ANALYSIS OF SHEAR TEST METHODS FOR COMPOSITE LAMINATES

by

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LIST OF SYMBOLS

[a]	Orthotropic elastic constant matrix (3x3)
[A]	Laminate extensional stiffness matrix (3x3)
[B]	Elemental strain-displacement matrix (6x3)
E_i	Moduli of elasticity
G_i	Moduli of rigidity
h	Lamina thickness
H	Laminate half thickness
[k]	Elemental stiffness matrix (6x6 or 8x8)
[K]	Finite element global stiffness matrix (variable size)
n	Total number of plies or lamina in a laminate
{N}	Inplane forces per unit length in laminate X-Y coordinates (1x3)
{Q}, $\{Q^T\}$	Elemental mechanical and thermal load vectors, respectively (1x6 or 1x8)
[Q], $[\overline{Q}]$	Lamina stiffness matrix in natural 1-2 coordinates and laminate X-Y coordinates, respectively (3x3)
{R}	Global load vector for finite element model (variable size)
[T]	Tensor transformation matrix (3x3)
u, v	Displacements in X and Y directions, respectively
{u}	Nodal displacement vector for an element (1x6 or 1x8)

U, W, Π	Strain energy, potential energy of external loads, and total potential energy, respectively
X, Y, Z	Global or laminate X, Y, and Z coordinates, respectively
$\{\alpha\}$, $\{\overline{\alpha}\}$	Lamina and average laminate coefficient of thermal expansion matrix (1x3), respectively
β_i	Generalized coordinate coefficients (i=1,2,3,...,6)
δ	Applied displacement
ΔT	Uniform laminate temperature change
$\{\varepsilon\}$, $\{\varepsilon^i\}$, $\{\varepsilon^\circ\}$	Total, thermal, and mechanical laminate midplane strains, respectively
γ	Engineering shear strain
$\overline{\sigma}$, $\overline{\tau}$	Average laminate normal and shear stress, respectively
$\{\sigma\}$, $\{\overline{\sigma}\}$	Lamina and average laminate stresses in natural 1-2 and laminate X-Y coordinates, respectively
θ	Angle of rotation from laminate coordinate system to lamina coordinate system
Subscripts	
1, 2	Natural lamina coordinate system
k	k^{th} ply of lamina
x, y	Laminate coordinate system

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1. INTRODUCTION

The use of composite materials for the primary structure of aerospace vehicles and other structures requiring high specific strength and stiffness materials dictates the need for precise knowledge of the mechanical properties of these materials. Fundamental material properties which are required for design and analysis include tension, compression, and shear response. This investigation is concerned with a finite element analysis of several test specimens which have been used to obtain experimental results for in-plane shear behavior of advanced composite materials.

Of the many shear test methods available, some provide qualitative data while others provide quantitative data. Qualitative shear test methods are used mainly for quality control and material screening. They generally provide comparative values of initial shear modulus and shear strength. The short beam shear test is a well known example of a qualitative test for obtaining comparative shear strength values. Quantitative shear test methods are used to accurately determine the complete shear stress-strain response for composite materials.

The "ideal" quantitative shear test should provide a region of pure, uniform shear stress; in addition, there should be a unique relationship between the applied load and the magnitude of the shear stress in the test section. Further, for accurate prediction of the ultimate shear stress, the test section should be one of maximum shear stress relative to all other regions of the specimen. All interlaminar, clamping, and other edge effects should be minimal and not contribute to the failure in the test section.

The specimen which has most of the attributes of the "ideal" shear specimen is the thin-walled tube loaded in torsion. Due to the relatively high cost of tubes as compared to flat laminates, several shear test methods which employ flat laminates have been used to obtain the in-plane shear stress-strain response of composite laminates. This study considers a finite element investigation of those flat specimens which either have been used extensively or appear to have potential for future use. A linear elastic analysis which has the capability to include thermal stresses and elastic fixtures is utilized.

The overall goal of this study is to determine the acceptability of the specimens analyzed for in-plane shear response of composite laminates.

2. LITERATURE REVIEW

2.1 The Slotted Shear Specimen

The slotted shear specimen has been used to measure shear properties of both metals and composites. Fenn and Clapper [1] used the geometry of Fig. 1a for their studies on metals. Breindel et al [2] suggested a standardized metal specimen thickness, width, and length of shear path (Fig. 1a). Iosipescu's photoelastic study [3] of this specimen indicated that the state of stress for a slotted specimen is generally unacceptable for quantitative shear behavior due to the lack of pure uniform shear stress in the test area.

Elkin et al [4] used the slotted specimen in Fig. 1b for shear tests on three unidirectional graphite-epoxies. They reported low strength values which were attributed to the stress concentrations at the edges of the holes. Their finite element analysis indicated a maximum stress concentration of 1.57 at the edge of the holes. Other variations of the slotted shear specimen shown in Figs. 1c and 1d have been used for composite materials [5].

2.2 The Cross Beam Shear Specimen

The cross beam shear specimen has been used for determining linear and nonlinear shear stress-strain response as well as ultimate shear strength values for graphite-epoxy and boron-epoxy laminates. As shown in Fig. 2a, the cross beam shear specimen is loaded in positive and negative bending to produce a biaxial state of tension and compression over the test area at the center of the specimen. Stresses on 45°

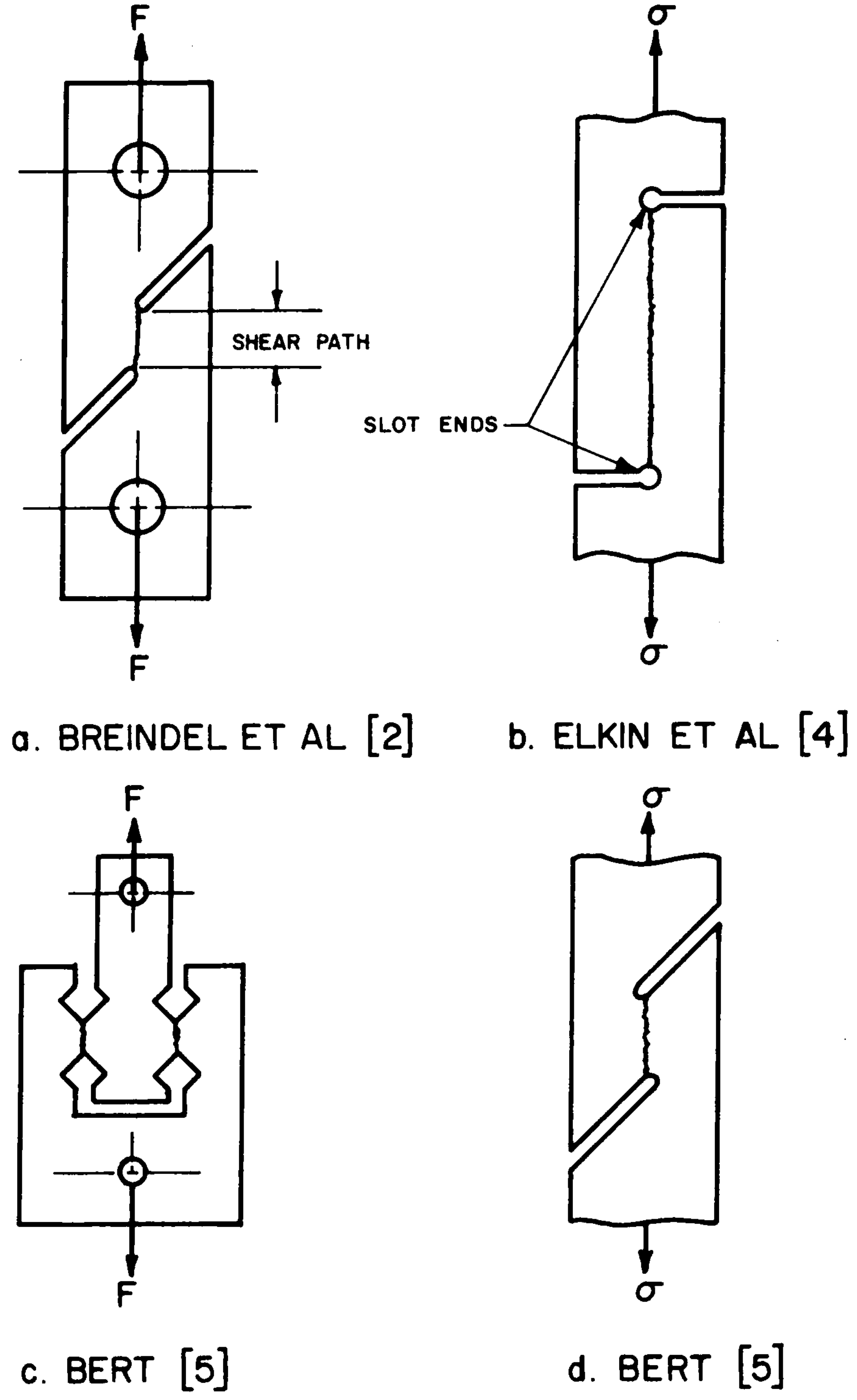


FIGURE 1. SLOTTED SHEAR SPECIMENS

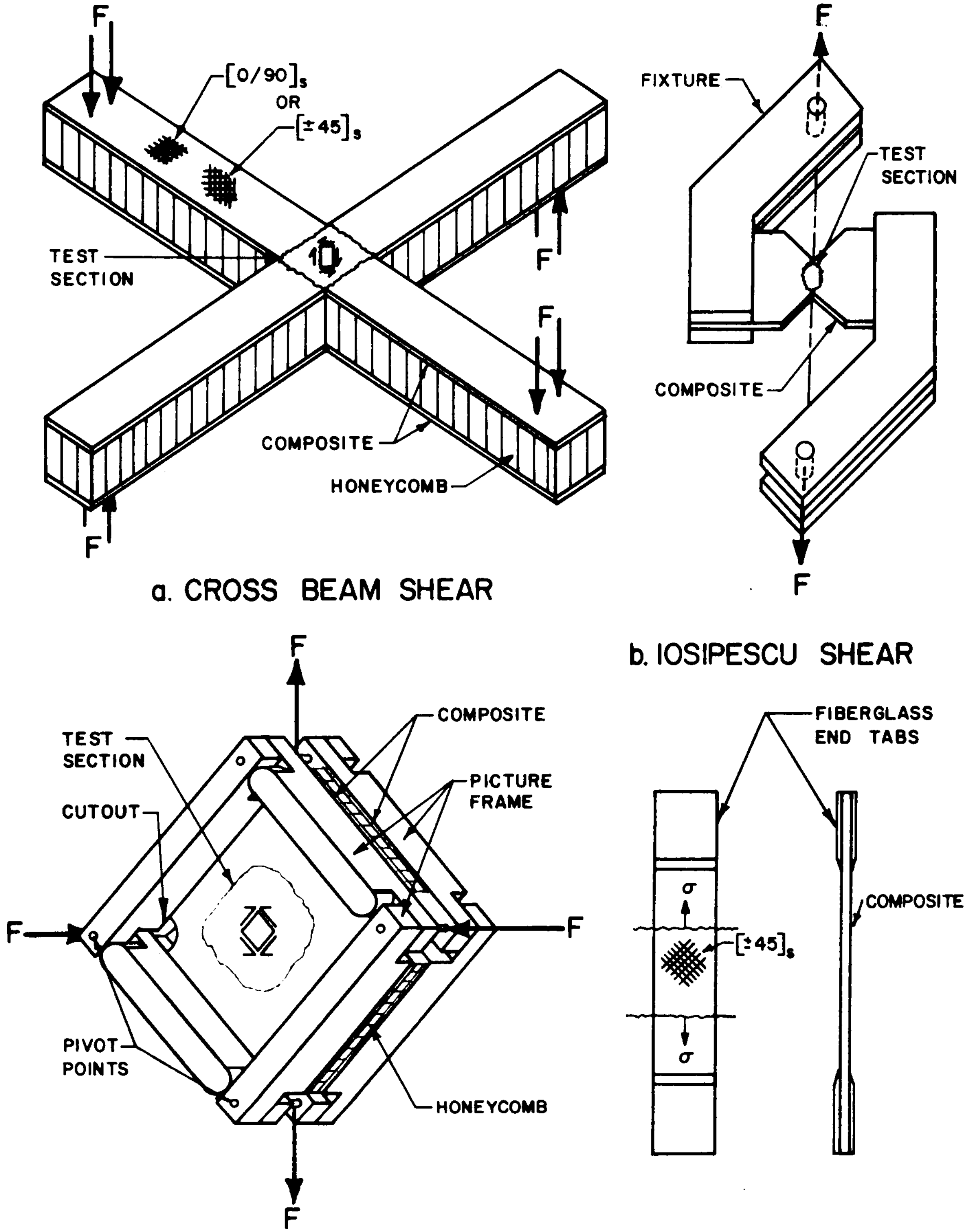
planes are presumed to be pure, uniform shear over the test section. Laminates commonly used with the cross beam are $[0/90]_s$ and $[\pm 45]_s$.

Petit [6] compared experimental results of this test method to those of the $[\pm 45]_s$ tension coupon and found good agreement for both linear and nonlinear shear stress-strain response of unidirectional laminates. Waddoups [7] compared experimental ultimate shear strength values obtained by this method to those of the short beam shear test and reported that the cross beam specimen produced considerably higher ultimate shear strength values than those of the short beam shear test.

2.3 The Iosipescu Shear Specimen

As mentioned previously, an extensive photoelastic study was conducted by Iosipescu [3] on some of the slotted shear specimens that had been used for shear testing. The results of this study generally indicated a need for a more precise shear test method. A new specimen (the Iosipescu specimen) was consequently developed as a test method for shear strength of metals.

Although the Iosipescu shear specimen has not been used extensively for shear testing of composite materials, the encouraging results from metals indicate that this specimen should be analyzed for composite materials to evaluate any potential for use as a test method. The effects of several modifications to the original Iosipescu specimen are studied for use with composite materials. While metal Iosipescu specimens are normally notched in two directions, the specimen in the present study is notched in only one direction (Fig. 2b) so that the laminate



a. CROSS BEAM SHEAR

b. IOSIPESCU SHEAR

c. PICTURE FRAME

d. $[\pm 45]_s$ TENSION COUPON

FIGURE 2. VARIOUS SHEAR SPECIMENS

stiffness in the test section is uniform and strain measurements may be taken at the center. The specimen is loaded to produce a shear stress acting between the notch ends with zero bending moment and zero normal stress. The reduced specimen area between the notch ends ensures failure in this region.

2.4 The Rail Shear Specimen

The rail shear specimen has been used quite often for composites with various laminate configurations and at elevated temperatures as well as room temperature. As shown in (Fig. 3) and in studies by Whitney et al [8], Rutherford et al [9], and The Advanced Composites Design Guide [10], this test method consists of bolting or bonding stiff rails to a composite plate to form a long, narrow test area between the rails. The load is introduced at the ends of the rails to displace the rails essentially parallel to one another.

A Fourier series plate solution and parametric study of this specimen was conducted by Whitney et al [8]. The study indicated that specimen dimensions can be standardized to produce an acceptable area of shear stress for most laminates. The only exceptions are laminates with a high Poisson's ratio ($\simeq 1.0$). Experimental results presented in this study showed excellent agreement with lamination theory for in-plane shear modulus. A double rail shear test procedure was investigated by Sims [11] who found good agreement with both the $[\pm 45]_s$ tension coupon and the torsion tube for linear and nonlinear shear stress-strain response. Bert [5] pointed out that the rail shear specimen may be readily sized

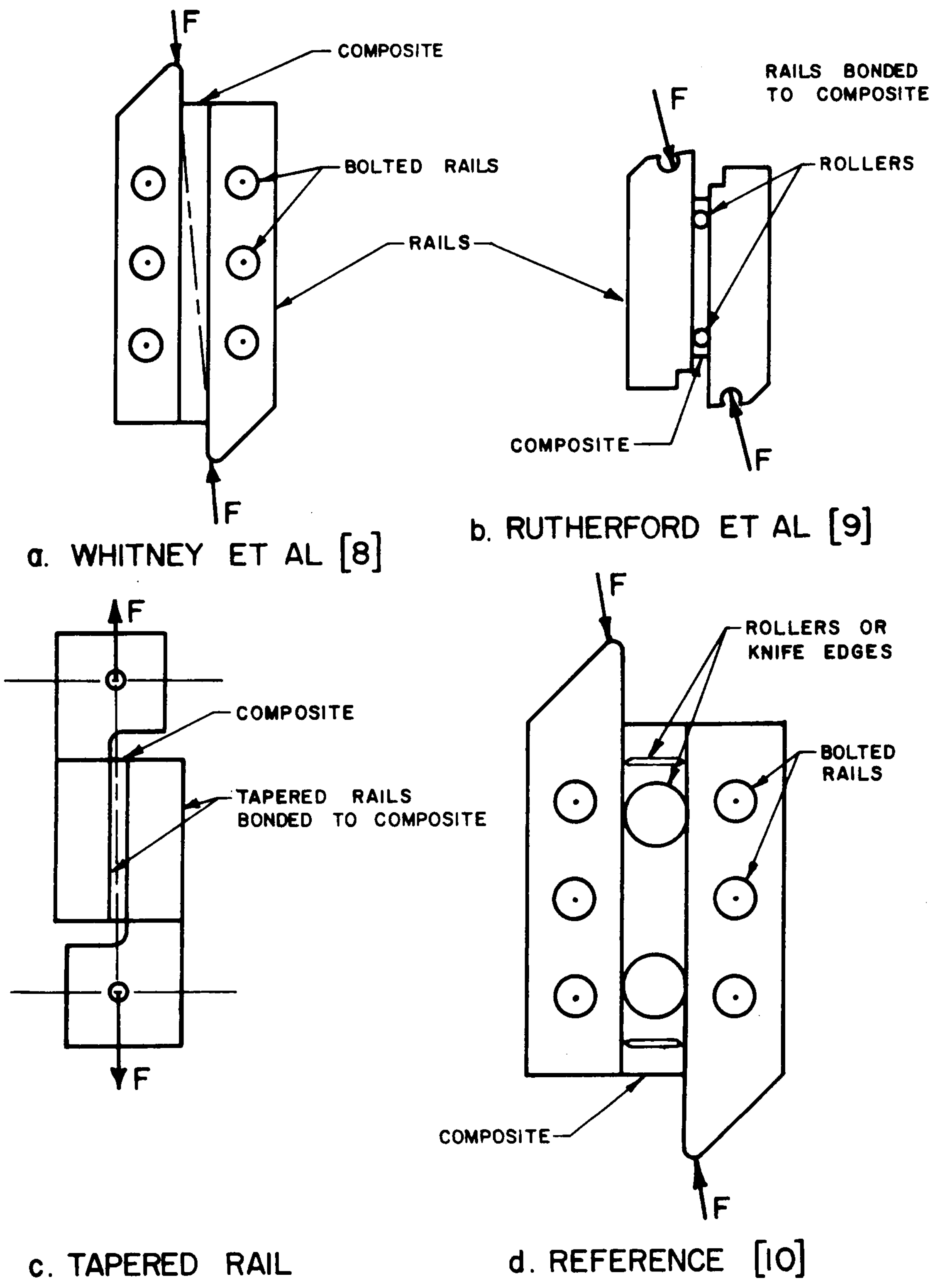


FIGURE 3. RAIL SHEAR SPECIMEN GEOMETRIES

to avoid buckling.

2.5 The Picture Frame Shear Specimen

The picture frame specimen shown in Fig. 2c consists of two thin square composite laminates bonded to a honeycomb core material for stability. A rigid four bar linkage frame is then bolted over the sandwiched composite for load introduction. In order to reduce high corner stresses these areas usually are cutout and have doublers bonded on near the resulting free edge. The load is introduced by a combination of tension and/or compression forces in pairs on opposite corners. Thus, the specimen is subjected to shear loading along planes at 45° to the diagonals.

A state of pure shear was not observed in a photoelastic study of this specimen by Bryan [12]. Dastin et al [13], Hyer and Douglas [14], and Hadcock and Whiteside [15] tested boron-epoxy and borsic-aluminum picture frame specimens with cutouts and/or doublers at the corners. Hadcock and Whiteside [15] and Douglas et al [16] reported that the corner stress concentrations nevertheless remained significantly high in both experiments and in the finite element analysis that included frame and nonlinear effects. However, Dastin et al [13] reported that reliable initial shear modulus and ultimate strength values could be obtained from the picture frame specimen for $[0/90]_s$ laminates.

2.6 The $[\pm45]_s$ Tension Coupon

The $[\pm45]_s$ tension coupon is a simple test for obtaining linear and nonlinear shear stress-stain response in the natural lamina coordinates.

As shown in figure 2d, the specimen is simply a $[\pm45]_s$ tension coupon, usually with tabs. During loading, incremental values of axial force and axial and transverse strain are recorded. Various transformation laws as employed by Petit [6], Adams and Thomas [17], Rosen [18], and Hahn [19] may be used to obtain the piecewise linear modulus of rigidity, G_{12}, in the natural lamina coordinates.

Petit compared test results of the $[\pm45]_s$ tension coupon to those of the cross beam and found close agreement with the exception of the highly nonlinear portion of the shear stress-strain curve. He also pointed out that the laminate Poisson's ratio must be exactly 1.0 in order to have a state of pure shear in the $[\pm45]_s$ tensile coupon. The measured value of Poisson's ratio for his tests was 0.85. The study also reported that specimen delamination may take place due to edge effects at high strain values.

2.7 The Off-axis Unidirectional Tension Coupon

The off-axis unidirectional tension coupon is a simple specimen similar in geometry to the $[\pm45]_s$ tension coupon shown in figure 2d. The laminate is unidirectional with fibers oriented at an angle usually between 10 and 75 degrees to the axis of loading. This specimen has been used to measure linear and nonlinear shear response in the natural lamina coordinates (with the aid of transformtion equations).

The state of stress in the off-axis unidirectional tension coupon has been determined by several methods. Rizzo [20] and Richards et al [21] analyzed this specimen by means of anisotropic finite elements;

geometry, boundary conditions, and influence of material properties were among the parameters studied. Pagano and Halpin [22] described the stress state by means of an elasticity solution supplemented with anisotropic rubber specimens subjected to large deformations.

Pipes and Cole [23] showed good agreement between the shear responses as obtained from 15°, 30°, 45°, and 60° off-axis coupons when corrections for Poisson effect are included. Richards et al [21] obtained good agreement between experiment and their finite element solution using long narrow 45° boron-epoxy coupons. Wu and Thomas [24] developed grips that allowed in-plane rotations of the anisotropic off-axis tension coupon. Chamis and Sinclair [25] suggested an optimum fiber orientation of 10° to minimize the effects of the biaxial state of stress present in the off-axis coupon.

2.8 The Torsion Tube

A desirable specimen for determining shear stress-strain response is the torsion tube (Bert [5,26]). In this test a thin-walled circular tube is subjected to pure torque resulting in a state of uniform pure shear away from the ends. The torsion tube has a very desireable shear stress distribution with minimal normal stresses in the test section. Torsion tubes have been successfully used to obtain both linear and nonlinear shear stress-strain response, however, Rizzo and Vicario [27] reported that strength values may be influenced by gripping the ends of the tube. The main drawback to this specimen is the difficulty associated with fabricating and testing tubular specimens.

2.9 Other Shear Test Methods

Various other test methods have been used to measure in-plane shear response of composite materials. See Bert [5], Adams and Thomas [17], Hancox [28], Greszczuk [29], Sattar and Kellogg [30], and Berg et al [31] for a description of these methods. These other methods generally provide qualitative data rather than accurate, quantitative shear stress-strain response.

2.10 Summary

While this literature review has discussed many test methods for the determination of in-plane shear response of composite laminates, it is evident that complete analyses including effects of specimen geometry and grips, laminate configuration, high temperature test requirements, and nonlinear behavior are lacking. This study will attempt to resolve some of these deficiencies for flat specimens.

3. THEORETICAL CONSIDERATIONS

3.1 Lamination Theory

This study is limited to linear, elastic, orthotropic laminates in plane stress due to mechanical and thermal loads. For such material behavior, the laminate moduli of elasticity (E_x,E_y), Poisson's ratios (ν_{xy},ν_{yx}) modulus of rigidity (G_{xy}), and the coefficients of thermal expansion ($\bar{\alpha}_x$,$\bar{\alpha}_y$) referenced to the laminate x-y coordinate system (Fig. 4) are the only material properties required for a finite element analysis. Experimental results have shown that these elastic and thermal constants of the laminate can be predicted given the lamina properties. The accepted method for doing this is called lamination theory [32].

The lamina, or individual ply, is the basis upon which the laminate is built. It is an orthotropic ply in a state of plane stress and is characterized by the constitutive relationship:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} \tag{3.1}$$

where the stresses σ_1, σ_2, τ_{12} and strains ε_1, ε_2, γ_{12} are referred to the 1-2 natural (or principal) lamina coordinate system shown in Fig. 4. The terms of the lamina stiffness matrix, [Q], can be expressed in terms of the engineering constants as follows:

$$Q_{11} = E_{11}/(1-\nu_{12}\nu_{21})$$

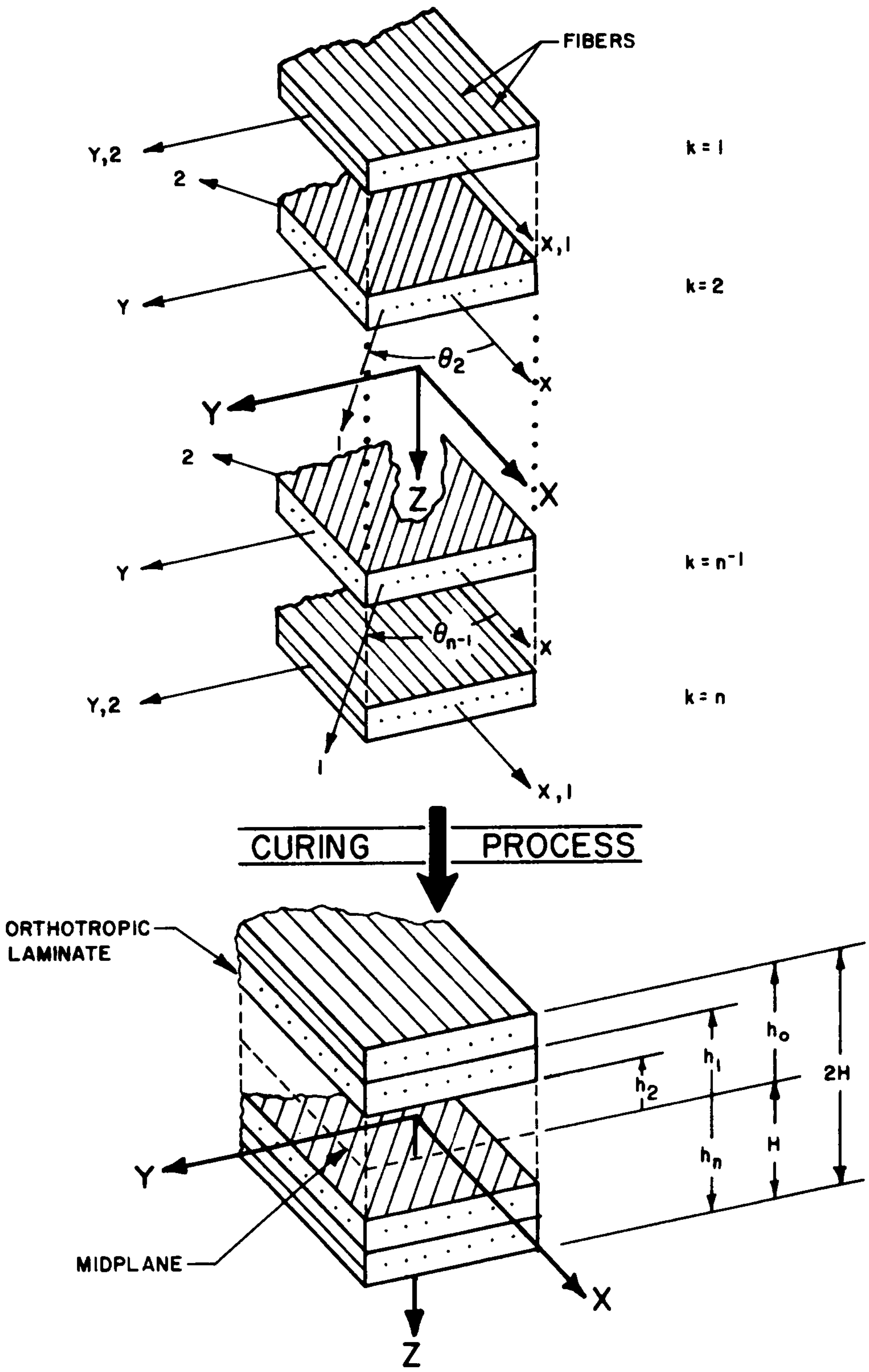


FIGURE 4. LAMINA & LAMINATE COORDINATES

$$Q_{22} = E_{22}/(1-\nu_{12}\nu_{21})$$
$$Q_{12} = \nu_{21}E_{11}/(1-\nu_{12}\nu_{21}) = \nu_{12}E_{22}/(1-\nu_{12}\nu_{21}) \tag{3.2}$$
$$Q_{66} = G_{12}$$
$$Q_{16} = Q_{26} = 0,$$

where all material properties are referred to the lamina principal directions. When the principal material directions of any lamina are not coincident with the laminate x-y coordinate system, the constitutive relations for each lamina must be referred to the laminate x-y coordinate system. This is accomplished using the transformation matrix [T], defined as

$$[T] = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2-n^2 \end{bmatrix}$$

and the inverse $[T]^{-1}$ defined as

$$[T]^{-1} = \begin{bmatrix} m^2 & n^2 & -2mn \\ n^2 & m^2 & 2mn \\ mn & -mn & m^2-n^2 \end{bmatrix} \tag{3.3}$$

where

$$m = \cos\theta$$
$$n = \sin\theta.$$

Theta (θ), the angle between the lamina natural coordinate system and the laminate x-y coordinate system, is shown in Fig. 4. Using the transformation matrix to relate the stresses and strains in the natural coordinate system to those in the x-y coordinate system leads to the relationship

$$\{\sigma_x\}_k = [\bar{Q}]_k\{\varepsilon_x\}_k. \tag{3.4}$$

Thus, for the k^{th} lamina, the stresses, $\{\sigma_x\}_k$, are related to the strains, $\{\varepsilon_x\}_k$, in the x-y coordinate system by the transformed lamina stiffness matrix, $[\bar{Q}]_k$, defined as

$$[\bar{Q}]_k = [T]_k^{-1}[Q][T]_k. \tag{3.5}$$

For symmetric laminates subjected to in-plane loading only, strains in each lamina are equal to the laminate midplane strain, $\{\varepsilon°\}$:

$$\{\varepsilon_x\}_k = \{\varepsilon°\}. \tag{3.6}$$

The stress resultants per unit length, $\{N\}$, are defined in terms of the stresses by the relationship

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \int_{-H}^{H} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} dz, \tag{3.7}$$

where 2H is the total laminate thickness. Combining equations (3.4), (3.6), and (3.7) and rewriting the integral in terms of a summation over all n plies, the force resultants are

$$\{N\} = (\sum_{k=1}^{n} [\bar{Q}]_k(h_k-h_{k-1}))\{\varepsilon°\}. \tag{3.8}$$

From equation (3.8), the extensional stiffness matrix A_{ij} is defined

$$A_{ij} = \sum_{k=1}^{n} (\bar{Q}_{ij})_k (h_k - h_{k-1}), \tag{3.9}$$

so that the force resultant relationship can be written

$$\{N\} = [A]\{\varepsilon^\circ\}. \tag{3.10}$$

The average laminate stresses, $\{\bar{\sigma}\}$, are defined

$$\{\bar{\sigma}\} = \frac{1}{2H}\{N\}. \tag{3.11}$$

Combining equations (3.10) and (3.11) and considering appropriate non-zero $\{\bar{\sigma}\}$ terms, the effective laminate engineering constants are:

$$\begin{aligned}
E_x &= \frac{A_{11}A_{22}-A_{12}^2}{2HA_{22}} \\
E_y &= \frac{A_{11}A_{22}-A_{12}^2}{2HA_{11}} \\
\nu_{xy} &= -\frac{A_{12}}{A_{22}} \\
\nu_{yx} &= -\frac{A_{12}}{A_{11}} \\
G_{xy} &= \frac{A_{66}}{2H}\ .
\end{aligned} \tag{3.12}$$

The laminate thermal coefficients of expansion, $\{\bar{\alpha}\}$, are derived in a similar manner. When the lamina thermal strains, $\{\varepsilon_x^T\}_k$, are included the lamina constitutive equation (3.4) becomes

$$\{\sigma_x\}_k = [\bar{Q}]_k(\{\varepsilon^\circ\}-\{\varepsilon_x^T\}) \tag{3.13}$$

and with zero force resultants, equation (3.8) becomes

$$0 = \sum_{k=1}^{n} [\bar{Q}]_k(\{\varepsilon^\circ\}-\{\varepsilon_x^T\}_k)(h_k-h_{k-1}). \tag{3.14}$$

Solving (3.14) for the laminate thermal strains $\{\varepsilon^i\}$ results in

$$\{\varepsilon^i\} = [A]^{-1} \sum_{k=1}^{n} [\bar{Q}]_k\{\varepsilon_x^T\}_k(h_k-h_{k-1}) \tag{3.15}$$

and for uniform temperature change ΔT, the lamina thermal strains are

$$\{\varepsilon_x^T\}_k = \{\alpha_x\}_k\Delta T. \tag{3.16}$$

Now defining the laminate thermal coefficients $\{\bar{\alpha}\}$ as

$$\{\bar{\alpha}\} = \frac{1}{\Delta T} \{\varepsilon^i\} \tag{3.17}$$

and combining (3.15), (3.16), and (3.17) gives the relationship

$$\{\bar{\alpha}\} = [A]^{-1} \sum_{k=1}^{n} [\bar{Q}]_k\{\alpha_x\}_k(h_k-h_{k-1}). \tag{3.18}$$

3.2 The Finite Element Formulation

The finite element method is a versatile technique for obtaining approximate solutions for structures subjected to loads. Text book accounts can be found in Desai and Abel [33] and Zienkiewicz [34], among others. This section describes the theory of the two-dimensional finite element program used in this study. The analysis considers orthotropic materials in plane stress due to mechanical and thermal loading.

The program uses both triangular and quadrilateral elements with the quadrilateral elements being composed of four triangles. All triangles are constant strain elements. Convergence requirements for the finite element solution are satisfied by using a linear displacement model for the in-plane displacements u and v in the x and y directions, respectively;

$$u = \beta_1+\beta_2 x+\beta_3 y$$
$$v = \beta_4+\beta_5 x+\beta_6 y, \tag{3.19}$$

where the generalized coordinate coefficients, β_i, are functions of the initial location of generic nodes 1, 2, and 3 and are given by

$$\begin{Bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{Bmatrix} = \frac{1}{D} \begin{bmatrix} x_2y_3-x_3y_2 & y_3-y_2 & x_3-x_2 \\ x_1y_3-x_3y_1 & y_3-y_1 & x_3-x_1 \\ x_1y_2-x_2y_1 & y_2-y_1 & x_2-x_1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$
$$\begin{Bmatrix} \beta_4 \\ \beta_5 \\ \beta_6 \end{Bmatrix} = \frac{1}{D} \begin{bmatrix} x_2y_3-x_3y_2 & y_3-y_2 & x_3-x_2 \\ x_1y_3-x_3y_1 & y_3-y_1 & x_3-x_1 \\ x_1y_2-x_2y_1 & y_2-y_1 & x_2-x_1 \end{bmatrix} \begin{Bmatrix} v_1 \\ v_2 \\ v_3 \end{Bmatrix} \tag{3.20}$$

where

$$D = \begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix} .$$

The general strain-displacement relationships are

$$\varepsilon_x = \frac{\partial u}{\partial x}$$

$$\varepsilon_y = \frac{\partial v}{\partial y} \qquad (3.21)$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

and combining equations (3.20) and (3.21) leads to the specific strain-displacement relationship

$$\{\varepsilon\} = [B]\{u\} \qquad (3.22)$$

where the strain-displacement matrix, [B], and the transposed nodal displacement matrix, $\{u\}^T$, are defined

$$[B] = \frac{1}{2A} \begin{bmatrix} y_2-y_3 & y_3-y_1 & y_1-y_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_3-x_2 & x_1-x_3 & x_2-x_1 \\ x_3-x_2 & x_1-x_3 & x_2-x_1 & y_2-y_3 & y_3-y_1 & y_1-y_2 \end{bmatrix} \qquad (3.23)$$

and

$$\{u\}^T = [u_1 \quad u_2 \quad u_3 \quad v_1 \quad v_2 \quad v_3], \qquad (3.24)$$

where A is the area of the element.

The thermoelastic constitutive relationship for an orthotropic laminate is

$$\{\bar{\sigma}\} = [a](\{\varepsilon\}-\{\varepsilon^i\}), \qquad (3.25)$$

where the strain that contributes to the laminate stress is the difference between the total midplane strain, $\{\varepsilon\}$, and the strain caused only by initial (free thermal) strains, $\{\varepsilon^i\}$ (eq. 3.15). The terms of the orthotropic laminate stiffness matrix, [a], referenced to the x-y coordinate system, are defined

$$a_{11} = E_x/(1-\nu_{xy}\nu_{yx})$$

$$a_{22} = E_y/(1-\nu_{xy}\nu_{yx})$$

$$a_{12} = \nu_{yx}E_x/(1-\nu_{xy}\nu_{yx}) = \nu_{xy}E_y/(1-\nu_{xy}\nu_{yx}) \tag{3.26}$$

$$a_{66}=G_{xy}$$

$$a_{16} = a_{26} = 0.$$

The total potential energy, Π, of a system is the sum of the strain energy, U, and the potential energy of the external loads, W:

$$\Pi = U + W. \tag{3.27}$$

Considering equations (3.22-3.25), U and W are, respectively

$$U = \frac{1}{2} \iiint\limits_V \{\varepsilon\}^T[a]\{\varepsilon\}dv - \iiint\limits_V \{\varepsilon\}^T[a]\{\varepsilon^i\}dv$$

$$+ \frac{1}{2} \iiint\limits_V \{\varepsilon^i\}^T[a]\{\varepsilon^i\}dv \tag{3.28}$$

$$W = - \{u\}^T\{Q\}$$

where V is the elemental volume and the transposed mechanical nodal load vector, $\{Q\}^T$, is defined as

$$\{Q\}^T = [q_1 \quad q_2 \quad q_3 \quad p_1 \quad p_2 \quad p_3]. \tag{3.29}$$

The variables q_i and p_i are mechanical loads applied in the x and y directions, respectively, at the i^{th} node of the element. Minimization of the total potential energy of an element (eq. 3.27) leads to the equilibrium equations

$$[k]\{u\} = \{Q\} + \{Q^i\} \tag{3.30}$$

where the relationships for the elemental stiffness matrix, [k], and the initial load vector, $\{Q^i\}$, are

$$[k] = \iiint_V [B]^T[a][B]dv \tag{3.31}$$

$$\{Q^i\} = \iiint_V [B]^T[a]\{\varepsilon^i\}dv \tag{3.32}$$

The total stiffness matrix [K] and the total load vector {R} for the entire structure is formed by combining the elemental stiffness matrices and the total load vector $(\{Q\}+\{Q^i\})$ for all the elements resulting in the system of linear equations

$$[K]\{X\} = \{R\}. \tag{3.33}$$

For N number of nodes the total stiffness matrix will be 2N x 2N and the total load vector and node displacement vector {X} will be 2N. These equations relate the stiffnesses and nodal displacements of the entire structure to the applied loads and hence this formulation is called the direct stiffness method. Geometric boundary conditions are then imposed and a solution is obtained for the node displacement vector, {X}.

4. THE SLOTTED SHEAR SPECIMEN

4.1 General

The slotted shear specimen was analyzed using the basic finite element representation shown in Fig. 5. The specimen was always loaded to simulate uniform axial and zero lateral displacement at the ends of the specimen. These boundary conditions are idealizations of machine grips loading a flat coupon specimen in tension. The effects of laminate properties were studied for three graphite-epoxy laminates. The [0], [90], and $[\pm 45]_s$ laminates were chosen because they offer substantial variations in orthotropic properties. The [0] laminate exhibits a low E_x/E_y value; the [90] laminate exhibits a high E_x/E_y value; and the $[\pm 45]_s$ laminate exhibits a high G_{xy}/E_y value.

Elementary rigid body statics assumes the shear stress distribution in the test section between the ends of the slots to be pure, uniform, and maximum and independent of the laminate properties. Actual stress contour plots for the laminates studied are shown in Figs. 6-8. All stresses have been normalized with respect to the average axial stress away from the test section. Since a limited parametric study on the slot spacing, S (ranging from 0.2W to W), and slot depth, D (ranging from 0.25W to 0.75W), showed no significant changes in the stress distributions, the slot spacing, slot depth, and slot width (T) were standardized to S = 0.6W, D = 0.5W, and T = 0.0625W for all subsequent studies. As indicated in the figures, the finite element results did exhibit the expected symmetries.

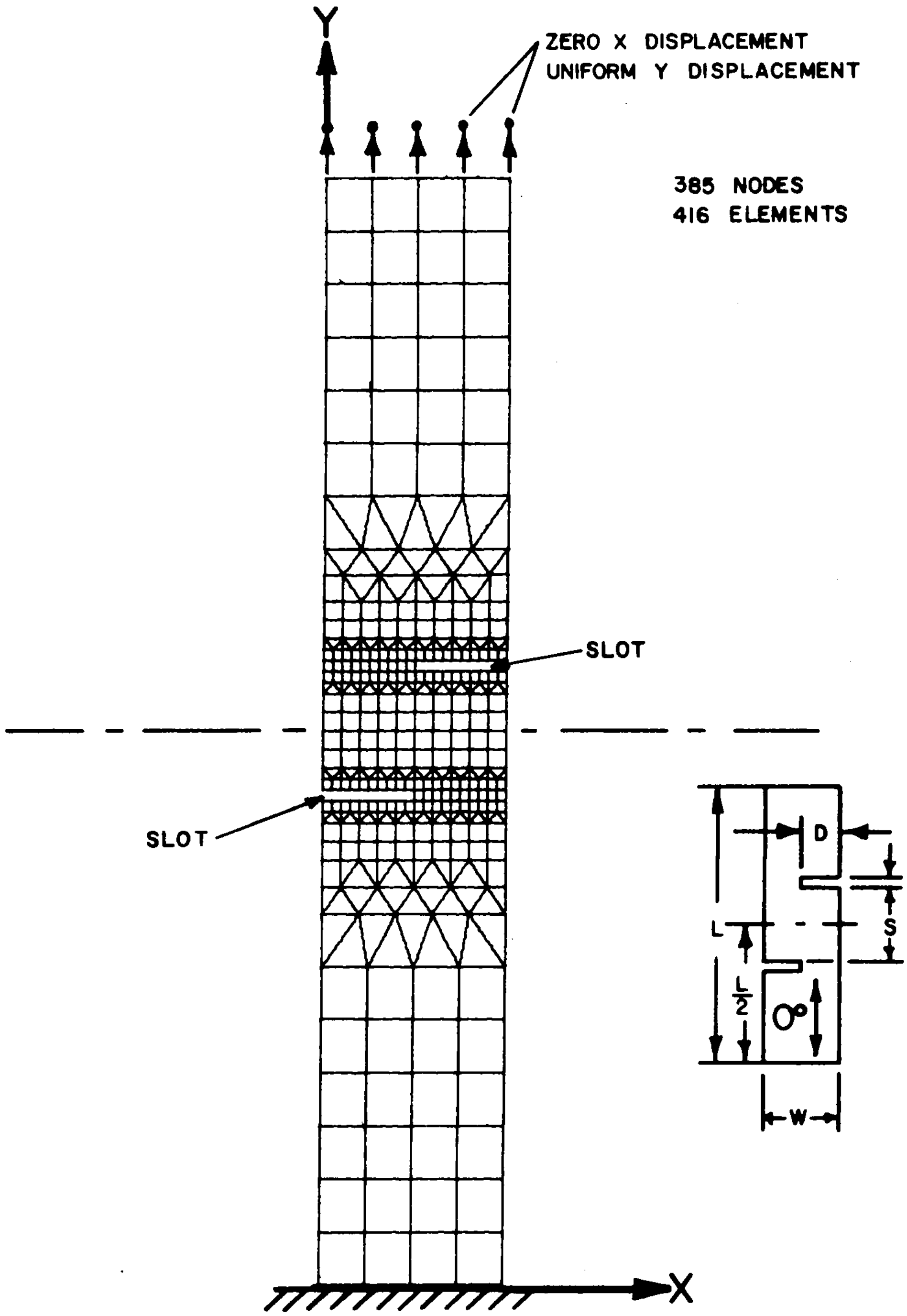


FIGURE 5. FINITE ELEMENT REPRESENTATION OF SLOTTED SHEAR SPECIMEN

4.2 [0] Laminate

The normalized stress contours for the [0] slotted shear specimen are shown in Fig. 6. The $\sigma_x/\bar{\sigma}$ and $\sigma_y/\bar{\sigma}$ contour plots clearly indicate stress concentrations at the slot ends ($\sigma_x/\bar{\sigma}$ = 0.4 and $\sigma_y/\bar{\sigma}$ = 8.0) with the $\sigma_y/\bar{\sigma}$ stress remaining at 4.0 along the centerline of the test section. The $\sigma_x/\bar{\sigma}$ stress is insignificant ($\sigma_x/\bar{\sigma}$ = -0.065) at the center of the specimen. The normalized shear stress remains essentially constant throughout the test section at -0.6; the maximum magnitude is -0.8 near the slot ends. Although this stress distribution is acceptable in principle, this region may be too narrow for practical strain measurements.

Since a region of uniform shear stress occurs in the test section, the [0] slotted specimen may be used for qualitative shear measurements. However, stress concentrations at the slot ends and a rather small area of uniform shear stress make this specimen unacceptable for quantitative shear measurements.

4.3 [90] and $[\pm 45]_s$ Laminates

The contour plots for [90] and $[\pm 45]_s$ graphite-epoxy laminates are shown in Figs. 7 and 8. The stress distributions for these laminates are clearly unacceptable for a shear specimen. As well as having $\sigma_x/\bar{\sigma}$ and $\sigma_y/\bar{\sigma}$ stress concentrations at or near the slot ends, the $\tau_{xy}/\bar{\sigma}$ contour plots indicate that the state of shear is not maximum or uniform (for the [90] laminate, $\tau_{xy}/\bar{\sigma}$ ranges from -2.0 to 0.25 and for the $[\pm 45]_s$ laminate, $\tau_{xy}/\bar{\sigma}$ ranges from -1.0 to -0.4).

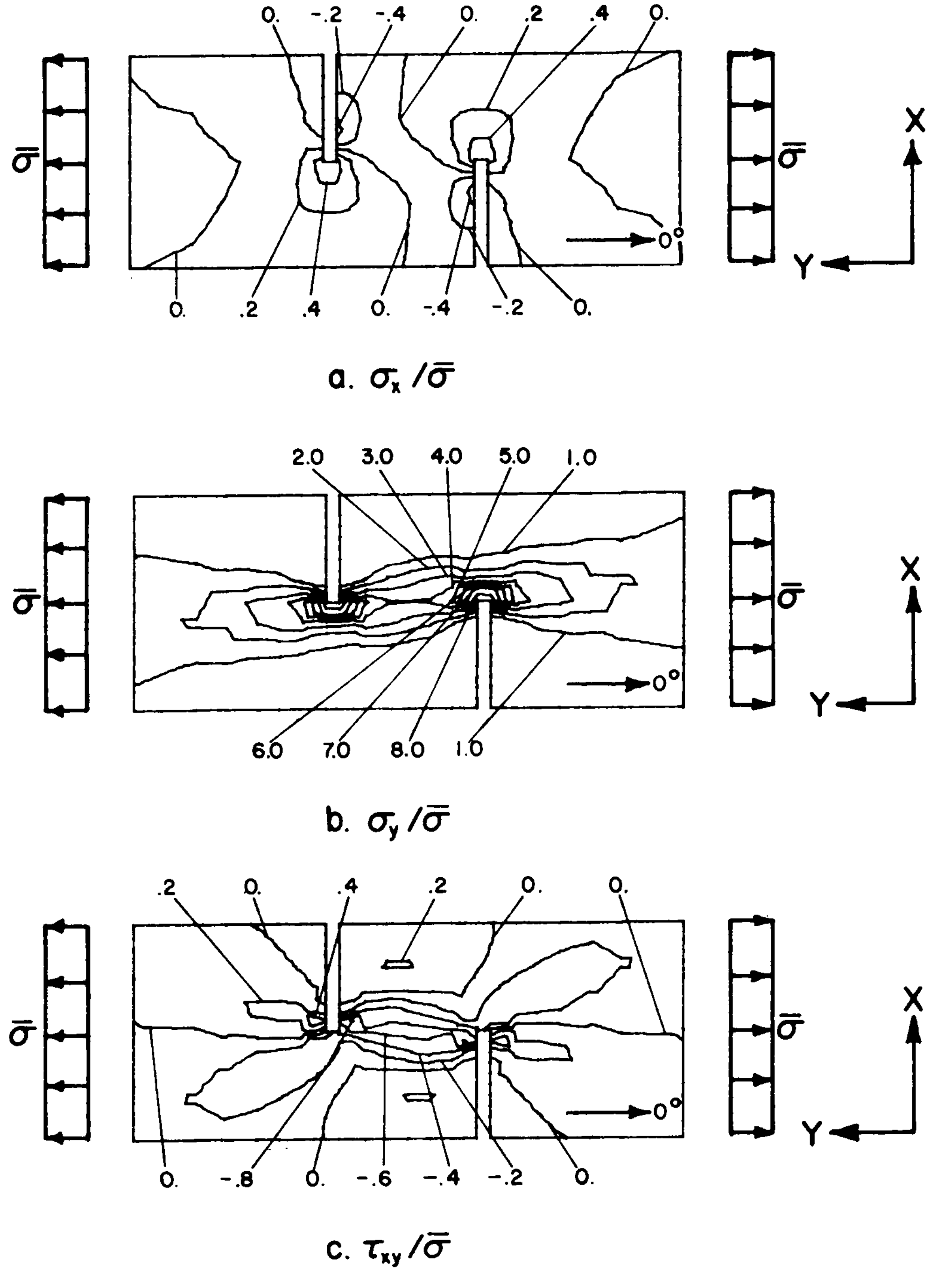
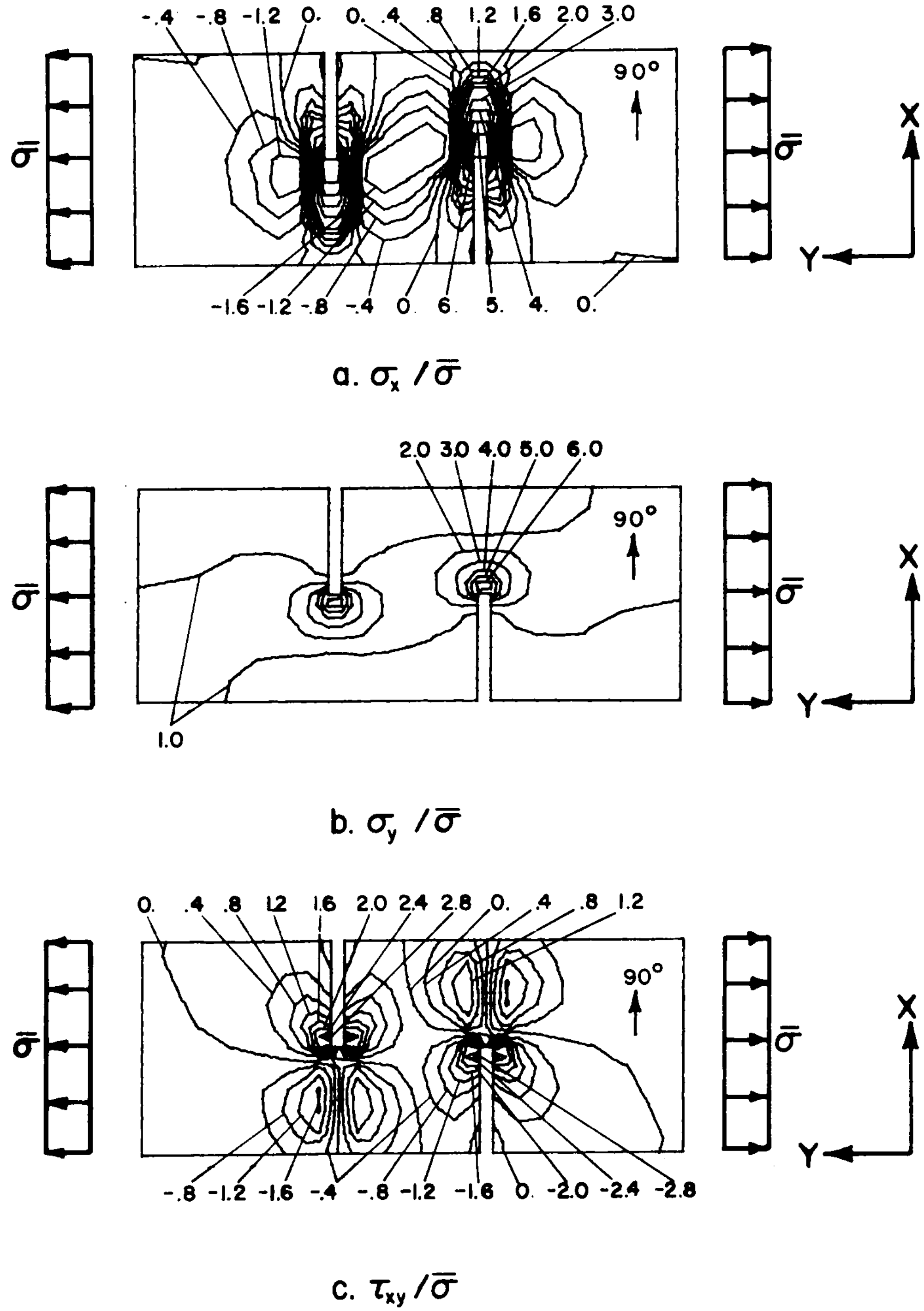


FIGURE 6. NORMALIZED STRESS CONTOURS FOR [0] SLOTTED SHEAR SPECIMEN, S=0.6W, D=0.5W, T=0.0625W



a. $\sigma_x / \bar{\sigma}$

b. $\sigma_y / \bar{\sigma}$

c. $\tau_{xy} / \bar{\sigma}$

FIGURE 7. NORMALIZED STRESS CONTOURS FOR [90] SLOTTED SHEAR SPECIMEN, S=0.6W, D=0.5W, T=0.0625W

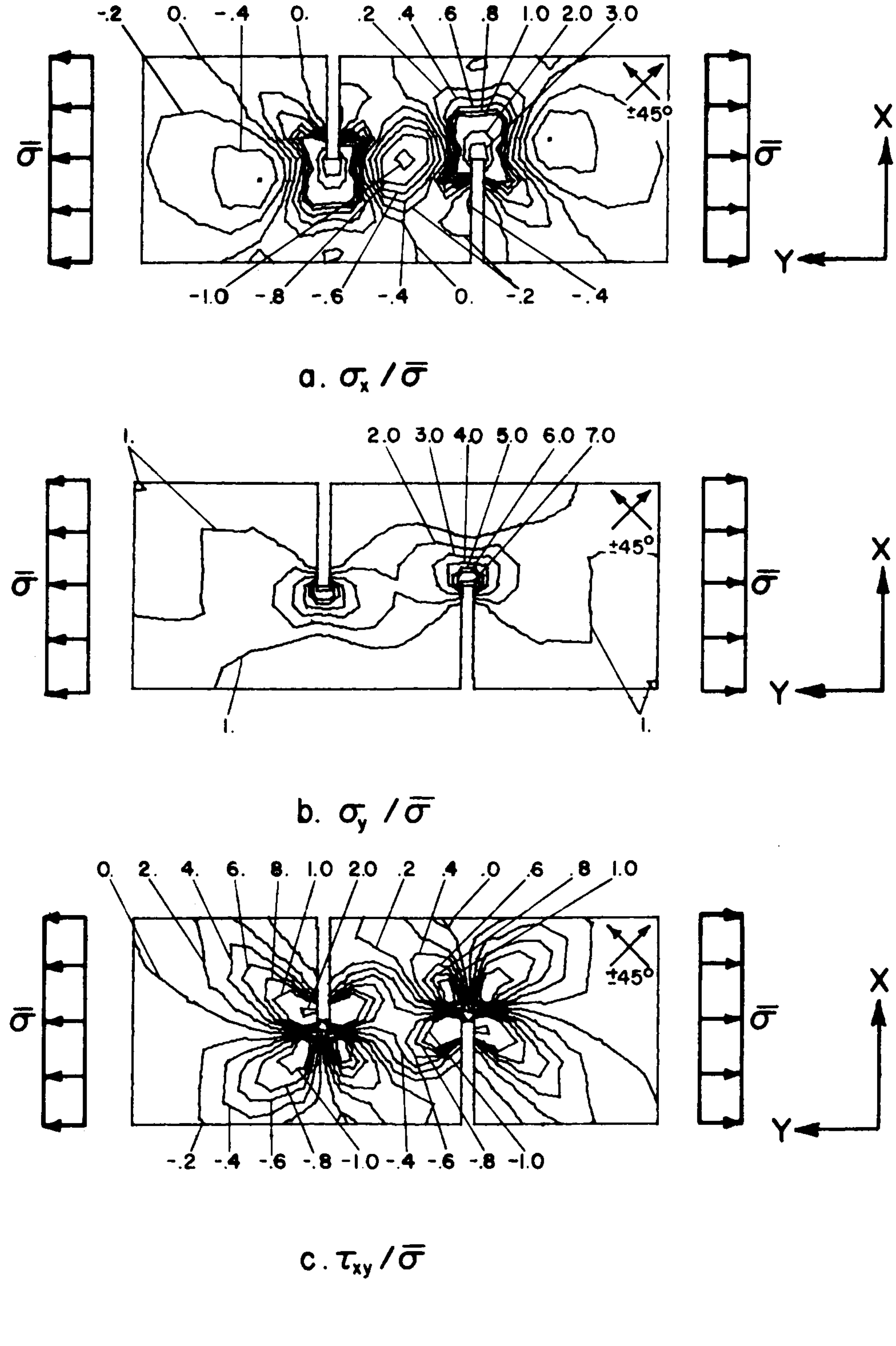


FIGURE 8. NORMALIZED STRESS CONTOURS FOR $[\pm 45]_s$ SLOTTED SHEAR SPECIMEN, S=0.6W, D=0.5W, T=0.0625W

4.4 Summary

For the [0], [90], and $[\pm 45]_s$ laminates, the slotted shear specimen exhibits σ_x, σ_y, and τ_{xy} stress concentrations in the vicinity of the slot ends as expected. As a result of such stress concentrations, high normal stresses were often found in the test section. For the [90] and $[\pm 45]_s$ laminates, $\tau_{xy}/\bar{\sigma}$ is very low in magnitude (for the [90] laminate $\tau_{xy}/\bar{\sigma}$ = 0.25 and for the $[\pm 45]_s$ laminate $\tau_{xy}/\bar{\sigma}$ = -0.4) and nonuniform in the test section. Obviously this slotted specimen does not provide acceptable shear data for these laminates. The normalized shear stress in the test section of the [0] laminate is somewhat uniform (with the exception of a small area in the vicinity of each slot end), however, this area of high shear stress is rather narrow for strain measurements. Also, the normalized σ_y is significant in the test section for the [0] laminate. In conclusion, the stress distributions of the slotted shear specimens analyzed were significantly influenced by laminate properties and none of the laminates studied were found to be acceptable for providing reliable quantitative shear data.

5. THE CROSS BEAM SHEAR SPECIMEN

5.1 General

As mentioned previously, the cross beam shear specimen shown in Fig. 2a has been used to obtain shear stress-strain response for both $[\pm 45]_s$ and $[0/90]_s$ laminates. When analyzing the results from such a test it is assumed that the test section is subjected to uniform biaxial normal stresses of equal magnitude and opposite sign resulting from the application of positive and negative bending moments to the cross beam. For this state of biaxial stress, stresses on 45° planes are pure shear and equal to the magnitude of the applied average normal stresses. The purpose of this analysis is to study the influence of corner effects, laminate properties, and, to a limited extent, core materials on the stress distribution in the test section.

The elastic analysis of the cross beam shear specimen involved the following assumptions. The composite material was assumed to be in a state of plane stress. This neglected any transverse stress components associated with the application of loads near the ends of the beam (Fig. 2a) and also interlaminar effects from the core in the test section. Loading and geometric symmetry allowed the analysis to be applied to only one quarter of the central portion of the specimen. These symmetry conditions were simulated by imposing zero x displacement along the y axis and zero y displacement along the x axis shown in Fig. 9.

Cross beam specimens usually are designed with two types of honeycomb core. Stiff, heavyweight core material is sandwiched between the

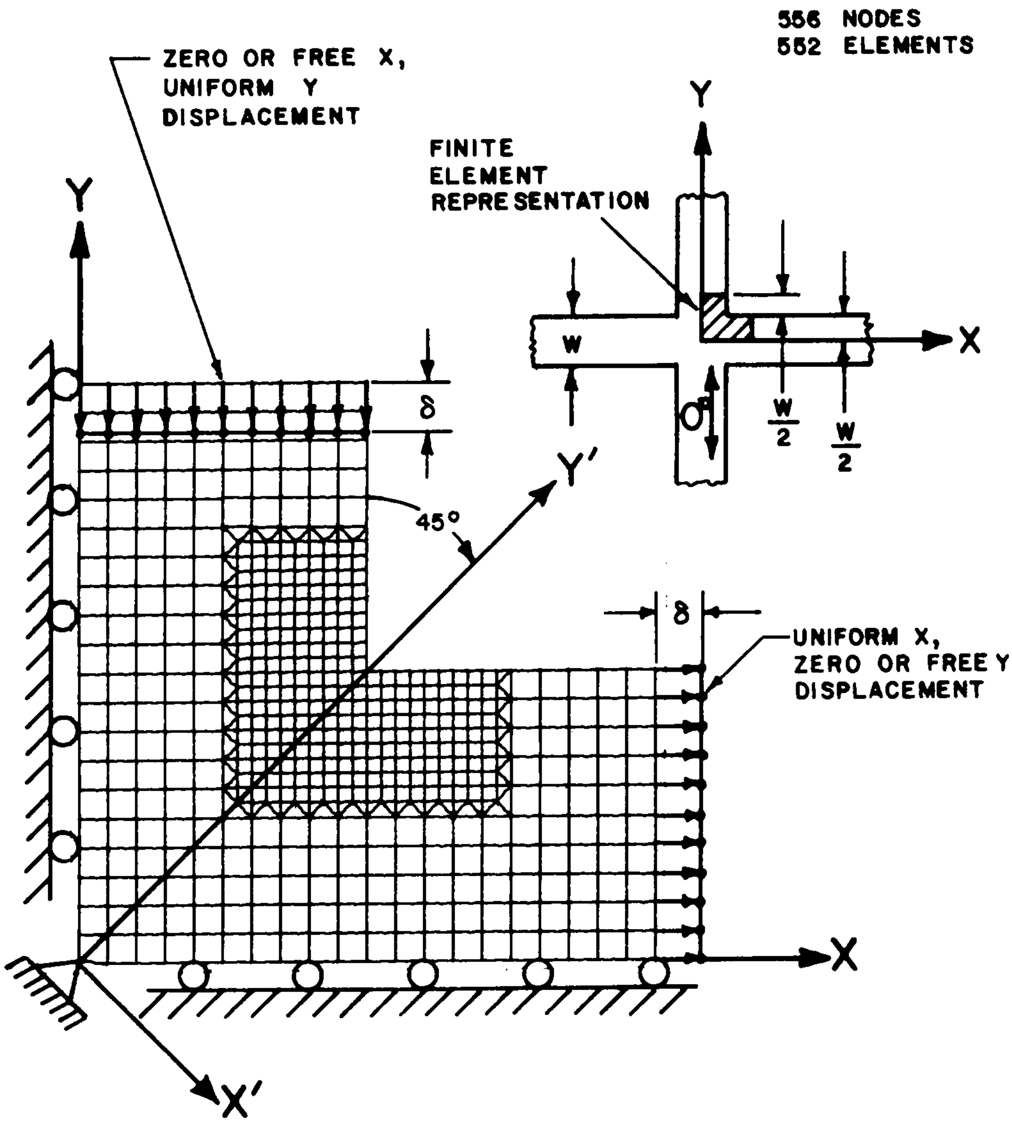


FIGURE 9. FINITE ELEMENT REPRESENTATION OF CROSS BEAM SHEAR SPECIMEN

composite material away from the central portion of the specimen (x and y $\geq$ W) to preclude local failure at the points of load application, while a more flexible, relatively light weight core material is used in the central portion (x and y < W). Because of the unknown net stiffening effects of core, displacements rather than forces were imposed as boundary conditions at x = W and y = W to represent the loading introduced by the bending moments. In order to study core stiffness effects away from the test section, two sets of lateral displacement boundary conditions were considered at x = W and y = W. "Rigid" core for x and y greater than W was simulated by fixing lateral displacements to zero across the width of the beam. "Flexible" core was simulated by allowing free lateral displacements across the width of the beam. The stiffness of the core material in the test section and all other core influences were always neglected.

Contour plots of the stresses on a 45° plane (x',y'), normalized with respect to the magnitude of the applied average normal stress, $\bar{\sigma}$, are shown in Figs. 10-14. These plots reflect the symmetric loading and geometry.

5.2 The $[\pm 45]_s$ Laminate

A cross beam with fibers at ±45° is used to determine shear stress-strain response of $[0/90]_s$ laminates. Such response is equivalent to the shear response of a lamina referenced to its natural coordinates. Figures 10-12 compare the rigid core results to the flexible core results for normalized σ_x', σ_y' and τ_{xy}' stress distributions.

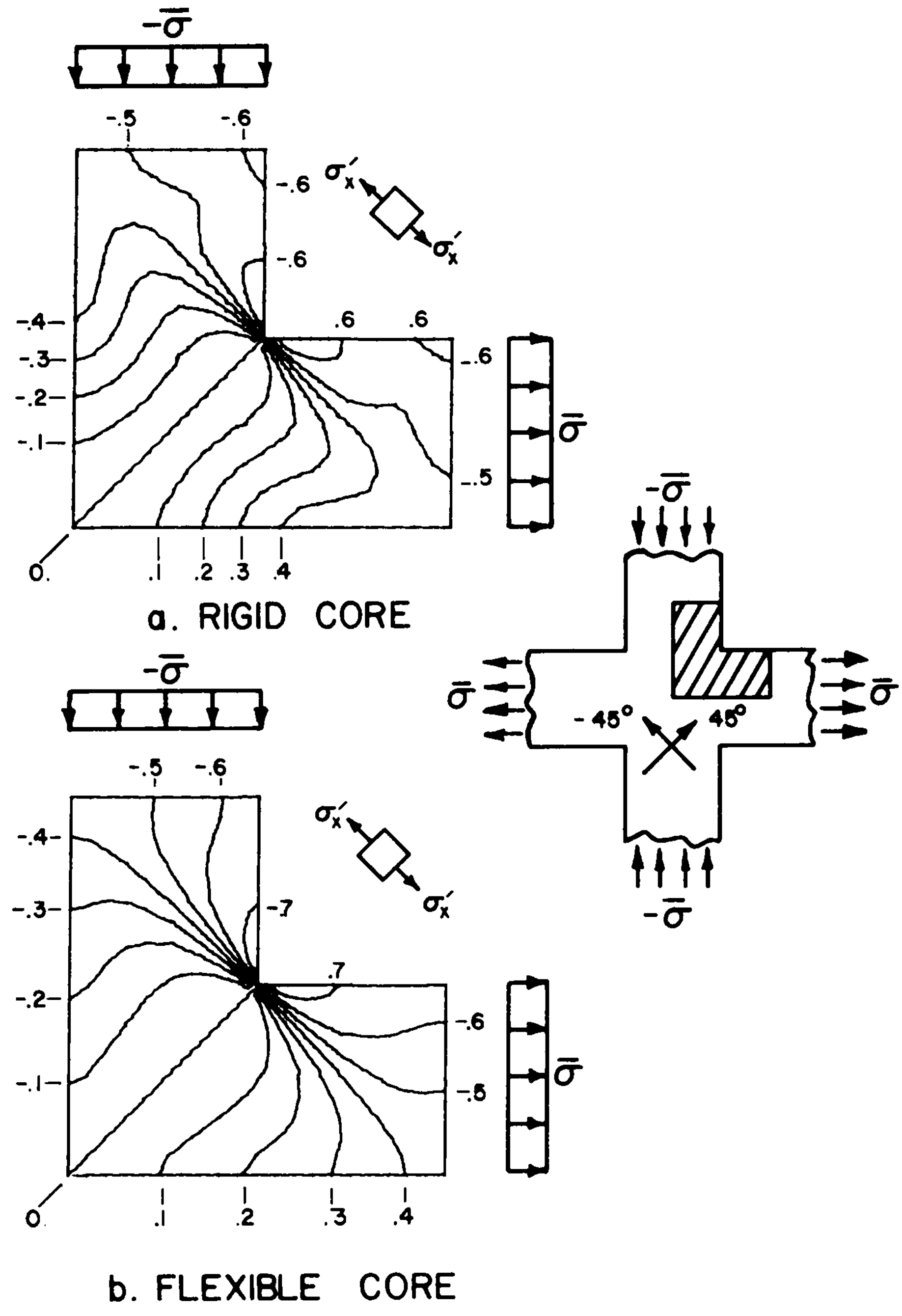


FIGURE 10. NORMALIZED σ_x' CONTOURS FOR $[\pm 45]_s$ CROSS BEAM

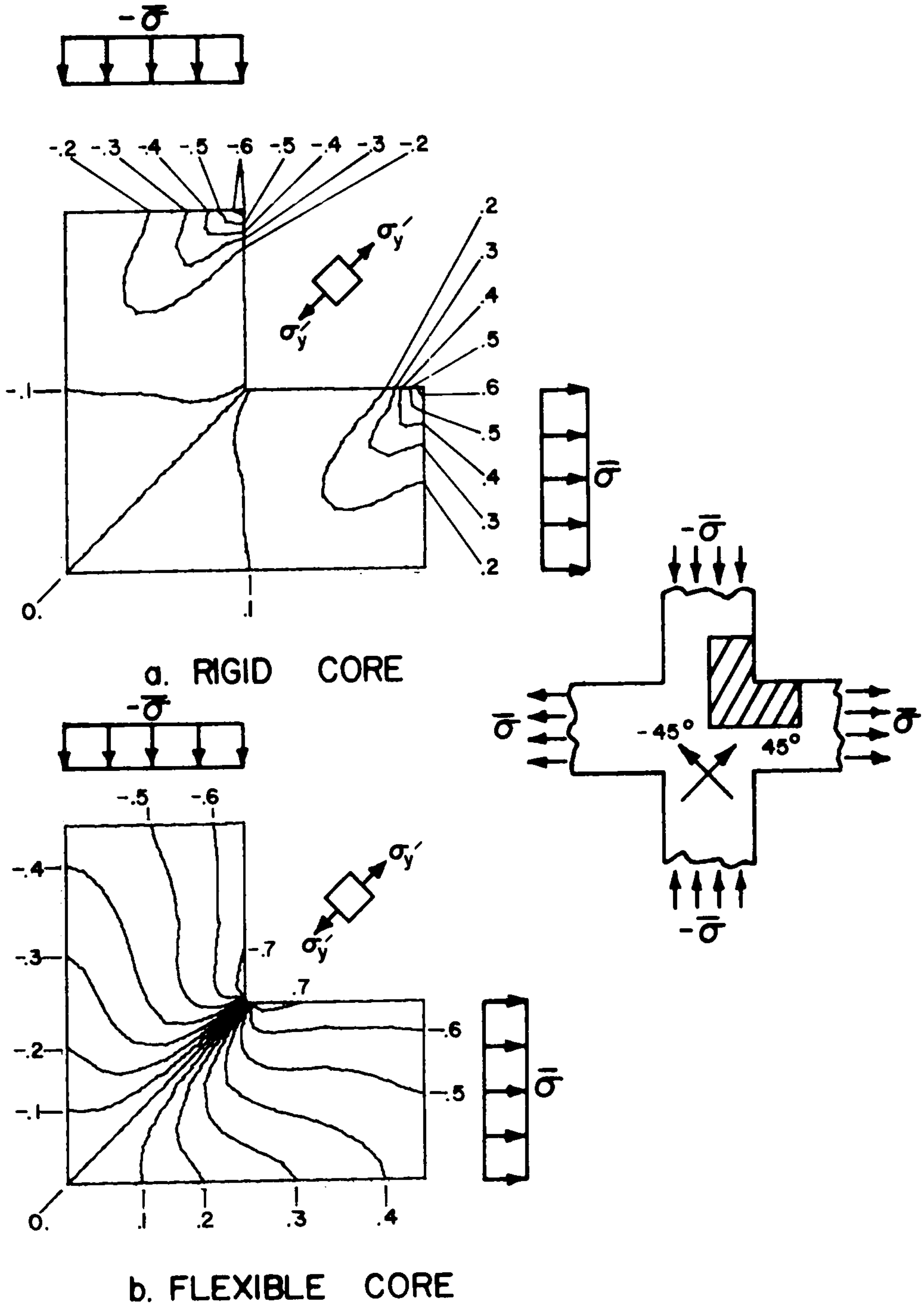


FIGURE 11. NORMALIZED σ_y' CONTOURS FOR $[\pm 45]_s$ CROSS BEAM

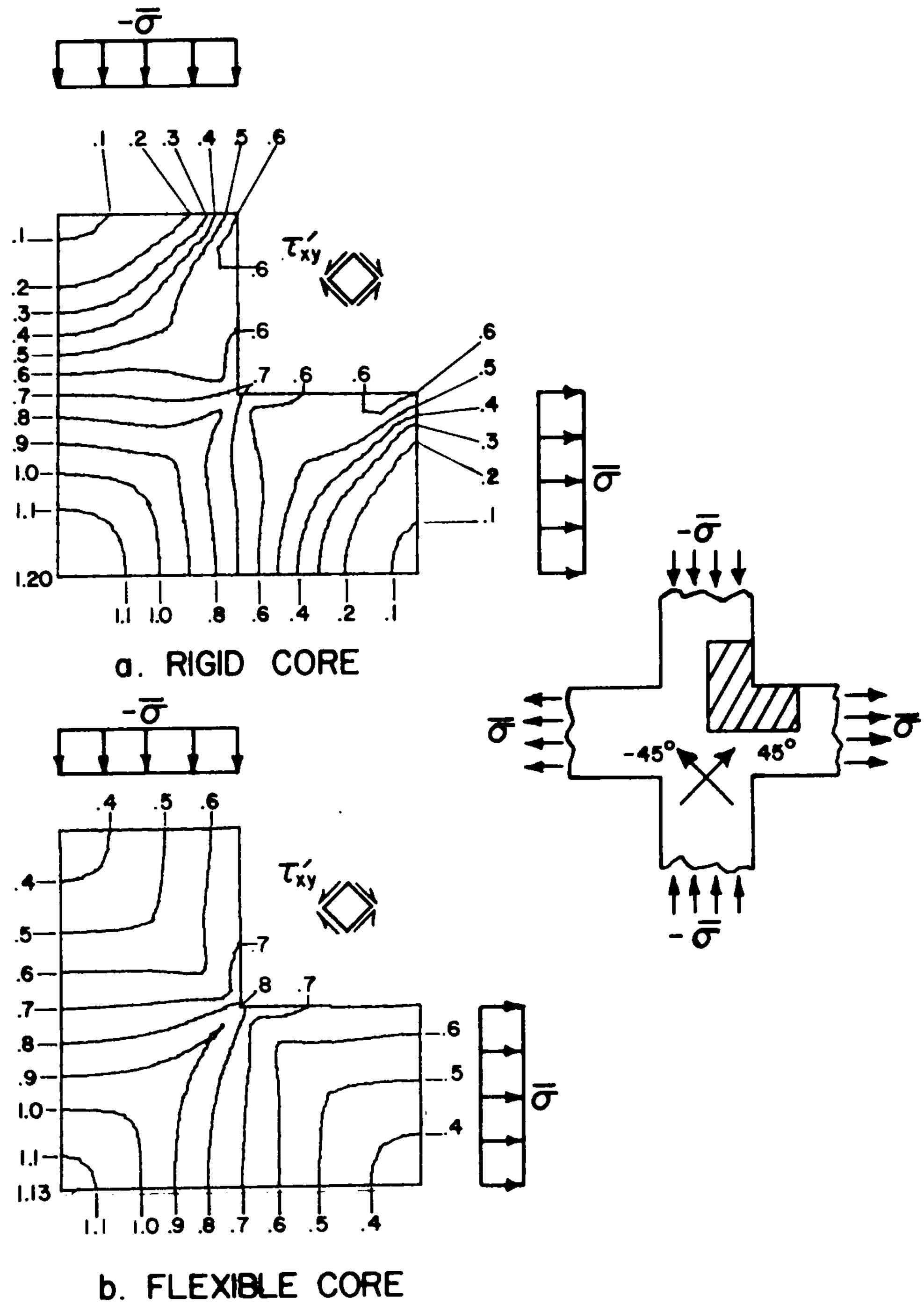


FIGURE 12. NORMALIZED τ'_{xy} CONTOURS FOR $[\pm 45]_s$ CROSS BEAM

For both cases the shear stress is maximum and pure over a finite region in the center of the specimen. This region is large enough for a strain gage to be used for accurate strain measurements. Although these are excellent features of the cross beam, the magnitude of $\tau'_{xy}/\bar{\sigma}$ at the center is significantly different from 1.0 because of the corner effects. Specifically, values of $\tau'_{xy}/\bar{\sigma}$ were found to be 1.20 and 1.13 for the rigid and flexible core boundary conditions, respectively.

Corner stresses for the rigid core boundary conditions were lower than those of the flexible core boundary conditions, especially for the $\sigma_y/\bar{\sigma}$ stress. With one exception (Fig. 11a), all normalized stress components are greater than 0.6 in the vicinity of the corner. This is significant and may contribute to premature failure in a mode different from pure shear.

In general the stress distributions in the $[\pm 45]_s$ cross beam are significantly more complex than those of the "ideal" cross beam. The magnitude of the shear stress in the test section is higher than 1.0 due to corner effects and stress concentrations at the corners undoubtedly contribute to any failures. However, this analysis indicates that the stress distribution in the center section is uniform pure shear. Thus, the elastic shear modulus may be determined knowing the magnitude of the shear stress in this region.

5.3 The $[0/90]_s$ Laminate

A cross beam with fibers at 0° and 90° is used to determine shear stress-strain response of a $[\pm 45]_s$ laminate. Finite element results for

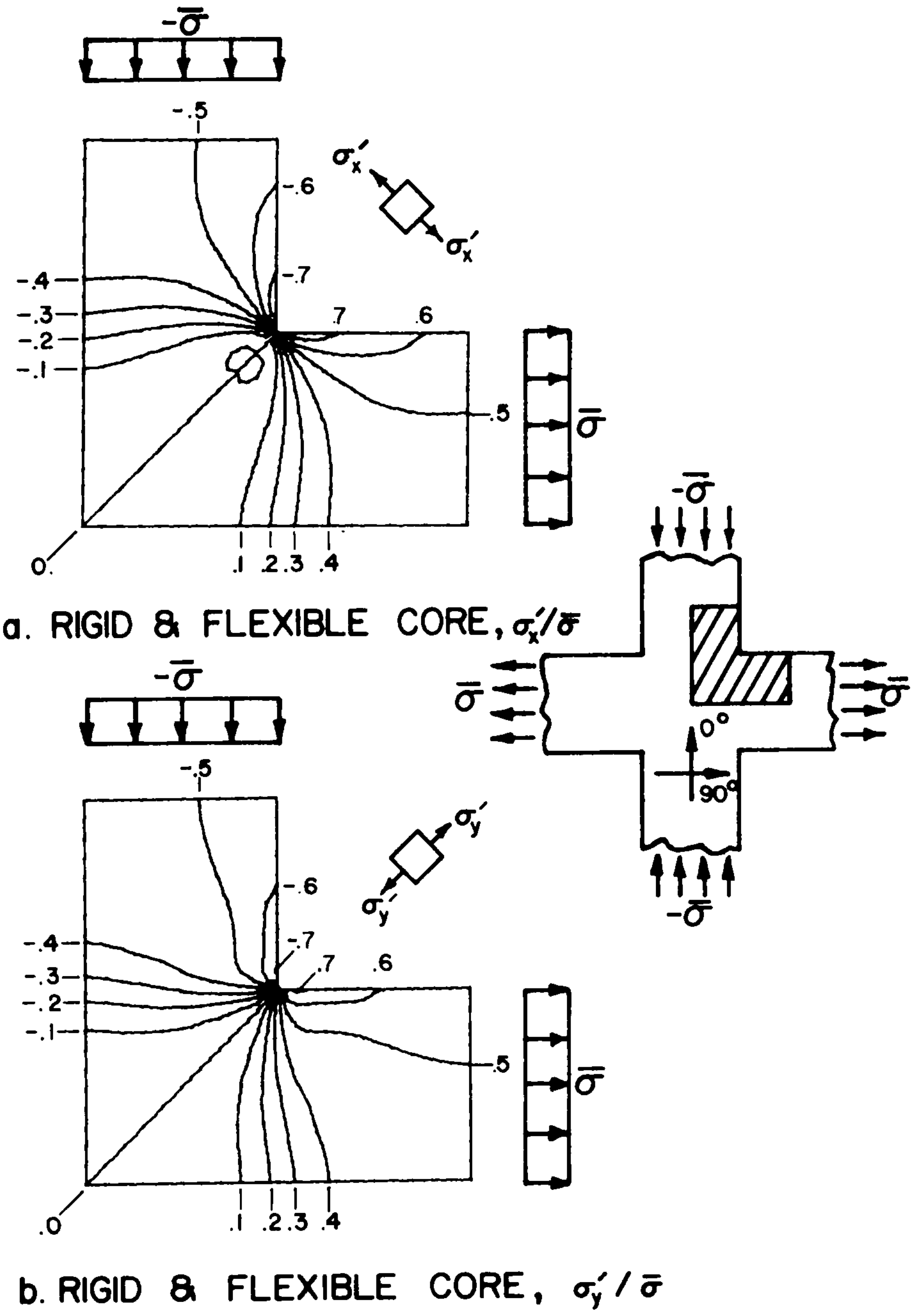


FIGURE 13. NORMALIZED σ_x' & σ_y' CONTOURS FOR $[0/90]_s$ CROSS BEAM

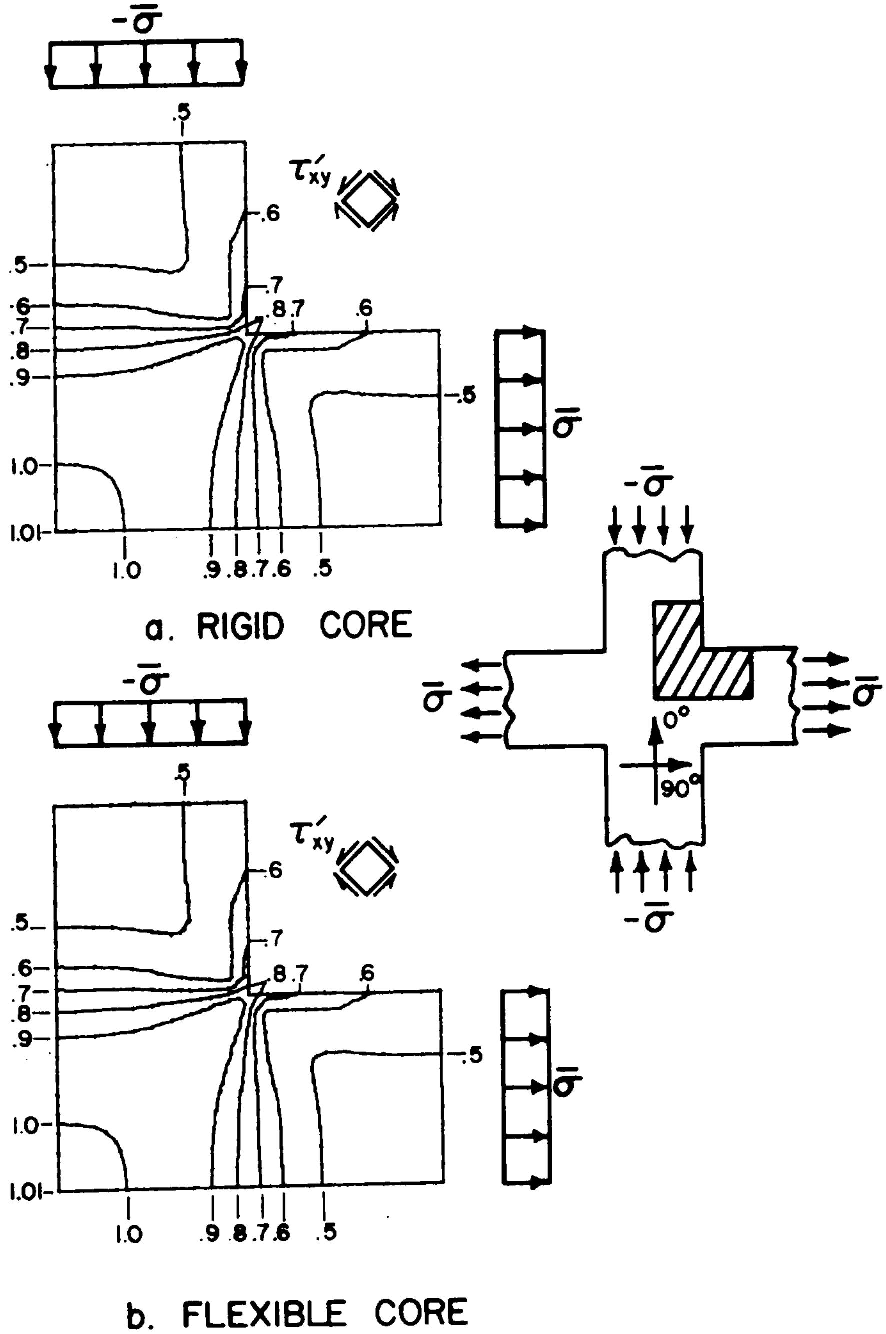


FIGURE 14. NORMALIZED τ'_{xy} CONTOURS FOR $[0/90]_s$ CROSS BEAM

both the rigid and flexible core boundary conditions were similar and thus specific results for both core types are shown only for the normalized τ'_{xy} stresses. The results are not affected appreciably by the lateral displacement boundary condition because the high lateral stiffness of the specimen results in very small lateral displacements.

As in the $[\pm 45]_s$ laminate, the $[0/90]_s$ laminate produces a maximum, pure shear region in the center of the specimen. This shear region has more characteristics of the "ideal" shear specimen. An area of maximum, uniform shear stress is present over nearly the entire test section and the maximum $\tau'_{xy}/\bar{\sigma}$ is very close to 1.00 (exactly 1.01). In the vicinity of the corner of the specimen all the normalized components of stress remain above 0.6, however.

The finite element analysis indicates that the $[0/90]_s$ laminate should provide reliable stress-strain response to a high degree of accuracy in the linear portion of the shear stress-strain curve. Although failures may initiate due to corner stresses, the core effects are minimal with the $[0/90]_s$ laminate.

5.4 Summary

For both the $[\pm 45]_s$ and the $[0/90]_s$ cross beam shear specimens, the maximum value of $\tau'_{xy}/\bar{\sigma}$ occurs at the center of the specimen and is suitable for strain measurements. However, the magnitude of $\tau'_{xy}/\bar{\sigma}$ at the center is dependent on the laminate properties, and for the $[\pm 45]_s$ laminate, $\tau'_{xy}/\bar{\sigma}$ is significantly higher than 1.0. For laminates that are relatively stiff across the width of the beam compared to their shear stiffness, the honeycomb core effects were shown to be minimal with this plane stress analysis.

Finite element results indicate the cross beam may be used with a $[0/90]_s$ laminate for linear response since its stress distribution is nearly "ideal" (with the exception of high corner stresses). The state of shear stress at the center of the $[\pm 45]_s$ laminate is influenced by the core stiffness and is significantly greater than 1.0 because of a low shear stress away from the center. Thus, the applied normal stress in the $[\pm 45]_s$ laminate is not simply related to the shear stress at the center of the specimen. However, initial shear modulus values may be determined if this relationship is known from a numerical study such as this one.

6. THE IOSIPESCU SPECIMEN

6.1 General

The specimen analyzed in this study is a modified Iosipescu specimen [3] in that the specimen is a flat laminate notched on two sides with the load introduced through fixtures bonded to the specimen. Iosipescu's original specimen was a bar notched on four sides and loads were applied directly to the specimen. This modified specimen was studied with both isotropic and orthotropic materials. In addition, several minor variations in the specimen configuration were studied as were the influence of elastic fixtures and thermal loads.

6.2 Rigid Fixture Results

6.2.1 General

Several laminates were studied assuming that the fixtures are rigid (Fig. 2b). The finite element mesh used to represent these specimens is shown in Fig. 15. Because a finite element analysis of the entire specimen indicated quarter symmetry of stresses and displacements, only one quarter of the specimen was modeled in subsequent studies. This was accomplished by applying uniform u and zero v displacements along the line $y = L/2$, and constraining the v displacements along $x = 0$ and u displacements along $y = 0$ as indicated in Fig. 15. The dimensions of the specimen were standardized to: notch depth, $A = 0.225W$; notch angle, $\theta = 90°$; and specimen length, $L = W$, where W is the width. Normalized stress contour plots for the rigid fixture assumption are shown in Figs. 16-21 and results are tabulated in Table 1. All stresses are normalized

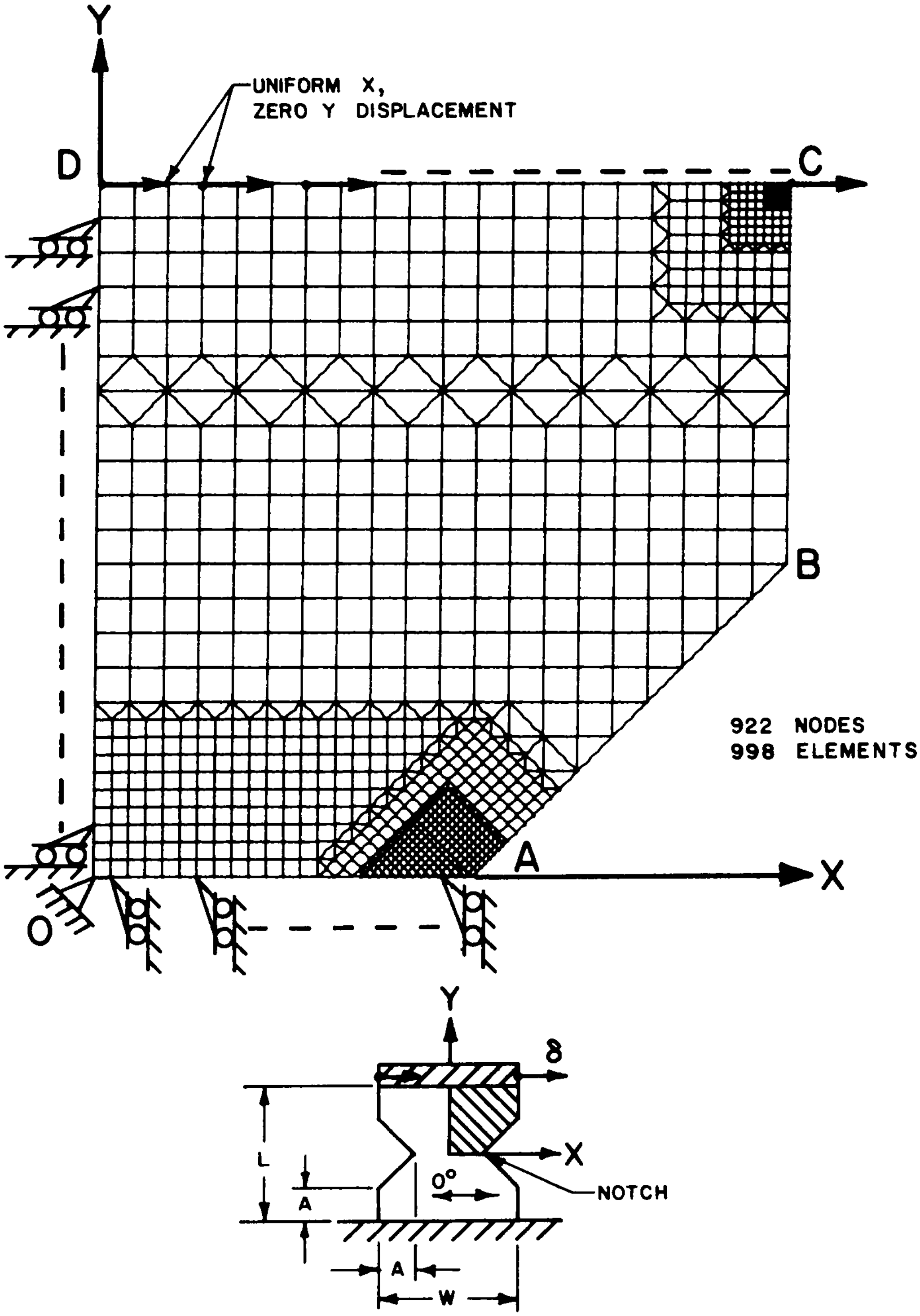


FIGURE 15. FINITE ELEMENT MODEL FOR IOSIPESCU SPECIMEN WITH RIGID FIXTURES

with respect to $\bar{\tau}$, the average shear stress along the line OA between the notches (Fig. 15).

6.2.2 Isotropic and Quasi-isotropic Materials

Normalized stress contours for an isotropic material (steel) and a quasi-isotropic material ($[0/90/\pm45]_s$ graphite-polyimide) are shown in Figs. 16 and 17, respectively. The two stress distributions are nearly identical exhibiting essentially uniform pure shear in the test section. The normalized shear stress along the centerline OA ranges from 0.97 at the center O to 1.12 in the vicinity of the notch tip A (Table 1). As expected, stress concentrations are present at the corner C. These linear elastic results indicate that this specimen produces a very satisfactory stress distribution for determining the linear shear response of isotropic and quasi-isotropic materials. They are in agreement with the photoelastic results obtained by Iosipescu. It is also evident from these results that failure will initiate at the notch tip A or the corner C due to the stress concentrations in these regions.

6.2.3 The [0] Laminate

Normalized stress contours for a [0] laminate are shown in Fig. 18. The shear stress is pure, uniform, and maximum at the center of the specimen (point O), but is not uniform along the centerline OA, ranging from 1.10 at the center O to 0.48 at the notch tip. Stress concentrations are again present at the corner C. These linear elastic results indicate that the stress distribution at the center of the [0] specimen is suitable for linear shear response, however, the magnitude of this normalized shear stress is greater than 1.0 due to the nonuniform shear

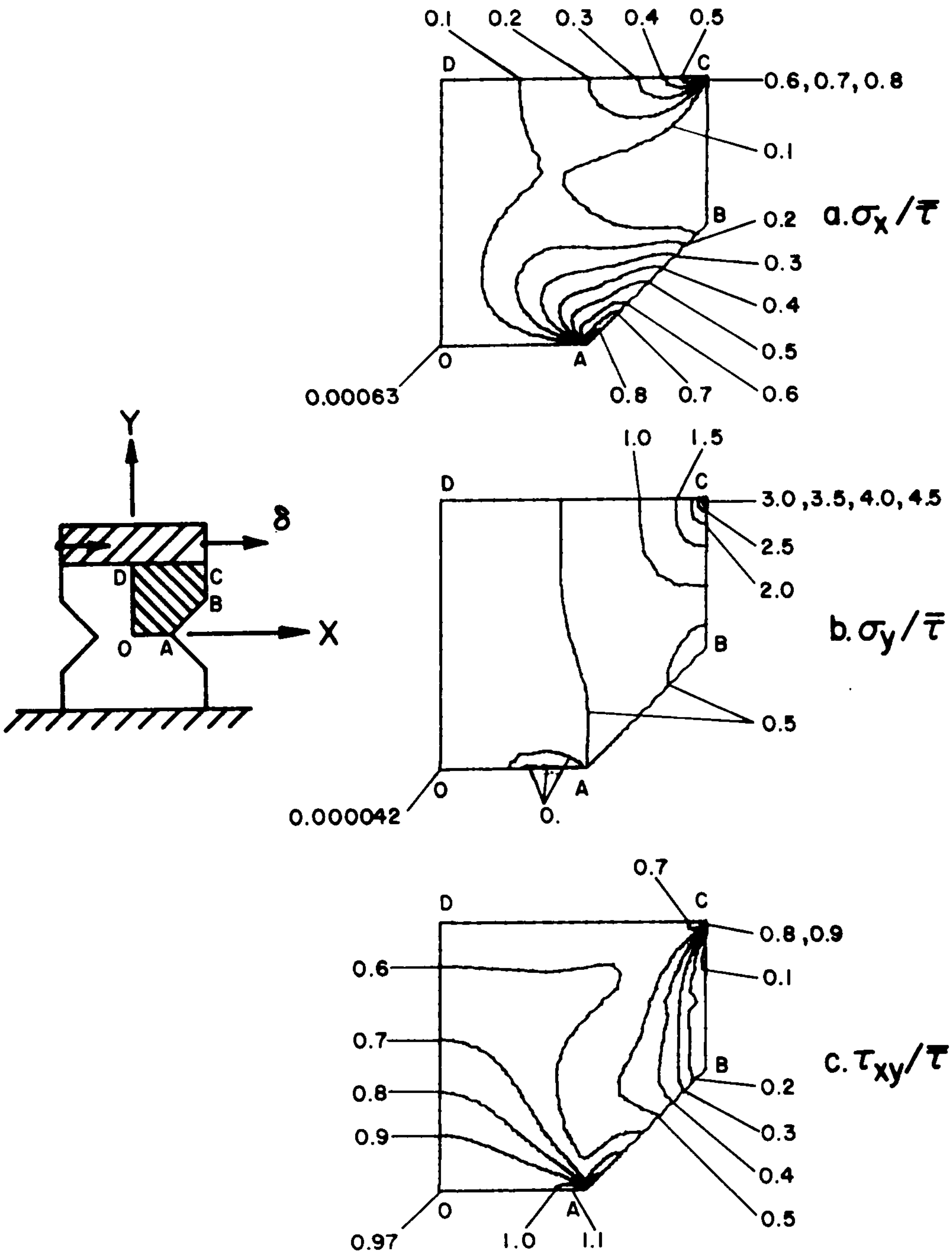


FIGURE 16. NORMALIZED STRESS CONTOURS FOR IOSIPESCU SPECIMEN WITH ISOTROPIC MATERIAL AND RIGID FIXTURES

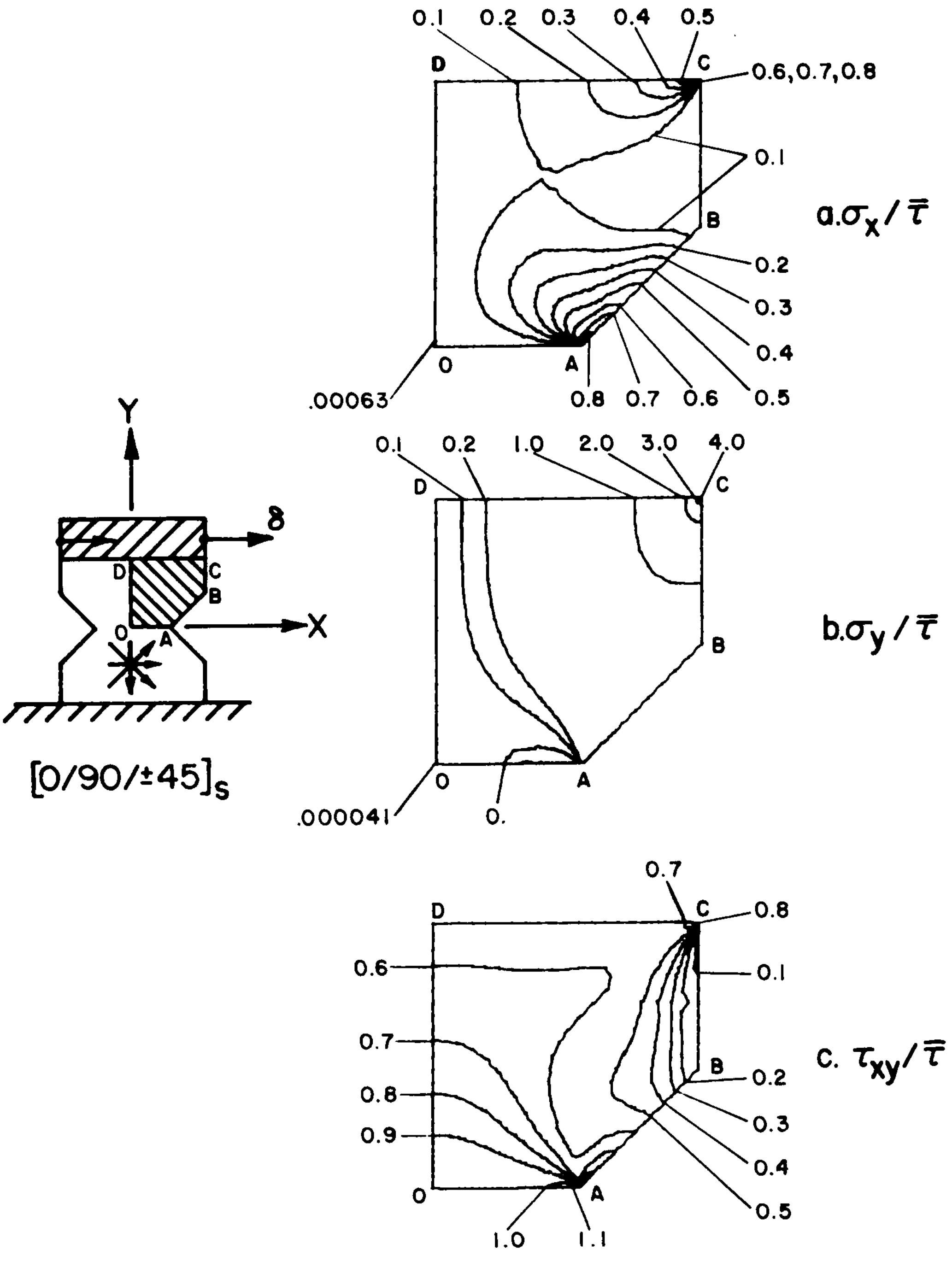


FIGURE 17. NORMALIZED STRESS CONTOURS FOR $[0/90/\pm 45]_s$ GRAPHITE-POLYIMIDE IOSIPESCU SPECIMEN WITH RIGID FIXTURES

stress distribution along the centerline OA. Since all stresses were rather low in magnitude at the notch tip, failure is expected at the corner C.

6.2.4 The [90] Laminate

Normalized stress contours for a [90] laminate are shown in Fig. 19. Although a pure, uniform shear stress at the center O is present, the distribution is very nonuniform along the centerline OA with normalized shear stress values ranging from 0.84 at the center to 1.94 at the notch tip A. Because this laminate exhibits a nonuniform shear stress distribution along OA, a stress factor relating the shear stress at the center of the specimen to the applied load (0.84) must be used in order to obtain accurate initial shear response. In addition, stress concentrations at the notch tip A will contribute to any failures.

6.2.5 The $[0/90]_s$ Laminate

Normalized stress contours for a $[0/90]_s$ laminate are shown in Fig. 20. The normalized shear stress along the centerline OA ranges from a maximum of 1.09 near the notch tip to a value of 0.96 at the center. As expected, stress concentrations are again present at the corner C. Since this laminate produces a very uniform region of pure shear in the test section, it will provide accurate linear shear response. These results also show that failure may initiate at the notch tip A or at the corner C due to the stress concentrations in these regions.

6.2.6 The $[\pm 45]_s$ Laminate

Normalized stress contours for a $[\pm 45]_s$ laminate are shown in Fig.

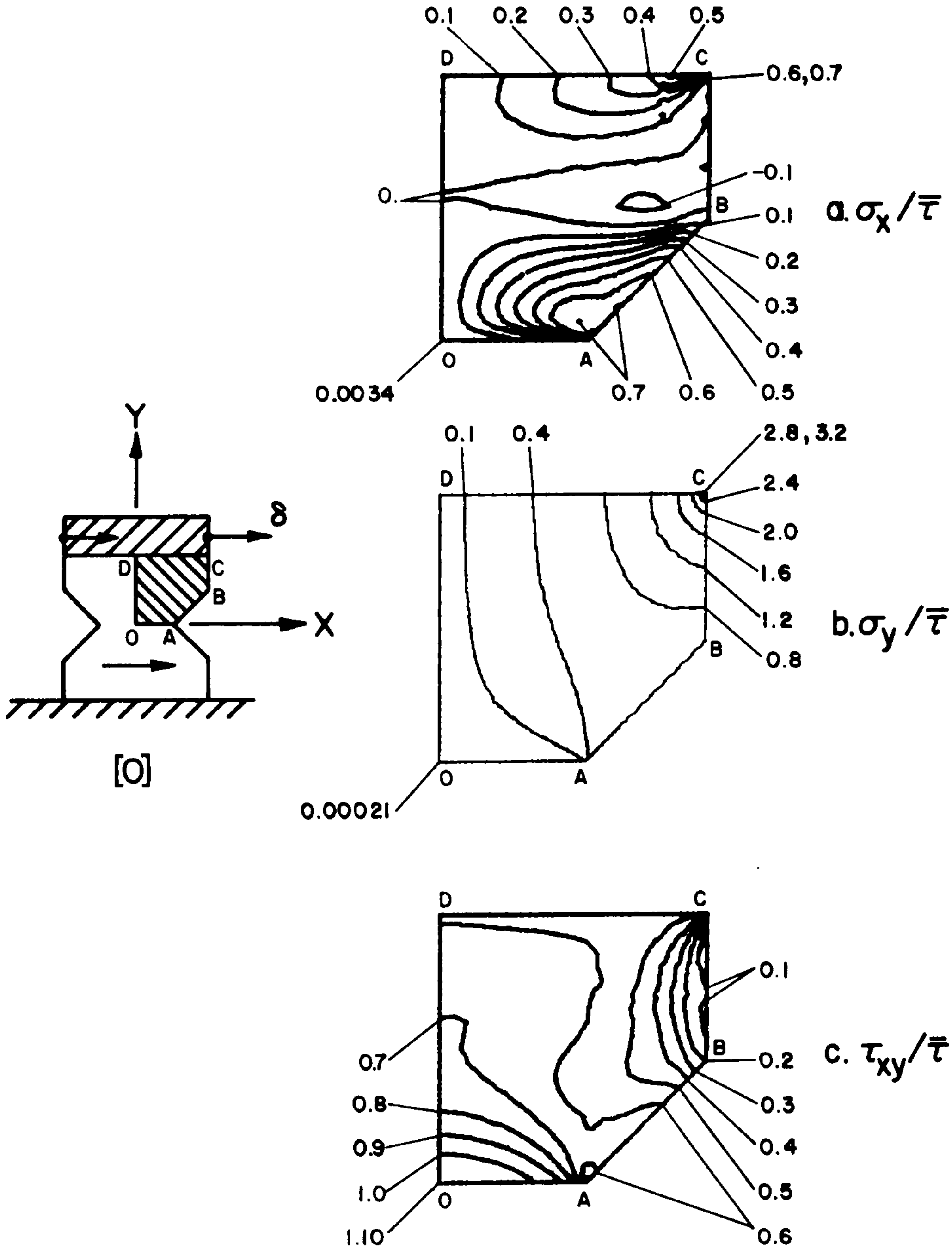


FIGURE 18. NORMALIZED STRESS CONTOURS FOR [0] GRAPHITE-POLYIMIDE IOSIPESCU SPECIMEN WITH RIGID FIXTURES

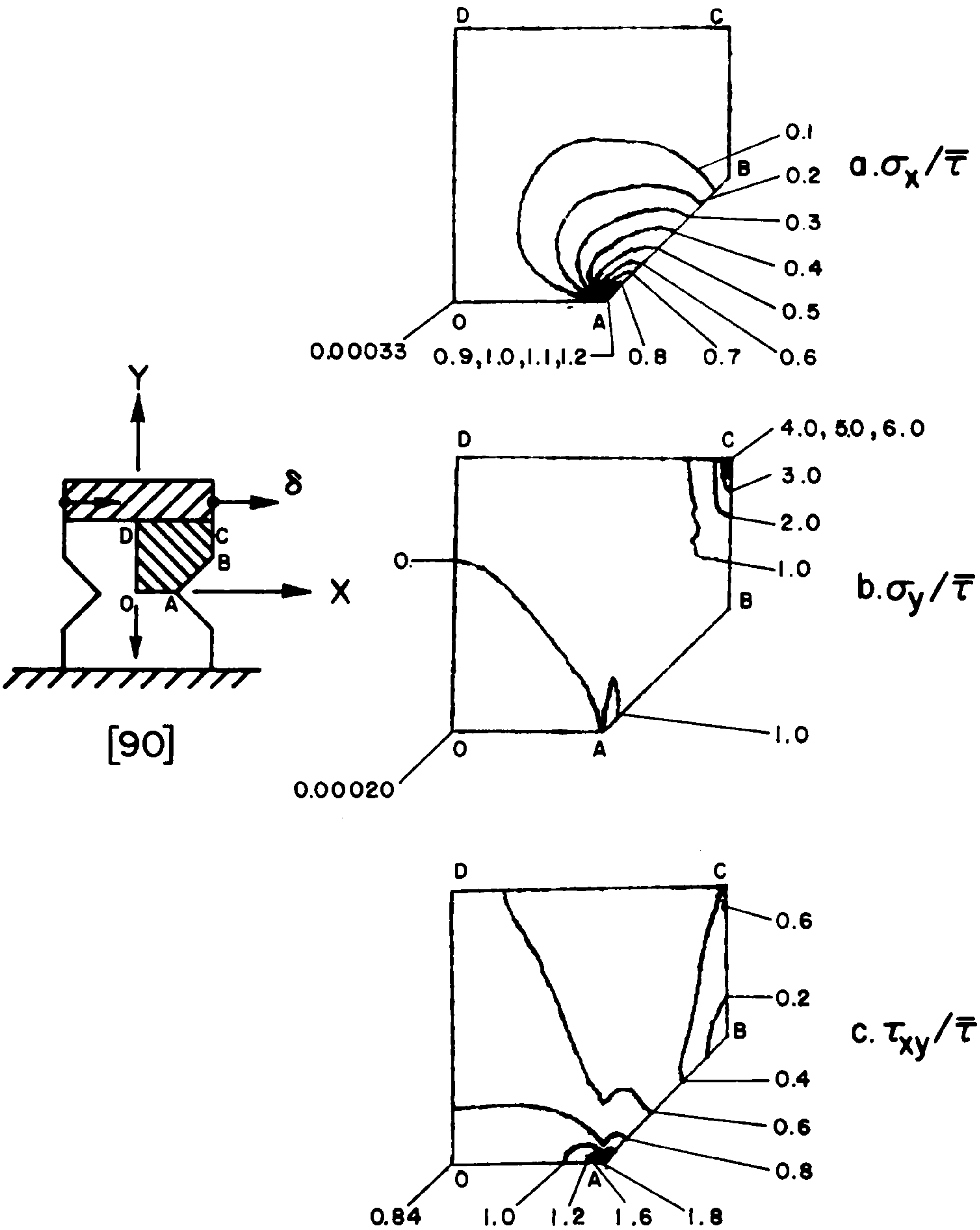


FIGURE 19. NORMALIZED STRESS CONTOURS FOR [90] GRAPHITE-POLYIMIDE IOSIPESCU SPECIMEN WITH RIGID FIXTURES

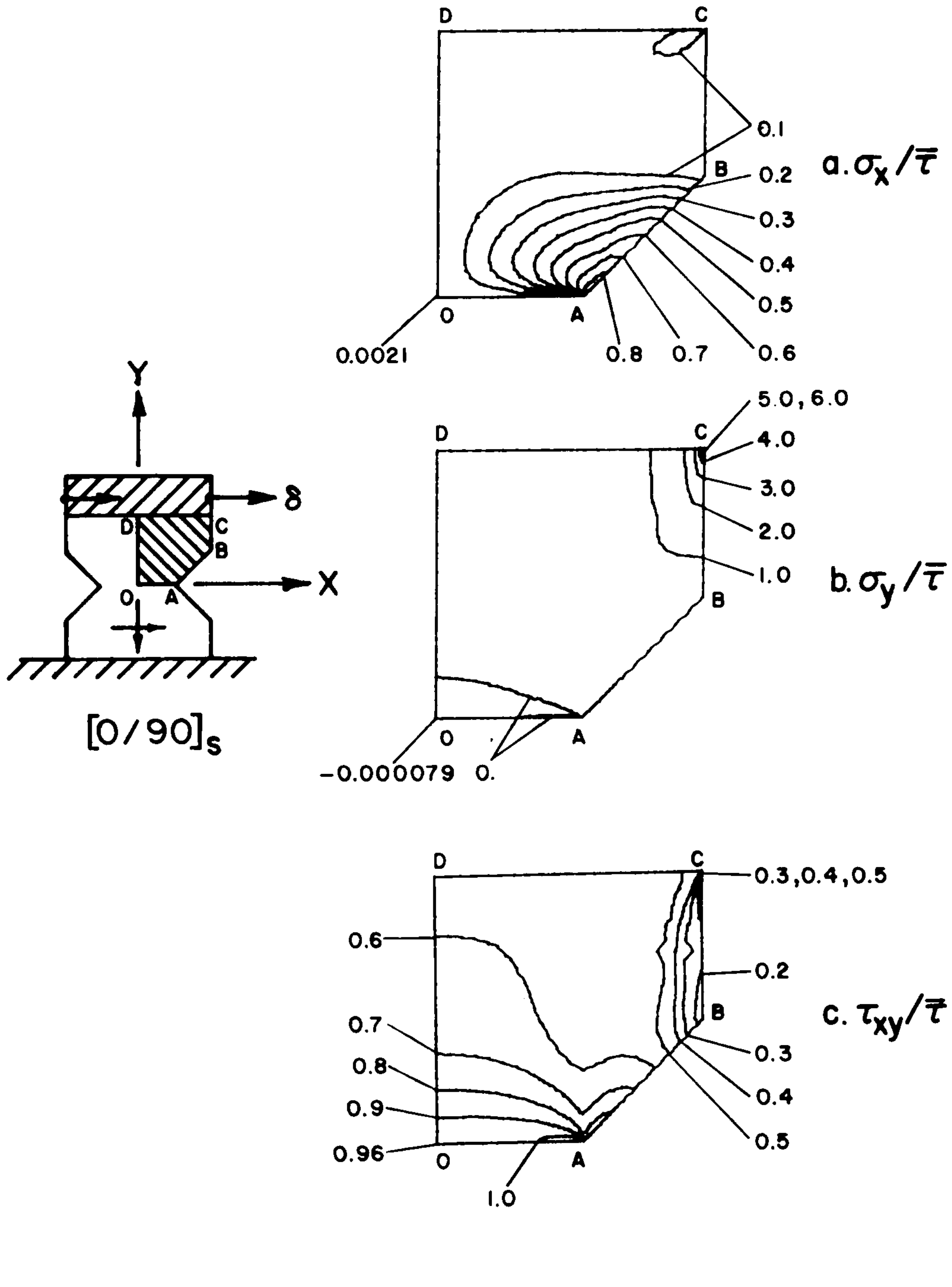


FIGURE 20. NORMALIZED STRESS CONTOURS FOR $[0/90]_s$ GRAPHITE-POLYIMIDE IOSIPESCU SPECIMEN WITH RIGID FIXTURES

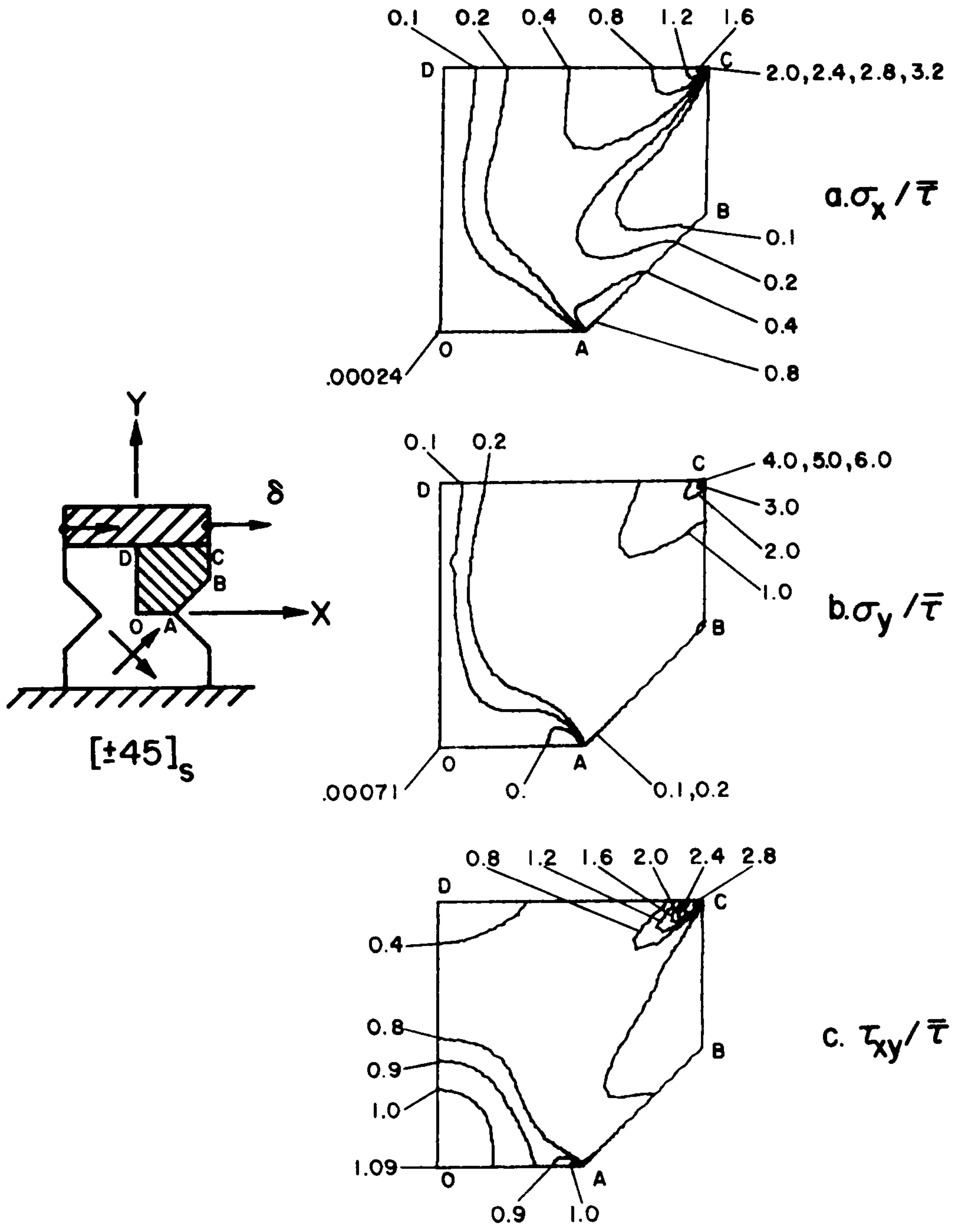


FIGURE 21. NORMALIZED STRESS CONTOURS FOR [±45] GRAPHITE-POLYIMIDE IOSIPESCU SPECIMEN WITH RIGID FIXTURES

TABLE 1. Normalized Stresses for Graphite-polyimide and Steel Iosipescu Specimen with Rigid Fixtures.

		O[1]	A[1]	C[1]
Steel	$\sigma_x/\bar{\tau}$	6.27×10^{-4}	0.42	0.89
	$\sigma_y/\bar{\tau}$	4.18×10^{-5}	0.35	4.70
	$\tau_{xy}/\bar{\tau}$	0.97	1.05	0.94
$[0/90/\pm45]_s$ Gr/Pi	$\sigma_x/\bar{\tau}$	6.28×10^{-4}	0.42	0.82
	$\sigma_y/\bar{\tau}$	4.08×10^{-5}	0.35	4.61
	$\tau_{xy}/\bar{\tau}$	0.97	1.05	0.88
[0] Gr/Pi	$\sigma_x/\bar{\tau}$	3.41×10^{-3}	0.25	0.57
	$\sigma_y/\bar{\tau}$	2.12×10^{-4}	0.18	3.26
	$\tau_{xy}/\bar{\tau}$	1.10	0.48	0.43
[90] Gr/Pi	$\sigma_x/\bar{\tau}$	3.34×10^{-4}	0.56	8.46×10^{-2}
	$\sigma_y/\bar{\tau}$	-2.02×10^{-4}	0.62	6.91
	$\tau_{xy}/\bar{\tau}$	0.84	1.95	0.38
$[0/90]_s$ Gr/Pi	$\sigma_x/\bar{\tau}$	2.06×10^{-3}	0.43	0.13
	$\sigma_y/\bar{\tau}$	-7.92×10^{-5}	0.34	6.26
	$\tau_{xy}/\bar{\tau}$	0.96	1.02	0.40
$[\pm45]_s$ Gr/Pi	$\sigma_x/\bar{\tau}$	2.38×10^{-4}	0.41	3.32
	$\sigma_y/\bar{\tau}$	7.18×10^{-4}	0.33	6.38
	$\tau_{xy}/\bar{\tau}$	1.09	1.06	3.17

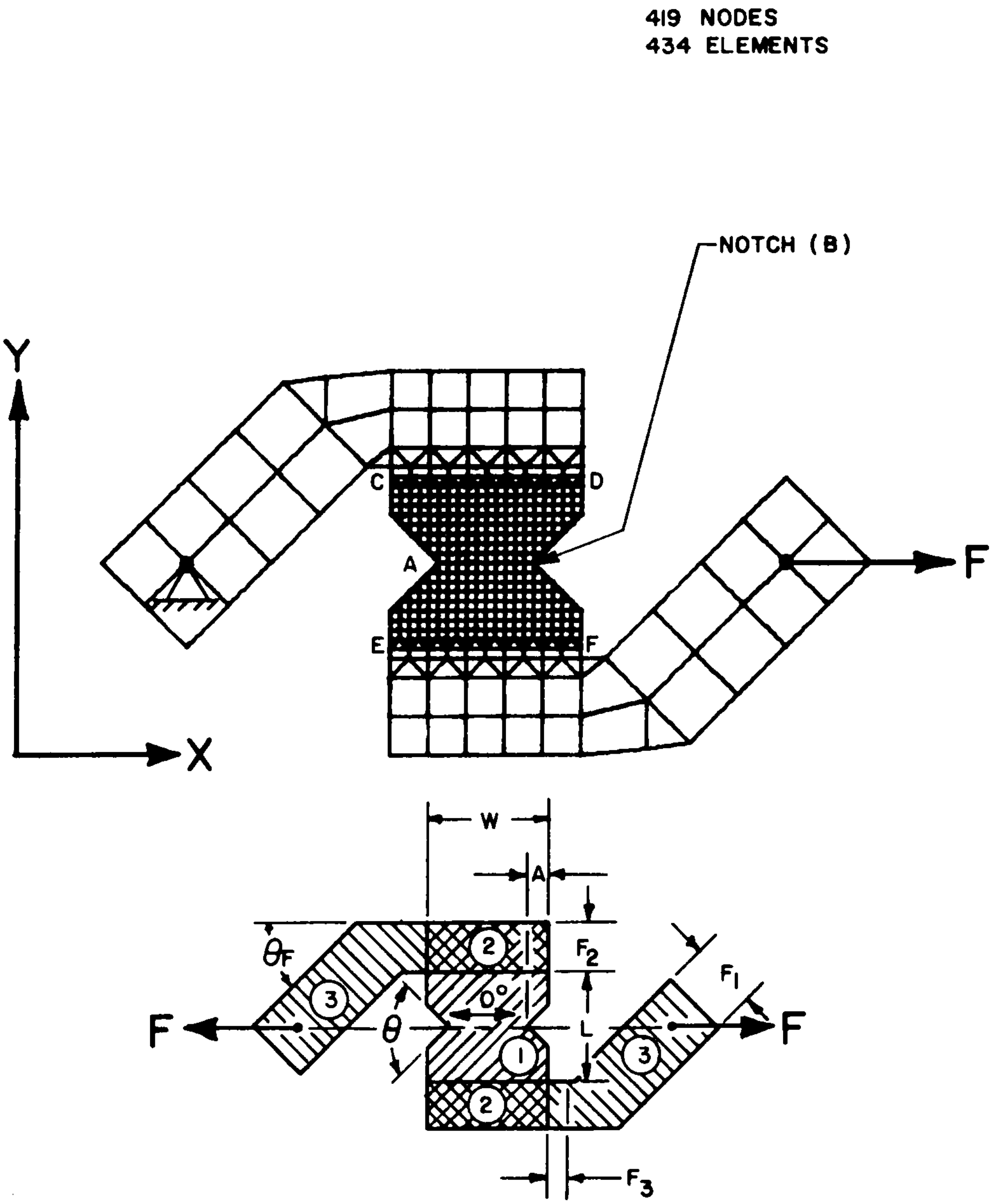
[1]See Figure 15.

21. The shear stress distribution in the vicinity of the center of the specimen is essentially pure and uniform. The normalized shear stress along the centerline OA ranges from a maximum of 1.10 near the notch tip to a minimum of 0.89 between O and A. The value at the center of the specimen is 1.09. All stress components are unusually high at the corner C. These linear elastic results indicate that this specimen produces a very satisfactory stress distribution for determining the linear shear response of a $[\pm 45]_s$ laminate. It is also evident that failure will initiate at the corner C due to the stress concentrations in this region.

6.3 Elastic Fixture Results

6.3.1 Mechanical Loading

The finite element representation for the specimen with elastic fixtures is shown in Fig. 22. This finite element representation is a two-dimensional elastic model of a specimen with four steel fixtures (each with a thickness $T_f = 0.25W$) bonded to the test material. Lamination theory was used to predict the elastic properties of the fixture - composite laminate. All dimensions were the same as those used in section 6.2 and in addition: $A = 0.25W$; fixture dimensions, $F_1 = 0.65W$, $F_2 = 0.25W$, $F_3 = 0.1W$; fixture angle, $\theta_f = 45°$; and specimen thickness, $T_C = 0.0384W$ (10 plies of graphite-polyimide, 0.00768"/ply). The stresses were normalized with respect to the average shear stress determined from the given applied load and the area of the specimen between the notch tips A and B.



(1) — COMPOSITE
(2) — COMPOSITE AND STEEL
(3) — STEEL

FIGURE 22. FINITE ELEMENT MODEL FOR IOSIPESCU SPECIMEN WITH ELASTIC FIXTURES

6.3.1.1 Isotropic Material

Normalized stress contours for an isotropic specimen (steel) are shown in Fig. 23. A pure uniform shear stress distribution exists along the centerline AB with normalized values ranging from 0.89 at the notch tips A and B to a maximum value of 0.93 at the center of the specimen. It is obvious that these results, which were obtained from a rather coarse finite element mesh, do not satisfy overall force equilibrium exactly since the total shear force transmitted across the test section is not equal to the applied load.

These results can nevertheless be compared to the rigid fixture solution (Fig. 16) in a general manner. With the exception of the lack of quarter symmetry near the elastic fixtures, the shape of the stress contours are very similar. The largest difference occurs near the corners C-F of Fig. 23 where the deformation of the elastic fixtures has the greatest effect on the specimen. Along the centerline AB (with rigid or elastic fixtures) the effects of bending in the fixtures are clearly diminished because of the symmetries that must exist there. This feature of the stress distributions is evident when the shapes of the 0.6 and 0.9 contour lines in Fig. 23c are compared. Therefore, although failure may initiate at the points A-F, this specimen produces a satisfactory stress distribution for determining linear shear response of isotropic materials.

6.3.1.2 The [0] Laminate

The normalized stress contours for a [0] laminate are shown in Fig. 24. The shear stress distribution along the centerline AB is essentially

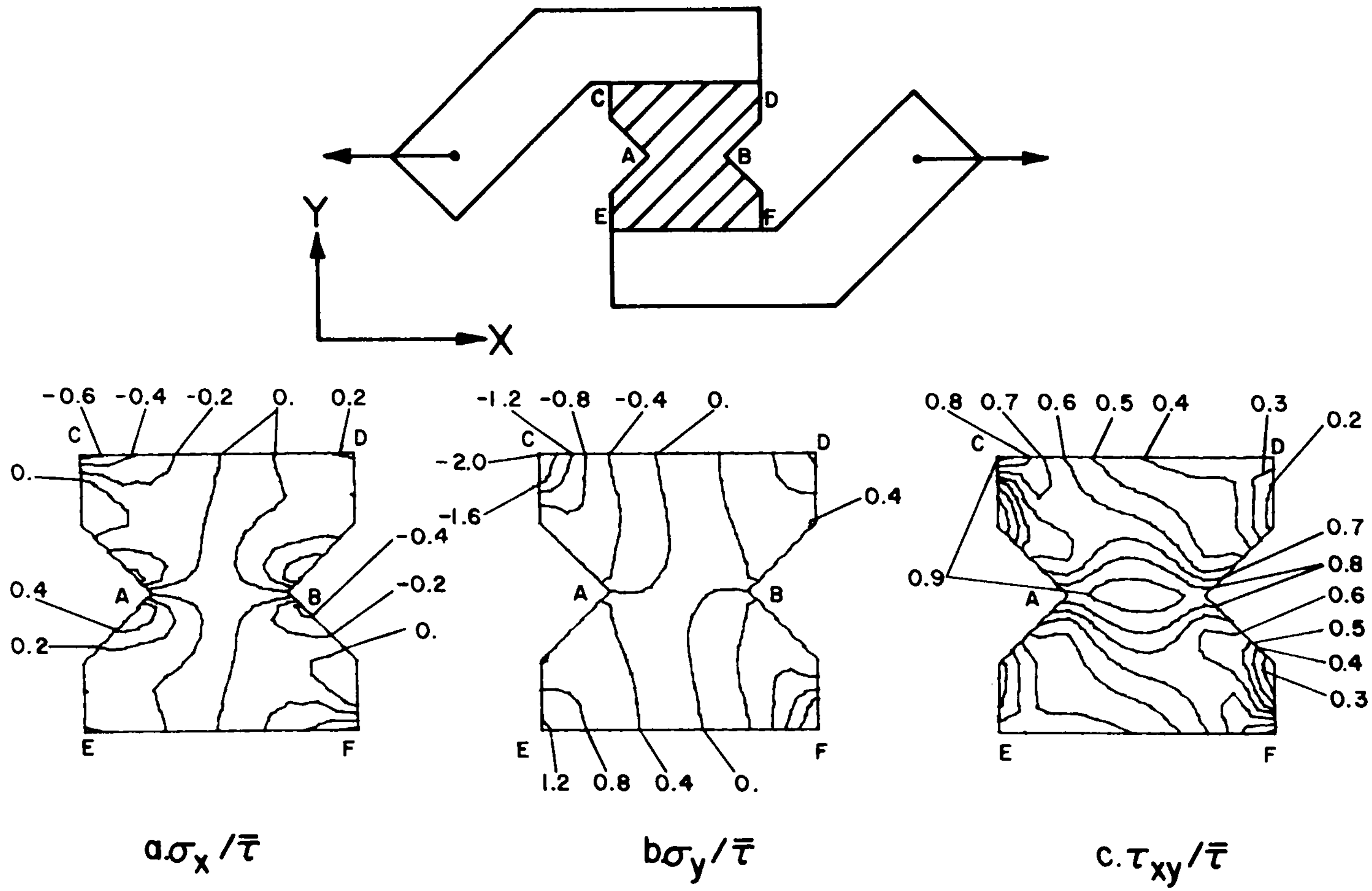


FIGURE 23. NORMALIZED STRESS CONTOURS FOR IOSIPESCU SPECIMEN WITH ISOTROPIC MATERIAL AND ELASTIC FIXTURES

pure, uniform, and maximum at the center of the specimen. The normalized values range from 0.72 at the notch tips A and B to 1.03 at the center of the specimen. Similar to the isotropic specimen, the fixture effects are minimal as a comparison of Figs. 18 and 24 shows. It is also evident from these results that failure is expected to initiate at the notch tips A or B or at the corners C, D, E, or F due to the stress concentrations in these regions.

6.3.2 Thermal Loading

In this section the effects of applying a temperature change of -600°F to a complete stress free specimen (laminate bonded to fixtures) are studied. This loading simulates the elastic behavior resulting from an elevated temperature bond of the fixtures to the specimen. The [0] laminate was used for this analysis with the mesh shown in Fig. 22 (F = 0). Contour plots for σ_x, σ_y, and τ_{xy} normalized with respect to their corresponding ultimate stress values, $(\sigma_x)_{ult}$, $(\sigma_y)_{ult}$, and $(\tau_{xy})_{ult}$ are shown in Fig. 25.

The most significant feature of the results is that the thermal stresses are negligible in the test section for all stress components. Normalized σ_y and τ_{xy} stresses are both very high at the corners with maximum values of -2.49 and 1.72, respectively. As expected, the stress in the fiber direction was quite low throughout the specimen with a maximum normalized value of -0.29 at the fixture-specimen interfaces CD and EF. If the specimen was mechanically loaded and the behavior was elastic, a superposition of the stresses depicted in Figs. 24 and 25

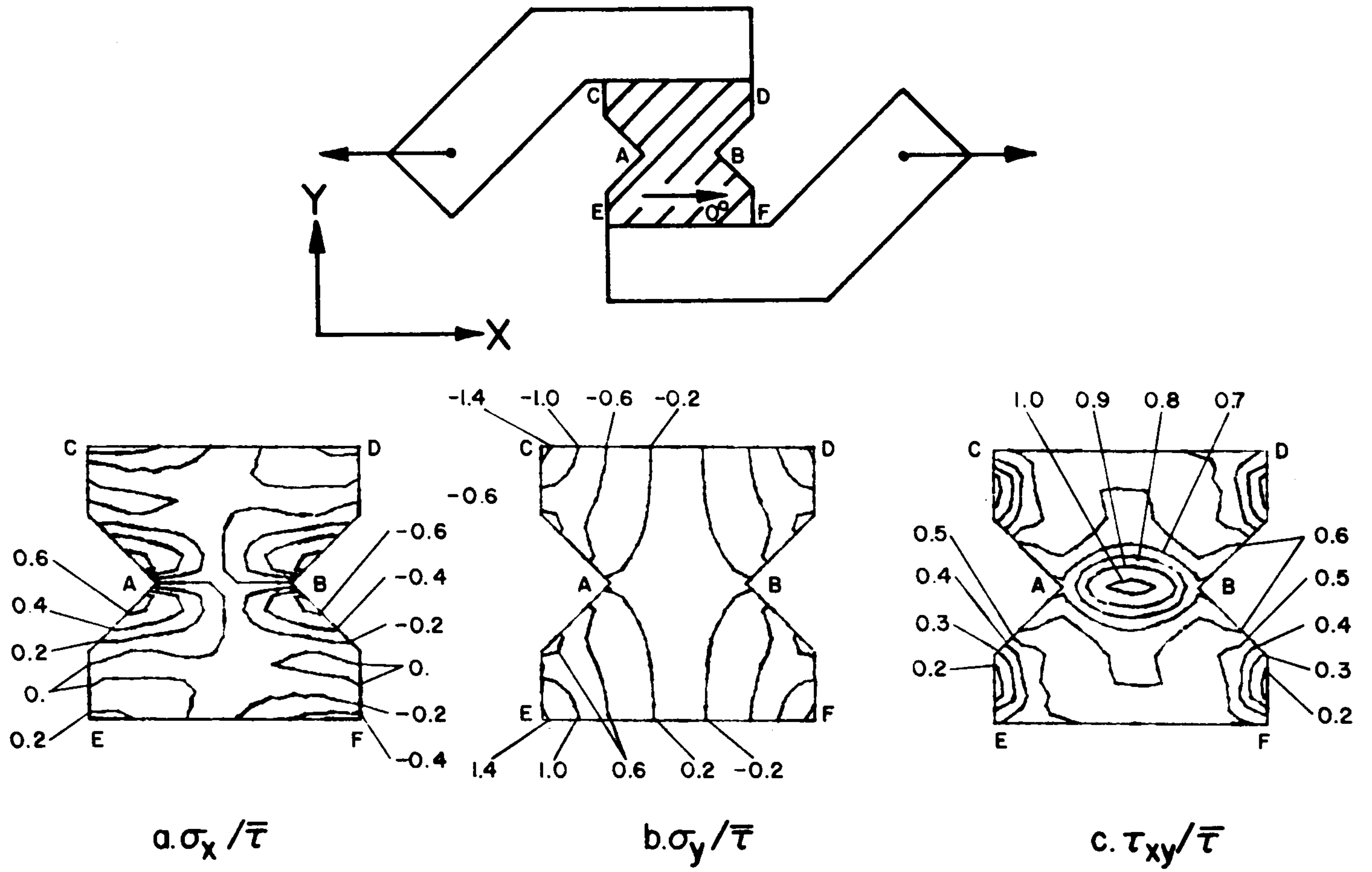
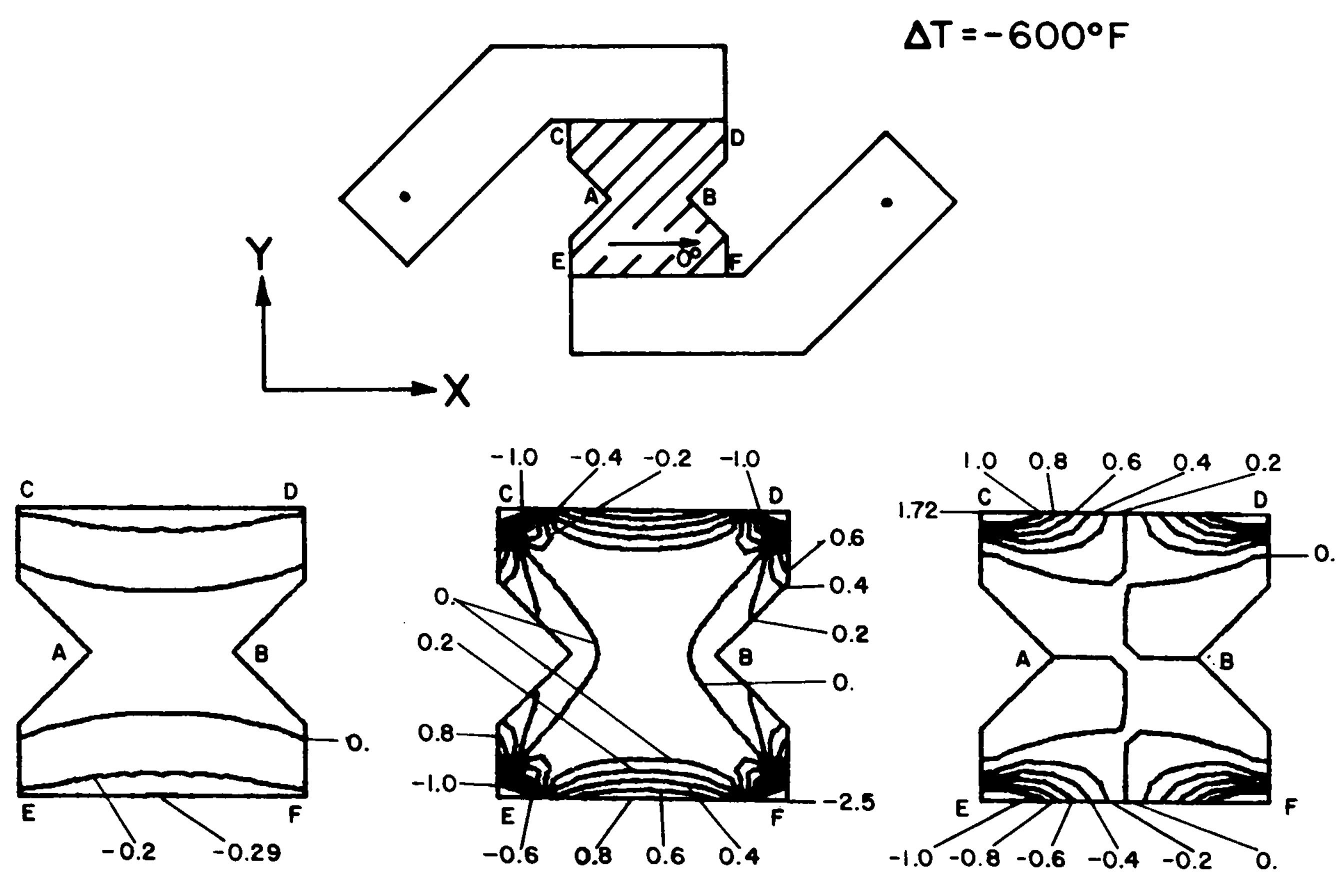


FIGURE 24. NORMALIZED STRESS CONTOURS FOR [0] GRAPHITE-POLYIMIDE IOSIPESCU SPECIMEN WITH ELASTIC FIXTURES



a. $\sigma_x/(\sigma_x)_{ult}$ b. $\sigma_y/(\sigma_y)_{ult}$ c. $\tau_{xy}/(\tau_{xy})_{ult}$

FIGURE 25. NORMALIZED STRESS CONTOURS FOR THERMALLY LOADED [0] GRAPHITE-POLYIMIDE IOSIPESCU SPECIMEN WITH ELASTIC FIXTURES

would give the combined stress distribution. Although the corner stresses at D and E would cancel one another for combined mechanical and thermal loading, the stresses at corners C and F would be additive. For this reason fixtures should be bonded to specimens at the desired test temperature in order to minimize the stress concentration at corners C and F.

6.4 Other Results

6.4.1 Doubler Effects

In addition to the elastic and rigid fixture cases described previously, the effect of doubling the stiffness (doubling the thickness in practice) of the test specimen in the vicinity of the corners was studied. The results showed that other things being equal (dimensions, loads, etc.), doubling the stiffness in the vicinity of the fixture/specimen interface reduced the stress concentrations in this region by approximately 50 percent (Table 2) while the stress distribution in the center of the specimen remained virtually unaltered. These results are significant because they indicate that the high corner stresses can be reduced to reasonable values through the use of doublers without significantly effecting the stress distribution in the test section.

6.4.2 Aspect Ratio Effects

The influence of specimen aspect ratio, $R = L/W$ (Fig. 22), was investigated for $R = 1.0$ and $R = 2.5$. The stresses in the test section of the specimen were not appreciably effected for the elastic and rigid fixture cases studied, however, the corner stresses were significantly

TABLE 2. Effects of Doublers and Aspect Ratios for the Iosipescu Specimen.

<table>
<tr><td rowspan="5">[0] Gr/Pi
R = 2.5
A = 0.225 W
Rigid Fixtures</td><td></td><td colspan="4">With Doublers</td><td colspan="4">Without Doublers</td></tr>
<tr><td></td><td>0[1]</td><td colspan="2">A[1]</td><td>C[1]</td><td>0</td><td colspan="2">A</td><td>C</td></tr>
<tr><td>$\sigma_x/\bar{\tau}$</td><td>1.81×10^{-2}</td><td colspan="2">0.48</td><td>0.60</td><td>1.81×10^{-2}</td><td colspan="2">0.48</td><td>0.30</td></tr>
<tr><td>$\sigma_y/\bar{\tau}$</td><td>1.87×10^{-3}</td><td colspan="2">0.27</td><td>5.19</td><td>1.86×10^{-3}</td><td colspan="2">0.27</td><td>2.60</td></tr>
<tr><td>$\tau_{xy}/\bar{\tau}$</td><td>1.10</td><td colspan="2">0.61</td><td>0.59</td><td>1.10</td><td colspan="2">0.61</td><td>0.30</td></tr>
<tr><td rowspan="5">[0] Gr/Pi
A = 0.225 W
Rigid Fixtures</td><td></td><td colspan="4">R = 1.0</td><td colspan="4">R = 2.5</td></tr>
<tr><td></td><td>0</td><td colspan="2">A</td><td>C</td><td>0</td><td colspan="2">A</td><td>C</td></tr>
<tr><td>$\sigma_x/\bar{\tau}$</td><td>3.41×10^{-3}</td><td colspan="2">0.25</td><td>0.57</td><td>3.35×10^{-3}</td><td colspan="2">0.25</td><td>1.30</td></tr>
<tr><td>$\sigma_y/\bar{\tau}$</td><td>2.12×10^{-4}</td><td colspan="2">0.18</td><td>3.26</td><td>2.38×10^{-4}</td><td colspan="2">0.18</td><td>6.10</td></tr>
<tr><td>$\tau_{xy}/\bar{\tau}$</td><td>1.10</td><td colspan="2">0.48</td><td>0.43</td><td>1.10</td><td colspan="2">0.48</td><td>0.66</td></tr>
<tr><td rowspan="5">[0] Gr/Pi
A = 0.225 W
Elastic
Fixtures</td><td></td><td colspan="4">R = 1.0</td><td colspan="4">R = 2.5</td></tr>
<tr><td></td><td>Center</td><td>A[2],B[2]</td><td>D[2],E[2]</td><td>C[2],F[2]</td><td>Center</td><td>A,B</td><td>D,E</td><td>C,F</td></tr>
<tr><td>$\sigma_x/\bar{\tau}$</td><td>-1.13×10^{-3}</td><td>-5.08×10^{-4}</td><td>0.39</td><td>-0.54</td><td>-3.07×10^{-5}</td><td>0.00</td><td>0.52</td><td>-0.80</td></tr>
<tr><td>$\sigma_y/\bar{\tau}$</td><td>3.53×10^{-4}</td><td>-4.71×10^{-4}</td><td>0.29</td><td>-1.52</td><td>8.45×10^{-6}</td><td>0.00</td><td>3.37</td><td>-3.43</td></tr>
<tr><td>$\tau_{xy}/\bar{\tau}$</td><td>1.03</td><td>0.72</td><td>1.47</td><td>0.48</td><td>1.04</td><td>0.72</td><td>0.37</td><td>0.46</td></tr>
</table>

[1]See Figure 15.

[2]See Figure 22.

TABLE 3. Graphite-epoxy and Graphite-polyimide Normalized Stresses for Iosipescu Specimen with Rigid Fixtures (R = 2.5).

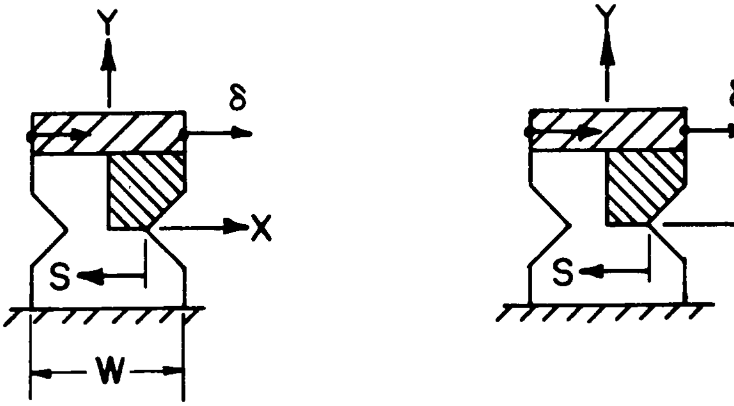
		Graphite-epoxy			Graphite-polyimide		
		0[1]	A[1]	C[1]	0	A	C
[0]	$\sigma_x/\bar{\tau}$	1.49×10^{-3}	0.39	0.54	1.33×10^{-3}	0.42	0.79
	$\sigma_y/\bar{\tau}$	1.07×10^{-4}	0.21	4.49	9.09×10^{-5}	0.24	4.56
	$\tau_{xy}/\bar{\tau}$	1.16	0.47	0.58	1.10	0.53	0.59
[90]	$\sigma_x/\bar{\tau}$	6.75×10^{-5}	0.70	4.20×10^{-2}	1.06×10^{-4}	0.71	8.67×10^{-2}
	$\sigma_y/\bar{\tau}$	-5.29×10^{-5}	0.72	6.78	-5.52×10^{-3}	0.68	7.02
	$\tau_{xy}/\bar{\tau}$	0.89	1.83	0.41	0.90	1.66	0.47
$[0/90]_s$	$\sigma_x/\bar{\tau}$	6.53×10^{-4}	0.60	7.50×10^{-2}			
	$\sigma_y/\bar{\tau}$	1.22×10^{-7}	0.41	6.06	Not Available		
	$\tau_{xy}/\bar{\tau}$	1.01	0.93	0.44			
$[\pm 45]_s$	$\sigma_x/\bar{\tau}$	1.40×10^{-5}	0.70	3.02	6.66×10^{-6}	0.71	3.29
	$\sigma_y/\bar{\tau}$	2.34×10^{-5}	0.56	6.24	2.51×10^{-5}	0.57	6.39
	$\tau_{xy}/\bar{\tau}$	0.93	1.27	2.83	0.93	1.30	3.06
$[0/90/\pm 45]_s$	$\sigma_x/\bar{\tau}$	2.32×10^{-4}	0.61	0.84	2.23×10^{-4}	0.61	0.92
	$\sigma_y/\bar{\tau}$	3.22×10^{-5}	0.45	5.20	3.22×10^{-5}	0.45	5.25
	$\tau_{xy}/\bar{\tau}$	0.98	1.04	0.87	0.98	1.04	0.92

[1]See Figure 15.

higher for the specimens with R = 2.5 (Table 2). These higher stresses were the result of the higher bending moments introduced from the fixtures into the specimen for R = 2.5. The effect of different materials (graphite-epoxy and graphite-polyimide) was also studied for R = 2.5 (Table 3) using a coarse mesh with rigid fixtures. The stress values for the graphite-epoxy and graphite-polyimide were virtually identical as indicated in the table. With the exception of the $[\pm 45]_s$ laminate, the shear stresses for R = 2.5 for these two materials also compare to the shear stresses at the center of the specimen for R = 1.0 (Table 2).

6.4.3 Rounded Notch Effects

The effects of rounding the notch tips were studied for both [0] [90] laminates. This was done by adding a "fillet", or round with a radius of 0.0325W to the notch tip. The normalized stress distributions along the line y = 0 in the vicinity of the notch tip are shown in Figs. 26 and 27 for the [0] and [90] laminates, respectively. For the [0] laminate all components of stress are considerably lower near the free edge with the rounded notch. However, the round causes the shear distribution to become more nonuniform, and as a result the normalized shear stress at the center of the specimen is higher than that of the V-notched specimen (Table 4). As indicated in Fig. 27, rounding the notch for a [90] laminate results in σ_x and τ_{xy} approaching zero near the free edge. However, σ_y remains very high. Since the shear stress distribution for the rounded notch is more uniform, the corresponding normalized shear stress at the center of the specimen is more nearly equal to 1.0. Thus while σ_x and τ_{xy} are always zero at the free edge of



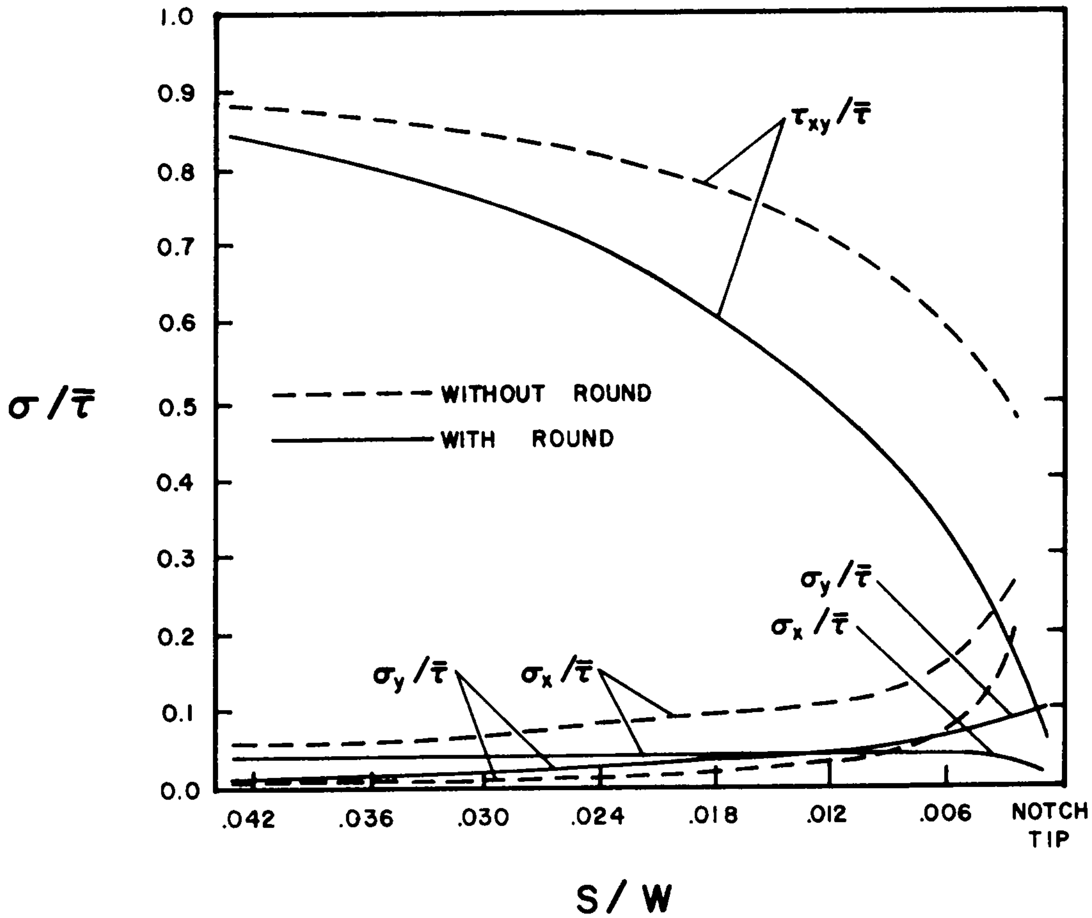


FIGURE 26. ROUNDED NOTCH EFFECTS FOR THE [0] IOSIPESCU SPECIMEN ALONG Y=0

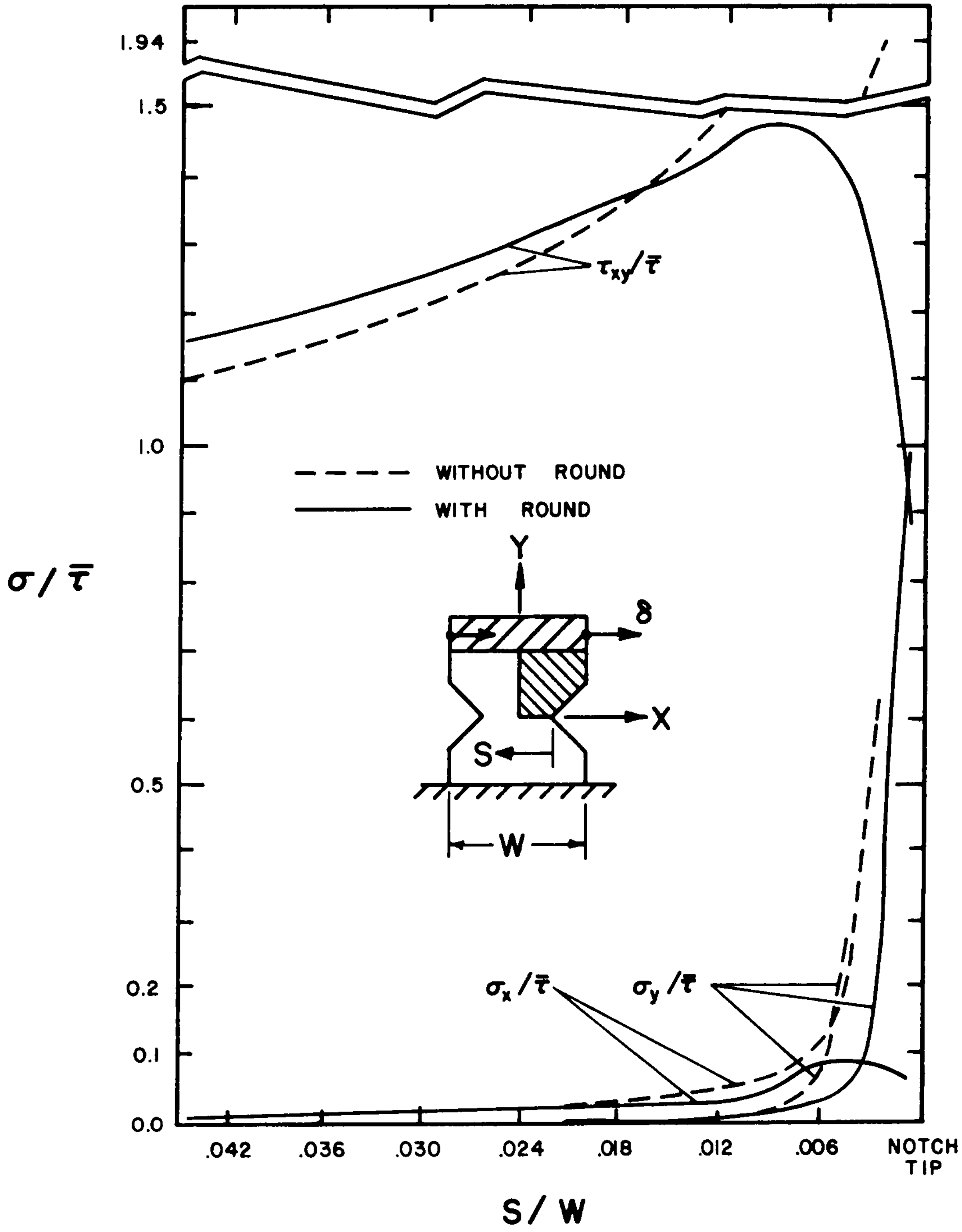


FIGURE 27. ROUNDED NOTCH EFFECTS FOR THE [90] IOSIPESCU SPECIMEN ALONG Y=0

TABLE 4. Normalized Stresses for V-notch and Rounded Notch Iosipescu Specimens with Rigid Fixtures.

		V-notch		Rounded Notch	
		0[1]	A[1]	0	A
$[0]_{Gr/Pi}$	$\sigma_x/\bar{\tau}$	3.41×10^{-3}	0.25	3.40×10^{-3}	1.24×10^{-2}
	$\sigma_y/\bar{\tau}$	2.12×10^{-4}	0.18	2.30×10^{-4}	9.71×10^{-2}
	$\tau_{xy}/\bar{\tau}$	1.10	0.48	1.14	7.28×10^{-2}
$[90]_{Gr/Pi}$	$\sigma_x/\bar{\tau}$	3.34×10^{-4}	0.56	3.28×10^{-4}	6.69×10^{-2}
	$\sigma_y/\bar{\tau}$	-2.02×10^{-4}	0.62	-1.98×10^{-4}	0.94
	$\tau_{xy}/\bar{\tau}$	0.84	1.94	0.87	0.91

[1]See Figure 15.

a rounded notch, the uniformity of the shear stress distribution is a function of the modulus ratio E_x/E_y.

6.5 Summary

While the Iosipescu specimen produces a region of uniform shear stress at the center of the specimen for isotropic and orthotropic materials, the uniformity of the shear stress between the notch tips is highly dependent upon the elastic properties of the laminate. This feature was most evident for the unidirectional laminates. Also, the complex state of stress at the notch tips and corners undoubtedly contributes to failure. This specimen can accurately provide initial shear modulus values for isotropic and orthotropic materials. Bonding suitable doublers to the corners and rounding the notch tips can reduce the stress concentrations in critical areas. Thermal stresses are not significant in the center of the specimen.

7. THE RAIL SHEAR SPECIMEN

7.1 General

As mentioned previously, the rail shear specimen has been used to obtain linear and nonlinear shear stress-strain response for composite laminates at both room temperature and elevated temperatures. For experimental work, it has been assumed that the specimen is subjected to pure, uniform shear stress with the magnitude of the shear stress directly related to the applied load through a simple P/A relationship. The finite element results presented here show to what extent these assumptions are valid. The effects of material properties, thermal loads, and rail configurations for both rigid and elastic rails are presented to more fully understand the elastic behavior of this specimen and to more accurately determine the relationship between the applied load and the stress distribution in the test section.

7.2 Rigid Rail Solutions

7.2.1 Fourier Series Representation

A Fourier series solution was presented [8] as the superposition of two problems with the boundary conditions (Fig. 29) for each problem being:

$$\text{problem 1.}\quad \begin{aligned} & u(x,0) = v(x,0) = v(x,b) = 0 \\ & N_x(0,y) = N_x(a,y) = 0 \\ & u(x,b) = \delta \\ & N_{xy}(0,y) = N_{xy}(a,y) = A_{66}\,\frac{\delta}{b} \end{aligned} \qquad (7.1)$$

problem 2. $u(x,0) = u(x,b) = v(x,0) = v(x,b) = 0$

$$N_x\,(0,y) = N_x(a,y) = 0 \tag{7.2}$$

$$N_{xy}\,(0,y) = N_{xy}(a,y) = -A_{66}\,\frac{\delta}{b}\,,$$

where N_i and A_{66} are defined from equations (3.7) and (3.9). These boundary conditions assume that the actual specimen is perfectly bonded to rigid rails and that uniform rail spacing is maintained during loading. The solution to problem 1 leads to a state of pure shear with the displacements

$$u = \frac{\delta}{b}\,y \;;\; v = 0. \tag{7.3}$$

The solution to the second problem was assumed to be of the form

$$u = \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} A_{mn} \sin \frac{(2m-1)\pi x}{a} \sin \frac{2n\pi y}{b}$$

$$(o<x<a;\; o\leq y\leq b) \tag{7.4}$$

$$v = \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} B_{mn} \cos \frac{(2m-1)\pi x}{a} \cos \frac{2n\pi y}{b}$$

$$(o\leq x\leq a;\; o\leq y\leq b)$$

The superposition of the two solutions result in an infinite m+n system of linear equations with unknown Fourier coefficients. Adequate convergence of the infinite series was obtained by truncating the system of equations to 24 terms. The Fourier coefficients were computed and stresses were determined as a function of position.

7.2.2 Comparison of Fourier Series and Finite Element Results

Fourier series solutions are compared to finite element results for $[0/\pm45]_s$ and $[\pm45]_s$ Thornel 50 graphite-epoxy in Fig. 28. The finite element representations used are shown in Fig. 29 along with boundary conditions for the problem. Results were obtained for a specimen aspect ratio a/b = 12. Fig. 28 shows the distribution of the shear stress, normalized with respect to the average shear stress, along the centerline of the specimen y = b/2.

As indicated in Fig. 28a, the two solutions exhibited very similar results for the $[0/\pm45]_s$ laminate. The normalized shear stress is uniform over almost 80 percent of the specimen at a value of 1.05. This value is necessarily greater than 1.0 to account for the shear stress approaching zero at the free edge.

The results for the $[\pm45]_s$ laminate shown in Fig. 28b do not exhibit the same high degree of correlation. The finite element results are very similar to those of the $[0/\pm45]_s$ laminate with the normalized shear stress uniform over almost 90 percent of the specimen at a value of 1.06. The Fourier series results exhibit a spike with a maximum value of 1.5 a small distance in from the free edge and then exhibit a continuously decreasing stress magnitude toward the center of the specimen. Because of the difference in these results, additional finite element results were obtained using the model shown in Fig. 29b. The results obtained with the refined mesh were so similar to those of the basic mesh that they cannot be distinguished in Fig. 28b. The finite element results are believed to be more accurate than the Fourier series

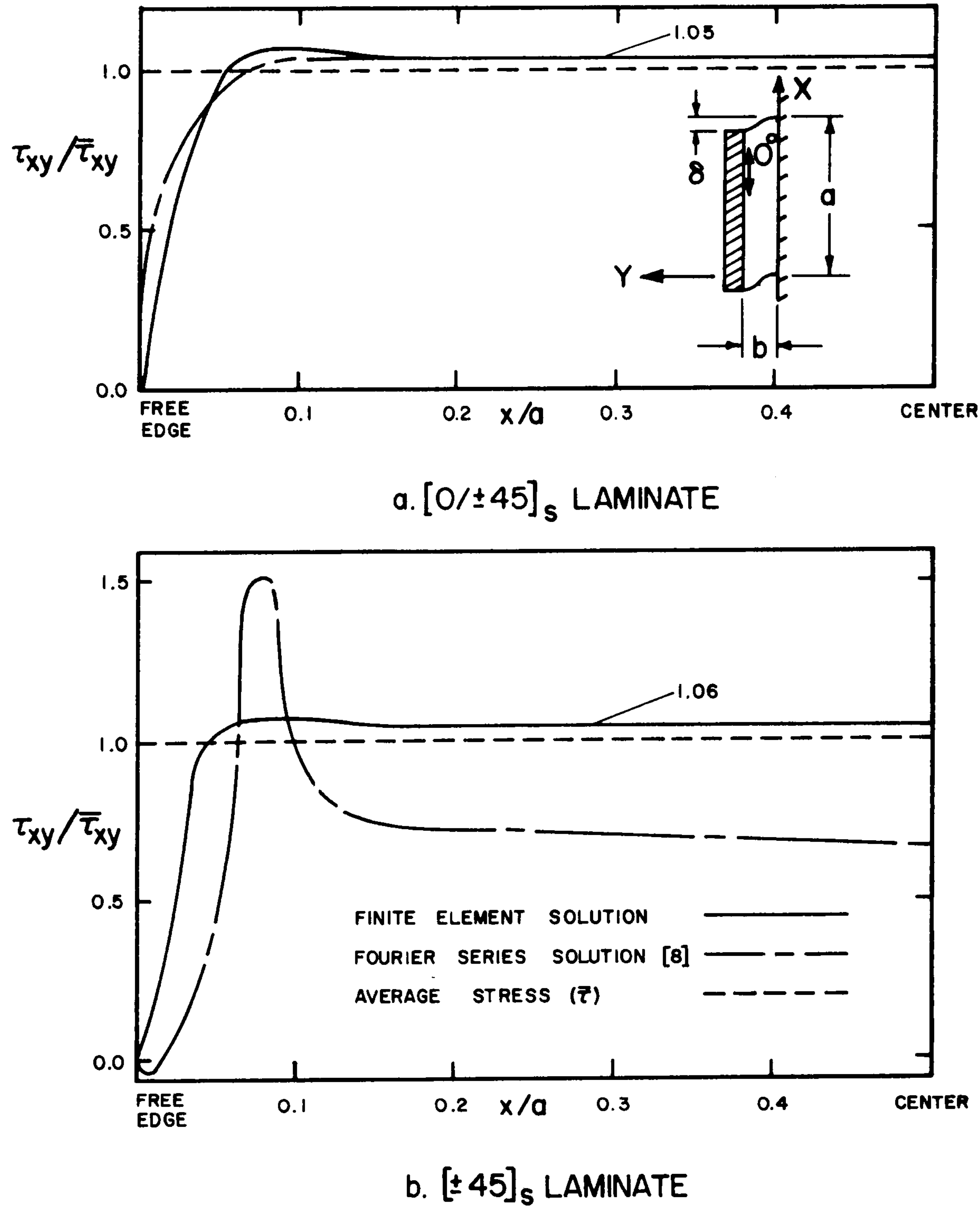


FIGURE 28. COMPARISON OF RIGID RAIL SOLUTIONS FOR THORNEL 50 GRAPHITE-EPOXY AT Y=b/2

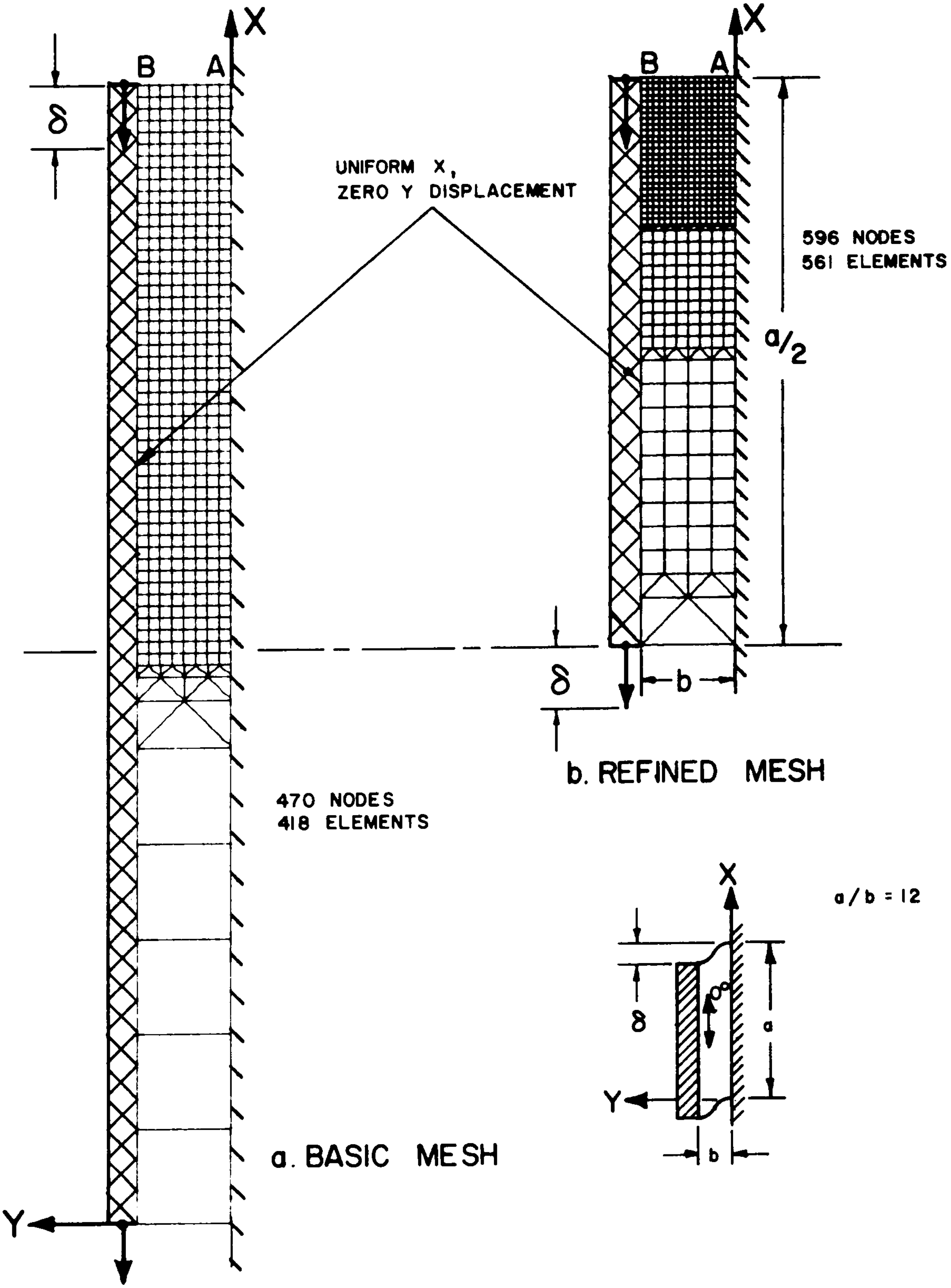


FIGURE 29. FINITE ELEMENT MODELS FOR RAIL SHEAR SPECIMEN WITH RIGID RAILS

results[1] for this $[\pm 45]_s$ laminate due to the proven capabilities of the method, the consistency with the $[0/\pm 45]_s$ laminate, and the more physically appealing stress distribution.

The finite element results indicated that both laminates exhibited high normal stresses in the vicinity of corners, but these normal stresses were negligible over 80 percent of the specimen. Thus, these results indicate that this specimen produces an essentially pure, uniform distribution of shear stress in the test section with the stress magnitude equal 1.05 and 1.06 times the average shear stress $\bar{\tau}$ for $[0/\pm 45]_s$ and $[\pm 45]_s$ laminates, respectively.

7.3 Elastic Rails

7.3.1 Effects of Offset Loading and Tapered Rails

The effects of nontapered and tapered rail configurations loaded along the centerline, and nontapered rails with offset loading for a [0] graphite-polyimide laminate were studied. The finite element mesh for the nontapered rail configuration with axial loading is shown in Fig. 30. The dimensions of the specimen were standardized to: specimen length, L = 10W; fixture dimensions, F_1 = 5.83W, F_2 = 3.33W, F_3 = 6.67W, F_4 = 1.23W; rail thickness, T_r = 3.33W; and composite thickness, T_c = 0.256W. Appropriate steel, steel and composite (from lamination theory), and composite properties represented the actual materials in the areas indicated (Fig. 30). The tapered rail configuration was represented by

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[1] Personal conversation with Dr. Whitney indicated that his solution appeared to be unstable for the $[\pm 45]_s$ laminate.

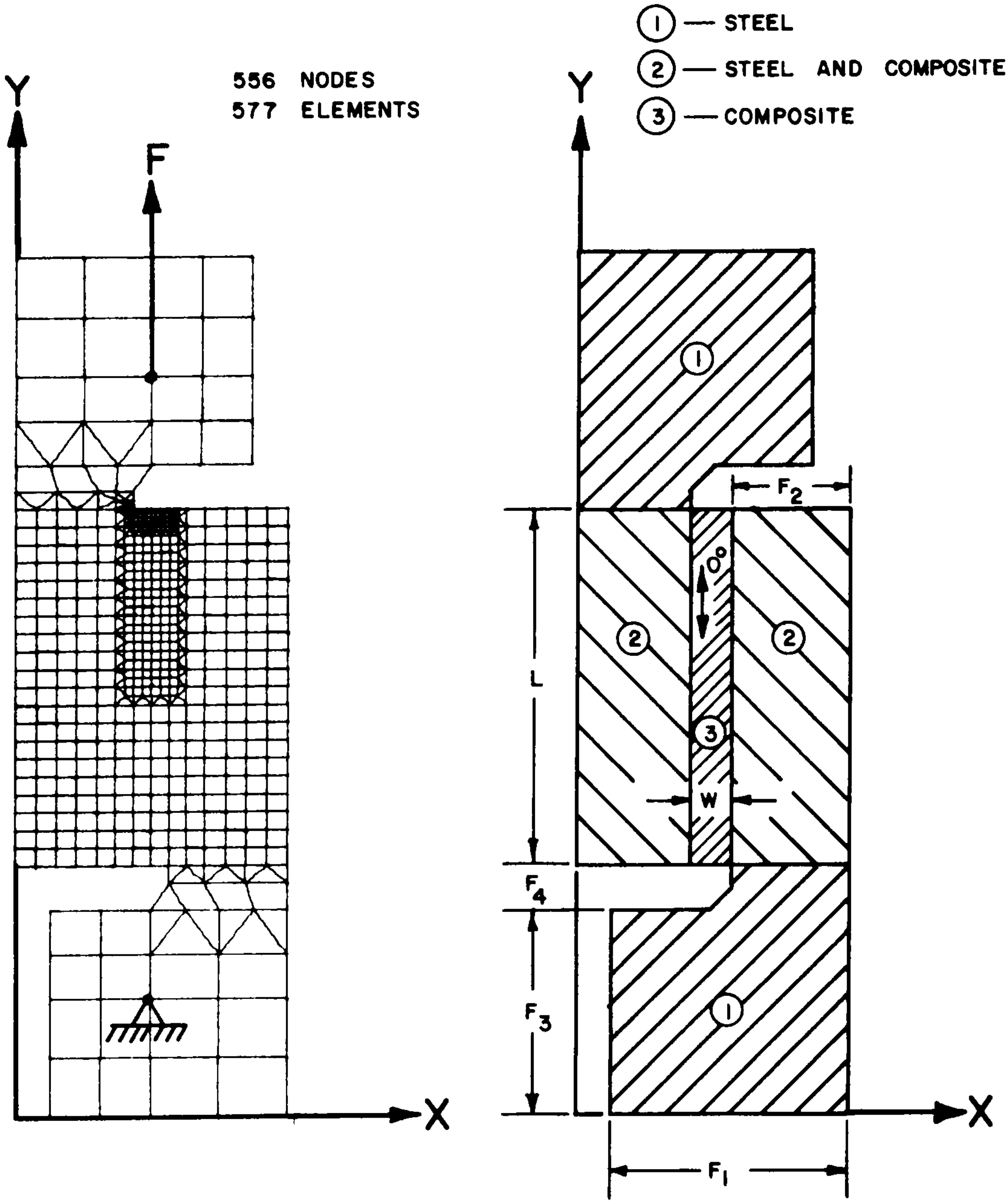


FIGURE 30. AXIALLY LOADED RAIL SHEAR SPECIMEN

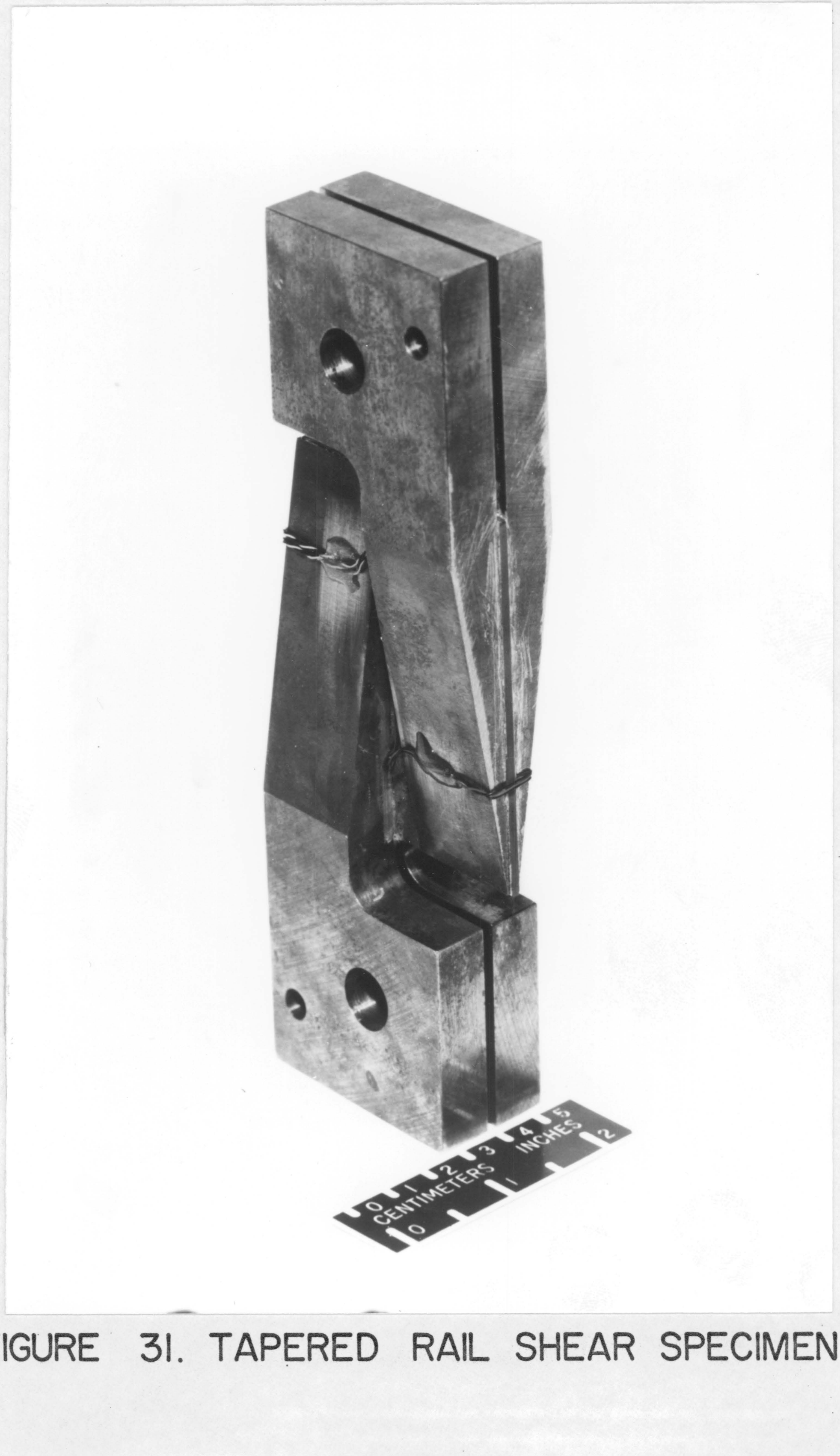


FIGURE 31. TAPERED RAIL SHEAR SPECIMEN

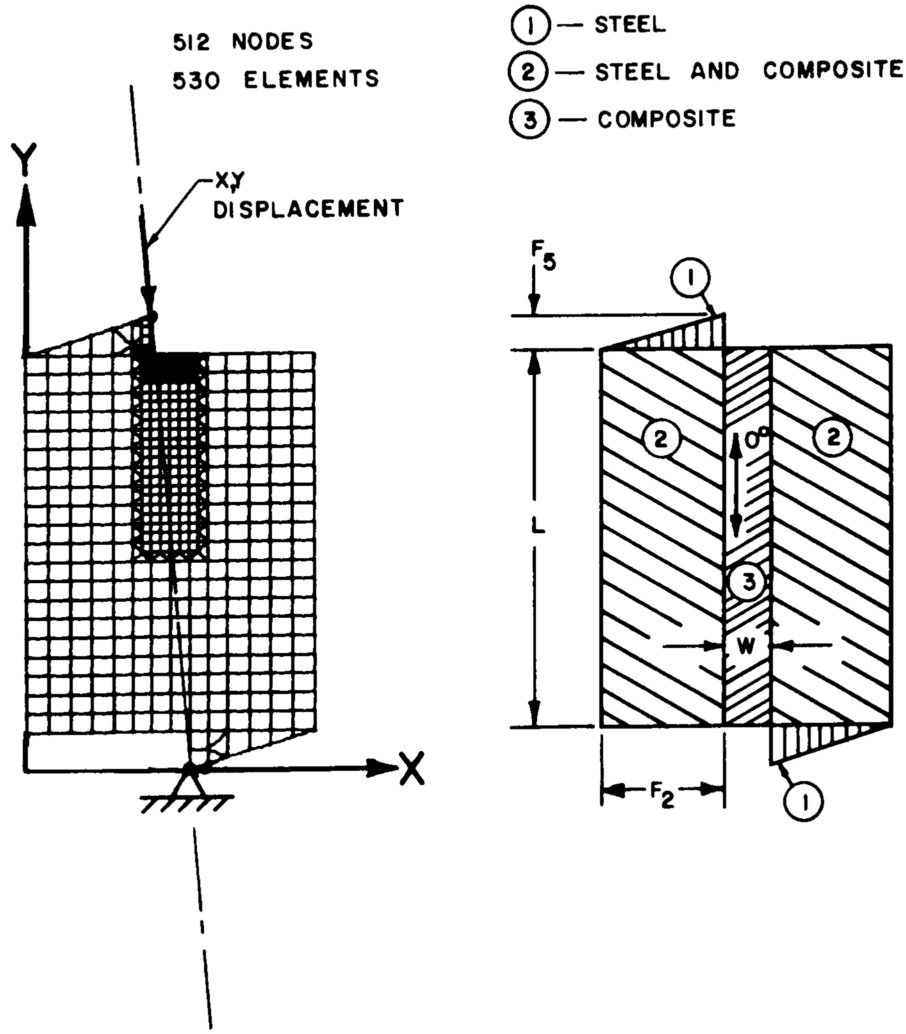


FIGURE 32. OFFSET LOADED RAIL SHEAR SPECIMEN

the same mesh as the nontapered rail configuration and the rail thickness, T_r, was tapered to zero at the free edge of the specimen (Fig. 31). Material properties for the varying rail thickness with uniform composite thickness laminate were obtained from lamination theory at appropriate intervals along the rails. The finite element representation for the shear specimen with offset loading is shown in Fig. 32. The dimensions C, F_2, T_r, and T_c are the same as those of the nontapered rail (Fig. 30) and in addition $F_5 = W$. The geometry and loading of this specimen are similar to the rail shear specimens used by Whitney et al [8].

The normalized shear stress distributions for the three rail configurations with a [0] laminate (Figs. 33c, 34c, and 35c) indicate a uniform shear stress over most of the specimen. The nontapered rail configuration produces a value of normalized τ_{xy} closest to 1.0 at the center of the specimen. In all cases the maximum shear stress (Table 5) did not occur at the center of the specimen.

The normal stress contour plots indicate that the highest stress concentrations occur in the vicinity of the free edge AB for all rail configurations studied. The σ_x stresses are maximum at the corners A and B. Because of the boundary condition $\sigma_y = 0$ along the free edge AB, the maximum value of σ_y is some distance in from the corner A along the rail-specimen interface AC for the nontapered and tapered rail configurations. For the offset loaded rail configuration σ_y is very low throughout the specimen, and, unlike the axially loaded rail configurations, it

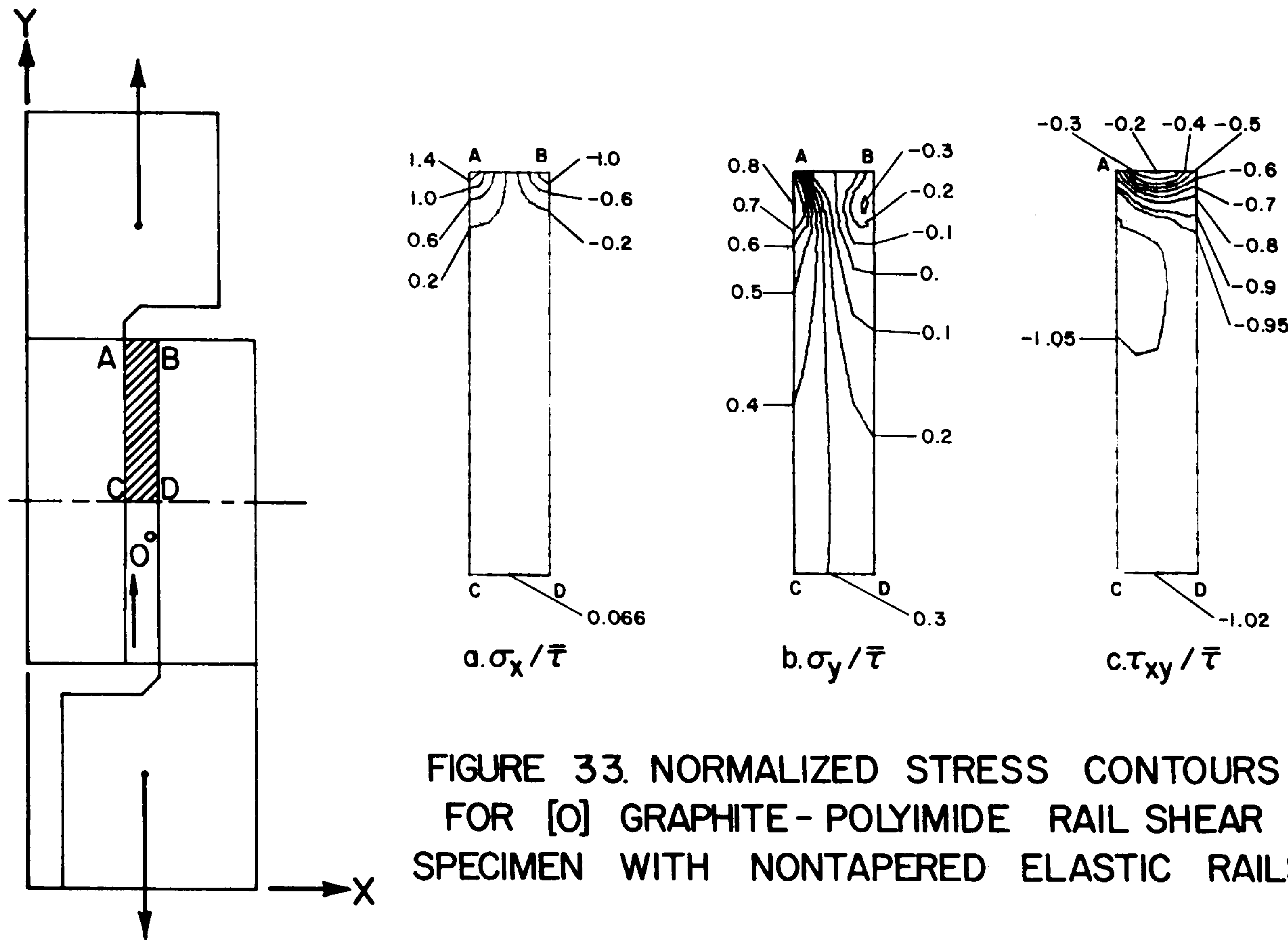


FIGURE 33. NORMALIZED STRESS CONTOURS FOR [0] GRAPHITE - POLYIMIDE RAIL SHEAR SPECIMEN WITH NONTAPERED ELASTIC RAILS

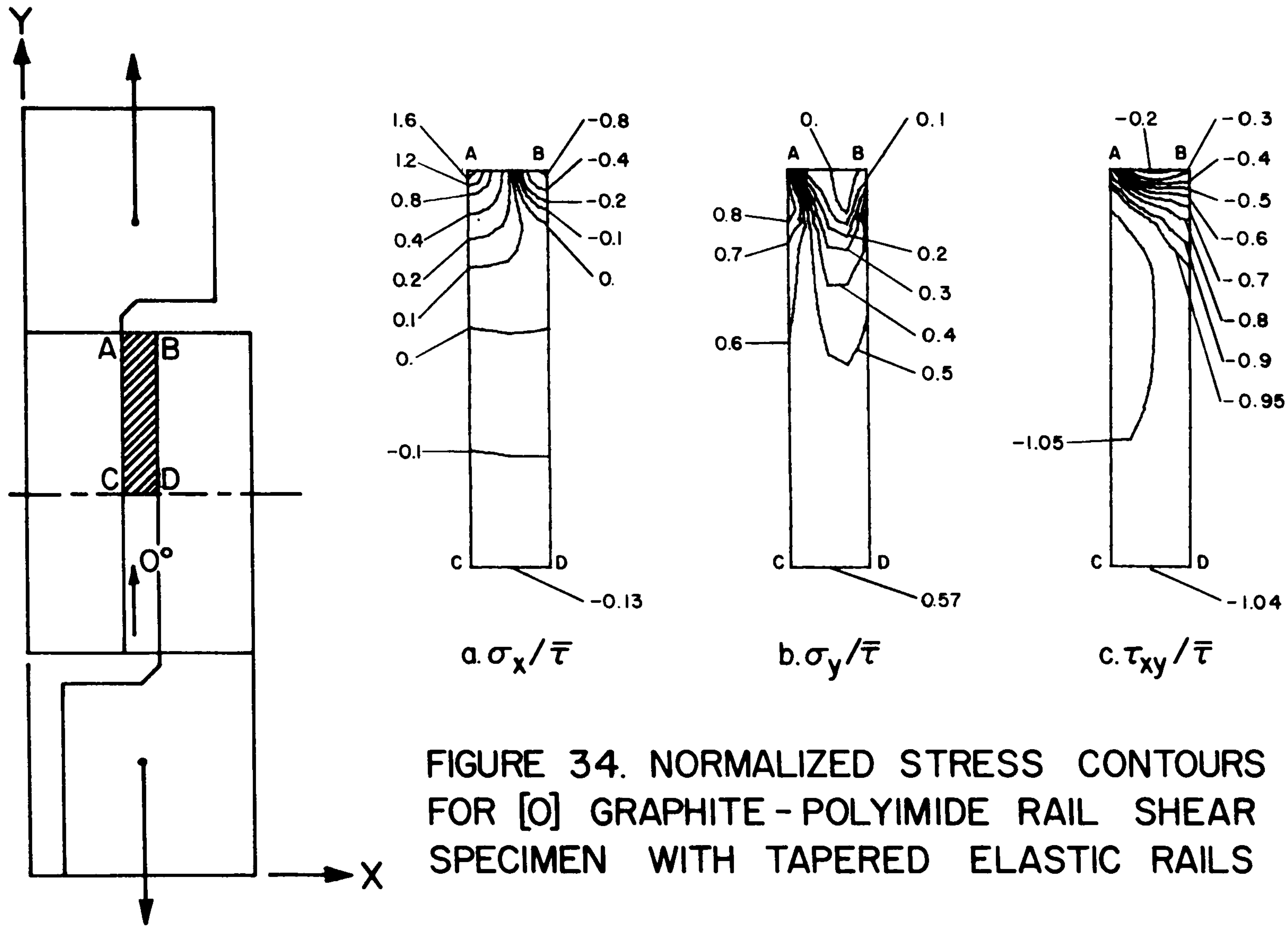


FIGURE 34. NORMALIZED STRESS CONTOURS FOR [0] GRAPHITE - POLYIMIDE RAIL SHEAR SPECIMEN WITH TAPERED ELASTIC RAILS

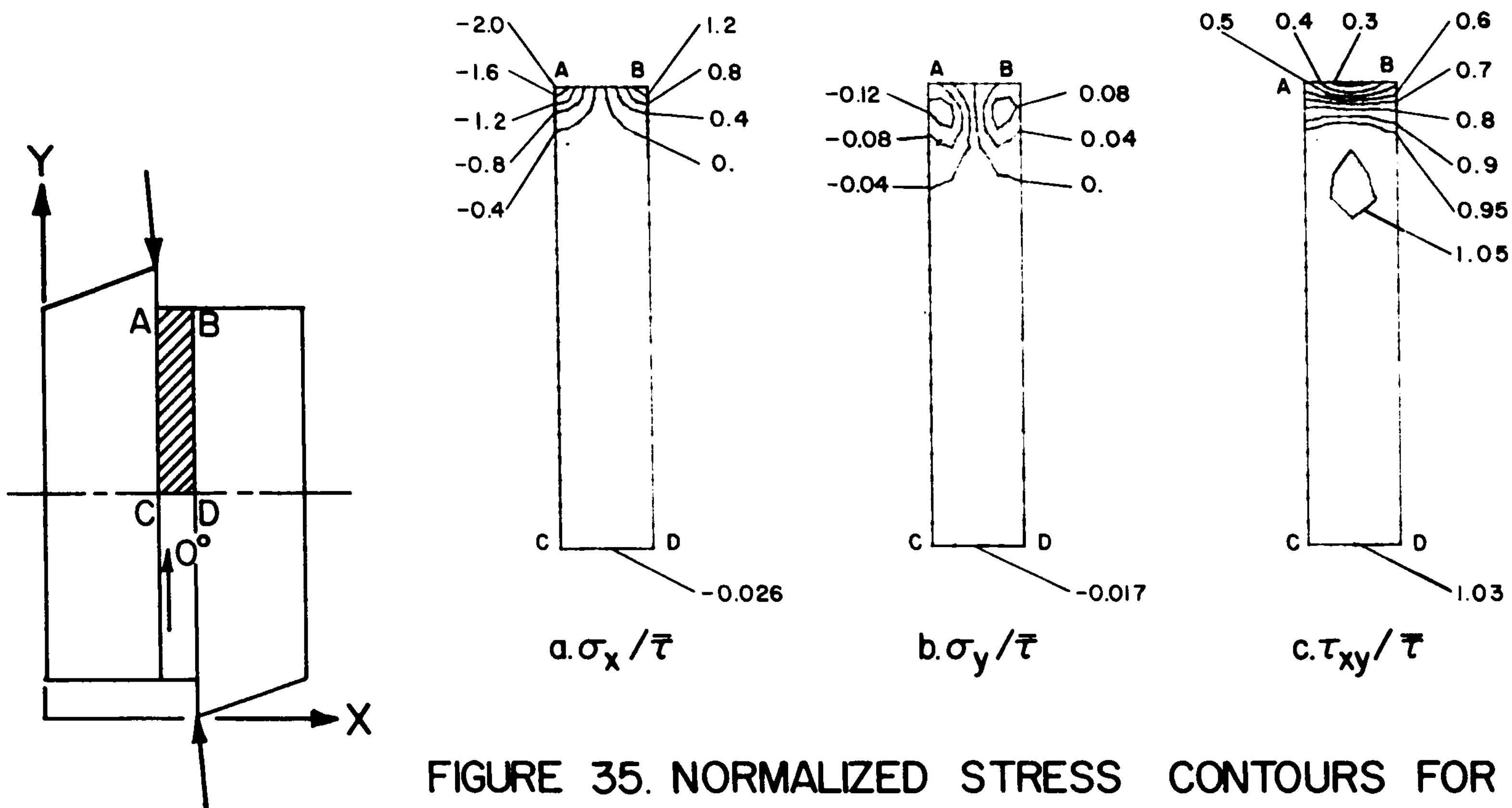


FIGURE 35. NORMALIZED STRESS CONTOURS FOR [0] GRAPHITE-POLYIMIDE RAIL SHEAR SPECIMEN WITH OFFSET LOADED ELASTIC RAILS

TABLE 5. Normalized Stresses for Graphite-polyimide Rail Shear Specimens with Elastic Rails.

			Center	A[1]	B[1]	Maximum
[0]	Non-Tapered Rails	$\sigma_x/\bar{\tau}$	-6.67×10^{-2}	1.77	-1.38	1.77
		$\sigma_y/\bar{\tau}$	0.30	0.76	-0.29	0.82
		$\tau_{xy}/\bar{\tau}$	-1.02	-0.74	-0.55	-1.08
	Tapered Rails	$\sigma_x/\bar{\tau}$	-0.13	1.91	-0.93	1.91
		$\sigma_y/\bar{\tau}$	0.57	0.79	1.68×10^{-2}	0.87
		$\tau_{xy}/\bar{\tau}$	-1.04	-0.76	-0.29	-1.08
	Offset Loaded Rails	$\sigma_x/\bar{\tau}$	-2.56×10^{-2}	-2.01	1.56	-2.01
		$\sigma_y/\bar{\tau}$	-1.74×10^{-2}	9.72×10^{-2}	5.20×10^{-2}	-0.14
		$\tau_{xy}/\bar{\tau}$	1.03	0.52	0.47	1.06
[90]	Tapered Rails	$\sigma_x/\bar{\tau}$	-0.85	5.69	-0.33	5.69
		$\sigma_y/\bar{\tau}$	9.64×10^{-3}	0.12	2.61×10^{-2}	0.12
		$\tau_{xy}/\bar{\tau}$	-0.99	-0.86	-0.52	-1.04

TABLE 5. (continued).

			Center	A[1]	B[1]	Maximum
$[\pm45]_s$	Tapered Rails	$\sigma_x/\bar{\tau}$	-0.53	2.47	-7.10×10^{-2}	2.47
		$\sigma_y/\bar{\tau}$	-0.30	1.39	-3.13×10^{-2}	1.39
		$\tau_{xy}/\bar{\tau}$	-1.01	-1.37	-6.50×10^{-2}	-1.37
[0]	Non-Tapered Rails Thermally Loaded	$\sigma_x/(\sigma_x)_{ult}$	-0.34	-5.45	-5.23	-5.45
		$\sigma_y/(\sigma_y)_{ult}$	-0.48	-0.28	-0.27	-0.51
		$\tau_{xy}/(\tau_{xy})_{ult}$	-2.70×10^{-3}	2.21	-2.14	2.28
	Tapered Rails Thermally Loaded	$\sigma_x/(\sigma_x)_{ult}$	-0.68	-5.03	-0.15	-5.03
		$\sigma_y/(\sigma_y)_{ult}$	-0.44	0.28	-2.69×10^{-2}	-0.48
		$\tau_{xy}/(\tau_{xy})_{ult}$	-0.12	2.19	-0.73	2.26

[1]See Figure 33.

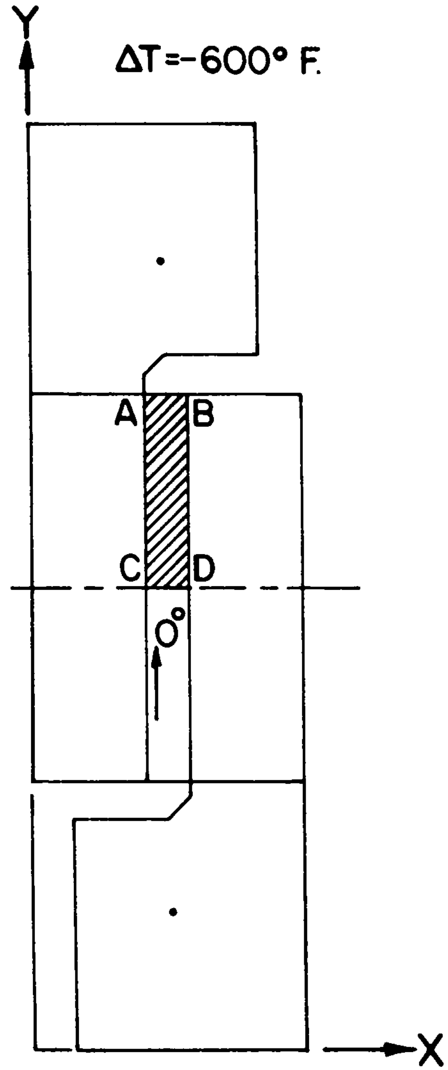
is a maximum away from the rail-specimen interface AC.

A characteristic of all rail shear specimens with elastic rails is the existance of nonzero σ_y normal stresses in the central portion of the specimen. These stresses must be nonzero in this region as a result of the nonzero axial strain present in both the fixture and the specimen as a result of the elastic behavior of the fixture. The effect of rail stiffness on σ_y at the center of the specimen is clearly illustrated by comparing normalized σ_y values for the nontapered and tapered rail configurations (Table 5). Since the tapered rail stiffness at the center of the specimen is half that of the nontapered rail, approximately twice the axial load (and hence stress) is transferred into the specimen with the tapered rails.

7.3.2 Thermal Load Effects

The thermal stresses in a specimen with a [0] graphite-polyimide laminate were determined for a temperature change $\Delta T = -600°F$ using the finite element mesh shown in Fig. 30 (with $F = 0$). Rail configurations studied included the nontapered and tapered rails described in section 7.3.1. The stress contour plots for σ_x, σ_y, and τ_{xy} normalized with respect to their corresponding ultimate stresses $(\sigma_x)_{ult}$, $(\sigma_y)_{ult}$, and $(\tau_{xy})_{ult}$ (Appendix C) are shown in Figs. 36 and 37.

This linear elastic analysis predicts stress concentrations at the corners A and B with both σ_x and τ_{xy} much greater than their respective ultimate values. While the magnitude of shear stress is zero in the center of the nontapered rail specimen, it is greater than zero in the tapered rail specimen. The nonzero shear stress in the center of the



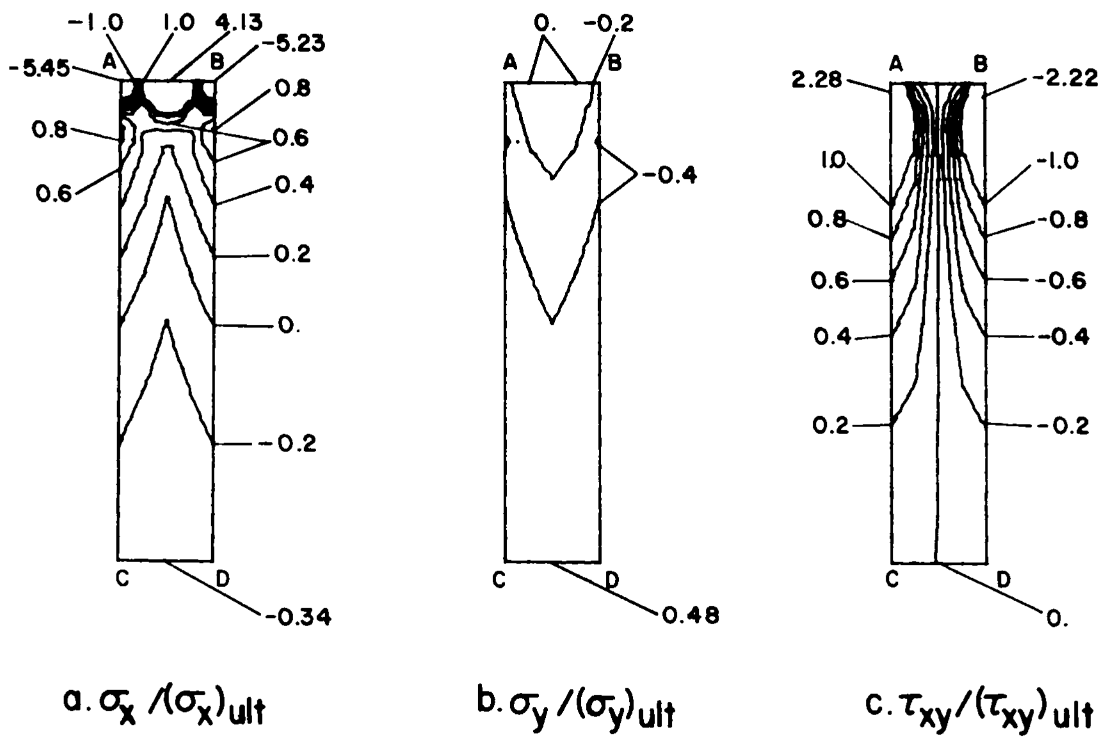


FIGURE 36. NORMALIZED STRESS CONTOURS FOR THERMALLY LOADED [0] GRAPHITE- POLYIMIDE RAIL SHEAR SPECIMEN WITH NONTAPERED RAILS

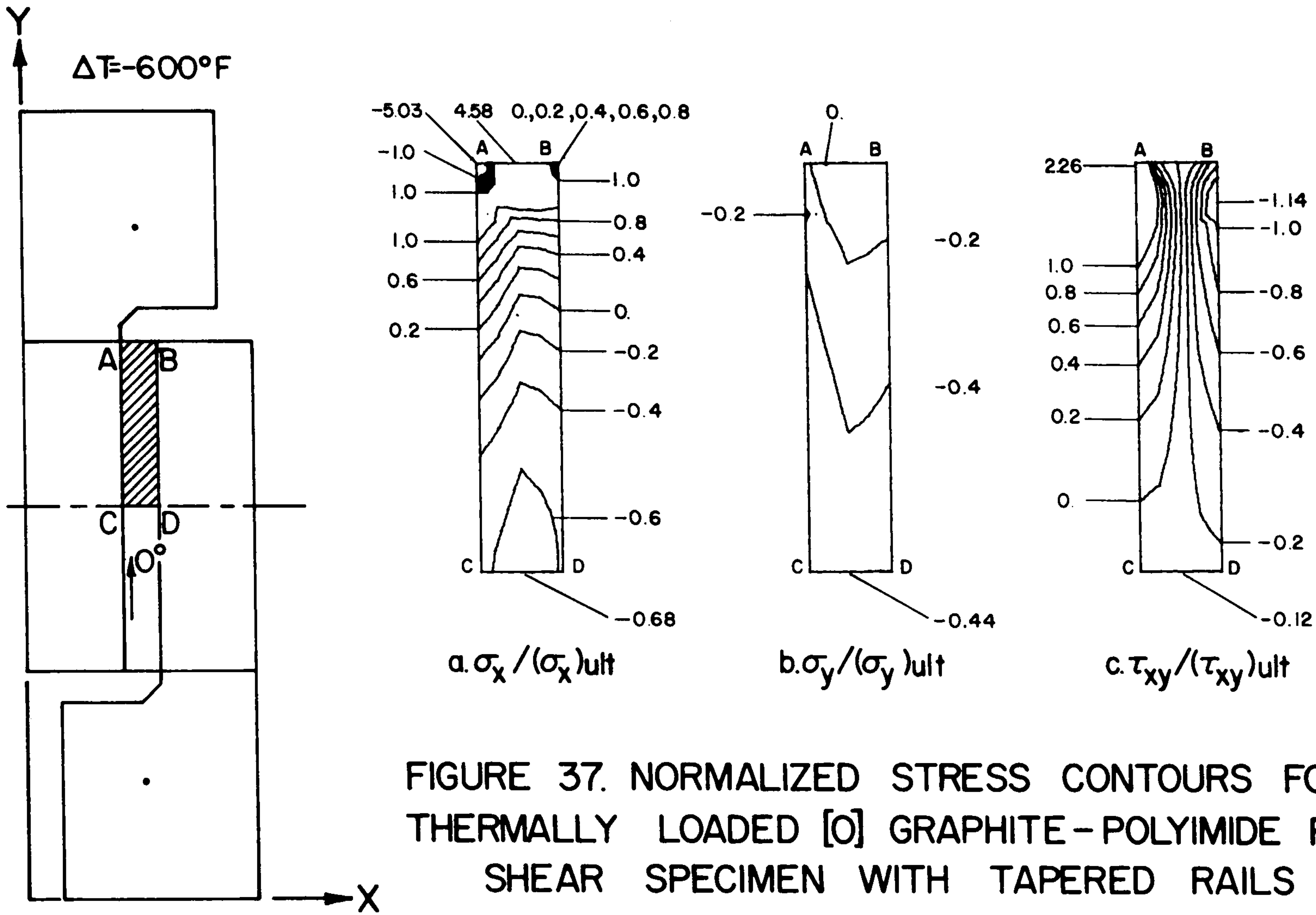


FIGURE 37. NORMALIZED STRESS CONTOURS FOR THERMALLY LOADED [0] GRAPHITE-POLYIMIDE RAIL SHEAR SPECIMEN WITH TAPERED RAILS

tapered rail specimen is attributed to the coarse finite element grid since the shear stress must be zero there to satisfy symmetry conditions. In general the normalized σ_y stresses are considerably lower than σ_x and τ_{xy}, however, σ_y is significant at the center of the specimen.

7.3.3 Other Mechanically Loaded Laminates

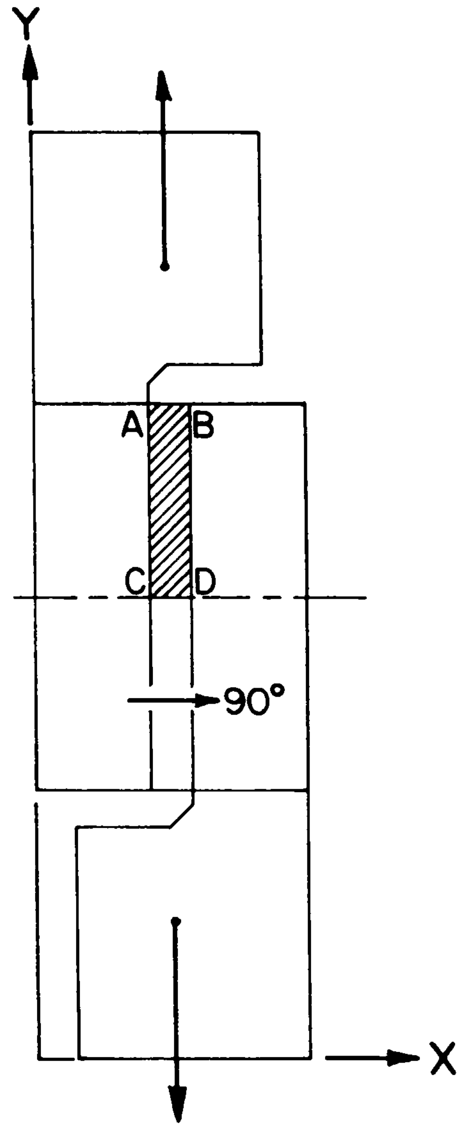
Normalized stress contour plots for [90] and $[\pm45]_s$ graphite-polyimide laminates using the tapered rail configuration are illustrated in Figs. 38 and 39, respectively. The finite element grid described in section 7.3.1 was used for these results.

7.3.3.1 The [90] Laminate

The normalized shear stress distribution for the [90] laminate (Fig. 38c) is nearly ideal in that the normalized shear stress reaches a value of -1.0 very close to the free edge AB and remains very uniform over the entire test section. However, the normal stresses are non-zero in the test section and high stress concentrations are present in the vicinity of corners A and B.

7.3.3.2 The $[\pm45]_s$ Laminate

The shear stress distribution for the $[\pm45]_s$ laminate is shown in Fig. 39c. As was the case for the rigid rail results (section 7.2), the tapered elastic rail solution indicates that the magnitude of the normalized shear stress at the center of the specimen is indeed close to -1.0 (Table 5). However, unlike all previous finite element results for both rigid and elastic rails, the maximum normalized shear stress was located at corner A where the value was -1.37. The contour plots of the normal stresses depict relatively low (but significant) values in the



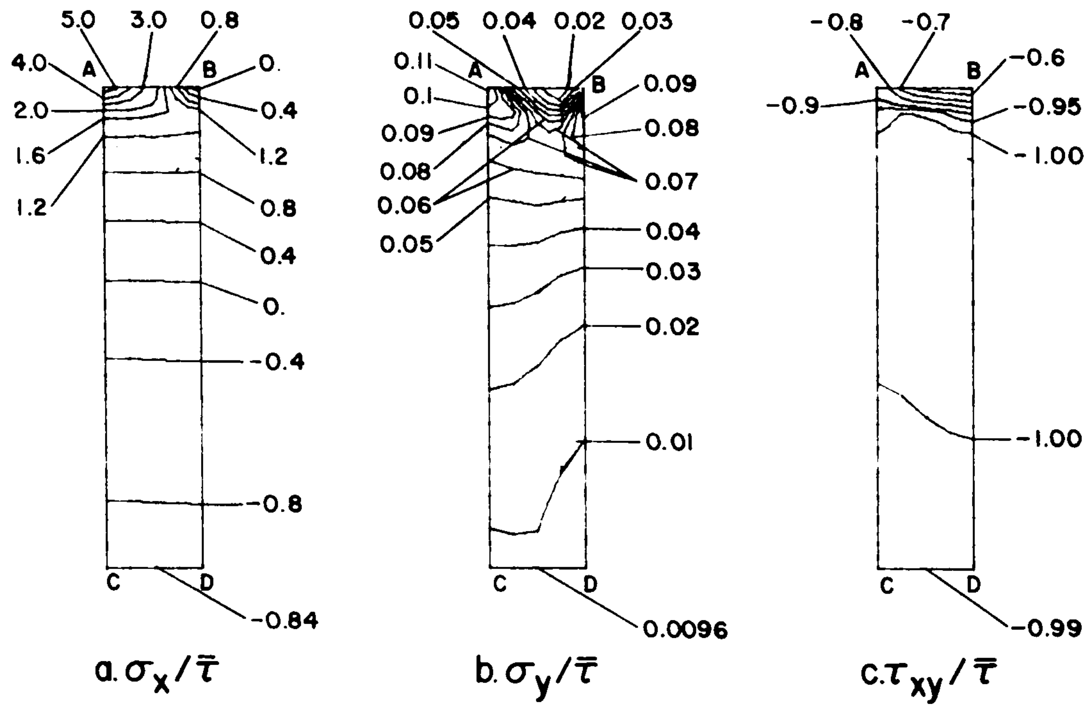


FIGURE 38. NORMALIZED STRESS CONTOURS FOR [90] GRAPHITE – POLYIMIDE RAIL SHEAR SPECIMEN WITH TAPERED RAILS

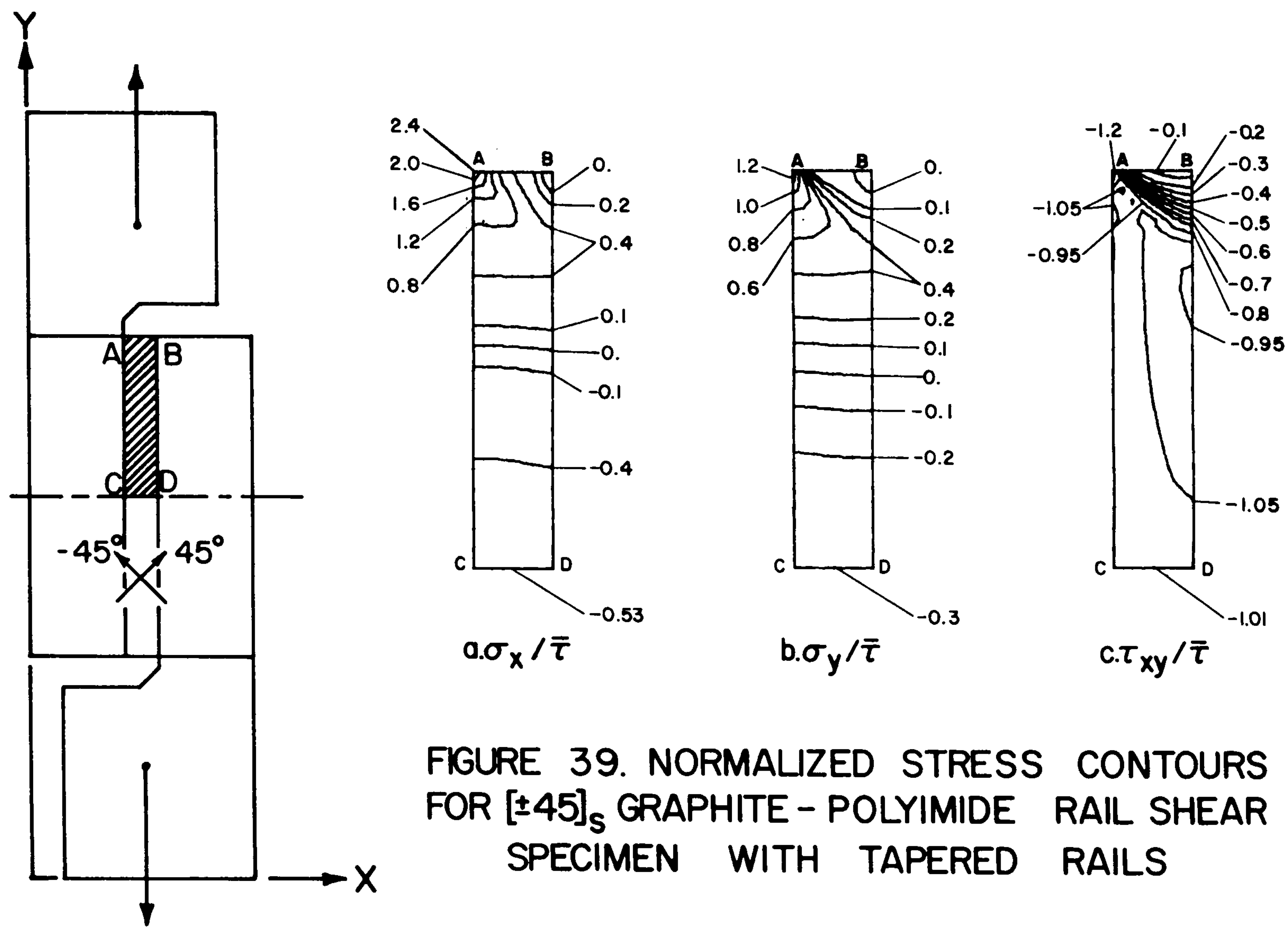


FIGURE 39. NORMALIZED STRESS CONTOURS FOR $[\pm 45]_s$ GRAPHITE - POLYIMIDE RAIL SHEAR SPECIMEN WITH TAPERED RAILS

test section and maximum values in the vicinity of the corners. The distributions of σ_x and σ_y were similar except that σ_y was generally lower in magnitude.

7.4 Summary

The rail configurations investigated produced regions of uniform shear stress over most of the test section for all laminates studied. However, this area of shear stress was often accompanied by significant normal stresses depending upon the stiffness of the rails, the method of load application, and the laminate properties. Significant axial stress was always present in the test section for elastic rails axially loaded. The magnitude of this axial stress was approximately inversely proportional to the stiffness of the rails. The offset loaded rail configuration minimized undesireable normal stresses in the center of the specimen. Stress concentrations in the vicinity of the corners were found in all specimens. Although tapered rails reduce stresses caused by thermal loads, this linear elastic analysis indicates that a non-linear analysis is required for a more accurate solution for a 600° temperature change.

8. CONCLUSIONS

A linear elastic finite element analysis has shown that the cross beam, Iosipescu, and rail shear specimens produce a region of shear stress in the test section. However, effects attributed to laminate properties, core material, and rail configurations significantly influenced the magnitude and the purity of the shear stress at the center of the specimens. In addition, all specimens exhibited stress concentrations in the vicinity of geometric discontinuities.

The rail shear specimen produced a region of uniform shear stress that was not significantly influenced by laminate properties or rail configurations. The axially loaded elastic rails resulted in significant normal stresses at the center of the specimen while the offset loaded elastic rail configuration substantially reduced these stresses. Significant stress concentrations in the vicinity of the free edges were found for all laminates and rail configurations; this was particularly true for the thermally loaded specimens.

The Iosipescu specimen produced uniform, pure shear at the central portion of the specimen for all laminates. Limited studies have shown that the stress concentrations at the notch tips and specimen corners can be substantially reduced by rounding the notch tips and using doublers in the specimen corners. The effects of elastic fixtures and thermal loads were minimal in the test section of this specimen. However, the magnitude of shear stress in the test section of the Iosipescu specimen was dependent upon the properties of the laminate. Although this specimen is relatively new (and unproven for composites), experience from its use with metals in addition to the analyses in this study indicate that the Iosipescu specimen has potential for future use in testing of composite materials.

The cross beam shear specimen was shown to be a viable specimen for the $[0/90]_s$ laminate. However, the $[\pm 45]_s$ laminate produced a non-uniform shear stress distribution in the test section and this distribution was influenced by the stiffness of the core material. A pure, uniform area of shear stress was not found in the slotted coupon. Therefore, it is not recommended for determining shear response of composites.

In summary, the linear elastic finite element results have indicated that several factors influence the stress distribution present in the test section of each specimen. The rail shear and Iosipescu specimens, despite specific disadvantages, were found to be the most suitable specimens for in-plane shear response of composite laminates.

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APPENDIX A.

COMPUTER PROGRAMS

A. COMPUTER PROGRAMS

A.1 Introduction

The present study required three distinct FORTRAN programs to obtain the levels of efficiency and analysis desired. The finite element program described in section A.2 is the basic program. This program performs the finite element solution independently of the other programs. However, for an accurate finite element solution, many intrinsically repetitive node and element cards are required which collectively represent the geometry of the structure. Since repetitive numerical procedures are simplified when programmed, a mesh generation program was developed for the requirements of the study as described in section A.3.

The number of elements in a finite element representation produce a proportional number of stress values which are provided in the form of tabular output by the basic finite element programs. The program described in section A.4 conveniently reduces this data by contour plotting the tabulated stress values.

In practice the finite element mesh is generated first and the resulting node and element card images are written on an indirect access file. Since the node and element card images constitute most of the input to the finite element program (see section A.2) the file is interactively edited to become an input data file for the finite element program. Similarly, the tabulated stress values from the finite element program may be stored on an indirect access file. This file constitutes a large portion of the input for the stress contour plotting program (see section A.4) and it also is edited to become an input file to the contour plotting program.

A.2 The Finite Element Program

A.2.1 General

The finite element method is a powerful tool for approximating the state of stress in virtually any structure subjected to loads. A requirement for utilizing such a method is the availability of a high speed digital computer and a program to carry out the solution procedure. This section describes such a FORTRAN program.

The use of the finite element method for solutions to structural problems has been well established by Desai and Abel [33], Zienkiewicz [34], and others. This particular program is an extensively modified version of the FORTRAN code by Desai and Abel. The program uses the displacement method for solution (see section 3.2). Both constant strain triangular elements and quadrilateral elements composed of four constant strain triangles are available for either plane stress or plane strain analysis. Capability added to the program by the author includes orthotropic material properties, orthotropic thermal effects, and the implementation of an equation solving subroutine developed by Wilson et al [35]. The improved equation solver allows the program to solve a very large system of linear equations in blocks residing on slow speed storage, as opposed to the more restricted, completely in core solution technique.

The job flow is one pass since linear elastic problems are directly solved. After material property, node, element, and load information is input, a stiffness matrix for each element is computed. All elemental stiffness matrices are then assembled into a global stiffness matrix, and the load vector is initialized. The unknown displacement vector is determined using Gauss elimination techniques on only non-zero stiffness terms. Once the global displacements are computed, elemental strains are determined. The elemental stresses are then computed from the elemental constitutive matrices and the elemental strains.

A.2.2 Input Description

Column		Contents
Title card (I5,9A8)		
1-5	NPROB	= Problem number. If NPROB > 100, stresses are written on unit 2. This option is used with the stress contour plotter (see section A.4). If NPROB = 0, no more problems are run.
6-77	A(I)	= Title information
Control card (4I5)		
1-5	NNP	= Number of nodes
6-10	NEL	= Number of elements
11-15	NMAT	= Number of materials
16-20	NOPT	= 1 for plane strain analysis
		= 2 for plane stress analysis

Laminate material property cards (7E10.0)

(for material I)

1-10	EX(I)	= Modulus of elasticity in X direction
11-20	EY(I)	= Modulus of elasticity in Y direction. IF (EY(I).EQ.0.) EY(I) = EX(I)
21-30	PRXY(I)	= Poisson's ratio, ν_{xy}. IF (PRXY(I).EQ.0.)PRXY(I) = EX(I)/2./GXY(I)-1.
31-40	GXY(I)	= Modulus of rigidity for material I. IF (GXY(I).EQ.0.) GXY(I) = EX(I)/(1.+PRXY(I))/2.
41-50	TH(I)	= Thickness for material I.
51-60	ALPHA(1,I)	= Coefficient of thermal expansion in X direction for material I.
61-70	ALPHA(2,I)	= Coefficient of thermal expansion in Y direction for material I.

Nodal location and mechanical load cards (2I5,4E10.0)

1-5	N	= node number
6-10	KODE(N)	= 0 for ULX(N) and VLY(N) prescribed loads
		= 1 for ULX(N) prescribed displacement and VLY(N) prescribed load.

		= 2 for ULX(N) prescribed load and VLY(N) prescribed displacement
		= 3 for ULX(N) and VLY(N) prescribed displacements
11-20	X(N)	= X coordinate
21-30	Y(N)	= Y coordinate
31-40	ULX(N)	= X displacement or load
41-50	VLY(N)	= Y displacement or load

Element and thermal load cards(6I5,E10.0)

1-5	M	= Element number
6-10	IE(M,1)	= First node of element
11-15	IE(M,2)	= Second node of element
16-20	IE(M,3)	= Third node of element
21-25	IE(M,4)	= Fourth node of element. For triangular elements, IE(M,3) = IE(M,4)
26-30	IE(M,5)	= Material properties associated with element M
31-40	TEMP(M)	= Change in temperature of element after displacement boundary conditions are imposed. IF (TEMP(M).EQ.0.)TEMP(M) = TEMP(M-1)

Comments

Node and element cards must be in ascending consecutive order. Nodes associated with triangular elements that are not ordered counterclockwise cause a warning message, but are reordered counterclockwise internally. Quadrilateral elements must be ordered counterclockwise. If the mesh generation program (section A.3) is used to supply node and element cards, these cards (or indirect access file) must be altered to include boundary condition (KODE), mechanical load (ULX(N),VLY(N)), elemental material (IE(M,5)), and elemental temperature (TEMP(M)) information. To terminate execution, a blank card is inserted after the last element and thermal load card of the last model. If NPROB is not zero the program expects to read a complete data set for another model.

A.3 The Mesh Generation Program

A.3.1 General

This program generates groups of elements and nodes. In a group, all elements are either identical to one another or have repetitive geometric characteristics. Many groups of elements make up the total model. A group of elements may be located anywhere in X-Y coordinates by several methods. The dimensions, orientation, number of elements, and type of elements for each group are specified by the user. Data for each element group is read sequentially so that data to generate specific element groups may be added or deleted from the entire data set. Specified groups of elements may be scaled, rotated, or repositioned so that element groups frequently needed in different meshes need only be checked once and can be inserted as needed into different meshes.

Input data to the mesh generator is comprised of four categories. The first category of input is the generation initialization and termination data. The initialization data is at the beginning of the data set and is used only once for the purpose of initializing title information and reference frames. At the end of the data is the card indicating that the mesh generation commands are complete so that the program can perform its final computations.

All other categories of input commands may be used anywhere at any time between the initialization and termination data. The second category of data consists of element group generation data sets. These data sets generate specific groups of elements. An element group generation data set can control every parameter associated with the generation of that particular group, or certain parameters may be passed to it from the preceeding group. The third category of input alters the reference frame of one, several, or all groups that constitute the complete model. General capabilities include translation, rotation, and scaling of any desired element groups. The last category of input data has the capacity to reposition nodes. While other input commands operate by element groups, the nodal repositioning commands operate on all nodes that have been previously generated. The only capacity for nodal repositioning in the present version is that of repositioning nodes to lie in a precise circular arc. This avoids the necessity of manually positioning individual nodes that will ultimately lie on some circular arc in the complete model.

A.3.2 Generation Initialization and Termination Data Input

Column		Contents
Title card (10A8)		
1-80	CARD(1)	= Title information
Coordinate Initialization Card (6F10.0,2I5)		
1-10	ORIGIN(1)	= Absolute X coordinate of initial global and local coordinate system origin
11-20	ORIGIN(2)	= Absolute Y coordinate or initial global and local coordinate system origin
21-30	SCALE(1)	= X scaling dimension for internal node numbering
31-40	SCALE(2)	= Y scaling dimension for internal node numbering
41-50	FACT(1)	= X global coordinate scale factor
51-60	FACT(2)	= Y global coordinate scale factor
61-65	MSCAN	= Node duplication elimination number
66-70	NSV	= 0 for numbering nodes by ascending X then Y location = 1 for numbering nodes by ascending Y the X location

Comments

The ORIGIN(I) variables initialize the origin of the global coordinates in terms of absolute (final) coordinates. Since the ORIGIN(I) fields have the same purpose as the global origin reset cards, the exact use of these fields are described in section A.3.4.

The node numbers are internally numbered by absolute X and Y location. Such node numbers are determined by the equations

```
        IX = X/SCALE(1)*10000.-.5
        IY = Y/SCALE(2)*10000.-.5
IF(NSV.EQ.0)NODE = IY*10000+IX
IF(NSV.EQ.1)NODE = IX*10000+IY
```

where X,Y are node locations.

NODE is the internal node number. The nodes are internally numbered in this manner for two reasons. Due to roundoff errors caused by reference frame alterations, a previously generated node does not necessarily have the exact coordinate location of another node intended to have the same location. For this reason, duplicate node positions are not eliminated by a check for X-Y absolute coordinate duplication. SCALE(I) must be slightly larger than the expected maximum X-Y node coordinates. NODE is then determined for each node, and any node whose internal IX or IY is equal to or one different from another IX or IY is eliminated. After all nodes and elements have been generated, the NODE numbers are put in ascending order and renumbered 1,2,3,...,NNP, where NNP is the total number of node points. This procedure minimizes the bandwidth of many meshes since nodes are renumbered by ascending X coordinate then ascending Y coordinate locations (for NSV=0).

The FACT(I) variables initialize the coordinate multiplication factors for all dimensions with respect to any global coordinate system. Since the FACT(I) fields have the same purpose as the coordinate scale reset cards, the exact use of these fields are described in section A.3.4.

The integer MSCAN is used to minimize scanning procedures for duplicate NODE numbers as element groups are processed. For example, if an element group with ten boundary nodes is being added to an existing model whose last ten generated nodes will coincide with these boundary nodes, MSCAN need only be ten. However, if one hundred new nodes are generated between the boundary nodes that have the same location, MSCAN should be at least one hundred and ten. Since all duplicate nodes are eliminated at the time the model is completed, MSCAN can be zero for any model. However, duplicate node information tends to fill up storage arrays rather quickly so the value of this variable should always be overestimated to minimize storage requirements.

Column		Contents
Termination card (I5)		
1-5	KIND	= 5. This stops the program from reading additional group generation, reference frame alteration, and nodal repositioning cards. At this point the pro-

gram initiates final processing of the completed mesh. This card may be last in the input deck (or file) or placed between any of the repeatable data sets for mesh debugging purposes

A.3.3 Element Generation Input

Type 1 Rectangular Element Generation Cards

Card 1 (I5,9A8)

1-5	KIND	= 1 for group type 1, rectangular element generation input to follow
6-77	CARD(I)	= Title for element group

Card 2 (3I5,F10.0)

1-5	M	= The number of rectangular elements in the X direction, see figure A-1a
6-10	N	= The number of rectangular elements in the Y direction
11-15	IRC	= The reset code for A(1) and A(2)
16-25	THETA	= The angle of rotation about the local origin

Card 3 (2F10.0,I5)

1-10	A(1) or AX	= Total reset or multiplying factor of A(1)
11-20	A(2) or AY	= Total reset or multiplying factor of A(2)
21-25	JRC	= Reset code for XY(1) and XY(2), the local origin location in terms of global coordinates

Card 4 (2F10.0)

1-10	XY(1) or TEMP(1)	= Total or incremental X local origin location in terms of global coordinates
11-20	XY(2) or TEMP(2)	= Total or incremental Y local origin location in terms of global coordinates

Comments

For the A(I) dimension reset option IRC, the following numbers are admissable

IRC	= 0 for no reset, A(1) and A(2) are used from the previous element group
	= 1 for a scalar reset of A(1), AX is read, A(1) = AX*A(1)*FACT(1)

= 2 for a scalar reset of A(2), AY is read, A(2) = AY*A(2)*FACT(2)

= 3 to reset both cases above

= -1 for a total reset of A(1), A(1) is read, A(1) = A(1)*FACT(1)

= -2 for a total reset of A(2), A(2), is read, A(2) = A(2)*FACT(2)

= -3 for a total reset of A(1) and A(2)

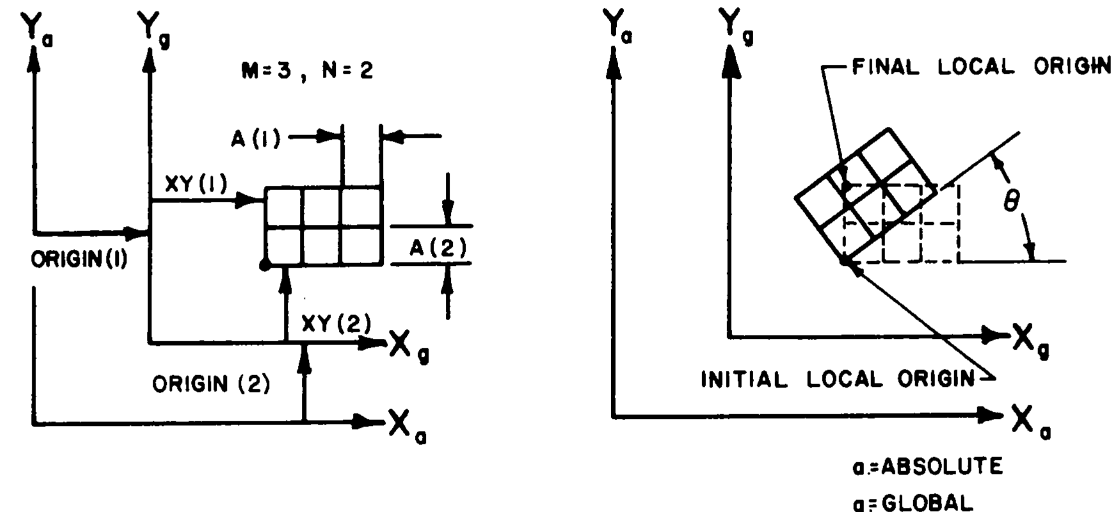
The program omits fields that are not applicable following resets. If IRC = 0, the following field on card 3 becomes (I5). If IRC = 1, the following field on card 3 becomes (F10.0,I5), and similarly for other cases.

For the local origin reset the following options are admissable

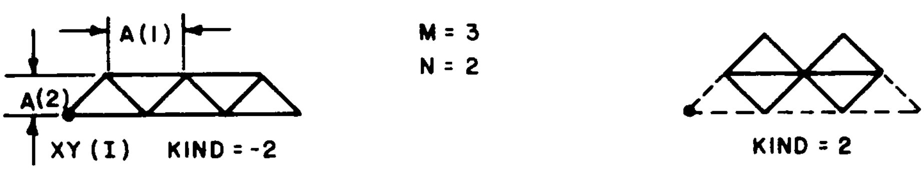
JRC = 0 for the default change in local origin with respect to the global coordinate system

= 1 for an incremental reset of XY(1), TEMP(1) is read, XY(1) = XY(1)+TEMP(1)*FACT(1)+ORIGIN(1)

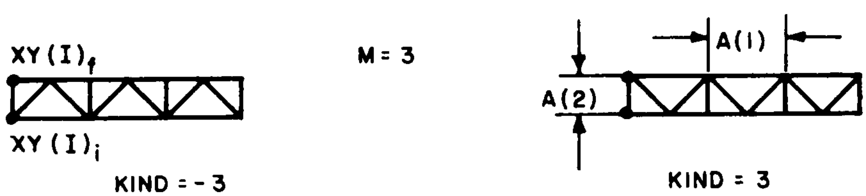
= 2 for an incremental reset of XY(2), TEMP(2) is read, XY(2) =



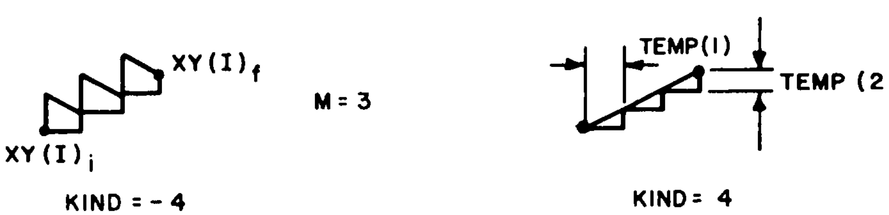
a. GROUP 1 - RECTANGLES



b. GROUP 2 - ISOSCELES TRIANGLES



c. GROUP 3 - TRANSITION "RECTANGLES"



d. GROUP 4 - IRREGULAR ELEMENTS

FIGURE A-1 ELEMENT GROUPS

XY(2)+TEMP(2)*FACT(2)+ORIGIN(2)

= 3 to reset both cases above

= -1 for a total reset of XY(1), XY(1) is read, XY(1) = XY(1)*FACT(1)+ORIGIN(1)

= -2 for a total reset of XY(2), XY(2) is read, XY(2) = XY(2)*FACT(2)+ORIGIN(2) where XY(I) become absolute coordinates.

= -3 for total reset of XY(1) and XY(2)

All element group generation cards may be used anywhere and any number of times after the initialization cards.

Type 2 Isosceles Triangle Element Generation Cards

Card (I5,9A8)

1-5	KIND	= 2 or -2, depending on the generation mode desired for isosceles triangle input, see figure A-1b
6-77	CARD(I)	= Title information for element group

Card 2 (3I5,F10.0)

1-5	M	= The number of isosceles triangles in the X direction

6-10	N	= The number of isosceles triangles in the Y direction
11-15	IRC	= The reset code for A(I), see type 1 description
16-25	THETA	= The angle of rotation about the local origin

Card 3 (2F10.0,I5)

1-10	A(1) or AX	= Total reset or multiplying factor of A(1)
11-20	A(2) or AY	= Total reset or multiplying factor of A(2)
21-25	JRC	= Reset code for XY(I), the absolute coordinate location for the local origin, see group 1

Card 4 (2F10.0)

1-10	XY(1) or TEMP(1)	= Total or incremental X local origin reset value
11-20	XY(2) or TEMP(2)	= Total or incremental Y local origin reset value, see group 1

Comments

See figure A-1b for isosceles triangle specifications. The description of cards 3 and 4 are identical to those of element group 1.

Type 3 Transition "Rectangle" Generation Cards

Card 1 (I5,9A8)

1-5	KIND	= 3 or -3, depending on the mode for transition rectangles, see figure A-1c
6-77	CARD(I)	= Title information for element group

Card 2 (2I5,F10.0)

1-5	M	= The number of transition rectangles in the X direction
6-10	IRC	= The reset code for A(I)
11-20	THETA	= The angle of rotation about the local origin

Card 3 (2F10.0,I5)

1-10	A(1) or AX	= Total reset or multiplying factor of A(1)
11-20	A(2) or AY	= Total reset or multiplying factor of A(2)
21-25	JRC	= Reset code for XY(1) and XY(2), the local origin location

Card 4

1-10	XY(1) or TEMP(1)	= Total or incremental X local origin reset value
11-20	XY(2) or TEMP(2)	= Total or incremental Y local origin reset value

Type 4 Irregular Element Generation Cards

Card 1 (I5,9A8)

1-5	KIND	= 4 for irregular triangle generation
		= -4 for irregular quadrilateral generation
6-77	CARD(I)	= Title information for element group

Card 2 (2I5)

1-5	NELODD	= The number of irregular elements to be generated, see figure A-1d
6-10	JRC	= The local origin reset code

Card 3 (2F10.0)

1-10	XY(1) or TEMP(1)	= Total or incremental X local origin reset value
11-20	XY(2) or TEMP(2)	= Total or incremental Y local origin reset value

Card 4 (6F10.0)

1-10	TEMP(1)	= Change in X coordinate from local origin for second node on element
11-20	TEMP(2)	= Change in Y coordinate from local origin for second node on element
21-30	TEMP(3)	= Same as TEMP(1) for third node
31-40	TEMP(4)	= Same as TEMP(2) for third node
41-50	TEMP(5)	= Same as TEMP(1) for fourth node
51-60	TEMP(6)	= Same as TEMP(2) for fourth node

Card 5 (2F10.0)

1-10	TEMP(1)	= Change in X coordinate of the local origin after each irregular element generation
11-20	TEMP(2)	= Change in Y coordinate of the local origin after each irregular element generation

A.3.4 Reference Frame Alteration Data Sets

Global Origin Reset Cards

Card 1 (I5,9A8)

1-5	KIND	= -1 for global origin reset
6-77	CARD(I)	= Title information

Card 2 (2F10.0)

1-10	ORIGIN(1)	= Location of X global origin in absolute coordinates and initial location of the local coordinate system
11-20	ORIGIN(2)	= Location of Y global origin in absolute coordinates and initial location of the local coordinate system

Comments

These cards may be inserted anywhere after the initialization cards and may be used any number of times. The local coordinate position is always initially set to the ORIGIN(I) position. All local origin changes and resets are done with respect to the global coordinate system.

Global Coordinate Scale Factor Reset Cards

Column Contents

Card 1 (I5,9A8)

1-5	KIND	= 6 for global scale factor reset
6-77	CARD(I)	= Title information for scaled element groups

Card 2 (2F10.0)

1-10	FACT(1)	= Scale factor for global coordinate in X direction
11-20	FACT(2)	= Scale factor for global coordinate in Y direction

Comments

FACT(I) scales all A(I), AX, AY, XY(I), and TEMP(I), input from the element group generation data sets (see section A.3.3).

Input for Rotating Several Element Groups

Column		Contents
Card 1 (I5,9A8)		
1-5	KIND	= 8 for element groups following to be rotated
6-77	CARD(I)	= Title information
Card 2 (5F10.0)		
1-10	XTH	= Rotation in degrees about the X axis at XPIVOT and YPIVOT
11-20	YTH	= Rotation in degrees about the Y axis at XPIVOT and YPIVOT
21-30	ZTH	= Rotation in degrees about the Z axis at XPIVOT and YPIVOT
31-40	XPIVOT	= Global X location for the

		point about which nodes are rotated
41-50	YPIVOT	= Global Y location for the point about which all nodes are rotated

Card to Terminate Element Groups to be Rotated (I5)

1-5	KIND	= -8, to rotate nodes of element groups generated between card 2 and this card

Comments

XPIVOT and YPIVOT are not scaled by FACT(I). Rotation commands may not be nested and for every KIND = 8 there must be a corresponding KIND = -8. All rotations executed with these commands are carried out independently of rotations of individual element groups.

A.3.5 Node Repositioning Data Sets

Column		Contents
Card 1 (I5,9A8)		
1-5	KIND	= 7 for node repositioning data to follow
6-77	CARD(I)	= Title information
Card 2 (7F10.0)		
1-10	XO	= Absolute X location for the

		center of the circular segment on which repositioned nodes are to lie
11-20	YO	= Absolute Y location for the center of the circular segment on which repositioned nodes are to lie
21-30	THI	= Initial angle from the X axis for circular segment
31-40	THF	= Final angle from the X axis for circular segment, THF > THI
41-50	TDEG	= Tolerance to include nodes located near angles THI and THF. Nodes that are repositioned lie between the angles THI-TDEG and THF+TDEG
51-60	RAD	= Radius of the circular segment about XO, YO to which nodes will be repositioned
61-70	TOL	= Tolerance to include nodes located between a radius of RAD-TOL and RAD+TOL to be repositioned along the radius RAD

A.4 The Plotting Program

A.4.1 General

In order to efficiently evaluate finite element meshes and stress fields, it is desirable to have a versatile computer program capable of plotting finite element meshes and stress contour plots either on paper or at an interactive graphics terminal. This section describes such a finite element mesh and stress contour plotting computer program.

The finite element mesh plotting algorithm simply plots the specified elements of the mesh to a specified plot width. Nodes, elements, or both may be numbered. Numbering schemes are chosen by the user.

The stress contour plotting algorithm plots the outline of the mesh and plots lines of constant σ_x, σ_y, and/or τ_{xy} throughout the interior of the mesh outline. The number of stress contours are specified by the user. Stress contour values may be input, internally generated, normalized by input values of stress, or internally normalized. Any stress contour values generated may be punched for future use.

A.4.2 Input Description

Column		Contents
Control Card (3I5)		
1-5	NNP	= Number of nodes
6-10	NEL	= Number of elements
11-15	NCASE	= Number of plot cases

Node Cards (10X,2F10.0)

11-20	XY(I,1)	= X coordinate of node I.
21-30	XY(I,2)	= Y coordinate of node I.

Element Cards (5X,4I5)

6-10	IELE(I,1)	= First node number of element I
11-15	IELE(I,2)	= Second node number of element I
16-20	IELE(I,3)	= Third node number of element I
21-25	IELE(I,4)	= Fourth node number of element I

Stress Cards (3E20.0)

1-20	SIG(I,1)	= σ_x at the centroid of element I
21-30	SIG(I,2)	= σ_y at the centroid of element I
31-40	SIG(I,3)	= τ_{xy} at the centroid of element I

Comments

There are NNP node cards, node numbers are in ascending order from 1 to NNP. There are NEL element cards in ascending order from 1 to NEL. IELE(I,3) = IELE(I,4) for triangular elements. If only a grid is to be plotted, NCASE is negative and the number of plot cases run are IABS(NCASE). No stresses are read for a negative NCASE. There are NEL stress cards, one associated with every element. Node and element cards (or records) are usually generated with the mesh generator and the stress cards are usually generated by the finite element program (NPROB > 100, see section A.2).

Plot Case Cards

Column		Contents
Title Card (10A8)		
1-80	TITLE(I)	= Title for plot case to follow
Control Card (5F10.0,3I5,6I1)		
1-10	XMIN	= Minimum X dimension for nodes plotted
11-20	XMAX	= Maximum X dimension for nodes plotted
21-30	YMIN	= Minimum Y dimension for nodes plotted
31-40	YMAX	= Maximum Y dimension for nodes plotted
41-50	WIDTH	= Width of the plotted mesh
51-55	NEON	= The labeling index for node numbering on mesh plots
56-60	NEOE	= The labeling index for element numbering on mesh plots
61-65	NCONT	= The number of stress contours

		to be plotted
66	IGRID	= 0 for no grid plot
		= 1 for grid plot
67	ISIGX	= 0 for no σ_x stress contour plot
		= 1 for σ_x stress contour plot
		= 2 for σ_x stress contour plot with all plotted stresses normalized by σ_x normalization elements
		= 3 for σ_x stress contour plot with all plotted stresses normalized by an input value of stress
68	ISIGY	= Same options as field 67 (ISIGX) for σ_y
69	ITAUXY	= Same options as field 67 (ISIGX) for τ_{xy}
70	ICOVAL	= 0 for elementwise stress contour value generation
		= 1 for input stress contour values
		= 2 for stress contour values of constant stress increment generation

71	ICOPUN	= 0 for no punched stress contour values = 1 for punched stress contour values

Normalizing Stress Card (F10.0)

1-10	AVE(4)	= Stress which all others will be normalized by. This card is needed only if ISIGX, ISIGY or ITAUXY is 3.

Stress Normalization Element Cards (I5,F10.0,12I5)

1-5	ICON	= 0 for no more elements to be added = +N for every Nth element to be averaged = -1 for each following element to be averaged
6-15	FACT	= Element's weighting factor
16-80	NOPAIR (I,1)	= Element numbers

Plotted Element Cards (14I5)

1-5	ICON	= 0 for no more elements to be added = +N for every Nth element to be

			plotted
		=	-1 for each following element to be plotted
6-75	NOPAIR (I,1)	=	Element numbers

Contour Stress Value Cards (3E20.0)

1-20	AVE(I,1)	=	σ_x stress value of contour I
21-40	AVE(I+1,1)	=	σ_x stress value of contour I+1
41-60	AVE(I+2,1)	=	σ_x stress value of contour I+2, repeat cards until NCONT values are read
1-20	AVE(I,2)	=	σ_y stress value of contour I
21-40	AVE(I+1,2)	=	σ_y stress value of contour I+1
41-60	AVE(I+2,2)	=	σ_y stress value of contour I+2, repeat cards until NCONT values are read
1-20	AVE(I,3)	=	τ_{xy} stress value of contour I
21-40	AVE(I+1,3)	=	τ_{xy} stress value of contour I+1
41-60	AVE(I+2,3)	=	τ_{xy} stress value of contour I+2, repeat cards until NCONT values are read. If ISIGX, ISIGY, and/or ITAUXY are 0, their corresponding stress contour values are not needed. Contour

stress value cards are only read for ICOVAL=1.

Comments

All plot case cards are repeated NCASE times. For automatic scaling, XMIN, XMAX, YMIN, and YMAX are zero. The smallest of XMAX-XMIN and YMAX-YMIN is plotted across the plotting drum. For nonzero values of XMIN, XMAX, YMIN, and YMAX the plot will be distorted accordingly. WIDTH is the maximum dimension across the plotting drum for the plotted elements when XMIN, XMAX, YMIN, and YMAX are zero. A poor choice of XMIN, XMAX, YMIN, and YMAX may cause WIDTH to be significantly different from the expected width.

The value of NEON determines which nodes are numbered according to the formula

$$\text{number of labeled node} = 1+K/\text{NEON} \quad (\text{NEON} \neq 0)$$
$$\text{where } K = 1,2,3,\ldots, \text{NNP}.$$

No nodes are numbered if NEON = 0. Element numbering is determined in the same manner with NEOE. Element numbers are underlined.

Elementwise stress contour value calculations provide stress contours in the critical, refined mesh areas. For ICOVAL = 0 or 2, the elemental stresses are first put in ascending order in array X. The stress contour values 1 through NCONT-1 are calculated with the equations

$$K = \text{NEL}/\text{NCONT}$$

AVE(I) = (X(I+L)+X(I+1+L)+...+X(I+K+L))/K

where L = K*(M-1) for M = 1,2,3,...,NCONT-1.

The last stress contour value is the average of the stresses at the remaining elements.

Stress contour values of constant stress increments for ICOVAL = 2 are generated with the equations

DX = (X(NEL)-X(1))/(NCONT+2)

AVE(I) = X(1)+DX*I for I = 1,2,3,...,NCONT.

The stress normalization cards determine a normalizing σ_x, σ_y, or τ_{xy} stress value from the stresses on one or several elements weighted by FACT. For example, the records

1	2.	1	3	6	8	10
2	1.	11	21			
-1	.5	17	18	35	42	56
0						

would cause the $(\tau_{xy})_{nom}$ (for ITAUXY = 2) stress value of the indicated element numbers to be weighted as follows

$$A = (\sigma_3+\sigma_4+\sigma_5+\sigma_6+\sigma_7+\sigma_8+\sigma_9+\sigma_{10})/2.$$

$$B = (\sigma_{11}+\sigma_{13}+\sigma_{15}+\sigma_{17}+\sigma_{19}+\sigma_{21})/1.$$

$$C = (\sigma_{17}+\sigma_{18}+\sigma_{35}+\sigma_{42}+\sigma_{56})/0.5$$

$$AVE(4) = (\tau_{xy})_{nom} = (A+B+C)/18.,$$

where σ_i are τ_{xy} stresses of the i^{th} element.

If neither ISIGX, ISIGY, nor ITAUXY are equal to 2, these cards are not included. Element numbers on individual cards must be in ascending order.

APPENDIX B.

PROGRAM LISTINGS

B.1 THE FINITE ELEMENT PROGRAM

```
C
C     >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>
C           PROGRAM FOR PLANE STRAIN OR STRESS FINITE ELEMENT SOLUTION
C     >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>
C
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/REAL/EX(120),EY(120),PRXY(120),PRYX(120),TH(120),
     1X(2000),Y(2000),ULX(2000),VLY(2000),
     2QK(10,10),Q(10),B(3,10),C(3,3),GXY(120),BT(3,5),XQ(5),YQ(5)
     3,TEMP(2000),ALPHA(2,120)
      COMMON/PARTIT/NBMAX,NEQB,NBLOCK,ILOW,IHIGH,NSTIF,NL,NR,IBAND
      COMMON/INTEG/NNP,NEL,NMAT,NOPT,MTYP,IE(2000,5),
     1KODE(2000),ISC(1),JSC(1),NIN,NOUT,MA,NEQ,NPROB,NTAPE,NSTORE
      DIMENSION A(5000)
C     CALL TRAPS(0,999,999,0,0)
      CALL ERRSET(208,999,-1,0)
      MAXEL=2000
      MAXNP=2000
      MAXMAT=120
      NSTORE=2000
      NBMAX=50
      NV=1
      NTAPE=2
      REWIND NTAPE
      NSTIF=12
      NRED=3
      NL=11
      NR=8
      NOUT=6
      NIN=15
C
C     PROBLEM IDENTIFICATION AND DESCRIPTION|  IF NPROB IS GT 100,
C     STRESSES ARE WRITTEN ON NTAPE=2
C
 9999 READ(NIN,100) NPROB,(A(I),I=1,9)
      IF(NPROB.LE.0) GO TO 999
 1020 WRITE(NOUT,200)NPROB,(A(I),I=1,9)
      CALL DATAIN(MAXEL,MAXNP,MAXMAT,ISTOP)
      MAXDOF=2*MAXNP
C
C     COMPUTE MAXIMUM NODAL DIFFERENCE AND SEMI-BANDWIDTH, REF. 33, EQ.
C
      MAXDIF=0
      DO 1 I=1,NFL
      DO 1 J=1,4
      DO 1 K=1,4
      LL=IABS(IE(I,J)-IE(I,K))
```

```
      IF(LL.GT.MAXDIF)MAXDIF=LL
    1 CONTINUE
      MA=2*(MAXDIF+1)
      NEQ=NNP*2
      NTEMP=NEQ*(MA+NV)+NV*NEQ*((MA-2)/NEQ+2)+NEQ+MA-1
      IF(NSTORE.LT.NTEMP)GO TO 304
      NEQB=NEQ
      M2=NEQ*(MA+NV)+1
      M3=M2+NV*NEQ*((MA-2)/NEQ+2)
      NBLOCK=1
      GO TO 303
  304 NEQB=(NSTORE-MA)/(2*(MA+NV)+1)
      NEQBT=(NSTORE-MA-NV*(MA-2))/(3*NV+MA+1)
      IF(NEQBT.LT.NEQB)NEQB=NEQBT
      M2=NEQB*(MA+NV)+1
      M3=NEQB*(MA+NV)+M2
      M3T=NV*NEQB*((MA-2)/NEQB+2)+M2
      IF(M3T.GT.M3)M3=M3T
      NBLOCK=(NEQ-1)/NEQB+1
  303 WRITE(NOUT,302)NBLOCK,NEQB
  302 FORMAT(///' THE NUMBER OF BLOCKS ARE',I10/
     1' THE NUMBER OF EQUATIONS PER BLOCK ARE',I10)
      MI=MA+NEQB-1
      IF(NEQB.LT.2.OR.NBLOCK.GT.NBMAX)WRITE(NOUT,19)
   19 FORMAT(' TERMINATION, CORE REQUIREMENTS EXCEEDED')
      IF(NEQB.LT.2.OR.NBLOCK.GT.NBMAX)STOP
      IBAND=MA+1
C
C     TRIANGULARIZE STIFFNESS MATRIX, REF. 33, EQ. (2-2)
C
      CALL ASEMBL(ISTOP,A(1))
      IF(ISTOP.GT.0) GO TO 999
C
C     SOLVE FOR DISPLACEMENTS CORRESPONDING TO LOAD VECTOR, REF. 33, EQ.
C
      CALL SESOL(NEQ,MA,NV,NBLOCK,NEQB,NAV,MI,NSTIF,NRED,NL,NR,
     1A(1),A(M2),A(M3))
      REWIND NL
      WRITE(NOUT,400)
      CALL STRESS(A)
      GO TO 9999
  100 FORMAT(I5,9A8)
  200 FORMAT(/8H1PROBLEM,I5,3H.. ,9A8/)
  400 FORMAT(37H1OUTPUT TABLE 1.. NODAL DISPLACEMENTS //
     113X,4HNODE,9X,11HU = X-DISP.,9X,11HV = Y-DISP,/)
  999 STOP
      END
```

```
      SUBROUTINE DATAIN(MAXEL,MAXNP,MAXMAT,ISTOP)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/REAL/EX(120),EY(120),PRXY(120),PRYX(120),TH(120),
     1X(2000),Y(2000),ULX(2000),VLY(2000),
     2QK(10,10),Q(10),B(3,10),C(3,3),GXY(120),BT(3,6),XQ(5),YQ(5)
     3,TEMP(2000),ALPHA(2,120)
      COMMON/INTEG/NNP,NEL,NMAT,NOPT,MTYP,IE(2000,5),
     1KODE(2000),ISC(1),JSC(1),NIN,NOUT,MA,NEQ,NPROB,NTAPE,NSTORE
      ISTOP=0
      READ(NIN,1)NNP,NEL,NMAT,NOPT
      WRITE(NOUT,100)NNP,NEL,NMAT,NOPT
C
C     CHECKS TO BE SURE INPUT DATA DOES NOT EXCEED STORAGE CAPACITY
C
      IF(NNP.LE.MAXNP) GO TO 201
      ISTOP=ISTOP+1
      WRITE(NOUT,251)MAXNP
  201 IF(NEL.LE.MAXEL) GO TO 202
      ISTOP=ISTOP+1
      WRITE(NOUT,252)MAXEL
  202 IF(NMAT.LE.MAXMAT) GO TO 204
      ISTOP=ISTOP+1
      WRITE(NOUT,253)MAXMAT
  204 IF(ISTOP.EQ.0) GO TO 205
      WRITE(NOUT,255)ISTOP
      STOP
205   WRITE(NOUT,101)
      DO 301 I=1,NMAT
      READ(NIN,2)EX(I),EY(I),PRXY(I),GXY(I),TH(I),ALPHA(1,I),ALPHA(2,I)
      IF(EY(I).EQ.0.)EY(I)=EX(I)
      IF(GXY(I).EQ.0.)GXY(I)=EX(I)/(1.+PRXY(I))/2.
      IF(PRXY(I).EQ.0.)PRXY(I)=EX(I)/2./GXY(I)-1.
      PRYX(I)=PRXY(I)*EY(I)/EX(I)
301   WRITE(NOUT,51)I,EX(I),EY(I),PRXY(I),GXY(I),TH(I),ALPHA(1,I),
     1ALPHA(2,I)
C
C     READ AND PRINT NODAL DATA
C
      WRITE(NOUT,103)
      N=1
    5 READ(NIN,3)M,KODE(M),X(M),Y(M),ULX(M),VLY(M)
      IF(M-N)4,6,7
    4 WRITE(NOUT,105)M
      WRITE(NOUT,52)M,KODE(M),X(M),Y(M),ULX(M),VLY(M)
      ISTOP=ISTOP+1
      GO TO 5
    7 DF=M+1-N
```

```
      RX=(X(M)-X(N-1))/DF
      RY=(Y(M)-Y(N-1))/DF
    8 KODE(N)=0
      X(N)=X(N-1)+RX
      Y(N)=Y(N-1)+RY
      ULX(N)=0.0
      VLY(N)=0.0
    6 WRITE(NOUT,52)N,KODE(N),X(N),Y(N),ULX(N),VLY(N)
      N=N+1
      IF(M-N)9,6,8
    9 IF(N.LE.NNP) GO TO 5
C
C     READ AND PRINT ELEMENT PROPERTIES, REF. 33, TABLE 6-4
C
      WRITE(NOUT,106)
   13 L=0
   14 READ(NIN,15)M,(IE(M,I),I=1,5),TEMP(M)
   16 L=L+1
      IF(M.NE.1.AND.TEMP(M).EQ.0.0)TEMP(M)=TEMP(M-1)
      IF(M-L)117,17,18
  117 WRITE(NOUT,118)M
      WRITE(NOUT,53)M,(IE(M,I),I=1,5)
      ISTOP =ISTOP+1
      GO TO 14
   18 IE(L,1)=IE(L-1,1)+1
      IE(L,2)=IE(L-1,2)+1
      IE(L,3)=IE(L-1,3)+1
      IE(L,4)=IE(L-1,4)+1
      IE(L,5)=IE(L-1,5)
17    IF(IE(L,5).EQ.0)IE(L,5)=IE(L-1,5)
      WRITE(NOUT,53)L,(IE(L,I),I=1,5),TEMP(M)
      IF(M-L)20,20,16
   20 IF(NEL-L)21,21,14
   21 CONTINUE
   31 IF(ISTOP.EQ.0) GO TO 999
      WRITE(NOUT,900) ISTOP
    1 FORMAT(6I5)
  100 FORMAT(35H0INPUT TABLE 1.. BASIC PARAMETERS    //
     15X, 40H NUMBER OF NODAL POINTS. . . . . . . . . .,I5/
     25X, 40H NUMBER OF ELEMENTS. . . . . . . . . . . .,I5 /
     35X, 40H NUMBER OF DIFFERENT MATERIALS . . . . . .,I5 /
     45X, 40H 1 = PLANE STRAIN, 2 = PLANE STRESS. . .,I5)
  251 FORMAT(////33H TOO MANY NODAL POINTS, MAXIMUM =,I5)
  252 FORMAT(////30H TOO MANY ELEMENTS, MAXIMUM = ,I5)
  253 FORMAT(//// 30H TOO MANY MATERIALS, MAXIMUM =,I5)
  255 FORMAT(////28H EXECUTION HALTED BECUASE OF,I5,13H FATAL ERRORS/)
    2 FORMAT(3E10.3)
```

```
101 FORMAT(36H0INPUT TABLE 2.. MATERIAL PROPERTIES //                  B1 01890
   110H  MATERIAL,5X,10H     EX     ,6X,9H     EY    ,7X,               B1 01900
   28H  PRXY  ,7X,  8H   GXY   ,7X,9HTHICKNESS,6X,                       B1 01910
   39H ALPHA X ,6X,9H ALPHA Y /)                                         B1 01920
 51 FORMAT(I10,7E15.4)                                                   B1 01930
103 FORMAT(34H1INPUT TABLE 3.. NODAL POINT DATA //                      B1 01940
   15X,5HNODAL,48X,7HX-DISP.,7HY-DISP./                                  B1 01950
   25X,5HPOINT,6X,4HTYPE,14X,1HX,14X,1HY,8X,7HOR LOAD,8X,7HOR LOAD)     B1 01960
  3 FORMAT(2I5,4E10.3)                                                   B1 01970
105 FORMAT(5X,17HERROR IN CARD NO.,I5,/)                                 B1 01980
 52 FORMAT(2I10,4E15.4)                                                  B1 01990
106 FORMAT(34H1INPUT TABLE 4.. ELEMENT DATA        //                    B1 02000
   111X,31HGLOBAL INDICES OF ELEMENT NODES/3X,7HELEMENT,                 B1 02010
   27X,1H1,7X,1H2,7X,1H3,7X,1H4,2X,8HMATERIAL,2X,11HTEMPERATURE)        B1 02020
113 FORMAT(5X, 25HERROR IN ELEMENT CARD NO.,I5/)                         B1 02030
 15 FORMAT(6I5,E10.3)                                                    B1 02040
 53 FORMAT(I10,4I8,I10,F10.2)                                            B1 02050
900 FORMAT(///45H ASSEMBLY AND SOLUTION WILL NOT BE PERFORMED., I5,     B1 02060
   122H  FATAL CARD ERRORS          )                                    B1 02070
999 RETURN                                                               B1 02080
    END                                                                  B1 02090
    SUBROUTINE ASEMBL(ISTOP,AK)                                          B1 02100
    IMPLICIT REAL*8 (A-H,O-Z)                                            B1 02110
    COMMON/PARTIT/NBMAX,NEQB,NBLOCK,ILOW,IHIGH,NSTIF,NL,NR,IBAND         B1 02120
    COMMON/REAL/EX(120),EY(120),PRXY(120),PRYX(120),TH(120),             B1 02130
   1X(2000),Y(2000),ULX(2000),VLY(2000),                                 B1 02140
   2QK(10,10),Q(10),B(3,10),C(3,3),GXY(120),BT(3,6),XQ(5),YQ(5)          B1 02150
   3,TEMP(2000),ALPHA(2,120)                                             B1 02160
    COMMON/INTEG/NNP,NEL,NMAT,NOPT,MTYP,IE(2000,5),                      B1 02170
   1KODE(2000),ISC(1),JSC(1),NIN,NOUT,MA,NEQ,NPROB,NTAPE,NSTORE          B1 02180
    DIMENSION AK(NEQB,IBAND),LP(8)                                       B1 02190
    REWIND 1                                                             B1 02200
    REWIND 4                                                             B1 02210
    REWIND 13                                                            B1 02220
    REWIND NR                                                            B1 02230
    REWIND NSTIF                                                         B1 02240
    ISTOP=0                                                              B1 02250
C   INITIALIZE PARTS OF MATRICES C AND BT (MATRIX C IN PROGRAM IS MATRB1 02260
C   A IN TEXT)                                                           B1 02270
    BT(1,4)=0.0                                                          B1 02280
    BT(1,5)=0.0                                                          B1 02290
    BT(1,6)=0.0                                                          B1 02300
    BT(2,1)=0.0                                                          B1 02310
    BT(2,2)=0.0                                                          B1 02320
    BT(2,3)=0.0                                                          B1 02330
    C(1,3)=0.0                                                           B1 02340
    C(2,3)=0.0                                                           B1 02350
```

```
      C(3,1)=0.0
      C(3,2)=0.0
      NBLOCK=0
      DO 63 I=1,NBMAX
      N=NEQ-NEQB*(I-1)
      IF(N.LE.0)GO TO 61
63    NBLOCK=NBLOCK+1
61    CONTINUE
      IF(NBLOCK.EQ.0)NBLOCK=1
C
C     COMPUTE ELEMENT STIFFNESSES AND LOADS ONE BY ONE
C
      DO 26 M=1,NEL
      IF(IE(M,5).GT.0) GO TO 11
      ISTOP=ISTOP+1
      GO TO 26
   11 CALL QUAD(M,AREA)
      IF(AREA.GT.0.0) GO TO 16
      ISTOP=ISTOP+1
      WRITE(NOUT,20)M
C
C     CONDENSE ELEMENT STIFFNESSES FROM 10X10 TO 8X8, REF 33, EQ. (5-64)
C     AND ELEMENT LOADS FROM 10X1 TO 8X1, REF. 33, EQ. (5-64D)
C
   16 IF(IE(M,3).EQ.IE(M,4)) GO TO 64
      DO 31 J=1,2
      IJ=10-J
      IK=IJ+1
      PIVOT=QK(IK,IK)
      DO 32 K=1,IJ
      F=QK(IK,K)/PIVOT
      QK(IK,K)=F
      DO 33 I=K,IJ
      QK(I,K)=QK(I,K)-F*QK(I,IK)
   33 QK(K,I)=QK(I,K)
   32 Q(K)=Q(K)-QK(IK,K)*Q(IK)
   31 Q(IK)=Q(IK)/PIVOT
C
C     STORE MULTIPLIERS, PIVOTS, CONDENSED LOADS, STRAIN-DISPLACEMENT, A
C     STRESS-STRAIN MATRICES ON SCRATCH TAPE, TO BE USED LATER FOR STRES
C     RECOVERY
C
64    WRITE(13)((QK(I,J),J=1,10),I=1,10),(Q(IZ),IZ=1,10)
      WRITE(1)((QK(I,J),J=1,10),I=9,10),Q(9),Q(10),
     1((B(I,J),J=1,10),I=1,3),((C(I,J),J=1,3),I=1,3),XQ(5),YQ(5)
26    CONTINUE
C
```

```
C        ASSEMBLE STIFFNESSES AND LOADS BY THE DIRECT STIFFNESS METHOD, REFB1 02830
C        SEC. 6-5                                                          B1 02840
C                                                                          B1 02850
         DO 10 KK=1,NBLOCK                                                 B1 02860
         REWIND 13                                                         B1 02870
         REWIND 4                                                          B1 02880
         ILOW=(KK-1)*NEQB+1                                                B1 02890
         IHIGH=KK*NEQB                                                     B1 02900
C        INITIALIZE OVERALL STIFFNESS MATRIX AK AND OVERALL LOAD VECTOR R  B1 02910
         DO 2 I=1,NEQB                                                     B1 02920
         DO 2 J=1,IBAND                                                    B1 02930
       2 AK(I,J)=0.0                                                       B1 02940
         DO 50 M=1,NEL                                                     B1 02950
         READ(13)((QK(I,J),J=1,10),I=1,10),(Q(IZ),IZ=1,10)                 B1 02960
         LIM=8                                                             B1 02970
         IF(IE(M,3).EQ.IE(M,4))LIM=6                                       B1 02980
         DO 40 I=2,LIM,2                                                   B1 02990
         IJ=I/2                                                            B1 03000
         LP(I-1)=2*IE(M,IJ)-1                                              B1 03010
      40 LP(I)=2*IE(M,IJ)                                                  B1 03020
         DO 50 LL=1,LIM                                                    B1 03030
         I=LP(LL)                                                          B1 03040
         IF(I.LT.ILOW.OR.I.GT.IHIGH)GO TO 62                               B1 03050
         II=I-ILOW+1                                                       B1 03060
         AK(II,IBAND)=AK(II,IBAND)+Q(LL)                                   B1 03070
62       DO 50 MM=1,LIM                                                    B1 03080
         J=LP(MM)-I+1                                                      B1 03090
         IF(I.LT.ILOW.OR.I.GT.IHIGH.OR.J.LE.0)GO TO 50                     B1 03100
         II=I-ILOW+1                                                       B1 03110
         AK(II,J)=AK(II,J)+QK(LL,MM)                                       B1 03120
      50 CONTINUE                                                          B1 03130
C                                                                          B1 03140
C        ADD EXTERNALLY APPLIED NODAL LOADS TO AK                          B1 03150
C                                                                          B1 03160
         DO 55 N=1,NNP                                                     B1 03170
         K=2*N                                                             B1 03180
         IK=K-1                                                            B1 03190
         IF(KODE(N).EQ.3) GO TO 55                                         B1 03200
         IF(IK.LT.ILOW.OR.K.GT.IHIGH)GO TO 55                              B1 03210
         IF(KODE(N).EQ.1) GO TO 57                                         B1 03220
         II=IK-ILOW+1                                                      B1 03230
         AK(II,IBAND)=AK(II,IBAND)+ULX(N)                                  B1 03240
         IF(KODE(N).NE.0) GO TO 55                                         B1 03250
57       JJ=K-ILOW+1                                                       B1 03260
         AK(JJ,IBAND)=AK(JJ,IBAND)+VLY(N)                                  B1 03270
C                                                                          B1 03280
C        TO TEMPORARILY STORE AK TERMS NEEDED IN SR GEOMBC                 B1 03290
```

```
C
   55 CONTINUE
      WRITE(NR)((AK(I,J),I=1,NEQB),J=1,IBAND)
10    CONTINUE
C
C     INTRODUCE KINEMATIC CONSTRAINTS (GEOMETRIC BOUNDARY CONDITIONS)
C     EQ. (6-18)
C
      REWIND NR
      DO 76 KK=1,NBLOCK
      READ(NR)((AK(I,J),I=1,NEQB),J=1,IBAND)
      ILOW=(KK-1)*NEQB+1
      IHIGH=KK*NEQB
      IT=IHIGH-1
      IF(IHIGH.GT.NNP*2)IT=NNP*2-1
   60 DO 70 IJ=ILOW,IT,2
      M=(IJ-1)/2+1
      IF(KODE(M).GE.0.AND.KODE(M).LE.3) GO TO 72
      ISTOP=ISTOP+1
      GO TO 70
   72 IF(KODE(M).EQ.0) GO TO 70
      IF(KODE(M).EQ.2) GO TO 71
      CALL GEOMBC(ULX(M),IJ,AK(1,1),MA)
      IF(KODE(M).EQ.1) GO TO 70
   71 CALL GEOMBC(VLY(M),IJ+1,AK(1,1),MA)
   70 CONTINUE
20    FORMAT(/5X,17H AREA OF ELEMENT  ,I5,14H  IS NEGATIVE  /)
C
C     FOR PUTTING BLOCKS ON TAPE
C
76    WRITE(NSTIF)AK
   31 RETURN
      END
      SUBROUTINE QUAD(M,TOTALA)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/REAL/EX(120),EY(120),PRXY(120),PRYX(120),TH(120),
     1X(2000),Y(2000),ULX(2000),VLY(2000),
     2QK(10,10),Q(10),B(3,10),C(3,3),GXY(120),BT(3,6),XQ(5),YQ(5)
     3,TEMP(2000),ALPHA(2,120)
      COMMON/INTEG/NNP,NEL,NMAT,NUPT,MTYP,IE(2000,5),
     1KODE(2000),ISC(1),JSC(1),NIN,NOUT,MA,NEQ,NPROB,NTAPE,NSTORE
      IC=0
17    I=IE(M,1)
      J=IE(M,2)
      K=IE(M,3)
      L=IE(M,4)
      MTYP=IE(M,5)
```

```
      TOTALA=0.0                                                        B1 03770
C                                                                       B1 03780
C     CONSTRUCT STRESS STRAIN MATRIX C, REF. 33, EQ. (3-16C)|  FOR PLANEB1 03790
C     STRAIN NOPT=1, AND FOR PLANE STRESS NOPT=2|  PRESENT CODE IS FOR  B1 03800
C     ISOTROPIC PLANE STRAIN ELEMENTS AND ORTHOTROPIC PLANE STRESS ELEMEB1 03810
C                                                                       B1 03820
      IF(NMAT.EQ.1.AND.M.GT.1) GO TO 5                                  B1 03830
      IF(NOPT.EQ.2) GO TO 2                                             B1 03840
      CF=EX(MTYP)/((1.0+PRXY(MTYP))*(1.0-2.0*PRXY(MTYP)))               B1 03850
      C(1,1)=CF*(1.0-PRXY(MTYP))                                        B1 03860
      C(1,2)=CF*PRXY(MTYP)                                              B1 03870
      C(2,1)=C(1,2)                                                     B1 03880
      C(2,2)=C(1,1)                                                     B1 03890
      C(3,3)=CF*(1.0-2.0*PRXY(MTYP))/2.0                                B1 03900
      GO TO 5                                                           B1 03910
    2 DEN=(1.-PRXY(MTYP)*PRYX(MTYP))                                    B1 03920
      C(1,1)=EX(MTYP)/DEN                                               B1 03930
      C(1,2)=PRXY(MTYP)*EY(MTYP)/DEN                                    B1 03940
      C(2,1)=C(1,2)                                                     B1 03950
      C(2,2)=EY(MTYP)/DEN                                               B1 03960
      C(3,3)=GXY(MTYP)                                                  B1 03970
    5 LIM=4                                                             B1 03980
      IF(K.EQ.L)LIM=3                                                   B1 03990
      XQ(5)=0.0                                                         B1 04000
      YQ(5)=0.0                                                         B1 04010
      DO 10 N=1,LIM                                                     B1 04020
      NN=IE(M,N)                                                        B1 04030
      XQ(N)=X(NN)                                                       B1 04040
      YQ(N)=Y(NN)                                                       B1 04050
      ALIM=LIM                                                          B1 04060
      XQ(5)=XQ(5)+X(NN)/ALIM                                            B1 04070
   10 YQ(5)=YQ(5)+Y(NN)/ALIM                                            B1 04080
C                                                                       B1 04090
C     INITIALIZE QUADRILATERAL STIFFNESS, LOAD, AND STRAIN-DISPLACEMENT B1 04100
C                                                                       B1 04110
      DO 13 II=1,10                                                     B1 04120
      Q(II)=0.0                                                         B1 04130
      DO 12 JJ=1,10                                                     B1 04140
   12 QK(II,JJ)=0.0                                                     B1 04150
      DO 13 JJ=1,3                                                      B1 04160
      B(JJ,II)=0.0                                                      B1 04170
   13 CONTINUE                                                          B1 04180
      IF(K.NE.L) GO TO 15                                               B1 04190
      CALL CST(1,2,3,TOTALA,M)                                          B1 04200
      GO TO 999                                                         B1 04210
   15 CALL CST(1,2,5,AREA,M)                                            B1 04220
      TOTALA=TOTALA+AREA                                                B1 04230
```

```
      CALL CST(2,3,5,AREA,M)
      TOTALA=TOTALA+AREA
      CALL CST(3,4,5,AREA,M)
      TOTALA=TOTALA+AREA
      CALL CST(4,1,5,AREA,M)
      TOTALA=TOTALA+AREA
C     FOR CHECKING AREAS
999   IF(TOTALA.GE.0..OR.IC.EQ.4)RETURN
      WRITE(NOUT,16)M,(IE(M,I),I=1,5)
16    FORMAT(' INPUT DATA AT ELEMENT NUMBER',I10,'  IS NOT NUMBERED CORR
     1ECTLY, FIXUP TAKEN'/' ELEMENTS MATRIX WAS',5I10)
      ITEMP=IE(M,2)
      IE(M,2)=IE(M,1)
      IE(M,1)=ITEMP
      IC=4
      GO TO 17
      END
      SUBROUTINE CST(I,J,K,AREA,M)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/REAL/EX(120),EY(120),PRXY(120),PRYX(120),TH(120),
     1X(2000),Y(2000),ULX(2000),VLY(2000),
     2QK(10,10),Q(10),B(3,10),C(3,3),GXY(120),BT(3,6),XQ(5),YQ(5)
     3,TEMP(2000),ALPHA(2,120)
      COMMON/INTEG/NNP,NEL,NMAT,NOPT,MTYP,IE(2000,5),
     1KODE(2000),ISC(1),JSC(1),NIN,NOUT,MA,NEQ,NPROB,NTAPE,NSTORE
      DIMENSION CB(3,6),LC(6),LT(3),TK(6,6)
      LT(1)=I
      LT(2)=J
      LT(3)=K
C
C     COMPUTE STRAIN-DISPLACEMENT MATRIX B FOR TRIANGLE, REF. 33, EQ. (5
C
      BT(1,1)=YQ(J)-YQ(K)
      BT(1,2)=YQ(K)-YQ(I)
      BT(1,3)=YQ(I)-YQ(J)
      BT(2,4)=XQ(K)-XQ(J)
      BT(2,5)=XQ(I)-XQ(K)
      BT(2,6)=XQ(J)-XQ(I)
      BT(3,1)=BT(2,4)
      BT(3,2)=BT(2,5)
      BT(3,3)=BT(2,6)
      BT(3,4)=BT(1,1)
      BT(3,5)=BT(1,2)
      BT(3,6)=BT(1,3)
      AREA=(BT(2,4)*BT(1,3)-BT(2,6)*BT(1,1))/2.0
C     COMPUTE C*B
      DO 10 II=1,3
```

```
      DO 10 JJ=1,6                                                      B1 04710
      CB(II,JJ)=0.0                                                     B1 04720
      DO 10 KK=1,3                                                      B1 04730
   10 CB(II,JJ)=CB(II,JJ)+C(II,KK)*BT(KK,JJ)                            B1 04740
C     COMPUTE B**T*C*B, REF. 33, EQ. (5-56A)                            B1 04750
      DO 12 II=1,6                                                      B1 04760
      DO 12 JJ=1,6                                                      B1 04770
      TK(II,JJ)=0.0                                                     B1 04780
      DO 12 KK=1,3                                                      B1 04790
   12 TK(II,JJ)=TK(II,JJ)+BT(KK,II)*CB(KK,JJ)                           B1 04800
C                                                                       B1 04810
C     ADD TRIANGLE STIFFNESS TO QUADRILATERAL STIFFNESS, REF. 33, EQ. (6B1 04820
C     ADD TRIANGLE STRAIN-DISPLACEMENT MATRIX TO QUADRILATERAL STRAIN   B1 04830
C     DISPLACEMENT MATRIX                                               B1 04840
C                                                                       B1 04850
      DO 15 II=1,3                                                      B1 04860
      LC(II)=2*LT(II)-1                                                 B1 04870
   15 LC(II+3)=2*LT(II)                                                 B1 04880
      DO 30 II=1,6                                                      B1 04890
      LL=LC(II)                                                         B1 04900
      FK=1.0/(4.0*AREA)                                                 B1 04910
      FB=2.0*FK                                                         B1 04920
      DO 20 JJ=1,6                                                      B1 04930
      MM=LC(JJ)                                                         B1 04940
   20 QK(LL,MM)=QK(LL,MM)+TK(II,JJ)*TH(MTYP)*FK                         B1 04950
      DO 30 JJ=1,3                                                      B1 04960
   30 B(JJ,LL)=B(JJ,LL)+BT(JJ,II)*FB                                    B1 04970
16    CONTINUE                                                          B1 04980
C                                                                       B1 04990
C     TO DEVELOPE NODAL FORCES FOR THERMAL EFFECTS, REF. 33, EQ. (7-38) B1 05000
C     ALPHA(3,MTYP)==0.                                                 B1 05010
C                                                                       B1 05020
      CB(2,1)=TEMP(M)*ALPHA(1,MTYP)                                     B1 05030
      CB(2,2)=TEMP(M)*ALPHA(2,MTYP)                                     B1 05040
      DO 17 II=1,3                                                      B1 05050
      CB(1,II)=0.0                                                      B1 05060
      DO 18 JJ=1,2                                                      B1 05070
18    CB(1,II)=CB(1,II)+CB(2,JJ)*C(II,JJ)                               B1 05080
17    CONTINUE                                                          B1 05090
      DO 19 II=1,3                                                      B1 05100
      JJJ=LT(II)                                                        B1 05110
      JJJ=JJJ*2                                                         B1 05120
      III=JJJ-1                                                         B1 05130
      KK=II+3                                                           B1 05140
      DO 19 JJ=1,2                                                      B1 05150
      Q(III)=Q(III)+BT(JJ,II)*CB(1,JJ)*TH(MTYP)/2.                      B1 05160
      Q(JJJ)=Q(JJJ)+BT(JJ,KK)*CB(1,JJ)*TH(MTYP)/2.                      B1 05170
```

```
19    CONTINUE
      RETURN
      END
      SUBROUTINE STRESS(R)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/REAL/EX(120),EY(120),PRXY(120),PRYX(120),TH(120),
     1X(2000),Y(2000),ULX(2000),VLY(2000),
     2QK(10,10),Q(10),B(3,10),C(3,3),GXY(120),BT(3,6),XQ(5),YQ(5)
     3,TEMP(2000),ALPHA(2,120)
      COMMON/PARTIT/NBMAX,NEQB,NBLOCK,ILOW,IHIGH,NSTIF,NL,NR,IBAND
      COMMON/INTEG/NNP,NEL,NMAT,NOPT,MTYP,IE(2000,5),
     1KODE(2000),ISC(1),JSC(1),NIN,NOUT,MA,NEQ,NPROB,NTAPE,NSTORE
      DIMENSION SIG(9),R(NSTORE)
      REWIND 1
      REWIND NL
      DO 6 J=1,NBLOCK
      NN=NBLOCK-J
      DO 6 I=1,NEQB
      N=NN*NEQB+I
6     READ(NL)R(N)
C
C     TO PRINT DISPLACEMENTS
C
      DO 1 I=1,NNP
      WRITE(NOUT,2)I,R((I-1)*2+1),R(I*2)
2     FORMAT (5X,I12,2F20.8)
1     CONTINUE
      WRITE(NOUT,300)
      NCLINE=38
C
C     RETRIEVE MULTIPLIERS FOR STRESS RECOVERY
C
      DO 5 M=1,NEL
      READ(1)((QK(I,J),J=1,10),I=1,2),Q(9),Q(10),
     1((B(I,J),J=1,10),I=1,3),((C(I,J),J=1,3),I=1,3),XC,YC
C
C     SELECT NODAL DISPLACEMENTS FOR THE ELEMENT
C
      LIM=4
      IF(IE(M,3).EQ.IE(M,4)) LIM=3
      DO 10 I=1,LIM
      II=2*I
      JJ=2*IE(M,I)
      Q(II-1)=R(JJ-1)
   10 Q(II)=R(JJ)
C
C     RECOVER CONDENSED DISPLACEMENTS FOR THE QUADRILATERAL, REF. 33,
```

```
C        EQ. (5-64G)
C
         IF(LIM.EQ.3) GO TO 16
         DO 15 K=1,2
         JK=K+8
         IK=JK-1
         DO 15 L=1,IK
      15 Q(JK)=Q(JK)-QK(K,L)*Q(L)
C
C        COMPUTE ELEMENT STRAINS, REF. 33, EQ. (5-35A)
C
         LIM=10
         FAC=0.25
         GO TO 17
      16 LIM=6
         FAC=1.0
      17 DO 20 I=1,3
         EX(I)=0.0
         DO 20 J=1,LIM
      20 EX(I)=EX(I)+B(I,J)*Q(J)*FAC
         DO 19 I=1,2
19       EX(I)=EX(I)-ALPHA(I,IE(M,5))*TEMP(M)
C
C        COMPUTE ELEMENT STRESSES, REF. 33, EQ. (5-35B)
C
         DO 30 I=1,3
         SIG(I)=0.0
         DO 30 J=1,3
      30 SIG(I)=SIG(I)+C(I,J)*EX(J)
C        COMPUTE PRINCIPAL STRESSES AND THE ANGLE WITH THE POSITIVE X AXIS
         SP=(SIG(1)+SIG(2))/2.0
         SM=(SIG(1)-SIG(2))/2.0
         DS=DSQRT(SM*SM+SIG(3)*SIG(3))
         SIG(4)=SP+DS
         SIG(5)=SP-DS
         SIG(6)=0.0
         IF(SIG(3).NE.0.0.AND.SM.NE.0.0)SIG(6)=28.648*DATAN2(SIG(3),SM)
C        PRINT STRESSES, 50 LINES PER PAGE
         IF(NOLINE.GT.0) GO TO 54
         WRITE(NOUT,1000)
         NOLINE=38
      54 NOLINE=NOLINE-1
         IF(NPROB.GT.100)WRITE(NTAPE,1020)(SIG(I),I=1,3)
 1020    FORMAT(3E20.10,20X)
       5 WRITE(NOUT,1010)M,XC,YC,(SIG(I),I=1,6)
     300 FORMAT(48H1OUTPUT TABLE 2 .. STRESSES AT ELEMENT CENTROIDS  //
        11X,7HELEMENT,9X,1HX,9X,1HY,4X,8HSIGMA(X),4X,8HSIGMA(Y),4X,
```

```
     2SHTAU(X,Y),4X,8HSIGMA(1),4X,8HSIGMA(2),7X,5HANGLE   )
1000 FORMAT(1H1,7HELEMENT,9X,1HX,9X,1HY,4X,8HSIGMA(X),4X,8HSIGMA(Y),
    14X,8HTAU(X,Y),4X,8HSIGMA(1),4X,8HSIGMA(2),7X,5HANGLE   )
1010 FORMAT (I8,2F10.2,   6E12.4)
     RETURN
     END
     SUBROUTINE GEOMBC(U,N,AK)
     IMPLICIT REAL*8(A-H,O-Z)
     COMMON/PARTIT/N3MAX,NEQB,NBLOCK,ILOW,IHIGH,NSTIF,NL,NR,IBAND
     DIMENSION AK(NEQB,IBAND)
C
C    THIS SUBROUTINE MODIFIES THE ASSEMBLAGE STIFFNESS AND LOADS FOR TH
C    PRESCRIBED DISPLACEMENT U AT DEGREE OF FREEDOM N
C
     II=N+1-ILOW
     AK(II,1)=1.0E50
     AK(II,IBAND)=1.0E50*U
     RETURN
     END
      SUBROUTINE SESOL (NEQ,MA,NV,NBLOCK,NEQB,NAV,MI,NSTIF,
    1                  NRED,NL,NR,A,B,MAXA)
     IMPLICIT REAL*8(A-H,O-Z)
C
C    THIS SUBROUTINE CALCULATES THE SOLUTION OF THE SIMULTANEOUS LINEAR
C    EQUATIONS
C
C                          K X = R
C
C    WHERE
C                 K  =  POSITIVE DEFINITE MATRIX
C                 R  =  SUPPLIED RIGHT HAND SIDE VECTORS
C                 X  =  CALCULATED SOLUTION VECTORS
C
C    THE MATRIX  K  AND THE RIGHT HAND SIDE VECTORS  R  ARE READ IN
C    BLOCKS FROM TAPE  NSTIF  .  THE SOLUTION VECTORS  X  ARE STORED
C    IN BLOCKS IN REVERSE ORDER ON TAPE  NL   .
C
C * * * * * * * * * * * * * *   I N P U T S   * * * * * * * * * * * * *
C
C                NEQ  =  NUMBER OF EQUATIONS
C
C                MA  =  MAXIMUM HALF BANDWIDTH OF K  (INCL. DIAGONAL)
C
C                NV  =  NUMBER OF LOAD VECTORS
C
C                NBLOCK  =  NUMBER OF BLOCKS
C
```

```
C                  NEQB  =  NUMBER OF EQUATIONS IN EACH BLOCK
C
C                  NAV  =  NEQB*(MA + NV)
C
C                  MI = MA + NEQB - 1
C
C                  NSTIF , NRED , NL , NR  =  TAPE NUMBERS
C
C                  A , B , MAXA  =  STORAGE ARRAYS
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     FOR MORE INFORMATION, SEE REF. 36
C
      DIMENSION A(NAV),B(NAV),MAXA(MI)
      MM=1
      MA2=MA - 2
      IF (MA2.EQ.0) MA2=1
      INC=NEQB - 1
      NWA=NEQB*MA
      NTB=(MA-2)/NEQB + 1
      NEB=NTB*NEQB
      NEBT=NEB + NEQB
      NWV=NEQB*NV
      NWVV=NEBT*NV
      N1=NL
      N2=NR
      REWIND NSTIF
      REWIND NRED
      REWIND N1
      REWIND N2
C
C     MAIN LOOP OVER ALL BLOCKS
C
      DO 600 NJ=1,NBLOCK
      IF (NJ.NE.1) GO TO 10
      READ (NSTIF) A
      IF (NEQ.GT.1) GO TO 100
      MAXA(1)=1
      WRITE (NRED) A,MAXA
      IF (A(1)) 1,174,3
    1 KK=1
      WRITE (6,1010) KK,A(1)
    3 DO 5 L=1,NV
    5 A(1+L)=A(1+L)/A(1)
      KR=1 + NV
      WRITE (NL) (A(KK),KK=2,KR)
      RETURN
   10 IF (NTB.EQ.1) GO TO 100
```

```
      REWIND N1
      REWIND N2
      READ(N1)A
C     FIND COLUMN HEIGHTS
 100  KU=1
      KM=MINO(MA,NEQB)
      MAXA(1)=1
      DO 110 N=2,MI
      IF (N-MA) 120,120,130
 120  KU=KU + NEQB
      KK=KU
      MM=MINO(N,KM)
      GO TO 140
 130  KU=KU + 1
      KK=KU
      IF (N-NEQB) 140,140,136
 136  MM=MM - 1
 140  DO 160 K=1,MM
      IF (A(KK)) 110,160,110
 160  KK=KK - INC
 110  MAXA(N)=KK
      IF (A(1)) 172,174,176
 174  KK=(NJ-1)*NEQB + 1
      IF (KK.GT.NEQ) GO TO 590
      WRITE (6,1000) KK
      STOP
 172  KK=(NJ-1)*NEQB + 1
      WRITE (6,1010) KK,A(1)
C
C     FACTORIZE LEADING BLOCK
C
 176  DO 200 N=2,NEQB
      NH=MAXA(N)
      IF (NH-N) 200,200,210
 210  KL=N + INC
      K=N
      D=0.
      DO 220 KK=KL,NH,INC
      K=K - 1
      C=A(KK)/A(K)
      D=D + C*A(KK)
 220  A(KK)=C
      A(N)=A(N) - D
      IF (A(N)) 222,224,230
 224  KK=(NJ-1)*NEQB + N
      IF (KK.GT.NEQ) GO TO 590
      WRITE (6,1000) KK
```

```
      STOP                                                              B1 07530
  222 KK=(NJ-1)*NEQB + N                                                B1 07540
      WRITE (6,1010) KK,A(N)                                            B1 07550
  230 IC=NEQB                                                           B1 07560
      DO 240 J=1,MA2                                                    B1 07570
      MJ=MAXA(N+J) - IC                                                 B1 07580
      IF (MJ-N) 240,240,280                                             B1 07590
  280 KU=MIN0(MJ,NH)                                                    B1 07600
      KN=N + IC                                                         B1 07610
      C=0.                                                              B1 07620
      DO 300 KK=KL,KU,INC                                               B1 07630
  300 C=C + A(KK)*A(KK+IC)                                              B1 07640
      A(KN)=A(KN) - C                                                   B1 07650
  240 IC=IC + NEQB                                                      B1 07660
      K=N + NWA                                                         B1 07670
      DO 430 L=1,NV                                                     B1 07680
      KJ=K                                                              B1 07690
      C=0.                                                              B1 07700
      DO 440 KK=KL,NH,INC                                               B1 07710
      KJ=KJ - 1                                                         B1 07720
  440 C=C + A(KK)*A(KJ)                                                 B1 07730
      A(K)=A(K) - C                                                     B1 07740
  430 K=K + NEQB                                                        B1 07750
  200 CONTINUE                                                          B1 07760
C                                                                       B1 07770
C     CARRY OVER INTO TRAILING BLOCKS                                   B1 07780
C                                                                       B1 07790
      DO 400 NK=1,NTB                                                   B1 07800
      IF ((NK+NJ).GT.NBLOCK) GO TO 400                                  B1 07810
      NI=N1                                                             B1 07820
      IF ((NJ.EQ.1).OR.(NK.EQ.NTB)) NI=NSTIF                            B1 07830
      READ (NI) B                                                       B1 07840
      ML=NK*NEQB + 1                                                    B1 07850
      MR=MIN0((NK+1)*NEQB,MI)                                           B1 07860
      IF (MA.EQ.1) ML=MR                                                B1 07870
      MD=MI - ML                                                        B1 07880
      KL=NEQB + (NK-1)*NEQB*NEQB                                        B1 07890
      N=1                                                               B1 07900
      DO 500 M=ML,MR                                                    B1 07910
      NH=MAXA(M)                                                        B1 07920
      KL=KL + NEQB                                                      B1 07930
      IF (NH-KL) 505,510,510                                            B1 07940
  510 K=NEQB                                                            B1 07950
      D=0.                                                              B1 07960
      DO 520 KK=KL,NH,INC                                               B1 07970
      C=A(KK)/A(K)                                                      B1 07980
      D=D + C*A(KK)                                                     B1 07990
```

```
      A(KK)=C
  520 K=K - 1
      B(N)=B(N) - D
      IF (MD) 580,580,530
  530 IC=NEQB
      DO 540 J=1,MD
      MJ=MAXA(M+J) - IC
      IF (MJ-KL) 540,550,550
  550 KU=MIN0(MJ,NH)
      KN=N + IC
      C=0.
      DO 575 KK=KL,KU,INC
  575 C=C + A(KK)*A(KK+IC)
      B(KN)=B(KN) - C
  540 IC=IC + NEQB
  580 KN=N + NWA
      K=NEQB + NWA
      DO 610 L=1,NV
      KJ=K
      C=0.
      DO 620 KK=KL,NH,INC
      C=C + A(KK)*A(KJ)
  620 KJ=KJ - 1
      B(KN)=B(KN) - C
      KN=KN + NEQB
  610 K=K + NEQB
  505 MD=MD - 1
  500 N=N + 1
      IF (NTB.NE.1) GO TO 560
      WRITE (NRED) A,MAXA
      DO 570 I=1,NAV
  570 A(I)=B(I)
      GO TO 600
  560 WRITE (N2) B
  400 CONTINUE
      M=N1
      N1=N2
      N2=M
  590 WRITE (NRED) A,MAXA
C
  600 CONTINUE
C
C     VECTOR BACKSUBSTITUTION
C
      DO 700 K=1,NWVV
  700 B(K)=0.
      REWIND NL
```

```
      DO 800 NJ=1,NBLOCK
      BACKSPACE NRED
      READ (NRED) A,MAXA
      BACKSPACE NRED
      K=NEBT
      DO 810 L=1,NV
      DO 820 I=1,NEB
      B(K)=B(K-NEQB)
820   K=K - 1
810   K=K + NEBT + NEB
      KN=0
      KK=NWA
      NDIF=NEQB
      IF (NJ.EQ.1) NDIF=NEQB - (NBLOCK*NEQB - NEQ)
      DO 855 L=1,NV
      DO 850 K=1,NDIF
850   B(KN+K)=A(KK+K)/A(K)
      KK=KK + NEQB
855   KN=KN + NEBT
      IF (MA.EQ.1) GO TO 915
      ML=NEQB + 1
      KL=NEQB
      DO 860 M=ML,MI
      KL=KL + NEQB
      KU=MAXA(M)
      IF (KU-KL) 860,870,870
870   K=NEQB
      KM=M
      DO 830 L=1,NV
      KJ=K
      DO 890 KK=KL,KU,INC
      B(KJ)=B(KJ) - A(KK)*B(KM)
890   KJ=KJ - 1
      KM=KM + NEBT
830   K=K + NEBT
860   CONTINUE
      N=NEQB
      DO 910 I=2,NEQB
      KL=N + INC
      KU=MAXA(N)
      IF (KU-KL) 910,920,920
920   K=N
      DO 930 L=1,NV
      KJ=K
      DO 940 KK=KL,KU,INC
      KJ=KJ - 1
940   B(KJ)=B(KJ) - A(KK)*B(K)
```

```
930     K=K + NEBT
910     N=N - 1
915     KK=0
        KN=0
        DO 950 L=1,NV
        DO 960 K=1,NEQB
        KK=KK + 1
960     A(KK)=B(KN+K)
950     KN=KN + NEBT
       DO 9991 K=1,NWV
9991   WRITE(NL)A(K)
800     CONTINUE
1000 FORMAT (// 46H   STOP  ***  ZERO DIAGONAL ENCOUNTERED DURING,
     1              18H EQUATION SOLUTION, /
     2          13X,18H EQUATION NUMBER =, I6 )
1010 FORMAT (/  50H WARNING ***  NEGATIVE DIAGONAL ENCOUNTERED DURING,
     1              18H EQUATION SOLUTION, /
     2          13X,18H EQUATION NUMBER =, I6, 5X, 7HVALUE =, E20.8 )
        RETURN
        END
```

B.2 THE MESH GENERATION PROGRAM

```
C
C      >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>
C                PROGRAM FOR GENERATING GENERAL TWO DIMENSIONAL GRIDS
C      >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>
C
       IMPLICIT REAL*8 (A-H,O-Z)
       COMMON/ITNODE/Z,XYCORD(1500,2),INODE(1500),NNP,ITRINO(1500,4),MB
       COMMON/RECTGR/XY(3),NIN,NOUT,SCALE(2),NRECT,NEL,MSCAN,NODE(4)
      1,NSV
       DIMENSION TEMP(2),A(2),XYT(6),ORIGIN(2),FACT(2),CARD(10)
C
C      UNIT NEFILE IS OUTPUT FOR NODE AND ELEMENT RECORDS
C      UNIT NIN IS FOR INPUT RECORDS
C      UNIT NOUT IS FOR PRINT RECORDS
C
       NEFILE=1
       NIN=9
       NOUT=6
       Z=0.
       NGA=0
       NNP=0
       NEL=1
       INODE(1)=0
C      READ DATA
       READ(NIN,37)(CARD(IZ),IZ=1,10)
37     FORMAT(10A8)
       WRITE(NOUT,38)(CARD(IZ),IZ=1,10)
38     FORMAT(10X,10A8)
       READ(NIN,24)ORIGIN(1),ORIGIN(2),SCALE(1),SCALE(2),FACT(1),
      1FACT(2),MSCAN,NSV
24     FORMAT(6F10.0,2I5)
       XY(1)=ORIGIN(1)
       XY(2)=ORIGIN(2)
       WRITE(NOUT,1)ORIGIN(1),ORIGIN(2),SCALE(1),SCALE(2),FACT(1),FACT(2)
1      FORMAT(' THE XY STARTING ORIGIN COORDS ARE',2E15.4/' THE XY SCALE
      1FACTORS FOR NODE NUMBERING ARE',2E15.4/
      2' THE DIMENSIONAL SCALE FACTORS ARE',2E15.4)
12     NGA=NGA+1
       READ(NIN,50)KIND,(CARD(IZ),IZ=1,9)
50     FORMAT(I5,9A8)
25     FORMAT(2I5)
       K=IABS(KIND)
       IF(K.EQ.5)GO TO 16
C
C      TO RESET GLOBAL ORIGIN
C
       IF(KIND.NE.-1)GO TO 2
```

```
      READ(NIN,24)ORIGIN(1),ORIGIN(2)                                   B2 00480
      XY(1)=ORIGIN(1)                                                   B2 00490
      XY(2)=ORIGIN(2)                                                   B2 00500
      WRITE(NOUT,36)(CARD(IZ),IZ=1,9),ORIGIN(1),ORIGIN(2)               B2 00510
36    FORMAT(9H >>>>>>>>,' ORIGIN RESET INTITLED',5X,9A8/               B2 00520
     1' TO X-Y COORDINATES',10X,2E20.7)                                 B2 00530
      NGA=NGA-1                                                         B2 00540
      GO TO 12                                                          B2 00550
2     IF(KIND.EQ.7)CALL CIRSEG(CARD)                                    B2 00560
      IF(KIND.EQ.7)NGA=NGA-1                                            B2 00570
      IF(KIND.EQ.7)GO TO 12                                             B2 00580
      IF(KIND.NE.8)GO TO 53                                             B2 00590
C     BEGIN CODE TO ROTATE FOLLOWING NODES AND ELEMENTS                 B2 00600
      READ(NIN,54)XTH,YTH,ZTH,XPIVOT,YPIVOT                             B2 00610
54    FORMAT(8F10.0)                                                    B2 00620
      XPIVOT=XPIVOT*FACT(1)+ORIGIN(1)                                   B2 00630
      YPIVOT=YPIVOT*FACT(2)+ORIGIN(2)                                   B2 00640
      WRITE(NOUT,55)(CARD(IZ),IZ=1,9),TH,YTH,ZTH,XPIVOT,YPIVOT          B2 00650
55    FORMAT(9H >>>>>>>>,' THE FOLLOWING ELEMENT GROUPS INTITLED',5X,9A8B2 00660
     1/' WILL BE ROTATED'/3E15.4/                                       B2 00670
     1' (THETA X,Y,Z)'/' WITH ABSOLUTE PIVOT COORDINATES'/2E15.4)       B2 00680
      KSTART=NNP+1                                                      B2 00690
      NGA=NGA-1                                                         B2 00700
      NSTART=NEL                                                        B2 00710
      MSCANT=MSCAN                                                      B2 00720
      MSCAN=0                                                           B2 00730
      GO TO 12                                                          B2 00740
53    IF(KIND.NE.-8)GO TO 52                                            B2 00750
C     END NODE ELEMENT GROUPS TO BE ROTATED                             B2 00760
      NGA=NGA-1                                                         B2 00770
      WRITE(NOUT,56)                                                    B2 00780
56    FORMAT(9H >>>>>>>>,' END NODE ELEMENT GROUPS TO BE ROTATED')      B2 00790
      NEND=NEL-1                                                        B2 00800
      KEND=NNP                                                          B2 00810
      MSCAN=MSCANT                                                      B2 00820
      IF(XTH.GE.90..AND.YTH.LE.-90..OR.XTH.LE.-90..AND.YTH.GE.90.)      B2 00830
     1GO TO 57                                                          B2 00840
      IF(XTH.GE.90..AND.YTH.GE.90..OR.XTH.LE.-90..AND.YTH.LE.-90.)      B2 00850
     1GO TO 57                                                          B2 00860
      IF(XTH.LE.90..AND.YTH.LE.90..AND.XTH.GE.-90..AND.YTH.GE.-90.)     B2 00870
     1GO TO 57                                                          B2 00880
C                                                                       B2 00890
C     TO FLIP ELEMENTS TO MAINTAIN PROPER CCW NUMBERING                 B2 00900
C                                                                       B2 00910
      DO 58 I=NSTART,NEND                                               B2 00920
      IT=ITRINO(I,3)                                                    B2 00930
      ITRINO(I,3)=ITRINO(I,4)                                           B2 00940
```

```
      ITRINO(I,4)=IT
      IT=ITRINO(I,1)
      ITRINO(I,1)=ITRINO(I,2)
      ITRINO(I,2)=IT
 56   CONTINUE
 57   CONTINUE
      IT=100000
      CALL ROTP(XTH,YTH,ZTH,0.0,0.0,0.0,KSTART,KEND,NSTART,NEND,IT,
     1XPIVOT,YPIVOT,1)
      GO TO 12
 52   CONTINUE
      IF(KIND.NE.6)GO TO 4
      READ(NIN,24)FACT(1),FACT(2)
      WRITE(NOUT,40)(CARD(IZ),IZ=1,8),FACT(1),FACT(2)
 40   FORMAT(9H >>>>>>>>,' DIMENSIONAL SCALE RESET INTITLED',5X,9A8/
     1' TO X-Y COORDINATES'/2E20.7)
      NGA=NGA-1
      GO TO 12
C
C     FOR GENERATION OF RECTANGLES, TRIANGES, AND TRANSITION REGIONS
C
 4    IF(K.EQ.4)READ(NIN,25)NELODD,JRC
      IF(K.EQ.4)GO TO 23
      IF(K.NE.3)READ(NIN,26)N,M,IRC,THETA
 26   FORMAT(3I5,F10.0)
      IF(K.EQ.3)READ(NIN,27)N,IRC,THETA
 27   FORMAT(2I5,F10.0)
      IF(IRC.EQ.0)READ(NIN,27)JRC
      IF(IRC.EQ.1)READ(NIN,28)AX,JRC
 28   FORMAT(F10.0,I5)
      IF(IRC.EQ.2)READ(NIN,28)AY,JRC
      IF(IRC.EQ.3)READ(NIN,29)AX,AY,JRC
      IF(IRC.EQ.1.OR.IRC.EQ.3)A(1)=A(1)*AX
      IF(IRC.EQ.2.OR.IRC.EQ.3)A(2)=A(2)*AY
 29   FORMAT(2F10.0,I5)

      IF(IRC.EQ.-1)READ(NIN,28)A(1),JRC
      IF(IRC.EQ.-2)READ(NIN,28)A(2),JRC
      IF(IRC.EQ.-3)READ(NIN,29)A(1),A(2),JRC
      IF(IRC.EQ.-1.OR.IRC.EQ.-3)A(1)=A(1)*FACT(1)
      IF(IRC.EQ.-2.OR.IRC.EQ.-3)A(2)=A(2)*FACT(2)
 23   IF(JRC.EQ.0)GO TO 8
      IF(JRC.NE.1)GO TO 7
      READ(NIN,29)TEMP(1)
      XY(1)=XY(1)+TEMP(1)*FACT(1)
      GO TO 8
 7    IF(JRC.NE.2)GO TO 9
```

```
      READ(NIN,29)TEMP(2)                                               B2  01420
      XY(2)=XY(2)+TEMP(2)*FACT(2)                                       B2  01430
      GO TO 3                                                           B2  01440
9     IF(JRC.NE.3)GO TO 5                                               B2  01450
      READ(NIN,29)TEMP(1),TEMP(2)                                       B2  01460
      XY(1)=XY(1)+TEMP(1)*FACT(1)                                       B2  01470
      XY(2)=XY(2)+TEMP(2)*FACT(2)                                       B2  01480
      GO TO 8                                                           B2  01490
5     IF(JRC.EQ.-1)READ(NIN,29)XY(1)                                    B2  01500
      IF(JRC.EQ.-2)READ(NIN,29)XY(2)                                    B2  01510
      IF(JRC.EQ.-3)READ(NIN,29)XY(1),XY(2)                              B2  01520
      IF(JRC.EQ.-1.OR.JRC.EQ.-3)XY(1)=XY(1)*FACT(1)+ORIGIN(1)           B2  01530
      IF(JRC.EQ.-2.OR.JRC.EQ.-3)XY(2)=XY(2)*FACT(2)+ORIGIN(2)           B2  01540
8     CONTINUE                                                          B2  01550
      IF(KIND.NE.1)GO TO 6                                              B2  01560
      IT=NNP+1                                                          B2  01570
      WRITE(NOUT,11)NGA,(CARD(IZ),IZ=1,9),IT,NEL                        B2  01580
11    FORMAT(' >>>>>>>> GENERATION AREA NUMBER',I5,'  INTITLED',5X,9A8/ B2  01590
     1' THE FIRST PRELIMINARY NODE NUMBER IN THIS GROUP IS ',I5/        B2  01600
     2' THE FIRST ELEMENT NUMBER IN THIS GROUP IS ',I5)                 B2  01610
      WRITE(NOUT,10)N,M,IRC,A(1),A(2),THETA,JRC,XY(1),XY(2)             B2  01620
10    FORMAT(' RECTANGLES',I5,'  X  ',I2,'  ARE BEING GENERATED'/       B2  01630
     1' THE DIMENSIONAL RC IS',I5,'  WITH XY SIDE DIMENSIONS',2E15.4/   B2  01640
     2' THE ANGLE OF ROTATION IS',E15.4/                                B2  01650
     3' THE LOCATION RC IS',I5,'  WITH BEGINNING XY COORDS',2E15.4)     B2  01660
      CALL RECT(A,N,M,THETA)                                            B2  01670
      GO TO 12                                                          B2  01680
6     IF(K.NE.2)GO TO 13                                                B2  01690
      IT=NNP+1                                                          B2  01700
      WRITE(NOUT,11)NGA,(CARD(IZ),IZ=1,9),IT,NEL                        B2  01710
      WRITE(NOUT,14)N,M,IRC,A(1),A(2),THETA,JRC,XY(1),XY(2)             B2  01720
14    FORMAT(' TRIANGLES',I5,'  X  ',I2,'  ARE BEING GENERATED'/        B2  01730
     1' THE DIMENSIONAL RC IS',I5,'  WITH XY SIDE DIMENSIONS',2E15.4/   B2  01740
     2' THE ANGLE OF ROTATION IS',E15.4/                                B2  01750
     3' THE LOCATION RC IS',I5,'  WITH BEGINNING XY COORDS',2E15.4)     B2  01760
      CALL TRIANG(A,N,M,KIND,THETA)                                     B2  01770
      GO TO 12                                                          B2  01780
13    IF(K.NE.3)GO TO 3                                                 B2  01790
      IT=NNP+1                                                          B2  01800
      WRITE(NOUT,11)NGA,(CARD(IZ),IZ=1,9),IT,NEL                        B2  01810
      WRITE(NOUT,15)N,IRC,A(1),A(2),THETA,JRC,XY(1),XY(2)               B2  01820
15    FORMAT(' TRANSITION TRIANGLES,',I5,'  RECTS, ARE BEING GENERATED'/B2  01830
     1' THE DIMENSIONAL RC IS',I5,'  WITH XY SIDE DIMENSIONS',2E15.4/   B2  01840
     2' THE ANGLE OF ROTATION IS',E15.4/                                B2  01850
     3' THE LOCATION RC IS',I5,'  WITH BEGINNING XY COORDS',2E15.4)     B2  01860
      CALL TRASIT(A,N,KIND,THETA)                                       B2  01870
      GO TO 12                                                          B2  01880
```

```
3     CONTINUE                                                          B2 01890
C                                                                       B2 01900
C     ANYTHING THAT GETS HERE INDICATES THAT IRREGULAR ELEMENTS ARE TO BB2 01910
C     GENERATED                                                         B2 01920
C                                                                       B2 01930
      XYT(1)=XY(1)                                                      B2 01940
      XYT(2)=XY(2)                                                      B2 01950
      IF(KIND.GT.0)GO TO 19                                             B2 01960
      READ(NIN,34)(XYT(I),I=3,8)                                        B2 01970
34    FORMAT(8F10.0)                                                    B2 01980
      DO 20 I=3,7,2                                                     B2 01990
      XYT(I)=FACT(1)*XYT(I)+XYT(1)                                      B2 02000
20    XYT(I+1)=FACT(2)*XYT(I+1)+XYT(2)                                  B2 02010
      GO TO 21                                                          B2 02020
19    READ(NIN,34)(XYT(I),I=3,6)                                        B2 02030
      DO 22 I=3,5,2                                                     B2 02040
      XYT(I)=FACT(1)*XYT(I)+XYT(1)                                      B2 02050
22    XYT(I+1)=FACT(2)*XYT(I+1)+XYT(2)                                  B2 02060
21    CONTINUE                                                          B2 02070
      READ(NIN,29)TEMP(1),TEMP(2)                                       B2 02080
      TEMP(1)=TEMP(1)*FACT(1)                                           B2 02090
      TEMP(2)=TEMP(2)*FACT(2)                                           B2 02100
      IT=NNP+1                                                          B2 02110
      WRITE(NOUT,11)NGA,(CARD(IZ),IZ=1,9),IT,NEL                        B2 02120
      IF(KIND.EQ.-4)WRITE(NOUT,17)TEMP(1),TEMP(2),NELODD,KIND,          B2 02130
     1(XYT(I),I=1,8)                                                    B2 02140
      IF(KIND.EQ.4)WRITE(NOUT,17)TEMP(1),TEMP(2),NELODD,KIND,           B2 02150
     1(XYT(I),I=1,6)                                                    B2 02160
17    FORMAT(' IRREGULAR ELEMENTS ARE BEING GENERATED, WITH XY TRANS'   B2 02170
     1,'LATION INCREMENTS',2E15.4/                                      B2 02180
     2' WITH',I5,'  ELEMENTS GENERATED'/' THE GENERATION CODE IS',I5/   B2 02190
     3' THE NODE XY COORDS ARE'/8E15.4)                                 B2 02200
      CALL IRRNE(TEMP,XYT,NELODD,KIND)                                  B2 02210
      GO TO 12                                                          B2 02220
15    CONTINUE                                                          B2 02230
C                                                                       B2 02240
C     CODE FOR GRID FINISH                                              B2 02250
C                                                                       B2 02260
18    NEL=NEL-1                                                         B2 02270
      CALL TRIM(NEFILE)                                                 B2 02280
      WRITE(NOUT,51)NNP,NEL                                             B2 02290
51    FORMAT(' THE TOTAL NUMBER OF NODES ARE',I6/                       B2 02300
     1' THE TOTAL NUMBER OF ELEMENTS ARE',I6)                           B2 02310
      STOP                                                              B2 02320
      END                                                               B2 02330
      SUBROUTINE TRIANG(A,N,L,MODE,ANGLE)                               B2 02340
      IMPLICIT REAL*8 (A-H,O-Z)                                         B2 02350
```

```
      COMMON/ITNODE/Z,XYCORD(1500,2),INODE(1500),NNP,ITRINO(1500,4),MB    B2 02360
      COMMON/VECTOR/XY(3),NIN,NOUT,SCALE(2),NRECT,NEL,MSCAN,NODE(4)       B2 02370
     1,NSV                                                                B2 02380
      DIMENSION A(2)                                                      B2 02390
      XO=XY(1)                                                            B2 02400
      YO=XY(2)                                                            B2 02410
      IELEO=NEL                                                           B2 02420
      NNPO=NNP+1                                                          B2 02430
      IF(ANGLE.EQ.0.)NSCAN=MSCAN                                          B2 02440
      IF(ANGLE.NE.0.)NSCAN=0                                              B2 02450
      IF(MODE.GT.0)IEVORO=1                                               B2 02460
      IF(MODE.LT.0)IEVORO=0                                               B2 02470
      DO 3 J=1,L                                                          B2 02480
      IC=0                                                                B2 02490
      IF(IEVORO.EQ.1)IC=1                                                 B2 02500
      IF(IC.EQ.1)IEVORO=0                                                 B2 02510
      IF(IC.EQ.1)GO TO 5                                                  B2 02520
      IF(IEVORO.EQ.0)IEVORO=1                                             B2 02530
    5 IF(IEVORO.EQ.0)NN=N-1                                               B2 02540
      IF(IEVORO.EQ.1)NN=N                                                 B2 02550
      DO 2 I=1,NN                                                         B2 02560
      IF(IEVORO.EQ.1)CALL NODENO(XY(1),XY(2),NODE(1),NSCAN)               B2 02570
      IF(IEVORO.EQ.1)GO TO 4                                              B2 02580
      Y=XY(2)+A(2)                                                        B2 02590
      X=XY(1)+A(1)*1.5                                                    B2 02600
      CALL NODENO(X,Y,NODE(1),NSCAN)                                      B2 02610
    4 XY(1)=XY(1)+A(1)                                                    B2 02620
      CALL NODENO(XY(1),XY(2),NODE(2),NSCAN)                              B2 02630
      X=XY(1)-A(1)/2.                                                     B2 02640
      Y=XY(2)+A(2)                                                        B2 02650
      CALL NODENO(X,Y,NODE(3),NSCAN)                                      B2 02660
      IF(IEVORO.NE.0)GO TO 6                                              B2 02670
      ITRINO(NEL,1)=NODE(2)                                               B2 02680
      ITRINO(NEL,2)=NODE(1)                                               B2 02690
      ITRINO(NEL,3)=NODE(3)                                               B2 02700
      ITRINO(NEL,4)=ITRINO(NEL,3)                                         B2 02710
      GO TO 2                                                             B2 02720
    6 DO 7 K=1,3                                                          B2 02730
    7 ITRINO(NEL,K)=NODE(K)                                               B2 02740
      ITRINO(NEL,4)=ITRINO(NEL,3)                                         B2 02750
    2 NEL=NEL+1                                                           B2 02760
    3 XY(1)=XY(1)-A(1)*NN                                                 B2 02770
      KSMAT=NNP                                                           B2 02780
      IF(ANGLE.NE.0.)CALL ROTR(0.,0.,ANGLE,0.,0.,0.,NNPO,KSMAT,IELEO,N    B2 02790
     1ELE-1,MSCAN,XO,YO,1)                                                B2 02800
      RETURN                                                              B2 02810
      END                                                                 B2 02820
```

```
      SUBROUTINE TRASIT(A,N,MODE,ANGLE)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/ITNODE/Z,XYCORD(1500,2),INODE(1500),NNP,ITRINO(1500,4),MB
      COMMON/RECTGR/XY(3),NIN,NOUT,SCALE(2),NRECT,NEL,MSCAN,NODE(4)
     1,NSV
      DIMENSION A(2)
      XO=XY(1)
      YO=XY(2)
      IELEO=NEL
      NNPO=NNP+1
      IF(ANGLE.NE.O.)NSCAN=0
      IF(ANGLE.EQ.O.)NSCAN=MSCAN
C     FOR LARGER TRIANGLES FIRST
      IF(MODE.GT.0)XY(1)=XY(1)-A(1)/2.
      IF(MODE.GE.0)CALL TRIANG(A,N+1,1,MODE,0.)
      IF(MODE.LT.0)CALL TRIANG(A,N,1,MODE,0.)
      IF(MODE.GT.0)GO TO 4
      DO 1 I=1,N
      CALL NODENO(XY(1),XY(2),NODE(1),NSCAN)
      Y=XY(2)+A(2)
      CALL NODENO(XY(1),Y,NODE(3),NSCAN)
      X=XY(1)+A(1)/2.
      CALL NODENO(X,Y,NODE(2),NSCAN)
      DO 10 K=1,3
   10 ITRINO(NEL,K)=NODE(K)
      ITRINO(NEL,4)=ITRINO(NEL,3)
      NEL=NEL+1
    1 XY(1)=XY(1)+A(1)
      XY(1)=XY(1)-N*A(1)
      DO 2 I=1,N
      X=XY(1)+A(1)
      CALL NODENO(X,XY(2),NODE(1),NSCAN)
      Y=XY(2)+A(2)
      CALL NODENO(X,Y,NODE(2),NSCAN)
      X=X-A(1)/2.
      CALL NODENO(X,Y,NODE(3),NSCAN)
      DO 11 K=1,3
   11 ITRINO(NEL,K)=NODE(K)
      ITRINO(NEL,4)=ITRINO(NEL,3)
      NEL=NEL+1
    2 XY(1)=XY(1)+A(1)
      XY(1)=XY(1)-N*A(1)
      XY(2)=XY(2)+A(2)
      GO TO 14
    4 DO 6 I=1,N
      X=XY(1)+A(1)
      CALL NODENO(X,XY(2),NODE(1),NSCAN)
```

```
      Y=XY(2)+A(2)
      X=X-A(1)/2.
      CALL NODENO(X,Y,NODE(2),NSCAN)
      CALL NODENO(X,XY(2),NODE(3),NSCAN)
      DO 12 K=1,3
12    ITRINO(NEL,K)=NODE(K)
      ITRINO(NEL,4)=ITRINO(NEL,3)
      NEL=NEL+1
6     XY(1)=XY(1)+A(1)
      XY(1)=XY(1)-N*A(1)
      DO 5 I=1,N
      X=XY(1)+1.5*A(1)
      CALL NODENO(X,XY(2),NODE(1),NSCAN)
      Y=XY(2)+A(2)
      CALL NODENO(X,Y,NODE(2),NSCAN)
      X=X-A(1)/2.
      CALL NODENO(X,XY(2),NODE(3),NSCAN)
      DO 13 K=1,3
13    ITRINO(NEL,K)=NODE(K)
      ITRINO(NEL,4)=ITRINO(NEL,3)
      NEL=NEL+1
5     XY(1)=XY(1)+A(1)
      XY(1)=XY(1)+A(1)/2.-N*A(1)
      XY(2)=XY(2)+A(2)
14    KSMAT=NNP
      IF(ANGLE.NE.0.)CALL ROTR(0.,0.,ANGLE,0.,0.,0.,NNPO,K
     1SMAT,IELEO,NEL-1,MSCAN,XO,YO,1)
      RETURN
      END
      SUBROUTINE ROTR(THETXX,THETYY,THETZZ,XTRANS,YTRANS,
     1ZTRANS,NMIN,NMAX,IEMIN,IEMAX,NSCAN,PIVOTX,PIVOTY,MODE)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/ITNODE/Z,XYCORD(1500,2),INODE(1500),NNP,ITRINO(1500,4),MB
      COMMON/RECTGR/XY(3),NIN,NOUT,SCALE(2),NRECT,NEL,MSCAN,NNNN(4)
     1,NSV
      DIMENSION NCHECK(1500)
      THETA=0.0174532925
      THETAX=-THETXX*THETA
      THETAY=-THETYY*THETA
      THETAZ=-THETZZ*THETA
C
C     BEGIN CHECKING FOR NODES, ITERATE OVER ALL ELEMENTS TO CHECK FOR M
C     NODE NUMBERS, AND CONVERT TO INTEGER NOTATION
C
      IF(MODE.NE.1)GO TO 4
      NNP=NMIN-1
      IT=IEMAX-IEMIN+1
```

```
      DO 5 I=1,IT
      DO 5 J=1,4
      K=(I-1)*4+J
    5 NCHECK(K)=0
    4 DO 18 I=NMIN,NMAX
      XRUN=(XYCORD(I,1)-PIVOTX)*DCOS(THETAZ)+(XYCORD(I,2)-PIVOTY)*DSIN
     1(THETAZ)
      YRUN=(XYCORD(I,2)-PIVOTY)*DCOS(THETAZ)-(XYCORD(I,1)-PIVOTX)*DSIN(T
     1HETAZ)
      ZRUN=Z*DCOS(THETAY)+XRUN*DSIN(THETAY)
      XF=XRUN*DCOS(THETAY)-Z*DSIN(THETAY)+PIVOTX
      YF=YRUN*DCOS(THETAX)+ZRUN*DSIN(THETAX)+PIVOTY
      ZF=ZRUN*DCOS(THETAX)-YRUN*DSIN(THETAX)
      XF=XF+XTRANS
      YF=YF+YTRANS
      ZF=ZF+ZTRANS
      NTEMP=INODE(I)
      CALL NODENO(XF,YF,NODE,NSCAN)
C
C     MODE IS ZERO FOR NEW ROTRED NODES AS WELL AS FOR KEEPING OLD NODES
C     MODE IS ONE FOR OLD ROTRED NODES TO BE ELIMINATED
C
      DO 1 K=IEMIN,IEMAX
      IT=K-IEMIN+1
      DO 1 J=1,4
      IZ=(IT-1)*4+J
      IC=0
      IF(ITRINO(K,J).EQ.NTEMP.AND.NCHECK(IZ).EQ.0)IC=1
      IF(ITRINO(K,J).EQ.NTEMP)NCHECK(IZ)=1
    1 IF(IC.EQ.1)ITRINO(K,J)=NODE
   18 CONTINUE
      RETURN
      END
      SUBROUTINE TRIM(NEFILE)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/ITNODE/Z,XYCORD(1500,2),INODE(1500),NNP,ITRINO(1500,4),MB
      COMMON/RECTGR/XY(3),NIN,NOUT,SCALE(2),NRECT,NEL,MSCAN,NNNN(4)
     1,NSV
      DIMENSION NODE(3)
      MAXBW=0
C
C     TO SEQUENCE NODES
C
      NN=NNP-1
      DO 18 I=1,NN
      K=NNP-I
      DO 18 J=1,K
```

```
      IF(INODE(J)-INODE(J+1)) 18,18,17                                  B2 04240
17    NTEMP=INODE(J)                                                    B2 04250
      XT=XYCORD(J,1)                                                    B2 04260
      YT=XYCORD(J,2)                                                    B2 04270
      XYCORD(J,1)=XYCORD(J+1,1)                                         B2 04280
      XYCORD(J,2)=XYCORD(J+1,2)                                         B2 04290
      XYCORD(J+1,1)=XT                                                  B2 04300
      XYCORD(J+1,2)=YT                                                  B2 04310
      INODE(J)=INODE(J+1)                                               B2 04320
      INODE(J+1)=NTEMP                                                  B2 04330
18    CONTINUE                                                          B2 04340
C                                                                       B2 04350
C     TO CHECK FOR MESH INSTABILITIES IN ONE DIRECTION                  B2 04360
C                                                                       B2 04370
      I=0                                                               B2 04380
10    I=I+1                                                             B2 04390
      IF(I.GE.NNP)GO TO 25                                              B2 04400
      IF(INODE(I).NE.INODE(I+1)-1.AND.INODE(I).NE.INODE(I+1))GO TO 10   B2 04410
      DO 14 J=1,NEL                                                     B2 04420
      DO 14 K=1,4                                                       B2 04430
14    IF(ITRINO(J,K).EQ.INODE(I))ITRINO(J,K)=INODE(I+1)                 B2 04440
      NNP=NNP-1                                                         B2 04450
      DO 16 J=I,NNP                                                     B2 04460
      XYCORD(J,1)=XYCORD(J+1,1)                                         B2 04470
      XYCORD(J,2)=XYCORD(J+1,2)                                         B2 04480
16    INODE(J)=INODE(J+1)                                               B2 04490
      GO TO 10                                                          B2 04500
C                                                                       B2 04510
C     TO INVERT NODE INTEGERS                                           B2 04520
C                                                                       B2 04530
25    DO 19 I=1,NNP                                                     B2 04540
      IX=INODE(I)/10000                                                 B2 04550
      IY=INODE(I)-IX*10000                                              B2 04560
19    INODE(I)=IY*10000+IX                                              B2 04570
      DO 26 I=1,NEL                                                     B2 04580
      DO 26 J=1,4                                                       B2 04590
      IX=ITRINO(I,J)/10000                                              B2 04600
      IY=ITRINO(I,J)-IX*10000                                           B2 04610
26    ITRINO(I,J)=IY*10000+IX                                           B2 04620
C                                                                       B2 04630
C     TO RESEQUENCE NODES IN OPPOSITE SENSE                             B2 04640
C                                                                       B2 04650
      NN=NNP-1                                                          B2 04660
      DO 20 I=1,NN                                                      B2 04670
      K=NNP-I                                                           B2 04680
      DO 20 J=1,K                                                       B2 04690
      IF(INODE(J)-INODE(J+1)) 20,20,21                                  B2 04700
```

```
21    NTEMP=INODE(J)
      XT=XYCORD(J,1)
      YT=XYCORD(J,2)
      XYCORD(J,1)=XYCORD(J+1,1)
      XYCORD(J,2)=XYCORD(J+1,2)
      XYCORD(J+1,1)=XT
      XYCORD(J+1,2)=YT
      INODE(J)=INODE(J+1)
      INODE(J+1)=NTEMP
20    CONTINUE
C
C     TO CHECK FOR MESH INSTABILITIES IN OPPOSITE DIRECTION
C
      I=0
22    I=I+1
      IF(I.GE.NNP)GO TO 15
      IF(INODE(I).NE.INODE(I+1)-1.AND.INODE(I).NE.INODE(I+1))GO TO 22
      DO 23 J=1,NEL
      DO 23 K=1,4
23    IF(ITRINO(J,K).EQ.INODE(I))ITRINO(J,K)=INODE(I+1)
      NNP=NNP-1
      DO 24 J=I,NNP
      XYCORD(J,1)=XYCORD(J+1,1)
      XYCORD(J,2)=XYCORD(J+1,2)
24    INODE(J)=INODE(J+1)
      GO TO 22
15    CONTINUE
      REWIND NEFILE
      DO 1 I=1,NNP
1     WRITE(NEFILE,2)I,XYCORD(I,1),XYCORD(I,2)
2     FORMAT(I5,5X,2F10.4)
      DO 6 K=1,NEL
      DO 5 J=1,4
      DO 8 I=1,NNP
      IF(ITRINO(K,J).NE.INODE(I))GO TO 8
      NNNN(J)=I
      IF(J.EQ.4)GO TO 9
8     CONTINUE
5     CONTINUE
9     WRITE(NEFILE,7)K,(NNNN(IZ),IZ=1,4)
7     FORMAT(5I5)
C     FOR MAXIMUM BANDWIDTH CALCS
      DO 12 IJ=1,4
      DO 12 JJ=1,4
      MTRIAL=IABS(NNNN(IJ)-NNNN(JJ))
12    IF(MTRIAL.GT.MAXBW)MAXBW=MTRIAL
6     CONTINUE
```

```
4     CONTINUE
      MAXBW=2*(MAXBW+1)
      WRITE(NOUT,13)MAXBW
13    FORMAT(///' THE MAXIMUM BAND WIDTH IS',I10)
      RETURN
      END
      SUBROUTINE RECT(A,N,L,ANGLE)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/ITNODE/Z,XYCORD(1500,2),INODE(1500),NNP,ITRINO(1500,4),MB
      COMMON/RECTGR/XY(3),NIN,NOUT,SCALE(2),NRECT,NEL,MSCAN,NODE(4)
     1,NSV
      DIMENSION A(2)
      XO=XY(1)
      YO=XY(2)
      IELEO=NEL
      NNPO=NNP+1
      IF(ANGLE.NE.0.)NSCAN=0
      IF(ANGLE.EQ.0.)NSCAN=MSCAN
      DO 4 JJ=1,L
      DO 3 J=1,N
      CALL NODENO(XY(1),XY(2),NODE(1),NSCAN)
      X=XY(1)+A(1)
      Y=XY(2)+A(2)
      CALL NODENO(X,Y,NODE(3),NSCAN)
      CALL NODENO(XY(1),Y,NODE(4),NSCAN)
      CALL NODENO(X,XY(2),NODE(2),NSCAN)
      DO 5 I=1,4
5     ITRINO(NEL,I)=NODE(I)
      NEL=NEL+1
3     XY(1)=XY(1)+A(1)
      XY(2)=XY(2)+A(2)
4     XY(1)=XO
      KSMAT=NNP
      IF(ANGLE.NE.0.)CALL ROTR(0.,0.,ANGLE,0.,0.,0.,NNPO,KSM
     1AT,IELEO,NEL-1,MSCAN,XO,YO,1)
      RETURN
      END
      SUBROUTINE NODENO(X,Y,NODF,M)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/ITNODE/Z,XYCORD(1500,2),INODE(1500),NNP,ITRINO(1500,4),MB
      COMMON/RECTGR/XY(3),NIN,NOUT,SCALE(2),NRECT,NEL,MSCAN,NNNN(4)
     1,NSV
      IX=X/SCALE(1)*10000.-.5
      IY=Y/SCALE(2)*10000.-.5
      NODE=IY+10000*IX
      IF(IX.GT.9999.OR.IY.GT.9999.OR.IX.LT.0.OR.IY.LT.0)WRITE(NOUT,4)
     1NEL,NODE
```

```
 4    FORMAT(////' >>>> ERROR >>>>'////' TRY AGAIN, A NODE COORDINATE '
     1,'LARGER THAN EXPECTED OR NEGATIVE AT ELEMENT',I5,'  ENCOUNTERED',
     2/' WITH NODE =',I10)
      IF(IX.GT.9999.OR.IY.GT.9999.OR.IX.LT.0.OR.IY.LT.0)STOP
      IF(NSV.EQ.1)NODE=IY*10000+IX
 C    TO CHECK FOR DUPLICATE NODE NUMBERS
      IF(NNP.EQ.0)GO TO 3
      IF(M.EQ.0)GO TO 3
      IF(NNP.LT.M)N=NNP
      IF(NNP.GE.M)N=M
      DO 1 I=1,N
      NN=NNP+1-I
      IF(NODE.EQ.INODE(NN)) GOTO 2
 1    CONTINUE
 3    NNP=NNP+1
      XYCORD(NNP,1)=X
      XYCORD(NNP,2)=Y
      INODE(NNP)=NODE
 2    RETURN
      END
      SUBROUTINE IRRNE(TRANS,XYT,NELODD,MODE)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/ITNODE/Z,XYCORD(1500,2),INODE(1500),NNP,ITRINO(1500,4),MB
      COMMON/RECTGR/XY(3),NIN,NOUT,SCALE(2),NRECT,NEL,MSCAN,NODE(4)
     1,NSV
      DIMENSION TRANS(2),XYT(8)
      IF(MODE.LT.0)N=7
      IF(MODE.GT.0)N=5
      DO 1 I=1,NELODD
      XY(1)=XY(1)+TRANS(1)
      XY(2)=XY(2)+TRANS(2)
      DO 2 J=1,N,2
      JJ=(J-1)/2+1
      CALL NODENO(XYT(J),XYT(J+1),NODE(JJ),MSCAN)
      ITRINO(NEL,JJ)=NODE(JJ)
      XYT(J)=XYT(J)+TRANS(1)
 2    XYT(J+1)=XYT(J+1)+TRANS(2)
      IF(N.EQ.5)ITRINO(NEL,4)=ITRINO(NEL,3)
 1    NEL=NEL+1
      RETURN
      END
      FUNCTION MODUL(I,J)
      IF(I.GE.J.AND.J.GT.0)MODUL=J
      IF(J.EQ.0)MODUL=I
      IF(J.GT.I)MODUL=J-I
      RETURN
      END
```

```
      SUBROUTINE CIRSEG(CARD)                                          B2 06120
      IMPLICIT REAL*8 (A-H,O-Z)                                        B2 06130
      COMMON/ITNODE/Z,XYCORD(1500,2),INODE(1500),NNP,ITRINO(1500,4),MB B2 06140
      COMMON/RECTGR/XY(3),NIN,NOUT,SCALE(2),NRECT,NEL,MSCAN,NODE(4)    B2 06150
     1,NSV                                                             B2 06160
      DIMENSION CARD(10)                                               B2 06170
      READ(NIN,1)XO,YO,THI,THF,TRAD,RAD,TOL                            B2 06180
    1 FORMAT(5F10.0)                                                   B2 06190
      WRITE(NOUT,2)(CARD(IZ),IZ=1,10),XO,YO,THI,THF,TRAD,RAD,TOL       B2 06200
    2 FORMAT(9H >>>>>>>>,' NODAL POINTS INTITLED',5X,9A8               B2 06210
     1/'HAVE BEEN ALTERED FOR A ROUND'/                                B2 06220
     2' THE ABSOLUTE CENTER IS',2E15.4/' THE INITIAL AND FINAL ANGLE IS'B2 06230
     3/' THE RADIAL TOLERANCE IS',F15.4/' THE ANGULAR TOLERANCE IS',   B2 06240
     4E15.4,' THE RADIUS IS',E15.4)                                    B2 06250
      RMIN=RAD-TOL                                                     B2 06260
      RMAX=RAD+TOL                                                     B2 06270
      THMIN=THI-TRAD                                                   B2 06280
      THMAX=THF+TRAD                                                   B2 06290
      DO 3 I=1,NNP                                                     B2 06300
      XDIF=XYCORD(1,I)-XO                                              B2 06310
      YDIF=XYCORD(2,I)-YO                                              B2 06320
      R=DSQRT(XDIF*XDIF+YDIF*YDIF)                                     B2 06330
      TH=DATAN2(YDIF,XDIF)/0.017453295                                 B2 06340
      IF(TH.LE.THMAX.AND.TH.GE.THMIN.AND.R.LE.RMAX.AND.R.GE.RMIN)GO TO 4B2 06350
      GO TO 3                                                          B2 06360
    4 DR=RAD-R                                                         B2 06370
      XYCORD(1,I)=XYCORD(1,I)+DR*XDIF/RAD                              B2 06380
      XYCORD(2,I)=XYCORD(2,I)+DR*YDIF/RAD                              B2 06390
    3 CONTINUE                                                         B2 06400
      END                                                              B2 06410
```

B.3 THE PLOTTING PROGRAM

```
C
C     >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>
C                 PROGRAM FOR PLOTTING TWO DIMENSIONAL FINITE ELEMENT
C                           GRIDS AND STRESS CONTOURS
C     >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>
C
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON XY(1000,3),X(1000),Y(1000),IELF(1000,4),NOPAIR(1000,3),
     1SIG(1000,3),AVE(20),TEMP(10)
      DIMENSION TITLE(10)
      NET=3
      NST=4
      REWIND NET
      REWIND NST
      NIN=5
      NOUT=6
      NPUNCH=1
      READ(NIN,72)NNP,NEL,NCASE
   72 FORMAT(15I5)
      WRITE(NOUT,14)NNP,NEL,NCASE
   14 FORMAT('1THE NUMBER OF PLOTTED NODES ARE',I10/
     1' THE NUMBER OF ELEMENTS ARE',I10/
     2' THE NUMBER OF RUN CASES ARE',I10)
      READ(NIN,2)(XY(I,1),XY(I,2),I=1,NNP)
    2 FORMAT(10X,2F10.0)
      DO 4 I=1,NEL
      READ(NIN,88)(IELE(I,J),J=1,4)
   88 FORMAT(5X,5I5)
    4 WRITE(NET)(IELE(I,J),J=1,4)
      IF(NCASE.LT.0)GO TO 3
      DO 86 I=1,NEL
      READ(NIN,20)(SIG(I,J),J=1,3)
   20 FORMAT(3E20.10,20X)
   86 WRITE(NST)(SIG(I,J),J=1,3)
    3 IF(NCASE.LT.0)NCASE=IABS(NCASE)
      DO 43 ICASE=1,NCASE
      READ(NIN,54)(TITLE(IZ),IZ=1,10)
   54 FORMAT(10A8)
      WRITE(NOUT,55)ICASE,(TITLE(IZ),IZ=1,10)
   55 FORMAT(///'1THE TITLE FOR CASE',I5,'  IS'//20X,10A8//)
      READ(NIN,1)XMIN,XMAX,YMIN,YMAX,WIDTH,NEON,NEOE,NCONT,IGRID,ISIGX,
     1ISIGY,ITAUXY,ICOVAL,ICOPUN
    1 FORMAT(5F10.0,3I5,6I1)
      IF(IGRID.EQ.0.AND.ISIGX.EQ.0.AND.ISIGY.EQ.0.AND.ITAUXY.EQ.0)STOP
C
C     FOR NOMINAL STRESS CALCULATIONS
C
```

```
      NCOMP=0                                                           B3 00480
      AVE(4)=0.                                                         B3 00490
      NSTR=0                                                            B3 00500
      IF(ISIGX.GT.0.OR.ISIGY.GT.0.OR.ITAUXY.GT.0)NCOMP=4                B3 00510
      IF(ISIGX.EQ.2)NCOMP=1                                             B3 00520
      IF(ISIGY.EQ.2)NCOMP=2                                             B3 00530
      IF(ITAUXY.EQ.2)NCOMP=3                                            B3 00540
      IF(ISIGX.EQ.3.OR.ISIGY.EQ.3.OR.ITAUXY.EQ.3)READ(NIN,1)AVE(4)      B3 00550
      IF(ISIGX.NE.2.AND.ISIGY.NE.2.AND.ITAUXY.NE.2)GO TO 96             B3 00560
      IF(NCOMP.EQ.0.OR.NCOMP.EQ.4)GO TO 96                              B3 00570
      NSTR=1                                                            B3 00580
95    READ(NIN,90)ICON,FACT,(NOPAIR(IZ,1),IZ=1,12)                      B3 00590
90    FORMAT(I5,F10.0,12I5)                                             B3 00600
      IC=1                                                              B3 00610
      REWIND NST                                                        B3 00620
      NREC=1                                                            B3 00630
      IF(ICON.EQ.0)GO TO 112                                            B3 00640
      IF(ICON.LT.0)GO TO 100                                            B3 00650
97    IF(NOPAIR(IC,1).EQ.NREC)GO TO 92                                  B3 00660
      READ(NST)AVE(1),AVE(2),AVE(3)                                     B3 00670
      NREC=NREC+1                                                       B3 00680
      GO TO 97                                                          B3 00690
92    II=NOPAIR(IC,1)                                                   B3 00700
      JJ=NOPAIR(IC+1,1)                                                 B3 00710
      KK=ICON-1                                                         B3 00720
      DO 98 I=II,JJ,ICON                                                B3 00730
      NOPAIR(NSTR,2)=NREC                                               B3 00740
      READ(NST)AVE(1),AVE(2),AVE(3)                                     B3 00750
      NREC=NREC+1                                                       B3 00760
      AVE(4)=AVE(4)+AVE(NCOMP)/FACT                                     B3 00770
      IF(KK.EQ.0)GO TO 105                                              B3 00780
      DO 104 J=1,KK                                                     B3 00790
      NREC=NREC+1                                                       B3 00800
104   READ(NST)AVE(1),AVE(2),AVE(3)                                     B3 00810
105   CONTINUE                                                          B3 00820
98    NSTR=NSTR+1                                                       B3 00830
      IC=IC+2                                                           B3 00840
      IF(IC.GT.11.OR.NOPAIR(IC,1).EQ.0)GO TO 95                         B3 00850
      GO TO 97                                                          B3 00860
100   IF(NOPAIR(IC,1).EQ.NREC)GO TO 99                                  B3 00870
      READ(NST)AVE(1),AVE(2),AVE(3)                                     B3 00880
      NREC=NREC+1                                                       B3 00890
      GO TO 100                                                         B3 00900
99    READ(NST)AVE(1),AVE(2),AVE(3)                                     B3 00910
      AVE(4)=AVE(4)+AVE(NCOMP)/FACT                                     B3 00920
      NOPAIR(NSTR,2)=NREC                                               B3 00930
      NREC=NREC+1                                                       B3 00940
```

```
      NSTR=NSTR+1
      IC=IC+1
      IF(IC.GT.12.OR.NOPAIR(IC,1).EQ.0)GO TO 95
      GO TO 100
112   CONTINUE
      WRITE(NOUT,113)
113   FORMAT(' THE FOLLOWING ELEMENTS WERE CHOSEN FOR NOMINAL STRESS',
     1' COMPUTATIONS'/)
      NSTR=NSTR-1
      WRITE(NOUT,72)(NOPAIR(IZ,2),IZ=1,NSTR)
96    CONTINUE
      NEL=1
75    READ(NIN,72)ICON,(NOPAIR(IZ,1),IZ=1,14)
      REWIND NET
      REWIND NST
      NREC=1
      IC=1
      IF(ICON.EQ.0)GO TO 76
      IF(ICON.LE.0)GO TO 80
77    IF(NOPAIR(IC,1).EQ.NREC)GO TO 82
      IF(NCOMP.GT.0)READ(NST)AVE(1),AVE(2),AVE(3)
      READ(NET)IJUNK,JJUNK,KJUNK,LJUNK
      NREC=NREC+1
      GO TO 77
82    II=NOPAIR(IC,1)
      JJ=NOPAIR(IC+1,1)
      KK=ICON-1
      DO 78 I=II,JJ,ICON
      NOPAIR(NEL,2)=NREC
      IF(NCOMP.GT.0)READ(NST)(SIG(NEL,J),J=1,3)
      READ(NET)(IELE(NEL,J),J=1,4)
      NREC=NREC+1
      IF(KK.EQ.0)GO TO 89
      DO 84 J=1,KK
      NREC=NREC+1
      IF(NCOMP.GT.0)READ(NST)AVE(1),AVE(2),AVE(3)
84    READ(NET)IJUNK,JJUNK,KJUNK,LJUNK
89    CONTINUE
78    NEL=NEL+1
      IC=IC+2
      IF(IC.GT.13.OR.NOPAIR(IC,1).EQ.0)GO TO 75
      GO TO 77
80    IF(NOPAIR(IC,1).EQ.NREC)GO TO 79
      IF(NCOMP.GT.0)READ(NST)AVE(1),AVE(2),AVE(3)
      READ(NET)IJUNK,JJUNK,KJUNK,LJUNK
      NREC=NREC+1
      GO TO 80
```

```
79     IF(NCOMP.GT.0)READ(NST)(SIG(NEL,J),J=1,3)                       B3 01420
       READ(NET)(IELE(NEL,J),J=1,4)                                    B3 01430
       NOPAIR(NEL,2)=NREC                                              B3 01440
       NEL=NEL+1                                                       B3 01450
       NREC=NREC+1                                                     B3 01460
       IC=IC+1                                                         B3 01470
       IF(IC.GT.14.OR.NOPAIR(IC,1).EQ.0)GO TO 75                       B3 01480
       GO TO 80                                                        B3 01490
76     CONTINUE                                                        B3 01500
       NEL=NEL-1                                                       B3 01510
       IF(ISIGX.EQ.3.OR.ISIGY.EQ.3.OR.ITAUXY.EQ.3)GO TO 110            B3 01520
       IF(NCOMP.EQ.0.OR.NCOMP.EQ.4)GO TO 85                            B3 01530
C                                                                      B3 01540
C      ALL ELEMENTS TO DETERMINE NOMINAL STRESS HAVE BEEN CHOSEN       B3 01550
C                                                                      B3 01560
       AVE(4)=AVE(4)/NSTR                                              B3 01570
C      TO NONDIMENSIONALIZE STRESSES                                   B3 01580
110    DO 101 I=1,NEL                                                  B3 01590
       DO 101 J=1,3                                                    B3 01600
101    SIG(I,J)=SIG(I,J)/AVE(4)                                        B3 01610
       WRITE(NOUT,102)NCOMP,NSTR,AVE(4)                                B3 01620
102    FORMAT(/' THE NOMINAL STRESS COMPONENT IS',I5,                  B3 01630
      1'  (0=GIVEN, 1=NORMAL X, 2=NORMAL Y, 3=SHEAR)'/                 B3 01640
      2' THE NUMBER OF ELEMENTS USED FOR NOMINAL STRESS COMPUTATIONS IS',B3 01650
      3I5/' THE COMPUTED NOMINAL/GIVEN STRESS IS',E20.7)               B3 01660
85     WRITE(NOUT,81)ICASE,NEL                                         B3 01670
81     FORMAT(/' THE TOTAL NUMBER OF ELEMENTS CONSIDERED IN CASE'      B3 01680
      1,I5,' ARE',I5/)                                                 B3 01690
       WRITE(NOUT,114)                                                 B3 01700
114    FORMAT(//' NEW ELEMENT',5X,'OLD ELEMENT',6X,'SIGMA X/NOM',      B3 01710
      29X,'SIGMA Y/NOM',11X,'TAU XY/NOM')                              B3 01720
       IF(NCOMP.EQ.0)WRITE(NOUT,115)(I,NOPAIR(I,2),I=1,NEL)            B3 01730
115    FORMAT(5X,I5,10X,I5)                                            B3 01740
       IF(NCOMP.NE.0)WRITE(NOUT,108)(I,NOPAIR(I,2),(SIG(I,J),J=1,3),   B3 01750
      1I=1,NEL)                                                        B3 01760
108    FORMAT(5X,I5,10X,I5,3E20.5)                                     B3 01770
       IF(IGRID.GT.0)WRITE(NOUT,15)                                    B3 01780
       IF(ISIGX.GT.0)WRITE(NOUT,16)                                    B3 01790
       IF(ISIGY.GT.0)WRITE(NOUT,17)                                    B3 01800
       IF(ITAUXY.GT.0)WRITE(NOUT,18)                                   B3 01810
       IF(ISIGX.EQ.0.AND.ISIGY.EQ.0.AND.ITAUXY.EQ.0)GO TO 65           B3 01820
       IF(ICOVAL.EQ.0)WRITE(NOUT,56)                                   B3 01830
       IF(ICOVAL.EQ.1)WRITE(NOUT,57)                                   B3 01840
       IF(ICOVAL.EQ.2)WRITE(NOUT,58)                                   B3 01850
65     CONTINUE                                                        B3 01860
15     FORMAT(' ELEMENT PLOT IS DESIRED')                              B3 01870
16     FORMAT(' NORMAL X STRESS CONTOUR PLOT IS DESIRED')              B3 01880
```

```
17    FORMAT(' NORMAL Y STRESS CONTOUR PLOT IS DESIRED')
18    FORMAT(' SHEAR STRESS CONTOUR PLOT IS DESIRED')
56    FORMAT(' CONTOURS ARE GENERATED ELEMENTWISE')
57    FORMAT(' CONTOUR LEVELS ARE INPUT')
58    FORMAT(' CONTOURS OF EQUAL STRESS INCREMENTS ARE GENERATED')
C
C     NOW REDUCE DATA SO THAT NO LINES ARE DRAWN TWICE
C
      N=4
      NOPAIR(1,1)=IELE(1,1)
      NOPAIR(1,2)=IELE(1,2)
      NOPAIR(2,1)=IELE(1,1)
      NOPAIR(2,2)=IELE(1,4)
      NOPAIR(3,1)=IELE(1,2)
      NOPAIR(3,2)=IELE(1,3)
      NOPAIR(4,1)=IELE(1,4)
      NOPAIR(4,2)=IELE(1,3)
      DO 19 I=1,4
19    NOPAIR(I,3)=1
      DO 7 K=2,NEL
      DO 7 J=1,4
      JJ=J+1
      IF(JJ.EQ.5)JJ=1
      N1=IELE(K,J)
      N2=IELE(K,JJ)
      IF(N1.EQ.N2)GO TO 7
      DO 6 I=1,N
      IF(N1.EQ.NOPAIR(I,1).AND.N2.EQ.NOPAIR(I,2).OR.N1.EQ.NOPAIR(I,2)
     1.AND.N2.EQ.NOPAIR(I,1))NOPAIR(I,3)=NOPAIR(I,3)+1
      IF(N1.EQ.NOPAIR(I,1).AND.N2.EQ.NOPAIR(I,2).OR.N1.EQ.NOPAIR(I,2)
     1.AND.N2.EQ.NOPAIR(I,1)) GOTO 7
6     CONTINUE
      N=N+1
      NOPAIR(N,1)=N1
      NOPAIR(N,2)=N2
      NOPAIR(N,3)=1
7     CONTINUE
C
C     FOR REDUCING AND CENTERING
C
      IF(XMAX.NE.0..OR.YMAX.NE.0.)GO TO 63
      XMIN=1.E50
      XMAX=-1.E50
      YMIN=1.E50
      YMAX=-1.E50
      DO 64 I=1,NEL
      DO 64 J=1,4
```

```
      IF(XY(IELE(I,J),1).LT.XMIN)XMIN=XY(IELE(I,J),1)
      IF(XY(IELE(I,J),1).GT.XMAX)XMAX=XY(IELE(I,J),1)
      IF(XY(IELE(I,J),2).LT.YMIN)YMIN=XY(IELE(I,J),2)
64    IF(XY(IELE(I,J),2).GT.YMAX)YMAX=XY(IELE(I,J),2)
63    HLET=.05
      HSPACE=HLET/5.
C     LETTER HEIGHT IS CONSTANT
      SHIFT=(11.-WIDTH)/2.
      IF(YMAX-YMIN.LT.XMAX-XMIN)GO TO 68
      IAXIS=0
      SCALEY=WIDTH/(XMAX-XMIN)
      DO 67 I=1,NNP
      X(I)=0.0
      T=(XY(I,2)-YMIN)*SCALEY
      XY(I,2)=(XMAX-XY(I,1))*SCALEY+SHIFT
67    XY(I,1)=T
      GO TO 69
68    SCALEY=WIDTH/(YMAX-YMIN)
      IAXIS=1
      DO 70 I=1,NNP
      X(I)=0.0
      XY(I,1)=(XY(I,1)-XMIN)*SCALEY
70    XY(I,2)=(XY(I,2)-YMIN)*SCALEY+SHIFT
69    CONTINUE
C
C     BEGIN PLOT COMMANDS, FIRST PLOT GRID
C
      IF(IGRID.EQ.0)GO TO 23
      DO 41 I=1,N
      CALL PLOT(XY(NOPAIR(I,1),1),XY(NOPAIR(I,1),2),3)
      CALL PLOT(XY(NOPAIR(I,2),1),XY(NOPAIR(I,2),2),2)
41    CONTINUE
C
C     BEGIN NODE NUMBERS
C
      IF(NEON.EQ.0)GO TO 13
      DO 109 I=1,NEL
      DO 109 J=1,4
109   X(IELE(I,J))=X(IELE(I,J))+1.
      DO 9 I=1,NNP,NEON
      IF(X(I).EQ.0.)GO TO 9
      VAR=I+.5
      CALL NUMBER(XY(I,1),XY(I,2),HLET,VAR,0.0,-1)
9     CONTINUE
13    CONTINUE
C
C     TO LABEL ELEMENTS
```

```
C
      IF(NEOE.EQ.0)GO TO 53
      DO 12 I=1,NEL,NEOE
      IF(IELE(I,3).NE.IELE(I,4))GO TO 40
      XAVE=(XY(IELE(I,1),1)+XY(IELE(I,2),1)+XY(IELE(I,3),1))/3.
      YAVE=(XY(IELE(I,1),2)+XY(IELE(I,2),2)+XY(IELE(I,3),2))/3.
      GO TO 11
40    XAVE=(XY(IELE(I,1),1)+XY(IELE(I,2),1)+XY(IELE(I,3),1)+
     1XY(IELE(I,4),1))/4.
      YAVE=(XY(IELE(I,1),2)+XY(IELE(I,2),2)+XY(IELE(I,3),2)+
     1XY(IELE(I,4),2))/4.
11    CONTINUE
      VAR=I+.5
      IF(I.GE.1)AL=HLET
      IF(I.GE.10)AL=HLET*2.
      IF(I.GE.100)AL=HLET*3.
      XAVE=XAVE-AL/2
      YAVE=YAVE-HLET/2.
      CALL NUMBER(XAVE,YAVE,HLET,VAR,0.0,-1)
      CALL PLOT(XAVE,YAVE-HSPACE,3)
      CALL PLOT(XAVE+AL,YAVE-HSPACE,2)
12    CONTINUE
53    CONTINUE
      CALL PLOT(12.,0.,-3)
C
C     CODE FOR CONTOUR PLOTS
C
23    IF(ISIGX.EQ.0.AND.ISIGY.EQ.0.AND.ITAUXY.EQ.0)GO TO 43
C     REDUCE DATA FROM NOPAIR FOR BORDER OF SPECIMEN
      KPAIR=0
      DO 21 I=1,N
      IF(NOPAIR(I,3).NE.1)GO TO 21
      KPAIR=KPAIR+1
      NOPAIR(KPAIR,1)=NOPAIR(I,1)
      NOPAIR(KPAIR,2)=NOPAIR(I,2)
21    CONTINUE
      DO 24 IGO=1,3
      IF(IGO.EQ.1.AND.ISIGX.GT.0)KS=1
      IF(IGO.EQ.1.AND.ISIGX.EQ.0)GO TO 24
      IF(IGO.EQ.2.AND.ISIGY.GT.0)KS=2
      IF(IGO.EQ.2.AND.ISIGY.EQ.0)GO TO 24
      IF(IGO.EQ.3.AND.ITAUXY.GT.0)KS=3
      IF(IGO.EQ.3.AND.ITAUXY.EQ.0)GO TO 24
C
C     TO PLOT THE BORDER OF THE SPECIMEN
C
      CALL RIMMER(KPAIR)
```

```
C
C     BEGIN ORGANIZING STRESS VALUES
C
      IF(ICOVAL.EQ.1)READ(NIN,20)(AVE(I),I=1,NCONT)
      IF(ICOVAL.EQ.1)WRITE(NOUT,51)
51    FORMAT(//' OUTPUT OF ASSIGNED STRESS CONTOUR LEVELS'//)
      IF(ICOVAL.EQ.1)WRITE(NOUT,30)(I,AVE(I),I=1,NCONT)
      IF(ICOVAL.EQ.1)GO TO 49
      IF(ICOVAL.NE.0)GO TO 59
C     PUT STRESSES IN ASCENDING ORDER
      DO 25 I=1,NEL
25    X(I)=SIG(I,KS)
      NN=NEL-1
      DO 26 I=1,NN
      K=NEL-I
      DO 26 J=1,K
      IF(X(J)-X(J+1))26,26,27
27    TEM=X(J)
      X(J)=X(J+1)
      X(J+1)=TEM
26    CONTINUE
      N=NEL/NCONT
      WRITE(NOUT,31)
31    FORMAT(//' OUTPUT OF STRESS VALUES AT THE CONTOURS'//)
      MX=NCONT-1
      DO 28 I=1,MX
      AVE(I)=0.
      ILOW=(I-1)*N+1
      IHIGH=ILOW+N-1
      DO 29 J=ILOW,IHIGH
29    AVE(I)=AVE(I)+X(J)
      AVE(I)=AVE(I)/N
28    WRITE(NOUT,30)I,AVE(I)
30    FORMAT(I10,E20.5)
      ILOW=IHIGH+1
      AVE(NCONT)=0.
      DO 42 I=ILOW,NEL
42    AVE(NCONT)=AVE(NCONT)+X(I)
      AVE(NCONT)=AVE(NCONT)/(NEL-IHIGH)
      WRITE(NOUT,30)NCONT,AVE(NCONT)
59    IF(ICOVAL.NE.2)GO TO 49
C     GET HIGHEST, LOWEST VALUE
      WRITE(NOUT,31)
      X(1)=1.E50
      X(NEL)=-1.E50
      DO 32 I=1,NEL
      IF(X(1).GT.SIG(I,KS))X(1)=SIG(I,KS)
```

```
62    IF(X(NEL).LT.SIG(I,KS))X(NEL)=SIG(I,KS)
      DS=(X(NEL)-X(1))/(NCONT+1)
      DO 61 I=1,NCONT
      AVE(I)=I*DS+X(1)
61    WRITE(NOUT,30)I,AVE(I)
      IF(ICOPUN.EQ.1)WRITE(NPUNCH,20)(AVE(IZ),IZ=1,NCONT)
C
C     DETERMINE THE NUMBER OF ELEMENTS JOINED AT EACH NODE, THEN AVERAGE
C     STRESSES ON NNP
C
49    CONTINUE
C     INITIALIZING
      DO 32 I=1,NNP
      X(I)=0.
32    Y(I)=0.
      DO 33 I=1,NEL
      DO 34 J=1,3
      X(IELE(I,J))=X(IELE(I,J))+SIG(I,KS)
34    Y(IELE(I,J))=Y(IELE(I,J))+1.
      IF(IELE(I,3).EQ.IELE(I,4))GO TO 33
      X(IELE(I,4))=X(IELE(I,4))+SIG(I,KS)
      Y(IELE(I,4))=Y(IELE(I,4))+1.
33    CONTINUE
      WRITE(NOUT,45)
45    FORMAT(//' OUTPUT OF STRESSES COMPUTED AT THE NNP'//
     1' NODE NUMBER',4X,'STRESS'//)
      DO 35 I=1,NNP
      IF(Y(I).EQ.0.)GO TO 35
      X(I)=X(I)/Y(I)
      WRITE(NOUT,46)I,X(I)
46    FORMAT(I5,E20.5)
35    CONTINUE
C
C     BEGIN CHECKING EVERY ELEMENT FOR CONTOURS THAT WILL CROSS IT,
C     CONTOURS DO NOT NEED TO BE ORDERED
C
      DO 36 I=1,NEL
      IF(IELE(I,3).EQ.IELE(I,4))JJ=3
      IF(IELE(I,3).NE.IELE(I,4))JJ=4
      DO 37 J=1,NCONT
      ICA=3
      ICB=0
      DO 38 K=1,JJ
      II=MODUL(K+1,JJ)
      IF(X(IELE(I,K)).GE.AVE(J).AND.X(IELE(I,II)).LE.AVE(J).OR.
     1X(IELE(I,II)).GE.AVE(J).AND.X(IELE(I,K)).LE.AVE(J))GO TO 39
      IF(K.EQ.4.AND.JJ.EQ.4)GO TO 50
```

```
      GO TO 38                                                          B3 04240
39    CONTINUE                                                          B3 04250
C     CALCULATE FIRST POINT ON ICA=3, SECOND ON ICA=2                   B3 04260
      IF(X(IELE(I,K)).LE.AVE(J))BOT=X(IELE(I,K))                        B3 04270
      IF(X(IELE(I,II)).LE.AVE(J))BOT=X(IELE(I,II))                      B3 04280
      IF(X(IELE(I,K)).LE.AVE(J))IC1=0                                   B3 04290
      IF(X(IELE(I,II)).LE.AVE(J))IC1=1                                  B3 04300
      DIF=DABS(X(IELE(I,K))-X(IELE(I,II)))                              B3 04310
      IF(DIF.NE.0.)FACT=(AVE(J)-BOT)/DIF                                B3 04320
      IF(DIF.EQ.0.)FACT=1.                                              B3 04330
      YPLOT=XY(IELE(I,MODUL(K+IC1,JJ)),1)+(XY(IELE(I,MODUL(II-IC1,JJ)),1B3 04340
     1)-XY(IELE(I,MODUL(K+IC1,JJ)),1))*FACT                             B3 04350
      XPLOT=XY(IELE(I,MODUL(K+IC1,JJ)),2)+(XY(IELE(I,MODUL(II-IC1,JJ)),2B3 04360
     1)-XY(IELE(I,MODUL(K+IC1,JJ)),2))*FACT                             B3 04370
      IF(JJ.NE.4)GO TO 44                                               B3 04380
      ICB=ICB+1                                                         B3 04390
      TEMP(ICB*2-1)=XPLOT                                               B3 04400
      TEMP(ICB*2)=YPLOT                                                 B3 04410
50    IF(K.EQ.4.AND.ICB.GE.4)GO TO 47                                   B3 04420
      IF(K.NE.4.OR.ICB.EQ.0)GO TO 38                                    B3 04430
      CALL PLOT(TEMP(2),TEMP(1),3)                                      B3 04440
      CALL PLOT(TEMP(4),TEMP(3),2)                                      B3 04450
      GO TO 38                                                          B3 04460
47    CONTINUE                                                          B3 04470
C                                                                       B3 04480
C     ANYTHING THAT GETS HERE IS LOOKED AT CAREFULLY SO THAT THE        B3 04490
C     TWO CONTOURS WITHIN THIS ELEMENT DON)T CROSS THE WRONG WAY        B3 04500
C     (FOR RECTANGULAR ELEMENTS ONLY)                                   B3 04510
C                                                                       B3 04520
      MAX=1                                                             B3 04530
      DO 48 IJ=2,4                                                      B3 04540
48    IF(X(IELE(I,IJ)).GT.X(IELE(I,MAX)))MAX=IJ                         B3 04550
      MIN=MODUL(MAX+1,JJ)                                               B3 04560
      CALL PLOT(TEMP(2*MAX),TEMP(2*MAX-1),3)                            B3 04570
      CALL PLOT(TEMP(2*MIN),TEMP(2*MIN-1),2)                            B3 04580
      MIN=2*MODUL(MAX+2,JJ)                                             B3 04590
      MAX=2*MODUL(MAX+3,JJ)                                             B3 04600
      CALL PLOT(TEMP(MIN),TEMP(MIN-1),3)                                B3 04610
      CALL PLOT(TEMP(MAX),TEMP(MAX-1),2)                                B3 04620
      GO TO 38                                                          B3 04630
44    CONTINUE                                                          B3 04640
      CALL PLOT(YPLOT,XPLOT,ICA)                                        B3 04650
      ICA=2                                                             B3 04660
38    CONTINUE                                                          B3 04670
37    CONTINUE                                                          B3 04680
36    CONTINUE                                                          B3 04690
      CALL PLOT(12.,0.,-3)                                              B3 04700
```

```
24      CONTINUE
22      CONTINUE
C       FOR TRANSLATING NNP AFTER A PLOT
        IF(IAXIS.EQ.1)GO TO 8
        DO 66 I=1,NNP
        T=XY(I,1)
        XY(I,1)=(SHIFT-XY(I,2))/SCALEY+XMAX
66      XY(I,2)=T/SCALEY+YMIN
        GO TO 43
8       DO 71 I=1,NNP
        XY(I,1)=XY(I,1)/SCALEY+XMIN
71      XY(I,2)=(XY(I,2)-SHIFT)/SCALEY+YMIN
43      CONTINUE
        CALL PLOT(0.0,0.0,-4)
        STOP
        END
        FUNCTION MODUL(I,J)
        IF(I.GT.J)MODUL=I-J
        IF(I.EQ.0)MODUL=J
        IF(I.LE.J.AND.I.GT.0)MODUL=I
        RETURN
        END
        SUBROUTINE RIMMER(K)
        IMPLICIT REAL*8 (A-H,O-Z)
        COMMON XY(1000,3),X(1000),Y(1000),IELE(1000,4),NOPAIR(1000,3),
       1SIG(1000,3),AVE(20),TEMP(10)
        N2=K*2-1
        DO 1 I=1,N2,2
        J=I+1
        JJ=J/2
        CALL PLOT(XY(NOPAIR(JJ,1),1),XY(NOPAIR(JJ,1),2),3)
        CALL PLOT(XY(NOPAIR(JJ,2),1),XY(NOPAIR(JJ,2),2),2)
1       CONTINUE
        RETURN
        END
```

APPENDIX C.

MATERIAL PROPERTIES

TABLE C. MATERIAL PROPERTIES

Material	E_{11} (msi)	E_{22} (msi)	ν_{12}	G_{12} (msi)	α_1 (μ in/in °F)	α_2 (μ in/in °F)
Graphite-epoxy	18.5	0.7	0.21	0.83	----	----
Thornel 50/E-798	19.6	1.2	0.30	0.59	----	----
Graphite-polyimide	17.7	0.99	0.33	0.71	0.19	21.0
Steel	29.0	29.0	0.32	11.0	9.60	9.60

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ANALYSIS OF SHEAR TEST METHODS FOR COMPOSITE LAMINATES

by

Henry W. Bergner, Jr.

(ABSTRACT)

An elastic plane stress finite element analysis of the stress distributions in four flat test specimens for in-plane shear response of composite materials subjected to mechanical or thermal loads is presented. The shear test specimens investigated include: slotted coupon, cross beam, Iosipescu, and rail shear. Results are presented in the form of normalized stress contour plots for all three in-plane stress components.

The slotted specimen is studied for three graphite-epoxy laminates ($[0]$, $[90]$, $[\pm 45]_S$); the cross beam is studied for two graphite-epoxy laminates ($[0/90]_S$, $[\pm 45]_S$); the Iosipescu and rail shear specimens are studied with several materials (steel, graphite-epoxy, and graphite-polyimide) and several laminates ($[0]$, $[90]$, $[0/90]_S$, $[\pm 45]_S$, $[0/90/\pm 45]_S$) with rigid and elastic fixtures loaded mechanically or thermally. Geometric alterations are also investigated.

The study shows that the cross beam, Iosipescu, and rail shear specimens have stress distributions which are more than adequate for determining linear shear behavior of composite materials. Laminate properties, core effects, and fixture configurations are among the factors which were found to influence the stress distributions.