

A METHODOLOGY FOR  
LOCATING FIRE STATIONS  
AT AIRPORTS

by

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## CHAPTER 1

### INTRODUCTION

Recently, a set of requirements were recommended to the Federal Aviation Administration (F.A.A.) for minimum fire fighting and rescue services at airports [8]. One of these regulations stipulates a maximum response time of three minutes for a fire fighting vehicle to reach the scene of a crash on any part of the runway system of an airport. This requirement will be in effect until January 1981 at which time a response period of two minutes is proposed. Another stipulation requires that if at least two fire stations are at an airport, they should be located so that the possibility of response routes being blocked from these stations to a potential crash site is minimized. The problem to be addressed in this research is the development of a methodology to aid airport planners in addressing these two requirements.

#### Description of Problem

There are several alternative means that can be utilized in meeting the recommended F.A.A. response guidelines. First, the fire department at the airport in question might obtain faster fire vehicles. Second, more access roadways can be built to connect runways and provide more direct vehicle response routes to potential crash sites. Also, an improved information system might be installed to decrease the period

of time between the sighting of an accident and the time at which fire and rescue services are alerted of its occurrence. A fourth alternative is to locate fire stations at strategic sites at the airport.

This fourth option is the approach that is taken in this research. In particular, the decision problem that is confronted is to determine the locations of the minimum number of fire stations necessary to meet the response time limit recommended to the F.A.A. In the determination, consideration is given to restrictions on the areas in which fire stations can be located. That is, stations must be located so that they will not be hazardous to moving aircraft. Therefore, areas such as the ends of runways or areas within a specified distance of runways or access roadways are not considered as possible location sites. Also, consideration is given to the second F.A.A. recommendation, so that stations are located in such a manner as to minimize the possibility of blockage of response routes to potential crashes by airline traffic either taxiing or landing on the runway system.

### Research Objectives

The objectives of the research are to formulate a mathematical model of the relevant decision problem, determine an effective solution approach, and apply the model and solution technique to the problem as it exists at the Atlanta airport.

### Decision Problem

A precise statement of the problem under investigation is as follows:

- GIVEN: (1) a set of existing fire stations (with one or more fire fighting vehicles each) at the airport in question;
- (2) the layout of the airport including the location of existing fire stations, the runway system, and the access roadways;
- DETERMINE: (1) the minimum number of fire stations required;
- (2) the locations of these stations;
- (3) the response time/response path relationship of the fire fighting vehicles on the runway and access roadway system;
- SUCH THAT: (1) at least one fire vehicle will be able to reach a potential crash anywhere on the runway system within a specified time period,  $t$ ;
- (2) each fire station is located no closer than a distance,  $d_1$ , from any runway and is a distance,  $d_2$ , from some access roadway, and is not in the approach path of any runway;
- (3) if an airport requires two or more fire stations, the possibility of fire vehicles' responding from station sites to potential crash sites on the same runways or access roadways is minimized.

### Importance of Research

The solution to the decision problem described above presents airport managers with an important planning aid. The most direct benefit is that a methodology is developed

whereby airports can save the cost of acquiring unnecessary fire stations to adhere to the previously mentioned recommendations to the F.A.A. It has been estimated that the cost of building and operating a single bay station is approximately \$20,000[8]. The same source has estimated the costs of two bay and three bay stations to be between \$30,000 - \$80,000 and \$70,000 - \$110,000 respectively. The apparent lack of economies of scale suggested by these figures is due to the fact that larger fire houses are frequently more elaborately constructed and furnished with more extensive facilities such as kitchen equipment.

Another aspect of the importance of the research is that a methodology is developed that provides airport regulatory agencies an aid in determining what an airport's safety standards should be. For example, the methodology can be used to determine fire vehicle response time requirements that are realistic from a capital investment standpoint.

Obviously, it would be favorable for fire vehicle response times to potential crash sites to be as small as possible. However, in order to reduce these times, a greater number of fire stations might be needed at an airport. In deciding upon what the maximum allowable response time should be, regulatory agencies must weigh the cost of building and operating a given number of fire stations against the amount of safety (i.e, the response times) that is provided by vehicles'

responding from them. For a given maximum response time the model can be used to determine how many stations are required so that this time is met. The cost of these stations can then be determined independently.

#### Nature of the Solution Procedure

A basic characteristic of the research problem is the requirement that a fire vehicle be capable of responding within a maximum response time to any point on the runway system where a crash might occur. This suggests that an infinite or continuous set of points (referred to as the demand space) must be covered by an airport's fire stations. In order for a point to be covered, it must be within the three (or two) minute response time capability of a fire vehicle responding from one of the stations at the airport. Another aspect of the problem that affects its solution is that there are an infinite number of possible alternative sites (i.e., the location space) to locate fire stations on the airport property. Therefore, the problem as it initially exists can be considered to be a facility location problem continuous in both demand and location space.

In the solution procedure developed in the research, this continuous location problem is transformed into a discrete location problem in such a way that the solution

to the discrete problem provides a solution to the original continuous one.<sup>1</sup> This transformation is made by changing the original continuous demand and location spaces into finite sets of points and structuring the problem into the form of a set covering model. The set covering model is appropriate since its structure involves determining the locations of the minimum number of facilities (from a finite set) that are required to provide acceptable service to a finite set of demand points. Alternative optima to this transformed set covering problem are found. Then constraint number 3 mentioned previously in the decision problem statement is considered. The solution to the cover problem that satisfies this constraint is considered to be the global "best" solution to the research problem. A more detailed description of the solution procedure is offered in Chapter 4.

#### Scope of Research

A third guideline was recommended to the F.A.A. along with those previously mentioned [8]. This guideline specifies that second-due fire vehicles should be capable of arriving at the scene of a crash within 60 second intervals following the response time limit set for the first arriving vehicle. It is hoped that a solution to the research problem

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<sup>1</sup>Because of the method used to determine if runway segments (i.e., the elements of the discrete demand space) are covered, there is a case in which the solution to the discrete problem is not the solution to the continuous problem solution. This case is discussed on page 120.

in which only the first two mentioned guidelines are considered, will also satisfy this third recommendation. However, no guarantee that this will occur is offered.

Also, no consideration is given to locating fire stations that will answer to structural fires on the airport property. In this case, it is assumed that if fire stations are located so that all points on the runway system are covered, then they will be capable of providing acceptable coverage for fires that might occur in the airport's buildings.

The methodology that is developed is general enough to apply to any airport. To illustrate its utility, it is used to solve the problem as it exists at the Atlanta Airport.

### Overview of the Thesis

The remainder of the thesis is divided into five chapters. Chapter 2 discusses literature related to the research problem, while the assumptions of the procedure and the formulation of a mathematical model are provided in Chapter 3. In Chapter 4, the research problem solution procedure is described in detail and a flow chart of this procedure is presented. Chapter 5 discusses the application of the methodology to the Atlanta Airport and the results that were determined. Finally, Chapter 6 summarizes the utility of the solution procedure and exposes its basic weaknesses or limitations. Included in this chapter are recommendations for further research.

## CHAPTER 2

### LITERATURE SURVEY

In this chapter, selected articles from the literature that is relevant to the research are discussed. The discussion is divided into three sections. The first section is concerned with a report on a study of fire fighting and rescue services at airports. The next section provides a discussion of literature that pertains to emergency facility location problems and approaches that have been used to analyze them. The articles that are presented in this section primarily deal with fire station location problems in urban areas. Finally, in the third section, selections from the literature concerning covering problem solution approaches are discussed.

#### Fire Fighting Service Needs at Airports

The literature on fire fighting services at airports is relatively extensive. However, the literature, with which the research is concerned, is well covered by a report that was presented by Cohn et.al.[8]. This report is a result of a study that was conducted of fire fighting services and the major needs for these services at airports across the United States. In the analysis, various existing F.A.A. safety regulations are presented along with basic information on the services that are currently offered by airport fire departments. Included in the report are summaries of such information as fire vehicle



capabilities and response speeds, equipment and manpower costs, and historical data on airplane crashes and fires.

As a result of the study, a number of recommendations concerning airport fire fighting and rescue needs were made to the F.A.A. Three of these recommendations that are of special relevance are as follows:

Sec. 12. Until January 1, 1981, a maximum response time of 3 min. shall be demonstrated at every airport by means of an unannounced drill for a simulated crash at the farthest point for aircraft movements, timed from the moment of "impact" to arrival at the crash scene of the first approved crash fire vehicle and the minimum number of trained personnel required to operate it. After January 1, 1981, a response time of 2 min. shall be demonstrated.

Sec. 14. Subsequent approved crash fire vehicles required as a minimum and the manpower required to operate each one shall arrive at the crash scene at not greater than 60 sec. intervals following the time limit set for the first vehicle.

Sec. 16. Fire stations housing the minimum required fire vehicles shall be located so as to assure response time in accordance with Sec. 12. At airports providing services under Index numbers VIIb through IX (i.e., airports serving larger airplanes ranging from 155 ft. to over 200 ft. in length), there shall be at least two fire stations so located as to minimize the possibility of response routes from both stations being blocked at the same time.

Two of these recommendations (i.e., sections 14 and 16) are the guidelines that provide an initial basis for the research decision problem of this thesis.

#### Emergency Facility Location

A considerable amount of attention has been given to problems concerning the location of emergency facilities.

Generally this work either deals with or can be applied to 1) locating or relocating fire stations, health care units, police patrol units, or rescue squad units in urban areas, 2) defining response areas for these emergency facilities, or 3) determining emergency travel times and their relationships to travel paths and travel distances. To attempt to discuss all of the literature concerning emergency facility location could be a rather lengthy task. Therefore, the majority of the references cited in this section deal only with selected articles that relate to fire station and fire vehicle location problems and approaches that have been taken to solve them. It should be pointed out at the beginning that the author has been unable to find any references that directly relate to fire station location problems at airports. Thus, the articles that are reviewed are concerned with location problems in urban areas.

One approach for locating fire stations in an urban area was developed by Hogg[11]. The approach involves an allocation procedure that is designed to minimize the total time spent by fire vehicles' traveling from  $n$  specified station sites to fires in a given city. The procedure can be separated into five steps. First, the city is divided into  $m$  sub-areas and a centroid point is defined for each area. Next, the average travel time,  $t_{ij}$ , from each station site  $i$  to the centroid point of each sub-area  $j$  is determined. In the third step, the number of fire vehicles that are required to fight fires within each sub-area are calculated from historical

data on past fires in the city. The fourth step involves the assignment of fire vehicles to protect the sub-areas. This is done by allocating vehicles to each sub-area, such that the vehicles that are assigned to protect a given sub-area  $j$ , respond from the station site  $i$  that is closest (in travel time) to the centroid point of  $j$ . Finally, in the fifth step, after all  $m$  sub-areas have been allocated vehicles, it is determined if each area has the number of vehicles allocated to it that it requires. If not, the additional number needed are assigned to the area from the next closest station site to its centroid point.

Once each sub-area has the number of vehicles allocated to it that it requires, the allocation procedure is complete, and the number of vehicles,  $d_{ij}$ , allocated from site  $i$  to sub-area  $j$  can be determined. The total time spent by vehicles' traveling to fires is represented by  $TD'$ , where

$$TD' = \sum_{i=1}^n \sum_{j=1}^m t_{ij} d_{ij}.$$

Hogg suggests that the allocation procedure might be applied in an iterative manner. On the first iteration,  $n$  station sites are allocated as just described. On the second iteration, the procedure is repeated for each combination of  $(n-1)$  sites, and the optimal combination, that provides the minimum value for  $TD'$  is determined. For the third iteration allocations are made for all combinations of  $(n-2)$  sites and the best combination is found. The iterations continue by

constantly decreasing the number of sites from which vehicles are allocated and determining the combination that minimizes TD'. At each iteration the minimum value for TD' will increase as fewer stations are considered. Hogg states that fire departments can use these minimum travel times in determining whether to build additional fire stations. For example, suppose that a city has  $r$  fire stations ( $r < n$ ). If the allocation procedure is used by the city's fire department, the department can determine what the reduction in travel times will be if additional stations are built, and decide if the times decrease enough to warrant the extra facilities.

Another development in the location of fire protection facilities is due to a SIMSCRIPT I.5 simulation, modeled by Carter and Ignall[4]. The model is designed for comparing and evaluating various operating policies and deployment strategies of the New York City Fire Department. Some of the policy decisions that can be made in the simulation are:

- 1) the number and type of fire vehicles to send to particular incidents;
- 2) whether or not to relocate fire vehicles to empty fire stations, and if so, which type of vehicles (i.e., engines, ladder trucks, etc.) should be relocated where;
- 3) where to relocate new fire stations and, if possible where to relocate existing ones.

Others who were concerned with similar policy decisions are Kolesar and Walker[16] who describe a real time information and control system that is used to relocate fire companies to

exposed regions (i.e., regions in which all the companies in the area are busy answering fire alarms) in New York City. Formerly, the system, used to perform this relocation, was devised based on the assumption that alarm rates would be low in a region and only one fire at a time would be in progress in the city. The new system is said to overcome the deficiencies of the previous system which failed at the time it was most needed, when alarm rates were high.

The objective of the new system is to ensure that a minimum coverage standard is met by the fire stations in the city. In order for this standard to be satisfied, at least one of the three closest fire engines and at least one of the two closest ladder trucks must be available for every alarm box in the city. In a circumstance when the standard is not met, the system determines an optimal strategy for relocating fire companies and satisfying the coverage requirement. The optimal relocation of companies is determined such that 1) the total travel response times of vehicles is minimized within the city and 2) all "first due" vehicle response times to areas in the city are balanced as equally as possible.

An algorithm used to specify the best relocation is broken down into three basic stages. In an emergency situation, it is first determined which fire houses should be filled to satisfy the minimum coverage criterion. Next, the procedure designates which available companies should be relocated. Finally, the specific relocation assignments of these companies

to the empty fire houses are made. Kolesar and Walker state that the best moves to make can be found by solving three separate zero-one integer programming problems, one for each stage. However, the computation time and computer memory required to determine solutions to these problems exceeds that which can be afforded by the on-line system that is used. Therefore, a more efficient heuristic algorithm is used to specify the relocation of companies.

An application of the set covering problem was developed by Walker[28] to locate two types of trucks in New York City's fire houses. The objective of the problem is to locate the fewest number of trucks in empty houses such that no ladder truck is assigned to a house that is a neighbor of a house with a tower truck located in it. Two additional constraints are imposed on the problem. First, some houses, due to physical characteristics, cannot receive tower trucks. Second, certain houses must be assigned ladder trucks. In order to solve the cover problem, Walker uses part of the heuristic procedure of the on-line engine relocation system that was just discussed.

Other research that is related to the location of fire department emergency facilities is presented by Carter, Chaiken, and Ignall[3]. A methodology is offered that specifies response areas for two urban emergency units within a designated region, when the spatial distribution of alarm rates and the home location of each unit is known. It is assumed in the methodology that a unit responds to alarms in its own district only when

it is available and not answering another call. If the unit is busy, the second unit responds to the alarm. If both units are busy the call is answered by another location. Under these assumptions a boundary line is determined between the home locations of the two units by minimizing the average response time to alarms within the designated region and by equalizing the workloads (i.e., the total time a unit is busy) of the units.

Finally, Kolesar[14] has formulated a model for predicting fire engine travel times. The model is based upon two earlier models that he helped to develop. The first of these models determines expected response distances for fire vehicles in a city[15] while the second relates expected vehicle travel time to response distance[17]. Through the use of these two models, Kolesar states that fire engine travel times can be predicted within a city. The expected travel time, ET, of the closest responding engine for a given region of a city is said to be

$$ET = \alpha + \beta [A / (n - \lambda ES)]^\gamma$$

where

A is the physical area of the region;

n is the number of fire engines stationed there;

$\lambda$  is the expected number of alarms received per hour;

ES is the expected total service time of fire engines traveling to and working at an alarm;

$\alpha$ ,  $\beta$ , and  $\gamma$  are estimated parameters that are dependent upon the physical characteristics of the region.

By using regression analysis and the simulation model mentioned in reference [4], the model was validated for New York City. It has been used in studies involved with fire engine deployment strategies within the city.

### Cover Problem Solution Approaches

The approaches that have been taken to solve the set covering problem can be grouped into four categories. In the discussion that follows an example solution procedure from each category is presented. First, however, it is helpful to provide a model of the cover problem.

Let

$$x_j = \begin{cases} 1, & \text{if a facility is located at site } j; \\ 0, & \text{otherwise (for } j=1, \dots, n); \end{cases}$$

$$a_{ij} = \begin{cases} 1, & \text{if demand point } i \text{ is covered by} \\ & \text{site } j; \\ 0, & \text{otherwise (for } i=1, \dots, m; j=1, \dots, n). \end{cases}$$

Then, generally, the set covering problem can be formulated as follows[10]:

$$(P1) \quad \text{Minimize } z = \sum_{j=1}^n c_j x_j$$

$$\text{subject to } \sum_{j=1}^n a_{ij} x_j \geq 1, \quad i=1, \dots, m$$

$$x_j = (0,1), \quad j=1, \dots, n.$$



A common variation of (P1) is the "total cover problem," (P2), in which  $c_j = 1$  for all  $j$  [10].

$$(P2) \quad \text{Minimize } z = \sum_{j=1}^n x_j$$

$$\text{subject to } \sum_{j=1}^n a_{ij} x_j \geq 1, \quad i=1, \dots, m$$

$$x_j = (0,1), \quad j=1, \dots, n.$$

One solution approach to the total cover problem is to add a supplementary single cut constraint to the problem and iteratively solve it through the use of linear programming. Toregas, Swain, ReVelle, and Bergman [27] have developed a cutting plane method using the MPS/360 package (an IBM mathematical programming code) to solve (P2) by adding the cut

$$\sum_{j=1}^n x_j \geq [m^0] + 1,$$

where  $[m^0]$  is the optimal fractional objective function value for (P2). In solving the problem with this constraint, the authors report that a non-optimum (0-1) solution has not been encountered. More recently however, Rao [23] has provided two examples to the contrary. In reply, Toregas, et.al. [26] offer certain remedies to these cases but fail to give any qualifiers that can be generally applied to the problem.

Another method for solving (P2) is the use of reduction techniques [1,19,25]. Roth [24] discusses a computerized procedure for selecting essential columns of the constraint

matrix of a cover problem, and for identifying dominant rows and columns. Once these selections and identifications are made, redundant rows and columns are deleted, resulting in a reduced cover matrix. Roth uses the following terminology to explain the reduction procedure:

Let

$$R_i = \{j: a_{ij} = 1\} = \text{ith row index set.}$$

$$C_j = \{i: a_{ij} = 1\} = \text{jth column index set.}$$

- (1) Essential Columns. Column  $j$  is essential iff there exists a row  $i$  such that  $R_i = \{j\}$ . In this situation, row  $i$  can be covered only by column  $j$ . Therefore column  $j$  may not be deleted from the matrix. However, all rows in  $C_j$  may be removed.
- (2) Row Dominance. If  $R_i \subseteq R_j$  for two rows  $i$  and  $j$  then row  $j$  may be deleted from the matrix since any column covering row  $i$  also covers row  $j$ .
- (3) Column Dominance. If  $C_\ell \subseteq C_j$  for two columns  $\ell$  and  $j$  then column  $\ell$  may be deleted from the matrix since column  $j$  covers at least all the rows that column  $\ell$  covers.

As an illustration of Roth's reduction procedure, consider an example cover matrix for (P2) as shown below.

		Facility			
		1	2	3	4
Demand Point	1	1	1	0	0
	2	0	0	1	0
	3	1	1	1	0
	4	1	1	0	1
	5	1	0	0	1

In this case,

$$\begin{array}{ll}
 C_1 = 1, 3, 4, 5 & R_1 = 1, 2 \\
 C_2 = 1, 3, 4 & R_2 = 3 \\
 C_3 = 2, 3 & R_3 = 1, 2, 3 \\
 C_4 = 4, 5 & R_4 = 1, 2, 4 \\
 & R_5 = 1, 4.
 \end{array}$$

Since column 3 is essential, rows 2 and 3 can be eliminated; since  $R_1 \subseteq R_4$ , row 4 can be deleted; since  $C_4 \subseteq C_1$  and  $C_2 \subseteq C_1$ , both columns 4 and 2 can be deleted from the matrix.

When no further deletions can be made, Roth randomly selects columns from the reduced matrix to determine feasible solutions to the problem. The feasible solution that consists of the minimum number of columns is identified as being the optimal solution to the problem.

The third category of approaches for solving the set covering problem is through the use of implicit enumeration procedures and bound methods [19,20,21,22]. Khumawala[13] provides a branch and bound procedure with simplifications that can significantly reduce the size of the search for solving (P2) (i.e., the evaluation of  $2^n$  combinations). In the algorithm, a branching decision is made to assign free facility "j" to be either open, by setting  $x_j = 1$ , or closed, by setting  $x_j = 0$ . A free facility is defined as being a facility that has not been designated as being either closed or open. The branches along which  $x_j = 1$  and  $x_j = 0$  are respectively called the open and closed branch.

Feasibility checks are made at closed branch nodes. When infeasibility is encountered, further branching ceases from that node. By utilizing a branching rule that may lead to an infeasible node, a situation is created in which further branching from that node is unnecessary.

Terminal nodes are defined as being nodes at which there are no more free facilities to assign. Non-terminal nodes, on the other hand, are nodes at which there is a branching decision that can be made. For any node, the bound is the number of facilities that have been designated as being free at that node. Upper bounds are updated by solutions at terminal nodes while at non-terminal nodes, lower bounds are updated. When the current lower bound is greater than or equal to the upper bound, the procedure stops with the current upper bound being the optimal solution.

The fourth category of cover problem solution approaches in the literature is the use of heuristic methods. Ignizio[12] has developed a heuristic procedure to provide "good" solutions to (P2). Basically, the procedure consists of two routines. The first routine iteratively selects one facility location at a time and adds it to a set,  $\theta$ , of previously selected facility sites. The site that is chosen at each iteration is the facility that covers the most demand points that are not covered by the set of locations in  $\theta$ . Sites continue to be added to  $\theta$  until all demand points are covered by the

facility sites in the set and a feasible (although possibly not optimal) solution to (P2) is found.

In the second routine, an attempt is made to improve upon the solution provided by the facility sites in the set  $\theta$ . Since facilities are added to  $\theta$  one at a time in the first routine, it is possible for some combination of facility sites in the set to eliminate the need for a site that previously had been added. To improve upon the solution provided by the sites in  $\theta$ , the second routine eliminates facilities from the set that are not necessary due to the existence of other combinations of sites.

The final set,  $\theta$ , of facility sites that is generated by employing the two routines is said to be the optimal solution to (P2). Ignizio reports that in applications of the procedure to many test problems, the true optimal solution was found 85% of the time. On the other hand, when the true optimum was not found, the derived solution differed from it on the average only by .2%.

## CHAPTER 3

### MATHEMATICAL MODEL FORMULATION

In Chapter 1 the decision problem being addressed in the research was stated and a general discussion of the methodology that is used to provide a solution to it was presented. The purpose of this chapter is to offer a mathematical representation of that problem. However, before this model is presented, the major assumptions made in the solution procedure and the underlying rationale involved in the development of the model are discussed.

#### Assumptions of the Solution Procedure

In formulating a mathematical representation of the research decision problem and developing a solution procedure, several assumptions were made. In general these assumptions are the author's interpretation of the fire fighting system at an airport. These assumptions are as follows:

1. Network Formulation of the Problem. It is assumed that an airport's runway system can be represented by a network with its nodes being selected critical points on the runway system and its branches being the segments of runway that lie between these points. The length of each branch is the length of the runway system that it represents.

It is also assumed that the shortest route a fire vehicle can travel from one critical point to another on the runway system is represented by the shortest path between the two network nodes that correspond to these two critical points. That is, by determining the shortest path between each pair of nodes of the network, the shortest route between each corresponding pair of critical points on the runway system is found.

2. Location Space Formulation. It is assumed, in formulating possible location areas for fire stations, that stations can be located only at points on the airport property that are some constant distance,  $d_2$ , from a taxiway or access roadway. These points must be at least a distance,  $d_1$ , from any runway and cannot be in the landing approach or take off paths of any runway. In addition, station locations cannot interfere with auxiliary airport facilities, such as navigation aids.
3. Fire Vehicle Response Route Determination. In determining the route a fire vehicle takes in responding from its station location to a potential crash site on the runway system, it is assumed that the vehicle travels only on the

paved runway system (i.e., runways, taxiways, and access roadways). In entering onto the runway system, a vehicle travels from its fire station site in a path perpendicular to the taxiway or access roadway to which this site is closest. It is also assumed that the route a vehicle takes to a crash site is the shortest route from its entry point onto the runway system to the crash site.

4. Fire Vehicle Response Speeds. In responding from any station location, it is assumed that fire vehicles travel a constant speed,  $s$ , to a potential crash anywhere on the runway system. This speed is constant for any distance a vehicle travels and for any number of turns it must make in transit to a crash site.
5. Determination of Minimal Blockage of Response Routes. For two or more station sites, it is assumed that the possible blockage of the response routes of vehicles' traveling from these sites to a potential crash on the runway system is dependent on the uniqueness of these routes. The uniqueness of these routes is based on the number of critical points that are common to the paths the vehicles take in traveling to the crash site. A critical point is common to



two response routes if vehicles, in transit from their respective fire stations to the same crash site, travel through that point on the runway system.

A major assumption of the model is that the fewer total common critical points in the response routes from two or more fire stations to the critical points on the runway system, the less chance that response routes will be blocked to potential crash sites anywhere on the system.

#### Transformations to Discrete Space

As mentioned in Chapter 1, the problem as it exists at an airport is one of locating the fewest number of fire stations in a continuous location space such that all of the points in a continuous demand space (i.e., the entire runway system) can be reached (or covered) within a specified time period,  $t$ . As part of the solution procedure, both the location space and the demand space are transformed into discrete spaces, each with a finite number of points. The rationale for these transformations is described below.

#### Demand Point Specification

In order to specify a finite set of demand points, critical points are designated in extreme areas of the runway system.

These extreme areas are defined to be regions at the ends of the runways and at intersections on the runway system (e.g., intersections of runways, taxiways, and access roadways). The runway system can be visualized as a network with the defined critical points being nodes and the segments of the system between these points being the arcs or branches connecting the nodes. Figure 1 illustrates such a network formed by the critical points of a simple runway system. The network branches, represented by the broken lines in the figure, connect the nodes as shown in Table 1.

By specifying critical points in this manner, a discrete demand space with a finite number of points can be defined. In particular, the runway segments represented by the branches of the network are considered to be the demand points of the location problem by designating each segment to be a "point" that must be covered. In terms of the decision problem, each segment formed by the critical extreme points of the runway system must be covered within two (or three) minutes by one or more vehicles responding from at least one of the airport's fire stations.

To illustrate two different ways in which a runway segment, bounded by critical points X and Y, can be covered within two minutes, Figures 2.a and 2.b are offered. Figure 2.a presents the simple case in which a single vehicle responds

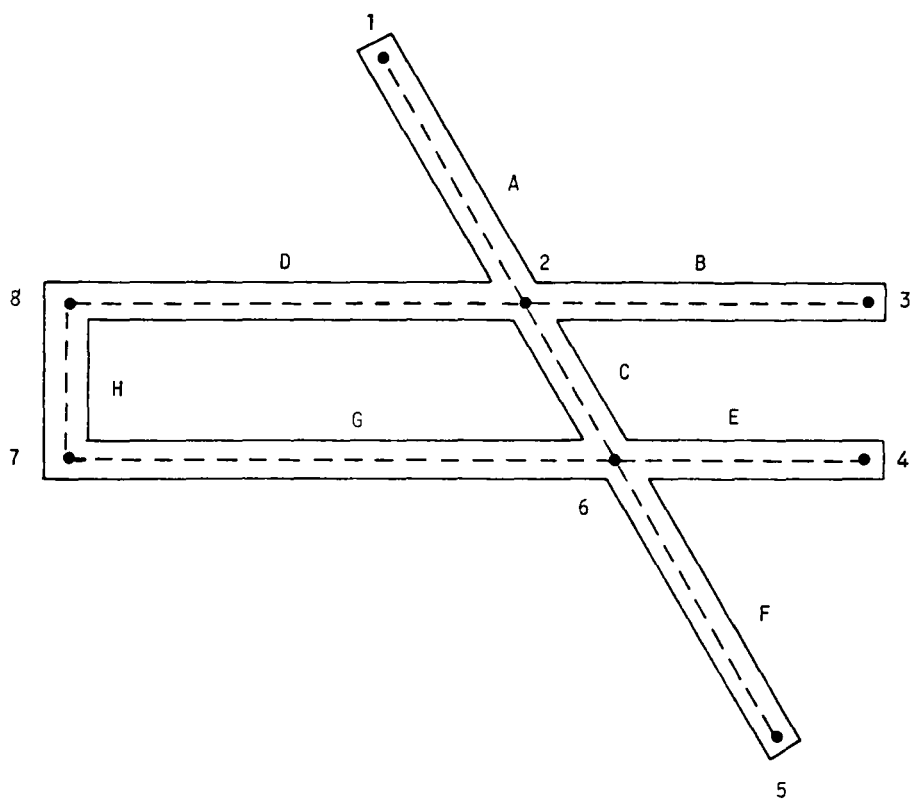
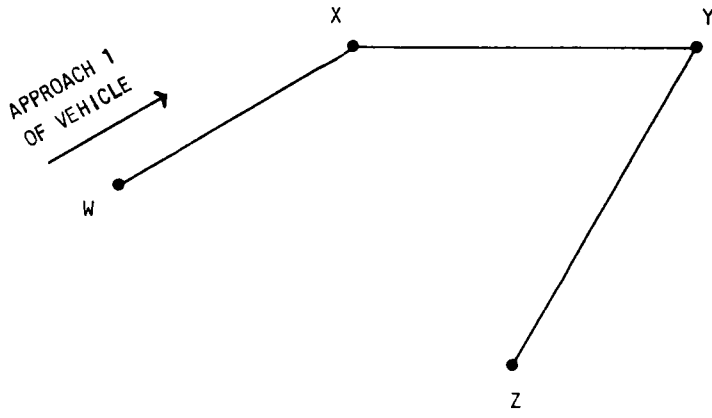


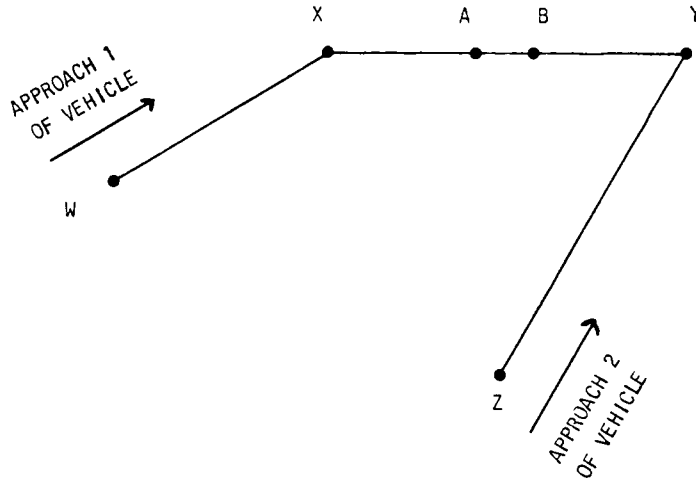
Figure 1. Network Interpretation of a Simple Runway System with 8 Critical Points

Table 1. Definition of the Branches of the Network in Figure 1.

<u>BRANCH</u>	CONNECTS	<u>NODE</u>	AND	<u>NODE</u>
A		1		2
B		2		3
C		2		6
D		2		8
E		4		6
F		5		6
G		6		7
H		7		8



(a)



(b)

Figure 2. Methods of Covering a Runway Segment  $\overline{XY}$ .

from the location of its station site and approaches runway segment  $\overline{XY}$  along segment  $\overline{WX}$ . Suppose a vehicle can travel from the location of its fire station and reach the point X within a period, say  $t < 2$  minutes, and can travel the distance between X and Y in some period,  $t' \leq (2 \text{ minutes} - t)$ . Runway segment  $\overline{XY}$  is then considered to be covered by the site from which the vehicle responds.

Figure 2.b illustrates an alternative method in which segment  $\overline{XY}$  can be covered. In this case suppose that a vehicle, responding from its fire station, can approach  $\overline{XY}$  either along segment  $\overline{WX}$  (i.e., approach 1) or  $\overline{ZY}$  (i.e., approach 2). Furthermore, suppose that by taking approach 1 the vehicle can only travel to point B within a 2 minute period and not reach point Y. Similarly, assume that by approaching  $\overline{XY}$  from point Z the vehicle can travel to point A but cannot reach point X within 2 minutes. Runway segment  $\overline{XY}$  is then considered to be covered by the vehicle's station since any point on the segment can be reached within 2 minutes by that vehicle.

#### Location Space Restrictions

In transforming the original continuous location space into one that is discrete, it is necessary to consider the restrictions as to where fire stations can be located on the airport property. It is recognized that there are certain areas on this property in which fire stations cannot be

located. In particular, it is undesirable to have such structures located in the approach paths of aircraft, landing at the ends of runways, or within a distance, say  $d_1$ , from any runway.<sup>2</sup> Fire stations located within these areas would cause unnecessary threats to the safe movement of aircraft in the process of landing or taking off. Another restriction on the location space is that fire stations can be located only at points that are a constant distance from a taxiway or access roadway. Furthermore, no station can be located where an auxiliary airport facility such as a navigational aid exists. (Recall that all of these restrictions were stated in the assumptions of the solution procedure.)

With these constraints taken into consideration, the location space for the decision problem can be defined by a restricted continuum. This restricted continuum is composed of the set of line segments that run parallel to and are a constant distance,  $d_2$ , from each taxiway or access roadway segment.<sup>3</sup> In determining this set, no part of the continuum can be in the approach paths of the runway system or within a distance,  $d_1$ , from any runway. In addition, no location on the airport where a navigational aid or other structure exists can be a part of the continuum.

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<sup>2</sup>This distance can vary from airport to airport and within the same airport. Therefore it can be left to the discretion of the planners of the airport in question to determine what  $d_1$  will be.

<sup>3</sup>The distance  $d_2$  can also vary (like the distance,  $d_1$ ) from airport to airport and it is left for an airport's planners to determine, what this distance will be.

As an illustration of the location continuum for a simple runway system, Figure 3 is presented. Notice in the illustration that the system consists of two runways and two taxiways connected by access roadways, with the distances  $d_1$  and  $d_2$  being 750 feet and 150 feet, respectively. The location space is represented by the broken line running parallel to the length of taxiway B and the broken line at one end of this part of the runway system. Notice that there are no other line segments that can be added to this restricted continuum without violating the definition for this space.

Once the restricted location space is defined for an airport, station sites can be located anywhere within this space (i.e., anywhere along the location lines). In choosing these locations, a discrete space is formed with a finite number of possible station sites for solving the decision problem. These sites are determined by considering the critical points of the runway that were defined before formulating the discrete demand space. For each critical point an attempt is made to locate a station site along each line segment within the restricted continuum such that the time it takes for a vehicle to travel from this site location to the critical point is exactly two minutes. The route that the vehicle takes is the shortest route from its station site to the critical point. A more detailed description of the manner in which sites are located and the location space is made discrete is provided in Chapter 4.



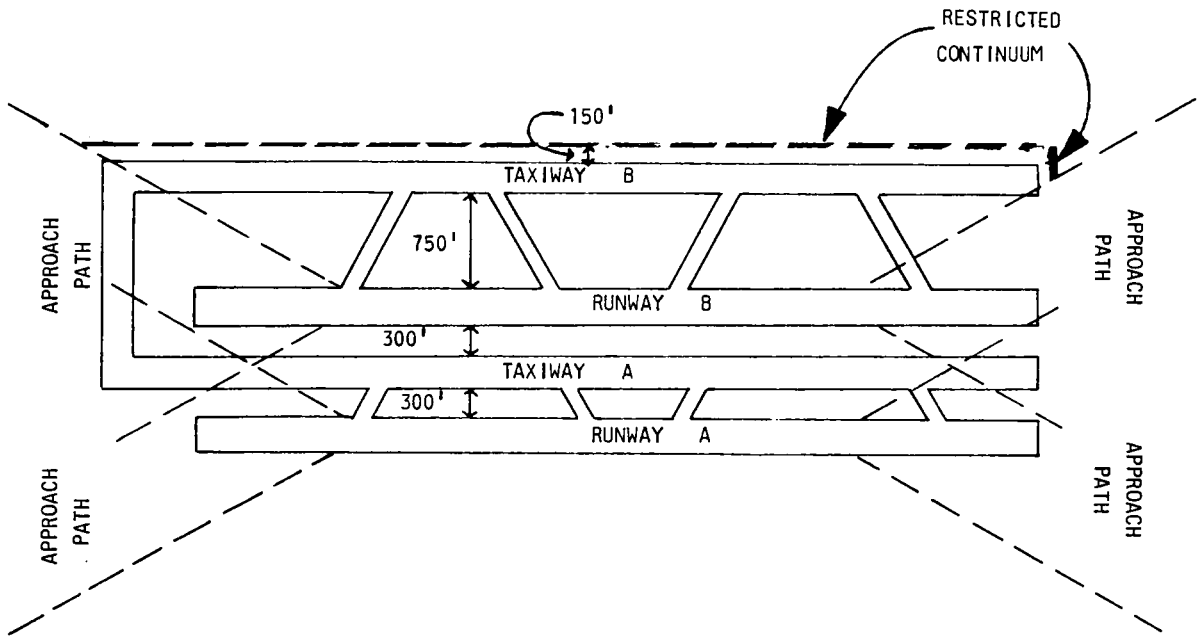


Figure 3. Formation of Restricted Continuous Location Space for a Simple Runway System.

### The Transformed Discrete Problem

After the original continuous demand and location spaces have been transformed into finite sets of points, it is possible to formulate a discrete version of the continuous location problem at an airport. This discrete version is structured in the form of a modified set covering model. The actual formulation of the cover model, using the transformed spaces is presented in the next chapter.

The solution to the discrete formulation solves the continuous problem in all cases except two. For the first case, this is due to the procedure that is used to formulate the set covering model and not a result of the manner in which the continuous spaces are made discrete. In the second case, it is possible for the continuous and discrete solutions to be different, however, this case is not expected to occur when the procedure is applied to a large airport. A more extensive discussion of these two cases will be presented later.

### Mathematical Model

As an aid in describing the decision problem of this research, a mathematical representation of the problem is now presented.

Let

$\{S_j\}$  = the set of possible location sites,  $j$ , on the restricted continuum. The restricted continuum consists of those lines that are a distance,  $d_2$ , from a taxiway or access roadway and at least a distance,  $d_1$ , from each runway on the airport property;

$\{S_1\}^c =$  the set of location sites not in  $\{S_1\}$ ;

$\{K\} =$  the finite set of critical points defined by the extreme points of the runway system (i.e., the set of nodes of the network representing the runway system);

$\{L\} =$  the set of segments of the runway system that connect the critical points of  $\{K\}$  (i.e., the set of branches of the network);

$x_j = \begin{cases} 1, & \text{if a fire station is located at site } j, \\ 0, & \text{otherwise} \end{cases}$

$a_{ij} = \begin{cases} 1, & \text{if a fire vehicle responding from station site } j \text{ can reach runway segment } i \text{ and traverse its entirety within } t \text{ minutes} \\ 0, & \text{otherwise (for } j \in \{S_1\} \text{ and } i \in \{L\}); \end{cases}$

$b_{jkl} = \begin{cases} 1, & \text{if a vehicle, responding from station site } j \text{ to critical point } k, \text{ travels through critical point } l \\ 0, & \text{otherwise (for } j \in \{S_1\}, k \in \{K\}, \text{ and } l \in \{K\}); \end{cases}$

then a formulation that mathematically represents the decision problem is as follows:

$$(M1) \text{ Minimize } z = \sum_{j \in \{S_1\} \cup \{S_1\}^c} x_j \quad (\text{Eq. 3.1})$$

and

$$\text{Min } y = \sum_{k \in \{K\}} \sum_{l \in \{K\}} \sum_{j \in \{S_1\}} \sum_{\substack{n \in \{S_1\} \\ n \neq j}} b_{jkl} b_{nkl} x_j x_n \quad (\text{Eq. 3.2})$$

subject to (1) all points on the runway system can be reached by a fire vehicle within  $t$  minutes, i.e.,

$$\sum_{j \in \{S_1\} \cup \{S_1\}^c} a_{ij} x_j \geq 1, \text{ for } i \in \{L\};$$

- (2) each fire station is no closer than a distance,  $d_1$ , from any runway and is located a distance,  $d_2$ , from either a taxiway or access roadway, i.e.,

$$x_j = 0 \text{ if } j \in \{S_1\}^c;$$

- (3)  $x_j = (0,1)$  for  $j \in \{S_1\} \cup \{S_1\}^c$ .

If Eq. 3.2 in (M1) is ignored, the formulation is reduced to a set covering problem, say model (M2), with discrete demand points and discrete facility sites. However, the second part of the objective function is included in the model formulation to satisfy the guideline for minimizing response route blockage from station sites. It is desirable that, in responding to areas of the runway system where a crash might occur, fire vehicles do not travel on the same parts of the runway system. That is, the chance of response routes being simultaneously blocked by other airport traffic is less if these paths to crash areas are unique.

This blockage consideration involves the use of the criterion given in Eq. 3.2. Specifically, for each alternative optima to (M2), the number of common critical points in the response routes from the station sites in the solution to each critical point is determined. However, with the solution sets generated for (M2), it may not be possible to have completely unique routes in all cases. Therefore, the alternative optimum that minimizes Eq. 3.2 is chosen to be the final solution

to the decision problem (M1). A more elaborate description of the procedure is provided in Chapter 4.

### Summary

In this chapter, the formulation of a mathematical model that represents the decision problem of the research was presented. As previously mentioned, the decision problem as it initially exists is a continuous facility location problem. This chapter has provided general discussions on the manner in which this problem is transformed into a discrete facility location problem. The mathematical model, itself, resembles a set covering problem that is discrete both in demand space and location space. In the discussions on these transformations from continuous to discrete space, mention was given to the procedure that is used to solve the decision problem along with some of the major assumptions upon which the procedure is based. These assumptions and the solution procedure are discussed in greater detail in the following chapter.

## CHAPTER 4

### SOLUTION PROCEDURE

Previously, in chapters 1 and 3, relatively general descriptions of the solution procedure utilized in this research were presented. The purpose of this chapter is to provide an in-depth discussion of this methodology. The procedure is divided into five major steps. In the following discussion, each of these steps is explained and a flow chart of the overall procedure is presented. Finally, a description of a computer program that affects the methodology is provided.

#### Procedure Description

In this section, the individual steps of the solution procedure are discussed. However, before these steps are listed, it is helpful to present, once again, some of the previously defined notations.

Let

$\{S_1\}$  = the set of possible location sites,  $j$ , on the restricted continuum. The restricted continuum consists of those lines that are a distance,  $d_2$ , from a taxiway or access roadway and at least a distance,  $d_1$ , from each runway on the airport property;

$\{K\}$  = the set of critical points defined by the extreme points of the runway system;

$\{L\}$  = the set of segments of the runway system that connect the critical points of  $\{K\}$ .

Using this notation, the basic steps involved in the procedure are as follows:

- Step 1. Transform the original continuous demand space into a discrete space; i.e., define  $\{K\}$  and  $\{L\}$ .
- Step 2. Transform the original continuous location space into a restricted continuum. Define the discrete location space (i.e.,  $\{S_1\}$ ) along this continuum.
- Step 3. Model the transformed problem as a set covering problem; i.e., obtain a numerical version of (M2).
- Step 4. Determine optimal solutions for the cover problem, (M2).
- Step 5. From these alternative optimal solutions of (M2), select the one that minimizes the possibility of blockage of response routes to all points in  $\{K\}$  to be the solution to the research decision problem, (M1).

A description of each of these steps and the logic that is involved in them follows.

#### Step 1 - Formation of a Discrete Demand Space, $\{L\}$

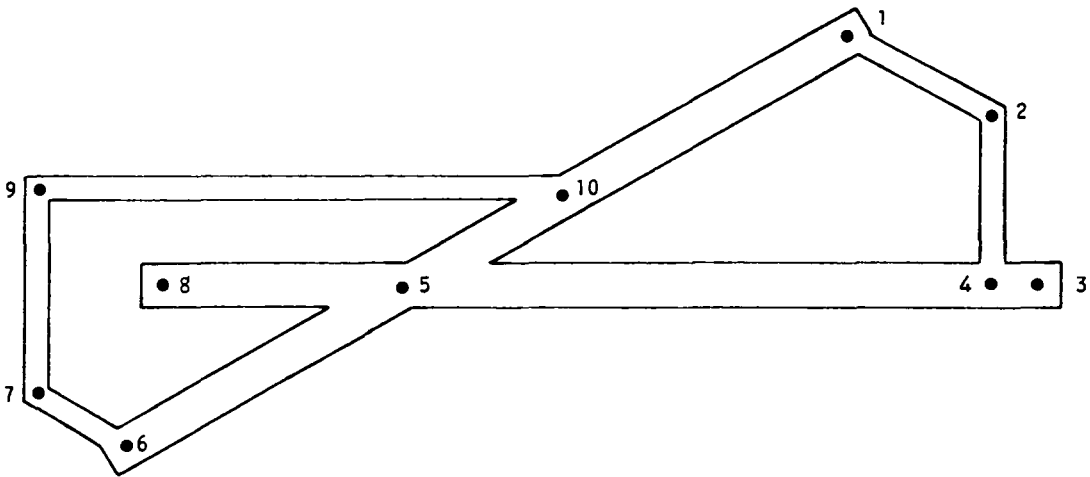
A basic part of the solution procedure is the creation of a network of nodes and branches that represents the runway system of an airport. As will be shown, characteristics of this network are used as an aid in the determination of a final solution to the decision problem. To define the nodes of this network, critical points are specified in the extreme areas of the runway system. An extreme area is considered to be any area that is either at the end of a runway

or at the intersections of runways, taxiways, or access roadways. Also, an extreme area is designated to be wherever a sharp turn occurs on a taxiway or access roadway. The branches of the network are defined to be those segments of the runway system that connect the specified critical points.

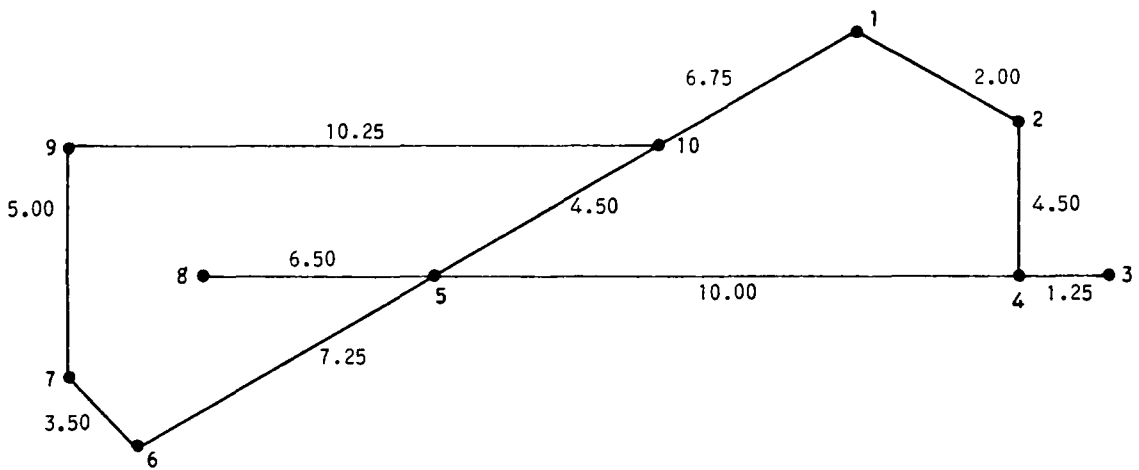
As an illustration of a network interpretation of a simple runway system, consider Figures 4.a and 4.b. Figure 4.a shows two runways that intersect at critical point 5. There are nine other extreme areas on the system in which critical points are designated. Figure 4.b is the network representation of Figure 4.a. The numbers along the branches of the network correspond to the length of the relevant runway segments. Critical points are represented by nodes 1 through 10.

The set  $\{K\}$  is formed from the critical points of a runway system. For any given airport, this is done by using a map of the airport and enumerating critical points in all extreme areas of the airport's runway system. A grid, representing a cartesian plane of any desired degree of accuracy, is then placed on the map and the X-Y coordinates of each critical point are determined. By enumerating the segments of the runway system that connect these points, the branches of the network are defined. The length of each branch can be determined by calculating the Euclidean distance between





(a)



(b)

Figure 4. Simple Runway System and Its Network Interpretation.

each pair of critical points that are connected by segments of the runway system.

With the branches of the network formed, a discrete demand space (i.e.,  $\{L\}$ ) can be defined. Although each actual branch represents a continuous or infinite set of points (i.e., all points on a runway segment), it is considered to be a discrete entity. In the eventual formulation of the cover problem, each branch is considered to be a demand point and is covered by a fire station only if all points of the branch can be reached within a specified time period by a vehicle responding from that station through the runway network system. More discussion of this approach is offered in the explanation of step 3.

#### Step 2 - Formation of a Discrete Location Space, $\{S_1\}$

After the runway system network is defined and the demand space is made discrete, the next step in the solution procedure is to specify the finite set of possible location sites for the problem. As part of this step, the lines that form the restricted continuous location space must be specified. For a particular airport, this is also done through the use of a map of the airport's runway system. From such a map it can be observed in which areas lines can be drawn such that each line is parallel to and a distance,  $d_2$ , from a taxiway or access roadway. The restricted location space can be

designated by specifying which of these lines are at least a distance,  $d_1$ , from any runway and not in the approach path of any runway. Other practical considerations are incorporated into the formation of these lines. In particular, no part of a line is included in the continuum if it is in close proximity<sup>4</sup> to navigational aids or other airport structures.

Figure 5.a illustrates the determination of the restricted location continuum and its relationship to the runway network system for the airport shown in Figure 4.a. Notice in 5.a that the broken lines parallel to the segments between critical points 9 and 10 and critical points 2 and 3 represent the location continuum for this particular system. It should be recognized that both broken lines are a distance,  $d_2$  from their associated runway segments and are at least a distance,  $d_1$ , from any runway. Also, no part of the location continuum is within the areas of any approach paths that are represented by the regions between slashed lines. Notice that no other parts of the system offer candidate locations for the restricted continuum. Figure 5.b is the network interpretation of 5.a.

The next part of step 2 concerns the determination of the shortest paths and shortest distances between the nodes of the network. With the length of each branch of the system

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<sup>4</sup>As in determining the distances  $d_1$  and  $d_2$ , it is left up to airport planners to decide how close the location space can be.

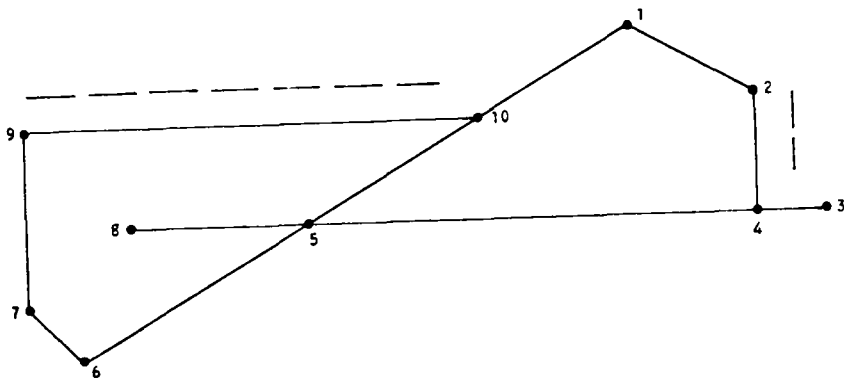
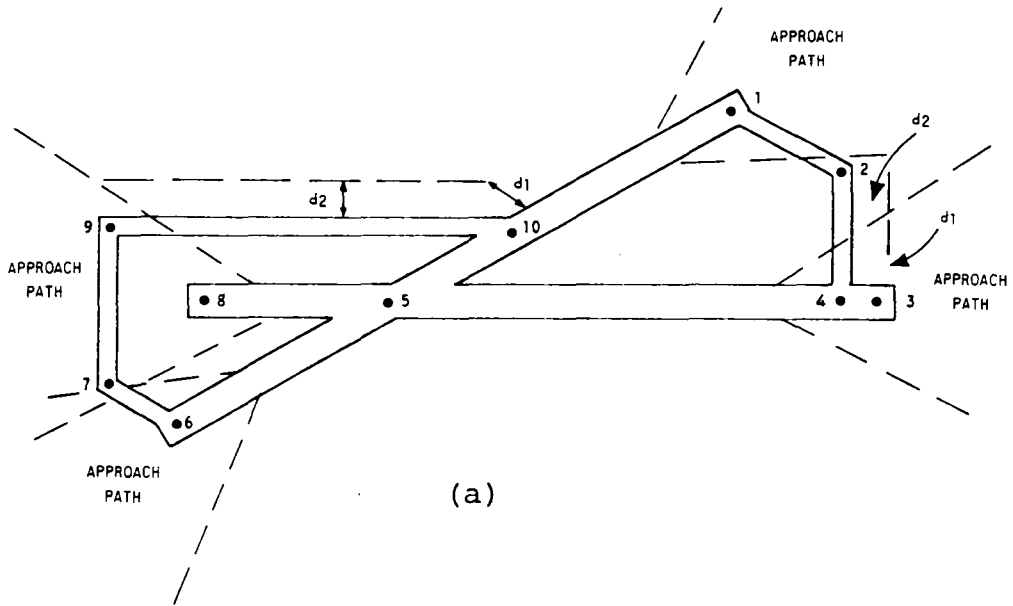


Figure 5. Restricted Location Continuum for a Simple Runway System and Its Network Interpretation.

known, an algorithm originally presented by Floyd [ 9 ] is implemented to invoke this task. The algorithm is designed to determine the shortest distance and associated path between each node and all other nodes in a network.

After these shortest paths are calculated for all pairs of nodes, it is possible to begin specifying possible station location sites and form  $\{S_1\}$ . The basic philosophy used in creating this set is to consider each node of the network by itself and determine those points on the location continuum whose shortest distance is  $\partial$  from the node being considered.<sup>5</sup> The specific manner in which  $\{S_1\}$  is formed is explained in the following paragraph by first giving the general procedure, and then offering an illustrative example for a simple runway system.

Before explaining the procedure it is helpful to define the following notation:

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<sup>5</sup>The distance,  $\partial$ , is the distance that a fire vehicle can travel at an average speed,  $s$ , within the specified time period,  $t$ . The constant,  $s$ , is chosen by airport officials and represents the average speed that a fire vehicle travels on the runway system. It should be recognized that in choosing a value for  $s$ , care should be taken to determine a value that can be said to be the best representative average speed. This is due to the fact that a fire vehicle's top speed and acceleration rates may vary. Also, in traveling to different points in the system an engine's average speed is related to the number of turns it must make and the length of straight roadway traversed.

The value for  $t$  is taken to be the guideline period of two minutes, less a period of 20 seconds in which a fire vehicle is manned and put into motion.

Let

$s$  = the average speed traveled over any path;

$\partial$  = the maximum distance that can be traveled in a time period,  $t$ , at an average speed,  $s$ ;

$m$  = the number of nodes in a network that represents a runway system;

$g_{ij}$  = the distance traveled by taking the shortest path between node  $i$  and node  $j$ ;

$\{C_i\}$  = the set of nodes,  $j$ , such that  $g_{ij} < \partial$ ;

$$f_{ij} = \begin{cases} 1, & \text{if there is a part of the restricted continuous location space that runs parallel to and is a distance, } d_2, \text{ from the network branch connecting node } i \text{ and node } j \\ 0, & \text{otherwise} \end{cases}$$

In the formulation of  $\{S_1\}$ , elements (i.e., candidate locations on the continuum) are added to the set for each node of the network system. To determine candidate locations to be added to  $\{S_1\}$  for a particular node  $i$ , the parts of the restricted continuum in the vicinities of the nodes that are elements of  $\{C_i\}$  are considered. Suppose node  $j \in \{C_i\}$ . If  $j$  is a terminal node with respect to node  $i$  (i.e., the only branch connecting  $j$  to the rest of the network is a branch that is in the shortest path from node  $i$  to node  $j$ ), it is not examined and the next node in  $\{C_i\}$  is considered. However, if  $j$  is not a terminal node, then the part of the location continuum parallel to the branch connecting node  $j$  and any node  $k$ , such that

$$f_{jk} = 1 \text{ and} \\ (\partial - g_{ij}) < g_{jk},$$

is examined as a possible area within which to define a new station location site to be added to  $\{S_1\}$  .

In this examination of the location continuum, an assumption is made concerning the paths that vehicles will travel from the possible location site to the runway segment to which the particular part of the continuum is parallel (see assumption number 3, p. 23 ). It is assumed that vehicles will obtain entry to the runway system as quickly as possible. Therefore, a vehicle's entry to the runway system will be on a route that is perpendicular to the adjacent branch of the system (i.e., the branch that is parallel to the part of the continuum on which the vehicle's station site is located). Furthermore, the vehicle must travel a distance  $d_1$  to reach the system. If

$$d_{crit} = \vartheta - (g_{ij} + d_1) > 0,$$

that is, if

$$\vartheta - g_{ij} > d_1$$

then a station site is added to  $\{S_1\}$  on the part of the restricted continuum being considered (i.e., the part parallel to the branch connecting nodes  $j$  and  $k$ ). In order to find the location of this site, the point, that is a distance,  $d_{crit}$ , from node  $j$  on the branch connecting  $j$  to node  $k$ , is determined. The site is located on the part of the relevant continuum segment that is a distance,  $d_2$ , from this point. The distance from the site to node  $i$  will be  $\vartheta$ .

This procedure, that was performed for node  $k$ , is repeated for all other parts of the restricted continuum that are parallel to branches emanating from node  $j$ . However, the branches (and the parts of the continuum that are parallel to them) that are in the shortest path from  $i$  to  $j$  are not considered except for one case. Only if

$$\partial - g_{ij} = d_1$$

is it possible for a site to be located on one of these parts of the continuum and be a distance,  $\partial$ , from node  $i$ .

When this procedure has been repeated for all nodes in  $\{C_i\}$ , location candidates are determined for the node  $i+1$  by considering  $\{C_{i+1}\}$  and similarly adding sites to  $\{S_1\}$ , where possible, in the vicinities of each element of  $\{C_{i+1}\}$ . The process continues for all nodes of the runway (i.e., until  $i=m$ ) at which point  $\{S_1\}$  will contain a finite number of location sites on the restricted continuum, each site being a distance  $\partial$  from at least one critical point on the runway system.

As an aid in explaining the selection procedure, an example of the generation of site locations and the formation of  $\{S_1\}$  for the runway network system shown in figure 5.b is presented below. In the example it is assumed that  $d_1$  is 2.5 units,  $d_2$  is 2.0 units, and  $\partial$  is 20.0 units of measure. The information regarding the shortest distances between



nodes in the network and the relative location of the restricted continuum to the runway system is provided in Tables 2 and 3, respectively. The site selection procedure is as follows:

(0) Initially, let

$$m = 10, \{S_1\} = \emptyset$$

(1) Node 1 ( $i = 1$ )

$$\{C_1\} = \{2, 3, 4, 5, 6, 8, 9, 10\}$$

- (a) Note that node 7 is not included in  $\{C_1\}$  since from table 1,  $g_{17} = 22.0 > \partial$ . Beginning with node 2, an effort is made to locate a candidate site on parts of the continuum that run parallel to branches emanating from that node. Since  $f_{21} = 0$ , the only alternative branch to consider is the branch connecting node 2 and node 4. Although  $f_{24} = 1$ , no site can be located on the part of the continuum parallel to that branch such that the distance from that site to node 1 is  $\partial$ . Try the next element in  $\{C_1\}$ .
- (b) Only one branch emanates from node 3, namely the branch connecting that node to node 4. However,  $f_{34} = 0$ . Therefore try the next element in  $\{C_1\}$ .
- (c) For node 4,  $f_{43} = 0$ , and  $f_{45} = 0$ . Therefore try the next element in  $\{C_1\}$ .
- (d) For node 5,  $f_{54} = f_{56} = f_{58} = f_{5,10} = 0$ . Again no part of the location continuum runs parallel to any branches emanating from the node under consideration (i.e., node 5). Try the next element of  $\{C_1\}$ .
- (e) For node 6,  $f_{65} = f_{67} = 0$ . Try the next element in  $\{C_1\}$ .
- (f) For node 8,  $f_{85} = 0$ . Try the next element in  $\{C_1\}$ .

Table 2. Shortest Distances Between the Nodes of the Runway Network System of Figure 6.

	$g_{ij}$									
	j = 1	2	3	4	5	6	7	8	9	10
i=1	0.00	2.00	7.75	6.50	11.25	18.50	22.00	17.75	17.00	6.75
2	2.00	0.00	5.75	4.50	13.25	20.50	24.00	19.75	19.00	8.75
3	7.75	5.75	0.00	1.25	11.25	18.50	22.00	17.75	24.75	14.50
4	6.50	4.50	1.25	0.00	10.00	17.25	20.75	16.50	23.50	13.25
5	11.25	13.25	11.25	10.00	0.00	7.25	10.75	6.50	14.75	4.50
6	18.50	20.50	18.50	17.25	7.25	0.00	3.50	13.75	8.50	11.75
7	22.00	24.00	22.00	20.75	10.75	3.50	0.00	17.25	5.00	15.25
8	17.75	19.75	17.75	16.50	6.50	13.75	17.25	0.00	21.25	11.00
9	17.00	19.00	24.75	23.50	14.75	8.50	5.00	21.25	0.00	10.25
10	6.75	8.75	14.50	13.25	4.50	11.75	15.25	11.00	10.25	0.00



- (g) For node 9,  $f_{97} = 0$ . Although  $f_{9,10} = 1$ , the branch connecting nodes 9 and 10 is in the shortest path from node 1 to node 9. Furthermore,  $g_{19} = 17.00 < \vartheta$  (i.e.,  $g_{19} \neq \vartheta$ ) and it is not possible to locate a site on the continuum running parallel to the branch connecting nodes 9 and 10 such that the distance from the site to node 1 is  $\vartheta$ . Try the final element in  $\{C_1\}$ .
- (h) For node 10,  $f_{10,1} = f_{10,5} = 0$ . Although  $f_{10,9} = 1$ , it is known that it is not possible to locate a site a distance  $\vartheta$  from node 1 on the part of the continuum running parallel to nodes 9 and 10.

## (2) Node 2

$$\{C_2\} = \{1, 3, 4, 5, 8\}$$

- (a) For node 1,  $f_{12} = f_{1,10} = 0$ . Try the next element in  $\{C_2\}$ .
- (b) For node 3,  $f_{34} = 0$ . Try the next element for  $\{C_2\}$ .
- (c) For node 4,  $f_{42} = 1$ . However no point on the part of the continuum, running parallel to the branch connecting nodes 2 and 4, is a distance  $\vartheta$  from node 4. Since the only other branches emanating from node 4 yield  $f_{43} = f_{45} = 0$ , try the next element in  $\{C_2\}$ .
- (d) For node 5,  $f_{54} = f_{56} = f_{58} = f_{5,10} = 0$ .
- (e) For node 8,  $f_{85} = 0$ .
- (f) For node 9,  $f_{97} = 0$ . Although  $f_{9,10} = 1$ , the branch connecting nodes 9 and 10 is in the shortest path from node 2 to node 9. Therefore, consider the next element in  $\{C_2\}$ .
- (g) For node 10,  $f_{10,1} = f_{10,5} = 0$ . However  $f_{10,9} = 1$ . Furthermore, since

$$\vartheta - g_{2,10} < g_{10,9}$$

and

$$\vartheta - g_{2,10} > d_2$$

an attempt is made to locate a site on the part of the continuum running parallel to the branch connecting nodes 9 and 10. It is found that it is possible to do so. In Figure 6.a, it is shown that the site is located at node 11 on the restricted continuum. At this point in the procedure  $\{S_1\}$  is no longer null but contains the element 11 (i.e.,  $\{S_1\} = \{11\}$ ). Notice that the bracketed portion of the branch connecting node 9 and node 10 has a unit length,

$$\begin{aligned} d_{\text{crit}} &= \vartheta - (g_{2,10} + d_2) \\ &= 9.25, \end{aligned}$$

Furthermore, the distance from node 11 to node 2 is  $\vartheta = 20.0$  units.

(3) Node 3

$$\{C_3\} = \{1,2,4,5,6,8,10\}$$

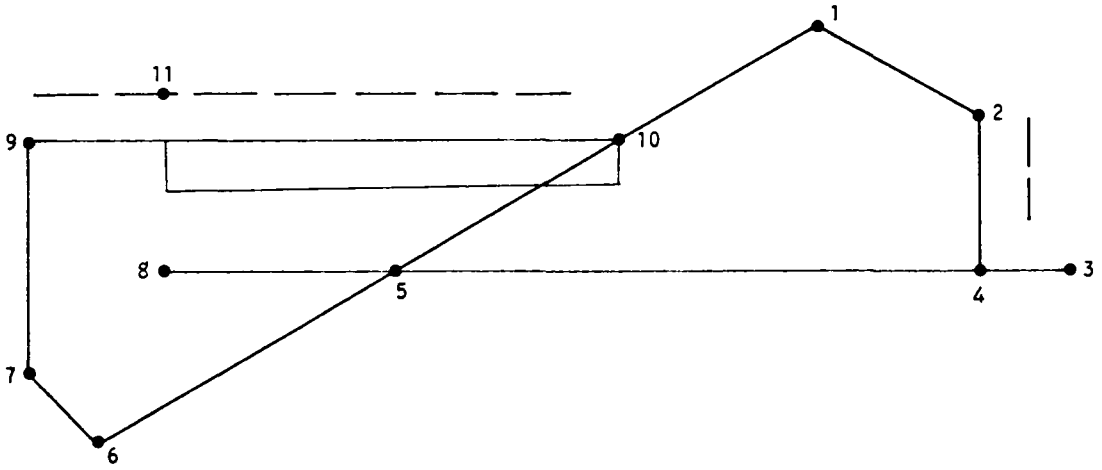
A similar procedure to that which has been described for  $\{C_1\}$  and  $\{C_2\}$  is repeated for  $\{C_3\}$ . No candidate station sites are determined until node 10 is considered. Node 12, in Figure 6.b represents the site that is a distance  $\vartheta$  from node 3 by taking the shortest path to node 10, traveling a distance,

$$\begin{aligned} d_{\text{crit}} &= \vartheta - (g_{3,10} + d_2) \\ &= 3.50 \text{ units,} \end{aligned}$$

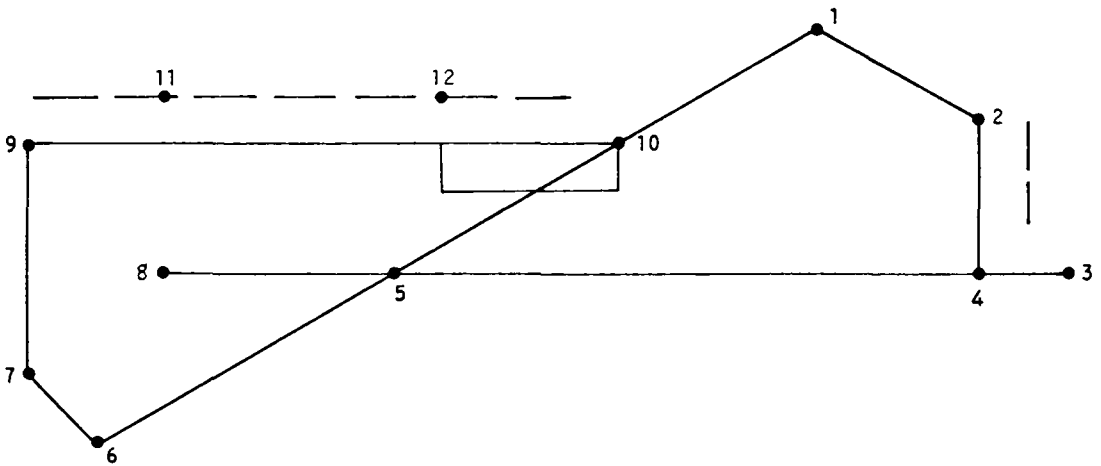
toward node 9, then locating the point that is a distance of  $d_2$  units away on the continuum. In Figure 6.b, the bracketed portion of the branch, connecting nodes 9 and 10, represents the distance,  $d_{\text{crit}}$ , for this case.

(4) Node 4

$$\{C_4\} = \{1,2,3,5,6,8,10\}$$

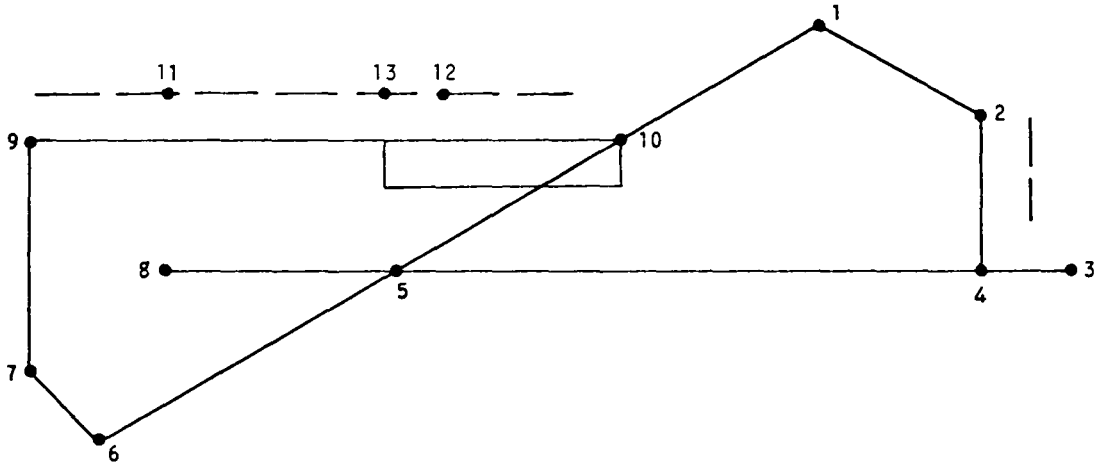


(a)

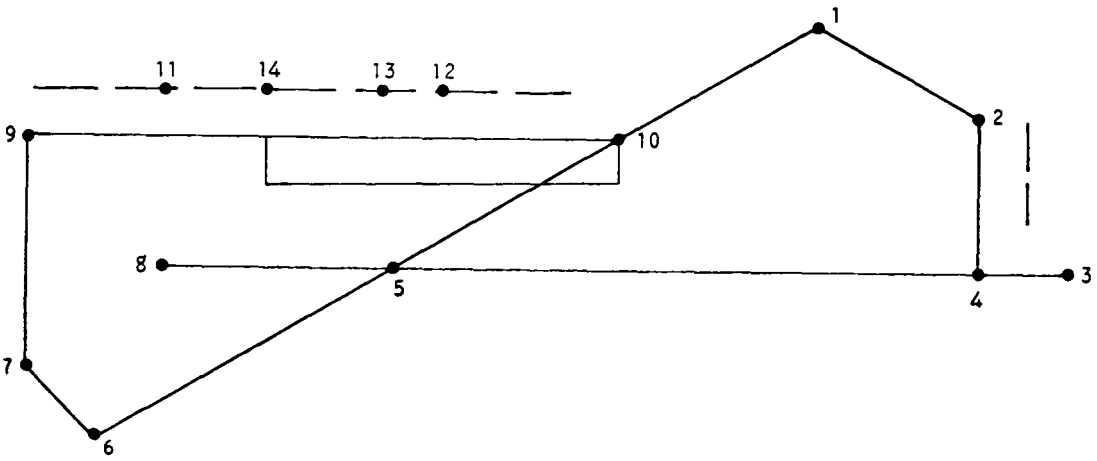


(b)

Figure 6. Potential Station Location Sites Generated for the Runway Network of Figure 5.b.



(c)



(d)

Figure 6: Continued.

Once again as the procedure is repeated, no candidate location sites are determined until node 10 is considered. During this step, a station site is located at node 13 as shown in Figure 6.c. The point that is a distance,  $d_2$ , from node 13 and is on the branch connecting nodes 9 and 10, is a distance,  $d_{crit}$ , of 4.75 units from node 10. This distance is shown as the bracketed portion of the relevant branch in the figure.

(5) Node 5

$$\{C_5\} = \{1, 2, 3, 4, 6, 7, 8, 9, 10\}$$

No station sites can be determined on the location continuum that are a distance,  $\partial$ , from node 5.

(6) Node 6

$$\{C_6\} = \{1, 3, 4, 5, 7, 8, 9, 10\}$$

The procedure is repeated for all elements of  $\{C_6\}$ . Again, no station sites can be determined on the location continuum that are a distance,  $\partial$ , from node 6. In considering node 4, it is found that

$$\partial - g_{64} = 2.75 \text{ units}$$

and

$$d_{crit} = \partial - (g_{64} + d_2) = .75 \text{ units}$$

Notice however that the location continuum parallel to the branch connecting nodes 2 and 4 is a distance,  $d_1$ , from the branch connecting nodes 4 and 3. Since  $.75 < d_1$  (the minimum distance a station can be from any runway), no point on this part of the continuum is a distance,  $\partial$ , from node 6.

In considering nodes 9 and 10, no point on the continuum parallel to the branch connecting these two nodes has a shortest distance,  $\partial$ , to node 6.



(7) Node 7

$$\{C_7\} = \{5,6,8,9,10\}$$

No stations can be determined on the continuum that are a distance,  $\partial$ , from node 7.

(8) Node 8

$$\{C_8\} = \{1,2,3,4,5,6,7,10\}$$

In considering node 10 a candidate site is located at node 14 as shown in Figure 6.d. The bracketed distance,  $d_{crit}$ , as shown in the figure is in this case 7.00 units.

(9) Node 9

$$\{C_9\} = \{1,2,5,6,7,10\}$$

No candidate sites can be found on the location continuum that are a distance,  $\partial$ , from node 9.

(10) Node 10

$$\{C_{10}\} = \{1,2,3,4,5,6,7,8,9\}$$

No station sites can be specified on the continuum that are a distance,  $\partial$ , from node 10. All  $m$  nodes have been studied and the formulation of  $\{S_1\}$  is complete for this example. Upon completion  $\{S_1\} = \{11,12,13,14\}$ .

Although this example represents a network system that is much smaller and less complex than a network model of an actual airport runway system, it describes the manner in which a discrete set of location sites is formulated in the solution methodology for the problem as it exists for any given airport. That is, if the relationship of the restricted location continuum to the runway system is known, it is possible to generate a finite number of candidate station sites. According to the assumptions that have been made,

it is guaranteed that a vehicle responding from each of these sites, and traveling at an average speed,  $s$ , can reach one or more of the critical points on the runway system within a time period,  $t$ . In the example problem only four candidate sites were generated, each site being a distance,  $\delta$ , from a node. It should be recognized in the candidate site selection process, that it is possible for more than one candidate site to be determined for a given node.

At this stage of the analysis, with a discrete location space defined, it is possible to begin the third step of the solution procedure: to model the location problem in the form of a set covering problem.

### Step 3 - Formulation of a Set Covering Model

As part of step 2, the shortest distances and shortest path relationships between all nodes of the runway network system were calculated. Using these calculations,  $\{S_1\}$  was formulated by selecting candidate location sites, each site being a distance,  $\delta$ , from some node in the network. The next step of the solution procedure is to find the shortest distances and paths from each element of  $\{S_1\}$  to all of the nodes of the runway network. That is, the third step involves calculating the response paths and distances from all potential fire station sites to all critical points in  $\{K\}$ . Using this information, a set covering model for the problem is formulated.

In determining these distance/path relationships, the original network system of critical points and runway segments is modified to include all of those station sites generated for  $\{S_1\}$  in step 2. Figures 7.a and 7.b help to illustrate the modification process for the station site at node 11 as given in the example of Figure 6.a (node  $11 \in \{S_1\}$ ). From step 2 recall that the shortest distance from node 11 to the branch connecting nodes 9 and 10 is  $d_2$ . Suppose that this shortest distance occurs at point x which is the intersection of the line perpendicular to node 11 and the branch connecting nodes 9 and 10. Also from step 2, it is known that the distance from node 10 to point x is

$$d_{\text{crit}} = \vartheta - (g_{2,10} + d_2).$$

Therefore, it is assumed in incorporating node 11 into the original network system that a branch connecting node 11 to node 10 is of length  $\vartheta - g_{2,10}$ . In a similar manner, it can be determined that the shortest distance a vehicle travels from node 11 to point x, then to node 9 is

$$d_{\text{crit}}' + d_1$$

where

$$d_{\text{crit}}' = g_{10,9} - d_{\text{crit}}.$$

Figure 7.b represents the original network that is modified to include node 11. With the distances from node 11

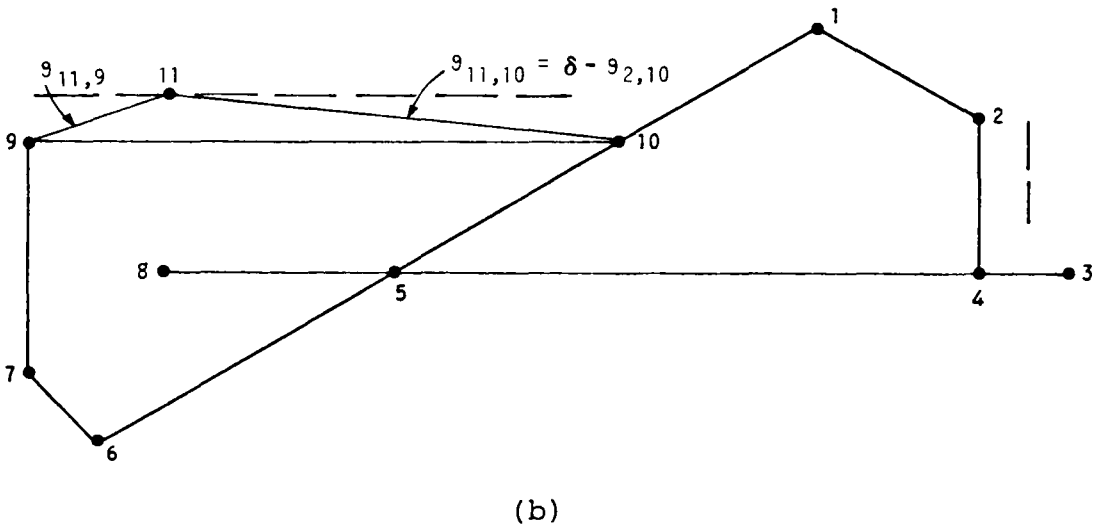
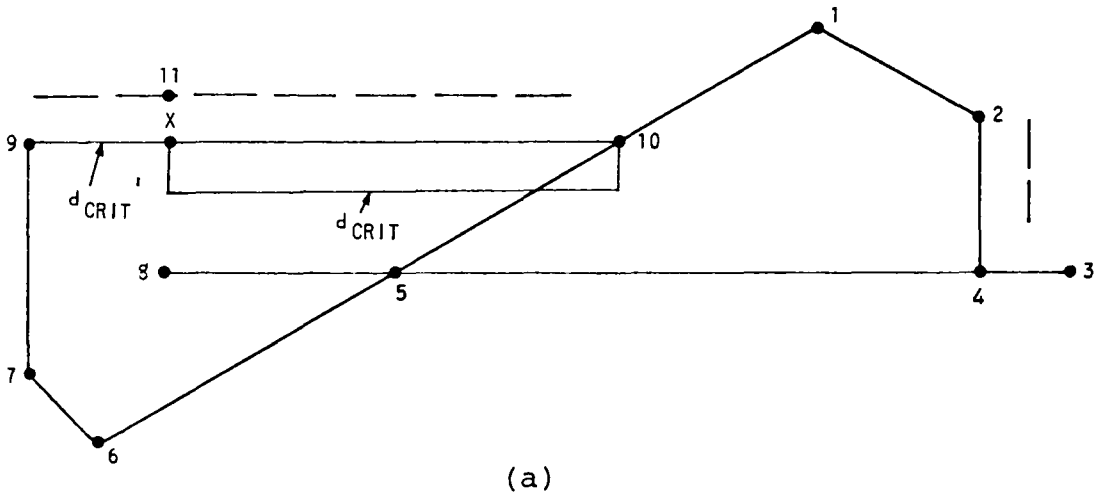


Figure 7. Incorporation of Node 11 to the Runway Network of Figure 5.b.

to nodes 10 and 9 known ( $g_{11,10}$  and  $g_{11,9}$ , respectively), it is possible to determine the shortest distance and shortest path from node 11 to every other node by again using Floyd's procedure.

In a similar manner as was described for node 11, the distances and paths from each station location in  $\{S_1\}$  to each node in the runway system network are determined. That is, for a given station site, it is found which runway system segment is closest to and parallel to the part of the continuum on which the site is located. Next, the distances from the site to the nodes that are the end points of this segment are calculated. Finally, the shortest paths and distances from the site to all the other network nodes, excluding those in  $\{S_1\}$ , are determined. Table 4 summarizes these shortest distances for the critical points and  $\{S_1\}$  determined in the example problem of step 2. With these distances determined, it is now possible to formulate a cover problem similar to (M2).

In modeling the research location problem as a cover problem, the discrete station sites of  $\{S_1\}$  are the alternative facility sites, and the branches of the network that connect the original critical points of the runway system are the demand points. The values of the cover coefficients,  $a_{ij}$ , are dependent upon whether a fire vehicle responding from site  $j$ ,  $j \in \{S_1\}$ , can reach each point on the runway segment

Table 4. Shortest Distances from Potential Station Site Nodes to Critical Points of the Runway Network System in Figure 6.d.

		$g_{ij}$									
		CRITICAL POINTS									
		j = 1	2	3	4	5	6	7	8	9	10
STATION SITE NODES	i=11	11.00	20.00	25.75	24.50	15.75	11.50	8.00	22.25	3.00	11.25
	12	12.25	14.25	20.00	18.75	10.00	17.25	12.25	16.50	8.75	5.50
	13	13.50	15.50	21.25	20.00	11.25	16.00	12.50	17.75	7.50	6.75
	14	15.75	17.75	23.50	22.25	13.50	13.75	10.25	20.00	5.25	9.00

represented by branch  $i$ ,  $i \in \{L\}$ , within the specified time period,  $t$ . If the vehicle can reach each of these points within this time period then  $a_{ij} = 1$ . Otherwise,  $a_{ij} = 0$ .

Figure 8 is provided as an aid in illustrating the manner in which the cover coefficients are determined for  $\{S_1\}$  that was formulated in step 2. Notice that the labeling scheme in the figure is different from that of Figures 6.a through 6.d. Rather than numbering the nodes, the branches (i.e., the actual demand points of the cover problem) are enumerated. Also, the location sites 11 through 14 have been labeled 1 through 4 respectively. Recalling that  $\theta = 20$  for this particular network, the associated cover problem is as follows:

$$\text{Minimize } z = \sum_{j=1}^4 x_j$$

$\underline{x}$

subject to

$$x_1 + x_2 + x_3 + x_4 \geq 1$$

$$x_1 + x_2 + x_3 + x_4 \geq 1$$

$$x_2 + x_3 \geq 1$$

$$x_2 \geq 1$$

$$x_2 \geq 1$$

$$x_1 + x_2 + x_3 + x_4 \geq 1$$

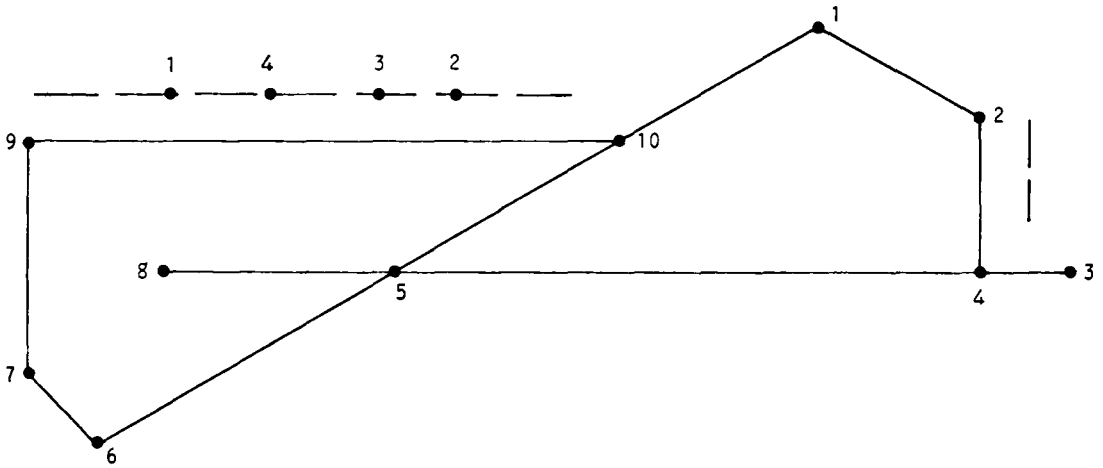


Figure 8. The Numbering of Potential Station Sites.



$$x_2 + x_3 + x_4 \geq 1$$

$$x_1 + x_2 + x_3 + x_4 \geq 1$$

$$x_1 + x_2 + x_3 + x_4 \geq 1$$

$$x_1 + x_2 + x_3 + x_4 \geq 1$$

$$x_1 + x_2 + x_3 + x_4 \geq 1$$

$$x_j = (0,1), j = 1,4$$

Once a set covering model has been formulated, the next step of the solution procedure is to find solutions to it. In the next section the technique used to determine these solutions is discussed.

#### Step 4 - Determination of Solutions to the Set Covering Model

In the preceding section, the method used to model the location problem of the research as a cover problem was discussed. The fourth step of the solution procedure is concerned with finding optimal solutions to this transformed problem.

In selecting a technique for solving the cover problem, it is desirable for the method to have two basic characteristics. First, the procedure should be capable of determining several alternative optimal solutions, if they exist, since

in step 5, the alternative optima are examined to resolve which solution offers the minimum blockage of response routes from station sites to potential crash areas on the runway system. Second, the procedure should be fast and should require a relatively short amount of time to determine these optimal solutions.

An easily programmable procedure that can be used to provide all the optimal solutions to the cover problem is one of complete enumeration of all possible solutions. For a large problem, especially with a large number of facility sites, this can prove, in general, to be computationally infeasible. Of those other cover problem solution procedures mentioned in Chapter 2, none readily generates alternative optimal solutions without some modification. However, if Roth's procedure (see p. 17) is used to reduce the size of the cover matrix and enumeration of solutions is performed in a strategic manner, then alternative optima (if they exist) to a rather large cover problem can be found relatively quickly. This is the procedure used to provide solutions to the cover problem generated in the research.

The procedure begins by eliminating all redundant rows of the cover matrix. Next, columns that are dominated by any other column are deleted from the matrix. As mentioned in Chapter 2, a column dominates another column if it covers all of the same demand points (i.e., rows of the cover matrix).

It should be noted that these column deletions can affect the number of alternative optima that are eventually found. That is, some columns might be eliminated (and not considered in the enumeration process), which, in combination with one another or with certain dominant columns, might offer different solutions. However, it should be recognized that column reduction will not affect the optimality of the cover problem.<sup>6</sup>

Once the cover matrix is in reduced form, the enumeration process begins. The strategy that is used at this point is based on the nature of the location problem. In particular, it is highly unlikely that it will take more than six stations to cover the runway system of any operating airport. Therefore, only solutions of six sites or less need be enumerated. The enumeration begins by determining if a single station site alone can cover the entire runway system. This is done by summing the coefficients of each column of the reduced cover matrix. If a column sum equals the number of rows in the matrix (i.e., there is a 1 in each row), then that particular column presents a feasible and an optimal solution site. The sums of all remaining columns are also examined to determine alternative optima.

---

<sup>6</sup>It is possible that column deletions will affect the optimality of the problem with respect to the second F.A.A. guideline. That is, an alternative optimal solution that minimizes the chance of possible blockage of vehicle response routes to potential crash areas might not be found.

If no one column can cover all the rows in the reduced matrix, the procedure continues by determining if the optimal solution requires two sites. This is done by considering all combinations of columns in the matrix taken two at a time. If there exist two columns such that there is a 1 in each row in either one or the other column, then this particular combination offers an optimal solution of two stations. All other combinations of two columns are then inspected for alternative optima. If no feasible solution exists for any two columns (i.e., there is a 0 in the same row of both columns) then the procedure continues in a similar manner by inspecting all combinations taken three at a time. If no feasible solution exists for three sites, all combinations of four, through six sites are iteratively inspected. In the procedure, once a feasible solution is found and all the alternative optima for the reduced cover matrix are determined, the enumeration terminates and step 4 is completed. No larger combinations need be examined since they necessarily lead to a larger value of the objective function.

#### Step 5 - Selection of a Solution that Minimizes the Chance of Response Route Blockage

At this point in the procedure, solutions to the cover problem, formulated in step 3, have been determined. Each one of these solutions specifies the minimum number of sites and their location on the airport property such that a vehicle

responding from at least one site at an average speed,  $s$ , can reach a potential crash anywhere on the runway system within a "t" minute time period. That is, each solution meets the first recommended F.A.A. guideline. The fifth and final step of the solution procedure is to select from these solutions the one that satisfies the second objective of (M1). Specifically, in this step, it is determined which of these sets of location sites meets the second mentioned F.A.A. guideline, thereby minimizing the possibility of the simultaneous blockage of response routes from these locations to potential crash sites on the runway system.

In the procedure, the fifth major assumption stated in Chapter 3 is used as a basis for determining which solution to the cover problem best satisfies the second guideline. The "best" set of location sites is assumed to be the one in which the response routes from the sites to each demand point in  $\{K\}$  are the "most unique." In determining the uniqueness of these response routes and ultimately the "best" set of sites, the number of common nodes in the response routes from the sets of sites to the points in  $\{K\}$  are calculated. A node is said to be common to two response routes if vehicles pass through that node in traveling these routes. The set of station sites that solves the cover problem and also has the fewest number of common nodes in the vehicle response routes from these sites to the critical points in  $\{K\}$

is chosen to be the final solution to the decision problem (M1).

The procedure begins by examining the response routes for the sites in the first solution to the cover problem. For the first critical point in  $\{K\}$ , the number of common nodes is calculated in the response routes from these sites to that point. As an example of this calculation, consider the following:

Let  $P_{ij}$  = the set of nodes in the response route from site  $j$  to critical point  $i$ .

Suppose that for a cover solution of sites 1, 2, and 3 and some critical point  $i$ ,

$$P_{i1} = \{1, 2, 3, 4\}$$

$$P_{i2} = \{2, 3, 7\}$$

$$P_{i3} = \{3, 4, 5, 6, 8\}.$$

The number of nodes common to both  $P_{i1}$  and  $P_{i2}$  is 2; to  $P_{i1}$  and  $P_{i3}$  is 2; and to  $P_{i2}$  and  $P_{i3}$  is 1. Therefore in the response routes from sites 1, 2, and 3 to critical point  $i$ , the number of common nodes is 5. (Notice that this is different from an alternative view of there being only one node common to the response routes; i.e., node 3.)

When the number of common nodes in the response routes to the first element of  $\{K\}$  is determined, the number of common nodes in the response routes to the second critical point in  $\{K\}$  is calculated. This number is added to the number for the first critical point. For the first solution to the cover problem the procedure continues for each point in  $\{K\}$ ,

accumulating the number of common nodes in the response routes to each point in  $\{K\}$  for the sites in the solution. When the response routes to the last element in  $\{K\}$  have been examined, the procedure is repeated for the second alternative optimal cover problem solution.

This process continues iteratively for all solutions to the cover problem. That is, for a given solution of station sites, the number of common nodes in the response routes to the first critical point in  $\{K\}$  is calculated. The number of common nodes in the response routes to the second critical point in  $\{K\}$  is then determined and is added to the number for the first point. After the last element in  $\{K\}$  has been considered an accumulated sum has been determined for this particular cover solution. The next cover solution is then examined. When all solutions to the cover problem have been examined, the solution procedure is finished. The cover solution that offers the smallest accumulated sum of common nodes in the response routes from its sites to the critical points in  $\{K\}$  is said to satisfy the second objective of (M1).

#### Procedure Flow Chart

In this chapter, the basic steps involved in the solution procedure of this research have been presented. In order to summarize the procedure, a flow chart of the total algorithm is presented in Figure 9.

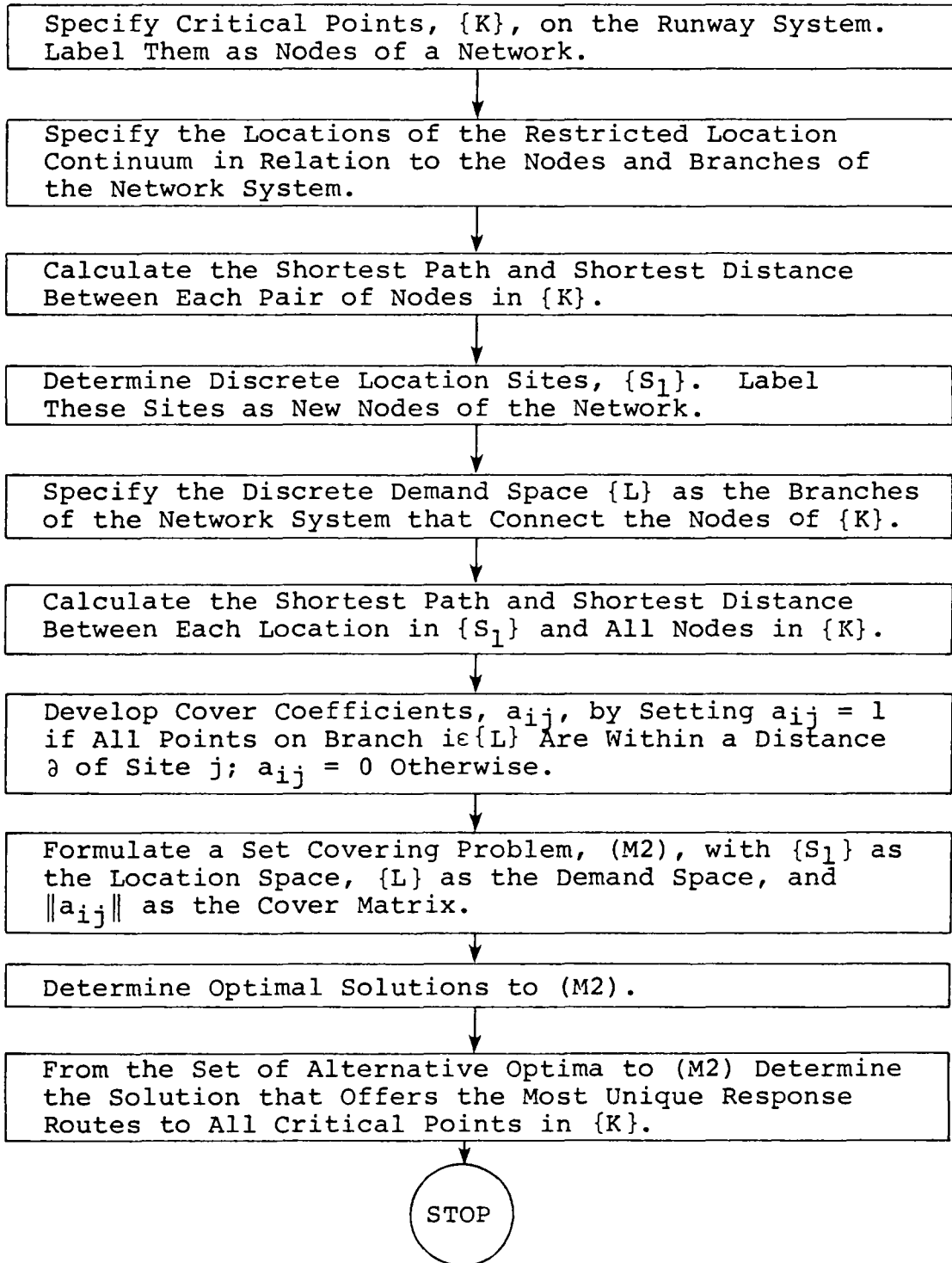


Figure 9. Solution Procedure Flow Chart



COMPUTER PROGRAM

A FORTRAN IV computer program has been developed to affect the solution procedure described in the previous section. This program basically follows the steps that are shown in the flow chart of Figure 9. In the development of such a program that is concerned with the analysis of a network system, problems can result, especially if the network consists of a large number of nodes and branches. The specific problem is one of exceeding the amount of memory that is readily available in the core storage of a computer. In order to store information about a network system (e.g., shortest distances between nodes, or the nodes in the shortest paths between nodes), arrays are utilized. When a network consists of too many nodes, the array area required to store this information can exceed the available core limits of the computer.

To circumvent this problem, two things have been programmed into the present computer code. First, all integer arrays are specified to be of half word length. This decreases the storage required for these arrays by one half. For the IBM 370/158 on which the model was programmed, care must be taken in this specification so that no integer value exceeds  $(2^{15} - 1)$ . No problem was encountered in this program since most of the integer variables were either 0 or 1 and none exceeded a value of 300.

The second feature of the program that reduces the amount of core storage needed is the use of direct file organization, using an on-line disk as work space. While this increases the amount of time in input and output exchanges to and from the work disk to execute the program, it allows an almost unlimited amount of storage.

No other distinct coding features were required. Results of applying the program to a specific airport are provided in Chapter 5, and a listing of the program is offered in Appendix A.

### Summary

This chapter has provided a detailed account of the steps involved in the solution procedure of the research. The purpose of this procedure is threefold. First it is designed to generate possible fire station sites in feasible locations on the airport property. Next, it determines combinations of these sites such that any point on the runway system can be reached by a vehicle responding from one of these sites within a "t" minute time period. The number of sites in each combination is the minimum number of fire stations necessary to provide this coverage. Finally, the procedure is structured to select from these combinations the set of sites that offers the minimal possibility of blockage of response routes of fire vehicles' traveling from

their station locations to potential crash areas on the runway system. With this set of sites determined, the research decision problem is solved. In the subsequent chapter, an application of the solution procedure is illustrated for an existing airport.

## CHAPTER 5

### APPLICATION OF THE SOLUTION PROCEDURE

#### Introduction

In the preceding chapter, a detailed discussion was presented in which a simple airport runway system was used as an illustrative aid in describing the steps of the solution procedure. To analyze the actual utility of the procedure as a planning tool for a large and more complex runway system, the methodology has been applied to the relevant fire station location problem as it will exist at the Atlanta Airport in 1978 to 1980. The results of this application are given in this chapter.

The presentation is divided into four sections. The first section deals with the application of the solution procedure for the location problem in which it was assumed that no fire stations currently exist at the airport (as in the case of a new airport design). The problem was to determine the best locations for the fewest number of fire stations required to meet the two F.A.A. guidelines. The next section presents the solution results of this application. The third section describes a second application of the procedure to the location problem in which three existing fire stations were assumed to be in operation at the airport. In this case, the locations of the fewest number of additional stations, if any, that are required to meet the two F.A.A. guidelines were determined.

The fourth section presents an analysis of the solution results obtained in this second situation.

Recall from the statement of the assumptions of the research given in Chapter 3 that it is assumed that vehicles travel an average speed,  $s$ , in responding to potential crash sites on the runway system. In order to determine the effect of changes in the value of  $s$  on the solutions provided by the procedure, each application was tested 3 times. Each time,  $s$  was assigned a different value. These were 40 m.p.h., 45 m.p.h., and 50 m.p.h. The sensitivity results are presented for each application situation.

#### Application 1 - No Existing Stations

To reiterate, the problem encountered in the case in which existing fire stations at the Atlanta Airport were ignored is one of determining the locations of the fewest number of fire stations required to meet the two relevant F.A.A. guidelines. A more precise statement of the problem is as follows:

- GIVEN: (1) the planned layout of the runway system at the Atlanta Airport for the years 1978 through 1980;
- DETERMINE: (1) the minimum number of fire stations required;
- (2) the locations of these fire stations;
- SUCH THAT: (1) at least one fire vehicle can reach a potential crash anywhere on the runway

system by traveling from one of these stations at an average speed,  $s$ , for a time period,  $t$ , of one minute and forty seconds;<sup>7</sup>

- (2) no fire station is located within a distance,  $d_1$ , of 750 feet from any runway;
- (3) no fire station is located in the landing approach area of any runway;
- (4) each fire station is located a distance,  $d_2$ , of 150 feet from a taxiway or access roadway;
- (5) the possibility of fire vehicles' responding to potential crash sites on the same runway is minimized.

In order to solve this problem, the solution procedure discussed in Chapter 4 was utilized. Recall that an integral part of this procedure was the formulation of a network that represents the runway system being studied. In the development, critical points on the runway system were established as nodes and the runway segments connecting these points were branches of the network. A later step of the procedure incorporated feasible fire station sites, that were generated on the location continuum, into the model.

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<sup>7</sup>In determining if a potential crash site could be reached within two minutes (and thereby meet the first F.A.A. guideline), twenty seconds were subtracted from the recommended response period to allow for the time elapsed between the sounding of an alarm and the time in which a vehicle can be manned and put into motion. Therefore, in terms of the definitions presented in Chapter 4,  $\delta$  was the distance that could be traveled by a vehicle traveling at an average speed,  $s$ , in a period of 1.66 minutes.

The discussion that follows explains the methodology used to define the layout of the Atlanta Airport. In the discussion, a description is provided of the first phase of the methodology in which the network, representative of the airport's runway system, was constructed. Included in this description is the manner in which the location continuum at the airport was identified.

The remainder of the discussion is a presentation of the second phase of the methodology. In this presentation the process that specified the required input for the solution procedure is explained. This input included the properties and relationships of the network system and location continuum that describe the layout of the airport.

### Phase 1

In order to construct a representative network and identify the location continuum at the Atlanta Airport, a map of the airport, as shown in Figure 10 was used. On this map nodes of the network were specified in the first phase of the methodology. These nodes are the 110 critical points enumerated on the runway system illustrated in Figure 11. Notice that these critical points are located in most of the areas of the system where runways, taxiways, or access roadways intersect. However, not all intersections have a critical point nor is every critical point centered in an intersection. The logic





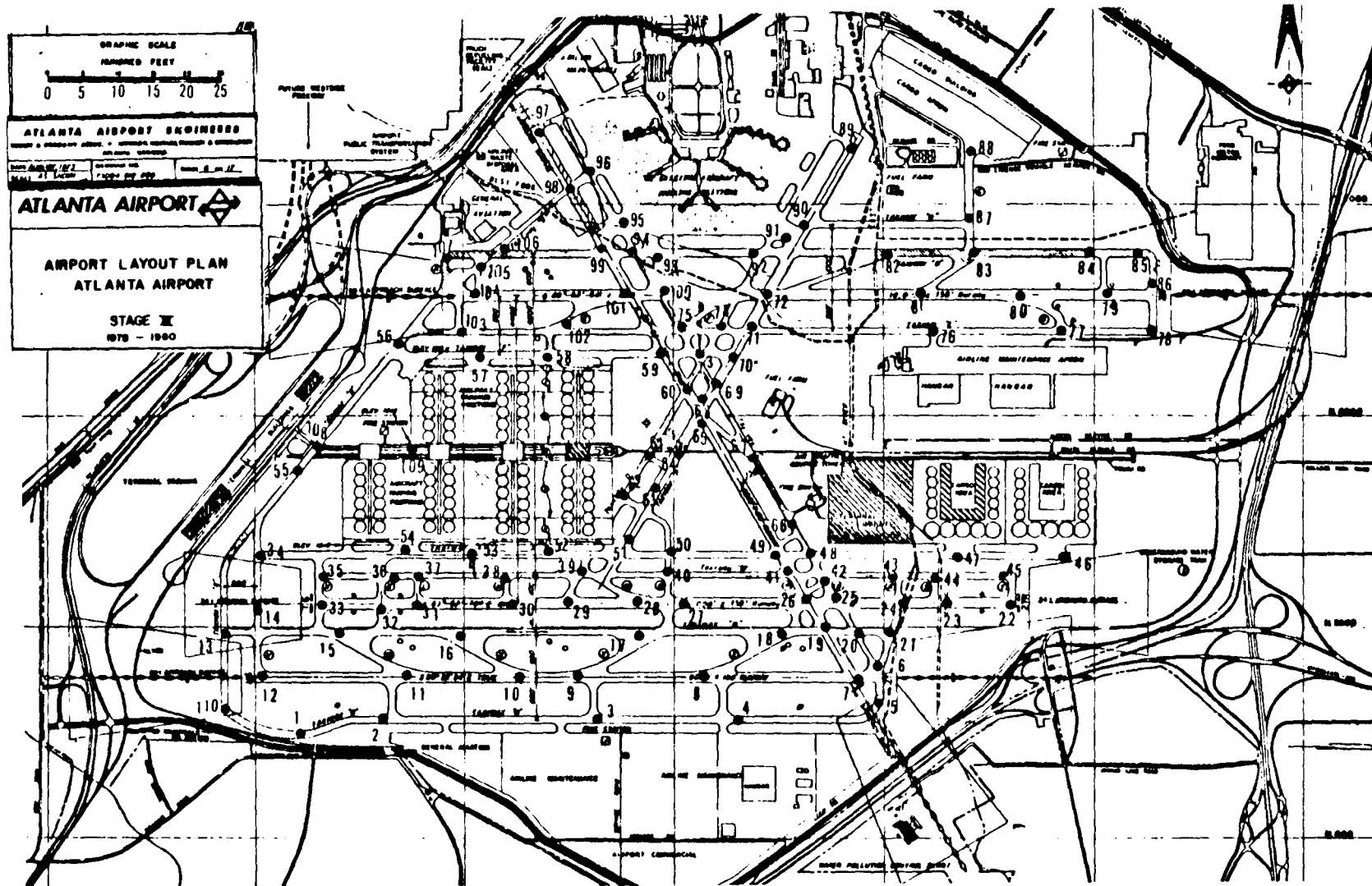


Figure 11. Enumeration of Critical Points for the Atlanta Airport.

involved in the placement of critical points is presented later, in the discussion of the second phase of defining the layout of the airport.

To identify the location continuum, suitable areas on the map of the airport property were inspected. In Figure 12, the location continuum is illustrated by the heavy black lines that run parallel to select taxiways or access roadways. The relationship of the location continuum to the runway system is provided in Table 5. Notice that according to the scale of the map these lines are a distance of 150 feet from the taxiways or runways that they parallel. Furthermore, no line is within a distance of 750 feet of any runway nor is any line in any runway landing approach.

Upon inspection, it is difficult to specify any additional areas for the continuum lines such that the lines would meet these location requirements. A subjective decision was made, however, for areas not specified in which location lines could be drawn and meet these conditions. As an example, note the area 150 feet northeast from the taxiway on which critical points 95 and 96 are designated. It is possible for a line in this area to be at least 750 feet from any runway and not in any landing approaches. However, no continuum segment was established there because a fire station located in that area might cause interference to the parking of taxiing aircraft (notice the relative proximity of aircraft parking positions).

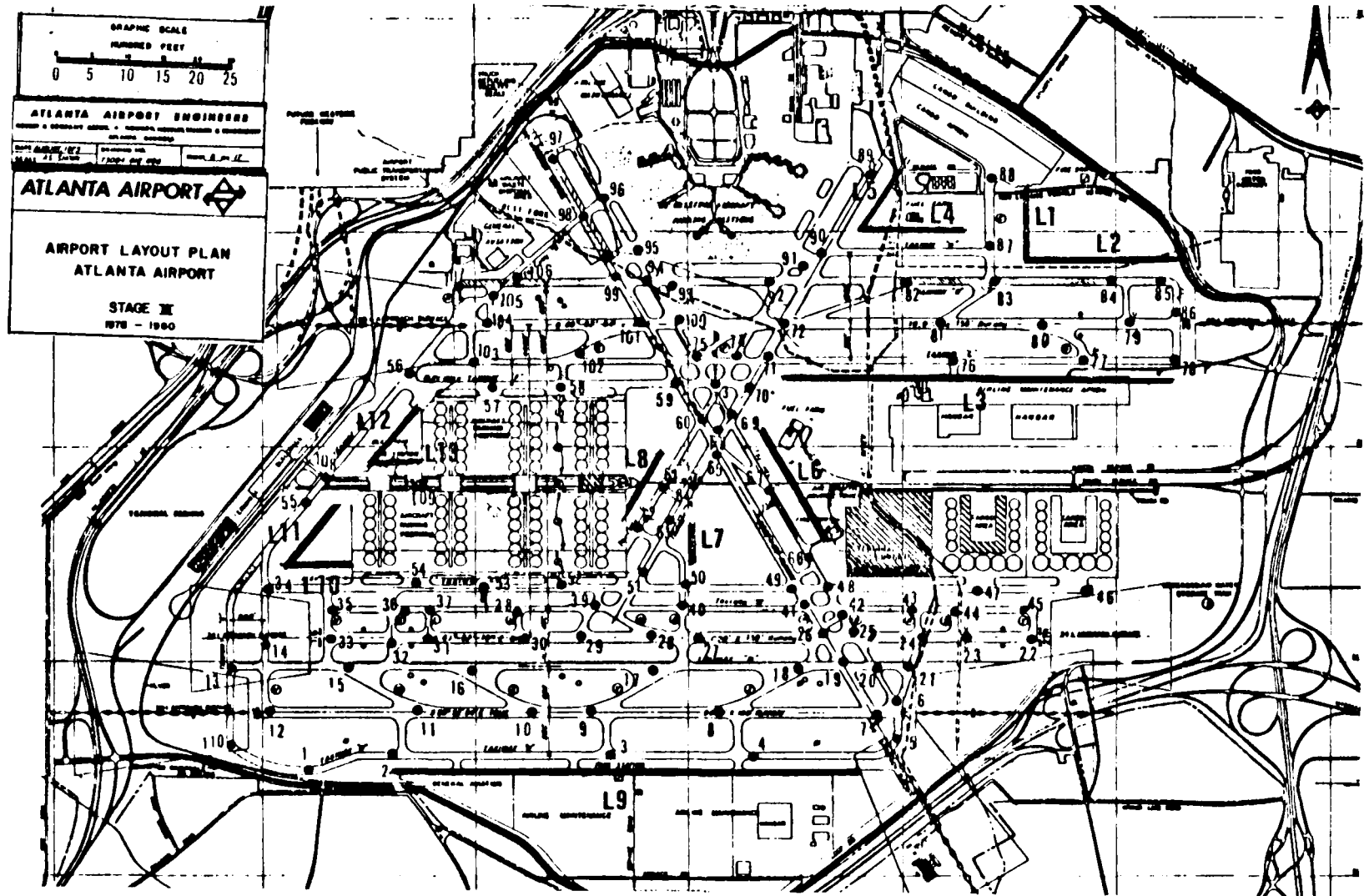


Figure 12. The Location Continuum at the Atlanta Airport.

Table 5. Relationship of Location Continuum  
to Runway System.

<u>LOCATION CONTINUUM LINE</u>	<u>PARALLELS RUNWAY CRITICAL POINTS</u>
L1	87,88
L2	83,84,85
L3	71,76,77,78
L4	90,87
L5	90,89
L6	69,67,66
L7	63,50
L8	62,61
L9	2,3,4,5
L10	34,54
L11	34,55,108
L12	108,56
L13	108,109

Phase 2

The second phase of the methodology dealt with the determination of information required to affect the solution procedure. This information was necessary to specify certain relationships of the network representation of the runway system and the location continuum. The relationships required were as follows:

- (1) The designation of the branches of the network system and the pairs of nodes each branch connects.
- (2) The relative location of each node within the network system.
- (3) The relative location of the line segments of the location continuum with respect to the transformed network system.

In order to specify the relevant information for the first requirement, Figure 12 was utilized. Starting with the runway segments connecting critical point 1 with critical point 2 and critical point 13 the branches of the network were enumerated. Table 6 depicts the branches that were enumerated and the nodes that are connected by each branch.

Notice that the table dictates the paths that a fire vehicle must travel through the system in order to get from one critical point to another. For example, suppose a vehicle is at critical point 2 and is to travel to critical point 12. In order to reach that point it must first pass either through points 1, 3, or 11.

Table 6. Enumeration of Branches  
Connecting Critical Points.

<u>BRANCH</u>	<u>CONNECTS CRITICAL POINT NUMBER</u>	<u>TO CRITICAL POINT NUMBER</u>
1	1	2
2	1	110
3	2	3
4	2	11
5	3	4
6	3	9
7	4	5
8	4	8
9	5	6
10	6	7
11	6	20
12	6	21
13	7	8
14	7	19
15	8	9
16	8	18
17	9	10
18	9	17
19	10	11
20	10	16
21	11	12
22	11	15
23	12	14
24	13	14
25	13	15
26	13	55
27	13	110
28	14	15
29	14	34
30	15	16
31	15	33
32	16	17
33	16	32
34	17	18
35	17	27
36	18	19
37	18	26
38	19	20
39	19	25
40	19	26
41	20	21
42	20	25
43	21	24
44	22	23
45	22	45

Table 6. Continued

<u>BRANCH</u>	<u>CONNECTS CRITICAL POINT NUMBER</u>	<u>TO CRITICAL POINT NUMBER</u>
46	23	24
47	23	44
48	24	25
49	24	43
50	25	26
51	25	27
52	25	42
53	26	27
54	26	41
55	26	42
56	26	49
57	27	28
58	27	40
59	28	29
60	28	39
61	29	30
62	29	39
63	30	31
64	30	38
65	31	32
66	31	37
67	32	33
68	32	36
69	33	35
70	34	35
71	34	54
72	34	55
73	35	36
74	36	37
75	36	54
76	37	38
77	38	39
78	39	40
79	39	51
80	39	52
81	40	41
82	40	50
83	41	42
84	41	48
85	41	49
86	42	43
87	42	48
88	43	44
89	44	45
90	46	47

Table 6. Continued

<u>BRANCH</u>	<u>CONNECTS CRITICAL POINT NUMBER</u>	<u>TO CRITICAL POINT NUMBER</u>
91	47	48
92	48	49
93	48	66
94	49	50
95	49	66
96	50	51
97	50	63
98	51	52
99	51	63
100	52	53
101	52	62
102	53	54
103	55	108
104	56	57
105	56	103
106	56	104
107	56	105
108	56	107
109	56	108
110	57	58
111	58	59
112	58	102
113	59	60
114	59	73
115	59	75
116	59	101
117	60	61
118	60	65
119	60	73
120	61	62
121	61	64
122	61	109
123	62	63
124	63	64
125	64	68
126	65	67
127	65	68
128	66	67
129	67	68
130	67	69
131	68	69
132	69	70
133	69	73
134	70	71
135	70	73



Table 6. Continued

<u>BRANCH</u>	<u>CONNECTS CRITICAL POINT NUMBER</u>	<u>TO CRITICAL POINT NUMBER</u>
136	70	74
137	71	72
138	71	74
139	71	76
140	72	75
141	72	81
142	72	82
143	72	90
144	72	91
145	72	92
146	72	100
147	73	74
148	73	75
149	74	75
150	74	92
151	75	100
152	76	77
153	77	78
154	77	80
155	78	79
156	78	86
157	79	80
158	79	84
159	79	86
160	80	81
161	81	83
162	82	83
163	82	91
164	83	84
165	83	87
166	84	85
167	85	86
168	87	88
169	89	90
170	90	91
171	91	92
172	91	94
173	91	95
174	92	93
175	93	94
176	93	100
177	94	95
178	94	99
179	94	100
180	95	96

Table 6. Continued

<u>BRANCH</u>	<u>CONNECTS CRITICAL POINT NUMBER</u>	<u>TO CRITICAL POINT NUMBER</u>
181	95	99
182	96	98
183	97	98
184	98	99
185	98	106
186	99	101
187	99	106
188	100	101
189	101	102
190	101	104
191	102	103
192	103	104
193	104	105
194	104	107
195	105	106
196	106	107
197	108	109

At this time, the location of certain critical points on the runway system should be observed in Figure 12. Notice that some points are not exactly centered in runway intersections. An example point is critical point 14. Suppose a vehicle must travel from point 13 to point 12. From Table 6 it can be seen that no runway segment connects point 13 and point 12. However a segment does connect points 13 and 14 and, similarly, points 14 and 12. Therefore the vehicle could travel from point 13 to point 14 then to point 12. Notice that since point 14 is not centered in the intersection where a vehicle would normally turn in moving from point 13 to point 12, the distance traveled in the network model exaggerates the real distance.

Point 14 is located such that the extra distance the vehicle must travel helps to offset the assumption that vehicles can maintain an average speed,  $s$ , while traveling on the runway system. That is, a vehicle possibly would have to slow down below the speed,  $s$ , to round the corner in traveling towards point 12. Therefore, since the average speed that the vehicle is assumed to travel, is higher than the actual, the network distance traveled from point 13 to point 12 is increased.

Other similar examples of critical point locations can be observed in Figure 12. It should be stressed that the exact locations of these points were specified in a subjective manner.

As an aid in satisfying the second mentioned requirement, a grid was overlaid on the airport map. This grid, as illustrated in Figure 13, takes the form of the first quadrant of a cartesian plane. Using it in this fashion, it was possible to estimate the X-Y coordinates of each critical point on the runway system.

A list of these coordinates for all 110 critical points is presented in Table 7. This list provided the relative location of each node with respect to all other nodes of the network, thereby satisfying the second requirement.

The grid overlay shown in Figure 13 was also used in determining the relevant information for requirement 3. In order to provide the relative locations of the line segments of the location continuum, the X-Y coordinates of the end points of each line segment were determined from the same grid. A list of these coordinates is presented in Table 8. This list, along with the relationships given in Table 5 provided the necessary information to satisfy the third requirement.

At this point in the methodology, the layout of the airport, defined by the relationships of the runway system network and the location continuum, was determined. The information provided in Tables 5, 6, 7, and 8 was used as input to the computer code, mentioned in Chapter 4, to affect the remainder of the solution procedure. The results of the procedure are discussed in the following section.

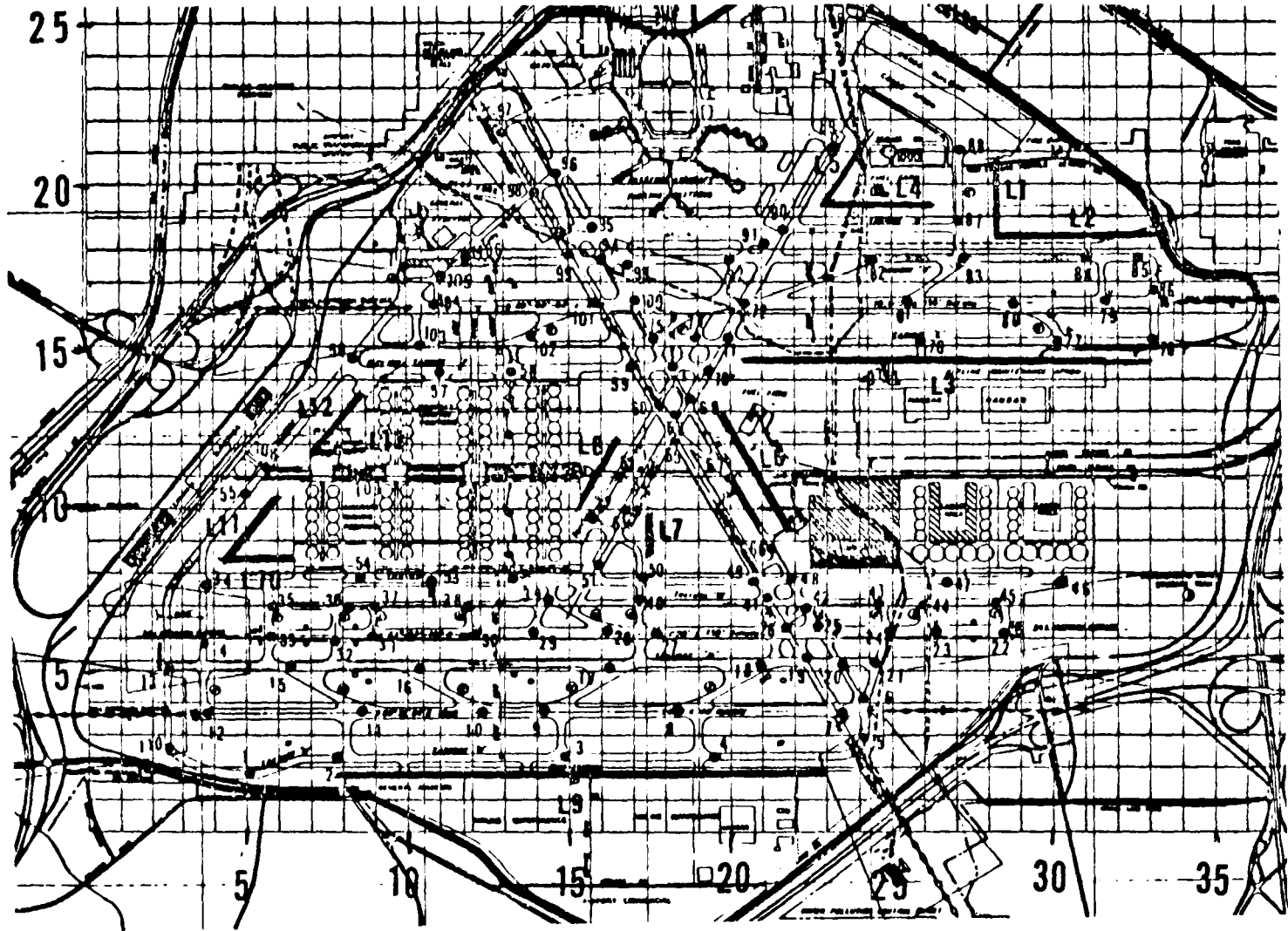


Figure 13. Grid Overlay of Airport Map for Application I.

Table 7. List of Critical Point X-Y Coordinate Positions.

<u>CRITICAL POINT</u>	<u>X</u>	<u>Y</u>	<u>CRITICAL POINT</u>	<u>X</u>	<u>Y</u>
1	5.00	1.50	24	25.25	6.00
2	8.00	2.25	25	23.00	6.25
3	15.50	2.25	26	21.75	6.50
4	19.75	2.25	27	18.00	5.75
5	24.25	2.50	28	16.50	6.00
6	24.50	3.75	29	13.75	5.75
7	23.50	3.50	30	12.50	5.75
8	18.25	3.50	31	9.50	5.50
9	14.25	3.50	32	7.75	5.75
10	13.00	3.50	33	5.75	6.00
11	9.50	3.50	34	4.00	7.50
12	4.00	3.25	35	6.25	6.75
13	2.25	4.75	36	8.25	6.75
14	4.00	5.50	37	9.00	7.00
15	6.50	5.00	38	12.00	7.00
16	10.00	4.75	39	14.50	7.00
17	17.25	5.00	40	17.00	7.00
18	21.50	5.00	41	20.75	7.00
19	22.50	5.50	42	22.00	7.00
20	23.50	5.25	43	24.50	7.00
21	24.70	5.00	44	26.00	7.00
22	28.75	5.75	45	29.00	7.00
23	26.50	6.00	46	30.50	7.50

Table 7. Continued

<u>CRITICAL POINT</u>	<u>X</u>	<u>Y</u>	<u>CRITICAL POINT</u>	<u>X</u>	<u>Y</u>
47	27.00	7.75	70	19.75	14.25
48	22.00	7.75	71	20.25	15.00
49	21.00	7.75	72	20.50	16.00
50	17.50	7.50	73	18.50	14.25
51	16.00	8.00	74	19.00	15.00
52	13.50	7.50	75	18.00	15.00
53	11.00	7.50	76	26.00	15.00
54	8.00	7.50	77	28.25	15.00
55	4.75	9.75	78	33.50	15.00
56	8.25	14.00	79	32.00	16.00
57	11.25	14.00	80	28.00	16.00
58	13.25	14.00	81	24.75	16.00
59	17.00	14.00	82	24.75	17.50
60	18.00	13.00	83	27.50	17.50
61	16.50	11.00	84	31.25	18.00
62	15.50	9.50	85	33.50	18.00
63	16.75	8.75	86	33.25	16.75
64	17.75	10.25	87	27.00	19.00
65	18.50	11.75	88	27.00	21.00
66	21.50	8.75	89	23.50	21.50
67	22.25	10.75	90	22.00	18.50
68	18.50	12.50	91	21.00	18.00
69	19.00	13.00	92	20.00	17.50

Table 7. Continued

<u>CRITICAL POINT</u>	<u>X</u>	<u>Y</u>
93	16.75	17.25
94	16.00	17.50
95	15.75	18.50
96	14.75	20.00
97	13.00	21.50
98	14.00	19.50
99	15.00	17.50
100	17.00	16.00
101	16.00	16.00
102	13.75	15.00
103	10.25	14.75
104	10.75	15.75
105	11.00	16.75
106	11.75	17.50
107	9.75	17.50
108	5.50	11.50
109	8.75	11.50
110	2.50	2.25



Table 8. X-Y Coordinates for Location Continuum Lines

<u>CONTINUUM LINE</u>	LEFT END POINT		RIGHT END POINT	
	<u>X</u>	<u>Y</u>	<u>X</u>	<u>Y</u>
L1	27.75	20.50	27.75	18.25
L2	27.75	18.25	33.5	18.25
L3	20.25	14.25	33.5	14.25
L4	22.50	18.75	26.50	18.75
L5	22.50	18.75	23.75	21.25
L6	19.75	13.50	22.25	9.25
L7	17.50	10.00	18.00	8.00
L8	15.75	10.00	16.75	11.75
L9	7.75	1.50	23.00	1.75
L10	4.50	8.00	6.25	8.00
L11	4.50	8.00	5.75	10.25
L12	7.00	11.50	8.25	13.25
L13	7.00	11.50	8.75	11.50

### Results of Application I

In this section, the results are presented for the application of the solution procedure in which it was assumed that no fire stations exist at the Atlanta Airport. As previously mentioned, in order to test the sensitivity of the solution procedure, the problem was solved three different times. Each time a different value was used for the average speed,  $s$ , that fire vehicles are assumed to be capable of maintaining in traveling on the runway system. The first time the problem was solved with  $s$  given a value of 40 m.p.h. while for the second and third times,  $s$  was assigned values of 45 m.p.h. and 50 m.p.h., respectively. In all three cases the procedure's execution time on the University's IBM 370/158 computer was less than 7.6 minutes.

On solving the problem for an  $s$  value of 40 m.p.h. it was found that five stations were required to satisfy the F.A.A. guidelines. Figure 14 illustrates the locations of these stations enumerated 1 through 5. Notice their relative locations with respect to the four existing stations sites currently planned for 1978 through 1980 at the airport. These existing sites are labeled A, B, C, and D. It can be seen that stations sites 2, 3, and 4 are in a close proximity to existing station sites A, B, and D, respectively. Station site 1 is in the general area of existing station D, however, it should be recognized that station D is not on the location continuum as defined in the solution procedure.

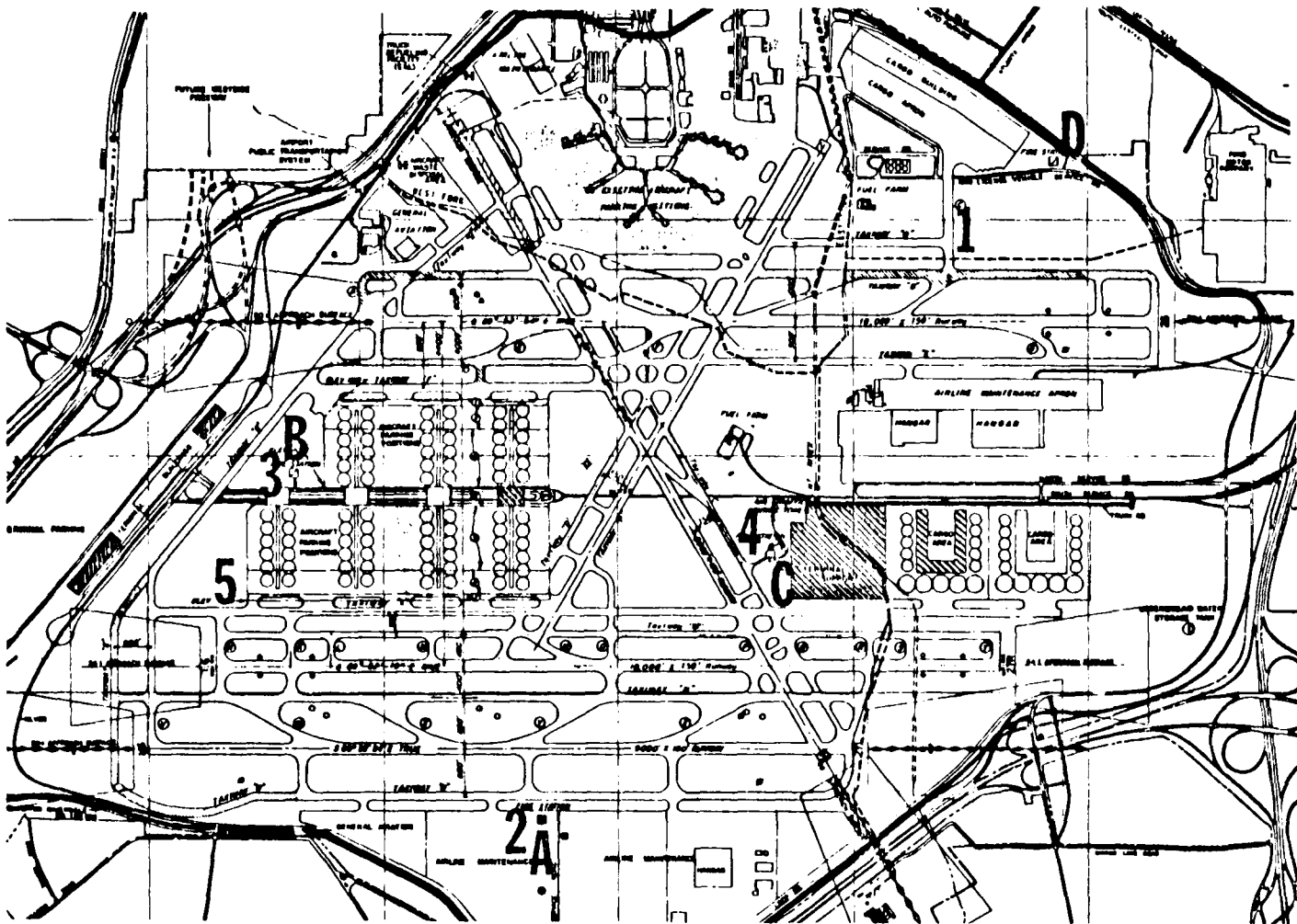


Figure 14. Application I - Station Locations Required for an Assumed  $s$  Value of 40 M.P.H.

As might be anticipated, the number of station sites required to solve the problem, when  $s$  was assumed to be 45 m.p.h., was less than the number needed for when  $s$  was assigned a value of 40 m.p.h. For the 45 m.p.h. case it was found that three station sites were required to meet the F.A.A. guidelines. The locations of these stations are depicted as sites 6, 7, and 8 in Figure 15. Notice that none of these three sites is relatively close to existing sites A, B, C, and D.

In determining the minimum number of stations required to cover the runway system for the case in which  $s$  was assumed to be 45 m.p.h., it should be pointed out that only one solution was found. It is possible that this was due to the columnwise reduction of the cover matrix that was used to expedite the solution procedure as discussed in Chapter 4 (see p. 66). Because this reduction process does not keep track of all sites deleted from the matrix, alternative solutions to the cover problem may have been eliminated. One of these alternatives may provide a solution that offers a smaller chance of possible blockage of routes of vehicles' responding to a potential crash site on the runway system. Therefore, although the solution that was determined (i.e., sites 6, 7, and 8) represents a solution to the problem of covering the runway system, another solution may exist that provides a smaller number of possible occurrences of vehicle response route blockage.

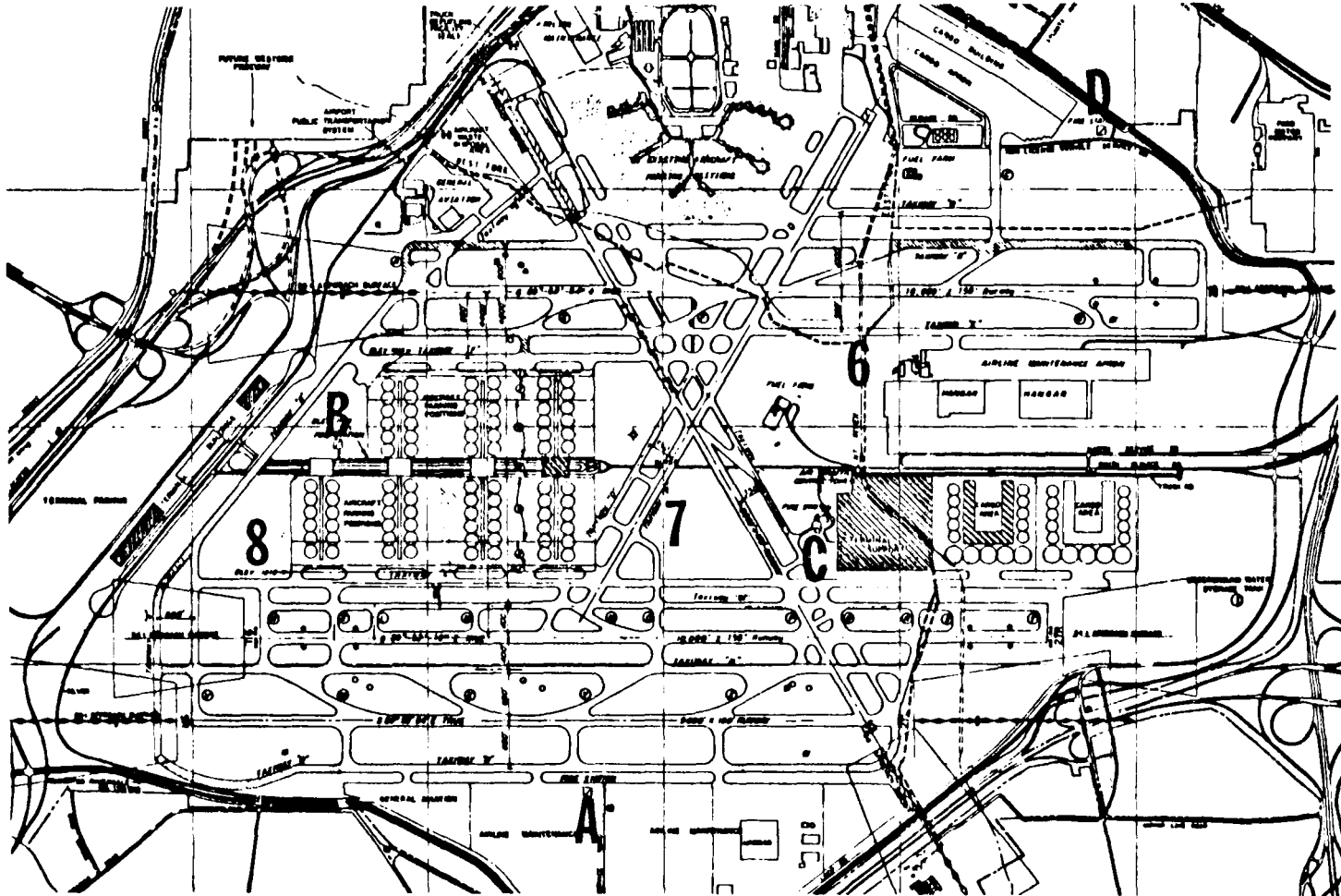


Figure 15. Application I - Station Locations Required for an Assumed  $s$  value of 45 M.P.H.

On solving the problem when  $s$  was assigned a value of 50 m.p.h. it was found once again that three station sites were required. Figure 16 illustrates the locations of these three sites enumerated 9, 10, and 11, respectively. Note in the figure that none of these three sites is in a close proximity to sites A, B, C, and D. Furthermore by comparing the three sites to sites 6, 7, and 8 in Figure 15, it can be seen that the two sets of site locations do not coincide with one another.

In Table 9, a summary of the results of Application I is presented. Included in the table are the number of alternative solutions generated in the procedure for each assumed value of  $s$ . These alternatives are sets of the minimum number of station sites that provided the required coverage of the runway system, prior to the selection of the set that minimizes the possibility of the blockage of response routes to potential crash sites.

#### Application II - Existing Stations Included

As previously mentioned, for the second application, three of the Atlanta Airport's fire stations, currently planned to be in existence in 1978 to 1980, were assumed to be in operation. The problem to be solved was to determine if these stations could meet the two relevant F.A.A. guidelines. If not, then the locations of the fewest number of additional stations required to satisfy the guidelines were found.

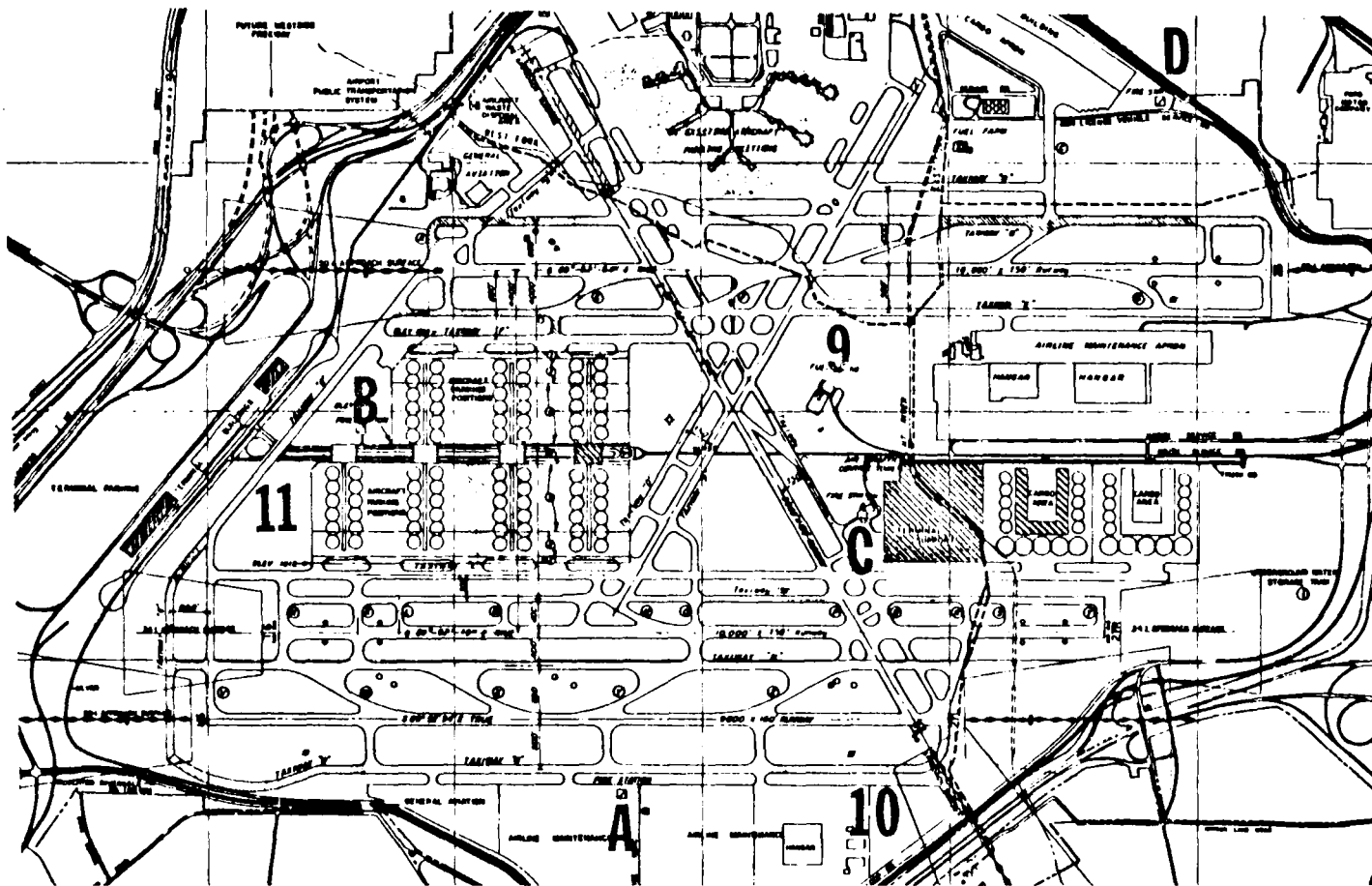


Figure 16. Application I - Station Locations Required for an Assumed  $s$  Value of 50 M.P.H.

Table 9. Summary of Results for Application I.

<u>AVERAGE VEHICLE RESPONSE SPEED, S</u>	<u>NUMBER OF STATIONS REQUIRED TO PROVIDE COVERAGE</u>	<u>NUMBER OF ALTERNATIVE SOLUTIONS PROVIDING COVERAGE</u>	<u>COORDINATES OF SITES PROVIDING MINIMAL RUNWAY BLOCKAGE</u>
40 m.p.h.	5	10	#1- (X,Y) = (28.37,18.25) #2- (X,Y) = (14.85,1.73) #3- (X,Y) = (7.43,12.10) #4- (X,Y) = (21.35,10.69) #5- (X,Y) = (6.13,8.00)
45 m.p.h.	3	1	#6- (X,Y) = (23.20,14.25) #7- (X,Y) = (17.58,9.70) #8- (X,Y) = (5.92,8.00)
50 m.p.h.	3	3	#11- (X,Y) = (20.60,14.25) #12- (X,Y) = (22.87,1.74) #13- (X,Y) = (5.39,9.77)



In solving this problem the input developed for the first application was used (see Tables 5, 6, 7, and 8). However some additional input was required. Although the layout of the airport and the network representation of the runway system are specified in Tables 5, 6, 7, and 8, the locations of the existing fire stations had to be determined. To make these determinations, a grid overlay as shown in Figure 17 was used once again. In this figure, as in the first application, the existing station sites are labeled A, B, C, and D, respectively. The X-Y coordinate positions of these fire stations were determined as follows:

Site A - (X, Y) = (15.25, 1.25)

Site B - (X, Y) = (8.00, 11.25)

Site C - (X, Y) = (22.00, 9.00)

Site D - (X, Y) = (30.25, 20.75)

Notice that station site D is 750 feet from any runway and is not in any runway landing approach. However, it is not a distance,  $d_2$ , of 150 feet from a taxiway or access roadway. It was therefore excluded from the input to the solution procedure and not considered to be in operation.

#### Results of Application II

As in Application I, the second application of the solution procedure was tested three times, a different value used for  $s$  each time. The procedure's execution time in all three cases was less than 7.2 minutes. In the results that are

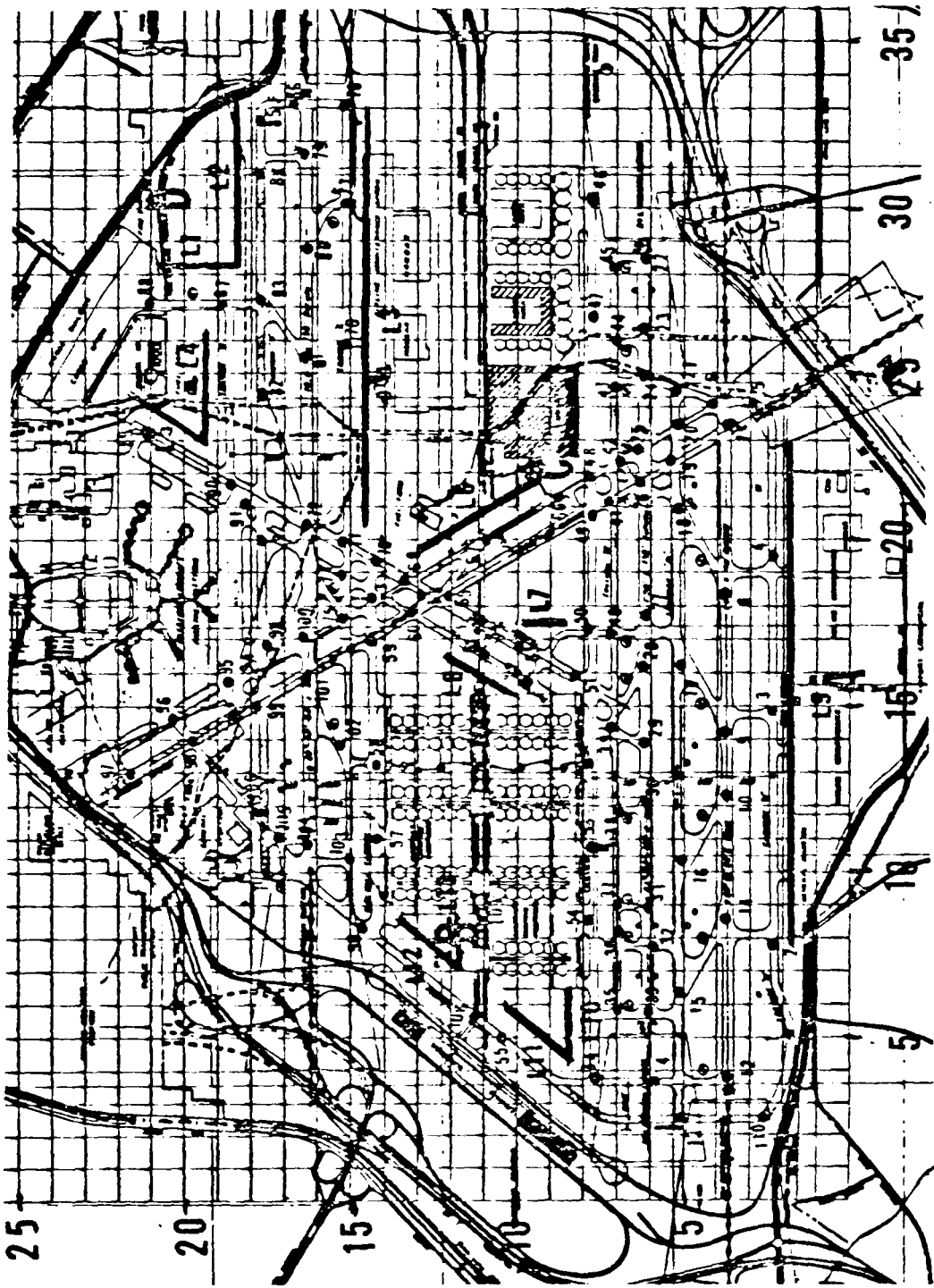


Figure 17. Grid Overlay of Airport Map for Application II.

presented in this section, areas of the runway system that are not covered by the three existing fire stations (i.e., sites A, B, and C) are identified. These uncovered areas are parts of the system that vehicles, are not able to reach, within a two minute time period, by responding from these sites at an average speed,  $s$ .<sup>8</sup>

In solving the problem for an  $s$  value of 40 m.p.h., it was determined that thirty-two runway segments were left uncovered by the three existing stations. These runway segments that were not covered are listed in Table 10. Included in the table are the segments of the runway system that were found to be uncovered when the procedure was applied for an  $s$  value of 45 m.p.h. and an  $s$  value of 50 m.p.h.

In order to meet the two F.A.A. guidelines, it was determined that three stations, additional to the three existing stations, were required when an average vehicle response speed of 40 m.p.h. was assumed. The locations of these stations are illustrated in Figure 18. In the figure, sites 1, 2, and 3 represent the locations of the additional stations required. Vehicles responding from one or more of these additional station sites are capable of reaching any point on the previously uncovered runway segments within a two

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<sup>8</sup>For this application as in the first application, the actual travel time period,  $t$ , was specified to be one minute and forty seconds (see footnote 7, p. 78).

Table 10. Runway Segments Not Covered by Three Existing Stations at the Atlanta Airport.

RUNWAY SYSTEM  
IS UNCOVERED:

<u>FROM CRITICAL POINT NUMBER</u>	<u>TO CRITICAL POINT NUMBER</u>	<u>ASSUMING AN AVERAGE VEHICLE SPEED, S (m.p.h.)</u>
1	110	40
53	54	40
71	76	40
72	81	40
72	82	40
82	91	40
91	94	40
91	95	40
95	96	40
96	98	40
98	99	40
98	106	40
99	106	40
101	104	40
76	77	40, 45
89	90	40, 45
97	98	40, 45
77	78	40, 45, 50
77	80	40, 45, 50
78	79	40, 45, 50
78	86	40, 45, 50
79	80	40, 45, 50
79	84	40, 45, 50
79	86	40, 45, 50
80	81	40, 45, 50
81	83	40, 45, 50
82	83	40, 45, 50
83	84	40, 45, 50
83	87	40, 45, 50
84	85	40, 45, 50
85	86	40, 45, 50
87	88	40, 45, 50

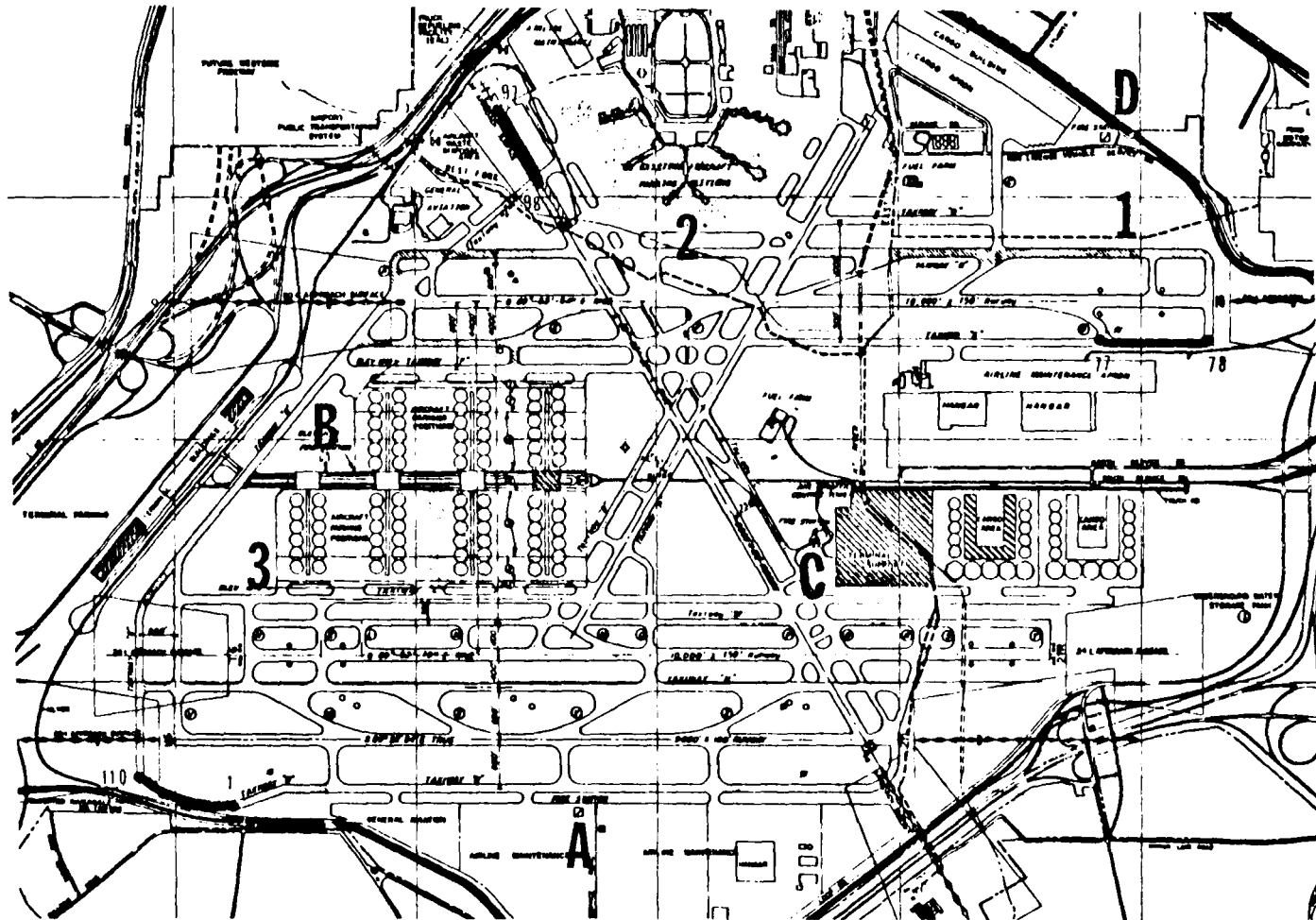


Figure 18. Application II - Station Locations Required for an Assumed  $s$  Value of 40 M.P.H.

minute time period. As an example, site 1 covers the shaded portion of runway between critical points 77 and 78.

In Figure 18 notice the shaded portions of the runway system between critical points 97 and 98 and between critical points 1 and 110. The list in Table 10 indicates that vehicles responding from existing stations A, B, and C are unable to reach all the points on either of these runway segments within two minutes. Furthermore, by inspecting the figure, it can be seen that if none of these sites cover the two shaded segments then certainly the omitted site D is not able to cover them. That is, if fire vehicles are assumed to travel an average speed of 40 m.p.h. in responding to potential crash sites from the four stations planned to be in existence by 1978 to 1980, these vehicles will not be able to reach every point (i.e., potential crash site) within a two minute time interval. However, vehicles responding from new sites 2 and 3 are able to cover these areas.

For an  $s$  value of 45 m.p.h. it was determined that one additional station was required. The location of this station is shown in Figure 19 as site 4. Eighteen areas of the runway system, identified in Table 10, are uncovered by stations A, B, and C in this case. These areas are covered by station 4.

In the solution procedure for this case only one optimal solution was found to satisfy the two minute coverage requirement. It should be emphasized once again that the column

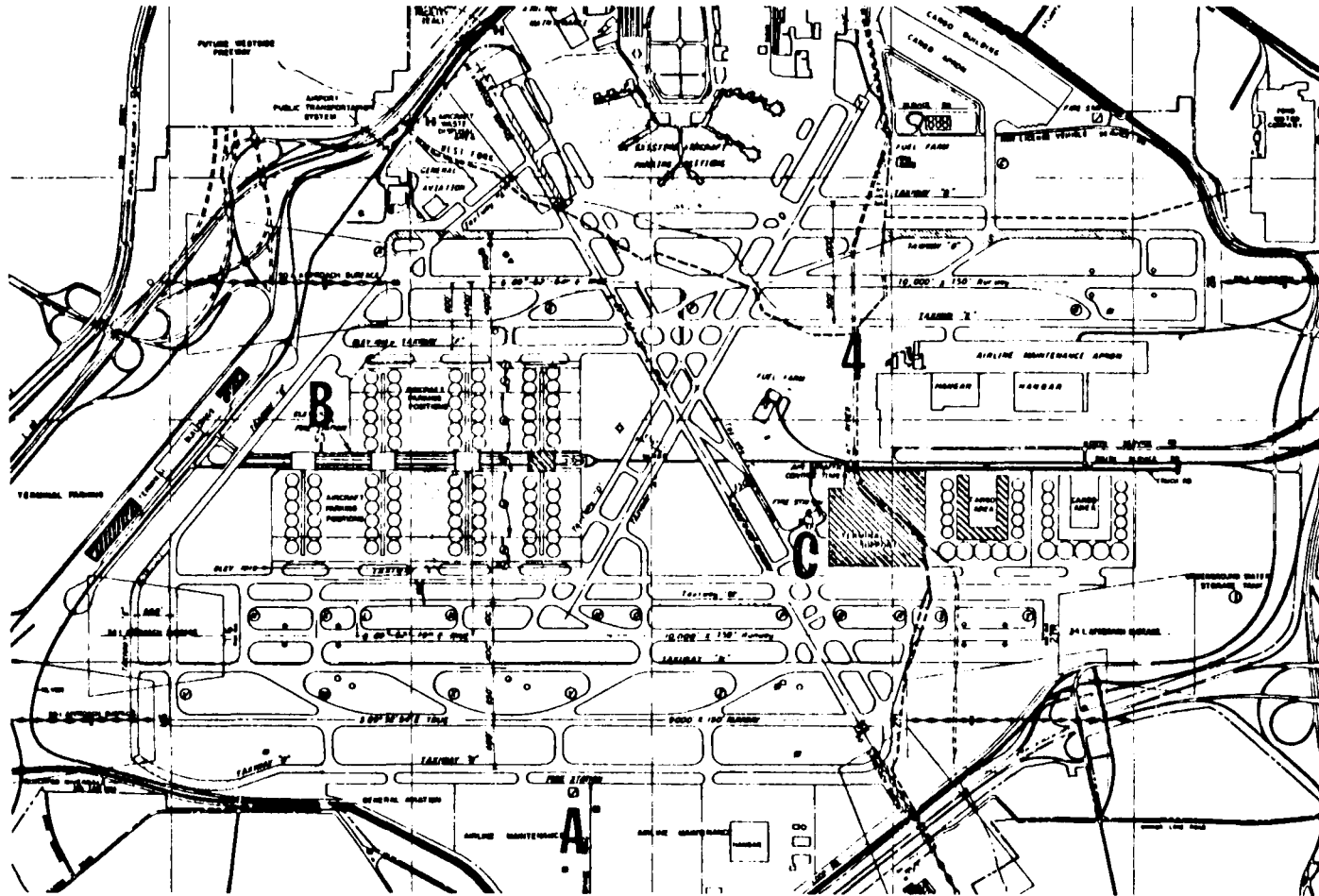


Figure 19. Application II - Station Locations Required for an Assumed  $s$  Value of 45 M.P.H.

reduction technique of the procedure may have eliminated possible alternative optimal solutions. As previously mentioned, some of these alternatives may provide better solutions that reduce the possibility of the blockage of response routes to potential crash sites on the runway system.

In the final test case of the procedure in which an average vehicle response speed of 50 m.p.h. was assumed, it was determined that one additional station was required to satisfy the guidelines. However, as for the previous case in which  $s$  was assigned a value of 45 m.p.h., only one optimal solution was found in the solution of the cover problem. The required additional station is shown in Figure 20 as site 5. It is possible for a vehicle, responding from this site at an average speed of 50 m.p.h., to reach any point on the fifteen uncovered segments, listed in Table 10, within a two minute time interval. The complete results of Application II are presented in Table 11.



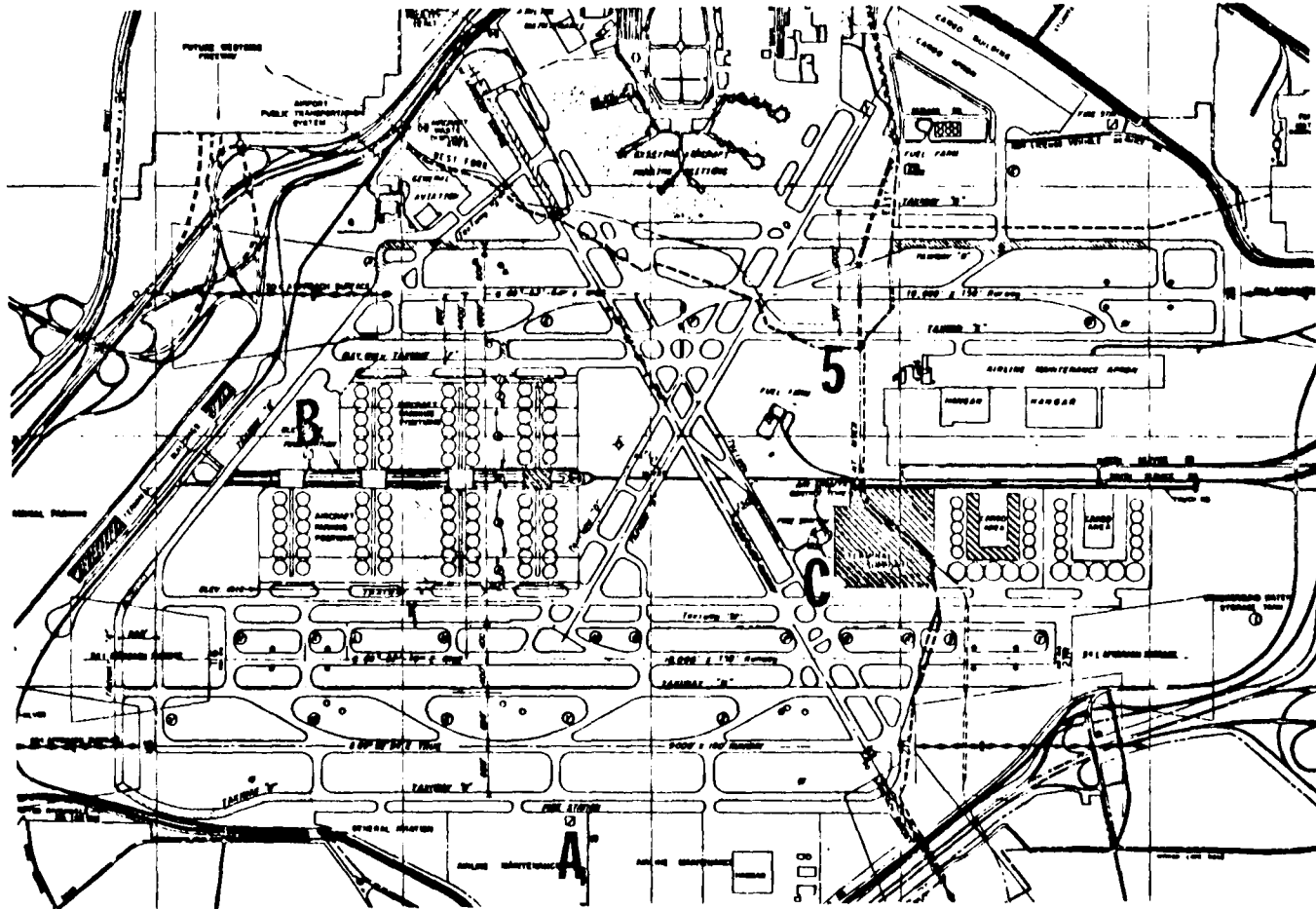


Figure 20. Application II - Station Locations  
 Required for an Assumed  $s$  Value of  
 50 M.P.H.

Table 11. Summary of Results for Application II.

<u>AVERAGE VEHICLE RESPONSE SPEED, S</u>	<u>NUMBER OF STATIONS REQUIRED TO PROVIDE COVERAGE</u>	<u>NUMBER OF ALTERNATIVE SOLUTIONS PROVIDING COVERAGE</u>	<u>COORDINATES OF SITES PROVIDING MINIMAL RUNWAY BLOCKAGE</u>
40 m.p.h.	3	2	#1- (X,Y) = (28.37,18.25) #2- (X,Y) = (18.27,18.00) #3- (X,Y) = (6.13,8.00)
45 m.p.h.	1	1	#4- (X,Y) = (23.20,14.25)
50 m.p.h.	1	1	#5- (X,Y) = (22.99,14.25)

## CHAPTER 6

### SUMMARY AND RECOMMENDATIONS

#### Summary

The major development of the research presented in this thesis was the formulation of a methodology that is of value to planners and fire departments at airports in addressing two recommendations to the Federal Aviation Administration. The first of the recommendations stipulates that by 1980, airport fire stations must be located such that any part of the runway system can be reached within a two minute time interval by the "first-due" vehicle responding from one of these stations. The second recommendation requires these stations to be located so that the chance of possible blockage of vehicle response routes to potential crash sites on the runway system is minimized. The methodology that has been developed is designed to determine the locations of the fewest number of stations required at an airport to meet these two recommendations. The methodology also allows planners to determine if the first recommendation is met at an existing airport with a fixed set of stations. If it is not met, the locations of the least number of additional stations, satisfying both recommendations, are specified.

#### Review of the Methodology and Its Application

In the development of the solution methodology, the problem that was initially faced was a location problem,

continuous in nature. That is, in its initial stage, the problem was one of determining the locations of the fewest number of stations from a set of restricted continuums on the airport grounds such that all points on the runway system (i.e., a continuous demand space) can be reached within a two minute time period by at least one vehicle responding from these station sites. Also, the locations of these stations had to meet the second recommendation to the F.A.A.

An important part of the solution procedure was the transformation of this continuous problem into a location problem with a discrete location space and a discrete demand space. The basic steps involved in the transformation were as follows:

- (1) Model the airport runway system as a network with nodes corresponding to critical points on the runway system and branches corresponding to the runway segments connecting these points.
- (2) Designate the individual branches of the network as the discrete elements for the location problem's demand space.
- (3) Specify feasible areas for locating fire stations on the airport property at least a distance,  $d_1$ , from any runway and a distance,  $d_2$ , from a taxiway or access roadway. No area can be in the landing approach of any runway.
- (4) Generate potential station sites in these feasible areas and designate the sites as discrete elements for the problem's location space.

A set covering model resulted when the problem's location and demand spaces were transformed into discrete spaces.

The demand space of the cover model was the set of branches of the runway network system, while the location space consisted of the potential station sites that were generated in the feasible areas of the airport property. The cover coefficients of the model were determined by using these two spaces and the speed/distance relationships that were either assumed or calculated in the development of the network system.

In finding solutions to the cover problem, an attempt was made to specify alternative optimal solutions. Upon determining alternative optima, each solution was inspected to find which set of station sites minimizes the chance of possible blockage of response routes to potential crash areas on the runway system. The set of sites that offered the most unique travel paths to these crash areas was selected to minimize this possible blockage.

To illustrate the utility of the methodology, it was applied to the problem as it exists at the Atlanta Airport. In the application, two situations were considered. First, it was assumed that no fire stations would be in existence at the airport for the years 1978 to 1980. The problem in this case was to determine the locations of the fewest number of stations required to meet the two recommendations to the F.A.A. For the second situation, it was assumed that three fire stations would be in operation, of the four planned to

be in existence at the airport in the years 1978 to 1980. In this case, it was first determined if the three stations satisfied the first recommendation. If they failed to do so, then the locations of the minimum number of additional stations required to meet both recommendations were found.

The solution procedure was applied to each of the situations three times. Each time the average speed that fire vehicles were assumed to be capable of traveling on the runway system, in responding to potential crash alarms, was changed. The first time this average speed was 40 m.p.h. The second and third times the average speed was 45 m.p.h., and 50 m.p.h., respectively.

It was revealed in these applications that if an average vehicle response speed of 40 m.p.h. is assumed, then the four station sites planned to be in existence at the airport fail to satisfy the first guidelines suggested to the F.A.A. That is, vehicles responding from these sites at an average speed of 40 m.p.h. cannot reach several areas of the runway system where crashes might occur, within a two minute time period.

In applying the methodology to the first mentioned situation, if the average vehicle response speed is 40 m.p.h. then five station locations were required to meet the guidelines. For response speeds of 45 m.p.h. and 50 m.p.h., it was determined in both cases that three station locations were needed.

In the second situation, for an assumed vehicle response speed of 40 m.p.h., the guidelines required three station locations in addition to the three said to be in operation. Also for this situation, it was determined that one additional station location was required to meet the recommendations for the 45 m.p.h. average speed and one additional station was needed for the 50 m.p.h. assumed speed.

### Weaknesses of the Methodology

Throughout the thesis it has been mentioned that due to certain characteristics, inherent to the methodology, there are cases in which an inexact solution may be determined. That is, in certain situations, the indicated solution to the representative discrete location problem may not be the actual solution to the continuous problem.

One of these situations is a result of the elimination of potential location sites in solving the cover problem, making it possible for some alternative optimal solutions not to be found. In order to expedite the determination of a solution to the cover problem that is formulated in the methodology, columns of the cover matrix that are dominated by other columns of the matrix are deleted. The chance exists that a deleted column, either by itself or in combination with other columns, may provide an alternative solution to the cover problem. Furthermore, it is possible that this alternative may offer a better solution to the second F.A.A. guideline

than the one that is actually specified by the solution procedure. That is, the alternative may actually offer a solution that provides a smaller chance of response routes being blocked to potential crash sites on the runway system. Therefore, it should be recognized that the solution procedure is capable of determining a "good" solution to meet the guidelines recommended to the F.A.A., with no guarantee that this "good" solution is in fact the best solution. However, it should be pointed out that the procedure requires less time to determine a "good" solution than the time required to obtain the exact solution.

Another case, in which an inexact solution may be determined, is a result of the procedure used to calculate the values of the cover coefficients for potential location sites in the formulation of the cover problem. The runway system in Figure 21 is used as an example in explaining this situation. In the figure, three potential station location sites are shown. These sites are denoted by the three circled numbers 1, 2, and 3. Also shown in the figure are the six runway segments of the system labeled 1 through 6, respectively.

Suppose that vehicles responding from the three sites can cover no other points on the runway system except as follows:

- (1) A vehicle responding from site 1 can reach any point on runway segments 3, 4, 5, and 6, but can only reach those points



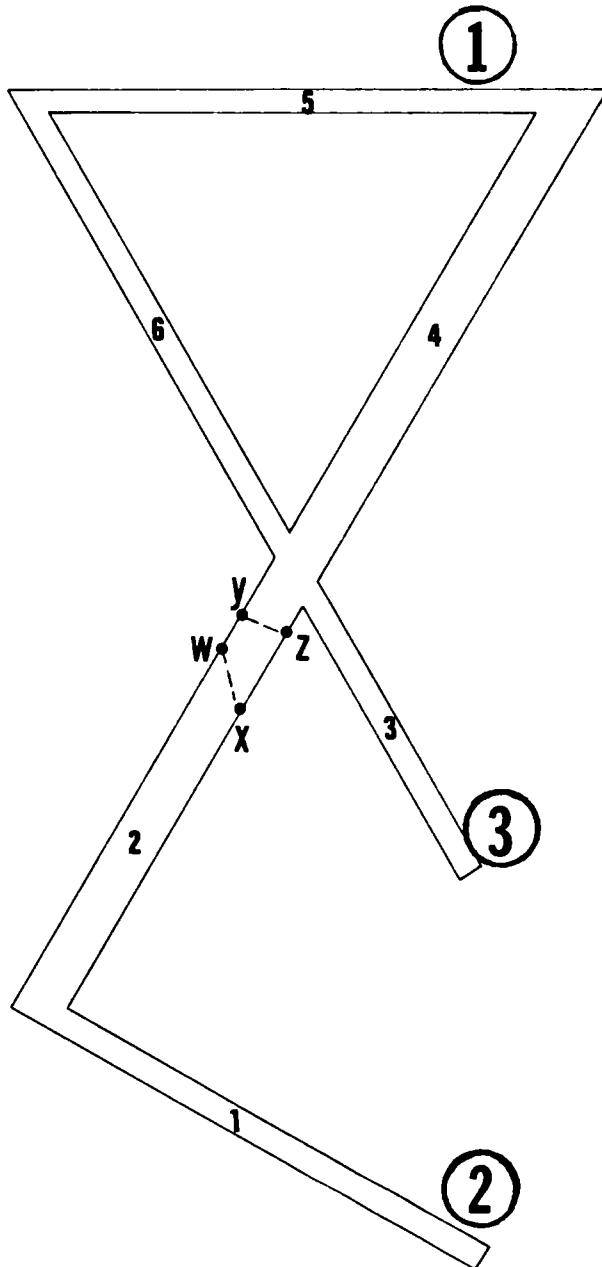


Figure 21. Example Runway System Used to Illustrate a Weakness in Cover Coefficient Determinations.

Table 12. Matrix of Cover Coefficients  
for Figure 21.

$$a_{ij}$$

		STATION SITES		
		j= 1	2	3
RUNWAY SEGMENTS	i= 1	0	1	0
	2	0	0	1
	3	1	0	1
	4	1	0	1
	5	1	0	0
	6	1	0	1

on segment 2 to the right of the broken line  $\overline{WX}$ , within the specified time period,  $t$ .

- (2) A vehicle responding from site 2 can reach any point on runway segment 1, but can only reach those points to the left of the broken line  $\overline{YZ}$ , within the specified time period,  $t$ .
- (3) A vehicle responding from site 3 can only reach those points on runway segments 2, 3, 4, and 6 within the specified time period,  $t$ .

With this information the matrix of cover coefficients can be formulated using the logic of the solution methodology. This matrix is shown in Table 11. Recall that a runway segment is defined to be covered by a station site only if a vehicle responding from that site can reach every point on the segment within the specified time period,  $t$ .

Upon inspection, this matrix indicates that all three station sites are required to cover every point of the runway system of Figure 21. In actuality, any point on the runway system can be reached either by a vehicle responding from site 1 or from site 2 within the specified time period,  $t$ .

This example illustrates a case in which the actual number of stations required to cover all points on a runway system is less than the number determined in solving the cover problem that is formulated in the solution procedure. It is possible that this may occur in applying the methodology to a given airport. It should be recognized however that if it does occur, the solution that is determined is one that provides "overcoverage" of certain areas of a runway system, (in contrast to leaving some areas uncovered).

A third situation that may arise, causing an inexact solution to be determined, is a result of the procedure used to generate potential station sites on the location continuum of an airport's runway system. The runway system in Figure 22 is used to illustrate this hypothetical case in which the number of stations required to solve the discrete problem formulation, again, is greater than the number needed to solve the continuous problem. On this runway system, there are five critical points labeled 1 through 5, respectively and only two feasible location continuum lines identified as  $L_1$  and  $L_2$ .

Suppose that two potential location sites have been generated, one at point A and one at point B. Also, suppose that these are the only two points that are an exact distance,  $\partial$ , from any critical point on the system (site A being a distance,  $\partial$ , from point 1 and site B being a distance,  $\partial$ , from point 2). Because of this, sites A and B are the only possible station sites for the discrete problem.

It is possible, in this case, that both of these sites are needed to cover all the points on the runway system. That is, in order for a vehicle, responding from one of these sites, to be able to reach any point on the runway system within a time period,  $t$ , both sites are required. At the same time, the chance exists that there is a single site, say at point  $x$  on  $L_2$ , that can cover the whole runway system

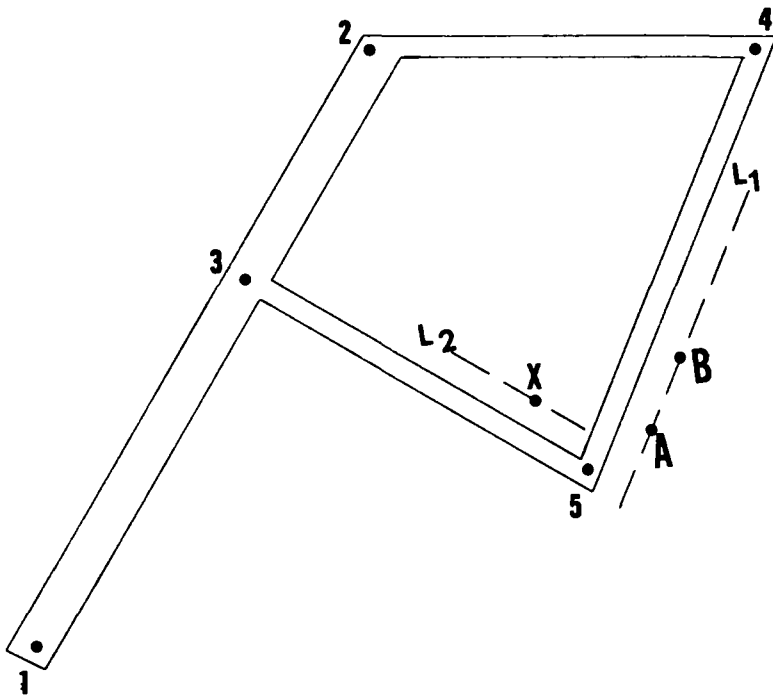


Figure 22. Example of a Procedure Weakness Due to the Manner in Which Potential Location Sites Are Generated.

and be less than a distance,  $\partial$ , from all of the critical points.

As shown in this example, there is a second case in which the continuous problem may require a fewer number of fire stations in its solution than the number needed to solve its discrete counterpart. This possibility exists, due to the site generation procedure of the methodology in which each potential fire station location that is specified, is a distance,  $\partial$ , from the critical point for which it is generated. It should be noted that when the solution methodology is applied to an existing airport, the chances are small that this situation will occur because the locations of critical points generally will be more evenly dispersed with respect to the continuum lines than those points shown in Figure 22. However, the case cannot be dismissed from consideration.

#### Recommendations for Further Research

There are several areas in which the current research can be extended. One recommended area concerns the development of a more efficient solution procedure to provide all the alternative optimal solutions to the cover problem that is formulated in the methodology. As previously mentioned, the procedure that is presently utilized may eliminate possible alternative solutions in order to expedite the solution process. An improvement in the methodology would

be to develop a procedure that is capable of rapidly determining all alternative optima at least in the same amount of time that is required by the current method.

Another research approach that could be taken is to incorporate the third mentioned recommendation to the F.A.A. into the solution procedure. This recommendation would require vehicles traveling to a potential crash area to arrive at the scene in one minute intervals after the first vehicle arrives. With the addition of this third guideline, the problem would be to determine the locations of the minimum number of stations required such that:

- (1) a vehicle responding from one of these stations can reach the scene of a potential crash anywhere on the runway system within a "t" minute time period;
- (2) backup vehicles must arrive at the scene within one minute intervals starting from the time the first vehicle arrives;
- (3) the chance of possible blockage of response routes of vehicles traveling from fire stations to any potential crash site on the runway system is minimized.

Another interesting possibility for further research is to consider hazardous areas of an airport in determining where the fewest number of stations should be located so that the two relevant F.A.A. guidelines of the research are met. An approach that could be used to solve this problem is through the use of a cover model with unequal cost coefficients in the objective function.

Using the notation and definitions presented in Chapter 3, this cover problem can be formulated as follows:

$$(M3) \quad \underset{\underline{x}}{\text{Minimize}} \quad z = \sum_{j \in \{S_1\} \cup \{S_1\}^c} c_j x_j$$

$$\text{subject to} \quad \sum_{j \in \{S_1\} \cup \{S_1\}^c} a_{ij} x_j \geq 1, \text{ for } i \in \{L\}.$$

In order to analyze the significance of the cost coefficients of the objective function of (M3), consider the following:

$$\underset{\underline{x}}{\text{Minimize}} \quad z = x_1 + 3x_2 + 4x_3 + 6x_4.$$

This objective function, disregarding any feasibility constraints, indicates that it is preferable for a station to be located at site 1 than at any other site. If two sites are considered, it is better to have stations located at sites 1 and 2 than at any other two sites. Similarly, sites 1, 2, and 3 offer the best solution over any other combinations of three sites. It should be recognized, however, that the constraint matrix, associated with the objective function above, determines which of these sites are feasible (i.e., which sites cover the runway system).

To illustrate the manner in which cost coefficients can be used to provide consideration to hazardous areas of the runway system at an airport, a technique that can be used to estimate their values is given as follows:



- (1) For each runway segment  $i \in \{L\}$ , assign a weight  $w_i$ ; the smaller the weight, the more hazardous the runway segment is and the greater the chance is that the area will require fire protection.
- (2) Determine a centroid point for each runway segment.
- (3) For each generated station site  $j \in \{S_1\}$ , determine the shortest response route distance,  $d(j,i)$ , from site  $j$  to the centroid point of runway segment  $i$ , for all  $i \in \{L\}$ .
- (4) Assign a value to each cost coefficient,  $c_j$ ,  $j \in \{S_1\}$ , where

$$c_j = \sum_{i \in \{L\}} d(j,i)w_i.$$

This procedure would assign a small cost value to a station site if its fire vehicles would have to respond relatively short distances to crashes in most of the hazardous areas of the airport. Station sites requiring longer travel distances to these areas would have larger cost coefficients. An optimal solution to (M3) represents the minimum number of fire stations covering the runway system that at the same time have the smallest sum total weighted distances (i.e., values for  $c_j$ ) to the hazardous areas of the runway system.

A final recommendation for further research is to perform various sensitivity analyses of the research procedure. While one analysis has already been demonstrated in the applications discussed in Chapter 5, another of particular interest is to determine how much of the runway system is left uncovered for a given number of stations.

As an example, suppose that  $m$  fire stations are required to cover all points on the runway system at an airport. It would be interesting for the procedure to reveal how many runway segments are left uncovered if each combination of  $(m-1)$  station sites were to be used. From these combinations of sites, it could then be determined which combination of  $(m-1)$  fire station sites provides the most coverage (i.e., the fewest uncovered runway segments) for the runway system.

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APPENDIX A  
COMPUTER CODE



C  
C YNODE(I)= Y COORDINATE OF RUNWAY SYSTEM MADE I  
C  
C XLOC(I)= X COORDINATE OF STATION SITE I  
C  
C YLOC(I)= Y COORDINATE OF STATION SITE I  
C  
C NLDC= NUMBER OF STATION SITE LOCATIONS  
C  
C TIME= DISTANCE A VEHICLE CAN TRAVEL WITHIN THE ALLOWED RESPONSE TIME  
C PERIOD LESS 20 SECONDS  
C  
C PATH(I)= STORAGE ARRAY FOR I/O OF DATA FROM DISK. ARRAY IS A LIST OF  
C THE NODES ENCOUNTERED IN TRAVELING BETWEEN TWO GIVEN NODES  
C  
C SPONS(I)= STORAGE ARRAY FOR I/O OF DATA FROM DISK. ARRAY IS A LIST OF  
C THE NODES ENCOUNTERED IN TRAVELING FROM A STATION SITE TO GIVEN  
C NODE  
C  
C X1PARA=  
C Y1PARA=  
C X2PARA= X AND Y COORDINATES OF LEFT-RIGHT ENDPOINTS OF PARALLEL OR  
C Y2PARA= PERPENDICULAR (W.P.T. A GIVEN BRANCH OF THE RUNWAY SYSTEM)  
C X1PERP= LINE SEGMENTS OF THE LOCATION CONTINUUM.  
C Y1PERP= -1. IMPLIES THAT THERE IS NO SEGMENT ADJACENT TO A RUNWAY  
C X2PERP= SEGMENT  
C Y2PERP=  
C  
C  
C KEY(I)= THE ORIGINAL COLUMN NUMBER OF ANY COLUMN IN THE REDUCED 'A'  
C MATRIX  
C



```

C IEXIST= THE NUMBER OF EXISTING FIRE STATIONS ASSUMED TO BE IN OPERATION
C
C DPERP= THE DISTANCE IN FEET A PERPENDICULAR LOCATION CONTINUUM LINE IS
C FROM A RUNWAY SEGMENT. -99999. IMPLIES THAT THERE IS NO
C PERPENDICULAR LINE FOR A RUNWAY SEGMENT.
C
C DPARA= THE DISTANCE IN FEET A PARALLEL LOCATION CONTINUUM LINE IS FROM
C A RUNWAY SEGMENT. -99999. IMPLIES THAT THERE IS NO PARALLEL
C LINE FOR A RUNWAY SEGMENT.
C
C M,NUN= THE NUMBER OF ROWS IN THE COVER MATRIX. ORIGINALLY THE NUMBER
C OF BRANCHES IN THE RUNWAY NETWORK SYSTEM.
C
C N,NLOC= THE NUMBER OF STATION LOCATIONS THAT ARE GENERATED

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COMMON /AREA1/ D(330,110),LABEL(330,110),BRANCH(330,110),
*SHORT(330,110),R(330,110),P(270,2),PI(270,2)
COMMON /AREA2/ X1PARA,Y1PARA,X2PARA,Y2PARA,X1PERP,Y1PERP,X2PERP,
*Y2PERP,XNODE(110),YNODE(110),XLOC(270),YLOC(270),NLOC,FRAC
COMMON /AREA3/ DPARA,DPERP,TIME,PATH(50),SPUNS(50)
COMMON /AREA4/ RSTUP,IPT,ICONT,STOP,N
COMMON /AREA5/ KEY(500),IEXIST
INTEGER*2 A(250,270),CPTS(7000,6),KA(6)
INTEGER*2 BRANCH,LABEL,P
INTEGER SPUNS,INTER(7000)

```

```

      INTEGER PATH
      INTEGER KSTOP,STOP
      DEFINE FILE 3(12100,10,U,NEXT8),2(12100,50,U,NEXT9),10(30625,50,U,
      *NEXT1)
150 FORMAT(/)
150 FORMAT(////)
      INF=1.E11
      IPT=0
      NLUC=1
C INPUT DATA
      CALL INPUT
      NNODE=N
      DO 1 I=1,N
      DO 1 J=I,N
600 FORMAT(50(1X,12))
      D(I,J)=INF
      D(J,I)=INF
      D(J,J)=0.
      IF (BRANCH(I,J).EQ.0) GO TO 1
C DETERMINE EUCLIDEAN DISTANCE BETWEEN CONNECTED NODES
      D(I,J)=((XNODE(I)-XNODE(J))**2+(YNODE(I)-YNODE(J))**2)**.5
      D(J,I)=D(I,J)
      1 CONTINUE
201 FORMAT(1X,20(1X,F4.1))
203 FORMAT(1X,10,1X,2(F3.2,1X))
205 FORMAT(12(F2.0))
206 FORMAT(1X,12(F2.0,1X))
209 FORMAT(1X,2(10,1X),10(F7.1,1X))
204 FORMAT(1X,4(F3.2,1X),4X,4(F3.2,1X),4X,2(F3.2,1X),4X,2(12,1X))
      IF (IEXIST.EQ.0) GO TO 205
      ICONT=IEXIST
C INPUT LOCATIONS OF EXISTING STATIONS -- FRAC IS THE SCALED COORDINATE

```

```

C DISTANCE, DCRIT (SEE STEP 4 OF SOLN. PROCEDURE) FROM NODE IZ
DO 361 I=1, IEXIST
  READ(5, 221) XLCC(I), YLFC(I), IZ, KNODE, FRAC
221  FORMAT(2F10.2, 2I3, F10.2)
  IL=(IZ-1)*H+KNODE
  READ(3, IL) X1PAPA, Y1PAPA, X2PARA, Y2PARA, X1PERP, Y1PERP, X2PERP,
  *Y2PERP, DPAPA, DPESP
  CALL PARA(IZ, KNODE)
361  CONTINUE
  ICNT=0
365  CONTINUE
C DETERMINE THE SHORTEST DISTANCE BETWEEN EACH PAIR OF CRITICAL POINTS
  ISTART=1
  NODE=N
  CALL ROUTE (ISTART, NODE)
C GENERATE POTENTIAL STATION LOCATION SITES ON THE LOCATION CONTINUUM
  ISTART=1
  CALL CONTOR(ISTART)
C DETERMINE THE SHORTEST DISTANCES BETWEEN EACH POTENTIAL STATION
C LOCATION SITE AND ALL CRITICAL POINTS OF THE RUNWAY SYSTEM
  ISTART=I+1
  NODE=N+NLDC-1
  CALL ROUTE (ISTART, NODE)
24  CONTINUE
  NLDC=NLDC-1
C SET UP THE PAI COVER MATRIX FOR POTENTIAL STATION LOCATION SITES J
C AND RUNWAY SEGMENT BRANCHES I
  NUM=0
DO 30 K=1, N
DO 30 L=K, N
  IF (BRANCH(K, L) .NE. 1) GO TO 30
  NUM=NUM+1

```

```

J=NUM4
DO 29 J=1, NLOC
  A(I,J)=0
  IF(SHORT(J+1,L).GT.TIME)GO TO 29
  IF(SHORT(J+N,K).GT.TIME)GO TO 29
  ARCK=TIME-SHORT(J+N,K)
  ARCL=TIME-SHORT(J+N,L)
  IF(ARCK+ARCL.GE.SHORT(K,L))A(I,J)=1
  IF(0(J+1,L).EQ.SHORT(J+N,L).AND.0(J+1,K).EQ.SHORT(J+N,K))
    *A(I,J)=1
29 CONTINUE
  WRITE(6,929)K,L,I
929 FORMAT(1X,2I5,AX,15,/)
  WRITE(6,600) (A(I,J),J=1,NLOC)
  WRITE(6,156)
30 CONTINUE
  WRITE(6,150)
DO 300 J=1,NLOC
  KEY(J)=J
300 CONTINUE
C DELETE REDUNDANT ROWS OF THE 'A' MATRIX THAT ARE COVERED BY EXISTING
C STATION SITES
  IF(1EXIST.GT.0)CALL EXIST(A,NUM,NLOC)
C PERFORM THE GAUSSIAN REDUCTION PROCEDURE
  CALL REDUCE(A,PUT,NLOC)
DO 31 J=1,NLOC
  WRITE(6,601)J,(A(I,J),I=1,NUM)
301 FORMAT(1X,'COL# ',I3,2X,30(1X,12))
  WRITE(6,156)
31 CONTINUE
7000 FORMAT(1H1)
  WRITE(6,7000)

```

```

WRITE(6,150)
C PERFORM THE COLUMN-WISE REDUCTION PROCEDURE
CALL ELIMIN(NUM,A,ALOC,N)
DO 32 J=1,NLBC
WRITE(6,601)J,((I,J),I=1,NUM)
WRITE(6,601)KEY(J)
981 FORMAT(1X,14)
WRITE(6,155)
32 CONTINUE
WRITE(6,150)
C DETERMINE OPTIMUM SOLUTIONS TO COVER PROBLEM (M2)
NEST=7000
CALL SELECT (A,OPTS,NUM,ALOC,FEAS,KA,NDPT,IFEAS)
WRITE(6,602)NDPT
602 FORMAT(1H1,1X,THE NUMBER OF OPTIMUM SOLUTIONS IS ',15)
IF(IFEAS.EQ.0)GO TO 84
DO 85 I=1,NDPT
DO 85 J=1,IFEAS
OPTS(I,J+LEXT)=OPTS(I,J)
35 CONTINUE
DO 86 I=1,NDPT
DO 86 J=1,IFEAS
OPTS(I,J)=J
66 CONTINUE
IFEAS=IFEAS+IFEAS
89 CONTINUE
40 FORMAT(3X,3013,/)
IDPROB=1
51 WRITE(6,55)IDPROB
55 FORMAT(/,1X,THE END OF SOLUTION FOR PROBLEM ',13,2X,')*#',/)
ISOL=NDPT
IF(NDPT.EQ.1)GO TO 254

```

```

1BEST=1
GO TO 960
954 CONTINUE
C FOR THE ALTERNATIVE SOLUTIONS TO THE COVER PROBLEM DETERMINE THE
C SOLUTION THAT MINIMIZES THE CHANCE OF POTENTIAL BLOCKAGE OF RESPONSE
C ROUTES
955 CALL ALTRPF(1BEST,ISOL,OPTS,1BEST,IFEAS,ITER)
960 CONTINUE
WRITE(5,900)
900 FORMAT(1L1,1X,2D(14.1), 1X, ' THE GLOBAL OPTIMAL SOLUTION HAS B
*BEEN DETERMINED ',2D(14.1),//)
IL=1BEST
DO 970 J=1,IFEAS
I=OPTS(IL,J)
WRITE(5,901)I,XLHC(I),YLHC(I)
970 CONTINUE
901 FORMAT(1X, 'STATION SITE = ',I3, ' LOCATED AT X = ',F8.3, ' Y = ',
*F8.3,//)
999 STOP
END

```

SUBROUTINE ROUTE(ISTART,NODE)

C  
C  
C  
C  
C  
C

THIS SUBROUTINE INCORPORATES FLOYD'S PROCEDURE FOR DETERMINING THE  
SHORTEST DISTANCE BETWEEN ALL NODES IN A NETWORK

```
COMMON /AREA1/ D(300,110), LABEL(300,110), BRANCH(300,110),  
*SHORT(300,110), P(300,110), P(270,2), D1(270,2)  
COMMON /AREA2/ X1PARA, Y1PARA, X2PARA, Y2PARA, X1PERP, Y1PERP, X2PERP,  
*Y2PERP, XNODE(110), YNODE(110), XLOC(270), YLOC(270), ALCC, FRAC  
COMMON /AREA3/ DPARA, DPERP, TIME, PATH(50), SPONS(50)  
COMMON /AREA4/ PSTOP, IPT, ICNT, STOP, N  
COMMON /AREA5/ XSY(300), IEXIST  
INTEGER*2 BRANCH, LABEL, R  
INTEGER PATH  
INTEGER PSTOP, STOP  
INTEGER SPONS  
INF=1.E11  
DO 2 I=ISTART, NODE  
IF(NODE.GT.N.AND.I.LE.N)GO TO 2  
DO 1 J=1, N  
IF(D(I, J).EQ.-1..OR.D(I, J).LE.INF)D(I, J)=J  
&BRANCH(I, J)=0  
IF(D(I, J).GT.0..AND.D(I, J).LT.INF)BRANCH(I, J)=1  
IF(D(I, J).EQ.-1.)D(I, J)=INF  
SHORT(I, J)=D(I, J)  
LABEL(I, J)=0  
1 CONTINUE  
2 CONTINUE  
DO 3 I=1, NODE  
DO 3 J=1, N
```

```

    LABEL(I,J)=0
3  CONTINUE
    DO 20 I=1,N
    DO 15 J=ISTART,NODE
    IF(J.EQ.I)GO TO 15
    IF(SHORT(J,I).GE.INF)GO TO 15
    DO 10 K=1,N
    IF(J.EQ.K)GO TO 10
    IF(K.EQ.I)GO TO 10
    IF(SHORT(I,K).GE.INF)GO TO 10
    DIST=SHORT(J,I)+SHORT(I,K)
    IF(SHORT(J,K).LE.DIST)GO TO 5
    SHORT(J,K)=DIST
    R(J,K)=R(J,I)
    LABEL(J,K)=1
    GO TO 10
5  IF(SHORT(J,I).GE.SHORT(J,K).OR.SHORT(I,K).GE.SHORT(J,K))GO TO 10
    IF(LABEL(J,K).EQ.1)GO TO 10
    R(J,K)=K
    LABEL(J,K)=1
10  CONTINUE
15  CONTINUE
20  CONTINUE
C  SHORTEST DISTANCE BETWEEN EACH PAIR OF NODES HAS BEEN FOUND
    DO 40 I=1,N
    R(I,I)=I
40  CONTINUE
    IP=0
    IK=0
    DO 30 I=ISTART,NODE
    DO 50 J=1,N
    IP=IP+1

```



```

L=I
M=0
IF(MDDE.GT.N)GO TO 70
44 M=M+1
   PATH(N)=R(L,J)
   IF(PATH(N).NE.J)GO TO 49
   STOP=N
C STORE THE LIST OF NODES IN THE SHORTEST PATH BETWEEN EACH PAIR OF
C CRITICAL POINTS. STOP IS THE NUMBER OF NODES IN THE PATH
   WRITE(9)IP,STOP,(PATH(IZ),IZ=1,N)
   GO TO 50
45 L=PATH(N)
   GO TO 44
70 IF(I.LE.N)GO TO 50
   IK=IK+1
74 M=M+1
   IT=I-N
   SPONS(M)=R(L,J)
   IF(SPONS(M).NE.J)GO TO 75
   RSTOP=M
C STORE THE LIST OF NODES IN THE SHORTEST PATHS BETWEEN EACH POTENTIAL
C STATION LOCATION SITE AND EACH CRITICAL POINT ON THE RUNWAY SYSTEM.
C RSTOP IS THE NUMBER OF NODES IN THE PATH
   WRITE(10)IK,RSTOP,(SPONS(IK),IK=1,N)
   GO TO 50
75 L=SPONS(M)
   GO TO 74
50 CONTINUE
60 CONTINUE
   RETURN
   END

```

```

SUBROUTINE CONTOR(ISTART)
C
C
C THIS SUBROUTINE GENERATES POTENTIAL STATION SITES ON THE LOCATION
C CONTINUUM
C
C
COMMON /AREA1/ D(300,110),LABEL(300,110),BRANCH(300,110),
*SHORT(330,110),X(300,110),P(270,2),NE(270,2)
COMMON /AREA2/X1PARA,Y1PARA,X2PARA,Y2PARA,X1PERP,Y1PERP,X2PERP,
*Y2PERP,XNODE(110),YNODE(110),XLOC(270),YLOC(270),NLOC,FRAC
COMMON /AREA3/ DPARA,DPERP,TIME,PATH(50),SPONS(50)
COMMON /AREA4/ RSTOP,IPT,ICONT,STOP,I
COMMON /AREA5/KEY(300),IEXIST
INTEGER*2 BRANCH,LABEL,R
INTEGER PATH
INTEGER SPONS
INTEGER RSTOP,STOP
ICOUNT =1
L=0
DO 80 I=ISTART,N
C CHECK EACH CRITICAL POINT J THAT IS A SHORTEST DISTANCE LESS THAN
C THE VALUE FOR TIME FROM CRITICAL POINT I
DO 80 J=1,N
L=L+1
IF(SHORT(I,J).GT.TIME.AND.BRANCH(I,J).NE.1.)GO TO 80
IF(SHORT(I,J).GE.TIME.AND.BRANCH(I,J).EQ.1.)GO TO 75
C CHECK ALL BRANCHES FROM J EXCEPT SHORTEST PATH BRANCH
READ(9'L)STOP,(PATH(I),I=1,STOP)
C CHECK ONLY THOSE BRANCHES EMANATING FROM CRITICAL POINT J TO
C CRITICAL POINTS K THAT ARE A SHORTEST DISTANCE GREATER THAN TIME FROM
C CRITICAL POINT I

```

```

D7 70 K=1,N
GATE=0.
IF(I.EQ.K)G7 TO 70
IF(STOP.EQ.1)G9 TO 10
IP=STOP
IF(K.EQ.PATH(IP-1)).AND.IP.GE.2)G9 TO 70
10 IF(BRANCH(J,K).NE.1)G9 TO 70
DIST=SHORT(I,J)+D(J,K)
IF(DIST.LT.FINE)G9 TO 70
15 IZ=J
KNODE=K
M=L
IP=STOP-1
20 FRAC=(TIME-SHORT(I,IZ))/D(IZ,KNODE)
21 CONTINUE
IL=(IZ-1)*N+KNODE
READ(8,IL)X1PANA,Y1PANA,X2PANA,Y2PANA,X1PERP,Y1PERP,X2PERP,Y2PERP,
*DPANA,OPER
205 FORMAT(1X,10F8.2,1X,13,1X,2(12,1X),1)(F7.1,1A)
IF(GATE.EQ.0.)FEAS=DPANA
IF(GATE.EQ.1.)FEAS=OPER
25 IF(FRAC.GE.FEAS)G7 TO 40
IF(IP.EQ.0)G9 TO 70
KNODE=IZ
IZ=PATH(IP)
IP=IP-1
201 FORMAT(1X,3(13,1X),2X,F8.2)
200 FORMAT(1X,3(13,1X),2X,F8.2,5X,13)
G9 TO 20
40 CONTINUE
IF(GATE.GE.1.)G9 TO 50

```

```

      IF(X1PARA.EQ.-1.)GO TO 45
      DIZ=FRAC+SHORT(IZ,I)
      DKN=FEAS*2.+SHORT(KNODE,I)-FRAC+SHORT(IZ,KNODE)
      IF(DKN.LT.DIZ)GO TO 45
C A POTENTIAL STATION SITE CAN BE LOCATED ON A PARALLEL CONTINUUM LINE
      CALL PARA(IZ,KNODE)
      IF(NLOC.GE.270)GO TO 81
45  GATE=GATE+1.
      IF (GATE.EQ.1.)GO TO 15
50  IF(X2PARA.EQ.-1.)GO TO 50
      DIZ=FRAC+SHORT(IZ,I)
      DKN=FEAS*2.+SHORT(KNODE,I)-FRAC+SHORT(IZ,KNODE)
      IF(DKN.LT.DIZ)GO TO 50
C A POTENTIAL STATION SITE CAN BE LOCATED ON A PERPENDICULAR CONTINUUM
C LINE
      CALL PERP(IZ,KNODE)
      IF(NLOC.GE.270)GO TO 81
70  CONTINUE
      GO TO 80
75  FRAC=TIME
      IF(X1PARA.EQ.-1.)GO TO 77
      CALL PARA(I,J)
77  IF(X2PARA.EQ.-1.)GO TO 80
      CALLPERP(I,J)
80  CONTINUE
      GO TO 999
81  WRITE(6,250)NLOC
230  FORMAT(1H1,10X,20(' '),2X,'NLOC= ',I3,2X,'HALT SELECTION OF FEASIB
      *LE LOCATION SITES',5(' '),1H1)
      WRITE(6,200)I,J,K,FRAC,IZ
999  RETURN
      END

```

```

SUBROUTINE PARA(IZ,KNODE)
C
C THIS SUBROUTINE LOCATES POTENTIAL STATION SITES ON PARALLEL CONTINUUM
C LINES
C
COMMON /AREAL/ 0(300,110),LABEL(260,110),BRANCH(300,110),
*SHORT(300,110),S(360,110),P(270,2),O(270,2)
COMMON /AREA2/X1PARA,Y1PARA,X2PARA,Y2PARA,X1PERP,Y1PERP,X2PERP,
*Y2PERP,XNODE(110),YNODE(110),XLOC(270),YLOC(270),NLOC,FRAC
COMMON /AREA3/ DPARA,DPERP,HPATH(50),SPONS(50)
COMMON /AREA4/ RSTOP,IOT,ICONT,STOP,N
COMMON /AREA5/KEY(300),LEXIST
INTEGER*2 BRANCH,LABEL,R
INTEGER PATH
INTEGER SPONS
INTEGER RSTOP,STOP
REAL LINE1,LINE2
LINE1=DPARA
LINE2=FRAC-LINE1
IF(ICONT.GT.0)GO TO 15
FRAC=(LINE1**2+LINE2**2)**.5
BOT=X2PARA-X1PARA
IF(BOT.EQ.0.)GO TO 10
TOP=Y2PARA-Y1PARA
SLOPE=TOP/BOT
10 IF(BOT.EQ.0.)SLOPE=1.E0
A=SLOPE**2+1.
E=Y2PARA-SLOPE*X2PARA
B=2.*SLOPE*E-2.*XNODE(IZ)-2.*YNODE(IZ)=SLOPE
C=E**2+XNODE(IZ)**2+YNODE(IZ)**2-E*YNODE(IZ)**2.-FRAC**2

```

```

    SQUARE=B**2-4.*A*C
    IF(SQUARE.LT.0.)GO TO 30
    XLOC(NLOC)=(-B+SQUARE**.5)/(2.*A)
    IF(XLOC(NLOC).LT.X1PARA.OR.XLOC(NLOC).GT.X2PARA)GO TO 2)
    YLOC(NLOC)=SLOPE*XLOC(NLOC)+E
15  CONTINUE
    P(NLOC,1)=IZ
    P(NLOC,2)=KNODE
    WRITE(6,200)NLOC,XLOC(NLOC),YLOC(NLOC),IZ,KNODE
200  FORMAT(' NLOC= ',I3,2X,'X= ',F8.3,2X,'Y= ',F8.3,' PARALLEL TO RUNW
1AY SEGMENT ',I3,'-',I3,2X)
    CALL LOCATE(LINE1,LIN2,IZ,KNODE)
    NLOC=NLOC+1
    IF(ICONT.GT.)GO TO 40
20  XLOC(NLOC)=(-B-SQUARE**.5)/(2.*A)
    IF(XLOC(NLOC).LT.X1PARA.OR.XLOC(NLOC).GT.X2PARA)GO TO 30
    YLOC(NLOC)=SLOPE*XLOC(NLOC)+E
    P(NLOC,1)=IZ
    P(NLOC,2)=KNODE
    WRITE(6,200)NLOC,XLOC(NLOC),YLOC(NLOC),IZ,KNODE
    CALL LOCATE(LINE1,LIN2,IZ,KNODE)
    NLOC=NLOC+1
30  CONTINUE
201  FORMAT(' NO INTERSECTION ',2X,'RUNWAY SEGMENT IS ',I3,'-',I3)
40  RETURN
    END

```

SUBROUTINE PERP (IZ,KNODE)

C  
C  
C  
C  
C  
C

THIS SUBROUTINE LOCATES POTENTIAL STATION SITES ON PERPENDICULAR  
CONTINUOUS LINES

```
COMMON /AREA1/ D(300,110), LABEL(300,110), BRANCH(300,110),  
*SHORT(300,110), R(300,110), P(270,2), D1(270,2)  
COMMON /AREA2/ X1PARA, Y1PARA, X2PARA, Y2PARA, X1PERP, Y1PERP, X2PERP,  
*Y2PERP, XNODE(110), YNODE(110), XLOC(270), YLOC(270), NLOC, FRAC  
COMMON /AREA3/ DPERP, TIME, PATH(50), SPONS(50)  
COMMON /AREA4/ RSTOP, IPT, ICONF, STOP, X  
COMMON /AREA5/ KFY(300), IFRIST  
INTEGER*2 BRANCH, LABEL, R  
INTEGER PATH  
INTEGER SPONS  
INTEGER RSTOP, STOP  
REAL LINE1, LINE2  
LINE1=DPERP  
LINE2=FRAC-LINE1  
FRAC=(LINE1**2+LINE2**2)**.5  
BOT=X2PERP-X1PERP  
IF(BOT.EQ.0.)GO TO 10  
TOP=Y2PERP-Y1PERP  
SLOPE=TOP/BOT  
10 IF(BOT.EQ.0.)SLOPE=1.E+9  
E=Y2PERP-SLOPE*X2PERP  
A=SLOPE**2+1.  
C=E**2+XNODE(IZ)**2+YNODE(IZ)**2-E*YNODE(IZ)*2.-FRAC**2  
B=2.*SLOPE*E-2.*XNODE(IZ)-2.*YNODE(IZ)*SLOPE  
SQUARE=B**2-4.*AC
```

```

IF(SQUARE.LT.0.)GO TO 30
XLCC(NLCC)=(-B+SQUARE**0.5)/(2.*A)
IF(XLCC(NLCC).LT.X1PERP.OR.XLCC(NLCC).GT.X2PERP)GO TO 20
YLCC(NLCC)=SLOPE*XLCC(NLCC)+F
P(NLCC,1)=IZ
P(NLCC,2)=KNODE
WRITE(6,200)NLCC,XLCC(NLCC),YLCC(NLCC),IZ,KNODE
200 FORMAT(' NLCC= ',I3,2X,'X= ',F8.3,2X,'Y= ',F8.3,' SECONDARY FEASIB
ILE LINE OF RUNWAY SEGMENT ',I3,'-',I3)
CALL LOCATE(LINE1,LINE2,IZ,KNODE)
NLCC=NLCC+1
20 XLCC(NLCC)=(-B-SQUARE**0.5)/(2.*A)
IF(XLCC(NLCC).LT.X1PERP.OR.XLCC(NLCC).GT.X2PERP)GO TO 30
YLCC(NLCC)=SLOPE*XLCC(NLCC)+F
P(NLCC,1)=IZ
P(NLCC,2)=KNODE
WRITE(6,200)NLCC,XLCC(NLCC),YLCC(NLCC),IZ,KNODE
CALL LOCATE(LINE1,LINE2,IZ,KNODE)
NLCC=NLCC+1
30 CONTINUE
201 FORMAT(' NO INTERSECTION ',2X,' RUNWAY SEGMENT IS ',I3,'-',I3)
40 RETURN
END

```



```

SUBROUTINE LOCATE(LINE1,LINE2,IZ,KNODE)
C
C
C THIS SUBROUTINE DETERMINES THE DISTANCE FROM A GENERATED STATION
C SITE TO NODES IZ AND KNODE THAT BOUND THE RUNWAY SEGMENT BRANCH
C ADJACENT TO THE SITE
C
C
COMMON /AREAL/ D(350,110),LABEL(350,110),BRANCH(350,110),
*SHRT(350,110),R(350,110),P(270,2),DI(270,2)
COMMON /AREA2/X1PARA,Y1PARA,X2PARA,Y2PARA,X1PERP,Y1PERP,X2PERP,
*Y2PERP,XNODE(110),YNODE(110),XLOC(270),YLOC(270),XLOC,FRAC
COMMON /AREA3/ DPARA,DPERP,TIME,PATH(50),SPOTS(50)
COMMON /AREA4/ RSTOP,IPT,ICONT,STOP,N
COMMON /AREA5/KEY(300),EXIST
INTEGER*2 BRANCH,LABEL,C
INTEGER PATH
INTEGER RSTOP,STOP
REAL LINE1,LINE2
L=NLDC-1
IF(L.EQ.0)GO TO 11
DO 1) K=1,L
IF(XLOC(NLDC).EQ.XLOC(K).AND.YLOC(NLDC).EQ.YLOC(K))GO TO 6)
10 CONTINUE
11 CONTINUE
DO 5) I=1,N
D(N+NLDC,I)=-1.
IF(I.EQ.12)D(N+NLDC,I)=LINE1+LINE2
IF(I.EQ.KNODE)D(N+NLDC,I)=LINE1+D(IZ,KNODE)-LINE2
50 CONTINUE
GO TO 99
60 CONTINUE

```

```
D(N+K, IZ)=LINE1+LINE2  
D(N+K, KNODE)=LINE1+D(IZ, KNODE)-LINE2  
NLOC=L  
99 RETURN  
END
```

```

SUBROUTINE SELECT(A,OPTS,N,K,NEST,K,ITEST,IFEAS)
C
C
C THIS SUBROUTINE SEARCHES FOR AN OPTIMAL SOLUTION TO THE COVER
C PROBLEM (M2) BY SELECTING COMBINATIONS OF COLUMNS OF THE 'A' MATRIX
C
C
COMMON /AREAD/KEY(300), IEXIST
INTEGER*2 A(250,N),K(6),OPTS(NEST,6)
DIMENSION RCOL(6)
ITEST=0
IFEAS=1
WRITE(6,157)
C DETERMINE IF SOLUTION IS 1
DO 4 J=1,N
DO 2 I=1,M
IF(A(I,J).EQ.1)GO TO 2
GO TO 4
2 CONTINUE
ITEST=ITEST+1
OPTS(ITEST,IFEAS)=KEY(N)
WRITE(6,200)OPTS(ITEST,IFEAS)
200 FORMAT(50(1X,11))
4 CONTINUE
WRITE(6,300)IFEAS
IF(ITEST.GT.0)GO TO 999
300 IFEAS=IFEAS+1
157 FORMAT(///)
C SEARCH FOR 2 SITES
IP1=N-1
DO 9 J1=1,IP1
K(1)=J1

```

```

IJ2=J1+1
DO 8 J2=IJ2,N
  K(2)=J2
  CALL BASIS(A,K,OPTS,ITEST,M,N,NEST,IFFAS)
  IF(ITEST.GE.NEST)GO TO 999
8 CONTINUE
9 CONTINUE
  WRITE(6,300) IFFAS
  IF(ITEST.GT.0)GO TO 999
C SEARCH FOR 3 SITES
  IFFAS=IFFAS+1
  IPI=N-2
  DO 20 J1=1,IPI
    K(1)=J1
    IJ2=J1+1
    IP2=IPI+1
    DO 15 J2=IJ2,IP2
      K(2)=J2
      IJ3=J2+1
      DO 10 J3=IJ3,M
        K(3)=J3
        CALL BASIS(A,K,OPTS,ITEST,M,N,NEST,IFFAS)
        IF(ITEST.GE.NEST)GO TO 999
10 CONTINUE
15 CONTINUE
20 CONTINUE
  WRITE(6,300) IFFAS
300 FORMAT(IX,'SEARCH FOR A SOLUTION OF ',I3,'2X','IS COMPLETED')
  IF(ITEST.GT.0)GO TO 999
C SEARCH FOR 4 SITES
  IFFAS=IFFAS+1
  IPI=N-3

```

```

00 40 J1=1, IPI
K(1)=J1
IJ2=J1+1
IP2=IPI+1
00 35 J2=IJ2, IP2
K(2)=J2
IJ3=J2+1
IP3=IP2+1
00 30 J2=IJ3, IP3
K(3)=J3
IJ4=J3+1
00 25 J4=IJ4, N
K(4)=J4
CALL BASIS(A,K,OPTS,ITEST,N,M,NEST,IFEAS)
IF(ITEST.GE.NEST)GO TO 999
25 CONTINUE
30 CONTINUE
35 CONTINUE
40 CONTINUE
IF(ITEST.GT.0)GO TO 999
WRITE(6,300)IFEAS
IFEAS=IFEAS+1
IPI=N-4
00 70 J1=1, IPI
K(1)=J1
IJ2=J1+1
IP2=IPI+1
00 65 J2=IJ2, IP2
K(2)=J2
IJ3=J2+1
IP3=IP2+1

```

C SEARCH FOR 5 SITES

```

DO 60 J3=IJ3,IP3
K(3)=J3
IJ4=J3+1
IP4=IP3+1
DO 55 J4=IJ4,IP4
K(4)=J4
IJ5=J4+1
DO 50 J5=IJ5,N
K(5)=J5
CALL BASIS(A,K,OPTS,ITEST,N,N,TEST,IFEAS)
IF(ITEST.GE.NEST)GO TO 999
50 CONTINUE
55 CONTINUE
60 CONTINUE
65 CONTINUE
70 CONTINUE
IF(ITEST.GT.0)GO TO 999
WRITE(6,300)IFEAS
C SEARCH FOR 6 SITES
IFEAS=IFCAS+1
IPI=N-5
DO 110 J1=1,IPI
K(1)=J1
IJ2=J1+1
IP2=IP1+1
DO 105 J2=IJ2,IP2
K(2)=J2
IJ3=J2+1
IP3=IP2+1
DO 100 J3=IJ3,IP3
K(3)=J3
IJ4=J3+1

```

```
IP4=IP3+1
D0 95 J4=IJ4, IP4
K(4)=J4
IJ5=J4+1
IP5=IP4+1
D0 90 J5=IJ5, IP5
K(5)=J5
IJ6=J5+1
D0 85 J6=IJ6, N
K(6)=J6
CALL BASIS(A,K,OPTS,ITEST,M,I,NEST,IFEAS)
IF(ITEST.GE.NEST)G) TO 999
85 CONTINUE
90 CONTINUE
95 CONTINUE
100 CONTINUE
105 CONTINUE
110 CONTINUE
999 WRITE(6,300) IFEAS
      RETURN
      END
```

SUBROUTINE BASIS(A,K,DPTS,ITEST,M,N,NEST,IFEAS)

C  
C  
C  
C  
C  
C

C THIS SUBROUTINE DETERMINES IF A SET OF COLUMNS SPECIFIED BY SUBROUTINE  
C SELECT PROVIDES A FEASIBLE COVER SOLUTION

COMMON /AREAS/KEY(300),IEXIST  
INTEGER\*2 A(250,N),K(6),DPTS(NEST,C)

200 FORMAT(50(1X,I1))

C A NEW TRIAL BASIS HAS BEEN FOUND. TEST FOR FEASIBLE SOLUTION

DO 40 I=1,M  
DO 39 IJ=1,IFEAS  
IK=K(IJ)  
IF(A(I,IK).EQ.1)GO TO 40

39 CONTINUE  
GO TO 999

40 CONTINUE

C A FEASIBLE SOLUTION HAS BEEN FOUND. ADD IT TO THE SET OF OPTIMUM SOLUTIONS

ITEST=ITEST+1  
DO 41 IP=1,IFEAS  
IZ=K(IP)  
DPTS(ITEST,IP)=KEY(IZ)

41 CONTINUE

WRITE(6,300)ITEST,(DPTS(ITEST,IP),IP=1,IFEAS)

300 FORMAT(//,1X,'OPTIMAL SOLUTIONS # ',I3,' IS ',/,1X,'LOCATIONS ',6(  
\*1X,I3),//)

999 RETURN  
END



SUBROUTINE ALTERN(NEST, ISOL, OPTS, IBEST, IFEAS, INTER)

C  
C  
C  
C  
C  
C  
C  
C  
C

C FROM THE SET OF ALTERNATIVE OPTIMAL SOLUTIONS TO THE COVER PROBLEM,  
C THIS SUBROUTINE DETERMINES WHICH SOLUTION MINIMIZES THE CHANCE OF  
C POSSIBLE RESPONSE ROUTE BLOCKAGE. IBEST IS THE ALTERNATIVE SOLUTION  
C NUMBER THAT MINIMIZES THIS CHANCE.

```
COMMON /AREA1/ D(300,110), LABEL(300,110), BRANCH(300,110),  
*SHORT(300,110), R(300,110), P(270,2), PI(270,2)  
COMMON /AREA2/ X1PARA, Y1PARA, X2PARA, Y2PARA, X1PEPP, Y1PEPP, X2PEPP,  
*Y2PEPP, XNODE(110), YNODE(110), XLOC(270), YLOC(270), NLOC, ERAC  
COMMON /AREA3/ DPARA, DPE-P, TIME, PATH(50), SPONS(50)  
COMMON /AREA4/ RSTOP, IPT, ICONT, STOP, N  
COMMON /AREA5/ KEY(300), IEXIST  
INTEGER RSTOP, STOP, PATH, SPECS, SPINS1(50), INTER(NEST)  
INTEGER*2 BRANCH, LABEL, R, OPTS(NEST,6)  
DO 50 IL=1, ISOL  
INTER(IL)=0  
KS=IFEAS-1  
DO 40 J=1, KS  
KJ=OPTS(IL, J)  
K=J+1  
DO 30 L=K, IFEAS  
LJ=OPTS(IL, L)  
DO 20 N=1, N  
I1=(4J-1)*N+M  
READ(10'I1)RSTOP, (SPONS1(I#), I#=1, RSTOP)  
IP=KSTOP  
I2=(4L-1)*N+M  
READ(10'I2)RSTOP, (SPONS(I#), I#=1, RSTOP)
```

```
IK=RSTJP
DO 15 IJ=1,IP
DO 10 IM=1,IK
IF(SPONS(IJ).EQ.SPONS(IM))INTER(IL)=INTER(IL)+1
10 CONTINUE
15 CONTINUE
20 CONTINUE
30 CONTINUE
40 CONTINUE
50 CONTINUE
K=INTER(L)
IBEST=I
DO 60 I=2,ISOL
IF(INTER(I).GE.K)GO TO 60
K=INTER(I)
IBEST=I
60 CONTINUE
RETURN
END
```

SUBROUTINE REDUCE(A0,NUM,NLGC)

C

C

C THIS SUBROUTINE PERFORMS A ROW REDUCTION PROCEDURE ON THE 'A' MATRIX

C

C

INTEGER \* 2 A0(250,270)

I=0

10 I=I+1

12 IF(I.GT.NUM)GO TO 99

K=0

15 K=K+1

IF(K.EQ.I)K=K+1

IF(K.GT.NUM)GO TO 10

DO 40 J=1,NLGC

IF(A0(K,J).EQ.1.AND.A0(I,J).EQ.0)GO TO 15

40 CONTINUE

NUM=NUM-1

DO 50 L=1,NUM

DO 50 J=1,NLGC

A0(L,J)=A0(L+1,J)

50 CONTINUE

GO TO 12

99 RETURN

END

```

C
C
C
C
C
SUBROUTINE ELIMIN(NUM,A,NLCC,NI)
C THIS SUBROUTINE PERFORMS A COLUMN REDUCTION PROCEDURE ON THE 'A' MATRIX
C
C
      COMMON /AREAS/KEY(200),IEXIST
      INTEGER*2 A(250,NI)
      J=J
      10 J=J+1
      15 IF(J.GT.NLCC)GO TO 99
      K=J
      20 K=K+1
      IF(K.EQ.J)K=K+1
      IF(K.GT.NLCC)GO TO 10
      DO 30 I=1,NUM
      IF(A(I,J).EQ.1.AND.A(I,K).EQ.0)GO TO 20
      30 CONTINUE
      NLCC=NLCC-1
      DO 40 L=J,NLCC
      KEY(L)=KEY(L+1)
      DO 40 I=1,NUM
      A(I,L)=A(I,L+1)
      40 CONTINUE
      GO TO 15
      99 RETURN
      END

```

```

SUBROUTINE EXIST(A,NUM,NLDC)
COMMON /AREAS/ KEY(300), IEXIST
INTEGER*2 A(250,270)
C THIS SUBROUTINE ELIMINATES ALL DEMAND PTS. COVERED BY EXISTING STATIONS FROM
C CONSIDERATION PRIOR TO CALLING SUBROUTINE SELECT
  I=0
  10 I=I+1
  15 CONTINUE
  IF(I.GT.NUM)GO TO 50
C DETERMINE IF ROW I CAN BE ELIMINATED
  DO 20 J=1,IEXIST
  IF(A(I,J).EQ.1)GO TO 30
  20 CONTINUE
  GO TO 10
  30 CONTINUE
  NUM=NUM-1
  IF(I.GT.NUM)GO TO 50
C ELIMINATE ROW I
  DO 40 K=I,NUM
  DO 40 J=1,NLDC
  A(K,J)=A(K+1,J)
  40 CONTINUE
  GO TO 15
  50 CONTINUE
C ALL ROWS HAVE BEEN INSPECTED. ELIMINATE COLUMNS CORRESPONDING TO EXISTING
C STATION SITES AND UPDATE KEY VECTOR
  K=NLDC-IEXIST
  DO 60 J=1,K
  KEY(J)=KEY(J+IEXIST)
  DO 60 I=1,NUM
  A(I,J)=A(I,J+IEXIST)
  60 CONTINUE

```

RETURN  
END

SUBROUTINE INPUT

C  
CC  
C  
C  
C  
C AS INPUT TO THE PROGRAM THE FOLLOWING SHOULD BE SPECIFIED:  
C  
C  
C 1. SCALE- THE NO. OF FEET/GRID BLOCK  
C  
C 2. CTIME,SPEED  
C  
C 3. THE X-Y COORDINATES OF EACH CRITICAL POINT(STATEMENT 5)  
C  
C 4. LAST1,LAST2- THE LARGEST TWO CRITICAL POINT NUMBERS THAT ARE  
C CONNECTED BY A RUNWAY SEGMENT AND HAVE A PART OF THE LOCATION  
C CONTINUUM ADJACENT TO THAT SEGMENT  
C  
C 5. THE NUMBERS OF CRITICAL POINTS THAT ARE CONNECTED BY RUNWAY  
C SEGMENTS(STATEMENT 30)  
C  
C 6. THE X-Y COORDINATES OF THE ENDPOINTS OF CONTINUUM LINES, THE  
C CRITICAL POINTS TO WHICH THEY ARE ADJACENT AND THE DISTANCE IN  
C FEET THEY ARE FROM THE RUNWAY SEGMENT CONNECTING THE CRITICAL  
C POINTS(STATEMENT 50)  
C  
C 7. THE NUMBER OF STATIONS IN OPERATION AT THE AIRPORT(STATEMENT 70)  
C  
C  
CC  
C

C

```

COMMON /AREA1/ D(300,110), LABEL(300,110), BRANCH(300,110),
*SHORT(300,110), R(300,110), F(270,2), DI(270,2)
COMMON /AREA2/ X1PARA, Y1PARA, X2PARA, Y2PARA, X1PERP, Y1PERP, X2PERP,
*Y2PERP, XNODE(110), YNODE(110), XLECC(270), YLECC(270), NLECC, FRAC
COMMON /AREA3/ DPARA, DPERP, TIME, PATH(50), SPONS(50)
COMMON /AREA4/ RSTSP, IPT, ICNT, STOP, N
COMMON /AREA5/ KEY(300), IEXIST
DIMENSION XP(10)
INTEGER*2 BRANCH, LABEL, R
INTEGER PATH

```

C

C

C\*\*\*\*\*

C\*\*\*\*\*

SPEED=41.

C\*\*\*\*\*

C\*\*\*\*\*

C

C

SCALE=434.7

CTIME=1.67

SPEED=SPEED\*5280.

TIME=(SPEED/60.)\*CTIME

TIME=TIME/SCALE

N=1

5 READ(5,20),END=10) XNODE(N), YNODE(N)

N=N+1

GO TO 5

10 N=N-1

DO 20 I=1,N

DO 20 J=1,N



```

    BRANCH(I, J)=0
20 CONTINUE
    READ(5, 201) LAST1, LAST2
    LAST=(LAST1-1)*N+LAST2
30 READ(5, 201, FMD=40) I, J
    BRANCH(I, J)=1
    BRANCH(J, I)=1
    GO TO 30
40 CONTINUE
    GATE=0.
    DO 60 I=1, N
    DO 60 J=1, N
    IP=(I-1)*N+J
    IF(GATE.EQ.1.) GO TO 56
    IF(IP.GE.LAST) GO TO 55
50 READ(5, 202) K, L, X1PARA, Y1PARA, X2PARA, Y2PARA, X1PERP, Y1PERP, X2PERP,
    *Y2PERP, DPASA, DPERP
    IF(DPARA.LT.99999.) DPASA=DPASA/475.
    IF(DPERP.LT.99999.) DPERP=DPERP/475.
54 IF((K-1)*N+L.EQ.IP) GO TO 55
55 CONTINUE
    DO 56 IJ=1, 8
    XP(IJ)=-1.
56 CONTINUE
    XP(9)=99999.
    XP(10)=99999.
    GATE=1.
    GO TO 59
58 CONTINUE
    GATE=0.
    XP(1)=X1PARA
    XP(2)=Y1PARA

```

```
XP(3)=X2PARA
XP(4)=Y2PARA
XP(5)=X1PEEP
XP(6)=Y1PEEP
XP(7)=X2PEEP
XP(8)=Y2PEEP
XP(9)=DPARA
XP(10)=DPEEP
59 WRITE(J,IP)(XP(N),N=1,10)
   IP=(J-1)*N+1
   WRITE(J,IP)(XP(N),N=1,10)
60 CONTINUE
70 READ(5,201)TEXT
200 FORMAT(2F10.2)
201 FORMAT(2I3)
202 FORMAT(2I3,4A,8F5.2,2F6.0)
   RETURN
   END
```

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A METHODOLOGY FOR LOCATING  
FIRE STATIONS AT AIRPORTS

by

Geoffrey Carter Burness

(ABSTRACT)

A methodology is developed to determine the locations of the fewest number of fire stations at an airport such that two guidelines, recommended to the Federal Aviation Administration, are met. The first guideline stipulates that airport fire vehicles must be capable of reaching any potential crash site on the runway system within a two minute time period. The second guideline requires stations to be located such that the chance of possible blockage of vehicle response routes to potential crash sites is minimized.

The methodology transforms an initial continuous facility location problem into a discrete problem. By modeling the discrete formulation as a modified set covering problem, a solution is determined that meets the two guidelines.

To illustrate the utility of the methodology, it is applied to two situations at the Atlanta Airport. In the first situation it is assumed that no fire stations exist at the airport, and in the second,

three of the four fire stations planned to be in existence at the airport are assumed to be in operation.

For both situations, the methodology is applied three times, changing the average speed vehicles are assumed to be capable of traveling on the runway system each time. For an average speed of 40 m.p.h., it is shown that fire vehicles, responding from the four stations at the airport, are not capable of reaching every potential crash site on the runway system within a two minute time period.