Analytical Unsteady aerodynamics model of a Horizontal axis wind turbines due to linear change in wind speed

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Small wind turbines work under unsteady conditions

Fig.1 University of Newcastle Australia 5 kW wind turbine, D= 5m

GENERAL FEATURES	
Rated Output and Rated Wind Speed	5 kW at 10.5 m/s
ROTOR	
Number of blades & diameter	2
Swept Area	19.64 m
Blade Material	Fibre glass reinforced epoxy – foam core
Rotor Position	Upwind
Aerofoil	SD7062
Rated tip Speed Ratio	8.0
Rotor Speed at Rated Output	320 rpm
Yaw System	"Free yaw" –delta wing tail fin
DRIVETRAIN	
Generator	Fan cooled 4 pole, 3 phase induction generator
Generator Output Voltage	80 - 500 V AC
Generator Frequency	20 to 70 Hz
Gearbox	8.02:1 Two-stage helical gearbox

The instantaneous power coefficient of small wind turbines can exceed the steady BETZ limit.

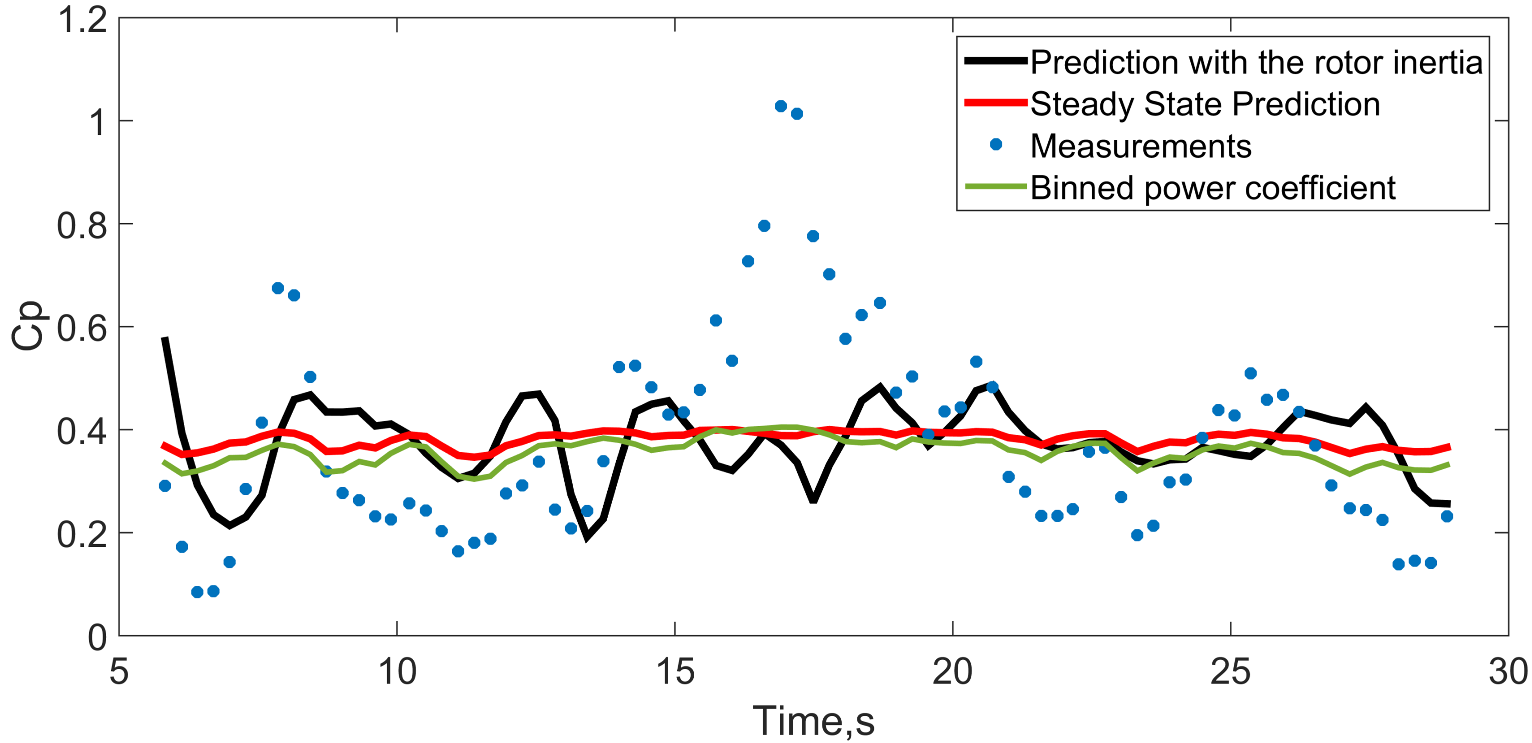


Fig.2 Measured electrical Power coefficient and the averaged "binned" rotor power coefficient of the wind turbine.

The power coefficient correlates with the tip speed ratio

$$J d\Omega/dt = Q\downarrow a - Q\downarrow g$$

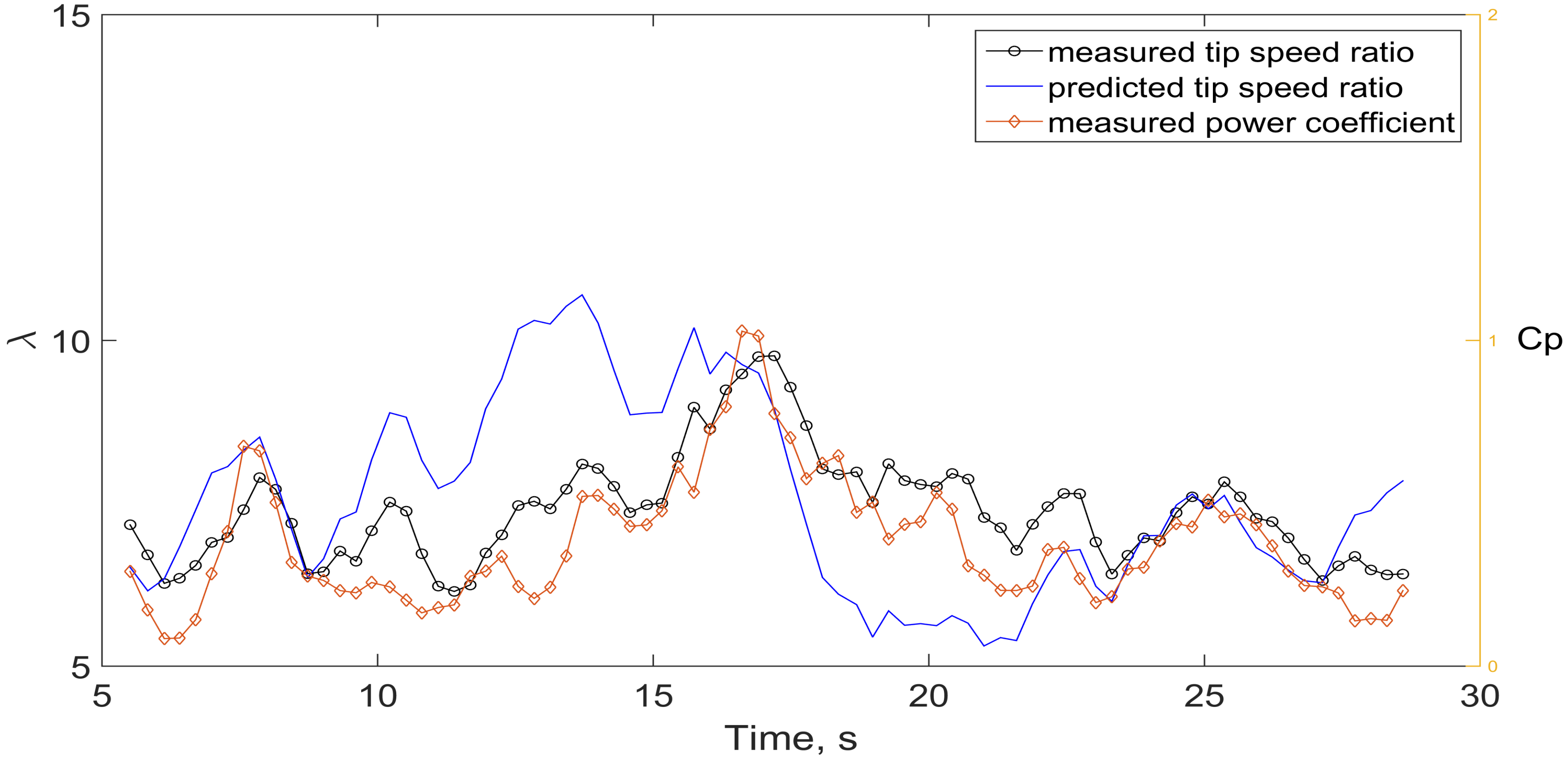


Fig.3 Measured and predicted tip speed ratio and measured power coefficient .

Optimum steady state control deviates from the actual torque

$Q_g = K\Omega^2$ $K = \rho A R^3 C_{P_{max}} / 2\lambda_0^3$ $C_{P_{max}} = 0.4$ & $\lambda_0 = 8$

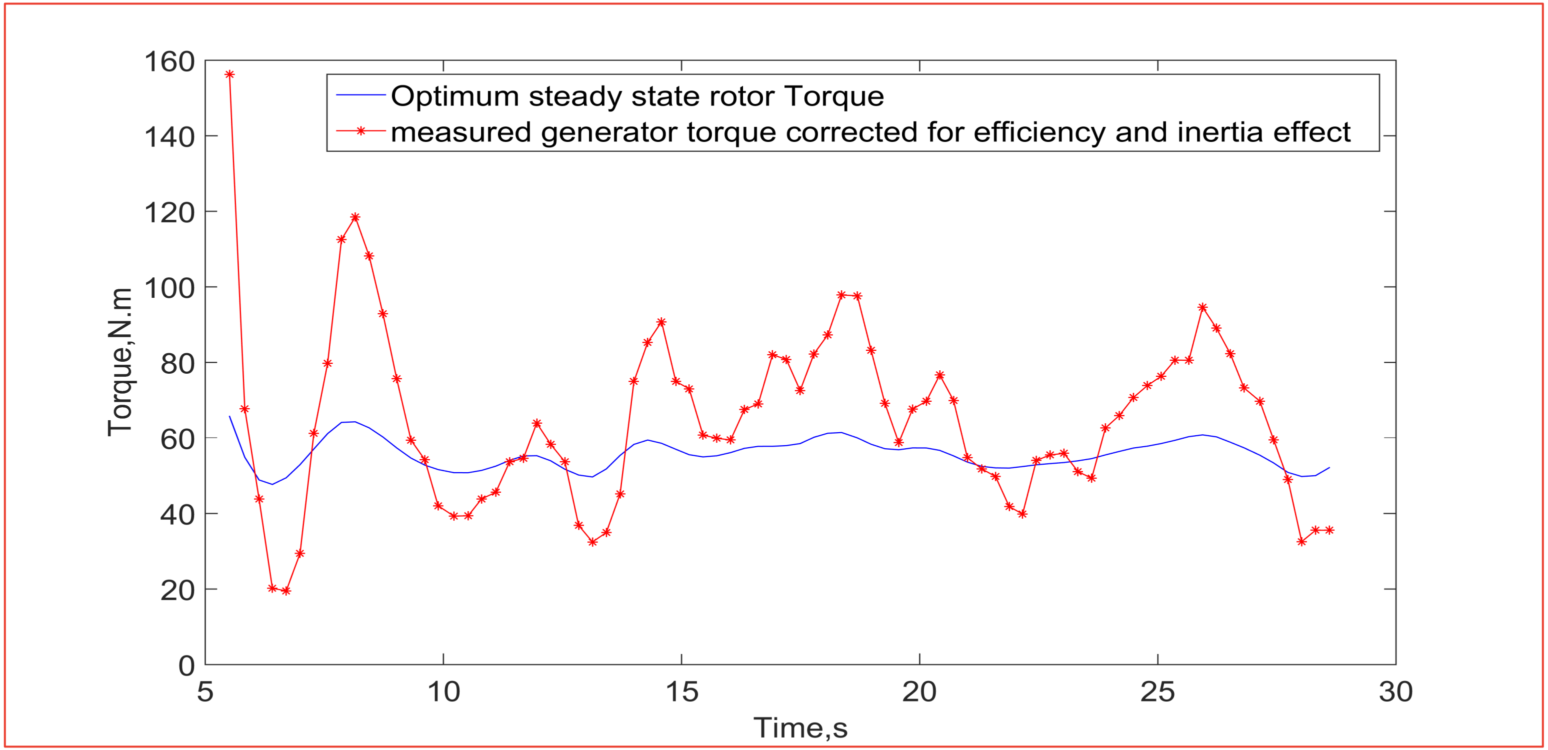


Fig.4 Measured rotor torque and the steady optimum torque.

Steady Control doesn't track the unsteady turbine torque

- Power coefficient of small wind turbines can exceed the steady Betz limit.
- The unsteady power coefficient correlates with the tip speed ratio .
- The controller change the generator torque and consequently rotor rpm based on steady optimal conditions.
- There is a large difference between actual turbine torque and the steady MPPT torque.
- This shows the need for a simple and fast unsteady wind turbine model to be used with real time control.

Two vortex systems are the main source of unsteady loading on wind turbines

Change in operating conditions change the strength of bound vorticity resulting in:

two vortex systems exist in the wake of wind turbines

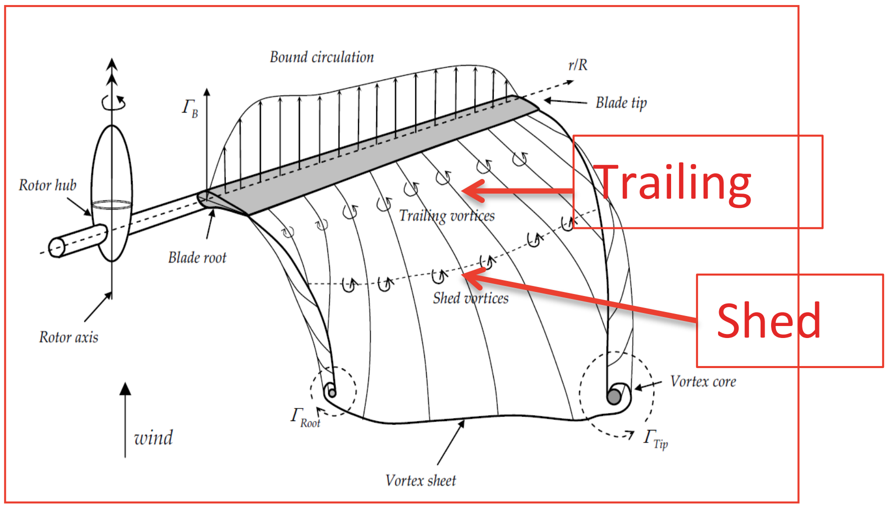


Fig.5a schematic of unsteady wind turbine vortices

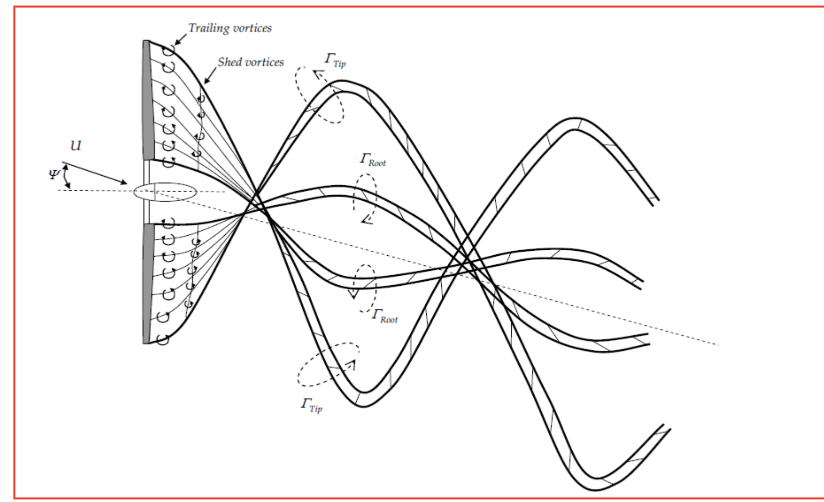


Fig.5b Roll up of the vortex wakes

The effect of unsteady trailing vortex system is called dynamic inflow.

The effect of unsteady shed vortex system is called dynamic stall.

How the unsteady aerodynamics fit in BEM?

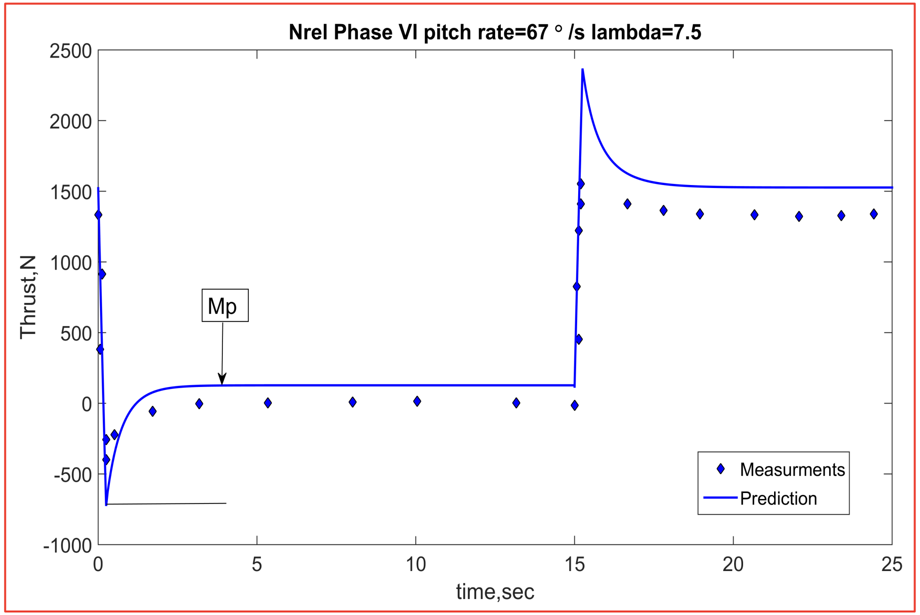
Unsteady Blade Element Method “UBEM”

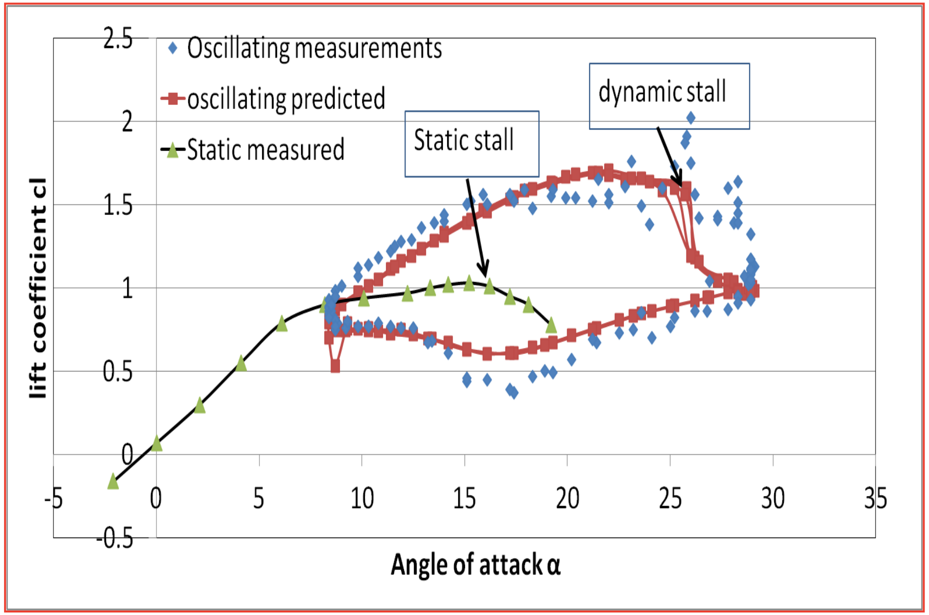
$$dCt=\sigma\left((1-a)^2+(\lambda_r)^2(1+a')^2\right)(c_l\cos\emptyset+c_d\sin\emptyset)=4a(1\text{-}a)-\tau\left[U_0/U_0-a\right]$$

The dynamic inflow results from dynamic change in the induced velocity at the rotor.

The dynamic stall results from the lag of the lift and drag coefficients to the angle of attack change.

UBEM is used to predict the forces on NREL turbine due to a change in Pitch angle





$$\alpha = \alpha_m + A \sin(2\pi f t) \quad k = \frac{\omega c}{2V} \qquad \alpha_m = 20°, A = 10° \ and \ k = 0.075$$

Fig.6 Measured and predicted thrust force for a linear change in pitch angle due to dynamic inflow

Fig.7 Measured and predicted Lift coefficient of a pitching S809 airfoil Using Leishman-Beddoes dynamic stall model

Existing dynamic inflow models have some shortcomings

ECN differential model:

$$du_i/dt = 4Rf(r/R)[C_t U^2 - 4u_i(U - u_i)]$$

ECN Integral model:

$$u_i(0,r,t) = 1/4\pi \int_0^{2\pi} (R - r\cos\varphi) \int_{-\infty}^{t} F(\gamma, u_i, I, t)\,dt\,d\varphi$$

DTU semi-empirical model:

$$u_{\text{int}} + \tau_1 \frac{du_{\text{int}}}{dt} = u_{q-steady} + g\tau_1 \frac{du_{q-steady}}{dt}$$

$$u_{dyn} + \tau_2 \frac{du_{dyn}}{dt} = u_{int} \qquad \tau_1 = \frac{1.1}{(1 - 1.3a)} \frac{R}{U_0} \qquad \tau_2 = \left[0.39 - 0.26\left(\frac{r}{R}\right)^2\right]\tau_1$$

The UBEM has the form of Ricatti nonlinear differential equation due to dynamic inflow at high tip speed ratio

$$dT = 0.5\rho c N c_{l_\alpha} (\Omega r)[(U(t) - u_i)/\Omega r - \theta_t - \theta_p]dr = (4\pi\rho r)[k_1 u_i^2 + k_2 U(t) u_i]dr - m[U(t) - u_i]$$

$$u_i = \alpha u_i^2 + \beta(t) u_i + \gamma(t)$$

Assumptions:

1- High tip speed ratio operation

$$W = \Omega r \sqrt{\frac{(1-a)^2}{\lambda^2} + 1} \approx \Omega r$$

2- The angle of attack is small , drag is neglected and lift is linear in angle of attack $c_l = 2\pi\alpha$

Riccati equation has analytical solution for a linear change in wind speed in the form of parabolic cylinder function

$U(t)=a_u t+b_u$

$u_i=\alpha u_i^2+(\beta_1+\beta_2 t)u_i+(\gamma_1+\gamma_2 t)$

$u_i=-1/\alpha_l\ [g+q\{C_1/C_2\ dU(v,\eta)/d\eta+dV(v,\eta)/d\eta\}]/C_1/C_2$

$U(v,\eta)=D_v(\eta)$

$V(v,\eta)=1/\pi\,\Gamma(v)[-\sin(\pi(v+0.5))\ D_v(\eta)+D_v(-\eta)]$

DUT Wind turbine Model is the only found controlled experiment for linear change in wind speed

Table4.2: DUT wind turbine model characteristics [14]

Diameter	1.2 m
No. of Blades	2
Airfoil section	NACA 0012
Chord	0.08 m “no taper”
Twist	6^{o} from $0.3<r/R<0.9$. outer 10% of the blade 0^{o}.
Rotor speed	Varies 10 to 16 Hz

Step down:	
$0<t<0.5$	5.7
$0.5<t<0.9$	$5.7 - 2(t - 0.5)$
$0.9<t<2$	4.9

Linear change in wind speed causes a very small over shoot

Tip speed ratio = 8 rpm =720   Re=1.5e5

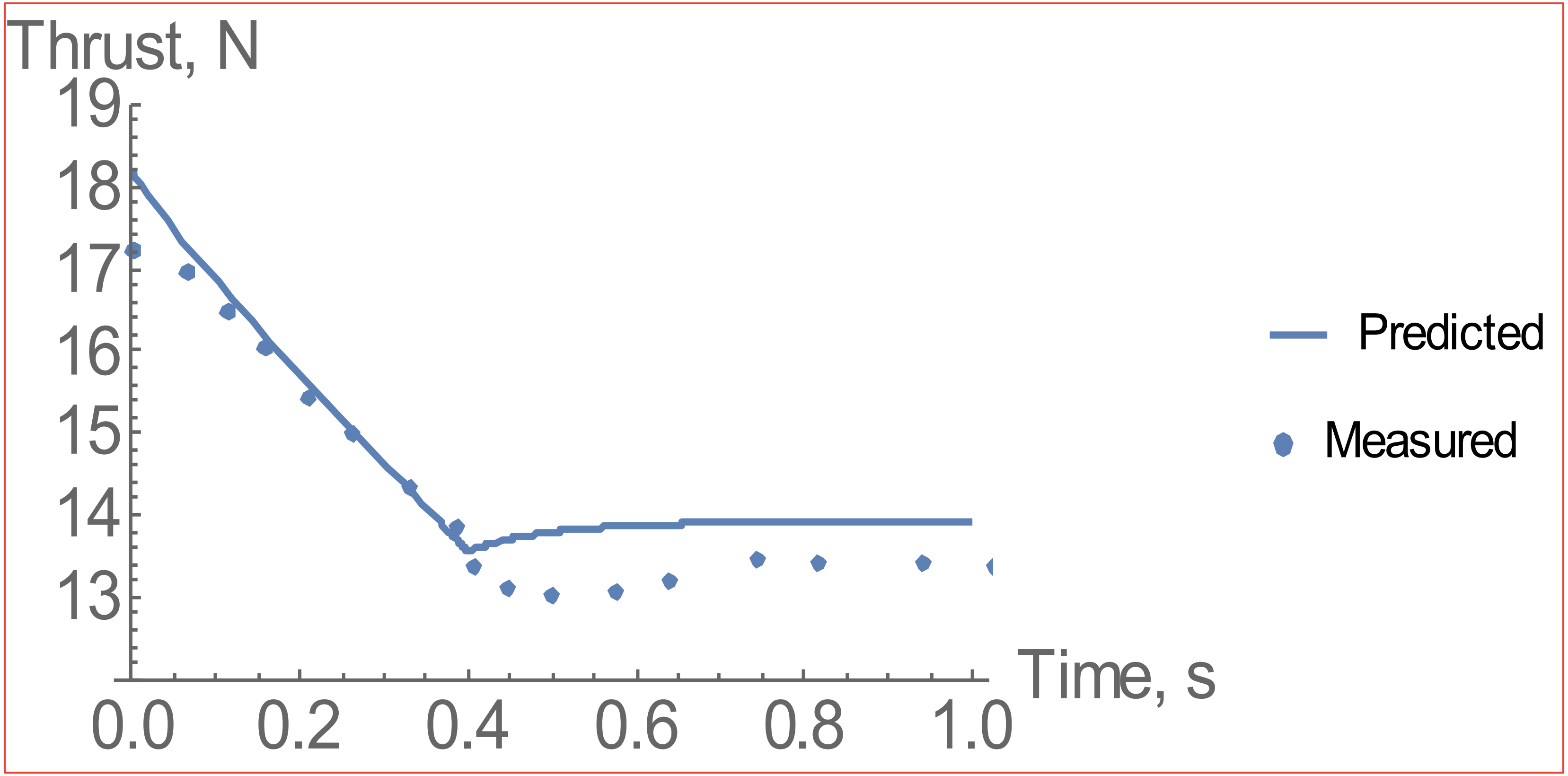
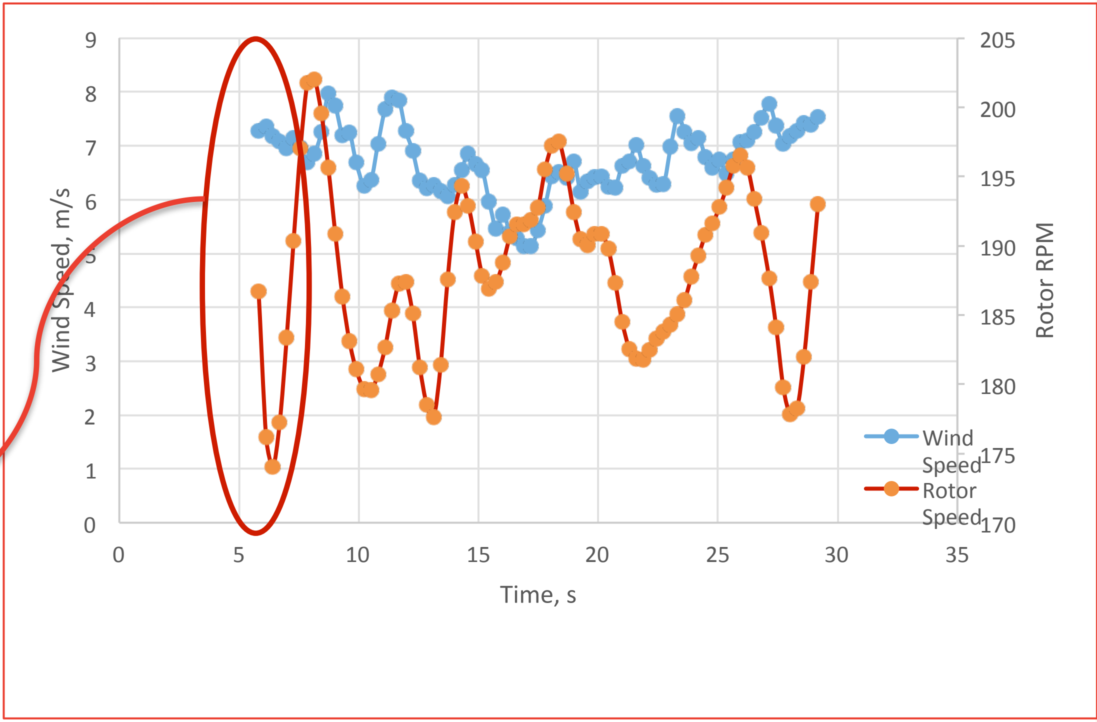


Fig.7 Measured and predicted thrust force for a linear change in wind speed

The wind series can be modeled as a sequence of linear wind velocity variation



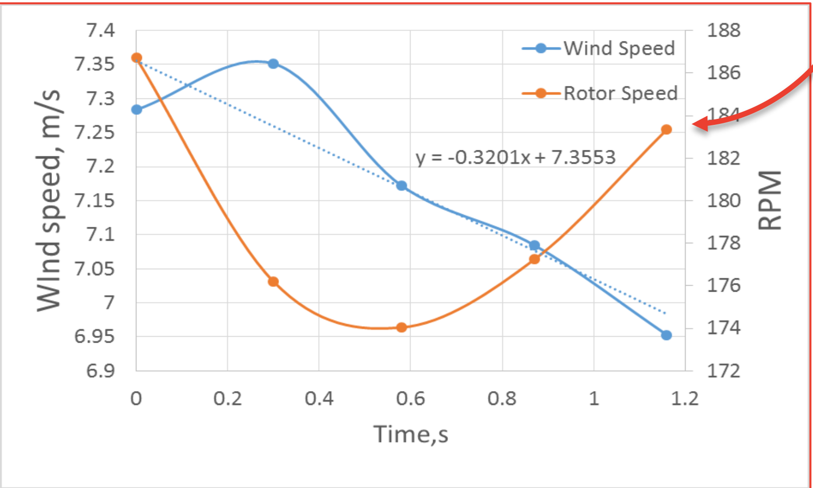
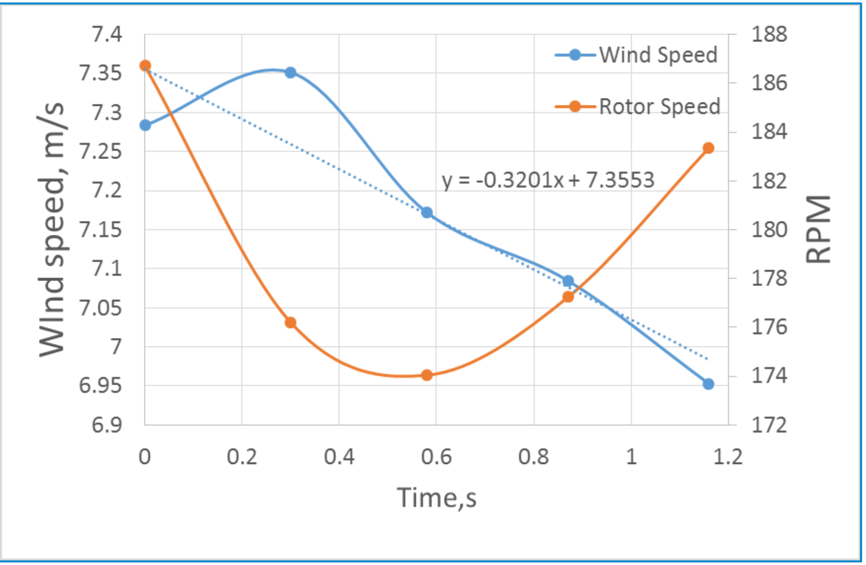


Fig.8 Simulating the wind field as a small step of linear change.

The change in torque is more correlated with change in rpm



Newcastle 5 kW wind turbine

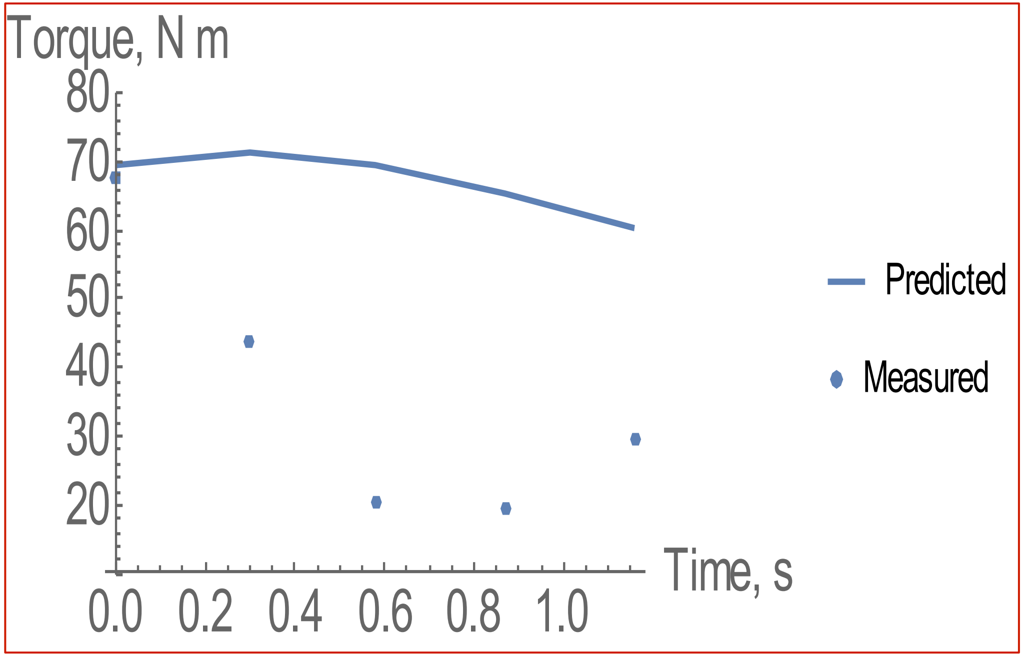


Fig.8 The measured and predicted torque for the conditions shown on the left.

Conclusion and Future work

- A closed form solution of the UBEM has been obtained for a linear change in wind velocity at high tip speed ratio.
- The model will be modified to include varying rpm.
- The solution need to be extended for small tip speed ratio and high angle of attack to simulate the starting conditions of wind turbines.

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Thank you