The role of flexibility on propulsive performance of flapping fins

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ABSTRACT

The versatility of the fish to adapt to diverse swimming requirements has attracted the attention of researchers in studying bioinspired propulsion for developing efficient underwater robotics. The tail/caudal fin is a major source of thrust generation and is believed that the fish modulates its fin stiffness to optimize the propulsive performance. Inspired by the stiffness modulation of fish fins, the objective of this research is to predict and evaluate the effect of flexibility on propulsive performance of flapping fins. The stiffness of the fins vary along their length and optimization studies have been performed to predict the stiffness profiles that maximize performance. Experiments performed on the real fish caudal fins to measure the stiffness variation along their length validate the theoretical optimal stiffness profiles and provide an insight about the evolution of fish fins for optimal performance. Along with the fin stiffness, the stiffness of the joint (caudal peduncle) connecting the fish body to the tail plays a major role in the generation of thrust. The numerical and experimental investigation has shown that there exists an optimal combination of fin and joint stiffness for each operating condition, thus providing the motivation for active stiffness control during locomotion to optimize efficiency.

Inspired by nature’s ability to modulate stiffness and shape for different operating conditions, an investigation has been carried out on active control of flapping foils for thrust tailoring using Macro Fiber Composites (MFCs). It has been observed that the performance can be enhanced by controlling the deformation, and distributed actuation along fin produces maximum performance through proper selection of the phase difference between heaving and voltage. Flapping fins produce forces which are oscillatory in nature causing center of mass (COM) oscillations of the attached bodies posing problems of control and maneuverability. Optimization studies have revealed that flexibility of the fin plays a major role in reducing the COM oscillations along with the other operating parameters. Based on these studies, the design principles and guidelines that control the performance have been proposed which aid in the development of aerial and underwater robotic vehicles. Additionally, these studies provide some insight in to how fish might modulate its stiffness based on the requirements.
To My Parents

Smt. Vijaya Kumari & Sri. Kanaka Das
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Chapter 1

1 Introduction

Through evolution for millions of years, nature has been a source of inspiration for the invention of numerous designs, mechanisms, technologies and processes that are robust, powerful and energy efficient [1, 2]. Some of them include biomimetic materials, structures, control, cognition, artificial muscles and organs, biosensors and interfaces between engineered and biological systems [1, 2]. Humans desire to fly and swim led to the study of morphology, kinematics and aero/hydrodynamic principles of various birds, insects, and fishes [3-10]. Engineers and researchers have drawn their attention towards bioinspired propulsion for performing multiple tasks (e.g. steering, propulsion, and braking) and achieving higher propulsive performance.

Screw propellers, which are considered to be more effective over other forms of propulsion, have limitations such as cavitation, small operable range of maximum efficiency, reliance on control surfaces for maneuverability and a detectable and identifiable acoustic signature [11]. According to Sir. James Lighthill pioneering works [12, 13], biomimetic approach could be an efficient alternative to conventional propellers. Using 2-D theoretical studies, Yamaguchi and Bose [14] predicted that the open water efficiencies of the oscillating propulsors were 17-25% higher than those of an optimal screw propeller. They reported that the propulsive efficiency of a partly chordwise flexible foil was 72% which was 5% higher than that of the screw propeller. By comparing the performance of flexible foils and rotary propellers, Prempraneerach et al.[15] reported that flexible foils can be very efficient propellers and are shown to be competitive against rotary propellers. Various studies predicted the propulsive efficiencies as high as 80% [6, 13, 16-19] instigating the interest in the field of aquatic propulsion. Apart from generating higher efficiencies, the potential benefits of biomimetic propulsion are increased maneuverability in confined spaces, reduced disturbance to the natural environment and thrust production without
excessive noise which is crucial for underwater defense surveillance and reconnaissance[20]. Fish, bird and insect locomotion is being studied extensively to develop bioinspired propulsive mechanisms depending on the type of application.

1.1 Bioinspiration from fish

The ability of fish to maneuver in tight places, perform stable high acceleration maneuvers, hover efficiently, and quickly brake is a major source of inspiration for developing biologically inspired underwater robots [21-23]. Fish power swimming by propagating bending waves that progress with increasing amplitude from head to the caudal fin (tail). This requires spatial and temporal coordination of muscle contraction in all the directions. Fish possess a two gear muscular system that controls the speed of locomotion [24]. The red fibers which comprise about 10% of the total muscle mass are actuated for slow speed movements while the white fibers constitutes 90% of the musculature are specialized for short duration bursts of high power output. The less known pink fibers are recruited at intermediate swimming speeds. For more details on the physiology of fish, refer to Shadwick et al.[24]. In recent years, inspired by fish, many fish-like robots have been developed using gears, pulleys and cable tendons [25-37] and smart materials [23, 38-50] to study the kinematics, optimal gaits and performance. The robotic fish developed at AOE department, Virginia Tech using flexible matrix composite (FMC) actuators is shown in figure 1.1. This fish is different from the previously built robotic fish in the sense that compliance has been built in to the system avoiding gears, pulleys and other rigid structures. An overview of the underwater robots developed by various groups can be found in [51, 52]. The morphology and kinematics have a large impact on propulsive performance explaining a reason for evolution of different shapes and swimming gaits.
Figure 1.1. Robotic fish developed at Virginia Tech using Flexible Matrix Composite (FMC) actuators

1.1.1 Morphology

Since most of the fish are neutrally buoyant, weight support is not a major problem and thus numerous propulsive systems which can function independently or together have evolved among fish [53]. The different fins located along the length of the body are used individually or in tandem for maneuverability, rapid starts, and braking. A layout of the fins and their position along the length of the body is shown in figure 1.2, and figure 1.3 shows the different swimming modes associated with these fins. The pectoral and pelvic fins are termed together as paired fins whereas the anal and dorsal fins are termed together as median fins. Different fish employ different propulsive modes grouped as BCF (body and caudal fin, figure 1.3(a)) and MPF (median and paired fins, figure 1.3(b)) for performing various swimming gaits. The power required for swimming increases with 2.5 to 2.8 power of velocity and so a given gait can perform efficiently only for a small range of speeds compared with a pedestrian or flyer [54]. Therefore, for a wide overall performance range, large number of gaits are performed by fish.
Figure 1.2. Outline of fins and their positions along the body. (Reproduced from [24].)

Figure 1.3. Swimming modes associated with (a) BCF propulsion and (b) MPF propulsion. Shaded areas contribute to thrust generation. (Reproduced from [20].)
1.1.2 Forces acting on the fish

Fish propels forward by transferring the momentum to the surrounding water and vice versa. The forces and the moments acting on the body during locomotion are as shown in figure 1.4. The equilibrium between thrust and drag is closely related to the equilibrium between weight, buoyancy, and the lift. The vertical forces are balanced using extended pectoral fins and the thrust to overcome the drag is produced mainly by the caudal, pectoral, and dorsal fins while all the fins are used for maneuverability and stability during locomotion [55]. Majority of the fish are negatively buoyant and are balanced by the vortex rings shed by the pectoral fins during swimming [56]. The swimming drag consists of three components namely, viscous or friction drag, form drag and vortex or induced drag [20]. The skin friction between the fish and the boundary layer of the water arises as a result of viscosity of water in areas of flow with large velocity gradients which depends on wetted area and swimming speed as well as the nature of boundary layer of the flow. The form drag is caused by the pressures formed in pushing the water aside during locomotion and depends on the shape. The vortex or induced drag is the energy lost in the vortices formed by the caudal and pectoral fins as they generate lift or thrust. The form and induced drag are together described as pressure drag. A detailed discussion of these individual drag components can be found in [17, 57, 58].

Figure 1.4. Forces acting on the fish. (Reproduced from [20].)
1.2 Literature Review

1.2.1 Hydrodynamics

The theory of oscillating airfoils date back to 1925 by Wagner [59] and other pioneering works by Theodorsen [60], Kussner [61] and Garrick [62]. In 1952, Sir. G. Taylor [4] published a ‘resistive theory’ by idealizing the long and narrow swimmers (such as eels) as a flexible cylinder with uniform cross section. According to this theory, the force of each element of the cylinder termed as resistive is assumed to be same as that which would act on corresponding element of a long straight cylinder moving at the same speed and inclination to the direction of motion. Hancock [63] generalized the resistive theory for undulations of large amplitudes by using Stokeslets and source doublets along the instantaneous center line of the body. The resistive theories are applicable to the bodies with undulating waves travelling from head to the tail and with cross-sectional forms which do not enhance the virtual-mass effect.

Later in 1960, Sir J. Lighthill [6] developed a ‘reactive theory’ popularly known as ‘slender-body theory’ considering inviscid flow for small amplitude motion of a slender fish. In this theory, the forces along the body are calculated by considering the inertial loading termed as virtual mass of the fluid accelerated around the fish body. The forces in the resistive theory depends on the instantaneous velocity of the body section relative to the fluid whereas the forces in the reactive theory depend on the inertia of the fluid and the rate of change of relative velocity of animal surface. The inertial forces are neglected in the resistive theory which are very important when the animal is much thinner in the direction of body displacements in which case the momentum of the virtual mass of the fluid exceeds the momentum of the associated animal mass. The total mean thrust $\bar{P}$ generated by a slender body of length $l$ and normal displacements $h(x,t)$ was derived by Lighthill as

$$\bar{P} = \frac{1}{2} \rho A(l) \left( \frac{\partial h}{\partial t} \right)^2 - U^2 \left( \frac{\partial h}{\partial x} \right)^2 \right)_{x=l}$$

(1.1)

where $\rho$ is the density of the fluid, $A$ is the area of the cross section, $U$ is the fluid velocity and $x$ is the coordinate along the length of the body. The mean thrust produced by the body solely
depends on the geometric and kinematic properties at the end of the tail \((x = l)\). The slender-body theory is appropriate to majority of the fish which use carangiform propulsion in which posterior 1/2 to 1/3rd of the body undulates and the caudal fins that are slender in nature. Wu [16] developed a different reactive theory based on wing theory using potential flow and this is appropriate for the fish propelling in carangiform mode with lunate or crescent-moon shaped fins with high aspect ratio.

Lighthill in 1971 [64], extended the reactive theory for fish motions of arbitrary large amplitude and Candelier et al.[65] extended it for three-dimensional elliptic bodies. Chopra [66] extended the two-dimensional theory of lunate-tail propulsion for arbitrary amplitudes, regular or irregular and Cheng et al.[67] developed a three-dimensional waving plate theory as an extension of two-dimensional theory by Wu [16] and found that undulatory motion could reduce three-dimensional effects.

Basu and Hancock [68] developed a panel method considering potential flow to calculate the forces acting on a two-dimensional rigid foil undergoing arbitrary unsteady motions. At high Reynolds (Re) numbers where the inertial forces dominate the viscous forces as in the case of fish propulsion, the panel methods can be used effectively to predict the propulsive performance of flapping foils. Katz and Weighs [69] analyzed large amplitude oscillation of a high aspect ratio airfoil considering inviscid incompressible fluid using a panel method. The two-dimensional unsteady discrete vortex panel method has been coupled with a linear beam theory to predict the effect of flexibility on propulsive performance of thin oscillating airfoils. Barrett et al.[21] reported that fish like locomotion produces less drag compared to a rigid hull using experimental and theoretical studies. The theoretical method based on potential theory employs a boundary integral method for arbitrary motions and modeling of shed wake by using an evolving desingularized dipole sheet. Using a three-dimensional numerical method based on potential flow and experimental studies, Wolfgang et al.[70] demonstrated that the interaction and manipulation of the vortices generated by the body and the oscillating caudal fin are fundamental to the propulsion and maneuverability of fish.

The immersed boundary method (IBM) developed by Peskin [71] to study flow patterns around heart valves has been further developed for solving complex fluid-structure interaction problems [72]. This method employs Eulerian – Lagrangian approach to solve the coupled fluid-structure
problem with non-body confirming grid. A detailed approach and modifications to this method are discussed in [73]. Using IBM method, Mittal et al. [74] successfully coupled the experimentally obtained kinematics with the computational fluid dynamics (CFD) model to predict the pectoral fin wake topologies and hydrodynamic forces. While several researchers studied the hydrodynamics of flexible foils/fins alone excluding the body [10, 75], Borazjani and Sotiropoulos [76] modeled a three-dimensional mackerel-like flexible body to numerically investigate the hydrodynamics of carangiform swimming in the transitional and inertial flow regimes. They observed that the drag increases with low Reynolds (Re) numbers and reduces significantly at Re = 4000 and also suggested that carangiform swimming is preferred by fast swimmers as the carangiform kinematics becomes a more efficient mode of locomotion in inertial regime. Using a three-dimensional lamprey-like flexible body, Borazjani and Sotiropoulos [77, 78] studied the hydrodynamics of anguilliform and carangiform swimming on mackerel and lamprey body shapes and observed that mackerel body shape produces larger swimming speeds in all the Re regimes compared to the lamprey model. They also showed that the wake topology depends less on the shape and kinematics but more on the Strouhal (St) number. In all these studies, the caudal fin is assumed to be rigid and follow the prescribed kinematics. Bergmann et al. [79] developed a three-dimensional model to mimic the geometry of RoboTuna [28] and allowed the caudal fin to deform owing to the fluid forces acting on the fin. The caudal fin has been modeled as a lumped parameter elastic medium composed of rigid struts connected by elastic links. The couple acting on each link is crudely approximated by a value that is proportional to the local tangential speed of the next junction. The results showed that the adding flexibility to the caudal fin improved performance compared to the rigid fin as predicted by the various studies without modeling the whole body.

Apart from the numerical studies, various experimental methods are being used to study the hydrodynamics of live fish and robotic models [80] and notable among them is the Digital Particle Image Velocimetry (DPIV). DPIV is a very powerful tool to visualize and quantify the three-dimensional flow around the bodies which works on the principle of high speed imaging techniques capturing the images of laser illuminated particles floating in the fluid. Drucker and Lauder [81] used DPIV to calculate the hydrodynamic forces in all the three directions generated by the pectoral fins of a blue gill sunfish by analyzing the wake. An illustration of the DPIV system configuration is shown in figure 1.5. By measuring the vorticity shed into the wake, the forces
generated by the fin are calculated and a detailed description is given in [81]. Lauder and his team at Harvard have been studying the hydrodynamics of live fishes and robotic models using DPIV [82-98] which would be otherwise very difficult using the numerical methods.

![DPIV System Diagram](image)

**Figure 1.5. Illustration of the DPIV system [81]**

### 1.2.2 Fin flexibility

The fins of fish and aquatic mammals are flexible in chordwise as well as spanwise which reconfigure to the fluid forces resulting in substantial drag reduction compared to the rigid foil which experience drag proportional to the square of its speed. A theoretical and experimental investigation on the drag reduction through self-similar bending of a flexible body can be found in
Numerous studies have been carried out theoretically and experimentally to find out the advantages of the flexibility and how nature exploits flexibility during propulsion.

1.2.2.1 Chordwise flexibility

Initial studies on the fish swimming [3-6, 8, 9, 12, 13, 16, 64, 66, 100-103] dealt with the kinematics and hydrodynamics of rigid fins and for the first time, Katz & Weihs [69, 104] examined pitching and heaving of a flexible slender airfoil in an inviscid flow and predicted an increase in the efficiency of chord wise flexible foils by 20% compared to the rigid foils. They covered both small and large aspect ratio propulsors with constant chordwise flexibility and the analysis is based on incompressible potential theory coupled with Euler-Bernoulli beam kinematics. A phase difference of 90° between pitching and heaving is found out to be optimal for all oscillating propulsors. Bose [105] studied the performance of chordwise flexible oscillating propulsors using a time-domain panel method and predicted that the propulsive efficiency varies strongly with changes in heaving amplitude, pitching amplitude and phase between heaving and pitching motions, but less strongly for variations in pitching axis position and flexibility.

Miao and Ho [106] studied the effect of chord wise flexure amplitude using the conformal hybrid meshes on unsteady aerodynamic characteristics and predicted that the efficiency increased relative to the rigid foil for certain values of Strouhal number. Pederzani and Haj-Hariri [107] performed the numerical analysis of heaving flexible foils in viscous flow and predicted that the heavier foils are more efficient than the lighter foils with in the analyzed parameter range. By studying the slender elastic filament with chord wise flexibility in 2-D inviscid flow, Alben [108] showed that the optimal efficiency approaches 100% as rigidity becomes small and decreases to 30-50% as rigidity becomes large. He also deduced power laws for optimized stiffness of the fin related to maximum efficiency and thrust with reduced frequency. Sousa et al. [109] carried out a numerical investigation coupling compact finite difference immersed boundary method and thin membrane structural kinematics and predicted that the optimal efficiency condition is somewhat independent of nature of the thrust producing device and more related to the wake topology. Alben et al.[98] studied thin foils oscillating at the leading edge and found resonant-like peaks in the swimming speed as a function of foil length and rigidity. They predicted that the foil speed is proportional to foil length to the -1/3 power and foil rigidity to the 2/15 power.
Prempraneerach et al.[15] showed experimentally that by properly selecting the chordwise flexibility, the efficiency of 2-D flapping foils can be increased by 36% compared to that of a rigid foil. Experiments were performed on five foils made of urethane with different shore A hardness values and the foil made with A60 produced larger efficiency compared to other foils. By using the Particle Image Velocimetry (PIV) and force measurements, Heathcote et al. [110, 111] found that characteristics of the vortices behind the flapping foils are dependent on the airfoil flexibility, plunge frequency and amplitude. Their studies suggested the existence of optimal airfoil stiffness that produced greater thrust for a given plunge frequency and amplitude. Tangorra et al. [112] developed a biomimetic pectoral fin to study the effect of fin rays flexibility on propulsive forces. Experiments conducted on seven different fins revealed that the fin rigidity has a significant impact on the direction, magnitude and time course of the propulsive forces.

1.2.2.2 Spanwise flexibility

Bose et al.[113] studied several cetacean flukes and determined that the center of the flukes is much stiffer than the edges and this spanwise flexibility causes a phase difference between the center and edges of the flukes during bending which would prevent the total loss of thrust at the end of the stroke. Using a time domain panel method, Liu and Bose [114] reported that span wise flexibility reduces the performance unless a careful active control of the phase of the spanwise flexibility relative to other motion parameters was applied to the time dependent motion. Zhu [115] discussed the effects of chord wise and span wise flexibility on the propulsive performance for fluid and inertial driven loading cases. Within a small range of structural parameters, he found that in the inertial driven loading case, chord wise flexibility reduces both thrust and efficiency and span wise flexibility increases thrust without reduction in efficiency. In the fluid driven loading case, he showed that chord wise flexibility increases efficiency but span wise flexibility compromises the performance of the reduction of both the thrust and efficiency. Zhu and Shoele [116] predicted the propulsive performance of a skeleton strengthened fin using a coupled boundary element hydrodynamic model and nonlinear Euler-Bernoulli beams. They demonstrated that the anisotropic deformability of the ray- reinforced fin significantly increased propulsion efficiency, reduced transverse force and reduced sensitivity to kinematic parameters.

Heathcote et al.[117] studied the effect of spanwise flexibility in a water tunnel using three different NACA0012 rectangular wings with varying spanwise stiffness heaving periodically. It
was observed that a limited degree of flexibility gives a thrust benefit of 50%. These findings suggest that birds, bats, insects and fish benefit aero/hydrodynamically from the flexibility of their wings and fins.

1.2.3 Fin and joint stiffness

Apart from the fin stiffness, the stiffness of the joint connecting the caudal fin to the body (seen in figure 2.1) has a significant effect on the propulsive performance [118]. Murray and Howle [119] investigated the effect of variations in translational spring stiffness at the quarter chord location on propulsor plunge and on the minimum oscillation frequency required to produce positive thrust. Guo and Yen [120] showed that by controlling the joint compliance, driving power can be reduced by 5-10%. Ziegler et al. [121] demonstrated that the static thrust generated by a swimming platform driven by a single actuator can be increased by optimizing the rotational stiffness of passive joints that follow the actuated joint. Anton et al. [122] used IPMCs as active joints for propelling the biomimetic robotic fish. They showed that a two link tail of the robotic fish could produce more thrust compared to a single link tail. Yong-Jai et al. [123] showed experimentally that the appropriate stiffness values of caudal fin and compliant joint can generate more thrust by using different materials for tail and compliant joint at different pitching frequencies.

1.2.4 Center of mass oscillations

Although the mean self-propelled speed (SPS) generated by the oscillatory fins is constant, the instantaneous velocity changes with time. As SPS is a direct product of thrust, the mean thrust will be close to zero at SPS but will have varying instantaneous values. This varying instantaneous SPS or thrust generate center of mass (COM) oscillations along the swimming direction. Similarly, the instantaneous magnitudes along the normal (lift) and rotational directions produces COM oscillations in those directions as well. According to Xiong & Lauder [124], fish have relatively low COM oscillations for their body size compared to terrestrial locomotion. By conducting experiments on three different species of fish (American eels (anguilliform), clown knife fish (gymnotiform), bluegill sunfish (labriform for speeds under 1.0 body lengths (BL)/s, carangiform)) which represent diversified modes of swimming in fishes, they found that the anguilliform swimming produces lowest COM oscillations in surge direction (swimming direction) while
carangiform COM oscillations being the highest of the three modes. The peak-to-peak COM magnitudes in surge (swimming) and sway (normal) directions for different swimming speeds by the three fishes as reported by Xiong & Lauder are shown in tables 1.1 and 1.2 respectively. While the anguilliform swimming showed no effect of speed on sway oscillations, both gymnotiform and carangiform modes showed greater sway COM oscillations with speed.

Table 1.1. Surge COM oscillation magnitudes

<table>
<thead>
<tr>
<th>Speed (BL/s)</th>
<th>American eel – surge (mm)</th>
<th>Clown knife fish – surge (mm)</th>
<th>Bluegill sunfish – surge (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.9</td>
<td>0.5</td>
<td>1.2</td>
</tr>
<tr>
<td>1.0</td>
<td>0.8</td>
<td>0.8</td>
<td>1.5</td>
</tr>
<tr>
<td>1.5</td>
<td>0.6</td>
<td>0.8</td>
<td>1.4</td>
</tr>
</tbody>
</table>

Table 1.2. Sway COM oscillation magnitudes

<table>
<thead>
<tr>
<th>Speed (BL/s)</th>
<th>American eel – sway (mm)</th>
<th>Clown knife fish – sway (mm)</th>
<th>Bluegill sunfish – sway (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>5.77</td>
<td>1.06</td>
<td>0.35</td>
</tr>
<tr>
<td>1.0</td>
<td>4.03</td>
<td>2.34</td>
<td>0.89</td>
</tr>
<tr>
<td>1.5</td>
<td>4.7</td>
<td>3.21</td>
<td>2.99</td>
</tr>
</tbody>
</table>

Wen & Lauder [125] studied the effect of surge oscillations on robotic fins using an inertia compensated flapping device. When the surge oscillation magnitudes of the order of 1 mm with a correct phase were imposed on the flapping foil, the oscillations in the swimming forces were significantly reduced which suggested that the imposed surge oscillations mimicked the COM oscillations that would be seen in a freely swimming foil or fish of equivalent area.
By conducting experiments on an FMC actuated stationary fish, Zhang & Philen [23] demonstrated that a nearly constant thrust with almost zero oscillation magnitudes can be obtained by tuning the oscillation frequency with the fish body elasticity. The variation of thrust with time for three different frequencies of 0.9Hz, 1Hz and 1.1Hz is shown in the figure 1.6. As seen in the figure 1.6, although the thrust has oscillations at operational frequencies of 0.9 and 1.1Hz, the oscillation magnitudes are almost negligible at 1Hz termed as a ‘sweet spot’. This study indicates that an optimal combination of frequency and stiffness produces reduced COM oscillations which would be of importance in the design of under-water robotics.

![Figure 1.6. Instantaneous thrust (N) produced by the stationary FMC actuated robotic fish](image)

1.2.5 Varying stiffness

The fish have streamlined bodies with varying dimensions along their length which reduce the drag during locomotion. The fins have varying thickness, muscle mass and bone density along their length and hence the stiffness is not constant from the joint to the tip. A cleared and stained
Bluegill sunfish caudal fin is shown figure 1.7. As seen in the figure 1.7, the bone density and thickness of the rays vary along the length of the caudal fin.

![Figure 1.7. Bluegill sunfish caudal fin](image)

Esposito et al.[126] designed a robotic caudal fin (see figure 1.8) with six individually movable fin rays based on the bluegill sunfish to test the effects of fin ray stiffness, frequency and kinematics on the propulsive performance. The fin rays were constructed with varying stiffness from root to the tip to mimic the caudal fin rays. It was found that all the applied kinematic motions produced thrust with cupping motion producing the largest of all.
Riggs et al.[127] reported that a biomimetic stiffness profile produces maximum thrust compared to the NACA profile in flexible fin propulsion. The body stiffness has been approximated as a third order curve from the work by McHenry et al.[128] to fabricate the biomimetic fins. The stiffness variation of a vinyl pumpkinseed sunfish model can be seen in figure 1.9. Walker [129] in his thesis reported the variation of optimal spanwise membrane tension for the MAV wings for different cases of maximum stroke-averaged thrust and efficiency using gradient based optimization techniques. It was shown that the optimal design has a stiffer leading edge and a flexible trailing edge.

Shoele and Zhu [130] developed a numerical model of ray reinforced fin using immersed boundary method to investigate the effects of ray flexibility especially the detailed distribution of ray stiffness on thrust generation. It was shown that the thrust can be increased significantly by using flexible rays and efficiency by strengthening the leading edge.
1.2.6 Active control

A remarkable feature of the fish fins compared to birds feathers and insect wings is that they can actively control the fin ray curvatures by differential muscle activity of the two hemitrichs (see figure 1.10) at the base of the fins [56, 131].

From the discussion in previous sections, it is known that the optimal stiffness of the oscillating fin is not constant and varies with the operating parameters. It therefore becomes important to develop fins that can vary the stiffness depending on operating conditions for optimal locomotion. Not many researchers have investigated real-time fin stiffness modulation. Kobayashi et al. [132] attempted to actively control the apparent stiffness of the pitching fins of paramecium like propulsive mechanism using ICPF (Ionic Conducting Polymer gel Film) actuators. The actuators were attached in a bimorph configuration which increase the apparent stiffness in the power stroke and decrease in the recovery stroke. It was shown that with the actuation of ICPF, the positive thrust force was greater compared to the case with no actuation. Kim et al. [48] demonstrated

![Graph of log₁₀(EI) vs Fractional axial position]

Figure 1.9. Vinyl Sunfish model stiffness variation along its length (Reproduced from [127])
experimentally that the camber generated by the MFCs produce sufficient aerodynamic benefit for the biomimetic flapping wings

Figure 1.10. Bilaminar structure of the fin rays and their muscle control (reproduced from [56])

Nakabayashi et al,[133] developed a variable stiffness joint connecting the fin with a variable effective length spring. A rectangular PET (polyethylene terephthalate) sheet attached with sliding rigid plates on either side is used as a spring and the apparent stiffness of the spring can be changed dynamically by changing the length of the rigid plates. This spring which acts as a joint is fixed to a chloride plate fin and the apparent stiffness of the joint is changed by reciprocating the rigid plates using a DC motor. The self-propelled speed improved by 47.2% for a pitching amplitude of 45° by carefully varying the apparent stiffness of the fin over a cycle.
1.2.7 Resonance effect

Some of the studies related to micro-air vehicles (MAV) reported that the maximum efficiency peaks are obtained at frequencies away from the resonant frequency [134-137]. In contrast to this, various numerical and experimental studies have shown that higher propulsive performances can be achieved by operating the fins close to resonance frequencies [109, 138-144]. Dewey et al.[141] have shown experimentally that operating at the resonance frequency is not a sufficient condition for maximized performance and the performance can be maximized if the oscillation frequency yields a Strouhal number in the optimal range (0.25 < St < 0.35) predicted by Triantafyllou et al.[145]. Quinn et al.[138] have reported that the propulsive performance of the heaving flexible panels depends strongly on structural resonance and the peaks in efficiency occur at the non-dimensional stiffness values where the trailing edge amplitude was maximal.

1.3 Motivation and Research Objectives

Researchers have shown that fish modulate the stiffness of the body and fins using the muscular system to execute the desired motions and swim efficiently [93, 146]. Fish minimize the mechanical cost of bending by increasing their body stiffness, which allows them to tune their body's natural frequency to match the tail beat frequency at a given swimming speed [147]. The myomeric muscles of fish actively change the stiffness of the body during bending to produce net positive mechanical work [148]. McHenry et al. [128] predicted that the sunfish increase their flexural body stiffness by a factor of two relative to their passive body stiffness in order to swim at faster speeds.

In tail dominant propulsive modes - subcarangiform, carangiform, thunniform, ostraciiform [20], the caudal fin plays an important role in the thrust generation [85] along with the control of orientation of the fish by active modulation of shape and stiffness [93, 146]. At higher speeds, fish increase the stiffness of the caudal fin to withstand the higher hydrodynamic loads [146]. Fish with a flexible tail fin, flexible vertebral column and broad muscular caudal peduncle show a wide range of swimming performances while the performance of the fish with stiff tail fin, stiff vertebral column and a slender caudal peduncle restricted to straight forward swimming [118].

Flapping or oscillating wings and fins deform owing to their geometry and flexibility as they propel their bodies through air or water. The aero/hydrodynamic characteristics of the flapping
wings/fins largely depend on their morphology and kinematics [7, 96, 97, 106, 115, 149, 150]. Various theoretical [69, 104, 106, 108, 109, 114, 115, 151] and experimental [15, 95, 96, 112, 126, 152, 153] studies showed that the flexibility of the fins has a significant impact on the propulsive performance. Zhang et al. [23] experimentally demonstrated that the sinusoidal fluctuation in the thrust could be reduced through hydroelastic tailoring of the tail stiffness with the actuation frequency using a biological-inspired artificial fish.

Esposito et al. [126] studied the effect of rayed fin stiffness on forces and wake flows and found that there are optimal fin stiffnesses for different scenarios of flapping frequencies, fin shape and flow speeds. Since the thrust depends on the stiffness of the fin, the self–propelled speed (SPS), which is calculated from thrust, is directly dependent on stiffness as well. Lauder et al. [96] showed experimentally that the SPS is dependent on the oscillation frequency, amplitude, stiffness, and the geometry of the fin. All these studies show that altering the stiffness along with the other parameters has a significant impact on the propulsive performance of the fin. Motivated by these studies, the objectives of this research are given below.

1. **Effect of combined fin and joint stiffness on propulsive performance.** Although the effect of fin stiffness on the propulsion performance is well reported by the previous studies, the combined effect of fin stiffness and the compliant joint stiffness has not received much attention. As far as authors’ knowledge, only Yong-Jai et al. [37] showed experimentally that the appropriate stiffness values of caudal fin and compliant joint can generate more thrust by using different materials for tail and compliant joint at different pitching frequencies. Their study has limitations in studying the combined fin and joint effects on propulsive efficiency as the tests were conducted in a fluid at rest. The combined effect of pitching and heaving amplitudes along with the fin and joint stiffnesses on the self–propelled speed and hydrodynamic efficiency is still an area which needs to be investigated. Therefore, an objective of this study is to investigate in detail the effect of stiffness of the compliant joint (which is caudal peduncle, the link between body and the caudal fin) and the self-propelled caudal fin on propulsive performance.

2. **Investigation of optimal parameters for reduced COM oscillations.** The bodies with flapping appendages produce center of mass (COM) oscillations as the flapping fins generate forces oscillatory in nature. The vehicles with larger COM oscillations pose
problems of control and maneuverability. Although various theoretical [69, 104, 106, 108, 109, 114, 115, 151, 154, 155] and experimental [15, 95, 96, 112, 126, 152, 153] studies proved that the flexibility of the fins has a significant impact on the propulsive performance, its role in minimizing the COM oscillations has not drawn much attention till date. Therefore, a detailed investigation is required to find out the effect of fin flexibility along with the operating parameters that reduce COM oscillations which is an objective of this research. The developed principles could aid in the design of aerial and underwater robotic vehicles with minimized COM oscillations.

3. **Optimal chordwise stiffness profiles of self-propelled flapping fins.** A quick observation of the fish fins reveal that the mass and stiffness are not constant along their span and length. The fish fins have evolved over the years to produce optimized performance and are a source of inspiration to develop fins related to aerial/underwater robotics. Walker[129] reported the advantages of varying stiffness by predicting the optimal spanwise tension values of membrane wings that maximize thrust for a fixed efficiency. Till date, very limited numerical and experimental studies have shown that the caudal fins with varying EI produce better performance compared to constant EI fins. A comprehensive investigation is necessary to predict the EI profiles that optimize propulsive performance and these profile variations for different operating conditions. Also, the predicted EI profiles needs be validated by conducting experimental studies to translate the advantages to a robotic application. Therefore, an objective of this research is to predict the optimal stiffness profiles for maximized performance and validate the predictions by performing experiments on robotic fins.

4. **Enhanced propulsive performance of flexible flapping fins using MFCs.** Recent studies on the role of body flexibility in propulsion suggest that fish have the ability to control the shape or modulate the stiffness of the fins for optimized performance. Many researchers have investigated novel actuation methods for developing biologically inspired underwater vehicles and propulsors. For example, ionic polymer-metal composites (IPMCs) [38-40], shape memory alloys (SMAs) [41, 42, 44], magnetostrictive films [45], flexible matrix composites (FMCs) [23, 156], matrix fiber composites (MFCs) [46, 47, 157] have been used in the past for developing underwater fish-like robots. All of these examples used the actuators as the primary means for generating the thrust for propulsion. Inspired by
nature’s ability to modulate stiffness for different operating conditions, the approach here is to investigate active deformation of flapping foils to tailor the thrust by employing MFC actuators. A comprehensive investigation of active deformation of flapping foils has not been performed till date and as per author’s knowledge only Kim et al. [48] demonstrated experimentally that the camber generated by the MFCs produce sufficient aerodynamic benefit for the biomimetic flapping wings. However, the effect of parameters such as oscillation frequency, voltage amplitude, phase between voltage and heaving, fin flexural stiffness on propulsive performance are areas that needs to be investigated. Therefore, a detailed theoretical and experimental investigation is required to study the effect of active control of flapping fins and the corresponding parameters on propulsive performance. The objective is to predict the advantages of active control using antagonistic pairs of MFCs attached to an artificial fin oscillating at its leading edge.
Chapter 2

2 Theoretical model

Prediction of hydrodynamic performance of flapping fins involves the study of fluid structure interaction between the fin and the surrounding fluid. The fluid structure interaction of all the fish fins along with its body is a very complex problem and so some assumptions are made in reducing the complex problem into a simplified one. This section discusses the assumptions made and the theoretical model developed to investigate the propulsive performance of flapping fins subjected to different operating conditions.

2.1 Problem formulation

2.1.1 Fin and compliant joint

For a slender fish, the flow can be assumed to be unidirectional passing along the length of the body and for simplicity; the effect of body has been ignored. Along with the stiffness of the fin, the stiffness of the joint connecting the fish body to the fin governs the passive deformation of the fin and the propulsive performance. The compliant joint has been modeled as a linear torsional spring at the leading edge of the fin (see figure 2.1). The coupled fluid structure interaction of the fin has been modeled considering 2-D unsteady discrete vortex panel method and a nonlinear Euler-Bernoulli beam theory. 2-D Unsteady panel methods coupled with Euler-Bernoulli beam theory have been used in the literature to understand the flapping kinematics and performance. Katz and Weihs [69] used potential flow coupled with Euler-Bernoulli beam kinematics to predict the effect of chordwise flexibility on propulsive performance. Qian and Zhang [158] used unsteady potential flow panel method coupled with Euler-Bernoulli beam theory to predict the aerodynamic characteristics of a plunging rigid airfoil with elastic trailing edge plate. Dynamic interactions of the flexible foils with the surrounding fluid have been studied using potential flow methods.
coupled with beam kinematics [98, 159]. In this paper, the fluid is assumed to be at rest and the forces exerted on the fin undergoing oscillatory motion are calculated based on ideal flow assumptions. The self-propelled speed (SPS) of the fin moving in otherwise quiescent fluid is calculated using Newton's second law [98, 160-162]. Using the coupled fluid structure interaction model, the SPS and propulsive efficiency are calculated for different parameters (oscillation frequency, heaving and pitching amplitudes) and reported.

![Simplified model of compliant joint and fin.](image)

Figure 2.1. Simplified model of compliant joint and fin.

To model the caudal fin, a thin slender fin with aspect ratio less than unity is considered. The foil has a uniform thickness, chord $c$ and span $s$. In a space-fixed Cartesian frame $(X,Y,Z)$ shown in figure 2.3, the fin moves in $-X$ direction at its self-propelled speed (SPS). The oscillatory pitching or/and heaving motion is employed at the leading edge of the fin. A prescribed heaving motion $h = H \sin(\Omega t)$ applied in $Z$ direction and the local pitching motion $\alpha = \alpha_0 \sin(\Omega t + \psi)$ applied around $y$ axis. $H$ is the heaving amplitude, $\alpha_0$ is the pitching amplitude, $\Omega$ is the oscillation frequency and $\psi$ is the phase difference between pitching and
heaving. The fin is allowed to deform passively based on the stiffness of the fin and the joint owing to the time dependent hydrodynamic forces.

2.1.2 Deformation control

The fin with uniform thickness and chord $c$ is assumed to move with a constant forward velocity and the forces exerted on the fin undergoing oscillatory motion are calculated based on ideal flow assumptions. The MFC’s attached on either side of the fin modulate the passive deformation of the fin. Three antagonistic pairs are used to produce distributed actuation and a schematic of the configuration is shown in figure 2.2. A coupled computational model which incorporates the linear piezoelectric constitutive model with fluid structure interaction has been developed. Fluid structure interaction of the fin has been modeled by coupling the 2-D unsteady discrete vortex panel method with the Euler-Bernoulli beam kinematics. The developed computational model is used to predict the thrust and efficiency with parameters such as heaving amplitude, oscillation frequency, flexibility of the fin and the voltage applied to the MFC’s.

In a space-fixed Cartesian frame $(X, Z)$ shown in figure 2.3, the fin moves in $-X$ direction at uniform velocity $U_\infty$. Combined heaving and pitching completely describe the kinematic behavior of the tail and this work focuses on the propulsive performance of MFC actuated fins in heaving. A heaving motion of $h = H \sin(\Omega t)$ is employed at the leading edge of the fin in $Z$ direction where $H$ is the heaving amplitude and $\Omega$ is the oscillation frequency and $t$ is the time period. A voltage signal $V = V_0 \sin(\omega t + \phi)$ is applied to control the deformation of the MFCs, where $V_0$ is the voltage amplitude, $\omega$ is the voltage frequency and $\phi$ is the phase difference between the heaving and voltage. The mathematical model used to simulate the piezohydroelastic behavior is discussed in the section 2.2.2.
2.2 Mathematical Model

Investigation of the effect of stiffness on the propulsive performance involves the study of structural response coupled with the fluid flow around the fin. This section deals with the mathematical modeling of fluid-structure interaction and the calculation of SPS and performance parameters.

2.2.1 Structural Dynamics

The nonlinear structural response of the fin with chord wise flexibility is modeled using Euler-Bernoulli beam kinematics. Since the foil executes oscillatory motion and the orientation is time dependent based on the path, the problem is solved in the path dependent non-inertial coordinate system ($x, y, z$). The beam kinematics and the strain in $x$ direction (neglecting the other strains) are given by,
\[ u_1 = u_0(x) - z \frac{\partial w_0(x,t)}{\partial x} \]
\[ u_2 = 0 \]
\[ u_3 = w_0(x,t) \] (2.1)

\[ \varepsilon_{i1} = \varepsilon_{xx} = \frac{\partial u_0}{\partial x} + \frac{1}{2} \left( \frac{\partial w_0}{\partial x} \right)^2 - z \frac{\partial^2 w_0}{\partial x^2} \]

where \( u_1, u_2, u_3 \) are the total displacements along \( x, y, z \) directions and \( u_0 \) and \( w_0(x,t) \) denote the axial and transverse displacements of a point on the neutral axis.

Using Hamilton's principle, the governing differential equations of motion in longitudinal and transverse direction [163] are obtained as

\[ \rho(x)A(x) \frac{\partial^2 u_0}{\partial t^2} - \frac{\partial N_{xx}}{\partial x} = Q(x,t) \] (2.2)

\[ \rho(x)A(x) \frac{\partial^2 w_0}{\partial t^2} + \frac{\partial}{\partial x} \left( \frac{\partial w_0}{\partial x} N_{xx} \right) + \frac{\partial^2 M_{xx}}{\partial x^2} = P(x,t) \] (2.3)

where \( Q(x,t) \) and \( P(x,t) \) are the distributed forces acting in longitudinal (neglected in this analysis) and transverse directions (lift per unit length) respectively. \( N_{xx} \) is the axial force per unit length and \( M_{xx} \) is the moment per unit length defined by considering the \( x \) axis along the geometric centroidal axis as,

\[ N_{xx} = EA(x) \left[ \frac{\partial u_0}{\partial x} + \frac{1}{2} \left( \frac{\partial w_0}{\partial x} \right)^2 \right] \] (2.4)

\[ M_{xx} = -EI(x) \frac{\partial^2 w_0}{\partial x^2} \] (2.5)
$EA(x)$ is the extensional stiffness and $EI(x)$ is the flexural stiffness of the beam. The boundary and initial conditions of the beam are given by,

$$u_0(0, t) = 0, \quad \frac{\partial u_0}{\partial x}(c, t) = 0$$

$$w_0(0, t) = H \sin(\Omega t), \quad \frac{\partial w_0}{\partial x}(0, t) = \alpha_0 \sin(\Omega t + \psi)$$

$$\frac{\partial^2 w_0}{\partial x^2}(c, t) = 0, \quad \frac{\partial^3 w_0}{\partial x^3}(c, t) = 0$$

$$u_0(x, 0) = \dot{u}_0 = w_0(x, 0) = \dot{w}_0 = 0$$

The torsional spring constant $K_{\theta}$ is added to the global stiffness matrix. Equations (2.2) and (2.3) are solved using finite elements by applying the boundary conditions given in equation (2.6). The vertical force acting on the foil is calculated based on unsteady panel method which is discussed in the section 2.2.3.

### 2.2.2 Structural Dynamics coupled with Linear Piezoelectric Model

The structural response of the fin with chord wise flexibility is modeled as a cantilever beam using Euler-Bernoulli beam kinematics. Since the foil executes oscillatory motion and the orientation is time dependent based on the path, the problem is solved in the path dependent non-inertial coordinate system $(x, y, z)$. The beam kinematics and the strain in $x$ direction are given by,

$$u_1 = -z \frac{\partial w(x, t)}{\partial x}$$

$$u_2 = 0$$

$$u_3 = w(x, t)$$

$$\epsilon_{11} = \epsilon_{xx} = -z \frac{\partial^2 w}{\partial x^2}$$

where $u_1, u_2, u_3$ are the displacements along $x, y, z$ directions and $w(x, t)$ denote the transverse displacement of a point on the neutral axis.
The constitutive equations for linear piezoelectric materials [164] can be written as

\[
\begin{bmatrix}
\tau \\
\bar{E}
\end{bmatrix} =
\begin{bmatrix}
Y & -h_{33} \\
-h_{33} & \beta_{33}
\end{bmatrix}
\begin{bmatrix}
\epsilon \\
D
\end{bmatrix}
\]

(2.8)

where \( \tau \) is the stress, \( \bar{E} \) is the electric field, \( Y \) is the elastic stiffness, \( h_{33} \) is the piezoelectric constant, \( \beta_{33} \) is the dielectric constant, \( D \) is the electric displacement and \( \epsilon \) is the mechanical strain.

The piezoelectric beam equations are derived using the Hamilton’s principle. A detailed derivation is given in ref. [165]. The structural system model can be derived as

\[
\rho_{b}A_{b}\ddot{w} + E_{b}I_{b}w''' + \left[ 2E_{p}I_{p}w''' + 2h_{33}I_{p}D' \right] \tilde{H}(x) + \left[ 4E_{p}I_{p}w''' + 4h_{33}I_{p}D' \right] \tilde{H}'(x) + \left[ 2E_{p}I_{p}w''' + 2h_{33}I_{p}D' \right] \tilde{H}''(x) = F(x,t)
\]

(2.9)

where \( \tilde{H}(x) \) is the Heaviside function defined for a piezo layer located between \( x_{1} \) and \( x_{2} \) as \( \tilde{H}(x-x_{1}) - \tilde{H}(x-x_{2}) \) and the subscripts \( b \) and \( p \) indicate properties of beam (fin) and piezo material respectively. \( \rho \) is the density, \( A \) is the cross sectional area, \( E \) is the Young’s modulus and \( F(x,t) \) is the hydrodynamic force acting on the beam. According to the configuration used in figure 2.2,

\[
E_{p}I_{p} = 2E_{p}b_{p}\left( \frac{t_{p}^{3}}{3} + \frac{t_{p}^{2}t_{b}}{2} + \frac{t_{p}t_{b}^{2}}{4} \right)
\]

(2.10)

\[
J_{p} = b_{p}(t_{p} + t_{b})\frac{t_{p}}{2}
\]

where \( t_{p} \) is the thickness of the MFC, \( b_{p} \) is the width of the MFC.
The derived actuator model can be written as

\[
\left[ h_{33} J_p w'' + A_p \beta_{33} D - V(t) b_p \right] \ddot{H}(x) = 0
\]  

(2.11)

The initial and boundary conditions of the beam are given by,

\[
\begin{align*}
  w(0, t) &= H \sin(\Omega t), \quad \frac{\partial w}{\partial x}(0, t) = 0 \\
  \frac{\partial^2 w}{\partial x^2}(c, t) &= 0, \quad \frac{\partial^3 w}{\partial x^3}(c, t) = 0 \\
  w(x, 0) &= \dot{w}(x, 0) = 0
\end{align*}
\]  

(2.12)

where \( c \) is the length of the fin.

Equations (2.9) and (2.11) are solved using finite elements by applying the boundary conditions given in equation (2.12). The electric charge \( Q \) is calculated from the electric displacement \( D \) and the power input to the MFC \( P_{MFC} \) is obtained as given below.

\[
Q = b_p L_p D \\
i = \frac{dQ}{dt} \\
P_{MFC} = iV
\]  

(2.13)

where \( L_p \) is the active length of the MFC, \( i \) is the electric current and \( V \) is the applied voltage.

The force acting on the foil is calculated based on 2-D unsteady discrete vortex panel method which is discussed in the next section.

2.2.3 Unsteady Fluid Flow

The fluid is assumed to be at rest and the disembodied fin moves in a prescribed path through an inviscid, incompressible fluid as shown in figure 2.3. A rigid link is assumed to be attached to
the torsional spring as shown in the figure 2.3 to which the heaving and pitching motion is prescribed. However, the effect of the geometry of the rigid link and the torsional spring is ignored in the calculation of hydrodynamic forces acting on the fin. The hydrodynamic forces acting on the fin are calculated based on the well-known 2-D unsteady discrete vortex panel method. A brief description of the flow assumptions and the model is given in this section and a detailed discussion is given in reference [166].

Figure 2.3. Discrete vortex model for the unsteady thin foil.

The fin is assumed to be a thin lifting airfoil and moving in $-X$ direction in an inertial frame of reference $(X, Z)$. The fin’s centerline is placed in a moving frame of reference $(x, z)$ with the leading edge at the origin. The flight path of the origin is prescribed as

$$
X_0 = -U_w t \\
Z_0 = -H \sin(\Omega t) \\
\theta = \alpha_0 \sin(\Omega t + \psi)
$$

(2.14)
The discrete vortex method used in this work is based on the lumped-vortex element. The velocity at an arbitrary point due to a vortex element of circulation $\Gamma_j$ located at $(x_j, z_j)$ is given by,

$$
\begin{pmatrix}
u \\
w
\end{pmatrix} = \frac{\Gamma_j}{2\pi r_j^2} \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{pmatrix} x - x_j \\
z - z_j
\end{pmatrix}
$$

(2.15)

where $r_j^2 = (x - x_j)^2 + (z - z_j)^2$

The centerline of the fin is discretized into N subpanels of equal length and N vortex points $(x_j, z_j)$ are placed at the quarter chord point of each subpanel and the zero normal flow conditions are fulfilled at three quarter point of each panel. Since the geometry of the fin changes at each time step, the normal $n_i$ and tangent $\tau_i$ vectors at each of the collocation points are found in the $(x, z)$ frame from the instantaneous surface shape $\eta(x, t)$ as shown in figure 2.3.

$$
n_i(t) = \frac{(-d\eta(x, t)/dx, 1)}{\sqrt{(d\eta(x, t)/dx)^2 + 1}} = (\sin \alpha_i(t), \cos \alpha_i(t))
$$

$$
\tau_i(t) = (\cos \alpha_i(t), -\sin \alpha_i(t))
$$

(2.16)

The lumped vortex panels are based on the exact solution where the flow normal to the panel vanishes only at three-quarter chord point when the vortex is placed at the quarter chord point thus allowing a single solution. Since the lumped vortex element is based on this condition, the last panel inherently fulfills the requirement of Kutta condition and no additional specification of this condition is made. At each time step a wake vortex is shed and it is placed at 0.25 of the distance covered by the trailing edge during the latest time step. The boundary condition acting on the surface of the fin is given by

$$
(\nabla \Phi_b + \nabla \Phi_w - V_0 - v_{rel} - \Omega \times r) \cdot n = 0
$$

(2.17)
where \( \Phi_B \) is the body potential, \( \Phi_W \) is the wake potential, \( V_0 \) is the prescribed velocity and \( v_{rel} = (0, \partial \eta / \partial t) \) limited to small amplitudes with in \((x, z)\) coordinate system. According to the Kelvin condition, the total circulation around the fin plus the strength of the shed vortex is equal to the total circulation around the fin in the previous time step.

\[
\Gamma(t) + \Gamma_W = \Gamma(t - \Delta t) \tag{2.18}
\]

and the instantaneous circulation \( \Gamma(t) \) is the sum of all fin’s vortices given by

\[
\Gamma(t) = \sum_{j=1}^{N} \Gamma_j \tag{2.19}
\]

Using the unsteady Bernoulli equation, the pressure difference \( \Delta P \) acting on the upper and lower surface of the fin is calculated as

\[
\Delta P_j = \rho_f \left[ (U(t) + u_w, W(t) + w_w)_j \cdot \frac{\Gamma_j}{\Delta l_j} + \frac{\partial}{\partial t} \sum_{k=1}^{j} \Gamma_k \right] \tag{2.20}
\]

where \( \rho_f \) is the density of the fluid, \( U(t), W(t) \) are the kinematic velocity components, \( u_w, w_w \) are the wake induced velocity components and \( \Delta l_j \) is the length of the \( j^{th} \) panel. The total lift and moment are obtained by integrating the pressure difference along the length of the fin given by

\[
F_z = \sum_{j=1}^{N} \Delta P_j \Delta l_j \cos \alpha_j \tag{2.21}
\]

\[
M = -\sum_{j=1}^{N} \Delta P_j \cos \alpha_j \Delta l_j x_j
\]
The drag produced by the fin due to its unsteady motion is given by

\[ F_x = \sum_{j=1}^{N} \rho \left( w_{W,j} \Gamma_j + \frac{\partial}{\partial t} \sum_{k=1}^{j} \Gamma_j \Delta l_k \sin \alpha_k \right) \]  

(2.22)

The fin produces thrust if \( F_x \) is negative since the fin is moving in the \(-X\) direction and drag if \( F_x \) is positive.

The rigid link and spring have been ignored for all the cases except in the study of combined fin and joint stiffness effects on propulsive performance.

### 2.2.4 Self–Propelled Speed

The flapping motion of the fin produces a net horizontal force \( F_x \), as given in equation (2.22), which propels the fin forward for a positive value (i.e. thrust). If the foil moves at its self-propelled speed or reaches its steady state velocity, the net thrust \( F_x \), produced balances the viscous drag \( D \) acting on the foil. The instantaneous horizontal force \( T \) acting on the foil is given by,

\[ T = F_x - D \]

(2.23)

It is assumed that the viscous drag is the same as that for a flat plate moving steadily at a speed equal to the instantaneous horizontal flow speed \( U(t) \) as used by Alben et al.[98]. The viscous force is given by,

\[ D = 1.328 \rho \sqrt{\nu U(t)^3} c \]

(2.24)

where \( \rho \) is the density and \( \nu \) is the kinematic viscosity of the fluid. The self-propelled speed of the foil is calculated using Newton's second law.
\[ \ddot{T} = m \frac{d\dot{U}}{dt} \]  \hspace{1cm} (2.25)

where \( m \) is the mass of the foil at its centroid and the velocity of the fin at time \( t \) is obtained by discretizing the equation (2.25) using a backward finite difference scheme.

\[ U' = \frac{T}{m} \Delta t + U'^{-1} \]  \hspace{1cm} (2.26)

where \( \Delta t \) is the time step, \( U' \) is the horizontal velocity at time \( t \) and \( U'^{-1} \) is the horizontal velocity at time \( t - \Delta t \).

### 2.3 Computational Procedure

The fluid-structure coupling is carried out by the loose coupling method. At the first time step \( \Delta t \), initial geometric configuration of the fin is given as input to the fluid solver and the calculated hydrodynamic load is fed into the structural solver to calculate the deformation of the fin. Then this deformation is used to modify the geometry of the fin and resolve the flow field to obtain the hydrodynamic forces. This process is repeated until the deformation converges for that time step. This converged deformation is used in the next time step and the process continues for all the time steps. The coupling procedure is shown as a flow chart in figure 2.4.
Equations (2.2) and (2.3) are nonlinear in nature and are solved using finite elements by employing Newton – Raphson iterative procedure [163]. The dynamic equations are solved using the Newmark's time integration scheme [167]. After the convergence of the coupled fluid-structure solver, the self-propelled speed of the fin is calculated from equation (2.26) and this updated SPS will be used for the next time step to calculate the forces acting on the fin. The analysis is run for few cycles until steady state velocity is reached.

The non-dimensional bending stiffness $\bar{E}$ of the foil is given by

$$\bar{E} = \frac{EI}{\rho_f U^2 \infty c^4}$$

(2.27)

where $EI$ is the bending stiffness of the fin and $c$ is the chord length of the fin.
Power input $P$ to the foil is given by

$$P = \frac{1}{T} \int_0^T \left( F_z(t) \frac{dh}{dt} \, dt + M_\gamma(t) \frac{d\alpha}{dt} \, dt \right)$$  \hspace{1cm} (2.28)

where $T$ is the total time period and the hydrodynamic propulsive efficiency $\eta_p$ is given by

$$\eta_p = \frac{-F_x U_{SPS}}{P}$$  \hspace{1cm} (2.29)

where $F_x$ is the mean force in the $X$ direction. Note that $F_x = D$ at SPS and therefore the thrust generated by the fin at SPS is calculated using the equation (2.24).
Chapter 3

3 Experimental Setup

The bioinspired fins are quite flexible and deform owing to the fluid dynamic forces acting on them which depends on the geometry, mass, frequency and magnitude of flapping and the free stream velocity or the self-propelled speed. To predict the performance and study the physics of these complex systems, high fidelity CFD simulations are required involving huge computational costs. Though experiments have their own limitations, they provide a very good understanding of the flow physics and evaluate performance of the complex fluid-structure interaction of flexible flapping fins. An experimental setup has been designed and fabricated to study the propulsive performance of flexible flapping foils in a water tunnel.

3.1 Flapping mechanism

Fish propels forward by passing undulated waves along its body as discussed in the previous sections and a combination of pitching and heaving motion completely describe the kinematics of a fish swimming. So, a combined pitching and heaving mechanism was fabricated to provide harmonic flapping motion to the flexible fins as shown in figure 3.1. The carriage made of aluminum frames housing a pitching motor and a scotch yoke mechanism slides over two parallel high precision stainless steel rods using linear bearings. The stainless steel rods are fixed firmly to the two Aluminum frames and the heaving motor is attached these frames which drives the carriage through a slider crank mechanism (figure 3.1). The two motors (CUI M223X0003 brushed DC motors) fixed to the mechanism provide heaving and pitching motions independent of each other. Encoders are attached to the motors and the feedback from these encoders is used to control the speed and phase difference between the two motors. The pitching motor is fixed on top of the carriage and is linked to the force sensor using a scotch yoke mechanism. The end of the yoke is connected to a rod which passes through two roller bearings. A six axis force sensor ATI MINI40
is used to measure the forces and torques generated by the flapping foil. One end of the sensor is connected to the rod linked to yoke and the other end is connected to the bar which mounts the flapping foils. This assembly is attached to linear stages which allow it to move in horizontal and vertical directions for accurate placement in the water tunnel prior to running the experiments. LabVIEW 2011 software is used to run the motors and collect the encoder and force data using a DAQ6211. The fins are actuated in a recirculating water tunnel of interior dimensions 6”x6”x18” as shown in figure 3.2.

![Figure 3.1](image_url)

Figure 3.1. Pitching and heaving mechanism for providing controlled flapping motion of flexible fins.

### 3.2 Digital image correlation

Digital image correlation (DIC) is being used for measuring the structural deformation of flapping wings in air [168] and here it is employed for the flexible fins in the water tunnel to measure the surface displacements and estimate 3D strains. This technique uses digital imaging processing methods to track the speckled pattern on the surface of the fin (see figure 4.8 for speckled fin). The setup consists of two Imager E-lite cameras mounted on a single tripod with a horizontal slider bar, two fluorescent lights, and a LaVision Programmable Timing Unit computer.
with DaVis 8.1 software. This setup can be seen in figure 3.2. The cameras focused on the fin captures the images at a frequency of 14Hz.

Figure 3.2. Experiment setup for evaluating flexible fins in water tunnel.

Figure 3.3. Custom calibration plate for the DIC experiments
A custom calibration plate as large as the fins was fabricated and calibration was performed in the entire range of expected deformation to increase accuracy of results. The calibration plate requires spherical dots of equal diameter and at equal distance to each other. The dots were made by removing the black paint on the plastic sheet using a laser cutting machine. Figure 3.3 shows the calibration plate submerged in the water tunnel for the calibration of the DIC before capturing the data for the fins.

The captured images are post processed using the DaVis software and a 3rd order polynomial fitting function was used for calibration since the fins were tested was underwater, causing diffraction.

3.3 Caudal fin EI Measurements

The stiffness variation along the length of different fish caudal fins has been measured by performing cantilever bending tests using Digital Image Correlation (DIC). The entire setup can be seen in figure 3.4. The setup consists of a linear slider, clamps to fix the six axis force transducer (ATI MINI40) and the fins, a plate and the DIC cameras. One end of the transducer is fixed to the clamp which slides over the linear slider and the other end is attached to a plate which applies equal displacement along the width of the fin as shown in the figure.

Figure 3.4. Experimental setup for caudal fin EI measurements
For all the measurements, the displacement is applied at 90% of the length of the fins. As the clamp with force transducer is moved manually along the slider for a certain displacement, the fin bends and the corresponding out of plane deformation is captured using DIC cameras at a frequency of 14Hz. The force transducer attached to the clamp is used to measure the time varying force acting on the fin in all the six directions although the force acting normal to the fin is used for the EI measurements. The EI variation along the length of the fin is calculated by assuming Euler-Bernoulli beam theory as given by Eq.(3.1). The deformation of the fin $\delta(x)$ and the normal force value $F$ at the end of the application of the displacement are employed in the following equation to calculate the EI variation along the length of the fin.

$$EI(x) = \frac{F(L-x)}{d^2 \delta(x)/dx^2}$$ (3.1)

where $L$ is the length of the fin. The term $d^2 \delta(x)/dx^2$ is calculated using finite difference method.
Chapter 4

4 Combined fin and joint stiffness effects

The geometric parameters and the range of motion parameters of the fin used in the theoretical studies are given in table 4.1. The phase difference between pitching and heaving is chosen as $90^0$ which is the optimal value for oscillating propulsors [69]. The stiffness of the fin is assumed to be constant $EI(x) = EI$ along the chord for all the simulations considered in this chapter.

<table>
<thead>
<tr>
<th>Table 4.1. Fin and motion parameters.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chord ($m$)</td>
</tr>
<tr>
<td>Thickness ($mm$)</td>
</tr>
<tr>
<td>Density ($kg/m^3$)</td>
</tr>
<tr>
<td>Width</td>
</tr>
<tr>
<td>$\alpha_0 (deg)$</td>
</tr>
<tr>
<td>$H$</td>
</tr>
<tr>
<td>$\psi (deg)$</td>
</tr>
<tr>
<td>$\Omega (rad/s)$</td>
</tr>
</tbody>
</table>

4.1 Theory

The procedure for the determination of self-propelled speed is discussed in section 2.2.4. Variation of SPS with time over two cycles for different foil stiffness is shown in figure 4.1. It is seen from the figure 4.1 that the foil with higher stiffness produce higher mean speed over a cycle
indicating that the rigid foils swim at faster speeds for the same input motion. This is largely due to the larger forces generated by the rigid fin when compared to flexible fins.

Figure 4.1. Variation of SPS over two cycles: $\alpha_0 = 3^0, H = 0.01c, \Omega = \pi, K_\theta = 100N \cdot m$.

Figure 4.2 shows the variation of SPS with fin stiffness $EI$ for two different joint stiffness $K_\theta$ values and two different oscillation frequencies $\Omega = \pi / 2$ and $\pi$ at $\alpha_0 = 3^0, H = 0.1c$. It is observed that SPS increases with increase of both joint and fin stiffness and converges after a certain stiffness value. This stiffness is defined as an optimized stiffness for thrust above which there is no change in SPS. However this optimal stiffness for thrust may not be optimal for efficiency and moreover, this values changes with operating parameters. It can also be inferred from this figure that oscillation frequency increases the SPS while maintaining the other parameters constant.
Figure 4.2. Comparison of mean SPS with stiffness: $H = 0.1c, \alpha_0 = 3^\circ$.

Figure 4.3. Comparison of mean SPS with stiffness: $\alpha_0 = 3^\circ, \Omega = \pi / 2$. 
The values of $H/c = 0.1$ and 0.2 are chosen for the simulations as used by Lauder et al. working with undulatory locomotion in fishes [96, 169]. Figure 4.3 shows the variation of SPS with fin stiffness $EI$ for three different joint stiffness $K_\theta$ values and two different heaving amplitudes $H = 0.1c, 0.2c$ at $\alpha_0 = 3^0, \Omega = \pi/2$. It is observed that higher heaving amplitudes produce larger speeds at the same stiffness of fin and the joint.

Figure 4.4 shows the variation of SPS with fin stiffness $EI$ for three different joint stiffness $K_\theta$ values and two different pitching amplitudes $\alpha_0 = 3^0, 7^0$ at $H = 0.1c, \Omega = \pi/2$. It is observed that higher pitching amplitudes produce larger speeds at the same stiffness of fin and the joint similar to heaving amplitudes.

![Figure 4.4. Comparison of mean SPS with stiffness: $H = 0.1c, \Omega = \pi/2$.](image)

The variation of $\eta_p$ as a function of $EI$ and $K_\theta$ for different heaving $H$ and pitching $\alpha_0$ amplitudes, oscillation frequency $\Omega$ are shown in figure 4.5. From the efficiency plots, it is
observed that the efficiency increases with flexibility which is in agreement with the previous studies [69, 104, 108] on chord wise flexibility. The increase of efficiency with flexibility is attributed to the bending of the foil which increases the component of hydrodynamic force in the direction of motion. The efficiency increases with flexibility, reaches a maximum value and then drops with further increase in flexibility.

Alben [108] reported a similar trend of efficiency with the stiffness of the fin for different reduced frequencies but for a fin attached to a body moving at a constant velocity. With the increase of oscillation frequency at the same pitching and heaving amplitudes, the value of maximum efficiency decreases as seen from figures 4.5(a) and (b). Also, it is observed that the stiffness of the joint and the fin at the occurrence of maximum efficiency changes with oscillation frequency. This behavior indicates that the stiffness of the fin and the joint needs to be altered with operational frequency and amplitudes to achieve maximum efficiency. It can be observed by comparing figures 4.5(a) and (c) that the optimal stiffness for maximum efficiency changes with increase in heaving amplitude and similarly for the pitching amplitude (compare figures 4.5(b) and (d)). Also notice that the maximum efficiency values change with the operating parameters. This is in line with the work by Flammang et al. [146] and McHenry et al. [128] which observed that fish increase the stiffness of the caudal fin to propel at higher speeds and efficiencies. For each configuration of parameters, there seems to be a particular set of $EI$ and $K_\theta$ at which the efficiency is maximum, which provides some insight into how fish may regulate stiffness for optimal swimming.

The values of $EI$ and $K_\theta$ at which the efficiency is maximum are defined as the optimal stiffnesses for efficiency and these optimal stiffnesses vary with the motion parameters. The optimized values of $EI$ and $K_\theta$ for two different heaving amplitudes of 0.1 and 0.2 at constant pitching amplitude of 30° are obtained using the MATLAB optimization routine fminsearch. The variation of optimal stiffnesses with oscillation frequency and heaving amplitudes are shown in figure 4.6. The red colored lines (round markers) refer to the optimized fin stiffness and the blue colored lines (square markers) refer to the optimized joint/spring stiffness. The increase in optimized stiffness with frequency is attributed to the requirement of higher stiffness of the fin and the joint due to the higher hydrodynamic loading. All the efficiency optimized stiffness curves
shown in figure 4.6 follow the power law $\sim \Omega^{2.3}$ with oscillation frequency for the range of values considered.

![Graphs showing variation of $\eta$ with stiffness](image)

**Figure 4.5.** Variation of $\eta$ with stiffness: a) $\alpha_0 = 3^0, H = 0.1c, \Omega = \pi / 2$  b) $\alpha_0 = 3^0, H = 0.1c, \Omega = 4\pi$

c) $\alpha_0 = 3^0, H = 0.2c, \Omega = \pi / 2$  d) $\alpha_0 = 7^0, H = 0.1c, \Omega = 4\pi$.
The SPS at the optimized stiffness values of $K_\theta$ and $EI$ with oscillation frequency is shown in the figure 4.7(a). SPS increases with increase in $\Omega$ and the higher heaving amplitudes produce larger speeds. The variation of maximum propulsive efficiency calculated at the optimized stiffness with oscillation frequency is shown in figure 4.7(b). Efficiency reduces with increase in $\Omega$ and the higher heaving amplitudes produce higher efficiency. From the figure 4.7, it is seen that higher heaving amplitudes not only produce larger speeds but also produce higher efficiencies. The generation of higher thrust and efficiency with higher heaving amplitudes is in line with what others [104, 170] have predicted but on a rigid foil moving with uniform velocity. From this study, it can be inferred that the joint and the fin stiffness need to be modulated for better efficiency when operating at different amplitudes and frequencies to achieve the desired propelling speeds.
Figure 4.7. a) SPS b) $\eta_p$ at optimized stiffness values of $K_0$ and $EI$
4.2 Experimental Results and Comparison with Theory

The experiments were conducted on trapezoidal foils of different thicknesses ranging from 0.1mm to 3.2mm having same surface area. The fins are of length 150mm with leading and trailing edge widths of 50 and 100mm respectively. To mimic the caudal joint, foils of length 20mm and thickness ranging from 0.2mm to 3.2mm were used and one of the speckled specimens is shown in figure 4.8. The experiments were performed for pure heaving and the foils were oscillated at 1Hz with heaving amplitude of 20mm.

4.2.1 Self – propelled speed (SPS)

A detailed experimental setup is discussed in chapter 3. Variation of thrust and efficiencies with the joint stiffness ($K_\theta$) for fins of thicknesses ranging from 0.1mm to 3.2mm for different free stream velocities are shown in figures 4.9 and 4.10 respectively. It can be seen that with the flexible joints, the stiffer fins produce lower thrusts and in this zone, flexible fins produce larger thrusts and efficiencies compared to stiffer ones. With stiffer joints, the stiffer fins produce larger thrusts and efficiencies. Also notice that for each fin, there exists a different joint stiffness value which produces maximum thrust and efficiency indicating that propulsive performance depends
on the combined stiffness values of the joint and the fin. This trend agrees with what was predicted by the theory discussed in section 4.1 and a comparison of the SPS values is discussed in this section.

Figure 4.9. Thrust variation with joint stiffness for different fin thickness values at four different free stream velocities.

Figure 4.10. Efficiency variation with joint stiffness for different fin thickness values at four different free stream velocities.
Using the experimental thrust data from figure 4.9, the SPS can be calculated for each configuration of fin and joint combination. For example, the variation of thrust with free stream velocity $U_\infty$ for a joint and fin thickness of 1.6mm is shown in figure 4.11. For certain values of thrust, the error bars are very small and they are not shown for the rest of the experimental results.

![Figure 4.11. Variation of thrust with free stream velocity for a joint and fin thickness of 1.6mm.](image)

The variation of thrust for all the fin thickness values is shown in figure 4.12. The speed at which the thrust crosses the zero line (i.e. thrust equals drag) is considered as the SPS for that configuration.
Contour plots of the experimental and computational self-propelled speed (SPS) variation with fin and joint stiffness are shown in figure 4.13. As seen in the figure, there is good agreement in the relationship between the joint and fin stiffness on the SPS response. However, the computational SPS values are higher than the estimated values determined from the experimentally measured thrust. It is predicted that the discrepancy is primarily due to the 2-D hydrodynamic model which assumes an infinite span and a simplified drag model (rigid plate) for the oscillating fins. Since the fins used in the experiments have an aspect ratio of 0.5, there is flow along the span which reduces the fluid momentum in the chord wise direction, thus reducing the thrust in forward direction. Alben et al.[98] have shown experimentally that the SPS increases with span of the foil and this could be one reason for higher theoretical SPS values which has infinite span. Other reasons for the discrepancy might be due to the negligence of viscosity, flow separation and the formation of leading edge vortices in the theoretical model. The timing of formation, convection and coalescence of the leading edge vortices with the trailing edge vortices influence the propulsive performance [171-173] which cannot be predicted by the present hydrodynamic model. A particle image velocimetry (PIV) study coupled with DIC measurements would aid in better understanding
of these phenomena. However, the experimental and computational SPS contours closely match as shown in figure 4.13.

![Image of SPS variation with fin joint stiffness](image)

**Figure 4.13.** SPS variation with fin joint stiffness a) Experiments b) Theory.

### 4.2.2 Digital Image Correlation Measurements

DIC measurements were used to compare the theoretical and experimental deformation patterns and to find a relation between the measured thrust and the deflection of the flexible fin. A comparison of the centerline deformation pattern between foils of thickness 0.2mm and 0.4mm measured using DIC for one cycle is shown in figure 4.14.
Figure 4.14. Deformation pattern of fins over a flapping cycle with a) 0.2mm thickness b) 0.4mm thickness.

Figure 4.15. Deformation of fin over a flapping cycle with 0.4mm thick joint and 0.2mm fin at $U_\infty = 0.1\text{m/s}$ a) DIC measurement b) Theoretical model.

A DIC and theoretical comparison of the deformation pattern of the foil with 0.4mm joint and 0.2mm fin is shown in figure 4.15. As seen from the figure 4.15, the theoretical deformation pattern and the trailing edge deflection values closely match with the measured DIC values.
As shown in figure 4.13, the SPS predicted by theory is higher than measured in the experiment. Therefore, to better understand the source of the discrepancies and any possible limitations in the hydrodynamic model, the experimentally obtained structural response of the flexible fin is coupled with the theoretical hydrodynamic model (labeled *DIC Coupled Hydrodynamic* in figure 4.16). The aim is to remove the theoretical beam model from the thrust prediction, allowing us to focus on the hydrodynamic model. A comparison of the thrust and efficiency measured using the force sensor for 0.4mm thick joint fins with the DIC deformation coupled hydrodynamic model is given in figure 4.16. Since the structural response from the experiment is used in the DIC coupled hydrodynamic analysis, the difference between the experimental and the DIC Coupled Hydrodynamic data is primarily due to the simplification of the hydrodynamic model using 2D panel method. It is expected that improvements can be achieved using a 3D panel method or a Navier-Stokes solution. Although the experimentally measured thrust values in figure 4.16(a) are smaller than the results from the DIC coupled hydrodynamic model, the trend matches well where there is an optimal fin thickness for thrust and efficiency.
Figure 4.16. Variation of (a) thrust and (b) efficiency with fin thickness at $U_\infty = 0.1\text{m/s}$ for 0.4mm joint fins.

4.3 Conclusions

The theoretical results indicate that SPS increases with both fin and joint stiffness in the case of combined pitching and heaving since the rigid fins produce more forces when compared to the flexible fins subjected to same oscillatory motion. But, this is not the case with pure heaving where an optimized stiffness value exists where thrust/SPS is a maximum. The mean SPS of the fin can be varied by changing the stiffness of the fin and the joint without changing the motion parameters.

Flexible foils produce higher efficiency but further increase in flexibility reduces the efficiency moving towards drag zone. The increase in efficiency with flexibility is attributed to the bending of the foil which increases the component of hydrodynamic force in the direction of motion. It is demonstrated that for each combination of oscillatory motion parameters, there exists a different combination of fin and joint stiffness which produce the maximum efficiency. These optimized stiffnesses increase with the oscillation frequency and heaving amplitude. Thus, the joint and the
fin stiffness need to be modulated to achieve better efficiencies when operating at different motion parameters.

It is observed from the experiments that the stiffer fins does not produce thrust with highly flexible joints suggesting a threshold joint stiffness above which they produce thrust. Experimental results also illustrate the existence of a combined fin and joint stiffness at which the propulsive performance is maximum for the operating parameters considered. The magnitude of theoretical SPS is higher compared to the experimental SPS due to the simplifications in the fluid-structure interaction model.

A DIC setup is used to measure the deformation of the heaving fins and the obtained deformation is compared with the theoretical deformations, which has good agreement in terms of deformation pattern and magnitude. Furthermore, to find out the discrepancies between the theoretical and experimental results, the deformation obtained using DIC is coupled with the 2D hydrodynamic model to predict the performance and to compare with the experimental results. By coupling the DIC structural response with the hydrodynamic model, a discrepancy due to the theoretical beam has been eliminated and the difference in the thrust compared to experimental data is primarily due to the simplified hydrodynamic model. The present fluid structure interaction model predicts the behavior reasonably well compared to the experimental data. This study provides some insight into how fish may regulate the combined fin and joint stiffness for optimal swimming performance and suggest the benefits of active stiffness modulation in bio-inspired underwater robotics.
Chapter 5

5 Investigation of COM Oscillations

One of the metric to measure the efficiency of self-propelled bodies is the cost of transport (COT) [174] which is given by the energy cost per unit distance per unit weight.

\[
COT = \frac{P_{\text{inp}}}{mU_{\text{Mean}}} \tag{5.1}
\]

The calculated \( U_{\text{Mean}} \) and COT are used to optimize the \( EI \) and \( \Omega \) that generates low COM oscillations. Although the mean SPS and lift are constant over time, the instantaneous SPS and lift have maximum and minimum peaks which generate COM oscillations in the axial and lateral directions as shown in figure 5.1. The COM oscillations in axial direction are characterized by the distance between the maximum and minimum peaks, which is the SPS cycle height \( h_{\text{SPS}} \) defined in figure 5.1 (a). The objective of the optimization problem is to predict the optimal stiffness \( EI \) and oscillation frequency \( \Omega \) which minimizes the \( h_{\text{SPS}} \) for given operating conditions.

A global search algorithm DIRECT acronym for DIviding RECTangles is employed to predict the optimal \( EI \) and \( \Omega \). DIRECT is a sampling algorithm which does not require objective function gradient and converges to the global minimum at the expense of exhaustive search over the domain. A detailed description of the theory and algorithm is given in ref.[175].

The problem of minimizing \( h_{\text{SPS}} \) and COT becomes a bi-objective optimization problem and Pareto fronts between the two objectives are generated using \( \epsilon \) - constraint method. Furthermore, optimization studies have been performed to predict the optimal parameters that reduce lift oscillations \( h_{\text{Lift}} \) (see figure 5.1(b)) which represent lateral COM oscillations. A comparison study
between the optimal parameters which reduce SPS and Lift oscillations reveal that optimizing either $h_{SPS}$ or $h_{Lift}$ minimize both SPS and Lift oscillation magnitudes as discussed in section 5.1.

The optimal parameters that reduce axial and lateral COM oscillations under different operating conditions have been predicted using theoretical studies and are discussed in section 5.1. Experiments have been performed to validate the theoretical results which are reported in section 5.2. The stiffness of the fin is assumed to be constant $EI(x) = EI$ along the chord for all the simulations reported in this chapter.

5.1 Theoretical results

Three different optimization studies have been performed to study the effect of stiffness on COM oscillations and COT. Section 5.1.1 discusses the optimization studies for minimizing axial COM oscillations while section 5.1.2 discusses the optimization studies for minimizing lateral COM oscillations. Section 5.1.3 discusses the optimization studies for minimizing the cost of transport. Minimizing axial and lateral COM oscillations comes with the cost of increased COT and this bi-objective problem has been investigated for different operating parameters and corresponding Pareto fronts have been reported in section 5.1.4.

5.1.1 Axial COM Oscillations

Optimization studies have been carried out to predict the optimal EI and frequency $\Omega$ combination which minimizes axial COM oscillations $h_{SPS}$ for different operating conditions. A constant mean SPS, $U_{Mean}$, can be obtained by various combinations of EI and $\Omega$ for a given heaving and pitching amplitude, and the instantaneous SPS variation for two combinations is shown in figure 5.1. Both the combinations produce a constant $U_{Mean}$ of 2BL/s, but the SPS with low oscillation magnitude $h_{SPS}$ represented by the dashed line has lower EI and higher $\Omega$ compared to the SPS represented by the solid line. This is in line with the experimental results reported by Park & Cho [176] showing that the robotic dolphin with flexible tail produced low COM oscillations compared to a stiff tail.
Figure 5.1. Instantaneous (a) self-propelled speed (SPS) and (b) Lift for two cycles: $\alpha = 8^0, H / c = 0.1$
The stiffer fin with EI of 15.13N-m² produced large axial and lateral forces compared to the flexible fin with EI of 0.018N-m². To maintain a constant $U_{\text{Mean}}$ of 2BL/s, the stiffer fin needs to be operated at lower frequency compared to the flexible fin. Although reducing the flexibility and increasing the oscillation frequency reduces the COM oscillations, lowering the stiffness beyond a particular limit poses a problem of generating sufficient thrust regardless of frequency. Also, increasing the frequency increases COT nullifying the effect of fin flexibility which is discussed in detail in the section 5.1.3. Therefore, it is important to study the optimal combination behavior of EI and $\Omega$ that produce low COM oscillations and low COT.

Figures 5.2(a) and (b) shows the optimal SPS oscillation magnitudes $h_{SPS}$ for different values of heaving and pitching amplitudes for $U_{\text{Mean}}$ values of 2BL/s and 4BL/s, respectively. As it can be observed, the optimal $h_{SPS}$ values are different for different operating conditions and their variation with $U_{\text{Mean}}$ is given in figure 5.3(a). It can be observed from figure 5.3(a) that higher $U_{\text{Mean}}$, $H/c$, and $\alpha$ produces larger forces acting on the fin, and subsequently the fluctuation magnitudes $h_{SPS}$. The corresponding optimal combinations of EI and $\Omega$ which produce minimum $h_{SPS}$ are shown in figures 5.3(c) and (d) respectively. The increased combination values of EI and $\Omega$ with $U_{\text{Mean}}$ can be attributed to the requirement of larger forces acting on the fin to maintain constant $U_{\text{Mean}}$. The variation of COT with $U_{\text{Mean}}$ for minimized $h_{SPS}$ can be seen in figure 5.3(b). The lower COT values of $\alpha = 8^0, H/c = 0.2$ compared to $\alpha = 8^0, H/c = 0.1$ is due to the requirement of lower optimal frequencies as seen in figure 5.3(d) for $\alpha = 8^0, H/c = 0.2$ to maintain a constant $U_{\text{Mean}}$. 

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Figure 5.2. Instantaneous self-propelled speed (SPS) for two cycles at $U_{Mean}$ values of (a) 2BL/s (b) 4BL/s
Figure 5.3. Variation of optimal (a) $h_{SPS}$, (b) COT, (c) EI and (d) $\Omega$ with SPS mean velocities for minimized $h_{SPS}$.
Optimizations studies have also been performed to investigate the effect of heaving and pitching amplitudes on COM oscillations. Optimized parameters for minimized $h_{SPS}$ are predicted with $H / c$ and $\alpha$ as the optimized variables along with EI, $\Omega$. As seen in figure 5.4, the $h_{SPS}$ has been further reduced for a given $U_{Mean}$ of 2.5BL/s. The pitching and heaving amplitudes converged to the lower bound values, thus increasing the $\Omega$ to maintain constant $U_{Mean}$ of 2.5BL/s. As seen from figure 5.4, reducing heaving and pitching amplitudes produces lower $h_{SPS}$ values and this is due to the displacement of lesser mass of water compared to large heaving amplitudes. These studies indicate that the COM oscillations can be reduced by choosing flexible fins, lower pitching and heaving amplitudes and higher oscillation frequencies for a given SPS. This could very well be the reason why fish increase their oscillation frequency instead of heaving amplitude to swim at faster speeds [177].

![Figure 5.4. Instantaneous self-propelled speed (SPS) for two cycles with optimized variables – EI, $\Omega, \alpha, H / c$](image)
5.1.2 Lateral COM Oscillations

Optimization studies have been carried out to predict the optimal EI and frequency $\Omega$ combination which minimizes lateral COM oscillations $h_{\text{Lift}}$ for different operating conditions. A comparison of $h_{\text{SPS}}$ for minimized $h_{\text{SPS}}$ and $h_{\text{Lift}}$ conditions is shown in figure 5.5(a). The solid lines represent the $h_{\text{SPS}}$ values related to studies of minimized $h_{\text{SPS}}$ and dotted lines represent the $h_{\text{SPS}}$ values related to studies of minimized $h_{\text{Lift}}$. As seen from figure 5.5(a), optimizing the $h_{\text{Lift}}$ produced slightly higher $h_{\text{SPS}}$ values but reduced corresponding COT values (see figure 5.5(b)). This is mainly due to the reduced $\Omega$ as seen in figure 5.5(d) for $h_{\text{Lift}}$ optimized cases represented by dotted lines. Although the EI values increased for $h_{\text{Lift}}$ optimized cases represented by dotted lines as seen in figure 5.5(c), the reduction of $\Omega$ has a greater effect on COT. These studies indicate that optimizing $h_{\text{Lift}}$ produces lower COT values which is very important but at the cost of a slight increase in the $h_{\text{SPS}}$ values.
Figure 5.5. Comparison of (a) $h_{SPS}$, (b) COT, (c) EI and (d) $\Omega$ for minimized $h_{SPS}$ and $h_{Lift}$ with SPS mean velocities

5.1.3 Cost of transport (COT)

Cost of transport is one of the parameters representing efficiency of self-propelled bodies and this section discusses the COT comparison of optimized $h_{SPS}$ and $h_{Lift}$ cases with optimal COT values for corresponding $U_{Mean}$ values. The studies have been carried out at a constant pitching and heaving amplitudes of $8^{0}$ and 0.1, respectively. A comparison of $h_{SPS}$ for three optimized cases is shown in figure 5.6(a). The optimal COT case produced large COM oscillations compared to optimized $h_{SPS}$ and $h_{Lift}$ cases and corresponding COT values are shown in figure 5.6(b). Of the three, $h_{SPS}$ optimized case produced larger COT values and the COT optimized case produced least COT values. This behavior could be explained by analyzing the corresponding EI and $\Omega$ values given in figures 5.6(c) & (d), respectively. The optimized COT case predicted larger EI and lower $\Omega$ combinations for a given $U_{Mean}$ value. As the COT values are dominated by $\Omega$ compared to EI,
smaller $\Omega$ values generate lower COT. But to maintain constant $U_{Mean}$, the stiffness EI of the fin has to be increased when reducing the $\Omega$ values for lowering COT. At the same time, larger EI and lower $\Omega$ combinations produce higher $h_{SPS}$ values as seen in figure 5.1. Hence, it is a compromise between EI and $\Omega$ that dictates COM oscillations and COT, thus making it a bi-objective problem. The Pareto fronts for this bi-objective problem and corresponding EI and $\Omega$ combinations are reported in the next section.
5.1.4 Bi-objective optimization

As seen in the previous sections, minimizing $h_{SPS}$ generates higher COT values, thereby making this a classical problem of bi-objective optimization. The two objectives of the problem are to minimize $h_{SPS}$ and COT for given $U_{Mean}$ values, and pitching and heaving amplitudes. The Pareto fronts between the two objectives are generated using $\epsilon$ - constraint method by constraining the COT. The Pareto fronts provide useful information on minimum possible $h_{SPS}$ and COT values for a given $U_{Mean}$. Figure 5.7 shows the Pareto optimal solutions between the two objectives for different values of $\alpha$, $H/c$, and four different $U_{Mean}$ values ranging from 1BL/s to 4BL/s. As seen from the figures 5.7(a) to (d), for all velocities, lower $h_{SPS}$ magnitudes are generated with lower operating amplitudes of pitching and heaving. In other words, for a given COT, lower pitching and
heaving amplitudes produce lower $h_{SPS}$ values and similarly for a given $h_{SPS}$, lower COT values can be achieved by operating at lower pitching and heaving amplitudes.

A comparison of the Pareto fronts of different velocities in objective and variable space for the fixed values of $\alpha = 8^0, H/ c = 0.2$ are given in figures 5.8(a) & (b) respectively. To have all the Pareto fronts on the same scale, objective1 $h_{SPS}$ has been normalized with corresponding $U_{Mean}$ values and objective2 COT has been normalized with the maximum COT value for given $U_{Mean}$. It can be observed from figure 5.8(a) for a given value of COT, larger $U_{Mean}$ values produce lower $h_{SPS} / U_{Mean}$ values although the $h_{SPS}$ values are higher. Also, from figure 5.8(b) that larger values of EI and $\Omega$ are required for increased $U_{Mean}$ values.
Figure 5.7. Comparison of Pareto fronts in objective space for different pitching and heaving amplitudes: (a) 1BL/s, (b) 2BL/s, (c) 3BL/s, (d) 4BL/s
Experimental results

Experiments have been performed to study the effect of fin stiffness and oscillation frequency on the COM oscillations in a water tank of dimensions: 1.2 m X 0.5 m X 0.5 m. Since the water is not recirculating, the free stream velocity is zero for these set of experiments and equipment used to test the fins is the same as discussed in section 3. The experiments have been conducted on three different trapezoidal foils A, B, and C of polycarbonate with thicknesses 0.7 mm, 1.2 mm and 1.6 mm respectively. The fins are of length 150mm with leading and trailing edge widths of 50 and 100 mm respectively. The dimensions of one of the specimens is shown in figure 5.10. The approach here is to determine the oscillation frequency of the fins which produce equal mean thrust values. The \( h'_{\text{Thrust}} \) and \( h'_{\text{Lift}} \) (see figure 5.12(b)) of instantaneous thrust and lift values are then compared for different combinations of fin thickness and oscillation frequency. Although the theoretical predictions are made for self-propelled speed, the COM oscillation behavior predicted for the thrust in experiments would be valid since the SPS is a direct product of thrust. The
experiments were performed in a fluid at rest under pure heaving and the foils were oscillated with heaving amplitude of 20 mm.

Figure 5.9. Experimental setup

Figure 5.10. Speckled fin specimen

Theoretical results discussed in the previous section have shown that the flexible fins operating at larger frequencies produce lower COM oscillations compared to stiffer fins operating at lower frequencies which produce equal mean SPS. Experiments under pure heaving conditions have been
carried out on three fins A, B, C with thicknesses of 0.7 mm, 1.2 mm and 1.6 mm respectively. The heaving amplitude of 20 mm is maintained constant for all the experiments reported here and the operating frequency is controlled by LabVIEW software. Although the experiments have been conducted in a water tank under zero free stream velocity conditions, the measured COM oscillation patterns would be valid which can be compared with the theoretical SPS predictions. Figure 5.11 shows the variation of mean thrust for all the three fins with oscillation frequency. It can be observed that the stiffer fins produced larger thrusts compared to the flexible fins.

Figure 5.11. Variation of mean thrust with $\Omega$ for three fins of different thicknesses and equal surface area

Fin A, which is the most flexible of the three fins, produced a mean thrust of 0.145N at a frequency of 3.3Hz while the fin B which is of medium flexibility produced 0.145N thrust at a frequency of 1.8Hz. A comparison of instantaneous thrust and lift values of fins A and B defined as Case-1 is shown in figure 5.12(a). The $h_{\text{thrust}}$ and $h_{\text{lift}}$ values of both the fins are given in table. Although both the fins produced same mean thrust of 0.145N, it can be observed that the $h_{\text{thrust}}$
and $h'_{Lift}$ values are lower for the flexible fin A compared to the fin B. Similarly, Fin B produced a mean thrust of 0.25N at a frequency of 4.1Hz while the fin C which is of higher stiffness produced 0.25N thrust at a frequency of 2.3Hz. A comparison of instantaneous thrust and lift values of fins B and C defined as Case-2 is shown in figure 5.12(b). The $h'_{Thrust}$ and $h'_{Lift}$ values of both the fins are given in table 5.1. Although both the fins produced same mean thrust of 0.25N, it can be observed that the $h'_{Thrust}$ and $h'_{Lift}$ values are lower for the medium flexible fin B compared to the stiffer fin C. This validates the theoretical predictions that the flexible fins operating at higher frequencies produce lower COM oscillations compared to stiffer fins operating at lower frequencies for a given mean thrust/SPS.
Figure 5.12. Instantaneous force comparison of fins (a) A and B (b) B and C

Table 5.1. Experimental thrust and lift oscillation magnitudes

<table>
<thead>
<tr>
<th>Force oscillation magnitudes (N)</th>
<th>Case – 1</th>
<th>Case – 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fin A</td>
<td>Fin B</td>
</tr>
<tr>
<td>( h'_{\text{Thrust}} )</td>
<td>0.235</td>
<td>0.277</td>
</tr>
<tr>
<td>( h'_{\text{Lift}} )</td>
<td>5.590</td>
<td>6.132</td>
</tr>
</tbody>
</table>

A comparison of the deformation pattern of the two fins for both the cases 1&2 can aid in understanding the physics behind the difference in generated COM oscillations. DIC has been used to measure the 3D deformations of the flapping fins and figure 5.13 shows the 3D deformation of the fin C operated at 2.3Hz.
The 2D centerline deformations are extracted from the 3D deformations for the fins in both the cases 1&2. As the DIC cameras captures only 14 frames/sec, the measurements are taken for about 100 cycles after the fin reaches its steady state so that deformations are more or less same for each cycle. The 2D centerline deformations from different cycles are extracted and compiled to have a plot with 14 frames representing one complete cycle. Figure 5.14(a) shows the centerline deformations of the fin A operated at 3.3Hz and 5.14(b) shows the centerline deformations of the fin B operated at 1.8Hz. Although the fin A is flexible compared to fin B, the trailing edge amplitude of fin A is 45mm while the trailing edge amplitude of fin B is 61mm. Also by careful observation, it can be identified that the flexible fin A has larger curvatures compared to fin B.

Figure 5.15(a) shows the centerline deformations of the fin B operated at 4.1Hz and 5.15(b) shows the centerline deformations of the fin C operated at 2.3Hz. Although the fin B is flexible compared to fin C, the trailing edge amplitude of fin B is 52mm while the trailing edge amplitude of fin C is 69mm. Even in this case, a careful observation reveals that the fin B has larger curvatures compared to fin C. Therefore, it can be deduced that the combination of deformation pattern and trailing edge amplitude play a role in the generation of COM oscillations. Moreover, the fins with
larger curvature patterns and lower trailing edge amplitudes produced lower COM oscillations. Lower trailing edge amplitudes displace less volume of water surrounding the fin, thereby reducing the force magnitudes and the larger curvatures aid in redirecting the forces in axial direction increasing thrust.

Figure 5.14. Comparison of DIC captured centerline deformation of fin (a) A (b) B for case – 1

Figure 5.15. Comparison of DIC captured centerline deformation of fin (a) B (b) C for case – 2
These experimental studies have validated the theoretical predictions that the flexible fins operating at larger frequencies produce lower COM oscillations compared to stiffer fins operating at lower frequencies for a given mean thrust/SPS, and that the trailing edge amplitude along with the deformation pattern play a role in the generation of COM oscillations.

5.3 Conclusions

It is quite reasonable that the bodies attached to the flapping foils have COM oscillations as the forces generated by the flapping foils are oscillatory in nature. These COM oscillations can be minimized by optimizing the operating parameters along with the fin stiffness. This study proposes optimal operating conditions and design thumb rules that would aid in the development of aerial and underwater robotic vehicles.

Theoretical optimization predictions which minimize $h_{SPS}$ showed that the COM oscillations can be reduced by choosing flexible fins, lower pitching and heaving amplitudes, and higher oscillation frequencies for a given SPS. This could explain why fish increase the oscillation frequency instead of heaving amplitude to swim at faster speeds. Theoretical predictions which minimize $h_{Lift}$ produced lower COT values but at the cost of slight increase in $h_{SPS}$ values. The optimal parameters which minimize $h_{SPS}$ and $h_{Lift}$ tend to increase COT and hence a compromise has to be made in choosing EI and $\Omega$ combination which dictates COM oscillations and COT. A bi-objective problem has been defined which minimizes both $h_{SPS}$ and COT, and the predicted Pareto fronts for this bi-objective problem have shown that larger optimal values of EI and $\Omega$ are required for increased $U_{Mean}$ values.

Experiments performed using three fins of different EI validated the theoretical predictions that flexible fins operating at larger frequencies produce lower COM oscillations compared to stiffer fins operating at lower frequencies for a given mean thrust/SPS. DIC has been used to measure the 3D deformations of the flapping fins and a comparison of the deformations demonstrated that the combination of deformation pattern and trailing edge amplitude play a role in the generation of COM oscillations. The fins having deformations with larger curvatures and lower trailing edge amplitudes produced lower COM oscillations. These larger curvatures and low trailing edge amplitudes can be obtained by using flexible fins operating at lower heaving amplitudes and higher frequencies.
It is well known that flexibility of the fin improves propulsive performance and this investigation demonstrated that the fin flexibility also minimizes COM oscillations. This study provides guidelines that aid the design of unmanned aerial and underwater vehicles to minimize COM oscillations as well as some insight into how fish might control or minimize COM oscillations during locomotion.
Chapter 6

6 Optimal chordwise stiffness profiles

The calculated $U_{\text{Mean}}$ and COT are used to optimize the $EI$ variation along the length of the fin. The objective of the optimization problem is to predict the optimal stiffness ($EI$) profile which minimizes $1/U_{\text{Mean}}$ which is maximizing $U_{\text{Mean}}$ for given operating conditions. The $EI$ variation along the length of the fin is approximated by a 4th order polynomial and the optimized variables are the coefficients ($T_i, i = 0, 4$) of this polynomial.

$$EI(x) = T_4x^4 + T_3x^3 + T_2x^2 + T_1x + T_0$$ (5.2)

A global search algorithm DIRECT acronym for DIviding RECTangles is employed to predict the optimal $EI$ profile. DIRECT is a sampling algorithm which does not require objective function gradient and converges to the global minimum at the expense of exhaustive search over the domain. A detailed description of the theory and algorithm is given in ref.[175].

The problem of maximizing mean SPS and minimizing COT becomes a bi-objective optimization problem which minimizes $1/U_{\text{Mean}}$ and COT. The Pareto fronts between the two objectives are generated using $\epsilon$-constraint method.

6.1 EI Profile Optimization

Optimization studies have been carried out to predict the optimal EI profile which maximizes SPS for different operating conditions. The optimized variables are the coefficients of the 4th order polynomial while the other parameters such as oscillation frequency, heaving and pitching amplitudes are maintained at a constant value. The figure 6.1 shows the optimal variation of EI in log scale with normalized chord for three different pitching amplitudes 4°, 8° and 12° degrees for...
maximized SPS. The heaving amplitude $H / c$ and operating frequency $\Omega$ are chosen as 0.1 and 1Hz respectively. The stiffness is higher at the leading edge and gradually reduces towards the trailing edge. These are similar to the spanwise tension profiles predicted by Walker [129] for a membrane wing. Comparing the three profiles for different pitching amplitudes, it can be seen that the optimal stiffness increases with increase in pitching amplitude due to the higher hydrodynamic forces acting on the fin. The optimal $U_{\text{Mean}}$ values are 3.4, 4 and 4.5BL/s for pitching amplitudes of $4^0$, $8^0$ and $12^0$ degrees respectively.

![Figure 6.1. Optimal EI variation along the length of the fin for different pitching amplitudes; $H / c = 0.1, \Omega = 1Hz$](image)

The figure 6.2 shows the optimal EI profiles for different oscillation frequencies operated at $H / c = 0.1$ and $\alpha = 4^0$. The profiles trend remains the same as that observed for pitching amplitude cases seen in figure 6.1. The corresponding optimal $U_{\text{Mean}}$ values are 1.5, 3.4 and 7.5BL/s for frequencies of 0.5, 1 and 2Hz respectively. The figure 6.3 show the optimal EI profiles for different heaving amplitudes operated at $\Omega = 1Hz$ and $\alpha = 8^0$. The corresponding optimal $U_{\text{Mean}}$ values are
4 and 5.2BL/s for $H/c$ values of 0.1 and 0.2 respectively. These predicted profiles validate the conclusions drawn by Shoele & Zhu [130] that the fins with stiffer leading edge produces better performance over a broad range of operating parameters. A stiff leading edge reduces the effective angle of attack leading to attached flow and gradual reduction of the stiffness towards the trailing edge increases the bending of the fin redirecting the forces towards the swimming direction and thus increasing thrust. These stiffness profile predictions also explain why the fish caudal fin rays gradually decrease in thickness towards the trailing edge and the grouping of the fin rays at the caudal peduncle [169].

Figure 6.2. Optimal EI variation along the length of the fin for different frequencies; $H/c = 0.1, \alpha = 0.4$
Figure 6.3. Optimal EI variation along the length of the fin for different heaving amplitudes; 
\[ \alpha = 8^0, \Omega = 1\text{Hz} \]

The SPS and COT are inversely proportional and maximizing the SPS comes at the cost of higher COT values making it a bi-objective problem. So, it is a compromise between SPS and COT and the Pareto fronts between \(1/U_{\text{Mean}}\) and COT are plotted and shown in figure 6.4 in objective and variable spaces respectively. A comparison of the plots show that for a given SPS, operating at lower heaving amplitudes and frequencies produce lower COT values. As an example, although the fin can be operated at different combinations of \(\Omega\) and \(H/c\) to propel at 3BL/s, the fin operating at \(\Omega = 1\text{Hz}\) and \(H/c = 0.1\) produces lower COT value of around 7.5 compared to other cases.
Figure 6.4. Pareto fronts in (a) objective space and (b) variable space.
The variation of EI profiles along the Pareto front for operating parameters $\Omega = 1Hz$ and $H/c = 0.1$ is shown in figure 6.5. As seen from the figure 6.5, higher values of EI are required to propel at higher SPS and to reduce COT, the fin stiffness should be on a lower side. This is the reason why fish increases its fin stiffness to operate at higher speeds and reduces the stiffness to operate at higher efficiencies (lower COT).

![Optimal EI profiles along the Pareto front for $H/c = 0.1, \Omega = 1Hz$](image)

Figure 6.5. Optimal EI profiles along the Pareto front for $H/c = 0.1, \Omega = 1Hz$

### 6.2 Experimental results

Experiments have been performed to compare the predicted profiles on real fish and robotic fins. The experimental setup used to measure EI variation of fish fins is discussed in section 3.3.

#### 6.2.1 Fish caudal fins

Three fish caudal fins (Trout, Vermilion snapper and Red snapper) of different shapes which represent carangiform swimming motion (tail dominant propulsion) are bought from the local fish
market to test the EI variation along their length. The caudal fins are shown in figure 6.6 and a speckled fin is shown in figure 6.7. As the fin gets stiffer with dried up speckle paint, the measurements are made when the paint is wet.

Figure 6.6. Fish caudal fins of different shapes and sizes

Figure 6.7. Speckled Vermilion snapper

The passive stiffness variation along the length of three different caudal fins is measured by employing the experimental procedure discussed in section 3.3. The DIC captured deformation of the Red snapper is shown in figure 6.8. The centerline deformation of the fins is calculated from
the 3D DIC plots and normalized with chord length to have all the fin deformations on same scale. The centerline deformations are shown in figure 6.9.

Figure 6.8. DIC Surface plot of caudal fin deformation
The normal force values measured by the transducer to generate corresponding deformations is shown in the table 6.1.

<table>
<thead>
<tr>
<th>Fin</th>
<th>Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trout</td>
<td>0.2185</td>
</tr>
<tr>
<td>Vermilion Snapper</td>
<td>0.8145</td>
</tr>
<tr>
<td>Red Snapper</td>
<td>1.2462</td>
</tr>
</tbody>
</table>

The EI variation along the length of the fin is calculated from the Eq.(3.1) and plotted as shown in the figure 6.10.
Figure 6.10. EI variation along the length of different caudal fins

The fish fin stiffness profiles are similar to the optimized profiles predicted by the theory with higher values at the leading edge that gradually reduce towards the trailing edge. The magnitudes of theoretical stiffness predictions are higher compared to the actual fish fins and this is due to the simplification of the fish fins by Euler Bernoulli beam kinematics and the surrounding fluid by 2D panel method. Although the magnitudes are different, the optimal profiles predicted by the theory closely resemble the fish fin stiffness profiles and act as a very good model to develop design principles. A discussion on this simplification of the theoretical model is given in ref.[153]. Also, the experiments performed on the fish fins measure only the passive stiffness and the fish in real environment actively modulate their stiffness which might be higher than the measured values. Although the real fin stiffness profiles and predicted profiles are similar, it is not appropriate to validate the predicted results by comparison as the fish fins have evolved over millions of years and maximizing thrust is one of the many objectives whereas the theoretical profiles are predicted only with the objective of maximizing the thrust.
6.2.2 Robotic fins

To translate the theoretical studies into robotic environment, fins with varying stiffness have been fabricated and tested in the water tunnel. The carbon fiber reinforced composite fins have been fabricated with varying stiffness along their length by controlling the number of layers in the layup from root to the tip as shown in figure 6.11. The fin in figure 6.11 has 10 layers at the root and 2 layer at the tip with decrement of 2 layers along its length.

![Carbon fiber reinforced composite fin with 10 layers at the root and 2 layers at the tip](image)

Figure 6.11. Carbon fiber reinforced composite fin with 10 layers at the root and 2 layers at the tip

The bending stiffness of different layers has been measured using four point bending tests on Advanced Machine Technology (ADMET) machine with the ASTM standards. The bending stiffness of the CFRC samples with number of layers is shown in figure 6.12(a).

Six different fins with layers 10,8,6,5,4,3 at the root and 2 or 1 layers at the tip are fabricated along with six other fins with constant number of layers 10,8,6,5,4,3. As the stiffness of different number of layers is known, the stiffness variation along the length of the fins is plotted in figure 6.12(b).
Figure 6.12. EI variation of (a) CFRC layers, (b) CFRC fins along their length
The fins are operated at a heaving amplitude of 20mm for all the cases discussed in this section. A performance comparison of the varying stiffness fins (termed VS) with constant stiffness fins (termed CS) at three different frequencies 0.5, 1 and 1.5Hz is shown in figure 6.13. The performance parameters thrust and propulsive efficiency ($\eta_p$) are plotted against leading edge stiffness which varies linearly towards trailing edge for varying stiffness fins and remains constant for constant stiffness fins (see figure 6.12(b)). It can be observed that the profiled fins perform better compared to constant stiffness fins at all the frequencies considered here. Also notice that the flexible fins (with lower leading edge stiffness) produce higher efficiencies when operated at 0.5Hz and there exists an optimal stiffness that produce higher $\eta_p$ for fins operating at 1Hz and 1.5Hz.
The better performance of profiled fins can be explained by comparing the deformation patterns and so DIC has been used to capture 3D deformation of flapping fins submerged under water. The 3D deformation of 6 layered varying stiffness fin is shown in figure 6.14(a). The centerline deformations are extracted from the 3D plots and a comparison of varying and constant 6 layered fins is shown in figure 6.14(b). The solid lines represent the constant stiffness fin and the dashed lines represent the deformation of the varying stiffness fin. As seen from figure 6.14(b), the profiled fin generates a larger trailing edge amplitude which increases the thrust generation [154]. Also, notice that the profiled fin generates larger curvatures compared to constant stiffness fin which redirect the forces acting on the fin towards the moving direction thus increasing thrust.
In summary, it is advantageous to use the profiled fins over constant EI fins to attain higher propulsive performance in the design of aerial/underwater robotics which has been proved theoretically and experimentally.
6.3 Conclusions

The chordwise optimal stiffness profiles have been predicted assuming a 4\textsuperscript{th} order polynomial for different operating conditions of pitching and heaving amplitudes and operating frequencies. The profiles were almost similar for all the cases but with different magnitudes. The profiles with higher magnitudes produced larger SPS values compared to the lower magnitude profiles which produced lower COT values. The generated Pareto fronts have shown that for a given SPS, operating at lower heaving amplitudes and frequencies produce lower COT values. These studies indicate that higher values of EI are required to propel at higher SPS and to reduce COT, the fin stiffness should be on a lower side. This is the reason why fish increases its fin stiffness to operate at higher speeds and reduces it to operate at higher efficiencies (lower COT).

Experiments on the real fish caudal fins have been performed to measure the EI variation along their length using cantilever bending tests. The measured chordwise EI profiles of the fish fins are similar to the theoretical predictions along their length. However, the predicted values are higher than the measured EI values and this discrepancy is due to the simplified model of Euler Bernoulli beam theory coupled with 2D unsteady hydrodynamics.

To translate the theoretical predictions into the robotic environment, fins with varying stiffness have been fabricated using carbon fiber reinforced composites and tested in the water tunnel. A comparison of the performance in terms of thrust and efficiency show that varying EI fins perform better over constant EI fins for all the three frequencies tested in the water tunnel. DIC has been used to measure the out of plane deformation of the flapping fins under water and compare the deformation patterns of constant and varying EI fins. Fins with varying EI have large curvatures and trailing edge deflection redirecting the forces acting on the fin towards the swimming direction, thus increasing thrust.

This theoretical and experimental investigation prove that propulsive performance can be improved and optimized by using profiled fins over constant stiffness fins. The optimal stiffness profiles predicted by the theoretical studies resemble that of real fish fins and provide an insight about the evolution of fish fins for optimal performance.
Chapter 7

7  Enhanced performance using MFCs

7.1  Analysis Results

The properties of the MFCs used in the simulations are taken from Smart Material Corp [178]. The values of \( H / c = 0.1 \& 0.2 \) are chosen for all the simulations as used by Lauder et al.[96, 169] working with undulatory locomotion in fishes.

7.1.1  Distributed actuation

It is believed that fish modulates its body and fin stiffness or deformation pattern by distributed actuation along its length and the advantages of it are discussed in this section. The passive stiffness of the fin will be higher at the location of the MFCs and the effect of the epoxy used to attach the MFC to the fin is neglected in the analysis. A comparison of the hydrodynamic performance for different MFC actuation configurations is given in table 7.1. The MFCs are numbered as 1, 2 and 3 from root to the tip as shown in figure 2.2. The MFCs are actuated with a sinusoidal voltage with a magnitude of 1500V and 0\(^{0}\) phase while the fin is operated at 1Hz with heaving amplitude of 0.1 \( c \) and the free stream velocity is 0.4m/s. A pronounced increase in mean thrust and hydrodynamic efficiency can be observed with MFC actuation compared to the passive configuration where no MFCs are actuated. Moreover, the MFC close to the root (MFC 1) produce maximum improvement compared to the MFCs at other locations and all the MFCs together produce maximum thrust compared to any other combinations.
As stated earlier, along with the voltage magnitude, phase difference between heaving and voltage has a significant effect on propulsive performance and a brief comparison for different combinations of phase values is given in table 7.2. For this particular fin, maximum enhancement is obtained by operating all the MFCs at 0° phase when compared to the other combinations. An important note here is that the propulsive performance reduces for certain values of phase depending on phase configuration. This is similar to the results in table 7.1, where different combinations of voltage magnitude can be employed to modify the passive deformation and enhance the propulsive performance. While this study highlights the importance of voltage magnitude and phase as well as the benefits of active fin control, it does not provide an understanding into relationship between fin stiffness, voltage magnitude, and phase for enhanced propulsive performance. Therefore a detailed discussion on these effects is provided in the following sections using a single pair of antagonistic MFC actuators.
Table 7.2. Effect of phase on hydrodynamic performance

<table>
<thead>
<tr>
<th>MFC Voltage Phase (degree)</th>
<th>Thrust (N)</th>
<th>Hydrodynamic Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>MFC 1  MFC 2  MFC 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0    0    0</td>
<td>0.3111</td>
<td>0.2547</td>
</tr>
<tr>
<td>90   90   90</td>
<td>0.1893</td>
<td>0.1561</td>
</tr>
<tr>
<td>180  180  180</td>
<td>0.0127</td>
<td>0.0151</td>
</tr>
<tr>
<td>0    90   180</td>
<td>0.1857</td>
<td>0.1831</td>
</tr>
<tr>
<td>90   180  0</td>
<td>0.1689</td>
<td>0.15</td>
</tr>
<tr>
<td>180  0    90</td>
<td>0.1245</td>
<td>0.1136</td>
</tr>
<tr>
<td>0    180  90</td>
<td>0.1852</td>
<td>0.1882</td>
</tr>
<tr>
<td>90   0    180</td>
<td>0.2091</td>
<td>0.1632</td>
</tr>
<tr>
<td>180  90   0</td>
<td>0.0873</td>
<td>0.0902</td>
</tr>
</tbody>
</table>

7.1.2 Phase Difference

The phase difference between the heaving and the applied voltage is an important parameter, which provides for the fin to change its apparent stiffness by aiding or opposing the passive deformation. Although distributed actuation produces maximum performance, only one pair of MFCs close the root are investigated for predicting the effects of phase on thrust and efficiency. This same configuration is used for the rest of the simulations and experimental studies. The variation of thrust with phase and voltage magnitude for two different stiffness values of $\bar{E} = 0.5$ and $\bar{E} = 12$ are shown in figure 7.1. As seen in figure 7.1, there is an optimal phase at which thrust is a maximum and this phase varies little with the voltage magnitude. However, the phase difference varies with $\bar{E}$, where the phase difference of a fin with $\bar{E} = 12$ is close to $54^0$ and of a fin with $\bar{E} = 0.5$ is close to $90^0$. 
Figure 7.1. Variation of mean thrust with phase and voltage magnitude, $\Omega = \pi$ a) $\bar{E} = 0.5$, b) $\bar{E} = 12$
The variation of \( \eta_p \) (efficiency) with phase and voltage magnitude for the stiffness values of \( \bar{E} = 0.5 \) and \( \bar{E} = 12 \) are shown in figure 7.2. As seen in figure 7.2, there is an optimal phase at which \( \eta_p \) is a maximum and similarly as thrust, it varies little with the voltage magnitude. However, the phase difference varies with \( \bar{E} \). The phase difference of a fin with \( \bar{E} = 0.5 \) is close to 72° and is close to 36° for a fin with \( \bar{E} = 12 \). From figures 7.1 and 7.2, it can be observed that the optimal phase difference for efficiency lags behind the optimal phase difference for thrust.
Figure 7.2. Variation of hydrodynamic efficiency with phase and voltage magnitude, $\varphi = \pi$ a) $\bar{E} = 0.5$, b) $\bar{E} = 12$

The optimized values of the phase difference for different $\bar{E}$ values are obtained by using the MATLAB optimization routine *fminsearch*. The variation of optimal phase difference for thrust with $\bar{E}$ for two different heaving amplitudes and the corresponding thrust values are shown in figure 7.3, and the variation of optimal phase difference for efficiency with $\bar{E}$ for two different heaving amplitudes and the corresponding efficiency values are shown in figure 7.4. As seen from the figure 7.3, at lower values of stiffness, the optimal phase is about $90^\circ$ and as the stiffness increases, the optimal phase reduces to $0^\circ$. A phase of $90^\circ$ for lower $\bar{E}$ values actually increases their apparent stiffness by opposing the passive deformation, thus producing larger thrusts. However, when the stiffness becomes large and approaches that of a rigid fin, the fins produce lower thrusts under pure heaving. This can be a reason for the reduction in the optimal phase difference to zero thus reducing the apparent stiffness of the fin by aiding the passive deformation.
Figure 7.3. Optimal phase difference for thrust with $\bar{E}$ and the corresponding thrust, $\Omega = \pi$
Figure 7.4. Optimal phase difference for efficiency with $\bar{E}$ and the corresponding efficiency, $\Omega = \pi$
As the optimal fin stiffness for efficiency is lower than the optimal fin stiffness for thrust [153], the optimal phase for efficiency lags behind the optimal phase for thrust as seen in figures 7.1 and 7.2. A phase variation of $5^\circ$-$10^\circ$ is seen with the increase in heaving amplitude from 0.1 to 0.2. However, large variation is seen between the optimal phase values for efficiency with the increase in heaving amplitude. This study demonstrates the importance of phase difference on propulsive performance and its variation with the stiffness of the fin.

### 7.2 Experimental Results and Comparison with Theory

The experiments were conducted on four different trapezoidal foils A, B, C and D and their configurations are given in table 7.3. The dimensions of the foils are shown in figure 7.5. The M4005-P1 and M5628-P1 MFC actuators from Smart Material Corp. [178] are used to control the deformation of the flexible fins. The MFCs are fixed on either side of the fin as an antagonistic pair. Although the MFCs are flexible, they are attached to the foils using epoxy which increases the stiffness at the bonding location. Fins A and B are fabricated using smaller MFCs (M4005-P1) attached to fins of total length of 180mm. Fins C and D with total length 150mm are fabricated using the larger MFCs (M5628-P1) and thus are stiffer than fins A and B. The size differences between the two types of actuators can be seen in figure 7.5.

<table>
<thead>
<tr>
<th>Fin</th>
<th>Length (mm)</th>
<th>Thickness (mm)</th>
<th>MFC Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>180</td>
<td>0.2</td>
<td>M4005-P1</td>
</tr>
<tr>
<td>B</td>
<td>180</td>
<td>0.4</td>
<td>M4005-P1</td>
</tr>
<tr>
<td>C</td>
<td>150</td>
<td>0.2</td>
<td>M5628-P1</td>
</tr>
<tr>
<td>D</td>
<td>150</td>
<td>0.4</td>
<td>M5628-P1</td>
</tr>
</tbody>
</table>

The operating voltage range of MFCs is -500V to 1500V. The MFC actuation voltage signal $V = V_0 \sin(\omega t + \phi)$ is generated using LabVIEW, converted to an analog signal using a NI DAQ6211, and amplified using a Trek high voltage amplifier. The foils are oscillated at 1Hz and
heaving amplitude of 20mm. The free stream velocity of the water is 0.05m/s and the frequency of the voltage $\omega$ is chosen to be the same as that of heaving frequency $\Omega$ which is 1Hz.

A set of experiments with fins A and B are performed to verify the improvement in propulsive performance as discussed in the previous section. The fins were actuated at a frequency of 1Hz (both heaving and voltage), heaving amplitude of 20mm and voltage of 1200V. The propulsive performance depends on the phase difference between heaving and applied voltage and the variation of instantaneous thrust ($F_x$) for a time period of two seconds is shown in figure 7.6. The
raw data obtained from the force transducer is filtered by using a low pass filter. As the fin is moved in −X direction, the negative value of the force $F_x$ indicates thrust.

It can be observed from figures 7.6(a) and (b) that the generated mean thrust forces are maximum at the phase difference of ~90° when compared to the values at 0° and 180°. The non-dimensional stiffnesses of the fins A and B are 0.0128 and 0.2 respectively, which represent the flexible foils. At 90° phase, the deformation caused by the MFCs oppose the passive deformation of the fin thus increasing the apparent stiffness to produce larger thrusts.

The mean thrust forces and the propulsive efficiency calculated from equation (2.29) are tabulated in table 7.4. From the values it can be observed that the thrust and efficiency are a maximum at a phase difference of ~90° for both the fins. Fin A produces larger efficiencies compared to fin B due to its higher flexibility value. The lower numerical values of efficiencies are due to the lower free stream velocity/forward velocity, which appears in the numerator of equation (2.29). These experiments demonstrate that MFCs can enhance the propulsive performance and that the thrust and efficiency is greater at 90° for the flexible fins.
Another set of experiments are performed using the fins C and D over a frequency range of 0.4 – 1.6Hz and voltage range of 0 – 1200V to study the effect of MFCs on propulsive performance. The optimal phase for fins C and D is determined to be 0° and therefore a constant phase difference.
of $0^\circ$ is maintained for all the tests performed on these fins. Note that the heaving frequency and the voltage frequency are kept same to maintain constant phase of $0^\circ$ between heaving and voltage. Figures 7.7(a) and (b) shows the experimental and theoretical thrust contours of fin C whereas figures 7.7(c) and (d) shows the experimental and theoretical thrust contours of fin D with voltage and oscillation frequency. As seen from the figure 7.7(a) in the frequency range of $0.8 - 1.2$Hz, there is an optimal voltage at which the thrust is a maximum. This optimal voltage corresponds to the optimal deformation or stiffness of the fin which produces maximum thrust. In all the figures 7.7(a) to (d), maximum thrust is produced at higher frequencies and higher actuation voltages. However, the effect of the MFCs is not as significant at higher frequencies in the experimental results. The fin D produced lower thrusts in both experimental and theoretical results compared to fin C which is 0.2mm thick. The magnitude of theoretical thrust values is higher compared to the experimental values and this discrepancy is primarily attributed to the simplification of the coupled system with 2-D hydrodynamic model, beam theory and the linear piezoelectric model. However, a good comparison of thrust variation can be observed between the experimental and theoretical results.

![Plot](https://via.placeholder.com/150)

**a)**

![Plot](https://via.placeholder.com/150)

**b)**
Figure 7.7. Thrust contours for heaving amplitude of 20mm and $U_\infty = 0.05\text{m/s}$ (a) fin C – Experimental, (b) fin C – Theoretical (c) fin D – Experimental, (d) fin D – Theoretical
Figure 7.8. Efficiency contours for heaving amplitude of 20mm and $U_\infty = 0.05\text{m/s}$ (a) fin C – Experimental, (b) fin C – Theoretical (c) fin D– Experimental, (d) fin D – Theoretical

Figure 7.8(a) and (b) shows the experimental and theoretical hydrodynamic efficiency contours of fin C whereas figure 7.8(c) and (d) shows the experimental and theoretical hydrodynamic efficiency contours of fin D with voltage and oscillation frequency. From figure 7.8(a), it can be observed that the MFC actuation of fin C at 0.4Hz transformed the mean thrust from negative (drag) to positive which also generated high hydrodynamic efficiency of about 20% operated at 1200V. Similar result is seen in the theoretical tests (figure 7.8(b)) that the fin produced maximum efficiency of about 40% at 0.4Hz and 1200V. At higher frequencies, the effect of MFCs is less in the experimental results which might be due to the higher hydrodynamic loading acting on the fin at higher frequencies. However, a good comparison of location of maximum efficiency and its variation can be observed between the experimental and theoretical results. From figure 7.8(c), the experimental hydrodynamic efficiency is high at 1Hz which is also depicted in the theoretical results in figure 7.8(d).
Figure 7.9. Deformation comparison of the fin C at 0.4Hz for one cycle: solid lines – 0V, dotted lines – 1200V

The enhanced propulsive performance is achieved by modifying the passive deformation pattern and a comparison of the theoretical deformation of fin C operated at 0V (solid lines) and 1200V (dotted lines) and 0.4Hz frequency is shown in figure 7.9. Clearly, the deformation pattern and the trailing edge amplitude are slightly different for the same fin operated passively and by MFC. The deformation pattern and the trailing edge amplitude plays an important role in the generation of high propulsive performance [138, 154] and as seen in figure 7.9, the improvement in thrust and hydrodynamic efficiency can be attributed to the change in the deformation pattern and trailing edge amplitude. This study explores the advantages of active control of the oscillating fins using MFCs and provides some insight on how fish might modulate its stiffness and deformation pattern for achieving high propulsive performance.
7.3 Conclusions

Theoretical results show that distributed actuation of the fin produces significant enhancement of the propulsive performance over single actuators by proper selection of the phase difference between voltage and heaving and the voltage magnitude. Also the MFCs actuated close to the root have larger effect on the performance compared to the MFC actuation at other locations.

The enhancement of the performance depends on a particular phase value and this phase is not constant for all the fin stiffness values. The theoretical results predicted an optimal phase at which the propulsive performance is a maximum for a particular set of operating parameters and fin stiffness and the optimal phase does not vary with the voltage magnitude. Furthermore, at lower values of stiffness, the optimal phase is about $90^0$ and as the stiffness increases, the optimal phase reduces to $0^0$. Note that these optimal phase values are obtained for the fins operating in mode-1 (first mode of cantilever beam) and will vary depending on the mode shape of the deforming fin. This change in optimal phase is attributed to the change in apparent stiffness which aids or opposes the passive deformation. Both the thrust and efficiency improved with the voltage magnitude for the parameters considered in the analysis.

The experiments carried out on fins A and B reveal that for lower $\bar{E}$ values, the optimal phase is around $90^0$ as predicted by the theoretical results. The MFCs showed significant enhancement of performance in the low frequency zone where the hydrodynamic forces are relatively lower compared to the high frequency zone. The effect of MFCs is not significant at higher oscillation frequencies and using the actuators which can overcome high hydrodynamic loads would improve the performance at these frequencies and higher amplitudes. Also, the magnitude of theoretical thrust values is higher compared to the experimental values and this discrepancy is primarily attributed to the simplification of the coupled system with 2-D hydrodynamic model, beam theory and the linear piezoelectric model. However, a good comparison of thrust and efficiency variation can be observed between the experimental and theoretical results.
Chapter 8

8 Summary and Future Work

The intelligent use of muscular system by the fish to perform stable high acceleration maneuvers, braking and efficient hover is the source of inspiration to develop highly efficient propulsive mechanisms with applications related to aerial/under water robotics. Choosing the optimal fin stiffness for different operating regimes is a challenge. This research addresses some of the issues and provides design principles and guidelines to be followed in the development of flapping fin robotics. The salient points of this research are as follows.

1. The combined fin and joint stiffness determines the propulsive performance and there exists different optimal combinations of fin and joint stiffness values that maximize performance for different operating conditions. This emphasizes the advantage of active stiffness modulation of flapping fins to switch between different swimming/flying regimes.

2. The center of mass oscillations of the bodies propelled by the flapping fins can be reduced by choosing lower fin stiffness values, lower heaving & pitching amplitudes and higher frequencies to maintain a given self-propelled speed. But operating at higher frequencies reduces efficiency and so the design is a compromise between efficiency and COM oscillations.

3. The stiffness variation along the length of the fin has a significant impact on the propulsive performance and the fins with varying stiffness similar to the fish fins perform better compared to constant stiffness fins in terms of thrust and efficiency. It is highly recommended to use the varying stiffness fins over constant stiffness fins for the swimming/ flying robotic applications.
4. The propulsive performance of the flapping fins can be enhanced by controlling the passive deformation and the phase difference between MFC actuation and heaving has to be chosen properly to achieve better results.

The body interaction and fin-fin interactions have been ignored in the current research. The interaction of the fish tail/caudal fin with other fins such as pectoral, dorsal and ventral fins would alter the performance and is a problem of future research. The actuation of different fins independently and in tandem could be exploited for better performance over a range of operating conditions and the actuation phase, shape, amplitude and frequency of different fins should be well understood.

The fin stiffness profile optimization design space consisted of only chordwise bending stiffness and a beam model was used. The theoretical model can be calibrated by considering a case that do not compare well with the experimental data for better predictions of performance parameters. The hydroelastic model has room for further improvement with incorporation of spanwise bending stiffness and 3D fluid dynamic model. The optimized geometric stiffness profile along chord and spanwise directions would be ideal for the design of underwater robotics opening up the possibility for further research.

This study highlights the advantages of active modulation of the fin stiffness which is a challenge and only open loop control method is employed in the current research. A closed loop control method would need to be developed to reap the benefits of active stiffness modulation of flapping fins under different operating regimes.
Bibliography


