EVALUATING RISK-ADJUSTED DISCOUNT RATES IN FOREST INVESTMENT DECISION MAKING

by

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(ABSTRACT)

One approach to risk in investment evaluation is to discount expected cash flows with a single risk-adjusted discount rate. When emphasis is placed on total (as opposed to systematic) risk there are no *a priori* criteria guiding the proper selection of the risk-adjusted discount rate. It is unlikely that a single rate will capture the risk differences between the investment alternatives considered.

This study evaluates risk-adjusted discount rates in the context of stand level investment decisions. The investment setting is a non-diversified risk averse individual facing mutually exclusive opportunities in forage hay, pine plantation, and mixed pine-hardwood management. These opportunities contrast differences in cash flow, objectives, capital requirement, and presumably risk. Risk-adjusted discount rate bias was defined as the tendency to incorrectly identify a suboptimal alternative as being the most preferred. The correct ranking and valuation of alternatives was conducted using an expected utility approach to risk. The scope of the analysis was to assess to what degree, if any, does risk-adjusted discount rate bias occur in an actual stand level investment setting. Therefore, the numerical results in the analysis pertain to a case study example and are not general enough to make definitive conclusions about the overall riskiness of forestry and hay investments.

The potential for risk-adjusted discount rate bias was demonstrated in a hypothetical investment context. However, when risk was empirically estimated through simulation, risk-adjusted discount rate bias was less pronounced in the ranking of alternatives. Instead, the influential parameter was the risk free discount rate. Based on an objective simulation of risk, which only accounted for historical variability in yields and prices, the estimated correct risk premiums in the discount rate were imperceptibly small, especially in the context of measurement error in specifying the risk free discount rate. The implication is not that risk can be ignored, but that treating risk *via* the risk-adjusted discount rate approach is inadequate. More general approaches to risk are recommended, implying much research is still needed in this area.

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INTRODUCTION

PROBLEM IDENTIFICATION

In general, the purpose of any stand level forest investment analysis is to evaluate the desirability of one silvicultural strategy versus another. Ultimately, the analysis must estimate the decision consequences in terms of the income flows produced from adopting different strategies. However, actual consequences will be determined by how the future unfolds. Therefore, decision consequences are uncertain. The economic environment that determines the future value of the timber is difficult to forecast, and the exact responses to silvicultural treatments, as well as harvest yields, are not known with certainty. Furthermore, regeneration establishment is subject to losses, while mature stands are subject to threats from wind, fire, and pests. Finally, differences between investment strategies may only focus on one aspect of management such as determining the rotation age of the stand, or the timing and composition of a particular type of thinning. Distinguishing the consequences between these marginal decisions becomes blurred under uncertainty since the possible consequences associated with each decision may overlap.

Ironically, the complex nature of forest investment uncertainty has led to analysis techniques that rely on the tidiness found in deterministic models. One common approach toward risk[1] in investment evaluation is to rely on a single adjustment in the discount rate. The idea is simple. Average (i.e., expected) estimates are used to represent each investment's cash flow and the choice of investment is based on a present

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1 While there have been formal distinctions between risk and uncertainty (e.g., see Leuschner, 1984), the terms will be used interchangeably. Uncertainty refers to future decision consequences not being known, while risk connotes a measurable reference to this notion.

value criterion such as net present value (NPV). Adding a risk premium to the discount rate in effect penalizes the entire cash flow for its uncertainty, since the magnitudes of future cash flow components are reduced in present value under the higher rate.[2] The larger the risk adjustment, the more difficult investment acceptance, but those that survive the higher rate are deemed worthy of their underlying risk. In essence, including a risk premium in the discount rate signals an unfavorable attitude toward investment risk.

This attitude is defined as risk aversion on the part of the decision maker. That is, the decision maker recognizes that the expected cash flow is just a summary of the stochastic nature of the investment's cash flow, whereby actual cash flow outcomes fall above or below this mean. Therefore, risk averse individuals are also sensitive to the dispersion of possible cash flow outcomes, and the nature of this dispersion defines an investment's risk. Conceptually, for each cash flow component there exists a certainty equivalent value which, if received with certainty, would give the same level of satisfaction (i.e., utility) as receiving the risky outcome. In this context, risk aversion implies accepting a certainty equivalent value lower than the expected value for each component in the cash flow. That is, a risk averse individual would be indifferent between a less than average outcome with certainty and the expected outcome with its associated risk.

In the strict sense, the risk-adjusted discount rate (RADR) approach parallels the certainty equivalent (CE) concept.[3] On the one hand, the expected cash flow is reduced

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[2] This is especially true when investment costs are certain and immediate, leaving the income flow produced as the uncertain component of the investment; the typical case in even-age forestry investments.

[3] See Clutter *et al.* (1983), or Brigham (1985), for a review of these two approaches to risk. In both reviews, the certainty equivalent approach to risk is cited as the theoretically preferred approach.

by the upward (risk) adjusted discount rate, while on the other, the expected cash flow components are each adjusted downward (to their certainty equivalents) and discounted with a risk free rate. Under the CE method, each cash flow component is directly assessed in terms of its risk, while the RADR approach is indirect in that it presupposes these risk adjustments to the expected cash flow. In essence, the correct risk adjustment in the discount rate yields a present value of the expected cash flow that is equal to the present value of the certainty equivalent cash flow discounted with the risk free rate (Robichek and Myers, 1966).

The advantage with the RADR approach is that it is easy to apply since all that is needed is a simple adjustment to the discount rate (Lewellen, 1977). This adjustment is assumed to embody the decision maker's attitude and perceptions regarding the investment's risk. However, the problem with the RADR approach is that there is no way to estimate the appropriate risk adjustment without first solving the investment decision using a certainty equivalent approach (Lintner, 1965a; Robichek and Myers, 1966). In essence, a single *a priori* specified risk-adjusted discount rate could over-or under-compensate for the riskiness in each cash flow component within an investment alternative, as well as, over-or under-compensate for the risk differences between alternatives. The likelihood that a single adjusted rate would apply to all cash flow components within all alternatives considered is very low.

This problem is magnified in forestry when one considers the lengthy time horizon underlying forestry investments. Small discrepancies in the risk adjustment may create a significant bias for or against particular alternatives since the longevity of forestry investments makes their valuation very sensitive to the magnitude of the discount rate. This is particularly true if a single rate is applied to all stand level alternatives being considered, since each alternative will differ in risk in as much as the uncertain economic and biological factors influence each alternative differently. In essence, the relationship between cash flow risk and the implied adjustment in the discount rate may be crucial to the proper selection of stand level investment alternatives. Herein, lies the ambiguity of using an *a priori* risk-adjusted discount rate in forest investment decision making as well as the impetus to evaluate its use more closely.

Risk-Adjusted Discount Rate Bias Defined

Risk-adjusted discount rate bias refers to incorrectly identifying the preferred alternative for the decision unit. Statements concerning whether decisions are incorrect or correct refer to the welfare position of the decision unit. The certainty equivalent approach identifies the welfare maximizing (correct) decision from the set of opportunities considered by explicitly recognizing risk preferences. If the RADR approach fails to identify this decision as the preferred decision in its analysis, the chosen rate is inconsistent with the risk preferences of the decision unit; and therefore, the identified decision is suboptimal (incorrect).

OBJECTIVES

The purpose of this research is to question the use of risk-adjusted discount rates in stand level investment analyses. The research objectives were to:

1) Analytically define the risk-adjusted discount rate problem in the context of stand level decision making.
2) Hypothetically demonstrate how the use of risk-adjusted discount rates may bias the true (i.e., welfare maximizing) ranking of investment alternatives.
3) Empirically assess to what degree the use of risk-adjusted discount rates may lead to suboptimal stand level decisions.
4) Provide recommendations concerning how the pitfalls associated with the use of risk-adjusted discount rates can be avoided.

The procedural requirements for the research pertain to objectives 2) and 3). To contrast the two objectives, the second is to purposely set up a hypothetical problem in which RADR bias occurs, while the third considers an empirical evaluation of risk and investigates whether RADR bias indeed actually occurs, and if so, to what degree.

Scope

The numerical results forthcoming in the analysis pertain to a specific case study set in the central mountain region of Virginia. The investment setting is a non-diversified risk averse individual facing mutually exclusive opportunities in forage hay, pine plantation, and mixed pine-hardwood management. Risk preferences are embodied in the individual's wealth utility function. However, rather than eliciting a function pertaining to a specific individual, classes of utility functions (e.g., logarithmic and negative exponential) are used to represent the risk preferences for some individual. Different degrees of risk aversion are represented by changing the parameters of the utility functions used in the analysis.

The numerical results are based on an empirical estimation of risk which only accounted for historical variation in yields and prices. Catastrophic events, such as losses from wind, fire, and pests, were not included. Exogenous shocks from shifts in policy or changes in international markets were not included in the modelling of prices. Therefore, definitive conclusions regarding the riskiness of forestry and hay investments must be conditioned in the context of these omissions.

One purpose of the empirical analysis was to illustrate some mechanics underlying the modelling of risk. However, rather than focusing on the sensitivity of the results to the model's assumptions and specification, the intent was to develop a stochastic representation of future events that would serve as a vehicle for assessing risk-adjusted discount rate bias in stand level decision making. In essence, if the risk-adjusted discount rate approach proves inadequate for this stochastic representation of the system, the implication would be that changing some of the specifics in the modelling aspects of the study would not alter this result.

NOTATION

The main text of the paper relies on the following notation.[4] Let:

A	=	the age of the timber stand (years).
AREA	=	the total acreage of the individual's land holdings suitable for either timber or hay production (acres).
c	=	Absolute risk aversion parameter for the negative exponential class of utility of wealth functions.
c_{pa}	=	coefficient of absolute risk aversion. Applicable to negative exponential utility of wealth. Scaled to reflect that wealth effects are distributed on a per acre basis (see appendix B).
CE_t	=	the certainty equivalent value for the uncertain cash flow component in year $t \in \{1, 2, 3, ...\}$ for a given investment opportunity. (Applicable units may be in total dollars ($), or in $/acre).
CF_t	=	the uncertain (i.e., random) cash flow component value in year $t \in \{1, 2, 3, ...\}$ for a given investment opportunity. (Applicable units may be in total dollars, or in $/acre). Note: possible outcomes are represented by cf_t.
COV(X, Y)	=	the covariance between random variables X and Y.
dbh	=	diameter breast height (inches).
e^X	=	the base of the natural logarithm raised to the power X.
E[•]	=	the expectations operator. Note: E[CF_t] denotes the expected cash flow component value in year $t \in \{1, 2, 3, ...\}$ (either in total dollars, or in $/acre), E[U(•)] denotes expected utility, E[W] denotes expected wealth (either in total dollars, or $/acre), and E[$NPV$] denotes expected net present value (either in total dollars, or in $/acre).

4 Additional notation used in the appendices is defined when first introduced.

f	= the risk free rate of discount (percent/100, assumed constant over all future time periods).
${}_t f_{t+1}$	= the one period forward rate of return from purchasing a bond at the beginning of year t and selling it one year hence (percent/100); $t \in \{0, 1, 2, \ldots, T\text{-}1\}$, where T denotes the last period considered.
g_t	= the risk-adjusted discount rate appropriate for discounting the expected cash flow component value in year $t \in \{1, 2, , 3 \ldots \}$ (percent/100).
$h(x)$	= the probability density function for x
(hay)	= superscript denoting the autoregressive model for hay price $P_{t'}^{(hay)}$.
HY_t	= uncertain hay yield (in period t) for a representative acre of the individual's holdings (tons/acre).
ΔHY_t	= change (over period $(t-1, t)$) in the uncertain hay yield for a representative acre of the individual's holdings (tons/acre).
(in)	= superscript denoting the autoregressive model for the economic index variable $\Delta IN_{t'}$.
$IN_{t'}$	= the economic index variable (see appendix C for candidate measures) representing the uncertain state of the overall economy (in period t') (real, 1967 = 100).
$\Delta IN_{t'}$	= the change (over period $(t'-1, t')$) in the economic index variable representing the uncertain state of the overall economy (real, 1967 = 100).
k	= the risk-adjusted discount rate appropriate for discounting the entire expected cash flow for a given investment opportunity (percent/100).
LOG(•)	= the natural logarithm function.
m	= the lag order for the lagged endogenous term in a given autoregression.
MBF	= thousand board feet, Scribner scale.
n	= the lag order for the explanatory variable in a bivariate autoregression.
NPV	= the uncertain net present value of the uncertain cash flow for a given investment opportunity. Note: the implied discount rate is the risk free rate. (Applicable units may either be in total dollars, or in $/acre).
NPV_{CE}	= the net present value of the certainty equivalent cash flow for a given investment opportunity. Note: the implied discount rate is the risk free rate. (Applicable units may be in total dollars, or $/acre).

NPV_{RADR} = the mean risk-adjusted net present value defined by discounting the expected cash flow for a given investment opportunity with a risk-adjusted rate. (Applicable units may be in total dollars, or in $/acre).

$\sim N(0,\sigma^2)$ = the normal distribution with mean zero and variance, σ^2.

$P_{t'}^{(hay)}$ = Virginia all hay (baled) price received by farmers in period t' (1967 $/ton).

$P_{t'}^{(pulp)}$ = Virginia southern yellow pine pulpwood stumpage price in period t' (1967 $/cord).

$P_{t'}^{(saw)}$ = Virginia southern yellow pine sawtimber stumpage price in period t' (1967 $/MBF).

(pulp) = superscript denoting the autoregressive model for pulpwood stumpage price $P_{t'}^{(pulp)}$.

${}_0R_t$ = the annual long-term rate of return (i.e., yield) on a bond purchased now (i.e., year 0) and matures t years hence (percent/100); $t \in \{1, 2, ..., T\}$, where T denotes the last period considered.

RP_{NPV} = the net present value risk premium associated with holding a claim to a given investment opportunity. (Applicable units may be in total dollars, or in $/acre).

rp_t = the risk premium in the discount rate for a given investment opportunity's uncertain cash flow component in year $t \in \{1, 2, 3, ...\}$ (percent/100).

RP_t = the cash flow component risk premium in year $t \in \{1, 2, 3, ...\}$. (Applicable units may be in total dollars, or in $/acre).

(saw) = superscript denoting the autoregressive model for sawtimber stumpage price $P_{t'}^{(saw)}$.

SKE[NPV] = the skewness of the empirical net present value probability distribution for a given investment opportunity ($/acre cubed).

t = an annual period. Note: explicit reference to uncertainty in the timing of cash flow components is denoted with t.

t' = a quarterly period.

$U(\cdot)$ = the individual's utility of wealth function. (Wealth units may be in total dollars, or in $/acre).

$U'(\cdot)$ = the partial derivative $\partial U(\cdot)/\partial(\cdot)$.

$U''(\cdot)$ = the second order partial derivative $\partial^2 U(\cdot)/\partial((\cdot))^2$.

$U^n(\cdot)$ = the nth order partial derivative $\partial^n U(\cdot)/\partial((\cdot))^n$, $n \in \{3, 4, ...\}$.

VAR[NPV] = the variance of the empirical net present value probability distribution for a given investment opportunity ($/acre squared).

W = represents an individual's uncertain wealth position in general. Note: possible outcomes are represented by w.

W_0 = the individual's wealth endowment (total dollars).

$W_0^{(pa)}$ = the individual's initial wealth endowment expressed on a per acre basis ($/acre).

W_t = the uncertain future individual wealth position from holding claim to a given investment opportunity with an uncertain cash flow component in year $t \in \{1, 2, 3, ...\}$ (total dollars). Note: possible outcomes are represented by w_t.

$W^{(pa)}$ = the uncertain individual wealth position from holding claim to a particular investment opportunity expressed on a per acre basis ($/acre). Note: possible outcomes are represented by $w^{(pa)}$.

x = an uncertain increment to wealth from holding claims to an uncertain investment opportunity. (Note: units in the same context as wealth (e.g., dollars)).

x! = denotes x factorial.

z = a uniform (0,1) random variate.

α_t = the certainty equivalent adjustment factor applicable to a given investment opportunity's cash flow component in year $t \in \{1, 2, 3, ...\}$.

$\beta_0^{(j)}$ = intercept parameter (i.e., deterministic component) in the autoregressive models defining the stochastic price process for the empirical simulation model, $j \in$ {hay, in, pulp, saw}.

$\beta_q^{(j)}$ = coefficient for lagged endogenous variables (the qth lag) in the autoregressive models defining the stochastic price process in the empirical simulation model, $j \in$ {hay, in, pulp, saw}.

$\varepsilon_t^{(hay)}$ = stochastic disturbance term (for period t) in the autogregressive model defining the stochastic hay price process for the empirical simulation model.

$\varepsilon_{t'}^{(j)}$ = stochastic disturbance term (for period t') in the autogregressive models defining the stochastic price process for the empirical simulation model, $j \in$ {in, pulp, saw}.

η_q = coefficients for the autoregressive process representing stochastic hay yields, ΔHY_t. (Note: $q = 0$ corresponds to the intercept parameter, $q \in \{1, 2, ...\}$ corresponds to the qth lagged term for ΔHY_t).

$\gamma_q^{(j)}$ = coefficient for contemporaneous ($q = 0$) and lagged effects ($q \in \{1, 2, \ldots\}$) from the explanatory variable (the qth lag) in the bivariate autoregressions defining the stochastic price process for the empirical simulation model, $j \in$ {hay, in, pulp, saw}.

$\int(\cdot)dx$ = denotes integration with respect to x.

$\lambda(w)$ = the Arrow Pratt measure of absolute risk aversion.

μ_t = stochastic disturbance term (for period t) in the autogregressive model defining the stochastic hay yield process.

∂ = denotes partial differentiation.

$\psi(w)$ = the Arrow Pratt measure of relative risk aversion.

$\rightarrow \infty$ = be read as "approaches infinity".

$\sigma^2_{CF_t}$ = the variance of the uncertain cash flow component value in year $t \in \{1, 2, 3, \ldots\}$ (dollars (or \$/acre) squared).

σ^2_{hay} = the variance of the stochastic disturbance term $\varepsilon_t^{(hay)}$ for all t.

$\sigma^2_{\Delta HY}$ = the variance of the stochastic disturbance term μ_t for all t.

σ^2_j = the variance of the stochastic disturbance term $\varepsilon_{t'}^{(j)}$ for all t', $j \in$ {in, hay, pulp, saw}.

$\sum_i$ = denotes summation with respect to i.

LITERATURE REVIEW

RISK AND UNCERTAINTY IN THE FINANCE LITERATURE

Portfolio Analysis: The Basis for Risk-Adjusted Discount Rates

The evolution of risk-adjusted discount rates in capital budgeting can be traced to the finance literature. Two important developments were Modigliani and Miller's (1958) definition of a firm's cost of capital; and the Capital Asset Pricing Model (CAPM), which relates an investment's risk to its expected return in a market equilibrium analysis (Sharpe 1964; Lintner 1965a; Lintner 1965b; Mossin, 1966; Rubinstein, 1973).[5] The CAPM model is based on the developments in portfolio theory, where investors are risk averse expected utility maximizers (Markowitz, 1952; Tobin, 1958; Markowitz, 1959).[6] Dispersion around the mean (e.g., standard deviation or variance) is the appropriate measure of portfolio risk; the higher the variance of portfolio returns for a given expected return, the lower the expected utility from holding the portfolio of risky assets. In this context, the risk associated with a single asset can be separated into non-diversifiable systematic risk (i.e., correlation with the market, or market risk), and diversifiable project specific risk (Brigham, 1985). Under the CAPM framework, the appropriate cost of capital (i.e., risk-adjusted discount rate) to use in capital budgeting

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5 For an excellent review of the Capital Asset Pricing Model, see Copeland and Weston (1983) or Brigham (1985). Fama and Miller (1972) provides a more rigorous derivation of the principles underlying CAPM.

6 Hakkanson (1970) formally defines the investor's decision model under uncertainty. Using a discrete time dynamic programming approach, Hakkanson shows that in general the individual's investment, financing, and consumption decisions are non-separable.

only accounts for the firm's systematic risk. Accordingly, the riskiness of individual projects should be assessed in terms of how project acceptance effects the firm's systematic risk (Brigham, 1985).

While the CAPM framework provides a theoretical justification for the use of risk-adjusted discount rates in project investment analysis, there are several cautions worth noting when applying a CAPM determined rate in capital budgeting.

1) The Capital Asset Pricing Model is a single period model, so the application of CAPM derived rates to multi-period investments needs questioning.
2) Using a single cost of capital to evaluate a set of risky project investments ignores the market risk effects underlying project acceptance.
3) For an individual project investment, the use of a single risk-adjusted discount rate has a specific implication regarding the risk adjustment to the cash flow.

Much attention has been given to the first caution. Fama (1970) and Fama (1977a) state that if investor opportunity sets evolve deterministically,[7] then a period by period application of the CAPM model is appropriate. This implies a known risk-adjusted discount rate for each period. However, if the risk free rate is unchanging, and the projects considered do not change the firm's systematic risk (i.e., consistent with the firm's line of business), then a single RADR applicable to all periods is appropriate (Fama, 1977a). In contrast, Merton (1973) and Bogue and Roll (1974) show that the risk adjustments are not as simple as proposed by Fama (1977a) when investor opportunities evolve stochastically. Constantinides (1980), who states the conditions under which stochastic investment opportunity sets are within the CAPM pricing framework, sums up the implications regarding capital budgeting;

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[7] This condition is met when investors' assessments of future events are only based on information available at the beginning of each period; unconditional of previous period events.

> . . . the sequential application of the SML (CAPM) model in the discounting of cash flows (while appropriate) becomes computationally complex and is of little practical use. (parenthetic added).

The second caution is a static concern. Lintner (1965a) rejects the notion that firms can be categorized into the broad risk classes that underlie Modigliani and Miller's (1958) cost of capital. In support of this claim, Martin, *et al.* (1979) found that grouping firms by industry was not an effective surrogate for defining equivalent risk classes. Lintner's (1965a) argument was that the systematic risk effects would be unique to the project being considered; namely due to differences between the systematic risk inherent in the cash flow. According to Lintner, there would be no single risk-adjusted discount rate appropriate for all project evaluation. Lintner argued that the appropriate capital budgeting technique would be to discount market determined certainty equivalents with the risk free rate (see also Rubinstein (1976a) and Rubinstein (1976b)). According to Lintner, solving for the appropriate RADRs for each project would be redundant since such an approach would ultimately have to rely on the certainty equivalent formulation of the problem.

Brigham (1985) suggests that in as much as the acceptance of the project will change the firm's systematic risk, the cost of capital should be adjusted accordingly. For example, a project that would increase the firm's systematic risk would be evaluated with a higher cost of capital than the firm's current cost of capital without the project. Accordingly, projects which would reduce systematic risk should be evaluated under a lower cost of capital. In this manner, the firm is taking into account investors' reaction to project acceptance. According to Fuller and Kerr (1981), this cost of capital adjustment can at best be made at the corporate divisional level. Fuller and Kerr advocate the pure play method in which the appropriate cost of capital is estimated for a non-integrated firm whose systematic risk is defined by projects similar to the one being considered. But as Zinkhan (1988) points out, this comparable company approach is not feasible for timberland investments since there are no publicly owned companies that solely deal with timberland management.[8]

Robichek and Myers (1966) address the third caution which is dynamic with respect to a specific project investment. Relying on a certainty equivalent formulation of the problem, Robichek and Myers demonstrated that using a single risk-adjusted rate to discount expected cash flows is consistent with the assumption that cash flows become more risky the farther into the future they occur.[9] Furthermore, the implied rate of increase in risk with respect to time is defined by the chosen risk-adjusted discount rate. Therefore, Robichek and Myers conclude the using risk-adjusted discount rates is only appropriate for a special class of investments where cash flow risk increases at a rate consistent with the risk adjustment in the discount rate. However, solving for the appropriate risk adjustment is redundant since a certainty equivalent formulation of the problem would have to be employed. For this reason Robichek and Myers favor the certainty equivalent approach since the risk adjustments are uniquely applied to each cash flow component, and therefore can account for any particular cash flow risk pattern.

In a similar analysis, Chen (1967) confirms the results of Robichek and Myers (1966). However, Chen criticized Robichek and Myers for the conclusion that the

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[8] As pointed out in Fortson (1986), publicly owned timber companies are usually integrated in that they have a large processing division.

[9] Robichek and Myers (1966) (and later in Chen (1967)) defined cash flow risk with respect to the certainty equivalent adjustments applied to the cash flow, not with respect to variance or systematic measures. Their claim was that if the individual's certainty equivalent adjustments for risk increases (at a constant rate) with respect to time, then a constant risk-adjusted discount rate would be implied. Rubinstein (1976a) formalizes the conditions under which investors' certainty equivalent adjustments (as defined by the market) for systematic risk will increase with respect to time.

results of their analysis define a conceptual problem with regard to the use of risk-adjusted discount rates. In essence, Chen sees no contradiction with the use of risk-adjusted rates and valuation with respect to risk.[10] Therefore, Chen sees no superiority in the certainty equivalent approach to risk. But Chen failed to recognize that these arguments rest on knowing, *a priori*, the true certainty equivalent adjustments to the expected cash flow. Again, given this information (and the risk free rate), the RADR approach to risk is redundant.

Risk Aversion

As indicated by the developments in finance theory, most discussion concerning risk and uncertainty implies risk aversion on the part of decision makers and investors. Furthermore, Sharpe (1965) and Arditti (1967) provide empirical evidence that investors are risk averse. Friedman and Savage (1948) also developed arguments that some tendency toward risk preferring behavior (e.g., gambling) was not a direct contradiction to the general assumption of risk aversion. By allowing individual preferences regarding wealth outcomes to be dependent on how the future unfolds (states of nature), Hirshleifer (1964) and Hirshleifer (1965) defined risk aversion in the more general sense of conservative behavior towards risk, which also allows for some tendency toward risk seeking behavior. Finally, Pratt (1964) formalizes measures of an individual's degree of risk aversion (e.g., absolute and relative risk aversion) based on properties of their utility function regarding wealth.

On the Appropriateness of Discounted Cash Flow Techniques

Hayes and Garvin (1982) suggest that evaluating project investments on the basis of discounted cash flow (DCF) techniques leads to a myopic view of the future. Hayes

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10 According to Chen (1967), this was the original intent of Robichek and Myers (1966).

and Garvin criticize the United States (US) corporate industry for relying on high discount rates that discourage long-term commitments to new capital investment and research and development (R&D) projects. While the authors concede that risk adjustments for systematic risk are warranted, they fault managers for employing discount rates with unreasonably high risk premiums.[11] The result is too much disinvestment in the firm's capital assets, which jeopardizes the future competitive position of the firm. According to Hayes and Garvin, the irony is that the necessary corrective measures (i.e., new investment in capital assets) are further discouraged by the same capital budgeting techniques that initially led to the firm's peril.

However, Hodder and Riggs (1985) suggest that it is not the DCF techniques which are at fault, but the technicians. Hodder and Riggs cite the inadequate treatment of inflation and uncertainty as the reason why DCF techniques lead to myopic decisions. Most notably, they argue that there is no *a priori* justification for using a single (risk-adjusted) discount rate in project investment evaluation. Using a hypothetical R&D project as an example, the author's show that merely recognizing that the project's risk varies (e.g., decreases) over the cash flow leads to a significant improvement (i.e., acceptance) in the project's evaluation. However, Hodder and Riggs concede that casual reliance on DCF techniques is inappropriate. In the same spirit as Hayes and Garvin (1981), Hodder and Riggs point out that DCF techniques should be viewed as tools, whereby managers ultimately make the final decision regarding investment acceptance.

The Case of Irreversible Decisions

The general notion that economic incentives lead to myopic decisions has called attention to the case where the uncertain investment opportunity represents an

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According to Hayes and Garvin (1981), these inflated premiums can be attributed to managerial risks. The most notable example is that high discount rates discourage subordinates from accepting projects solely for personal prominence within the firm.

irreversible decision. According to Henry (1974), an irreversible decision implies long-term reductions regarding investment opportunities in the asset.[12] According to Henry, any objective treatment of risk will systematically (and incorrectly) favor irreversible decisions. While Henry relied on a risk neutrality assumption in the analysis, simply ignoring risk is not responsible for this bias (Henry, 1974). Arrow and Fisher (1974) suggest that the correct tendency should be to err on the side of underinvestment (e.g., preservation) when payoffs from investment (e.g., development) are uncertain since development is irreversible.[13] However, Henry (1974) points out that the significance of this irreversibility effect depends on the specification of the risk free discount rate since higher rates imply that the future (and its uncertainty) is of lesser importance.

The Term Structure of Interest Rates

One aspect of DCF techniques that is often taken for granted is that the discount rate is non-stochastic and constant in a multiperiod setting. In general, interest rates are not constant through time (Copeland and Weston, 1983).[14] Furthermore, this generalization goes beyond the fact that risk adjustments in the discount rate may vary across time periods. In essence, for every period there is a term structure of interest rates

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Most notably, Henry (1974) uses the harvest of a forest stand as an example of an irreversible decision. However, the definition becomes clear when preservation versus development issues are at stake (Arrow and Fisher, 1974).

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Arrow and Fisher (1974) also assume risk neutrality since irreversible decisions are common to assets held in the public domain. According to Arrow and Fisher, the tendency to err on the side of preservation appeals to a feel for risk aversion (i.e., conservative behavior).

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Merton (1973) and Bogue and Roll (1974) target the case where the market rate of interest should be treated stochasticly; that is, expectations regarding the interest rate can no longer be assumed to evolve deterministicly, given the market information available at the beginning of each period. The implication is that investors' risk premiums are in themselves stochastic (Bogue and Roll, 1974).

that relates the periodic (e.g., annual) rate of return (i.e., yield) to the length of time (i.e., the number of periods) to maturity (Copeland and Weston, 1983). For example, the annual yield on a bond with 1 year left to maturity will most likely differ from the yield on a bond that has 10 years left before maturity; the difference reflecting the time left to maturity.

In general, the term structure of interest rates is defined with respect to nominal rates of return which include a component for inflation. As such, the term structure is very volatile from period to period and the relationship between long-term yields (i.e., the annual rate of return on a 10 year maturity) and short-term yields (i.e., the annual rate of return on a 1 year maturity) can be explained in terms of market expectations regarding inflation.[15]

However, regardless of the hypothesis concerning the existence of the term structure, the long-term rate of return on a given maturity can be decomposed into a sequence (i.e., product) of periodic forward rates of return. For example, if ${}_0R_T$ denotes the annual long-term yield on a bond purchased now (i.e., year 0) that matures T years hence, then the relationship between ${}_0R_T$ and the forward rates can be expressed as (Copeland and Weston, 1983)

$$(1 + {}_0R_T)^T = (1 + {}_0f_1)(1 + {}_1f_2) \cdot \cdot \cdot (1 + {}_{T-1}f_T) \qquad [1]$$

where ${}_tf_{t+1}$ is the one period forward rate of return from purchasing a bond in year t and selling it one year hence ($t \in \{0, 1, 2, \ldots, T-1\}$). The implication regarding capital budgeting decisions (either deterministic or under uncertainty) is that cash flow

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15 Other hypotheses concerning the term structure of interest rates pertain to liquidity differences between long and short-term maturities, as well as the hypothesis that bonds of different maturities may be priced in separate markets (Copeland and Weston, 1983). In regard to the liquidity preference hypothesis, investors demand an illiquidity premium for holding longer maturities since their value is more sensitive to short-term fluctuations in the market interest rate; namely due to inflation.

components occurring in year t should be discounted with the long-term rate ${}_0R_t$ which in turn is a product of the forward rates ${}_0f_1$, ${}_1f_2$, ..., ${}_{t-1}f_t$. That is, it would be inappropriate to discount the entire cash flow for an investment with intermittent payoffs, where the last payoff ends T years hence, with the long-term interest rate ${}_0R_T$ (Copeland and Weston, 1983). In essence, it is the forward interest rates that define the sequential (i.e., year to year) opportunity cost of invested capital.[16]

The Real Risk Free Interest Rate

The problem of identifying the appropriate forward rates to apply in DCF techniques can be reduced if inflationary effects are ignored and the analysis conducted in real terms; a common approach in forestry. However, the problem regarding the constancy of the interest rate still exists as long as there is a term structure regarding the real rate of return on investment. Fama (1975) argues for a constant short-term real rate of return on (1 month) US Treasury Bills, while others suggests that the short-term real rate of return varies over time (Carlson, 1977; Elliott, 1977; Joines, 1977; Nelson and Schwert, 1977).[17] Nelson and Schwert (1977) provide the clearest explanation for this discrepancy; namely, it is difficult to make assessments about *ex ante* (i.e., before-the-fact, or expected) real rates based on *ex post* (i.e., after-the-fact) assessments due to measurement error in the analysis. Using more powerful tests than Fama (1975) (and the same data) Nelson and Schwert reject Fama's joint hypothesis that *ex ante* real rates are constant and inflationary expectations efficient (i.e., utilize and summarize all market information). However, Nelson and Schwert concede that the rejection of the

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Baumol (1967) also stresses the importance of selecting the appropriate long-term rate of discount. Baumol's concern targets the practice of estimating the long-term rate on the basis of original maturity, instead of on the basis of the time left to maturity.

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All of these studies target US Treasury Bills, so the implication is that the real risk free rate of return is the subject of the debate.

joint hypothesis only implies that either real rates are variable, or that inflationary expectations are inefficient, or both.

It should be noted that these studies do not address the question regarding the term structure for the real risk free interest rate. However, if the short-term risk free rate of return is assumed to remain constant in a multiperiod setting[18], then the implicit assumption is that there is no term structure of interest rates for US Treasury Bills. That is, the forward rate applicable to each period is constant.[19] Therefore, if the real rate of return on US Treasury Bills (say 9-12 month issues, see Carlson, 1977) is assumed constant and representative of the forward rate, then applying this rate as the risk free rate in a multiperiod setting is justified.[20]

Simulation Issues

Lewellen and Long (1972) question the use of simulation models in evaluating risky project investments,[21] since these models focus on project specific risks. Instead, Lewellen and Long support the RADR approach because this approach is grounded on the relevant market risks associated with project investments. Simulation is an

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Note that this was one of the requirements for justifying the use of a constant CAPM derived risk-adjusted discount rate in a multiperiod setting (Fama, 1977a).

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Rubinstein (1976a) defined this as an unbiased term structure for the risk free interest rate. Rubinstein formalizes the conditions under which the risk free interest rate term structure will be unbiased; namely investors exhibit constant relative risk aversion.

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In spite of evidence to the contrary (e.g, Carlson, 1977), Fama (1977b) maintains that treating the real rate of return as if it were a constant is still a useful (and not unrealistic) representation of the real world.

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The use of simulation in capital budgeting was first introduced by Hertz (1964) (and later updated in a reprint; Hertz, 1979).

unnecessary and costly undertaking since the only information needed for the RADR approach is an assessment of the mean cash flows; not the entire probability distribution. In essence, the authors are relying on an equilibrium model (such as CAPM) to provide the appropriate risk-adjusted discount rate. In this context, a subjective evaluation of the mean cash flows, without regard to the specific events that define this mean, is sufficient for capital budgeting decisions. However, this argument ignores the usefulness of simulation in estimating what the mean cash flows might be.

One ambiguity in using simulation to evaluate investment performance in the presence of uncertainty is what discount rate (risk-adjusted versus risk free) to specify in present value calculations (Brigham, 1985). The choice of the discount rate will depend on how simulation will be used. If the purpose of the simulation is just to get a better estimate of the mean cash flows, the appropriate rate to specify is a risk-adjusted rate if the decision maker is risk averse. In this case, the estimated mean NPV from the simulation represents what Lewellen and Long (1972) call a mean risk-adjusted NPV; that is, a certainty equivalent valuation of the investment. In essence, the mean risk-adjusted NPV is equivalent to discounting the mean cash flows with the risk-adjusted rate (Lewellen and Long, 1972). Therefore, in this context the only relevant decision parameter is mean risk-adjusted NPV, since the discount rate has been adjusted for risk.

However, if the simulation is used to estimate the entire probability distribution of investment results, then the specified discount rate should be risk-less. In this case, the simulation results (the entire distribution of outcomes) become input into other risk assessment techniques such as expected utility or stochastic dominance models. Since these other techniques rely on the probability distributions of investment returns to measure (and adjust for) risk; using a risk-adjusted discount rate would double count for risk (Lewellen and Long, 1972). Furthermore, according to Lintner (1965a), the risk free rate is the only rate that preserves the distributional characteristics of uncertain present value outcomes.

PUBLIC INVESTMENT AND RISK

Hirshleifer's State Preference Model

The issue of risk and the public rate of discount was opened up when Hirshleifer (1964) and Hirshleifer (1965) developed a state preference model that rationalized intertemporal consumption and investment decisions under uncertainty. Under this framework, a risk free (or pure) rate of interest reveals marginal time preference alone; that is, certain intertemporal consumption decisions. However, in as much as future consumption opportunities are gained through risky investment in real assets, future consumption is risky. Ultimately, it depends on the state of the world that determines the rate of return on investment. Therefore, individual (household) claims to future consumption represent both time and state contingent claims. Conceptually, each real asset yielding income in some future period can be thought of as representing several commodities (objects of choice), one commodity for each state contingent claim to consumption in that period. Therefore, the trading of real assets represents individual attempts to move toward a preferred combination of time-and-state consumption claims (Hirshleifer, 1965).

The major appeal of Hirshleifer's state preference approach is that it is a logical extension of consumption/investment decision theory under certainty (Hirshleifer, 1965). Expected rate of return differentials can be rationalized in terms of the uncertainty underlying state contingent claims to consumption. As Hirshleifer (1961) and Hirshleifer (1964) point out, these observed rate differentials can no longer be dismissed as reflecting market imperfections like capital rationing, transaction costs, and differential tax treatment. As such, Hirshleifer concludes that the public rate of discount depends on the nature of the investment (the time-state consumption claims it represents) and therefore, should embody the market rates of discount for these time-and-state contingent claims.[22] In as much as these rates include premia for risk, so does the public discount rate (Hirshleifer, 1964; Hirshleifer, 1966).

Government's Ability to Handle Risk

Hirshleifer's (1964) conclusion that the public discount should be adjusted for risk startled other economists; namely, Samuelson (1964) and Vickrey (1964). Both Samuelson and Vickrey maintain that private risk markets are imperfect, that is, the government has a risk pooling ability superior to that found in the private sector. Therefore, risk premia represent a private cost that need not be considered from the standpoint of public investment. To do so would simply be in error since the government's ability to handle risk better is ignored (Samuelson, 1964). Therefore, the public discount rate should represent a risk free measure of the social opportunity cost of public investment.

Hirshleifer (1966) concedes that if private risk market imperfections exist, then it is possible that private risks are socially irrelevant, and therefore the market rates of return reflecting private risk aversion should not influence the government's choice of discount rate. However, Hirshleifer then demonstrates that the imperfection must cause the single price law of perfect (as well as, efficient) markets to be violated; that is, individuals cannot effectively trade their time-and-state consumption claims. But this is

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[22] In essence, these time-and-state discount rates reflect the price of a dollars worth of future consumption at a given time under a given state of the world. When individuals are risk averse and state contingent claims are risky (i.e., more than one possible future state), the time-and-state discount rates will exceed the pure rate of interest (which reflects the price to a certain claim to future consumption); the difference being the risk premium required for holding a consumption claim contingent on that state (Hirshleifer, 1966).

tantamount to saying that the competition underlying the exchange of real assets in private markets is completely blocked. On the other hand, if competition is not blocked, then the single price law must hold for time-and-state consumption claims. Any time-and-state contingent claim, regardless of whether it is through public or private investment, should be discounted at the appropriate market time-and-state discount rate. As Hirshleifer (1966) points out, making a distinction between private and public investment would be equivalent to charging different prices for the same commodity. Therefore, a public investment project with some time-and-state distribution of returns should be evaluated with a risk-adjusted discount rate applicable to a comparable project (in terms of its distribution of time-and-state returns) undertaken by the private sector.

The Arrow-Lind Theorem

Arrow and Lind (1970) suggest that as long as risk aversion can be identified as a private cost, but not a social cost, the government need not concern itself with risk. Arrow and Lind show that if the government's project returns are independent of national income, and that if both project benefits and costs are borne by a large number of individuals, then the social cost of risk, even in the presence of private sector risks, is effectively zero. In essence, their theorem states the conditions under which the government can effectively spread[23] private sector risks such that in the social aggregate, the aversion to this risk is zero. The proof relies on the assumption that as each

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[23] Arrow and Lind (1970) make a clear distinction between risk spreading and risk pooling. Risk spreading refers to reducing the amount of risk borne by individuals by decreasing the share of the project's benefits. On the other hand, risk pooling refers to keeping the individual's share of a project intact, but diversifying away some of the project's risk by holding it in combination with other assets (i.e., portfolio diversification). As Bailey and Jensen (1972) point out, the government's ability to pool (diversify) risks can be no better than the diversification ability of the private sector. That is, even if all real assets existing in the market were held in a portfolio, the level of systematic (non-diversifiable or market) risk could not be lowered by government control of these assets.

taxpayer's claim to expected project benefits (and the time-and-state consumption opportunities they represent) goes to zero (as benefits are spread over a larger number of taxpayers), the individual required risk premium for holding this claim goes to zero as well. That is, households do not perceive the government project as having any effect on their current holding of time-and-state contingent claims to future consumption. Therefore, public investment decisions can ignore risk and the public discount rate need not be adjusted for risk.

The Public Benefit Disbursement Issue

The Arrow-Lind theorem is somewhat restricted. It only applies to the case where claims to the public investment's benefits are marketable; that is, beneficiaries of the project pay for the benefits received and the government redistributes this revenue lump sum[24] (Tresch, 1981). However, even if the government has this ability, Bailey and Jensen (1972) point out that the majority of public investment projects (e.g., public highways) do not meet the criterion that claims to the benefits are marketable. In most cases, benefits accrue to a targeted beneficiary without reimbursement. According to Bailey and Jensen, this is analogous to dumping more of the represented asset into the portfolio of assets held by the targeted group. Therefore, the uncertainty underlying the project benefits adds risk to the targeted beneficiary's wealth. Arrow and Lind (1970) also concede this point, but differ with Bailey and Jensen's view regarding the frequency of its occurrence in application. According to Arrow and Lind, in this case the project benefits (not the costs[25]) should be discounted with a risk-adjusted rate. This is

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[24] Lump sum refers to the government's ability to collect or disburse revenues without creating any distortions in the economy; namely, that the relative prices for all goods remains unchanged.

[25] That is, since costs are spread over all taxpayers, they should still be discounted with the risk free rate. However, the work of Arrow and Lind (1970) has been criticized because it fails to account for the fact that taxes represent compulsive investment (McKean and Moore, 1972;

consistent with Hirshleifer (1966) and Bailey and Jensen (1972); that is, if individuals perceive that they are bearing risk associated with the project, the public investment should cover the required premium for taking on this risk.

Nichols (1972) in a comment on Arrow and Lind (1970) makes an interesting point. The implication of the Arrow-Lind Theorem is that there will be a tendency for government to shift into activities already being provided by the private sector. That is, when project benefits are marketable, benefits should be discounted with a risk free rate. However, if benefits cannot be marketed (due to market imperfections), then benefits should be discounted with a higher risk-adjusted rate. Therefore, marketable benefits can cover costs more easily, since the discount rate is less stringent. Furthermore, since government can accept projects with lower expected returns than the private sector (since the government can ignore risk), there would be a tendency for the government to take on more investment in a particular area than the private sector. On the other hand, projects which target activities that are presumably best suited for government investment suffer from a more stringent acceptance guideline since benefits are discounted at a rate higher than the rate applicable to the costs.

Risk as an Opportunity Cost

In general, the proposition that the government should evaluate public projects with a risk free public discount rate has been criticized because it suggests that government is better suited to take on all risky investment than the private sector (Bailey and Jensen, 1972; Mckean and Moore, 1972). However, as Arrow and Lind (1972) point out, this criticism is purely reactionary and should not be used to attack the merits underlying the Arrow-Lind Theorem. According to Arrow and Lind, justifying public

Wellington, 1972). Arrow and Lind (1972) sidestep this issue by assuming that taxes represent a voluntary consent by society for government to go about its business.

investment on the basis of the government's ability to handle risk is not the issue. Instead, the question being addressed is how should risk be accounted for in the social opportunity cost of public investment, given defined bounds for considering the public investment to start. For the most part, the risk free proponents favor a single opportunity cost measure applicable to all government investment. On the other hand, the risk-adjusted proponents favor an opportunity cost measure that depends on the actual type of investment undertaken. While this difference can be attributed to the different views regarding risk, it is not the all or none issue that is at stake, but rather a more subtle issue of how risk translates into an opportunity cost.

Using Harberger's (1969) derivation of the social opportunity cost of public investment[26], Cathcart (1988) shows that risk need only be considered as an opportunity cost from the standpoint of displaced household consumption. In contrast, the opportunity cost measure for displaced private investment is a certainty equivalent measure (Cathcart, 1988). However, some perceive the high social opportunity cost of displaced private investment as implying an adjustment for risk (Mishan, 1972). But as Cathcart (1988) demonstrates, the social opportunity cost is higher on the displaced private investment side only because it is a weighted average of before-corporate-tax (risk free, or certainty equivalent) returns. In contrast, the social opportunity cost from induced household saving is defined on an after-personal-tax (risk-adjusted) return basis. According to Cathcart, once this distinction has been made, the conditions under which the public discount rate should include an adjustment for risk become more apparent.

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[26] Harberger (1969) defines the social opportunity cost of public investment in terms of the private investment displaced, and the household savings induced (i.e., displaced current consumption), as a result of government borrowing from capital markets.

RISK AND UNCERTAINTY IN THE FORESTRY LITERATURE

Overview

The early developments in risk and uncertainty in the forestry literature can be described as conceptual brainstorming (e.g., see Lundgren and Thompson, 1972). Marty's (1964) call for complete problem identification, system evaluation, sensitivity testing, and subjective choice rings true of the most sophisticated treatment of uncertainty, only lacking in the analytical aspects of probability estimation, simulation, and utility preference. Furthermore, Marty's sensitivity testing is probably the most common applied approach to uncertainty (see Rose, 1983; Dutrow and Kaiser, 1984; Kurtz *et al.*, 1984; Adams and Haynes, 1985; and Baughman, 1985 for examples).[27] Teeguarden (1969) stressed that economic guidelines from deterministic analyses provide little operational help for land managers considering whether to reforest or not in the face of long-term uncertainties. Flora (1964) argued that the commitment to long term forestry investments was a sign of indifference to uncertainty and argued that ignoring risk (risk neutrality) led to proper stand level decisions. In contrast, Thompson (1968) notes that forestry decision makers face incomplete information as a result of uncertainty, and therefore should rely on decision models that explicitly recognize this uncertainty. One strategy is to formulate a decision tree, whereby future decisions and their consequences explicitly recognize the outcome of uncertain events (Bailey and

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[27] Adams and Haynes (1985) sensitivity analysis targeting the assumptions underlying the long-range assessment for the US forestry sector provides the best example. Relying on small deviations from the assumptions, Adams and Haynes construct a future outlook markedly different from the traditional view of increasing scarcity in forest products markets. As the authors point out, such sensitivity calls for more research in ascertaining the likelihood of plausible views. In this manner, policy decisions can be based upon the spectrum of possible outlooks (i.e., address uncertainty), rather than on a single projection.

Kaiser, 1967; Talerico *et al.*, 1978). Fight and Bell (1977) argued that social attitudes toward uncertainty should be incorporated into harvest scheduling decisions.

Later developments in forestry have focused on modelling risk; that is incorporating probabilistic outcomes into decision models. Hool (1966) defines the uncertain stand level decision environment as an Markov decision process; where the decision process is defined as a dynamic programming problem. The advantage with this formulation is that it provides the optimal stand prescription for each possible realization of the future. In essence, the optimal policy defines a set of decisions conditional on future outcomes. Kaya and Buongiorno (1987) apply the approach to the harvesting decision for uneven-aged hardwood timber stands. Kao (1982) and Kao (1984) use dynamic programming to define the optimal stocking level for forest stands under risk and uncertainty respectively.[28] Lembersky and Johnson (1975) and Lembersky (1976) use a Markov decision process to target optimal stand policies based upon economic and biological decision criteria respectively.

At the stand level, simulation has been an important tool in evaluating investment performance under possible future scenarios (Row, 1973; Hassler and Sinclair, 1982; Engelhard and Anderson, 1983; Anderson *et al.*, 1985; Chambers *et al.*, 1986; Anderson *et al.*, 1987). At the forest level, research has focused on accounting for probabilistic outcomes in forest harvest scheduling (linear programming) models (Thompson and Haynes, 1971; Dixon and Howitt, 1980; Johnson, 1984; Hoganson and Rose, 1987). The risk associated with fire, especially with to the optimal rotation age decision, has also received considerable attention (Martell, 1980; Reed, 1984; Caulfield, 1988). Reed and

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28 The two studies make a formal distinction between risk (Kao, 1982) and uncertainty (Kao, 1984). Under risk, the probability distribution underlying possible future outcomes is known, while under uncertainty this information is incomplete. In the latter case, probability information is updated using a Bayesian approach (what Kao (1984) calls adaptive optimization).

Errico (1985) and Reed and Errico (1986) assess the impact of fire risk at the stand and forest decision levels respectively. Their results indicate that projected forest harvest levels will be markedly overestimated if fire occurrence is not taken into account. Blattenberger *et al.* (1984) provides an overview of accounting for fire risk in management decisions; while Wilson Jr. (1987) defines the fire risk problem in a Bayesian framework.

Analyses which target Risk Neutrality

Most studies which emphasize risk modelling assume risk neutrality on the part of the decision maker by focusing on the mean consequence from decisions (e.g., see Hool, 1966; Thompson, 1968; Kao, 1982; Johnson, 1984; Kao, 1984; Knapp *et al.*, 1984; Burkhart *et al.*, 1986; Hoganson and Rose, 1987; Kaya and Buongiorno, 1987).[29] Other studies recognize the usefulness of simulation in estimating a probability distribution of decision consequences, but do not emphasize how this additional information enters the decision process (e.g., see Hassler and Sinclair, 1982; Engelhard and Anderson, 1983; Chambers *et al.*, 1986; and Anderson *et al.*, 1987). Finally, some studies are ambiguous in their assumption concerning the risk attitudes of decision makers.[30]

The risk neutrality assumption provides a convenient starting point from which to address the uncertainty underlying forestry decisions. In most cases, accounting for risk aversion would merely imply re-specifying the objective function used in the analysis

29 The tendency of forestry research to imply a risk neutrality assumption can be partly explained by the large amount of forest investment activity taken on by the federal government.

30 One example of this is in Lembersky and Johnson (1975). In this analysis the discount rate was specified as 10 percent real; however the authors did not state whether this rate was risk free or adjusted for risk. If the rate is considered as risk-adjusted, then the authors are assuming risk aversion and their probability assessments are for risk modelling purposes; that is, estimating mean risk-adjusted decision consequences. However, if the rate is considered as risk free, an assumption of risk neutrality would be assumed if preferred decisions are identified on the basis of mean consequences only.

(e.g., maximizing the expected utility of outcomes, rather than the expected outcome (Bullard, 1985)). Bullard (1985) addresses the question of bidding for timber when the potential profit under the bid is uncertain. Routledge (1980) introduces the effect of uncertain catastrophic loss and unpredictable prices on the optimal rotation age decision. While catastrophic losses can have a strong impact on the rotation decision, unpredictable price fluctuations (around an expected trend) can be ignored. Ironically, the less the decision maker knows about the stochastic process defining the price fluctuations, the better off the decision maker is in ignoring these fluctuations (Routledge, 1980). In a similar analysis, Routledge (1987) measures the expected economic loss due to uncertain long-term soil degradation impacts from timber harvest activity. Based on the effect that soil degradation has on timber revenues alone (i.e., ignoring externalities such as aesthetic loss and stream degradation), Routledge shows that these losses can be significant.

In an innovative analysis, Brazee and Mendelsohn (1988) emphasize that the rotation age decision need not be made today. In contrast to Routledge (1980), Brazee and Mendelsohn show that the rotation age decision with respect to uncertain prices should actually represent a decision policy based on a set of reservation prices. Using a dynamic programming approach, the reservation prices were determined under the objective of maximizing expected return. Therefore, by adopting a policy of not harvesting the stand until the realized price exceeds the reservation price, the timber (as it nears maturity) will never be sold during poor price years. In essence, the policy is designed to ensure that the timber is never sold when prices are depressed; a distinct possibility if the rotation age decision is considered fixed.

Analyses which target Risk Aversion

Recently, research has begun to explicitly focus on the assumption that the decision maker is risk averse. In an exhaustive treatment of southern pine management under uncertainty, Row (1973) targets risk aversion. However, Row suggests that the best the analyst can do is to identify efficient management decisions; that is, sort out those decisions which would be unacceptable (i.e., inefficient) to any risk averse investor. In a similar vein, Thompson *et al.* (1979) used simulation to generate policy contingent probability distributions of outcomes and employed stochastic dominance rules to weed out undesirable decision policies. McQuilkin (1984) appealed to a safety first principle in developing pruning guidelines when pruning consequences were considered uncertain. Employing expected utility maximization for a risk averse owner, Johansson and Löfgren (1985) analyzed the stock supply response underlying the harvest decision in a two period model (harvest now, or harvest in the uncertain future). Koskela (1989a) generalizes this model to show that the harvest decision under price uncertainty cannot be made independent of the individual's consumption preferences.[31]

Cathcart (1987) applied expected utility maximization to the optimal stand rotation age decision in order to compare the respective decisions made under assumptions of risk neutrality versus risk aversion. The result was that the rotation age decision under risk aversion could either be longer or shorter (or perhaps even the same as) than the decision made under risk neutrality. In a more general analysis, Caulfield (1988) looked at the rotation age decision under various techniques (e.g., stochastic dominance) that target risk aversion in the decision maker. However, rather than focussing on a single optimum age, the techniques target identifying rotation ages which are efficient (i.e., could be acceptable to some risk averse individual).

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31 Koskela (1989b) performs the same analysis under the restriction of credit rationing in capital markets.

Applications of Portfolio Theory in Forest Investment Analysis[32]

Perhaps the earliest application of portfolio theory to forestry was done by Dowdle (1962). Dowdle applied Markowitz's (1952) mean-variance efficient frontier to the financial maturity of individual trees. As such, Dowdle showed that many maturity ages were efficient; that is, harvesting at a higher age implied a higher return with a corresponding increase in risk.[33] Dowdle concluded that the optimal maturity age would ultimately depend on the risk attitude of the decision maker. Field (1988) defines the forest level investment problem as being one of managing a portfolio of stands. Therefore, diversification in management can reduce risk. Field's point is subtle, but the implication is that for a given stand type, focus should be placed on identifying efficient strategies from which to diversify from; rather than focus on identifying a single optimum.

Several studies have applied CAPM to investment decisions in a natural resource context. Mills (1980) and Mills and Hoover (1982) relied on CAPM principles to demonstrate the desirability of holding forestry investments, in spite of their low expected returns, in a well diversified portfolio of forest and non-forest assets. Dunn (1982) used the principles of portfolio diversification to determine the optimal species composition in a mixed hardwood stand based on each individual species' non-diversifiable risk. Thomson (1987) relied on the CAPM approach to explain why

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The application of financial asset pricing models in forest investment evaluation has recently (*circa* 1989) received much attention from forest economists. The forthcoming Proceedings of the 1989 Southern Forest Economics Workshop (San Antonio, Texas, March 1-3) will contain recent developments in this area (Binkley, C. S. 1989. Professor of Forestry and Resource Management, School of Forestry and Environmental Studies, Yale University, New Haven, Connecticut. Personal communication).

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However, Caulfield (1988) concluded that the mean-variance criterion was weak in narrowing down the size of the set of efficient rotation ages. Caulfield favored more robust techniques such as stochastic dominance.

mid-western forest investments, in spite of their lower expected returns, persist when southern investments appear more attractive in terms of their expected returns.

Green (1987) describes southern timberland investments as being high quality-low risk investments that should be welcomed in any portfolio. Redmond and Cubbage (1988) confirm this appeal by demonstrating that sawtimber investments can reduce portfolio risk dramatically since the returns on these investments are negatively correlated with market returns.[34] The assumption underlying their analysis was that changes in stumpage prices were representative of potential timberland investment returns. The implication from their results is that based on the systematic risk underlying timber investments, the required rate of return on sawtimber investment would be less than the risk free rate of return on US Treasury Bills (i.e., negative premium for risk). Furthermore, this result was confirmed for sawtimber investments in various regions of the US. Stevens (1987) also cites that the low risk associated with timber returns in Maine will make these investments attractive to risk averse investors.[35]

CAPM Derived Risk-Adjusted Discount Rates for Timberland Investments

Zinkhan (1988) provides a straightforward approach to deriving the appropriate (risk-adjusted) discount rate for timberland investments. The procedure is to estimate the systematic risk for timberland investments and then to determine the discount rate from CAPM's security market line.[36] Like Redmond and Cubbage (1988), Zinkhan found

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[34] Pulpwood investments were also found to be appealing for reducing portfolio risks, but in this case the return to pulpwood investments were positively correlated with the market.

[35] However, Stevens' (1987) analysis was not conducted in a CAPM framework. Stevens defined market risk as being synonymous with stumpage price risk, but did not measure the correlation between changes in stumpage prices and the market rate of return.

[36] The security market line in CAPM relates an asset's expected (i.e., risk-adjusted) rate of return to its systematic risk (see Copeland and Weston, 1983; Brigham, 1985).

that the returns from southern timber investments were negatively correlated with the market.[37]

Zinkhan (1988) notes that a discount rate derived using a CAPM approach is only applicable to capital budgeting decisions made by individuals who hold well diversified portfolios, or by firms whose shareholders hold well diversified portfolios. Furthermore, Zinkhan points out that in general the systematic risk for a forest products firm will not reflect the systematic risk for timberland investments since forest products firms are usually dominated by a timber processing division. According to Fortson (1986), the failure to recognize this discrepancy (as well as the debt advantage ascribed to holding timberlands) has led many firms to reject land and timber acquisitions which should have been accepted had the discount rate been properly adjusted for risk (and debt capacity), the implication being that the systematic risk for the processing division is much higher than for the timberland division. It is no small wonder that as forest products firms divest themselves of their timberland holdings, institutional investors (e.g., insurance and pension funds) are eager to add them to their portfolios.[38]

<u>Capital Budgeting Techniques in the Forest Products Industry</u>

Redmond and Cubbage (1985) surveyed the US forest products industry to assess what techniques were being used by managers in capital budgeting decisions. Almost all companies surveyed relied on some type of DCF technique, with the internal rate of

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In both Redmond and Cubbage (1988) and Zinkhan (1988), the Standard and Poor 500 market index was used to represent the market rate of return. However, instead of focusing on stumpage prices, Zinkhan (1988) used a timberland value index as a proxy for the rate of return from timberland investments.

38

Bush III, C. E. 1986. Vice president, Continental Forest Investments, Inc. Savannah, Georgia. Keynote Address to the Sixteenth Forestry Forum of the School of Forestry and Wildlife Resources: How is Southern Forestry Changing? April 24-25. Virginia Polytechnic Institute and State University, Blacksburg, Virginia.

return (IRR) criterion being the predominant favorite. With respect to the choice of the discount rate (or the hurdle rate in the IRR case), most companies use a weighted average cost of capital. However, respondents were ambiguous in specifying whether the cost of capital measure contained an explicit adjustment for risk. For those firms that did consider the discount rate as being risk-adjusted, there was still ambiguity regarding whether the adjustment was subjective or relied on a formal analysis techniques like CAPM. Furthermore, few firms considered the risk effects of project acceptance when selecting the discount rate. While most firms viewed timber investments as being less risky than other investment opportunities (58 percent[39]), they predominantly used the same discount rate across all divisions. Finally, 90 percent of the respondents relied on a single discount rate for all timber investment evaluation. However, two respondents (out of 40) lowered the discount when evaluating long-term investments.

In general, most firms relied on subjective approaches to risk. Of the firms surveyed, 90 percent never measured the covariances between projects, 77 percent never used CAPM, 48 percent did not estimate the expected cash flow[40], and 43 percent never performed simulation analyses (Redmond and Cubbage, 1985). The most frequently stated approaches (in order of use) were sensitivity analysis, raising the discount rate, and subjective adjustments to the cash flow. Finally, the most frequent definition of risk was the probability of not meeting a target rate of return from the project investment; while more traditional definitions such as variance and systematic risk were infrequently used (Redmond and Cubbage, 1985).

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39 Only 11 percent considered timber investments as being more risky.

40 However, there was some ambiguity in the survey regarding the definition of the expected cash flow. Redmond and Cubbage (1985) defined expected cash flow estimates as being the same as using certainty equivalents.

Discount Rate Issues in Forestry

Not surprisingly, there are divergent views concerning the choice of discount rate to use in the evaluation of forestry investments. In essence, the concerns parallel the argument by Hayes and Garvin (1982) that DCF techniques may be myopic (Klemperer, 1976). Samuelson (1976) argues that the forester's passion for the future has led to the use of discount rates that are too low. Casual inspection of the forestry literature indicated that the real before-tax discount rates used by research analysts ranged from 2 to 12 percent (Cathcart and Klemperer, 1988a). Furthermore, analysts paid little attention to whether the chosen rate was risk-adjusted or not[41], or whether it is appropriate for the decision environment being addressed. Only recently have analysts begun to stress the importance of selecting a discount rate consistent with the decision environment the investment is being considered under (Fortson, 1986; Zinkhan, 1988). If one accepts that the risk free discount rate is around 2 to 3 percent (real before-tax), the conclusion is that most analysts implicitly include a risk premium in the discount rate.

Manning (1977) advocates a dual rate approach. According to Manning, long-term stand level investment decisions should employ a relatively low social opportunity cost measure, while harvest scheduling decisions should employ a relatively higher private opportunity cost measure. Manning's underlying premise is that public investment should maximize social welfare; therefore a social opportunity cost measure should be used.[42] Foster (1979a) introduces the notion of duration dependent discount

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[41] The only exception being a study by de Naurois and Buongiorno (1986) in which the 3 percent real before tax discount rate included an explicit 0.5 percent adjustment for risk.

[42] Manning's (1977) argument targets forestry in British Columbia, Canada; where timberland assets are primarily held in the public domain.

rates. Foster's argument is that earlier cash flows should be assessed at higher rates to represent the short term opportunity costs of invested funds. However, distant cash flows should be evaluated under lower rates which assure the permanence of long term goals that provide social benefits beyond the limited time horizon underlying the opportunity cost of capital. Binkley (1981) questioned whether the declining discount rate schedule proposed by Foster was due to declining social attitudes toward meeting short term opportunity costs the farther in the future the cash flow is. Binkley felt that the declining schedule could be explained in terms of how investors react to uncertainty. While both studies are ambiguous in terms of the empirical evidence supporting the existence of their respective schedules, the ideas (as well as those in Manning, 1977) at least break the tenet that forestry investment cash flows should be discounted with a single rate applicable to all periods.

Foster (1979b) introduces the concept of risk-adjusted discount rates in the context of risk neutrality on the part of the decision maker.[43] Under Foster's approach, the risk adjustment is applicable to discounting the best outcome in a win or lose (receive nothing) situation. However, Foster failed to recognize that the risk adjustments are unnecessary since his approach is equivalent to discounting expected cash flows with a risk free rate. Similar arguments apply to the risk-adjusted discount rate in Reed (1984).[44] In essence, the risk adjustments in Foster (1979b) and Reed (1984) are just refinements in the discount rate that sets the present value of the best possible cash flow outcome equal to the cash flow's expected present value under a risk free rate.

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[43] Foster (1979b) was criticized by Chang (1980) for his assumption of risk neutrality on the part of the decision maker. Chang (1980) argued that a risk averse attitude was a more appropriate assumption for investment decision making under uncertainty.

[44] However, Reed's (1984) analysis was designed to show that the risk of fire had the same effect on the optimal rotation age decision as an increase in the discount rate (i.e., shorten the optimal rotation age).

The Discount Rate for Public Forest Investment

Row *et al.* (1981) argued for a 4 percent real before-tax discount rate for use by the USDA[45] Forest Service in economic and social analysis; as opposed to the 10 percent (real before-tax) rate suggested by the Office of Management and Budget (OMB). In contrast, Baumol (1967), Stockfish (1967), and Hanke and Anwyll (1980) would argue that a 4 percent discount rate for public investment evaluation was too low. In essence, the arguments by Baumol and Stockfish, who support the notion that the public rate of discount should reflect private sector opportunity costs, led to OMB's 10 percent rate. Furthermore, Stockfish argues the risk premium component should be as high as 3 percent. However, the USDA Forest Service is currently (*circa* 1989) using the 4 percent figure, primarily on the recommendation of Row *et al.* (1981).

The premise underlying Row *et al.*'s (1981) arguement was that the 4 percent rate reflected the marginal (as opposed to average) real before-tax rate of return on long-term Aaa corporate bonds over the historical period 1960-1978. Furthermore, Row *et al.* felt that these securities were essentially risk-less since they are issued by large corporations with stable earning records (i.e., blue chip securities). However, the contention was not that forestry investments were less risky than other investment opportunities, but instead that the discount rate should represent a risk free opportunity cost with allowances for risk made elsewhere in the analysis. This latter point is often overlooked by many analysts who adopt the 4 percent rate and apply it in a private (i.e., corporate) context without making any other explicit adjustments for risk.

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[45] United States Department of Agriculture

RISK ASSESSMENT TECHNIQUES IN GENERAL

Many risk assessment techniques have received attention by researchers, especially those in agricultural economics (Barry, 1984). These models can generally be classified as expected utility, safety first, lexicographic ordering, stochastic dominance, risk programming, or econometric models. Bosch *et al.* (1986) combined stochastic dominance rules with expected utility preferences to assess the required compensation to irrigators as a result of policies targeting the restriction of water use. Roy (1952), Charnes and Cooper (1959), and Kataoka (1963) provide examples of the application of safety first rules to decision analysis. In general, the approach is either to: subject the objective function (e.g., expected profit) to a safety constraint (e.g., probability of loss) (Charnes and Cooper, 1959); optimize the safety rule (e.g., minimize the probability of loss) (Roy, 1952); or some combination of the two approaches (Kataoka, 1963). Plye and Turnovsky (1971) show that assumptions regarding relative risk aversion (i.e., increasing, constant, or decreasing with respect to wealth) implies the adoption of a specific safety first rule.

Lexicographic ordering relies on an explicit ranking of objectives in the order (highest to lowest) that they must be met; the solution is then to optimize the lowest ranked objective subject to the other objectives being met. However, Lin *et al.* (1974) found that the subjective nature of this approach (i.e., the ranking of the objectives) made it difficult for analysts to explain rational behavior on the basis of lexicographic ordering. Just and Pope (1978) review properties of stochastic production functions and the econometric procedures necessary to estimate them. Finally, Freund (1956) introduced quadratic programming in the context of a farm production control problem. Freund showed that diversification was a rational choice for expected utility maximizers. Not surprisingly, Markowitz (1959) used quadratic programming as the solution technique to the mean-variance portfolio problem. Mapp Jr. and Helmers (1984) discuss linear programming approximation methods for solving quadratic programming problems, while Musser *et al.* (1984) discuss applications of risk programming models in general. Finally, Raiffa (1968) gives a complete overview of decision making under uncertainty.

The expected utility model is the broadest category and is based on a well defined set of preference ordering axioms (Markowitz, 1959; Anderson *et al.*, 1977).[46] For example, in a mean-variance application of expected utility,[47] the axiomatic conditions imply that for a given expected (mean) outcome, the opportunity with the lower variance of outcomes (risk) is the opportunity that gives the highest expected utility. Risk averse individual's are formally classified as individuals who exhibit positive, but declining, marginal utility with respect to wealth.[48] Applications of the expected utility model are diverse. Expected utility provides the basis for CAPM and underlies the risk-adjusted discount rate and certainty equivalent approaches to risk. Sandmo (1971) applied the model to the theory of the firm under uncertainty, while Brookshire *et al.* (1985) used the model to explain the existence of price differentials between houses located in and out of earthquake hazard zones. Machina (1987) provides an up to date summary of the issues underlying the general expected utility model.

Other techniques target specific steps underlying a more general approach to risk. Spetzler and Staël von Holstein (1975), and Norris and Kramer (1986), discuss

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[46] The basis for these axioms was first derived in Neumann and Morgenstern (1953).

[47] The mean-variance case of the expected utility model holds when the probability distribution controlling outcomes is normal; or when the individual's utility function for wealth is quadratic (Tobin, 1958; Markowitz, 1959).

[48] This condition is also consistent with Hirshleifer's (1965) definition of risk conservative behavior.

techniques designed to elicit an individual's subjective probability distribution regarding the likelihood of future events. Anderson *et al.* (1977) discuss the elicitation of an individual's wealth utility function, as well as multi-attribute utility preferences. Hillier (1963) identifies the conditions under which the probability distribution underlying present value (e.g., NPV) outcomes may be considered normal. Gupta and Rosenhead (1968) stress that future decisions should not be treated as if they have to be made today. Instead, decisions should be defined as policies whereby appropriate actions are guided by the realization of future events. Markowitz (1959) contains an excellent theoretical model that defines the adaptive nature inherent in a sequential decision process. Finally, the development of artificial intelligence systems for natural resource management (Coulson *et al.*, 1987) will open up new applications for decision making under uncertainty; primarily because expert systems include the latest in simulation technology that allows decisions to adapt to, and interact with, changing environments.

ANALYTICAL TREATMENT OF THE RADR PROBLEM

THE INDIVIDUAL MODEL

As indicated in the literature review, the predominant use of risk-adjusted discount rates pertains to capital budgeting in a corporate context. For the most part, analysts rely on a single rate for discounting all components in the cash flow. But as Fama (1977a) points out, this is only appropriate under a strict set of restrictive assumptions. Most analysts tend to agree that there is no *a priori* reason to rely on a single discount rate in a multiperiod setting. Furthermore, there is really no *a priori* reason to apply the same rate to all investment projects being considered. Therefore, the problem with RADRs does not lie within the theoretical basis for their justification, but in their use in application. The reason for this is that the theoretically correct adjustments can at best be subjectively approximated in application, which reinforces the tendency to rely on a single rate. Therefore, questioning the application of risk-adjusted discount rates can only be assessed by solving the risk problem using an alternative method. This is the approach taken here.

The theoretical model from which to question the use of risk-adjusted discount rates is the certainty equivalent approach to risk. Lintner (1965a) demonstrated this in a market equilibrium context, while Robichek and Myers (1966) and Chen (1967) provided a more general illustration. In this chapter, the key elements of their analyses are presented in the context of an individual. Expected utility is the criterion measuring individual welfare and is used to define the certainty equivalent adjustments to the cash flow.

UTILITY OF WEALTH

The individual's utility of wealth function maps realized wealth positions to a measure of satisfaction (i.e., utility). For risk averse individuals, more wealth is better, but less so as one gets more and more (i.e., diminishing marginal utility with respect to wealth). Utility functions which are concave from below (i.e., $U'(\cdot) > 0$, $U''(\cdot) < 0$) exhibit this property. Two classes of utility functions are considered below.[49]

LOGARITHMIC UTILITY:

$$U(w) = LOG(w), \quad w > 0 \qquad [2a]$$

NEGATIVE EXPONENTIAL UTILITY:

$$U(w) = 1 - e^{-cw} \qquad [2b]$$

Measuring the Degree of Risk Aversion

Pratt (1964) formalized measures of the individual's degree of risk aversion. In essence, these measures describe the individual's risk preferences in the neighborhood of the current wealth position. Two measures were presented.[50]

Absolute Risk Aversion:

$$\lambda(w) = -\left(\frac{U''(w)}{U'(w)}\right) > 0 \qquad [3]$$

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[49] Lower case notation is used to denote some realized outcome for wealth.

[50] These measures are commonly referred to the Arrow and Pratt measures of risk aversion (e.g., see Barry, 1984) since K. J. Arrow independently derived similar measures in unpublished work (see Pratt, 1964). Pratt (1964) refers to absolute risk aversion as local risk aversion and relative risk aversion as local proportional risk aversion.

Relative Risk Aversion:

$$\psi(w) = w\lambda(w) \qquad [4]$$

where for

LOGARITHMIC UTILITY:

$$\lambda(w) = \frac{1}{w} \qquad [3a]$$

$$\psi(w) = 1 \qquad [4a]$$

NEGATIVE EXPONENTIAL UTILITY:

$$\lambda(w) = c \qquad [3b]$$

$$\psi(w) = wc \qquad [4b]$$

As indicated in (3a) and (4a) the logarithmic class of utility functions exhibits decreasing absolute risk aversion with respect to wealth (i.e., $\partial\lambda(w)/\partial w < 0$), but constant relative risk aversion (= 1).[51] In contrast, the negative exponential class of utility functions exhibits constant absolute risk aversion with respect to wealth (= c), but increasing relative risk aversion (i.e., $\partial\psi(w)/\partial w > 0$).

The property of decreasing absolute risk aversion is considered to be an intuitive representation of risk aversion (Arrow, 1970).[52] Furthermore, the constant relative risk aversion property is also considered to be a plausible representation of risk aversion (Rubinstein, 1976b). However, Arrow (1970) does not dismiss the case of increasing relative risk aversion, while Graves (1979) favors decreasing relative risk aversion. In a review of the literature, Ryu and Vora (1988) found empirical estimates for relative risk

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51 Rubinstein (1976b) shows that the logarithmic class of utility functions can be generalized to exhibit either increasing or decreasing relative risk aversion.

52 In contrast, the property of increasing absolute risk aversion with respect to wealth (i.e., $\partial\lambda(w)/\partial w > 0$), which occurs for the quadratic class of utility functions, is targeted as being counter-intuitive (Arrow, 1970; Anderson *et al.*, 1977).

aversion to range from 0.1 to 7.0, with most analysts settling on a value in the neighborhood of 4.0 (e.g., see Litzenberger and Ronn, 1986). For typical wealth positions, this implies that absolute risk aversion will be well below 1. Using stochastic dominance techniques, Wilson and Eidman (1983) elicited absolute risk aversion coefficients in the neighborhood of 0.0002.

EXPECTED UTILITY

In the face of uncertainties, the individual's wealth position from holding claims to the investment opportunity is uncertain; that is, depends on the realization of investment returns. However, for each possible outcome there is a well defined utility ascribed to that outcome. Therefore, the individual's expected utility from holding claims to the asset can be formally defined as

$$\mathrm{E}[\mathrm{U}(W)] = \int (\mathrm{U}(\mathrm{W}_o + x)h(x))\mathrm{d}x \qquad [5]$$

where

$$W = \mathrm{W}_o + x \qquad [6]$$

represents the uncertain wealth position from holding claims to the investment opportunity with uncertain return x. In essence, (5) defines the expected utility as the expected value of all possible utility outcomes from holding claims to the uncertain investment opportunity. Therefore, expected utility provides a measure for the ranking of uncertain investment opportunities; the higher the expected utility, the more preferred the opportunity. Furthermore, the axiomatic principles regarding the preference ordering of opportunities under expected utility hold for any positive linear transformation of the utility of wealth function (Anderson *et al.*, 1977).

Moment Distribution Approximation of Expected Utility

As an approximation to (5), Anderson *et al.* (1977) describes a moment method to express expected utility as a function of the distributional moments for $h(x)$. This turns out to be a much more convenient specification of expected utility for empirical analysis since it is the moments of $h(x)$ that are commonly estimated. Furthermore, the moment distribution approximation method allows risk adjustments to be defined with respect to specific properties of $h(x)$ (e.g., variance and skewness).

Under the moment distribution approximation method, the utility of wealth function is expanded around the mean wealth position from holding claims to the uncertain investment opportunity using a Taylor series expansion (Anderson *et al.*, 1977).[53]

$$\mathrm{U}(W) \equiv \mathrm{U}(\mathrm{E}[W]) + \mathrm{U}'(\mathrm{E}[W])(W - \mathrm{E}[W]) + \left(\frac{\mathrm{U}''(\mathrm{E}[W])(W - \mathrm{E}[W])^2}{2!} \right)$$

$$+ \ldots + \left(\frac{\mathrm{U}^n(\mathrm{E}[W])(W - \mathrm{E}[W])^n}{n!} \right) + \ldots \qquad [7]$$

where: $\mathrm{U}(\mathrm{E}[W])$ - is the function $\mathrm{U}(\,\bullet\,)$ evaluated at $\mathrm{E}[W]$.
$\mathrm{U}'(\mathrm{E}[W])$ - is the partial $\mathrm{U}'(\,\bullet\,)$ evaluated at $\mathrm{E}[W]$.
$\mathrm{U}''(\mathrm{E}[W])$ - is the partial $\mathrm{U}''(\,\bullet\,)$ evaluated at $\mathrm{E}[W]$.
$\mathrm{U}^n(\mathrm{E}[W])$ - is the partial $\mathrm{U}^n(\,\bullet\,)$ evaluated at $\mathrm{E}[W]$; $n \in \{3, 4, \ldots\}$.

Taking the expectation of (7) results in[54]

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53 Note that $\mathrm{U}(W)$ implies that utility is being evaluated with respect to some wealth position from holding claims to the investment; the exact position being uncertain. Furthermore, an identity is used in (7) to highlight that (7) is not an approximation as $n \rightarrow \infty$.

54 Note that $\mathrm{E}[W]$ is a given parameter. Therefore, $\mathrm{U}(\mathrm{E}[W])$, $\mathrm{U}'(\mathrm{E}[W])$, $\mathrm{U}''(\mathrm{E}[W])$ and, $\mathrm{U}^n(\mathrm{E}[W])$ ($n = 3, 4, \ldots$) are non-stochastic.

$$E[U(W)] \equiv U(E[W]) + \left(\frac{U''(E[W])E[(W - E[W])^2]}{2!}\right)$$

$$+ \ldots + \left(\frac{U^n(E[W])E[(W - E[W])^n]}{n!}\right) + \ldots \qquad [8]$$

since $E[(W - E[W])] = 0$. However, by substituting in (6) for W and noting $E[(W - E[W])^n] = E[(x - E[x])^n]$ for all $n \in \{2, 3, \ldots\}$ (since W_0 is deterministic), then (8) can be written as

$$E[U(W)] \equiv U(W_0 + E[x]) + \left(\frac{U''(W_0 + E[x])E[(x - E[x])^2]}{2!}\right)$$

$$+ \ldots + \left(\frac{U^n(W_0 E[x])E[(x - E[x])^n]}{n!}\right) + \ldots \qquad [9]$$

Therefore, according to Anderson *et al.* (1977) a useful (and close) approximation for expected utility in (9) is[55]

$$E[U(W)] = U(W_0 + E[x]) + \left(\frac{U''(W_0 + E[x])E[(x - E[x])^2]}{2!}\right)$$

$$+ \left(\frac{U^3(W_0 E[x])E[(x - E[x])^3]}{3!}\right) \qquad [10]$$

where $E[x]$ defines the mean, $E[(x - E[x])^2]$ the variance, and $E[(x - E[x])^3]$ the skewness of $h(x)$.

Since $U''(\bullet) < 0$ for any risk averse individual, any variance (for $E[x]$ fixed) will decrease expected utility. Furthermore, for the logarithmic (with $W > 0$) and negative exponential (with $c > 0$), $U^3(\bullet) > 0$. This implies that for a fixed mean and variance,

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55 Anderson *et al.* (1977) notes that the effects of higher moments beyond skewness can be considered negligible in most applications.

$h(x)$ positively skewed (i.e., $\mathbf{E}[(x - \mathbf{E}[x])^3] > 0$) increases expected utility, while $h(x)$ negatively skewed (i.e., $\mathbf{E}[(x - \mathbf{E}[x])^3] < 0$) decreases expected utility. With respect to skewness, the tradeoff is that while the most likely outcome (i.e., the mode) is smaller for the positively skewed distribution, this is compensated by the higher probability of extreme gains. However, even though the negatively skewed distribution has a larger mode, this is offset by the greater probability of a extreme losses (figure 1). In essence, risk aversion implies favoring investment opportunities with probability distributions skewed towards larger gains, and disfavoring investment opportunities with probability distributions skewed towards extreme losses.

THE PARALLELISM BETWEEN THE RADR AND CE APPROACHES TO RISK

The expected utility model just presented provides the theoretical basis for both the certainty equivalent (CE) and risk-adjusted discount rate (RADR) approaches to risk. Note further that the model applies to any asset with uncertain returns x. However, the implicit assumption underlying the expected utility model's direct application to project investment cash flows is that the investment opportunities in the asset are mutually exclusive, and that the chosen opportunity defines the individual's portfolio.[56] Under these assumptions, cash flow realizations represent increments to future wealth and the parameters (i.e., variance and skewness) of the cash flow component probability distributions define each component's risk. It is also convenient (but not necessary) to assume that the individual's utility of wealth is both time and state independent; that is, regardless of how far in the future and under what course of events it may follow, the same utility of wealth function applies.

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[56] For example, think of a non-industrial private land owner whose only investment opportunity is to manage his or her land for timber or agricultural production. In this case, both diversifiable and non-diversifiable risks become relevant to the decision.

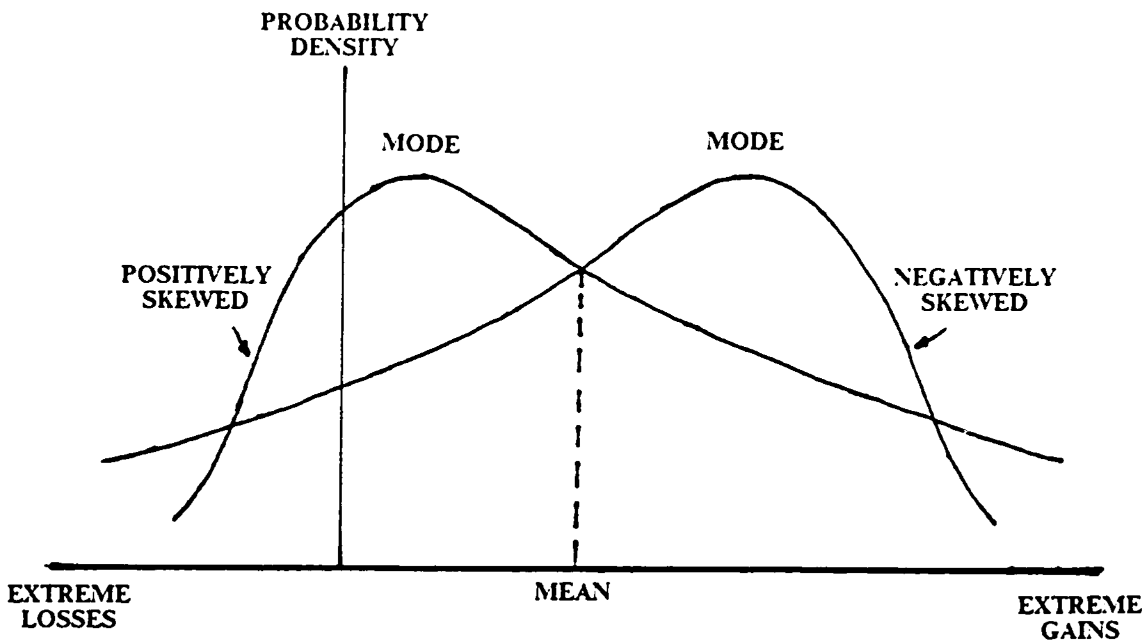

Figure 1. Comparison of skewed probability distributions with a common mean and variance.

Under these conditions, the certainty equivalent value for an uncertain cash flow component occurring t years hence is defined by the following relationship[57]

$$U(W_o + CE_t) = E[U(W)] \qquad [11a]$$

$$W = W_o + CF_t \qquad [11b]$$

where $E[U(W)]$ is defined by the approximation in (10). The certainty equivalent value CE_t is such that the certain utility $U(W_o + CE_t)$ equals the expected utility $E[U(W_o + CF_t)]$ from holding future claims to the uncertain cash flow component occurring t years hence. That is, (11) defines indifference between the certain and risky alternatives.

Adjusting Uncertain Revenue Cash Flow Components for Risk

An uncertain revenue inflow in year t implies $E[CF_t] > 0$. Applying the relationship in (11) results in (due to the concavity of $U(\bullet)$)

$$CE_t < E[CF_t] \qquad [12]$$

since risk averse individuals are willing to accept lower expected cash flows for lower risk, and there is no risk in accepting the certain amount CE_t. Rearranging (12), the cash flow component risk premium is defined as

$$E[CF_t] - CE_t = RP_t > 0 \qquad [13]$$

In essence, the risk premium RP_t is the extra amount of expected cash flow value necessary to compensate the investor for the cash flow component's underlying risk such that the investor is indifferent between the risky cash flow and the certain alternative.

Noting (12), the following then must also hold

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57 While not necessary, (11) assumes that the individual's wealth position at the beginning of each period does not vary (i.e., equal to W_o) over time.

$$\left(\frac{CE_t}{(1+f)^t}\right) < \left(\frac{E[CF_t]}{(1+f)^t}\right) \qquad [14]$$

Furthermore, since (12) and (14) hold for any $t \in \{1, 2, \ldots\}$, then

$$NPV_{CE} < E[NPV] \qquad [15]$$

where:

$$NPV_{CE} = \sum_t \left(\frac{CE_t}{(1+f)^t}\right) \qquad [15c]$$

$$E[NPV] = \sum_t \left(\frac{E[CF_t]}{(1+f)^t}\right) \qquad [15d]$$

Therefore, the individual's risk premium (in present value terms) necessary to compensate the investor for the risk underlying the investment's entire cash flow is defined as

$$RP_{NPV} = E[NPV] - NPV_{CE} > 0 \qquad [16]$$

As an alternative to the net present value risk premium in (16), the discount rate can be adjusted upward such that the risk premium in (16) vanishes. That is, the risk adjustment is now made in the discount rate such that[58]

$$NPV_{RADR} = NPV_{CE} \qquad [17]$$

Conceptually, there will be more than one RADR (i.e, one for each uncertain revenue component in the cash flow). Therefore, for (17) to hold, the following must also hold

$$\left(\frac{CE_t}{(1+f)^t}\right) = \left(\frac{E[CF_t]}{(1+g_t)^t}\right) \text{ for any } t \in \{1, 2, \ldots\} \qquad [18]$$

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[58] Note that NPV_{RADR} in (17) should be interpreted as a different measure of the certainty equivalent net present value NPV_{CE} (Hirshleifer, 1961).

Furthermore, since $CE_t < E[CF_t]$ holds for any $t \in \{1, 2, \ldots\}$, then for (18) to hold, $g_t > f$, and g_t (as applied to uncertain revenues) can be structured as

$$(1 + g_t) = (1 + f)(1 + rp_t), \quad rp_t > 0 \qquad [19a]$$

or,

$$g_t = f + rp_t + f(rp_t), \quad rp_t > 0 \qquad [19b]$$

In (19a), the risk-adjusted discount rate factor $(1 + g_t)$ is decomposed into the risk free factor $(1 + f)$ and the factor $(1 + rp_t)$ representing the adjustment for risk. The proper discount rate adjustments to explicitly account for the individual's attitude toward the risk underlying the revenue cash flow (which is measured by the present value risk premium defined in (16)), are the discount rate risk premiums $rp_t > 0$ such that the equality in (18) holds for all relevant $t \in \{1, 2, \ldots\}$. Solving (18) for g_t and noting (19b) results in[59]

$$g_t = \left(\frac{E[CF_t]}{CE_t}\right)^{\frac{1}{t}}(1 + f) - 1 \qquad [20a]$$

or,

$$f + rp_t + f(rp_t) = \left(\frac{E[CF_t]}{CE_t}\right)^{\frac{1}{t}}(1 + f) - 1 \qquad [20b]$$

or,

$$rp_t = \left[\left[\frac{1}{\alpha_t}\right]^{\frac{1}{t}} - 1\right] > 0 \qquad [20c]$$

where:

$$\alpha_t = \frac{CE_t}{E[CF_t]} \qquad [20d]$$

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59 See Chen (1967) for a similar derivation of (20c). However, Chen's result differs slightly from (20c) in that Chen considers the cross product term, $f(rp_t)$ to be negligible.

since for the risk averse case,[60] $1/\alpha_t > 1$. Note further that the ratio, $1/\alpha_t = \mathrm{E}[CF_t]/\mathrm{CE}_t$ is directly related to the cash flow component risk premium RP_t defined in (13). For a fixed expected cash flow component value $\mathrm{E}[CF_t]$ the larger RP_t, the larger the ratio $\mathrm{E}[CF_t]/\mathrm{CE}_t$, and accordingly, the larger the risk-adjusted discount rate g_t; or equivalently, the larger the discount rate risk premium, rp_t. In fact, the inverse ratio α_t (as defined in (20d)) is Robichek and Myers (1966) certainty equivalent adjustment factor and provides a scale free measure of the cash flow component's risk (the smaller α_t, the more risky the cash flow component is considered to be).

Equation (20) explicitly shows that the appropriate risk adjustment in the discount rate is unique to each cash flow component. Two relations appear in (20c). The appropriate risk premium rp_t, is a function of

1) The individual's certainty equivalent adjustment to the expected cash flow, α_t.
2) The timing of the cash flow component, t.

The implication is that a separate discount rate adjustment should be made for each revenue flow component in the investment. That is, the net present value under the RADR approach (as applied to revenue inflows) should be calculated as follows

$$\mathrm{NPV}_{\mathrm{RADR}} = \sum_t \left(\frac{\mathrm{E}[CF_t]}{(1+g_t)^t} \right), \quad g_t > f \qquad [21]$$

However (with $0 < \mathrm{CE}_t < \mathrm{E}[CF_t]$), there will be a single risk-adjusted discount rate $k > f$ appropriate for discounting the entire expected revenue cash flow such that (Chen, 1967)

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This condition only holds if $\mathrm{CE}_t > 0$. For the case $\mathrm{CE}_t < 0 < \mathrm{E}[CF_t]$, (20c) becomes ambiguous in interpretation. In essence, $\mathrm{CE}_t < 0$ implies that the revenue component, in spite of its positive mean value, is valued as representing a loss once adjusted for risk. In this case, the RADR method cannot be correctly applied unless $g_t < -1$. For more on the interpretation of certainty equivalent adjustments for cases like this (with respect to $\mathrm{E}[NPV]$), see appendix A.

$$\text{NPV}_{\text{RADR}} = \sum_t \left(\frac{\text{E}[CF_t]}{(1+\text{k})^t} \right) = \sum_t \left(\frac{\text{E}[CF_t]}{(1+\text{g}_t)^t} \right), \quad \text{g}_t,\ \text{k} > \text{f} \qquad [22]$$

The implication in (22) is that k can only be determined by knowing the correct values for g_t for all relevant t. However, given a different investment alternative, with different risk underlying its revenue cash flow, the solution to (22) will most likely result in a different value for k. In essence, knowing the correct specification for the risk-adjusted discount rate implies already solving the problem *via* the certainty equivalent approach.

Adjusting Uncertain Cost Cash Flow Components for Risk

An uncertain cost outflow in year t implies $\text{E}[CF_t] < 0$. In the case of uncertain costs, the relationships in (12)-(17) will still hold for all relevant $t \in \{1, 2, \ldots\}$. The result $\text{CE}_t < \text{E}[CF_t]$ implies that a risk averse individual will be indifferent to paying a higher certain cost (in absolute value), or incurring the uncertain cost (with expected value $|\text{E}[CF_t]|$) with its risk. Therefore, in absolute value terms $|\text{CE}_t| > |\text{E}[CF_t]|$. Therefore in the case of uncertain costs, for (18) to hold, $g_t < f$. That is, for uncertain cost cash flow components the correct specification for g_t becomes

$$\text{g}_t = \text{f} + \text{rp}_t + \text{f}(\text{rp}_t), \quad \text{rp}_t < 0 \qquad [23]$$

since,[61] $1/\alpha_t < 1$ in (20c). As was the case for uncertain revenues, the risk adjustment for uncertain costs is a function of the individual's certainty equivalent adjustment to the expected cash flow and the timing of the cost cash flow component. However, the striking result is that for uncertain costs, the correct risk-adjusted discount rate to apply

61 Note that with $\text{CE}_t < \text{E}[CF_t] < 0$, $\alpha_t = \text{CE}_t/\text{E}[CF_t] > 1$ since $|\text{CE}_t| > |\text{E}[CF_t]|$. For a more general discussion of this result (with respect to certainty equivalent adjustments to $\text{E}[NPV]$), see appendix A.

will be less than the risk free rate (i.e., $g_t < f$ for all relevant $t \in \{1, 2, \ldots\}$ in (21), or $k < f$ in (22)).[62] Therefore, applying a risk-adjusted discount rate with a positive premium for risk will represent a under-compensation for cost risk, even for the case when costs are considered certain.

THE PROBLEM WITH RISK-ADJUSTED DISCOUNT RATES

In the strict sense, there is no conceptual problem with applying risk-adjusted discount rates in uncertain capital budgeting decisions (Chen, 1967). Furthermore, given a well-functioning capital market in equilibrium where individuals hold well diversified portfolios of assets, the correct risk-adjusted discount rate (or rates) to apply in capital budgeting decisions are well defined. The consensus in the finance literature seems to be that as long as proper judgmental decisions are made regarding their selection, the RADR approach provides a useful means for guiding project decisions.

However, when one abstracts from the general equilibrium framework, and focuses on project investment decisions in a partial equilibrium framework, the correct application of the RADR approach is strictly conceptual. That is, when emphasis is placed on project investment risk, where both diversifiable and non-diversifiable risks become implicitly relevant, there are no *a priori* criteria for guiding the proper selection of the risk-adjusted discount rate (or rates). As the analytical results underlying (12) through (23) indicate, the RADR problem can be summarized as

1) The appropriate risk adjustment in the discount rate will be unique to each component in the investment's cash flow.

62

However, Lewellen (1977) points out that when the risk adjustments pertain to systematic risks, then increasing the discount rate to account for cost risk may be appropriate when costs outcomes are negatively correlated with the market return. However, Arrow and Lind (1970) favor a downward adjusted rate (below the level of the risk free rate) for discounting uncertain costs incurred by risk averse individuals.

2) While in most cases 1 above can be accounted for with a single risk-adjusted discount rate applicable to the entire cash flow, invariably it will be unique to each alternative considered. Furthermore, the rate applicable to uncertain revenue inflows will most likely differ from the rate applicable to uncertain cost outflows.

3) In order to determine the appropriate risk-adjusted discount rate (or rates) in 2 above, the problem must already have been solved *via* the certainty equivalent approach to risk.

This study explicitly addresses the implications arising from these concerns in the context of stand level investment decisions set in a partial equilibrium framework.

METHODS

THE DECISION ENVIRONMENT

As indicated earlier, analyses addressing risk must be explicit about the decision environment they address. The problem analyzed in this study was a risk averse individual facing opportunities in either agricultural or timber production on a given site. The alternatives considered contrasted differences in cash flow (e.g., duration and payoff pattern), objectives, capital requirement, and presumably risk; providing distinct areas in which RADR biases may occur. The individual model was chosen because it defines a closed environment surrounding the investment decision. That is, the individual's opportunity set was restricted to the land asset which was assumed to be small enough to preclude diversification across uses. Finally, the analysis was static in that the investment decision was made today (i.e., year zero of the cash flow). These assumptions parallel the assumptions underlying deterministic stand level analyses conducted on a per acre basis; namely, a partial equilibrium analysis focusing on the returns to the land treating all other economic effects as exogenous.

The correct benchmark ranking of alternatives was based on the expected utility model as defined in equations (5) through (10) (Anderson *et al.*, 1977). The individual's expected utility from holding claims to a particular investment opportunity was a weighted function of the mean NPV for the alternative and the investment's risk. Risk was measured by the variance and skewness (toward losses) of the probability distribution of NPV outcomes. The individual's certain opportunity cost of invested capital was defined by his or her marginal rate of time preference and represented by the risk free rate of discount (Hirshleifer, 1965).

DEMONSTRATING RISK-ADJUSTED DISCOUNT RATE BIAS: THE HYPOTHETICAL CASE

The purpose of the hypothetical analysis was to demonstrate the source of RADR bias, namely cash flow components ultimately differ in risk, which implies a unique risk-adjusted discount rate for each component within the cash flow (i.e., the g_t in (18)). However, this also implies that different alternatives will differ in overall risk, that is each alternative will have a unique risk-adjusted discount rate (i.e., the k in (22)) that best captures the riskiness underlying the entire cash flow.

For the hypothetical analysis, the individual's wealth utility function was logarithmic (see equation (2a)). Two cases were considered; the first with $W_0 = \$20000$, and the second with $W_0 = \$60000$. Since logarithmic utility exhibits decreasing absolute risk aversion with respect to wealth, the $W_0 = \$20000$ case portrayed an individual who was more sensitive to risk. Wealth effects and investment valuation were in total dollars. Finally, the cash flow component probability distributions were all considered to be normal, therefore variance was the only relevant of risk. This assumption was necessary since logarithmic utility in general implies addressing risk beyond a mean-variance formulation of expected utility (e.g., considering skewness).[63] The risk free rate was assumed to be 2.0 percent.

Investment Alternatives Considered

Three classes of investments, which differ in duration (e.g., short versus long investment life) and payoff pattern (e.g., annual versus delayed), were considered (figure

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63 If the probability distributions were not normal, quadratic utility would of been implied by the mean-variance approach to risk (Tobin, 1958; Markowitz, 1959).

2).[64] The sawtimber investment represented an investment opportunity with a single initial cost and a delayed uncertain payoff in year 40. The pulpwood investment represented the same type of investment but under a shorter duration (20 years). The crop investment represented an even shorter duration investment (10 years) with annual (beginning in year 1) uncertain payoffs. As indicated in figure 2, the short duration investments were repeated such that all investment alternatives had a common life.

Within each class of investments, opportunities were defined by an assumed risk trend underlying the revenue cash flow. Cash flow risk was constructed to increase, decrease, or remain constant with respect to time (table 1).[65] A total of 15 alternatives were considered (defined by the 5 risk trends for each investment class). True indifference between all 15 alternatives was defined by setting the true certainty equivalent NPV (i.e., NPV_{CE}) for all alternatives equal to \$5,000. This was accomplished by holding the present value of the certain investment costs equal to \$1,000, and the present value of the certainty equivalent revenue cash flow equal to \$6,000.[66] Therefore, any explicit ranking of alternatives under a chosen RADR would reflect bias in the

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64

A fourth class, used only to illustrate RADR bias within a given class of alternatives, was also considered. Here, the revenue cash flow components occurred every 10 years for 60 years, and defined the return to a single up front investment cost in year zero.

65

The starting risk values used to generate the cash flow variances were scaled such that they were consistent with the magnitude of the expected cash flow. This was accomplished by selecting a starting risk value such that the probability of incurring a loss was around 0.16. An initial expected cash flow starting value (used only for scaling the starting risk value) was estimated by assuming a certainty equivalent adjustment factor of 0.75.

66

All present value targets were met under the 2 percent risk free discount rate. For the sawtimber investments, this gave a single certainty equivalent payoff in year 40 equal to \$13,247.80. For pulpwood investments, this gave two certainty equivalent payoffs in years 20 and 40 equal to \$5,329.13. For the crop investment, this gave an annual certainty equivalent payoff (beginning in year 1) equal to \$219.34 per year. Accordingly, sawtimber investments had an investment cost of \$1000.00, pulpwood investments had an investment cost of \$597.74 (repeated in year 20), and crop investments had an investment cost of \$328.36 (repeated in years 10, 20, and 30) (figure 2).

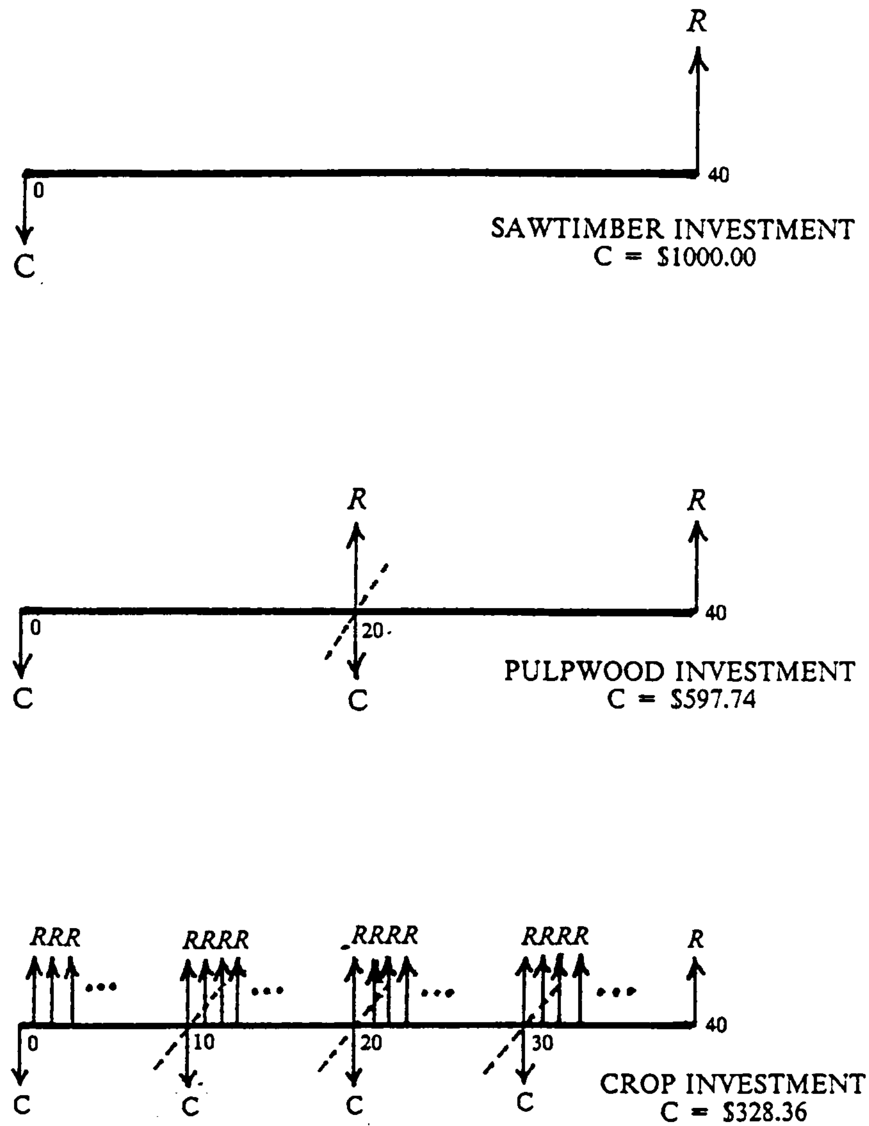

Figure 2. Three hypothetical investment types that differ in duration and payoff pattern.

C - denotes certain investment cost (dollars).
R - denotes uncertain payoff (dollars).

/ indicates that the investment is repeated.

••• indicates an annual payoff flow.

Table 1. Cash flow risk trends used in the hypothetical analysis.

I) Decreasing cash flow risk with respect to time[1]

Linear trend: $\sigma^2_{CF_t} = \sigma^2_{SV}(1 - MD(t - 1))$

Geometric trend: $\sigma^2_{CF_t} = \sigma^2_{SV}(1 - GD)^{t-1}$

II) Constant cash flow risk with respect to time

$$\sigma^2_{CF_t} = \sigma^2_{SV}$$

III) Increasing cash flow risk with respect to time

Linear trend: $\sigma^2_{CF_t} = \sigma^2_{SV}(1 + MI(t - 1))$

Geometric trend: $\sigma^2_{CF_t} = \sigma^2_{SV}(1 + GI)^{t-1}$

Notes:

(1) When risk is decreasing under the linear trend, cash flow component risk can conceivably go to zero and remain at zero thereafter.

$\sigma^2_{CF_t}$ - variance of the uncertain cash flow component in year t (dollars squared).

σ^2_{SV} - starting cash flow variance value (implied cash flow component risk in year 1 of the cash flow) (dollars squared).

MD - annual rate of linear decrease (%/100).

GD - annual rate of geometric decrease (%/100).

MI - annual rate of linear increase (%/100).

GI - annual rate of geometric increase (%/100).

ranking since the true ranking was one of indifference. Finally, the entire investment horizon (40 years) was considered to be uncertain, that is, when the shorter duration investments were repeated, the risk trends underlying the revenue cash flow remained in effect. For example, under an increasing risk trend, the payoff for the second (repeated) pulpwood investment is riskier (i.e., has a higher payoff variance) than the first.

The Certainty Equivalent (CE) Approach

For the hypothetical analysis, adjusting each cash flow component for risk (the CE approach) defined the true valuation of the investment alternatives considered.[67] However, since the cash flow certainty equivalent and the risk values were taken as given, the application of the certainty equivalent approach was backwards. For each revenue component in the cash flow, the problem was to solve for the expected value such that when adjusted for risk the predetermined certainty equivalent value would result.[68]

The utility of a given wealth outcome was given by[69]

$$U(w_t) = LOG(w_t), \qquad w_t > 0 \qquad [24]$$

where:

$$w_t = W_o + cf_t \qquad [24a]$$

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67 The underlying assumption being that the individual's utility of wealth function is applicable in all future time periods, and the initial wealth endowment remains constant over time. The risk free rate was set at 2 percent.

68 The necessary expected cash flow values themselves will be a function of the risk preferences of the individual as specified by the utility function for wealth. The analysis was restricted to revenue components as all investment costs were considered to be certain.

69 Lower case notation is used to denote realized outcomes for the random variables considered; in this case W_t and CF_t.

Using Anderson *et al.*'s (1977) moment distribution approximation of expected utility in (10)[70], the certainty equivalent-expected cash flow component value relationship is given by

$$U(W_o + CE_t) = E[U(W_o + CF_t)] \quad [25]$$

where:

$$U(W_o + CE_t) = LOG(W_o + CE_t) \quad [25a]$$

$$E[U(W_o + CF_t)] = LOG(W_o + E[CF_t]) - \frac{\sigma^2_{CF_t}}{2(W_o + E[CF_t])^2} \quad [25b]$$

That is, (25) specifies that the certainty equivalent value for the uncertain cash flow component is the value CE_t such that the utility from the certain wealth position $W_o + CE_t$ (given by $U(W_o + CE_t)$ in (25a)) equals the expected utility from holding claims to the uncertain cash flow component that defines the uncertain future wealth position $W_o + CF_t$ (given by $E[U(W_o + CF_t)]$ in (25b)). The relationship in (25b) explicitly defines the expected utility from holding claims to the uncertain cash flow component as a function of the mean outcome $E[CF_t]$ and its variance $\sigma^2_{CF_t}$.

The backwards approach taken in this analysis was to find the solution for $E[CF_t]$ in (25b) such that the expected utility $E[U(W_o + CF_t)]$ equaled the certain utility from the certainty equivalent value $U(W_o + CE_t)$ in (25a), thus satisfying the relationship in (25). For example, if W_o = \$20,000, $\sigma^2_{CF_t}$ = 1,397,034 dollars squared, CE_t = 1,889.96 dollars, then the expected cash flow value, $E[CF_t]$ for year t would be 1,921.82 dollars, since in (25b)

$$E[U(W_o + CF_t)] = LOG(21921.82) - \frac{1397034}{2(21921.82)^2} = 9.993784$$

70

See equation (10). Equation (25b) assumes that each cash flow component's skewness is zero due to the normality assumption regarding each component's probability distribution.

which approximately equals

$$U(W_0 + CE_t) = LOG(21889.96) = 9.993783$$

in (25a).[71]

Investment Valuation using Risk-Adjusted Discount Rates

The mean risk-adjusted net present value (i.e., NPV_{RADR} as defined in (22)) was calculated by discounting the derived expected revenue values for each cash flow component with the risk-adjusted rate.[72] The risk free rate was set at 2 percent, and risk-adjusted rates ranging from 2.5 to 6.0 percent were considered (i.e., approximating risk premiums of 0.5, 1.0, 1.5, ..., 4.0 percent respectively).[73]

Performing the Analysis

A computer program (in FORTRAN) was written to perform the analysis sequentially as follows:

1) Set up the investment cost and certainty equivalent revenue cash flows for each investment class considered such that the true certainty equivalent net present value (i.e., NPV_{CE}) target of $5,000 was met.
2) Determine the starting risk values for each investment class considered.

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71 The relationship in (25) was solved iteratively, with convergence defined by taking the antilog of (25a) and (25b) and having them be within 0.001 antilog units of each other.

72 In order to restrict the source of RADR bias to the evaluation of risk in the uncertain revenue cash flow, all costs (considered to be certain) were discounted with the risk free rate.

73 These approximations consider the risk premium as the difference between the risk-adjusted and risk free discount rates. But as indicated in (19b), the actual risk premium represents both additive and multiplicative adjustments to the risk free discount rate. In the strict sense, the actual premium is slightly less than the difference between the risk-adjusted discount and risk free discount rates due to the multiplicative adjustment. For example, if the risk-adjusted discount rate is 3.0 percent and the risk free discount rate is 2.0 percent, the actual risk premium is 0.9804 percent rather than 1.0 percent since $0.03 = 0.02 + 0.009804 + 0.02(0.009804)$.

3) Generate the revenue cash flow variances for the constant, increasing, and decreasing cash flow risk trends defining each alternative within a given class.

4) Derive the expected revenue cash flow component values for each alternative (defined by the underlying risk trend) within a given class of investments such that the relationship in (25b) holds.

5) Taking the expected cash flow component values from step 4 as given, evaluate and rank each alternative by mean risk-adjusted net present value (i.e., NPV_{RADR}) under the RADR approach.

Again, any explicit ranking of alternatives for a given *a priori* choice for the risk-adjusted discount rate signals bias, since the true ranking was one of indifference. The program also solved iteratively for the correct RADR to apply to each expected revenue component within a given alternative (i.e., the g_t in (18)), as well as for the overall correct single RADR to discount the entire expected revenue cash flow with (i.e., the k in (22)). The certainty equivalent adjustment factor (as defined in (20d)), standard deviation, and probability of a negative revenue outcome (i.e., net loss) was also provided for every component defining the uncertain revenue cash flow for each alternative.

EMPIRICAL ANALYSIS: THE LANDOWNER'S DECISION

Investment Alternatives Considered

The individual landowner was assumed to own 30 acres of cutover forest land in the central mountains of Virginia.[74] The land was considered to be marginal for agricultural production, but forage hay (alfalfa (*Medicago sativa*) - orchardgrass (*Dactylis glomerata*) mix) was a viable crop alternative (Deaton, 1988).[75] The land was also

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74 Alleghany, Bath, Botetourt, Craig, and Rockbridge counties. Central to this region is a large sawtimber and pulping facility in Covington (Alleghany County), Virginia.

75 According to Deaton (1988), producing forage hay crops for sale on the open market is

assumed to be suitable for loblolly pine (*Pinus taeda* L.) management (site index 60, base age 25 years). If left unmanaged, the site was assumed to naturally regenerate (at no cost) to a mixed hardwood-pine stand.[76]

The decision problem was to choose the best management use of the site. The candidate alternatives were:

1) Sawtimber production (i.e., long rotation, timber)
2) Pulpwood production (i.e., short rotation, timber)
3) Custodial management (i.e., mixed pine-hardwoods)
4) Forage hay production (i.e., agricultural production)

Each alternative was evaluated as if each each use would be continued in perpetuity. This assumption was made in order to resolve the problem that the different strategies represent investment opportunities that differ in life (i.e., rotation). In essence, the chosen investment horizon is infinite. However, the assumption precludes the ability to switch between land uses at some point in the future. Therefore, the investment problem should be viewed as identifying which land use the landowner should choose for the first rotation. In this context, forage hay production is the most liquid opportunity and custodial management represents doing nothing.

Sawtimber Investments

These investment opportunities were in loblolly pine plantations. Sawtimber (trees 8 inches dbh and larger) was assumed to be the primary timber commodity, with

becoming (*circa* 1988) a viable production outlet for small landowners in the mountain region of Virginia since many livestock operations are scaling back their feed operations.

[76] Most likely, the pine will consist of a loblolly-Virginia pine (*Pinus virginiana* Mill.) mix. If the surrounding area is assumed to consist of loblolly pine plantations, or the site recently harvested of a loblolly pine plantation, the greater the tendency toward natural loblolly pine regeneration (Deaton, S. 1988. Vice president, Old Dominion Timber Company, Manakin-Sabot, Virginia. Personal communication).

pulpwood (trees 5 to 7 inches dbh) being a secondary product. Three silvicultural alternatives were considered:[77]

Alternative A:

Plant 500 trees/acre
No thinning

Alternative B:

Plant 700 trees/acre
No thinning

Alternative C:

Plant 700 trees/acre
Thinning from below at age 20
(residual stand basal area target = 76 square feet/acre)
(hardwood removal)

For all three alternatives, planting success was considered to be uncertain. In essence, there was an assumed probability of inadequate stocking 2 years following planting in which case the stand was replanted and establishment delayed by 2 or more years (depending on whether the second planting was successful or not). Alternatives A and B implied a lower management intensity in which the stand was planted and allowed to grow. The higher planting density for alternative B was designed to suppress natural hardwood regeneration. On the other hand, alternative A contain a lower planting density and therefore a lower establishment cost, the tradeoff being a larger competitive influence from hardwoods. Alternative C represented a more intensively managed stand by targeting a thinning at age 20. The thinning operation was also assumed to effectively release the stand from hardwood competition.

Assuming a constant hardwood basal area component throughout the rotation has been shown to be a realistic way of representing hardwood competition in loblolly pine

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[77] Candidate rotation ages were 28, 32, 36, 40, 44, 48, and 52 years.

plantations (Burkhart *et al.*, 1987). For alternative A, hardwood basal area was assumed to occupy 20 percent of the stand's basal area throughout the rotation. However, for alternative B, the hardwood basal area was assumed to be 15 percent. Finally, for alternative C, hardwood basal area was initially 15 percent, but reduced to zero following the thinning.[78]

Pulpwood Investment

This investment opportunity targeted pulpwood as the only timber commodity produced. Therefore, the rotation age was fixed at 20 years. This alternative was included because its duration lies between the short term hay investment and the long term sawtimber investments. The planting density was 800 trees per acre, with hardwoods occupying 4.8 percent of total stand basal area.[79] All harvested timber (including volume from the sawtimber diameter class (8 inches dbh and larger) was valued as pulpwood.

Custodial Management

With custodial management, the stand was assumed to regenerate naturally at no cost (i.e., no site preparation) to one of two possible states:

STATE A: Hardwood basal area occupies 65 percent of the stand
Pine basal area occupies 35 percent of the stand
(probability of occurrence = 0.40)

STATE B: Hardwood basal area occupies 40 percent of the stand
Pine basal area occupies 60 percent of the stand
(probability of occurrence = 0.60)

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[78] Deaton, S. 1988. Vice president, Old Dominion Timber Company, Manakin-Sabot, Virginia. Personal communication.

[79] This value represents an endemic level of hardwood competition in loblolly pine plantations (Burkhart *et al.*, 1987).

The probabilities of occurrence were arbitrarily set. It was felt that the on the average, a 50-50 hardwood-pine mix would occur.[80] In essence, state A corresponds to a bad seed year for pine.

Due to the anticipated low quality of timber for this stand, all pine volume was valued as pulpwood. The candidate rotation ages were 12, 16, 20, 24, 28, 32, and 36 years. Hardwoods were assumed to have no commercial value.[81] For state A, the pine density was assumed to be analogous to a plantation with an initial density of 200 trees per acre, while for state B the density was assumed to be 400 trees per acre. These planting densities refer to the established seedlings used in the stochastic yield projections of the stand.

Forage Hay Production

This alternative represented the only agricultural alternative considered. While the decision to produce forage hay was a realistic alternative to timber production, it was also considered to provide a sharp contrast (i.e., 1 year payoffs) to the longer duration timber investments. Table 2 summarizes the management regime and investment costs for the hay investment. As indicated in table 2, the hay investment had a rotation of 6 years after which crop establishment would have to be repeated to continue production.[82] Liming occurred every other year to ensure proper soil ph. Due to the shorter growing season in the mountain region, only 3 cuttings per year were assumed.

80

Deaton, S. 1988. Vice president, Old Dominion Timber Company, Manakin-Sabot, Virginia. Personal communication.

81

In all the timber investments, hardwood felling and yarding costs during timber harvests (including thinnings) were assumed to be completely offset by the intrinsic value hardwood had as firewood.

82

Childress III, F. S. 1989. Vice president, Childress Farms Inc., Christiansburg, Virginia. Personal communication.

Table 2. Management regime and investment costs for the alfalfa-orchardgrass hay alternative.

TREATMENT	COST[1] (1967 $/acre)	SOURCE
ONE TIME STUMP REMOVAL COST	94.35	Childress III, F.S[2]
YEAR 0[3]: Establishment		
Site preparation (herbicide or mechanical)	4.72	Childress III, F. S.
Liming[4]	6.29	Deaton (1988)[5]
Initial fertilization[6]	17.87	Deaton (1988)[5] Childress III, F. S.
Seed acquisition[7]	14.16	Childress III, F. S.
Sowing (no till drill)	7.86	Childress III, F. S.
TOTAL:	**50.90**	
YEARS 1, 3, 5:		
End year harvest cost[8]	23.59	Childress III, F. S
Beginning year fertilization[9]	15.37	Deaton (1988)[5] Childress III, F. S.
TOTAL:	**38.96**	
YEARS 2, 4:		
End year harvest cost[8]	23.59	Childress III, F. S
Liming[6]	6.29	Deaton (1988)[5]
Beginning year fertilization[9]	15.37	Deaton (1988)[5] Childress III, F. S.
TOTAL:	**45.25**	
YEAR 6:		
End year harvest cost[8]	23.59	Childress III, F. S

Notes:

(1) Original cost estimates in 1988 $/acre. Converted to 1967 $/acre using the Bureau of Labor Statistics annual producer price index (all commodities, 1967 = 1.0) (see appendix D). Deflator = 3.179.

(2) Vice president, Childress Farms Inc., Christiansburg, Virginia. Personal communication, 1989.

(3) Rotational cash flow year.

(4) One ton/acre application (Childress III, F. S. 1989. Vice president, Childress Farms Inc., Christiansburg, Virginia. Personal communication.)

(5) Agricultural enterprise budget, grass-clover hay.

(6) 140 pounds/acre phosphorous, 140 pounds/acre potassium (VCES, 1987). Phosphorous at 0.22 1988 $/pound, potassium at 0.15 $/pound (Deaton, 1988). Includes 5.00 1988 $/acre application cost (Childress III, F. S.).

(7) 15 pounds/acre alfalfa, 5 pounds/acre orchardgrass (VCES, 1987). Alfalfa at 2.50 1988 $/pound, orchardgrass at 1.50 1988 $/pound.

(8) Includes mowing, raking, and baling cost for 3 cuttings per year.

(9) Top dressing, 80 pounds/acre phosphorous, 175 pounds/acre potassium (VCES, 1987). Phosphorous at 0.22 1988 $/pound, potassium at 0.15 $/pound (Deaton, 1988). Includes 5.00 1988 $/acre application cost (Childress III, F. S.).

The rotational costs in table 2 assume that the land was cleared and was ready for planting. In contrast, this study assumed that the land was cutover.[83] Therefore, to make the hay investment directly comparable to the timber investments, a one time stump removal cost of 94.35 1967 $/acre was assumed at the beginning (year 0) of the entire infinite time horizon cash flow (table 2). That is, since the beginning asset for all investments was bare land (with stumps), the cost of removing the stumps for agricultural purposes had to be accounted for.

Investment Performance Criterion

Simulation of NPV Outcomes

The performance of each investment alternative was simulated over time, treating output yields and prices as stochastic. Catastrophic losses from wind, fire, or pests were not considered. Wealth effects and investment valuation were on a per acre basis (i.e., with respect to the performance of a representative acre of the individual's land holding). For each simulated outcome, a NPV was computed by discounting the realized cash flow with the risk free rate. On the basis of these simulated outcomes, an empirical probability distribution for NPV was estimated for each alternative-rotation age combination considered. A multiple rotation approach was adopted. That is, for each simulation, the investment alternative's cash flow was repeated for one rotation following another. However, the simulated outcome for each rotation would be stochastic in itself (i.e., the cash flow realization for the first rotation would likely be different from the second, the second likely different from the third, and so on).

83

The reason for this assumption was because the growth and yield projections for the timber investments assumed cutover sites (Burkhart *et al.*, 1987). Relaxing this assumption (i.e., assuming an old field site) would result in a marked under-estimation of the competitive influence from hardwoods (Amateis, R. L. 1989. Research Associate, Department of Forestry, Virginia Polytechnic Institute and State University, Blacksburg, Virginia. Personal communication).

Two stopping criteria were used to end each simulation.

1) Extend the time horizon until the increment in the realized NPV from the last rotation considered was less than 1 percent of the total NPV from all rotations considered.
2) In the event that the returns from two successive rotations were negative (i.e., increment to NPV < 0), end the simulation under the assumption of land abandonment.

The implicit assumption underlying the first criterion was that from the cutoff point onward, the impact of all future rotations on NPV, regardless of the outcome, was negligible. However, under land abandonment, the returns from the cutoff point onward were implicitly zero.[84]

Utility Assumptions

In the empirical analysis, two classes of utility of wealth functions were considered.

1) Logarithmic Utility
2) Negative Exponential Utility

For each utility class, two cases regarding the degree of risk aversion were considered.

LOGARITHMIC UTILITY:

CASE 1: Low degree of absolute risk aversion
CASE 2: High degree of absolute risk aversion

NEGATIVE EXPONENTIAL UTILITY:

CASE 3: Low degree of relative risk aversion
CASE 4: High degree of relative risk aversion

Appendix B summarizes the parameter assumptions used to represent each case.

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[84] Actually, land abandonment implies that for that particular simulation, the strategy should be to switch to an alternative land use at the time of abandonment. However, in this study land abandonment was simply an escape mechanism to abandon the investment after an *ex post* assessment of future events. Two successive rotations incurring losses were considered to be convincing enough that for this particular course events the investment should be abandoned.

<u>Expected Utility and Certainty Equivalent NPV Valuation</u>

With net present value defined on a per acre basis, the wealth position from a given net present value outcome is given by[85]

$$w = W_o + npv(AREA) \qquad [26]$$

where the term npv(AREA) defines the increment to wealth if the alternative is adopted and that particular net present value outcome occurs. However, it is convenient to express the individual's wealth effect on a per acre basis by distributing the wealth across all acreage. That is,

$$w^{(pa)} = \frac{w}{AREA} = W_o^{(pa)} + npv \qquad [27]$$

where:

$$W_o^{(pa)} = \frac{W_o}{AREA} \qquad [27a]$$

represents the individual's wealth endowment distributed on a per acre basis. In this case, the utility of wealth from a given NPV outcome is given by[86]

LOGARITHMIC UTILITY:

$$U(w^{(pa)}) = LOG(W_o^{(pa)} + npv) \qquad [28a]$$

NEGATIVE EXPONENTIAL UTILITY:

$$U(w^{(pa)}) = 1 - e^{-c_{pa}(W_o^{(pa)} + npv)} \qquad [28b]$$

85

The lower case notation represents some outcome for uncertain *NPV*.

86

Appendix B discusses the conversion from total wealth to per acre wealth effects with respect to the proper specification of the utility function such that the transformation has no effect on the expected utility valuation of the investment (as measured by the certainty equivalent NPV, NPV_{CE}). For logarithmic utility, the conversion implies a positive linear transformation of the utility of total wealth function. For negative exponential utility, the coefficient of absolute risk aversion acts as the scale parameter.

Using Anderson *et al.*'s (1977) moment distribution approximation of expected utility as defined in (10), the expected utility from holding claims to the risky investment alternative is given by

LOGARITHMIC UTILITY:

$$\mathbf{E}\big[\mathrm{U}(W^{(\mathrm{pa})})\big] = \mathrm{LOG}(\mathrm{W}_{\mathrm{o}}^{(\mathrm{pa})} + \mathbf{E}[NPV]) - \left(\frac{1}{2}\right)\frac{\mathbf{VAR}[NPV]}{(\mathrm{W}_{\mathrm{o}}^{(\mathrm{pa})} + \mathbf{E}[NPV])^{2}} + \left(\frac{1}{3}\right)\frac{\mathbf{SKE}[NPV]}{(\mathrm{W}_{\mathrm{o}}^{(\mathrm{pa})} + \mathbf{E}[NPV])^{3}} \quad [29a]$$

NEGATIVE EXPONENTIAL UTILITY:

$$\mathbf{E}\big[\mathrm{U}(W^{(\mathrm{pa})})\big] = 1 - e^{-c_{\mathrm{pa}}\left(\mathrm{W}_{\mathrm{o}}^{(\mathrm{pa})} + \mathbf{E}[NPV]\right)} \cdot \left(1 + \left(\frac{1}{2}\right)c_{\mathrm{pa}}^{2}(\mathbf{VAR}[NPV]) - \left(\frac{1}{6}\right)c_{\mathrm{pa}}^{3}(\mathbf{SKE}[NPV])\right) \quad [29b]$$

where:

$$\mathbf{VAR}[NPV] = \mathbf{E}\big[(NPV - \mathbf{E}[NPV])^{2}\big] \quad [29c]$$

$$\mathbf{SKE}[NPV] = \mathbf{E}\big[(NPV - \mathbf{E}[NPV])^{3}\big] \quad [29d]$$

The certainty equivalent net present value is then given by the value $\mathrm{NPV}_{\mathrm{CE}}$ which satisfies

$$\mathrm{U}(\mathrm{W}_{\mathrm{o}}^{(\mathrm{pa})} + \mathrm{NPV}_{\mathrm{CE}}) = \mathbf{E}\big[\mathrm{U}(W^{(\mathrm{pa})})\big] \quad [30a]$$

where:

LOGARITHMIC UTILITY:

$$\mathrm{U}(\mathrm{W}_{\mathrm{o}}^{(\mathrm{pa})} + \mathrm{NPV}_{\mathrm{CE}}) = \mathrm{LOG}(\mathrm{W}_{\mathrm{o}}^{(\mathrm{pa})} + \mathrm{NPV}_{\mathrm{CE}}) \quad [30b]$$

NEGATIVE EXPONENTIAL UTILITY:

$$\mathrm{U}(\mathrm{W}_{\mathrm{o}}^{(\mathrm{pa})} + \mathrm{NPV}_{\mathrm{CE}}) = 1 - e^{-c_{\mathrm{pa}}(\mathrm{W}_{\mathrm{o}}^{(\mathrm{pa})} + \mathrm{NPV}_{\mathrm{CE}})} \quad [30c]$$

which gives,

LOGARITHMIC UTILITY:

$$NPV_{CE} = e^{E[U(W^{(pa)})]} - W_o^{(pa)} \qquad [31a]$$

NEGATIVE EXPONENTIAL UTILITY:

$$NPV_{CE} = -\left(\frac{LOG(1 - E[U(W^{(pa)})])}{c_{pa}}\right) - W_o^{(pa)}, \quad E[U(W^{(pa)})] < 1 \qquad [31b]$$

Equation (29) defines expected utility with respect to the mean, variance, and the skewness of the NPV probability distribution. Skewness was added to improve the approximation of expected utility since it is not known whether the NPV distribution will be normal. However, adjustments for higher order moments beyond the distribution's skewness were considered negligible (Anderson *et al.*, 1977).

As defined in (30), the certainty equivalent net present value is the certain gain NPV_{CE} that equates the certain utility $U(W_o^{(pa)} + NPV_{CE})$ with the expected utility $E[U(W^{(pa)})]$ from holding claims to the risky alternative being considered. That is, the individual is indifferent between the risky alternative and the certain amount NPV_{CE}. Therefore, NPV_{CE} in (31) provides the true valuation of the investment alternative. Finally, alternatives can be ranked by NPV_{CE}, since the higher the expected utility from holding claims to the alternative in (29), the higher NPV_{CE} in (31).[87]

The formulation in equations (29) through (31) differs from the hypothetical analysis in that the expected utility approximation is defined with respect to the probability distribution for NPV, rather than with respect to each probability

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87 The implicit assumption being that the necessary capital for each alternative can be borrowed at the risk free rate. That is, in this partial equilibrium framework, the risk perceptions in the capital market for these investments are the same as the individual's perceptions. Therefore, the individual's risk premium for taking on the investment will be the same premium required by the capital market for the borrowed funds.

distribution defining cash flow component outcomes. That is, in the empirical analysis, the certainty equivalent NPV valuation for the alternative considered was defined with respect to the NPV distribution rather than adjusting each expected cash flow component for its risk and discounting the certainty equivalent cash flow component values with the risk free rate. While the formulation in (29) through (31) above sidesteps the certainty equivalent cash flow adjustment, the implication is that the certainty equivalent NPV can still be defined with respect to the individual's certainty equivalent cash flow for the alternative.

Simulation Model

As indicated in the previous section, simulation was used to estimate the empirical NPV probability distribution for each alternative-rotation age combination considered. This necessitated developing stochastic yield and output price models for sawtimber, pulpwood and hay. All economic variables were defined in real terms, with 1967 taken as the base year of valuation. All investment costs were considered certain and to remain constant in 1967 dollars.

Stochastic Price Model

Timber prices were assumed to be correlated with the general state of the economy (as measured by an economic index), which was also considered as stochastic. Furthermore, timber yield outcomes had no effect on the price received (i.e., the individual was a price taker). However, in spite of being a price taker in hay markets, hay price was related to hay yield outcomes. This was because state wide average data was used in estimating the hay price model (appendix D). The relationship was that when yields were low, hay prices were high to reflect state wide shortages in supply (figure 3). Conversely, prices were suppressed when yields were relatively high.[88] Therefore, the hay yield realization's for the individual's crop were assumed to reflect the type of year hay producers in general were experiencing.

The stochastic price model was represented by a vector autoregressive (VAR) time series process. VAR models are appealing for simulation because emphasis is placed on the relative movement of prices over time, with no requirement for an *a priori* specification of the market structure (e.g., supply and demand relationships) which may motivate dynamic price behavior. In essence, VAR models let the data speak by looking for the model specification that best fits the data. The weakness with the approach is that only historical relationships are used to determine the appropriate model specification. Therefore unforeseen events (e.g., new product substitutions, policy shifts) that may change the historical relationship are implicitly ignored.

The VAR model considered was as follows:[89]

Economic Index (quarterly):

$$\Delta IN_{t'} = \beta_0^{(in)} + \beta_1^{(in)} \Delta IN_{t'-1} + \quad \dots \quad + \beta_m^{(in)} \Delta IN_{t'-m} + \varepsilon_{t'}^{(in)} \qquad [32a]$$

$$IN_{t'} \equiv IN_{t'-1} + \Delta IN_{t'} \qquad [32b]$$

Sawtimber Stumpage Price (quarterly):

$$\begin{aligned} P_{t'}^{(saw)} = {} & \beta_0^{(saw)} + \beta_1^{(saw)} P_{t'-1}^{(saw)} + \quad \dots \quad + \beta_m^{(saw)} P_{t'-m}^{(saw)} \\ & + \gamma_0^{(saw)} IN_{t'} + \gamma_1^{(saw)} IN_{t'-1} + \quad \dots \quad + \gamma_n^{(saw)} IN_{t'-n} + \varepsilon_{t'}^{(saw)} \end{aligned} \qquad [32c]$$

88

Unlike timber, the entire hay crop must be brought to market each year.

89

The critical assumption being that the entire process is stationary; that is, each variable has a constant mean and variance independent of t′ (or t). Furthermore, the error terms $\varepsilon_{t'}^{(j)}$, ($j \in$ {in, pulp, saw}) and $\varepsilon_t^{(hay)}$ in each autoregression were assumed to be covariance stationary; that is, non-contemporaneous autocorrelations within each autoregression depend only on the lag difference, and not t′ (or t) itself. However, for the purpose of the simulation model, these autocorrelations were assumed to be zero. Stationarity concerns and the diagnostic for serial correlation in the model are discussed in appendix C.

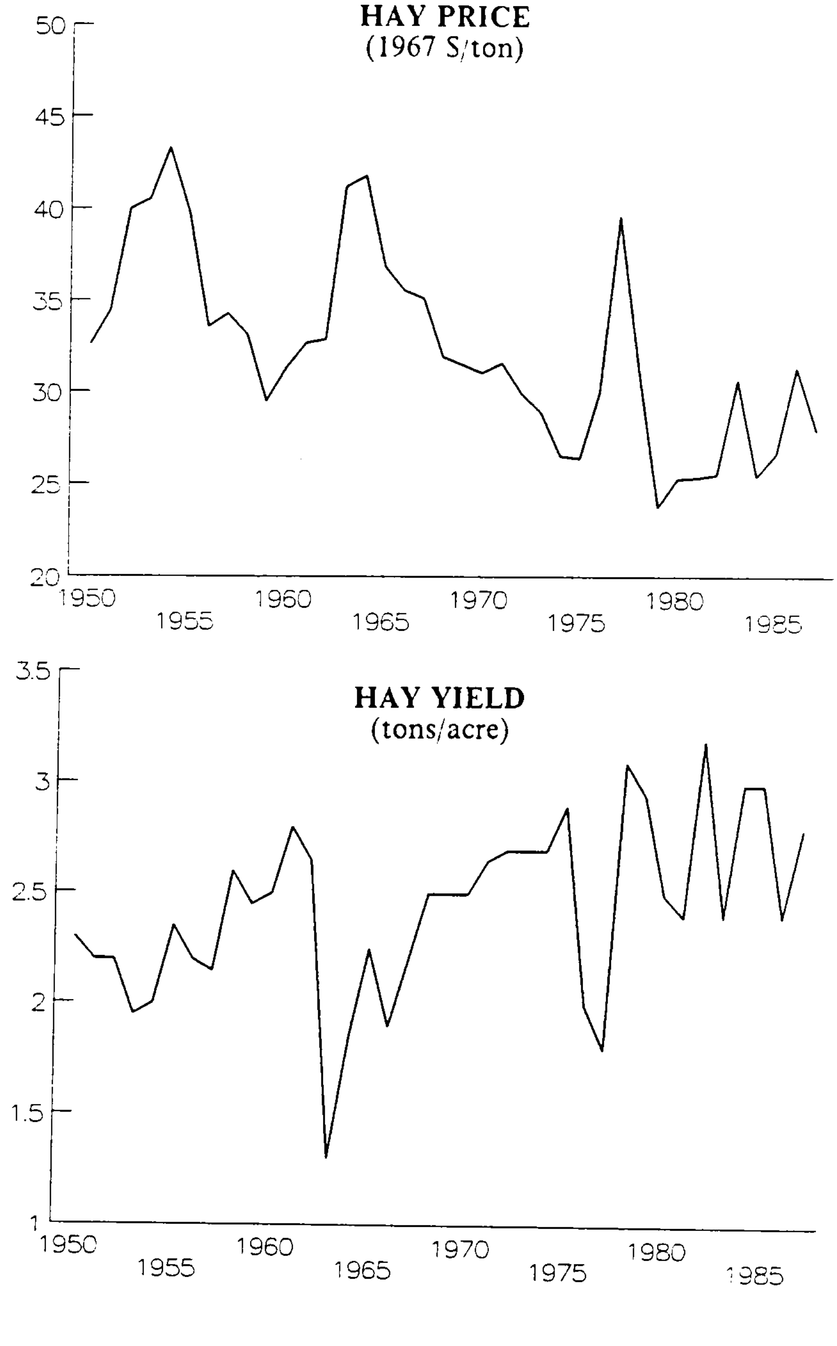

Figure 3. Historical relationship between hay yield and price in Virginia, 1950-1987.

Hay yield reflects average per acre yields for alfalfa and alfalfa mixtures in Virginia. Hay price reflects Virginia all hay (baled) price received by farmers. (Source: Ransom, D, and D. Mellon. 1988. Agricultural statisticians, National Agricultural Statistical Service, Washington DC. Personal communication.).

Pulpwood Stumpage Price (quarterly):

$$P_{t'}^{(pulp)} = \beta_0^{(pulp)} + \beta_1^{(pulp)} P_{t'-1}^{(pulp)} + \ \dots \ + \beta_m^{(pulp)} P_{t'-m}^{(pulp)}$$
$$+ \gamma_0^{(pulp)} IN_{t'} + \gamma_1^{(pulp)} IN_{t'-1} + \ \dots \ + \gamma_n^{(pulp)} IN_{t'-n} + \varepsilon_{t'}^{(pulp)} \qquad [32d]$$

Hay Price (annual):

$$P_t^{(hay)} = \beta_0^{(hay)} + \beta_1^{(hay)} P_{t-1}^{(hay)} + \ \dots \ + \beta_m^{(hay)} P_{t-m}^{(hay)}$$
$$+ \gamma_0^{(hay)} HY_t + \gamma_1^{(hay)} HY_{t-1} + \ \dots \ + \gamma_n^{(hay)} HY_{t-n} + \varepsilon_t^{(hay)} \qquad [32e]$$

The first restriction underlying the specification of the VAR process was that the timber price autoregressions in (32c) and (32d) include contemporaneous and lagged effects of the economic index variable, while the economic index variable autoregression in (32a) was restricted to be a univariate process.[90] That is, this restriction represented the hypothesis that timber price movements are determined in part by the cyclical nature of the economy. However, timber prices were assumed to have no corresponding feedback effects. The intent was to avoid modelling sawtimber and pulpwood prices independently of one another.[91] In essence, the VAR model specified in (32) above defined a system in which regional timber prices were tied to the general state of the economy; treating this effect as purely exogenous.

However, preliminary analysis indicated that including contemporaneous and lagged effects of the economic index variable did not lead to a satisfactory specification

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90 First differences for the economic index variable were used to alleviate non-stationarity concerns for the indexes considered since all indexes followed an increasing trend (see appendix C). Actual index value realizations were then defined by the identity in (32b).

91 A natural relaxation of the specification of (32c) and (32d) would have been to include pulpwood stumpage price in the sawtimber autoregression, and sawtimber stumpage price in the pulpwood autoregression, since sawtimber and pulpwood markets are most likely linked. However, data limitations, namely missing observations for pulpwood prices (appendix D), limited the performance of this specification.

for the hay price model. Therefore, the hay price model only included contemporaneous and lagged effects of hay yield.

Two areas had to be considered in the estimation of (32). The first involved selecting the appropriate economic index variable, while the second involved choosing the appropriate lag order for each variable in the VAR.[92] Hsiao's (1979) procedure was used to address these areas. This procedure, as well as the VAR estimation, is summarized in appendix C.

Given the fitted model in (32), the simulation in a given forecast quarter t′ (or year t) involved generating random variates for each of the stochastic disturbance terms $\varepsilon_{t'}^{(j)}$, ($j \in$ {in, pulp, saw}) and $\varepsilon_t^{(hay)}$ under the assumption that[93]

$$\varepsilon_{t'}^{(j)} \sim N(0, \sigma_j^2) \quad \text{for all } t', \text{ for all } j \in \{\text{in, pulp, saw}\} \qquad [33a]$$

$$COV(\varepsilon_{t'}^{(j)}, \varepsilon_{t'-s}^{(j)}) = 0 \quad \begin{array}{l}\text{for all } s \in \{1, 2, \ldots\}, \text{ for all } t', \\ \text{for all } j \in \{\text{in, pulp, saw}\}\end{array} \qquad [33b]$$

$$COV(\varepsilon_{t'}^{(i)}, \varepsilon_{t'-s}^{(j)}) = 0 \quad \begin{array}{l}\text{for all } i,j \in \{\text{in, pulp, saw}\},\ i \neq j; \\ \text{for all } s \in \{0, 1, 2, \ldots\}, \text{ for all } t'\end{array} \qquad [33c]$$

$$\varepsilon_t^{(hay)} \sim N(0, \sigma_{hay}^2) \quad \text{for all } t \qquad [33d]$$

$$COV(\varepsilon_t^{(hay)}, \varepsilon_{t-s}^{(hay)}) = 0 \quad \text{for all } s \in \{1, 2, \ldots\}, \text{ for all } t \qquad [33e]$$

$$COV(\varepsilon_t^{(hay)}, \varepsilon_{t'-s}^{(j)}) = 0 \quad \begin{array}{l}\text{for all } j \in \{\text{in, pulp, saw}\}, \text{ for all } t \\ \text{for all } s \in \{0, 1, 2, \ldots\}, \text{ for all } t'\end{array} \qquad [33f]$$

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92 In (32) the lag order for lagged endogenous variables is represented by m, while the lag order for the economic index variable in the price relationships is represented by n. However, the lag order for each variable was allowed to vary across autoregressions, that is neither m, nor n need to be the same value in all relationships.

93 The assumptions is (33a), (33b), (33d), and (33e) follow from the Gaussian assumptions regarding the distribution of the error term in the ordinary least squares (OLS) procedure used to estimate (32) (appendix C). Furthermore, the assumptions in (33c) and (33f) follow from a correct specification of the VAR model, namely all inter-dependencies between variables were accounted for by the introduced contemporaneous and lagged effects.

and then updating the realized values for each variable by adding the stochastic disturbance outcome to the corresponding fitted relationship defined by previous simulated outcomes.[94]

Volatility in Regional Hay Markets

Perhaps the most distinct difference between the hay alternative and the timber investment is that the former represents a perishable crop that must be harvested each year. While there is uncertainty in the price received, farmers seem to be more concerned with whether a market for the crop will exist or not. In many instances, the result is selling the crop at a loss.[95]

The state wide hay price data used in this analysis does not take into account the volatility found in regional hay markets. Therefore, it was felt that the estimated hay price autoregression would under estimate the degree of uncertainty inherent in regional hay markets. This volatility in regional hay markets was modelled exogenous to the hay price autoregression. The approach taken was to represent the presence or absence of a market outlet for the hay crop as a Bernoulli trial of success or failure (probability of success = 0.85). That is, in any given year, there was a 15 percent probability that the crop would be lost due to lack of an available market (i.e., price received equals zero) regardless of the simulated outcome for hay price from the stochastic price model.[96] In

[94] The simulation was initialized using the historical observations ending in 1987. That is, the first uncertain period in the investment horizon corresponded to 1988.

[95] Childress III, F. S. 1989. Vice president, Childress Farms Inc., Christiansburg, Virginia. Personal communication.

[96] For example, the individual could expect (on the average) 3 poor market years over the course of a 20 year period.

essence, this exogenous perturbation was designed to increase the variability in the hay investment's cash flow.

The simulation model accounted for market loss years by generating a uniform (0,1) random variate z. If $z > 0.85$ then a market loss year occurred and the revenue received for the crop was zero.

Stochastic Timber Yield Model

Timber yields were modelled using a time homogeneous Markov chain approach. Stand states were defined with respect to the following volume classes[97]

Pulpwood Volume (cords)

Low volume
High volume

Sawtimber Volume (MBF Scribner)

Low volume
Medium volume
High volume

Furthermore, the stand state definitions implicitly considered the age of the stand, beginning with age 8, and continuing every 4 years to a maximum age of 52 (e.g., ages $A \in \{8, 12, 16, ,20, \ldots, 52\}$). Therefore, there were six states applicable to each age

STATE 1: pulpwood volume low
sawtimber volume low

STATE 2: pulpwood volume low
sawtimber volume medium

STATE 3: pulpwood volume low
sawtimber volume high

STATE 4: pulpwood volume high
sawtimber volume low

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97 The diameter class defining pulpwood volume in the sawtimber investments was 5 to 7 inches dbh. Sawtimber included all trees greater the 8 inches dbh inclusive. Finally, for the pulpwood and custodial management investments, sawtimber volume units were measured in cords.

STATE 5: pulpwood volume high
sawtimber volume medium

STATE 6: pulpwood volume high
sawtimber volume high

for a maximum total of 73 states (6 for the 12 stand ages considered plus one additional state denoting stand establishment).

The volume class definitions (e.g., low, medium, and high) were age dependent and were defined by dividing the range of likely stand yields into equal intervals defining each class. For example, if the range of likely outcomes for pulpwood volume (in the 5 to 7 inch dbh class) at age 24 was between 6 and 12 cords per acre, then the low pulpwood volume class captured all volume outcomes which were less than 9 cords/acre, and the high pulpwood volume class captured all volume outcomes which were greater than 9 cords/acre inclusive. Accordingly, if the range of likely outcomes for sawtimber volume (in dbh classes 8 inches and up) at age 24 was between 3 and 6 MBF/acre, then the low volume class captured all outcomes which were less that 4 MBF/acre, the medium volume class captured all outcomes between 4 and 5 MBF/acre inclusive, and the high volume class captured all outcomes greater than 5 MBF/acre. Therefore, if the volume outcome for the 24 year stand was 8.6 cords/acre of pulpwood, and 4.2 MBF/acre of sawtimber, then the realized state would be STATE 2: low pulpwood volume, medium sawtimber volume.

Figure 4 diagrams the transition probability matrix for the Markov chain representing stochastic timber yields. The matrix is applicable to a stand harvested at age 24, therefore there are a total of 31 stand states. Rows define the starting state for the stand, while columns define possible stand states the stand can progress to over a single transition. The shaded regions in figure 4 define possible transitions with non-zero probability. As indicated in figure 4, the possible transitions for are defined for a 4 year growth projection of the stand, beginning with age 8 and ending at age 20. For example,

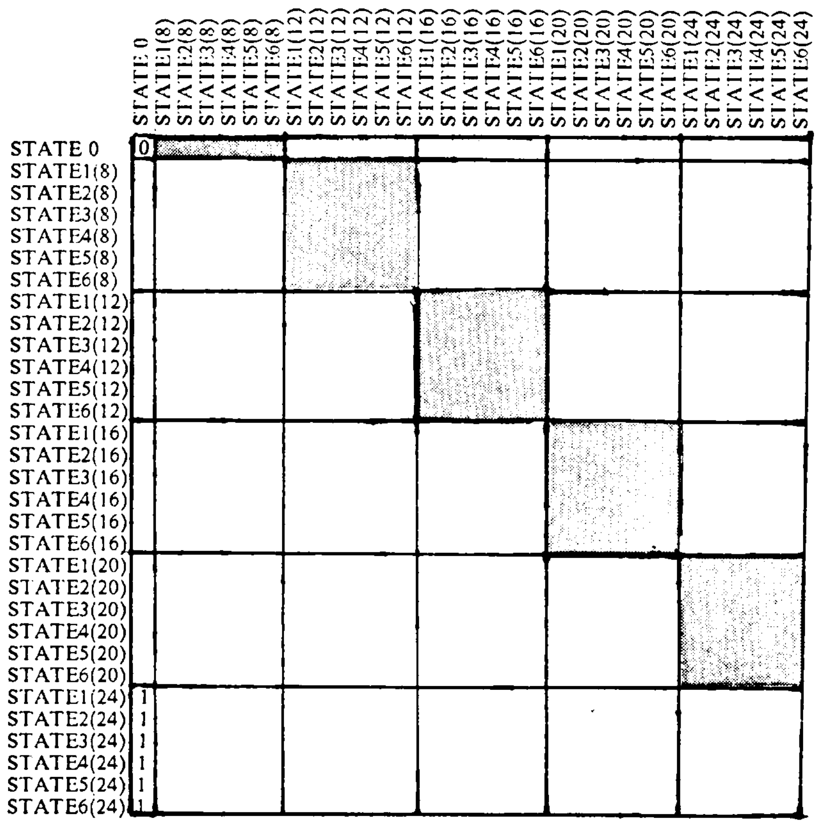

Figure 4. Example of a timber stand transition probability matrix for an even-age stand harvested at age 24.

Shaded regions define state transitions with non-zero probability of occurrence.

Stand state definitions (for stand ages, A $\in$ {8, 12, 16, ... , 24} years):

STATE1(A): pulpwood volume low, sawtimber volume low
STATE2(A): pulpwood volume low, sawtimber volume medium
STATE3(A): pulpwood volume low, sawtimber volume high
STATE4(A): pulpwood volume high, sawtimber volume low
STATE5(A): pulpwood volume high, sawtimber volume medium
STATE6(A): pulpwood volume high, sawtimber volume high
STATE 0: Seedling establishment following planting

if the stand is in one of the six states applicable to age 16 (i.e., the stand is 16 years old), the only possible transitions are to one of the six stand states applicable to the stand at age 20. Accordingly, if the stand is just established (i.e., age 0), then the only possible transitions are to stand states applicable to age 8. Finally, if the stand is 24 years old (i.e., ready for harvest), the stand reverts to the initial establishment state with probability 1.[98]

In the dynamic sense, the stand's stochastic growth and yield projection followed the state transitions defined by the shaded regions along the diagonal in figure 4. This is consistent with Hool's (1966) conception of the transition probability matrix for Markov chains for even-age stands. Furthermore, the time homogeneity property of the Markov chain is defined with respect to rotation age. For example, regardless of whether the stand was at age 16 in the first, second, or nth rotation in the cash flow, the transition probabilities applicable to a 16 year old stand states were fixed. However, the length of each transition period was dependent on the starting state due to the age dependency implicit in the stand state definitions. With respect to the example in figure 4, if the starting state was initial establishment, the transition period covered the 8 year juvenile growth phase for the stand. Accordingly, if the starting states pertained to ages 8, 12, 16, or 20 years, the transition period was 4 years long. However, if the starting state pertained to a 24 year old stand, the transition period length was instantaneous for naturally regenerated stands (due to assumed certain establishment), but stochastic for plantations due to the possibility of planting failures.

Appendix E summarizes the procedure used to estimate the transition probability matrix for each timber alternative considered and discusses the mechanics of how yield outcomes for a representative acre were simulated from the estimated Markov chain.

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98 Establishment failure is handled exogenous to the timber yield Markov chain.

Plantation Establishment Failure

Since the growth and yield simulator (appendix E) used to estimate the stand state transition probabilities only accounted for regeneration mortality in established stands, planting failures had to be modelled exogenous to the Markov chains representing stochastic timber yields.[99] For this study, a planting failure was defined as the event in which there was insufficient stocking of established free to grow seedlings two years following planting (Shores, 1970). However, an actual minimum stocking level defining planting success was not explicitly defined, instead a planting failure represented some planting loss large enough such that future projection of the stand under the Markov chain approach was not warranted. That is, the stand had to be replanted to replace losses due to factors not considered in the estimation of the Markov chain models (e.g., frost, animal damage, poor planting stock, improper planting). In this manner, once successful planting is identified, the growth and yield simulation of the stand using the Markovian representation of yield commenced.

Each planting was defined as a Bernoulli trial representing success or failure. That is, the probability of planting success was the same for the first, second, or any subsequent replanting operation (Shores, 1970; Thompson *et al.*, 1973). Furthermore, the planting and site preparation cost for each planting operation was considered constant and certain.[100] However, the cost of actually achieving an established stand is uncertain in as much as planting success is uncertain (Thompson *et al.*, 1973). The planting and site preparation cost for each operation and the probability of planting

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[99] Burkhart, H. 1988. Thomas M. Brooks professor, Department of Forestry, Virginia Polytechnic Institute and State University, Blacksburg, Virginia. Personal communication.

[100] While a planting failure may not require complete site preparation and replanting, the simulation model implicitly assumed complete failure. Thompson *et al.* (1973) implies that establishment costs following plantation failures (which may only represent insufficient partial stocking of the site) are of the same order of magnitude as the initial planting cost.

success was taken from Shores (1970) (table 3).[101] The establishment operation represented a mechanical (i.e., brush removal) and burn site preparation treatment followed by hand planting.

The simulation model accounted for planting failure by generating a uniform (0,1) random variate z at the beginning of each rotation considered in the simulation. If $z > 0.75$ (the probability of planting success, table 3) then the planting operation failed and a subsequent planting took place 2 years hence. Accordingly, a second establishment cost was incurred at that time. This second establishment operation was then subject to another Bernoulli trial and the process repeated if the second planting failed as well. Once a given trial indicated planting success (i.e., $z \leq 0.75$), then stand yield was simulated using the appropriate Markov chain. However, the timing of any thinning and final harvest in the cash flow took account of any delays in establishment. Therefore, not only was the cost of achieving stand establishment uncertain, but the time of establishment as well.

Stochastic Hay Yield Model

The stochastic hay model was represented by the following autoregressive process

$$\Delta HY_t = \eta_0 + \eta_1 \Delta HY_{t-1} + \ \dots \ + \eta_m \Delta HY_{t-m} + \mu_t \qquad [34a]$$

$$HY_t \equiv HY_{t-1} + \Delta HY_t \qquad [34b]$$

101

Shores' (1970) analysis for rugged terrain in the Virginia Piedmont was assumed to best reflect conditions for more favorable sites in Virginia's central mountain zone. The data in table 3 represent averages of Shores' survey results. However, Shores' analysis reflected 1969 cost estimates. Therefore, Shores' cost estimates were converted to 1967 dollars using the Bureau of Labor Statistic's annual producer price index (all commodities, 1967 = 1.0) (see appendix D). Deflator = 1.065. Finally, Shores' analysis implicitly assumed an initial planting density of 600 trees/acre, so the cost estimates were scaled to reflect differences in planting density.

Table 3. Regeneration establishment costs and probability of success for plantation investments.

PLANTING DENSITY (trees/acre)	ESTABLISHMENT COST(1) (1967 $/acre)	PROBABILITY OF PLANTING SUCCESS(2)
500	34.23	0.75
700	47.93	0.75
800	54.77	0.75

Notes:

(1) Adapted from Shores (1970). Assumes of per seedling establishment cost of 0.0685 1967 dollars. Based on a per acre establishment cost of 41.08 1967 $/acre for 600 trees per acre (average of Shores' survey results for unfavorable topography in the Virginia Piedmont). Cost figures applicable to each establishment operation incurred.

(2) From Shores (1970). Average of Shores' survey results (chop and burn site preparation, hand planting) for plantations established on unfavorable topography in the Virginia Piedmont. Assumed applicable to all planting densities.

The data underlying the estimation was the average per acre hay yield for alfalfa and alfalfa mixtures in Virginia (tons/acre), annual 1919-1988.[102] In essence, the Virginia state averages were assumed to be reflective of the yield for a representative acre of the individual's land holding. However, this most likely underestimated the true yield variability since hay yield averaged on a regional basis will be more variable than yield average on a state wide basis. Most notably, regional impacts such as drought will not be properly reflected in the state wide data. Therefore, the exogenous market loss model for hay prices can also be considered as a proxy designed to counteract the under-representation of hay yield variability.

The lag order choice and estimation of (34) followed Hsiao's (1979) procedure and was analogous to the estimation of the VAR model representing stochastic prices in (32).[103] Since the historical yield followed a upward trend (most likely due to technological innovations in cultivation), first differences were used to alleviate any non-stationarity concerns.[104] The fitted model is summarized in table 4.

Since (34a) was estimated using ordinary least squares (OLS), the simulation proceeded under the the Gaussian assumptions for μ_t, namely

$$\mu_t \sim N(0, \sigma^2_{\Delta HY}) \text{ for all t} \qquad [35a]$$

$$COV(\mu_t, \mu_{t-s}) = 0 \text{ for all } s \in \{1, 2, \ldots\}, \text{ for all t} \qquad [35b]$$

102

Ransom, D. 1988. Agricultural statistician, National Agricultural Statistical Service, Washington DC. Personal communication.

103

This procedure is discussed in appendix C. The maximum lag order considered was m = 20.

104

See appendix C for a discussion concerning stationarity as applied to the VAR model representing stochastic prices.

Table 4. Fitted autoregressive model for for hay yield.

HAY YIELD AUTOREGRESSION			
COEFFICIENT[(1)]	ESTIMATED VALUE	TSTAT[(2)]	SIGNIFICANCE LEVEL[(3)]
η_0	0.118	2.348	0.05
η_1	-0.805	-5.464	(< 0.001)
η_2	-0.776	-4.356	(< 0.001)
η_3	-0.593	-3.006	0.005
η_4	-0.547	-2.695	0.01
η_5	-0.708	-3.549	(< 0.001)
η_6	-0.589	-2.778	0.01
η_7	-0.558	-2.636	0.02
η_8	-0.671	-3.343	0.002
η_9	-0.493	-2.430	0.02
η_{10}	-0.437	-2.234	0.05
η_{11}	-0.161	-0.884	0.50
η_{12}	-0.219	-1.521	0.20

Virginia average per acre hay yield for alfalfa and alfalfa mixtures (tons/acre).
Estimation range (annual): 1932-1988
Degrees of Freedom = 44
Coefficient of Determination ($R^{(2)}$ adjusted) = 0.434
Q(12)[(4)] = 13.319, α = 0.897
Standard error of residuals = 0.329 (tons/acre)

Notes:

(1) See introduction for definition of notation.

(2) Test statistic for test:

Ho: COEFFICIENT = 0
Ha: COEFFICIENT ≠ 0

(3) Significance level for TSTAT. Gives probability of observing TSTAT given, Ho: COEFFICIENT = 0, is true.

(4) Q(p) is the Ljung-Box statistic based of p autocorrelations (Doan and Litterman, 1986). Test statistic for (Harvey, 1981b):

Ho: No serial correlation
Ha: Serial correlation

α gives probability of observing Q(p) given, Ho: No serial correlation, is true.

The procedure for each simulated year t was to generate a random variate representing a realized outcome for μ_t and then adding this result to the fitted relationship defined by previous simulated outcomes. Each simulation was initialized using the historical yield data ending in year 1987.

Flowchart of the Model

Figure 5 summarizes the empirical simulation model (written in FORTRAN). The model was constructed to consider more than one candidate rotation age for each timber alternative considered. Given the rotation age, each simulation started at year zero (1988) of the multiple rotation cash flow. Beginning with the first rotation, establishment success or failure was checked. For successful establishment, revenue outcomes were determined from the simulated yield and price outcomes applicable to that year. After all revenue outcomes were determined for the rotation, the entire rotational cash flow (including costs) were discounted to present value. If the increment to net present value (under the risk free discount rate) from the just completed rotation was small enough (or land abandonment assumed), the simulation was stopped; otherwise the next rotation in the cash flow was considered. At the end of all the simulations, the mean, variance, and skewness values for the empirical NPV probability distribution were estimated and the expected utility analysis (including the estimation of NPV_{CE}) was performed. Accordingly, the alternative was also evaluated under the RADR approach. After all the candidate rotation ages had been considered, the program stopped.

Random Variate Generators

The uniform (0,1) random variate generator used in the program was a prime

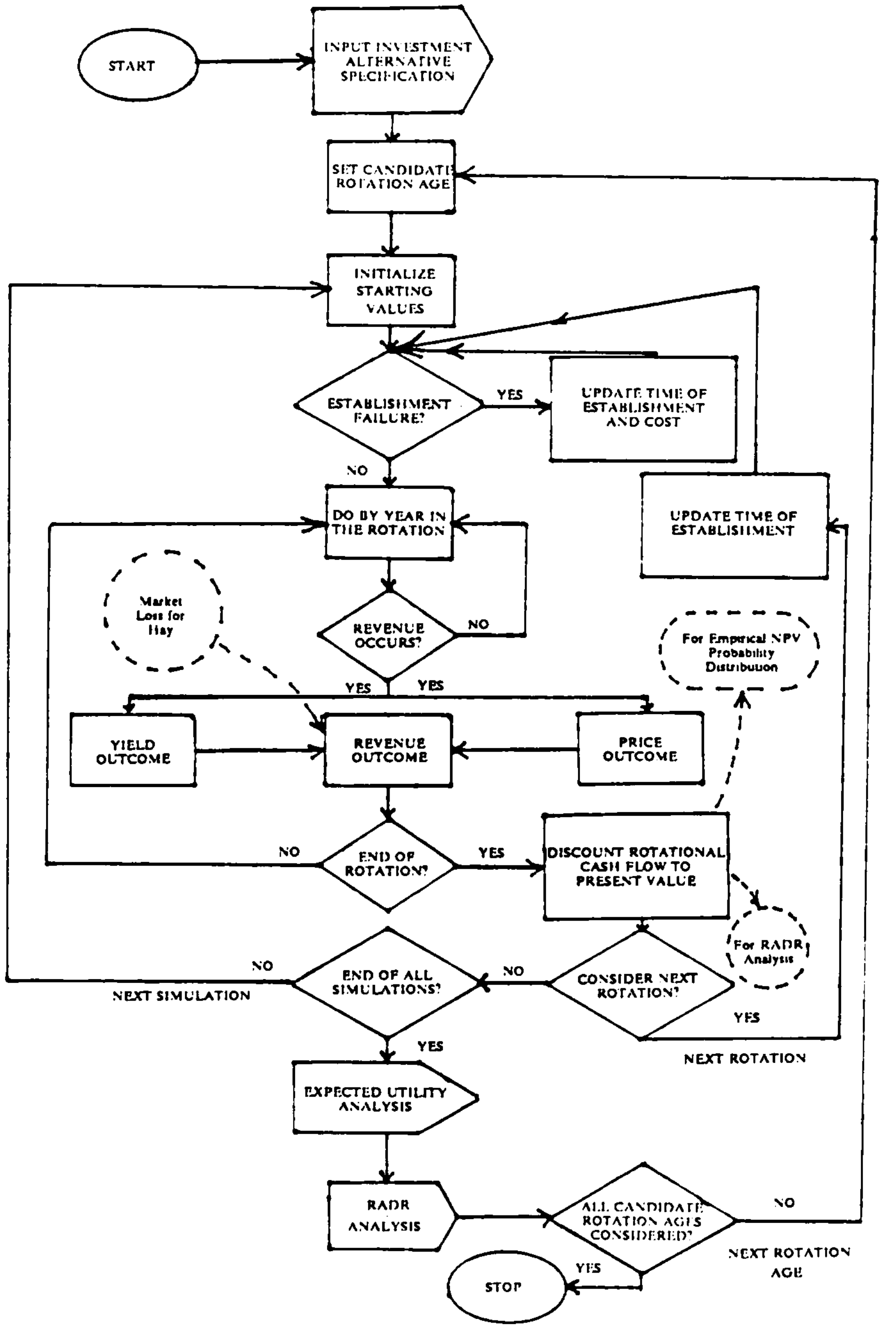

Figure 5. Flowchart for the empirical simulation model.

NPV - net present value

RADR - risk-adjusted discount rate

modulus multiplicative linear congruential generator taken from Schrage (1979).[105] Random variates representing the stochastic disturbances in the VAR stochastic price model, as well as in the autoregressive process for hay yields, were generated using the standard normal generator in PTAEDA2 (Burkhart *et al.*, 1987).[106] However, to be explicit about the assumptions underlying (10c) and (10f), a separate generator was used for each autoregressive relationship in the VAR model.

Risk-Adjusted Discount Rate Analysis

Under the RADR approach, the mean risk-adjusted NPV was earlier defined (in equation (22)) as

$$\mathrm{NPV}_{\mathrm{RADR}} = \sum_t \frac{\mathrm{E}[CF_t]}{(1+\mathrm{k})^t} \qquad [36]$$

However, (36) can be written as

$$\mathrm{NPV}_{\mathrm{RADR}} = \mathrm{E}\left[\sum_t \frac{CF_t}{(1+\mathrm{k})^t}\right] \qquad [37]$$

105 This generator, called RAND, is discussed in Law and Kelton (1982).

106 The generator, which utilizes the sum of 12 independent uniform (0,1) random variates, was extensively tested and was found to be the most expedient and reliable of all the candidate generators considered for PTAEDA2 (Burkhart, H. 1988. Thomas M. Brooks Professor, Department of Forestry, Virginia Polytechnic Institute and State University. Blacksburg, Virginia. Personal communication). However, the Box-Muller generator for a standard normal variate (see Law and Kelton, 1982) has been found to be more expedient and statistically reliable than the above approach (Tew, J. D. 1989. Assistant Professor, Department of Industrial Engineering and Operations Research, Virginia Polytechnic Institute and State University, Blacksburg, Virginia. Personal communication.)

Therefore, an equivalent approach to estimating NPV_{RADR} is to discount each cash flow realization with the risk-adjusted rate, and then take the mean value of the net present value results.[107]

The actual relationship between (36) and (37) is that (36) follows from (37) when the timing of the cash flow components t is certain. However, for the plantation investments considered in this study, t is uncertain due to the possibility of delays in establishment from planting failure. With t random (denoted now as t), (36) is no longer an applicable specification for NPV_{RADR} since (14) now defines a joint expectation with respect to both the cash flow component valuing and its timing.[108] Furthermore, the realization of CF_t and t will not be independent since CF_t ultimately depends on t due to the stochastic price process being time dependent, namely through the lagged terms in the VAR model. Therefore, (37) provides a more general representation of NPV_{RADR} that is equally applicable to the case of certain t or random t.

For each simulated cash flow realization, the realization of the timing of the cash flow components is known. Therefore, the empirical RADR analysis was to discount each cash flow realization with the risk-adjusted rate and then estimate NPV_{RADR} by taking the average value of the results. In order to preserve a common ground between

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107 Note that (37) does not define the mean outcome for the empirical NPV distribution. This parameter is only defined with respect to the risk free rate (Lintner, 1965a; Lewellen and Long, 1972). With a positive risk premium in the discount rate (i.e., the risk aversion case), the relationship is

$$E[NPV] = E\left[\sum_t \frac{CF_t}{(1+f)^t}\right] > E\left[\sum_t \frac{CF_t}{(1+k)^t}\right] = NPV_{RADR}$$

since $k > f$. Therefore, the specification of NPV_{RADR} in (37) should still be interpreted as a mean risk-adjusted net present value (Lewellen and Long, 1972).

108 Lewellen and Long (1972) discuss this ambiguity with respect to uncertain project life, given that the revenue value is known.

the expected utility and RADR approaches to risk, the RADR analysis used the same cash flow realizations underlying the estimation of the empirical NPV probability distribution.

The analysis considered two cases concerning the treatment of investment costs in the RADR analysis:

1) Investment costs were discounted with the risk free rate.
2) Investment costs were discounted with the risk-adjusted rate.

The first case isolates the source of RADR bias (if any) to the uncertain revenue cash flow for each alternative. That is, since costs were assumed to be relatively certain when compared to the revenue cash flow, discounting costs with the risk-adjusted discount rate would introduce an additional (and deliberate) source of bias. Therefore, the second case analyzed the possible effects of doing so.

The analysis targets various risk-adjusted discount rates for a fixed specification for the risk free rate. For example, if the risk free rate was 3 percent real, real risk adjusted discount rates of 3.5, 4.0, 4.5, and 5.0 percent were used (i.e., 4 risk-adjusted rates containing approximate risk premiums of 0.5, 1.0, 1.5, and 2.0 percent respectively).[109]

Risk Free Discount Rate Assumptions

Sensitivity analysis targeted 3 possible specifications for the risk free rate (applicable to both the expected utility and RADR analysis): 2, 3, and 4 percent (real).

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[109] As indicated in the discussion of the risk-adjusted discount rates considered in the hypothetical analysis, the difference between the risk-adjusted and risk free discount rates can be thought of as a close approximation of the actual risk premium. In the strict sense, the risk-adjustment will be slightly less than the difference between the two rates.

These rates bracket Carlson's (1977) estimated real rate of return on Treasury Bills (9-12 month issues) during the late 1960s.[110]

Using a market risk free rate of return is not inconsistent with the assumption that the risk free rate represents the individual's marginal rate of time preference. The assumption is that there was an exogenous (and risk-less) capital market[111] that the individual could either borrow from or lend to in order to meet their consumption preferences.[112] If the individual's subjective time preference is less than the risk free rate, the individual will be a lender in this market; if the subjective time preference is greater than the risk free rate, the individual will be a borrower. In market equilibrium (i.e., borrowing equals lending) all individuals make investment decisions based on the same marginal rate of time preference, which is set by the market clearing interest rate (Copeland and Weston, 1983). This in turn defines the risk free opportunity cost of investment.

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[110] However, Carlson (1977) observed much lower estimates for the real risk free rate of return during earlier periods. Furthermore, Ibbotson and Sinquefield (1977) also have estimated lower estimates for the real risk free rate. This study chose the 2 percent value as the lower benchmark primarily out of concern that under lower rates, the simulated discounted cash flow may not represent a declining geometric series, implying an infinite valuation (in present value) for the alternative.

[111] The risk premium required for supplying capital for each alternative is the same for the individual as for the capital market in this partial equilibrium framework. In essence, the expected utility approach in this study is designed to determine the correct premium, while the RADR approach presupposes this information. Therefore, the risk free discount rate measures the certain opportunity cost of invested funds.

[112] This is the Fisher consumption-investment model (see Copeland and Weston, 1983).

RESULTS

DEMONSTRATING RISK-ADJUSTED DISCOUNT RATE BIAS: THE HYPOTHETICAL CASE[113]

RADR Bias within a Class of Investment Alternatives

In order to illustrate RADR bias within a given class of investment alternatives, the hypothetical analysis considered an investment opportunity with a certain cost (in year zero) of $1,000 and uncertain payoffs in years $t = 10, 20, \ldots, 60$, where the certainty equivalent NPV (i.e., NPV_{CE}) was $5,000 under the 2 percent risk free discount rate.[114] Given the variance[115]) and certainty equivalent cash flow component values, tables 5 ($W_0 = \$20000$) and 6 ($W_0 = \$60000$) show the necessary expected cash flow values that satisfy the relationships in equation (25), where wealth utility was logarithmic. Accordingly, the certainty equivalent adjustment factors (as defined in (20d)) give a scale free measure of each cash flow component's risk to the individual.

Table 7 shows the correct risk-adjusted discount rate to apply to each cash flow component, and the single correct overall risk-adjusted discount rate to apply to the entire expected revenue cash flow such that the proper valuation of each alternative

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113 The results of the hypothetical analysis are also contained in Cathcart and Klemperer (1988b).

114 In this case the certainty equivalent revenue cash flow was $1,889.96 in years $t = 10, 20, \ldots, 60$.

115 These were generated using a starting variance value of 6,501,146 dollars squared. Also shown in tables 5 and 6 are the cash flow component standard deviations and probability of a non-positive cash flow outcome to give a better feel for the magnitude of the variances for each alternative (alternatives 1, 2, and 3 defined by the cash flow risk trend).

Table 5. Cash flow component values regarding risk under different cash flow risk trends (W_o = $20000, hypothetical analysis).

WEALTH ENDOWMENT = $20,000.00
RISK FREE RATE OF DISCOUNT = 0.02

YEAR	EXP CF	RISK	CE CF	CE ADJ FAC	ST DEV	PLOSS
ALTERNATIVE 1: Decreasing Cash Flow Risk: Linear Trend MD = 0.02						
10	2007.94	5207121	1889.96	0.9412	2281.91	0.1894
20	1979.33	3937092	1889.96	0.9548	1984.21	0.1593
30	1950.63	2667063	1889.96	0.9689	1633.11	0.1162
40	1921.82	1397034	1889.96	0.9834	1181.96	0.0520
50	1892.86	127006	1889.96	0.9985	356.38	0.0000
60	1889.96	0	1889.96	1.0000	0.00	0.0000
ALTERNATIVE 2: Increasing Cash Flow Risk: Geometric Trend GI = 0.03						
10	2076.82	8285440	1889.96	0.9100	2878.44	0.2353
20	2139.99	11134841	1889.96	0.8832	3336.89	0.2607
30	2224.10	14964183	1889.96	0.8498	3868.36	0.2827
40	2335.64	20110416	1889.96	0.8092	4484.46	0.3012
50	2483.04	27026496	1889.96	0.7611	5198.70	0.3165
60	2676.83	36321024	1889.96	0.7060	6026.69	0.3285
ALTERNATIVE 3: Increasing Cash Flow Risk: Geometric Trend GI = 0.06						
10	2131.02	10728346	1889.96	0.8869	3275.42	0.2576
20	2316.31	19212640	1889.96	0.8159	4383.22	0.2986
30	2637.31	34406576	1889.96	0.7166	5865.71	0.3265
40	3181.59	61616368	1889.96	0.5940	7849.61	0.3426
50	4075.87	110344336	1889.96	0.4637	10504.49	0.3490
60	5486.27	197608080	1889.96	0.3445	14057.31	0.3482

Notes:

YEAR	- year in which the uncertain revenue cash flow component occurs.
EXP CF	- expected cash flow component value (dollars).
CE CF	- certainty equivalent cash flow component value (dollars).
RISK	- variance of the uncertain cash flow component (dollars squared).
CE ADJ FAC	- certainty equivalent adjustment factor (see equation (20d)).
ST DEV	- standard deviation of the uncertain cash flow component value (dollars).
PLOSS	- probability that the cash flow component value outcome is less than zero.
MD	- annual rate of linear decrease in cash flow variance (%/100).
GI	- annual rate of geometric increase in cash flow variance (%/100).

Table 6. Cash flow component values regarding risk under different cash flow risk trends (W_0 = $60000, hypothetical analysis).

WEALTH ENDOWMENT = $60,000.00
RISK FREE RATE OF DISCOUNT = 0.02

YEAR	EXP CF	RISK	CE CF	CE ADJ FAC	ST DEV	PLOSS
ALTERNATIVE 1: Decreasing Cash Flow Risk: Linear Trend MD = 0.02						
10	1932.02	5207121	1889.96	0.9782	2281.91	0.1986
20	1921.82	3937092	1889.96	0.9834	1984.21	0.1664
30	1911.51	2667063	1889.96	0.9887	1633.11	0.1209
40	1901.31	1397034	1889.96	0.9940	1181.96	0.0539
50	1891.06	127006	1889.96	0.9994	356.38	0.0000
60	1889.99	0	1889.96	1.0000	0.00	0.0000
ALTERNATIVE 2: Increasing Cash Flow Risk: Geometric Trend GI = 0.03						
10	1956.83	8285440	1889.96	0.9658	2878.44	0.2483
20	1979.75	11134841	1889.96	0.9546	3336.89	0.2765
30	2010.57	14964183	1889.96	0.9400	3868.36	0.3016
40	2051.83	20110416	1889.96	0.9211	4484.46	0.3236
50	2107.19	27026496	1889.96	0.8969	5198.70	0.3426
60	2181.34	36321024	1889.96	0.8664	6026.69	0.3587
ALTERNATIVE 3: Increasing Cash Flow Risk: Geometric Trend GI = 0.06						
10	1976.51	10728346	1889.96	0.9562	3275.42	0.2731
20	2044.63	19212640	1889.96	0.9244	4383.22	0.3204
30	2166.09	34406576	1889.96	0.8725	5865.71	0.3560
40	2381.91	61616368	1889.96	0.7935	7849.61	0.3808
50	2762.95	110344336	1889.96	0.6840	10504.49	0.3963
60	3428.78	197608080	1889.96	0.5512	14057.31	0.4036

Notes:

- YEAR - year in which the uncertain revenue cash flow component occurs.
- EXP CF - expected cash flow component value (dollars).
- CE CF - certainty equivalent cash flow component value (dollars).
- RISK - variance of the uncertain cash flow component (dollars squared).
- CE ADJ FAC - certainty equivalent adjustment factor (see equation (20d)).
- ST DEV - standard deviation of the uncertain cash flow component value (dollars).
- PLOSS - probability that the cash flow component value outcome is less than zero.
- MD - annual rate of linear decrease in cash flow variance (%/100).
- GI - annual rate of geometric increase in cash flow variance (%/100).

Table 7. Example of correct risk adjusted discount rates to use in investment evaluation (hypothetical analysis).

	RISK FREE RATE OF DISCOUNT = 0.02			
	W_o = $20000.00		W_o = $60000.00	
YEAR	CE ADJ FAC	CF RADR	CE ADJ FAC	CF RADR
ALTERNATIVE 1: Decreasing Cash Flow Risk: Linear Trend MD = 0.02				
10	0.9412	0.0262	0.9782	0.0223
20	0.9548	0.0223	0.9834	0.0209
30	0.9686	0.0211	0.9887	0.0204
40	0.9834	0.0204	0.9940	0.0202
50	0.9985	0.0200	0.9994	0.0200
60	1.0000	0.0200	1.0000	0.0200
	INV RADR = 0.0212		INV RADR = 0.0204	
ALTERNATIVE 2: Increasing Cash Flow Risk: Geometric Trend GI = 0.03				
10	0.9100	0.0297	0.9658	0.0236
20	0.8832	0.0264	0.9546	0.0224
30	0.8498	0.0255	0.9400	0.0221
40	0.8092	0.0254	0.9211	0.0221
50	0.7611	0.0256	0.8969	0.0222
60	0.7060	0.0259	0.8664	0.0224
	INV RADR = 0.0261		INV RADR = 0.0224	
ALTERNATIVE 3: Increasing Cash Flow Risk: Geometric Trend GI = 0.06				
10	0.8869	0.0323	0.9562	0.0246
20	0.8159	0.0304	0.9244	0.0240
30	0.7166	0.0314	0.8725	0.0246
40	0.5940	0.0334	0.7935	0.0259
50	0.4637	0.0358	0.6840	0.0278
60	0.3445	0.0383	0.5512	0.0302
	INV RADR = 0.0341		INV RADR = 0.0266	

Notes:

- W_o - individual's wealth endowment.
- YEAR - year in which the uncertain cash flow component occurs.
- CE ADJ FAC - certainty equivalent adjustment factor.
- CF RADR - implied correct risk-adjusted rate to discount the expected cash flow component value with (%/100).
- INV RADR - implied correct risk-adjusted rate for discounting the entire expected cash flow (%/100).
- MD - annual rate of linear decrease in cash flow risk (%/100).
- GI - annual rate of geometric increase in cash flow risk (%/100).

(defined by the cash flow risk trend, alternatives 1, 2, and 3 in table 7) was achieved (i.e., NPV_{RADR} = $5000). Inspection of table 7 shows that the correct cash flow component RADRs (CF RADR in table 7) vary with the individual's perception of cash flow component risk as measured by the certainty equivalent adjustment factor. Furthermore, note that the increasing cash flow risk (with respect to the variability in the cash flow) did not always imply a constant (or even an increasing) risk-adjusted discount rate with respect to time.[116] Finally, the overall implied RADR (INV RADR in table 7) varied with the overall riskiness of the alternative. In both cases, the higher the perceived risk, the larger the risk adjustment in the discount rate.

If the results are compared across wealth endowment assumptions, the more endowed individual (W_0 = $60000) was less conservative in his or her adjustment for risk than the less endowed individual (W_0 = $20000) due to decreasing absolute risk aversion with respect to wealth under logarithmic utility. In other words, in order for the two individuals to have the same valuation for the three alternatives shown, the more endowed individual requires a lower expected cash flow for all three alternatives (table 6) than the less endowed individual (table 5).

RADR Bias in the Ranking and Valuation of Investment Alternatives

The hypothetical analysis considered three broad classes of investments (figure 2). Figure 6 illustrates the RADR bias in the ranking (due to valuation bias) of these

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116 This result is not inconsistent with earlier observations in Robichek and Myers (1966) and Chen (1967). Their contention that increasing cash flow risk with respect to time justifies the use of a constant RADR refers to certainty equivalent adjustment factor; not the variability in the cash flow. As Chen (1967) demonstrated, if the certainty equivalent adjustment factor decreases (with respect to time) at a constant rate (i.e., the adjustment for risk with respect to time increases at a constant rate), the cash flow component risk premiums in the discount rate will be constant. However, it is the variability in the cash flow that is observable. As the results here demonstrate, the likelihood that the dynamic relationship underlying cash flow variability will meet Chen's condition for constant RADRs will be (at best) remote.

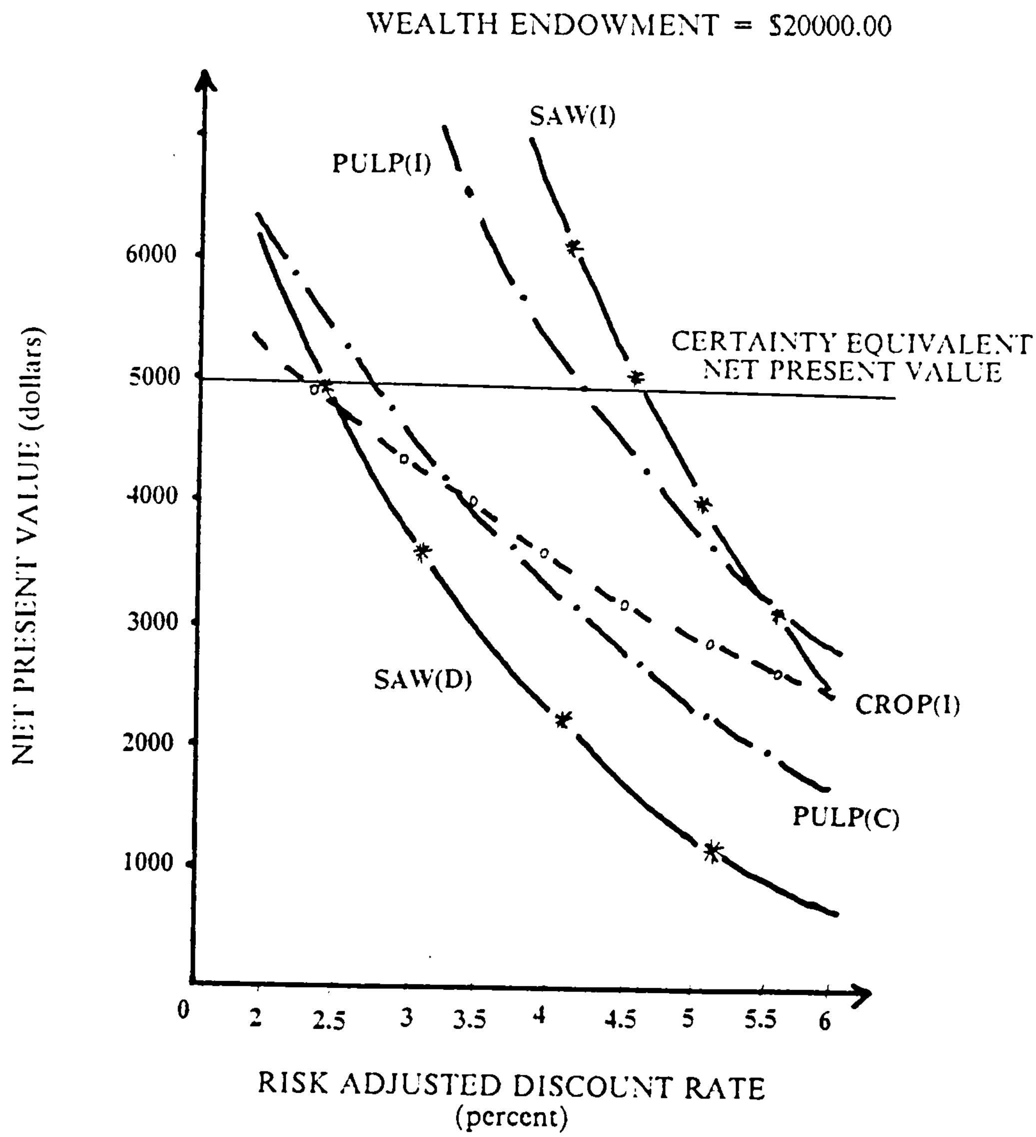

Figure 6. Mean risk-adjusted NPV results for the 3 investment types considered in the hypothetical analysis (W_o = $20000).

NPV	- net present value
SAW(I)	- sawtimber investment with geometrically increasing cash flow risk (rate of increase is 6 %/year).
PULP(I)	- pulpwood investment with geometrically increasing cash flow risk (rate of increase is 6 %/year).
CROP(I)	- crop investment with geometrically increasing cash flow risk (rate of increase is 6 %/year).
PULP(C)	- pulpwood investment with constant cash flow risk.
SAW(D)	- sawtimber investment with geometrically decreasing cash flow risk (rate of decrease is 2 %/year).

alternatives for the case where the wealth endowment to the individual was $20,000, while figure 7 diagrams the results of the analysis for the case where the wealth endowment was $60,000.[117]

Indicated in figures 6 and 7 is the certainty equivalent NPV of $5,000. The curves in figures 6 and 7 represent the corresponding NPV estimates under the various risk adjusted discount rates. The intersection of these curves with the certainty equivalent NPV line gives the correct single RADR for discounting the entire cash flow. For example, in figure 6 the correct single RADR for the sawtimber investment with geometrically increasing cash flow risk (GI=0.06, SAW(I) curve in figure 6) was 4.5 percent. The correct single RADR for the crop investment with geometrically increasing cash flow risk (GI=0.06, CROP(I) curve in figure 6) was 2.2 percent. Comparison of these rates implies that while risk was increasing geometrically for both alternatives, the crop investment turned out to be less risky because the annual payoffs in effect diluted the risk. However, in the sawtimber investment, the risk was concentrated in the single uncertain cash flow payoff in year 40.

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117 Figures 6 and 7 diagram the results for 5 of the 15 investment alternatives considered. They are

SAW(I) - sawtimber investment with geometrically increasing cash flow risk (rate of increase is 6 %/year).

PULP(I) - pulpwood investment with geometrically increasing cash flow risk (rate of increase is 6 %/year).

CROP(I) - crop investment with geometrically increasing cash flow risk (rate of increase is 6 %/year).

PULP(C) - pulpwood investment with constant cash flow risk.

SAW(D) - sawtimber investment with geometrically decreasing cash flow risk (rate of decrease is 2 %/year).

The alternatives were chosen because they highlight the biases encountered in all 15 alternatives considered. Note that figures 6 and 7 are similar, indicating that the biases incurred are insensitive to the wealth endowment assumption.

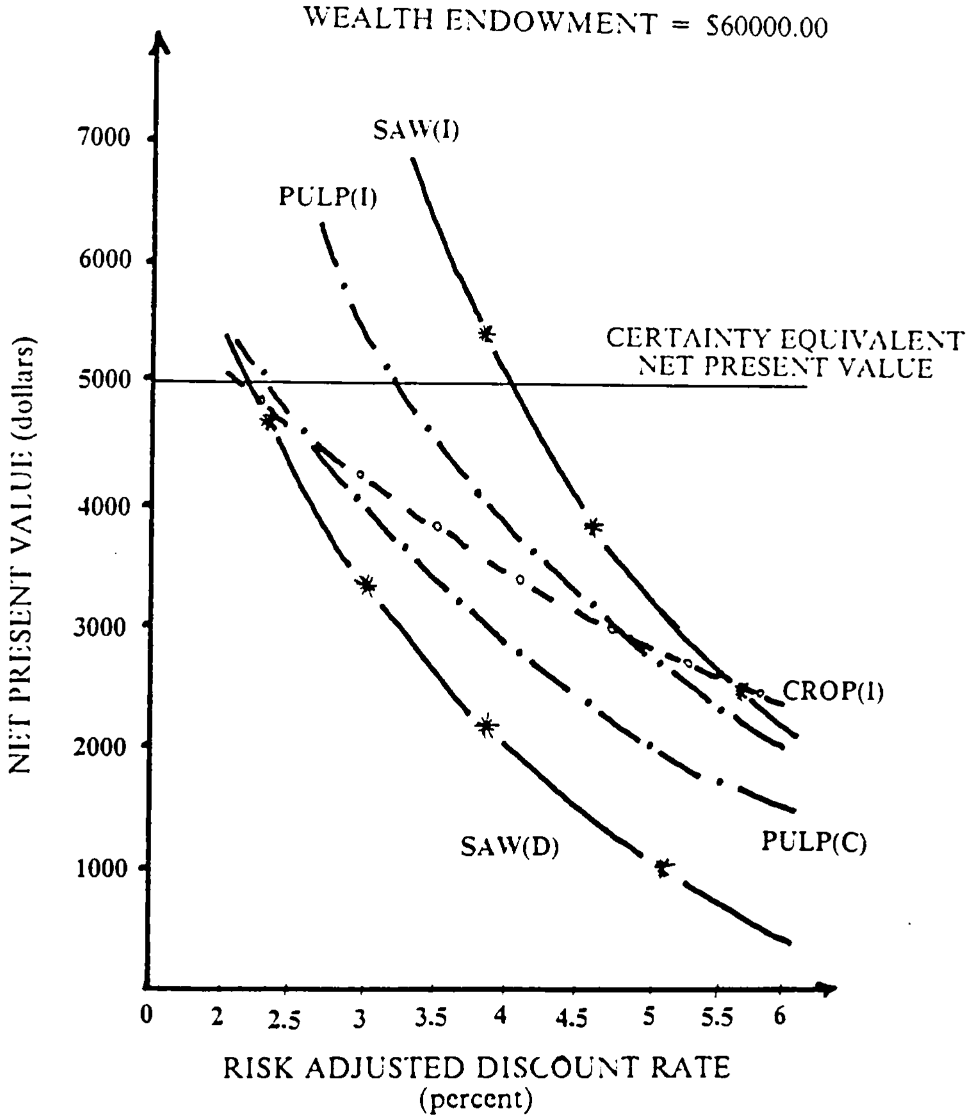

Figure 7. Mean risk-adjusted NPV results for the 3 investment types considered in the hypothetical analysis (W_0 = $60000).

NPV	- net present value
SAW(I)	- sawtimber investment with geometrically increasing cash flow risk (rate of increase is 6 %/year).
PULP(I)	- pulpwood investment with geometrically increasing cash flow risk (rate of increase is 6 %/year).
CROP(I)	- crop investment with geometrically increasing cash flow risk (rate of increase is 6 %/year).
PULP(C)	- pulpwood investment with constant cash flow risk.
SAW(D)	- sawtimber investment with geometrically decreasing cash flow risk (rate of decrease is 2 %/year).

The amount of deviation of the NPV curves from the certainty equivalent NPV line reflects the valuation bias from using an incorrectly specified RADR. For every rate considered, some alternatives were incorrectly valued. For example, if the chosen RADR was 4 percent in figure 6, all the investment alternatives with an implied correct RADR less than 4 percent (the CROP(I), PULP(C), and SAW(D) alternatives) were under-valued, reflecting too high an adjustment for risk. However, the SAW(I) and PULP(I) alternatives (with implied correct RADRs greater than 4 percent) were over-valued, reflecting too low an adjustment for risk. The result was that the SAW(I) alternative would be incorrectly ranked as the preferred alternative.

The slope of the NPV curves in figures 6 and 7 indicate how sensitive the investment's valuation is to the magnitude of the risk adjustment in the discount rate. The steeper the slope, the more sensitive the alternative was to the magnitude of the risk-adjusted discount rate. For example, the sawtimber and pulpwood investments (which have delayed payoffs) were more sensitive to an incorrect specification of the RADR than the crop investment (which had an annual payoff). This was consistent with traditional sensitivity analyses with respect to the magnitude of the discount rate; low rates tend to favor delayed payoff investments while higher discount rates tend to have a harsher effect on these types of investments. The result was that under different RADRs, there were differences in the ranking of the alternatives (albeit, an incorrect ranking, recall that the true ranking was one of indifference). For example, in figure 7, under the 2.5 percent RADR, the CROP(I) alternative was ranked relatively low, while the PULP(I) and SAW(I) investments were ranked relatively high. However, when the RADR was chosen to be 6 percent, the CROP(I) investment outranks all alternatives (figure 7).

EMPIRICAL ANALYSIS: THE LANDOWNER'S DECISION

The hypothetical analysis demonstrated that a single *a priori* specified RADR will not only bias the valuation of each investment alternative, but the relative desirability between alternatives as well. The purpose of the empirical analysis was to assess to what degree these biases occur in a typical stand level decision environment. The results of the empirical analysis presented here emphasize the ranking and the valuation of the alternatives considered. Underlying these results was the identification of the preferred rotation age for timber investments, as well as the interaction between the estimated empirical NPV probability distribution and the risk adjustments from the expected utility analysis.

Summary of the Empirical Analysis Results by Alternative Type

Appendices F through K detail the results of the empirical analysis for the land use alternatives A through F respectively.[118] In general, the appendices detail the identification of the preferred rotation age[119] for timber investments, as well as highlight the results of the expected utility analysis based on the estimated empirical NPV probability distribution for each alternative. Finally, differences between the true

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118 The alternatives were:

ALTERNATIVE A: Sawtimber, 500 trees/acre, no thinning (appendix F).
ALTERNATIVE B: Sawtimber, 700 trees/acre, no thinning (appendix G).
ALTERNATIVE C: Sawtimber, 700 trees/acre, thinning at age 20 (appendix H).
ALTERNATIVE D: Pulpwood investment (appendix I).
ALTERNATIVE E: Custodial management (appendix J).
ALTERNATIVE F: Forage hay investment (appendix K).

119 The preferred rotation age was defined as the age which maximized NPV_{CE} under the expected utility analysis, or maximized NPV_{RADR} under the RADR analysis, for the candidate rotation ages considered.

certainty equivalent NPV valuation and the mean risk-adjusted NPV valuation under the RADR approach are discussed.

The results underlying alternative A were typical of the results for the other sawtimber investments (appendix F). For the most part, the simulated yields for the timber investments fell within a narrow range of possible outcomes. In general, the relative risk of sawtimber investments increased under higher specifications of the risk free discount rate; primarily due to the sensitivity of these investments to establishment costs. Furthermore, the empirical NPV distributions were distinctly skewed negatively, due to the sensitivity to establishment costs combined with the possibility of repetitive planting failure. Thinning the stand did not result in any deviation from the above generalizations (appendix H).

For the most part, the preferred rotation age was insensitive to the assumption regarding the individual's risk preferences. That is, the same preferred rotation age was identified under each set of utility assumptions considered.[120] The only exception was for alternative B, where differences in the degree of relative risk aversion (negative exponential utility) resulted in differences in the preferred rotation age (appendix G). More importantly, assuming risk neutrality still led to the same identified preferred rotation age as for the risk averse cases considered. Finally, there was essentially no observed risk-adjusted discount rate bias in the identification of the preferred rotation age for the timber investments (alternative's A through C, and E in appendices F

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120 These cases were (see appendix B):

LOGARITHMIC UTILITY OF WEALTH (per acre basis)

CASE 1: Low degree of absolute risk aversion.
CASE 2: High degree of absolute risk aversion.

NEGATIVE EXPONENTIAL UTILITY OF WEALTH (per acre basis)

CASE 3: Low degree of relative risk aversion.
CASE 4: High degree of relative risk aversion.

through H, and J respectively). That is, the RADR approach identified the same preferred rotation age as the age from the expected utility analysis.

The pulpwood investment (alternative D, appendix I) was the only alternative that was not viable under any of the risk free discount rates considered (2, 3, and 4 percent). In essence, targeting pulpwood as the only timber commodity in the plantation did not justify the establishment cost. Therefore, alternative D represented the case where a bad investment was a bad investment, regardless of risk. On the other hand, the custodial management alternative (alternative E, appendix J) was viewed as being essentially risk free regardless of the assumptions concerning the degree of risk aversion. This was because the variability of NPV outcomes was extremely low, and all possible outcomes were positive due to the absence of any investment costs. The empirical NPV probability distribution for this alternative was essentially symmetric under all the risk free discount rates considered; suggesting that the negative skewness for the other plantation investments can be attributed to establishment costs and the possibility of planting failure, and not due to the probability distribution underlying timber harvest revenues.

Finally, the forage hay investment (alternative F, appendix K) was the most volatile of all alternatives considered. The variability of NPV outcomes was markedly higher than the case for timber investments, and skewness was positive. Furthermore, unlike the plantation investments, the hay investment remain viable (with respect to the mean NPV) under the 4 percent risk free discount rate. However, negative NPV outcomes were very likely, indicating that the hay investment was sensitive to a sustained annual profit margin over the course of each rotation.[121] Therefore, the

121 In contrast, the viability of the plantation investments were sensitive to the success or failure of stand establishment.

expected utility adjustments for variance under risk aversion were significant enough such that the hay investment was not viewed as a viable alternative (i.e., $NPV_{CE} < 0$) in spite of its positive mean NPV.

Efficiency of Alternatives

Table 8 summarizes the parameter estimates for the empirical NPV probability distributions for each alternative. For timber investments, only the results for the preferred rotation age are shown. Note that alternative A completely dominates alternative B. That is, increasing the planting density from 500 trees per acre (alternative A) to 700 trees per acre (alternative B), but not thinning, resulted in a lower mean NPV accompanied by higher variance and more (negative) skewness. This result held regardless of the specification of the risk free discount rate (table 8). Therefore, alternative B will never be preferable to alternative A for any risk averse individual. However, thinning the stand planted at 700 trees/acre (alternative C) was not dominated by alternative A since the mean NPV for alternative C exceeds the mean NPV for alternative A (table 8).[122] In this case, some risk averse individuals may prefer alternative A because the lower mean is accompanied by lower risk in terms of the variance and skewness. However, other risk averse individuals may be willing to accept the higher risk associated with alternative C because it is accompanied by a higher mean as well.

With the exception of the pulpwood investment (alternative D), all remaining alternatives were efficient. That is, out of the set of alternatives {A, C, E, F}, none dominates the other in the sense that a lower mean was accompanied by higher variance and more (negative) skewness. The forage hay investment, while having the highest variance, was not dominated by any other alternative because it had positive skewness.

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[122] Note further, alternative C dominates alternative B. Therefore, thinning not only had the effect of improving the mean NPV, but lowered the variance and skewness of the empirical NPV probability distribution as well.

Table 8: Empirical NPV probability distribution parameter results by alternative type (empirical analysis).

ALT[1]	RISK FREE DISCOUNT RATE (percent/100) 0.02			0.03			0.04		
	MEAN[2]	VARIANCE[3]	SKEWNESS[4]	MEAN	VARIANCE	SKEWNESS	MEAN	VARIANCE	SKEWNESS
A (r=32)	72.27	1114	-42000	22.65	780	-32178	-1.96	619	-26230
B (r=36)	57.62	2015	-134728	5.55	1411	-98537	-19.92	1121	-77900
(r=40)	53.54	1744	-69097	1.07	1203	-51204	-24.09	961	-43200
C (r=32)	87.21	1696	-52493	26.03	1174	-46143	-4.65	939	-41068
D (r=20)	-43.63	1703	-96888	-52.40	1415	-75016	-57.99	1213	-61644
E (r=16)	38.51	38	-18	26.08	21	-10	18.87	13	-6
F (r=6)	49.22	19288	931348	31.37	13693	352050	16.11	10239	125468

Notes:

[1] Alternative (r denotes the preferred rotation age in years).

A: Sawtimber, 500 trees/acre, no thinning.
B: Sawtimber, 700 trees/acre, no thinning.
C: Sawtimber, 700 trees/acre, thinning at age 20.
D: Pulpwood investment.
E: Custodial management.
F: Forage hay investment.

[2] in 1967 dollars/acre

[3] in 1967 $(\$/acre)^2$

[4] in 1967 $(\$/acre)^3$

NPV - net present value.

Therefore, depending on the specific risk preferences of the risk averse individual, and how they define mean-variance and mean-skewness tradeoffs (appendix B), any one of the set of alternatives {A, C, E, F} could be the most preferred. However, in a mean-variance context, the hay investment would be dominated by alternatives A, C, and E when the risk free discount rate is 2 percent, and by alternative E when the risk free rate is 4 percent. In as much as the expected utility adjustment for variance out weights the adjustment for skewness, most risk averters would most likely prefer timber investment over hay under these two specifications of the risk free rate.

RADR Bias in the Valuation of Investments

A striking result in the empirical analysis was that any noticeable risk premium (i.e., 0.5 percent or higher) represented an under-valuation of the alternative. For the most part, this result held regardless of whether investment costs were discounted with the risk free rate or the risk-adjusted rate. The valuation bias for each alternative followed the pattern in figure 8. As indicated in figure 8, in order for the alternative to be over-valued, the risk premium in the RADR would have to be imperceptibly small (roughly on an order or magnitude less than 0.1 percent; i.e., < 0.001 percent/100). Therefore, in contrast to the hypothetical demonstration (figures 6 and 7), essentially any adjustment for risk in the discount rate will under-value the investment due to an over-compensation for cash flow risk.

The only exception to this result was for the forage hay investment (alternative F). If the investment costs were discounted with the RADR, over-valuation of the investment resulted under perceptible specifications of the discount rate risk premium (see appendix K). But discounting with a RADR containing a positive risk premium is

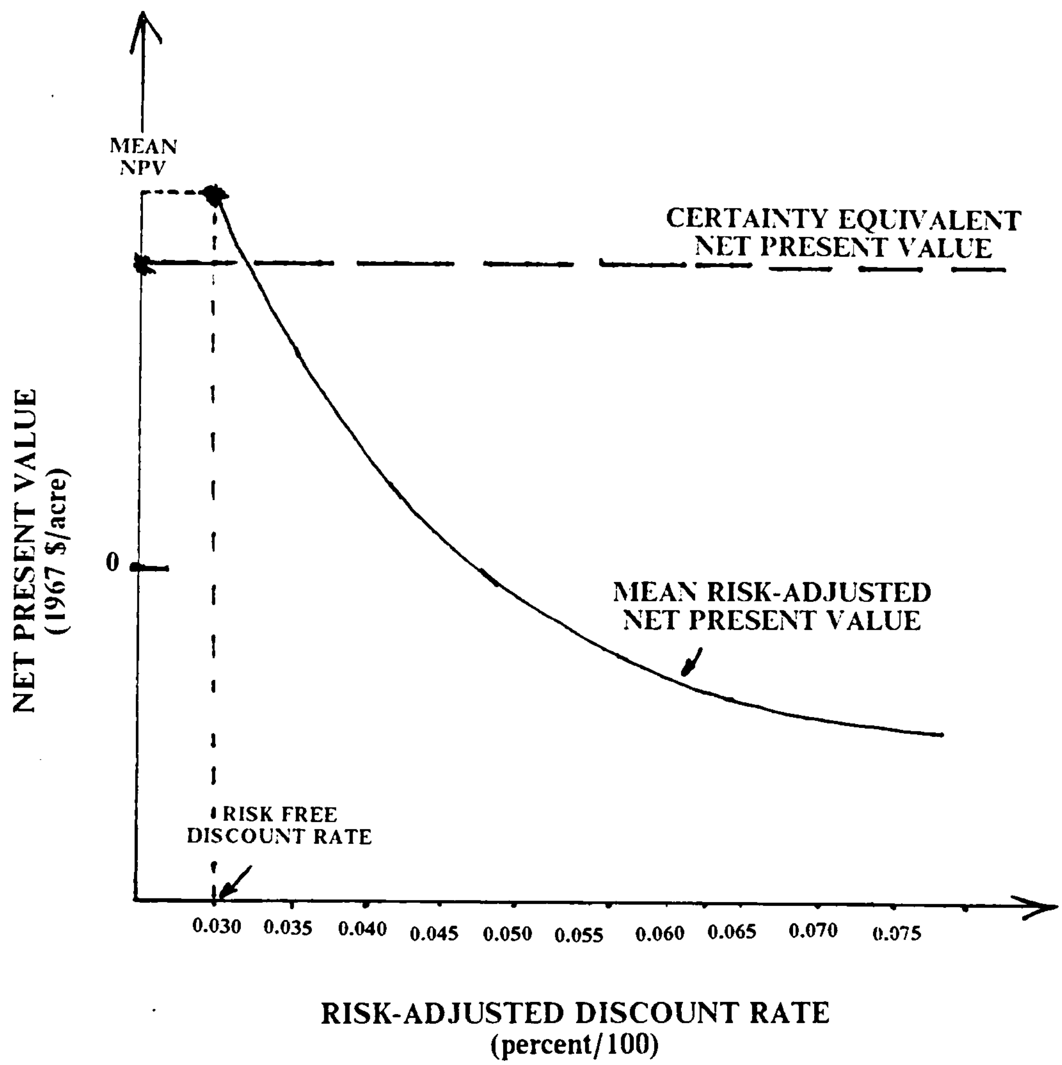

Figure 8. Illustration of RADR valuation bias for each alternative considered in the empirical analysis.

The pattern was representative of the results given in appendices F through K. The 3 percent risk free discount rate was used as a benchmark. The pattern also reflects the results when the risk free rate was 2 or 4 percent. However, the range of risk-adjusted discount rates over which the mean risk-adjusted NPV remains positive will vary (i.e., lower under higher risk free discount rates).

NPV - net present value

RADR - risk-adjusted discount rate

a deliberate under-compensation for cost risk.[123] In essence, since the hay investment was cost intensive (i.e., annual cost cash flow), the under-compensation for cost risk more than offset the over-compensation for revenue risk. By treating costs as relatively certain (the assumption in this study) and discounting them with the risk free rate in the RADR analysis, the valuation bias for the forage hay investment followed the pattern shown in figure 8.

The True Ranking of the Alternatives under the Expected Utility Analysis

Risk Free Discount Rate = 2 percent

Table 9 summarizes the ranking of the alternatives for the different utility cases (including risk neutrality) when the risk free discount rate was specified to be 2 percent. In all cases, the preferred investment was the sawtimber investment with thinning (alternative C). That is, the preferred alternative identified under risk neutrality was also the preferred investment under all 4 risk averse cases considered. For the most part the ranking of the timber alternatives by themselves was the same under all utility cases (including risk neutrality). The only exception was under a high degree of relative risk aversion (CASE 4, table 9). In this case the custodial management alternative rose in ranking above alternative B.

The most dramatic reversal in ranking occurred for the forage hay alternative when the assumption switched from one of risk neutrality to one of risk aversion. Under risk neutrality, the hay alternative was preferred to the essentially risk-less custodial management alternative. However under risk aversion, the custodial management alternative was preferable to hay in spite of its lower mean NPV. Furthermore,

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123 As indicated in the analytical treatment of the problem, the risk premium in the discount rate for uncertain costs will be negative. Therefore, applying the RADR with a positive risk premium will under-compensate for risk, even for the case in which investment costs are considered certain.

Table 9. True ranking of investment alternatives, empirical analysis (risk free discount rate = 2 percent).

RISK FREE DISCOUNT RATE = 2 PERCENT						
ALTERNATIVE	MEAN[1] NPV	CERTAINTY EQUIVALENT NET PRESENT VALUE (1967 $/acre)				
	RISK NEUTRALITY	LOGARITHMIC[2] UTILITY		NEGATIVE EXPONENTIAL[3] UTILITY		
		CASE 1	CASE 2	CASE 3	CASE 4	
A: Sawtimber 500 trees/acre no thinning	2)[4] 72.27 (r[5] = 32)	2) 69.95 (r = 32)	2) 64.80 (r = 32)	2) 69.16 (r = 32)	2) 56.60 (r = 32)	
B: Sawtimber 700 trees/acre no thinning	3) 57.62 (r = 36)	3) 52.86 (r = 36)	3) 39.40 (r = 36)	3) 51.77 (r = 36)	4) 30.88 (r = 40)	
C: Sawtimber 700 trees/acre thinning at age 20	1) 87.21 (r = 32)	1) 83.94 (r = 32)	1) 78.08 (r = 32)	1) 82.55 (r = 32)	1) 65.99 (r = 32)	
D: Pulpwood investment	6) -43.63 (r = 20)	6) -50.79 (r = 20)	6) na[6] (r = 20)	6) -48.50 (r = 20)	6) -67.63 (r = 20)	
E: Custodial management	5) 38.51 (r = 16)	4) 38.42 (r = 16)	4) 38.17 (r = 16)	4) 38.41 (r = 16)	3) 38.00 (r = 16)	
F: Forage Hay investment	4) 49.22 (r = 6)	5) 16.72 (r = 6)	5) 3.85 (r = 6)	5) 7.60 (r = 6)	5) -10.73 (r = 6)	

Notes:

(1) Mean net present value (NPV) from the empirical NPV probability distribution.

(2)

CASE 1: Low degree of absolute risk aversion (see appendix B).
CASE 2: High degree of absolute risk aversion (see appendix B).

(3)

CASE 3: Low degree of relative risk aversion (see appendix B).
CASE 4: High degree of relative risk aversion (see appendix B).

(4) Denotes ranking of alternatives (1 is most preferred, 6 is least preferred).

(5) r - preferred rotation age in years.

(6) na - not available, mean NPV loss exceeds initial wealth endowment of 18.96 1967 $/acre.

accepting the hay investment over any of the sawtimber investments (alternatives A through C) would imply accepting a lower mean NPV with more risk. In essence, the risk adjustments for the hay alternative (as measured by NPV_{CE}) had little impact regarding the hay investment's ranking relative to the sawtimber investments since in terms of the mean alone, the sawtimber investments dominated.

Risk Free Discount Rate = 3 percent

Table 10 summarizes the ranking of the alternatives when the risk free discount rate was specified to be 3 percent. Under the higher risk free discount rate, the plantation investments became more sensitive to the cost impacts relative to the discounted value of revenue outcomes due to the delayed payoff pattern for these investments. Under risk aversion, the result was a lower ranking relative to custodial management (alternatives E) than was the case when the risk free discount rate was 2 percent. Under risk neutrality, sawtimber investments were ranked lower than forage hay (alternative F) and custodial management.

Unlike the case when the risk free discount rate was 2 percent, the identified preferred alternative depended on whether the individual was neutral or averse to risk. Under risk neutrality the preferred alternative was forage hay, while under risk aversion custodial management was the preferred (table 10). For the most part, adjusting the hay investment for risk affected its desirability. As indicated in table 10, the CASE 2-4 individuals valued the alternative as not being viable in spite of its positive mean NPV. That is, the mean NPV was not large enough to compensate for the alternative's underlying risk.

Risk Free Discount Rate = 4 percent

Table 11 summarizes the ranking of the alternatives when the risk free discount rate was 4 percent. In this case, the cost impacts on the sawtimber investments

Table 10. True ranking of investment alternatives, empirical analysis (risk free discount rate = 3 percent).

RISK FREE DISCOUNT RATE = 3 PERCENT						
ALTERNATIVE	MEAN[1] NPV	CERTAINTY EQUIVALENT NET PRESENT VALUE (1967 $/acre)				
	RISK NEUTRALITY	LOGARITHMIC[2] UTILITY		NEGATIVE EXPONENTIAL[3] UTILITY		
		CASE 1	CASE 2	CASE 3	CASE 4	
A: Sawtimber 500 trees/acre no thinning	4)[4] 22.65 (r[5] = 32)	3) 20.58 (r = 32)	2) 9.66 (r = 32)	3) 20.46 (r = 32)	2) 10.72 (r = 32)	
B: Sawtimber 700 trees/acre no thinning	5) 5.55 (r = 36)	5) 1.12 (r = 36)	5) -18.15 (r = 36)	4) 1.42 (r = 36)	4) -16.16 (r = 40)	
C: Sawtimber 700 trees/acre thinning at age 20	3) 26.03 (r = 32)	2) 23.00 (r = 32)	3) 9.47 (r = 32)	2) 22.75 (r = 32)	3) 9.43 (r = 32)	
D: Pulpwood investment	6) -52.40 (r = 20)	6) -58.73 (r = 20)	6) na[6] (r = 20)	6) -56.44 (r = 20)	6) -72.99 (r = 20)	
E: Custodial management	2) 26.08 (r = 16)	1) 26.03 (r = 16)	1) 25.84 (r = 16)	1) 26.02 (r = 16)	1) 25.80 (r = 16)	
F: Forage Hay investment	1) 31.37 (r = 6)	4) 4.75 (r = 6)	4) -10.49 (r = 6)	5) -0.30 (r = 6)	5) -27.19 (r = 6)	

Notes:

[1] Mean net present value (NPV) from the empirical NPV probability distribution.

[2]

CASE 1: Low degree of absolute risk aversion (see appendix B).
CASE 2: High degree of absolute risk aversion (see appendix B).

[3]

CASE 3: Low degree of relative risk aversion (see appendix B).
CASE 4: High degree of relative risk aversion (see appendix B).

[4] Denotes ranking of alternatives (1 is most preferred, 6 is least preferred).

[5] r - preferred rotation age in years.

[6] na - not available, mean NPV loss exceeds initial wealth endowment of 18.96 1967 $/acre.

Table 11. True ranking of investment alternatives, empirical analysis (risk free discount rate = 4 percent).

RISK FREE DISCOUNT RATE = 4 PERCENT						
ALTERNATIVE	MEAN[1] NPV	CERTAINTY EQUIVALENT NET PRESENT VALUE (1967 $/acre)				
	RISK NEUTRALITY	LOGARITHMIC[2] UTILITY		NEGATIVE EXPONENTIAL[3] UTILITY		
		CASE 1	CASE 2	CASE 3	CASE 4	
A: Sawtimber 500 trees/acre no thinning	3)[4] -1.96 (r[5] = 32)	2) -3.84 (r = 32)	3) -17.98 (r = 32)	2) -3.30 (r = 32)	2) -11.77 (r = 32)	
B: Sawtimber 700 trees/acre no thinning	5) -20.93 (r = 36)	5) -24.65 (r = 36)	5) na (r = 36)	5) -23.92 (r = 36)	4) -37.24 (r = 36)	
C: Sawtimber 700 trees/acre thinning at age 20	4) -4.65 (r = 32)	4) -7.56 (r = 32)	4) -18.95 (r = 32)	3) -7.30 (r = 32)	3) -18.79 (r = 32)	
D: Pulpwood investment	6) -57.99 (r = 20)	6) -63.66 (r = 20)	5) na (r = 20)	6) -61.44 (r = 20)	6) -76.05 (r = 20)	
E: Custodial management	1) 18.87 (r = 16)	1) 18.84 (r = 16)	1) 18.69 (r = 16)	1) 18.84 (r = 16)	1) 18.70 (r = 16)	
F: Forage Hay investment	2) 16.11 (r = 6)	3) -6.45 (r = 6)	2) -17.52 (r = 6)	5) -8.62 (r = 6)	5) -38.10 (r = 6)	

Notes:

(1) Mean net present value (NPV) from the empirical NPV probability distribution.

(2)

CASE 1: Low degree of absolute risk aversion (see appendix B).
CASE 2: High degree of absolute risk aversion (see appendix B).

(3)

CASE 3: Low degree of relative risk aversion (see appendix B).
CASE 4: High degree of relative risk aversion (see appendix B).

(4) Denotes ranking of alternatives (1 is most preferred, 6 is least preferred).

(5) r - preferred rotation age in years.

(6) na - not available, mean NPV loss exceeds initial wealth endowment of 18.96 1967 $/acre.

dominated their valuation as all alternatives were not viable. Furthermore, as was the case under the 2 percent risk free discount rate, there was no discrepancy between the preferred alternative under risk neutrality versus risk aversion. In both cases, custodial management (alternative E) was preferred (table 11). In essence, the effect of the higher risk free rate dominated. Even before risk is taken into account, the costs associated with the plantation investments were not justified (i.e., $E[NPV] < 0$) under the 4 percent risk free discount rate. This favored the cost free custodial management option.

Discussion

With respect to mean net present value, these results were consistent with sensitivity analyses around the discount found in deterministic analyses. As was the case in the hypothetical analysis (figures 6 and 7), low discount rates in general favor the delayed payoff plantation investments, while high discount rates in general have a harsher effect on these investments. For example, plantation investments were viable under the 2 percent risk free discount rate, but not viable under the 4 percent risk free discount rate. In contrast, the hay investment remained viable under all three risk free discount rate assumptions (2, 3 and 4 percent). It was presumed in the hypothetical analysis that this sensitivity would be magnified as a result of errors in specifying the risk premium in the risk-adjusted discount rate. But as the results of the empirical analysis indicate, this sensitivity can be attributed to the risk free discount rate. Furthermore, the sensitivity can be attributed to discounted revenues not covering discounted costs. The result was that under risk neutrality, the preferred alternative switched from sawtimber with thinning (alternative C), to forage hay (alternative F), to custodial management (alternative E) under risk free discount rates of 2, 3, and 4 percent respectively.

There were also some risk effects associated with the magnitude of the risk free discount rate. Under the 2 percent rate, the alternative with the lowest risk (custodial management) was ranked relatively low (CASES 1-4, table 9). But under risk free discount rates of 3 and 4 percent, custodial management was the preferred alternative (tables 10 and 11). In essence, under higher risk free discount rates, risk averse individuals become more sensitive to the risk differences between alternatives since the higher rate lowers the mean NPV relative to its effect on variance and skewness. As indicated in table 12, each alternative becomes relatively more risky (as measured by the coefficient of variation and relative skewness[124]) the higher the risk free discount rate.[125]

Ranking of the Alternatives under the RADR Approach to Risk

For the purpose of assessing the RADR bias in the ranking of the alternatives, one of the utility cases needed to be chosen as a benchmark representing the true ranking of the alternatives. Since the RADR analysis tended to under-estimate the true certainty equivalent NPV valuation of each alternative, the utility CASE 4 (high degree of relative risk aversion, negative exponential utility) was chosen since for the most part under this case the strongest risk adjustments to each alternative occurred (i.e, the lowest values for NPV_{CE}). Therefore, if RADR bias occurs relative to this case, it should also occur relative to CASES 1-3 since under these cases the risk adjustments were even less perceptible than that suggested by the RADR valuation.

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Table 12 should be interpreted by fixing the alternative (including the rotation age) and looking at the relative risk differences across different values for the risk free discount rate. While the relative risk measures are comparable across alternatives (for a fixed value of the risk free discount rate), doing so has little interpretative significance since the efficiency of alternatives is defined by absolute risk differences (table 8), not relative risk differences.

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The only exception was alternative D, in which case the absolute value of the mean NPV increases under higher risk free discount rates because the alternative is not viable based on the mean alone (table 8). Similarly for alternative B under a 4 percent risk free discount rate. For the hay investment, increasing relative skewness implies less relative risk since in this case the NPV probability distributions were skewed positively. However, in the absolute sense the hay investment became more risky in terms of skewness as positive skewness decreased under higher risk free discount rates (table 8).

Table 12. The effect of the risk free discount rate on the relative riskiness of an alternative (empirical analysis).

ALT[1]	RISK FREE DISCOUNT RATE (percent/100) 0.02 COEF[2] VAR	0.02 REL[3] SKEW	0.03 COEF VAR	0.03 REL SKEW	0.04 COEF VAR	0.04 REL SKEW
A (r=32)	0.4618	0.4810	1.2336	1.4043	12.6939	15.1593
B (r=36)	0.7789	0.8897	6.7676	32.4216	1.6807	2.1440
(r=40)	0.8897	0.7664	8.3222	34.7043	1.2864	1.4566
C (r=32)	0.4722	0.4293	1.3166	1.3779	6.5892	7.4196
D (r=20)	0.9459	1.0527	0.7179	0.8049	0.6006	0.6812
E (r=16)	0.1607	0.0681	0.1768	0.0825	0.1929	0.0938
F (r=6)	2.8216	1.9811	3.7303	2.2509	6.2812	3.1075

Notes:

[1] Alternative (r denotes the preferred rotation age in years).

A: Sawtimber, 500 trees/acre, no thinning.
B: Sawtimber, 700 trees/acre, no thinning.
C: Sawtimber, 700 trees/acre, thinning at age 20.
D: Pulpwood investment.
E: Custodial management.
F: Forage hay investment.

[2] (standard deviation)/mean (absolute value).

[3] $(\text{skewness})^{1/3}$/mean (absolute value).

Risk Free Discount Rate = 2 percent

The benchmark true ranking (most preferred to least preferred) of the alternatives was as follows (CASE 4, table 9):

1) Alternative C: Sawtimber, 700 trees/acre, thinning at age 20.
2) Alternative A: Sawtimber, 500 trees/acre, no thinning.
3) Alternative E: Custodial management.
4) Alternative B: Sawtimber, 700 trees/acre, no thinning.
5) Alternative F: Forage hay investment.
6) Alternative D: Pulpwood investment.

Table 13 summarizes the ranking of the alternatives (by mean risk-adjusted NPV) under the RADR analysis. Regardless of whether costs were discounted with the risk free or risk-adjusted rate, the preferred alternative (alternative C) was almost never identified as such. The only exception was under a 2.5 percent RADR. However, even in this case the remaining ranking was not preserved since alternative E is incorrectly ranked as being preferable to alternative A. Furthermore, the bias in ranking carries over to the other risk averse cases (CASES 1-3) serving as the benchmark representing the true ranking since alternative C was always the most preferred and alternative A was always preferable to alternative E (table 9).

Instead of selecting alternative C as being preferred alternative, the RADR analysis tended to favor custodial management (costs discounted with the risk free rate), or forage hay (costs discounted with the risk-adjusted rate) (table 13).

Risk Free Discount Rate = 3 percent

The benchmark true (most preferred to least preferred) ranking of the alternatives was as follows (CASE 4, table 10):

1) Alternative E: Custodial management.
2) Alternative A: Sawtimber, 500 trees/acre, no thinning.
3) Alternative C: Sawtimber, 700 trees/acre, thinning at age 20.
4) Alternative B: Sawtimber, 700 trees/acre, no thinning.
5) Alternative F: Forage hay investment.
6) Alternative D: Pulpwood investment.

Table 13. **Ranking of investment alternatives by mean risk-adjusted NPV (empirical analysis, risk free discount rate = 2 %).**

RISK FREE DISCOUNT RATE = 2 PERCENT

MEAN RISK-ADJUSTED NET PRESENT VALUE
(1967 $/acre)

		COSTS DISCOUNTED WITH RISK FREE RATE				COST DISCOUNTED WITH RISK-ADJUSTED RATE			
	RA[1] =	0.025	0.030	0.035	0.040	0.025	0.030	0.035	0.040
TR[2]	AL[3]								
2)	A	3)[4] 30.01 (r[5] = 32)	2) 1.50 (r = 32)	2) -18.70 (r = 32)	2) -33.17 (r = 36)	2) 42.63 (r = 32)	4) 22.65 (r = 32)	3) 8.48 (r = 32)	3) -1.96 (r = 32)
4)	B	4) 11.07 (r = 36)	4) -20.16 (r = 36)	4) -42.17 (r = 36)	4) -58.26 (r = 36)	5) 26.44 (r = 36)	5) 5.55 (r = 36)	5) -9.17 (r = 36)	5) -19.92 (r = 36)
1)	C	1) 33.06 (r = 32)	3) -3.60 (r = 32)	3) -29.12 (r = 40)	3) -47.08 (r = 40)	1) 50.74 (r = 32)	3) 26.03 (r = 32)	4) 8.41 (r = 32)	4) -4.65 (r = 32)
6)	D	5) -54.28 (r = 20)	5) -63.28 (r = 20)	5) -70.93 (r = 20)	5) -77.45 (r = 20)	6) -48.51 (r = 20)	6) -52.40 (r = 20)	6) -55.51 (r = 20)	6) -57.99 (r = 20)
3)	E	2) 31.37 (r = 16)	1) 26.08 (r = 16)	1) 22.04 (r = 16)	1) 18.87 (r = 16)	4) 31.37 (r = 16)	2) 26.08 (r = 16)	2) 22.04 (r = 16)	1) 18.97 (r = 16)
5)	F	6) -70.88 (r = 6)	6) -173.50 (r = 6)	6) -263.72 (r = 6)	6) -412.77 (r = 6)	3) 39.94 (r = 6)	1) 31.37 (r = 6)	1) 23.45 (r = 6)	2) 16.11 (r = 6)

Notes:

(1) RA - risk-adjusted discount rate (percent/100).

(2) TR - true ranking of alternatives (CASE 4: Negative exponential utility, high degree of relative risk aversion, table 9)

(3) Alternatives:

ALTERNATIVE A: Sawtimber, 500 trees/acre, no thinning.
ALTERNATIVE B: Sawtimber, 700 trees/acre, no thinning.
ALTERNATIVE C: Sawtimber, 700 trees/acre, thinning at age 20.
ALTERNATIVE D: Pulpwood investment.
ALTERNATIVE E: Custodial management.
ALTERNATIVE F: Forage hay investment.

(4) Denotes ranking of alternatives (1 is most preferred, 6 is least preferred).

(5) r - preferred rotation age in years.

NPV - net present value.

Table 14 summarizes the ranking of the alternatives (by mean risk-adjusted NPV) under the RADR analysis. Unlike the case when the risk free discount rate was 2 percent, the RADR analysis for the most part correctly ranked custodial management (alternative E) as being the preferred alternative, even when when costs were discounted with the risk-adjusted rate.

More importantly, when costs were discounted with the risk free rate, the 3.5 percent RADR preserved the entire true ranking of alternatives. This was also true for the other RADRs being considered, the exception being that when costs were discounted with the risk free rate, the hay investment was incorrectly ranked lower the the pulpwood investment under RADRs greater than 3.5 percent (table 14). Finally, with the exception of incorrectly ranking the hay alternative, discounting the costs with the RADR preserved the true ranking of the timber investments (table 14). In essence, the RADR bias favoring the hay investment relative to the plantation investments can be attributed to the hay investment being biased toward positive mean risk-adjusted NPVs under this treatment of costs (see appendix K).

With respect to the ranking of the hay investment relative to the sawtimber investments, these results also hold when the other utility cases (CASES 1-3) are taken as the benchmark defining the true ranking of alternatives. That is, restricting the source of bias to the uncertain revenue cash flow (by discounting costs with the risk free rate), the RADR approach correctly targeted the preference of the sawtimber investments over hay. However, deliberately discounting the certain investment costs with the RADR created the distortion of ranking hay above the sawtimber investments.

It should be noted that while the true ranking of the alternatives under the RADR approach was more or less preserved (especially with respect to identifying the preferred alternative) when the risk free discount rate was 3 percent, biases still existed with respect to valuation. Most notably, the RADR analysis tended to incorrectly identify

Table 14. Ranking of investment alternatives by mean risk-adjusted NPV (empirical analysis, risk free discount rate = 3 %).

RISK FREE DISCOUNT RATE = 3 PERCENT

MEAN RISK-ADJUSTED NET PRESENT VALUE
(1967 $/acre)

		COSTS DISCOUNTED WITH RISK FREE RATE				COST DISCOUNTED WITH RISK-ADJUSTED RATE			
	RA[1] =	0.035	0.040	0.045	0.050	0.035	0.040	0.045	0.050
TR[2]	AL[3]								
2)	A	2)[4] 2.44 (r[5] = 32)	2) -12.44 (r = 32)	2) -23.73 (r = 32)	2) -32.49 (r = 32)	3) 8.48 (r = 32)	3) -1.96 (r = 32)	3) -9.87 (r = 28,32)	3) -15.40 (r = 28)
4)	B	4) -16.46 (r = 36)	4) -32.55 (r = 36)	4) -44.66 (r = 36)	4) -53.97 (r = 36)	5) -9.17 (r = 36)	5) -19.92 (r = 36)	5) -27.99 (r = 36)	5) -34.17 (r = 36)
3)	C	3) -0.06 (r = 32)	3) -19.35 (r = 32)	3) -34.04 (r = 32)	3) -45.48 (r = 32)	4) 8.41 (r = 32)	4) -4.65 (r = 32)	4) -14.61 (r = 32)	4) -22.38 (r = 32)
6)	D	6) -60.04 (r = 20)	5) -66.56 (r = 20)	5) -72.15 (r = 20)	5) -76.95 (r = 20)	6) -55.51 (r = 20)	6) -57.99 (r = 20)	6) -59.97 (r = 20)	6) -61.56 (r = 20)
1)	E	1) 22.04 (r = 16)	1) 18.87 (r = 16)	1) 16.34 (r = 16)	1) 14.27 (r = 16)	2) 22.04 (r = 16)	1) 18.87 (r = 16)	1) 16.34 (r = 16)	1) 14.27 (r = 16)
5)	F	5) -58.85 (r = 6)	6) -138.02 (r = 6)	6) -207.90 (r = 6)	6) -269.89 (r = 6)	1) 23.45 (r = 6)	2) 16.11 (r = 6)	2) 9.30 (r = 6)	2) 2.99 (r = 6)

Notes:

(1) RA - risk-adjusted discount rate (percent/100).

(2) TR - true ranking of alternatives (CASE 4: Negative exponential utility, high degree of relative risk aversion, table 10)

(3) Alternatives:

ALTERNATIVE A: Sawtimber, 500 trees/acre, no thinning.
ALTERNATIVE B: Sawtimber, 700 trees/acre, no thinning.
ALTERNATIVE C: Sawtimber, 700 trees/acre, thinning at age 20.
ALTERNATIVE D: Pulpwood investment.
ALTERNATIVE E: Custodial management.
ALTERNATIVE F: Forage hay investment.

(4) Denotes ranking of alternatives (1 is most preferred, 6 is least preferred).

(5) r - preferred rotation age in years.

NPV - net present value.

alternatives A and C as not being viable (i.e., $NPV_{RADR} < 0$, table 14), while these alternatives were essentially identified as being viable in the expected utility analysis (i.e., $NPV_{CE} > 0$, table 10).

Risk Free Discount Rate = 4 percent

Table 15 summarizes the ranking of the alternatives (by mean risk-adjusted NPV) under the RADR analysis. Holding the magnitude of the risk premium in the discount rate constant, the ranking of the alternatives was essentially the same as for the case when the RADR was defined with respect to a 3 percent risk free component.

Under the 4 percent risk free discount rate, the benchmark preferred alternative under all the risk aversion cases considered was custodial management (alternative E).[126] As was the case when the risk free discount rate was 3 percent, the RADR analysis was consistent with identifying custodial management as the preferred alternative, regardless of the treatment of costs (table 15). Even more so, the RADR approach tended to preserve the entire ranking of alternatives. Moreover, the RADR approach also identified custodial management as being the only viable investment for risk averse individuals. Unlike the case when the risk free discount rate was 3 percent, this result held when costs were discounted with the risk free or risk-adjusted rate.

Discussion

While the pattern was not distinct, there seems to be a tendency toward preserving the true ranking of the alternatives the higher the value of the risk free discount rate. When the risk free rate was 2 percent, the ranking of the alternatives under the RADR approach distorted the true ranking of the alternatives. However, when the risk free discount rate was 3 or 4 percent, the RADR analysis for the most part preserved the true

[126] No explicit benchmark ranking is considered since custodial management represented the only viable investment alternative under risk aversion.

Table 15. **Ranking of investment alternatives by mean risk-adjusted NPV (empirical analysis, risk free discount rate = 4 %).**

RISK FREE DISCOUNT RATE = 4 PERCENT

MEAN RISK-ADJUSTED NET PRESENT VALUE
(1967 $/acre)

		COSTS DISCOUNTED WITH RISK FREE RATE				COST DISCOUNTED WITH RISK-ADJUSTED RATE			
RA[1] =		0.045	0.050	0.055	0.060	0.045	0.050	0.055	0.060
TR[2]	AL[3]								
2)	A	2)[4] -13.24 (r[5] = 32)	2) -22.00 (r = 32)	2) -28.91 (r = 32)	2) -34.45 (r = 32)	3) -9.87 (r = 28,32)	3) -15.40 (r = 28)	3) -19.81 (r = 28)	3) -23.37 (r = 28)
4)	B	4) -32.03 (r = 36)	4) -41.34 (r = 36)	4) -48.62 (r = 36)	4) -54.39 (r = 36)	5) -27.99 (r = 36)	5) -34.17 (r = 36)	5) -38.99 (r = 36)	5) -42.77 (r = 32)
3)	C	3) -19.34 (r = 32)	3) -30.78 (r = 32)	3) -39.86 (r = 32)	3) -47.16 (r = 32)	4) -14.61 (r = 32)	4) -22.38 (r = 32)	4) -28.54 (r = 32)	4) -33.50 (r = 32)
6)	D	6) -63.57 (r = 20)	5) -68.38 (r = 20)	5) -72.52 (r = 20)	5) -76.12 (r = 20)	6) -59.97 (r = 20)	6) -61.56 (r = 20)	6) -62.82 (r = 20)	6) -63.82 (r = 20)
1)	E	1) 16.34 (r = 16)	1) 14.27 (r = 16)	1) 12.57 (r = 16)	1) 11.14 (r = 16)	1) 16.34 (r = 16)	1) 14.27 (r = 16)	1) 12.57 (r = 16)	1) 11.14 (r = 16)
5)	F	5) -53.77 (r = 6)	6) -115.76 (r = 6)	6) -171.03 (r = 6)	6) -220.53 (r = 6)	2) 9.30 (r = 6)	2) 2.99 (r = 6)	2) -2.89 (r = 6)	2) -8.37 (r = 6)

Notes:

(1)RA - risk-adjusted discount rate (percent/100).

(2)TR - true ranking of alternatives (CASE 4: Negative exponential utility, high degree of relative risk aversion, table 11)

(3)Alternatives:

ALTERNATIVE A: Sawtimber, 500 trees/acre, no thinning.
ALTERNATIVE B: Sawtimber, 700 trees/acre, no thinning.
ALTERNATIVE C: Sawtimber, 700 trees/acre, thinning at age 20.
ALTERNATIVE D: Pulpwood investment.
ALTERNATIVE E: Custodial management.
ALTERNATIVE F: Forage hay investment.

(4)Denotes ranking of alternatives (1 is most preferred, 6 is least preferred).

(5)r - preferred rotation age in years.

NPV - net present value.

ranking of the alternatives. The primary reason for this was simply because under the higher risk free discount rates, most of the alternatives were non-viable (i.e., $NPV_{CE} < 0$) under the assumptions regarding risk neutrality or risk aversion.[127]

Perhaps the most troublesome alternative for the RADR analysis to handle was the forage hay investment (alternative F). When costs were correctly treated as being relatively certain, and therefore discounted with the risk free rate, the RADR analysis markedly under-valued this alternative. However, deliberately under-compensating for cost risk offset this effect. But in almost all cases this counteracting effect would not lead to a proper valuation of the hay alternative (appendix K). For example, when the risk free discount rate was 3 percent, discounting costs with the risk-adjusted discount rate incorrectly ranked the hay investment as being the most preferred to risk averse individuals (table 14), implying too little an adjustment for risk. The paradox is that the hay investment was by far the riskiest, implying that this alternative should be the most amicable to the RADR approach. Instead, the essentially risk-less custodial management investment (alternative E) was the most amicable to the RADR approach. In general, the fact that this alternative had no associated costs made it relatively insensitive to the magnitude of the discount rate.

It is becoming apparent that RADR bias in itself may be less of a problem in the relative ranking of alternatives. Instead, the magnitude of the risk free discount rate seems to be the major cause of discrepancies in ranking, primarily due to its effect on the relative impacts of discounted costs and revenues in investment valuation.

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127 This was especially true when the risk free discount rate was 4 percent (table 11). However, when the risk free rate was 3 percent, the RADR analysis incorrectly identify some alternatives as being non-viable (table 14) when their true valuation identified a viable investment (table 10).

DISCUSSION

THE SIGNIFICANCE OF RADR BIAS

With respect to the certainty equivalent valuation of each alternative, the RADR approach markedly undervalued each investment alternative (i.e., $NPV_{RADR} < NPV_{CE}$). Regardless of the magnitude of the risk free discount rate, the implied correct risk premium in the discount rate would be much less than one half of one percent. In essence, the analysis results indicate that trying to select, *a priori*, the correct risk premium to include in the discount rate is analogous to measuring the width of a needle with a yardstick. The implication is that in the context of measurement error regarding the risk free discount rate, the risk premiums for the alternatives considered in this study were imperceptibly small. As the empirical results indicate, the mean NPV (i.e., $E[NPV]$) provided a closer approximation of true certainty equivalent net present value (i.e., NPV_{CE}) than any estimate (i.e., NPV_{RADR}) relying on risk-adjusted discount rates considered in this study. The implication is not that risk can be ignored, but that treating risk *via* the RADR approach is inadequate.

However, the low discount rate premiums for the investments considered in this study are based on an objective assessment of risk which only accounted for the historical variations in yields and prices. It should be noted that if individual's subjectively believe that the risks associated with these investments are much larger than what was modelled in this study, larger risk premiums would be implied. But as the hypothetical analysis demonstrated, the RADR approach to risk would still be inadequate since it would be difficult to ascertain, *a priori*, the correct risk-adjusted discount rate to apply to each alternative considered. In essence, the analysis results indicate that the valuation biases due to small risk adjustments to the discount rate are much larger than the correct certainty equivalent net present value adjustments for each alternative. The risk-adjusted discount rate valuation biases can be attributed to the cost sensitivity in the alternatives considered; small adjustments to the discount rate markedly increased the present value influence of costs relative to revenues. This was especially true when the risk-adjusted discount rate was only applied to the revenue cash flow. However, even when the certain costs were incorrectly discounted with the risk-adjusted discount rate, under-valuation still resulted in spite of the effect of offsetting the over-compensation for revenue cash flow risk. For the case of custodial management (alternative E), which incurred no investment costs, the RADR under-valuation can be attributed to the fact that this investment was essentially risk free (see appendix J).

When compared to the hypothetical demonstration of the RADR problem, the empirical analysis results regarding biases in the ranking of alternatives were less dramatic. The reasons for this are twofold. The hypothetical analysis was deliberately set up such that the true ranking of the alternatives was one of indifference. This essentially removed any effect attributed to the risk free discount rate. That is, any explicit ranking of the alternatives other than indifference could be attributed to the risk premium in the discount rate. However, in the empirical analysis there was never any indifference between two alternatives. Secondly, in the hypothetical analysis, the expected cash flow component values were solved for given the certainty equivalent cash flow and the underlying risk measures (i.e., variance). The result was that each revenue component could actually be a loss (i.e., negative outcome).[128] In contrast, only the

[128] The probability of this occurring ranged from 0.05 to 0.33 depending on the timing of the revenue and the cash flow risk trend. For the most part, alternatives with increasing cash flow risk trends incurred a probability of loss for a revenue outflow in the neighborhood of 0.20.

revenue component for the forage hay investment could incur a loss in any given year (due to the volatility in the hay market). Furthermore, the timber investments in the empirical analysis could never receive negative stumpage revenues from harvest. Therefore, it was necessary to consider highly volatile cash flows, and start from a position of indifference, in order to explicitly show RADR bias in the ranking of alternatives.

When compared to the forage hay investment, timber investments were less risky. This contradicts the intuition that the long-term nature of forestry implies taking on a lot of risk. In contrast to Flora (1964), investing in forestry does not imply indifference to risk, but implies taking on a low level of risk. Part of this can be attributed to the fact that a representative acre for the stand was targeted in the empirical analysis. As discussed in appendix E, yield variability in the timber stands tends to decline when defined with respect to a representative acre.[129] Furthermore, by only considering historical price fluctuations, this study did not account for sudden shocks to the system such as unanticipated changes in market relationships, or sudden swings in policy. The multiple rotation approach underlying the simulation model introduced another natural tendency to dampen risk; namely that poor performance in the first rotation could be offset by better performance in the second, performance in the second rotation could be offset by performance in the third, and so on.[130] In essence, the RADR approach could not effectively handle the low level of risk estimated in the empirical analysis.

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129 The simulated yield outcomes for each timber alternative considered in this study fell within a fairly narrow range of possible outcomes (see appendices F through J).

130 Shutting off the alternative under land abandonment when it became readily apparent that under that particular course of events the investment was a bad idea further reduced the variability in the investment's performance.

Rotation Age Bias

In the empirical analysis there were no observed risk-adjusted discount rate biases in determining the preferred rotation age for timber investments. That is, regardless of the specified risk premium (i.e., 0.5, 1.0 , 1.5, or 2.0 percent), the RADR approach (under all risk free discount rate assumptions; 2, 3, or 4 percent) in almost every case identified the same rotation age as the age identified in the expected utility analysis. Furthermore, regardless of the risk free discount rate the rotation age decision was invariant across specific assumptions concerning the specification of the individual's wealth utility function, as well as invariant across the risk neutral versus risk aversion assumption in general (see appendices F through H, and appendix J).

This is not to say that the rotation age in general is invariant to the magnitude of the discount rate. This result holds at the margin, especially in a deterministic context (Chang, 1984). Therefore, the insensitivity of the rotation age to the discount rate in this study can be partly attributed to the 4 year time interval between successive candidate rotation ages. However, the comparative static result that the rotation age will always decrease the higher the discount rate becomes ambiguous under uncertainty.[131] While the result will hold with respect to the risk free discount rate under risk neutrality, increasing the discount rate for risk adds another dimension to the analysis. As indicated in Cathcart (1987), the rotation age decision under risk aversion goes beyond the rotation age's effects on mean net present value. Under risk aversion, the discount rate's effect on the entire probability distribution of net present value outcomes must be considered. As indicated in appendices G and H, the rotation age decision will focus on mean-variance and mean-skewness tradeoffs. Therefore, increasing the risk free discount

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131 For a good illustration of how deterministic comparative static results may not hold under risk aversion see Koskela (1989a) in the context of the harvest decision, and Sandmo (1971) in the context of the theory of the firm.

rate may well lead to the preference for a longer rotation in as much as these tradeoffs are preferred by the individual (appendix G). Adding a risk premium in the discount rate should not distort the true rotation age decision in as much as the premium reflects the true preferences of the individual. This study's results indicate that insofar as the rotation age decision is concerned, there was no RADR bias in its selection.

THE IMPORTANCE OF THE RISK FREE DISCOUNT RATE

The hypothetical analysis presumed that any bias in the ranking of alternatives could be attributed to errors in specifying the risk premium in the discount rate. However in the empirical analysis, discrepancies in ranking were more easily attributed to the risk free discount rate assumption than to the risk premium in the discount rate. For the most part, valuation biases (i.e., $NPV_{RADR} < NPV_{CE}$) had little impact on the relative ranking of alternatives. Only when the risk free discount rate was low (i.e., 2 percent) did valuation bias translate into ranking bias.

As indicated in table 12, the relative riskiness of each alternative decreased under lower risk free discount rates; primarily due to its effect on the mean of the NPV probability distribution. In essence, low risk free rates imply lower risk adjustments relative to the utility ascribed to the mean NPV alone, making the risk adjustments even more subtle for the RADR approach to effectively handle. This may seem counter intuitive in as much as a higher risk free discount rate implies less attachment to the importance of future events. But as the results of the empirical analysis indicate, it is the risk free discount rate's effect on the mean NPV that signals the effect of time preference, not its effect on the moments defining risk (e.g., variance and skewness).

That the preferred alternative was more or less invariant to the assumption of risk neutrality versus risk aversion can also be ascribed to risk free discount rate effects. Under the 2 percent risk free discount rate, ranking the alternatives by mean NPV (i.e,

$E[NPV]$) essentially accounted for the relatively small certainty equivalent adjustments for risk (as measured by the difference $E[NPV] - NPV_{CE}$) (table 9). In contrast, under the 3 percent risk free rate, the certainty equivalent adjustments to $E[NPV]$ were large enough that ranking the alternatives by NPV_{CE} made a difference. That is, when the results in table 10 are compared to tables 9 and 11, the differences between the estimated mean net present values were small enough such that ranking by NPV_{CE} made a difference. As indicated in table 10, risk averters preferred the essentially risk-less custodial management alternatives, while risk neutral individuals would ignore the risk associated with hay and chose this alternative because it had the highest mean NPV.

When the risk free discount rate was 4 percent, the sheer magnitude of this rate essentially dominated. In all cases, the plantation investments were just not viable (i.e., $E[NPV] < 0$), risk considerations notwithstanding (table 11). Furthermore, the risk averter's increased sensitivity toward risk under the 4 percent rate resulted in the hay investment being negatively valued (i.e., $NPV_{CE} < 0$) in spite of its positive mean net present value. While a risk neutral individual would accept hay production as being a viable investment, the mean net present value under the 4 percent risk free discount rate was not large enough to make this alternative preferable to custodial management (table 11). In essence, under the 4 percent risk free rate, the custodial management investment dominated, risk notwithstanding.

Combined, these results show that the risk free discount rate is perhaps the most important parameter in any approach to risk. Not only does the risk free rate affect the the probability distribution of NPV outcomes (see tables 8 and 12), but also how risk averters will adjust for risk. As indicated by the 3 percent risk free discount rate case, these effects can be pronounced enough such that adjusting for risk can make a difference in selecting the preferred alternative.

Efficiency of Alternatives

While the identification of the true preferred alternative was sensitive to the risk free discount rate assumption, the efficiency[132] of alternatives was not. For the alternatives considered in this study, the following were efficient (table 12)[133]

ALTERNATIVE A: Sawtimber, 500 trees/acre, no thinning
ALTERNATIVE C: Sawtimber, 700 trees/acre, thinning at age 20
ALTERNATIVE E: Custodial management
ALTERNATIVE F: Forage hay production

That is, alternatives B (pulpwood) and D (sawtimber, 700 trees/acre, no thinning) were dominated by at least one of the above alternatives. Furthermore, the hay alternative was never dominated because it was the only alternative that exhibited positive skewness in the probability distribution for net present value outcomes.

However, in the context of mean-variance tradeoffs there was some sensitivity in the efficiency of the hay alternative. Under the 2 percent risk free discount rate, hay was dominated by the timber investments (alternatives A, C, and E). In contrast, since the hay alternative had the highest mean NPV under the 3 percent risk free discount rate, it was still efficient in a mean-variance context. But under the 4 percent risk free discount rate, the hay alternative would be dominated by custodial management in a mean-variance context, since the mean NPV for hay no longer exceeded the mean NPV for custodial management (table 12).

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In this study, efficiency was defined with respect to mean-variance and mean-skewness tradeoffs. An alternative was considered to dominate another when acceptance of the dominating alternative implied accepting a higher mean net present value under lower risk (i.e., both variance and negative skewness would decrease).

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However, this set is defined with the rotation age decision notwithstanding. As previously indicated, the rotation age decision was invariant to the risk aversion assumptions considered in this study. However, this does not imply there was only one efficient rotation age (e.g., see appendix G).

MISCONCEPTIONS REGARDING THE USE OF RISK-ADJUSTED DISCOUNT RATES

One misconception regarding the use of risk-adjusted discount rates is that there is essentially no difference between the choice of the risk premium and the decision environment it pertains to. However, as indicated in the literature review and in the procedures underlying this study, any analysis which considers risk must be very explicit about the decision environment.[134] Therefore, if the individual could be characterized as not being well diversified, applying an opportunity cost of capital based on the systematic risk for timberland investments would imply a contradiction; the individual is not diversified, but the discount rate is only adjusted for systematic risk. While this study indicates a very small positive premium for total (revenue) cash flow risk, other studies have shown that, based on systematic risks, the risk premium may well be negative (Redmond and Cubbage, 1988; Zinkhan, 1988). That is, while timber investments are less risky in the absolute sense, they may even be better than risk-less in the systematic sense. Not only can the risk associated with timber investments be completely diversified away, but some extraneous sources of portfolio risk as well.

A second misconception regarding the use of risk-adjusted discount rates is that the same rate should be applied equally to uncertain costs and revenues. However, as the analytical treatment of the RADR problem indicated, for risk averse individuals the premium for total risk will be positive for uncertain revenues, but negative for uncertain costs. In essence, if costs and revenues are both considered to be uncertain, the RADR

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134 This study targeted an individual who did not hold a well diversified portfolio of assets simply because this assumption allowed the stand level investment problem to be defined in a partial equilibrium framework where both diversifiable and non-diversifiable (i.e., total) risk became relevant. Had the individual held a well diversifiable portfolio, the entire analysis would have to be reformulated such that the investment decision considered the portfolio effects of acceptance. Increasing the size of the individual's land holding would imply a similar modification since diversification in alternative selection becomes relevant.

used to discount the expected revenue cash flow should exceed the risk free discount rate, while the RADR to discount the expected cost cash flow should be less than the risk free rate. Unfortunately, this is tantamount to measuring the respective widths of two needles with a yardstick. Differences between cost and price uncertainties also have a strong implication regarding the estimation of systematic risk for timberland investments. It is the distributional properties (i.e., covariance with the market) of net returns (i.e., net of capital outlay) that will define the systematic risk. Therefore, the correlation of both prices and costs with the market should be considered when estimating systematic risk.

Another often overlooked aspect of using risk-adjusted discount rates is that the cash flow to be discounted is the expected (i.e., mean) cash flow. It is not the most likely cash flow (unless cash flow component probability distributions are all symmetric), nor is it a single (i.e., point) estimate of the cash flow. For example, applying the RADR approach to optimistic cash flow estimates would be misleading, if not simply wrong, since the optimistic scenario represents one possible outcome out of a set of many possible outcomes. Simply recognizing that the optimistic scenario may not actually be realized implies that the analysis should consider the probability distribution of outcomes, not a single outcome. For the RADR approach to risk, this implies estimating the mean cash flow. If a point estimate is used to represent the cash flow, the correct approach would be to condition the analysis results on the scenario being true and discount the cash flow with the risk free rate. That is, relying on a point estimate of the cash flow implies that the analysis is essentially deterministic. Therefore, adjusting the discount rate for point estimate uncertainty would be even more dubious than adjusting the discount rate for well defined risk.

On Identifying the Preferred Alternative

The use of risk-adjusted discount rates implies that the objective of the analysis is to identify a single preferred (i.e., optimal) alternative. In this manner, the RADR method is no different from an expected utility-certainty equivalent approach to risk. However, risk can be reduced through diversification. Therefore, from an applied research perspective it may be prudent to identify efficient sets of alternatives, rather than focus on a single optimum. The advantage with targeting efficiency is that it addresses a general class of decision units (e.g. risk averters) rather than some hypothetical entity. With respect to stand level analyses conducted in a partial equilibrium framework there are two aspects of diversification worth noting.

Since diversification reduces risk, the preferred stand management strategy will most likely be to diversify across silvicultural alternatives. Accordingly, forests can be viewed as representing a portfolio of stands. Therefore, diversification also implies diversifying across land use.[135] For example, farmers considering timber management on woodlots will most likely base their decision on how timber management offsets (or reinforces) the risk associated with the agricultural enterprise. Therefore, analyses which weed out inefficient alternatives identify the set of alternatives to diversify from. Furthermore, the analysis could be extended to identify efficient combinations of alternatives using portfolio analysis techniques. However, this would necessitate knowing not only the mean and variance (and perhaps skewness) for each alternative, but the covariances between alternatives as well.

Even if the land base is too small to make diversification feasible, the preferred alternative for any risk averter, or even those with a risk neutral attitude, will be contained in the efficient set. The challenge is to identify the efficiency criterion (e.g.,

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[135] Note that this has a general appeal in the context of multiple use management.

stochastic dominance or mean-variance comparisons) that effectively reduces the size to the efficient set (e.g., see Caulfield, 1988). The appeal to the approach is that the analysis does not need to explicitly measure the risk preferences of the individual. For example, in order to explicitly define the individual's risk preferences, this study had to specify the functional form for wealth utility and the appropriate parameters as well. Therefore, the alternative rankings in the results only pertain to those individuals whose risk preferences are consistent with these utility assumptions. Furthermore, tables 9 through 11 indicate that the ranking of alternatives can be sensitive to the specification of the wealth utility function. However, efficiency criterion only assume general assumptions regarding risk preferences that target a given class of individuals (e.g., risk aversion), not a specify entity. In this manner, individuals can make their own decisions according to their subjective risk preferences.

THE APPROPRIATENESS OF SIMULATION

As the empirical analysis indicated, simulation is general enough to model the uncertainty underlying cash flow outcomes, as well as modelling the uncertainty in the timing of outcomes. For example, simply accounting for plantation failure defines uncertainty regarding the timing of stand establishment. Without simulation, an expected time of establishment would have to be assessed, as well as an expected cost.[136] With simulation this is not necessary if all cash flow realizations are discounted to present value and risk assessed in present value terms; the approach taken in this study.

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[136] The expected cost estimate would also have to account for the timing differences between actual cost outcomes. For this reason, a simple weighted (with respect to probability of occurrence) average across time would be inadequate.

However, simulation is very costly and the information requirements enormous, even for relatively simple models. Therefore, if simulation is employed, the most should be made of the advantages of the approach, namely estimation of the entire probability distribution of investment returns. This in itself calls for the use of more robust approaches to risk than the RADR method. However, it should be noted that simulation does not imply that objectively modelling risk (as was done in this study) is preferable to subjective models. In fact, most decision analysts agree that the decision maker's subjective models of the system are the correct models to use no matter how inconsistent they may be with objective representations of the system (Markowitz, 1959; Raiffa, 1968). In essence, the true role of objective models is to provide a basis upon which to update subjective representations of the system. However, objective models are useful in that they allow the analysis to be conducted in general terms regarding the decision maker being targeted.

The tradeoff underlying simulation is that while uncertainty can be modelled, the model itself must be built upon a larger set of assumptions. For example, the simulation model used in this study is only as good as the Markovian representation of stand yield, the VAR representation of commodity prices, the autoregressive process for hay yield, and in that costs could be considered as being relatively certain. In the same vein, the model is only as good as the data used in estimation. In many cases the modelling error in simulation may be counterproductive to the objective of reducing the uncertainty underlying the analysis.

Scenario Analysis

Recognizing that decision consequences are uncertain simply implies there is more than one possible consequence for each action taken. Therefore, simply identifying a set of plausible scenarios (e.g., worst case, best case, most likely, and perhaps the average case) leads to a significant improvement in defining the problem (see Adams and Haynes, 1985 for an example). Weighting each scenario by the likelihood of it being true would essentially define a probability distribution of outcomes.[137] Sensitivity analysis could then target the relative weights (i.e., probability of occurrence) placed on each scenario. Therefore detailed simulation models, while useful, are not necessary to capture the uncertainties underlying the decision. The tradeoff between simulation and scenario analysis is that scenario analysis can account for a wider array of uncertainties in a more subjective manner, while simulation must reduce the scope of the problem in order to address the remaining elements at a more detailed level.

CONCLUDING REMARKS

This study evaluated the use of risk-adjusted discount rates in the context of an individual who does not hold a well diversified portfolio of assets. In as much as this provides an adequate description of non-industrial private forest (NIPF) landowners, economic analyses which target NIPF landowners should not rely on the use of risk-adjusted discount rates. This conclusion also applies to any analyses targeting the responses of NIPF owners to policy (e.g., timber supply studies). In essence, if the analyst recognizes that risk is important enough to be adjusted for, the analyst must also recognize that the RADR approach to risk is inadequate. Furthermore, the results of the study indicate that the choice of the risk free discount rate will influence the analysis no matter what the approach toward risk. For both policy and economic analyses this

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137 A more formal approach would be to represent the probability distribution of events using a triangular distribution. Similarly for the beta distribution. In this context, the problem could be analyzed using a combined scenario-simulation approach. See Law and Kelton (1982) for a general discussion regarding the use of triangular and beta distributions; see Hassler and Sinclair (1982), and Chambers *et al.* (1986) for applications.

calls for sensitivity analyses regarding the specific assumptions underlying the choice of the risk free rate.

Policy analysts should heed the principles underlying efficiency. That is, the analysis should explicitly recognize that risk averters facing the same uncertainties will not all respond in the same fashion. This alone may account for many failed policies. Unfortunately, this makes policy analyses that much more difficult since the analysis must explicitly account for how individuals react to risk. As the finance literature indicates, this may call for a general equilibrium approach to the problem.

While adjusting discount rates for systematic risk seems to be well grounded in the finance literature, this does not imply, *a priori*, that firms should use a single risk-adjusted discount rate in all capital budgeting decisions. Just like the case in the individual model, there are no exact guidelines in selecting the discount rate to apply to project evaluation. However, unlike the NIPF landowner, corporations have a well defined cost of capital to provide a benchmark for adjustments.

In the context of implementing forestry decisions (i.e., harvest scheduling), owners of large tracts of forest land (including public agencies) should recognize the importance of diversification. That is, in the face of uncertain markets, technological change, and physical threats to growing stock, diversification in management makes good sense. This goes beyond diversifying across stand types; it may also imply diversifying across silvicultural alternatives (and age classes) within a given stand type.

When one becomes familiar with the arguments made against the use of risk-adjusted discount rates, namely that a single RADR applied to all alternatives will not capture the risk differences between alternatives, the notion that the USDA Forest Service (or any federal agency for that matter) should rely on a single risk-adjusted discount rate (such as OMB's 10 percent mandate) in all economic and policy analyses is imprudent at best. In this vein, at least the Forest Service has a head start in as much as their 4 percent discount rate represents a low risk measure of social opportunity costs. But since it is not clear when the government can ignore risk, the prudent approach would be to tackle uncertainties head on, and then choose to ignore risk if appropriate.

Interpreting the Numerical Results

Perhaps the most surprising result in the empirical analysis was that the implied discount rate premium for total (as opposed to systematic) revenue risk was around one tenth of one percent for all the alternatives considered. The result is counter intuitive in that the risk premiums for financial assets, as well as for other real assets, are usually considered to be much larger. Therefore, it seems natural to question the numerical results of this study.

The low level of risk estimated in this study was grounded by the fact that risks were modelled using objective data. This was probably an adequate representation of yield uncertainty in as much as the biological processes controlling growth and yield outcomes will not markedly change in the future. However, the price model was essentially a stochastic extrapolation of historical data. Furthermore, catastrophic losses from wind, fire, and pests were ignored. One could easily argue that changes in the historical relationships between prices (e.g., due to policy shifts and exogenous shocks to markets) adds considerable uncertainty to the investment environment. Therefore, the weakness in the empirical model is that the objective representation of the system may not actually be in line with individual subjective perceptions regarding the riskiness of forestry and hay investments. If these perceptions consider the alternatives to be more risky than modelled in this analysis, larger risk premiums would be implied.

The danger with case studies is the tendency to make deductive generalizations from the results. This study did so only with respect to the appropriateness of the RADR approach to risk. That is, changing the assumptions and modelling techniques will inevitably change the numerical results, but it would still be unlikely that there would be a set of empirical results that unequivocally supports the RADR approach to risk. However, some generalizations can still be made from the numerical results of this study. That the timber investments were low in risk does not contradict previous works and confirms the notion that forestry is not really a risky venture. Most notably, estimates of systematic risks suggest that timberland investments may be better than risk-less due to their diversification appeal when held in a portfolio of assets (e.g., see Redmond and Cubbage, 1988). There was also a distinct difference between the nature of the risk in forestry and hay investments. The timber investments had negatively skewed NPV probability distributions of outcomes, while the hay alternative had a positively skewed distribution. The hay investment was also considerably more risky than the timber alternatives. In essence, the simulated outcomes for timber investments were sensitive to establishment losses, while the simulated outcomes for the hay alternative was sensitive to the occurrence of a sustainable annual profit margin. That these differences could not be discerned with any perceptible discount rate risk premium highlights the inadequacy of the RADR approach.

Therefore, one should not conclude that the appropriate discount rate risk premium to apply to forestry investments should be in the neighborhood of one tenth of one percent. Instead, the overall conclusion is that the RADR approach is inadequate since alternatives will most likely differ in risk. This in itself calls for more robust techniques that can account for these differences in the evaluation of risky investments.

SUMMARY

The purpose of this research was to evaluate the use of risk-adjusted discount rates in the context of stand level decisions set in a partial equilibrium framework, the premise being that the risk-adjusted discount rate approach to risk would be inadequate. This necessitated targeting an individual whose ability to diversify was limited. Furthermore, the individual's land holding was assumed to be small enough (i.e., 30 acres) such that diversification across land use was not feasible. In essence, the chosen opportunity would represent the individual's portfolio of investment. In this context, both diversifiable and non-diversifiable risks (i.e., total risk) became relevant to the decision. The alternatives considered contrasted differences in cash flow (e.g., duration and payoff pattern), objectives, capital requirement, and presumably risk; providing distinct areas in which risk-adjusted discount rate bias may occur. Finally, the analysis was static in that the investment decision was being made today and investment performance (measured by net present value) was defined with respect to a representative acre for the stand. The underlying intent was to parallel the assumptions underlying deterministic stand level analyses conducted on a per acre basis.

Risk-adjusted discount rate bias referred to incorrectly identifying the preferred alternative for the individual. The true (i.e., correct) preferred alternative was identified using a certainty equivalent approach to risk based on the individual's expected utility from holding claims to the uncertain asset. The higher the individual's expected utility, the more preferred the investment. Analytically, the risk-adjusted discount rate problem was defined as follows.

1) The appropriate risk adjustment in the discount rate will be unique to each component in the investment's cash flow.

2) While in most cases 1 above can be accounted for with a single risk-adjusted discount rate applicable to the entire cash flow, invariably it will be unique to each alternative considered. Furthermore, the rate applicable to uncertain revenue inflows will most likely differ from the rate applicable to uncertain cost outflows.

3) In order to determine the appropriate risk-adjusted discount rate (or rates) in 2 above, the problem must already have been solved *via* the certainty equivalent approach to risk.

In other words, given 3 above there are really no criteria guiding the *a priori* selection of the appropriate risk-adjusted discount rate for each alternative considered.

Using a hypothetical set of investment alternatives which paralleled the differences between long-term timber investments and short term crop investments, the above three problem areas were demonstrated with respect to uncertainties in the revenue cash flow. It was found that the long-term timber investments were more sensitive to errors in the specification of the risk premium in the discount rate; namely specifying too high a risk-adjusted discount rate (i.e, over-compensating for risk) biased the analysis toward the selection of shorter-term crop alternatives. It was also demonstrated that increasing cash flow risk with respect to time (i.e., increasing cash flow variability with respect to time) did not imply a constant, or even an increasing, risk-adjusted discount rate with respect to time. In essence, a constant risk-adjusted discount rate with respect to time is only implied when the certainty equivalent adjustment to the cash flow increases at a constant rate with respect to time (Robichek and Myers, 1966; Chen, 1967); not when observable measures of risk (i.e., variance) increases with respect to time.

In order to dramatize the demonstration of risk-adjusted discount rate bias, the hypothetical analysis had to assume volatile cash flows and started from a point of true indifference between all alternatives. In this manner, biases could be attributed directly to the magnitude of the risk premium in the discount rate. Since these conditions may not likely hold in an actual investment setting, the problem was readdressed empirically.

The intent was to assess to what degree, if any, the risk-adjusted discount rate problem exists in an actual stand level investment setting.

The empirical analysis targeted the individual model where yields and commodity prices were considered to be uncertain. In order to simplify the scope of the uncertainties, investment costs were considered to be certain. Therefore, applying a risk-adjusted discount rate with a positive premium for risk would represent a deliberate under-compensation for cost risk. However, costs were also correctly discounted with the risk free rate in the risk-adjusted discount rate analysis in order to assess the effects of this anomaly. This also isolated the empirical effect of any risk-adjusted discount rate bias to the true source of uncertainty: the revenue cash flow.

The expected utility model (defined with respect to the probability distribution of net present value outcomes) was used to identify the correct ranking of alternatives. Unlike the hypothetical analysis, which only targeted logarithmic utility of wealth, the empirical analysis also considered the negative exponential class of utility functions. For logarithmic utility, absolute risk aversion decreases with respect to wealth, but relative risk aversion is constant. For negative exponential utility, absolute risk aversion is constant, but relative risk aversion increases with respect to wealth. Most analysts agree that these properties provide intuitive representations of risk aversion. The analysis considered different specifications for each class of utility functions to determine the effect of differences in the degree of risk aversion. Wealth effects were defined with respect to a representative acre for the stand.

In the empirical analysis, simulation was used to estimate the net present value probability distribution for each alternative. The following alternatives were considered.

ALTERNATIVE A: Sawtimber, 500 trees/acre, no thinning
ALTERNATIVE B: Sawtimber, 700 trees/acre, no thinning
ALTERNATIVE C: Sawtimber, 500 trees/acre, thinning at age 20
ALTERNATIVE D: Pulpwood production

ALTERNATIVE E: Custodial management
ALTERNATIVE F: Forage hay production

All plantation investments were subject to planting failure, while the forage hay investment was subject to the risk of having no available market outlet in any given year. The natural regeneration underlying custodial management was subject to uncertainty regarding the type of seed year (i.e., good or bad) for pine. Catastrophic losses from wind, fire, and pests were not considered. Timber investments targeted loblolly pine, while the hay crop was an alfalfa-orchardgrass mixture. The decision problem was assumed to be set in the central mountain region of Virginia.

Stochastic timber yields were modelled using a Markov chain, where stand states for each relevant age were defined with respect to yield. Stochastic prices were represented by a vector autoregressive process. Pulpwood and sawtimber stumpage prices were assumed to be correlated with the general state of the economy as measured by an economic index (e.g., coincident index of cyclical indicators). However, it was found that correlating hay price with hay yield provided the best model specification for hay prices. In essence, the hay yield outcome for the individual was assumed to reflect the type of year the region as a whole was having. Stochastic hay yield was modelled as an univariate autoregressive process. The simulation of each alternative assumed a multiple rotation cash flow, where the outcome for the first rotation could differ from the second, the second from the third, and so on. The analysis was conducted in real before-tax terms, with 1967 taken as the base year. The analysis considered risk free discount rates of 2, 3, and 4 percent.

When compared to the hypothetical treatment of the problem, risk-adjusted discount rate bias in ranking alternatives was less dramatic, especially when the risk free discount rate was 4 percent. The risk-adjusted discount rate approach tended to preserve the true ranking of alternatives. However, any risk adjustment (i.e., as low as one half of a percent) to the risk free discount rate markedly underestimated the true certainty equivalent net present value for each alternative. Furthermore, this result held for all three risk free discount rates considered (i.e., 2, 3, and 4 percent). The counter intuitive result arising from the empirical analysis was that the risk averse individual's sensitivity toward risk increased, the higher the risk free discount rate. However, with the exception of the 3 percent risk free discount rate case, ignoring risk altogether (i.e., assuming risk neutrality) led to identifying the same preferred alternative as for the case under risk aversion. In essence, differences in the mean net present value for each alternative (especially between the best and second best) were large enough such that the certainty equivalent adjustment did not markedly change the ranking of alternatives.

The results of the empirical analysis indicated that the preferred rotation age decision (i.e., the rotation age which maximized expected utility) was invariant across assumptions regarding the individual's degree of risk aversion and the functional form of the wealth utility assumption. Furthermore, the same rotation age would be identified under risk neutrality. Finally, there was no observed risk-adjusted discount rate bias with respect to determining the preferred rotation age.

There was a distinct difference between the shape of the net present value probability distributions for timber investments and the shape of the distribution for the hay alternative. Timber investments had negatively skewed distributions (regardless of the risk free discount rate), while the hay investment had a positively skewed distribution. The viability of the hay investment was sensitive to a sustained annual profit margin over the course of a rotation, while the viability of the plantation investments were sensitive to success or failure of stand establishment. In contrast, custodial management, which incurred no investment costs, had a net present value probability distribution which was more or less symmetric. In terms of identifying the efficient set of alternatives, alternatives B and D were inefficient; that is, would never be preferred by any risk averse (or risk neutral) individual.

For the most part, the risk-adjusted discount rate approach to uncertainty is analogous to measuring a needle with a yardstick. In essence, the approach is not refined enough to capture the subtle nature underlying risk averters' adjustments for cash flow risk. In as much as non-industrial private forest landowners can be characterized as not holding a well diversified portfolio of assets, this research suggests that economic and policy analyses targeting this group should not rely on risk-adjusted discount rates as an approach to uncertainty.

RECOMMENDATIONS

The literature review indicated that there is no one approach to addressing uncertainty and risk. Techniques vary as much as the decision environments they address. Therefore, specific recommendations regarding the choice of analysis technique are not forthcoming. Instead, the following recommendations address those considerations that should be made when it is recognized that uncertainty should be accounted for in the analysis.

I) The analysis must be very explicit about the decision environment it addresses (i.e., individual, corporation, or public agency). Furthermore, opportunities for diversification should be identified and taken advantage of. Finally, recognizing whether the decision unit is risk averse, risk neutral, or even risk preferring will dictate the choice of analysis technique.

II) The risk free discount rate is perhaps the most important parameter in the analysis. Therefore, sensitivity analyses should be conducted on the assumptions underlying the choice of the risk free discount rate.

III) For the most part, the risk-adjusted discount rate approach is inadequate, especially for analyses addressing individual landowners. However, if the risk-adjusted discount rate approach is to be employed, the following should be noted.

a) The use of risk-adjusted discount rates implies risk aversion on the part of the decision maker.

b) The discounted cash flow should be the expected (i.e., mean) cash flow. Therefore, estimation of the cash flow should focus on mean values.

c) Opportunity cost measures for firms will not in general reflect the appropriate risk adjustments in the discount rate for non-diversified individuals.

d) There is no *a priori* reason to rely on a single risk-adjusted discount rate in the analysis. Most notably, the risk adjustment applicable to uncertain revenues will invariably be different from the risk adjustments for uncertain costs.

e) Corporate capital budgeting decisions should concentrate more on the systematic risk effects of project acceptance using the overall cost of capital as a benchmark. This implies that the cost of capital measure applicable to timberland and processing divisions will most likely differ (e.g., see Fortson, 1986; Redmond and Cubbage, 1988; Zinkhan, 1988).

f) Sensitivity analyses should make a distinction between the risk free and risk premium components in the discount rate. In this manner, the analysis will not focus on the adoption of a single overall rate. Instead, the decision maker can choose the appropriate risk free rate, and rely on subjective perceptions regarding the relative riskiness of each alternative when choosing the appropriate risk premium (e.g., see Cathcart and Klemperer, 1988b).

g) The use of risk-adjusted discount rates implies that the objective of the analysis is to identify a single preferred alternative. In many cases, this objective alone may restrict the usefulness of results.

IV) The analysis should estimate the probability distribution of net present value outcomes for each alternative considered. However, there is a tradeoff between the level of detail and the scope of the problem when simulation is used to model uncertainty. In many instances, scenario analysis (i.e., identifying worst case, best case, and most likely cases) may capture the essence of the uncertainties. Emphasis should be placed on ascertaining the relative likelihood of each scenario being realized. The subjective nature of this approach should not be considered as a disadvantage.

V) For analyses which target risk averse decision units, the efficient set of alternatives should be identified before proceeding with the selection of the preferred alternative (if desirable).

VI) As indicated in the literature review, government agencies such as the USDA Forest Service should address uncertainty in the context of how public investment decisions impose risk on the private sector. This may imply relying on a social opportunity cost measure that is adjusted for risk (e.g., see Bailey and Jensen, 1972). However, this risk adjustment (if any) will invariably be specific to the decision problem being addressed. Relying on a single (risk-adjusted) discount rate for all analyses, especially those addressing policy, would be imprudent.

Directions for Further Research

As these recommendations indicate, much work remains to be done in implementing superior approaches to risk-adjusted discount rates. The following areas should be given the highest priority.

I) Criteria guiding the selection of the risk free discount rate need to be developed. Particular emphasis should be placed on the term structure of the risk free interest rate and how this defines the year to year opportunity cost in a multiperiod setting. Furthermore, the consequences of constraining the supply of available capital (i.e., credit rationing) need to be explored. The implication may well be that the discount rate in general should be treated as being uncertain, just like yields, prices, and costs.

II) Research should continue along the lines of portfolio applications to forest investment decision making. Most notably, the pricing of forest assets, and the identification of the management regimes (or portfolio of regimes) that define their value, should consider the portfolio effects of accepting the investment. That is, given a specific silvicultural alternative, what are the portfolio effects of adopting this alternative? The irony here is that the economic desirability of specific stand level (i.e., silvicultural) decisions can only be addressed at the forest level. That is, addressing uncertainty in a partial equilibrium framework may be insufficient.

III) More research is needed in ascertaining how decisions vary with assumptions concerning the risk attitudes of decision makers (e.g., risk neutrality versus risk aversion). Focus should be placed on identifying decisions unaffected by these assumptions. Furthermore, policy analysts need to know the predicted responses of groups that the policy pertains to (e.g., see Bosch *et al*, 1986).

IV) Research should continue along the lines of adaptive (i.e., sequential) decision making, whereby decision policies recognize that future decision actions will be conditional on the outcome of future events. Decision tree analysis and dynamic programming provide a starting point for this research.

V) More research is needed in developing stochastic growth and yield models for an average (i.e., representative) acre in the stand. As the application in this study indicated (appendix E), localized growth and yield models will tend to over-estimate the true yield variability on an average stand acre.

VI) More attention needs to be placed on defining the role in the decision process for objective representations of the system. That is, since decisions may actually be based on subjective representations of the system, does objective modelling misrepresent the decision process? For example, do the historical relationships underlying commodity prices completely represent the decision maker's expectations regarding future prices?

VII) Research is needed in the area of eliciting measures of forest landowner risk preferences. Most notably, do analytical representations, such as logarithmic and negative exponential wealth utility functions, properly account for NIPF landowner sensitivity toward risk?

VIII) A logical area of future research would be to develop other case studies applicable to different timber types and regions such that the riskiness of forestry investments can be assessed. Most notably, can the case be developed that forestry investments indeed imply taking on low levels of risk?

IX) Finally, research should explore the application of alternatives (e.g., expected utility, stochastic dominance, portfolio analysis) to the risk-adjusted discount rate approach to risk. Focus should be placed on identifying those techniques tailored for application at the individual, corporate, or public decision levels.

In this manner, a better understanding and less ambiguous treatment of uncertainty in forest investment analysis will help ensure a better awareness of the investment opportunities available to all ownerships.

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APPENDIX A

CERTAINTY EQUIVALENT ADJUSTMENT FACTORS

THE NET PRESENT VALUE CASE

For the purpose of comparing risk adjustments for a given alternative under different individual utility functions for wealth, a scale free measure of risk adjustment is necessary. The chosen measure was the certainty equivalent adjustment factor $\alpha_{\mathrm{NPV}}^{(l)}$, $l \in \{\mathrm{CE}, \ \mathrm{RADR}\}$ which is defined as[138]

$$\alpha_{\mathrm{NPV}}^{(\mathrm{CE})} = \left(\frac{\mathrm{NPV_{CE}}}{\mathrm{E}[NPV]} \right), \quad \mathrm{E}[NPV] \neq 0 \qquad [\mathrm{A.1a}]$$

$$\alpha_{\mathrm{NPV}}^{(\mathrm{RADR})} = \left(\frac{\mathrm{NPV_{RADR}}}{\mathrm{E}[NPV]} \right), \quad \mathrm{E}[NPV] \neq 0 \qquad [\mathrm{A.1b}]$$

or,

$$\mathrm{NPV_{CE}} = \alpha_{\mathrm{NPV}}^{(\mathrm{CE})}(\mathrm{E}[NPV]) \qquad [\mathrm{A.2a}]$$

$$\mathrm{NPV_{RADR}} = \alpha_{\mathrm{NPV}}^{(\mathrm{RADR})}(\mathrm{E}[NPV]) \qquad [\mathrm{A.2b}]$$

where net present value is in 1967 \$/acre. For interpretative purposes, three cases must be considered.

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138 For the case $\mathrm{E}[NPV] = 0$ the certainty equivalent adjustment factor is not defined. As indicated in (A.1), this factor is defined with respect to the true certainty equivalent adjustment from the expected utility analysis (A.1a), or with respect to the mean risk-adjusted NPV from a given *a priori* specification of the risk-adjusted discount rate (A.1b).

CASE A: $E[NPV] > 0$, NPV_{CE} (or NPV_{RADR}) ≥ 0

This represents the typical case for a viable investment alternative which represents a wealth gain for the individual. In this case

$$0 \leq \alpha_{NPV}^{(l)} < 1, \quad l \in \{CE, RADR\} \tag{A.3}$$

since NPV_{CE} (or NPV_{RADR}) $< E[NPV]$ under risk aversion. Note that in this case, the smaller $\alpha_{NPV}^{(l)}$ is, the larger the adjustment for risk.

CASE B: $E[NPV] < 0$, NPV_{CE} (or NPV_{RADR}) < 0

This represents the undesirable case in which the investment alternative is not viable. In this case,

$$\alpha_{NPV}^{(l)} > 1, \quad l \in \{CE, RADR\} \tag{A.4}$$

since NPV_{CE} (or NPV_{RADR}) $< E[NPV] < 0$ under risk aversion.[139] Unlike **CASE A** above, the larger $\alpha_{NPV}^{(l)}$, the larger the adjustment for risk.

CASE C: $E[NPV] > 0$, NPV_{CE} (or NPV_{RADR}) < 0

This represents the case that the alternative is unacceptable in terms of its risk, in spite of its positive mean NPV. That is, the investment has enough risk such that when the mean value is adjusted, it represents a non-viable alternative. Therefore, while a risk neutral individual would accept this alternative as representing a viable investment, the risk averse individual being considered in this case would not.[140]

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139 In this case, $NPV_{CE} < E[NPV] < 0$ implies that the individual would be willing to pay the amount $|NPV_{CE}|$ to forgo claim to the investment. That is, the individual would be indifferent to paying this amount, or instead accepting the alternative. Obviously, the individual is better off by simply not accepting the alternative.

140 Like in **CASE B** above, the preferred strategy for the risk averse individual would be to simply not accept the alternative.

In this case

$$\alpha_{\mathrm{NPV}}^{(l)} < 0, \quad l \in \{\mathrm{CE},\ \mathrm{RADR}\} \qquad \text{[A.5]}$$

Like **CASE A** above, the smaller $\alpha_{\mathrm{NPV}}^{(l)}$ (or the larger $|\alpha_{\mathrm{NPV}}^{(l)}|$) is, the larger the adjustment for risk.

In summary (for $l \in \{\mathrm{CE},\ \mathrm{RADR}\}$), $\{0 \leq \alpha_{\mathrm{NPV}}^{(l)} < 1\}$ reflects that the risk adjustment pertains to a viable alternative, while $\{\alpha_{\mathrm{NPV}}^{(l)} < 0\}$ or $\{\alpha_{\mathrm{NPV}}^{(l)} > 1\}$ reflects that the risk adjustment pertains to a non-viable alternative. Finally, $\alpha_{\mathrm{NPV}}^{(l)} = 1$ implies risk neutrality (i.e., no adjustment for risk).

APPENDIX B

EXPECTED UTILITY OF TOTAL VERSUS PER ACRE WEALTH

For the empirical analysis, the modelled asset was a representative acre for the investment alternative considered. That is, net present value was defined on a per acre basis analogous to the convention used in deterministic stand level analysis. However, utility is usually defined with respect to the total wealth position for the individual. Therefore, it was necessary to derive the appropriate specification of the utility wealth function, with wealth distributed on a per acre basis, such that the expected utility ranking was preserved and the certainty equivalent net present value satisfied the conversion.

In this study, the individual was assumed to hold claims to land suitable for either timber or agricultural production of fixed size (denoted here as AREA). The assumption underlying the definition of the representative acre was that the total wealth effect was simply the per acre wealth effect multiplied by the size of the land holding. That is, for a given net present value outcome, the total wealth gain was[141]

$$npv^{(T)} = npv^{(pa)}(AREA) \qquad [B.1]$$

Accordingly, the distributional moments for the total net present value probability distribution are given by

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141 For the purposes of this appendix, the notation used will be explicit in whether a total, versus a per acre, wealth effect is being used. Superscripts (T) refer to total wealth effects, while superscripts (pa) refer to per acre wealth effects. Furthermore, $U_T(\cdot)$ explicitly refers to the total wealth specification of utility, while $U_{pa}(\cdot)$ explicitly refers to the per acre wealth specification of utility.

$$\text{MEAN VALUE:} \quad \mathrm{E}[NPV^{(T)}] = (\text{AREA})\mu \qquad [B.2a]$$

$$\text{VARIANCE:} \quad \mathrm{VAR}[NPV^{(T)}] = ((\text{AREA})^2)\sigma^2 \qquad [B.2b]$$

$$\text{SKEWNESS:} \quad \mathrm{SKE}[NPV^{(T)}] = ((\text{AREA})^3)\delta^3 \qquad [B.2c]$$

where: μ = the mean of the per acre net present value probability distribution (= $\mathrm{E}[NPV^{(pa)}]$).

σ^2 = the variance of the per acre net present value probability distribution (= $\mathrm{VAR}[NPV^{(pa)}]$).

δ^3 = the skewness of the per acre net present value probability distribution (= $\mathrm{SKE}[NPV^{(pa)}]$).

CONVERSION RELATIONSHIPS

Logarithmic Utility

For logarithmic utility, the moment distribution approximation of expected utility, with wealth considered in total, is given by

$$\mathrm{E}[\mathrm{U_T}(W^{(T)})] = \mathrm{LOG}\left(\mathrm{W_o^{(T)}} + \mathrm{E}[NPV^{(T)}]\right) - \left(\frac{1}{2}\right)\left(\frac{\mathrm{VAR}[NPV^{(T)}]}{(\mathrm{W_o^{(T)}} + \mathrm{E}[NPV^{(T)}])^2}\right) + \left(\frac{1}{3}\right)\left(\frac{\mathrm{SKE}[NPV^{(T)}]}{(\mathrm{W_o^{(T)}} + \mathrm{E}[NPV^{(T)}])^3}\right) \qquad [B.3]$$

Noting (B.2) above and substituting in the per acre conversion $\mathrm{w^{(T)}} = \mathrm{w^{(pa)}}(\text{AREA})$, (B.3) can be written as

$$\begin{aligned}\mathrm{E}[\mathrm{U_T}(W^{(T)})] &= \mathrm{LOG}(\mathrm{W_o^{(pa)}}(\text{AREA}) + \mu(\text{AREA})) - \left(\frac{1}{2}\right)\left(\frac{\sigma^2(\text{AREA})^2}{(\mathrm{W_o^{(pa)}}(\text{AREA}) + \mu(\text{AREA}))^2}\right) \\ &\quad + \left(\frac{1}{3}\right)\left(\frac{\delta^3(\text{AREA})^3}{(\mathrm{W_o^{(pa)}}(\text{AREA}) + \mu(\text{AREA}))^3}\right) \\ &= \mathrm{LOG}(\text{AREA}) + \mathrm{LOG}(\mathrm{W_o^{(pa)}} + \mu) - \left(\frac{1}{2}\right)\left(\frac{\sigma^2}{(\mathrm{W_o^{(pa)}} + \mu)^2}\right) \\ &\quad + \left(\frac{1}{3}\right)\left(\frac{\delta^3}{(\mathrm{W_o^{(pa)}} + \mu)^3}\right)\end{aligned}$$

$$= \text{LOG(AREA)} + E[U_{pa}(W^{(pa)})] \qquad [B.4]$$

where $W^{(pa)}$ defines the individual's uncertain wealth position on a per acre basis.

The result in (B.4) is consistent with specifying the per acre conversion directly in the utility of total wealth function. That is, for a given realized per acre wealth position, the utility of total wealth is given by

$$\begin{aligned} U_T(w^{(T)}) &= \text{LOG}(w^{(pa)}(\text{AREA})) \\ &= \text{LOG(AREA)} + U_{pa}(w^{(pa)}) \qquad [B.5a] \end{aligned}$$

or,

$$U_{pa}(w^{(pa)}) = -\text{LOG(AREA)} + U_T(w^{(T)}) \qquad [B.5b]$$

where $U_{pa}(w^{(pa)}) = \text{LOG}(w^{(pa)})$. Noting that $w^{(T)}$ and $w^{(pa)}$ are random, taking the expectation of (B.5a) results in (B.4) above. In essence, the per acre conversion simply implies a positive linear transformation of the individual's total wealth utility function $U_T(\bullet)$, which therefore perserves the axiomatic principles regarding the preference ordering of risky alternatives (Anderson *et al.*, 1977).

Accordingly, the certainty equivalent net present value for the investment NPV_{CE} also satisfies the conversion relationship. With respect to total wealth, $NPV_{CE}^{(T)}$ is found in the following relationship

$$\begin{aligned} U_T(W_o^{(T)} + NPV_{CE}^{(T)}) &= E[U_T(W^{(T)})] \\ &= \text{LOG(AREA)} + E[U_{pa}(W^{(pa)})] \qquad [B.6] \end{aligned}$$

due to (B.5a). Accordingly, the per acre certainty equivalent net present value $NPV_{CE}^{(pa)}$ is found in the following relationship

$$U_{pa}(W_o^{(pa)} + NPV_{CE}^{(pa)}) = E[U_{pa}(W^{(pa)})] \qquad [B.7]$$

Substituting (B.7) into (B.6) results in

$$\begin{aligned} U_T(W_o^{(T)} + NPV_{CE}^{(T)}) &= LOG(AREA) + U_{pa}(W_o^{(pa)} + NPV_{CE}^{(pa)}) \\ &= LOG(AREA) + LOG(W_o^{(pa)} + NPV_{CE}^{(pa)}) \\ &= LOG(W_o^{(pa)}(AREA) + NPV_{CE}^{(pa)}(AREA)) \\ &= U_T(W_o^{(T)} + NPV_{CE}^{(pa)}(AREA)) \end{aligned} \qquad [B.8a]$$

Therefore,

$$NPV_{CE}^{(T)} = NPV_{CE}^{(pa)}(AREA) \qquad [B.8b]$$

Negative Exponential Utility

For the negative exponential utility function, applying the conversion from total wealth to per acre wealth does not result in a positive linear transformation of the total wealth utility function $U_T(\bullet)$. However, in this case the coefficient of absolute risk aversion, c, can be used as a scale parameter for conversion purposes. Consider the total wealth utility specification given by

$$U_T(w^{(T)}) = 1 - e^{-c_T(w^{(T)})} \qquad [B.9]$$

Substituting in the per acre conversion $w^{(T)} = w^{(pa)}(AREA)$, (B.9) can be written as

$$U_T(w^{(T)}) = 1 - e^{-c_T(w^{(pa)}(AREA))} \qquad [B.10]$$

If $c_{pa} = c_T(AREA)$, then (B.10) becomes

$$U_T(w^{(T)}) = 1 - e^{-c_{pa}(w^{(pa)})} \qquad [B.11]$$

Note that (B.9) and (B.11) are equivalent specifications of $U_T(w^{(T)})$.[142] That is,

$$U_{pa}(w^{(pa)}) = 1 - e^{-c_{pa}(w^{(pa)})} = U_T(w^{(T)}) \qquad [B.12]$$

142 Note: $U_T(w^{(T)}) \neq U_T(w^{(pa)})$.

as long as $c_{pa} = c_T(\text{AREA})$.

Accordingly, adjusting the coefficient of absolute risk aversion for scale defines an equivalent conversion in the moment distribution approximation of expected utility. For total wealth effects, this approximation is given by

$$E[U_T(W^{(T)})] = 1 - \left(e^{-c_T(W_o^{(T)} + E[NPV^{(T)}])}\right) \cdot \left(1 + \left(\frac{c_T^2}{2}\right)\text{VAR}[NPV^{(T)}] - \left(\frac{c_T^3}{6}\right)\text{SKE}[NPV^{(T)}]\right) \quad \text{[B.13]}$$

Noting (B.2) and substituting the scale adjustment $c_T = (c_{pa}/(\text{AREA}))$, (B.13) becomes

$$\begin{aligned} E[U_T(W^{(T)})] &= 1 - \left(e^{-\left(\frac{c_{pa}}{\text{AREA}}\right)(W_o^{(T)} + E[NPV^{(T)}])}\right) \cdot \left(1 + \left(\frac{1}{2}\right)\left(\frac{c_{pa}^2}{\text{AREA}^2}\right)\text{VAR}[NPV^{(T)}] - \left(\frac{1}{6}\right)\left(\frac{c_{pa}^3}{\text{AREA}^3}\right)\text{SKE}[NPV^{(T)}]\right) \\ &= 1 - e^{-c_{pa}(W_o^{(pa)} + \mu)}\left(1 + \left(\frac{1}{2}\right)c_{pa}^2(\sigma^2) - \left(\frac{1}{6}\right)c_{pa}^3(\delta^3)\right) \\ &= E[U_{pa}(W^{(pa)})] \end{aligned} \quad \text{[B.14]}$$

That is, the expected utility ranking is preserved under the per acre conversion as long as $c_{pa} = c_T(\text{AREA})$.

Finally, the certainty equivalent net present value NPV_{CE} also satisfies the conversion. This follows directly from (B.14) since

$$\begin{aligned} U_T(W_o^{(T)} + NPV_{CE}^{(T)}) &= E[U_T(w^{(T)})] \\ &= E[U_{pa}(W^{(pa)})] \\ &= U_{pa}\left(W_o^{(pa)} + NPV_{CE}^{(pa)}\right) \end{aligned} \quad \text{[B.15a]}$$

or,

$$1 - e^{-c_T(W_o^{(T)} + NPV_{CE}^{(T)})} = 1 - e^{-c_{pa}(W_o^{(pa)} + NPV_{CE}^{(pa)})}$$

$$= 1 - e^{-(c_T(AREA))(W_o^{(pa)} + NPV_{CE}^{(pa)})}$$

$$= 1 - e^{-c_T(W_o^{(T)} + NPV_{CE}^{(pa)}(AREA))} \qquad [B.15b]$$

or,

$$NPV_{CE}^{(T)} = NPV_{CE}^{(pa)}(AREA) \qquad [B.15c]$$

PARAMETER ASSUMPTIONS

In this study, two cases (representing differences in the degree of risk aversion)[143] for each utility function class were considered. The two cases for the logarithmic utility specification represented total wealth endowments of 20,000 and 2,000 (1988) dollars (or 5,688 and 569 (1967) dollars) respectively.[144] The second case represented an extremely high degree of absolute risk aversion. Accordingly, the two cases for the negative exponential class represented different degrees of risk aversion as measured by the relative risk aversion coefficient. Under CASE 3, the relative risk aversion coefficient was 1, while under CASE 4 (representing a higher degree of risk aversion) the relative risk aversion coefficient was 5.[145]

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143 Theses measures were first introduced in the analytical treatment of the risk-adjusted discount rate problem. For logarithmic utility, the relative risk aversion coefficient is constant (= 1) and absolute risk aversion decreases under higher wealth assumptions. For negative exponential utility, absolute risk aversion is constant (i.e, c_T fixed) and relative risk aversion increases the higher the wealth assumption.

144 The consumer price index (all commodities, 1982-84 = 100) was used as the deflator (US Department of Commerce, 1988). Adjusted (1967 = 1.00), deflator = 3.516.

145 However, CASES 3 and 4 below contrast different relative risk aversion assumptions by varying the absolute risk aversion coefficient c_T and holding the wealth assumption constant. Therefore, CASES 3 and 4 also contrasted different degrees of absolute risk aversion as well (CASE 3: low degree, CASE 4: high degree).

Specifically, the parameter assumptions were:

LOGARITHMIC UTILITY

CASE 1: Low degree of absolute risk aversion
$W_o^{(T)} = 5688.30$ 1967 dollars, which gives
$W_o^{(pa)} = 189.61$ 1967 \$/acre, with AREA = 30 acres.

CASE 2: High degree of absolute risk aversion
$W_o^{(T)} = 568.83$ 1967 dollars, which gives
$W_o^{(pa)} = 18.96$ 1967 \$/acre, with AREA = 30 acres.

NEGATIVE EXPONENTIAL UTILITY

CASE 3: Low degree of relative and absolute risk aversion
$W_o^{(T)} = 5688.30$ 1967 dollars, $c_T = 0.0001758$; which gives
$W_o^{(pa)} = 189.61$ 1967 \$/acre, $c_{pa} = 0.005274$, with AREA = 30 acres.

CASE 4: High degree of relative and absolute risk aversion
$W_o^{(T)} = 5688.30$ 1967 dollars, $c_T = 0.0008790$; which gives
$W_o^{(pa)} = 189.61$ 1967 \$/acre, $c_{pa} = 0.02637$, with AREA = 30 acres.

CASES 1 and 3 contrast different specifications of the utility function, by holding the degree of relative and absolute risk aversion fixed (since $1/5688.30 = c_T = 0.0001758$).

DECOMPOSITION OF EXPECTED UTILITY

For the purposes of reporting the expected utility analysis results, it is convenient to decompose the expected utility approximation into those parts attributed to the empirical NPV probability distribution mean, variance, and skewness. That is,

$$E[U_{pa}(w^{(pa)})] = \text{MEAN COMPONENT} + \text{VARIANCE COMPONENT} + \text{SKEWNESS COMPONENT} \qquad [B.16]$$

These components are given by:

LOGARITHMIC UTILITY

$$\text{MEAN COMPONENT} = \text{LOG}(W_o + E[NPV^{(pa)}]) \qquad [B.17a]$$

$$\text{VARIANCE COMPONENT} = -\left(\frac{1}{2}\right)\left(\frac{\text{VAR}[NPV^{(pa)}]}{(W_o^{(pa)} + E[NPV^{(pa)}])^2}\right) \qquad [B.17b]$$

$$\text{SKEWNESS COMPONENT} = +\left(\frac{1}{3}\right)\left(\frac{\text{SKE}[NPV^{(pa)}]}{(W_o^{(pa)} + E[NPV^{(pa)}])^3}\right) \qquad [B.17c]$$

NEGATIVE EXPONENTIAL UTILITY

$$\text{MEAN COMPONENT} = 1 - e^{-c_{pa}\left(W_o + E[NPV^{(pa)}]\right)} \qquad [B.18a]$$

$$\text{VARIANCE COMPONENT} = -e^{-c_{pa}\left(W_o + E[NPV^{(pa)}]\right)}\left(\left(\frac{c_{pa}^2}{2}\right)\text{VAR}[NPV^{(pa)}]\right) \qquad [B.18b]$$

$$\text{SKEWNESS COMPONENT} = e^{-c_{pa}\left(W_o + E[NPV^{(pa)}]\right)}\left(\left(\frac{c_{pa}^3}{6}\right)\text{SKE}[NPV^{(pa)}]\right) \qquad [B.18c]$$

It should be noted that the mean effect cannot be completely isolated because the mean also appears in the variance and skewness component terms. Furthermore, this distortion may cloud the isolation of the variance and skewness effects on expected utility as well. In essence, the variance and skewness components account for the mean-variance and mean-skewness tradeoffs underlying the expected utility valuation of the investment.

APPENDIX C

ESTIMATION OF THE VECTOR AUTOREGRESSION (VAR) STOCHASTIC PRICE MODEL

CANDIDATE ECONOMIC INDEX VARIABLES

One hypothesis underlying the specification of the VAR model for commodity prices was that timber prices had a common correlation through the general state of the economy. One can think of the economy as being in one of the following states:

1) Near a trough
2) Expansion
3) Near a peak
2) Recession

Historically, the US Department of Commerce Bureau of Economic Analysis has defined peaks and troughs for the state of the economy. Accordingly, the Bureau keeps track of economic indexes that either lead, lag, or are roughly coincident to the fluctuations between peaks and troughs. Therefore, these indexes can be used as proxies for the state of the economy.

The indexes considered in this study were the following (US Department of Commerce, 1988).

1) Composite Index of 12 Leading Indicators (1967 = 100)
2) Composite Index of 6 Lagging Indicators (1967 = 100)
3) Composite Index of 4 Roughly Coincident Indicators (1967 = 100)

In general these indexes represent real changes (base year 1967) in the economy based on the changes in the variables that make up each index.[146] The question pertaining to this part of the analysis was which index best explained the dynamic behavior in sawtimber and pulpwood prices. This question was addressed by treating each index variable as a separate candidate in model estimation and then choosing the model that gave the best fit.

MODEL ESTIMATION

Hsiao's Procedure

Model estimation was based on the procedure outlined in Hsiao (1979). Hsiao's approach is appealing because it is designed to reduce the number of parameters needed to be estimated in the VAR model. In general, the approach is to treat each autoregression in the VAR model as a single equation with lagged endogenous and lagged exogenous (i.e., other explanatory) variables as the regressors. Furthermore, Hsiao (1979) suggests that the explanatory variables used in each autoregression need only be a subset of all the variables included in the VAR model. Finally, Hsiao's approach allows the lag order (i.e., the number of lags) to vary across the autoregressions for each variable considered.[147] According to Hsiao, this not only reduces

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Examples of these variables are the index of stock prices for common stocks and the new housing unit index for the leading index, industrial production index and personal income (less transfer payments) for the coincident index, and mean unemployment duration and the mean prime rate for the lagging index (US Department of Commerce, 1988).

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VAR models were originally defined as having all variables being joint dependent (each autoregressive relationship contains every variable considered); where the lag order for the entire process is fixed (Sims 1977 in Hsiao, 1979). However, Hsiao's procedure allows for deviations from the unrestricted model by adding restrictions based on *a priori* knowledge.

the number of parameters that need to be estimated, but also allows the influence of each variable to be felt at different time lags.

For the VAR model considered in this study, there were 4 autoregressions that needed to be estimated (equations (32a), (32c)-(32f)). However, the specification of the VAR model in (32) differed from Hsiao's (1979) approach in that contemporaneous effects were also considered by including the current value (i.e., lag zero) for the index variable in the sawtimber and pulpwood autoregressions. Similarly, for the specification of hay yield in the hay price autoregression.

Two important concepts underlie Hsiao's (1979) procedure: Akaike's final prediction error (FPE) criterion (Akaike (1969a,b) in Hsiao, 1979), and Granger's definition of causality (Granger (1969) in Hsiao, 1979). The FPE criterion is used to determine the appropriate lag order for each variable in the autoregression, while Granger's causality defines whether an explanatory variable should actually be included in the model.

Choosing the lag order

Assuming that the stochastic disturbance term has no autocorrelations, ordinary least squares (OLS) can be used to estimate each autoregression (Harvey, 1981a).[148] Hsiao's (1979) criterion to choose the lag order for each regressor is to introduce lags up to the point that Akaike's FPE criterion is minimized.

For example, consider the following bivariate autoregression.[149]

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148 That is, the OLS estimators are BLUE (best, linear, unbiased estimators). This result, which follows from the Mann-Wald theorem, also holds when the disturbance term may not be distributed independently of the regressors (Harvey, 1981a). For the case where serial correlation in the error term is a problem, generalized least squares (GLS) is the appropriate estimation procedure (Harvey, 1981a).

149 In this example, the intercept (i.e., deterministic component) is suppressed for convenience.

$$y_t = \beta_1 y_{t-1} + \dots + \beta_m y_{t-m} + \gamma_1 x_{t-1} + \dots + \gamma_n x_{t-n} + \varepsilon_t \qquad [C.1]$$

where t denotes the period, $\beta_1, \dots, \beta_m$ and $\gamma_1, \dots, \gamma_n$ are the parameters to be estimated, and ε_t denotes the stochastic disturbance term. In (C.1) y_t is endogenous and x_t is the candidate explanatory regressor. The problem is to choose m and n for y_t and x_t respectively. For this model, Akaike's FPE criterion is defined as (Hsiao, 1979)

$$\text{FPE} = \left(\frac{T+m+n+1}{T-m-n-1}\right)\sum_{t=1}^{T}\frac{(y_t-\hat{y}_t)^2}{T} \qquad [C.2]$$

where T denotes the total number of observations used in estimation, and $\hat{y}_t$ is the fitted value for y_t. The criterion can be decomposed into two parts. The term $(T+m+n+1)/(T-m-n-1)$ represents estimation error and increases with higher lag orders m and n. The second term, $\sum_{t=1}^{T}(y_t-\hat{y}_t)^2/T$ represents modelling (i.e., fitted or prediction) error (Hsiao, 1979).[150] Simply adding lag terms to the model will reduce $\sum_{t=1}^{T}(y_t-\hat{y}_t)^2/T$, so the first term acts as a penalty term for increasing the lag order.

Given the definition for FPE in (C.2), Hsiao (1979) defines a sequential procedure for estimating the model in (C.1).[151]

STEP 1: Consider the univariate model (i.e., $n = 0$)

$$y_t = \beta_1 y_{t-1} + \dots + \beta_m y_{t-m} + \varepsilon_t^{(1)}$$

Pick $\{m: 1 \le m \le \text{MMAX}\}$ that minimizes FPE, where MMAX denotes the maximum lag order to consider. Call this value m^*.

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150 The term prediction error refers to estimation error over the range of data used to fit the model, and not prediction error in forecasts using an independent validation data set.

151 The stochastic disturbance for the model in STEP p will be denoted as $\varepsilon_t^{(p)}$.

STEP 2: Consider the bivariate model

$$y_t = \beta_1 y_{t-1} + \dots + \beta_{m^*} y_{t-m^*} + \gamma_1 x_{t-1} + \dots + \gamma_n x_{t-n} + \varepsilon_t^{(2)}$$

Pick {n: $1 \leq n \leq NMAX$} that minimizes FPE, where m^* denotes the lag order for the endogenous variable from STEP 1, and NMAX denotes the maximum lag order to consider. Call this value n^*.

If $FPE^{(STEP\ 2)} < FPE^{(STEP\ 1)}$, then the bivariate model from STEP 2 is the appropriate specification. However, if $FPE^{(STEP\ 2)} > FPE^{(STEP\ 1)}$, then the appropriate specification is the univariate model identified in STEP 1.

Finally, when $FPE^{(STEP\ 2)} < FPE^{(STEP\ 1)}$, Hsiao (1979) suggests the following

STEP 3: Consider the bivariate model

$$y_t = \beta_1 y_{t-1} + \dots + \beta_{m^*} y_{t-m^*} + \gamma_1 x_{t-1} + \dots + \gamma_{n^*} x_{t-n^*} + \varepsilon_t^{(3)}$$

Pick {m: $1 \leq m \leq m^*$} that minimizes FPE, where m^* denotes lag order from STEP 1 and n^* denotes the lag order from STEP 2.

If $m = m^*$ then the model identified in STEP 2 is the appropriate specification. However, if $m < m^*$ then the specification in STEP 3 is appropriate. In essence, STEP 3 is designed to check whether the lag order for the lagged endogenous variable from STEP 1 represents over-fitting the model given the lag order for the introduced explanatory variable from STEP 2.

Justification of the Introduced Explanatory Variables

According to Hsiao (1979), the condition $FPE^{(STEP\ 2)} < FPE^{(STEP\ 1)}$ supports the hypothesis that x_t has non-contemporaneous Granger causality effects on y_t. For the timber price autogressions in (32c) and (32d), this provided a test of the hypothesis that the general state of the economy (as represented by the economic index variable) indeed influenced the determination of sawtimber and pulpwood stumpage prices. For both the

the sawtimber and pulpwood autoregressions, the coincident index was the best variable to represent the general state of the economy. That is, from the set of candidate indexes considered, the autoregressions including the coincident index had the lowest FPE in STEP 2. Furthermore, the bivariate specification for the pulpwood autoregression had the lowest overall FPE. For the sawtimber autoregression, the univariate model had the lowest overall FPE, but not markedly lower than the FPE result for the bivariate model containing the coincident index. However, it was felt that the difference was not large enough to abandon the original bivariate specification for the sawtimber autoregression.[152]

For the hay price autoregression in (32e), the bivariate model had the lowest FPE supporting the hypothesis that hay yield influences the determination of hay price. Finally, since the economic index model in (32a) was restricted to be a univariate model, only STEP 1 of Hsiao's procedure applied.

Stationarity Concerns

According to Masani (1966) (in Hsiao, 1979), a vector stationarity time series process is fundamental to the specification of the process as a VAR model. However, it is difficult to test for stationarity in the VAR. However, one simple diagnostic is to test for stationarity in the univariate AR(1) specification for each variable considered. This AR(1) specification in general is,

$$y_t = \beta_0 + \beta_1 y_{t-1} + \varepsilon'_t \qquad [C.3]$$

where in this example the intercept term β_0 is not suppressed and ε'_t represents the stochastic disturbance term. If $|B_1| < 1$, then the AR(1) process in (C.3) will be

152 The absolute difference was less than 2 percent of the FPE value for the best univariate model.

stationary given that ε'_t is a white noise stationary process (i.e., constant mean, constant variance-covariance matrix independent of t) (Harvey, 1981b).[153] For the variables considered in this study (including hay yield), the condition $\beta_0 > 0$ was also important because this condition implies a non-negative mean value.[154] Therefore, the model in (C.3) was estimated for the three price variables (as well as for the index variable candidates and hay yield), and the hypotheses: H_o: $\beta_0 = 0$ and H_o: $|\beta_1| \geq 1$ independently tested. The respective alternative hypotheses were: H_a: $\beta_0 > 0$ and H_a: $|\beta_1| < 1$.

For the price variables (and hay yield), both hypotheses were rejected in favor of the alternative hypotheses at the 0.02 level of significance, supporting (for each variable) a stationary AR(1) process with non-negative means.[155] Furthermore, adding more lag terms to the AR(1) model will tend to induce stationarity (Harvey, 1981b). Based on these results, the bivariate autoregressions for the price variables were assumed to be stationary.[156]

However, the hypothesis test failed to reject H_o: $|\beta_1| \geq 1$ for the AR(1) specification of the coincident index variable. Non-stationarity for this variable is evident by the positive trend in the observed data (appendix D). Therefore, for the

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153 These conditions will hold under the Gaussian assumptions regarding ε'_t in OLS estimation.

154 However, the intercept term does not represent the actual mean value, instead the mean value will be an function of the intercept term. The actual mean would be identified by the intercept term in the moving average representation of the autoregressive process (Harvey, 1981b).

155 The hypothesis, H_o: $\beta_0 = 0$ was also rejected (at a 0.02 level of significance) if favor of H_a: $\beta_0 > 0$ for all economic index variable candidates as well.

156 It use to be convention to assume that stationarity in the univariate model implied stationarity in multivariate models. But recent feelings are that this may not always be the case (Orden, D. 1988. Assistant professor, Department of Agricultural Economics, Virginia Polytechnic Institute and State University, Blacksburg, Virginia. Personal communication).

purposes of model estimation, first differences were used.[157] Using differenced data, the AR(1) representation of the coincident index variable satisfied the stationarity diagnostic. Furthermore, a positive mean for the differenced data was also implied, again indicating the upward trend for the undifferenced process.

Accounting for Contemporaneous Effects in the Calculation of FPE

This study deviated from Hsiao's (1979) specification of VAR models in that contemporaneous effects of the index variable were allowed in the timber price autoregressions. Similarly, contemporaneous effects of hay yield were allowed in the hay price autoregression. It was felt that introducing contemporaneous terms (i.e., lag = 0) should be treated just like adding another lag term to the model for the purpose of calculating FPE.[158] Therefore, when contemporaneous terms were introduced in the model, the FPE criterion was defined as

$$\text{FPE} = \left(\frac{\text{T}+\text{m}+(\text{n}+1)+1}{\text{T}-\text{m}-(\text{n}+1)-1}\right)\sum_{t=1}^{\text{T}}\frac{(y_t-\hat{y}_t)^2}{\text{T}} \qquad \text{[C.2]}$$

where the adjustment (n+1) accounted for the extra parameter γ_0 associated with the contemporaneous term x_t. In this case (with respect to STEP 2 only) n = 0 implies no lagged terms.[159]

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157 According to Harvey (1981b), a stochastic process which is initially non-stationary will most likely be stationary if first differences are used. Therefore, in spite of satisfying the stationarity diagnostic, the hay yield autoregression was also fitted using differenced data since the historical data also followed a upward trend.

158 Orden, D. 1988. Assistant professor, Department of Agricultural Economics, Virginia Polytechnic Institute and State University, Blacksburg, Virginia. Personal communication.

159 A similar approach was used to account for the case in which intermediate lags were dropped. In general, the values for m and n represented the total number of lagged endogenous and explanatory (including any contemporaneous term) in the model.

Estimation Range

Hsiao's (1979) procedure requires that each candidate autoregression targeted in STEPS 1-3 be estimated over the same range of data. This necessitated setting aside an initialization data set large enough to account for the largest lag order considered. For the sawtimber and pulpwood autoregressions, this promoted some problems because the available data was limited to the period 1978:1 (i.e., first quarter, 1978) to 1988:3. Therefore, the maximum lag order for these variables was restricted to be 4 (i.e., $M = 4$ quarters). Any larger lag order would not allow the estimation to cover the 1979-1980 period, during which sawtimber and pulpwood stumpage prices fell dramatically (appendix D). Furthermore, the pulpwood price data had missing observations from 1985:3 to 1986:2. This required a second initialization data at the start of the 1986:2 period to allow for lags. For similar reasons, the maximum lag order for the hay price variable was set at 8 years. However, plenty of data was available for the estimation of the coincident index autoregression (as well as for the hay yield autoregression). In this case, the maximum lag order considered was 16 periods.

After the best model specification was identified for each autoregression, the chosen models were re-estimated using all available data; that is the initialization data set was consistent with the chosen lag orders.

Software

The autoregressions were estimated using the OLS procedure in the time series statistical package, RATS (Doan and Litterman, 1986). The data underlying the estimation is documented in appendix D.

RESULTS

The final estimated autoregressions representing the stochastic price VAR model are summarized in tables 16-19. For the sawtimber stumpage prices (table 17), the coincident index had contemporaneous as well as a first lag effect. When compared to the pulpwood autoregression, sawtimber price movements tended to be opposite those for pulpwood. That is, the contemporaneous effect of the economy on sawtimber was positive, while this effect was negative for pulpwood (table 18). On the other hand, the lagged effect on sawtimber was negative, while the lagged effect of the economy on pulpwood was positive. In general, if the economy was expanding (i.e., contemporaneous value for the index variable exceeded the lagged values), the effect would be a rise in sawtimber stumpage price accompanied by a decline in pulpwood prices. Accordingly, during periods of economic decline (i.e., lagged values for the coincident index exceeded the contemporaneous value), the effect would be a decline in sawtimber prices and a rise in pulpwood prices.[160] One explanation for this result may be that when the market is good for sawtimber, the availability of sawmill residue as a source of chips may soften the demand for pulpwood; while when the market is bad the lack of mill residues increases the demand for pulpwood.[161]

For the hay price model, the inverse relationship between hay yield and hay price holds since the sign of the coefficients representing contemporaneous and lagged effects are negative (table 19). That is, when the contemporaneous and prior year hay yields

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160 Redmond and Cubbage (1988) also found that sawtimber stumpage prices tend to move counter cyclical to pulpwood stumpage prices.

161 However, there would be countervailing supply effects as well. For example, when the sawtimber market is good, the tendency would be to shift small diameter logs into sawtimber; restricting the availability of pulpwood. Accordingly, during poor market periods for sawtimber, small diameter logs would shift into pulpwood; increasing supply.

Table 16. Fitted autoregressive model for the coincident index (first differences).

COINCIDENT INDEX AUTOREGRESSION			
COEFFICIENT[1]	ESTIMATED VALUE	TSTAT[2]	SIGNIFICANCE LEVEL[3]
$\beta_0^{(in)}$	0.440	3.181	0.002
$\beta_1^{(in)}$	0.711	9.086	(< 0.001)
$\beta_2^{(in)}$	-0.248	-2.599	0.02
$\beta_3^{(in)}$	0.234	2.453	0.02
$\beta_4^{(in)}$	-0.234	-2.967	0.005

Composite index of four roughly coincident indicators (1967 = 100).
Estimation range (quarterly): 1949:2-1988:3
Degrees of Freedom = 153
Coefficient of Determination ($R^{(2)}$ adjusted) = 0.392
$Q(36)^{(4)}$ = 42.494, α = 0.212
Standard error of residuals = 1.527

Notes:

(1) See introduction for definition of notation.

(2) Test statistic for test:

Ho: COEFFICIENT = 0
Ha: COEFFICIENT ≠ 0

(3) Significance level for TSTAT. Gives probability of observing TSTAT given, Ho: COEFFICIENT = 0, is true.

(4) Q(p) is the Ljung-Box statistic based of p autocorrelations (Doan and Litterman, 1986). Test statistic for (Harvey, 1981b):

Ho: No serial correlation
Ha: Serial correlation

α gives probability of observing Q(p) given, Ho: No serial correlation, is true.

Table 17. **Fitted autoregressive model for sawtimber stumpage price.**

SAWTIMBER STUMPAGE PRICE AUTOREGRESSION			
COEFFICIENT[1]	ESTIMATED VALUE	TSTAT[2]	SIGNIFICANCE LEVEL[3]
$\beta_0^{(saw)}$	4.418	0.567	(> 0.50)
$\beta_1^{(saw)}$	0.569	3.585	0.002
$\beta_2^{(saw)}$	0.195	1.072	0.50
$\beta_3^{(saw)}$	-0.384	-2.140	0.05
$\beta_4^{(saw)}$	0.401	2.435	0.05
$\gamma_0^{(saw)}$	0.443	1.748	0.10
$\gamma_1^{(saw)}$	-0.444	-1.669	0.20

Virginia southern yellow pine sawtimber stumpage price (1967 \$/MBF).
(MBF is thousand board feet, Scribner).

Estimation range (quarterly): 1979:1-1988:3

Degrees of Freedom = 32

Coefficient of Determination ($R^{(2)}$ adjusted) = 0.534

$Q(18)^{(4)} = 7.769, \alpha = 0.982$

Standard error of residuals = 3.333 (1967 \$/MBF)

Notes:

(1) See introduction for definition of notation.

(2) Test statistic for test:

Ho: COEFFICIENT = 0
Ha: COEFFICIENT ≠ 0

(3) Significance level for TSTAT. Gives probability of observing TSTAT given, Ho: COEFFICIENT = 0, is true.

(4) Q(p) is the Ljung-Box statistic based of p autocorrelations (Doan and Litterman, 1986). Test statistic for (Harvey, 1981b):

Ho: No serial correlation
Ha: Serial correlation

α gives probability of observing Q(p) given, Ho: No serial correlation, is true.

Table 18. Fitted autoregressive model for pulpwood stumpage price.

PULPWOOD STUMPAGE PRICE AUTOREGRESSION			
COEFFICIENT[1]	ESTIMATED VALUE	TSTAT[2]	SIGNIFICANCE LEVEL[3]
$\beta_0^{(pulp)}$	1.969	2.651	0.02
$\beta_1^{(pulp)}$	0.855	4.154	(< 0.001)
$\beta_2^{(pulp)}$	-0.523	-2.575	0.02
$\gamma_0^{(pulp)}$	-0.013	-1.933	0.10
$\gamma_3^{(pulp)}$	0.011	1.530	0.20

Virginia southern yellow pine pulpwood stumpage price (1967 $/cord).
Estimation range (quarterly): 1979:1-1985:2, 1987:1-1988:3
Degrees of Freedom = 30
Coefficient of Determination ($R^{(2)}$ adjusted) = 0.499
Q(15)[4] = 6.036, α = 0.979
Standard error of residuals = 0.222 (1967 $/cord)

Notes:
(1) See introduction for definition of notation.
(2) Test statistic for test:
Ho: COEFFICIENT = 0
Ha: COEFFICIENT $\neq$ 0
(3) Significance level for TSTAT. Gives probability of observing TSTAT given, Ho: COEFFICIENT = 0, is true.
(4) Q(p) is the Ljung-Box statistic based of p autocorrelations (Doan and Litterman, 1986). Test statistic for (Harvey, 1981b):
Ho: No serial correlation
Ha: Serial correlation
α gives probability of observing Q(p) given, Ho: No serial correlation, is true.

Table 19. Fitted autoregressive model for hay price.

HAY PRICE AUTOREGRESSION			
COEFFICIENT[1]	ESTIMATED VALUE	TSTAT[2]	SIGNIFICANCE LEVEL[3]
$\beta_0^{(hay)}$	36.617	5.501	(< 0.001)
$\beta_1^{(hay)}$	0.474	3.019	0.01
$\beta_2^{(hay)}$	-0.251	-2.462	0.05
$\beta_6^{(hay)}$	0.221	2.370	0.05
$\beta_7^{(hay)}$	-0.287	-2.188	0.05
$\beta_8^{(hay)}$	0.345	3.379	0.005
$\gamma_0^{(hay)}$	-6.457	-7.796	(< 0.001)
$\gamma_1^{(hay)}$	-2.299	-1.657	0.20

Virginia all hay (baled) price received by farmers (1967 \$/ton).
Estimation range (annual): 1958-1987
Degrees of Freedom = 22
Coefficient of Determination ($R^{(2)}$ adjusted) = 0.873
Q(15)[4] = 13.823, α = 0.539
Standard error of residuals = 1.669 (1967 \$/ton)

Notes:

(1)See introduction for definition of notation.

(2)Test statistic for test:

Ho: COEFFICIENT = 0
Ha: COEFFICIENT ≠ 0

(3)Significance level for TSTAT. Gives probability of observing TSTAT given, Ho: COEFFICIENT = 0, is true.

(4)Q(p) is the Ljung-Box statistic based of p autocorrelations (Doan and Litterman, 1986). Test statistic for (Harvey, 1981b):

Ho: No serial correlation
Ha: Serial correlation

α gives probability of observing Q(p) given, Ho: No serial correlation, is true.

are low, the hay price tends to be higher than when the contemporaneous and prior year yields are high. The first lag term also indicates that there are some carry over effects of hay yield on hay price as farmers. Perhaps buyers take advantage of good (yield) years by storing hay for the following season. This would have the effect of softening the demand for hay in the subsequent year. Somewhat surprising is the high lag order for the endogenously lagged hay price, suggesting effects are felt as far back as 8 years. However, inspection of the hay price data (appendix D) suggests that historically, hay prices have been following a 8 to 10 year cyclical pattern.

As indicated by the Ljung-Box statistic, serial correlation in the error term was only evident in the coincident index model (table 1), but absent in the other autoregressions. Coefficients of determination (adjusted) are reported for reference, but little meaning should be attributed to them. Low values simply imply that more variability in the process can be attributed to random disturbance. In this case, the simulated outcomes will be driven by the outcome for the stochastic disturbance term rather than the outcome for the fitted relationship.

In general, Hsiao's (1979) procedure resulted in choosing lag orders approaching the maximum order considered. The large lag orders for hay price (table 19) and hay yield (table 4) greatly reduced the the amount of serial correlation in the model. Finally, intermediate lags for the coincident index variable in the pulpwood autoregression (table 18) were dropped since they were insignificant and removing these terms improved model performance by lowering FPE. Similarly for endogenous lags in the hay price autoregression (table 19).

Model Performance

Figures 9 and 10 provide an example of the results from a given simulation. In figure 9, sawtimber and pulpwood outcomes are compared to the outcomes for the coincident index. Figure 10 compares the hay price outcomes to the simulated outcomes for hay yield. For the most part, the simulations were consistent with the observed historical relationships (see appendix D). Figures 11 through 15 summarize the lower and upper bounds, as well as the mean values, from a complete set of 500 simulations for the coincident index, sawtimber stumpage price, pulpwood stumpage price, hay price, and hay yield variables respectively. The results illustrated are typical of the simulation runs for each investment alternative considered. Considering the long-term nature of the projections, the model behaved reasonably well. The only weakness was that the simulations of pulpwood and hay prices (figures 13 and 14) did not seem to satisfy the stationarity condition. In both cases, the tendency was to simulate a steady downward trend in prices. Variability (as indicated by the range of simulated outcomes) was for the most part constant; the exception being the coincident index variable (figure 11).

The reason for the downward trend in pulpwood and hay prices can be attributed to counter-cyclical movements to the explanatory variables introduced in their respective bivariate autoregressions (coincident index for pulpwood, hay yield for hay prices). That is, in as much as the coincident index and hay yield followed upward trends (figures 11 and 15), the effect was to respectively drive pulpwood and hay prices along a downward trend, random (i.e., stochastic disturbance) effects notwithstanding.[162] For the hay and pulpwood investments (alternatives F and D respectively), the effect was to drive the returns from distant rotations to well below zero, thus having most simulations end on the land abandonment assumption. This was especially true for the hay alternative, where 427 out of 500 simulations ended under land abandonment after 4 or 5 rotations

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162 Trends in coincident index (figure 16, appendix D) and hay yield (figure 19, appendix D) were originally avoided in the estimation since first differenced data was used. However, the observed downward trend in hay prices (figure 18, appendix D) could not be effectively handled by considering first differences. Preliminary analysis indicated that model specifications for hay price based on first differences performed poorly in all aspects of estimation.

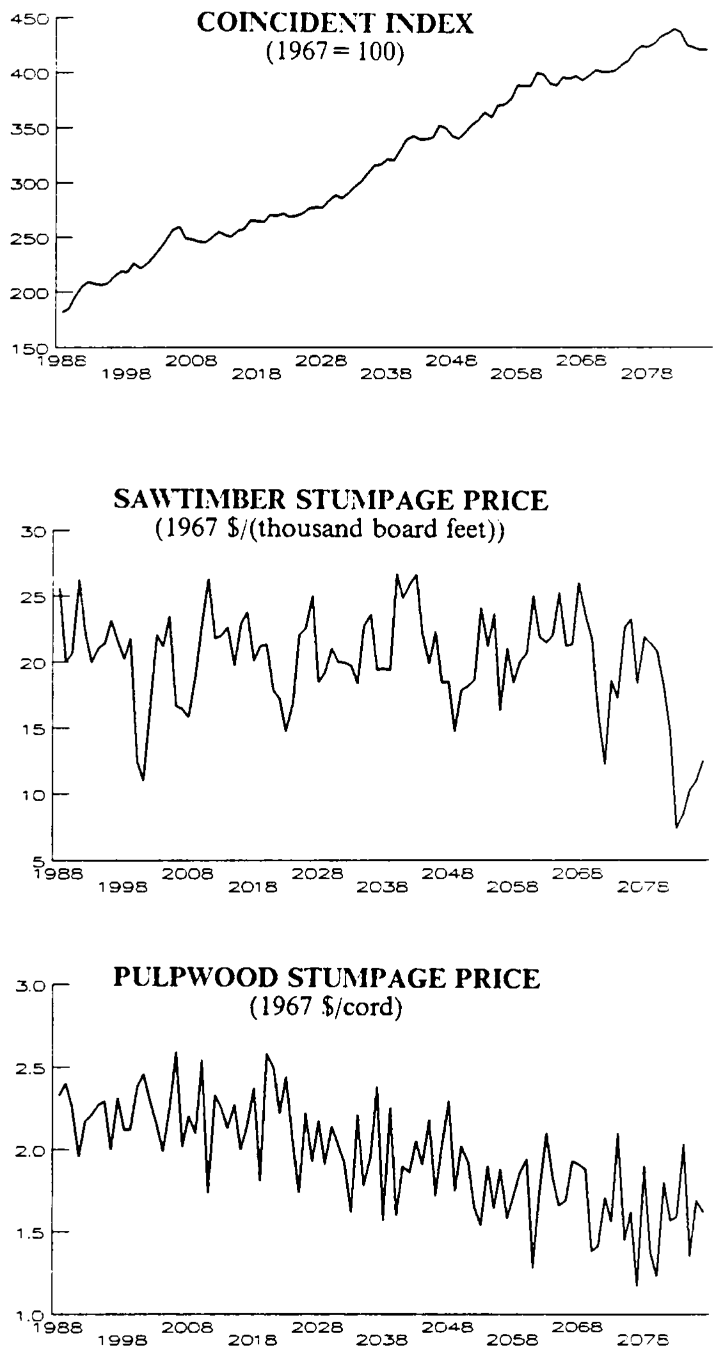

Figure 9. Example of the simulated outcomes for the coincident index, sawtimber, and pulpwood variables (1st 100 years of projection).

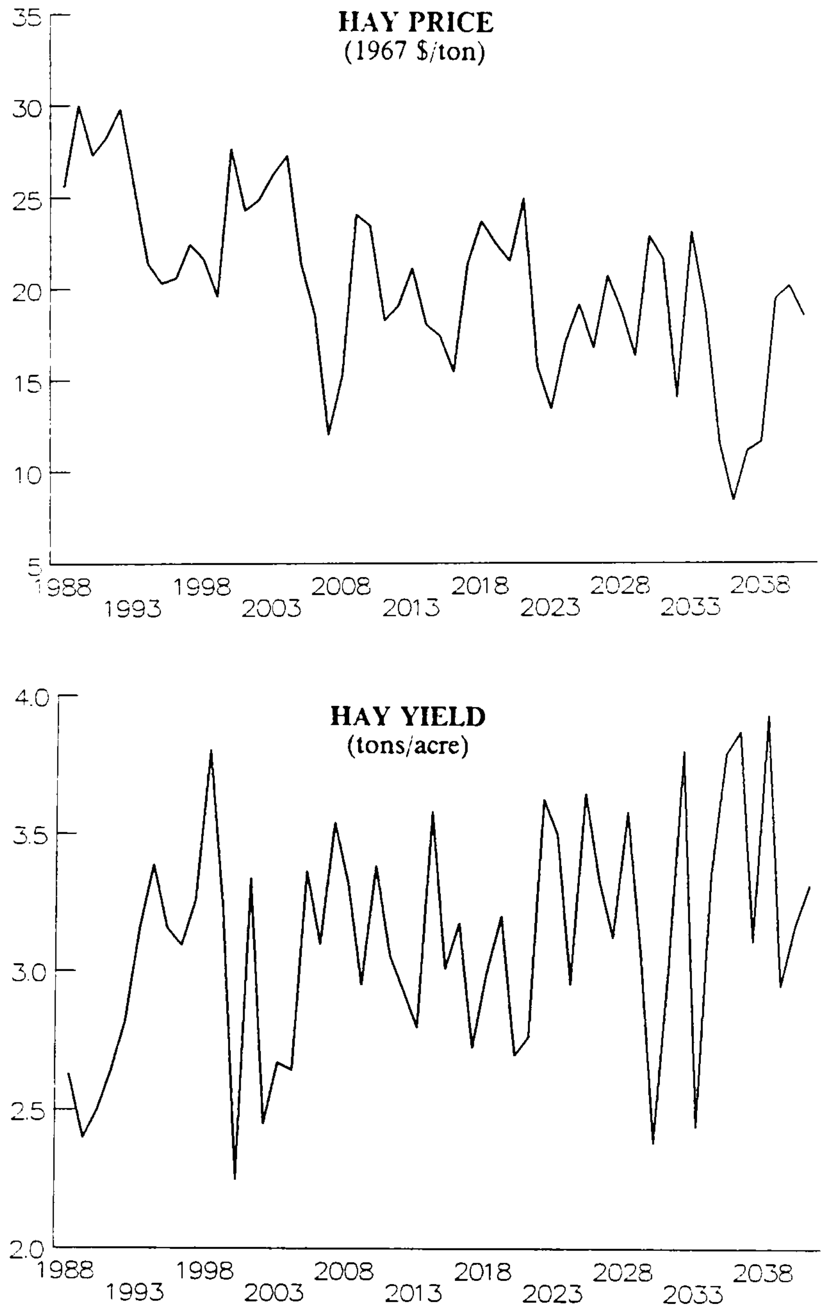

Figure 10. Example of the simulated outcomes for hay prices and hay yields (1st 54 years of projection).

COINCIDENT INDEX
(1967 = 100)

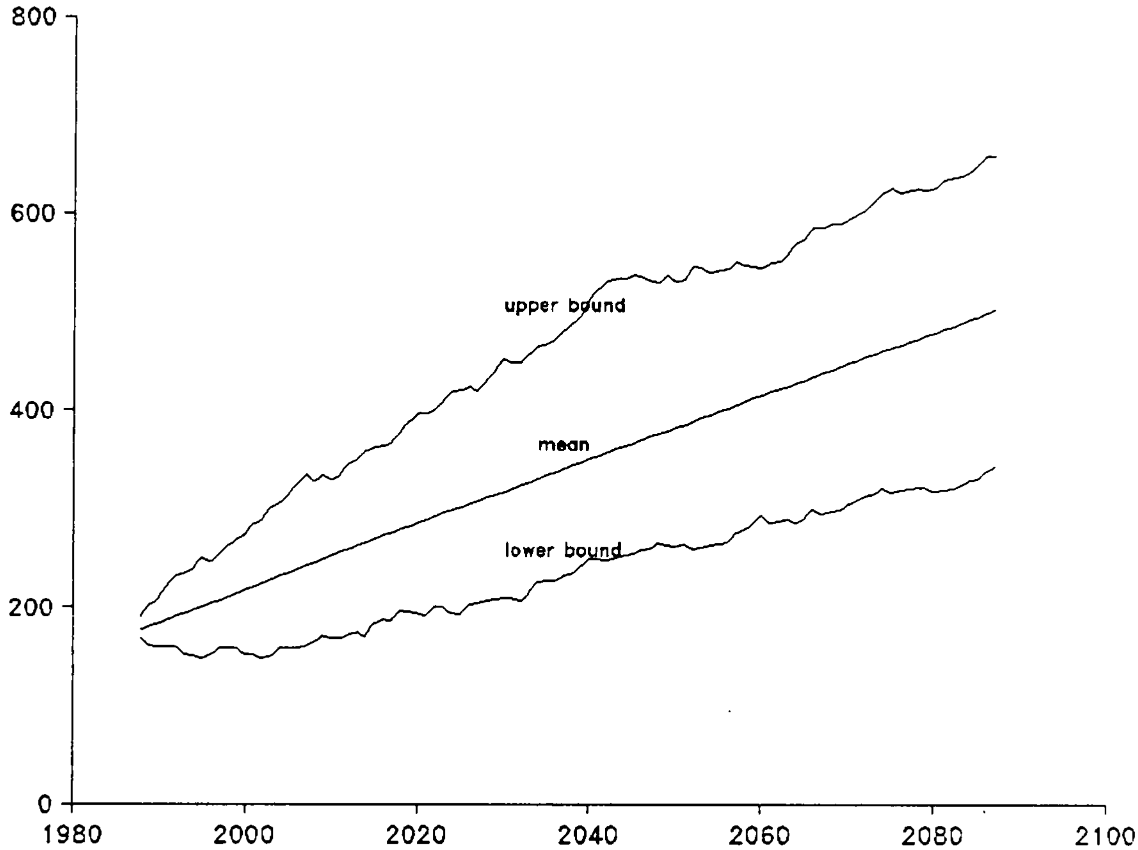

Figure 11. Lower bound, mean value, and upper bound of simulated outcomes for the coincident index (1st 100 years of projection).

SAWTIMBER STUMPAGE PRICE
(1967 $/(thousand board feet))

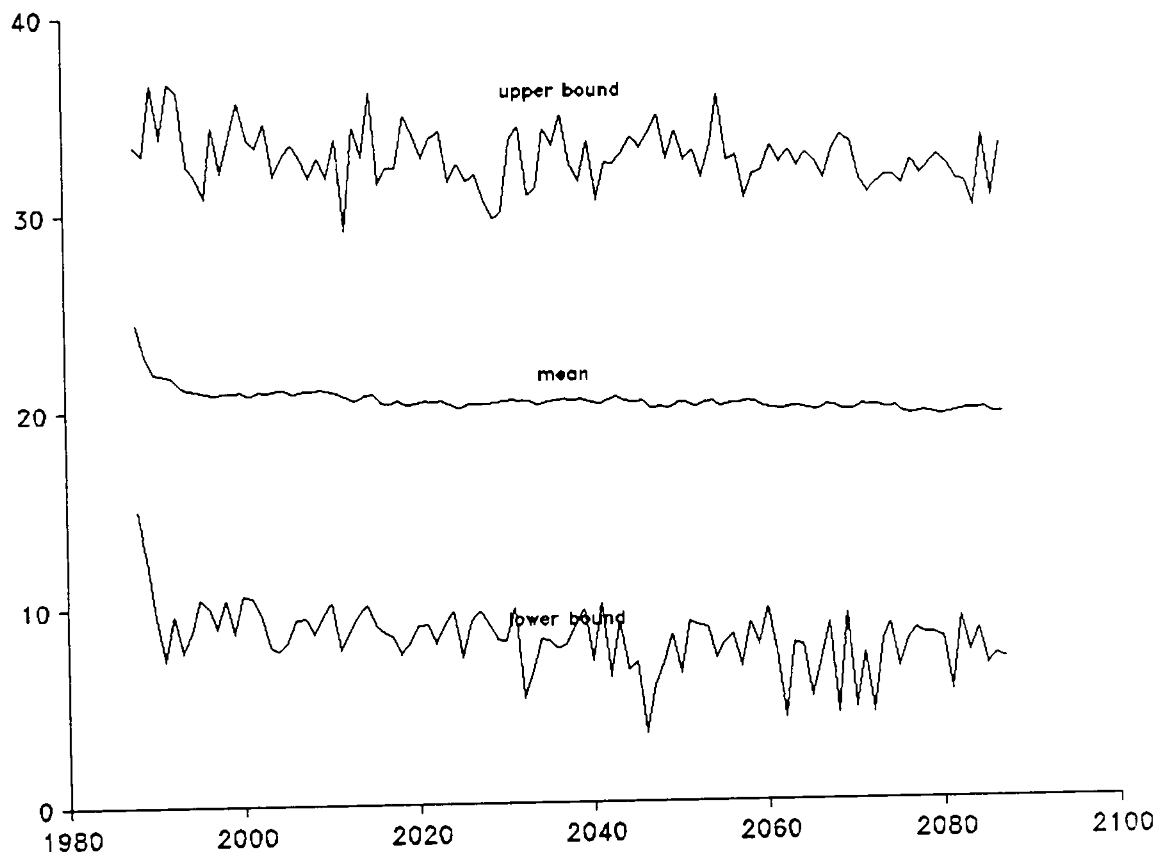

Figure 12. Lower bound, mean value, and upper bound of simulated outcomes for sawtimber stumpage price (1st 100 years of projection).

PULPWOOD STUMPAGE PRICE
(1967 $/cord)

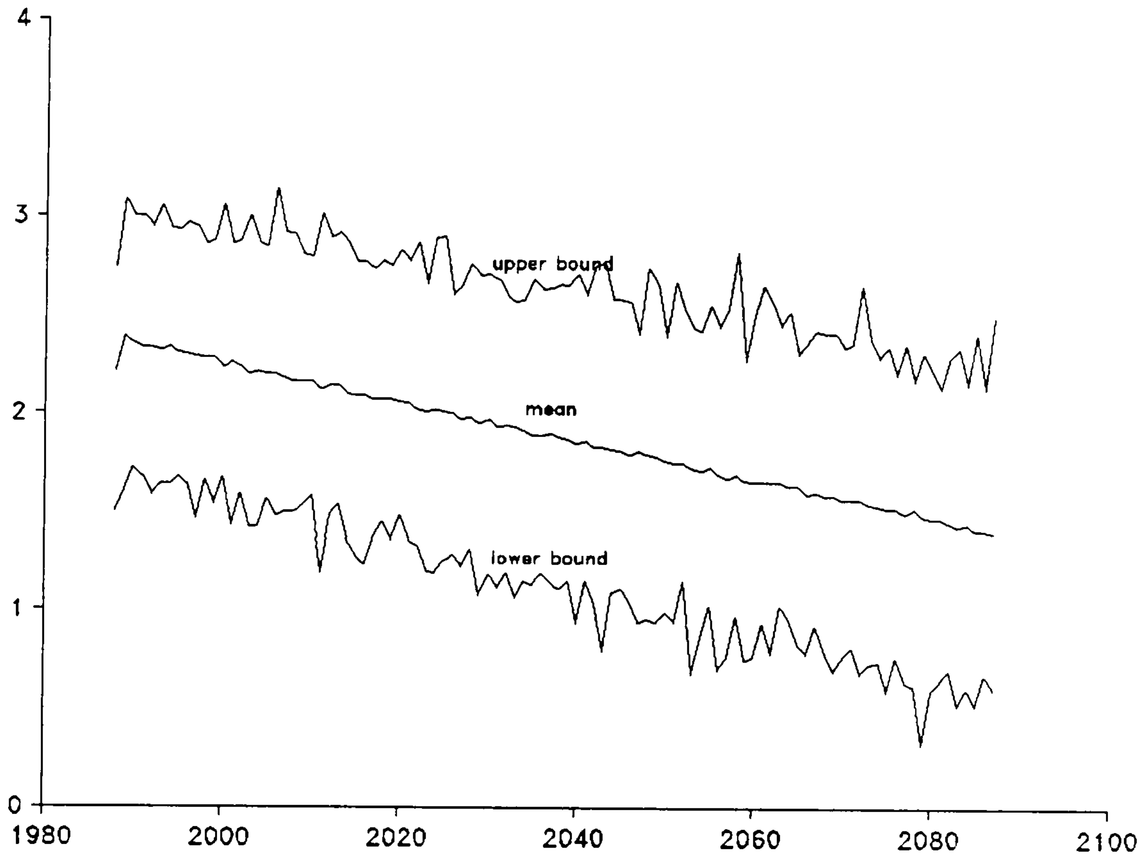

Figure 13. Lower bound, mean value, and upper bound of simulated outcomes for pulpwood stumpage price (1st 100 years of projection).

HAY PRICE
(1967 $/ton)

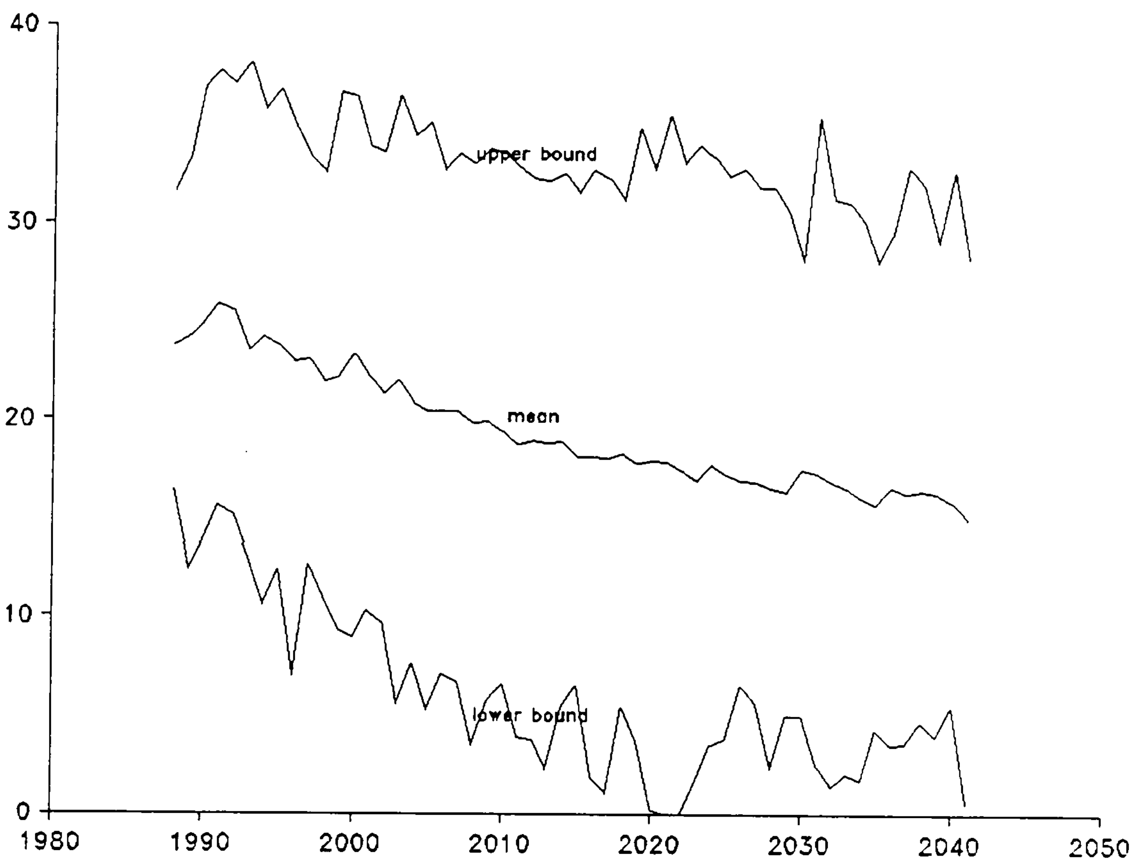

Figure 14. Lower bound, mean value, and upper bound of simulated outcomes for hay price (1st 54 years of projection).

HAY YIELD
(tons/acre)

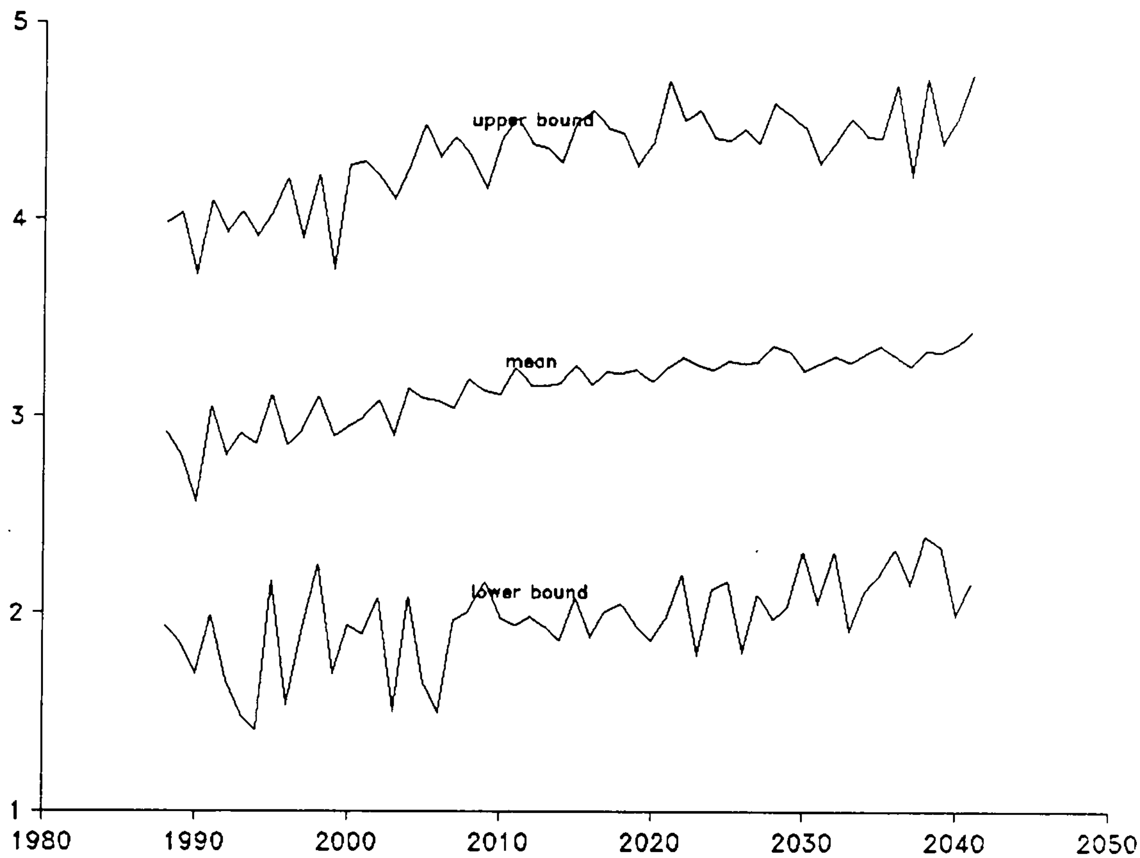

Figure 15. Lower bound, mean value, and upper bound of simulated outcomes for hay yield (1st 100 years of projection).

(i.e, 25 or 30 years) into each simulation.[163] In contrast, the other timber investments (alternatives A, B, C, and E) almost never ended under land abandonment. In this respect, the hay investment was under-valued relative to the sawtimber investments. Accordingly, the riskiness of the hay investment was also enhanced relative to the riskiness of timber investments. However, this was not considered to be a problem in as much as those simulations signaling poor performance could be considered as another proxy for the volatile markets hay producers typically face. But if future markets are stable such that the historical price level can be sustained (on the average), the hay investment would be that much more attractive. However, historically real hay prices have followed a slight downward trend (appendix D).

These results point to another problem in coping with uncertainties; modelling error in itself may add additional uncertainty to the analysis. However, for this study's purposes the error should not be considered distracting. Even if the estimated VAR model was truly stationary, risk-adjusted discount rate bias in handling the modelled risk would still occur. That is, if the hay alternative were dropped from the analysis, the conclusions concerning the use of risk-adjusted discount rates in stand level investment evaluation would still follow from the RADR approach's inability to effectively handle the valuation and ranking of timber investments, namely alternatives A, B, C, and E.

163 However, the poor performance of the pulpwood investment (see appendix I) cannot be entirely attributed to the tendency to simulate a downward trend in pulpwood stumpage price. For the most part, land abandonment occurred after the first two rotations (i.e., 40 years into the projection) indicating that this investment was not viable under earlier (and higher) simulated prices.

APPENDIX D

DATA USED TO ESTIMATE THE STOCHASTIC PRICE VECTOR AUTOREGRESSION MODEL

Figures 16-19 plot the historical data used in the estimation of the stochastic price VAR model. Figure 16 plots the historical date for the coincident index. In figure 17, the historical relationships between pulpwood and sawtimber stumpage price, along with the coincident index, are compared. Figure 18 plots the historical hay price data, while figure 19 plots the data for hay yield.

ECONOMIC INDEX VARIABLES

Original data monthly: 1948:1-1987:9 (i.e., September, 1987); quarterly: 1987:4-1988:3 (i.e., third quarter, 1988). Candidate index variables were:

1) Composite Index of 12 Leading Indicators (1967 = 100)
2) Composite Index of 6 Lagging Indicators (1967 = 100)
3) Composite Index of 4 Roughly Coincident Indicators (1967 = 100)

Monthly data converted to quarterly data using simple average approach in Doan and Litterman (1986).

Source: US Department of Commerce (1988). Note: 1948:1-1986:7 data taken from Citibase (1978).

SAWTIMBER AND PULPWOOD STUMPAGE PRICES

Original data in nominal dollars, monthly: 1978:1-1987:12; quarterly: 1988:1-1988:3. Monthly data converted to quarterly data using simple average approach in Doan and Litterman (1986). Series converted to real terms using Bureau of Labor Statistics annual producer price index (all commodities) (1967 = 1.0) (see documentation below). Missing observations pulpwood: quarterly, 1985:3-1986:2.

Source: Virginia southern yellow pine sawtimber stumpage price (\$/(MBF Scribner)) and southern yellow pine pulpwood stumpage price (\$/cord): State of Virginia, AREA 1 (mountain zone). Timber Mart South (Norris, 1988).

COINCIDENT INDEX
(1967 = 100)

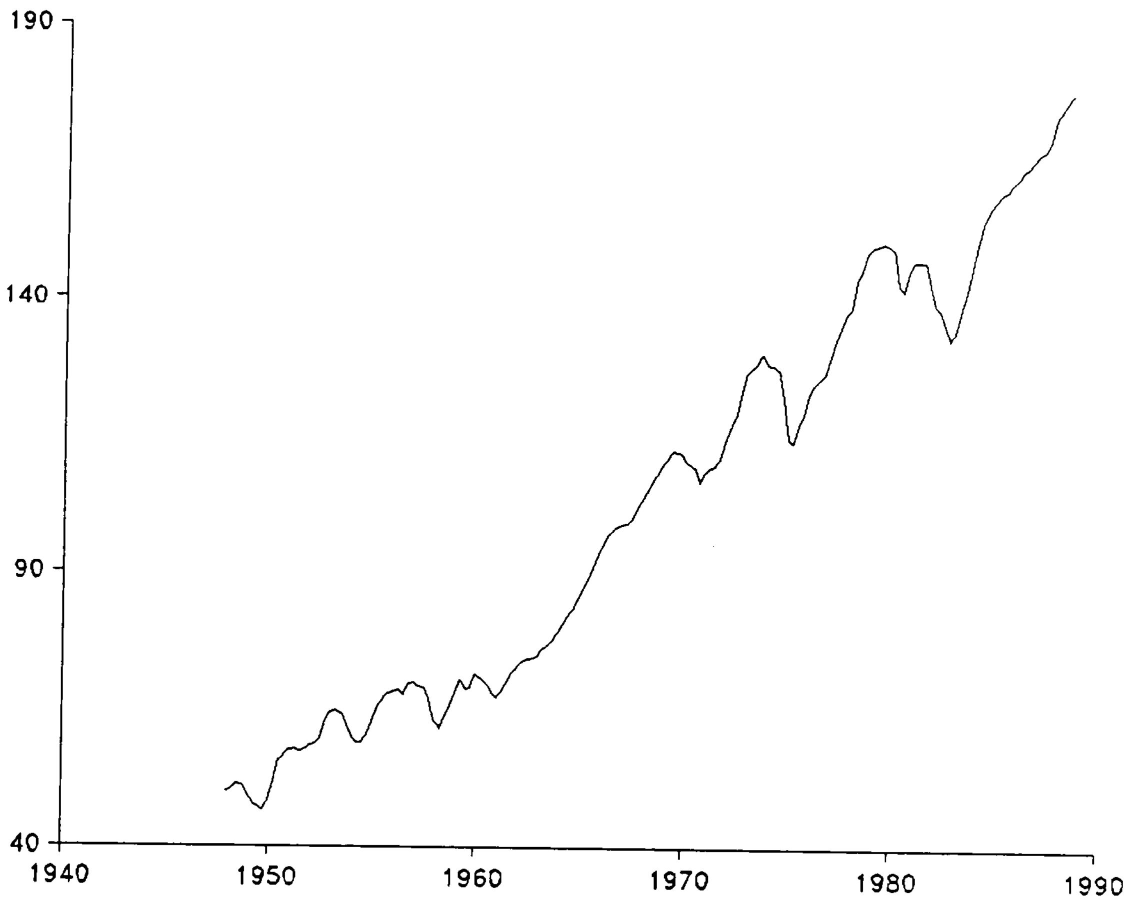

Figure 16. Historical data for the coincident index variable (quarterly, 1948-1988).

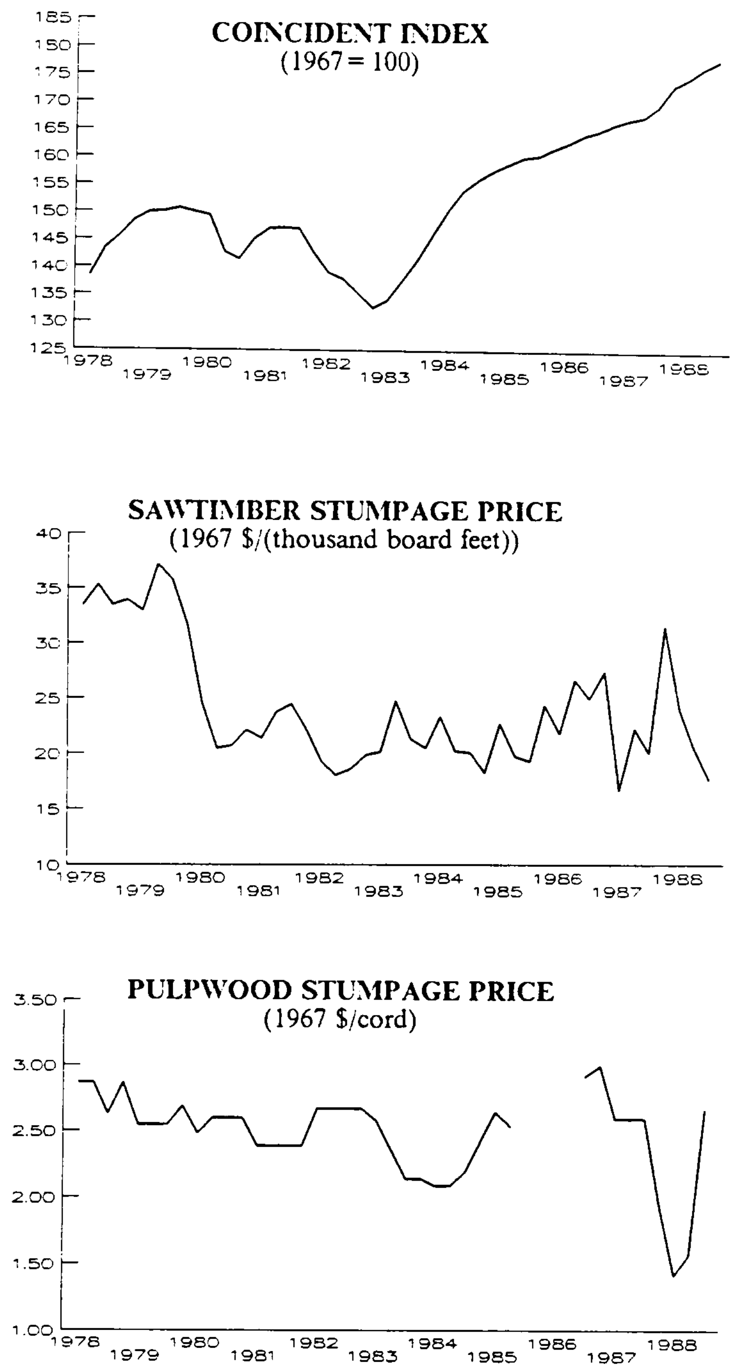

Figure 17. Historical relationships between the coincident index and sawtimber and pulpwood stumpage prices (quarterly, 1978-1988).

HAY PRICE
(1967 \$/acre)

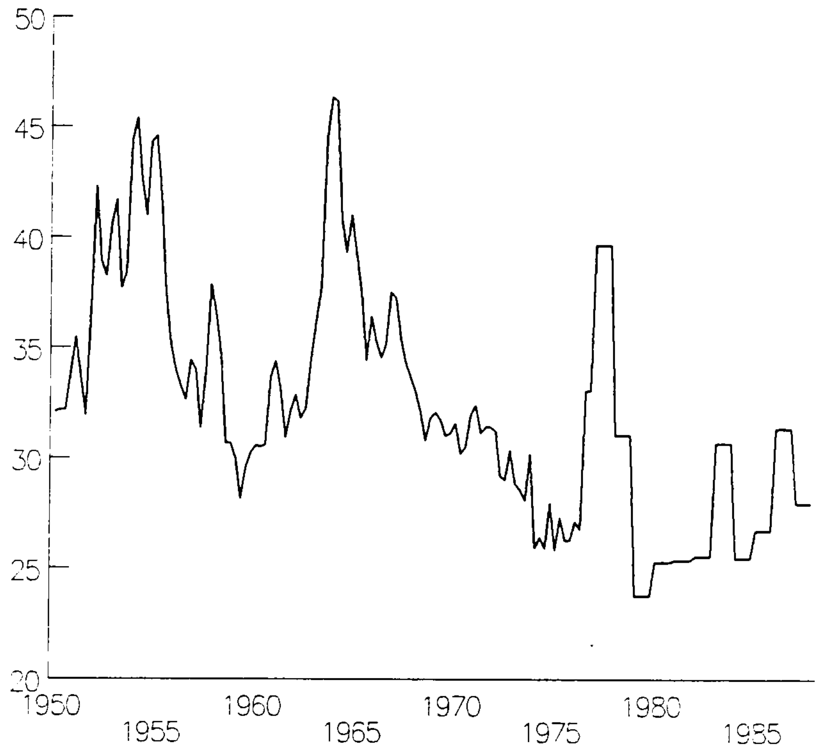

Figure 18. Historical data for hay price (quarterly, 1949-1987).

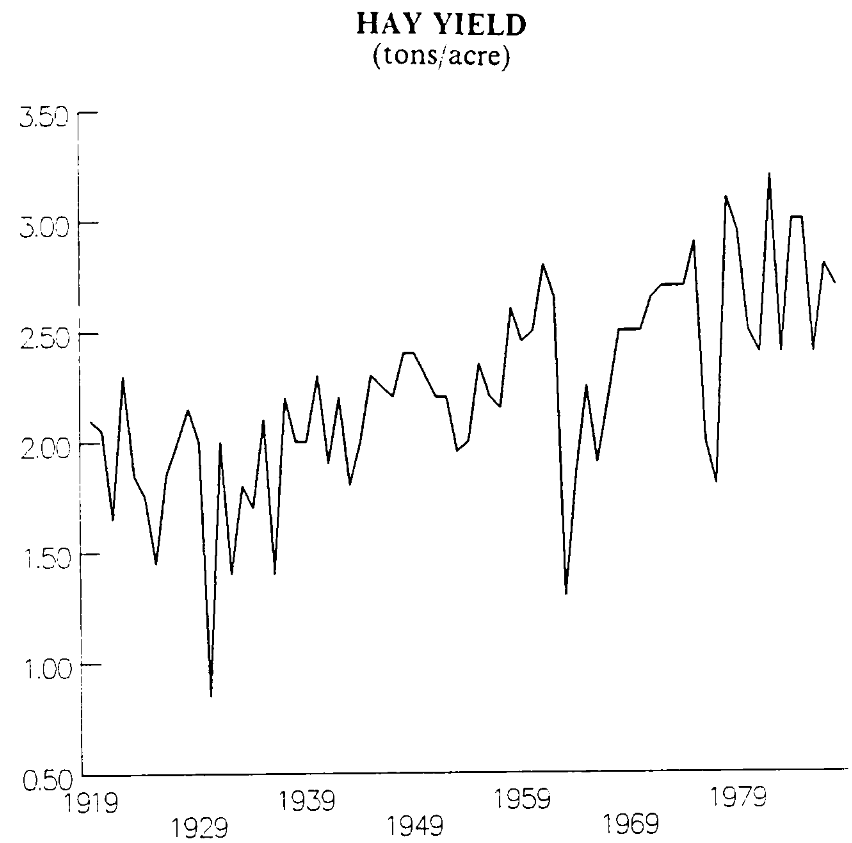

Figure 19. Historical data for hay yield (annual, 1919-1988).

HAY PRICE

Original data in nominal dollars, monthly: 1949:1-1976:4; annual (season average) 1977-1987. Monthly data converted to annual data using simple average approach in Doan and Litterman (1986). Series converted to real terms using Bureau of Labor Statistics annual producer price index (all commodities) (1967 = 1.0) (see documentation below).

Source: Virginia all hay (baled) price received by farmers ($/ton). Mellon D. 1988. Agricultural Statistician, National Agricultural Statistical Service, Washington DC. Personal communication.

HAY YIELD

Original data annual: 1919-1988 (1988 value preliminary estimate).

Source: Virginia average per acre hay yield for alfalfa and alfalfa mixtures (tons/acre). Ransom, D. 1988. Agricultural Statistician, National Agricultural Statistical Service, Washington DC. Personal communication.

PRODUCER PRICE INDEX (1967 = 1.00)

Original data annual: 1950-1985; monthly: 1986:1-1988:9. (1988 data originally (1982 = 100). Converted to 1967 = 100 using 1982 value from index (1967 = 1.0), then adjusted (1967 = 1.0)). (1986-1987 data originally (1967 = 100). Adjusted (1967 = 1.0)). Monthly data converted to annual data using simple average approach.

Source: Bureau of Labor Statistics producer price index (all commodities). Annual data (1950-1985, 1967 = 1.0) from Adams *et al.* (1988). Monthly data, 1986:1-1987:12 (1967 = 100) and 1988:1-1988:9 (1982 = 100) from US Department of Commerce (1988).

APPENDIX E

ESTIMATION OF MARKOVIAN STOCHASTIC TIMBER YIELD MODEL

DEFINING STAND VARIABILITY ON A REPRESENTATIVE ACRE

The stochastic growth and yield model used to estimate the transition probabilities and yield outcomes for timber investments was PTAEDA2 (Burkhart *et al.*, 1987). PTAEDA2 is an individual tree, distance dependent simulator for loblolly pine plantations established on recently cutover sites. In general, PTAEDA2 simulates all trees on a given plot, and then converts the volume projections to a per acre basis by dividing the results by plot size (measured in acres).[164] This promoted some problems in defining a representative acre for the timber stand.

As defined in this study, a representative acre should represent average stand conditions expressed on a per acre basis. Therefore, multiplying projected results for the representative acre by stand size gives the estimate of total yield for the entire stand. However, PTAEDA2 is a localized yield model which simulates the volume projections for a given plot in the stand. For a fixed plot size, the variability in PTAEDA2 projections represents the variability across numerous plots of this size contained in the

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164 Plot size is determined by the number of trees represented in the projection and their spacing at planting. For plantation investments, this study assumed exact spacing since this speeds up the projection time considerably. However, the custodial management projections relied on PTAEDA2's variable spacing procedure. In this case, tree location could vary by as much as 50 percent from its exact position on the spacing grid (Burkhart *et al.*, 1987). In this manner, the random spacing of seedlings in natural stands was approximated.

same stand.[165] In this context, the variability of total stand yield (and therefore, the per acre average yield) will be somewhat less than the individual plot variability since low yields on some plots will be offset by higher yields on other plots. Therefore, the direct conversion of plot yield to per acre yield in PTAEDA2 will over-estimate the variability on a representative acre for the stand.[166]

As an example, consider an even-age stand of fixed size equal to AREA, with total stand yield equal to *TY*. Therefore, the yield on a representative acre, *Y*, is given by

$$Y = \frac{TY}{\text{AREA}} \qquad \text{[E.1]}$$

Denote the variance of *TY* as σ^2_{TY}. The variance of *Y* (denoted as σ^2_Y) will then be

$$\sigma^2_Y = \left(\frac{1}{\text{AREA}}\right)^2 \sigma^2_{TY} \qquad \text{[E.2]}$$

As indicated in (E.2), the larger the area of the stand, the lower the yield variance for a representative acre. Again, this is due to the fact that larger stands have a naturally buffering effect with respect to yield variability. That is, localized areas that have poor yields can be offset by higher yields in other locations. Therefore, when converted to an average per acre basis, this localized variability washes out (Pedan *et al.*, 1973).

For simplicity, think of the stand as being made up of N independently and identically distributed (IID) (with respect to plot yields) plots of size AREA/N. Denote the variance of the plot yield as σ^2_N. Then under the IID assumption, total stand yield variability can be approximated by

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165 Reynolds, M. 1988. Associate Professor. Department of Statistics. Virginia Polytechnic Institute and State University, Blacksburg, Virginia. Personal communication.

166 I am greatly indebted to M. Reynolds, Associate Professor, Department of Statistics, Virginia Polytechnic Institute and State University for developing the following illustration.

$$\sigma_{TY}^2 = \sum_{i=1}^{N} \sigma_N^2 = N\sigma_N^2 \qquad [E.3]$$

By substituting (E.3) into (E.2), the yield variance on a representative acre under the IID assumption can be approximated by

$$\sigma_Y^2 = \left(\frac{N}{AREA^2}\right)\sigma_N^2 \qquad [E.4]$$

Now, if a single plot is repeatedly projected in PTAEDA2, with each projection directly converted to a per acre basis (by dividing the plot yield by plot size = AREA/N), the variance of these (i.e., PTAEDA2) per acre yield projections (denoted as σ_{py}^2) is

$$\begin{aligned}\sigma_{py}^2 &= \left(\frac{N}{AREA}\right)^2 \sigma_N^2 \\ &= N\sigma_Y^2 \qquad [E.5]\end{aligned}$$

by noting (E.4). The implication in (E.5) is that the estimate of per acre plot variability in PTAEDA2 over-approximates the variability for a representative acre; the smaller the plot size (i.e., the larger N), the larger the bias. Only if the entire stand was simulated in PTAEDA2 (i.e., N = 1) will the per acre conversion in PTAEDA2 result in an unbiased approximation of the variance for a representative acre.

To avoid this problem, PTAEDA2 was modified such that the per acre conversion of the simulated plot outcome was removed. With this adjustment, the yield variability from the PTAEDA2 projections estimate σ_N^2. The next step was then to determine how many plots to project independently of one another in order to represent the entire stand. For the stand size considered in this study (30 acres), 60 one half acre plots would provide a complete representation of the stand under the IID assumption underlying (E.4). However, the IID assumption is an oversimplification, since undoubtedly there is some common positive correlation in yield across all areas of the stand (e.g., due to climatic effects); especially in smaller stands.[167] If this is indeed the case, then (E.4) most likely under-approximates the yield variance for a representative acre.

The paradox is that representing the stand with a single plot over-approximates σ_y^2, while a total representation of the stand using 60 one half acre plots under-approximates σ_y^2 under the IID assumption. This suggests using $\{N^*: 1 < N^* < 60\}$ plots. This study settled on using $N^* = 20$ plots; the assumption being that the projections on each plot were independent of one another. That is, any positive correlation between plot yields was ignored. However, by using a smaller number of plots (than say using 60 plots), the variance for the representative acre is over-approximated under the IID assumption. Therefore, this over-estimation can be thought of as a proxy for the positive covariance effects ignored in this representation of the stand.

To see this, let σ_{ij} denote the covariance in yield outcomes between plot i and plot j, $(i \neq j)$ in the stand. When the correlations between plot yields are positive (e.g., $\sigma_{ij} > 0$ for all $i,j \in \{1, 2, \ldots, N\}$), then the true total stand yield variance based on a complete plot representation of the stand (i.e, $N = 60$) becomes

$$\sigma_{TY}^2 = \sum_{i=1}^{N}\sum_{j=1}^{N}\sigma_{ij} = \sum_{i=1}^{N}\sigma_i^2 + 2\left(\sum_{i=1}^{N}\sum_{\substack{j=1 \\ j\neq i}}^{N}\sigma_{ij}\right) \qquad [E.6]$$

where $\sigma_{ii} = \sigma_i^2$ denotes the variance for the ith plot. For simplicity, assume $\sum_{i=1}^{N}\sigma_i^2$ is roughly equal to $\sum_{i=1}^{N}\sigma_N^2 = N\sigma_N^2$. Then, σ_{TY}^2 as defined in (E.3) under-approximates the true value for σ_{TY}^2 in (E.6) since for $i \neq j$, $2\left(\sum_{i=1}^{N}\sum_{j=1}^{N}\sigma_{ij}\right) > 0$. Now, representing the 30 acre stand with 20 independent one half acre plots is equivalent to representing a 10 acre stand in

[167] I am indebted to Marc McDill (Graduate Research Assistant, Department of Forestry, Virginia Polytechnic Institute and State University, Blacksburg, Virginia) for emphasizing this point.

its entirety under the IID assumption. Furthermore, this representation of the 10 acre stand results in a under-approximation of the variance for a representative acre for this stand, since covariance effects are being ignored (i.e, total stand variance is under-approximated). However, from (E.2) the variance of a representative acre for the 10 acre stand will be greater than the variance of a representative acre for the 30 acre stand. Therefore, the proxy being used in this study is that the under-estimating the yield variance of a representative acre for the 10 acre stand under the IID assumption closely approximates the true yield variance of a representative acre for the 30 acre stand.

ESTIMATION OF THE MARKOV TRANSITION PROBABILITY MATRIX FOR A GIVEN PLOT

Since plot simulations formed the basis of the growth and yield projection of the stand, the Markov Chain representing stochastic timber yields was defined with respect to a single plot in the stand. In essence, all the plot simulations for a given timber alternative followed the same Markov chain. However, the estimated Markov chain was unique to each alternative considered. It is convenient to decompose the Markov Chain transition probability matrix for the even-aged stand in figure 4 by the shaded transition components. That is, within a rotation, one can think of each shaded element as a separate transition matrix defining stand state transitions from one age to another (figures 20 and 21). In figure 20, the initial state is stand establishment and the probability of going from establishment to stand state[168] $s_j^{(8)}$ at age 8 is denoted as

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168 These states were defined as (for A $\in$ {8, 12, 16, 20, ..., 52} years):

$s_1^{(A)}$ - pulpwood volume low, sawtimber volume low
$s_2^{(A)}$ - pulpwood volume low, sawtimber volume medium
$s_3^{(A)}$ - pulpwood volume low, sawtimber volume high
$s_4^{(A)}$ - pulpwood volume high, sawtimber volume low

$P_o(0, j)$ ($j \in \{1, 2, \ldots, 6\}$). Figure 21 illustrates the transition probability matrix from going from stand state $s_i^{(A)}$ at age A to stand state $s_j^{(A')}$ at age $A' = A + 4$. Let $P_A(i, j)$ denote these probabilities ($i,j \in \{1, 2, \ldots, 6\}$).[169]

The procedure underlying the estimation of the transition probabilities for each timber investment (alternatives A through E) was as follows:[170]

Step 1: Based on 50 PTAEDA2 projections, estimate the range of likely volume outcomes for pulpwood and sawtimber at each stand age considered in the projection (i.e., $A \in \{8, 12, 16, 20, \ldots, 52 \text{ years}\}$).

Step 2: From the results in Step 1, define the 6 stand states (i.e., $s_j^{(A)}$, $j \in \{1, 2, \ldots, 6\}$) applicable to each age (i.e., $A' \in \{8, 12, 16, 20, \ldots, 52 \text{ years}\}$).

Step 3: Based on 1000 PTAEDA2 projections:[171]

a) Estimate the transition probabilities based on the following relative frequency of outcomes:

$$P_o(0, j) = \left(\frac{C_o(0, j)}{1000} \right), \quad \sum_{j=1}^{6} P_o(0, j) = 1$$

$s_5^{(A)}$ - pulpwood volume high, sawtimber volume medium
$s_6^{(A)}$ - pulpwood volume high, sawtimber volume high

169 These stand state transitions were defined over the growth periods: 0-8, 8-12, 12-16, 16-20, 20-24, ..., 48-52 years. The following notation will be used to represent the beginning and ending ages of a transition period. Beginning stand ages are denoted with $A \in \{8, 12, 16, 20, \ldots, 48 \text{ years}\}$ (or stand establishment; i.e., age $A = 0$), while ending stand ages are denoted with $A' \in \{8, 12, 16, 20, \ldots, 52 \text{ years}\}$. The only exception is when one is looking back accross the transition in which case ending stand ages are denoted with $A \in \{8, 12, 16, 20, \ldots, 52 \text{ years}\}$ and beginning stand ages are denoted with $A'' \in \{8, 12, 16, 20, \ldots, 48 \text{ years}\}$ (or stand establishment; i.e., age $A'' = 0$).

170 PTAEDA2 was modified by adding subroutines that performed the above procedure.

171 Preliminary analysis indicated that 1000 PTAEDA2 projections gave reliable estimates of the transition probabilities and mean volumes. However, the estimated probabilities were still somewhat sensitive to the initial seed used to initialize the PTAEDA2 projections, especially for those transitions which occurred infrequently. For this study, the default seed value in PTAEDA2 was used to initialize the projections.

	$s_1^{(8)}$	$s_2^{(8)}$	$s_3^{(8)}$	$s_4^{(8)}$	$s_5^{(8)}$	$s_6^{(8)}$
STAND ESTABLISHMENT	$P_o(0,1)$	$P_o(0,2)$	$P_o(0,3)$	$P_o(0,4)$	$P_o(0,5)$	$P_o(0,6)$

Figure 20. Transition probability vector for the juvenile transition period (stand establishment to stand age 8 years).

$P_o(0, j)$ - probability of going from stand establishment to stand state $s_j^{(8)}$ at age 8, $j \in \{1, 2, ..., 6\}$.

Stand state definitions at age 8:

$s_1^{(8)}$ - pulpwood volume low, sawtimber volume low
$s_2^{(8)}$ - pulpwood volume low, sawtimber volume medium
$s_3^{(8)}$ - pulpwood volume low, sawtimber volume high
$s_4^{(8)}$ - pulpwood volume high, sawtimber volume low
$s_5^{(8)}$ - pulpwood volume high, sawtimber volume medium
$s_6^{(8)}$ - pulpwood volume high, sawtimber volume high

	$s_1^{(A')}$	$s_2^{(A')}$	$s_3^{(A')}$	$s_4^{(A')}$	$s_5^{(A')}$	$s_6^{(A')}$
$s_1^{(A)}$	$P_A(1,1)$	$P_A(1,2)$	$P_A(1,3)$	$P_A(1,4)$	$P_A(1,5)$	$P_A(1,6)$
$s_2^{(A)}$	$P_A(2,1)$	$P_A(2,2)$	$P_A(2,3)$	$P_A(2,4)$	$P_A(2,5)$	$P_A(2,6)$
$s_3^{(A)}$	$P_A(3,1)$	$P_A(3,2)$	$P_A(3,3)$	$P_A(3,4)$	$P_A(3,5)$	$P_A(3,6)$
$s_4^{(A)}$	$P_A(4,1)$	$P_A(4,2)$	$P_A(4,3)$	$P_A(4,4)$	$P_A(4,5)$	$P_A(4,6)$
$s_5^{(A)}$	$P_A(5,1)$	$P_A(5,2)$	$P_A(5,3)$	$P_A(5,4)$	$P_A(5,5)$	$P_A(5,6)$
$s_6^{(A)}$	$P_A(6,1)$	$P_A(6,2)$	$P_A(6,3)$	$P_A(6,4)$	$P_A(6,5)$	$P_A(6,6)$

Figure 21. Transition probability matrix for stand state transitions over the growth period: stand age A years to age A′ years.

$P_A(0, j)$ - probability of going from stand state $s_i^{(A)}$ at age A, $i \in \{1, 2, \ldots, 6\}$ to stand state $s_j^{(A')}$ at age A′, $j \in \{1, 2, \ldots, 6\}$; $A \in \{8, 16, 20, \ldots, 48\}$, $\{A'\colon A' = A + 4\}$.

Stand state definitions at age $X \in \{A, A'\}$:

$s_1^{(X)}$ - pulpwood volume low, sawtimber volume low
$s_2^{(X)}$ - pulpwood volume low, sawtimber volume medium
$s_3^{(X)}$ - pulpwood volume low, sawtimber volume high
$s_4^{(X)}$ - pulpwood volume high, sawtimber volume low
$s_5^{(X)}$ - pulpwood volume high, sawtimber volume medium
$s_6^{(X)}$ - pulpwood volume high, sawtimber volume high

$$P_A(i, j) \; = \; \left(\frac{C_A(i, j)}{NUM_A(i)} \right); \quad \sum_{j=1}^{6} P_A(i, j) = 1, \quad \text{for all } i \in \{1,\; 2,\; \ldots,\; 6\}$$

where:

$C_o(0,j)$ - counts the number of times the stand went from stand establishment to stand state $s_j^{(8)}$ ($j \in \{1, 2, \ldots, 6\}$) at age 8 years. Note: $\sum_{j=1}^{6} C_o(0,j) = 1000$.

$C_A(i,j)$ - counts the number of times the stand went from stand state $s_i^{(A)}$ at age $A \in \{8, 12, 16, 20, \ldots, 48\}$ to stand state $s_j^{(A')}$ at age $A' = A + 4$ ($i,j \in \{1, 2, \ldots, 6\}$). Note: $\sum_{j=1}^{6}\sum_{i=1}^{6} C_A(i,j) = 1000$ for all $A \in \{8, 12, 16, 20, \ldots, 48\}$.

$NUM_A(i)$ - counts the number of times the stand reached stand state $s_i^{(A)}$ ($i \in \{1, 2, \ldots, 6\}$) at age $A \in \{8, 12, 16, 20, \ldots, 48\}$. Note: $NUM_8(i) = C_o(0,i)$ and $NUM_A(i) = \sum_{j=1}^{6} C_{A''}(j,i)$ for all $i \in \{1, 2, \ldots, 6\}$; for all $A \in \{12, 16, 20, \ldots, 48\}$ $(A'' = A - 4)$. Furthermore, $\sum_{i=1}^{6} NUM_A(i) = 1000$ for all $A \in \{8, 12, 16, \ldots, 48\}$.

$P_o(0,j)$ - is the transition probability of going from stand establishment to stand state $s_j^{(8)}$ ($j \in \{1, 2, \ldots, 6\}$) at age 8 years.

$P_A(i,j)$ - is the transition probability of going from stand state $s_i^{(A)}$ at age $A \in \{8, 12, 16, 20, \ldots, 48\}$ to stand state $s_j^{(A')}$ at age $A' = A + 4$ ($i,j \in \{1, 2, \ldots, 6\}$).

b) Estimate the mean pulpwood and sawtimber volume outcomes for state transitions $S_o(0, j)$ and $S_A(i, j)$ as:

$$\overline{V}_o^{(pw)}(0, j) \; = \; \left(\frac{\sum_{\{h \rightarrow C_o(0,j)\}} V_o^{(pw)}(0,j,h)}{C_o(0, j)} \right)$$

$$\overline{V}_o^{(sa)}(0, j) \; = \; \left(\frac{\sum_{\{h \rightarrow C_o(0,j)\}} V_o^{(sa)}(0,j,h)}{C_o(0, j)} \right)$$

$$\overline{V}_A^{(pw)}(i,j) = \left(\frac{\sum_{\{h \to C_A(i,j)\}} V_A^{(pw)}(i,j,h)}{C_A(i,j)} \right)$$

$$\overline{V}_A^{(sa)}(i,j) = \left(\frac{\sum_{\{h \to C_A(i,j)\}} V_A^{(sa)}(i,j,h)}{C_A(i,j)} \right)$$

where:

$\{h \to C_o(0,j)\}$ - denotes that the outcome from simulation h increments $C_o(0,j)$ (i.e., the state transition was from stand establishment to stand state $s_j^{(8)}$ ($j \in \{1, 2, \ldots, 6\}$) at age 8 years).

$\{h \to C_A(i,j)\}$ - denotes that the outcome from simulation h increments $C_A(i,j)$ (i.e., the state transition was from stand state $s_i^{(A)}$ at age $A \in \{8, 12, 16, 20, \ldots, 48\}$ to stand state $s_j^{(A')}$ at age $A' = A + 4$ ($i,j \in \{1, 2, \ldots, 6\}$)).

$S_o(0,j)$ - denotes the state transition from stand establishment to stand state $s_j^{(8)}$ ($j \in \{1, 2, \ldots, 6\}$) at age 8 years.

$S_A(i,j)$ - denotes the state transition from stand state $s_i^{(A)}$ at age $A \in \{8, 12, 16, 20, \ldots, 48\}$ to stand state $s_j^{(A')}$ at age $A' = A + 4$ ($i,j \in \{1, 2, \ldots, 6\}$).

$V_o^{(pw)}(0,j,h)$ - denotes the PTAEDA2 pulpwood volume outcome at stand age 8 years for each simulation $h \to C_o(0,j)$; that is, for each simulation in which state transition $S_o(0,j)$ occurred ($j \in \{1, 2, \ldots, 6\}$).

$V_o^{(sa)}(0,j,h)$ - denotes the PTAEDA2 sawtimber volume outcome at stand age 8 years for each simulation $h \to C_o(0,j)$; that is, for each simulation in which state transition $S_o(0,j)$ occurred ($j \in \{1, 2, \ldots, 6\}$).

$V_A^{(pw)}(i,j,h)$ - denotes the PTAEDA2 pulpwood volume outcome at stand age $A' \in \{12, 16, 20, \ldots, 52\}$ for each simulation $h \to C_A(i,j)$; that is, for each simulation in which state transition $S_A(i,j)$ occurred ($i,j \in \{1, 2, \ldots, 6\}$).

$V_A^{(sa)}(i,j,h)$ - denotes the PTAEDA2 sawtimber volume outcome at stand age $A' \in \{12, 16, 20, \ldots, 52\}$ for each simulation $h \to C_A(i,j)$; that is, for each simulation in which state transition $S_A(i,j)$ occurred ($i,j \in \{1, 2, \ldots, 6\}$).

$\overline{V}_0^{(pw)}(0,j)$ - is the mean pulpwood volume estimate at stand age 8 for state transitions $S_0(0,j)$, $j \in \{1, 2, \ldots, 6\}$.

$\overline{V}_0^{(sa)}(0,j)$ - is the mean sawtimber volume estimate at stand age 8 for state transitions $S_0(0,j)$, $j \in \{1, 2, \ldots, 6\}$.

$\overline{V}_A^{(pw)}(i,j)$ - is the mean pulpwood volume estimate at stand age $A' \in \{12, 16, 20, \ldots, 52\}$ for state transitions $S_A(i,j)$, $i,j \in \{1, 2, \ldots, 6\}$ $(A = A' - 4)$.

$\overline{V}_A^{(sa)}(i,j)$ - is the mean sawtimber volume estimate at stand age $A' \in \{12, 16, 20, \ldots, 52\}$ for state transitions $S_A(i,j)$, $i,j \in \{1, 2, \ldots, 6\}$ $(A = A' - 4)$.

Pulpwood volume units were in cords, while sawtimber volume units were in board feet, Scribner scale; except for the case for alternatives D and E (pulpwood and custodial management investments respectively) in which case the sawtimber volume was also in cords.[172]

Each PTAEDA2 projection ended at stand age 52 years, the maximum candidate rotation age considered. In essence, under shorter rotation ages, the transition matrices and mean volume estimates pertaining to higher stand ages than the rotation age being considered were ignored.

Thinning Removal Volumes

No modifications were necessary to estimate the Markov Chain for thinned stands. However in this case, the range of likely volume outcomes (Step 1) for stand ages subsequent to the age at thinning were based on the projection of the thinned stand. Thinnings could be scheduled anywhere in the projection; however each transition period

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172 All volumes were based on the simulated plot yield (one half acre in size); that is not converted to a per acre basis.

could only contain one thinning.[173] If the thinning coincided with a stand age defining a transition boundary (i.e., at thinning ages $B \in \{8, 12, 16, 20, \ldots, 48 \text{ years}\}$), then the thinning was considered to occur at the beginning of the next transition. For example, if the thinning was scheduled to occur at age $B = 20$ years, then it was considered to occur during the transition period covering the projection of the stand from age 20 to age 24 years.

The estimated mean pulpwood and sawtimber removal volumes (for thinnings occurring under stand state transitions $S_A(i,j)$, $A \in \{8, 12, 16, 20, \ldots, 48\}$) were calculated as follows:

$$\overline{RV}_A^{(pw)}(B{:}\ i,j) = \left(\frac{\sum_{\{h \rightarrow C_A(i,j)\}} RV_A^{(pw)}(B{:}\ i,j,h)}{C_A(i,j)} \right)$$

$$\overline{RV}_A^{(sa)}(B{:}\ i,j) = \left(\frac{\sum_{\{h \rightarrow C_A(i,j)\}} RV_A^{(sa)}(B{:}\ i,j,h)}{C_A(i,j)} \right)$$

where:

B - denotes the thinning age (years).

$RV_A^{(pw)}(B{:}\ i,j,h)$ - denotes the PTAEDA2 pulpwood removal volume outcome at stand age $B \in \{A, A+1, \ldots, A+3\}$ ($A \in \{8, 12, 16, 20, \ldots, 48\}$) for each simulation $h \rightarrow C_A(i,j)$; that is, for each simulation in which state transition $S_A(i,j)$ occurred ($i,j \in \{1, 2, \ldots, 6\}$).

$RV_A^{(sa)}(B{:}\ i,j,h)$ - denotes the PTAEDA2 sawtimber removal volume outcome at stand age $B \in \{A, A+1, \ldots, A+3\}$ ($A \in \{8, 12, 16, 20, \ldots, 48\}$) for each simulation $h \rightarrow C_A(i,j)$; that is, for each simulation in which state transition $S_A(i,j)$ occurred ($i,j \in \{1, 2, \ldots, 6\}$).

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[173] PTAEDA2 does not allow any thinning to occur during the juvenile growth phase of the projection (i.e., from stand establishment to stand age 8 years). Furthermore, no thinning could occur at the end of the projection (i.e., stand age = 52 years).

$\overline{RV}_A^{(pw)}(B\colon i,j)$ - is the mean pulpwood removal volume estimate at stand age $B \in \{A, A+1, \dots, A+3\}$ ($A \in \{8, 12, 16, 20, \dots, 48\}$) for state transitions $S_A(i,j)$, $i,j \in \{1, 2, \dots, 6\}$.

$\overline{RV}_A^{(sa)}(B\colon i,j)$ - is the mean sawtimber removal volume estimate at stand age $B \in \{A, A+1, \dots, A+3\}$ ($A \in \{8, 12, 16, 20, \dots, 48\}$) for state transitions $S_A(i,j)$, $i,j \in \{1, 2, \dots, 6\}$.

SIMULATING OFF THE MARKOV CHAIN

At the beginning of each rotation in the simulation, the starting state for each plot was stand establishment.[174] Given stand establishment, the first step of the Markov Chain simulation was to determine to which state $s_j^{(8)}$ ($j \in \{1, 2, \dots, 6\}$) the plot reached at age 8 years. The transition probability vector defining these transitions (figure 20) represents a discrete probability density (i.e., mass) function for transitions to state $s_j^{(8)}$ (i.e., j random, now denoted as j). Therefore, the procedure was to generate a uniform (0,1) random variate z and find j satisfying (Law and Kelton, 1982)

$$\sum_{i=0}^{j-1} P_o(0,i) \;\le\; z \;<\; \sum_{i=0}^{j} P_o(0,i) \qquad \text{[E.7]}$$

where $P_o(0,0) \equiv 0$. That is, the plot is progressed to state $s_j^{(8)}$ ($j \in \{1, 2, \dots, 6\}$) at age 8 years if $\left\{ j\colon \sum_{i=0}^{j-1} P_o(0,i) \le z < \sum_{i=0}^{j} P_o(0,i) \right\}$.

Accordingly, each row of the transition probability matrix in figure 21 (applicable to some stand age $A \in \{8, 12, 16, 20, \dots, 48 \text{ years}\}$) defines a discrete probability

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174 For plantations, regeneration establishment was modelled exogenous to the Markov Chain simulation. For custodial management, there was a Markov Chain applicable to the stand regenerated under STATE A (high hardwood basal area, poor seed year for pine), as well as a Markov Chain applicable under STATE B (low hardwood basal area, good seed year for pine). That is, the Markov Chain used in the simulation of each custodial management rotation depended on whether the stand regenerated under STATE A or STATE B. Furthermore, all plot simulations were consistent with the assumed regeneration state, that is all plots either regenerated under STATE A or STATE B.

density function representing transitions to state $s_j^{(A')}$ ($j \in \{1, 2, \ldots, 6\}$) at stand age $A' = A + 4$ years, given that the plot is in state $s_i^{(A)}$ at stand age A years, where i defines the appropriate row vector in the transition matrix. Therefore, if the previous simulated transition progressed the plot to state $s_{i^*}^{(A)}$ ($i^* \in \{1, 2, \ldots, 6\}$) at age $A \in \{8, 12, 16, 20, \ldots, 48 \text{ years}\}$, the next transition was simulated by generating a new uniform (0,1) random variate z' and finding j satisfying

$$\sum_{i=0}^{j-1} P_A(i^*, i) \leq z' < \sum_{i=0}^{j} P_A(i^*, i) \qquad \text{[E.8]}$$

where $P_A(i^*,0) \equiv 0$ for all $i^* \in \{1, 2, \ldots, 6\}$ and $A \in \{8, 12, 16, 20, \ldots, 48\}$. Again, the result is to progress the plot to state $s_j^{(A')}$ ($j \in \{1, 2, \ldots, 6\}$) at age $A' = A + 4$ years if $\left\{j: \sum_{i=0}^{j-1} P_A(i^*, i) \leq z' < \sum_{i=0}^{j} P_A(i^*, i)\right\}$.

The simulation is continued in this manner until A equals the rotation age being considered.[175] At this point the stand (i.e., all plots) was assumed to be clearcut and the estimated mean plot yields for pulpwood and sawtimber stored based on the last simulated transition. That is, if the last simulated state transition was $S_{(A'')}(i^*, j^*)$ (where $A'' = A - 4$, $A \in \{8, 12, 16, 20, \ldots, 52\}$ equals the rotation age) then the applicable yield outcomes for the plot correspond to $\overline{V}_{A''}^{(pw)}(i^*, j^*)$ and $\overline{V}_{A''}^{(sa)}(i^*, j^*)$ for pulpwood and sawtimber respectively.

Accordingly, if the state transition $S_{(A'')}(i^*, j^*)$ (ending at age $A \in \{12, 16, 20, \ldots, 52\}$, $A'' = A - 4$) covered a period containing a thinning (i.e., $B \in \{A'', A'' + 1, \ldots, A'' + 3\}$) then the applicable plot removal volume outcomes would be given by $\overline{RV}_{A''}^{(pw)}(B: i^*, j^*)$ and $\overline{RV}_{A''}^{(sa)}(B: i^*, j^*)$ for pulpwood and sawtimber respectively.

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175 Note that the candidate rotation (i.e., harvest) age for the stand must coincide with an age $A' \in \{8, 12, 16, 20, \ldots, 52 \text{ years}\}$ defining the end of a transition period.

Conversion to Per Acre Yield Outcomes

For every rotation considered in the investment's cash flow, the Markov Chain simulation was performed independently for each of the 20 plots considered. In essence, the simulated yield (and removal volume) outcome for each rotation in the cash flow was based on the 20 plot simulations. Per acre pulpwood and sawtimber yields (i.e., the yield outcome for the representative acre for the stand) were calculated by summing up the plot yield outcomes for each product and dividing by the total area represented by all plots (e.g., 20 one half acre plots represented 10 acres). Furthermore, sawtimber yields from the sawtimber investments (alternatives A through C) were converted to MBF/acre. These values were then stored for the purpose of determining the revenue outcome for the harvest.

The procedure was the same for thinning removal volumes. That is, the simulated removal volume outcomes for pulpwood and sawtimber where summed by plot and then converted to a per acre basis (sawtimber in MBF/acre) for the purpose of determining the revenue outcome for the thinning.

APPENDIX F

EMPIRICAL ANALYSIS RESULTS

ALTERNATIVE A

Sawtimber Investment
500 trees/acre
No Thinning

ROTATION AGE DETERMINATION

The candidate rotation ages for alternative A were 28, 32, 36, 40, 44, 48, and 52 years. Table 20 summarizes the results of the stochastic timber yield model. While each rotation age was treated as a separate alternative in the empirical analysis, the yield results were not noticeably different from the results for the longest rotation age considered (52 years, table 20). Also shown in table 20 are the results of the establishment failure model for each rotation age, as well as the average number of rotations (representing the infinite time horizon cash flow) per simulation. Furthermore, only one simulation ended under the the assumption of land abandonment (rotation age = 52 years, table 20).

Table 21 summarizes the rotation age determination for the investment under the expected utility analysis. The preferred rotation age, which maximizes expected utility (i.e., maximizes NPV_{CE}), was 32 years regardless of the assumptions underlying the specification of the utility of wealth function. More importantly, assuming risk neutrality led to the same preferred rotation age as for the case under risk aversion. Table 22 summarizes the rotation age determination under the risk-adjusted discount rate approach (however, investment costs are discounted with the risk free rate). For the

Table 20. Timber yield summary, Alternative A: Sawtimber, 500 trees/acre, no thinning.

STAND AGE (years)	PULPWOOD VOLUME (5-7" dbh(1) class) (cords/acre)			SAWTIMBER VOLUME (8" dbh and up) (MBF/acre)(2)		
	LB(3)	MV(5)	UB(5)	LB	MV	UB
8	1.81	1.96	2.09	0.00	0.00	0.00
12	7.16	7.44	7.69	0.00	0.00	0.00
16	8.33	8.74	9.25	0.29	0.38	0.46
20	8.40	8.83	9.31	1.84	2.08	2.26
24	7.76	8.11	8.56	3.71	3.98	4.24
28	6.49	6.94	7.35	5.35	5.70	6.02
32	5.32	5.74	6.23	6.65	7.09	7.48
36	4.24	4.63	5.16	7.70	8.14	8.56
40	3.35	3.72	4.12	8.32	8.82	9.30
44	2.56	2.96	3.30	8.65	9.20	9.78
48	2.04	2.33	2.66	8.81	9.38	9.97
52	1.56	1.81	2.18	8.73	9.40	10.07

	ROTATION AGE (years)						
	28	32	36	40	44	48	52
SKPLNT(6)	1316	1227	1135	1047	1001	904	869
TNROT(7)	3928	3660	3434	3191	3027	2819	2688
AVPLNT(8)	1.335	1.335	1.330	1.328	1.331	1.321	1.323
AVNROT(9)	7.86	7.32	6.87	6.38	6.05	5.64	5.38
LABMNT(10)	0	0	0	0	0	0	1

Notes:

(1)dbh - diameter breast height (inches).

(2)MBF - thousand board feet, Scribner.

(3)LB - lowest simulated outcome.

(4)MV - mean value of simulated outcomes.

(5)UB - highest simulated outcome.

(6)SKPLNT - number of planting failures (all simulated rotations). Note: the total number of planting attempts is SKPLNT+TNROT.

(7)TNROT - total number of simulated rotations.

(8)AVPLNT - average number of plantings per rotation.

(9)AVNROT - average number rotations per simulation.

(10)LABMNT- indicates the number of simulations (out of 500) ending under the land abandonment assumption.

Table 21. Certainty equivalent net present value results by rotation age, Alternative A: Sawtimber, 500 trees/acre, no thinning.

	MEAN NET PRESENT VALUE(1) (1967 $/acre)						
	ROTATION AGE (years)						
	28	32	36	40	44	48	52
RF(2) = 0.02	61.63	72.27**	71.22	66.03	54.96	45.97	33.11
RF = 0.03	18.89	22.65**	19.95	15.47	7.98	2.45	-5.64
RF = 0.04	-2.79	-1.96**	-4.86	-8.69	-13.98	-17.50	-23.03
	CERTAINTY EQUIVALENT NET PRESENT VALUE(3) (1967 $/acre)						
	ROTATION AGE (years)						
	28	32	36	40	44	48	52
	LOGARITHMIC UTILITY(4)						
CASE 1:							
RF = 0.02	59.14	69.95**	68.58	63.60	52.51	44.13	30.95
RF = 0.03	16.69	20.58**	17.71	13.36	6.19	0.92	-7.54
RF = 0.04	-4.78	-3.84**	-6.85	-10.61	-15.62	-18.88	-24.78
CASE 2:							
RF = 0.02	52.85	64.80**	62.12	57.63	47.53	38.38	22.35
RF = 0.03	4.16	9.66**	5.27	-0.08	-6.78	-12.38	-18.90
RF = 0.04	-18.47	-17.98**	-18.85	-18.96	-18.96	-18.96	na(5)
	NEGATIVE EXPONENTIAL UTILITY(6)						
CASE 3:							
RF = 0.02	58.48	69.16**	67.71	62.87	52.38	43.75	30.68
RF = 0.03	16.60	20.46**	17.60	13.32	6.20	0.98	-7.36
RF = 0.04	-4.62	-3.70**	-6.66	-10.38	-15.41	-18.67	-24.44
CASE 4:							
RF = 0.02	45.54	56.60**	53.57	49.69	42.19	34.52	20.48
RF = 0.03	6.50	10.72**	7.27	3.55	-1.56	-5.64	-15.32
RF = 0.04	-13.90	-11.77**	-14.97	-18.35	-21.91	-24.15	-31.19

Notes:

(1) From the empirical net present value probability distribution estimation. Represents risk neutral case.

(2) RF - risk free discount rate.

(3) From the expected utility analysis results.

(4) CASE 1: Low degree of absolute risk aversion (see appendix B).
CASE 2: High degree of absolute risk aversion (see appendix B).

(5) na - not available, expected net present value loss exceeds wealth endowment of 18.96 1967 $/acre.

(6) CASE 3: Low degree of relative risk aversion (see appendix B).
CASE 4: High degree of relative risk aversion (see appendix B).

** Denotes the preferred rotation age.

Table 22. Mean risk-adjusted net present value results by rotation age, Alternative A (costs discounted with risk free rate).

INVESTMENT COSTS DISCOUNTED WITH THE RISK FREE RATE

MEAN RISK-ADJUSTED NET PRESENT VALUE[1]
(1967 $/acre)

	ROTATION AGE (years)						
	28	32	36	40	44	48	52
RF[2] = 0.02							
RA[3] = 0.020[4]	61.63	72.27**	71.22	66.03	54.96	45.97	33.11
RA = 0.025	21.74	30.01**	29.07	25.65	17.50	11.31	2.00
RA = 0.030	-5.50	1.50**	0.96	-1.11	-7.07	-11.24	-18.05
RA = 0.035	-25.00	-18.70**	-18.78	-19.77	-24.02	-26.65	-31.64
RA = 0.040	-39.50	-33.59	-33.17**	-33.27	-36.16	-37.59	-41.18
RF = 0.03							
RA = 0.030[4]	18.89	22.65**	19.95	15.47	7.98	2.45	-5.64
RA = 0.035	-0.62	2.44**	0.22	-3.18	-8.97	-12.97	-19.22
RA = 0.040	-15.12	-12.44**	-14.17	-16.69	-21.11	-23.90	-28.76
RA = 0.045	-26.22	-23.73**	-24.97	-26.74	-30.04	-31.86	-35.63
RA = 0.050	-34.91	-32.49**	-33.26	-34.38	-36.76	-37.77	-40.67
RF = 0.04							
RA = 0.040[4]	-2.79	-1.96**	-4.86	-8.69	-13.98	-17.50	-23.03
RA = 0.045	-13.89	-13.24**	-15.66	-18.74	-22.92	-25.46	-29.90
RA = 0.050	-22.58	-22.00**	-23.95	-26.38	-29.64	-31.37	-34.94
RA = 0.055	-29.51	-28.91**	-30.42	-32.29	-34.76	-35.83	-38.69
RA = 0.060	-35.12	-34.45**	-35.54	-36.91	-38.72	-39.23	-41.51

Notes:

(1) From the risk-adjusted discount rate analysis, investment costs discounted with the risk free rate.

(2) RF - risk free discount rate (also used to discount investment costs).

(3) RA - risk-adjusted discount rate (only applied to the expected revenue cash flow).

(4) Represents the risk neutral case. Mean risk-adjusted net present value equals the expected net present value from the empirical net present value probability distribution.

** Denotes the preferred rotation age.

Alternative A: Sawtimber, 500 trees/acre, no thinning.

most part, the same preferred rotation age of 32 years was identified regardless of the *a priori* specification of the RADR. Discounting the investment costs with the risk-adjusted discount rate also led to the same result (i.e., preferred rotation age of 32 years), with the exception that for higher rates (i.e., 4.5 percent and higher) a shorter preferred rotation age was identified (table 23).

EMPIRICAL NET PRESENT VALUE DISTRIBUTION

Table 24 summarizes the empirical net present value probability distribution results for the preferred rotation age of 32 years. As expected, the mean value declined the higher the value of the specified risk free rate. However, relative variability (as measured by the coefficient of variation) and relative skewness increased for higher risk free discount rates. This is because the investment was cost sensitive, therefore using a higher risk free rate lowers the mean relative to the variance. Figures 22-24 diagram the empirical NPV probability distribution for the three risk free discount rates considered.

EXPECTED UTILITY ANALYSIS RESULTS

Table 25 details the expected utility analysis results for the preferred rotation of 32 years. While the expected utility values for each utility case considered are not directly comparable in the absolute sense, they show that the majority of the expected utility value was attributed to the mean component; with minor adjustments for variance and skewness.[176] Also shown in table 25 is the net present value risk premium for each

[176] The only exception was for the low wealth position (18.96 1967 \$/acre) under logarithmic utility (CASE 2, table 25). At the 4 percent risk free discount rate, the individual is very sensitive to the expected NPV loss of -1.96 1967 \$/acre, with the variance and skewness adjustments overriding the mean component.

Table 23. **Mean risk-adjusted net present value results by rotation age, Alternative A (costs discounted with the risk-adjusted rate).**

INVESTMENT COSTS DISCOUNTED WITH THE RISK-ADJUSTED RATE

MEAN RISK-ADJUSTED NET PRESENT VALUE[1]
(1967 $/acre)

	ROTATION AGE (years)						
	28	32	36	40	44	48	52
RF[2] = 0.02							
RA[3] = 0.020[4]	61.63	72.27**	71.22	66.03	54.96	45.97	33.11
RA = 0.025	36.22	42.63**	40.44	35.61	26.57	19.57	9.52
RA = 0.030	18.89	22.65**	19.95	15.47	7.98	2.45	-5.64
RA = 0.035	6.46	8.48**	5.59	1.45	-4.83	-9.23	-15.88
RA = 0.040	-2.79	-1.96**	-4.86	-8.69	-13.98	-17.50	-23.03
RF = 0.03							
RA = 0.030[4]	18.89	22.65**	19.95	15.47	7.98	2.45	-5.64
RA = 0.035	6.46	8.48**	5.59	1.45	-4.83	-9.23	-15.88
RA = 0.040	-2.79	-1.96**	-4.86	-8.69	-13.98	-17.50	-23.03
RA = 0.045	-9.87**	-9.87	-12.69	-16.22	-20.71	-23.50	-28.16
RA = 0.050	-15.40**	-16.00	-18.68	-21.92	-25.74	-27.93	-31.90
RF = 0.04							
RA = 0.040[4]	-2.79	-1.96**	-4.86	-8.69	-13.98	-17.50	-23.03
RA = 0.045	-9.87**	-9.87	-12.69	-16.22	-20.71	-23.50	-28.16
RA = 0.050	-15.40**	-16.00	-18.68	-21.92	-25.74	-27.93	-31.90
RA = 0.055	-19.81**	-20.83	-23.34	-26.30	-29.55	-31.26	-34.65
RA = 0.060	-23.37**	-24.68	-27.01	-29.71	-32.48	-33.77	-36.70

Notes:

(1) From the risk-adjusted discount rate analysis, investment costs discounted with the risk-adjusted rate.

(2) RF - risk free discount rate.

(3) RA - risk-adjusted discount rate (applied to the entire cash flow).

(4) Represents the risk neutral case. Mean risk-adjusted net present value equals the expected net present value from the empirical net present value probability distribution.

** Denotes the preferred rotation age.

Alternative A: Sawtimber, 500 trees/acre, no thinning.

Table 24. Empirical net present value probability distribution summary, Alternative A (rotation age = 32 years).

NET PRESENT VALUE PROBABILITY DISTRIBUTION SUMMARY[1]

ROTATION AGE = 32 YEARS

	RISK FREE RATE OF DISCOUNT (percent/100)		
	0.02	0.03	0.04
MEAN	72.27	22.65	-1.96
VARIANCE	1114.32	780.41	619.06
(Standard Deviation)	(33.38)	(27.94)	(24.88)
SKEWNESS	-42000.45	-32177.87	-26230.38
LOWER BOUND[2]	-127.51	-151.34	-156.26
UPPER BOUND[3]	147.61	75.40	37.12
COEFFICIENT OF VARIATION[4]	0.4618	1.2336	12.6939
RELATIVE SKEWNESS[5]	0.4810	1.4043	15.1593

Notes:

(1) All net present value figures in 1967 \$/acre ($(\$/\text{acre})^2$ for variance, $(\$/\text{acre})^3$ for skewness).

(2) Lowest simulated outcome.

(3) Highest simulated outcome.

(4) = (standard deviation)/mean (absolute value).

(5) = $(\text{skewness})^{1/3}$/mean (absolute value).

Alternative A: Sawtimber, 500 trees/acre, no thinning.

ROTATION AGE = 32 YEARS
RISK FREE RATE OF DISCOUNT = 2 PERCENT

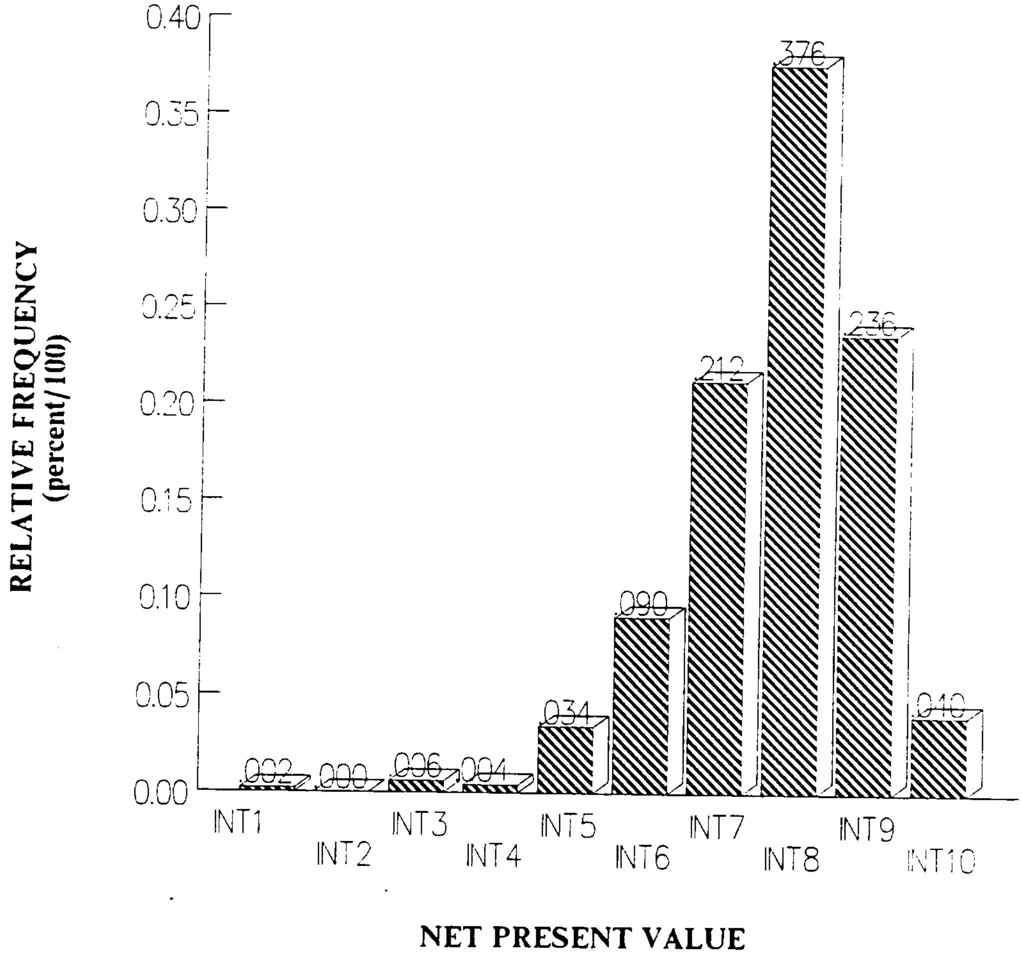

Figure 22. Empirical net present value relative frequency plot, Alternative A (2 % risk free discount rate, rotation age = 32 years).

INT1:	Between	-127.51	and	-100.00	1967 $/acre
INT2:	Between	-100.00	and	-72.48	1967 $/acre
INT3:	Between	-72.48	and	-44.97	1967 $/acre
INT4:	Between	-44.97	and	-17.46	1967 $/acre
INT5:	Between	-17.46	and	10.05	1967 $/acre
INT6:	Between	10.05	and	37.56	1967 $/acre
INT7:	Between	37.56	and	65.08	1967 $/acre
INT8:	Between	65.08	and	92.59	1967 $/acre
INT9:	Between	92.59	and	120.10	1967 $/acre
INT10:	Between	120.10	and	147.61	1967 $/acre

MEAN VALUE = 72.27 (1967 $/acre)

Alternative A: Sawtimber, 500 trees/acre, no thinning

ROTATION AGE = 32 YEARS

RISK FREE RATE OF DISCOUNT = 3 PERCENT

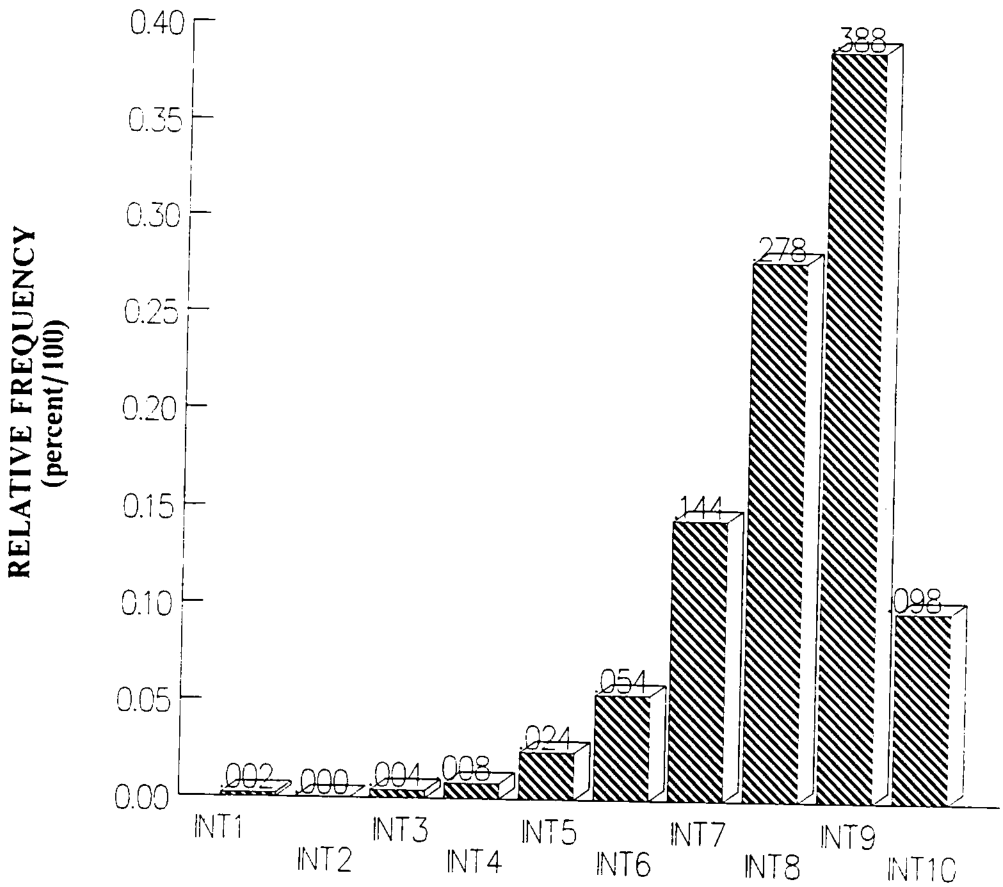

Figure 23. Empirical net present value relative frequency plot, Alternative A (3 % risk free discount rate, rotation age = 32 years).

INT1:	Between	-151.34	and	-128.26	1967 $/acre
INT2:	Between	-128.26	and	-105.99	1967 $/acre
INT3:	Between	-105.99	and	-83.32	1967 $/acre
INT4:	Between	-83.32	and	-60.64	1967 $/acre
INT5:	Between	-60.64	and	-37.97	1967 $/acre
INT6:	Between	-37.97	and	-15.30	1967 $/acre
INT7:	Between	-15.30	and	7.38	1967 $/acre
INT8:	Between	7.38	and	30.05	1967 $/acre
INT9:	Between	30.05	and	52.73	1967 $/acre
INT10:	Between	52.73	and	75.40	1967 $/acre

MEAN VALUE = 22.65 (1967 $/acre)

Alternative A: Sawtimber, 500 trees/acre, no thinning

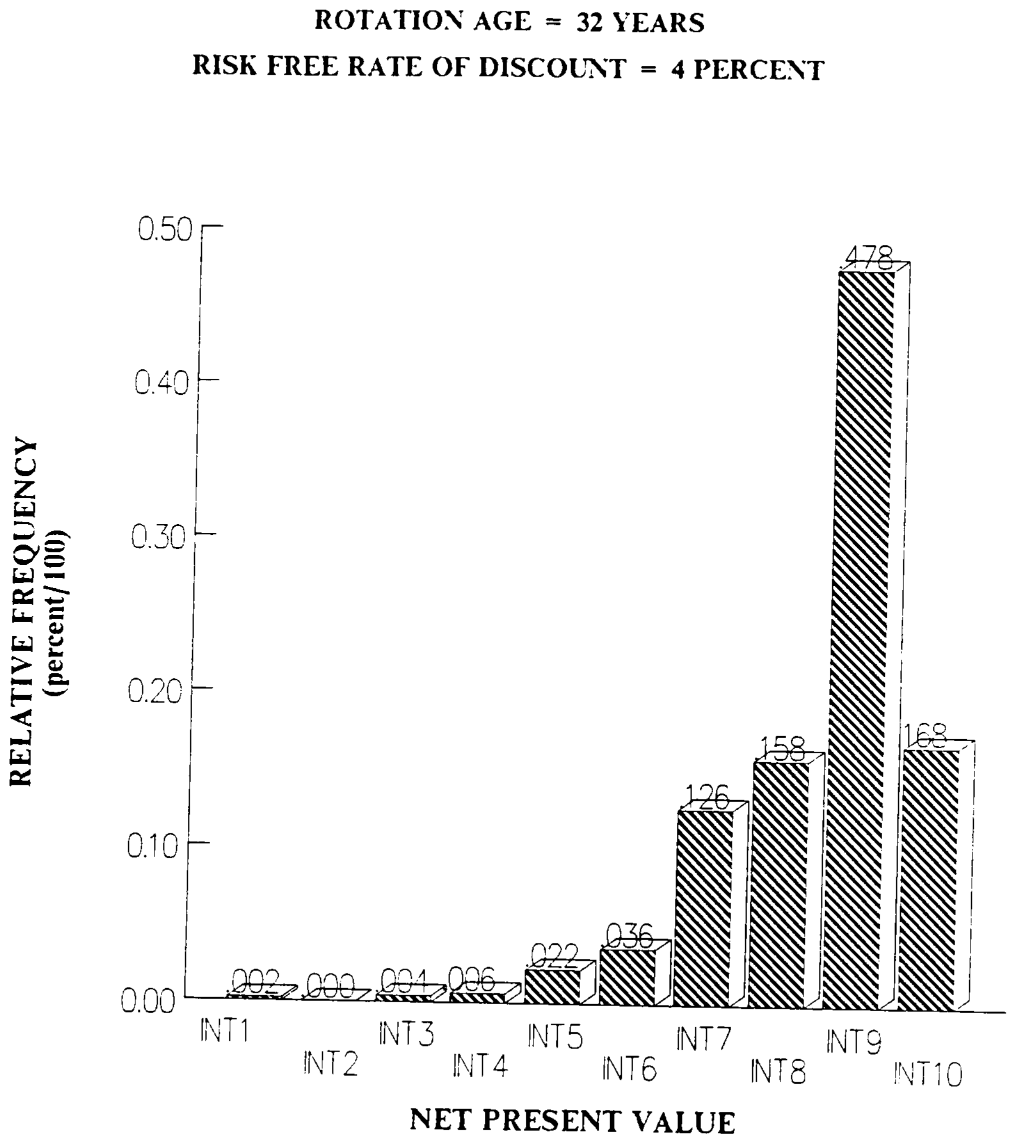

Figure 24. Empirical net present value relative frequency plot, Alternative A (4 % risk free discount rate, rotation age = 32 years).

INT1:	Between	-156.26	and	-136.92	1967 $/acre
INT2:	Between	-136.92	and	-117.58	1967 $/acre
INT3:	Between	-117.58	and	-98.25	1967 $/acre
INT4:	Between	-98.25	and	-78.91	1967 $/acre
INT5:	Between	-78.91	and	-59.57	1967 $/acre
INT6:	Between	-59.57	and	-40.23	1967 $/acre
INT7:	Between	-40.23	and	-20.89	1967 $/acre
INT8:	Between	-20.89	and	-1.55	1967 $/acre
INT9:	Between	-1.55	and	17.78	1967 $/acre
INT10:	Between	17.78	and	37.12	1967 $/acre

MEAN VALUE = -1.96 (1967 $/acre)

Alternative A: Sawtimber, 500 trees/acre, no thinning

Table 25: Expected utility analysis summary, Alternative A: sawtimber, 500 trees/acre, no thinning (rotation age = 32 years).

	EXPECTED UTILITY (Total)	MEAN[1] COMP EU	VAR[2] COMP EU	SKEW[3] COMP EU	MEAN[4] NPV (1967 \$/acre)	CE[5] NPV (1967 \$/acre)	RP[6] NPV (1967 \$/acre)	CE[7] ADJUST FACTOR	VAR[8] NPV	SKEW[9] NPV
LOGARITHMIC UTILITY										
CASE 1:[10]										
RF[11] = 0.02	5.55898	5.56789	-0.00812	-0.00078	72.27	69.95	2.32	0.9679	1114	-42000
RF = 0.03	5.34803	5.35781	-0.00866	-0.00112	22.65	20.58	2.07	0.9086	780	-32178
RF = 0.04	5.22449	5.23460	-0.00879	-0.00132	-1.96	-3.84	1.88	1.9592	619	-26230
CASE 2:[12]										
RF = 0.02	4.42801	4.51338	-0.06694	-0.01844	72.27	64.80	7.47	0.8966	1114	-42000
RF = 0.03	3.35409	3.72834	-0.22537	-0.14888	22.65	9.66	12.99	0.4265	780	-32178
RF = 0.04	-0.01520	2.83348	-1.07046	-1.77822	-1.96	-17.98	16.02	9.1735	619	-26230
NEGATIVE EXPONENTIAL UTILITY										
CASE 3:[13]										
RF = 0.02	0.74456	0.74871	-0.00389	-0.00026	72.27	69.16	3.11	0.9570	1114	-42000
RF = 0.03	0.66974	0.67354	-0.00354	-0.00026	22.65	20.46	2.19	0.9033	780	-32178
RF = 0.04	0.62487	0.62831	-0.00320	-0.00024	-1.96	-3.70	1.74	1.8876	619	-26230
CASE 4:[14]										
RF = 0.02	0.99848	0.99900	-0.00039	-0.00014	72.27	56.60	15.67	0.7832	1114	-42000
RF = 0.03	0.99492	0.99629	-0.00101	-0.00036	22.65	10.72	11.93	0.4733	780	-32178
RF = 0.04	0.99081	0.99291	-0.00153	-0.00057	-1.96	-11.77	9.81	6.0051	619	-26230

Notes:

(1) Mean component of expected utility (see appendix B).
(2) Variance component of expected utility (see appendix B).
(3) Skewness component of expected utility (see appendix B).
(4) Mean net present value (NPV) for the empirical NPV (probability) distribution.
(5) Certainty equivalent net present value.
(6) Net present value risk premium.
(7) Certainty equivalent adjustment factor (see appendix A).
(8) Variance for the empirical NPV distribution (1967 \$/acre)^2.
(9) Skewness of empirical NPV distribution (1967 \$/acre)^3.
(10) Low degree of absolute risk aversion (see appendix B).
(11) Risk free discount rate (%/100).
(12) High degree of absolute risk aversion (see appendix B).
(13) Low degree of relative risk aversion (see appendix B).
(14) High degree of relative risk aversion (see appendix B).

case considered. In all cases, the premium was small in magnitude which implies a very small correct risk adjustment in the discount rate.

Comparing the certainty equivalent NPVs in table 21 with the mean-risk adjusted NPVs in tables 22 and 23 shows that including a risk premium of only 0.5 percent represents too high an adjustment for risk. For example, under the 3 percent risk free rate (rotation age = 32 years, table 21) the lowest certainty equivalent NPV (corresponding to logarithmic utility, CASE 2, table 21) is 9.66 1967 \$/acre. However, under a 3.5 *a priori* specified RADR, the mean risk-adjusted NPV is 2.44 1967 \$/acre (investment costs discounted with the 3 percent risk free rate, table 22). Furthermore, discounting the investment costs with the 3.5 percent RADR resulted in a mean risk-adjusted NPV of 8.48 1967 \$/acre (table 23, rotation age = 32 years). In this case, under-compensating for cost risk[177] offsets the over-compensation for revenue risk. However, the net result was still an over-compensation for cash flow risk.

Also shown in table 25 is the certainty equivalent adjustment factor, which represents the necessary adjustment to the expected net present value for risk (see appendix A). Since the adjustment factor is a scale free measure of risk, it is comparable across the utility cases considered. Holding the degree of absolute risk aversion constant (i.e., comparing CASES 1 and 3 in table 25), specifying the utility of wealth function as either logarithmic (CASE 1) or negative exponential (CASE 3) showed very little differences in the risk adjustment. As expected, increasing the degree of absolute risk aversion by lowering the initial wealth endowment (CASE 1 ($W_o^{(pa)} = 189.61$ 1967 \$/acre) versus CASE 2 ($W_o^{(pa)} = 18.96$ 1967 \$/acre)) or increasing the degree of relative risk

177 Recall that for uncertain costs, the risk premium in the discount rate for a risk averse individual will be negative. Therefore, applying the RADR with a positive risk premium will under-compensate for cost risk, even for the case in which investment costs are considered certain.

aversion from 1 to 5 (CASE 3 versus CASE 4 respectively) resulted in higher risk adjustments.

APPENDIX G

EMPIRICAL ANALYSIS RESULTS

ALTERNATIVE B

Sawtimber Investment
700 trees/acre
No Thinning

ROTATION AGE DETERMINATION

The candidate rotation ages for alternative B were 28, 32, 36, 40, 44, 48, and 52 years. Table 26 summarizes the results of the stochastic timber yield model. As was the case for alternative A (appendix F), the yield results for the longest rotation age considered (52 years) were reflective of the yield results for shorter rotations. When compared to alternative A, the yields from the higher planting density were noticeably higher. As was the case with alternative A, very few simulations ended under the assumption of land abandonment (table 26).

Table 27 summarizes the rotation age determination for the investment under the expected utility analysis. Unlike the case for alternative A (appendix F), the preferred rotation age differed for different assumptions regarding the utility of wealth assumptions. For the case of high relative risk aversion (CASE 4, table 27), under low risk free discount rates the preferred rotation age was 40 years versus 36 years for the other cases (including risk neutrality). However, using an *a priori* specification of the RADR identified the 36 year rotation age regardless of whether costs were discounted with the risk free rate (table 28), or discounted with the risk-adjusted rate (table 29). Finally, comparing table 27 with tables 28 and 29 indicates that including a risk premium

Table 26. **Timber yield summary, Alternative B: Sawtimber, 700 trees/acre, no thinning.**

STAND AGE (years)	PULPWOOD VOLUME (5-7" dbh[1] class) (cords/acre)			SAWTIMBER VOLUME (8" dbh and up) (MBF/acre)[2]		
	LB[3]	MV[4]	UB[5]	LB	MV	UB
8	2.03	2.18	2.34	0.00	0.00	0.00
12	8.98	9.29	9.63	0.00	0.00	0.00
16	10.97	11.44	11.87	0.17	0.24	0.31
20	10.99	11.68	12.29	1.71	1.88	2.06
24	10.50	11.08	11.71	3.71	4.00	4.28
28	9.32	9.86	10.45	5.53	5.94	6.23
32	7.85	8.38	8.83	7.17	7.61	8.00
36	6.44	6.89	7.37	8.44	8.92	9.41
40	5.06	5.56	6.15	9.38	9.86	10.35
44	4.00	4.43	4.90	9.90	10.46	11.03
48	3.12	3.48	3.89	10.21	10.82	11.48
52	2.38	2.69	3.09	10.17	10.96	11.62

	ROTATION AGE (years)						
	28	32	36	40	44	48	52
SKPLNT[6]	1223	1204	1104	1040	972	895	856
TNROT[7]	3652	3609	3319	3150	2945	2786	2632
AVPLNT[8]	1.335	1.330	1.333	1.330	1.330	1.321	1.325
AVNROT[9]	7.30	7.22	6.64	6.32	5.89	5.57	5.26
LABMNT[10]	6	1	0	0	0	3	5

Notes:

(1)dbh - diameter breast height (inches).

(2)MBF - thousand board feet, Scribner.

(3)LB - lowest simulated outcome.

(4)MV - mean value of simulated outcomes.

(5)UB - highest simulated outcome.

(6)SKPLNT - number of planting failures (all simulated rotations). Note: the total number of planting attempts is SKPLNT+TNROT.

(7)TNROT - total number of simulated rotations.

(8)AVPLNT - average number of plantings per rotation.

(9)AVNROT - average number rotations per simulation.

(10)LABMNT- indicates the number of simulations (out of 500) ending under the land abandonment assumption.

Table 27. Certainty equivalent net present value results by rotation age, Alternative B: Sawtimber, 700 trees/acre, no thinning.

	28	32	36	40	44	48	52
MEAN NET PRESENT VALUE[1] (1967 $/acre) — ROTATION AGE (years)							
RF[2] = 0.02	33.24	50.33	57.62**	53.54	45.15	35.67	20.94
RF = 0.03	-5.06	2.73	5.55**	1.07	-5.25	-11.60	-20.75
RF = 0.04	-24.65	-20.93	-19.92**	-24.09	-28.95	-33.30	-39.34
CERTAINTY EQUIVALENT NET PRESENT VALUE[3] (1967 $/acre) — ROTATION AGE (years)							
	28	32	36	40	44	48	52
LOGARITHMIC UTILITY[4]							
CASE 1:							
RF = 0.02	28.02	46.02	52.86**	49.60	41.18	31.99	16.85
RF = 0.03	-9.91	-1.22	1.12**	-2.52	-8.91	-15.09	-24.65
RF = 0.04	-29.20	-24.65	-24.07**	-27.49	-32.43	-36.68	-43.11
CASE 2:							
RF = 0.02	8.75	33.22	39.40**	38.86	28.52	18.48	-1.92
RF = 0.03	-18.96	-18.39.	-18.15**	-18.43	-18.43	-18.96	na[5]
RF = 0.04	na	na	na	na	na	na	na
NEGATIVE EXPONENTIAL UTILITY[6]							
CASE 3:							
RF = 0.02	27.49	45.15	51.77**	48.68	40.48	31.51	16.67
RF = 0.03	-9.35	-0.97	1.42**	-2.31	-8.53	-14.61	-23.88
RF = 0.04	-28.18	-23.92	-23.21**	-26.81	-31.60	-35.80	-41.97
CASE 4:							
RF = 0.02	6.12	25.59	29.26	30.88**	22.55	15.33	-0.16
RF = 0.03	-27.17	-16.55	-16.57	-16.16**	-22.62	-27.63	-37.79
RF = 0.04	-43.73	-37.24	-38.39**	-38.60	-43.59	-47.15	-54.23

Notes:

(1) From the empirical net present value probability distribution estimation. Represents risk neutral case.

(2) RF - risk free discount rate.

(3) From the expected utility analysis results.

(4) CASE 1: Low degree of absolute risk aversion (see appendix B).
CASE 2: High degree of absolute risk aversion (see appendix B).

(5) na - not available, expected net present value loss exceeds wealth endowment of 18.96 1967 $/acre.

(6) CASE 3: Low degree of relative risk aversion (see appendix B).
CASE 4: High degree of relative risk aversion (see appendix B).

** Denotes the preferred rotation age.

Table 28. **Mean risk-adjusted net present value results by rotation age, Alternative B (costs discounted with risk free rate).**

INVESTMENT COSTS DISCOUNTED WITH THE RISK FREE RATE

MEAN RISK-ADJUSTED NET PRESENT VALUE(1)
(1967 $/acre)

	ROTATION AGE (years)						
	28	32	36	40	44	48	52
RF = 0.02(2)							
RA(3) = 0.020(4)	33.24	50.33	57.62**	53.54	45.15	35.67	20.94
RA = 0.025	-8.95	4.24	11.07**	8.09	2.21	-4.41	-15.00
RA = 0.030	-38.01	-26.94	-20.16**	-22.09	-26.08	-30.53	-38.18
RA = 0.035	-58.94	-49.10	-42.17**	-43.15	-45.66	-48.43	-53.88
RA = 0.040	-74.56	-65.44	-58.26**	-58.41	-59.70	-61.13	-64.91
RF = 0.03							
RA = 0.030(4)	-5.06	2.73	5.55**	1.07	-5.25	-11.60	-20.75
RA = 0.035	-25.99	-19.43	-16.46**	-20.00	-24.83	-29.50	-36.46
RA = 0.040	-41.62	-35.77	-32.55**	-35.25	-38.87	-42.20	-47.48
RA = 0.045	-53.61	-48.17	-44.66**	-46.62	-49.23	-51.46	-55.42
RA = 0.050	-63.02	-57.80	-53.97**	-55.26	-57.02	-58.33	-61.25
RF = 0.04							
RA = 0.040(4)	-24.65	-20.93	-19.92**	-24.09	-28.95	-33.30	-39.34
RA = 0.045	-36.64	-33.33	-32.03**	-35.46	-39.31	-42.56	-47.20
RA = 0.050	-46.05	-42.96	-41.34**	-44.10	-47.10	-49.44	-53.10
RA = 0.055	-53.57	-50.57	-48.62**	-50.78	-53.05	-54.63	-57.44
RA = 0.060	-59.65	-56.66	-54.39**	-56.01	-57.65	-58.88	-60.70

Notes:

(1) From the risk-adjusted discount rate analysis, investment costs discounted with the risk free rate.

(2) RF - risk free discount rate (also used to discount investment costs).

(3) RA - risk-adjusted discount rate (only applied to the expected revenue cash flow).

(4) Represents the risk neutral case. Mean risk-adjusted net present value equals the expected net present value from the empirical net present value probability distribution.

** Denotes the preferred rotation age.

Alternative B: Sawtimber, 700 trees/acre, no thinning.

Table 29. **Mean risk-adjusted net present value results by rotation age, Alternative B (costs discounted with the risk-adjusted rate).**

INVESTMENT COSTS DISCOUNTED WITH THE RISK-ADJUSTED RATE

MEAN RISK-ADJUSTED NET PRESENT VALUE(1)
(1967 $/acre)

	ROTATION AGE (years)						
	28	32	36	40	44	48	52
RF = 0.02(2)							
RA(3) = 0.020(4)	33.24	50.33	57.62**	53.54	45.15	35.67	20.94
RA = 0.025	10.54	21.91	26.44**	21.99	14.74	7.01	-4.47
RA. = 0.030	-5.06	2.73	5.55**	1.07	-5.25	-11.60	-20.75
RA = 0.035	-16.28	-10.89	-9.17**	-13.53	-19.06	-24.31	-31.70
RA = 0.040	-24.65	-20.93	-19.92**	-24.09	-28.95	-33.30	-39.34
RF = 0.03							
RA = 0.030	-5.06	2.73	5.55**	1.07	-5.25	-11.60	-20.75
RA = 0.035	-16.28	-10.89	-9.17**	-13.53	-19.06	-24.31	-31.70
RA = 0.040	-24.65	-20.93	-19.92**	-24.09	-28.95	-33.30	-39.34
RA = 0.045	-31.06	-28.54	-27.99**	-31.94	-36.22	-39.82	-44.79
RA = 0.050	-36.08	-34.43	-34.17**	-37.88	-41.65	-44.63	-48.75
RF = 0.04							
RA = 0.040	-24.65	-20.93	-19.92**	-24.09	-28.95	-33.30	-39.34
RA = 0.045	-31.06	-28.54	-27.99**	-31.94	-36.22	-39.82	-44.79
RA = 0.050	-36.08	-34.43	-34.17**	-37.88	-41.65	-44.63	-48.75
RA = 0.055	-40.08	-39.07	-38.99**	-42.46	-45.78	-48.23	-51.66
RA = 0.060	-43.30	-42.77**	-42.78	-46.01	-48.93	-50.94	-53.81

Notes:

(1) From the risk-adjusted discount rate analysis, investment costs discounted with the risk-adjusted rate.

(2) RF - risk free discount rate.

(3) RA - risk-adjusted discount rate (applied to the entire cash flow).

(4) Represents the risk neutral case. Mean risk-adjusted net present value equals the expected net present value from the empirical net present value probability distribution.

** Denotes the preferred rotation age.

Alternative B: Sawtimber, 700 trees/acre, no thinning.

of only 0.5 percent in the discount rate represented a gross over-compensation for risk (i.e., $NPV_{RADR} < NPV_{CE}$ in all cases).

A Closer Look at Differences in the Preferred Rotation Age

As indicated above, for the case of negative exponential utility, differences in the degree of relative risk aversion led to differences in the identified rotation age when the risk free discount rate was either 2 or 3 percent. Somewhat counter intuitive, at the higher level of relative risk aversion, a longer rotation age was preferred. Table 30 and figures 25 and 26 compare the empirical NPV probability distributions for the candidate rotation ages of 36 and 40 years. In the absolute sense, both variance and skewness for the longer rotation investment (40 years) were lower than the values for the 36 rotation age investment (table 30). Therefore, the 40 year rotation age investment was still efficient in that it represented a lower mean, accompanied by a lower variance and lower (negative) skewness (in absolute value). In essence, the expected utility mean-variance and mean-skewness tradeoff adjustments under CASE 4 indicated preference for the 40 year rotation age, while under CASE 3 these adjustments led to preference for the 36 year rotation.

For example, under the 2 percent risk free discount rate (the most interesting case since the investment under either rotation age is viable)[178], the two rotation ages were similar in mean and variability (as measured by the coefficient of variation), but differed in terms of skewness. Since the two rotation ages were similar in mean and variance (RF = 0.02, table 31), the lower negative skewness for the longer rotation age (40 years) was enough to make this age preferred, in spite of the lower mean, for the more risk

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[178] Under the 3 percent risk free discount rate, the 40 year rotation age represented a viable investment for the CASE 3 individual, but was not viable to the CASE 4 individual (table 31).

Table 30. Empirical net present value probability distribution summary, Alternative B (rotation ages: 36 and 40 years).

NET PRESENT VALUE PROBABILITY DISTRIBUTION SUMMARY[1]

	RISK FREE RATE OF DISCOUNT (percent/100)			
	0.02		0.03	
	R[2] = 36	R = 40	R = 36	R = 40
MEAN	57.62	53.54	5.55	1.07
VARIANCE	2014.54	1744.03	1410.79	1203.27
(Standard Deviation)	(44.88)	(41.76)	(37.56)	(34.69)
SKEWNESS	-134728.09	-69091.68	-98536.68	-51203.93
LOWER BOUND[3]	-177.15	-114.18	-186.18	-141.75
UPPER BOUND[4]	136.36	133.16	60.00	59.85
COEFFICIENT OF VARIATION[5]	0.7789	0.7829	6.7676	32.4216
RELATIVE SKEWNESS[6]	0.8897	0.7664	8.3222	34.7043

Notes:

(1) All net present value figures in 1967 \$/acre ($(\$/\text{acre})^2$ for variance, $(\$/\text{acre})^3$ for skewness).

(2) Denotes the rotation age (years).

(3) Lowest simulated outcome.

(4) Highest simulated outcome.

(5) = (standard deviation)/mean (absolute value).

(6) = $(\text{skewness})^{1/3}$/mean (absolute value).

Alternative B: Sawtimber, 700 trees/acre, no thinning.

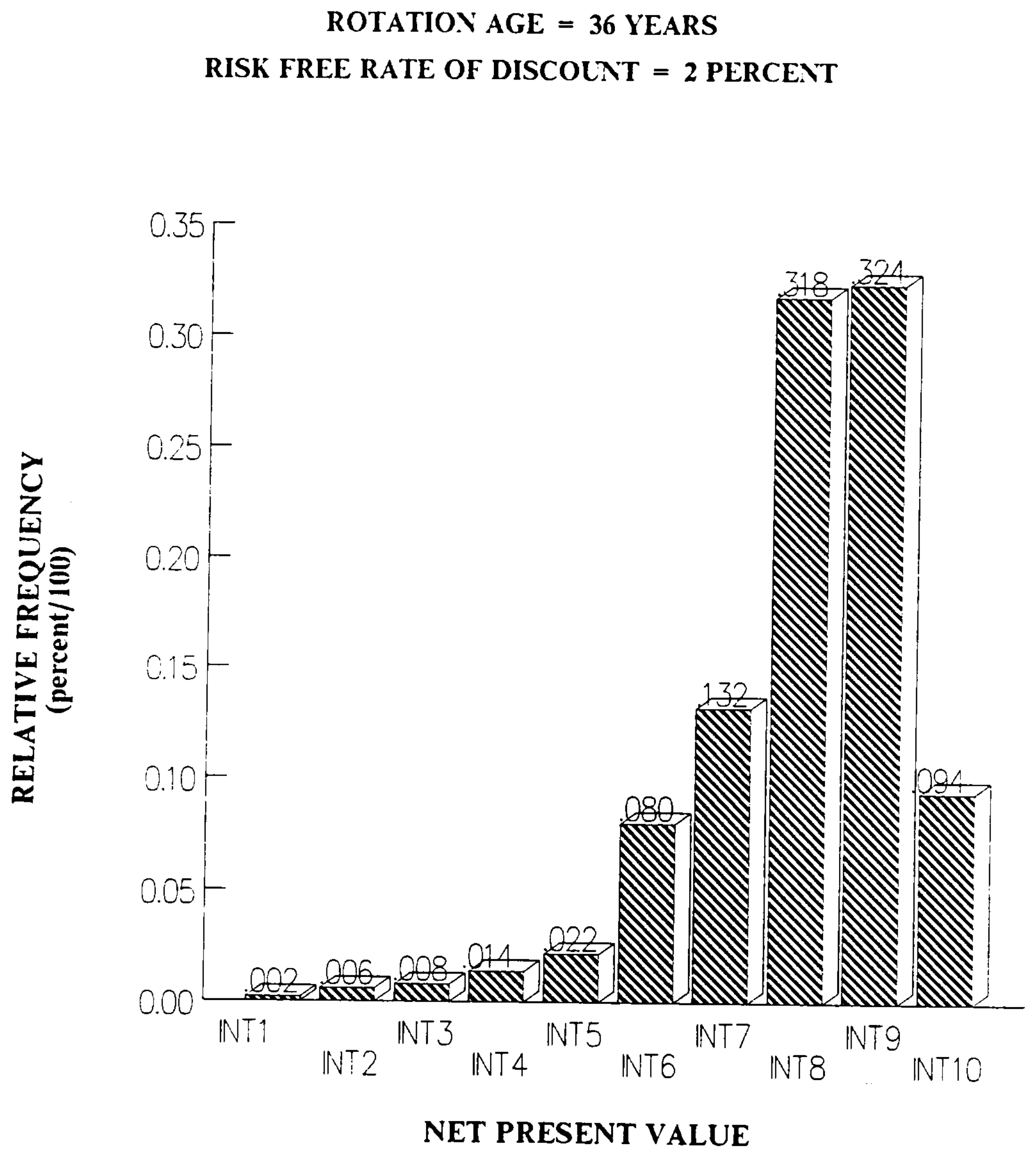

Figure 25. Empirical net present value relative frequency plot, Alternative B (2 % risk free discount rate, rotation age = 36 years).

INT1:	Between	-177.15	and	-145.80	1967 $/acre
INT2:	Between	-145.80	and	-114.45	1967 $/acre
INT3:	Between	-114.45	and	-83.10	1967 $/acre
INT4:	Between	-83.10	and	-51.74	1967 $/acre
INT5:	Between	-51.74	and	-20.39	1967 $/acre
INT6:	Between	-20.39	and	10.96	1967 $/acre
INT7:	Between	10.96	and	42.31	1967 $/acre
INT8:	Between	42.31	and	73.66	1967 $/acre
INT9:	Between	73.66	and	105.01	1967 $/acre
INT10:	Between	105.01	and	136.36	1967 $/acre

MEAN VALUE = 57.62 (1967 $/acre)

Alternative B: Sawtimber, 700 trees/acre, no thinning

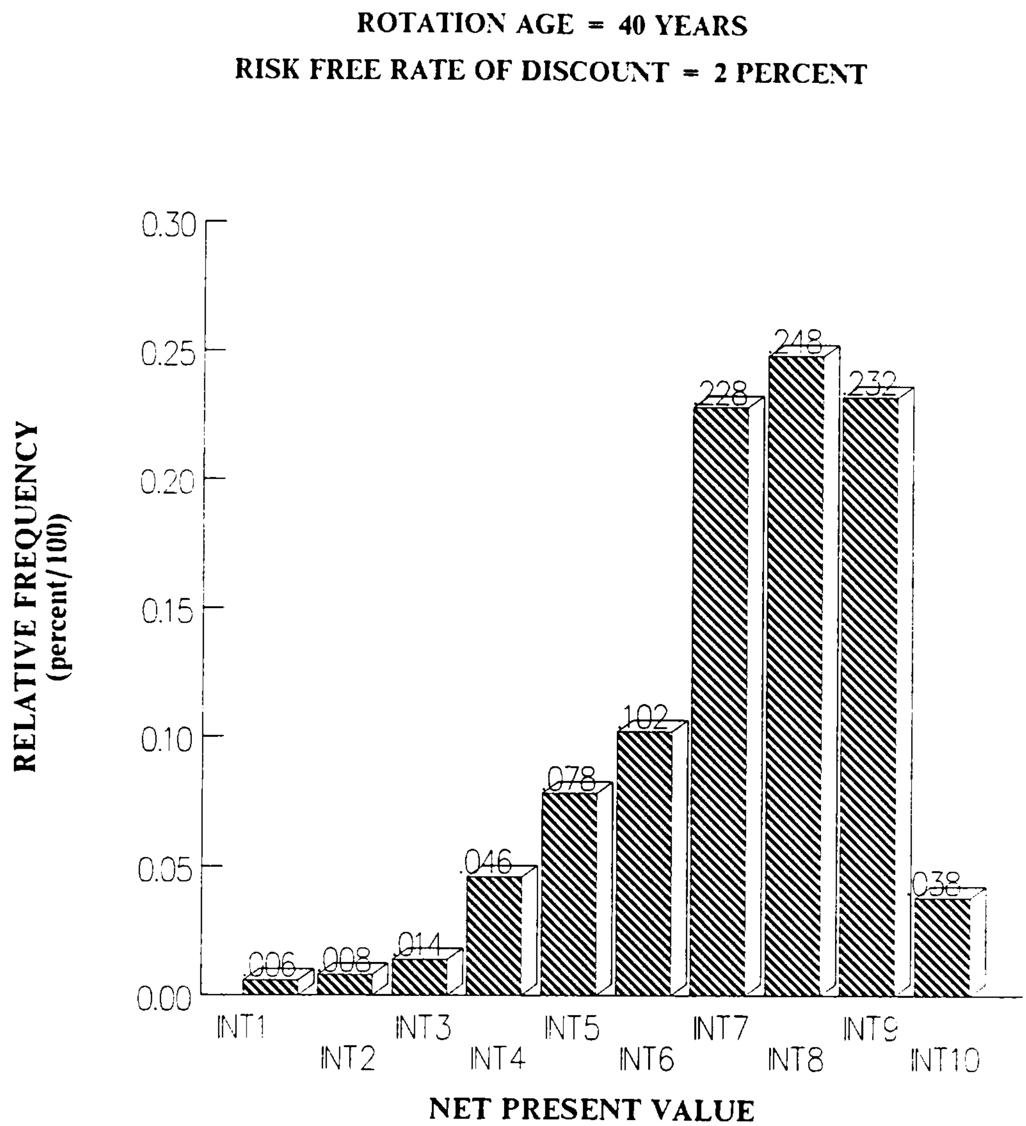

Figure 26. Empirical net present value relative frequency plot, Alternative B (2 % risk free discount rate, rotation age = 40 years).

INT1:	Between	-114.18	and	-89.45	1967 $/acre
INT2:	Between	-89.45	and	-64.71	1967 $/acre
INT3:	Between	-64.71	and	-39.38	1967 $/acre
INT4:	Between	-39.38	and	-15.24	1967 $/acre
INT5:	Between	-15.24	and	9.49	1967 $/acre
INT6:	Between	9.49	and	34.22	1967 $/acre
INT7:	Between	34.22	and	58.96	1967 $/acre
INT8:	Between	58.96	and	83.69	1967 $/acre
INT9:	Between	83.69	and	108.43	1967 $/acre
INT10:	Between	108.43	and	133.16	1967 $/acre

MEAN VALUE = 53.54 (1967 $/acre)

Alternative B: Sawtimber, 700 trees/acre, no thinning

Table 31: Expected utility analysis summary, Alternative B (negative exponential utility, rotation ages: 36 and 40 years).

	EXPECTED UTILITY (Total)	MEAN[1] COMP EU	VAR[2] COMP EU	SKEW[3] COMP EU	MEAN[4] NPV (1967 $/acre)	CE[5] NPV (1967 $/acre)	RP[6] NPV (1967 $/acre)	CE[7] ADJUST FACTOR	VAR[8] NPV	SKEW[9] NPV
					NEGATIVE EXPONENTIAL UTILITY (CASE 3)[10]					
RF[11] = 0.02										
ROT[12] = 36	0.72003	0.72853	-0.00761	-0.00089	57.62	51.77	5.85	0.8985	2015	-134728
ROT = 40	0.71543	0.72262	-0.00673	-0.00047	53.54	48.68	4.86	0.9092	1744	-69092
RF = 0.03										
ROT = 36	0.63486	0.64273	-0.00701	-0.00086	5.55	1.42	4.13	0.2559	1411	-98536
ROT = 40	0.62761	0.63419	-0.00612	-0.00046	1.07	-2.31	3.38	-2.1589	1203	-51204
					NEGATIVE EXPONENTIAL UTILITY (CASE 4)[13]					
RF = 0.02										
ROT = 36	0.99689	0.99853	-0.00103	-0.00061	57.62	29.26	28.36	0.5078	2015	-134728
ROT = 40	0.99702	0.99836	-0.00100	-0.00035	53.54	30.88	22.66	0.5768	1744	-69091
RF = 0.03										
ROT = 36	0.98957	0.99418	-0.00286	-0.00175	5.55	-16.57	22.12	-2.9856	1411	-98537
ROT = 40	0.98968	0.99345	-0.00274	-0.00103	1.07	-16.16	17.23	-15.1029	1203	-51204

Notes:

(1) Mean component of expected utility (see appendix B).
(2) Variance component of expected utility (see appendix B).
(3) Skewness component of expected utility (see appendix B).
(4) Mean net present value (NPV) for the empirical NPV (probability) distribution.
(5) Certainty equivalent net present value.
(6) Net present value risk premium.
(7) Certainty equivalent adjustment factor (see appendix A).
(8) Variance for the empirical NPV distribution (1967 $/acre)^2.
(9) Skewness for the empirical NPV distribution (1967 $/acre)^3.
(10) Low degree of relative risk aversion (see appendix B).
(11) Risk free discount rate (%/100).
(12) Rotation age (years).
(13) High degree of relative risk aversion (see appendix B).

averse individual (CASE 4). As indicated in table 31, under CASE 4 the expected utility adjustment for (negative) skewness was lower (in absolute value) for the investment under a 40 year rotation than for the investment under a 36 year rotation age. Even though the mean component of expected utility was higher for the shorter rotation age (36 years, CASE 4, table 31), the net effect was a higher level of total expected utility for the 40 year rotation. As indicated by the certainty equivalent adjustment factor, the 40 year rotation age had a lower risk adjustment (0.5768) to a smaller mean (53.54 1967 $/acre), which resulted in a smaller NPV risk premium (22.66 1967 $/acre); the net effect being a larger certainty equivalent NPV (30.88 1967 $/acre) than the 36 year rotation age investment (CASE 4, RF=0.02,table 31).[179]

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Note that the certainty equivalent adjustment factor for the 40 year rotation age need not have been lower than the adjustment factor for the 36 year rotation. This is because the certainty equivalent adjustment factor is a relative measure which does not take into account where the mean-risk tradeoffs are being made. Therefore, it is conceivable that the 40 year rotation age investment could of been relatively more risky than the 36 year rotation age investment, yet still be preferred. This was exactly the case when the risk free discount rate was 3 percent (CASE 4, table 31).

APPENDIX H

EMPIRICAL ANALYSIS RESULTS

ALTERNATIVE C

Sawtimber Investment
700 trees/acre
Thinning at age 20

ROTATION AGE DETERMINATION

The candidate rotation ages for alternative C were 28, 32, 36, 40, 44, 48, and 52 years. Table 32 summarizes the results of the stochastic timber yield model for the longest rotation age considered (52 years). These results also reflect the results for shorter rotation ages. When compared to the no thinning case (alternative B), the thinning had the desired effect of increasing the sawtimber yields. Like alternatives A and B, very few of the simulations ended under the land abandonment assumption (table 32).

Table 33 summarizes the rotation age determination under the expected utility analysis. Like the case for alternative A (appendix F), the preferred rotation age (32 years) was the same for the risk neutrality case, as well as for any of the risk averse cases considered (CASES 1-4, table 33). While the rotation age of 36 years was very similar to the 32 rotation age in terms of the mean NPV (table 34), this age was dominated by the 32 year investment. That is, increasing the rotation age from 32 to 36 years would incur a slightly lower mean accompanied by a higher variance and (negative) skewness (i.e., more risk) (table 34). This is confirmed by comparing figures 27 and 28. The 32 year rotation investment was less skewed (negatively) and had a range of outcomes

Table 32. Timber yield summary, Alternative C: Sawtimber, 700 trees/acre, thinning at age 20.

STAND AGE (years)	PULPWOOD VOLUME (5-7" dbh[1] class) (cords/acre)			SAWTIMBER VOLUME (8" dbh and up) (MBF/acre)[2]		
	LB[3]	MV[4]	UB[5]	LB	MV	UB
8	2.05	2.20	2.37	0.00	0.00	0.00
12	9.00	9.29	9.64	0.00	0.00	0.00
16	10.87	11.42	11.85	0.18	0.25	0.33
20	10.99	11.63	12.19	1.73	1.90	2.07
(Removal)	(6.27)	(6.83)	(7.35)	(0.00)	(0.00)	(0.00)
24	1.25	2.08	2.96	4.31	4.59	4.91
28	0.20	0.51	0.95	6.84	7.17	7.45
32	0.03	0.09	0.26	8.76	9.05	9.39
36	0.00	0.02	0.08	10.02	10.38	10.74
40	0.00	0.00	0.03	10.84	11.34	11.75
44	0.00	0.00	0.01	11.44	12.00	12.52
48	0.00	0.00	0.00	11.87	12.44	13.10
52	0.00	0.00	0.00	12.05	12.70	13.45

	ROTATION AGE (years)						
	28	32	36	40	44	48	52
SKPLNT[6]	1295	1211	1118	1045	983	898	859
TNROT[7]	3840	3629	3360	3179	2966	2804	2648
AVPLNT[8]	1.337	1.337	1.333	1.329	1.331	1.320	1.324
AVNROT[9]	7.68	7.26	6.72	6.36	5.93	5.61	5.30
LABMNT[10]	1	0	0	0	0	0	1

Notes:

(1)dbh - diameter breast height (inches).

(2)MBF - thousand board feet, Scribner.

(3)LB - lowest simulated outcome.

(4)MV - mean value of simulated outcomes.

(5)UB - highest simulated outcome.

(6)SKPLNT - number of planting failures (all simulated rotations). Note: the total number of planting attempts is SKPLNT+TNROT.

(7)TNROT - total number of simulated rotations.

(8)AVPLNT - average number of plantings per rotation.

(9)AVNROT - average number rotations per simulation.

(10)LABMNT- indicates the number of simulations (out of 500) ending under the land abandonment assumption.

Table 33. Certainty equivalent net present value results by rotation age, Alternative C: Sawtimber, 700 trees/acre, age 20 thinning.

	MEAN NET PRESENT VALUE[1] (1967 $/acre) ROTATION AGE (years)						
	28	32	36	40	44	48	52
RF[2] = 0.02	61.49	87.21**	87.15	83.70	75.46	65.16	51.84
RF = 0.03	12.30	26.03**	22.84	18.55	13.19	6.22	-2.76
RF = 0.04	-12.70	-4.65**	-8.66	-12.98	-16.42	-21.24	-27.78

	CERTAINTY EQUIVALENT NET PRESENT VALUE[3] (1967 $/acre) ROTATION AGE (years)						
	28	32	36	40	44	48	52
	LOGARITHMIC UTILITY[4]						
CASE 1:							
RF = 0.02	56.39	83.94**	82.12	79.10	71.81	61.81	47.46
RF = 0.03	7.45	23.00**	18.12	14.17	9.86	3.11	-7.05
RF = 0.04	-17.31	-7.56**	-13.13	-17.18	-19.58	-24.26	-31.93
CASE 2:							
RF = 0.02	42.63	78.08**	72.31	70.13	64.31	53.93	34.41
RF = 0.03	-15.11	9.47**	-3.27	-6.30	-8.28	-14.99	-18.96
RF = 0.04	-18.96	-18.95**	-18.96	-18.96	-18.96	na[5]	na
	NEGATIVE EXPONENTIAL UTILITY[6]						
CASE 3:							
RF = 0.02	55.14	82.55**	80.14	77.31	70.52	60.81	46.56
RF = 0.03	7.60	22.75**	17.96	14.06	9.84	3.22	-6.59
RF = 0.04	-16.52	-7.30**	-12.50	-16.55	-19.07	-23.68	-30.90
CASE 4:							
RF = 0.02	31.08	65.99**	55.38	55.06	52.79	44.53	26.27
RF = 0.03	-12.31	9.43**	-1.88	-3.79	-4.00	-9.58	-23.27
RF = 0.04	-33.71	-18.79**	-29.29	-31.76	-30.77	-34.74	-45.33

Notes:

(1) From the empirical net present value probability distribution estimation. Represents risk neutral case.

(2) RF - risk free discount rate.

(3) From the expected utility analysis results.

(4) CASE 1: Low degree of absolute risk aversion (see appendix B).
CASE 2: High degree of absolute risk aversion (see appendix B).

(5) na - not available, expected net present value loss exceeds wealth endowment of 18.96 1967 $/acre.

(6) CASE 3: Low degree of relative risk aversion (see appendix B).
CASE 4: High degree of relative risk aversion (see appendix B).

** Denotes the preferred rotation age.

Table 34. Empirical net present value probability distribution summary, Alternative C (rotation ages: 32 and 36 years).

NET PRESENT VALUE PROBABILITY DISTRIBUTION SUMMARY[1]				
	RISK FREE RATE OF DISCOUNT (percent/100)			
	0.02		0.03	
	R[2] = 32	R = 36	R = 32	R = 36
MEAN	87.21	87.15	26.03	22.84
VARIANCE	1695.61	2443.52	1174.39	1685.30
(Standard Deviation)	(41.18)	(49.43)	(34.27)	(41.05)
SKEWNESS	-52493.17	-150939.33	-46143.11	-109070.78
LOWER BOUND[3]	-86.32	-158.95	-135.59	-174.03
UPPER BOUND[4]	182.72	180.49	92.16	88.43
COEFFICIENT OF VARIATION[5]	0.4722	0.5672	1.3166	1.7973
RELATIVE SKEWNESS[6]	0.4293	0.6109	1.3779	2.0919

Notes:

(1) All net present value figures in 1967 \$/acre ($(\$/\text{acre})^2$ for variance, $(\$/\text{acre})^3$ for skewness).

(2) Denotes the rotation age (years).

(3) Lowest simulated outcome.

(4) Highest simulated outcome.

(5) = (standard deviation)/mean (absolute value).

(6) = $(\text{skewness})^{1/3}$/mean (absolute value).

Alternative C: Sawtimber, 700 trees/acre, thinning at age 20.

ROTATION AGE = 32 YEARS
RISK FREE RATE OF DISCOUNT = 2 PERCENT

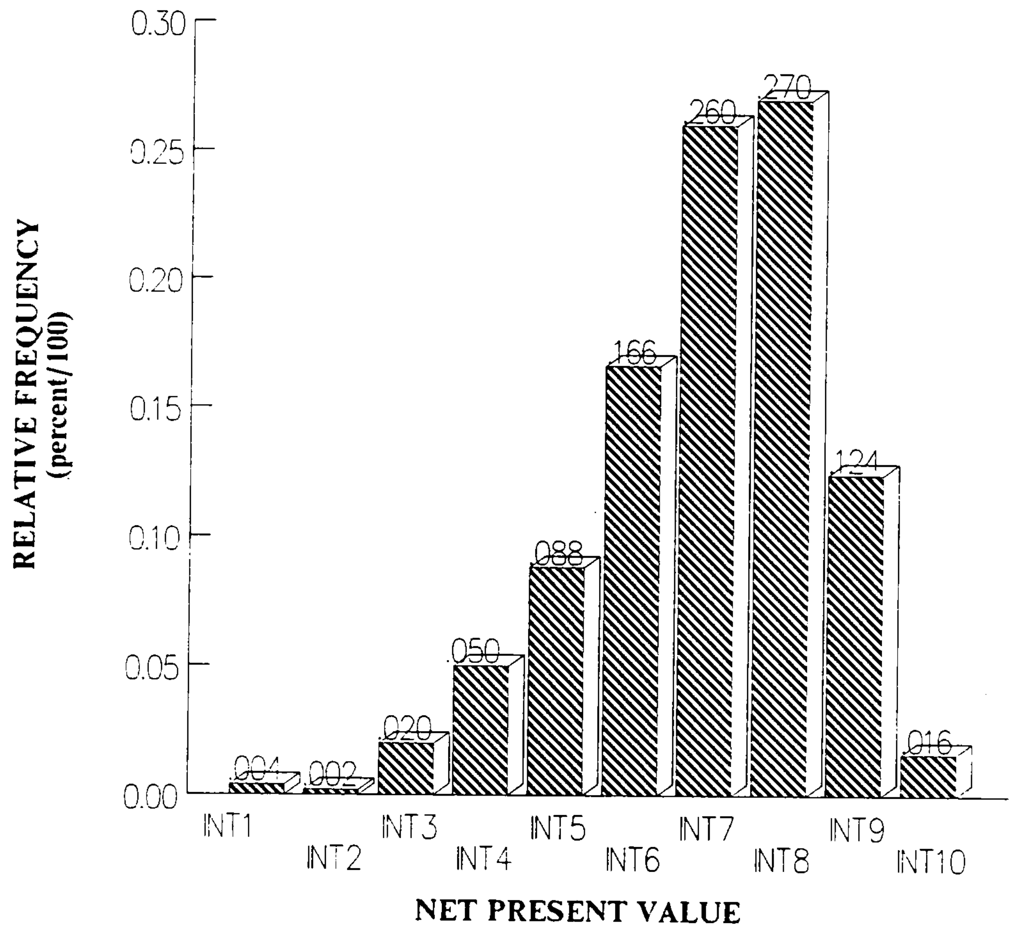

Figure 27. Empirical net present value relative frequency plot, Alternative C (2 % risk free discount rate, rotation age = 32 years).

INT1:	Between	-86.21	and	-59.42	1967 $/acre
INT2:	Between	-59.42	and	-32.51	1967 $/acre
INT3:	Between	-32.51	and	-5.61	1967 $/acre
INT4:	Between	-5.61	and	21.30	1967 $/acre
INT5:	Between	21.30	and	48.20	1967 $/acre
INT6:	Between	48.20	and	75.10	1967 $/acre
INT7:	Between	75.10	and	102.01	1967 $/acre
INT8:	Between	102.01	and	128.91	1967 $/acre
INT9:	Between	128.91	and	155.82	1967 $/acre
INT10:	Between	155.82	and	182.72	1967 $/acre

MEAN VALUE = 87.21 (1967 $/acre)

Alternative C: Sawtimber, 700 trees/acre, thinning at age 20

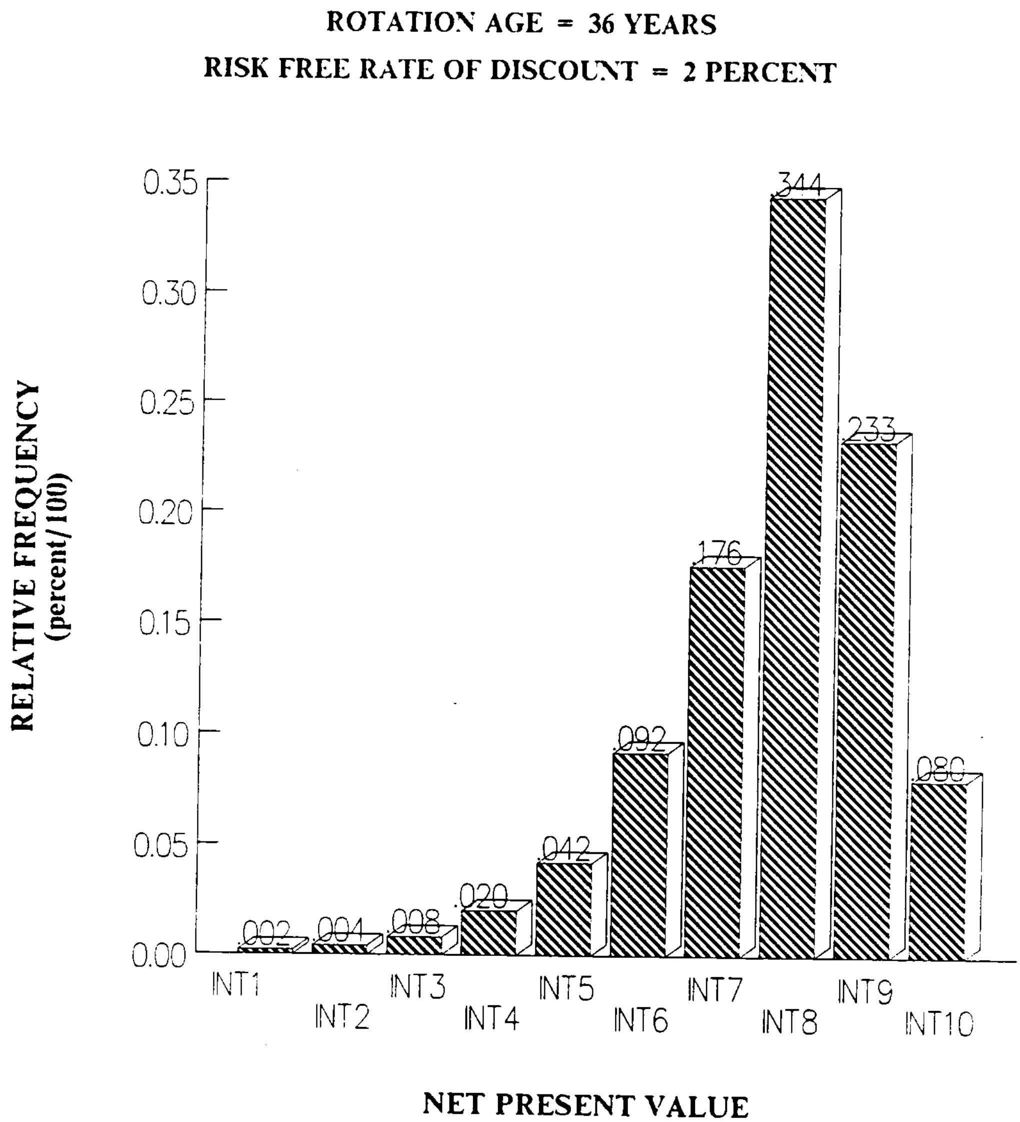

Figure 28. Empirical net present value relative frequency plot, Alternative C (2 % risk free discount rate, rotation age = 36 years).

INT1:	Between	-158.95	and	-125.01	1967 $/acre
INT2:	Between	-125.01	and	-91.06	1967 $/acre
INT3:	Between	-91.06	and	-57.12	1967 $/acre
INT4:	Between	-57.12	and	-23.17	1967 $/acre
INT5:	Between	-23.17	and	10.77	1967 $/acre
INT6:	Between	10.77	and	44.71	1967 $/acre
INT7:	Between	44.71	and	78.66	1967 $/acre
INT8:	Between	78.66	and	112.60	1967 $/acre
INT9:	Between	112.60	and	146.55	1967 $/acre
INT10:	Between	146.55	and	180.49	1967 $/acre

MEAN VALUE = 87.15 (1967 $/acre)

Alternative C: Sawtimber, 700 trees/acre, thinning at age 20 contained within the range of outcomes for the 36 year rotation investment. For example, the lower bound for the 32 year investment under a 2 percent risk free discount rate falls within the third relative frequency interval (INT3, figure 28) for the 36 year rotation investment.

As was the case for alternatives A and B (appendices F and G respectively), the preferred rotation age under the risk-adjusted discount rate analysis for the most part was the same as the identified rotation age in the expected utility analysis (32 years, tables 35 and 36).

Table 35. Mean risk-adjusted net present value results by rotation age, Alternative C (costs discounted with risk free rate).

INVESTMENT COSTS DISCOUNTED WITH THE RISK FREE RATE

MEAN RISK-ADJUSTED NET PRESENT VALUE[1]
(1967 $/acre)

	ROTATION AGE (years)						
	28	32	36	40	44	48	52
RF = 0.02[2]							
RA[3] = 0.020[4]	61.49	87.21**	87.15	83.70	75.46	65.16	51.84
RA = 0.025	12.29	33.06**	32.78	30.79	25.14	18.00	8.21
RA = 0.030	-21.37	-3.60**	-3.68	-4.44	-8.05	-12.83	-20.11
RA = 0.035	-45.52	-29.69	-29.40	-29.12**	-31.11	-34.06	-39.45
RA = 0.040	-63.51	-48.98	-48.25	-47.08**	-47.73	-49.22	-53.14
RF = 0.03							
RA = 0.030[4]	12.30	26.03**	22.84	18.55	13.19	6.22	-2.76
RA = 0.035	-11.85	-0.06**	-2.88	-6.13	-9.87	-15.00	-22.10
RA = 0.040	-29.84	-19.35**	-21.73	-24.08	-26.49	-30.16	-35.79
RA = 0.045	-43.64	-34.04**	-35.95	-37.52	-38.81	-41.30	-45.75
RA = 0.050	-54.47	-45.48**	-46.92	-47.80	-48.15	-49.65	-53.15
RF = 0.04							
RA = 0.040[4]	-12.70	-4.65**	-8.66	-12.98	-16.42	-21.24	-27.78
RA = 0.045	-26.50	-19.34**	-22.88	-26.42	-28.75	-32.38	-37.74
RA = 0.050	-37.33	-30.78**	-33.85	-36.70	-38.09	-40.73	-45.14
RA = 0.055	-45.99	-39.86**	-42.47	-44.70	-45.28	-47.10	-50.72
RA = 0.060	-53.01	-47.16**	-49.34	-51.01	-50.90	-52.02	-54.99

Notes:

(1) From the risk-adjusted discount rate analysis, investment costs discounted with the risk free rate.

(2) RF - risk free discount rate (also used to discount investment costs).

(3) RA - risk-adjusted discount rate (only applied to the expected revenue cash flow).

(4) Represents the risk neutral case. Mean risk-adjusted net present value equals the expected net present value from the empirical net present value probability distribution.

** Denotes the preferred rotation age.

Alternative C: Sawtimber, 700 trees/acre, thinning at age 20.

Table 36. Mean risk-adjusted net present value results by rotation age, Alternative C (costs discounted with the risk-adjusted rate).

INVESTMENT COSTS DISCOUNTED WITH THE RISK-ADJUSTED RATE

MEAN RISK-ADJUSTED NET PRESENT VALUE[1]
(1967 $/acre)

	ROTATION AGE (years)						
	28	32	36	40	44	48	52
RF = 0.02[2]							
RA[3] = 0.020[4]	61.49	87.21**	87.15	83.70	75.46	65.16	51.84
RA = 0.025	32.27	50.74**	48.63	44.60	37.94	29.51	18.71
RA. = 0.030	12.30	26.03**	22.84	18.55	13.19	6.22	-2.76
RA = 0.035	-2.02	8.41**	4.66	0.30	-4.01	-9.80	-17.41
RA = 0.040	-12.70	-4.65**	-8.66	-12.98	-16.42	-21.24	-27.78
RF = 0.03							
RA. = 0.030[4]	12.30	26.03**	22.84	18.55	13.19	6.22	-2.76
RA = 0.035	-2.02	8.41**	4.66	0.30	-4.01	-9.80	-17.41
RA = 0.040	-12.70	-4.65**	-8.66	-12.98	-16.42	-21.24	-27.78
RA = 0.045	-20.89	-14.61**	-18.70	-22.91	-25.62	-29.64	-35.30
RA = 0.050	-27.31	-22.38**	-26.74	-30.49	-32.59	-35.92	-40.87
RF = 0.04							
RA = 0.040[4]	-12.70	-4.65**	-8.66	-12.98	-16.42	-21.24	-27.78
RA = 0.045	-20.89	-14.61**	-18.70	-22.91	-25.62	-29.64	-35.30
RA = 0.050	-27.31	-22.38**	-26.74	-30.49	-32.59	-35.92	-40.87
RA = 0.055	-32.44	-28.54**	-32.50	-36.37	-37.94	-40.70	-45.06
RA = 0.060	-36.59	-33.50**	-37.32	-40.99	-42.11	-44.38	-48.23

Notes:

(1) From the risk-adjusted discount rate analysis, investment costs discounted with the risk-adjusted rate.

(2) RF - risk free discount rate.

(3) RA - risk-adjusted discount rate (applied to the entire cash flow).

(4) Represents the risk neutral case. Mean risk-adjusted net present value equals the expected net present value from the empirical net present value probability distribution.

** Denotes the preferred rotation age.

Alternative C: Sawtimber, 700 trees/acre, thinning at age 20.

APPENDIX I

EMPIRICAL ANALYSIS RESULTS

ALTERNATIVE D

Pulpwood Investment

Table 37 summarizes the stochastic timber yield results for the pulpwood investment alternative. Table 38 summarizes the net present value results for the different approaches to valuation. As indicated in table 38, regardless of the assumptions underlying the valuation, the pulpwood investment was not a viable investment alternative. In essence, targeting pulpwood as the only timber commodity did not justify the high investment costs associated with plantation establishment (table 3). Part of this can be attributed to the tendency to simulate pulpwood stumpage prices along a downward trend (see appendix C: Model performance). As indicated in table 37, most simulations ended with dropping the investment after two unsuccessful rotations earning negative returns (i.e., land abandonment assumed). However, this implies that the first two rotations in most simulations resulted in negative net present values, indicating that the original pulpwood price level did not support managing a plantation solely for pulpwood production.

Table 37. Timber yield summary, Alternative D: Pulpwood investment

STAND AGE (years)	TOTAL STAND VOLUME (cords/acre)		
	LB(1)	MV(2)	UB(3)
8	2.80	2.92	3.10
12	12.85	13.43	13.98
16	22.93	24.24	25.56
20	31.78	33.27	34.72

SKPLNT(4)	345
TNROT(5)	1077
AVPLNT(6)	1.320
AVNROT(7)	2.15
LABMNT(8)	493

Notes:

(1)LB - lowest simulated outcome.

(2)MV - mean value of simulated outcomes.

(3)UB - highest simulated outcome.

(4)SKPLNT - number of planting failures (all simulated rotations). Note: the total number of planting attempts is SKPLNT+TNROT.

(5)TNROT - total number of simulated rotations.

(6)AVPLNT - average number of plantings per rotation.

(7)AVNROT - average number rotations per simulation.

(8)LABMNT - indicates the number of simulations (out of 500) ending under the land abandonment assumption.

Table 38. Net present value results, Alternative D: Pulpwood investment.

DISCOUNT RATE[1] (%/100)	MEAN NPV[2]	CERTAINTY EQUIVALENT NET PRESENT VALUE[3] (1967 $/acre)				MEAN RISK-ADJUSTED NET PRESENT VALUE[4] (1967 $/acre)	
		CASE 1	CASE 2	CASE 3	CASE 4	COSTS RFDR	COSTS RADR
RF = 0.02	-43.63	-50.79	na[5]	-48.50	-67.73		
RA = 0.025						-54.28	-48.51
RA = 0.030						-63.28	-52.40
RA = 0.035						-70.93	-55.51
RA = 0.040						-77.45	-57.99
RF = 0.03	-52.40	-58.73	na	-56.44	-72.99		
RA = 0.035						-60.04	-55.51
RA = 0.040						-66.56	-57.99
RA = 0.045						-72.15	-59.97
RA = 0.050						-76.95	-61.56
RF = 0.04	-57.99	-63.66	na	-61.44	-76.05		
RA = 0.045						-63.57	-59.97
RA = 0.050						-68.38	-61.56
RA = 0.055						-72.52	-62.82
RA = 0.060						-76.12	-63.82

Notes:

(1)

RF risk free discount rate.
RA risk-adjusted discount rate.

(2) From the empirical net present value (NPV) probability distribution estimation (1967 $/acre). Represents risk neutral case.

(3)

CASE 1: Logarithmic utility of wealth, low degree of absolute risk aversion.
CASE 2: Logarithmic utility of wealth, high degree of absolute risk aversion.
CASE 3: Negative exponential utility of wealth, low degree of relative risk aversion.
CASE 4: Negative exponential utility of wealth, high degree of relative risk aversion.

(4)

COSTS RFDR: Investment costs discounted with the risk free rate.
COSTS RADR: Investment costs discounted with the risk-adjusted rate.

(5) na - not available, expected net present value loss exceeds wealth endowment of 18.96 1967 $/acre.

APPENDIX J

EMPIRICAL ANALYSIS RESULTS

ALTERNATIVE E

Custodial Management

ROTATION AGE DETERMINATION

The candidate rotation ages for alternative E were 12, 16, 20, 24, 28, 32, and 36 years. Table 39 summarizes the results of the stochastic timber yield model. These results also reflected the results for shorter rotation ages. The lower bounds reflect the case where the stand was established following a poor seed year. Also shown in table 39 is the percentage of the time the stand resulted from a poor seed year. For the most part, the results reflect the 0.40 probability of incurring a poor seed year at the time of establishment. Finally, since there were no costs associated with this investment, no simulations could end under land abandonment (table 39).

Table 40 summarizes the rotation age determination under the expected utility analysis. Like the case for the other timber alternatives (appendices A through I), the preferred rotation age (16 years) was invariant to the risk neutrality assumption, or any of the risk averse cases considered (CASES 1-4, table 40). Furthermore, the preferred rotation age under the risk-adjusted discount rate analysis was the same (16 years) as the identified rotation in the expected utility analysis (table 41).[180]

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[180] Only one RADR case was considered since custodial management did not incur any investment costs.

Table 39. Timber yield summary, Alternative E: Custodial management

STAND AGE (years)	TOTAL STAND VOLUME (cords/acre)		
	LB(1)	MV(2)	UB(3)
8	1.25	1.37	1.51
12	3.27	4.98	6.28
16	4.64	8.02	10.59
20	5.17	10.03	13.72
24	5.30	11.16	15.71
28	5.07	11.63	16.69
32	4.77	11.62	17.16
36	4.25	11.27	16.73

	ROTATION AGE (years)						
	12	16	20	24	28	32	36
PSEEDY(4)	2299	1923	1567	1387	1292	1194	1073
TNROT(5)	5878	4740	4082	3588	3224	2949	2734
PERPSY(6)	0.391	0.406	0.384	0.387	0.401	0.405	0.392
AVNROT(7)	11.76	9.48	8.16	7.18	6.45	5.90	5.47
LABMNT(8)	0	0	0	0	0	0	0

Notes:

(1)LB - lowest simulated outcome.

(2)MV - mean value of simulated outcomes.

(3)UB - highest simulated outcome.

(4)PSEEDY - number of rotations which were established under State A: poor seed year and high hardwood basal area component.

(5)TNROT - total number of simulated rotations.

(6)PERPSY - percentage of rotations established under State A: poor seed year and high hardwood basal area component.

(7)AVNROT - average number rotations per simulation.

(8)LABMNT - indicates the number of simulations (out of 500) ending under the land abandonment assumption.

Table 40. Certainty equivalent net present value results by rotation age, Alternative E: Custodial management.

	MEAN NET PRESENT VALUE(1) (1967 $/acre)						
	ROTATION AGE (years)						
	12	16	20	24	28	32	36
RF(2) = 0.02	33.36	38.51**	36.65	32.82	27.19	22.15	18.05
RF = 0.03	23.14	26.08**	24.10	21.21	17.03	13.53	10.72
RF = 0.04	17.12	18.87**	16.96	14.65	11.41	8.83	6.79

	CERTAINTY EQUIVALENT NET PRESENT VALUE(3) (1967 $/acre)						
	ROTATION AGE (years)						
	12	16	20	24	28	32	36
	LOGARITHMIC UTILITY(4)						
CASE 1:							
RF = 0.02	33.34	38.42**	36.54	32.70	27.08	22.04	17.97
RF = 0.03	23.12	26.03**	24.03	21.14	16.98	13.48	10.68
RF = 0.04	17.11	18.84**	16.92	14.61	11.37	8.80	6.78
CASE 2:							
RF = 0.02	33.25	38.17**	36.19	32.29	26.67	21.60	17.60
RF = 0.03	23.06	25.84**	23.77	20.85	16.70	13.20	10.47
RF = 0.04	17.06	18.69**	16.72	14.40	11.19	8.64	6.66
	NEGATIVE EXPONENTIAL UTILITY(5)						
CASE 3:							
RF = 0.02	33.33	38.41**	36.52	32.68	27.06	22.03	17.97
RF = 0.03	23.12	26.02**	24.03	21.13	16.97	13.47	10.68
RF = 0.04	17.11	18.84**	16.91	14.60	11.37	8.80	6.78
CASE 4:							
RF = 0.02	33.20	38.00**	35.98	32.10	26.57	21.55	17.62
RF = 0.03	23.05	25.80**	23.73	20.83	16.72	13.25	10.52
RF = 0.04	17.06	18.70**	16.74	14.43	11.24	8.69	6.70

Notes:

(1) From the empirical net present value probability distribution estimation. Represents risk neutral case.

(2) RF - risk free discount rate.

(3) From the expected utility analysis results.

(4) CASE 1: Low degree of absolute risk aversion (see appendix B).
CASE 2: High degree of absolute risk aversion (see appendix B).

(5) CASE 3: Low degree of relative risk aversion (see appendix B).
CASE 4: High degree of relative risk aversion (see appendix B).

** Denotes the preferred rotation age.

Table 41. Mean risk-adjusted net present value results by rotation age, Alternative E: Custodial management.

	MEAN RISK-ADJUSTED NET PRESENT VALUE[1] (1967 $/acre)						
	ROTATION AGE (years)						
	12	16	20	24	28	32	36
RF[2] = 0.02							
RA[3] = 0.020[4]	33.36	38.51**	36.65	32.82	27.19	22.15	18.05
RA = 0.025	27.52	31.37**	29.41	26.11	21.30	17.13	13.77
RA = 0.030	23.14	26.08**	24.10	21.21	17.03	13.53	10.72
RA = 0.035	19.77	22.04**	20.08	17.51	13.85	10.86	8.48
RA = 0.040	17.12	18.87**	16.96	14.65	11.41	8.83	6.79
RF = 0.03							
RA = 0.030[4]	23.14	26.08**	24.10	21.21	17.03	13.53	10.72
RA = 0.035	19.77	22.04**	20.08	17.51	13.85	10.86	8.48
RA = 0.040	17.12	18.87**	16.96	14.65	11.41	8.83	6.79
RA = 0.045	14.99	16.34**	14.48	12.38	9.49	7.25	5.50
RA = 0.050	13.24	14.27**	12.48	10.56	7.97	6.01	4.49
RF = 0.04							
RA = 0.040[4]	17.12	18.87**	16.96	14.65	11.41	8.83	6.79
RA = 0.045	14.99	16.34**	14.48	12.38	9.49	7.25	5.50
RA = 0.050	13.24	14.27**	12.48	10.56	7.97	6.01	4.49
RA = 0.055	11.79	12.57**	10.83	9.07	6.74	5.01	3.68
RA = 0.060	10.57	11.14**	9.47	7.83	5.73	4.20	3.04

Notes:

(1) From the risk-adjusted discount rate analysis. Note: There were no investment costs associated with custodial management.

(2) RF - risk free discount rate.

(3) RA - risk-adjusted discount rate.

(4) Represents the risk neutral case. Mean risk-adjusted net present value equals the expected net present value from the empirical net present value probability distribution.

** Denotes the preferred rotation age.

EXPECTED UTILITY AND EMPIRICAL NPV PROBABILITY DISTRIBUTION RESULTS

Table 42 summarizes the results of the estimated empirical NPV probability distributions. Relative to the other timber investments, custodial management exhibited very little skewness and variance. The alternative was essentially risk free since there was no chance of incurring any loss and the variability of NPV outcomes was low. Therefore, the expected utility adjustments for variance and skewness were essentially zero and the NPV risk premiums never exceeded 1.00 1967 dollars per acre (table 43). In essence, the risk adjustment was essentially the same as the case for risk neutrality (i.e., the certainty equivalent adjustment factors in table 43 were for the most part equal to 1). As was the case for the other timber alternatives, any risk adjustment in the discount rate under-valued the investment (table 41).

For the most part, the empirical NPV probability distributions were symmetric (figures 29-31) with skewness being slightly negative (table 42). This confirms that the negative skewness of the plantation investments was primarily due to establishment costs. As was the case with the other timber alternatives, the relative risk (as measured by the coefficient of variation and relative skewness) increased under a higher specification of the risk free discount rate (table 42). However, in the absolute sense, lower mean NPVs were accompanied by lower variance and lower (negative) skewness as the risk free discount rate increased (table 42).

Table 42. Empirical net present value probability distribution summary, Alternative E (rotation age = 16 years).

NET PRESENT VALUE PROBABILITY DISTRIBUTION SUMMARY(1)

ROTATION AGE = 16 YEARS

	RISK FREE RATE OF DISCOUNT (percent/100)		
	0.02	0.03	0.04
MEAN	38.51	26.08	18.87
VARIANCE	38.28	21.26	13.23
(Standard Deviation)	(6.19)	(4.61)	(3.64)
SKEWNESS	-18.02	-9.95	-5.55
LOWER BOUND(2)	20.99	14.29	10.34
UPPER BOUND(3)	57.43	38.91	27.95
COEFFICIENT OF VARIATION(4)	0.1607	0.1768	0.1929
RELATIVE SKEWNESS(5)	0.0681	0.0825	0.0938

Notes:

(1) All net present value figures in 1967 \$/acre ($(\$/\text{acre})^2$ for variance, $(\$/\text{acre})^3$ for skewness).

(2) Lowest simulated outcome.

(3) Highest simulated outcome.

(4) = (standard deviation)/mean (absolute value).

(5) = $(\text{skewness})^{1/3}$/mean (absolute value).

Alternative E: Custodial management.

Table 43: Expected utility analysis summary, Alternative E: Custodial management (rotation age = 16 years).

	EXPECTED UTILITY (Total)	MEAN[1] COMP EU	VAR[2] COMP EU	SKEW[3] COMP EU	MEAN[4] NPV (1967 $/acre)	CE[5] NPV (1967 $/acre)	RP[6] NPV (1967 $/acre)	CE[7] ADJUST FACTOR	VAR[8] NPV	SKEW[9] NPV
LOGARITHMIC UTILITY										
CASE 1:[10]										
RF[11] = 0.02	5.42949	5.42986	-0.00037	0.00000	38.51	38.42	0.09	0.9977	38.28	-18.02
RF = 0.03	5.37361	5.37384	-0.00023	0.00000	26.08	26.03	0.05	0.9981	21.26	-9.95
RF = 0.04	5.33969	5.33984	-0.00015	0.00000	18.87	18.84	0.03	0.9984	13.23	-5.55
CASE 2:[12]										
RF = 0.02	4.04538	4.05121	-0.00580	-0.00003	38.51	38.17	0.34	0.9912	38.28	-18.02
RF = 0.03	3.80226	3.80745	-0.00524	-0.00004	26.08	25.84	0.24	0.9908	21.26	-9.95
RF = 0.04	3.62846	3.63311	-0.00462	-0.00003	18.87	18.69	0.18	0.9905	13.23	-5.55
NEGATIVE EXPONENTIAL UTILITY										
CASE 3:[13]										
RF = 0.02	0.69957	0.69973	-0.00016	0.00000	38.51	38.41	0.10	0.9974	38.28	-18.02
RF = 0.03	0.67930	0.67940	-0.00009	0.00000	26.08	26.02	0.06	0.9977	21.26	-9.95
RF = 0.04	0.66691	0.66697	-0.00006	0.00000	18.87	18.84	0.03	0.9984	13.23	-5.55
CASE 4:[14]										
RF = 0.02	0.99753	0.99756	-0.00003	0.00000	38.51	38.00	0.51	0.9868	38.28	-18.02
RF = 0.03	0.99659	0.99661	-0.00003	0.00000	26.08	25.80	0.28	0.9893	21.26	-9.95
RF = 0.04	0.99588	0.99590	-0.00002	0.00000	18.87	18.70	0.17	0.9910	13.23	-5.55

Notes:

(1) Mean component of expected utility (see appendix B).
(2) Variance component of expected utility (see appendix B).
(3) Skewness component of expected utility (see appendix B).
(4) Mean net present value (NPV) for the empirical NPV (probability) distribution.
(5) Certainty equivalent net present value.
(6) Net present value risk premium.
(7) Certainty equivalent adjustment factor (see appendix A).
(8) Variance for the empirical NPV distribution (1967 $/acre)^2.
(9) Skewness for the empirical NPV distribution (1967 $/acre)^3.
(10) Low degree of absolute risk aversion (see appendix B).
(11) Risk free discount rate (%/100).
(12) High degree of absolute risk aversion (see appendix B).
(13) Low degree of relative risk aversion (see appendix B).
(14) High degree of relative risk aversion (see appendix B).

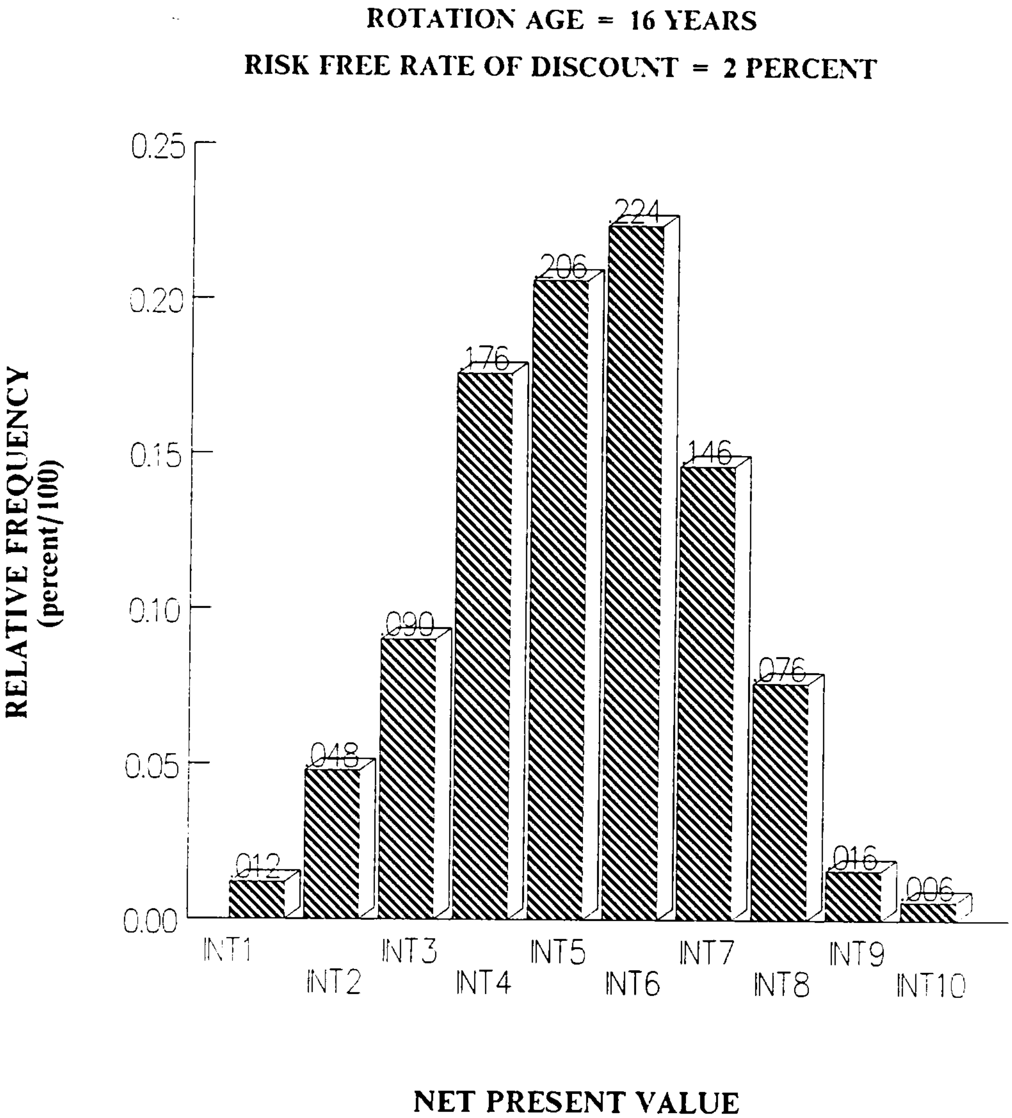

Figure 29. Empirical net present value relative frequency plot, Alternative E (2 % risk free discount rate, rotation age = 16 years).

INT1:	Between	20.99	and	24.63	1967 $/acre
INT2:	Between	24.63	and	28.28	1967 $/acre
INT3:	Between	28.28	and	31.92	1967 $/acre
INT4:	Between	31.92	and	35.57	1967 $/acre
INT5:	Between	35.57	and	39.21	1967 $/acre
INT6:	Between	39.21	and	42.86	1967 $/acre
INT7:	Between	42.86	and	46.50	1967 $/acre
INT8:	Between	46.50	and	50.15	1967 $/acre
INT9:	Between	50.15	and	53.79	1967 $/acre
INT10:	Between	53.79	and	57.43	1967 $/acre

MEAN VALUE = 38.51 (1967 $/acre)

Alternative E: Custodial management.

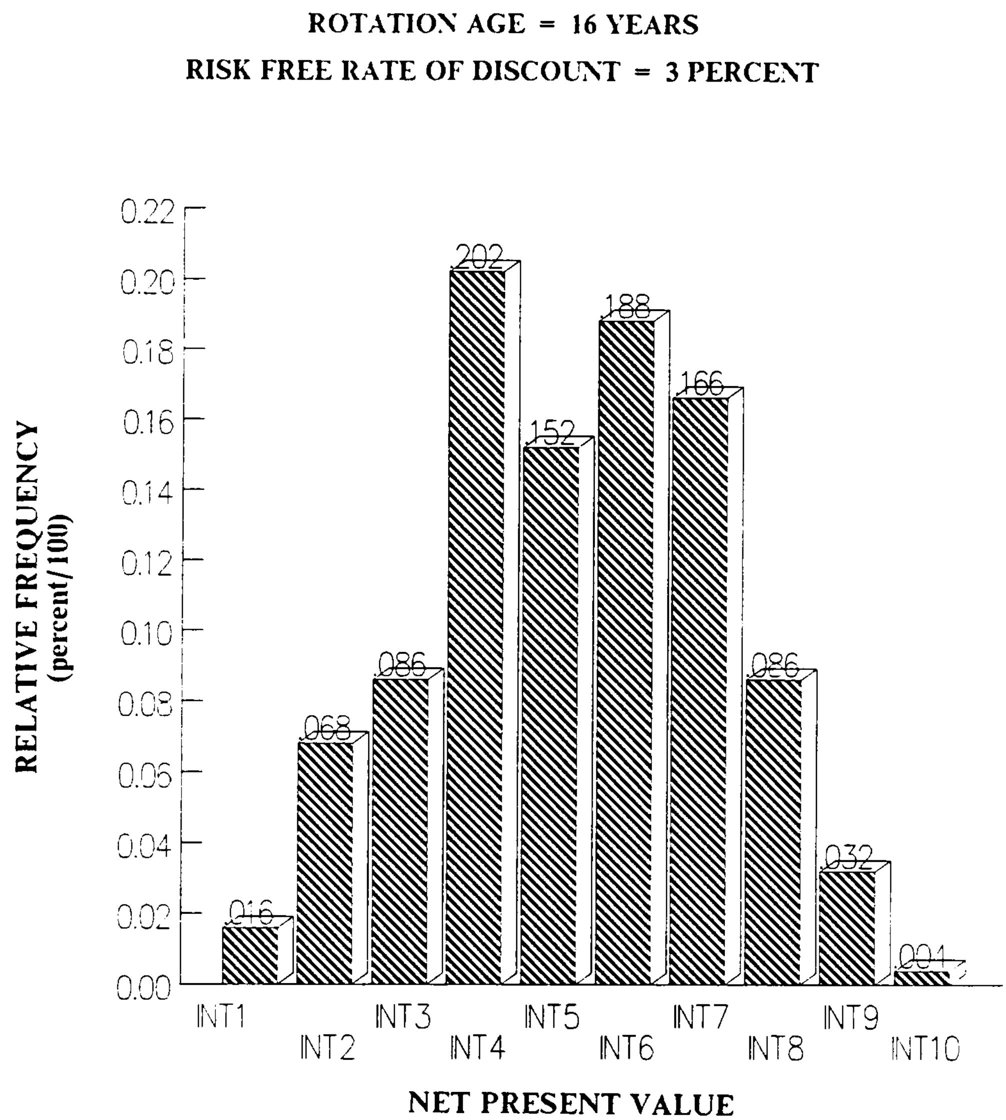

Figure 30. Empirical net present value relative frequency plot, Alternative E (3 % risk free discount rate, rotation age = 16 years).

INT1:	Between	14.29	and	16.75	1967 \$/acre
INT2:	Between	16.75	and	19.21	1967 \$/acre
INT3:	Between	19.21	and	21.68	1967 \$/acre
INT4:	Between	21.68	and	24.14	1967 \$/acre
INT5:	Between	24.14	and	26.60	1967 \$/acre
INT6:	Between	26.60	and	29.06	1967 \$/acre
INT7:	Between	29.06	and	31.53	1967 \$/acre
INT8:	Between	31.53	and	33.99	1967 \$/acre
INT9:	Between	33.99	and	36.45	1967 \$/acre
INT10:	Between	36.45	and	38.91	1967 \$/acre

MEAN VALUE = 26.08 (1967 \$/acre)

Alternative E: Custodial management.

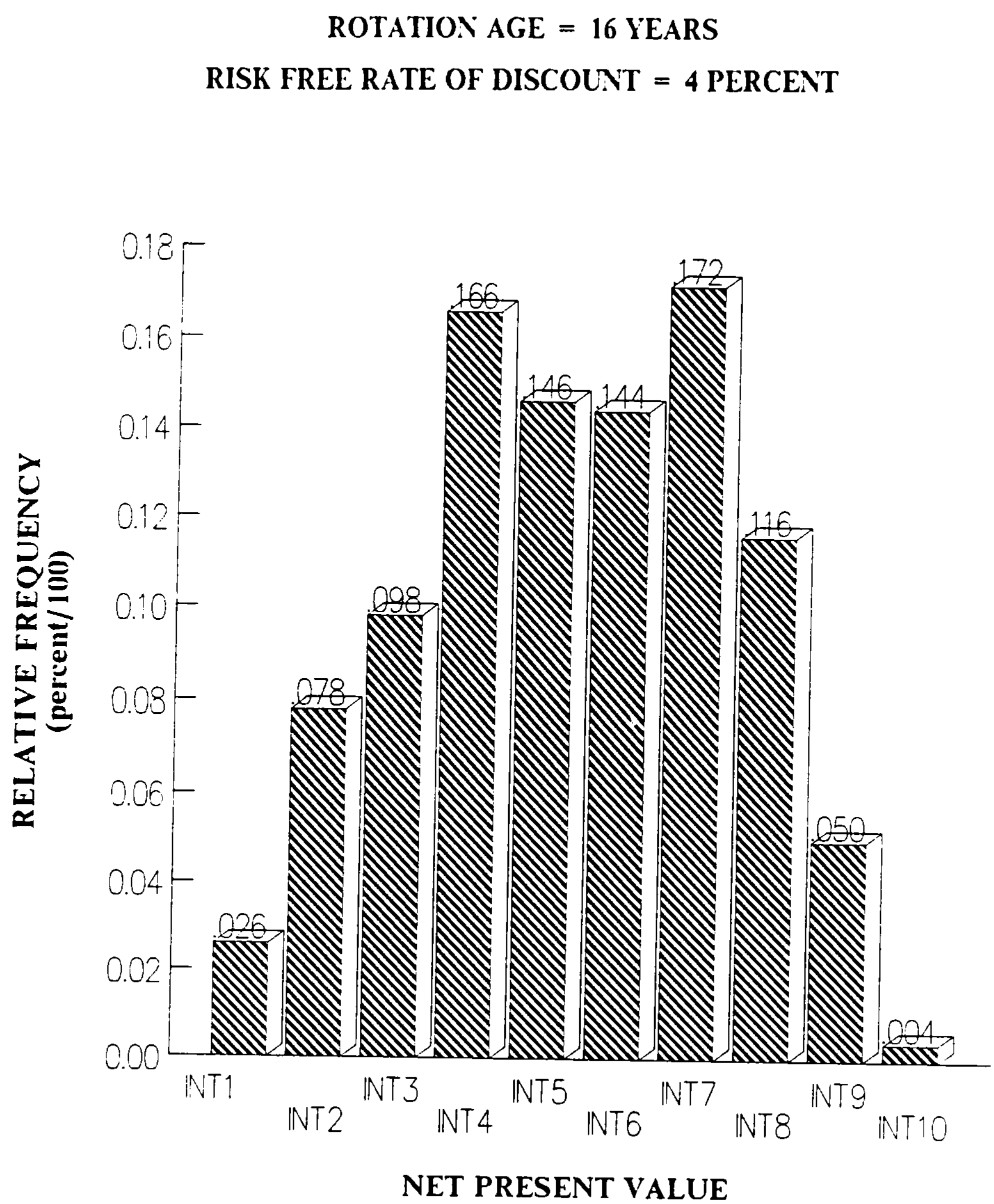

Figure 31. Empirical net present value relative frequency plot, Alternative E (4 % risk free discount rate, rotation age = 16 years).

INT1:	Between	10.34	and	12.10	1967 $/acre
INT2:	Between	12.10	and	13.86	1967 $/acre
INT3:	Between	13.86	and	15.62	1967 $/acre
INT4:	Between	15.62	and	17.39	1967 $/acre
INT5:	Between	17.39	and	19.15	1967 $/acre
INT6:	Between	19.15	and	20.91	1967 $/acre
INT7:	Between	20.91	and	22.67	1967 $/acre
INT8:	Between	22.67	and	24.43	1967 $/acre
INT9:	Between	24.43	and	26.19	1967 $/acre
INT10:	Between	26.19	and	27.95	1967 $/acre

MEAN VALUE = 18.87 (1967 $/acre)

Alternative E: Custodial management.

APPENDIX K

EMPIRICAL ANALYSIS
RESULTS
ALTERNATIVE F
Forage Hay Investment

The simulation results for the autoregressive process defining the stochastic hay yield model are summarized in appendix C (figure 15). Of all the cash flow years simulated, 13 percent of the time the absence of a market outlet occurred; a slight under-representation of the assumed 15 percent probability of occurrence. Unlike the timber investment alternatives (alternative D excluded), most of the simulations ended under the assumption of land abandonment (427 simulations). Therefore, the number of rotations per simulation (6.93) defined a shorter simulated time horizon (i.e., roughly 42 years) than the timber investments. The reason for this was because the hay price simulations tended to follow a downward trend (see appendix C: Model performance). As discussed in appendix C this was not considered to be much of a problem (though regrettable) in as much as it represented an additional proxy for the volatility that producers typically experience in hay markets. However, the land abandonment assumption dampened the effect of the downward trend in simulated prices since it did not allow negative returns to accumulate and completely offset the positive gains from earlier rotations.

EXPECTED UTILITY AND EMPIRICAL NPV PROBABILITY DISTRIBUTION RESULTS

Table 44 summarizes the certainty equivalent and mean risk-adjusted NPV results for the forage hay investment. Table 45 summarizes the empirical NPV probability distribution parameters for the different risk free discount rates considered. In general, the distributions had a very high variance, but positive skewness. In contrast to the timber alternatives (appendices F through J), table 46 shows that variance adjustments lowered expected utility to a level well below that attributed to the mean. Furthermore, the variance adjustments overshadowed the expected utility gains from positive skewness. For example under a high degree of absolute risk aversion (CASE 2, RF = 0.04, table 46), total expected utility (0.36413) was markedly lower than the mean component (3.55732) due to the variance adjustment (-4.16290), whereas the expected utility gain from positive skewness was only 0.96971. Therefore, while the range of possible outcomes indicated that the there was the possibility of extremely good returns[181], the variance of the NPV probability distribution still defined the possibility for highly negative returns (table 45).

The net result was that the certainty equivalent NPVs were markedly lower than the mean NPV for all utility cases considered (tables 44 and 46), suggesting a large net present value premium for risk (table 46). Furthermore, the the mean net present value for the investment under risk free discount rates of 3 and 4 percent were not high enough to compensate the individuals for this premium, that is the investment was not considered viable (i.e., $NPV_{CE} < 0$) in spite of its postive mean NPV. The extreme case occurred under the 4 percent risk free discount rate, where high degrees of absolute or

181 For the most part, these outcomes correspond to the case when hay prices for one reason or another did not decline during the simulation.

Table 44. Net present value results, Alternative F: Forage hay investment.

DISCOUNT RATE[1] (%/100)	MEAN NPV[2]	CERTAINTY EQUIVALENT NET PRESENT VALUE[3] (1967 $/acre)				MEAN RISK-ADJUSTED NET PRESENT VALUE[4] (1967 $/acre)	
		CASE 1	CASE 2	CASE 3	CASE 4	COSTS RFDR	COSTS RADR
RF = 0.02	49.22	16.72	3.85	7.60	-10.73		
RA = 0.025						-70.80	39.94
RA = 0.030						-173.50	31.37
RA = 0.035						-263.72	23.45
RA = 0.040						-412.77	16.11
RF = 0.03	31.37	4.75	-10.49	-0.30	-27.19		
RA = 0.035						-58.85	23.45
RA = 0.040						-138.02	16.11
RA = 0.045						-207.90	9.30
RA = 0.050						-269.89	2.99
RF = 0.04	16.11	-6.45	-17.52	-8.62	-38.10		
RA = 0.045						-53.77	9.30
RA = 0.050						-115.76	2.99
RA = 0.055						-171.03	-2.89
RA = 0.060						-220.53	-8.37

Notes:

(1)

RF risk free discount rate.
RA risk-adjusted discount rate.

(2) From the empirical net present value (NPV) probability distribution estimation (1967 $/acre). Represents risk neutral case.

(3)

CASE 1: Logarithmic utility of wealth, low degree of absolute risk aversion.
CASE 2: Logarithmic utility of wealth, high degree of absolute risk aversion.
CASE 3: Negative exponential utility of wealth, low degree of relative risk aversion.
CASE 4: Negative exponential utility of wealth, high degree of relative risk aversion.

(4)

COSTS RFDR: Investment costs discounted with the risk free rate.
COSTS RADR: Investment costs discounted with the risk-adjusted rate.

Table 45. Empirical net present value probability distribution summary, Alternative F: Forage hay investment.

NET PRESENT VALUE PROBABILITY DISTRIBUTION SUMMARY[(1)]

	RISK FREE RATE OF DISCOUNT (percent/100)		
	0.02	0.03	0.04
MEAN	49.22	31.37	16.11
VARIANCE	19288.17	13692.64	10239.37
(Standard Deviation)	(138.88)	(117.02)	(101.19)
SKEWNESS	931347.88	352049.75	125468.05
LOWER BOUND[(2)]	-282.43	-261.53	-245.19
UPPER BOUND[(3)]	462.66	363.51	297.39
COEFFICIENT OF VARIATION[(4)]	2.8216	3.7303	6.2812
RELATIVE SKEWNESS[(5)]	1.9841	2.2509	3.1075

Notes:

(1) All net present value figures in 1967 \$/acre ($(\$/\text{acre})^2$ for variance, $(\$/\text{acre})^3$ for skewness).

(2) Lowest simulated outcome.

(3) Highest simulated outcome.

(4) = (standard deviation)/mean (absolute value).

(5) = $(\text{skewness})^{1/3}$/mean (absolute value).

Table 46: Expected utility analysis summary, Alternative F: Forage hay investment.

	EXPECTED UTILITY (Total)	MEAN[1] COMP EU	VAR[2] COMP EU	SKEW[3] COMP EU	MEAN[4] NPV (1967 $/acre)	CE[5] NPV (1967 $/acre)	RP[6] NPV (1967 $/acre)	CE[7] ADJUST FACTOR	VAR[8] NPV	SKEW[9] NPV
				LOGARITHMIC UTILITY						
CASE 1:[10]										
RF[11] = 0.02	5.32948	5.47576	-0.16907	0.02279	49.22	16.72	32.50	0.3397	19288	931348
RF = 0.03	5.26877	5.39809	-0.14020	0.01087	31.37	4.75	26.62	0.1514	13693	352050
RF = 0.04	5.21034	5.32651	-0.12097	0.00480	16.11	-6.45	22.56	-0.4004	10239	125468
CASE 2:[12]										
RF = 0.02	3.12712	4.22219	-2.07448	0.97941	49.22	3.85	45.37	0.0782	19288	931348
RF = 0.03	2.13659	3.91868	-2.70233	0.92035	31.37	-10.49	41.86	-0.3344	13693	352050
RF = 0.04	0.36413	3.55732	-4.16290	0.96971	16.11	-17.52	33.63	-1.0875	10239	125468
				NEGATIVE EXPONENTIAL UTILITY						
CASE 3:[13]										
RF = 0.02	0.64657	0.71623	-0.07612	0.00646	49.22	7.60	41.62	0.1544	19288	931348
RF = 0.03	0.63154	0.68822	-0.05937	0.00268	31.37	-0.30	31.67	-0.0096	13693	352050
RF = 0.04	0.61500	0.66209	-0.04812	0.00104	16.11	-8.62	24.73	0.5351	10239	125468
CASE 4:[14]										
RF = 0.02	0.99106	0.99816	-0.01234	0.00524	49.22	-10.73	59.95	-0.2180	19288	931348
RF = 0.03	0.98620	0.99705	-0.01402	0.00317	31.37	-27.19	58.56	-0.8668	13693	352050
RF = 0.04	0.98160	0.99559	-0.00317	0.00169	16.11	-38.10	54.21	-2.3650	10239	125468

Notes:

(1) Mean component of expected utility (see appendix B).
(2) Variance component of expected utility (see appendix B).
(3) Skewness component of expected utility (see appendix B).
(4) Mean net present value (NPV) for the empirical NPV (probability) distribution.
(5) Certainty equivalent net present value.
(6) Net present value risk premium.
(7) Certainty equivalent adjustment factor (see appendix A).
(8) Variance for the empirical NPV distribution (1967 $/acre)^2.
(9) Skewness for the empirical NPV distribution (1967 $/acre)^3.
(10) Low degree of absolute risk aversion (see appendix B).
(11) Risk free discount rate (%/100).
(12) High degree of absolute risk aversion (see appendix B).
(13) Low degree of relative risk aversion (see appendix B).
(14) High degree of relative risk aversion (see appendix B).

relative risk aversion (CASES 2 and 4 respectively in table 44) assigned a very negative certainty equivalent net present value to the alternative. Therefore, while a risk neutral individual would consider the hay investment as being viable under the risk free discount rates of 2, 3, or 4 percent, risk averters (for the most part) would not.

Comparison to Timber Investment Alternatives

When the empirical NPV relative frequency plots for the hay investment (figures 32-34) are compared to the plots for timber investments (appendices F through I), the differences between the NPV probability distributions for the two investment types can be seen. For timber investments, the distributions were distinctly skewed negatively, namely due to those outcomes incurring a high number of plantation failure. On the other hand, the annual revenue flow for the hay investment had the effect of diluting the impact of market losses in any given year. The result were NPV distributions which were skewed positively (table 45). In essence, the viability of the hay investment was sensitive to a sustained annual profit margin over the course of a rotation[182], while the viability of the plantation investments were sensitive to the success or failure of stand establishment.

RISK-ADJUSTED DISCOUNT RATE ANALYSIS RESULTS

Unlike the case for timber investments, applying a RADR with a positive risk premium to the certain costs allowed for the case where the mean risk-adjusted NPV represented an over-valuation of the investment. For example, under the 2 percent risk

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This sensitivity can be seen in the results of the RADR analysis (table 44). If investment costs were discounted with the risk free rate, but revenues with the risk-adjusted rate, the higher present value impact of the discounted costs drives the mean risk-adjusted NPVs to well below zero. However, if investment costs were (albeit incorrectly) discounted with a risk-adjusted rate, the mean-risk adjusted NPVs remain positive.

RISK FREE RATE OF DISCOUNT = 2 PERCENT

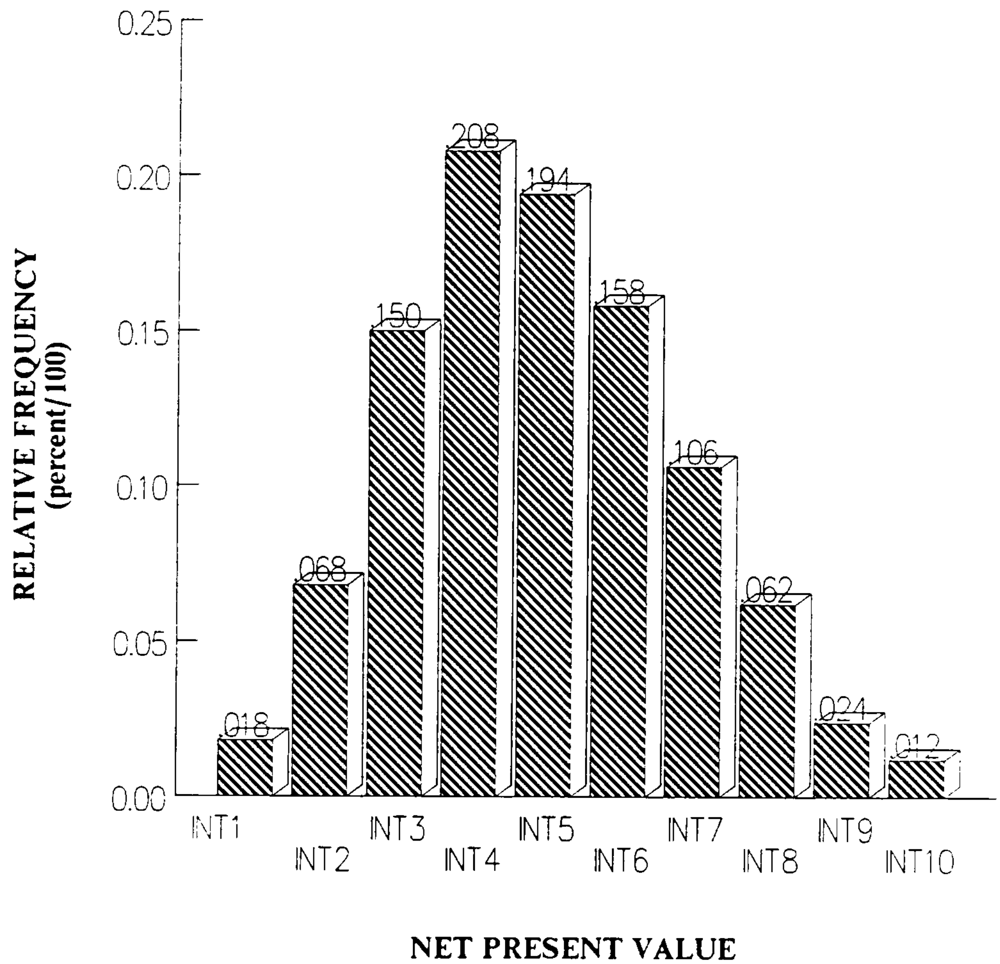

Figure 32. Empirical net present value relative frequency plot, Alternative F: Forage hay investment (risk free discount rate = 2%).

INT1:	Between	-282.43	and	-207.92	1967 $/acre
INT2:	Between	-207.92	and	-133.41	1967 $/acre
INT3:	Between	-133.41	and	-58.90	1967 $/acre
INT4:	Between	-58.90	and	15.61	1967 $/acre
INT5:	Between	15.61	and	90.11	1967 $/acre
INT6:	Between	90.11	and	164.62	1967 $/acre
INT7:	Between	164.62	and	239.13	1967 $/acre
INT8:	Between	239.13	and	313.64	1967 $/acre
INT9:	Between	313.64	and	388.15	1967 $/acre
INT10:	Between	388.15	and	462.66	1967 $/acre

MEAN VALUE = 49.22 (1967 $/acre)

RISK FREE RATE OF DISCOUNT = 3 PERCENT

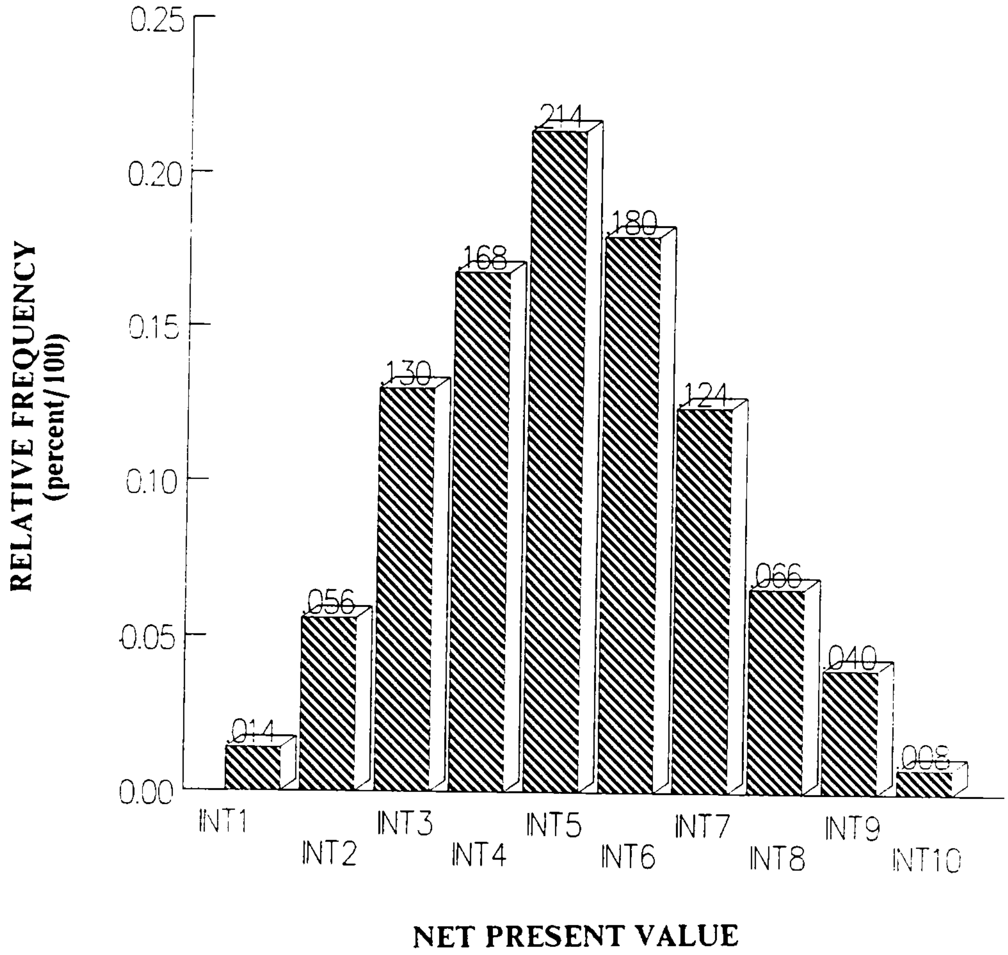

Figure 33. Empirical net present value relative frequency plot, Alternative F: Forage hay investment (risk free discount rate = 3%).

INT1:	Between	-261.53	and	-199.03	1967 $/acre
INT2:	Between	-199.03	and	-136.52	1967 $/acre
INT3:	Between	-136.52	and	-74.02	1967 $/acre
INT4:	Between	-74.02	and	-11.51	1967 $/acre
INT5:	Between	-11.51	and	50.99	1967 $/acre
INT6:	Between	50.99	and	113.50	1967 $/acre
INT7:	Between	113.50	and	176.00	1967 $/acre
INT8:	Between	176.00	and	238.51	1967 $/acre
INT9:	Between	238.51	and	301.01	1967 $/acre
INT10:	Between	301.01	and	363.51	1967 $/acre

MEAN VALUE = 31.37 (1967 $/acre)

RISK FREE RATE OF DISCOUNT = 4 PERCENT

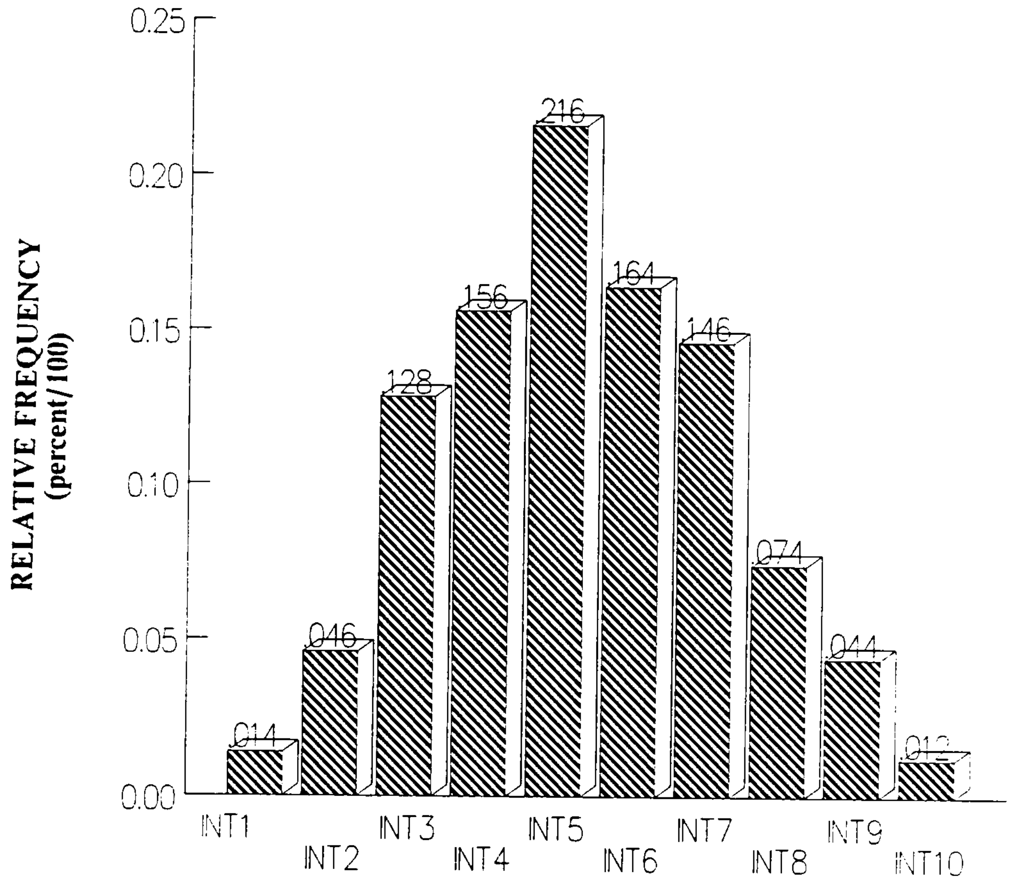

Figure 34. Empirical net present value relative frequency plot, Alternative F: Forage hay investment (risk free discount rate = 4%).

INT1:	Between	-245.19	and	-190.93	1967 $/acre
INT2:	Between	-190.93	and	-136.67	1967 $/acre
INT3:	Between	-136.67	and	-82.42	1967 $/acre
INT4:	Between	-82.42	and	-28.16	1967 $/acre
INT5:	Between	-28.16	and	26.10	1967 $/acre
INT6:	Between	26.10	and	80.36	1967 $/acre
INT7:	Between	80.36	and	134.62	1967 $/acre
INT8:	Between	134.62	and	188.88	1967 $/acre
INT9:	Between	188.88	and	243.14	1967 $/acre
INT10:	Between	243.14	and	297.39	1967 $/acre

MEAN VALUE = 16.11 (1967 $/acre)

free discount rate the true certainty equivalent valuation for the hay investment was 16.72 1967 $/acre (CASE 1, table 44), while applying risk-adjusted discount rates of 2.5 and 3.0 percent resulted in mean risk-adjusted NPVs of 39.94 and 31.37 1967 $/acre respectively (costs discounted with RADR, table 44). In essence, since the hay investment was cost intensive (i.e., annual cost cash flow), the under-compensation for cost risk[183] more than offset the over-compensation for revenue risk.

Therefore, the hay investment alternative represented the only case where a single RADR may capture the entire riskiness of the cash flow. For the example considered above, this rate would be roughly 4.0 percent (RF = 0.02, table 44) for the CASE 1 individual since under the 4.0 percent RADR the mean risk-adjusted NPV was 16.11 1967 $/acre, while the true certainty equivalent NPV was 16.72 1967 $/acre. However, it should be noted that the implied RADR of 4.0 percent for this example only occurred if the over-compensation of revenue risk was offset by a deliberate under-compensation for cost risk. That is, if costs were correctly treated as being relatively certain when compared to revenues, applying any RADR distinctly under-values the investment (costs discounted with the risk free rate, table 44). Therefore, revenue risk alone was not significant enough to justify any distinct risk adjustment to the discount rate. Furthermore, it is highly unlikely that *a priori* one would of been perceptive enough to realize that applying a 4.0 percent RADR (risk free component equal to 2.0 percent) to both costs and revenues would of resulted in the correct valuation of the alternative for the CASE 1 individual. Finally, even if this rate could of been identified, it still would of not represented the appropriate risk adjustment for the other risk averse individuals since it would of over-valued the investment (CASES 2-4, table 44).

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As indicated in the analytical treatment of the RADR problem, discounting certain costs with an RADR containing a positive risk premium implies an under compensation for cost risk since when costs are uncertain, the implied premium in risk-adjusted discount rate will be negative.

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