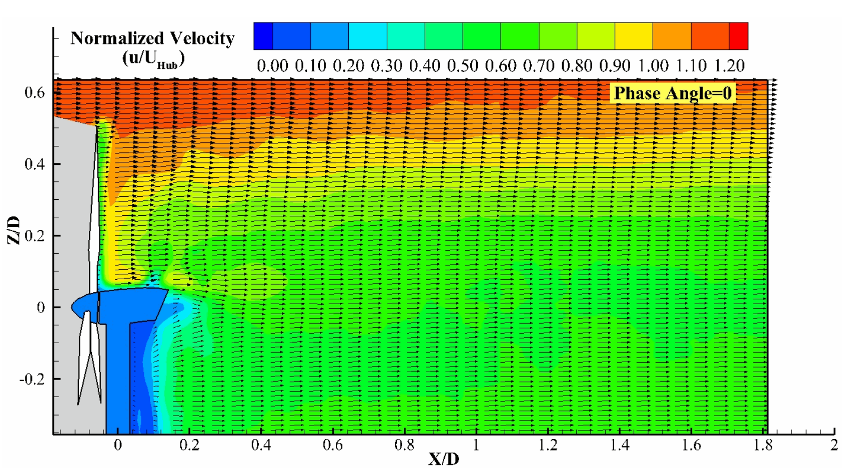


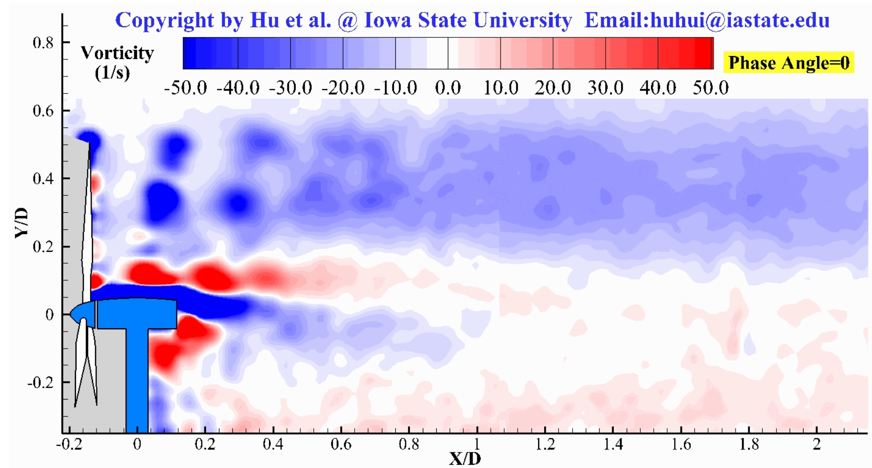


# *Analysis of Turbine Wake Characteristics using Proper Orthogonal Decomposition and Triple Decomposition Methods*

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# Contents

# Motivation and Objectives

- The study of wake behind a wind turbine is crucial in determining the wind farm layouts.
- The study focuses on understanding the momentum recovery process through quantitative and qualitative analysis.

Image courtesy:

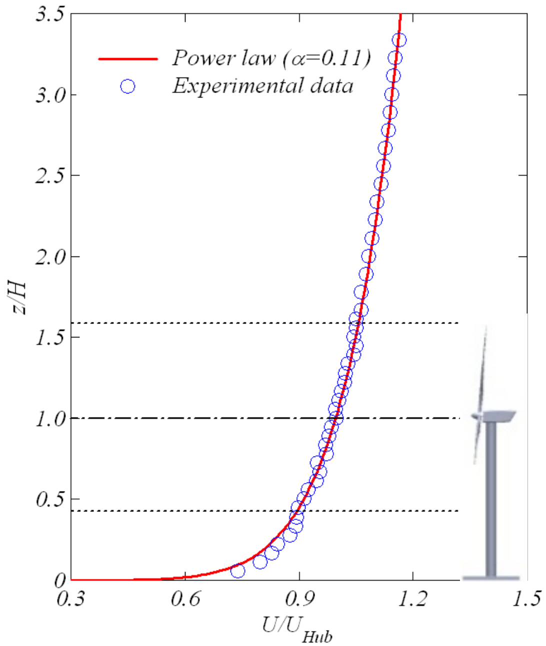
***http://nstergiannis.blogspot.com/***

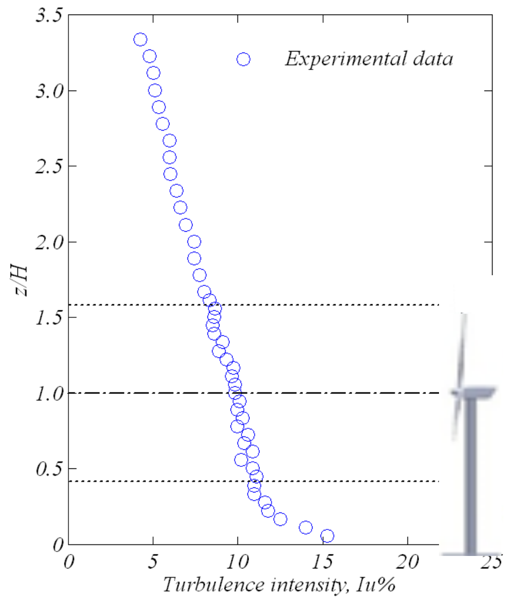
# Simulation of Atmospheric Boundary Layer (ABL) Winds


| Terrain Category | Terrain description | Gradient height, $Z_G$ (m) | Roughness length, $Z_O$ (m) | Wind Speed exponent, $\alpha$ |
|---|---|---|---|---|
| 1 | Open sea, ice, tundra desert | 250 | 0.001 | 0.11 |
| 2 | Open country with low scrub or scattered trees | 300 | 0.03 | 0.15 |
| 3 | Suburban area, small towns, well wooded areas | 400 | 0.3 | 0.25 |
| 4 | Tall buildings, city centers, well developed industrial areas | 500 | 3.0 | 0.36 |

*Offshore wind farm (9.5% at hub height)*

***Architecture Institute of Japan***

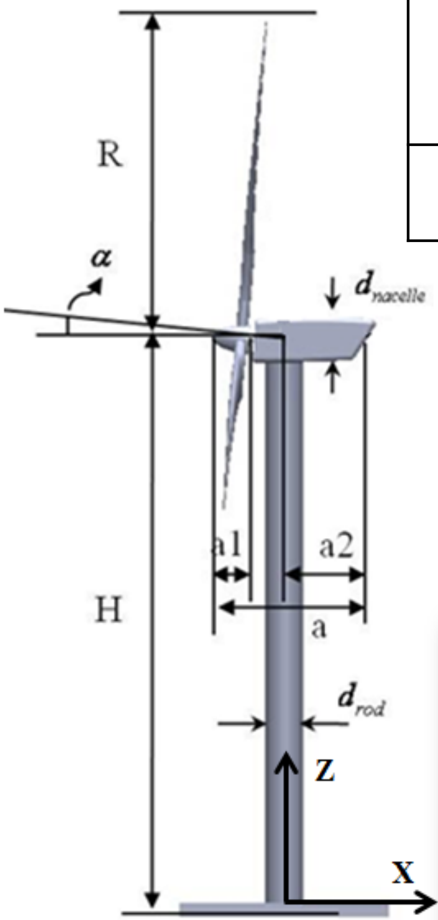


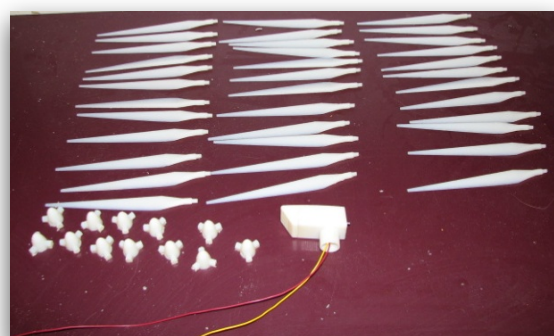



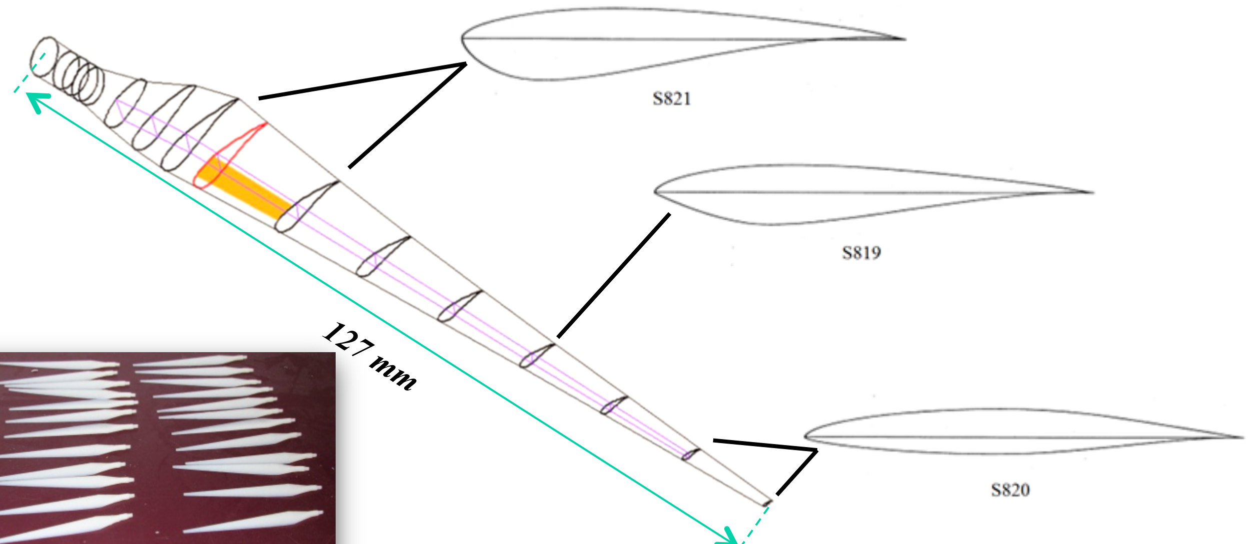

***Hansen & Barthelmie, Wind Energ.2012;15:183–196***

# Wind Turbine Models

| *Parameter* | *R (mm)* | *H (mm)* | *$d_{pole}$ (mm)* | *$d_{nacelle}$ (mm)* | *α (deg.)* | *a (mm)* | *a1 (mm)* | *A2 (mm)* |
|---|---|---|---|---|---|---|---|---|
| *Dimension* | *127* | *226* | *18* | *18* | *5°* | *68* | *20* | *35* |





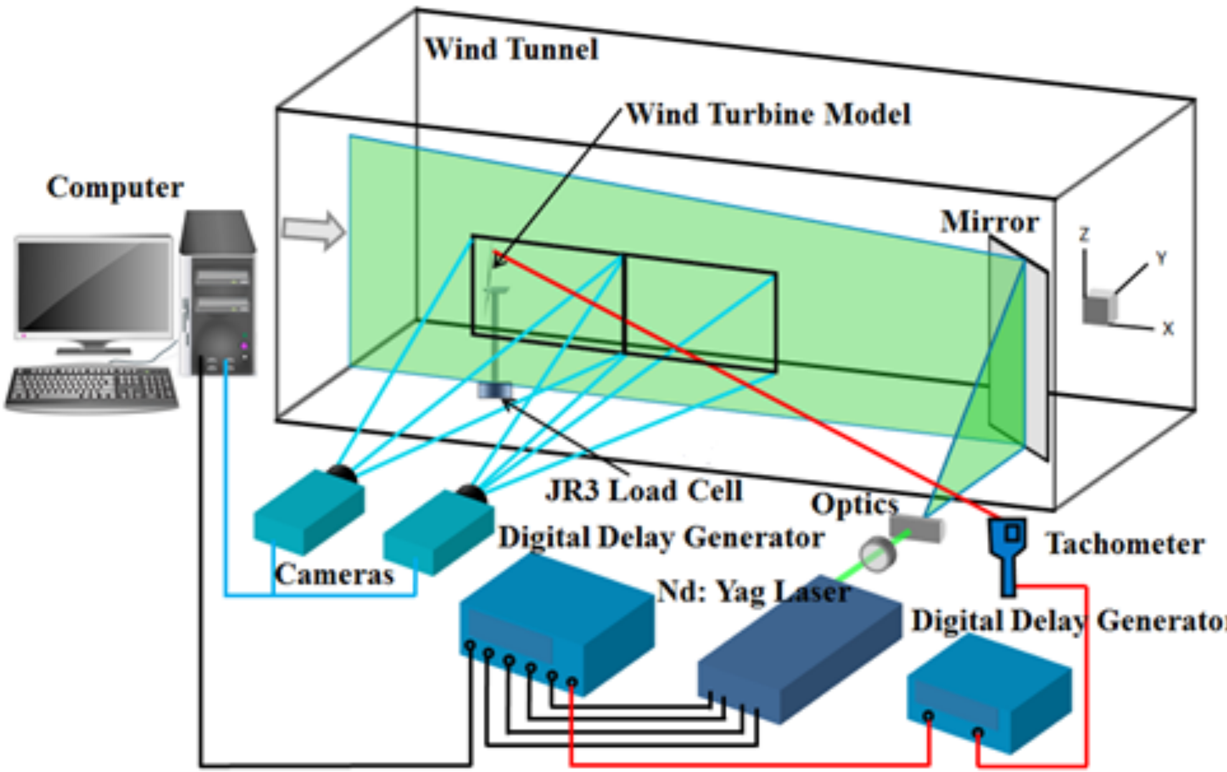

*1:350 scaled model to simulate a 2MW wind turbine*

*ERS-100 prototype of wind turbine blade developed by TPI*

# Experimental Setup

***Test conditions:***

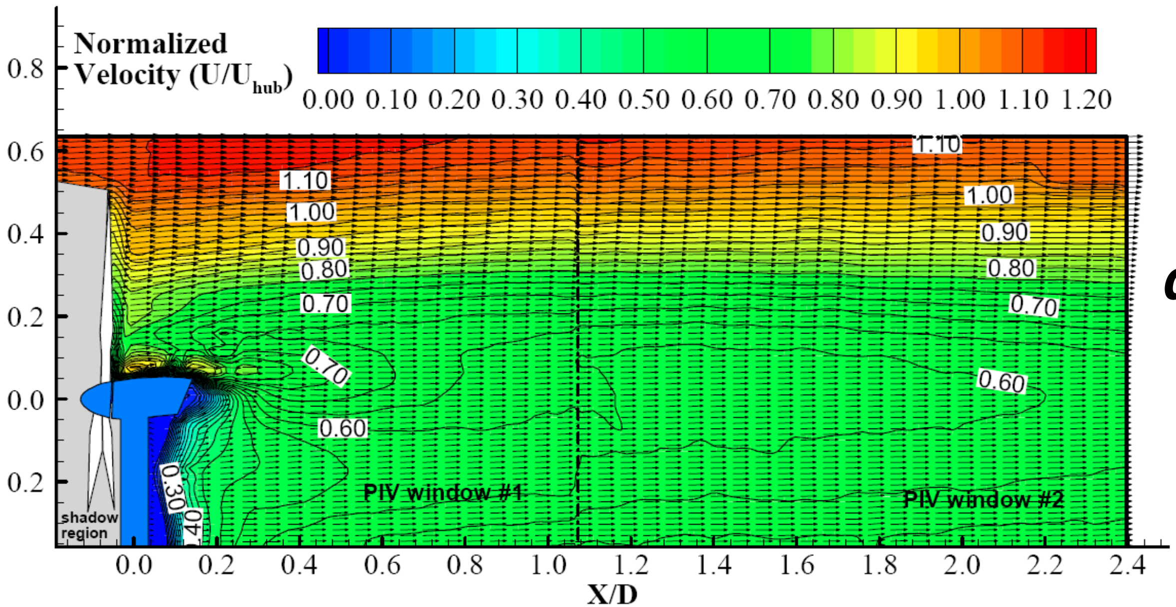
- *Velocity at hub height $U_{Hub} = 4.5$ m/s*
- *Chord Reynolds number, $Re \approx 7{,}000$*
- *Rotation speed of wind turbine:*
- *$\varpi = 0 \sim 2200$ rpm*
- *Tip-speed-ratio, $\lambda = 0 \sim 6.0$*



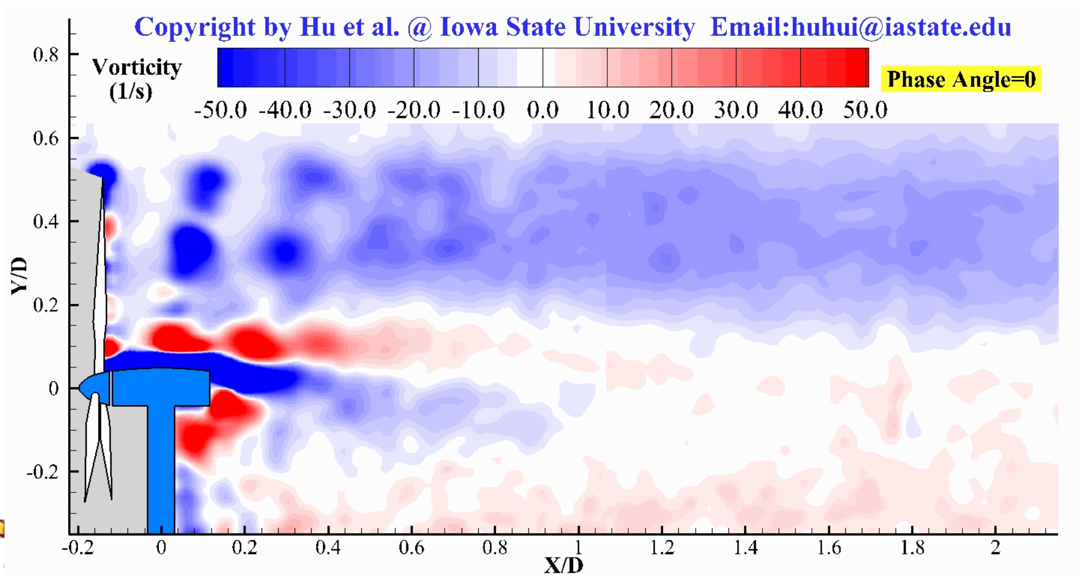

***Measured parameters:***

- *Wake flow field distributions*
- *1000 snapshots for free run case*
- *300 snapshots for phase lock case*

# Velocity and Vorticity Distributions

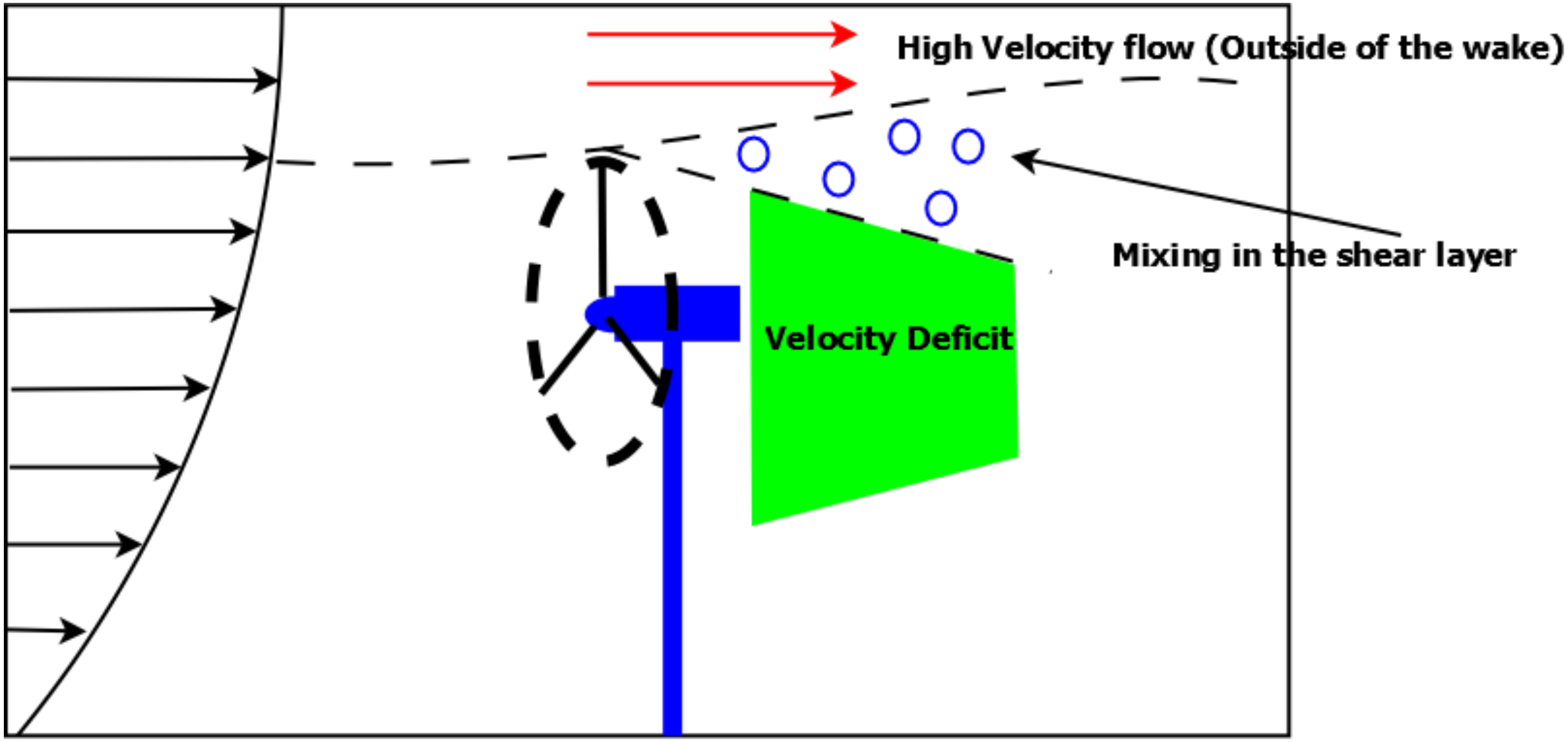



*Normalized Velocity distribution of ensemble average*




*Vorticity distribution of Phase Locked*

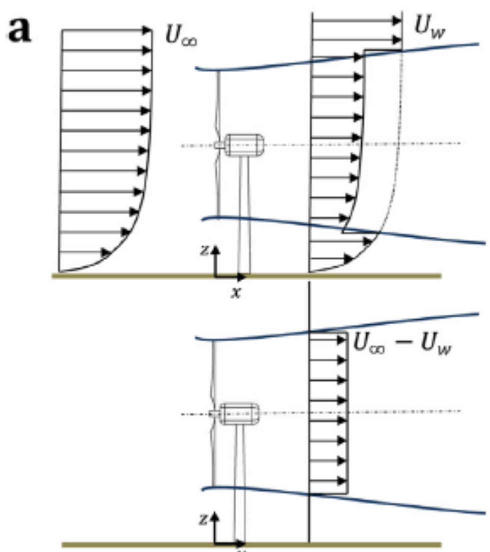
# Wake Aerodynamics



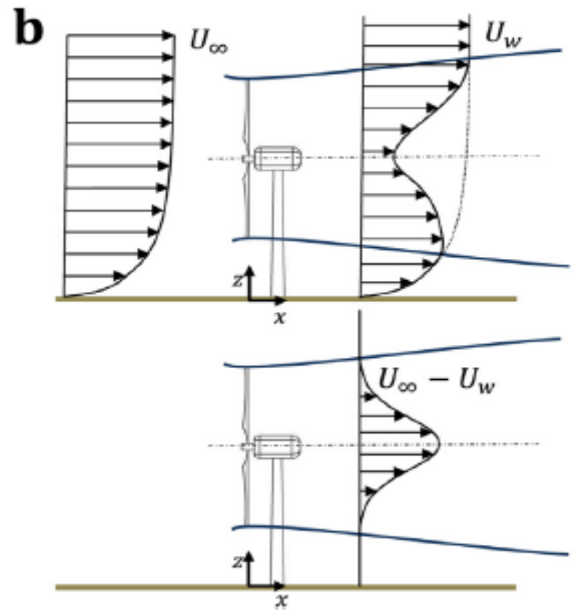

- Velocity deficit results in a shear layer
- Turbulent mixing can be observed
- High turbulence mixing shortens momentum recovery distance

*Sanderse, Aerodynamics of wind turbine wakes:pg13-14*

# Analytical Wake Models




Top hat distribution

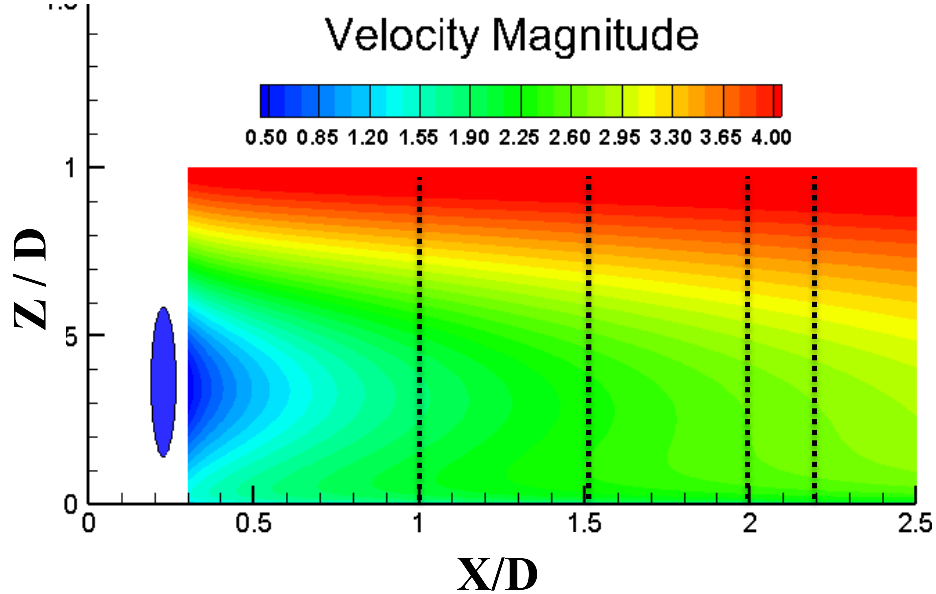


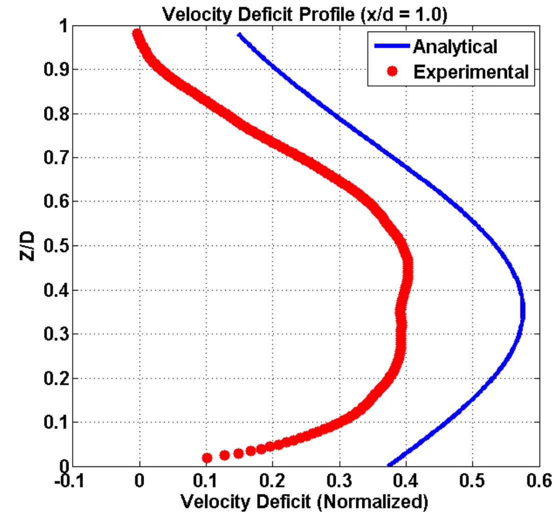

Gaussian distribution

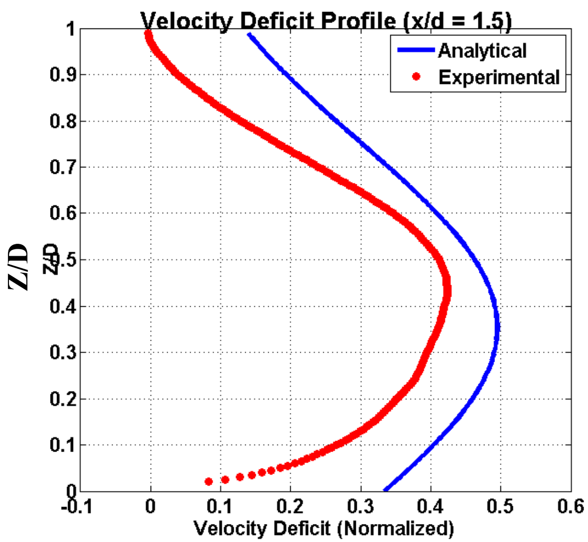
$$\Delta U/U_\infty = \left(1 - \sqrt{1 - \frac{C_T}{8\left(k^* x/d_0 + 0.2\sqrt{\beta}\right)^2}}\right) \times \exp\left(-\frac{1}{2\left(k^* x/d_0 + 0.2\sqrt{\beta}\right)^2}\left\{\left(\frac{z-z_h}{d_0}\right)^2 + \left(\frac{y}{d_0}\right)^2\right\}\right)$$

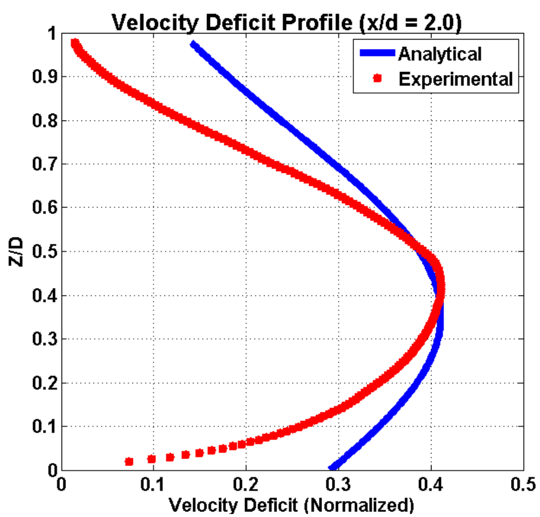
*Bastankhah & Porte-Agel, A new analytical mode for wind-turbine wakes, Renewable Energy.2014;70:116-123*

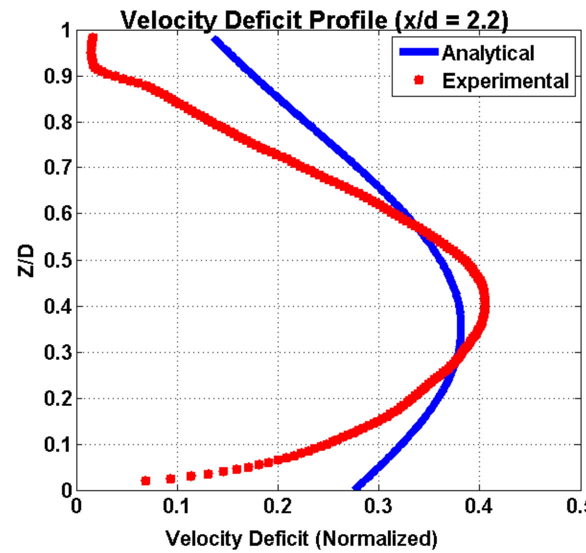
# Predicting Velocity Deficit



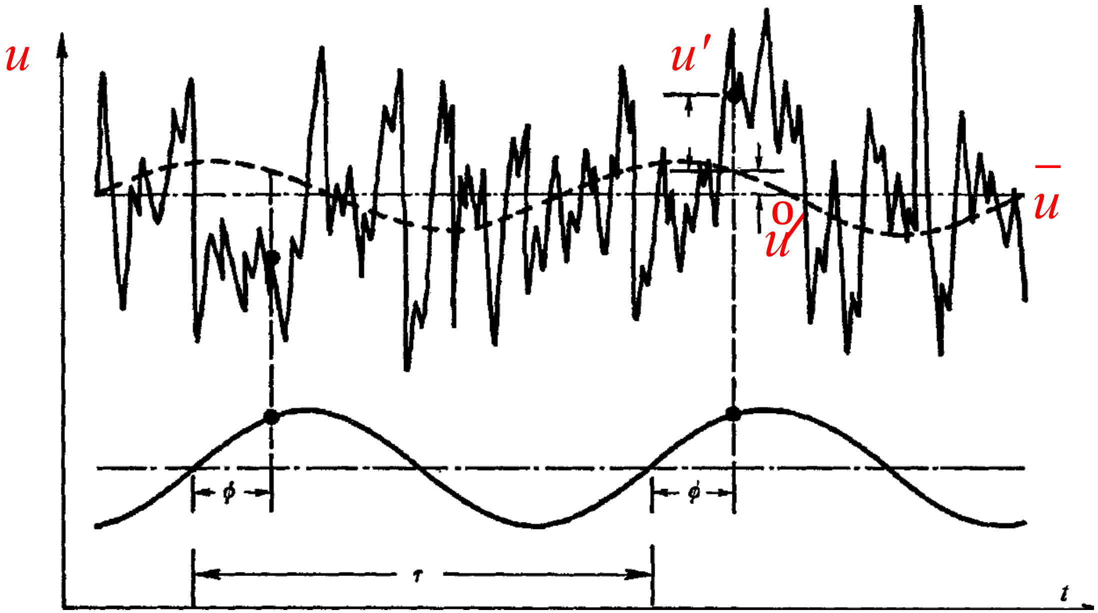



# Phase-locked PIV measurement results in turbine wake

**Triple decomposition method:**

**Hussain (J. Fluid Mech. vol. 41, part 2, pp. 241-258, 1970)**




$$u = \bar{u} + \tilde{u} + u'$$

***u*** is instantaneous velocity, the right terms correspond to mean value, contribution of organized wave and random fluctuating respectively.

Then the phase-averaged velocity is obtained:

$$\langle u \rangle = \bar{u} + \tilde{u} = u - u'$$

The phase averaging process is equivalent to a low-pass filter which keeps the mean flow and the **organized wave at a single frequency** and removes the other higher frequency fluctuations.

# Phase-locked PIV measurement results-Reynolds shear stress

The Reynolds stress contribute a major part of the momentum transfer in turbine wake flow

Triple decomposition method to Reynolds stress:

$$r_{ij}/-\rho = (u + u')(v + v') = u\,v + u'v + u\,v' + u'v'$$

Where: $\tilde{u} = \langle u \rangle - \bar{u}$ $\quad u' = u - \langle u \rangle$

Additionally, coherent structures are uncorrelated with random parts :

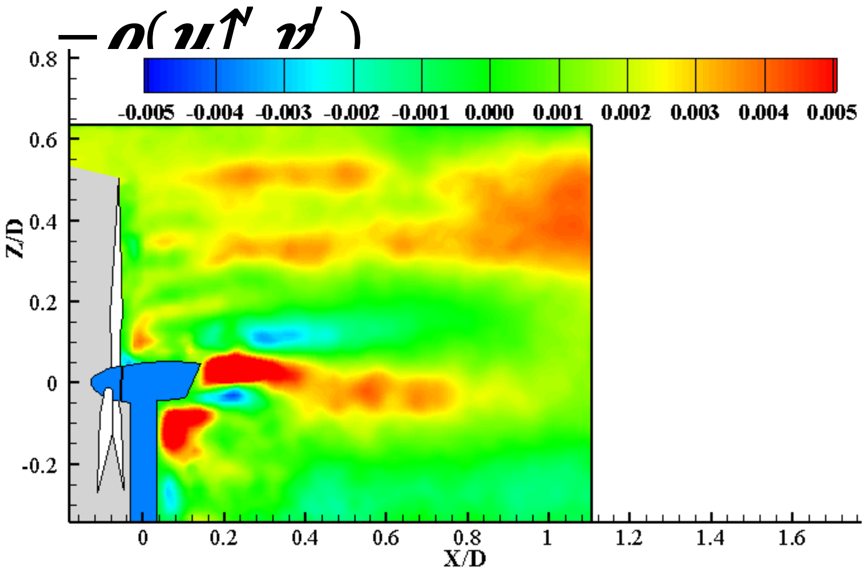
$$u'v = 0 \qquad u\,v' = 0$$

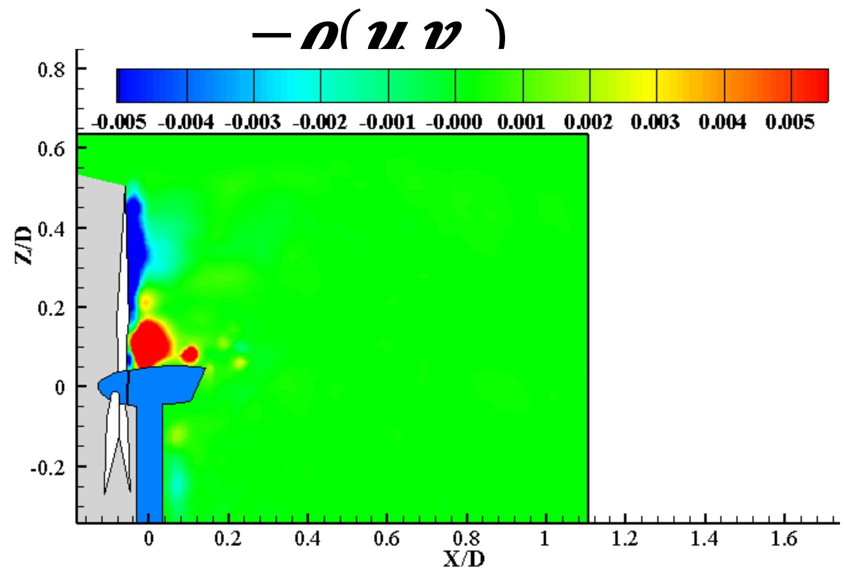
So: $$r_{ij}/-\rho = u\,v + u'v'$$

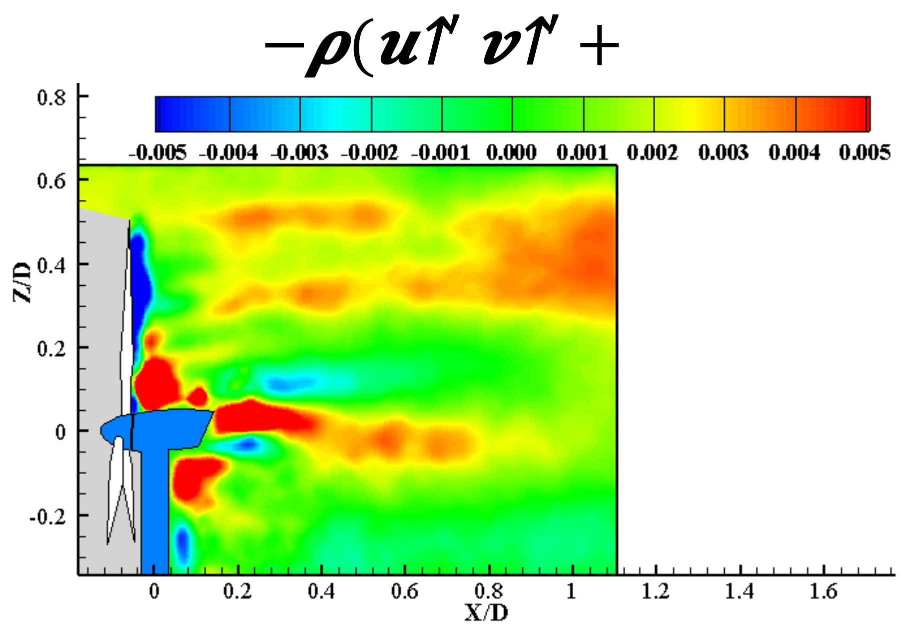
Therefore the overall Reynolds stress consists of the contributions from the

coherent structures $u\,v$ and random fluctuating $u'v'$

# Ensemble Averaged PIV measurement results-Reynolds stress

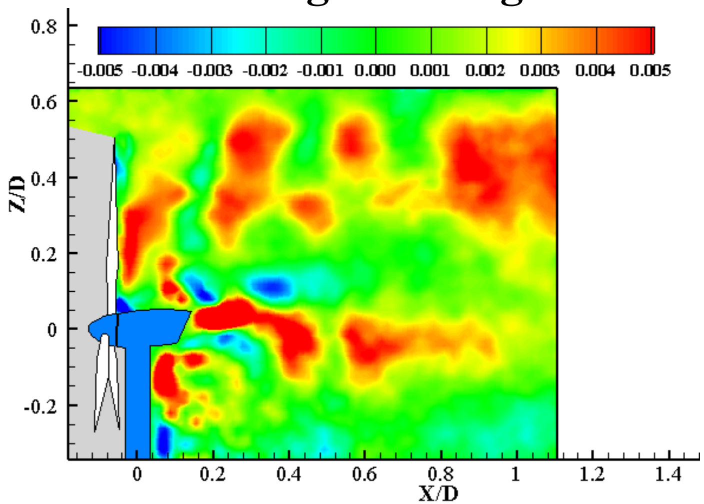


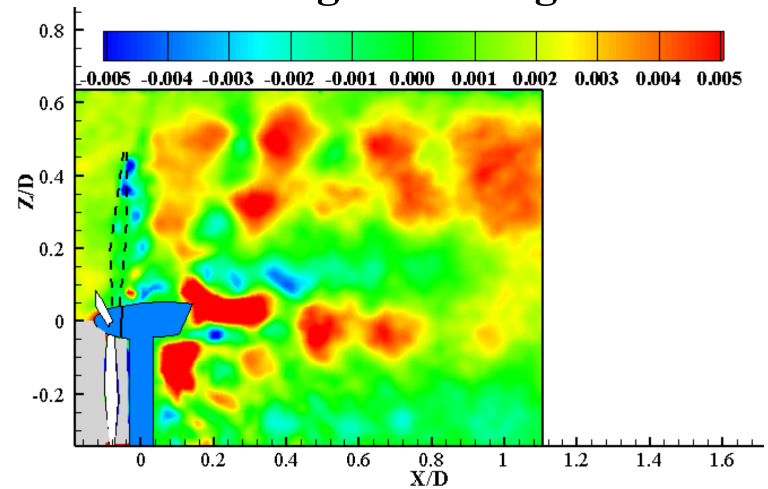



# Decomposed Reynolds Stress $(-\rho \langle u' v' \rangle)$

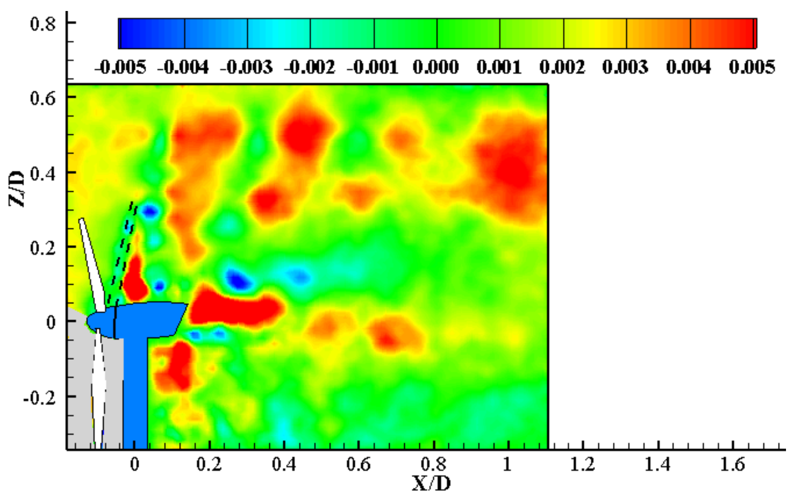
## Phase angle = 0 deg.



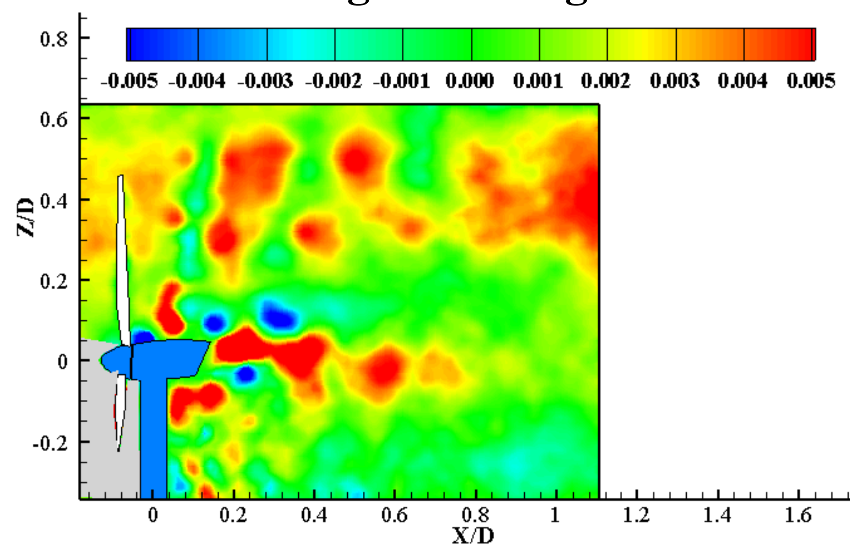

## Phase angle = 30 deg.




## Phase angle = 60 deg.




## Phase angle = 90 deg.



# POD (Proper Orthogonal Decomposition)

- POD is used to extract dominant large scale structures buried in a flow.
- The flow is decomposed to modes using linear decomposition and reconstructed using singular value decomposition (SVD).
- The magnitude of the eigenvalues represent the kinetic energy which the modes are ranked based on.
- Large scale structures can be represented with the first few modes.

# Method of Snapshots

(1) Arrange all of fluctuating velocity components in a matrix U as:

$$\mathbf{U} = [\mathbf{u}^1 \quad \mathbf{u}^2 \quad \cdots \quad \mathbf{u}^N] = \begin{bmatrix} u_1^1 & u_1^2 & \cdots & u_1^N \\ \vdots & \vdots & \ddots & \vdots \\ u_M^1 & u_M^2 & \cdots & u_M^N \\ v_1^1 & v_1^2 & \cdots & v_1^N \\ \vdots & \vdots & \ddots & \vdots \\ v_M^1 & v_M^2 & \cdots & v_M^N \end{bmatrix}$$

where M is the number of spatial discrete points and N is the number of the PIV snapshots, which represent the spatial and temporal resolutions of the PIV data respectively.

(2) **The eigenvalues and eigenvectors** of the auto-covariance matrix are calculated as:

$$\tilde{\mathbf{C}} \cdot \mathbf{A}^i = \lambda^i \cdot \mathbf{A}^i, \text{with } \tilde{\mathbf{C}} = \mathbf{U}^T \cdot \mathbf{U}$$

$$\lambda_1 > \lambda_2 > \cdots > \lambda_N = 0$$

*Ma, Geisler, Agocs & Schroeder, "Investigation of coherent structures in active flow control over a backward-facing step by PIV", 2014. 16th International symposium on Flow Visualization, Okinawa, Japan.*

# Free run PIV results analysis-Proper orthogonal decomposition (POD)

(3) **Each mode** is obtained by projecting matrix U onto each eigenvector and then normalized by its norm as:

$$\phi^i = \frac{\sum_{n=1}^{N}\left(A_n^i \cdot \mathbf{u}^n\right)}{\left\|\sum_{n=1}^{N}\left(A_n^i \cdot \mathbf{u}^n\right)\right\|}, i = 1, \cdots, N$$

So:

$$\Phi = [\phi^1 \quad \phi^2 \quad \cdots \quad \phi^N]$$

**Reconstruct the snapshot:**

(4) **The coefficients** of each mode can be obtained:

$$\mathbf{a}^n = \mathbf{\Phi}^T \cdot \mathbf{u}^n$$

(5) **A snapshot** can be reconstructed as:

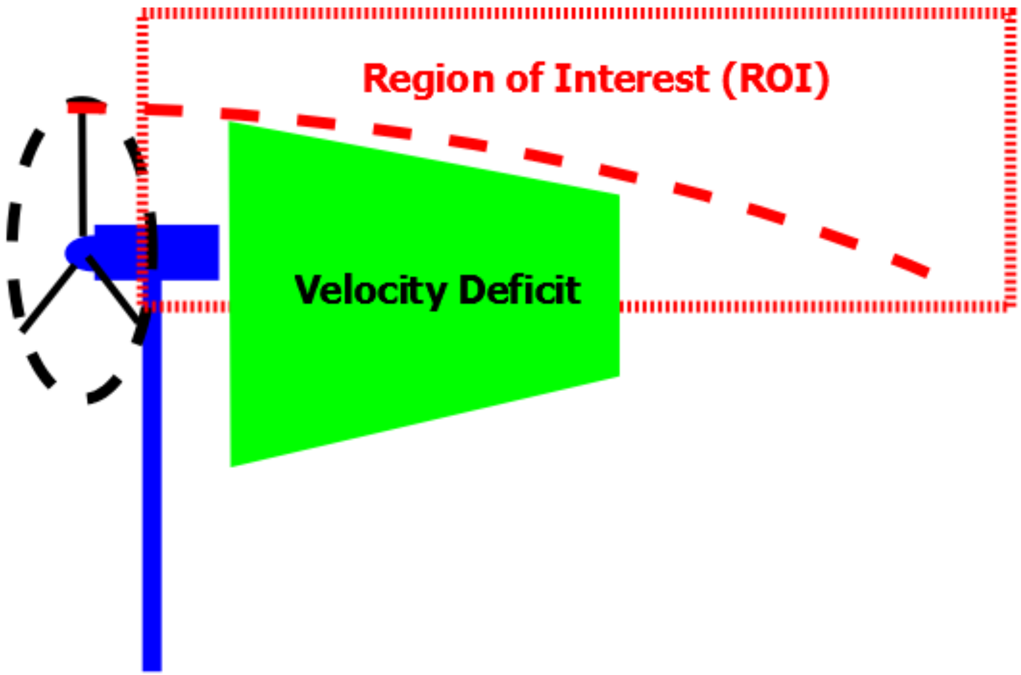
$$\mathbf{u}^n = \sum_{i=1}^{N} a_i^n \phi^i = \mathbf{\Phi} \cdot \mathbf{a}^n$$

(6) **An instantanous velocity fields** can be reconstructed using the first Lth modes as:

$$U^n = \bar{U} + u^n = \bar{U} + \sum_{i=1}^{L} a_i^n \phi_i$$ where $\bar{U}$ is the ensemble-averaged velocity

# Selecting a Region of Interest (ROI)

- A region in the wake was selected for  POD analysis to minimize the outliers and measurement noise.
- The region encompasses includes the shear layer, located between the wake and high momentum flow outside the wake.

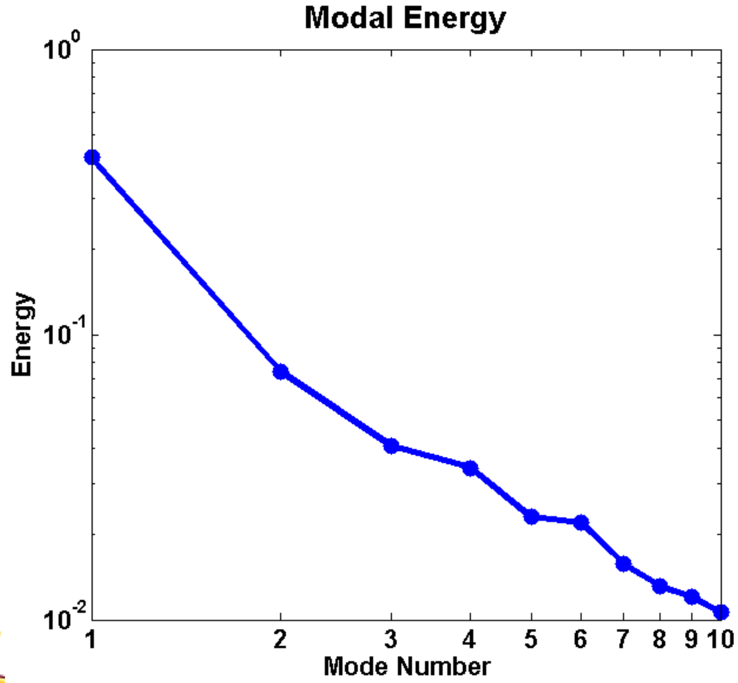


# Free run PIV results analysis-Proper Orthogonal Decomposition (POD)

1. Each eigenvalue is proportional to its total kinetic energy of velocity fluctuations

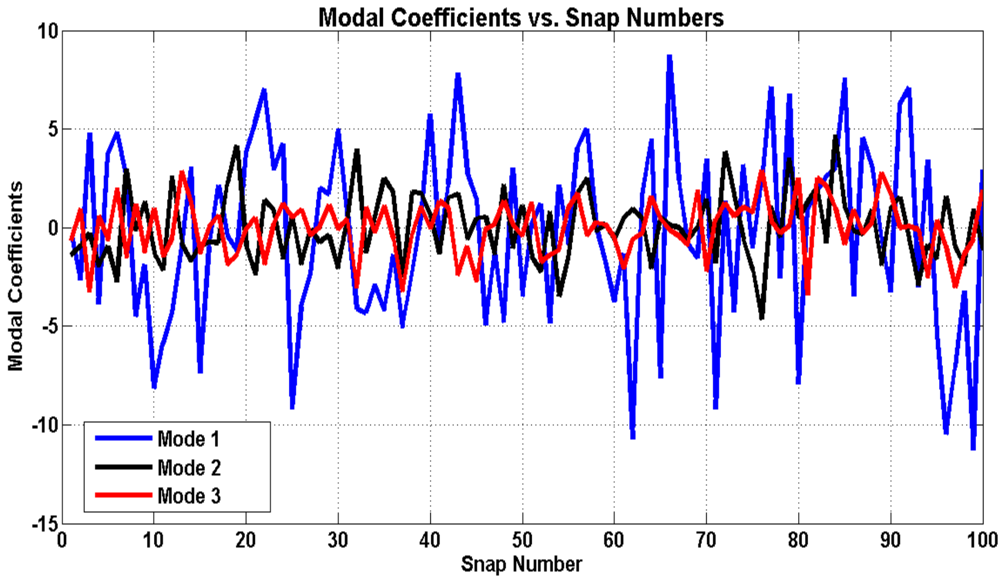
2. The most important modes containing higher energy are in the first few modes, which correspond to large-scale flow structures

3. Other further modes containing lower energy are less dominant, sometime are merged into measurement noise.

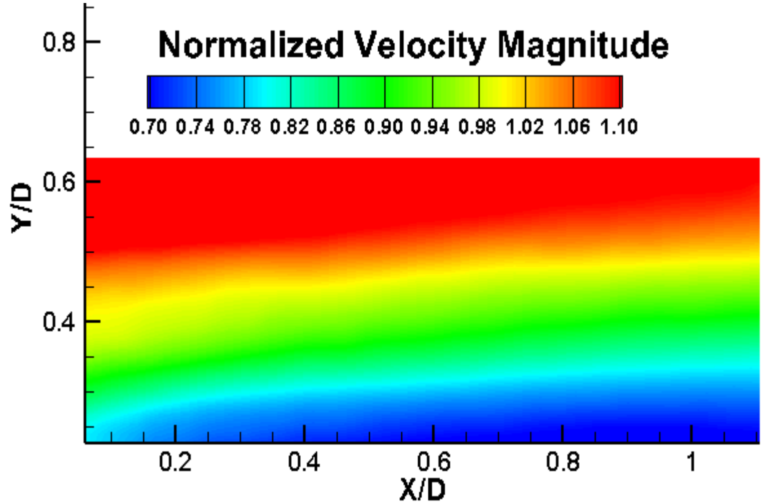


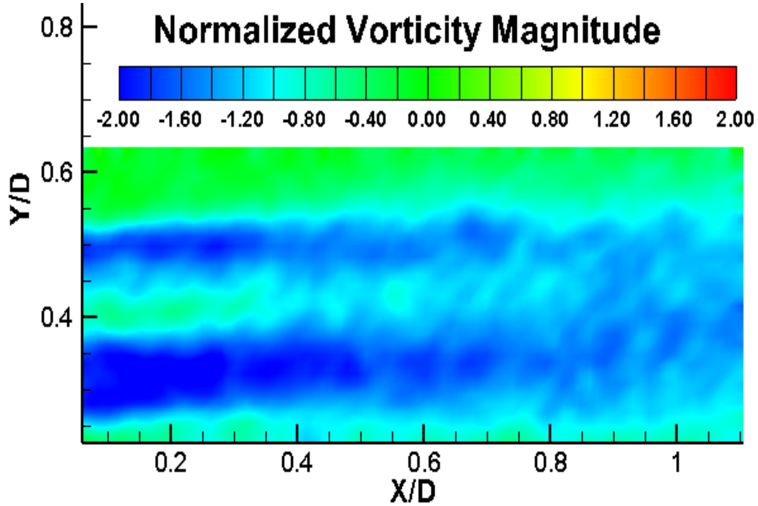

| Mode Number | Energy % |
|---|---|
| 1 | 41.7 |
| 2 | 7.4 |
| 3 | 4.0 |
| 4 | 3.4 |
| 5 | 2.3 |
| 6 | 2.2 |
| 7 | 1.6 |
| 8 | 1.3 |
| 9 | 1.2 |
| 10 | 1.0 |

# Proper Orthogonal Decomposition (POD)



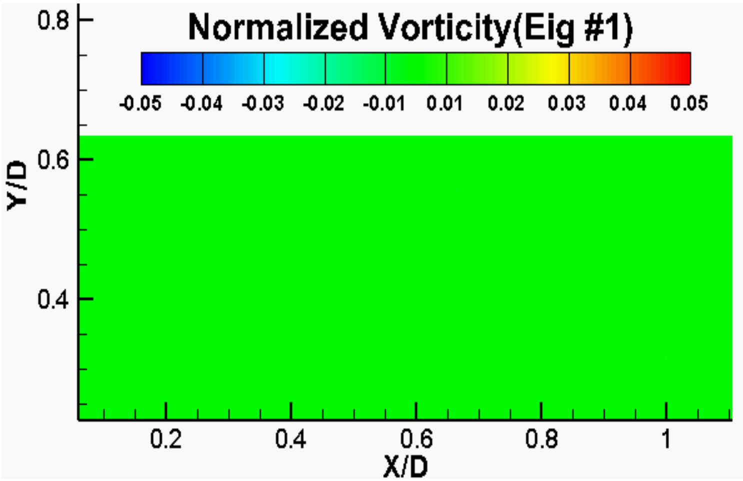

| Mode | Std. Dev |
|---|---|
| 1 | 4.31 |
| 2 | 1.72 |
| 3 | 1.37 |



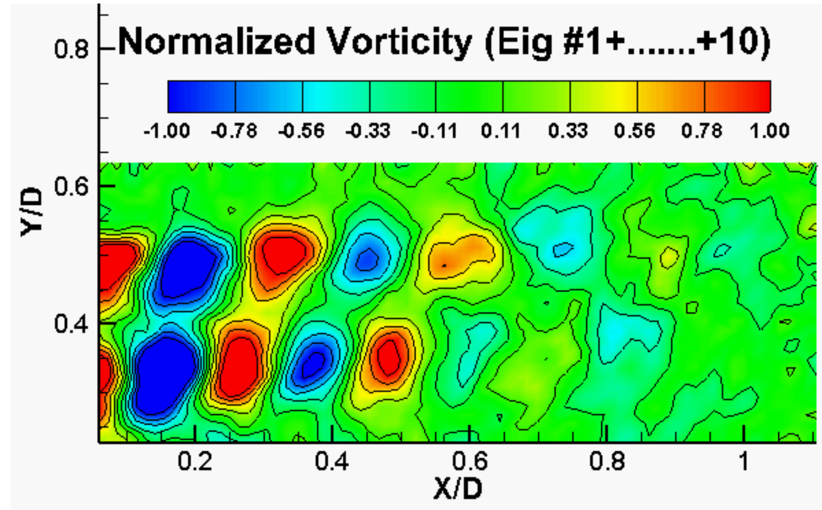




**Time Averaged Solutions (Mode 0)**

# Fluctuations and Velocity Reconstructions

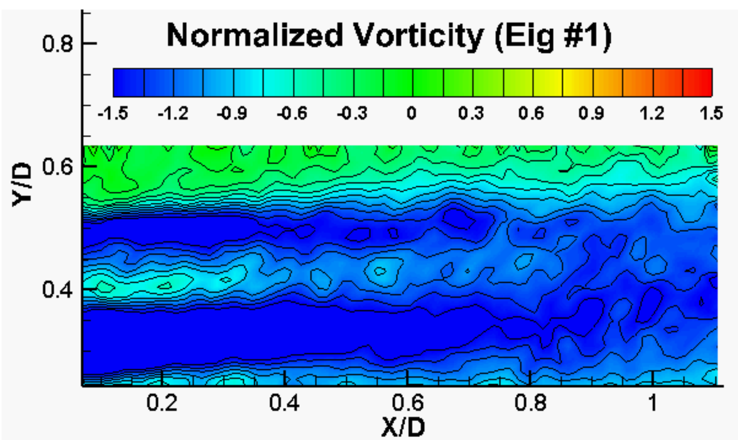


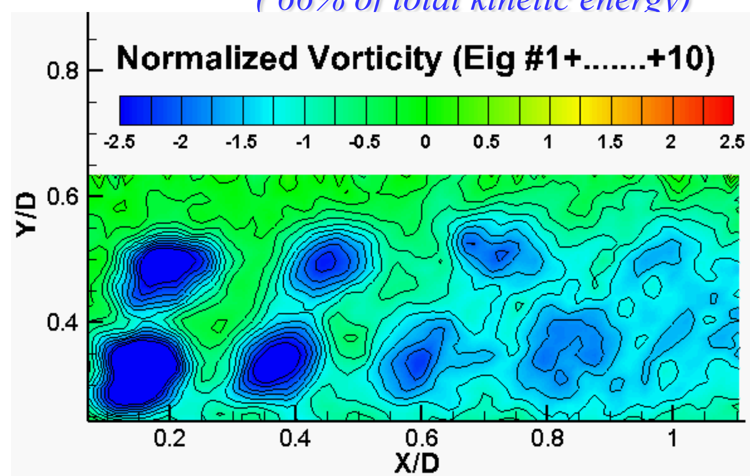

$$u^n = \sum_{i=1}^{L} a_i^n \phi_i$$




*Reconstructed by using the first 1 mode*
*( 41% of total kinetic energy)*

*Reconstructed by using the first 5 mode*
*( 66% of total kinetic energy)*



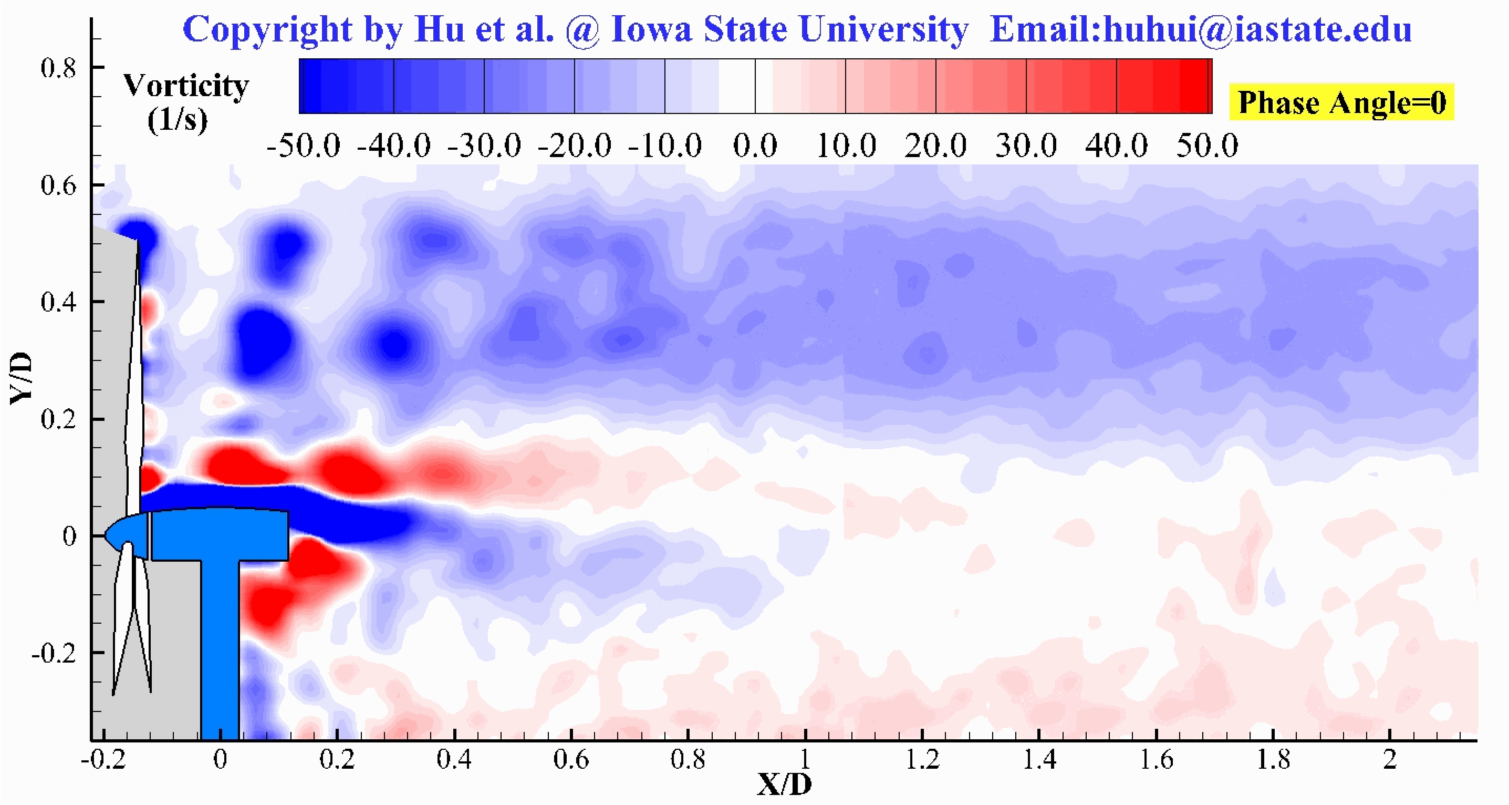




$$U^n = \bar{U} + u^n = \bar{U} + \sum_{i=1}^{L} a_i^n \phi_i$$

# Conclusions

- Proper Orthogonal Decomposition (POD) can be successfully used to identify principal components of the flow.
- The principal components carry most of the kinetic energy and the low order reconstructions aid the design process of blades, rotor assemblies and vortex generators
- Reynolds stress obtained from TD indicates vertical momentum transfer in the recovering wake.
- The wind farm designer can use this information to determine the distance between turbines for max. performance.

# Thank you for your attention!

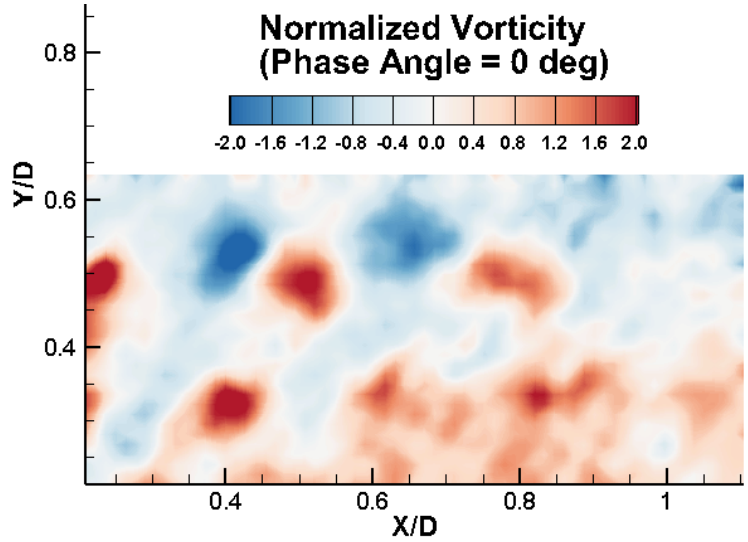


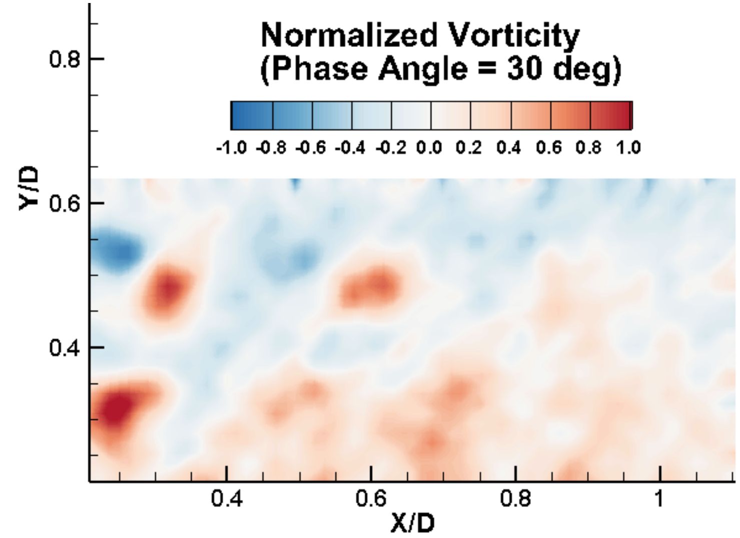
# APPENDIX

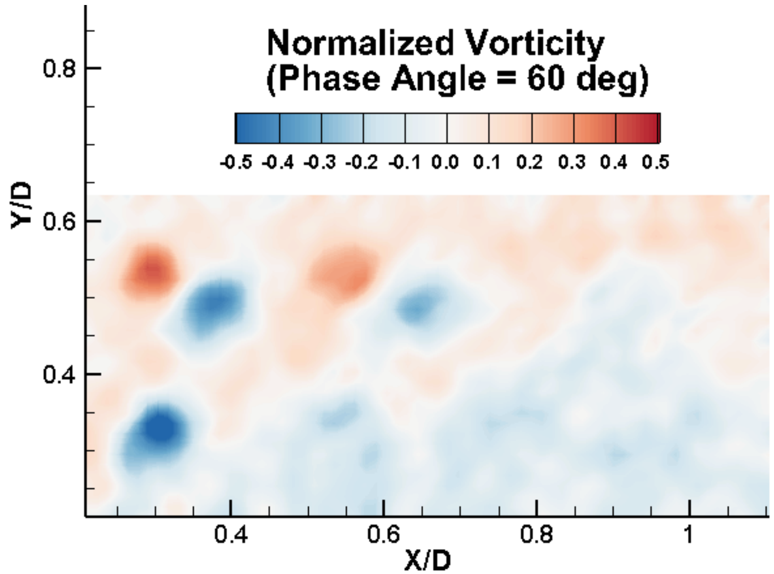
# Future Work - DMD

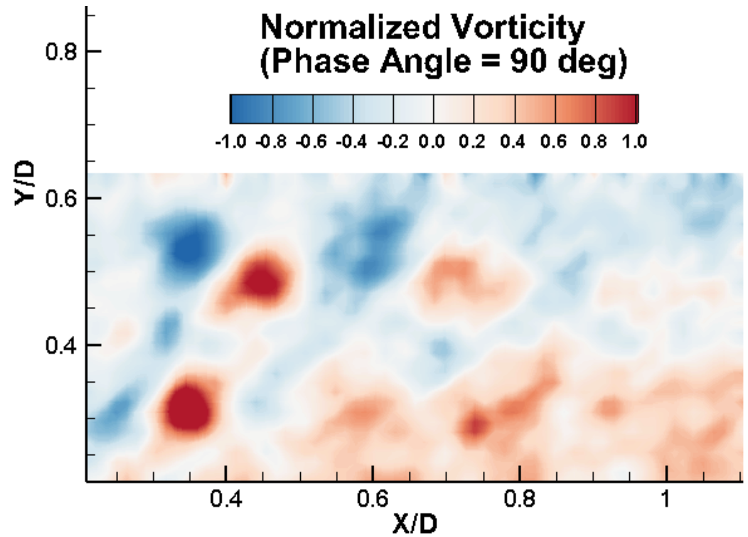
- Dynamic Mode Decomposition (DMD) is a new tool in dynamical systems that are used to investigate flow features of unsteady flows.
- DMD assigns frequencies to the large scale structures while POD modes are ranked based on kinetic energy of the flow.
- DMD is based on a temporal orthogonal framework while POD is based on a spatial orthogonal one.

# POD reconstructions on phase locked data

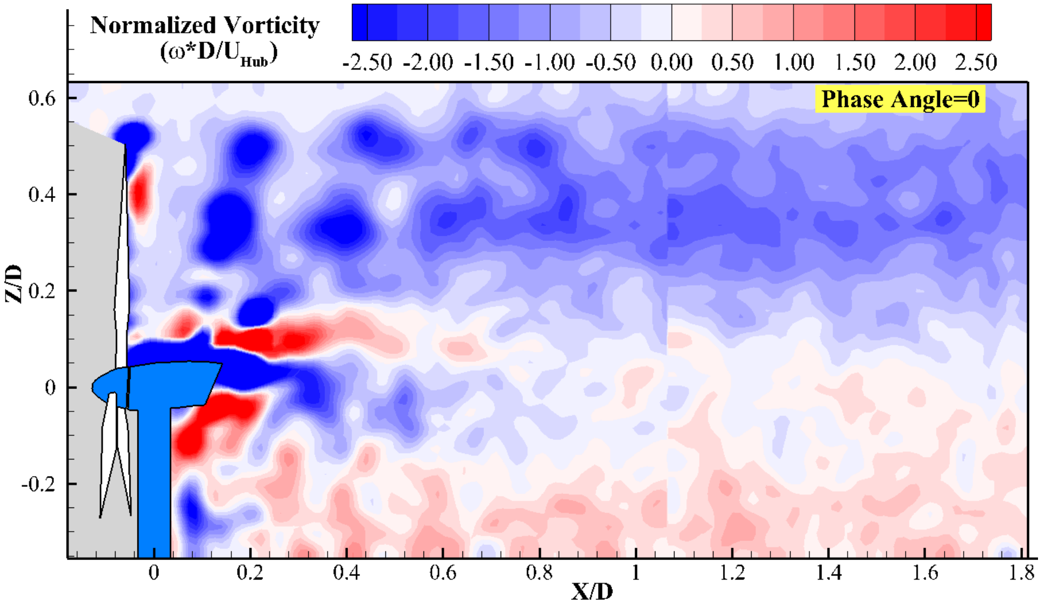


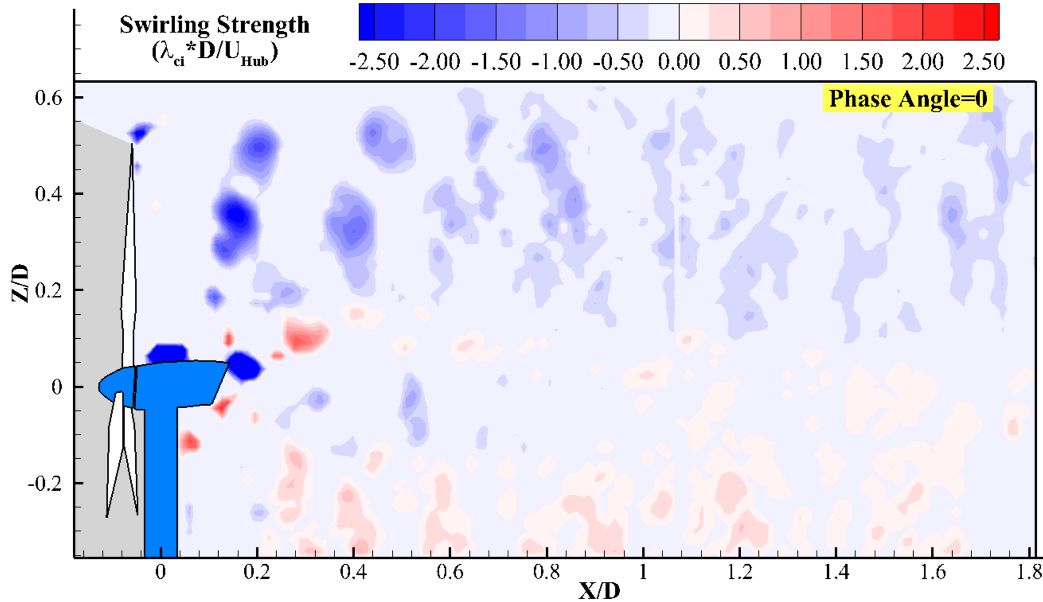



# Swriling strength distributions in the turbine wake

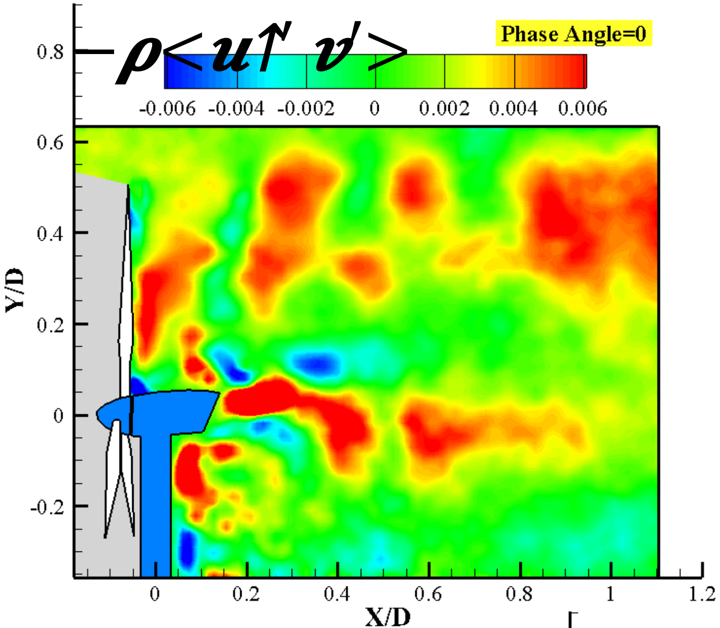


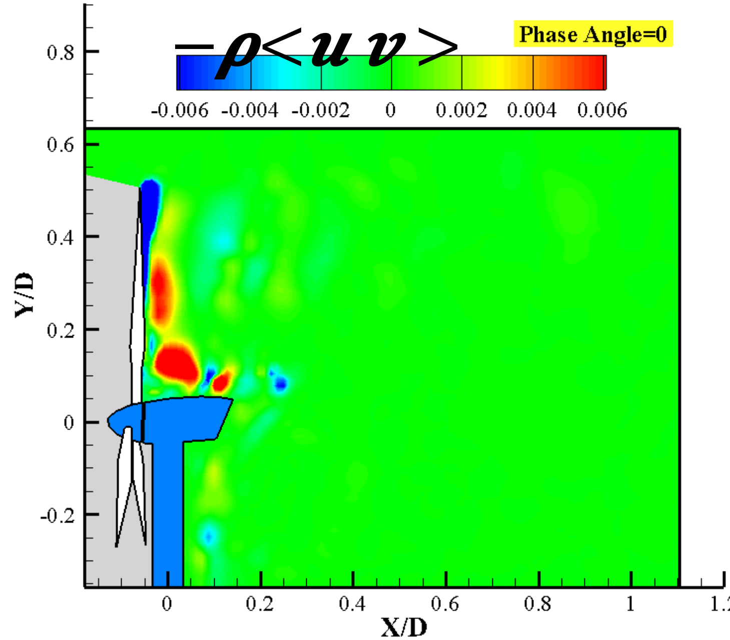

***Low turbulence inflow***

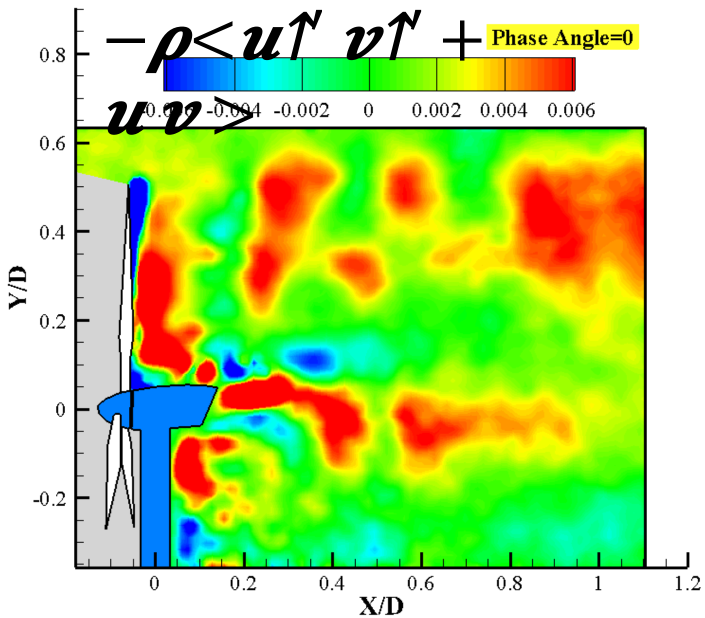
***Phase angle=0***



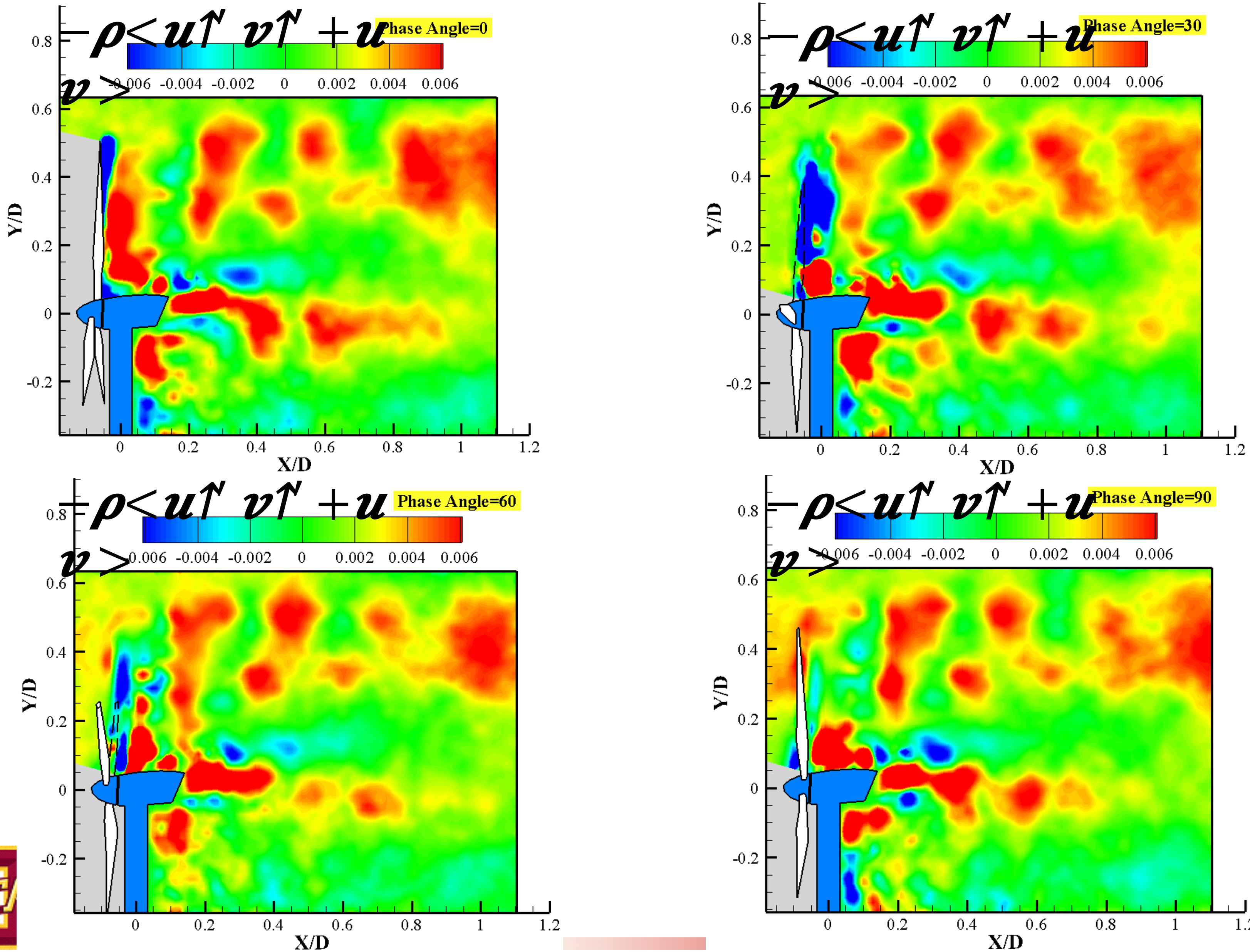
# Phase-locked PIV measurement results-Reynolds stress



# Phase-locked PIV measurement results-Reynolds stress



# Observations

- **Reynolds stresses are distributed within the shear layer.**
- **The shed vortices also contain turbulent mixing components.**
- **Presence of a tower and a nacelle also contributes to T.K.E and Reynolds Stress in downstream wake.**