THE ROBUSTNESS OF LISREL ESTIMATES IN STRUCTURAL EQUATION MODELS WITH CATEGORICAL DATA

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(ABSTRACT)

This study was an examination of the effect of type of correlation matrix on the robustness of LISREL maximum likelihood and unweighted least squares structural parameter estimates for models with categorical manifest variables. Two types of correlation matrices were analyzed; one containing Pearson product-moment correlations and one containing tetrachoric, polyserial, and product-moment correlations as appropriate. Using continuous variables generated according to the equations defining the population model, three cases were considered by dichotomizing some of the variables with varying degrees of skewness.

When Pearson product-moment correlations were used to estimate associations involving dichotomous variables, the structural parameter estimates were biased when skewness was present in the dichotomous variables. Moreover, the degree

of bias was consistent for both the maximum likelihood and unweighted least squares estimates. The standard errors of the estimates were found to be inflated, making significance tests unreliable.

The analysis of mixed matrices produced average estimates that more closely approximated the model parameters except in the case where the dichotomous variables were skewed in opposite directions. However, since goodness-of-fit statistics and standard errors are not available in LISREL when tetrachoric and polyserial correlations are used, the unbiased estimates are not of practical significance. Until alternative computer programs are available that employ distribution-free estimation procedures that consider the skewness and kurtosis of the variables, researchers are ill-advised to employ LISREL in the estimation of structural equation models containing skewed categorical manifest variables.

I am perplexed by my own data.

Dostoyevsky

The Possessed

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Preface

The use of models to represent and explain phenomena is pervasive in every field of study. In the social sciences, a class of models that has become widely applied is the structural equation model, for its use allows the specification and testing of hypothesized relationships among variables within a theoretical framework. Causal modeling techniques originated with the work of the geneticist, Sewall Wright (1921, 1934), but have only relatively recently been introduced to the social sciences (e.g., Blalock, 1961, 1964; Boudon, 1965; Land, 1969). Duncan (1966) illustrated the use of causal modeling techniques with several empirical models, illustrating the advantages of these techniques over ordinary least squares regression analyses.

The first applications of causal models in the social sciences employed only manifest measures of the variables in the model, but the works of Joreskog (1970) and Goldberger (1971) extended the versatility of causal modeling by developing techniques that allowed the inclusion of latent, theoretical variables (e.g., attitudes, intelligence, self-concept). While causal modeling is well grounded in statistical theory and its application has become widespread, rigorous tests of the techniques are just now appearing in the literature. For instance, the asymptotic distribution of the indirect effects in causal models was not known until published independently by Bobko and Rieck (1980) and Sobel (1982).

The most widely used method for the estimation of latent variable causal models is the maximum likelihood procedure in LISREL (Joreskog and Sorbom, 1983). Inherent in this procedure is the assumption of a multivariate normal distribution for the manifest indicators of the latent factors. In causal models incorporating only manifest variables, the assumption of multivariate normality is also made since the path coefficients are estimated by partial regression coefficients. However, regression analysis has been shown to be relatively robust against non-normality (e.g., Bartlett, 1935; Boneau, 1960; Gayen, 1949; Srivastava, 1958), but not against measurement and specification error (Bohrnstedt and Carter, 1971; Walker and Lev, 1953). LISREL recognizes and controls for measurement error in the estimation of structural parameters, but the robustness of LISREL estimates from the analysis of non-normal data has not been conclusively determined since there has only been one study conducted to date (Boomsma, 1983).

The research proposed here adds to this literature by addressing the question of the robustness of the LISREL maximum likelihood and unweighted least squares estimates for models containing dichotomous manifest variables.

CHAPTER 1

INTRODUCTION TO STRUCTURAL EQUATION MODELING

One of the goals of science is to find, describe, and predict relationships among events. In an effort to accomplish this goal, mathematical formulas or equations, called mathematical or statistical models, are developed to represent in a simplified manner events in the real world. Examples of these models range from the precise equations found in physics, such as Newton's laws of force and acceleration, to complex econometric models such as Klein's predictive model of the economy (Klein, 1970).

Such conceptual population models define relationships with a vector function $\sigma(\underset{\sim}{\theta},\underset{\sim}{x})$, where $\underset{\sim}{\theta}$ is a vector of unknown population parameters and $\underset{\sim}{x}$ is a matrix of measures on the events of interest. The objective is to estimate these unknown parameters. For statistical models (those containing a stochastic component), this is accomplished by defining a sample model based on the conceptual population model and collecting data on a representative sample drawn from the population. The data matrix, $\underset{\sim}{x}$, may be reduced to a set of sample statistics, $\underset{\sim}{s}$, such as means, correlations, covariances, or moments. The relationships are then defined as a function of the unknown parameters and the set of sample statistics. Making some basic assumptions concerning the stochastic components of the model, these data are then used to draw inferences about the parameters of the model by applying appropriate estimation procedures. When the probability density function of the random variables is known, maximum likelihood estimators have optimal properties for drawing valid inferences. If distributions are not known, either asymptotic theory or a distribution-free estimation procedure should be applied.

A statistical model commonly applied in the social sciences is the general linear model, $\underset{\sim}{Y} = \underset{\sim}{X}\underset{\sim}{\beta} + \underset{\sim}{\varepsilon}$, which encompasses linear regression, analysis of variance, and analysis of covariance models. Another often applied model is the factor analysis model, $\underset{\sim}{Y} = \underset{\sim}{\nu} + \underset{\sim}{\Lambda}\underset{\sim}{\eta} + \underset{\sim}{\varepsilon}$ (equivalently, $\underset{\sim}{\Sigma} = \underset{\sim}{\Lambda}\underset{\sim}{\Phi}\underset{\sim}{\Lambda}' + \underset{\sim}{\Psi}$). These models were generally applied independently until the seminal works of Joreskog (1970) and Goldberger (1971), wherein the relationships among the variants of these models were explicated and indications made of the advantage to be gained by merging the models to provide a general structural equation.

<u>A General Structural Equation Model</u>

The first general structural equation model to be proposed for the analysis of covariance structures was the ACOVS model (Joreskog, 1970, 1973b, 1974; Joreskog, Gruvaeus, and van Thillo, 1970). Three years later, Joreskog (1973a) developed an even more general model that has become known as the Lisrel (linear structural relations) model. The Lisrel model combines confirmatory factor analysis with a simultaneous equation system for the factors, and an extensive amount of literature has been devoted to the specification, estimation, and testing of variations of the model (e.g., Joreskog, 1977, 1978, 1979; Joreskog and Sorbom, 1976a, 1977; Sorbom, 1974, 1976, 1978; Sorbom and Joreskog, 1981; Werts, et al., 1977). Several other models incorporating different sets of assumptions and estimation procedures have subsequently appeared in the literature that define a specific covariance or moment structure (Bentler, 1982a, 1983a, 1983b; Bentler and Weeks, 1979, 1980; Browne, 1974, 1982; Lohmoller, 1981; McDonald, 1978; Muthen, 1976, 1977, 1979, 1981, 1983b, 1984; Wold, 1977, 1978, 1980, 1982). However, each of these models may be viewed as a generalization, simplification, or different representation of the Lisrel model (Bentler, 1983b; Boomsma, 1983).

<u>The Lisrel Model</u>

The Lisrel model consists of two parts, the measurement model defined by:

$$\underset{\sim}{x} = \underset{\sim}{\Lambda}_x \underset{\sim}{\xi} + \underset{\sim}{\delta}$$

$$\underset{\sim}{y} = \underset{\sim}{\Lambda}_y \underset{\sim}{\eta} + \underset{\sim}{\varepsilon}$$

and structural model defined by:

$$\boldsymbol{\eta} = \mathbf{B}\boldsymbol{\eta} + \boldsymbol{\Gamma}\boldsymbol{\xi} + \boldsymbol{\zeta}$$

with the assumptions that,

(i) $\boldsymbol{\zeta}$ is uncorrelated with $\boldsymbol{\xi}$

(ii) $\boldsymbol{\varepsilon}$ is uncorrelated with $\boldsymbol{\eta}$

(iii) $\boldsymbol{\delta}$ is uncorrelated with $\boldsymbol{\xi}$

(iv) $\boldsymbol{\zeta}$, $\boldsymbol{\varepsilon}$ and $\boldsymbol{\delta}$ are mutually uncorrelated

(v) $\mathbf{B}$ has zeroes in the diagonal and $\mathbf{I} - \mathbf{B}$ is nonsingular.

The measurement model specifies how the observed variables, $\mathbf{x}$ and $\mathbf{y}$, are are determined through $\boldsymbol{\Lambda}_x$ and $\boldsymbol{\Lambda}_y$ by the latent variables, $\boldsymbol{\xi}$ and $\boldsymbol{\eta}$, respectively; the $\boldsymbol{\delta}$ and $\boldsymbol{\varepsilon}$ terms represent residuals in $\mathbf{x}$ and $\mathbf{y}$ unexplained by $\boldsymbol{\xi}$ and $\boldsymbol{\eta}$. The structural model specifies the causal relationships among the latent endogenous variables in $\boldsymbol{\beta}$, between the exogenous and endogenous variables in $\boldsymbol{\Gamma}$, and describes unexplained residuals of the latent factors in $\boldsymbol{\zeta}$ (Joreskog and Sorbom, 1983). The elements of $\boldsymbol{\beta}$ and $\boldsymbol{\Gamma}$ are regression coefficients resulting from the regression of the endogenous latent factors on their respective antecedent causal factors.

By way of example, a simple three factor model taken from Cuttance (1983) is shown in Figure 1. The first exogenous variable, ξ_1, has a single manifest indicator, x_1.

The variance in x_1 unexplained by ξ_1 is represented by δ_1. The second exogenous variable, ξ_2, is indexed by three manifest variables, x_2, x_3, and x_4, with their respective residuals measured by δ_2, δ_3, and δ_4. The factor loadings, λ_{11}^{x}, λ_{22}^{x}, λ_{32}^{x}, and λ_{42}^{x}, are the slopes of the equations relating the observed variables to their respective latent factors. When standardized, the squares of these slopes give the reliabilities of the observed variables in terms of the extent to which their variances are explained by the latent factor. The covariance of the two latent exogenous variables is given by Φ_{21}.

Insert Figure 1 About Here

The single latent endogenous variable, η_1, is indexed by two observed variables, y_1 and y_2, with respective factor loadings of λ_{11}^{y} and λ_{21}^{y}. The residuals in y_1 and y_2 unexplained by η_1 are ε_1 and ε_2. The causal influences of ξ_1 and ξ_2 on η_1 are represented by γ_{11} and γ_{12}, respectively, with the variance in η_1 unexplained by ξ_1 and ξ_2 given by ζ_1.

When a Lisrel model such as the one described above is applied in a substantive area, it should be based on a theory formulated from sources such as previous scholarly work, exploratory analyses of other data, or mere common sense. Such a theory specifies which variables measure the

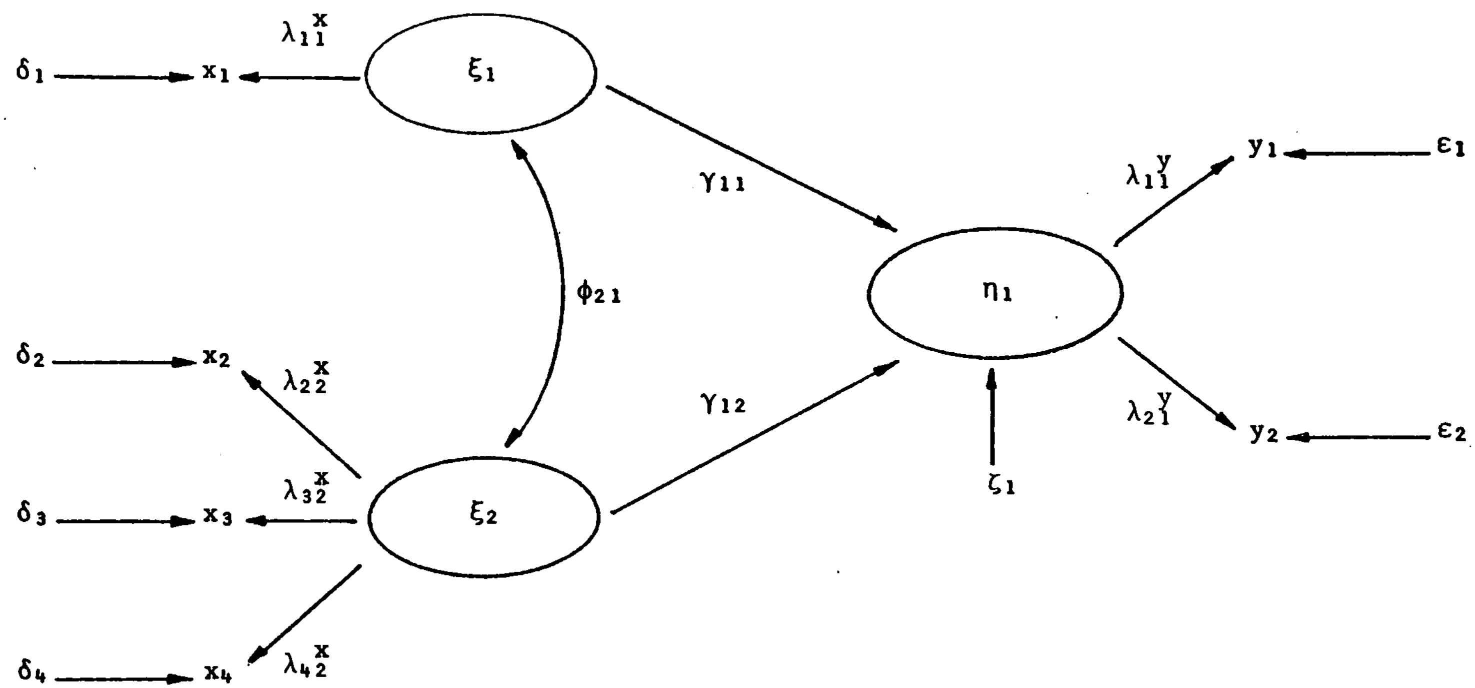

Figure 1. A Three Factor Structural Equation Model

latent factors, and the causal relationships among the factors. The models are tested to see if the structure imposed by the model is evidenced in the covariance structure of the data.

The Estimation of Lisrel Models

The theoretical development of a general structural equation model was followed by the writing of a computer program, LISREL (Joreskog and van Thillo, 1972), now in its sixth edition (Joreskog and Sorbom, 1983), designed for the estimation of Lisrel models. With the assumption that the observed variables have a multivariate normal distribution, a modification of an iterative minimization procedure described by Fletcher and Powell (1963) is used to fit the estimated covariance matrix, $\underset{\sim}{\Sigma}$, implied by the model to the sample covariance matrix, $\underset{\sim}{S}$. The covariance structure that the model implies for the observed variables is defined in terms of the model parameters as

$$\underset{\sim}{\Sigma} = \begin{vmatrix} \underset{\sim}{\Lambda}_y(\underset{\sim}{I}-\underset{\sim}{\beta})^{-1}(\underset{\sim}{\Gamma}\underset{\sim}{\Phi}\underset{\sim}{\Gamma}'+\underset{\sim}{\Psi})(\underset{\sim}{I}-\underset{\sim}{\beta}')^{-1}\underset{\sim}{\Lambda}_y' + \underset{\sim}{\Theta}_\varepsilon & \underset{\sim}{\Lambda}_y(\underset{\sim}{I}-\underset{\sim}{\beta})^{-1}\underset{\sim}{\Gamma}\underset{\sim}{\Phi}\underset{\sim}{\Lambda}_x' \\ \underset{\sim}{\Lambda}_x\underset{\sim}{\Phi}\underset{\sim}{\Gamma}'(\underset{\sim}{I}-\underset{\sim}{\beta}')^{-1}\underset{\sim}{\Lambda}_y' & \underset{\sim}{\Lambda}_x\underset{\sim}{\Phi}\underset{\sim}{\Lambda}_x' + \underset{\sim}{\Theta}_\delta \end{vmatrix}$$

The estimates of the parameters of the hypothesized model are those values minimizing the maximum likelihood fitting function:

$$F = \log|\underset{\sim}{\Sigma}| + \mathrm{tr}(\underset{\sim}{S}\underset{\sim}{\Sigma}^{-1}) - \log\underset{\sim}{\Sigma} - (p + q)$$

where F is the value to be minimized, $\underset{\sim}{\Sigma}$ and $\underset{\sim}{S}$ are as previously defined, and p and q are the number of variables in $\underset{\sim}{y}$ and $\underset{\sim}{x}$, respectively. A likelihood-ratio chi-square statistic comparing $\underset{\sim}{\Sigma}$ and $\underset{\sim}{S}$ is computed, and gives an indication of the goodness of fit of the whole model.

Applying the Lisrel Model

With the public availability of LISREL, the combining of confirmatory factor analysis with the analysis of structural equations has become an important analytical tool for social scientists. Many applications of the Lisrel model can be found in the psychology, sociology, economics, and education literature, and reviews of the applied literature by Bentler (1980) and Bielby and Hauser (1977) contain hundreds of references. Among the earliest applications in education were studies by Mason, et al. (1976), who analyzed response errors in student reports of parental socioeconomic characteristics, and by Werts, Linn, and Joreskog (1977), who developed a simplex model of academic growth. More recent applications are those of Hauser, Tsai, and Sewell (1983), who proposed a revision of the Wisconsin model of the process of educational and occupational attainment (Sewell, Haller and Portes, 1969), and by Wolfle and Robertshaw (1983), who examined measurement error differences between blacks and whites in educational achievement models.

Despite the fact that the maximum-likelihood procedure in Lisrel assumes a multivariate-normal distribution among the observed variables, many of the studies that have appeared in the literature have included variables that were necessarily measured on dichotomous or ordered polychotomous scales with assumed underlying continuities. For example, Wolfle and Robertshaw (1982) examined the effect of college attendance (a dummy variable) on locus of control using four categorical variables as manifest indicators of locus of control. These variables had four categories representing responses ranging from "disagree strongly" to "agree strongly". Studies employing categorical variables such as these were conducted on the often untested assumption that the data did not deviate substantially from multivariate normality. Hauser, Tsai, and Sewell (1983), however, noted that their use of dichotomous variables as social psychological indicators resulted in a conceptually defective analysis. They concluded that since none of the dichotomous variables were highly skewed they could be reasonably confident in their results, with the possible exception of the underestimation of the validity and reliability of those variables. Yet, questions concerning the validity of conclusions drawn from studies incorporating categorical variables are raised. These questions arise from the appropriateness of the application of the Lisrel model itself, the zero-order measures of association, and the estimation procedures used in the analyses.

Appropriateness of the Lisrel Model

The inclusion of dichotomous or ordered polychotomous variables as indices of latent factors in a Lisrel model necessarily alters the model since these categorical variables are imperfect measures of the underlying continuous latent response variables denoted by y*. Note that while the y* variables are latent, unobserved variables, they are not of the same class of variables as ξ and η. These latter variables may be thought of as latent constructs, such as intelligence, that are only indirectly observed by a number of manifest variables. The y* variables, in contrast, represent latent, theoretically measurable continuous variables that are assumed to underlie the actually measured categorical variables.

In the population, the measurement equation for the latent response variables is

$$\underset{\sim}{y}^* = \underset{\sim}{\Lambda}_y \underset{\sim}{\eta} + \underset{\sim}{\varepsilon}.$$

The latent response variable, y^*, is then related to the observed y by a step function defined by threshold values. For a categorical variable y with c categories this relationship is defined as:

$$
\begin{aligned}
y &= c-1 \text{ if } \tau_{c-1} < y^*, \\
&= c-2 \text{ if } \tau_{c-2} < y^* <= \tau_{c-1}, \\
&\;\;\vdots \\
&= 1 \quad \text{if } \tau_1 < y^* <= \tau_2, \\
&= 0 \quad \text{if } \qquad y^* <= \tau_1
\end{aligned}
$$

where the τ_i represent threshold values. When the observed variables are continuous, the relationship is simply the identity

$$y = y^*.$$

The use of Pearson product-moment correlations to measure the associations involving dichotomous and ordered polychotomous variables used as indicator variables requires the assumption that this identity holds, and that the categorical variable perfectly measures the latent response variable when in fact it often does not.

<u>Appropriateness of Measures of Association</u>

In the absence of more appropriate estimation techniques, these models have generally been estimated by the application of the LISREL maximum likelihood procedure to analyze an observed correlation or covariance matrix computed from Pearson product-moment correlations. Product-moment correlations used as measures of associations between dichotomous and ordered polychotomous variables with underlying continuities, however, are in general not free to vary between -1 and +1, but are dependent on the marginal distributions of the variables. These correlations underestimate the true degree of association, particularly for variables with highly skewed distributions (Carroll, 1961; Lancaster and Hamdan, 1964; Muthen, 1983a, 1983b; Pearson, 1900, 1904, 1913; Pearson and Pearson, 1922). On the assumption that the underlying y* variables are normally distributed, associations among these variables are more appropriately measured by tetrachoric and polychoric correlations (Carroll, 1961; Lancaster and Hamdan, 1964; Muthen, 1983a, 1983b; Olsson, 1979a; Pearson, 1900, 1904, 1913; Ritchie-Scott, 1918). These correlations are estimates of population correlations among latent response variables (Brown and Benedetti, 1977; Muthen, 1983b; Pearson, 1900) and are computed from estimates of the threshold values. The use of these correlations may be thought of as robustifying the correlations against categorization and skewness, or "stretching" the correlations to assume values between -1 and +1 (Muthen, 1983b).

Muthen (1983a) vividly illustrated the underestimation of the association between dichotomous variables with underlying continuities. Table 1, taken from Muthen (1983a), shows the Pearson correlations for varying degrees and combinations of skewness when the true correlation is .50. As can be seen, the greatest underestimation occurs when the two variables are highly skewed in opposite directions.

Insert Table 1 About Here

A related instance of inadequate measures of association is the use of product-moment correlations to measure the degree of association between categorical variables with underlying continuities and continuous variables measured on interval scales. Pearson product-moment correlations also underestimate the true relationship between these variables, and the more appropriate measures of association are polyserial correlations (Jaspen, 1946; Olsson, Drasgow, and Dorans, 1982; Pearson, 1913). Again, the degree of underestimation is greatest for associations among variables whose distributions are highly skewed.

Appropriateness of Estimation Procedures

Additional questions are raised by the application of maximum likelihood procedures to the estimation of Lisrel

Table 1. Pearson Correlations for True Correlations of .50.

	10/90*	20/80	30/70	40/60	50/50	60/40	70/30	80/20	90/10
10/90	.25								
20/80	.26	.30							
30/70	.26	.30	.32						
40/60	.24	.30	.32	.33					
50/50	.23	.28	.31	.33	.33				
60/40	.20	.26	.30	.32	.33	.33			
70/30	.18	.23	.27	.30	.31	.32	.32		
80/20	.15	.20	.23	.26	.28	.30	.30	.30	
90/10	.10	.15	.18	.20	.22	.24	.26	.26	.25

*Column headings represent the percent of cases in each category of the dichotomy; i.e., 10/90 means 10% in first category and 90% in the second.

models that include dichotomous or ordered polychotomous variables. As noted previously, the maximum likelihood procedure in LISREL assumes a multivariate normal distribution for the observed variables and minimizes a fitting function that is based on the multivariate normal probability density function. This assumption of a multivariate normal distribution for the data is the weakest part of the LISREL program (de Leeuw, et al., 1983), for in many instances the distributions of variables are unknown or suspected to be far from normal. This is particularly true for models containing dichotomous and ordered polychotomous variables.

Methodological Developments

The applied researcher in the past has been compelled to overlook the problems mentioned above because of the lack of availability of alternative methodologies. For example, the limitations in the use of Pearson product-moment correlations as zero-order measures of association between dichotomous, ordered polychotomous, and continuous variables have been known for a long time, but the complexities of the estimation procedures for tetrachoric, polychoric, and polyserial correlations have prevented their application in practice. In addition, the early versions of LISREL gave no alternatives for choice of estimation procedure. These problems facing the applied researcher did not go unnoticed by theorists, and recently, efficient procedures for computing these correlations have been developed that now permit the relatively easy computation of these correlations (Cox, 1974; Divgi, 1979; Joreskog and Sorbom, 1981, 1983; Kirk, 1973; Olsson, 1979a; Olsson, et al., 1982).

There has also been an extensive amount of research devoted to the development of limited information and distribution free procedures for the estimation of Lisrel-type models (Anderson, 1973; Bentler, 1982a, 1983a, 1983b; Bentler and Weeks, 1980; Browne, 1974, 1982, 1984; Christoffersson, 1975; Joreskog and Goldberger, 1972; Joreskog and Sorbom, 1981, 1983; Lee and Bentler, 1980; Muthen, 1977, 1978, 1981, 1984; Muthen and Christoffersson, 1981). These are generalized least squares procedures that minimize the general fitting function

$$F = (\utilde{s} - \utilde{\sigma}(\utilde{\theta}))'\utilde{W}(\utilde{s} - \utilde{\sigma}(\utilde{\theta}))$$

where $\utilde{W}$ is a consistent estimator of the asymptotic covariance matrix of $\utilde{s}$, the sample statistics, and $\utilde{\sigma}(\utilde{\theta})$ is the matrix of parameters implied by the model. These procedures make no distributional assumptions concerning the observed variables, but do assume that categorical variables are imperfect measures of underlying continuous latent response variables with normal distributions. In response to these developments, the fifth edition of LISREL (Joreskog and Sorbom, 1981) included generalized least squares and unweighted least squares estimation procedures. The fact remains, however, that many Lisrel models reported in the applied literature were estimated, and continue to be estimated, with maximum likelihood procedures when other estimation procedures would be more appropriate.

Robustness Studies

The development of limited information procedures and efficient algorithms for the computation of tetrachoric, polychoric, and polyserial correlations have allowed some of the questions concerning the validity of conclusions drawn from the estimation of Lisrel-type models containing categorical variables to be be addressed. These studies have examined the impact of choice of correlation type in factor analysis models, and differences among various estimation procedures for both factor analysis models and structural equation models.

The studies of factor analysis models containing categorical variables have implications concerning the measurement portion of Lisrel-type models. Using their own computer programs, Muthen (1983a) and Olsson (1979b) found that the analysis of Pearson product-moment matrices resulted in downwardly biased estimates of the factor loadings of the categorical variables and inflated values for the chi-square goodness-of-fit statistics. As for the estimation of Lisrel models, these results suggest that the reliabilities of categorical variables would be underestimated with product-moment correlations and the models too often rejected for lack of fit.

In studies of the robustness of various estimation procedures, including some not necessarily found in LISREL, for factor analysis models containing categorical variables (Boomsma, 1982, 1983; Muthen, 1978; Muthen and Kaplan, 1984; Olsson, 1979b; Tanaka, 1984), the maximum likelihood (ML), generalized least squares (GLS), and asymptotically distribution free (ADF) procedures were all found to perform well for data that do not deviate too drastically from normality. In cases of extreme skewness, distortions in the ML and GLS chi-squares and standard errors were found; the chi-squares were inflated and the standard errors were biased downwards. However, the parameter estimates were generally unbiased for large sample sizes ($N > 400$). An exception was found by Tanaka (1984) wherein both ML and ADF were found to underestimate not only the standard errors, but the factor loadings as well.

The case studies by Browne (1982), Huba and Bentler (1983), Huba and Harlow (1984), and Joreskog and Goldberger (1972) provided comparisons of the ML, GLS, and ADF estimators in factor analyses of categorical variables. No large differences were found among estimated factor loadings for the procedures, although Browne (1982) and Joreskog and Goldberger (1972) reported that in general the GLS and ADF parameter estimates were lower than those obtained with ML. However, the ADF chi-square was consistently lower than either the GLS or ML, and the estimated standard errors were generally smaller for GLS and ML.

Results from the above studies suggest in terms of the measurement portion of the Lisrel model, that the choice of estimation procedure is not as important as choice of correlation type when the concern is with the robustness of parameter estimates. However, for valid hypothesis testing and assessment of fit, ADF seems to be the preferred estimation procedure, which is not currently available in LISREL.

The extension of these kinds of studies to the structural portion of Lisrel-type models has just begun. To date there has been only one study of the robustness of the structural parameter estimates against non-normality (Boomsma, 1983). Using an adaptation of LISREL-III (Joreskog and Sorbom, 1976b) which he called LISREP, Boomsma examined the robustness of the maximum likelihood estimates for models in which all indicator variables for both the exogenous and endogenous latent factors were categorical variables. The robustness of the estimates for three different structural models was determined for varying degrees of categorization and skewness. Specifying that the structure should hold for the observed categorical variables, random normal variables were generated and subsequently categorized. Multinomial correlation matrices (the multinomial correlation is the extension of the phi coefficient to polychotomous variables), and their corresponding covariance matrices were computed using FORTRAN programs written by Boomsma. Parameter estimates were then obtained from the analysis of the multinomial covariance matrices. For one of the models, a comparison was made between the analyses of covariance and correlation matrices.

In the models estimated, Boomsma found no bias in the parameter estimates or estimates of standard errors. However, he found both a categorization effect and a skewness effect on the standard deviations of the estimates. They were found to be generally too small for model variations with zero or small skewnesses, and generally too large for variations with moderate and large skewnesses. This means, on the average, that the estimates are not too far from the population values, but for a single sample, the parameter estimates may deviate substantially. Boomsma also reported only a minor effect of categorization on the chi-square goodness-of-fit statistic, but with increased skewness the model was rejected too often. Thus, the number of categories of variables had less effect than the skewness of the variables. In the comparisons of the estimates resulting from the analyses of covariance and correlation matrices, the standard deviations of the standardized parameter estimates were smaller when the correlation matrix was analyzed; this was more marked with relatively large skewnesses.

In summary, a number of studies have shown the factor loadings in the measurement portion of Lisrel-type models containing categorical variables to be biased when Pearson product-moment correlations are used to measure associations among these variables. The use of tetrachoric and polychoric correlations produce more accurate parameter estimates regardless of estimation procedure, but generalizations should be made with caution, particularly when the variables exhibit large degrees of skewness. When the data are appropriately analyzed, there is no consistent bias in the maximum likelihood parameter estimates of measurement models with categorical variables. However, the variances of the estimates are affected by non-normality as is the chi-square statistic. From the studies reviewed above, it is evident that the LISREL maximum likelihood measurement parameter estimates are probably non-robust when skewness is present in the observed variables and product-moment correlations are used to estimate the model. Clearly, substantive conclusions appearing in the literature should be reviewed where they have been based on the LISREL estimation of Pearson product-moment correlations for models with categorical indicators. For such models the reported reliabilities for the categorical variables are probably downwardly biased. Cross-validations are advisable as a test of the generalizations made from these studies.

Raison d'etre de la recherche

Some of the questions concerning the LISREL estimation of structural models with categorical variables have been addressed by the studies reviewed, but with the exception of Boomsma (1982, 1983), LISREL was not the program used in the analyses. New methodological developments hold promise for the future; programs such as EQS (Bentler, 1982b) BENWEE (Browne and Cudek, 1983), and LACCI (Muthen, 1982) have been written, and are designed for the estimation of these models. However, they are still in the developmental and testing stages and generally not available to the public. LISREL remains the only program generally available to the applied researcher today.

The inclusion in LISREL-V and LISREL-VI of procedures for the estimation of tetrachoric, polychoric, and polyserial correlations as well as options for estimation procedures other than maximum likelihood now gives alternative methodologies to researchers who question the assumption of multivariate normality for their data. What is not known at present is the influence of the choice of correlation type on LISREL latent variable structural parameter estimates, or the robustness of the LISREL limited information procedures. This is of particular concern where models with categorical variables are estimated because of the necessary assumption that the observed categorical variables perfectly measure the latent continuous response variables (i.e., $y = y^*$).

The purpose of this research is to examine the effect of correlation type on the robustness of LISREL maximum likelihood and unweighted least squares estimates of the structural parameters of models containing categorical variables as indicators of latent factors.

CHAPTER 2

METHODOLOGY

Ideally, this study would examine the robustness of the three estimation procedures in LISREL (maximum likelihood, generalized least squares, and unweighted least squares) by the analysis of covariance matrices. However, there are several restrictions placed on the methodology used in this study by the limitations of the LISREL-VI program. LISREL does not provide estimates of variances and covariances corresponding to the tetrachoric, polychoric, and polyserial correlations, and only the standardized parameter estimates are reported since this study was of necessity restricted to the analysis of correlation matrices.

Furthermore, in generalized least squares procedures the weight matrix, $\underset{\sim}{W}$, is the inverse of the sample covariance matrix of the statistics in $\underset{\sim}{s}$. Thus, when correlation matrices are analyzed, the weight matrix consists of the variances and covariances of the correlations. The algorithm in LISREL, however, assumes the input correlation matrix is a product-moment matrix, and the algebraic formulas for the variances and covariances of tetrachoric, polychoric, and polyserial correlations are not the same as that for product-moment matrices. Thus, the GLS procedure in LISREL is not appropriate for the analysis of matrices containing other than Pearson product-moment correlations. In contrast, an unweighted least squares (ULS) procedure fits a function in which the weight matrix is the identity matrix; this procedure reduces to an iterative ordinary least squares method which is not restricted to a particular correlation type. As a result, analyses reported here employed only the ML and ULS procedures, and comparisons of the structural parameter estimates for the choices of correlation type were made between them.

Another area addressed in this study is the robustness of the LISREL chi-square statistic and the standard errors of parameter estimates. Past studies have shown them to be affected by the categorization and skewness of the variables and the choice of estimation procedure. However, the technical and computational problems of computing these statistics have not been solved for the ULS procedures in LISREL or for the input of tetrachoric or polyserial correlations. Consequently, comparisons between procedures and correlation type can only be made for the parameter estimates and not their standard errors or the chi-square goodness-of-fit statistic, and the robustness of the standard errors can only be determined for the maximum likelihood analyses of product-moment matrices. Additionally, results are restricted to the standard errors of the unscaled estimates since the LISREL output does not include the standard errors of the standardized estimates. While it would obviously be more desirable to examine the standard errors for the parameter estimates reported, the significance tests would be the same for the standardized and unscaled estimates.

The Model

In conducting this study on the effects of correlation type on the ML and ULS structural parameter estimates, simulated data were generated using a model with known parameters. There are, of course, a wide variety of models that can be estimated with LISREL, and ideally, a cross section of the different types of models would be chosen for analysis. However, because simulations are time consuming and costly, it was decided to propose only one model. Variations of the model chosen appear often in the literature (e.g., Bielby, Hauser, and Featherman, 1977; Cuttance, 1983; Ethington and Wolfle, 1985; Fox, 1980; Wiley, 1973; Wolfle and Robertshaw, 1982), and the parameters of the model chosen were adapted from the standardized results of Bielby, et al. (1977), whose model was well-defined and good-fitting.

The Lisrel model (Figure 2) used in this study is a fully recursive structural model containing five latent factors, two exogenous and three endogenous, each indexed by two measured variables. The measurement portion of the model is therefore defined by ten equations, four in x and six in y, and the structural portion by three equations (see Appendix A). For these equations, the parameters defining the model specify the covariance structure that holds for the model. This structure can be specified to hold for the latent response variables (i.e., the y*) or for the observed categorical variables (i.e., the y).

Insert Figure 2 About Here

The present study arose from concerns about the validity of conclusions drawn from the estimation of Lisrel models containing categorical indicators; these conclusions are generally couched in terms of the latent response variables. Thus, in order to examine the distortion in the true structural relationship caused by the use of artificially categorized variables, the model was specified to hold for the latent response variables and the data were generated as continuous variables and subsequently categorized. Since Boomsma (1983) found minor effects of number of categories on the parameter estimates and Muthen (1983a) showed that the effect of categorization on the product-moment correlations was greatest for dichotomous variables, it was decided to categorize the continuous variables as dichotomies.

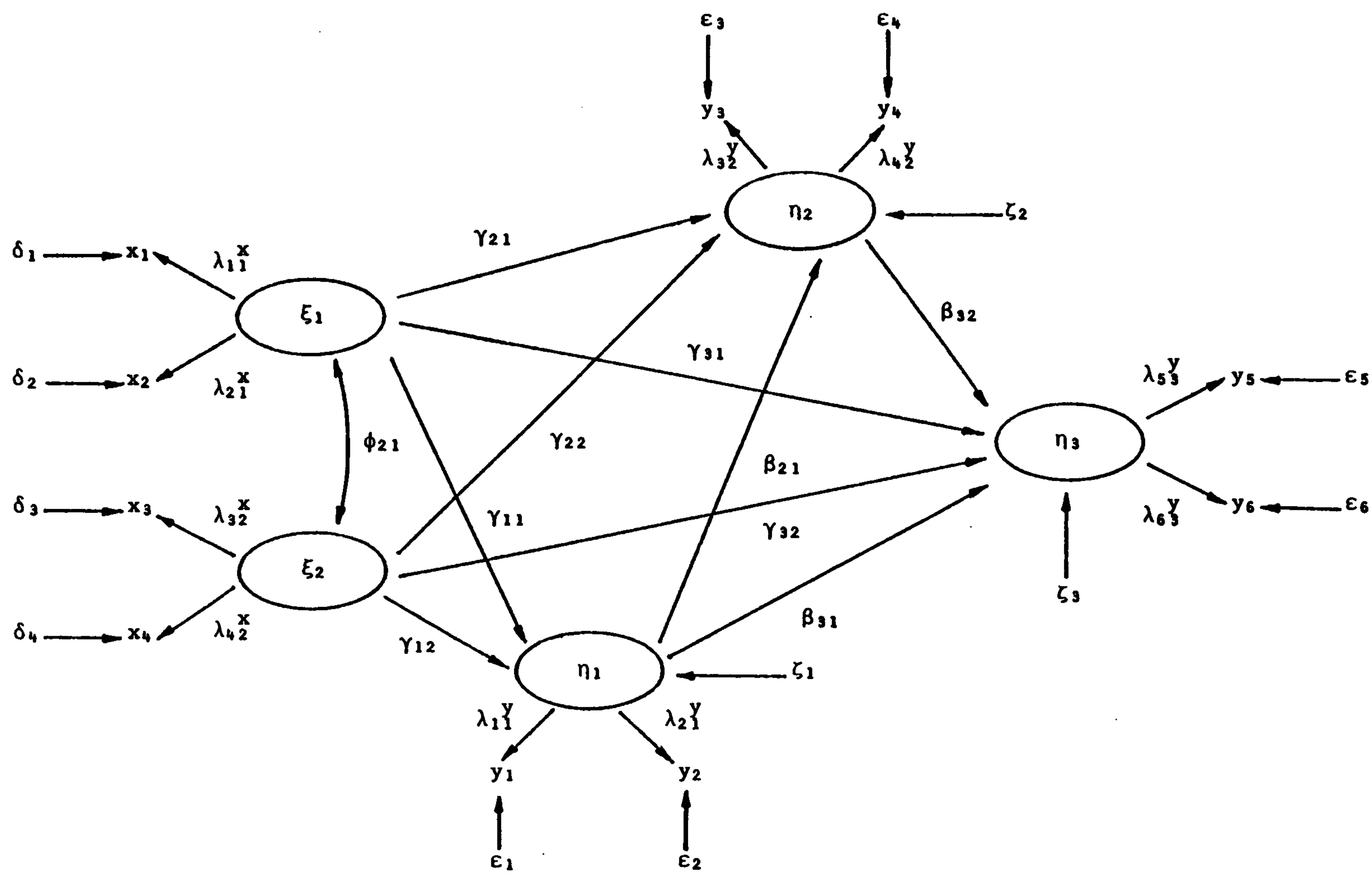

Figure 2. Structural Equation Model Used for Data Generation

The Data

In a study of the robustness of LISREL parameter estimates against small sample size, Boomsma (1982, 1983) found that a sample of at least 200 was necessary in order to obtain reliable parameter estimates and to avoid non-convergence problems. However, for this study a sample size of 500 was used in order to avoid a possible confounding of the effects of correlation type and estimation procedure by small sample size. Because of the randomness involved in the generation of the data, parameter estimates from any single data set can vary unpredictably from the fixed parameters, and the extent of this variation is an important consideration in the determination of robustness. Therefore, it was decided to generate observations on the ten variables for 50 data sets of 500 cases each. These data were generated according to the parameters of the thirteen equations defining the model (see Appendix A).

A SAS PROC MATRIX (SAS Institute Inc., 1982) program (see Appendix B) incorporating these equations was written to generate the data. Values for the two latent ξ variables were generated from a standard normal distribution and fixed for all replications. The η latent variables were allowed to vary. Values for each of the ten variables were then determined from the generation of standard normal errors for each equation. The errors were multiplied by an appropriate constant so that each of the ten variables would have an approximate standard normal distribution. These data represent the y* variables (latent response variables) and meet the assumptions necessary for the application of the maximum likelihood procedure to the estimation of Lisrel models.

Pearson product-moment correlations measuring the associations among these variables were computed, and both maximum likelihood and unweighted least squares estimates obtained for the model using these data. These estimates are the appropriate assessment of the structural relationships implied by the model. Deviations from the specified parameters that resulted from the estimation of the model with these data are due to the randomness involved in the data generation and can be considered random error. The distortion in the true structural relationship caused by the categorization of the continuous variables is then assessed by comparing the estimates resulting from the use of the continuous data to those estimates resulting after the continuous variables have been categorized.

Three cases of non-normality were considered by dichotomizing with varying degrees of skewness, the data generated for indicators of the two endogenous latent factors used as independent variables. Thus, y_1 and y_2, the indicators of η_1, and y_3 and y_4, the indicators of η_2 were dichotomized in each case, leaving the remaining six variables continuous. The first case examines only the effect of categorization by dichotomizing the variables so that a symmetrical distribution with no skewness results. Thus, fifty percent of the cases were in each category. The degrees of skewness used in the second and third cases were suggested by the results of previous studies reviewed above. In particular, the bias in Pearson product-moment correlations was greatest for the more highly skewed variables and for variables skewed in opposite directions (Muthen, 1983a); moreover, Boomsma (1983) reported that when the absolute value of the skewnesses of the variables was larger than one, maximum likelihood estimation was not to be recommended. Considering these results, it was decided to dichotomize each of four variables, y_1, y_2, y_3, and y_4, with a skewness of 1.5 (80% vs. 20%) for the second case. In the third case, the skewness for the indicators of η_1 (y_1 and y_2) was set at 1.5 (80% vs. 20%) and for η_2 (y_3 and y_4) was -1.5 (20% vs. 80%).

Two types of correlation matrices were then computed for each case; one consisting only of Pearson product-moment correlations and one consisting of tetrachoric, polyserial, and product-moment correlations as appropriate. That is, tetrachoric correlations were calculated between two categorical variables; polyserial correlations were calculated between a categorical and a continuous variable; product-moment correlations were calculated between two continuous variables. Since the observed variables were generated as standard normal variables and then dichotomized, the tetrachoric and polyserial correlations are estimates of the associations between the latent response variables and other variables in the model, while the product-moment correlations are biased estimates of these associations.

Determination of Robustness

For each case, the two types of correlation matrices were used to estimate the model, first with the maximum likelihood procedure and then with the unweighted least squares procedure. This was repeated for each sample, giving fifty sets of parameter estimates for each combination of correlation type and procedure within each case. After the variables were dichotomized, the normality assumption for the observed variables was no longer met. Therefore, differences between the model parameters and the parameter estimates cannot be attributed just to random error, but also to correlation type, skewness, or estimation procedure.

Of primary interest in this study were the effects of correlation type and estimation procedure on the structural parameter estimates; specifically, the Lisrel gamma and beta estimates. The average estimates of these parameters resulting from the analyses of matrices containing the tetrachoric and polyserial correlations were expected to more closely approximate both the model parameters and the estimates from the use of the continuous variables than those obtained from the analyses of Pearson product-moment matrices. For greater degrees of skewness, the Pearson product-moment estimates were expected to be more biased, regardless of estimation procedure.

As a first step in the determination of the robustness of the LISREL structural parameter estimates, several statistics associated with the individual estimates were computed. For each combination of correlation type and estimation procedure, the parameter estimates were averaged across replications and the variances about their own mean computed. Those averages closely approximating the parameters were considered robust estimates, but the proximity of the estimates to the parameters should only be considered in conjunction with the variances of the estimates. Unbiased estimates with large variances indicate that the estimates approximate the population values on the average, but for a single sample, the estimate may be far from the true parameter.

The robustness of the parameter estimates was determined by the examination of bias. Three measures of bias were computed. The first measure of bias was computed from the mean square error of the estimate (MSE = σ^2 + $(\text{Bias})^2$). This bias represents the inflation in the mean square error caused by differences between the parameter estimate and the parameter itself. Comparisons between the MSE and the variance of the estimate indicate the degree to which the variance estimates the true error variance.

The next two measures reflect bias in the parameter estimate itself. The relative bias of the average estimate for parameter ω across the 50 replications was computed by $|(\bar{\hat{\omega}} - \omega)|/\omega$, giving an indication of whether there is a consistent overestimation or underestimation of the population parameters. When multiplied by 100, the bias in this instance is expressed as a percent of the parameter. Values greater than .10 for this measure of bias were considered substantial enough to warrant concern. The degree of bias with respect to the sampling distribution was given by $(\bar{\hat{\omega}} - \omega)/(s_{\hat{\omega}}/\sqrt{n})$. This latter statistic is distributed as Student's t with 49 degrees of freedom and tests whether the average estimate is significantly different from the parameter. The actual significance tests were not carried out, but when this measure of bias approached $|1.8|$, the parameter was considered to be in the extremes of the sampling distribution.

The robustness of the standard errors of the estimates can be examined only for the maximum likelihood estimates resulting from the analyses of the product-moment matrices. As mentioned previously, ULS estimates of standard errors are not available, and the variances and covariances of the tetrachoric and polyserial correlations used in the computation of the maximum-likelihood standard errors are incorrect. Three measures of bias of the ML product-moment standard errors were computed as before but with the parameter ω replaced by $\bar{\hat{\omega}}_c$, the mean estimated standard error for the continuous data.

The preceding analyses examine the individual parameter estimates in relation to their corresponding parameters. The results indicate the degree of confidence that can be placed on the estimates in terms of their approximation of population parameters. As mentioned previously, differences between the structural parameter estimates resulting from the estimation of the model with the continuous variables used as the indicator variables and the model parameters are due to the randomness involved in the data generation procedure. However, some of these estimates differed substantially from their corresponding parameters, and these differences affect the parameter estimates obtained after the data were dichotomized.

Since the continuous variables were generated as standard normal variables, the estimation of the model with these data does represent the best sample estimate of the structure implied by the model, regardless of the degree of approximation of the model parameters. This is analogous to the estimation of a proposed model using data precisely meeting the assumptions underlying the application of the maximum likelihood procedure (i.e., multivariate normality). Any distortion in the structural relationship implied by the model caused by the subsequent categorization of some of the variables is then assessed by comparing the estimates from the continuous data to those resulting after dichotomization. These comparisons also indicate whether substantive conclusions are affected by the way in which the data are measured and analyzed.

Comparisons of structural parameter estimates were made using a 2x3 factorial analysis of variance. The rows were defined by estimation procedure -- maximum likelihood versus unweighted least squares. The columns were defined by correlation type -- (1) continuous data estimated with product-moment correlations; (2) dichotomized variables estimated with product-moment correlations; and (3) dichotomized variables estimated with a mixed matrix of tetrachoric, polyserial, and product-moment correlations. Based on the findings of earlier studies (e.g., Huba and

Bentler, 1983; Huba and Harlow, 1984), no differences were expected to be found between estimation procedures (i.e., maximum likelihood versus unweighted least squares). However, differences were expected between correlation types. The estimation of dichotomized variables with a mixed matrix should more closely approximate the estimates from the continuous data than would the dichotomized variables estimated with product-moment correlations. Once these overall assessments were made, specific post hoc comparisons were made using Tukey's Highest Significant Difference method.

While the maximum likelihood estimates are known to be normally distributed, the assumption of normality for the distribution of the unweighted least squares estimates can be questioned because of the skewness and categorization of the variables. In order to verify the appropriateness of the use of analysis of variance with these data, a random selection of sets of the unweighted least squares parameter estimates were tested for normality by a series of Pearson's χ^2 goodness-of-fit tests.

CHAPTER 3

ROBUSTNESS OF LISREL MAXIMUM LIKELIHOOD AND UNWEIGHTED LEAST SQUARES ESTIMATES

The purpose of this research was to examine the effect of correlation type on the robustness of LISREL maximum likelihood and unweighted least squares estimates of the structural parameters of models containing categorical variables as indicators of latent factors. The robustness of the estimates was determined using data generated to fit the equations (see Appendix A) defining the Lisrel model shown in Figure 2. Fifty sets of data were generated using the SAS PROC MATRIX (SAS Institute, 1982) program given in Appendix B, each set containing 500 observations on 10 standard normal variables. The average Pearson product-moment correlations for these continuous variables are given in Table 2. These averages will be compared to the average correlations computed after the variables have been dichotomized.

Insert Table 2 About Here

The robustness of the estimates of the model parameters was examined for three cases of non-normality determined by the dichotomization of the manifest indicators of η_1 and η_2 with varying degrees of skewness. In the first case, the

Table 2. Average Correlations for Continuous Variables

	X_1	X_2	X_3	X_4	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6
X_1	--									
X_2	.697	--								
X_3	.345	.344	--							
X_4	.337	.335	.802	--						
Y_1	.115	.117	.108	.110	--					
Y_2	.124	.128	.120	.120	.664	--				
Y_3	.258	.261	.323	.318	.519	.553	--			
Y_4	.234	.234	.288	.282	.472	.509	.753	--		
Y_5	.068	.069	.214	.213	.527	.566	.504	.458	--	
Y_6	.065	.067	.204	.209	.506	.547	.479	.433	.731	--

skewness of each variable was zero (50% of the data were in each category); in the second case, each variable had a skewness of 1.5 (80% vs. 20%); and in the third case, the indicators of η_1 had a skewness of 1.5 (80% vs. 20%) and the indicators of η_2 had a skewness of -1.5 (20% vs. 80%). Two types of correlation matrices were computed for each case; one consisting only of Pearson product-moment correlations, and one consisting of tetrachoric, polyserial, and product-moment correlations as appropriate. Tables 3, 4, and 5 give the averages of each type of correlation matrix for Cases 1, 2, and 3, respectively. It is evident, by comparing these matrices to the matrix of correlations among the continuous variables given in Table 2, that in every case the mixed matrix more closely approximates the correlations while the product-moment matrix underestimates the associations involving dichotomous variables.

Insert Tables 3, 4, and 5 About Here

The focus of this investigation was on the effect of correlation type on the LISREL structural parameter estimates (betas and gammas), their standard errors, and the maximum likelihood chi-squares. These estimates were obtained by using the two types of correlation matrices to estimate the model, first with the maximum likelihood

Table 3. Average Correlations for Case 1*

	X_1	X_2	X_3	X_4	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6
X_1	---	.697	.345	.337	.095	.094	.203	.187	.068	.065
X_2	.697	---	.344	.335	.092	.098	.210	.186	.069	.067
X_3	.345	.344	---	.802	.087	.101	.255	.232	.214	.204
X_4	.337	.335	.802	---	.089	.100	.257	.227	.213	.209
Y_1	.119	.115	.109	.112	---	.462	.342	.305	.419	.405
Y_2	.117	.123	.127	.125	.663	---	.366	.337	.456	.434
Y_3	.254	.263	.320	.322	.512	.544	---	.538	.405	.382
Y_4	.234	.232	.291	.284	.460	.504	.746	---	.371	.346
Y_5	.068	.069	.214	.213	.542	.571	.507	.464	---	.731
Y_6	.065	.067	.204	.209	.507	.545	.480	.434	.731	---

*Product-moment matrix is above the diagonal and mixed matrix below.

Table 4. Average Correlations for Case 2*

	X_1	X_2	X_3	X_4	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6
X_1	---	.697	.345	.337	.079	.083	.179	.161	.068	.065
X_2	.697	---	.344	.335	.085	.088	.181	.161	.069	.067
X_3	.345	.344	---	.802	.075	.079	.234	.201	.214	.204
X_4	.337	.335	.802	---	.077	.079	.227	.196	.213	.209
Y_1	.113	.121	.107	.110	---	.419	.305	.270	.364	.351
Y_2	.118	.125	.113	.113	.657	---	.333	.297	.393	.381
Y_3	.253	.255	.332	.322	.511	.550	---	.506	.361	.339
Y_4	.229	.227	.285	.278	.462	.500	.750	---	.317	.302
Y_5	.068	.069	.214	.213	.520	.559	.516	.454	---	.731
Y_6	.065	.067	.204	.209	.503	.544	.485	.433	.731	---

*Product-moment matrix is above the diagonal and mixed matrix below.

Table 5. Average Correlations for Case 3*

	X_1	X_2	X_3	X_4	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6
X_1	---	.697	.345	.337	.079	.083	.182	.163	.068	.065
X_2	.697	---	.344	.335	.085	.088	.183	.160	.069	.067
X_3	.345	.344	---	.802	.075	.079	.223	.202	.214	.204
X_4	.337	.335	.802	---	.077	.079	.222	.199	.213	.209
Y_1	.113	.121	.107	.110	---	.419	.201	.190	.364	.351
Y_2	.118	.125	.113	.113	.657	---	.207	.199	.393	.381
Y_3	.263	.266	.322	.322	.521	.546	---	.512	.351	.334
Y_4	.235	.232	.291	.288	.481	.517	.757	---	.323	.304
Y_5	.068	.069	.214	.213	.520	.559	.503	.463	---	.731
Y_6	.065	.067	.204	.209	.503	.544	.477	.435	.731	---

*Product-moment matrix is above the diagonal and mixed matrix below.

procedure and then with the unweighted least squares procedure. Estimates were also obtained for the measurement parameters (lambdas). While no measures of bias were computed for these estimates, the averages, variances, and mean square errors are given in Appendices C through F. No further analyses were conducted on the measurement parameters, but the impact of correlation type on these measurement-model estimates was found to be consistent with the results of previous studies (e.g., Muthen, 1983a; Olsson, 1979b). That is, with both maximum likelihood and unweighted least squares estimation procedures, the factor loadings for the dichotomized variables (y_1, y_2, y_3, and y_4) were consistently underestimated when product-moment matrices were analyzed. The analysis of matrices containing tetrachoric, polyserial, and product-moment matrices produced measurement parameter estimates with no discernable differences from the model parameters. There were also no apparent differences between the ML and ULS estimates. These results were consistent for each of the three cases, with slightly more bias appearing in the product-moment estimates in Case 3 where the variables were skewed in the opposite direction.

Analyses Using y^* Variables

The quality of the data generation procedure can be checked by examining the results of the analysis of the continuous data (the latent response variables prior to categorization) since any deviations of these estimates from the model parameters are due to the randomness involved in the data generation process. Statistics for the structural parameter estimates using these data are shown in Table 6. The maximum likelihood chi-square statistic gives an indication of how well the generated data fit the model. For each of the 50 estimations of the model, this statistic was compared with the critical value of $\chi^2_{.05;25} = 37.6526$. With an alpha level of .05, probability theory indicates that for the 50 data sets used to estimate the model, 2.5 rejections should occur. That is, 2.5 times out of 50 the model would be rejected as not fitting the data by chance alone. Since only two rejections occurred, overall the data fit the model very well.

Insert Table 6 About Here

In Table 6, the measures of variability and bias are given for each parameter estimate. For example, the population value for β_{21} is .663, and the average maximum likelihood estimate of this parameter is .667. Both the variance and mean square error are given, and comparisons of the two indicate the accuracy of the sample variance in estimating the error variance. Since the variance and mean

Table 6. Structural Parameter Estimates, Sampling Variability, and Bias Using Continuous Variables.

Parameter	ML Estimate	ML Variability*	ML Bias**	ULS Estimate	ULS Variability*	ULS Bias**
β_{21} = .663	.667	.0013 .0013	0 .0063 .8342	.668	.0013 .0013	0 .0074 .9760
β_{31} = .669	.689	.0057 .0061	.0200 .0294 1.8402	.691	.0059 .0064	.0224 .0326 2.0072
β_{32} = .127	.109	.0071 .0074	.0173 .1386 -1.4816	.106	.0074 .0079	.0224 .1693 -1.7678
γ_{11} = .132	.134	.0034 .0034	0 .0182 .2901	.134	.0031 .0031	0 .0182 .3052
γ_{12} = .105	.097	.0034 .0035	.0100 .0790 -1.0032	.097	.0033 .0034	.0100 .0800 -1.0366
γ_{21} = .126	.115	.0016 .0017	.0100 .0873 -1.9543	.115	.0015 .0016	.0100 .0841 -1.9621
γ_{22} = .238	.234	.0017 .0017	0 .0181 - .7362	.232	.0017 .0017	0 .0252 -1.0399
γ_{31} = -.153	-.155	.0025 .0025	0 .0144 - .3118	-.153	.0024 .0024	0 0 0
γ_{32} = .188	.193	.0023 .0023	0 .0266 .7350	.194	.0024 .0024	0 .0309 .8439
χ^2		24.1654				
Var (χ^2)		52.1977				
Number rejections		2				

*The entries under variability are variance and mean square error.
**The entries under bias are error bias, relative bias, and bias with respect to sampling distribution.

square error of this estimate are the same, .0013, there is no inflation of the mean square error caused by differences between the individual estimates and the parameter, and the variance is an unbiased estimator of true error variance. The first entry under Bias, 0, reflects the absence of this type of bias. The second entry under bias, .0063, indicates that the parameter is overestimated by .63%, and the last entry, .8342, shows that the average estimate is .8342 standard deviations from the center of the sampling distribution. The entries for the ULS estimates are interpreted in the same manner.

Three of the parameter estimates exhibit enough bias to warrant concern. The estimates for β_{31}, .689 (ML) and .691 (ULS), compared to the parameter of .669, and γ_{21}, .115 (ML) and .115 (ULS), compared to the parameter of .126, show biases with respect to sampling distributions of 1.8402, 2.0072, -1.9543, and -1.9621, respectively. This indicates that these estimates lie in the extremes of their sampling distributions. However, when this bias is considered in conjunction with the bias expressed as a percent of the parameter, it seems not to be serious. For β_{31}, these relative biases are .0294 and .0326 for the ML and ULS estimates, respectively, and for γ_{21} are .0873 and .0841. The estimates for β_{32}, .109 (ML) and .106 (ULS), have biases relative to the parameter itself of .1386 and .1693, indicating that the parameter is underestimated by as much as 16.93%. The bias present in these estimates is probably a function of the randomness in the data generation procedure and the use of only 50 replications, but should be kept in mind in the interpretation of subsequent results.

The variances of the estimates in general are unbiased estimates of the population error variance. The variances of the estimates of β_{31}, .0057 and .0059 for ML and ULS, respectively, and of β_{32}, .0071 and .0074 for ML and ULS, respectively, slightly underestimate the corresponding mean square errors, .0061, .0064, .0074, and .0079, but this is not considered substantial.

Analyses Using Dichotomized Variables

Maximum Likelihood Chi-square

Results of the analyses for each of the three cases of dichotomization are given in Tables 7, 8, and 9. The likelihood ratio chi-square statistic shows little effect of skewness or categorization in terms of number of rejections for lack of fit when product-moment matrices are analyzed, although the rejection of 4 of the 50 estimations of the model in Case 1 is slightly more than the expected 2.5. This is in contrast to previous studies (e.g., Boomsma, 1983; Muthen, 1983a; Olsson, 1979b) where inflation was found in the maximum likelihood chi-square for increasing skewness. The empirical distribution of the chi-square statistic does appear to be affected. For the model used in this study, this statistic is distributed as χ^2 with 25 degrees of freedom. In Cases 1 and 2, the variance is slightly larger than expected, and in Case 3, both the mean and variance are considerably smaller than expected (22.7272 and 36.2031, respectively), indicating that the distribution of the chi-square statistic in Case 3 deviates considerably from χ^2 with 25 degrees of freedom.

Insert Tables 7, 8, and 9 About Here

The highly inflated chi-squares occurring in the analyses of the mixed matrices cannot be compared with previous studies that calculated goodness-of-fit measures without using LISREL, for, in LISREL, this statistic is computed on the assumption that the correlations used in the analysis are product-moment correlations. Based on these chi-squares, the population model was rejected 45, 49, and 48 times for Cases 1, 2, and 3, respectively. This should be an important consideration for researchers estimating substantive Lisrel models for it indicates that the rejection of an estimated model using tetrachoric or polyserial correlations would probably not be a function of the data not fitting the model, but of the type of correlation used in the analysis. In any event, the user would not know the cause of rejection.

Table 7. Structural Parameter Estimates and Sampling Variability for Case 1 (50/50).

	Pearson Matrix				Mixed Matrix			
	ML		ULS		ML		ULS	
Parameter	Estimate	Variability*	Estimate	Variability*	Estimate	Variability*	Estimate	Variability*
β_{21} = .663	.626	.0030 .0044	.625	.0031 .0045	.661	.0027 .0027	.660	.0028 .0028
β_{31} = .669	.636	.0075 .0086	.638	.0077 .0087	.678	.0107 .0108	.679	.0113 .0114
β_{32} = .127	.146	.0119 .0122	.142	.0127 .0130	.131	.0168 .0168	.129	.0175 .0175
γ_{11} = .132	.120	.0047 .0048	.121	.0044 .0045	.125	.0051 .0052	.127	.0047 .0047
γ_{12} = .105	.102	.0054 .0054	.101	.0052 .0052	.106	.0061 .0061	.106	.0056 .0056
γ_{21} = .126	.116	.0022 .0023	.117	.0022 .0023	.120	.0025 .0026	.120	.0025 .0025
γ_{22} = .238	.219	.0031 .0034	.219	.0030 .0033	.231	.0038 .0039	.230	.0035 .0035
γ_{31} = -.153	-.146	.0037 .0038	-.145	.0035 .0036	-.152	.0042 .0042	-.152	.0039 .0039
γ_{32} = .188	.187	.0039 .0039	.188	.0038 .0038	.180	.0046 .0047	.182	.0044 .0044
χ^2		24.8112				57.5588		
Var (χ^2)		63.9152				445.2100		
Number Rejections		4				45		

*The entries under variability are variance and mean square error.

Table 8. Structural Parameter Estimates and Sampling Variability for Case 2 (80/20).

	Pearson Matrix				Mixed Matrix			
	ML		ULS		ML		ULS	
Parameter	Estimate	Variability*	Estimate	Variability*	Estimate	Variability*	Estimate	Variability*
β_{21} = .663	.614	.0069 .0093	.612	.0067 .0093	.665	.0063 .0063	.665	.0058 .0058
β_{31} = .669	.599	.0143 .0192	.599	.0138 .0187	.693	.0238 .0244	.697	.0216 .0223
β_{32} = .127	.111	.0133 .0136	.111	.0127 .0130	.111	.0266 .0268	.107	.0235 .0239
γ_{11} = .132	.120	.0056 .0058	.120	.0056 .0057	.135	.0070 .0070	.136	.0069 .0069
γ_{12} = .105	.078	.0055 .0062	.080	.0054 .0061	.089	.0069 .0071	.092	.0069 .0071
γ_{21} = .126	.091	.0044 .0056	.093	.0040 .0051	.102	.0064 .0070	.103	.0056 .0062
γ_{22} = .238	.213	.0026 .0032	.212	.0026 .0033	.247	.0034 .0035	.244	.0035 .0035
γ_{31} = −.153	−.133	.0044 .0048	−.130	.0042 .0047	−.158	.0061 .0061	−.157	.0057 .0057
γ_{32} = .188	.217	.0034 .0042	.216	.0036 .0043	.196	.0053 .0054	.197	.0053 .0054
χ^2		25.9206				78.3846		
Var (χ^2)		59.3208				689.6611		
Number Rejections		2				49		

*The entries under variability are variance and mean square error.

Table 9. Structural Parameter Estimates and Sampling Variability for Case 3 (80/20 vs. 20/80).

	Pearson Matrix				Mixed Matrix			
	ML		ULS		ML		ULS	
Parameter	Estimate	Variability*	Estimate	Variability*	Estimate	Variability*	Estimate	Variability*
β_{21} = .663	.381	.0030 .0825	.384	.0031 .0811	.690	.0122 .0129	.684	.0115 .0119
β_{31} = .669	.557	.0053 .0179	.556	.0052 .0180	.787	.0571 .0709	.774	.0519 .0629
β_{32} = .127	.283	.0042 .0285	.281	.0041 .0279	-.011	.0685 .0875	.010	.0648 .0785
γ_{11} = .132	.122	.0058 .0059	.121	.0056 .0057	.139	.0077 .0077	.137	.0069 .0069
γ_{12} = .105	.077	.0056 .0064	.079	.0054 .0061	.083	.0072 .0077	.091	.0069 .0071
γ_{21} = .126	.126	.0042 .0042	.127	.0042 .0042	.103	.0098 .0104	.111	.0082 .0084
γ_{22} = .238	.224	.0034 .0036	.222	.0035 .0038	.242	.0061 .0061	.239	.0065 .0065
γ_{31} = -.153	-.158	.0030 .0030	-.156	.0029 .0029	-.148	.0080 .0080	-.141	.0097 .0099
γ_{32} = .188	.179	.0032 .0033	.178	.0034 .0035	.228	.0088 .0104	.222	.0078 .0089
χ^2		22.7272				103.2198		
Var (χ^2)		36.2031				1983.9096		
Number Rejections		0				48		

*The entries under variability are variance and mean square error.

Parameter Estimates

An examination of the structural parameter estimates indicates that the analyses of the product-moment matrices generally underestimated the model parameters, particularly for the beta coefficients. For example, the maximum likelihood estimation of the product-moment matrices produced average estimates for β_{21} of .626, .614, and .381 in Cases 1, 2, and 3, respectively. Each of these underestimates the true parameter of .663, and the average estimate of .667 calculated from the continuous data.

The analyses of the mixed matrices produced estimates that are, on the average, closer to the parameters. However, they do in general exhibit greater variability with increasing skewness. The increased variation is particularly evident in Case 3 in which the indicators of the two η variables were skewed in opposite directions. Again, for example, notice the estimates of β_{21}. The maximum likelihood analyses of mixed matrices produced parameter estimates of .661, .665, and .690 for the three cases. Each of these is closer to the parameter, .663, than the corresponding estimates of .626, .614, and .381 obtained from the analysis of the product-moment matrices. The variabilities of these estimates were comparable to that of their corresponding product-moment estimates in Cases 1 and 2, but in Case 3, the product-moment estimate (.381) had a variance of .0030 as opposed to a variance of .0122 for the estimate obtained when mixed matrices were analyzed (.690). The increased variability of the estimates for increasing skewness has also been reported in the studies of Boomsma (1983) and Muthen and Kaplan (1984).

In Case 1, comparisons of the measures of variability for each parameter estimate indicate that the variances of the estimates in general are close approximations of the mean square errors, regardless of estimation procedure or correlation type. However, the biases present in the product-moment estimates in Case 2 cause inflation in the mean square errors of these estimates and underestimation of the mean square errors by the sample variances. The variances of the estimates from the analyses of the mixed matrices more closely approximate the corresponding mean square errors. In Case 3, the variances of the beta parameter estimates considerably underestimate the error variances regardless of estimation procedure or correlation type, but the variances of the gamma estimates are not similarly affected.

These results are consistent for both estimation procedures; that is, mixed matrices in general provided estimates closer to the parameters but with slightly greater variability than did product-moment matrices regardless of whether the parameters were estimated with maximum likelihood or unweighted least squares. The question now is whether the differences between any of the estimates and their corresponding parameters are substantial enough to cause misleading inferences to be made concerning the population parameters. This question, and the robustness of the estimates, is addressed separately for each of the three cases described above.

Robustness of Estimates in Case 1

The first case of non-normality examines the effect of categorization only. The manifest indicators for the two independent endogenous variables, η_1 and η_2, were dichotomized with fifty percent of the cases in each category, giving a symmetrical distribution with no skewness. The three indications of bias (error bias, relative bias, and bias with respect to sampling distribution) of the structural parameter estimates for each combination of correlation type and estimation procedure are given in Table 10.

Insert Table 10 About Here

When the matrix used in the analysis is the tetrachoric, polyserial, and product-moment correlation matrix (mixed matrix) both the maximum likelihood (ML) and unweighted least squares (ULS) parameter estimates are

Table 10. Bias of Structural Parameter Estimates for Case 1 (50/50).

	Pearson Matrix				Mixed Matrix			
	ML		ULS		ML		ULS	
Parameter	Estimate	Bias*	Estimate	Bias*	Estimate	Bias*	Estimate	Bias*
β_{21} = .663	.626	.0374	.625	.0374	.661	0	.660	0
		.0557		.0569		.0035		.0041
		-4.7440		-4.7774		-.3128		-.3609
β_{31} = .669	.636	.0332	.638	.0316	.678	.0100	.679	.0100
		.0487		.0465		.0141		.0151
		-2.6588		-2.5047		.6422		.6725
β_{32} = .127	.146	.0173	.142	.0173	.131	0	.129	0
		.1457		.1213		.0291		.0165
		1.2012		.9654		.2019		.1122
γ_{11} = .132	.120	.0100	.121	.0100	.125	.0100	.127	0
		.0894		.0818		.0500		.0402
		-1.2199		-1.1518		-.6518		-.5455
γ_{12} = .105	.102	0	.101	0	.106	0	.106	0
		.0324		.0371		.0124		.0076
		- .3275		- .3835		.1182		.0754
γ_{21} = .126	.116	.0100	.117	.0100	.120	.0100	.120	0
		.0817		.0738		.0500		.0468
		-1.5562		-1.4112		-.8874		-.8344
γ_{22} = .238	.219	.0173	.219	.0173	.231	.0100	.230	0
		.0790		.0807		.0286		.0319
		-2.3996		-2.4911		-.7755		-.9124
γ_{31} = -.153	-.146	.0100	-.145	.0100	-.152	0	-.152	0
		.0490		.0529		.0059		.0039
		.8708		.3697		.0985		.0683
γ_{32} = .188	.187	0	.188	0	.180	.0100	.182	0
		.0032		0		.0404		.0319
		- .0678		0		-.7915		-.6390

*The entries under bias are error bias, relative bias, and bias with respect to sampling distribution.

robust against categorization, exhibiting no substantial bias by any of the three measures. However, the analysis of product-moment matrices produced substantial bias in three of the parameters (β_{21}, β_{31}, and γ_{22}) when considered with respect to their sampling distributions. For the maximum likelihood estimates, this bias is -4.7440 for β_{21}, -2.6588 for β_{31}, and -2.3996 for γ_{22}, with the parameter underestimated in each instance. There are also comparable biases in the unweighted least squares estimates (-4.7774, -2.5047, and -2.4911, respectively). The magnitude of this bias is probably a function of the small variance of the estimates, for when the bias of these estimates is considered as relative bias (relative to the parameter), none appears to be too serious. The bias of these three parameters in this instance is .0557, .0487, and .0790, respectively for the ML estimates, and .0569, .0465, and .0807 for the ULS estimates. In contrast, the large relative bias of the estimates of β_{32} (.1475 and .1213 for the ML and ULS estimates, respectively) did not appear as substantial bias because of the larger variability of those estimates. In the analysis of the continuous data, the estimate for β_{32} also showed the largest variability and greatest relative bias, but underestimated the model parameter.

Results pertaining to the standard errors of the parameter estimates from the maximum likelihood analyses of product-moment matrices are given in Table 11. Any bias detected in the standard errors has impact on the significance tests of the parameter estimates. The standard errors reported here are not those of the standardized parameter estimates given in Table 7, but for the corresponding unscaled estimates. As mentioned previously, since the significance tests are the same for both standardized and unscaled estimates, the effects of correlation type should be the same. The bias reported for the standard errors was computed with respect to the mean standard error resulting from the analysis of the continuous data.

Insert Table 11 About Here

In Case 1, the standard errors of the beta coefficients exhibit serious bias by all three measures, and the seriousness of this bias is primarily reflected in the relative bias. For example, the standard error for the estimate of β_{31} is .1534 in Case 1. When compared to the corresponding standard error of .0914 for the estimate obtained when the continuous data was analyzed, the bias is .6783 indicating an overestimation of 67.83%. This

Table 11. Standard Errors and Bias of Standard Errors for Unstandardized Structural Parameter Estimates.

Parameter Estimated	Continuous	Case 1		Case 2		Case 3	
	Standard* Error	Standard* Error	Bias**	Standard* Error	Bias**	Standard* Error	Bias**
β_{21}	.0568	.0913	.0346	.1054	.0480	.0932	.0361
		(.0001)	.6074	(.0004)	.8556	(.0002)	.6408
		(.0013)	23.0000	(.0027)	18.0000	(.0015)	18.2000
β_{31}	.0914	.1534	.0616	.1821	.0900	.1191	.0283
		(.0014)	.6783	(.0062)	.9923	(.0007)	.3031
		(.0052)	11.9231	(.0143)	8.1712	(.0015)	7.4865
β_{32}	.0783	.1236	.0458	.1353	.0566	.0852	.0100
		(.0009)	.5785	(.0025)	.7280	(.0001)	.0881
		(.0030)	10.5349	(.0057)	8.0282	(.0002)	4.6000
γ_{11}	.0494	.0463	.0031	.0458	.0036	.0461	.0033
		(4.00 E-6)	.0628	(1.16 E-5)	.0729	(9.00 E-6)	.0668
		(1.36 E-5)	-10.3333	(2.46 E-5)	-7.2000	(1.99 E-5)	-8.2500
γ_{12}	.0476	.0448	.0028	.0440	.0036	.0443	.0033
		(3.24 E-6)	.0588	(1.02 E-5)	.0756	(7.29 E-6)	.0693
		(1.08 E-5)	- 9.3333	(2.30 E-5)	-7.2000	(1.81 E-5)	-8.2500
γ_{21}	.0422	.0469	.0047	.0488	.0096	.0495	.0097
		(4.84 E-6)	.1114	(7.84 E-6)	.1564	(6.25 E-6)	.1730
		(2.72 E-5)	15.6667	(.0001)	16.5000	(.0001)	18.2500
γ_{22}	.0409	.0456	.0047	.0472	.0063	.0481	.0098
		(4.84 E-6)	.1149	(5.29 E-6)	.1540	(3.61 E-6)	.1760
		(2.65 E-5)	15.6667	(4.51 E-5)	21.0000	(.0001)	24.0000
γ_{31}	.0422	.0478	.0056	.0507	.0084	.0486	.0064
		(8.41 E-6)	.1327	(2.92 E-5)	.2014	(6.76 E-6)	.1517
		(3.98 E-5)	14.0000	(.0001)	10.6250	(4.80 E-5)	16.0000
γ_{32}	.0434	.0497	.0086	.0525	.0078	.0479	.0045
		(2.60 E-5)	.1452	(3.84 E-5)	.2097	(6.25 E-6)	.1037
		(.0001)	9.0000	(.0001)	10.1111	(2.64 E-5)	11.2500

*Standard errors are the maximum-likelihood estimates for unstandardized structural parameter estimates resulting from the analyses of product-moment matrices. Entries in parentheses are their variance and mean square error, respectively.

**The entries under bias are error bias, relative bias, and bias with respect to sampling distribution.

inflation in the standard errors would lead far too often to a conclusion that the parameters were zero in the population. These results are contradictory to those reported by Boomsma (1983) who found no bias in the LISREL standard errors. However, this inconsistency may be a function of Boomsma's specification of the structure holding for the observed categorical variables, whereas in this study, the structure is seen to hold for the latent continuous variables.

The bias of the standard errors of the gamma coefficients appears to be as substantial as that of the betas when the average standard error is considered relative to its sampling distribution. However, this appears to be a function of the very slight variation of the estimates, rather than the magnitude of the bias itself. In fact, the standard errors for γ_{11} (.0463) and γ_{12} (.0448) reflect little relative bias (.0628 and .0588, respectively), and conclusions concerning the significance of the estimates of γ_{11} and γ_{12} would probably not be affected by this slight inflation. The inflation in the standard errors of the estimates of the remaining gammas is large enough to warrant caution in the interpretation of significance tests, although comparisons of the estimated standard errors to the average standard error using the continuous data show little differences.

It can be concluded that both the maximum likelihood and unweighted least squares structural parameter estimates for this model are robust against an effect of categorization when the appropriate correlations are used in the analyses. That is, when using tetrachoric and polyserial correlations for categorical variables with no skewness, one can expect to obtain robust estimates of the population parameters. However, the likelihood ratio chi-square statistic is highly inflated with the analyses of mixed matrices, but this inflation is due to the assumption in LISREL that the correlations are product-moment correlations. Practitioners are cautioned to ignore this statistic as a measure of goodness-of-fit when using tetrachoric or polyserial correlations to estimate their models. When product-moment matrices are analyzed, some of the parameter estimates appear to have substantial bias. However, when the measures of bias are considered simultaneously and in conjunction with the estimates from the continuous data, the estimates are generally robust with both estimation procedures. The likelihood ratio chi-square statistic also appears to be robust with the analysis of product-moment matrices. The standard errors of the beta estimates are clearly non-robust with their inflation resulting in conservative significance tests.

<u>Robustness of Estimates in Case 2</u>

The second case estimated the model with each of the manifest indicators of η_1 and η_2 dichotomized with a skewness of 1.5 (80% in the first category and 20% in the second). Results here show the effects of correlation type and estimation procedure on the parameter estimates when each of the dichotomized variables are skewed in the same direction.

The underestimation by product-moment correlations of the associations involving these variables is greater here than in Case 1 (see Table 4). The impact of this underestimation is evident in the bias that appears in the estimates resulting from the analyses of product-moment matrices (see Table 12). All of these estimates, with the exception of the estimates for γ_{11} (.120 for both the ML and ULS estimates), have large biases either in terms of percentage of the parameter or relative to the sampling distribution. This is true for both the maximum likelihood and unweighted least squares estimates. Each of the parameters were underestimated with the exception of γ_{32}, leading to a general underestimation of the magnitude of the influence of one latent variable on another.

Insert Table 12 About Here

Table 12. Bias of Structural Parameter Estimates for Case 2 (80/20).

Parameter	Pearson Matrix				Mixed Matrix			
	ML		ULS		ML		ULS	
	Estimate	Bias*	Estimate	Bias*	Estimate	Bias*	Estimate	Bias*
β_{21} = .663	.614	.0490 .0733 -4.1255	.612	.0510 .0766 -4.3753	.665	0 .0032 .1870	.665	0 .0030 .1851
β_{31} = .669	.599	.0700 .1054 -4.1716	.599	.0700 .1051 -4.2378	.693	.0245 .0365 1.1192	.697	.0265 .0419 1.3487
β_{32} = .127	.111	.0173 .1283 - .9996	.111	.0173 .1252 - .9958	.111	.0141 .1283 - .7071	.107	.0200 .1598 - .9364
γ_{11} = .132	.120	.0141 .0924 -1.1502	.120	.0100 .0886 -1.1075	.135	0 .0197 .2202	.136	0 .0295 .3327
γ_{12} = .105	.078	.0265 .2552 -2.5643	.080	.0265 .2400 -2.4211	.089	.0141 .1505 -1.3493	.092	.0141 .1267 -1.1290
γ_{21} = .126	.091	.0346 .2754 -3.7008	.093	.0332 .2643 -3.7199	.102	.0245 .1921 -2.1390	.103	.0245 .1865 -2.2127
γ_{22} = .238	.213	.0245 .1059 -3.5000	.212	.0265 .1080 -3.5703	.247	.0100 .0378 1.0860	.244	0 .0252 .7167
γ_{31} = -.153	-.133	.0200 .1333 2.1757	-.130	.0224 .1477 2.4548	-.158	0 .0327 - .4539	-.157	0 .0235 - .3363
γ_{32} = .188	.217	.0283 .1548 3.5539	.216	.0265 .1473 3.2864	.196	.0100 .0420 .7673	.197	.0100 .0452 .8256

*The entries under bias are error bias, relative bias, and bias with respect to sampling distribution.

Three of the parameter estimates resulting from the analyses of the mixed matrices exhibit enough bias to cause concern, with the degree of bias consistent for both estimation procedures. The estimates of γ_{21} (.102 for ML and .103 for ULS) have large biases both in terms of the sampling distribution and relative to the parameter. The parameter is underestimated by 19.21% and 18.65% with maximum likelihood and unweighted least squares, respectively, and both estimates are in the extremes of their sampling distributions (-2.1390 for ML and -2.2127 for ULS). The estimates of β_{32} and γ_{12} are within an acceptable range in terms of their sampling distributions, but the percentage of underestimation is quite large. The discrepancies between these two measures of bias again appear to be a function of the large variances of the parameter estimates. The bias relative to the parameter for the estimates of β_{32} is .1283 (ML) and .1598 (ULS), and for γ_{12} is .1505 (ML) and .1267 ULS). The estimates for the other six parameters appear to be robust when a mixed matrix is used to estimate the zero-order associations among the manifest variables.

The bias in the maximum likelihood standard errors of the parameter estimates for Case 2 (see Table 11) is quite large when the standard errors are considered relative to their sampling distributions. This bias can be misleading, however, because the extremely large values found here are caused by the small variances. For example, the bias relative to the sampling distribution for the standard error of γ_{11} is -7.2000 but the variance of this estimate is only .0000116. In such situations, a better indication of the degree of bias in the standard errors is the relative bias; that is, relative to the standard error of the estimates obtained from the analysis of the continuous data.

As with Case 1, the standard errors of the beta parameter estimates are seriously inflated, overestimated by as much as 99.23% in the case of β_{31} (see Table 11). The impact of this inflation on significance tests is serious, for with highly inflated standard errors and underestimated parameters, the tests are not just conservative, but extremely unreliable. The relative bias in the standard errors of the gamma estimates is not as severe, but is larger than in Case 1. Again, the combination of the inflated standard errors and downwardly biased parameter estimates make significance tests unreliable.

In summary, both the maximum likelihood and unweighted least squares parameter estimates resulting from the analysis of product-moment correlations are clearly non-robust against categorization and skewness, generally underestimating the parameters. The maximum likelihood standard errors of the estimates are severely inflated for several of the estimates which, when combined with the underestimation of the parameters, leads to extremely unreliable significance tests. The likelihood ratio chi-square statistic is generally robust with the analysis of product-moment matrices, although the variance of this statistic is slightly larger than expected. However, obtaining a good-fitting model with biased parameter estimates and unreliable significance tests is of little practical use. In contrast, the analysis of matrices containing tetrachoric, polyserial, and product-moment correlations result in more robust estimates, regardless of estimation procedure. Three of the parameter estimates did show a degree of bias, however, but with the exception of estimates of β_{32}, the parameter estimates were much closer to the population values than the corresponding product-moment estimates.

Robustness of Estimates in Case 3

The final case of non-normality considered in this study had the manifest indicators of η_1 (y_1 and y_2) and of η_2 (y_3 and y_4) skewed in opposite directions. The skewness for the indicators of η_1 was set at 1.5 (80% vs. 20%) and for η_2 was set at -1.5 (20% vs. 80%). The underestimation of the associations between these variables is quite large in this case (see Table 5). For example, the correlation between y_2 and y_3 measured as continuous variables is .553, but the product-moment estimate of this correlation after the variables are dichotomized and skewed in opposite directions is only .207 while the tetrachoric estimate is .546. The large attenuation in the correlations was expected to substantially affect the parameter estimates, and the analyses of the mixed matrices were expected to produce more robust estimates. In the event, however, the results were not as expected.

The biases of the parameter estimates for this case are shown in Table 13. When product-moment matrices are analyzed, as expected, the beta parameter estimates are severely biased by each of the three measures, and the degree of bias is consistent for both estimation procedures. All three measures of bias for these estimates are considerably larger than the corresponding estimates in Case 2. The estimates for the gammas, however, do not exhibit the same increase in bias. In fact, with the exception of the estimate for γ_{12}, they are no more biased than the estimates in Case 1 where the dichotomized variables each had no skew and the zero-order associations between the variables were much better estimated.

Insert Table 13 About Here

Table 13. Bias of Structural Parameter Estimates for Case 3 (80/20 vs. 20/80).

	Pearson Matrix				Mixed Matrix			
	ML		ULS		ML		ULS	
Parameter	Estimate	Bias*	Estimate	Bias*	Estimate	Bias*	Estimate	Bias*
β_{21} = .663	.381	.2820 .4250 -36.2296	.384	.2793 .4213 -35.7134	.690	.0265 .0409 1.7373	.684	.0200 .0309 1.3535
β_{31} = .669	.557	.1122 .1679 -10.8778	.556	.1131 .1692 -11.0865	.787	.1175 .1761 3.4867	.774	.1049 .1567 3.2516
β_{32} = .127	.283	.1559 1.2260 16.9119	.281	.1543 1.2126 16.9354	-.011	.1378 1.0843 -3.7192	.010	.1170 .9205 -3.2467
γ_{11} = .132	.122	.0100 .0758 - .9292	.121	.0100 .0826 - 1.0277	.139	0 .0515 .5495	.137	0 .0364 .4084
γ_{12} = .105	.077	.0283 .2667 - 2.6469	.079	.0265 .2438 - 2.4528	.083	.0224 .2057 -1.7990	.091	.0141 .1324 -1.1813
γ_{21} = .126	.126	0 0 0	.127	0 .0048 .0655	.103	.0245 .1849 -1.6625	.111	.0141 .1230 -1.2137
γ_{22} = .238	.224	.0141 .0576 - 1.6616	.222	.0173 .0672 - 1.9079	.242	0 .0151 .3251	.239	0 .0025 .0525
γ_{31} = -.153	-.158	0 .0307 - .6076	-.156	.0 .0176 - .3542	-.148	0 .0307 .3717	-.141	.0141 .0797 .8749
γ_{32} = .188	.179	.0100 .0495 - 1.1619	.178	.0100 .0553 - 1.2571	.228	.0400 .2112 2.9960	.222	.0332 .1787 2.6937

*The entries under bias are error bias, relative bias, and bias with respect to sampling distribution.

Turning to the estimates resulting from the analyses of the mixed matrices, substantial bias is also found in both the ML and ULS estimates of many of the parameters. Only four of the nine parameter estimates (β_{21}, γ_{11}, γ_{22}, and γ_{31}) do not show substantial bias, and, surprisingly, the estimates of γ_{21} and γ_{32} are extremely biased while the corresponding product-moment estimates are not. This was not expected, for the closer approximation of the correlations between the underlying variables was expected to result in more robust estimates from the analyses of the mixed matrices than from the product-moment analyses. Evidently the extreme skewness in opposite directions that is present among some of the variables is not entirely compensated for by the closer approximations of the correlations involving these variables by the tetrachoric and polyserial correlations.

The biases in the standard errors of the maximum likelihood estimates from the analyses of product-moment matrices for this case are also given in Table 11. The biases in the standard errors of the beta estimates are not as severe as in Case 2. In fact, the standard error of β_{32}, .0783, is only slightly overestimated by the observed standard error of .0852. The biases in the standard errors of the gamma estimates are generally less than in Case 2, but still larger than that of Case 1. The overall impact remains the same, however, with extremely conservative significance tests resulting.

The unexpected results in this case lead to the conclusion that both the maximum likelihood and unweighted least squares estimates are non-robust, regardless of the type of correlation matrix analyzed. The appearance of robustness in the product-moment estimates of the gamma parameters is probably misleading given the attenuation in the correlations. When product-moment matrices are analyzed, the distribution of the chi-square statistic is clearly affected, with too few cases rejected and too little variance. This is in contrast to previous studies that found this statistic to be inflated in cases of extreme skewness. This statistic is again extremely inflated with the analysis of the mixed matrices. The standard errors are not as seriously biased as in Case 2, but the bias is substantial enough in many cases to conclude again that significance tests would be misleading. In general, it appears that both the ML and ULS estimates for this model are unreliable when variables have extreme skews in opposite directions, regardless of the type of correlation used in the analyses.

Comparisons to y* Estimates

The results discussed above involved comparisons of the structural parameter estimates to their corresponding parameters, and the robustness of the estimates was determined relative to the parameters. As noted previously, the bias that appeared in some of the estimates resulting from the analysis of the continuous data indicated that the quality of the generated data in terms of its ability to reproduce the parameters of the model was not as good as might be expected. However, these data were generated as approximate standard normal variables, and as such, are ideal data for the estimation of a Lisrel model, regardless of what parameters were used to generate the data. This would be analogous to the estimation of a proposed model with sample data that had an approximately multivariate normal distribution.

The estimation of a Lisrel model with these data would be the best possible test of a model. The χ^2 would be influenced only by the extent to which the correlations between the variables as implied by the structure of the model could produce the sample correlation matrix and not by deviations from normality. The estimates of the parameters of the model are the best sample estimates of the population parameters; thus, even though the estimates from these data do not exactly reproduce the parameters because of the randomness of the data generation procedure, they are the best estimators of the population parameters for the model used in this study. Any differences between these estimates and the estimates obtained after some of these variables were dichotomized and skewed would represent inaccuracies in the assessment of the true structure caused by the dichotomization and skewness.

These comparisons were made by a series of 2x3 factorial analyses of variance in which the rows were defined by estimation procedure (i.e., maximum likelihood versus unweighted least squares) and the columns by the way in which the data were measured (i.e., continuous data, dichotomized variables with associations measured by product-moment correlations, and dichotomized variables with associations measured by tetrachoric, polyserial, or product-moment correlations). No mean differences were expected between analyses of the continuous data and the analyses of the mixed matrices. However, differences were expected between the average estimate from the analyses of the product-moment matrices computed for the dichotomized variables and that from the continuous data. No differences were expected between estimation procedures. In those instances where mean differences were found between correlation types, Tukey's HSD procedure was used to determine which pairs of estimates differed.

Distribution of Unweighted Least Squares Estimates

Prior to undertaking the analyses of variance, the distribution of the estimates was examined to verify the appropriateness of parametric tests for these data. Boomsma (1983) examined the distribution of LISREL maximum likelihood estimates for models with skewed categorical variables, and found no substantial deviations from normality. The distribution of the LISREL unweighted least squares estimates has not yet been established. Twelve of the 54 sets of ULS estimates were randomly selected and tested for normality by Pearson's χ^2 goodness-of-fit test. The number of class intervals chosen for the test was six with equal probability of intervals in order for departures from normality in the middle range to be more easily detected. The results of these tests are given in Table 14. Only three of the sets of estimates were found to deviate from normality at the .20 level of significance, and this was not considered serious enough to warrant switching to non-parametric tests.

Insert Table 14 About Here

Analyses of Variance

The results of the ANOVAs for the estimates of the nine parameters are given in Tables 15 through 23. There were no differences found between the maximum likelihood and unweighted least squares estimates for any of the parameters; this was true for each combination of

Table 14. Normality Tests for ULS Estimates

Parameter Estimated	Case	Matrix	χ^2
γ_{12}	2	mixed	4.40
γ_{12}	3	mixed	3.12
β_{31}	1	mixed	10.80*
γ_{11}	1	Pearson	4.08
γ_{11}	3	Pearson	4.08
γ_{22}	3	mixed	1.52
β_{21}	2	Pearson	11.44*
γ_{31}	1	Pearson	4.40
γ_{32}	1	mixed	6.64
β_{21}	2	mixed	9.20*
β_{32}	2	Pearson	4.72
γ_{21}	2	mixed	6.00

*Significant at α = .20

correlation type and degree of skewness. Thus, conclusions drawn from the estimation of Lisrel models by the maximum likelihood procedure would be the same as those resulting from the unweighted least squares procedure. The manner in which the data are measured, however, does have an effect on the parameter estimates, with the estimates of the gamma parameters less affected than the beta parameter estimates.

Insert Tables 15 to 23 About Here

<u>Differences in Gamma Parameter Estimates</u>

Of the eighteen gamma estimates obtained after categorization and skewness were introduced, the average estimate for γ_{22} in Case 2 and γ_{32} in Case 3 were the only ones differing across correlation type. The results of the post hoc tests for the analyses of variance on these parameter estimates are shown in Table 24. For γ_{22} in Case 2, differences were found between the average estimate from the analysis of the product-moment matrix and that from the analysis of the mixed matrix. However, neither of these differed significantly from the estimate using the continuous data. The post hoc tests for the estimates of γ_{32} in Case 3 reflect the anomalies seen in this case. When the data were dichotomized, the average of the estimates from the analyses of product-moment matrices did not differ

Table 15. Analysis of Variance for Estimates of γ_{11}

Case 1

Source	SS	df	MS	F
Procedure (P)	4.4E-5	1	4.4E-5	.0102
Matrix Type (M)	.0095	2	.0048	1.1036
P x M	2.3E-5	2	1.2E-5	.0027
Error	1.2700	294	.0043	

Case 2

Source	SS	df	MS	F
Procedure (P)	2.7E-5	1	2.7E-5	.0050
Matrix Type (M)	.0146	2	.0073	1.3574
P x M	2.2E-5	2	1.1E-5	.0020
Error	1.5800	294	.0053	

Case 3

Source	SS	df	MS	F
Procedure (P)	7.0E-5	1	7.0E-5	.0127
Matrix Type (M)	.0147	2	.0073	1.3290
P x M	5.0E-5	2	2.5E-5	.0045
Error	1.6250	294	.0055	

Table 16. Analysis of Variance for Estimates of γ_{12}

Case 1

Source	SS	df	MS	F
Procedure (P)	1.0E-5	1	1.0E-5	.0020
Matrix Type (M)	.0044	2	.0022	.4479
P x M	2.7E-6	2	1.3E-6	.0003
Error	1.4500	294	.0049	

Case 2

Source	SS	df	MS	F
Procedure (P)	.0001	1	.0001	.0250
Matrix Type (M)	.0160	2	.0080	1.5014
P x M	8.7E-5	2	4.4E-5	.0082
Error	1.5700	294	.0053	

Case 3

Source	SS	df	MS	F
Procedure (P)	.0008	1	.0008	.1541
Matrix Type (M)	.0170	2	.0085	1.5737
P x M	.0008	2	.0004	.0733
Error	1.5900	294	.0054	

Table 17. Analysis of Variance for Estimates of γ_{21}

Case 1

Source	SS	df	MS	F
Procedure (P)	2.7E-5	1	2.7E-5	.0127
Matrix Type (M)	.0012	2	.0006	.2884
P x M	6.0E-6	2	3.0E-6	.0014
Error	.6250	294	.0021	

Case 2

Source	SS	df	MS	F
Procedure (P)	5.2E-5	1	5.2E-5	.0130
Matrix Type (M)	.0271	2	.0135	3.3844
P x M	1.3E-5	2	6.6E-6	.0016
Error	1.1750	294	.0040	

Case 3

Source	SS	df	MS	F
Procedure (P)	.0007	1	.0007	.1437
Matrix Type (M)	.0190	2	.0095	1.8935
P x M	.0008	2	.0004	.0832
Error	1.4750	294	.0050	

Table 18. Analysis of Variance for Estimates of γ_{22}

Case 1

Source	SS	df	MS	F
Procedure (P)	7.0E-5	1	7.0E-5	.0245
Matrix Type (M)	.0112	2	.0056	1.9557
P x M	2.2E-5	2	1.1E-5	.0039
Error	.8400	294	.0029	

Case 2

Source	SS	df	MS	F
Procedure (P)	.0002	1	.0002	.0759
Matrix Type (M)	.0558	2	.0279	10.5830*
P x M	9.8E-5	2	4.9E-5	.0186
Error	.7750	294	.0026	

* $p < .01$

Case 3

Source	SS	df	MS	F
Procedure (P)	.0004	1	.0004	.1048
Matrix Type (M)	.0145	2	.0072	1.8571
P x M	2.0E-5	2	1.0E-5	.0027
Error	1.1450	294	.0039	

Table 19. Analysis of Variance for Estimates of γ_{31}

Case 1

Source	SS	df	MS	F
Procedure (P)	3.6E-5	1	3.6E-5	.0107
Matrix Type (M)	.0046	2	.0023	.6633
P x M	5.6E-5	2	2.8E-5	.0081
Error	1.0100	294	.0034	

Case 2

Source	SS	df	MS	F
Procedure (P)	.0002	1	.0002	.0565
Matrix Type (M)	.0398	2	.0199	4.6268
P x M	8.0E-6	2	4.0E-6	.0009
Error	1.2650	294	.0043	

Case 3

Source	SS	df	MS	F
Procedure (P)	.0011	1	.0011	.2195
Matrix Type (M)	.0083	2	.0041	.8543
P x M	.0005	2	.0003	.0540
Error	1.4250	294	.0048	

Table 20. Analysis of Variance for Estimates of γ_{32}

Case 1

Source	SS	df	MS	F
Procedure (P)	7.0E-5	1	7.0E-5	.0193
Matrix Type (M)	.0075	2	.0037	1.0235
P x M	1.6E-5	2	8.0E-6	.0022
Error	1.0700	294	.0036	

Case 2

Source	SS	df	MS	F
Procedure (P)	4.4E-16	1	4.4E-16	1.2E-13
Matrix Type (M)	.0315	2	.0157	4.1524
P x M	7.4E-5	2	3.7E-5	.0098
Error	1.1150	294	.0038	

Case 3

Source	SS	df	MS	F
Procedure (P)	.0003	1	.0003	.0719
Matrix Type (M)	.1124	2	.0562	11.8421*
P x M	.0006	2	.0003	.0669
Error	1.3950	294	.0047	

* $p < .01$

Table 21. Analysis of Variance for Estimates of β_{21}

Case 1

Source	SS	df	MS	F
Procedure (P)	2.0E-6	1	2.0E-6	.0009
Matrix Type (M)	.1004	2	.0502	20.7882*
P x M	3.0E-5	2	1.5E-5	.0062
Error	.7100	294	.0024	

* p < .01

Case 2

Source	SS	df	MS	F
Procedure (P)	2.1E-5	1	2.1E-5	.0044
Matrix Type (M)	.1876	2	.0938	19.4870*
P x M	.0001	2	5.6E-5	.0117
Error	1.4150	294	.0048	

* p < .01

Case 3

Source	SS	df	MS	F
Procedure (P)	9.6E-5	1	9.6E-5	.0175
Matrix Type (M)	5.8094	2	2.9047	527.1477*
P x M	.0012	2	.0006	.1054
Error	1.6200	294	.0055	

* p < .01

Table 22. Analysis of Variance for Estimates of β_{31}

Case 1

Source	SS	df	MS	F
Procedure (P)	.0002	1	.0002	.0186
Matrix Type (M)	.1539	2	.0770	9.2745*
P x M	2.5E-5	2	1.2E-5	.0015
Error	2.4400	294	.0083	

* p < .01

Case 2

Source	SS	df	MS	F
Procedure (P)	.0003	1	.0003	.0229
Matrix Type (M)	.5890	2	.2944	23.2121*
P x M	.0001	2	7.2E-5	.0057
Error	3.7300	294	.0127	

* p < .01

Case 3

Source	SS	df	MS	F
Procedure (P)	.0012	1	.0012	.0520
Matrix Type (M)	2.5407	2	1.2703	56.9760*
P x M	.0032	2	.0016	.0717
Error	6.5550	294	.0223	

* p < .01

Table 23. Analysis of Variance for Estimates of β_{32}

Case 1

Source	SS	df	MS	F
Procedure (P)	.0006	1	.0006	.0494
Matrix Type (M)	.0678	2	.0339	2.7152
P x M	6.8E-5	2	3.4E-5	.0027
Error	3.6700	294	.0125	

Case 2

Source	SS	df	MS	F
Procedure (P)	.0005	1	.0005	.0304
Matrix Type (M)	.0006	2	.0003	.0198
P x M	.0003	2	.0002	.0102
Error	4.5300	294	.0154	

Case 3

Source	SS	df	MS	F
Procedure (P)	.0019	1	.0019	.0725
Matrix Type (M)	4.0545	2	2.0272	76.3622*
P x M	.0093	2	.0047	.1760
Error	7.8050	294	.0265	

* $p < .01$

significantly from the average estimate using continuous data. Differences were found between the mean estimates of the analyses of continuous data and the analyses of mixed matrices.

Insert Table 24 About Here

Differences in Beta Parameter Estimates

The choice of correlation coefficient has a greater effect on the beta parameter estimates, for in only two instances were mean differences not found among the estimates. Tables 25, 26, and 27 give the results of the post hoc tests for the analyses of variance on the beta parameter estimates. In every instance, differences were found between the mean estimate obtained when product-moment correlations were used to measure the associations between the dichotomous variables and the mean estimate when continuous variables were used. When tetrachoric or polyserial correlations were used, the average estimate was closer to that produced by the continuous data, with significant differences occurring only for β_{31} and β_{32} in Case 3.

Insert Tables 25, 26, and 27 About Here

Table 24. Tukey's HSD Tests for γ Parameter Estimates

γ_{22} in Case 2

Matrix	Pearson	Continuous	Mixed
ω	.2124	.2329	.2455
Pearson	--	.0205	.0331*
Continuous		--	.0126
Mixed			--

HSD = .0212, α = .01

γ_{32} in Case 3

Matrix	Pearson	Continuous	Mixed
ω	.1782	.1934	.2247
Pearson	--	.0152	.0465*
Continuous		--	.0313*
Mixed			--

HSD = .0284, α = .01

Table 25. Tukey's HSD Test for β_{21} Parameter Estimates

β_{21} in Case 1

Matrix	Pearson	Mixed	Continuous
ω	.6257	.6605	.6676
Pearson	--	.0348*	.0419*
Mixed		--	.0071
Continuous			--

HSD = .0202, α = .01

β_{21} in Case 2

Matrix	Pearson	Mixed	Continuous
ω	.6133	.6605	.6676
Pearson	--	.0518*	.0543*
Mixed		--	.0025
Continuous			--

HSD = .0286, α = .01

β_{21} in Case 3

Matrix	Pearson	Continuous	Mixed
ω	.3825	.6676	.6868
Pearson	--	.2851*	.3043*
Continuous		--	.0192
Mixed			--

HSD = .0306, α = .01

Table 26. Tukey's HSD Test for β_{31} Parameter Estimates

β_{31} in Case 1

Matrix	Pearson	Mixed	Continuous
ω	.6372	.6788	.6898
Pearson	--	.0416*	.0526*
Mixed		--	.0110
Continuous			--

HSD = .0375, α = .01

β_{31} in Case 2

Matrix	Pearson	Continuous	Mixed
ω	.5986	.6898	.6952
Pearson	--	.0912*	.0966*
Continuous		--	.0054
Mixed			--

HSD = .0464, α = .01

β_{31} in Case 3

Matrix	Pearson	Continuous	Mixed
ω	.5563	.6898	.7803
Pearson	--	.1335*	.2240*
Continuous		--	.0905*
Mixed			--

HSD = .0615, α = .01

Table 27. Tukey's HSD Test for β_{32} Parameter Estimates

	β_{32} in Case 3		
Matrix	Mixed	Continuous	Pearson
ω	-.0003	.1075	.2819
Mixed	--	.1078*	.2822*
Continuous		--	.1744*
Pearson			--

HSD = .0671, α = .01

From these tests it can be concluded that there are substantial mean differences between the estimates from the analyses of matrices containing product-moment correlations used to measure associations involving dichotomous variables and the best estimators of the population parameters. Unless the dichotomous variables are highly skewed in opposite directions, there are no mean differences between the parameter estimates from the analyses of matrices containing tetrachoric, polyserial, or product-moment correlations and the best estimates of those parameters. No mean differences were found between the maximum likelihood and unweighted least squares estimates. These results indicate that correlation type is more important than estimation procedure when the concern is approximating the best estimates of the population parameters.

CHAPTER 4

DISCUSSION

The development of procedures for the analysis of covariance structures has profoundly influenced the way in which social science researchers examine hypothesized relationships among theoretical variables. While the robustness of the traditional statistical procedures against violations of assumptions has been tested, the estimation procedures used in the analysis of covariance structures have only recently begun to be examined for robustness. These robustness studies have primarily been concerned with the effects of non-normality as evidenced by the inclusion of categorical indicator variables in Lisrel-type models.

Muthen (1983a) and Olsson (1979b) found that the analysis of Pearson product-moment matrices resulted in downwardly biased estimates of factor loadings for categorical variables, but the analysis of tetrachoric or polychoric correlations produced robust estimates. Their results imply that the lambda coefficients in the measurement portion of Lisrel models would be underestimated when product-moment correlations were analyzed, and the use of more appropriate measures of association results in more accurate lambda parameter estimates. Boomsma (1983) extended these studies to the structural portion of the LISREL model by examining the robustness of LISREL-III maximum likelihood estimates in models containing only categorical manifest variables. Analyzing multinomial covariance matrices, computed from multinomial correlations, Boomsma (1983) concluded that both the measurement and structural parameter estimates were robust against categorization but not against skewed distributions for the indicator variables. What was not addressed in his study was the impact of correlation type on the structural parameter estimates.

The present study adds to this literature by having examined the effect of correlation type on the robustness of the LISREL-VI maximum likelihood and unweighted least squares structural parameter estimates. Ten approximate standard normal variables were generated to fit the parameters of the model shown in Figure 2. Three cases were considered by dichotomizing four of the continuous variables with varying degrees of skewness. For each case, two types of correlation matrices were used to estimate the model; the first contained only product-moment correlations, while the second contained tetrachoric, polyserial, and product-moment correlations, as appropriate. Both the maximum likelihood and the unweighted least squares procedures in LISREL-VI were used to analyze each matrix. Resulting estimates were then compared to the population parameters and to the estimates resulting from the analysis of the continuous data.

From the results reported in Chapter 3, a general qualitative conclusion can be drawn: the analysis of product-moment correlation matrices resulted in biased estimates of the structural parameters of the model used in this study when skewness was present in the dichotomous variables. Moreover, the degree of bias was consistent for both the maximum likelihood and unweighted least squares procedures. The bias was particularly evident in the beta parameter estimates, which represent the effects of the factors having the dichotomized indicator variables. The attenuation present in the product-moment correlations involving the dichotomous variables appeared to produce estimates that generally underestimated the population parameters. These estimates were found to differ on the average from both the population parameters and the estimates resulting from the analysis of the continuous data. The impact of this attentuation was further manifested in the inflation of the maximum likelihood standard errors. This combination of underestimation of parameters and inflated standard errors resulted in unreliable significance tests, particularly in cases of extreme skewness.

These results extend the findings of Muthen (1983a) and Olsson (1979b) concerning the impact of the use of Pearson product-moment correlations to measure associations involving categorical variables with underlying continuities. In their studies of factor analysis models, they found downwardly biased estimates of the factor loadings for the categorical variables when product-moment correlations were analyzed. This study has found that the analyses of these correlations not only produced underestimates of the parameters of the measurement portion of the Lisrel model, but of the structural portion as well.

With the exception of Case 3, the analysis of mixed matrices produced average estimates that more closely approximated the model parameters and that did not differ significantly from the best estimates of those parameters (estimates from the use of the continuous data). These unbiased estimates did not, however, come without costs. These estimates, on the average, were closer to the population values, but were slightly more variable. Perhaps of greater importance was the inflation in the maximum likelihood chi-square. In the present version of LISREL, this statistic is computed on the assumption that the correlations used in the analysis are product-moment correlations, and when tetrachoric or polyserial correlations are used the chi-square is highly inflated. On the one hand, the researcher who is unaware of this would be led to reject a model as not fitting the data when in fact the large chi-square is mainly a function of the type of correlation used. On the other hand, the inflated chi-square might be dismissed as being a function of correlation type when the data really do not fit the proposed model.

Prior to this research, there had been only one study on the robustness of LISREL maximum likelihood estimates. Boomsma (1983) reported no bias in the parameter estimates or estimates of standard errors, but both a categorization and skewness effect on the variances of the estimates. Additionally, he found the population model to be rejected too often with increasing skewness. Comparisons of the results reported here and those of Boomsma should be made only in the context of the differences in methodology.

The major difference in methodology was in the specification of the model structure. In this study, the model was specified to hold for the continuous latent response variables, and the data generated as continuous variables using equations incorporating the model parameters. Boomsma, in contrast, specified the model in terms of the observed categorical variables. He then generated continuous variables according to the parameters of the covariance matrix specified by the structure. In both studies the continuous variables were subsequently dichotomized and correlation matrices computed. Two types of correlation matrices were used in this study; one consisting only of Pearson product-moment correlations, and one with tetrachoric, polyserial, and product-moment correlations, as appropriate. Boomsma (1983) analyzed only multinomial correlation matrices since all manifest variables were categorical.

This study found the parameter estimates and the estimates of the standard errors to be biased when product-moment matrices were analyzed, while the analysis of the mixed matrices, on the average, produced estimates closer to the population parameters. Boomsma (1983) found the parameter estimates and standard errors to be relatively unbiased with the analysis of multinomial covariance matrices. Whether these differences are a result of the specification of the structure of the model, the model itself, or the mix of continuous and dichotomous manifest variables used in this study is not apparent. However, both studies found increasing variability of the estimates for greater degrees of skewness indicating that even when the estimates are on the average robust, those from a single sample may deviate substantially from the population parameter.

Unlike Boomsma (1983), in this study the maximum likelihood chi-square statistic was not found to be inflated when product-moment correlations were analyzed. However, finding a good fitting model with biased parameter estimates and inflated standard errors is not of practical usefulness.

The lack of an appropriate goodness-of-fit measure when tetrachoric and polyserial correlations are used adds to the problems facing the researcher who uses LISREL to estimate structural equation models.

Implications for Users of LISREL

While it is noted that the results of studies of the effect of non-normality in structural equation modeling are not completely independent of the models used in the studies, they do pose potentially serious problems for the applied researcher. It is clear that LISREL is non-robust when observed categorical variables have skewed distributions. At present, researchers do not have alternative estimation procedures available for the analysis of latent variable structural equation models. Programs such as LACCI by Muthen (1982) and EQS by Bentler (1982b) that employ distribution-free estimation procedures that consider the skewness and kurtosis of the variables hold promise for the future and should be available for public distribution shortly. Until that time, researchers are ill-advised to employ LISREL in the estimation of models containing skewed categorical manifest variables.

In cases where the categorical variables have approximately symmetric distributions, there are choices to be made. A conservative approach would be to analyze product-moment matrices. The parameter estimates, while biased, have less variability and the maximum likelihood chi-square statistic appears generally reliable. Additionally, both standardized and unscaled estimates may be obtained since covariance matrices could be analyzed. However, any inferences concerning the population parameters should be exercised with caution. The parameters are likely to be underestimated and the standard errors inflated when the structure of the model is seen to hold for the underlying continuous latent response variables.

The use of tetrachoric or polyserial correlations produce unbiased parameter estimates on the average, but the greater variability of the estimates and lack of standard errors and goodness-of-fit measures detract from their use. Cross-validation of models could help overcome concern about the variability of the estimates, but again, inferences about the population parameters could not be made. The absence of a reliable goodness-of-fit statistic results in not knowing whether the data in fact fit the proposed model.

Perhaps the best recommendation for researchers who wish to use LISREL to estimate latent variable structural equation models is to avoid the use of categorical variables, particularly dichotomies. Careful consideration of the nature of the data to be analyzed is of utmost importance. Consideration given to how the variables are to be measured while the research is in the conceptual stage would avoid the problems that have been identified in this study.

Implications for Future Research

Some of the considerations for future research arise from the limitations of the present study. Foremost among these is the determination of how the data are to be generated and analyzed, for any robustness study is only as good as its data. As could be seen from the results of the analysis of the continuous data, the data used in this study fit the model well, but three of the parameter estimates deviated substantially from the parameters used in the generation. This could possibly have been improved by reducing sources of variation in the generation of the data. The values for the two latent ξ variables were fixed for all replications, but the η latent variables were allowed to vary. Fixing values for the η variables would reduce the sources of variation and would fix them as standard normal variables. Additionally, only fifty replications were used in this study. Thus, if the only source of variation was in the error term and a greater number of replications were used, the parameter estimates from the analysis of the continuous data may have better reproduced the parameters.

Subsequent versions of LISREL should allow investigations of the robustness of the estimates to be made in a more appropriate manner. The statistical theory of analyzing covariance structures is based on the Wishart distribution of the covariance matrix, and accordingly LISREL analyses should ideally be conducted using covariance matrices. This study was restricted to the analysis of correlation matrices because of limitations of the LISREL program. Once the problems are solved concerning the computation of variances and covariances when tetrachoric and polyserial correlations are used as input, studies can be conducted in which covariance matrices are analyzed. Admittedly, these investigations could be done now by writing programs that would correctly compute the covariance matrix, as Boomsma (1983) has done. Should results support the use of LISREL, they would still be of little practical use to social scientists who are unable themselves to compute these covariance matrices. It is best to wait for improvements in the LISREL program and for studies to be conducted in the manner in which LISREL is used by the applied researcher.

The two computer programs with the greatest potential for structural equation analyses with non-normal data are LACCI (Muthen, 1982) and EQS (Bentler, 1982b). These programs contain asymptotically distribution free procedures that consider moments higher than the second. Both programs are now in testing stages by distributors and should soon be available for public distribution. Once these programs are available, comparable studies of the robustness of LACCI, EQS, and LISREL should be conducted.

Perhaps of more importance is the investigation of the robustness of LISREL against non-normality evidenced by the use of non-symmetric continuous distributions with skewnesses comparable to that used in studies employing categorical variables. The best approach researchers could take in structural equation analyses is to consider *a priori* how the constructs of interest will be measured, and great care should be taken to use interval scales of measurement. However, this does not assure that the data gathered from a sample will be symmetrically distributed. The impact of skewness in continuous distributions on LISREL estimates is thus of great importance.

Summary

The development of efficient procedures for the analysis of covariance structures has substantially influenced the methodologies used in social science research. This is evident in the number of references contained in the reviews of the applied literature by Bentler (1980) and Bielby and Hauser (1977). From the results of the recent studies on the robustness of these procedures against non-normality introduced by skewed categorical variables, it is evident that conclusions drawn from the applied studies incorporating categorical manifest variables should be reexamined. It is also evident that researchers using LISREL for the analysis of structural equation models should avoid blindly applying these procedures without careful consideration of whether their data meet the assumptions underlying the application of LISREL. Results of applied research are only as reliable as the data analyzed, and the nature of the data determines the appropriate methodology.

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Appendix A

Equations for the measurement model:

$$x_1 = .844\xi_1 + \delta_1$$

$$x_2 = .841\xi_1 + \delta_2$$

$$x_3 = .917\xi_2 + \delta_3$$

$$x_4 = .892\xi_2 + \delta_4$$

$$y_1 = .783\eta_1 + \varepsilon_1$$

$$y_2 = .847\eta_1 + \varepsilon_2$$

$$y_3 = .906\eta_2 + \varepsilon_3$$

$$y_4 = .829\eta_2 + \varepsilon_4$$

$$y_5 = .868\eta_3 + \varepsilon_5$$

$$y_6 = .833\eta_3 + \varepsilon_6$$

Equations for the structural model:

$$\eta_1 = .132\xi_1 + .105\xi_2 + \zeta_1$$

$$\eta_2 = .663\eta_1 + .126\xi_1 + .238\xi_2 + \zeta_2$$

$$\eta_3 = .669\eta_1 + .127\eta_2 - .153\xi_1 + .188\xi_2 + \zeta_3$$

Appendix B

Data Generation Program

```
PROC MATRIX;
KSI1 = NORMAL(J(1,500,0));
KSI2 = NORMAL(J(1,500,0));
KSI = KSI1//KSI2;
BETA = 0 0 0/.663 0 0/.669 .127 0;
GAMMA = .132 .105/.126 .238/-.153 .188;
ZETA1 = .98472595#NORMAL(J(1,500,0));
ZETA2 = .62332537#NORMAL(J(1,500,0));
ZETA3 = .61721971#NORMAL(J(1,500,0));
ZETA = ZETA1//ZETA2//ZETA3;
ETA = INV(I(3)-BETA)*(GAMMA*KSI + ZETA);
TD1 = .5363432#NORMAL(J(1,500,0));
TD2 = .5410351#NORMAL(J(1,500,0));
TD3 = .3988872#NORMAL(J(1,500,0));
TD4 = .4520354#NORMAL(J(1,500,0));
DELTA = TD1//TD2//TD3//TD4;
TE1 = .62202174#NORMAL(J(1,500,0));
TE2 = .53159289#NORMAL(J(1,500,0));
TE3 = .42327769#NORMAL(J(1,500,0));
TE4 = .55924865#NORMAL(J(1,500,0));
TE5 = .496564198#NORMAL(J(1,500,0));
TE6 = .55327298#NORMAL(J(1,500,0));
EPSILON=TE1//TE2//TE3//TE4//TE5//TE6;
LX = .844 0/.841 0/0 .917/0 .892;
LY = .783 0 0/.847 0 0/0 .906 0/0 .829 0/0 0 .868/0 0 .833;
X = LX*KSI+DELTA;
Y = LY*ETA+EPSILON;
```

Appendix C. Measurement Parameter Estimates for Continuous Variables.

	ML		ULS	
Parameter	Estimate	Variability*	Estimate	Variability*
$\lambda^{y}_{11} = .783$	.787	.0005 .0006	.786	.0006 .0006
$\lambda^{y}_{21} = .847$	.845	.0005 .0005	.845	.0006 .0006
$\lambda^{y}_{32} = .906$	.910	.0004 .0005	.911	.0005 .0005
$\lambda^{y}_{42} = .829$	.827	.0007 .0007	.827	.0007 .0007
$\lambda^{y}_{53} = .868$	.873	.0003 .0003	.874	.0003 .0004
$\lambda^{y}_{63} = .833$	.838	.0004 .0004	.837	.0005 .0005
$\lambda^{x}_{11} = .844$	.836	.0016 .0017	.836	.0018 .0019
$\lambda^{x}_{21} = .841$	.836	.0017 .0018	.837	.0018 .0018
$\lambda^{x}_{32} = .917$	.905	.0006 .0007	.904	.0007 .0008
$\lambda^{x}_{42} = .892$	.887	.0006 .0006	.888	.0007 .0007

*The entries under variability are variance and mean square error.

Appendix D. Measurement Parameter Estimates for Case 1 (50/50).

	Pearson Matrix				Mixed Matrix			
	ML		ULS		ML		ULS	
Parameter	Estimate	Variability*	Estimate	Variability*	Estimate	Variability*	Estimate	Variability*
$\lambda^y_{11} = .783$	.653	.0009 .0179	.652	.0010 .0181	.783	.0010 .0010	.783	.0010 .0010
$\lambda^y_{21} = .847$	.707	.0015 .0210	.708	.0017 .0210	.847	.0017 .0017	.847	.0019 .0019
$\lambda^y_{32} = .906$	.770	.0016 .0202	.770	.0016 .0200	.906	.0018 .0018	.906	.0016 .0016
$\lambda^y_{42} = .829$	.699	.0019 .0188	.699	.0019 .0190	.824	.0019 .0019	.824	.0020 .0020
$\lambda^y_{53} = .868$	.876	.0004 .0005	.876	.0005 .0005	.873	.0006 .0006	.875	.0005 .0006
$\lambda^y_{63} = .833$	.835	.0007 .0007	.835	.0007 .0007	.837	.0007 .0007	.835	.0007 .0007
$\lambda^x_{11} = .844$	.835	.0014 .0015	.836	.0016 .0017	.835	.0016 .0016	.835	.0018 .0019
$\lambda^x_{21} = .841$	.835	.0017 .0017	.836	.0017 .0017	.837	.0017 .0017	.837	.0019 .0019
$\lambda^x_{32} = .917$	.904	.0006 .0008	.902	.0007 .0009	.902	.0006 .0008	.902	.0007 .0009
$\lambda^x_{42} = .892$	.888	.0006 .0006	.890	.0007 .0007	.890	.0007 .0007	.891	.0008 .0008

*The entries under variability are variance and mean square error.

Appendix E. Measurement Parameter Estimates for Case 2 (80/20).

	Pearson Matrix				Mixed Matrix			
	ML		ULS		ML		ULS	
Parameter	Estimate	Variability*	Estimate	Variability*	Estimate	Variability*	Estimate	Variability*
$\lambda^{y}_{11} = .783$	.621	.0023 .0284	.623	.0026 .0282	.779	.0026 .0026	.783	.0029 .0029
$\lambda^{y}_{21} = .847$	.675	.0022 .0320	.673	.0024 .0325	.844	.0026 .0026	.841	.0027 .0027
$\lambda^{y}_{32} = .906$	.759	.0020 .0237	.758	.0019 .0237	.920	.0022 .0024	.919	.0020 .0021
$\lambda^{y}_{42} = .829$	.667	.0024 .0285	.668	.0023 .0283	.817	.0025 .0026	.818	.0024 .0025
$\lambda^{y}_{53} = .868$	.873	.0006 .0006	.873	.0006 .0006	.871	.0006 .0007	.873	.0005 .0006
$\lambda^{y}_{63} = .833$	.838	.0007 .0008	.838	.0007 .0008	.840	.0007 .0007	.839	.0006 .0006
$\lambda^{x}_{11} = .844$	.838	.0018 .0018	.837	.0017 .0018	.837	.0024 .0024	.837	.0021 .0021
$\lambda^{x}_{21} = .841$	.835	.0019 .0020	.835	.0018 .0018	.835	.0023 .0023	.835	.0020 .0020
$\lambda^{x}_{32} = .917$	.906	.0007 .0008	.905	.0005 .0007	.907	.0008 .0009	.905	.0006 .0008
$\lambda^{x}_{42} = .892$	.887	.0006 .0006	.888	.0006 .0006	.886	.0007 .0008	.887	.0007 .0007

*The entries under variability are variance and mean square error.

Appendix F. Measurement Parameter Estimates for Case 3 (80/20 vs. 20/80).

	Pearson Matrix				Mixed Matrix			
	ML		ULS		ML		ULS	
Parameter	Estimate	Variability*	Estimate	Variability*	Estimate	Variability*	Estimate	Variability*
λ^{y}_{11} = .783	.624	.0020 .0273	.626	.0022 .0268	.781	.0031 .0031	.784	.0030 .0030
λ^{y}_{21} = .847	.671	.0020 .0329	.669	.0020 .0337	.843	.0026 .0026	.838	.0026 .0027
λ^{y}_{32} = .906	.748	.0012 .0262	.748	.0012 .0262	.911	.0035 .0035	.909	.0021 .0021
λ^{y}_{42} = .829	.685	.0018 .0227	.685	.0018 .0225	.836	.0035 .0035	.835	.0024 .0024
λ^{y}_{53} = .868	.873	.0005 .0005	.873	.0006 .0006	.871	.0005 .0005	.873	.0005 .0006
λ^{y}_{63} = .833	.838	.0006 .0007	.838	.0007 .0008	.840	.0006 .0006	.838	.0006 .0007
λ^{x}_{11} = .844	.837	.0015 .0016	.837	.0016 .0016	.838	.0019 .0019	.836	.0019 .0020
λ^{x}_{21} = .841	.835	.0017 .0017	.835	.0017 .0017	.835	.0020 .0020	.837	.0019 .0020
λ^{x}_{32} = .917	.904	.0008 .0010	.903	.0007 .0009	.902	.0017 .0019	.902	.0009 .0011
λ^{x}_{42} = .892	.888	.0007 .0007	.890	.0007 .0007	.892	.0016 .0016	.891	.0009 .0009

*The entries under variability are variance and mean square error.

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