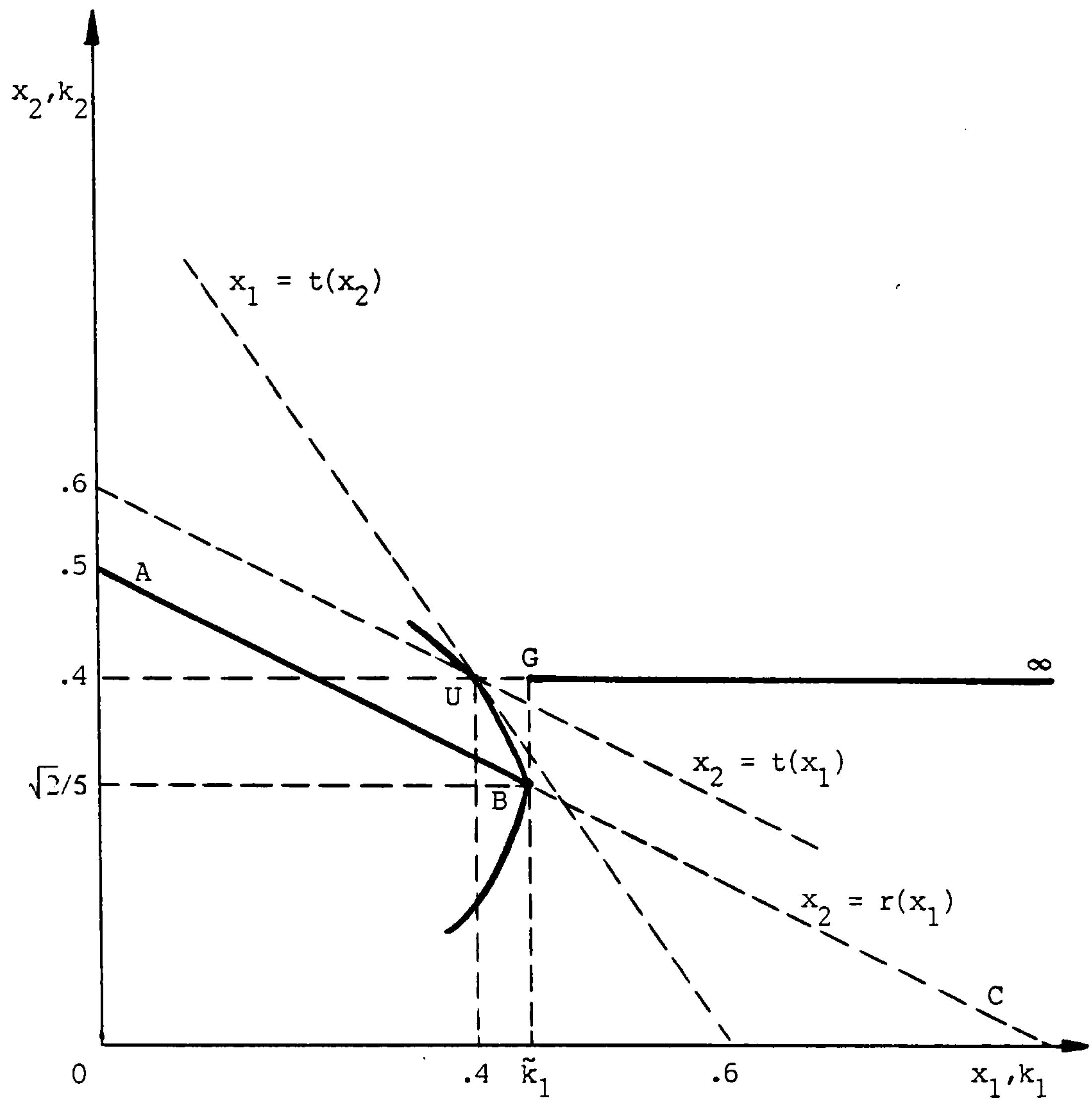

FIGURE 2.1

Firm 2's Subgame Perfect Reaction Correspondence and

Nonexistence of Equilibrium

$k_1 = .4$. It is easy to show that the profit of firm 1 is increasing in k_1 on the interval $[0, \tilde{k}_1)$: it increases as we move down from point A to point B along the segment [A, B) in Figure 2.1. Indeed, it reaches a maximum when firm 1 chooses $\tilde{k}_1$ and firm 2 chooses $r(\tilde{k}_1) = \sqrt{2}/5$, which corresponds to point B. Thus if firm 1 expects firm 2 to choose a capacity of $r(\tilde{k}_1)$ when confronted with $\tilde{k}_1$, then it will choose $\tilde{k}_1$. However, if firm 1 expects firm 2 to choose .4 instead of $r(\tilde{k}_1)$ as a response to $\tilde{k}_1$, then it cannot find its best capacity and we would not have an equilibrium. To avoid this problem, we introduce the following assumption.

Assumption 1. In the k^m subgame, if $E(k^m) > 0$, then firm m+1 chooses the smallest of its best response capacities, min $A(k^m)$.

By Definition 7 and Assumption 1, we can write $\bar{k}_{m+1}$ as a function of k^m, $\bar{k}_{m+1}(k^m)$, in the k^m subgame.

Definition 9. Let $\Pi_i(k^{m+1}) = \bar{x}_i(k^{m+1})\, f(\bar{X}(k^{m+1})) - C_i(\bar{x}_i(k^{m+1}), k_i)$, for $i \in M$. The capacity-output combination $(\bar{k}^{m+1}, \bar{x}^{m+1})$ is a subgame perfect Nash equilibrium of the full game if

$$\Pi_i(\bar{k}^m, \bar{k}_{m+1}(\bar{k}^m)) \geq \Pi_i((\bar{k}^m_{-i}, k_i), \bar{k}_{m+1}(\bar{k}^m_{-i}, k_i)) \text{ for all } k_i \geq 0 \text{ and for all } i \in M,$$

$$\bar{k}_{m+1} = \bar{k}_{m+1}(\bar{k}^m), \text{ and}$$

$$\bar{x}_i = \bar{x}_i(\bar{k}^m, \bar{k}_{m+1}(\bar{k}^m)) \text{ for all } i \in \bar{M}.$$

We show that in all subgame perfect equilibria all firms are constrained. Thus in this model, incumbent firms cannot deter entry by holding capacities that will be unused after entry. For future reference, we first establish the following.

Lemma 14. Given a capacity combination of the other incumbent firms k^m_{-i}, firm i with $i \in M$ never chooses a capacity k_i such that it is nonconstrained in the equilibrium of k^m subgame.

Proof. Suppose that given k^m_{-i}, firm i chooses a capacity, k_i, such that it is nonconstrained in the k^m subgame ($i \notin L(k^m, \bar{k}_{m+1}(k^m))$). We show that the capacity is not its best response to the capacity vector of the other incumbent firms: there exists $\tilde{k}_i > 0$ such that $\Pi_i((k^m_{-i}, \tilde{k}_i), \bar{k}_{m+1}(k^m_{-i}, \tilde{k}_i)) > \Pi_i(k^m, \bar{k}_{m+1}(k^m))$. Indeed, let $\tilde{k}_i = \bar{x}_i(k^m, \bar{k}_{m+1}(k^m))$. Then $\tilde{k}_i < k_i$ since $i \notin L(k^m, \bar{k}_{m+1}(k^m))$.

We first show that $\bar{k}_{m+1}(k^m_{-i}, \tilde{k}_i) \leq \bar{k}_{m+1}(k^m)$. Suppose on the contrary that $\bar{k}_{m+1}(k^m_{-i}, \tilde{k}_i) > \bar{k}_{m+1}(k^m)$. Since $i \notin D((k^m_{-i}, \tilde{k}_i)), \bar{k}_{m+1}(k^m))$, we have $\bar{x}_j((k^m_{-i}, \tilde{k}_i), \bar{k}_{m+1}(k^m)) = \bar{x}_j(k^m, \bar{k}_{m+1}(k^m))$ for all $j \in \overline{M}$ (see Lemma 4 (b)) and thus $\Pi((k^m_{-i}, \tilde{k}_i), \bar{k}_{m+1}(k^m)) = \Pi(k^m, \bar{k}_{m+1}(k^m))$. We also have $\Pi((k^m_{-i}, \tilde{k}_i), \bar{k}_{m+1}(k^m_{-i}, \tilde{k}_i)) > \Pi((k^m_{-i}, \tilde{k}_i), \bar{k}_{m+1}(k^m))$. It follows from the facts that $\bar{k}_{m+1}(k^m_{-i}, \tilde{k}_i)$ is the mininum of best responses of firm m+1 to $(k^m_{-i}, \tilde{k}_i)$ and $\bar{k}_{m+1}(k^m_{-i}, \tilde{k}_i) > \bar{k}_{m+1}(k^m)$. Next, $i \notin D((k^m_{-i}, \tilde{k}_i), \bar{k}_{m+1}(k^m))$ and $\bar{k}_{m+1}(k^m_{-i}, \tilde{k}_i) > \bar{k}_{m+1}(k^m)$ result in $i \notin D((k^m_{-i}, \tilde{k}_i), \bar{k}_{m+1}(k^m_{-i}, \tilde{k}_i))$ (see Lemma 4 (c)), which implies with $\tilde{k}_i < k_i$ that $\bar{x}_j((k^m_{-i}, \tilde{k}_i), \bar{k}_{m+1}(k^m_{-i}, \tilde{k}_i)) = \bar{x}_j(k^m, \bar{k}_{m+1}(k^m_{-i}, \tilde{k}_i))$ for all $j \in \overline{M}$ (see Lemma 4 (b)) and thus $\Pi((k^m_{-i}, \tilde{k}_i), \bar{k}_{m+1}(k^m_{-i}, \tilde{k}_i)) = \Pi(k^m, \bar{k}_{m+1}(k^m_{-i}, \tilde{k}_i))$.

Hence $\Pi(k^m, \bar{k}_{m+1}(k^m_{-i}, \tilde{k}_i)) > \Pi(k^m, \bar{k}_{m+1}(k^m))$, which contradicts that $\bar{k}_{m+1}(k^m)$ is a best response of firm m+1 to k^m.

Second, $\bar{X}((k^m_{-i}, \tilde{k}_i), \bar{k}_{m+1}(k^m_{-i}, \tilde{k}_i)) \leq \bar{X}(k^m, \bar{k}_{m+1}(k^m))$ since $\tilde{k}_i < k_i$ and $\bar{k}_{m+1}(k^m_{-i}, \tilde{k}_i) \leq \bar{k}_{m+1}(k^m)$ (see Lemma 4 (e)).

Third, $i \in L((k^m_{-i}, \tilde{k}_i), \bar{k}_{m+1}(k^m_{-i}, \tilde{k}_i))$ since $i \in L((k^m_{-i}, \tilde{k}_i), \bar{k}_{m+1}(k^m))$ and $\bar{k}_{m+1}(k^m_{-i}, \tilde{k}_i) \leq \bar{k}_{m+1}(k^m)$ (see Lemma 4 (c)).

Thus, $\bar{x}_i(k^m, \bar{k}_{m+1}(k^m)) = \bar{x}_i((k^m_{-i}, \tilde{k}_i), \bar{k}_{m+1}(k^m_{-i}, \tilde{k}_i))$.

The last two results imply that

$\Pi_i((k^m_{-i}, \tilde{k}_i), \bar{k}_{m+1}(k^m_{-i}, \tilde{k}_i)) > \Pi_i(k^m, \bar{k}_{m+1}(k^m))$. Q.E.D.

Proposition 1. In each subgame perfect equilibrium, all firms are constrained.

Proof. The proposition is established by Lemma 9 and Lemma 14.

Consider a market situation where m incumbent firms, each with a constant marginal cost of V+R, engage in Cournot competition. Since $f'(X)$ exists and is negative on the interval $[0, \hat{X})$, the Cournot equilibrium is unique (see Szidarovszky and Yakowitz (1982)). By symmetry, the Cournot equilibrium output of firm i, $i \in M$, is the same as that of firm j, $j \in M$. Let x^c be the Cournot output of firm i, $i \in M$, and $X^c = mx^c$.

We show in the following proposition that whenever there is an entry-preventing equilibrium, there always exists a unique symmetric equilibrium. It is obvious that in the case where $E(k^m_\infty) > 0$, there is no entry-preventing equilibrium.

Proposition 2. Consider the case where $E(k_\infty^m) \leq 0$. (a) If $Y \leq X^c$, then $((k^c)^m, 0)$ is a unique symmetric entry-preventing equilibrium (capacity vector) where $(k^c)^m = k^m$ with $k_i = x^c$ for all $i \in M$.[6] (b) If $Y > X^c$ and there is an entry-preventing equilibrium, then $((Y/m)^m, 0)$ is the unique symmetric entry-preventing equilibrium.

Proof. Proof of part (a). We show that $((k^c)^m, 0)$ is an equilibrium. First, $\bar{k}_{m+1}((k^c)^m) = 0$ since $E(k_\infty^m) \leq 0$ and $Y \leq X^c$ imply that $E((k^c)^m \leq 0$. Second, it is easy to show that for $i \in M$, $\Pi_i((k^c)^m, 0) \geq \Pi_i(((k^c)_{-i}^m, k_i), 0) \geq \Pi_i(((k^c)_{-i}^m, k_i), \bar{k}_{m+1}((k^c)_{-i}^m, k_i))$ for all $k_i \geq 0$. It is easy to show that there is no other symmetric entry-preventing equilibrium.

Proof of part (b). Let $(k^m, 0)$ be an equilibrium and $k_i = \max(k_1, \cdots, k_m)$. Then $L(k^m, 0) = \bar{M}$ by Proposition 1 and $\sum_{j \varepsilon M} k_j = Y$ since $Y > X^c$. Suppose that $((Y/m)^m, 0)$ is not an equilibrium. Since $\bar{k}_{m+1}((Y/m)^m) = 0$ (see Lemma 11), it implies that there exists $\tilde{k}_i$ such that $\Pi_i(((Y/m)_{-i}^m, \tilde{k}_i), \bar{k}_{m+1}((Y/m)_{-i}^m, \tilde{k}_i)) > \Pi_i((Y/m)^m, 0)$. Trivially, $Y/m > \tilde{k}_i$ since $Y > X^c$. By Lemma 13, $\bar{k}_{m+1}((Y/m)_{-i}^m, \tilde{k}_i) = r((m-1)Y/m + \tilde{k}_i)$ and $L(((Y/m)_{-i}^m, \tilde{k}_i), \bar{k}_{m+1}((Y/m)_{-i}^m, \tilde{k}_i)) = \bar{M}$. Next, let $\hat{k}_i$ be such that $\sum_{j \varepsilon M \backslash \{i\}} k_j + \hat{k}_i = (m-1)Y/m + \tilde{k}_i$. Then $\hat{k}_i < k_i$ since $\tilde{k}_i < Y/m$. Since $\hat{k}_i < k_i$, $\sum_{j \varepsilon M} k_j = Y$, and $\bar{k}_{m+1}(k^m) = 0$, by Lemma 12 $\bar{k}_{m+1}(k_{-i}^m, \hat{k}_i) = r(\sum_{j \varepsilon M \backslash \{i\}} k_j + \hat{k}_i)$ and $L((k_{-i}^m, \hat{k}_i), \bar{k}_{m+1}(k_{-i}^m, \hat{k}_i)) = \bar{M}$. Thus we have the following: $\bar{X}(((Y/m)_{-i}^m, \tilde{k}_i), \bar{k}_{m+1}((Y/m)_{-i}^m, \tilde{k}_i)) = \bar{X}((k_{-i}^m, \hat{k}_i), \bar{k}_{m+1}(k_{-i}^m, \hat{k}_i)) = \bar{X}$, since all firms are constrained in both cases, $\sum_{j \varepsilon M \backslash \{i\}} k_j + \hat{k}_i = (m-1)Y/m + \tilde{k}_i$, and $\bar{k}_{m+1}((Y/m)_{-i}^m, \tilde{k}_i) = \bar{k}_{m+1}(k_{-i}^m, \hat{k}_i)$. Notice that $k_i - Y/m = \hat{k}_i - \tilde{k}_i > 0$.

We are now prepared to show that $\Pi_i((k^m_{-i}, \hat{k}_i), \bar{k}_{m+1}(k^m_{-i}, \hat{k}_i)) > \Pi_i(k^m, 0)$, which contradicts that $(k^m, 0)$ is an equilibrium. The profit of firm i in the equilibrium of the $(k^m_{-i}, \hat{k}_i)$ subgame is $\Pi_i((k^m_{-i}, \hat{k}_i), \bar{k}_{m+1}(k^m_{-i}, \hat{k}_i))$ $= \hat{k}_i(f(\bar{X})-V-R) = \tilde{k}_i(f(\bar{X})-V-R) + (\hat{k}_i - \tilde{k}_i)(f(\bar{X})-V-R)$ $= \Pi_i(((Y/m)^m_{-i}, \tilde{k}_i), \bar{k}_{m+1}((Y/m)^m_{-i}, \tilde{k}_i)) + (\hat{k}_i - \tilde{k}_i)(f(\bar{X})-V-R)$ and $\Pi_i(k^m, 0) = k_i(f(Y)-V-R) = Y/m(f(Y)-V-R) + (k_i - Y/m)(f(Y)-V-R) = \Pi_i((Y/m)^m, 0) + (k_i - Y/m)(f(Y)-V-R)$. Since we have been assumed that $\Pi_i(((Y/m)^m_{-i}, \tilde{k}_i), \bar{k}_{m+1}((Y/m)^m_{-i}, \tilde{k}_i)) > \Pi_i((Y/m)^m, 0)$ which also implies that $f(Y) < f(\bar{X})$ with the facts that $L((Y/m)^m, 0) = \bar{M}$ (see Lemma 11), $L(((Y/m)^m_{-i}, \tilde{k}_i), \bar{k}_{m+1}((Y/m)^m_{-i}, \tilde{k}_i)) = \bar{M}$, and $Y/m > \tilde{k}_i$, thus $\Pi_i((k^m_{-i}, \hat{k}_i), \bar{k}_{m+1}(k^m_{-i}, \hat{k}_i)) > \Pi_i(k^m, 0)$. It is easy to show that there is no other symmetric entry-preventing equilibrium. Q.E.D.

Consider now a two-stage game where in the first stage m incumbent firms decide independently how much to produce and in the second stage firm m+1 chooses an output, taking the incumbents' outputs as given. Let $F = 0$ and let $V+R$ be the constant marginal cost of each firm. Notice that firm m+1 produces $r(Q)$ where Q is the total output of the incumbent firms. Thus $s(Z)$, the best response of an incumbent firm when the other incumbent firms produce a total output of Z, is defined as follows.[7]

Definition 10. Let $s(Z) = \operatorname{argmax}_{x \geq 0} \; xf(Z+x+r(Z+x)) - (V+R)x$ for $Z \in [0, \infty)$.

It is easy to show that given our assumptions, $s(\cdot)$ is continuously differentiable and $-1 < s' < 0$ for Z such that $s(Z) > 0$. Let $g(Q) = f(Q + r(Q))$.

Since $g'(Q)$ exists and is negative on $[0, \hat{X})$, the equilibrium of the two-stage game is unique (see Szidarovszky and Yakowitz (1982)). By symmetry, the equilibrium output of firm i, $i \in M$, is the same as that of firm j, $j \in M$. Let x^s be the equilibrium output of firm i, $i \in M$. Then $r(mx^s)$ is the equilibrium output of firm m+1.

s(Z) can be interpreted, in our three-stage game, as the best response (capacity) of firm i, $i \in M$, to a capacity vector of the other incumbent firms, $(k_j)_{j \in M\setminus\{i\}}$ with $Z = \sum_{j \in M\setminus\{i\}} k_j$, under the conditions that $\bar{k}_{m+1}(k^m) = r(Z+k_i)$ and $L(k^m, \bar{k}_{m+1}(k^m)) = \bar{M}$ where $k_i = s(Z)$.

Proposition 3. Consider the case where $E(k^m_\infty) \leq 0$. If $Y > X^c$, then either $((Y/m)^m, 0)$ or $((k^s)^m, k^s_{m+1})$ is an equilibrium where $(k^s)^m = k^m$ with $k_i = x^s$ for all $i \in M$ and $k^s_{m+1} = r(mx^s)$.

Proof. It is obvious that when $Y \leq mx^s$, $((Y/m)^m, 0)$ is an equilibrium. Consider the case where $Y > mx^s$. Suppose that neither is an equilibrium. Since $((Y/m)^m, 0)$ is not an equilibrium and $\bar{k}_{m+1}((Y/m)^m) = 0$ (see Lemma 11), for some $i \in M$ there exists $\tilde{k}_i$ such that $\Pi_i(((Y/m)^m_{-i}, \tilde{k}_i), \bar{k}_{m+1}((Y/m)^m_{-i}, \tilde{k}_i)) > \Pi_i((Y/m)^m, 0)$. We have $Y/m > \tilde{k}_i$ since $Y > X^c$. By Lemma 13, $\bar{k}_{m+1}((Y/m)^m_{-i}, \tilde{k}_i) = r((m-1)Y/m + \tilde{k}_i)$ and $L(((Y/m)^m_{-i}, \tilde{k}_i), \bar{k}_{m+1}((Y/m)^m_{-i}, \tilde{k}_i)) = \bar{M}$. Since $\Pi_i(((Y/m)^m_{-i}, \tilde{k}_i), \bar{k}_{m+1}((Y/m)^m_{-i}, \tilde{k}_i)) > \Pi_i((Y/m)^m, 0)$ and $Y/m > \tilde{k}_i$, $\bar{X} = \bar{X}(((Y/m)^m_{-i}, \tilde{k}_i), \bar{k}_{m+1}((Y/m)^m_{-i}, \tilde{k}_i)) < Y$. We show that $L((k^s)^m, k^s_{m+1}) = \bar{M}$. It is trivial that $m+1 \in L((k^s)^m, k^s_{m+1})$. Since $mk^s < Y$ and thus $\Pi((k^s)^m, k^s_{m+1}) > 0$, if $j \notin L((k^s)^m, k^s_{m+1})$ for some $j \in M$ and thus all incumbent firms are nonconstrained (see Lemma 3 (b)), then $E(K^m_\infty) \geq \Pi((k^s)^m, k^s_{m+1}) > 0$ (see Lemma which contradicts that $E(k_\infty^m) \leq 0$. Now that $((k^s)^m, k_{m+1}^s)$ is not an equilibrium and $L((k^s)^m, k_{m+1}^s) = \overline{M}$, we should have the following: $L(((k^s)_{-i}^m, k_i), \bar{k}_{m+1}((k^s)_{-i}^m, k_i)) = \overline{M}$ and $\Pi_i(((k^s)_{-i}^m, k_i), 0) > \Pi_i((k^s)^m, k_{m+1}^s)$ where $k_i = Y-(m-1)k^s$.

Choose $\hat{k}_i$ such that $(m-1)k^s + \hat{k}_i = (m-1)Y/m + \tilde{k}_i$. Then $\hat{k}_i > \tilde{k}_i$ since $k^s < Y/m$. Since $k_i > \hat{k}_i$, $(m-1)k^s + k_i = Y$, and $\bar{k}_{m+1}((k^s)_{-i}^m, k_i) = 0$, by Lemma 12 we have $\bar{k}_{m+1}((k^s)_{-i}^m, \hat{k}_i) = r((m-1)k^s + \hat{k}_i) = r((m-1)Y/m + \tilde{k}_i)$ and $L(((k^s)_{-i}^m, \hat{k}_i), \bar{k}_{m+1}((k^s)_{-i}^m, \hat{k}_i)) = \overline{M}$.
Notice that $\overline{X} = \overline{X}(((k^s)_{-i}^m, \hat{k}_i), \bar{k}_{m+1}((k^s)_{-i}^m, \hat{k}_i))$.

We are prepared to show that $\Pi_i(((k^s)_{-i}^m, \hat{k}_i), \bar{k}_{m+1}((k^s)_{-i}^m, \hat{k}_i)) >$ $\Pi_i(((k^s)_{-i}^m, k_i), 0)$. The profit of firm i in the equilibrium of the $((k^s)_{-i}^m, \hat{k}_i)$ subgame is $\Pi_i(((k^s)_{-i}^m, \hat{k}_i), \bar{k}_{m+1}((k^s)_{-i}^m, \hat{k}_i)) = \hat{k}_i(f(\overline{X})-V-R) = \tilde{k}_i(f(\overline{X})-V-R) + (\hat{k}_i-\tilde{k}_i)(f(\overline{X})-V-R) = \Pi_i(((Y/m)_{-i}^m, \tilde{k}_i), \bar{k}_{m+1}((Y/m)_{-i}^m, \tilde{k}_i)) + (\hat{k}_i-\tilde{k}_i)(f(\overline{X})-V-R)$ and $\Pi_i(((k^s)_{-i}^m, k_i), 0) = k_i(f(Y)-V-R) = Y/m(f(Y)-V-R) +$ $(\hat{k}_i-\tilde{k}_i)(f(Y)-V-R) = \Pi_i((Y/m)^m, 0) + (\hat{k}_i-\tilde{k}_i)(f(Y)-V-R)$ since $k_i = Y - (m-1)k^s = Y - ((m-1)Y/m + \tilde{k}_i - \hat{k}_i) = Y/m + (\hat{k}_i - \tilde{k}_i)$. Since $\Pi_i(((Y/m)_{-i}^m, \tilde{k}_i), \bar{k}_{m+1}((Y/m)_{-i}^m, \tilde{k}_i)) > \Pi_i((Y/m)^m, 0)$ and $f(\overline{X}) > f(Y)$, we have $\Pi_i(((k^s)_{-i}^m, k_i), 0) < \Pi_i(((k^s)_{-i}^m, \hat{k}_i), \bar{k}_{m+1}((k^s)_{-i}^m, \hat{k}_i)) \leq \Pi_i((k^s)^m, k_{m+1}^s)$, which is a contradiction. Q.E.D.

Now consider the case where $E(k_\infty^m) > 0$. Incumbent firms have no choice but to accommodate firm m+1. Let $Z^* \geq 0$ be such that $r(Z^*)f(Z^*+r(Z^*)) - (V+R)r(Z^*) - F = E(k_\infty^m)$. By Lemma 1 (d), it is unique. In our three-stage game, Z^* can be interpreted as the sum of incumbents' capacities, $\sum_{i \in M} k_i$, which yields the profit of $E(k_\infty^m)$ to firm m+1, if firm m+1 chooses $k_{m+1} = r(Z^*)$ and $L(k^{m+1}) = \overline{M}$. Let $k^E = Z^*/m$ and $k^E_{m+1} = r(Z^*)$.

Lemma 15. Let $(k^E)^m = k^m$ where $k_i = k^E$ for all $i \in M$. In the equilibrium of the $(k^E)^m$ subgame, firm m+1 chooses k^E_{m+1} and all firms are constrained.

Proof. It is easy to show that $k^E_{m+1} = \min A((k^E)^m)$. Then it follows from Assumption 1 that $\bar{k}_{m+1}((k^E)^m) = k^E_{m+1}$.

We show that in equilibrium all firms are constrained. We have $m+1 \in L((k^E)^m, k^E_{m+1})$ since $k^E_{m+1} = r(Z^*) < t(Z^*) \leq t(m\bar{x})$ where $\bar{x} = \bar{x}_i((k^E)^m, k^E_{m+1})$ for all $i \in M$ (see Lemma 3 (b)). Suppose that $i \notin L((K^E)^m, k^E_{m+1})$ for some $i \in M$. Then $j \notin L((k^E)^m, k^E_{m+1})$ for all $j \in M$ (see Lemma 3 (b)) and $m\bar{x} < mk^E = Z^*$. Hence $\Pi(k^m_\infty, k^E_{m+1}) = \Pi((k^E)^m, k^E_{m+1}) = k^E_{m+1}(f(m\bar{x}+k^E_{m+1}) - V - R) - F > k^E_{m+1}(f(Z^*+k^E_{m+1}) - V - R) - F = E(k^m_\infty)$, which is a contradiction. Notice that the first equality comes from Lemma 4 (b). Q.E.D.

Hence the profit of firm m+1 in the equilibrium of the $(k^E)^m$ subgame equals that in the equilibrium of the k^m_∞ subgame, i.e. $\Pi((k^E)^m, k^E_{m+1}) = E(k^m_\infty)$.

Proposition 4. Consider the case where $E(k^m_\infty) > 0$. (a) If $k^S < k^E$, then $((k^S)^m, k^S_{m+1})$ is a unique symmetric (entry-allowing) equilibrium. (b) If $k^S \geq k^E$, then under Assumption 1, $((k^E)^m, k^E_{m+1})$ is a unique symmetric (entry-allowing) equilibrium.

Proof. Proof of part (a). We have $L((k^S)^m, k^S_{m+1}) = \overline{M}$. It follows from the facts that $(k^S)^m < (k^E)^m$, $k^S_{m+1} = r(mk^S)$, $k^E_{m+1} = r(mk^E)$, and by Lemma 15

$L((k^E)^m, k^E_{m+1}) = \overline{M}$, and from Lemma 5.

We first show that $\bar{k}_{m+1}((k^s)^m) = k^s_{m+1}$. Obviously, $\Pi((k^s)^m, k^s_{m+1}) > \Pi((k^s)^m, k_{m+1})$ for $k_{m+1} \geq 0$ such that $k_{m+1} \neq k^s_{m+1}$ and $L((k^s)^m, k_{m+1}) = \overline{M}$. For k_{m+1} such that $\bar{x} < k^s$ where $\bar{x} = \bar{x}_i((k^s)^m, k_{m+1})$ for all $i \in M$ (see Lemma 3 (b)), $\Pi((k^s)^m, k_{m+1}) = \Pi(k^m_\infty, k_{m+1}) \leq E(k^m_\infty) = \Pi((k^E)^m, k^E_{m+1}) < \Pi((k^s)^m, k^s_{m+1})$. The first equality comes from Lemma 4 (b) and the last inequality follows from the fact that $L((k^E)^m, k^E_{m+1}) = L((K^s)^m, k^s_{m+1}) = \overline{M}$ and $k^s < k^E$, and from Lemma 1 (d).

Next, consider firm i, $i \in M$. For $k_i < k^s$, $\Pi_i(((k^s)^m_{-i}, k_i), \bar{k}_{m+1}((k^s)^m_{-i}, k_i)) < \Pi_i((k^s)^m, k^s_{m+1})$ since $\bar{k}_{m+1}((k^s)^m_{-i}, k_i) = r((m-1)k^s + k_i)$ by Lemma 11 and $L(((k^s)^m_{-i}, k_i), \bar{k}_{m+1}((k^s)^m_{-i}, k_i)) = \overline{M}$ by Lemma 5 (and the definition of k^s). By Lemma 14, firm i will not choose k_i such that $i \notin L(((k^s)^m_{-i}, k_i), \bar{k}_{m+1}((k^s)^m_{-i}, k_i))$. For $k_i > k^s$ such that $i \in L(((k^s)^m_{-i}, k_i), \bar{k}_{m+1}((k^s)^m_{-i}, k_i))$ and thus all firms are constrained (see Lemma 3 (c) and Lemma 8),
$\Pi_i((k^s)^m, k^s_{m+1}) > \Pi_i(((k^s)^m_{-i}, k_i), \bar{k}_{m+1}((k^s)^m_{-i}, k_i))$ by the definition of k^s.

It is easy to show that there is no other symmetric equilibrium.

Proof of part (b). By Lemma 15, $L((k^E)^m, k^E_{m+1}) = \overline{M}$ and $\bar{k}_{m+1}((k^E)^m) = k^E_{m+1}$. We must show that firm i, $i \in M$, has no incentive to choose a capacity other than k^E, given $(k^E)^m_{-i}$. Indeed, for $k_i < k^E$, since $\bar{k}_{m+1}((k^E)^m_{-i}, k_i) = r((m-1)k^E + k_i)$ by Lemma 11 and thus $L(((k^E)^m_{-i}, k_i), \bar{k}_{m+1}((k^E)^m_{-i}, k_i)) = \overline{M}$ by Lemma 5, we have $\Pi_i((k^E)^m, k^E_{m+1}) > \Pi_i(((k^E)^m_{-i}, k_i), \bar{k}_{m+1}((k^E)^m_{-i}, k_i))$ from the fact that $k^E \leq k^s$. Suppose that $k_i > k^E$. Then $\bar{k}_{m+1}((k^E)^m_{-i}, k_i) = \bar{k}_{m+1}(k^m_\infty)$ and $\bar{x} < k^E$ where $\bar{x} = \bar{x}_j(((k^E)^m_{-i}, k_i), \bar{k}_{m+1}((k^E)^m_{-i}, k_i))$ for all $j \in M$. Since $\overline{X}((k^E)^m, k^E_{m+1}) < \overline{X}((k^E)^m, \bar{k}_{m+1}(k^m_\infty)) = \overline{X}(((k^E)^m_{-i}, k_i), \bar{k}_{m+1}((k^E)^m_{-i}, k_i))$

(see Lemma 4 (e) and (b)), we have $\Pi_i((k^E)^m, k^E_{m+1}) >$ $\Pi_i(((k^E)^m_{-i}, k_i), \bar{k}_{m+1}((k^E)^m_{-i}, k_i))$. It is easy to show that there is no other symmetric equilibrium. Q.E.D.

Propositions 2, 3 and 4 established the existence of an equilibrium of the full game by constructing symmetric equilibria. As we will see in the sequel, if $E(k^m_\infty) \leq 0$ there might exist two disconnected symmetric equilibria where one is entry-allowing and the other is entry-preventing. It can also be shown that when $E(k^m_\infty) \leq 0$, there might exist a continuum of entry-preventing equilibria, each yielding the same total output. This result can also be found in Bernheim (1984) and Gilbert and Vives (1986). What is new in this essay is that in this strategic entry deterrence model there might exist a continuum of entry-allowing equilibria. It happens when $E(k^m_\infty) > 0$ and $k^E < k^s$. In the following example, there is a continuum of entry-allowing equilibria among which the total output of incumbent firms (and the total output of all firms) is not constant. Furthermore, we show in the example that in some equilibria the profit (market share) of an entrant is larger than that of an incumbent firm.

Example 2. There are two incumbent firms, called firm 1 and firm 2, and one potential entrant, firm 3. Hence $\overline{M} = \{1, 2, 3\}$. The inverse demand function is given as follows: $p = f(X) = 2 - X$ for $0 \leq X \leq 2$ and $p = 0$ for $X > 2$. Let $V = .9$, $R = .1$, and $F \in [0, 77/1600)$.

It is easy to show that given k^2_∞, $A(k^2_\infty) = \{.275\}$. Since $\bar{x}_i(k^2_\infty, k_3) = \bar{x}_i(k^2_\infty, .275) = .275$ for all $i \in \overline{M}$, $E(k^2_\infty) = \Pi(k^2_\infty, .275) = 77/1600 - F > 0$ and thus $\bar{k}_3(k^2_\infty) = .275$. $E(k^2_\infty) > 0$ implies that $E(k^2) > 0$ for every capacity vector k^2 and thus $\bar{k}_3(k^2) > 0$: that is, firm 3 always enters.

To get the subgame perfect reaction correspondence of firm 1, focus first on the maximum capacity, denoted by $\tilde{k}_1$, that firm 1 can choose, given k_2, satisfying that $L(\tilde{k}_1, k_2, \bar{k}_3(\tilde{k}_1, k_2)) = \overline{M}$. Notice that $L(\tilde{k}_1, k_2, \bar{k}_3(\tilde{k}_1, k_2)) = \overline{M}$ implies that $\bar{k}_3(\tilde{k}_1, k_2) = r(\tilde{k}_1+k_2)$. Hence $\tilde{k}_1$ is implicity computed as follows. For $k_2 \in [0, .275]$, $E(\infty, k_2) = \Pi(\tilde{k}_1, k_2, \bar{k}_3(\tilde{k}_1, k_2)) = r(\tilde{k}_1+k_2)[f(\tilde{k}_1+k_2+r(\tilde{k}_1+k_2)) - 1] - F$. Since $E(\infty, k_2) = (.8-k_2)(1.1-k_2)/9 - F$ and $\Pi(\tilde{k}_1, k_2, \bar{k}_3(\tilde{k}_1, k_2)) = (1-\tilde{k}_1-k_2)^2/4 - F$, then $(.8-k_2)(1.1-k_2)/9 = (1-\tilde{k}_1-k_2)^2/4$. The relation is represented by the locus AB in Figure 2.2. That is, AB shows $\tilde{k}_1$ given $k_2 \in [0, .275]$. For $k_2 \in [.275, (29-2\sqrt{77})/40]$, $E(\infty, k_2) = E(k_\infty^2) = 77/1600 - F = (1-\tilde{k}_1-k_2)^2/4 - F$. We have $77/1600 = (1-\tilde{k}_1-k_2)^2/4$, which is represented by the locus BC in Figure 2.2. For $k_2 \in [(29-2\sqrt{77})/40, (15-2\sqrt{22})/15]$, $E(\tilde{k}_1, \infty) = (1-\tilde{k}_1-k_2)^2/4 - F$. Since $E(\tilde{k}_1, \infty) = (.8-\tilde{k}_1)(1.1-\tilde{k}_1)/9$, it gives us $(.8-\tilde{k}_1)(1.1-\tilde{k}_1)/9 = (1-\tilde{k}_1-k_2)^2/4$. The relation is represented by the locus CDE in Figure 2.2. For $k_2 > (15-2\sqrt{22})/15$, $\tilde{k}_1$ does not exist, but for convenience of exposition let $\tilde{k}_1 = 0$.

Next, notice that for $k_2 > (29-2\sqrt{77})/40$, firm 1 is constrained in the third stage if it chooses a capacity between $\tilde{k}_1$ and .275. But firm 2 is nonconstrained in that case. $\Pi_1(.275, k_2, \bar{k}_3(k^2)) > \Pi_1(\hat{k}_1, k_2, \bar{k}_3(\hat{k}_1, k_2))$ for all $\hat{k}_1 \in (\tilde{k}_1, .275)$.

Now we are prepared to derive firm 1's reaction correspondence. First, for $k_2 \in [0, (15-2\sqrt{22})/15]$, $\Pi_1(\tilde{k}_1, k_2, \bar{k}_3(\tilde{k}_1, k_2)) > \Pi_1(k_1, k_2, \bar{k}_3(k_1, k_2))$ for all $k_1 \in [0, \tilde{k}_1)$ since $x_1 = s(x_2) = .5 - .5x_2 > \tilde{k}_1$, given $k_2 = x_2$. Notice also that by Lemma 14, firm 1 never chooses a capacity which results in excess capacity in the third stage. Thus, for $k_2 \in [0, (29-2\sqrt{77})/40]$, since $\tilde{k}_1$ is also

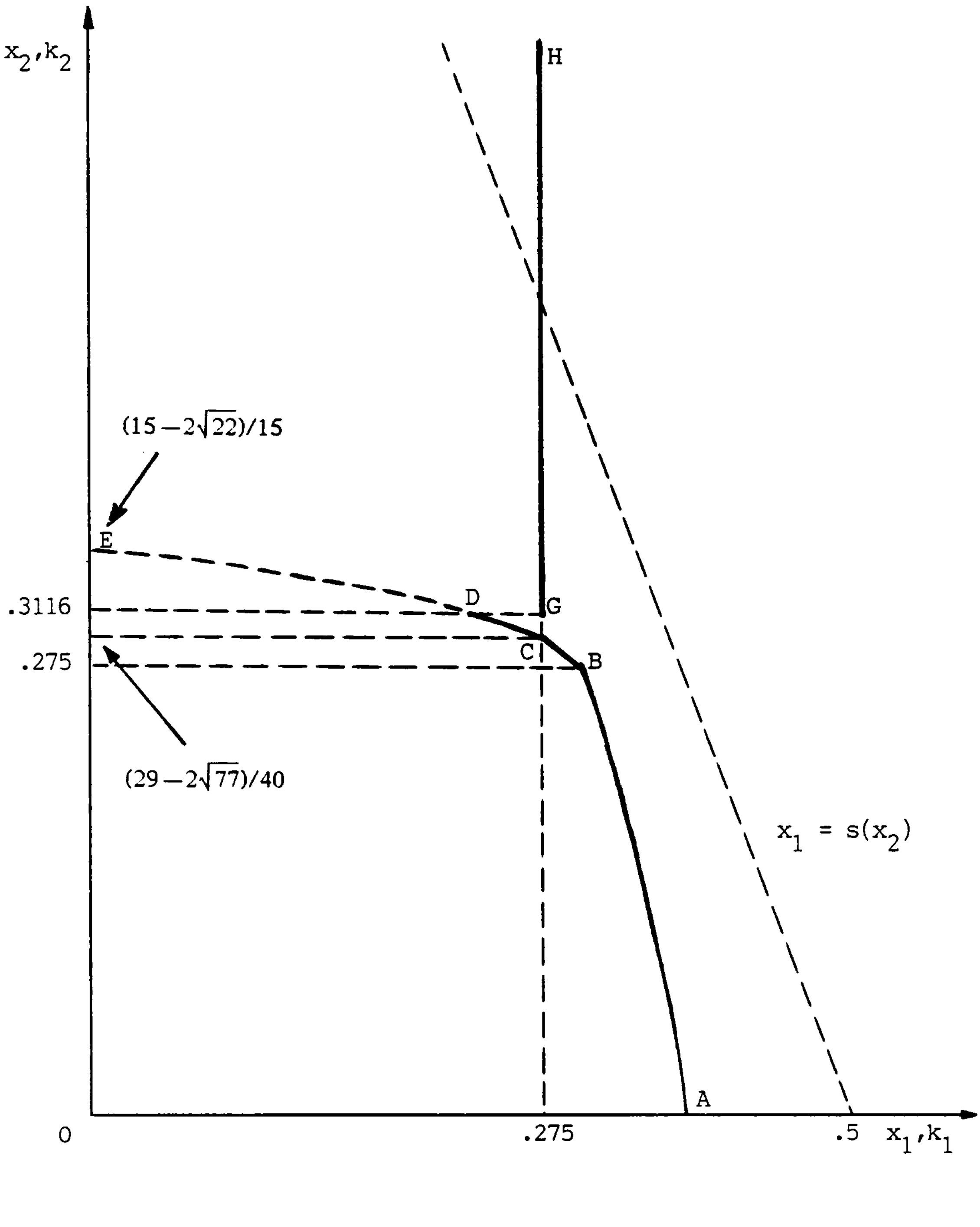

FIGURE 2.2

Firm 1's Subgame Perfect Reaction Correspondence the maximum capacity firm 1 can choose without resulting in excess capacity, $\tilde{k}_1$ is the best response of firm 1. For $k_2 > (29-2\sqrt{77})/40$, firm 1 compares $\Pi_1(\tilde{k}_1, k_2, \bar{k}_3(\tilde{k}_1, k_2))$ with $\Pi_1(.275, k_2, \bar{k}_3(k^2))$ and chooses that capacity which yields the highest profit. Firm 1's profit decreases as we move from right to left along CDE and $\Pi_1(\tilde{k}_1, k_2, \bar{k}_3(\tilde{k}_1, k_2)) = \Pi_1(.275, k_2, \bar{k}_3(k^2))$ when k_2 is equal to .3116 (point D in Figure 2.2). Hence, for $k_2 \in [(29-2\sqrt{77})/40, .3116]$, firm 1's best response is $\tilde{k}_1$ and for $k_2 \geq .3116$ it is .275. In Figure 2.2, ABCD and GH are firm 1's reaction correspondence.

The subgame perfect reaction correspondence of firm 2 is obtained in a similar way. Let $\tilde{k}_2$ be the maximum capacity firm 2 can choose, given k_1, satisfying that $L(k_1, \tilde{k}_2, \bar{k}_3(k_1, \tilde{k}_2)) = \overline{M}$. For $k_1 \in [0, .275]$, $(.8-k_1)(1.1-k_1)/9 = (1-k_1-\tilde{k}_2)^2/4$, for $k_1 \in [.275, (29-2\sqrt{77})/40]$, $77/1600 = (1-k_1-\tilde{k}_2)^2/4$, and for $k_1 \in [(29-2\sqrt{77})/40, (15-2\sqrt{22})/15]$, $(.8-\tilde{k}_2)(1.1-\tilde{k}_2)/9 = (1-k_1-\tilde{k}_2)^2/4$. For $k_1 > (15-2\sqrt{22})/15$, $\tilde{k}_2$ does not exist. Firm 2's best response is $\tilde{k}_2$ for $k_1 \in [0, .3116]$ and .275 for $k_1 \geq .3116$. Hence, the subgame perfect reaction correspondence of firm 2 and that of firm 1 are symmetric about the line $x_2 = x_1$.

Equilibrium capacity (and output) vectors of firm 1 and firm 2 are shown in Figure 2.3. Several remarks are in order. First, the total output of the incumbent firms is in general different across the equilibria. It is the same and maximal for equilibrium capacity (and output) vectors on BC in Figure 2.3. Since $\bar{k}_3(\bar{k}_1, \bar{k}_2) = r(\bar{k}_1+\bar{k}_2)$, the total output of all firms is also in general different across the equilibria. It is the same and maximal for $(\bar{k}_1, \bar{k}_2)$ on BC in Figure 2.3. Second, the capacity (output, or market share) of firm 3 is larger than that of firm 1 on AG in Figure 2.3 and is larger than that of firm 2 on

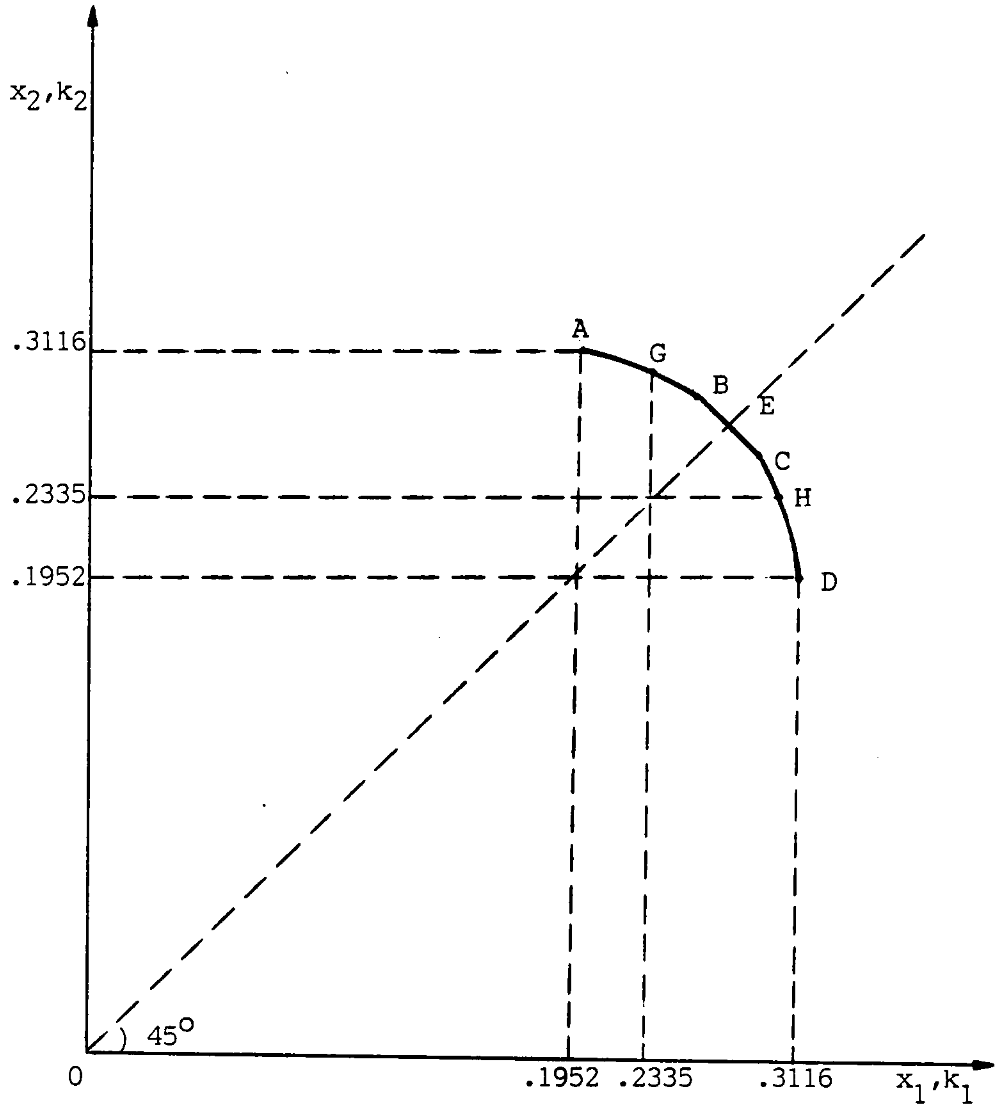

FIGURE 2.3

A Continuum of Entry-Allowing Equilibria

HD in Figure 2.3. It can be easily verified that the profit of firm 3 can be larger than that of firm 1 or firm 2. For example, if $F = 0$, the profit of firm 3 is larger than that of firm 1 on AG and that of firm 2 on HD in Figure 2.3. Third, the capacity vector $(\bar{k}_1, \bar{k}_2, \bar{k}_3(\bar{k}_1, \bar{k}_2))$ when $(\bar{k}_1, \bar{k}_2)$ is at E in Figure 2.3, is equivalent to $((k^E)^m, k^E_{m+1})$ in Proposition 4.

5. POSSIBLE UNDERINVESTMENT IN ENTRY DETERRENCE

Entry deterrence has the characteristics of a public good. Entry prevention by a group of incumbent firms protects all incumbent firms from the new competitor and makes all incumbent firms better off. That is, "consumption" of entry deterrence is not exclusive. Now, the incumbent firms in this model cannot collude on an investment in entry deterrence. That is, they cannot coordinate their choice of capacities in the first stage. Thus, the free rider problem of a public good suggests that in our model there would be an underinvestment in entry deterrence.

Gilbert and Vives (1986) consider a two-stage model in which in the first stage incumbent firms decide independently how much to produce and in the second a potential entrant decides whether or not to enter the industry and if it enters how much to produce, taking the total output of the incumbent firms as given. They show that even if the incumbent firms cannot collude when they decide on quantities they will produce and thus the free rider problem suggests that the incumbent firms would tend to underinvest in entry deterrence, there is no underinvestment in entry deterrence in their model.

They associate an underinvestment in entry deterrence with one or more of the following:

(a) Incumbents' total profits are higher when they prevent rather than allow entry, but the (unique) industry equilibrium allows entry.

(b) Either entry prevention or entry may be an industry equilibrium, but incumbents' profits are higher when entry is prevented.

(c) An established monopoly (or colluding incumbents) prevents entry in more situations than an established, noncooperating, oligopoly. (Gilbert and Vives (1986), p. 77)

At a glance, one may argue that since all incumbent firms are constrained in a subgame perfect equilibrium of our three-stage game (see Proposition 1), our three-stage model reduces to the two-stage model of Gilbert and Vives (1986), and may conclude that there is no incentive to underinvest in entry deterrence in our model. However, this reasoning is not correct. Notice that while it is always possible, in their model, for each incumbent firm to prevent entry, given the other incumbent firms' outputs, this is not the case in our model. There is no capacity vector of the incumbent firms that prevents entry when $E(k_\infty^m)>0$. Even when $E(k_\infty^m)\leq 0$, there are situations where an incumbent firm would be better off keeping a potential entrant out, given the other incumbent firms' capacities, but it is not feasible for it to do so. This outcome follows from the fact that a potential entrant ignores capacities that are not going to be used after it enters. That is, it ignores noncredible threats made by the incumbent firms. Indeed, the following example shows that there may be an underinvestment in entry deterrence in our model. In the example, there are two disconnected symmetric equilibria: one is entry-preventing and the other is entry-allowing. Incumbents' profits are higher when entry is prevented (see (b) in the previous page), which also implies that colluding incumbents prevent entry but noncooperating incumbents may not do so (see (c) in the previous page).

Example 3. There are two incumbent firms, firm 1 and firm 2, and a single potential entrant, firm 3. The inverse demand function is given by: $p = f(X) = 1.2 - X$ for $0 \leq X \leq 1.2$ and $p = 0$ for $X > 1.2$. Let $V = .92$, $R = .08$, and $F = (.013)^2$.

Suppose that in the first stage both incumbent firms choose infinite capacities. Let k_3 be firm 3's capacity chosen in the second stage. Firm 3 does not choose a capacity which is not going to be used fully in the third stage. In the third stage, since the incumbent firms are nonconstrained and firm 3 is constrained, $x_1 = t(x_2+k_3) = .14-.5x_2-.5k_3$, $x_2 = t(x_1+k_3) = .14-.5x_1-.5k_3$, and $x_3 = k_3$, which imply that $\bar{x}_1(k_\infty^2, k_3) = \bar{x}_2(k_\infty^2, k_3) = 7/75-k_3/3$. Firm 3, exercising perfect foresight with respect to the outputs of the incumbent firms, chooses a capacity which is an element in $A(k_\infty^2)$. $A(k_\infty^2) = \{1/50\}$. Since $\bar{x}_1(k_\infty^2, 1/50) = \bar{x}_2(k_\infty^2, 1/50) = 13/150$ and $\bar{x}_3(k_\infty^2, 1/50) = 1/50$, $E(k_\infty^2) = \Pi(k_\infty^2, 1/50) = 1/7500 - F < 0$. Hence, if both incumbent firms choose "sufficiently large" capacities, firm 3 will not enter the industry.

The limit output Y has the value of .174 (see Definition 8) and is larger than the sum of the full cost two-firm Cournot equilibrium outputs: $X^c = 2x^c = 2/15$. There is then at least one symmetric equilibrium.

Either $(Y/2, Y/2, 0)$ or (k^s, k^s, k_3^s) is an equilibrium (see Proposition 3), where $(k^s, k^s, k_3^s) = (1/15, 1/15, 1/30)$ (see Definition 10).[8]

In fact, both are equilibria. We first show that $(Y/2, Y/2, 0)$ is an equilibrium. Since $E(k_\infty^2) \leq 0$, if both incumbent firms choose the capacity vector of $(Y/2, Y/2)$, then firm 3 will not enter the industry (see Lemma 9). Now focus on the best response capacity of firm 1 to the capacity of firm 2 when $k_2 = Y/2$. Firm 1 will not choose a capacity larger than $Y/2$ since $Y/2 > r(Y/2) = .0565$. Notice that $r(Y/2)$ can be interpreted as the best response capacity of firm 1 to $k_2 = Y/2$ when there is no potential entrant. If firm 1 chooses a capacity smaller than $Y/2$, given $k_2 = Y/2$, then firm 3 will enter the industry and will choose $k_3 = r(k_1+Y/2) = .0565-.5k_1$ and all firms will be constrained in the third stage (see Lemma 13). Thus, firm 1's best choice among capacities smaller than $Y/2$, given $k_2 = Y/2$, is $s(Y/2) = .0565$, and its profit resulting from that choice is .001596. However, if firm 1 chooses $Y/2$, then its profit increases to .002262. Hence firm 1's best response to $k_2 = Y/2$ is $Y/2$. By symmetry, firm 2's best response capacity to $k_1 = Y/2$ is $Y/2$. Next, we show that $(1/15, 1/15, 1/30)$ is also an equilibrium. Given $(k_1, k_2) = (1/15, 1/15)$, firm 3 will enter the industry, since the total output of the incumbent firms will be smaller than Y, and will choose $k_3 = r(k_1+k_2) = 1/30$ (see Lemma 13). Consider then firm 1's best response capacity to $k_2 = 1/15$. It is easily seen that if firm 1 chooses an infinite capacity, then firm 3 will choose $k_3 = 2/75$ and $E(\infty, 1/15) = 2/5625 - F > 0$. That is, given $k_2 = 1/15$, there is no capacity for firm 1 that will prevent firm 3 from entering. Since firm 1 never chooses a capacity which will result in excess capacity (see Lemma 14), firm 1's best response to $k_2 = 1/15$ is $s(1/15) = 1/15$.

By symmetry, firm 2's best response to $k_1 = 1/15$ is 1/15.

The profit of each incumbent firm is .002262 at the symmetric entry-preventing equilibrium and 1/450 at the entry-allowing equilibrium. It is higher when entry is prevented than when entry is allowed.

6. COMPARATIVE STATICS

In this model an increase in the number of incumbent firms, m, makes entry prevention more plausible. This is so since by increasing the number of incumbent firms, on one hand entry prevention may become feasible in the sense that $E(k_\infty^m)$ becomes nonpositive (see Lemma 4 (e) and Definition 5), and on the other hand competition among incumbent firms becomes more intensive and reduces the profitability of allowing entry, relative to preventing entry. An increase in the fixed cost of entry, F, also increases the possibility of entry prevention since it may make entry prevention feasible by reducing the value of $E(k_\infty^m)$ and it decreases the limit output, Y. Let α be the proportion of the capacity cost to the full cost: $\alpha = R/(V+R)$. We then have $0<\alpha\leq 1$. Notice that entry prevention becomes more likely with an increase in α when $V+R$ is constant, since this decreases $E(k_\infty^m)$. The parameter α can be interpreted as a measure of the degree of strategic advantage held by incumbent firms (or first movers).

In the sequel, we first show that when there is a single potential entrant the feasibility of preventing entry does not imply the existence of an entry-preventing equilibrium.[9] We then examine how an increase in the number of incumbent firms and a decrease in the cost of entry affect both equilibrium prices and welfare. It is shown that an increase in the number of incumbent firms can raise the equilibrium price and lower (consumer) welfare. This result was also found in the model of Gilbert and Vives (1986, p. 80). However, in their model if we focus only on the equilibria which yield higher profits to incumbent firms, then the market price is monotone nonincreasing in the number of incumbent firms. As we will see, this is not the case in our model. Reducing the cost of entry may also increase the equilibrium price. In this case, a decrease in the cost of entry causes a switch from an entry-preventing equilibrium to an entry-allowing one. Welfare declines due to the cost of entry incurred, as well as lower total output. Finally, we show that a decrease in demand may raise the total equilibrium output.

The inverse demand function is assumed to be linear: $p = f(X) = a - bX$ for $0 \leq X \leq a/b$ and 0 otherwise, with $a > 1$. Let $V+R = 1$. Then $R = \alpha$ and $V = 1-\alpha$. We focus on symmetric equilibria.[10] Notice, however, that our conclusions below do not depend on this choice. The process of finding symmetric equilibria to this specific model is described below (see the last three propositions). First, we compute $E(k_\infty^m)$. If $E(k_\infty^m) > 0$, then $((k^s)^m, k_{m+1}^s)$ is the unique symmetric equilibrium if $k^s < k^E$ and $((k^E)^m, k_{m+1}^E)$ is the unique symmetric equilibrium if $k^s \geq k^E$. For $E(k_\infty^m) \leq 0$, we derive the value of the limit output Y. If $Y \leq X^c = mx^c$, then $((k^c)^m, 0)$ is the unique symmetric equilibrium. Otherwise, both $((Y/m)^m, 0)$ and $((k^s)^m, k_{m+1}^s)$ should be investigated and at least one of them is an equilibrium. Notice that in this specific model where the inverse demand function is linear, $k^s = x^c$. In Appendix B and C explicit solutions, when $m = 1$ and $m = 2$, are provided.

Let a = 2 and b = 1. Figure 2.4 depicts subgame perfect equilibria in the parameter space (F, α) when there is a single incumbent firm. The incumbent firm is firm 1 and a potential entrant is firm 2. In Figure 2.4, the locus ABC represents $E(k_\infty^1) = 0$. $E(k_\infty^1) < 0$ for the values of F and α above the locus. The locus DBC is the boundary between entry-preventing and entry-allowing equilibria. For the values of F and α on BC, entry is prevented and for (F, α) on DB with F $\simeq$ .0054, the incumbent firm is indifferent between preventing and allowing entry. For (F, α) in region S, $\bar{k}_1 = .5$ and $\bar{k}_2 = .25$. Figure 2.5 presents subgame perfect symmetric equilibria in the (F, α) space when there are two incumbent firms. Firm 1 and 2 are the incumbent firms and firm 3 is a potential entrant. In Figure 2.5, the locus ABCD depicts $E(k_\infty^2) = 0$ and above that locus $E(k_\infty^2) < 0$. Notice that the set of all (F, α) which make entry prevention feasible when m = 1 is a proper subset of that set when m = 2. For (F, α) in region I, entry is allowed: $\bar{k}_1 = \bar{k}_2 = 1/3$ and $\bar{k}_3 = 1/6$. For (F, α) in regions II, III, and IV, there are two symmetric equilibria: $(\bar{k}_1, \bar{k}_2, \bar{k}_3) = (1/3, 1/3, 1/6)$ is an entry-allowing equilibrium and $(\bar{k}_1, \bar{k}_2, \bar{k}_3) = (1/2-\sqrt{F}, 1/2-\sqrt{F}, 0)$ is an entry-preventing equilibrium. For (F, α) in regions III and IV, the profits of incumbent firms are higher when entry is prevented than when entry is allowed. These are situations we have focused on in Section 5. For (F, α) in region IV, total output corresponding to an entry-allowing equilibrium is larger than that for an entry-preventing equilibrium. This was not the case in the model of Gilbert and Vives (1986). For (F, α) on GCD, entry is prevented and for (F, α) on BC, both entry prevention and allowing entry are equilibria. For (F, α) on EB, either allowing entry alone or both entry prevention and allowing entry are equilibria and for (F, α) on FG,

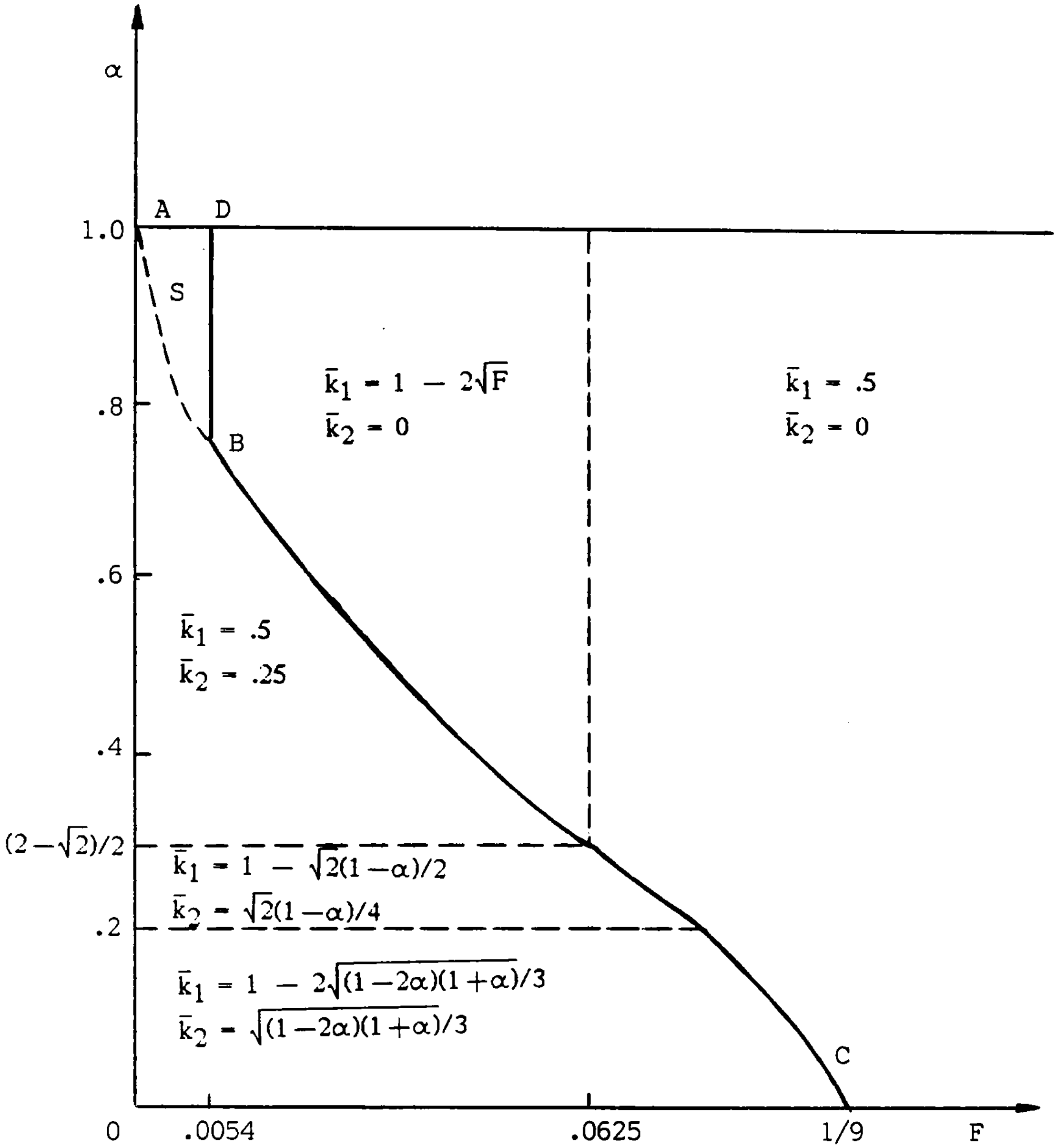

FIGURE 2.4

Subgame Perfect Equilibria in (F, α) Space with m = 1

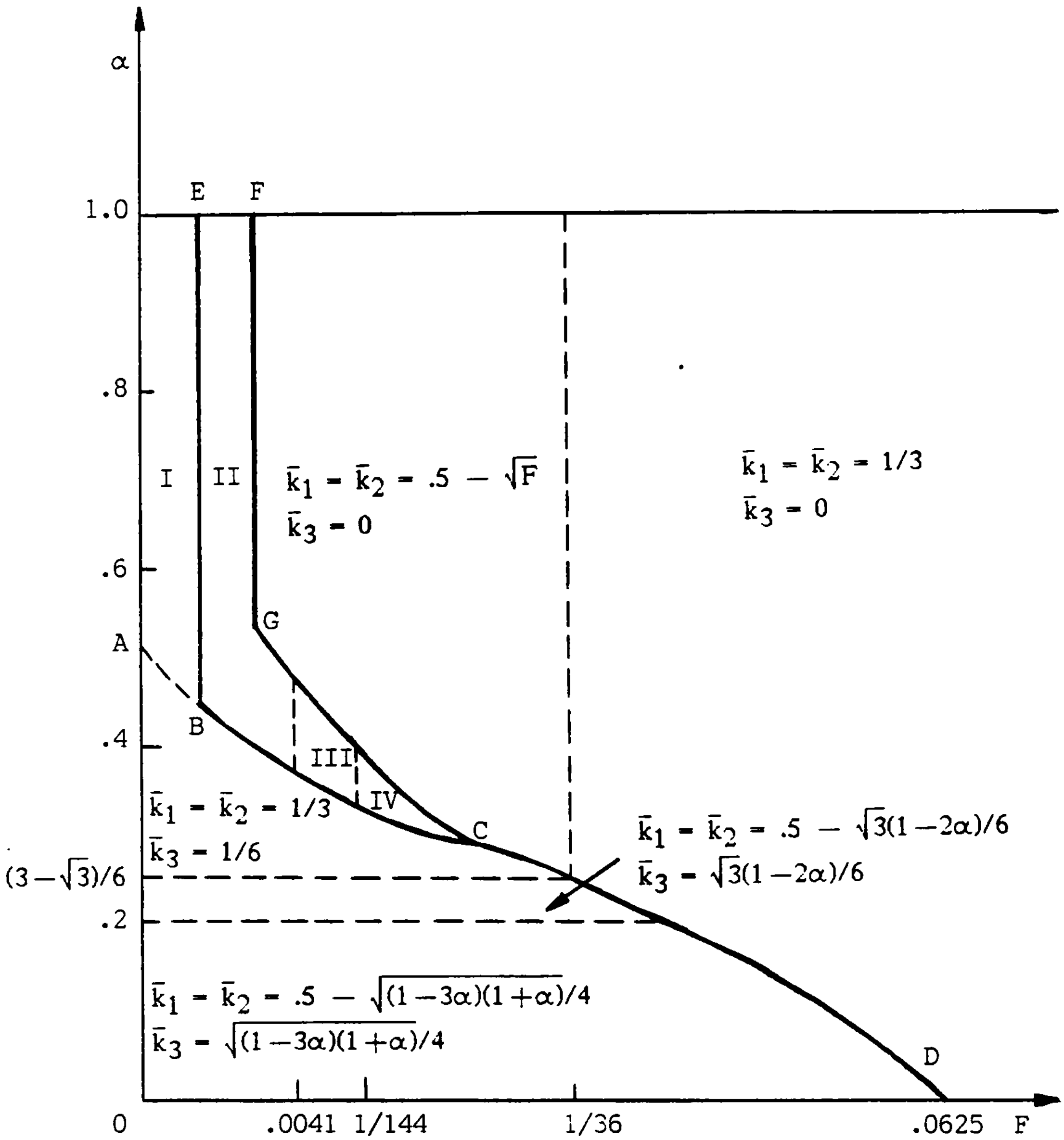

FIGURE 2.5

Subgame Perfect Symmetric Equilibria in (F, α) Space with m = 2 either entry prevention alone or both entry prevention and allowing entry are equilibria.

In Figures 2.4 and 2.5, the effect of a change in α on equilibrium capacities is noteworthy. For a given level of F, the capacities of the incumbent firms are nondecreasing in α and the capacity of the potential entrant is nonincreasing in α. A change in the parameter α, the proportion of the capacity cost to the full cost, has no effect on the full cost but does affect the credibility of investments in entry deterrence—capacities chosen by incumbent firms. As α decreases, incumbent firms lose more and more credibility of their capacities and thus the difference in capacities between incumbent firms and the entrant becomes smaller and smaller.

In Figure 2.4, preventing entry is profitable (or desirable) for (F, α) when F $>$ .0054 and feasible for (F, α) on and above ABC. Neither the profitability nor the feasibility of entry deterrence implies the other. In region S, $E(k^1_\infty) \leq 0$ but accommodation is the unique equilibrium. This is also the case in region I in Figure 2.5.

Now consider the effect of a change in the number of incumbent firms m on the market price and welfare. In our specified model, increasing the number of incumbent firms from one to two reduces the total equilibrium output over the wide range in the (F, α) space. In such cases, the market price rises and thus consumer welfare falls. This is illustrated in Figure 2.6 which draws on Figures 2.4 and 2.5. In Figure 2.6, $\bar{X}_1$ is the total output when m = 1 and $\bar{X}_2$ is the total output when m = 2. For (F, α) on BC, $\bar{X}_1 < \bar{X}_2$ and for (F, α) on AB or on AD, $\bar{X}_1 = \bar{X}_2$. For (F, α) on DGH, $\bar{X}_1 > \bar{X}_2$. For (F, α) in the shaded area, $\bar{X}_1 > \bar{X}_2$ if we restrict attention to equilibria which yield higher

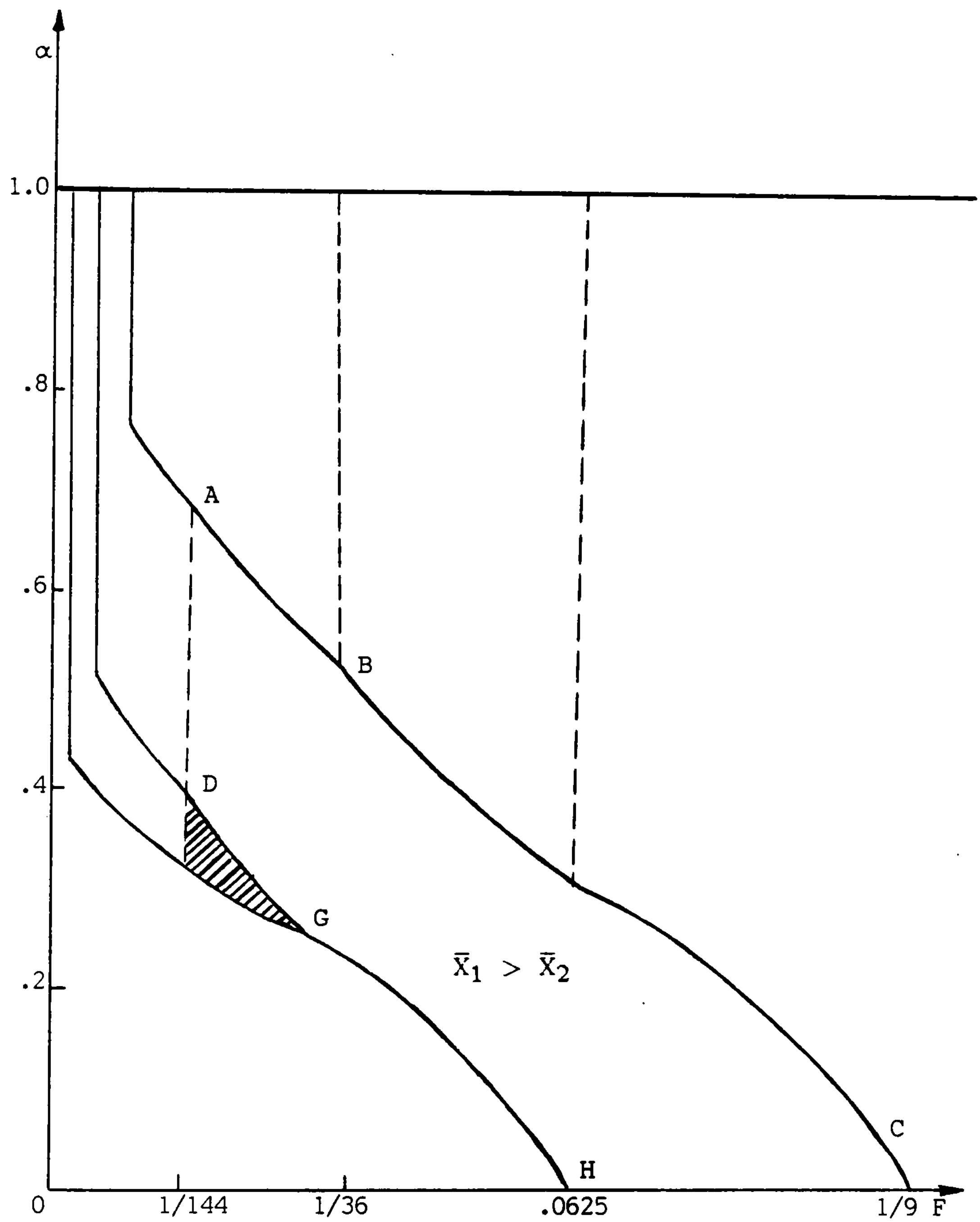

FIGURE 2.6

A Decrease in the Total Equilibrium Output

When m Increases from One to Two

profits to the incumbent firms. For (F, α) in other regions, $\bar{X}_1 \leq \bar{X}_2$. The reason why increasing the competitiveness of the industry has such a perverse effect is that in our model credibility of capacities increases with an increase in the number of incumbent firms. For example, when $E(k_\infty^1) > 0$, there are situations where a single incumbent firm has no choice but to allow entry, but two incumbent firms prevent entry simply by choosing capacities equal to the two-firm full cost Cournot output.

In Figures 2.4 and 2.5, a decrease in the cost of entry, F, has no effect on the total equilibrium output and the market price if entry is allowed at both values of F. If entry is prevented at both values of F, the total equilibrium output is nondecreasing and thus the market price is nonincreasing when F decreases. However, it is easily seen in both figures that a decrease in F can cause a decrease in the total equilibrium output and raise the market price. For example, consider the case where $\alpha = .9$ and F decreases from .0064 to .0053 in Figure 2.4. When F = .0064, entry is prevented and the total equilibrium output is .84 but when F = .0053, entry is allowed and the total equilibrium output is .75.

Finally, we show that a decrease in demand can raise the total equilibrium output. Consider the case where m = 1. Let b = 1, $\alpha = .8$, and F = .006. If a = 2.1, then $E(k_\infty^1) > 0$ and entry is allowed. The total equilibrium output is .825 (see Appendix B). But if a = 2, then entry is prevented and the total equilibrium output is .8451 (see Figure 2.4).

7. THE CASE OF MULTIPLE POTENTIAL ENTRANTS

In this section, we assume that there are multiple potential entrants who choose capacities sequentially, in place of a single potential entrant in the original model. We first establish that the feasibility of preventing entry implies the existence of an entry-preventing equilibrium, which was not the case when there is only one potential entrant. Then, we show in Example 4 that even in the case of multiple potential entrants there may be an under-investment in entry deterrence.

The m incumbent firms and n potential entrants now form the set of players in the (n+2)-stage noncooperative strategic entry deterrence game. In the first stage, m incumbent firms choose capacities simultaneously and independently, in the second through the (n+1)st stages n potential entrants choose capacities sequentially, after learning the capacities predetermined, and in the last stage the firms engage in capacity-constrained Cournot competition, given the capacities chosen in the first n+1 stages. The potential entrant which decides a capacity in the (i+1)st stage is called firm m+i for i = 1, ... , n. The number of potential entrants, n, is finite and is assumed to be large enough so that if all m+n firms choose positive capacities, then at least one firm earns a negative profit.

Let $E_{m+i}(k^{m+i-1})$ be the maximum profit that firm m+i can earn if it enters, given the capacities chosen in the first i stages, k^{m+i-1}, and given that no other potential entrants enter after that firm (see Definition 5).

Lemma 16. Firm m+i enters if and only if $E_{m+i}(k^{m+i-1}) > 0$.

The proof is similar to that of Proposition 6 in Eaton and Ware (1987). Each potential entrant exercises perfect foresight about the actions of the potential entrants following it when it chooses its capacity. However, Lemma 16 states that as far as entry decision is concerned, each potential entrant can ignore the existence of the following potential entrants.

Lemma 17. (a) If $E_{m+1}(k_\infty^m) > 0$, then there is no capacity vector of the incumbent firms that makes all potential entrants stay out. (b) If $E_{m+1}(k_\infty^m) \leq 0$ and all the capacities chosen by the incumbent firms are larger than or equal to Y/m, then all potential entrants stay out.

The proof is immediate and is omitted. It follows from Lemma 17 that preventing entry is feasible if and only if $E_{m+1}(k_\infty^m) \leq 0$. We show in the following proposition that whenever preventing entry is feasible there is an equilibrium in which all potential entrants stay out.

Proposition 5. If preventing entry is feasible, then $((k^c)^m, (k_i = 0)_{i \varepsilon N})$ is an equilibrium if $Y \leq X^c$ and otherwise $((Y/m)^m, (k_i = 0)_{i \varepsilon N})$ is an equilibrium, where N is the set of potential entrants.

Proof. The poof of the first part is trivial and is omitted. For the proof of the second part, notice first that if each of the incumbent firms chooses the capacity Y/m, then all potential entrants stay out (see Lemma 17 (b)). Next, consider firm i, $i \in M$, which chooses a capacity, given that all the other incumbent firms choose capacities equal to Y/m. We must show that given

$(k_j = Y/m)_{j \varepsilon M_{-i}}$, firm i chooses the capacity Y/m. First, it follows from Lemma 16 that if firm i chooses a capacity less than Y/m, then entry will continue until total output is greater than or equal to Y. But if it chooses a capacity larger than or equal to Y/m, then all potential entrants stay out (see Lemma 17 (b)). Thus the profit of firm i when it chooses a capacity less than Y/m is smaller than that with $k_i = Y/m$. Second, firm i will not choose a capacity larger than Y/m since $r((m-1)Y/m) < Y/m$. Q.E.D.

Hence for the values of F and α in region S in Figure 2.4, preventing entry is the only equilibrium when there are multiple potential entrants, while allowing entry is the equilibrium when there is a single potential entrant. For (F, α) in region I in Figure 2.5, there is no entry-preventing equilibrium in the case of a single potential entrant but there is such an equilibrium in the case of multiple potential entrants.

In the following example, we show that even when there are multiple potential entrants, an underinvestment in entry deterrence may arise.

Example 4. There are two incumbent firms, firm 1 and firm 2, and n potential entrants, firms 3 through n+2. The inverse demand function is given by: $p = 1.2 - X$ for $0 \leq X \leq 1.2$ and $p = 0$ for $X > 1.2$. Let V = .92, R = .08, and F = .000289.

Under these specifications, we have $E_3(k_\infty^2) = 1/7500 - F < 0$ (see Appendix C) and thus preventing entry is feasible. The limit output, Y, is .166 (see Appendix C) and is larger than the sum of the full cost two-firm Cournot equilibrium outputs which is 2/15 (see Appendix B). It then follows from Proposition 5 that there is an equilibrium in which both incumbent firms choose capacities equal to .083 and all potential entrants stay out.

Next, we show that there is another equilibrium in which both incumbent firms choose capacities equal to 1/15, firm 3 chooses the capacity 1/30, and the rest of the potential entrants stay out. Notice first that if the capacities chosen in the first two stages are $(k_1, k_2, k_3) = (1/15, 1/15, 1/30)$, then firms 4 through n+2 will not enter. It follows from Lemma 16 and from the fact that the sum of the capacities chosen in the first two stages, 1/6, is larger than the limit output .166 and $E_3(k_\infty^2) < 0$ implies that if both incumbent firms are nonconstrained after some potential entrants do enter, then the firms entered earn negative profits. Next, consider firm 3. Given $(k_1, k_2) = (1/15, 1/15)$, it is natural for firm 3 to choose $r(k_1+k_2) = 1/30$ since by choosing that capacity further entry is prevented. Finally, consider the incumbent firms which choose capacities in the first stage. Focus on firm 1's best response to firm 2's capacity 1/15. Given $k_2 = 1/15$, even if firm 1 chooses an infinite capacity, firm 3 will enter since $E_3(\infty, 1/15) = 2/5625 - F > 0$. Firm 1 exercises perfect foresight about the actions in the future stages and knows that if it chooses $s(1/15) = 1/15$, then firm 3 will choose $r(2/15) = 1/30$ and the rest of the potential entrants will stay out. Hence $s(1/15) = 1/15$ is firm 1's best response to $k_2 = 1/15$. By symmetry, firm 2's best response to $k_1 = 1/15$ is 1/15.

The profit of each incumbent firm is .002822 when entry is prevented and is 1/450 when entry is allowed, and is higher when entry is prevented.

Eaton and Ware (1987) show that if there is only one incumbent firm in this framework, then an underinvestment in entry deterrence never arises.

One may argue that an underinvestment in entry deterrence can be more plausible when the incumbent firms make investment decisions simultaneously rather than when they do so sequentially.

8. CONCLUSIONS

Gilbert and Vives (1986) investigated the free rider problem of entry deterrence in a model where (i) multiple incumbent firms confront a single potential entrant and commit noncooperatively to outputs that they *must* produce after the entrant's action and (ii) the entrant must pay a fixed cost of entry to enter the market. They argue that even if the incumbent firms cannot collude on their output levels, there is no underinvestment in entry deterrence. Waldman (1987a) showed that even after the introduction into their model of uncertainty about the cost of entry, there is no evidence of underinvestment in entry deterrence.

We have considered a three-stage model where in the first stage m incumbent firms face a single potential entrant and choose capacities simultaneously and independently, in the second stage the potential entrant chooses a capacity, and in the third stage the firms play a capacity-constrained Cournot game.

We showed that in our model the free rider problem of entry deterrence does arise. The reason why we obtain a result different from that of Gilbert and Vives is that while in their model it is always feasible for an incumbent firm to prevent entry, given output levels of the others, in our model there are situations where an incumbent firm cannot prevent entry even with an infinite capacity, given capacities of the others.

In the literature of strategic entry deterrence, it is generally felt that both a coordinated cartel and a single established firm, facing a potential entrant, yield the same result with respect to entry deterrence. However, in our model it is no longer true. There are situations where it is feasible for a coordinated cartel to prevent entry while it is not feasible for a single incumbent firm.

Salant, Switzer, and Reynolds (1983) and Perry and Porter (1985) showed in their static frameworks that a merger always increases the industry price and thus hurts consumers. In this chapter, however, we showed that consumers might benefit from a merger of incumbent firms. That is, there are situations where decreasing the number of incumbent firms causes the equilibrium price to fall. It can happen when preventing entry is feasible before a merger but not feasible after the merger.

Decreasing the cost of entry, which facilitates entry, may increase the equilibrium price and reduce consumer welfare, while a decrease in demand can raise the total equilibrium output.

When there is a single potential entrant, the feasibility of preventing entry does not imply the existence of an entry-preventing equilibrium. However, if there are "enough" potential entrants which choose capacities sequentially, then whenever preventing entry is feasible there is an entry-preventing equilibrium.

FOOTNOTES

1. See Section 5 for their description of an underinvestment in entry deterrence.

2. Waldman (1987a) also shows that in the model of Bernheim (1984) if incumbent firms are uncertain about the exact investment in entry deterrence needed to deter entry, then there is a strong tendency to underinvest in entry deterrence.

3. Gilbert (1987) provides an excellent discussion on the Bain-Sylos-Modigliani limit pricing model.

4. $k^m > \tilde{k}^m$ means that $k_i \geq \tilde{k}_i$ for all $i \in M$ and $k^m \neq \tilde{k}^m$.

5. A value of F in [0, .08) yields the same situation.

6. For simplicity, by an equilibrium we mean an equilibrium capacity vector.

7. r(Z) is assumed to be convex, which is a sufficient condition for s(Z) to be unique for all $Z \in [0, \infty)$.

8. For the case of linear demand, r(Z) = s(Z) for $Z \in [0, \infty)$. Hence $x^c = x^s$.

9. In Section 7 we show that when there are multiple potential entrants the feasibility of preventing entry implies the existence of an entry-preventing equilibrium.

10. In the case of a single incumbent firm, the equilibrium is unique except where the incumbent firm is indifferent between preventing and allowing entry (see Appendix B).

CHAPTER III

SUNK CAPACITY COSTS, HORIZONTAL MERGER, AND INDUSTRY PRICE

1. INTRODUCTION

Several recent papers theoretically analyze the profitability of a horizontal merger and its effects on the outsiders' profits and industry prices. Is a merger profitable to the merging parties (insiders)? The answers are different in the the studies. Szidarovszky and Yakowitz (1982) show in a Cournot oligopoly model that either when the demand function is strictly decreasing or when cost functions are strictly convex, if some firms merge, the profit of the merged firm can fall below the joint profits of the participating firms before the merger. Salant, Switzer, and Reynolds (1983) demonstrate, in a static Cournot model with a linear demand and identical firms producing a homogeneous product with a constant average cost, that a merger always reduces the joint profits of the merging parties if it includes less than 80 percent of the firms. They also observe that even mergers which create efficiency gains through scale economies may be unprofitable. Using a model with a homogeneous product and a linear demand, where total supply of capital is fixed for the industry, each firm has an increasing marginal cost function, and the firms compete in quantities, Perry and Porter (1985) characterize the circumstances under which an incentive to merge exists. Even though they show that more mergers are profitable than in the model of Salant *et al.*, mergers may still be unprofitable in their model. Deneckere and Davidson (1985) demonstrate in a product differentiation model with price competition that mergers of any size are beneficial and mergers of larger size yield higher profits than those of smaller size.

These studies, however, agree on the following observations: (i) mergers, profitable or not, always increase price(s) and (ii) a merger to a monopoly is always profitable.[1]

In this chapter, introducing a model which is typical in the literature on strategic entry deterrence, we present both an example in which a merger causes industry price to decrease and an example in which a merger to a monopoly is not profitable.[2]

We consider a market for a homogeneous product where firms have sunk capacity costs. Firms can increase their capacities instantaneously but cannot decrease them in some appropriately defined time period. Starting with a premerger equilibrium in which all incumbent firms have no incentive to increase their capacities and potential entrants stay out, if some of the incumbent firms merge at the beginning of some period, we have a potential three-stage game in the period. In the first stage, the merged entity with the combined capacity of the participating firms decides whether or not to increase its capacity and then the other incumbent firms (outsiders) choose their capacities simultaneously and independently. In the second stage, after observing the capacity vector of the incumbent firms, potential entrants (if any) choose capacities simultaneously and independently. In the third stage, knowing how much capacity each firm has, the firms engage in capacity-constrained Cournot competition. Assuming that all firms have perfect foresight about actions in future stages, we adopt as the equilibrium concept of the game that of a subgame perfect Nash equilibrium.

In the example in Section 3, a merger decreases industry price. The reason is that the outsider increases its capacity (output in the third stage) much more than in a usual Cournot model (*à la* Salant *et al.*) since the merged firm "has been committed" to the combined capacity of the participating firms. A decrease in industry output by the insiders is more than offset by an increase in output by the outsider.

In an earlier paper, Davidson and Deneckere (1984) have demonstrated that consumers can benefit from a merger when *ongoing* tacit collusion becomes unsustainable after the merger. However, under their assumption that firms have a strong incentive to collude and the fact that as the number of firms declines collusion becomes more likely (with an exception in the price-setting game), the possibility of renegotiations for tacit collusion after mergers makes their arguments less interesting.[3]

In Section 4 we present an example where a merger of incumbent firms into a single firm is unprofitable. This is so because the merged firm cannot prevent a potential entrant from entering, while that potential entrant stayed out before the merger.

Several authors (e.g. Deneckere and Davidson (1985)) argue that even if a merger to a monopoly is profitable, its realization is not obvious due to the "free rider problem". In our example, however, firms will not merge all the way to a monopoly, not because of the free rider problem but because of the *feasibility* of entry deterrence.

In the next section we present our general model. In Sections 3 and 4 we discuss the different possible outcomes. In Section 5 we offer several conclusions.

2. THE MODEL

Consider a market for a homogeneous product with a linear demand. The inverse demand function in each period is given by: $p = a - bX$ for $0 \leq X \leq a/b$ and $p = 0$ for $X > a/b$, where p is industry price, a and b are positive constants, and X is the total industry output. We assume that the production costs of a firm in the industry depend on the installed capacity, its output, and a quasi-fixed cost (for brevity, a fixed cost). Capacity and output are measured in the same units. A firm with k units of capacity can produce up to k units of the product. If firm i has capacity k_i and is producing output x_i in a given period, then its cost in that period is

$$C_i(x_i, k_i) = \begin{cases} F_i + Vx_i + Rk_i & \text{for } 0 < x_i \leq k_i \\ Rk_i & \text{for } x_i = 0 \end{cases}$$

where F_i is a fixed cost, V is a constant unit variable cost, and R is a constant unit capacity cost. We assume that capacity costs, once capacities are installed, are sunk but fixed costs are not sunk. We also assume that capital used to build capacity is infinitely durable. Hence each firm may increase but cannot reduce capacity over time.

The demand function and the cost functions of firms, in each period, are common knowledge to the firms. We assume that each firm tries to maximize its own profit.

Consider a premerger equilibrium in which m competing incumbent firms have no incentive to increase their capacities and n potential entrants stay out. The industry structure has been developed according to the following rules. In the first stage of *each* period, firms in the industry decide simultaneously and independently whether or not to increase their capacities. In the second stage, potential entrants, if any, choose capacities simultaneously and independently, after learning how much capacity each incumbent firm has. We assume that each potential entrant enters if and only if it can make a positive profit. In the third stage of the period, the firms with positive capacity engage in capacity-constrained Cournot competition. We assume that all firms have perfect foresight about actions in future stages. Thus a relevant equilibrium concept for the three-stage game is that of a subgame perfect Nash equilibrium.

Now suppose that some firms merge in a certain period after the premerger equilibrium has been attained. The merged entity is treated as a single firm whose initial capacity is the sum of the capacities of the participating firms and whose fixed cost is assumed to have a value between the smallest and the sum of those of the insiders. We then allow the merged firm to decide whether or not to increase capacity before the other incumbent firms do so in the first stage of the period. This is natural because the merged firm can determine postmerger capacity before the announcement of the merger is made. The same rules as in the previous paragraph are applied to the second and the third stages of the period.

We compare the outcomes of the premerger equilibrium with those of the postmerger equilibrium. In particular, we examine the effects of the merger on industry price and the profit of the merged firm.

To simplify the exposition, we assume in the sequel that $V+R = 1$. Fixed costs of the potential entrants are assumed to be infinite in Section 3, which is equivalent to saying that there is no potential entrant. In Section 4, we consider an example in which all incumbent firms merge into a single entity and fixed costs of the incumbent firms and a potential entrant are the same before the merger.

3. A PRICE-DECREASING MERGER

We demonstrate in this section that mergers in our model may decrease industry price. Consider a premerger market situation where there are three incumbent firms and the fixed costs of potential entrants are infinite.[4] Each firm has capacity equal to $.25(a-1)/b$ and the parameters in the demand function and the cost functions of the firms have values in the range where the firms make positive profits with these capacities.

To make sure that the market is in equilibrium, we have to show that no firm has an incentive to increase its capacity, given the capacities of the other two firms. Notice first that the capacity vector is equal to the equilibrium output vector in the usual three-firm Cournot competition with a constant marginal cost $V+R = 1$. Since output levels are chosen by capacity-constrained Cournot competition with a constant marginal cost V $(0 \leq V \leq 1)$, the firms are *constrained* in equilibrium, i.e. they produce up to their capacities.[5] As a firm increases capacity *ceteris paribus* starting from the present capacity, it will first be constrained and then *nonconstrained* (it will produce less than capacity) beyond some capacity (border line capacity) while the other firms are still constrained. The profit of the firm with the border line capacity is greater than or equal to that with capacity beyond it since outputs are the same in both cases. We also see that its profit with the present capacity is greater than or equal to that with the border line capacity since it is constrained in both cases and $.25(a-1)/b$ is the best response to total output of the other firms $.5(a-1)/b$ in the usual Cournot competition with $V+R$.

Industry output in this premerger equilibrium is $.75(a-1)/b$ and the industry price is $.25(a+3)$.

We assume that two of the three firms merge at the beginning of some period.[6] Before anything done, the capacity of the merged firm (firm 1) is $.5(a-1)/b$ and that of an outsider (firm 2) is $.25(a-1)/b$. This initial capacity vector is represented by point S in Figure 3.1. In the figure G_iG_i, $i = 1, 2$, is firm i's Cournot reaction function computed with a constant marginal cost $V+R$ and H_iH_i, $i = 1, 2$, is firm i's Cournot reaction function with V (assuming $F_i = 0$). As the unit capacity cost R decreases, H_iH_i moves toward G_iG_i which remains unchanged throughout.[7] If $R = 0$, then G_iG_i and H_iH_i coincide. In particular, when $R = .25(a-1)$, H_1H_1 passes through point S.

It is easy to show that the industry price will not decrease after the

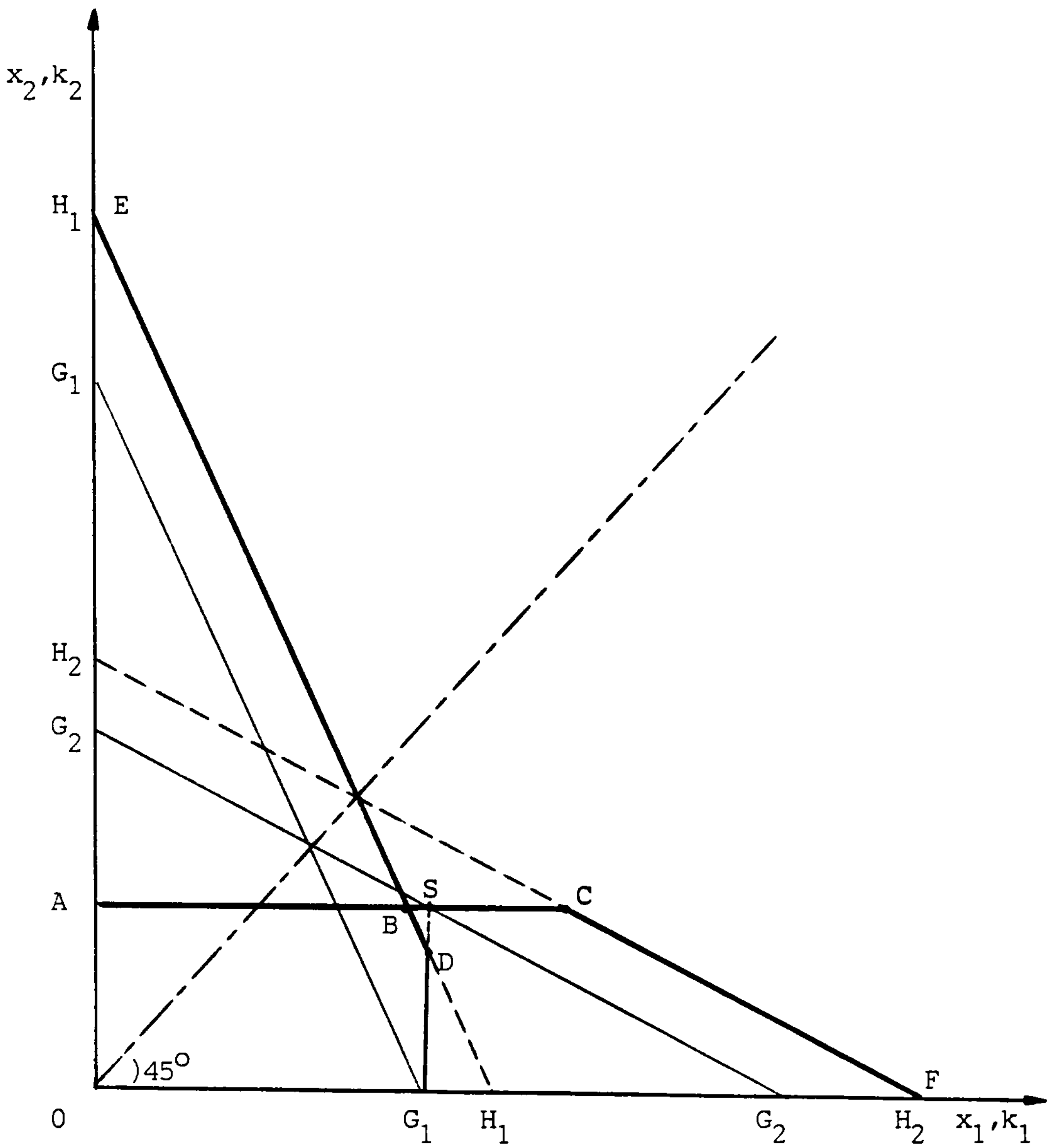

FIGURE 3.1

The Equilibrium Outputs with the Initial Capacity Vector merger if no firm increases its capacity. Consider first a value of R in the interval $[0, .25(a-1))$. With the initial capacity vector, firm 1's capacity-constrained Cournot reaction function in the third stage is represented by EDG_1 and firm 2's by ACF in Figure 3.1. Point B is the intersection of both reaction functions and is located to the left of point S. Postmerger industry output (at point B) is smaller than that prior to the merger which is equal to output level at point S. Industry price then increases after the merger. When the value of R is in the interval $[.25(a-1), 1]$, point D will be vertically above point S so that the intersection of both firms' capacity-constrained Cournot reaction functions will be point S. In that case the output and the price after the merger are the same as those before the merger.

However, there are values of R such that firm 2 will increase its capacity and the resulting price will be lower than that before the merger. To illustrate such a case, let $R = .2(a-1)$ and $1 < a \leq 6$. The corresponding lines G_iG_i and H_iH_i are shown in Figure 3.2. Point S represents the initial capacity vector and is located to the right of H_1H_1, which implies that firm 1's best output choice in response to firm 2's output (greater than or equal to the initial capacity of firm 2) is always less than its initial capacity, i.e. firm 1 will be nonconstrained with the initial capacity whatever capacity firm 2 chooses. Hence firm 1 never increases its capacity.

Now firm 2 makes a capacity decision. It can implement a third stage equilibrium anywhere on the segment UT of H_1H_1. Since when $R = .2(a-1)$ firm 2's iso-profit contour (with $V+R$) is tangent to H_1H_1 at point U and since its profit decreases as we move from left to right along G_2G_2, point U maximizes firm 2's profit on UT. Thus, firm 2 will increase its capacity to

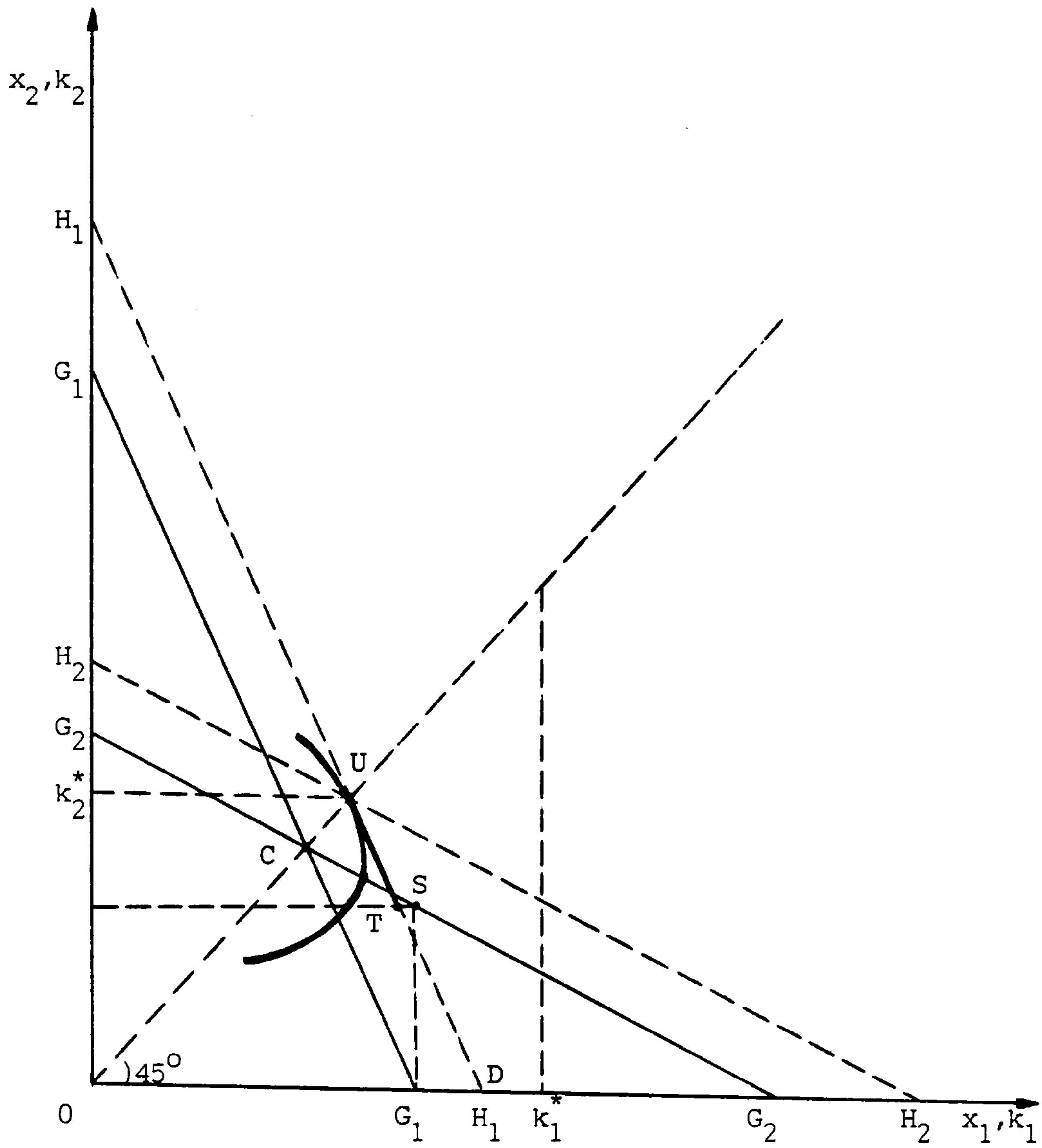

FIGURE 3.2

Firm 2's Postmerger Capacity

$k_2^* = .4(a-1)/b$.

The final capacity vector is then $(k_1, k_2) = (.5(a-1)/b, .4(a-1)/b)$ and each firm's output is $.4(a-1)/b$. Output and price in the postmerger equilibrium are $.8(a-1)/b$ and $.2(a+4)$, respectively. Since the premerger price was $.25(a+3)$ and $a > 1$, we see that postmerger price is lower than the price before the merger.

In this example firm 1 never increases its capacity, regardless of the value of R. Firm 2, however, will increase its capacity for $R \in [0, 1]$ if $a > 3+\sqrt{2}$ and for $R \in [0, (2-\sqrt{2})(a-1)/2))$ if $a \leq 3+\sqrt{2}$.[8] Both firms produce at the same level of output for $R \in [0, \alpha]$ where $\alpha = \min(.2(a-1), 1)$. If $a \geq 9$, the postmerger price is always greater than or equal to the price prior to the merger. If $3+\sqrt{2} < a < 9$, the postmerger price is lower than the price before the merger for $R \in (.125(a-1), 1]$. Finally, if $1 < a \leq 3+\sqrt{2}$, the postmerger price is lower than the price before the merger for $R \in (.125(a-1), (2-\sqrt{2})(a-1)/2))$.

4. AN ENTRY-INDUCING MERGER TO A MONOPOLY

This section presents an example where a merger of the incumbent firms to a monopoly is not profitable for the firms. Furthermore, this is another example where the postmerger price is lower than the premerger price. These phenomena arise because entry prevention becomes *nonfeasible* as the firms merge into a single entity.[9] Consider a market situation with two incumbent firms, each with capacity $(a-1)/3b$, and one potential entrant. The fixed cost of the potential entrant is assumed to be $\epsilon+.03(a-1)^2/b$ with $\epsilon > 0$ very small and let $R = .2(a-1)$ (and thus $V = .2(6-a)$) with $1 < a \leq 6$.

As in Section 3, we start by examining whether the premerger market situation is in equilibrium. We first show that the potential entrant decides not to enter in the second stage of every premerger period given the capacities of the incumbent firms.

As a preliminary step, suppose that each of the incumbent firms has infinite capacity. They will be nonconstrained in the third stage whatever capacity the entrant chooses in the second stage. Thus the equilibrium output vector (in the third stage) is on the intersection of Cournot reaction functions of the incumbent firms computed with V. If the entrant chooses a capacity k in the open interval $(.3(a-1)/b, \infty)$, it is also nonconstrained and the output levels of the firms are the same at $.3(a-1)/b$. If it chooses k in the interval $[0, .3(a-1)/b]$, it is constrained and each of the incumbent firms produces at the level of $.4(a-1)/b - k/3$. The maximum profit the potential entrant can earn by entering is attained when it chooses a capacity of $.3(a-1)/b$. The corresponding output level of each incumbent firm is then $.3(a-1)/b$ and the corresponding gross profit of the entrant is $.03(a-1)^2/b$.[10]

Return to the original potential entrant's situation. It decides whether or not to enter given that each incumbent firm has a capacity of $(a-1)/3b$. Suppose that it chooses a positive capacity. Depending on the entrant's capacity, the incumbent firms are either nonconstrained or constrained. As we saw in the previous paragraph, the entrant's profit-maximizing capacity, given that the incumbent firms are nonconstrained, is $.3(a-1)/b$ and its corresponding gross profit is $.03(a-1)^2/b$. We can easily compute its profit-maximizing capacity, given that the incumbent firms are constrained. It is $(a-1)/6b$ which equals the Cournot best response to the sum of the incumbents' outputs with the marginal cost of 1. The corresponding gross profit of the entrant is $(a-1)^2/36b$. Comparing the gross profits, we see that the entrant will choose $.3(a-1)/b$ if it enters. However, its gross profit in that case is lower than its fixed cost of $\epsilon+.03(a-1)^2/b$. Hence, the potential entrant will not enter.

It is easy to show that no incumbent firm has an incentive to increase its capacity in every premerger period. With foresight that the potential entrant will not enter, each incumbent firm in the first stage keeps its current capacity which is the best response to its rival's current capacity.

The model in this section can also be explained by Figure 3.2 in Section 3, where G_iG_i and H_iH_i are firm i's Cournot reaction functions with $V+R$ and V, respectively. The premerger capacity vector of the incumbent firms (firm 1 and firm 2) is represented by point C in the figure. Since H_iH_i is drawn with $R = .2(a-1)$ and is located outside point C, the incumbent firms are constrained. Hence, in this premerger equilibrium, the gross profit of each incumbent firm is $(a-1)^2/9b$ and the market price is $(a+2)/3$.

We now assume that the incumbent firms merge at the beginning of a given period. Notice first that since capacity costs are sunk, the merged firm cannot decrease its capacity below the combined capacity of the incumbent firms. We have to analyze in that period a three-stage game where in the first stage the merged incumbent firm (firm 1) decides whether or not to increase its capacity, in the second stage the potential entrant (firm 2) chooses a capacity, and in the third stage the firms engage in capacity-constrained Cournot competition. Firm 1's initial capacity is $2(a-1)/3b$, which is denoted by k_1^* in Figure 3.2. Since the equilibrium output vector in the third stage is on UD (and it depends on firm 2's capacity), firm 1 will always be nonconstrained, even with its initial capacity. Thus firm 1 never increases its capacity and if some output vector on UD yields a positive profit to firm 2, firm 1 cannot prevent firm 2 from entering. As shown in Figure 3.2, the output vector at point U gives firm 2 the maximum profit and firm 2's gross profit at that point is $.08(a-1)^2/b$. Since the gross profit is greater than the fixed cost, firm 2 enters with the capacity $k_2^* = .4(a-1)/b$.

The final capacity vector in the postmerger and postentry equilibrium is then $(k_1, k_2) = (2(a-1)/3b, .4(a-1)/b)$ where each firm produces at the same level of output $.4(a-1)/b$. The gross profit of the merged firm is then $2(a-1)(17-2a)/75b$. Since the premerger joint gross profits of the insiders was $2(a-1)^2/9b$, even the merger to a monopoly hurts the participating firms if a is greater than $76/31$. The postmerger and postentry market price is $(a+4)/5$ and is lower than the premerger market price of $(a+2)/3$.

Deneckere and Davidson (1985) argue that a merger to a monopoly is most profitable but the free rider problem makes it less likely. However, in our example, firms do not merge all the way to a monopoly, not because the free rider problem prevents them from merging, but because the merger will cause the potential entrant to enter and thus the insiders will be hurt seriously from the merger. Our example provides a more plausible explanation for the existence of an industry that is not a pure monopoly. Indeed, we believe that one cannot properly assess the effects of a merger without taking into account the possibility of entry.

5. CONCLUSIONS

The previous papers addressing the question of the incentives to merge have never found that mergers can decrease industry prices, except Davidson and Deneckere (1984) who have demonstrated in quantity-setting and price-setting supergames that consumers can benefit from a merger. However, we have shown in this chapter that mergers may decrease industry price. The main reason for the result is that capacity costs for firms are sunk. Since the merged firm cannot decrease its capacity below the combined capacity of the insiders and thus is forced to commit to that capacity level, there are situations where the outsiders increase their capacities much more than they increase their outputs in a regular Cournot model.

We have also demonstrated that a merger to a monopoly is not always beneficial, and this is a striking contrast to the results in the previous papers. That is, there are situations, in our model, where if incumbent firms merge into a single entity, entry prevention becomes impossible and allowing entry hurts the insiders seriously.

We conclude that one cannot examine properly the effects of a merger without considering the possibility of entry.

FOOTNOTES

1. The papers have another observation in common. They all conclude that a merger never hurts the firms which are not participating in the merger. We will discuss this outcome in Chapter IV.

2. There are many papers on strategic entry deterrence which introduce sunk capacity costs as a credible entry deterrent (e.g., Dixit (1980), Ware (1984)).

3. They also enumerate three potential problems with the quantity-setting supergame analysis (see pp. 121–122).

4. We are in fact assuming that there is no potential entrant. In the rest of this section we use "firm" instead of "incumbent firm".

5. Imagine the usual Cournot competition with either V or $V+R$. The best response of a firm to total output of the other firms for the former is always greater than (if $R > 0$) or equal to that for the latter.

6. We assume an exogenous merger. Even though the postmerger price is lower than the premerger price, the merger may be profitable to the insiders since they can save fixed costs.

7. Since we assume that $V+R = 1$, we can interpret R as the proportion of R to $V+R$.

8. If $R = (2-\sqrt{2})(\alpha-1)/2$, firm 2 is indifferent between increasing capacity and staying put. We assume that in this case firm 2 stays put.

9. In our model, a merger of some incumbent firms typically makes entry prevention nonfeasible.

10. We define *gross profit* = total revenue — (total cost — a fixed cost).

CHAPTER IV

SUNK CAPACITY COSTS AND HORIZONTAL MERGER WITH CAPACITY-CONSTRAINED BERTRAND COMPETITION

1. INTRODUCTION

Several recent papers have investigated the issue of the existence of incentives to merge. Davidson and Deneckere (1984), Perry and Porter (1985), Salant, Switzer, and Reynolds (1983), and Szidarovszky and Yakowitz (1982) have all demonstrated that mergers may reduce the joint profits of the constituent firms (insiders) in quantity-setting games. On the other hand, Davidson and Deneckere (1984) and Deneckere and Davidson (1985) have observed that mergers are never disadvantageous in static price-setting games.

These studies, however, share the observation that the firms (outsiders) which do not participate in a merger, benefit (or more accurately are never hurt) from the merger. This observation is dated back to Stigler (1950).[1] Davidson and Deneckere (1984) further show that in both the price-setting and the quantity-setting supergames mergers in the industry with firms practicing tacit collusion tend to be beneficial to the outsiders since the threat point implicit in the collusion becomes more favorable for the outsiders.

In this chapter, adopting a model in which firms engage in capacity-constrained price competition after they set capacities, we present an example where a merger does hurt the outsiders.

We consider a market for a homogeneous product where capacity costs are sunk in some appropriately defined time period once firms have installed capacities. We assume that firms can immediately increase their capacities. Starting with a premerger equilibrium in which no firm has an incentive to increase its capacity, if some firms merge at the beginning of a certain period, we have a three-stage game in the period. In the first stage, the merged firm with the combined capacity of the insiders decides whether or not to increase its capacity. In the second stage, after observing the capacity of the merged firm, the outsiders choose their capacities simultaneously and independently. To provide the advantage of being a first mover for the merged firm is natural because the merged firm can commit itself to its postmerger capacity before it announces the merger. In the third stage, after learning how much capacity each firm has, the firms engage in capacity-constrained price competition. We adopt the efficient-rationing rule. Our equilibrium concept of the game is that of a subgame perfect Nash equilibrium.

In the example of Section 4, a merger hurts the outsider. Davidson and Deneckere (1984) present a model very similar to ours. The reason why they get a different result is that they do not consider the possibility that firms increase their capacities after mergers. Indeed, in our example, both the merged firm and the outsider increase their capacities.

Another interesting observation in the example is that the premerger price is higher than the lowest price in the support of the postmerger (mixed strategy) equilibrium. Interpreting mixed strategies as Edgeworth price cycles, we may state that there are periods after the merger in which prices are below the premerger price.[2]

Finally, we demonstrate in the example that unlike the previous static price competition models, a merger reduces the joint profits of the participating firms. However, firms have incentives to merge *because a merger hurts the outsider more severely than the merged firm.*

The chapter is divided into five parts. The basic model is presented in the next section. In Section 3 we review and study further a duopoly with capacity-constrained price competition. Section 4 contains an example which carries out all the arguments. In Section 5 we present several conclusions.

2. THE MODEL

Consider a market for a homogeneous product with a linear demand. The market demand function in each period is given by: $D(p) = a - p$ for $0 \leq p \leq a$ and $D(p) = 0$ for $p > a$, where $D(p)$ is the quantity demanded at price p and a is a positive constant. We assume that the production costs of a firm in the industry depend on its installed capacity and a quasi-fixed cost (for brevity, a fixed cost).[3] Capacity and output are measured in the same units. A firm with k units of capacity can produce up to k units of the product at zero marginal cost. If firm i has capacity k_i and is producing output x_i in a given period, then its cost in that period is

$$C_i(x_i, k_i) = \begin{cases} F_i + Rk_i & \text{for } 0 < x_i \leq k_i \\ Rk_i & \text{for } x_i = 0 \end{cases}$$

where F_i is a fixed cost and R is a constant unit capacity cost. We assume that capacity costs, once installed, are sunk but fixed costs are not. There is no rental or resale market for idle capacity already in place. Hence, over time, each firm may increase but cannot reduce its capacity.

The demand function and the cost functions of firms in each period are common knowledge to the firms. We assume that in each period firms engage in price competition, subject to the capacity constraints chosen before the stage of price competition when each firm learns how much capacity its opponents have. We assume that each firm seeks to maximize its own expected profit.

Consider a premerger equilibrium in which n competing firms have no incentive to increase their capacities and there is no potential entrant. Now consider a period where some firms merge. We then have a three-stage game in that period. We treat the merged entity as a single firm whose initial capacity is the sum of the capacities of the participating firms and whose fixed cost is assumed to have a value between the smallest and the sum of those of the insiders. In the first stage, the merged firm decides whether or not to increase capacity above its initial level. In the second stage, after observing the final capacity of the merged firm, the outsiders decide simultaneously and independently whether or not to increase their capacities above their premerger levels. In the third stage, after learning how much capacity each firm has, the firms simultaneously and independently name prices. We assume that all firms have perfect foresight about actions in the following stages. Thus a relevant equilibrium concept for the three-stage game is that of a subgame perfect Nash equilibrium.

We compare the premerger and the postmerger equilibria. In particular, we examine the effects of the merger on industry prices and the profits of the outsiders as well as the joint profits of the insiders.

In order to complete the description of the model, we have to elaborate on our rationing rule. We adopt the rationing rule considered in Levitan and Shubik (1972), Kreps and Scheinkman (1983), and Osborne and Pitchik (1986).[4] We assume that consumers first seek to buy from the firms quoting the lowest price. If the firms with the lowest price cannot satisfy the quantity demanded at that price and if the quantity demanded at the next lowest price is larger than total capacity of the firms with the lowest price, then the firms naming the second lowest price sell their outputs to consumers. If the firms with the second lowest price cannot satisfy the residual quantity demanded at their price and if the quantity demanded at the third lowest price is larger than total capacity of the firms with the first and the second lowest prices, then unsatisfied consumers purchase from the firms choosing the third lowest price, and so on. Assuming that income effects are absent, we have in each "round" a residual demand function parallel to the market demand function. Finally, if the residual quantity demanded at the price of a certain round can be fully supplied and if there are more than one firm quoting that price, then each of the firms sells a quantity equal to the residual quantity demanded at the price multiplied by the proportion of its capacity to total capacity of the firms.

3. DUOPOLY WITH CAPACITY-CONSTRAINED PRICE COMPETITION

The example we will present in the next section is of a duopoly in which the merged firm and the outsider decide sequentially whether or not to increase their capacities and then the two firms quote prices. Hence it is helpful to study in this section a duopoly with capacity-constrained price competition. We first discuss the pure strategy equilibrium or the support of the mixed strategy equilibrium, and the corresponding expected revenue of each firm, given their capacities, and then focus on their sequential capacity choices.

There are two firms, firm 1 and firm 2. Their capacities are given by k_1 and k_2. We assume, for simplicity, that $F_i = 0$ and $k_1 \geq k_2 > 0$. Suppose that the firms charge prices p_i and p_j in the interval $[0, a]$. (When we use i and j at the same time, we mean that $j \neq i$.) With our rationing rule described in Section 2, the quantity sold by firm i is equal to $\min(k_i, a-p_i)$ if $p_i<p_j$, $\min(k_i, k_i(a-p_i)/k)$ if $p_i=p_j$, and $\min(k_i, \max(0, a-k_j-p_i))$ if $p_i>p_j$, where $k = k_1+k_2$.

It is straightforward to check that the (k_1, k_2) subgame has a unique pure strategy equilibrium when (k_1, k_2) is in region I or IV of Figure 4.1.[5] (The line AB can be interpreted as the usual Cournot reaction function of firm 1 in the case of zero marginal cost.) In region I both firms name the price of $a-k_1-k_2$ and in region IV each charges the price of zero.

We do not have a pure strategy equilibrium with (k_1, k_2) in regions II and III.[6] Let Φ_j be a mixed strategy of firm j chosen from the set of cumulative distribution functions on $[0, a]$. Then the expected revenue of

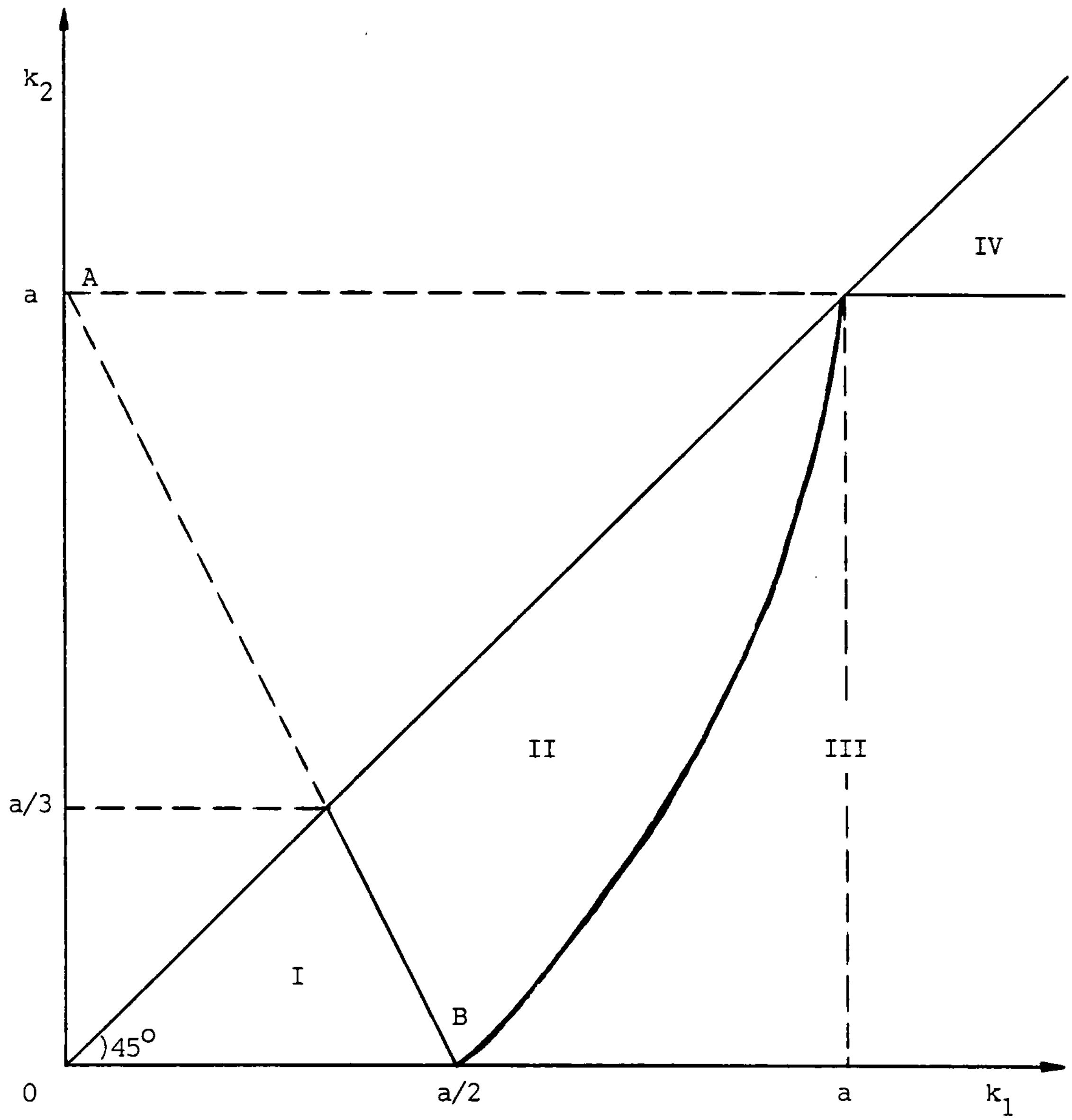

FIGURE 4.1

The Equilibrium of the (k_1, k_2) Subgame firm i, charging a price p_i, is

$$H_i(p_i, \Phi_j) = p_i \{(1 - \Phi_j(p_i)) \min (k_i, a - p_i) + \alpha_j(p_i) \min (k_i, k_i(a - p_i)/k)$$

$$+ (\Phi_j(p_i) - \alpha_j(p_i)) \min (k_i, \max (0, a - k_j - p_i))\} \quad (1)$$

where $\alpha_j(p_i)$ is the size of the atom in Φ_j at p_i. Since the revenue function $pD(p)$ is concave, the support of the equilibrium strategy of a firm is the same *closed* interval as that of the other firm, denoted by $[p_L, p_H]$, and the equilibrium strategy of firm 2 is atomless while that of firm 1 may have an atom only at the highest price of the support p_H.[7] Then from (1) we have

$$H_1(p_H, \Phi_2) = p_H \min (k_1, \max (0, a - k_2 - p_H)) \quad (2)$$

$$H_1(p_L, \Phi_2) = p_L \min (k_1, a - p_L) \quad (3)$$

$$H_2(p_L, \Phi_1) = p_L \min (k_2, a - p_L) \quad (4)$$

where Φ_i is now the equilibrium strategy of firm i.

We are interested in the common support $[p_L, p_H]$, not in the equilibrium strategy of each firm. The highest price of the support p_H is the price at which firm 1 (the firm with larger capacity) earns the maximum revenue *when it is undercut.* If firm 1 charges a higher price than firm 2, then its revenue is $p_1 \min (k_1, \max (0, a - k_2 - p_1))$, which is maximized in regions II and III when the value of p_1 is equal to $(a - k_2)/2$. Thus p_H is equal to $(a - k_2)/2$.

To compute p_L, we utilize the fact that in a mixed strategy equilibrium of a two-player game a player is indifferent to the choice of any pure strategy in the support when the other player uses his equilibrium mixed strategy. In our context, this implies that when firm 2 uses its equilibrium strategy Φ_2, firm 1's revenue is the same regardless of whether it charges p_H or p_L. From (2) and (3), we have

$$(a - k_2)^2/4 = p_L \min(k_1, a - p_L). \tag{5}$$

Hence $p_L = (a-k_2)^2/4k_1$ if $k_1 \le a-p_L$ and $p_L = (a-\sqrt{2ak_2-k_2^2})/2$ if $k_1 \ge a-p_L$. In region II, k_1 is lower than $a-p_L$ and in region III it is greater than $a-p_L$. For the capacity vector on the border line between the two regions which is expressed by $k_1 = (a+\sqrt{2ak_2-k_2^2})/2$, $k_1 = a-p_L$. Hence, in region II the lowest price of the support $p_L = (a-k_2)^2/4k_1$ and in region III $p_L = (a-\sqrt{2ak_2-k_2^2})/2$.

Next, focus on the equilibrium expected revenues of the firms. In region I of Figure 4.1 the revenue of firm i equals $k_i(a-k_1-k_2)$ while in region IV it is zero. It is obvious from the above that in regions II and III the equilibrium expected revenue of firm 1 is equal to $(a-k_2)^2/4$. To compute that of firm 2, notice first that in regions II and III $k_2 \le a-p_L$. Then from (4) we have $H_2(p_L, \Phi_1) = p_L k_2$. Utilizing the fact that the equilibrium expected revenue of a firm is equal to the expected revenue computed with a pure strategy against the other firm's equilibrium mixed strategy, we show that the equilibrium expected revenue of firm 2 is $k_2(a-k_2)^2/4k_1$ in region II and $k_2(a-\sqrt{2ak_2-k_2^2})/2$ in region III.

Now consider a three-stage game where firm 1 and firm 2 sequentially choose capacity levels k_1 and k_2 and then engage in price competition in the (k_1, k_2) subgame. Based on our analysis above, we can derive firm 2's subgame perfect reaction correspondence. Assuming $R > 0$, it is easy to check that given k_1, firm 2 will not choose a capacity level in regions IV and V of Figure 4.2. For example, given $k_1 \geq a/3$, since any $k_2 > k_1$ yields the same equilibrium expected revenue as when $k_2 = k_1$ and since $R > 0$, firm 2 will not choose a capacity greater than k_1. If firm 1 chooses a capacity level k_1 in the interval $[0, \tilde{k}_1)$, then firm 2 will choose a capacity $k_2 = ((a-R)-k_1)/2$ (represented by AB in the figure) which maximizes $(a-k_1-k_2)k_2 - Rk_2$. (The segment AB coincides with the part of Cournot reaction function with a constant marginal cost R.) In region II, firm 2's (expected) profit with k_2 given k_1 is equal to $k_2(a-k_2)^2/4k_1 - Rk_2$ and the capacity level maximizing its profit is

$$k_2 = (2a - \sqrt{a^2 + 12Rk_1})/3, \tag{6}$$

which is represented by GH. In region III firm 2 solves

$$\operatorname*{argmax}_{k_2>0} \; k_2\,(a - \sqrt{2ak_2 - k_2^2})/2 - Rk_2. \tag{7}$$

Notice that in region III firm 2's optimal level of capacity does not depend on k_1. The closed-form solution of (7) is cumbersome to compute but the relative position can be drawn by SF in the figure. Given $k_1 = \tilde{k}_1$, firm 2 is indifferent between choosing point B and point C. Also given $k_1 = k_1^*$, firm 2's

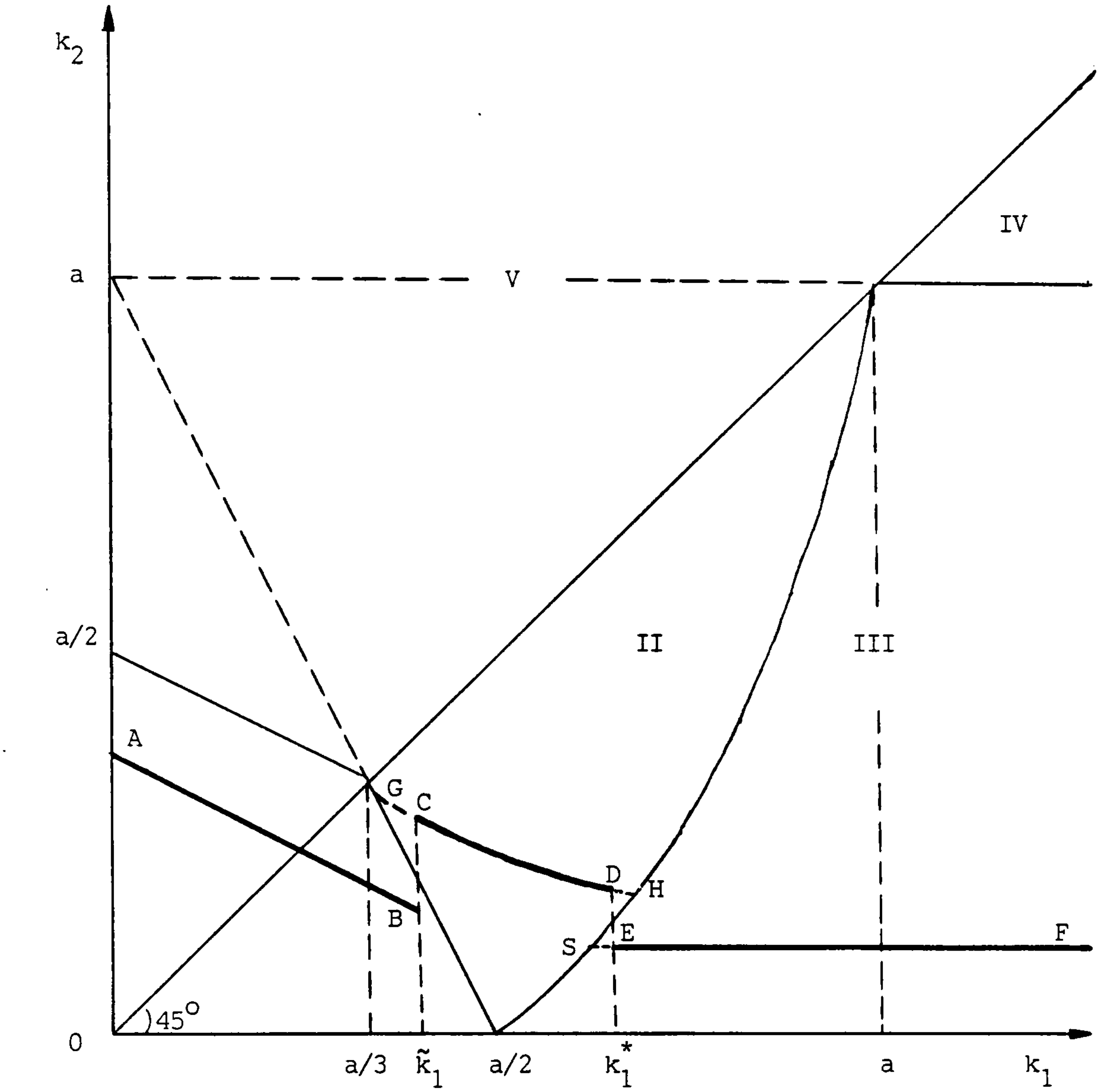

FIGURE 4.2

Firm 2's Subgame Perfect Reaction Correspondence expected profit at point D is equal to that at point E. As a consequence, firm 2's subgame perfect correspondence comprises the union of $[A, B]$, $[C, D]$, and $[E, F]$. Firm 2's profit decreases as we move down from point A to point B along the segment $[A, B]$ and from point C to point D along $[C, D]$, and is constant on $[E, F]$.

As a digression, we can transform our setting into a strategic entry deterrence model. Let firm 1 be an incumbent and firm 2 a potential entrant. Assuming that firm 2 enters if and only if it can earn a positive expected profit, we first observe that preventing entry is not always feasible even with $F_2 > 0$. Given values of parameters F_2 and R, if firm 2's expected profit is positive at the points on EF in Figure 4.2, it is impossible for firm 1 to deter entry. A striking observation is that firm 1 can prevent entry with excess capacity. That is, there are situations where firm 1 chooses a capacity level greater than $a/2$ (the output level maximizing a monopolist's profit) and firm 2 decides not to enter, which implies that firm 1 has an idle capacity in post-entry periods. The main reason for this result is that in Figure 4.2 firm 2's profit decreases as we move down from point C to point D along the segment CD. This observation is a contrast to those in Dixit (1980), Ware (1984), and others, who use quantity-setting models in which each firm's marginal revenue is always decreasing in the other's output (i.e., the Cournot reaction function of each firm is everywhere downward sloping).[8]

4. AN OUTSIDER-HURTING MERGER

We show in this section that a merger may hurt outsiders as well as insiders and the postmerger prices may be lower than the premerger prices. Consider a premerger market with three firms, each having the same fixed cost F, and no potential entrant. Each firm has a capacity level of $(a-R)/4$. (Notice that the capacity is the same as a three-firm Cournot equilibrium output with constant marginal cost R.) The parameters, a, $R>0$, and F, have values such that the firms make positive profits with their capacitites and furthermore any firm's predatory increase in capacity cannot succeed in driving the rivals out of the industry. Under these conditions, it is easy to check that the market is in equilibrium. That is, no firm has an incentive to increase its capacity given the others' capacities.

We have a unique pure strategy equilibrium in the price competition subgame with these capacities. Each firm charges the price of $(a+3R)/4$ and earns the profit of $(a-R)^2/16 - F$.[9]

Suppose now that two of the firms merge at the beginning of some period. The initial capacity of the merged firm (called firm 1) is $(a-R)/2$ and that of the outsider (called firm 2) is $(a-R)/4$. If no firm increases its capacity, postmerger prices are higher or equal to the premerger price. It is immediate when a pure strategy equilibrium occurs in the (k_1, k_2) subgame (region I in Figure 4.1). In the case of a mixed strategy equilibrium (region II in Figure 4.1), the lowest price in the support $p_L = (a-k_2)^2/4k_1 > (a+3R)/4$ (see Section 3). One can also verify with the explicit solutions in Section 3 that if both firms keep their initial capacity levels, they benefit from the

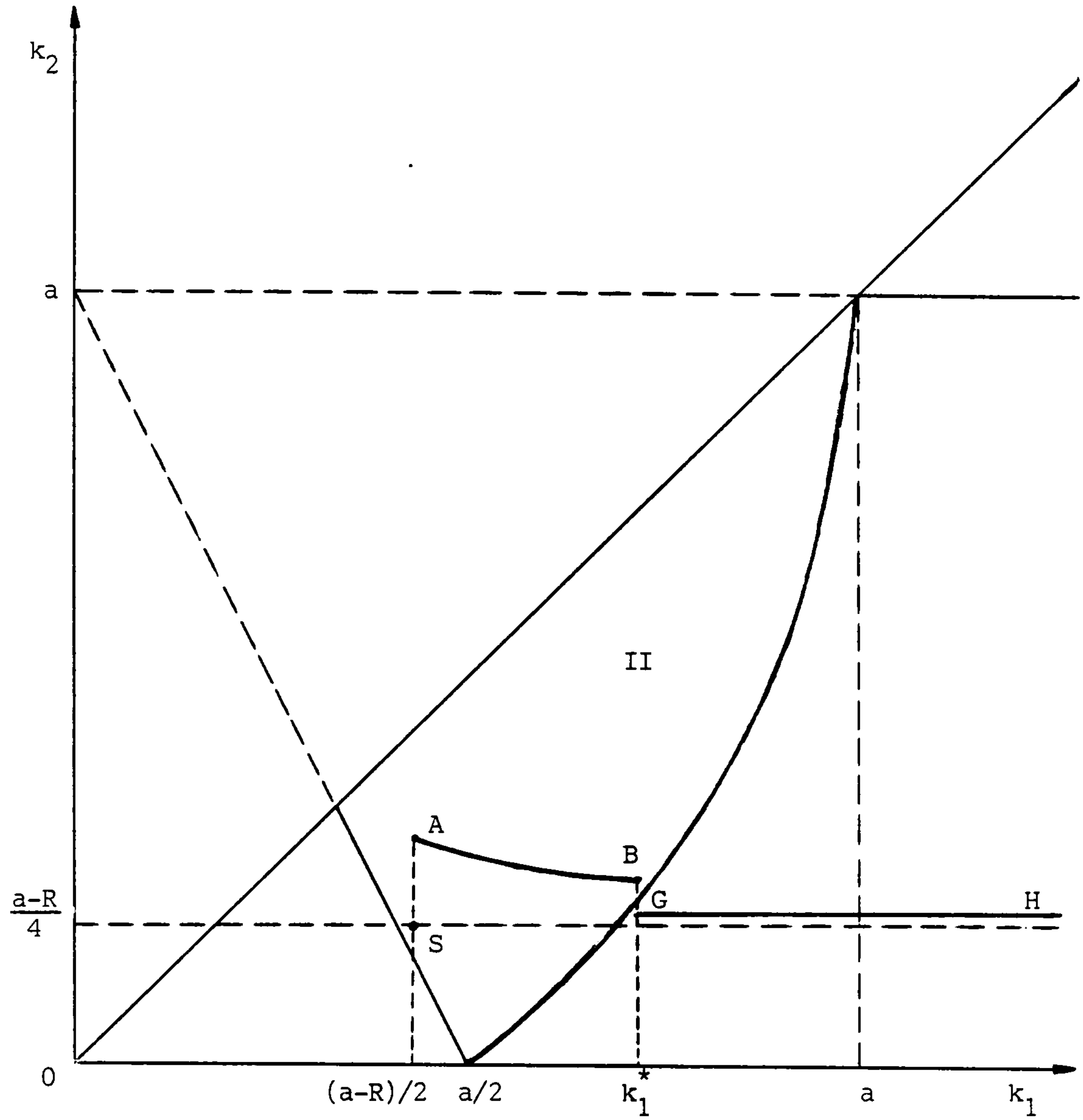

FIGURE 4.3

Increases in Capacity Levels after the Merger

merger.

However, in our setting the firms may increase capacity above their initial levels. Indeed, we are interested in such a case. We have in mind the situation in Figure 4.3. The initial capacity vector is represented by point S. Since the firms cannot decrease capacity levels (capacity costs are sunk), after the first two stages in the period the final capacity vector is a point in the union of $[A, B]$ and $[G, H]$, which are the parts of firm 2's subgame perfect reaction correspondence (see Figure 4.2). The exact location on the locus is determined by firm 1's capacity decision in the first stage. If firm 1 stays put at the initial capacity level, the final capacity vector is point A. If it increases its capacity to k_1^*, then the final capacity vector is either point B or point G, *etc.* From the fact that firm 1's equilibrium expected revenue is $(a - k_2)^2/4$ in region II and from expression (6), both studied in Section 3, we have that firm 1's expected profit along the segment $[A, B]$ is

$$\Pi_1 = \left(a + \sqrt{a^2 + 12Rk_1} \right)^2/36 - Rk_1 - F. \tag{8}$$

We can easily see that Π_1 is decreasing in k_1. Hence, the only possible position of the final capacity vector on $[A, B]$ is point A. Next, firm 1's expected profit along the segment $[G, H]$ is also decreasing in k_1. This is so since while capacity costs are increasing in k_1, its expected revenue is constant throughout since along the segment its expected revenue is $(a - k_2)^2/4$ and k_2 is constant. Thus point G is the only potential location for the final capacity vector on the segment $[G, H]$. However, firm 2 is indifferent between point B and point G when firm 1 chooses k_1^*. To avoid difficulties, we assume that firm 2 chooses point G in that case. Therefore, to find the final capacity vector we only need to compare firm 1's profits at points A and G.

Since we do not have a closed-form solution for expression (7) in Section 3, we proceed by choosing particular values of the parameters. Let $a = 100$, $R = 1$, and $F = 0$. Then at point A firm 1's capacity equals 49.5 and firm 2's capacity is 32.3576 (computed from expression (6)). Firm 1's expected profit at this point is 1094.3731 (computed from expression (8)). Firm 2's capacity at point G solves for expression (7) with the values of the parameters and is 27.6410, and its profit is 400.4766. Since firm 2's profit at point B is equal to that at point G and by using expression (6) firm 2's expected profit at point B is expressed only with k_1, we can get firm 1's capacity at point G which is 85.5421. Then firm 1's expected profit at point G is $(a-k_2)^2/4 - Rk_1 \simeq$ 1223.4124. With the values of the paramters, firm 1's expected profit is greater at point G than at point A. Hence, the final capacity vector is (k_1, k_2) = (27.6410, 85.5421).

The premerger price is 25.75 and the range of postmerger prices is [15.4885, 36.1795]. We may say that in some postmerger periods firms charge prices below the premerger price. The profit of each firm before the merger is 612.5625 while after the merger the expected profit of the merged firm is 1223.4124 and that of the outsider is 400.4766. The outsider's profit is lowered by 212.0859 while the joint profits of the insiders is lowered by only 1.7126.[10] We cannot find the "free rider problem" discussed in some models of a horizontal merger (e.g. Deneckere and Davidson (1985)). Rather, firms seek to merge to avoid the disadvantage of being the outsider. Finally, we have shown that the merged firm can get hurt from the merger even in a model of price competition.

5. CONCLUSIONS

Previous papers focusing on an incentive to merge never found that outsiders can suffer (profitwise) from mergers. Rather, they found that outsiders benefit from mergers and thus each firm is better off waiting for its rivals to merge. However, we have demonstrated in this chapter that a merger can hurt outsiders more than insiders. In such a case the free rider problem never occurs. We may expect that firms will either seek to merge or increase capacities to make mergers unattractive.

In addition, we have shown that in a strategic entry deterrence model where an incumbent and one potential entrant sequentially choose capacity levels and then engage in capacity-constrained price competition, the incumbent can prevent entry with excess capacity even in the case of a linear demand. This observation is in contrast to those in Dixit (1980) and Ware (1984), while Bulow, Geanakoplos, and Klemperer (1985) derived the same result.

FOOTNOTES

1. The papers share two other observations: (i) mergers, profitable or not, always increase price(s) and (ii) a merger to a monopoly is always profitable. We discussed them in Chaper III.

2. This observation is also a contrast to those in the previous papers. For explanation about Edgeworth cycle, see Tirole (1988), p. 211, 234.

3. We assume, for simplicity, that each firm can produce up to its capacity with zero marginal cost. We can generalize our arguments to the case of a positive constant marginal cost by considering the price in the text as the margin over the cost.

4. We adopt the efficient-rationing (parallel-rationing) rule for the analysis. For explanation of another often considered rationing rule, called the proportional-rationing (randomized-rationing), see Davidson and Deneckere (1986) or Tirole (1988), pp. 212–214.

5. The stage in which the two firms compete in prices, given their capacity vector (k_1, k_2), is defined as the (k_1, k_2) subgame.

6. However, the mixed strategy equilibrium of the (k_1, k_2) subgame always exists and is always unique. See Osborne and Pitchik (1986).

7. See Osborne and Pitchik (1986), pp. 244–245.

8. Bulow, Geanakoplos, and Klemperer (1985) demonstrate that the incumbent may prevent entry with excess capacity even with a quantity-setting model, if the Cournot reaction functions slope up.

9. See Brock and Scheinkman (1985), p. 373.

10. Notice that even with a small fixed cost the merger is profitable.

CHAPTER V

SUMMARY AND CONCLUSIONS

We have examined the free rider problem of entry deterrence, the profitability of a horizontal merger, and the effects of a horizontal merger on the outsiders' profits and industry prices, in markets where firms' capacity costs are sunk.

We have considered, in Chapter II, a three-stage entry deterrence model in which in the first stage all incumbent firms facing a single potential entrant choose their capacities simultaneously and independently, in the second stage the potential entrant, after observing the incumbents' capacity vector, chooses its capacity, and in the third and final stage the firms engage in capacity-constrained Cournot competition. Capacity costs are assumed to be sunk. If the potential entrant enters the market (chooses a positive capacity), it must sink in the second stage a fixed cost of entry as well as capacity costs. Studying the subgame perfect Nash equilibria of the game, we have shown that the free rider problem of entry deterrence can occur. That is, there are situations where both entry prevention and allowing entry are equilibria but entry prevention is mutually more profitable than allowing entry. This observation is to be contrasted with that of Gilbert and Vives (1986). They showed that the free rider problem never arises in a two-stage model which is simply an extension of the Bain-Sylos-Modigliani limit pricing model to the case of multiple incumbent firms.

One may argue that our observation about the free rider problem of entry deterrence in our model is not significant and thus not interesting in that if both entry prevention and allowing entry are equilibria and entry prevention is mutually more profitable than allowing entry, then even noncooperative incumbent firms will rather prevent entry. The criticism may be overcome by modifying our model. Consider a three-stage game which is the same as our model except that the incumbent firms are uncertain about the cost of entry. We conjecture that there are situations where if the incumbent firms could collude on their capacity decisions, entry prevention would be optimal, but the unique subgame perfect equilibrium allows entry, which implies that the free rider problem is significant. This conjecture is based on two recent studies. Bernheim (1984) found the same phenomena as in our original model but Waldman (1987a) demonstrated that in Bernheim's model the free rider problem of entry deterrence is significant if incumbent firms are uncertain about the exact investment in entry deterrence needed to deter entry. If our conjecture is correct, it is another contrast to Gilbert and Vives's limit pricing model because Waldman (1987a) observed that in Gilbert and Vives's model there is no evidence of underinvestment in entry deterrence even if the incumbent firms are uncertain about the fixed cost of entry.

In Chapter II we have also shown the following. Increasing the number of incumbent firms may cause the equilibrium price to increase and thus consumer welfare to decrease. In all subgame perfect equilibria no firm has excess capacity. There may exist a continuum of entry-allowing equilibria and the total equilibrium output may be different across the equilibria. A decrease in the cost of entry may increase the equilibrium price and hurt consumers. A decrease in demand may result in an increase in total output. Modifying the original model into a model with multiple potential entrants which choose capacities sequentially, we have established that whenever preventing entry is feasible, there is an entry-preventing equilibrium. We have also demonstrated that even with multiple potential entrants the free rider problem of entry deterrence can occur.

Several recent papers which theoretically analyze the profitability of a horizontal merger and its effects on the outsiders' profits and industry prices, all observed that a merger never decreases industry prices, a merger to a monopoly is always profitable, and a merger never hurts the outsiders. However, we have shown that these observations can be reversed even in the models which are prevalent in the industrial organization literature.

Considering a market for a homogeneous product where firms with sunk capacities compete in quantities and there are potential entrants, we have illustrated in Chapter III that a merger may decrease industry price and a merger to a monopoly is not always profitable. The latter observation is striking because no previous study explained explicitly the real life observation of the existence of an industry that is not a pure monopoly. In our model, firms may not merge all the way to a monopoly because then they cannot prevent entry. Indeed, we believe that in order to properly assess the effects of a horizontal merger, the possibility of entry should be taken into consideration.

In Chapter IV we have considered a market for a homogeneous product in which firms with sunk capacities engage in capacity-constrained price competition and presented an example where a merger hurts the outsider.

Another interseting observation in that example is that the premerger price is higher than the lowest price in the support of the postmerger mixed strategy price equilibrium. Interpreting mixed strategies as Edgeworth price cycles, we may say that there are periods after the merger in which prices are below the premerger price. We have also demonstrated in the example that unlike the previous static price competition models, a merger reduces the joint profits of the participating firms. In the example firms still have incentives to merge because a merger hurts the outsider more severely than the merged firm, which differs from the previous studies.

We have also shown in Chapter IV that excess capacity can prevent entry in a model where an incumbent and one potential entrant sequentially choose capacity levels and then engage in capacity-constrained price competition.

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APPENDICES

APPENDIX A

This appendix provides proofs of lemmas in Section 3 of Chapter II.

Proof of Lemma 4. Proof of part (c). We show that $D(k^{m+1}) \subseteq D(\tilde{k}^{m+1})$. Suppose $i \notin D(\tilde{k}^{m+1})$. Then $D(k^{m+1}) = D(\tilde{k}^{m+1})$, since it follows from part (b) that the equilibrium outputs of each firm are the same in the two subgames.

Consider the case where $i \in D(\tilde{k}^{m+1})$. Let firm j be a firm which has the largest capacity among the firms in $D(k^{m+1})\backslash\{i\}$. If it does not exist, then we are done. In the case where it does exist, we are only required to show that $j \in D(\tilde{k}^{m+1})$ by Lemma 3 (c). On the contrary, suppose that $j \notin D(\tilde{k}^{m+1})$. Then $t(\bar{X}_{-j}(k^{m+1})) > \bar{x}_j(k^{m+1}) = k_j = \tilde{k}_j \geq \bar{x}_j(\tilde{k}^{m+1}) = t(\bar{X}_{-j}(\tilde{k}^{m+1}))$.

First, we show that $\bar{x}_h(k^{m+1}) > \bar{x}_h(\tilde{k}^{m+1})$ for all $h \in \bar{M}\backslash\{i\}\backslash D(k^{m+1})$. By Lemma 3 (e), $\bar{x}_h(k^{m+1}) > \bar{x}_j(k^{m+1})$. Since $\bar{x}_j(k^{m+1})$ is equal to k_j, the capacity of firm h is larger than that of firm j, $k_h > k_j$. It implies with $j \notin D(\tilde{k}^{m+1})$ that $h \notin D(\tilde{k}^{m+1})$ (see Lemma 3 (c)). Since both firms h and j are not strictly constrained, we get $\bar{x}_h(\tilde{k}^{m+1}) = \bar{x}_j(\tilde{k}^{m+1})$ by Lemma 3 (d). Hence $\bar{x}_h(k^{m+1}) > \bar{x}_j(k^{m+1}) \geq \bar{x}_j(\tilde{k}^{m+1}) = \bar{x}_h(\tilde{k}^{m+1})$.

Second, $\bar{x}_h(k^{m+1}) \geq \bar{x}_h(\tilde{k}^{m+1})$ for all $h \in D(k^{m+1})\backslash\{i\}$, since firm h is strictly constrained in the k^{m+1} subgame and its capacities are the same in the two subgames.

Third, we show that $\bar{x}_i(k^{m+1}) > \bar{x}_i(\tilde{k}^{m+1})$. Suppose that $i \in L(k^{m+1})$. Then $\bar{x}_i(k^{m+1}) = k_i$. We get $\bar{x}_i(\tilde{k}^{m+1}) = \tilde{k}_i$ from the hypothesis that

$i \in D(\tilde{k}^{m+1})$. Since $k_i > \tilde{k}_i$, we are done. Suppose next that $i \notin L(k^{m+1})$. Then since firm j is strictly constrained and firm i is nonconstrained in the k^{m+1} subgame, we have $\bar{x}_i(k^{m+1}) > \bar{x}_j(k^{m+1}) = k_j$ by Lemma 3 (e). We also have $k_j = \tilde{k}_j \geq \bar{x}_j(\tilde{k}^{m+1}) > \bar{x}_i(\tilde{k}^{m+1})$. It follows from the fact that $j \notin D(\tilde{k}^{m+1})$ but $i \in D(\tilde{k}^{m+1})$, and from Lemma 3 (e). Both relations yield $\bar{x}_i(k^{m+1}) > \bar{x}_i(\tilde{k}^{m+1})$.

Hence from the three results above we have $\bar{X}_{-j}(k^{m+1}) > \bar{X}_{-j}(\tilde{k}^{m+1})$, which implies with Lemma 1 (a) that $t(\bar{X}_{-j}(k^{m+1})) \leq t(\bar{X}_{-j}(\tilde{k}^{m+1}))$. This leads to a contradiction.

We can show $L(k^{m+1}) \subseteq L(\tilde{k}^{m+1})$ by arguments similar to those of the first part. Q.E.D.

Proof of part (d). By Lemma 3 (d), $\bar{x}_i(\tilde{k}^{m+1}) = \bar{x}_j(\tilde{k}^{m+1}) = \bar{x}(\tilde{k}^{m+1})$ and $\bar{x}_i(k^{m+1}) = \bar{x}_j(k^{m+1}) = \bar{x}(k^{m+1})$ for $i, j \in \bar{M} \backslash L$. Let h be the cardinality of $\bar{M} \backslash L$. Then $\bar{x}(\tilde{k}^{m+1}) = t(\sum_{j \varepsilon L} \tilde{k}_j + (h-1)\bar{x}(\tilde{k}^{m+1}))$, $\bar{x}(k^{m+1}) = t(\sum_{j \varepsilon L} k_j + (h-1)\bar{x}(k^{m+1}))$, $\bar{X}(\tilde{k}^{m+1}) = \sum_{j \varepsilon L} \tilde{k}_j + h\bar{x}(\tilde{k}^{m+1})$, and $\bar{X}(k^{m+1}) = \sum_{j \varepsilon L} k_j + h\bar{x}(k^{m+1})$. Since firm i belongs to L, to complete the proof we need only to show that $\bar{x}(z) = t(z+(h-1)\bar{x}(z))$ is decreasing in z and $\bar{X}(z) = z + h\bar{x}(z)$ is increasing in z where z is a variable between $\sum_{j \varepsilon L} \tilde{k}_j$ and $\sum_{j \varepsilon L} k_j$. Indeed, $d\bar{x}(z)/dz = t'(\cdot)/[1-t'(\cdot)(h-1)] < 0$ and $d\bar{X}(z)/dz = 1 + ht'(\cdot)/[1-t'(\cdot)(h-1)] > 0$. Q.E.D.

Proof of part (e). Since $i \in D(\tilde{k}^{m+1})$, it follows from part (a) that there exists $\hat{k}_i > \tilde{k}_i$ such that $\hat{k}_i \notin D(\tilde{k}_{-i}^{m+1}, \hat{k}_i)$ but $\hat{k}_i \in L(\tilde{k}_{-i}^{m+1}, \hat{k}_i)$. Let k_i^* be min $(\hat{k}_i, k_i)$. Then $\bar{X}(k^{m+1}) \equiv \bar{X}(\tilde{k}_{-i}^{m+1}, k_i) = \bar{X}(\tilde{k}_{-i}^{m+1}, k_i^*)$. Notice that when $k_i > k_i^* = \hat{k}_i$, the equality comes from the fact in part (b). Hence, we need only to show that $\bar{X}(\tilde{k}_{-i}^{m+1}, k_i^*) > \bar{X}(\tilde{k}^{m+1})$.

In the first step, choose $k_i^1 \in (\tilde{k}_i, k_i^*]$ such that it is the largest capacity of firm i satisfying $D(\tilde{k}^{m+1}) = L(\tilde{k}^{m+1}\backslash k_i^1)$. It is possible from the facts in part (c). If $k_i^1 < k_i^*$, then in the second step choose $k_i^2 \in (k_i^1, k_i^*]$ such that it is the largest capacity of firm i satisfying $D(\tilde{k}_{-i}^{m+1}, k_i^1) = L(\tilde{k}_{-i}^{m+1}, k_i^2)$. Repeat steps until k_i^* is chosen. It follows from part (d) that at each step the total output increases.

Hence $\bar{X}(\tilde{k}_{-i}^{m+1}, k_i^*) > \bar{X}(\tilde{k}^{m+1})$. Q.E.D.

<u>Proof of Lemma 5</u>. It is obvious that firm m+1 is constrained in the $\tilde{k}^{m+1}$ subgame. It follows from the fact that $t(\sum_{i\varepsilon M} \tilde{k}_i) \geq r(\sum_{i\varepsilon M} \tilde{k}_i) - \tilde{k}_{m+1}$ (see Lemma 1 (c)), and from Lemma 3 (f).

Second, we show that firms with the same capacities in the two subgames are constrained in the $\tilde{k}^{m+1}$ subgame. Let firm i be a firm with $k_i = \tilde{k}_i$ where $i \in M$. Then we have $\sum_{j\varepsilon \overline{M}_{-i}} k_j > \sum_{j\varepsilon \overline{M}_{-i}} \tilde{k}_j$. It follows from the fact that the total capacity in the k^{m+1} subgame is larger than that in the $\tilde{k}^{m+1}$ subgame (see Lemma 1 (b)) and from the fact that $k_i = \tilde{k}_i$. We also have $\sum_{j\varepsilon \overline{M}_{-i}} k_j = \sum_{j\varepsilon \overline{M}_{-i}} \bar{x}_j(k^{m+1})$ from the fact that all firms are constrained in the k^{m+1} subgame. Now we are prepared to show $\tilde{k}_i \leq t(\sum_{j\varepsilon \overline{M}_{-i}} \bar{x}_j(\tilde{k}^{m+1}))$, which means that $i \in L(\tilde{k}^{m+1})$. Indeed, $\tilde{k}_i = k_i \leq t(\sum_{j\varepsilon \overline{M}_{-i}} \bar{x}_j(k^{m+1})) =$ $t(\sum_{j\varepsilon \overline{M}_{-i}} k_j) \leq t(\sum_{j\varepsilon \overline{M}_{-i}} \tilde{k}_j) \leq t(\sum_{j\varepsilon \overline{M}_{-i}} \bar{x}_j(\tilde{k}^{m+1}))$. The first inequality comes from the hypothesis that firm i is constrained in the k^{m+1} subgame and the last one follows from the fact that the output of each firm cannot exceed its capacity and from Lemma 1 (a).

Finally, we show that $i \in L(\tilde{k}^{m+1})$ for $i \in M$ with $k_i > \tilde{k}_i$. Suppose first that $\sum_{j\varepsilon\overline{M}_{-i}} \tilde{k}_j \leq \sum_{j\varepsilon\overline{M}_{-i}} k_j$. Then Lemma 1 (a) implies that $t(\sum_{j\varepsilon\overline{M}_{-i}} \tilde{k}_j) \geq t(\sum_{j\varepsilon\overline{M}_{-i}} k_j)$. Next since all firms are constrained in the k^{m+1} subgame, we have $k_i \leq t(\sum_{j\varepsilon\overline{M}_{-i}} \bar{x}_j(k^{m+1})) = t(\sum_{j\varepsilon\overline{M}_{-i}} k_j)$. Thus we derive the following: $\tilde{k}_i < t(\sum_{j\varepsilon\overline{M}_{-i}} \tilde{k}_j) \leq t(\sum_{j\varepsilon\overline{M}_{-i}} \bar{x}_j(\tilde{k}^{m+1}))$, which means that $i \in L(\tilde{k}^{m+1})$. Consider the case where $\sum_{j\varepsilon\overline{M}_{-i}} \tilde{k}_j > \sum_{j\varepsilon\overline{M}_{-i}} k_j$. From the fact that the total capacity in the k^{m+1} subgame is larger than that in the $\tilde{k}^{m+1}$ subgame, we get $\sum_{j\varepsilon\overline{M}_{-i}} \tilde{k}_j - \sum_{j\varepsilon\overline{M}_{-i}} k_j < k_i - \tilde{k}_i$. Then by Lemma 1 (b) we have $t(\sum_{j\varepsilon\overline{M}_{-i}} k_j) - t(\sum_{j\varepsilon\overline{M}_{-i}} \tilde{k}_j) < k_i - \tilde{k}_i$. Since $k_i \leq t(\sum_{j\varepsilon\overline{M}_{-i}} k_j)$, we obtain $\tilde{k}_i < t(\sum_{j\varepsilon\overline{M}_{-i}} \tilde{k}_j) \leq t(\sum_{j\varepsilon\overline{M}_{-i}} \bar{x}_j(\tilde{k}^{m+1}))$. Q.E.D.

Proof of Lemma 6. Since firm i is constrained in the k^{m+1} subgame, we have $t(\overline{X}_{-i}(k^{m+1})) \geq k_i$. We also have $t(\overline{X}_{-i}(\tilde{k}^{m+1})) < \tilde{k}_i$ from the hypothesis that firm i is nonconstrained in the $\tilde{k}^{m+1}$ subgame. Together with $k_i \geq \tilde{k}_i$, they yield $t(\overline{X}_{-i}(k^{m+1})) > t(\overline{X}_{-i}(\tilde{k}^{m+1}))$. Then it implies with Lemma 1 (a) that $\overline{X}_{-i}(k^{m+1}) < \overline{X}_{-i}(\tilde{k}^{m+1})$. Hence $\overline{X}(k^{m+1}) = \overline{X}_{-i}(k^{m+1}) + k_i \leq \overline{X}_{-i}(k^{m+1}) + t(\overline{X}_{-i}(k^{m+1})) < \overline{X}_{-i}(\tilde{k}^{m+1}) + t(\overline{X}_{-i}(\tilde{k}^{m+1})) = \overline{X}(\tilde{k}^{m+1})$. The strict inequality results from Lemma 1 (b). Q.E.D.

Proof of Lemma 7. Suppose on the contrary that there exists firm i, for $i \in M$, which is nonconstrained in the $\tilde{k}^{m+1}$ subgame. Then since it is constrained in the k^{m+1} subgame and nonconstrained in the $\tilde{k}^{m+1}$ subgame and since $k_i \geq \tilde{k}_i$, it follows from Lemma 6 that $\overline{X}(k^{m+1}) < \overline{X}(\tilde{k}^{m+1})$, which contradicts the hypothesis that $\overline{X}(k^{m+1}) = \overline{X}(\tilde{k}^{m+1})$. Q.E.D.

Proof of Lemma 8. Let k_{m+1} be a capacity in $A(k^m)$ which firm m+1 chooses in the k^m subgame. In the case where it is zero, the proof is trivial. Consider the case where $k_{m+1} > 0$. We must show that firm m+1 is constrained. Suppose on the contrary that it is nonconstrained. Then there exists a $\tilde{k}_{m+1} > 0$ such that $\Pi(k^m, \tilde{k}_{m+1}) > \Pi(k^m, k_{m+1})$, which contradicts the hypothesis that $k_{m+1} \in A(k^m)$. Indeed, let $\tilde{k}_{m+1} = \bar{x}_{m+1}(k^m, k_{m+1})$. Then $\tilde{k}_{m+1} < k_{m+1}$ and firm m+1 is not strictly constrained in the $(k^m, \tilde{k}_{m+1})$ subgame. Thus by Lemma 4 (b) we get $\bar{x}_i(k^m, \tilde{k}_{m+1}) = \bar{x}_i(k^m, k_{m+1})$ for all $i \in \overline{M}$, which implies with $R > 0$ and $\tilde{k}_{m+1} < k_{m+1}$ that $\Pi(k^m, \tilde{k}_{m+1}) > \Pi(k^m, k_{m+1})$. Q.E.D.

Proof of Lemma 10. We first show that in the $\tilde{k}^m$ subgame any capacity of firm m+1, $\hat{k}_{m+1}$, such that $i \notin L(\tilde{k}^m, \hat{k}_{m+1})$ for some $i \in \overline{M}$, cannot be an element of $A(\tilde{k}^m)$. By Lemma 8, if $m+1 \notin L(\tilde{k}^m, \hat{k}_{m+1})$, then $\hat{k}_{m+1} \notin A(\tilde{k}^m)$. Suppose next that there exists a $\hat{k}_{m+1}$ such that $m+1 \in L(\tilde{k}^m, \hat{k}_{m+1})$ but $i \notin L(\tilde{k}^m, \hat{k}_{m+1})$ for some $i \in M$. Then $\hat{k}_{m+1} > k_{m+1}$ (see Lemma 4 (c)) and $\overline{X}(\tilde{k}^m, \hat{k}_{m+1}) > \overline{X}(k^{m+1})$ by Lemma 6. Let k'_{m+1} satisfy $\overline{X}(k^m, k'_{m+1}) = \overline{X}(\tilde{k}^m, \hat{k}_{m+1})$ and $m+1 \in L(k^m, k'_{m+1})$. Such a k'_{m+1} exists since $\overline{X}(k^{m+1}) < \overline{X}(\tilde{k}^m, \hat{k}_{m+1}) \leq \overline{X}(k^m, \hat{k}_{m+1})$ (see Lemma 4 (e)) and $\overline{X}(\cdot)$ is continuous in k_{m+1} (see Lemma 3 (a)). Notice that $\hat{k}_{m+1} \geq k'_{m+1}$ and $\hat{k}_{m+1} - k'_{m+1} \leq \sum_{i \in M}(k_i - \tilde{k}_i)$. Let k''_{m+1} be such that $\overline{X}(\tilde{k}^m, k''_{m+1}) = \overline{X}(k^{m+1})$. Then $m+1 \in L(\tilde{K}^m, k''_{m+1})$ since $\hat{k}_{m+1} > k''_{m+1}$ (see Lemma 4 (e)) and $m+1 \in L(\tilde{k}^m, \hat{k}_{m+1})$. And by Lemma 7, $M \subset L(\tilde{k}^m, k''_{m+1})$. We are prepared to show that $\Pi(\tilde{k}^m, k''_{m+1}) > \Pi(\tilde{k}^m, \hat{k}_{m+1})$, which implies $\hat{k}_{m+1} \notin A(\tilde{k}^m)$. The profit of firm m+1 with k''_{m+1} is $\Pi(\tilde{k}^m, k''_{m+1}) = k''_{m+1}[f(\overline{X}(k^{m+1})) - V - R] - F = \Pi(k^{m+1}) + [f(\overline{X}(k^{m+1})) - V - R](k''_{m+1} - k_{m+1})$ and that with $\hat{k}_{m+1}$ is $\Pi(\tilde{k}^m, \hat{k}_{m+1}) = \hat{k}_{m+1}[f(\overline{X}(k^m, k'_{m+1})) - V - R\ ] - F =$

$\Pi(k^m,\ k'_{m+1})\ +\ [(f(\bar{X}(k^m,\ k'_{m+1}))-V-R](\hat{k}_{m+1}-k'_{m+1})$. From the fact that $k_{m+1}\in A(k^m)$, we have $\Pi(k^{m+1}) \geq \Pi(k^m,\ k'_{m+1})$. We also get $[f(\bar{X}(k^{m+1}))-V-R](k''_{m+1}-k_{m+1}) > [f(\bar{X}(k^m,\ k'_{m+1}))-V-R](\hat{k}_{m+1}-k'_{m+1})$ since $\bar{X}(k^m,\ k'_{m+1}) > \bar{X}(k^{m+1})$ and $k''_{m+1} - k_{m+1} = \sum_{i\varepsilon M} (k_i - \tilde{k}_i) \geq \hat{k}_{m+1} - k'_{m+1}$. Both facts yield the inequality we want to establish.

We have shown that $A(\tilde{k}^m)$ contains only a capacity (or capacities), $\hat{k}_{m+1}$, satisfying $L(\tilde{k}^m,\ \hat{k}_{m+1}) = \bar{M}$. Notice then that $\Pi(\tilde{k}^{m+1}) > \Pi(\tilde{k}^m,\ \hat{k}_{m+1})$ for all $\hat{k}_{m+1} > 0$ such that $\hat{k}_{m+1} \neq \tilde{k}_{m+1}$ and $L(\tilde{k}^m,\ \hat{k}_{m+1}) = \bar{M}$. It follows from the fact that Lemma 5 implies $L(\tilde{k}^{m+1}) = \bar{M}$, and from the fact that $\tilde{k}_{m+1} = r(\sum_{i\varepsilon M} \tilde{k}_i)$. Hence we have the following: if $\tilde{k}_{m+1} > 0$, then $\tilde{k}_{m+1} = \operatorname{argmax}_{k_{m+1}>0} \Pi(\tilde{k}^m,\ k_{m+1})$ and thus $A(\tilde{k}^m) = \{\tilde{k}_{m+1}\}$; if $\tilde{k}_{m+1} = 0$, then $\operatorname{argmax}_{k_{m+1}>0} \Pi(\tilde{k}^m,\ k_{m+1})$ does not exist and thus $A(\tilde{k}^m) = \{\ 0\ \}$. Q.E.D.

<u>Proof of Lemma 11</u>. We first show that in the $(Y/m)^m$ subgame, any positive capacity of firm m+1 ends up with nonpositive profit to the firm and thus $\tilde{k}_{m+1} = 0$. By Lemma 3 (b), for any $k_{m+1} > 0$, we have $\bar{x}_i((Y/m)^m,\ k_{m+1}) = \bar{x}((Y/m)^m,\ k_{m+1})$ for all $i\in M$. Suppose that $k_{m+1} > 0$ satisfies $\bar{x}((Y/m)^m,\ k_{m+1}) < Y/m$. That is, $i\notin L((Y/m)^m,\ k_{m+1})$ for all $i\in M$. Then since $\bar{x}_{m+1}((Y/m)^m,\ k_{m+1}) = \bar{x}_{m+1}(k^m_\infty,\ k_{m+1})$ and $\bar{x}((Y/m)^m,\ k_{m+1}) = \bar{x}(k^m_\infty,\ k_{m+1})$ (see Lemma 4 (b)), $\Pi((Y/m)^m,\ k_{m+1}) = \Pi(k^m_\infty,\ k_{m+1}) \leq E(k^m_\infty) \leq 0$. Suppose next that $k_{m+1} > 0$ satisfies $\bar{x}((Y/m)^m,\ k_{m+1}) = Y/m$. Then the total output of incumbent firms is just the limit output Y and thus $\Pi(\ (Y/m)^m,\ k_{m+1}) \leq 0$.

We now show that $L((Y/m)^m,\ 0) = \bar{M}$. Trivially, firm m+1 is constrained. We must show that all incumbent firms are constrained. Suppose on the contrary that $\bar{x}((Y/m)^m,\ 0) < Y/m$. Then there exists $k_{m+1} > 0$ such that

$\Pi((Y/m)^m, k_{m+1}) > 0$. Indeed, let $k_{m+1} = r(Y)$. In the $((Y/m)^m, r(Y))$ subgame firm m+1 is constrained by Lemma 3 (f) but all incumbent firms are nonconstrained by Lemma 4 (c). We have then $m\bar{x}((Y/m)^m, r(Y)) < Y$. Thus the profit of firm m+1, $\Pi((Y/m)^m, r(Y))$, is greater than zero:

$r(Y)[f(m\bar{x}((Y/m)^m, r(Y))+r(Y))-V-R] - F > r(Y)[f(Y+r(Y))-V-R] - F = 0.$

Since all incumbent firms are nonconstrained in the $((Y/m)^m, r(Y))$ subgame, Lemma 4 (b) implies $\bar{X}(k_\infty^m, r(Y)) = \bar{X}((Y/m)^m, r(Y))$. Hence $\Pi(k_\infty^m, r(Y)) = \Pi((Y/m)^m, r(Y)) > 0$, which contradicts the hypothesis that $E(k_\infty^m) \leq 0$. Q.E.D.

<u>Proof of Lemma 12</u>. Let $k_{m+1} = r(\sum_{i\varepsilon M} k_i) = r(Y)$. We first show that $L(k^{m+1}) = \bar{M}$. From Lemma 3 (f) it follows that firm m+1 is constrained. Suppose on the contary that $i \notin L(k^{m+1})$ for some $i \in M$. Then $\sum_{j\varepsilon M} \bar{x}_j(k^{m+1}) < Y$ implies $\Pi(k^{m+1}) = k_{m+1}[f(\bar{X}(k^{m+1}))-V-R] - F >$

$k_{m+1}[f(Y+k_{m+1})-V-R] - F = 0$. This contradicts the hypothesis that firm m+1 stays out in the equilibrium of the k^m subgame.

Next we show that $k_{m+1} \in A(k^m)$. Suppose that $k_{m+1} \notin A(k^m)$. Then there should exist $\hat{k}_{m+1} > 0$ such that $\Pi(k^m, \hat{k}_{m+1}) > \Pi(k^{m+1}) = 0$. This again leads to a contradiction to the hypothesis that $\bar{k}_{m+1} = 0$ in the k^m subgame.

Then we have $A(\tilde{k}^m) = \{r(\sum_{i\varepsilon M} \tilde{k}_i)\}$. It follows from the fact that $L(k^{m+1}) = \bar{M}$ and $k_{m+1} \in A(k^m)$, and from Lemma 10.

Hence $E(\tilde{k}^m) > 0$ and $A(\tilde{k}^m) = \{r(\sum_{i\varepsilon M} \tilde{k}_i)\}$ imply that in the $\tilde{k}^m$ subgame, $\bar{k}_{m+1} = r(\sum_{i\varepsilon M} \tilde{k}_i)$. The fact that all firms are constrained in the equilibrium of the $\tilde{k}^m$ subgame comes from the results above and from Lemma 5. Q.E.D.

APPENDIX B

This appendix provides subgame perfect equilibria for different values of F and α in the case of a single incumbent firm. The incumbent firm is called firm 1 and a potential entrant is called firm 2.

(1) Compute first $E(k^1_\infty)$.

(i) If $0 < \alpha \leq (a-1)/5$, then $E(k^1_\infty) = (a-1-2\alpha)(a+\alpha-1)/9b - F$.

(ii) If $(a-1)/5 \leq \alpha < \min(a-1, 1)$ or $\alpha = 1$ with $2 < a \leq 6$, then $E(k^1_\infty) = (a-\alpha-1)^2/8b - F$.

(iii) If $a-1 \leq \alpha \leq 1$ with $1 < a \leq 2$, then $E(k^1_\infty) < 0$.

(2) Subgame perfect equilibria in the case of $E(k^1_\infty) > 0$.

(i) If $0 < \alpha \leq (a-1)/5$, then $\bar{k}_1 = k^E = (a-1)/b - 2\sqrt{(a-1-2\alpha)(a+\alpha-1)}/3b$ and $\bar{k}_2 = k^E_2 = \sqrt{(a-1-2\alpha)(a+\alpha-1)}/3b$.

(ii) If $(a-1)/5 \leq \alpha \leq (2-\sqrt{2})(a-1)/2$, then $\bar{k}_1 = k^E = (a-1)/b - \sqrt{2}\,(a-\alpha-1)/2b$ and $\bar{k}_2 = k^E_2 = \sqrt{2}\,(a-\alpha-1)/4b$.

(iii) If $(2-\sqrt{2})(a-1)/2 \leq \alpha < \min(a-1, 1)$ or $\alpha = 1$ with $2 < a \leq 3+\sqrt{2}$, then $\bar{k}_1 = k^S = (a-1)/2b$ and $\bar{k}_2 = k^S_2 = (a-1)/4b$.

(3) Subgame perfect equilibria in the case of $E(k^1_\infty) \leq 0$.

The limit output, Y, is equal to $(a-1)/b - 2\sqrt{F/b}$.

(i) If $Y \leq X^c$ or equivalently $(a-1)^2/16b - F \leq 0$,

then $\bar{k}_1 = k^c = (a-1)/2b$ and $\bar{k}_2 = 0$.

(ii) If $Y > X^c$ or equivalently $(a-1)^2/16b - F > 0$ and

$((a-1)(\sqrt{2}-1))^2/32b - F < 0$, then $\bar{k}_1 = Y$ and $\bar{k}_2 = 0$.

(iii) If $((a-1)(\sqrt{2}-1))^2/32b - F > 0$, then entry is allowed:

$\bar{k}_1 = k^s = (a-1)/2b$ and $\bar{k}_2 = k^s_2 = (a-1)/4b$.

(iv) If $((a-1)(\sqrt{2}-1))^2/32b - F = 0$, then firm 1 is indifferent

between preventing and allowing entry: both (Y, 0) and

$((a-1)/2b, (a-1)/4b)$ are equilibria.

APPENDIX C

This appendix provides subgame perfect symmetric equilibria for different values of F and α when there are two incumbent firms. The incumbent firms are labeled as firm 1 and firm 2 respectively and a potential entrant is labeled as firm 3.

(1) Compute first $E(k_\infty^2)$.

(i) If $0 < \alpha \leq (a-1)/5$, then $E(k_\infty^2) = (a-3\alpha-1)(a+\alpha-1)/16b - F$.

(ii) If $(a-1)/5 \leq \alpha < \min((a-1)/2, 1)$ or $\alpha = 1$ with $3 < a \leq 6$, then $E(k_\infty^2) = (a-2\alpha-1)^2/12b - F$.

(iii) If $(a-1)/2 \leq \alpha \leq 1$ with $1 < a \leq 3$, then $E(k_\infty^2) < 0$.

(2) Subgame perfect symmetric equilibria in the case of $E(k_\infty^2) > 0$.

(i) If $0 < \alpha \leq (a-1)/5$, then $\bar{k}_1 = \bar{k}_2 = k^E = (a-1)/2b - \sqrt{(a-3\alpha-1)(a+\alpha-1)}/4b$ and $\bar{k}_3 = k_3^E = \sqrt{(a-3\alpha-1)(a+\alpha-1)}/4b$.

(ii) If $(a-1)/5 \leq \alpha \leq (3-\sqrt{3})(a-1)/6$, then $\bar{k}_1 = \bar{k}_2 = k^E = (a-1)/2b - \sqrt{3}(a-2\alpha-1)/6b$ and $\bar{k}_3 = k_3^E = \sqrt{3}(a-2\alpha-1)/6b$.

(iii) If $(3-\sqrt{3})(a-1)/6 \leq \alpha < \min((a-1)/2, 1)$ or $\alpha = 1$ with $3 < a \leq 4+\sqrt{3}$, then $\bar{k}_1 = \bar{k}_2 = k^S = (a-1)/3b$ and $\bar{k}_3 = k_3^S = (a-1)/6b$.

(3) Subgame perfect symmetric equilibria in the case of $E(k_\infty^2) \le 0$.

The limit output, Y, is equal to $(a-1)/b - 2\sqrt{F/b}$.

(i) If $Y \le X^c$ or equivalently $(a-1)^2/36b - F \le 0$,
then $\bar{k}_1 = \bar{k}_2 = k^c = (a-1)/3b$ and $\bar{k}_3 = 0$.

(ii) If $Y > X^c$ or equivalently $(a-1)^2/36b - F > 0$ and $(34\sqrt{bF} - 7(a-1))^2 \le 32(a-1)^2$, then $\bar{k}_1 = \bar{k}_2 = Y/2$ and $\bar{k}_3 = 0$ form a symmetric equilibrium.

(iii) If $(2a-2-3\alpha)^2/72b - F > 0$ and $0 < \alpha < 2(a-1)/3$, then $\bar{k}_1 = \bar{k}_2 = k^s = (a-1)/3b$ and $\bar{k}_3 = k_3^s = (a-1)/6b$ form a symmetric equilibrium.

(iv) If $((a-1)(\sqrt{2}-1))^2/72b - F \ge 0$, then $\bar{k}_1 = \bar{k}_2 = k^s = (a-1)/3b$ and $\bar{k}_3 = k_3^s = (a-1)/6b$ form a symmetric equilibrium.

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