Large Eddy simulation of Trailing Edge Acoustic Emissions of an Airfoil

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Main Topics

❖ Computational Aeroacoustics (CAA)

❖ The role of Large Eddy Simulation (LES) in CAA

❖ Results and Discussion
General Acoustic Problem

Acoustic Wave Equation

\[ \frac{\partial^2 p}{\partial t^2} - c_0^2 \nabla^2 p = 0 \]

Assumptions
- medium at rest
- irrotational flow

Wave propagation

Source

Receiver
Aeroacoustic Problem
Computational Aeroacoustic (CAA)

- Methods with high fidelity:
  - Direct simulation (DNS/LES)
  - LES + Integral methods
Acoustic Analogy

Lighthill’s Equation

\[
\frac{\partial^2 \rho}{\partial t^2} - c_0^2 \nabla^2 \rho = \frac{\partial^2}{\partial x_i \partial x_j} T_{ij}
\]

Acoustic Wave Equation

\[
\frac{\partial^2 p}{\partial t^2} - c_0^2 \nabla^2 p = 0
\]

\[
T_{ij} = \rho u_i u_j + (p - c_0^2 \rho) \delta_{ij} - \tau_{ij}
\]

- momentum flux
- entropy production
- viscous stress
Acoustic Analogy

\[ p(\vec{x},t) \cong \frac{x_i x_j}{4\pi \|x\|^3} \frac{\partial^2}{\partial t^2} \int_V \left[ T_{ij} \right]_{te} dV(\vec{y}) \]

\[ -\frac{x_j}{4\pi \|x\|^2} \frac{\partial}{\partial t} \int_S \left[ p n_i \right]_{te} dS \]

\[ p(\vec{x},t) \cong -\frac{x_j}{4\pi \|\vec{x}\|^2} \frac{\partial}{\partial t} \int_S p n_i dS \]

Simplifications can be made if Mach number is small (Acoustic Compactness)
Problem Description

\[ U = 40 \text{ m/s} \]

\[ \text{Re} \approx 400,000 \]

\[ Ma \approx 0.115 \]

- The Mach number is around 0.1, which indicates that the space integral can be neglected.
- The unsteady pressure around the airfoil needs to be obtained before acoustic calculation.
Large Eddy Simulation (LES)

- The role of LES is to provide unsteady pressure information around the airfoil.
- Two main prerequisites of LES are required:
  - The design of a high-quality and affordable mesh, especially in the near wall region;
  - The generation of turbulent boundary layer in numerical simulation.
This state-of-the-art simulation focuses on the influence of space integral in acoustic prediction.

For the engineering application, such cost is too high to be feasible currently.

What if we only put high resolution mesh in the near wall region and use relatively coarse mesh out of the region of interest?
It turns out that if the aspect ratio is too high, the information communication is unbalanced along different directions and leads to the pressure convergence problem.
Pressure Convergence

<table>
<thead>
<tr>
<th></th>
<th>original mesh</th>
<th>new mesh</th>
</tr>
</thead>
<tbody>
<tr>
<td>mesh</td>
<td>5.5 M</td>
<td>6 M</td>
</tr>
<tr>
<td>aspect ratio</td>
<td>~150</td>
<td>~75</td>
</tr>
<tr>
<td>pressure iterations for each time step</td>
<td>~1000</td>
<td>~10</td>
</tr>
<tr>
<td>total time cost estimation</td>
<td>~180 days</td>
<td>~5 days</td>
</tr>
</tbody>
</table>

- It is shown that the convergence problem can be solved if the highest aspect ratio of mesh is reduced.
- It also indicates that the mesh design will influence not only the quality of LES result, but also the time cost.
It seems that the mean flow information around the airfoil is captured in the present simulation.
It is shown that an obvious offset of boundary layer development between our result and benchmark.
The offset of boundary layer development is even more obvious near the trailing edge, which is the exact region of interest in our work. It indicates that the numerical tripping needs to be adjusted.
Friction Coefficient

Old tripping

New tripping
Reynolds Stress

\[ R_{xy} \]

\[ R_{yy} \]

Old tripping

New tripping
Conclusions

❖ Unsteady flow information can be obtained by LES, which can be used for the further acoustic calculation.

❖ Mesh design will influence not only the quality of LES result, but also the convergence and time cost consequently.

❖ Numerical tripping will directly influence the development of boundary layer, which is the region of interest for CAA.

❖ Since that the turbulent boundary layer exits for many engineering applications, finding a suitable numerical tripping is important for obtaining a reliable acoustic result.
Thank you!