THE DEMAND FOR GASOLINE AND DIESEL FUEL IN AGRICULTURAL USE IN VIRGINIA

by

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To my grandfather,
Arthur Charles Struck

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CHAPTER 1

INTRODUCTION

The agricultural industry, not unlike other industries, has become increasingly dependent upon energy resources, namely electricity, fossil fuels, chemicals, and fertilizers, due in large measure to relatively low energy prices. For the past two and a half decades, fossil fuels, pesticides, herbicides, fertilizers, and other petroleum products have been substituted for other inputs, particularly land and labor, in agricultural production. In 1974, approximately 3.7 billion gallons of gasoline, 2.6 billion gallons of diesel fuel, 1.5 billion gallons of LP gas, 0.3 billion gallons of fuel oil, 164 billion cubic feet of natural gas, 33,000 tons of coal, and 22 billion kilowatt hours of electricity were used in crop and livestock production in the U.S. The utilization of energy factors in agricultural production over the past 25 years was unequivocally a major reason for the salient agricultural labor and land reduction. From 1950-1973, there was a 6 percent decrease in land use for crops and a concomitant 59 percent decline in the number of on-farm work hours.1/ Moreover, the agricultural use of energy resources over the past 25 years helped to account for the substantial 52 percent increase

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1/ "Energy in Agriculture", Proceedings of the Conference-Workshop Southern Regional Education Board, October 1975, p. 97.

in total farm production.[2] Finally, tractors, trucks, fertilizer, combines, feed-handling devices, milking machines, irrigation systems, and crop drying systems almost totally rely upon energy inputs.

Fossil fuels play an important role in present energy-intensive agricultural enterprises. This study focuses on the demand relationships for two fossil fuels in agricultural use, namely gasoline and diesel fuel. In 1974 and 1975, crop and livestock production required on the average about six billion gallons of these fossil fuels in the U.S.,[3] and in addition, farmers used an average of 16.8 gallons of diesel fuel, gasoline, and LP gas per acre for all crops in the U.S.[4] Finally, data from the Economic Research Service, USDA, for 1974 indicate that 65 to 70 percent of the utilization of fossil fuels in the U.S. for agricultural production purposes is attributable to gasoline and diesel fuel.[5] In sum, the selection of these fossil fuel inputs rests upon their relative importance in agricultural production.

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[2] Ibid.

[3] "Energy to Keep Agriculture Going", The Office of Communication, U.S. Department of Agriculture, December 1973, p. 3.

[4] G.L. Casler, and J.H. Erickson, "Energy Requirements for New York State Agriculture", Agricultural Economics Extension Bulletin 74-24, Cornell University, 1974, p. 3.

[5] "Potential for Cooperative Distribution of Petroleum Products in the South", Farmer Cooperative Service, USDA, July 1973, p. 1.

Problem and Objectives

Historically, the agricultural sector, like other sectors, has enjoyed relatively low prices of nearly all types of energy. In the middle 1970's however, prices for energy related inputs rose sharply as a result of continuously rightward shifting energy demands and leftward shifting energy supplies due to dwindling domestic reserves and oil price increases by OPEC nations. Although initially the rapidly rising energy prices may have been viewed as a temporary phenomenon, most now agree that the U.S. has entered an era of high energy prices.

In terms of energy use and any type of national energy policy, agriculture faces a dilemma. Although agricultural production alone uses only three to four percent of the total U.S. energy budget, the production, processing, and distribution of food and fiber together utilize almost twenty percent. On the surface, agricultural production uses too large a proportion of energy to be neglected from a national policy viewpoint but too small a proportion to receive serious consideration. Energy use in agricultural production, however, differs from energy use in non-agricultural production in terms of seasonality and the need for uninterrupted services. Poultry houses, unlike schools and steel factories, cannot be closed on weekends. Crop planting, harvesting, curing, and drying have to be done during a certain period of time. Due to the biological nature of agriculture, the interseasonal rate of substitution for fuels in many production activities is very low, and the impact of interrupted service is relatively large. One way or another, energy will be allocated to the agricultural sector. If the future entails politically imposed limitations on the quantities of energy purchased, then information concerning energy use in agricultural production and food processing and distribution is needed to facilitate an efficient and equitable allocation.

In order to assess the impacts which higher energy prices and restrictions on quantities of energy may have in the agricultural sector, information about the economic factors that influence the demand for different types of energy in different types of agriculture is needed. Little is known about these major factors and how sensitive the quantities demanded of various types of energy in different types of agriculture are to price changes in both the short and long run. Estimates of elasticities and cross-elasticities may be very useful in providing insights as to the relative responsiveness of farmers and producers of energy and related inputs to relative price changes and changes in certain other measurable variables. For example, if the quantities demanded of energy inputs are not responsive to price changes, then the prices of energy factors will not be very effective allocators of limited quantities. On the other hand, if price changes affect quantities demanded, then the different effects in different types of agriculture need to be known. A quantitative analysis of factors affecting the demand for energy in various types of agricultural use is of paramount importance in developing or modifying allocation programs for energy distribution. Whether allocation of energy is done by legislative or administrative procedures or through market forces depends in part on the nature of the demands for energy inputs. An examination of the demand for different types of energy may bring to light new opportunities for energy conservation and more efficient methods of energy allocation. In summary, agricultural economists, producers of energy and related inputs, and farmers presently do not fully comprehend the magnitude and influence of the economic factors that affect the demand for various types of energy in the agricultural sector. A quantitative analysis of the demand for different types of energy may provide a useful guide for the direction of future agricultural policy concerning energy.

The objectives of this study are: (1) to determine the usage patterns of gasoline, diesel fuel and other fossil fuels in different types of agriculture; (2) to determine the demand relationships for gasoline and diesel fuel in agricultural use and to identify and assess major factors that affect these relationships; and (3) to determine seasonal, short-run, and long-run differences in the demand relations for gasoline and diesel fuel in agricultural use.

Scope

This study focuses on the demand relationships for gasoline and diesel fuel in agricultural use for seven rather homogeneous crop reporting districts of Virginia (Figure 1). At various locations in these seven districts, private service agencies, local cooperatives, and retail branches make fossil fuels available to

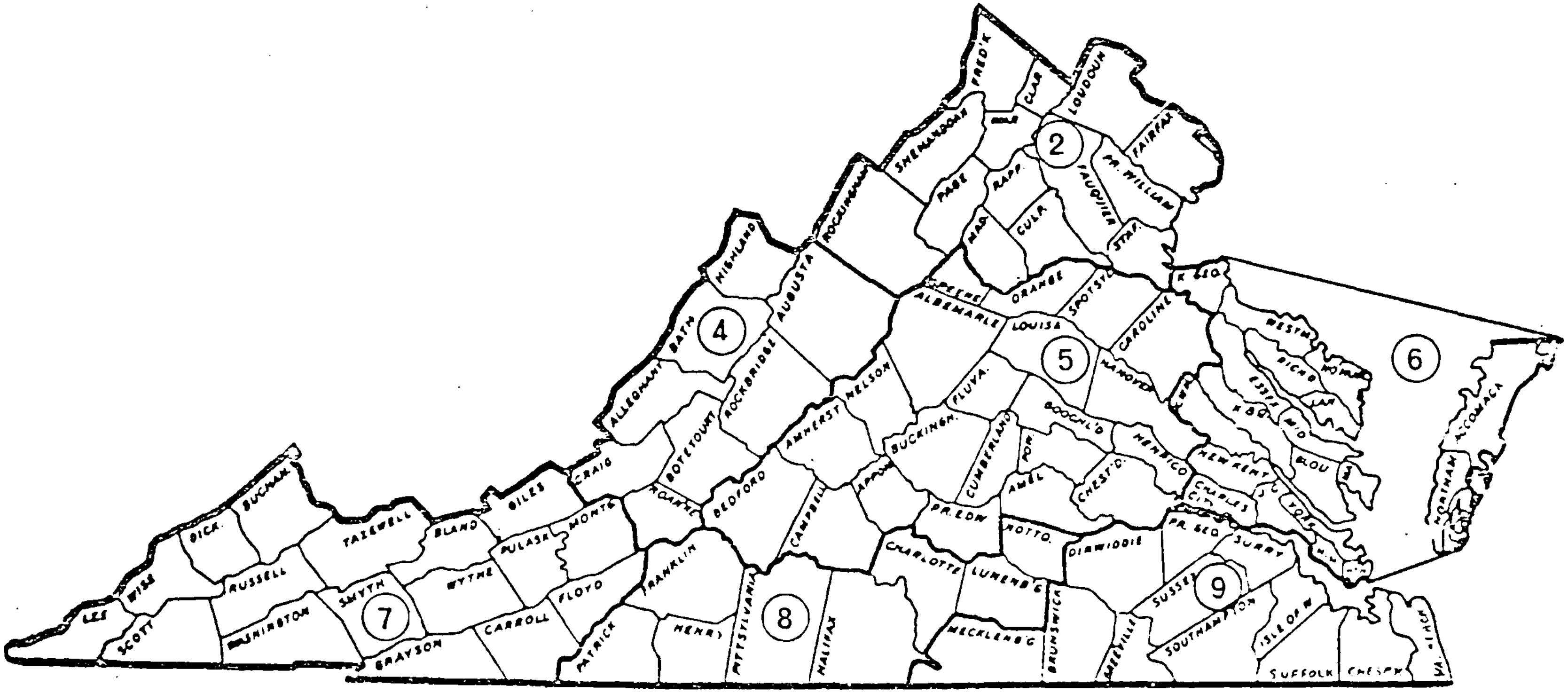

Figure 1. Virginia Crop Reporting Districts

farmers, particularly gasoline, diesel fuel, fuel oil, and LP gas[6] (Appendix A). The seven crop reporting districts reflect major differences in soil types, in types of farm operations, and in types of farm outputs (Table 1.1). Estimates of the demand relationships for gasoline and diesel fuel are applicable not only to Virginia but also to other areas which possess similar characteristics in climate, types of agriculture, and geography.

Literature Review

There is little information dealing specifically with the usage patterns of and the demand relationships for gasoline and diesel fuel in the agricultural sector. The direct and indirect impacts of high energy prices and quantity restrictions on agricultural activities are also virtually unknown. However, several studies have dealt with the demand for fertilizer, labor, and other inputs in the agricultural sector and the demand for fossil fuels in the residential, commercial, private, and industrial sectors. In addition, some studies have investigated the effect of increased energy prices and decreased quantities of energy inputs on the agricultural sector. A review of several of these studies was undertaken in order to put this work in proper perspective.

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[6] However, others besides farmers may purchase these fossil fuels from the private service agencies, local cooperatives, and retail branches. In this study, the wholesale handlers of gasoline, diesel fuel, heating oil, and LP gas are members of the Southern States Cooperative, Inc.

Table 1.1. Crops and Principal Producing Areas, Virginia

Crop	Principal Producing Areas and Counties (Figure 1)
Barley	2, 4, 5, 6
Corn:	
Grain	Statewide
Silage	2, 4, 5, 7, some in all counties
Cotton	9
Hay:	
Alfalfa	2, 4, 5, 7, 8
Clover-timothy	2, 4, 5, 7
Lespedeza	5, 8
Peanuts	9
Rye	6, 9
Sorghum:	
Grain	9
Silage	2, 5
Soybeans	5, 6, 9
Tobacco:	
Type 11 - Flue-cured	8, 9
Type 21 - Fire	5, 8
Type 31 - Burley	7
Type 37 - Sun	5
Winter Wheat	2, 5, 6, 8, 9
Potatoes	Accomack, Northampton, City of Virginia Beach
Sweet potatoes	Accomack, Northampton, City of Virginia Beach, James City
Orchardgrass	Loudoun, Fauquier, Page, Rappahannock, Clarke, Prince William
Fruit:	
Apples	Frederick, Clarke, Shenandoah, Rockingham, Augusta, Warren
Peaches	Frederick, Rockingham, Shenandoah, Augusta, Page, Clarke, Warren

Table 1.1 continued.

Crop	Principal Producing Areas and Counties (Figure 1)
Vegetables	Accomack, Northampton, City of Chesapeake, City of Virginia Beach, Carroll, Floyd, Washington, James City, Wythe, Hanover, City of Suffolk, Lancaster, Northumberland, Richmond, Westmoreland

Source: *Virginia Agricultural Statistics*, Virginia Cooperative Crop Reporting Service, Bulletin No. 41, August, 1976.

Heady and Tweeten[7/] estimated demand relationships for a large number of resources, including fertilizer, labor, machinery, plant and equipment, and operating inputs. General input demand functions derived from production functions were estimated. These types of functions were single equation, short-run relationships. Experimental data from Iowa, Kansas, Michigan, North Carolina, and Tennessee were used as the basis for inferences about static input demand curves and elasticities.

An analysis by Daniel and Havlicek[8/] used data from a farm panel to estimate fertilizer demand functions. Observations were obtained from 900 Illinois farmers by months for the years 1960-1965. These demand equations expressed the quantity of each fertilizer purchased as a function of the price of fertilizer, the prices of other fertilizer nutrients, farm product prices, the price of land, the price of labor, lagged variables, and several variables associated with characteristics of fertilizers purchased. On the basis of their study: (1) the prices of nitrogen, phosphate, and mixed fertilizers were important to the farmer when purchasing fertilizers; (2) nitrogen price elasticities were -2.05 in the spring and -1.37 in the fall; (3) phosphate price elasticities were -.89 in the spring and -.38 in the fall; (4) potash price elasticities were not significantly

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[7/] E.O. Heady, and L.G. Tweeten, Resource Demand and Structure of the Agricultural Industry, Iowa State University Press, Ames, Iowa, 1963.

[8/] Raymond Daniel, and Joseph Havlicek, Jr., "Econometric Analysis of the Farmer Demand for Fertilizer Nutrients", unpublished Ph.D. dissertation, Purdue University, 1970.

different from zero; (5) crop prices influenced the fall purchases of nitrogen and phosphate; (6) farmers located in certain areas of the state purchased different quantities of fertilizer nutrients; and (7) demographic characteristics of the operator, particularly education, age, residence, and farm ownership were not consistently related to purchases of all fertilizer nutrients.

Balestra and Nerlove[9/] estimated the parameters of a single-equation dynamic demand function for natural gas in the residential and commercial sectors. The economic variables in the model were the price of natural gas, income, and the lagged quantity of natural gas. The estimation technique, using pooled time series data from different cross-sections, was the error component or variance component procedure. The coefficient of determination of the model was .99, and the long-run price elasticity of demand for natural gas in these respective sectors was -.63. Nerlove maintained that dynamic models had advantages over the traditional static models in that there was a better explanation of the data, and the coefficients were more reasonable in sign and magnitude.

Houthakker, Verleger, and Sheehan[10/] presented a quarterly dynamic

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[9/] Pietro Balestra, and Marc Nerlove, "Pooling Cross Section and Time Series Data in the Estimation of a Dynamic Model: The Demand for Natural Gas", Econometrica, July, 1966.

[10/] H.S. Houthakker, P.K. Verleger, and D.P. Sheehan, "Dynamic Demand Analysis for Gasoline and Residential Electricity", American Journal of Agricultural Economics, May, 1974.

demand analysis for gasoline in the private sector. The single-equation dynamic demand function was estimated using pooled time series data from different states without any attempt to differentiate by region. The economic variables in the model were the price of gasoline, income, and the lagged quantity of gasoline. Parameter estimates were obtained by employing the error component technique used previously by Balestra and Nerlove. The coefficient of determination of the model was .92, and the short-run and long-run price elasticities of demand for gasoline were respectively -.075 and -.24; price was defined as the ratio of the price at the pump, including all taxes, deflated by the consumer price index.

Ramsey, Rasche, and Allen[11/] investigated the private and commercial (truck) demand relationships for gasoline. The approach was to formulate a static model in terms of a simultaneous equation system, to pay particular attention to relative prices in addition to own price and income, and to separate the demand by households from that by commercial (truck) users. On the basis of their study, small relative changes between gasoline and diesel fuel prices had pronounced effects on the demand for gasoline. The price elasticities of demand in the private sector and in the commercial sector were respectively -.70 and -2.8. The price of gasoline in the private sector was measured by the ratio of the nominal retail price of

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[11/] J. Ramsey, R. Rasche, and B. Allen, "An Analysis of the Private and Commercial Demand for Gasoline", The Review of Economics and Statistics, November, 1975.

gasoline to the consumer price index. The price of gasoline in the commercial sector was measured by the retail price of gasoline deflated by the price index of truck freight rates.

Erickson, Spann, and Ciliano[12/] presented an econometric analysis of substitution and usage effects on industrial sector energy demands. The industrial energy factors included residual and distillate fuel oil, natural gas, and electricity. The approach was to formulate a static model using a single equation for each industrial energy demand relation. Each of the equations regressed the quantity of energy input i used by industries in state j in year t on the following variables: (1) the price of energy factor i to industrial users in state j in year t; (2) the prices of alternative energy factors, namely, fuel oil, coal, natural gas, and electricity, in year t in state j; (3) a measure of economic activity in state j in year t; and (4) a set of variables describing the industrial composition of state j. All data used in the analysis were pooled time series, cross-sectional values taken from the period 1960 through 1970. The model was estimated by ordinary least squares. On the basis of their study, the industrial demand for each energy factor was sensitive to the price of that factor. The price elasticities of demand for residual and distillate fuel oil, natural gas, and electricity were respectively -.65, -2.53, and -1.02. In addition, the cross-price

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[12/] E.W. Erickson, R.M. Spann, and R. Ciliano, "An Econometric Analysis of Substitution and Usage Effect on Industrial Sector Energy Demands", in Energy Policy Evaluation, (D.C. Heath & Company), 1974.

elasticities between various energy factors were for the most part consistent with economic theory and often statistically significant.

Wilson[13/] developed monthly forecasting equations for the consumption of motor gasoline, aviation gasoline, distillate oil, diesel fuel, and liquified petroleum gas for the U.S. and the U.S. agricultural sector using data from 1974 and 1975. The agricultural consumption of motor gasoline, aviation gasoline, diesel fuel, and LP gas was forecasted at 81, 1, 51, and 18 million barrels per year in 1974 and 84, 1, 61, and 19 million barrels per year in 1975. On the basis of these forecasts, agricultural consumption of petroleum products was anticipated to grow in the future at a relatively limited rate. No emphasis, however, was placed on economic structural relationships or causality.

Casler and Erickson,[14/] Coble and LePori,[15/] Cervinka, Chancellor, Coffelt, Curley, and Dobie,[16/] and Robinson[17/] estimated diesel fuel,

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[13/] R.R. Wilson, "Petroleum for U.S. Agriculture", American Journal of Agricultural Economics, December, 1974.

[14/] G.L. Casler, and J.H. Erickson, "Energy Requirements for New York State Agriculture", Agricultural Economics Extension Bulletin 74-24, Cornell University, 1974.

[15/] C.G. Coble, and W.A. LePori, "Energy Consumption, Conservation, and Projected Needs for Texas Agriculture", Department of Agricultural Engineering, Texas Agricultural Experiment Station, Texas A and M University, 1974.

[16/] V. Cervinka, W.J. Chancellor, R.J. Coffelt, R.G. Curley, and J.B. Dobie, "Energy Requirements for Agriculture in California", California Department of Food and Agriculture, University of California at Davis, 1974.

[17/] B.H. Robinson, "Estimated Fuel Consumption for Agricultural gasoline, and other fossil fuel requirements from agricultural engineering data in different types of agricultural production in New York, Texas, California, and South Carolina for 1974. The estimation of these energy requirements indicated the percentage of fossil fuel use in different types of agriculture and the quantity of fossil fuel needed per unit of output in different types of agriculture. The Economic Research Service, United States Department of Agriculture, under a jointly funded cooperative agreement with the Federal Energy Administration, developed estimates of the utilization of fossil energy by crops and livestock for 1974 for all fifty states of the U.S.

Carter and Youde[18/] discussed some of the impacts of the changing energy situation on U.S. agriculture, particularly the direct and indirect impacts of high energy prices. They argued that the major long-term adjustment problems for agriculture would result indirectly from the impact of energy prices on general price levels and economic growth rather than from direct price increases of energy-based farm inputs.

In both the short run and the long run, rising energy input prices directly affect an increase in agricultural production costs. Tweeten

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Production in South Carolina", College of Agricultural Sciences, Clemson University, 1974.

[18/] H.O. Carter, and J.G. Youde, "Some Impacts of the Changing Energy Situation on U.S. Agriculture", American Journal of Agricultural Economics, December, 1974.

and Quance[19/] estimated the impacts of input price changes on U.S. farm costs and revenues from 1958 to 1967. Results of their study suggested that for factors with elastic demands, notably fertilizer, price increases would increase net farm income, while for inputs with inelastic demands, rising prices would decrease net farm income.

Burton[20/] conducted sensitivity analyses of the impacts of increased energy input prices and decreased quantities of energy inputs on representative Virginia dairy farms. The energy factors included gasoline, diesel fuel, and LP gas. Results indicated that the use of energy inputs was not responsive to price increases. On the basis of Burton's study, should the government be faced with the choice of an energy conservation policy based on large energy price increases or on a strict rationing of energy inputs, the strict rationing policy would cause greater reductions in net farm income. Dvoskin and Heady[21/] argued that even if energy prices doubled there would be little change in energy use in agricultural production,

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[19/] L. Tweeten, and L. Quance, "The Impact of Input Price Inflation on the U.S. Farming Industry", Canadian Journal of Agricultural Economics, February, 1971.

[20/] R.O. Burton, "Adjustments in a Farm Business in Response to an Energy Crisis", unpublished Masters thesis, Virginia Polytechnic Institute and State University, 1976.

[21/] D. Dvoskin, and E.O. Heady, "U.S. Agricultural Production Under Limited Energy Supplies, High Energy Prices, and Expanding Agricultural Exports", The Center for Agricultural and Rural Development, Iowa State University, 1976.

particularly crop production, and there would be little effect on the level of output. On the other hand, an outright restriction in quantities of energy inputs would decrease output levels, and given the inelastic nature of the demands for most farm products, net farm incomes would rise. In short, depending on the type of agricultural production, the changing energy situation in the agricultural sector may increase or decrease net farm income.

Lehrmann, Black, and Connor[22/] investigated the effects of increased prices of crude oil and natural gas on corn and soybean production on cash grain farmers in southeastern Michigan. Sensitivity analyses indicated that neither natural gas nor crude oil price increases alone had a significant effect on the optimal corn and soybean crop mix. On the basis of their study, a governmental policy to reduce energy use in agriculture by increasing energy input prices would have a minor effect on agricultural production.

Penn and Irwin[23/] conducted constrained input-output (I/O) simulations of energy restrictions in the food and fiber system in the context of the national economy. Specific simulations in the

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[22/] J.A. Lehrmann, J.R. Black, and L.J. Connor, "Direct Economic Effect of Increased Energy Prices on Corn and Soybean Production on Cash Grain Farms in Southeastern Michigan", Department of Agricultural Economics, Michigan State University, 1976.

[23/] J.B. Penn, and G.D. Irwin, "Constrained Input-Output Simulations of Energy Restrictions in the Food and Fiber System", Economic Research Service, U.S. Department of Agriculture, Washington, D.C., February, 1977.

I/O model used to study effects of alternate energy availabilities on the U.S. food and fiber system were: (1) reduction of crude petroleum imports; (2) a shortage of refined petroleum; (3) restricted natural gas supplies; and (4) energy requirements for expanded agricultural exports. Results of the simulations suggested that fuel allocations to agricultural production would be of little assistance if other needed inputs were unobtainable. The fertilizer and chemical sectors were relatively more severely affected by energy restrictions than others.

In sum, in the commercial, industrial, private, and residential sectors, the quantity demanded of fossil energy was sensitive to own price. In addition, the cross-price elasticities between various energy factors were often statistically significant. Further, two technical approaches have been used in examining the usage patterns and demand relationships for fossil fuels in agriculture: (1) projecting total agricultural energy requirements, allocating these requirements among different agricultural subsectors, and estimating fossil energy use in different agricultural enterprises; and (2) linear programming (LP) or constrained input-output (I/O) analyses to assess impacts of high energy prices and quantity restrictions on agricultural activities. However, these conditionally normative approaches fail to take into account, in most cases, substitution among energy sources and changes in relative prices. The models usually require some stringent assumptions which limit the applicability of the results, and they provide little information about the economic factors that influence the demand for fossil energy. In short, the projection and LP or I/O approaches may be too restrictive to portray adequately the range of opportunity and response open to the agricultural sector. Simply put, many of the analyses concerned with energy use in agriculture have yielded results with limited usefulness, and emphasis needs to be given to more positive approaches which can provide information about the economic structure of energy use in U.S. agriculture.

Thesis Organization

The theoretical input demand relationships for gasoline and diesel fuel are developed in Chapter 2. Chapter 3 focuses on the use of fossil fuels in agricultural production in Virginia, the South, and the U.S. The statistical models and statistical procedures used in this study are presented in Chapter 4. Chapter 5 contains the statistical results and the economic analyses. The summary and conclusions of this study are presented in Chapter 6.

CHAPTER 2

THEORETICAL FRAMEWORK

In this chapter, the theoretical input demand relationships for gasoline and diesel fuel are developed in order to suggest the economic factors affecting the quantities purchased of these fossil fuels. The first section is a discussion of the static theoretical development of derived input demand under the assumption that the firm operates with a capital constraint. In the second section, which also deals with some basic concepts of derived input demand in a static environment, the firm, by assumption, no longer operates with a capital constraint. Later, the dynamic concepts and the technological adjustment concepts of derived input demand are considered. Further, since quantities and prices of fossil fuels which clear the market are determined by the intersection of the supply and demand relationships for fossil fuels, attention is given to the supply relationships of gasoline and diesel fuel. Finally, the static theoretical development of derived input demand, the dynamic concepts, and the technological adjustment concepts are brought together in developing the economic models for gasoline and diesel fuel.

The Theory of Derived Input Demand

Static Theoretical Development

The static theoretical development of derived input demand is discussed by Mosak,[24/] Hicks,[25/] Samuelson,[26/] Henderson and Quandt,[27/] and others. The mathematical development of the derived demand for an input for a firm operating with or without a capital constraint is presented in Appendix B.

Limited Capital

Consider a multi-input, multi-output firm with a production process that involves h outputs and n-h inputs. By assumption, the firm operates with a capital constraint. In addition, the firm operates under the assumptions of perfect competition in the output and input markets. So, the decision unit of the firm makes rational choices, consistent choices, and possesses perfect knowledge. The technical relationships between the inputs and the outputs of the firm may be expressed in implicit form by a homogeneous production function of degree k.

$$F(tx_1, tx_2, \dots, tx_n) = t^k F(x_1, x_2, \dots, x_n) = 0 \qquad (2\text{-}1)$$

[24/] J.L. Mosak, "Interrelations of Production, Price, and Derived Demand", Journal of Political Economy, December, 1938, pp. 761-787.

[25/] J.R. Hicks, Value and Capital (London: Oxford University Press, 1965), pp. 320-323.

[26/] P.A. Samuelson, Foundations of Economic Analysis, (New York: Atheneum, 1965), pp. 60-78.

[27/] J.M. Henderson and R.E. Quandt, Microeconomic Theory, (McGraw-Hill Book Company, New York, 1971), pp. 95-98.

where k is a constant, t is any positive real number, and the x_i's, i=1,..., n, represent the (1,..., h) independent output variables and the (h+1,..., n) independent input variables. Further, it is assumed that (1) may be written in such a way that the partial derivatives for outputs are positive and the partial derivatives for inputs are negative.[28/] Time, by assumption, has limited influence on the production process. The production process begins and ends with the given period, and no conditions outside this period have any relevance to the production plan. Finally, the firm, by assumption, makes instantaneous adjustments to changes in prices, and complete divisibility of inputs and outputs exists.

From the static theoretical development presented in Appendix B, under the above assumptions, the quantity of an input purchased by a firm depends upon the following economic factors: (1) the price of the particular input, (p_i); (2) the prices of complementary or substitute inputs, (p_j); (3) the price(s) of the output(s), (p_o); and (4) the initial capital outlay of the firm (c). The static theoretical input demand relationship is given by

$$x_i = f(p_i, p_j, p_o, c). \tag{2-2}$$

Under the capital constraint assumption, the signs of $\frac{\partial x_i}{\partial p_i}$, $\frac{\partial x_i}{\partial p_j}$, $\frac{\partial x_i}{\partial p_o}$,

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[28/] J.M. Henderson and R.E. Quandt, Microeconomic Theory, (McGraw-Hill Book Company, New York, 1971), p. 95.

and $\frac{\partial x_i}{\partial c}$ are determinate. However, in most cases, $\frac{\partial x_i}{\partial p_i} < 0$, where $\frac{\partial x_i}{\partial p_i}$ is the change in the quantity purchased of an input with respect to a change in the price of input i, all other factors nonvariant. Further $\frac{\partial x_i}{\partial p_j} > 0$, <u>ceterus paribus</u>, if inputs i and j are substitutes for each other. If inputs i and j are complementary factors, then $\frac{\partial x_i}{\partial p_j} < 0$, <u>ceterus paribus</u>, where $\frac{\partial x_i}{\partial p_j}$ is the change in the quantity purchased of input i with respect to a change in the price of input j, all other factors nonvariant. Finally $\frac{\partial x_i}{\partial p_o} > 0$, <u>ceterus paribus</u>, for the case of normal inputs. For the case of inferior inputs, the relationship between the quantity of an input purchased and the price of the output is negative. Finally, $\frac{\partial x_i}{\partial c} > 0$, where $\frac{\partial x_i}{\partial c}$ is the change in the quantity purchased of input i with respect to a change in capital, all other factors nonvariant.

Unlimited Capital

Under the assumption that the firm has unlimited capital, the static theory of derived input demand (Appendix B) states that the quantity of an input purchased depends upon the price of the particular input, (p_i), the prices of complementary or substitute inputs, (p_j), and the price(s) of the output(s), (p_o). The theoretical static input demand relationship is given by:

$$x_i = f(p_i, p_j, p_o) \qquad (2\text{-}3)$$

As in the limited capital case, $\frac{\partial x_i}{\partial p_i} < 0$, $\frac{\partial x_i}{\partial p_o} \gtrless 0$, and $\frac{\partial x_i}{\partial p_j} \gtrless 0$ depending upon whether input i is a normal or inferior factor or whether inputs i and j are substitutes or complements, all other factors nonvariant.

Dynamic Concepts

Due to lags in the production process, imperfect knowledge, rigidities and stickiness in the economy, and other factors, the firm does not respond immediately to input and output price changes. Lagged responses or adjustments are generally attributable to psychological factors, technical factors, and institutional factors.

Several dynamic models have been developed to explain the lagged response or adjustment of firms to price changes. Chen, Courtney, and Schmitz[29/] utilized a polynomial distributed lag model to analyze the response of milk production to changes in administered milk prices. Distributed lag models arise in theory when economic stimuli produce their effects only after some lag in time and when the effects are distributed over time. Nerlove[30/] championed the partial adjustment

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[29/] D. Chen, R. Courtney, and A. Schmitz, "A Polynomial Lag Formulation of Milk Response", American Journal of Agricultural Economics, February, 1972, pp. 77-79.

[30/] M. Nerlove, "Distributed Lags and Estimation of Long-Run Supply and Demand Elasticities, Theoretical Considerations", Journal of Farm Economics, May, 1958, pp. 301-311.

model and the adaptive expectation model so as to permit the estimation of long-run supply and demand elasticities. These Nerlovian distributed lag models are based on the notion of an expected normal value, the level about which future values of a variable(s) are expected to fluctuate. As a method for considering the expected adjustments of production to given price changes, Harlow[31/] developed a recursive model to estimate quarterly hog supply functions. Finally, Waugh[32/] showed that the current output of a firm was related to past prices.

Technological Adjustment Concepts

Technological advancement and changes in the quality of inputs create questions and problems as to how a technological change may influence the demand for an input or how that change may be incorporated into the demand function.[33/] In the case of fossil fuels, a change from gasoline to diesel engines in tractors and combines has taken place in the agricultural sector (Table 2.1). Estimates indicated that by 1980 over 80 percent of farm tractors and 90 percent of

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[31/] A.A. Harlow, "Factors Affecting the Price and Supply of Hogs", U.S.D.A. Technical Bulletin 1274, 1962.

[32/] F.V. Waugh, "Cobweb Models", Journal of Farm Economics, November, 1964, pp. 732-751.

[33/] W.G. Tomek and K.L. Robinson, "Agricultural Price Analysis and Outlook: A Review of Literature", p. 58.

Table 2.1. Production of Wheel Tractors in the U.S. by Fuel Type (1955-1975)

Year	Gasoline	Diesel	LP Gas	Total	Ratio of Diesel to Gasoline Tractors
1955	277,105	41,506	11,530	330,141	0.150
1956	177,077	26,762	10,815	214,654	0.151
1957	179,142	37,352	12,556	229,050	0.209
1958	173,040	55,864	12,365	241,269	0.323
1959	167,961	79,548	12,407	259,916	0.474
1960	84,306	62,033	5,848	152,187	0.736
1961	84,102	80,920	6,395	171,417	0.962
1962	99,414	80,293	8,394	188,101	0.808
1963	96,108	98,609	8,732	203,449	1.026
1964	90,004	115,439	7,778	312,221	1.283
1965	98,970	138,247	6,833	244,050	1.397
1966	107,683	157,352	5,652	270,687	1.461
1967	80,630	158,741	2,844	242,215	1.969
1968	61,978	147,744	3,477	213,199	2.384
1969	59,111	135,118	1,475	195,704	2.286
1970	48,981	122,622	(1)	171,603*	2.503
1971	40,642	126,859	(1)	167,501*	3.121
1972	42,389	154,827	(1)	197,216*	3.653
1973	37,860	173,646	(2)	211,506	4.587
1974	33,860	176,214	(3)	210,074	5.204
1975	28,737	184,058	(3)	212,795	6.405

Source: Current Industrial Reports, Series M355, Bureau of the Census.

(1) Withheld to avoid disclosing the operations of individual companies.

(2) For 1973, gasoline and LP gas powered tractors 90 horsepower and over have been included with diesel tractors to avoid disclosure of individual company data. These tractors amount to less than 1% of wheel tractors produced.

(3) For 1974 and 1975, LP gas and gasoline fuel type tractors are combined.

* Production totals exclude LP gas powered tractors which amount to less than 1% of wheel tractors produced.

self-propelled combines will be diesel powered.[34/] In addition, there have been improvements in horsepower capacity, hydraulic systems, independent power takeoffs, and other factors in agricultural machinery and equipment in the past ten years.

Technological change is a broad and indistinct category that includes among other things changes and improvements in the quality of productive factors. Griliches, Court, Dhrymes, and Fettig have demonstrated the possibility of measuring quality change by generating price indices that take explicit account of available information on quality-change factors.[35/] The "hedonic" characteristics or Lancasterian approach to the construction of price indices is based on the assumption that the multitude of varieties of a particular product can be comprehended in terms of a much smaller number of characteristics or basic attributes.

Consider a set of prices p_{it}, where i designates the variety of a commodity and t stands for the time period of observation. The reason why these different varieties sell at different prices may be due to some differences in their properties, dimensions, or other factors. In short, p_{it} is a function of a set of characteristics, X_{jit}, j=1,...., k, and some random factors, u_{it}. Mathematically,

$$P_{it} = f(X_{1it}, \ldots, X_{kit}, u_{it}) \qquad (2\text{-}4)$$

[34/] G.L. Casler and J.H. Erickson, "Energy Requirements for New York State Agriculture", Agricultural Economics Extension Bulletin, 74-24, Cornell University, 1974, p. 2.

[35/] Zvi Griliches, "Hedonic Price Indexes Revisited: Some Notes on the State of the Art", Report 6502, University of Chicago, January, 1968, p. 2.

To estimate such a function, additional assumptions about the number and kind of relevant characteristics and the form in which they affect the price of the product need to be made. There is no *a priori* reason to expect price and commodity attributes to be related in any particular fixed fashion.[36/]

Weather

Weather plays a role in agricultural production. Weather conditions affect the quantities of fossil fuels used in the drying of corn, the irrigation of crops, the curing of tobacco, the heating of poultry houses, and the operation of agricultural machinery and equipment.

Behavioral Adjustment Concepts

Input demand relationships which incorporate the behavioral characteristics of the entrepreneur are much broader relationships than those specified by the economic theory of the firm.[37/] However, the behavioral characteristics of the management factor were ignored due to the lack of data.

Input Supply and Demand Relationships

Input supply and demand relationships together determine the equilibrium price-quantity combination of an input. However,

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[36/] Ibid.

[37/] Raymond Daniel, "Econometric Analyses of the Farmer Demand For Fertilizer Nutrients", unpublished Ph.D. dissertation, Purdue University, 1970, p. 16.

depending upon the characteristics of the factor market, the demand and supply functions for an input may or may not be estimated independently of each other. If the factors affecting the equilibrium input price-quantity combination may be described by a simultaneous or recursive system of supply and demand equations, then the demand and the supply relationships for an input cannot be estimated independently of each other. On the other hand, if the actions of a firm or a group of firms have little influence on the price of the input, then the factor price is determined by forces outside the system being examined. In this instance, the input demand relationship can be estimated independently of the input supply relationship.

In relation to the U.S., Virginia uses only 0.9 percent of gasoline and 1.0 percent of diesel fuel in agricultural production. The prices of fossil fuels are not dependent upon the quantities purchased. So, in Virginia, the actions of a farmer or a group of farmers have an imperceptible influence on the prices of gasoline and diesel fuel. Therefore, it is assumed that the supply relationships of gasoline and diesel fuel are perfectly elastic. Thus, gasoline and diesel fuel prices are exogenous, and the need for considering the factors influencing the supply of these fossil fuels is eliminated. The demand relationships for gasoline and diesel fuel can be estimated independently of the supply relationships. A single equation approach rather than a simultaneous equation approach is warranted.

The Demand Relationships for Gasoline and Diesel Fuel

The static theoretical development of derived input demand, dynamic concepts, technological adjustment concepts, and the influence of weather have been discussed. This section will bring together all these factors in developing the economic models for gasoline and diesel fuel.

An economic model is an ideal representation of a combination of behavioral, technical, institutional, and definitional relationships. In addition, the economic model includes the number of equations, the general form of the equations, and the a priori restrictions of the parameters of the relationships. Finally, the variables in the model are ideal in the sense that they represent exactly what they are supposed to represent.

The following theoretical demand relationships are suggested as a representation of farmers' purchases of gasoline and diesel fuel for agricultural use:

$$Q_G^D = f(P_G, P_{DF_{(t-k)}}, P_{G_{(t-k)}}, P_i, P_{LD}, P_{LB}, P_F, C, T, W, A, L, S) \qquad (2\text{-}5)$$

$$Q_{DF}^D = g(P_{G_{(t-k)}}, P_{DF}, P_{DF_{(t-k)}}, P_i, P_{LD}, P_{LB}, P_F, C, T, W, A, L, S) \qquad (2\text{-}6)$$

where:

Q_G^D = the quantity of gasoline purchased

Q_{DF}^D = the quantity of diesel fuel purchased

P_G = the price of gasoline

$P_{G(t-k)}$ = the price of gasoline at time t-k

P_{DF} = the price of diesel fuel

$P_{DF(t-k)}$ = the price of diesel fuel at time t-k

P_i = the expected price(s) for farm products (i=1,..., h)

P_{LD} = the price of land

P_{LB} = the price of labor

P_F = the price of fertilizer

C = capital

T = technology

W = weather

A = the number of acres of cropland

L = location within the state

S = the season (spring, summer, fall, winter)

From equations (24) and (45) in Appendix B, in most instances, the sign on the slope of the firm's demand function for an input i with respect to the price of input i, $\frac{\partial x_i}{\partial p_i}$ all other factors non-variant, is negative, a priori. Hence, the quantity of gasoline purchased by the farmer Q_G^D, is hypothesized to vary inversely with the price of gasoline P_G. Similarly, the quantity of diesel fuel purchased by the farmer Q_{DF}^D, is hypothesized to vary inversely with the price of diesel fuel, P_{DF}.

A priori, the expected price(s) for farm products (P_i), the price of land (P_{LD}), and the price of labor (P_{LB}) are acsumed to have a positive effect on Q^D_{DF} and Q^D_G. From equations (32) and (48), excluding the possibility of an inferior input, the relationship between the quantity of an input i and the price of an output j, $\frac{\partial x_i}{\partial p_j}$, ceterus paribus, is positive. Gasoline and diesel fuel are considered to be substitutes for land and labor. For the past 25 years, fossil fuels have been substituted for land and labor in agricultural production. From 1950-1973, there was a 6 percent decline in land use for crops and a 59 percent decline in the number of on-farm workhours. However, the farmer typically purchases gasoline and diesel fuel not to substitute directly for land by reducing acreage but to use on the given acreage. From equations (26) and (46) in Appendix B, the relationship between the quantity of an input i and the price of input j, $i \neq j$, $\frac{\partial x_i}{\partial p_j}$, ceterus paribus, is positive in most cases if inputs i and j are substitutes for each other.

In agricultural use, gasoline and diesel fuel are considered to be substitutes for each other. Diesel engines have been substituted for gasoline engines in tractors and combines. By 1980, over 80 percent of farm tractors and 90 percentof self-propelled combines will be diesel powered.[38/] However, farmers are unable to make an

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[38/] G.L. Casler and J.H. Erickson, "Energy Requirements for instantaneous adjustment to a change from gasoline to diesel fuel tractors. A certain amount of stickiness exists in response to changes in the prices of inputs. A diesel fuel tractor may not immediately replace a gasoline tractor for the following reasons: (1) even if every farmer knows about the efficiency gains of the diesel engine, a major capital expenditure is involved; (2) uncertainty exists in the farmer's decision-making process; and (3) gasoline tractors that are still in existence may have remaining life, and they may continue to be used for a while. Hence, $P_{G_{(t-k)}}$ is assumed to have a positive effect on Q^D_{DF} and a negative effect on Q^D_G, all other factors nonvariant. $P_{DF_{(t-k)}}$ is assumed to have a positive effect on Q^D_G and a negative effect on Q^D_{DF}, all other factors nonvariant.

A priori, the price of fertilizer (P_F) is hypothesized to have a negative effect on Q^D_G and Q^D_{DF}, ceterus paribus. For the past 25 years, fossil fuels and fertilizers have been substituted for other inputs, particularly land and labor, in agricultural production. Therefore, fertilizer is hypothesized to be a complementary input to gasoline and diesel fuel in agricultural use. From equations (26) and (46), the relationship between the quantity of an input i and the price of input j, i≠j, $\frac{\partial x_i}{\partial p_j}$, ceterus paribus, is negative in

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New York State Agriculture", Agricultural Economics Extension Bulletin, 74-24, Cornell University, 1974, p. 3.

most cases if inputs i and j are complementary factors.

Technology (T), number of acres of cropland (A), and capital (C) are assumed a priori to have a positive effect on Q_G^D and Q_{DF}^D, ceterus paribus. Technological change is a broad and indistinct category that includes among other things changes and improvements in the quality of productive factors. As previously discussed, a change from gasoline to diesel engines in tractors and combines has taken place in the agricultural sector. In addition, there have been improvements in feed-handling devices, irrigation systems, crop drying systems, and other factors in the past 25 years. Simply put, changes and improvements in agricultural machinery and equipment over the past two and a half decades have not only made the agricultural sector more specialized and mechanized but also have resulted in an increased utilization of gasoline and diesel fuel. Further, farmers may obtain capital (C) for any number of agricultural purposes. Some purposes include the maintenance of crop land, the purchase of agricultural equipment, land and livestock, and the construction or repair of buildings. By assumption, as the farmer accumulates capital, farm output and the use of farm inputs increase. Finally, cropland acreage measures the size of a farm. As farm size increases, farm output and the use of farm inputs increase.

A priori, weather is hypothesized to have a positive effect on Q_G^D and Q_{DF}^D, ceterus paribus. In wet or damp weather, the quantities of gasoline and diesel fuel used in the curing of tobacco and the drying of corn and other crops increase. In addition, when the farm operator uses his tractor(s), combine(s), and other agricultural machinery and equipment in field operations on wet land, the quantities of gasoline and diesel fuel in agricultural use increase. In dry weather, however, between the usual planting and harvesting dates, the quantities of gasoline and diesel fuel used in crop irrigation systems increase. In short, a measure of weather is developed so that an increase in the value of that measure reflects an increase in inclement weather. The agricultural use of gasoline and diesel fuel is hypothesized to increase (decrease) as the value of the measure of weather increases (decreases).

The agricultural use of gasoline and diesel fuel differs among different districts of Virginia (L) which may be attributed to several factors associated with different soil types and different types of farm operations. The variable L, location within the state, represents a combined measure of these differences. Pinpointing the exact districts which purchase more or less gasoline and diesel fuel is a difficult task. However, the areas which attribute most of their income to crops are hypothesized to purchase the largest quantities of gasoline and diesel fuel. Crop reporting districts 5, 6, 8, and 9 of Virginia (Figure 1), are hypothesized to purchase relatively more gasoline and diesel fuel. Crop reporting districts 2, 4, and 7 are hypothesized to purchase relatively less gasoline and diesel fuel.

The agricultural use of gasoline and diesel fuel also differs among various seasons (S) of the year. Since the usual planting and harvesting dates are in the spring and the fall, farmers are hypothesized to purchase more gasoline and diesel fuel, <u>ceterus</u>

paribus, in these seasons. Farmers are hypothesized to purchase less gasoline and diesel fuel in the winter and the summer.

CHAPTER 3

FOSSIL FUEL USE IN AGRICULTURAL PRODUCTION IN VIRGINIA, THE SOUTH, AND THE U.S.

In this chapter, attention is devoted to the use of fossil fuel energy, namely gasoline, diesel fuel, LP gas, fuel oil, natural gas, and coal, in agricultural production in Virginia, the South, and the U.S. The South includes the following thirteen states: Alabama, Arkansas, Florida, Georgia, Kentucky, Louisiana, Mississippi, North Carolina, Oklahoma, South Carolina, Tennessee, Texas, and Virginia. The ensuing analyses are based on cross-sectional data provided by the Economic Research Service, United States Department of Agriculture. Under a jointly funded cooperative agreement with the Federal Energy Administration, estimates of the use of fossil energy by crops and livestock for 1974 were developed for all states from budget data.

Energy Use in Crop and Livestock Production[39]

The usage patterns of fossil fuels are important since different kinds of agriculture, namely crop and livestock production, require different quantities of fossil fuel. Analyses of fossil fuel use in Virginia, Southern, and U.S. agricultural production are conducted on

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[39] Refers to the direct use of energy on the farm for crop and livestock production: mechanized feeding, space heating, farm business auto use, field operations, irrigation, fertilizer application, and crop drying. The energy required to manufacture fertilizers, pesticides, and herbicides is not included.

aggregate enterprises, on major livestock enterprises, and on major crop enterprises. Additionally, the per acre use of gasoline, diesel fuel, and LP gas for major crop enterprises is analyzed. Finally, in each case, the use of fossil fuel in Virginia agriculture is compared to the use of fossil fuel in Southern and U.S. agriculture.

Total Crop and Livestock Use

In 1974, approximately 33 million gallons of gasoline, 27 million gallons of diesel fuel, 8 millin gallons of fuel oil, 29 million gallons of LP gas, 96 million cubic feet of natural gas, and 5,000 tons of coal were used in crop and livestock production in Virginia. A breakdown of Virginia fossil fuel use in crop and livestock production as a percentage of Southern and U.S. use for 1974 is presented in Table 3.1. In total agricultural production, Virginia accounts for 1.2 percent of the total agricultural receipts in the U.S. and 4.4 percent of the total agricultural receipts in the South. Virginia uses 0.9, 1.0, 2.8, 2.0, 15.0, and less than 0.1 percent of the gasoline, diesel fuel, fuel oil, LP gas, coal, and natural gas in total agricultural production in the U.S. and 3.4, 3.2, 4.0, 5.9, 23.0, and 0.1 percent of these fossil fuels in total agricultural production in the South. In crop and livestock production, the usage patterns of the six fossil fuels are similar to that in total agricultural production with few exceptions.

Large differences exist in the percentage of fossil fuel use

Table 3.1. Total Value of Production, Acreage, and Fossil Fuel Use in Total Agricultural, Livestock, and Crop Production in Virginia as a Percentage of the South and the U.S. (1974)

	% Total Agriculture[1]		% Total Livestock		% Total Crops	
ITEM	South	US	South	US	South	US
Total Value of Production	4.4	1.2	4.5	1.3	4.4	1.1
Acreage	N/A[2]	N/A[2]	N/A[2]	N/A[2]	3.7	0.9
Gasoline	3.4	0.9	3.5	1.2	3.4	0.8
Diesel Fuel	3.2	1.0	7.0	1.2	2.9	1.0
Fuel Oil	4.0	2.8	54.1	5.2	3.8	2.7
LP Gas	5.9	2.0	4.8	2.0	6.3	2.0
Natural Gas	0.1		1.4	0.8		
Coal	23.0	15.0	23.0	15.0	N/A[2]	N/A[2]

Source: J. Havlicek, Jr., "The Energy Problem: Its Effect Upon Agriculture and Some Potential Responses", Department of Agricultural Economics, Virginia Polytechnic Institute and State University, Blacksburg, Virginia, February, 1977, p. 5.

J. Havlicek, Jr. and O. Capps, Jr., "Southern Agriculture's Usage of Critical Fossil Fuel Inputs and Needed Economic Research", paper presented at the Southern Agricultural Economics Association Annual Meetings, February, 1977, p. 5.

Note: Blank spaces in table denot less than 0.1 percent usage.

[1] Refers to total livestock and total crops

[2] Not applicable

between crops and livestock in Virginia, the South, and the U.S. As exhibited in Table 3.2, except for coal, crop production requires a larger percentage of fossil fuel energy than livestock production. In Virginia, the South, and the U.S., almost all of the fuel oil use is attributable solely to crop production. In addition, 63 to 97 percent of the gasoline, diesel fuel, LP gas, and natural gas use is attributable to crop production enterprises. Although all of the coal use is designated to livestock production, coal does not play an important role in agricultural production. Furthermore, the percentage of fossil fuel use in crop production in Virginia, the South, and the U.S. exceeds the percentage of total value of production attributable to crops. Conversely, the percentage of the total value of production attributable to livestock exceeds the percentage of fossil fuel usage in livestock production. Finally, Virginia uses a larger percentage of fossil fuels in livestock production than either the South or the U.S.

Livestock Enterprises

The percentage of total value of production and fossil fuel use in Virginia, the South, and the U.S. for selected livestock enterprises for 1974 is presented in Table 3.3. The types of livestock include beef cows and calves, beef feedlots, milk cows, broilers, layers, pullets, hogs, turkeys, sheep and lambs, and miscellaneous poultry. Although the percentages differ, in terms of value of production, milk cows, beef cows and calves, hogs, broilers, pullets, and layers are the major livestock enterprises in Virginia, the South,

Table 3.2. The Percentage of Total Value of Production and Fossil Fuel Use Between Crops and Livestock in Virginia, the South, and the U.S. (1974)

	US %	South %	Virginia %
Total value of production attributable to crops	62.0	59.0	58.4
livestock	38.0	41.0	41.6
Gasoline use attributable to crops	77.9	70.6	69.8
livestock	22.1	29.4	30.2
Diesel fuel use attributable to crops	86.6	92.8	84.3
livestock	13.4	7.2	15.7
Fuel oil use attributable to crops	97.1	99.6	94.5
livestock	2.9	0.4	5.5
LP gas use attributable to crops	77.5	72.1	77.5
livestock	22.5	27.9	22.5
Natural gas use attributable to crops	97.2	96.8	63.5
livestock	2.8	3.2	36.5
Coal use attributable to crops	0.0	0.0	0.0
livestock	100.0	100.0	100.0

Source: J. Havlicek, Jr., "The Energy Problem: Its Effect Upon Agriculture and Some Potential Responses", Department of Agricultural Economics, Virginia Polytechnic Institute and State University, Blacksburg, Virginia, February, 1977, p. 6.

J. Havlicek, Jr. and O. Capps, Jr., "Southern Agriculture's Usage of Critical Fossil Fuel Inputs and Needed Economic Research", paper presented at the Southern Agricultural Economics Association Annual Meetings, February, 1977, p. 7.

Table 3.3. The Percentage of Total Value of Production and Fossil Fuel Use in Virginia, the South, and U.S. for Selected Livestock Enterprises (1974)

TYPE OF LIVESTOCK	Total Value of Production			Gasoline Use			Diesel Fuel Use			Fuel Oil Use			LP Gas Use			Natural Gas Use			Coal Use		
	VA	South	US	VA	South	US	VA	South	US	VA	South	US	VA	South	US	VA	South	US	VA	South	US
Beef Cows & Calves	27.1[1]	42.0[1]	39.2[1]	37.0	58.7	38.4	72.3	66.0	49.8				7.0	0.3	3.4						
Beef Feedlots				0.8	1.0	9.4	3.0	11.3	24.5									.02			
Milk Cows	31.6	16.4	25.5	29.7	15.1	26.7							18.4	14.4	23.0						
Broilers	12.3	16.1	6.4	5.8	6.0	2.8				50.7	61.1	72.6	49.9	68.4	36.7	65.7	79.7	48.4	58.2	78.2	57.7
Layers	8.5[2]	12.4[2]	8.0[2]	1.7	2.1	1.7		0.7	0.5	1.3	2.5	5.9	0.8	2.0	1.5		2.4	5.4			
Pullets				3.5	3.6	2.8		0.5	0.3	5.0	6.2	13.7	5.3	6.4	7.1	2.9	7.0	11.8	6.1	7.6	18.7
Hogs	12.7	5.9	18.1	13.7	9.8	14.1	17.1	20.6	22.6				1.8	5.3	15.1						
Turkeys	5.5	2.1	1.8	4.3	1.2	1.2	0.1	0.6	0.5	42.6	29.7	5.4	16.1	5.9	12.7	25.7	10.1	25.1	34.8	13.9	19.0
Sheep & Lambs	1.0	0.4	0.7	2.9	1.2	2.7	7.5	0.9	1.9												
Misc. Poultry	1.2	0.3	0.3	0.5	0.1	0.2				0.4	0.5	2.4	0.7	0.4	0.5	2.9	0.7	9.2	0.9	0.3	4.7

Source: J. Havlicek, Jr., "The Energy Problem: Its Effect Upon Agriculture and Some Potential Responses", Department of Agricultural Economics, Virginia Polytechnic Institute and State University, Blacksburg, Virginia, February, 1977, p. 8.

J. Havlicek, Jr. and O. Capps, Jr., "Southern Agriculture's Usage of Critical Fossil Fuel Inputs and Needed Economic Research", paper presented at the Southern Agricultural Economics Association Annual Meetings, February, 1977, p. 9.

[1] Includes Beef Feedlots

[2] Includes Layers and Pullets

Note: Blank spaces in table denote less than 0.1 percent usage.

and the U.S. In Virginia, the largest percentage of livestock receipts is attributable to dairy enterprises, but in the South and the U.S., the largest percentage of livestock receipts is attributable to beef enterprises.

Although differences exist in the percentage of use, beef, dairy, and hog enterprises are the major users of gasoline in Virginia, the South, and the U.S., while beef and hog enterprises are the major users of diesel fuel. Beef, dairy, and hog activities account for approximately 80 percent of the gasoline use in all livestock production. Similarly, beef cows and calves, beef feedlots, and hogs use 92 to 98 percent of the diesel fuel. Beef enterprises alone account for 75 percent of the diesel fuel usage in all livestock production.

Poultry enterprises, namely broilers, and turkeys, dominate the use of fuel oil, LP gas, natural gas, and coal in Virginia, Southern, and U.S. livestock production. These two livestock enterprises use 78 to 93 percent of the fuel oil, 49 to 74 percent of the LP gas, 73 to 91 percent of the natural gas, and 76 to 93 percent of the coal. Again, the percentages vary among Virginia, the South, and the U.S., but the key users remain the same.

Crop Enterprises

In Virginia, the South, and the U.S., the prominent users of fossil fuels in agricultural production are crops. The percentage of total value of production and fossil fuel use in Virginia, the South, and the U.S. for selected crop enterprises for 1974 is exhibited in Table 3.4. The percentages of value of production and

Table 3.4. The Percentage of Total Value of Production and Fossil Fuel Use in Virginia, the South, and U.S. for Selected Crop Enterprises (1974)

	Total value of production			Gasoline Use			Diesel Fuel Use			Fuel Oil			LP Gas			Natural Gas		
TYPE OF CROP	VA	South	US	VA	South	US	VA	South	US	VA	South	US	VA	South	US	VA	South	US
Soybeans	10.8	15.0	15.3	10.4	16.3	13.4	19.4	18.3	15.0				0.7	4.0	3.3		1.8	1.2
Corn	21.4	10.3	26.3	20.3	10.4	23.8	21.7	9.8	20.6	0.9	3.8	3.8	9.9	8.5	50.9	100.0	8.5	16.2
Cotton		10.2	4.2		11.4	3.7		21.7	8.9					6.9	2.4		16.2	12.8
Flue-cured Tobacco	17.1	8.2	1.6	11.1	3.8	0.9	5.7	1.9	0.6	82.9	31.1	21.9	75.6	47.3	14.9			
Grain Sorghum	0.2	6.6	3.1	0.4	8.6	3.4	0.5	7.9	4.5					6.0	3.5		27.5	18.2
Winter Wheat	5.1	6.2	9.1	5.2	11.8	10.1	7.9	9.8	8.6				0.3	3.4	2.0		12.2	8.9
Hay-other	8.3	5.8	2.0	9.2	4.8	4.2			0.4				2.8	2.3	3.1			1.1
Burley Tobacco	4.5	4.2	1.3	2.9	1.7	0.5		0.2	0.1				0.8	1.8	0.7			
Peanuts	7.6	4.2	1.1	4.6	2.3	0.6	10.6	4.0	1.4				6.5	2.4	0.8		1.3	0.7
Apples	4.8			3.9			6.4			16.2			0.4					
Corn Silage	8.7	2.0	4.3	8.2	1.4	3.4	8.6	1.4	6.5				0.6	0.4	1.3		0.2	2.4
Vegetables (Fresh)	2.1	1.9	3.0	2.5	1.9	1.8	1.8	1.2	1.6				0.2	0.5	0.3			0.3
Alfalfa	1.6	1.5	5.9	8.0	3.6	13.1	0.3	0.7	5.3				1.4	1.6	6.2		3.9	17.1
Potatoes	3.0	0.9	2.5	3.0	0.4	1.2	1.6	0.2	1.6						0.1			0.1

Table 3.4. Continued

TYPE OF CROP	Total value of production			Gasoline Use			Diesel Fuel Use			Fuel Oil Use			LP Gas Use			Natural Gas Use		
	VA	South	US	VA	South	US	VA	South	US	VA	South	US	VA	South	US	VA	South	US
Barley	1.5	0.2	1.4	1.7	0.3	1.7	3.8	0.4	1.6						0.3			0.3
Sweet Potatoes	1.0	0.6	0.2	0.9	0.4	0.1	0.4	0.1	0.1				0.2	0.1				
Vegetables (Process)	0.9	0.3	1.5	2.1	0.6	1.7	1.4	0.3	2.0				0.2	0.1	0.7			
Peaches	0.5	0.3	0.4	0.5	0.3	0.2	1.0	0.5	0.4		1.2	3.1						
Oats	0.4	0.3	1.7	1.0	1.9	3.3	2.6	1.9	1.9					0.2	0.8			
Sorghum Silage	0.4	0.4	0.3	0.3	0.3	0.2	0.7	0.3	0.5					0.2	0.1		0.2	0.5
Rye	0.2	0.1	0.1	2.8	0.8	0.5	5.2	0.9	0.5				0.2	0.1	0.1			

Source: J. Havlicek, Jr., "The Energy Problem: Its Effect Upon Agriculture and Some Potential Responses", Department of Agricultural Economics, Virginia Polytechnic Institute and State University, Blacksburg, Virginia, February, 1977, p. 9.

J. Havlicek, Jr. and O. Capps, Jr., "Southern Agriculture's Usage of Critical Fossil Fuel Inputs and Needed Economic Research", paper presented at the Southern Agricultural Economics Association Annual Meetings, February, 1977, p. 13.

Note: Blank spaces in table denote less than 0.1 percent usage.

fossil fuel use for crop activities, not unlike livestock activities, differ among the three regions. In terms of value of production, and gasoline use, corn, flue-cured tobacco, soybeans, and hay are the major crop enterprises in Virginia. These enterprises account for 68 percent of the crop receipts and 67 percent of the gasoline use in all crop production. In the South, soybeans, corn, cotton, and flue-cured tobacco are the notable crop activities, in terms of value of production and gasoline use, and in the U.S., corn, soybeans, winter wheat, and hay are the notable crop activities.

Corn, soybeans, peanuts, and winter wheat use a large portion of diesel fuel, nearly 70 percent, in Virginia crop production. In the South and U.S., cotton, soybeans, corn, and winter wheat require the most diesel fuel in crop production.

Nearly all the fuel oil usage in Virginia crop production is attributable to flue-cured tobacco and apples. In Southern and U.S. crop production, only oranges and flue-cured tobacco require large percentages of fuel oil. Together, these crop enterprises account for 80 percent of the fuel oil use in the South and 65 percent of the fuel oil use in the U.S.

In Virginia, flue-cured tobacco, peanuts, and corn utilize a large percentage of LP gas, roughly 92 percent, in crop production. In the South, and the U.S., flue-cured tobacco, corn, rice, hay, cotton, grain sorghum, and soybeans account for 80 to 84 percent of the LP gas use in crop production. Flue-cured tobacco by far uses the most LP gas in Virginia and Southern crop production, but in the

U.S., corn uses the most LP gas in crop production.

Corn accounts for 100 percent of the natural gas use in crop production in Virginia. In Southern crop production, grain sorghum, cotton, rice, winter wheat, and corn use approximately 80 percent of the natural gas. In crop production in the U.S., grain sorghum, alfalfa, corn, cotton, rice, and winter wheat account for 72 percent of the natural gas use. In Southern and U.S. crop production, grain sorghum requires the largest percentage of natural gas.

The per acre use of gasoline, diesel fuel, and LP gas in Virginia, the South, and the U.S. for selected crop enterprises for 1974 is presented in Table 3.5. The selection of these 3 fossil fuels rests upon their relative importance in crop production. In Virginia, tobacco, apples, and vegetables require the most gasoline on a per acre basis, while in the South and U.S., citrus fruits, namely oranges, lemons, and grapefruits, require the most gasoline per acre. In terms of per acre use of diesel fuel, apples, peaches, peanuts, vegetables, and flue-cured tobacco are the major crop enterprises in Virginia. In the South and the U.S., oranges, apples, and tobacco use the most gallons of diesel fuel per acre. In Virginia, the South, and the U.S., nothing compares with flue-cured tobacco in the use of LP gas per acre. Simply put, the usage patterns of fossil fuels in crop production differ according to total or per acre use. Further, in many cases, the gasoline, diesel fuel, and LP gas per acre use in Virginia crop production is very similar to that in Southern and U.S. crop production.

Table 3.5. Gallons Per Acre Use of Gasoline, Diesel Fuel, and LP Gas in Virginia, the South, and the U.S. for Selected Crop Enterprises (1974)

	Gasoline Use			Diesel Fuel Use			LP Gas Use		
CROP	VA	South	US	VA	South	US	VA	South	US
Soybeans	5.4	6.3	7.2	10.0	8.2	6.4	0.4	0.8	0.7
Corn	8.3	8.5	10.5	8.8	9.2	7.2	4.0	3.7	9.0
Cotton	7.0	6.8	7.8	10.5	14.9	14.8	0.5	2.2	2.0
Flue-Cured Tobacco	41.7	41.7	41.7	21.3	23.8	23.7	280.0	277.7	277.7
Grain Sorghum	6.4	8.3	7.1	8.9	8.9	7.3	0.9	3.1	2.9
Winter Wheat	4.1	5.2	5.6	6.1	5.0	3.7	0.2	0.8	0.6
Hay-Other	2.4	3.3	3.6			0.3	0.7	1.1	1.1
Burley Tobacco	41.9	44.4	43.3	0.9	5.7	5.0	12.1	25.3	23.7
Peanuts	10.4	10.6	11.1	23.7	20.9	20.9	14.6	5.8	5.8
Apples	35.3			57.1			3.7		
Corn Silage	11.2	10.7	9.2	11.6	12.1	13.8	0.8	1.6	1.4

Table 3.5. Continued

	Gasoline Use			Diesel Fuel Use			LP Gas Use		
CROP	VA	South	US	VA	South	US	VA	South	US
Vegetables (Fresh)	32.4	33.8	32.7	22.8	24.8	23.5	2.8	4.7	2.4
Alfalfa	23.2	20.4	14.2	0.9	4.7	5.0	3.9	4.8	2.7
Potatoes	21.9	22.1	24.2	12.1	12.1	26.4	0.6	0.2	0.8
Barley	3.2	4.1	5.4	7.0	6.1	4.0		0.2	0.4
Sweet Potatoes	26.8	26.5	27.2	10.5	9.8	10.0	4.4	4.4	4.5
Vegetables (Process)	33.2	33.8	28.4	22.2	23.5	25.8	3.5	4.0	4.8
Peaches	25.2	27.3	25.6	45.4	47.8	43.0		0.8	1.0
Oats	2.8	4.1	5.3	7.0	4.5	2.4		0.3	0.5
Sorghum Silage	7.2	9.4	6.7	15.5	10.5	14.1		2.5	1.6
Rye	3.2	3.6	4.8	6.1	5.1	3.8	0.2	0.3	0.4

Source: J. Havlicek, Jr., "The Energy Problem: Its Effect Upon Agriculture and Some Potential Responses", Department of Agricultural Economics, Virginia Polytechnic Institute and State University, Blacksburg, Virginia, February, 1977, p. 11.

J. Havlicek, Jr. and O. Capps, Jr., "Southern Agriculture's Usage of Critical Fossil Fuel Inputs and Needed Economic Research", paper presented at the Southern Agricultural Economics Association Annual Meetings, February, 1977, p. 16.

Note: Blank spaces in table denote less than 0.1 percent usage.

SUMMARY

Virginia uses 0.9, 1.0, 2.8, 2.0, 15.0 and less than 0.1 percent of the gasoline, diesel fuel, fuel oil, LP gas, coal, and natural gas in total agricultural production in the U.S. and 3.4, 3.2, 4.0, 5.9, 23.0 and 0.1 percent of these fossil fuels in total agricultural production in the South. In crop and livestock production, the usage patterns of the six fossil fuels, with few exceptions, are similar to that in total agricultural production. However, large differences exist in the percentage of fossil fuel use between crops and livestock in Virginia, the South, and the U.S. Except for coal, crop production requires a larger percentage of fossil fuel energy than livestock production. Crop production accounts for approximately 75 percent of the fossil fuel use in Virginia agricultural production, and livestock production accounts for the remaining 25 percent. In Southern and U.S. agricultural production, crop production accounts for approximately 90 percent of the fossil fuel use, and livestock production accounts for the remaining 10 percent. Hence, Virginia uses a larger percentage of fossil fuels in livestock production than either the South or the U.S. Nevertheless, in Virginia, the South, and the U.S., the prominent users of fossil fuels in agricultural production are crops.

In livestock production, beef, dairy, and hog enterprises are the major users of gasoline in Virginia, the South, and the U.S, while beef and hog enterprises are the major users of diesel fuel. Poultry enterprises, namely broilers and turkeys, dominate the use of fuel oil, LP gas, natural gas, and coal in Virginia, Southern, and

U.S. livestock production. Differences exist in the percentage of fossil fuel use among Virginia, the South, and the U.S. but the key users remain the same.

In Virginia crop production, corn, flue-cured tobacco, soybeans, hay, peanuts, winter wheat, and apples are the major users of gasoline, diesel fuel, fuel oil, LP gas, and natural gas. In Southern and U.S. crop production, soybeans, corn, cotton, flue-cured tobacco, winter wheat, hay, oranges, rice, and grain sorghum use relatively large portions of these five fossil fuels. The percentages of fossil fuel use for crop activities, not unlike livestock activities, differ among Virginia, the South, and the U.S. Differences in total livestock and crop use among production enterprises in the three regions may be due to differences in livestock and crop mix, temperature, climate, fuel prices, land prices, labor prices, interest rates, and fertilizer prices.

On a per acre basis in Virginia, tobacco, apples, vegetables, peaches, and peanuts require the most gasoline, diesel fuel, and LP gas. In the South and the U.S., citrus fruits, apples, and tobacco use the most gallons of the three fossil fuels per acre. The usage patterns of fossil fuels in crop production differ according to total or per acre use. Finally, in many cases, the gasoline, diesel fuel, and LP gas per acre use in Virginia crop production is similar to that in Southern and U.S. crop production.

CHAPTER 4

THE STATISTICAL MODELS AND STATISTICAL PROCEDURES

The economic models for the gasoline and diesel fuel demand relationships have been presented in Chapter 2. The next step is to translate the economic models into statistical models and obtain numerical estimates of the parameters of the empirical variables. Information from the estimated statistical models then provides a basis for conclusions and inferences about the flow of causal forces in the economic models. In the first section of this chapter, the statistical models are presented. In the second section, attention is given to the statistical models of estimation.

The Statistical Models

A statistical model is a stochastic representation of the economic model. The statistical model specifies the relationships of the economic model in a specific algebraic form and identifies specific measures for the variables in the economic model. However, measurement problems with respect to the functional forms and the empirical variables may occur. In most cases, the "true functional form" of an economic phenomenon is never known, and the probability of choosing the exact functional form of the statistical model is practically zero. Further, the measures chosen for the variables in the statistical model may not measure precisely the economic phenomenon of interest. As an explicit recognition that the relationships of the statistical model are not expected to hold exactly, the statistical model contains a stochastic disturbance term.

Data

Quantities of gasoline and diesel fuel purchased for agricultural use are endogenous variables whose values are determined by the specified models. These observations, expressed in gallon units, were obtained from Southern States Cooperative, Inc. (SSC) for Virginia by months for the years 1970-1976. At various locations in the seven Virginia crop reporting districts, private service agencies, local cooperatives, and retail branches of SSC make gasoline, diesel fuel, and other fuels available, predominantly to farmers. For each quarterly time period from 1971 to 1976, the quantities of gasoline and diesel fuel purchased were aggregated for each crop reporting district. In addition, estimates of SSC market share of these fuels in each crop reporting district were developed. Farm expenditure for gasoline and diesel fuel was reported for each county in the state in the Census of Agriculture[40/] for 1974. From the data provided by SSC and Agricultural Prices,[41/] farm expenditures for gasoline and diesel fuel purchased from Southern States

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[40/] Census of Agriculture, U.S. Department of Commerce, Social and Economics Statistics Administration, Bureau of the Census, 1974.

[41/] Agricultural Prices, U.S. Department of Agriculture, U.S. Government Printing Office, 1971-1976.

was estimated for 1974. The estimate of Southern States' market share of these fuels for 1974 for each crop reporting district were then obtained by

$$\frac{FE_{GDFSS_i}}{FE_{GDFCA_i}} \quad i = 2, 4, 5, 6, 7, 8, 9 \qquad (4\text{-}1)$$

where

FE_{GDFSS_i} = an estimate of farm expenditure for gasoline and diesel fuel purchased from Southern States in crop reporting district i

FE_{GDFCA_i} = farm expenditure for gasoline and diesel fuel reported in the Census of Agriculture in crop reporting district i

The estimates of Southern States' market share of gasoline and diesel fuel for each crop reporting district are exhibited in Table 4.1. These market share estimates were assumed to remain the same from 1971 to 1976.

The independent or exogenous variables, variables whose values are determined outside the specified models, included: (1) the number of acres of cropland; (2) the real weighted average price for farm output; (3) the real average price of gasoline paid by farmers in the current time period; (4) the real average price of gasoline paid by farmers in time period t-3; (5) the real average price of gasoline paid by farmers in time period t-5; (6) the real average price of diesel fuel paid by farmers in the current time period; (7) the real average price of diesel fuel paid by farmers in time period t-3; (8) the real average price of diesel fuel paid by farmers in time period t-5; (9) the real weighted average price of fertilizer;

Table 4.1. Estimates of Southern States' Market Share of Gasoline and Diesel Fuel by Virginia Crop Reporting District, 1971-1976

District	Market Share (%)
2	77
4	68
5	45
6	15
7	14
8	6
9	12

(10) the average interest rate charged farmers by Production Credit Associations; (11) the real average price of farm labor; (12) the real average price of farm land and buildings; (13) the average number of inches of rainfall; and (14) the ratio of diesel fuel tractors to gasoline tractors in the U.S. The exogenous variables influence the values of the endogenous variables, but, it is assumed the reverse does not hold. Real prices were obtained by deflating actual prices by the wholesale price index (1967=100).

The number of cropland acres was not available on a crop reporting district basis. However, this information was reported for each county in the state in the Census of Agriculture for 1969 and 1974.[42/] Extrapolation and interpolation estimates for the years not included in the Census of Agriculture were based on the district trend in crop acreage. The empirical variable in the statistical models is given by the product of the market share estimates of SSC with the number of cropland acres.[43/]

SSC did not provide data concerning the prices paid by farmers for gasoline and diesel fuel. Quarterly average prices paid by farmers in the state for these fuels were reported in Agricultural

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[42/] Census of Agriculture, U.S. Department of Commerce, Social and Economics Statistics Administration, Bureau of the Census, 1969 and 1974.

[43/] A one-to-one correspondence between the number of cropland acres and farmers' purchases of gasoline and diesel fuel was assumed. It was assumed that farmers' sources of fossil energy did not influence the quantities purchased.

Prices.[44] Nevertheless, the U.S.D.A. price information was considered representative of the prices farmers paid for gasoline and diesel fuel.

Land values were not available on a crop reporting district basis. However, the per acre average value of farm land and buildings for each county in the state was reported in the Census of Agriculture for 1974.[45] In addition, an index of the per acre value of farm land and buildings for March and November (March, 1967=100) was included in Farm Real Estate Market Developments.[46] A quarterly estimate of land values for each crop reporting district was then developed. Quarterly average farm wage rates for field workers in Virginia were reported in Farm Labor.[47]

Monthly rainfall data were reported in Climatological Data for six regions of Virginia: (1) the Tidewater region; (2) the Eastern Piedmont region; (3) the Western Piedmont region; (4) the Northern region; (5) the Central Mountain region; and (6) the Southwestern

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[44] Agricultural Prices, U.S. Department of Agriculture, U.S. Government Printing Office, 1971-1976.

[45] Census of Agriculture, U.S. Department of Commerce, Social and Economic Statistics Administration, Bureau of the Census, 1974.

[46] Farm Real Estate Market Developments, Economic Research Service, Washington, D.C., 1971-1976.

[47] Farm Labor, Crop Reporting Board, Statistical Reporting Service, U.S. Department of Agriculture, Washington, D.C., 1971-1976.

Mountain region.[48/] From these data, a monthly estimate of precipitation for each county was developed. The average number of inches of rainfall per quarter for each crop reporting district was then obtained.

Monthly interest rates charged farmers by Production Credit Associations, prices of short and intermediate term capital, in Virginia were provided by the Federal Intermediate Credit Bank of Baltimore. The Production Credit Associations included: (1) the Farmville Association; (2) the Richmond Association; (3) the Roanoke Association; (4) the Central Valley Association; (5) the Warrenton Association; (6) the Southwest Association; (7) the Staunton Association; (8) the Waverly/Southside Association; and (9) the Salisbury/ Marva Association. From these data, a monthly interest rate charged farmers for each county was developed. The average interest rate charged farmers by Production Credit Associations per quarter for each crop reporting district was then obtained.

The average prices paid per ton by farmers for ammonium nitrate (33.3% N), superphosphate (20% P_2O_5), and muriate of potash (60% K_2O) were reported in <u>Agricultural Prices</u> for April and October.[49/] Prior to 1975, the average prices per ton paid by farmers for

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[48/] <u>Climatological Data</u>, U.S. Department of Commerce, National Oceanic and Atmospheric Administration, Environmental Data Service, Washington, D.C., 1971-1976.

[49/] <u>Agricultural Prices</u>, U.S. Department of Agriculture, U.S. Government Printing Office, 1971-1976.

fertilizer materials were reported for April and September. Fertilizer consumption by total nutrients (nitrogen, phosphate, and potash) for each county for 1974 and 1975 was included in Virginia Agricultural Statistics.[50] A weighted fertilizer price for each county was developed for 1974 and 1975 for April, September, and October by

$$\frac{(P_N \times Q_N) + (P_{SP} \times Q_{SP}) + (P_{MP} \times Q_{MP})}{Q_N + Q_{SP} + Q_{MP}} \tag{4-2}$$

where

P_N = the average price per ton paid by farmers for ammonium nitrate

P_{SP} = the average price per ton paid by farmers for superphosphate

P_{MP} = the average price per ton paid by farmers for muriate of potash

Q_N = the quantity of nitrogen purchased by farmers in county i

Q_{SP} = the quantity of superphosphate purchased by farmers in county i

Q_{MP} = the quantity of muriate of potash purchased by farmers in county i

In addition, a monthly index of prices paid by farmers for mixed fertilizers (1967=100) was reported by the Bureau of Labor Statistics. The average weighted price of fertilizer per quarter for each crop reporting district was then obtained.

The season average prices of corn, winter wheat, soybeans, peanuts, flue-cured, fire-cured, sun-cured, and burley tobacco, and

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[50] Virginia Agricultural Statistics, Virginia Cooperative Crop Reporting Service, Bulletins 40-41, 1975-1976.

hay were reported in Virginia Agricultural Statistics.[51] The selection of these crops rested on their importance to Virginia agriculture. In addition, these crops were the key users of gasoline and diesel fuel in agricultural production (See Chapter 3). Crop production figures for each crop reporting district were also included in Virginia Agricultural Statistics.[52] A season average weighted price of farm output was developed for each crop reporting district by

$$[(P_C Q_{Ci}) + (P_S Q_{Si}) + (P_{WW} Q_{WWi}) + (P_P Q_{Pi}) + (P_H Q_{Hi}) + (P_{FCT} Q_{FCTi}) + (P_{BT} Q_{BTi}) + (P_{FT} Q_{FTi}) + (P_{SCT} Q_{SCTi})] / [Q_{Ci} + Q_{Si} + Q_{WWi} + Q_{Pi} + Q_{Hi} + Q_{FCTi} + Q_{BTi} + Q_{FTi} + Q_{SCTi}] \qquad (4\text{-}3)$$

where

P_C = real average price of corn for the previous year

P_S = real average price of soybeans for the previous year

P_{WW} = real average price of winter wheat for the previous year

P_P = real average price of peanuts for the previous year

P_H = real average price of hay for the previous year

P_{FCT} = real average price of flue-cured tobacco for the previous year

P_{BT} = real average price of burley tobacco for the previous year

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[51] Ibid.

[52] Ibid.

P_{FT} = real average price of fired-cured tobacco for the previous year

P_{SCT} = real average price of sun-cured tobacco for the previous year

Q_{Ci} = pounds of corn harvested in the previous year in crop reporting district i

Q_{Si} = pounds of soybeans harvested in the previous year in crop reporting district i

Q_{WWi} = pounds of winter wheat harvested in the previous year in crop reporting district i

Q_{Pi} = pounds of peanuts harvested in the previous year in crop reporting district i

Q_{Hi} = pounds of hay harvested in the previous year in crop reporting district i

Q_{FCTi} = pounds of flue-cured tobacco harvested in the previous year in crop reporting district i

Q_{BTi} = pounds of burley tobacco harvested in the previous year in crop reporting district i

Q_{FTi} = pounds of fire-cured tobacco harvested in the previous year in crop reporting district i

Q_{SCTi} = pounds of sun-cured tobacco harvested in the previous year in crop reporting district i
$i = 2, 4, 5, 6, 7, 8, 9$

In addition, a monthly index of prices received by farmers for Virginia crops was reported in _Virginia Agricultural Statistics_.[53] The average weighted price of farm output per quarter for each crop reporting district was then obtained.

The ratio of diesel fuel to gasoline tractors, reported for the U.S. in _Current Industrial Reports_, was a proxy variable for a very

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[53] Ibid.

particular type of technological change.[54] Since gasoline and diesel fuel tractor numbers were not available for Virginia, it was assumed that the diesel fuel-to-gasoline tractor ratios for the U.S. and Virginia were identical.

Functional Form

The choice of a functional form of a quantitative relationship is in most cases not a theoretical issue but an empirical issue. The statistical model may assume a number of alternative functional forms. Linear, quadratic, hyperbolic, semilogarithmic, and double logarithmic equation forms are the most common. No definite rule cites the appropriate form for an economic phenomenon. However, a general criteria for selecting a specific functional form of the statistical model exists: (1) select a simple form rather than a complex form; (2) select a form not inconsistent with economic behavior; and (3) select a form with predictive power.[55] These three criteria are not necessarily consistent and may contradict each other. In short, the choice among alternative functional forms involves a compromise among economic theory, goodness of fit, and simplicity. Both economic and statistical considerations influence the choice of the algebraic formulation to be adopted. The choice of the empirical variables rests in large part on how closely they measure the economic

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[54] Current Industrial Reports, Series M355, Bureau of the Census.

[55] Arthur Goldberger, Econometric Theory, (John Wiley and Sons, Inc., New York, 1964), pp. 213-227.

variables.

Some experimentation was involved in the choice of mathematical form and the selection of the empirical variables. The criteria for the selection of a functional form, as suggested above, involved a subjective weighting of consistency of signs and magnitudes of the estimated parameters, compared to a priori reasoning and previous studies, statistical significance of the estimates, and explanatory power of the estimated relationships.[56] The mathematical form chosen for the gasoline and diesel fuel demand relationships was the double logarithmic form. Linear and semilogarithmic functional forms were also considered. The empirical variables were chosen to approximate as closely as possible the theoretical variables specified by the economic models. The following statistical models were formulated and estimated:

GASOLINE

$$\begin{aligned} \mathrm{LnQ}^{D}_{G_{it}} &= \mathrm{LnA}_{o} + \beta_1 \mathrm{LnACRE}_{it} + \beta_2 \mathrm{LnRPCR}_{it} + \beta_3 \mathrm{LnRPGS}_{it} + \\ &\beta_4 \mathrm{LnRPDFL}_{i,\ (t-5)} + \beta_5 \mathrm{LnRWPFR}_{it} + \beta_6 \mathrm{LnINTER}_{it} + \\ &\beta_7 \mathrm{LnRPLABO}_{it} + \beta_8 \mathrm{LnRPLND}_{it} + \beta_9 \mathrm{LnPRECIP}_{it} + \beta_{10} \mathrm{LnTE}_{it} + \\ &\beta_{21} \mathrm{LnRPGS}_{i,\ (t-5)} + \varepsilon_{it}. \qquad (4\text{-}4) \end{aligned}$$

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[56] J.W. Freebairn, and G.C. Rausser, "Effects of Changes in the Level of U.S. Beef Imports", American Journal of Agricultural Economics, November, 1975, p. 678.

DIESEL FUEL

$$LnQ^{D}_{DF_{it}} = LnA_{1} + \beta_{11}LnACRE_{it} + \beta_{12}LnRPCR_{it} +$$
$$\beta_{13}LnRPGS_{i,\ (t-3)} + \beta_{14}LnRPDFL_{it} + \beta_{15}LnRWPFR_{it} +$$
$$\beta_{16}LnINTER_{it} + \beta_{17}LnRPLABO_{it} + \beta_{18}LnRPLND_{it} +$$
$$\beta_{19}LnPRECIP_{it} + \beta_{20}LnTE_{it} + \beta_{22}LnRPDFL_{i,\ (t-3)} + u_{it} \tag{4-5}$$

where:

$Q^{D}_{G_{it}}$ = the quantity of gasoline purchased from SSC (gallons)

$Q^{D}_{DF_{it}}$ = the quantity of diesel fuel purchased from SSC (gallons)

$ACRE_{it}$= the number of acres of cropland times the market share estimates of SSC (acres)

$RPCR_{it}$= the real weighted average price of farm output (cents per pound)

$RPGS_{it}$= the real average price of gasoline paid by farmers in the current time period (bulk delivery) (cents per gallon)

$RPGS_{i,\ (t-3)}$= the real average price of gasoline paid by by farmers in time period t-3 (bulk delivery) (cents per gallon)

$RPGS_{i,\ (t-5)}$= the real average price of gasoline paid by farmers in time period t-5 (bulk delivery) (cents per gallon)

$RPDFL_{it}$= the real average price of diesel fuel paid by farmers

$RPDFL_{i,\ (t-3)}$= the real average price of diesel fuel paid by farmers in time period t-3 (cents per gallon)

$RPDFL_{i,\ (t-5)}$= the real average price of diesel fuel paid by farmers in time period t-5 (cents per gallon)

$RWPFR_{it}$ = the real weighted average price of fertilizer (dollars per ton)

$INTER_{it}$ = the average interest rate charged farmers by Production Credit Associations (percent)

$RPLABO_{it}$ = the real average price of farm labor (field workers) (dollars per hour)

$RPLND_{it}$ = the real average price of farm land and buildings (dollars per acre)

$PRECIP_{it}$ = the average number of inches of rainfall

TE_{it} = the ratio of diesel fuel tractors to gasoline tractors in the U.S. (no units)

i = a subscript denoting crop reporting district in the state ($i = 2, 4, 5, 6, 7, 8, 9$) (Figure 1)

t = a subscript denoting time period (quarter)

Ln = a prefix denoting transformation to natural logarithms

β_j = the coefficient of the jth empirical variable ($j = 1, \ldots 22$)

Under the assumption that the supply relationships of gasoline and diesel fuel are perfectly elastic, a single equation approach rather than a simultaneous equation approach was warranted.

The Parameters

On the basis of economic theory, the following *a priori* hypotheses were made regarding the parameters in the statistical models. In Chapter 2 and Appendix B, it was shown that the sign on the slope of the firm's demand function for an input i with respect to the price of input i, $\frac{\partial x_i}{\partial p_i}$, all other factors nonvariant, was negative, *a priori*. Therefore, $\beta_3 < 0$, $\beta_{21} < 0$, $\beta_{22} < 0$, and $\beta_{14} < 0$. Excluding the possibility of an inferior input, the relationship between the quantity purchased of an input i and the price of an output j, $\frac{\partial x_i}{\partial p_j}$, *ceterus paribus*, was shown to be positive, *a priori*. Hence, $\beta_2 > 0$ and $\beta_{12} > 0$. Further, gasoline and diesel fuel were considered to be substitutes for land and labor. The relationship between the quantity purchased of an input i and the price of input j, $i \neq j$, $\frac{\partial x_i}{\partial p_j}$, *ceterus paribus*, was shown to be positive if inputs i and j were substitutes for each other. Thus, $\beta_7 > 0$, $\beta_8 > 0$, $\beta_{17} > 0$, and $\beta_{18} > 0$, *a priori*. In agricultural use, gasoline and diesel fuel were considered to be substitutes for each other. Therefore, $\beta_4 > 0$ and $\beta_{13} > 0$, *a priori*. Furthermore, fertilizer was considered to be a complementary factor to gasoline and diesel fuel in agricultural use. The relationship between the quantity purchased of an input i and the price of input j, $i \neq j$, $\frac{\partial x_i}{\partial p_j}$, *ceterus paribus* was shown to be negative if inputs i and j were complementary factors. Hence, $\beta_5 < 0$ and $\beta_{15} < 0$, *a priori*. Moreover, cropland acreage was a measure of farm size. It was hypothesized that as farm size increased, the use of farm inputs increased. Thus, $\beta_1 > 0$ and $\beta_{11} > 0$, *a priori*. The influence of the weather measure on the quantity purchased of an input was hypothesized to be positive, *a priori*. Therefore, $\beta_9 > 0$ and $\beta_{19} > 0$. Interest rates charged farmers by Production Credit Associations are prices of short and intermediate term capital. It was assumed that as interest rates increased, the use of farm inputs decreased and conversely, as interest rates decreased, the use of farm inputs increased. Hence, $\beta_6 < 0$ and $\beta_{16} < 0$, *a priori*. Finally, the ratio of diesel fuel tractors to gasoline tractors was a proxy variable for a very particular type of technological change. It was hypothesized that as this ratio increased, the quantity of diesel fuel purchased increased, and the quantity of gasoline purchased decreased. Thus, $\beta_{10} < 0$ and $\beta_{20} > 0$, *a priori*. In sum, hypotheses made concerning the parameters in the statistical models permitted the use of one-tail statistical testing procedures.

Methods of Estimation

Pooling Cross-Sectional and Time-Series Data

As shown by equations (4-4) and (4-5), a statistical model for cross-sectional and time-series data may be written as

$$Y_{it} = \beta_0 + \beta_1 X_{it,\,1} + \dots + \beta_K X_{it,\,K} + \varepsilon_{it}$$

$$(i = 1, \dots, N)\ (t = 1, \dots, T). \tag{4-6}$$

The sample data are represented by T time-series and N cross-sectional observations for a total of N x T observations. In matrix notation, the statistical model can be written as

$$Y = X\beta + \varepsilon \qquad (4\text{-}7)$$

where

$$Y = \begin{bmatrix} Y_{11} \\ \vdots \\ Y_{1T} \\ \vdots \\ Y_{N1} \\ \vdots \\ Y_{NT} \end{bmatrix}, \quad X = \begin{bmatrix} 1 & X_{11,\,1} \ldots X_{11,\,K} \\ \vdots & \vdots \qquad \vdots \\ 1 & X_{1T,\,1} \ldots X_{1T,\,K} \\ \vdots & \vdots \qquad \vdots \\ 1 & X_{N1,\,1} \ldots X_{N1,\,K} \\ \vdots & \vdots \qquad \vdots \\ 1 & X_{NT,\,1} \ldots X_{NT,\,K} \end{bmatrix}, \quad \beta = \begin{bmatrix} \beta_0 \\ \\ \beta_1 \\ \vdots \\ \beta_K \end{bmatrix}, \quad \varepsilon = \begin{bmatrix} \varepsilon_{11} \\ \vdots \\ \varepsilon_{1T} \\ \vdots \\ \varepsilon_{N1} \\ \vdots \\ \varepsilon_{NT} \end{bmatrix}$$

$$(N \times T) \times 1 \qquad (N \times T) \times (K + 1) \qquad (k + 1) \times 1 \qquad (N \times T) \times 1$$

$$(4\text{-}8)$$

The full description of the statistical model is given by (4-7), (4-8), and the following assumptions:

(i) $E(\varepsilon\varepsilon') = \Omega$, where

$$\Omega = \begin{bmatrix} E(\varepsilon_{11}^{2}) & \ldots & E(\varepsilon_{11}\varepsilon_{1T}) & \ldots & E(\varepsilon_{11}\varepsilon_{N1}) & \ldots & E(\varepsilon_{11}\varepsilon_{NT}) \\ \vdots & & \vdots & & \vdots & & \vdots \\ E(\varepsilon_{1T}\varepsilon_{11}) & \ldots & E(\varepsilon_{1T}^{2}) & \ldots & E(\varepsilon_{1T}\varepsilon_{N1}) & \ldots & E(\varepsilon_{1T}\varepsilon_{NT}) \\ \vdots & & \vdots & & \vdots & & \vdots \\ E(\varepsilon_{N1}\varepsilon_{11}) & & E(\varepsilon_{N1}\varepsilon_{1T}) & \ldots & E(\varepsilon_{N1}^{2}) & \ldots & E(\varepsilon_{N1}\varepsilon_{NT}) \\ \vdots & & \vdots & & \vdots & & \vdots \\ E(\varepsilon_{NT}\varepsilon_{11}) & & E(\varepsilon_{NT}\varepsilon_{1T}) & \ldots & E(\varepsilon_{NT}\varepsilon_{11}) & \ldots & E(\varepsilon_{NT}^{2}) \end{bmatrix} \quad (4\text{-}9)$$

$$(N \times T) \times (N \times T).$$

(ii) each of the exogenous variables is nonstochastic, and the vector of values for any $X_{it,\,K}$ is independent of the disturbance terms ($i = 1, \ldots, N$, $t = 1, \ldots, T$, $K = 1, \ldots, K$.)

(iii) the number of observations, $n = N \times T$, exceeds the number of exogenous variables plus the intercept, $(k + 1)$. So, $n > k + 1$.

(iv) no exact linear relation exists between any of the exogenous variables and the intercept.

This specification provides a general framework for the discussion of different models designed to deal with pooled cross-sectional and time-series observations.[57/]

Various specifications of the behavior of the disturbance terms when dealing with cross-sectional and time-series data have been discussed and used by Kuh,[58/] Balestra and Nerlove,[59/] Wallace and Hussain,[60/] Fuller and Battese,[61/] Parks,[62/] and others. Wallace and Hussain and Fuller and Battese assumed that each error term in (4-8) is the sum of three independent components of variation: (1) a

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[57/] J. Kmenta, Elements of Econometrics, (Macmillan Publishing Company, Inc., New York, 1971), pp. 508-517.

[58/] E. Kuh, "The Validity of Cross-Sectionally Estimated Behavior Equations in Time Series Applications", Econometrica, 1959, pp. 197-214.

[59/] P. Balestra and M. Nerlove, "Pooling Cross Section and Time Series Data in the Estimation of a Dynamic Model: The Demand for Natural Gas", Econometrica, 1966, pp. 585-612.

[60/] T.D. Wallace and A. Hussain, "The Use of Error Components Models in Combining Cross Section with Time Series Data", Econometrica, 1969, pp. 55-72.

[61/] W.A. Fuller and G.E. Battese, "Estimation of Linear Models With Cross-Error Structure", Journal of Econometrics, 1974, pp. 67-78.

[62/] R.W. Parks, "Efficient Estimation of a System of Regression Equations When Disturbances Are Both Serially and Contemporaneously Correlated", Journal of the American Statistical Association, June, 1967, pp. 500-509.

cross-sectional component; (2) a time component; and (3) a random component. Kuh posited that each disturbance term in (4-8) consists of two additive parts, the first representing a constant cross-sectional effect and the second symbolizing a random variable varying through time. Balestra and Nerlove discussed the specification of the disturbance terms when lagged endogenous variables were incorporated in (4-6). They assumed that each error term is composed of two statistically independent parts: a cross-sectional effect and a remainder. A different approach to the specification of the behavior of the disturbance terms proposed by Parks when dealing with cross-sectional and time-series data involves combining the assumptions frequently made about cross-sectional observations with those usually made concerning time-series observations. Regarding time-series data, it is often assumed that the disturbance terms are autoregressive though not necessarily heteroscedastic. Concerning cross-sectional data, it is frequently assumed that the disturbance terms are mutually independent but heteroscedastic. However, when the cross-sectional units are geographical regions with arbitrarily drawn boundaries, such as the seven Virginia crop reporting districts, the assumption of mutual independence may not be satisfied. Dropping the assumption of mutual independence, Parks developed a cross-sectionally correlated and time-wise autoregressive model.

The Parks Model

Parameter estimates of the statistical models in (4-4) and (4-5) were obtained by using the Parks model in combining cross-sectional and time-series data. The specification of the behavior of the disturbance terms is given by[63/]

(i) $E(\varepsilon_{it}^{2}) = \gamma_{ii}$

(ii) $E(\varepsilon_{it}\varepsilon_{jt}) = \gamma_{ij} \quad (i \neq j)$

(iii) $\varepsilon_{it} = \rho_i \varepsilon_{i,\ t-1} + U_{it}$

where

$U_{it} \sim N(0, \phi_{ii})$

$E(\varepsilon_{i,\ t-1} U_{jt}) = 0$ for all i, j

$E(U_{it}, U_{jt}) = \phi ij \quad (i \neq j)$

$E(U_{it}, U_{js}) = 0 \quad (t \neq s)$

$i, j = 1, \ldots, N$

$t = 1, \ldots, T$

Condition (i) allows cross-sectional heterosedasticity. Condition (ii) permits a cross-sectional mutual correlation of disturbance terms. Condition (iii) allows a different first order autoregression scheme for each cross section. The autoregressive disturbances, U_{it}, are assumed to be normally distributed with constant variance and zero covariance between time periods for the same cross-sectional unit. Further, the autoregressive disturbances at the same time period in different cross sections are correlated. Finally, independence is assumed between U_{it} and U_{js} for $t \neq s$ and between $\varepsilon_{i,\ t-1}$

[63/] J. Kmenta, Elements of Econometrics, (Macmillan Publishing Company, Inc., New York, 1971), p. 512.

and U_{jt} for all i, j = 1,..., N.[64]

From these specifications, the following relationships may be deduced:

$$\text{(i)}\quad E(\varepsilon_{it}^2) = \frac{\phi_{ii}}{1-\rho_i^2} = \gamma_{ii}$$

$$\text{(ii)}\quad E(\varepsilon_{it}\varepsilon_{jt}) = \frac{\phi_{ij}}{1-\rho_i\rho_j} = \gamma_{ij}$$

$$\text{(iii)}\quad E(\varepsilon_{it}\varepsilon_{is}) = \rho_i^{t-s}\gamma_{ii} \quad (t \geq s), \text{ t, s are positive integers}$$

$$\text{(iv)}\quad E(\varepsilon_{it}\varepsilon_{js}) = \rho_i^{t-s}\gamma_{ij} \quad (i \neq j)$$

$$i, j = 1,\ldots, N; \quad t, s = 1,\ldots, T$$

The initial value of ε is assumed to have the following properties:[65]

$$\varepsilon_{io} \sim N\left(o, \frac{\phi_{ii}}{1-\rho_i^2}\right)$$

$$E(\varepsilon_{io}\varepsilon_{jo}) = \frac{\phi_{ij}}{1-\rho_i\rho_j}$$

The autoregressive schemes are assumed to be appropriate even at the initial observation.

The matrix Ω for the Parks model is given by

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[64] J.L. Murphy, Introductory Econometrics, (Richard D. Irwin, Inc., Homewood, Illinois, 1973), pp. 363-364.

[65] J. Kmenta, Elements of Econometrics, (Macmillan Publishing, Company, Inc., New York, 1971), p. 513.

$$\Omega_{(NxT)x(NxT)} = \begin{bmatrix} \gamma_{11}P_{11} \cdot & \cdot \cdot \gamma_{1N}P_{1N} \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \gamma_{N1}P_{N1} \cdot & \cdot \cdot \gamma_{NN}P_{NN} \end{bmatrix} \qquad (4\text{-}10)$$

where

$$P_{ij} = \begin{bmatrix} 1 & \rho_j & \cdots & \rho_j^{T-1} \\ \rho_i & 1 & \cdots & \rho_j^{T-2} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \rho_i^{T-1} & \rho_i^{T-2} & \cdots & 1 \end{bmatrix}_{TxT} \qquad (4\text{-}11)$$

Estimation of the Parks Model

Under the usual assumptions, a straight forward application of ordinary least squares (OLS) to the Parks model yields unbiased and consistent parameter estimates. Such estimates may be inefficient however, due to the assumptions of autoregression and heteroscedasticity. A variance-covariance matrix of the disturbances, $E(\varepsilon\varepsilon') = \Omega$, can be developed, and the Parks model can be estimated via generalized least squares (GLS).

To obtain consistent estimates of the elements of Ω, apply the ordinary least squares method to all (NxT) pooled observations and calculate the corresponding residuals e_{it}. From these residuals, obtain estimates of ρ_i by

$$\hat{\rho}_i = \frac{\sum_{t=2}^{T} e_{it} e_{i,\ t-1}}{\sum_{t=2}^{T} e_{i\ ,\ t-1}^{2}} \qquad (t = 2, \ldots, T) \qquad (4\text{-}12)$$

Next, using (4-12), transform the variables and form

$$Y^*_{it} = \beta^*_0 + \beta^*_1 X_{1t,1} + \ldots + \beta^*_k X^*_{it,\ k} + U^*_{it} \tag{4-13}$$

where

$$Y^*_{it} = Y_{it} - \hat{\rho}_i Y_{i,\ t-1}$$

$$X^*_{it,k} = X_{it,k} - \hat{\rho}_i X_{i,\ t-1,\ k} \quad (k = 1, \ldots, K)$$

$$\beta^*_0 = \beta_0(1 - \hat{\rho}_i)$$

$$U^*_{it} = \varepsilon_{it} - \hat{\rho}_i \varepsilon_{i,\ t-1}$$

$$t = 2, \ldots, T$$

$$i = 1, \ldots, N$$

The ordinary least squares method is applied to (4-13), and the residuals $\hat{U}^*_{it}$ are calculated. From these residuals, obtain estimates of γ_{ii}, γ_{ij}, ϕ_{ii}, and ϕ_{ij} by

$$\hat{\gamma}_{ii} = \frac{\hat{\phi}_{ii}}{1-\hat{\rho}_i^{\,2}}, \quad \hat{\gamma}_{ij} = \frac{\hat{\phi}_{ij}}{1-\hat{\rho}_i\hat{\rho}_j} \tag{4-14}$$

$$\hat{\phi}_{ii} = \frac{1}{T-K-1} \sum_{t=2}^{T} \hat{U}^{*2}_{it}, \quad \hat{\phi}_{ij} = \frac{1}{T-K-1} \sum_{t=2}^{T} \hat{U}^*_{it} \hat{U}^*_{jt} \tag{4-15}$$

$$i, j = 1, \ldots, N$$

Using this procedure, consistent estimates of ρ_i, γ_{ii}, γ_{ij}, and therefore of Ω are obtained. Finally, using the GLS estimation procedure,

$$\hat{\beta} = (X'\hat{\Omega}^{-1}X)^{-1}(X'\hat{\Omega}^{-1}Y), \tag{4-16}$$

where $\hat{\Omega}$ is a consistent estimator for Ω. Under general conditions, coefficients obtained in this manner have the same properties as the

Aitken estimator. The estimator, $\hat{\beta}$, has the properties of consistency, asymptotic unbiasedness, asymptotic efficiency, and asymptotic normality. The asymptotic variance-covariance matrix of $\hat{\beta}$ is given by

$$\text{Var-Cov} \quad (\hat{\beta})=(X'\hat{\Omega}^{-1}X)^{-1} \qquad (4\text{-}17)$$

Multicollinearity

Multicollinearity is the statistical problem of high correlation among the explanatory variables. The independent variables in a statistical model may not be causally related, but they may exhibit a similar trend or pattern over time or among cross-sectional observations. If the correlation coefficient between any pair of exogenous variables is ± 1, then perfect multicollinearity exists in the sample. However, perfect correlation between two independent variables is a sufficient but not a necessary condition for the presence of perfect multicollinearity when the number of independent variables exceeds two.[66/] In the case of perfect multicollinearity, estimation of the parameters in a statistical model is not possible. If no correlation exists among the exogenous variables, then no multicollinearity exists. However, multicollinearity is a question of degree, not of kind. The meaningful distinction is not between the presence or absence of multicollinearity but between its various degrees.[67/]

[66/] Ibid., p. 384.

[67/] Ibid., p. 380.

A recognition of the problems attributable to multicollinearity is worthwhile. The existence of multicollinearity presents some difficulties in testing hypotheses about parameters in statistical models. The estimators have the desirable properties of asymptotic efficiency, asymptotic unbiasedness, asymptotic normality, and consistency. However, "imprecise" parameter estimates arise due to large variances of the estimators. The sizes of the variances about the expected values of the estimators are greater than in the absence of multicollinearity. In this case, multicollinearity may lead to Type II errors. Test statistics are concluded not to be significant when in fact they may be. In addition, the inclusion or exclusion of an exogenous variable highly collinear with some other exogenous variables may affect the magnitude of the parameter estimates. The inclusion of an independent variable correlated with the previously included variables may change the size of both the estimated parameter and the estimate of its standard error. Similarly, the exclusion of a collinear variable from a statistical model may result in a specification error, causing the estimators of the remaining parameters to be biased. Finally, when multicollinearity occurs, each independent variable in the collinear set share in the explanatory role of any and all independent variables in the set. Consequently, interpretation difficulties with the parameter estimates exist.[68/]

[68/] J.L. Murphy, Introductory Econometrics, (Richard D. Irwin, Inc., Homewood, Illinois, 1973), pp. 368-369.

CHAPTER 5

STATISTICAL RESULTS AND ECONOMIC ANALYSES

This chapter deals with the statistical results and economic analyses in order to: (1) identify and assess the major factors that affect the demand relationships for gasoline and diesel fuel; and (2) determine differences in the demand relations for gasoline and diesel fuel in agricultural use. First, the estimated parameters of the empirical variables in the statistical models for gasoline and diesel fuel are presented. In addition, the statistical models are evaluated to determine how well they conform to the *a priori* assumptions and hypotheses stated in previous chapters. Second, to determine the degree and seriousness of correlation among the exogenous variables in both statistical models, a measure of multicollinearity is included. Third, economic analyses are performed on the estimated statistical models. Finally, a comparison of the estimated coefficients in the statistical models with previous studies is made.

The Estimated Demand Relationships

Parameter estimates of the statistical models were obtained by using generalized least squares (GLS) on the Parks model in combining cross-sectional and time-series data. The t-test was used to determine the significance of the parameters of the exogenous variables in the gasoline and diesel fuel demand relationships. The coefficient of determination, R^2, and the adjusted coefficient of determination, $\bar{R}^2$, were used to determine the amount of variation in the quantities of gasoline and diesel fuel purchased accounted for by the set of exogenous variables in the statistical models. Finally, the F-test was used to determine whether a significant amount of variation in the quantities of gasoline and diesel fuel purchased was accounted for by the statistical models. However, the t-tests, F-tests, R^2, and $\bar{R}^2$ were not strictly valid for the estimated demand relationships.[69/] Nevertheless, they may serve as useful approximations.

The estimated coefficients and standard errors of the gasoline and diesel fuel demand relationships are exhibited in Table 5.1 and Table 5.2. As mentioned previously, some experimentation was involved in the selection of the statistical models for gasoline and diesel fuel.[70/] The criteria for selection involved a subjective weighting of consistency of signs and magnitudes of the estimated parameters, compared to *a priori* reasoning and previous studies, statistical significance of the estimates, and explanatory power of the estimated relationships.

[69/] The estimated variances and standard errors of the estimated coefficients were estimates of asymptotic variances and standard errors. The algorithm developed by Drummond and Gallant used to obtain parameter estimates of the statistical models did not calculate R^2, $\bar{R}^2$, and F-values. These values were calculated by the researcher using the information provided by the algorithm. However, not enough information was available to calculate the exact R^2, $\bar{R}^2$, and F-values.

[70/] Two other statistical models for gasoline and diesel fuel were considered. The estimated coefficients and standard errors of

Table 5.1. The Estimated Coefficients and Standard Errors of the Gasoline Demand Relationship

Variable	Estimated Coefficient	Estimated Standard Error
Intercept	0.987060	1.0800
Acres (Cropland)	0.954426[a]	0.021757
Real Price of Output	0.516849[a]	0.048623
Real Price of Gasoline	-0.168451	0.26039
Real Price of Gasoline in Period t-5	-1.05951[a]	0.40088
Real Price of Diesel Fuel in Period t-5	0.785196[a]	0.25920
Real Price of Fertilizer	-0.443836[a]	0.14411
Interest Rate	-0.198895	0.19888
Real Price of Labor	0.990572[a]	0.29288
Ratio of Diesel Fuel Tractors to Gasoline Tractors	0.282943[a]	0.094320
Real Price of Land and Buildings	0.573520[a]	0.022086
Precipitation	0.118197[a]	0.037200

[a] Significant at .10 level

R^2 = .973

$\bar{R}^2$ = .971

DW = 2.213

F = 488.138 (significant at .10 level)

Table 5.2. The Estimated Coefficients and Standard Errors of the Diesel Fuel Demand Relationship

Variable	Estimated Coefficient	Estimated Standard Error
Intercept	-10.7893[a]	2.3719
Acres (Cropland)	1.51204[a]	0.10021
Real Price of Output	1.81707[a]	0.20630
Real Price of Diesel Fuel	0.288363	0.52946
Real Price of Diesel Fuel in Period t-3	-1.09013[a]	0.52289
Real Price of Gasoline in Period t-3	1.92170[a]	0.73696
Real Price of Fertilizer	-0.461010	0.50214
Interest Rate	-1.51714[a]	0.36776
Real Price of Labor	1.50450[a]	0.50726
Ratio of Diesel Fuel Tractors to Gasoline Tractors	0.889023[a]	0.19369
Real Price of Land and Buildings	0.910776[a]	0.15255
Precipitation	-0.0888608[a]	0.065843

[a] Significant at .10 level

R^2 = .841
$\bar{R}^2$ = .829
DW = 2.099
F =71.646 (significant at .10 level)

The coefficients of determination, R^2, were .973 and .841 respectively in the gasoline and diesel fuel demand relationships. The adjusted coefficients of determination, $\bar{R}^2$, were respectively .971 and .829. The level of significance chosen for the F-tests and the t-tests was the .10 level. Since the F-tests were statistically significant, the amount of variation in the quantities of gasoline and diesel fuel purchased accounted for by the set of exogenous variables was judged to be significantly different from zero. In the gasoline demand relationship, the estimated coefficient of the real price of gasoline in the current period and the estimated coefficient of the interest rate were judged to be not significantly different from zero. All other factors were statistically significant in accounting for the variation in the quantity of gasoline purchased. The sign of the estimated coefficient of the ratio of diesel fuel tractors to gasoline tractors was positive and thus inconsistent with the *a priori* hypotheses and assumptions previously stated. The signs of the remaining estimated coefficients, however, conformed to the *a priori* assumptions and hypotheses. In the diesel fuel demand relationship, the estimated coefficient of the real price of diesel fuel in the current period and the estimated coefficient of the real price of fertilizer were judged to be not significantly different from zero. All other factors were statistically significant in accounting for the variation in the quantity of diesel fuel purchased.

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the empirical variables in these statistical models are exhibited in Appendix C.

The sign of the estimated coefficient of precipitation was negative, and the sign of the estimated coefficient of the real price of diesel fuel in the current period was positive. The signs of these estimated coefficients were thus inconsistent with the *a priori* hypotheses and assumptions previously stated. The signs of the remaining estimated coefficients conformed to the *a priori* assumptions and hypotheses.

A Multicollinearity Measure

Given that some multicollinearity almost always exists and in light of the problems attributable to multicollinearity, a measure of its effect is in order. For this study, a measure of the extent of multicollinearity was obtained in the following manner:[71]

(i) calculate the incremental contribution of each exogenous variable in explaining the variation of the endogenous variable, given the inclusion of all other exogenous variables, by

$$\Theta_k = \frac{(1-R^2)t_k^{\,2}}{n-k-1} \tag{5-1}$$

where

t_k = the student's t-value for significance of the parameter β_k

k = the number of exogenous variables k = 1,..., K.

n = the number of observations

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[71] J.L. Murphy, *Introductory Econometrics*, (Richard D. Irwin, Inc., Homewood, Illinois, 1973), p. 375.

R^2 = the coefficient of determination

(ii) $\sum_{k=1}^{K} \Theta_k$, the sum of the $\Theta_{k's}$, $k = 1, \ldots, K$

(iii) compare (ii) to the collective contribution of all the variables simultaneously, (R^2).

The multicollinearity effect is denoted by $\bar{M}$ and is given by

$$\sum_{k=1}^{K} \Theta_K - R^2 = \bar{M} \tag{5-2}$$

The sign of $\bar{M}$ may be positive or negative. "Large" absolute values of $\bar{M}$ relative to the value of R^2 indicate severe multicollinearity. From Tables 5.3 and 5.4, the values of $\bar{M}$ in the gasoline and diesel fuel demand relationships were -.494 and -.431, respectively. The ratios of $\bar{M}$ to R^2 in absolute value were .508 and .512 respectively. It appears that the multicollinearity effects on the statistical models were not serious.

Economic Analyses

Since the double logarithmic mathematical form was used in estimating the gasoline and diesel fuel demand relationships, the estimated parameters of the empirical variables represent elasticities. By definition, in the statistical model $Y_{it} = \beta_0 + \beta_1 X_{it,\,1} + \ldots + \beta_k X_{it,\,k} + \varepsilon_{it}$, the elasticity of Y_{it} with respect to $X_{it,\,j}$ $(j = 1, \ldots, k)$ is given as $\eta = \frac{\partial Y_{it}}{\partial_{it,\,j}} \times \frac{X_{it,\,j}}{Y_{it}}$, the percentage change in Y_{it}, $\frac{\partial Y_{it}}{Y_{it}}$, relative to a percentage change in X, $\frac{\partial X_{it,\,j}}{X_{it,\,j}}$.

Table 5.3. A Measure of Multicollinearity in the Gasoline Demand Relationship

Variable	F-Value	Incremental Contribution (Θ_k)
LACRE	1924.3136	.3329062
LRPCR	112.9969	.0195484
LPDFLAG5	9.1766584	.0015875
LPGSLAG5	6.9849204	.0012083
LRPGS	0.4185184	.0000724
LRWPFR	9.485168	.0016409
LINTER	1.0002	.000173
LRPLABO	11.439276	.0019789
LTE	8.9988	.0015567
LRPLND	674.33702	.1166603
LPRECIP	10.095235	.0017464

$\Sigma\Theta_k = .479; \; R^2 = .973$

$\bar{M} = \Sigma\Theta_k - R^2 = -.494$

$\frac{|\bar{M}|}{R^2} = .508$

Table 5.4. A Measure of Multicollinearity in the Diesel Fuel Demand Relationship

Variable	F-Value	Incremental Contribution (Θ_k)
LACRE	227.67792	.2320493
LRPCR	77.57734	.0790668
LPDFLAG3	4.346391	.0044298
LRWPFR	0.8428892	.000859
LINTER	17.018925	.0173456
LRPLABO	8.7965628	.0089654
LTE	21.067182	.0214716
LRPLND	35.643288	.0363276
LPRECIP	1.8214201	.0018563
LPGSLAG3	6.7995777	.0069301
LRPDFL	0.2966327	.0003023

$\Sigma\Theta_k = .410; \; R^2 = .841$

$\bar{M} = \Sigma\Theta_k - R^2 = -.431$

$\frac{|\bar{M}|}{R^2} = .512$

The double logarithmic transformation corresponds to the assumption of a constant elasticity of Y_{it} with respect to $X_{it,\ j}$. The estimates of the elasticities should be interpreted with some degree of caution. The interpretation of any coefficient in a multiple regression equation assumes that ceterus paribus conditions hold with respect to all other empirical variables in the statistical model. In addition, the parameter estimates are applicable for the most part only within the range of data used in this study. Any projections outside the range of these data must be made with extreme circumspection. Nevertheless, the estimates of the elasticities are very useful in providing insights as to the relative responsiveness of farmers to relative price changes and changes in certain other measurable variables.

Gasoline and diesel fuel purchases in the current period were not influenced by changes in the real price of gasoline and diesel fuel. It appears that a change in the real price of gasoline and diesel fuel does not immediately induce farmers to alter decisions once made because of the capital costs and production costs involved. On the other hand, in the planning stage, the real price of the different fuels had an effect on the decision-making process. The aggregate market response of farmers to a one percent increase (decrease) in the real price of gasoline was a 1.05951 percent decrease (increase) in the quantity of gasoline purchased in time period t+5, an elastic response of farmers to changes in the real price of gasoline. The aggregate market response of farmers to a one percent decrease (increase) in the real price of diesel fuel was a 0.785196 percent decrease (increase) in the quantity of gasoline purchased in time period t+5. The adjustment period required for farmers to generate responses of these magnitudes was fifteen months (5 quarters) which apparently reflects changes in management practices. A major change in management practices appears to be the substitution of diesel fuel tractors for gasoline tractors. The aggregate market response of farmers to a one percent increase (decrease) in the real price of diesel fuel was a 1.09013 percent decrease (increase) in the quantity of diesel fuel purchased in time period t+3, an elastic response of farmers to changes in the real price of diesel fuel. The aggregate market response of farmers to a one percent increase (decrease) in the real price of gasoline was a 1.92170 percent increase (decrease) in the quantity of diesel fuel purchased in time period t+3. The adjustment period required for farmers to generate responses of these magnitudes was nine months (3 quarters) which also probably reflects changes in management practices.

A one percent increase (decrease) in the real price of output generated a 0.516849 percent increase (decrease) in the quantity of gasoline purchased and a 1.81707 percent increase (decrease) in the quantity of diesel fuel purchased. Since the real price of output had a statistically significant positive influence upon the quantities of gasoline and diesel fuel purchased, these fossil fuels, as expected, were not inferior inputs.

A one percent increase (decrease) in the real price of fertilizer generated a 0.443836 percent decrease (increase) in the quantity of gasoline purchased. Since the real price of fertilizer had a statistically significant negative effect upon the quantity of gasoline purchased, fertilizer was a complementary factor to gasoline in agricultural use. The real price of fertilizer was not a statistically significant factor in accounting for the variation in the quantity of diesel fuel purchased, suggesting inexplicably that fertilizer and diesel fuel were independent factors in agricultural use.

A one percent increase (decrease) in the real price of labor was associated with a 0.990572 percent increase (decrease) in the quantity of gasoline purchased and a 1.50450 percent increase (decrease) in the quantity of diesel fuel purchased. Since the real price of labor had a statistically significant positive influence upon the quantities of gasoline and diesel fuel purchased, labor was a substitute for gasoline and diesel fuel in agricultural use.

A one percent increase (decrease) in the real price of land and buildings was associated with a 0.573520 percent increase (decrease) in the quantity of gasoline purchased and a 0.910776 percent increase (decrease) in the quantity of diesel fuel purchased. Since the real price of land and buildings had a statistically significant positive effect upon the quantities of gasoline and diesel fuel purchased, land was a substitute for gasoline and diesel fuel in agricultural use.

A one percent increase (decrease) in the number of cropland acres generated a 0.954426 percent increase (decrease) in the quantity of gasoline purchased and a 1.51204 percent increase (decrease) in the quantity of diesel fuel purchased. It appears that if farmers increase their scale of operation, then their purchases of gasoline and diesel fuel for agricultural use increase.

A one percent increase (decrease) in the interest rate charged farmers by Production Credit Associations generated a 1.51714 percent decrease (increase) in the quantity of diesel fuel purchased. The interest rate was not a statistically significant factor in accounting for the variation in the quantity of gasoline purchased. A partial explanation for this result may lie in the fact that a change from gasoline to diesel fuel engines in tractors and combines involves a major capital expenditure on the part of the farmer. It appears that farmers are more willing to substitute diesel fuel tractors and combines for gasoline tractors and combines when interest rates are relatively low than when interest rates are relatively high.

A one percent increase (decrease) in precipitation was associated with a 0.118197 percent increase (decrease) in the quantity of gasoline purchased. Similarly, a one percent increase (decrease) in precipitation generated a 0.888608 percent decrease (increase) in the quantity of diesel fuel purchased, inconsistent with the a priori hypothesis stated previously.

A one percent increase (decrease) in the ratio of diesel fuel tractors to gasoline tractors generated a 0.889023 percent increase (decrease) in the quantity of diesel fuel purchased. Similarly, a one percent increase (decrease) in the ratio of diesel fuel tractors to gasoline tractors generated a 0.282943 percent increase (decrease) in the quantity of gasoline purchased, inconsistent with the a priori hypothesis stated previously. A partial explanation for this inconsistency may lie in the fact that farmers use gasoline in their trucks, combines, and automobiles as well as in their tractors.

In sum, with few exceptions, the estimates of the elasticities in the diesel fuel demand relationship exceeded the estimates of the elasticities in the gasoline demand relationship. In addition, in terms of magnitude of response and elasticities, the real price of gasoline in period t-3, the real price of output, the interest rate, the number of cropland acres, the real price of labor, the real price of diesel fuel in period t-3, the real price of land and buildings, the ratio of diesel fuel tractors to gasoline tractors, and precipitation were the statistically significant factors accounting for the variation in the quantity of diesel fuel purchased. Similarly, in terms of magnitude of response and elasticities, the real price of gasoline in period t-5, the real price of labor, the number of cropland acres, the real price of diesel fuel in period t-5, the real price of land and buildings, the real price of output, the real price of fertilizer, the ratio of diesel tractors to gasoline tractors, and precipitation were the statistically significant factors accounting for the variation in the quantity of gasoline purchased.

Comparison With Previous Studies

As previously mentioned, there is little information dealing specifically with the demand relationships for gasoline and diesel fuel in the agricultural sector. However, several studies have dealt with the demand for fossil fuels in the residential, commercial, private, and industrial sectors. To point out differences in the demands for various fossil fuels among the private, residential, commercial, industrial, and agricultural sectors, comparisons of

Table 5.5. A Comparison of Estimated Price Elasticities for Various Fossil Fuels Among the Private, Residential, Commercial, Industrial, and Agricultural Sectors

Fossil Fuel	Private Sector	Residential Sector	Commercial Sector	Industrial Sector	Agricultural Sector
Diesel Fuel					-1.09[e]
Gasoline	-.075[b], -.24[b], -.70[c]		-2.8[c]		-1.06[e]
Natural Gas		-.63[a]	- .63[a]	-2.53[d]	
Residual and Distillate Fuel Oil				- .65[d]	

[a] Balestra and Nerlove

[b] Houthakker, Verleger, and Sheehan

[c] Ramsey, Rasche, and Allen

[d] Erickson, Spann, and Ciliano

[e] Capps

estimated price elasticities of demand for gasoline, diesel fuel, natural gas, and residual and distillate fuel oil are made (Table 5.5).

Balestra and Nerlove estimated the parameters of a single-equation dynamic demand function for natural gas in the residential and commercial sectors. The estimation technique, using pooled time-series data from different cross-sections, was the error component procedure. The long-run price elasticity of demand for natural gas in these respective sectors was -.63.

Houthakker, Verleger, and Sheehan presented a quarterly dynamic demand analysis for gasoline in the private sector. The single-equation dynamic demand function was estimated using pooled time-series data from different states without any attempt to differentiate by regions. Parameter estimates were obtained by employing the error component technique used by Balestra and Nerlove. The short-run and long-run price elasticities of demand for gasoline were respectively, -.075 and -.24.

Ramsey, Rasche, and Allen investigated the private and commercial demand relationships for gasoline. The approach was to formulate a static model in terms of a simultaneous equation system. Small relative changes between gasoline and diesel fuel prices had pronounced effects on the demand for gasoline in these sectors. The price elasticities of demand in the private sector and in the commercial sector were respectively -.70 and -2.8.

Erickson, Spann, and Ciliano presented an economic analysis of substitution and usage effects on industrial sector energy demands. The industrial energy factors included residual and distillate fuel oil and natural gas. The approach was to formulate a static model using a single equation for each industrial energy demand relation. All data used in the analysis were pooled time-series, cross-sectional values taken from the period 1960 through 1970. The model was estimated by ordinary least squares. The price elasticities of demand for residual and distillate fuel oil and natural gas were respectively -.65 and -2.53.

In this study, demand relationships for gasoline and diesel fuel in agricultural use for Virginia were investigated. All data used were pooled quarterly, time-series, cross-sectional observations taken from the period 1971 through 1976. Parameter estimates were obtained by using generalized least squares (GLS) on the Parks model in combining cross-sectional and time-series data. The own price elasticity of demand for gasoline and diesel fuel were respectively, -1.06 and -1.09. The adjustment periods required for farmers to generate responses of these magnitudes were fifteen months and nine months, respectively.

In most cases, single-equation functions were estimated using pooled time-series data from different cross-sections. However, different estimation techniques were employed. In addition, with the exception of this study, all the price elasticities of demand dealt with the current time period.

The demand for gasoline is elastic in the commercial and agricultural sectors but inelastic in the private sector. The demand for natural gas is inelastic in the residential and commercial sectors but elastic in the industrial sector. Further, the demand for diesel fuel in the agricultural sector is elastic, but the demand for residual and distillate fuel oil is inelastic in the industrial sector. In short, differences appear to exist in the demands for various fossil fuels among the private, residential, commercial, industrial, and agricultural sectors.

CHAPTER 6

SUMMARY AND CONCLUSIONS

The objectives of this study were: (1) to determine the usage patterns of gasoline, diesel fuel and other fossil fuels in different types of agriculture; (2) to identify and assess the major factors that affect the demand relationships for gasoline and diesel fuel; and (3) to determine differences in the demand relations for gasoline and diesel fuel in agricultural use. A summary of the methodology, statistical procedures, and statistical results of the previous chapters is presented. Further, conclusions and implications based on an analysis of the statistical results are given. Finally, suggestions for further research are discussed.

Summary

The usage patterns of gasoline, diesel fuel, and other fossil fuels in different types of agriculture were developed from cross-sectional data provided by the Economic Research Service. Under a jointly funded cooperative agreement with the Federal Energy Administration, estimates of the use of fossil energy by crops and livestock for all fifty states were developed. In Virginia, the South, and the U.S., the prominent users of fossil fuels in agricultural production are crops. In livestock production, beef, dairy, and hog enterprises are the major users of gasoline in Virginia, the South, and the U.S., while beef and hog enterprises are the major users of diesel fuel. Poultry enterprises dominate the use of fuel oil, LP gas, natural gas, and coal in Virginia, Southern, and U.S. livestock production. In Virginia crop production, corn, flue-cured tobacco, soybeans, hay, peanuts, winter wheat, and apples are the major users of gasoline, diesel fuel, fuel oil, LP gas, and natural gas. In Southern and U.S. crop production, soybeans, corn, cotton, flue-cured tobacco, winter wheat, hay, oranges, rice, and grain sorghum use relatively large portions of these five fossil fuels. Finally, on a per acre basis, in Virginia, tobacco, apples, vegetables, peaches, and peanuts require the most gasoline, diesel fuel, and LP gas. In the South and the U.S., citrus fruits, apples, and tobacco use the most gasoline, diesel fuel, and LP gas per acre.

From the static theoretical development and the dynamic and technological adjustment concepts of derived input demand, the economic models for gasoline and diesel fuel were developed. The economic models were translated into statistical models and numerical estimates of the parameters of the empirical variables were obtained. The estimated statistical models provided a basis for conclusions and inferences about the economic models. All data used were pooled time-series, cross-sectional observations taken from the period 1971 through 1976. The disturbance terms in the statistical models were assumed to follow a different first-order autoregressive scheme for each cross-section and were specified as cross-sectionally heteroscedastic and mutually correlated. A generalized least squares (GLS) procedure was employed to obtain asymptotically efficient, asymptotically normal, asymptotically unbiased, and consistent estimates of the parameters. Some experimentation was involved in the selection of the statistical models for gasoline and diesel fuel. The criteria for selection involved a subjective weighting of consistency of signs and magnitudes of the estimated parameters, compared to *a priori* reasoning and previous studies, statistical significance of the estimates, and explanatory power of the estimated relationships.

Approximately 97 percent of the variation in the quantity of gasoline purchased and 84 percent of the variation in the quantity of diesel fuel purchased was accounted for by the set of exogenous variables. The F-tests were statistically significant at the .10 level, and with few exceptions, the signs of the estimated coefficients conformed to the *a priori* hypotheses and assumptions previously stated. The estimated coefficients of the empirical variables were, in most cases, judged to be significantly different from zero at the .10 level.

Conclusions and Implications

The estimates of the elasticities in the diesel fuel demand relationship exceeded the estimates of the elasticities in the gasoline demand relationship. The quantity of diesel fuel purchased appears to be more sensitive to changes in economic factors and other variables than the quantity of gasoline purchased. In terms of magnitude of response and elasticities, the real price of gasoline in period t-5, the real price of labor, the number of cropland acres, the real price of diesel fuel in period t-5, the real price of land and buildings, the real price of output, the real price of fertilizer, the ratio of diesel fuel tractors to gasoline tractors, and precipitation were the statistically significant factors accounting for the variation in the quantity of gasoline purchased. The estimated coefficient of the real price of gasoline in the current period and the estimated coefficient of the interest rate charged farmers by Production Credit Associations were judged to be not significantly different from zero. Similarly, in terms of magnitude of response and elasticities, the real price of gasoline in period t-3, the real price of output, the interest rate, the number of cropland acres, the real price of labor, the real price of diesel fuel in period t-3, the real price of land and buildings, the ratio of diesel fuel tractors to gasoline tractors, and precipitation were the statistically significant factors accounting for the variation in the quantity of diesel fuel purchased. The estimated coefficient of the real price of diesel fuel in the current period and the estimated coefficient of the real price of fertilizer were judged to be not statistically different from zero.

Dvoskin and Heady and Lehrmann, Black, and Connor argued that the use of energy inputs was not responsive to energy price increases. It appears that a change in the real price of gasoline and diesel fuel does not immediately induce farmers to alter the decisions once made because of the capital costs and production costs involved. However, while farmers appear to be somewhat passive to increases in the real prices of gasoline and diesel fuel in the current period, they are quite responsive to such increases when given time to adjust their usage patterns. A one percent increase in the real price of gasoline and diesel fuel, ceterus paribus, generated a 0,274314 percent decrease in the quantity of gasoline purchased in time period t+5 and a 0.83157 percent increase in the quantity of diesel fuel purchased in time period t+3. The adjustment periods required for farmers to generate responses of these magnitudes were nine months and fifteen months, respectively, which probably reflect changes in management practices. A major change in management practices appears to be the substitution of diesel fuel tractors for gasoline tractors. All data used in the analysis were taken from the period 1971 to 1976, a period of transition for gasoline and diesel fuel tractors. Once the shift from gasoline tractors to diesel fuel tractors has been completely made, the above results and conclusions may not hold. Nevertheless, the allocation of gasoline and diesel fuel for agricultural use may be accomplished through market forces only after a period of nine to fifteen months, an important implication for public policy.

A potential increase in the nominal price of gasoline by as much as 50 cents per gallon over a period of ten years was suggested by President Carter. Assuming that Carter's proposal results in a 2 to 4 percent increase in the real price of gasoline and diesel fuel per year for the next ten years, ceterus paribus, by 1987, the quantity of gasoline purchased for agricultural use may decrease by 5 to 10 percent, and the quantity of diesel fuel purchased for agricultural use may increase by 15 to 30 percent. Doubling the real prices of these fuels, all other factors nonvariant, may generate a 27 percent decrease in the quantity of gasoline purchased in fifteen months and a 83 percent increase in the quantity of diesel fuel purchased in nine months. In short, producers and distributors of fossil energy may change future production and distribution levels of gasoline and diesel fuel for the agricultural sector when changes in the real prices of these fuels occur.

A one percent increase in the real price of output, all other factors nonvariant, generated a .516849 percent increase in the quantity of gasoline purchased and a 1.81707 percent increase in the quantity of diesel fuel purchased. It appears that supporting the price for farm output above the market clearing price augments the quantities of gasoline and diesel fuel purchased for agricultural use. In short, price support programs for farm products appear to be in conflict with energy conservation programs.

A one percent increase in the real price of fertilizer, the real price of labor, the real price of land and buildings, the interest rate charged farmers, precipitation, cropland acreage, and the ratio of diesel fuel tractors to gasoline tractors, ceterus paribus, generated a 2.3 percent increase in the quantity of gasoline purchased and a 2.7 percent increase in the quantity of diesel fuel purchased. It appears that changes in factors other than real energy prices also affect the quantities of gasoline and diesel fuel purchased. In addition, it appears that changes in cropland acreage, in real factor prices, and in the interest rate charged farmers have a greater influence on the quantities of gasoline and diesel fuel purchased than a change in the ratio of diesel fuel tractors to gasoline tractors.

In sum, the estimated statistical models provided a basis for conclusions and implications for policy makers, for fossil energy producers and distributors, and for farmers. The conclusions and implications derived were inevitably limited by the choice of the mathematical form and the choice of the empirical variables. Notwithstanding, the agricultural sector in Virginia appears to adjust to changes in economic factors and other variables influencing the demand for gasoline and diesel fuel.

Suggestions for Further Research

Since it appears that real price changes affect the quantities of gasoline and diesel fuel purchased for agricultural use, the effects in different types of agriculture merit investigation. The focus in this study was on the demand relationships for gasoline and diesel fuel primarily in agricultural production in Virginia. Estimation of the demand relationships for fossil energy in food processing and distribution in Virginia and for fossil energy in the agricultural sector in other regions of the U.S. may be worthwhile. In addition, it may be of interest to incorporate the behavioral characteristics of the management factor in the gasoline and diesel fuel demand relationships. Finally, the forecasting of gasoline and diesel fuel consumption in the agricultural sector in Virginia warrants attention.

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APPENDICES

APPENDIX A

THE SOUTHERN STATES COOPERATIVE SYSTEM[1/]

Southern States is a family of cooperative associations owned and controlled by thousands of farmers in Delaware, Kentucky, Maryland, Virginia, and West Virginia. The family includes Southern States Cooperative, Inc., a subsidiary corporation, affiliated local cooperative associations, privately-owned franchised retail service agencies, independent local cooperatives, departmentalized retail branches, and others. Southern States performs two basic functions: a purchasing function and a marketing function.

With respect to petroleum, Southern States provides on-the-farm delivery of gasoline, kerosene, heating oils, motor oils, diesel fuel, and at some points, LP gas. The organization does not attempt to be a price pace-setter on petroleum. The policy is to price fuel competitively with the "going" prices in the industry. Southern States has an important affiliation in petroleum as a result of the part ownership of a refinery at Texas City, Texas. This refinery has a capacity of approximately 80,000 barrels per day. In addition, Southern States is a partner in some crude oil exploration efforts through the LVO International Corporation. The petroleum distribution

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[1/] Southern States Cooperative, "Beyond Their Line Fences", Richmond, Virginia, 1971. F.M. Armbrecht, J.H. Buchholz, and P.E. Mullinix, "The Southern States Family", Richmond, Virginia, May, 1976.

of Southern States is made up principally of (1) local cooperative associations, many of which are affiliated with Southern States through a Management Agreement; (2) private service agencies, franchised by Southern States to serve farmers in their area; and (3) retail branches operating on a departmentalized basis.

Table A.1. Agency and Regional Location of Wholesale Handlers of Gasoline, Diesel Fuel, Heating Oil, and LP Gas in the Southern States Cooperative System in Virginia, 1970-1976

Agency	Regional Location	Type of Agency	Fuel Type
Amelia Cooperative	5	LC	G, DF, HO
Amelia Lumber Company	5	POA	DF
Augusta Cooperative Farm Bureau	4	POA	G
Augusta Petroleum	4	LC	G, DF, HO
Bedford Cooperative	5	LC	G, DF, HO
Bridgewater Home	2	POA	HO
Brookneal Cooperative	5	LC	G, DF, HO
Burkeville Service	8	RB	G, DF, HO
Carlton R. Brooks, Inc.	5	POA	G, DF, HO
Chesapeake Cooperative	9	LC	G
Chesapeake Fertilizer	9	RB	G, DF, HO
Chesapeake Petroleum Service	9	RB	G, DF, HO
Cities Service Oil Company	5	POA	DF, HO
Cooperative Mills Richmond	5	POA	HO
Culpeper Petroleum Service	2	LC	G, DF, HO
Dayton Transport Company	2	POA	DF, HO
E. Garland Payne	6	POA	G
Fairfax Petroleum Service	2	RB	G, DF, HO
Fairfax Petroleum Service - Woodbridge	2	RB	G, DF, HO
Fredericksburg Petroleum Service	5	RB	G, DF, HO
Fredericksburg Petroleum Service - King George	6	RB	G, DF, HO
Front Royal Cooperative	2	LC	LP
Gloucester Service	6	RB	G
Graham Oil and Heating Service	7	POA	G, DF, HO
Grain Marketing Richmond	5	POA	HO
Hubbard's Service	4	POA	G

Table A.1. Continued

Agency	Regional Location	Type of Agency	Fuel Type
Leesburg Cooperative	2	LC	G
Leesburg Petroleum Service	2	RB	G, DF, HO
Luray Service	2	RB	LP
Marion Cooperative	7	LC	G, DF, HO
Mays Farmer Service	5	POA	G, DF, HO
Messick Gas, Inc.	6	POA	G, HO
Nansemond Cooperative Association	9	POA	G
P.L. Duncan and Sons, Inc.	5	POA	DF, HO
R.E. Charlton, Jr.	5	POA	DF, HO
Roanoke Cooperative	4	LC	G, DF, HO
Roanoke Cooperative-Troutville	4	LC	G, DF, HO
Rockbridge Farmers Cooperative	4	POA	G, DF, HO
Rockingham Cooperative Farm Bureau	2	POA	G, DF, HO
Rockingham Petroleum Cooperative	2	LC	G. DF, HO
Rockingham Petroleum Cooperative - Marathon	2	LC	G
Rockingham Petroleum Cooperative - Timberville	2	LC	G, DF, HO
Rockingham Poultry Marketing Cooperative	2	POA	HO
Russell County Cooperative	7	POA	G, DF, HO
Scott Pallets, Inc.	5	POA	DF
Seay Milling and Machinery Company	5	POA	G, DF, HO
South Boston Petroleum Service	8	RB	G, DF, HO
Sterling Supply	2	POA	G
Tappahannock Service	6	RB	G, DF, HO

Table A.1. Continued

Agency	Regional Location	Type of Agency	Fuel Type
Tidewater Petroleum Cooperative	9	LC	G, DF, HO, LP
Tricounty Petroleum Service-Glen Allen	5	RB	G, DF, HO
Tricounty Petroleum Service-South Richmond	5	RB	G, DF, HO
Washington Farmers Cooperative	7	LC	G, DF, HO
Watts Petroleum Service, Inc.	6	POA	G, DF, HO
Whitsel Brothers, Inc.	2	POA	HO
Winchester Cooperative	2	LC	G, DF, HO

KEY

LC - Local Cooperative
RB - Retail Branch
POA - Privately Owned Agency or Local Cooperatives Without Management Contracts
G - Gasoline
DF - Diesel Fuel
LP - LP Gas
HO - Heating Oil (Kerosene and #2 Fuel Oil)

Source: Retail Operations Department, Southern States Cooperative, Inc.

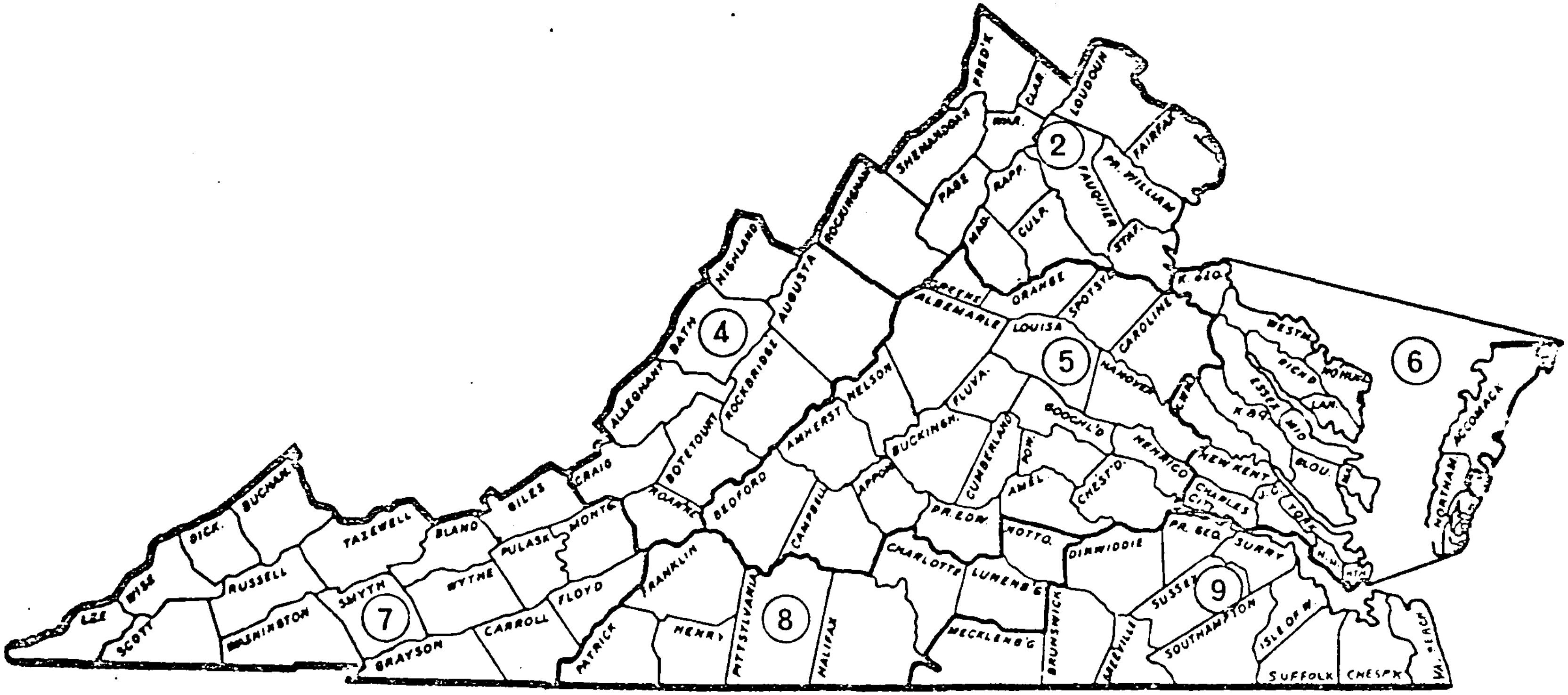

Figure A.1. Virginia Crop Reporting Districts

APPENDIX B

MATHEMATICAL FORMULATION OF THE STATIC THEORY OF DERIVED INPUT DEMAND

Limited Capital

The technical relationships between the input and the outputs of the firm may be expressed in implicit form by a homogeneous production function of degree k.

$$F(tx_1,\ldots, tx_n) = t^k F(x_1,\ldots, x_n) = 0 \tag{1}$$

where k is a constant, t is any positive real number, and the x_i's, i=1,..., n, represent the (1,..., h) independent output variables and the (h+1,..., n) independent input variables.

The net revenue (profit), the difference between the total revenue from the sale of all h outputs and the expenditure upon all n-h inputs, for the firm is given by

$$\Pi = \sum_{i=1}^{h} p_i x_i - \sum_{i=h+1}^{n} p_i x_i \tag{2}$$

where the p_i's, i=1,..., n, represent the (1,..., h) output prices and the (h+1,..., n) input prices. By assumption, the firm maximizes net revenue subject to a production function and a capital constraint. The production function is given by (1), and the capital constraint is given by

$$C - S - \sum_{i=h+1}^{n} p_i x_i = 0^{1/}, \quad C \geq 0 \tag{3}$$

[1/] Raymond Daniel, "Econometric Analysis of the Farmer Demand for Fertilizer Nutrients", unpublished Ph.D. dissertation, Purdue University, 1970, p. 166.

where C is the initial capital outlay of the firm, S is a slack variable, and p_ix_i, i=h+1,..., n represents the input costs to the firm. Solving for S in (3),

$$S = C - \sum_{i=h+1}^{n} p_ix_i \tag{4}$$

The slack variable (S) represents the difference between the initital capital outlay of the firm and the expenditure upon all n-h inputs.

In order to mathematically solve for a maximum net revenue subject to (1) and (3), form the Lagrangian expression

$$Z = \sum_{i=1}^{h} p_ix_i - \sum_{i=h+1}^{n} p_ix_i + \lambda_1F(x_1, x_2, \ldots, x_n) + \lambda_2(C-S-\sum_{i=h+1}^{n} p_ix_i) \tag{5}$$

where λ_1 and λ_2 are Lagrangian multipliers. Including S in (5) permits the firm to maximize profit without exhausting its initial capital outlay. If the firm reaches a net revenue maximization point before draining its capital outlay, then the capital constraint is no longer binding.

Differentiating Z in (5) with respect to x_i, i=1,..., n, λ_1, and λ_2, the necessary or first-order conditions for a firm to maximize net revenue subject to a production function and a capital constraint are given by

$$\begin{array}{llll} \frac{\partial Z}{\partial x_1} = p_1 + \lambda_1F_1 & = 0 & & \\ \vdots \qquad \vdots & & \text{h equations} & \\ \frac{\partial Z}{\partial x_n} = p_h + \lambda_1F_n & = 0 & & \end{array} \tag{6}$$

$$\begin{array}{ll}
\frac{\partial Z}{\partial x_{h+1}} = -p_{h+1} + \lambda_1 F_{h+1} - \lambda_2 p_{h+1} = 0 & \\
\vdots \qquad\qquad \vdots & \text{n-h equations} \\
\frac{\partial Z}{\partial x_n} = -p_n + \lambda_1 F_n - \lambda_2 p_n = 0 & \\
& \qquad\qquad (6) \\
\frac{\partial Z}{\partial \lambda_1} = F(x_1, x_2, \ldots, x_n) = 0 & \\
\frac{\partial Z}{\partial \lambda_2} = C - S - \sum_{i=h+1}^{n} p_i x_i = 0 &
\end{array}$$

The second Lagrangian, λ_2, does not occur in the first-order conditions for net revenue maximization with respect to the independent output variables x_i, i=1,..., h. The capital outlay constraint C does not affect total revenue $\sum_{i=1}^{h} p_i x_i$. The F_i (i=1,..., h, h+1, ..., n) is the partial derivative of (1) with respect to its ith argument. In short, the necessary conditions for profit maximization subject to two constraints involve n+2 partial derivatives and n+2 equations.

Select any two of the first h equations of (6). Then,

$$\frac{P_e}{P_f} = \frac{F_e}{F_f} = \frac{\partial x_f}{\partial x_e} = -\lambda_1 \qquad (7)$$

where e and f refer to outputs. The rate of product transformation, RPT for every pair of outputs, holding the levels of all other outputs and all inputs constant, must equal the ratio of their prices. In addition, from (7), λ_1=-RPT for every pair of outputs.

Select any two of the n-h equations of (6). Then,

$$\frac{P_k}{P_\ell} = \frac{F_k}{F_\ell} = \frac{\partial x_\ell}{\partial x_k} = \frac{\lambda_1}{1+\lambda_2} \tag{8}$$

where k and ℓ refer to inputs. The rate of technical substitution, RTS, for every pair of inputs, holding the levels of all outputs and all other inputs constant, must equal the ratio of their prices. In addition, from (7) and (8), $\lambda_2 = -(1+\frac{RPT}{RTS})$ for every pair of inputs and outputs.

Select one equation from the first h equations of (6) and one equation from the n-h equations of (6). Then,

$$\frac{P_e}{P_k(\lambda_2+1)} = \frac{F_e}{F_k} = \frac{\partial x_k}{\partial x_e} = -\lambda_1 \tag{9}$$

or

$$P_k(1+\lambda_2) = P_e \frac{\partial x_e}{\partial x_k} \tag{10}$$

where e refers to outputs and k refers to inputs. The value of the marginal product of each input with respect to the relevant output, VMP, is equal to the input price times $(1+\lambda_2)$.

The second-order or sufficient conditions for the maximization of net revenue subject to a production function and a capital constraint require that the quadratic form of second-order partials,

$$\sum_{i=1}^{n} \sum_{j=1}^{n} f_{ij}\, dx_i\, dx_j$$

where

$$f(x_1, x_2, \ldots, x_n) = \sum_{i=1}^{h} p_i x_i - \sum_{i=h+1}^{n} p_i x_i,$$ be negative for a maximum for all nontrivial sets of values of the dx_i's that satisfy

$$F_1 dx_1 + \ldots + F_n dx_n = 0 \qquad (11)$$

Form the determinants

$$\begin{vmatrix} Z_{11} & Z_{12} & F_1 \\ Z_{21} & Z_{22} & F_2 \\ F_1 & F_2 & 0 \end{vmatrix}, \quad \begin{vmatrix} Z_{11} & Z_{12} & Z_{13} & F_1 \\ Z_{21} & Z_{22} & Z_{23} & F_2 \\ Z_{31} & Z_{32} & Z_{33} & F_3 \\ F_1 & F_2 & F_3 & 0 \end{vmatrix}, \ldots \begin{vmatrix} Z_{11} & \cdots & Z_{1n} & F_1 \\ \vdots & & & \\ Z_{n1} & \cdots & Z_{nn} & F_n \\ F_1 & \cdots & F_n & 0 \end{vmatrix}$$

$$Z_{ij} = \frac{\partial^2 Z}{\partial x_i \partial x_j} \qquad i, j = 1, \ldots, n. \qquad (12)$$

by bordering the principal minors of the Hessian determinant of second partial derivatives of Z by a row and a column containing the first partial derivatives of the production function constraint. The element in the southeast corner of each of these arrays is zero.

Simply put, the second-order conditions for a constrained maximum are satisfied if these bordered determinants alternate in sign, starting with plus. If

$$(-1)^m \begin{vmatrix} Z_{11} & \cdots & Z_{1m} & F_1 \\ \vdots & & & \\ Z_{m1} & \cdots & Z_{mm} & F_m \\ F_1 & \cdots & F_m & 0 \end{vmatrix} > 0 \qquad (13)$$

where m = (2,..., n), then the sufficient conditions for the constrained maximization of net revenue are satisfied.

If the necessary and sufficient conditions for profit maximization subject to the constraints of a production function and capital hold, then the quantity of each input the firm uses and the quantity of each output the firm produces may be expressed uniquely in terms of the input and output prices and the level of capital by solving the system of n+2 equations in (6) simultaneously. In mathematical terms,

$$X_i = g_i(p_1, \ldots, p_n, C) \qquad (14)$$

where (14) represents a supply function for the independent output variables (i = 1,..., h) or a demand function for the independent input variables (i = h+1,..., n).[2/]

The decision unit of the firm, by assumption, makes rational choices, consistent choices, and possesses perfect knowledge. In order to maximize profit, the goal of the firm, the decision unit responds to changes in input and output prices by varying input and output levels to satisfy (6). By differentiating (6) with respect to $p_1, \ldots, p_n$, and C one at a time with each of the other variables held constant, the theoretical nature of the relationships in (14) may be examined. By assumption, the firm may make instantaneous adjustments to changes in prices, and complete divisibility of inputs and outputs exists.

[2/] Raymond Daniel, "Econometric Analysis of the Farmer Demand for Fertilizer Nutrients", unpublished Ph.D. dissertation, Purdue University, 1970, p. 170.

Let

$$L = \begin{bmatrix} \lambda_1 F_{11} & \cdots & \lambda_1 F_{1h} & \lambda_1 F_{1,h+1} & \cdots & \lambda_1 F_{1n} & F_1 & 0 \\ \vdots & & \vdots & \vdots & & \vdots & \vdots & \\ \lambda_1 F_{h1} & \cdots & \lambda_1 F_{hh} & \lambda_1 F_{h,h+1} & \cdots & \lambda_1 F_{hn} & F_h & 0 \\ \lambda_1 F_{h+1,1} & \cdots & \lambda_1 F_{h+1,h} & \lambda_1 F_{h+1,h+1} & \cdots & \lambda_1 F_{h+1,n} & F_{h+1} & -p_{h+1} \\ \vdots & & \vdots & \vdots & & \vdots & \vdots & \\ \lambda_1 F_{n1} & \cdots & \lambda_1 F_{nh} & \lambda_1 F_{n,h+1} & \cdots & \lambda_1 F_{nn} & F_n & -p_n \\ F_1 & \cdots & F_h & F_{h+1} & \cdots & F_n & 0 & 0 \\ 0 & \cdots & 0 & p_{h+1} & \cdots & -p_n & 0 & 0 \end{bmatrix}_{(n+2)\times(n+2)}$$

(15)

Let

$$M = \begin{bmatrix}
\frac{\partial x_1}{\partial p_1} & \cdots & \frac{\partial x_1}{\partial p_h} & \frac{\partial x_1}{\partial p_{h+1}} & \cdots & \frac{\partial x_1}{\partial p_n} & 0 \\
\vdots & & \vdots & \vdots & & \vdots & \\
\frac{\partial x_h}{\partial p_1} & \cdots & \frac{\partial x_h}{\partial p_h} & \frac{\partial x_h}{\partial p_{h+1}} & \cdots & \frac{\partial x_h}{\partial p_n} & 0 \\
\frac{\partial x_{h+1}}{\partial p_1} & \cdots & \frac{\partial x_{h+1}}{\partial p_h} & \frac{\partial x_{h+1}}{\partial p_{h+1}} & \cdots & \frac{\partial x_{h+1}}{\partial p_n} & \frac{\partial x_{h+1}}{\partial C} \\
\vdots & & \vdots & \vdots & & \vdots & \vdots \\
\frac{\partial x_n}{\partial p_1} & \cdots & \frac{\partial x_n}{\partial p_h} & \frac{\partial x_n}{\partial p_{h+1}} & \cdots & \frac{\partial x_n}{\partial p_n} & \frac{\partial x_n}{\partial C} \\
\frac{\partial \lambda_1}{\partial p_1} & \cdots & \frac{\partial \lambda_1}{\partial p_h} & \frac{\partial \lambda_1}{\partial p_{h+1}} & \cdots & \frac{\partial \lambda_1}{\partial p_n} & 0 \\
0 & \cdots & 0 & \frac{\partial \lambda_2}{\partial p_{h+1}} & \cdots & \frac{\partial \lambda_2}{\partial p_n} & 0
\end{bmatrix}_{(n+2)x(n+1)} \qquad (16)$$

Let

$$N = -\begin{bmatrix} 1 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ \vdots & & \vdots & \vdots & & \vdots & \vdots \\ 0 & \cdots & 1 & 0 & \cdots & 0 & 0 \\ 0 & \cdots & 0 & -(1+\lambda_2) & \cdots & 0 & 0 \\ \vdots & & \vdots & \vdots & & \vdots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & -(1+\lambda_2) & 0 \\ 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ x_1 & \cdots & x_h & -x_{h+1} & \cdots & -x_n & 1 \end{bmatrix}_{(n+2)x(n+1)} \qquad (17)$$

Using matrix notation, LM = N. To solve for M, L must be a nonsingular (n+2)x(n+2) matrix. Under the assumption that $|L| \neq 0$, L^{-1} exists and

$$L^{-1}LM = L^{-1}N$$

$$LM = L^{-1}N$$

$$M = L^{-1}N$$

The (i,j)th element of L^{-1}, usually written as ℓ^{ij}, is equal to $\frac{L_{ji}}{|L|}$ where $|L|$ is the determinant of L and L_{ji} is the cofactor of ℓ_{ji} in L.

Thus,

$$\begin{vmatrix}
\frac{\partial x_1}{\partial p_1} & \cdots & \frac{\partial x_1}{\partial p_h} & \frac{\partial x_1}{\partial p_{h+1}} & \cdots & \frac{\partial x_1}{\partial p_n} & 0 \\
\vdots & & \vdots & \vdots & & \vdots & \vdots \\
\frac{\partial x_h}{\partial p_1} & \cdots & \frac{\partial x_h}{\partial p_h} & \frac{\partial x_h}{\partial p_{h+1}} & \cdots & \frac{\partial x_h}{\partial p_n} & 0 \\
\frac{\partial x_{h+1}}{\partial p_1} & \cdots & \frac{\partial x_{h+1}}{\partial p_h} & \frac{\partial x_{h+1}}{\partial p_{h+1}} & \cdots & \frac{\partial x_{h+1}}{\partial p_n} & \frac{\partial x_{h+1}}{\partial C} \\
\vdots & & \vdots & \vdots & & \vdots & \vdots \\
\frac{\partial x_n}{\partial p_1} & \cdots & \frac{\partial x_n}{\partial p_h} & \frac{\partial x_n}{\partial p_{h+1}} & \cdots & \frac{\partial x_n}{\partial p_n} & \frac{\partial x_n}{\partial C} \\
\frac{\partial \lambda_1}{\partial p_1} & \cdots & \frac{\partial \lambda_1}{\partial p_h} & \frac{\partial \lambda_1}{\partial p_{h+1}} & \cdots & \frac{\partial \lambda_1}{\partial p_n} & 0 \\
0 & \cdots & 0 & \frac{\partial \lambda_2}{\partial p_{h+1}} & \cdots & \frac{\partial \lambda_2}{\partial p_n} & 0
\end{vmatrix} = -L^{-1}$$

$$\begin{vmatrix}
1 & \cdots & 0 & 0 & \cdots & 0 & 0 \\
\vdots & & \vdots & \vdots & & \vdots & \vdots \\
0 & \cdots & 1 & 0 & \cdots & 0 & 0 \\
0 & \cdots & 0 & -(1+\lambda_2) & \cdots & 0 & 0 \\
\vdots & & \vdots & \vdots & & \vdots & \vdots \\
0 & \cdots & 0 & 0 & \cdots & -(1+\lambda_2) & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\
x_1 & \cdots & x_h & -x_{h+1} & \cdots & -x_n & 1
\end{vmatrix} \tag{18}$$

where

$$L^{-1} = \frac{1}{|L|} \begin{vmatrix} L_{11} & \cdots & L_{1h} & L_{1,(h+1)} & \cdots & L_{1n} & L_{1(n+1)} & L_{1(n+2)} \\ \vdots & & \vdots & \vdots & & \vdots & \vdots & \vdots \\ L_{h1} & \cdots & L_{hh} & L_{h,(h+1)} & \cdots & L_{hn} & L_{h(n+1)} & L_{h(n+2)} \\ L_{(h+1),1} & \cdots & L_{(h+1),h} & L_{(h+1),(h+1)} & \cdots & L_{(h+1),n} & L_{(h+1),(n+1)} & L_{(n+1),(n+2)} \\ \vdots & & \vdots & \vdots & & \vdots & \vdots & \vdots \\ L_{n1} & \cdots & L_{nh} & L_{n,(h+1)} & \cdots & L_{nn} & L_{n,(n+1)} & L_{n,(n+2)} \\ L_{(n+1),1} & \cdots & L_{(n+1),h} & L_{(n+1),(h+1)} & \cdots & L_{(n+1),n} & L_{(n+1),(n+1)} & L_{(n+1),(n+2)} \\ L_{(n+2),1} & \cdots & L_{(n+2),h} & L_{(n+2),(h+1)} & \cdots & L_{(n+2),n} & L_{(n+2),(n+1)} & L_{(n+2),(n+2)} \end{vmatrix} \qquad (19)$$

From (18) and (19), the *a priori* relationship between the quantity of output i and the price of output i, i = 1, 2,..., h is given by

$$\frac{\partial x_i}{\partial p_i} = \frac{-1}{|L|}[L_{ii} + x_i L_{i(n+2)}] \qquad (20)$$

L_{ii} represents the principal minor of L obtained by deleting the ith row and ith column from L. In order for a maximum to exist, the principal minors must alternate in sign. So, $\frac{L_{ii}}{|L|} < 0.$ In addition, from (18) for i = 1,..., n, $\frac{L_{i(n+2)}}{|L|} = 0.$ The relationship between the quantity of an output and the capital constraint (C) is zero. So,

$$\frac{\partial x_i}{\partial p_i} = \frac{L_{ii}}{|L|} > 0 \qquad (21)$$

The sign of the slope of the firm's supply function of output i with respect to the price of output i, all other factors nonvariant, is always positive.

From (18) and (19) the *a priori* relationship between the quantity of an input j and the price of input j, j = h+1,..., n is given by

$$\frac{\partial x_j}{\partial p_j} = \frac{-1}{|L|}[-(1+\lambda_2)L_{jj} - x_j L_{(n+2)}] \qquad (22)$$

In order for a maximum to exist, the principal minors must alternate in sign. $\frac{L_{jj}}{|L|} < 0.$ Further, the relationship between the quantity of an input and the capital constraint (C) is not zero.

$$\frac{\partial x_j}{\partial C} = \frac{L_{j(n+2)}}{|L|} \neq 0 \quad j = h+1, \ldots, n. \tag{23}$$

However, $L_{j(n+2)} \gtrless 0$ and $|L| \gtrless 0$. So, $\frac{L_{j(n+2)}}{|L|} \gtrless 0$, and thus, the sign on (23) is indeterminate. By substituting (23) into (22),

$$\frac{\partial x_j}{\partial p_j} = (1+\lambda_2) \frac{L_{jj}}{|L|} + x_j \frac{\partial x_j}{\partial C} \gtrless 0. \tag{24}$$

Since $\frac{\partial x_j}{\partial C} \gtrless 0$, the sign on the slope of the firm's demand function for an input i with respect to the price of input i, all other factors nonvariant, is indeterminate.[3/] The sign of $\frac{\partial x_j}{\partial p_j}$ depends upon the sign and magnitude of $\frac{\partial x_j}{\partial C}$ and the magnitude of $(1+\lambda_2) \frac{L_{jj}}{|L|}$. However, in most instances $\frac{\partial x_j}{\partial p_j} < 0$.

From (18) and (19), the *a priori* relationship between the quantity of an input i, i = h+1,..., n and the price of input j, j = h+1,..., n i≠j, is given by

$$\frac{\partial x_i}{\partial p_j} = \frac{-1}{|L|} [-(1+\lambda_2) L_{ij} - x_j L_{i(n+2)}] \tag{25}$$

[3/] In consumer demand theory, the notion of a positive sloping demand curve has long been recognized (Griffin good), while in the theory of derived input demand, the notion of a positive sloping input demand curve has been neglected. Nevertheless, *a priori*, the sign on the slope of the firm's demand function for an input i with respect to the price of input i, all other factors nonvariant, may be positive.

By substituting (23) into (25),

$$\frac{\partial x_i}{\partial p_j} = (1+\lambda_2)\frac{L_{ij}}{|L|} + x_j\frac{\partial x_i}{\partial C} \gtrless 0 \qquad (26)$$

No information exists concerning the sign of L_{ij}, the cofactor of the off-diagonal elements, and the sign of the determinant L. Since $\frac{L_{ij}}{|L|} \gtrless 0$ and $\frac{\partial x_i}{\partial C} \gtrless 0$, the sign on the slope of the input demand function with respect to the price of other inputs, ceterus paribus, is indeterminate. The sign of $\frac{\partial x_i}{\partial p_j}$ depends upon the sign and magnitude of $\frac{L_{ij}}{|L|}$ and $\frac{\partial x_i}{\partial C}$. In most cases, $\frac{\partial x_i}{\partial p_j} > 0$, ceterus paribus, if inputs i and j are substitutes for each other. If inputs i and j are complementary factors, then $\frac{\partial x_i}{\partial p_j} < 0$, ceterus paribus.

From (26), an important relationship between the inputs in the production process may be shown.

$$(1+\lambda_2)\frac{L_{ij}}{|L|} = \frac{\partial x_i}{\partial p_j} - x_j\frac{\partial x_i}{\partial C} \qquad (27)$$

The substitution effect, $(1+\lambda_2)\frac{L_{ij}}{|L|}$, between two inputs is equal to the change in the quantity of input i with respect to a change in the price of input j, i≠j, minus the use of input j times the change in the quantity of input i with respect to a change in capital, all other factors nonvariant. In addition, by symmetry of second order partial derivatives, Young's Theorem,

$$L_{ij} = L_{ji} \tag{28}$$

So,

$$(1+\lambda_2) \frac{L_{ij}}{|L|} = (1+\lambda_2) \frac{L_{ji}}{|L|} \tag{29}$$

and

$$\frac{\partial x_i}{\partial p_j} - x_j \frac{\partial x_i}{\partial C} = \frac{\partial x_j}{\partial p_i} - x_i \frac{\partial x_j}{\partial C} \tag{30}$$

The change in the quantity of input i with respect to a change in the price of input j, i≠j, minus the use of input j times the change in the quantity of input i with respect to a change in capital must be equal to the change in the quantity of input j with respect to a change in the price of input i, i≠j, minus the use of input i times the change in the quantity of input j with respect to a change in capital.

From (18) and (19), the <u>a priori</u> relationship between the quantity of an input i, i = h+1,..., n and the price of an output j, j = 1,..., h is given by

$$\frac{\partial x_i}{\partial p_j} = \frac{-1}{|L|} [L_{ij} + x_j L_{i(n+2)}] \tag{31}$$

By substituting (23) into (31),

$$\frac{\partial x_i}{\partial p_j} = \frac{-L_{ij}}{|L|} - x_j \frac{\partial x_i}{\partial C} \gtrless 0 \tag{32}$$

Again, since $\frac{L_{ij}}{|L|} \gtrless 0$ and $\frac{\partial x_i}{\partial C} \gtrless 0$, the sign on the slope of the demand function with respect to the output price, ceterus paribus, is indeterminate. The sign of $\frac{\partial x_i}{\partial p_j}$ depends upon the sign and magnitude of $\frac{L_{ij}}{|L|}$ and $\frac{\partial x_i}{\partial C}$. For the case of inferior inputs, the relationship between the quantity of an input and the price of the output is negative. For the case of normal inputs, $\frac{\partial x_i}{\partial p_j} > 0$.

In sum, under the assumptions that the firm is subject to a capital constraint and a production function constraint, and the necessary and sufficient conditions in (6) and (13) hold, the static theory of derived input demand states that the quantity of an input purchased by a firm depends upon the price of the particular input, (p_i) the price of other inputs (p_j), the price(s) of the output(s) (p_o), and the initial capital outlay of the firm (C). The static theoretical input demand relationship is given by

$$X_i = f(p_i,\ p_j,\ p_o,\ C) \tag{33}$$

Unlimited Capital

Under the assumption that the firm has no capital constraint, the firm maximizes net revenue subject to only a production function. In order to mathematically solve for a maximum net revenue subject to (1), form the Lagrangian expression

$$Z = \sum_{i=1}^{h} p_i x_i - \sum_{i=h+1}^{n} p_i x_i + \lambda F(x_1,\ x_2, \ldots, x_n) \tag{34}$$

Note that (31) and (5) are similar except that under the condition of unlimited capital, λ_2 in (5) is equal to zero. Differentiating Z with respect to x_i, i = 1,. . . . n, and λ, the first order conditions for a firm with no constraint on capital to maximize net revenue subject to a production function are given by

$$\begin{aligned} \frac{\partial Z}{\partial x_i} &= p_i + \lambda F_i = 0 \quad i = 1, \ldots h \\ \frac{\partial Z}{\partial X_i} &= -p_i + \lambda F_i = 0 \quad i = h+1,\ldots, n \\ \frac{\partial Z}{\partial \lambda} &= F(x_1, x_2,\ldots, x_n) = 0 \end{aligned} \tag{35}$$

The $F_i (i = 1, \ldots n)$ is the partial derivative of (1) with respect to its ith argument. The necessary conditions for profit maximization subject to a production function constraint involve n+l partial derivatives and equations.

Select any two of the first h equations of (35). Then,

$$\frac{P_e}{P_f} = \frac{F_e}{F_f} = \frac{\partial x_f}{\partial x_e} = -\lambda \tag{36}$$

where e and f refer to outputs. The rate of product transformation, RPT, for every pair of outputs, holding the levels of all other outputs and all inputs constant, must equal the ratio of their prices. From (36), λ = -RPT for every pair of outputs. In addition, note that (36) and (7) are precisely the same equations.

Select any two of the n-h equations of (35). Then,

$$\frac{P_k}{P_\ell} = \frac{F_k}{F_\ell} = \frac{\partial x_e}{\partial x_k} = \lambda \tag{37}$$

where k and ℓ refer to inputs. The rate of technical substitution, RTS, for every pair of inputs, holding the levels of all outputs and all other inputs constant, must equal the ratio of their prices. From (37), λ = RTS for every pair of inputs. In addition, from (36) and (37), the RTS for every pair of inputs is equal to minus the RPT for every pair of outputs. Note that when $\lambda_2=0$, the case of unlimited capital, (37) and (8) are the same equations.

Select one equation from the first h equations of (35) and one equation from the n-h equations of (35). Then,

$$\frac{P_e}{P_k} = \frac{F_e}{F_k} = \frac{\partial x_k}{\partial x_e} = -\lambda \tag{38}$$

or

$$P_k = P_e \cdot \frac{\partial x_e}{\partial x_k} \tag{39}$$

where e refers to outputs and k refers to inputs. The value of marginal product of each input with respect to the relevant output is equal to the price of the input. Note that again when $\lambda_2=0$, no constraint on capital, (39) and (10) are the same equations.

The second-order conditions for the maximization of net revenue subject to a production function constraint and no capital constraint and the second-order conditions for the maximization of net revenue subject to a production function and a capital constraint are exactly the same. These sufficient conditions are given in (11), (12), and (13).

If the necessary and sufficient conditions for profit maximization subject to a production function constraint hold, then the quantity of each input the firm uses and the quantity of each output the firm produces may be expressed uniquely in terms of the input and output prices by solving the system of (n+1) equations in (35) simultaneously.

$$x_i = g_i(p_1,\ldots, p_n) \tag{40}$$

where (40) represents a supply function for the output variables (i = 1,... h) or a demand function for the input variables (i = h+1, ..., n).

In order to maximize net revenue, the decision unit responds to changes in input and output prices by varying input and output levels to satisfy (35). The decision unit of the firm, by assumption, makes rational choices, consistent choices, and possesses perfect knowledge. By differentiating (35) with respect to p_1, p_2,..., p_n, one at a time with each of the other price variables held constant, the theoretical nature of the relationships in (40) may be examined. Further, it is assumed that the firm may make instantaneous adjustments to changes in prices, and complete divisibility of inputs and outputs exists. In final matrix form similar to (15), (16), (17), (18), and (19),

$$\underset{\displaystyle E \qquad\qquad (n+1)\text{x } n}{\begin{vmatrix}
\frac{\partial x_1}{\partial p_1} & \cdots & \frac{\partial x_1}{\partial p_h} & \frac{\partial x_1}{\partial p_{h+1}} & \cdots & \frac{\partial x_1}{\partial p_n} \\
\vdots & & \vdots & \vdots & & \vdots \\
\frac{\partial x_h}{\partial p_1} & \cdots & \frac{\partial x_h}{\partial p_h} & \frac{\partial x_h}{\partial p_{h+1}} & \cdots & \frac{\partial x_h}{\partial p_n} \\
 & & & & & \\
\frac{\partial x_{h+1}}{\partial p_1} & \cdots & \frac{\partial x_{h+1}}{\partial p_h} & \frac{\partial x_{h+1}}{\partial p_{h+1}} & \cdots & \frac{\partial x_{h+1}}{\partial p_n} \\
\vdots & & \vdots & \vdots & & \vdots \\
\frac{\partial x_n}{\partial p_1} & \cdots & \frac{\partial x_n}{\partial p_h} & \frac{\partial x_n}{\partial p_{h+1}} & \cdots & \frac{\partial x_n}{\partial p_n} \\
 & & & & & \\
\frac{\partial \lambda}{\partial p_1} & & \frac{\partial \lambda}{\partial p_h} & \frac{\partial \lambda}{\partial p_{h+1}} & & \frac{\partial \lambda}{\partial p_n}
\end{vmatrix}}
= \frac{-1}{|F|}
\underset{\displaystyle F^{-1} \qquad\qquad (n+1)\text{x}(n+1)}{\begin{vmatrix}
F_{11} & \cdots & F_{1h} & F_{1,(h+1)} & \cdots & F_{1n} & F_{1,(n+1)} \\
\vdots & & & & & & \\
F_{h1} & \cdots & F_{hh} & F_{h,(h+1)} & \cdots & F_{hn} & F_{h,(n+1)} \\
F_{(h+1),1} & \cdots & F_{(h+1),h} & F_{(h+1),(h+1)} & \cdots & F_{(h+1),n} & F_{(h+1),(n+1)} \\
\vdots & & \vdots & \vdots & & \vdots & \vdots \\
F_{n1} & \cdots & F_{nh} & F_{n,(h+1)} & \cdots & F_{nn} & F_{n,(n+1)} \\
F_{(n+1),1} & \cdots & F_{(n+1),h} & F_{(n+1),(h+1)} & \cdots & F_{(n+1),n} & F_{(n+1),(n+1)}
\end{vmatrix}}
\text{ X }
\underset{\displaystyle G \qquad\qquad (n+1)\text{x } n}{\begin{vmatrix}
1 & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & & \vdots & \vdots & & \vdots \\
0 & & 1 & 0 & \cdots & 0 \\
0 & \cdots & 0 & -1 & \cdots & 0 \\
\vdots & & \vdots & \vdots & & \vdots \\
0 & \cdots & 0 & 0 & & -1 \\
0 & \cdots & 0 & 0 & \cdots & 0
\end{vmatrix}}
\qquad (41)$$

In matrix notation, $E = F^{-1}G$.

where the (i, j)th element of F^{-1}, written f^{ij}, is equal to $F_{ji}/|F|$, where $|F|$ is the determinant of F and F_{ji} is the cofactor of f_{ji} in F.

From (38), the a priori relationship between the quantity of an output i and the price of output i, i = 1,... h is given by

$$\frac{\partial x_i}{\partial p_i} = \frac{-1}{|F|} F_{ii} \tag{42}$$

F_{ii} represents the principal minor of F obtained by deleting the ith row and ith column from F. In order for a maximum to exist, the principal minors must alternate in sign. So, $\frac{F_{ii}}{|F|} < 0$. Thus,

$$\frac{\partial x_i}{\partial p_i} > 0 \tag{43}$$

The sign on the slope of the firm's supply function of output i with respect to the price of output i, all other factors nonvariant, is always positive. Note that the slope of the firms supply curve is positive whether or not capital is a constraint.

From (41), the a priori relationship between the quantity of an input j and the price of input j, j = h+1,..., n is given by

$$\frac{\partial x_j}{\partial p_j} = \frac{1}{|F|} F_{jj} \tag{44}$$

In order for a maximum to exist, the principal minors must alternate in sign. $\frac{F_{ii}}{|F|} < 0$. Thus,

$$\frac{\partial x_i}{\partial p_j} < 0 \qquad (45)$$

The sign on the slope of the firm's demand function for an input i with respect to the price of input i, all other factors nonvariant, is unequivocally negative in the unlimited capital case. In the case where capital is a constraint, the slope of the firm's input demand function with respect to own price is indeterminate.

From (41), the a priori relationship between the quantity of an input i, i = h+1,..., n and the price of input j, j = h+1,..., n, i≠j is given by

$$\frac{\partial x_i}{\partial p_j} = \frac{-1}{|F|} F_{ij} \gtrless 0 \qquad (46)$$

Since $\frac{F_{ij}}{|F|} \gtrless 0$, the sign on the slope of the input demand function with respect to the price of other inputs, ceterus paribus, is indeterminate in the unlimited capital case. The sign on $\frac{\partial x_i}{\partial p_j}$ depends upon the sign of $\frac{F_{ij}}{|F|}$. So, similar to the case of a firm that operates under a capital constraint, the price of other inputs (p_j) has either a positive or negative effect on the quantity of an input (x_i). In most cases, $\frac{\partial x_i}{\partial p_j} > 0$, ceterus paribus, if inputs i and j are substitutes for each other. If inputs i and j are complementary factors, then $\frac{\partial x_i}{\partial p_j} < 0$, <u>ceterus paribus</u>.

Under the assumption of unlimited capital, from (46), an important relationship between the inputs in the production process may be shown. Since $F_{ij} = F_{ji}$ by symmetry of second order partial derivatives,

$$\frac{\partial x_i}{\partial p_j} = \frac{\partial x_j}{\partial p_i} \quad i \neq j, \; i, j = h+1, \ldots, n \tag{47}$$

The substitution effect on the ith input due to a change in the price of the jth input is precisely equal to the substitution effect on the jth input due to a change in the price of the ith input. Note that under the assumption of a capital constraint, (47) does not hold. When the firm operates with limited capital,

$$\frac{\partial x_i}{\partial p_j} - x_j \frac{\partial x_i}{\partial C} = \frac{\partial x_j}{\partial p_i} - x_i \frac{\partial x_j}{\partial C} \tag{30}$$

From (41), the *a priori* relationship between the quantity of an input i, i = h+1,..., n and the price of an output j = 1,..., h is given by

$$\frac{\partial x_i}{\partial p_j} = \frac{-1}{|F|} F_{ij} \gtrless 0 \tag{48}$$

Again, since $\frac{F_{ij}}{|L|} \gtrless 0$, the sign on the slope of the input demand function with respect to output price, *ceterus paribus*, is indeterminate. The sign $\frac{\partial x_i}{\partial p_j}$ depends upon the sign of $\frac{F_{ij}}{|F|}$. In the unlimited capital cases, the sign is positive for normal inputs and negative for inferior inputs.

In summary, under the assumptions that the firm has unlimited capital and the first-order and second-order conditions in (35) and (13) hold, the static theory of derived input demand states that the quantity of an input purchased by a firm depends upon the price of the particular input (p_i), the price of other inputs (p_j), and the price(s) of the output(s) (p_o). The theoretical static input demand relationship is given by

$$X_i = f(p_i, p_j, p_o) \tag{49}$$

APPENDIX C

SUPPLEMENTARY TABLES

This appendix contains relevant tables which supplement the discussion in Chapter 5. Two other statistical models for gasoline and diesel fuel were considered. The estimated coefficients and standard errors of the empirical variables in these statistical models are exhibited in Tables C.1, C.2, C.3, and C.4. The correlation matrices of the exogenous variables in the gasoline and diesel fuel demand relationships in Tables 5.1 and 5.2 are exhibited in Tables C.5 and C.6.

Table C.1. The Estimated Coefficients and Standard Errors of the Empirical Variables in the Gasoline Demand Relationship

Variable	Estimated Coefficient	Estimated Standard Error
INT	0.760277	0.98236
LACRE	0.953824[a]	0.021661
LRPCR	0.514571[a]	0.048247
LPDFLAG5	0.780065[a]	0.25732
LRWPFR	-0.487586[a]	0.12615
LINTER	-0.198637	0.19697
LRPLABO	0.918697[a]	0.29029
LTE	0.281016[a]	0.093424
LRPLND	0.570971[a]	0.021860
LPRECIP	0.122472[a]	0.037172
LPGSLAG5	-1.09319[a]	0.39544

[a] Significant at .10 level.

Table C.2. The Estimated Coefficients and Standard Errors of the Empirical Variables in the Diesel Fuel Demand Relationship

Variable	Estimated Coefficient	Estimated Standard Error
INT	-10.8726[a]	2.2685
LACRE	1.52060[a]	0.10097
LRPCR	1.84428[a]	0.20935
LPDFLAG3	-1.14595[a]	0.49728
LRWPFR	-0.290790	0.34277
LINTER	-1.52906[a]	0.35230
LRPLABO	1.57557[a]	0.47552
LTE	0.949474[a]	0.14530
LRPLND	0.956637[a]	0.16529
LPRECIP	-0.0918598[a]	0.062321
LPGSLAG3	1.91819[a]	0.69401

[a] Significant at .10 level.

Table C.3. The Estimated Coefficients and Standard Errors of the Empirical Variables in the Gasoline Demand Relationship

Variable	Estimated Coefficient	Estimated Standard Error
INT	0.447221	0.67353
LACRE	0.942288[a]	0.022114
LRPCR	0.485638[a]	0.047901
LRPGS	-0.554233[a]	0.24879
LPDFLAG1	0.838424[a]	0.19203
LRWPFR	-0.686934[a]	0.15164
LINTER	-0.560791[a]	0.13635
LRPLABO	0.845112[a]	0.25028
LTE	0.337335[a]	0.061312
LRPLND	0.565178[a]	0.024032
LPRECIP	0.107938[a]	0.032119

[a] Significant at .10 level.

Table C.4. The Estimated Coefficients and Standard Errors of the Empirical Variables in the Diesel Fuel Demand Relationship

Variable	Estimated Coefficient	Estimated Standard Error
INT	-7.35677[a]	1.8506
LACRE	1.45705[a]	0.10295
LRPCR	1.67863[a]	0.21367
LPGSLAG1	0.959064[a]	0.55889
LRPDFL	-0.535314	0.56765
LRWPFR	-0.670353[a]	0.36270
LINTER	-1.12338[a]	0.40454
LRPLABO	1.21880[a]	0.54746
LTE	0.957425[a]	0.16276
LRPLND	0.807445[a]	0.16360
LPRECIP	-0.0707770	0.076408

[a] Significant at .10 level.

Table C.5. Correlation Matrix of the Exogenous Variables in the Gasoline Demand Relationship

	INT	LACRE	LRPCR	LRPGS	LPDFLAG5	LRWPFR	LINTER	LRPLABO	LTE	LRPLND	LPRECIP	LPGSLAG5
INT	1.000	-0.153	-0.162	-0.396	0.263	0.029	-0.163	-0.051	0.099	-0.104	0.131	-0.651
LACRE		1.000	0.955	-0.021	0.067	-0.115	-0.021	0.036	-0.073	0.367	-0.033	0.006
LRPCR			1.000	-0.016	0.107	-0.166	0.039	0.034	-0.111	0.480	-0.010	-0.013
LRPGS				1.000	-0.005	-0.461	0.031	-0.148	-0.031	-0.010	-0.022	-0.116
LPDFLAG5					1.000	-0.344	0.579	0.305	-0.733	0.097	0.215	-0.765
LRWPFR						1.000	-0.569	0.300	0.080	-0.143	-0.119	-0.217
LINTER							1.000	0.014	-0.555	0.067	0.183	-0.230
LRPLABO								1.000	-0.104	0.008	0.094	-0.217
LTE									1.000	-0.105	-0.007	0.404
LRPLND										1.000	0.166	-0.067
LPRECIP											1.000	-0.301
LPGSLAG5												1.000

Table C.6. Correlation Matrix of the Exogenous Variables in the Diesel Fuel Demand Relationship

	INT	LACRE	LRPCR	LPDFLAG3	LRWPFR	LINTER	LRPLABO	LTE	LRPLND	LPRECIP	LPGSLAG3	LRPDFL
INT	1.000	-0.307	-0.371	0.523	-0.099	0.166	-0.374	0.082	-0.368	-0.158	-0.861	-0.232
LACRE		1.000	0.922	0.012	-0.181	-0.237	0.046	0.043	0.178	0.001	0.080	0.076
LRPCR			1.000	-0.006	-0.166	-0.254	0.059	0.019	0.426	0.003	0.091	0.044
LPDFLAG3				1.000	-0.625	0.372	-0.345	-0.525	-0.021	0.081	-0.736	0.163
LRWPFR					1.000	-0.290	0.459	0.559	-0.048	-0.231	0.175	-0.706
LINTER						1.000	-0.229	-0.140	-0.090	0.180	-0.226	-0.174
LRPLABO							1.000	0.348	0.021	-0.083	0.299	-0.228
LTE								1.000	-0.129	-0.081	0.120	-0.639
LRPLND									1.000	0.062	0.045	-0.048
LPRECIP										1.000	0.051	0.183
LPGSLAG3											1.000	0.217
LRPDFL												1.000

The exogenous variables listed in Tables C.1, C.2, C.3, C.4, C.5, and C.6 are:

INT = intercept

LACRE = number of cropland acres times the market share estimates of SSC

LRPCR = real price of output

LPDFLAG5 = real price of diesel fuel in period t-5

LRWPFR = real price of fertilizer

LINTER = interest rate

LRPLABO = real price of labor

LTE = ratio of diesel fuel tractors to gasoline tractors in the U.S.

LRPLND = real price of land and buildings

LPRECIP = precipitation

LPGSLAG5 = real price of gasoline in period t-5

LPDFLAG3 = real price of diesel fuel in period t-3

LPGSLAG3 = real price of gasoline in period t-3

LRPGS = real price of gasoline in the current period

LPDFLAG1 = real price of diesel fuel in period t-1

LRPDFL = real price of diesel fuel in the current period

LPGSLAG1 = real price of gasoline in period t-1

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THE DEMAND FOR GASOLINE AND DIESEL FUEL IN AGRICULTURAL USE IN VIRGINIA

by

Oral Capps, Jr.

(ABSTRACT)

The objectives of this study were: (1) to determine the usage patterns of gasoline, diesel fuel, and other fossil fuels in different types of agriculture; (2) to determine the demand relationships for gasoline and diesel fuel in agricultural use and to identify and assess the major factors that affect these relationships; and (3) to determine differences in the demand relations for gasoline and diesel fuel in agricultural use.

The usage patterns of gasoline, diesel fuel, and other fossil fuels in agricultural production in Virginia, the South, and the U.S. were developed from cross-sectional data provided jointly by the Economic Research Service and the Federal Energy Administration. From the static theoretical development and the dynamic and technological adjustment concepts of derived input demand, the theoretical demand relationships for gasoline and diesel fuel were formulated. Asymptotically efficient, asymptotically normal, asymptotically unbiased, and consistent parameter estimates were obtained by employing a generalized least squares (GLS) procedure on the Parks model in combining cross-sectional and time-series data. All data were from the period 1971 through 1976.

The agricultural sector in Virginia appears to adjust to changes in economic factors and other variables influencing the demand for gasoline and diesel fuel. While farmers appear to be somewhat passive to increases in the real prices of gasoline and diesel fuel in the current period, they are quite responsive to such increases when given time to adjust their usage patterns.