SCHEDULING FUEL-SHUFFLING OPERATIONS FOR A NUCLEAR POWER REACTOR

by

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I. INTRODUCTION

A. Problem Statement

Approximately once a year, a pressurized water nuclear reactor must undergo refueling operations. Activities undertaken during a specific refueling outage include various maintenance and inspection chores plus what is termed the fuel shuffle. During the course of any reactor operating cycle, the energy available within the fuel assemblies is only partially consumed. The rate at which the energy dissipates from the fuel components is affected by factors such as location in the reactor core and the presence of neutron-absorbing control rods. During the fuel shuffle itself, assemblies with the greatest fissionable uranium depletion are removed from the core and fresh fuel is inserted. Typically about one-third of the core is replaced in this manner, and the remaining fuel assemblies must also be transferred to new positions within the core. Similar shuffling activities are usually required for the control components suspended within the fuel elements.

The identity of the fuel and control assemblies to be replaced and the new locations of the partially spent assemblies are determined by nuclear engineers through consideration of the physics of the reactor core and the economics of fuel management and utilization. The problem area to be dealt with here is that of scheduling all the operations required to remove, relocate, or insert the various fuel elements and control rods. The present analysis is directed toward the specific task of refueling a pressurized water reactor utilizing a given set of equipment in hopes of establishing efficient scheduling rules for use with other types of reactors and differing equipment capabilities.

B. Objectives and Motivation

The measure of effectiveness to be employed in the construction of schedules for the fuel shuffle is that of total time for completion. The entire refueling outage may last anywhere from four to seven weeks, and the fuel-shuffling segment can be completed in approximately six to ten days. An estimate of the cost of a reactor being unavailable for use is in the neighborhood of $100,000 to $200,000 per day. Thus minimization of the total shuffle completion time seems to be an appropriate objective in terms of cost reduction.

Although the time taken up by the fuel shuffle itself seems rather small in comparison with the total refueling downtime, there have been instances in the past in which the shuffling operations became a part of the critical path for the completion of all outage activities. In other words, at a certain point in the whole refueling process, the rapid performance of the shuffle became a prime concern in terms of ultimately allowing the reactor to be put back into operation on time.

An additional motivation for attempting to construct an efficient shuffle schedule lies in the fact that a decrease in the time required for the refueling operations will correspond to a similar reduction in the period during which the supporting workers are exposed to radiation. Since there exist limitations on the exposure levels experienced by these individuals during given periods, a rapid completion of the shuffling activities allows for the effective utilization of the reactor personnel in completing other refueling duties.

C. Solution Approach

Standard operations research techniques in the area of scheduling and sequencing cannot be readily applied to the problem of generating effective schedules for the fuel-shuffling process. Rather, the unique characteristics of the shuffle problem dictate the use of a heuristic solution procedure in lieu of more general optimization methods. As a first step in this heuristic technique, the problem is broken down into workable groups of related operations. Effective scheduling procedures are then devised for each of the various groups.

The general mode of implementation for such a procedure is that of a computer program package in which representations of reactor core configurations before and after the shuffling process are given as input. The various groups of operations are then identified, and the appropriate scheduling procedures are applied to produce the complete shuffle schedule. The computerization of this process reduces the amount of time required for determining the initial plan of action for a given refueling outage and also allows for the rapid construction of a revised schedule in the event of fuel-handling equipment failure during the course of the shuffle.

This general methodology for determining fuel shuffle schedules also corresponds to the techniques currently employed by the operations research group at the Babcock and Wilcox Company [12]. In the present work, the main focus is upon the development of improved, near-optimal schedules for the individual groups of shuffle operations through the application of heuristic techniques. Consideration is also extended to the possibility of obtaining improvements through the combination of two or more of these groups. An integer linear programming model is formulated to provide assistance in the task of effectively integrating the various groups of operations present in a given shuffle.

The actual analysis undertaken here involves the examination of data from three representative refueling instances. The various subgroups of operations are identified, individual scheduling procedures are developed, and effective means of combining the groups are determined. The nature and exact form of the aforementioned integer programming model used in the integration procedures is presented in each case, along with the results of the application of a standard integer programming computer code.

D. The Nature of the Results

From this in-depth consideration of the shuffling operations required in the sample refueling instances, it is seen that shuffle schedule improvements can be made primarily by eliminating any superfluous transfers of fuel or control components and by allowing for the simultaneous operation of the equipment whenever possible. The combination of the various groups of shuffle operations frequently provides the means by which the above general scheduling rules can be applied in an efficient manner. The integer programming model used in this integration process is beneficial in determining the optimal combination of the various groups of operations involved in complex shuffling schemes.

The effectiveness of these heuristic scheduling techniques is measured mainly in terms of the improvements made over the most efficient of the past or existing methods employed in industry. For the most part, the magnitude of such time reductions is fairly small in comparison with the actual completion times for all the shuffle operations. The possibility exists for further improvement through the development of optimization techniques for scheduling the individual groups of transfer operations. However, the benefits gained with these optimization procedures are shown to be rather small in light of the additional effort required for the development and implementation of such techniques.

Important information is provided to decision makers in industry by these bounds on the magnitude of the improvements that might be made over the heuristic scheduling methods proposed here. The benefits gained from the determination of optimal schedules must be weighed against the associated costs. Based upon the cases examined here, this expense does not appear to be justified.

II. BACKGROUND AND RELATED RESEARCH

A. A Review of the Literature in the Nuclear Engineering Field

The problem of effectively scheduling the operations comprising a fuel shuffle has not really surfaced in the nuclear engineering literature to date. Recent work in related areas is summarized in the following paragraphs. Some of the topics discussed here will be considered further in the presentation of possible extensions of the present problem.

The related problem which seems to be addressed most frequently in the nuclear science literature is that of in-core fuel management. An effective and economically viable method of fuel utilization is sought which determines both short-term fuel shuffling policies and long-range fuel consumption patterns. In going from one reactor cycle to the next, consideration must be given to the number and location of the fuel assemblies to be replaced, the positioning of the partially spent elements remaining in the core, and the general control rod patterns. Various engineering and physics constraints are found to be binding. Factors such as heat flux, burnup, and reactivity control must be taken into account.

There are a variety of objective functions associated with these in-core fuel management studies. Broad energy cost minimization criteria are prevalent along with specific reactor physics measures such as minimum radial power peaking ratio and minimum amount of fresh fuel required. The corresponding solution techniques employed are also fairly diverse and include classical calculus, dynamic programming, linear programming, and direct search procedures. Two representative solution approaches to this fuel management problem are cited among the references [5,19].

Another related issue tackled by energy systems researchers involves the determination of the frequency and specific times of occurrence of refueling and maintenance operations. For example, Gruhl [11] has developed a technique to calculate such a reactor maintenance schedule which minimizes total system costs including those of operation, maintenance, refueling, and contract obligations. Here certain engineering and environmental constraints are also taken into account.

Finally, there remains the task of scheduling all of the maintenance and refueling activities including the fuel shuffle which constitute an outage. This research would seem to be directed toward qualitative activity and critical path analysis as evidenced in Metropolitan Edison's experiences in planning a refueling outage at its Three Mile Island station [20].

B. A Closer Look at the Fuel-Shuffling Process

Before proceeding with a discussion of work undertaken in the field of operations research which may be applicable to the fuel shuffle problem, it would be instructive to provide a detailed description of the shuffling operations themselves and the various means by which such tasks may be completed. A schematic overview of the nature and configuration of the equipment involved in the refueling process is shown in Figure 1. This particular representation is indicative of Babcock and Wilcox's refueling capabilities [12] and will be considered as the primary system representation. As previously stated, the results obtained here will provide the foundation for future considerations of the shuffle scheduling problem under the assumption of different configurations of equipment. Referring to Figure 1, the reactor is seen to be situated in one large room which adjoins another room containing the spent fuel storage pool. This latter area houses both the totally spent fuel assemblies and the fresh fuel which will be transferred into the core. Communication between these two rooms is accomplished by means of the fuel transfer mechanisms pictured as the east and west upenders in the figure.

Within most of the fuel assemblies are various types of neutron-absorbing rods for the purpose of reactivity and flow control. These components are inserted from the top of the fuel elements and are full-length control assemblies, partial rods for axial power shaping within the core, fast-consuming "lumped-burnable-poison" rods, or orifice rods which regulate the core's coolant flow. The various control rods are also involved in the fuel-shuffling process since they may either be moved to a new location along with the fuel assembly on which they are currently mounted or be relocated independently of their assemblies.

The transfer of fuel and control elements within the primary reactor room is carried out by two bridges termed the main and the

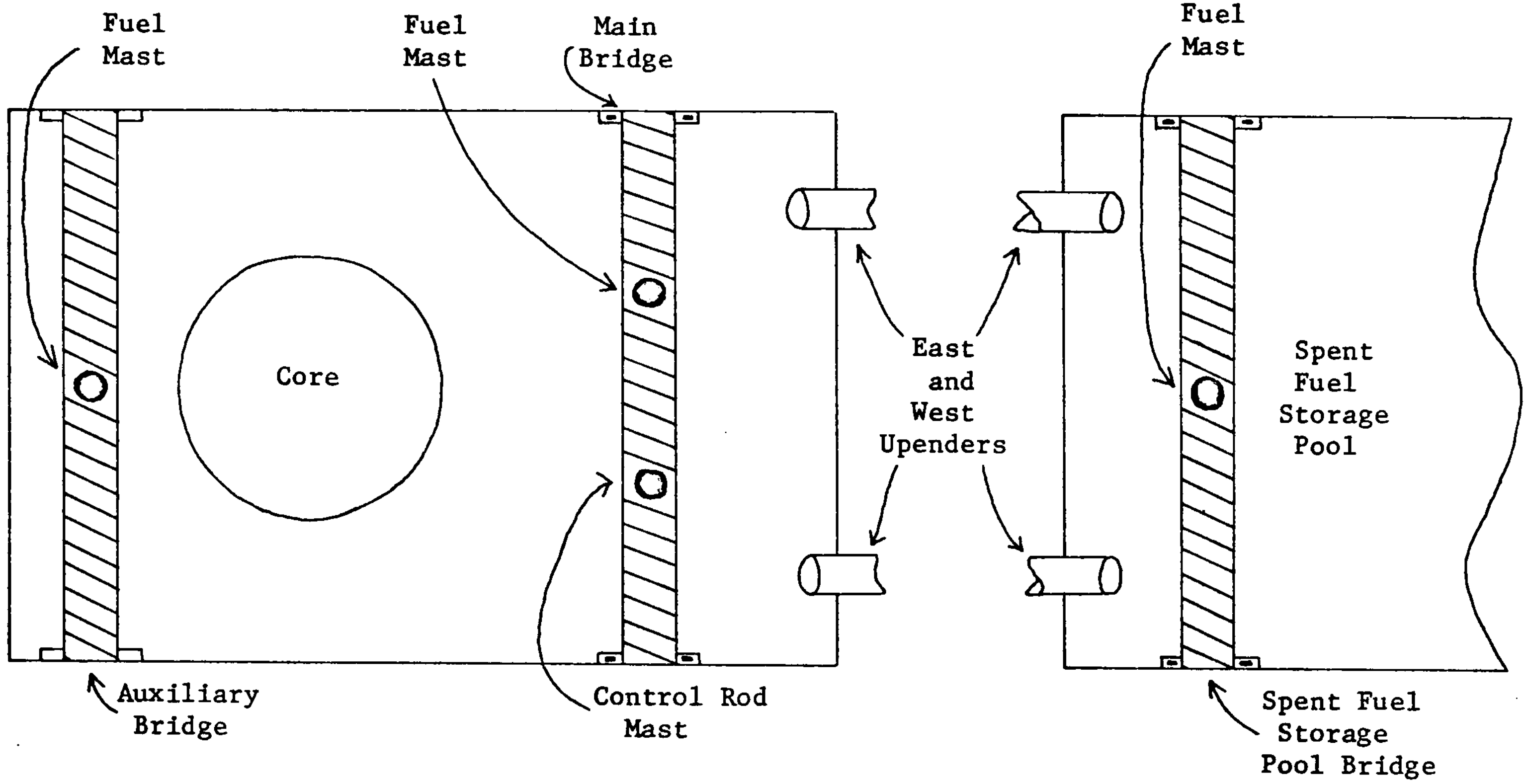

FIGURE 1
THE PRIMARY EQUIPMENT INVOLVED IN THE REFUELING PROCESS[a]

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[a] Courtesy of the Babcock and Wilcox Company.

auxiliary. The bridges move across the breadth of the core on a single set of tracks. Hence, the bridges do not operate in a totally independent manner; they cannot pass one another or be in the same relative core location simultaneously. Positioned on the bridges are masts resembling block-and-tackle type lifting devices. A mast can be moved along the bridge itself to access different sections of the core and can be raised up from and lowered into the core area to accomplish the actual transfer of the fuel and control components. The auxiliary bridge possesses one single-function mast capable of handling either a fuel assembly or a fuel element with its control component attached on top. Two single-function masts are situated on the main bridge. One mast is identical to that of the auxiliary bridge while the other is capable of transporting only control components by themselves.

Another distinction between the main and auxiliary bridges lies in the fact that the removal of totally spent assemblies and the insertion of fresh fuel can only be accomplished by the main bridge. The main bridge interacts in this regard with the upenders to transfer assemblies to and from the reactor room. With two such fuel transfer mechanisms present in the system, schedules could be constructed in which the main bridge inserts a spent fuel assembly into one upender and then picks up a fresh component at the other. Fuel movement within the spent fuel storage pool is carried out by an analogous bridge and mast system.

A brief description of the procedures involved in a fuel shuffle would serve to perhaps highlight and clarify the interactions of the aforementioned refueling equipment. A much more detailed examination of the operating characteristics of a representative set of fuel-handling equipment is given by Dossett [8]. First, it should be pointed out that the reactor core is under approximately fifteen feet of water throughout the shuffling process. Hence, inherent difficulties arise in the identification and verification of assemblies as they are removed from their positions in the core or received at the upenders. The fuel-handling bridges travel approximately three feet above the water. The bridge operator positions the bridge and corresponding mast over the appropriate core location, and the mast is slowly lowered to access a given fuel or control assembly.

The vertical position of the mast as it is lowered down into the core is monitored by a numbered tape attached to the mast assembly. A spotter walking alongside the reactor pool also observes the lowering of the mast to the proper position. A final location verification is provided as the bridge operator peers through a telescope which extends down into the water itself. The mast then engages and the assembly is raised and transferred to its new location within the core if appropriate. Similar location checking activities are performed when a fuel or control assembly is lowered into the reactor.

The removal of spent fuel assemblies and the insertion of new components are performed by the main bridge and the upenders. The mast on the bridge will lower an assembly into the upender's fuel basket rotating frame, which alters the assembly's inclination from vertical to horizontal. The component then travels through the fuel transfer tube and is received by the fuel-handling bridge in the spent fuel storage pool. These fuel transfer mechanisms are designed to handle either fuel assemblies alone or assemblies paired with their control components. Control rod assemblies are also stored within the reactor room itself in a storage area located between the two upenders.

Fuel transfer operations in the spent fuel storage area are also carried out under fifteen feet of water, and a bridge which is identical to the reactor room's auxiliary bridge is present. In this storage room there is an elevator which serves as a temporary storage area in which manual adjustments can be made to assemblies and control rods. In addition, there are various fuel storage racks, which are allocated to fuel elements that are entering or leaving the reactor. It is usually the case that certain areas of these racks will be assigned as the final destinations for the spent fuel. The specific schedule employed will dictate the exact final locations of the spent components. Thus, the fundamental operations of the fuel-handling bridge in the spent fuel pool will include transfers of components among the storage pool upenders, the storage racks, the elevator, and the spent fuel shipping cask area.

Supervision of the fuel-shuffling operations is maintained by a team of several reactor operators, which is headed by an individual with the title of senior reactor operator. These people translate the general shuffle schedule into specific instructions to be relayed to the bridge operators and supporting personnel. The course of the refueling process is monitored through the use of a control board containing a map of the reactor room and the storage areas. Little chips are affixed to the board to denote the various fuel assemblies and control rods and their positions as they are transferred about in the system.

In the event of mechanical difficulties which may interrupt the outlined shuffle schedule, this board is used to investigate possible alternate plans of action. The senior reactor operator has the authority to make temporary changes in the overall schedule, and the nature of these alterations is usually such that every effort is made to eventually get back on the original schedule. The amount of time involved in the manual generation of a completely new plan of action in the event of equipment failure would be far out of the realm of practical consideration. Recently, however, individuals at the Babcock and Wilcox Company have succeeded in implementing an interactive computer program to assist the reactor operators in making such schedule revisions [12].

At this point, a description has been presented for what can be termed the base system representation. Here there are two fuel-handling bridges in the primary reactor room: the main and the auxiliary bridges. The auxiliary bridge possesses one single-function fuel mast and the main bridge is equipped with a similar mast plus a single-function mast for control components. There exist two fuel transfer mechanisms between the reactor room and the storage pool, and the bridge in the spent fuel pool has a single-function fuel mast.

Modifications and simplifications of this basic equipment configuration are in operation at various reactor installations [12]. The characteristics of these different equipment options are as follows:

Alternative 1. This option is the same as the base system except that the spent fuel pool bridge has two functional masts: one for control rods and one for fuel assemblies.

Alternative 2. This option is the same as the base system except that there is only one fuel transfer system between the main reactor area and the fuel storage room.

Alternative 3. This option is the same as the base system except that there are two main bridges each with a fuel mast and a control rod mast moving over the core.

Alternative 4. In this equipment configuration, there exists only one main bridge in the reactor room and this bridge possesses one multiple-function mast. This lifting device can handle either fuel assemblies alone, fuel assemblies paired with control components, or control components alone. A single upender is present and the storage pool bridge possesses a single-function fuel mast.

Alternative 5. This option is the same as the fourth alternative above with the addition of an auxiliary bridge with a single-function mast for shuffling operations within the reactor room and a second upender for fuel transfer.

A truly general and effective technique for determining the best sequence for the operations making up the fuel shuffle segment of the refueling outage would need to account for variations in equipment configurations. The specific equipment on hand for a given refueling operation could perhaps be given as input to the solution procedure. In this manner, a revised schedule could be generated quickly in the event of an equipment failure during the course of the shuffle.

C. A Description of the Problem in the Vocabulary of Production Scheduling

In this section, the results of a survey of the operations research literature in the area of traditional production scheduling and sequencing are presented. The details of the representation of the present problem in terms of the standard scheduling models are given below as a preface to a review of the work undertaken in this field. This description will also provide additional insight into the nature of the fuel shuffle itself.

The "jobs" to be processed by the system would include the basic unit operations of the shuffle: (1) the removal of a totally spent fuel element or fuel and control component pair, (2) the insertion of a new fuel assembly or fuel and control element pair, and (3) the transfer of a partially spent component or a control rod to a new predetermined position within the core. The processing times for the individual operations are assumed to be deterministic in nature. For the most part, the four quarters of the core are symmetrical in terms of specific fuel and control component transfers. Hence, it may be possible in general to construct a complete shuffle schedule by considering just one-fourth of the actual number of jobs.

Referring to the previously described primary equipment configuration, the shuffle scheduling problem can be seen to possess the additional properties outlined in the next paragraphs.

1. Two "semi-parallel" machines will be considered: the main and auxiliary bridges in the reactor room. Each job will be processed on one and only one machine. Further, any of the jobs may be performed by the main bridge, but the auxiliary bridge may only complete the in-core transfers of fuel assemblies or fuel and control component pairs.

No mention has yet been made of the scheduling of the operations to be carried out by the bridge in the spent fuel pool. Initially, attention will be focused on the development of sequences of operations to be completed by the main and auxiliary bridges. The spent fuel pool bridge's schedule will be dictated by the final sequence constructed. However, the possibility definitely does exist for extending the scope of the model to include the generation of an integrated schedule for all three machines.

Secondly, the number of tasks to be performed solely by the main bridge is usually far greater than the number of operations which the auxiliary bridge can complete. In other words, it is seldom the case that the former piece of equipment is assigned to undertake any of the in-core fuel component transfers. However, for the present, the option will still be left open for the main bridge to carry out some of these transfers in case this particular assignment of activities should prove advantageous for shuffles of a more general nature.

2. Fairly simply structured precedence relations exist among the jobs. For example, an assembly cannot be inserted into a given core location until the component presently occupying that slot has been removed.

3. There exist certain machine set-up times, which can be seen to be sequence dependent. Specifically, these set-up times are embodied primarily in the transport times as the bridges move from one area of the reactor room to another after the completion of one job and in preparation for the next job. Here it should also be mentioned that due to the relative size of the bridges, all moves from one sector of the reactor core to another will take approximately the same amount of time. The labeling of the set-up times as sequence dependent above stems from the fact that if the main bridge is scheduled, for example, to complete an in-core transfer in the next step, then the travel time will be greater if this bridge were presently at the upenders than if it were positioned over the core.

4. Rather unique restrictions are in effect for the simultaneous operation of the two machines. The bridges may not cross paths or be situated over the core at the same time. This latter restriction is due to the rather large differences in size between the core positions and the bridge structures.

One final comment is in order before proceeding with a discussion of the specific work on scheduling techniques present in the literature. In the base system representation, the main bridge is seen to possess two functional masts and would thus appear to have the capability for transferring a single control component plus either a fuel assembly or fuel and control component pair at essentially the same time. In most of the traditional scheduling methods, the assumption is made that each machine may process just one job at a time. Solutions obtained through the use of such methods would be directly applicable to systems characterized under equipment alternative 5 of the previous section, in which the main bridge possesses one multiple-function mast. For the present case, improvements in the schedule generated under this assumption may possibly be made by considering the performance of two separate transfers in the same step.

D. A Survey of the Scheduling Literature

From the results of a fairly comprehensive literature search, it would appear that no research has been conducted to date toward the solution of scheduling problems with characteristics identical to those listed for the fuel-shuffling task. The related literature can be categorized on the basis of the specific types of system constraints considered as outlined in the next paragraphs.

Sequence-Dependent Set-Up Times. Here the single-machine case may be tackled as a traveling-salesman problem to minimize total tour length [6], or a broader interpretation can be given to changeover costs [13].

Parallel Processors. Representative articles are cited among the references [10,16].

Parallel Processors and Sequence-Dependent Set-Up Times. The optimization objectives in this case include the minimization of set-up costs while also balancing the workload on the machines involved [7], and the minimization of the sum of set-up, operating, and waiting costs [1].

Precedence Relations. The solution techniques employed here are primarily based on foundations of graph theory [2,3].

General. This research deals with the development of general classes of solution procedures which could be applied to the area of scheduling and perhaps to the fuel shuffle problem itself [17,18].

None of the representative articles cited above can deal effectively with all the special properties of the fuel shuffle problem. Optimal schedules could perhaps be generated through the construction of mixed integer programming models, as in the case of Conway, Maxwell, and Miller [6]. Decision variables, a number of which would be of a zero-one nature, would be employed to determine the optimal sequencing of the jobs of the fuel shuffle and perhaps also the machines to which the tasks would be assigned. Numerous constraints would be required to depict the precedence relations among the jobs and the realistic operation of the bridges.

The number of variables and constraints stemming from such an integer programming formulation places severe limitations on the practical application of such a technique. In the case of a complete shuffle consisting of about two hundred separate operations, it is anticipated that the constraints and decision variables would both number in the tens of thousands. The presence of quarter core symmetry would, however, allow for the construction of a schedule for just one-fourth of the jobs. Here again, the size of the resulting problem would be too large to be dealt with effectively by existing pure or mixed integer solution procedures. Furthermore, certain nonlinear constraints must be included in the model. Thus, additional difficulties may be encountered in the search for a feasible solution technique.

A second mathematical programming technique which might be employed for the problem of constructing optimal shuffle schedules is embodied in the work of Eastman [9]. A procedure for solving linear programming problems with added pattern constraints is presented and applied to a scheduling task involving precedence relations among the jobs, choice of parallel machines, and sequence-dependent set-up times. The objective function to be minimized through the use of this branch-and-bound method corresponds to the sum of the processing and set-up times. However, this approach would appear to be infeasible due to (1) the lack of explicit consideration of the restrictions on the simultaneous operation of the main and auxiliary bridges, (2) the absence in the objective function of any measure of the waiting time that might be involved as one bridge is delayed in the performance of its duties by another bridge, and (3) suspected inefficiency in dealing with large problems.

In most areas of analysis, when a given problem is not amenable to solution through existing optimization techniques, attention is usually focused on the application of heuristic methods. One such heuristic scheduling procedure is that of Muhlemann and Lockett [15]. Here a sequencing task in the pharmaceutical industry is seen to have the characteristics of sequence-dependent changeover times, parallel usage of production facilities, and flexibility in the choice of such facilities for processing. At each stage of this schedule construction procedure, the job which will be delayed the least by the other jobs already in the sequence is assigned the next position in the schedule.

Another solution technique for the shuffle problem which attempts to find good, although not necessarily optimal, schedules is that of simulation. In the work of Conway, Maxwell, and Miller [6], the results of a simulation of a general job shop are presented. Another illustrative work is that of Wilbrecht and Prescott [21], in which descriptions are given of experimental investigations into the effectiveness of various dispatching rules in situations involving sequence-dependent set-up times.

Although it might be possible to make use of modified versions of either Muhlemann and Lockett's procedure or a simulation technique, better results could perhaps be obtained through the development of a heuristic method which explicitly considers the special characteristics of the fuel shuffle problem. A description of such a heuristic schedule generation procedure is presented in the next chapter.

III. GENERAL SOLUTION APPROACH

In the shuffle scheduling techniques employed by the Babcock and Wilcox Company [12] and in the approach proposed here, the problem is broken down into units of operation termed chains. A chain begins with the removal of a spent fuel assembly followed by the in-core transfer of a partially spent element to the now-empty position of the initial assembly. Additional in-core moves follow and the chain is terminated with the insertion of a fresh fuel component into the final empty slot that has been created. From three to six fuel elements are usually involved in an individual chain, and all the necessary precedence constraints are specified by the chain's composition.

Consideration must also be extended to the question of the shuffling of the control rods in each of the chains. The control components are usually either transferred to the same location as their corresponding fuel elements or shuffled within their designated chain. The movement of a control rod outside of its original chain occurs fairly infrequently.

The structuring of the shuffle problem data into such logical units allows for the decomposition of the scheduling analysis itself into workable units. In the early manual procedures employed at Babcock and Wilcox, different combinations of chains were examined by hand to determine a feasible schedule with minimal flow time. Time-consuming verification procedures had to be included to ensure that each combination of chains allowed for the proper completion of all operations required to obtain the final core configuration.

In the present computerized version of this process, the chains for a given shuffle are analyzed and classified according to certain general type designations. Predetermined scheduling rules are then applied to each group of chains to produce the best possible sequence for that type. The approach taken here is that of conducting further analyses into the effective scheduling of groups of identical chains. Consideration is also given to the possibility of achieving time reductions through the combination of chains of different types.

Three pairs of initial and final core maps provided by the Babcock and Wilcox Company are used in the investigations into the nature of the chains involved in sample reactor refueling instances. In the present analysis, each set of maps is examined in detail and the characteristics of the chains are recorded. It is hoped that from this limited set of actual operating data, generalizations can be made concerning the nature and extent of possible improvements over past or existing scheduling techniques. Furthermore, the knowledge gained through the analysis of the present data can be applied to the task of generating good shuffle sequences for future refueling periods.

As outlined in the detailed examination of the specific chains, schedule flow time can be reduced in certain instances through the disruption of the assumed chain structure and the subsequent integration of two or more chains. For complex data sets, the appropriate manner in which to combine the chains for the complete shuffle so as to achieve the maximum possible time savings is not readily identifiable. The following integer linear programming model is designed to offer some assistance in this chain integration task. Let

N = the total number of different types of chains.

n_i = the number of chains of type i present, $i=1,2,\ldots,N$.

x_{ij} = the number of chains of type i which are combined with chains of type j. $i=1,2,\ldots,N$ and $j=i,i+1,\ldots,N$.

s_{ij} = the expected time savings per chain if i and j are integrated. $i=1,2,\ldots,N$ and $j=i,i+1,\ldots,N$.

Then the problem will take the form

$$\text{maximize} \sum_{i=1}^{N} \sum_{j=i}^{N} s_{ij} x_{ij}$$

subject to

$$\sum_{j=1}^{N} x_{ij} + \sum_{j=1}^{i-1} x_{ji} \le n_i \quad \text{for } i=1,2,\ldots,N$$

and $x_{ij} \ge 0$ and integer for $i=1,2,\ldots,N$ and $j=i,i+1,\ldots,N$.
The sum $\sum_{j=1}^{i-1} x_{ji}$ appearing in the constraint equation above is taken to be zero when i assumes the value of one.

Presented in the next chapter are the results of a very close examination of the types of chains which are part of the three specific refueling instances. Consideration is first given to the development of the best refueling procedure for each of the individual groups of chains, and then the question of achieving time reductions for the entire shuffle through proper combination of the groups is addressed. With respect to this latter task, a discussion will be presented in each case concerning the application of the aforementioned integer programming procedure.

IV. AN IN-DEPTH CONSIDERATION OF THE SCHEDULING AND COMBINATION OF CHAINS

A. Introduction

Throughout the following analysis the notation presented in Table 1 will be employed to represent the various components of the fuel-shuffling process. Individual chains will be independently represented in terms of their basic configurations and required fuel and control component transfers. Providing reproductions of the entire core maps would serve no practical purpose as the chains are definitely not discernible through a casual inspection of the maps.

An essential input to the analysis and comparison of shuffle schedules is a listing of the performance times for each of the various operations undertaken by the main, auxiliary, and spent fuel pool bridges. These numbers are considered fixed and are assumed to be expected values of the completion times. Under Babcock and Wilcox's current status of operation, the completion times are:

FA or FA/CR pick up by the MB or AB	5 minutes
FA or FA/CR insert	5 minutes
CR pick up	30 minutes
CR insert	30 minutes
In-core bridge move	2 minutes
Core-to-upender bridge move	4 minutes
Positioning of the bridge/masts	1.5 minutes
FA travel time, core to spent fuel pool	10 minutes
FA or FA/CR pick up or insert by the SFB	5 minutes
SFB storage rack-to-storage rack move	2 minutes.

TABLE 1

EXPLANATION OF NOTATION FOR THE FUEL-SHUFFLING PROCESS

1. Major Equipment Involved in a Shuffle

MB = main bridge
AB = auxiliary bridge
SFB = spent fuel pool bridge
REU = reactor building east upender
RWU = reactor building west upender
FEU = east upender in the spent fuel storage pool
FWU = west upender in the spent fuel storage pool

2. Fuel and Control Components

FA = fuel assembly
CR = any of the various control rods and assemblies
FAi = a specific component designated fuel assembly i
CRi = a specific component designated control rod i
FAi(j) = fuel assembly i in instance j of a chain
CRi(j) = control rod i in instance j of a chain

The control rod moves do appear to be the dominating factor in the shuffling process, since the time corresponding to a CR insert or removal clearly overshadows most of the other operations -- especially the bridge travel times. Further, it should be noted that the MB will be the key focus of the analysis in terms of equipment scheduling, since it is the MB alone which must perform all removals and insertions of totally spent and fresh fuel and also all CR transfers. The AB may only operate when the MB is away from the core, and the SFB accepts spent fuel and brings out fresh assemblies according to the schedule of operations of the MB. Hence, in a "perfect" shuffle sequence the MB would be continuously busy, performing its tasks in a minimal number of moves and never being delayed by the AB or SFB. The nature of the chains making up a given refueling process will dictate whether or not such an optimal schedule can be achieved.

B. The First Data Set

The initial pair of old and new core maps to be considered in detail provides a good starting point since the chains involved here are particularly simple in nature. In addition, no control assemblies are removed from or brought into the core, and several FA/CR pairs need not be transferred at all.

In the following sections, descriptions of the configurations of the four primary classes of chains for this data set will be presented, and effective scheduling techniques will be outlined for each individual type.

1. Chains of Type A1

The specific form of the eighteen chains of this type present in the first data set is represented in Figure 2. Clearly FA2 and CR2 may be moved together from b to a by the AB once the MB creates an empty slot at a. In order to remove the spent assembly FA1, CR1 must be transferred to another location until the MB returns with the fresh assembly FA3. This control component may be stored temporarily in the core with a fuel element which is not paired with a CR.

A schedule which allows for the completion of an A1 chain in this manner is given in Cycle A-I of Table 2. All such activity descriptions will list only the operations of the MB. The activities of the AB will usually be scheduled to take place when the MB is at the upenders. In addition, the choice of the east and west upenders here, and in all subsequent sequences of operations, is arbitrary.

The MB must wait just two minutes for the AB in the third step of Cycle A-I. The temporary storage and return of CR1 constitute about 78% of the total time to perform this chain. This fact would seem to indicate that proper handling of such independent CR moves would be one source for reductions in the overall chain performance time.

Referring to Table 2, Cycle A-II shows an improved schedule which does not require the in-core storage of control components. Here the MB is absent from the core a total of 48.5 minutes and hence is not delayed at all by the AB. The effect of the time-consuming CR transfers is greatly reduced by the initial removal of CR1 along with

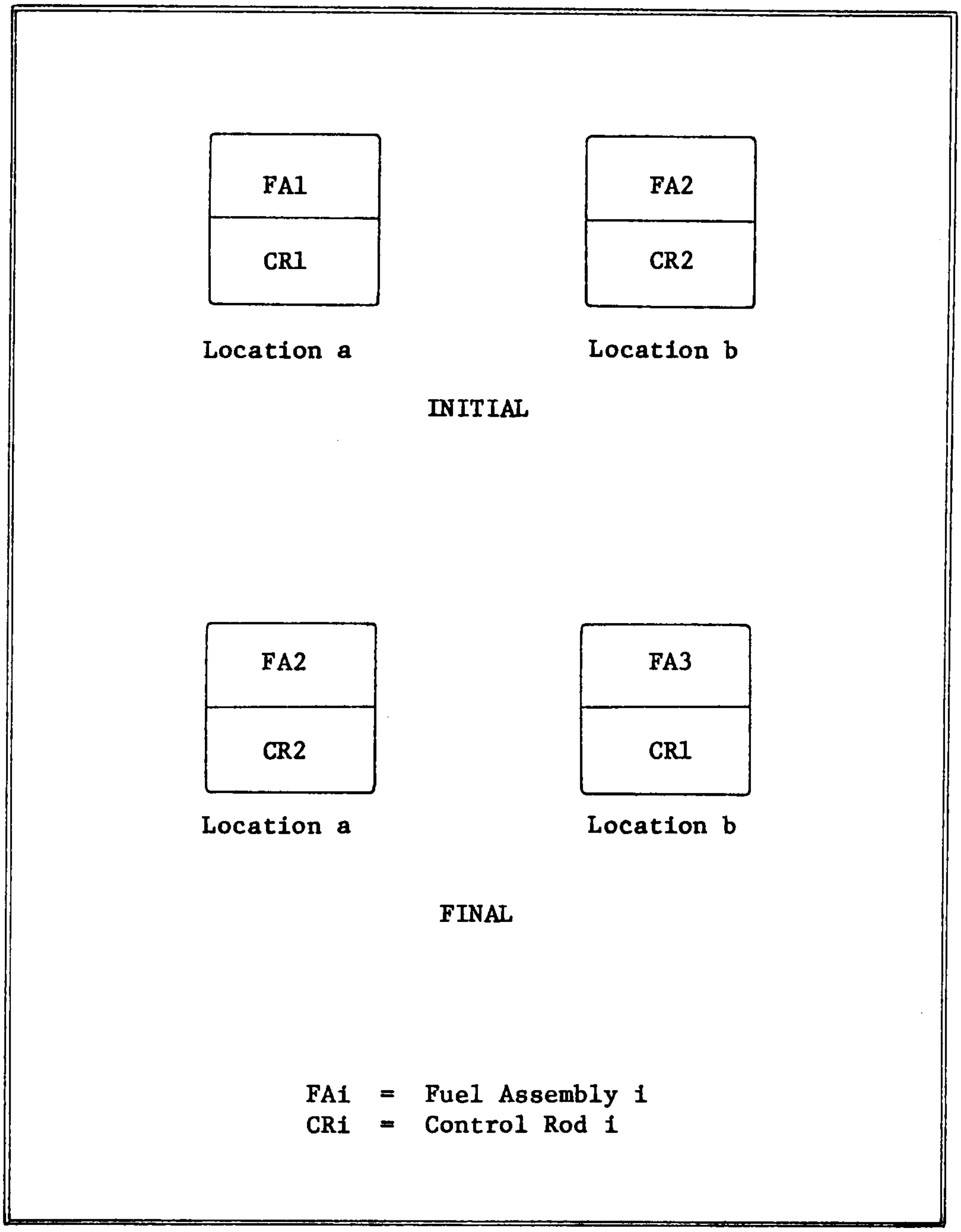

FIGURE 2
REPRESENTATIVE CONFIGURATION OF AN A1 CHAIN

TABLE 2

SCHEDULING PROCEDURES FOR A1 CHAINS

Cycle A-I

- transfer CR1 to a temporary location
- remove FA1 and insert into REU
- remove FA3 from RWU and insert into b
- return CR1 to b

Total Completion Time: 172 minutes

Cycle A-II

- remove FA1/CR1 and insert into REU
- remove CR1 from REU
- remove FA3 from RWU
- travel back to core and insert FA3 and then CR1 into b

Total Completion Time: 99 minutes

Cycle A-III

- remove FA1/CR1 and insert into REU
- remove CR1 from REU and insert on top of FA3 in RWU
- remove FA3/CR1 from RWU and take back to core

Total Completion Time: 99 minutes

FA1. Cycle A-III in Table 2 represents another scheduling procedure for type A1 chains, which also has a flow time of 99 minutes. Here the MB is away from the core for a total of 80 minutes. This equivalent procedure is mentioned because it provides the AB with a rather significant amount of time to work while the MB is busy at the upenders.

2. Chains of Type A2

Thirty-two chains of this class are present in the first data set and Figure 3 provides a representation of the basic structure of each chain. In this case the temporary storage of CR1 can be avoided altogether since either CR1 can be moved on top of FA2 at the very outset or FA1 and CR1 may be taken out as a pair to the upenders. The activities of the MB for the first of these alternatives may be outlined as follows:

Cycle A-IV

- transfer CR1 from a to b
- remove FA1 and take to REU
- remove FA3 from RWU and return to core.

The total amount of time taken up here is 105 minutes, and the MB is away from the core for 17 minutes.

The second schedule option is similar to the previously described Cycle A-II except for the fact that FA3 and CR1 are returned to different core locations using both masts of the MB in the final move. Hence, the total completion time for the cycle here would be 101 minutes,

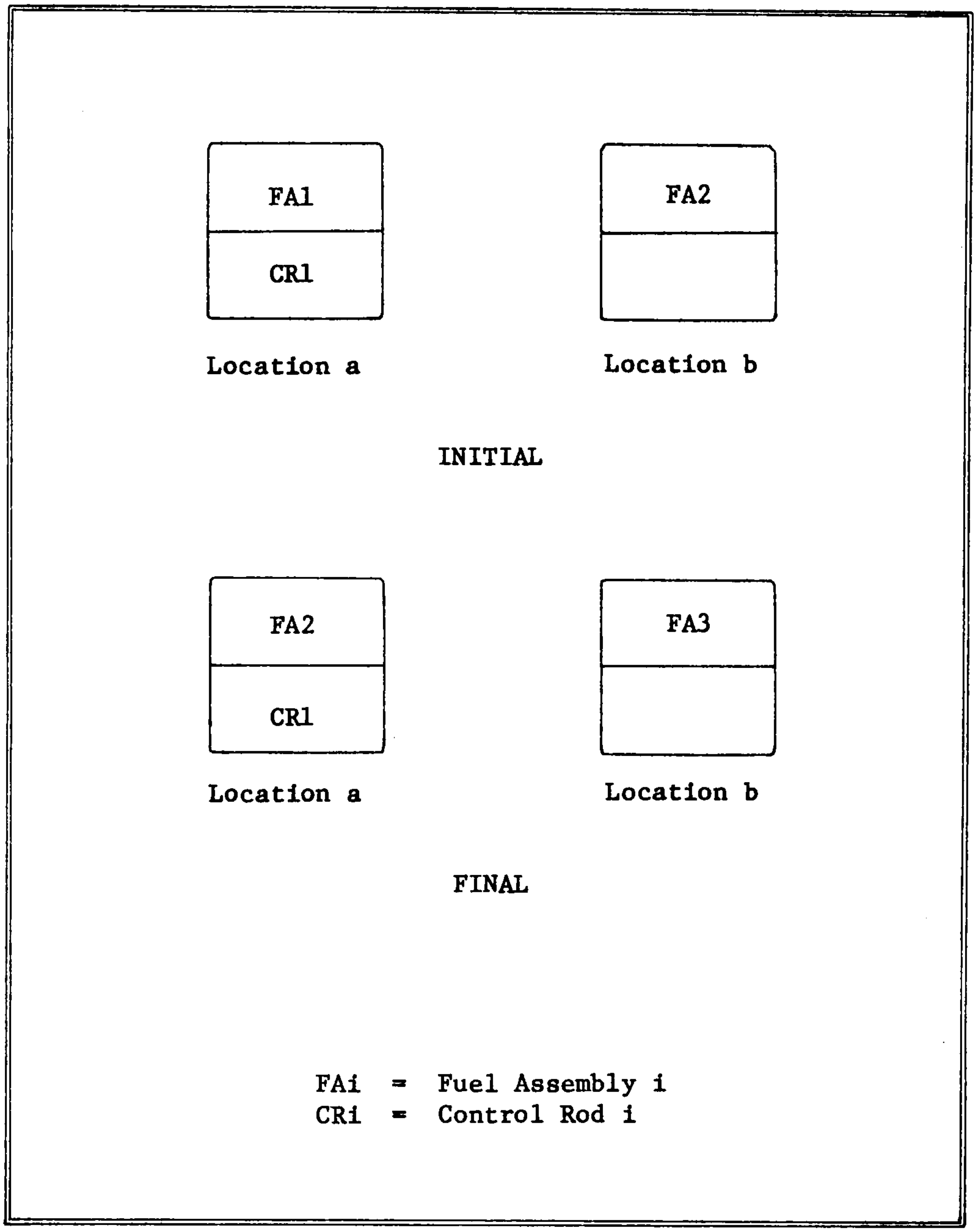

FIGURE 3
REPRESENTATIVE CONFIGURATION OF AN A2 CHAIN

including a span of 48.5 minutes for the AB to be employed during the absence of the MB. The difference between these two procedures for type A2 chains is slight, and a savings of approximately two hours can be realized through the use of the latter alternative for each of the thirty-two chains. Also, no delay is caused to the MB by either the AB or SFB in the latter schedule.

3. Chains of Type A3

Four A3 chains are contained in the first set of shuffle data. An analysis of the current locations and final destinations of the control components, as depicted in Figure 4, reveals the fact that all CR moves may be completed by the MB prior to the FA removal. It is further noted that between the time the MB removes FA1 and comes back with FA7, the AB must perform five FA/CR transfers taking up a total of 87 minutes. This chain can be scheduled as a sequence of four initial CR transfers followed by an FA removal and insertion. The total completion time will be 374 minutes, which includes 70 minutes of idle time on the part of the MB.

To reduce this rather large idle period, a schedule similar to Cycle A-II could be employed in which FA1 and CR1 are removed initially as a pair from the core and CR1 is brought back along with a fresh fuel assembly. Completion time in this case will be 340.5 minutes, including 38.5 minutes of idle waiting time for the MB. However, the question may now be posed as to whether this remaining idle time can be reduced further by means of scheduling some of the work of the

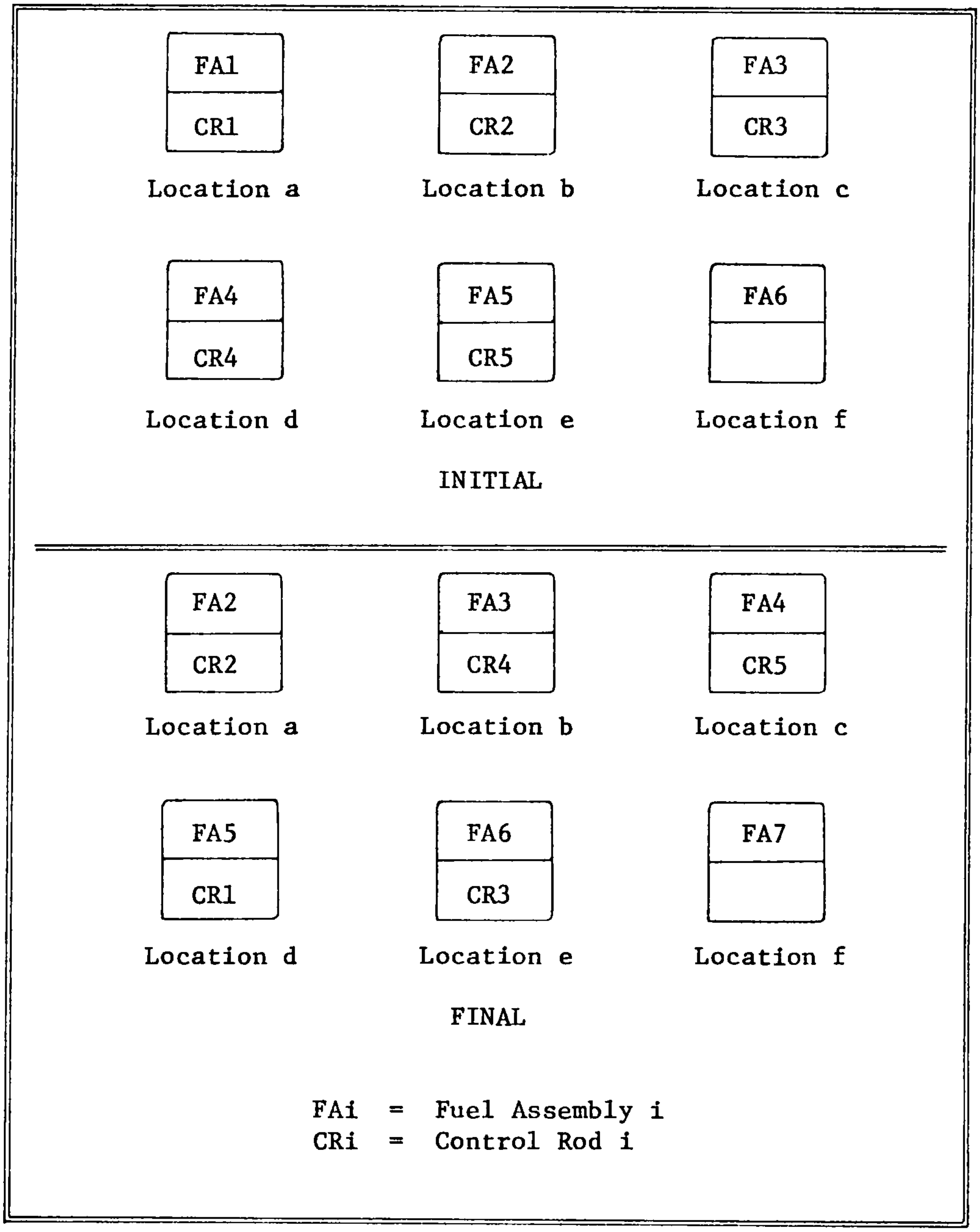

FIGURE 4
REPRESENTATIVE CONFIGURATION OF AN A3 CHAIN

AB to be done during the course of another separate chain.

From the list of operations labeled Cycle A-III for use in performing type A1 chains, it is seen that the MB is at the upenders for a span of 80 minutes. During this period the AB could execute a total of four complete FA and FA/CR moves without delaying the MB. Referring again to Figure 4, a type A3 chain may be scheduled in combination with two type A1 chains as in Cycle A-V of Table 3. If the schedule outlined in the second paragraph of this section were used, 538.5 minutes would be required to complete two type A1 and one type A3 chains. This corresponds to a 5.7% savings per A1/A3 combination.

In order to compensate for the time savings lost through the extra trips to and from the upenders necessitated in Cycle A-V, an integration procedure for an arbitrary number, n, of chains could be employed as outlined in Cycle A-VI of Table 3. In this case, a total of (6n-6) minutes could be saved over the technique described in Cycle A-V. Hence, for the four A3 chains of this data set, an additional reduction in completion time of 18 minutes would be achieved.

One of the four type A3 chains present in this specific set of reactor core maps has a different structure as shown in Figure 5. Only CR1 and CR3 may be relocated initially. In order to transfer CR4 and CR5 to their final positions, one CR out of this pair must be removed and stored temporarily within the core while the other CR is being moved to its new location. A modified version of Cycle A-V

TABLE 3

INTEGRATED SCHEDULING PROCEDURES FOR A3 CHAINS

Cycle A-V

- transfer CR1, CR3, CR4, and CR5
- remove FA1 from the core and come back empty
- perform one Cycle A-III
  here AB will also transfer FA2/CR2, FA3/CR4, FA4/CR5
- perform one Cycle A-III
  here AB will also transfer FA6/CR3
- travel to upender and bring back FA7

Total Completion Time: 508 minutes

Cycle A-VI

- perform initial CR transfers for first chain
- remove FA1(1) from the core and come back empty
- perform intermediate Cycle A-III's for first chain
- perform initial CR transfers for second chain
- remove FA1(2) from the core and return with FA7(1)
- perform intermediate Cycle A-III's for second chain
- perform initial CR transfers for third chain
- remove FA1(3) from the core and return with FA7(2)

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- travel to the upenders and bring back FA7(n)

Total Completion Time for n A3 Chains plus 2n A1 Chains:
(502n + 6) minutes

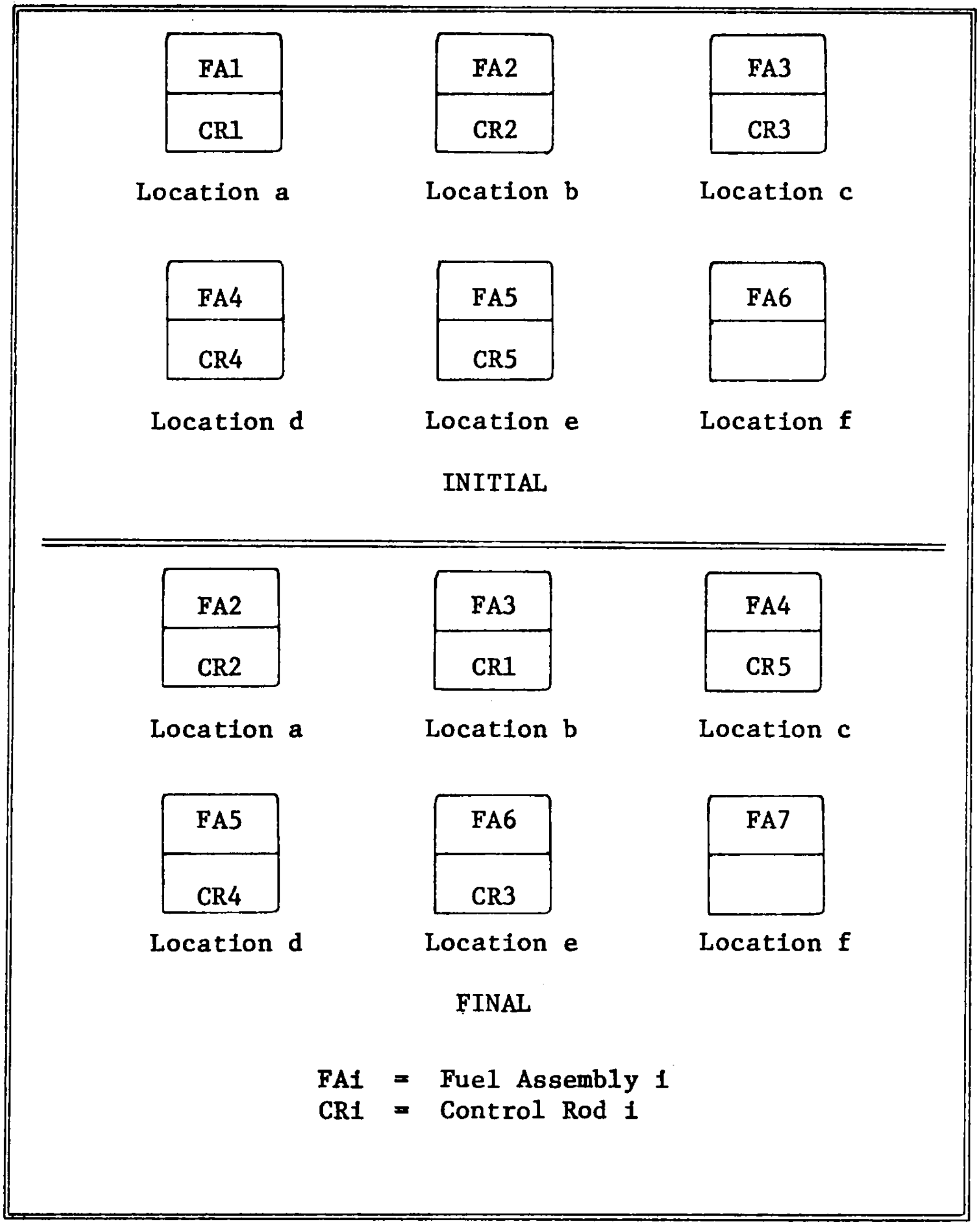


FIGURE 5
REPRESENTATIVE CONFIGURATION OF A SPECIAL A3 CHAIN

could be employed for this unique A3 chain. Here CR1 and CR3 would be relocated prior to the removal of FA1, and the chain would end with the transfer of CR4 to a temporary location within the core, the movement of CR5 from position d to c, and the return of CR4 to d.

4. Chains of Type A4

The first data set contains a total of six such chains. From Figure 6, it is seen that CR1 and CR2 remain at the same core positions during the shuffle but are mounted on different fuel elements at the conclusion of the transition. A feasible technique for performing this particular chain would involve the initial transfer of CR2 to a temporary location, the performance of a sequence of operations similar to Cycle A-II, and the return of CR2 to position b. The total amount of time required here is 235 minutes. The in-core storage operations constitute over one-half of the time involved.

An alternate scheduling procedure in which CR2 is stored temporarily at the upenders rather than within the core may be described as follows:

Cycle A-VII

- remove FA1/CR1 plus CR2 and insert CR2 on top of FA3 in REU and FA1/CR1 into RWU
- remove CR1 from RWU
- remove FA3/CR2 from REU
- return to core and place CR1 at a and FA3/CR2 at b.

A flow time of only 166 minutes is required, and no MB idle time due to delay by the AB or SFB is involved. Thus, a savings of 69

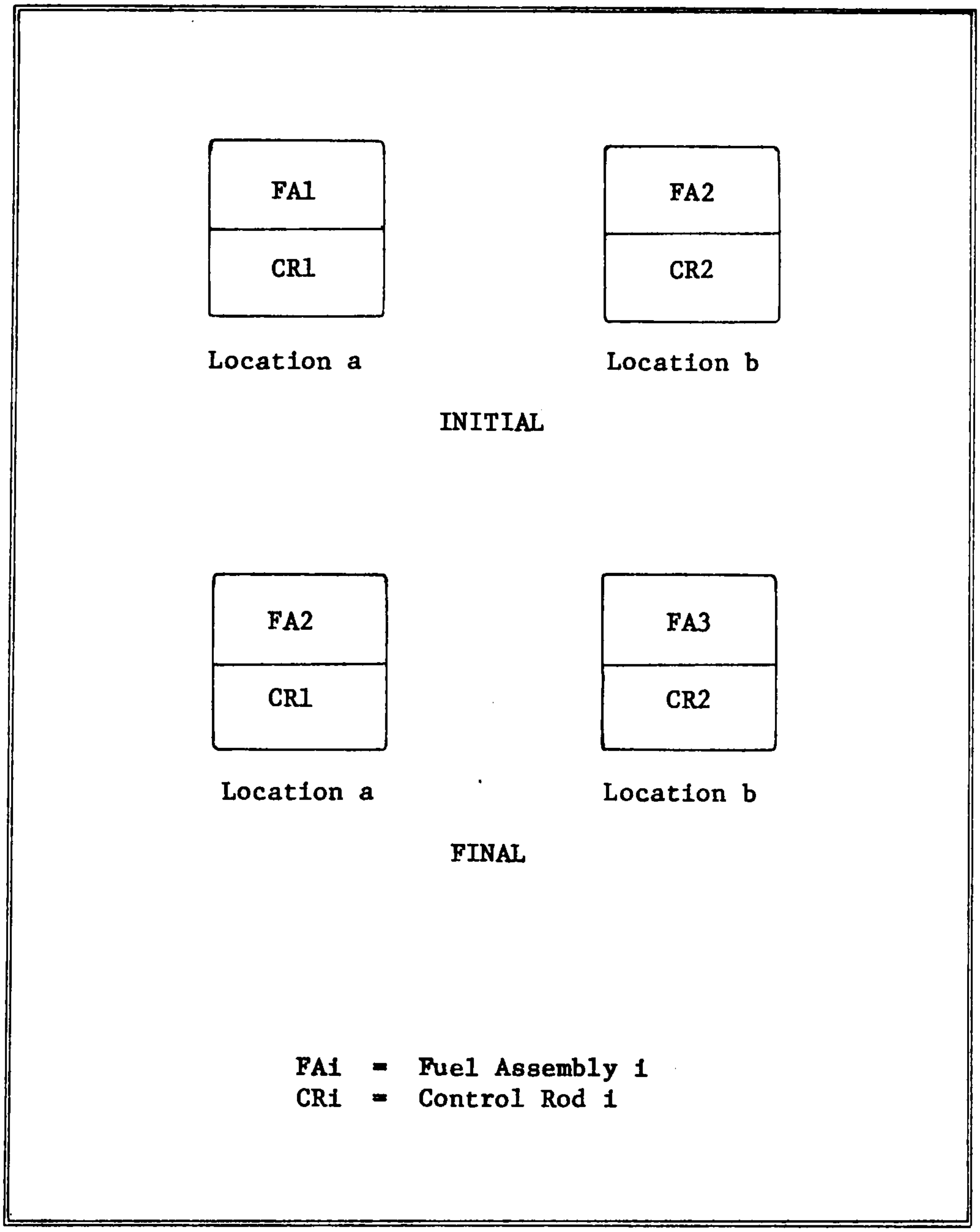


FIGURE 6
REPRESENTATIVE CONFIGURATION OF AN A4 CHAIN

minutes per chain can be realized over the aforementioned Cycle A-II technique.

5. Comments on Effective Scheduling Rules and Chain Integration

From the analysis of the chains present in this initial data set, it is seen that temporary in-core storage of control components does not constitute an effective scheduling alternative. When a CR has to be relocated in order to remove a spent fuel assembly, as in the case of type A1 chains, the option of temporarily transporting the given CR to the upenders provides fairly substantial reductions in total completion time. The temporary relocation of CR's within the core may however be unavoidable, as in the case of the A3 chain represented by Figure 5.

Flow time reductions can also be achieved through the scheduling of AB moves required for one type of chain to be done during the completion of chains of a different type. Activities corresponding to chains of the same class may also be combined in an attempt to reduce the number of superfluous trips to and from the upenders.

It is noted that of the four basic classes of chains presented here, three sets may be processed in what would seem the optimal manner independently. Only in the case of type A3 chains can any time reductions be brought about through combination with any of the other groups of chains. Each of the four A3 chains can be combined with two A1 chains, and in the current data set, there exist sufficient chains of the latter type to allow for the integration of all the A3 chains. Therefore, an explicit use of the integer programming model previously described is not required for the chains occurring in this first set of sample shuffle data.

C. The Second Data Set

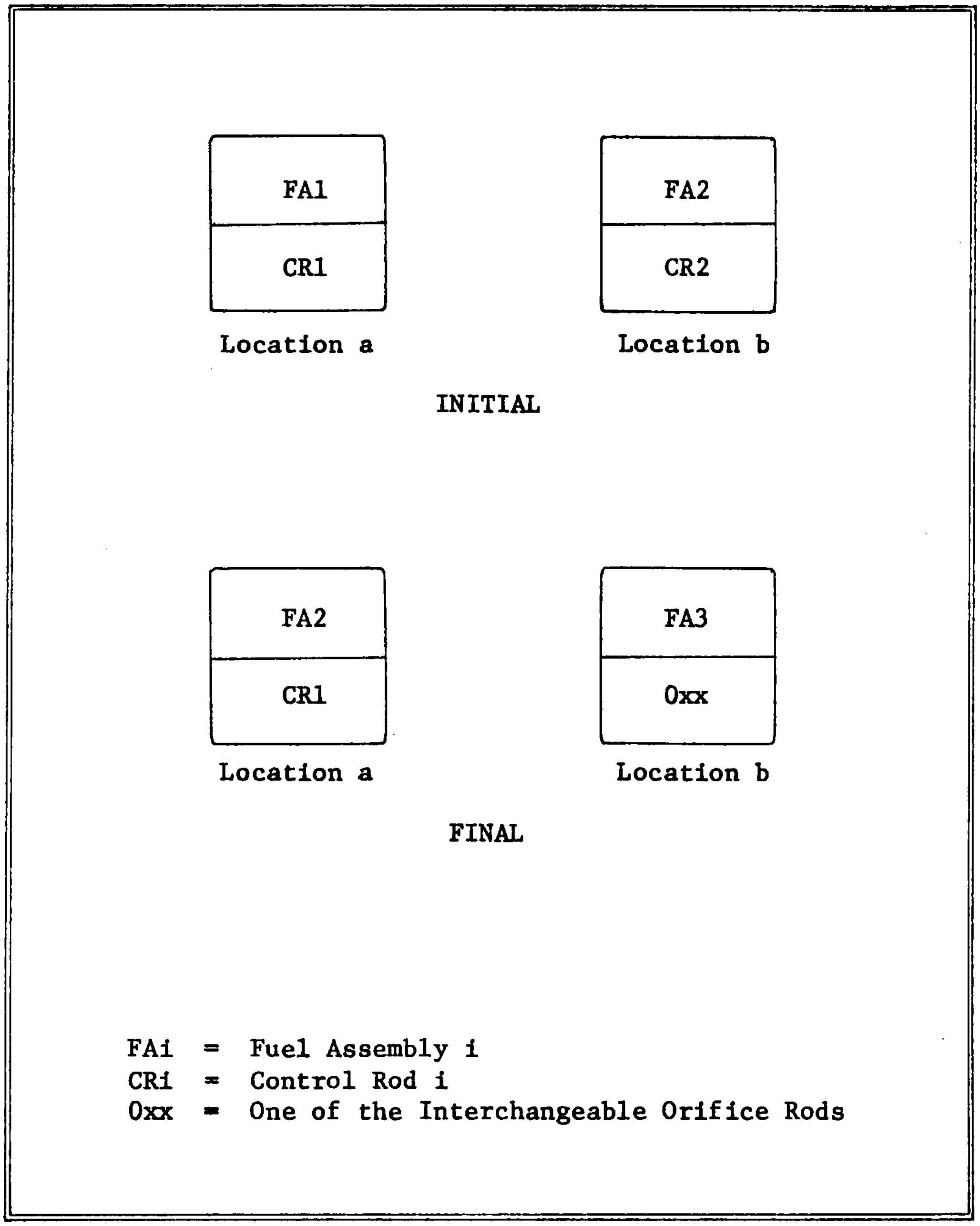
The various chains present in this set of original and final core maps are quite a bit more complex in nature than those of the previous set. This added difficulty stems primarily from the following three observations:

1. Control rods as well as fuel assemblies are removed from and brought into the reactor core.
2. In general, the number of control components leaving the core is greater than both the number of control assemblies coming in and the number of fuel elements being discharged.
3. Control assemblies are transferred within pairs of chains.

The new control elements which are transferred into the core are in the form of interchangeable orifice rods. In the chain descriptions that follow, a new CR will be given the general designation Oxx. Further, the extra CR removals and insertions may in certain cases be accomplished through the use of dummy fuel assemblies or spent fuel components from prior refueling cycles. Such an element will be designated as a DFA.

1. Chains of Type B1

Figure 7 provides a representation of the basic configuration of



FAi = Fuel Assembly i
CRi = Control Rod i
Oxx = One of the Interchangeable Orifice Rods

FIGURE 7
REPRESENTATIVE CONFIGURATION OF A B1 CHAIN

each of the four B1 chains in this data set. Here one FA plus one CR are removed and one fresh FA and one new CR are brought in. A procedure for completing chains of this sort, which corresponds to the basic technique employed in the early manual schedules of Babcock and Wilcox, is described in Cycle B-I of Table 4. For the first B1 chain executed in this manner, a DFA must be positioned at the RWU to accept CR2. For the following B1 chain, FA1 will serve as the extra component at the REU. In Cycle B-I the MB is delayed by the SFB for 11 minutes out of a total completion time of 177 minutes. Hence, idle time accounts for approximately 6.2% of the processing period.

In an attempt to reduce the time the MB spends waiting at the upenders, a procedure could be employed in which CR1 is stored temporarily within core instead of being transported out along with FA1. The remaining FA and CR removals and insertions could then be completed by Cycle B-II as shown in Table 4. Here, the MB is delayed only two minutes by the AB, and no waiting time for the SFB is involved. Adding in the time for the in-core storage of CR1, the flow time for the entire B1 chain in this case will be 237 minutes.

These results would seem to point to the superiority of the scheduling procedure represented as Cycle B-I. The in-core relocations of CR1 necessitated through the use of Cycle B-II take up too much time for this procedure to be effective. However, if several chains were integrated as described in Cycle B-III of Table 4, these CR storages could be eliminated.

TABLE 4

SCHEDULING PROCEDURES FOR B1 CHAINS

Cycle B-I

- remove FA1/CR1 and CR2 and deposit at REU and RWU respectively
- remove CR1 from REU
- remove FA3/Oxx from RWU
- return to core and insert CR1 and FA3/Oxx

Total Completion Time: 177 minutes

Cycle B-II

- transfer CR2 from b to a
- take FA1/CR2 to REU
- remove FA3/Oxx from RWU
- return to core and insert FA3/Oxx

Total Completion Time: 103 minutes

Cycle B-III

An Integrated Schedule for the Four B1 Chains

- take CR2(1) to REU
- transfer CR1(1) from a to b
- perform 1 Cycle B-II removing FA1(1)/CR2(2) and returning with FA3(1)/Oxx(1)
- transfer CR1(2) from a to b
- perform 1 Cycle B-II removing FA1(2)/CR2(3) and returning with FA3(2)/Oxx(2)
- transfer CR1(3) from a to b
- perform 1 Cycle B-II removing FA1(3)/CR2(4) and returning with FA3(3)/Oxx(3)
- transfer CR1(4) from a to b
- perform 1 Cycle B-II removing FA1(4)/extra CR and returning with FA3(4)/Oxx(4)

Total Completion Time: 12 hours and 31 minutes

This schedule allows for the removal of an extra single CR from the core and involves only 8 minutes of MB idle time. For a series of n B1 chains plus one CR removal performed in this manner, (170n + 71) minutes would be required. If the extra CR removal were not incorporated into the integrated schedule, the flow time for the same set of operations above would increase to (170n + 77) minutes. The use of the CR removal in the former case primarily compensates for the extra trip to and from the upenders necessitated by the initial removal of CR2(1) alone. Both procedures above offer improvement for all values of $n \geq 1$ over the use of Cycle B-I since in the latter case, (177n + 71) minutes are taken up in processing n B1 chains plus an extra CR removal.

A few comments should be made before proceeding to a discussion of the other types of chains. First of all, it should be noted that for the given chains there exist sets of fixed MB and AB moves that must be performed regardless of the scheduling technique used. Time savings may only be accomplished through the elimination of all superfluous equipment operations and of any unnecessary idle periods, particularly for the MB. These time reductions may consequently be small in magnitude in comparison with the total chain completion time. In addition, when attempts are made at constructing new scheduling procedures to circumvent some of the idle periods, the alternative of temporary in-core storage should be avoided. As first discovered in the case of the initial data set, extra CR relocations are very time consuming and would usually offset any reductions obtained through idle time elimination.

2. Chains of Type B2

There are twenty chains of this type in the second data set. The configuration of the fuel and control components in positions a and b of Figure 8 is identical to that of type A2 chains. Also, it is noted that for these two locations alone, no control components must be removed or brought in. Therefore, a modified version of the Cycle A-II procedure, which has a completion time of 101 minutes, could be applied.

In the entire shuffling scheme currently under consideration, there exist certain separate CR transfers which are not paired with FA removals and insertions as in type B1 chains. For example, the second data set contains twenty subchains involving a single core location. Position aa of Figure 8 provides a representation of the structure of each of these units.

If the transformation operations for locations a, b, and aa in Figure 8 were completed separately, 237 minutes would be required. On the other hand, these two sets of moves could be combined using the basic scheduling techniques found in either Cycle B-I or B-II. If Cycle B-I were employed, an extra transfer of Oxx to its final destination at aa would be required, and the total flow time would be increased to 242 minutes. For Cycle B-II this additional Oxx transfer plus the initial movement of CR1 from a to b would be included. The total time involved here would amount to 235 minutes.

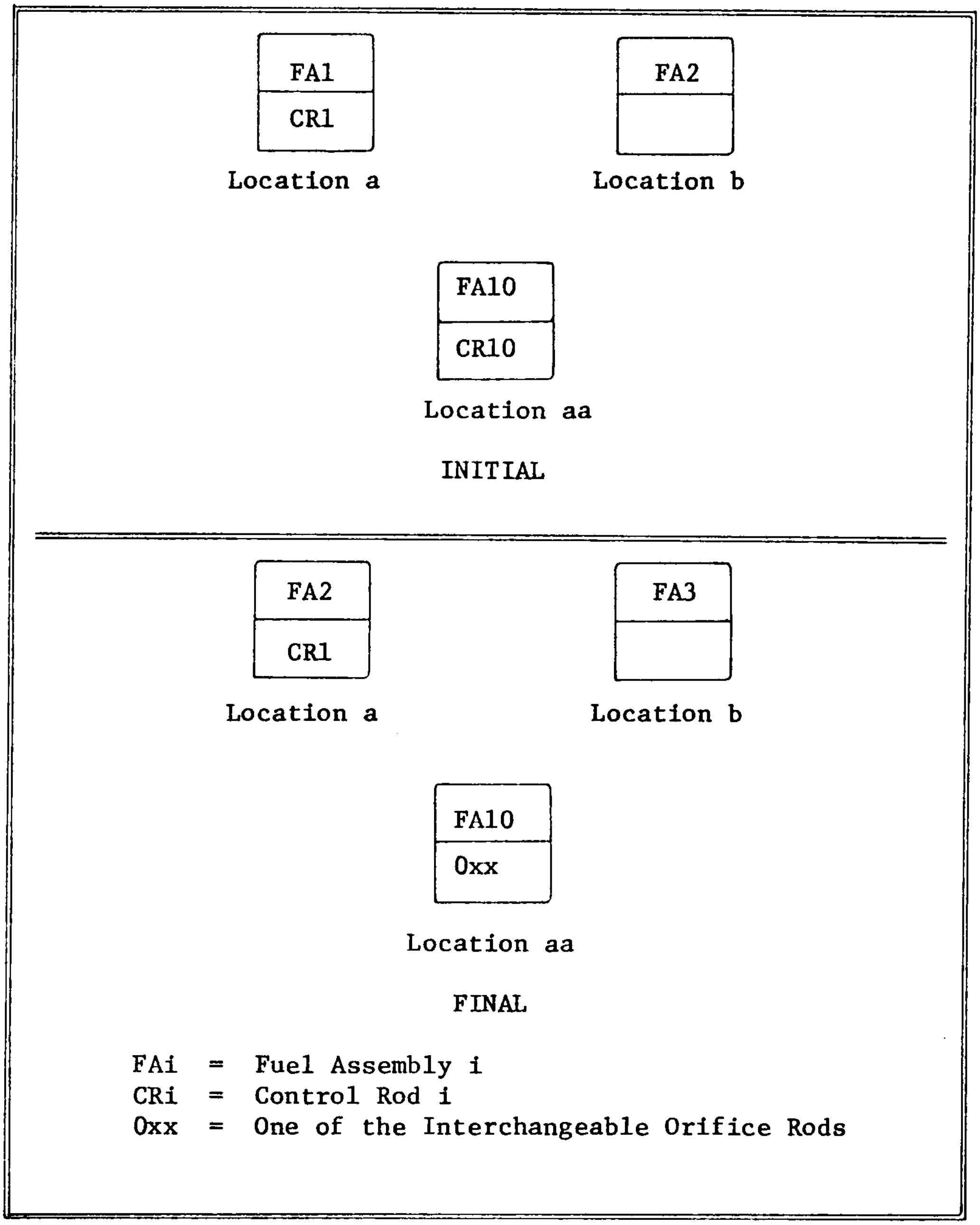


FIGURE 8
REPRESENTATIVE CONFIGURATION OF A B2 CHAIN

All three procedures outlined above for processing a complete B2 chain require approximately the same amount of time. The removal of the extra control assembly CR1 in Cycle B-I is clearly not necessary here since CR1 may just be transferred to its new position at the beginning of the schedule. Further, the technique using Cycle B-II offers only a slight improvement over the separate completion of the two sets of operations. However, the storage arrangements in the spent fuel pool are such that each slot contains an FA along with a 0xx component. This fact reaffirms the advisability of integrating all the transfers represented in Figure 8.

3. Chains of Type B3

There are eight chains of this class and their structure is depicted in Figure 9. It is noted that in this case, two control assemblies must be removed from the reactor core and two fresh components brought in. Furthermore, the AB must perform two FA transfers during the absence of the MB. These in-core moves could be completed in 36 minutes. This latter fact would seem to rule out the use of a scheduling technique resembling Cycle B-II since in that sequence of operations, the MB is away from the core only 17 minutes. For B3 chains, Cycle B-II would involve at least 19 minutes of MB idle time.

Cycle B-I would seem to be a bit more promising since the MB is scheduled to remain at the upenders for a period of 91 minutes. By summing the Cycle B-I flow time and the time corresponding to the removal of a CR and the insertion of a 0xx element, the completion of

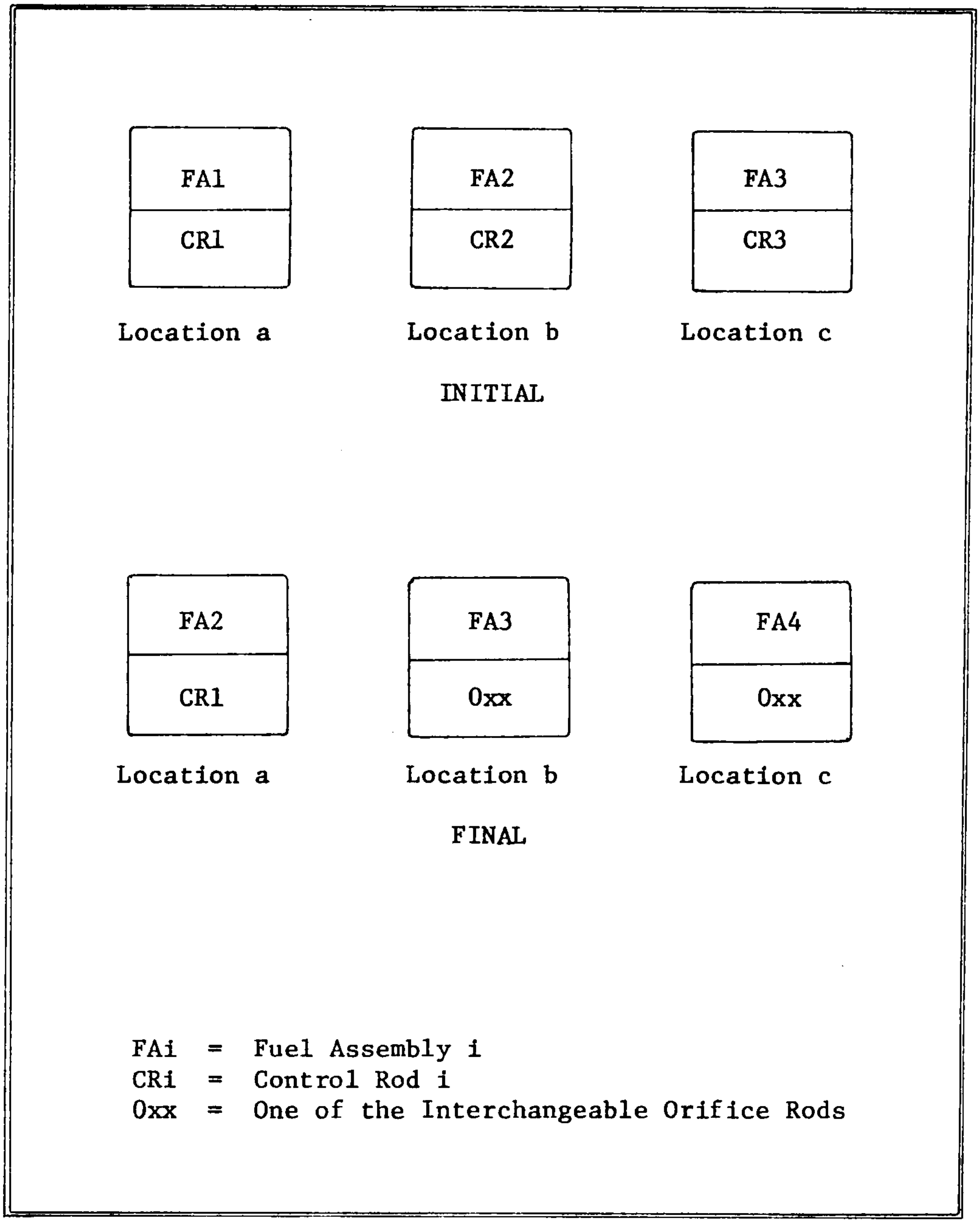


FIGURE 9
REPRESENTATIVE CONFIGURATION OF A B3 CHAIN

a B3 chain can be seen to take (177 + 136) or 313 minutes. In this case, 11 minutes of MB idle time corresponding to delay by the SFB will be present for each chain.

Through a proper combination of FA and CR removals and insertions plus an integration of several chains, the Cycle B-I waiting time can be eliminated almost entirely. Cycle B-IV represents such an integrated schedule and is outlined in Table 5.

During the execution of each separate chain, the MB leaves the core twice and is absent for a total of 42 minutes each time. During each of these periods the AB could undertake two FA or FA/CR moves. The flow time for each subchain may be computed to be 304 minutes, and the completion time for a set of n chains plus an extra CR removal would be (304n + 71) minutes. If the CR move were not used in Cycle B-IV above, this set of operations could be performed in (304n + 77) minutes. These results may be compared with the requirement of (313n + 71) minutes for the performance of n B3 chains and one CR removal through successive application of Cycle B-I. For the eight chains present in this set of core maps, a maximum savings of 72 minutes or 2.8% may thus be achieved.

To illustrate the character and scope of the operations of the SFB, a schedule of tasks which this bridge must complete in compliance with Cycle B-IV is also given in Table 5. From a detailed examination of the SFB processing times and the timing of the moves listed in the table, it is seen that in the execution of Cycle B-IV, the MB is not delayed at all by the SFB.

TABLE 5

INTEGRATED SCHEDULE FOR B3 CHAINS WITH A DESCRIPTION OF RELATED OPERATIONS IN THE STORAGE POOL

Cycle B-IV

- take CR2(1) to REU
- transfer CR1(1) from a to b
- transfer CR3(1) from c to a
- take FA1(1)/CR3(1) to REU and return with Oxx(1) for location b
- take CR2(2) to RWU and return with FA4(1)/Oxx(1)

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●
●

- transfer CR1(n) from a to b
- transfer CR3(n) from c to a
- take FA1(n)/CR3(n) to REU and return with Oxx(n) for location b
- take extra CR to RWU and return with FA4(n)/Oxx(n)

Total Completion Time for n B3 Chains: (304n + 71) minutes

SFB Schedule

- take DFA to FEU
- take DFA/Oxx(1) to FWU
- remove DFA/CR2(1) from FEU and transport to storage rack
- remove FA1(1)/CR3(1) from FEU and transport to storage rack
- take FA4(1)/Oxx(1) to FEU
- remove DFA/CR2(2) from FWU and transport to storage rack
- take DFA/Oxx(2) to FWU
- remove FA1(2)/CR3(2) from FEU and transport to storage rack

●
●
●

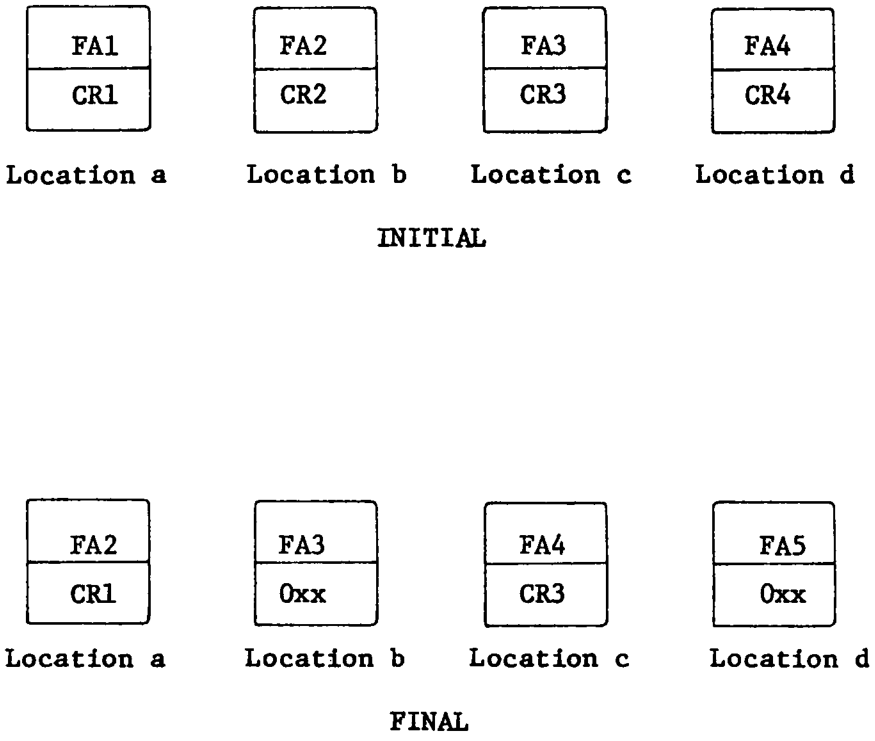
etc.

4. Chains of Type B4

One chain of this type is present in the second data set and its basic composition is represented in Figure 10. A B4 chain may be readily scheduled with a slight variation of Cycle B-IV as follows:

- take CR2 to REU
- transfer CR1 from a to b, CR4 from d to a, and CR3 from c to d
- take FA1/CR4 to REU and return with Oxx for FA3
- take an extra CR to RWU and return with FA5/Oxx.

The flow time here for n B4 chains and one extra CR withdrawal may be computed as (373n + 71) minutes. If schedules along the lines of Cycle B-I were employed and the CR removals and insertions were independently completed, (380n + 71) minutes would be involved. Now, if the extra CR removal were not used in the last step of Cycle B-IV, a savings over the Cycle B-I method of operation would still be achieved since the flow time would be reduced to (373n + 77) minutes. However, if the surplus CR removal were eliminated from Cycle B-IV, the AB would not be able to complete its work in the final step without delaying the MB. Hence, there must exist within the remainder of the shuffle schedule, periods of MB absence from the core during which the B4 chains could be completed. A discussion of the availability and effective use of such instances of AB idleness which occur while the MB is working at the upenders will be undertaken in a later section as part of a treatment of the integration of the chains from this data set.



FAi = Fuel Assembly i
CRi = Control Rod i
Oxx = One of the Interchangeable Orifice Rods

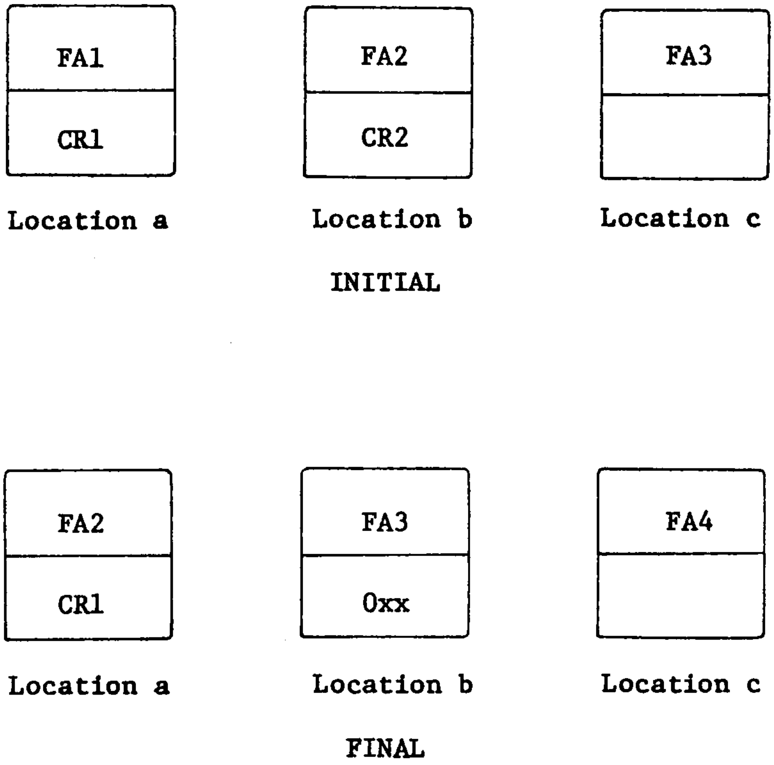
FIGURE 10
REPRESENTATIVE CONFIGURATION OF A B4 CHAIN

5. Chains of Type B5

Four B5 chains are present in the set of sample shuffle data. Referring to Figure 11, it is seen that these chains bear a strong resemblance to those previously described as type B1. In this case, however, the AB must perform two FA transfers. The Cycle B-II integration procedure could be applied here but due to the added workload on the AB, 19 minutes of MB idle time corresponding to delay by the AB would occur in each subchain. The Cycle B-I method would necessitate only 11 minutes of MB idle time in each chain and would thus seem to represent the better technique in this case.

As depicted in Figure 11, this chain involves only one FA/CR pair to be withdrawn from the core and one FA/Oxx pair to enter. If, in the overall set of moves to be completed, there were several extra control assemblies to be removed or inserted, a schedule similar to Cycle B-IV could be employed. Such a sequence is characterized in Cycle B-V of Table 6. For n B5 chains, (n + 1) extra CR removals and n extra Oxx insertions would have to be available. Completion time for this schedule may be determined as (369n + 71) minutes. This technique provides a savings of 9 minutes over an equivalent set of operations involving successive applications of Cycle B-I. This savings may be translated into a 2.3% reduction for the four chains of this data set.

If only a limited number of additional Oxx components must be brought into the core, a type B5 chain may be processed in conjunction with only CR removals through the use of Cycle B-VI listed in Table 6.



FAi = Fuel Assembly i
CRi = Control Rod i
Oxx = One of the Interchangeable Orifice Rods

FIGURE 11
REPRESENTATIVE CONFIGURATION OF A B5 CHAIN

TABLE 6

SCHEDULING PROCEDURES FOR B5 CHAINS

Cycle B-V

- take CR2(1) to REU
- transfer CR1(1) from a to b and extra CR to a
- take FA1(1)/extra CR to REU and return with extra Oxx
- take CR2(2) to RWU and return with FA4(1)/Oxx(1)
- transfer Oxx(1) from c to b

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•

•

- transfer CR1(n) from a to b and extra CR to a
- take FA1(n)/extra CR to REU and return with extra Oxx
- take extra CR to RWU and return with FA4(n)/Oxx(n)
- transfer Oxx(n) from c to b

Cycle B-VI

- take CR2(1) to REU
- transfer CR1(1) from a to b and extra CR to a
- take FA1(1)/extra CR to REU and return empty
- take CR2(2) to RWU and return with FA4(1)/Oxx(1)
- transfer Oxx(1) from c to b

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•

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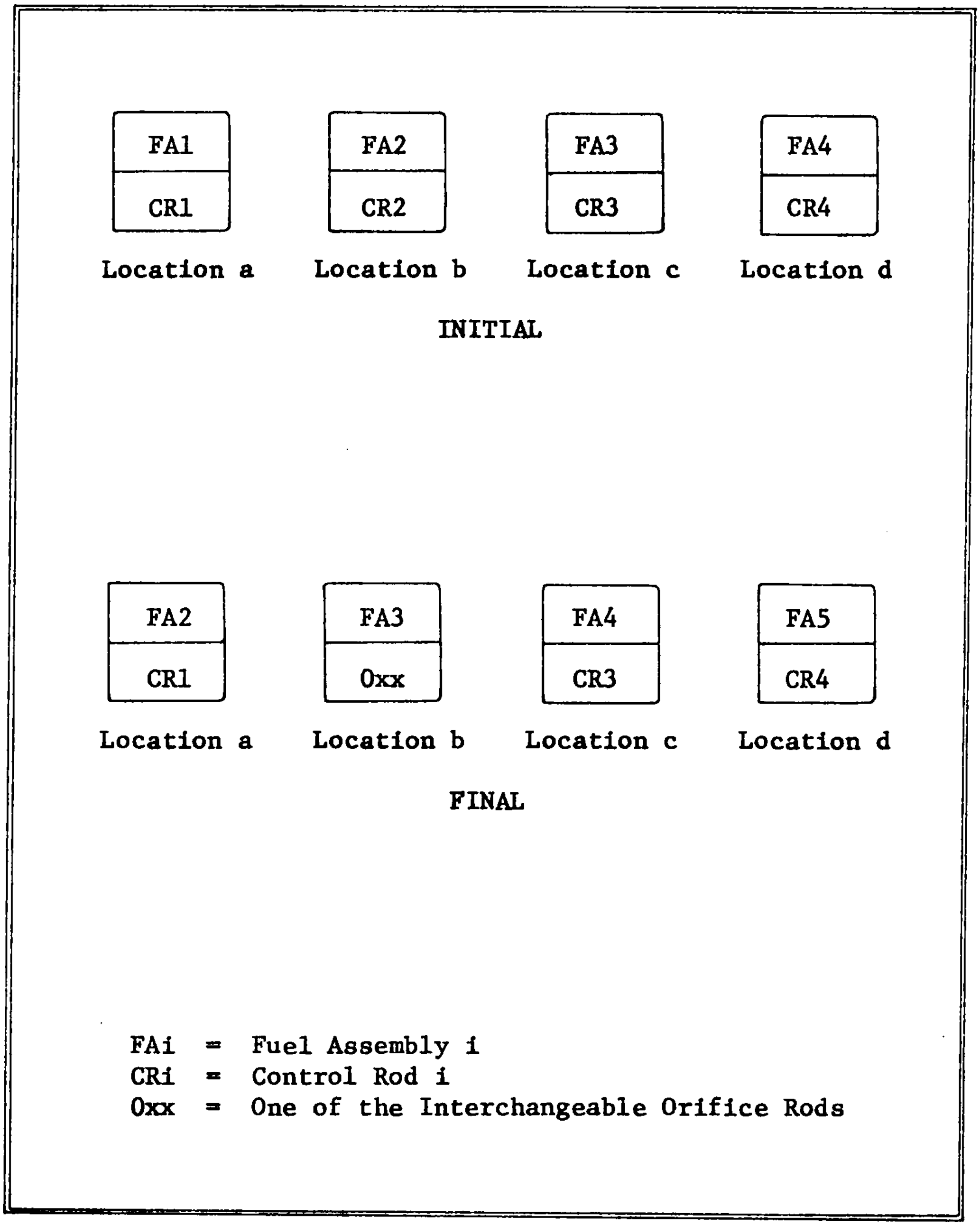
etc.

To schedule n such chains in this manner, (n + 1) extra CR removals must be available. The completion time in this case would be (304n + 71) minutes. Consequently, a savings of 36 minutes or 2.7% could be realized for the four chains of this data set.

A schedule which involves the integration of only one surplus CR removal for the completion of n chains may be constructed by modifying Cycle B-VI. After eliminating the operations involved with the extra CR withdrawal, as in the second and third steps of the list of operations, the flow time becomes (239n + 71) minutes for n chains and one CR removal. Only a 3-minute improvement per chain can be realized over a procedure involving applications of Cycle B-I. Finally, if all the extra CR removals were eliminated from Cycle B-VI, the time to complete n B5 chains alone would be (239n + 6) minutes. This latter procedure would offer an improvement over a technique employing Cycle B-I for $n \geq 3$ only. Further, this modified form of Cycle B-VI may be used only when sufficient opportunities exist for performing the AB work regularly done during the next to last step of the Cycle B-VI procedure during another separate period of MB absence from the core.

6. Chains of Type B6

The primary configuration of the three B6 chains of this data set is depicted in Figure 12. The only major difference between type B5 and B6 chains is that in the latter case, the AB must accomplish three intermediate FA transfers instead of two. Hence,



FAi = Fuel Assembly i
CRi = Control Rod i
Oxx = One of the Interchangeable Orifice Rods

FIGURE 12
REPRESENTATIVE CONFIGURATION OF A B6 CHAIN

either of the Cycle B-I or B-V procedures could be applied. However, under the Cycle B-VI method, most of the incremental time savings would be lost due to the 8.5-minute delay of the MB by the AB in the third step listed. This added idle time may be avoided if a method similar to Cycle B-VI were used in which the extra AB move would be performed during a period of MB absence from the core in the execution of another chain. The various modifications of Cycle B-VI discussed in the previous section may also be applied here. The same precautions must be taken for the completion of the extra AB move in each case.

Unfortunately, there are certain difficulties associated with the application of any of these methodologies per se. After the insertion of FA5/Oxx into position d and prior to the final transfer of Oxx, the pairs FA3/CR3 and FA4/CR4 would occupy b and c respectively. Thus, CR3, CR4, and Oxx must all switch locations, and the first move may only be accomplished after the temporary relocation of one of the three control components. This time-consuming in-core storage may be avoided, however, by using the Oxx component transported in with FA5 for another chain. A type B5 chain would appear to be the prime recipient for the Oxx since the FA which eventually receives the fresh component in the B5 chain is not paired with a CR initially. The Oxx element required in the B6 chain could be brought in as a part of the B5 chain following the relocation of CR3 and CR4 in B6.

7. Chains of Type B7

There are eight chains of this class. Referring to Figure 13, it

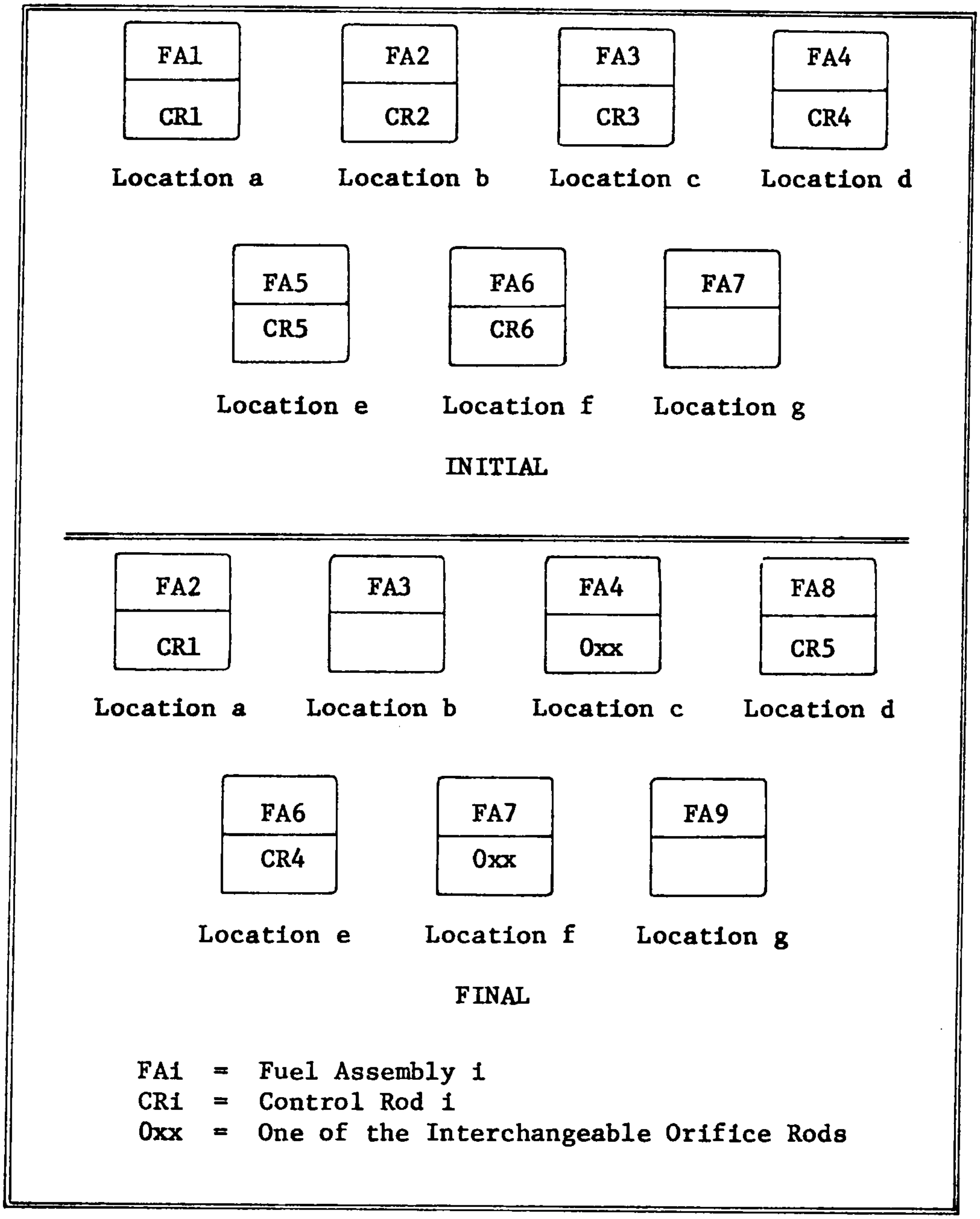


FIGURE 13
REPRESENTATIVE CONFIGURATION OF A B7 CHAIN

is seen that two subchains corresponding to the removal of FA1 and FA5 actually occur. In addition, CR4 and CR5 are transferred among the chains, and there exists one extra CR to be removed. One scheduling technique which allows this extra CR move to be processed independently of the B7 chain is described in Cycle B-VII of Table 7.

Before discussing any alternate procedures for executing B7 chains, it would be worthwhile to mention the importance of this extra CR removal in perhaps bringing about completion time reductions in other segments of the overall shuffle schedule. If the operations involved in chains of type B1, B3, B4, B5, and B6 are combined effectively with extra CR removals and Oxx insertions, reductions in flow time can be achieved. Further, the number of extra CR moves needed for combination with the different groups of chains varies somewhat among the sets. The point to be made here is that for the present data set, type B7 chains appear to offer the only instance in which additional CR removals occur. Hence, the relative savings involved in using these extra moves in combination with other groups of chains and with the B7 chains themselves must be carefully considered.

One way in which the additional CR withdrawal can be used as a part of the B7 chain is outlined in Cycle B-VIII found in Table 7. The final two AB moves are performed during the seventh step of the sequence. A savings of 11 minutes or 1.8% may be realized over the Cycle B-VII technique.

If CR3 were not removed in the seventh step of Cycle B-VIII, a feasible schedule would still result if the two AB moves that would

TABLE 7

SCHEDULING PROCEDURES FOR B7 CHAINS

Cycle B-VII

- perform 1 Cycle B-I removing FA1/CR1 plus CR2 and returning with FA8/Oxx plus CR1
- transfer Oxx from d to g
- perform 1 Cycle B-I removing FA5/CR5 plus CR6 and returning with FA9/Oxx plus CR5
- transfer CR4 from c to e
- transfer Oxx from g to c
- remove CR3 from the core

Total Completion Time: 624 minutes

Cycle B-VIII

- perform 1 Cycle B-I removing FA1/CR1 plus CR2 and returning with FA8/Oxx
- transfer Oxx from d to g
- transfer CR5 from e to d
- transfer CR6 from f to e
- take FA5/CR6 to REU and return empty
- transfer CR4 from c to e
- take CR3 to RWU and return with FA9/Oxx
- transfer Oxx from g to c

Total Completion Time: 613 minutes

normally take place then were completed during a separate period of MB absence from the core. This technique would have only a 5-minute advantage over Cycle B-VII, which amounts to less than a 1% improvement.

8. Comments on Effective Scheduling Rules and Chain Integration

In the case of the second set of sample shuffle data, improved schedules can be devised primarily through the process of combining the operations involved in different types of chains. The time required for temporary in-core storage of control components plus MB idle time can be reduced by (1) the integration of the moves involved in several chains of the same general class, (2) the scheduling of AB transfers required for one chain during certain segments of other chains, and (3) the combination of separate CR removals with different types of chains.

The analysis of the effective combination of the chains making up the second shuffle is focused mainly on the integration of the surplus CR removals present in the B7 chains and on the scheduling of AB activities for one chain during the periods of MB absence from the core as a part of another chain. Table 8 provides a summary of the requirements and expected savings for each of the seven types of chains. The benefits are expressed in terms of the savings gained through the use of the improved procedure over the most effective technique which does not involve integration. In a majority of the cases, comparisons are made with the standard Cycle B-I method.

TABLE 8

INTEGRATION REQUIREMENTS FOR THE SECOND DATA SET

	(1) Chain Type	(2) Number Present	(3) Number Spaces Needed for n Chains	(4) Benefit from Spaces for n Chains (minutes)	(5) CR Moves Needed for n Chains	(6) Benefit from CR Moves for n Chains (minutes)	(7) Spaces Available in n Chains	(8) Extra Spaces per Chain if No Integration Involved
1.	B1	4	0	0	1	6	1	0
2.	B3	8	0	0	1	6	1+2(n-1) (plus 2 if CR move allocated)	0
3.	B4	1	1	7n-6	1	7n	0	2
4.	B5	4	2	3n-6	a) 1 b) n+1	a) 3n b) 9n	0	3
5.	B6	3	n+2	3n-6	a) 1[a] b) n+1[a]	a) 3n b) 9n	0	2
6.	B7	8	2n	5n	n	11n	2n	3

---

[a] Under both alternative a) and b), n extra spaces are required.

Further, a space is defined as the span of time necessary for the AB to perform one complete in-core transfer.

Referring to the entries in columns three and seven of Table 8, it is noted that at the outset, there exist sufficient unscheduled spans of MB absence from the core to allow each of the benefits in column four to be realized. Thus the time savings given in column six may be rescaled to represent the expected improvement gained through the use of some of the extra CR removals over the best technique without integration of these moves. In other words, each of the entries in column six should be decreased by the corresponding figure in column four to give the true relative savings in the event of the allocation of a CR removal. In this manner, the expected time reduction per combination with a CR move becomes a constant 6 minutes for each type of chain. Hence, the eight surplus CR removals can be arbitrarily allocated among the groups of chains. A total relative savings of 48 minutes can be obtained in this manner.

Here again, due to the nature of the data for this second refueling instance, the use of the initial integer programming procedure for chain combination is not really necessary. The simply structured chains of both the first and second data sets can be combined effectively through an inspection of the data on the relative benefits of the various integration alternatives. However, the integer programming model for the second data set is formulated and solved as a test problem for the integer programming code employed for the third set of shuffle data. The details of the models for both the second and third data sets are presented in the next chapter.

D. The Third Data Set

As a general rule, the chains present in this final pair of core maps are similar in nature to those of the previous section and can be scheduled using the same fundamental rules. In-core CR transfers among the chains are far more prevalent in this final case. In one instance, six separate chains are linked through CR exchanges. Some of the chains from the third set of core maps are identical to those present in the second set. For each of the type designations listed below, the number of chains contained in the third data set is specified.

Type B1:	4
Type B2:	18
Type B3:	4
Type B5:	3
Type B7:	3

The chains unique to the present case are discussed in the next sections.

1. Chains of Type C1

In the third set of shuffling data, there are four chains of this basic type. Figure 14 provides a representation of the structure of the C1 chain. All comments regarding effective scheduling techniques made in the section on type B6 chains may be readily applied here. However, the difficulties associated with the movement of CR3, CR4,

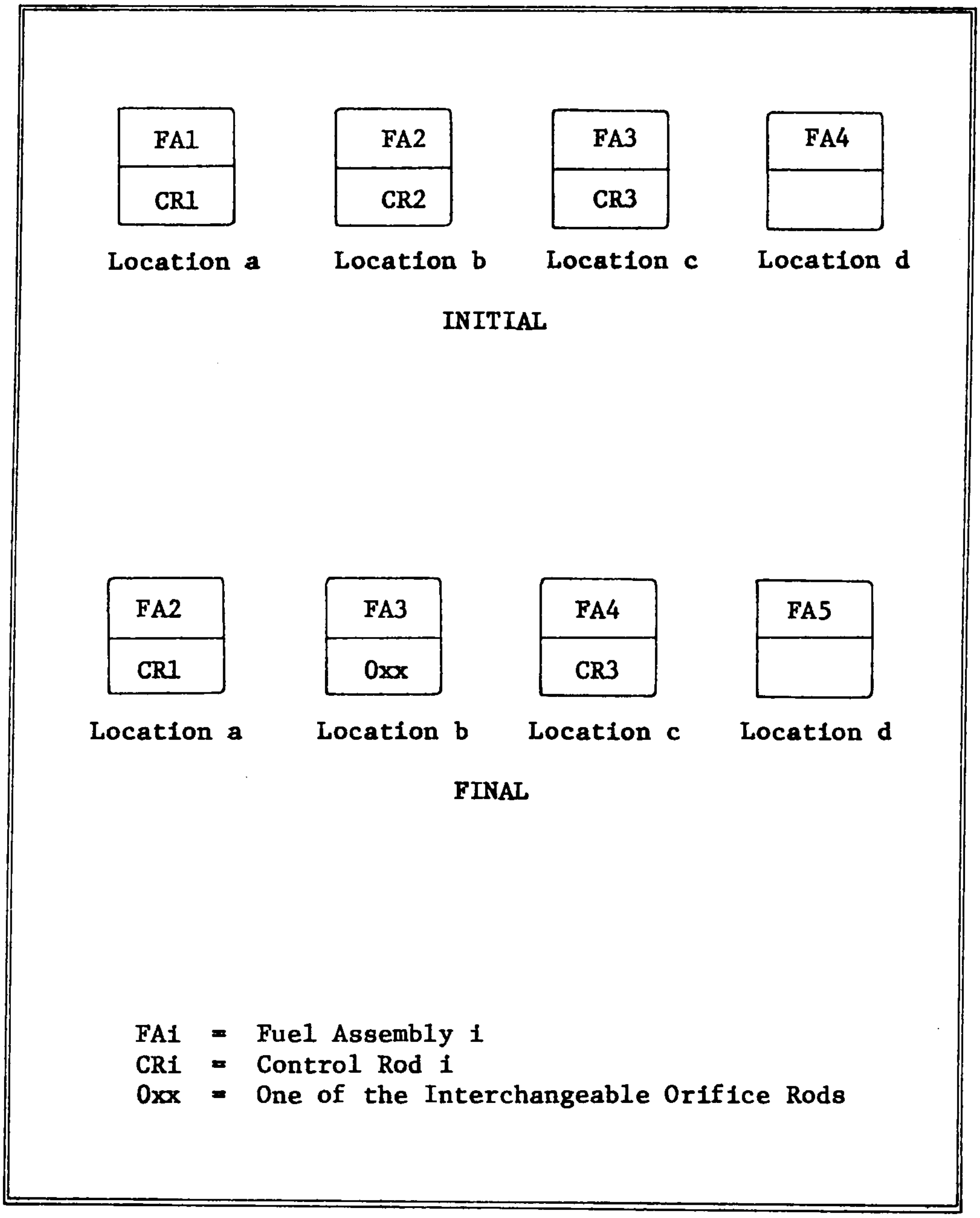


FIGURE 14
REPRESENTATIVE CONFIGURATION OF A C1 CHAIN

and the new Oxx at the end of the B6 schedules will not be encountered in the present case. Here, CR3 may even be inserted on top of FA4 before work on the main part of the chain commences.

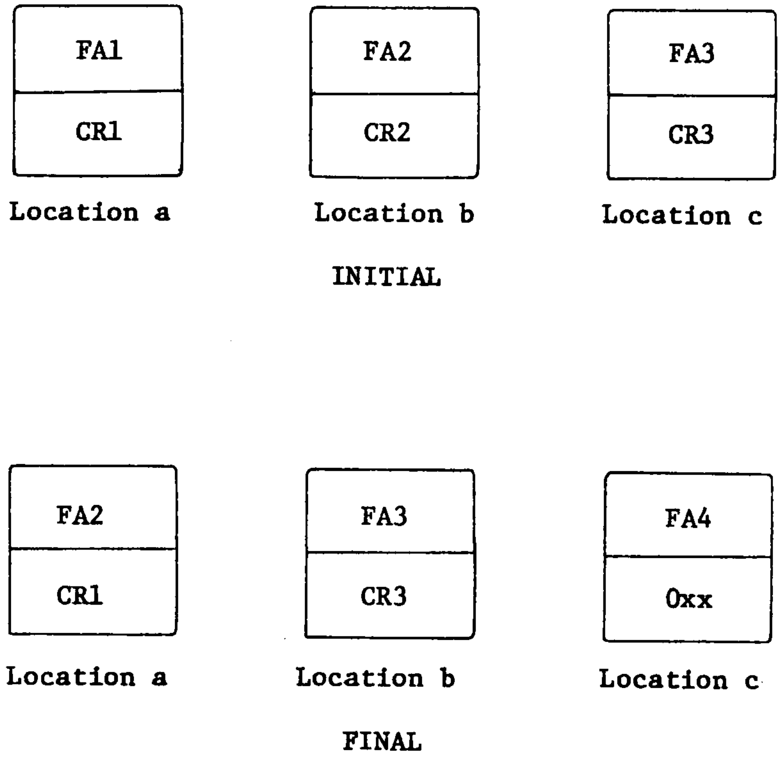
2. Chains of Type C2 and C3

Two C2 chains along with one C3 chain are present in the third data set. Referring to Figures 15 and 16, it is seen that these two types of chains are very similar in nature, and both may be scheduled through the application of any of the procedures outlined in the section on B5 chains. The new CR designations present in Figure 16 represent components of a different nature from the Oxx elements. This distinct notation is employed here to clarify the limitations on interchangeability for the P and S components.

In the case of the C3 chain, any of the proposed scheduling alternatives will involve the switching of the components P and S1 at the end of the cycle. The option of temporarily storing one of these components within the core may be avoided by scheduling the Oxx element for another chain to be brought in along with FA4. The P component for the C3 chain could then be transferred in at a later point. However, the specific arrangement of the fuel and control components within the spent fuel storage pool must be taken into account in determining the feasibility of this latter scheduling alternative.

3. Chains of Type C4

The single chain of this type present in the third set of shuffle



FAi = Fuel Assembly i
CRi = Control Rod i
Oxx = One of the Interchangeable Orifice Rods

FIGURE 15
REPRESENTATIVE CONFIGURATION OF A C2 CHAIN

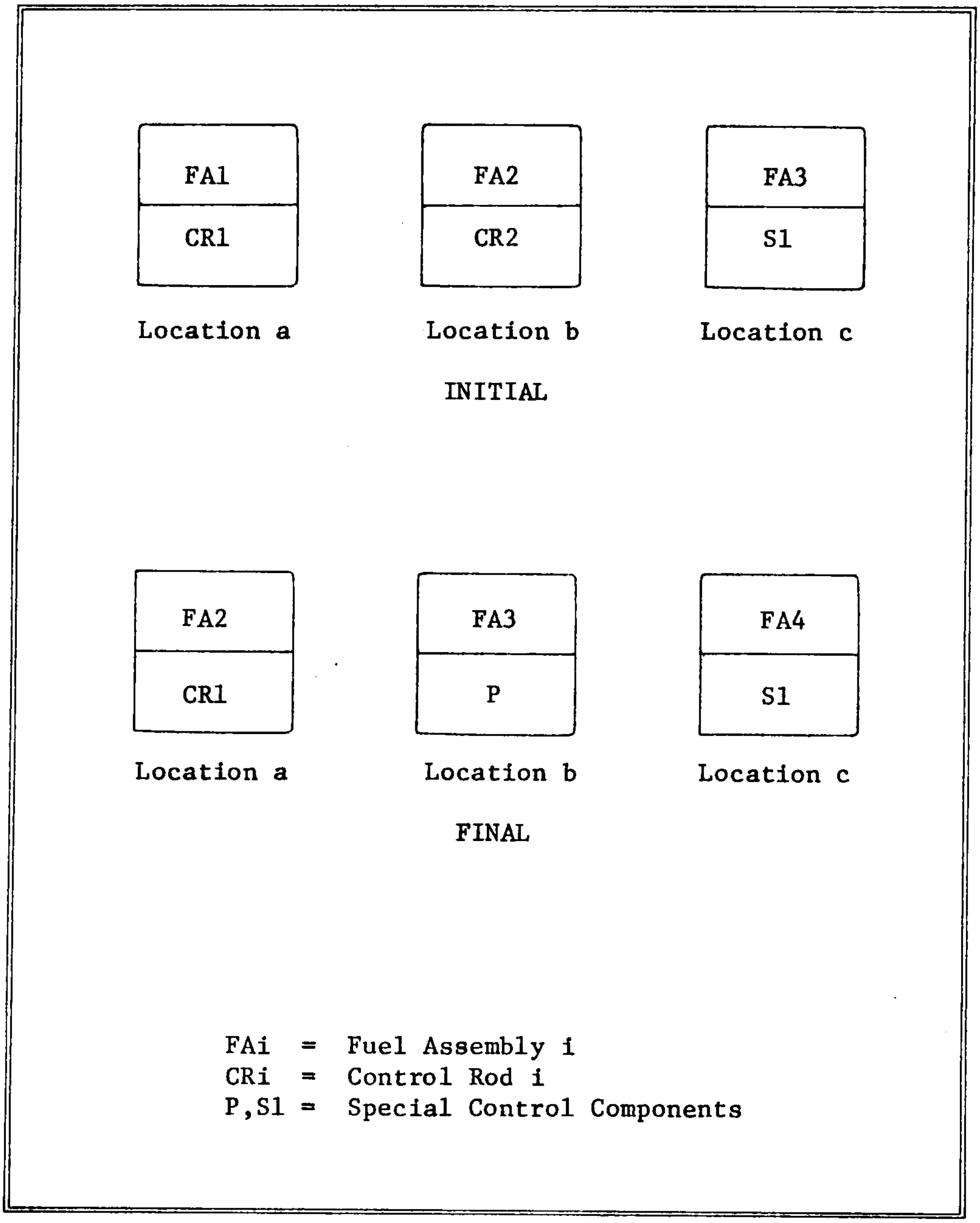


FIGURE 16
REPRESENTATIVE CONFIGURATION OF A C3 CHAIN

data is represented in Figure 17. Here, no control assemblies enter or leave the core. This chain could be effectively combined with the CR removal and insertion operations corresponding to a single core location such as position aa in Figure 8. A feasible schedule for the above combination would involve two instances of a procedure similar to either Cycle B-I or B-II. A 14-minute or 2.9% savings could be obtained through the use of Cycle B-II.

4. Chains of Type C5

The third data set contains one chain of this class. Its basic structure is given in Figure 18. Here, two subchains corresponding to the removal of FA1 and FA4 are present, and no extra solitary CR removals or insertions are involved. An initial scheduling technique for C5 chains which involves Cycle B-I has the following form:

- perform 1 Cycle B-I removing FA4/CR4 plus CR2 and returning with FA8/0xx
- transfer 0xx from f to c
- transfer S2 from e to f
- perform 1 Cycle B-I removing FA1/CR1 plus CR5 and returning with FA7/P plus CR1
- transfer P from c to e.

Twenty-two minutes of idle time would result if the above method were applied.

All comments concerning alternate scheduling methods made in the sections on B5 and C3 chains are appropriate for each of the instances of Cycle B-I in the given schedule. Again, these time-saving techniques may only be used when surplus control component transfers

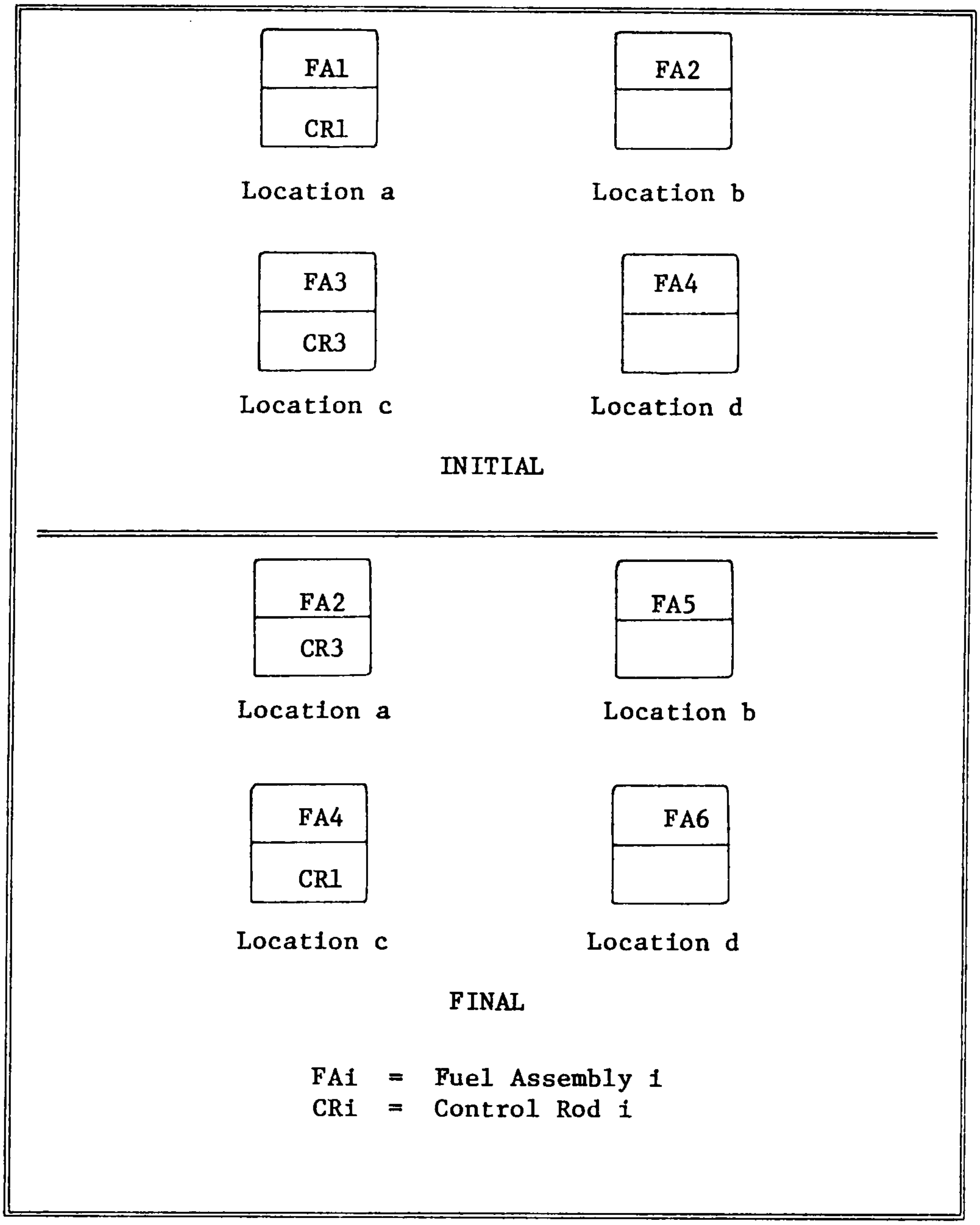
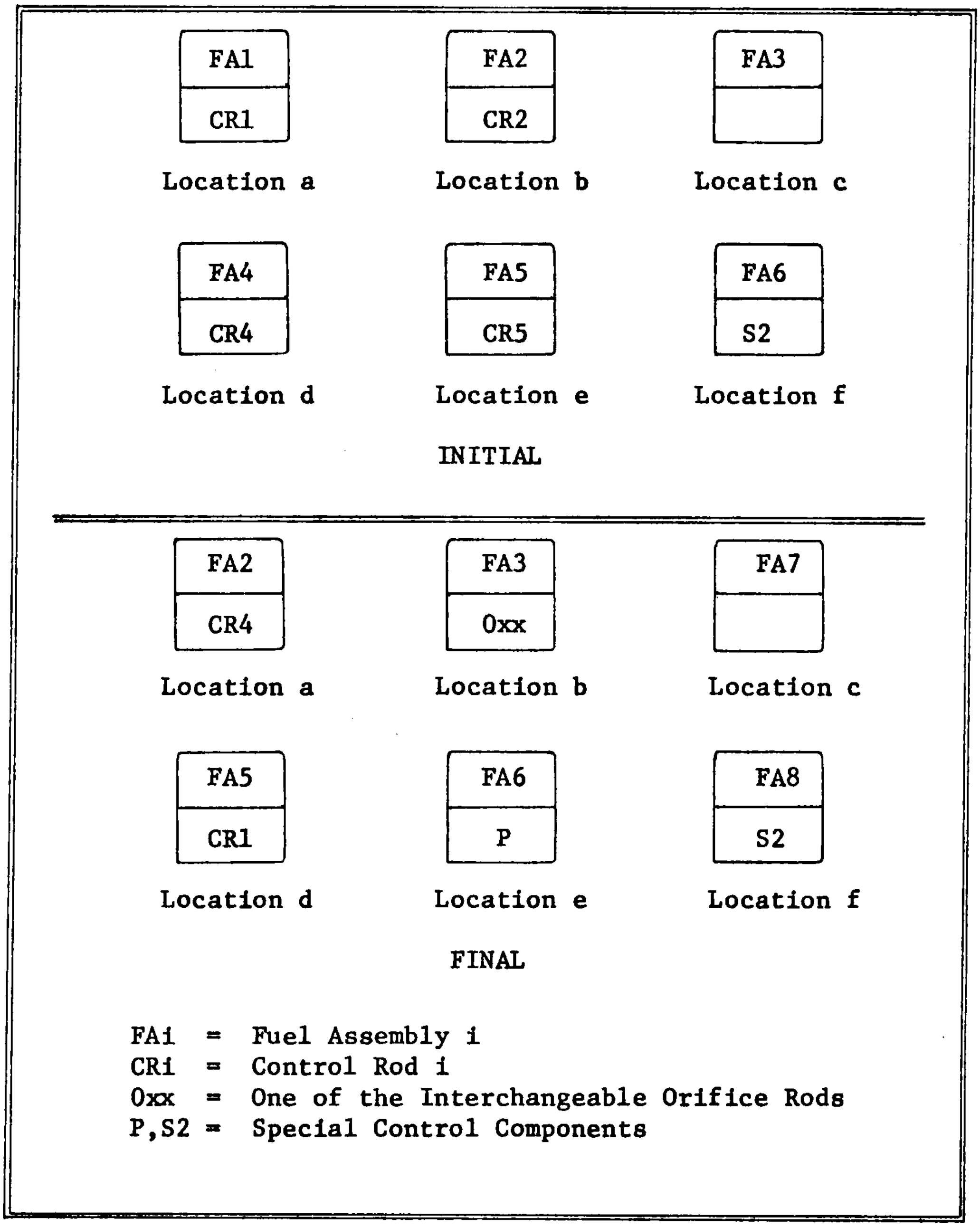


FIGURE 17
REPRESENTATIVE CONFIGURATION OF A C4 CHAIN



FAi = Fuel Assembly i
CRi = Control Rod i
Oxx = One of the Interchangeable Orifice Rods
P,S2 = Special Control Components

FIGURE 18
REPRESENTATIVE CONFIGURATION OF A C5 CHAIN

are present in the remainder of the data set.

5. Chains of Type C6

The third data set contains one C6 chain. Four distinct subchains are actually present and are labeled A through D in Figure 19. Two individual CR removals are also required. The nature of the exchange of control assemblies among the various chains is rather complex. In fact, for the CR1 and CR12 components, which are initially mounted on FA's to be removed, the final destinations are fresh fuel elements. This latter fact has a bearing on the order in which the individual subchains are processed. An initial schedule consisting of four cycles of the type B-I is outlined in Cycle C-I of Table 9.

The removals of CR9 and CR6 may be considered independent of the remainder of the C6 chain, although the CR9 withdrawal must be undertaken before the chain itself. Efforts to reduce or eliminate the periods of MB idle time in Cycle C-I would be of the same nature as those prescribed under type B7 chains. One procedure which would not necessitate the integration of the two surplus CR removals takes each general application of Cycle B-I of the form

- remove FAi/CRi plus CRj and return with FAk/0xx plus CRi
  and transforms it into the following sequence:
- remove FAi/CRj and return empty
- go to upenders empty and return with FAk/0xx.

In the latter case, all the intermediate AB moves would be performed during the absence of the MB from the core in dealing with

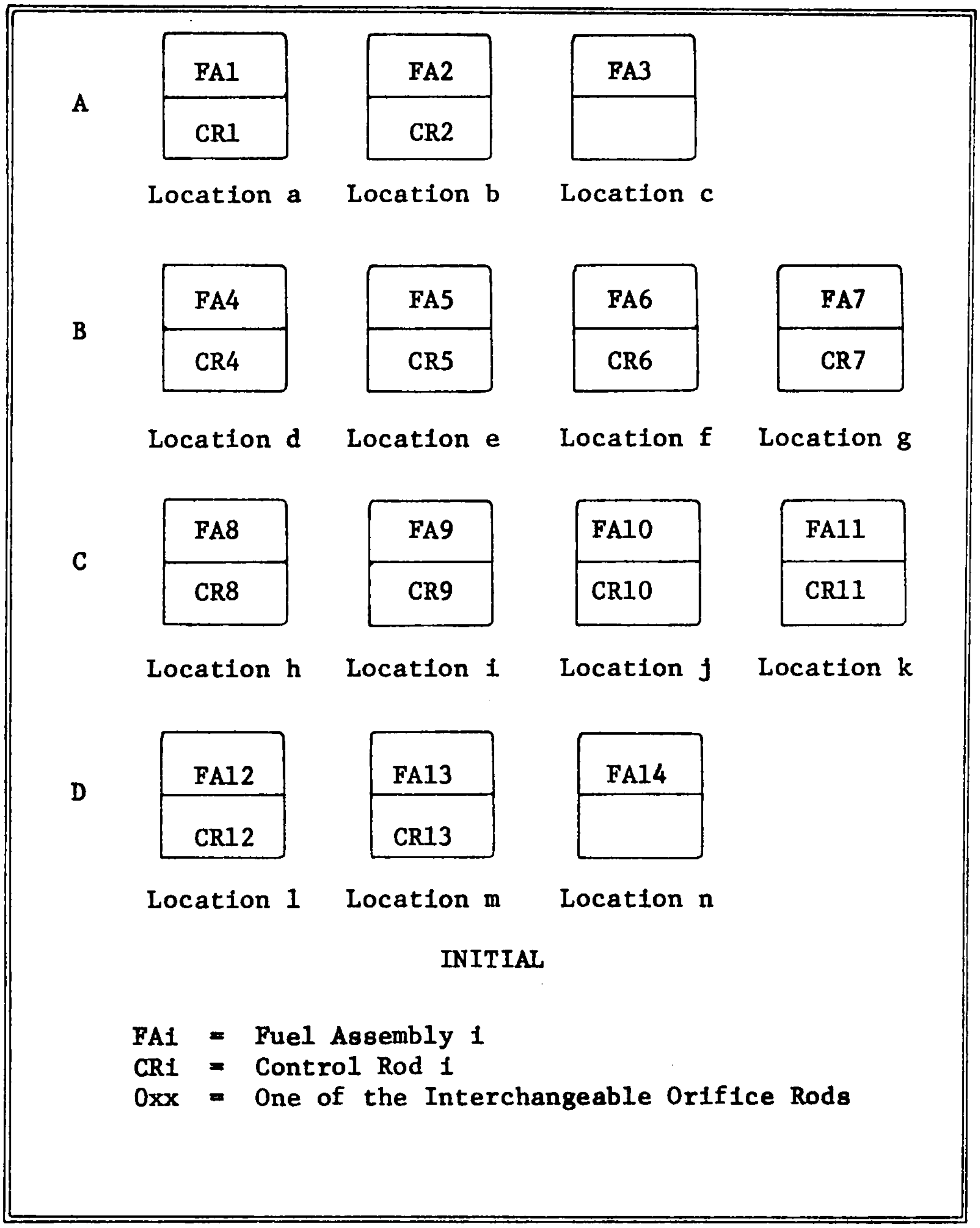
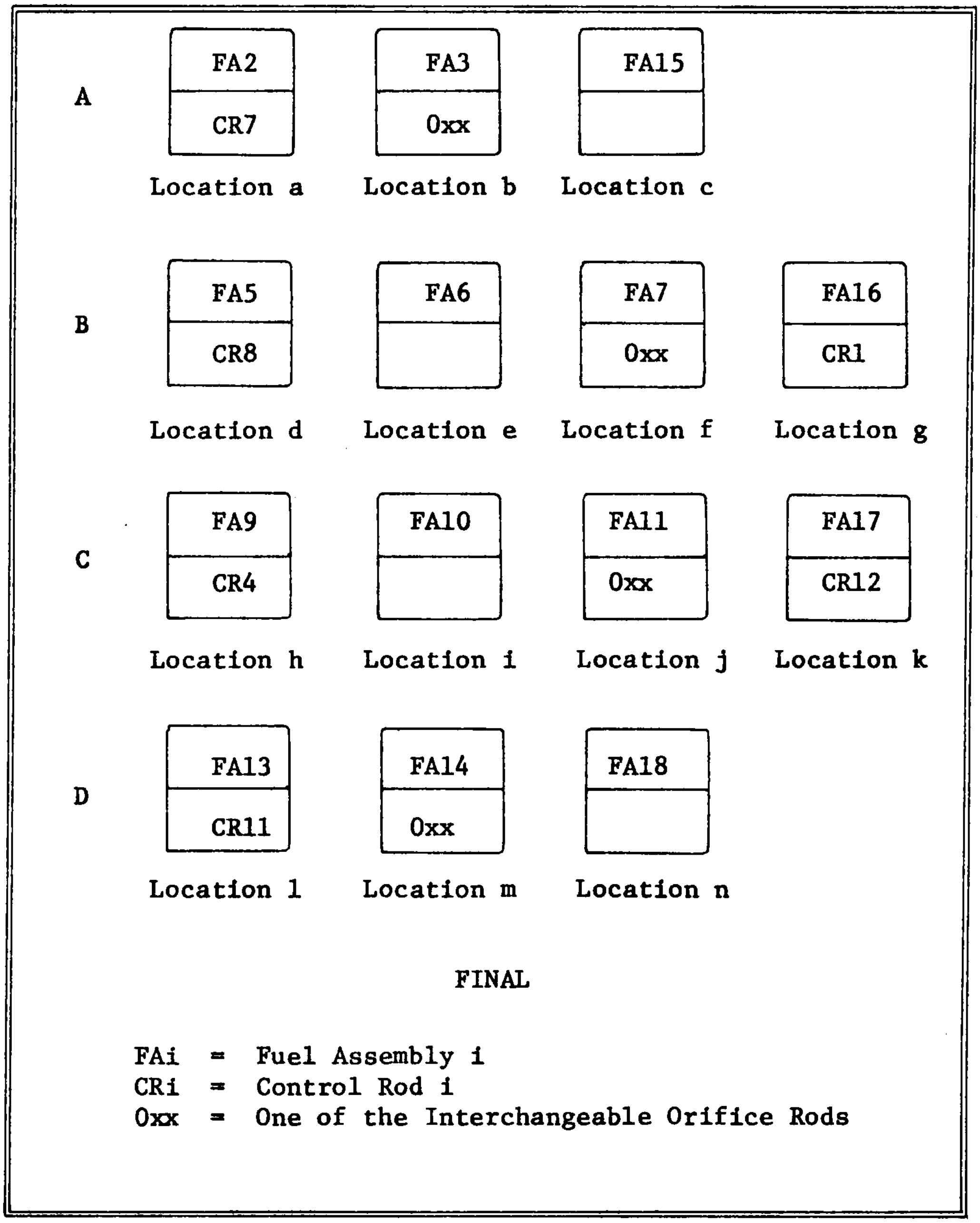


FIGURE 19
REPRESENTATIVE CONFIGURATION OF A C6 CHAIN



FAi = Fuel Assembly i
CRi = Control Rod i
0xx = One of the Interchangeable Orifice Rods

FIGURE 19
CONTINUED

TABLE 9

A SCHEDULING PROCEDURE FOR C6 CHAINS

Cycle C-I

Action by the MB	Subchain(s) Involved
• remove CR9 from the core	C
• perform 1 Cycle B-I removing FA4/CR4 plus CR5 and returning with FA16/Oxx plus CR4	B,C
• transfer Oxx from g to c	B,A
• perform 1 Cycle B-I removing FA1/CR1 plus CR2 and returning with FA15/Oxx plus CR1	A,B
• transfer Oxx from c to n	A,D
• transfer CR7 from f to a	B,A
• perform 1 Cycle B-I removing FA8/CR8 plus CR13 and returning with FA17/Oxx plus CR8	C,D,B
• transfer Oxx from k to f	C,B
• transfer CR11 from j to m	C,D
• perform 1 Cycle B-I removing FA12/CR12 plus CR10 and returning with FA18/Oxx plus CR12	D,C
• transfer Oxx from n to j	D,C
• remove CR6 from the core	B

another chain. Furthermore, CRi above would be moved to its new location at the outset. The savings per application of Cycle B-I would be just 3 minutes. Less than a 1% reduction can be achieved for the entire chain. Also, this method may only be used when sufficient opportunities are available for the AB to carry out its tasks for a C6 chain during portions of other chains.

A surplus CR removal may also be integrated into the sequence of operations outlined above. In this case, individual instances of Cycle B-I given as

- remove FAi/CRi plus CRj and return with FAk/Oxx plus CRi

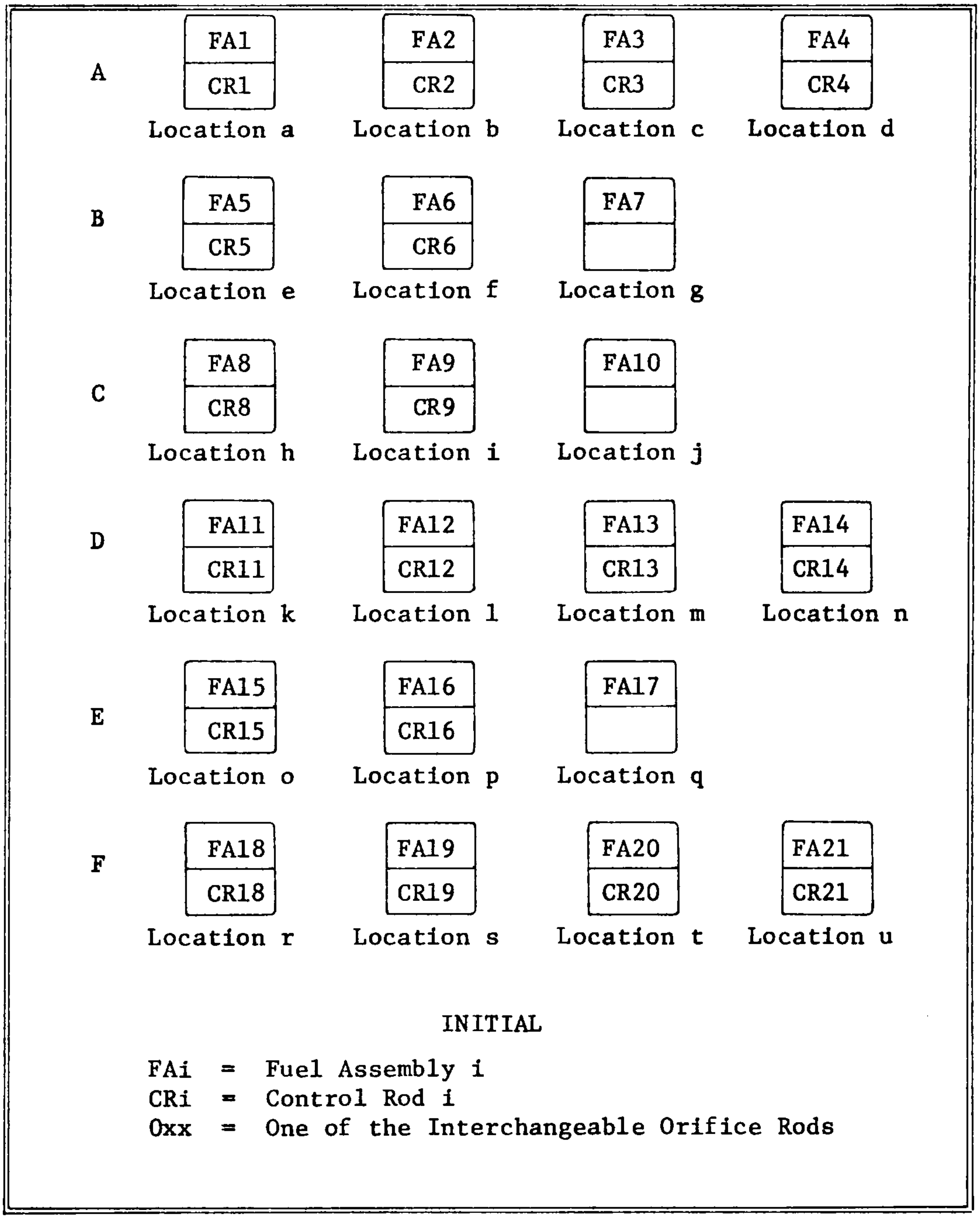
would be transformed to

- remove FAi/CRj and return empty
- remove extra CR and return with FAk/Oxx.

Nine minutes could be saved through each use of an extra CR move as outlined above. For subchains involving more than two intermediate AB moves, the additional work would have to be scheduled as part of some other chain.

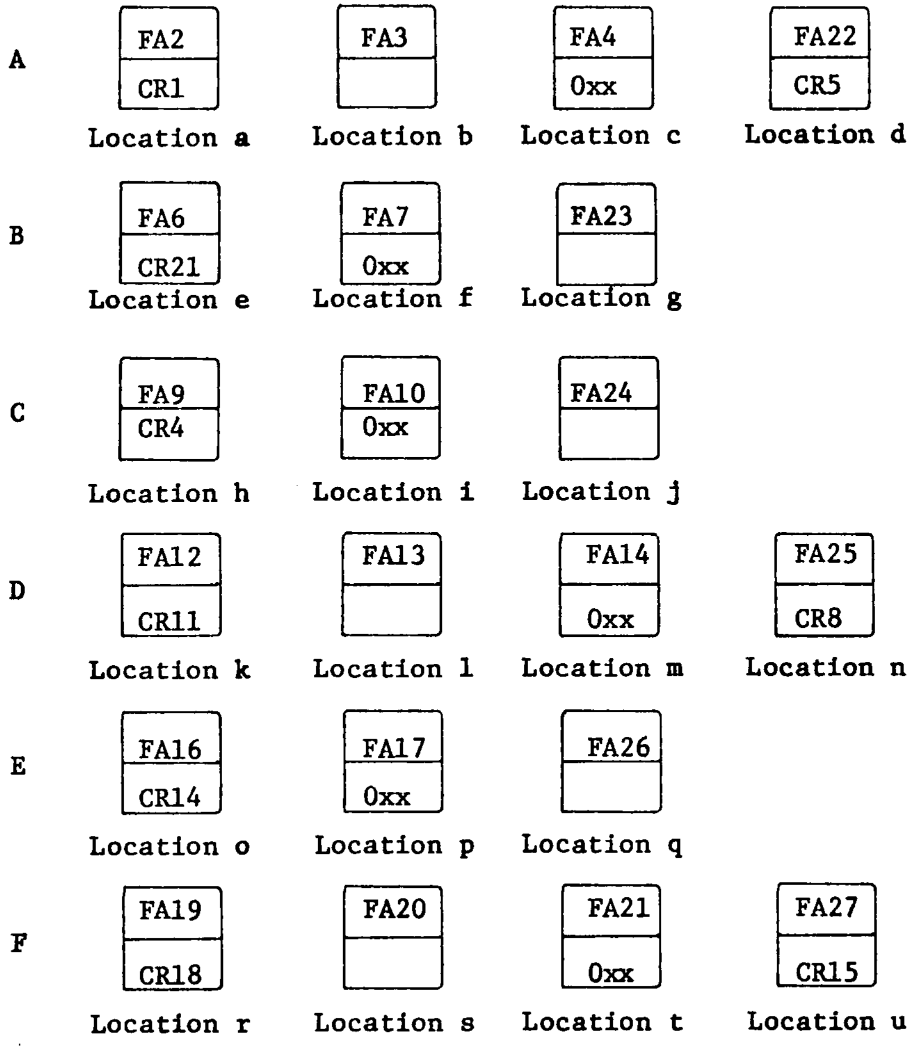
6. Chains of Type C7

The basic structure of the one C7 chain in the present set of shuffling data is depicted in Figure 20. Cycle C-II in Table 10 represents a preliminary schedule for completing the operations involved with the six subchains. This sequence allows for the independent completion of the three surplus CR moves and hence for



FAi = Fuel Assembly i
CRi = Control Rod i
Oxx = One of the Interchangeable Orifice Rods

FIGURE 20
REPRESENTATIVE CONFIGURATION OF A C7 CHAIN



FINAL

FAi = Fuel Assembly i
CRi = Control Rod i
0xx = One of the Interchangeable Orifice Rods

FIGURE 20
CONTINUED

TABLE 10

A SCHEDULING PROCEDURE FOR C7 CHAINS

Cycle C-II

Action by the MB	Subchain(s) Involved
• perform 1 Cycle B-I removing FA1/CR1 plus CR2 and returning with FA22/0xx plus CR1	A
• transfer 0xx from d to g	A,B
• perform 1 Cycle B-I removing FA5/CR5 plus CR6 and returning with FA23/0xx plus CR5	B,A
• transfer 0xx from g to j	B,C
• transfer CR21 from u to e	F,B
• perform 1 Cycle B-I removing FA11/CR11 plus CR12 and returning with FA25/0xx plus CR11	D
• transfer 0xx from n to u	D,F
• perform 1 Cycle B-I removing FA8/CR8 plus CR9 and returning with FA24/0xx plus CR8	C,D
• transfer 0xx from j to q	C,E
• transfer CR4 from c to h	A,C
• perform 1 Cycle B-I removing FA18/CR18 plus CR19 and returning with FA27/0xx plus CR18	F
• transfer 0xx from u to c	F,A
• perform 1 Cycle B-I removing FA15/CR15 plus CR16 and returning with FA26/0xx plus CR15	E,F
• transfer CR14 from m to o	D,E
• transfer 0xx from q to m	E,D
• remove CR3, CR13, and CR20 from the core	A,D,F

their integration into schedules for other chains. Methods of improving Cycle C-II may be sought along the lines discussed for type C6 chains.

7. Integration of the Chains of the Third Data Set

A summary of the integration requirements for the chains present in the third set of core maps appears in Table 11. An examination of the entries in columns three and seven of Table 11 reveals the fact that sufficient unscheduled periods of MB absence from the core are not available to achieve all improvements indicated. Hence, the integration problem here cannot actually be solved by inspection. Rather, both the spaces for extra AB work and the surplus CR removals must be viewed as resources to be allocated among the chains.

This chain integration task can be accomplished through the use of a modified version of the initial integer programming model in which only zero-one variables are employed. The details of such a representation and the nature of the solution obtained are discussed in the next chapter.

TABLE 11

INTEGRATION REQUIREMENTS FOR THE THIRD DATA SET

	(1) Chain Type	(2) Number Present	(3) Number Spaces Needed for n Chains	(4) Benefit from Spaces for n Chains (minutes)	(5) CR Moves Needed for n Chains	(6) Benefit from CR Moves for n Chains (minutes)	(7) Spaces Available in n Chains	(8) Extra Spaces per Chain if No Integration Involved
1.	B1	4	0	0	1	6	1	0
2.	B3	4	0	0	1	6	1+2(n-1) (plus 2 if CR move allocated)	0
3.	B5 also C2,C3, C5	8	2	3n-6	a) 1 b) n+1	a) 3n b) 9n	0	3
4.	B7	3	2n	5n	n	11n	2n	3
5.	C1	4	n+2	3n-6	a) 1^a b) $n+1^a$	a) 3n b) 9n	0	2
6.	C6	1	10n	12n	$4n^b$	36n	0	2 or 3 per subchain
7.	C7	1	6n	11n	3n	29n	6n	3 per sub-chain

---

[a] Under both alternative a) and b), n extra spaces are required.

[b] Here 2n extra spaces are required.

V. IMPLEMENTATION OF THE INTEGER PROGRAMMING PROCEDURE

A. The Nature of the Model and Solution Method

A modified version of the initial integer programming model for use in the integration of the various types of chains is developed here to fit the special requirements of the second and third sets of sample shuffle data. In the present case, zero-one variables are employed to determine the most effective means of combining the given chains. Specifically, the information given in Tables 8 and 11 is represented by variables of the basic form:

$$x_i = \begin{cases} 1 & \text{if the required CR moves are allocated for chain } i \\ 0 & \text{otherwise,} \end{cases}$$

$$y_i = \begin{cases} 1 & \text{if the required spaces are allocated for chain } i \\ 0 & \text{otherwise.} \end{cases}$$

In addition, constraints of the type

$$x_i + y_i \leq 1$$

are present in most of the cases considered.

Models are presented here for the integration of the chains contained in both the second and third shuffles. The second data set serves primarily as an initial test problem for the implementation of the specific zero-one programming code used. The details of the models constructed for both sets of sample data are given in Tables 12 through 15.

TABLE 12

INTEGER PROGRAMMING FORMULATION FOR THE SECOND DATA SET--VARIABLE DEFINITIONS

		Variable Will Equal 1 If and Only If These Allocations Are Made		
Variable	Corresponding Chain	CR Moves	Spaces	Related Constraints
x_1	B1	1	0	-
x_2	B3	1	0	-
x_3	B4	1	0	$x_3+y_3 \leq 1$
y_3	B4	0	1	
x_{4i} $(i=1,2,...,5)$	B5	i	0	$\sum_{i=1}^{5} x_{4i}+y_{40}+y_{42} \leq 1$
y_{4i} $(i=0,2)$	B5	i	2	
x_{51}	B6	1	3	$\sum_{i=1}^{4} x_{5i} + y_{51} \leq 1$
x_{52}	B6	2	1	
x_{53}	B6	3	2	
x_{54}	B6	4	3	
y_{51}	B6	0	5	
x_{6i} $(i=1,2,...,8)$	B7	1	0	$x_{6i}+y_{6i} \leq 1$ $(i=1,2,...,8)$
y_{6i} $(i=1,2,...,8)$	B7	0	2	

TABLE 13

INTEGER PROGRAMMING FORMULATION FOR THE SECOND DATA SET--OBJECTIVE FUNCTION AND MAJOR CONSTRAINTS

Objective Function Coefficients[a]

Variable	x_1	x_2	x_3	y_3	x_{41}	x_{42}	x_{43}	x_{44}	x_{45}	y_{40}	y_{42}	x_{51}	x_{52}	x_{53}	x_{54}
Coefficient	6	6	7	1	12	9	18	27	36	6	12	9	9	18	27

Variable	y_{51}	x_{61} through x_{68}	y_{61} through y_{68}
Coefficient	3	11	5

Principal Constraints

1. Availability of Surplus CR Moves:

$$x_1 + x_2 + x_3 + \sum_{i=1}^{5} ix_{4i} + 2y_{42} + \sum_{i=1}^{4} ix_{5i} + \sum_{i=1}^{8} x_{6i} \leq 8$$

2. Requirements for and Availability of Extra Spaces[b]:

$$y_3 + 2y_{40} + 2y_{42} + 3x_{51} + x_{52} + 2x_{53} + 3x_{54} + 5y_{51} + 2\sum_{i=1}^{8} y_{6i} \leq$$

$$2x_2 + 2(1-x_3-y_3) + 2x_3 + y_3 + 12(1 - \sum_{i=1}^{5} x_{4i} - y_{40} - y_{42}) + \sum_{i=1}^{5} x_{4i} +$$

$$y_{40} + y_{42} + 3\sum_{i=2}^{4} (5-i)x_{4i} + \sum_{i=1}^{4} x_{5i} + y_{51} + 6(1 - \sum_{i=1}^{4} x_{5i} - y_{51}) +$$

$$4x_{52} + 2x_{53} + 3\sum_{i=1}^{8} (1-x_{6i}-y_{6i}) + 32$$

---

[a] The problem involves the maximization of the stated objective function.

[b] Simplification of this constraint can be carried out by transferring all variables to the left-hand side of the inequality.

TABLE 14

INTEGER PROGRAMMING FORMULATION FOR THE THIRD DATA SET-- VARIABLE DEFINITIONS

Variable	Corresponding Chain	Variable Will Equal 1 If and Only If These Allocations Are Made		Related Constraints
		CR Moves	Spaces	
x_1	B1	1	0	-
x_2	B3	1	0	-
x_{3i} (i=1,2,...,8)	B5	i	0	$\sum_{i=1}^{8} x_{3i} + y_{30} + \sum_{i=2}^{6} y_{3i} \le 1$
y_{3i} (i=0,2,...,6)	B5	i	2	
x_{4i} (i=1,2,3)	B7	1	0	$x_{4i}+y_{4i} \le 1$ (i=1,2,3)
y_{4i} (i=1,2,3)	B7	0	2	
x_{51}	C1	1	4	$\sum_{i=1}^{5} x_{5i} + \sum_{i=1}^{3} y_{5i} \le 1$
x_{52}	C1	2	1	
x_{53}	C1	3	2	
x_{54}	C1	4	3	
x_{55}	C1	5	4	
y_{51}	C1	0	5	
y_{52}	C1	0	6	
y_{53}	C1	2	6	
x_{6i} (i=1,2)	C6 subchains	1	0	$x_{6i}+y_{6i} \le 1$ (i=1,2,3,4)
x_{6i} (i=3,4)	C6 subchains	1	1	
y_{6i} (i=1,2)	C6 subchains	0	2	
y_{6i} (i=3,4)	C6 subchains	0	3	
x_{7i} (i=1,2,3)	C7 subchains	1	0	$x_{7i}+y_{7i} \le 1$ (i=1,2,3)
y_{7i} (i=1,2,3)	C7 subchains	0	2	

TABLE 15

INTEGER PROGRAMMING FORMULATION FOR THE THIRD DATA SET--OBJECTIVE FUNCTION AND MAJOR CONSTRAINTS

Objective Function Coefficients[a]

Variable	x_1	x_2	x_{31}	x_{32}	x_{33}	x_{34}	x_{35}	x_{36}	x_{37}	x_{38}	y_{30}	y_{32}	y_{33}	y_{34}	y_{35}
Coefficient	6	6	24	9	18	27	36	45	54	63	18	24	30	36	42

Variable	y_{36}	x_{41}	x_{42}	x_{43}	y_{41}	y_{42}	y_{43}	x_{51}	x_{52}	x_{53}	x_{54}	x_{55}	y_{51}	y_{52}
Coefficient	48	11	11	11	5	5	5	12	9	18	27	36	3	6

Variable	y_{53}	x_{61}	x_{62}	x_{63}	x_{64}	y_{61}	y_{62}	y_{63}	y_{64}	x_{71}	x_{72}	x_{73}	y_{71}	y_{72}	y_{73}
Coefficient	12	9	9	9	9	3	3	3	3	9	9	11	3	3	5

Principal Constraints

1. Availability of Surplus CR Moves:

$$x_1 + x_2 + \sum_{i=1}^{8} ix_{3i} + \sum_{i=2}^{6} iy_{3i} + \sum_{i=1}^{3} x_{4i} + \sum_{i=1}^{5} ix_{5i} + 2y_{53} + \sum_{i=1}^{4} x_{6i} +$$

$$\sum_{i=1}^{3} x_{7i} \leq 8$$

---

[a] The problem involves the maximization of the stated objective function.

TABLE 15
CONTINUED

Principal Constraints

2. Requirements for and Availability of Extra Spaces[b]:

$$2y_{30} + 2\sum_{i=2}^{6} y_{3i} + 2\sum_{i=1}^{3} y_{4i} + 4x_{51} + \sum_{i=2}^{5} (i-1)x_{5i} + 5y_{51} + 6y_{52} + 6y_{53}$$

$$+ 2y_{61} + 2y_{62} + 3y_{63} + 3y_{64} + x_{63} + x_{64} + 2\sum_{i=1}^{3} y_{7i} \le$$

$$2x_2 + 3\sum_{i=2}^{8} (9-i)x_{3i} + 24(1 - \sum_{i=1}^{8} x_{3i} - y_{30} - \sum_{i=2}^{6} y_{3i}) + \sum_{i=1}^{8} x_{3i} + y_{30}$$

$$+ \sum_{i=2}^{6} y_{3i} + 3\sum_{i=1}^{3} (1-x_{4i}-y_{4i}) + 6x_{52} + 4x_{53} + 2x_{54} + 2y_{51} + \sum_{i=1}^{5} x_{5i}$$

$$+ \sum_{i=1}^{3} y_{5i} + 8(1 - \sum_{i=1}^{5} x_{5i} - \sum_{i=1}^{3} y_{5i}) + 3(1-x_{61}-y_{61}) + 3(1-x_{62}-y_{62})$$

$$+ 2(1-x_{63}-y_{63}) + 2(1-x_{64}-y_{64}) + 3\sum_{i=1}^{3} (1-x_{7i}-y_{7i}) + 20$$

---

[b] Simplification of this constraint can be carried out by transferring all variables to the left-hand side of the inequality.

These chain integration problems are solved through an application of a branch and bound zero-one programming algorithm developed by Balas [4]. With this technique, all possible solution vectors are implicitly enumerated. Certain sets of solutions are not explicitly considered on the basis of optimality and feasibility criteria. A listing of the computer code used in implementing Balas' procedure is presented in the appendix.

Referring to the information given in Tables 8 and 11 on the integration of the chains in the second and third data sets, it is seen that the maximum savings can be achieved when sufficient spaces are available for allocation in the improved schedules for each type of chain and when the limited CR moves are assigned to a subset of the chains. Considering the entries in columns four and six of both Table 8 and Table 11, these maximum time reductions are computed to be 98 minutes and 110 minutes for the second and third data sets, respectively. From Table 16, it is seen that the solutions obtained through the use of the given algorithm correspond to the aforementioned maximum time savings.

B. The Computational Efficiency of the Solution Procedure

In terms of statistics on the computation time involved, it is noted that for both data sets, fairly good solutions can be obtained rather quickly at the start of the implicit enumeration process. In the case of the second set of shuffle data, the solution listed in Table 16 is found in less than 15 seconds of execution time with

TABLE 16

OPTIMAL SOLUTIONS TO THE CHAIN INTEGRATION PROBLEMS

The Second Data Set

Variables Having a Value of 1 in the Final Solution

x_1 x_2 y_3
x_{41} y_{51} x_{64}
x_{65} x_{66} x_{67}
x_{68} y_{61} y_{62}
y_{63}

All other variables are 0.
The value of the objective function is 98.

The Third Data Set

Variables Having a Value of 1 in Solution 1

x_1 x_2 y_{30}
y_{41} y_{42} y_{43}
y_{52} x_{62} x_{63}
x_{64} y_{61} x_{71}
x_{72} x_{73}

All other variables are 0.
The value of the objective function is 110.

Variables Having a Value of 1 in Solution 2

x_1 x_2 y_{30}
x_{41} x_{42} y_{43}
x_{51} y_{61} y_{62}
y_{63} y_{64} x_{71}
x_{72} x_{73}

All other variables are 0.
The value of the objective function is 110.

an IBM System/370 Model 158 computer. However, due to the fairly large number of variables involved in that representation, no final verification of the optimality of that solution is provided by the Balas routine after 30 minutes of execution. The exploration and backtracking steps, which are performed within the program as a part of a search for solutions superior to the one obtained initially, require a rather significant amount of time.

In the case of the third data set, a good solution is determined in less than 15 seconds of execution time. The addition of two variables with values of one to this preliminary solution yields the maximum possible value of the objective function. If the program is executed for a total of 75 minutes, a solution is found which is slightly superior to the one given initially and which can be transformed to an optimal result through the addition of a single variable with a value of one. Here again, no actual verification of this optimal result is provided by the algorithm. Two solution vectors for the third data set which correspond to the maximum value of the objective function are presented in Table 16.

A rather significant amount of time is required by the Balas algorithm in the implicit enumeration of the extremely large number of possible solutions for the sample integration problems. A reduction in the amount of execution time required for these problems might be achieved through the use of a more efficient zero-one programming algorithm. Furthermore, the formulations for both data sets involve numerous constraints in which the sum of two variables is required to be less than or equal to one. A solution procedure which allows for the explicit consideration of such special constraints may provide improvements over the results obtained with a more general integer programming technique.

VI. RESULTS

A. A Comparison of Scheduling Methods

In order to gain insight into the effectiveness of the heuristic scheduling techniques described in the previous chapters, the nature and extent of the improvements made over past or existing procedures are considered here. In this analysis, two shuffle schedules for the second data set provided by the Babcock and Wilcox Company are reviewed. One of the schedules had been generated by hand; the other had been constructed through the use of one of the earlier computerized shuffle-scheduling routines. It should be noted that the latter results have undergone numerous revisions. No concise documentation on these changes is presently available for inspection. However, the basic sequencing rules embodied in the manual schedule are used as the bases for comparison throughout the analysis of the individual chains in which the principal objective is that of determining the best possible schedule in each case.

As discussed in the chain-by-chain analysis of the sample shuffle data, the manual schedule involves successive applications of the standard Cycle B-I technique outlined in Table 4. Most of the effort in improving this basic procedure involves the elimination of the 11-minute period of idle time which occurs when the MB is delayed at the upenders by the SFB. The comparisons of the Cycle B-I method and the improved techniques made throughout the analysis indicate that time reductions in the range of 2% to 4% can be achieved in each instance. Additional improvements of a more substantial nature can be made through the elimination of all unnecessary in-core storages of control components as in the case of the B6 chains. Such superfluous CR transfers have a significant effect on the total completion time since the movement of a CR from one location within the core to another takes over one hour under the assumed operating status.

The principal sequencing rule used in the schedule generated by the computer routines is that of Cycle B-II given in Table 4. This basic technique requires extensive use of the option of temporarily storing control components at positions within the core during the execution of related fuel assembly moves. As stated above, such CR transfers are very time consuming, and the analysis of the second set of shuffle data reveals the fact that these extra moves can be avoided. The Cycle B-II technique is the superior scheduling method only in the case of a B3 chain.

Further comments on the effectiveness of the scheduling rules employed in this preliminary version of the computer program package are not really warranted due to the fact that subsequent revisions of this procedure have been made. The main goal of the analysis described in the preceding chapters is that of determining the best possible schedule for each of the chains. The Cycle B-I method of the manual schedule serves as an adequate starting point for the consideration of improved procedures.

All comments concerning the effectiveness of the scheduling techniques presented above for the second data set are applicable to the third set of sample shuffle data as well. The chains present in these two data sets are very similar in nature, and comparable scheduling techniques can be applied in each case. A few general comments concerning the effectiveness of the proposed scheduling techniques for the chains of the first data set may also be made. Here again, the temporary in-core storage of control assemblies can be avoided in almost every instance, and all MB idle time is eliminated in the resulting schedules. Only A3 chains can be combined with chains of other classes, and a 6.5% reduction in total completion time can be achieved in this case.

B. The Near-Optimal Nature of the Proposed Techniques

A second and perhaps more meaningful method of analyzing the efficiency of the scheduling rules presented here would involve an investigation of the relationship between these techniques and what would appear to be optimal procedures. Again using the second data set as an example, only two of the improved schedules involve any periods of idle time for the MB. For B1 and B2 chains, a 2-minute period of such idle time is present for each chain, and a total of 48 minutes is included in the complete shuffle. Secondly, all unnecessary CR transfers are avoided through the use of the improved schedules.

When attempts are made at eliminating the MB idle time in the standard Cycle B-I technique, it is often the case that the additional travel time required for the MB will offset some of the idle time reductions obtained. These extra moves are in the form of trips to and from the upenders, and the flow time is increased by 6 minutes in each case. The integration of single CR removals into some, but not all, of the individual schedules of the second data set serves to compensate for this extra travel time.

Improvements in the heuristic scheduling techniques proposed here could possibly be made through the elimination of the remaining MB idle time and all surplus moves. However, the benefits to be gained through the development and implementation of optimization techniques for the construction of schedules superior to those presented here would be overshadowed by the added expense and time required for this continued research.

VII. CONCLUSIONS AND RECOMMENDATIONS FOR EXTENSION

A. Concluding Remarks

The heuristic scheduling rules developed for the chains present in the three sets of sample shuffle data appear to be fairly effective. Improvements are made over techniques employed in industry, and these completion time reductions are generally in the range of 2% to 4%. Thus, substantial reductions in the shuffle completion time are brought about only through changes in the actual processing times for the individual transfer operations themselves.

Improvements in the scheduling methods proposed here can be achieved primarily through the development of optimizing procedures guaranteed to produce the best possible schedule either for the complete shuffle, individual chains, or groups of two or more chains. However, the benefits to be realized with such optimization techniques are small in light of the additional effort required in their development and application.

B. Possible Extensions

The basic structure of the heuristic solution technique presented here involves the initial consideration of effective scheduling methods for the individual chains followed by the determination of the best means of combining the units, possibly through the use of an integer programming model. The present analysis involves a specific equipment configuration and a fixed set of processing times for the individual shuffle operations. Similar chain-by-chain studies can be conducted for the various equipment alternatives and different processing time data.

In the case of only one main bridge available for use over the reactor core, the focus of attention would be on the elimination of MB idle time corresponding to delay by the SFB since no real choices exist for the assignment of the fuel and control component transfers to the single bridge. When only one upender is present for transfers between the reactor room and the spent fuel storage pool, delay of the MB by the SFB is unavoidable. Hence, in the case of two bridges in the reactor room, effective scheduling techniques for the one-upender configuration allow for the completion of AB moves and also CR transfers by the MB during these periods of delay.

Finally, in the event that changes are made in the processing times of the transfer operations, corresponding modifications of the scheduling techniques presented here might be required. Specifically, the number of in-core transfers which can be performed by the AB while the MB is at the upenders may be different. Further changes in the scheduling procedures might also be made so that the variable nature of the completion time data is explicitly considered.

In line with the development of modified scheduling techniques to deal with the various equipment configurations, an analysis can be conducted to determine the most effective set of equipment. The cost of a given set of fuel-handling equipment can be contrasted with the relative time required to perform a complete representative shuffle under the best schedule of operations. Equipment failure and repair time data might also be incorporated into the analysis.

Another specific area of extension of the basic shuffle problem involves the consideration of the inspection operations which are undertaken in conjunction with the shuffle. Certain fuel and control rod elements are removed from the core temporarily to undergo various inspections. Presently at the Babcock and Wilcox Company, a shuffle schedule is first constructed without the explicit consideration of these inspection operations. The moves are then incorporated into the complete plan of action. Some of the assumed chain structures would be disrupted if these additional fuel and control component transfers were explicitly considered during the construction of a shuffle schedule. In this case, some of the scheduling techniques proposed here would probably have to be modified.

One final area of extension involves the combination of the in-core fuel management studies with a consideration of core configurations which might lead to effective shuffling schedules. Alternate locations for fuel assemblies and control rods might be designated from the reactor physics and fuel utilization point of view. In this manner, the scheduler might be given time-saving options which still meet the optimization criteria of the fuel management studies.

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APPENDIX

COMPUTER PROGRAM FOR THE INTEGER PROGRAMMING ALGORITHM

A listing of the FORTRAN computer program for the Balas integer programming algorithm [4] is presented in this section. The program was provided by Dr. Lester C. Frair of the Department of Industrial Engineering and Operations Research at Virginia Polytechnic Institute and State University. The computer code presented here was implemented on the IBM System/370 Model 158 computer at Virginia Polytechnic Institute and State University. The units 5 and 6 specified in the input and output statements correspond to the card reader and line printer, respectively.

For this procedure, the integer programming problem must contain only zero-one variables, and the objective function must be of the minimization type. In addition, all constraints should be inequalities of the form less than or equal to. The dimensions of the arrays within the program are such that a problem involving up to 50 variables and 49 constraints can be solved.

The assumed form of the problem to be solved by the Balas algorithm is given below.

$$\text{minimize} \sum_{J=1}^{N} A(1,J) \cdot X(J)$$

subject to

$$\sum_{J=1}^{N} A(I,J) \cdot X(J) \leq B(I) \qquad \text{for } I=2,3,\ldots,M.$$

$$X(J) = 0 \text{ or } 1 \qquad \text{for } J=1,2,\ldots,N.$$

The data that must be given as input to the procedure has the following form:

Card	Variable	Definition	Format
1	JAC	number of problems to be solved	I3
2	M,N	number of constraints + 1, number of variables	2I3
3, etc.	A(1,J) J=1,2,...,N	objective function coefficients	16I5
	A(I,J) I=2,3,...,M J=1,2,...,N	constraint matrix	16I5
	B(I) I=2,3,...,M	right-hand-side vector	16I5

The elements of the A matrix are read in one row at a time, and the first entry of each row should be punched at the beginning of a new card. The first element of the B vector should also be placed at the beginning of a new card.

Within the program itself, the problem may be modified so that all the coefficients of the objective function will be positive. A variable X(J) which has a negative coefficient in the objective function will be replaced by the quantity [1-X(J)]. Corresponding changes will be made in the coefficients and constants involved in the constraints.

The data for the original and modified versions of the problem will be printed along with the various feasible solutions found. For each solution, the number of the iteration at which the result was determined and the corresponding value of the objective function for the modified version of the problem will also be given. These latter values will be labeled PIVOT and ZMOD, respectively.

The optimal solution will not be given any special designation. Rather, a sequence of solutions with decreasing values of the objective function will be printed. At the conclusion of all the backtracking and exploration steps undertaken within the program, the last vector listed will correspond to the optimal solution.

```
C     BALAS ZERO-ONE INTEGER PROGRAMMING CODE
C     ******************************************************************
      IMPLICIT INTEGER (A-H,O-Z)
      COMMON A(50,50),B(50),X(50),V(50),ENTER(50),Z(50),ZT,M,N,ZMIN,IVT
      DIMENSION SOL(50)
C  HERE THE DATA IS READ IN.
      READ 1111,JAC
 1111 FORMAT (I3)
      DO 224 KA=1,JAC
      PRINT 225, KA
  225 FORMAT('1',15X,'TEST PROBLEM',I3)
      READ 100,M,N
  100 FORMAT(2I3)
      DO 1 I=1,M
    1 READ 101,(A(I,J),J=1,N)
      READ 101,(B(I),I=2,M)
  101 FORMAT(16I5)
      PRINT 2000
 2000 FORMAT('0',15X,'ORIGINAL PROBLEM MATRIX')
      DO 2002 I=1,M
      PRINT 2003, (A(I,J),J=1,N)
 2003 FORMAT (13I10)
 2002 CONTINUE
      PRINT 2004
 2004 FORMAT('0',15X,'RIGHT HAND SIDE')
      PRINT 2005, (B(I),I=2,M)
 2005 FORMAT (13I10)
C  THE BALAS ZERO-ONE ALGORITHM REQUIRES THAT THE PROBLEM BE IN A
C  SPECIAL FORM.
```

```
C   ALL THE COEFFICIENTS OF THE OBJECTIVE FUNCTION MUST BE POSITIVE.
C   ALL THE NEGATIVE A(1,J) MUST BE TRANSFORMED TO POSITIVE.
C   THE VALUE OF THE OBJECTIVE EQUATION MUST BE ADJUSTED FOR THE
C   CHANGES MADE.
C   THE CONSTRAINT MATRIX AND THE RIGHT-HAND-SIDE VECTOR MUST ALSO
C   BE ADJUSTED.
C   THE X(J) WHICH HAVE BEEN CHANGED ARE IDENTIFIED THROUGH THE
C   SOL(J) SEQUENCE.
       DO 3 K=1,N
       IF(A(1,K).GT.0) GO TO 18
       A(1,K)=-A(1,K)
       DO 19 J=2,M
       B(J)=B(J)-A(J,K)
       A(J,K)=-A(J,K)
    19 CONTINUE
       SOL(K)=1
       GO TO 3
    18 SOL(K)=0
     3 CONTINUE
C   THIS SECTION INITIALIZES THE PROBLEM.  ZMIN IS SET AT THE MAXIMUM
C   VALUE OF THE EQUATION.
       DO 10 J=1,N
    10 X(J)=0
       ZMIN=0
       DO 15 I=1,N
       ZMIN=ZMIN+A(1,I)
    15 CONTINUE
       LAST=0
       ZT=0
```

```
      PRINT 2006
 2006 FORMAT('0',15X,'MODIFIED PROBLEM MATRIX')
      DO 85 I=1,M
      PRINT 80, (A(I,J),J=1,N)
   80 FORMAT (13I10)
   85 CONTINUE
      PRINT 2004
      PRINT 2009, (B(I),I=2,M)
 2009 FORMAT (13I10)
      PIVOT=0
   50 PIVOT=PIVOT+1
      DO 52 I=2,M
      IF(B(I).LT.0) GO TO 53
   52 CONTINUE
      GO TO 42
   53 DO 51 J=1,N
      IF(X(J).LT.0)X(J)=0
   51 CONTINUE
      CALL FEAS1
C  OPTIMALITY CONDITION 1
C  THIS ELIMINATES ALL X(J) WHOSE ENTRY WOULD CAUSE A VALUE OF Z
C  GREATER THAN ZMIN.
      CALL OPT1
C  TEST A
C  THIS TEST DETERMINES IF ANY VARIABLES X(J)=0, THAT IS, IF THERE
C  ARE ANY FREE X(J)'S.
      CALL FREEX(FLAG)
      IF(FLAG.GE.1) GO TO 122
C  FEASIBILITY CONDITION 2
```

```
C   THIS CHECKS TO SEE IF FROM THE CURRENT VARIABLE ASSIGNMENTS,
C   A FEASIBLE SOLUTION CAN EVER BE OBTAINED.  IF NOT, BACKTRACKING
C   OPERATIONS ARE PERFORMED.
        CALL FEAS2(FLAG4)
        IF(FLAG4.EQ.1) GO TO 122
C   OPTIMALITY CONDITION 2
C   THE X(J) WHICH GIVES THE GREATEST INCREASE IN FEASIBILITY IS
C   CHOSEN AS THE ENTERING VARIABLE.  IN CASE OF TIES, THE X(J) WITH
C   THE SMALLEST OBJECTIVE FUNCTION COEFFICIENT ENTERS.
        CALL OPT2
C   HERE THE PROBLEM IS UPDATED.
        LAST=LAST+1
        ENTER(LAST)=IVT
        X(IVT)=2
        DO 41 I=2,M
        B(I)=B(I)-A(I,IVT)
    41  CONTINUE
        ZT=ZT+A(1,IVT)
        GO TO 50
C   THIS CHECKS TO SEE IF THERE IS A FEASIBLE SOLUTION.  IF SO, THE
C   SOLUTION IS PRINTED.
    42  IF(PIVOT.LE.1) GO TO 77
        ZMIN=ZT
        ZPRIN=0
  4000  PRINT 4001
        WRITE(6,601) PIVOT,ZMIN
   601  FORMAT('0',15X,'PIVOT IS',I7,5X,' ZMOD=',I6)
  4001  FORMAT('0',15X,'**********BEST*****SOLUTION*****SO*****FAR********
       ,**')
```

```
      DO 604 J=1,N
      IF(SOL(J).EQ.0) GO TO 606
      IF(X(J).LT.2) GO TO 701
  603 WRITE(6,608) J
  608 FORMAT('0',15X,'X(',I3,') = 0')
      GO TO 604
  701 ZPRIN=ZPRIN-A(1,J)
  607 WRITE(6,609) J
  609 FORMAT('0',15X,'X(',I3,') = 1 ')
      GO TO 604
  606 IF(X(J).LT.2) GO TO 603
      ZPRIN=ZPRIN+A(1,J)
      GO TO 607
  604 CONTINUE
      WRITE(6,602) ZPRIN
  602 FORMAT('0',15X,'THE OBJECTIVE FUNCTION =',I6)
      WRITE(6,4001)
      GO TO 895
  122 IF(PIVOT.EQ.1) GO TO 1001
C  THIS IS THE BACKTRACKING PROCEDURE.  IT IS USED WHEN THE PRESENT
C  COURSE CANNOT LEAD TO A FEASIBLE SOLUTION AND WHEN ANOTHER
C  BRANCH MUST BE CHECKED.
  895 ALTER=ENTER(LAST)
   71 IF(X(ALTER).EQ.2) GO TO 70
      X(ALTER)=0
      LAST=LAST-1
      IF(LAST.EQ.0) GO TO 1000
      ALTER=ENTER(LAST)
      GO TO 71
```

```
  70 X(ALTER)=1
     ZT=ZT-A(1,ALTER)
     DO 76 I=2,M
     IF(A(I,ALTER))73,76,73
  73 B(I)=B(I)+A(I,ALTER)
  76 CONTINUE
     GO TO 50
  77 WRITE(6,4001)
     WRITE(6,601) PIVOT,ZMIN
     ZPRIN=0
     DO 78 J=1,N
     IF(SOL(J).EQ.0) GO TO 78
     ZPRIN=ZPRIN-A(1,J)
     WRITE(6,609) J
  78 CONTINUE
     WRITE(6,602)ZPRIN
     WRITE(6,4001)
     GO TO 1000
1001 WRITE(6,3500)
3500 FORMAT('0',15X,'***** NO FEASIBLE SOLUTION *****')
1000 CONTINUE
 224 CONTINUE
     STOP
     END
```

```
      SUBROUTINE FREEX(FLAG)
      IMPLICIT INTEGER (A-H,O-Z)
      COMMON A(50,50),B(50),X(50),V(50),ENTER(50),Z(50),ZT,M,N,ZMIN,IVT
      FLAG=0
      DO 20 J=1,N
      IF(X(J).EQ. 0) RETURN
   20 CONTINUE
      FLAG=1
      RETURN
      END


      SUBROUTINE FEAS1
      IMPLICIT INTEGER (A-H,O-Z)
      COMMON A(50,50),B(50),X(50),V(50),ENTER(50),Z(50),ZT,M,N,ZMIN,IVT
      DO 20 J=1,N
      IF (X(J).NE.0) GO TO 20
      DO 10 I=2,M
      IF(B(I).GT.0) GO TO 10
      IF(A(I,J).LT.0) GO TO 20
   10 CONTINUE
      X(J)=-1
   20 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE OPT1
      IMPLICIT INTEGER (A-H,O-Z)
      COMMON A(50,50),B(50),X(50),V(50),ENTER(50),Z(50),ZT,M,N,ZMIN,IVT
      DO 20 J=1,N
      IF(X(J).NE.0) GO TO 20
      IF(A(1,J)+ZT.LT.ZMIN) GO TO 20
      X(J)=-2
   20 CONTINUE
      RETURN
      END


      SUBROUTINE FEAS2(FLAG4)
      IMPLICIT INTEGER (A-H,O-Z)
      COMMON A(50,50),B(50),X(50),V(50),ENTER(50),Z(50),ZT,M,N,ZMIN,IVT
      FLAG4=0
      DO 20 I=2,M
      SUM=0
      IF(B(I).GE.0) GO TO 20
      DO 15 J=1,N
      IF(X(J).EQ.0.AND.A(I,J).LT.0) SUM=SUM+A(I,J)
   15 CONTINUE
      IF(SUM.LE.B(I)) GO TO 20
      FLAG4=1
      RETURN
   20 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE OPT2
      IMPLICIT INTEGER (A-H,O-Z)
      COMMON A(50,50),B(50),X(50),V(50),ENTER(50),Z(50),ZT,M,N,ZMIN,IVT
      VMAX=-9999999
      DO 30 J=1,N
      IF(X(J).NE.0) GO TO 30
      VT=0
      DO 20 I=2,M
      VTT=B(I)-A(I,J)
      IF(VTT.GT.0)VTT=0
   20 VT=VT+VTT
      IF(VT.LE.VMAX) GO TO 30
      VMAX=VT
      IVT=J
   30 CONTINUE
      IF (VMAX.LE.-9999999) STOP
      RETURN
      END
```

SCHEDULING FUEL-SHUFFLING OPERATIONS FOR A NUCLEAR POWER REACTOR

by

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(ABSTRACT)

During the refueling of a pressurized water nuclear reactor, a group of operations termed the fuel shuffle is performed. Certain fuel and control components are removed from or inserted into the reactor core. Specific assemblies are transferred to new, predetermined locations within the core. An analysis of the problem of scheduling these fuel-shuffling operations is presented. The objective is to minimize the shuffle completion time in order to ensure that the fuel shuffle does not delay reactor startup.

A heuristic scheduling procedure currently employed in industry involves the decomposition of the problem into groups of related operations. A computer program is used for the identification of the groups in a given shuffle and for the application of scheduling rules for the individual groups.

To provide improvements over existing techniques, attention is focused on the development of heuristic methods for scheduling these groups of operations. These groups are also combined, and an integer programming model for this integration process is presented. Three sample shuffles are considered, and an analysis of effective scheduling techniques is conducted for a representative set of fuel-handling equipment.

Efforts are made to eliminate the unnecessary equipment moves and idle time present in existing schedules. Improved, near-optimal schedules are developed, and the potential benefits gained from future efforts to reduce the shuffling time are shown to be insignificant.