Performing Network Level Crash Evaluation Using Skid Resistance

Ross J. McCarthy

Thesis submitted to the faculty of the Virginia Polytechnic Institute and State University in partial fulfillment of the requirements for the degree of

Master of Science
In
Civil Engineering

Gerardo Flintsch
Edgar de León Izeppi
Kevin K. McGhee
Tony Parry

July 10, 2015
Blacksburg, Virginia

Keywords: Skid Resistance, Poisson, Poisson-Gamma, Negative Binomial, Safety Performance Function, Empirical Bayes

Copyright © 2015, Ross J. McCarthy
Performing Network Level Crash Evaluation Using Skid Resistance

Ross J. McCarthy

ABSTRACT

Evaluation of crash count data as a function of roadway characteristics allows Departments of Transportation to predict expected average crash risks in order to assist in identifying segments that could benefit from various treatments. Currently, the evaluation is performed using negative binomial regression, as a function of average annual daily traffic (AADT) and other variables.

For this thesis, a crash study was carried out for the interstate, primary and secondary routes, in the Salem District of Virginia. The data used in the study included the following information obtained from Virginia Department of Transportation (VDOT) records: 2010 to 2012 crash data, 2010 to 2012 AADT, and horizontal radius of curvature (CV). Additionally, tire-pavement friction or skid resistance was measured using a continuous friction measurement, fixed-slip device called a Grip Tester. In keeping with the current practice, negative binomial regression was used to relate the crash data to the AADT, skid resistance and CV. To determine which of the variables to include in the final models, the Akaike Information Criterion (AIC) and Log-Likelihood Ratio Tests were performed.

By mathematically combining the information acquired from the negative binomial regression models and the information contained in the crash counts, the parameters of each network’s true average crash risks were estimated using the Empirical Bayes (EB) approach. The new estimated average crash risks were then used to prioritize segments according to their expected crash reduction if a friction treatment were applied.
ACKNOWLEDGEMENTS

I want to thank the Virginia Tech Transportation Institute Faculty, which includes my advisor, Dr. Gerardo Flintsch, and Senior Research Associates Dr. Edgar de León Izeppi and Dr. Samer Katicha for all of their support and guidance. I would also like to acknowledge all participating parties at Virginia Tech and the Virginia Tech Transportation Institute for providing the necessary equipment and resources for collecting, processing and discussing the data for this project. In addition to Virginia Tech faculty, I want to thank all supporting faculty from the University of Nottingham, including but not limited to Dr. Tony Parry and Andrew Dawson. Lastly, I want to thank all participating members at the Virginia Department of Transportation for all of their support and guidance, including but not limited to, Associate Principal Research Scientist, Kevin McGhee, and Highway Safety Improvement Program Planning Manager, Stephen Read.
# Table of Contents

1. Introduction ...................................................................................................................................... 1  
   1.1. Problem Statement ......................................................................................................................... 1  
   1.2. Thesis Objectives .......................................................................................................................... 2  
   1.3. Significance .................................................................................................................................. 2  
   1.4. Scope and Overview ....................................................................................................................... 3  

2. Literature Review .............................................................................................................................. 4  
   2.1. Infrastructure Safety Management ................................................................................................. 4  
   2.2. Skid Resistance ............................................................................................................................. 5  
   2.3. Measurement of Skid Resistance .................................................................................................... 9  
   2.4. Related Studies ............................................................................................................................. 10  
      2.4.1. United Kingdom Skid Resistance Strategy ............................................................................... 11  
      2.4.2. Ontario Ministry of Transportation ......................................................................................... 11  
      2.4.3. University of Connecticut ....................................................................................................... 11  
      2.4.4. Texas Department of Transportation ....................................................................................... 12  
   2.5. Summary .................................................................................................................................... 12  

3. Data ................................................................................................................................................. 14  
   3.1. Data Collection ............................................................................................................................. 14  
   3.2. Data Processing ............................................................................................................................ 16  
   3.3. Distribution of Network Data ........................................................................................................ 16  
      3.3.1. Annual Average Daily Traffic ................................................................................................. 16  
      3.3.2. Crash Observations ................................................................................................................... 17  
      3.3.3. Horizontal Radius of Curvature ............................................................................................... 21  
      3.3.4. Skid Resistance ....................................................................................................................... 21  

4. Methodology ........................................................................................................................................ 23  
   4.1. Modeling Approach ....................................................................................................................... 23  
      4.1.1. Standard Poisson Model ........................................................................................................... 23  
      4.1.2. Negative Binomial Model ........................................................................................................ 24  
      4.1.3. Modeling Technique Selection ................................................................................................. 27  
      4.1.4. Influential Variable Selection ..................................................................................................... 28  
      4.1.5. Safety Performance Functions .................................................................................................. 26  
      4.1.6. Pearson Chi-Square Test .......................................................................................................... 27  
      4.1.7. Dispersion Parameter ................................................................................................................. 28  
      4.1.8. Akaike Information Criterion .................................................................................................... 28  
      4.1.9. Log-Likelihood Ratio Test .......................................................................................................... 29  
   4.2. High Crash Risk Location Identification ...................................................................................... 29  
      4.2.1. Empirical Bayes Method .......................................................................................................... 30  
      4.2.2. Prioritization of Locations for Improvement ........................................................................... 31  


5. Results and Discussion ................................................................. 32
   5.1. Safety Performance Functions ..................................................... 32
       5.1.1. Testing the Standard Poisson Distribution .................................. 32
       5.1.2. Negative Binomial Distribution .............................................. 33
       Interstate Routes ........................................................................... 34
       Primary Routes ........................................................................... 35
       Secondary Routes ......................................................................... 37
       Model Fit Verification .................................................................. 38
       Summary .................................................................................... 42
   5.2. High Crash Risk Locations .......................................................... 42
       5.2.1. Benefit Assessment ................................................................ 44
       5.2.2. Summary ............................................................................ 46

6. Summary, Conclusions and Recommendations .......................... 47
   6.1. Findings .................................................................................. 47
   6.2. Conclusions ............................................................................ 47
   6.3. Recommendations .................................................................... 48

References ....................................................................................... 49

Appendix A: Creating Figures Using Poisson and NB Regression .......... 54
List of Figures

Figure 2.1: Interaction of Forces on a Rotating Tire .......................................................... 6
Figure 2.2: Macro-texture versus Micro-texture................................................................. 7
Figure 2.3: Coefficient of Friction versus the Percentage of Tire Slip............................... 8
Figure 3.1: The Equipment Used to Collect Skid Resistance .............................................. 15
Figure 3.2: The Cumulative Distribution of Annual Average Daily Traffic for Each Network Category ...................................................................................................................... 17
Figure 3.3: The Distribution of the Observed Number of Crashes for the Interstate Network .... 18
Figure 3.4: The Distribution of the Observed Number of Crashes for the Primary Network ...... 18
Figure 3.5: The Distribution of the Observed Number of Crashes for the Secondary Network .. 19
Figure 3.6: Percentage of the Network with or without Crashes .......................................... 20
Figure 3.7: Percentage of Crash Sites with Observed Number of Crashes ............................. 20
Figure 3.8: The Cumulative Distribution of Horizontal Radius of Curvature for Interstate and Primary Routes .......................................................................................................................... 21
Figure 3.9: The Cumulative Distribution of Skid Resistance for Each Network Category ........ 22
Figure 4.1: Observed and Expected Crash Counts for Interstate Routes Using Poisson Regression; (a) Complete Detail (b) Detail for Crash Counts 5 to 11 ................................. 25
Figure 5.1: Observed and Expected Crash Count for Interstate Routes Using SPF Regression; (a) Complete Detail (b) Detail for Crash Counts 5 to 11.................................................. 39
Figure 5.2: Observed and Expected Crash Count for Primary Routes Using SPF Regression; (a) Complete Detail (b) Detail for Crash Counts 5 to 24................................. 40
Figure 5.3: Observed and Expected Crash Count for Secondary Routes Using SPF Regression; (a) Complete Detail (b) Detail for Crash Counts 5 to 17 ........................................ 41
Figure 5.4: A Complete High Risk Assessment of Crashes for I-81 North; (a) Complete Detail (b) Detail for Mile Post 163 to 173 ................................................................. 43
Figure 5.5: Comparing EB Estimates for GN Improvements for I-81 North ......................... 45
Figure 5.6: Comparing the Reduction of EB Estimates for I-81 North .................................. 46
Figure A.1: Cumulative Density Plot for Quantile Analysis ................................................. 54
List of Tables

Table 3.1: Miles of Virginia State Roadway Measured............................................................... 14
Table 3.2: Summary of 2010 to 2012 Crash Data for Virginia Salem District ...................... 15
Table 5.1: Chi-Square Test for Poisson Distribution................................................................. 32
Table 5.2: Parameter Estimates for Interstate Route Regression Models................................. 33
Table 5.3: Parameter Estimates for Primary Route Regression Models................................... 33
Table 5.4: Parameter Estimates for Secondary Route Regression Models............................... 34
Table 5.5: Akaike Information Criterion Test Results for Interstate Routes ............................. 34
Table 5.6: Log-Likelihood Ratio Test Results for Interstate Routes ........................................... 35
Table 5.7: Akaike Information Criterion Test Results for Primary Routes............................... 36
Table 5.8: Log-Likelihood Ratio Test Results for Primary Routes ............................................ 36
Table 5.9: Akaike Information Criterion Test Results for Secondary Routes............................ 37
Table 5.10: Log-Likelihood Ratio Test Results for Secondary Routes........................................ 37
1. INTRODUCTION

Infrastructure shapes and drives economies, from the fast movement of goods and people to the spread of ideas, and a key component in this is the level of service the transportation infrastructure is able to provide. As the transportation infrastructure ages, it deteriorates, resulting in reduced pavement performance, of which an important aspect includes the safety and comfort of its users. The safety performance of the pavement requires an effective methodology for measuring and maintaining properties of a network of roadways, in order to simplify the process of prioritizing locations with higher crash risk according to different manageable roadway properties.

When evaluating highway safety, safety analysts should consider how various road network properties interact to result in crashes. The interactions that lead to a crash include roadway (geometric design, texture, surface condition, etc.), human and vehicle factors. According to the National Highway Traffic Safety Administration (NHTSA) (1), skid resistance “is a key input for highway geometric design, as it is used in determining the adequacy of the minimum stopping sight distance, minimum horizontal radius, minimum radius of crest of vertical curves, and maximum super-elevation in horizontal curves.” Skid resistance, or friction, is a characteristic of the pavement surface that provides the driver with the ability to accelerate, brake, and steer the vehicle. Skid resistance is reduced as a consequence of aggregate texture loss due to polishing, surface contamination from water or pollution, roadway geometry (cross-slope, radius of curvature, etc.), and driver/vehicle characteristics (excessive speed, inadequate tire properties, etc.). At any point in time, low skid resistance may result in a driver losing control of a vehicle, resulting in a crash of random severity (property damage, injury or fatality). In particular, when a pavement surface is wet, the lubricating effect of water can result in a reduction in the amount of friction from that which is available when the pavement surface is dry.

1.1. PROBLEM STATEMENT

In the U.S, the Federal Highway Administration (FHWA) currently assesses the safety performance of a roadway based on its estimated average crash frequency, as a function of the annual average daily traffic (AADT) and the length of the road segments (2). However, the models do allow for additional variables to be included in the evaluation. As part of a highway safety improvement initiative to reduce friction-related accidents, the FHWA issues guidance to the Departments of Transportation (DOTs) to identify locations that have an “elevated wet-weather crash” risk, using three possible approaches (3). A common approach is for DOTs to compute a “wet crash ratio (WCR),” where the number of wet crashes is divided by the number of total (wet and dry) crashes. For the first approach using WCR, a DOT compares their computed WCR to a specified value (0.25 to 0.50). If that value is exceeded, the location is deemed as having an “elevated wet-weather crash” risk (3). A second approach compares the WCR of a highway location to an average WCR for locations of similar design characteristics. If the WCR of the location exceeds the average WCR by a DOT specified percentage, the location is deemed to have an “elevated wet-weather crash” risk (3). For the third approach, instead of computing a WCR, “a minimum number of wet-weather or total crashes within a road segment” of a specific type (i.e., rural or urban) and length (generally between 0.2 to 2.0 mile) is chosen as...
a criterion. If this number is reached or exceeded, the location is deemed to have an “elevated wet-weather crash” risk (3).

Once a location is identified as having an “elevated wet-weather crash” risk, friction testing is performed to determine the amount of available skid resistance. If the measured skid resistance is below a specified amount, it is considered to be a contributing factor, and further investigation is conducted to determine possible remedies (3). In Virginia, as part of a Wet Accident Reduction Program (WARP), the Virginia Department of Transportation (VDOT) refers to locations having “elevated wet-weather crash” risks as Potential Wet Accident Hot Spots (PWAHS) (4). VDOT tests the friction at a PWAHS using a locked-wheel skid tester. The locked-wheel skid tester reports skid resistance as the coefficient of friction multiplied by 100, resulting in measurements that typically range from 0 to 100 (5). If the friction measurement is below 20, then skid resistance is considered a contributing factor (4).

Several problems arise from the current practice. First, it may be limiting if the FHWA analyzes crashes using only AADT and section length. Second, in Virginia, locked-wheel skid testers are not used to measure skid resistance as part of an annual, routine, network level survey. Instead, they’re used as an investigative tool in response to identified PWAHS (4). There may be potential for improvement to current crash analysis if skid resistance would be measured routinely and used as an additional factor in the Department of Transportation (DOT) crash analysis models.

1.2. THESIS OBJECTIVES

The broad scope of this research is to establish a methodology of collecting and evaluating network-level skid resistance data, with an understanding of its potential effect on crash risk. The specific objectives of this thesis are:

1. Determine whether consideration of skid resistance can improve the current practice for evaluating crashes.
2. Propose a methodology of ranking road segments according to their expected crash risk.

1.3. SIGNIFICANCE

Various characteristics of the roadway (e.g., AADT, surface properties, road geometry) can influence driver safety. More specifically, the amount of achievable grip at the tire-pavement contact surface is reliant on the amount of skid resistance. Skid resistance has been shown to vary with different tire properties, pavement properties, and pavement functional condition. This includes, but is not limited to, pavement surface condition (i.e. wet or dry), surface texture, speed, and tire and tread condition. However, the characteristic of the pavement surface that is directly responsible for the skid resistance is the pavement surface texture. Currently, DOTs screen their road networks for segments of roadway with higher risks of crashes, based on past crash history and AADT, and on segment length. The goal of this study is to show that collecting surface friction data can be beneficial to routine network pavement management, allowing better comprehension of the direct impact of skid resistance on safety, and a proactive approach to prioritizing locations of a network for skid resistance improvements based on the greatest reduction in crash risk.
1.4. **Scope and Overview**

This thesis is organized into five chapters. Chapter 1, *Introduction*, establishes the purpose of the thesis, including the *Problem Statement*, the *Objectives*, its *Significance*, and its *Scope and Overview*.

Chapter 2, *Literature Review*, explains the key concepts of skid resistance and its fundamental importance to road safety performance. It begins with a brief history of network-level road safety management in the U.S. The characteristics and terminology of skid resistance are discussed. State-of-the-art methods for measuring skid resistance are listed. The results of four studies examining skid resistance are briefly summarized.

Chapter 3, *Data*, explains how all of the data was collected and processed, and how it’s distributed for each network category. It lists the types of data obtained from VDOT records, and the steps taken to process the data into segments. It describes the equipment used to measure skid resistance and the approach used to process the skid data.

Chapter 4, *Methodology*, discusses how to model the processed data, and identify locations with high crash risk. This chapter discusses the types of regression models to use, followed by the goodness-of-fit tests to use. It explains how to identify high crash locations along a network using the Empirical Bayes (EB) approach.

Chapter 5, *Results and Discussion*, discusses the results of the analysis discussed in Chapter 4, and in the same respective order.

Chapter 6, *Conclusions and Recommendations*, briefly outlines the major findings from Chapter 5, and addresses the questions in the *Thesis Objective*. Recommended changes and work are explained.
2. LITERATURE REVIEW

2.1. INFRASTRUCTURE SAFETY MANAGEMENT

Every day, crashes occur on the National Highway System, which result in property damage, injury, or a fatality. In 2011, there were more than 5.3 million crashes (1). Of these crashes, 1.5 million resulted in injury, and 29.7 thousand in death (1). In the U.S., “motor vehicle crashes remain the leading cause of death among ages 5 to 34” (6). In addition to these physical damages, crashes also have a profound economic impact. In 2012, the U.S. Department of Transportation claimed that the cost of losing one life is equivalent to $9.1 million (7). In order to reduce the physical and economic repercussions of crashes, it’s imperative that state DOTs be encouraged to construct and manage highway safety guidelines that abide by the policies set forth by the federal government.

In the U.S., the history of implementing safety in transportation policy has evolved considerably since it was first pursued with the 1966 Highway Safety Act. In 1966, the U.S. Congress enacted the Highway Safety Act which promulgated eighteen uniform guidelines (standards) with the purpose of reducing traffic accidents (8). The Act required each state to develop a series of safety programs, which included the development of statewide systems to log traffic accidents. States were also required to investigate the cause of the accidents, in order to receive appropriate Federal funding for the application of corrective measures. As time progressed, new improvement acts were passed. Following the 1966 Highway Safety Act, each subsequent act improved upon the prior, with one such change being the establishment of new safety improvement program areas. In 1978, one of these program areas encouraged State DOTs to develop systems of identifying hazardous locations through use of crash record systems, including but not limited to The Hazard Elimination Program established via the Surface Transportation Assistance Act of 1978, which included improvement projects such as pavement grooving and skid-resistance overlays (9).

In 1991, Congress passed the Intermodal Surface Transportation Efficiency Act, which instituted new safety guidelines, one of which required State DOTs to develop, establish, and implement pavement, bridge, and safety management systems (10). This helped in making cost effective maintenance decisions for the road network (10). In 1998, the Transportation Equity Act for the 21st Century was enacted, which incorporated “safety and security of the transportation system for motorized and non-motorized users at the metropolitan and statewide level” (11).

Seven years after the enactment of the Transportation Equity Act, the drive to improve infrastructure safety led legislators to increase funding for state-run improvement programs. A new “core federal-aid program,” entitled the Highway Safety Improvement Program (HSIP), was established by the Safe, Accountable, Flexible, Efficient Transportation Equity Act – A Legacy for Users (SAFETEA-LU). The HSIP doubled the previous infrastructure safety funding, and required state-wide “data-driven, performance-based” programs with goals of reducing traffic related fatalities and serious injuries on state managed roads (9). For states to receive infrastructure funding, they had to submit annual reports that described at least five percent of potentially hazardous locations, listing potential “remedies, costs, and impediments” that might resolve the safety concerns (12).
State HSIPs needed to follow a three step process of planning, implementation, and evaluation, with each step driven by Strategic Highway Safety Plans (SHSP). In the planning phase, locations that were potentially higher safety concerns were identified and prioritized. During the implementation phase, the high priority locations identified in the planning phase were considered for scheduling and implementation of maintenance and repair projects. Following this, the maintenance and repair projects underwent performance evaluations to assess their abilities to effectively resolve the safety concerns associated with each high priority location (9). The establishment of HSIPs marked a turning point for legislative control over U.S. infrastructural safety issues. However, in 2012, the American Traffic Safety Services Association (ATSSA) developed a conceptual strategy known as Toward Zero Deaths: A National Strategy on Highway Safety (TZD), which posited a long-term goal of gradually reducing all crash-related fatalities (approximately 43,000 per year) on all U.S. roadways (13).

The concept of TZD was derived from a similar policy, called Vision Zero which was originally implemented in Sweden, and adopted by several other European countries. In 2012, the Center for Excellence in Rural Safety (CERS) identified thirty states that indicated some level of commitment to developing SHSPs (14). Each of these committed states defined target goals for the reduction of crash related fatalities, which they planned to reach via TZD. In 2006, one of Virginia’s SHSPs included the pursuance of TZD, with a goal of reducing “deaths and severe injuries by half by 2030” (15). In 2011, using statewide crash data from 2006 to 2008, the Louisiana DOT also committed to TZD, with a similar goal of reducing fatality and severe injury crashes by fifty percent by 2030 (16).

In 2012, the Moving Ahead for Progress in the 21st Century Act (MAP-21) changed project funding to include the development of performance- and outcome-driven goals (17). MAP-21 constructed several new “formula” funding programs, one of which was called Transportation Alternatives (TA), which incorporated, improved upon and funded pre-existing core highway programs (17). These programs included the HSIPs, the National Highway Performance Program (NHPP), Surface Transportation Program (STP), Congestion Mitigation and Air Quality Improvement Program (CMAQ), and Metropolitan Planning. To improve the HSIP, one of the performance goals was to enhance the safety of the infrastructure by setting severe injury and fatality crash-reduction standards based on crash rates, or the probability of these occurrences per set number of vehicle miles travels (17). Another HSIP improvement extended the infrastructure funding to include state-wide and tribal-owned lands (18).

2.2. Skid Resistance

As a tire travels over a pavement surface, a force called tire-pavement friction develops at the contacting surfaces, hindering the directional motion of the tire (19). The degree of tire-pavement friction is quantitatively measured using a dimensionless quantity called the coefficient of friction, $\mu$. Expressed in Equation 2.1, $\mu$ is the ratio of the friction force tangent to the contact surfaces ($F_F$) over the normal force ($F_N$) (20). A visual illustration of these forces is shown in Figure 2.1.
The amount of available tire-pavement friction is dependent on properties of the vehicle tire, the pavement surface and the pavement operational conditions. The contribution of the pavement surface to tire-pavement friction or skid resistance is attributed to the characteristics of the surface texture. The key constituents of surface texture necessary for the development of skid resistance are micro- and macro-texture (Figure 2.2) (21). The macro-texture is dependent on the size and shape of the aggregate, and assists in providing channels for water to flow as the tire and the pavement come into contact (21). The wavelengths of macro-texture typically range from 0.5 mm to about 50 mm (22). Meanwhile, the micro-texture defines the texture along the surface of the aggregate (21), with wavelengths of 1 µm to 0.5 mm (22).
The complex interaction resulting from the behavioral responses of the tire rubber in contact with the pavement surface give rise to skid resistance. Resulting from a molecular-kinetic thermal process in the tire, the rubber both shears and deforms against the texture of the pavement surface allowing two fundamental force components of skid resistance to form: hysteresis and adhesion (20). As a tire slips over a pavement surface, adhesion forces occur due to molecular bonding between the tire rubber and the micro-texture (19). Simultaneously, as the tire slips over the pavement surface, the macro-texture aids by producing stresses that deform the tire rubber through the storing and recovering of strain energy in the rubber tread. Because of the viscoelastic behavior of the rubber in the tire, as the tire relaxes, not all of the strain energy is recovered, resulting in losses in the form of heat, a process also referred to as hysteresis, which is converted into friction (20).

As drivers maneuver their vehicles (i.e. braking, accelerating, or changing their vehicle’s direction of travel), tire-pavement friction is produced at the tire-pavement contact patch. In the situation where a driver applies the brakes, the relative difference between the peripheral speed of the tire and the velocity of the vehicle result in tire slipping over the pavement surface. Literature commonly refers to this slippage as (longitudinal) slip speed, $S$ which is the relative difference between the directional velocity of a vehicle, $V$, and the average peripheral velocity of the tire, $V_P$, during constant braking or free rolling (19).

$$S = V - V_P = V - (0.68 \times \omega \times r)$$

(2.2)

Where:
$S = \text{Slip speed (mph)}$

$V = \text{Vehicle velocity (mph)}$

$V_P = \text{Average peripheral velocity of the tire (mph)}$

$\omega = \text{Angular Velocity of the tire (rad/sec)}$

$r = \text{Tire radius (ft.)}$

While a tire is in a free rolling state (no applied brakes), $V$ is equal to $V_P$, such that $S$ is equal to zero. However, when the brakes are fully engaged, $V_P$ equals zero and $S$ equals $V$ (24). Figure 2.3 illustrates the process of slip speed as a result of applied braking which is also expressed as the percentage of slip calculated by taking the ratio of $S$ over $V$, multiplied by 100 (19). When a wheel is fully locked ($S$ equals $V$), the condition is referred to as 100 percent slip, but when the wheel is free rolling, the condition is referred to as zero percent slip (24). During the transition from free rolling to fully locked, the slip speed initially increases to a maximum point, called critical slip (between 18 to 30 percent slip) (24), where skid resistance is at its peak, then decreases gradually until the tire is fully sliding (100 percent slip), where skid resistance is as much as half its peak value (19).

![Figure 2.3: Coefficient of Friction versus the Percentage of Tire Slip (24). Adapted from Flintsch, G.W., McGhee, K.K., and Najafi, S. The Little Book of Tire Pavement Friction, Volume 1. For Pavement Surface Consortium, 2012, Used under fair use, 2015.](image)

When traversing a tangent section of roadway, the available skid resistance will correspond to longitudinal slip speed. However, when a vehicle travels around a horizontal curve, or cross-slope effects, lateral friction forces develop at the tire-contact patch, allowing the vehicle to travel along a curved path (19). The angular difference between the original direction of travel and the direction of the tire is called the slip angle. As a result of the angular slip, lateral friction forces develop, resulting in a centripetal force (inward pull) countering a
centrifugal force (outward pull), preventing the vehicle from slipping off the roadway (25). The relationship of lateral friction, $f$, to radius of curvature, $R$, and vehicle speed $V$ is shown in Equation 2.3 (26). If the centrifugal force (the outward pull) exceeds $f$, the tire will slip sideways, eventually resulting in insufficient lateral friction to keep the vehicle on the road, leading to a roadway departure (25).

$$f = \frac{V^2}{15R} - 0.01e$$

(2.3)

Where:

$f$ = Side “lateral” friction demand  
$V$ = Vehicle speed (mph)  
$R$ = Horizontal radius of curvature (ft.)  
$e$ = Rate of roadway super-elevation (%)  

2.3. MEASUREMENT OF SKID RESISTANCE

In the context of roadway safety management, there are numerous methods for measuring skid resistance, the majority of which obtain measurements by moving a tire or slider over a wetted pavement surface (25). The American Society for Testing and Materials (ASTM) sets the standards for operating and calibrating the equipment used for measuring skid resistance for most of the methods used in the U.S. The methods can be grouped into two categories: high-speed equipment, and low-speed or stationary equipment (19). The decision of which method to use may depend on the size of the network, the purpose of the measurement, the level of detail, and the availability of the equipment.

For network-level management, an optimal method for measuring skid resistance could be the use of high-speed equipment. The high-speed equipment is often subcategorized into four groups: locked-wheel (longitudinal friction force), fixed-slip (longitudinal friction force), sideway-force (sideway “lateral” friction factor), and variable slip (19). In the U.S., most state DOTs employ the locked-wheel skid tester, following ASTM E274 (22). The locked-wheel skid tester is a trailer (of constant load and operated at a constant speed of 40 to 60 mph) that hitches onto the back of a vehicle, and consists of two full-scale wheels, one of which is used for measuring (5). The test wheel is equipped with either a standard ribbed-tire (ASTM E501) or a standard smooth-tire (ASTM E524). Many studies have shown that depending on the type of test tire used, the measurement of skid resistance obtained will relate strongly to macro-texture or micro-texture (20). The tire tread of the standard ribbed-tire is better for measuring skid resistance relative to macro-texture, whereas the standard smooth-tire is better for measuring micro-texture related skid resistance (19).

While a locked-wheel skid tester is in operation, an apparatus in front of the test wheel sprays water on the pavement to simulate a wetted surface condition. Simultaneously as the pavement surface is wetted, the test wheel fully locks up and measures the coefficient of friction for an interval of one to three seconds (5). These measurements are averaged over this time interval to provide a single measurement called a Skid Number (SN) (5).

The second subcategory, which was used to obtain skid resistance measurements for this report, is the fixed-slip method. The fixed-slip device used in this study consists of a trailer that
hitches on the back of a vehicle, and operated under a constant load and at a constant speed of 40 to 60 mph (27). The fixed-slip method continuously reads and measures skid resistance, rather than periodically locking up and measuring skid resistance. Fixed-slip devices use a single testing wheel, equipped with a standard tire (ASTM E1551 or E1844), which during operation is kept at a constant slip speed (slip ratio of 12 to 20 percent) using a connected chain, or a hydraulic braking system (19).

Outside of the U.S., some countries (e.g., Great Britain) utilize side-way-force measuring equipment to measure the side-way-force coefficient (SFC), also referred to as lateral friction. Two commonly used side-way-force equipment are the Mu-Meter (ASTM E670) and the Sideway-Force Coefficient Routine Investigation Machine (SCRIM). Like the fixed-slip equipment, side-way-force measurement equipment continuously measure skid resistance. A side-way-force device is comprised of a standardized testing tire placed on a free-rolling test wheel, which is oriented at a small, fixed angle apart from the direction of travel, called a slip-angle or yaw-angle (25). The yaw-angle of the test wheel is between 7.5 to 20 degrees (19). The small yaw-angle combined with low slip-speeds results in sensitivity to micro-texture, but often an insensitivity to macro-texture (19).

The fourth sub-category is the “Variable Slip Technique,” the standards of which are established by ASTM E1859. This equipment utilizes a test wheel, capable of measuring longitudinal friction with a full range of speeds, from free rolling to fully locked. During operation, this equipment works by reducing the free-rolling velocity of the test wheel until it achieves a fully-locked condition, while simultaneously recording the frictional forces as the tire progresses through the range of percent slip (0 to 100) (19). An example of variable-slip equipment is the ROAR, which is used in Denmark and the Netherlands (25).

The slow-moving and static test methods, also referred to as laboratory methods, can be used in the field or in a lab. Two devices that are typical for industrial and research use are the Dynamic Friction Tester (DFT) and the British Pendulum Tester (BPT). The BPT obtains a measure of skid resistance by dropping a pendulum, equipped with a rubber slider, and measuring the difference in the height of the pendulum before and after it contacts the pavement surface, which corresponds to the kinetic energy lost as a result of pavement friction (28). Furthermore, when pendulum is dropped, the slip-speed is generally slow, which results in a measurement of skid resistance that is closely related to micro-texture (22).

The DFT is also a static device. However, its method of measurement is different. The DFT obtains a measurement of skid resistance as a function of speed, by dropping a spinning disk, with three spring-loaded rubber sliders, onto a wetted pavement surface (29). In operation, while water is fed to the pavement surface, the disk’s spin is accelerated to a speed of 55 mph, and then it is released onto the surface (22). After the disk contacts the surface, the rubber sliders decelerate the disk, while the device simultaneously records skid resistance at four different speeds (20, 40, 60, and 80 kph) (19).

2.4. RELATED STUDIES

Many researchers have attempted to empirically determine the relation between crashes and skid resistance. With increasing improvement to and availability of field measurement equipment, the ability to adequately collect skid resistance (and other roadway characteristics)
necessary for quantitative modeling of crashes has also been enriched. This section discusses several studies attempting to explain this relationship.

2.4.1. United Kingdom Skid Resistance Strategy

In 2005, a study was conducted in the U.K. which examined the relationship of skid resistance (and other factors) on crash risk for different site categories (i.e., highways, divided roads, two-lane undivided roads), in order to determine “investigatory levels of skid resistance,” and to aid in determining “financial costs and benefits” of improving skid resistance (30). To analyze the crashes, crash risks were computed using two approaches. The first approach computed crash risk as the total number of crashes per 100 million vehicle kilometers driven, and the second approach computed crash risk using generalized linear modeling (GLM). For the highways, using the first approach, no relationship was found between skid resistance and crash risk. Similarly, when using GLM models, skid resistance was not found to be statistically significant. For divided roadways, the first approach showed that for all crashes, the risk increased with a decrease in skid resistance, and GLM modeling determined skid resistance to be statistically significant. On two-lane undivided roads, the first approach showed a strong trend between skid resistance and crash risk, and GLM models showed that skid resistance was statistically significant.

2.4.2. Ontario Ministry of Transportation

In Ontario, a study was conducted at the University of Waterloo, with support from the Ontario Ministry of Transportation (31). The researchers devised a methodology for evaluating the impact of various roadway characteristics on crash risk at the network level. These characteristics included skid resistance, AADT, annual average daily truck traffic (AADTT), and pavement surface condition (wet, dry, snowy, icy, or other). The team developed a “multi-step approach” for assessing the crash risk along any segment of roadway, with specific interest in skid resistance. In the initial phase of their analysis, simple linear regression was used to directly relate skid resistance with crash counts. Unfortunately, the results did not indicate skid resistance to be significant. In the next step, “logarithmic, power and exponential” regressions were tested using best fit analysis. The results of analysis indicated that the condition of the pavement surface (wet, dry, etc.) influenced driver safety. When the pavement is wet, the risk of having an accident is higher than if the pavement is dry. The probability of a crash occurring was determined to increase with a decrease in skid resistance.

2.4.3. University of Connecticut

Researchers at the University of Connecticut conducted a study (1) to construct a methodology for assessing the association between skid resistance and crash occurrences, and (2) to determine which types of road conditions are more likely to experience a high frequency of skid resistance related crashes (32). The road conditions considered were: geometrics (horizontal & vertical curvature, and shoulder width), presence of intersections and driveways, rural routes, urban routes, skid resistance, and speed limit. To form a model that relates their road conditions to their crash data, while also considering the randomization of crash occurrences, the researchers tested the standard Poisson regression and the negative binomial
regression. Their investigation confirmed the significance of skid resistance in estimating crash risk. In general, as skid resistance increases, the number of expected crashes decreases.

Locations where a higher demand for braking is required were found to have a number of friction-related crashes. For example, when the impact of skid resistance in the presence of horizontal curvature was analyzed, the expected number of crashes increased more when curves were present than when they were not. In general, the effect that some of their road conditions had on crash expectancy were in line with what would be expected. However, some of their results suggest possible erroneous responses for some of the road conditions. For example, when comparing the effect of increasing skid resistance along urban routes versus rural routes, the expected number of crashes was found to increase for urban areas, whereas the expected number of crashes decrease for rural routes. Logical review of the relationship found in urban areas suggested potential problems within their statistical analysis.

2.4.4. Texas Department of Transportation

A study conducted by the University of Texas (33) tried to determine threshold value for friction by statistically comparing crash rates (crashes/year) to roadway characteristics (skid resistance and other factors). The skid resistance data was collected by the Texas Department of Transportation from 2008 to 2011, using a locked-wheeled skid test (ASTM E274) utilizing with a smooth test tire (ASTM E524). The crash data (and other roadway properties) was also obtained for the same four year period using a statewide database. After collecting the data, the team grouped the entire set into two groups, highways and state roads. For the highways, data was separated into groups, considering design speed (high, >55 mph; low, <55 mph), horizontal curvature, and AADT (0 to 2500; 2500 to 4500; >4500). For both highways and state roads, when the pavements were wet, the crash rates increased at lower levels of skid resistance than for dry pavements, suggesting a higher risk for crashes during or immediately after wet weather. In addition, comparing locations with the same skid resistance, wet weather crash rates were generally higher where speed limits were higher. These findings suggest that in order to maintain equivalent safety for sites with high speeds, greater skid resistance would be required than for those with low speeds.

2.5. SUMMARY

The amount of skid resistance is a property of the pavement surface texture, but measurement also varies based on geometric design properties (curvature, speed limit, etc.), water-film thickness and tire properties (i.e. ribbed or smooth) (19). As part of a FHWA advisory to reduce wet-weather related crashes, skid resistance is measured using high-speed investigation equipment (4). The high-speed equipment used for measuring skid resistance can be separated into four sub-groups: locked-wheel (commonly used in the U.S.), fixed-slip, side-way-force and variable-slip (19).

To explore the impact of skid resistance on accident risk, agencies have performed various studies. When general linear regression (i.e. exponential, logarithmic, etc.) was used to evaluate the safety of a pavement as a function of skid resistance, the safety performance of the pavement was found to decrease when the pavement surface was wet, which directly reduced the available skid resistance and indirectly increased the risk of a crash (31). For example, a study at the University of Texas found that, for highways and state roads, when the pavements were wet,
the crash rates increased at lower levels of skid resistance than for similar pavements that were dry \((33)\). Furthermore, when horizontal curvature was present, the effect of skid resistance on crash risk is higher than for tangent sections with similar skid resistance \((32)\). Nevertheless, regardless of the geometric design, an increase in skid resistance resulted in a decrease in the expected number of crashes \((32)\).
3. DATA

This chapter discusses what data was collected, how it was collected, where it was collected, how it was prepared for analysis, and how it was distributed for each network.

3.1. DATA COLLECTION

In Virginia, there are approximately 57,867 lane-miles of state-maintained roads (interstate, primary, secondary and frontage), which are managed by nine highway districts: Bristol, Culpeper, Fredericksburg, Hampton Roads, Lynchburg, Northern Virginia, Richmond, Salem, and Staunton (34). This study collected network data for the Salem District, which contains 9,200 lane-miles of roads spread over twelve counties: Bedford, Botetourt, Carroll, Craig, Floyd, Franklin, Giles, Henry, Montgomery, Patrick, Pulaski, and Roanoke (35). This study measured all of the interstate and primary routes, but only a portion of the secondary routes (identified by the Salem District Traffic and Safety Engineering Division as being most critical due to higher AADT and crash occurrences). Table 3.1 shows that a total of 1,993 lane-miles of roadway were measured, comprised of 232 lane-miles of the interstate, 1,120 lane-miles of the primary, and 641 lane-miles of the secondary.

Table 3.1: Miles of Virginia State Roadway Measured

<table>
<thead>
<tr>
<th>Route Type</th>
<th>State</th>
<th>Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interstate</td>
<td>1,118</td>
<td>232</td>
</tr>
<tr>
<td>Primary</td>
<td>8,111</td>
<td>1,120</td>
</tr>
<tr>
<td>Secondary</td>
<td>48,305</td>
<td>641</td>
</tr>
<tr>
<td>Frontage</td>
<td>333</td>
<td>0</td>
</tr>
<tr>
<td>TOTAL</td>
<td>57,867</td>
<td>1,993</td>
</tr>
</tbody>
</table>

The data used for this investigation included crash counts from 2010 to 2012, AADT from 2010 to 2012, horizontal radius of curvature (CV) and skid resistance. The data for crash counts, AADT and CV were obtained from VDOT records. Crash counts were received as three separate files, separated by year, but combined into one large set. The records categorized the crashes according to various descriptions, of which the categories of interest included the name of the route in which they occurred, the route mile post, crash severity, and pavement surface condition. The severity of any crash could assume one of five possible outcomes: fatality (K), minor injury (A), moderate injury (B), severe injury (C) or property damage only (O). When provided, the pavement surface condition would be categorized as either dry, wet, snowy, icy, muddy, oil/other fluids, other, natural debris or flooded. In keeping with the main Problem Statement for this report, only the data for dry and wet pavements were considered. Likewise, A, B and C crashes were grouped into one category called Injury. Table 3.2 shows that for the portion of the Salem district evaluated in this study, there were a total of 8,630 crashes.
Table 3.2: Summary of 2010 to 2012 Crash Data for Virginia Salem District

<table>
<thead>
<tr>
<th>Route Type</th>
<th>Dry</th>
<th>Wet</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fatalities</td>
<td>Injury</td>
<td>Property Damage</td>
</tr>
<tr>
<td>Interstate</td>
<td>22</td>
<td>485</td>
<td>1,181</td>
</tr>
<tr>
<td>Primary</td>
<td>72</td>
<td>1,526</td>
<td>2,597</td>
</tr>
<tr>
<td>Secondary</td>
<td>17</td>
<td>510</td>
<td>783</td>
</tr>
<tr>
<td>TOTAL</td>
<td>111</td>
<td>2,521</td>
<td>4,561</td>
</tr>
</tbody>
</table>

The other two sets of data obtained from the VDOT records were AADT and CV (not available for secondary routes). The data for AADT and CV were extracted according to several criterion: route type, route name, and route mile post start and finish. Meanwhile, the Grip Tester (designed by Findlay Irvine, but distributed in the U.S. by AeroGroup), shown in Figure 3.1, was used to measure skid resistance. The Grip Tester measures longitudinal friction using the ASTM fixed-slip method (E2340), and is often used on highway and airport runway pavements (36). Its structure contains a three-wheeled system, although only one wheel (fitted with an ASTM E1844-08 standard smooth-tread tire) was used for measuring. The axle containing the test wheel is connected to a chain-system that controls the wheel’s angular velocity in order to produce a constant sixteen percent slip (36). A hose, attached to the Grip Tester and the back of a 250 gallon water tank (located on the truck bed), fed water in front of the test wheel, producing a water-film thickness of 0.5 mm (approximately 0.02 inches) (36). The constant slip of the test wheel, coupled with the wetting of the pavement surface, allowed the Grip Tester to continuously measure wet pavement friction. However, the system reported an average measurement of skid resistance, referred to as a Grip Number (GN) approximately every three feet.

Figure 3.1: The Equipment Used to Collect Skid Resistance
3.2. DATA PROCESSING

Out of the 8,630 crashes, a greater portion resulted in property damage or occurred when the pavement was dry (greater number of days without precipitation). Unfortunately, disproportion between the different crash categories resulted in small sample sets for wet crashes, fatalities and injuries. To avoid bias due to these small crash counts, larger sample sizes (greater number of observations) were necessary. To achieve larger samples, all accidents (wet and dry), and all severity were combined into one large sample set, separated only by route type (interstate, primary and secondary), route name and route mile post.

The measurements of CV were modified as follows. First, the measurements in feet were converted into miles. Second, VDOT records assigned tangent segments values of zero, which is potentially problematic when using regression software. Theoretically, as radius of curvature approaches infinite, a curve should approximate a straight line. Thus, for the tangent sections, the zeros were converted to a very high value (ten miles) that exceeds the greatest recorded measure.

In Virginia, VDOT uses locked-wheel skid testers to measure friction (4). As part of WARP, VDOT typically performs a test at the beginning of each 0.1 mile (4). The tests are performed at 40 mph over a one second interval. Since the tests are conducted at 40 mph over a one second interval, the value of SN would be an average for approximately 58.7 feet. Nevertheless, the value of SN is used as a representative of the entire 0.1 mile section (4).

To match VDOT’s practice, for the measurements collected using the Grip Tester, this study adopted an average of length similar to that used by the locked-wheel skid tester. Variations of the traveling speed were accounted for using a GN correction factor of ±0.007 for each mph above or below 40 mph. Since the values of skid resistance obtained using a locked-wheel skid tester were an average of about 58.7 feet, to closely approximate this with three foot measurements, a moving average of 60 feet was chosen for each 0.1 mile section.

For each network classification (interstate, primary, and secondary), a file was created that aggregated all of the data (crash counts, AADT, GN, and CV), indexed by route name and route mile post. However, prior to joining all of the data, each set of data was averaged into 0.1 mile segments. In explaining the process of averaging the data, the extremes of each segment will be referred to as nodes, with node 1 referring to the segment start, and node 2 referring to the point just prior to the start of the next segment. To accumulate the crash data for each segment, all the crash counts between node 1 and node 2 were summed together and assigned to node 1. If the AADT or CV measurements between node 1 and node 2 varied, they were averaged and assigned to node 1, otherwise averaging was not performed.

3.3. DISTRIBUTION OF NETWORK DATA

This section shows and explains the distribution of AADT, Crash Observations, CV, and GN for the network categories.

3.3.1. Annual Average Daily Traffic

Figure 3.2 presents the cumulative distribution of AADT for the three network categories. The figure shows how the AADT varies according to the level of service provided by the network.
category. Higher service level roads (e.g., interstate routes) have higher $AADT$. To demonstrate this relationship, we will review the percentage of each network that has $AADT$ greater than 10,000. The percentage of the interstate, primary and secondary networks meeting this criterion are 97.6%, 38.1% and 0.2%, respectively.

![Figure 3.2: The Cumulative Distribution of Annual Average Daily Traffic for Each Network Category](image)

3.3.2. Crash Observations

Histograms shown in Figure 3.3, Figure 3.4, and Figure 3.5 show that the crash data follow a right skewed distribution.
Figure 3.3: The Distribution of the Observed Number of Crashes for the Interstate Network

Figure 3.4: The Distribution of the Observed Number of Crashes for the Primary Network
Figure 3.5: The Distribution of the Observed Number of Crashes for the Secondary Network

Based on the information from Figure 3.2, as the AADT for a network increases, the percentage of the network experiencing accidents should also be expected to increase. The increase in the number of crash sections results in fewer sections without accidents. In Figure 3.6, the percentage of each network without crashes increases with a shift from the interstate network to the secondary network. For the interstate, primary and secondary network, the percentage of the networks that did not experience accidents was 49%, 73%, and 81%, respectively. Furthermore, for the interstate network, more sections had accidents (51%) than had no accidents (49%).
Figure 3.6: Percentage of the Network with or without Crashes

Of the sections with accidents, Figure 3.7 shows that sites with a single crash are most common. For sites with multiple crashes occurring, the number of sites decreases with every unit increase in accidents (i.e. 2, 3, 4, …, n).

Figure 3.7: Percentage of Crash Sites with Observed Number of Crashes
3.3.3. Horizontal Radius of Curvature

Figure 3.8 presents the cumulative distribution of CV for all non-tangent sections of the Interstate and Primary networks. In a like manner to the other measured properties, higher service level roads should be expected to have fewer ‘severe’ curves (CV less than 1,600 feet). To demonstrate this, 23.3% of the Primary network has CV less than 1,600 feet, while 5.4% of the Interstate network has CV less than a 1,600 feet.

Figure 3.8: The Cumulative Distribution of Horizontal Radius of Curvature for Interstate and Primary Routes

3.3.4. Skid Resistance

Figure 3.9 presents the cumulative distribution of the average GN measurements for the three network categories. This figure illustrates that higher service level roads (interstate networks) generally have higher measured skid resistance than on lower service level roads (primary or secondary networks). A greater percentage of sites on the secondary networks have lower skid values than sites on interstate or primary networks. However, while there is a difference between the interstate and primary networks, the difference is small. For the proceeding example, we will assume that when GN is below 0.4, there is a higher risk for an accident. The percentages of the interstate, primary and secondary networks with GN measurements below 0.4 are 7.2%, 8.8% and 17.7%, respectively.
Figure 3.9: The Cumulative Distribution of Skid Resistance for Each Network Category
4. METHODOLOGY

In 1995, in light of the SAFETEA-LU legislation, the FHWA introduced the Highway Safety Improvement Program (HSIP), whereby three levels of government (federal, state, and local) would work together to reduce highway crashes. The HSIP process was comprised of three phases: planning, implementation, and evaluation (37). In the planning phase, crash data was collected and modeled in a way that allowed for the prioritization of safety countermeasures (37).

Interpreting the safety performance of pavement networks requires the statistical analysis of crash data as a function of various pavement properties. Before beginning the analysis, it’s critical to know how crashes are distributed, which requires understanding the intrinsic nature of crash occurrences (recorded as crash counts). Crash counts are discrete, non-negative integers. Crashes are also rare. Given the large volume of traffic flowing through each segment of roadway, not every vehicle, nor every segment of roadway, will have an accident. In addition to their rarity, they are also random, because the probability of any segment of roadway experiencing any number of crashes is dependent on the properties of that section of pavement at that specific time. Finally, crashes result from the complex interaction of road properties, which include road user reactions (human factors), vehicle characteristics, and environmental conditions (38).

4.1. MODELING APPROACH

Crashes occur randomly. Therefore the probability of a crash occurring in one segment will not increase or decrease the probability of a crash occurring in the same segment or another segment at any point in time (39). This type of occurrence is referred to as the “law of rare events,” which results in crash counts that follow a Poisson distribution (40). In Poisson regression it can be assumed that for each road segment, the risk of any number of crashes occurring will have some probability of occurrence regardless of the actual number of crashes that have occurred for that segment.

4.1.1. Standard Poisson Model

At each road segment, crashes are assumed to occur according to a Poisson process. Therefore, it should be assumed that all of the crashes for each network will follow a standard Poisson distribution, where the relationship between crashes and various road properties is established using Poisson regression (41). In Poisson regression, for each road segment \((i)\), the probability of any number of crashes occurring is computed using the probability mass function shown in Equation 4.1 (42). In this equation, the probability of any random number of crashes \((y)\) occurring is conditional on the Poisson mean \((\lambda)\) (42).

\[
P(y | \lambda) = \frac{\lambda^y e^{-\lambda}}{y!}
\]  

(4.1)

Where:

\(\lambda = \text{Poisson condition mean (expected number of crashes per year)}\)
y = observed number of crashes, y = 0, 1, 2, …, n

When using a Poisson model, several assumptions must be satisfied. The first assumption is that the network crash data is equi-dispersed, where \( \lambda \) equals the conditional variance. In order for the crash data to be equi-dispersed, for each road segment, the independent variables (the measured road properties) \( (X_j) \) should account for all variation in \( \lambda \) (43). In Equation 4.2 (42), the relationship between the \( \lambda \) and the \( X_j \)’s can be expressed using a log-linear model, where the natural log of \( \lambda \) is linearly related to the product of \( X_j \) and the regression coefficients, \( B_j \) (41).

\[
\ln(\lambda) = \beta_0 + \sum_{j=1}^{n} X_j \beta_j \]  
\( (4.2) \)

Where:

\( \lambda \) = Poisson conditional mean (expected number of crashes per year)
\( X_j \) = independent variables
\( B_j \) = regression coefficients

The second assumption requires each crash observation, \( y \), to be independent of other crash observations. If this assumption is satisfied, then \( B_j \) are estimated using the maximum log-likelihood “Newton-Raphson” method. This method requires iteratively solving the first-order condition (Equation 4.4 (44)) of the log-likelihood function (Equation 4.3 (42)), until Equation 4.3 yields a maximum value (42).

\[
\ln( L) = \sum [ \lambda + y \ln(\lambda) - \ln( y)! ] \]  
\[
\sum_{j=1}^{n} (y - e^{\beta_0 + X_j \beta_j}) X_j = 0 \]  
\( (4.4) \)

For any road segment, crashes result from complex interactions, some of which are measured or measurable, and others that are unmeasurable. Therefore, the road segment properties selected for \( X_j \) (measured or measurable) will account only for a small percentage of all possible road properties contributing to the observed crashes. For each road segment, the dismissal of the unmeasured properties results in an inaccurate prediction of \( \lambda \). The inaccuracy results in over-dispersion of the crash data, where the conditional variance exceeds the conditional mean (39). Thus the standard Poisson assumption of equi-dispersion (conditional mean equals the conditional variance) is violated (39).

4.1.2. Negative Binomial Model

Since the crash models accounted only for a small portion of all responsible road properties, the standard Poisson model will fail to account for the remaining variation in the estimation of \( \lambda \) (39). Furthermore, the influence of any road property naturally varies from one segment to another, which also results in a Poisson mean that will change accordingly. The overall result is over-dispersion of all network crash data. An example of how this over-dispersion affects the crash estimation is shown in Figure 4.1, where confidence intervals for all observed crash counts are modeled using the assumptions of the standard Poisson model. Because of the over-dispersion, crashes for some road segments are under-predicted, and others are over-predicted. A countermeasure for over-dispersion is to use an extension of the standard
Poisson model called the “Poisson-gamma” (NB2), also referred to as the negative binomial model (45).

Figure 4.1: Observed and Expected Crash Counts for Interstate Routes Using Poisson Regression; (a) Complete Detail (b) Detail for Crash Counts 5 to 11
Each section of pavement will experience a random number of crashes that follow a Poisson distribution, but due to over-dispersion, together, all of the Poisson estimates follow a gamma distribution (46). The NB2 allows for a prior prediction of the average number of crashes without bias due to over-dispersion. In Equation 4.5 (46), a gamma distributed, random error (intercept) term \( \nu \) enters Equation 4.2. In Equation 4.5, the \( \lambda \) for each pavement section is treated as a random continuous variable, whose section to section variability depends on \( \nu \) (47).

\[
\mu = \lambda \nu = e^\beta_0 \sum_{i=1}^{n} x_i \beta_i e^\nu
\]  

(4.5)

Where:

\( \mu = \) NB2 conditional mean (expected number of crashes)

\( \lambda = \) Poisson conditional mean (expected number of crashes)

\( X_i = \) independent variables

\( B_j = \) regression coefficients

\( \nu = \) Random error term

After mixing the Poisson mean and the gamma distributed error, and integrating out the error, an over-dispersion parameter (\( \alpha \)) is estimated. With \( \alpha \) and \( \lambda \), the probability of a random variable (\( Y \)) assuming any possible number of observed crash counts (\( y \)) can be computed using a probability density function shown in Equation 4.6 (47).

\[
P(y | \lambda, \alpha) = \frac{\Gamma(\alpha^{-1} + y)}{\Gamma(\alpha^{-1}) \Gamma(y+1)} \left( \frac{\alpha^{-1}}{\alpha^{-1} + \lambda} \right)^{\alpha^{-1}} \left( \frac{\lambda}{\alpha^{-1} + \lambda} \right)^y
\]  

(4.6)

Where:

\( \lambda = \) Poisson conditional mean (expected number of crashes)

\( \alpha = \) over-dispersion conditional parameter

From Equation 4.6, the first two moments yield the NB2 conditional mean (Equation 4.7) with a Poisson distribution and an NB2 conditional variance expressed as a “quadratic variance function” (Equation 4.8), which allows the conditional variance to exceed the conditional mean through the employment of a gamma distributed, over-dispersion parameter, \( \alpha \) (47).

\[
E[y | \lambda, \alpha] = \lambda
\]  

(4.7)

\[
\text{VAR}[y | \lambda, \alpha] = \lambda + \alpha \lambda^2
\]  

(4.8)

**Safety Performance Functions**

In the U.S., the same negative binomial regression is used regularly to develop safety performance functions (SPFs). The SPF (Equation 4.9) embraces the NB2 model, requiring the evaluation of \( AADT \) as a mandatory variable for every DOT application, while other factors (roadway geometry, traffic control features, etc.) are left to the discretion of state and local agencies (2). The FHWA’s mandatory inclusion of \( AADT \) stems from research establishing daily traffic volumes as the “most important contributor to crashes”. In Equation 4.9, the natural
log transformation of AADT accounts for the hypothetical situation where no traffic exists on a given road segment, which results in zero expected number of crashes (2).

\[ \lambda = L \times e^{\beta_0 + \ln(AADT)\beta_1 + \sum_{j=2}^{n} X_j \beta_j} \] (4.9)

Where:

\( \lambda \) = conditional mean (expected number of crashes)
\( L \) = road segment length (mi.)
\( AADT \) = average annual daily traffic
\( X_j \) = independent variables
\( \beta_j \) = regression coefficients

In Virginia, the current structure for Equation 4.9, accounts only for AADT (48). Equation 4.10 shows the complete SPF model used in this report. In this model, in addition to AADT, two additional variables are considered: the skid resistance (\( GN \)), and the horizontal radius of curvature (\( CV \)) (interstate and primary routes only). Since the road segment length is set to 0.1 miles, \( L \) was excluded from the model. The decision to take the inverse of radius of curvature was based on the relationship of minimum radius of curvature and the maximum allowable lateral friction (Equation 2.3), used for designing horizontal curves (26).

\[ \lambda = e^{\beta_0 + \ln(AADT)\beta_1 + GN\beta_2 + CV^{-1}\beta_3} \] (4.10)

Where:

\( \lambda \) = conditional mean (expected number of crashes)
\( AADT \) = average annual daily traffic
\( GN \) = skid resistance
\( CV \) = radius of curvature (mi.)
\( \beta_j \) = regression coefficients

### 4.1.3. Modeling Technique Selection

**Pearson Chi-Square Test**

In order to model non-negative, discrete, random events such as crash data, a modeling technique should be selected from the Poisson family of models. From this family of models, literature recommends the initial evaluation of the standard Poisson distribution (45). To comply with this evaluation, the dependent variable (crash counts) alone, is used to generate a base model, of which the distribution is tested for being Poisson, using the Pearson Chi-Square goodness-of-fit (49). According to (50), proper application of the Pearson Chi-Square test requires the count data to be discrete, Poisson or binomially distributed, and grouped by the total number of crashes experienced on any road segment (i.e., \( y = 0, 1, 2, \ldots, n \) crashes).

For this report, the null-hypothesis (\( H_0 \)) claims that the total observed crash counts (\( O \)), for each crash group (\( y \)) is Poisson distributed. To test this claim, a Chi-Square test statistic (\( X^2 \)), is computed using Equation 4.11, and then compared to a Chi-Square critical value with \( n-p-1 \)
degrees of freedom \((n)\) is equal to the largest crash group \(y\), and \(p\) is the number of regression coefficients \((50)\). If the Chi-Square exceeds the critical value, then the null-hypothesis is rejected (the distribution of the crash data follows another distribution other than standard Poisson) \((50)\).

\[
X^2 = \sum \frac{(O - E)^2}{E}
\]

(4.11)

Where:

\(X^2 = \text{Chi-Square test statistic}\)
\(O = \text{total observed crash count for each crash group } y\)
\(E = \text{expected number of crashes}\)

**Dispersion Parameter**

The Chi-Square test is a recommended starting point for determining if the crash data is Poisson distributed, but, it will not conclude whether the data is over- or under-dispersed. If the crash data fails to be Poisson distributed due to over-dispersion, then negative binomial regression is a valuable method to explore \((45)\). To determine how the crash data is dispersed, Equation 4.12 can be used to compute a dispersion parameter \((\tau)\) \((41)\). The value of \(\tau\) can be above zero, zero, or below zero. If \(\tau\) is above zero, the crash data is considered to be over-dispersed, but if the value is below zero, the crash data under-dispersed.

\[
\tau = \frac{1}{n - p - 1} \sum \left(\frac{y - \lambda}{\lambda} - 1\right)
\]

(4.12)

4.1.4. Influential Variable Selection

In the process of setting up the models, for each route type (interstate, primary, and secondary), several models were created, each utilizing a different array of variables:

1. Intercept
2. Intercept + AADT
3. Intercept + AADT + GN
4. Intercept + AADT + GN + CV

To determine which model incorporated the “optimal” combination of variables, two evaluation techniques were explored: the Akaike Information Criterion (AIC) and the log-likelihood ratio test (LLRT).

The FHWA SPF assumes AADT is always statistically significant, so for this report, all models competing with Model 1 will have AADT.

**Akaike Information Criterion**

AIC assesses the fitness of a model, with respect to the parameters selected, based on the log-likelihood value of the model \((L)\) and a penalty term related to the number of estimated
Coefficient parameters \( p \) (46). First, \( AIC \) is computed for each model \( i \) using Equation 4.13 (51).

\[
AIC_i = -2 \ln(L) + 2p
\]  

(4.13)

Next, the model with the lowest \( AIC \) becomes \( AIC_{\text{min}} \) with which all other model \( AIC \)’s are compared. Using Equation 4.14, the difference between \( AIC_i \) for each model and \( AIC_{\text{min}} \) yields a \( \Delta AIC_i \) (51).

\[
\Delta AIC_i = AIC_i - AIC_{\text{min}}
\]  

(4.14)

Using \( \Delta AIC_i \), the probability of each model being the best model is determined by computing the Akaike Weight \( (W_i) \) shown in Equation 4.15 (51).

\[
W_i = \frac{e^{-\frac{\Delta AIC_i}{2}}}{\sum_{i=1}^{n} e^{-\frac{\Delta AIC_i}{2}}}
\]  

(4.15)

After computing \( W_i \), the model with the highest value is taken as the best model. The best model is then compared to the other models using an evidence ratio \( (ER_i) \) calculation in Equation 4.16 (51). The value of \( ER_i \) expresses how likely it is that the best model will perform better than the other models (51).

\[
ER_i = \frac{W_{\text{Best}}}{W_i}
\]  

(4.16)

**Log-Likelihood Ratio Test**

LLR tests are used to compare two models using stepwise comparison. For each model comparison (each step), a hypothesis test is used to determine the significance of adding additional parameters to a model with fewer parameters. However, the LLR test is potentially vulnerable to over-fitting problems associated with adding too many parameters, and therefore it’s recommended to compare the responses of the LLR test to those acquired using AIC (41).

Using Equation 4.17 (41), an LLR test statistic \( (LL) \) is equal to twice the difference between a model with fewer parameters \( (L_R) \) and a model with additional parameters \( (L_U) \) (52). If \( LL \) approximates a Chi-Square distribution, then \( LL \) is compared to a Chi-Square critical value with degrees of freedom \( (DOF) \) equal to the difference in the \( n-p-1 \) of both models. The null-hypothesis typically claims with 95\% confidence (p-value \( \leq 0.05 \)) that \( L_U \) has equal or less predictive capacity as \( L_R \) (52). To reject the null-hypothesis, \( LL \) must exceed the critical value (41).

\[
X^2 \approx LL = 2 \times (L_U - L_R)
\]  

(4.17)

**4.2. High Crash Risk Location Identification**

The HSIPs suggest that in order to manage and improve the safety of a pavement network, it’s necessary to have a method for identifying segments that would benefit most from safety improvements, also referred to as “locations with promise” (2). To identify “locations
with promise”, the FHWA uses a process called “network screening”. First, the parameters (the average expected number of crashes mean and the over-dispersion) of the prior distribution are estimated from the observed number of crashes as a function of various pavement properties using the SPF. Then, these parameters are used to define a posterior distribution using the Empirical Bayes (EB) approach to compute an estimate of the mean number of crashes that is close to the true long-term estimate of crashes (53). Finally, the EB estimates can be utilized as a way to rank segments according to their risk.

4.2.1. Empirical Bayes Method

Due to the error associated with the collected data, in addition to the fluctuating influence of the individual site characteristic that result in the observed crashes, on average, the estimated number of crashes calculated using negative binomial regression will experience regression-to-the-mean (RTM) bias. A common problem associated with RTM bias is an estimate of annual crashes that may not be a true estimation of the average number of crashes (53). For example, the negative binomial estimate for some random site may be high (i.e. eleven crashes) one year, but low (i.e. three crashes) the following year. Years of investigation of this bias has led researchers to propose the EB method (55).

With EB, the estimated number of crashes at site $i$ ($\lambda_i$) and the amount of associated over-dispersion ($\alpha$) are computed using an SPF regression shown in Equation 4.9, and then combined to produce a weighted measurement ($W_i$) using Equation 4.18 (56). Then, using $W_i$, $\lambda_i$, $\alpha$ and the observed number of crashes ($O_i$), a new overall estimate of crash risk, nearly approximating the true average number of crashes, is produced using the EB theorem in Equation 4.19 (56). The standard deviation of $EB_i$ can be estimated using Equation 4.20 (56).

$$W_i = \frac{1}{1 + \lambda_i \alpha} \quad (4.18)$$

$$EB_i = W_i \times \lambda_i + (1 - W_i) \times O_i \quad (4.19)$$

$$\sigma_{EB_i} = \sqrt{(1 - W_i) \times EB_i} \quad (4.20)$$

Overall, the EB method produces an estimate that approximates the true average crash risk. However, it should be noted that there are limitations based on specific assumptions made regarding the distribution of the SPF and EB estimates. The purpose of performing the EB method is to generate a posterior distribution that will closely approximate the true mean distribution, by reducing the “expected total mean squared error” of all the data for the network (57). Furthermore, if we apply the concepts provided by (57) to the data for this study, as long as the total number of crash observations are greater than or equal to three, and if the prior average crash rate and the posterior true average crash rates of each network follow a gamma distribution, then $EB$ (posterior estimate) should be a closer estimate of the true average crash rate than the SPF rate.

In conclusion, an advantage of the EB approach to estimation is that its overall estimation of safety for the entire network is closer to the true network average. However, the theory behind Equation 4.19 indicates that the precision of the estimates at individual sites is governed by the size of $O_i$ (57). An example of this limitation is that for any road segment with an $O_i$ that
deviates far from $\lambda_i$ (referred to as “extreme values”), the $EB_i$ estimate for that segment will have greater bias, resulting in an $EB_i$ that is either far below or far above the true average estimate for that segment.

4.2.2. Prioritization of Locations for Improvement

Locations identified as having a high accident risk using the EB methodology could be ranked specifically by their EB estimates. Ideally, all locations with EB estimates exceeding some designated threshold could benefit from receiving some form of safety improvement, if unconstrained highway improvement funds were available. Realistically, DOTs have limited available funding, and therefore they have to select some portion of a road network to improve. In addition, they would need to choose which type of improvement (i.e. a high friction surface course or a simple overlay) would provide the best return on the spending (highest reduction in the long-term expected number of crashes).

To illustrate how the models can be used, a simplified implementation simulation was performed. For simplicity, two types of surface treatments are tested: a high friction surface (HFS) course and a hot mix asphalt overlay (OLA). For the purpose of this study, an HFS treatment will be assumed to last the longest and provide the highest improvement to $GN$ (increasing the current level of $GN$ to 0.95). An OLA is a short-term, low-cost option that produces a lower improvement to $GN$ (increasing the current level of $GN$ to 0.7). In addition to testing only two types of surface treatments, for each treatment type, all network values below the level provided by the treatment tested will be improved.

The first step is to compute the current EB estimates, $EB_{CURRENT}$, for the network under evaluation, using the instruction from Section 4.4.2. Next, for each treatment, compute a new set of EB estimates, $EB_{NEW}$. Lastly, rank the entire network based on the differences between the $EB_{CURRENT}$ and $EB_{NEW}$. This methodology is outlined in the following order:

1. Compute two new sets of SPF estimates (one set for each treatment), $SPF_{NEW}$, using the modified $GN$ data
2. Next compute an SPF ratio: $SPF_{RATIO} = \frac{SPF_{NEW}}{SPF_{CURRENT}}$
3. Compute $EB_{NEW}$: $EB_{NEW} = EB_{CURRENT} \times SPF_{RATIO}$
4. Compute the difference: $\Delta EB = EB_{CURRENT} - EB_{NEW}$
5. Rank the network using $\Delta EB$
5. RESULTS AND DISCUSSION

5.1. SAFETY PERFORMANCE FUNCTIONS

5.1.1. Testing the Standard Poisson Distribution

Crashes occur as a result of a various road properties (some that are measured and others that are not measured or unmeasurable). Due to road properties that are unaccounted for, in addition to segment-to-segment variation of the properties, the standard Poisson assumption of equi-dispersion is often violated. This violation results in over-dispersion of the network crash data, where the “differences between the crash counts and model predictions” exceed what is expected with the standard Poisson distribution (55). If the data is over-dispersed, the Chi-Square test and the test for over-dispersion should offer validation.

Accordingly, the crash distributions were tested using a Chi-Square test and a Dispersion parameter ($\tau$). The results of both tests are shown in Table 5.1, with Chi-Square results provided at the top, and the values of $\tau$ provided at the bottom. The null- and alternative- hypotheses of the Chi-Square test are respectively listed as $H_0$ and $H_a$. $H_0$ claims the crash data is standard Poisson distributed, where the conditional variance equals the conditional mean, and $H_a$ suggests the data follows a distribution that is not standard Poisson. The following four rows present the Critical Value with DOF, the Chi-Square value ($X^2$), the p-value, and the Chi-Square Final Result. The Final Result of the Chi-Square test determined that for network classifications, the difference between the total number of observed crashes and the predicted number of crashes is larger than should be expected if the crash counts were standard Poisson distributed, hence for each case, $H_0$ is rejected with a significance level of 0.05. For the final test, if the value for $\tau$ is greater than zero, then the data can be considered over-dispersed (41). For the interstate, primary, and secondary networks, the values for $\tau$ are, respectively, 41.7, 489.8 and 444.5, thus indicating that for each network, the crash data is over-dispersed. In conclusion, for each network classification, the results of both tests indicate that the crash data does not follow a standard Poisson distribution, and the data is over-dispersed.

Table 5.1: Chi-Square Test for Poisson Distribution

<table>
<thead>
<tr>
<th>Route</th>
<th>Interstate</th>
<th>Primary</th>
<th>Secondary</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0$</td>
<td>Standard Poisson Distributed</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H_a$</td>
<td>Other</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DOF</td>
<td>10</td>
<td>22</td>
<td>15</td>
</tr>
<tr>
<td>Critical Value</td>
<td>18.3</td>
<td>33.9</td>
<td>25</td>
</tr>
<tr>
<td>$X^2$</td>
<td>432.5</td>
<td>5.27E+28</td>
<td>8.53E+20</td>
</tr>
<tr>
<td>p-value</td>
<td>0E+00</td>
<td>0E+00</td>
<td>0E+00</td>
</tr>
<tr>
<td>FINAL RESULT</td>
<td>Reject $H_0$</td>
<td>Reject $H_0$</td>
<td>Reject $H_0$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>41.7</td>
<td>489.8</td>
<td>444.5</td>
</tr>
</tbody>
</table>
5.1.2. Negative Binomial Distribution

In the last section, the crash data for each network classification failed to comply with the standard Poisson assumption of equi-dispersion, and was found to be over-dispersed. The literature recommends that when crash data is found to be over-dispersed, the data should be assumed to follow Negative Binomial distribution. The FHWA methodology for determining an SPF with additional variables is employed for this research. For this report, the negative binomial regression was performed using Matlab, with a code accessed at the website provided in (59). The modifications include the grouping of all the collected data into uniform 0.1 miles segments, and the inclusion of $G_N$ and $C_V$. Since $C_V$ was only available for the interstate and primary routes, it was not included in the model for the secondary network.

Assuming the crash data followed a negative binomial distribution, negative binomial regression was used to develop crash risk models as a function of up to three potential site characteristics. The site characteristics included in each model were selected based on the results of goodness-of-fit testing. For each network classification, up to four models were created, where Model 1 was the base model with only an intercept, and the consecutive models were gradual adaptations (change of the array of road segment properties) as presented in Section 4.1.4. The results for the interstate, primary and secondary routes are provided in Table 5.2, Table 5.3, and Table 5.4 respectively. In each of these tables, the regression coefficients for the model variables are provided under their related column title: the Intercept, AADT, $G_N$, and $C_V$. In like manner, the estimation of over-dispersion variable ($\alpha$) and log-likelihood values are also provided.

Table 5.2: Parameter Estimates for Interstate Route Regression Models

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameter Estimates</th>
<th>DOF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha$</td>
<td>LL</td>
</tr>
<tr>
<td>1</td>
<td>0.71</td>
<td>-3081.20</td>
</tr>
<tr>
<td>2</td>
<td>0.63</td>
<td>-3032.52</td>
</tr>
<tr>
<td>3</td>
<td>0.62</td>
<td>-3025.61</td>
</tr>
<tr>
<td>4</td>
<td>0.62</td>
<td>-3025.38</td>
</tr>
</tbody>
</table>

Table 5.3: Parameter Estimates for Primary Route Regression Models

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameter Estimates</th>
<th>DOF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha$</td>
<td>LL</td>
</tr>
<tr>
<td>1</td>
<td>1.91</td>
<td>-9731.92</td>
</tr>
<tr>
<td>2</td>
<td>1.66</td>
<td>-9538.47</td>
</tr>
<tr>
<td>3</td>
<td>1.64</td>
<td>-9527.07</td>
</tr>
<tr>
<td>4</td>
<td>1.62</td>
<td>-9510.40</td>
</tr>
</tbody>
</table>
Table 5.4: Parameter Estimates for Secondary Route Regression Models

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameter Estimates</th>
<th>DOF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha$</td>
<td>LL</td>
</tr>
<tr>
<td>1</td>
<td>1.99</td>
<td>-4047.10</td>
</tr>
<tr>
<td>2</td>
<td>0.95</td>
<td>-3735.68</td>
</tr>
<tr>
<td>3</td>
<td>0.95</td>
<td>-3733.02</td>
</tr>
</tbody>
</table>

Interstate Routes

For interstate routes, four NB2 models were tested. The array of potentially significant variables include $AADT$, $GN$, and $CV$. Interstate pavement systems are often credited as having lower volumes of fatal accidents despite the fact that they service the highest volumes of vehicular traffic. This is attributed to the higher design standards the interstate system are required to meet; this includes geometric design (larger horizontal radius of curvature, and a greater number of tangent sections) \(^{(60)}\). Looking at Table 5.5, the model with the lowest $AIC$ is \textit{Model 3}. The results from computing $W$ indicate that \textit{Model 3} has a 68% chance of being the best model, while \textit{Model 4} has approximately a 32% chance of being the best model. Despite \textit{Model 4} having the second highest chance of being the best, evidence still supports \textit{Model 3} as the best fit. Furthermore, the evidence ratio for \textit{Model 4} indicates that \textit{Model 3} is 2.14 times more likely of being a better fit than \textit{Model 4}. In conclusion, \textit{Model 3} is the best fit model according to $AIC$.

Table 5.5: Akaike Information Criterion Test Results for Interstate Routes

<table>
<thead>
<tr>
<th>Model</th>
<th>AIC</th>
<th>$\Delta AIC$</th>
<th>$W_i$</th>
<th>$ER_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6,164.40</td>
<td>107.17</td>
<td>0.00%</td>
<td>1.87E+23</td>
</tr>
<tr>
<td>2</td>
<td>6,069.04</td>
<td>11.81</td>
<td>0.19%</td>
<td>367.39</td>
</tr>
<tr>
<td>3</td>
<td>6,057.23</td>
<td>0.00</td>
<td>68.04%</td>
<td>1.00</td>
</tr>
<tr>
<td>4</td>
<td>6,058.75</td>
<td>1.52</td>
<td>31.77%</td>
<td>2.14</td>
</tr>
</tbody>
</table>

For the $LLR$ test, \textit{Model 3} is also the best fit. For this test, three stepwise comparisons were performed. For \textit{Comparison 1}, \textit{Model 1 (Lr, the base model)} was compared to \textit{Model 2 (Lr, containing AADT)}. Using Equation 4.17, the value for $LL$ exceeded the Critical Value by an amount of 91.37, which resulted in a rejection of the null-hypothesis. Stemming from the initial comparison, for \textit{Comparison 2}, \textit{Model 2 was compared to Model 3 (AADT and GN)}. The estimation of $LL$ was nearly 2.5 times greater than the Critical Value, again resulting in a rejection of the null-hypothesis. For the final comparison, \textit{Model 3 was compared to Model 4 (AADT, GN, and CV)}. The $LL$ for \textit{Comparison 3} was 0.48, which was lower than the critical value. Similarly, the p-value was estimated to be 0.49, which exceeded the significance level. As a result, for \textit{Comparison 3}, the null-hypothesis failed to be rejected, and \textit{Model 3} was chosen as the preferred model for interstate routes.
Table 5.6: Log-Likelihood Ratio Test Results for Interstate Routes

<table>
<thead>
<tr>
<th>Comparison</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>L_R</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>L_U</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>DOF</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Critical Value</td>
<td>5.99</td>
<td>5.99</td>
<td>5.99</td>
</tr>
<tr>
<td>LL</td>
<td>97.36</td>
<td>13.81</td>
<td>0.48</td>
</tr>
<tr>
<td>p-value</td>
<td>1E-05</td>
<td>0E+00</td>
<td>0.49</td>
</tr>
<tr>
<td>Test Result</td>
<td>Reject H_0</td>
<td>Reject H_0</td>
<td>Fails to Reject H_0</td>
</tr>
<tr>
<td>Best Fit</td>
<td>Model 2</td>
<td>Model 3</td>
<td>Model 3</td>
</tr>
<tr>
<td>FINAL RESULT</td>
<td>Model 3 is the Best Fit</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In light of the initial description of interstate design, the rejection of the model containing CV may be related to several factors. Due to overall higher level of geometric design, very few segments have severe horizontal curvature (less than 1,600 feet). For our data, roughly 5.4% of the network had CV less than 1,600 feet. In addition to higher service level geometric design, the measurements of CV may contain errors associated with the equipment used. Together, the two factors may contribute to CV having little or no significance to the models. On the other hand, AADT and GN were, as expected, found to be statistically significant. Finding AADT to be significant, and the impact it has predicting the risk of crashes, supports the current view of the FHWA, which reports that if AADT increases, on average, so should the expected number of crashes. Meanwhile, the negative regression coefficient estimated for GN (Table 5.2) implies that as the amount of available GN along any segment of roadway decreases, the average expected number of crashes increases. Together, the results of both goodness-of-fit tests, coupled with the estimated regression coefficients for this report’s data, culminate into a final model shown in Equation 5.1.

$$\lambda = e^{-0.35+1.25\ln(AADT)-1.19GN} \quad (5.1)$$

**Primary Routes**

The process applied for primary routes followed a similar methodology as that used for evaluating interstates, where the same type of roadway characteristics were tested, and four different models were constructed. In contrast to the interstate routes, the outcome resulting from the two tests led to a different conclusion. The first fit test performed was the AIC. Reviewing the output from Table 5.7, Model 4 was found to have the lowest AIC, and therefore it was set as AIC_{min}. Next, the calculations of W_i indicated that Model 4 had a 100% chance of being the best model. In conclusion, Model 4 is the best fit model according to AIC.
Table 5.7: Akaike Information Criterion Test Results for Primary Routes

<table>
<thead>
<tr>
<th>Model</th>
<th>AIC</th>
<th>ΔAIC</th>
<th>( W_i )</th>
<th>( ER_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>19,465.84</td>
<td>437.04</td>
<td>0.00%</td>
<td>7.96E+94</td>
</tr>
<tr>
<td>2</td>
<td>19,080.95</td>
<td>52.15</td>
<td>0.00%</td>
<td>2.11E+11</td>
</tr>
<tr>
<td>3</td>
<td>19,060.14</td>
<td>31.34</td>
<td>0.00%</td>
<td>6.39E+06</td>
</tr>
<tr>
<td>4</td>
<td>19,028.80</td>
<td>0.00</td>
<td>100.00%</td>
<td>1.00</td>
</tr>
</tbody>
</table>

**BEST FIT**

Model 4

Next, the LLR test inputs and outputs from Table 5.8 are discussed. For *Comparison 1*, *Model 1* was compared to *Model 2* (AADT). The estimated *LL* was approximately 65 times greater than the *Critical Value*, and thus the null-hypothesis was rejected and *Model 2* was chosen to be the best fit. Continuing from the first comparison, the second comparison examined the difference between *Model 2* and *Model 3* (AADT and GN). Resulting from this, *LL* was estimated to be nearly four times greater than the *Critical Value*, and therefore the null-hypothesis was again rejected, and *Model 3* was chosen to be the best fit for *Comparison 2*. Finally, for the third comparison, *Model 3* was compared to *Model 4* (AADT, GN, and CV). For *Comparison 3*, *LL* was approximately six times greater than the *Critical Value*, and therefore *Model 4* was selected as the preferred model for the primary network.

Table 5.8: Log-Likelihood Ratio Test Results for Primary Routes

<table>
<thead>
<tr>
<th>Comparison</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>LR</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>LU</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>DOF</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Critical Value</td>
<td>5.99</td>
<td>5.99</td>
<td>5.99</td>
</tr>
<tr>
<td>LL</td>
<td>386.89</td>
<td>22.81</td>
<td>33.34</td>
</tr>
<tr>
<td>p-value</td>
<td>0E+00</td>
<td>0E+00</td>
<td>8E-09</td>
</tr>
<tr>
<td>Test Result</td>
<td>Reject H₀</td>
<td>Reject H₀</td>
<td>Reject H₀</td>
</tr>
<tr>
<td>Best Fit</td>
<td>Model 2</td>
<td>Model 3</td>
<td>Model 4</td>
</tr>
<tr>
<td>FINAL RESULT</td>
<td>Model 4 is the Best Fit</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In conclusion, the outcome of both tests proved *AADT*, *GN*, and *CV* to be statistically significant in assessing the probability of crashes within the primary road network. As with interstates, for primary routes, the average expected number of crashes per year should increase with increases of *AADT* and decreases in *GN*. Unlike the interstate network, the data evaluated in the study for the primary network showed that the effect of *CV* was determined to be statistically significant. However, interpreting its statistical significance relative to regression analysis from Table 5.3 requires an understanding of the inverse transformation of *CV* assumed for the model shown in Equation 5.2. Recalling Equation 2.3, the amount of available *GN*, and
therefore the ability of a vehicle to remain safely on the roadway, is inversely related to $CV$. In addition, in a report published in 2014 by the FHWA, the combined effect of horizontal curvature and pavement grade on crash risk were evaluated, and in the study, the risk of a crash was found to be inversely related to $CV$ (61). As the inverse of $CV$ increases, the actual measure of $CV$ decreases. Therefore, in the final model selected for this report (Equation 5.2), as the inverse of $CV$ increases (the horizontal curves become increasingly sharp), the expected number of crashes per year should increase.

$$\lambda = e^{-0.25+0.37\ln(AADT)+1.00GN+0.04CV^{-1}}$$ (5.2)

Secondary Routes

For the secondary network, data for $CV$ was not available. The exclusion of this variable from the final model should not reflect its potential impact on crash risk had the data been available. For this network, the road characteristics considered were $AADT$ and $GN$. With these two variables, three models were tested. Referring to the results of $AIC$ shown in Table 5.9, Model 3 had the lowest $AIC$, therefore establishing it as $AIC_{min}$. Next, the results of $W_i$ indicated that Model 3 had an 84% chance of being the best model, while Model 2 had the second highest support, with approximately a 16% chance. Despite Model 2 having the second highest chance, evidence favored Model 3 as a best fit. Furthermore, the evidence ratio for Model 2 indicated that Model 3 was 5.31 times more likely to be a better fit than Model 2. In conclusion, Model 3 is the best fit model according to $AIC$.

Table 5.9: Akaike Information Criterion Test Results for Secondary Routes

<table>
<thead>
<tr>
<th>Model</th>
<th>AIC</th>
<th>AAI C</th>
<th>$W_i$</th>
<th>ERi</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8,096.21</td>
<td>624.18</td>
<td>0.00%</td>
<td>3.45E+135</td>
</tr>
<tr>
<td>2</td>
<td>7,475.37</td>
<td>3.34</td>
<td>15.86%</td>
<td>5.31</td>
</tr>
<tr>
<td>3</td>
<td>7,472.03</td>
<td>0.00</td>
<td>84.14%</td>
<td>1.00</td>
</tr>
<tr>
<td>BEST FIT</td>
<td>Model 3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Following the $AIC$, the set of models were tested using $LLR$. For this, two comparisons were needed to select the best fit model. For Comparison 1, Model 1 was compared to Model 2 (only $AADT$). In this step of the test, $LL$ was nearly 104 times greater than the Critical Value, and so the null-hypothesis was rejected and Model 2 was selected. For Comparison 2, Model 2 was compared to Model 3 ($AADT$ and $GN$). In this comparison, the estimated value of $LL$ was approximately eleven percent smaller than the Critical Value. Nevertheless, the $p$-value is close to 0.02, which is below the significance level of 0.05; ergo the null-hypothesis was rejected and Model 3 was chosen as the optimal fit.

Table 5.10: Log-Likelihood Ratio Test Results for Secondary Routes

<table>
<thead>
<tr>
<th>Comparison</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_R$</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>$L_U$</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>DOF</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Critical Value</td>
<td>5.99</td>
<td>5.99</td>
</tr>
<tr>
<td></td>
<td>LL</td>
<td>p-value</td>
</tr>
<tr>
<td>----------------</td>
<td>------</td>
<td>---------</td>
</tr>
<tr>
<td></td>
<td>622.84</td>
<td>0E+00</td>
</tr>
</tbody>
</table>

For the secondary network under evaluation for this research, unfortunately, data for CV was not available. Therefore, an interpretation cannot be made regarding the impact of horizontal curvature and crash risk. Meanwhile, the results of both tests concluded that AADT and GN have a significant role in predicting crash risks. Using the regression coefficients from Table 5.4, an equation for Model 3 was constructed (Equation 5.3). Interpretation of Equation 5.3 suggests that with increases in AADT, the average expected risk of crashes should also increase. Additionally, as the amount of GN available along any road segment decreases, the average risk of crashes should increase.

\[
\lambda = e^{-0.55+0.75\ln(AADT)–0.56GN}
\]  

(5.3)

Model Fit Verification

To provide visual support for using a NB distribution for modeling crash data, plots (Figure 5.1, Figure 5.2, and Figure 5.3) were constructed for each route with a similar structure as Figure 4.1. Each plot demonstrates the accuracy of using an NB regression to predict the mean number of crashes for each road segment relative to the true observed number of crashes. In each figure, the number of crashes observed (represented using blue bars) are plotted against the maximum and minimum threshold (along with the upper and lower 95 percent confidence interval) of the crash risks associated with the different road segments (road segments with zero crashes, one crash, two crashes, etc.). (Note: for an explanation of how the figures are made, please refer to Appendix A.) For each type of route, there are two figures. The first figure captures the predictions for all road segments, while the second figure provides a window that captures segments that are troublesome to visualize using the first figure. For each figure constructed using the NB regression, when their predictions are compared to predictions made using Poisson regression (Figure 4.1), over-dispersion appears to be minimized.
Figure 5.1: Observed and Expected Crash Count for Interstate Routes Using SPF Regression; (a) Complete Detail (b) Detail for Crash Counts 5 to 11
Figure 5.2: Observed and Expected Crash Count for Primary Routes Using SPF Regression; (a) Complete Detail (b) Detail for Crash Counts 5 to 24
Figure 5.3: Observed and Expected Crash Count for Secondary Routes Using SPF Regression; (a) Complete Detail (b) Detail for Crash Counts 5 to 17
Summary

The results of the AIC and LLR testing indicated that for the interstate network, *Model 3* was the best fit, which implies that AADT and GN are both statistically significant in estimating crash risk. For the primary network, the two tests revealed *Model 4* to be the best fit, specifying that AADT, GN, and CV are statistically significant. For the secondary network, *Model 3* was selected (data for CV was not available to test), which indicates that AADT and GN are statistically significant.

5.2. HIGH CRASH RISK LOCATIONS

As was explained in Section 4.2, the EB approach was used for estimating crash risks that are close approximations of the true crash risk. Figure 5.4 (a) plots the observed number of crashes and the estimated number of crashes (SPF and EB) for Virginia Interstate-81 North from mile post 88 to 174, while Figure 5.4 (b) zooms on the same estimates from mile post 163 to 173. The blue dots individually represent the total number of crash observations for the three year crash period, for the 0.1 mile segments of roadway. From this, we see that a much greater number of road segments experienced zero crashes than experienced one or more crashes. However, although there are sections that did not experience crashes between the first and third years of the study, this does not conclude that the sections are completely safe, nor is a high crash count at some segments an indication of being unsafe. Instead, it’s more likely that all sections randomly experience crashes, which could vary from one year to the next. If this assumption is true, every road segment for this network (including those with zero observed crashes) should have some probability of experiencing any random number of crashes.
The roadway characteristics used in this study account only for a small fraction of the possible roadway site properties that result in conditions that lead to crashes. However, there are also other properties that were not measured or were unmeasurable. Neglecting the other site
properties, coupled with variation of the site properties from one segment to another, the crash data for each network is over-dispersed. Therefore, while crashes for each segment are Poisson distributed, due to over-dispersion, the expected average crash rates for each network will follow a (prior) gamma distribution. To predict the parameters of the (prior) gamma distribution, negative binomial (SPF) regression was used. Using these parameters, SPF models were constructed for each route network, which are shown in Equation 5.1, Equation 5.2, and Equation 5.3.

Examining Figure 5.4, we see that the SPFs result in every segment of roadway having a small probability of experiencing a random number of crashes. Simply stated, if the assumptions for the regression resulting in the SPFs are true, the sum of all SPF estimates shown in Figure 5.4 should closely approximate the actual number of crashes observed for the three year period. To verify this, for this stretch of I-81 North, there were a total of 829 crashes observed. However, using the SPF from Equation 5.1, the estimated number of crashes for the three year period was nearly 884. While this number is higher than our true measure, we refer back to Figure 5.1, Figure 5.2, and Figure 5.3, where it can be seen that with any prediction, there is associated variation due to assumptions made in the modeling process. However, the estimations generally should be similar to the observed crash counts.

According to (2) and (53), to acquire a closer approximation of the true expected average crash risk, the EB approach was used to empirically estimate the mean of the (posterior) gamma distribution by combining the information from the SPF estimates and the information contained in the observed crash counts. Using the equations for computing EB, the estimated total number of crashes for this corridor of I-81 was approximately 863 crashes, which is less than the total estimated using the SPF, and closer to the true total number of crashes. Using Figure 5.4, the locations with potentially higher risks for crashes become easier to identify, if it is assumed that these locations are represented by the highest peaks of the Empirical Bayes curve.

5.2.1. Benefit Assessment

In Section 4.2.2, Prioritization of Locations for Improvement, a methodology was constructed that could potentially demonstrate the benefit of using a modified version (adding $GN$ as an independent variable) of the FHWA SPF (Equation 4.10), in combination with an EB method to identify road segments with above acceptable annual average crash estimates that could benefit most from receiving some form of $GN$ improvement. However, it’s important to note that for some sections, the skid resistance is not low, but the crash risk is still high because of other contributing factors. For these sections, improving skid resistance may not be beneficial. Nevertheless, in general, the SPFs in Equation 5.1 through Equation 5.3, illustrate that the average crash risk should be expected to increase with decreases in $GN$. Therefore, it’s possible that if an HFS is placed (increasing $GN$ to 0.95), the expected average crash risk should decrease more than if an OLA is placed (increasing $GN$ to 0.7).

For this demonstration, the mean estimated number of crashes resulting from the surface friction treatments are examined along Virginia I-81 North (mile post 88 to 174), although for visual purposes, only a small window (ten miles) of this corridor is shown. Furthermore, for simplicity, HFS and OLA treatments are applied to the entire network, with the exception of road segments with current $GN$ equal or greater than the potential improvement ($GN$ of 0.7 for OLA, and $GN$ of 0.95 for HFS). For I-81 North from mile post 163 to 173, Figure 5.5 compares the
average estimated number of crashes that would be expected to occur after placing an OLA or an HFS. In Figure 5.6, the same ten mile stretch is used to illustrate the expected crash reduction for each treatment.

The results shown in Figure 5.5 and Figure 5.6 agree with the initial assumption regarding accident reduction. Both OLA and HFS treatments reduced the average expected number of crashes, although using an HFS treatment yielded a higher reduction in accidents. For the entire stretch of I-81 North from mile post 88 to 174, the $EB_{CURRENT}$ was approximately 863 crashes for the three year period. When placing an OLA, the new total average expected number of crashes was approximately 681 crashes (approximately 182 crashes prevented), whereas placing an HFS resulting in 506 crashes remaining (approximately 357 crashes prevented).

Figure 5.5: Comparing EB Estimates for GN Improvements for I-81 North.
5.2.2. Summary

If it is assumed that the posteriori distribution (gamma) acquired using the EB approach follows the same prior distribution (gamma) of the estimated SPF, it can be assumed that EB will estimate, with higher precision, the average number of crashes at the individual 0.1 mile segment along the I-81 Northbound corridor shown in Figure 5.4. In this figure, it’s visibly evident that the EB estimate is achieved by assuming that the true number of crashes to expect at any site will be somewhere between the values indicated by the SPF rate and the observed number of crashes, by reducing the observed number of crashes to a value closer to that provided by the SPF rate. If the EB frequency is considered a more reliable estimate, and if the corridor of I-81 shown in Figure 5.4 were then ranked in accordance with these estimates, mile post 168 would be a high priority area, with the highest level of crash risk.

Using the same EB approach and its governing assumptions regarding the prior and posterior distributions, the same corridor of I-81 can be evaluated according to the possible benefits to be received by applying one of two possible GN improvements used for this example, where the locations with the highest potential reductions in the expected number of crashes become the highest priority, and treatments can be selected according to a proceeding cost analysis (not covered in this report). Accordingly, based upon Figure 5.6, the pavement at mile post 168 should benefit from one of the two possible treatments.
6. SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

In order to monitor and manage the safety performance of any pavement network, there needs to be a method for utilizing crash history in order to identify road segments with a high crash risk. However, to utilize the crash history, the nature of crashes should be considered. Crashes are complex to model, because they occur as a result of interactions between human, vehicle, roadway and environmental factors. Crashes also occur randomly, which results in crash counts that may vary from segment to segment, or year to year, without influencing one another. Because of these two traits, crashes are often modeled as a function of several measurable roadway characteristics, using Poisson generalized linear modeling (41). Currently, the FHWA uses SPF's to calculate the expected number of annual crashes as a function of road segment length and AADT. However, surface texture has been identified as a key component of the pavement for providing skid resistance (GN), which is necessary to provide drivers with the ability to accelerate, brake, and steer their vehicles (1). This study: (1) extends the scope of the current SPF models to also consider the effect of GN, and (2) proposes a method for ranking and prioritizing road segments according to their expected crash risk.

6.1. FINDINGS

The main finding of this study is that crashes are Poisson distributed at each section, but at the network level, the variation of site characteristics results in data that is not Poisson distributed due to over-dispersion. Together, all of the Poisson estimates are gamma distributed, with parameters estimated using NB2 (SPF) regression. To construct the SPF models, goodness-of-fit testing can be performed using AIC and LLR. For all three network classifications, the results found AADT and GN to be statistically significant. For each of these network models (Equation 5.1, Equation 5.2, and Equation 5.3), as AADT increases, so should the expected number of crashes, and as GN decreases, the average number of crashes should be expected to increase. CV was not statistically significant for the interstate roads, but was found to be statistically significant for the primary roads. For the primary roads, the effect of CV was similar to that of GN, where a decrease in CV results in an increase in expected number of crashes.

6.2. CONCLUSIONS

When using AIC and LLR tests to evaluate the precision of model predictions, three or more models are compared, to determine the set of independent variables that are the best fit. Using AIC and LLR, GN was found to be statistically significant to the SPF models. An interpretation of this result suggests that, regardless of the route classification (interstate, primary, or secondary), when assessing the crash risk of any roadway the accuracy of the model prediction can be improved by adding GN as a variable.

When negative binomial regression is used to determine a prior distribution of the true crash rates, the EB approach allows for estimation of a posterior distribution (assuming the prior distribution is the same) to acquire a closer approximation of the true crash risk. This EB estimate (posterior estimate) can be used for preliminary screening of networks in order to identify high priority crash sites (locations with a high EB estimate) for further investigation.

When comparing the difference of the EB estimates of road segments before and after receiving a GN treatment, the potential benefit of receiving a treatment can be interpreted as the
difference in the EB estimates before the treatment and after the treatment. Following this, the locations with the highest difference in EB estimates (highest expected reduction in crash risk) should be labeled a higher priority for GN improvements.

6.3. Recommendations

In this study, the volume of crashes occurring when the pavement was wet, or those resulting in fatalities or injuries was too low to make accurate predictions, resulting in the need to assess all crashes combined. However, if the size of the network is increased, the amount of available data for different crash types should also increase. This increase in data could allow the crash rates for various crash types (i.e. wet pavement crashes, fatality crashes, injury crashes, etc.) to be evaluated.

Although the data for AADT and GN were available for the three route classifications, unfortunately, the data for CV was not available for the secondary routes. Nevertheless, if the quality of design for each route network is governed by the route classification (i.e., interstate or local road), the number of segments with horizontal radius of curvature should increase as one shifts from interstate routes to primary routes, and to secondary routes. Then for routes with lower design standards (e.g., secondary routes), the accident rates influenced by CV should be higher. Additionally, due to lower speeds and lower AADT, a higher number of the curves should have CV considered ‘severe’ (radius of curvature less than 1,600 feet). The results of our model regression demonstrated that for interstate routes, CV was not statistically significant, although CV was statistically significant in assessing crash risk for Primary routes. Although CV was not statistically significant to the interstate models, based on engineering reasoning, this conclusion does not absolutely confirm that CV doesn’t have an effect on accidents. Given the stated assumptions regarding the level of service decreasing based on network importance, if CV were collected for secondary routes, a higher number of sections should be expected to have horizontal curves, with radius below 1,600 feet. Therefore, for secondary routes, CV could be expected to have a greater impact on crash risk. In future research, if CV data was available for secondary routes, we believe that the precision of the SPF models could be improved.

For this report, the accuracy of the SPF models was improved when GN and CV were considered. However, if additional pavement properties (super-elevation, gradient, lane width, etc.) were collected, we believe that our current understanding of the relationship of common design features and crashes could be expanded. Using this enhanced database and an EB approach discussed in Section 5.2.1, combined with cost analysis (examined in future work), decision makers should be capable of performing a more in-depth preliminary assessment of their networks, in order to identify which treatments (for design characteristics apart from skid resistance) are most applicable for different crash sites.
REFERENCES


APPENDIX A: CREATING FIGURES USING POISSON AND NB REGRESSION

This section discusses how to construct Figure 4.1, Figure 5.1, Figure 5.2, and Figure 5.3. To model the Observed crashes group the crash counts into bins, and use a histogram to graphically illustrate the total number (‘the frequency’) of crashes for each bin. To create the thresholds for the model predictions, two parameters are used: the mean expected number of crashes ($\lambda$) and over-dispersion ($\alpha$). (Note: for Poisson regression, the over-dispersion is assumed to be zero). For each crash ‘bin’ segment (0, 1, 2, …, n crash observed at any segment), numbers are randomly generated using $\lambda$ and $\alpha$. For each set of randomly generated numbers (10,000 sets total), the frequency for each bin is computed. As a result, for each bin, there should be 10,000 different frequencies. For example, for this study, there were 12 bins (either 0 through 11 crashes were observed at any segment), and for each bin there should be 10,000 frequencies, and therefore 12 x 10,000 total frequencies.

Next, the total set of randomly generated frequencies are evaluated to produce the upper (Maximum) and lower (Minimum) limits, and the upper and lower 95% confidence interval. First, the highest and smallest frequency (from each randomly generated numbers) from each bin are selected as the Maximum and Minimum limits of the model predictions. Next, the Upper and Lower 95% confidence intervals are selected. When assessing frequency data, a direct approach to acquiring these intervals is to use quantiles (62). As an example, the quantile method will be demonstrated using the data for the Interstate network. The quantile method is performed in the following order:

1. For each bin, arrange the frequencies in ascending order.
2. Number the ordered bins 1 to 10,000, with the smallest being 1 and the largest being 10,000. See Figure A.1 for a visual example of this step performed on the bin associated with segments with zero crash observations.

Figure A.1: Cumulative Density Plot for Quantile Analysis
3. Compute the Upper and Lower Quantile. Where \( n \) is equal to 10,000. (Note: when \( \%\text{C.I.} = (1 - \theta) \times 100 \), then for a 95\% C.I., \( \theta = 0.05 \) (62).

\[
Q_{\text{Upper}} = \frac{n \times \left(1 - \frac{\theta}{2}\right)}{n + 1}; Q_{\text{Lower}} = \frac{n \times \left(\frac{\theta}{2}\right)}{n + 1}
\]  \( \text{(A.1)} \)

4. Locate the Upper and Lower C.I., by referencing the frequency associated with a numbered value equal to \( n \times Q_{\text{Upper}} \) (Upper 95\% C.I.) and \( n \times Q_{\text{Lower}} \).