Essays in Social Security: Net of Benefits Tax Rates, Labor Supply, Savings and Welfare

by

Robert R. Beach, Jr.

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APPROVED:

Firouz Gahvari, Chairman

Helmuth Cremer

Jacques Cremer

Yannis Ioannides

Richard Steinberg

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(ABSTRACT)

In the standard case in which the interest rate is assummed to be greater than the rate of population growth, implementation of a social security program leads to a reduction in capital formation and a loss of welfare of the representative individual. This dissertation asks whether the parameters of a stylized social security program can be manipulated to reduce this welfare loss. By attaching weights to the earnings used in computing the average monthly earnings, an instrument is created which the social security administrator can use to manipulate the net marginal tax rates and the relative cost of leisure between years. If, as a result, aggregate savings increase, then steady state welfare may also increase.

The effect of changing the weights in the benefit formula is considered first in a simple three period partial equilibrium model. Individuals work for two periods and are retired in the third. It is shown, under assumptions of separability, that first period labor supply must go up and second period labor supply must go down in response to an increase in the earnings weight attached to the first period. Furthermore,

although there is an element of ambiguity, a strong case can be made that aggregate savings must increase. It is also shown that, contrary to intuition, a zero net tax is not neutral and in fact must lead to a reduction in capital formation and welfare.

These same issues are then considered in a many-period model in which interest rates and wage rates are allowed to respond to changes in aggregate savings. It is found that alternatives to the current program that provide more weight to earnings of younger workers can reduce the welfare loss by a small amount. Because of the intractability of the many-periods case a computer simulation is used to perform the analysis.

In addition, the adjustment costs of a public savings program are considered. (Feldstein, among others, has suggested that social security be used as a vehicle for a public savings program to increase private investment in the economy.) It is shown that while such a program would adversely affect that welfare of a number of generations, these welfare losses are quite small: less than 0.05% for all the cases considered.

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PREFACE

This dissertation is composed of two essays. Each addresses the issue of social security reform. In the first, the social security benefit formula is used to reduce the welfare loss that a pay-as-you-go program imposes. In the second, the adjustment costs of a public savings program created through an increase in the social security payroll tax are considered.

The first five chapters and appendices A and B compose the first essay: Net Social Security Taxes and Life-Cycle Decisions. The next four chapters and appendices C and D compose the second essay: The Adjustment Costs of a Public Savings Program.

I: NET SOCIAL SECURITY TAXES AND LIFE-CYCLE DECISIONS

1. INTRODUCTION

A number of theoretical and empirical studies have concluded that implementation of a pay-as-you-go social security program reduces, through what Feldstein (1974a) refers to as an asset substitutiion effect, the aggregate capital stock.[1] Hence, in the subsequent steady state the average worker-consumer is made worse off. This paper does not dispute this conclusion. Instead, the issue addressed here is whether or not the social security benefit formula can be adjusted to reduce the welfare loss the program imposes.

The framework employed is an overlapping generations growth model. Individuals work for a fixed number of years and are retired for a fixed number of years. They must decide how much to consume during each year of life and how much to work during each of the working years. This framework is consistent with the suggestion by Diamond (1977) that social security is best analyzed in terms of an individual's life-cycle decisions rather than as an annual tax-transfer system.

Within the life-cycle framework the net effect of social security can be considered. If individuals are allowed to make a labor-leisure decision, then not only does the wage tax affect a worker's decisions but these decisions are also affected by the manner in which benefits are computed.

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[1] See Feldstein (1974a), Samuelson (1975a, 1975b), or Kotlikoff (1979), among others.

Thus, in the life-cycle framework it is useful to think in terms of a net social security tax: an additional hour worked during a certain year of life is not only taxed at a certain rate but also generates benefits that are received in the future. By taking into account the present value of these marginal benefits a net marginal social security tax can be computed. This net tax can be negative as well as positive and can raise or lower the opportunity cost of leisure relative to other years and thus affect the labor supply decisions of individuals.

Papers by Burkhauser and Turner (1985) and Browning (1985), among others, compute the net marginal social security tax across working cohorts at different points in time. For instance, a married retired worker who was 65 in 1982 faced a net tax of -20.90 percent (a subsidy) as computed by Burkhauser and Turner. Browning's results are similar in that he computes a net tax that declines for older workers (although his net rates are higher than Burkhauser and Turner's due to choosing a higher discount rate).

By attaching weights to the earnings used in computing the average indexed monthly earnings, an instrument is created by which the social security administration can manipulate the net marginal tax rates and the relative cost of leisure between years. As these weights change, labor supply and aggregate savings change. If the response to aggregate savings is positive, in the long run the interest rate must go down and the wage rate must go up. This has a positive effect on life-cycle full income and can result in an increase in the utility of a representative individual.

The effect of changing the weights in the benefit formula is considered first in a simple three-period, partial-equilibrium model. Individuals work for two periods and are retired in the third. It is shown, under assumptions of strong separability, that first period labor supply must go up and second period labor supply must go down in response to an increase in the first period earnings weight. However, the response of aggregate savings is ambiguous. Furthermore, it is shown that the optimal weights, assuming the interest rate and wage rate do not respond to changes in aggregate savings, depend on the elasticity of labor supply with respect to the earnings weights in the two working periods.

These same issues are then considered in a many-period model in which interest rates and wage rates are allowed to respond to changes in aggregate savings. The advantage of the many-periods model is that it allows for what Summers (1981) refers to as "human wealth" effects (that is, the effect of changes in the interest rate on the present value of life-cycle income). It is also more likely to produce an interest elasticity of savings that is positive. The shortcoming of the many periods model is its intractability and the fact that with leisure in the utility function a closed-form solution is unattainable. For these reasons a numerical simulation is used to perform the analysis.

It is found that alternatives to the current program (i.e. the program defined by setting all weights to one) can reduce the welfare loss by a small amount. The two alternatives considered are the uniform net tax (the weights are set so that the net tax is the same across all years) and an exempted net tax (the weights of the early working years are set to one and the weights of the later years are set to zero). Solutions are computed for a representative worker who works for 40 periods and is retired for 10 periods. The consistent result is that the uniform net tax reduces the welfare loss by a small amount and, that the exempted net tax usually reduces it by an additional small amount.

In addition, the optimal weights are computed for a worker who works for 5 periods and is retired for 3 periods. The results suggest that the pattern of weights under a uniform net tax is similar to the optimal pattern in that the weights applied to the youngest workers are the highest. However, it is shown that the optimal weights are much greater for the youngest workers and much lower for the oldest workers than in the case of uniform net taxes. The optimal weights for the oldest workers are actually negative in the most inelastic case.[2]

It is also shown that a zero net tax is not sustainable with a pay-as-you-go constraint. A zero net tax has the same property as a fully-funded tax in that--with wages and interest rates fixed--it has no effect on an individual's budget constraint. This is often taken as proof of the neutrality of a given policy. However, this is essentially a partial equilibrium result. When wages and interest rates are allowed to respond to changes in capital formation then it can be shown that a zero net tax is not neutral in the standard case in which $r > n$. In this

[2] A negative weight for a given period means that this periods earnings are subtracted out of the averaging formula.

case if the weights are set so that net taxes are zero, the social security program must incur a deficit. This deficit adversely affects capital formation. Consequently, a zero net tax must result in a loss of steady state welfare.

2. THE LITERATURE

The literature on social security as it relates to this paper is divided into two parts. First, the literature on the dynamic effects of taxes and social security on consumption, savings, and labor supply decisions is considered. Second, the literature on net social security taxes is considered.

The Dynamic Effects of Social Security

There is no attempt made here to provide a comprehensive survey of the literature on dynamic taxation or even a comprehensive survey of the life-cycle effects of social security. Rather the objective here is the following. First, to provide a representative sampling of the effects of social security and taxation in a life-cycle framework. And second, to provide a theoretical basis for the computer simulation experiments discussed below. Throughout, the emphasis will be on how changes in the economic environment dynamically affect savings and consumption. Notably absent is an extensive discussion on the literature in which social security is placed in an environment of uncertain lives and imperfect annuities markets as in Abel (1985) or Hubbard (1987). Also notably absent is any discussion in which social security is analyzed as an intergenerational voting game as in Browning (1975) or Veall (1985).

If utility is maximized over an individual's lifetime subject to a budget constraint; what pattern of life-cycle consumption results? In this framework, of course, it is the interaction of the interest rate, the subjective rate of discount (so-called time impatience), and the population growth rate that is pivotal in the intertemporal decisions of individuals. This is discussed in Tobin (1967). For simplicity, he assumes preferences are such that each individual attempts to equalize the present value of consumption over time. If it is assumed the real interest rate is zero, then consumption in all periods will be the same. If the real interest rate, r, is greater than zero, then consumption is shifted to the future because the price of future consumption has gone down. This occurs even if individuals are intertemporally neutral and display no time impatience. If a subjective rate of discount, δ, is allowed for, then consumption is spread evenly with respect to the difference between the market interest rate and the subjective discount factor (i.e. $r-\delta$). Thus as r goes down or δ goes up savings will be negatively affected. Individuals will have less need of savings since they will wish to consume less in the future and more now.

In the no-bequest version of the life-cycle model all that is saved is dissaved. Savings during the working years must provide for consumption in the retirement years. Tobin argues that positive savings occur in the aggregate not because of time impatience but because of positive rates of population and technical growth.

When aggregate savings are such that output, capital stock, and labor all grow at the same rate then the economy is in a steady state or golden age. In a one-asset world with no government, physical capital is the only kind of wealth held by individuals. If the government also provides assets--either in the form of money or public debt--then the golden age level of capital will be less than private wealth. If the government owns a share of the real capital than private wealth will be less than the level of real capital.

The golden age path then depends on (ignoring for the moment technical progress) the rate of growth of the labor force, n, and the rate of return on capital, r. As n grows faster capital must rise accordingly to maintain the steady state. With more capital the interest rate drops and the wage rate goes up. As the interest rate goes down consumption is shifted backward in time since consumption tomorrow has become more expensive relative to consumption today. Desired wealth relative to income goes up as the interest rate grows since now the price of future goods relative to current goods has gone down.

Summers (1981) decomposes these effects into intertemporal substitution effects and what he terms a "human wealth" effect. In particular he considers the shortcomings of the two-period life-cycle model. The two-period model obscures two things. First, all savings are eventually dissaved. Net positive savings arise in the aggregate because the young who save are more numerous than the retired old who dissave. In a two-period model the savings decision is equivalent to the first period consumption decision and does not adequately model the dissaving of older generations. Second, the role of future labor income is obscured. As the interest rate changes the present value of life-cycle earned income is affected. This is what Summers refers to as the human wealth effect. For instance, if the interest rate were to go up (say because of some government policy that adversely affects capital formation), future income is discounted more and people save more due to income effects. That is, individuals consume less and work more across their life-cycle. It is this discounting or human wealth effect across many periods that leads to the positive response of aggregate savings to the interest rate. It is easy to demonstrate this positive response does not necessarily occur in a two-period model (for example, in the case of log utility). Furthermore, Summers demonstrates that for most reasonable parameterizations of his model the human wealth effect that arises from a change in the interest rate is quantitatively much more important than the intertemporal substitution effects that this change creates. In other words, the elasticity of intertemporal substitution has little effect on savings.

The Summers paper also highlights the importance of considering these matters in a general equilibrium framework. For example, taxation of labor income is often assumed to be the same as taxation of consumption. This arises from the fact that in a partial equilibrium in which wages and interest rates do not change, the budget constraints under either of the two taxes can be shown to be the same. This result, however, does not hold in a general equilibrium framework. The timing of the two taxes is different. Consumption taxation extracts revenue later in an individuals lifetime than does wage taxation. As a result consumption taxation causes more savings in the younger years. Since capital formation is different under the two taxes, interest rates and wages evolve differently and thus intertemporal substitution and human wealth effects are not the same in the two cases. This is demonstrated in Summers' simulations in which he considers two alternatives to taxing capital: a wage tax and a consumption tax. It is assumed that in each period the government raises the same amount of revenue under each tax (differential tax analysis). A shift from a capital tax to a labor tax increases life-cycle income by 14% and welfare by 5%; whereas a shift to a consumption tax increases life-cycle income by 18% and welfare by 12%. The fact that a shift to a consumption tax increases income and reduces the welfare loss more than a shift to a labor tax is primarily due to the fact that its burden is shared by both workers and retirees and thus the impact on savings of the younger, more numerous generations is not as great. A wage tax has a more adverse effect on capital accumulation.

It is important to understand the underlying dynamics of the steady state analysis here and why such relatively large changes in steady state welfare occur. By removing the 50% tax on capital initial real interest rates rise substantially and substantially reduce initial "human wealth" (that is, the present value of lifetime income). It is this rather sharp change in human wealth that brings forth the reduction in consumption that results in the jump in savings of the young. There are also secondary effects at work. On the one hand, there are the substitution effects that arise from the decrease in the relative price of future consumption. This also has a positive effect on savings as individuals attempt to finance the desired increase in future consumption. On the other hand, there is an additional income effect as the price of consumption across each year goes down as the interest rate rises. This has a negative effect on savings since for any given level of human wealth individuals can afford more consumption in each year. On net, though, the human wealth effect would be expected to dominate and a substantial increase in savings should result. This means in the subsequent steady state, interest rates will be lower, wages higher, human wealth higher, and hence steady state welfare will increase on the order of the 12% reported by Summers.

The Tobin and Summers papers consider life-cycle decisions with perfect capital markets and no uncertainty. Although there are certain inherent limitations to this approach[3] nonetheless much of the discussion as to the effects of social security on the economy and capital accumulation is couched in just such a framework. How then does social security affect the life-cycle decisions of individuals? Kotlikoff (1977) identifies three separate effects of social security on life-cycle savings. These are:

- A replacement effect that is negative.
- A retirement effect that can be positive or negative.
- A yield effect that can be positive or negative.

[3] The most notable shortcoming is that if $r>n$ the implementation of a pay-as-you-go social security program usually results in a loss of welfare. Why would a rational society impose this on itself?

The replacement effect represents a substitution of social security for private savings. If the social security program offers an implicit yield on paid-in taxes equal to the market rate of interest, lifetime wealth is unaffected by the social security program and consumption is unaltered across the life-cycle (this is, of course, assuming interest rates and wages are unaltered by the program). If it is further assumed that the retirement decision is unaffected, then social security simply replaces private savings dollar for dollar. This is the essence of the replacement effect.

Because of disincentives such as the earnings test and incentives such as early retirement at age 62, social security, it is argued, induces individuals to retire before they would have otherwise. Feldstein (1974a) argues this effect is positive since people work fewer years they need to save more for retirement. Kotlikoff, though, argues it could also be negative since individuals who retire early earn less so that savings could be less.

Further, social security affects an individual's lifetime income. This Kotlikoff refers to as the yield effect. The yield effect can be positive or negative depending on the relationship between the interest rate and the sum of the rate of population growth and technical growth (in a pay-as-you-go program this sum represents the implicit rate of return of social security). If $r>n+g$, then the rate of return on social security taxes is less than the market interest rate. Thus the impact of social security in terms of the yield effect is to reduce the present value of the individual's life-cycle income. As a result people consume less and work more and savings are increased over each working period. If $r<n+g$ then the market rate of return is less than the implicit return on social security taxes and the present value of an individual's life-cycle income is increased. People consume more and work less and the impact of the yield effect is to reduce savings.

The net effect of social security on savings depends on these three effects. Kotlikoff first considers partial equilibrium effects. He computes a positive retirement effect and finds that on net the retirement and replacement effects reduce capital accumulation by 35% to 44% which is very close to the results in Feldstein discussed below. In a general equilibrium framework further effects occur as the interest rate goes up and wages down in response to the reduction in capital. These responses mitigate the reduction in capital since they have positive effects on savings. Thus in a general equlibrium the effect on savings is not as dramatic. Kotlikoff finds a reduction of 15.8% to 21.3%. He concludes that it is the response of savings to higher interest rates that plays a major role in damping the partial equilibrium effects.

In a general equilibrium framework, to ask what effect social security has on welfare is to ask what effect it has on capital accumulation. This has been central to the debate as typified in the exchange between Feldstein (1974a) and Barro (1974) and their subsequent communications.

Feldstein argues that the effect of social security on capital accumulation is theoretically ambiguous. The replacement effect causes workers to save less because of the benefits they receive. The retirement effect causes workers to save more because social security provides various incentives to retire earlier than they would have otherwise (although these effects could be negative as discussed in the Kotlikoff paper). The ambiguity, Feldstein claims, can only be resolved empirically. This is done by adding a social security wealth variable to a standard Modigliani-style regression of the consumption function. Feldstein finds a significant positive coefficient on the social security wealth variable and concludes social security has reduced the rate of total private saving by 38%.

On the other hand, Barro argues that the Ricardian Equivalence Principle applies to social security. Given a world of no uncertainty and perfect capital markets, if the welfare of each generation is somehow linked to the welfare of the succeding generation, then the welfare of those currently alive is linked to the welfare of those in the infinite future. Thus rational individuals do not consider government debt issue to be any different than an increase in taxes. This is the Ricardian Equivalence Principle. This is formalized in Barro's model by what he refers to as the "net bequest effect." If the level of debt is increased by additional debt issue current generations who buy the bonds realize their children and their children's children will be taxed to pay off the interest and principle on the debt. Current generations respond by increasing their gross bequests so that on net (that is, including the increased tax burden) they pass on to their children the same bequest in either case. It can be shown that given the assumptions of no uncertainty and perfect capital markets (among others) the consumption and capital accumulation of current and future generations is the same no matter whether an increase in government expenditures is financed by issuing debt or increasing taxes.

The net bequest argument can be applied to an increase in social security benefits. Any increase in benefits to the current generation must be financed by an increase in the payroll taxes of those currently working and future generations. Since the welfare of each generation is linked to that of succeding generations, those that receive an increase in benefits respond by increasing their gross bequests. They save more. Again, it can be shown that consumption and capital accumulation remain as if the increase in benefits had not occurred.

Barro (1978) tests this empirically in essentially the same manner as Feldstein and finds the coefficient of the social security wealth variable to be insignificant. The empirical evidence, he concludes, supports his theoretical model: social security has little effect on capital accumulation. The contradiction between Feldstein's results and Barro's results has been the subject of some debate and is related to the debate on whether or not actual savings in the economy conform to a strict (no bequests) model. However, these issues do not bear directly on this paper since an economy with no bequests is assumed. For this reason they are not discussed further.*

Even without bequests, it is still possible within a strict life-cycle model for a social security program to be neutral or to even increase welfare. Of course it depends on the type of program implemented. The regressions run by Barro and Feldstein deal with the current U.S. program which is usually assumed to be pay-as-you-go. Fully-funded programs and forced-savings programs have different effects on the economy. In an overlapping generations growth model the effect on welfare is very closely related to the relationship between the interest rate, the rate of population growth, and the so-called golden rule.

Much of this evolved out of earlier ad hoc growth models such as the one proposed by Solow (1956). Samuelson (1975a) discusses briefly the relationship between the overlapping generations growth model and the Solow growth model. It is a property of the Solow model that the lower the rate of population growth the greater is per capita consumption. In Samuelson's (1958) consumption-loans model on the other hand, the greater the rate of population growth the better, since having more children means more support for their parents. If the Solow model is combined with the consumption-loans model of Samuelson, as in Diamond (1965) then there is

* For a further discussion see Feldstein's (1976a) and Buchanan's (1976) replies to Barro's (1974) paper and Barro's (1976) response to these. In addition, the Leimer and Lesnoy (1981) paper and the Kotlikoff and Summers (1981) paper are related to this issue; as are papers by Koskela and Viren (1983) and Briden and Zedella (1986).

an optimal rate of population growth: what Samuelson refers to as the goldenest golden age.

In general, though, since the rate of population growth is usually considered to be determined exogenously, to attain the golden-rule level of capital the economy must resort to some form of public debt if there is too much capital ($r<n$) or some form of public saving program (such as surplus-funded social security) if there is not enough capital ($r>n$).

It is in this sense that Samuelson (1975b) considers optimal social security programs. If the laissez-faire economy is such that there is too much capital, steady state welfare can be increased by saving less and consuming more. In this case implementation of a pay-as-you-go social security program can increase steady state welfare. In Samuelson's model individuals live two periods. They work the first period and are retired in the second period. Individuals save less during the working period since they know they will receive social security payments next period when they are retired. As a result the capital-labor ratio is lowered and the economy moves closer to the golden-rule steady state.

If the state of the economy is such that there is not enough capital in the economy, implementation of a social security program in which a surplus is allowed to build up (a stock of public capital is created) can result in a steady state that is closer to the golden rule steady state. And, in fact, there are optimum social security programs that result in a new steady state at the golden rule level. However, in this case there are those along the transistion path who end up worse off due to the tax burden necessary to build up the stock of public capital. This is because a build-up necessarily implies a significant reduction in benefits and a significant increase in payroll taxes until the desired level of surplus is achieved.

Furthermore, in the case of a fully-funded social security program in which the implicit rate of return on social security payroll taxes is equal to the market rate of return, there will be no effect on capital formation since private capital is replaced by public capital dollar for dollar.

Computer simulations by Burbidge (1983) confirm Samuelson's theoretical results. If it is assumed that there is not enough capital in the economy, then a one-time payroll tax to increase public capital results in an increase in steady state utility, although the generation that is taxed and those immediately after suffer a welfare loss. A pay-as-you-go program results in a steady state characterized by a lowered capital-labor ratio and a loss of welfare, although generations immediately after implementation of the program experience an increase in welfare.

In considering social security policies, more often than not primary consideration is given to the resultant steady states. As discussed above, any change in policy not only affects generations alive at some future steady state but also the generations along the transistion path to the new steady state. Leimer and Petri (1988) compute the effect of four different prescriptions for social security reform on specific cohorts. Reform is necessary, they argue, since legislated benefits will significantly exceed legislated taxes even if scheduled payroll taxes are implemented. The four policies they consider are:

- Increase taxes to cover current legislated benefits.
- Decrease benefits so they will be covered by legislated taxes.
- Adjust both taxes and benefits with the burden split equally between the two.
- Increase taxes as needed to establish trust funds equal to five times expenditures by the year 2000 and subsequently maintain funds at this level.

Their model computes the effects of the current system and the four alternatives across 75 years. Under the current system, which incorporates the 1983 amendments, the revenue gap in 2030 will be 34% of scheduled benefits or 51% of scheduled taxes. Furthermore, the population growth rate shrinks from the post war average of 3.4% to 2.7%. Under all of the four policy alternatives capital formation will be greater. This is especially true of the benefit reduction case. Different cohorts, however, are affected by each policy differently. A tax increase to meet current legislated benefits is clearly preferred by the earliest generations since they will be less likely to have to pay the full amount of the tax increase. Amassing a trust fund (i.e. a forced saving program) is preferred by those born between 1953 and 1984. Yet, a decrease in benefits to meet legislated tax rates is preferred by those born after

1984. Under benefit reductions workers anticipate significant reductions in benefits and as a result savings are increased sharply and subsequently the stock of capital is greater than it would have been otherwise.

Put another way, those who are retired or are about to retire will be better off from the tax increase since they will escape most of the burden of this increase. Those in the middle of their working careers have enough time remaining until retirement to benefit from the higher wages and lower interest rates (i.e. the increased present value of their lifetime income) that result from the increased public capital created by a sizable trust fund. That the youngest workers are better off by a reduction in benefits can be given the following interpretation. In the long run the increase in capital formation is greater as a result of the benefit reduction than as a result of the trust fund. Creation of the trust fund brings capital into the economy more quickly, but as more and more young workers are faced with the reduction in benefits eventually more capital is created in this manner than by the trust fund.

Which policy is best overall depends to a large extent on how policy-makers weigh the welfare of different generations. If generations born before the year 2000 receive more weight (are discounted less) then funding is the best policy alternative. If future cohorts receive more weight than the benefit reduction policy is the best. For this to be the case the intergenerational discount rate needs to be fairly low and the time horizon long enough to include those born in 2080.

Since the model used in this paper assumes no uncertainty and no capital market imperfections, the literature on uncertain lives and liquidity constraints as it relates to social security has been ignored to this point. A brief discussion follows to provide another theoretical perspective to these issues and, more to the point, because the Hubbard and Judd paper discussed below contains a policy prescription that can be implemented (in a modified way) in the computer simulation developed in this paper.

Briefly then, Abel (1985) and Hubbard (1987), among others, have shown that when individuals have uncertainty as to their lifespans that an actuarially fair social security program can generate increases in the lifetime welfare of individuals. This is due principally to failures in the annuities market brought about by the problem of adverse selection. Optimizing individuals will not purchase annuities actually offered on the market. (This is discussed in Eckstein (1985).)

Hubbard and Judd (1987) incorporate this into their model, along with the possibility of binding liquidity constraints, and along with bequests (but these are not planned bequests but rather accidental bequests of the type discussed in Yaari's (1965) paper). They find a pay-as-you-go social security program changes welfare from between -14.7% to -41.8% and changes the capital-output ratio from between -21% to -68% as the intertemporal elasticity of substitution ranges from 1.10 to 0.20. For moderate rates of the elasticity of substitution (β= 0.50) welfare drops by 28% and the capital output ratio drops by 46%. The payroll tax rate used here is a flat 6% for all workers. These results mean, for one thing, that even though actuarially fair social security can increase welfare upon its initial implementation it still causes workers to save less and the subsequent adverse effect on the level of capital results in a steady state with a lower level of capital and lower welfare.

Hubbard and Judd then propose the following experiment. What happens to welfare if income earned during the early working years is exempted from the payroll tax but not from the benefit formula? Specifically, the first fifteen years of employment are exempted and a flat tax is applied to the remaining years of employment. To preserve comparability, the "exempted flat tax" is such that benefits are the same as they would have been for a flat tax of 6% on all working generations. In this model the point is to lessen the probability that the liquidity constraints apply. They calculate the welfare changes that come about as a result but only for the partial equilibrium case. For the moderate case (r=4% and $\beta = 0.50$) the welfare gain is 5.3%.

In the Hubbard and Judd paper the increase in welfare comes about because the exemption reduces the probability that the liquidity constraint is binding and allows for more saving when young. This increase in the rate of capital accumulation results in a higher level of steady state welfare. In a general equilibrium model the savings behavior of the younger generations is more critical than that of the older generations. Any policy that increases the saving of the younger generations at the expense of the older generations has the potential of increasing steady state welfare. This is the reasoning behind the exempted net tax used in this paper. However, since the payroll tax is taken as fixed, the exemption is implemented by adjusting the weights of the benefit formula. Thus, in this paper the exemption is not full because the exempted workers still face a positive net tax.

Net Social Security Taxes

If social security benefits can be linked to wages earned then the net or effective tax rate will not be the legislated rate but must be adjusted to account for the benefits received. Despite the fact that tax rates and benefit formulas are subject to congressional mandate; historically, social security taxes have been considered to be of the nature of compulsory contributions towards retirement. Though there is uncertainty as to what the benefit formula will actually be when a worker goes to retire and uncertainty as to life expectancy, to assume that workers perceive a link between wages earned and benefits received does not seem so extreme.

Seater (1985) employs the taxable ceiling to compute a time series for an aggregate effective social security tax for the self-employed and those employed by others. He defines the effective rate to be the legislated rate times the ratio of income or wages below the taxable ceiling over total income or wages. He finds in the early years of the social security program an effective tax of about 1/2% for both groups. For the later years he finds an effective tax of 4% for those employed by others and

3.5% for the self-employed. From the perspective of the life-cycle model, this is a poor measure of the effective social security tax since it completely ignores benefits.

Gordon (1983) points out that workers decisions are not only distorted by the payroll tax, but by the other features of social security such as the earnings test, the acturial adjustment for early retirement, and the formulas for retirement benefits. He computes an effective tax rate taking all these distortions into consideration and in the presence of an income tax. He computes a net tax rate for those turning 65 in 1975, 1985, and in 2005. He does this across the life-cycles of these workers and takes into account the various amendments to the social security law including the 1983 amendment. For workers age 65 in 1985 the net tax goes from -29% at age 45 to -43% at age 62. For workers age 65 in 2005, the net taxes are much higher (reflecting, perhaps, the maturation of the program) going from -2.4% at age 45 to -20.7% at age 62. In both cases the net tax represents a subsidy.

These computations illustrate one of the basic features of the net tax. It should go down as the worker grows older since the older worker does not discount the stream of benefits as much.

Burkhauser and Turner (1985) and Browning (1985) also compute a net tax that decreases over the worker's life-cycle. The main difference here is that Burkhauser and Turner compute net taxes that are substantially lower than Browning's. This is due to the discount rates assumed in each paper: 1% in the first case and as much as 6% in the second. Of course the higher the discount rate the more future benefits are discounted and the higher the net tax.

Burkhauser and Turner compute net marginal taxes for those aged 65 in 1982, for a cross-section of workers in 1983, and for those aged 22 in 1983 (before and after the 1983 amendment). Considering the results for a cross-section of cohorts in 1983 (reproduced here as Table 2.01); the social security tax has in fact been a subsidy for all cohorts except those under 25 (which get taxed the full legislated rate of 9.55%). Burkhauser and Turner conclude that during the start-up phase of the U.S. program the marginal benefits exceed the taxes paid in. For those born after 1948, the 1983 amendments to the social security act substantially increased the true marginal tax rates of younger workers through increased payroll taxes and reduced benefits. However, they still compute negative net tax rates for a substantial number of years across the life-cycles of even the youngest workers.

Browning (1985) computes a net tax in much the same manner as Burkhauser and Turner. However, he argues that the low, often negative, net taxes that they compute are due to a discount rate that is unrealistically low at 1%. The net tax rate as mentioned above is highly sensitive to the discount rate since the higher it is the less is the present value of the stream of benefits and the higher the net tax rate.

Browning discusses several different candidates for the discount rate. Lind's (1982) suggestion of 2% as an approximation of the real rate of time preference for long-term government bonds is dismissed because most households do not hold long-term government bonds and because this is a riskless rate. For purposes of computing a net tax, Browning argues, the rate should include risks since workers seldom know what their true benefits will be until shortly before retirement. For a risky discount factor Lind suggests a rate of 4.6% based on average after-tax real returns on common stocks over the long run. Hollands and Myers (1972) estimated this to be 5.77%. Since these refer to a balanced portfolio and since many people do not own such a portfolio Browning argues a still higher rate may be appropriate. Considering that the empirical estimate of discount rates actually employed by workers often exceed 6%, he suggests that using a discount rate of 6% would not be an overstatement. Browning then computes net taxes using discount rates of 1%, 1.75%, 4%, and 6%. As can be seen in Browning's Table 2 (reproduced here as Table 2.02); as the discount rate goes up the net tax rate goes up across all cohorts.

Summary of the Literature

In a life-cycle model of social security with no bequests, no uncertainty, and perfect capital markets, it is only natural to link the stream of benefits to the payroll taxes paid. This allows a net social security tax to be computed as in Burkhauser and Turner or as in Browning. How consumption and leisure decisions respond intertemporally to changes in the net tax depend not only on the parameters of the model but also on how the wage and interest rates respond to changes in the savings behavior of individuals. Allowing for a labor supply decision, these responses occur in much the same manner as described in Tobin.

For example, if the net tax for a particular year goes down, then the opportunity cost of leisure for that particular year goes up. It is entirely possible that individuals will substitute out of leisure for that year into consumption for that year and into consumption and leisure in other years.[5] The fact that individuals work more and save more in that particular year changes the intertemporal pattern of savings and aggregate capital formation. Wages and the interest rate change. Hence, the present value of life-cycle income changes and brings into play further income effects which Summers refers to as human wealth effects and Kotlikoff refers to as yield effects. These effects are potentially rather substantial and it is essential that they be considered in dynamic tax analysis.

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[5] How individuals respond as relative prices change intertemporally is discussed in more detail in the following chapter.

Table 2.01
Net Social Security Tax Rates for
Males of Various Ages in 1983.

Age	True Payroll Tax Rate Worker	True Payroll Tax Rate Worker with Dependent Spouse
25	9.55	9.55
26	4.67	-.07
30	4.26	-.86
41	1.77	-5.63
44	2.74	-5.52
47	1.94	-5.25
50	1.38	-4.67
53	1.95	-5.04
56	1.25	-6.25
59	-0.77	-9.86
61	-2.15	-12.22

Source: Burkhauser and Turner (1985)
Note: Earnings of those 25 or younger are assumed not to be in the 35 highest years of earnings.

Table 2.02
Net Taxes for Single Workers
(Marginal Replacement Factor = 0.32)

r =	.01	.0175	.04	.06
Age				
25	3.4	6.2	10.7	12.2
35	4.1	6.2	10.1	11.7
45	4.8	6.2	9.2	10.8
55	5.4	6.2	8.2	9.5
Source: Browning (1985)				

3. THEORETICAL MODEL AND RESULTS

Given an economy that suffers from too little capital and a pay-as-you-go social security program, this paper asks whether or not the social security benefit formula can be manipulated to improve the steady state utility of a representative individual. Specifically, whether or not a benefit formula that favors earnings in the younger years can bring about an increase in aggregate savings and hence an increase in steady state utility. The instruments used here are weights attached to each period's earnings.

This question is considered within the framework of an overlapping generations growth model in which population growth occurs but technical growth does not. There is no uncertainty. Workers live for many periods. Most of the periods are spent working and the rest are spent in retirement. Workers are free to choose consumption and leisure during the working periods and consumption during retirement. However, there is no retirement decision in the sense that all workers retire at the same age. Consumption in retirement is provided for by savings accumulated during the working periods and by social security benefits which are financed by revenues raised from taxing all working generations. Each worker over each generation is exactly alike in preferences and lives for exactly the same number of years. Each generation is different from the preceding generation in that there are 1+n times more of them. In the computer simulations discussed below individuals work for 40 periods and are retired for 10 periods. In this chapter individuals work for two periods and are retired for one period. This allows for consideration of the basic tradeoff that must occur if the favored treatment of the younger workers is to bring about the desired increased in steady state utility.

The analysis in this chapter considers the initial impact of a change in the weight applied to the earnings while young. That is, a partial equilibrium in which the gross wage rate and interest rate do not respond to changes in aggregate savings is assumed. Under assumptions of separability, comparative statics results are obtained which accord with the intuition that incentives provided to younger, more numerous workers will result in more work effort and more savings from these generations, and hence an increase in aggregate savings. This increase may occur despite the fact that savings for older workers may actually go down.

Two other issues are considered within the context of this three-period model. First, the optimal weights are characterized. It is shown here that the optimal weights are not necessarily such that the net tax is uniform. Instead, they depend upon the own-price and cross-price elasticities of labor supply in the two working periods. The other issue considered is the infeasibility of a zero net tax. It is shown that although a zero net tax has the same property as a fully-funded net tax in that the individual's budget constraint is the same as it would be without the tax; a zero net tax is not feasible. This is because it can be shown that in the case in which there is too little capital in the economy, a zero net tax must result in a social security deficit.

Social Security in the Three-Period Model

Government intervention here occurs solely in the form of a pay-as-you-go social security program. This program is characterized by a fixed payroll tax and a benefit formula which allows each period's earnings to receive different weights. As these weights change the present value of the benefits received from working an additional hour changes. That is, the net tax--defined by taking into account these benefits--changes. In the three-period model the benefit formula can be expressed as:

$$SSB = R_f\left[\frac{\sigma_1 w(\bar{L} - v_1) + \sigma_2 w(\bar{L} - v_2)}{2}\right] \tag{3.01}$$

where

SSB = the annual social security benefit.

w = the steady state gross wage.

R_f = the marginal replacement rate.

$\bar{L}$ = the most an individual can work in a period.

v_i = the leisure consumed in period i.

σ_i = the weight applied to earnings in year i.

The term in brackets represents the formula for averaging eligible earnings and R_f, the marginal replacement rate, represents the percentage of that which is returned to the worker in the form of retirement benefits.[6]

From the benefit formula a wage net of social security taxes and the present value of future benefits can be defined for each working year:

$$w_n^1 = \left[(1 - \theta) + \frac{R_f \sigma_1}{2(1 + r)^2} \right] w \tag{3.02}$$

and

$$w_n^2 = \left[\frac{(1 - \theta)}{(1 + r)} + \frac{R_f \sigma_2}{2(1 + r)^2} \right] w \tag{3.03}$$

where

w_n^i = the net wage for period i.
θ = the social security payroll tax rate.

From 3.02 and 3.03 the marginal net tax can be defined as:

$$NT_1 = \theta - \frac{R_f \sigma_1}{2(1 + r)^2} \tag{3.04}$$

and

$$NT_2 = \theta - \frac{R_f \sigma_2}{2(1 + r)} \tag{3.05}$$

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[6] The social security program is discussed in more detail in the following chapter.

where

NT_t = the net tax in period t.

If σ_1 and σ_2 are set to 1, then the current program has been defined. If σ_1 and σ_2 are set so that NT_1 and NT_2 are the same, a uniform net tax has been defined.[7]

Note that 3.03 represents the present value of the net wage for period two. Thus, the net tax for the older workers, NT_2, is the term in brackets in 3.03 with the discount rate, (1+r), factored out. Because cohorts of different ages must wait different periods until retirement the present value of the marginal benefits varies across cohorts and hence the net marginal tax rate varies across cohorts.

This can be illustrated more clearly by considering the life-cycle budget constraint for two workers. As before, both live for three periods. Both work for two periods and are retired in the third. The first is a young worker and the second is an old worker. The budget constraint for the young worker can be expressed as:

$$c_1 + \frac{c_2}{1+r} + \frac{c_3}{(1+r)^2} = (1-\theta)w(\bar{L} - v_1) + \frac{(1-\theta)w(\bar{L} - v_2)}{(1+r)} + \frac{R_f}{(1+r)^2}\left[\frac{\sigma_1 w(\bar{L} - v_1) + \sigma_2 w(\bar{L} - v_2)}{2}\right] \tag{3.06}$$

---

[7] The social security program is discussed in more detail in the next chapter.

where

c_i = consumption in period i.

r = the real interest rate.

and where everything else is defined above.

The budget constraint for the two year old is similar except the first period decisions are history. The two year old budget constraint can be expressed as:

$$c_2 + \frac{c_3}{1+r} = (1+r)s_1 + (1-\theta)w(\bar{L} - v_2) + \frac{R_f}{(1+r)}\left[\frac{\sigma_1 w(\bar{L} - v_1) + \sigma_2 w(\bar{L} - v_2)}{2}\right] \tag{3.07}$$

where

s_1 = first period savings.

and where everything else is defined above.

If the young worker works an additional hour in the current period the present value of the change in benefits can be expressed as:

$$\Delta b_1 = \left[\frac{R_f \sigma_1}{2(1+r)^2}\right] w \tag{3.08}$$

If the old worker works an additional hour the present value of the change in benefits can be expressed as:

$$\Delta b_2 = \left[\frac{R_f \sigma_2}{2(1 + r)} \right] w \tag{3.09}$$

The change in taxes paid is θw in both cases. Thus the present value of the marginal change in net taxes paid for the young worker can be expressed as:

$$\theta w - \Delta b_1 = \left[\theta - \frac{R_f \sigma_1}{2(1 + r)^2} \right] w \tag{3.10}$$

For the old worker the marginal changes in net taxes paid can be expressed as:

$$\theta w - \Delta b_2 = \left[\theta - \frac{R_f \sigma_2}{2(1 + r)} \right] w \tag{3.11}$$

Thus the net tax rate for each worker at a given point in time (i.e across cohorts) can be expressed as:

$$\text{Young worker's net tax} = \theta - \frac{R_f \sigma_1}{2(1 + r)^2}$$

$$\text{Old worker's net tax} = \theta - \frac{R_f \sigma_2}{2(1 + r)}$$

The two expressions above, of course, are the same as 3.04 and 3.05.

Finally, the social security program is subject to a pay-as-you-go constraint. This simply means that taxes paid into the social security fund by the young and middle-aged workers must equal the benefits received by the retired workers. For the three-period model this can be expressed as:

$$\theta w(\bar{L} - v_1)(1 + n)^T + \theta w(\bar{L} - v_2)(1 + n)^{T-1} = R_f\left[\frac{\sigma_1(\bar{L} - v_1)w + \sigma_2(\bar{L} - v_2)w}{2}\right](1 + n)^{T-2} \tag{3.12}$$

or:

$$\left[(1 + n)^2\theta - \frac{R_f\sigma_1}{2}\right]w(\bar{L} - v_1) + \left[(1 + n)\theta - \frac{R_f\sigma_2}{2}\right]w(\bar{L} - v_2) = 0 \tag{3.13}$$

where

n = the population growth rate.
T = calendar time.

The Worker's Decision Problem

Each individual works for two periods and is retired for an additional period. There is no retirement decision since all individuals retire after the second period. Individuals choose consumption and leisure during the two working periods and consumption during retirement. Consumption in retirement is provided for by savings accumulated during the working years and by social security benefits that are based on earnings during the working periods. That is, individuals solve the following:

$$\max_{c_t, v_t} U = U(c_1, v_1, c_2, v_2, c_3) \tag{3.14}$$

subject to:

$$c_1 + w_n^1 v_1 + \frac{c_2}{1+r} + w_n^2 v_2 + \frac{c_3}{(1+r)^2} = (w_n^1 + w_n^2)\bar{L} \tag{3.15}$$

where w_n^t is the net wage for period t defined in 3.02 and 3.03, and where:

r = the real interest rate.

$\bar{L}$ = the most an individual can work in a given period.

c_t = consumption in period t.

v_t = leisure in period t.

In addition, the following expressions are defined for the purpose of comparative statics exercise immediately below:

$$U_1 \equiv \frac{\partial U}{\partial c_1} ; \; U_2 \equiv \frac{\partial U}{\partial v_1} : \; U_3 \equiv \frac{\partial U}{\partial c_2} ; \; U_4 \equiv \frac{\partial U}{\partial v_2} ; \; U_5 \equiv \frac{\partial U}{\partial c_3} \tag{3.16}$$

This notation is extended in the obvious manner to the second-partial and cross-partial derivatives.

Comparative Statics with a Pay-as-You-Go Constraint

The labor-leisure and savings decisions are now considered in the presence of a pay-as-you-go constraint. For reasons of tractability it is assumed that preferences are additively separable with respect to to c_t and v_t.

In addition, it is assmmed that the initial equilibrium is such that certain terms discussed below must be zero. The equations for this exercise consist of the first-order conditions of the consumer's optimization problem, 3.14 and 3.15, and the pay-as-you-go constraint for the social security program, 3.13. The first order conditions consist of the following plus the budget constraint 3.15:

$$U_1 - \alpha = 0 \tag{3.17}$$

$$U_2 - w_n^1\alpha = 0 \tag{3.18}$$

$$U_3 - \frac{\alpha}{1 + r} = 0 \tag{3.19}$$

$$U_4 - w_n^1\alpha = 0 \tag{3.20}$$

$$U_5 - \frac{\alpha}{(1 + r)^2} = 0 \tag{3.21}$$

where the notation used here with respect to the partial derivatives of the utility function is the same notation used above, and where α represents the Lagrange multiplier of the worker's decision problem.

From 3.17 α can be expressed in terms of U_1. Substituting this into 3.18 through 3.21 yields the following system of equations:

$$U_2 - w_n^1 U_1 = 0 \tag{3.22}$$

$$U_3 - \frac{U_1}{1 + r} = 0 \tag{3.23}$$

$$U_4 - w_n^2 U_1 = 0 \tag{3.24}$$

$$U_5 - \frac{U_1}{(1 + r)^2} = 0 \tag{3.25}$$

The equations 3.22 through 3.25 plus the budget constraint 3.15 plus the pay-as-you-go constraint 3.13 represent six equations in six unknowns: c_1, v_1, c_2, v_2, c_3, σ_2. Taken as exogenous are : r, w, R_f, σ_1. Of interest here is how the six endogenous variables respond to a change in the first period earnings weight, σ_1.

Calculating the total differentials of these six equations (3.22-3.25, 3.15, and 3.13) as σ_1 changes, and imposing the separability assumption yields the following equations:

$$- w_n^1 U_{11} dc_1 + U_{22} dv_1 - U_1 \frac{R_f w}{2(1 + r)^2} d\sigma_1 = 0 \tag{3.26}$$

$$- \frac{1}{1 + r} U_{11} dc_1 + U_{33} dc_2 = 0 \tag{3.27}$$

$$- w_n^2 U_{11} dc_1 + U_{44} dv_2 - U_1 \frac{R_f w}{2(1 + r)^2} d\sigma_2 = 0 \tag{3.28}$$

$$- \frac{U_{11}}{(1 + r)^2} dc_1 + U_{55} dc_3 = 0 \tag{3.29}$$

$$dc_1 + w_n^1 dv_1 + \frac{1}{1+r} dc_2 + w_n^2 dv_2 + \frac{1}{(1+r)^2} dc_3 - \frac{R_f w}{2(1+r)^2} (\bar{L} - v_2) d\sigma_2 = \frac{R_f w}{2(1+r)^2} (\bar{L} - v_1) d\sigma_1 \tag{3.30}$$

$$-\left[(1+n)^2\theta - \frac{R_f\sigma_1}{2}\right] wdv_1 - \left[(1+n)\theta - \frac{R_f\sigma_2}{2}\right] wdv_2 - \frac{R_f w}{2} (\bar{L} - v_2)(\bar{L} - v_2) d\sigma_2 = \frac{R_f w}{2} (\bar{L} - v_1) d\sigma_1 \tag{3.31}$$

At this point it is assumed σ_1 is chosen initially so that:

$$(1+n)^2\theta - \frac{R_f\sigma_1}{2} = 0 \tag{3.32}$$

That is:

$$\sigma_1 = \frac{2(1+n)^2\theta}{R_f} \tag{3.33}$$

This means for the pay-as-you-go constraint to hold that:

$$\sigma_2 = \frac{2(1+n)\theta}{R_f} \tag{3.34}$$

This in turn means that:

$$(1+n)\theta - \frac{R_f\sigma_2}{2} = 0 \tag{3.35}$$

This allows these terms to be set to zero in equation 3.31.[*] Doing this allows 3.26 through 3.31 to be expressed in the following matrix form:

---

[*] For reasonable values of n, θ, and R_f this neighborhood is very close to the current program.

$$
\begin{bmatrix}
-w_n^1 U_{11} & U_{22} & 0 & 0 & 0 & 0 \\
-\frac{1}{1+r} U_{11} & 0 & U_{33} & 0 & 0 & 0 \\
-w_n^2 U_{11} & 0 & 0 & U_{44} & 0 & -U_1 \frac{R_f w}{2(1+r)^2} \\
-\frac{1}{(1+r)^2} U_{11} & 0 & 0 & 0 & U_{55} & 0 \\
1 & w_n^1 & \frac{1}{1+r} & w_n^2 & \frac{1}{(1+r)^2} & -\frac{R_f w}{2(1+r)^2}(\bar{L} - v_2) \\
0 & 0 & 0 & 0 & 0 & -\frac{R_f w}{2}(\bar{L} - v_2)
\end{bmatrix}
\begin{bmatrix} dc_1 \\ dv_1 \\ dc_2 \\ dv_2 \\ dc_3 \\ d\sigma_2 \end{bmatrix}
=
\begin{bmatrix}
U_1 \frac{R_f w}{2(1+r)^2} d\sigma_1 \\
0 \\
0 \\
0 \\
\frac{R_f w}{2(1+r)^2}(\bar{L} - v_1) d\sigma_1 \\
\frac{R_f w}{2}(\bar{L} - v_1) d\sigma_1
\end{bmatrix}
\quad (3.36)
$$

To determine expressions for the changes in the endogeneous variables with respect to a change in σ_1 Cramer's Rule is applied. First, it is noted that the determinant of the six-by-six matrix in 3.36 can be expressed as:

$$\Delta = -\frac{R_f w}{2}(\bar{L} - v_2)\Big[w_n^1 w_n^1 U_{11}U_{33}U_{44}U_{55} + \frac{1}{1+r}U_{11}U_{22}U_{44}U_{55} + w_n^2 w_n^2 U_{11}U_{22}U_{33}U_{55} + U_{22}U_{33}U_{44}U_{55} + \frac{1}{(1+r)^4}U_{11}U_{22}U_{33}U_{44}\Big] \tag{3.37}$$

Given the assumption above with respect to separability and if it is further assumed that leisure and consumption in all periods are normal, then $U_{tt} < 0$ and this expression must be negative:[9]

$$\Delta < 0$$

The Response of Consumption and Leisure: By substituting the column matrix on the right hand side of equation 3.36 into the appropriate column of the gradient matrix on the left hand side, Cramer's rule can be applied. The determinant of the matrix obtained in this manner divided by the determinant of the original six-by-six matrix yields the response of the endogenous variable that corresponds to this column to a change in the first period earnings weight. This method is used to obtain the following results.

$$\frac{dc_1}{d\sigma_1} = \frac{1}{\Delta}\left\{U_1 \frac{R_f w}{2(1+r)^2}\frac{R_f w}{2}U_{33}U_{55}[w_n^1(\bar{L} - v_2)U_{44} - w_n^2(\bar{L} - v_1)U_{22}]\right\} \tag{3.38}$$

If the term in brackets is positive then first period consumption goes down unambiguously since the determinant of the six-by-six matrix is

[9] This is shown in Gahvari (1986).

negative and U_{tt} is assumed to be negative. Otherwise, first period consumption could go up or down.

The response of first period leisure can be expressed as:

$$\frac{dv_1}{d\sigma_1} = \frac{-1}{\Delta}\left\{U_1 \frac{R_f w}{2(1+r)^2} \frac{R_f w}{2}\left[w_n^1 w_n^2(\bar{L} - v_1)U_{11}U_{33}U_{55} + \frac{1}{(1+r)^2}(\bar{L} - v_2)U_{11}U_{44}U_{55} + w_n^2 w_n^2(\bar{L} - v_2)U_{11}U_{33}U_{55} + \frac{1}{(1+r)^4}(\bar{L} - v_2)U_{11}U_{33}U_{44} + (\bar{L} - v_2)U_{33}U_{44}U_{55}\right]\right\} \tag{3.39}$$

Given the assumptions on preferences and the fact that the determinant of the original six-by-six matrix is negative, this expression must be negative. That is:

$$\frac{dv_1}{d\sigma_1} = \frac{>0}{<0} < 0$$

The response of second period consumption can be expressed as:

$$\frac{dc_2}{d\sigma_1} = \frac{1}{\Delta}\left\{\frac{U_1}{1+r} \frac{R_f w}{2(1+r)^2} \frac{R_f w}{2}\left[w_n^1(\bar{L} - v_2)U_{44} - w_n^2(\bar{L} - v_1)U_{22}\right]\right\} \tag{3.40}$$

As in the case of first period consumption, if the term in brackets is positive then second period consumption must go down unambiguously. Otherwise, second period consumption could go up or down.

The response of second period leisure can be expressed as:

$$\frac{dv_2}{d\sigma_1} = \frac{1}{\Delta}\left\{U_1 \frac{R_f w}{2(1+r)^2}\frac{R_f w}{2}\left[\frac{1}{(1+r)^2}(\bar{L}-v_1)U_{11}U_{22}U_{55} \right.\right.$$
$$+ w_n^1 w_n^2(\bar{L}-v_2)U_{11}U_{33}U_{55} + w_n^1 w_n^1(\bar{L}-v_1)U_{11}U_{33}U_{55} \quad (3.41)$$
$$\left.\left. + \frac{1}{(1+r)^4}(\bar{L}-v_1)U_{11}U_{22}U_{33} + (\bar{L}-v_1)U_{22}U_{33}U_{55}\right]\right\}$$

From this it should be apparent that second period leisure goes up unambiguously given the assumptions on preferences and the negativity of the determinant of the gradient. That is:

$$\frac{dv_2}{d\sigma_1} = \frac{\leq 0}{< 0} > 0$$

Consumption in the third period can be expressed as:

$$\frac{dc_3}{d\sigma_1} = \frac{1}{\Delta}\left\{\frac{U_1}{(1+r)^2}\frac{R_f w}{2(1+r)^2}\frac{R_f w}{2}U_{11}U_{33}[w_n^1\,(\bar{L}-v_2)U_{44} - w_n^2(\bar{L}-v_1)U_{22}]\right\}$$

(3.42)

Again, if the term in brackets is positive then the response of third period consumption must be positive. Otherwise, third period consumption could go up or down.

Recall that the second period earnings weight is also considered endogenous here. As would be expected it can be shown that the response of the second period weight, σ_2, is unambiguously negative. That is, to maintain the pay-as-you-go constraint, the second period weight must go down as the first period weight goes up. This can be seen from the following expression:

$$\frac{d\sigma_2}{d\sigma_1} = \frac{1}{\Delta}\left\{\frac{R_f w}{2}(\bar{L} - v_1)\left[U_{22}U_{33}U_{44}U_{55} + w_n^1 w_n^1 U_{11}U_{33}U_{44}U_{55} + \frac{1}{(1+r)^4}U_{11}U_{22}U_{44}U_{55} + w_n^2 w_n^2 U_{11}U_{22}U_{33}U_{55} + \frac{1}{(1+r)^4}U_{11}U_{22}U_{33}U_{44}\right]\right\} \quad (3.43)$$

That is:

$$\frac{d\sigma_2}{d\sigma_1} = \frac{>0}{<0} < 0$$

However, substituting 3.37, the expression for the determinant of the six-by-six matrix above, into 3.43 above results in the more convenient expression:

$$\frac{d\sigma_2}{d\sigma_1} = -\frac{(\bar{L} - v_1)}{(\bar{L} - v_2)} \quad (3.44)$$

Full Income: The response of full income to a change in σ_1 can be derived in the following manner. Full income is defined as:

$$\begin{aligned} FI &= (w_n^1 + w_n^2)\bar{L} \\ &= \left[(1-\theta) + \frac{R_f\sigma_1}{2(1+r)^2} + \frac{(1-\theta)}{1+r} + \frac{R_f\sigma_2}{2(1+r)^2}\right]w\bar{L} \end{aligned}$$

where FI is full income and all other terms are defined above.

The change in full income due to a change in σ_1 can then be expressed as:

$$
\begin{aligned}
\frac{dFI}{d\sigma_1} &= \left[\frac{R_f}{2(1+r)^2} + \frac{R_f}{2(1+r)^2}\frac{d\sigma_2}{d\sigma_1}\right]w\bar{L} \\
&= \frac{R_f w\bar{L}}{2(1+r)^2}\left[1 + \frac{d\sigma_2}{d\sigma_1}\right]
\end{aligned}
\tag{3.45}
$$

Substituting 3.44 into 3.45 yields the following:

$$
\frac{dFI}{d\sigma_1} = \frac{R_f w\bar{L}}{2(1+r)^2}\left[1 - \frac{(\bar{L}-v_1)}{(\bar{L}-v_2)}\right] \tag{3.46}
$$

Note here that if the initial labor supply in both periods is the same full income does not change at all. That is, the increase in net wages that occurs in the first period as a result of the increase in the first period earnings weight is offset by the decrease in the net wage that occurs in the second period as a result of the decrease in the second period earnings weight. Moreover, if $\bar{L} - v_2 < \bar{L} - v_1$ then the response of full-income is positive, and if $\bar{L} - v_1 > \bar{L}v -_2$ then the response of full-income is negative.

Savings: Can the increase in σ_1 bring about an increase in aggregate savings? This is the relevant question since an increase in savings is necessary for an increase in steady state utility. Increased savings, by raising real wages and lowering interest rates, results in a steady state in which life-cycle income is greater and hence so is individual utility. Although the comparative statics answer to this question is ambiguous, an example with log preferences is provided in which the response is indeed positive.

Let s_1 and s_2 be defined as individual savings out of wages for the first and second periods respectively. These can be expressed as:

$$s_1 = (1-\theta)w(\bar{L} - v_1) - c_1 \tag{3.47}$$

$$s_2 = (1-\theta)w(\bar{L} - v_2) - c_2 \tag{3.48}$$

Taking the total derivatives with respect to σ_1:

$$\frac{ds_1}{d\sigma_1} = -(1-\theta)\frac{dv_1}{d\sigma_1} - \frac{dc_1}{d\sigma_1} \tag{3.49}$$

$$\frac{ds_2}{d\sigma_1} = -(1-\theta)\frac{dv_2}{d\sigma_1} - \frac{dc_2}{d\sigma_1} \tag{3.50}$$

Substituting 3.38 and 3.39 into 3.49 and substituting 3.40 and 3.41 into 3.50 the following expressions are obtained:

$$\frac{ds_1}{d\sigma_1} = \frac{1}{\Delta} U_1 \frac{R_f w}{2(1+r)^2} \frac{R_f w}{2} \Bigg\{ (1-\theta) w \Bigg[w_n^1 w_n^1 (\bar{L} - v_1) U_{11} u_{33} U_{55} + \frac{1}{(1+r)^2} (\bar{L} - v_2) U_{11} U_{44} U_{55} + w_n^2 w_n^2 (\bar{L} - v_2) U_{11} U_{33} U_{55} + \frac{1}{(1+r)^4} (\bar{L} - v_2) U_{11} U_{33} U_{44} + (\bar{L} - v_2) U_{33} U_{44} U_{55} \Bigg] - U_{33} U_{55} \Bigg[w_n^1 (\bar{L} - v_2) U_{44} - w_n^1 (\bar{L} - v_1) U_{22} \Bigg] \Bigg\} \tag{3.51}$$

and

$$\frac{ds_2}{d\sigma_1} = \frac{-1}{\Delta} U_1 \frac{R_f w}{2(1+r)^2} \frac{R_f w}{2} \Bigg\{ (1-\theta) w \Bigg[\frac{1}{(1+r)^4} (\bar{L} - v_1) U_{11} U_{22} U_{55} + w_n^1 w_n^2 (\bar{L} - v_2) U_{11} U_{33} U_{55} + w_n^1 w_n^1 (\bar{L} - v_1) U_{11} U_{22} U_{33} + (\bar{L} - v_1) U_{22} U_{33} U_{55} \Bigg] + \frac{1}{1+r} U_{11} U_{55} [w_n^1 (\bar{L} - v_2) U_{44} - w_n^2 (\bar{L} - v_1) U_{22}] \Bigg\} \tag{3.52}$$

From this, it is apparent that if:

$$w_n^1 (\bar{L} - v_2) U_{44} - w_n^2 (\bar{L} - v_1) U_{22} > 0$$

then:

$$\frac{ds_1}{d\sigma_1} > 0 \text{ and } \frac{ds_2}{d\sigma_1} \text{ is ambiguous.}$$

On the other hand if:

$$w_n^1(\bar{L} - v_2)U_{44} - w_n^2(\bar{L} - v_1)U_{22} < 0$$

then:

$$\frac{ds_1}{d\sigma_1} \text{ is ambiguous and } \frac{ds_2}{d\sigma_1} < 0$$

Now let s_{12} be defined as the aggregate savings per young. Then:

$$s_{12} = s_1 + \frac{1}{1+n}\left[(1+r)s_1 + s_2\right] \tag{3.53}$$

This can be expressed as:

$$s_{12} = \frac{(2+r+n)}{1+n} s_1 - \frac{1}{1+n} s_2 \tag{3.54}$$

Taking the total derivative with respect to σ_1 yields:

$$\frac{ds_{12}}{d\sigma_1} = \frac{(2+r+n)}{1+n}\frac{ds_1}{d\sigma_1} - \frac{1}{1+n}\frac{ds_2}{d\sigma_1} \tag{3.55}$$

Substituting 3.51 and 3.52 into 3.55 and collecting similar terms, the following is obtained:

$$
\begin{aligned}
\frac{ds_{12}}{d\sigma_1} = \frac{1}{\Delta} U_1 \frac{R_f w}{2(1+r)^2} \frac{R_f w}{2} \Bigg\{ & (1-\theta)w\Bigg[\left(\frac{(2+r+n)}{1+n} - \frac{1}{1+n}\right) U_{11}U_{33}U_{55} \\
& \frac{1}{(1+r)^2} U_{11}U_{55}\left(\frac{(2+r+n)}{1+n} U_{44} - \frac{1}{1+n}(\bar{L}-v_1)U_{22}\right) \\
& + U_{11}U_{33}U_{55}\left(\frac{(2+r+n)}{1+n} w_n^2 w_n^2 - \frac{1}{1+n} w_n^1 w_n^2\right) \\
& + U_{11}U_{33}\left(\frac{(2+r+n)}{1+n}(\bar{L}-v_2)U_{44} - \frac{1}{1+n}\frac{(\bar{L}-v_1)}{1+n} U_{22}\right) \qquad (3.56) \\
& + U_{33}U_{55}\left(\frac{(2+r+n)}{1+n}(\bar{L}-v_2)U_{44} - \frac{1}{1+n}(\bar{L}-v_1)U_{22}\right)\Bigg] \\
& - \frac{(2+r+n)}{1+n} U_{33}U_{55}(w_n^1(\bar{L}-v_2)U_{44} - w_n^2(\bar{L}-v_1)U_{22}) \\
& - \frac{1}{1+n}\frac{1}{1+r} U_{11}U_{55}(w_n^1(\bar{L}-v_2)U_{44} - w_n^2(\bar{L}-v_1)U_{22})\Bigg\}
\end{aligned}
$$

Here, the result is essentially ambiguous. However, the terms on the first 5 lines of this expression that contribute in a positive manner to aggregate savings might well dominate since they are multiplied by a factor that is (2+r+n) times greater than the terms that affect the expression in a negative manner. In this case the following conclusion holds:

$$
\text{If } [w_n^1(\bar{L}-v_2)U_{44} - w_n^2(\bar{L}-v_1)U_{22}] > 0 \text{ then } \frac{ds_{12}}{d\sigma_1} > 0 \qquad (3.57)
$$

An Example: The example below illustrates that, indeed, an increase in σ_1 can lead to an increase in aggregate savings. To simplify the calculations the following normalizations are made:

$w = 1$ and $\bar{L} = 1$

Preferences are specified as follows:

$$U = \sum_{t=1}^{3} \rho^{t-1} u_t \tag{3.58}$$

where

$$\begin{aligned} u_t &= \ln c_t + \delta \ln v_t \quad \text{for } t=1,2 \\ u_t &= \ln c_t + \delta \ln \bar{L} \quad \text{for } t=3 \\ \rho &= \text{a discount factor.} \end{aligned} \tag{3.59}$$

Individuals maximize U subject to the following budget constraint:

$$c_1 + w_n^1 v_1 + \frac{1}{1+r} c_2 + w_n^2 v_2 + \frac{c_3}{(1+r)^2} = (w_n^1 + w_n^2)\bar{L} \tag{3.60}$$

Solution of this problem yields the following:

$$\begin{aligned} c_1 &= \frac{(w_n^1 + w_n^2)\bar{L}}{(1 + \delta + \rho + \rho\delta + \rho^2)} \\ v_1 &= \frac{\delta(w_n^1 + w_n^2)\bar{L}}{w_n^1(1 + \delta + \rho + \delta\rho + \rho^2)} \\ c_2 &= \frac{\rho(1 + r)(w_n^1 + w_n^2)\bar{L}}{(1 + \delta + \rho + \rho\delta + \rho^2)} \\ v_2 &= \frac{\rho\delta(w_n^1 + w_n^2)\bar{L}}{w_n^2(1 + \delta + \rho + \rho\delta + \rho^2)} \\ c_3 &= \frac{\rho^2(1 + r)^2(w_n^1 + w_n^2)\bar{L}}{(1 + \delta + \rho + \rho\delta + \rho^2)} \end{aligned} \tag{3.61}$$

Also note:

$$U_{44} = -\frac{\rho\delta}{v_2^2}$$
$$U_{22} = -\frac{\delta}{v_1^2} \tag{3.62}$$

Consider first whether the following expression holds:

$$w_n^1(\bar{L} - v_2)U_{44} - w_n^2(\bar{L} - v_1)U_{22} > 0$$

Substituting the expressions for U_{44} and U_{22} into the inequality above:

$$w_n^1(\bar{L} - v_2)\left(-\frac{\rho\delta}{v_2^2} \right) - w_n^2(\bar{L} - v_1)\left(-\frac{\delta}{v_1^2} \right) > 0$$

This simplifies further to the following:

$$\frac{(\bar{L} - v_1)}{w_n^2} > \frac{(\bar{L} - v_2)}{w_n^1} \tag{3.63}$$

Since w has been normalized to one:

$$w_n^1 = 1 - \theta + \frac{R_f\sigma_1}{2(1 + r)^2} \quad \text{and} \quad w_n^2 = \frac{1 - \theta}{1 + r} + \frac{R_f\sigma_2}{(1 + r)^2}$$

And also recall that it was assumed:

$$\sigma_1 = \frac{2(1 + n)^2\theta}{R_f} \quad \text{and} \quad \sigma_2 = \frac{2(1 + n)\theta}{R_f}$$

The model is parameterized as follows:

$$\theta = 0.10$$
$$n = 0.50$$
$$r = 0.60$$
$$\rho = 0.60$$
$$\delta = 1.00$$

Under this parameterization the following values are computed:

$$\begin{aligned}
w_n^1 &= 0.98789 \\
\frac{1}{w_n^1} &= 1.01226 \\
w_n^2 &= 0.62109 \\
\frac{1}{w_n^2} &= 1.61006 \\
c_1 &= 0.45196 \\
v_1 &= 0.45750 \\
v_2 &= 0.43661 \\
U_{44} &= -3.14748 \\
U_{22} &= -4.7777
\end{aligned} \tag{3.64}$$

That is:

$$\frac{\bar{L} - v_1}{w_n^2} = 1.61006 - 0.73661 = 0.873452$$

and

$$\frac{\bar{L} - v_2}{w_n^1} = 1.012258 - 0.441965 = 0.57029$$

This means from the analysis of 3.52 and 3.53 that:

$$\frac{ds_1}{d\sigma_1} > 0 \text{ and } \frac{ds_2}{d\sigma_1} \text{ is ambiguous.}$$

It also means with respect to the expression for aggregate savings that the last two lines of 3.56 must have a positive effect on aggregate savings. If this is true for the first 5 lines of 3.56 then aggregate savings must increase in response to an increase in σ_1. With this in mind each line is analyzed under the parameterization discussed above.[10]

For line 1 to have a positive effect it must be the case that:

$$2 + r + n = 3.1 > 1$$

This is obviously the case.

For line 2 to have a positive effect it must be the case that:

$$(2 + r + n)(1 - v_2)U_{44} - (1 - v_1)U_{22} < 0$$

That is:

$$(3.1)(0.56339)(-3.14749) - (0.54250)(-4.77777) < 0$$

or:

$$-5.49712 + 2.59194 < 0$$

[10] For a term to have a positive effect on savings it must be negative since Δ, the determinant of the six-by-six matrix in 3.36, is negative.

This is clearly the case and line 2 must have a positive effect upon savings. Note, also, that this means lines 4 and 5 must have a positive effect.

It remains to show that line 3 has a positive effect. For this to be true it must be the case that:

$$(2 + r + n)(w_n^2 w_n^2) - (w_n^1 w_n^2) > 0$$

or:

$$(3.1)(0.38575) - (0.61357) > 0$$

or:

$$1.19583 - 0.61357 > 0$$

Line 3 also has a positive effect. Thus, it is shown that under this parameterization, at least, all terms in 3.63 affect aggregate savings in a positive manner so that aggregate savings must respond positively to a change in σ_1.

Optimal Weights

Here it is assumed that changes in aggregate savings have no effect on the gross wage rate or interest rate. It is shown that even in this case, neither the current program nor a zero net tax is optimal. The planner's problem in this case is to choose the earnings weights so as to maximize the indirect utility of a representative individual subject to a pay-as-you-go constraint imposed on the social security program. If individuals work two periods and are retired for the third then the planner's problem can be expressed as:

$$\max_{\sigma_1,\sigma_2} V = U(c_1(\sigma_1,\sigma_2),\ v_1(\sigma_1,\ \sigma_2),\ c_2(\sigma_1,\sigma_2),\ v_2(\sigma_1,\sigma_2),\ c_3(\sigma_1,\sigma_2)) \tag{3.65}$$

subject to:

$$\left[(1+n)^2\theta w - \frac{R_f w\sigma_1}{2}\right](\bar{L} - v_1(\sigma_1,\sigma_2)) + \left[(1+n)\theta w - \frac{R_f w\sigma_2}{2}\right](\bar{L} - v_2(\sigma_1,\sigma_2)) = 0 \tag{3.66}$$

where all variables are defined above and where the pay-as-you-go constraint simply expresses the fact that taxes paid in by the young and middle-aged must equal the benefits paid out to the old.

The first order conditions for this problem are:

$$\frac{\partial V}{\partial \sigma_1} + \lambda\left[\frac{R_f w(\bar{L} - v_1)}{2} + \left((1+n)^2\theta w - \frac{R_f\sigma_1}{2}\right)\frac{\partial v_1}{\partial \sigma_1} + \left((1+n)\theta w - \frac{R_f w\sigma_2}{2}\right)\frac{\partial v_2}{\partial \sigma_1}\right] = 0 \tag{3.67}$$

$$\frac{\partial V}{\partial \sigma_2} + \lambda\left[\frac{R_f w(\bar{L} - v_2)}{2} + \left((1+n)^2\theta w - \frac{R_f w\sigma_1}{2}\right)\frac{\partial v_1}{\partial \sigma_2} + \left((1+n)\theta w - \frac{R_f w\sigma_2}{2}\right)\frac{\partial v_2}{\partial \sigma_2}\right] = 0 \tag{3.68}$$

By the envelope theorem for constrained optimization problems:[11]

$$\frac{\partial V}{\partial \sigma_1} = \alpha\left[\frac{R_f w(\bar{L} - v_1)}{2(1 + r)^2} \right] \tag{3.69}$$

and

$$\frac{\partial V}{\partial \sigma_2} = \alpha\left[\frac{R_f w(\bar{L} - v_2)}{2(1 + r)^2} \right] \tag{3.70}$$

where α = the marginal utility of income. Substituting 3.69 and 3.70 into 3.67 and 3.68 the following expressions are obtained:

$$\begin{aligned}(\alpha + \lambda)&\left[\frac{R_f w(\bar{L} - v_1)}{2} \right] + \\ &\lambda\left[\left((1 + n)^2\theta w - \frac{R_f w\sigma_1}{2}\right) \frac{\partial v_1}{\partial \sigma_1} + \left((1 + n)\theta w - \frac{R_f w\sigma_2}{2}\right) \frac{\partial v_2}{\partial \sigma_1} \right] = 0\end{aligned} \tag{3.71}$$

$$\begin{aligned}(\alpha + \lambda)&\left[\frac{R_f w(\bar{L} - v_2)}{2} \right] + \\ &\lambda\left[\left((1 + n)^2\theta w - \frac{R_f w\sigma_1}{2}\right) \frac{\partial v_1}{\partial \sigma_2} + \left((1 + n)\theta w - \frac{R_f w\sigma_2}{2}\right) \frac{\partial v_2}{\partial \sigma_2} \right] = 0\end{aligned} \tag{3.72}$$

From 3.71 and 3.72 the following can be derived:

---

[11] In this case the partial derivative of the indirect utility function with respect to σ_1 or σ_2 is equal to the Lagrange multiplier of the utility maximization problem times the partial of the budget constraint with respect to σ_1 or σ_2, respectively. See equations 3.14 and 3.15 for a statement of the individual's optimization problem.

$$\left[\left((1+n)^2\theta w - \frac{R_f w\sigma_1}{2}\right)\frac{\partial v_1}{\partial\sigma_1} + \left((1+n)\theta w - \frac{R_f w\sigma_2}{2}\right)\frac{\partial v_2}{\partial\sigma_1}\right]/(\bar{L}-v_1) = \left[\left((1+n)^2\theta w - \frac{R_f w\sigma_1}{2}\right)\frac{\partial v_1}{\partial\sigma_2} + \left((1+n)\theta w - \frac{R_f w\sigma_2}{2}\right)\frac{\partial v_2}{\partial\sigma_2}\right]/(\bar{L}-v_2) \quad (3.73)$$

From the pay-as-you-go constraint 3.66:

$$\left[(1+n)\theta w - \frac{R_f w\sigma_2}{2}\right] = \frac{(\bar{L}-v_1)}{(\bar{L}-v_2)}\left[(1+n)^2\theta w - \frac{R_f w\sigma_1}{2}\right] \quad (3.74)$$

Substituting this into 3.73:

$$\left[\left((1+n)^2\theta w - \frac{R_f w\sigma_1}{2}\right)\left(\frac{\partial v_1}{\partial\sigma_1} - \frac{(\bar{L}-v_1)}{(\bar{L}-v_2)}\frac{\partial v_2}{\partial\sigma_1}\right)\right]/(\bar{L}-v_1) = \left[\left((1+n)^2\theta w - \frac{R_f w\sigma_1}{2}\right)\left(\frac{\partial v_1}{\partial\sigma_2} - \frac{(\bar{L}-v_1)}{(\bar{L}-v_2)}\frac{\partial v_2}{\partial\sigma_2}\right)\right]/(\bar{L}-v_2) \quad (3.75)$$

or:

$$\frac{1}{(\bar{L}-v_1)}\left[\frac{\partial v_1}{\partial\sigma_1} - \frac{(\bar{L}-v_1)}{(\bar{L}-v_2)}\frac{\partial v_2}{\partial\sigma_1}\right] = \frac{1}{(\bar{L}-v_2)}\left[\frac{\partial v_1}{\partial\sigma_2} - \frac{(\bar{L}-v_1)}{(\bar{L}-v_2)}\frac{\partial v_2}{\partial\sigma_2}\right] \quad (3.76)$$

Multiply both sides by $\sigma_1\sigma_2$ and note that:

$$-\frac{\partial(\bar{L}-v_i)}{\partial\sigma_i} = \frac{\partial v_i}{\partial\sigma_i}$$

The following expression is then obtained:

$$- \sigma_2 \frac{\partial(\bar{L} - v_1)}{\partial \sigma_1} \frac{\sigma_1}{(\bar{L} - v_1)} + \sigma_2 \frac{\partial(\bar{L} - v_2)}{\partial \sigma_1} \frac{\sigma_1}{(\bar{L} - v_2)} = - \sigma_1 \frac{(\bar{L} - v_1)}{(\bar{L} - v_2)} \frac{\partial(\bar{L} - v_1)}{\partial \sigma_2} \frac{\sigma_2}{(\bar{L} - v_1)} + \sigma_1 \frac{(\bar{L} - v_1)}{(\bar{L} - v_2)} \frac{\partial(\bar{L} - v_2)}{\partial \sigma_2} \frac{\sigma_2}{(\bar{L} - v_2)} \tag{3.77}$$

Define the elasticity of labor supply in year i with respect to the earnings weight for year j as:

$$\xi_{ij} \equiv \frac{\partial(\bar{L} - v_i)}{\partial \sigma_j} \frac{\sigma_j}{(\bar{L} - v_i)} \tag{3.78}$$

Then 3.77 can be expressed as:

$$\sigma_2 = \frac{(\bar{L} - v_1)}{(\bar{L} - v_2)} \frac{(\xi_{22} - \xi_{12})}{(\xi_{21} - \xi_{11})} \sigma_1 \tag{3.79}$$

From 3.79 it can be seen that in general σ_1 is not equal to σ_2 and that this relationship depends on the elasticity of labor supply with respect to the earnings weights applied to both periods.

The Fallacy of Zero Net Taxes

It is easy to show from equations 3.04 and 3.05 that the weights, σ_1 and σ_2, can be set so that the net tax is zero. A zero net tax would appear to be even more appealing than the uniform net tax.[12] The zero net tax has the same property as a fully-funded net tax in that the budget

[12] In the subsequent chapter it is shown that a uniform net tax results in a welfare gain when compared to the current program.

constraint reduces to the original budget constraint with no taxes. This argument is often used to "prove" the neutrality of a fully funded social security program. However, this neutrality really depends on another property of the fully funded program: for every dollar of private savings lost a dollar of public savings is created. Does the zero net tax retain this neutrality? It is argued below that it does not and that in fact the zero net tax in the standard case (r>n) must result in a social security deficit. (Actually, in the standard case any amount of public debt is not sustainable in this model since the debt grows at the rate of interest, r, and output grows at the rate of the population growth, n.)

A zero net tax can be derived by setting NT_1 and NT_2 equal to zero in equations 3.04 and 3.05 solving for the following:

$$\begin{aligned} \sigma_1 &= \frac{2\theta(1 + r)^2}{R_f} \\ \sigma_2 &= \frac{2\theta(1 + r)}{R_f} \end{aligned} \tag{3.80}$$

To show the budget constraint for a zero net social security tax reduces to the budget constraint in the case of no tax, an individual who works two periods and is retired for the third is considered. The budget constraint for this individual without a social security program is the following:

$$c_1 + \frac{c_2}{1 + r} + \frac{c_3}{(1 + r)^2} = w(\bar{L} - v_1) + \frac{w(\bar{L} - v_2)}{1 + r} \tag{3.81}$$

With a social security program the individual's budget constraint can be expressed in the following form:

$$c_1 + \frac{c_2}{1+r} + \frac{c_3}{(1+r)^2} = (1-\theta)w(\bar{L} - v_1) + \frac{(1-\theta)w(\bar{L} - v_2)}{1+r} + \left(\frac{1}{(1+r)^2}\right)\left(\frac{R_f w}{2}\sum_{t=1}^{2}\sigma_t(\bar{L} - v_t)\right) \tag{3.82}$$

Substituting the zero net tax expressions (3.80) for σ_t:

$$\begin{aligned} c_1 + \frac{c_2}{1+r} + \frac{c_3}{(1+r)^2} &= (1-\theta)w(\bar{L} - v_1) + \frac{(1-\theta)w(\bar{L} - v_2)}{1+r} \\ &\quad + \left(\frac{1}{(1+r)^2}\right)\left(\frac{R_f}{2}\sum_{t=1}^{2}\frac{2\theta w(\bar{L} - v_t)}{(1+r)^{t-1}R_f(1+r)^{-2}}\right) \\ &= (1-\theta)w(\bar{L} - v_1) + \frac{(1-\theta)w(\bar{L} - v_2)}{1+r} \\ &\quad + \frac{w\theta}{(1+r)^2}\sum_{t=1}^{2}\frac{(1+r)^2(\bar{L} - v_t)}{(1+r)^{t-1}} \\ &= w(\bar{L} - v_1) + \frac{w(\bar{L} - v_2)}{1+r} \end{aligned} \tag{3.83}$$

That is, the budget constraint reduces to the budget constraint of the case without a social security tax just as in the fully-funded case. This is, however, essentially a partial equilibrium result. When wages and interest rates are allowed to respond, as in an overlapping generations growth model, the fully-funded result holds up but the zero net tax result does not.

It can be shown only in the case in which the interest rate equals the rate of population growth will a pay-as-you-go constraint be consistent with a zero net tax. If r>n, as in the standard case, then social security must run a deficit. To see why, again consider the pay-as-you-go constraint, 3.12, which for this case can be expressed as:

$$\sum_{t=1}^{2} \theta w(\bar{L} - v_t) N_{t,T} = \frac{R_f w}{2} \left[\sum_{j=1}^{2} \sigma_j (\bar{L} - v_j) \right] N_{3,T} \tag{3.84}$$

where all variables are defined above, and where, recall:

$N_{t,T}$ = population of cohort t at calendar time T.

Note that:

$$N_{1,T} = (1+n)^2 N_{3,T}$$

$$N_{2,T} = (1+n) N_{3,T}$$

Thus 3.84 can be expressed as:

$$\theta w(\bar{L} - v_1)(1+n)^2 N_{3,T} + \theta w(\bar{L} - v_2)(1+n) N_{3,T} = \tag{3.85}$$

$$\frac{R_f w}{2} \left[\frac{2\theta(\bar{L} - v_1)}{R_f\left(\frac{1}{(1+r)}\right)^2} + \frac{2\theta(\bar{L} - v_2)}{R_f\left(\frac{1}{(1+r)}\right)} \right] N_{3,T}$$

or if both sides are divided by $(1+n)^2 N_{3,T}$:

$$\theta w(\bar{L} - v_1)\left(1 - \frac{(1 + r)^2}{(1 + n)^2}\right) + \theta w(\bar{L} - v_2)\left(1 - \frac{(1 + r)}{(1 + n)}\right) = 0 \tag{3.86}$$

3.86 obviously holds only if r=n. If r>n then these terms are negative and the equality does not hold. This simply means that taxes collected are less than benefits paid out. From 3.86 the reason is clear. The social security tax base grows at the rate n yet benefits are discounted at the rate r. For the net tax to be zero the present value of the stream of marginal benefits must be such that a social security deficit is created. (This is shown for the general case in Appendix B.)

Again, the point is that the zero net tax is like fully-funded social security in that, given a fixed wage and interest rate, the budget constraint reduces to the original budget constraint without a tax. This is often used as a "proof" that a given tax is neutral or, if the argument is modified a little, that a given tax is equivalent to some other tax. However, in a general equilibrium growth model this is not sufficient. Summers (1981) makes this point with respect to wage and consumption taxes. They are equivalent in the sense that for a fixed w and r the budget constraints are equivalent. But because of dynamic considerations -- the consumption tax is imposed across all years, whereas a wage tax is imposed only on the working years -- savings are affected differently, the steady state levels of capital are different, and hence the welfare losses are different under the two tax policies. The same type of analysis applies to the fully-funded social security program and a zero net tax program. They are equivalent in the "budget constraint" sense but result in two different steady states. Because each dollar of private savings is replaced by a dollar of public savings, capital formation is unaffected in the fully-funded case. Because in the zero net tax case a deficit is created, capital formation decreases as does welfare.

4. SIMULATION MODEL AND RESULTS

The simulations discussed below are based on a many-periods overlapping generations growth model. This model is discussed in detail in this chapter. Also discussed is the procedure used to compute the solutions. First, however, the pay-as-you-go social security program discussed in the previous chapter is reconsidered for the more general case in which individuals work for many years and are retired for a lesser number of years. The current U.S. program is discussed in some detail. Finally, the selection of values for the parameters of the model and the simulation results are considered.

Social Security in the Many-Periods Model

The government exists only to administer the social security program. It is free to change the tax and benefit parameters of the system so as to minimize any welfare loss it might create. The stylized social security program used in this study represents a simplified version of the U.S. social security program. Both the current U.S. program and the simplified program used in this study are discussed below.

The Current U.S. Program: As a result of the 1983 law the current wage tax is 7.51%. Employers also contribute 7.51%. However, part of this goes toward health and disability insurance funds. If these are subtracted out then the worker's and employer's shares become 5.53%. The incidence of this is usually considered to fall upon the worker. If this were the case then the tax would be about 11%. This depends, however, on labor being supplied inelastically. Since in this model labor is supplied endogenously this does not hold in the strict sense. Nevertheless, a 10% payroll tax is assumed to be representative of the current U.S. program.

Benefits are determined by several factors; the most important of which are: (i) computation of average indexed monthly earnings (AIME), (ii) the formula which relates monthly benefits to the workers AIME, and (iii) the difference in treatment between married and single workers. The AIME is a measure of average covered earnings over a specified number of years and each year's earnings is wage-indexed to the wage-level at the year of retirement.

Once the AIME has been determined then the primary insurance amount (PIA) can be computed. As discussed above, the PIA is the level of monthly retirement benefits for a single worker retiring at age 62. The PIA is calculated from the following formula:

- 90% of the first $339 of the AIME plus

- 32% of the AIME between $339 and $1705 plus

- 15% of the AIME in excess of $1705.

$339 and $1705 are referred to as the bend points and are wage-indexed. The 90%, 32%, and 15% in the above formula are referred to as the marginal replacement rates and are not scheduled to change. $48,000 ($4000 per month) is the ceiling on taxable earnings and is also wage-indexed.

For married retirees one spouse can claim to be the other's dependent and receive 50% of the working spouse's benefits. It is usually considered advantageous to do this if the spouse's income is less than 30% of that of the spouse that earns the higher income.

The Simplified Program: The current system can be simplified in the following way. In a steady state without technical growth no wage-indexing occurs since each period's wage is the same. Thus, the AIME for a representative worker can be expressed by the following:

$$\text{AIME} = \frac{w}{m} \sum_{j=1}^{M} (\bar{L} - v_j)$$

where recall

m = the number of working years.

w = the gross wage rate.

$\bar{L}$ = the maximum time available for work.

v_j = leisure in period j.

In computing the primary insurance amount, a simple proportional formula is substituted for the bend points specified in the current U.S. program. Instead of having marginal replacement rates for each bend point, a single "representative" marginal replacement rate is used. For example, a representative marginal replacement rate can be derived in the following manner.[13] Ignore the benefits of children, assume all those that receive benefits are married, and use the average benefits of a retired worker of $479 (as reported in the statistical supplement of the social security bulletin for 1986). Since it is assumed that all retirees are married their spouses receive in benefits an additional 50% of this $479. Total benefits, then, equal 1.5 x $479 = $719. This represents the monthly benefits of the representative worker (the PIA). The AIME can be derived from this as follows. The marginal replacement rate is 90% for the first $230 of the AIME. Thus, out of the $479 in benefits received 0.9 x $230 = $207 came out of the first $230 of the AIME. From $230 to $1388 of the AIME, the marginal replacement rate is 32%. From this the amount of the AIME that falls between $230 and $1388, call it X, can be computed:

$$.32X = \$479 - \$207$$

$$X = \$850$$

Since $850 is less than $1158 (i.e. $1388-$230) the AIME that results in an average of $479 in benefits must be $1080 (i.e. $850+$230). Since the average PIA of a married worker is $719 and the average AIME is $1080, a simple proportional marginal replacement rate can be computed which in

[13] This example is based on the bend points used in 1986.

this case is equal to 0.67 (the ratio: \$719/\$1080). The point here is that for any given wage under the current system with marginal replacement rates at each bend point, there is an equivalent simple portional benefit formula which yields the same level of benefits. A simple proportional benefit formula is sufficient for the purposes of this paper since all workers are exactly alike.

The simplified version of the U.S. program used in this paper can be characterized as follows:

The wage tax = 10%.

$$\text{The AIME} = \frac{w}{m}\left[\sum_{j=1}^{m}(\bar{L} - v_j)\right] \tag{4.01}$$

$$\text{The PIA} = \frac{R_f w}{m}\left[\sum_{j=1}^{m}(\bar{L} - v_j)\right] \tag{4.02}$$

where:

The AIME and PIA are as discussed above.

and where:

R_f = a proportional marginal replacement factor.

The formula for the PIA can be generalized further:

$$PIA = \frac{R_f w}{m}\left[\sum_{j=1}^{m} \sigma_j(\bar{L} - v_j)\right] \tag{4.03}$$

where:

σ_j = a factor that is applied to each year's earnings.

The Worker's Decision Problem over Many Periods.

The workers' decision problem is now considered in further detail. This discussion is simplified by assuming there is no technical growth in the economy so that in the steady state not only are interest rates constant, but wage rates are also constant (since the efficiency of labor is not changing).

It is assumed utility is such that there is constant elasticity of substitution between utility in different periods:

$$U^{-\gamma} = \sum_{t=1}^{Q} \frac{u_t^{-\gamma}}{(1+\rho)^{t-1}} \tag{4.04}$$

where

γ = the intertemporal substitution parameter.

ρ = the rate of time preference.

u_t = the elementary utility fuction.

and

$$u_t = \left(\delta c_t^{-\beta} + (1-\delta)v_t^{-\beta}\right)^{\frac{-1}{\beta}} \text{ for } t \leq m \tag{4.05}$$

where

δ = distribution parameter.

c_t = consumption at time t.

v_t = leisure at time t.

β = contemporaneous substitution parameter.

and

$$u_t = \left(\delta c_t^{-\beta} + (1-\delta)\bar{L}^{-\beta}\right)^{\frac{-1}{\beta}} \text{ for } t > m \tag{4.06}$$

where

$\bar{L}$ = the most an individual can work in a given period.

Workers maximize this intertemporal utility function subject to their life-cycle income. This consists of their income net of social security taxes during the working periods and a social security benefit payment during the retired years. The optimization problem can be expressed as:

$$\max_{c_t, v_t} U^{-\gamma} = \sum_{t=1}^{m} \frac{\left[(\delta c_t^{-\beta} + (1-\delta)v_t^{-\beta})^{\frac{-1}{\beta}}\right]^{-\gamma}}{(1+\rho)^{t-1}} + \sum_{t=m+1}^{Q} \frac{\left[(\delta c_t + (1-\delta)\bar{L}^{-\beta})^{\frac{-1}{\beta}}\right]^{-\gamma}}{(1+\rho)^{t-1}} \tag{4.07}$$

subject to

$$c_1 + \sum_{t=2}^{Q} \frac{c_t}{(1+r)^{t-1}} + \sum_{t=1}^{m} \frac{w_n v_t}{(1+r)^{t-1}} + \sum_{j=m+1}^{Q} \frac{\frac{R_f w}{m} \sum_{t=1}^{m} \sigma_t v_t}{(1+r)^{j-1}} =$$

$$\sum_{t=1}^{m} \frac{(1-\theta)w\bar{L}}{(1+r)^{t-1}} + \sum_{j=m+1}^{Q} \frac{R_f w\bar{L}}{m} \frac{\sum_{t=1}^{m} \sigma_t}{(1+r)^{j-1}}$$

From the solution to this problem the following is obtained:

$$c_1 = \left\{ w_n \bar{L} \sum_{t=1}^{m} \frac{1}{(1+r)^{t-1}} + \frac{R_f w \bar{L}}{m} \left(\sum_{t=1}^{m} \sigma_t \right) \sum_{j=m+1}^{Q} \frac{1}{(1+r)^{j-1}} - \sum_{t=m+1}^{Q} \frac{\left(\frac{1+r}{1+\rho} \right)^{\frac{t-1}{1+\beta}} \left(\frac{\delta c_t^{-\beta} + (1-\delta)\bar{L}^{-\beta}}{D_1} \right)^{\frac{\gamma-\beta}{\beta(1+\beta)}} c_1^{\frac{1+\gamma}{1+\beta}}}{(1+r)^{t-1}} \right\} \Bigg/ \tag{4.08}$$

$$\left\{ 1 + \sum_{t=2}^{m} \frac{\left(\frac{1+r}{1+\rho} \right)^{\frac{t-1}{1+\gamma}} \left(\frac{D_t}{D_1} \right)^{\frac{\gamma-\beta}{\beta(1+\gamma)}}}{(1+r)^{t-1}} + \sum_{t=1}^{m} \frac{w_n \left(\frac{1+r}{1+\rho} \right)^{\frac{t-1}{1+\gamma}} \left(\frac{1-\delta}{\delta H_t} \right)^{\frac{1}{1+\beta}} \left(\frac{D_t}{D_1} \right)^{\frac{\gamma-\beta}{\beta(1+\gamma)}}}{(1+r)^{t-1}} + \sum_{j=m+1}^{Q} \sum_{t=1}^{m} \frac{\frac{R_f w}{m} \sigma_t \left(\frac{1+r}{1+\rho} \right)^{\frac{t-1}{1+\gamma}} \left(\frac{1-\delta}{\delta H_t} \right)^{\frac{1}{1+\beta}} \left(\frac{D_t}{D_1} \right)^{\frac{\gamma-\beta}{\beta(1+\gamma)}}}{(1+r)^{j-1}} \right\}$$

where

$$D_t = \delta + (1-\delta) \left(\frac{1-\delta}{\delta H_t} \right)^{\frac{-\beta}{1+\beta}}$$

and

$$H_t = w_n + \frac{(1+r)^{t-1} R_f w \sigma_t}{m} \sum_{j=m+1}^{Q} \frac{1}{(1+r)^{j-1}}$$

For $t \leq m$ consumption and leisure at time t can be expressed in terms of c_1 as:

$$c_t = \left(\frac{1+r}{1+\rho}\right)^{\frac{t-1}{1+\gamma}} \left(\frac{D_t}{D_1}\right)^{\frac{\gamma-\beta}{\beta(1+\gamma)}} c_1 \tag{4.09}$$

$$v_t = \left(\frac{1-\delta}{\delta H_t}\right)^{\frac{1}{1+\beta}} c_t \tag{4.10}$$

For $t > m$ consumption at time t can be expressed in terms of c_1 and itself c_t as:

$$c_t = \left(\frac{1+r}{1+\rho}\right)^{\frac{t-1}{1+\beta}} \left(\frac{\delta c_t^{-\beta} + (1-\delta)\bar{L}^{-\beta}}{D_1}\right)^{\frac{\gamma-\beta}{\beta(1+\beta)}} (c_1)^{\frac{1+\gamma}{1+\beta}} \tag{4.11}$$

The Steady State Economy

Production: There is one good in this economy which can either be consumed or used in the production of next period's output (that is, next period's capital). It is also assumed that this capital does not depreciate. This good is produced by a constant-returns-to-scale (Cobb-Douglas) technology without technical growth. That is:

$$Y_T = AK_T^{\Psi} L_T^{(1-\Psi)} \tag{4.12}$$

where

T = calendar time subscript.

Y_T = output of the consumption good at T.

K_T = aggregate capital at T.

L_T = aggregate labor force at T.

Ψ = capital's share in output.

A = a scaling constant.

Perfect competition is assumed which ensures that gross factor returns equal their respective marginal products:

$$w = (1-\Psi)A\left(\frac{K_T}{L_T}\right)^{\Psi} \tag{4.13}$$

$$r = \Psi A\left(\frac{K_T}{L_T}\right)^{-(1-\Psi)} \tag{4.14}$$

where

A, K_T, L_T, and Ψ are defined above.

w = the gross real wage rate.

r = the gross real interest rate.

Since in a steady state without technical growth the capital-labor ratio is constant, the real wage and the real interest rate need not be subscripted.

Aggregate Consumption, Labor, and Savings: From the consumption and leisure equations (4.08 through 4.11) and the marginal product of capital and labor equations (4.13 and 4.14), equilibrium consumption, labor, and savings at calendar time T can be determined. Aggregate consumption can be expressed as:

$$C_T = \sum_{t=1}^{Q} c_t N_{t,T} \tag{4.15}$$

where

C_T = aggregate consumption at time T.

c_t = consumption of an age t individual.

$N_{t,T}$ = the population of those age t at calendar time T.

Aggregate labor can be expressed as:

$$L_T = \sum_{t=1}^{m} (\bar{L} - v_t) N_{t,T} \tag{4.16}$$

where

$v_t, \bar{L}$, and $N_{t,T}$ are defined above.

L_T = aggregate level of labor at time T.

Savings is assumed to be life-cycle in the sense that the only wealth in the economy is the accumulated savings of those currently alive at calendar time T:

$$A_T = \sum_{t=1}^{Q} a_t N_{t,T} \tag{4.17}$$

where

$N_{t,T}$ is defined above.

A_T = accumulated wealth in the economy at time T.

$a_t = (1+r)a_{t-1} + (1-\theta)w(L - v_t) - c_t$ for $t \le m$

$a_t = (1+r)a_{t-1} + b - c_t$ for $t > m$

b = the yearly social security benefit.

The Steady State: In this economy, since there is no government expenditure the GNP identity can be expressed as:

$$\begin{aligned} Y_T &= C_T + I_T \\ &= C_T + K_{T+1} - K_T \end{aligned} \tag{4.18}$$

where I_T represents investment.

With constant returns to scale labor's share plus capital's share equals total output. Thus (4.18) can be expressed as:

$$\begin{aligned} wL_T + rK_T &= C_T + K_{T+1} - K_T \\ K_{T+1} &= (1+r)K_T + w_n L_T - C_T + B_T \end{aligned} \tag{4.19}$$

where

$w_n = (1-\theta)w$ = wages net of taxes.

B_T = aggregate social security benefits paid out at T.

At this point note that with a pay-as-you-go social security program the following equation must hold:

$$\Theta_T = B_T \tag{4.20}$$

where

$$\Theta_T = \sum_{t=1}^{m} \theta w(\bar{L} - v_t)N_{t,T} \tag{4.21}$$

and

$$B_T = \frac{R_f w}{m}\left[\sum_{j=1}^{m} \sigma_j(\bar{L} - v_j)\right]\left[\sum_{t=m+1}^{Q} N_{t,T}\right] \tag{4.22}$$

and where

θ, v_t, L, R_f, w, m, and $N_{t,T}$ are defined above.

Equations (4.20) through (4.22) simply mean that for a given calendar year T total social security taxes paid in equal total benefits paid out.

Since in this model all wealth is held in the form of next period's capital stock, $K_{T+1} = A_T$. Since also in a steady state with no technical growth $K_{T+1} = (1+n)K_T$ the following equation must hold:

$$K_T = \frac{1+r}{1+n} K_T + \frac{1}{(1+n)} (w_n L_T - C_T + B_T) \tag{4.23}$$

where, recall, n is the rate of population growth and all other variables are defined above.

The GNP identity (4.16) can also be expressed as:

$$K_T = \frac{1}{(1+n)} \sum_{t=1}^{Q-1} a_t N_{t,T} \tag{4.24}$$

where

a_t and $N_{t,T}$ are defined above.

The advantage of this equation over (4.16) is that it avoids the well-known problem of a trivial golden-rule solution at r=n.

The Solution Procedure: Since a_t is a function of net wages and the interest rate which are in turn functions of K_T and L_T, and since this is also true for v_t; equations (4.10) and (4.37) represent K_T and L_T as functions of themselves. This is convenient in that these equations are expressed in the form necessary to apply the method of linear iteration for two variables. This method is used to obtain a numerical solution to the above model. More specifically, an initial guess is made on K_t and L_T. This allows the gross real wage and interest rate to be computed and hence all the terms of the right hand side of the aggregate capital and the right hand side of the aggregate labor equations can be determined.

If the right hand side of these two equations is within some epsilon of their respective left hand sides (i.e. the current guesses for K_T and L_T) the solution is said to converge. Otherwise, another iteration is necessary. If this is the case then the right hand values computed for this iteration are used as the guesses for K_T and L_T for the next iteration. The procedure continues as described here until the solution converges or it is apparent the solution will not converge.

This same procedure is used in a nested manner to get solutions for first period and retirement periods consumption that are consistent with 4.08 and 4.11, given the current guess for K_T and L_T. That is, an initial guess is made on c_1. Given this, initial guesses are made on the c_t for the retired periods. When the right hand sides of equations 4.11 are within some epsilon of the current guesses for the c_t, the solutions for equations 4.11, given the initial guess for c_1, are obtained. In turn, this is used to compute the right hand side of 4.08. When this is within some epsilon of the current guess for c_1 then a solution is obtained for 4.08 and 4.11 given the current guess for K_T and L_T

Selecting the Parameters

There is no attempt made here to "calibrate" the model by selecting sets of parameters that yield values of the endogenous variables close to those of the real U.S. economy. Rather, the only guideline is that parameters lie within or near acceptable limits. A sensitivity analysis of sorts is performed by considering high and low values for the intertemporal and contemporaneous substitution parameters, and also high and low values for the distribution parameter.[14]

Intertemporal Elasticity of Substitution (γ): Most studies of this parameter do not include leisure in the utility function, but estimates do not appear sensitive to this. Values range from 0.07 estimated by Grossman and Shiller (1981) to 0.75 estimated by Weber (1975). Typically, values range from 0.15 to 0.40. With leisure in the utility function Ghez and Becker (1975) estimate a value of 0.28 whereas MaCurdy (1981) estimates a value between 0.10 and 0.45. Auerbach and Kotlikoff settle on a value of 0.25 for their model. In this paper values of 0.40 and 1.33 are used ($\gamma = 1.50$ and $\gamma = -.25$) to represent an inelastic and an elastic case respectively.

Contemporaneous Elasticity of Substitution (β): Evidently, there have been few studies as to the appropriate values of this parameter. Auerbach

[14] In their 1987 book Auerbach and Kotlikoff survey much of the empirical work with respect to the values of the parameters used in this model. Much of the discussion below is based on this survey.

and Kotlikoff use a value of 0.8 which is based on the value of 0.83 reported by Ghez and Becker (1983). In this paper values of 0.56 and 1.33 are used (β = 0.75 and β = −.25) to represent an inelastic and an elastic case respectively.

The Distribution Parameter (δ): The distribution parameter determines the weight consumption and leisure receive in the utility function. Evidently, there are no estimates of this parameter. Auerbach and Kotlikoff use a value of 0.40 because it yields reasonable results in their model with respect to labor supply decisions. However, any value under 0.50 gives more weight to leisure than to consumption. Given the composite nature of the consumption good this seems somewhat questionable. The values used in this paper are 0.45 and 0.70.

The Subjective Rate of Time Preference (ρ): Auerbach and Kotlikoff claim there is little evidence as to what this value should be. They use a value of 1.5% since it leads to realistic consumption profiles and labor supply decisions. Browning (1985) however suggests that discount rates higher than 6% have been empirically estimated. Nonetheless the value used in this paper of 2% is close to Auerbach and Kotlikoff's and also Summer's (1981) and Burbidge's (1983) values.

Capital's Share (Γ): Studies of the elasticity of substitution between capital and labor in the U.S. usually obtain values approximately equal to 1. (Nerlove (1967), Bendt and Christianson (1973)). Given a CES production function this of course implies the Cobb-Douglas subcase. It is well-known that for a Cobb-Douglas production function, factor shares of income are constant and equal to the capital intensity parameter, Γ. Historically, capital's share of national income has been estimated to be 0.25 in the U.S. This is the value used in this paper.

Rate of Population Growth: The rate of population growth is usually assumed to be between 1% and 2%. For example, Summers (1981) uses a rate of 1.5% and the implied rate in Burbidge (1983) is 2.3%. Also, Kotlikoff (1979) uses a rate of 2%. In this paper a rate of 2% is used.

Although no attempt was made to calibrate the model to the real U.S. economy, marginal replacement rates close to the "representative" rate of 67% are obtained under most parameterizations. Also, in most cases, implementation of a social security program reduces the capital-labor ratio by approximately 20% which is consistent with the result in Kotlikoff (1979).

Current Program versus Uniform Net Tax and Exempted Net Tax

Adjusting the Benefit Formula: Recall, if weights are attached to each year's earnings then the yearly social security benefits (the PIA) can be computed using the following formula:

$$\text{PIA} = \frac{R_f w}{m} \sum_{t=1}^{m} (\sigma_t(\bar{L} - v_t))$$

where recall that

PIA = the primary insurance amount.

R_f = a proportional marginal replacement factor.

m = the number of working years.

σ_t = the weights applied to working year t.

w = the gross real wage.

$\bar{L}$ = full employment.

v_t = leisure consumption in year t.

Note that if $\sigma_t = 1$ then this defines the current benefit formula. As σ_t changes so does the present value of the marginal stream of benefits and consequently the net tax for this period. This can be seen more clearly from the formula for the net tax:[15]

$$NT_t = \theta - \frac{(1+r)^{t-1} R_f \sigma_t}{m} \sum_{j=m+1}^{Q} \frac{1}{(1+r)^{j-1}} \tag{4.25}$$

where recall that

θ = the payroll tax rate.

Q = the worker's lifespan.

From 4.25 it is clear that as σ_t changes so does the net tax. If:

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[15] This is a generalization of 3.03 and 3.04.

$$\sigma_t = \frac{1}{(1 + r)^{t-1}} \tag{4.26}$$

then the age dependent term of the net tax formula can be removed and, since the other terms depend only on the interest rate, a uniform net tax can be derived. Furthermore, this uniform net tax is consistent with a pay-as-you-go constraint.[16] More generally, σ_t can be set to impose any level of uniform net tax as in the following formula for σ_t:

$$\sigma_t = \frac{(\theta - NT)m}{(1 + r)^{t-1} R_f \sum_{j=m+1}^{Q} \frac{1}{(1 + r)^{j-1}}} \tag{4.27}$$

for all t.

where NT is the uniform net tax and all other variables are defined above.

In this case for a given θ and NT, the resultant uniform net tax may or may not be consistent with a pay-as-you-go constraint. It could result in either a deficit or surplus in the program. However, the pay-as-you-go case is the only one of interest here.

Again, the object here is to manipulate the yearly weights in the benefit formula to determine if the welfare losses of social security can be reduced. The no-tax or laissez-faire case is considered as the base case

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[16] The marginal replacement factor is allowed to "float" to determine the yearly level of benefits and maintain a pay-as-you-go constraint.

against which the alternative social security programs are compared. These alternatives consist of the current program, a program in which the benefit formula imposes a uniform net tax, and a program in which the first fifteen working years receive full weight and the remaining working years receive zero weights (referred to in the tables as an exempted net tax).

The appeal of uniform net taxes is derived from the static case in which labor supply is fixed exogenously. In this case it is easy to show that a proportional uniform tax on all goods is optimal. This is simply due to the fact that this type of tax is equivalent to a tax on the inelastic good which in this case is leisure. In addition there are cases of the Ramsey Rule when this also holds.

The optimality of uniform taxes, then, applies generally in the static case when labor is supplied inelastically. It cannot be argued that uniform net taxes are optimal in this case. Indeed, the analysis in the previous chapter showed that in general uniform net taxes would not be optimal. However, there is a theoretical basis for employing a uniform net tax. Since in the current program the net tax is higher for the younger workers, a uniform net tax would be expected to lower the net tax for the younger workers and raise it for the older workers. The opportunity cost of leisure goes up for the younger workers, they work more, and since they are more numerous than the older workers (who are now working less) aggregate savings could increase.

Like the uniform net tax, the exempted net tax is designed to reduce the disincentives that a social security program imposes on younger workers. The object again is to get the younger, more numerous workers to work more and hence save more. This should result in more capital in the steady state and should reduce the welfare loss.

The comparisons between the no tax steady state, the current program, the uniform net tax, and the exempted net tax are summarized in tables B.01 through B.40 for a number of different parameterizations. The consistent result that is observed is that a switch from the current program to a uniform net tax results in a slight gain in welfare. In some cases further gains can be realized by switching to an exempted net tax. (Although in other cases this results in a slight reduction of welfare.) These results hold for both elastic and inelastic values of the substitution parameters, and for high and low values of the distribution parameter. The welfare losses under each of the parameterizations are summarized in tables 4.01 and 4.02. Table 4.01 represents cases in which consumption receives more weight in utility ($\delta = 0.70$). Table 4.02 represents cases in which leisure receives more weight ($\delta = 0.45$).

For example, in the relatively inelastic case, represented as column 1 in Table 4.01, it can be seen that implementation of the current program results in a 5.73% loss in welfare (in terms of the present value of full income in the no-tax case). A switch to a uniform net tax reduces the welfare loss to 5.68%. A switch to an exempted net tax results in a further reduction to 5.66%.

Similar results hold for the more elastic case, represented as column 4 in Table 4.01, The current program results in a welfare loss of 3.97%. A switch to a uniform net tax reduces the welfare loss to 3.94%, but in this case a switch to an exempted net tax decreases the welfare loss slightly to 3.95%. (In this case a uniform net tax proves to be more effcient; however this is due, principally, to the fact that no attempt is made to find the optimal number of exempted years.)

If leisure receives more weight, it is observed that the initial welfare loss is not as great. This is because the social security payroll tax lowers the opportunity cost of leisure, which results in individuals substituting into leisure which is now more desirable than is the case in Table 4.01. However, the gains from switching to alternative tax schemes is much the same as when consumption receives more weight.

The simulations then have the predicted results qualitatively. Wage incentives for the young created by manipulating the social security benefit formula result in increased work effort and savings by the young. This results in a net increase in aggregate savings despite the fact that savings by the older workers must decrease. This increase in net savings leads to lower interest rates and higher wages resulting in a higher level of full income and a higher level of steady state utility.

Quantitatively, though, the welfare gains achieved through the alternative programs are relatively small. This is, however, consistent with Summers (1981) in which it is argued that full income effects are quantitatively more important than intertemporal substitution effects. Policy decisions which substantially affect full income such as switching from a tax on capital to a tax on consumption as in Summer's paper can substantially reduce steady state welfare losses. In the simulations in this paper the pay-as-you-go constraint acts to inhibit the full income effect in two closely related ways. First, any increase in net wages that occurs in the early working years must be offset by a reduction in net wages in the later working years. This is because any increase in the earnings weights for the early years must be met with a decrease in the weights for the later years to maintain the pay-as-you-go constraint. Thus, at the individual level--with wages and interest rates held fixed--full income will change very little. Second, at the aggregate level, the increase in savings that comes about from the higher net wages of the young must be offset to a large extent by a reduction in savings of the older generations since their net wages will be lower.[17] The changes in welfare are relatively small because in all cases the net result on full income is relatively small.

Optimal Weights

In the simulations discussed above the current program in which each year's earnings are weighted identically is "suboptimal" in the general

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[17] This is substantiated by comparing full income under the current program to full income under, say, the uniform net tax program. In the most inelastic case this represents a 0.7% increase. Similar results apply to the other cases.

equilibrium case in the sense that either of the other two programs considered results in a higher level of steady-state utilty. In addition, each of these programs suggests that the optimal pattern of weights is one in which higher weights are applied to earnings when young than to earnings when old. In this section the question of the pattern of optimal weights is addressed from the general equilibrium perspective. Recall, in the previous chapter this was addressed from the perspective of a partial equilibrium. It was shown that the optimal earnings weights depended on the own-price and cross-price elasticities of labor supply. When the wage rate and interest rate are allowed to respond to changes in aggregate savings, another potential source of welfare gain is introduced. This occurs because in an economy with a positive rate of population growth if the young respond to incentives brought about by changes in the benefit formula, then it is possible for aggregate savings to increase.[18] As a result, the interest rate goes down and the gross wage rate goes up. Both responses have a positive effect on full income and as a result it is possible for steady state utility to increase.

Due to complexities in the calculations, the optimal weights were computed for an individual who works for 5 periods and is retired for 3 periods. This was sufficient to establish the basic result relating the pattern

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[18] As discussed in the previous chapter, although this is not assured by economic theory, there is reason to suspect that this indeed is the case.

of the optimal weights to the elasticity parameters. A quasi-Newton maximization method was employed to compute these numerically.[19]

The results are displayed in Tables 4.03 through 4.06. The suspicion above that at the optimum the weights for the younger workers would be greater turns out to indeed be true for all the cases considered. The reason for this is the obvious one. Aggregate savings will increase must come at the cost of a reduction in savings of the old due to the pay-as-you-go constraint.

What is interesting here is that as the intertemporal substitution becomes more inelastic, the pattern of optimal weights becomes more exaggerated with respect to the difference between weights applied to the younger working years and the weights applied to the older working years. This can be observed by comparing Table 4.03 to Table 4.06. The intuitive explanation of this is the following. In the relatively inelastic case the savings behavior of the old will be less affected by changes in the earnings weights. This allows the weights for the young to be set much higher than in the case in which intertemporal substitution is relatively elastic.

If preferences are such that the elasticity of substitution between current consumption and current leisure is greater than one (1.33 in this

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[19] The computer program that performs the maximization simply calls a modified version of the program used above to compute the steady state equilibrium, the arguments in the call statement being the weights in the benefit formula.

case) then the above result is mitigated somewhat. This can be seen by comparing Table 4.03 to Table 4.04 and Table 4.05 to Table 4.06.

Tables 4.03 and 4.04 represent the cases in which the intertemporal substitution is inelastic ($\gamma = 1.50$). In Table 4.03, substitution between current consumption and leisure is inelastic (β = 0.75); in Table 4.04 substitution between current consumption and leisure is elastic (β = -0.25). In the latter case it is apparent that the optimal pattern for the earnings weights does not give as much weight to the younger years and does not penalize the older years as in Table 4.03. The reason is when the substitution between current consumption and leisure is elastic the old are more willing to substitute into leisure contemporaneously as the net wages of the old go down with the decrease in the earnings weights for these years. That is, the potential reduction in the savings of the old means it is optimal not to penalize old age earnings as much as in the case in which substitution between current consumption and leisure is inelastic.

Tables 4.05 and 4.06 exhibit a similar property for the case in which intertemporal substitution is elastic (γ = -0.25). In Table 4.06 substitution between current consumption and leisure is elastic (β = -0.25). It can be seen that the optimal pattern for the earnings gives more weight to the younger years and does not penalize the older years as much as in Table 4.05. The reason is the same as above. When the elasticity of substitution between current consumption and leisure is elastic the old are more willing to substitute into leisure as their net wage rates go down.

Table 4.01
Summary of Welfare Losses
(δ = 0.70)

	γ = 1.50		γ = -.25	
Tax	β = 0.75	β = -.25	β = 0.75	β = -.25
CP	5.73%	7.14%	3.12%	3.97%
UNT	5.68%	7.10%	3.08%	3.94%
ENT	5.66%	7.08%	3.08%	3.95%

CP = Current Program
UNT = Uniform Net Tax
ENT = Exempted Net Tax
β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

Table 4.02
Summary of Welfare Losses
(δ = 0.45)

	γ = 1.50		γ = -.25	
Tax	β = 0.75	β = -.25	β = 0.75	β = -.25
CP	4.07%	3.56%	2.23%	2.15%
UNT	4.02%	3.50%	2.18%	2.09%
ENT	3.99%	3.48%	2.19%	2.13%

CP = Current Program
UNT = Uniform Net Tax
ENT = Exempted Net Tax
β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

Table 4.03
Optimal Weights versus Uniform Net Tax
(γ = 1.50; β = 0.75; δ = 0.70)

YR	1	2	3	4	5	ΔW
CP	1.000 (0.181)	1.000 (0.181)	1.000 (0.181)	1.000 (0.181)	1.000 (0.181)	-5.426%
UNT	1.000 (0.268)	0.792 (0.212)	0.628 (0.168)	0.497 (0.133)	0.394 (0.106)	-5.403%
OPT	4.163 (0.716)	2.604 (0.448)	0.752 (0.129)	-1.174 (-.202)	-2.048 (-.352)	-5.333%

(...) = Replacement factor x weight
CP = Weights for Current Program
UNT = Weights for Uniform Net Tax
OPT = Optimal Weights
β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

Table 4.04
Optimal Weights versus Uniform Net Tax
(γ = 1.50; β = -.25;δ = 0.70)

YR	1	2	3	4	5	ΔW
CP	1.000	1.000	1.000	1.000	1.000	-6.828%
	(0.181)	(0.181)	(0.181)	(0.181)	(0.181)	
UNT	1.000	0.767	0.589	0.452	0.347	-6.801%
	(0.282)	(0.216)	(0.166)	(0.127)	(0.098)	
OPT	2.579	1.607	0.478	-0.690	-1.218	-6.728%
	(0.714)	(0.445)	(0.132)	(-.191)	(-.337)	

(...) = Replacement factor x weight
CP = Weights for Current Program
UNT = Weights for Uniform Net Tax
OPT = Optimal Weights
β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

Table 4.05
Optimal Weights versus Uniform Net Tax
(γ = -.25; β = 0.75; δ = 0.70)

YR	1	2	3	4	5	ΔW
CP	1.000	1.000	1.000	1.000	1.000	-3.574%
	(0.181)	(0.181)	(0.181)	(0.181)	(0.181)	
UNT	1.000	0.867	0.751	0.651	0.564	-3.561%
	(0.232)	(0.201)	(0.174)	(0.151)	(0.131)	
OPT	2.100	1.644	1.016	0.325	-0.070	-3.544%
	(0.353)	(0.276)	(0.171)	(0.055)	(-.012)	

(...) = Replacement factor x weight
CP = Weights for Current Program
UNT = Weights for Uniform Net Tax
OPT = Optimal Weights
β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

Table 4.06
Optimal Weights versus Uniform Net Tax
(γ = -.25; β = -.25; δ = 0.70)

YR	1	2	3	4	5	ΔW
CP	1.000	1.000	1.000	1.000	1.000	-4.245%
	(0.181)	(0.181)	(0.181)	(0.181)	(0.181)	
UNT	1.000	0.856	0.733	0.628	0.538	-4.233%
	(0.238)	(0.204)	(0.174)	(0.149)	(0.128)	
OPT	1.963	1.539	0.993	0.402	0.097	-4.222%
	(0.340)	(0.266)	(0.161)	(0.070)	(0.017)	

(...) = Replacement factor x weight
CP = Weights for Current Program
UNT = Weights for Uniform Net Tax
OPT = Optimal Weights
β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

5. SUMMARY

This study argues that the social security benefit formula provides a policy instrument that can be used to change the intertemporal pattern of net earnings. This is accomplished by applying weights to each year's earnings. In effect, wage incentives can be created to induce the younger, more numerous workers to provide more labor and save more, and hence increase aggregate savings. This, despite the fact that savings of the older, less numerous workers may actually go down.

This is not a straightforward proposition as the results of the three-period partial equilibrium analysis indicate. In general, the initial effect of an increase in the first period's earnings weight is ambiguous with respect to labor supply and savings. However, by assumming intertemporal and contemporaneous separability, the response of labor supply can be shown to be positive in the first period and negative in the second. Further, although there is still an element of ambiguity, it is argued that the terms that affect savings in a positive manner may well outweigh the terms that have a negative effect. An example is provided to substantiate this claim.

As it turns out these results hold for the more general cases considered in the simulations. In all cases considered, policies such as the uniform net tax which give more weight to the earnings of the young, when compared to the current program of equal weights, result in more labor supply when young and in an increase in aggregate savings. This increase in aggregate savings results in a lower interest rate and a higher wage rate in the long run. This means an increase in life-cycle full income and an increase in steady state utility.

Qualitatively then, the results are as predicted. Quantitatively, however, these changes are anything but dramatic. The typical increase in steady state welfare is less than .1 percent. This requires some explanation in light of other studies in which alternative policies designed to stimulate capital formation result in gains of up to 20 percent in steady state welfare. To obtain results of this magnitude requires a substantial initial jolt to the intertemporal pattern of relative prices. For instance, in Summers (1981) this is accomplished by removing the fifty percent tax on capital and replacing it with a wage or consumption tax that generates the same amount of revenue. In a rough sense, the switch in policy has little effect on full income initially since each individual's tax bill would be approximately the same under either policy; but removing the tax on capital doubles the interest rate and drastically lowers the price of future consumption across the board. Individuals will have strong incentives to consume less and save more now to provide for a planned increase in future consumption. The sharp rise in savings that occurs brings a sharp increase in life-cycle income through the human wealth effect and hence an increase in steady state utility.

Why doesn't this occur in the model of social security in this paper? There are two reasons offered. First, there is no initial effect on the interest rate and hence no across-the-board reduction in the price of future consumption. The initial effects are all due to changes in the in the intertemporal pattern of net wages. The rate of population growth and the pay-as-you-go constraint combine to prevent the changes in the net wages from being any greater than 2 or 3 percent. Second, the changes in the net wages that occur across time tend to offset their effect on savings; unlike the case in Summers in which the changes across time reinforce the effect on savings. Evidence for both arguments can be found by comparing the uniform net tax in, say, Table B.07 (of Appendix B) to the net taxes under the current program. Two points are readily observable. First, in the early years, which are more critical due to present value considerations, the difference in the net tax is at most 2 percent. Second, in the later years the net tax goes up which just means that the savings of the older workers would be expected to go down. The simulation results reflect the fact that on net aggregate savings will increase by only a small amount. This means life-cycle income will go up by only a small amount and the increase in steady welfare will indeed be modest.

Although the uniform net tax and the exempted net tax lead to increases in steady state welfare, they are not chosen optimally. When the weights are chosen optimally to maximize steady state utility, what is the result? It is found that although the pattern of the weights is much more distorted than in the case of, say, the uniform net tax; the additional increase in steady state welfare is slight. This increase is no more than 0.05 percent as can be observed in Tables 4.03 through 4.06.

These results suggest that the potential gains of such a policy would be far outweighed by other considerations. In a more realistic setting the change from the status quo to a policy favoring younger workers would no doubt engender additional administrative costs. More clerks and managers would have to be hired, more pamphlets produced explaining the new policy. More importantly, a policy change that penalizes the earnings of older workers and favors the earnings of younger workers would be politically feasible only if the long run gains were dramatic and indisputable.

II. THE ADJUSTMENT COSTS OF A PUBLIC SAVINGS PROGRAM

1. INTRODUCTION AND LITERATURE

Diamond (1965), among others, has shown that in an overlapping generations growth model steady state utility is optimized when the capital-labor ratio of the economy attains the golden rule level. Samuelson (1975a, 1975b) and Feldstein (1974b, 1977) suggest that social security be used as a public savings program to attain this golden-rule level. It is assummed by these authors that this is a desirable social objective. That is, the long run gains of raising the capital-labor ratio to the golden rule level outweigh the short-run costs of increased taxes necessary to create the government savings.

This issue is addressed here in a simple framework. It is assummed that moving to the golden-rule steady state is a desirable social objective. The planner's task is simple: to find the single tax rate, applied to all generations, that ensures private and government savings will converge to the golden-rule levels. The theoretical model employed here is an overlapping generations growth model in which individuals live for two periods.[20] Two initial steady states are assummed: a laissez-faire steady state with no social security program and a steady state that represents a pay-as-you-go social security program. Computer simulations are used to compute these steady states and to compute recursively the transition path to the golden-rule steady state.

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[20] Similar to that used in Diamond (1965), Samuelson (1975a, 1975b), or Burbidge (1983), among others.

The main result of the computer simulations is that, although a number of generations suffer a loss in welfare, the maximum welfare loss in all cases is very small. In the laissez-faire cases the maximum welfare loss is less than 0.001%. In the pay-as-you-go cases the maximum welfare loss is close to 0.04%. Such small welfare losses are primarily due to the fact that the increase in the tax rate necessary to achieve the golden-rule steady state is indeed quite small.

Taken literally, this suggests that such a policy could be implemented with only a small effect on the earlier generations that must pay for the buildup of government savings; contrary to Feldstein's (1974b) claim that such a transition would be quite expensive in terms of the welfare losses to these generations.

2. THE THEORETICAL MODEL

The model employed here is a two-period overlapping generations growth model much like the model discussed in Diamond (1965). It represents a dynamic version of the model employed by Samuelson (1975a, 1975b). It allows for the possibility of a public savings program arising from tax revenues collected from the young workers.

The Worker's Problem: Each worker lives for two periods. The first period is spent working a fixed amount. The second period is spent in retirement. Preferences can be characterized by a CES utility function. That is, workers solve the following:

$$\max_{c_t^1,\, c_t^2} U^{-\gamma} = (c_t^1)^{-\gamma} + \frac{(c_t^2)^{-\gamma}}{(1+\rho)} \tag{2.01}$$

subject to:

$$c_t^1 + \frac{c_t^2}{1 + r_{t+1}} = (1-\theta)w_t + \frac{R_f w_t}{1 + R_{t+1}} \tag{2.02}$$

where

c_t^1 = first period consumption of generation t.

c_t^2 = second period consumption of generation t.

$\frac{1}{1+\gamma}$ = the (constant) elasticity of intertemporal substitution.

ρ = the subjective rate of time discount.

θ = the social security tax on the young.

R_f = the marginal replacement rate.

w_t = earnings of the young in year t.

r_{t+1} = the interest rate paid on savings held from period t to t+1.

The solution to this problem results in the following expressions for c_t^1 and c_t^2 :

$$c_t^2 = \frac{(1 + r_{t+1})^{\frac{2+\gamma}{1+\gamma}}(1 - \theta + \frac{R_f}{1 + r_{t+1}})w_t}{(1 + r_{t+1})(1 + \rho)^{\frac{1}{1+\gamma}} + (1 + r_{t+1})^{\frac{1}{1+\gamma}}} \qquad (2.03)$$

$$c_t^1 = \left[\frac{1 + \rho}{1 + r_{t+1}} \right]^{\frac{1}{1+\gamma}} c_t^2 \qquad (2.04)$$

Production: Production is characterized by constant returns to scale technology and the assumption of competitive firms. This means factor returns are equal to their marginal products. A Cobb-Douglas production function is assummed. Production can be summarized as follows:

$$y_t = Ak_t^{\alpha} \qquad (2.05)$$

$$r_t = \alpha A k_t^{-(1-\alpha)} \tag{2.06}$$

$$w_t = (1-\alpha) A k_t^{\alpha} \tag{2.07}$$

where

- A = a scaling factor.
- α = capital's share.
- k_t = capital-labor ratio.
- y_t = output per young.
- r_t = return to capital at time t.
- w_t = wage rate at time t.

Market Clearing Condition: Next period's capital-labor ratio is equal to private savings this period plus the accumulated government savings this period in terms of this period's young:

$$(1+n)k_{t+1} = ((1-\theta)w_t(k_t) - c_t^1(k_t, k_{t+1})) + g_t \tag{2.08}$$

where

- n = the rate of population growth.
- g_t = government savings per worker at time t.

Government Savings: An expression for government savings per worker, g_t, can be derived as follows. Let G_t be defined as government savings at time t. Then:

$$G_1 = \theta w_1 N_1 - R_f w_0 N_0$$

where

$$N_t = (1+n)^t = \text{ total workers at time t.}$$

In other words, government savings in period 1 equals taxes collected from the first generation of young.

Define the following:

$$g_t = \frac{G_t}{N_t}$$

so that:

$$\frac{G_1}{N_1} = g_1 = \theta w_1 - \frac{R_f w_0}{1+n}$$

Government savings for the second period equals principle plus interest on first period savings plus tax revenues collected from the second period young. That is:

$$G_2 = (1 + r_2)G_1 + \theta w_2 N_2 - R_f w_1 N_1$$

So that:

$$\frac{G_2}{N_2} = \frac{(1+r_2)}{(1+n)}\frac{G_1}{N_1} + \theta w_2 - \frac{R_f w_1}{1+n}$$

And in general:

$$\frac{G_t}{N_t} = \frac{(1+r_t)}{(1+n)}\frac{G_{t-1}}{N_{t-1}} + \theta w_t - \frac{R_f w_{t-1}}{1+n}$$

Or in terms of government savings per young:

$$g_t = \frac{(1+r_t)}{(1+n)} g_{t-1} + \theta w_t - \frac{R_f w_{t-1}}{1+n} \tag{2.09}$$

Note: it is assumed that $g_t \geq 0$.

Laissez-Faire versus Pay-as-you-Go: A laissez-faire steady state is defined by setting the payroll tax, θ, and the marginal replacement rate, R_f, to zero. A pay-as-you-go steady state is defined by specifying a payroll tax. This automatically implies a marginal replacement rate through the pay-as-you-go constraint. Further, it is assummed that this marginal replacement rate is the same for all generations.

The Planner's Problem: In the laissez-faire case the planner must simply find the tax rate, θ, that ensures the economy will, over time, converge to the golden-rule steady state. In the pay-as-you-go case the planner must find the increase in the current payroll tax, given the marginal replacement rate that satisfies the pay-as-you-go constraint at the original payroll tax, that ensures the economy will converge to the golden-rule steady state.

Solution Procedure: For a given payroll tax and a given initial laissez-faire or pay-as-you-go steady state, it is a simple matter to compute recursively the dynamic equilibrium by utilizing equations 2.03 through 2.09. However, it should be noted that because the equation for next period's capital (2.08) above is expressed as a nonlinear function of itself, the method of linear iteration is used to obtain a solution. This involves making an initial guess of k_{t+1} (k_t is used), computing the right hand side of 2.08, and then comparing the two. If they are not within some epsilon, the current right-hand-side value becomes the current guess and the iteration is repeated. The desired tax rate, θ, is found by using a computerized search technique that is completed when the interest rate for the last period is within some epsilon of the rate of population growth.

3. SIMULATION RESULTS

Selecting Parameters

The interpretation that is often given to the two-period model is that each period represents 35 years. This means that the population growth rate and the discount parameters need to be compounded over 35 years. Another consideration is that the planning horizon, T, has to represent a period long enough so that a steady state is approximated. In the simulations discussed below, setting T=60 was found to be sufficient for this purpose. Below the values used for each parameter are discussed.[21]

Capital's Share (α): Historically, capital's share of national income has been estimated to be 0.25 in the U.S.. This is the value used in this paper.

The Rate of Population Growth (n): The population growth rate is assummed to be 2%. If this is compounded for 35 years a growth rate of close to 100% is obtained.

The Intertemporal Elasticity of Substitution (γ): Two values are used in the simulations representing an elastic and an inelastic case. For

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[21] For a discussion of the appropriate range of values for these parameters see the discussion in Chapter 4 of the previous part of this dissertation.

the elastic case a value of 1.33 (γ = -0.25) is chosen. For the inelastic case a value of 0.67 (γ = 0.50) is used.

The Individual Rate of Time Preference (ρ): Two rates are used. A rate of 135% which is implied by a discount rate of 2.5% compounded over 35 years and a rate of 200% which is implied by a discount rate of 3.25% compounded over 35 years.

Results

The emphasis here is on the welfare loss that occurs on the transition path to the golden rule steady state. Parameters are assummed such that the interest rates in the initial laissez-faire and pay-as-you-go steady states are greater than the rate of population growth. Two measures of this welfare loss are considered: the number of generations that suffer a welfare loss and the extent of the welfare loss (measured by the greatest loss of welfare).

The Laissez-Faire Case: Four subcases are considered. First, the intertemporal elasticity of substitution is set to the relatively high value of 1.33 (γ = -.25). Given this relatively high value, two values for the individual's rate of time preference are considered: one that is relatively low (ρ = 1.35) and one that is relatively high (ρ = 2.00). Then, the intertemporal elasticity of substitution is set to the relatively low value of 0.67 (γ = 0.50). Given this, the two values above for the individual's rate of time preference are considered.

The results of the simulations for these four cases are tabulated in appendix C. These results are summarized in Table 3.01. The number of generations that suffer a welfare loss, the maximum welfare loss, the tax needed to reach the golden-rule steady state, and the interest rate at the laissez-faire steady state are presented for each of these four cases.

The basic result that emerges is that, although in each case a number of generations is affected adversely, in no case does the welfare loss exceed 0.001%. This is due mostly to the fact that in no case is the tax required to reach the golden-rule steady state greater than 0.00001%.

By holding the intertemporal elasticity of substitution, γ, fixed, the response to a change in the individual's rate of time preference can be analyzed. In both cases ($\gamma = -.25$ and $\gamma = 0.50$), both measures of welfare loss go down as the rate of time preference goes up (from $\rho = 1.35$ to $\rho = 2.00$). The reason is the following. If the representative individual has a high rate of time preference (discounts the future more), aggregate savings in the economy will be less, all else the same, than if the representative individual were to have a lower rate of time preference. This means the initial laissez-faire steady state will be further from the golden rule and there will be more generations along the transition path that gain from moving towards the golden-rule steady state.

By holding the rate of time preference, ρ, constant, the response to changes in the intertemporal elasticity of substitution can be analyzed. In the case of a low rate of time discount ($\rho = 1.35$), as the elasticity parameter becomes less elastic (γ goes from -.25 to 0.50) both measures of welfare loss go down. In the case of a high rate of time discount (ρ = 2.00), both measures of welfare loss also go down, but the measures are actually much closer. The reason is the following. With a low rate of time discount, all else the same, the more elastic case means that the representative individual is more likely to substitute into future consumption and save more now. That is, the less elastic case will be further from the golden rule. (The interest rates in Table 3.01 indicate this.) There will be more generations along the transition path that will gain from a move towards the golden-rule steady state. A higher rate of time discount brings the steady states for the elastic and inelastic cases closer together since, all else the same, individuals prefer future consumption less.[22]

The Pay-as-you-Go Case: The same four subcases are considered in the pay-as-you-go case. The results of these simulations are tabulated in appendix D and summarized in Table 3.02.

The response to changes in the intertemporal elasticity of substitution and in the individual's rate of time discount fall along the same lines as is in the laissez-faire case. The basic question that must be answered in comparing the two cases is why in the pay-as-you-go case more generations are affected and affected to a larger extent.

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[22] It is quite possible with even higher rates of time discount that the inelastic case will be closer to the golden rule than the elastic case.

The reason this occurs is the following. In the pay-as-you-go case the stock of government savings in the early periods is much lower. Also, in the pay-as-you-go case private savings decrease more sharply in the early periods. This is because the initial effects of the pay-as-you-go program (at a payroll tax of 10%) is great enough to reduce the net wage to the point where the increase in current consumption that occurs in response to the decrease in the interest rate can actually lower the final level of private savings. This occurs in the two cases in which intertemporal substitution is relatively elastic ($\gamma = -.25$). However, even in the inelastic cases ($\gamma = 0.50$) the initial decrease in private savings lasts for a longer period of time.

Thus, in the pay-as-you-go case it takes longer for the combination of private savings and public savings to create enough capital for the human wealth effects to offset the negative effects that occur as the price of future consumption goes up.

Table 3.01
Summary of Welfare Losses
Laissez-Faire Case

	γ = -.25		γ = 0.50	
Measure	ρ = 1.35	ρ = 2.00	ρ = 1.35	ρ = 2.00
Gens	12	11	11	10
ΔW(%)	0.001497	0.000211	0.000514	0.000209
Tax(%)	0.00000669	0.00000109	0.00000213	0.00000077
r	2.10	2.55	2.45	2.84

Gens = Number of generations that suffer a welfare loss.
ΔW = Maximum percentage change in welfare.
Tax = Tax needed to reach the golden rule.
r = Interest rate for the laissez-faire steady state.
γ = Intertemporal elasticity of substitution
ρ = Rate of time preference.
Labor's Share = 0.75
Rate of Population Growth = 100.0
Golden Rule Interest Rate = 1.0

Table 3.02
Summary of Welfare Losses
Pay-as-you-Go Cases

	γ = -.25		γ = 0.50	
Measure	ρ = 1.35	ρ = 2.00	ρ = 1.35	ρ = 2.00
Gens	17	17	16	16
ΔW(%)	0.021098	0.026274	0.037467	0.046382
Tax(%)	0.10000049	0.10000008	0.10000007	0.10000003
r	2.58	3.18	3.38	4.01

Gens = Number of generations that suffer a welfare loss.
ΔW = Maximum percentage change in welfare.
Tax = Tax needed to reach the golden rule.
r = Interest rate for the laissez-faire steady state.
γ = Intertemporal elasticity of substitution
ρ = Rate of time preference.
Labor's Share = 0.75
Rate of Population Growth = 100.0
Golden Rule Interest Rate = 1.0

4. SUMMARY

In this study the transition to a golden-rule steady state was considered in a simple framework. The planner simply had to find the tax rate (or change in the tax rate in the pay-as-you-go case) which results in the golden-rule steady state. Taken literally, these results suggest that the transition costs to a golden-rule steady state are very small. Indeed, if the planner were free to choose separate tax rates for each generation, welfare losses should be even lower.

It should be noted, however, as the problem is posed here, the length of time it takes to reach the golden-rule is of no concern to the planner. Given the interpretation that each period in the two-period model represents 35 years, the sixty periods allowed for in these simulations represent 2100 years. Reserved for future consideration is a problem very similar to the one here: What if the planner is given some finite period of time in which to achieve the golden-rule? This could be achieved by imposing a public savings program and then returning to the initial laissez-faire or pay-as-you-go policy once the golden-rule level of capital has been attained. In this case transition costs would be greater than those reported here and would go up as the allowed-for transition phase becomes shorter.

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APPENDIX A: THE FALLACY OF A ZERO NET TAX

The pay-as-you-go constraint, equation 4.20 through 4.22, can be expressed as:

$$\sum_{t=1}^{m} \theta w(\bar{L} - v_t)N_{t,T} = \frac{r_f w}{m}\left[\sum_{j=1}^{m} \sigma_j \bar{L} - v_j)\right]\left[\sum_{t=m+1}^{Q} N_{t,T}\right]$$

Recall also equation 4.20 for a zero net tax:

$$\sigma_t = \frac{\theta m}{(1+r)^{t-1} R_f \displaystyle\sum_{j=m+1}^{Q} \frac{1}{(1+r)^{j-1}}}$$

Note, as before, the following relationship between the populations of cohorts:

$$N_{j,T} = (1+n)^{m-j+1} N_{m+1,T} \qquad \text{(A.01)}$$

where $N_{j,T}$ is defined above.

Substituting the zero net tax equation into the pay-as-you-go constraint and using the relationship A.01 the following is obtained:

$$\sum_{t=1}^{m} \theta w(\bar{L} - v_t)(1 + n)^{m+1-t} N_{m+1,T} =$$

$$\frac{R_f w}{m}\left[\sum_{t=1}^{m} \frac{\theta m(\bar{L} - v_j)}{R_f(1 + r)^{j-1} \sum\limits_{j=m+1}^{Q} \frac{1}{(1 + r)^{j-1}}}\right]\left[\sum_{j=1}^{Q-m} (1 + n)^{-(j-1)}\right] N_{m+1,T}$$

Divide both sides by $(1 + n)^m N_{m+1,T}$ and get:

$$\sum_{t=1}^{m} \theta w(\bar{L} - v_t)(1 + n)^{-}(t - 1) =$$

$$\frac{R_f w}{m}\left[\sum_{t=1}^{m} \frac{\theta m(\bar{L} - v_j)}{R_f(1 + r)^{t-1} \frac{(1 + n)^m}{(1 + r)^m}}\right]\left[\frac{\sum\limits_{j=1}^{Q-m} (1 + n)^{-(j-1)}}{\sum\limits_{j=1}^{Q-m} (1 + r)^{-(j-1)}}\right]$$

or

$$\sum_{t=1}^{m} \theta w(\bar{L} - v_t)\left[\frac{1}{(1 + n)^{t-1}} - \frac{(1 + r)^{m-t+1}}{(1 + n)^{m-t+1}}\left(\frac{\sum\limits_{j=1}^{Q-m} (1 + n)^{-(j-1)}}{\sum\limits_{j=1}^{Q-m} (1 + r)^{-(j-1)}}\right)\right] = 0$$

or

$$\sum_{t=1}^{m} \frac{\theta w(\bar{L} - v_t)}{(1+n)^{t-1}} \left[1 - \frac{(1+r)^{m-t+1}}{(1+n)^{m-t+1}} \left(\frac{\sum_{j=1}^{Q-m} (1+n)^{-(j-1)}}{\sum_{j=1}^{Q-m} (1+r)^{-(j-1)}} \right) \right] = 0 \qquad \text{(A.02)}$$

At issue here is whether or not the equality above holds. That is, whether or not the pay-as-you-go constraint holds. Consideration of this leads to the following proposition:

If the weights, σ_i, are set so that the net taxes are zero (see equation 4.25) then it must be the case that:

(i) The pay-as-you-go constraint holds if and only if $r = n$.

(ii) A deficit occurs if and only if $r > n$.

(iii) A surplus occurs if and only if $r < n$.

Proof:

Only (ii) is proved below. The arguments for (i) and (iii) are essentially the same.

If $r > n$ then:

$$\frac{(1+r)^{m-t+1}}{(1+n)^{m-t+1}} > 1 \text{ for all } t. \qquad \text{(A.03)}$$

and:

$$\frac{\displaystyle\sum_{j=1}^{Q-m} \frac{1}{(1+n)^{j-1}}}{\displaystyle\sum_{j=1}^{Q-m} \frac{1}{(1+r)^{j-1}}} > 1 \text{ for all t.} \tag{A.04}$$

This means that in equation A.02 the expression in brackets must be negative for all t. Since the expression outside the brackets must always be positive, the program must suffer a deficit.

If A.02 is negative, can it be shown that $r > n$? First note that if the left hand side of equation A.03 is greater than 1, then the left hand side of equation A.04 is greater than 1. Also, if one of these left hand sides is equal to 1 (or less than 1), than the other left-hand-side must be equal to 1 (or less than 1). This means that in A.02 the term in brackets must be of the same sign for all t.

Since:

$$\frac{\theta w(\bar{L} - v_t)}{(1+n)^{t-1}} > 0 \text{ for all t,}$$

the expression in brackets must be negative for all t. That is, A.03 and A.04 must hold. But these can only hold if $r > n$.

APPENDIX B: TABLES OF RESULTS

These tables detail the results of the simulations discussed in Chapter 4. The variables decribed in these tables are defined below:

w_g = the gross wage rate at time t.
w_n = the wage net of taxes (ignoring benefits).
r = the gross real wage rate.
SSB = individual social security benefits per period.
C_t = aggregate consumption at time t.
K_t = aggregate capital at time t.
L_t = aggregate labor supply at time t.
K/N = capital per person at time t.
K/L = capital-labor ratio at time t.
ΔW = the percentage change in individual welfare.

All values are computed at time T = 200. This means that the total population at time t, given a population growth rate of 2 %, can be set as follows for all tables:

N = 52.48

Table B.01
Comparisons of Steady States: β = 0.75; γ = 1.50
(δ = 0.70)

Var	No Tax	Current Program	Uniform Net Tax	Exempt. Net Tax
w_g	1.000	0.936	0.939	0.941
w_n	1.000	0.843	0.845	0.846
r	5.43%	6.61%	6.55%	6.52%
SSB	-------	0.388	0.387	0.387
C_t	34.080	32.470	32.370	32.320
K_t	172.860	132.970	133.810	134.270
L_t	28.150	28.160	28.020	27.940
K/N	3.290	2.530	2.550	2.560
K/L	6.140	4.720	4.770	4.800
ΔW	-------	-5.73%	-5.68%	-5.66%

β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

Table B.02
Comparisons of Net Taxes: $\beta = 0.75$; $\gamma = 1.50$
($\delta = 0.70$)

Age	Current Program	Uniform Net Tax	Exempt. Net Tax
1	8.98%	7.44%	7.55%
5	8.68%	7.44%	6.85%
10	8.19%	7.44%	5.68%
15	7.51%	7.44%	4.07%
20	6.56%	7.44%	10.00%
25	5.27%	7.44%	10.00%
30	3.48%	7.44%	10.00%
35	1.03%	7.44%	10.00%
40	-2.36%	7.44%	10.00%

β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution Parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

Table B.03
Consumption Across the Life-Cycle: β = 0.75; γ = 1.50
(δ = 0.70)

Age	No Tax	Current Program	Uniform Net Tax	Exempt. Net Tax
1	0.4924	0.4207	0.4225	0.4232
5	0.5191	0.4516	0.4531	0.4539
10	0.5546	0.4935	0.4945	0.4954
15	0.5925	0.5394	0.5397	0.5409
20	0.6330	0.5897	0.5889	0.5876
25	0.6763	0.6447	0.6427	0.6409
30	0.7225	0.7051	0.7014	0.6990
35	0.7719	0.7715	0.7654	0.7624
40	0.8247	0.8447	0.8353	0.8316
41	0.7787	0.8057	0.8012	0.7986
45	0.8249	0.8703	0.8646	0.8613
50	0.8865	0.9587	0.9513	0.9471

β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution Parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

Table B.04
Leisure Across the Life-Cycle: β = 0.75; γ = 1.50
(δ = 0.70)

Age	No Tax	Current Program	Uniform Net Tax	Exempt. Net Tax
1	0.3034	0.2840	0.2821	0.2825
5	0.3199	0.3043	0.3025	0.3016
10	0.3417	0.3315	0.3301	0.3269
15	0.3651	0.3608	0.3603	0.3535
20	0.3901	0.3921	0.3932	0.3983
25	0.4167	0.4254	0.4290	0.4344
30	0.4452	0.4603	0.4682	0.4737
35	0.4756	0.4965	0.5110	0.5167
40	0.5082	0.5332	0.5576	0.5635

β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution Parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

Table B.05
Savings out of Wages: β = 0.75; γ = 1.50
(δ = 0.70)

Age	No Tax	Current Program	Uniform Net Tax	Exempt. Net Tax
1	0.2042	0.1828	0.1842	0.1841
5	0.1610	0.1347	0.1364	0.1373
10	0.1036	0.0699	0.0717	0.0743
15	0.0424	-0.0007	0.0010	0.0063
20	-0.0231	-0.0773	-0.0760	-0.0783
25	-0.0930	-0.1604	-0.1601	-0.1621
30	-0.1677	-0.2503	-0.2519	-0.2536
35	-0.2475	-0.3472	-0.3521	-0.3533
40	-0.3328	-0.4513	-0.4614	-0.4621

β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution Parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

Table B.06
Comparisons of Steady States: β = −.25; γ = 1.50
(δ = 0.70)

Var	No Tax	Current Program	Uniform Net Tax	Exempt. Net Tax
w_g	1.000	0.940	0.943	0.944
w_n	1.000	0.846	0.848	0.849
r	6.01%	7.23%	7.18%	7.15%
SSB	-------	0.462	0.460	0.459
C_t	41.820	38.960	38.750	38.650
K_t	189.760	144.870	145.180	145.310
L_t	34.210	33.410	33.170	33.050
K/N	3.620	2.760	2.770	2.770
K/L	5.550	4.340	4.380	4.400
ΔW	-------	-7.14%	-7.10%	-7.08%

β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

Table B.07
Comparisons of Net Taxes: $\beta = -.25$; $\gamma = 1.50$
($\delta = 0.70$)

Age	Current Program	Uniform Net Tax	Exempt. Net Tax
1	9.22%	7.85%	8.05%
5	8.97%	7.85%	7.43%
10	8.54%	7.85%	6.37%
15	7.92%	7.85%	4.87%
20	7.06%	7.85%	10.00%
25	5.83%	7.85%	10.00%
30	4.09%	7.85%	10.00%
35	1.62%	7.85%	10.00%
40	-1.88%	7.85%	10.00%

β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution Parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

Table B.08
Consumption Across the Life-Cycle: $\beta = 0.75$; $\gamma = 1.50$
($\delta = 0.70$)

Age	No Tax	Current Program	Uniform Net Tax	Exempt. Net Tax
1	0.5898	0.4865	0.4900	0.4905
5	0.6273	0.5274	0.5303	0.5316
10	0.6776	0.5835	0.5856	0.5882
15	0.7319	0.6458	0.6465	0.6516
20	0.7906	0.7153	0.7138	0.7098
25	0.8539	0.7929	0.7881	0.7833
30	0.9224	0.8801	0.8702	0.8645
35	0.9963	0.9784	0.9608	0.9541
40	1.0762	1.0900	1.0608	1.0529
41	0.8522	0.8822	0.8772	0.8746
45	0.9218	0.9765	0.9701	0.9668
50	1.0165	1.1080	1.0995	1.0952

β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution Parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

Table B.09
Leisure Across the Life-Cycle: β = −.25; γ = 1.50
(δ = 0.70)

Age	No Tax	Current Program	Uniform Net Tax	Exempt. Net Tax
1	0.1906	0.1941	0.1910	0.1915
5	0.2027	0.2097	0.2068	0.2057
10	0.2189	0.2305	0.2283	0.2242
15	0.2365	0.2529	0.2521	0.2432
20	0.2554	0.2766	0.2783	0.2852
25	0.2759	0.3013	0.3073	0.3147
30	0.2980	0.3263	0.3393	0.3473
35	0.3219	0.3507	0.3746	0.3832
40	0.3477	0.3729	0.4136	0.4229

β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution Parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

Table B.10
Savings out of Wages: β = −.25; γ = 1.50
(δ = 0.70)

Age	No Tax	Current Program	Uniform Net Tax	Exempt. Net Tax
1	0.2197	0.1955	0.1962	0.1961
5	0.1700	0.1415	0.1425	0.1429
10	0.1035	0.0678	0.0690	0.0706
15	0.0316	-0.0135	-0.0121	-0.0089
20	-0.0460	-0.1031	-0.1017	-0.1027
25	-0.1298	-0.2016	-0.2006	-0.2013
30	-0.2204	-0.3099	-0.3098	-0.3102
35	-0.3182	-0.4289	-0.4303	-0.4303
40	-0.4239	-0.5593	-0.5634	-0.5628

β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution Parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

Table B.11
Comparisons of Steady States: β = 0.75; γ = –.25
(δ = 0.70)

Var	No Tax	Current Program	Uniform Net Tax	Exempt. Net Tax
w_g	1.000	0.956	0.960	0.965
w_n	1.000	0.860	0.864	0.868
r	3.54%	4.05%	4.00%	3.94
SSB	-------	0.418	0.417	0.416
C_t	33.870	33.180	33.090	32.960
K_t	278.650	233.460	236.150	239.150
L_t	29.580	29.680	29.530	29.330
K/N	5.310	4.450	4.500	4.560
K/L	9.420	7.870	8.000	8.150
ΔW	-------	-3.12%	-3.08%	-3.08%

β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

Table B.12
Comparisons of Net Taxes: β = 0.75; γ = -.25
(δ = 0.70)

Age	Current Program	Uniform Net Tax	Exempt. Net Tax
1	7.00%	4.45%	3.06%
5	6.49%	4.45%	1.90%
10	5.72%	4.45%	0.17%
15	4.78%	4.45%	-1.93%
20	3.63%	4.45%	10.00%
25	2.23%	4.45%	10.00%
30	-0.52%	4.45%	10.00%
35	-1.56%	4.45%	10.00%
40	-4.10%	4.45%	10.00%

β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution Parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

Table B.13
Consumption Across the Life-Cycle: β = 0.75; γ = 1.50
(δ = 0.70)

Age	No Tax	Current Program	Uniform Net Tax	Exempt. Net Tax
1	0.3978	0.3371	0.3373	0.3385
5	0.4309	0.3742	0.3741	0.3731
10	0.4761	0.4263	0.4258	0.4210
15	0.5261	0.4855	0.4847	0.4746
20	0.5813	0.5525	0.5518	0.5576
25	0.6423	0.6283	0.6281	0.6324
30	0.7097	0.7139	0.7149	0.7171
35	0.7842	0.8104	0.8138	0.8133
40	0.8665	0.9188	0.9264	0.9223
41	1.0708	1.1200	1.1076	1.0917
45	1.1326	1.2061	1.1908	1.1715
50	1.2142	1.3217	1.3023	1.2782

β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution Parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

Table B.14
Leisure Across the Life-Cycle: β = 0.75; γ = -.25
(δ = 0.70)

Age	No Tax	Current Program	Uniform Net Tax	Exempt. Net Tax
1	0.2451	0.2221	0.2184	0.2168
5	0.2655	0.2459	0.2422	0.2373
10	0.2934	0.2788	0.2757	0.2651
15	0.3242	0.3157	0.3138	0.2953
20	0.3582	0.3568	0.3572	0.3725
25	0.3958	0.4024	0.4066	0.4224
30	0.4373	0.4528	0.4629	0.4791
35	0.4832	0.5079	0.5269	0.5433
40	0.5339	0.5678	0.5998	0.6161

β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution Parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

Table B.15
Savings out of Wages: β = 0.75; γ = -.25
(δ = 0.70)

Age	No Tax	Current Program	Uniform Net Tax	Exempt. Net Tax
1	0.3570	0.3321	0.3380	0.3414
5	0.3036	0.2746	0.2805	0.2890
10	0.2305	0.1941	0.1999	0.2170
15	0.1498	-0.1033	0.1080	0.1371
20	0.0605	-0.0009	0.0035	-0.0129
25	-0.0380	-0.1142	-0.1155	-0.1310
30	-0.1470	-0.2431	-0.2509	-0.2649
35	-0.2674	-0.3871	-0.4051	-0.4168
40	-0.4004	-0.5470	-0.5806	-0.5890

β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution Parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

Table B.16
Comparisons of Steady States: β = −.25; γ = −.25
(δ = 0.70)

Var	No Tax	Current Program	Uniform Net Tax	Exempt. Net Tax
w_g	1.000	0.960	0.963	0.965
w_n	1.000	0.864	0.866	0.869
r	3.78%	4.27%	4.24%	4.20%
SSB	-------	0.495	0.493	0.490
C_t	40.840	39.560	39.380	39.120
K_t	311.480	262.440	263.560	264.550
L_t	35.300	35.000	34.790	34.500
K/N	5.940	5.000	5.020	5.040
K/L	8.824	7.498	7.575	7.669
ΔW	-------	-3.97%	-3.94%	-3.95%

β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

Table B.17
Comparisons of Net Taxes: $\beta = -.25$; $\gamma = -.25$
($\delta = 0.70$)

Age	Current Program	Uniform Net Tax	Exempt. Net Tax
1	7.31%	4.76%	3.48%
5	6.82%	4.76%	2.31%
10	6.08%	4.76%	0.56%
15	5.17%	4.76%	-1.59%
20	4.04%	4.76%	10.00%
25	2.66%	4.76%	10.00%
30	0.95%	4.76%	10.00%
35	-1.15%	4.76%	10.00%
40	-3.74%	4.76%	10.00%

β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution Parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

Table B.18
Consumption Across the Life-Cycle: β = −.25; γ = 1.50
(δ = 0.70)

Age	No Tax	Current Program	Uniform Net Tax	Exempt. Net Tax
1	0.4603	0.3791	0.3816	0.3840
5	0.5048	0.4263	0.4284	0.4302
10	0.5664	0.4936	0.4950	0.4959
15	0.6356	0.5715	0.5720	0.5716
20	0.7131	0.6618	0.6609	0.6589
25	0.8002	0.7663	0.7637	0.7594
30	0.8979	0.8874	0.8824	0.8754
35	1.0075	1.0275	1.0197	1.0090
40	1.1305	1.1898	1.1782	1.1630
41	1.1568	1.2252	1.2128	1.1965
45	1.2684	1.3777	1.3615	1.3405
50	1.4233	1.5933	1.5732	1.5451

β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution Parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

Table B.19
Leisure Across the Life-Cycle: β = −.25; γ = −.25
(δ = 0.70)

Age	No Tax	Current Program	Uniform Net Tax	Exempt. Net Tax
1	0.1487	0.1431	0.1385	0.1363
5	0.1631	0.1598	0.1554	0.1503
10	0.1830	0.1831	0.1796	0.1692
15	0.2054	0.2093	0.2075	0.1895
20	0.2304	0.2385	0.2398	0.2567
25	0.2586	0.2710	0.2771	0.2958
30	0.2901	0.3066	0.3202	0.3410
35	0.3255	0.3452	0.3700	0.3930
40	0.3653	0.3865	0.4275	0.4530

β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution Parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

Table B.20
Savings out of Wages: β = $-.25$; γ = $-.25$
(δ = 0.70)

Age	No Tax	Current Program	Uniform Net Tax	Exempt. Net Tax
1	0.3909	0.3614	0.3648	0.3665
5	0.3321	0.2998	0.3033	0.3081
10	0.2506	0.2123	0.2157	0.2261
15	0.1591	0.1117	0.1146	0.1327
20	0.0564	-0.0039	-0.0023	-0.0129
25	-0.0588	-0.1364	-0.1374	-0.1475
30	-0.1880	-0.2882	-0.2935	-0.3027
35	-0.3331	-0.4617	-0.4739	-0.4815
40	-0.4958	-0.6597	-0.6823	-0.6876

β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution Parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

Table B.21
Comparisons of Steady States: β = 0.75; γ = 1.50
(δ = 0.45)

Var	No Tax	Current Program	Uniform Net Tax	Exempt. Net Tax
w_g	1.000	0.939	0.944	0.946
w_n	1.000	0.845	0.849	0.852
r	5.03%	6.07%	5.99%	5.94%
SSB	-------	0.297	0.296	0.295
C_t	25.760	24.620	24.520	24.450
K_t	142.080	110.550	111.750	112.460
L_t	21.450	21.450	21.290	21.190
K/N	2.710	2.110	2.130	2.140
K/L	6.620	5.150	5.250	5.310
ΔW	-------	-4.07%	-4.02%	-3.99%

β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

Table B.22
Comparisons of Net Taxes: $\beta = 0.75$; $\gamma = 1.50$
($\delta = 0.45$)

Age	Current Program	Uniform Net Tax	Exempt. Net Tax
1	8.71%	7.03%	7.07%
5	8.36%	7.03%	6.31%
10	7.80%	7.03%	5.08%
15	7.05%	7.03%	3.43%
20	6.04%	7.03%	10.00%
25	4.68%	7.03%	10.00%
30	2.86%	7.03%	10.00%
35	0.41%	7.03%	10.00%
40	-2.88%	7.03%	10.00%

β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution Parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

Table B.23
Consumption Across the Life-Cycle: β = 0.75; γ = 1.50
(δ = 0.45)

Age	No Tax	Current Program	Uniform Net Tax	Exempt. Net Tax
1	0.3845	0.3333	0.3356	0.3365
5	0.4030	0.3550	0.3569	0.3579
10	0.4273	0.3841	0.3853	0.3865
15	0.4531	0.4157	0.4161	0.4176
20	0.4805	0.4500	0.4493	0.4477
25	0.5095	0.4873	0.4852	0.4830
30	0.5402	0.5279	0.5239	0.5211
35	0.5728	0.5722	0.5657	0.5621
40	0.6074	0.6207	0.6108	0.6064
41	0.5766	0.5959	0.5907	0.5873
45	0.6087	0.6408	0.6343	0.6301
50	0.6516	0.7022	0.6937	0.6884

β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution Parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

Table B.24
Leisure Across the Life-Cycle: β = 0.75; γ = 1.50
(δ = 0.45)

Age	No Tax	Current Program	Uniform Net Tax	Exempt. Net Tax
1	0.4313	0.4082	0.4056	0.4062
5	0.4520	0.4338	0.4313	0.4300
10	0.4792	0.4677	0.4658	0.4609
15	0.5082	0.5039	0.5029	0.4931
20	0.5388	0.5421	0.5431	0.5504
25	0.5714	0.5822	0.5864	0.5937
30	0.6058	0.6239	0.6332	0.6405
35	0.6424	0.6667	0.6838	0.6910
40	0.6812	0.7098	0.7383	0.7454

β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution Parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

Table B.25
Savings out of Wages: $\beta = 0.75$; $\gamma = 1.50$
($\delta = 0.45$)

Age	No Tax	Current Program	Uniform Net Tax	Exempt. Net Tax
1	0.1842	0.1669	0.1691	0.1690
5	0.1450	0.1236	0.1260	0.1275
10	0.0934	0.0658	0.0683	0.0725
15	0.0387	0.0036	0.0060	0.0140
20	-0.0193	-0.0630	-0.0613	-0.0649
25	-0.0808	-0.1342	-0.1340	-0.1371
30	-0.1460	-0.2100	-0.2124	-0.2149
35	-0.2152	-0.2905	-0.2971	-0.2990
40	-0.2886	-0.3754	-0.3886	-0.3896

β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution Parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

Table B.26
Comparisons of Steady States: β = −.25; γ = 1.50
(δ = 0.45)

Var	No Tax	Current Program	Uniform Net Tax	Exempt. Net Tax
w_g	1.000	0.949	0.955	0.959
w_n	1.000	0.854	0.859	0.863
r	5.04%	5.89%	5.79%	5.72%
SSB	-------	0.261	0.259	0.257
C_t	23.460	21.630	21.400	21.240
K_t	129.220	100.440	101.300	101.850
L_t	19.530	18.710	18.430	18.240
K/N	2.460	1.910	1.930	1.940
K/L	6.620	5.340	5.500	5.580
ΔW	-------	-3.56%	-3.50%	-3.48%

β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

Table B.27
Comparisons of Net Taxes: β = -.25; γ = 1.50
(δ = 0.45)

Age	Current Program	Uniform Net Tax	Exempt. Net Tax
1	8.60%	6.89%	6.94%
5	8.24%	6.89%	6.17%
10	7.65%	6.89%	4.95%
15	6.88%	6.89%	3.33%
20	5.84%	6.89%	10.00%
25	4.46%	6.89%	10.00%
30	2.63%	6.89%	10.00%
35	0.18%	6.89%	10.00%
40	-3.07%	6.89%	10.00%

β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution Parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

Table B.28
Consumption Across the Life-Cycle: β = 0.75; γ = 1.50
(δ = 0.45)

Age	No Tax	Current Program	Uniform Net Tax	Exempt. Net Tax
1	0.3560	0.2950	0.3008	0.3025
5	0.3731	0.3139	0.3189	0.3217
10	0.3957	0.3394	0.3430	0.3480
15	0.4196	0.3675	0.3690	0.3773
20	0.4450	0.3984	0.3969	0.3901
25	0.4719	0.4328	0.4269	0.4191
30	0.5004	0.4712	0.4592	0.4502
35	0.5307	0.5147	0.4940	0.4837
40	0.5628	0.5643	0.5314	0.5196
41	0.4648	0.4789	0.4714	0.4661
45	0.5037	0.5305	0.5208	0.5141
50	0.5567	0.6023	0.5894	0.5806

β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution Parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

Table B.29
Leisure Across the Life-Cycle: β = −.25; γ = 1.50
(δ = 0.45)

Age	No Tax	Current Program	Uniform Net Tax	Exempt. Net Tax
1	0.4652	0.4659	0.4598	0.4603
5	0.4876	0.4931	0.4875	0.4843
10	0.5171	0.5288	0.5244	0.5149
15	0.5484	0.5662	0.5641	0.5458
20	0.5815	0.6049	0.6067	0.6206
25	0.6167	0.6444	0.6527	0.6667
30	0.6540	0.6841	0.7021	0.7162
35	0.6935	0.7228	0.7552	0.7694
40	0.7354	0.7593	0.8124	0.8266

β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution Parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

Table B.30
Savings out of Wages: $\beta = -.25$; $\gamma = 1.50$
($\delta = 0.45$)

Age	No Tax	Current Program	Uniform Net Tax	Exempt. Net Tax
1	0.1787	0.1613	0.1634	0.1631
5	0.1392	0.1191	0.1215	0.1232
10	0.0872	0.0631	0.0657	0.0704
15	0.0320	0.0031	0.0056	0.0146
20	-0.0265	-0.0609	-0.0590	-0.0628
25	-0.0886	-0.1290	-0.1285	-0.1316
30	-0.1544	-0.2014	-0.2032	-0.2054
35	-0.2242	-0.2779	-0.2836	-0.2848
40	-0.2982	-0.3586	-0.3702	-0.3700

β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution Parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

Table B.31
Comparisons of Steady States: β = 0.75; γ = −.25
(δ = 0.45)

Var	No Tax	Current Program	Uniform Net Tax	Exempt. Net Tax
w_g	1.000	0.955	0.962	0.970
w_n	1.000	0.860	0.866	0.873
r	3.41%	3.92%	3.83%	3.74
SSB	-------	0.320	0.320	0.319
C_t	25.850	25.310	25.210	25.060
K_t	221.700	185.040	188.900	193.220
L_t	22.710	22.760	22.580	22.350
K/N	4.230	3.530	3.600	3.680
K/L	9.760	8.130	8.370	8.640
ΔW	-------	-2.23%	-2.18%	-2.19%

β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

Table B.32
Comparisons of Net Taxes: β = 0.75; γ = −.25
(δ = 0.45)

Age	Current Program	Uniform Net Tax	Exempt. Net Tax
1	6.74%	4.29%	3.13%
5	6.20%	4.29%	2.05%
10	5.40%	4.29%	0.45%
15	4.43%	4.29%	-1.48%
20	3.25%	4.29%	10.00%
25	1.82%	4.29%	10.00%
30	0.08%	4.29%	10.00%
35	-2.02%	4.29%	10.00%
40	-4.56%	4.29%	10.00%

β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution Parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

Table B.33
Consumption Across the Life-Cycle: β = 0.75; γ = 1.50
(δ = 0.45)

Age	No Tax	Current Program	Uniform Net Tax	Exempt. Net Tax
1	0.3216	0.2766	0.2775	0.2795
5	0.3461	0.3048	0.3051	0.3045
10	0.3794	0.3439	0.3436	0.3387
15	0.4159	0.3877	0.3870	0.3762
20	0.4559	0.4368	0.4358	0.4416
25	0.4997	0.4917	0.4908	0.4943
30	0.5478	0.5528	0.5527	0.5533
35	0.6005	0.6206	0.6224	0.6194
40	0.6582	0.6957	0.7009	0.6933
41	0.7529	0.7811	0.7689	0.7539
45	0.7867	0.8286	0.8138	0.7957
50	0.8309	0.8916	0.8730	0.8508

β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution Parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

Table B.34
Leisure Across the Life-Cycle: β = 0.75; γ = -.25
(δ = 0.45)

Age	No Tax	Current Program	Uniform Net Tax	Exempt. Net Tax
1	0.3607	0.3314	0.3262	0.3248
5	0.3882	0.3639	0.3587	0.3516
10	0.4255	0.4086	0.4040	0.3875
15	0.4664	0.4581	0.4549	0.4257
20	0.5113	0.5125	0.5123	0.5352
25	0.5605	0.5720	0.5770	0.5990
30	0.6144	0.6367	0.6497	0.6705
35	0.6735	0.7064	0.7317	0.7505
40	0.7382	0.7807	0.8240	0.8401

β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution Parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

Table B.35
Savings out of Wages: β = 0.75; γ = -.25
(δ = 0.45)

Age	No Tax	Current Program	Uniform Net Tax	Exempt. Net Tax
1	0.3177	0.2983	0.3060	0.3100
5	0.2657	0.2421	0.2501	0.2616
10	0.1951	0.1646	0.1725	0.1961
15	0.1177	0.0782	0.0850	0.1252
20	0.0328	-0.0177	-0.0135	-0.0358
25	-0.0602	-0.1237	-0.1244	-0.1442
30	-0.1622	-0.2404	-0.2494	-0.2656
35	-0.2739	-0.3682	-0.3900	-0.4015
40	-0.3965	-0.5072	-0.5485	-0.5536

β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution Parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

Table B.36
Comparisons of Steady States: β = −.25; γ = −.25
(δ = 0.45)

Var	No Tax	Current Program	Uniform Net Tax	Exempt. Net Tax
w_g	1.000	0.960	0.968	0.978
w_n	1.000	0.864	0.871	0.880
r	3.40%	3.83%	3.74%	3.63%
SSB	-------	0.288	0.286	0.283
C_t	23.630	22.700	22.460	22.090
K_t	203.940	170.140	173.180	176.190
L_t	20.780	20.390	20.090	19.650
K/N	3.890	3.240	3.300	3.560
K/L	9.820	8.340	8.620	8.970
ΔW	-------	-2.15%	-2.09%	-2.13%

β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

Table B.37
Comparisons of Net Taxes: $\beta = -.25$; $\gamma = -.25$
($\delta = 0.45$)

Age	Current Program	Uniform Net Tax	Exempt. Net Tax
1	6.61%	4.18%	3.24%
5	6.06%	4.18%	2.20%
10	5.24%	4.18%	0.67%
15	4.25%	4.18%	-1.15%
20	3.06%	4.18%	10.00%
25	1.63%	4.18%	10.00%
30	-0.11%	4.18%	10.00%
35	-2.20%	4.18%	10.00%
40	-4.73%	4.18%	10.00%

β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution Parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

Table B.38
Consumption Across the Life-Cycle: β = −.25; γ = 1.50
(δ = 0.45)

Age	No Tax	Current Program	Uniform Net Tax	Exempt. Net Tax
1	0.3006	0.2511	0.2561	0.2607
5	0.3232	0.2762	0.2803	0.2838
10	0.3539	0.3111	0.3138	0.3156
15	0.3875	0.3503	0.3514	0.3508
20	0.4242	0.3946	0.3934	0.3901
25	0.4644	0.4444	0.4404	0.4337
30	0.5085	0.5005	0.4931	0.4822
35	0.5567	0.5638	0.5521	0.5361
40	0.6095	0.6349	0.6181	0.5961
41	0.6207	0.6502	0.6322	0.6088
45	0.6674	0.7151	0.6920	0.6627
50	0.7307	0.8054	0.7747	0.7368

β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution Parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

Table B.39
Leisure Across the Life-Cycle: β = −.25; γ = −.25
(δ = 0.45)

Age	No Tax	Current Program	Uniform Net Tax	Exempt. Net Tax
1	0.3929	0.3797	0.3699	0.3669
5	0.4224	0.4141	0.4049	0.3938
10	0.4625	0.4610	0.4533	0.4289
15	0.5063	0.5121	0.5076	0.4654
20	0.5543	0.5674	0.5683	0.6045
25	0.6069	0.6266	0.6362	0.6721
30	0.6645	0.6895	0.7123	0.7472
35	0.7275	0.7554	0.7975	0.8307
40	0.7965	0.8236	0.8928	0.9235

β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution Parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

Table B.40
Savings out of Wages: $\beta = -.25$; $\gamma = -.25$
($\delta = 0.45$)

Age	No Tax	Current Program	Uniform Net Tax	Exempt. Net Tax
1	0.3065	0.2851	0.2929	0.2962
5	0.2544	0.2301	0.2382	0.2496
10	0.1837	0.1547	0.1625	0.1869
15	0.1062	0.0713	0.0777	0.1195
20	0.0215	-0.0207	-0.0172	-0.0421
25	-0.0714	-0.1218	-0.1235	-0.1452
30	-0.1730	-0.2322	-0.2424	-0.2598
35	-0.2842	-0.3524	-0.3756	-0.3872
40	-0.4060	-0.4825	-0.5247	-0.5288

β = Contemporaneous Elasticity of Substitution Parameter
γ = Intertemporal Elasticity of Substitution Parameter
δ = Distribution Parameter
Labor's Share = 0.75
Subjective Rate of Time Preference = 0.02
Rate of Population Growth = 0.02

APPENDIX C: LAISSEZ-FAIRE SIMULATIONS

Appendices C and D contain the results of the computer simulations that compute the adjustment costs for the public savings program. In all simulations the dynamic solution was computed for 60 periods. The first page of each case is indicated by the page that contains the parameter values and the results of the steady state solution. The headings for the various results are self-explanatory with the following possible exceptions:

NT = the net tax.
PS = private savings.
GS = government savings.

PARAMETERS

```
CAPITALS SHARE                 (ALPHA) =    0.2500
POPULATION GROWTH                 (ZN) =    1.0000
INTERTEMPORAL ELASTICITY       (GAMMA) =   -0.2500
SUBJ RATE OF TIME DISC           (RHO) =    1.3500
```

STEADY STATE VALUES.

```
(LAISSEZ-FAIRE)
WAGES              =  2.000000
INTEREST RATE      =  2.095802
SCALE              =  3.550824
CAPITAL/LABOR      =  0.318096
GOLDEN RULE K/L    =  0.853150
UTILITY            =  6.307529
SOC UTILITY        = 18.922587
C1                 =  1.363809
C2                 =  1.969521
PVCONS             =  2.000000
PVINC              =  2.000000
END STEADY STATE COMPUTATIONS.  (ITER   =  12)
```

DYNAMIC SOLUTION

YR	TAX	BEN	NT	UTILITY	WEL GAIN	PS	GS
0	0.000000000000	0.000000000000	0.000000000000	6.307528943300	0.000000000000	0.636190922202	0.000000000000
1	0.000006693437	0.000000000000	0.000006693437	6.307473544035	-0.000878303776	0.636185633573	0.000013386875
2	0.000006693437	0.000000000000	0.000006693437	6.307459937298	-0.001094025921	0.636185196293	0.000034108356
3	0.000006693437	0.000000000000	0.000006693437	6.307456892090	-0.001142304856	0.636186424147	0.000066182240
4	0.000006693437	0.000000000000	0.000006693437	6.307456223000	-0.001152912649	0.636188751933	0.000115825823
5	0.000006693437	0.000000000000	0.000006693437	6.307456109896	-0.001154705823	0.636192451437	0.000192658319
6	0.000006693437	0.000000000000	0.000006693437	6.307456184567	-0.001153521977	0.636198201054	0.000311558033
7	0.000006693437	0.000000000000	0.000006693437	6.307456443171	-0.001149422046	0.636207108486	0.000495527431
8	0.000006693437	0.000000000000	0.000006693437	6.307457095552	-0.001139079164	0.636220904203	0.000780104824
9	0.000006693437	0.000000000000	0.000006693437	6.307458686301	-0.001113859311	0.636242274601	0.001220137741
10	0.000006693437	0.000000000000	0.000006693437	6.307462527950	-0.001052953539	0.636275390382	0.001900134980
11	0.000006693437	0.000000000000	0.000006693437	6.307471744590	-0.000906832300	0.636326734601	0.002949975646
12	0.000006693437	0.000000000000	0.000006693437	6.307493753328	-0.000557904240	0.636406406086	0.004568480617
13	0.000006693437	0.000000000000	0.000006693437	6.307545970631	0.000269952485	0.636530176549	0.007058152882
14	0.000006693437	0.000000000000	0.000006693437	6.307668870672	0.002218418242	0.636722761486	0.010874908664
15	0.000006693437	0.000000000000	0.000006693437	6.307954900207	0.006753150263	0.637023024349	0.016695869719
16	0.000006693437	0.000000000000	0.000006693437	6.308609812281	0.017136171570	0.637492180909	0.025504039717
17	0.000006693437	0.000000000000	0.000006693437	6.310073979559	0.040349180836	0.638226277297	0.038676731200
18	0.000006693437	0.000000000000	0.000006693437	6.313236542589	0.090488673791	0.639373465203	0.058039093051
19	0.000006693437	0.000000000000	0.000006693437	6.319744381092	0.193664395401	0.641152760461	0.085802721553
20	0.000006693437	0.000000000000	0.000006693437	6.332285777748	0.392496565159	0.643860952360	0.124269742799
21	0.000006693437	0.000000000000	0.000006693437	6.354501850333	0.744711716046	0.647839279716	0.175206737510
22	0.000006693437	0.000000000000	0.000006693437	6.390061731788	1.308480535412	0.653369355118	0.238971788968
23	0.000006693437	0.000000000000	0.000006693437	6.440892104964	2.114348786393	0.660511835720	0.313794756603
24	0.000006693437	0.000000000000	0.000006693437	6.505571318624	3.139777512000	0.668983969056	0.395767938107
25	0.000006693437	0.000000000000	0.000006693437	6.579289784600	4.308515168830	0.678192882640	0.479747799049
26	0.000006693437	0.000000000000	0.000006693437	6.655587034391	5.518137042575	0.687426911790	0.560708282652
27	0.000006693437	0.000000000000	0.000006693437	6.728566226595	6.675154201920	0.696074544928	0.634817836594
28	0.000006693437	0.000000000000	0.000006693437	6.794238903004	7.716333354625	0.703746206386	0.699856393708
29	0.000006693437	0.000000000000	0.000006693437	6.850712262760	8.611665905032	0.710279144120	0.755067988584
30	0.000006693437	0.000000000000	0.000006693437	6.897698041924	9.356581696727	0.715677713646	0.800747182399
31	0.000006693437	0.000000000000	0.000006693437	6.935873457896	9.961817381179	0.720042921002	0.837802875230
32	0.000006693437	0.000000000000	0.000006693437	6.966366747056	10.445260095972	0.723517656937	0.867415755163
33	0.000006693437	0.000000000000	0.000006693437	6.990426599711	10.826706663580	0.726252393989	0.890812799770
34	0.000006693437	0.000000000000	0.000006693437	7.009241898396	11.125005709906	0.728387051087	0.909139644535
35	0.000006693437	0.000000000000	0.000006693437	7.023860211759	11.356765460747	0.730043265994	0.923401136552
36	0.000006693437	0.000000000000	0.000006693437	7.035163263013	11.535964816883	0.731322555999	0.934443911352
37	0.000006693437	0.000000000000	0.000006693437	7.043871789797	11.674030402657	0.732307436066	0.942962096807
38	0.000006693437	0.000000000000	0.000006693437	7.050563503455	11.780121293697	0.733063788162	0.949513974270
39	0.000006693437	0.000000000000	0.000006693437	7.055695224814	11.861479958954	0.733643563107	0.954542416541
40	0.000006693437	0.000000000000	0.000006693437	7.059624716998	11.923778399737	0.734087364634	0.958395225820

YR	TAX	BEN	NT	UTILITY	WEL GAIN	PS	GS
41	0.000006693437	0.000000000000	0.000006693437	7.062630222189	11.971427886847	0.734426724652	0.961343519140
42	0.000006693437	0.000000000000	0.000006693437	7.064927037983	12.007841763264	0.734686015085	0.963597469894
43	0.000006693437	0.000000000000	0.000006693437	7.066681131338	12.035651280596	0.734884008192	0.965319333377
44	0.000006693437	0.000000000000	0.000006693437	7.068020083823	12.056879125883	0.735035125659	0.966633981984
45	0.000006693437	0.000000000000	0.000006693437	7.069041764152	12.073076916463	0.735150425342	0.967637292327
46	0.000006693437	0.000000000000	0.000006693437	7.069821130743	12.085433048281	0.735238373558	0.968402747330
47	0.000006693437	0.000000000000	0.000006693437	7.070415523445	12.094856591273	0.735305445018	0.968986590795
48	0.000006693437	0.000000000000	0.000006693437	7.070868769582	12.102042386869	0.735356587564	0.969431827743
49	0.000006693437	0.000000000000	0.000006693437	7.071214343028	12.107521132184	0.735395579639	0.969771315011
50	0.000006693437	0.000000000000	0.000006693437	7.071477797249	12.111697953625	0.735425305322	0.970030141172
51	0.000006693437	0.000000000000	0.000006693437	7.071678631777	12.114881998107	0.735447965219	0.970227454565
52	0.000006693437	0.000000000000	0.000006693437	7.071831722029	12.117309101545	0.735465237977	0.970377864770
53	0.000006693437	0.000000000000	0.000006693437	7.071948413299	12.119159133017	0.735478403812	0.970492515539
54	0.000006693437	0.000000000000	0.000006693437	7.072037357008	12.120569252728	0.735488438921	0.970579905305
55	0.000006693437	0.000000000000	0.000006693437	7.072105149479	12.121644039246	0.735496087592	0.970646514156
56	0.000006693437	0.000000000000	0.000006693437	7.072156819612	12.122463221109	0.735501917239	0.970697282614
57	0.000006693437	0.000000000000	0.000006693437	7.072196201045	12.123087577079	0.735506360407	0.970735977085
58	0.000006693437	0.000000000000	0.000006693437	7.072226215210	12.123563423720	0.735509746738	0.970765468690
59	0.000006693437	0.000000000000	0.000006693437	7.072249091457	12.123926105316	0.735512327711	0.970787945988
60	0.000006693437	0.000000000000	0.000006693437	7.072266526688	12.124202524679	0.735514294807	0.970805077129

YR	WG	WN	RG	C1	C2	K/L
0	2.000000000000	2.000000000000	2.095801612340	1.363809077798	1.969520882710	0.318095702500
1	1.999999114633	1.999985727764	2.095804395674	1.363800094190	1.969494313115	0.318095702500
2	2.000005099790	1.999991712881	2.095785580201	1.363806516588	1.969461074373	0.318099510224
3	2.000021041350	2.000007654334	2.095735465988	1.363821230188	1.969412536395	0.318109652325
4	2.000047212699	2.000033825508	2.095653196672	1.363845073575	1.969338070306	0.318126303193
5	2.000088054184	2.000074666720	2.095524820561	1.363882215283	1.969222989903	0.318152288878
6	2.000151334983	2.000137947095	2.095325932176	1.363939746042	1.969044995596	0.318192554878
7	2.000249270656	2.000235882112	2.095018174873	1.364028773627	1.968769728587	0.318254879543
8	2.000400783614	2.000387394057	2.094542172719	1.364166489854	1.968344250746	0.318351317958
9	2.000635100589	2.000621709463	2.093806312431	1.364379434863	1.967687121805	0.318500504514
10	2.000997285665	2.000983892115	2.092669567046	1.364708501733	1.966673483053	0.318731206171
11	2.001556667275	2.001543269981	2.090915521652	1.365216535380	1.965112915216	0.319087762681
12	2.002419540849	2.002406137779	2.088213662675	1.365999731693	1.962717421733	0.319638355123
13	2.003748026437	2.003734614475	2.084062956813	1.367204437926	1.959056947633	0.320487443351
14	2.005787378135	2.005773952522	2.077712606451	1.369051191036	1.953502124378	0.321794164715
15	2.008903953504	2.008890507031	2.068057629091	1.371867482682	1.945160675662	0.323798835075
16	2.013634360514	2.013620882379	2.053517068561	1.376128701470	1.932830365021	0.326859447034
17	2.020740832591	2.020727306889	2.031927955472	1.382501029592	1.915022070361	0.331498110313
18	2.031255083121	2.031241487042	2.000537799975	1.391868021840	1.890147569430	0.338451504248
19	2.046469587040	2.046455889124	1.956249628546	1.405303128663	1.856981065171	0.348706279127
20	2.067806080357	2.067792239627	1.896316103273	1.423931287266	1.815401549103	0.363477741007
21	2.096484394932	2.096470362245	1.819555281537	1.448631082529	1.767124536748	0.384065347580
22	2.132990314787	2.132976037749	1.727720581146	1.479606682631	1.715803860198	0.411523008613
23	2.176534640435	2.176520071936	1.626085610016	1.516008236217	1.666054421265	0.446170572043
24	2.224880882437	2.224865990335	1.522368762885	1.555882021280	1.621826174599	0.487153296161
25	2.274809119900	2.274793893608	1.424312464777	1.596601010968	1.585240503451	0.532375953581
26	2.323027933696	2.323012384653	1.337447862072	1.635585472864	1.556531220541	0.578970340844
27	2.367000018935	2.366984175568	1.264286128338	1.670909630640	1.534742681495	0.624067597221
28	2.405296330923	2.405280231223	1.204853908209	1.701534024837	1.518496524908	0.665446190761
29	2.437495906696	2.437479591470	1.157733158247	1.727200447350	1.506466485935	0.701801300047
30	2.463871101388	2.463854609620	1.120949908455	1.748176895975	1.497563663397	0.732673566352
31	2.485066875579	2.485050241939	1.092511244511	1.765007320937	1.490958158963	0.758212448022
32	2.501865571199	2.501848825118	1.070651783914	1.778331168181	1.486038513682	0.778922898116
33	2.515045548684	2.515028714384	1.053907737246	1.788776320395	1.482359857548	0.795466706050
34	2.525310220205	2.525293317199	1.041108393961	1.796906266111	1.479599056507	0.808532596879
35	2.533261082262	2.533244126037	1.031336298836	1.803200860043	1.477520546421	0.818763347811
36	2.539394958866	2.539377961585	1.023880796942	1.808055405586	1.475951608624	0.826722201273
37	2.544112916365	2.544095887504	1.018195113673	1.811788451438	1.474764803608	0.832883233675
38	2.547733670082	2.547716616986	1.013860205812	1.814652828824	1.473865543985	0.837634766436
39	2.550507707301	2.550490635637	1.010555654246	1.816847072531	1.473183257506	0.841288881216
40	2.552630336888	2.552613251016	1.008036779384	1.818525886382	1.472665056972	0.844092989824

YR	WG	WN	RG	C1	C2	K/L
41	2.554252967191	2.554235870458	1.006116884785	1.819809145806	1.472271164616	0.846241295227
42	2.555492475975	2.555475370946	1.004653582575	1.820789355861	1.471971575198	0.847885121896
43	2.556438801480	2.556421690116	1.003538307588	1.821537681924	1.471743602614	0.849141742489
44	2.557160989010	2.557143872813	1.002688296742	1.822108747154	1.471570063285	0.850101670784
45	2.557711950854	2.557694830969	1.002040463044	1.822544405627	1.471437923017	0.850834553821
46	2.558132181967	2.558115059270	1.001546720680	1.822876685712	1.471337284909	0.851393858835
47	2.558452642944	2.558435518101	1.001170419276	1.823130073083	1.471260624912	0.851820560444
48	2.558696986726	2.558679860247	1.000883625169	1.823323272684	1.471202223213	0.852146017906
49	2.558883273171	2.558866145446	1.000665048400	1.823470565806	1.471157726930	0.852394207653
50	2.559025285493	2.559008156817	1.000498462847	1.823582851495	1.471123822710	0.852583447325
51	2.559133539501	2.559116410101	1.000371501843	1.823668444882	1.471097987736	0.852727723247
52	2.559216056139	2.559198926187	1.000274740221	1.823733688210	1.471078300678	0.852837709892
53	2.559278952208	2.559261821834	1.000200994682	1.823783418023	1.471063298037	0.852921551374
54	2.559326891721	2.559309761027	1.000144790548	1.823821322105	1.471051864902	0.852985459675
55	2.559363430557	2.559346299618	1.000101955360	1.823850212026	1.471043151832	0.853034172113
56	2.559391279512	2.559374148386	1.000069309131	1.823872231148	1.471036511599	0.853071300874
57	2.559412505008	2.559395373741	1.000044428274	1.823889013334	1.471031451021	0.853099599926
58	2.559428682188	2.559411550812	1.000025465687	1.823901804074	1.471027593505	0.853121168746
59	2.559441011639	2.559423880181	1.000011013658	1.823911552470	1.471024654369	0.853137607714
60	2.559450408543	2.559433277021	0.999999999216	1.823918982214	1.471022414400	0.853150136849
			0.999991604717			0.853159685968

PARAMETERS

```
CAPITALS SHARE                 (ALPHA) =    0.2500
POPULATION GROWTH                 (ZN) =    1.0000
INTERTEMPORAL ELASTICITY       (GAMMA) =   -0.2500
SUBJ RATE OF TIME DISC           (RHO) =    2.0000
```

STEADY STATE VALUES.

```
(LAISSEZ-FAIRE)
WAGES              =  2.000000
INTEREST RATE      =  2.556395
SCALE              =  3.731630
CAPITAL/LABOR      =  0.260784
GOLDEN RULE K/L    =  0.911559
UTILITY            =  4.951251
SOC UTILITY        = 14.853753
C1                 =  1.478433
C2                 =  1.854898
PVCONS             =  2.000000
PVINC              =  2.000000
END STEADY STATE COMPUTATIONS.  (ITER   =  12)
```

DYNAMIC SOLUTION

YR	TAX	BEN	NT	UTILITY	WEL GAIN	PS	GS
0	0.000000000000	0.000000000000	0.000000000000	4.951250914931	0.000000000000	0.521566882591	0.000000000000
1	0.000001094375	0.000000000000	0.000001094375	4.951243336498	-0.000153060986	0.521566087965	0.000002188750
2	0.000001094375	0.000000000000	0.000001094375	4.951241077906	-0.000198677585	0.521565878581	0.000006080778
3	0.000001094375	0.000000000000	0.000001094375	4.951240577734	-0.000208779500	0.521565879554	0.000013001533
4	0.000001094375	0.000000000000	0.000001094375	4.951240467797	-0.000210999889	0.521565964005	0.000025307790
5	0.000001094375	0.000000000000	0.000001094375	4.951240445479	-0.000211450655	0.521566132529	0.000047189933
6	0.000001094375	0.000000000000	0.000001094375	4.951240447785	-0.000211404073	0.521566436554	0.000086097871
7	0.000001094375	0.000000000000	0.000001094375	4.951240477440	-0.000210805133	0.521566979529	0.000155274452
8	0.000001094375	0.000000000000	0.000001094375	4.951240569091	-0.000208954072	0.521567948176	0.000278253488
9	0.000001094375	0.000000000000	0.000001094375	4.951240859711	-0.000203084436	0.521569680904	0.000496836385
10	0.000001094375	0.000000000000	0.000001094375	4.951241776646	-0.000184565190	0.521572794214	0.000885207456
11	0.000001094375	0.000000000000	0.000001094375	4.951244679050	-0.000125945569	0.521578432779	0.001574817164
12	0.000001094375	0.000000000000	0.000001094375	4.951253819058	0.000058654407	0.521588779685	0.002797949416
13	0.000001094375	0.000000000000	0.000001094375	4.951282455906	0.000637030416	0.521608175650	0.004963069812
14	0.000001094375	0.000000000000	0.000001094375	4.951371464914	0.002434737906	0.521645731889	0.008782249821
15	0.000001094375	0.000000000000	0.000001094375	4.951644368078	0.007946540250	0.521721741007	0.015477846058
16	0.000001094375	0.000000000000	0.000001094375	4.952461934350	0.024458857766	0.521883633563	0.027091522263
17	0.000001094375	0.000000000000	0.000001094375	4.954817695362	0.072037965607	0.522243954901	0.046871426604
18	0.000001094375	0.000000000000	0.000001094375	4.961190486373	0.200748691849	0.523059121628	0.079554243753
19	0.000001094375	0.000000000000	0.000001094375	4.976841703047	0.516855004026	0.524847176042	0.131025705153
20	0.000001094375	0.000000000000	0.000001094375	5.010381102640	1.194247448259	0.528437273369	0.206548837326
21	0.000001094375	0.000000000000	0.000001094375	5.070853061485	2.415594535776	0.534693593125	0.307400983582
22	0.000001094375	0.000000000000	0.000001094375	5.160756492125	4.231366593883	0.543845106652	0.428028123293
23	0.000001094375	0.000000000000	0.000001094375	5.271463250822	6.467301726213	0.555031460078	0.557057704551
24	0.000001094375	0.000000000000	0.000001094375	5.387478538233	8.810452768342	0.566712263458	0.682062146457
25	0.000001094375	0.000000000000	0.000001094375	5.495021892861	10.982496893660	0.577517030203	0.793975179601
26	0.000001094375	0.000000000000	0.000001094375	5.586602876983	12.832150358918	0.586703879708	0.888455796411
27	0.000001094375	0.000000000000	0.000001094375	5.660409895621	14.322824532101	0.594098747579	0.964930762019
28	0.000001094375	0.000000000000	0.000001094375	5.717861901767	15.483177887906	0.599849423672	1.025014111838
29	0.000001094375	0.000000000000	0.000001094375	5.761616706269	16.366890009433	0.604225787354	1.071235116812
30	0.000001094375	0.000000000000	0.000001094375	5.794478408950	17.030595065895	0.607510738983	1.106264317957
31	0.000001094375	0.000000000000	0.000001094375	5.818935281909	17.524548480483	0.609954467031	1.132529900845
32	0.000001094375	0.000000000000	0.000001094375	5.837026636355	17.889938050856	0.611761574153	1.152074235205
33	0.000001094375	0.000000000000	0.000001094375	5.850353906940	18.159107818538	0.613092491776	1.166537357860
34	0.000001094375	0.000000000000	0.000001094375	5.860143483953	18.356827085480	0.614069949940	1.177197791386
35	0.000001094375	0.000000000000	0.000001094375	5.867319954665	18.501769663314	0.614786406063	1.185032737682
36	0.000001094375	0.000000000000	0.000001094375	5.872573313049	18.607871302575	0.615310820940	1.190779048256
37	0.000001094375	0.000000000000	0.000001094375	5.876414979641	18.685461120940	0.615694287821	1.194987113038
38	0.000001094375	0.000000000000	0.000001094375	5.879222247164	18.742159267955	0.615974489169	1.198065306571
39	0.000001094375	0.000000000000	0.000001094375	5.881272551041	18.783569083629	0.616179128331	1.200315190962
40	0.000001094375	0.000000000000	0.000001094375	5.882769429416	18.813801410776	0.616328526550	1.201958692707

YR	TAX	BEN	NT	UTILITY	WEL GAIN	PS	GS
41	0.000001094375	0.000000000000	0.000001094375	5.883861962296	18.835867205844	0.616437566333	1.203158730329
42	0.000001094375	0.000000000000	0.000001094375	5.884659212567	18.851969202781	0.616517134418	1.204034690843
43	0.000001094375	0.000000000000	0.000001094375	5.885240902337	18.863717542361	0.616575188283	1.204673948281
44	0.000001094375	0.000000000000	0.000001094375	5.885665269671	18.872288453855	0.616617540717	1.205140387764
45	0.000001094375	0.000000000000	0.000001094375	5.885974839614	18.878540812049	0.616648436045	1.205480688204
46	0.000001094375	0.000000000000	0.000001094375	5.886200653732	18.883101560875	0.616670972383	1.205728939527
47	0.000001094375	0.000000000000	0.000001094375	5.886365365836	18.886428237456	0.616687410668	1.205910028839
48	0.000001094375	0.000000000000	0.000001094375	5.886485505638	18.888854690972	0.616699400607	1.206042120008
49	0.000001094375	0.000000000000	0.000001094375	5.886573132812	18.890624489671	0.616708145776	1.206138467399
50	0.000001094375	0.000000000000	0.000001094375	5.886637045014	18.891915319051	0.616714524189	1.206208741506
51	0.000001094375	0.000000000000	0.000001094375	5.886683659800	18.892856793985	0.616719176323	1.206259997287
52	0.000001094375	0.000000000000	0.000001094375	5.886717657910	18.893543450965	0.616722569308	1.206297381190
53	0.000001094375	0.000000000000	0.000001094375	5.886742454655	18.894044268753	0.616725044012	1.206324647252
54	0.000001094375	0.000000000000	0.000001094375	5.886760540023	18.894409537413	0.616726848922	1.206344533686
55	0.000001094375	0.000000000000	0.000001094375	5.886773730412	18.894675942596	0.616728165316	1.206359037732
56	0.000001094375	0.000000000000	0.000001094375	5.886783350669	18.894870242120	0.616729125412	1.206369616126
57	0.000001094375	0.000000000000	0.000001094375	5.886790367079	18.895011951966	0.616729825646	1.206377331362
58	0.000001094375	0.000000000000	0.000001094375	5.886795484401	18.895115306076	0.616730336352	1.206382958373
59	0.000001094375	0.000000000000	0.000001094375	5.886799216645	18.895190685892	0.616730708827	1.206387062357
60	0.000001094375	0.000000000000	0.000001094375	5.886801938700	18.895245663027	0.616730980487	1.206390055540

YR	WG	WN	RG	C1	C2	K/L
0	2.000000000000	2.000000000000	2.556395031206	1.478433117409	1.854897869688	0.260783578964
1	1.999999384113	1.999997195363	2.556397392885	1.478431107399	1.854894131172	0.260783578964
2	2.000000456635	1.999998267885	2.556393280197	1.478432389304	1.854886327616	0.260784138357
3	2.000003986984	2.000001798230	2.556379742807	1.478435918676	1.854873057350	0.260785979680
4	2.000010622413	2.000008433651	2.556354299013	1.478442469646	1.854849601998	0.260789440544
5	2.000022500448	2.000020311673	2.556308753099	1.478454179144	1.854807927821	0.260795635898
6	2.000043638229	2.000041449431	2.556227703681	1.478475012877	1.854733843136	0.260806661231
7	2.000081225184	2.000079036345	2.556083591019	1.478512056816	1.854602159136	0.260826267212
8	2.000148050215	2.000145861303	2.555827402994	1.478577913128	1.854368132092	0.260861126991
9	2.000266835464	2.000264646422	2.555372098891	1.478694965518	1.853952431997	0.260923100832
10	2.000477922699	2.000475733426	2.554563267898	1.478902939212	1.853214642192	0.261033258644
11	2.000852844143	2.000850654460	2.553127508495	1.479272221680	1.851907170058	0.261229000835
12	2.001518158789	2.001515968377	2.550582337654	1.479927188692	1.849596238293	0.261576624972
13	2.002696900902	2.002694709201	2.546081342596	1.481086533551	1.845530720029	0.262193364550
14	2.004779384109	2.004777190129	2.538155284636	1.483131458239	1.838436831504	0.263285622731
15	2.008440217890	2.008438019903	2.524301491301	1.486716278896	1.826234184882	0.265213990855
16	2.014819847057	2.014817642089	2.500398853076	1.492934008526	1.805751128776	0.268599793533
17	2.025771586242	2.025769369288	2.460064679603	1.503525414387	1.772745472829	0.274487577913
18	2.044101154738	2.044098917725	2.394477723576	1.521039796097	1.722925142230	0.284557690753
19	2.073537970843	2.073535701615	2.293939574929	1.548688525573	1.654718933760	0.301306682690
20	2.117908636064	2.117906318278	2.152763334470	1.589469044909	1.572905270857	0.327936440597
21	2.179074536204	2.179072151479	1.976522190460	1.644378558354	1.489016548145	0.367493055348
22	2.254459927875	2.254457460650	1.784803425675	1.710612353998	1.415569208681	0.421047288354
23	2.336709410827	2.336706853590	1.602890403693	1.781675393512	1.359157149764	0.485936614973
24	2.416780482717	2.416777837853	1.448792992256	1.850065574395	1.319392757772	0.556044582315
25	2.487844892760	2.487842170125	1.328152826220	1.910325139921	1.292475524957	0.624387204958
26	2.546833999748	2.546831212556	1.237986859151	1.960127332849	1.274406580005	0.685746104902
27	2.593654126301	2.593651287871	1.172146160758	1.999552540292	1.262179452309	0.737579838059
28	2.629758615602	2.629755737660	1.124528047486	2.029906313989	1.253793515170	0.779514754799
29	2.657091596279	2.657088688424	1.090180411344	2.052862901070	1.247964424513	0.812431767755
30	2.677539443644	2.677536513412	1.065394179789	2.070025774429	1.243866001258	0.837730452083
31	2.692717278454	2.692714331611	1.047479857426	2.082759864580	1.240958001103	0.856887528470
32	2.703924278725	2.703921319618	1.034509243841	2.092159745465	1.238880087069	0.871242183938
33	2.712169586158	2.712166618027	1.025102817306	2.099074126252	1.237387381490	0.881917904679
34	2.718220747959	2.718217773206	1.018271954517	2.104147823266	1.236310787309	0.889814924818
35	2.722653843403	2.722650863799	1.013306118562	2.107864457735	1.235532005416	0.895633870663
36	2.725897487952	2.725894504798	1.009693111824	2.110583683858	1.234967419706	0.899909571873
37	2.728268707732	2.728265721983	1.007062736011	2.112571434162	1.234557457799	0.903044934598
38	2.730001042260	2.729998054615	1.005146843601	2.114023565446	1.234259420339	0.905340700429
39	2.731266042748	2.731263053719	1.003750872927	2.115083925388	1.234042562694	0.907019897870
40	2.732189473140	2.732186483100	1.002733468205	2.115857956551	1.233884672836	0.908247159647

YR	WG	WN	RG	C1	C2	K/L
41	2.732863399051	2.732860408274	1.001991826927	2.116422841941	1.233769663428	0.909143609628
42	2.733355148131	2.733352156815	1.001451128892	2.116835022398	1.233685859529	0.909798148331
43	2.733713920685	2.733710928977	1.001056889365	2.117135740694	1.233624779371	0.910275912630
44	2.733975651202	2.733972659207	1.000769415640	2.117355118490	1.233580253283	0.910624568282
45	2.734166574972	2.734163582769	1.000559782478	2.117515146724	1.233547790462	0.910878964240
46	2.734305840665	2.734302848309	1.000406906506	2.117631875926	1.233524120379	0.911064562124
47	2.734407421697	2.734404429230	1.000295417717	2.117717018562	1.233506860258	0.911199955955
48	2.734481513417	2.734478520869	1.000214109872	2.117779120262	1.233494273609	0.911298719754
49	2.734535553799	2.734532561192	1.000154811812	2.117824415416	1.233485094666	0.911370760308
50	2.734574968757	2.734571976107	1.000111565063	2.117857451918	1.233478400645	0.911423306587
51	2.734603716206	2.734600723524	1.000080024456	2.117881547201	1.233473518732	0.911461632848
52	2.734624683111	2.734621690406	1.000057021202	2.117899121098	1.233469957854	0.911489586805
53	2.734639975188	2.734636982466	1.000040244372	2.117911938455	1.233467361322	0.911509975249
54	2.734651128355	2.734648135621	1.000028008547	2.117921286699	1.233465467631	0.911524845632
55	2.734659262801	2.734656270058	1.000019084603	2.117928104743	1.233464086521	0.911535691304
56	2.734665195560	2.734662202811	1.000012576099	2.117933077398	1.233463079242	0.911543601524
57	2.734669522538	2.734666529784	1.000007829245	2.117936704138	1.233462344605	0.911549370769
58	2.734672678358	2.734669685601	1.000004367213	2.117939349249	1.233461808813	0.911553578504
59	2.734674980009	2.734671987249	1.000001842241	2.117941278422	1.233461418044	0.911556647362
60	2.734676658683	2.734673665922	1.000000000697	2.117942685435	1.233461133043	0.911558885592
			0.999998657598			0.911560518013

PARAMETERS

```
CAPITALS SHARE                       (ALPHA) =   0.2500
POPULATION GROWTH                       (ZN) =   1.0000
INTERTEMPORAL ELASTICITY             (GAMMA) =   0.5000
SUBJ RATE OF TIME DISC                 (RHO) =   1.3500
```

STEADY STATE VALUES.

```
(LAISSEZ-FAIRE)
WAGES              =  2.000000
INTEREST RATE      =  2.446654
SCALE              =  3.690920
CAPITAL/LABOR      =  0.272480
GOLDEN RULE K/L    =  0.898324
UTILITY            =  0.770133
SOC UTILITY        =  2.310398
C1                 =  1.455041
C2                 =  1.878286
PVCONS             =  2.000000
PVINC              =  2.000000
END STEADY STATE COMPUTATIONS.  (ITER   =  18)
```

DYNAMIC SOLUTION

YR	TAX	BEN	NT	UTILITY	WEL GAIN	PS	GS
0	0.000000000000	0.000000000000	0.000000000000	0.770132560306	0.000000000000	0.544959241707	0.000000000000
1	0.000002131875	0.000000000000	0.000002131875	0.770130267779	-0.000297679616	0.544957593057	0.000004263750
2	0.000002131875	0.000000000000	0.000002131875	0.770129068621	-0.000453387533	0.544959199605	0.000011611586
3	0.000002131875	0.000000000000	0.000002131875	0.770128724627	-0.000498054492	0.544963647143	0.000024274154
4	0.000002131875	0.000000000000	0.000002131875	0.770128626506	-0.000510795203	0.544971793238	0.000046095044
5	0.000002131875	0.000000000000	0.000002131875	0.770128600265	-0.000514202504	0.544985968361	0.000083696352
6	0.000002131875	0.000000000000	0.000002131875	0.770128597983	-0.000514498890	0.545010431542	0.000148484968
7	0.000002131875	0.000000000000	0.000002131875	0.770128614637	-0.000512336443	0.545052584348	0.000260103066
8	0.000002131875	0.000000000000	0.000002131875	0.770128670966	-0.000505022251	0.545125180656	0.000452353567
9	0.000002131875	0.000000000000	0.000002131875	0.770128841026	-0.000482940295	0.545250138414	0.000783349714
10	0.000002131875	0.000000000000	0.000002131875	0.770129346346	-0.000417325554	0.545465033267	0.001352822790
11	0.000002131875	0.000000000000	0.000002131875	0.770130842607	-0.000223039410	0.545834034886	0.002331408947
12	0.000002131875	0.000000000000	0.000002131875	0.770135241472	0.000348143456	0.546466015470	0.004009540122
13	0.000002131875	0.000000000000	0.000002131875	0.770148056462	0.002012141314	0.547543626481	0.006877130422
14	0.000002131875	0.000000000000	0.000002131875	0.770184848137	0.006789458534	0.549367472757	0.011747946939
15	0.000002131875	0.000000000000	0.000002131875	0.770287996334	0.020183022433	0.552416392913	0.019938420577
16	0.000002131875	0.000000000000	0.000002131875	0.770566311692	0.056321652720	0.557412126700	0.033483868451
17	0.000002131875	0.000000000000	0.000002131875	0.771273828474	0.148191133125	0.565345360199	0.055295855059
18	0.000002131875	0.000000000000	0.000002131875	0.772921311812	0.362113180141	0.577372220642	0.089011479228
19	0.000002131875	0.000000000000	0.000002131875	0.776324515080	0.804011554941	0.594474676827	0.138152063457
20	0.000002131875	0.000000000000	0.000002131875	0.782384237919	1.590853087350	0.616902414885	0.204447527058
21	0.000002131875	0.000000000000	0.000002131875	0.791531747289	2.778636832913	0.643697591898	0.286095104174
22	0.000002131875	0.000000000000	0.000002131875	0.803253506348	4.300681174770	0.672756203271	0.377495831838
23	0.000002131875	0.000000000000	0.000002131875	0.816238247111	5.986720881694	0.701534696888	0.471084356199
24	0.000002131875	0.000000000000	0.000002131875	0.829031352464	7.647877156958	0.727923851812	0.559922934198
25	0.000002131875	0.000000000000	0.000002131875	0.840587658094	9.148437738981	0.750718434174	0.639341290854
26	0.000002131875	0.000000000000	0.000002131875	0.850414468569	10.424427222037	0.769573635562	0.707178841474
27	0.000002131875	0.000000000000	0.000002131875	0.858439010081	11.466396088910	0.784708969904	0.763200290400
28	0.000002131875	0.000000000000	0.000002131875	0.864819254319	12.294856612114	0.796613180532	0.808329217797
29	0.000002131875	0.000000000000	0.000002131875	0.869803561613	12.942057827895	0.805847730673	0.844026563415
30	0.000002131875	0.000000000000	0.000002131875	0.873651774409	13.441739700211	0.812944125263	0.871886974627
31	0.000002131875	0.000000000000	0.000002131875	0.876599137835	13.824448285441	0.818361953541	0.893416495065
32	0.000002131875	0.000000000000	0.000002131875	0.878844031602	14.115942747951	0.822479326243	0.909931997434
33	0.000002131875	0.000000000000	0.000002131875	0.880547197100	14.337094999540	0.825598180701	0.922532254142
34	0.000002131875	0.000000000000	0.000002131875	0.881835747212	14.504410365589	0.827955108407	0.932106442900
35	0.000002131875	0.000000000000	0.000002131875	0.882808642010	14.630738591101	0.829733192576	0.939359273954
36	0.000002131875	0.000000000000	0.000002131875	0.883542122018	14.725979338740	0.831072906599	0.944841147031
37	0.000002131875	0.000000000000	0.000002131875	0.884094503126	14.797704796996	0.832081390810	0.948977467105
38	0.000002131875	0.000000000000	0.000002131875	0.884510165587	14.851677643004	0.832840016957	0.952094547726
39	0.000002131875	0.000000000000	0.000002131875	0.884822762227	14.892267621403	0.833410397710	0.954441308983
40	0.000002131875	0.000000000000	0.000002131875	0.885057744733	14.922779577195	0.833839081243	0.956206859641

YR	TAX	BEN	NT	UTILITY	WEL GAIN	PS	GS
41	0.000002131875	0.000000000000	0.000002131875	0.885234325504	14.945708197549	0.834161177373	0.957534435338
42	0.000002131875	0.000000000000	0.000002131875	0.885366986798	14.962933971490	0.834403136552	0.958532283041
43	0.000002131875	0.000000000000	0.000002131875	0.885466633973	14.975872935576	0.834584867870	0.959282071207
44	0.000002131875	0.000000000000	0.000002131875	0.885541472704	14.985590578312	0.834721346802	0.959845338930
45	0.000002131875	0.000000000000	0.000002131875	0.885597673378	14.992888110816	0.834823832500	0.960268414155
46	0.000002131875	0.000000000000	0.000002131875	0.885639874493	14.998367831755	0.834900786590	0.960586149179
47	0.000002131875	0.000000000000	0.000002131875	0.885671561486	15.002482317335	0.834958566699	0.960824749563
48	0.000002131875	0.000000000000	0.000002131875	0.885695352842	15.005571572003	0.835001948612	0.961003911636
49	0.000002131875	0.000000000000	0.000002131875	0.885713215380	15.007890982841	0.835034519289	0.961138434984
50	0.000002131875	0.000000000000	0.000002131875	0.885726626236	15.009632352618	0.835058972489	0.961239437392
51	0.000002131875	0.000000000000	0.000002131875	0.885736694668	15.010939716044	0.835077331012	0.961315269412
52	0.000002131875	0.000000000000	0.000002131875	0.885744253612	15.011921228154	0.835091113720	0.961372202357
53	0.000002131875	0.000000000000	0.000002131875	0.885749928483	15.012658097503	0.835101461032	0.961414945580
54	0.000002131875	0.000000000000	0.000002131875	0.885754188854	15.013211297211	0.835109229182	0.961447035253
55	0.000002131875	0.000000000000	0.000002131875	0.885757387386	15.013626619524	0.835115060945	0.961471126494
56	0.000002131875	0.000000000000	0.000002131875	0.885759788523	15.013938401754	0.835119439116	0.961489212816
57	0.000002131875	0.000000000000	0.000002131875	0.885761591145	15.014172468234	0.835122725959	0.961502790902
58	0.000002131875	0.000000000000	0.000002131875	0.885762944431	15.014348189411	0.835125193493	0.961512984443
59	0.000002131875	0.000000000000	0.000002131875	0.885763960383	15.014480108538	0.835127045943	0.961520637063
60	0.000002131875	0.000000000000	0.000002131875	0.885764723087	15.014579144002	0.835128436629	0.961526382120

YR	WG	WN	RG	C1	C2	K/L
0	2.000000000000	2.000000000000	2.446653985049	1.455040758293	1.878285942120	0.272480027506
1	1.999998258856	1.999993995110	2.446660375026	1.455036402053	1.878280436599	0.272480027506
2	1.999999911996	1.999995648246	2.446654308022	1.455036448641	1.878269542633	0.272480928403
3	2.000008127552	2.000003863784	2.446624157332	1.455040216641	1.878253476869	0.272485405596
4	2.000023825616	2.000019561815	2.446566547574	1.455047768577	1.878226570769	0.272493960649
5	2.000051318633	2.000047054774	2.446465656307	1.455061086412	1.878180437605	0.272508944141
6	2.000098817914	2.000094553953	2.446291361020	1.455084122411	1.878101031874	0.272534832357
7	2.000180688849	2.000176424714	2.445990980211	1.455123840366	1.877964323467	0.272579458255
8	2.000321719773	2.000317455338	2.445473659355	1.455192274682	1.877729069321	0.272656343707
9	2.000564554011	2.000560289058	2.444583251717	1.455310150644	1.877324647272	0.272788767111
10	2.000982404561	2.000978138716	2.443052118069	1.455513105449	1.876630648438	0.273016744064
11	2.001700612191	2.001696344815	2.440423369522	1.455862309929	1.875443392067	0.273408928029
12	2.002932730866	2.002928460864	2.435922406762	1.456462445394	1.873422928460	0.274082721916
13	2.005039624033	2.005035349539	2.428251480219	1.457491723059	1.870015088543	0.275237777796
14	2.008622486953	2.008618204821	2.415280527102	1.459250732064	1.864353022735	0.277210378451
15	2.014658780334	2.014654485333	2.393635616508	1.462238092421	1.855177379397	0.280557709848
16	2.024672503830	2.024668187481	2.358295304605	1.467256060781	1.840895713223	0.286177406745
17	2.040874087963	2.040869737074	2.302575641864	1.475524376875	1.820018394564	0.295447997575
18	2.066086991020	2.066082586380	2.219303670922	1.488710365738	1.792186258790	0.310320607629
19	2.103146576437	2.103142092792	2.104039636472	1.508667415965	1.759454694408	0.333191849935
20	2.153570738083	2.153566146939	1.959679802065	1.536663732054	1.726505072687	0.366313370142
21	2.216003835506	2.215999111263	1.798668040133	1.572301519365	1.698666057674	0.410674970971
22	2.285782916825	2.285778043822	1.638919389581	1.613021840550	1.679073073406	0.464896348036
23	2.356469554091	2.356464530392	1.495812100535	1.654929833504	1.667649277917	0.525126017554
24	2.422298647369	2.422293483331	1.377144402234	1.694369631520	1.662328504564	0.586309526544
25	2.479730590812	2.479725304336	1.283657143976	1.729006870162	1.660734744619	0.643923393005
26	2.527532887239	2.527527498855	1.212193903965	1.757953863293	1.661063872186	0.695029862514
27	2.566051551648	2.566046081147	1.158420968330	1.781337111244	1.662208079268	0.738376238518
28	2.596419275665	2.596413740423	1.118247838014	1.799800559892	1.663575238635	0.773954630152
29	2.620012193813	2.620006608274	1.088309959095	1.814158877601	1.664885059642	0.802471199164
30	2.638160199352	2.638154575124	1.066004526852	1.825210449861	1.666025794708	0.824937147044
31	2.652024565949	2.652018912165	1.049373066187	1.833656958624	1.666969629313	0.842415549945
32	2.662565734510	2.662560058253	1.036958856710	1.840080732010	1.667726877111	0.855889224303
33	2.670553012666	2.670547319381	1.027682427827	1.844949138680	1.668322531449	0.866205661839
34	2.676590354163	2.676584648007	1.020743952440	1.848629539600	1.668784897180	0.874065217422
35	2.681145692359	2.681139976491	1.015549973377	1.851406783915	1.669140518621	0.880030775654
36	2.684578349154	2.684572625969	1.011659330022	1.853499719370	1.669412267912	0.884546233265
37	2.687162536272	2.687156807578	1.008743462099	1.855075416768	1.669618958967	0.887957026815
38	2.689106595957	2.689100863118	1.006557263771	1.856260846161	1.669775634762	0.890529428957
39	2.690568323990	2.690562588034	1.004917631987	1.857152190325	1.669894103238	0.892467282342
40	2.691666958691	2.691661220394	1.003687628236	1.857822139151	1.669983517738	0.893925853346

YR	WG	WN	RG	C1	C2	K/L
41	2.692492451007	2.692486710949	1.002764748290	1.858325533576	1.670050912126	0.895022970442
42	2.693112574021	2.693106832642	1.002072210225	1.858703696089	1.670101658092	0.895847806356
43	2.693578344184	2.693572601812	1.001552468928	1.858987733942	1.670139839589	0.896467709797
44	2.693928138155	2.693922395037	1.001162378870	1.859201048235	1.670168551884	0.896933469539
45	2.694190809745	2.694185066066	1.000869580817	1.859361233567	1.670190133492	0.897283342866
46	2.694388044813	2.694382300714	1.000649799412	1.859481514124	1.670206350611	0.897546123327
47	2.694536137196	2.694530392782	1.000484820586	1.859571826083	1.670218533805	0.897743467884
48	2.694647326883	2.694641582232	1.000360976175	1.859639633620	1.670227684880	0.897891658131
49	2.694730807123	2.694725062294	1.000268008313	1.859690543005	1.670234557553	0.898002930124
50	2.694793481989	2.694787737027	1.000198217949	1.859728764537	1.670239718582	0.898086477137
51	2.694840535935	2.694834790872	1.000145826221	1.859757459860	1.670243593964	0.898149204940
52	2.694875861845	2.694870116707	1.000106495358	1.859779002986	1.670246503801	0.898196300212
53	2.694902382648	2.694896637453	1.000076969206	1.859795176421	1.670248688566	0.898231658039
54	2.694922292910	2.694916547672	1.000054803467	1.859807318491	1.670250328879	0.898258203306
55	2.694937240288	2.694931495019	1.000038163249	1.859816434074	1.670251560889	0.898278132218
56	2.694948461773	2.694942716480	1.000025671137	1.859823277364	1.670252485324	0.898293093720
57	2.694956886134	2.694951140823	1.000016293007	1.859828414864	1.670253179365	0.898304325966
58	2.694963210564	2.694957465239	1.000009252632	1.859832271746	1.670253700418	0.898312758431
59	2.694967958498	2.694962213163	1.000003967259	1.859835167220	1.670254091595	0.898319088968
60	2.694971522907	2.694965777564	0.999999999405	1.859837340935	1.670254385266	0.898323841503
			0.999997020642			0.898327409374

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PARAMETERS

CAPITALS SHARE                    (ALPHA) =    0.2500
POPULATION GROWTH                    (ZN) =    1.0000
INTERTEMPORAL ELASTICITY          (GAMMA) =    0.5000
SUBJ RATE OF TIME DISC              (RHO) =    2.0000


STEADY STATE VALUES.

(LAISSEZ-FAIRE)
WAGES               =  2.000000
INTEREST RATE       =  2.837771
SCALE               =  3.830327
CAPITAL/LABOR       =  0.234926
GOLDEN RULE K/L     =  0.943846
UTILITY             =  0.895656
SOC UTILITY         =  2.686969
C1                  =  1.530149
C2                  =  1.803179
PVCONS              =  2.000000
PVINC               =  2.000000
END STEADY STATE COMPUTATIONS.  (ITER   =  19)
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DYNAMIC SOLUTION

YR	TAX	BEN	NT	UTILITY	WEL GAIN	PS	GS
0	0.000000000000	0.000000000000	0.000000000000	0.895656269552	0.000000000000	0.469850773066	0.000000000000
1	0.000000672500	0.000000000000	0.000000672500	0.895655380340	-0.000099280444	0.469850027675	0.000001345000
2	0.000000672500	0.000000000000	0.000000672500	0.895654678940	-0.000177591705	0.469850485679	0.000003925903
3	0.000000672500	0.000000000000	0.000000672500	0.895654474664	-0.000200399202	0.469852186455	0.000008878341
4	0.000000672500	0.000000000000	0.000000672500	0.895654415515	-0.000207003113	0.469855689014	0.000018381351
5	0.000000672500	0.000000000000	0.000000672500	0.895654398945	-0.000208853142	0.469862479243	0.000036615817
6	0.000000672500	0.000000000000	0.000000672500	0.895654395792	-0.000209205183	0.469875527969	0.000071602716
7	0.000000672500	0.000000000000	0.000000672500	0.895654402633	-0.000208441386	0.469900566738	0.000138727107
8	0.000000672500	0.000000000000	0.000000672500	0.895654432706	-0.000205083783	0.469948592766	0.000267487839
9	0.000000672500	0.000000000000	0.000000672500	0.895654544324	-0.000192621604	0.470040667768	0.000514403565
10	0.000000672500	0.000000000000	0.000000672500	0.895654956594	-0.000146591664	0.470217045494	0.000987609356
11	0.000000672500	0.000000000000	0.000000672500	0.895656466780	0.000022020522	0.470554379303	0.001893437889
12	0.000000672500	0.000000000000	0.000000672500	0.895661963796	0.000635762255	0.471197615037	0.003623560267
13	0.000000672500	0.000000000000	0.000000672500	0.895681754902	0.002845438713	0.472417198414	0.006914140250
14	0.000000672500	0.000000000000	0.000000672500	0.895751566851	0.010639941057	0.474705157836	0.013122928656
15	0.000000672500	0.000000000000	0.000000672500	0.895988889681	0.037137029091	0.478915339470	0.024665759268
16	0.000000672500	0.000000000000	0.000000672500	0.896745234256	0.121582881868	0.486405791675	0.045558927779
17	0.000000672500	0.000000000000	0.000000672500	0.898914348673	0.363764452071	0.499017594446	0.081676030953
18	0.000000672500	0.000000000000	0.000000672500	0.904235334155	0.957852347456	0.518581223629	0.139706741875
19	0.000000672500	0.000000000000	0.000000672500	0.914885265477	2.146916912142	0.545808809983	0.223785024798
20	0.000000672500	0.000000000000	0.000000672500	0.931843588298	4.040313229201	0.579196371401	0.331201398255
21	0.000000672500	0.000000000000	0.000000672500	0.953491344253	6.457284637782	0.615150075078	0.451747938523
22	0.000000672500	0.000000000000	0.000000672500	0.976455537248	9.021236208906	0.649604805182	0.572390891215
23	0.000000672500	0.000000000000	0.000000672500	0.997710511761	11.394353579526	0.679729230224	0.682757089678
24	0.000000672500	0.000000000000	0.000000672500	1.015680712790	13.400726072974	0.704427467314	0.777329098303
25	0.000000672500	0.000000000000	0.000000672500	1.030048184630	15.004853943096	0.723853456148	0.854725084235
26	0.000000672500	0.000000000000	0.000000672500	1.041159926677	16.245479663582	0.738745302159	0.916067563256
27	0.000000672500	0.000000000000	0.000000672500	1.049586370279	17.186291880078	0.749982874710	0.963609026719
28	0.000000672500	0.000000000000	0.000000672500	1.055901028470	17.891323308476	0.758380053957	0.999876662973
29	0.000000672500	0.000000000000	0.000000672500	1.060598243778	18.415767279628	0.764615477039	1.027234527338
30	0.000000672500	0.000000000000	0.000000672500	1.064075667526	18.804021553753	0.769226548754	1.047705825280
31	0.000000672500	0.000000000000	0.000000672500	1.066641903514	19.090541737455	0.772626907557	1.062935351956
32	0.000000672500	0.000000000000	0.000000672500	1.068531620953	19.301528642010	0.775129624438	1.074217765653
33	0.000000672500	0.000000000000	0.000000672500	1.069921079990	19.456661708563	0.776969182817	1.082550591170
34	0.000000672500	0.000000000000	0.000000672500	1.070941634222	19.570606562913	0.778320013701	1.088691293189
35	0.000000672500	0.000000000000	0.000000672500	1.071690666027	19.654235945155	0.779311284314	1.093209234619
36	0.000000672500	0.000000000000	0.000000672500	1.072240119099	19.715582366901	0.780038344668	1.096529324940
37	0.000000672500	0.000000000000	0.000000672500	1.072643014553	19.760565634194	0.780571427548	1.098967047015
38	0.000000672500	0.000000000000	0.000000672500	1.072938361213	19.793541081325	0.780962185060	1.100755774998
39	0.000000672500	0.000000000000	0.000000672500	1.073154823930	19.817709138280	0.781248562448	1.102067684414
40	0.000000672500	0.000000000000	0.000000672500	1.073313448192	19.835419533115	0.781458413396	1.103029554974

YR	TAX	BEN	NT	UTILITY	WEL GAIN	PS	GS
41	0.000000672500	0.000000000000	0.000000672500	1.073429675834	19.848396346418	0.781612172131	1.103734608338
42	0.000000672500	0.000000000000	0.000000672500	1.073514831585	19.857903983894	0.781724823769	1.104251320530
43	0.000000672500	0.000000000000	0.000000672500	1.073577218577	19.864869489982	0.781807353797	1.104629952978
44	0.000000672500	0.000000000000	0.000000672500	1.073622922733	19.869972357895	0.781867813998	1.104907377453
45	0.000000672500	0.000000000000	0.000000672500	1.073656404156	19.873710557947	0.781912104930	1.105110632265
46	0.000000672500	0.000000000000	0.000000672500	1.073680931036	19.876448983404	0.781944550173	1.105259538979
47	0.000000672500	0.000000000000	0.000000672500	1.073698897949	19.878454988881	0.781968317517	1.105368625515
48	0.000000672500	0.000000000000	0.000000672500	1.073712059268	19.879924449800	0.781985727782	1.105448538232
49	0.000000672500	0.000000000000	0.000000672500	1.073721700255	19.881000865713	0.781998481201	1.105507078101
50	0.000000672500	0.000000000000	0.000000672500	1.073728762466	19.881789361461	0.782007823317	1.105549960700
51	0.000000672500	0.000000000000	0.000000672500	1.073733935765	19.882366960054	0.782014666487	1.105581373429
52	0.000000672500	0.000000000000	0.000000672500	1.073737725150	19.882790044881	0.782019679263	1.105604383981
53	0.000000672500	0.000000000000	0.000000672500	1.073740500931	19.883099960708	0.782023351187	1.105621239632
54	0.000000672500	0.000000000000	0.000000672500	1.073742534220	19.883326977372	0.782026040909	1.105633586649
55	0.000000672500	0.000000000000	0.000000672500	1.073744023623	19.883493269168	0.782028011154	1.105642630996
56	0.000000672500	0.000000000000	0.000000672500	1.073745114623	19.883615079300	0.782029454375	1.105649256079
57	0.000000672500	0.000000000000	0.000000672500	1.073745913789	19.883704306172	0.782030511545	1.105654109017
58	0.000000672500	0.000000000000	0.000000672500	1.073746499184	19.883769665517	0.782031285930	1.105657663837
59	0.000000672500	0.000000000000	0.000000672500	1.073746927990	19.883817541703	0.782031853173	1.105660267771
60	0.000000672500	0.000000000000	0.000000672500	1.073747242093	19.883852611339	0.782032268682	1.105662175173

YR	WG	WN	RG	C1	C2	K/L
0	2.000000000000	2.000000000000	2.837770629854	1.530149226934	1.803179497287	0.234925630163
1	1.999998790107	1.999997445108	2.837775779959	1.530147417432	1.803178821429	0.234925630163
2	1.999998909665	1.999997564665	2.837775271041	1.530147078986	1.803174109496	0.234925686338
3	2.000002143557	2.000000798556	2.837761505487	1.530148612101	1.803166475201	0.234927205791
4	2.000009223608	2.000007878602	2.837731368490	1.530152189588	1.803152238992	0.234930532398
5	2.000023063330	2.000021718315	2.837672459456	1.530159239071	1.803125043787	0.234937035182
6	2.000049692257	2.000048347224	2.837559117522	1.530172819255	1.803072903487	0.234949547530
7	2.000100804268	2.000099459201	2.837341584016	1.530198892462	1.802972915742	0.234973565342
8	2.000198858946	2.000197513812	2.836924324041	1.530248921046	1.802781237516	0.235019646922
9	2.000386906423	2.000385561162	2.836124339331	1.530344893395	1.802414105278	0.235108040302
10	2.000747341440	2.000745995937	2.834591830415	1.530528950443	1.801712121207	0.235277535666
11	2.001437473330	2.001436127363	2.831660585589	1.530881748060	1.800374225824	0.235602327425
12	2.002756249492	2.002754902639	2.826070487383	1.531557287602	1.797840095234	0.236223908596
13	2.005266754070	2.005265405528	2.815469427745	1.532848207114	1.793095999892	0.237410587652
14	2.010011720950	2.010010369217	2.795577309217	1.535305211381	1.784406357085	0.239665669332
15	2.018860607959	2.018859250275	2.758978232788	1.539943910806	1.769109456965	0.243914043246
16	2.034965211372	2.034963842858	2.693992045470	1.548558051183	1.743983963434	0.251790549369
17	2.063052792431	2.063051405028	2.585450647363	1.564033810582	1.707120531187	0.265982359727
18	2.108756326570	2.108754908431	2.420962603266	1.590173684802	1.661336477665	0.290346812700
19	2.175923450964	2.175921987655	2.203618644107	1.630113177672	1.615584516968	0.329143982752
20	2.262585652652	2.262584131063	1.959982482878	1.683387759662	1.580004488604	0.384796917390
21	2.359649131015	2.359647544151	1.727925389438	1.744497469073	1.558857292497	0.455198884828
22	2.455106638060	2.455104987001	1.534108919354	1.805500181819	1.549714307225	0.533449006801
23	2.539844060889	2.539842352844	1.385626298782	1.860113122620	1.547760175914	0.610997848198
24	2.609892813373	2.609891058221	1.277024596015	1.905463590906	1.549126528998	0.681243159951
25	2.665224819822	2.665223027458	1.199128512589	1.941369571310	1.551603822054	0.740878282809
26	2.707735148974	2.707733328022	1.143533070926	1.968988025863	1.554165454735	0.789289270192
27	2.739850436241	2.739848593692	1.103790643443	1.989865718981	1.556417508170	0.827406432708
28	2.763862822788	2.763860964090	1.075270730245	2.005480910133	1.558257706082	0.856795950715
29	2.781699453572	2.781697582879	1.054718736192	2.017082105840	1.559703516481	0.879128358465
30	2.794892105240	2.794890225675	1.039853446963	2.025663676921	1.560813392031	0.895925002189
31	2.804621924495	2.804620038387	1.029068542332	2.031993130830	1.561652998135	0.908466187017
32	2.811783731380	2.811781840455	1.021225228331	2.036652216017	1.562282046955	0.917781129756
33	2.817048074890	2.817046180425	1.015510693118	2.040076997609	1.562750263095	0.924673695046
34	2.820913921664	2.820912024599	1.011341372873	2.042592010899	1.563097186278	0.929759886994
35	2.823750825756	2.823748926784	1.008296277348	2.044437642470	1.563353415859	0.933505653445
36	2.825831618901	2.825829718529	1.006070548700	2.045791373861	1.563542230178	0.936260259466
37	2.827357277570	2.827355376172	1.004442782555	2.046783948624	1.563681139082	0.938283834804
38	2.828475615220	2.828473713070	1.003251827864	2.047511528010	1.563783212485	0.939769237282
39	2.829295223706	2.829293321005	1.002380195035	2.048044758557	1.563858154213	0.940858980029
40	2.829895816581	2.829893913476	1.001742120686	2.048435500080	1.563913141835	0.941658123431

YR	WG	WN	RG	C1	C2	K/L
41	2.830335874970	2.830333971569	1.001274943106	2.048721799438	1.563953470498	0.942243984185
42	2.830658285015	2.830656381397	1.000932848477	2.048931557628	1.563983037636	0.942673390235
43	2.830894487048	2.830892583271	1.000682324148	2.049085229474	1.564004709999	0.942988072150
44	2.831067525108	2.831065621215	1.000498846844	2.049197807218	1.564020592780	0.943218653388
45	2.831194286555	2.831192382577	1.000364466448	2.049280277647	1.564032231115	0.943387595726
46	2.831287145474	2.831285241433	1.000266041550	2.049340691260	1.564040758467	0.943511368598
47	2.831355168099	2.831353264013	1.000193949831	2.049384946496	1.564047005987	0.943602044576
48	2.831404996651	2.831403092531	1.000141144945	2.049417364749	1.564051582968	0.943668471516
49	2.831441497215	2.831439593070	1.000102466507	2.049441111870	1.564054935975	0.943717133007
50	2.831468234559	2.831466330396	1.000074135095	2.049458507080	1.564057392257	0.943752779651
51	2.831487820075	2.831485915899	1.000053382578	2.049471249413	1.564059192055	0.943778892008
52	2.831502166694	2.831500262509	1.000038181482	2.049480583245	1.564060509997	0.943798019958
53	2.831512675785	2.831510771593	1.000027046670	2.049487420406	1.564061475441	0.943812031622
54	2.831520373803	2.831518469605	1.000018890411	2.049492428696	1.564062182652	0.943822295410
55	2.831526012669	2.831524108468	1.000012915934	2.049496097314	1.564062700697	0.943829813779
56	2.831530143186	2.831528238982	1.000008539611	2.049498784607	1.564063080173	0.943835321075
57	2.831533168821	2.831531264614	1.000005333937	2.049500753069	1.564063358144	0.943839355227
58	2.831535385120	2.831533480913	1.000002985766	2.049502194982	1.564063561760	0.943842310281
59	2.831537008576	2.831535104367	1.000001265719	2.049503251194	1.564063710912	0.943844474884
60	2.831538197768	2.831536293558	1.000000005775	2.049504024876	1.564063820166	0.943846060472
			0.999999082859			0.943847221927

APPENDIX D: PAY-AS-YOU-GO SIMULATIONS

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PARAMETERS

CAPITALS SHARE              (ALPHA) =    0.2500
POPULATION GROWTH              (ZN) =    1.0000
INTERTEMPORAL ELASTICITY    (GAMMA) =   -0.2500
SUBJ RATE OF TIME DISC        (RHO) =    1.3500


STEADY STATE VALUES.

(PAY-AS-YOU-GO)
SOC SEC TAX      =  0.100000
SOC SEC BEN      =  0.200000
NET WAGE         =  1.679384
WAGES            =  1.865983
INTEREST RATE    =  2.580582
SCALE            =  3.550824
CAPITAL/LABOR    =  0.241028
GOLDEN RULE K/L  =  0.853150
UTILITY          =  5.896052
SOC UTILITY      = 17.688157
C1               =  1.197329
C2               =  2.099235
PVCONS           =  1.783612
PVINC            =  1.783612
END STEADY STATE COMPUTATIONS.  (ITER   =  15)
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DYNAMIC SOLUTION

YR	TAX	BEN	NT	UTILITY	WEL GAIN	PS	GS
0	0.100000000000	0.200000000000	0.044143155610	5.896052454045	0.000000000000	0.482055400969	0.000000000000
1	0.100000494687	0.200000000000	0.044143735202	5.896047669867	-0.000081142049	0.482055063164	0.000000923078
2	0.100000494687	0.200000000000	0.044143734932	5.896045271872	-0.000121813246	0.482054759868	0.000002576076
3	0.100000494687	0.200000000000	0.044143650396	5.896044404595	-0.000136522703	0.482054466834	0.000005665617
4	0.100000494687	0.200000000000	0.044143475242	5.896043668604	-0.000149005470	0.482053993609	0.000011336773
5	0.100000494687	0.200000000000	0.044143149683	5.896042528018	-0.000168350388	0.482053142708	0.000021722030
6	0.100000494687	0.200000000000	0.044142552529	5.896040486387	-0.000202977468	0.482051588259	0.000040733713
7	0.100000494687	0.200000000000	0.044141459170	5.896036768931	-0.000266027378	0.482048744540	0.000075534730
8	0.100000494687	0.200000000000	0.044139457731	5.896029986533	-0.000381060238	0.482043541125	0.000139234421
9	0.100000494687	0.200000000000	0.044135794425	5.896017618788	-0.000590823385	0.482034020580	0.000255818145
10	0.100000494687	0.200000000000	0.044129090405	5.895995144166	-0.000972004217	0.482016609599	0.000469150550
11	0.100000494687	0.200000000000	0.044116824889	5.895954553577	-0.001660440931	0.481984794826	0.000859385994
12	0.100000494687	0.200000000000	0.044094394851	5.895882110025	-0.002889119812	0.481926750984	0.001572771120
13	0.100000494687	0.200000000000	0.044053412261	5.895755675002	-0.005033521076	0.481821149446	0.002875408893
14	0.100000494687	0.200000000000	0.043978650025	5.895544596226	-0.008613522737	0.481630000205	0.005249053892
15	0.100000494687	0.200000000000	0.043842657732	5.895224563881	-0.014041431456	0.481287193788	0.009557878046
16	0.100000494687	0.200000000000	0.043596577429	5.894849906406	-0.020395809717	0.480682614233	0.017326128971
17	0.100000494687	0.200000000000	0.043155464329	5.894808502818	-0.021098035284	0.479647822126	0.031160715524
18	0.100000494687	0.200000000000	0.042377878993	5.896557513063	0.008566053678	0.477967491279	0.055275472151
19	0.100000494687	0.200000000000	0.041046523875	5.904288483199	0.139687175752	0.475473943628	0.095806662558
20	0.100000494687	0.200000000000	0.038875430336	5.927331262299	0.530504239873	0.472285341496	0.160064705514
21	0.100000494687	0.200000000000	0.035595433838	5.980350023641	1.429728962786	0.469077353975	0.253535752391
22	0.100000494687	0.200000000000	0.031150279076	6.076026143134	3.052443825632	0.466944952280	0.375017314794
23	0.100000494687	0.200000000000	0.025885957552	6.212499895775	5.367106961763	0.466632578248	0.514041098959
24	0.100000494687	0.200000000000	0.020476046873	6.370380750910	8.044845268283	0.467939301961	0.654731409465
25	0.100000494687	0.200000000000	0.015570947796	6.524991811933	10.667126230481	0.470055105210	0.783140367382
26	0.100000494687	0.200000000000	0.011522754475	6.659432315545	12.947304445648	0.472244532002	0.891554441200
27	0.100000494687	0.200000000000	0.008382783779	6.767595964635	14.781814059205	0.474129342808	0.978189032448
28	0.100000494687	0.200000000000	0.006036889012	6.850540021871	16.188586775066	0.475610332559	1.044865679284
29	0.100000494687	0.200000000000	0.004321708264	6.912327903744	17.236540170235	0.476720497520	1.094891860479
30	0.100000494687	0.200000000000	0.003082975815	6.957554618418	18.003607882512	0.477532437575	1.131781318593
31	0.100000494687	0.200000000000	0.002194633793	6.990301609487	18.559013237595	0.478118517186	1.158662907391
32	0.100000494687	0.200000000000	0.001560212585	7.013849608737	18.958399088272	0.478538530579	1.178091801186
33	0.100000494687	0.200000000000	0.001108274346	7.030706843208	19.244306220251	0.478838306486	1.192054364661
34	0.100000494687	0.200000000000	0.000786838509	7.042738353440	19.448366654350	0.479051752165	1.202048626841
35	0.100000494687	0.200000000000	0.000558453330	7.051308230647	19.593716060131	0.479203507088	1.209182423509
36	0.100000494687	0.200000000000	0.000396290725	7.057403961361	19.697102703338	0.479311301727	1.214264426290
37	0.100000494687	0.200000000000	0.000281200534	7.061735670439	19.770570656897	0.479387824959	1.217879734499
38	0.100000494687	0.200000000000	0.000199543602	7.064811778831	19.822742994501	0.479442127274	1.220449119701
39	0.100000494687	0.200000000000	0.000141619896	7.066995212583	19.859775123522	0.479480650988	1.222273903659
40	0.100000494687	0.200000000000	0.000100537463	7.068544510800	19.886051996556	0.479507975932	1.223569232982

YR	TAX	BEN	NT	UTILITY	WEL GAIN	PS	GS
41	0.100000494687	0.200000000000	0.000071402690	7.069643589977	19.904692929369	0.479527355146	1.224488407053
42	0.100000494687	0.200000000000	0.000050742417	7.070423153674	19.917914719768	0.479541097948	1.225140498110
43	0.100000494687	0.200000000000	0.000036092383	7.070976024236	19.927291681146	0.479550842969	1.225603031241
44	0.100000494687	0.200000000000	0.000025704533	7.071368091292	19.933941334606	0.479557753064	1.225931068907
45	0.100000494687	0.200000000000	0.000018339036	7.071646108614	19.938656647507	0.479562652725	1.226163699287
46	0.100000494687	0.200000000000	0.000013116629	7.071843244315	19.942000167651	0.479566126793	1.226328660602
47	0.100000494687	0.200000000000	0.000009413799	7.071983024626	19.944370911664	0.479568590017	1.226445631719
48	0.100000494687	0.200000000000	0.000006788414	7.072082134685	19.946051867856	0.479570336502	1.226528571253
49	0.100000494687	0.200000000000	0.000004926972	7.072152406812	19.947243718303	0.479571574793	1.226587379049
50	0.100000494687	0.200000000000	0.000003607184	7.072202231936	19.948088777331	0.479572452897	1.226629075723
51	0.100000494687	0.200000000000	0.000002671436	7.072237558518	19.948687933841	0.479573075358	1.226658639683
52	0.100000494687	0.200000000000	0.000002007985	7.072262605433	19.949112742054	0.479573516679	1.226679601109
53	0.100000494687	0.200000000000	0.000001537593	7.072280363992	19.949413936096	0.479573829578	1.226694463095
54	0.100000494687	0.200000000000	0.000001204081	7.072292955001	19.949627485910	0.479574051427	1.226705000440
55	0.100000494687	0.200000000000	0.000000967619	7.072301882147	19.949778894776	0.479574208719	1.226712471537
56	0.100000494687	0.200000000000	0.000000799965	7.072308211571	19.949886244977	0.479574320240	1.226717768618
57	0.100000494687	0.200000000000	0.000000681097	7.072312698601	19.949962347257	0.479574399153	1.226721524296
58	0.100000494687	0.200000000000	0.000000596823	7.072315880353	19.950016311350	0.479574455246	1.226724187138
59	0.100000494687	0.200000000000	0.000000537068	7.072318136421	19.950054575394	0.479574495029	1.226726075114
60	0.100000494687	0.200000000000	0.000000494701	7.072319736004	19.950081705120	0.479574523236	1.226727413694

YR	WG	WN	RG	C1	C2	K/L
0	1.865982582056	1.679384323850	2.580581792352	1.197328922881	2.099235308027	0.241027990965
1	1.865981270239	1.679382220138	2.580587234934	1.197327156974	2.099236448971	0.241027990965
2	1.865981274412	1.679382223893	2.580587217623	1.197327464025	2.099232752135	0.241027993121
3	1.865982580544	1.679383399411	2.580581798623	1.197328932578	2.099226554471	0.241028667972
4	1.865985286769	1.679385835012	2.580570570843	1.197331841403	2.099215342054	0.241030066225
5	1.865990316859	1.679390362091	2.580549701817	1.197337219383	2.099194850608	0.241032665191
6	1.865999543245	1.679398665834	2.580511423621	1.197347077575	2.099157342213	0.241037432369
7	1.866016436238	1.679413869520	2.580441340369	1.197365124980	2.099088699250	0.241046160986
8	1.866047359444	1.679441700389	2.580313057138	1.197398159264	2.098963084158	0.241062139635
9	1.866103958883	1.679492639857	2.580078279473	1.197458619276	2.098733251123	0.241091387773
10	1.866207537029	1.679585860136	2.579648705239	1.197569250537	2.098312935970	0.241144919363
11	1.866397036331	1.679756409414	2.578863033185	1.197771614588	2.097544902022	0.241242880075
12	1.866743558996	1.680068279642	2.577427160625	1.198141528657	2.096143643877	0.241422090410
13	1.867376647274	1.680638058778	2.574806608764	1.198816909332	2.093594160265	0.241749761052
14	1.868531377059	1.681677315014	2.570035960179	1.200047314810	2.088978895458	0.242348279170
15	1.870631244358	1.683567194544	2.561390730913	1.202280000756	2.080699898100	0.243439527048
16	1.874429108133	1.686985270063	2.545853013768	1.206302655830	2.066089471094	0.245422535917
17	1.881231113762	1.693107071764	2.518337464865	1.213459249639	2.041036395832	0.249004371602
18	1.893204150684	1.703882799071	2.470859446207	1.225915307793	2.000134630518	0.255404268825
19	1.913657423264	1.722290734275	2.392477168950	1.246816790647	1.938472810645	0.266621481715
20	1.946907334911	1.752215638309	2.271980195405	1.279930296813	1.855990876564	0.285640303093
21	1.996973491250	1.797275154247	2.105346029669	1.328197800272	1.761997124881	0.316175023505
22	2.064711466483	1.858239298448	1.904856553061	1.391294346168	1.673005353619	0.361306553183
23	2.145142109797	1.930626837642	1.698525926623	1.463994259394	1.602585965184	0.420981133537
24	2.228507195375	2.005655373422	1.514949873860	1.537716071461	1.554174288023	0.490336838604
25	2.305133112566	2.074618660989	1.368838959400	1.604563555779	1.523565300228	0.561335355713
26	2.369395806052	2.132455053336	1.260455562266	1.660210521334	1.504781434206	0.626597736296
27	2.420028613091	2.178024554624	1.182984032419	1.703895211816	1.493182285077	0.681899486601
28	2.458376349253	2.212537498199	1.128483667281	1.736927165640	1.485856635154	0.726159187628
29	2.486725250661	2.238051495443	1.090327516424	1.761330997923	1.481110426857	0.760238005921
30	2.507374875819	2.256636147870	1.063610401151	1.779103710295	1.477965408536	0.785806179000
31	2.522278813750	2.270049684636	1.044867231590	1.791931167449	1.475843708609	0.804656878084
32	2.532973131346	2.279674565182	1.031688610877	1.801136034602	1.474392758167	0.818390712289
33	2.540617664082	2.286554640862	1.022403777663	1.807716334376	1.473390462587	0.828315165883
34	2.546068299433	2.291460209982	1.015851523918	1.812408457817	1.472692980304	0.835446335574
35	2.549947990971	2.294951930446	1.011221785768	1.815748423358	1.472205023314	0.840550189503
36	2.552706253035	2.297434364940	1.007947376160	1.818123063213	1.471862338846	0.844192965298
37	2.554665636940	2.299197809485	1.005629920441	1.819809984526	1.471621016324	0.846787864008
38	2.556056731346	2.300449793762	1.003988918189	1.821007666488	1.471450740808	0.848633779729
39	2.557043966691	2.301338305084	1.002826493064	1.821857654096	1.471330427664	0.849945623488
40	2.557744394207	2.301968689502	1.002002859234	1.822460713571	1.471245332362	0.850877277324

YR	WG	WN	RG	C1	C2	K/L
41	2.558241238179	2.302415848831	1.001419166353	1.822888493686	1.471185103458	0.851538604457
42	2.558593622332	2.302732994394	1.001005459805	1.823191896446	1.471142453137	0.852007881099
43	2.558843524465	2.302957906191	1.000712207442	1.823407063221	1.471112239538	0.852340798029
44	2.559020736497	2.303117396932	1.000504324058	1.823559643868	1.471090831656	0.852576937105
45	2.559146395997	2.303230490420	1.000356950660	1.823667837695	1.471075660007	0.852744410985
46	2.559235496904	2.303310681192	1.000252470699	1.823744554399	1.471064906558	0.852863176006
47	2.559298673780	2.303367540349	1.000178398141	1.823798950332	1.471057283977	0.852947393698
48	2.559343468472	2.303407855550	1.000125882447	1.823837519048	1.471051880365	0.853007110868
49	2.559375229136	2.303436440131	1.000088649610	1.823864865338	1.471048049596	0.853049453877
50	2.559397748118	2.303456707204	1.000062251860	1.823884254307	1.471045334412	0.853079476921
51	2.559413714503	2.303471076943	1.000043535918	1.823898001585	1.471043408833	0.853100764310
52	2.559425034829	2.303481265230	1.000030266412	1.823907748552	1.471042043610	0.853115857520
53	2.559433061064	2.303488488838	1.000020858329	1.823914659260	1.471041075682	0.853126558894
54	2.559438751746	2.303493610449	1.000014187974	1.823919559023	1.471040389427	0.853134146337
55	2.559442786491	2.303497241718	1.000009458668	1.823923032999	1.471039902875	0.853139525933
56	2.559445647159	2.303499816318	1.000006105569	1.823925496078	1.471039557910	0.853143340128
57	2.559447675395	2.303501641729	1.000003728204	1.823927242577	1.471039312593	0.853146044429
58	2.559449113371	2.303502935906	1.000002042709	1.823928480660	1.471039139356	0.853147961724
59	2.559450132973	2.303503853548	1.000000847603	1.823929358519	1.471039016565	0.853149321192
60	2.559450855883	2.303504504166	1.000000000261	1.823929980930	1.471038929511	0.853150285071
			0.999999399494			0.853150968465

PARAMETERS

```
CAPITALS SHARE                 (ALPHA) =   0.2500
POPULATION GROWTH                 (ZN) =   1.0000
INTERTEMPORAL ELASTICITY       (GAMMA) =  -0.2500
SUBJ RATE OF TIME DISC           (RHO) =   2.0000
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STEADY STATE VALUES.

```
(PAY-AS-YOU-GO)
SOC SEC TAX      =  0.100000
SOC SEC BEN      =  0.200000
NET WAGE         =  1.673013
WAGES            =  1.858903
INTEREST RATE    =  3.183816
SCALE            =  3.731630
CAPITAL/LABOR    =  0.194619
GOLDEN RULE K/L  =  0.911559
UTILITY          =  4.554418
SOC UTILITY      = 13.663255
C1               =  1.283775
C2               =  2.000279
PVCONS           =  1.761874
PVINC            =  1.761874
END STEADY STATE COMPUTATIONS.  (ITER   =  14)
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DYNAMIC SOLUTION

YR	TAX	BEN	NT	UTILITY	WEL GAIN	PS	GS
0	0.100000000000	0.200000000000	0.052196749220	4.554418376013	0.000000000000	0.389237677565	0.000000000000
1	0.100000075000	0.200000000000	0.052196940403	4.554416293824	-0.000045717986	0.389237787428	0.000000139418
2	0.100000075000	0.200000000000	0.052196972722	4.554414803362	-0.000078443615	0.389237646689	0.000000376016
3	0.100000075000	0.200000000000	0.052196966003	4.554414399540	-0.000087310215	0.389237547657	0.000000937456
4	0.100000075000	0.200000000000	0.052196933591	4.554414195649	-0.000091786985	0.389237383458	0.000002155702
5	0.100000075000	0.200000000000	0.052196859710	4.554413920805	-0.000097821656	0.389237043309	0.000004774797
6	0.100000075000	0.200000000000	0.052196699971	4.554413361699	-0.000110097781	0.389236314186	0.000010399918
7	0.100000075000	0.200000000000	0.052196356797	4.554412170161	-0.000136260018	0.389234749472	0.000022479421
8	0.100000075000	0.200000000000	0.052195619794	4.554409620141	-0.000192250048	0.389231390548	0.000048418478
9	0.100000075000	0.200000000000	0.052194037148	4.554404152446	-0.000312302588	0.389224178372	0.000104116279
10	0.100000075000	0.200000000000	0.052190638970	4.554392450288	-0.000569243382	0.389208696443	0.000223701054
11	0.100000075000	0.200000000000	0.052183343658	4.554367531737	-0.001116372533	0.389175481756	0.000480395321
12	0.100000075000	0.200000000000	0.052167686677	4.554314987983	-0.002270059994	0.389104299996	0.001031136368
13	0.100000075000	0.200000000000	0.052134107195	4.554206545251	-0.004651104581	0.388952100072	0.002211539413
14	0.100000075000	0.200000000000	0.052062195465	4.553993659783	-0.009325367038	0.388628275055	0.004735911902
15	0.100000075000	0.200000000000	0.051908677245	4.553625863096	-0.017400968715	0.387946491738	0.010109081816
16	0.100000075000	0.200000000000	0.051583132259	4.553221752148	-0.026273911760	0.386542384339	0.021432604816
17	0.100000075000	0.200000000000	0.050902478668	4.553901618858	-0.011346282043	0.383778724173	0.044807060591
18	0.100000075000	0.200000000000	0.049520319822	4.560713419620	0.138218386790	0.378801153768	0.091094394599
19	0.100000075000	0.200000000000	0.046871584648	4.589077904606	0.761008887006	0.371161694121	0.175892039865
20	0.100000075000	0.200000000000	0.042305699740	4.667885140685	2.491355762782	0.362049948206	0.312417433683
21	0.100000075000	0.200000000000	0.035651996576	4.819229632613	5.814381436600	0.354231684717	0.496388438565
22	0.100000075000	0.200000000000	0.027833607391	5.025425993499	10.341773166189	0.349431707722	0.700576424564
23	0.100000075000	0.200000000000	0.020400424231	5.238309165988	15.015985215079	0.347063256169	0.892342926340
24	0.100000075000	0.200000000000	0.014371728001	5.420288671724	19.011654710334	0.345874496676	1.052196150309
25	0.100000075000	0.200000000000	0.009921887637	5.559657789217	22.071740674925	0.345145680569	1.175684092527
26	0.100000075000	0.200000000000	0.006788075049	5.660444909730	24.284693289094	0.344624075679	1.266742962294
27	0.100000075000	0.200000000000	0.004626718358	5.731283584686	25.840076855284	0.344235246597	1.331999651598
28	0.100000075000	0.200000000000	0.003148961128	5.780367253314	26.917792264280	0.343948503055	1.377935877715
29	0.100000075000	0.200000000000	0.002142083002	5.814123293225	27.658963520941	0.343741353422	1.409902480436
30	0.100000075000	0.200000000000	0.001456936884	5.837241597813	28.166565209665	0.343594350222	1.431981294796
31	0.100000075000	0.200000000000	0.000990925160	5.853035763488	28.513352974215	0.343491395150	1.447155102512
32	0.100000075000	0.200000000000	0.000673995544	5.863809954454	28.749918657846	0.343419952273	1.457548796387
33	0.100000075000	0.200000000000	0.000458453012	5.871152714312	28.911141436503	0.343370690210	1.464652341518
34	0.100000075000	0.200000000000	0.000311856955	5.876153823238	29.020949287094	0.343336869119	1.469499930702
35	0.100000075000	0.200000000000	0.000212148508	5.879558667834	29.095708439967	0.343313717525	1.472804643091
36	0.100000075000	0.200000000000	0.000144328343	5.881876122794	29.146592104339	0.343297901280	1.475055984909
37	0.100000075000	0.200000000000	0.000098196696	5.883453178306	29.181219039792	0.343287110972	1.476588997029
38	0.100000075000	0.200000000000	0.000066816997	5.884526252980	29.204780218975	0.343279756325	1.477632542709
39	0.100000075000	0.200000000000	0.000045471536	5.885256344217	29.220810613557	0.343274746424	1.478342747161
40	0.100000075000	0.200000000000	0.000030951522	5.885753051540	29.231716667469	0.343271335443	1.478826019004

YR	TAX	BEN	NT	UTILITY	WEL GAIN	PS	OS
41	0.100000075000	0.200000000000	0.000021074360	5.886090966713	29.239136169691	0.343269013672	1.479154837423
42	0.100000075000	0.200000000000	0.000014355440	5.886320847968	29.244183603552	0.343267433608	1.479378550433
43	0.100000075000	0.200000000000	0.000009784891	5.886477231806	29.247617276649	0.343266358452	1.479530747493
44	0.100000075000	0.200000000000	0.000006675764	5.886583615498	29.249953111499	0.343265626928	1.479634287365
45	0.100000075000	0.200000000000	0.000004560771	5.886655984864	29.251542103983	0.343265129239	1.479704724179
46	0.100000075000	0.200000000000	0.000003122037	5.886705215386	29.252623043806	0.343264790785	1.479752640721
47	0.100000075000	0.200000000000	0.000002143329	5.886738704849	29.253358361930	0.343264560411	1.479785236881
48	0.100000075000	0.200000000000	0.000001477562	5.886761486249	29.253858566288	0.343264403684	1.479807410932
49	0.100000075000	0.200000000000	0.000001024670	5.886776983481	29.254198834378	0.343264297065	1.479822495117
50	0.100000075000	0.200000000000	0.000000716587	5.886787525597	29.254430304457	0.343264224536	1.479832756298
51	0.100000075000	0.200000000000	0.000000507012	5.886794696951	29.254587763721	0.343264175197	1.479839736564
52	0.100000075000	0.200000000000	0.000000364448	5.886799575315	29.254694876516	0.343264141633	1.479844484949
53	0.100000075000	0.200000000000	0.000000267467	5.886802893624	29.254767735618	0.343264118684	1.479847715075
54	0.100000075000	0.200000000000	0.000000201498	5.886805151060	29.254817301480	0.343264103183	1.479849912424
55	0.100000075000	0.200000000000	0.000000156619	5.886806686810	29.254851021471	0.343264092644	1.479851407183
56	0.100000075000	0.200000000000	0.000000126090	5.886807731518	29.254873959819	0.343264085476	1.479852423996
57	0.100000075000	0.200000000000	0.000000105322	5.886808442183	29.254889563666	0.343264080600	1.479853115682
58	0.100000075000	0.200000000000	0.000000091195	5.886808925612	29.254900178185	0.343264077283	1.479853586202
59	0.100000075000	0.200000000000	0.000000081585	5.886809254768	29.254907405359	0.343264075178	1.479853906273
60	0.100000075000	0.200000000000	0.000000075045	5.886809478501	29.254912317789	0.343264073604	1.479854123961

YR	WG	WN	RG	C1	C2	K/L
0	1.858902960201	1.673012664181	3.183815885706	1.283774986616	2.000279370752	0.194619193969
1	1.858900981214	1.673010743675	3.183826054195	1.283772956247	2.000284495417	0.194619193969
2	1.858900430700	1.673010248213	3.183828882868	1.283772601524	2.000283567189	0.194618963423
3	1.858900545150	1.673010351217	3.183828294799	1.283772803560	2.000282068314	0.194619011352
4	1.858901097235	1.673010848094	3.183825458053	1.283773464636	2.000278972492	0.194619242556
5	1.858902355696	1.673011980709	3.183818991785	1.283774937400	2.000272365885	0.194619769580
6	1.858905076598	1.673014429520	3.183805011226	1.283778115334	2.000258165163	0.194620909053
7	1.858910922034	1.673019690412	3.183774976430	1.283784940940	2.000227680099	0.194623357052
8	1.858923475677	1.673030988690	3.183710475055	1.283799598143	2.000162232520	0.194628614447
9	1.858950433317	1.673055250564	3.183571970970	1.283831072192	2.000021714351	0.194639904513
10	1.859008314668	1.673107343776	3.183274612898	1.283898647333	1.999720113359	0.194664147326
11	1.859132572690	1.673219175986	3.182636378206	1.284043694230	1.999073216478	0.194716198749
12	1.859399235651	1.673459172631	3.181267275460	1.284354872635	1.997687569901	0.194827938538
13	1.859971074154	1.673973827241	3.178333984878	1.285021727169	1.994728067586	0.195067718182
14	1.861195354690	1.675075679631	3.172066055934	1.286447404576	1.988446095690	0.195581819743
15	1.863807456324	1.677426570906	3.158747912028	1.289480079168	1.975285195133	0.196682093478
16	1.869339864820	1.682405738138	3.130785396170	1.295863353798	1.948455788737	0.199027786777
17	1.880878426852	1.692790443101	3.073519172904	1.309011718928	1.896700986650	0.203987494578
18	1.904196580568	1.713776779696	2.961984349862	1.334975625928	1.806820561411	0.214292892382
19	1.948510152701	1.753658991292	2.764458554631	1.382497297171	1.676349790574	0.234947774184
20	2.023996872215	1.821597033194	2.466542433248	1.459547084988	1.530085340872	0.273526866993
21	2.132762903726	1.919486453396	2.108095919829	1.565254768679	1.408259726906	0.337233690944
22	2.260144746267	2.034130102130	1.771370230892	1.684698394407	1.330001915714	0.425310061641
23	2.382322245149	2.144089841960	1.512573837539	1.797026585791	1.287091462861	0.525004066143
24	2.483166009895	2.234849222668	1.335675124071	1.888974725992	1.264576045660	0.619703091254
25	2.559084461098	2.303175823057	1.220293345746	1.958030142488	1.252377508437	0.699035323493
26	2.613499924395	2.352149735943	1.145646484408	2.007525660264	1.245384155458	0.760414886548
27	2.651556949503	2.386401055686	1.097021715937	2.042165809088	1.241165881291	0.805683518987
28	2.677848850533	2.410063764641	1.065025295049	2.066115261586	1.238524114362	0.838117449098
29	2.695897257971	2.426307329982	1.043777885855	2.082565976560	1.236825903586	0.860942190385
30	2.708243369056	2.437418829032	1.029568002642	2.093824478810	1.235714526807	0.876821916929
31	2.716671553122	2.445004194060	1.020015325075	2.101512798910	1.234978231355	0.887787822509
32	2.722417982475	2.450175980046	1.013569860953	2.106756027773	1.234486321791	0.895323248831
33	2.726332900664	2.453699406122	1.009209775828	2.110328715912	1.234155793035	0.900484374330
34	2.728998707465	2.456098632044	1.006255141509	2.112761762925	1.233932827854	0.904011515864
35	2.730813351280	2.457731811341	1.004250484306	2.114418093817	1.233782019306	0.906418399910
36	2.732048329153	2.458843291334	1.002889234682	2.115545390054	1.233679829602	0.908059180308
37	2.732888685542	2.459599612022	1.001964361393	2.116312501050	1.233610498482	0.909176943094
38	2.733460460579	2.460114209512	1.001335731432	2.116834453187	1.233563420590	0.909938054000
39	2.733849468195	2.460464316337	1.000908343085	2.117189569913	1.233531434288	0.910456149517
40	2.734114117930	2.460702501078	1.000617721177	2.117431165635	1.233509694131	0.910808746792

YR	WG	WN	RG	C1	C2	K/L
41	2.734294159126	2.460864538141	1.000420075416	2.117595524469	1.233494913692	0.911048677223
42	2.734416638572	2.460974769633	1.000285649587	2.117707336025	1.233484863101	0.911211925547
43	2.734499958413	2.461049757484	1.000194216670	2.117783399032	1.233478027926	0.911322992020
44	2.734556638371	2.461100769442	1.000132024002	2.117835142514	1.233473379089	0.911398552972
45	2.734595195776	2.461135471104	1.000089719439	2.117870341865	1.233470217076	0.911449957147
46	2.734621424923	2.461159077335	1.000060942605	2.117894286550	1.233468066787	0.911484926709
47	2.734639267619	2.461175135759	1.000041367445	2.117910575348	1.233466603642	0.911508715753
48	2.734651405210	2.461186059590	1.000028051628	2.117921655907	1.233465608341	0.911524898646
49	2.734659661901	2.461193490611	1.000018993575	2.117929193546	1.233464931297	0.911535907308
50	2.734665278578	2.461198545620	1.000012831828	2.117934321084	1.233464470740	0.911543396091
51	2.734669099363	2.461201984327	1.000008640283	2.117937809130	1.233464157448	0.911548490417
52	2.734671698479	2.461204323531	1.000005788969	2.117940181898	1.233463944330	0.911551955881
53	2.734673466546	2.461205914791	1.000003849349	2.117941796107	1.233463798906	0.911554313291
54	2.734674669241	2.461206997217	1.000002529961	2.117942894034	1.233463700404	0.911555916880
55	2.734675487435	2.461207733591	1.000001632381	2.117943640947	1.233463633421	0.911557007803
56	2.734676044019	2.461208234516	1.000001021796	2.117944149041	1.233463587858	0.911557749914
57	2.734676422636	2.461208575272	1.000000606445	2.117944494672	1.233463556864	0.911558254736
58	2.734676680190	2.461208807070	1.000000323902	2.117944729788	1.233463535780	0.911558598141
59	2.734676855391	2.461208964751	1.000000131703	2.117944889573	1.233463522024	0.911558831742
60	2.734676974629	2.461209072065	1.000000000896	2.117944998461	1.233463512115	0.911558990726
			0.999999911991			0.911559098783

PARAMETERS

```
CAPITALS SHARE               (ALPHA) =   0.2500
POPULATION GROWTH               (ZN) =   1.0000
INTERTEMPORAL ELASTICITY     (GAMMA) =   0.5000
SUBJ RATE OF TIME DISC         (RHO) =   1.3500
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STEADY STATE VALUES.

```
(PAY-AS-YOU-GO)
SOC SEC TAX       =  0.100000
SOC SEC BEN       =  0.200000
NET WAGE          =  1.616411
WAGES             =  1.796012
INTEREST RATE     =  3.378583
SCALE             =  3.690920
CAPITAL/LABOR     =  0.177195
GOLDEN RULE K/L   =  0.898324
UTILITY           =  0.696781
SOC UTILITY       =  2.090342
C1                =  1.262021
C2                =  1.910926
PVCONS            =  1.698447
PVINC             =  1.698447
END STEADY STATE COMPUTATIONS.  (ITER   =  23)
```

DYNAMIC SOLUTION

YR	TAX	BEN	NT	UTILITY	WEL GAIN	PS	GS
0	0.100000000000	0.200000000000	0.054323120489	0.696780726792	0.000000000000	0.354389449990	0.000000000000
1	0.100000071719	0.200000000000	0.054323304650	0.696780559084	-0.000024068866	0.354388837022	0.000000128808
2	0.100000071719	0.200000000000	0.054323388952	0.696780040418	-0.000098506458	0.354388542510	0.000000267610
3	0.100000071719	0.200000000000	0.054323400567	0.696779864238	-0.000123791241	0.354388535889	0.000000694958
4	0.100000071719	0.200000000000	0.054323369185	0.696779784640	-0.000135214893	0.354388733929	0.000001703585
5	0.100000071719	0.200000000000	0.054323279180	0.696779706763	-0.000146391613	0.354389253148	0.000004011339
6	0.100000071719	0.200000000000	0.054323068319	0.696779561990	-0.000167168985	0.354390457350	0.000009268943
7	0.100000071719	0.200000000000	0.054322586343	0.696779243890	-0.000212821794	0.354393205683	0.000021239639
8	0.100000071719	0.200000000000	0.054321488492	0.696778523759	-0.000316173075	0.354399464338	0.000048491629
9	0.100000071719	0.200000000000	0.054318989130	0.696776888487	-0.000550862601	0.354413710967	0.000110526689
10	0.100000071719	0.200000000000	0.054313300247	0.696773185887	-0.001082249354	0.354446129541	0.000251714277
11	0.100000071719	0.200000000000	0.054300355872	0.696764856429	-0.002277669565	0.354519851183	0.000572914512
12	0.100000071719	0.200000000000	0.054270923798	0.696746407773	-0.004925368521	0.354687253584	0.001302954147
13	0.100000071719	0.200000000000	0.054204112850	0.696707037086	-0.010575738214	0.355066127178	0.002958685251
14	0.100000071719	0.200000000000	0.054053014134	0.696630572670	-0.021549694985	0.355917307186	0.006695825325
15	0.100000071719	0.200000000000	0.053714130529	0.696519665159	-0.037466827433	0.357798932819	0.015040491629
16	0.100000071719	0.200000000000	0.052967957788	0.696543384953	-0.034062629657	0.361820295129	0.033239618701
17	0.100000071719	0.200000000000	0.051388237078	0.697593549736	0.116654051016	0.369875231047	0.071029345778
18	0.100000071719	0.200000000000	0.048294605290	0.702422316383	0.809664988518	0.384353023524	0.142508652477
19	0.100000071719	0.200000000000	0.043010722065	0.715636199055	2.706084071856	0.406685044285	0.258698921573
20	0.100000071719	0.200000000000	0.035617068227	0.739474901419	6.127347239283	0.435306969839	0.413730114857
21	0.100000071719	0.200000000000	0.027391481840	0.769737032800	10.470482779302	0.465509186733	0.583017928007
22	0.100000071719	0.200000000000	0.019925028420	0.799305512695	14.714067419149	0.492403432822	0.740195010003
23	0.100000071719	0.200000000000	0.014036699970	0.823741339596	18.221028212009	0.513642422606	0.870778314243
24	0.100000071719	0.200000000000	0.009743086725	0.842140674069	20.861648677677	0.529278220768	0.972003718907
25	0.100000071719	0.200000000000	0.006725018613	0.855368646987	22.760089953320	0.540388532568	1.047251044900
26	0.100000071719	0.200000000000	0.004634524240	0.864677707199	24.096099956827	0.548156650413	1.101772224179
27	0.100000071719	0.200000000000	0.003193617296	0.871166228924	25.027314250699	0.553550223695	1.140644759568
28	0.100000071719	0.200000000000	0.002201516225	0.875668953093	25.673532493562	0.557284016863	1.168072824037
29	0.100000071719	0.200000000000	0.001518272744	0.878787003193	26.121026228648	0.559865495984	1.187292937417
30	0.100000071719	0.200000000000	0.001047470258	0.880943822379	26.430566820579	0.561649265730	1.200699250695
31	0.100000071719	0.200000000000	0.000722873924	0.882434828721	26.644551835444	0.562881498834	1.210021110165
32	0.100000071719	0.200000000000	0.000498977686	0.883465188988	26.792426228004	0.563732617078	1.216489082528
33	0.100000071719	0.200000000000	0.000344487537	0.884177061694	26.894592186109	0.564320454841	1.220970326480
34	0.100000071719	0.200000000000	0.000237861034	0.884668822671	26.965168331245	0.564726439037	1.224071977167
35	0.100000071719	0.200000000000	0.000164255813	0.885008499223	27.013917749751	0.565006822086	1.226217272903
36	0.100000071719	0.200000000000	0.000113438931	0.885243111288	27.047588610042	0.565200459409	1.227700387877
37	0.100000071719	0.200000000000	0.000078351896	0.885405149462	27.070843870556	0.565334187649	1.228725378827
38	0.100000071719	0.200000000000	0.000054124168	0.885517060170	27.086904979034	0.565426541534	1.229433596889
39	0.100000071719	0.200000000000	0.000037394096	0.885594349214	27.097997284192	0.565490321734	1.229922864176
40	0.100000071719	0.200000000000	0.000025841060	0.885647726753	27.105657877555	0.565534368672	1.230260834503

YR	TAX	BEN	NT	UTILITY	WEL GAIN	PS	GS
41	0.100000071719	0.200000000000	0.000017862883	0.885684590139	27.110948406615	0.565564787677	1.230494276327
42	0.100000071719	0.200000000000	0.000012353316	0.885710048430	27.114602108526	0.565585795153	1.230655510267
43	0.100000071719	0.200000000000	0.000008548483	0.885727630152	27.117125387510	0.565600302984	1.230766867612
44	0.100000071719	0.200000000000	0.000005920896	0.885739772209	27.118867981268	0.565610322135	1.230843775456
45	0.100000071719	0.200000000000	0.000004106299	0.885748157580	27.120071426022	0.565617241389	1.230896890197
46	0.100000071719	0.200000000000	0.000002853143	0.885753948555	27.120902530372	0.565622019843	1.230933572317
47	0.100000071719	0.200000000000	0.000001987715	0.885757947825	27.121476494215	0.565625319856	1.230958905530
48	0.100000071719	0.200000000000	0.000001390050	0.885760709734	27.121872875576	0.565627598854	1.230976400924
49	0.100000071719	0.200000000000	0.000000977303	0.885762617116	27.122146617752	0.565629172735	1.230988483388
50	0.100000071719	0.200000000000	0.000000692259	0.885763934360	27.122335664872	0.565630259661	1.230996827615
51	0.100000071719	0.200000000000	0.000000495407	0.885764844052	27.122466221320	0.565631010294	1.231002590182
52	0.100000071719	0.200000000000	0.000000359461	0.885765472288	27.122556383939	0.565631528684	1.231006569835
53	0.100000071719	0.200000000000	0.000000265576	0.885765906149	27.122618650469	0.565631886685	1.231009318198
54	0.100000071719	0.200000000000	0.000000200740	0.885766205775	27.122661651887	0.565632133921	1.231011216226
55	0.100000071719	0.200000000000	0.000000155963	0.885766412770	27.122691359309	0.565632304661	1.231012527011
56	0.100000071719	0.200000000000	0.000000125040	0.885766555648	27.122711864742	0.565632422576	1.231013432242
57	0.100000071719	0.200000000000	0.000000103685	0.885766654321	27.122726025891	0.565632504009	1.231014057398
58	0.100000071719	0.200000000000	0.000000088937	0.885766722464	27.122735805618	0.565632560247	1.231014489132
59	0.100000071719	0.200000000000	0.000000078752	0.885766769524	27.122742559520	0.565632599084	1.231014787288
60	0.100000071719	0.200000000000	0.000000071718	0.885766802023	27.122747223778	0.565632625906	1.231014993195

YR	WG	WN	RG	C1	C2	K/L
0	1.796012076077	1.616410868470	3.378582822243	1.262021418479	1.910925973328	0.177195048027
1	1.796010166143	1.616409020721	3.378593600933	1.262020183699	1.910929589974	0.177195048027
2	1.796008734178	1.616407731953	3.378601682235	1.262019189443	1.910928407637	0.177194482915
3	1.796008536897	1.616407554400	3.378602795594	1.262019018511	1.910927275698	0.177194405060
4	1.796009069949	1.616408034146	3.378599787310	1.262019300217	1.910925192665	0.177194615424
5	1.796010598763	1.616409410079	3.378591159444	1.262020156931	1.910920610287	0.177195218757
6	1.796014180448	1.616412633595	3.378570946309	1.262022176245	1.910910223461	0.177196632244
7	1.796022367268	1.616420001733	3.378524744836	1.262026796050	1.910886600736	0.177199863147
8	1.796041015204	1.616436784874	3.378419510281	1.262037320536	1.910832835732	0.177207222661
9	1.796083468580	1.616474992909	3.378179952501	1.262061281941	1.910710483547	0.177223977984
10	1.796180095760	1.616561957364	3.377634784827	1.262115827822	1.910432227813	0.177262118828
11	1.796399947221	1.616759823664	3.376394826426	1.262239972481	1.909800249995	0.177348921909
12	1.796899768148	1.617209662462	3.373578102673	1.262522408878	1.908369263771	0.177546382847
13	1.798034040253	1.618230507275	3.367197563690	1.263164380097	1.905151380655	0.177995103865
14	1.800597645816	1.620537752098	3.352835861855	1.264620444912	1.898026282043	0.179012406214
15	1.806339082973	1.625705045127	3.320966471830	1.267906112308	1.882776860636	0.181306566256
16	1.818942011151	1.637047679584	3.252413580532	1.275227384455	1.852398320772	0.186419712224
17	1.845457880893	1.660911960449	3.114224478006	1.291036729403	1.799792263096	0.197529956915
18	1.896813134630	1.707131685130	2.868062969224	1.322778661605	1.728221759564	0.220452288413
19	1.983181894103	1.784863562461	2.509427659969	1.378178518176	1.659966830187	0.263430838001
20	2.102357068166	1.892121210571	2.106409908706	1.456814240732	1.619522299089	0.332691982929
21	2.234447018826	2.011002156692	1.754495030588	1.545492969959	1.609571726659	0.424518542348
22	2.355501108505	2.119950828721	1.497657094663	1.627547395899	1.616712411883	0.524263557370
23	2.452696437116	2.207426617500	1.326572305518	1.693784194895	1.628716953463	0.616299221413
24	2.524965337173	2.272468622368	1.215894980462	1.743190401601	1.639869335303	0.692210368424
25	2.576641287501	2.318976973958	1.144196045352	1.778588441390	1.648627539801	0.750640969837
26	2.612921579882	2.351629234498	1.097193433978	1.803472584085	1.655063867658	0.793819788734
27	2.638181692576	2.374363334112	1.065977947374	1.820813110416	1.659657606001	0.824964437296
28	2.655701411299	2.390131079706	1.045019980004	1.832847062843	1.662890560356	0.847097491632
29	2.667829859514	2.401046682229	1.030832113981	1.841181186246	1.665149149565	0.862678420450
30	2.676217884995	2.408595904560	1.021169701932	1.846946638830	1.666720448246	0.873579216700
31	2.682015885505	2.413814104603	1.014561293452	1.850932605770	1.667810835093	0.881174258212
32	2.686022314460	2.417419890376	1.010028150381	1.853687273298	1.668566286135	0.886451304499
33	2.688790215936	2.419911001506	1.006912122811	1.855590546664	1.669089137336	0.890110849803
34	2.690702220460	2.421631805440	1.004767122014	1.856905366403	1.669450753751	0.892645390661
35	2.692022881401	2.422820400192	1.003289082032	1.857813578106	1.669700739372	0.894399208102
36	2.692935039077	2.423641342036	1.002269917580	1.858440882627	1.669873499710	0.895612047495
37	2.693565027367	2.424208331451	1.001566830070	1.858874143802	1.669992865700	0.896450423643
38	2.694000122449	2.424599916993	1.001081633627	1.859173375460	1.670075326707	0.897029783238
39	2.694300611372	2.424870357003	1.000746726238	1.859380035269	1.670132287340	0.897430069212
40	2.694508135068	2.425057128315	1.000515519664	1.859522759642	1.670171630615	0.897706592955

YR	WG	WN	RG	C1	C2	K/L
41	2.694651453959	2.425186115306	1.000355886608	1.859621327630	1.670198804094	0.897897601588
42	2.694750431531	2.425275195113	1.000245662123	1.859689399961	1.670217571554	0.898029532002
43	2.694818786249	2.425336714355	1.000169549648	1.859736411371	1.670230533069	0.898120652710
44	2.694865992452	2.425379199935	1.000116990396	1.859768877800	1.670239484639	0.898183585298
45	2.694898593298	2.425408540693	1.000080694858	1.859791299305	1.670245666764	0.898227048795
46	2.694921107580	2.425428803546	1.000055630030	1.859806783702	1.670249936226	0.898257065793
47	2.694936656026	2.425442797146	1.000038320657	1.859817477290	1.670252884761	0.898277796080
48	2.694947393834	2.425452461172	1.000026366981	1.859824862318	1.670254921043	0.898292112693
49	2.694954809395	2.425459135177	1.000018111848	1.859829962442	1.670256327311	0.898301999889
50	2.694959930603	2.425463744263	1.000012410883	1.859833484602	1.670257298487	0.898308828061
51	2.694963467322	2.425466927310	1.000008473807	1.859835917016	1.670257969186	0.898313543638
52	2.694965909789	2.425469125531	1.000005754865	1.859837596847	1.670258432372	0.898316800238
53	2.694967596563	2.425470643627	1.000003877162	1.859838756943	1.670258752250	0.898319049259
54	2.694968761454	2.425471692029	1.000002580419	1.859839558107	1.670258973159	0.898320602441
55	2.694969565930	2.425472416057	1.000001684886	1.859840111396	1.670259126162	0.898321675074
56	2.694970121502	2.425472916072	1.000001066430	1.859840493496	1.670259231383	0.898322415836
57	2.694970505182	2.425473261384	1.000000639323	1.859840757375	1.670259304049	0.898322927409
58	2.694970770153	2.425473499858	1.000000344361	1.859840939611	1.670259354233	0.898323280703
59	2.694970953142	2.425473664548	1.000000140660	1.859841065464	1.670259388890	0.898323524689
60	2.694971079515	2.425473778284	0.999999999984	1.859841152378	1.670259412824	0.898323693186
			0.999999902833			0.898323809551

PARAMETERS

```
CAPITALS SHARE              (ALPHA) =   0.2500
POPULATION GROWTH              (ZN) =   1.0000
INTERTEMPORAL ELASTICITY    (GAMMA) =   0.5000
SUBJ RATE OF TIME DISC        (RHO) =   2.0000
```

STEADY STATE VALUES.

```
(PAY-AS-YOU-GO)
SOC SEC TAX       =  0.100000
SOC SEC BEN       =  0.200000
NET WAGE          =  1.603996
WAGES             =  1.782218
INTEREST RATE     =  4.010374
SCALE             =  3.830327
CAPITAL/LABOR     =  0.148133
GOLDEN RULE K/L   =  0.943846
UTILITY           =  0.796992
SOC UTILITY       =  2.390975
C1                =  1.307730
C2                =  1.840846
PVCONS            =  1.675137
PVINC             =  1.675137
END STEADY STATE COMPUTATIONS.  (ITER   =  23)
```

DYNAMIC SOLUTION

YR	TAX	BEN	NT	UTILITY	WEL GAIN	PS	GS
0	0.100000000000	0.200000000000	0.060082818531	0.796991671018	0.000000000000	0.296265778081	0.000000000000
1	0.100000032500	0.200000000000	0.060082973955	0.796991400124	-0.000033989549	0.296265164047	0.000000057922
2	0.100000032500	0.200000000000	0.060083071616	0.796990693379	-0.000122666108	0.296264832249	0.000000021440
3	0.100000032500	0.200000000000	0.060083101404	0.796990463438	-0.000151517199	0.296264735136	0.000000056247
4	0.100000032500	0.200000000000	0.060083106443	0.796990385326	-0.000161318108	0.296264729854	0.000000189462
5	0.100000032500	0.200000000000	0.060083096095	0.796990348612	-0.000165924637	0.296264796732	0.000000551803
6	0.100000032500	0.200000000000	0.060083061379	0.796990307774	-0.000171048712	0.296264997705	0.000001504849
7	0.100000032500	0.200000000000	0.060082968040	0.796990218435	-0.000182258196	0.296265532234	0.000004001413
8	0.100000032500	0.200000000000	0.060082722879	0.796989989152	-0.000211026714	0.296266934663	0.000010538020
9	0.100000032500	0.200000000000	0.060082080762	0.796989392368	-0.000285906352	0.296270606758	0.000027651049
10	0.100000032500	0.200000000000	0.060080399684	0.796987832374	-0.000481641558	0.296280219391	0.000072450412
11	0.100000032500	0.200000000000	0.060075999209	0.796983766787	-0.000991758174	0.296305374697	0.000189710376
12	0.100000032500	0.200000000000	0.060064483725	0.796973244048	-0.002312065513	0.296371155398	0.000496508539
13	0.100000032500	0.200000000000	0.060034371931	0.796946519522	-0.005665240572	0.296542841669	0.001298369033
14	0.100000032500	0.200000000000	0.059955787912	0.796882106883	-0.013747212010	0.296988719294	0.003388403606
15	0.100000032500	0.200000000000	0.059751752517	0.796749695540	-0.030361104935	0.298132138070	0.008797466550
16	0.100000032500	0.200000000000	0.059228918019	0.796622009856	-0.046382060836	0.300975182805	0.022545168245
17	0.100000032500	0.200000000000	0.057932329547	0.797392224021	0.050258116578	0.307575058598	0.055974663999
18	0.100000032500	0.200000000000	0.054952339668	0.803143179266	0.771840970468	0.321053586497	0.129647929501
19	0.100000032500	0.200000000000	0.049080085287	0.822874928970	3.247619629417	0.343845519250	0.264964031429
20	0.100000032500	0.200000000000	0.040059054486	0.862091325941	8.168172553243	0.374478546186	0.457494592783
21	0.100000032500	0.200000000000	0.029792562874	0.911998583617	14.430127287595	0.406729621180	0.668959634732
22	0.100000032500	0.200000000000	0.020782571283	0.958388632093	20.250771362431	0.434269955509	0.859764478063
23	0.100000032500	0.200000000000	0.014065783993	0.994327030363	24.760027804803	0.454806616223	1.012123882132
24	0.100000032500	0.200000000000	0.009428797004	1.019841773993	27.961409269246	0.469122318823	1.125695087123
25	0.100000032500	0.200000000000	0.006313960294	1.037330516992	30.155753781881	0.478840512259	1.207174784632
26	0.100000032500	0.200000000000	0.004234511508	1.049174201624	31.641802515206	0.485385025023	1.264353048708
27	0.100000032500	0.200000000000	0.002845095693	1.057167448757	32.644729825948	0.489786463606	1.303943023831
28	0.100000032500	0.200000000000	0.001914472501	1.062558634732	33.321171772753	0.492748354037	1.331123623596
29	0.100000032500	0.200000000000	0.001289707307	1.066195325049	33.777473946257	0.494743326102	1.349682312380
30	0.100000032500	0.200000000000	0.000869523635	1.068649197416	34.085365792026	0.496088085590	1.362308287069
31	0.100000032500	0.200000000000	0.000586562748	1.070305384881	34.293170656877	0.496995086639	1.370877456673
32	0.100000032500	0.200000000000	0.000395836586	1.071423413310	34.433451725093	0.497607088648	1.376683963125
33	0.100000032500	0.200000000000	0.000267198904	1.072178258885	34.528163577483	0.498020159345	1.380614239818
34	0.100000032500	0.200000000000	0.000180400071	1.072687949680	34.592115412052	0.498299017166	1.383272617747
35	0.100000032500	0.200000000000	0.000121814926	1.073032129544	34.635300287870	0.498487295605	1.385069826314
36	0.100000032500	0.200000000000	0.000082264782	1.073264555524	34.664463199998	0.498614428719	1.386284438009
37	0.100000032500	0.200000000000	0.000055561322	1.073421518742	34.684157661419	0.498700279478	1.387105129895
38	0.100000032500	0.200000000000	0.000037530026	1.073527522345	34.697458127030	0.498758255461	1.387659574096
39	0.100000032500	0.200000000000	0.000025353782	1.073599111925	34.706440602369	0.498797408426	1.388034108480
40	0.100000032500	0.200000000000	0.000017131015	1.073647460466	34.712506982105	0.498823850144	1.388287094158

YR	TAX	BEN	NT	UTILITY	WEL GAIN	PS	GS
41	0.100000032500	0.200000000000	0.000011577922	1.073680113300	34.716603992779	0.498841707640	1.388457969812
42	0.100000032500	0.200000000000	0.000007827671	1.073702165806	34.719370960970	0.498853767852	1.388573381805
43	0.100000032500	0.200000000000	0.000005294927	1.073717059341	34.721239679971	0.498861912872	1.388651331170
44	0.100000032500	0.200000000000	0.000003584416	1.073727117965	34.722501753847	0.498867413740	1.388703977496
45	0.100000032500	0.200000000000	0.000002429200	1.073733911252	34.723354119960	0.498871128847	1.388739534028
46	0.100000032500	0.200000000000	0.000001649006	1.073738499234	34.723929782531	0.498873637915	1.388763548217
47	0.100000032500	0.200000000000	0.000001122088	1.073741597823	34.724318568059	0.498875332463	1.388779766861
48	0.100000032500	0.200000000000	0.000000766225	1.073743690519	34.724581142458	0.498876476910	1.388790720538
49	0.100000032500	0.200000000000	0.000000525885	1.073745103865	34.724758477596	0.498877249836	1.388798118368
50	0.100000032500	0.200000000000	0.000000363567	1.073746058398	34.724878244631	0.498877771848	1.388803114666
51	0.100000032500	0.200000000000	0.000000253942	1.073746703063	34.724959131843	0.498878124400	1.388806489029
52	0.100000032500	0.200000000000	0.000000179905	1.073747138450	34.725013760744	0.498878362503	1.388808767980
53	0.100000032500	0.200000000000	0.000000129902	1.073747432499	34.725050655538	0.498878523311	1.388810307120
54	0.100000032500	0.200000000000	0.000000096131	1.073747631091	34.725075573226	0.498878631916	1.388811346612
55	0.100000032500	0.200000000000	0.000000073324	1.073747765214	34.725092401920	0.498878705265	1.388812048655
56	0.100000032500	0.200000000000	0.000000057920	1.073747855733	34.725103759532	0.498878754797	1.388812522795
57	0.100000032500	0.200000000000	0.000000047517	1.073747916930	34.725111438042	0.498878788255	1.388812843017
58	0.100000032500	0.200000000000	0.000000040491	1.073747958262	34.725116624001	0.498878810852	1.388813059286
59	0.100000032500	0.200000000000	0.000000035746	1.073747986176	34.725120126459	0.498878826113	1.388813205347
60	0.100000032500	0.200000000000	0.000000032541	1.073748005029	34.725122491914	0.498878836420	1.388813303993

YR	WG	WN	RG	C1	C2	K/L
0	1.782217829711	1.603996046740	4.010373794890	1.307730268659	1.840845856761	0.148133214711
1	1.782215544101	1.603993931769	4.010389224274	1.307728767722	1.840850524266	0.148133214711
2	1.782213728215	1.603992297471	4.010401482769	1.307727465222	1.840849607604	0.148132610984
3	1.782213174358	1.603991799000	4.010405221697	1.307727063864	1.840849197070	0.148132426844
4	1.782213080657	1.603991714669	4.010405854247	1.307726984815	1.840848770533	0.148132395692
5	1.782213273054	1.603991887827	4.010404555424	1.307727091095	1.840847851399	0.148132459658
6	1.782213918557	1.603992468779	4.010400197822	1.307727471074	1.840845517486	0.148132674267
7	1.782215654084	1.603994030754	4.010388481814	1.307728498520	1.840839428780	0.148133251277
8	1.782220212514	1.603998133340	4.010357709476	1.307731198677	1.840823485597	0.148134766823
9	1.782232151785	1.604008878684	4.010277113198	1.307738271926	1.840781762910	0.148138736341
10	1.782263408753	1.604037009954	4.010066122661	1.307756790563	1.840672561739	0.148149128903
11	1.782345226507	1.604110645930	4.009513907179	1.307805271233	1.840386928313	0.148176334901
12	1.782559318931	1.604303329104	4.008069405251	1.307932173706	1.839640889519	0.148247542536
13	1.783119050968	1.604807087920	4.004296117040	1.308264246251	1.837699779375	0.148433831969
14	1.784579145181	1.606121172664	3.994475537209	1.309132453370	1.832699240271	0.148920605351
15	1.788365691317	1.609529064063	3.969156448063	1.311396925993	1.820140586652	0.150188561450
16	1.798039843846	1.618235801025	3.905433725475	1.317260618221	1.790516536434	0.153464802310
17	1.821860070663	1.639674004386	3.754241043875	1.332098945788	1.729925009816	0.161760175525
18	1.875774254301	1.688196767908	3.439739028270	1.367143181411	1.636167889045	0.181774861299
19	1.979301122841	1.781370946230	2.927733843922	1.437525426980	1.543140536203	0.225350757999
20	2.133830804953	1.920447655108	2.336615561274	1.545969108923	1.493543095675	0.304404775339
21	2.307102780202	2.076392427201	1.848699733313	1.669662806021	1.488289165457	0.415986569484
22	2.460148506575	2.214133575963	1.524695905783	1.779863620454	1.502732565703	0.537844627956
23	2.576475840881	2.318828173057	1.327360784246	1.864021556835	1.519602139346	0.647017216786
24	2.658532664962	2.392679312064	1.208206600083	1.923556993241	1.533183716775	0.733465249177
25	2.714671965890	2.443204681074	1.134789038448	1.964364168815	1.542961348990	0.797408702973
26	2.752675350951	2.477407726394	1.088434312558	1.992022701372	1.549732930544	0.843007648445
27	2.778323912456	2.500491430915	1.058567547592	2.010704967309	1.554357074221	0.874869036865
28	2.795624599003	2.516062048245	1.039036123185	2.023313694207	1.557497385888	0.896864743718
29	2.807295981581	2.526566292186	1.026130494541	2.031822966084	1.559624809929	0.911935988816
30	2.815171803137	2.533654531330	1.017542351903	2.037566445740	1.561064279078	0.922212819241
31	2.820487657522	2.538438800104	1.011799814441	2.041443713464	1.562037592257	0.929198186330
32	2.824076298104	2.541668576511	1.007947538396	2.044061487863	1.562695437460	0.933936271656
33	2.826499248048	2.543849231382	1.005357641893	2.045829072037	1.563139948220	0.937145525887
34	2.828135308559	2.545321685788	1.003613869670	2.047022668623	1.563440257137	0.939317199581
35	2.829240104967	2.546316002520	1.002438618323	2.047828706915	1.563643121874	0.940785817456
36	2.829986183120	2.546987472834	1.001645999189	2.048373044115	1.563780151147	0.941778560960
37	2.830490031042	2.547440935947	1.001111193464	2.048740656469	1.563872705981	0.942449433364
38	2.830830300924	2.547747178830	1.000750231830	2.048988923369	1.563935219044	0.942902704686
39	2.831060102806	2.547954000516	1.000506553903	2.049156592090	1.563977440480	0.943208914778
40	2.831215301382	2.548093679230	1.000342028780	2.049269829086	1.564005956496	0.943415758453

YR	WG	WN	RG	C1	C2	K/L
41	2.831320116675	2.548188012990	1.000230935096	2.049346305350	1.564025216197	0.943555472151
42	2.831390905296	2.548251722746	1.000155915574	2.049397954894	1.564038223426	0.943649838726
43	2.831438713615	2.548294750232	1.000105254086	2.049432837359	1.564047008216	0.943713574829
44	2.831471001855	2.548323809647	1.000071040841	2.049456395907	1.564052941250	0.943756622021
45	2.831492808350	2.548343435491	1.000047935139	2.049472306644	1.564056948269	0.943785695618
46	2.831507535806	2.548356690201	1.000032330641	2.049483052286	1.564059654502	0.943805331437
47	2.831517482296	2.548365642042	1.000021792005	2.049490309579	1.564061482219	0.943818593066
48	2.831524199866	2.548371687855	1.000014674601	2.049495210944	1.564062716608	0.943827549662
49	2.831528736718	2.548375771021	1.000009867751	2.049498521185	1.564063550280	0.943833598724
50	2.831531800777	2.548378528674	1.000006621361	2.049500756826	1.564064113318	0.943837684102
51	2.831533870154	2.548380391114	1.000004428850	2.049502266714	1.564064493578	0.943840443257
52	2.831535267752	2.548381648952	1.000002948095	2.049503286449	1.564064750395	0.943842306714
53	2.831536211650	2.548382498460	1.000001948037	2.049503975149	1.564064923842	0.943843565242
54	2.831536849132	2.548383072193	1.000001272627	2.049504440277	1.564065040983	0.943844415216
55	2.831537279668	2.548383459677	1.000000816475	2.049504754411	1.564065120097	0.943844989264
56	2.831537570441	2.548383721372	1.000000508403	2.049504966574	1.564065173181	0.943845376960
57	2.831537766818	2.548383898111	1.000000300342	2.049505109856	1.564065209378	0.943845638796
58	2.831537899448	2.548384017478	1.000000159821	2.049505206626	1.564065233826	0.943845815636
59	2.831537989022	2.548384098095	1.000000064917	2.049505271982	1.564065250337	0.943845935069
60	2.831538049518	2.548384152542	1.000000000822	2.049505316121	1.564065261489	0.943846015730
			0.999999957534			0.943846070206

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