Modifications to RANS Turbulence Model for Use in Urban Wind Resource Assessment

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8th June 2015
The people’s choice: The $k - \epsilon$ model

- CFD seen as a pecuniary palliative for wind resource assessment.
- Why the $k - \epsilon$? Tabrizi et al. (2013) Van Hoof and Blocken (2010), etc.
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The modeled terms in $k - \epsilon$ model

**Production of $k$**

$$\frac{\partial k}{\partial t} + U_j \frac{\partial k}{\partial x_j} = 2\nu_t S_{ij} S_{ij} + \frac{\partial}{\partial x_j} \left( (\nu + \nu_t) \frac{\partial k}{\partial x_j} \right) + \epsilon$$

$\nu_t = C_\mu \frac{k^2}{\epsilon}$
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- **Turbulent diffusion**

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  \]

- Turbulent diffusion

- Dissipation rate of $k$

Go to Strain Rate

\[\nu_t = C_\mu \frac{k^2}{\epsilon}\]
Deficiency 1 of $k - \epsilon$ model

- Over-prediction of $k$ in stagnation region

Is the dissipation rate of $k$ unable to catch up with the production of $k$?

Is the production of $k$ not modeled correctly?

Is the gradient diffusion model not valid?
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- Is the dissipation rate of $k$ unable to catch up with the production of $k$?
- Is the production of $k$ not modeled correctly?
- Is the gradient diffusion model not valid?
Meanwhile, at the inlet ... 

- Production of $k$ is balanced by its dissipation rate.
- When and to what extent is this valid?
- What about buoyancy?

**Neutrally-Stratified Atmosphere**

\[
U = \frac{u_\tau}{\kappa} \ln \left( \frac{z - y_0}{z_0} \right),
\]

\[
k = \frac{u_\tau^2}{\sqrt{C_\mu}}
\]

\[
\epsilon = \frac{u_\tau^3}{\kappa Z}
\]
The stubbornness of $k - \epsilon$ modelers. Part 1.

- Over-prediction of $k$ in stagnation regions. ⇒ Buildings.

$k - \epsilon$

SST

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NAWEA 2015 Graduate Student Symposium
8th June 2015
The stubbornness of $k - \epsilon$ modelers. Part 2.

- Negative normal stresses. $\Rightarrow$ Realizability.
Over-prediction of $k$ and under-prediction of recirculation region at the lee of hills. ⇒ Terrain.
Previous works

Durbin’s Model

\[ \nu_t = \min \left( \frac{C_\mu k^2}{\epsilon}, \frac{k}{\sqrt{6S}} \right). \]  

(4)

Yaps’s Model

\[ S_\epsilon = 0.83 \frac{\epsilon^2}{k} \left( \frac{k^{1.5}}{\epsilon l_e} - 1 \right) \left( \frac{k^{1.5}}{\epsilon l_e} \right)^2 \]  

(5)

where \( l_e = C_\mu^{-0.75} \kappa y_n \) with \( y_n \) being the normal distance to the nearest wall and \( \kappa \) is the Karman constant in the logarithmic law for the mean velocity.
Figure: The computed $S_{ij}S_{ij}$ at $x/b=-0.75$ using the $k-\epsilon$ turbulence model.
Figure: The computed $S_{ij}S_{ij}$ at $x/b=-0.75$ using the $k - \epsilon$ turbulence model.

$$\nu_t = C_\mu \frac{k}{S_{ij}S_{ij}} \sum |S_{ij}|, \quad G_k = 2C_\mu k \sum |S_{ij}|$$ (6)
Strain rate immediately upstream of a building.

Figure: The computed $S_{ij}S_{ij}$ at $x/b=-0.75$ using the $k-\epsilon$ turbulence model.

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The birth of the MW turbulence model!
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The birth of the MW turbulence model!

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(6)
Prediction of $k$.

(a) $k$ profile at position $x/b = -0.75$

(b) $k$ profile at position $x/b = -0.5$

(c) $k$ profile at position $x/b = -0.25$

(d) $k$ profile at position $x/b = -0.25$
Will the MW work in the lee of a hill?

Re-circulation region.

Production of $k$ smears out the steep velocity profile resulting in a shorter attachment length.
2D ridges. The Rushil experiments.

(a) Original $k - \epsilon$.
(b) Modified $k - \epsilon$.
(c) MW.
(d) SST.

Re-attachment point

<table>
<thead>
<tr>
<th>Method</th>
<th>Measured: $x/a$</th>
<th>Standard $k - \epsilon$: $x/a$</th>
<th>Modified $k - \epsilon$: $x/a$</th>
<th>MW: $x/a$</th>
<th>SST $k - \omega$: $x/a$</th>
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Figure: Profile of $k$ at the crest.
Conclusions

- Development of a RANS turbulence model to reduce $k$ in stagnation region.
- Re-formulation of the eddy viscosity.
- Tested on an isolated building and achieved good results relative to other conventional models.
- Model applied to complex topography. Extended re-circulation region compared to the $k - \epsilon$.
- Computationally less expensive than the SST.
This study reports on work sponsored by the Natural Science and Engineering Research Council and the ENMAX Corporation under the NSERC Industrial Research Chairs scheme.

Questions?