

STATISTICAL STUDIES OF THE RELATIONSHIPS  
Between  
TERMINAL GROWTH AND YIELD IN THE YORK APPLE

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Major Thesis in Horticulture  
For  
The Degree of Master of Science

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Submitted  
to  
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by  
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Approved:

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Robert C. Moore

April 1932

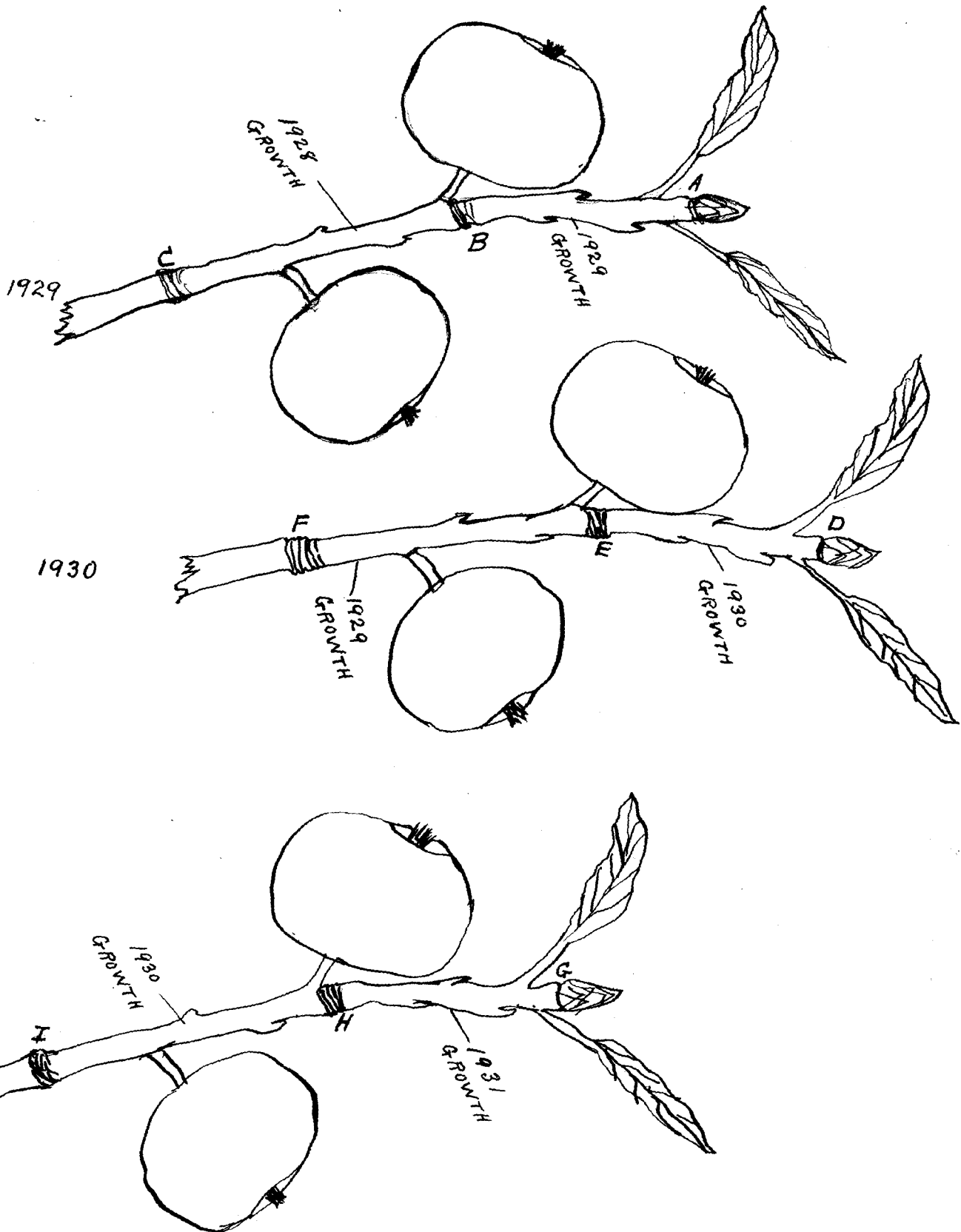


PLATE I

1929 Notes on Terminal Growth length and diameter and weight of fruit on 1928 Terminal Wood of the York Apple

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No. of Limb	: inches of : 1929	: inches of : 1928	: inches of : 1929	: inches of : 1928	: Ounces of Apple : 1928	: Number of Apples : 1928	: Ounces : average : 1928-29	: Inches : Average : Terminal : 1928
1	12	14	4	8	9	2	5	13
2	4	7	3	4	7	2	4	6
3	12	16	4	8	9	2	5	14
4	7	8	4	6	5	1	5	8
5	10	5	4	6	4	1	4	8
6	9	5	4	6	6	1	6	7
7	11	6	4	6	6	1	6	9
8	11	15	5	6	4	1	4	13
9	5	7	4	5	4	1	4	6
10	9	10	4	6	6	1	6	10
11	10	13	4	8	6	2	3	12
12	9	14	5	7	7	2	4	12
13	9	6	4	6	4	1	4	8
14	10	4	4	5	5	1	5	7
15	6	8	3	6	7	2	4	7
16	9	6	4	5	7	2	4	8
17	5	10	3	5	6	2	3	8
18	15	19	6	9	16	4	4	17
19	8	5	4	3	3	1	3	7
20	12	19	5	8	16	5	3	16
21	9	3	4	5	8	2	4	6
22	4	9	4	6	10	2	5	7
23	11	4	5	5	8	2	4	8
24	12	7	5	6	6	1	6	10
25	7	7	4	6	7	1	7	7
26	5	6	3	6	8	2	4	6
27	8	4	5	6	12	2	6	6
28	10	13	5	8	11	2	6	12
29	7	7	3	6	3	1	3	7
30	8	6	4	6	4	1	4	7

No. : inches; inches; 32nd ; 32nd ; Ounces; Number; Ounces ; Inches  
 of ; length; length; inches; inches; of ; of ; average; average  
 Limb; of : of : dia. of; dia. of; Apple : Apples; weight : combined  
 : 1928 : 1928 : 1928 : 1928 : on : on : per Ap.: 1928-29  
 : Term. : Term.; Term. : Term. : 1928 : 1928 : on 1928; Terminal  
 : Growth: Growth; Growth; Growth; Ter. G.; Ter. G. ; Ter. G.; Gro. Length

31	11	12	5	8	11	2	6	13
32	11	4	5	8	6	1	6	8
33	14	4	5	6	8	1	8	9
34	11	11	5	6	12	2	6	11
35	10	9	5	6	7	1	7	10
36	12	20	4	10	9	2	5	16
37	11	15	5	10	15	4	5	13
38	12	17	4	10	2	7	5	15
39	5	15	6	8	15	3	5	15
40	5	10	4	7	8	1	8	8
41	12	6	5	6	9	3	3	9
42	15	14	6	10	12	2	6	15
43	13	12	6	8	11	2	6	13
44	11	11	4	6	4	1	4	11
45	6	13	4	6	12	3	4	10
46	10	7	4	6	11	2	2	9
47	6	8	3	5	15	3	5	7
48	6	9	4	6	11	2	6	8
49	10	13	4	8	9	3	3	12
50	8	13	4	8	5	2	3	11
51	11	16	6	8	9	3	3	14
52	8	20	5	8	17	5	5	14
53	8	14	5	8	6	2	3	11
54	6	5	4	5	7	2	4	6
55	12	13	5	7	15	4	4	13
56	12	15	6	7	12	2	6	14
57	15	17	5	10	28	6	5	16
58	8	12	4	7	8	2	8	10
59	14	8	2	3	8	1	8	11
60	8	7	1	2	7	2	4	8

1929

No. : inches: inches: 32nd : 32nd : Ounces: Number: Ounces : Inches  
of : length: length: inches: inches: of : of : average: average  
Limb: of : of : dia. of: dia. of: Apple : Apples: weight : combined  
: 1929 : 1928 : 1929 : 1928 : on : on : per Ap.: 1928-29  
: Term. : Term. : Term. : \*Term. : 1928 : 1929 : on 1928: Term. G.  
: Growth: Growth: Growth: Growth: Ter. G : Ter. G : Ter. G : Length

61	5	4	3	3	8	1	8	5
62	11	11	3	5	5	1	5	11
63	5	6	1	2	13	2	6	6
64	7	10	2	3	13	2	6	9
65	11	8	2	4	4	1	4	10
66	6	2	2	3	8	2	4	4
67	13	8	2	3	8	1	8	11
68	8	4	3	5	8	2	4	6
69	10	2	3	5	7	2	4	6
70	6	2	3	4	8	1	8	4
71	10	3	2	5	20	2	10	7
72	13	4	2	6	5	1	5	9
73	13	8	2	5	7	1	7	11
74	13	7	3	4	3	1	3	10
75	11	5	3	4	8	1	8	8
76	9	7	2	5	6	1	6	8
77	13	7	2	3	20	4	5	10
78	12	9	2	4	8	1	8	11
79	7	2	1	6	8	1	8	5
80	13	17	4	8	16	4	4	15
81	9	14	4	8	20	5	4	12
82	10	11	4	6	16	5	3	11
83	4	2	4	4	5	1	5	3
84	14	19	7	10	16	2	8	17
85	16	17	6	9	16	3	5	17
86	5	9	6	6	7	4	2	7
87	7	4	4	10	7	4	2	6
88	10	8	3	4	3	1	3	9
89	13	8	4	10	10	2	5	10
90	10	14	4	8	23	5	5	12

1939

No : inches: inches: 32nd: 32nd: Ounces: Number: Ounces: Inches  
of : length: length: inches: inches: of : of : average: average  
Limb: of : of : dia. of: dia. of : Apple : Apples: weight : combined  
: 1939: 1938 : 1939: 1938 : on : on : per Ap : 1938-39  
: Term. : Term. : Term. : Term. : 1938 : 1938 : on 1938: Term.G.  
: Growth: Growth : Growth: Growth : Ter.G : Ter.G : Ter.G : Length

No	inches	inches	32nd	32nd	Ounces	Number	Ounces	Inches
of	length	length	inches	inches	of	of	average	average
Limb	of	of	dia. of	dia. of	Apple	Apples	weight	combined
	: 1939:	: 1938 :	: 1939:	: 1938 :	: on :	: on :	: per Ap :	: 1938-39
	: Term. :	: Term. :	: Term. :	: Term. :	: 1938 :	: 1938 :	: on 1938:	: Term.G.
	: Growth:	: Growth :	: Growth:	: Growth :	: Ter.G :	: Ter.G :	: Ter.G :	: Length
91	10	20	4	11	30	7	3	15
92	6	8	3	4	6	3	3	7
93	9	14	3	4	8	3	3	12
94	7	10	3	6	1	1	1	8
95	11	19	3	8	2	1	2	15
96	4	4	2	3	6	2	3	4
97	10	6	4	6	11	3	4	8
98	13	11	4	5	9	2	5	13
99	10	17	3	8	2	7	5	14
100	10	8	3	4	9	1	9	9
100	13	19	6	8	20	4	5	16
102	14	14	6	10	8	2	4	14
103	14	16	4	6	4	2	2	15
104	15	17	4	8	4	2	2	16
105	12	10	4	5	8	1	8	11
106	13	12	5	5	12	1	13	12
107	7	6	3	4	3	1	3	6
108	6	2	1	3	8	1	8	4
109	13	16	5	8	1	4	4	15
110	11	20	6	11	29	9	3	16
111	11	19	4	9	24	6	4	15
112	13	18	5	10	17	4	4	16
113	18	24	8	16	22	6	4	21
114	8	8	4	5	8	3	3	8
115	11	14	4	8	21	6	4	13
116	9	4	4	6	12	2	6	7
117	12	9	5	8	10	3	3	11
118	10	8	4	6	7	1	7	9
119	7	5	4	6	8	1	8	6
120	10	5	4	5	12	2	6	8

1929

No. : inches: inches: 32nd : 32nd : Ounces: Number: Ounces : Inches  
of : length: length: inches: inches: of : of : average: Average  
Limb: of : of : dia. of: dia. of: Apple : Apples: weight : Combined  
: 1929 : 1928 : 1928: 1928: on : on : Per Ap.: 1928-29  
: Term. : Term. : Term. : Term. : 1928 : 1928 : on 1928: Term. G.  
: Growth: Growth: Growth: Growth: Ter. G : Ter. G : Ter. G : Length

No.	1929	1928	1928	1928	1928	1928	1928	1928	1928-29
Limb	of	of	dia. of	dia. of	Apple	Apples	weight	Per Ap.	1928-29
	Term.	Term.	Term.	Term.	1928	1928	on 1928	Term. G.	
	Growth	Growth	Growth	Growth	Ter. G	Ter. G	Ter. G	Length	
131	10	8	4	6	4	1	4	9	
132	9	9	4	6	8	2	4	9	
133	18	14	5	8	5	1	5	16	
134	13	8	5	7	10	2	5	11	
135	7	10	4	6	6	2	3	9	
136	8	6	4	5	5	1	5	7	
137	12	12	4	8	8	1	8	12	
138	7	7	4	6	8	1	8	7	
139	13	16	5	9	16	3	5	15	
130	7	9	4	6	12	2	6	8	
131	8	8	4	5	6	1	6	8	
132	11	13	5	8	6	1	6	12	
133	5	5	4	6	7	1	7	5	
134	10	17 <sup>n</sup>	6	10	10	3	3	14	
135	11	6	4	5	9	1	9	9	
136	13	14	4	8	14	3	5	14	
137	11	16	4	4	9	6	2	14	
138	8	6	4	4	5	1	5	7	
139	11	10	5	8	16	3	5	11	
140	5	3	3	3	7	1	7	4	
141	12	18	5	7	6	1	6	15	
142	14	12	4	6	8	2	4	13	
143	10	13	4	7	5	2	3	12	
144	9	9	6	4	4	1	4	9	
145	3	6	3	5	10	2	5	5	
146	10	10	4	6	8	1	6	10	
147	10	12	3	6	8	1	8	11	
148	8	5	3	5	12	2	6	7	
149	9	22	4	8	17	4	4	16	
150	6	15	4	8	17	4	4	11	



1939

No. : inches: inches: 32nd : 32nd : Ounces: Number: Ounces : Inches  
of : length: length: inches: inches: of : of : Average: Average  
Limb: of : of : dia. of: dia. of: Apples: Apples: weight : Combined  
: 1939: 1938 : 1929 : 1928 : on : on : Per Ap.: 1938-39  
: Term.: Term. : Term. : Term. : 1938 : 1928 : on 1938: Term.G  
: Growth: Growth: Growth: Growth: Ter. G: Ter. G : Ter. G. : Length

151	10	13	4	7	16	4	4	12
152	12	21	3	9	9	2	5	17
153	12	11	3	6	13	2	7	12
154	12	19	4	9	8	2	4	16
155	19	21	5	10	16	4	4	20
156	3	5	3	4	4	1	4	4
157	2	4	6	4	12	1	12	3
158	9	6	4	5	7	1	7	8
159	9	15	4	8	24	7	3	12
160	11	17	5	5	17	4	4	14
161	11	20	5	8	8	3	3	16
162	13	12	4	8	11	3	4	13
163	14	17	4	10	7	3	2	16
164	16	16	6	8	16	3	5	16
165	9	16	4	8	12	2	6	13
166	9	11	4	7	8	2	4	10
167	4	12	3	5	8	2	4	8
168	10	15	4	8	9	2	5	13
169	13	15	4	8	8	2	4	14
170	14	16	5	7	12	3	4	15
171	9	14	7	8	12	3	4	12
172	9	20	3	8	12	3	4	15
173	12	24	5	10	24	5	5	19
174	10	9	4	6	9	2	5	10
175	8	10	4	6	8	1	8	9
176	10	11	5	7	7	1	7	11
177	11	16	6	8	23	5	5	14
178	11	14	4	7	16	3	5	13
179	9	13	4	6	12	3	4	11
180	7	10	4	6	10	2	5	9

1929

No. : inches: inches: 32nd : 32nd : Ounces: Number: Ounces : Inches  
of : length: length: inches: inches: of : of : Average: Average  
Limb: of : of : dia. of: dia. of: Apple : Apples: weight : Combined  
: 1929 : 1928 : 1929: 1928 : on : on : per Ap.: 1928-29  
: Term. : Term. : Term. : Term. : 1928 : 1928 : on 1928: Term. G  
: Growth: Growth: Growth: Growth: Ter. G : Ter. G: Ter. G : Length

181	13	9	5	7	7	3	4	11
182	11	16	4	9	7	1	7	14
183	7	8	4	7	5	1	5	8
184	8	9	4	7	7	3	4	9
185	11	8	4	6	7	3	4	10
186	10	14	4	6	9	3	5	13
187	9	10	4	6	5	1	5	10
188	9	13	4	8	37	6	5	11
189	8	7	4	5	16	4	4	8
190	9	8	4	6	9	3	5	9
191	9	9	4	6	11	3	6	9
192	13	10	5	8	9	1	8	13
193	13	9	4	7	7	3	4	11
194	13	17	4	8	16	4	4	15
195	10	20	4	8	16	5	3	15
196	8	10	3	5	8	3	4	9
197	7	8	3	4	7	3	4	8
198	10	11	4	6	16	3	5	11
199	14	16	4	8	20	5	4	15
200	9	15	4	6	6	3	3	13
201	7	6	4	5	7	3	4	7
202	11	15	4	6	9	3	3	13
203	8	14	4	7	8	3	4	11
204	5	8	3	4	11	3	4	7
205	13	15	4	8	16	5	3	14
206	12	10	4	7	16	4	4	11
207	15	17	5	10	23	7	3	16
208	13	13	4	8	16	3	5	13
209	8	13	4	7	18	5	4	10
210	13	11	5	6	8	3	4	13

1939

No. : inches; inches: 32nd : 32nd : ounces; number; ounces : inches  
of : length; length; inches; inches: of : of : average; average  
Limb: of : of : dia. of; dia. of; Apple : apples: weight : combined  
: 1939 : 1938 : 1939 : 1938 : on : on : per Ap.: 1938-39  
: Term. : Term. : Term. : Term. : 1938 : 1938 : on 1938: Term. G  
: Growth; Growth; Growth; Growth; Ter. G : Ter. G: Ter. G; length

No.	1939	1938	1939	1938	1939	1938	1939	1938
211	9	14	4	8	24	8	3	12
212	14	13	4	7	13	3	4	14
213	15	16	6	10	21	5	4	16
214	13	15	5	9	24	6	4	14
215	13	17	6	10	30	8	4	15
216	14	15	4	9	21	5	4	15
217	17	20	6	10	24	5	5	19
218	12	20	6	10	29	9	3	18
219	16	27	7	12	24	5	5	22
220	6	6	3	4	4	1	4	6
221	8	18	4	7	15	4	4	13
222	10	5	4	6	8	1	8	8
223	9	10	5	6	10	2	5	10
224	8	10	4	6	11	3	4	9
225	8	9	4	8	9	2	5	9
226	11	12	4	8	13	3	4	12
227	8	14	4	8	10	4	3	11
228	11	18	4	9	18	6	3	15
229	12	16	4	8	12	3	4	14
230	7	11	3	6	18	5	4	9
231	11	11	4	6	9	2	5	11
232	8	6	4	6	6	1	6	7
233	6	9	4	6	8	2	4	8
234	7	9	4	6	6	1	6	8
235	6	8	3	5	8	3	3	7
236	8	5	4	5	7	1	7	7
237	9	7	5	6	5	1	5	8
238	8	11	4	6	11	1	11	10
239	9	13	4	7	9	2	5	11
240	9	10	4	6	8	2	4	10

1939

No. : inches: inches: 32nd : 32nd : ounces: number: ounces : inches  
of : length: length: inches: inches: of : of : average: average  
Limb: of : of : dia. of: dia. of: apple : apples: weight : combined  
: 1939 : 1938 : 1939 : 1938 : on : on : Per Ap.: 1938-39  
: Term. : Term. : Term. : Term. : 1938 : 1938 : on 1938: Term. G  
: Growth: Growth: Growth: Growth: Ter. G.: Ter. G : Ter. G : length

241	10	14	5	8	17	4	4	13
242	7	13	4	6	6	3	3	10
243	10	17	5	8	14	4	4	14
244	8	8	4	6	7	3	4	8
245	10	10	4	7	6	3	3	10
246	8	10	5	6	7	3	4	9
247	8	10	4	6	7	1	7	9
248	9	11	4	6	7	1	7	10
249	5	3	4	6	13	3	6	4
250	9	13	4	8	9	3	5	11
251	6	11	4	6	11	3	4	9
252	8	6	4	6	11	3	4	7
253	5	6	4	5	8	7	8	6
254	9	11	4	6	7	1	7	10
255	3	2	4	4	4	1	4	3
256	8	5	4	4	6	1	6	7
257	5	4	3	5	5	3	3	5
258	8	7	4	6	13	3	7	8
259	6	11	4	6	6	1	6	9
260	10	6	4	6	6	1	6	8
261	10	10	5	7	6	3	3	10
262	10	10	4	7	14	3	5	10
263	6	6	4	4	9	3	3	6
264	7	5	4	4	9	3	5	6
265	6	7	4	6	12	3	6	7
266	10	13	4	8	10	3	5	12
267	7	8	4	7	9	1	9	8
268	9	12	4	6	9	3	5	11
269	9	10	4	6	13	3	4	10
270	9	9	4	6	9	3	5	9

1929

No. : inches: inches: 32nd : 32nd : ounces: number: ounces : inches  
of : length: length: inches: inches: of : of : average: average  
Limb: of : of : dia. of: dia. of: apple : apples: weight : combined  
: 1929 : 1928 : 1929 : 1928 : on : on : per Ap.: 1928-29  
: Term. : Term. : Term.: Term : 1928 : 1928 : on: 1928: Term.G  
: Growth: Growth: Growth: Growth: Ter. G : Ter. G : Ter. G. : length

No.	1929	1928	1929	1928	1928	1928	1928	1928
271	12	11	5	8	15	4	4	12
272	14	6	6	6	9	1	9	10
273	13	8	5	6	8	1	8	11
274	13	5	5	5	10	1	10	9
275	11	6	4	6	7	1	7	9
276	8	11	4	6	12	3	4	10
277	11	13	4	8	12	4	5	12
278	5	7	4	7	12	1	12	6
279	8	4	5	6	4	1	4	6
280	8	6	4	6	9	2	5	7
281	9	8	4	6	12	1	12	9
282	10	12	4	7	9	3	3	11
283	11	17	6	8	13	3	4	9
284	8	9	4	6	8	1	6	9
285	7	12	4	6	9	2	5	10
286	5	12	3	5	7	2	4	9
287	9	8	5	7	6	1	6	9
288	4	12	3	5	12	2	6	8
289	5	8	3	5	8	1	8	7
290	8	18	6	10	14	3	5	13
291	5	12	3	6	15	4	4	9
292	7	8	4	5	5	1	5	8
293	11	13	4	6	12	2	6	12
294	11	20	5	9	28	6	5	16
295	6	7	4	5	6	1	6	7
296	8	10	4	6	8	2	4	9
297	11	6	4	5	10	2	5	9
298	6	6	4	5	5	1	5	6
299	10	8	4	5	5	1	5	9
300	9	9	4	6	9	2	5	9

1930 Notes on Terminal Growth Length and Diameter and the  
 Weight and number of fruits on 1929 Terminal Growth Wood of  
 the York Apple

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No. : inches: inches: 32nd : 32nd : ounces: number: ounces : inches  
 of : length: length: inches: inches: of : of : average : average  
 Limb: of : of : dia. of: dia. of: apples: apples: wght. pr. : combined  
 : 1930 : 1929 : 1930 : 1929 : on : on : apple : 1929-30  
 : Term.: Term.: Term. : Term.: 1929 : 1929 : on 1929 : Term.G.  
 : Growth: Growth: Growth: Growth: Term.G: Term.G: Term. G. : length

No.	1930 Limb of	1929 Limb of	1930 32nd dia. of	1929 32nd dia. of	1929 on apples	1929 on apples	1929 on apple	1929-30 Term.G. length
1	4	1	3	5	4	1	3	
2	7	10	2	5	2	2	9	
3	6	8	3	4	5	1	7	
4	6	4	3	5	5	1	5	
5	9	14	3	8	18	5	12	
6	5	7	3	5	8	3	0	
7	9	14	4	6	6	1	12	
8	8	7	4	6	8	3	8	
9	10	15	3	7	7	2	13	
10	9	14	4	7	9	3	12	
11	11	14	4	8	17	6	13	
12	3	10	3	7	9	2	7	
13	4	7	3	5	3	1	6	
14	5	7	3	5	5	1	6	
15	6	5	4	5	4	1	6	
16	3	3	3	4	6	2	3	
17	6	6	4	6	4	1	6	
18	4	6	3	6	4	1	5	
19	9	15	4	8	18	6	12	
20	4	8	3	6	4	1	6	
21	7	9	4	6	5	1	8	
22	4	10	3	5	4	1	7	
23	4	7	2	4	4	1	6	
24	5	5	3	5	3	1	5	
25	5	12	3	6	8	3	4	
26	4	9	3	5	5	1	7	
27	6	9	3	6	8	5	8	
28	6	8	3	6	8	2	7	
29	4	5	3	4	4	2	5	
30	4	5	3	5	5	2	5	

1930

No. : inches: inches: 32nd : 32nd : ounces: number: ounces: : inches  
of : length: length: inches: inches: of : of : average: average  
Limb: of : of : dia. of: dia. of: apples: apples: wt. per : combined  
: 1930 : 1929 : 1930 : 1929 : on : on : apple : 1929-30  
: Term.: Term.: Term.: : Term : 1929 : 1929 : on 1929: Term. G  
: Growth: Growth: Growth: Growth: Term. G: Term. G: Term. G: length

31	8	10	3	6	3	1	3	9
32	4	7	3	5	4	2	2	6
33	5	3	3	5	2	1	2	4
34	5	5	3	5	7	2	4	5
35	4	10	3	6	6	3	2	7
36	5	1	3	5	4	2	2	3
37	6	7	3	4	5	2	3	7
38	6	8	3	5	4	2	2	7
39	5	7	3	4	3	1	3	6
40	5	7	3	5	4	2	2	6
41	9	1	3	5	5	1	5	5
42	10	10	4	6	7	1	7	10
43	6	5	3	5	5	1	5	6
44	9	25	5	10	14	5	3	17
45	4	1	3	5	5	2	3	3
46	9	7	4	6	4	2	2	8
47	5	6	3	6	5	2	3	6
48	4	9	3	5	3	1	3	7
49	6	5	3	5	4	1	4	6
50	6	13	3	5	6	1	6	10
51	4	9	3	5	5	1	5	7
52	6	16	3	6	7	2	4	11
53	4	8	3	5	5	3	2	6
54	5	7	3	6	3	1	3	6
55	3	3	3	4	6	2	3	3
56	10	9	4	7	10	2	5	10
57	9	6	3	5	8	1	2	8
58	9	12	4	9	10	2	5	11
59	7	7	3	6	13	2	7	7
60	9	9	4	6	11	2	6	9

1930

No. : inches; inches: 32nd : 32nd : ounces; number; ounces: : inches  
of : length; length; inches; inches: of : of : average; average  
Limb: of ; of : dia. of; dia. of; apples; Apples; wt. per : combined  
: 1930 : 1929 : 1929 : 1929 : on : on : apple : 1929-30  
: Term : Term : Term : Term.: 1929 : 1929 : on 1929; Term.G  
: Growth; Growth; Growth; Growth; Term.G; Term.G; Term.G : length

No.	inches	inches	32nd	32nd	ounces	number	ounces	inches
61	6	8	3	6	8	1	8	7
62	6	3	4	5	7	1	7	5
63	9	6	4	7	11	1	11	8
64	8	3	4	7	5	1	5	6
65	8	7	3	6	7	1	7	8
66	6	7	3	6	7	1	7	7
67	3	4	3	4	5	1	5	4
68	9	4	3	6	6	1	6	6
69	5	4	3	4	7	1	7	5
70	7	12	3	6	8	1	8	10
71	9	3	4	5	10	1	10	6
72	4	6	3	5	10	2	5	5
73	7	4	3	6	8	1	8	6
74	8	3	4	6	9	2	5	6
75	3	1	2	5	5	1	5	2
76	4	5	3	4	10	2	5	5
77	6	4	3	6	6	1	6	5
78	6	13	2	5	11	2	6	10
79	4	3	3	4	7	1	7	4
80	9	3	3	5	9	1	9	6
81	7	4	4	7	12	2	6	6
82	8	2	4	5	8	1	8	5
83	10	6	4	6	8	1	8	8
84	6	7	3	6	8	1	8	7
85	5	7	3	5	14	2	7	6
86	6	3	3	4	7	1	7	5
87	3	2	3	5	6	1	6	3
88	5	2	3	5	4	1	4	4
89	3	1	3	4	4	1	4	2
90	3	1	3	4	4	1	4	2



1930

No. : inches: inches: 32nd: 32nd : ounces: number: ounces: : inches  
of : length: length: inches: inches: of : of : average: average  
Limb: of : of : dia. of: dia. of: apples: apples: wt. per.: combined  
: 1930 : 1929 : 1928 : 1928 : on : on : apple : 1928-30  
: Term : Term : Term. : Term.: 1928 : 1928 : on 1928: Term. G.  
: Growth: Growth: Growth: Growth: Term. G: Term. G: Term. G : length

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91	4	3	3	5	8	1	8	3
92	3	3	3	4	10	1	10	3
93	7	8	3	6	9	1	9	8
94	9	10	4	7	7	1	7	10
95	5	4	3	4	4	1	4	5
96	4	4	3	5	6	1	6	4
97	4	3	4	1	4	1	4	3
98	9	12	3	7	9	1	9	11
99	8	11	4	8	9	1	9	10
100	4	10	3	5	6	2	3	7

1931 Notes on Length and Diameter of Terminal Growth (1930 and 1931 Growth) and yield on 1930 growth of York Apple Trees.

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no. : inches; inches; 16th : 16th : ounces; number; ounces : inches  
of : length; length; inches; inches; fruit : fruits; average : average  
Limb: of : of : dia. of; dia. of: on : on : wt. per : combined  
: 1931 : 1930 : 1930 : 1931: 1930 : 1930 : apple on: 1930-31  
: Term. : Term.; Termin; Termin; Termin; Termin: 1930 : Terminal  
: Growth; Growth; Growth; Growth; Growth; Growth; Ter. Gro; Growth

no.	inches	inches	16th	16th	ounces	number	ounces	inches
of	length	length	inches	inches	fruit	fruits	average	average
Limb:	of	of	dia. of	dia. of	on	on	wt. per	combined
:	1931	1930	1930	1931	1930	1930	apple on	1930-31
:	Term.	Term.	Termin	Termin	Termin	Termin	1930	Terminal
:	Growth	Growth	Growth	Growth	Growth	Growth	Ter. Gro	Growth
1	3	4	4	3	6	2	3	4
2	2	10	5	3	12	4	3	6
3	2	7	4	3	7	3	2.25	5
4	1	3	3	1	2	1	2	2
5	3	8	4	3	3	1	3	6
6	10	14	8	4	25	9	2.75	12
7	9	13	7	3	18	7	2.5	11
8	11	13	7	4	19	11	1.75	12
9	7	7	4	3	12	4	3	7
10	10	8	5	3	10	5	2	9
11	2	4	3	3	4	1	4	3
12	8	7	5	3	6	2	3	8
13	2	5	3	2	4	2	2	4
14	3	8	5	3	7	3	2.25	6
15	4	9	4	4	4	3	1.25	7
16	9	9	6	4	12	3	4	9
17	5	6	4	3	4	2	2	6
18	1	6	3	1	4	2	2	4
19	14	8	6	4	14	3	4.75	11
20	7	10	6	4	8	2	4	9
21	5	9	5	4	5	1	5	7
22	4	7	5	5	4	1	4	6
23	5	11	6	4	11	2	5.5	8
24	6	16	6	3	10	3	3.25	11
25	4	9	5	4	6	2	3	7
26	7	9	4	3	4	2	2	8
27	6	12	6	3	12	4	3	9
28	6	7	4	3	8	3	2.75	7
29	9	6	4	3	6	1	6	8
31	3	10	3	3	6	2	3	7

1931

no. : length: inches: 16th : 16th : ounces: number: ounces : inches  
of : inches: length: inches: inches: fruit : fruits: average: average  
Limb: 1931: of : dia. of: dia. of: on : on : wt. per : combined  
: Term. : 1930 : 1930 : 1931 : 1930 : 1930 : apple : 1930-31  
: Growth: Term. G: Term. G: Term. G: Term. G: Term. G: 1930 TG: Ter. G.

32	10	12	6	4	22	8	2.75	11
33	7	5	5	3	8	4	3	6
34	7	10	6	3	21	12	2.35	9
35	2	4	5	3	3	1	3	3
36	3	8	5	3	9	3	3	6
37	10	11	7	4	12	4	3	11
38	9	14	8	4	23	8	3	12
39	12	14	10	5	35	12	3	13
40	12	15	8	4	17	8	2	14
41	4	7	4	3	6	3	2	8
42	4	5	4	3	4	1	4	5
43	4	2	4	2	10	3	3.35	3
44	1	4	3	1	4	1	4	3
45	2	3	3	2	3	1	3	3
46	2	3	4	3	4	1	4	3
47	1	3	3	2	4	1	4	2
48	3	4	4	2	6	2	3	4
49	2	4	3	2	4	1	4	3
50	8	8	5	3	6	1	6	8
51	6	9	6	3	8	2	4	8
52	3	7	4	3	6	2	3	5
53	2	5	4	3	4	1	4	4
54	5	2	4	2	4	1	4	4
55	1	4	3	2	5	1	5	3
56	1	6	3	1	4	1	4	4
57	14	13	8	5	12	3	4	14
58	6	10	5	3	10	4	2.5	8
59	7	9	6	3	6	2	3	8
60	6	10	5	4	12	4	3	8
61	9	10	6	4	10	3	3.25	10
62	4	9	5	3	9	3	3	7
63	7	8	5	3	11	4	2.75	8
64	12	16	8	4	11	3	3.75	14
65	9	10	6	3	12	5	2.5	10
66	6	8	5	3	9	4	2.25	7
67	9	4	6	3	11	3	3.75	7
68	8	7	5	3	9	2	4.5	8
69	10	11	7	4	12	3	4	11
70	9	7	5	4	12	5	2.5	8

1931

No. of Limb:	inches: 1931:	inches: 1930:	inches: dia. of: 1930:	inches: dia. of: 1931:	ounces: fruit on 1930:	number: fruits on 1930:	ounces: average: 1930:	inches: average: 1930-31:
:	:Growth:	:Growth:	:Term. G:	:Term. G:	:Term. G:	:Term. G:	:1930 TG:	:Term. G:
71	10	8	6	4	8	3	2.75	9
72	8	8	4	3	7	3	2.25	7
73	6	5	3	2	5	2	2.5	6
74	4	12	4	2	12	4	3	8
75	7	10	5	3	10	3	3.25	9
76	6	10	6	3	5	1	5	8
77	3	7	4	3	5	3	1.75	5
78	5	8	4	3	6	2	3	7
79	3	5	4	3	4	1	4	4
80	8	12	6	3	20	7	3	10
81	8	11	7	3	7	4	1.75	10
82	4	15	8	3	22	9	2.5	10
83	7	12	8	3	17	10	1.75	10
84	6	7	5	3	9	5	1.75	7
85	3	9	4	3	10	2	5	6
86	3	4	4	3	6	1	6	4
87	2	3	4	3	4	1	4	3
88	1	6	4	2	4	1	4	4
89	7	6	4	2	7	1	7	7
90	6	5	3	2	6	1	6	6
91	1	6	5	2	6	1	6	3
92	11	9	7	4	12	4	3	10
93	1	10	5	2	5	2	2.5	6
94	13	8	8	5	11	3	3.75	11
95	7	7	5	3	11	4	2.75	7
96	7	8	6	3	16	2	3	8
97	8	9	7	4	8	3	2.75	9
98	5	16	6	3	16	8	2	11
99	3	10	4	3	12	5	2.5	7
100	6	6	4	3	6	2	3	6
101	8	7	5	4	7	3	2.25	8
102	2	5	4	3	4	1	4	4
103	6	6	5	4	6	2	3	6
104	9	9	6	4	7	3	2.25	9
105	7	17	7	3	12	3	4	12
106	10	12	6	3	12	4	3	11
107	1	11	5	3	9	2	4.5	6
108	9	14	7	4	20	7	3	12
109	7	18	7	3	23	8	3	13
110	1	13	5	2	10	2	5	7
111	5	13	5	2	10	2	5	9
112	5	5	5	3	8	3	2.75	5
113	5	6	4	3	8	3	2.75	6
114	7	8	5	3	14	5	2.75	8

1931

No. : inches : inches : 16th : 16th : ounces : number : ounces : inches  
of : length : length : inches : inches : fruit : fruits : average : average  
limb : 1931 : 1930 : dia. of : dia. of : on : on : wt. per : combined  
: Term. : Term. : 1930 : 1931 : 1930 : 1930 : apple : 1930-31  
: Growth : Growth : Term. G : Term. G : Term. G : Term. G : 1930 TG : Term. Gr

No.	1931	1930	1930 dia. of	1931 dia. of	1930 on	1931 on	1930 wt. per	1930-31 combined
115	6	11	6	2	11	4	3.75	9
116	5	11	4	3	13	4	3.25	8
117	5	10	4	2	10	4	2.5	8
118	3	10	4	2	11	4	2.75	7
119	9	14	6	4	18	6	3	12
120	4	6	4	2	6	2	3	5
121	7	6	6	3	11	4	2.75	7
122	7	4	5	4	8	3	2.75	6
123	5	4	4	3	6	2	3	5
124	6	6	4	3	6	2	3	6
125	4	5	4	3	6	2	3	5
126	2	6	4	3	4	1	4	4
127	7	5	5	3	6	2	3	6
128	7	7	4	3	8	3	2.75	7
129	5	7	4	3	6	2	3	6
130	9	8	5	4	10	4	2.5	9
131	6	5	5	3	6	2	3	6
132	5	6	4	3	12	4	3	6
133	8	13	7	4	13	4	3.25	11
134	6	7	5	3	8	3	2.75	7
135	3	7	4	3	6	2	3	5
136	8	10	6	3	7	2	3.5	9
137	7	4	4	4	5	1	5	6
138	4	6	4	3	4	1	4	5
139	7	3	4	3	9	2	4.5	5
140	6	6	5	3	12	5	2.5	6
141	8	6	6	4	8	3	2.75	7
142	13	13	8	5	18	5	3.5	13
143	6	9	6	4	14	4	3.5	8
144	5	7	6	4	12	5	2.5	6
145	5	5	4	3	9	3	3	5
146	8	8	6	4	9	4	2.25	8
147	4	4	4	3	5	2	2.5	4
148	3	3	5	3	6	2	3	3
149	6	4	4	3	5	2	2.5	5
150	7	7	5	3	8	3	2.75	7
151	4	5	4	3	5	2	2.5	5
152	9	5	5	3	8	3	2.75	5
153	4	6	4	3	6	2	3	5
154	9	5	4	3	10	4	2.5	7
155	7	6	4	3	14	5	2.75	7
156	9	9	6	3	15	8	2	9
157	4	10	4	3	6	3	2	7

1931

No. : inches: inches: 16th : 16th : ounces: number: ounces : inches  
of : length: length: inches: inches: fruit : fruits: average: average  
limb: 1931: 1930 : dia. of: dia. of: on : on : wt. per : combined  
: Termin: Termin: 1930 : 1931: 1930 : 1930: apple : 1930-31  
: Growth: Growth: Term. G: Term. G: Term. G Ter. G: 1930 TG: Term. G

No.	inches: 1931	inches: 1930	16th : inches: 1930	16th : inches: 1931	ounces: fruit on 1930	number: fruits on 1930	ounces : average wt. per apple 1930	inches : average combined 1930-31
158	6	4	4	3	10	4	3.5	5
159	5	7	4	3	7	3	3.35	6
160	4	5	4	3	6	2	3	5
161	8	7	5	3	14	6	2.25	8
162	2	7	4	3	7	2	3.5	5
163	7	8	4	3	12	5	3.5	8
164	5	5	4	3	7	3	2.25	5
165	5	10	6	4	14	5	2.75	8
166	12	17	8	4	17	4	4.25	15
167	4	7	4	3	8	3	2.75	6
168	5	6	4	3	7	3	2.25	6
169	2	4	4	2	6	2	3	3
170	9	7	4	3	8	3	2.75	8
171	11	17	8	4	19	6	3	14
172	4	3	4	4	5	1	5	4
173	3	5	5	3	6	2	3	4
174	11	4	4	4	9	2	4.5	8
175	6	3	4	3	5	1	5	5
176	3	5	4	3	6	1	6	4
177	7	6	4	3	9	2	4.5	7
178	4	4	4	3	6	1	6	4
179	3	4	4	3	6	1	6	4
180	7	11	5	3	14	4	3.5	9
181	4	7	5	3	12	3	4	6
182	3	4	4	3	7	1	7	4
183	7	8	6	4	16	5	3.25	8
184	2	6	4	3	4	1	4	4
185	5	5	4	3	9	2	4.5	5
186	5	3	4	3	5	1	5	4
187	3	4	4	3	4	1	4	4
188	11	14	7	4	20	5	4	13
189	9	20	6	3	17	4	4.25	15
190	7	3	4	3	4	1	4	5
191	6	7	5	3	11	4	2.75	7
192	6	2	5	3	6	2	3	4
193	4	4	4	3	4	1	4	4
194	4	8	5	3	10	4	2.5	6
195	5	6	5	4	10	3	3.25	6
196	9	5	5	4	12	5	2.5	7
197	8	5	5	4	7	2	3.5	7
198	7	5	5	4	8	3	2.75	6
199	7	5	4	3	8	3	2.75	6
200	9	3	5	3	10	3	3.25	6

## Terminal Growth and Yield Correlations in the York Apple

### Purpose

This problem purposes not only to show the relationships, if any, that exist between terminal growth and yield, but also to ascertain the statistical methods that may be best adaptable.

### Importance of the Problem

Relationships between vegetative growth and fruiting have been the subject of much study, and any results giving information on this important phase of nutrition should be of value. Valuable data may have been summarily discarded because at first sight no results were apparent. The worker in most branches of science finds quite frequently that in his raw observations there is evidence of a certain tendency, but because of the fluctuations, it may be difficult for him to draw conclusions. Some statistical method may therefore be necessary to reveal the amount of fluctuation or variation present in order to show more clearly the net tendency. In this thesis the net tendency that is proposed to be shown finally is the relationship that may exist between the aforementioned factors.

The statistical approach of this problem combines the technic of several authors, and it is the hope of the writer that other workers may be spared some time and effort by following this problem.

### Material

A random selection was made of trees of the same age, and of only those branches which bore fruit that year. Such sampling was employed in order that selection as representative as possible be secured. Records were made in the same manner each year for three consecutive years. Plate I shows the manner in which the measures were taken. In 1929, the first year of the project, the first set of notes was taken, which comprised the length and diameter of AB, the current (1929) terminal growth, also the length and diameter of BC, the last year's (1928) terminal growth, and the weight in ounces, and the number of apples on BC. The following year (1930) another set of observations was made, including the length and diameter of DE (the 1930 growth), also the length and diameter of EF (the 1929 growth), and the ounces and the number of apples on EF. In a like manner the 1931 set of notes included the length and diameter of GH (1931 growth), also the length and diameter of HI (1930 growth), along with the ounces yield and the number of apples on HI. The length of terminal growth in each case represents inches, and the diameter one thirty-second inches except that diameter in the 1931 notes is in one sixteenth inches. Observations were made about September 1st of each year. The following tables give the observed data for each year.



In glancing at the notes it is seen that in 1929, which was a heavy crop year for the York variety, a total of 300 branches were measured, there being a plentiful supply of branches, which bore fruit on their last year's terminal. However, in 1930 this was not the case. Most of the York trees were in the "off year", and what little fruit they bore, was found mostly on the older wood. The writer was unable to find more than 100 branches suitable for measuring. In 1931, 200 branches were measured.

### Procedure

#### The Correlation Table.

After assembling the raw data or orchard notes, relationships between two factors may be worked out.

If, for example, the length of terminal growth and the ounces of yield are the two variable factors considered, it is desired to know if a longer terminal growth is associated with a larger yield. This relationship may be computed statistically resulting in a figure between  $-1$  and  $+1$  called the coefficient of correlation and represented by "r". With data of the type found in the problem, where a large number of cases (number of branches) is used, the best method of computing "r" is by a frequency or correlation table. The method used here is that of Odell (1A).

Chart No. 1 shows a correlation table worked out for terminal growth and ounces yield. It will be noted that at the top of the table there is a row of consecutive figures starting at 1 in the left hand column of the table and going to 25 on the right hand side of the table. The figures in the columns under these consecutive numbers are called the X values and represent terminal growth measures. Similarly the columns of consecutive numbers at the extreme left side of the table starting at 1 in the lower corner and going to 24 in the upper corner are called the Y's and the figures in the rows to the right of these consecutive numbers are the Y values and represent yield.

To assemble the raw data into a frequency table of this kind, the first step was that of putting down the vertical and horizontal consecutive numbers. Then turning to the data and using the figures recorded under the terminal growth and yield headings, a tally mark is made on the frequency table in the square where the corresponding yield line intersects this terminal growth column, for example:

Raw Data.

Y					
6					
5				///	
4					
3				/	
2		/	//		
1					
	1	2	3	4	5

X Terminal Growth

Terminal Growth	Yield
5	5
3	3
5	5
2	2
3	2
4	3
5	5
etc.	etc.

All the figures were arranged in this manner, after which a count was made of the number of tally marks in each square and a number placed in each square to denote the number of marks therein. The marks were first made lightly with a pencil and later erased as they were counted and the numbers inserted in their places. The rows and columns are then totaled and the totals marked "f" or frequencies. In the chart the totals or frequencies in the row at the bottom of the table are the X frequencies and the first frequency 7 shows that there were 7 branches having a terminal growth 1 inch in length; the second frequency 7 shows that there were 7 branches having a terminal growth 2 inches long, etc. The frequencies at the right of the table are the Y frequencies and the top figure 3 shows that three branches had a yield of 24 ounces. The next 0 shows that there were no branches with a yield of 23 ounces, etc.

These frequency columns are then totaled, and, if the data has been correctly transferred to the correlation table, the sum of the X frequencies should equal the sum of the Y frequencies, and this figure should equal the total number of branches measured. This is a partial check on the correctness in transferring the data to the table and on the totaling of the lines and columns.

CHART I

DATA FOR 1929 and 1930 COMBINED TO DETERMINE THE RELATIONSHIP BETWEEN LAST YEARS

TERMINAL GROWTH AND CURRENT YIELD ON LAST YEARS TERMINAL GROWTH

Length of 1928 Terminal Growth)  
Length of 1929 Terminal Growth) combined-

arrayed against

Yield (in 1929) on 1928 Terminal Growth)  
Yield (in 1930) on 1929 Terminal Growth) combined

COMBINED TERMINAL GROWTH LENGTH X

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	f	dy	fdy	fd <sup>2</sup> y	Sx	Sxy	Means of x's		
24														1	1					1							3 +	17 +	51	867 +	28 +	476	16.55	
23																											0	16	0	0	0	0	0	
22																											0	15	0	0	0	0	0	
21														1													1	14	14	196 +	7	98	14.00	
20																1				1							2	13	26	338 +	22 +	266	18.00	
19																											0	12	0	0	0	0	0	
18													1	1	1												3	11	33	565 +	21 +	231	14.00	
17														2	1	1		1	1								5	10	50	500 +	48 +	480	16.60	
16										1	1		2	1	1			1									7	9	63	567 +	44 +	396	13.29	
15											1		1		1												3	8	24	192 +	18 +	144	13.00	
14							1			1				1											1		4	7	28	196 +	28 +	196	14.00	
13							1				1							1									3	6	18	108 +	15 +	90	12.00	
12			2	1			1		1	1	1	1				1											10	5	50	250 +	28 +	140	9.80	
11							1		1	1	1	2	2							1							9	4	36	144 +	26 +	104	9.89	
10			2		2	2		1	2	1	1	1	2	2													11	3	33	99 -	1 -	3	6.91	
9			2			1		3	2	1	2	2	2	1													16	2	32	64 +	48 +	96	10.00	
8		4	2			3	3	5	1	3	1	5		1					1								29	1	29	29 +	36 +	36	8.24	
7		3	1	2	1		3	3	3	3	1		1	1	1												23						8.50	
6		1	2	3	1	2	2		3	1		1	1														17	-	1 -	17	0	0	7.00	
5		3	1	2	2	2	2	4	3	1																	20	2	40	80 -	21 -	42	5.95	
4		4	2	1	3	4		3	1						1												23	3	69	207 -	32 +	96	5.61	
3							3	1	1	1																	8	4	32	128 +	2 -	8	7.25	
2			1												1												1	5	5	25 -	4 +	20	3.00	
1										1						1											2	6	12	72 +	12 -	72	13.00	
	7	7	11	11	13	17	21	20	12	19	7	15	9	10	7	5	4	1	1	4	0	0	0	0	0	1	200		487	4442	2848			
f	7	7	11	11	13	17	21	20	12	19	7	15	9	10	7	5	4	1	1	4	0	0	0	0	1	200		487	4442	2848				
dx	6	5	4	3	2	1	1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	200								
fdx	42	35	44	33	26	17	20	24	57	28	65	54	70	56	45	40	11	12	52	0	0	0	0	18	18	4779								
fd <sup>2</sup> x	252	175	176	99	52	17	20	48	171	112	325	324	490	448	405	400	121	144	676	0	0	0	0	324	324	4779								
Sy	108	15	4	6	22	11	7	12	39	13	155	288	567	384	81	250	110	12	585	0	0	0	0	7	7	2848								
Sxy	108	15	4	6	22	11	7	12	39	13	155	288	567	384	81	250	110	12	585	0	0	0	0	7	7	2848								
y's	4.45	6.57	7.09	7.18	6.15	7.06	6.76	6.65	7.50	7.68	11.14	9.38	12.33	15.10	13.86	8.80	13.25	17.00	8.00	18.25	0	0	0	0	14.00	14.00	14.00	14.00	14.00	14.00	14.00	14.00		

True Mean = Assumed Mean - correction  
 Mean of X = 7. + 1.775 = 8.775  
 Mean of Y = 7. + 1.56 = 8.56  
 Probable Error (E) of Mean = .6745  
 E of Mx = .6745 \* 4.5546 / 14.1421 = .218  
 E of My = .6745 \* 4.447 / 14.1421 = .212

Correction x (cx) =  $\frac{\sum fdx}{N} = \frac{+555}{200} = + 1.775$  (cx)<sup>2</sup> = 3.1506 cx.cy = 2.769

Correction y (cy) =  $\frac{\sum fdy}{N} = \frac{+312}{200} = + 1.56$  (cy)<sup>2</sup> = 2.4336

S<sup>2</sup><sub>x</sub> =  $\frac{\sum fd^2x}{N} = \frac{4779}{200} = 23.895$  σ<sup>2</sup><sub>x</sub> = S<sup>2</sup><sub>x</sub> - (cx)<sup>2</sup> = 23.895 - 3.1506 = 20.7444

σ<sub>x</sub> =  $\sqrt{\sigma^2_x} = \sqrt{20.7444} = 4.5546$

S<sup>2</sup><sub>y</sub> =  $\frac{\sum fd^2y}{N} = \frac{4442}{200} = 22.21$  σ<sup>2</sup><sub>y</sub> = S<sup>2</sup><sub>y</sub> - (cy)<sup>2</sup> = 22.21 - 2.4336 = 19.7764

σ<sub>y</sub> =  $\sqrt{\sigma^2_y} = \sqrt{19.7764} = 4.447$

Correlation Coefficient (r) =  $\frac{S_{xy} - cx.cy}{N} = \frac{2848 - 2.769}{200} = + .5664$

Probable Error of r (Er) =  $.6745 \frac{1. - r^2}{\sqrt{N}} = .6745 \frac{1. - (.5664)^2}{\sqrt{200}} = .6745 \frac{1. - .32080896}{14.1421356} = .0324$

r = + .5664 ± .0324 r is 17.5 x its

The frequency table having been completed, the computations were then started, which led up to securing the value for "r". The method used here employs what is known as the "assumed mean". This is a guess made as to the middle point of the values in each of the "f" columns. The closer this guess to the actual mid point or mean, the smaller the figures in the computation following. In chart I the mean or mid point of the X value was assumed to be at 7, and the mean of the Y value also at 7. This was then considered the zero or starting point, and for convenience, lines were drawn through the table just above and just below the line 7 in which the assumed mean of the Y value lies. Also lines were drawn just to the left and just to the right of the vertical column 7 in which the assumed mean of X lies. The column headed "dy" and the row headed "dx" show the difference or deviations from the assumed mean as the zero point. Since the class numbers, or consecutive numbers at the margin of the table, increase by 1 consecutively 1, 2, 3, etc. the deviation from the assumed mean also increased by 1. With the Y values, the deviation above the assumed mean are positive (+), while those below are negative (-). Similarly in the dx row, the deviations on the right of the assumed mean are positive, while those on the left are negative.

The next step was the multiplication of each  $f$  value by its corresponding  $d$  value to get the  $fd$  values both of  $X$  and  $Y$ . Then the  $fd$  values were each multiplied by their corresponding  $d$  values to get  $fd^2$  values. Part of the figures of the  $fdy$  columns and of the  $fdx$  rows were positive and part were negative. This column and the row were each totaled algebraically.

Since there were a number of frequencies in each class it is necessary that these frequency values be collected together. Hence the  $SX$  column is used, and each figure in this column is the sum of the  $X$  values of the measures in the same row. For example the first figure under  $SX$  is  $+28$ . This figure is secured in the following way: There are 3 frequencies in this row. One lies in the  $X$  column 14, one under 15, and the third under 20. The one under 14 is multiplied by the  $dx$  value of  $+7$ ; the one under 15 is multiplied by  $dx$  value  $+8$ ; and the one under 20 is multiplied by the  $dx$  value  $+13$ . Thus we have:  
 $1 \times +7 = +7$ ;  $1 \times +8 = +8$ ;  $1 \times +13 = +13$ ,  $+7 +8 +13 = +28$ .

The next two entries in the  $SX$  column are obviously zero. The fourth figure is  $+7$  and is secured by multiplying 1 by its  $dx$  value  $+7$ . Each other entry in the  $SX$  column is computed in a like manner, but when we come to Row 13, which has an  $SX$  value of  $+28$ , it is seen that some of the frequencies lie on the negative side of the assumed mean, and the computation becomes:

$$\begin{aligned} 1 \times +13 &= +13; & 1 \times +9 &= +9; & 1 \times +5 &= +5; & 1 \times +4 &= +4; \\ 1 \times +3 &= +3; & 1 \times +2 &= +2; & 1 \times 0 &= 0; & 1 \times -2 &= -2; & 2 \times -3 &= -6 \\ & +13 & +9 & +5 & +4 & +3 & +2 & = & +36 \\ & & & & & -2 & -6 & = & -8 \\ \text{Algebraic Sum} & = & +28 \end{aligned}$$

Hence it will be seen that the sums must be made algebraically.

The entries in the  $S_{xy}$  column are had by multiplying the  $S_x$  values by their corresponding  $dy$  values as

$$\begin{aligned} + 28 \times +17 &= + 476 \\ + 7 \times +14 &= + 98 \text{ etc. down to class 10, which is} \\ - 1 \times +3 &= - 3 \text{ etc. down to class 4, which is} \\ - 33 \times -3 &= + 96 \\ + 2 \times -4 &= - 8 \text{ etc.} \end{aligned}$$

Similarly the entries on the  $S_y$  Row are found by multiplying each frequency number by its corresponding  $dy$  value and summing the results algebraically as follows:

$$\begin{aligned} 3 \times -2 &= -6; & 4 \times -3 &= -12; & & & -12 & -6 & = & -18 \\ 2 \times -3 &= -6; & 1 \times -1 &= -1; & 4 \times +1 &= +4; & -6 & -1 & = & -7 \\ & & & & & & & & & +4 \\ & & & & & & & & = & -3 \end{aligned}$$

$$\begin{aligned} 1 \times -5 &= -5; & 1 \times -2 &= -2; & 2 \times -1 &= -2; & 3 \times 0 &= 0; & 2 \times -2 &= -4 \\ 2 \times +3 &= +6; & -5 & -2 & -2 & = & -9; & +4 & +6 & = & +10; & +10 & -9 & = & +1 \text{ etc.} \end{aligned}$$

This was continued for each  $S_y$  value through all 25 classes. The  $S_{xy}$  entries are had by multiplying each  $S_y$  entry by its corresponding  $dx$  value as  $-18 \times -6 = +108$ ;  $-3 \times -5 = +15$ , etc.

The columns  $fdy$ ,  $fd^2y$  and  $S_{xy}$ , and the rows  $fdx$ ,  $fd^2x$  and  $S_{xy}$  were each totaled algebraically. The columns  $S_{xy}$  and the row  $S_{xy}$  should be equal if the computations

have been correct. This is a check on the accuracy of the  $\Sigma x$  values. In calculating  $r$  only the totals of the columns and rows are used.

The next step was that of applying a correction to the error due to the guess in assuming a mean. This correction is represented by the letter "c" and is found, as shown just beneath the frequency table, by dividing the sum of the  $fd$  columns by the total number of cases  $N$ .

Before actually finding the value of "r" it is first necessary to compute the standard deviation. Odell (1B) defines this as the square root of the mean of the squares of the deviations from the mean of the distribution. As its name implies it is the most standard or absolute measure of deviation.

Standard deviation is simply the absolute measure of the deviation of the individual cases from the arithmetic mean of all cases and is represented by the Greek letter  $\sigma$  as the definition states

$$\sigma = \sqrt{\frac{\Sigma fd^2}{N}}$$

In chart I it is noted that this formula applies, except that correction for the assumed mean is also applied so that

$$\sigma = \sqrt{\frac{\Sigma fd^2}{N} - c^2}$$



Standard deviations were worked out for both x and y values. These are not comparable with each other since the x values are in inches length of terminal growth, while the Y values are in ounces of yield. Correction of x is multiplied by correction of Y to get  $oxoy = 2.769$

### The Correlation Coefficient

All the necessary figures are at hand to compute the formula for the coefficient of correlation r:

$$r = \frac{\sum xy - oxoy}{\sqrt{\sum x^2 \sum y^2}}$$
 and by substituting the values

from the correlation table it is found that  $r = .5564$ . As mentioned previously the value of r must lie somewhere between -1 and +1. A correlation coefficient of +1 shows a perfect positive correlation between the two factors considered and a coefficient of -1 shows a perfect negative correlation. The nearer the value approaches 1, most obviously the closer the association or relationship between the factors considered.

The Probable Error

To determine the degree of reliability that may be attached to the calculated  $r$ , it is necessary to secure another calculation called the "probable error". Babcock and Clausen (5) define this as "an arbitrary term used to denote the amount that must be added to or subtracted from the observed value to obtain two limiting figures of which it may be said that there is an even chance that the true value lies within or without these limits. These limiting figures include about 50% of the observed values of the distribution. The remaining 50% lie somewhere outside these limiting figures, and there are even chances that the true value of  $r$  lies outside or inside. The greater the difference between the value of  $r$  and the value of its probable error, the greater the chances that the true value of  $r$  lies within the limits described by the probable error. In Chart I the formula as given by Odell (1-c) for calculating the probable error of  $r$  is

$$E_r = .6745 \frac{1-r^2}{N}$$

and by substituting  $E_r$  is .0324 and  $r = .5664 \pm .0324$ , which means that the chances are even that the true value of  $r$  lies somewhere between .5664 - .0324 and .5664 + .0324, or between .5988 and .5340. In order to be reasonably sure that  $r$  denotes a true correlation between

the factor involved, the  $r$  value should be at least 4 times its probable error. Some workers consider 3 times  $E$  as denoting correlation and others think that the multiple should be 5. If  $r$  is 3 times its  $E$  then-

$$3 \times \pm .0324 = \pm .0972 \quad \text{and} \quad .5664 + .0972 = .6636 \\ .5664 - .0972 = .4692$$

Hence the limiting figures are expanded, and 95.7% of the distribution lies within the limits of .6636 and .4692. The remaining 4.3% lies somewhere without these limits. The chances then become  $95.7 \div 4.3 = 22$  to 1 that the true value of  $r$  lies within these limits.

If  $r$  is 4 times its  $E$  then 99.3% of the distribution lies between the limits described by the probable error and the remaining .7% lie outside the limits. Hence the chances are  $99.3 \div .7$  or 142 to 1 that  $r$  lies between these limits. Thus it will be seen that, as the value of  $r$  increases in proportion to its probable error, the limiting figures increase and the chances become greater that the true value of  $r$  will lie within the expanded limits.

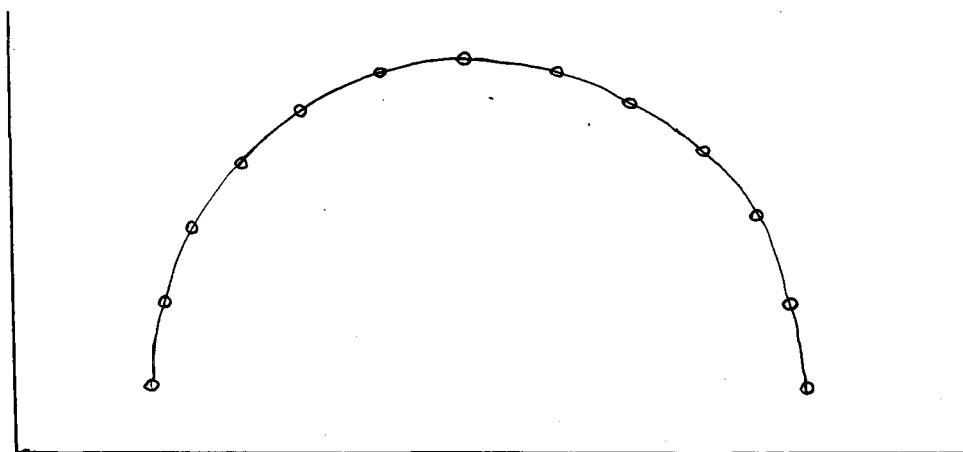
In the problem under consideration,  $r$  or .5664 is 17.5 times  $E$  or .0324. When  $r$  is 10 times  $E$ , the chances are over one million to one, which is practically certainty. Thus 17.5 times  $E$  means that it is practically certain that the true value of  $r$  lies within the limits described by the probable error, and that the correlation is a true relation

and is not due to chance or experimental error. Also since  $r$  is positive, the relation is positive showing the terminal growth and yield to increase or decrease together.

### Non-Linear Relationship

Garrett (2) states that "The relation existing between the paired values of two sets of measures  $X$  and  $Y$  may be described in a general way as 'linear' or 'non-linear'. When the means of the arrays of successive columns or rows in a correlation table follow straight lines (exactly or approximately) the regression is called 'linear', and the relation between the two sets of measures is a 'straight line' relationship". On the other hand Holzinger (3) states "when the means of the arrays do not lie fairly closely on a straight line, the regression is regarded as non-linear, and the correlation coefficient will give an understatement of the degree of association present for these curvilinear trends and is therefore inapplicable". In the latter case a curve of some kind is necessary to properly describe the drift or trend. If  $r$  is used to measure the relation where curvilinearity exists the value for  $r$  may be very low, and yet the relation be high. An extreme case of this is given in the figure below in which all the observations lie on a half circle, and the trend is perfect curvilinearity. The

value of  $r$ , however, is zero because the straight line representing the "r" relation will be perfectly horizontal and  $S_{xy}$  will equal zero.



The Correlation Ratio

Prof. Karl Pearson has devised a measure for the correlation of non-linear frequency table. He called this decimal "correlation ratio". The correlation ratio may also be used for linear relations, since here the ratio and coefficient will be equal. In general the correlation ratio ( $\eta$ -eta) will be greater than  $r$ . The greater the curvilinear tendency the wider the gap between  $\eta$  and  $r$ .  $\eta$  is always positive and lies somewhere between zero and 1. A positive or negative expression of  $\eta$  is indicated by the direction taken by the curve.

The correlation ratio  $\eta$  must be calculated for both the  $x$  or terminal growth factor and for  $y$  the yield factor. The correlation ratio for the means of the rows

is expressed as  $\eta_{xy}$ , and that for the means of the columns as  $\eta_{yx}$ . There are several methods for calculating these ratios, the one used for the problem in Chart I, being found in Mills (4A). The formulae are as follows:

$$\eta_{xy} = \frac{\sum mx}{\sum x}$$

$$\eta_{yx} = \frac{\sum my}{\sum y}$$

The calculation of  $\eta_{xy}$  is found in Table A. The first column headed Y contains the y or yield classes found at the left of Chart I. The column headed mx contains the means of the X's or terminal growth classes found at the extreme right of Chart I. The means of X's are computed in the following manner. The first entry is 16.33 which is the sum of each corresponding frequency multiplied by its X class number and divided by the number of frequencies.

That is,  $1 \times 20 = 20$ ;  $1 \times 15 = 15$ ;  $1 \times 14 = 14$

$$20 + 15 + 14 = 49; \quad 49 \div 3 = 16.33$$

also  $1 \times 14 = 14$ ;  $14 \div 1 = 14$  the fourth entry, etc. each entry is calculated in like manner.

The 13th entry is 9.80 which is found by-

$$1 \times 20 = 20; 1 \times 16 = 16; 1 \times 13 = 13; 1 \times 11 = 11; 1 \times 10 = 10$$

$$1 \times 9 = 9; 1 \times 7 = 7; 1 \times 5 = 5; 2 \times 4 = 8$$

$$20 + 16 + 13 + 11 + 10 + 9 + 7 + 5 + 8 = 98$$

$$98 \div 10 = 9.80$$

TABLE A

CORRELATION RATIO  $\eta_{XY}$

Yield	Mean Terminal Growth	Deviation from Mean of all x's (8.775)	$d^2$	f	$fd^2$
Y	MX	d	$d^2$	f	$fd^2$
24	16.33	7.555	57.078	3	171.234
23	0	0	0	0	0
22	0	0	0	0	0
21	14.00	5.225	27.301	1	27.301
20	18.00	9.225	85.101	2	170.202
19	0	0	0	0	0
18	14.00	5.225	27.301	3	81.903
17	16.60	7.825	61.231	5	306.155
16	13.29	4.515	20.385	7	142.695
15	13.00	4.225	17.851	3	53.553
14	14.00	5.225	27.301	4	109.204
13	12.00	3.225	10.401	3	31.203
12	9.80	1.025	1.051	10	10.510
11	9.87	1.115	1.243	9	11.187
10	6.91	1.865	3.478	11	38.258
9	10.00	1.225	1.501	16	24.016
8	8.24	.535	.286	29	8.294
7	8.30	.475	.226	23	5.198
6	7.00	1.775	3.151	17	53.567
5	5.95	2.825	7.981	20	159.620
4	5.61	3.165	10.017	23	230.391
3	7.25	1.525	2.326	8	18.608
2	3.00	5.775	33.351	1	33.351
1	13.00	4.225	17.851	2	35.702
					<u>1722.1207</u>

$$\sigma_{mx} = \sqrt{\frac{\sum fd^2}{N}}$$

$$\sigma_{mx} = \sqrt{\frac{1722.1207}{200}} = 2.9343$$

$$\eta_{xy} = \frac{\sigma_{mx}}{\sigma_x} = \frac{2.9343}{4.5546} = .6444$$

Criterion of Linearity = Zeta ( $\zeta$ )

$$\zeta \eta_{xy} = \eta^2 = r^2 = .6444^{(2)} - .5664^{(2)} = .0945$$

$$\text{Standard error } (\sigma \zeta) = 2 \sqrt{\frac{\zeta}{N}} \sqrt{(1-\eta^2)^2 - (1-r^2)^2} + 1$$

$$\sigma \zeta = 2 \sqrt{\frac{.0945}{200}} \sqrt{(1-.6444^{(2)})^2 - (1-.5664^{(2)})^2} + 1$$

$$\sigma \zeta = .0204$$

$$\zeta = 4.6 \text{ times } \sigma \zeta$$

After the Y values and mx values have been arrayed against each other, we wish to find the deviation of each mx value from the mean of all X's (8.775). The figure 8.775 is found from Chart I by Assumed mean of  $x \pm$  correction of x.

$$\begin{array}{rcl} \text{Assumed mean of X} & = & 7 \\ \text{Correction of X} & = & +1.775 \\ \text{Mean of X's} & = & \underline{8.775} \end{array}$$

The deviations in column d represent the differences between 8.775 and each mx entry or

$$\begin{array}{rcl} 16.33 - 8.775 & = & 7.555 \\ 14. & - & 8.775 & = & 5.225 \\ \text{etc.} & & & & \end{array}$$

The d values are each squared to get the entries in column  $d^2$ .

The column marked f contains the frequencies found in the frequency column of Chart 1.

Obviously the f value multiplied by its corresponding  $d^2$  value gives the  $fd^2$  value found in the right hand column. It will be seen then that  $\sigma_{mx} = \sqrt{\frac{\sum fd^2}{N}}$

substituting the values in Table A we find  $\sigma_{mx} = 3.9343$ .

Then  $r_{xy} = \frac{\sigma_{mx}}{\sigma_x}$  and substituting values in Table A we find  $r_{xy} = .6444$ , which compared with the r value .5864 of Chart I, is seen to be greater than r. The spread or difference



between the values of  $\eta$  and  $r$  denote the departure from linearity as shown by the formula for criterion of linearity of  $\eta xy$  or  $\xi \eta xy$  given by Mills (4B)

$\xi \eta xy = \eta^2 - r^2$  substituting in Table A we find

$$\xi \eta xy = .0945$$

How large must this difference be to indicate that the relation is non-linear, and that  $r$  does not adequately express the measure of correlation?

In order to establish significance the error must be calculated, and according to Blakeman's formula from Mills (4-B) -

$$\text{Standard error of } \xi (\sigma \xi) = 2 \sqrt{\frac{\xi}{n} \sqrt{(1-\eta^2)^2 - (1-r^2)^2} + 1}$$

Substituting in Table A it is seen that-

$$\sigma \xi = .0204 \text{ and that } \xi \eta xy \text{ is } 4.6 \text{ times } \sigma$$

It is generally conceded that 2 or more times  $\sigma$  denotes significance, and since this problem shows 4.6 times, indications are that the relation is curvilinear and that  $r$  will not serve for this array of factors.

Having found that the relationship of terminal growth on yield is non-linear, it is desired to determine whether  $\eta y x$  or the relationship of yield on terminal growth is linear or non-linear.

Table B shows these calculations which are made in the same manner as those in Table A except that the value arrayed against each other are reversed. The column headed X contains the class numbers of the X or terminal growth row found at the top of Chart I. The column headed my contains the means of the y's found at the bottom of the frequency table of Chart I. These means of y's are calculated similarly to the means of x's thus for the first entry 4.43, the column in which 4.43 is found contains 7 frequencies, i.e. 4 in class 4 and 3 in class 5. Hence-

$$4 \times 4 = 16; \quad 3 \times 5 = 15 \\ 16 + 15 = 31; \quad 31 \div 7 = 4.43$$

For the second entry 6.57-

$$3 \times 4 = 12; \quad 1 \times 6 = 6; \quad 4 \times 8 = 32 \\ 12 + 6 + 32 = 50; \quad 50 \div 7 = 7.14 \\ \text{etc.}$$

The mean of all Y's = assumed mean of Y + correction of Y =  $7 + 1.56 = 8.56$

The calculations from there on are similar to those in Table A, and will not be explained further than to point out that  $\Sigma nyx$  is 3 times  $\sigma^2$  and hence the relation is non-linear.

TABLE B

CORRELATION RATIO  $\eta_{yx}$

Terminal Growth	Mean Yield	Deviation from Means of all y's (8.56)	$d^2$	f	$fd^2$
x	my	d	$d^2$	f	$fd^2$
25	14.00	5.44	29.594	1	29.594
24	0	0	0	0	0
23	0	0	0	0	0
22	0	0	0	0	0
21	0	0	0	0	0
20	18.25	9.69	93.896	4	375.584
19	8.00	.56	.314	1	.314
18	17.00	8.44	71.234	1	71.234
17	13.25	4.69	21.986	5	87.984
16	8.80	.24	.058	5	.290
15	13.86	5.30	28.090	7	196.630
14	15.10	6.54	42.772	10	427.720
13	12.33	3.77	14.213	9	127.917
12	9.38	.82	.672	13	8.736
11	11.14	2.58	6.656	7	46.592
10	7.68	.88	.774	19	14.706
9	7.50	1.06	1.124	12	13.488
8	6.65	1.91	3.648	20	72.960
7	6.76	1.80	3.240	21	68.040
6	7.06	1.50	2.250	17	38.250
5	8.15	2.41	5.808	13	75.504
4	7.18	1.38	1.904	11	20.944
3	7.09	1.47	2.161	11	23.771
2	6.57	1.99	3.960	7	27.720
1	4.43	4.13	17.057	7	119.399
					<u>1847.377</u>

$$\sigma_{my} = \sqrt{\frac{\sum fd^2}{N}} = \sqrt{\frac{1847.377}{200}} = 3.0393$$

$$\eta_{yx} = \frac{\sigma_{my}}{\sigma_y} = \frac{3.0393}{4.447} = .6834$$

$$\rho_{\eta_{yx}} = \eta^2 - r^2 = .6834^2 - .5664^2 = .1462$$

$$\sigma_{\rho} = 2 \sqrt{\frac{\rho}{N}} \sqrt{(1-\eta^2)^2 - (1-r^2)^2} + 1$$

$$\sigma_{\rho} = 2 \sqrt{\frac{.1462}{200}} \sqrt{(1-.6834^2)^2 - (1-.5664^2)^2} + 1$$

$$\sigma_{\rho} = .0491$$

$\rho_{\eta_{yx}}$  is 3 times  $\sigma_{\rho}$

Hence curvelinear

Fitting a Curve To The Data

Since the correlation ratios show that the relation is non-linear, and that a curve rather than a straight line gives the best description of the relation, the next calculation will necessarily be that of finding the particular curves which best fit the observations. Pearl's (6-A) logarithmic method of least squares was chosen as this method arrives at a third degree parabola with somewhat less calculation than is involved in the ordinary method of least squares.

The same x and y values are arrayed against each other as in Tables A and B. Table I shows the arrangement of the data for the curve X on Y or terminal growth on yield. The remaining columns are headed xy,  $x^2$ , and  $y (\log x)$ . The column xy contains figures found by multiplying each x value by its corresponding y, thus  $1 \times 13 = 13$ ;  $3 \times 3 = 6$ , etc. The column headed  $x^2$  contains the squares of each of the x values. In the column  $y (\log x)$  each y value is multiplied by the logarithm of its corresponding x thus,

$13 \times 0 = 0$ ;  $3 \times .30103 = .9031$ ;  $7.35 \times .47713 = 3.4591$ , etc. Each of these columns is then totaled and the totals listed to the right. It will be noted that some of the entries in the y column are marked zero. However, these are not true

TABLE 1

Logarithmic curve fitted to 1929 and 1930 combined data on :-

Length of 1928 terminal growth }  
 " " 1929 " " " } combined

arrayed against: yield (in 1929) on 1928 terminal growth }  
 " ( " 1930) " 1929 " " " } combined

when yield remains constant and its corresponding terminal growth is calculated

Yield	Term.				
x	Growth				
	y	xy	x <sup>2</sup>	y(log x)	
1	13.	13.00	1	0	
2	3.	6.00	4	.9031	(Length of 1928 T.G.)
3	7.25	21.75	9	3.4591	( " " 1929 " ) combined
4	5.61	22.44	16	3.3776	arrayed against:-
5	5.95	29.75	25	4.1589	
6	7.	42.00	36	5.4471	(yield (in 1929) on 1928 TG)
7	8.30	58.10	49	7.0143	( " ( " 1930) " 1929 " )
8	8.24	65.92	64	7.4415	
9	10.	90.	81	9.5424	
10	6.91	69.10	100	6.9100	
11	9.89	108.79	121	10.2993	N = 21
12	9.80	117.60	144	10.5760	SX = 236
13	12.	156.00	169	13.3673	SY = 226.17
14	14.	196.00	196	16.0458	SXY = 2984.21
15	13.	195.00	225	15.2892	SX = 3526
16	13.29	212.64	256	16.0028	S(y log x) = 232.3021
17	16.60	282.20	289	20.4255	
18	14.	252.00	324	17.5738	S(log x) = 19.8098010
no cases	0	0	-	0	S(x log x) = 266.5983459
20	18.	360.00	400	23.4185	S(log x) <sup>2</sup> = 21.3075031
21	14.	294.00	441	18.5111	
no cases	0	0	-	0	
--	0	0	-	0	
24	16.33	391.92	576	22.5388	
236	226.17	2984.21	3526	232.3021	

TABLE 2

Calculations of a, b and c values necessary for computing Logarithmic curve values.

$$\begin{aligned}
 na + bSx &+ cS \log x = S y & (1) \\
 aSx + bSx^2 &+ cS (x \log x) = S xy & (2) \\
 aS(\log x) + bS(x \log x) &+ cS (\log x)^2 = S (y \log x) & (3)
 \end{aligned}$$
  

$$\begin{aligned}
 21a + 236b + 19.809801 c &= 226.17 & (1) \\
 236a + 3526b + 266.59835 c &= 2984.21 & (2) \\
 49.809801a + 266.59835b + 21.307503 c &= 232.3021 & (3)
 \end{aligned}$$
  


---


$$\begin{aligned}
 236 [21a + 236b + 19.809801 c &= 226.17] & (1) \\
 21 [236a + 3526b + 266.59835 c &= 2984.21] & (2) \\
 4956a + 55696b + 4675.1130 c &= 53,376.12 \\
 4956a + 74046b + 5598.5654 c &= 62,668.41 \\
 \hline
 18,350b + 923.4524 c &= 9292.29 & (4)
 \end{aligned}$$
  

$$\begin{aligned}
 19.809801 [21a + 236b + 19.809801 c &= 226.17] & (1) \\
 21 [19.809801a + 266.59835b + 21.307503 c &= 232.3021] & (3)
 \end{aligned}$$
  

$$\begin{aligned}
 416.005821a + 4675.1130b + 392.42822c &= 4480.3827 \\
 416.005821a + 5598.5654b + 447.45758c &= 4878.3441 \\
 \hline
 923.4524b + 55.02936c &= 397.9614 & (5)
 \end{aligned}$$
  

$$\begin{aligned}
 923.4524 [18,350b + 923.4524 c &= 9292.29] & (4) \\
 18,350 [923,4524b + 55.02936c &= 397.9614] & (5)
 \end{aligned}$$
  

$$\begin{aligned}
 16,945,351.54b + 852,764.535c &= 8,580.987.50^3 \\
 16,945,351.54b + 1,009,788.756c &= 7,302,591.69 \\
 \hline
 157,024.421c &= -1,278,395.81^2 \\
 c &= -8.141
 \end{aligned}$$
  

$$\begin{aligned}
 923.4524b - 447.9940 &= 397.9614 \\
 923.4524b &= 845.9554 \\
 b &= +.916 \\
 21a + 216.176 - 161.2716 &= 226.17 \\
 21a &= 171.2656 \\
 a &= + 8.156
 \end{aligned}$$

TABLE 3

Calculating the y values for each x class from the equation

$$y = a + bx + c(\log x)$$

$$y = 8.156 + .916x - 8.141 \log x$$

x	a	+ bx	+ (-) c(log x)	= y
1	8.156	+ .916	- 0	= 9.072
2	8.156	+ 1.832	- 2.451	= 7.537
3	8.156	+ 2.748	- 3.884	= 7.020
4	8.156	+ 3.664	- 4.801	= 6.919
5	8.156	+ 4.580	- 5.690	= 7.046
6	8.156	+ 5.496	- 6.335	= 7.317
7	8.156	+ 6.412	- 6.880	= 7.688
8	8.156	+ 7.328	- 7.352	= 8.132
9	8.156	+ 8.244	- 7.768	= 8.692
10	8.156	+ 9.160	- 8.141	= 9.175
11	8.156	+ 10.076	- 8.478	= 9.754
12	8.156	+ 10.992	- 8.786	= 10.362
13	8.156	+ 11.908	- 9.069	= 10.995
14	8.156	+ 12.824	- 9.331	= 11.649
15	8.156	+ 13.740	- 9.575	= 12.321
16	8.156	+ 14.656	- 9.803	= 13.009
17	8.156	+ 15.572	- 10.017	= 13.711
18	8.156	+ 16.488	- 10.219	= 14.425
19	8.156	+ 17.404	- 10.410	= 15.150
20	8.156	+ 18.320	- 10.592	= 15.884
21	8.156	+ 19.236	- 10.764	= 16.628
22	8.156	+ 20.152	- 10.929	= 17.379
23	8.156	+ 21.068	- 11.086	= 18.138
24	8.156	+ 21.984	- 11.236	= 18.904

TABLE 4

Calculation of values necessary for plotting a straight line for the data.

$$na + b \sum x = \sum y \quad (1)$$

$$a \sum x + b \sum x^2 = \sum xy \quad (2)$$

$$236 \left[ \begin{array}{l} 21a \\ 236a \end{array} \right. + \begin{array}{l} 236b \\ 3536b \end{array} = \begin{array}{l} 226.17 \\ 2984.21 \end{array} \quad \begin{array}{l} (1) \\ (2) \end{array}$$

$$\begin{array}{l} 4956a + 55696b = 53376.12 \\ 4956a + 74046b = 62668.41 \\ \hline 18350b = 9292.29 \\ b = .506 \end{array}$$

$$\begin{array}{l} 21a + 119.516b = 226.17 \\ 21a = 106.654 \\ a = 5.079 \end{array}$$

$$y = a + bx$$

$$y = 5.079 + .506 x$$

x	a	+ bx	= y
1	5.079	+ .506	= 5.585
12	5.079	+ 6.072	= 11.151
24	5.079	+ 12.144	= 17.223

TABLE 5

Tables 5-9 inclusive are calculations for the curves showing the relationship of y on x, in which the x or Terminal Growth values are constant, and their y or yield values are calculated

(Length of 1928 Terminal Growth )  
 ( " " 1929 " " ) combined

arrayed against

(Yield (in 1929) on 1928 Terminal Growth)  
 ( " (in 1930) " 1929 " " ) combined

Terminal Growth	Yield	XY	X <sup>2</sup>	Y(log x)
X	Y			
1	4.43	4.43	1	0
2	6.57	13.14	4	1.9778
3	7.09	21.27	9	3.3828
4	7.18	28.72	16	4.3228
5	6.15	30.75	25	4.2987
6	7.06	42.36	36	5.4937
7	6.76	47.32	49	5.7129
8	6.65	53.20	64	6.0055
9	7.50	67.50	81	7.1568
10	7.68	76.80	100	7.6800
11	11.14	122.54	121	11.6011
12	9.38	112.56	144	10.1227
13	12.33	160.29	169	13.7349
14	15.10	211.40	196	17.3066
15	13.86	207.90	225	16.3006
16	8.80	140.80	256	10.5963
17	13.25	225.25	289	16.3035
18	17.00	306.00	324	21.3396
19	8.00	152.00	361	10.2300
20	18.25	365.00	400	23.7438
-	0	0	-	0
-	0	0	-	0
-	0	0	-	0
-	0	0	-	0
25	14.00	350.00	625	19.5712
325	208.18	2739.23	5525	216.8813

N = 21  
 Sx = 235  
 Sy = 208.18  
 Sxy = 2739.23  
 Sx<sup>2</sup> = 3495  
 S(y logx) = 216.8813  
 S(log x) = 19.7840646  
 S(x logx) = 264.9518043  
 S(log x)<sup>2</sup> = 21.2237048



TABLE 6

Y on X

Calculations of a, b and c necessary for computing the logarithmic curve values

$$\begin{aligned}
 na + b \sum x_i^2 + c \sum \log x_i &= \sum y_i \\
 a \sum x_i + b \sum x_i^2 + c \sum (x_i \log x_i) &= \sum x_i y_i \\
 A \sum (\log x_i) + b \sum (x_i \log x_i) + c \sum (\log x_i)^2 &= \sum y_i \log x_i
 \end{aligned}$$

y on x Logarythmic curve fitted

$$\begin{aligned}
 21a + 235 b + 19.7840646c &= 208.18 \\
 235a + 3495 b + 264.9518043c &= 2739.23 \\
 19.7840646a + 264.9518043b + 21.2237048c &= 216.8813
 \end{aligned}$$

$$\begin{aligned}
 235 \left[ \begin{array}{l} 21a \\ 21 \end{array} \right. + 235 b + 19.7840646c &= 208.18 \\
 \left. \begin{array}{l} 235a \\ 21 \end{array} \right] + 3495 b + 264.9518043c &= 2739.23
 \end{aligned}$$

$$\begin{aligned}
 4935a + 55225 b + 4649.2553 c &= 48,922.3 \\
 4935a + 73395 b + 5563.9878 c &= 57,523.83 \\
 \hline
 18,170 b + 914.73252 c &= 8,601.53
 \end{aligned}$$

$$\begin{aligned}
 19.7840646 \left[ \begin{array}{l} 21a \\ 21 \end{array} \right. + 235 b + 19.7840646c &= 208.18 \\
 \left. \begin{array}{l} 19.7840646 a \\ 21 \end{array} \right] + 264.9518043b + 21.2237048c &= 216.8813
 \end{aligned}$$

$$\begin{aligned}
 415.465365a + 4649.255275b + 4391.40923c &= 4118.64665 \\
 415.465365a + 5563.9878 b + 445.69781c &= 4554.5073 \\
 \hline
 914.732525b + 54.28858c &= 435.86065
 \end{aligned}$$

$$\begin{aligned}
 914.73252 \left[ \begin{array}{l} 18.170b \\ 18,170 \end{array} \right. + 914.73252c &= 8,601.53 \\
 \left. \begin{array}{l} 18,170 \\ 18,170 \end{array} \right] + 54.28858c &= 435.86065
 \end{aligned}$$

$$\begin{aligned}
 16,670,689,8884b + 836,735.58315c &= 7,868,099.2128 \\
 16,670,689,8884b + 986,423.48860c &= 7,912,528.0105 \\
 \hline
 149,687.91545c &= 51,428.7977 \\
 c &= .344
 \end{aligned}$$

$$\begin{aligned}
 914.7325 b + 18.6753 &= 435.86065 \\
 914.7325b &= 417.18535 \\
 b &= + .456 \\
 21a + 107.16 + 6.8057 &= 208.18 \\
 21 a &= 94.2143 \\
 a &= 4.486
 \end{aligned}$$

$$\begin{aligned}
 a &= + 4.486 \\
 b &= + .456 \\
 c &= + .344
 \end{aligned}$$

TABLE 7

Y on X

Calculations of Y value for each X class from the equation  
 $Y = a + bx + c (\log x)$

$$Y = a + bx + c (\log x) = 4.486 + .456x + .344 \log x$$

x	a	+	bx	+	c(log x)	=	y
1	4.486	+	.456	+	0	=	4.942
2	4.486	+	.912	+	.104	=	5.502
3	4.486	+	1.368	+	.164	=	6.018
4	4.486	+	1.824	+	.207	=	6.517
5	4.486	+	2.280	+	.240	=	7.006
6	4.486	+	2.736	+	.268	=	7.480
7	4.486	+	3.192	+	.291	=	7.969
8	4.486	+	3.648	+	.311	=	8.445
9	4.486	+	4.104	+	.328	=	8.918
10	4.486	+	4.560	+	.344	=	9.390
11	4.486	+	5.016	+	.358	=	9.861
12	4.486	+	5.472	+	.371	=	10.329
13	4.486	+	5.928	+	.383	=	10.797
14	4.486	+	6.384	+	.394	=	11.261
15	4.486	+	6.840	+	.405	=	11.731
16	4.486	+	7.296	+	.414	=	12.196
17	4.486	+	7.752	+	.423	=	12.661
18	4.486	+	8.208	+	.432	=	13.126
19	4.486	+	8.664	+	.440	=	13.590
20	4.486	+	9.120	+	.448	=	14.054
21	4.486	+	9.576	+	.455	=	14.417
22	4.486	+	10.032	+	.462	=	14.980
23	4.486	+	10.488	+	.468	=	15.442
24	4.486	+	10.944	+	.475	=	15.905
25	4.486	+	11.400	+	.481	=	16.367

TABLE 8  
Y on X

Computations for fitting a straight line to the data

Y on X Straight line fitting.

	na	+	b Sx	=	S y	
	ASx	+	b Sx <sup>2</sup>	=	S xy	
235	[31a	+	235 b	=	208.18]	
21	[35a	+	3495 b	=	2739.25]	
	4935a	+	55225 b	=	48,922.3	
	4935a	+	73395 b	=	57,523.83	
			<hr style="width: 100%; border: 0.5px solid black;"/>			
			18,170 b	=	48,601.63	y = a + bx
			b	=	+ .4734	y = 4.6158 + .4734x

$$\begin{aligned}
 21a + 111.249 &= 208.18 \\
 21a &= 96.931 \\
 a &= 4.6158
 \end{aligned}$$

x	a	+	bx	=	y
1	4.6158	+	.4734	=	5.0892
13	4.6158	+	6.1542	=	10.7700
25	4.6158	+	11.835	=	16.4508

zero's, but merely a lack of cases. Obviously it would be unfair to give these a zero value in the calculations, so that they were left out of the sums of the columns in which they were found and N or the number of Cases became 21 instead of 24. The values, of  $S(\log x)$ ,  $S(x \log x)$ , and  $S(\log x)^2$  then became changed as follows:-

In Pearl's (6-B) Tables of sums of logarithms of consecutive numbers the sums of consecutive x values from 1 to 18 were used. To these were added the log. values of 20, 21 and 24, as shown following:

Sums at 18=

$S(\log x) = 15.8063410$ ;	$S(x \log x) = 179.6863859$ ;	$S(\log x)^2 = 15.9415788$
$\log. 20 = 1.30103$ ;	$(20 \log 20) = 26.02030$ ;	$(\log 20)^2 = 1.6926791$
$\log. 21 = 1.32222$ ;	$(21 \log 21) = 27.76662$ ;	$(\log 21)^2 = 1.7482656$
$\log. 24 = 1.38021$ ;	$(24 \log 24) = 33.12504$ ;	$(\log 24)^2 = 1.9249794$
$S(\log x) \underline{19.809801}$ ;	Total <u>266.5983459</u> ;	Total <u>21.3075031</u>

Thus it will be seen that the sums were taken from the tables as far as the x items went consecutively before encountering a zero. Separate logarithmic values were then calculated for 20, 21 and 24 and these were added to the sums taken from the table at 18. This made a total sum of the values which were actually represented in the data and totally disregarded the zero items, in fitting a third degree logarithmic curve to the observations, the normal equation for the curve is  $y = a + bx + c \log x$ . It therefore remains to find the values for a, b, and c. Equations (1), (2) and (3) in table 2 are solved as ordinary simultaneous

equations the method being shown in detail. The values are found to be  $a = +8.156$ ;  $b = +.916$ ;  $c = -8.141$ , and since  $y = a + bx + c \log x$ , substitution is made as shown in table 3. A calculated Y value is obtained for each of the consecutive x figures from 1 to 24. These Y values are plotted with the x values on cross section paper to give the curve shown in Chart 2.

It is interesting to compare the curve of calculated values with the observed values x and y in Table 1. Hence a plot is made on the same sheet with the curve, showing the corresponding x and y values listed in Table 1. Inasmuch as there are fluctuations between the observed values, this plotted graph will be a zigzag line consisting of peaks and valleys.

As a comparison it is interesting to note whether a calculated straight line will closely fit the curve and actual observations. There is also a partial check on the accuracy of the calculations. In fitting a straight line, the equation is  $y = a + bx$  and the equations are shown in Table 3, for finding a and b. The fitting of a straight line involves much less calculating as there are only two unknowns a and b, and only two equations. It was found that  $a = +5.079$  and  $b = +.506$  and since  $y = a + bx$  the y values for the first x (or 1), the middle x (or 12) and the last x (or 24) are found. The calculated Y value when  $x = 1$  gives one end

of the straight line, and the calculated Y value when  $x = 24$  gives the other end of the straight line. A plot of this line on Chart 3 is shown as the dotted (or broken) lines, which divides the values in the curve and the values in the zig-zag of observed values, in two. In comparing the observed values with the straight line and curve it will be seen that the plotted line of observed values has many peaks and valleys, representing fluctuations from the straight line. The straight line represents the mid values between the fluctuations on either side of the straight line, and these fluctuations are ironed out into the straight line. This may be somewhat compared with a much kinked string, which, when pulled at both ends, follows a straight line.

As shown before, however, this straight line does not adequately describe the situation, and it is readily seen that the curve more nearly follows the trend of the observed values. It will be noted that the base line or co-ordinate is used for the x values in every case. By x values is meant the values in the x column of Table 1, which in this case is yield. However when the curve for x on y is calculated the situation will be reversed. The entries in the x column of Table 5 will be the terminal growth classes and those in the y column will be the means of yield.

Similarly, in plotting the curve, the base line or co-ordinate will be the terminal growth or x classes, and the ordinate or vertical line will represent the calculated yield or y values.

In observing the relative position of the plotted observation, the straight line and the curve in Chart 2, it is readily seen that each goes in the same direction and that the straight line irons out the fluctuations but does not follow the means of the arrays as closely as the curve. Therefore in order to show up the true curvilinear relationship between terminal growth and yield the curve is necessary.

There was considerable fluctuation in the observed data which undoubtedly produced an unfavorable effect on the results. The writer is of the opinion that had the raw data been more selective the correlations would have been higher. Roberts (8) mentions that - - - - "Quite a number of fruits may be borne on terminal spurs but it is seldom that any large percentage of lateral buds set fruits." His observations were made with the Wealthy variety of apple. The York apple, however, has a tendency for the fruits to hang in a ropy fashion toward the ends of the branches, and the yields recorded herein, being nothing more than the lateral and terminal fruits on the one year old terminal wood, were quite heavy on some of the longer growth. It is possible, though, that some of the fluctuations may have been caused by dropping of the lateral bloom.

A calculating machine and Barlow's Table of Squares, Square Roots and Cube Roots as well as a set of Common Logarithms of numbers were found to be of much value in the calculations in this problem.

### Discussion of Results.

As mentioned before the project includes 3 years notes, starting with 1929 and ending with 1931. The notes for each individual year were correlated as shown in Tables 11, 12 and 13. It will be noted that in 1929 all correlations were significant except length and diameter of current seasons terminal growth with the average weight per apple on last year's terminal growth. All correlations were positive except those involving average weight per apple. It might be mentioned here that average weight per apple is simply a measure of size of apple, and that reference from here on will be made to size of apple rather than average weight. Since correlation involving size of apple are negative, it may be said that as length and diameter of the terminal growth becomes greater the size of apple becomes smaller. This is probably due to the fact that as the length of terminal growth becomes greater there is an increase in the number of apples on the terminal growth, and with increased number of apples the size becomes smaller.



In 1930 all correlations were significant except those involving size of apple, and one in which length of 1929 terminal growth was correlated with diameter of 1930 terminal growth. All were positive except one in which length of 1929 terminal growth was correlated with size of apple.

In 1931, all correlations were significant except those involving size of apple, which were negative and insignificant.

Since 1929 and 1931 were the "on years" and 1930 the "off year", it is possible that size of apple was affected by the small number of apples per tree in 1930.

Correlations of terminal growth with average weight per apple in the "on year" were found to be negative, and although these correlations did not show mathematical significance, there was biological significance. This means that with increase in length of terminal growth there is some decrease in size of apple. It was noted in making the observations in the orchard that the shorter terminal growths had a smaller proportion of lateral apples than did the longer growth. Some of the shorter growths had only terminal apples and no lateral fruit. It was noted also that terminal apples were generally larger than lateral fruits, and that the larger lateral fruits were nearer the

TABLE 11---CORRELATION OF DATA FOR THE YEAR 1929---YORK IMPERIAL VARIETY OF APPLE

Factors Involved	Factors	Mean and	Probable error	Standard deviation	Correlation Coefficient(r)	Probable Error of r	Significance of r
Length of 1928 terminal growth with 1929 yield on 1928 terminal growth	Yield	10.3133	± .2158	5.5276	+ .5249	± .0282	18.6 X E
	Length	10.7733	± .1787	4.8589			
Length of 1929 terminal growth with 1928 yield on 1928 terminal growth	Yield	10.3133	± .2158	5.5276	+ .3322	± .0346	9.6 X E
	Length	9.6067	± .1147	2.9437			
Diameter of 1928 terminal growth with 1929 yield on 1928 Term. Gro.	Yield	10.3133	± .2158	5.5276	+ .5246	± .0282	18.6 X E
	Diameter	6.5267	± .0737	1.8927			
Diameter of 1929 terminal growth with 1929 yield on 1928 Ter. Gro.	Yield	10.3133	± .2158	5.5276	+ .3325	± .0346	9.6 X E
	Diameter	4.0933	± .0393	1.0090			
Length of (1928-29 Ave) terminal Gro. with 1929 yield on 1928 T.G.	Combined length	10.42	± .1349	3.4520	+ .5091	± .0288	17.7 X E
	Yield	10.3333	± .2152	5.5248			
Length of 1928 terminal growth with average wt. per apple on 1928 T.G.	Ave. wt. per apple	4.98	± .0712	1.8293	- .3013	± .0354	8.5 X E
	Length	10.7733	± .1787	4.8589			
Length of 1929 terminal growth with ave. wt. per apple on 1928 ter. gro.	Ave. wt. per apple	4.98	± .0712	1.8293	- .0590	± .0388	1.5 X E
	Length	9.6067	± .1147	2.9437			
Diameter of 1928 terminal growth with ave. wt. per apple on 1928 TG.	Diameter	6.5267	± .0737	1.8927	- .2175	± .0371	5.9 X E
	Ave. wt. per apple	4.98	± .0712	1.8293			
Diameter of 1929 terminal growth with ave. wt. per apple on 1928 TG.	Diameter	4.0933	± .0393	1.009	- .0567	± .0388	1.5 X E
	Ave. wt. per apple	4.98	± .0712	1.8293			
Length of (1928-29 ave) ter. growth with av. wt. per apple on 1928 T.G.	ave. wt. per apple	4.98	± .0712	1.8293	- .2309	± .0369	6.3 X E
	Combined length	10.42	± .1349	3.4521			
Length of 1928 Terminal growth with Length of 1929 Terminal growth	Length 1929	9.6067	± .1147	2.9437	+ .5263	± .0282	18.7 X E
	Length 1928	10.7733	± .1787	4.8589			
Diameter of 1928 terminal growth with Diameter of 1929 terminal growth	Diameter 1929	4.0933	± .0393	1.009	+ .6688	± .0235	28.5 X E
	Diameter 1928	6.5267	± .0737	1.8927			
Length of 1928 terminal growth with diameter of 1929 terminal growth	Length	10.7733	± .1787	4.8589	+ .4538	± .0308	14.7 X E
	Diameter	4.0933	± .0393	1.009			
Length of 1928 terminal growth with diameter of 1928 terminal growth	Length	9.6067	± .1147	2.9437	+ .5511	± .0271	20.3 X E
	Diameter	6.5267	± .0737	1.8927			
Length of 1928 terminal growth with Diameter of 1928 terminal growth	Length	10.7733	± .1787	4.8589	+ .7593	± .0165	46. X E
	Diameter	6.5267	± .0737	1.8927			
Length of 1929 terminal growth with diameter of 1929 terminal growth	Diameter	4.0933	± .0393	1.009	+ .4464	± .0312	14.3 X E
	Length	9.6067	± .1147	2.9437			

TABLE 12---CORRELATION OF DATA FOR THE YEAR 1930---YORK IMPERIAL VARIETY OF APPLE

Factors Involved	Factors	Mean and	Probable Error	Standard Deviation	Correlation Coefficient (r)	Probable Error of r	Significance of r
Length of 1929 Terminal growth with 1930 yield on 1929 terminal growth	Yield	6.88	± .215	3.1820	+ .4236	± .0554	7.6 X E
	Length	6.91	± .278	4.1185			
Length of 1930 terminal growth with 1930 yield on 1929 terminal growth	Length	6.04	± .143	2.1256	+ .4945	± .0509	9.7 X E
	Yield	6.88	± .215	3.1820			
Diameter of 1929 terminal growth with 1930 yield on 1929 terminal growth	Diameter	5.48	± .083	1.2286	+ .5494	± .0471	11.7 X E
	Yield	6.88	± .215	3.1820			
Diameter of 1930 term. growth with 1930 yield on 1929 terminal growth	Diameter	3.22	± .035	.5212	+ .3053	± .0612	5. X E
	Yield	6.88	± .215	3.1820			
Length of (1929-30 ave) T.G. with 1930 yield on 1929 terminal growth	Yield	6.88	± .215	3.1820	+ .4569	± .0534	8.6 X E
	Length	6.73	± .184	2.7345			
Length of 1929 terminal growth with ave. wt. per apple on 1929 T.G.	ave. wt. per apple	4.97	± .144	2.1421	- .1205	± .0664	1.8 X E
	Length	6.91	± .278	4.1185			
Length of 1930 terminal growth with ave. wt. per apple on 1929 T. G.	Ave. wt. per apple	4.97	± .159	2.3643	+ .2869	± .0626	4.3 X E
	Length	6.04	± .143	2.1256			
Diameter of 1929 terminal growth ave. wt. per apple on 1929 ter. G	Ave. wt. per apple	4.97	± .159	2.3643	+ .0807	± .0629	1.3 X E
	Diameter	5.48	± .083	1.2286			
Diameter of 1930 Ter. Growth with ave. wt. per apple on 1929 Ter. G	Ave. wt. per apple	4.97	± .159	2.3643	+ .1027	± .0657	1.6 X E
	Diameter	3.22	± .035	.5212			
Length of (1929-30 ave) Ter. Gro. Ave. wt. per apple on 1929 T. G.	Ave. wt. per apple	4.97	± .159	2.3643	+ .0390	± .0673	.6 X E
	Combined length	6.73	± .184	2.7345			
Length of 1929 terminal growth with Length of 1930 terminal growth	Length 1930	6.04	± .143	2.1256	+ .4242	± .0553	7.7 X E
	Length 1929	6.91	± .278	4.1185			
Diameter of 1929 ter. growth with Diameter of 1930 terminal growth	Diameter 1929	5.48	± .083	1.2286	+ .4754	± .0522	9.1 X E
	Diameter 1930	3.22	± .035	.5212			
Length of 1929 terminal growth with diameter of 1930 terminal growth	Diameter	3.22	± .035	.5212	+ .2575	± .0657	3.7 X E
	Length	6.91	± .278	4.1185			
Length of 1930 terminal growth with Diameter of 1929 terminal growth	Diameter	5.48	± .083	1.2286	+ .6321	± .0405	15.6 X E
	Length	6.04	± .143	2.1256			
Length of 1929 terminal growth with diameter of 1929 terminal growth	Diameter	5.48	± .083	1.2286	+ .6410	± .0397	16.2 X E
	Length	6.91	± .278	4.1185			
Length of 1930 terminal growth with diameter of 1930 terminal growth	Diameter	3.22	± .035	.5212	+ .5697	± .0455	12.5 X E
	Length	6.04	± .143	2.1256			

TABLE 13 CORRELATION OF DATA FOR THE YEAR 1931--YORK IMPERIAL VARIETY OF APPLE

tors Involved	Factors	Mean and	Probable Error	Standard Deviations	Coefficient of Correlation (r)	Probable Error (E) of (r)	Significance
f 1930 terminal growth with ld on 1930 terminal growth	Length	7.695	± .169	3.5344	+ .7003	± .0243	28.8 X E
	Yield	9.16	± .239	5.0085			
f 1931 terminal growth with ld on 1930 terminal growth	Length	5.875	± .138	2.8878	+ .5801	± .0316	18.4 X E
	Yield	9.16	± .239	5.0085			
of 1930 terminal growth with ld on 1930 terminal growth	Diameter	4.94	± .062	1.2947	+ .7340	± .022	33.4 X E
	Yield	9.16	± .239	5.0085			
of 1931 terminal growth with ld on 1930 terminal growth	Diameter	3.125	± .034	.7066	+ .4111	± .0396	10.4 X E
	Yield	9.16	± .239	5.0085			
f (1930-31 average) ter. gro. with ld on 1930 terminal growth	Combined length	7.04	± .131	2.7456	+ .7540	± .0206	36.6 X E
	Yield	9.16	± .239	5.0085			
f 1930 terminal growth with per apple on 1930 terminal growth	Length	7.695	± .169	3.5344	- .1538	± .0466	3.3 X E
	Av. wt. per apple	4.155	± .201	4.2080			
f 1931 terminal growth with per apple on 1930 terminal growth	Length	5.875	± .138	2.8878	- .1206	± .0470	2.4 X E
	Av. wt. per apple	4.155	± .201	4.2080			
of 1930 terminal growth with per apple on 1930 terminal growth	Diameter	4.94	± .062	1.2947	- .1577	± .0459	3.4 X E
	Av. wt. per apple	4.155	± .201	4.2080			
of 1931 terminal growth with per apple on 1930 terminal growth	Diameter	3.125	± .034	.7066	- .057	± .0476	1.2 X E
	Av. wt. per apple	4.155	± .201	4.2080			
f (1930-31 ave) terminal growth with per apple on 1930 terminal growth	Combined Length	7.04	± .131	2.7456	- .1636	± .0464	3.5 X E
	Av. wt. per apple	4.155	± .201	4.2080			
f 1930 terminal growth with f 1931 terminal growth	Length 1931	5.875	± .138	2.8878	+ .4420	± .0384	11.5 X E
	Length 1930	7.695	± .169	3.5344			
of 1930 terminal growth with of 1931 terminal growth	Diameter 1930	4.94	± .062	1.2947	+ .6204	± .0293	21.2 X E
	Diameter 1931	3.125	± .034	.7066			
f 1930 terminal growth with of 1931 terminal growth	Diameter	3.125	± .034	.7066	+ .3056	± .0432	7.1 X E
	Length	7.695	± .169	3.5344			
f 1931 terminal growth with of 1930 terminal growth	Diameter	4.94	± .062	1.2947	+ .6929	± .0248	27.9 X E
	Length	5.875	± .138	2.8878			
f 1930 terminal growth with of 1930 terminal growth	Diameter	4.94	± .062	1.2947	+ .7020	± .0241	29.1 X E
	Length	7.695	± .169	3.5344			
f 1931 terminal growth with of 1931 terminal growth	Diameter	3.125	± .034	.7066	+ .6344	± .0285	22.3 X E
	Length	5.875	± .138	2.8878			

TABLE 14--RESULTS OF COMBINED 1929 and 1930 DATA--YORK IMPERIAL VARIETY OF APPLE

Factors Compared		Mean	Probable error	Standard Deviation	Correlation Coefficient r	Probable error of r	Correlation Ratio	Criterion of Linearity	Standard error of error	Significance of
year's terminal growth length with yield on previous year's terminal growth	X Combined Length	8.775	± .218	4.5546	+ .5664	± .0324	$r_{xy} = .6444$	$.0945$	$.0204$	$4.6 X \sigma$
	Y Combined Yield	8.560	± .212	4.4470			$r_{yx} = .6834$	$.1462$	$.0431$	$5.1 X \sigma$
year's terminal growth length with yield on previous year's terminal growth	X Combined Length	7.805	± .147	3.0818	+ .5080	± .0355	$r_{xy} = .6523$	.1674	.0511	3.3 X $\sigma$
	Y Combined Yield	8.560	± .212	4.4471			$r_{yx} = .61$	.114	.0439	2.6 X $\sigma$
year's terminal growth length with ave. wt. per apple on previous years terminal growth	X Com. Ave. wt. per apple	5.105	± .099	2.0795			$r_{xy} = .4425$	.1549	.0266	5.8 X $\sigma$
	Y Length	8.775	± .218	4.5546	- .2023	± .0457	$r_{yx} = .2948$	.0460	.0290	1.6 X $\sigma$
year's terminal growth length with ave. wt. per apple on previous years terminal growth	X Comb. av.wt. per apple	5.105	± .099	2.0795	+ .0219	± .0475	$r_{xy} = .3929$	.1540	.047	3.3 X $\sigma$
	Y Combined Length	7.805	± .147	3.0818			$r_{yx} = .1968$	.0383	.0166	2.3 X $\sigma$

terminal fruits. Since the longer terminal growth had more of these smaller lateral fruits, it seems logical that the average weight per apple should be less than for the shorter growth, which have the larger terminal fruits. This may give a clue to some phase of thinning studies.

In general the following conclusions may be drawn from the analysis of the data.

- 1- Length of terminal growth is associated with yield on one year terminal wood.
- 2- Diameter of terminal growth is associated with yield on one year terminal wood.
- 3- Length of terminal growth is associated with diameter of terminal growth.
- 4- Length of terminal growth in one year is associated with length of terminal growth the following year.
- 5- Diameter of terminal growth in one year is associated with diameter of terminal growth the following year.
- 6- There is biological significance to the relationship between length of terminal growth and size of apple and between diameter of terminal growth and size of apple in the "on year".

These factors are inversely related, in that the greater the growth the smaller the size of apple.

The coefficients of correlation are not mathematically significant but are considered biologically significant because they continue to have the same relationship for both "on years". However in 1930 the "off year", this inverse or negative relation did not hold.

7- The average length of the current season's and last season's terminal growth is associated with yield on one year terminal wood.

When an "on year" (1929) and an "off year" (1930) have their data combined the results given in table 14 are found.

- a- The length of the combined previous seasons terminal growth is associated with the combined yield on one year terminal wood.
- b- The length of the combined current seasons terminal growth is associated with the combined yield on one year terminal wood.
- c- The relation between previous seasons combined terminal growth length and size of apple might be considered significant and is inverse or negative.
- d- Length of current seasons terminal growth has a positive association with size of apple, but the association is insignificant.

In going still farther and examining the curves for the combined two years data it will be noted that the following results are apparent:

- 1- With an increase in length of the previous years terminal growth, there is an increase in the current yield on this last year's terminal growth.
- 2- With an increase in current yield on last years terminal growth, there is an increase in the length of this last years terminal growth.
- 3- As current season's terminal growth increases there is an increase in the current yield on last season's terminal growth.
- 4- As the current yield on last year's terminal growth increases there is an increase in the length of the current year's terminal growth.
- 5- As the length of the previous season's terminal growth increases up to 4 inches, there is an increase in size of apple on this previous season's terminal growth. As the length increases from 4 inches to 25 inches there is a decrease in size of apple. The general tendency is toward a decrease in size of apple as terminal growth increases.
- 6- With an increase in size of apple there is a decrease in the length of the previous season's terminal growth.
- 7- With increase in length of current season's terminal growth up to 3 inches there is also an increase in size of apple on last years terminal growth. However with

a further increase in terminal growth between 3 and 17 inches there is a decrease in size of apple.

- 8- With increase in size of apple there is also an increase in length of current season's terminal growth. These curves give a graphic description of the relationship between the factors involved and need no further explanation.

Charts 3-9 inclusive give these curves, from which the last enumerated conclusions were drawn.

# CHART 2

Two Years Combined

LAST YEARS TERMINAL GROWTH WITH THE  
CURRENT YIELD ON LAST YEARS TERMINAL GROWTH

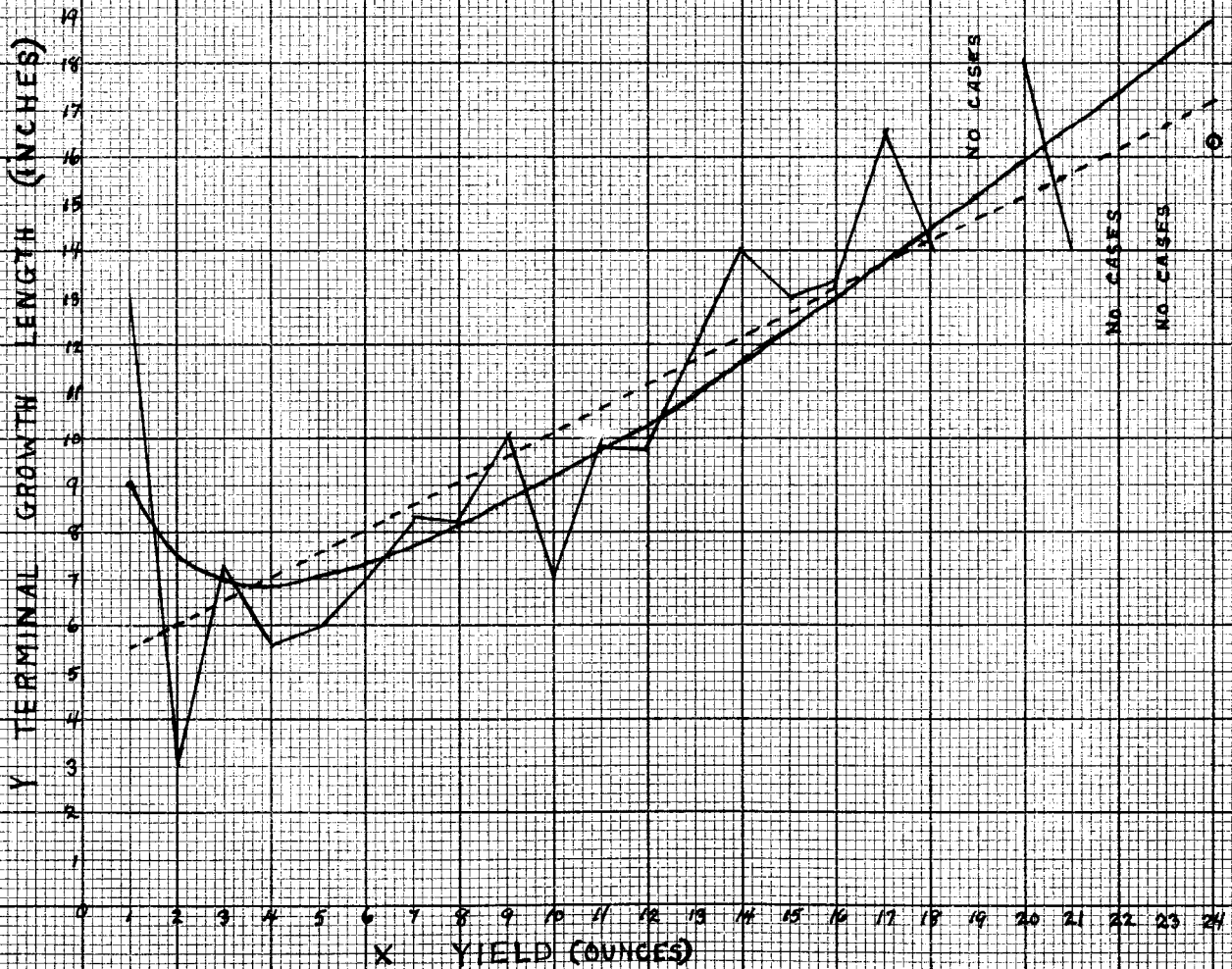
Terminal growth on yield

Plots of

(~) Observed values

(---) Straight line relationship

(~) Curvilinear relationship





# CHART 3

## TWO YEARS COMBINED

### LAST YEARS TERMINAL GROWTH WITH THE CURRENT YIELD ON LAST YEARS TERMINAL GROWTH

Yield on terminal growth

Plots of

- (~~~~) Observed values
- (----) Straight line relationship
- (——) Curvilinear relationship

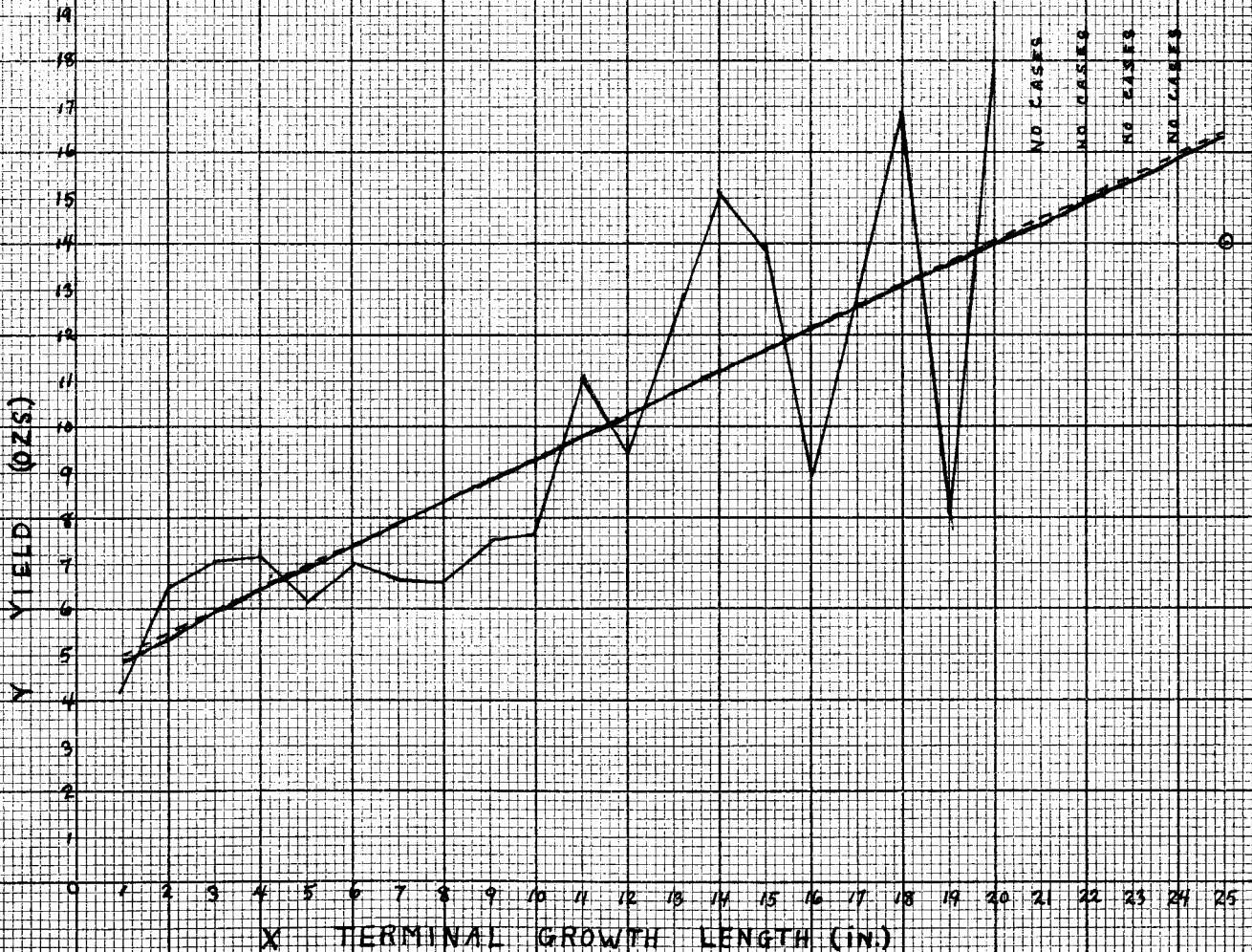


CHART 4

TWO YEARS COMBINED

CURRENT SEASONS TERMINAL GROWTH WITH THE  
CURRENT YIELD ON LAST YEARS TERMINAL GROWTH

Terminal growth on yield

Plots of

- (—) Observed values
- (---) Straight line relationship
- (~) Curvilinear relationship

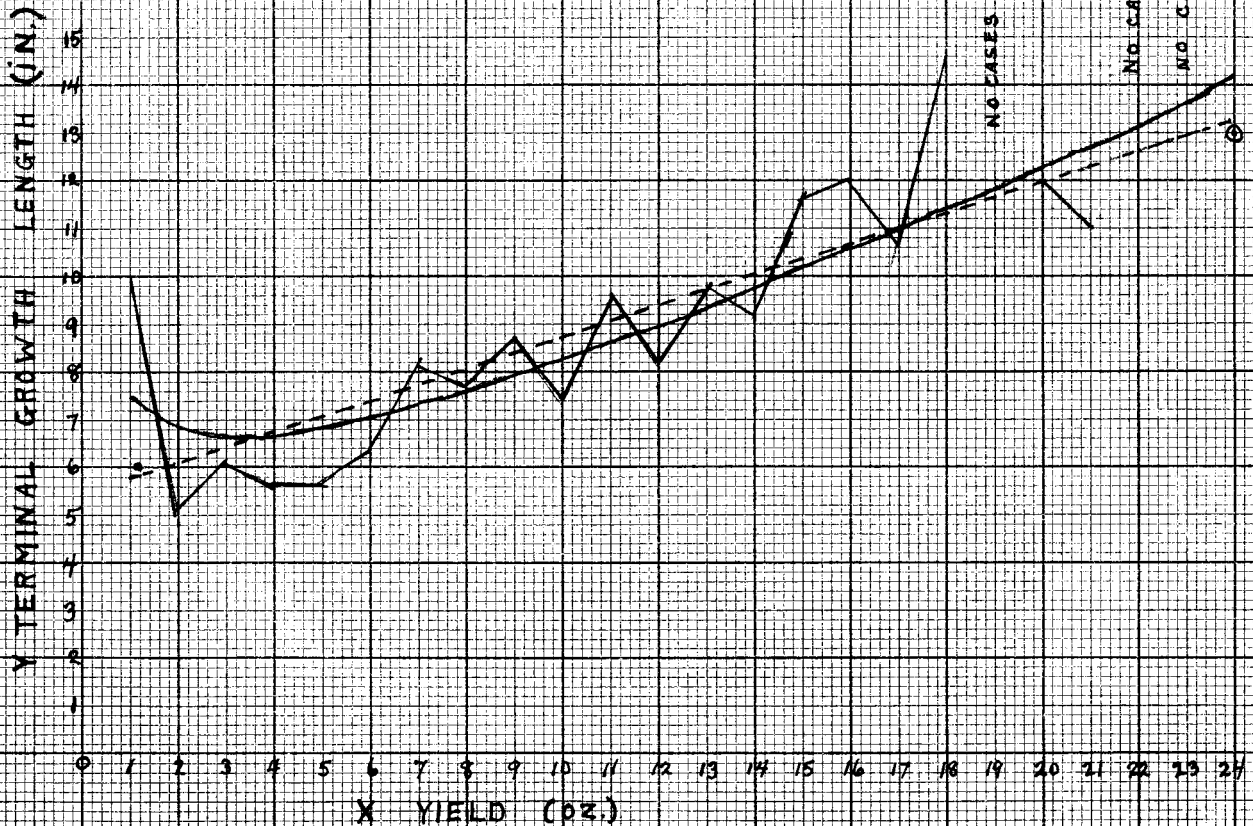


CHART 5

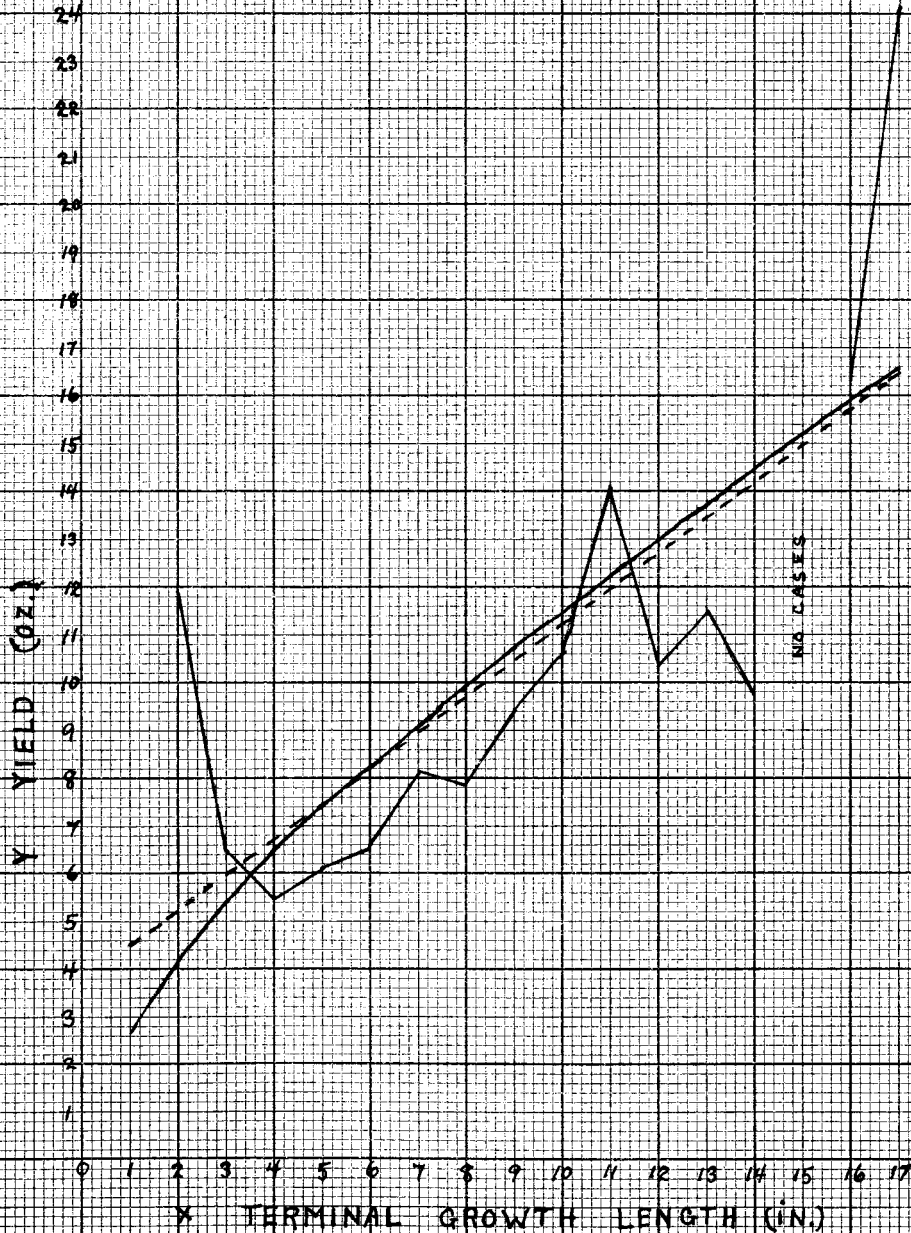
TWO YEARS COMBINED

CURRENT SEASONS TERMINAL GROWTH WITH THE CURRENT  
YIELD ON LAST YEARS TERMINAL GROWTH

Yield on terminal growth

Plots of

- (—) Observed values
- (---) Straight line relationship
- (~) Curvilinear relationship



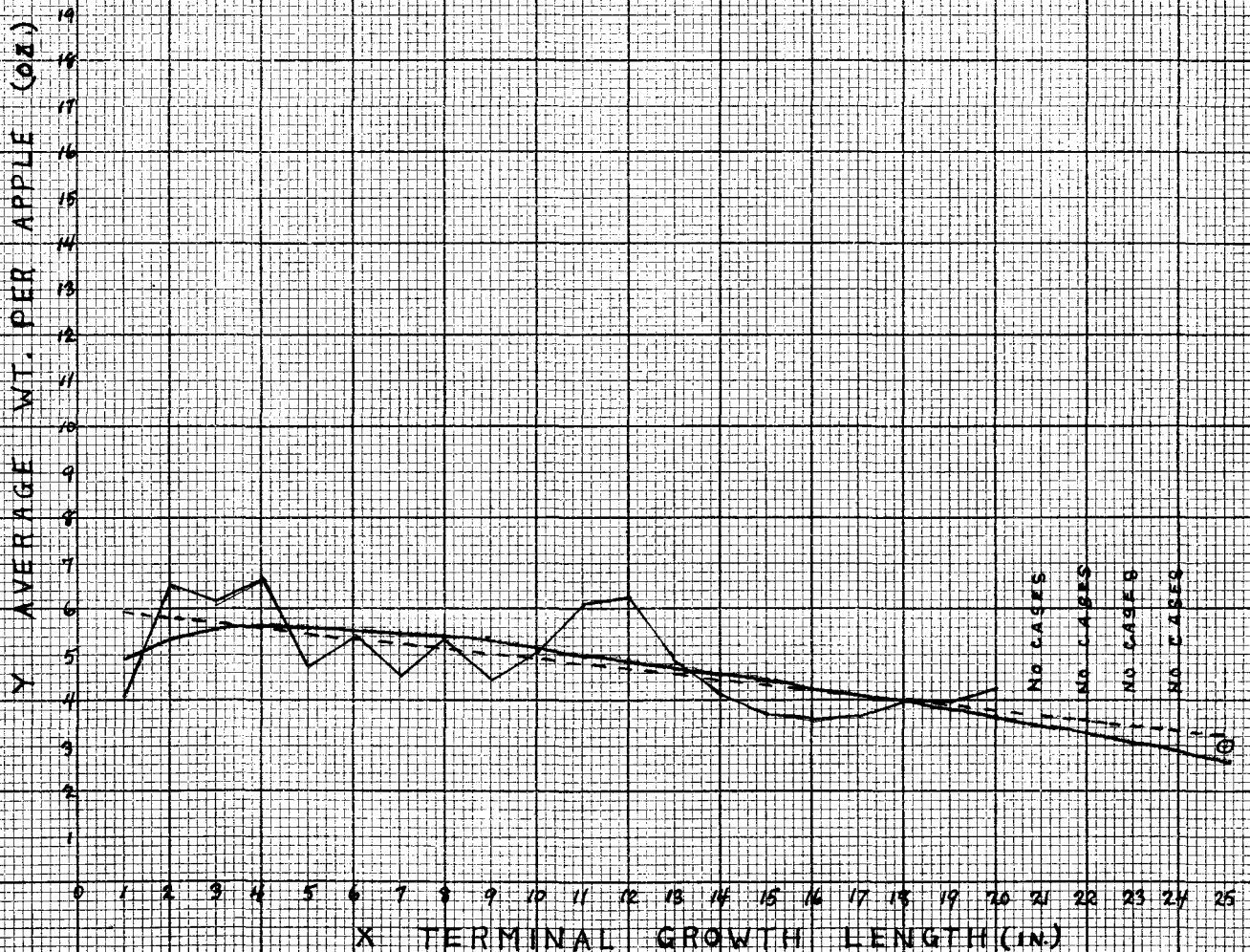
# CHART 6

TWO YEARS COMBINED  
 LAST YEARS TERMINAL GROWTH WITH THE CURRENT AVERAGE  
 WEIGHT PER APPLE ON LAST YEARS TERMINAL GROWTH.

Average weight per apple on terminal growth

Plots of

- (—) Observed values
- (---) Straight line relationship
- (~) Curvilinear relationship



# CHART 7

## TWO YEARS COMBINED

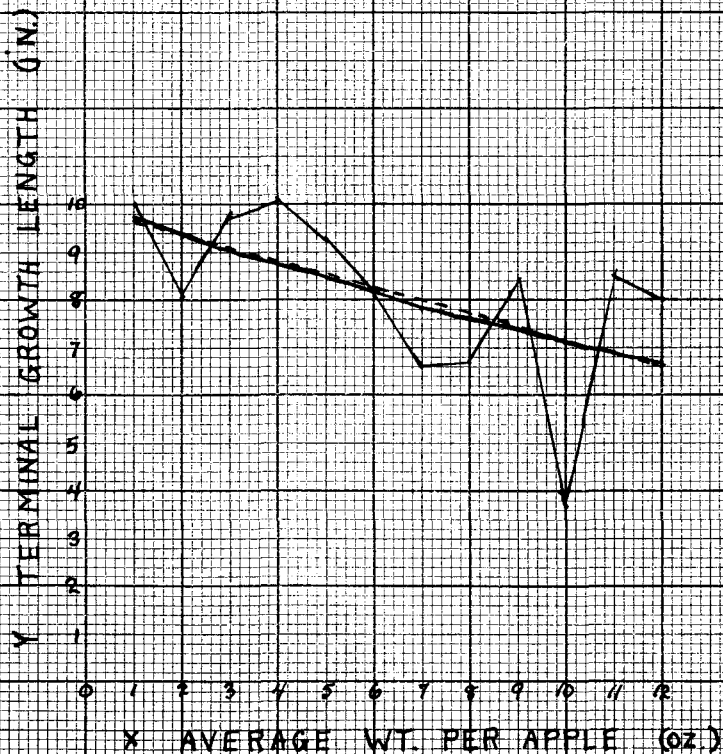
LAST YEARS TERMINAL GROWTH WITH THE CURRENT

AVERAGE WEIGHT PER APPLE ON LAST YEARS TERMINAL GROWTH

Terminal growth on average weight per apple

Plots of

- (—) Observed values
- (---) Straight line relationship
- (—) Curvilinear relationship



Two Years Combined

CURRENT SEASONS TERMINAL GROWTH WITH THE CURRENT  
AVERAGE WEIGHT PER APPLE ON LAST YEARS TERMINAL GROWTH

Average weight per apple on terminal growth

Plots of

- (—) Observed values
- (---) Straight line relationship
- (—) Curvilinear relationship

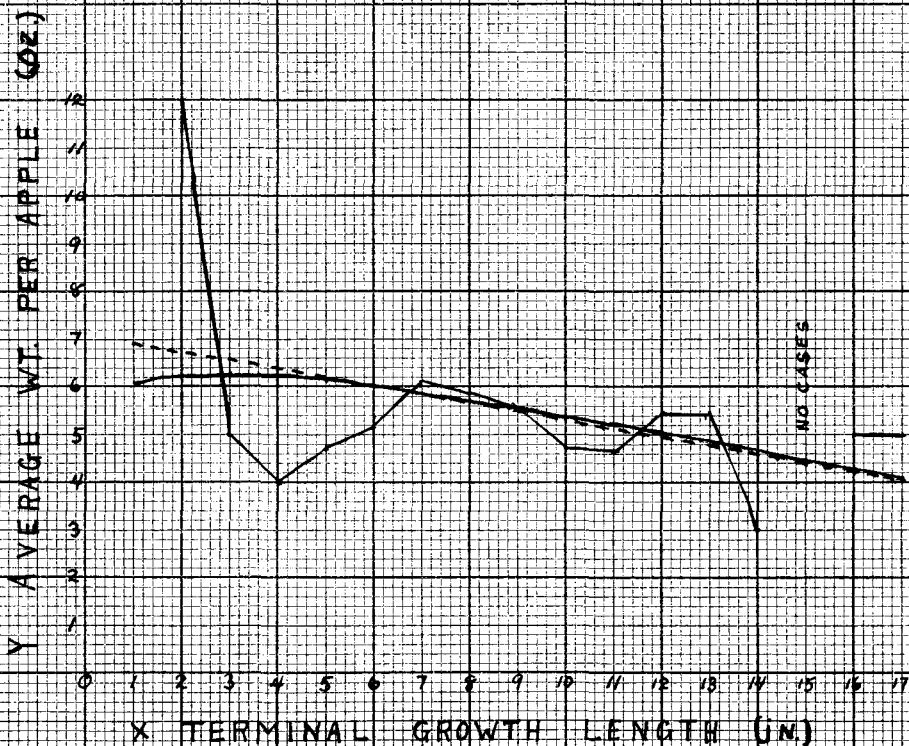


CHART 9

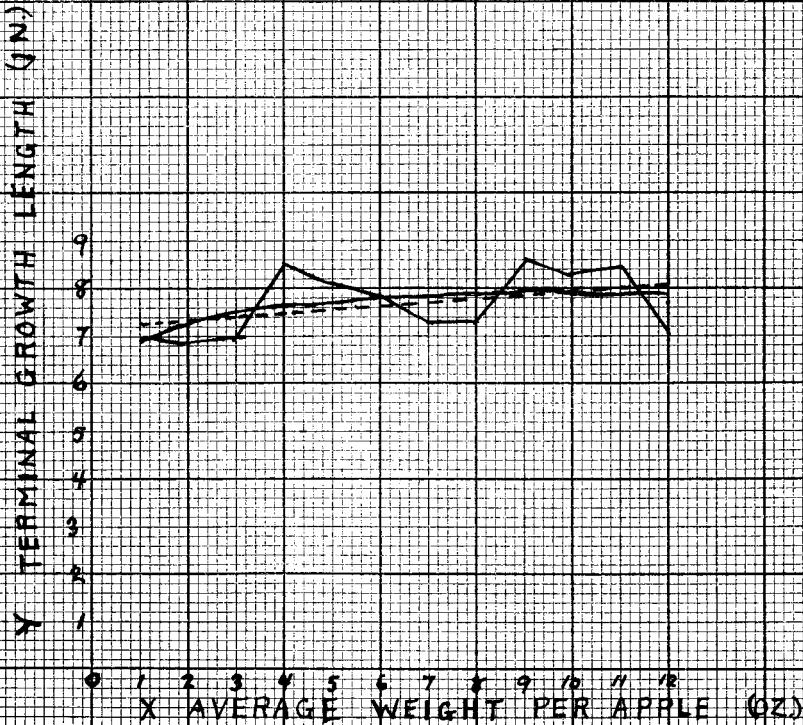
TWO YEARS COMBINED

CURRENT SEASONS TERMINAL GROWTH WITH THE CURRENT  
AVERAGE WEIGHT PER APPLE ON LAST YEARS TERMINAL GROWTH

Terminal growth on average weight per apple

Plots of

- (—) Observed values
- (---) Straight line relationship
- (~) Curvilinear relationship



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