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An Analytical Procedure for Evaluating Aerodynamics of Wind Turbines in Yawed Flow

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Yawed Flow Aerodynamics

- Turbines are subjected to changing wind directions leading to yaw error and reduced power output

- Zero yaw in free stream, yet turbines in middle of farm see yawed flow

- Analytical prediction based on yaw error is helpful for onboard computations.

Analytical Formulation

Analytical Formulation

- Based on momentum theory
- **$Cp = f(\gamma, v)$**

where;

Cp = Coefficient of power for Horizontal Axis Wind Turbine (HAWT)

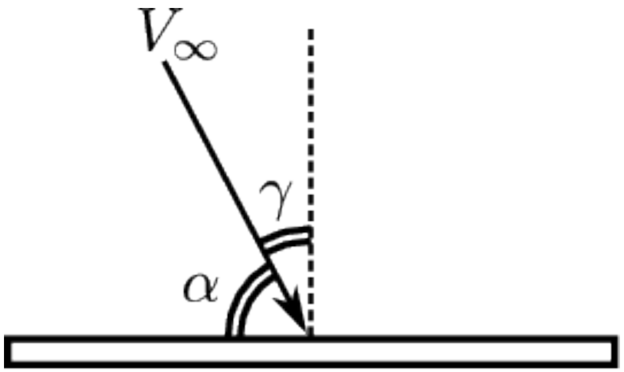
γ = Yaw error angle

v = deficit velocity at rotor disk

Analytical Formulation

- Yaw error angle and Tip-path-plane angle:

$$\gamma = 90^\circ - \alpha$$



- Inflow ratio:

$$\lambda = \frac{V_\infty \sin\alpha - v}{\Omega R}$$

- Advance ratio:

$$\mu = \frac{V_\infty \cos\alpha}{\Omega R}$$

where:

γ = Yaw error angle

V_∞ = Free stream velocity

α = angle between rotor plane and horizontal: ***Tip-path-plane (TPP) angle***

Ω = rotor angular velocity

R = rotor radius

v = induced velocity on the rotor plane

Analytical Formulation

- Power Coefficient:

$$k_P = \frac{P}{\rho A (\Omega R)^3}$$

- Thrust Coefficient:

$$k_T = \frac{T}{\rho A (\Omega R)^2}$$

- Relation between k_P and k_T:

$$k_P = \lambda k_T$$

Note:

✓ k_P and k_T are simply manipulations of generally accepted definitions of C_P and C_T, where;

$$C_P = \frac{P}{\frac{1}{2}\rho A {V_\infty}^3} \qquad C_T = \frac{T}{\frac{1}{2}\rho A {V_\infty}^2}$$

Analytical Formulation

- By momentum conservation in rotor normal direction:

$$T = \dot{m}(2v)$$

$$\dot{m} = \rho A[(V_\infty \sin\alpha - v)^2 + (V_\infty \cos\alpha)^2]^{1/2}$$

- Non-dimensionalizing *T*:

$$k_T = 2(\mu \tan\alpha - \lambda)\sqrt{\mu^2 + \lambda^2}$$

- Replacing $\mathrm{k_T}$ with $\mathrm{k_P}$ using: $k_P = \lambda k_T$

$$\lambda = \frac{k_P}{2\left(\mu \tan\alpha - \lambda\right)\sqrt{\mu^2 + \lambda^2}}$$

- Re-arrange above Eq. in a form solvable by Newton-Raphson's iterative solution technique

Analytical Formulation

k_P – Inflow Equation

$$\lambda_{n+1} = \lambda_n - \frac{\lambda_n - \frac{k_P}{2g}}{1 + \frac{k_P}{2g}\frac{g'}{g}}$$

where:

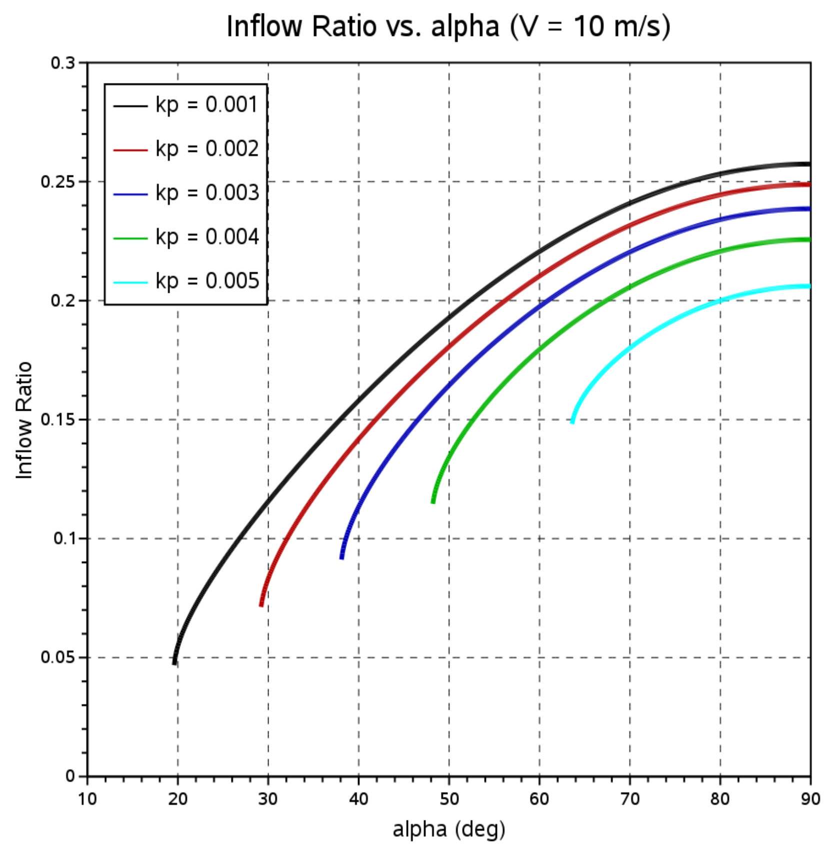
$$g = (\mu \tan\alpha - \lambda)\sqrt{\lambda^2 + \mu^2}$$

$$g' = \frac{\mu\lambda\tan\alpha - \lambda^2}{\sqrt{\lambda^2 + \mu^2}} - \sqrt{\lambda^2 + \mu^2}$$

•Solve using Newton-Raphson's iterative solution technique

•Therefore, k_P = *f(*λ *,* α*)* = *f(inflow, yaw-error)*

Analytical Formulation



Solution of k_P – Inflow equation for $V_\infty = 10$ m/s

Numerical Method

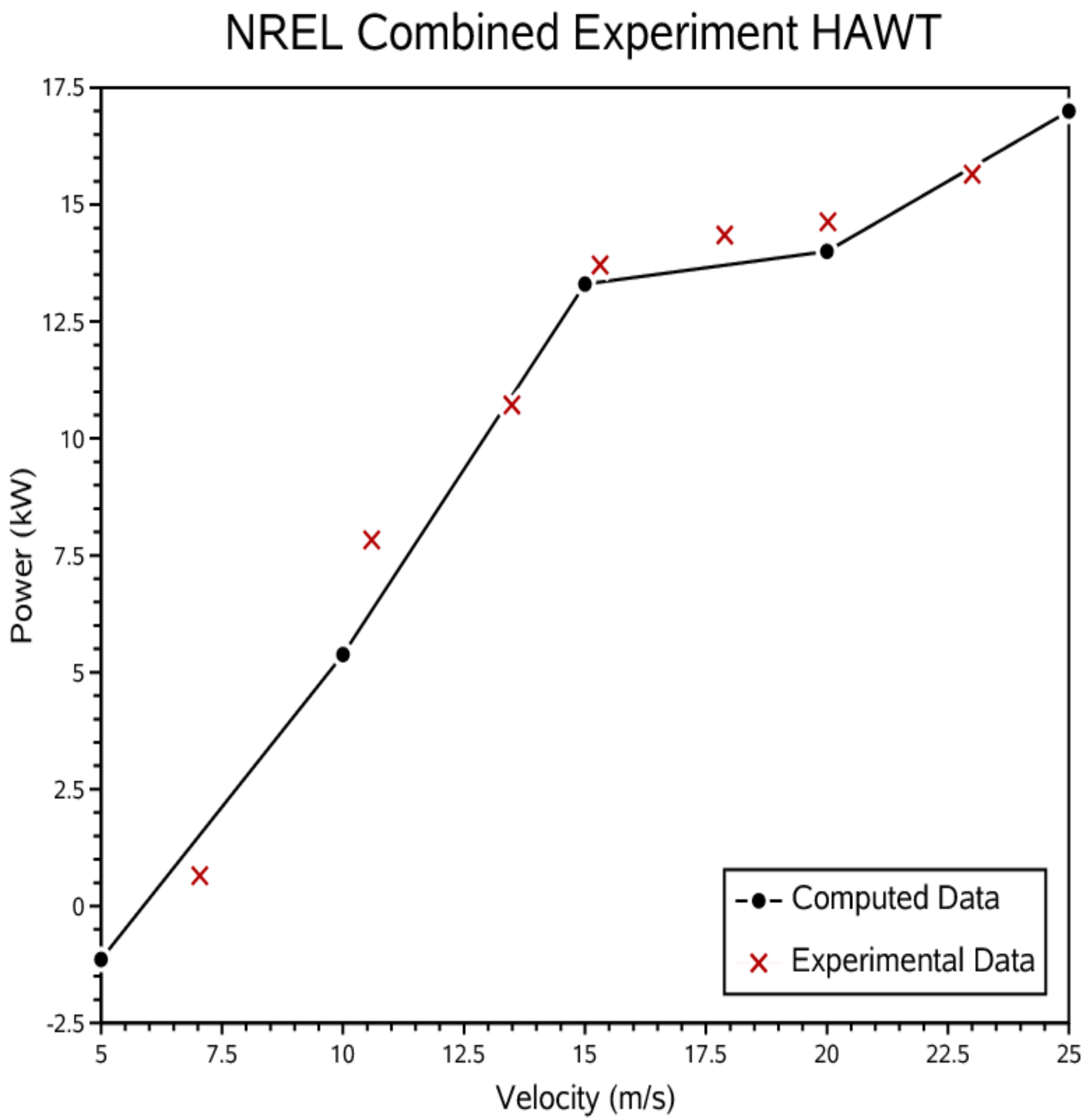
Rot3DC

- Structured finite volume solver with turbine treated as momentum sources.
- Solves 3D, unsteady, incompressible RANS Navier-Stokes equations
- Rotor momentum source depends on:
  - local flow properties
  - turbine rotor geometry
  - 2D aerodynamic characteristics of blade cross-section

Rot3DC Validation

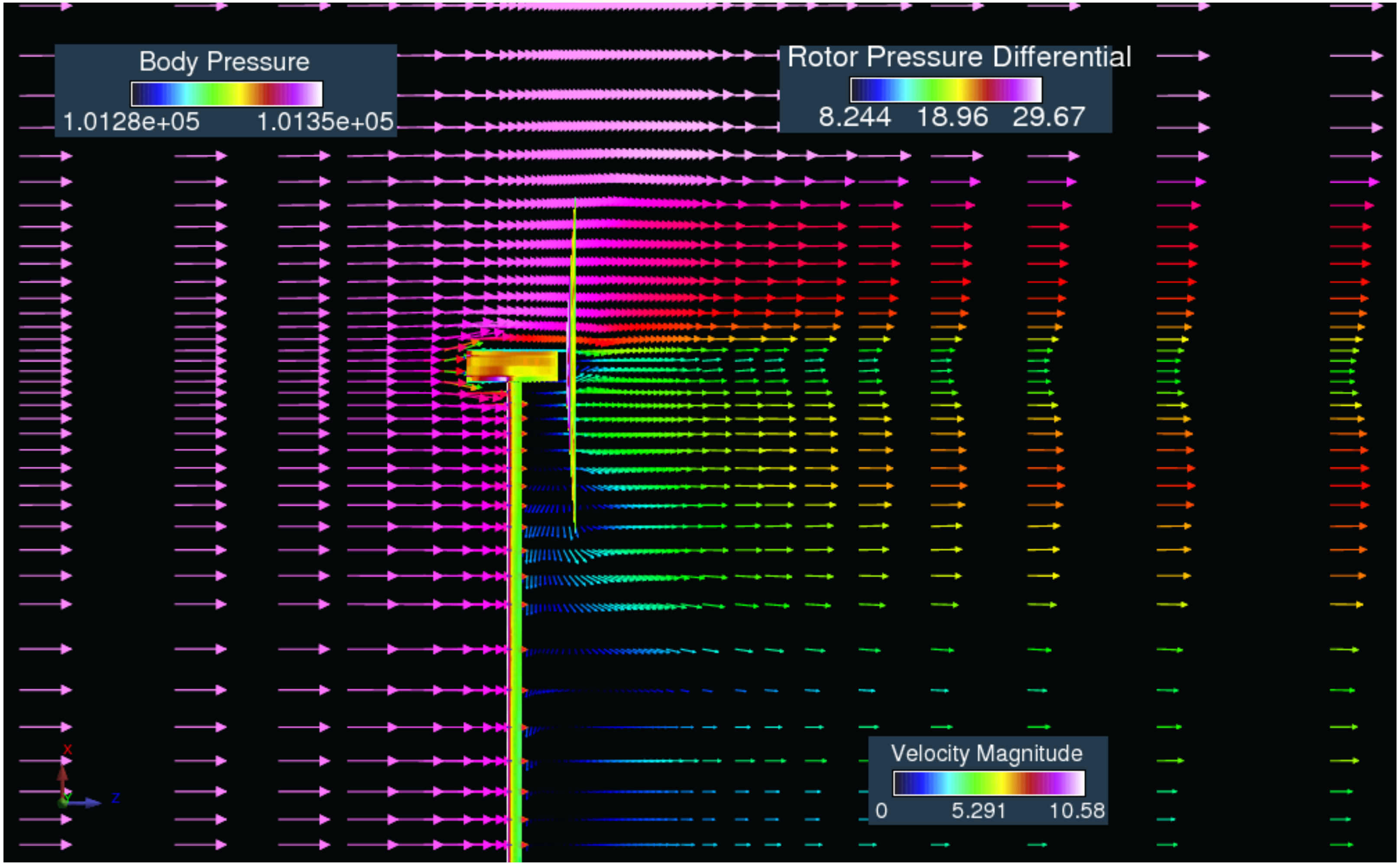
NREL Combined Experiment

NREL Combined Experiment : Power Comparison



Power vs. Windspeed

NREL Combined Experiment : Flow Solution



Y-plane through rotor center
$V_\infty = 10$ m/s

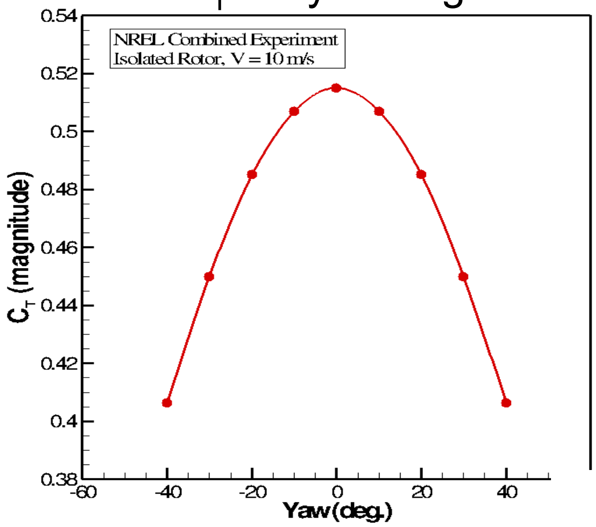
NREL Isolated Rotor: Yaw Study

- NREL rotor without tower and nacelle, in upwind position
- Free stream at angles of [-40^0, 40^0]
- Relation between Yaw and TPP angle:

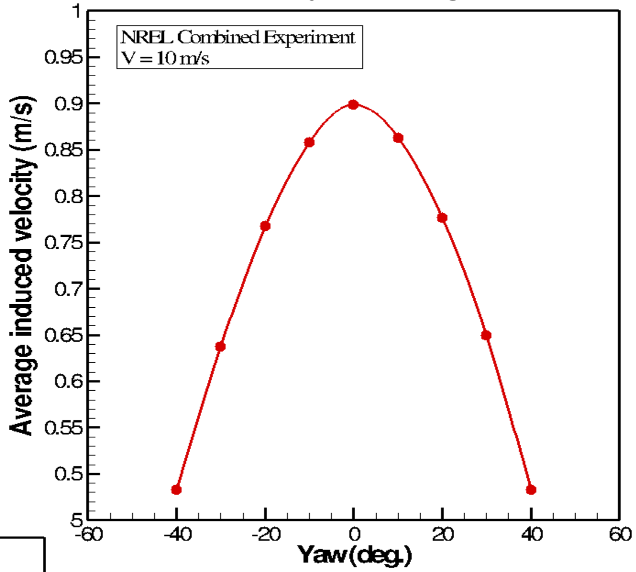
$$\gamma = 90^0 - \alpha$$

NREL Isolated Rotor:
Yawed Free Stream (V_∞ = 10 m/s)

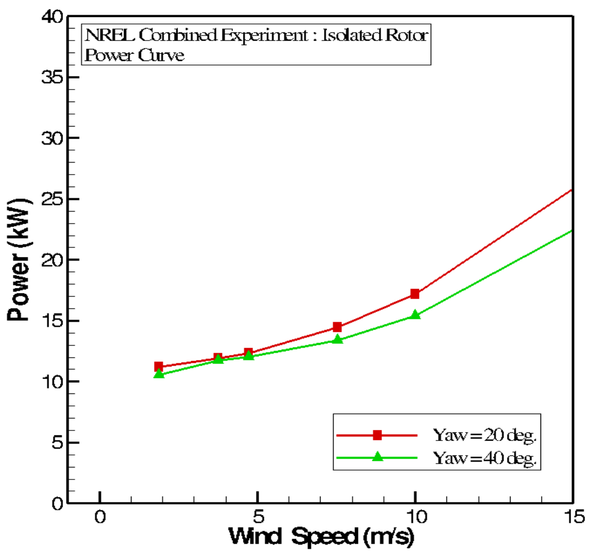
C_T vs. yaw angle



Average induced velocity vs. yaw angle



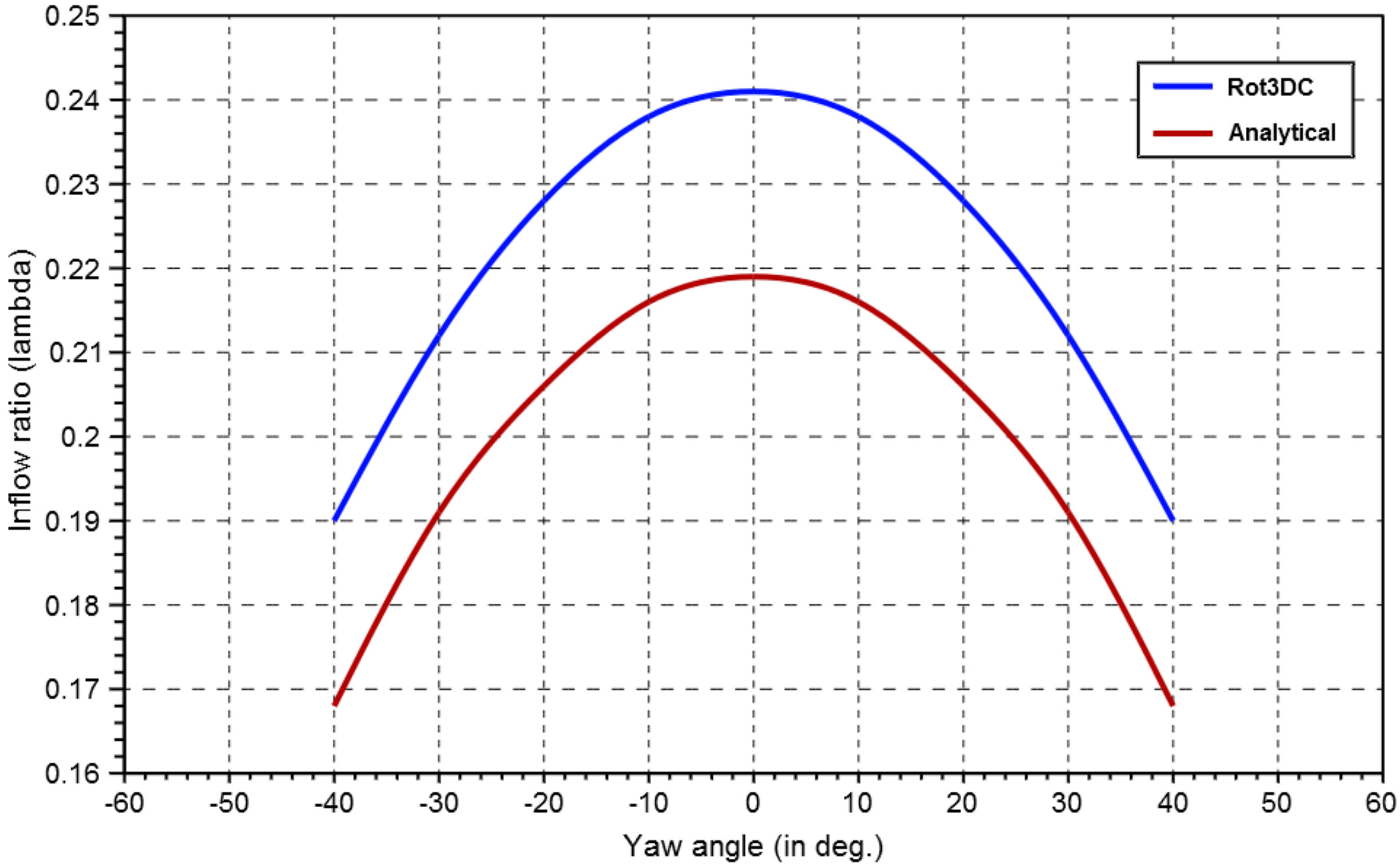
Power vs. Wind-speed



Note: Rot3DC calculated solution

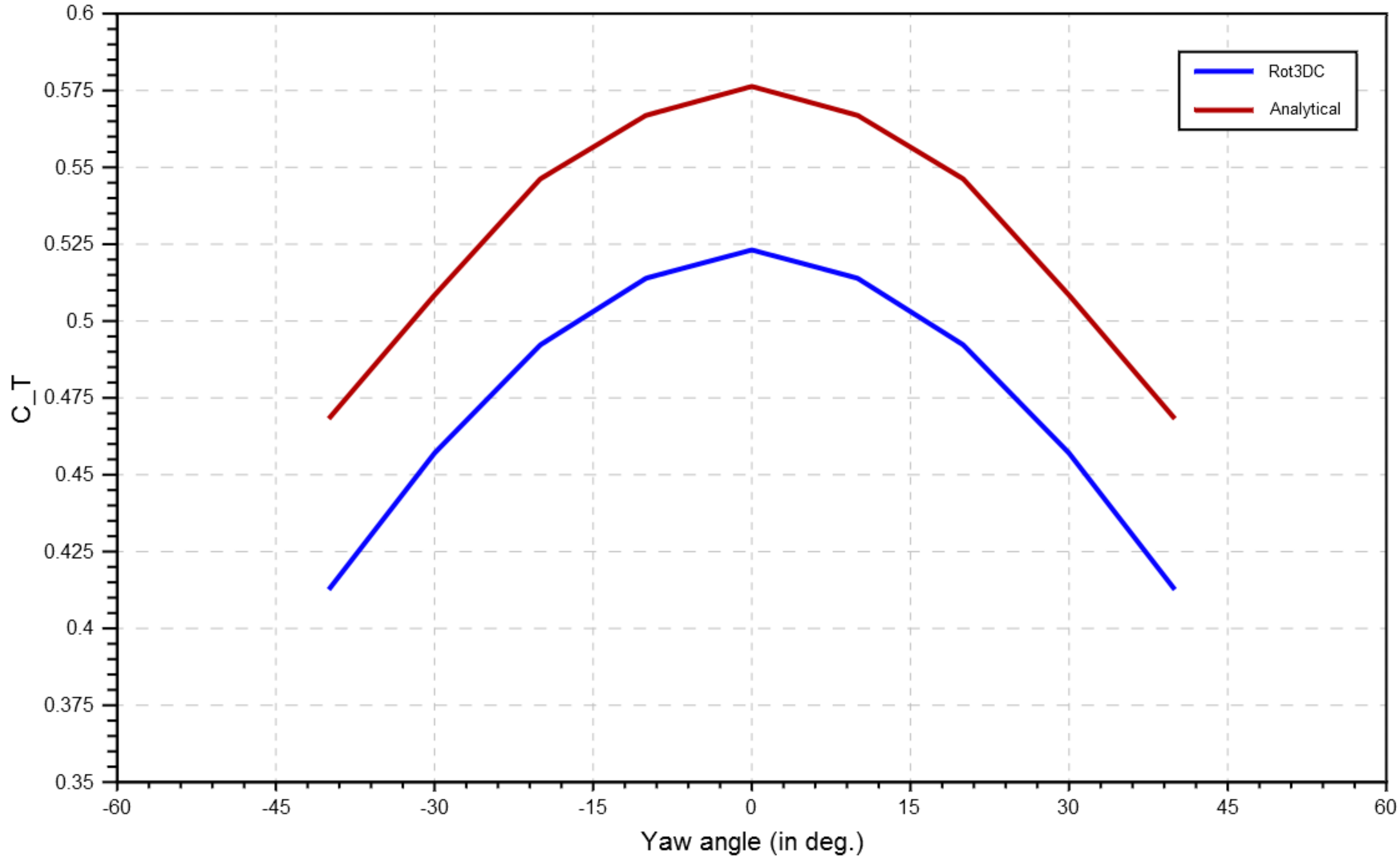
Analytical Method and Rot3DC Correlations

Correlations:
Comparison of Inflow Ratio (V_∞ = 10 m/s)



Inflow Ratio (λ) vs. Yaw Angle

Correlations:
Comparison of C_T (V_∞ = 10 m/s)



Coefficient of Thrust vs. Yaw Angle

Correlations:
Yawed Free stream (V_∞ = 10 m/s)

γ	C_P	λ_{num}	λ_{analy}	% error for λ	$C_{T num}$	$C_{T analy}$	% error for C_T
0°	0.4764	0.241	0.219	9.13	0.5230	0.5763	9.24
10°	0.4622	0.238	0.216	9.24	0.5139	0.5669	9.34
20°	0.4247	0.228	0.206	9.65	0.4922	0.5462	9.88
30°	0.3665	0.212	0.191	9.91	0.4570	0.5084	10.1
40°	0.2969	0.190	0.168	11.6	0.4126	0.4682	11.9

Comparison between Analytical Solution and Rot3DC

Note: $\alpha = 90^0 - \gamma$

Conclusions

- Simple analytical solution procedure for evaluating wind turbine performance in yawed flow
- Analytical solution within 10% error margin of computational fluid dynamics (Rot3DC) simulations
- CFD results compare well with experiments and adequately predict turbine performance under conditions of yaw
- Simplicity of the developed analytical expression can be exploited to provide input to onboard yaw control feedback systems

Thank You!

Questions ?