

AN EXPERIMENT IN PROGRAMMED BUSINESS MATHEMATICS
AT EASTERN MENNONITE HIGH SCHOOL
HARRISONBURG, VIRGINIA

by

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CHAPTER I

THE PROBLEM

Introduction

Our contemporary world with its rapidly expanding population, its rapid advances in many fields of learning, and its tremendous demands upon young people to fill responsible positions in business and the professions requires well-trained students equipped with the essential tools of communication, computation, the ability to get along well with other people, and a keen awareness of the importance of keeping abreast with the times.

It is to the point of sharpening the basic mathematical tools that this thesis speaks. The rapidly increasing school population, the lack of qualified teachers, the crowded classrooms, and the tight schedules call for paring courses of nonessentials and packing the most "meat" into the courses that are offered. There must be maximum efficiency in subject-matter presentation.

Educators are interested in this challenging problem. They have introduced ideas such as team teaching, the pooling of teacher-skills for the enrichment of courses; more extensive use of audio-visual media; and, one of the newest ideas of all, programmed instruction.

The lead article in Programed Instruction,¹ May, 1961, declares

The school year 1960-1961 can be marked down as the year when programed instruction got out of the psychological laboratories and into the classrooms. In school systems all across the nation, children and teachers received their first exposure to an embryonic but immensely promising teaching technique.²

A Definition of Programmed Instruction

Briefly stated, programmed instruction is a teaching technique in which a body of information is broken down into small parts which are presented to the student one at a time in a carefully constructed sequence. The student must make a response by writing the answer on an answer sheet or tape. The student finds out immediately whether his answer is correct by turning a page of the programmed text, moving a mask, or flicking a lever on a teaching machine. This immediate reinforcement helps fix the fact in his mind. Then another step is presented, and he must apply what he has learned earlier in the sequence and respond to the new question.

Thus, by presenting small steps in careful sequence, the student is enabled to move along at his own pace; he is not embarrassed by the fact that he may be slower than his classmates, nor is the brighter student hampered by his slow-learning classmates.

¹The Center for Programed Instruction, Inc., consistently uses only one "m" in the word "programed." All quotations taken from its official organ, Programed Instruction, will follow that spelling, but otherwise the word will be spelled "programmed."

²"Out of the Laboratory, into the Classroom," Programed Instruction I (May, 1961), 1.

The technique of programming is intended to enable the student to be right most of the time and to correct him immediately when he is wrong.

Psychologists are convinced that when a student's right answer is immediately confirmed, he will be more likely to make that right answer again the next time the question comes around. You are able to help the student also by immediately correcting a wrong answer (which you can't do immediately in a test or a homework assignment)--but you help him a great deal more if he finds the right answer at the first crack.³

This is obviously superior to the procedure followed in most classrooms of giving tests or homework assignments in which the student does not find out until the next day or week what he had wrong. It is physically impossible for a classroom teacher to give immediate confirmation of all test questions. Programmed material can and does provide that immediate confirmation.

Furthermore, programmed instruction requires the student to make a response. He cannot remain impassive to the material. He must think about it and act upon it.

Determining the Value of Programmed Instruction

Douglas Porter wrote in 1957:

Modern mass education has to minister an expanded curriculum to increasing numbers of students with a wide range of individual differences in scholastic ability. . . . If one of the values of American education is to remain the achievement of individual excellence, some method has to be found to permit more attention to the individual student.

³"Programing for Your Classroom," Ibid., p. 3.

One partial solution . . . is concerned with increasing efficiency of the teaching process itself. . . . Teaching efficiency might be increased by two closely related measures: (a) the application of some demonstrably valid psychological principles derived from "learning" experiments and (b) the use of teaching aids and devices.⁴

The years since 1957 have, indeed, proved that educators are taking seriously the challenge of finding and using new teaching aids and devices. But each new teaching technique must first prove its worth.

Programmed instruction is moving rapidly into the educational picture, but it is still new and it must be tested. Kenneth Komoski asks a few pertinent questions that must be faced with reference to the ultimate value and place of programmed instruction in education:

Is programed instruction better than what we now have? . . . Even if it can produce immediate results will its long range effects be desirable? Furthermore, how should it be used? Where does programed instruction fit in a total educational program? As fascinating, valid, and important as such questions may be, they are clearly going to be answered, not through debate, but by judicious use of and careful experimentation with PROGRAMS in schools.⁵

A writer from the Center for Programed Instruction, Inc., has this to say about programmed instruction:

. . . Programed instruction is the systematization of an excellent teaching procedure. But it is not a gimmick, or a superficial technique for applying something teachers

⁴Douglas Porter, "A Critical Review of a Portion of the Literature on Teaching Devices," Teaching Machines and Programed Learning, A Source Book, ed. A. A. Lumsdaine and Robert Glaser (Washington: Department of Audio-Visual Instruction, N. E. A., 1960), p. 114.

⁵Kenneth Komoski, "Programed Instruction--Expandable or Expensible?" Programed Instruction I (October, 1961), 3.

have known for a long time. In many ways, programing is fundamentally new. It originated, not in classrooms, but in the laboratories of experimental psychologists, and its foundations lie deep in the development of learning theory.⁶

The technique of programming corroborates Bernard's theory of the facilitation of insight as it relates to problem solving: "Give the pupil some help. Offer suggestions, hints, clues. Show him how to take a specific step. Arrange a sequential approach."⁷

The Center for Programed Instruction, Inc., has some words of warning about programming:

Programing, like all good teaching, requires great patience. There are no perfect programs yet. Even the published programs will go through a lot of revision in years to come, as their authors readily admit. Programing, despite some publicity to the contrary, is not a fully matured technology; it is still growing.⁸

The purpose of this thesis, then, is to contribute a small share to the immense amount of testing and refining of self-instructional material that must be done before programmed instruction comes fully into its own.

⁶"Programing for Your Classroom," Programed Instruction I (May, 1961), 3.

⁷Harold W. Bernard, Psychology of Learning and Teaching (New York: McGraw-Hill Book Company, Inc., 1954), pp. 142, 143.

⁸"Programing for Your Classroom," Programed Instruction I (May, 1961), 4.

Statement of the Problem

The main concern of this thesis is to find out what errors are made and what difficulties are encountered by individual high school students as they complete a specified section of the text, Programmed Business Mathematics⁹ which has been specifically prepared for post high school and college freshman students. If the program is adjusted successively after testing several students individually so that each group of students makes fewer errors in learning, what changes must be made? How are these changes classified?

Need for the Study

A study of current population and employment trends makes two facts clear; namely, the number of high school students is growing and will continue to increase, and the number of clerical workers is also increasing. Therefore, new and better techniques need to be developed to help the high school student gain and retain the maximum amount of basic mathematical knowledge.

It is obvious that the students who will need and profit most from business mathematics are those who will become clerical and sales workers. A sizable number of high school graduates go immediately into clerical-type jobs or to other semi-skilled labor.

⁹Harry Huffman, Programmed Business Mathematics (New York: Gregg Publishing Division, McGraw-Hill Book Company, Inc., 1962)

In most of these beginning positions, they are expected to have reasonable facility in handling arithmetical computations and in understanding and using basic mathematical terms and concepts.¹⁰

The Counselor's Business Education Newsletter, May, 1962, reports:

Applicants for unskilled jobs are overabundant; there is a shortage of qualified people in technical, scientific, engineering, and clerical fields. These trends will increase in the next decade. Skilled, trained and educated workers will be in demand.¹¹ (Joint statement of the United States Department of Labor, the Office of Education, and the Department of Defense.)

As further proof of the need for high schools to train more clerical workers, this item from U. S. News & World Report cites these expected employment trends for the remainder of this decade.

Clerical workers are expected to increase 30 per cent, to 12.8 million; sales workers, increase of 23 per cent, to 5.4 million workers; and service workers to increase by 34 per cent, to 11.1 million.¹²

These nearly 30 million workers will depend daily on a workable knowledge of arithmetical fundamentals.

A noted business educator, Herbert A. Tonne, points out that much general clerical work can best be learned on the job,

¹⁰Leslie J. Whale, "The Importance of Business Arithmetic in the Clerical Program," The Clerical Program in Business Education, The American Business Education Yearbook, Vol. 16 (Somerville, New Jersey: Somerset Press, 1959), pp. 115-119.

¹¹Gregg Publishing Division of McGraw-Hill Book Company, Inc., (New York), p. 4

¹²U. S. News & World Report, March 11, 1963, p. 99.

and that there is such a large variety of duties that are possible that "it is impossible for the school to give complete and specific training on a pre-employment basis."¹³ But having said that, he makes clear that there are some specific areas of skills that should be learned in school; and one of them is the area of arithmetical skills:

It is incumbent upon the school to train students who are likely to obtain clerical positions in the skills most frequently needed by such workers. . . . Students who are prospective clerks should be more adept in fundamental arithmetic than other students since most of them will use arithmetic constantly in their daily work. The arithmetical training given these students need not be highly diversified, but the simple processes should be thoroughly mastered. (*Italics mine.*)¹⁴

The National Business Education Association in an official pronouncement had this to say about the business education curriculum in high schools:

We believe that the high school curriculum should include sequences of courses that adequately prepare youth for initial employment in business.¹⁵

Tonne mentions another angle of the problem; namely, the possibility of wasting time on the secondary level by presenting training in reading, writing, and arithmetic as if the student

¹³Herbert A. Tonne, Principles of Business Education (Third ed.; New York: Gregg Publishing Division, McGraw-Hill Book Company, Inc., 1961), p. 319.

¹⁴Ibid., p. 326.

¹⁵"This We Believe about Business Education in the High School," Business Education Forum, January, 1963, p. 2.

had never had them before. He says that "this relearning from scratch is not quite as unwise as ignoring the problem altogether; but it is wasteful."¹⁶

Programmed teaching, in a very unique way, offers a solution to the problem of reinforcing the arithmetical skills which will surely be needed in any clerical job that the high school graduate undertakes. Without forcing all students into a lockstep situation where everyone has to cover the same ground whether or not he needs it, programmed instruction provides an opportunity for students to learn or relearn arithmetical fundamentals on an individual basis. Students who are already skilled in the arithmetical fundamentals would find programmed material a good review if they need review; but, in any case, students work through the programmed material at their own rate of speed.

Writing in the same vein, J. Lloyd Trump, a member of The National Association of Secondary-School Principals, asserts:

The present high standards of American junior and senior high schools result from experimentation and adaptation of new ideas. Continued changes are essential to insure superiority in a competitive world.

The needs of schools should be worked out in great detail so that facilities and equipment

¹⁶Tonne, p. 53.

can be designed to satisfy the specific requirements:
. . . Greater use of automatic devices to perform all
or part of a considerable number of teaching activi-
ties. . . Provide teaching machines for independent
use by students.¹⁷

In light of the projected figures showing a great increase in clerical and related jobs during this decade, the need for this study may be summarized as follows:

1. The need of students for adequate, usable mathematical knowledge to enable them to make their initial entrance into clerical and related jobs requiring basic mathematical skills.

2. The need for a method of self-instruction that will enable students to move along at their own pace in learning or relearning these skills; thus releasing teachers for other teaching activities.

3. The need for a significantly large amount of testing and experimentation with the new teaching technique known as programmed business mathematics to see how it can help to fill the need of training future clerical workers on the secondary level.

¹⁷New Directions to Quality Education, The Secondary School Tomorrow (Washington), pp. 3, 11.

Objectives of the Study

The specific objectives of this study are:

1. To try out Programmed Business Mathematics with individual high school students at Eastern Mennonite High School in order to obtain data about their responses.

2. To ascertain student reactions and comments that would help to pinpoint causes of errors and thus serve as a guide to the writer in editing the frames.

3. To adapt the frames for use on the high school level by rewording frames, by deletions or additions of explanatory material, or by rearranging the sequence of frames if it seems advisable.

4. To determine where new frames should be added to simplify the introduction of new concepts, or to determine which, if any, frames are superfluous and should be dropped entirely.

5. To continue the above processes of testing, adapting, deleting, or adding until the error limit has fallen to five per cent or less.

Delimitations of the Study

1. The portion of the programmed material that was used was limited to Business Fundamentals of addition, subtraction, multiplication, and division, (the first 239 frames of the set).

2. High School students of Eastern Mennonite High School (excluding eighth graders) were chosen for testing.

3. There was to be only one student working on any one

part of the test at a time and he or she was carefully supervised by the writer.

4. There were no limitations upon age, I. Q., or class of the participating students except the limitation listed in point 2 above.

5. The final basis of participation was that each participating student had to take the Schorling, Clark, Potter Hundred-Problem Arithmetic Test,¹⁸ to work through all of the 239 more or less frames, and to have a conference with the teacher at the conclusion of the testing.

Definition of Terms

1. Participating students are referred to as student-editors or students when general reference is made to them. They are referred to by first name only, for the sake of clarity, when it is necessary to differentiate between them.

2. Programmed Instruction or program refers to a sequence of items arranged in easy steps leading gradually from the simple to the more difficult and always building upon and referring to what has gone before. As soon as the student makes a response to the question, he is immediately reinforced by looking at the correct answer.

¹⁸Raleigh Schorling, John R. Clark, Mary A. Potter, Hundred-Problem Arithmetic Test, Form W, (Yonkers-on-Hudson: World Book Company, 1944).

3. Frames, A frame is one segment of programmed information consisting of: one small step of information, or a statement, or a question to be answered, or a problem to be solved, or a combination of these. It may call for one response or several responses.

4. Cue or prompt refers to a device, such as underlining, that suggests the correct answer for the purpose of assisting the student-editor in choosing the right answer.

5. Reinforcement or answer is used interchangeably to refer to the correct answer or response to the question or problem.

6. Writer refers to the author of this thesis. At times, for the purpose of clarity, she is called the teacher.

7. Student-writers refers to students of Dr. Harry Huffman who participated in the Directed Study during the summer of 1961 at Virginia Polytechnic Institute.

CHAPTER II

REVIEW OF LITERATURE

Early Attempts to Produce Self-Instructional
and Test-Scoring Devices

In 1926 Professor Sidney L. Pressey of the Ohio State University published an article describing "a simple apparatus which gives tests and scores--and teaches."¹ He had exhibited his device at meetings of the American Psychological Association in 1924 and 1925. Pressey was ahead of his time in some respects. He saw that teachers were overburdened with teaching chores such as drill, paper grading, and giving tests. He wished to free the teacher from some of this burdensome, time-consuming drill and testing by inventing a mechanical device that would handle testing and scoring and relieve the teacher for the more creative, inspiring activities that are her real function.² He felt that labor-saving devices have as much place in education as in any business enterprise.

¹S. L. Pressey, "A Simple Apparatus Which Gives Tests and Scores--and Teaches," Teaching Machines and Programmed Learning, ed. A. A. Lumsdaine, and Robert Glaser (Washington: National Education Association of the United States), p. 35.

²Ibid., p. 41.

Therefore, the mechanical device that Pressey designed in the 1920's was to administer and score objective tests. He hoped that teachers could be relieved of some of their routine drill activities, that students would benefit by having immediate knowledge of results of taking a test, and that the machines would implement several laws of learning.

Pressey's Multiple-Choice and Test-Scoring Devices

The multiple-choice, immediate feedback device of Sidney Pressey was a very simple one. It was about the size of a portable typewriter. On one side was a window in which multiple-choice questions appeared one at a time. On the other side were four keys. The student pressed the key corresponding to the answer that he thought was correct. The pressing of the key operated to turn up a new question. The apparatus also counted the number of correct responses. A test sheet in the device recorded the scores. This was the testing phase of the machine.

To use as a teaching mechanism, the machine could be reset so that a new question would not roll up until the correct answer to the present question was made. Thus, the student always had to end up with the correct response. The machine could even be set to reward the student with a piece of candy if he attained a prearranged number of correct responses.³

³Ibid., pp. 35-37.

Pressey's Device for Automatic Teaching of Drill Material

Pressey was not satisfied to stop with his first machine, but applied himself next to inventing a device which would "omit a question from further presentation as soon as the subject has obtained the correct answer in succession."⁴ This device consisted of a revolving drum and four keys to press for indicating the answers. The questions were on a paper which was fastened inside the drum. When the right key was pressed, the drum moved to the next question. When the wrong key was pressed, the drum did not move and would not until the right key was chosen. When a question was answered correctly the first time on the first two rounds, the item was thrown out, and the learner's attention was concentrated on items which were more difficult for him.

Dr. Pressey defended the soundness of his new device by pointing out its significance in relation to the laws of learning. The "law of recency" operated to establish the correct answer in the mind of the learner since always the last answer chosen (before the drum revolved to the next question) was the right answer. The "law of frequency" provided for many correct responses because making a correct response was the only way the learner could go on to the next question, since whenever a wrong response was made it had to be compensated for by a correct response. The "law of

⁴S. L. Pressey, "A Machine for Automatic Teaching of Drill Material," Ibid., p. 42.

exercise" automatically functioned to establish the right response. The "law of effect" operated by penalizing wrong responses and by rewarding two consecutive right responses by the elimination of that question. Again, as with the earlier device, Pressey emphasized that "most important of all--there is that individual and exact adjustment to difficulty."⁵

Pressey's Test-Scoring and Tabulating Device

In 1932 Dr. Pressey reported a new mechanical device for very rapid tabulating by item as well as scoring and totaling tests. In this there was an answer unit on which the student could indicate his answers. The answer unit was a strip of thin cardboard $1\frac{3}{4}$ inches wide by 11 inches long. There was a space at the top where the student could write his name and below that were 30 rows which each had five $\frac{3}{16}$ -inch circles. The student had a hand-punch and answered the questions by punching through the appropriate circle. In scoring these cards, they were placed on a metal platen of the same size which also had 150 holes corresponding to the ones on the card. Pins were inserted in the "right" holes and were released by spring action to project up from the platen if the student punched the right hole. If the student did not punch the right hole, the cardboard held down the pin so it could not come up when

⁵Ibid., pp. 44, 45.

the card was scored. The cards could be scored by placing them on the platen, where projecting pins activated a counter to count the number of correct answers. If the student had not answered correctly, the pin was held down and could not trip the pinion and engage the mechanism that operated the counter.⁶

Pressey's Punchboard

In the late 1940's Dr. Pressey, working under the sponsorship of the Special Devices Center of the Office of Naval Research, in cooperation with The Ohio State University Research Foundation, developed several new devices, one of which was the "punchboard."

The punchboard was 3 by 5 inches in size and about one-half inch thick. It had a plyboard center one-fourth inch thick with two thin sheets of pressboard fastened on top with sufficient space between to insert slips of paper. Between the top sheet and the second sheet, the student could insert a paper slip, and between the second sheet of the pressboard and the plyboard center, he could insert a punched key. The student was then ready to answer the questions which were on a mimeographed or duplicated sheet. Two columns of 15 four-hole items made possible 30 questions on a side or a total of 60 questions using both sides, and each question had four choices.

⁶S. L. Pressy, "A Third and Fourth Contribution Toward the Coming 'Industrial Revolution' in Education," Ibid., pp. 48, 49.

To take the test, the student used a pencil to punch a hole at the spot he chose as the right answer. If he was right, his pencil went through the paper and down into the hole in the key sheet. If he was wrong, his pencil came against the key sheet, but not through a hole; so the student knew he was wrong and tried another hole, proceeding in this way until he found the right answer.

After taking the test, the student could withdraw his answer sheet and see which questions he had to punch more than once to find the right answer, and this sheet could be saved as a record of his work. The keys could be turned over or end for end thus having four answer patterns.⁷ So much for the mechanics of the punchboard. But what about its value in line with Pressey's objective of learning?

Experimentation was carried on with a variety of subject matter--Russian vocabulary (as an example of rote material more valuable than nonsense syllables, and indicative of possible values of self-instructional tests in learning a foreign vocabulary), hard English vocabulary items, and subject matter of two psychology courses. Psychology classes at Ohio State University were used for experimentation. It is beyond the scope of this thesis to present a detailed report of Pressey's work with punchboards, but Dr. Pressey

⁷S. L. Pressey, "Development and Appraisal of Devices Providing Immediate Automatic Scoring of Objective Tests and Concomitant Self-Instruction," Ibid., pp. 70-72.

lists some major conclusions that were significant:

1. It has demonstrated a simple way to telescope into one single, simultaneous process the taking of a test, the scoring of it, the informing of students as to their errors, and their guidance in the finding of the right answers.
2. The investigation has shown that such a telescoped testing process, which informs each student immediately as he answers each question whether his answer is correct and which guides him to the right answer when he is wrong, does indeed transform test taking into a form of systematically directed self-instruction. . . . As with other teaching aids, self-instructional tests must be made a carefully planned part of the total instructional program if their values are to be adequately realized. Occasional, incidental use of these tests must not be expected to show their real potentialities.
3. The investigation has shown that when the self-instructional tests were used systematically in college courses as an integral part of the teaching method, gains were substantial and sufficiently generalized to improve understanding of a topic as a whole--even help on related topics. Thus used, the punchboard test brought better work in regular classes, as shown by higher scores on midterm tests and final examinations. . . . The device was found especially valuable with superior students.
4. The punchboard was found to be simple and convenient even for student self-use and could readily be made yet more compact and convenient. However, the total project has shown that there are various promising means for automatic scoring and self-instruction.⁸

Pressey's Drum Tutor

This device was a small box with a window showing the number of the test question, four keys, and an error window which showed the

⁸Ibid., pp. 86, 87.

cumulative count of errors made. If the student answered a question correctly, the number of the question changed in the number window. If he was incorrect, the error count increased while the question number remained the same. The Drum Tutor had two apparent advantages over the punchboard: "(a) Each student constantly has before him a cumulative count of his errors and a record of the number of errors for each trial, and (b) repeated runs through a test are easy because the students quickly learned to reset their machines for the next trial."⁹

This quotation well summarizes Pressy's point of view:

If test materials or simple testing devices could be developed so that as a student answered each question, that answer was immediately and automatically scored and recorded as right or wrong, then clearly much trouble would be saved. . . . If he is weak on certain points, the test should locate them and aid in the remedying of these weaknesses. And this should be done promptly. Self-scoring devices inform the student immediately--when he indicates his answer to an objective question--whether that answer is correct; if it is wrong, he is at once automatically guided to the discovery of the correct answer.¹⁰

Later Contributions to the Idea of Self-Instruction

Dr. B. F. Skinner, Professor of Psychology at Harvard University, published a paper in 1954, telling about his teaching

⁹Avery Livingston Stephens, "Certain Special Factors Involved in the Law of Effect," Ibid., p. 92.

¹⁰Pressey, Ibid., p. 69.

machine. The Skinner philosophy of learning is based on laboratory experiments in which scientists and psychologists studied behavior in animals. Skinner found that in teaching animals to respond "the animal has to be active, not passive; it has to be rewarded the moment it makes the correct response; and it must be taught by a progression of minute easy steps."¹¹ Skinner planned his teaching machines with this data in mind.

Skinner put great stress on the importance of discovering and controlling the variables of which learning is a function. An organism learns or can be taught by producing changes in its environment. Skinner discovered that by Arranging appropriate "contingencies of reinforcement," specific forms of behavior could be brought about by specific kinds of stimuli. Douglas Porter states this idea well when he writes: "The relationship between behavior and a reinforcement, by virtue of which the behavior is strengthened is called a 'contingency of reinforcement.' In order for contingencies of reinforcement to be effective, however, certain conditions must be met:

- (a) Reinforcement must take place immediately after a response has been made. A time interval of more than a few seconds between response and reinforcement greatly reduces the effectiveness of the reinforcement.

¹¹Gay Gaer Luce, "Can Machines Replace Teachers?" Reader's Digest (December, 1960), p. 101.

- (b) The reinforcement must be made precisely contingent upon performance of the behavior that is being taught. . . .
- (c) A sufficient number of reinforcements must be given."¹²

In his 1954 paper, Skinner stated:

The whole process of becoming competent in any field must be divided into a very large number of very small steps, and reinforcement must be contingent upon the accomplishment of each step . . . by making each successive step as small as possible, the frequency of reinforcement can be raised to a maximum. . . .¹³
Mechanical and electrical devices must be used.¹³

James Holland, also in the psychology department of Harvard, attempted to show how teaching machines are an application of principles from the laboratory. He calls this "a new technology-- a behavioral engineering of teaching procedures," and further asserts that "the old, defunct explanatory concepts of knowledge, meaning, mind, or symbolic processes have never offered the possibility of manipulation or control; but behavior, verbal or otherwise, can be controlled with ease and precision."¹⁴ Holland feels that machines, as such, should not obscure the really important part of this new technology which is the application of these methods of behavioral control to developing programs for teaching. Holland lists what he considers the important principles of learning:

¹²Douglas Porter, "Teaching Machines," Teaching Machines and Programmed Learning, Ibid., pp. 207, 208.

¹³Skinner, "The Science of Learning and the Art of Teaching," Ibid., pp. 108, 109.

¹⁴James G. Holland, "Teaching Machines: An Application of Principles from the Laboratory," Ibid., pp. 215, 216.

1. Immediate reinforcement.
2. Behavior is learned only when it is emitted and reinforced. (a high density of correct items is necessary to avoid punishing effect of errors).
3. Gradual progression to establish complex repertoires. (Making the student correct as often as possible, is the fastest way to develop a complex repertoire).
4. Gradual withdrawal of stimulus support, "fading."
5. Control of the student's observing and echoic behavior.
6. Discrimination training. This is used in abstraction or concept formation.
7. Let the student write the program.¹⁵

Combining the Skinner and Holland principles, it is possible to describe the Skinner teaching machines as to mode of operational characteristics which they possess: (key ideas are underlined)

1. They present a sequence of problem materials one item at a time.
2. The machines require that the student compose his response (constructed response) rather than select it from a set of alternatives.
3. They provide some means by which a student can indicate or record his solution to each item.
4. They immediately confirm the correctness of each response.
5. The student must pass through a carefully designed sequence of steps, programming, each step being so small that it

¹⁵Ibid., pp. 219-225

can always be taken; and each moving toward a desired end-product behavior.

These teaching-machine characteristics have some important implications for the classroom. Skinner and Holland would like to see these effects coming about in the classroom.

1. Constant interchange between program and student.
2. Machine induces sustained activity.
3. A very carefully graded sequence of problem materials can be presented with assurance that a student will have mastered earlier prerequisite items before attempting the more complex parts.
4. Machine helps the student to come up with the right answer by the orderly construction of the program and in part by techniques of hinting, prompting, suggestion.
5. Since the machines automatically score each response, the teacher is freed from having to correct many papers and may concentrate on arranging discussion, guiding social learning, helping students who have special difficulties and other activities which the machine cannot do.
6. The student is not limited in his study by the amount of time a teacher has to guide his learning activities. . . . A large number of responses can be made to a given body of content, and the content can be enriched with topics which there would otherwise be no time to cover.¹⁶

¹⁶Douglas Porter, "Teaching Machines," Ibid., pp. 208-210.

Skinner Teaching Machines

One representative kind of Skinner teaching machines has two windows. The material for study is printed in 30 radial frames on a twelve-inch disk. The student inserts the disk and closes and locks the machine. After the student begins working, the machine cannot be unlocked. One frame (except for one corner) is visible through a window. The student must make a response by writing his answer on a tape which is exposed through a second opening. He then lifts a lever on the front of the machine, moving what he has written under a transparent cover and moving into view the correct response in the remaining corner of the frame. If he has responded correctly, he moves the lever horizontally. This movement punches a hole in the tape opposite his response, recording the fact that he called it correct. Also this regulates the machine so that the frame will not come into view the second time the disk is used.

Even if his response was not correct, a second frame appears when the lever is returned to its starting position, and he proceeds with the next frame. On the second round of the disk, the student works only with those frames he missed the first time. This cycle is repeated until the disk revolves without stopping; this is a signal that the assignment is finished.¹⁷

¹⁷B. F. Skinner, "Teaching Machines," Ibid., pp. 141-143.

Card-Sort Device

J. Leslie Briggs invented a Card-Sort Device in which items were printed on separate cards. Items appear in the window and then the card is sorted into one of two retaining wells depending on whether it was answered correctly or not. Each item may have from two to four possible answers. The device continuously discriminates right from wrong answers; and on each succeeding trial, the student works only with those items previously missed until no cards are left in the "wrong" deck. Red and green lights above each answer key furnish the knowledge of results after each response. If the red light goes on, it stays on until the next card is presented.¹⁸

There are many other teaching machines. One, the Subject-Matter Trainer, is designed to permit practice under several modes of operation. Some forms of machines combine written-response tape synchronized with slide projector presentations. All of these devices come under the classification of stimulus-response devices, and they also have the important characteristic of being "immediate reinforcers."

Programmed Textbooks

Although teaching machines have become popular in the past decade, there is still another very promising medium of presenting

¹⁸Leslie J. Briggs, "Two Self-Instructional Devices," Ibid., pp. 302.

programmed instruction; namely, the programmed textbook. Although machines will continue to be important, it appears obvious that a programmed textbook has some advantages that teaching machines do not possess: they can be used anywhere (are not limited to the classroom), they are relatively inexpensive, and they have no mechanical parts to get out of adjustment.

The outside of a programmed textbook looks much the same as any other text, but inside the difference becomes apparent.

Each page consists of four or five panels, but the page is not read from top to bottom as in most books. Instead one reads from page to page, top panels first to the end of the chapter; then the student is directed to go back to the first page again and read the second panels of each page to the end of the chapter; then go back and pick up the third panel of each page and so on.

Beginning on page 1, the student reads and responds to the first frame. However, he must turn the page to see the answer or confirmation of his response which is on page 2. Right beside it is Frame 2, but its answer is on page 3 and beside it is Frame 3.

The interesting feature of some books is that they are printed on the right-hand side and the left page appears to be printed upside down. When the text has been covered entirely--all right-hand pages read--the book is turned upside down, and the student works through it the other way. In this way the entire book utilized, but all the writing that must be done when responding to frames is on the flat right leaves of the book.

Glaser, Homme, and Evans have this to say about programmed texts:

With respect to the main functions of presently developed teaching machines as we have described them, the programmed text is clearly capable of fulfilling these functions which depend on the structure of the program rather than upon the technique of presenting it. This means that one is faced with the same problems in constructing a program, whether it be for a machine or a programmed textbook.

Following from both these lines of thought, i.e., textbook redesign and the development of teaching machines, the important basic problem obviously concerns the properties of the learning sequences that lead to the acquisition of knowledge. In the face of present relationships between the science of learning and the art of teaching, the solution to this problem requires experimental investigation.¹⁹

This brief look at the history of self-instructional and mechanical testing devices, teaching machines, and programmed textbooks makes clear that the concept of programming is only beginning. Exciting new possibilities lie in the future, possibilities that John Dewey would probably have approved of thoroughly in his search for "positive and humane methods"²⁰ of educational practices.

¹⁹Robert Glaser, Lloyd E. Homme, and J. L. Evans, "An Evaluation of Textbooks in Terms of Learning Principles," Ibid., pp. 438, 443.

²⁰Quoted by B. F. Skinner, "Teaching Machines," Ibid., p. 158.

CHAPTER III

PROCEDURE FOR THE STUDY

The Pilot Study

In order to apply the teaching machine concept to business mathematics, Professor Harry Huffman developed a textbook, Programmed Business Mathematics. In preparation for this text, Professor Huffman carried on comprehensive study and research. For two years he had the full-time and intensive help of an experimental psychologist, Stuart Margulies. The entire project was supervised by the Center for Programed Instruction. Dr. Huffman supervised many individual students or groups of students as they went through the Program.¹

In the summer of 1961 at Virginia Polytechnic Institute, the writer participated in a Directed Study, Bus. Ed. 600, "Programmed Instruction in Business Education," under the direction of Dr. Huffman. This directed study served as an introduction to the basic philosophy and concepts embodied in the idea of programming.

All of the students enrolled in the directed study first worked through part of Section One of the fundamentals frames. This proved to be an excellent way to get acquainted with the distinctive

¹Harry Huffman, "Some Points of View About Programming Business Mathematics," (Mimeographed paper), p. 1.

features of programming. Each student-writer was encouraged to write in comments as he went along concerning points he did not understand, wording that was not clear, copy errors that needed correction, and even suggestions for rewording frames.

Dr. Huffman had frequent conferences with his student-writers. In these sessions, the student-writers pooled their ideas for revision and evaluated the proposed changes.

The student-editors also had a taste of the painstaking work that goes into frame construction. Some of the tasks were working out problems to double check on their correctness, reading frames for grammatical correctness and coherence, and then finally retyping the frames on masters for duplication.

Each student-writer was assigned three or four college freshmen who were currently taking an introductory math course at Virginia Polytechnic Institute. The college students received 451 Fundamentals frames to work through under the supervision of the student-writers. Each college student was asked to write down his comments on the frames sheets as he went along.

The student-writers tabulated errors for their own assignees at the conclusion of the period of testing. Then, to make a more careful study possible, each student editor was assigned a certain number of frames for a more detailed study of error patterns. For this, each student-writer had access to the other student-writers' tabulations and, therefore, had a wider range of responses to work with.

A final report was required in which each student was expected to do some rewriting of the frames on which he had made a detailed study. The college students' comments were used as guides in the rewriting.

The following section is taken from the writer's evaluation of her assignees:

It was interesting to watch the students as they worked. Their reactions were not always verbal, but I could sense that they became more interested in the frames the longer they worked on them (with one exception).

It seems quite evident that the students feel that the frames are overly simple when they begin, but when they begin making mistakes, they realize that they must go more slowly. They seemed to feel that the steps, as a whole, are not too large.

The suggestions for improvement that they made had to do mostly with the mechanics of the programming. They wished for clearer print, more legible illustrations, and occasionally the layout of a frame was confusing because they didn't see all the blanks that needed to be filled in until they looked at the answers and saw they hadn't finished the frame.

I felt that the project would have meant more to Mr. L. and Mr. E. if they hadn't "prompted" themselves so often.

Since the work is so concentrated, it seems that the students should work in blocks of time not more than one hour long; then take a break and go on for a second hour if they wish. For younger students, I believe that one hour would be long enough at a time.

²Miriam L. Weaver, "Evaluation of Three Students' Work," (Typewritten report of Directed Study), p. 1.

Since one of the purposes of this thesis is to study the kinds of errors that high school students make and the difficulties they encounter in the Fundamentals frames, it would be informative to classify the kinds of errors made by college students on the same program for the purpose of comparison.

Frames 97 to 174, all in the multiplication section, were assigned to the writer for special study. Out of these, there were 16 frames missed by three or more assignees. The following table indicates the types of errors made by college students. The table shows that most errors for college students come under the heading of "careless" errors, with "computational" errors running a close second. For the most part, these students did not admit to making errors because of a lack of comprehension, but it seems likely that this must have accounted for some of the errors.

TABLE I

Tabulation of Errors made by College Students
on Fundamentals Frames 97 to 174

(Multiplication)

Classification of Errors	Number of Errors
Apparently Prompted Themselves	7
Didn't understand, so responded differently than expected	7
Careless errors	24
Added or Omitted part or all of Questions on Frame	7
Computational Errors	
Addition	1
Subtraction	4
Multiplication	9
Division	9
Decimal or comma placement errors	8

Preparations Made by the Writer

Frame Arrangement.--Before administering the set of frames to students, the writer took the duplicated sheets of the program, cut the frames apart and pasted them on five- by eight-inch cards, one frame to a card. The answer or reinforcement was written on the back of the card. The reason for placing the frames on cards was to facilitate the insertion of new cards into the sequence when new frames were added.

Securing Student-Editors.--The writer teaches Business Education courses at Eastern Mennonite High School, Harrisonburg, Virginia, and all the testing for the experiment was carried out at this place.

Students chosen for the experiment were all high school students of this school. Three freshmen, three sophomores, and five seniors participated. Three of the eleven students tested were boys and eight were girls.

These eleven students entered the experiment as an interesting and profitable out-of-class project since the writer was not able to use any one of her own classes for testing. The students gave up their only free periods of the school day in order to cooperate in the testing sessions. For this reason, the writer did not feel justified in asking them to give up additional study hall time to take retention tests although it undoubtedly would have added to the value of this experiment to administer retention tests at intervals in the program.

Preliminary Arithmetic Test.--In order to determine the students' skill in basic arithmetic fundamentals, the writer administered one form (either V or W) of the Schorling, Clark, Potter "Hundred-Problem Arithmetic Test" to each participating student just before he began working on the frames. The test consists of five sections: I. Addition; II. Subtraction; III. Multiplication; IV. Division; and V. Fractions, Decimals, and Per Cents. The results of these tests are shown on the "Student-Editor Data Summary" sheets in Chapter IV.

The Answer Tablet.--The writer provided answer tablets for the students. Spiral shorthand pads with a line down the center of the page were used for this purpose. A sample answer sheet is illustrated on the next page. Frame numbers and comments were written in the right-hand column of the page and answers were written on the left-hand side of the page. If a response was incorrect, the student placed an "x" beside the number of the frame.

Directions to Student-Editors.--The writer oriented the student-editors into the special features of the programmed material during the initial testing session. The students were told that they were important members of a team that was testing the value of a promising new teaching-learning technique.

Specific directions such as these were given: dotted lines on the frame mean filling in as many words as are needed to complete

a statement; pay especial attention to underlined words because they give hints as to how to answer questions; it is permissible to review previous frames, but at no time is one to look ahead; and finally, the writer tried to impress on the students the importance of reading the frame carefully and responding to it before turning over the card to see the answer. Because there was only one set of cards available for use, each student was instructed to make no marks or write no answers on the frames, but to write all responses on the answer tablet pages.

ILLUSTRATION I

Sample Answer Sheet

	<u>Name of Student-Editor</u>
A	1
A	2
c	3
15,540	
b	4
no	
2,400	5
25	6
hundreds	7 X Didn't understand.
\$30,000	8

Testing ProceduresProcedure of Student-Editors

Taking Arithmetic Test.--The first step in the process for the student-editor was to take the Hundred-Problem Arithmetic Test. This test was administered by the writer and scored immediately. It provided an on-the-spot picture of the student-editor's ability to solve addition, subtraction, multiplication, division, decimal, fraction, and per cent problems.

Working through the Program.--The student-editor received his answer tablet and the set of frames on cards. The writer worked through the first several frames with the student-editor to be sure that he understood the procedure he was to follow. The student read the frame, responded by writing the answer in the answer tablet, turned over the card to see if his answer corresponded to the reinforcement on the back, then placed the card face down on the table and picked up the next card and responded to that one. If the student-editor made an incorrect response, he put an "x" beside the number of that frame in the answer tablet. He was invited to write comments in the right-hand portion of the answer sheet.

At the conclusion of each testing session, the student turned in his answer tablet and the frames. These were kept in a safe place by the teacher until the next testing session.

When the student completed the program, he had a conference with the teacher. This will be discussed more fully later in the chapter.

Procedures of Writer while
Collecting the Data

The Plan of Action.--The writer felt that it would be desirable to test several students with the program just as it was before making any changes or revisions. Two students worked with the original frames on the first round of testing. The writer calculated percentage of errors and then revised the frames.

During the second round of testing, three student-editors worked through the program. Again percentages of error were computed. Four students were tested in the third round, and this was followed by a third revision. Two students were tested in the final round. By this time, the percentage of errors of the student-editors came within the error limit goal set up at the beginning of the experiment, so there were no further revisions.

Supervision and Encouragement of Student-Editors.--Since the students who agreed to enter this experiment did so voluntarily, the problem of initial motivation was not great. However, it soon became apparent that the student-editors found that the program was more difficult than they had first thought. Also, they found that they made errors on what appeared to be easy questions.

It was at this point that the writer tried to encourage them to read more carefully and to reread a frame if necessary before making a response. In this way the student-editors would be less likely to make "unnecessary" mistakes, and they would feel a greater sense of satisfaction in the work they did. The writer assured them

that if they made mistakes, it was an indication that the program needed clarification or revision and that it was not a reflection upon the student-editors.

Conference of Writer and Student-Editors.--A conference was held with one student-editor at a time upon his completion of the program. This proved to be a valuable and rewarding feature--especially for the writer. The student-editor had his answer tablet returned to him for the duration of the conference so that he could refer to it freely, and the writer had another pad handy for jotting down pertinent items. In this conference, the writer tried to ascertain the student-editor's reaction to the frames in general and how valuable he felt the experience was. Then the writer asked the student to be as frank as he wished in commenting about specific frames. Together, the conferees discussed every frame on which the student-editor had made an incorrect response. The student-editors were very alert in observing errors on the frames such as lines omitted, a zero omitted in an answer, and the like.

The students' comments on their answer sheets and during the conference are the basis for detailed reports of each student's errors. Included in these comments are general observations and remarks about errors they found on the frames. (These comment sheets are a part of Chapter IV.)

Tabulating Errors.--The writer formulated error tabulation sheets which were simple to maintain and which proved to be most helpful in the work of revision. Each sheet was blocked off with

half-inch blocks except for a numbered column down the left side of the page and space at the top for the names of the students. There was room for 20 numbers down each page and the lines were numbered consecutively from 1 to 239.

It was an easy matter for the writer to fill in a student's name above one of the columns on each page and then run down the page placing an "x" in each block which corresponded to the numbers of the frames he had missed. There was room also to jot down a brief notation about the cause of the error. This chart provided a convenient, cumulative record of incorrect responses. One advantage of the chart was the immediate visual information indicated by the pattern of "x's" which showed trouble spots that needed revision.

Frame Revision.--The writer studied the frames which were missed by student-editors to analyze the causes of errors and to determine how frames could be changed, rewritten, added, or deleted. Especial attention was given to those frames which were missed by more than one person.

CHAPTER IV

ANALYSIS OF THE DATA

Introduction to the Analysis

This chapter is divided into three main sections. The first section consists of information about the students who participated in the experiment. Most of the information is displayed in table form.

In the second section of the chapter there is a study of the kinds of errors that students made and an attempt to classify them into four categories. A classification-of-errors table is an important part of this section.

The third section of the chapter deals with the clarification, revision, addition, and deletion of frames which the writer carried out in an effort to help the students reduce the percentage of errors they made on the frames. In addition to stating the reasons for revision, the writer has shown the frames that were changed or revised--showing first the original frame, if it is needed for clarity, and then the revision. In each case that a new frame was constructed, the new frame is shown also.

Information about Student-Editors

Student-Editor Data Summary

This experiment had two basic essentials. One was the programmed material; the other was the subjects, the high school students who were willing to participate in the experiment. Since high school students are not automatons, it will add to the value of this study to know something about their individual interests and abilities. Therefore the writer compiled a limited variety of information that may help to interpret the results of the experiment. Permission was obtained from the principal of Eastern Mennonite High School to secure some information from the permanent records of the students for this report.

For the sake of clarity and uniformity, data sheets were compiled. These are displayed on the following pages and are arranged in table form. Included on each sheet are representative English and mathematics grades since both of these skills are essential for working through a program in business mathematics.

The intelligence quotient derived from the California Short-Form Test of Mental Maturity tells something of the student's capacity to learn and apply new concepts. For eight of the students, it was possible to obtain information from the Kuder Preference Record Profile Sheets. Percentiles in two areas were chosen as being relevant to this report: the computational and the clerical. This, of course, is not a test of actual ability; but as the name implies it shows the student's

interest in and preference for occupations within these vocational areas. The raw score and percentile rank on the Hundred-Problem Arithmetic Test taken just before the program testing began show the student's ability in basic computational skills.

The last category on this sheet is the record of the student's score in the business mathematics program. This shows at a glance the percentage of errors and the corresponding percentage of accuracy obtained by the student-editor.

TABLE 2

STUDENT-EDITOR DATA SUMMARY

Name of Student: Joe
 Age: 14-8
 Classification: Freshman

Scholastic Cross-Section

Year	Subject	Semester Grades	
1960-61	Algebra I	A	A
1960-61	English I	A	A

Test Scores

Date	Name of Test	Score	Percentile
9/14/60	California Short-Form Test of Mental Maturity, I. Q.	126	
3/5/62	Kuder Preference Vocational Record (Profile Sheet for Boys)		Comput. 78%-ile Cler. 11%-ile
11/4/61	Hundred-Problem Arithmetic Test	93	97%-ile

Score in Programmed Material

Frame Sequence	Number of Errors	Per Cent	
		Errors	Accuracy
Original	45	10%	90%

TABLE 3

STUDENT-EDITOR DATA SUMMARY

Name of Student: John
 Age: 17-6
 Classification: Senior

Scholastic Cross-Section

Year	Subject	Semester Grades	
1961-62	Bookkeeping I	C	B
1961-62	English IV	B	C

Test Scores

Date	Name of Test	Score	Percentile
11/61	California Short-Form Test of Mental Maturity, I. Q.	115	
12/8/61	Hundred-Problem Arithmetic Test	76	61 $\frac{1}{2}$ -ile

Score in Programmed Material

Frame Sequence	Number of Errors	Per Cent	
		Errors	Accuracy
Original	14	3.12%	96.9%

TABLE 4

STUDENT-EDITOR DATA SUMMARY

Name of Student: Kathy
 Age: 18-1
 Classification : Senior

Scholastic Cross-Section

Year	Subject	Semester Grades	
1960-61	Bookkeeping I	B	B
1961-62	English IV	B	C

Test Scores

Date	Name of Test	Score	Percentile
9/14/60	California Short-Form Test of Mental Maturity, I. Q.	85	
3/7/61	Kuder Preference Vocational Record (Profile Sheet for Girls)		Cler. 74%-ile Comput. 33%-ile
2/6/62	Hundred-Problem Arithmetic Test	82	74%-ile

Score in Programmed Material

Frame Sequence	Number of Errors	Per Cent	
		Errors	Accuracy
First Revision	51	11.28%	88.7%

TABLE 5

STUDENT-EDITOR DATA SUMMARY

Name of Student: Mary Ethyl
 Age: 17-11
 Classification: Senior

Scholastic Cross-Section

Year	Subject	Semester Grades	
1961-62 1961-62	Bookkeeping I English IV	B A	B E

Test Scores

Date	Name of Test	Score	Percentile
11/61	California Short-Form Test of Mental Maturity, I. Q.	106	
2/6/62	Hundred-Problem Arithmetic Test	72	54%-ile

Score in Programmed Material

Frame Sequence	Number of Errors	Per Cent	
		Errors	Accuracy
First Revision	66	14.60%	85.4%

TABLE 6

STUDENT-EDITOR DATA SUMMARY

Name of Student: Judy
 Age: 15-1
 Classification: Freshman

Scholastic Cross-Section

Year	Subject	Semester Grades	
1960-61	Eighth-Grade Math	A	A
1961-62	English I	A	A

Test Scores

Date	Name of Test	Score	Percentile
11/61	California Short-Form Test of Mental Maturity, I. Q.	112	
3/14/62	Kuder Preference Vocational Record (Profile Sheet for Girls)		Comput. 88%-ile Cler. 97%-ile
5/4/62	Hundred-Problem Arithmetic Test	66	45%-ile

Score in Programmed Material

Frame Sequence	Number of Errors	Per Cent	
		Errors	Accuracy
First Revision	40	8.85%	91.2%

TABLE 7

STUDENT-EDITOR DATA SUMMARY

Name of Student: David
 Age: 15-3
 Classification: Sophomore

Scholastic Cross-Section

Year	Subject	Semester Grades
1961-62	Algebra I	A
1961-62	English I	A

Test Scores

Date	Name of Test	Score	Percentile
2/6/63	California Short-Form Test of Mental Maturity, I. Q.	120	
10/18/62	Hundred-Problem Arithmetic Test	93	97%-ile

Score in Programmed Material

Frame Sequence	Number of Errors	Per Cent	
		Errors	Accuracy
Second Revision	3	.63%	99.3%

TABLE 8

STUDENT-EDITOR DATA SUMMARY

Name of Student: Carol Ann
 Age: 14-3
 Classification: Freshman

Scholastic Cross-Section

Year	Subject	Semester Grades	
1961-62	Eighth-Grade Math	B	A
1961-62	Eighth-Grade English	B	A

Test Scores

Date	Name of Test	Score	Percentile
11/61	California Short-Form Test of Mental Maturity, I. Q.	113	
3/2/62	Kuder Preference Vocational Record (Profile Sheet for Girls)		Comput. 36%-ile Cler. 40%-ile
8/7/62	Hundred-Problem Arithmetic Test	86	92%-ile

Score in Programmed Material

Frame Sequence	Number of Errors	Per Cent	
		Errors	Accuracy
Secnd Revision	38	8.07%	92%

TABLE 9

STUDENT-EDITOR DATA SUMMARY

Name of Student: Phyllis
 Age: 15-11
 Classification: Sophomore

Scholastic Cross-Section

Year	Subject	Semester Grades
1961-62	Algebra I	B A
1961-62	English I	B A

Test Scores

Date	Name of Test	Score	Percentile
9/14/62	California Short-Form Test of Mental Maturity, I. Q.	121	
3/2/62	Kuder Preference Vocational Record (Profile Sheet for Girls)		Comput. 29%-ile Cler. 80%-ile
11/30/62	Hundred-Problem Arithmetic Test	88	91%-ile

Score in Programmed Material

Frame Sequence	Number of Errors	Per Cent	
		Errors	Accuracy
Second Revision	7	1.48%	98.5%

TABLE 10

STUDENT-EDITOR DATA SUMMARY

Name of Student: Lois
 Age: 15-8
 Classification: Sophomore

Scholastic Cross-Section

Year	Subject	Semester Grades	
1961-62	Algebra I	B	A
1961-62	English I	A	A

Test Scores

Date	Name of Test	Score	Percentile
9/14/60	California Short-Form Test of Mental Maturity, I. Q.	119	
3/2/62	Kuder Preference Vocational Record (Profile Sheet for Girls)		Comput. 89%-ile Cler. 99%-ile
11/30/62	Hundred-Problem Arithmetic Test	84	85%-ile

Score in Programmed Material

Frame Sequence	Number of Errors	Per Cent	
		Errors	Accuracy
Second Revision	11	2.33%	97.7%

TABLE 11

STUDENT-EDITOR DATA SUMMARY

Name of Student: Esther
 Age: 18-2
 Classification: Senior

Scholastic Cross-Section

Year	Subject	Semester Grades	
1959-60	Algebra I	B	A
1961-62	English III	B	B

Test Scores

Date	Name of Test	Score	Percentile
9/14/60	California Short-Form Test of Mental Maturity, I. Q.	106	
5/2/62	Kuder Preference Vocational Record (Profile Sheet for Girls)		Comput. 40%-ile Cler. 40%-ile
3/19/63	Hundred-Problem Arithmetic Test	83	79%-ile

Score in Programmed Material

Frame Sequence	Number of Errors	Per Cent	
		Errors	Accuracy
Third Revision	20	4.10%	95.9%

TABLE 12

STUDENT-EDITOR DATA SUMMARY

Name of Student: Rachel
 Age: 17-5
 Classification: Senior

Scholastic Cross-Section

Year	Subject	Semester Grades	
1960-61	Geometry	B	B
1961-62	English III	B	B

Test Scores

Date	Name of Test	Score	Percentile
9/14/60	California Short-Form Test of Mental Maturity, I. Q.	122	
1/4/62	Kuder Preference Vocational Record (Profile Sheet for Girls)		Comput. 60%-ile Cler. 57%-ile
3/19/63	Hundred-Problem Arithmetic Test	75	64%-ile

Score in Programmed Material

Frame Sequence	Number of Errors	Per Cent	
		Errors	Accuracy
Third Revision	5	1.02%	99%

Teacher-Student Conference Comments

As indicated in Chapter III, the conference of the teacher and the student-editor at the conclusion of the testing sessions was a valuable part of the experiment. The student had his answer tablet and the teacher had a pad for jotting down notes. Since most of the students had not written down comments as they went along, the conference provided a good opportunity to complete this phase of the testing.

The comments that students made about difficulties they faced with various frames suggested to the writer possible reasons for these difficulties. One significant fact that the writer noticed was that when the teacher and student-editor together reviewed his trouble spots, he was often able to identify the cause of his error and to see at a glance what he should have had. Frequently, students remarked that they did not see why they had missed a particular frame in the first place since it seemed so clear to them when looking over it a second time.

The students' comments are compiled into individual reports on the following pages. Items enclosed in parentheses in these reports are teacher comments. At the end of each student-comment section, there is a brief statement of the teacher's evaluation of the student-editor's work and attitudes.

STUDENT COMMENTS ON FUNDAMENTALS OF MATHEMATICS

Joe (Original Sequence)

FramesStudent-Editor's Comments

ADDITION

- 31 Wrong estimation
- 39 Used "sum of 10" instead of "combination of 10."
- 40 Mistake in addition.
- 42 I didn't read carefully; thought the question said "which seems easier."
- 45 I didn't read carefully.
- 49 I was confused by previous question.
- 54 Made a mistake in addition; second answer correct.
- 57 Mistake in addition.

SUBTRACTION

- 63 Rounded to nearest 50 instead of 100.
- 70 Mistake in addition.

MULTIPLICATION

- 93 Mistake in addition.
- 98 I didn't understand the term "extension." I thought numbers go from right to left when multiplied.
- 100 Wrong addition.
- 126 The wording is misleading. The frame said "upper and lower estimates" and the answer had "lower to upper."
- 129 Had "reasonable"; don't know why.
- 132 I rounded numbers to 1,200 x 80 for lower estimate and 1,300 x 90 for upper estimate.

Joe's Comments on Fundamentals of Mathematics, (Continued)

- 133 The wording is not too clear; I had only "c" for answer and didn't multiply.
- 134 "Finding" instead of "estimating."
- 138 I omitted "to the other factor."
- 139 I goofed.
- 140 Wrong division.
- 143 Isn't there an error on card? 500 is $1/2$ of 1000, isn't it?
- 145 Wrong division.
- 149 I had one too many zeros on answer for 250×39 .
- 150 The answer was misleading. This time it omitted the words, "to other factor."
- 153 There is an incomplete answer on frame.
- 154 7,500 instead of 3,500 on third answer.
- 166 "Annexing zeros" instead of "verification."
- 167 Frame needs lines.
- 169 I didn't write down factors, but just the answer.

DIVISION

- 182 Faulty division sign on frame; it looked like an addition sign.
- 185 Frame needs answer to problem.
- 187 Error of division.
- 194 I didn't read carefully; had 32 instead of 30.
- 196 Had "estimation" instead of "trial divisor."
- 205 I had 49 instead of 4,900.
- 209 Had 214 instead of 213.

Joe's Comments on Fundamentals of Mathematics, (Continued)

- 212 Had 243 instead of 24.
215 "Quotient" instead of "dividend."
216 Had 23 instead of 24.
219 Had 300 instead of 30.
226 Error in estimation.
230 Error in finding lower and upper estimates.
237 Error in estimation.

Writer's Evaluation

Joe worked very fast and not always too accurately. Since he was the first student-editor working through the program, he cooperated very well in editing the frames for errors, omissions, poor copy, and the like. He said he enjoyed working through the program and learned some new points about estimation and shortcuts.

STUDENT COMMENTS ON FUNDAMENTALS OF MATHEMATICS

John (Original Sequence)

<u>Frames</u>	<u>Student-Editor's Comments</u>
ADDITION	
5	Too high an estimation
14	First answer was 3 instead of 2.
29	\$78 instead of \$80.
42	No comment
45	Didn't get the word "angular."
54	\$5.66 instead of \$5.96.
MULTIPLICATION	
95	Said "j" was incorrect.
111	\$3,700 instead of \$3,600.
149	3,362,500 instead of 33,625. Decimal point error.
163	Error in using the shortcut method.
175	Error in estimation.
DIVISION	
190	(The student-editor skipped this frame and simply wrote a "?" in the answer space.)
196	Had the word "estimate" instead of "trial divisor."
211	Error in estimation.

Writer's Evaluation

Although the teacher supervised John closely, she felt fairly sure that he was "prompting" himself much of the time. This student-editor had very little to offer in the way of comments.

STUDENT COMMENTS ON FUNDAMENTALS OF MATHEMATICS

Kathy (First Revision)

<u>Frames</u>	<u>Student-Editor's Comments</u>
ADDITION	
14	I didn't understand this one.
22	Estimated wrong.
39	I didn't read the last part.
44	I thought it meant only the "tens" part of the sum. I had 4 instead of 54 and thought the 5 would be carried.
45	I didn't know; couldn't remember.
48	I rounded it to thousands instead of ten thousands; added wrong.
49	I had "sub-addition" instead of "sub-total."
SUBTRACTION	
68	Didn't read carefully.
59	Had "same" instead of "same extent."
75	Didn't read carefully; added them all; didn't withdraw.
MULTIPLICATION	
91	Error in multiplication.
92	I didn't finish the problem; (she didn't show correct answer).
95	Missed two that were incorrect.
109	I had "rounding" instead of "lower"; just didn't think.
110	I had "numbers" instead of "factors"; gave lower estimate of each number instead of multiplying them.

Kathy's Comments on Fundamentals of Mathematics, (Continued)

- 114 Same error as 110.
- 116 I didn't understand this problem.
- 118 I thought it should be lower than the exact number; had 30,000 instead of 35,000.
- 128 Error in multiplication.
- 130 Didn't answer the last question on this one.
- 143 A line needs to be filled in on the frame.
- 148 Omitted last zero.
- 158 Had 10 instead of 100.
- 159 Error in multiplication.
- 162 Error in multiplication.
- 166 (She was confused and thought it was talking about checking subtraction by addition of remainder and smaller number.)
- 175 Wrong estimation.
- 177 I marked one wrong that is correct.

DIVISION

- 185 Error in canceling zeros; had 200 instead of 20.
- 189 Had "similar" instead of "measuring."
- 194 Error in estimation; had 200 instead of 30.
- 198 I thought "8 into 16" instead of "2 into 16."
- 210 Rounded the dividend incorrectly; had 32,000 instead of 320,000.
- 212 I had 20 instead of 24 on an estimated answer.
- 219 Had 31 instead of 30 in estimation.

Kathy's Comments on Fundamentals of Mathematics, (Continued)

- 221 Had "estimate" instead of "remainder."
- 223 Had "reasonable estimate" instead of "trial divisor."
- 224 I got all confused; didn't know what they wanted.
(She had trial divisor which wasn't too bad a guess.)
- 226 I estimated too closely. I thought the questions
are not consistent.
- 229 Had error in remainder.
- 230 (Student-editor did not understand this apparently;
she had "upper," in answer to the question of whether
the product lies between the upper and lower estimates.)
- 231 I thought since there is a remainder, you could make
it a decimal.
- 239 Had 8 instead of 800 for the remainder. (Actually
this is the reduced form of the remainder.)

Writer's Evaluation

Kathy worked as carefully and conscientiously as she could, but her main difficulty was with comprehension. Her low I. Q. undoubtedly made it more difficult for her to understand what she was reading.

The writer administered Form W of the Hundred-Problem Test to Kathy at the conclusion of the testing sessions on which she scored 87 and ranked in the 87th percentile as compared with her raw score of 82 and rank in the 74th percentile preceding the testing with the program.

STUDENT COMMENTS ON FUNDAMENTALS OF MATHEMATICS

Mary Ethyl (First Revision)

<u>Frames</u>	<u>Student-Editor's Comments</u>
ADDITION	
9	I misunderstood this; didn't think through it.
14	Had wrong estimation on last part.
15	I understood it, but had wrong estimation.
23	I can't remember why I got it wrong. (She did it correctly when we had our conference together.)
29	Wrong estimation.
48	Missed both additions.
57	Errors in addition.
SUBTRACTION	
62	I didn't know exactly what it was calling for; had "estimating" instead of "rounding" and "hundred" instead of "two."
68	I didn't round the thousands correctly; had 3,200 instead of 3,300.
75	I didn't read carefully and added them all instead of doing what it asked for.
76	I thought it was calling for a different word; I had "subtrahend" instead of "larger number."
MULTIPLICATION	
85	I didn't understand what was meant by 10 89's.
90	I thought it was asking for the product or answer instead of labeling these numbers as partial products.
95	Figured product for "i" incorrectly.
103	I didn't understand what was called for; had "estimation" instead of "rounding."

Mary Ethyl's Comments on Fundamentals of Mathematics, (Continued)

- 108 I rounded the larger number up instead of down.
- 109 Had "nearest" instead of "lower."
- 110 Had "numbers" instead of "factors."
- 114 I didn't understand the question and didn't work it correctly.
- (This conference was never completed, so the following comments are by the writer.)
- 115 Error in upper estimate; she rounded the amount up, but rounded the weeks down to 40; had \$4,000 instead of \$5,000.
- 116 Same type of error here as in 115.
- 117 Did not understand to what numbers are to be rounded; had tens and units instead of hundreds and tens.
- 133 Error in multiplication.
- 135 She had "numbers" instead of "factors."
- 140 Error in shortcut multiplying; she did not annex a zero.
- 144 Error in division when using multiplying shortcut.
- 146 Error in stating rule for shortcut of multiplying by 25.
- 148 Error in stating rule for shortcut of multiplying by 250. Did not answer the last question on frame.
- 149 Error in division on one part of frame.
- 154 Omitted one part.
- 163 Error in using shortcut for multiplying by 98.
- 164 Error in using shortcut for multiplying by 99.
- 165 Had "simplified" instead of "many individual."

Mary Ethyl's Comments on Fundamentals of Mathematics, (Continued)

- 166 Had "annexing zeros" instead of "verification."
167 Had "verified" instead of "reversed."
174 Marked one of the extensions wrong which is correct.
175 Error in estimation.
176 Careless error; added a withdrawal instead of subtracting.
177 Marked an extension wrong which is correct, and did not correct one that was incorrect.

DIVISION

- 187 Division error.
188 Had "add" instead of "count."
210 She rounded the dividend incorrectly.
211 Error of estimation in both problems on this frame.
219 Error of division.
223 Had "estimate" instead of "trial divisor."
225 Estimation error.
226 Error in estimation and in exact balance because she added the withdrawals.
227 Error in estimation of division.
229 Error in estimation of division.
230 Errors of estimation on both parts of answer.
231 Error of estimation.
233 Error in estimating answer and in finding the trial divisor.
238 Error in division.

Mary Ethyl's Comments on Fundamentals of Mathematics, (Continued)

239 Had 8 for remainder instead of 800 (this is right when canceling out zeros).

Writer's Evaluation

Mary Ethyl encountered the most difficulty of any of the student-editors in working through the program. She said in her comments that she was not very good in mathematics. Most of her errors could be classified as errors of estimation or rounding and not applying new concepts being introduced throughout the program. Mary had a good attitude and worked as carefully as she could, but she was weak in comprehension. There seems to be a relationship between her low score on the Hundred-Problem test and her low score on the program.

The writer administered Form W to Mary at the conclusion of the testing and she did better having a raw score of 83 and ranking in the 79th percentile.

STUDENT COMMENTS ON FUNDAMENTALS OF MATHEMATICS

Judy (First Revision)

FramesStudent-Editor's Comments

ADDITION

- 5 Probably an error in choosing an estimate.
- 17 Error in estimation.
- 31 I didn't understand; error in estimation.
- 37 Omitted the word "reverse" both places it was called for; had "addition."
- 48 Error in addition; 7 instead of 8.
- 54 Error in addition; 586 instead of 596.
- 57 Error in addition; 52 instead of 53.

SUBTRACTION

- 62 I didn't get the idea or else I didn't think; had estimating instead of "rounding."
- 70 Error in addition on first problem.

MULTIPLICATION

- 85 Read the problem wrong; "read in" the frame number.
- 92 Error in multiplication.
- 99 I didn't understand the terms "right and "left" as used, or what extended to right or left means.
- 103 I didn't understand; had "extent" instead of "rounding."
- 128 Error in multiplication in finding exact product.
- 140 Made two errors in using shortcut for multiplying by 5.
- 147 Error in dividing; annexed too many zeros.

Judy's Comments on Fundamentals of Mathematics, (Continued)

- 154 Error in multiplying 9×827 by shortcut method.
- 161 I was thinking of the "three" zeros instead of 1000 in stating rule.
- 162 Error in subtraction in using shortcut for multiplying by 999.
- 166 I didn't understand this; had "estimating products" instead of "verification."
- 175 Error in estimation.
- 176 Error in estimation.
- 177 Error in multiplication.

DIVISION

- 187 Error in division; one too many zeros in answer.
- 201 Error in division problem with zeros; 34 instead of 340 .
- 202 Wrong estimation.
- 209 Rounded dividend to 1 digit instead of 2 digits with zeros.
- 213 Chose the wrong incorrect average.
- 214 I chose the wrong incorrect average here too.
- 219 Had 31 instead of 20 for estimated answer.
- 226 Wrong estimation, but had exact answer correct.
- 227 Had an error in the estimation, but the exact answer was correct.
- 228 Error in estimated answer, but exact answer was correct.
- 229 Same as 228.
- 230 Did not get the point of the question.

Judy's Comments on Fundamentals of Mathematics, (Continued)

231 Error in estimated answer.

Writer's Evaluation

Judy said she had not had a math course since eighth grade and that she forgot how to do a lot of the simple arithmetical procedures—especially decimals, fractions and per cents. The majority of her errors on the program were in estimation and rounding.

STUDENT COMMENTS ON FUNDAMENTALS OF MATHEMATICS

David (Second Revision)

Because David missed only three, the writer had no conference with him since he indicated that he understood everything very well and had no questions or remarks.

Frames

ADDITION

- 42 In this question he checked "B" as being more difficult than "A."

MULTIPLICATION

- 97 To the question of whether dollar signs are omitted in extensions, he responded "no."
- 138 In stating the rule for shortcut in multiplying by 5, student-editor omitted the words "to the other factor."

Writer's Evaluation

David had very few comments to make other than that he enjoyed working through the program, and that he learned some new things about shortcuts that he had not known before. Although his score is the highest of anyone the writer tested, the writer feels sure that this student-editor did not "prompt" himself, but worked fast and accurately, following directions all the way through.

STUDENT COMMENTS ON FUNDAMENTALS OF MATHEMATICS

Carol Ann (Second Revision)

FramesStudent-Editor's Comments

ADDITION

- 16 I thought "barely sufficient" meant "not sufficient."
- 23 Error in estimation; had \$20,000 instead of \$2,000.
- 29 Error in rounding two different numbers to tens.
- 37 Omitted the word "reverse" in "reverse addition."
- 48 Error in one addition.
- 57 Error in addition.

SUBTRACTION

- 73 Wrong estimation; I didn't know what numbers to round to.

MULTIPLICATION

- 92 Wrong total in addition.
- 94 Error in multiplication.
- 95 I think there are too many parts to this frame. Had an error in one multiplication.
- 98 I didn't understand what "extension" means.
- 100 I forgot to add the extensions.
- 101 (Student-editor had the answer "too many beakers," which could be a valid observation, although it is wrong.)
- 132 I didn't understand the question (didn't remember how to find lower and upper estimates).
- 140 Error in using shortcut for multiplying by 5.
- 142 Careless--didn't put in decimal.

Carol Ann's Comments on Fundamentals of Mathematics, (Continued)

- 143 Careless, I don't know why I put in 300 instead of 3 zeros (The writer thinks she can see why student-editor made this mistake.)
- 149 Error in division when using shortcut for multiplying by 250.
- 159 Error in subtraction in using shortcut for multiplying by 98.
- P 161-1 I didn't know what to put. I should have gotten that right, I suppose. Error in stating rule for multiplying by 999.
- 164 Error in multiplication.
- 172 Error in estimation.

DIVISION

- 183 I don't know what I was trying to do; didn't understand how to complete the rule.
- 185 I was confused by the arrow. (Student-editor showed solution of "a" instead of "b".)
- 201 Chose the wrong answer to this estimation--too many zeros.
- 206 I tried to estimate in tens rather than in ones.
- 210 Didn't round the dividend correctly.
- 211 Error in estimating answer--too many zeros.
- 212 Estimated answer had one digit plus a zero;
- 215 Had "complete answer" instead of "dividend."
- 226 Error in rounding, but exact answer was correct.
- 230 Didn't read the question correctly. Careless answer.
- 231 Didn't read the frame carefully. Error in estimation.

Carol Ann's Comments on Fundamentals of Mathematics, (Continued)

Writer's Evaluation

Carol was not able to work consecutively through the program due to several factors beyond her control following minor surgery. The resulting nervous tension worked adversely upon her--she could not concentrate as she should have. She became discouraged about making so many "stupid" mistakes. Her good score in the Hundred-Problem test and high percentile ranking indicate that she might have done better on the program had she felt better physically.

STUDENT COMMENTS ON FUNDAMENTALS OF MATHEMATICS

Phyllis (Second Revision)

FramesStudent-Editor's Comments

ADDITION

29 Rounded to nearest ones instead of tens.

31 Error in addition.

SUBTRACTION

70 Error in addition.

76 Had "correct answer" instead of "larger number."

MULTIPLICATION

140 Error in division in using shortcut for multiplying
by 5 mentally.

DIVISION

210 Didn't round the dividend correctly.

Writer's Evaluation

Phyllis was glad for the opportunity to work through the program and said she enjoyed it and learned "a lot." She worked with a high degree of accuracy which compares favorably with her grades and test scores.

STUDENT COMMENTS ON FUNDAMENTALS OF MATHEMATICS

Lois (Second Revision)

FramesStudent-Editor's Comments

ADDITION

- | | |
|----|-------------------------------------------------------------------------------|
| 7 | Rounded off and added the numbers instead of supplying the word. |
| 17 | I made a wrong assumption and thought he would be worth \$23,000. |
| 31 | Rounded off the first number incorrectly, so the estimated sum was wrong too. |

SUBTRACTION

- | | |
|----|--------------------|
| 70 | Error in addition. |
|----|--------------------|

MULTIPLICATION

- | | |
|-----|-------------------------------------------------------------------------|
| 87 | Used the word "added" instead of "annexed." |
| 101 | I didn't understand the question and goofed on the answer. |
| 147 | Multiplied instead of dividing in using shortcut on all three problems. |
| 170 | Multiplication error. |

Writer's Evaluation

Lois said she enjoyed working through the frames. She likes math and did well on the program. Lois was one of the students who was eager to work through the program.

STUDENT COMMENTS ON FUNDAMENTALS OF MATHEMATICS

Esther (Third Revision)

FramesStudent-Editor's Comments

ADDITION

- | | |
|----|-----------------------------------------------------------------------------------------|
| 17 | Error in estimation. |
| 32 | Error in estimation. |
| 39 | Error in one of the additions. |
| 42 | I must have read the question wrong. |
| 48 | Careless error; had 45,000 instead of 450,000.
Error in addition in the second part. |

SUBTRACTION

- | | |
|----|------------------------------------------------------------------------------------|
| 65 | Careless answer; added instead of subtracting
because all were addition before. |
| 73 | Didn't know what rounding was called for, chose
the wrong estimate. |

MULTIPLICATION

- | | |
|-----|--------------------------------------------------------------------------------------------|
| 94 | Multiplied wrong on the first problem. |
| 98 | I didn't understand the question; didn't read it
carefully. |
| 104 | Error in multiplication; had 1,200 instead of
120,000. |
| 112 | I didn't know that you ever round a number like
426 up. |
| 129 | I thought 3,000 times \$5 is \$15,000 and thought
that is near enough to be reasonable. |
| 143 | Careless; didn't think. Had 100 and 2 instead of
1000 and 3. |
| 172 | Error in estimation. |
| 175 | Error in estimation. |

Esther's Comments on Fundamentals of Mathematics, (Continued)

DIVISION

- 209 Error in division--rounded off answer too much.
- 217 I just put the wrong thing down; had "dividend"
 instead of "trial divisor."
- 231 I thought that because there is a remainder, the
 answer would be "decimals and whole numbers."

Writer's Evaluation

Esther did not work as carefully as she might have and therefore had too many careless errors. She had a tendency to answer quickly and then think how she should have answered the question, rather than being sure before she responded to a frame.

Because she is a Senior, her raw score of 83 on the Hundred-Problem test put her in a much lower percentile accordingly than a sophomore would have earned.

STUDENT COMMENTS ON FUNDAMENTALS OF MATHEMATICS

Rachel (Third Revision)

FramesStudent-Editor's Comments

ADDITION

- 24 I don't know why I rounded incorrectly; had 3,000 instead of 2,000.
- 29 Rounded 15.83 to nearest ones instead of nearest tens.

MULTIPLICATION

- 92 Error in multiplying in one partial product.
- 116 Estimated the upper range incorrectly.

DIVISION

- 214 Error in estimation.

Writer's Evaluation

Rachel said she likes mathematics. She worked through the program carefully and methodically taking plenty of time to think through the questions and problems. Her accuracy on the program and her I. Q. compare favorably; her low score in the Hundred-Problem Arithmetic Test was caused by missing and omitting too many problems in the Division section and the Fractions, Decimals, and Per Cents section.

Classification of Errors

Faulty or Incomplete Frames

As might be expected, there were a number of mechanical errors: lines indicating where an answer should be placed were lacking, errors in the answers, a zero missing in an answer, printing on the frames that was too pale to read, a division sign that looked too much like a plus sign, and similar items. Only four errors were attributed to this cause, however,

Carelessness

Into this category fall a variety of wrong responses. "I just didn't think," "I didn't write down the last answer," "I didn't read it carefully," or "I had the wrong word," are some of the common reasons given for this kind of errors. It is true that some errors come under two categories, and so they were counted two times each on the chart which follows this section. This was often the case in the simple computational errors where through careless addition, for example, the response was incorrect. This counted as a computational error and also as a careless error.

Some of the careless errors were caused specifically by too rapid reading, omitting part or all of the responses on a frame, or by failure to make the desired response. In this kind of error, there was often a lack of understanding of some new concept also, but the error came about because of the student's overlooking or not following cues that were clearly given on the frames; so here, again,

some errors fit under two headings: "Failed to make desired response," and "Did not apply concept."

Computational Errors

A total of 66 errors come under this heading. The simple, basic processes of addition, subtraction, multiplication, and division belong in this group.

Lack of Comprehension of New Concepts

There were 165 errors which fit into this category. The concepts that seemed to cause the most trouble to student-editors were those of estimation and rounding. These two alone were responsible for 82 errors. Fifty-nine errors were caused by not applying a new concept other than estimation or rounding. Some of these occurred at the initial introduction of a new concept, indicating the need for more buildup on the concept before asking the student to state a rule or work a problem using the concept.

Twenty-four errors came under the general statement, "I didn't understand." This referred most often to not knowing exactly what the frame was asking for, and the reason behind that was usually an unfamiliar concept that just had not become clear to the student. The statement, "I didn't understand," sometimes meant not knowing how to apply the concept.

The table on the following page summarizes this section by displaying a breakdown of causes of errors and listing the times each student-editor had errors in each classification.

TABLE 13

CLASSIFICATION OF ERRORS

Causes of Errors	Joe	John	Kathy	Mary	Judy	David	Carol	Phyllis	Lois	Esther	Rachel	Totals
Faulty or Incomplete Frames	3		1									4
Carelessness:												92
Didn't think through	1	2	3	6	1		1			2		16
Too rapid reading	4		3	2	1		2			2		14
Omitted part or all of responses		1	3	1			2					7
Failed to make desired response	9	2	10	14	4	2	5	1	4	4		55
Computational:												66
Addition, Subtraction, Multiplication, Division	14	3	8	14	9		8	3	2	4	1	
Lack of Comprehension of Concepts												165
Estimation, rounding	11	5	6	19	16		12	2	1	6	4	82
Did not apply concept	3	2	10	18	9	1	7	2	5	2		59
Did not understand	5	1	8	6	1				1	2		24

Frame Clarification, Revision,
Addition, and Evaluation

As mentioned in the preceding section, a number of mechanical defects of the frames came to light throughout the testing. These were quickly cleared up by filling in the lines that had been omitted, retyping some frames that were too pale, double spacing some frames that appeared to be too cluttered and crowded in their single-spaced form, and making corrections of numbers on several of the answers. The writer does not feel that there is any need to elaborate on these minor corrections.

Very early in the testing, the writer had to decide upon some system for numbering frames so that new ones could be added without destroying the original numbering. The system that was devised is not foolproof, but it did allow for the retention of original frame numbers and provided a special numbering for revised or inserted frames. There is one exception to this: Original frames 98 and 99 were exchanged and their numbers were also exchanged.

The illustration on the following page shows the symbols used and an explanation of the symbols.

The First Round of Testing

The first two students tested present a somewhat conflicting picture. Joe, who had a high score on the Hundred-Problem Arithmetic test and ranked in the 97th percentile, made 45 errors on the frames; while John with an Arithmetic test score of 76 and percentile ranking

ILLUSTRATION 2

Symbols Used on Revised Frames

Symbols	Meaning of Symbols
*	One asterisk following the number signifies that the change on the card was made in the first revision. By itself, it indicates that a minor prompt was added.
**	Two asterisks following the number denote the second revision. They also denote a simple prompt added.
***	Three asterisks following the number denote the third revision or a minor prompt added.
R	A capital R appearing to the left of the number means that a revision was made of the original frame either on the original card or by removing the original card and retyping the frame.
P -1 -2	A capital P preceding the number indicates the insertion of a new frame. If more than one frame was inserted, a hyphen and number follow the original frame number, thus: P 54-1 ** and P 54-2 ** show that two frames were inserted before the original frame on the second revision.
a b	If one frame was revised by breaking it into smaller parts and putting the information on several frames, it is marked thus: R 95 ***, R 95a ***, R 95b *** etc.

of 61 had only 14 errors. The most logical answer for this disparity seems to be that John prompted himself quite liberally when the teacher was not beside him. The writer had not planned to revise the frames at this point because the test scores of only two students were hardly sufficient to indicate weak spots, but the large number of errors that Joe made suggested that some changes were necessary before any more students went through the program.

The First Revision

Building Speed and Accuracy in Addition

Frame 45--Angular Addition to Improve Accuracy.--This frame produced some interesting guesses. Joe said he didn't read carefully, and he put "singular" for the first blank and left the last blank empty. John left the first blank empty and wrote "angle" in the last blank. Since the word "angular" had not been used before, it seemed that a prompt should be added. For this reason, the word "angle" was underscored in the second sentence of the frame. Illustration 3 shows Frame 45* as it appeared after the prompt was added.

Building Speed and Accuracy in Multiplication

Frames 98 and 99--Computing Extensions on Invoices.--Joe was completely confused by Frame 98, not knowing what the term "extension" meant. Also, he thought that the last part of the frame was hard to understand. The last part calls for the product of 5 and \$462 to be

ILLUSTRATION 3

Angular Addition Prompt Added

REVISED FRAME

45*

98,765
 96,865
 76,854
 99,876
 86,974
 97,774
 67,989
87,987
 44
 54
 65
 56
65
 713,084

Each column of the addition problem at the left
 is added separately. The sums for each column
 are set down in an angular fashion, which leads to
 the designation of angular addition. The amount
 of "carrying" in this type of addition is
 (increased-reduced). _____ This
 method of addition is known as _____
 _____ addition.

angular reduced angular

extended to right or left, and Joe said "left" because he thought that when you multiply you set down the numbers in the answer from right to left. Since the term "extension" had not been used previously, Frames 98 and 99 were exchanged, and the words "extending" and "extension" were cued by underscoring them on Frame 98*. Frame 98* now showed a completed extension. It seemed more logical to show an invoice with the extension already worked out and then to ask the student to compute an extension on the next frame. Illustration 4 shows the frames in their original order and then reversed and cued.

Frame 111--Rounding Down Both Factors.--John apparently made an error in multiplication because his answer of \$3,700 could not be the product of two whole-number factors. This frame had also been missed by 7 out of 11 college boys working on the frames at Virginia Polytechnic Institute. Five of the seven boys rounded \$93.78 down to 90.00 but did not round down 42 weeks to 40. To prevent a similar thing from happening for the high school students, the frame was revised.

The necessary steps that needed to be added to this procedure were to have the student round down both the factors before estimating the take-home pay. In the revision, the student had to respond by writing down his lower estimate of the weekly pay and the time, before computing the final answer.

Illustration 5 shows the original Frame 111 and Frame R 111*. This change seemed to remove the difficulty, because no one else

ILLUSTRATION 4

Computing Extensions on Invoices

ORIGINAL FRAME

98

Computation on an invoice.

5 Adding machines 462.00

Put in extension here _____ ↗

The product of 5 and \$462 is extended to the (right/left). _____

2,310 right

ORIGINAL FRAME

99

The process of calculating the amount of each item on an invoice is known as extending the invoice. Underscore the extension below.

Invoice

25 boxes Letterheads 5.10 127.50

127.50

ILLUSTRATION 4 (Continued)

ORDER OF FRAMES EXCHANGED

98*

The process of calculating the amount of each item on an invoice is known as extending the invoice. Underscore the extension below.

Invoice

25 boxes Letterheads 5.10 127.50

127.50

99*

Computation on an invoice.

5 Adding machines 462.00 _____

Put in extension here

The product of 5 and \$462 is extended to the (right/left). _____

2,310 right

ILLUSTRATION 5

Rounding Down Both Factors

ORIGINAL FRAME

111

Obenchain has weekly take-home pay of \$93.78. What would be the lower estimate of his total take-home pay for 42 weeks? _____

\$3,600

REVISED FRAME

R 111*

Obenchain has weekly take-home pay of \$93.78 for 42 weeks.

Round down both factors: Weekly pay _____
 Time _____

What would be the lower estimate of his total take-home pay? _____

\$90 \$40 \$3,600

missed the questions.

Frame 126--Obtaining Upper and Lower Estimates.--On this frame the question called for the complete thought process for obtaining the upper and lower estimates, but in the answer the numbers were listed in reverse order--from lower to upper. This is rather minor, but the wording was misleading to Joe, so on the answer on the back of the card, two possibilities were shown: upper to lower estimates and then lower to upper estimates in case anyone had answered it that way. Illustration 6 shows Frame 126 with its original answer and the revised answer. There was no further trouble or confusion with this frame.

Frame 129--Determining Whether an Extension is Reasonable or Unreasonable.--This question is too relative. How much is reasonable or unreasonable? If both factors are rounded "up," the answer would be 15,000.00. Depending upon his understanding of the words, the student might answer either way and still be right in his own thinking. Joe said the answer was reasonable, and he was surprised when he found out he was wrong.

If the answer is to be obviously incorrect, it should be a thousand or two more. To make the answer really "unreasonable," it was revised upward two thousand dollars. Illustration 7 shows the original and revised Frames 129 and R 129*. Only one more student-editor missed this frame after the revision.

ILLUSTRATION 6

Obtaining Upper and Lower Estimates:
A Corrected Answer

ORIGINAL FRAME

126

Show the complete thought process for obtaining the upper and lower estimates for this problem:

341 is between _____ and _____
~~X 29~~ is between _____ and _____

The product is between _____ and _____

ORIGINAL WORDING OF ANSWER

REVISED WORDING OF ANSWER

300 400
 20 30
 6,000 12,000

400 300
 30 20
 12,000 6,000

or

300 400
 20 30
 6,000 12,000

ILLUSTRATION 7

Determining Whether an Extension
is Reasonable or Unreasonable

ORIGINAL FRAME 129

Computation on an invoice:

2,869 A-1516 tubes 4.86 15,943.34

By estimation, determine whether the extension is (reasonable/
unreasonable). _____

unreasonable

REVISED FRAME R 129*

Computation on an invoice:

2,869 A-1516 tubes 4.86 17,943.34

By estimation, determine whether the extension is
(reasonable/unreasonable.) _____

unreasonable

Frame 133--Correcting a Frame on Estimation of Products--

Again, a minor technicality was confusing to Joe. The frame asked which area in a series of four was incorrect. Two answers appeared on the back of the frame--the incorrect area and also the corrected number of square feet. Joe had picked out the area which was obviously incorrect, but he had not supplied the corrected number of square feet. This was easily remedied by adding another question to the frame. Illustration 8 shows the revised form of Frame R 133*.

Building Speed and Accuracy in Division

Frame 196--Introducing the Concept of Trial Divisor.--Both Joe and John filled in the word "estimation" instead of "trial divisor." The words "trial divisor" were cued by underscoring to help elicit the correct response. Illustration 9 shows Frame 196* as it appeared after the prompt was added. No one else missed this frame.

The Second Round of Testing

Following the first revision three students, Kathy, Mary Ethyl and Judy, worked through the frames. Their I. Q.'s were 85, 106, and 112 respectively. On the Hundred-Problem Arithmetic test were the lowest of any students in the experiment with the exception of one. Kathy had a raw score of 82 and ranked in the 74th percentile, Mary Ethyl's raw score was 72 and she ranked in the 54th percentile, and Judy had a raw score of 66 which placed her in the 45th percentile.

ILLUSTRATION 8

Correcting a Frame on Estimation of Products

REVISED FRAME

R 133*

	<u>Length</u>		<u>Width</u>	<u>Area (Sq. ft.)</u>
(a)	97'	X	62'	6,014
(b)	282'	X	146'	41,172
(c)	329'	X	189'	82,181
(d)	868'	X	719'	624,092

Which of these areas is obviously incorrect? _____

*What is the correct answer? _____

(c) should be 62,181

* An asterisk within the body of the frame indicates the prompt that was added. The internal asterisks did not appear on the frame cards.

ILLUSTRATION 9

Introducing the Concept of Trial Divisor

PROMPT ADDED TO FRAME

196*

In the problem $2,592 \div 81$, the divisor is _____. Because numbers composed of two or more digits are difficult to use as a divisor, it is customary to use a trial divisor* of one digit. When a divisor is composed of two or more digits, a/an..... is used to obtain the answer.

81 trial divisor

On the program, Kathy had 51 errors, 11.23 per cent; Mary Ethyl with 66 errors which amounted to 14.60 per cent of errors, and Judy missed 40 which was 8.85 per cent of errors. None of the three students came close to being within the five per cent limit. The abundance of errors made by these students called for more thorough study and revision than was done following the first testing.

The Second Revision

Building Speed and Accuracy in Addition

Frame 5--Estimating the Approximate Sum.---Judy had an error in estimation on this frame; the writer felt that it may have been due to her taking too seriously the direction which specified that the estimation was to be done without adding. Obviously, there must be some mental process of adding going on, so the wording was changed to read "By adding mentally, estimate . . ." The original and revised frames are shown in Illustration 10.

Frame 29--Estimating a Sum in Tens.---Part of this frame was confusing to the student-editors because they had not had previous practice in rounding numbers to tens. In Frame 28, the rounding was done for them, but in Frame 29, the students were to supply the rounded figures. John and Mary Ethyl rounded 15.83 to 16.00 and 2.43 to 2.00. This gave them a total of \$78. The place they needed help was in rounding numbers of less than twenty to the nearest tens. A new frame was added just preceding Frame 29. Both of these are shown in Illustration 11. On this frame, the students were asked to

ILLUSTRATION 10

Estimating the Approximate Sum

ORIGINAL FRAME

5

Without adding, estimate the approximate sum of these numbers:

\$375.49, \$815.75, \$1,276.43. Estimated sum is: \$ _____

Any answer between 2,300 and 2,500 is acceptable

REVISED FRAME

R 5**

By adding mentally, estimate the approximate sum of these numbers:

\$375.49, \$815.75, \$1,276.43. Estimated sum is: \$ _____.

Any answer between 2,300 and 2,500 is acceptable

ILLUSTRATION 11

Estimating a Sum in Tens

ADDED FRAME

P 29**

Round these figures to the nearest tens:

\$50.00	_____
15.83	_____
2.43	_____
9.85	_____

50 20 0 10

ORIGINAL FRAME

29

The estimated sum in tens of \$50.00 + \$15.83 + \$2.43 + \$9.85 is _____.

\$80

round each figure to the nearest tens as an intermediate step before going on to the original frame where they were to write the sum. Even this was not sufficient help in explaining the principle of rounding numbers of less than ten to the nearest tens, so there was a further revision which is shown later.

Frame 31--Rounding Numbers to the Nearest Thousands.--In spite of the buildup on rounding to thousands on previous frames, this frame was confusing to the student-editors. Joe and Kathy rounded incorrectly and Judy said that she did not understand the frame. The part that was not clear for the students was deciding how to round 125,000 to thousands. Actually, it does not need to be rounded, but the student-editors were not sure about it. The writer felt that this frame should be revised by requiring the student-editors to write down the rounded numbers before estimating the answer and by under-scoring the word "thousands" as a prompt. The original and the revision of Frame 31 are shown in Illustration 12. This revision was not sufficient, and further addition is shown in the next section of this chapter.

Frame 34--Verification by Reverse Addition.--Although no one had missed Frames 34 or 35, an extra frame was inserted here for review or reinforcement purposes to strengthen the sequence. Frames 34, P 35**, and 35 are shown in Illustration 13.

Frame 39--Adding by Combinations of Ten to Build Speed.--Two persons missed this frame. Joe had the right idea, but did not express it accurately and Kathy had overlooked the last question on the frame. To pinpoint attention on the words "combinations of 10,"

ILLUSTRATION 12

Rounding Numbers to the Nearest Thousands

ORIGINAL FRAME

31

Shriver Road Construction Company has these debts. Estimate the total in thousands:

Building	\$125,000
Land	17,500
Machinery	1,200
Trucks	<u>8,900</u>

Estimated sum _____

\$153,000

REVISED FRAME

R 31**

Shriver Road Construction Company has these debts. Round the numbers to the nearest thousands and estimate the total in thousands.

Building	\$125,000	_____
Land	17,500	_____
Machinery	1,200	_____
Trucks	<u>8,900</u>	_____

Estimated sum _____

125,000	18,000	1,000	9,000	153,000
---------	--------	-------	-------	---------

ILLUSTRATION 13

Verification by Reverse Addition

ORIGINAL FRAME

34

The comparison of the sums by adding both from left to right and from right to left when adding a row of numbers is a way of verification known as reverse addition. Verify the sum below by reverse addition.

_____ = 7 + 4 + 3 + 5 + 6 + 8 + 7 + 9 + 6 + 8 = _____

63 63

ADDED FRAME

P 35**

The purpose of reverse addition is to verify the sum, or we could say that the process of verification is done by r _____

a _____.

reverse addition

ORIGINAL FRAME

35

The process of adding both from left to right and from right to left to verify the sum is known as _____ addition.

reverse

they were cued by underlining. The same kind of cue was added to Frame 40. (Illustration 14)

Frame 41--Verification by Reverse Addition.--No one had missed this frame, but it seemed to the writer that the process of reverse addition could be explained more clearly by having equals signs at the beginning as well as at the end of the rows of numbers to help get across the idea of verifying by adding in both directions.

Illustration 15 shows how this was done.

Frame 45--Angular Addition to Improve Accuracy.--This frame had been revised once, but the first person tested after the revision found it just as confusing as ever. This was Kathy who filled in the first and last blanks with the word "regular" even though the word "angle" had been underlined as a cue on the first revision. The writer felt that the word, "angular," should be used at least once before it could be elicited as a response. The original Frame 45 was removed and a revised writing substituted. The revision of Frame 45 is shown in Illustration 16. There were no further misses on this frame.

Frame 48--Estimating the Sum by Rounding to Ten Thousands.--Kathy, Mary Ethyl, and Judy missed this frame because of errors they made in rounding to ten thousands and errors in addition. In checking over the frames, the writer found that there had been no previous rounding to ten thousands, so the frame was revised by showing specifically how to round to ten thousands. The problem on the frame was not changed, however. The revised frame is shown in

ILLUSTRATION 14

Adding by Combinations of Ten to Build Speed

PROMPT ADDED TO FRAME

39**

Add by combination of 10.**

2 5	2 7	26	51
5 5	5 3	54	29
5 5	8 7	15	78
5 5	2 3	95	32
5 5	7 8	22	47
<u>9 5</u>	<u>3 2</u>	<u>88</u>	<u>43</u>

These problems are easy to add because there are many pairs of numbers that form

Combinations of 10

PROMPT ADDED TO FRAME

40**

Observing pairs of numbers that form 10's increases accuracy in adding. Add these by watching for combinations of 10.**

8 5	5 6	57	59	83
5 3	8 5	56	58	19
5 5	5 4	84	81	67
<u>9 7</u>	<u>7 5</u>	<u>33</u>	<u>82</u>	<u>41</u>

290 270 230 280 210

ILLUSTRATION 15

Verification by Reverse Addition

 PROMPT ADDED TO FRAME

41**

Add these rows of numbers by watching for combinations of 10.
Verify by reverse addition.

$$\underline{\quad} = 7 + 3 + 6 + 5 + 4 + 1 + 8 + 7 + 2 + 3 = \underline{\quad}$$

$$\underline{\quad} = 6 + 5 + 5 + 9 + 6 + 1 + 4 + 7 + 6 + 3 = \underline{\quad}$$

$$\underline{\quad} = 9 + 9 + 1 + 8 + 2 + 6 + 5 + 4 + 5 + 9 = \underline{\quad}$$

46 46

52 52

58 58

ILLUSTRATION 16

Angular Addition to Improve Accuracy

REVISED FRAME

R 45**

98,765
 96,865
 76,854
 99,876
 86,974
 97,774
 67,989
87,987
 44
 54
 65
 56
65
 713,084

Each column of the addition problem at the left is added separately. The sums for each column are set down in an angle fashion, which leads to the designation of angular addition.

The amount of "carrying" in this type of addition is (increased/reduced). _____.

This method of addition is known as _____ addition.

reduced angular

Illustration 17. Following this revision, there were two students who had addition errors on the frame, but no more errors of rounding to ten thousands.

Frame 49--Introducing the Concept of Sub-Total Addition.--

Joe said he was confused by this question, and Kathy seemed to be also, because she said "sub-addition" instead of "sub-total." This called for some revision to provide for more buildup of the term "sub-total addition," to prevent further errors. A very simple cue was added as seen in Illustration 18. There were no more errors made on this frame.

Frame 54--Clarifying the Concept of Horizontal Addition.--

Joe, John, and Judy had errors in addition on this frame which would not necessarily call for revision, but the frame seemed to lack a step; namely, telling how to carry out the horizontal addition. The writer thought that the sequence would be strengthened by the insertion of a very easy example of horizontal addition presented in step fashion before the longer problem was introduced. The three steps are: (1) Add mentally, (2) Write sums in angular fashion, (3) Find total. These explanatory steps are on Frame P 54**. On Frame 54, the words "without recopying," were under-scored to remind the students that Step 1 is to be a mental process. Illustration 19 shows both of these frames.

ILLUSTRATION 17

Estimating the Sum by Rounding to Ten Thousands

REVISED FRAME

R 48**

(a) Estimate the sum in ten thousands.	\$62,432.63
(b) Obtain exact answer by the	24,371.97
angular method.	15,600.84
	59,124.85
** (Hint: when rounding to ten thousands,	75,576.48
62,432.63 becomes 60,000.00;	64,383.66
24,371.97 becomes 20,000.00;	52,948.90
15,600.84 becomes 20,000.00;	79,002.01
59,124.85 becomes 60,000.00 and so on.)	<u>18,239.71</u>

\$450,000 \$451,681.05

ILLUSTRATION 18

Introducing the Concept of Sub-Total Addition

PROMPT ADDED TO FRAME

49**

When preparing a temporary financial report, an accountant frequently obtains sub-totals. Therefore the name applied to this process would be s** -t** addition.

sub-total

ILLUSTRATION 19

Clarifying the Concept of Horizontal Addition

ADDED FRAME

P 54**

Amounts are sometimes arranged in horizontal fashion for addition. This kind of addition is horizontal addition.

Follow these steps in horizontal addition:

Step 1: Add mentally, (beginning with the unit's, then ten's, then hundred's places.)

$$23 + 77 + 56 + 34 =$$

Step 2: Write totals in angle fashion. 20
17

Step 3: Find the total: 190

Addition of horizontal rows of numbers is called _____ addition.

horizontal

CUE ADDED TO FRAME

54**

Amounts horizontally arranged require skill in horizontal addition. Without recopying add the ten amounts below:
**

.47 .23 .50 .65 .75 .96 .84 .23 .47 .86 Total

This method of addition is known as _____ addition.

5.96 horizontal

Building Speed and Accuracy in Subtraction

Frame 61--Approximating Answers in Subtraction by Rounding Numbers.--No one missed Frame 61, but Mary Ethyl and Judy missed Frame 62 which follows, because they said they didn't understand what was being called for. The writer felt that the teaching value of Frame 61 could be reinforced by having the student write out the numbers he rounded and this in turn would make clear what responses are expected on Frame 62. There were no more errors made on this Frame. Both frames appear in Illustration 20.

Frame 68--Estimating the Difference by Rounding.--The original frame was not very clear in specifying how many responses were expected. Kathy and Mary Ethyl missed the import of the frame, so the writer removed this frame and substituted a new one with blanks for responses clearly indicated in the order in which they should be given. Thus, the first blank is asking for the rounded smaller number; the second blank, for the rounded larger number; and the last blank calls for the estimated answer. Illustration 21 shows the original and revised copies of Frame 68.

Frame 76--Verification of Subtraction.--Mary Ethyl missed this because she said she didn't read it carefully and she thought it was calling for a different answer. She had 51 for the difference which is correct, but had the word "subtrahend" instead of "larger number." She was really making the problem more difficult by trying to supply a more difficult term (and choosing the wrong term--subtrahend instead of minuend), and she did not understand that she was to

ILLUSTRATION 20

Approximating Answers in Subtraction
by Rounding

REVISED FRAME

R 61**

Burman had \$5,245.67 in his checking account. After writing a check for \$1,995.39, he had approximately:

- (a) \$2,000
- (b) \$3,000
- (c) \$4,000

Answer → _____

**This answer is found by rounding
\$5,245.67 to \$ _____ and
\$1,995.39 to \$ _____.

b \$5,000 \$2,000

ORIGINAL FRAME

62

Estimates of differences in subtraction are made by _____ numbers as you did in addition. Since only two numbers are involved, care must be taken in choosing how to round the _____ numbers.

(No.)

rounding two

ILLUSTRATION 21

Estimating the Difference by Rounding

ORIGINAL FRAME 68

Complete the estimate of the difference for the problem at the right. The number first rounded is the

	Problem	Rounded
	3,264.45	_____
	- 576.38	_____

Estimated answer

smaller number 600 3,300 2,700

REVISED FRAME R 68**

Complete the estimate of the difference for the problem at the right. The number first rounded is

	Problem	Rounded
	3,264.45	_____
	- 576.38	_____

Rounded smaller number is _____

Rounded larger number is _____

Estimated answer is _____

smaller number 600 3,300 2,700

to use the term "larger number." Illustration 22 shows the original frame and the writer's attempt to clarify the frame by adding guide words.

Building Speed and Accuracy in Multiplication

Frame 85--Introducing Multiplication Concepts.--Mary Ethyl didn't understand what was meant by 10 89's and Judy read the frame number, 85, into the problem which understandably was confusing to her. The original frame was removed, and on the revision the frame number was moved to a position where it would not be "read into" the problem. To clarify the concept of finding 10 89's, another example was added preceding it: ten 49's is 490. Frame 85 and its revision appear on Illustration 23. No one else had trouble understanding this concept.

Frame 90--Clarifying the Wording on a Frame.--On this frame the factors, partial products, and product are shown and labeled. The question on the frame was worded like this: 3,432, 2,145, and 2,574 are The term which should be elicited is "partial products." Mary Ethyl misunderstood the point of the question and supplied the number of the product, so the writer added one word to the statement: "3,432, 2,145, and 2,574 are called"; and there were no further misses on this frame. The entire frame is not illustrated.

Frame 146--Cueing a Multiplication Shortcut Rule.--This frame partially states the rule for a shortcut to multiplying by 25. However, unless a student is very alert, he may not get the idea of

ILLUSTRATION 22

Verification of Subtraction

ORIGINAL FRAME

76

Complete this subtraction:

$$\begin{array}{r} 215 \\ -164 \\ \hline \end{array}$$

Difference

If you add the difference and the smaller number you should obtain 215, which is the

51 larger number

CLARIFIED FRAME

76**

Complete this subtraction:

$$\begin{array}{r} \text{Larger number} \quad 215 \\ \text{Smaller number} \quad -164 \\ \hline \text{Difference} \end{array}$$

If you add the difference and the smaller number you should obtain 215, which is the

51 larger number

ILLUSTRATION 23

Introducing Multiplication Concepts

ORIGINAL FRAME

The number of the frame was here:

If ten 1's is 10 and ten 2's is 20, then ten 89's = _____ 85
and ten 137's = _____.

890 1,370

REVISED FRAME

R 85**

If ten 1's is 10 and ten 2's is 20, then ten 49's is 490
and ten 89's is _____ and ten 137's is _____.

890 1,370

dividing by 4 because 25 is one fourth of 100. Therefore, the words "one fourth" were cued by underlining them to make more clear the principle of the shortcut: "Because 25 is one fourth of 100, a shortcut to multiplying by 25 is to annex _____ zeros and divide by _____." (Answers: two 4)

Frame 149--Moving Decimals when Annexing Zeros.---Joe and John both had trouble with decimal placement in this problem. Mary had a division error. To forestall further errors of decimal placement, it seemed advisable to add an explanatory sentence to Frame 149 reminding the -student-editors to move their decimals one place to the right for each zero annexed. Refer to Illustration 24 revised form of Frame 149. There were no further errors involving decimals on this frame.

Frame 150--Shortcut Rules for Multiplying by 25 and 250.---No one missed the idea of this frame, but in stating the rule Kathy and Judy used the word "add" instead of "annex." To help fix the correct terminology firmly in mind, the frame was revised so that the rules were partially stated and the student-editors had to fill in the strategic words. Illustration 25 shows Frame 150 and the frame which replaced it.

ILLUSTRATION 24

Moving Decimals when Annexing Zeros


REVISED FRAME

R 149**

Cost Sheet for Printing Shop

<u>Hours</u>		<u>Cost of machine per hour</u>	<u>Total</u>
25	X	\$ 62.	_____
25	X	87.	_____
250	X	39.	_____
250	X	134.50	_____

**(Be sure to move your decimal one place to the right for each zero you annex.)

Complete calculations 

1,550 2,175 9,750 33,625

ILLUSTRATION 25

Shortcut Rules for Multiplying by 25 and 250

ORIGINAL FRAME

150

What are the shortcut rules for multiplying by 25 and 250?.....
.....

For 25 annex two zeros to other factor and divide by 4
For 250 annex three zeros to other factor and divide by 4

REVISED FRAME

R 150**

The shortcut rule for multiplying by 25 is to annex.....
to the other factor and divide by _____.

The shortcut rule for multiplying by 250 is to annex.....
to the other factor and

two zeros 4 three zeros divide by 4

Frame 156--Providing More Guidance for Stating the Shortcut for Multiplying by 99.--The frame here said simply, "Describe an easy way to multiply by 99." Here again the student-editors probably knew what they meant when they stated the rule for multiplying by 99; but their statements, though correct, were somewhat ambiguous. For example, Joe said, "Multiply other factor by 100 and subtract the factor from the product of 100 x the factor." The writer restated the frame to read: "The rule for an easy way to multiply by 99 is to _____ the other _____ from _____ times itself." (Answers: subtract, factor, 100) No one missed the frame before it was revised or afterward.

Frames 157 and 159--Adding Explanations to Frames.--Both Frames 157 and 159 contained multiplication problems, but no word of explanation of any kind. The implication was that the shortcut method for multiplying by 99 was to be used on Frame 157 and the shortcut rule for multiplying by 98 on Frame 159. In order to remind the students to use the shortcut methods for multiplying by 99 and 98 respectively, a statement to that effect was added to each frame.

Frame 161--Adding a Frame to Review and Strengthen the Sequence of Shortcut Multiplication of 9, 99, and 999.--Preceding Frame 161, there is a series of frames applying the concept of using a shortcut for multiplying by 9 and 99. Frame 161 introduces the next logical step, which is the rule for multiplying by 999.- The rule is partially stated, and there is one blank to fill.

The writer felt that just before the student has to respond to ". . . subtract the other factor from _____ times itself," a review frame would be in order to reinforce the rules for shortcut multiplying by 9 and 99 and then lead on in logical progression to the similar rule for multiplying by 999. Frame P161** was added for this purpose, and this is shown in Illustration 26.

Frames 165, 166--Verification of Multiplication Problems.--

Joe, Kathy, Mary Ethyl, and Judy missed Frame 166 on which they were to supply the term "verification" in this sentence: ". . . another technique for increasing accuracy in multiplication is that of _____." The word is used on Frame 165, but previous to that there had been no mention of "verification" since Frame 80.

The writer decided to back up and cue the word "verification" by underscoring. The words "many individual" were cued the same way. Frame 166 is exactly the same as it was except for one letter of the word, "v" for verification, being placed at the beginning of the blank. This seemed to clear the difficulty because no one else missed either frame after the prompts were added. Illustration 27 shows Frames 165 and 166 with prompts added.

Frame 175--Estimating the Difference in Ten Thousands.--This frame contains a review subtraction problem in which the student-editors are to estimate the difference in ten thousands and choose one of three answers. John, Kathy, Mary Ethyl, and Judy all rounded their numbers to thousands and chose the wrong answer. A prompt was

ILLUSTRATION 26

Adding a Frame to Review and Strengthen
the Sequence of Shortcut Multiplication
of 9, 99, and 999

ADDED FRAME

P 161**

The shortcut for multiplying by 9 is to _____ the other
factor from _____ times itself.

The shortcut for multiplying by 99 is to

The shortcut for multiplying by 999 is to

subtract 10

subtract the other factor from 100 times itself

subtract the other factor from 1000 times itself

ORIGINAL FRAME

161

The shortcut for multiplying of 999, similar to those for 9 and 99,
is to subtract the other factor from _____ times itself.
No.

1,000

ILLUSTRATION 27

Verification of Multiplication
Concept Prompted

PROMPT ADDED TO FRAME (Three words underscored)

165**

As you will remember, many individual steps are involved in most multiplication problems. So, you will expect the verification procedure also to have steps.

many individual

PROMPT ADDED TO FRAME

166**

We have previously studied estimation of the product in multiplication. As in addition and subtraction, another technique for increasing accuracy in multiplication is that of v.....

verification

added to this frame by underscoring the word, ten, in the sentence:

"Estimate the difference in ten thousands." No other student-editors encountered any trouble with this frame.

Building Speed and Accuracy in Division

Frame 209--Rounding Divisor and Dividend for Estimation of Answers in Division.--The student-editors encountered some difficulty in rounding off their numbers correctly and obtaining the answers mentally when they had to have two digits in the answer. An example of this is Kathy's response on Frame 212. Here the problem reads: "1,674 ÷ 71 = 23 and remainder 41. What is the estimated answer? _____" Kathy said the answer is 20.

Therefore, in order to help the students round numbers and estimate answers more accurately, the writer added cues to Frame 209 by underlining important key words: "round divisor to one digit" and "round dividend to two digits (or less) followed by 0's" to try to clear the difficulty. The complete frame is shown in Illustration 28. However, this change was not sufficient, and a further revision was made following the next round of testing. This is reported in the next section of this chapter.

Frame 226--Determining a Balance by Estimation.--Joe, Kathy, Mary Ethyl, and Judy had trouble with this frame. In the first place, they did not take into account that some of the items are withdrawals; and secondly there was nothing to indicate whether they were to round to hundreds or to tens in determining the estimated balance of the account, so they were not sure how to round the numbers.

ILLUSTRATION 28

Rounding Divisor and Dividend for Estimation
of Answers in Division

PROMPTS ADDED TO FRAME (Two words underscored)

209**

The steps for estimating answers in division are:

- (1) round divisor to one digit followed by 0's
- (2) round dividend to two digits (or less) followed by 0's
- (3) obtain answer mentally

Problem

$321 \overline{)64,256}$

Rounded Form



$300 \overline{)64,000}$

Answer: $2,10\cancel{0}$ or $2,13\cancel{3}$

The writer tried to remedy these two specific points by adding two explanatory sentences above the problem on the frame reminding them that a minus sign means a withdrawal and that they need to round to hundreds. (See Illustration 29) This was sufficient to help all the following student-editors, except Carol, to answer correctly.

Frame 227--Rounding the Divisor and Dividend.--On this frame there is another division problem. The students needed to remember to round the dividend to two digits plus zeros and the divisor to one digit plus zero, so a statement in parentheses was added to the frame: "(Round the divisor to one digit, the dividend to two digits)." Illustration 30 shows the complete frame.

The Third Round of Testing

After the second revision, four student-editors worked through the program. The I. Q.'s of these four students were very close--ranging from 113 to 121. Their raw scores in the Hundred-Problem Arithmetic Test ranged from 84 to 93. One member working with this second revision had a total of only 3 errors on the entire set of frames which was the lowest of anyone. The highest number of errors was 38 for Carol. The per cent of errors fell between .63% for David to 6.07% for Carol. The average percentage of errors of these four students is 3.13%. Although this group as a whole came well within the five per cent error limit goal, the writer felt that there were still some weak spots in the program which needed further clarification.

ILLUSTRATION 29

Determining a Balance by Estimation

PROMPT ADDED TO FRAME

226**

**The items with a minus sign are withdrawals.

**Round to hundreds.

\$125.42 +	\$300.00 +
79.60 +	50.00 +
100.00 -	100.00 -

Determine the estimated balance of this account. \$ _____

Calculate the exact balance. \$ _____

\$400	\$355.02
-------	----------

ILLUSTRATION 30

Rounding the Divisor and Dividend

PROMPT ADDED TO FRAME

227**

The estimated answer for $63/\overline{3717}$ is _____ and the actual answer is _____.

**(Round the divisor to one digit, the dividend to two digits)

60 or 61	59
----------	----

The Third Revision

The specific areas which continued to cause trouble were: rounding numbers of five or less to the nearest tens, rounding five-place numbers to thousands, and rounding six-place numbers to ten thousands. All these are in the Addition section of the program. In the Multiplication section, the weakest spot for the student-editors was the part of the program where the concept of annexing zeros and dividing by 4 as a shortcut for multiplying by 25 or 250 was introduced. The third troublesome area was in the Division section where the students had to round dividends and divisors and estimate the answers.

The third revision, although not as extensive as the second, was aimed particularly at these three trouble spots. Several other minor changes were made at other places.

Building Speed and Accuracy in Addition

Frame 29--Rounding Numbers of Less than Twenty to Tens.--Even though Frame P 29** had been added requiring that figures be rounded to the nearest tens as a buildup to Frame 29 which called for the estimated sum of the rounded figures, Carol and Phyllis made the same error that had been made previously; namely, they rounded 15.83 to 16.00 and 2.43 to 2.00. So a new frame was inserted before P 29**. Illustration 31 shows the new frame. Of the two students who worked through the final revision, one of them got the idea and had the four figures rounded to tens correctly on P 29**, but the other student-

editor rounded 15.83 to 16.00. She did round 2.42 to 0, however, so that point got across. The writer would add another line to this frame if another revision were to be made: "17.28 rounds to 20.00," in an effort to help the students to know how to round numbers between 15 and 19 to the nearest tens.

Frame 31--Rounding Numbers to Thousands.--A total of five student-editors had had trouble with this frame. Lois, for example, rounded 125,000 to 100,000. With so much fuzzy thinking still in evidence, the writer decided to add four frames preceding Frame R 31**. These frames were designed to provide thorough practice in rounding to thousands. Illustration 32 shows this series of four frames and R 31**.-- This took care of the problem which had come up so often in Frame 31. Both succeeding student-editors rounded the numbers correctly on Frame R 31**.

Building Speed and Accuracy in Multiplication

Frame 95--Building Accuracy by Auditing Products.--In evaluating the results of testing following the second revision, the writer jotted down an observation to the effect that Frame 95 would be improved by omitting about half of the problems, because it is so long that students can easily become bored by repetition of multiplication problems. Therefore, it might be better to make several frames out of this one if more practice is desired. Carol objected to marking the whole frame wrong in her answer tablet when she had made only one error in one of the problems.

ILLUSTRATION 31

Rounding Numbers of Less than Twenty to Tens

ADDED FRAME

P 29-1***

When rounding a number to the nearest tens,

\$43.75 rounds to \$40.00

8.50 rounds to 10.00

3.98 rounds to 0.00

6.20 rounds to _____

\$10

ADDED FRAME (Second Revision)

P 29**

Round-these figures to the nearest tens:

\$50.00 _____

15.83 _____

2.43 _____

9.85 _____

50 20 0 10

ILLUSTRATION 31 (Continued).

ORIGINAL FRAME

29

The estimated sum in tens of $\$50.00 + \$15.83 + \$2.43 + \9.65
is _____.

\$80

ILLUSTRATION 32

Rounding Numbers to Thousands

ADDED FRAME

P 31-1***

To round a number to the nearest thousands, any number
between 500 and 1,499 becomes 1 thousand.

Therefore, 527 rounds to _____ thousand.

1,325 rounds to _____ thousand.

8,550 rounds to _____ thousand.

1 1 9

ILLUSTRATION 32 (Continued)

ADDED FRAME

P 31-2***

Similarly, much larger numbers can be rounded to thousands, also.

22,000 rounded to thousands remains the same—22,000.

22,550 rounded to thousands becomes 23,000

Round these numbers to thousands:

15,250 rounds to _____

251,875 rounds to _____

15,000

252,000

ADDED FRAME

P 31-3***

Rounding to thousands

125,000 remains 125,000

125,500 rounds to 126,000

125,400 rounds to _____

125,000

ILLUSTRATION 32 (Continued)

ADDED FRAME

P 31-4***

Rounded to thousands, 128,750 is _____

251,324 is _____

129,000

251,000

REVISED FRAME (Second Revision)

R 31**

Shriver Road Construction Company has these debts. Round the numbers to the nearest thousands and estimate the total in thousands.

Building \$125,000 _____

Land 17,500 _____

Machinery 1,200 _____

Trucks 8,900 _____

Estimated sum _____

125,000

18,000

1,000

9,000

153,000

For this reason, the frame was divided into three frames each having four problems with slightly varied format. No more errors of any kind were made on these 12 problems. Illustration 33 shows the breakdown of Frame 95 into three frames.

Frame 130.--Finding Lower and Upper Estimates by Rounding Down and Up.--Two review frames were added just preceding Frame 130 to help build accuracy in the process of finding lower and upper estimates. Illustration 34 shows these two frames.

Frames 145, 147, 148--Using the Shortcut Method for Finding Extensions.--In order to pinpoint more clearly the shortcut procedure that is necessary in multiplying by 50, one frame was inserted just before Frame 145. Similarly, a frame was inserted just before Frame 147 to remind the student-editors of the necessary steps in using the shortcut for multiplying by 25. A newly-written frame reviewing the rule for the shortcut method of multiplying by 250 was substituted for Frame 148. These three review frames and the original Frame 148 are shown in Illustration 35.

Building Speed and Accuracy in Division

Frame 210--Rounding the Dividend and Divisor to Estimate Answers in Division.--Carol and Phyllis had trouble with Frame 210 because of errors in rounding. Carol rounded the dividend 315,789 to 300,000, and Phyllis rounded it to 316,000. This figure should be rounded to two digits and zeros. So even though the rules for

ILLUSTRATION 33

Building Accuracy by Auditing Products

FRAME DIVIDED INTO THREE FRAMES

R 95***

Audit these products

Write the letters (a) to (d) in your answer tablet.

Place a check mark beside each correct answer in your answer tablet. Show corrected answers for incorrect items.

(a) 17 $\begin{array}{r} \times 5 \\ \hline 85 \end{array}$	(b) 92 $\begin{array}{r} \times 7 \\ \hline 644 \end{array}$	(c) 87 $\begin{array}{r} \times 4 \\ \hline 348 \end{array}$	(d) 75 $\begin{array}{r} \times 4 \\ \hline 300 \end{array}$
----------------------------------------------------------------	-----------------------------------------------------------------	-----------------------------------------------------------------	-----------------------------------------------------------------

all correct

FRAME DIVIDED INTO THREE FRAMES

R 95a***

Audit these products. Write the letters (e) to (h) in your answer tablet. Place a check mark beside those that are correct and write the corrected answer for incorrect items.

(e) 125 $\begin{array}{r} \times 15 \\ \hline 1,125 \end{array}$	(f) 135 $\begin{array}{r} \times 8 \\ \hline 1,180 \end{array}$	(g) 250 $\begin{array}{r} \times 13 \\ \hline 3,250 \end{array}$	(h) 175 $\begin{array}{r} \times 12 \\ \hline 2,100 \end{array}$
---------------------------------------------------------------------	--------------------------------------------------------------------	---------------------------------------------------------------------	---------------------------------------------------------------------

(e) 1,875 (f) 1,080

ILLUSTRATION 33 (Continued)

FRAME DIVIDED INTO THREE FRAMES

R 95b***

Audit these products:

- (i) 98 x 12 = 976
- (j) 59 x 19 = 1,121
- (k) 50 x 80 = 400
- (l) 33 x 55 = 1,815

Write the letters from (i) to (l) in your answer tablet.
 Check the letters having correct answers.
 Write the corrected answer for those incorrect.

(i) 1,176 (k) 4,000

ILLUSTRATION 34

Finding Lower and Upper Estimates
 by Rounding Down and Up

ADDED FRAME

P 130-1***

1,247 rounds down to _____ (thousands)

87 rounds down to _____ (tens)

1,000 30

ILLUSTRATION 34 (Continued)

ADDED FRAME

P 130-2***

1,247 rounds up to _____ (thousands)

87 rounds up to _____ (tens)

2,000 90

ILLUSTRATION 35

Using the Shortcut Method for Finding Extensions

ADDED FRAME

P 145-1***

Using the shortcut method for finding the extensions on these problems.

$$50 \times 3.70$$

$$50 \times 1.75$$

you need to add _____ zero(s) to the cost and then divide by ____ (No.)

two 2

ILLUSTRATION 35 (Continued)

ADDED FRAME

P 147-1***

The shortcut for multiplying by 25 is to annex _____ zero(s)
and divide by _____ (no.)

two 4

REVISED FRAME

R 148***

Because 250 is one quarter of 1,000, you can use a shortcut to
multiplying by 250 by annexing three zeros and dividing by 4.

$$250 \times 763 = \underline{\hspace{2cm}}$$

190,750

ORIGINAL FRAME

148

Because 250 is one quarter of 1,000, you can use a shortcut to
multiplying by 250 by annexing _____ (No.) zeros and dividing
by _____.

$$250 \times 763 = \underline{\hspace{2cm}}$$

three 4 190,750

estimating answers in division are shown on Frame 209, the writer added a new frame just preceding 210 to help the students to round the numbers before doing the estimation that is required on Frame 210. The added frame and revised forms of Frame 210 are shown in Illustration 36.

Frame 212—Using Two Digits in Estimating an Answer.—In this section on rounding the dividends and divisors and estimating the answers in division, guidance is given on how to round dividends and divisors. However, no specific guidance is given on how many digits there should be in the quotient. In most of the answers there are two digits; in a few, there is only one digit followed by a zero. This was confusing to the students as evidenced by the number of times they had errors charged against them for this cause. Therefore, this explanatory sentence was added to Frame 212: "Use 2 digits in the answer."

The last two student-editors had a combined error score of only one error on one frame in the division section compared with a total of 40 errors on this same section made by Kathy, Mary Ethyl, and Judy following the first revision, and 11 errors made by David, Carol, Phyllis and Lois following the second revision.

Results of the Third Revision

The two student-editors who worked on the program following the third revision both had less than five per cent of errors. Esther had 4.10% of errors, and Rachel had 1.02%; this averages 2.56% for the final round of testing.

ILLUSTRATION 36

Rounding the Dividend and Divisor
to Estimate Answers in Division

ADDED FRAME

P 210-1***

In the problem $795/\overline{315,789}$

we must round 795 to _____ (1 digit followed by zeros)

we must round 315,789 to _____ (2 digits followed by zeros)

800 320,000

REVISED FRAME

R 210**

Estimate the answer of $795/\overline{315,789}$

Show the rounded form: $\overline{\hspace{2cm}}$

**The answer is _____.

Answer 400

Rounded form: $800/\overline{320,000}$

CHAPTER V

SUMMARY AND CONCLUSIONS

Summary

A study to discover the means of reducing the frequency of learning errors in programmed business mathematics was carried out at Eastern Mennonite High School, Harrisonburg, Virginia, between November, 1961, and March, 1963.

The Subjects

Eleven high school students of Eastern Mennonite High School--three boys and eight girls--from grades nine to twelve composed the four groups of test subjects. The students were selected by the writer, but they were not compelled to work through the program.

The students ranged in age from 14 to 18; their I. Q.'s ranged from 85 to 125. The testing took place either in the writer's office or in a vacant classroom, but at no time was it part of a regularly organized class. Only one student at a time worked on any given section of the program because only one set of frames was available.

The Programmed Material

The writer used a set of materials which now are part of Dr. Harry Huffman's text, Programmed Business Mathematics: Concepts, Skills, and Applications, published in 1962 by Gregg Publishing Division of McGraw-Hill Book Company, Inc., and designed especially for

post high school and college freshman students. The portion of the text selected for this study was the part of Section One on "Rebuilding the Fundamentals" which covered these areas: Speed and Accuracy in Addition, Speed and Accuracy in Subtraction, Speed and Accuracy in Multiplication, and Speed and Accuracy in Division.

Instead of the programmed textbook style of displaying several frames on one page, the writer placed each frame on a separate card with the answers on the back of the cards to facilitate the insertion and removal of frames as the testing proceeded.

Objectives of the Experiment

The objectives of the study were to try out Programmed Business Mathematics in order to obtain data about the responses made by high school students at Eastern Mennonite High School, to ascertain causes of errors, to adapt frames for use on the high school level by revision, deletion, or addition, and to continue testing until the error limit fell to five per cent or less on the complete set of frames used in the testing.

Method of Testing

Students were tested individually, but the frames were not changed or revised after each student-editor had gone through them. To provide a more accurate picture of the effectiveness of each revision, several students were tested with each revision because weak spots in the program would tend to show up more clearly if there were

clusters of errors on certain frames. Also, one student's performance might vary from day to day according to his mood, the time of day, and the like, which would tend to distort the record of his progress.

Original Round of Testing

The first two students worked with the original frames.

Joe, a beginning freshman, missed 45 questions out of 449 possible responses which was a 10 per cent rate of errors. Joe was the most enthusiastic of all the student-editors, but his lack of specific training in business arithmetic and the fact that he worked very fast may have contributed to his large number of errors.

John, a senior, was tested next. He was just the opposite of Joe. He had very little to say and showed no great liking or dislike for the experiment. Because of his pattern of behavior in other classes, the writer could not be positively sure that John did not prompt himself as he went along. He missed 14 out of 449 responses, an average of 3.12 per cent.

The average percentage of errors for the first two student-editors was 6.56 per cent

The First Revision

At this point there was no clear indication that a revision should take place because although Joe had many errors, John had so few that error patterns were not yet apparent, and there needed to be further corroboration of difficulties on certain frames to warrant revising them. However, there were five frames that both student-

editors had missed and some other frames that needed clarification, so the writer rewrote two frames, added prompts by underlining on four frames, and corrected mechanical errors on three frames, a total of nine changes.

Second Round of Testing

Kathy, a senior who had previously taken Bookkeeping I, had 51 errors out of 452 possible responses or 11.28 per cent of errors.

Mary Ethyl, also a senior, currently taking bookkeeping, missed 66 questions out of 452. Her 14.60 percentage of errors was the highest percentage of errors that any student-editor had. Because Mary Ethyl had many errors, the writer administered Form W of the Hundred-Problem Arithmetic Test to her after she had worked on the program. With a raw score 11 points higher than she had gotten on Form V just before the testing, Mary Ethyl moved from the 54th percentile to the 79th percentile on the Hundred-Problem Test.

Judy, a freshman who was taking no mathematics course at the time, had 40 incorrect responses out of 452. This was an error rate of 8.85 per cent.

The average percentage of errors for these three student-editors was 11.58 per cent. It should be observed that this represents a difference of 5.02 per cent more errors than the average of the first two student-editors. This can most logically be explained by the fact that individual differences in students' abilities became

apparent and that the number of changes in the first revision was not sufficient to clarify the frames.

The Second Revision

The writer made 29 changes on the frames following the second round of testing. Thirteen prompts were added by supplying important words or sentences, five prompts were added by underlining key words, five frames were revised and rewritten, one frame was reworded for clarity, and one frame received a mechanical correction. Four new frames were constructed to help build clearer understanding of concepts.

Third Round of Testing

David, a sophomore, had the best score of any student-editor working on the frames. Out of a total of 471 responses, he had only three errors, an average of .63 per cent.

Carol, a freshman, worked on the program before David began but finished after he did due to a temporary illness. This may have contributed to her high number of errors which totaled 38 or 8.07 per cent.

Phyllis and Lois, both sophomores, missed 7 and 11 questions respectively. Their percentages of errors amounted to 1.48 per cent for Phyllis and 2.33 per cent for Lois.

The average percentage of errors for these four students was 3.13, a definite improvement over the preceding group. This average was within the five per cent limit, but the writer wished to revise

and strengthen the sequence of frames at several specific places in order to remove still more of the error hazards.

The Third Revision

The main thrust of the third revision was adding new frames at strategic places to provide more practice in rounding numbers. Six new frames dealing with rounding to thousands, three frames on rounding to tens, and one each on rounding to hundred thousands and hundreds were added. In addition to this, four new frames were added at other places. Five prompts were added, one frame was rewritten and one mechanical error was corrected. A total of 22 changes were made.

Fourth Round of Testing

Esther and Rachel, both seniors currently taking Bookkeeping II, worked on the frames following the third revision. Esther missed 20 out of a total of 488 responses with 4.10 per cent of errors. Rachel missed only 5 responses out of 488, a percentage of 1.02. The average percentage of errors for these two student-editors was 2.56, the lowest of any group tested. However, their percentage of errors average was not as low as the average of the last two students tested in the third round of testing in which Phyllis with 1.48 per cent of errors and Lois with 2.33 per cent, averaged 1.90 per cent. This leads the writer to the conclusion that although it is possible to adjust the frames to remove most of the error potentials,

individual differences are still operative and they do account for variations in the percentage of errors.

Error Percentages

A table summarizing the students' achievement in terms of percentage of errors in working through the frames is shown here:

TABLE 14
DIFFERENCES IN PERCENTAGE OF ERRORS
FOR THREE REVISIONS

Round of Testing	Average Percentage by Groups	Difference in Percentage of Errors from Original	Difference in Percentage of Errors from Revision 1	Difference in Percentage of Errors from Revision 2
Original	6.56%	-----	-----	-----
First	11.58%	+ 5.02%	-----	-----
Second	3.13%	- 3.43%	- 8.45%	-----
Third	2.56%	- 4.00%	- 9.02%	- .57%

Causes of Errors

The total number of errors made by the eleven student-editors was an even 300. Table 13 on page 96 showed the classification of errors made by individual student-editors. Table 15 summarizes the classification of errors according to the round of testing. Inspection of these tables shows that estimation and rounding account for 82 errors; computational processes account for 66 errors; not applying a concept, 59; and failure to make the desired response, 55.

TABLE 15

A SUMMARY OF THE CLASSIFICATION OF ERRORS
MADE BY FOUR GROUPS

Causes of Errors	First Round of Testing	Second Round of Testing	Third Round of Testing	Fourth Round of Testing
Faulty or Incomplete Frames	3	1	---	---
Carelessness				
Didn't think through question carefully	3	10	1	2
Too rapid reading	4	6	2	2
Omitted part or all of responses	1	4	2	---
Failed to make desired response	11	28	12	4
Computational				
Addition, Subtraction	17	31	13	5
Multiplication, Division				
Lack of Comprehension of Concepts				
Estimation, rounding	16	41	15	10
Did not apply concept	5	37	15	2
Did not understand	6	15	1	2
TOTALS (By Groups)	66	173	61	27

In the tabulation of errors made by college students on page 46, on a portion of the multiplication frames, there were 23 computational errors and 7 attributed to lack of understanding, roughly a proportion of three to one.

In the tabulation of errors of high school students, 165 errors were attributed to a lack of comprehension of concepts and 66 to errors of computation, a proportion of 5 to 2, but just in the reverse order. A possible explanation for this reversal might be that high school students have not learned how to read a rule and then apply it to the problems they must solve.

Individual Differences

In summarizing the students' work in the various revisions, it became apparent that no matter how careful the writer was in revising frames the error rates did not move progressively downward. There had been no attempt to control two variables, I. Q. and previous training in business mathematics. These variables undoubtedly affected the error percentages of the respective student-editors.

Inasmuch as potential error stimuli were being removed in the revisions and since I. Q. and previous training were not controlled, the error percentages on the original frames and the percentages on the final revision are not exactly comparable; but they do show that error potentiality has been reduced.

A third factor causing variation in errors is carelessness. No matter how well matched student editors might be in I. Q. and

previous training in business mathematics, the manner in which they work must be taken into account. One student may be extremely cautious in his work; another may be very careless and this will affect the work he does.

Frame Revision

Table 16 shows a summary of the kinds of changes carried out by the writer. Nine changes were made in the first revision, 29 changes in the second revision and 22 in the third revision. The writer feels that the changes were effective in reducing the error potential even though the errors of each successive student-editor were not lower than the preceding student-editor in every instance, due to the variables which were not controlled.

Table 17 shows the number of frames comprising the testing material on the several rounds of testing.

General Observations

The work of eleven student-editors at Eastern Mennonite High School using the programmed business mathematics fundamentals frames leads the writer to these observations:

1. The three main causes of errors ranked in this order: lack of comprehension, 165 errors; carelessness, 92 errors; and computational errors, 66.

2. Table 15 shows how these errors were distributed among the four rounds of testing.

TABLE 16

SUMMARY OF THE KINDS OF CHANGES
MADE ON THE PROGRAM

Type of Changes	First Revision	Second Revision	Third Revision	Totals
Mechanical Changes:				
Lines or dots added	2	1	1	4
Frames exchanged	1			1
Frame reworded		1		1
				6
Revisions:				
Rewrite frame	2	5	1	8
Prompt by underlining	4	5		9
Prompt by adding a word or sentence		12	5	17
Prompt on rounding to 10,000's		1		1
				35
New Frames:				
Rounding to 10's		1	3	4
Rounding to 100's			1	1
Rounding to 1000's			6	6
Rounding to 100,000's			1	1
Other concepts		3	4	7
				19
Totals	9	29	22	60

TABLE 17

NUMBER OF FRAMES AND RESPONSES

Total Frames

Section	Original Program	First Revision	Second Revision	Third Revision
Addition	157	57	60	65
Subtraction	22	22	22	22
Multiplication	99	99	100	107
Division	58	58	58	59
Totals	236	236	240	253

Total Responses

Section	Original Program	First Revision	Second Revision	Third Revision
Addition	105	105	114	123
Subtraction	30	30	32	32
Multiplication	219	222	230	236
Division	95	95	95	97
Totals	449	452	471	488

3. Table 15 also shows that the errors decreased following the revision made after the second round of testing.

4. The writer made sixty changes in the frames--9 changes in the first revision, 29 changes in the second revision, and 22 in the third revision.

5. It is possible to adapt Programmed Business Mathematics for use on the high school level and for students to work through the program with less than five per cent of errors. However, the writer believes that no matter how much the program would be refined, revised, or added to, the program will probably never result in a perfect performance because of factors such as fatigue, carelessness, or similar causes.

Recommendations

1. The program should be tried with an entire class to observe the effectiveness of the revisions with a larger number of individuals.

2. Retention tests should be given to motivate students to work carefully, to determine mastery of facts, and to indicate areas where further teaching is needed.

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ABSTRACT

This thesis is the report of an experiment carried on under the direction of Harry Huffman, Professor of Business Education at Virginia Polytechnic Institute, using material in Programed Business Mathematics which was written by Dr. Huffman especially for post high school students and college freshmen.

The programmed material used in the experiment consisted of the first 239 frames covering the basic fundamentals of addition, subtraction, multiplication, and division. The main emphasis of the program is on building speed and accuracy in applying the skills and concepts related to these basic processes.

The subjects of the experiment were eleven high school students from grades 9, 10, and 12, at Eastern Mennonite High School.

The purpose of the experiment was to obtain these data:

1. What kind of errors do high school students make on the program?
2. What changes need to be made in the program to adapt it for use on the high school level?
3. Can the program be adjusted so that high school students can work through it with five per cent or less of errors?

The writer found that errors were grouped under these major headings and in this order: Lack of comprehension of concepts introduced, carelessness, and computational (errors of addition,

subtraction, multiplication, and division).

The frames were revised three times during the experiment. Revision consisted of adding prompts of underlining or new words, rewriting frames, or constructing new frames.

Two students went through the original frames: average errors, 6.56 per cent. Three students worked with the first revision: average errors, 11.58 per cent. Four students worked with the second revision: average errors, 3.13 per cent. Two students worked with the third revision: average errors, 2.56.

Although the error rate did not drop consistently, it did drop from a high of 11.58 per cent on the second round of testing to 2.56 per cent for the final round, a decrease of 9.02 in per cent.

Individual differences in I. Q., previous training, and the manner in which the student-editors worked undoubtedly account for part of the variation in error percentages.

The results of the experiment show that it is possible for high school students to work on the fundamentals of business mathematics frames with an error limit of five per cent or less.