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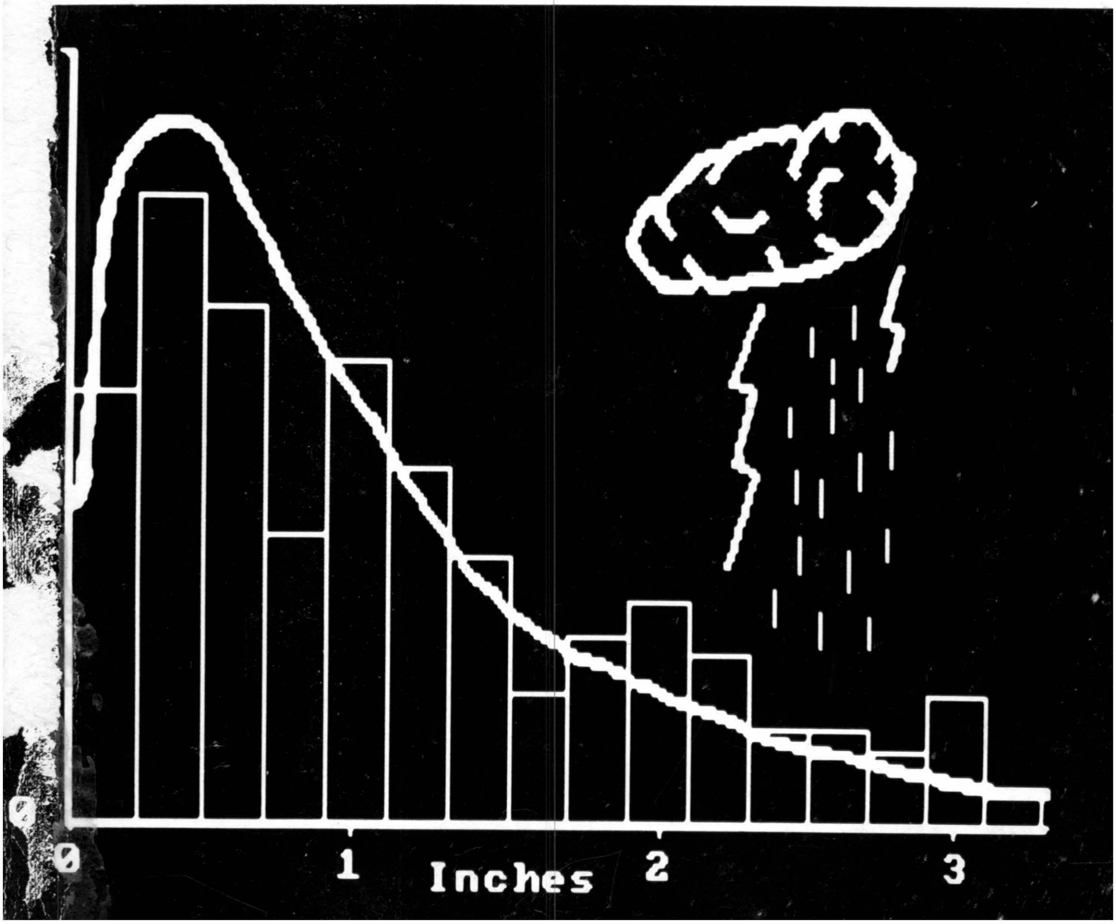

James R. Nichols, Dean and Director
College of Agriculture and Life Sciences
Virginia Agricultural Experiment Station
Virginia Polytechnic Institute and State University
Blacksburg, Virginia 24061

The Virginia Agricultural and Mechanical College came into being in 1872 upon acceptance by the Commonwealth of the provisions of the Morrill Act of 1862 "to promote the liberal and practical education of the industrial classes in the several pursuits and professions of life." Research and investigations were first authorized at Virginia's land-grant college when the Virginia Agricultural Experiment Station was established by the Virginia General Assembly in 1886.

The Virginia Agricultural Experiment Station received its first allotment upon passage of the Hatch Act by the United States Congress in 1887. Other related Acts followed, and all were consolidated in 1955 under the Amended Hatch Act which states "It shall be the object and duty of the State agricultural experiment stations . . . to conduct original and other researches, investigations and experiments bearing directly on and contributing to the establishment and maintenance of a permanent and effective agricultural industry of the United States, including the researches basic to the problems of agriculture and its broadest aspects and such investigations as have for their purpose the development and improvement of the rural home and rural life and the maximum contributions by agriculture to the welfare of the consumer . . . "

In 1962, Congress passed the McIntire-Stennis Cooperative Forestry Research Act to encourage and assist the states in carrying on a program of forestry research, including reforestation, land management, watershed management, rangeland management, wildlife habitat improvement, outdoor recreation, harvesting and marketing of forest products, and "such other studies as may be necessary to obtain the fullest and most effective use of forest resources."

In 1966, the Virginia General Assembly "established within the Virginia Polytechnic Institute a division to be known as the Research Division . . . which shall encompass the now existing Virginia Agricultural Experiment Station . . . "

STATISTICAL STUDY OF RAINFALL DISTRIBUTIONS AND POINT-SPECIFIC RAINFALL SIMULATION MODELS

K. W. Molten, T. B. Brumback Jr., and J. C. Baker

Department of Agronomy
Virginia Polytechnic Institute and State University
Blacksburg, Virginia 24061

Virginia Agricultural Experiment Station
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ABSTRACT

The dual objectives of this study were to statistically test commonly assumed rainfall distributions against historical data and to use these distributions to statistically test and compare point-specific rainfall simulation models for the growing season at Blacksburg, Virginia. The distributions tested were selected for ease of parameter estimation and frequency of use in rainfall models. Two of the four rainfall models compared were the independent exponential and the independent gamma models. Independent models used daily rainfall probabilities to determine the occurrence of a rainfall event and the corresponding distributions to simulate an amount. Two other models tested use a first-order Markov chain to determine both occurrence and amount of rainfall. The first of these models, termed the Haan 7X7 model, used cumulative transition probabilities of 7 rainfall classes in a 7X7 matrix. Each of the 7 classes was constructed over defined ranges within the rainfall distribution curve. The 7X7 matrix assumed no rain in class 1, uniform distributions in classes 2 through 6, and an exponential distribution in class 7. The second Markov chain model, termed the Markov 5X5 model, used cumulative transition probabilities of 5 classes defined in a 5X5 matrix. Each of these 5 classes was also constructed over defined ranges within the rainfall distribution curve. The 5X5 matrix assumed no rain in class 1, a gamma distribution in classes 2 through 4, and an exponential distribution in class 5. The results of the distribution study showed that, for Blacksburg, Virginia, the assumption of monthly rainfall following an exponential distribution, as used in the independent exponential model, is invalid, as is the assumption of monthly uniform distributions in classes 2 through 5 of the Haan 7X7 model. The assumption of monthly rainfall following a gamma distribution, as used in the independent gamma and in classes 2 through 4 of the Markov 5X5 model, was valid as was the assumption of an exponential distribution in class 5 of the Markov 5X5 and class 7 of the Haan 7X7 models. The results of the model comparisons on a seasonal basis showed that the independent exponential model can accurately estimate rainfall but can not accurately estimate runs of wet and dry days. The independent gamma model underestimates rainfall amounts and can not accurately

estimate runs of wet and dry days. The Haan 7X7 and the Markov 5X5 models accurately estimate both amounts and runs of wet and dry days. Overall the best model to use with any assurance of actually representing rainfall during the growing season in Blacksburg, Virginia, was the Markov 5X5 model. This model was termed the best because the distributions assumed were valid and the model results compared well statistically with the historical data, indicating that the model maintained the integrity of the statistical properties of the modeled population.

ACKNOWLEDGEMENTS

The authors would like to express their appreciation to Dr. Osman Balci, of the Virginia Tech Computer Science Department, for his assistance in developing portions of the computer programs and for his assistance in testing and interpreting the results of the distribution study.

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1. INTRODUCTION

Adequate simulation of rainfall is an important aspect of hydrologic models. Hydrologic models can be classified into two broad categories based on the size of an area and the purpose of the model under consideration. The first category, point-specific (small scale) models, includes models of field-scale runoff and of water movement within a soil-plant-atmosphere continuum. The second category, watershed (large scale) models includes models for streamflow, peakflow, stream hydrograph, and watershed runoff. Within each of these categories rainfall over the area is simulated differently. Point-specific models require rainfall amounts be modeled in time only, given the assumption that rainfall is received equally in all parts of the area. Watershed models require that rainfall amounts be modeled in time and in space. Time changes in rainfall require modeling the occurrence and amount associated with a rainfall event. Spatial changes in rainfall require the additional modeling of intensity and persistence of an event over a defined area. In this study, point-specific rainfall models are considered in order to determine the validity of distributions used to describe rainfall and to determine the accuracy and validity of the models themselves.

Point-specific rainfall models involve simulating the occurrence of a rainfall event and simulating an amount to be associated with an event. Both of these steps require study of historical rainfall records. In the past, two methods have been used to simulate the occurrence of a rainfall event. The first method involves determining each day's probability of an event from historical records, and is referred to as the independent method. The second method involves use of chain-dependence to determine the probability of an event based on the previous sequence of wet and dry days. Given the occurrence of a rainfall event, many methods have been used to simulate an amount of rainfall based on a prescribed distribution. Rainfall distributions which have been used in rainfall models include the one-parameter exponential, the two-parameter gamma, and the three-parameter, mixed-exponential probability density functions (Richardson, 1982; Richardson and Wright, 1984; Todorovic and Woolhiser, 1975; Woolhiser and Roldan, 1982).

One of the most important factors in modeling rainfall is the definition of the "season" under consideration. The "season" in a rainfall model represents the period of time for which the parameters that describe the occurrence probabilities and distributions are applicable. Thom (1966) refers to the "season" as a climatological series and defines this series so that it is the sample of a population. He points out that "... the individual climatological series and populations must first be defined so that the exact meaning of the mixture of populations is defined in advance of statistical analysis." In other words, the "season" must be defined for a specific area and for a specific time period and cannot be used outside the bounds of the population the "season" represents.

Many methods of modeling rainfall have been developed, tested, and compared (Haan, et al. 1976; Richardson, 1982; Richardson and Wright, 1984; Todorovic and Woolhiser, 1975; Woolhiser and Roldan, 1982). These methods vary in their identification of the season and in the method of simulating occurrence and amount of rainfall; however, general conclusions can be drawn from the results of this previous work.

Richardson (1982) uses one of the simplest approaches in simulating rainfall. He assumes the "season" as a day for generating the occurrence of a rainfall event and then assumes the "season" as a month to generate and compare rainfall according to a one-parameter exponential model, a two-parameter gamma model, and a three-parameter mixed-exponential model. He then compares the results of each of these models to historical data and concludes that the three-parameter mixed-exponential model is slightly better at estimating rainfall than is the two-parameter gamma model. He points out, however, that, due to the complexity of estimating the parameters in the mixed-exponential model and only a minor difference in results, the two-parameter gamma model is the more appropriate choice in most applications.

As stated earlier, rainfall occurrence processes have also been modeled as chain-dependent events. The simplest method used to determine chain dependence is a first-order Markov chain. A first-order Markov chain determines the probability of the occurrence of a rainfall event based on whether or not the previous day was wet or dry. As the order of a Markov chain increases, the chain dependence also increases. Thus a second-order Markov chain depends on the occurrence of a rainfall event within the previous two days. Chin (1977) compared orders of Markov chains in predicting rainfall occurrence at 100 stations in the conterminous United States. He compared only occurrence processes without consideration of the amounts and found, that, for Virginia, July and August rainfall followed a first-order Markov chain occurrence process while January and February rainfall followed a second-order Markov chain occurrence process. Todorovic and Woolhiser (1975) compared a binomial counting process with a first-order Markov chain to simulate rainfall occurrence with an exponential distribution of rainfall amount over an n-day period, or "season". They concluded that the first-order Markov chain is slightly better than the binomial counting process. Roldan and Woolhiser (1982) compared first-order Markov chain occurrence processes with an alternating renewal process of wet-day intervals and a truncated negative binomial distribution of dry-day intervals, and also found the Markov chain process to be the better method. Woolhiser and Roldan (1982) then compared models using chain-dependent vs.

independent occurrence methods within exponential, gamma, and mixed-exponential models for 14-day periods. They found that the independent mixed-exponential model gave the best fit followed by the independent gamma and chain-dependent gamma, respectively. Richardson and Wright (1984) used a first-order Markov chain gamma model over monthly "seasons" with a correction factor and found good representation of historical data.

The above-mentioned models separately simulate the occurrence and amount processes. Two other methods which combine the interaction of these factors should also be mentioned. Chang et al. (1984) use the discrete auto-regressive moving average (DARMA) family of models to simulate rainfall. They state that the first-order Markov chain process can be shown to be a special case within the DARMA family. They define four non-overlapping "seasons" of 90 days each within each year beginning January 1, and find that the DARMA model described by a first-order Markov chain is the best fit for seasons two, three, and four.

The second model which attempts to combine the interaction of occurrence and amount processes was developed by Haan et al. (1976). They used a first-order Markov chain in a 7X7 matrix of transition probabilities to describe rainfall occurrence and amount. The 7X7 matrix is constructed by first breaking the rainfall frequency distribution of each month into 7 classes (Table 1). Class 1 is assumed to be a dry day and does not follow a distribution. Within classes 2 through 6 they assume a

TABLE 1

Upper and lower limits of rainfall classes and distributions assumed by Haan et al. (1976)

Class	Limits (inches)	Distribution
1	0	-------
2	0.01-0.03	uniform
3	0.03-0.07	uniform
4	0.07-0.15	uniform
5	0.15-0.31	uniform
6	0.31-0.63	uniform
7	>0.63	exponential

uniform distribution. Class 7 is assumed to follow an exponential distribution. Haan constructs the 7X7 transition probability matrix by determining the cumulative probability of each of the 7 classes following each of the 7 classes. He constructs a 7X7 matrix for each month of the year and uses the matrix as a Markov chain. In other words, if yesterday's rainfall class was 1, todays rainfall class is based on the cumulative probability of any of the 7 classes following class 1 in the prescribed month. Haan et al. (1976) found good agreement between the statistical properties of the simulated and historical data.

In summary, the above literature shows that methods of modeling point-specific rainfall vary widely in the assumptions of "season", occurrence processes, and distributions. "Season" definitions vary depending on whether an occurrence or an amount process is being modeled. The "season" has been defined as a day for independent occurrence processes and a month for chain-dependent occurrence processes. Amount processes have assumed "seasons" of 14 days and one month. Combination models, which model both occurrence and amount, have assumed "seasons" of one month and 90 days. Occurrence processes have assumed independent daily probabilities, chain-dependent event probabilities, and binomial counting processes. Distributions which have been assumed for modeling rainfall amounts include the three-parameter mixed-exponential, the two-parameter gamma, and the one-parameter exponential probability density functions. The combination models have assumed uniform and exponential distributions within rainfall range classes and have used discrete autoregressive moving average methods for modeling rainfall.

Though there is wide variation within the literature, three overall statements can be made about the previous work. First, the selection of the "season" for occurrence and amount processes is extremely important because this definition determines how parameters are estimated from historical data and thus defines a particular population. Second, a Markov chain is the best overall method of modeling rainfall occurrence, and third, the mixed-exponential probability density function is the best distribution for modeling rainfall amount followed by the gamma and exponential probability density functions, respectively. None of the above methods of simulating point-specific rainfall can be defined as the best for a specific area since the models vary widely in their definition of the "season" and because it is not clear in the literature if the assumed probability density functions actually describe the historical rainfall distributions.

The dual objectives of this study were to statistically test commonly used rainfall distributions with historical records of Blacksburg, Virginia, and to compare the statistical properties of four point-specific rainfall models with the historical data the models were meant to represent. The rainfall distributions chosen for testing are the one-parameter exponential and the two-parameter gamma distributions. The three-parameter mixed-exponential distribution was not considered because of the complexity of estimating the three parameters and because, as stated by Richardson (1982),there is very little difference in the performance of the mixed-exponential and the gamma models. Exponential, gamma, and uniform distributions were also tested within the classes defined by Haan et al. (1976). The point-specific rainfall models chosen were the independent exponential, independent gamma, Haan's 7X7 model, and a first-order Markov chain 5X5 model. The Haan 7X7 model follows the same approach as Haan et al. (1976). The first-order Markov chain 5X5 model is a modification of the Haan 7X7 model, which assumes no rain in class 1, a gamma distribution in classes 2 through 4, and an exponential distribution in class 5. Class 2 of this model combined Haan's classes 2 and 3, and class 4 combined Haan's classes 5 and 6. Classes 3 and 5 are identical to Haan's classes 4 and 7, respectively (Table 2).

TABLE 2

Upper and lower limits of rainfall classes and distributions used in the Markov 5X5 model

Class	Limits (inches)	Distribution
1	0	------
2	0.01-0.07	gamma
3	0.07-0.15	gamma
4	0.15-0.63	gamma
5	>0.63	exponential

The "season" assumed for this study was a day for independent occurrence processes and a month for chain-dependent occurrence processes and all rainfall distributions. Rainfall was generated for the months March through October. These months were chosen because the literature shows that winter months may not follow a first-order Markov chain in Virginia and because the months March and April represent periods of soil moisture accumulation immediately preceeding the growing season months, May through October, at Blacksburg, Virginia, where the results of this study will be used at a later time to study the effects of water stress on corn yields.

2. MATERIALS AND METHODS

2.1 RAINFALL DISTRIBUTION STUDY

Daily historical rainfall records, for the months of March through October at Blacksburg, Virginia, in years 1953 through 1981, were accessed from the Hydrologic Information Storage and Retrieval System (HISARS) (Johnson et al., 1975) and output to an on-line Statistical Analysis System (SAS) (Helwig and Council, 1979) data set for statistical analyses, parameter estimation, and model comparisons. From the daily historical records frequency tables were generated for monthly amounts. Historical records of monthly rainfall amounts were input into UNIFIT (Law and Vincent, 1983) to test exponential and gamma distributions within each month. The monthly rainfall records were separated into Haan's classes, and UNIFIT was used to test uniform, exponential, and gamma distributions in classes 2 through 6. For Haan's class 7, exponential and gamma distributions were tested. Uniform distributions were not tested in class 7 because of the lack of an adequate upper endpoint for the rainfall range. Classes 2 and 4 of the Markov 5X5 model were constructed by combining classes 2 and 3 and classes 5 and 6 of the Haan 7X7 model and gamma distributions were tested within the combined classes. UNIFIT performed a χ-square goodness-of-fit test to compare simulated against historical data.

2.2 RANDOM NUMBER AND RANDOM VARIATE GENERATION

In the following rainfall models occurrence probabilities are assumed to follow a uniform probability distribution, and rainfall amounts are assumed to follow either a uniform, exponential, or gamma probability distribution. The probability density functions (PDF) and the corresponding cumulative distribution functions (CDF) of each distribution are given in Table 3. The CDF is the equation used to generate a random variate for rainfall amount according to the prescribed distribution. In order to generate a random variate that follows a particular distribution, it is first necessary to generate a uniformly distributed random number between 0 and 1, U≈U(0,1). This number, U, is equal to F(x) in the CDF and the assumption is

Table 3: Probability density functions and associated cumulative distribution functions used in the rainfall models.

Distribution	Probability Density Function	Cumulative Distribution Function	Limits
Uniform (x(u))	$f(x) - \frac{1}{b-a}$	$F(x) = \frac{x-a}{b-a}$	$a \leq x \leq b$
Exponential (x(e))	$f(x) = \frac{1}{\beta} e^{-x/\beta}$	$F(x) = 1-e^{-x/\beta}$	$x \geq 0$
Gamma (x(g))	$f(x) = \frac{\beta^{-\alpha} x^{\alpha-1} e^{-x/\beta}}{\int_0^\infty t^{\alpha-1} e^t d^t}$	F(x) has no closed form	$x > 0$

made that any cumulative probability value between 0 and 1 has an equal chance of occurrence throughout a simulation.

In the point-specific rainfall models described later a multiplicative congruential random number function (MCRNF) is used to generate $U \approx U(0,1)$ (Balci, 1984). The MCRNF used is

$$U = z(i) \ / \ b$$

where b is a constant and z(i) is determined from

$$z(i)=\mathrm{MODulo}((a*z(i-1)),b)$$

where a and b are constants and z(i-1) is the previous value of z. This MCRNF requires an initial seed value z(0), which must be a 5-digit, odd-integer number so that the remainder of the two numbers (a*z(i-1)) and b can be found by the MODulo function.

This method of generating random numbers allows repetition of random number sequences when the same initial seed number, z(0), is used to start a random number sequence. This study used two MCRNF's in each model. The random number generated by the first MCRNF is used to determine the occurrence of a rainfall event. If rain is to be generated, the second MCRNF returns a random number to the CDF describing the assumed rainfall distribution so that a random variate for rainfall amount can be determined. The initial seed number, z(0), to each MCRNF is different for each of the two functions within each model, but the same z(0) values are used in all four simulation models so that the models run on the same sequences of numbers; therefore, their results can be compared.

In order to generate a random variate according to a prescribed distribution, the CDF must be solved for the variable x. For the uniform and exponential CDF's x is easily determined by inverse-transform because the equations have closed-form solutions. For the gamma CDF, there is no closed-form solution for real values of α, and an acceptance-rejection method is used to generate a value for x. Law and Kelton (1982) discuss generation of random numbers and random variates, and only the techniques used in the following rainfall models are discussed here.

The inverse-transform of the uniform CDF is

$$x(u) = a = (b-a)U$$

where x(u) is a uniformly distributed random variate, a is the lower bound and b the upper bound of the distribution's range, and U is U(0,1).

The inverse-transform of the exponential CDF is

$$x(e) = -\beta \ln(1-U)$$

where x(e) is an exponentially distributed random variate and β, the scale parameter, is equal to the sample mean. Because U is U(0,1), the calculation (1-U) is often considered to be unnecessary and the following equation is used to generate x(e)

$$x(e) = -\beta \ln(U)$$

This is the form of the equation used in the independent exponential rainfall model. For generating rainfall within class 7 of the Haan 7X7 model and class 5 of the Markov 5X5 model, where rainfall ranges from 0.63 inches to ∞, the form used is

$$x(e) = [-(\beta-0.63)\ \ln(U)] + 0.63$$

where β is the mean rainfall greater than 0.63 inches. The algorithm used in generating rainfall amounts which follow a gamma distribution (Balci, personal communication) is an acceptance-rejection technique dependent on the value of the given shape parameter α.

Given that α and z are less than 1.0, with z determined from

$$z = U_1{**}(1/\alpha)\ /\ [U_1{**}(1/\alpha) + U_2{**}(1/(1-\alpha))]$$

where U_1 and U_2 are U(0,1), the gamma random variate, x(g), is calculated from

$$x(g) = z * \ln(U_3) * \beta$$

where U_3 is U(0,1) and β is the given scale parameter.

Given α greater than or equal to 1.0 and less than 5.0, x(g) is determined from

$$x(g) = -\ln[U(\alpha i)!] * \alpha/\alpha i$$

where αi is the integer portion of α, $U(\alpha i)!$ is the product of αi generated random numbers U(0,1). This equation is valid when

$$U[(\alpha i)+1] < (Y/\alpha i){**}(\alpha-\alpha i) * EXP[-(\alpha-\alpha i)*Y/(\alpha-1)]$$

where $Y=-\ln[U(\alpha i)!]$.

For α greater than or equal to 5.0

$$x(g) = -\ln[U(\alpha i)!] * \beta$$

2.3 PROGRAM LOGIC

Listings of the four point-specific rainfall simulation models are given in Appendices A through D. All four of the models are written in FORTRAN IV and follow the same basic approach.

In the first step, the probability data and distribution parameters are input. SAS was used to generate the daily dry-day probabilities needed as input into the independent exponential and gamma models, and the probability matrices needed as input into the Haan 7X7 and Markov 5X5 models. The distribution parameters needed to model a uniform distribution are the upper and lower bounds of the distribution's range. For classes 2 through 6 of the Haan 7X7 model, the lower bound is calculated in the program as:

$$LOW = [2**(k-1)-1]/100$$

where k is Haan's class number. The upper bound is calculated in the program as:

$$UP = 0.02*[2**(k-1)-1]+0.01$$

The distribution parameter needed for modeling an exponential distribution is the scale parameter β, which is equivalent to the distributions mean. SAS was used to determine β for the independent exponential model and classes 7 and 5 of the Haan 7X7 and Markov 5X5 models, respectively. The distribution parameters needed for modeling a gamma distribution are the shape, α, and scale, β, parameters. These parameters were estimated by UNIFIT using a maximum likelihood technique, and the monthly values obtained from the distribution study were used in the independent gamma model and in classes 2 through 4 of the Markov 5X5 model.

In the second step the occurrence of a rainfall event is determined by calling the first MCRNF to generate U≈U(0,1). The random number U returned from the MCRNF is equal to x(u), given the lower and upper bounds of 0 and 1, respectively, and is taken to represent the probability of a rainfall event. In all four models, the same seed number is used to begin the first MCRNF so that the same sequence of numbers is produced and the results of the models can be compared. In the independent exponential and gamma models, if x(u) is less than or equal to the current day's known dry-day probability, then there is no rain for that day. If x(u) is greater than the dry-day probability, then the parameters which define the distribution to be used are passed to the third step of the program. For the Haan 7X7 and the Markov 5X5 models, x(u) is used to determine the present day's class placement based on the previous day's class placement. For the beginning of each year simulated, the previous day to March 1 is assumed to be a dry day, class 1. Class placement is determined by comparing the returned probability of a rainfall event to the cumulative probability of occurrence corresponding to the previous day's class placement. The new class placement is determined when the cumulative occurrence probability exceeds x(u). If the new class placement is 1, then there is 0 rain, and the program continues to the next day. For classes 2 through 6 in the Haan 7X7 model, the upper and lower limits of the class range are calculated and passed to the third step of the program. For classes 2 through 4 of the Markov 5X5 model, the shape and scale parameters for the month are passed to the third step of the program. For class 7 of the Haan 7X7 model and class 5 of the Markov 5X5 model, the mean monthly rainfall greater than 0.63 inches is passed to the third step of the program.

The third step of each program is to determine a rainfall amount. The second MCRNF is called to return U≈U(0,1). Again the same seed number is used in all four models to allow comparison of the results. The rainfall amount is calculated using the methods described previously, given the parameters which describe the distribution under consideration.

The last step for each model, after daily rainfall occurrence and amount have been determined, is the accumulation of monthly and growing season totals. Twenty-nine simulations were generated for statistical comparison with the 29 years of historical data, and the seasonal mean for the 29 simulations is calculated.

2.4 STATISTICAL TESTS AND COMPARISONS OF RAINFALL MODELS

Calculation of 90% confidence limits on the monthly and seasonal totals was performed for each model and for the historical data. A paired T-test statistic was determined for each model by pairing simulated daily rainfall amounts with daily historical records. The number of runs of wet and dry days over the 29 years simulated was determined for all models and compared to the historical records. A paired T-test statistic was determined for wet-day runs of between 1 and 7 days and for dry-day runs of between 1 and 15 days by pairing simulated seasonal runs with historical seasonal runs. A paired T-test of runs greater than 7 wet days or greater than 15 dry days would not give valid data because of the low frequency of occurrence of runs greater than these periods.

3. RESULTS AND DISCUSSION

3.1 RESULTS OF RAINFALL DISTRIBUTION STUDY

Graphs of the monthly frequency of occurrence of historical rainfall amounts between 0.01 and 0.63 inches (Figures 1-8) show that there may be bias in recording low amounts of rainfall at a non-automated station such as Blacksburg, Virginia. Therefore, the low α levels determined in the χ-square test performed by UNIFIT are not considered reason enough to reject any distribution unless α is less than 0.001.

Results of the χ-square goodness of fit tests of the monthly exponential and gamma distributions (Table 4) show that, for Blacksburg, Virginia, an exponential distribution is an invalid assumption in the months April through October and the distribution is only marginally acceptable in the month of March. The gamma distribution, on the other hand, is a reasonable assumption in all months.

Some results of the χ-square goodness-of-fit tests of distributions tested within Haan's classes 2 through 7 (Tables 5-10) produced invalid results due to the small size of some of the data sets. However, general statements can be made about the distributions within each class. First, no distribution describes the variation in class 2 rainfall

(Table 5), probably because of the narrow range of rainfall amounts within class 2. Second, in classes 3 through 5 the best overall estimate on a seasonal basis is made by the gamma probability density function (Tables 6-8), although in any one month this assumption may be marginal. Third, class 6 rainfall is best described by a uniform probability density function (Table 9), and fourth, class 7 is best described by an exponential probability density function (Table 10).

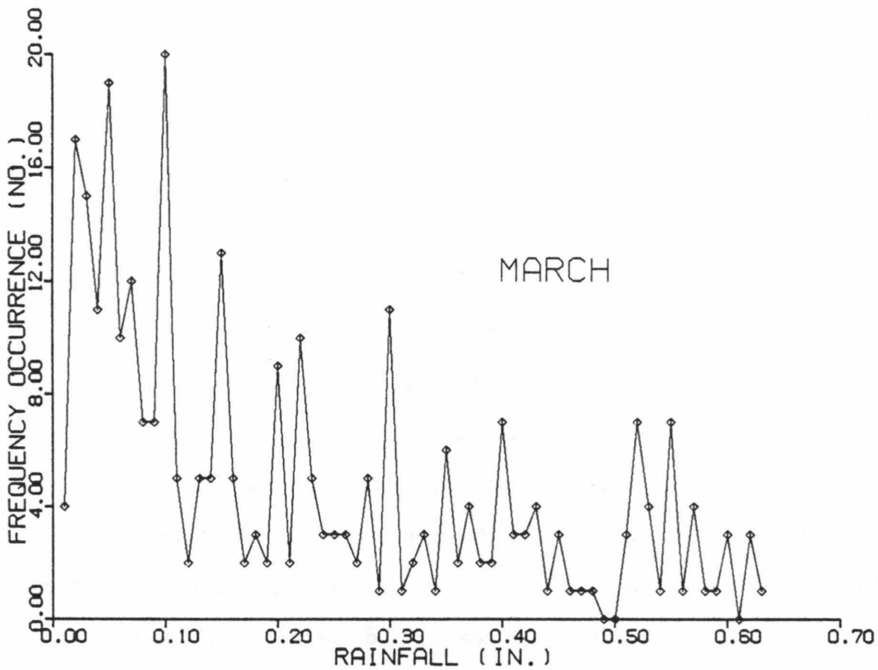

Figure 1: Frequency of occurrence of rainfall amounts between 0.01 and 0.63 in. recorded in March at Blacksburg, Va.

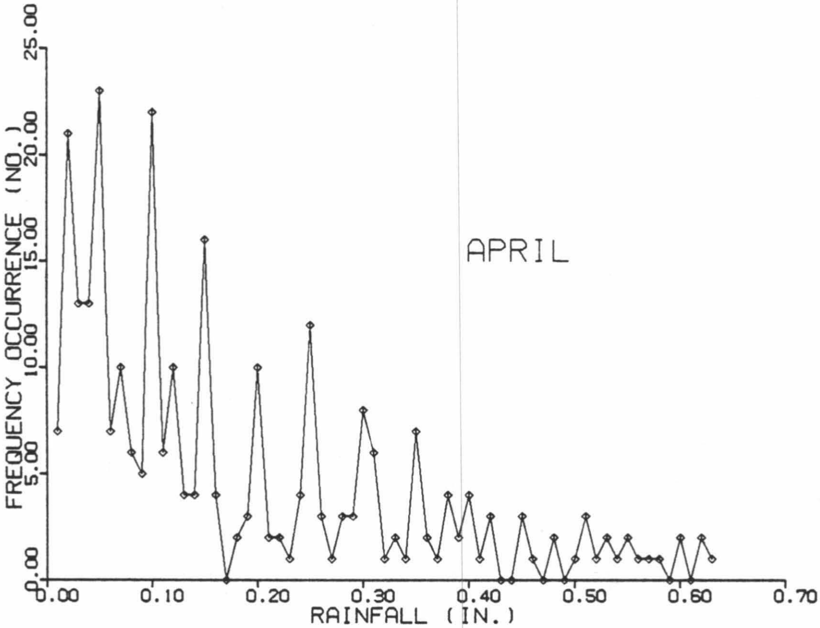

Figure 2: Frequency of occurrence of rainfall amounts between 0.01 and 0.63 in. recorded in April at Blacksburg, Va.

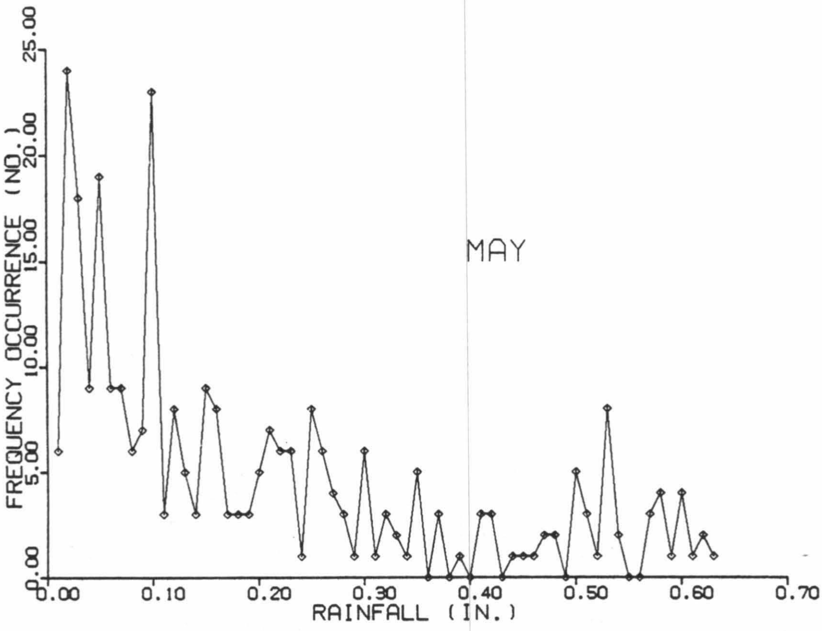

Figure 3: Frequency of occurrence of rainfall amounts between 0.01 and 0.63 in. recorded in May at Blacksburg, Va.

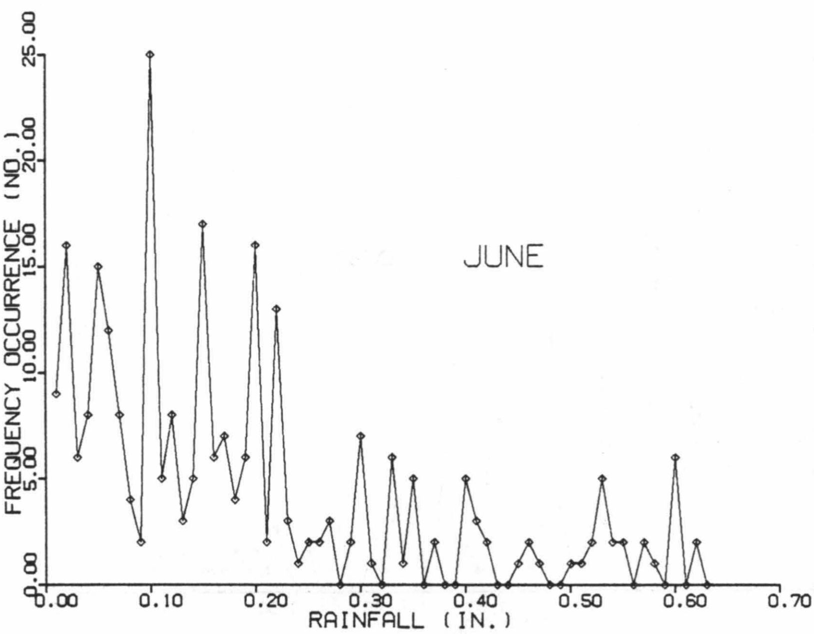

Figure 4: Frequency of occurrence of rainfall amounts between 0.01 and 0.63 in. recorded in June at Blacksburg, Va.

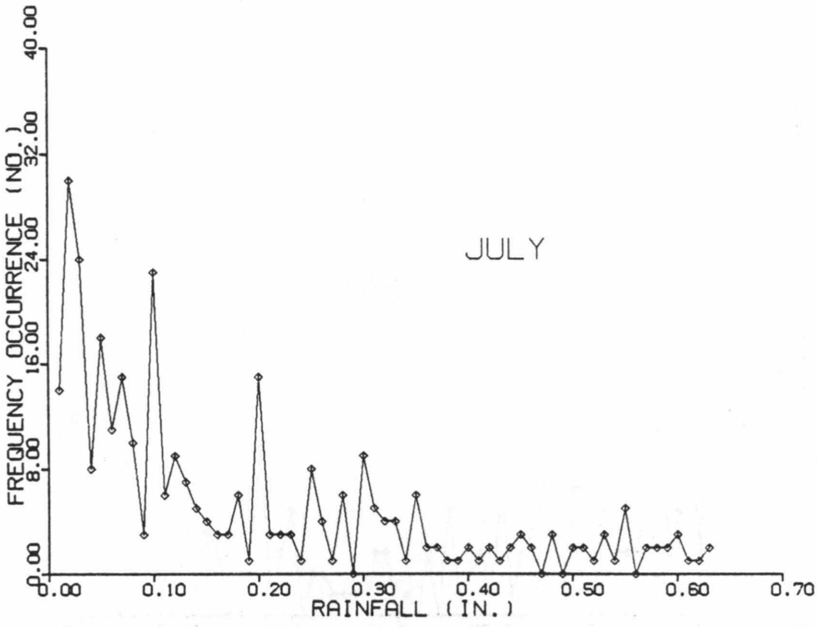

Figure 5: Frequency of occurrence of rainfall amounts between 0.01 and 0.63 in. recorded in July at Blacksburg, Va.

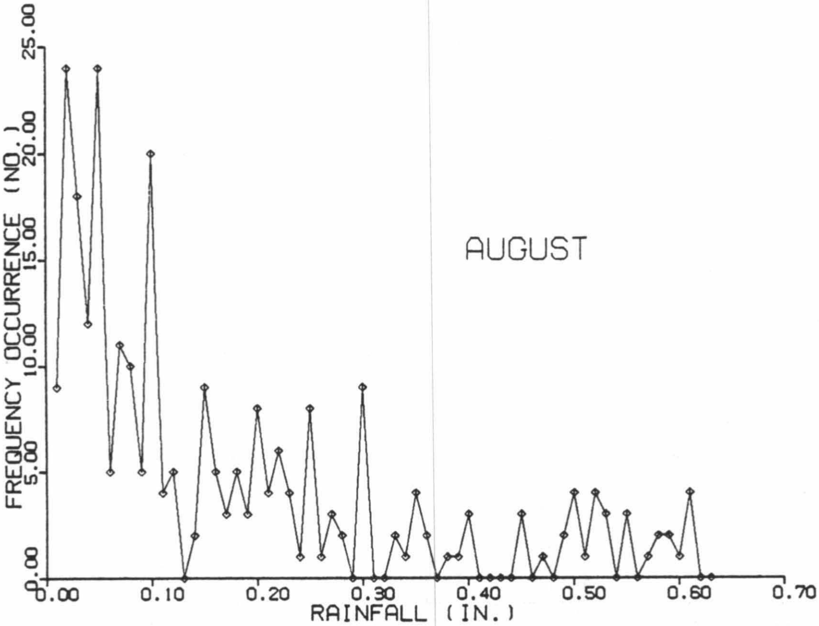

Figure 6: Frequency of occurrence of rainfall amounts between 0.01 and 0.63 in. recorded in August at Blacksburg, Va.

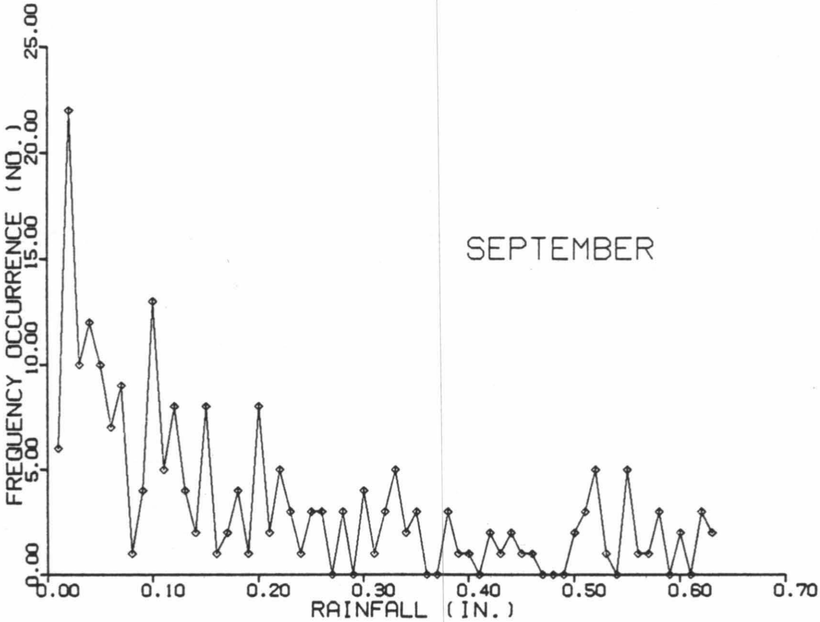

Figure 7: Frequency of occurrence of rainfall amounts between 0.01 and 0.63 in. recorded in September at Blacksburg, Va.

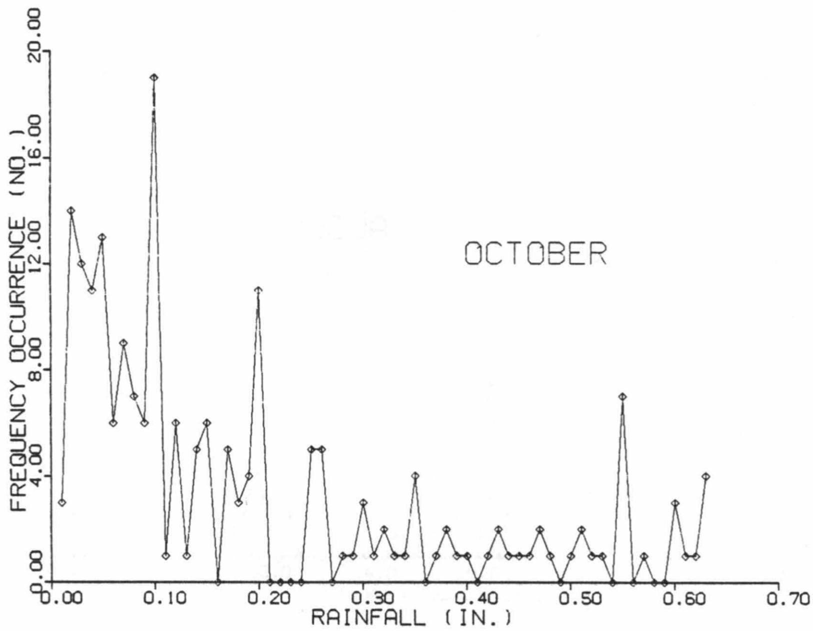

Figure 8: Frequency of occurrence of rainfall amounts between 0.01 and 0.63 in. recorded in October at Blacksburg, Va.

TABLE 4

Results of chi-square goodness-of-fit test of monthly exponential and gamma distributions

	Exponential		Gamma	
Month	χ^2	α	χ^2	α
3	19.38	0.040	14.32	0.160
4	27.70	0.001	6.55	0.680
5	37.79	<0.001	23.91	0.004
6	28.52	<0.001	24.04	0.004
7	41.39	<0.001	5.62	0.780
8	62.48	<0.001	12.92	0.170
9	42.44	<0.001	17.31	0.044
10	61.19	<0.001	17.63	0.040

TABLE 5

Results of chi-square goodness-of-fit test of exponential, gamma, and uniform distributions in Haan's class 2

	Exponential		Gamma		Uniform	
Month	χ^2	α	χ^2	α	χ^2	α
3	19.69	<0.001	23.20	<0.001	15.08	<0.001
4	18.89	<0.001	25.92	<0.001	14.88	<0.001
5	23.71	<0.001	21.33	<0.001	19.88	<0.001
6	12.00	0.002	10.37	0.016	8.18	0.017
7	20.23	<0.001	24.73	<0.001	20.29	<0.001
8	19.13	<0.001	19.92	<0.001	17.21	<0.001
9	24.41	<0.001	17.00	<0.001	16.21	<0.001
10	Not Valid		13.44	<0.001	12.67	0.002

TABLE 6

Results of chi-square goodness-of-fit test of exponential, gamma, and uniform distributions in Haan's class 3

	Exponential		Gamma		Uniform	
Month	χ^2	α	χ^2	α	χ^2	α
3	9.01	0.029	1.69	0.64	11.17	0.011
4	12.00	0.007	3.49	0.32	16.33	<0.001
5	11.67	0.008	1.14	0.77	11.79	0.008
6	12.64	0.005	0.53	0.91	8.24	0.041
7	11.42	0.010	2.35	0.50	12.25	0.006
8	15.59	0.001	6.75	0.08	20.14	<0.001
9	1.84	0.610	2.34	0.51	6.99	0.072
10	3.37	0.340	2.17	0.54	8.26	0.041

TABLE 7

Results of chi-square goodness-of-fit test of exponential, gamma, and uniform distributions in Haan's class 4

	Exponential		Gamma		Uniform	
Month	χ^2	α	χ^2	α	χ^2	α
3	13.33	0.040	7.36	0.06	14.93	0.002
4	Not Valid		Not Valid		6.42	0.040
5	16.02	0.001	2.15	0.54	14.12	0.002
6	28.45	<0.001	9.91	0.02	24.26	<0.001
7	24.33	<0.001	4.64	0.20	13.72	0.003
8	13.32	0.004	7.44	0.06	19.68	<0.001
9	18.28	<0.001	0.77	0.86	12.53	0.006
10	9.06	0.02	4.32	0.23	8.89	0.030

TABLE 8

Results of chi-square goodness-of-fit test of exponential, gamma, and uniform distributions in Haan's class 5

	Exponential		Gamma		Uniform	
Month	χ^2	α	χ^2	α	χ^2	α
3	14.98	0.002	4.91	0.179	8.53	0.036
4	21.08	<0.001	5.68	0.128	4.31	0.230
5	16.79	<0.001	4.88	0.180	4.15	0.246
6	17.45	<0.001	6.26	0.100	22.15	<0.001
7	13.89	0.004	3.85	0.280	4.39	0.222
8	9.48	0.236	1.34	0.720	3.46	0.330
9	11.67	0.009	2.17	0.540	7.59	0.060
10	6.02	0.110	2.01	0.570	2.10	0.550

TABLE 9

Results of chi-square goodness-of-fit test of exponential, gamma, and uniform distributions in Haan's class 6

	Exponential		Gamma		Uniform	
Month	χ^2	α	χ^2	α	χ^2	α
3	48.12	<0.001	32.44	<0.001	19.51	0.006
4	4.17	0.240	2.79	0.430	6.05	0.110
5	19.67	<0.001	10.66	0.140	3.18	0.360
6	7.07	0.070	2.58	0.460	0.25	0.970
7	8.80	0.030	6.13	0.110	3.25	0.360
8	23.68	<0.001	11.29	0.010	5.79	0.120
9	11.92	0.008	9.50	0.020	4.17	0.240
10	8.26	0.040	3.94	0.270	2.69	0.440

TABLE 10

Results of chi-square goodness-of-fit test of exponential, gamma, and uniform distributions in Haan's class 7

	Exponential		Gamma	
Month	χ^2	α	χ^2	α
3	1.31	0.52	3.01	0.39
4	3.72	0.16	not valid	
5	not valid		not valid	
6	5.60	0.06	not valid	
7	not valid		not valid	
8	0.83	0.66	1.71	0.63
9	3.51	0.32	5.89	0.12
10	1.51	0.68	1.63	0.65

Results of the χ-square goodness-of-fit tests for gamma distributions in classes 2 and 4 of the Markov 5X5 model (Tables 11 and 12) show that the gamma distribution describes the historical data in these classes. Classes 3 and 5 of the Markov 5X5 model are identical to classes 5 and 7 of the Haan 7X7 model and yield the same results.

TABLE 11

Results of chi-square goodness-of-fit test of gamma distribution in class 2 of the Markov 5X5 model

	Gamma	
Month	χ^2	α
3	7.26	0.202
4	14.62	0.023
5	10.62	0.101
6	18.64	0.005
7	10.34	0.111
8	15.65	0.016
9	5.58	0.472
10	2.96	0.705

TABLE 12

Results of chi-square goodness-of-fit test of gamma distribution in class 4 of the Markov 5X5 model

	Gamma	
Month	χ^2	α
3	6.73	0.081
4	0.76	0.860
5	6.79	0.079
6	8.48	0.037
7	2.88	0.411
8	5.66	0.129
9	5.64	0.131
10	4.06	0.255

3.2 MODEL COMPARISONS

Results of simulated rainfall (Tables 13-16) give the monthly and seasonal accumulations and seasonal mean of the independent exponential, independent gamma, Haan 7X7, and Markov 5X5 models, respectively. The monthly and seasonal accumulations and the seasonal mean of the historical records are given in Table 17.

Monthly and seasonal 90% confidence limits for each model and the historical data (Figures 9-17) show that no one model predominates in all months. However, the best overall seasonal estimate is made by the Markov 5X5 model, followed by the Haan 7X7 model, the independent exponential, and the independent gamma models, respectively.

The paired T-test statistic of simulated daily rainfall amounts paired with historical daily records (Table 18) again shows that the Markov 5X5 model is the best method of rainfall simulation on a seasonal basis, followed by the Haan 7X7, independent exponential, and independent gamma respectively.

Results of the paired T-tests on simulated vs. historical wet- and dry-day runs (Tables 19 and 20) show a significant difference between one-day runs of wet-days (Table 19) and one- and two-day runs of dry-days (Table 20) between the historical data and the independent exponential and gamma models. The chain dependent models, on the other hand, do not vary significantly. Dry-day runs of between 3 and 10 days and wet-day runs of between 2 and 7 days do not seem to vary dramatically from model to model. However, only the Markov 5X5 and Haan 7X7 models seem able to predict long wet or dry periods that compare favorably with historical records.

TABLE 13

Results of independent exponential rainfall simulation

Simulated Year	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	TOTAL
1	3.93	2.66	4.42	3.89	2.92	2.18	3.85	3.16	27.01
2	4.26	2.47	2.23	4.89	3.52	2.65	4.79	1.54	26.36
3	3.06	3.91	6.11	1.56	3.88	5.72	2.36	3.01	29.61
4	3.69	3.41	3.51	2.48	5.43	1.55	4.90	4.35	29.31
5	1.52	2.38	0.92	3.77	3.87	2.76	3.69	2.31	21.23
6	3.99	2.78	4.68	2.28	1.97	2.85	4.65	2.32	25.52
7	4.40	4.03	3.62	2.23	6.03	2.31	5.17	1.46	29.24
8	3.56	4.34	3.50	1.21	3.36	3.34	1.72	1.35	22.37
9	4.61	5.15	2.44	2.30	4.97	3.72	2.64	1.98	27.82
10	3.89	4.50	3.39	1.28	2.53	1.63	2.84	3.28	23.35
11	2.81	2.99	3.35	4.09	6.54	3.95	0.74	3.76	28.23
12	3.14	3.33	4.81	3.08	3.01	4.01	2.32	5.34	29.04
13	1.83	2.41	4.68	4.97	4.11	2.50	1.89	6.09	28.47
14	3.99	4.16	2.94	1.50	3.43	3.32	7.44	2.01	28.80
15	1.10	4.35	4.27	3.29	4.38	2.82	3.09	2.50	25.80
16	4.34	2.53	2.65	3.87	5.25	3.88	1.64	2.28	26.43
17	2.54	0.64	1.31	4.24	1.77	4.53	3.89	2.25	21.16
18	6.12	3.08	1.39	1.11	4.89	3.34	2.72	3.54	26.18
19	2.45	2.37	3.80	4.63	4.63	6.24	4.54	1.21	29.87
20	2.42	2.88	4.49	2.13	4.95	4.19	5.76	2.32	29.14
21	3.13	4.42	3.26	1.77	3.56	2.66	3.05	1.48	23.34
22	3.43	4.11	2.62	3.17	2.28	2.83	0.52	2.18	21.15
23	4.35	3.76	5.67	3.99	4.61	3.73	1.89	2.88	30.88
24	5.71	5.47	4.46	2.74	6.06	4.27	3.48	3.06	35.24
25	4.50	5.14	3.44	2.29	4.50	2.59	3.64	4.64	30.75
26	2.82	1.11	8.03	3.88	2.68	2.45	3.51	4.22	28.70
27	5.24	3.74	4.60	3.04	3.51	1.80	2.48	2.25	26.66
28	3.68	2.26	2.73	3.68	5.28	2.82	2.83	3.50	26.79
29	2.69	4.52	6.85	2.83	2.59	4.92	2.92	4.60	31.92

Average for 29 simulations is 27.25 inches

TABLE 14

Results of independent gamma rainfall simulation

Simulated Year	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	TOTAL
1	3.33	3.98	3.13	1.26	5.49	3.21	1.98	5.56	27.94
2	4.76	4.66	3.73	2.17	4.40	3.32	5.21	5.54	33.79
3	5.48	2.28	5.54	1.28	2.36	3.81	1.06	3.11	24.91
4	3.78	2.41	3.25	2.10	2.82	3.65	1.13	1.48	20.63
5	2.65	4.95	0.70	2.02	4.84	3.88	5.10	1.55	25.68
6	3.14	4.71	4.90	4.29	2.81	1.33	1.69	3.42	26.29
7	0.62	1.70	1.39	4.65	5.14	0.77	3.10	5.41	22.79
8	3.65	3.15	3.07	4.24	5.12	3.46	2.82	1.30	26.81
9	3.81	5.05	2.18	3.51	2.80	3.70	4.30	2.72	28.06
10	3.60	4.95	3.60	1.86	3.42	4.51	4.89	2.81	29.64
11	3.02	1.58	5.12	2.71	3.78	2.08	1.43	2.92	22.64
12	5.46	3.64	2.87	3.10	5.02	3.88	3.51	5.06	32.54
13	5.32	2.22	5.24	2.82	5.15	1.95	3.18	7.53	33.41
14	5.30	5.65	3.53	3.57	6.19	4.48	5.59	2.34	36.65
15	2.91	4.71	3.54	3.47	4.15	3.13	5.75	5.48	33.14
16	2.51	1.84	2.65	1.68	3.85	2.64	4.88	3.04	23.09
17	1.39	2.34	3.10	3.38	1.21	2.58	2.66	2.79	19.44
18	3.32	3.05	2.75	1.33	0.94	4.40	2.51	4.31	22.60
19	1.78	2.81	3.90	5.04	2.33	3.46	3.39	0.35	23.05
20	0.78	5.25	4.74	1.61	5.25	3.30	6.53	0.97	28.44
21	2.38	2.67	8.32	4.20	3.01	3.11	6.63	2.46	32.79
22	1.56	3.43	2.03	2.62	2.15	2.85	0.15	4.98	19.76
23	2.70	3.25	4.99	1.98	5.03	3.32	3.64	3.10	28.02
24	5.69	6.24	3.99	1.89	1.38	1.72	5.45	3.65	30.02
25	2.72	5.61	3.83	2.10	3.51	1.66	0.60	4.69	24.73
26	1.97	2.22	5.78	3.55	3.42	0.97	2.98	2.42	23.31
27	2.19	5.19	4.26	2.88	5.26	4.64	3.76	2.24	30.42
28	2.68	1.54	1.36	2.79	1.60	2.14	0.77	5.25	18.13
29	4.05	3.32	3.32	3.58	3.77	3.64	3.91	0.50	26.09

Average for 29 simulations is 26.72 inches

TABLE 15

Results of Haan 7X7 rainfall simulation

Simulated Year	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	TOTAL
1	3.21	2.07	5.71	2.39	4.14	1.12	2.48	1.19	22.31
2	3.75	5.37	6.83	1.39	4.37	2.13	8.50	0.58	32.93
3	4.97	2.86	6.14	1.65	2.63	2.52	2.77	2.54	26.08
4	2.93	3.21	3.74	3.10	0.85	2.34	1.13	2.64	19.94
5	0.61	3.44	1.68	3.48	3.58	2.68	6.49	2.69	24.64
6	2.19	3.36	3.82	1.72	3.51	2.29	2.08	1.52	20.50
7	4.42	2.33	2.50	2.47	5.21	2.26	1.89	6.44	27.51
8	3.55	4.44	3.65	0.89	3.21	2.11	1.27	2.48	21.60
9	3.38	6.81	4.74	2.42	6.76	2.17	4.59	2.30	33.17
10	4.01	3.39	3.50	1.78	4.12	1.14	5.49	3.20	26.64
11	6.15	4.96	3.65	0.76	4.42	2.18	3.00	3.89	29.02
12	3.17	2.40	3.15	6.58	2.88	5.21	3.44	6.53	33.36
13	3.81	5.60	5.63	7.02	3.20	3.89	2.37	6.11	37.63
14	3.12	8.08	3.98	2.31	4.64	8.43	5.48	3.27	39.32
15	2.44	8.18	3.52	3.75	6.92	3.37	5.26	2.18	35.61
16	2.47	1.60	1.67	1.51	7.31	5.79	2.14	2.65	25.14
17	2.47	2.43	4.81	2.61	1.30	1.89	11.30	5.62	32.44
18	4.40	4.55	1.27	1.08	2.62	2.01	2.94	3.30	22.16
19	1.91	3.37	2.49	5.80	1.75	4.34	5.10	0.67	25.43
20	2.79	1.97	4.60	0.98	6.60	2.34	5.25	1.64	26.18
21	3.91	4.78	5.74	1.70	2.41	2.34	2.64	1.39	24.91
22	1.57	4.44	1.77	3.80	4.91	2.05	0.60	2.99	22.12
23	7.27	4.51	3.19	3.21	3.46	3.51	1.18	0.86	27.17
24	6.36	4.37	4.51	4.42	3.61	3.03	2.02	2.38	30.69
25	3.96	5.35	2.03	2.66	2.57	1.54	0.48	2.33	20.91
26	5.56	1.62	7.34	2.95	0.80	6.41	1.15	3.07	28.89
27	2.73	8.17	3.40	1.84	4.50	2.96	0.68	1.68	25.95
28	4.91	3.28	1.48	2.46	2.50	4.70	2.02	2.67	24.02
29	3.50	5.10	3.54	2.98	1.95	6.31	3.63	1.68	28.69

Average for 29 simulations is 27.41 inches

TABLE 16

Results of Markov 5X5 rainfall simulation

Simulated Year	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	TOTAL
1	2.47	2.43	4.45	1.54	3.37	0.98	2.50	1.53	19.28
2	5.53	4.95	5.31	1.46	6.59	1.62	6.67	0.46	32.58
3	5.53	3.02	6.51	1.55	2.42	3.31	5.34	1.86	29.54
4	2.31	3.78	3.78	4.61	0.77	2.54	0.95	3.43	22.18
5	0.52	3.54	1.64	3.82	4.34	2.54	8.26	1.98	26.63
6	3.14	4.76	4.01	1.76	3.09	2.86	1.94	1.69	23.24
7	5.14	2.71	2.01	1.84	5.04	1.43	2.33	7.20	27.70
8	4.12	4.08	3.35	1.01	3.35	2.87	2.37	2.19	23.35
9	2.71	6.33	4.65	2.12	4.19	2.15	4.23	1.42	27.80
10	3.78	4.33	3.98	2.00	2.82	1.35	5.40	3.65	27.31
11	6.39	4.18	3.69	1.03	5.74	2.45	7.73	3.41	34.63
12	3.58	4.23	4.50	4.59	3.51	5.47	3.38	9.04	38.30
13	3.54	5.54	6.09	5.67	3.63	2.68	2.51	4.53	34.18
14	4.47	9.35	4.69	1.83	5.10	5.15	7.18	4.20	41.97
15	2.72	6.05	4.17	5.02	6.34	2.97	5.80	2.02	35.09
16	3.61	2.74	2.00	1.90	5.54	6.22	2.45	1.55	26.01
17	2.04	2.22	5.18	4.06	1.78	3.26	5.68	5.76	29.99
18	4.90	5.68	2.11	0.93	3.05	1.96	1.55	2.77	22.95
19	1.49	3.98	2.64	6.82	2.25	3.83	5.89	0.23	27.12
20	2.75	1.96	4.63	1.16	7.92	2.68	6.51	2.30	29.91
21	4.70	6.08	5.69	1.57	1.98	2.88	2.43	1.56	26.88
22	0.93	4.37	2.68	4.85	5.45	2.58	0.68	2.66	24.18
23	6.80	3.63	2.98	2.76	4.27	3.37	1.21	1.27	26.29
24	6.64	4.68	3.45	5.44	3.55	2.68	2.23	3.39	32.05
25	4.55	7.12	2.07	3.09	3.40	1.50	0.75	1.89	24.36
26	5.53	2.27	5.68	2.36	1.19	4.09	0.75	2.08	23.95
27	2.59	7.46	2.74	1.64	4.90	2.60	0.86	1.05	23.83
28	5.21	4.16	1.22	2.57	3.66	3.38	2.13	1.53	23.86
29	5.77	7.70	4.30	3.06	1.75	4.04	3.36	1.63	31.63

Average for 29 simulations is 28.17 inches

TABLE 17

Historical rainfall data

Historical Year	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	TOTAL
1953	4.25	2.73	4.25	5.75	1.86	2.18	1.57	0.50	23.09
1954	3.52	2.22	3.99	1.48	3.98	3.02	1.52	4.22	23.95
1955	8.08	2.48	1.66	3.87	2.14	3.46	0.45	1.56	23.70
1956	3.30	4.14	2.59	4.44	4.21	1.96	5.00	3.31	28.95
1957	2.65	4.55	2.10	4.47	3.39	1.24	10.29	2.11	30.80
1958	4.36	4.12	5.75	2.54	2.64	2.46	1.87	1.35	25.09
1959	3.37	4.78	2.20	1.77	2.83	4.22	5.41	4.94	29.52
1960	3.79	2.95	4.69	1.66	2.09	4.52	2.46	2.32	24.48
1961	4.15	2.91	2.92	4.74	2.19	4.39	1.10	3.74	26.14
1962	3.83	2.91	3.72	2.43	5.36	1.55	3.89	2.10	25.79
1963	4.30	1.07	2.70	1.69	3.04	2.22	3.79	0.12	18.93
1964	2.59	4.00	2.09	1.31	4.43	4.38	3.03	3.40	25.23
1965	3.96	3.14	2.60	3.38	5.08	3.13	2.93	4.05	28.27
1966	1.34	2.65	3.15	0.39	4.94	4.61	5.09	3.66	25.83
1967	3.93	2.28	4.13	2.34	2.93	5.75	2.23	2.76	26.35
1968	3.64	3.88	3.45	3.22	2.38	3.73	1.18	4.78	26.26
1969	2.29	2.10	0.92	3.92	2.55	4.25	5.35	1.77	23.15
1970	2.00	3.32	2.09	3.80	7.25	5.44	2.78	3.52	30.20
1971	2.14	2.51	9.75	5.63	4.56	2.57	4.49	6.26	37.91
1972	2.11	6.48	4.63	6.58	5.70	3.57	6.31	3.26	38.64
1973	6.33	5.21	6.20	3.06	3.58	3.67	1.75	4.80	34.60
1974	4.62	4.00	4.86	3.33	2.27	4.14	4.88	1.15	29.25
1975	8.33	2.40	7.79	3.93	3.02	1.81	5.49	3.99	36.76
1976	2.98	1.21	2.62	8.66	3.26	3.43	3.71	9.14	35.01
1977	2.11	4.73	2.64	5.73	1.88	6.36	4.13	6.29	33.87
1978	4.79	6.63	3.90	1.78	5.15	3.10	2.43	1.23	29.01
1979	3.38	4.27	2.71	5.19	4.32	2.49	6.19	3.21	31.76
1980	5.70	4.65	2.77	2.38	6.34	2.10	0.96	2.93	27.83
1981	2.84	2.86	4.81	2.31	5.66	2.91	3.53	4.35	29.27

Average for 29 years of historical data is 28.61 inches

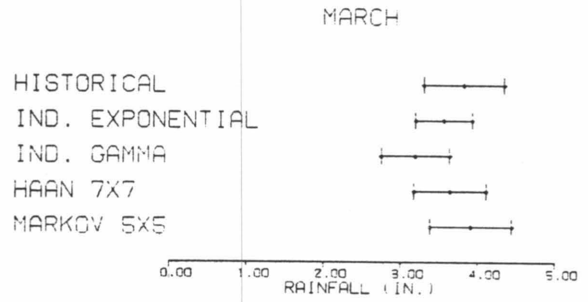

Figure 9: 90 percent confidence limits of simulated and historical (1953-1981) rainfall amounts in March at Blacksburg, Va.

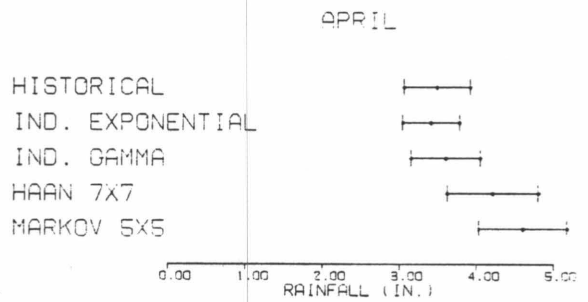

Figure 10: 90 percent confidence limits of simulated and historical (1953-1981) rainfall amounts in April at Blacksburg, Va.

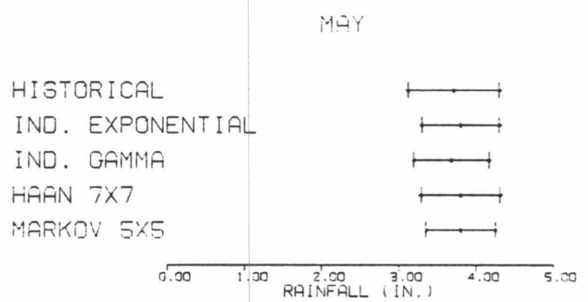

Figure 11: 90 percent confidence limits of simulated and historical (1953-1981) rainfall amounts in May at Blacksburg, Va.

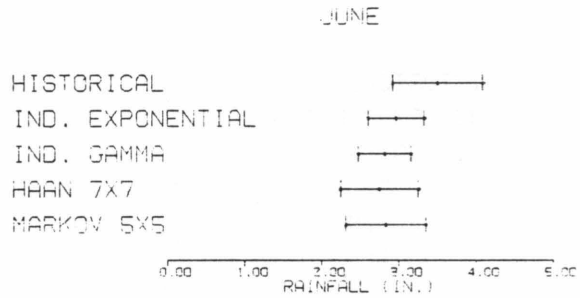

Figure 12: 90 percent confidence limits of simulated and historical (1953-1981) rainfall amounts in June at Blacksburg, Va.

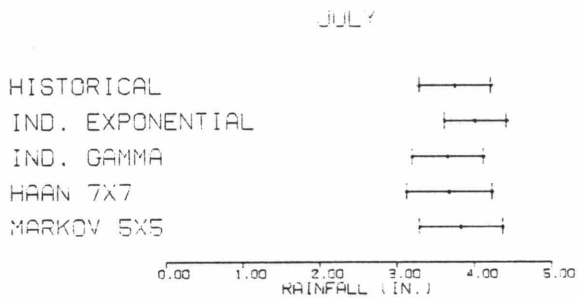

Figure 13: 90 percent confidence limits of simulated and historical (1953-1981) rainfall amounts in July at Blacksburg, Va.

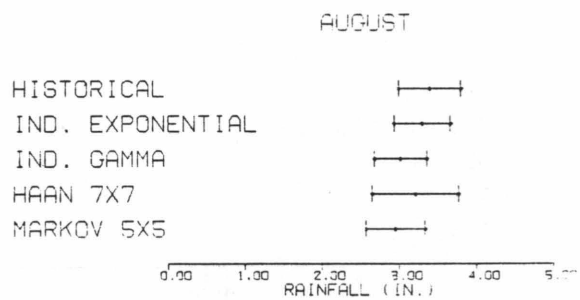

Figure 14: 90 percent confidence limits of simulated and historical (1953-1981) rainfall amounts in August at Blacksburg, Va.

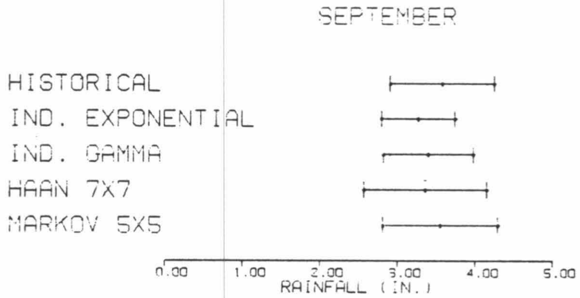

Figure 15: 90 percent confidence limits of simulated and historical (1953-1981) rainfall amounts in September at Blacksburg, Va.

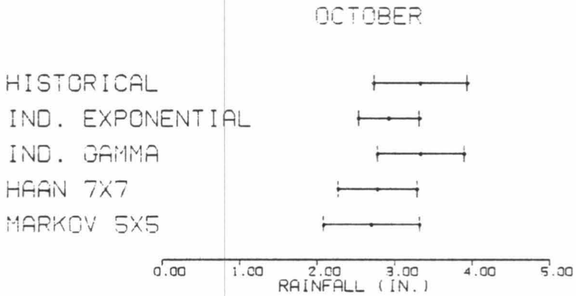

Figure 16: 90 percent confidence limits of simulated and historical (1953-1981) rainfall amounts in October at Blacksburg, Va.

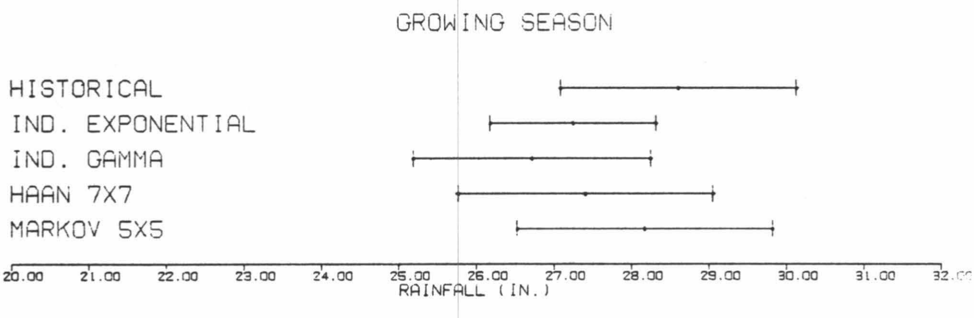

Figure 17: 90 percent confidence limits of simulated and historical rainfall amounts over the growing season at Blacksburg, Va.

TABLE 18

Results of paired T-test of daily historical vs. daily simulated data

	Exponential		Gamma		Markov 5X5		Haan 7X7	
Month	T	α	T	α	T	α	T	α
3	-0.71	0.48	-1.76	0.08	0.25	0.80	-0.49	0.63
4	-0.20	0.84	0.29	0.77	2.43	0.02	1.65	0.10
5	0.21	0.84	-0.09	0.93	0.21	0.83	0.22	0.83
6	-1.45	0.15	-1.90	0.06	-1.73	0.08	-2.01	0.04
7	0.68	0.49	-0.25	0.80	0.17	0.87	-0.20	0.84
8	-0.27	0.78	-0.98	0.33	-1.12	0.26	-0.44	0.66
9	-0.73	0.46	-0.40	0.69	-0.05	0.96	-0.48	0.63
10	-1.01	0.31	-0.01	0.99	-1.51	0.13	-1.31	0.19
season	-1.25	0.21	-1.67	0.09	-0.37	0.71	-1.02	0.31

Table 19 - Results of paired T-test of historical vs. simulated runs of wet days

	Historical	Exponential			Gamma			Haan 7x7			Markov 5x5		
Runs	occur	occur	T	a	occur	T	a	occur	T	a	occur	T	a
1	583	1024	-11.55	0.0001	1038	-12.95	0.0001	619	- 0.90	0.3771	622	- 0.94	0.3553
2	346	359	- 0.53	0.6022	343	0.13	0.8949	299	2.52	0.0176	305	2.00	0.0556
3	155	144	2.90	0.0072	106	3.51	0.0015	152	0.20	0.8427	147	0.60	0.5558
4	91	56	2.82	0.0087	47	3.72	0.0009	87	0.32	0.7502	84	0.55	0.5854
5	26	22	0.55	0.5881	22	0.60	0.5558	41	- 2.14	0.0408	41	- 2.10	0.0448
6	20	7	3.08	0.0046	6	3.31	0.0026	29	- 1.36	0.1843	30	- 1.78	0.0863
7	14	3	2.37	0.0250	4	2.07	0.0479	11	0.62	0.5410	12	0.35	0.7303

Table 20 - Results of paired T-test of historic vs. simulated runs of dry days

	Historical	Exponential			Gamma			Haan 7x7			Markov 5x5		
Runs	occur	occur	T	a	occur	T	a	occur	T	a	occur	T	a
1	347	565	- 6.87	0.0001	543	- 6.14	0.0001	337	0.41	0.6839	346	0.04	0.9661
2	232	364	- 5.01	0.0001	347	- 4.70	0.0001	271	- 1.78	0.0856	268	- 1.67	0.1070
3	200	215	- 0.75	0.4578	223	- 1.17	0.2504	148	3.33	0.0024	142	3.62	0.0012
4	131	159	- 1.47	0.1520	149	- 0.99	0.3319	144	- 0.80	0.4306	145	- 0.90	0.3734
5	91	93	- 9.20	0.8407	102	- 1.17	0.2505	98	- 0.52	0.6054	101	- 0.77	0.4500
6	78	71	- 0.73	0.4728	80	- 0.20	0.8407	67	1.13	0.2664	67	1.19	0.2450
7	52	41	1.09	0.2862	45	0.79	0.4347	47	0.54	0.5920	45	0.75	0.4574
8	28	22	1.00	0.3249	24	0.72	0.4749	42	- 1.61	0.1192	44	- 1.84	0.0767
9	18	21	- 0.46	0.8692	19	- 0.17	0.8692	22	- 0.63	0.5365	21	- 0.55	0.5864
10	18	18	0.00	1.0000	18	0.00	1.0000	24	- 1.00	0.3259	24	- 1.00	0.3259
11	16	9	1.19	0.2434	9	1.19	0.2434	19	- 0.46	0.6476	19	0.46	0.6476
12	13	5	2.12	0.0433	3	3.02	0.0053	3	2.42	0.0225	4	2.20	0.0365
13	5	3	0.70	0.4892	3	0.70	0.4892	14	- 1.97	0.0591	11	- 1.44	0.1609
14	10	3	1.89	0.0698	5	1.15	0.2584	3	1.89	0.0698	4	1.80	0.0831
15	6	1	1.98	0.0572	3	1.36	0.1843	7	- 0.33	0.7452	8	- 0.63	0.5365

3.3 DISCUSSION OF RESULTS

The purpose of this study was to evaluate methods of modeling point-specific rainfall based on appropriate easily determined rainfall distributions and to test commonly used methods of modeling point-specific rainfall against the historical data the models were designed to represent.

For the growing season March through October at Blacksburg, Virginia, the most appropriate model for simulating point-specific rainfall is the Markov chain 5X5 model. All statistical comparisons of this model with the historical data are adequate. The seasonal confidence limits of this model approximate the historical confidence limits (Figure 17). The model compares well with the historical data on a day to day basis as reflected in the seasonal paired T-test statistic (Table 18). The model has the ability to simulate sequences of wet- and dry-day runs that compare favorably with the historical data (Tables 19 and 20), and most important, the model assumes valid rainfall distributions in classes 2 through 5 (Tables 11, 7, 12, and 10, respectively).

The Haan 7X7 model is the second best overall predictor of rainfall. As found by Haan et al. (1976) and by this study, the model compares well statistically with the historical data. However, the assumption of uniformity within classes 2 through 5 is not statistically valid for Blacksburg, Virginia. Because these distributions are not valid, the model cannot be considered valid.

The independent exponential model, the third best overall model, is also invalid and should not be used to simulate rainfall at Blacksburg, Virginia, for two reasons. First, the model does not adequately simulate runs of wet- and dry-days (Tables 19 and 20) and, second, the historical data do not statistically fit an exponential distribution (Table 4).

The independent gamma model is the least appropriate model. Though this model assumes a valid distribution, it underestimates total seasonal rainfall by approximately 2 inches (Figure 17), and it does not adequately model runs of wet and dry days (Tables 19 and 20).

Two significant conclusions can be drawn from the results of this study. First, care must be taken in assuming rainfall distributions. As shown by the above results, a model may accurately simulate rainfall even when invalid distributions are assumed, as shown by the results of the Haan 7X7 model. For any simulation, the assumptions made must be shown to be valid either on a quantitative or qualitative basis before the results of the simulation can be considered valid. Second, model results should be adequately tested against the population being modeled to ascertain that the integrity of the population is maintained. The results show that neither the independent exponential nor the gamma models compare favorably with the historical records of wet- and dry-day runs. These models estimate almost twice the number of 1-day runs of both wet-and dry-days and they do not have the ability to construct long wet or dry periods that compare favorably with the historical data. For any hydrologic model to adequately represent rainfall the model should have the ability to simulate long periods of wet or dry weather. For example, a soil-plant-atmosphere continuum model that may be used to predict yield deficits due to drought stress, must be able to model long periods of dry weather. On the other hand, large scale simulation models that predict flooding, need the ability to predict long periods of wet weather. The independent models do not adequately simulate wet- and dry-day runs as found in the historical data and, therefore, they do not maintain the integrity of the population.

The results of this study show that care must be taken in making assumptions and in developing models for simulating point-specific rainfall.

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APPENDIX A

LISTING OF INDEPENDENT EXPONENTIAL RAINFALL MODEL

```
C*************************************************************
C*                                                           *
C*          INDEPENDENT EXPONENTIAL RAINFALL MODEL           *
C*                                                           *
C*                    BY    K. W. MOLTEN                     *
C*                                                           *
C*  THIS PROGRAM SIMULATES RAINFALL ACCORDING TO AN          *
C*  EXPONENTIAL DISTRIBUTION OF RAINFALL.  MEANS FOR EACH    *
C*  MONTH TO BE SIMULATED MUST BE DETERMINED FROM            *
C*  HISTORICAL DATA.                                         *
C*                                                           *
C*************************************************************
C
      DIMENSION PROBS(245), RAIN(29,8), TOTAL(29), YMON(10)
      INTEGER I, J, K, L, CLASS, NDY, MO, YR, DAY, COUNT
      REAL LOW, UP, AVER
      DATA TOTAL/29*0.0/, RAIN/232*0.0/
C
C* SET MONTHLY MEANS NEEDED FOR EXPONENTIAL DISTRIBUTION
C* FUNCTION.
C*
      YMON(1)=0
      YMON(2)=0
      YMON(3)=0.319
      YMON(4)=0.308
      YMON(5)=0.313
      YMON(6)=0.319
      YMON(7)=0.294
      YMON(8)=0.318
      YMON(9)=0.389
      YMON(10)=0.394
C*
C*  READ DAILY DRY DAY PROBABILITIES OF A
C*  RAINFALL EVENT
C*
      DO 1 I=1,245
      READ(5,100) PROBS(I)
  100 FORMAT (F6.3)
    1 CONTINUE
C*
C*  DEFINE THE FIRST YEAR AS 1952 FOR LATER COMPARISONS.
C*
      YR=1952
```

```
C*
C*   CREATE 29 SIMULATIONS OF SEASONAL RAINFALL
C*
       DO 99 L=1,29
C*
C*   DEFINE MONTH AS FEBRUARY AND INCREASE YEARLY COUNTER
C*
       MO=2
       YR=YR+1
C*
C*   INITIALIZE THE DAY COUNTER FOR THIS YEARS SEASON
C*
       COUNT=0
C*
C*   FOR EACH MONTH OF THE YEAR THIS PORTION OF THE PROGRAM
C*   SIMULATES THE DAILY RAINFALL BASED ON THE PROBABILITY
C*   OF TODAY BEING A DRY DAY
C*
       DO 3 I=1,8
C*
C*   INCREASE MONTH COUNTER AND DETERMINE NUMBER OF
C*   DAYS IN THE MONTH
C*
         MO=MO+1
         NDY=31
         IF (I.EQ.2 .OR. I.EQ.4 .OR. I.EQ.7) NDY=30
         DO 4 J=1,NDY
C*
C*   DETERMINE THE DAY OF THE SEASON
C*
         COUNT=COUNT+1

C*
C*   DETERMINE THE UNIFORMLY DISTRIBUTED RANDOM NUMBER
C*   TO BE USED IN DETERMINING RAINFALL OCCURRENCE
C*
           RN1=UNIFR(0.0,1.0)
C*
C*   DETERMINE IF RAINFALL OCCURRED
C*
             IF (RN1 .LE. PROBS(COUNT)) GO TO 9
C*
C*   GENERATE A SECOND RANDOM NUMBER TO GENERATE AN
C*   AMOUNT OF RAINFALL ACCORDING TO AN EXPONENTIAL
C*   DISTRIBUTION
C*
           RN2 = EXPON(YMON,MO)
C*
C*   ACCUMULATE THE DAILY RAINFALL INTO A MONTHLY TOTAL
C*
           RAIN(L,I)=RAIN(L,I) + RN2
           GO TO 6
    9      RN2=0.0
    6      CONTINUE
C          WRITE (22,32) YR,MO,J,RN2
```

```
C  32      FORMAT (3I5,F5.2)
    4   CONTINUE
C*
C*  ACCUMULATE MONTHLY TOTALS INTO YEARLY TOTALS
C*
        TOTAL(L) = TOTAL(L) + RAIN(L,I)
    3 CONTINUE
C*
C*  DETERMINE AVERAGE RAINFALL FOR SEASONAL SIMULATIONS
C*
      AVER = (AVER*(L-1) + TOTAL(L))/L
   99 CONTINUE
C*
C*  OUTPUT MONTHLY YEARLY AND AVERAGE DATA
C*
      WRITE(6,200)
  200 FORMAT(5X,'MAR',4X,'APR',4X,'MAY',4X,'JUN',4X,
     C'JUL',4X,'AUG',4X,'SEP',4X,'OCT',2X,'TOTAL')

      DO 300 I=1,29
      WRITE(6,400) ((RAIN(I,J),J=1,8), TOTAL(I))
  400 FORMAT(1X,9F7.2)
  300 CONTINUE

      WRITE(6,500) L, AVER
  500 FORMAT(3X,'AVERAGE FOR ',I2,' SIMULATIONS IS ',
     CF6.2,' INCHES')
      RETURN
      END
C*
C*  FUNCTION TO DETERMINE THE RANDOM VARIATE RN1
C*  FOR RAINFALL OCCURRENCE BASED ON A UNIFORM
C*  DISTRIBUTION
C*
      FUNCTION UNIFR(LOW,UP)
      REAL LOW, UP
      INTEGER ISEED
      DATA ISEED /39873/
C*
C*  DETERMINE RANDOM VARIATE FOR RAINFALL OCCURRENCE
C*
      UNIFR=LOW + (UP-LOW)*RAND1(ISEED)
      RETURN
      END
C*
C*      EXPONENTIAL DISTRIBUTION FUNCTION
C*
      FUNCTION EXPON(YMON,MO)
      DIMENSION YMON(10)
      INTEGER ISEED, MO
      DATA ISEED /74321/
C*
C* DETERMINE RANDOM VARIATE FOR RAINFALL AMOUNT
C*
      EXPON=-(YMON(MO)*ALOG(RAND2(ISEED)))
```

```
      RETURN
      END
C*
C*
C*    FUNCTION FOR GENERATING RAINFALL OCCURRENCE
C*
C*   GENERATE A RANDOM NUMBER USING A MULTIPLICATIVE
C*   CONGRUTIONAL RANDOM NUMBER FUNCTION (BALCI, 1984)
C*
      FUNCTION RAND1(ISEED)
      DOUBLE PRECISION Z, D2P31M
      DOUBLE PRECISION DMOD
      DATA D2P31M /2147483647.DO/
      Z=ISEED
      Z=DMOD(16807.DO*Z,D2P31M)
      ISEED=Z
      RAND1=Z/D2P31M
      RETURN
      END
C*
C*    FUNCTION FOR GENERATING RAINFALL AMOUNT
C*
C*   GENERATE A RANDOM NUMBER USING A MULTIPLICATIVE
C*   CONGRUTIONAL RANDOM NUMBER FUNCTION (BALCI, 1984)
C*
      FUNCTION RAND2(ISEED)
      DOUBLE PRECISION Z, D2P31M
      DOUBLE PRECISION DMOD
      DATA D2P31M /2147483647.DO/
      Z=ISEED
      Z=DMOD(16807.DO*Z,D2P31M)
      ISEED=Z
      RAND2=Z/D2P31M
      RETURN
      END
```

APPENDIX B

LISTING OF INDEPENDENT GAMMA RAINFALL MODEL

```
C***********************************************************
C*                                                         *
C*                INDEPENDENT GAMMA RAINFALL MODEL         *
C*                                                         *
C*                       BY    K. W. MOLTEN                *
C*                                                         *
C*   THIS PROGRAM SIMULATES RAINFALL ACCORDING TO A GAMMA  *
C*   DISTRIBUTION SHAPE AND SCALE PARAMETERS FOR THE GAMMA *
C*   DISTRIBUTION ARE ESTIMATED FROM THE UNIFIT PROGRAM,   *
C*   COMPLIMENTS OF OSMAN BALCI, VA TECH COMPUTER SCIENCE  *
C*   DEPARTMENT.                                           *
C*                                                         *
C***********************************************************
C*
      DIMENSION PROBS(245),RAIN(29,8),TOTAL(29),SHAPE(10),
     CSCALE(10)
      INTEGER I, J, K, L, CLASS, NDY, MO, YR, DAY, COUNT
      REAL LOW, UP, AVER, B, A
C
C SET SHAPE PARAMETER FOR EACH MONTH
C
      SHAPE(1)=0
      SHAPE(2)=0
      SHAPE(3)=0.832
      SHAPE(4)=0.692
      SHAPE(5)=0.726
      SHAPE(6)=0.771
      SHAPE(7)=0.614
      SHAPE(8)=0.597
      SHAPE(9)=0.631
      SHAPE(10)=0.644
C
C SET SCALE PARAMETER FOR EACH MONTH
C
      SCALE(1)=0
      SCALE(2)=0
      SCALE(3)=0.371
      SCALE(4)=0.433
      SCALE(5)=0.417
      SCALE(6)=0.391
      SCALE(7)=0.463
      SCALE(8)=0.514
      SCALE(9)=0.601
```

```
      SCALE(10)=0.596
C
C  READ IN DAILY DRY DAY PROBABILITIES
C  PROBABILITIES
C
      DO 1 I=1,245
      READ(5,100) PROBS(I)
  100 FORMAT (F6.3)
    1 CONTINUE
C
C  DEFINE THE FIRST YEAR AS 1952 FOR LATER COMPARISONS.
C
      YR=1952
C
C  CREATE 29 SIMULATIONS OF SEASONAL RAINFALL
C
      DO 99 L=1,29
C
C  DEFINE MONTH AS FEBRUARY AND INCREASE YEARLY COUNTER
C
      MO=2
      YR=YR+1
C
C   INITIALIZE THE DAY COUNTER FOR THIS YEARS SEASON
C
      COUNT=0
C
C  FOR EACH MONTH OF THE YEAR THIS PORTION OF THE PROGRAM
C  SIMULATES THE DAILY RAINFALL BASED ON THE PROBABILITY
C  OF TODAY BEING A DRY DAY.
C
      DO 3 I=1,8
C
C  INCREASE THE MONTH COUNTER AND DETERMINE THE NUMBER OF
C  DAYS IN THE MONTH
C
        MO=MO+1
        NDY=31
        IF (I.EQ.2 .OR. I.EQ.4 .OR. I.EQ.7) NDY=30
        DO 4 J=1,NDY
C
C   DETERMINE THE DAY OF THE SEASON
C
        COUNT=COUNT+1
C
C  DETERMINE UNIFORMLY DISTRIBUTED RANDOM NUMBER TO BE USED
C  AS THE DETERMINING PROBABILITY OF RAINFALL
C
          RN1=UNIFR(0.0,1.0)
C
C  DETERMINE IF TODAY IS A DRY DAY OR A WET DAY
C  BASED ON THE RETURNED RANDOM VARIATE RN1
C
            IF (RN1 .LE. PROBS(COUNT)) GO TO 9
C
```

```
C  GENERATE A SECOND RANDOM NUMBER TO BE USED IN DETERMINING
C  THE AMOUNT OF RAINFALL FOR TODAY ACCORDING TO A GAMMA
C  DISTRIBUTION
C
           B=SCALE(MO)
           A=SHAPE(MO)
           RN2 = GAMA(B,A)
C
C  ACCUMULATE THE DAILY RAINFALL INTO A MONTHLY TOTAL
C
           RAIN(L,I)=RAIN(L,I) + RN2
           GO TO 6
    9      RN2=0.0
    6      CONTINUE
C          WRITE (22,32) YR,MO,J,RN2
C  32      FORMAT (3I5,F5.2)
    4    CONTINUE
C
C  ACCUMULATE MONTHLY TOTALS INTO YEARLY TOTALS
C
         TOTAL(L) = TOTAL(L) + RAIN(L,I)
    3 CONTINUE
C
C  DETERMINE AVERAGE RAINFALL FOR SEASONAL SIMULATIONS
C
        AVER = (AVER*(L-1) + TOTAL(L))/L
   99 CONTINUE
C
C  OUTPUT MONTHLY, YEARLY AND AVERAGE DATA
C
        WRITE(6,200)
  200 FORMAT(5X,'MAR',4X,'APR',4X,'MAY',4X,'JUN',4X,
     C'JUL',4X, 'AUG',4X,'SEP',4X,'OCT',2X,'TOTAL')

        DO 300 I=1,29
        WRITE(6,400) ((RAIN(I,J),J=1,8), TOTAL(I))
  400 FORMAT(1X,9F7.2)
  300 CONTINUE

        WRITE(6,500) (L, AVER)
  500 FORMAT(3X,'AVERAGE FOR ',I2,' SIMULATIONS IS ',
     CF6.2,' INCHES')
        RETURN
        END
C
C  FUNCTION TO DETERMINE THE RANDOM VARIATE RN1
C  FOR RAINFALL OCCURRENCE BASED ON A UNIFORM
C  DISTRIBUTION
C
        FUNCTION UNIFR(LOW,UP)
        DIMENSION SHAPE(10)
        REAL LOW, UP
        INTEGER ISEED, MO
        DATA ISEED /39873/
C
```

```
C DETERMINE RANDOM VARIATE FOR RAINFALL OCCURRENCE
C
      UNIFR=LOW + (UP-LOW)*RAND1(ISEED)
      RETURN
      END
C
C          GAMMA DISTRIBUTION FUNCTION
C
C COMPLIMENTS OF OSMAN BALCI VA TECH COMPUTER SCIENCE DEPT.
C
C     B    = SCALE PARAMETER
C     A    = SHAPE PARAMETER
C     ISEED= RANDOM NUMBER GENERATOR SEED
C
      FUNCTION GAMA(B,A)
      REAL  B,A
      INTEGER  ISEED
      DATA ISEED /74321/
      IF (A.GE.1.0) GO TO 10
1     X=RAND2(ISEED)**(1./A)
      Y=RAND2(ISEED)**(1./(1.-A))
      Z=X+Y
      IF (Z.GT.1.) GO TO 1
      W=X/Z
      GAMA=W*(-ALOG(RAND2(ISEED)))*B
      RETURN
10    KA=A
      AK=KA
      C=A-AK
      IF (A.LT.5.) GO TO 20
      IF (RAND2(ISEED).LT.C) KA=KA+1
20    PR=1.0
      DO 30 I=1,KA
30    PR=PR*RAND2(ISEED)
      GAMA=-ALOG(PR)
      IF(A.GE.5.0) GO TO 40
      TEST=(GAMA/AK)**C*EXP(-C*(GAMA/A-1.))
      IF(RAND2(ISEED).GT.TEST) GO TO 20
      GAMA=GAMA*(A/AK)
40    GAMA=GAMA*B
      RETURN
C
      END
C
C*
C*   FUNCTION FOR GENERATING RAINFALL OCCURRENCE
C*
C  GENERATE A RANDOM NUMBER USING A MULTIPLICATIVE
C  CONGRUTIONAL RANDOM NUMBER FUNCTION (BALCI, 1984)
C
      FUNCTION RAND1(ISEED)
      DOUBLE PRECISION Z, D2P31M
      DOUBLE PRECISION DMOD
      DATA D2P31M /2147483647.DO/
      Z=ISEED
```

```
      Z=DMOD(16807.D0*Z,D2P31M)
      ISEED=Z
      RAND1=Z/D2P31M
      RETURN
      END
C
C*
C*    FUNCTION FOR GENERATING RAINFALL AMOUNT
C*
C   GENERATE A RANDOM NUMBER USING A MULTIPLICATIVE
C   CONGRUTIONAL RANDOM NUMBER FUNCTION (BALCI, 1984)
C
      FUNCTION RAND2(ISEED)
      DOUBLE PRECISION Z, D2P31M
      DOUBLE PRECISION DMOD
      DATA D2P31M /2147483647.D0/
      Z=ISEED
      Z=DMOD(16807.D0*Z,D2P31M)
      ISEED=Z
      RAND2=Z/D2P31M
      RETURN
      END
```

APPENDIX C

LISTING OF HAAN 7X7 RAINFALL MODEL

```
C***********************************************************
C*                                                         *
C*   HAANS 7X7 FIRST ORDER MARKOV CHAIN RAINFALL MODEL     *
C*                                                         *
C*              BY    K. W. MOLTEN                         *
C*                                                         *
C*  THIS PROGRAM SIMULATES RAINFALL ACCORDING TO HAANS 7X7 *
C*  PROBABILITY MATRIX.  FOR EACH MONTH AND THE FOLLOWING  *
C*  CLASSES AND DISTRIBUTIONS:                             *
C*                                                         *
C*     CLASS 1   0 INCHES                                  *
C*     CLASS 2   0.01 TO 0.03 IN. UNIFORM DISTRIBUTION     *
C*     CLASS 3   0.03 TO 0.07 IN. UNIFORM DISTRIBUTION     *
C*     CLASS 4   0.07 TO 0.15 IN. UNIFORM DISTRIBUTION     *
C*     CLASS 5   0.15 TO 0.31 IN. UNIFORM DISTRIBUTION     *
C*     CLASS 6   0.31 TO 0.63 IN. UNIFORM DISTRIBUTION     *
C*     CLASS 7   >0.64 IN.  EXPONENTIAL DISTRIBUTION       *
C*                                                         *
C***********************************************************
C*
      DIMENSION PROBS(8,7,7),RAIN(30,8),TOTAL(30),YMON(10)
      INTEGER I, J, K, L, CLASS, NDY, MO, YR, DAY
      REAL LOW, UP, AVER
C*
C* SET MONTHLY MEANS FOR CLASS 7 EXPONENTIAL DISTRIBUTION
C* FUNCTION.
C*
      YMON(1)=0
      YMON(2)=0
      YMON(3)=1.000
      YMON(4)=1.091
      YMON(5)=0.971
      YMON(6)=1.026
      YMON(7)=1.042
      YMON(8)=1.182
      YMON(9)=1.182
      YMON(10)=1.332
C*
C*  READ IN MONTHLY FIRST ORDER MARKOV CHAIN
C*  PROBABILITIES, 8 7X7 MATRISES.
C*
      DO 1 I=1,8
      DO 2 J=1,7
      READ(5,100) (PROBS(I,J,K),K=1,7)
```

```
  100 FORMAT (F5.3,5F6.3,F4.1)
    2 CONTINUE
    1 CONTINUE
C*
C*  DEFINE FIRST YEAR AS 1952 FOR LATER COMPARISONS.
C*
      YR=1952
C*
C*  CREATE 29 SIMULATIONS OF SEASONAL RAINFALL
C*
      DO 99 L=1,29
C*
C*  DEFINE PREVIOUS DAY TO MARCH 1 AS A DRY DAY
C*
      CLASS=1
C*
C*  DEFINE THE MONTH TO BEGIN AT END OF FEBRUARY
C*
      MO=2
C*
C*  INCREASE YEAR COUNTER
C*
      YR=YR+1
C*
C*  FOR EACH MONTH OF THE YEAR SIMULATE DAILY RAINFALL
C*  BASED ON THE PROBABILITY OF RAIN TODAY BASED
C*  ON WHETHER OR NOT THERE WAS RAIN YESTERDAY
C*
      DO 3 I=1,8
C*
C*  INCREASE THE MONTHLY COUNTER
C*
        MO=MO+1
C*
C*  SET THE NUMBER OF DAYS IN EACH MONTH TO 31
C*
        NDY=31
C*
C*  FOR APRIL, JUNE, AND SEPTEMBER THE NUMBER OF
C*  DAYS IN THE MONTH IS SET TO 30
C*
        IF (I.EQ.2 .OR. I.EQ.4 .OR. I.EQ.7) NDY=30
C*
C*  SIMULATE RAINFALL FOR EACH DAY OF THE MONTH
C*
        DO 4 J=1,NDY
C*
C*  DETERMINE UNIFORMLY DISTRIBUTED RANDOM NUMBER FOR
C*  DETERMINING RAINFALL OCCURRENCE RN1
C*
          RN1=UNIFR(0.0,1.0,Q)
C*
C*  DETERMINE THE CLASS TO WHICH THE RANDOM VARIATE RN1
C*  IS A MEMBER BASED ON MONTH AND PREVIOUS CLASS
C*
```

```
          DO 5 K=1,7
            IF (RN1 .LE. PROBS(I,CLASS,K)) GO TO 6
    5     CONTINUE
C*
C*   DETERMINE THE UPPER AND LOWER BOUNDS OF
C*   RAINFALL CLASS IF CLASS = 2, 3, 4, OR 5
C*   FOR THE GENERATION OF A RAINFALL AMOUNT
C*
    6     P=2**(K-1)-1
          LOW=P/100
          UP=0.02*P+0.01
C*
C*   CHANGE THE UPPER BOUND OF CLASS 1 TO 0
C*
          IF (K .EQ. 1) UP=0
C*
C*   GENERATE A SECOND RANDOM NUMBER FOR DETERMINING
C*   THE AMOUNT OF RAINFALL FOR TODAY
C*
          RN2 = HAAN(LOW,UP,K,YMON,MO)
C*
C*   ACCUMULATE DAILY RAINFALL INTO A MONTHLY TOTAL
C*
          RAIN(L,I)=RAIN(L,I) + RN2
C*
C*   SET NEW PREVIOUS CLASS TO THE CURRENT CLASS FOR
C*   GENERATION OF TOMORROWS RAINFALL.
C*
    7     CLASS=K
C         WRITE (22,32) YR,MO,J,RN2
C  32     FORMAT (3I5,F5.2)
    4   CONTINUE
C*
C*   ACCUMULATE MONTHLY TOTALS INTO YEARLY TOTALS
C*
        TOTAL(L) = TOTAL(L) + RAIN(L,I)
    3 CONTINUE
C*
C*   DETERMINE AVERAGE RAINFALL FOR SEASONAL SIMULATIONS
C*
      AVER = (AVER*(L-1) + TOTAL(L))/L
   99 CONTINUE
C*
C*   OUTPUT MONTHLY YEARLY AND AVERAGE DATA
C*
      WRITE(6,200)
  200 FORMAT(5X,'MAR',4X,'APR',4X,'MAY',4X,'JUN',4X,
     C'JUL',4X,'AUG',4X,'SEP',4X,'OCT',2X,'TOTAL')

      DO 300 I=1,29
      WRITE(6,400) ((RAIN(I,J),J=1,8), TOTAL(I))
  400 FORMAT(1X,9F7.2)
  300 CONTINUE
```

```
      WRITE(6,500) (L, AVER)
  500 FORMAT(3X,'AVERAGE FOR ',I2,' SIMULATIONS IS ',
     CF6.2,' INCHES')
      RETURN
      END
C*
C*  FUNCTION TO DETERMINE THE RANDOM VARIATE RN1
C*  FOR RAINFALL OCCURRENCE BASED ON A UNIFORM
C*  DISTRIBUTION
C*
      FUNCTION UNIFR(LOW,UP)
      REAL LOW, UP
      INTEGER ISEED
      DATA ISEED /39873/
C*
C* DETERMINE RANDOM VARIATE FOR RAINFALL OCCURRENCE
C*
      UNIFR=LOW + (UP-LOW)*RAND1(ISEED)
      RETURN
      END
C*
C*  FUNCTION FOR DETERMINING RN2 RAINFALL AMOUNT
C*  BASED ON A UNIFORM DISTRIBUTION IN CLASSES
C*  2, 3, 4, 5, OR 6 OR ON AN EXPONENTIAL DISTRIBUTION
C*  IF CLASS = 7.
C*
      FUNCTION HAAN(LOW,UP,K,YMON,MO)
      DIMENSION YMON(10)
      REAL LOW, UP
      INTEGER ISEED, MO
      DATA ISEED /74321/
C*
C* DETERMINE RANDOM VARIATE FOR RAINFALL AMOUNT
C*
      IF (K .EQ. 7) GO TO 77
      HAAN=LOW + (UP-LOW)*RAND2(ISEED)
      GO TO 78
   77 HAAN=(-(YMON(MO)-0.63)*ALOG(RAND2(ISEED))) + 0.63
   78 RETURN
      END
C*
C*
C*   FUNCTION FOR GENERATING RAINFALL OCCURRENCE
C*
C*  GENERATE A RANDOM NUMBER USING A MULTIPLICATIVE
C*  CONGRUTIONAL RANDOM NUMBER FUNCTION (BALCI, 1984)
C*
      FUNCTION RAND1(ISEED)
      DOUBLE PRECISION Z, D2P31M
      DOUBLE PRECISION DMOD
      DATA D2P31M /2147483647.D0/
      Z=ISEED
      Z=DMOD(16807.D0*Z,D2P31M)
      ISEED=Z
```

```
      RAND1=Z/D2P31M
      RETURN
      END
C*
C*
C*   FUNCTION FOR GENERATING RAINFALL AMOUNT
C*
C*  GENERATE A RANDOM NUMBER USING A MULTIPLICATIVE
C*  CONGRUTIONAL RANDOM NUMBER GENERATOR. (BALCI, 1984)
C*
      FUNCTION RAND2(ISEED)
      DOUBLE PRECISION Z, D2P31M
      DOUBLE PRECISION DMOD
      DATA D2P31M /2147483647.D0/
      Z=ISEED
      Z=DMOD(16807.D0*Z,D2P31M)
      ISEED=Z
      RAND2=Z/D2P31M
      RETURN
      END
```

APPENDIX D

LISTING OF MARKOV 5X5 RAINFALL MODEL

```
C*************************************************************
C*                                                           *
C*     5X5 FIRST ORDER MARKOV CHAIN RAINFALL MODEL           *
C*                                                           *
C*                  BY    K. W. MOLTEN                       *
C*                                                           *
C*  THIS PROGRAM SIMULATES RAINFALL ACCORDING TO A 5X5       *
C*  PROBABILITY MATRIX. MATRIX USING THE FOLLOWING CLASSES   *
C*  AND DISTRIBUTIONS:                                       *
C*                                                           *
C*          CLASS 1   0 INCHES                               *
C*          CLASS 2   0.01 TO 0.07 IN. GAMMA DISTRIBUTION    *
C*          CLASS 3   0.07 TO 0.15 IN. GAMMA DISTRIBUTION    *
C*          CLASS 4   0.15 TO 0.63 IN. GAMMA DISTRIBUTION    *
C*          CLASS 5   >0.63 IN. EXPONENTIAL DISTRIBUTION     *
C*                                                           *
C*  (SHAPE AND SCALE PARAMETERS FOR THE GAMMA DISTRIBUTION   *
C*  ARE ESTIMATED FROM UNIFIT PROGRAM, COMPLIMENTS OF        *
C*  OSMAN BALCI MEANS USED AS THE SCALE PARAMETER IN         *
C*  THE EXPONENTIAL DISTRIBUTION WERE ESTIMATED FROM         *
C*  HISTORICAL DATA.                                         *
C*                                                           *
C*************************************************************
C*
      DIMENSION PROBS(10,5,5),RAIN(29,8),TOTAL(29),
     CSHAPE(10,5),SCALE(10,5),ALOCA(5)
C
      INTEGER I, J, K, L, CLASS, NDY,  MO, YR, DAY
C
      REAL LOW, UP, AVER, B, A, LCA, MEAN,RN1,RN2
C
C  READ LOCATION, SHAPE, AND SCALE PARAMETERS FOR
C  EACH MONTH AND EACH CLASS.
C
      DO 16 I=3,10
      DO 15 J=1,5
      READ(5,600) (ALOCA(J), SCALE(I,J), SHAPE(I,J))
  600 FORMAT (3F6.3)
   15 CONTINUE
   16 CONTINUE
C
C  READ IN MONTHLY FIRST ORDER MARKOV CHAIN TRANSITION
C  PROBABILITIES, 8 5X5 MATRISES
C
```

```
      DO 21 I=3,10
      DO 22 J=1,5
      READ(5,100) (PROBS(I,J,K),K=1,5)
  100 FORMAT (5F8.2)
      DO 24 K=1,5
      PROBS(I,J,K)=PROBS(I,J,K)/100
   24 CONTINUE
      DO 23 K=2,5
      L=K-1
      PROBS(I,J,K)=PROBS(I,J,L) + PROBS(I,J,K)
   23 CONTINUE
   22 CONTINUE
   21 CONTINUE
C
C  DEFINE THE FIRST YEAR AS 1952 FOR LATER COMPARISONS.
C
      YR=1952
C
C  CREATE 29 SIMULATIONS OF SEASONAL RAINFALL
C
      DO 99 L=1,29
C
C  DEFINE THE DAY PREVIOUS TO MARCH 1ST AS A DRY DAY
C
      CLASS=1
C
C  DEFINE MONTH AS FEBRUARY AND INCREASE YEARLY COUNTER
C
      MO=2
      YR=YR+1
C
C  FOR EACH MONTH OF THE YEAR SIMULATE DAILY RAINFALL
C  BASED ON THE PROBABILITY OF RAIN TODAY BASED
C  ON WEITHER OR NOT THERE WAS RAIN YESTERDAY
C
      DO 3 I=1,8
C
C  INCREASE THE MONTH COUNTER AND DETERMINE NUMBER OF
C  DAYS IN THE MONTH
C
        MO=MO+1
        NDY=31
        IF (I.EQ.2 .OR. I.EQ.4 .OR. I.EQ.7) NDY=30
        DO 4 J=1,NDY
C
C  DETERMINE UNIFORMLY DISTRIBUTED RANDOM VARIATE RN1
C  FOR DETERMINING RAINFALL OCCURRENCE
C
          RN1=UNIFR(0.0,1.0)
C
C  DETERMINE THE CLASS TO WHICH THE RN1 IS A MEMBER
C
          DO 5 K=1,5
            IF (RN1 .LE. PROBS(MO,CLASS,K)) GO TO 6
    5     CONTINUE
```

```
C
C  DETERMINE IF RAINFALL OCCURRED. IF YES GENERATE AMOUNT
C
   6      IF (K .EQ. 1) GO TO 9
C
C  IF CLASS IS 2, 3 OR 4 THEN GENERATE A RANDOM AMOUNT
C  ACCORDING TO A GAMMA DISTRIBUTION IF CLASS IS 7
C  GENERATE A RANDOM AMOUNT ACCORDING TO AN EXPONENTIAL
C  DISTRIBUTION.
C
          B=SCALE(MO,K)
          A=SHAPE(MO,K)
          LCA=ALOCA(K)
          RN2 = GAMEX(B,A,LCA,K)
C         R=0
          GO TO 8
C
C  ACCUMULATE THE DAILY RAINFALL INTO A MONTHLY TOTAL
C
   8      RAIN(L,I)=RAIN(L,I) + RN2
   9      CLASS=K
          IF (K .EQ. 1) RN2=0.0
C         WRITE (22,32) YR,MO,J,RN2
C  32     FORMAT (3I5,F5.2)
   4    CONTINUE
C
C  ACCUMULATE MONTHLY TOTALS INTO YEARLY TOTALS
C
        TOTAL(L) = TOTAL(L) + RAIN(L,I)
   3 CONTINUE
C
C  DETERMINE AVERAGE RAINFALL FOR SEASONAL SIMULATIONS
C
      AVER = (AVER*(L-1) + TOTAL(L))/L
  99 CONTINUE
C
C  OUTPUT MONTHLY, YEARLY AND AVERAGE DATA
C
      WRITE(6,200)
 200  FORMAT(5X,'MAR',4X,'APR',4X,'MAY',4X,'JUN',4X,
     C'JUL',4X,'AUG',4X,'SEP',4X,'OCT',2X,'TOTAL')

      DO 300 I=1,29
      WRITE(6,400) ((RAIN(I,J),J=1,8), TOTAL(I))
 400  FORMAT(1X,9F7.2)
 300  CONTINUE

      WRITE(6,500) (L, AVER)
 500  FORMAT(3X,'AVERAGE FOR ',I2,' SIMULATIONS IS ',
     CF10.2,' INCHES')
      RETURN
      END
```

```
C
C  FUNCTION TO DETERMINE THE RANDOM VARIATE RN1
C  FOR RAINFALL OCCURRENCE BASED ON A UNIFORM
C  DISTRIBUTION
C
      FUNCTION UNIFR(LOW,UP)
      REAL LOW, UP
      INTEGER ISEED
      DATA ISEED /39873/
C     IF (Q .EQ. 1) ISEED=39873
C     IF (Q .EQ. 2) ISEED=54891
C     IF (Q .EQ. 3) ISEED=49349
C
C DETERMINE RANDOM VARIATE FOR RAINFALL OCCURRENCE
C
      UNIFR=LOW + (UP-LOW)*RAND1(ISEED)
      RETURN
      END
C
C     GAMMA OR EXPONENTIAL DISTRIBUTION FUNCTION
C
C COMPLIMENTS OSMAN BALCI VA TECH COMPUTER SCIENCE DEPT.
C MODIFIED TO INCLUDE EXPONENTIAL DISTRIBUTION.
C
C     B    = SCALE PARAMETER
C     A    = SHAPE PARAMETER
C     ISEED= RANDOM NUMBER GENERATOR SEED
C
      FUNCTION GAMEX(B,A,LCA,K)
      REAL  B,A,LCA
      INTEGER ISEED,K
      DATA ISEED /74321/
C
C IF CLASS = 2, 3, OR 4 GENERATE GAMMA RANDOM VARIATE
C IF CLASS = 5 GENERATE EXPONENTIAL RANDOM VARIATE

      IF (K .EQ. 5) GO TO 56
      IF (A.GE.1.0) GO TO 10
    1 X=RAND2(ISEED)**(1./A)
      Y=RAND2(ISEED)**(1./(1.-A))
      Z=X+Y
      IF (Z.GT.1.) GO TO 1
      W=X/Z
      GAMEX=W*(-ALOG(RAND2(ISEED)))*B + LCA
      RETURN
   10 KA=A
      AK=KA
      C=A-AK
      IF (A.LT.5.) GO TO 20
      IF (RAND2(ISEED).LT.C) KA=KA+1
   20 PR=1.0
      DO 30 I=1,KA
   30 PR=PR*RAND2(ISEED)
      GAMEX=-ALOG(PR)
```

```
      IF(A.GE.5.0) GO TO 40
      TEST=(GAMEX/AK)**C*EXP(-C*(GAMEX/A-1.))
      IF(RAND2(ISEED).GT.TEST) GO TO 20
      GAMEX=GAMEX*(A/AK)
   40 GAMEX=GAMEX*B + LCA
      GO TO 57
   56 GAMEX=(-((B-LCA)*ALOG(RAND2(ISEED)))) + LCA
   57 RETURN
C
      END
C
C*
C*    FUNCTION FOR GENERATING RAINFALL OCCURRENCE
C*
C   GENERATE A RANDOM NUMBER USING A MULTIPLICATIVE
C   CONGRUTIONAL RANDOM NUMBER FUNCTION (BALCI, 1984)
C
      FUNCTION RAND1(ISEED)
      DOUBLE PRECISION Z, D2P31M
      DOUBLE PRECISION DMOD
      DATA D2P31M /2147483647.D0/
      Z=ISEED
      Z=DMOD(16807.D0*Z,D2P31M)
      ISEED=Z
      RAND1=Z/D2P31M
      RETURN
      END
C
C*
C*    FUNCTION FOR GENERATING RAINFALL AMOUNT
C*
C   GENERATE A RANDOM NUMBER USING A MULTIPLICATIVE
C   CONGRUTIONAL RANDOM NUMBER FUNCTION (BALCI, 1984)
C
      FUNCTION RAND2(ISEED)
      DOUBLE PRECISION Z, D2P31M
      DOUBLE PRECISION DMOD
      DATA D2P31M /2147483647.D0/
      Z=ISEED
      Z=DMOD(16807.D0*Z,D2P31M)
      ISEED=Z
      RAND2=Z/D2P31M
      RETURN
      END
```

Virginia's Agricultural Experiment Stations

1 —— Blacksburg
Virginia Tech
Main Station

2 —— Steeles Tavern
Shenandoah Valley Research Station
Beef, Sheep, Fruit, Forages, Insects

3 —— Orange
Piedmont Research Station
Small Grains, Corn, Alfalfa, Crops

4 —— Winchester
Winchester Fruit Research Laboratory
Fruit, Insect Control

5 —— Middleburg
Virginia Forage Research Station
Forages, Beef

6 —— Warsaw
Eastern Virginia Research Station
Field Crops

7 —— Suffolk
Tidewater Research and Continuing Education Center
Peanuts, Swine, Soybeans, Corn, Small Grains

8 —— Blackstone
Southern Piedmont Research and Continuing Education Center
Tobacco, Horticulture Crops, Turfgrass, Small Grains, Forages

9 —— Critz
Reynolds Homestead Research Center
Forestry, Wildlife

10 —— Glade Spring
Southwest Virginia Research Station
Burley Tobacco, Beef, Sheep

11 —— Hampton
Seafood Processing Research
and Extension Unit
Seafood