Wind Farm Layout Optimization
Considering Commercial Turbine
Selection and Hub Height Variation

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500 MW x $2 million/MW (in average) = $1 billion

400 MW x $2 million/MW (in average) = $0.8 billion

400 MW for only $0.65 billion or $1.625 million/MW

100 MW for $0.35 billion or $3.5 million/MW

HOW ABOUT EXPLORING TRADE-OFF RANGE?

Providing that

ALL CALCULATIONS ARE BASED ON DATA OFFERED

BY MANUFACTURERS & DEVELOPERS
1. WFLO - a background.
2. Research Objectives.
3. Wake Modelling
5. Power Calculations.
7. Optimization.
8. Results and Discussion.
10. Further Work.
WFLO

It is the problem of how to design a wind farm so that desirable quantity ($P$, $CF$, etc.) is maximized and/or undesirable quantity (cost, noise, etc.) is minimized.

**Design Variables**
- $N$
- Turbines’ siting
- Turbines’ sizes
- Turbines’ heights
- Owners’ Decision

**Constraints**
- $N$
- Farm Area
- Total Cost
- Noise Level

**Optimization Methodology**
- GA
- Other Bio-Inspired
- MILP & MINLP
- MCO
- PSO
- other

**Objective Function(s)**
- $P$
- Cost Of Energy
- $CF$
- Noise Level
- Land Usage
- Multi-Objective
The first WFLO work has been published in 1994,

1994-2005: no significant contributions have been added,

2005-2009: few remarkable contributions,

2009-2014: wide awareness and variety in approaches,

Very few studies considered turbine selection and/or hub height variation,

Nobody implemented COMMERCIAL turbine selection,

Nobody implemented general realistic $C_T$ representation,

Nobody considered more than TWO objective functions.
“The proposed work aims to add the commercial turbine selection and general realistic $C_T$ representation to the WFLO, combined with hub height variation and considering three objective functions”

The investigated parameters:

- Selection among 61 HAWT (1.5 ~ 3 MW)
- Hub height ($80 \ m \leq H \leq 140 \ m$)
- Average spacing ($3.5 \ D \leq S \leq 6 \ D$)
- Reference wind speed ($8 \ m/s \leq U_{\text{ref}} \leq 12 \ m/s$) @ 60 m
3- Wake Modelling

Jensen’s Wake Model: Jensen (1983), Katic et al. (1986), and Frandsen (1992)

\[
R_{\downarrow w,ij} = R_j \left[1 - \frac{a_{\downarrow j}}{1 - 2a_{\downarrow j}}\right]^{1/2} + (0.5/ln(\frac{H_j}{z_{\downarrow o}}))
\]

\[
\delta U_{\downarrow ij} = \left\{1 - \sqrt{1 - \frac{C_{\downarrow ij}}{1 + a_{\downarrow j}} \Delta y_{\downarrow ij}} / R_j \right\}^{1/2}
\]

\[
\delta U_{\downarrow i} = \sqrt{\sum_{j=1}^{j=i-1}(\delta U_{\downarrow ij})^2}
\]

\[
U_{\downarrow i} = U_{\downarrow i,0} (1 - \delta U_{\downarrow i})
\]

\[
U_{\downarrow i,0} = U_{\text{ref}} \left[\ln(\frac{H_i}{z_{\downarrow o}})\right]^{1/3}
\]

A Schematic front view, parallel to the wind direction, Y.
61 numerical power curves are fitted with 9th degree polynomial, 8 \( C_T-C_P,c \) and 3 \( C_T-C_P,a \), could be found in the manuals, Neither Frandsen’s formula nor \( C_T = 0.88 \) is accurate, Each of \( C_T-C_P,c \), and \( C_T-C_P,a \) has almost a general curve, \( C_T \) should be related to \( C_P \) instead of \( U \).
5- Power Calculations

- **Total output power**

\[ P = \sum_{i=1}^{N} P_i = \sum_{i=1}^{N} \sum_{k=0}^{k} K \cdot a_{ik} [U_{i0}, (1 - \sqrt{\sum_{j=1}^{N} j = i - 1} \cdot \left(1 - \frac{\alpha_{ij} \Delta y_{ij}}{R_{ij}} \right)^{1/2} \right)^{1/2} \right)^{1/2} \] \left(\frac{A_{ij}}{A_{i}}\right)^2 \right)^{1/2} \] \left(\sum_{i=1}^{N} P_{R,i} \right) \]

- **Farm capacity factor**

\[ CF = \frac{P}{IP} = \sum_{i=1}^{N} \sum_{k=0}^{k} K \cdot a_{ik} [U_{i0}, (1 - \sqrt{\sum_{j=1}^{N} j = i - 1} \cdot \left(1 - \frac{\alpha_{ij} \Delta y_{ij}}{R_{ij}} \right)^{1/2} \right)^{1/2} \right)^{1/2} \right)^{1/2} \right)^{1/2} \] \left(\frac{A_{ij}}{A_{i}}\right)^2 \right)^{1/2} \] \left(\sum_{i=1}^{N} P_{R,i} \right) \]

\[ IP \quad \text{Installed Power} \quad P_R \quad \text{Rated Power} \]
Only the ICC is considered,

Turbines’ cost is the major cost component,

The ICC of 1 MW at $H = 80$ m is considered unity and denoted Capital Cost Index ($CCI$),

The tower cost $= 0.15/0.68 = 0.2206$ of the $CCI$,

An increase in $H$ by 1 m costs $0.2206/80 = 0.0027575$ of the $CCI$, 

6- Simple Cost Analysis (2)

- Capital Cost Index per Installed Power

\[ CCIIP = \frac{CCI}{IP} = \sum_{i=1}^{N} P_{R,i} \left[ 1 + 0.0027575 (H_{i} - H_{min}) \right] / \sum_{i=1}^{N} P_{R,i} \]

- Capital Cost Index per Output Power

\[ CIOIP = \frac{CCI}{P} = \sum_{i=1}^{N} P_{R,i} \left[ 1 + 0.2757 (H_{i} - H_{min}) \right] / \sum_{i=1}^{N} \sum_{k=0}^{K} a_{ik} [U_{o,i} \left( 1 - \sqrt{1 - \left( C_{T,j} \left[ 1 + a_{j} \Delta y_{ij} / R_{j} \left[ 1 - 2 a_{j} / a_{ij} \right] \right]^{1/2} \right]^{2} \right) ]^{k} \]
The 3 objective functions are scaled, adapted, weighted, and combined into one Total Objective Function:

\[ TOF = \omega \downarrow P \frac{f\downarrow P}{P} + \omega \downarrow CF \frac{f\downarrow CF}{CF} + \omega \downarrow C \frac{f\downarrow C}{CCI/P} \]

\[ \omega \downarrow P + \omega \downarrow CF + \omega \downarrow C = 1.0 \]

Scaling: turning all terms in to the same order of magnitude,

Minimum turbines’ proximity = 3 \(D\)

\(\text{TolFun} = 10^{-15}\) (default = \(10^{-6}\)),

\(\text{ConFun} = 10^{-9}\) (default = \(10^{-6}\)),

\(\text{PopulationSize} = 10 \sim 50 \text{ nvars} \& \text{Generations} = 3,000.\)
Case 1: Turbines In Line (parallel to wind direction), $N = 6$

Normalized $P$ & $CCI$ for case 1, $U_{ref} = 8$ m/s @ 60 m.
Case 1: Turbines In Line (parallel to wind direction), $N = 6$

Normalized $P$ & $CCI$ for case 1, $U_{ref.} = 10$ m/s @ 60 m.
Case 1: Turbines In Line (parallel to wind direction), $N = 6$

Normalized $P$ & $CCI$ for case 1, $U_{ref.} = 12$ m/s @ 60 m.
Case 2: Small Rectangular Wind Farm, $N = 18$

Normalized $P$ & $CCI$ for 3x6 WF, $U_{ref.} = 8$ m/s @ 60 m.
Case 2: Small Rectangular Wind Farm, $N = 18$

Normalized P & CCI for 3x6 WF, $U_{\text{ref.}} = 12$ m/s @ 60 m.
The dependence of $P$ and $CCI$ on $U$ and $S$ is not smooth, which is expected, because the problem is not continuous, as the turbines’ data are not. So, the results should be understood qualitatively not necessarily quantitatively.

There is a wide margin of trade-off between power output and capital cost, so the weighting factors should be adjusted according to the design priorities in order to obtain the desirable optimum layout.

At high wind speeds, all optimizations (except for minimum $CCIOp$) tend to develop almost the same output power as the reference case while costing less $ICC$. 
The range of trade-off between power and cost can be summarized as:

<table>
<thead>
<tr>
<th>Case 1</th>
<th>( P/P_{\text{ref}} )</th>
<th>( CCI/CCI_{\text{ref}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>from</td>
<td>to</td>
</tr>
<tr>
<td>CF</td>
<td>0.88</td>
<td>1.00</td>
</tr>
<tr>
<td>CCIOP</td>
<td>0.65</td>
<td>1.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case 2</th>
<th>( P/P_{\text{ref}} )</th>
<th>( CCI/CCI_{\text{ref}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>from</td>
<td>to</td>
</tr>
<tr>
<td>CF</td>
<td>0.91</td>
<td>1.10</td>
</tr>
<tr>
<td>CCIOP</td>
<td>0.68</td>
<td>0.91</td>
</tr>
</tbody>
</table>
9- Conclusions

1. Wind farm design with identical turbines or even with different turbines from one manufacturer should be abandoned in favour of the turbine selection optimizations described in this proposal.

2. A wide band of optimum designs can be obtained according to the optimization preferences and priorities.

3. The representation of $C_T$ in terms of the wind speed is not the right way.

4. The lack in $C_T$ data could be overcome for multi-MW HAWT by generalization of the available data.
4. The proposed methodology is suitable for large scale WFs as well as for compact designs.

5. Taller towers are needed, not only to reach higher wind speeds, but also to reduce the wake effects in the compact WF designs.

6. The manufacturers should show more flexibility and accept the fair competition by providing more wind turbine designs and more accurate technical data.
10- Further Work

- Real case large wind farm.
- Modified wake model.
- More realistic wind profile.
- Noise Level minimization.
- TOF with different weighting factors.
- Optimization.
REFERENCES

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QUESTIONS ?