SIMPLIFIED "MOMENT DISTRIBUTION METHOD"

by

Yen Wu

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TABLE OF CONTENTS

FIGURES AND TABLES

CHAPTER 2

NOTATIONS

M_{AB}	End moment of branch (member) AB at joint A
$^{F}M_{AB}$	Fixed end moment at branch AB at joint A
K	Stiffness (Stiff.) = I/L
L	Length of a member
E	Young's Modulus
I	Moment of inertia of a cross-section about the axis of bending
k	Stiffness factor
S	Modified stiffness (M. S.)
s	Modified stiffness factor (M. S. F.)
β	Modified stiffness coefficient (M. S. C.)
C	Modified carry-over factor (M. C. O. F.)
θ_A	Rotation of joint A
R	Relative lateral displacement of branch divided by the length of the branch
τ_{AB}	End shear of branch AB at joint A
M_A	Internal moment
H_A	Internal horizontal force
V_A	Internal vertical force
w	Loading function
Q	Coefficient of the loading function

M_A, H_A, V_A: Reactions at joint A between two "component frames" or between two "part frames"

CHAPTER 3

INTRODUCTION AND REVIEW OF LITERATURE

The analysis of statically indeterminate frames becomes more and more involved as the number of their bays and stories increases. Finite approximations, such as the widely used Hardy-Cross(1) method of moment distribution, are no exceptions from this rule. Many successful attempts have been made during the last thirty years to simplify its procedure. Many modified versions of the conventional moment distribution method were introduced, although their treatment in textbooks is often neglected or incomplete. The author feels that with this thesis he may contribute his small share to the continuous effort of the structural engineer to improve its methods and procedures.

With the development of the computer techniques and the increasing popularity of using the computers for analysis and design, the simplification of methods may seem to have lost some of its significance and challenge. It is true that for very complex frames, with a large number of branches, the computer analysis may be the best approach. One should think, however, of structures which are too small to justify the use of computers on the basis of economy and still sufficiently complex to make the simplification of the conventional methods desirable. Of course, as a final consideration, simplified methods should readily be adaptable also for use on computers and in such a case they may simplify and economize even the programming and automatic computation. Still, the author feels that the major significance of the methods presented herein lays in their simplicity for conventional calculations. For this type of analysis they already proved themselves to be time-saving to a great extent.

The main subjects of the thesis will be presented in three chapters. Chapter four reviews basic definitions of frequently used concepts of the conventional moment distribution method and of one of its simplified versions, named "Synthetic Method" by Professor Yu[2]. It also defines several new concepts pertinent to the development of the theory to the simplified methods presented thereafter.

Chapter five will develop a simplified method applicable to single-bay, multiple-story rigid frames consisting of prismatic members.

Chapter six extends the simplifications to multiple-bay, multiple-story rigid frames also consisting of prismatic members. It develops a new, simplified version of the Hardy-Cross method to the above described frames.

The development of the theories in Chapters five and six will be followed by the presentation of two illustrative examples, one for each of these chapters. The thesis is then concluded by a general discussion in Chapter seven.

CHAPTER 4

DEFINITIONS

This chapter presents the definition of some of the frequently used concepts as they are interpreted throughout the thesis. Furthermore, it aims to clarify the meaning of several other items of the nomenclature which were introduced specifically for this thesis or which are used inconsistently elsewhere in the technical literature. It is hoped that a brief study of this chapter will contribute to a clear understanding of the principles and methods introduced in the following two chapters.

It should be pointed out here that for all definitions involving the Young's modulus, E, it is assumed that E is constant throughout the thesis.

4-1. Superposition. Superposition, as generally defined, refers also here to loading systems only. Two or more loading systems acting simultaneously on the same structure may be superimposed, together with their effects, under certain conditions. These conditions are stated in standard texts and it is assumed throughout this thesis that they are satisfied.

4-2. "Combination" of Structures. In this thesis, two or more structures are said to be combined into an "equivalent" single structure, if their physical characteristics (i.e., stiffnesses) and their loading systems are both integrated into a single system. This can be done only if the configurations of the involved structures are compatible, furthermore, their physical characteristics and the applied loading systems are related to each other in such a manner that all deflections and rotations of every structure involved in this process become identical, as will be shown in chapter 5.

4-3. "Reduction" of a Frame. A hypothetical process which is the inverse of the "combination" of structures, as outlined above.

4-4. "Reduced Frame" or "Subframe". When an arbitrary frame is resolved into a symmetrically stiff frame and, say, a column, the obtained symmetrically stiff frame is sometimes called in this thesis a "reduced frame" or "subframe".

4-5. Symmetrically Stiff Frame. A frame whose geometrical and material properties are symmetrical about its center line is called a symmetrically stiff frame.

4-6. Symmetrical Frame. A frame whose geometrical and material properties are symmetrical about its center line and which is furthermore symmetrically loaded is called a symmetrical frame.

4-7. Antisymmetrical Frame. A frame whose geometrical and material properties are symmetrical about its center line and which is furthermore antisymmetrically loaded is called an antisymmetrical frame.

4-8. Branch. Any member (beam or column) of a frame is called a branch.

4-9. Prismatic Branch. A branch with uniform cross-section is called a prismatic branch.

4-10. Branch End Moment. The moments acting on the ends of the free body diagram of a branch are called branch end moments.

4-11. Joint Branch Moment. Moments applied to the free body diagram of a joint by individual branches which are in fact fixed to this joint are called joint branch moments. If a symbol (M) designates a joint branch moment, it always has two subscripts. The first subscript then refers to the joint and the two subscripts together refer to the specific branch. A joint branch moment is equal in magnitude and opposite in direction to the corresponding branch end moment.

4-12. Stiffnesses: 4EK. Stiffness is the moment at a hinged end of a member necessary to produce a unit rotation of this end while the other end of the member is fixed. For instance for a member ab:

$$4EK_{ab} = M_{ab}\Big|_{\substack{\theta_a = 1 \\ \theta_b = 0 \\ R_{ab} = 0}}$$

where R_{ab} is relative lateral displacement of two ends a and b of a member ab.

<u>4-13. Modified Stiffness: S</u>. The term "Modified Stiffness" applies to several different cases. It may represent the moment which must be applied to a hinged or free end of a member in order to produce a unit rotation of this end while the other end of the member remains hinged or fixed. On the other hand, it also may represent the magnitude of one of two moments which are of equal magnitude and are applied in identical or opposing direction to the hinged ends of a member, in order to produce a unit rotation of both of these ends.

$$S_{ab} = M_{ab}\Big|_{\theta_a = 1}$$

or

$$S_{ab} = M_{ab}\Big|_{\substack{\theta_a = 1 \\ \theta_b = \pm 1 \\ R_{ab} = 0}}$$

For calculation S, a "modified stiffness coefficient", (see Art. 4-14 below), will be determined for each joint branch from Table 1. S may be calculated then by the relation:

$$S = \beta EK.$$

4-14. Modified Stiffness Coefficient: β. Modified stiffness coefficient is the ratio of the modified stiffness of a member to the stiffness of the same member, i.e.

$$\beta_{ab} = S_{ab} / EK_{ab}$$

may be determined for each joint branch by the use of Table 1.

4-15. Stiffness Factor: k. Stiffness factors are the relative stiffnesses of two or more members which are connected. Notice that k is a pure number, not containing the EK factor and it should be the lowest possible integer, i.e.

$$k_{ab} = K_{ab} / (H.C.F.),$$

where (H.C.F.) is the highest common factor of the stiffnesses of all branches at joint "a". This "stiffness factor" is used in the conventional moment distribution method, but will not be used in this thesis. It is shown only to compare it with the "Modified stiffness factor" shown in Art. 4-16 below.

4-16. Modified Stiffness Factor: s. Modified stiffness factors are the relative stiffnesses of two or more members which are connected at the same joint. Notice that s is a pure number, not containing the EK factor and it should be the lowest possible integer, i.e.

$$s_{ab} = S_{ab} / (\text{H.C.F.})$$

where (H.C.F.) is the highest common factor of the stiffnesses of all branches at joint "a".

4-17. Carry-over Factor: C.O.F. If the hinged end of a member is rotated while the other end is fixed, the ratio of the moment at the fixed end to the moment producing rotation at the hinged end is called the carry-over factor.

$$r_{ab} = M_{ba} / M_{ab} \Big|_{\substack{\theta_b = 0 \\ R_{ab} = 0}}$$

This conventional carry-over factor is not used in this thesis.

4-18. Modified Carry-over Factor: M.C.O.F. or MCOF or C. The term "modified carry-over factor" applies to several cases. Consider a member AB which is hinged or free at the end A while its other end B is fixed or hinged. If now end A is rotated, the ratio of the moment at end B to the moment at end A is herein called the modified carry-over factor. On the other hand, if two moments of equal magnitude are applied in identical or opposing directions to the two hinged ends of a member AB, the modified carry-over factor is equal to zero (see Table 1).

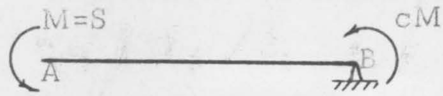
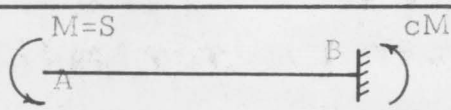
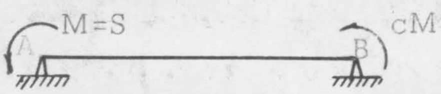
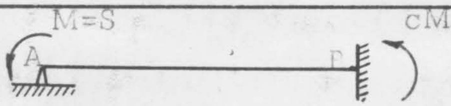
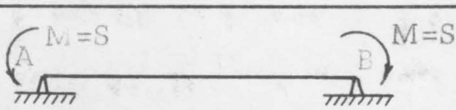
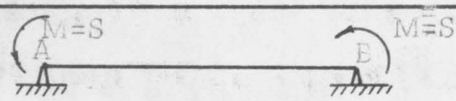
$$C_{ab} = \frac{M_{ba}}{M_{ab}}$$

or

$$C_{ab} = 0 \Big|_{\substack{\theta_a = \pm\theta_b \\ R_{ab} = 0}}$$

4-19. Remarks. The definitions of Art. 4-12, 15 and 17 were introduced by Professor Hardy Cross[1] in 1931. The definitions of Art. 4-13, 14, 16, and 18 were adopted by Professor Yu[2] in 1956.

TABLE 1 for $\theta_A = 1$

case	Beam	$\beta_{AB} = S_{AB}/EK$	C_{AB}	Remark
1		0	0	
2		1	-1	
3		3	0	$\beta_{AB} = \beta_{BA}$
4		4	1/2	
5		2	0	$\beta_{AB} = \beta_{BA}$
6		6	0	$\beta_{AB} = \beta_{BA}$

CHAPTER 5

SIMPLIFIED "MOMENT DISTRIBUTION" METHOD FOR SINGLE-BAY FRAMES

5-1. Basic Theory. Consider two beams A'B' and A"B" of equal length and each having a uniform cross-section as shown in Figure 1. If their elastic curves coincide under certain loads then these two beams can be combined into a single beam AB which has the same elastic curve as either beam A'B' or beam A"B". This can be accomplished by adding the two loads and adding the two stiffnesses. By an inverse process, a single beam can be also resolved into two "component" beams which have the same elastic curve as the original. It will be shown below that linear relationships can be set up between loads, reactions, and stiffnesses of the two individual beams and of the equivalent single beam, if all three have identical deflection curve.

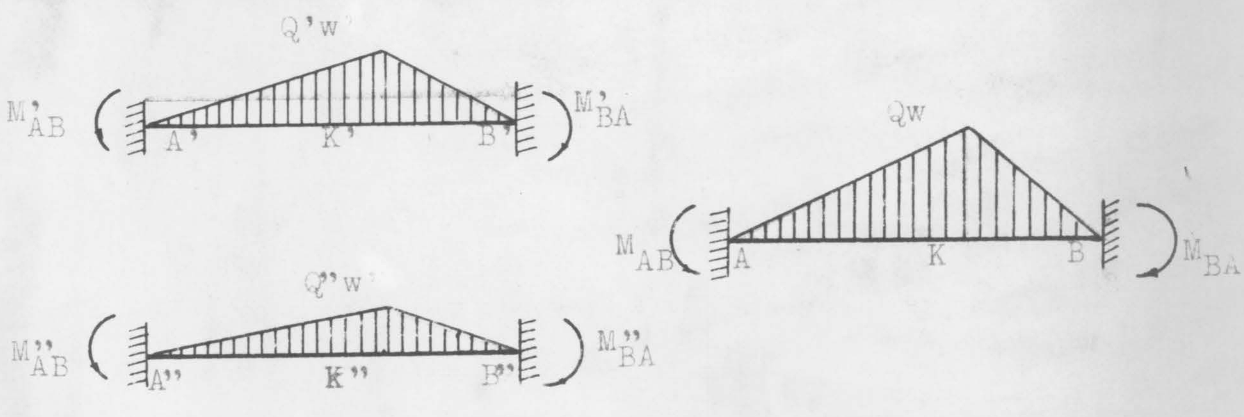


Fig. 1

As an example, let us consider three "fixed" beams, A'B', A"B", and AB loaded by Q'w, Q"w, and Qw respectively, where Q', Q", and Q are arbitrary scaling constants and w is an arbitrary loading function of x, as shown in Figure 1. Define the stiffness of beams A'B', A"B", and AB as K', K", and K respectively. Let us impose now the conditions that the loading constants Q', Q", and Q are related to the stiffnesses K', K", and K in the following manner:

$$Q' : Q'' : Q = K' : K'' : K \qquad (1)$$

and that all three beams have identical deflection curves. Then, the ratios of the fixed and moments at the left support are equal to the ratios of the fixed and moments at the right support, furthermore to the ratios of the stiffnesses, and also to the ratios of the loading constants (equation 1) of the three beams, i.e.,

$$\begin{aligned} F^{M'}_{AB} : F^{M''}_{AB} : F^{M}_{AB} &= F^{M'}_{AB} : F^{M''}_{BA} : F^{M}_{BA} \\ &= K' : K'' : K \qquad (2) \\ &= Q' : Q'' : Q \,. \end{aligned}$$

The total end moments of the members can be expressed by the slope deflection equations as follows:

$$
\begin{aligned}
M'_{AB} &= 2EK'(\ 2\theta'_A + \theta'_B - 3R'\ ) + F^{M'}AB \\
M'_{BA} &= 2EK'(\ 2\theta'_B + \theta'_A - 3R'\ ) + F^{M'}BA \\
M''_{AB} &= 2EK''(\ 2\theta''_A + \theta''_B - 3R''\ ) + F^{M''}AB \\
M''_{BA} &= 2EK''(\ 2\theta''_B + \theta''_A - 3R''\ ) + F^{M''}BA \\
M_{AB} &= 2EK\ (\ 2\theta_A + \theta_B - 3R\ ) + F^{M}AB \\
M_{BA} &= 2EK\ (\ 2\theta_B + \theta_A - 3R\ ) + F^{M}BA
\end{aligned}
\qquad (3)
$$

where θ is the rotation of a cross-section and R is the relative lateral displacement per unit length of the beam between the two ends. However, by hypethesis, the deflection curves are identical, therefore,

$$
\begin{aligned}
\theta'_A &= \theta''_A = \theta_A \\
\theta'_B &= \theta''_B = \theta_B \\
R' &= R'' = R \quad .
\end{aligned}
$$

Hence, M''_{AB} can be expressed in terms of M'_{AB} and the stiffnesses as follows:

$$
\begin{aligned}
M''_{AB} &= 2EK''(\ 2\theta''_A + \theta''_B - 3R''\ ) + F^{M''}AB \\
&= 2EK'\frac{K''}{K'}\ (\ 2\theta'_A + \theta'_B - 3R'\ ) + F^{M'}{}_{AB}\frac{K''}{K'} \\
&= [2EK'\ (\ 2\theta'_A + \theta'_B - 3R'\ ) + F^{M'}AB\ ]\ \frac{K''}{K'} \quad ,
\end{aligned}
$$

or $$M''_{AB} = M'_{AB}\frac{K''}{K'} \ .$$

Similarly, we have

$$M''_{BA} = M_{BA}\frac{K''}{K}$$

and $$M''_{BA} = M'_{BA} \frac{K''}{K'}$$

$$M''_{AB} = M_{AB} \frac{K''}{K}$$

or $$M'_{AB} : M''_{AB} : M_{AB} = M'_{BA} : M''_{BA} : M_{BA} \qquad (4)$$

$$= K' : K'' : K.$$

The above relations indicate that the end moments on beams A'B', A"B", and AB are proportional to the corresponding stiffnesses of the beams.

Also, since the stiffness factors are proportional to the stiffnesses all corresponding loads on the three beams must be proportional to the stiffness factors. For instance:

$$M'_{AB} : M''_{AB} : M_{AB} = k' : k'' : k \qquad (5)$$

where k is defined in Art. 4-15.

Looking at Figure 1, let us combine now beams A'B' and A"B" in order to obtain an equivalent single beam AB, by imposing the following conditions:

$$Q' + Q'' = Q$$
$$K' + K'' = K. \qquad (6)$$

Since fixed end moments are always proportional to the loads, the following relationships will exist:

$${}^{F}M'_{AB} + {}^{F}M''_{AB} = {}^{F}M_{AB}$$

and $${}^{F}M'_{BA} + {}^{F}M''_{BA} = {}^{F}M_{BA}. \qquad (7)$$

Furthermore, substituting Equation 4 into Equation 6:

$$
\begin{aligned}
M'_{AB} + M''_{AB} &= M_{AB} \\
M'_{BA} + M''_{BA} &= M_{BA}.
\end{aligned}
\tag{8}
$$

Looking at Figure 2a, consider now a beam AB as a branch of an arbitrary frame. Define K as the stiffness of beam AB. Applying the above developed principle, beam AB may be resolved into two component beams A'B' and A"B", as indicated in Figures 2a and 2b. These figures show three free body diagrams, each consisting of a beam, its end joints, and a short part of all other branches connected to these joints. In order to maintain equilibrium on the free body diagrams of Figure 2b, it is assumed that equal and opposite reactions (moments M_A and M_B and forces V_A, V_B, and H_A) are acting between the corresponding joints of these two component structures. By adding the stiffnesses and all loads of these two component beams, the equivalent beam of Figure 2a will be obtained.

The end moments of the two component beams of Figure 2b can be expressed now in the following ways:

$$
\begin{aligned}
M'_{AB} &= M_{AC} + M_A \quad \text{and} \quad M'_{BA} = M_{BE} + M_B \\
M''_{AB} &= M_{AB} \mp M_A \quad \text{and} \quad M''_{BA} = M_{BF} - M_B
\end{aligned}
\tag{9}
$$

where M_A and M_B are unknown moments.

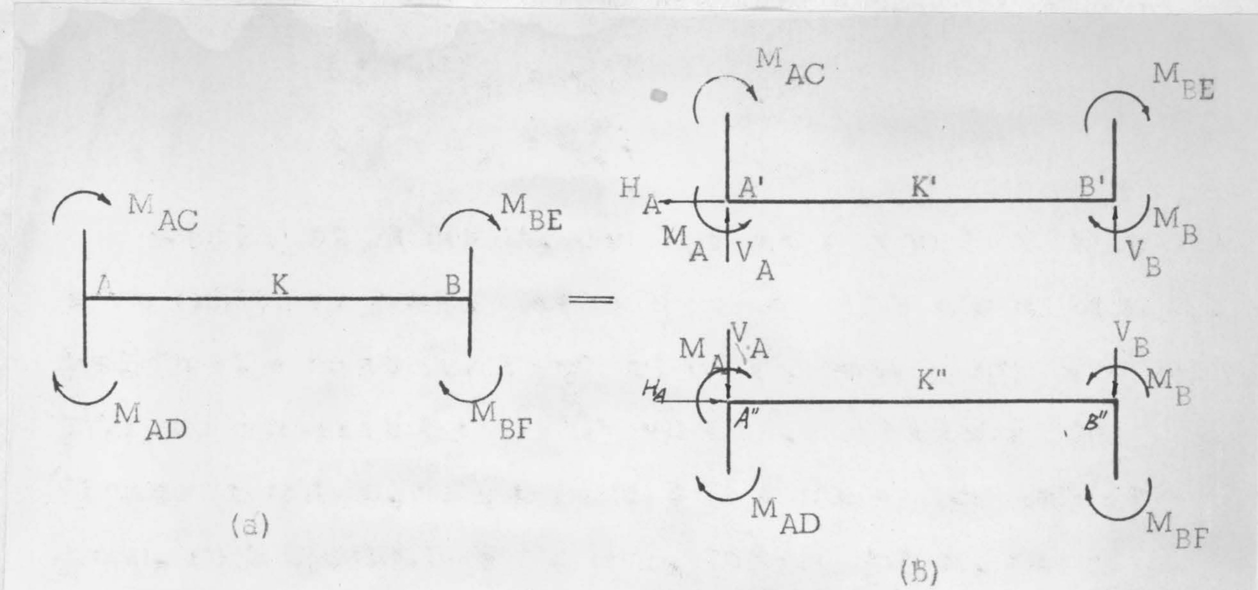


Fig. 2

By hypothesis, beam A'B' and A"B" have the same elastic curve, therefore, according to Equation 4 the following relations apply:

$$M'_{AB} : M''_{AB} = M'_{BA} : M''_{BA} = K' : K''. \qquad (10)$$

Substituting Equations 9 into Equation 10:

$$(M_{Ac} + M_A) : (M_{AD} - M_A) = K' : K''$$
$$(M_{BE} + M_B) : (M_{BF} - M_B) = K' : K'' .$$

and finally the unknown moments M_a and M_b may be expressed as

$$M_A = \frac{K''M_{AC} - K'M_{AD}}{K' + K''}$$
$$(11)$$
$$M_B = \frac{K''M_{BE} - K'M_{BF}}{K + K''} \quad .$$

M_A and M_B of Equation 11 are internal joint moments, which will occur externally at the ends of the component branches whenever a branch is resolved into two components.

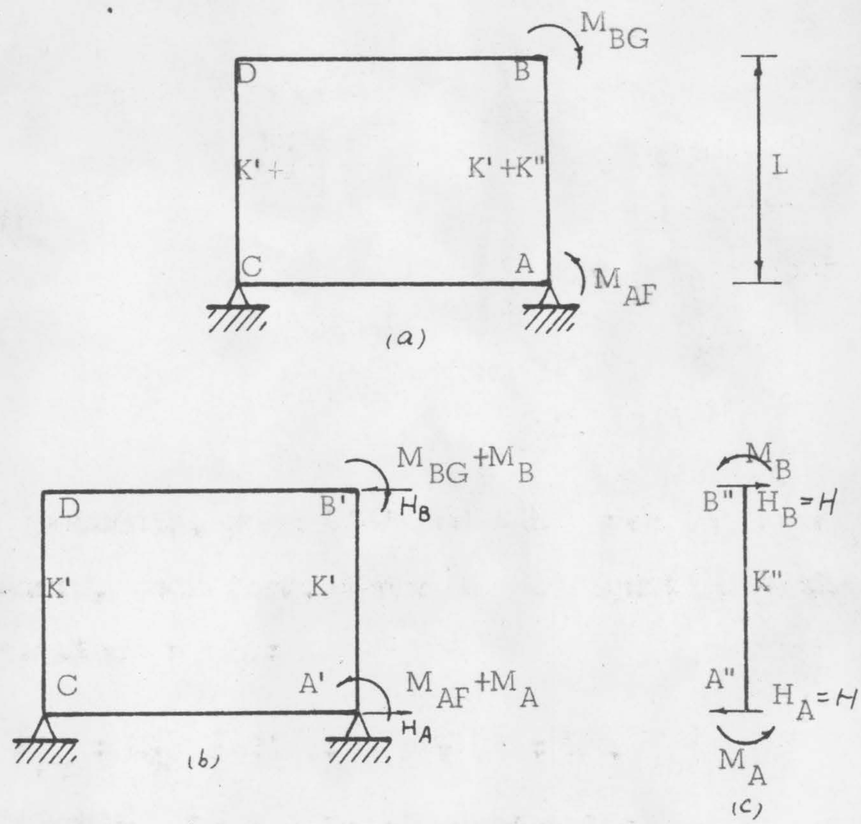


Fig. 3

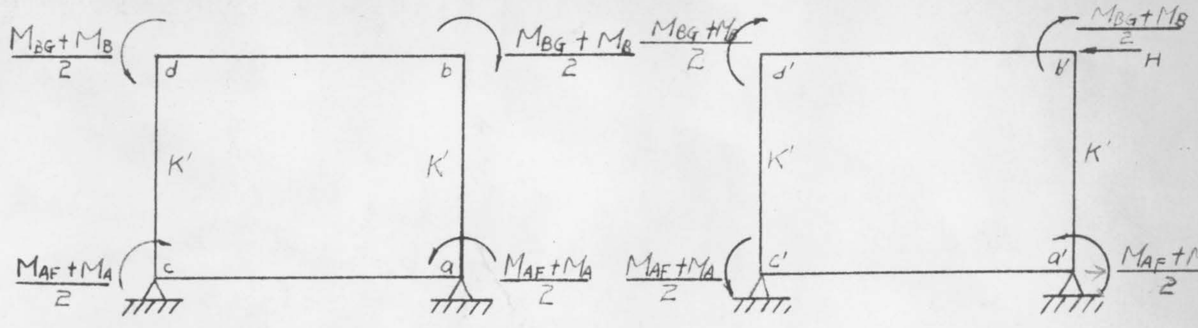


Fig. 4

5-2. Extension of the Basic Theory to Frames. Let us extend now these principles to a closed frame ABCD loaded by two moments M_{BG} and M_{AF}, and supported as shown in Figure 3a. Consider Figure 3b, where internal moments M_A and M_B and internal forces H_A and H_B are developed by M_{BG} and M_{AF} as internal reactions between the joints A' and B' of the reduced frame A'B'CD (which also carries the originally applied moments) and joints A" and B" of the otherwise unloaded separated column A"B". Then the loads on reduced frame A'B'CD, as shown in Figure 3, will be considered as the superposition of two separated loading systems applied to the same reduced frame as shown in Figure 4a and 4b. Both a'b'c'd' and abcd designate in Figure 4 the reduced frame A'B'CD of Figure 3a and the notations a'b'c'd' and abcd refer actually to the two separated loading systems applied to this reduced frame. In this process of separation, moments M_A and M_B will be divided equally between joints A' and A", and B' and B" respectively. Furthermore, opposite moments of equal magnitudes will be applied at joints c, c', and d, d' respectively in order to transform a'b'c'd' and abcd into a symmetrical and antisymmetrical frame, as illustrated in Figure 4. H_A and H_B will not be resolved but are applied in their original magnitudes to the antisymmetrical frame.

The original frame ABCD is now represented by the reduced, symmetrically stiff frame A'B'CD and "subcolumn" A"B", while the original loads M_{AF} and M_{BG} are obtained by superimposing the two separated loading systems a'b'c'd' and abcd of the subframe and the loads on the subcolumn A"B".

The magnitudes of the branch end moments of systems a'b'c'd' and abcd can be determined now by a simplified method granted by symmetry and antisymmetry. It is apparent that these branch end moments in the symmetrically loaded frame will appear in terms of M_A and M_B while in the antisymmetrically loaded frames then they will be expressed as functions of M_A and M_B and H_A and H_B.

It is now apparent from Figure 3c that $H_A = H_B = H$ and from Figure 5 that the sum of the shears of columns CD and A'B' is also equal to H, i.e.

$$\tau'_{AB} + \tau_{CD} = H \; .$$

Figure 3b is the superposition of Figures 4a and 4b. Consequently, comparing Figure 5b, which shows the columns of Figure 3b, and Figure 4,

$$\tau'_{AB} + \tau_{CD} = \tau_{ab} + \tau_{cd} + \tau'_{ab} + \tau'_{cd} ,$$

where the right hand side represents the shears in the vertical branches of Figure 4. But, for the symmetrical frame of Figure 4a,

$$\tau_{ab} + \tau_{cd} = 0.$$

Hence $\qquad \tau'_{ab} + \tau'_{cd} = \tau'_{AB} + \tau_{CD}$

or $\qquad \tau'_{ab} + \tau'_{cd} = H.$

Furthermore, equilibrium requires that on Figure 5b

$$M'_{AB} + M_{CD} + M'_{BA} + M_{DC} = -HL$$

and by superposition

$$M'_{AB} + M_{CD} + M'_{BA} + M_{DC} = M_{ab} + M_{cd} + M_{ba} + M_{dc} + M'_{ab} + M'_{cd} + M'_{ba} + M'_{dc} \; .$$

Consequently,

$$M_{ab} + M_{cd} + M_{ba} + M_{dc} + M'_{ab} + M'_{cd} + M'_{ba} + M'_{dc} = -HL$$

But, by symmetry,

$$M_{ab} + M_{cd} = 0$$

and $$M_{ba} + M_{dc} = 0\ ,$$

therefore,

$$M'_{ab} + M'_{cd} + M'_{ba} + M'_{dc} = -HL.$$

Furthermore, by antisymmetry

$$M'_{ab} = M'_{cd}$$

and $$M'_{ba} = M'_{dc}.$$

Therefore,

$$M'_{ab} + M'_{ba} = M'_{cd} + M'_{dc} = \frac{1}{2}HL. \tag{12}$$

From Figure 3c

$$M''_{AB} + M''_{BA} = HL.$$

However, by superposition

$$M'_{AB} + M'_{BA} = M_{ab} + M'_{ab} + M'_{ba} + M_{ba}$$

or, by Equation 12

$$M'_{AB} + M'_{BA} = M_{ab} + M_{ba} - \frac{1}{2}HL.$$

Furthermore, by the basic theory, Art. 5-1,

$$M'_{AB} + M'_{BA} = \frac{K'}{K''}(M''_{AB} + M''_{BA})$$
$$= \frac{K'}{K''}(\ HL\ ).$$

Therefore,

$$M_{ab} + M_{ba} - \frac{1}{2}HL = \frac{K'}{K''}(\ HL\ )$$

so $$M_{ab} + M_{ba} = \frac{1}{2} HL + \frac{K'}{K''} HL$$

or $$HL = \frac{2K''}{2K'+K''}(M_{ab} + M_{ba}). \tag{13}$$

Now, since H is expressed in terms of the moments of the symmetrical frame only, one unknown remains only for each story, namely the moments between the subframe and the subcolumn, M_A and M_B.

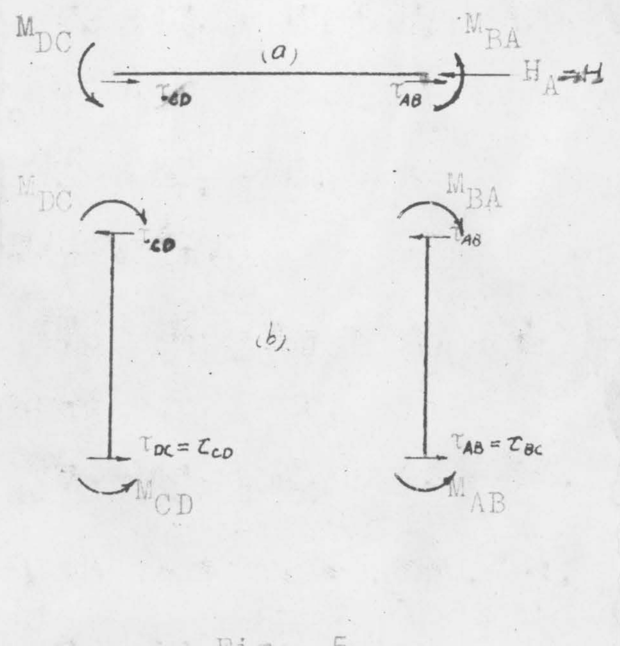


Fig. 5

5-3. Illustrative Example. Given a single-bay three-story rigid frame, consisting of prismatic branches, and loaded by three horizontal forces as shown in Figure 6. Find the M-diagram.

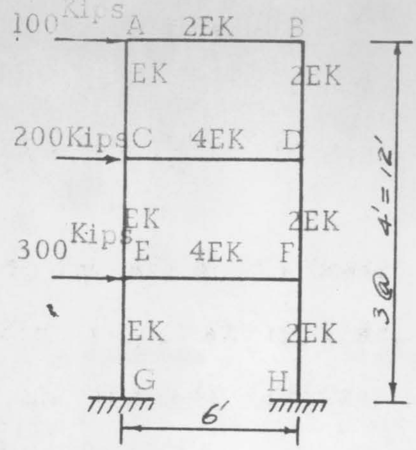


Fig. 6

Solution: In order to analyze this frame by a simplified moment distribution method, we resolve the frame into a "symmetrically stiff", "reduced" frame and a column, as shown in Figure 7. The loading system of the reduced frame is then resolved into a symmetrical and an antisymmetrical system, as shown in Figure 8. Thus, the original frame may be expressed as the superposition of a symmetrical and an antisymmetrical frame and then the combination of this superimposed frame with a column, according to the principles developed earlier in this article.

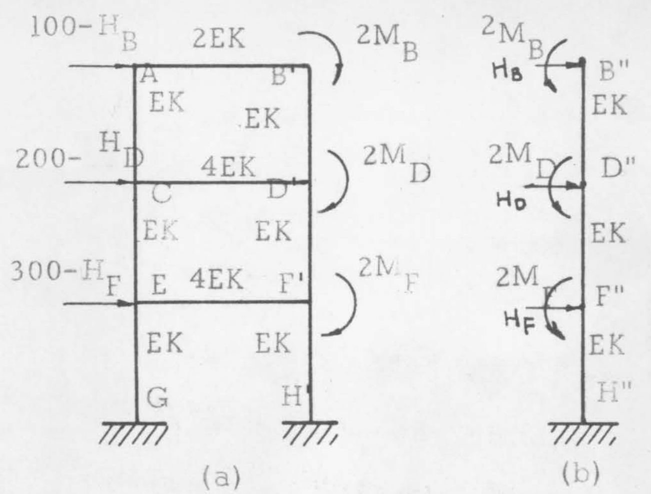


Fig. 7

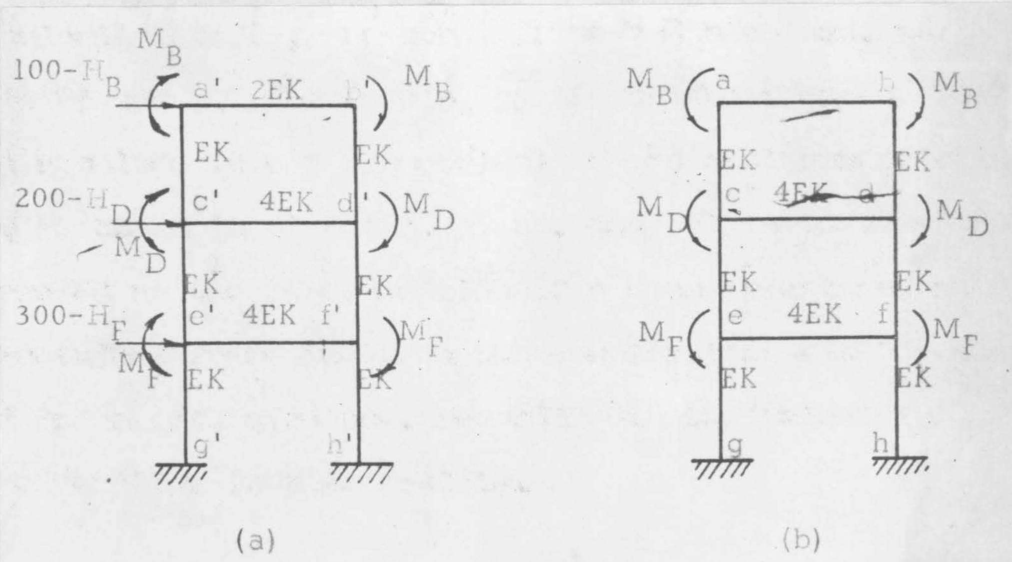


Fig. 8

First, we consider the analysis of the symmetrical frame. The modified stiffnesses and modified carry-over factors of its branches can be found from Table 1.

For example, in Figure 8b, due to symmetrical loading, the end a of the branch ac has no relative displacement with respect to end c. Joint a is rotated by a moment applied to a while joint c remains fixed. Furthermore, the deflection of beam ab is symmetrical. So, the needed properties of the branches meeting at joint a can be determined as follows:

TABLE 2

Branch	ac	ab
Case	4	5
Stiffness	EK	2EK
M.S.C. (β)	4	2
M.S. (S)	4EK	4EK
M.S.F. (s)	1	1
M.C.O. (C)	1/2	0

By similar procedure, all the modified constants can be determined for the left half of the frame.

Let us now cut the frame along its center line which is the line of symmetry. Because of symmetry only a half of the frame, say the left half, needs to be analyzed. The half frame considered in this example is shown in Figure 9. Notice that because of symmetry the slope of the branches at the center line must be zero, therefore, we may introduce fixed ends there for the branches of the half frame. The final moments in the right half frame will be equal in magnitude and opposite in direction to those obtained in this analysis for the left half.

The only loads applied to this half frame are the moments applied to the joints as shown in Figure 9. Moment distribution can be carried out now on this half frame. The calculations are shown in Table 3.

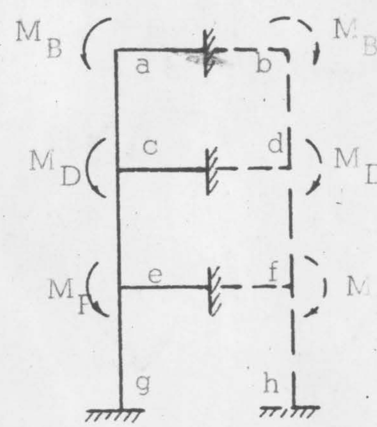


Fig. 9

In Table 3, the second line lists the joints and the third line lists the branches of the half frame as needed for the moment distribution. From the fourth to the seventh line, the calculation of the modified stiffness factors is shown, and the eighth line lists the modified carry-over factors. The calculation of these constants was illustrated in Table 2. In lines nine through eleven the "moment distribution" is carried out. This procedure is different than the conventional moment distribution and is explained below.

Lines twelve states the equation to be solved for the unknown moment X. Lines thirteen through sixteen list the coefficients of the applied moments (M_B, M_D, and M_F) for the equations expressing the branch moments (M_{ab}, etc.) in terms of these applied moments.

The moment distribution (lines 9 through 11) in Table 3 is carried out by the following procedure.

First we consider the joint <u>a</u> of this frame. Since the modified stiffness factors of branches <u>ab</u> and <u>ac</u> are both equal to unity, the "distribution" moments of the branches <u>ab</u> and <u>ac</u> are equal to each other and are called X, where X is an arbitrary unknown moment. Since the carry-over moment in branch ab (col. 5) is equal to zero, the total moment of this branch (line 12) is also equal to X. For equilibrium, the sum of the total moments (line 12) of branches <u>ab</u> and <u>ac</u> and the external moment applied at joint <u>a</u> should vanish. I.e.

$$M_{ac} + X + M_B = 0$$

Therefore, $$M_{ac} = - (X + M_B),$$

a relation stated in line 12 of column 6. Adding vertically in column 6, line 12 must be equal to the sum of lines 10 and 11. Therefore, the distributed moment of branch ac (Col. 6) must be equal to $-2X-M_B$, as indicated in line 11 of column 6.

Now we consider the joint c. The carry-over factor of branch ca (col. 7) is equal to 1/2, and the moment which was "carried-over" from column 7 to the branch ac (col. 6) was equal to $-2X-M_B$. But since this carry-over moment is equal to the distributed moment of column 7 times the carry-over factor of col. 7, therefore, the distributed moment of col. 7 can be found as follows:

$$(\text{C.O.M.})_{ac} = (\text{C.O.F.})_{ca} \times (\text{D.M.})_{ca} \tag{14}$$

or $$(\text{D.M.})_{ca} = (\text{C.O.M.})_{ac} / (\text{C.O.F.})_{ca} \ , \tag{15}$$

But $(\text{C.O.M.})_{ac} = -2X - M_B$

and $(\text{C.O.F.})_{ca} = 1/2$

thus $(\text{D.M.})_{ca} = 2(-2X - 2M_B)$

$= -4X-2M_B$,

which is the value to be listed in column 7.

All the other distributed moments in line 9 of branches ca, cd, and ce can be found from $(\text{D.M.})_{ca}$ in the ratios of the modified stiffness factors, S, i.e.

$$(\text{D.M.})_{ca} : (\text{D.M.})_{cd} : (\text{D.M.})_{ce} = S_{ca} : S_{cd} : S_{ce}$$

But it was shown above that $(\text{D.M.})_{ca} = -4X - 2M_B$, therefore

$(\text{D.M.})_{cd} = -4M_B-8X$ . . . col. 8 } in line 9

$(\text{D.M.})_{ce} = -2M_B-4X$ . . . col. 10 }

The carry-over moment (line 11) of branch ca (col. 7) is equal to the product of the distributed moment (line 12) of ac (col. 6) and the carry-over factor (line 8) of ac (col. 6). Also the carry-over moment (line 12) of branch cd (col. 8) is equal to zero. Since branch cd is "fixed" at the center line as shown in Figure 9, line 11, col. 8 must be zero, and the total moment of branches ca and cd can be determined now by adding col. 7 and 8. For equilibrium, the total branch moment (line 12) of ce (col. 10) can be determined by setting the sum of col. 7, 8, 9, and 10 in line 11 equal to zero. This enforces $\Sigma M_c=0$, and all quantities related to joint c may now be determined in terms of S, M_B, and M_D by the same procedure used for joint a above.

By identical procedure as used for joint c, the distributed moments (line 10) carry-over moments (line 11) and total branch moments (line 12) can be determined for all the branches of joint e (col. 11-15). In case of more than three stories, identical procedure will be repeated for all joints of the left half of the symmetrical frame.

The only deviation for the above typical procedure occurs with regard to the very last branch (col. 14 and 15 in this example), ahd the last joint (joint e of this example). Since joint g is fixed, the carry-over moment (line 11) of branch eg (col. 14) is equal to zero and the total branch moment (line 12) of eg (col. 14) is equal to the distributed moment (line 10) only. For equilibrium of the last joint, the sum of the total branch moments (line 12) and the external, applied, joint moment at joint e (col. 11-14) must be equal to zero. Therefore, the external unknown moment X can be determined from this relation, i.e.

$$29X - 15M_B - 2M_D - 62X - 32M_B$$
$$- 4M_D - M_F - 31X - 16M_B - 2M_D = 0$$
$$X = \frac{1}{122} (- 63M_B - 8M_D - M_F) .$$

All branch moments of the antisymmetrical frame can be obtained by similar procedure. The only difference is that one assumes all fixed end moments to be known and represents them by a symbol. For instance, in this particular example there is no fixed end moment in the horizontal branches. In the vertical branches, due to sidesway, fixed end moments N, M, and L are introduced as shown in Table 4. Actually, N, M, and L are unknowns and represent the product of the story shear and the story height. They can be eliminated by Equation 13, leaving only three unknowns, M_B, M_D and M_F, for the given three-story frame.

TABLE 3

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Structure	2	Joint		a		c				e				g
	3	Branch		ab	ac	ca	cd		ce	ec	ef		eg	ge
Mod. M.D. Coef.	4	Stiffness		2EK	EK	EK	4EK		EK	EK	4EK		EK	EK
	5	M.S.C.		2	4	4	2		4	4	2		4	4
	6	M.S.		4EK	4EK	4EK	8EK		4EK	4EK	8EK		4EK	4EK
	7	M.S.F.		1	1	1	2		1	1	2		1	
	8	MCOF.			1/2	1/2			1/2	1/2			1/2	
Calculation of Branch Mom.	9	A.M. & F.E.M.	M_B					M_D				M_F		
	10	D. M.		X	X	$-4X-2M_B$	$-8X-4M_B$		$-4X-2M_B$	$31X+16M_B$ $-2M_D$	$62X+32M_B$ $-4M_D$		$31X+16M_B$ $-2M_D$	
	11	C. O. Mom.			$-2X-M_B$	$\frac{1}{2}X$			$\frac{31}{2}X+8M_B$ $-M_D$	$-2X-M_B$				$\frac{31}{2}X+8M_B$ $-M_D$
	12	Branch Mom.	M_B	X	$-X-M_B$	$-\frac{7}{2}X-2M_B$	$-8X-4M_B$	M_D	$-\frac{23}{2}X+6M_B$ $-M_D$	$29X+15M_B$ $-2M_D$	$62X+16M_B$ $-2M_D$	M_F	$31X+16M_B$ $-2M_D$	$\frac{31}{2}X+8M_B$ $-M_D$
	13	$\therefore\ \Sigma M_e = 0$ we have $X = \frac{1}{122}[-63M_B + 8M_D - M_F]$												
Sum of Branch Mom.	14													
	15	M_B	1	-0.516	-0.484	-0.193	0.131	0	0.062	0.024	-0.016	0	-0.008	-0.004
	16	M_D	0	0.066	-0.066	-0.229	-0.525	1	-0.246	-0.098	0.066	0	0.033	0.016
	17	M_F	0	-0.008	0.008	-0.029	0.065	0	-0.094	-0.238	-0.508	1	-0.254	-0.127
	18		-30.8	10.3	20.5	25.5	40.7	-114.2	47.9	65.4	110.3	-230.9	55.2	27.6

TABLE 4

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Structure	2	Joint	a'			c'				e'				g'
	3	Branch		a'b'	a'c'	c'a'	c'd'		c'e'	e'c'	e'f'		e'g'	g'e'
M.M.D. Const.	4	Stiffness		2EK	EK	EK	4EK		EK	EK	4EK		EK	EK
	5	M. S. C.		6	1	1	6		1	1	6		1	1
	6	M. S.		12EK	EK	EK	24EK		EK	EK	24EK		EK	EK
	7	M. S. F.		12	1	1	24		1	1	24		1	
	8	MCOF.			-1	-1	-		-1	-1			-1	
Calulations of Branch Mom.	9	A. M. & F. E. M.	$-M_B$		$+N$	$+N$		$-M_D$	$+M$	$+M$		$-M_F$	$+L$	$+L$
	10	Distrib. Mom.		$12X$	X	$+13X+N$ $-M_B$	$312X$ $+24N$ $-24M_B$		$13X+N$ $-M_B$	$337X+M$ $+27N+M_D$ $+26M_B$	$8088X-624M_B$ $-24M_D+24M$ $+648N$		$337X+M$ $+27N+M_D$ $+26M_B$	
	11	C.O. Mom.			$-13X$ $+M_B$ $-N$	$-X$			$-337X-M$ $-27N+M_D$ $+26M_B$	$-13X-N$ $+M_B$				$-337X+M$ $-27N+M_D$ $-26M_B$
	12	Branch Mom.	$-M_B$	$12X$	$-12X+M_B$	$12X$ $-M_B$ $+2N$	$312X$ $+24N$ $-24M_B$	$-M_D$	$-324X+M_D$ $+25M_B$ $-26N$		$8088X+24M$ $-24M_D$ $-624M_B$ $+648N$	$-M_F$	$337X+M$ $+27N+M_D$ $+26M_B$ $+L$	$-337X-M$ $-27N-M_D$ $-26M_B$ $+L$
	13	$X=\frac{1}{8749}\left[675M_B+26M_D+M_F-731N-27M-L\right]$												
Eqn. of Branch Mom.	14	Loading	0	-72.05	72.05	61.28	-273.40	0	212.12	187.78	-564.26	0	367.48	423.51
	15	M_B	-1	1.034	-0.034	-0.192	0.170	0	-0.022	0.007	-0.005	0	0.002	-0.002
	16	M_D	-0	0.085	-0.085	-0.013	1.031	-1	-0.018	-0.097	0.085	0	0.012	0.005
	17	M_F	0	-0.002	0.002	0.010	0.085	0	-0.095	-0.016	1.036	-1	-0.020	-0.107
	18	F.B.E.M.	30.8	-113.1	82.3	66.4	-416.1	114.2	235.4	202.3	-813.2	230.9	379.9	447.7

Now, we have three unknowns in this example, M_B, M_D, and M_F. From each joint of the left half frame, we can get an equilibrium equation. Therefore, three unknowns can be determined in the following way:

$$\sum M_B = 0$$

$$2.450M_B - 0.019M_D - 0.005M_F + 72.05 = 0$$

$$\sum M_D = 0$$

$$-0.39M_B - 2.444M_D - 0.019M_F + 273.40 = 0$$

$$\sum M_F = 0$$

$$0.012M_B + 0.020M_D - 2.456M_F - 564.36 = 0$$

Solving these equations, we obtain

$$M_B = -30.86 \text{ ft-kips}$$

$$M_D = -114.19 \text{ ft-kips}$$

$$M_F = -230.86 \text{ ft-kips}$$

Finally, the branch end moments (end moments) in kips-ft are shown in Table 5 as follows.

TABLE 5

M_{AB}	M_{AC}	M_{CA}	M_{CD}	M_{CE}	M_{EC}	M_{EF}	M_{EG}	M_{GE}
-113.1	82.3	66.4	-416.6	235.4	202.3	-813.2	379.9	447.7
10.3	20.5	25.5	40.7	47.9	65.4	110.3	55.2	27.6
-102.8	102.8	91.9	375.3	283.3	367.7	-702.9	435.1	475.3

M_{BA}	M_{BD}	M_{DB}	M_{DF}	M_{FD}	M_{FE}	M_{DC}	M_{FH}	M_{HF}
-113.1	164.5	132.8	470.8	404.6	-813.2	-416.1	759.8	895.4
-10.3	41.0	-51.0	95.8	-130.8	-110.3	-40.7	-110.4	-55.2
-123.5	123.5	81.8	375.0	273.8	-923.5	-456.8	649.4	840.2

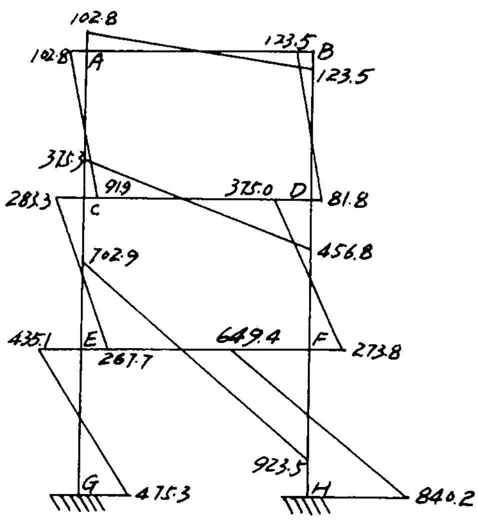


Fig. 10

CHAPTER 6

SIMPLIFIED MOMENT DISTRIBUTION METHOD FOR MULTIPLE-BAY FRAMES

6-1. Theory. The "exact" method introduced in Chapter 5 may become very complex in case of multiple-bay multistory frames. Therefore, a simple approximation method will be introduced.

Resolve a given, arbitrary, multiple-bay, multistory frame into two or more symmetrically stiff, single-bay frames, as illustrated later in Art. 6-3. This article explains how the horizontal loads must be distributed among single-bay part frames in proportion to their corresponding column stiffnesses. The loading systems on each part frame will be resolved into components which are either symmetrical or antisymmetrical.

After the fixed end moments are determined, the first cycle of moment distribution begins with a single modification outlined as follows. Distribute moments at each joint and carry-over. Then, according to our theory developed in Art. 5-1, the ratio of the end moments of component branches must be equal to the ratio of their stiffnesses. This principle applies to the "inside" component branches of two part frames. Therefore, in this modified moment distribution, after each carry-over step, balancing moments must be added at the joints of the inside component branches in order to satisfy the above theory.

This balancing principle will be illustrated now on two branches (Figure 11) which are components of a single equivalent branch. The basic theory requires that the relation

$$M'_{DE} : M''_{DE} = K' : K'' \qquad (14)$$

must be satisfied.

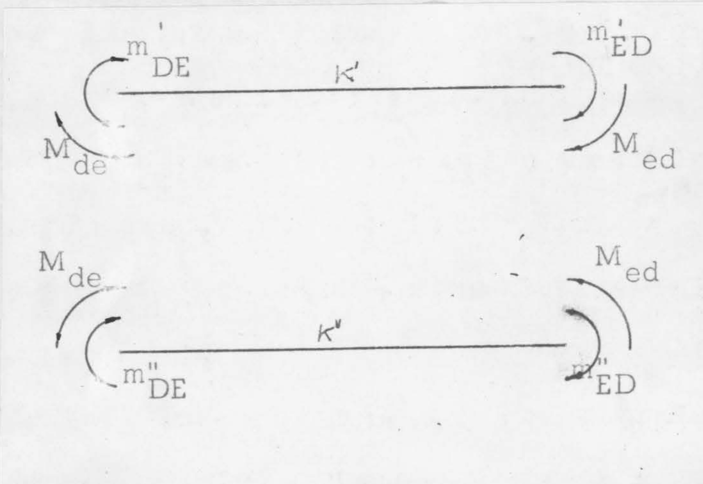


Fig. 11

Suppose, during a moment distribution process the branch end moments of a cycle (distribution and carry-over) are m'_{DE} and m''_{DE}, and it is found that

$$m'_{DE} : m''_{DE} \neq K' : K'' .$$

Then, to satisfy the theory, equal and opposite balancing moments, M_{de}, must be introduced at the ends between the two branches. M_{de} can be determindd now by the relations

$$M'_{DE} = m'_{DE} + M_{de} , \quad M''_{DE} = m''_{DE} - M_{de}$$

and Equation (4)

$$(m'_{DE} + M_{de}) : (m''_{DE} - M_{de}) = K' : K'' .$$

Solving for M_{de}:

$$M_{de} = \frac{K'M''_{DE} - K''M'_{DE}}{K' + K''} \qquad (15)$$

If now this M_{de} is applied to branch D'E' and simultaneously, with opposite sign, to D"E", the ratio of the moments at end D of the two component branches is corrected for this cycle.

Accordingly, in this method one complete cycle will consist of three steps, namely

1. distribution
2. carry-over
3. balancing

In Art. 6-2 an illustrative example will be introduced and then the procedure of its solution is shown in Art. 6-3.

<u>6-2. Example</u>. A double bay two-story frame is loaded by two horizontal forces and one uniform vertical load as shown in Figure 12. Find the end moments of each branch, and sketch the M-diagram of the frame.

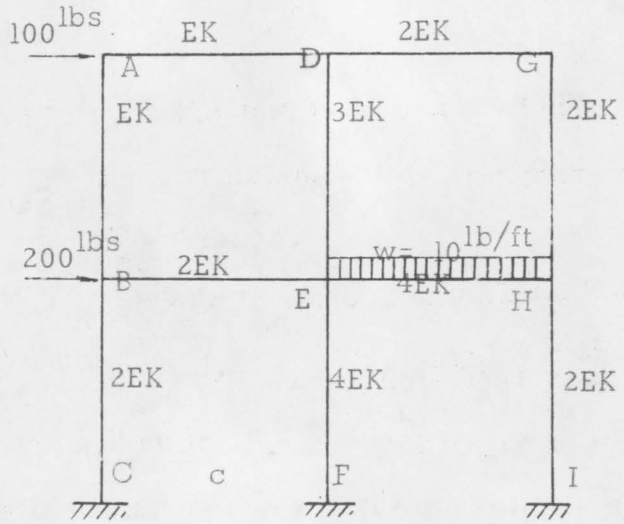


Fig 12

The complete solution of this example is shown and discussed in Art. 6-3 below.

6-3. Solution Procedure. The following solution procedurerefers to example 2 as well as to a more general multistory frame. It will be discussed in four parts.

Part 1. Preliminaries

Step 1. Break up the given frame into two symmetrically stiff single-bay frames called part frames as shown in Figure 13. If the ratio of column stiffnesses does not permit doing this directly, introduce additional single columns, whenever required, as was done in example 1.

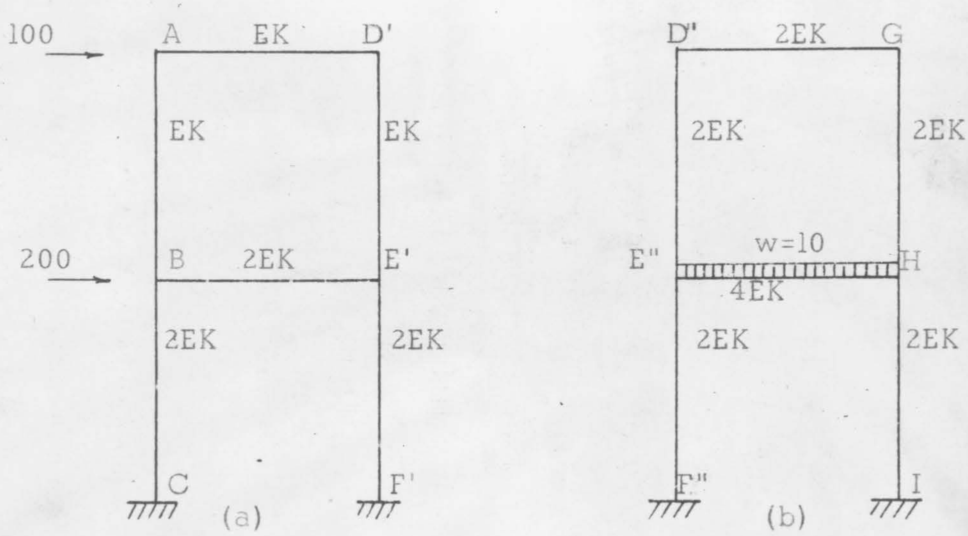
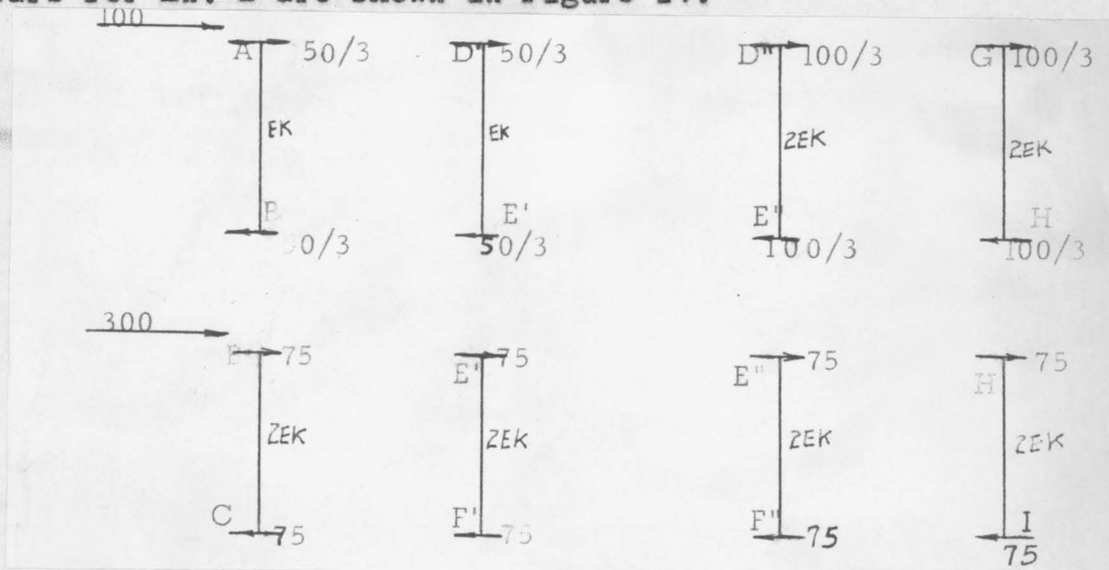


Fig. 13

Step 2. Branch loads (loads acting directly on branches) remain unchanged on their branches, as illustrated by the uniform load on branch EH of example 2. If the branch is resolved, the component branches carry the branch load in proportion to their stiffnesses.

Step 3. The initial story shears in the part frames must add up for each story to the horizontal forces applied to this story. Their value must be proportional to the story column stiffnesses of these symmetrically stiff part frames in the corresponding story. Values of all initial story shears for Ex. 2 are shown in Figure 14.



Step 4. The above defined total initial load system on each of the part frames will now be separated into two superimposable forces systems, one of which is symmetrical while the other one is antisymmetrical. For instance, in Ex. 2, the uniform load on beam EH becomes part of the symmetrical load system of the right part frame, while, in general, story shears and column loads create the antisymmetrical systems. Having no single columns in this example, we obtain a total of four force systems, a symmetrical and an antisymmetrical one for each of the two part frames, as it may be seen in Table 6 (6a and 6b).

Step 5. By reasons explained in Ex. 1 (symmetry and antisymmetry), only half of the frame joints are needed in Table 6 for each force system. The columns of Table 6 may be set up now accordingly. As it will be seen later, the procedure can be followed easier, if for each part frame the joints of rather the "inside" half frame are listed, since these are the joints where the balancing moments actually occur.

Step 6. Calculate K, β, S, s, and C for each branch, as explained in Ex. 1, and list these constants in Table 6 as shown.

Step 7. Calculate the fixed end moments due to branch loads and initial sidesways, and list them in Table 6 according to symmetry and antisymmetry. In Ex. 2, branch EH is symmetrically loaded and the fixed end moments,

$$_F M_{EH} = -_F M_{HE} = \frac{WL^2}{12} = 120 \text{ ft-lb} ,$$

are shown in the table columns of the symmetrical system. All fixed end moments due to sidesway effect the antisymmetrical system. For instance, in Ex. 2, the fixed end moments on branches AB and D'E' will be calculated by the relations

$$_F M_{AB} + _F M_{DE} + _F M_{BA} + _F M_{ED} = \frac{100}{3} \times 12 = 400$$

and $$_F M_{AB} = _F M_{BA} = _F M'_{DE} = _F M'_{ED} \quad .$$

Therefore, from Figure 14,

$$_F M'_{AB} = {_F M}_{BA} = {_F M'}_{DE} = {_F M'}_{ED} = \frac{HL}{4} = \frac{(100/3)12}{4}$$

Similarly,

$$_F M''_{DE} = {_F M''}_{ED} = {_F M}_{GH} = {_F M}_{HG} = 200 \text{ ft-lb}$$

By similar calculations it becomes apparent, that in this example the fixed end moments in all columns of the first story are equal, since story shears and column height are both equal for the part frames. Therefore,

$$_F M_{BC} = {_F M}_{CB} = {_F M}_{EF} = {_F M}_{FE} = 450 \text{ ft-lb}$$

Fixed end moments due to sidesways are to be listed in the table columns of the antisymmetrical force systems as explained before and as shown in Table 6.

Part II. First Cycle of Moment Distribution and Correction.

Step 1. Distribute moments at all joints simultaneously by the conventional method.

Step 2. Carry over simultaneously at each joint.

Step 3. Calculate the balancing moments for the ends of the component (inside) branches, i.e., find the balancing joint branch moments for DE, ED, and EF. Assign half of the balancing moments to the symmetrical systems and the other half to the antisymmetrical systems and list these values in Table 6a and 6b as shown.

For instance,

$$M'_{DE} = -14.3 + 36.7$$

$$M''_{DE} = -28.6 - 46.4$$

$$M_{de} = \frac{K'M''_{DE} - K''M'_{DE}}{K' + K''} = -2.5 \text{ ft-lb}$$

The other two balancing equations for this particular example are

$$\frac{1}{3}\left[-46.4 + 28.6 - 40-2\ (14.3-36)\right] = 22.3$$

$$\frac{1}{2}\left[-46.4 + 40-(-73.3)\right] = 33.45 \doteq 33.4\ .$$

Consider now joint branch DE. Add $\frac{M_{de}}{2} \doteq -1.2$, -1.1 in the two (symmetrical and antisymmetrical) DE columns of Table 6a and add $\frac{M_{de}}{2} = 1.2$ in the corresponding columns of Table 6b.

Remarks to Part II. Steps 1, 2 and 3 of Part II are considered as one complete cycle. Notice that in this method each cycle starts with "distribution" and ends with "balancing". The final cycle will be an incomplete one which consists of distribution only.

Part III. Iteration.

Repeat the cycle outlined in Part II as many times as necessary. When the carry-over moments and balancing moments are very small, carry out the last (incomplete) cycle consisting of distribution only, then stop.

Part IV. Results.

Step 1. Adding all moments in the columns of Tables 6a and 6b the component branch end moments (C. B. E. M.) will be obtained.

Step 2. Summing the two or four C. B. E. M.'s of each branch end, we obtain the final branch end moments, and the analysis is completed. The results of Ex. 2 are shown in Table 7, and are illustrated by a moment diagram of the complete frame in Figure 16.

TABLE 6a

	Antisymmetry						Symmetry					
Joint	D		E			F	D		E			F
Branch	DA	DE	ED	EB	EF	FE	DA	DE	ED	EB	EF	FE
Stiff. K	1	1	1	2	2	2	1	1	1	2	2	2
M. S. C. β	6	1	1	6	1	1	2	4	4	2	4	4
M. S. S	6	1	1	12	2	2	2	4	4	4	8	8
M. S. F. s	6	1	1	6	1		1	2	1	1	2	
M. C. O. F.		-1	-1		-1	-1		1/2	1/2		1/2	
F. E. M.		100	100		450	450						
Distribute	-85. 7	-14. 3	-36. 7	-440. 0	-73. 3							
C. O.		36. 7	14. 3			73. 3						
B. M.		-1. 2 *	11. 2 #		16. 7 Λ			-1. 1*	11. 1#		16. 7 Λ	
Distribute	-30. 4	-5. 1	-2. 8	-33. 8	-5. 6		0. 4	0. 7	-7. 0	-6. 9	-13. 9	
C. O.		2. 8	5. 1			5. 6		-3. 5	0. 4			-7. 0
B. M.		-1. 0	2. 9		5. 6			-1. 0	3. 0		5. 7	
Distribute	-1. 6	-0, 2	-0. 9	-10. 9	-1. 8		1. 5	3. 0	-2. 3	-2. 2	-4. 5	
C. O.		0. 9	0. 2			1. 8		-1. 1	1. 5			-2. 2
B. M.		-1. 1	1. 1		2. 9			-1. 1	1. 1		2. 9	
Distribute	0. 2	0	-0. 3	-3. 3	-0. 6		0	0	-1. 4	-1. 4	-2. 7	
B. E. M.	-11. 7	11. 7	94. 1	-488. 0	393. 9	530. 7	1. 9	-1. 9	6. 3	-10. 5	4. 2	-9. 2

$* \frac{1}{3}\left[46.4 - 28.6 + 20 - (36.7 - 14.3)\right] = - 2.3$

$\# \frac{1}{3}\left[-46.4 + 28.6 + 40 - 2(14.3 - 36.7)\right] = 22.3$

$\Lambda \frac{1}{2}\left[-46.4 + 40 - (- 73.3) \right] = 33.4$

TABLE 6b

	Antisymmetry						Symmetry					
Joint	D		E			F	D	D	E			F
Branch	DG	DE	ED	EH	EF	FE	DG	DE	ED	EH	EF	FE
Stiff. K	2	2	2	4	2	2	2	2	2	4	2	2
M. S. C. β	6	1	1	6	1	1	2	4	4	2	4	4
M. S. S	12	2	2	24	2	2	4	8	8	8	8	8
M. S. F. s	6	1	1	12	1		1	2	1	1	1	
M. C. O. F.		-1	-1		-1			1/2	1/2.		1/2	
F. E. M.		200	200		450	450				-120		
Distribute	-171. 4	-28. 6	-46. 4	-557. 2	-46. 4				40. 0	40. 0	40. 0	
C. O.		46. 4	28. 6			46. 4		20. 0				20. 0
B. M.		1.2^{*}	$-11.2^{\#}$		-16.7^{Λ}			1.2^{*}	$-11.2^{\#}$		-16.8^{Λ}	
Distribute	-40. 8	-6. 8	0	-0. 7	0		-7. 1	-14. 1	9. 3	9. 4	9. 3	
C. O.		0	6. 8			0		4. 7	-7. 1			4. 7
B. M.		1. 0	-3. 0		-5. 6			1. 0	-3.0		-5. 7	
Distribute	-0. 9	-0. 1	0. 1	1. 6	0. 1		-1. 9	-3. 8	5. 3	5. 2	5. 3	
C. O.		-0. 1	0. 1			-0. 1		2. 6	-1. 9			2. 6
B. M.		1. 1	-1. 1		3. 0			1. 1	-1. 0		-2. 9	
Distribute	-0. 9	-0. 1	0. 3	3. 4	0. 3		-1. 2	-2. 5	1. 9	2. 0	1. 9	
B. E. M.	-214. 0	214. 0	174. 2	-552. 9	378. 7	496. 3	-10. 2	10. 2	32. 3	-63. 4	31. 1	27. 3

TABLE 7

AD	AB	DA	BA	BE	BC	CB	DE	ED	EB
-117.5	117.5	117.5	94.1	-488.0	393.9	530.7	117.5	94.1	-488.0
-1.9	1.9	1.9	-6.3	10.5	-4.2	9.2	-1.9	6.3	-10.5
							214.0	174.2	
							10.2	32.3	
-119.4	119.4	-115.6	87.8	477.5	389.7	539.9	340.8	306.9	-498.5

EF	FE	DG	GD	GH	HG	HI	IH	EH	HE
393.9	530.7	-214.0	-214.0	214.0	174.2	378.7	496.3	-552.9	-552.9
4.2	-9.2	-10.2	10.2	-10.2	-32.3	-31.1	-27.3	-63.4	63.4
378.7	496.3								
31.1	27.3								
807.9	1045.1	-224.2	-203.8	203.8	141.9	247.6	469.0	-616.3	-489.5

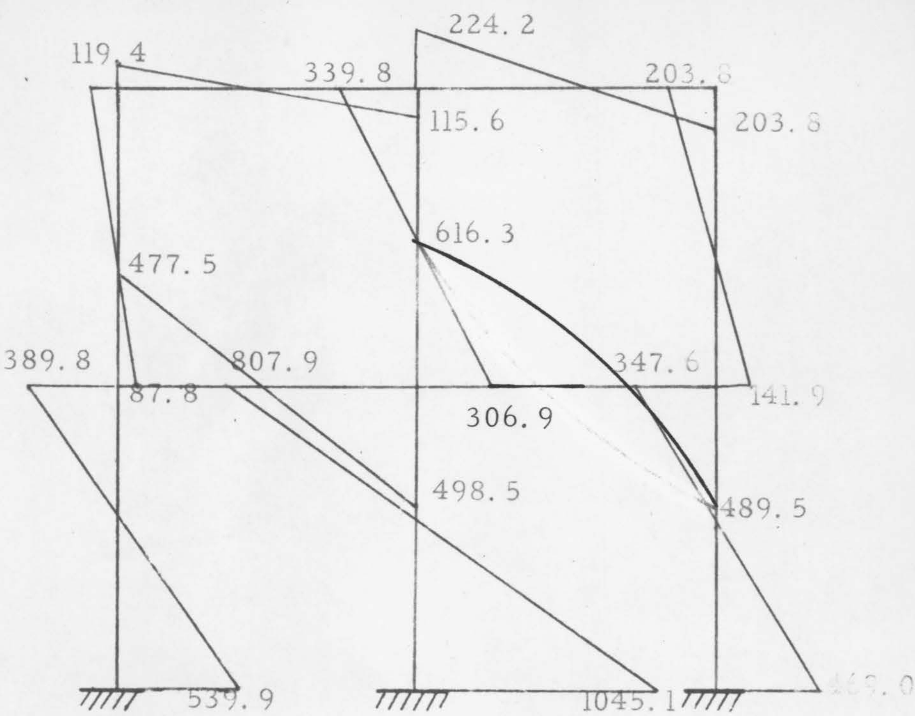

224.2
119.4
339.8
203.8
115.6
203.8
616.3
477.5
389.8
807.9
347.6
87.8
141.9
306.9
498.5
489.5
539.9
1045.1
465.0

Fig. 15

CHAPTER 7

DISCUSSION AND CONCLUSIONS

Two methods were introduced in this thesis. The first one applies to single-bay frames, with an arbitrary number of stories. This first method incorporates certain principles of both the moment distribution and slope-deflection methods. The tabulation form and other details were taken from versions of the Hardy-Cross method while the solution of a set of simultaneous equations in a single cycle of calculations reassembles features of the slope-deflection method. However, the concept of primary unknowns (slopes and deflections) of this later method is eliminated and the number of unknowns per story is cut to one third, i.e., to one unknown moment per story. This fact implies the superiority of the new procedure over both of the conventional methods.

The second method introduced in this thesis was developed for completely general multiple-bay, multiple-story frames. The only imposed specialization was the requirement of prismatic members, a restriction which may be removed by further development of the method. It employs a quite different approach than the first method and is a genuine "moment distribution method" with its iteration procedure. It takes full advantage of symmetry and anti-symmetry, furthermore of the introduced modified constants. On this account it converges more rapidly to the true moment values than the conventional method, and it also cuts down on the number of joints to be analyzed. Notice in Example 2 that after only three cycles of iteration the errors of the branch end moments become very small, i.e., less than two per cent.

The modified constants introduced in Chapter four are adopted from the "Synthetic Analysis" of Professor Yu[2] and they are suitable for frames consisting of prismatic members. But their concept may readily be developed into the concept of generalized constants if someone desires to employ this new method to frames consisting of non-prismatic members.

The basic principle of the two methods, the separation of the given frame into symmetrical stiff "reduced" frames (and single columns) and the resolutions of the force system into symmetrical and anti-symmetrical ones should prove itself adoptable for the analysis of a great variety of frames besides those shown in the illustrations. The author believes that they will permit significant time-saving and increased accuracy in the analysis or design of statically indeterminate frames.

CHAPTER 8

BIBLIOGRAPHY

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the scanned document

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ABSTRACT

Moment Distribution Method was modified so that simple techniques applicable to symmetrical and anti-symmetrical frames may be applied to non-symmetrical rigid frames consisting of prismatic members. This approach simplifies considerably the calculations.

Using the above approach, two different "Simplified Moment Distribution Methods" were introduced. Method No. 1, an "exact" method, makes it possible to execute moment distribution in a single cycle. The "exact" values of the unknown moments are obtained by solving a set of simultaneous equations. This method is applicable to single-bay frames having an arbitrary number of stories. In the solution there is one unknown moment and one equation for each story.

Method No. 2 simplifies the analysis of multiple-bay, multiple-story frames. It is a modified version of the standard moment distribution. Only half of the total number of joints has to be considered in this analysis and the convergence of the iteration process is accelerated.

The presentation of the theory is preceded by the definition of a set of modified constants pertinent to the two methods. Illustrative examples for the analysis of single-bay as well as multiple-bay frames are included.