Low Frequency Noise Reduction using Novel Poro-Elastic Acoustic Metamaterials

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(ABSTRACT)

Low frequency noise is a common problem in aircraft and launch vehicles. New technologies must be investigated to reduce this noise while contributing minimal weight to the structure. This thesis investigates passive and active control methods to improve low frequency sound absorption and transmission loss using acoustic metamaterials. The acoustic metamaterials investigated consist of poro-elastic acoustic heterogeneous (HG) metamaterials and microperforated (MPP) acoustic metamaterials. HG metamaterials consist of poro-elastic material with a periodic arrangement of embedded masses acting as an array of mass-spring-damper systems. MPP acoustic metamaterials consist of periodic layers of micro-porous panels embedded in poro-elastic material. This thesis examines analytically, experimentally, and numerically the behavior of acoustic metamaterials compared to a baseline poro-elastic sample. The development of numerical techniques using finite element analysis will aid in understanding the physics behind their functionality and will influence their design. Design studies are performed to understand the effects of varying the density, size, shape, and placement of the embedded masses as well as the location and distribution of microperforated panels in poro-elastic material. An active HG metamaterial is investigated, consisting of an array of active masses embedded within poro-elastic material. Successful tonal and broadband noise control is achieved using a feedforward, filtered-x LMS control algorithm to minimize the downstream sound pressure level. Low-frequency absorption and transmission loss is successfully increased in the critical frequency range below 500 Hz. Acoustic metamaterials are compact compared to conventional materials and find applications in controlling low-frequency sound radiation in aircraft and launch vehicles.
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1. INTRODUCTION

1.1. Motivation

Recently, much work has been done on reducing the environmental impact imposed by next generation aircraft while improving fuel consumption and saving weight. A large aspect of the environmental impact is the need to decrease radiated noise from aircrafts. Programs such as the Environmentally Responsible Aviation (ERA) program have been initiated by NASA to address these issues. The ERA program is working on technologies that will simultaneously reduce aircraft weight by 10% while reducing aircraft noise by 1/8 compared to current standards. Other work on controlling exterior aircraft noise has been developed under NASA’s Subsonic Fixed Wing Project, including the next generation Hybrid Wing Body (HWB) aircraft.

To address these issues, there is a need to develop absorptive materials that are lighter, thinner, and have increased broadband absorption capabilities at low frequencies compared to conventional standards. There remains opportunity to improve this technology for quieting aircraft in the areas of engine noise and airframe noise. Some examples include the control of airframe noise by placing absorptive material on a landing gear recess, and the control of engine inlet noise by placing the material on the wing surface.

Noise and vibration inside aircraft interior are some of the leading causes of health and performance risk among crew and contribute to discomfort among passengers. This structural borne noise can cause vibrations due to engine imbalances which radiate into the aircraft interior [1]. There is a focus on reducing structural noise as more engines are being mounted directly on the rear fuselage wall as implemented in the Hybrid Wing Body aircraft [2]. To suppress this noise introduced to the aircraft interior, acoustic treatment is placed between the fuselage and trim panel of an aircraft. This treatment, however, adds weight which reduces the efficiency of next generation aircraft. Advanced absorptive materials which are thinner and lightweight are highly needed for this type of application.

Furthermore, the transmission of noise into a launch vehicle’s payload fairing (PLF) has become a critical issue in the design and success of its mission. The increase in size and the use of lightweight and cost-effective composite structures have lowered the acoustic transmission loss in the PLF, leading to increased vibration responses and the potential for structural damages.
One example where the acoustic transmission was a significant problem is in the Cassini spacecraft, developed at the Jet Propulsion Laboratory for NASA to explore to planet Saturn and its moons [3].

The current method to reduce the interior acoustic vibration response is to use acoustic blankets within the PLF of expendable launch vehicles (ELV). These blankets perform three main functions. They reduce the acoustic transmission of energy into the PLF as measured by the transmission loss, increase the acoustic absorption of the PLF as measured by the absorption coefficient, and add additional damping to the PLF as measured by the structural damping factor [4]. Many acoustic blankets used in launch vehicles perform well at high frequencies, but are limited at low frequencies due to size and weight restrictions imposed by launch vehicle conditions.

A recent example showing the need to attenuate low-frequency noise is the Space Launch System (SLS). The SLS is America’s next heavy-lift launch vehicle, and will be the most powerful and loudest rocket in history. High acoustic environments are thus expected in the engine compartments, stage and launch vehicle adaptors, and the payload bay. Currently, the payload bay is designed in accordance with the overall sound pressure level of the Atlas V rocket. However, compared to the Atlas V which has 285 thousand pounds of thrust, the SLS has a heavy 9.2 million pounds of thrust, or 32 times the amount for the Atlas V [5]. This could result in a much louder liftoff with a significant low frequency component in the critical range below 500 Hz. Currently there are no great solutions for reducing this low frequency noise. There is a need to improve these passive techniques by combining different noise control methods, including the introduction of composite absorptive material and active noise control.

The goal of this research is to design and develop an advanced blanket concept for the application of launch vehicles at lift-off and next generation aircraft to further increase the absorption and transmission loss in the critical low frequency range below 500 Hz. Acoustic heterogeneous (HG) metamaterials and microperforated panels (MPP) will be used in solving this problem by attenuating low frequency noise in structures. These techniques will be investigated because they show promise in increasing low frequency absorption and transmission loss and can be implemented in a blanket system within the PLF of expendable launch vehicles. Furthermore, these materials can be passively or actively tuned to achieve attenuation at target
bandwidths. The different types of noise control are introduced, followed by an introduction to acoustic metamaterials and heterogeneous metamaterials. Finally, the objectives and outline of this paper are given.

1.2. Introduction to noise control

Passive, reactive, and active [6] control are the three common methods of noise control. Passive noise control dissipates vibration and acoustic energy into heat through friction. Passive treatments are seen in mufflers, noise barriers, damping materials, and acoustic absorbent material, and do not require additional control energy. Acoustic poro-elastic foam is a passive control method effective at absorbing high frequencies. Tiny air molecules within poro-elastic foam oscillate at the frequency of sound excitation which results in frictional losses. Changes in flow through irregular pores result in a loss of momentum in the direction of wave propagation. These two phenomena account for a loss in energy in the high frequency range [7]. Acoustic foam is not as effective at low frequencies because the wavelengths are much larger than the thickness of the foam. Controlling low frequency noise is therefore a challenge due to the thickness restrictions of passive absorbers, and is generally impractical due to the added mass and bulk which would be required. In the aircraft fuselage where the thickness is limited to a few inches, passive treatments are ineffective below 500Hz [8]. Passive noise control is generally an inexpensive and reliable method to reduce noise in structures and vehicles, and can be combined with active approaches. Reactive noise control methods, such as Helmholtz resonators, serve as acoustic attenuators in a narrow frequency bandwidth. These devices have shown to reduce low frequency noise when placed on lightweight vibrating structures [9]. Active noise control methods generate an out of phase signal to create destructive interference with a noise source. These methods generally require additional equipment and space for functioning. To control noise across the entire bandwidth, an integrated control scheme is needed.

A desirable noise control system is compact and overcomes the restrictions imposed by a purely active or passive device. Active-passive hybrids take advantages of both methods of noise control. The active elements enhance the performance of the passive system by adapting to changes in the noise source and improving low frequency attenuation. The acoustic metamaterial design proposed in this paper utilizes a combination of passive and active control methods to increase low frequency absorption and transmission loss.
1.3. Introduction to acoustic metamaterials

Metamaterials are artificially engineered materials that have properties that cannot be found in nature [10]. Acoustic metamaterials are tuned to the acoustic wavelength and can be categorized into non-resonant and resonant materials. Non-resonant metamaterials consist of a periodic arrangement of elements, such as spheres or cylinders, embedded within a material matrix and are typically spaced less than a wavelength apart. These materials disrupt the propagation of waves by scattering and refraction effects. Cervera et al. [11] arranged periodic cylinders to act like a material with a low impedance value in air to attenuate sound at frequencies where the wavelength is smaller than the spacing between the cylinders. Popa et al. [12] arranged perforated plates in such a way to transmit a sound wave around a material as a 2D acoustic cloak.

Resonant metamaterials are generally heterogeneous materials containing a periodic arrangement of elements smaller than the acoustic wavelength of the material that can be dynamically tuned by changing the spacing, arrangement, and density of their interior elements. By selectively tuning the material properties of the metamaterial, the elastic or acoustic behavior can be significantly altered from conventional material properties. Lui et al. [13] utilized the resonant effects of a cube with small lead balls covered in a thin layer of silicon to increase the absorption coefficient. Resonant metamaterials can be applied to aircraft interior, airframe noise in naval vessels, and controlling noise in automobiles.

1.4. Heterogeneous (HG) metamaterial

Work in studying the properties of a heterogeneous material has been carried out at Virginia Tech in the past two decades, and has evolved into what is now termed a heterogeneous (HG) metamaterial. First, the previous work at Virginia Tech is investigated.

Wright et al. [14] investigated the use of de-tuned vibration absorbers to reduce the sound radiated from a structure. The maximum sound reduction occurred when the each tuned vibration absorber (TVA) was detuned from the excitation frequency by varying its stiffness. Fuller and Cambou [15] extended this approach by investigating a continuous vibration absorber with tunable properties that vary with location. These distributed vibration absorbers (DVA) were
designed by varying the mass distribution using a genetic optimization algorithm. Experimental tests of a DVA on a vibrating beam showed improvement in global noise reduction compared to a TVA of the same weight. Joshi and Jangrid [16] implemented multiple TVA’s tuned to slightly different frequencies for controlling vibration levels of flexible structures. They presented the optimized values for the damping ratio, the tuned frequencies and the frequency bandwidth to minimize the root mean square displacement of the structure. Cambou [17] developed a distributed active vibration absorber consisting of a sinusoidal PVDF layer with an optimal mass distributed on top. Good attenuation was obtained for an active and passive version. However, the PVDF layer acted as an elastic layer with a high stiffness, and a large mass was required for obtaining low frequency attenuation. Marcotte et al. [18] considered applying DVA’s to lower frequencies by investigating materials with a suitable spring stiffness. Acoustic foam was found to provide the necessary stiffness to achieve low frequency attenuation. The DVA’s were designed to have tunable frequencies of 100Hz, 122Hz and 140Hz by varying the thickness of the foam and/or weight of the top mass. Active control using a DVA concept was also investigated. Gentry [19] embedded a layer of piezoelectric actuator in foam to excite the structural and acoustic phases of the foam when driven by an externally supplied control voltage. An experiment was performed by lining a section of duct wall with an array of smart foam and sound was minimized at several downstream error microphones. Successful harmonic and broadband noise control was achieved. In 1999 Mathur et al. [20] tested DVA’s on aircraft and helicopter panels. The results demonstrated that DVA’s provided broadband attenuation of panel vibration and transmitted sound. It was found that the masses of the DVA’s could be directly embedded into an acoustic blanket treatment. The heterogeneous blanket concept was thus created.

A sample of HG material was constructed from a 4 ft. by 4 ft. section of 2 inch thick melamine foam with 50 spherical 6gm individual masses embedded in randomly distributed depths [21]. The results showed close to a doubling of the absorption coefficient at low frequencies compared to a sample of foam with no masses from 50 to 200 Hz due to the resonant frequencies of the masses. The sample was also tested in an acoustic transmission loss (TL) facility at Virginia Tech by locating it on a panel representative of an aircraft fuselage. Increased TL on the order of 6 dB was shown over a low frequency range of 60 to 180 Hz with added masses. The addition of the masses to the panel showed an increase of 6% in weight. Gautam
presented the development of a finite element model to understand the behavior of distributed vibration absorbers and heterogeneous blankets. The variation in material properties and geometrical configurations was also studied on improving vibration attenuation capability of heterogeneous material. Idrisi [23] further studied the modeling and optimization of heterogeneous blankets for the improvement of sound transmission through a double-panel system. A genetic algorithm was used to optimize the design of the heterogeneous blankets. A full-scale fuselage experiment was performed on a Gulfstream section and the results indicated that the proper tuning of heterogeneous blankets can result in broadband noise reduction below 500 Hz with less than 10% added mass. Fuller and Saux [24] investigated the application of acoustic metamaterial concepts to improving the sound performance of poro-elastic foams. Experimental testing of different configurations of HG material in which the masses are periodically arranged are performed, and the results show promise for a material system with low frequency sound absorption capabilities. Further work must be done to understand the behavior of a HG material arranged in a periodic configuration by employing a finite element model.

This research seeks to improve upon the work at Virginia Tech by further investigating a heterogeneous material with periodically spaced embedded masses in a poro-elastic material by developing a finite element model to compare with analytical and experimental results. This concept is unique from previous studies of HG material in that a periodic arrangement of masses within a poro-elastic material is investigated.

A heterogeneous (HG) metamaterial is a new class of acoustic metamaterial. It is defined as a composite system consisting of multiple small masses embedded within a passive poro-elastic matrix material. The embedded masses create an array of resonant mass-spring-damper systems within the material that operate at low frequencies where the passive poro-elastic material is no longer effective. By employing the poro-elastic material to provide the stiffness for the embedded masses, the HG metamaterial utilizes two passive control schemes: damping at high frequencies, and dynamic absorption at low frequencies, into a single device for broadband noise reduction. The displacement of the masses against the foam stiffness at their low frequency resonance leads to an increase in mechanical damping losses and absorption. An increased effect of the embedded mass on the poro-elastic material is due to a mismatch in the impedance between the two materials. For optimum absorption a larger impedance mismatch is desired [25].
HG metamaterials can be used for controlling low frequency sound radiation, improving low frequency transmission loss when attached to vibrating structures, and is a lighter and thinner replacement to conventional materials [26; 27]. Embedding spherical masses in varying depths of melamine foam showed an increased absorption as a function of depth. These materials have shown to significantly reduce interior noise with only a marginal increase in the overall mass of the structure. It has been demonstrated that HG metamaterials can be used as lightweight blanket treatments for effectively controlling low frequency sound radiating from structures [28; 29]. Kidner et al. concluded that HG metamaterial is more efficient when placing the masses to target certain modes by varying the depth, weight, or shape [30]. Proper tuning will result in a mode split of the targeted resonance into two damped peaks above and below the original peak [31]. It was also demonstrated that porous materials having porous inclusions, called composite porous materials, show increased performance in sound absorption and sound insulation [32].

The development of numerical finite element models will allow for an advanced understanding of the physics behind the material functionality. These models will be used for conducting parametric studies in order to develop more advanced and effective designs.

Figure 1: HG metamaterial schematic

In this paper, multiple studies will be performed to investigate the effect of each parameter on the acoustic performance of a HG metamaterial. These studies will be utilized to gain more knowledge about the physics of HG metamaterials and will aid in the design and manufacturing for specific applications. Figure 1 illustrates the arrangement of periodically
distributed masses acting as a series of mass-spring-damper systems embedded in a poro-elastic material.

There is a wide range of applications available for poro-elastic HG metamaterials. One broad application includes the placement of these materials on aircraft for the damping of sound and vibration. This application is largely dictated by the choice of the poro-elastic matrix material. In this paper, two poro-elastic foams are investigated. The first poro-elastic material, melamine foam, is a commonly used as acoustic foam that does not have stringent flammability requirements. This foam can be used on launch vehicles due to the lack of this requirement. The second, polyimide foam, has stringent flammability requirements and can therefore be applied to aircraft where there are flammability requirements.

HG metamaterials can be extended to active techniques that use a control system to attenuate sound. Fuller and Kidner [28] demonstrated an active and passive version of a HG blanket by utilizing active masses. 5 active spherical 6gm individual weight masses were embedded within a foam treatment and placed on top of a vibrating beam. The results show that the active version of a HG blanket demonstrated an additional broadband performance over the passive version. In this study, an active heterogeneous metamaterial consists of a distributed array of linear actuators embedded within a poro-elastic matrix. The actuators function as the active noise control elements which can actively modify the acoustic impedance at its surface, resulting in a net decrease in sound power through the material. Some or all of the elements of the passive HG metamaterial can be replaced by active elements to create a composite hybrid active-passive HG metamaterial system. This study uses a similar form of active input as studied by Fuller and Kidner, but is used to control sound as opposed to structural vibration.

The goal of the active HG metamaterial is to yield narrowband and broadband sound attenuation. Passive poroelastic materials are effective at attenuating sound at high frequencies, while the active elements are effective at attenuating sound at low frequencies. A feedforward, filtered-x LMS control algorithm is used to minimize the downstream pressure using one error microphone located downstream of the sample to achieve sound attenuation at a point. An active-passive hybrid is used in this research to develop an active HG metamaterial. Due to its compact nature, active metamaterial has many advantages over common active noise control techniques that use secondary acoustic sources around the primary noise source. It is shown in
the literature that active control has the potential to provide broadband sound control, increased controller reliability, and decreased control spillover. Many applications arise in interior aircraft noise control where size and weight are critical issues [33; 34].

1.5. Objectives

The main objectives of this research are to investigate new forms of acoustic metamaterial using analytical, experimental, and numerical methods. Analytic formulations and experimental tests will be used to validate a numerical finite element model using COMSOL Multiphysics. In order to understand the physical behavior of acoustic metamaterials, a numerical study is performed by adjusting key acoustic parameters. This study is continued into a real world test using a reverberation room at NASA Langley Research Center.

Two key acoustic performance metrics of interest are the absorption coefficient and transmission loss. The absorption coefficient quantifies the ratio of the absorbed and incident energy of a material, while the transmission loss quantifies the amount of energy that is not transmitted through the material. Using HG metamaterial, these quantities can be decreased or tuned at specific frequencies while having a mass or volume smaller than conventional acoustic material. The acoustic absorption coefficient and transmission loss will be studied in a normal incident impedance tube. HG metamaterial will be designed by embedding masses periodically within a poro-elastic foam. An extensive study of HG metamaterial is performed by studying the effects of the poro-elastic properties, as well as the number, shape, and distribution of the embedded masses.

Microperforated panels (MPP) is also studied for absorption and transmission loss and compared to HG metamaterials. Designs are performed combining MPP with HG metamaterial by layering the MPP within a poro-elastic material or adding an air cavity backed by HG metamaterial downstream of the MPP. MPP as a form of metamaterial is investigated by embedding periodic layers of MPP within a poro-elastic material.

Active feedforward control is implemented into acoustic HG metamaterial by utilizing actively responding elements within the poro-elastic matrix. Samples are tested in a duct with active vibrating masses using an active control algorithm to achieve desired attenuation of tonal and broadband frequencies.
1.6. Outline

Chapter 1 provides the motivation behind the work presented in this thesis along with an introduction to noise control and acoustic metamaterials. It also provides an introduction to heterogeneous (HG) metamaterials and active noise control, and lists the objectives of this research.

Chapter 2 provides a detailed description of the analytical and numerical formulation of an acoustic HG metamaterial. The theoretical modeling of sound propagation through a poro-elastic material is presented. Experimental testing is performed to measure the absorption coefficient and transmission loss of a test sample using an impedance tube. The numerically computed results are presented and validated using experimental results. An extensive numerical parametric study is performed to explore design strategies and configurations of different HG properties.

Chapter 3 introduces microperforated panels (MPP) and provides a description of the analytical modeling and theory. A numerical model is built and results are validated using experimental tests performed in an impedance tube.

Chapter 4 develops a microperforated panel as a form of acoustic metamaterial. Studies are performed with multiple MPP embedded periodically within an air cavity and embedded periodically within poro-elastic material. A HG metamaterial is then combined with MPP and additional experimental and numerical parametric studies are performed.

Chapter 5 investigates new arrangements of an active acoustic HG metamaterial and a brief outline of feedforward control theory is presented. Tonal and broadband noise control is studied using a SISO feedforward, filtered-x LMS controller. Sound reduction is measured using an error microphone located downstream of the sample in an impedance tube and results are presented.

Chapter 6 summarizes the major conclusions and offers future work of this research.
2. HETEROGENEOUS (HG) ACOUSTIC METAMATERIAL

This chapter describes the analytical and numerical modeling of an acoustic heterogeneous (HG) metamaterial developed as part of this thesis. The theoretical modeling of sound propagation through a poro-elastic material is presented. Experimental testing is performed to measure the absorption coefficient and transmission loss of a test sample using an impedance tube. A numerical model using COMSOL Multiphysics is built and the results are presented and validated using experimental results. A numerical parametric study is performed to explore different configurations of a HG metamaterial and conclusions are presented. Using the knowledge obtained from the numerical parametric studies, a HG metamaterial panel is tested in a reverberation room to measure the diffuse field absorption coefficient compared to a poro-elastic panel. Results are presented and discussed.

The goal of a HG metamaterial design is to increase the absorption coefficient and transmission loss of acoustic foam at low frequencies. The absorption coefficient ($\alpha$) is the ratio of absorbed and incident energy in a material backed by a rigid plate. The transmission loss (TL) is the reciprocal of the transmission coefficient defined as the ratio of sound energy transmitted through a material to sound energy incident on the material, expressed in decibels (dB).

2.1. Heterogeneous metamaterial vibration study

The purpose of the heterogeneous metamaterial vibration study is to understand how the resonant frequency and damping is affected by varying the depth of a 6gm spherical steel mass in a 2in sample of melamine foam. This model will be approximated to a 1 degree-of-freedom mass-spring-damper system to analytically determine the resonant frequency of the system excited by a vertical harmonic displacement. A numerical model is also built using COMSOL Multiphysics to determine the damping ratio and resonant frequency as a function of mass depth. Finally, an experimental test is performed where the system is placed on a shaker and a transfer function of the plate motion to vertical mass motion will be recorded to determine the damping ratio and resonant frequency of the system.
2.1.1. HG metamaterial analytical vibration model

A simple 1 degree-of-freedom mass-spring-damper system is built to approximate a spherical mass embedded in a sample of melamine foam. As shown in Figure 2, a mass is attached to a spring and damper connected in parallel and connected to a plate that is being forced in a harmonic motion. The mass represents the spherical steel mass, the spring represents the equivalent stiffness of melamine foam, the damper represents the equivalent damping of the melamine foam, and the base represents a vibrating structure. Table 1 shows the values used to calculate the resonant frequency of this system.

Table 1: Material properties for analytic vibration study calculations

<table>
<thead>
<tr>
<th>Material</th>
<th>Material Property</th>
<th>Property Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Melamine</td>
<td>Young’s modulus, $E$</td>
<td>400 kPa</td>
</tr>
<tr>
<td></td>
<td>Damping ratio, $\zeta$</td>
<td>0.0625</td>
</tr>
<tr>
<td>Steel</td>
<td>Density, $\rho$</td>
<td>7850 kg/m$^3$</td>
</tr>
<tr>
<td></td>
<td>Radius, $r$</td>
<td>7/32 in</td>
</tr>
</tbody>
</table>

The stiffness of the foam is calculated using the stiffness model for a material in tension or compression by the formula

$$ k = \frac{AE}{L} $$  \hspace{1cm} (1)

where $E$ is the Young’s Modulus of melamine foam, $A$ is the cross-sectional area of the foam below the supported mass, and $L$ is the depth of the embedded mass in the foam. The cross-sectional area of the foam is the projected area of the sphere, or $A = \pi r^2$.

The damping constant $c$ is determined by the formula

$$ c = 2\zeta \sqrt{mk} $$  \hspace{1cm} (2)
The transfer function of the single degree of freedom mass spring damper system is

\[ TF = \frac{1}{ms^2 + cs + k} \]  

(3)

Using MATLAB, the magnitude of the frequency response function is shown below in Figure 3.

![Figure 3: Transfer function between rigid plate and mass vertical motion as a function of depth embedded in 2in melamine - Analytical](image)

The natural frequency of the system is represented by the formula

\[ \omega_n = \sqrt{\frac{k}{m}} \]  

(4)

where \( m \) is the mass of the steel sphere, calculated from the density using the formula

\[ m = \rho V = \rho \left( \frac{4}{3} \pi r^3 \right) = 0.00564 kg \]  

(5)

and the damped natural frequencies are calculated by the formula

\[ \omega_d = \omega_n \sqrt{1 - \zeta^2} \]

The damped natural frequencies are calculated as a function of depth of the embedded mass, ranging from 0.4in to 2in, where the depth is measured from the base. Figure 4 shows the
analytical damped natural frequencies of a 6gm steel mass as a function of depth embedded in 2in melamine foam.

![Graph showing damped natural frequency as a function of depth]

Figure 4: Damped natural frequency of a 7/16in diameter spherical steel mass as a function of depth embedded in 2in melamine - Analytical

2.1.2. HG metamaterial numerical vibration model

![Figure 5 showing 2in melamine foam with 7/16in diameter spherical steel mass]

Figure 5: 2in melamine foam embedded with 7/16in diameter spherical steel mass

A numerical model in COMSOL is built to calculate the damping ratio and resonant frequency of a spherical mass embedded in foam and attached to a vibrating structure. To model this system, melamine foam is modeled as a poro-elastic material with a length, width, and height of 2in. Since the model is for structural analysis, only the Young’s modulus, Poisson’s ratio, and density of melamine are taken into account. The spherical mass is modeled as a linear elastic material and embedded in the center of the foam. Table 2 shows the material properties used for melamine and steel.
Table 2: Material properties for vibration study numerical calculations

<table>
<thead>
<tr>
<th>Material</th>
<th>Material property</th>
<th>Property Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Melamine Foam</td>
<td>Young’s modulus, E</td>
<td>(400+50i) kPa</td>
</tr>
<tr>
<td></td>
<td>Poisson’s ratio, ν</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>Density, ρ</td>
<td>9.85 kg/m³</td>
</tr>
<tr>
<td>Steel</td>
<td>Young’s modulus, E</td>
<td>205e6 kPa</td>
</tr>
<tr>
<td></td>
<td>Poisson’s ratio, ν</td>
<td>0.28</td>
</tr>
<tr>
<td></td>
<td>Density, ρ</td>
<td>7850 kg/m³</td>
</tr>
</tbody>
</table>

A prescribed displacement is applied to the base of the foam, where the motion is prescribed only in the z-direction (vertical motion). A frequency domain study is performed over the range of 1-250Hz in 1Hz increments. In post-processing, the vertical displacement of the steel mass is recorded and the vertical displacement of the base is recorded. To calculate the transfer function between the rigid plate and mass vertical motion, the vertical displacement of the sphere is divided by the vertical displacement of the base and converted to dB. Figure 6 shows the transfer function between the rigid plate and mass vertical motion as a function of depth in 2in melamine using COMSOL.

Figure 6: Transfer function between rigid plate and mass vertical motion as a function of depth in 2in melamine – COMSOL
From these transfer functions, the resonant frequencies and the damping ratio at each depth can be calculated. The resonant frequency is the frequency where the peak magnitude of the transfer function exists, and the damping ratio in percentage is calculated from the half power bandwidth equation

\[ \zeta = 100 \left( \frac{\omega_2 - \omega_1}{2\omega_n} \right) \]  \hspace{1cm} (6)

where \( \omega_n \) is the resonant frequency, and \( \omega_1 \) and \( \omega_2 \) are the frequencies to the left and right of the resonant frequency where the magnitude drops by 3dB, respectively. Figure 7 shows the resonant frequency, Figure 8 shows the equivalent stiffness, and Figure 9 shows the damping ratio of a 6gm steel mass as a function of depth embedded in 2in melamine.

![Resonant frequency graph](image)

Figure 7: Resonant frequency of a 7/16in diameter spherical steel mass as a function of depth embedded in 2in melamine – COMSOL
Figure 8: Equivalent stiffness of a 7/16in diameter spherical steel mass as a function of depth embedded in 2in melamine – COMSOL

Figure 9: Damping ratio of a 7/16in diameter spherical steel mass as a function of depth embedded in 2in melamine – COMSOL
2.1.3. HG metamaterial experimental vibration study

![Experimental setup of calculating resonant frequency and damping ratio of a 7/16 diameter spherical steel mass embedded in 2in melamine foam – open and closed](image)

Experimental tests are performed to compare to analytical results and to verify the COMSOL model. As shown in Figure 10, a 2in cube sample of melamine foam is cut along its cross-sectional area and a depth of 1in is cut into the foam where the embedded mass will be placed. The diameter of the mass is precision cut into the foam, and a 7/16in diameter 6gm spherical steel mass is embedded in the foam. A small ICP accelerometer is then attached to the top of the spherical mass with a piece of wax. The remaining top half of the foam is further cut to contain the accelerometer and is placed on top. The base of the foam is then glued to the base of a shaker, and a second accelerometer is placed on the base of the shaker close to the sample to measure the vertical motion of the base.

A 400Hz bandwidth random signal is sent to the shaker to induce vibrations to the sample. Using B&K Pulse Data Acquisition Software, a frequency response, H1, is calculated using the signal from the mass accelerometer to the reference signal from the base accelerometer to calculate a transfer function between the two.

Three different depths are performed to analyze the transfer function over a range of different cases. The different depths of the embedded masses are 0.6in, 1.0in (as shown in Figure 10), and 1.8in. This data is presented and compared against numerical and analytical results to validate the COMSOL model.
2.1.4. Results and validation of HG metamaterial vibration study

Figure 11 as shown below compares the transfer functions calculated from the analytical, experimental and numerical results using COMSOL. Results show that the same approximate resonant frequency and damping exists between the three tests. The embedded masses act as resonant systems to increase the structural impedance of the material. The resonant frequency of the system is moves to lower frequencies as the mass depth is moved closer to the base.

![Graph showing transfer function comparison](image)

**Figure 11**: Transfer function between rigid plate and mass vertical motion as a function of depth in 2in melamine – Experimental vs. COMSOL

Figure 12 shows the damping ratio compared against experimental and numerical results. Results show that the damping ratio is approximately the same for the three different tests.
Figure 12: Damping ratio of a 7/16in diameter spherical steel mass as a function of depth embedded in 2in melamine – Experimental vs. COMSOL

Figure 13 shows the resonant frequency compared against experimental, numerical, and analytical results. Results show that the resonant frequency is approximately the same for all tests. Generally, as the mass is moved closer to the base, the resonant frequency increases.

Figure 13: Resonant frequency of a 7/16in diameter spherical steel mass as a function of depth embedded in 2in melamine – Analytical vs. Experimental vs. COMSOL
2.2. Modeling of HG metamaterial in an impedance tube

2.2.1. Analytical modeling of a poro-elastic material

The theory describing sound propagation in isotropic, poro-elastic material can be explained by the Biot theory as presented by Allard [35]. The Biot theory assumes the poro-elastic material to be homogeneous and isotropic. In order to satisfy this condition, the pore size must be much smaller than the macroscopic elementary volume of the material being evaluated. The macroscopic elementary volume is assumed to be much smaller than the wavelength of propagating sound so the acoustic properties can be considered constant throughout the element. Deformations are assumed to be small which guarantees linearity in the mechanical process, and viscous damping at the pore walls is considered. The following sections use an equivalent fluid representation of a poro-elastic material to calculate the absorption coefficient and transmission loss using the characteristic impedance and complex wave number for many types of poro-elastic material.

2.2.1.1. Analytical modeling of the absorption coefficient of a poro-elastic material

The normal incident absorption coefficient of a poro-elastic material backed by a rigid wall is calculated using the laws of Delany and Bazley [35]. The normal flow resistivity of a 0.0254 m sample of melamine is assumed to be 10,000 Nm⁻⁴s. Assuming the density and speed of sound in air to be 1.225 kg/m³ and 343.2 m/s, respectively, the complex wave number \( k \) and the characteristic impedance \( Z_c \) are calculated for a large range of frequencies in many fibrous materials with porosity close to 1. The quantities \( k \) and \( Z_c \) depend on the angular frequency \( \omega \) and the flow resistivity \( \sigma \) of the material. These quantities are calculated using the following equations, where \( X = \rho_0 f / \sigma \) is a dimensionless parameter, \( \rho_0 \) is the density of air, and \( f \) is the frequency.

\[
Z_c = \rho_0 c_0 \left[ 1 + 0.057X^{-0.754} - j0.087X^{-0.732} \right] \tag{7}
\]

\[
k = \frac{\sigma}{c_0} \left[ 1 + 0.0978X^{-0.700} - j0.189X^{-0.595} \right] \tag{8}
\]
In Figure 14, an acoustic plane wave is incident on an infinite poro-elastic material backed by a rigid wall. A second wave is shown reflecting off the surface \((x = -d)\). If the acoustic field is a superposition of these two waves, the total pressure \(p_T\) and the total velocity \(v_T\) are described by equations 9 and 10, respectively.

\[
p_T(x, t) = Ae^{j(kx + \omega t)} + A'e^{j(kx + \omega t)} \quad (9)
\]
\[
v_T(x, t) = \frac{A}{Z_c}e^{j(kx + \omega t)} - \frac{A'}{Z_c}e^{j(kx + \omega t)} \quad (10)
\]

where \(A\) and \(A'\) are the amplitudes of the incident and reflecting waves and \(Z_c\) is the characteristic impedance as seen in equation 7. The impedance at \(M_1\) \((x = 0)\) can be written as

\[
Z(M_1) = \frac{p_T(M_1)}{v_T(M_1)} = Z_c \frac{Ae^{-jkm_1} + A'e^{jkm_1}}{Ae^{-jkm_1} - A'e^{jkm_1}} \quad (11)
\]

At location \(M_2\) \((x = -d)\), the impedance can be written as

\[
Z(M_2) = \frac{p_T(M_2)}{v_T(M_2)} = Z_c \frac{Ae^{-jkm_2} + A'e^{jkm_2}}{Ae^{-jkm_2} - A'e^{jkm_2}} \quad (12)
\]

Equation 11 can be rewritten to solve for the ratio \(A'/A\)

\[
\frac{A'}{A} = \frac{Z(M_1) - Z_c e^{-2jkm_1}}{Z(M_1) + Z_c} \quad (13)
\]
Equation 13 can be substituted into equation 12 to eliminate the dependence on amplitude, where $d$ is equal to $x(M_1) - x(M_2)$. This is known as the impedance translation theorem.

$$Z(M_2) = Z_c \frac{-jZ(M_1) \cot kd + Z_c}{Z(M_1) - jZ_c \cot kd} \quad (14)$$

Assuming the impedance at location $M_1$ is infinite because it is a rigid impervious plate, the impedance at location $M_2$ can be rewritten as

$$Z(M_2) = -jZ_c \cot kd \quad (15)$$

From the impedance equation, the reflection factor $R$ can be calculated, and the absorption coefficient $\alpha$ can then be solved.

$$R = \frac{Z(M_2) - \rho_0 c_0}{Z(M_2) + \rho_0 c_0} \quad (16)$$

$$\alpha = 1 - |R|^2 \quad (17)$$

The MATLAB codes used to calculate the analytical absorption coefficient are found in Appendix B.1.

2.2.1.2. Analytical modeling of the transmission loss of a poro-elastic material

The sound pressure and normal particle velocity at the rear surface of a poro-elastic sample [36] of thickness $d$ can be calculated from the sound pressure and normal particle velocity at the front surface using a four pole matrix shown by

$$[P]_{x=0} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} [P]_{x=d} \quad (18)$$

The transmission coefficient, $T$, of a poro-elastic sample is a function of frequency and is the ratio of the sound energy transmitted through the material to the sound energy incident on the material. The transmission coefficient and transmission loss are expressed by

$$T = \frac{2e^{jkd}}{T_{11} + T_{12} + T_{21} + T_{22}} \quad (19)$$

$$TL = -20 \log_{10}|T| \quad (20)$$

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \begin{bmatrix} \cosh(kd) & Z_c \sinh(kd) \\ \sinh(kd)/Z_c & \cosh(kd) \end{bmatrix} \quad (21)$$
Zc and k are the characteristic impedance and wave number of the poro-elastic sample, calculated from equations 7 and 8. The MATLAB codes used to calculate the analytical transmission loss are found in Appendix B.2.

2.2.2. Numerical modeling using COMSOL Multiphysics

Finite element analysis is a useful numerical technique for optimizing new designs, predicting performance and evaluating new concepts. The basic concept of finite element (FE) modeling is the subdivision of the domain of a problem into simpler parts or elements. Using finite elements, complex partial differential equations describing the behavior of a system can be reduced to a set of linear equations that can easily be solved. FE modeling allows for the accurate representation of complex geometry with the inclusion of dissimilar material properties with a reasonable degree of confidence.

Two finite element models are built using COMSOL Multiphysics to calculate the absorption coefficient and transmission loss of test samples. Numerical predictions are compared against experimental and analytical results to validate the model.

2.2.2.1. Absorption coefficient COMSOL modeling

Figure 15: COMSOL model of (a) absorption coefficient and (b) transmission loss
A 3D cylindrical model using the Poroelastic Waves physics module in COMSOL Multiphysics is used to predict absorption [37]. The COMSOL model is shown in Figure 15a. The geometry consists of two cylindrical domains stacked on top of each other. The top cylinder is modeled as a linear elastic fluid (air) in the pressure acoustics domain. The bottom cylinder is modeled in the poroelastic domain as a porous sample. The porous material is described as a drained isotropic matrix with the fluid properties of air. Biot’s low frequency range approximation is used to determine the fluid viscosity. The transition between the low and high frequency range approximation is defined by the reference frequency $f_c$ given by the expression

$$
f_c = \frac{\rho_p \mu}{2\pi \kappa \rho_f} \quad (22)
$$

where $\rho_f$ is the fluid (air) density, 1.2 kg/m$^3$, and $\mu$ is the dynamic viscosity of air, 1.8·10$^{-5}$ Pa·s, and the porosity and permeability are defined for the poro-elastic materials melamine and polyimide as seen in Table 3. The reference frequency $f_c$ for melamine and polyimide calculated to be 1432Hz and 5372Hz, respectively.

The heterogeneous sample consists of 36 spheres embedded within foam in 3 equidistant layers. Figure 16a shows a cross-section view of one layer of heterogeneous material. As shown in Figure 16b, within each periodic layer 12 spheres are periodically spaced with a center-to-center distance equal to a quarter of the outer radius of $d = 0.9365$in. Three equidistant layers are spaced a distance equal to a quarter of the total height of the sample of 1in. The spheres are modeled as linear elastic materials with a 7/16in diameter.

Figure 16: (a) Cross-section view of heterogeneous material and (b) experimental cross-section view of 12 periodically spaced steel masses in melamine
The boundary conditions include a sound hard boundary on the outer walls of the air domain, and a porous, fixed constraint on the outer walls of the porous material. This condition makes the porous matrix fully constrained at the edges where the displacements are zero in all directions, and sets a sound-hard boundary for fluid pressure. Free and continuity boundaries exist between the different domains.

Plane wave radiation is applied to the top boundary, with an incident pressure field $p_{\text{inc}} = 1\text{Pa}$. Plane wave radiation adds a radiation boundary condition for a plane wave. A plane wave travels towards the poro-elastic material, and a reflected wave travels in the opposite direction.

A direct frequency analysis is performed from 20-2,000Hz in 20Hz increments. The average pressure $p$ on the surface of the sample is measured at each frequency, and the absorption coefficient $\alpha$ is calculated as

$$\alpha = 1 - \left| \frac{p_{\text{scat}}}{p_{\text{inc}}} \right|^2$$  \hspace{1cm} (23)

where $p_{\text{scat}} = p - p_{\text{inc}}$.

### 2.2.2.2. Transmission loss COMSOL modeling

A second model is built to calculate the transmission loss of a poro-elastic material with embedded masses. The model is similar to the absorption coefficient model, but a pressure acoustics domain is placed both above and below the poro-elastic sample [38]. Plane wave radiation is applied to the top and bottom boundaries, with an incident pressure field, $p_{\text{inc}} = 1\text{Pa}$, applied to the top boundary. The tube lengths are both 2in long. The transmission loss is calculated as

$$TL(f) = 20 \log \left( \frac{|p_{\text{inc}}|}{|p_{\text{out}}|} \right)$$  \hspace{1cm} (24)

where the average pressure $p_{\text{out}}$ is taken from the outlet of the sample. Figure 15b shows the COMSOL model for calculating transmission loss, and Table 3 shows the material properties used.
Table 3: Material Properties of acoustic HG metamaterials

<table>
<thead>
<tr>
<th>Material</th>
<th>Density [kg/m³]</th>
<th>Poisson’s ratio</th>
<th>Young’s modulus [kPa]</th>
<th>Biot-Willis coefficient</th>
<th>Porosity</th>
<th>Permeability [m²]</th>
<th>Tortuosity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Melamine</td>
<td>9.85</td>
<td>0.4</td>
<td>400 + 50i</td>
<td>0.99</td>
<td>0.99</td>
<td>1.5e-9</td>
<td>1.0059</td>
</tr>
<tr>
<td>Polyimide</td>
<td>9.6</td>
<td>0.45</td>
<td>60 + 20i</td>
<td>0.45</td>
<td>0.45</td>
<td>0.2e-9</td>
<td>3.25</td>
</tr>
<tr>
<td>Steel</td>
<td>7850</td>
<td>0.28</td>
<td>205e6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aluminum</td>
<td>2700</td>
<td>0.33</td>
<td>70e6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Polypropylene</td>
<td>927.8</td>
<td>0.45</td>
<td>1.55e6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2.2.3. Sensitivity study on the properties of a poro-elastic material

The choice of the type of poro-elastic material used in the COMSOL studies has a large effect on the absorption and transmission loss responses of the material. The acoustic properties of an isotropic poro-elastic material are specified by seven macroscopic parameters. The three structural parameters are the density, Poisson’s ratio, and Young’s modulus, the three acoustic properties are the Biot-Willis coefficient, porosity, and permeability, and the geometric parameter is the tortuosity. These seven parameters are varied on measuring the normal incidence absorption coefficient and transmission loss as measured in a standing wave tube modeled in COMSOL Multiphysics. By performing a parameter sweep on each property while holding the others constant, the sensitivity of each parameter on the absorption performance of a poroelastic material is further understood. This model is based on the finite element implementation of the Biot theory for wave propagation in poroelastic material.

2.2.3.1. Sensitivity of poro-elastic material to density

Density is a structural property of the foam, defined as the mass per unit volume. Figure 17 shows the sensitivity of the absorption coefficient and transmission loss of 4in melamine to density. As the density increases, the natural frequency of the foam moves to lower frequencies. As the results show, this corresponds to a moving of the peak absorption to lower frequencies. The peak transmission loss is also moved to lower frequencies. Furthermore, foam with a higher density results in a higher transmission loss at higher frequencies because the added mass of the foam impedes the acoustic waves transmitting through the material.
Figure 17: Sensitivity of (a) absorption coefficient and (b) transmission loss of 4in melamine to density
2.2.3.2. Sensitivity of poro-elastic material to Poisson’s ratio

The Poisson’s ratio is a structural property defined as the ratio of transverse to axial strain. Figure 18 shows the sensitivity of the absorption coefficient and transmission loss of 4in melamine to Poisson’s ratio. The results show that the absorption and transmission loss is largely unaffected by changes in the Poisson’s ratio. This can be explained because a standing wave is normally incident upon the sample, and due to the edge effects of the tube, all radial and solid phase displacements are zero at the circumferential edges of the tube. Therefore, any acoustic waves traveling normal to the axial direction of the tube have little effect on the normal incidence absorption and transmission loss.

Figure 18: Sensitivity of (a) absorption coefficient and (b) transmission loss of 4in melamine to Poisson’s ratio
2.2.3.3. Sensitivity of poro-elastic material to Young’s modulus

The Young’s modulus, or elastic modulus, is a measure of the stiffness of an elastic isotropic material. This property has the opposite effect of the density of the material. As the Young’s modulus increases, the natural frequency of the foam increases. Figure 19 shows the sensitivity of the absorption coefficient and transmission loss of 4in melamine to the Young’s modulus. As the Young’s modulus increases, the absorption peak is moved to higher frequencies. The peak transmission loss is also moved to higher frequencies. As seen at high frequencies, the foam is not very sensitive to changes in the Young’s modulus.

![Figure 19: Sensitivity of (a) absorption coefficient and (b) transmission loss of 4in melamine to Young’s modulus](image-url)
2.2.3.4. Sensitivity of poro-elastic material to Biot-Willis coefficient

The Biot-Willis coefficient is a dimensionless acoustic property that relates the bulk modulus, or compressibility, of the drained porous matrix to a block of solid material. A rigid porous material has a Biot-Willis coefficient approximately equal to the porosity, and a soft or limp porous material has a Biot-Willis coefficient approximately equal to 1.

Figure 20 shows the sensitivity of the absorption coefficient and transmission loss of 4in melamine to the Biot-Willis coefficient. The absorption coefficient results show that changing the Biot-Willis coefficient does not largely affect the absorption. The transmission loss is increased at certain frequencies, but it decreases at others. This parameter is largely not sensitive to the absorption and transmission loss through the foam.

![Figure 20: Sensitivity of (a) absorption coefficient and (b) transmission loss of 4in melamine to Biot-Willis coefficient](image)
2.2.3.5. Sensitivity of poro-elastic material to porosity

The porosity is a dimensionless acoustic property that is defined as the amount of void volume inside the porous material and varies from 0, where there is only fluid material, and 1, where there is only solid material. Figure 21 shows the sensitivity of the absorption coefficient and transmission loss of 4in melamine to the porosity. In general, porous materials with higher porosity have a smoother absorption versus frequency, and as the porosity decreases to a fluid material the absorption peaks are more pronounced. This shows that the solid phase of the foam is able to effectively absorb a broadband range of frequencies, and a predominantly fluid material is only effective at absorbing frequencies near resonance. The transmission loss of the material is not as sensitive to changes in porosity. As the dimensions of the pores are smaller, losses occur in the foam due to thermal conduction and viscous friction.

Figure 21: Sensitivity of (a) absorption coefficient and (b) transmission loss of 4in melamine to porosity
2.2.3.6. Sensitivity of poro-elastic material to permeability

The permeability is an acoustic property that is a measure of the ability of the porous material to allow fluid to pass through it. It is similarly related to the porosity of the porous material. Figure 22 shows the sensitivity of the absorption coefficient and transmission loss of 4in melamine to the permeability. The results show that as the permeability decreases, less fluid is allowed to pass through the material and transmission loss is higher at low frequencies. Conversely, the absorption coefficient is lower at low frequencies. All of the cases studied appeared to share the same resonant peak of absorption near 1100 Hz.

Figure 22: Sensitivity of (a) Absorption coefficient and (b) transmission loss of 4in melamine to permeability
2.2.3.7. Sensitivity of poro-elastic material to tortuosity

The tortuosity, or the structural form factor, is a dimensionless geometrical property that is related to the complexity of the propagation path through the material. It is conventionally defined as the deviation from cylindrical passages within the foam.

Figure 23 shows the absorption coefficient and transmission loss of 4in melamine to tortuosity. It is shown that the absorption peak can be shifted by controlling the tortuosity. As the tortuosity increases, the absorption peak moves to lower frequencies and the levels of transmission loss increases. Generally, the absorption and transmission loss is increased as the tortuosity increases, signifying that the acoustic waves are more likely to be absorbed in a foam that has a complex path of propagation.

Figure 23: Sensitivity of (a) absorption coefficient and (b) transmission loss of 4in melamine to tortuosity
2.2.3.8. Sensitivity of small tube vs. large tube

Measurements in standing wave tubes can be affected by the edge effects of the tube. The diameter of the tube, therefore, will affect the absorption and transmission loss results.

To understand the edge effects in a tube, a small tube is modeled with a cylinder of radius 1.873 inches and a fixed boundary condition is applied to the circumferential edges. By this constraint, the solid phase displacement at the edge is set to zero in the radial and axial directions, as well as the fluid phase displacement in the radial direction. These conditions assume the foam is bonded to the inner surface of the tube. The large tube is modeled by applying a roller boundary condition to the circumferential edges of the cylinder. By applying this constraint, the solid phase axial displacement is no longer zero and the edge effects of the small tube no longer apply.

Figure 24 compares the absorption coefficient and transmission loss measured in a small tube and a large tube. It is shown that there is a significant difference between small and large tube results. The first absorption peak appears at a lower frequency in the large tube than it does in the small tube.

The tight contact between the foam and tube alters the acoustically induced motion of the solid part of the porous material, thus altering the frequency dependence of the absorption coefficient [39]. The edge effects also result in a higher transmission loss at low frequencies compared to the large tube. The large tube results are more representative of the foam because due to the relatively small thickness, it is not able to absorb long wavelengths of sound waves corresponding to low frequencies.
Figure 24: (a) Absorption coefficient and (b) transmission loss of 4in melamine in small tube vs. large tube
2.3. Experimental setup of impedance tube

Experiments are conducted to investigate the effects of utilizing HG metamaterials to increase the acoustic absorption coefficient and transmission loss of poro-elastic materials. The goal of the experimental tests is to verify the numerical results and to characterize the properties of the materials. The initial HG metamaterial sample configuration consists of 12 spherical masses embedded within a poro-elastic foam in 3 equidistant layers.

The foam is precision cut using a rotating rig mounted on a band saw, and the blade distance from the center of foam is adjusted to perform a 3.786in diameter cut on a block of 2in thick foam. This is performed to ensure an airtight fit in an impedance tube. To position the arrangement of embedded masses, a template is printed and mounted on the surface of the foam, and the locations of each mass are precisely marked. A circular rotary cutting tool with a diameter equal to the diameter of the embedded mass is drilled the length of the mass radius into the surface of the foam, and a drop of superglue followed by the mass is placed in the cavity. This step is repeated for the other half of the foam and the two pieces are then glued together. An experimental cross-section view of 12 periodically placed steel masses in melamine is shown in Figure 25. Cylindrical samples of two poro-elastic foams, melamine and polyimide, are tested. Different types of embedded 7/16in diameter spheres are also tested, including 6gm steel mass and 1gm polypropylene mass.

Figure 25: Experimental cross-section view of 12 periodically spaced steel masses in melamine
An impedance tube designed and constructed at Virginia Tech is used to measure the absorption coefficient and transmission loss of a sample under normal acoustic wave incidence [40]. The impedance tube can be mounted in two configurations to measure these two acoustic properties. Two B&K ½in capacitor microphones spaced 2.9in and 10.9in apart upstream and downstream the sample are used for measurements in the 233-2070Hz and 84-760Hz bandwidths, respectively. The signals are acquired by a NI data acquisition system and the absorption coefficients, complex impedance, and TL are obtained from VT developed software implemented in MATLAB. See Appendix A.1 for impedance tube requirements. See Figure 26 for the test configurations and Table 4 for a list of the equipment used for performing tests.

![Image of impedance tube setup]

Figure 26: (a) Absorption coefficient and (b) transmission loss impedance tube configuration

<table>
<thead>
<tr>
<th>Table 4: Equipment used for impedance tube experiments</th>
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</thead>
<tbody>
<tr>
<td><strong>Equipment used for Impedance Tube Experiments:</strong></td>
</tr>
<tr>
<td>MATLAB</td>
</tr>
<tr>
<td>Pulse Labshop</td>
</tr>
<tr>
<td>B&amp;K Type 3160-A-042 4ch. Input 2ch. Output Generator Module 50 Khz</td>
</tr>
<tr>
<td>National Instruments NI cDAQ-9178 with 2 chassis</td>
</tr>
<tr>
<td>Rane MA 6 Multi-Channel Amplifier</td>
</tr>
<tr>
<td>4 ½” B&amp;K Condenser Microphones</td>
</tr>
<tr>
<td>BNC Cables</td>
</tr>
<tr>
<td>Impedance Tube with Speaker, Test Holder, Steel Backing Plate and Microphone Slots</td>
</tr>
</tbody>
</table>
2.3.1. Absorption coefficient experiment overview

The test sample is mounted at one end of a straight, rigid, smooth and airtight 48in impedance tube with a 3.786in diameter and cut-off frequency of 2090Hz. The closest microphone is spaced 21in from the sample. A plane wave is sent downstream the tube as random filtered white noise to take account of the speaker dynamics. The complex acoustic transfer function of the two microphone signals is determined and used to compute the normal incident absorption coefficient.

![Figure 27: Absorption coefficient schematic](image)

The absorption coefficient $\alpha$ can be determined from the measuring the transfer function between two microphone positions in front of the test material. The sound pressures of the incident wave $p_I$ and the reflected wave $p_R$, ignoring the time dependent component, $e^{i\omega t}$, are

$$p_I = \hat{p}_I e^{jk_0x}$$  \hspace{1cm} (25)

$$p_R = \hat{p}_R e^{-jk_0x}$$  \hspace{1cm} (26)

The sound pressures $p_1$ and $p_2$ in the two microphone positions are

$$p_1 = \hat{p}_I e^{jk_0x_1} + \hat{p}_R e^{-jk_0x_1}$$  \hspace{1cm} (27)

$$p_2 = \hat{p}_I e^{jk_0x_2} + \hat{p}_R e^{-jk_0x_2}$$  \hspace{1cm} (28)

The transfer function for the incident wave alone $H_I$ is

$$H_I = \frac{p_{2I}}{p_{1I}} = e^{-jk_0(x_1-x_2)} = e^{-jk_0s}$$  \hspace{1cm} (29)
where \( s = x_1 - x_2 \). The transfer function for reflected wave alone \( H_R \) is

\[
H_R = \frac{p_{2R}}{p_{1R}} = e^{j k_0 (x_1 - x_2)} = e^{j k_0 s}
\]  

(30)

The transfer function \( H_{12} \) for the total sound field can be obtained,

\[
H_{12} = \frac{p_2}{p_1} = \frac{e^{j k_0 x_2} + r e^{-j k_0 x_2}}{e^{j k_0 x_1} + r e^{-j k_0 x_1}}
\]  

(31)

where \( \hat{p}_R = r \hat{p}_I \). Transposing to yield \( r \) is

\[
r = \frac{H_{12} - H_I}{H_R - H_{12}} e^{2j k_0 x_1}
\]  

(32)

Solving for \( \alpha \) yields the absorption coefficient

\[
\alpha = 1 - |r|^2
\]  

(33)

The experimental overview is found in Appendix A.2.

### 2.3.2. Transmission loss experiment overview

Transmission Loss is a key indicator of the effectiveness of acoustical treatments for engineering applications. Transmission loss is the decibel ratio of the ratio of sound energy incident on a material to the sound energy transmitted through the material. The experimental setup to calculate transmission loss is represented in Figure 28.

![Figure 28: Transmission loss schematic](image-url)
A speaker sends white noise through the tube as a plane wave, and two sets of microphones measure the acoustic pressure upstream and downstream of the sample. The four acoustic waves related to the material sample are an incident \((A_1)\) and transmitted \((B_2)\) component of the forward traveling wave, and an incident \((B_1)\) and transmitted \((A_2)\) component of the backward travelling wave.

Two sets of tests are run with an open ended and closed (anechoic) termination [41]. To account for a slight air gap between the edge of the foam and the walls of the impedance tube, a thin layer of Vaseline is applied to the outer edges of foam to ensure an airtight fit. This is important because it reduces the amount of sound that may travel through the interior walls if a proper seal is not achieved. Time domain data of the microphones are recorded using an NI Data Acquisition System and post-processed in MATLAB. See Appendix A.3 for a detailed description on how to determine the coefficients of the Transmission Loss Matrix and the experimental overview used to perform transmission loss measurements.

2.4. Results and validation using impedance tube

2.4.1. Melamine COMSOL test validation

Figure 29 shows the absorption coefficient and transmission loss of melamine using numerical and analytical results. For absorption coefficient tests the sample is backed by a rigid plate, and for transmission loss tests the sample is not backed by a rigid plate to allow a transmitted wave to pass through the material. The sample of melamine is varied from 1in to 4in in 1in increments. The increased thickness of melamine greatly improves low frequency absorption and transmission loss. The absorption and transmission is also increased as the input frequency is increased.

It is shown in the numerical calculation that there exists a dip in the transmission loss near 700 Hz that is not seen in the analytical calculation. This is because the numerical model is representative of a sample of foam mounted in a circular duct, where fixed edge conditions are applied to the outer walls to simulate friction in a duct. This results in zero displacement and sound radiation at the walls. The first mode of the structure is activated near 700 Hz, resulting in the most efficient transfer of noise radiated through the structure at this frequency. Song et al. [42] found that the edge constraint results in a shearing resonance of the sample at which
frequency the transmission loss is a minimum. The analytical model, on the other hand, does not experience this resonant behavior as it does not account for friction at the outer walls in a duct. Overall, the numerical results calculated in COMSOL Multiphysics compare well to the analytical results.

Figure 29: (a) Absorption coefficient and (b) transmission loss of melamine with various thickness – COMSOL vs. analytical
Figure 30 shows the effects of adding 36 embedded masses to a 4in sample of melamine foam using numerical and experimental results in an impedance tube. 1gm polypropylene and 6gm individual weight steel spheres of diameter 7/16in are chosen as the material for the respective embedded masses in the following tests. The numerical results calculated in COMSOL Multiphysics compare well to experimental results in an impedance tube. The addition of the embedded masses in melamine does not overall increase the absorption of the material at high frequencies because it is shown that melamine foam is already an effective absorber of sound at high frequencies, thus masking the behavior of the embedded masses. It is also shown that the addition of embedded masses yields an increase in broadband transmission loss, which according to the mass law shows that increasing the weight of the material will result in an increase in transmission loss through the material. Furthermore, the acoustic wave speed is lowered through the material due to scattering effects off of the embedded masses.
Figure 30: (a) Absorption coefficient and (b) transmission loss of 4in melamine – experimental vs. COMSOL

The COMSOL model is also compared to NASA’s Normal Incidence Tube (NIT) to independently validate the results. The NIT tests for the normal incidence absorption coefficient of a sample with a 2in square cross-section from 400-3000Hz in 50Hz increments. Figure 31
compares the absorption coefficient results using experimental, numerical, and analytical methods. The results agree well with each other.

![Graph showing absorption coefficient results using different methods.]

Figure 31: Absorption coefficient of 2in melamine square foam in NIT – analytical vs. experimental vs. COMSOL

2.4.2. Polyimide COMSOL test validation

Figure 32 shows the effects of adding 36 embedded masses to a 4in sample of polyimide foam using numerical and experimental results in an impedance tube. For absorption coefficient tests the sample is backed by a rigid plate, and for transmission loss tests the sample is not backed by a rigid plate to allow for a transmitted wave to pass through the material. 1gm polypropylene and 6gm individual weight steel spheres of diameter 7/16in are chosen as the material for the respective embedded masses in the following tests. The numerical results calculated in COMSOL Multiphysics compare well to experimental results in an impedance tube. The addition of embedded masses introduces a low frequency peak in absorption as well as a broadband increase in transmission loss. It is shown that the low frequency peak in absorption is shifted to lower frequencies as heavier masses are embedded, indicating that this behavior is due to a resonant effect of the embedded masses. Therefore the peak resonant displacement of the masses corresponds to the peak acoustic absorption of the system. This is advantageous because it allows for the tuning of the peak absorption frequency by changing the properties of the embedded masses.
Figure 32: (a) Absorption coefficient and (b) transmission loss of 4in polyimide – experimental vs. COMSOL

2.4.3. Impedance tube test conclusions

The effects of the embedded masses on the absorption coefficient are more evident in polyimide as compared to melamine in Figures 31a and 32a. Melamine has a higher Young’s
modulus resulting in a higher stiffness and a higher resonant frequency. Also melamine already has a high absorption coefficient at low frequencies, which may lead to a masking of the contributions of the embedded masses at low frequencies. The higher stiffness reduces the ability of the embedded masses to displace against the foam, leading to a marginal increase in absorption at lower frequencies.

As seen in Figure 32a, the peak absorption frequency in polyimide moves to lower frequencies as heavier masses are embedded. This effect is due to a dynamic resonance behavior as the masses heavily displace against the stiffness of the foam at their resonant frequency, leading to an increase in damping losses. The frequency of maximum displacement coincides with the frequency of maximum absorption, determined by the stiffness of the foam and the mass of the spheres.

As seen in Figure 32b, the addition of embedded masses increases the broadband transmission loss of the material. A higher impedance mismatch between the two materials leads to a lowering of the acoustic wave speed through the material due to scattering effects. As heavier masses are embedded, the transmission loss increases.

2.5. Numerical parametric studies with HG properties

Parametric design studies of an acoustic HG metamaterial are performed. Each test varies one HG property while keeping all others constant and studies the effects on absorption and transmission loss. The shape, size, material, depth, and spacing of the embedded masses are varied to determine which parameters are most sensitive to the acoustic performance. The baseline material chosen for these studies is polyimide with embedded 1gm polypropylene masses because this combination yields increased absorption at low frequencies as seen in Figure 32a.

2.5.1. Changing the material of the embedded masses

In this test a 4in sample of polyimide is embedded with 36 periodic spherical masses. The material of the embedded masses is varied using polypropylene, aluminum, and steel. The absorption coefficient and transmission loss of these samples are presented in Figure 33.
Figure 33: (a) Absorption coefficient and (b) transmission loss of 4in polyimide with 36 embedded masses of various material - COMSOL

As seen in Figure 33a, as the densities of the embedded masses is increased, the absorption peak moves to lower frequencies. This peak absorption frequency corresponds to the natural frequency of the system where the embedded masses experience peak displacement. It is therefore beneficial to add heavier masses when low frequency sound absorption is desired. As
seen in Figure 34, a power law \( y = x^n \) is fitted to the data to relate the density of the embedded masses to the natural frequency of the system. This indicates a power-law distribution between the density and natural frequency. This function will allow the user to interpolate between densities. High agreement is shown by the \( R^2 \) value of 0.9962, indicating a precise fit to the data.

As seen in Figure 33b, as the densities of the embedded masses is increased, the broadband transmission loss greatly increases, especially at low frequencies. It is expected that the transmission loss is increasing due to diffraction effects off of the masses as well as the mass law which states that for a doubling of the mass the transmission loss will increase by 5 dB.

Figure 34: Estimating the natural frequency of embedded masses in polyimide foam as a function of density

As seen in Figure 35, the HG metamaterial splits the baseline absorption frequency into a low and high absorption peak at 220Hz and 920Hz. Below the baseline frequency, the absorption peak is due to the resonant behavior of the masses as high levels of displacement within the foam lead to an increase in damping losses. Above the baseline frequency, the masses do not displace and the absorption peak is due to an effective stiffening of the foam which raises the natural frequency \( \omega_n = \sqrt{k/m} \), where \( k \) and \( m \) are the effective stiffness and mass of the system. The impedance mismatch between the masses and foam also slows down the acoustic wave in the
material, leading to a higher transmission loss due to diffraction effects between the acoustic wave and impinging masses.

Figure 35: Absorption coefficient of polyimide with 36 polypropylene masses – COMSOL

Figure 36 illustrates the sound pressure level (SPL) within the HG metamaterial and the effective displacements at 220Hz and 920Hz and compared to a baseline polyimide sample at its peak absorption frequency at 460 Hz. The color gradient shows the damping of the acoustic wave as it travels through the material. Furthermore, the effective displacement at low frequencies is shown to be much higher than the displacement at high frequencies. This shows that the resonant effect of the embedded masses leads to a dynamic vibration absorption within the foam and increased absorption at resonance. The increase in absorption at high frequencies is therefore not due to the dynamic vibration absorption of the masses, but it is due to diffraction effects caused by the impedance mismatch between the embedded masses and the poro-elastic material.
A test is performed in COMSOL to further determine what effect the displacements of the masses have on the dynamic vibration absorption of the HG metamaterial. In the following test, a 4in sample of polyimide foam is embedded with 36 periodically distributed polypropylene masses. In the first test, the masses are free to move and displace against the stiffness of the foam. In the second test, a fixed boundary constraint is applied to the boundaries of the spheres so that they are not allowed to displace within the foam.

Figure 37 shows the absorption coefficient and transmission loss of the masses with free and fixed boundary constraints. The results show that the low frequency absorption peak is only found in the case where the masses are allowed to move within the foam. Therefore, the masses act as dynamic vibration absorbers within the foam to dissipate sound energy at their peak displacement. Conversely, the low frequency transmission loss is higher when the masses are fixed because the magnitude of the reflected wave is large at low frequencies.

It is also shown that each sample has a second absorption peak above 800Hz. This corresponds to the quarter wave resonance of the sample. The quarter wave resonance of a 4 inch resonator in air is shown in equation 34, which corresponds closely to this absorption peak.

\[ f_{1/4} = \frac{1}{4} \left( \frac{c}{\lambda} \right) = \frac{1}{4} \left( \frac{343.2}{0.0254} \right) = 844.49 \text{Hz} \]  

(34)

As shown in Figure 37, the high frequency absorption peak of each sample is explained by the quarter wave resonance of the 4 inch resonator. The absorption at this frequency is
increased by the presence of both fixed and free masses, showing that the acoustic waves are scattered by the rigid masses at high frequencies. The displacements of the embedded masses result in low frequency increase in absorption due to their dynamic vibration at low frequencies.

Figure 37: (a) Absorption coefficient and (b) transmission loss of 4in polyimide with 36 free vs. fixed polypropylene masses – COMSOL
2.5.2. Changing the size of the embedded masses

In this test the radius of 36 periodic polypropylene spheres embedded within a 4in sample of polyimide is varied in multiples of $x$ as seen in Figure 38, where $x = 7/32$in.

![Figure 38: Changing the size of the embedded masses](image)

As seen in Figure 39a, the peak absorption is moved to lower frequencies as the radius of the embedded masses is increased. The transmission loss in Figure 39b is similar to the results found with increasing the density of the masses. As the radius of the masses is increased, the broadband transmission loss increases due to the increasing weight of the structure and diffraction off of the masses within the foam. As seen in Figure 40, a power law is fitted to the data to relate the radius of the embedded masses to the natural frequency of the system. This indicates a power-law distribution between the radius and the natural frequency of the system. This function will allow the user to interpolate between radii. High agreement is shown by the $R^2$ value of 0.988.
Figure 39: (a) Absorption coefficient and (b) transmission loss of 4in polyimide with various sizes of 36 periodically spaced spherical polypropylene masses - COMSOL
Figure 40: Estimating the natural frequency of embedded polypropylene masses in polyimide foam as a function of sphere radius - COMSOL

Increasing the radius of the masses reduces the separation distance between each mass, which allows the masses to operate more closely together in series. By equating the poro-elastic material to a distributed spring, each mass will have a region of influence where the adjacent material moves with the same displacement. As the separation distance decreases, the masses interact and behave more closely to a single bulk material. As this effective mass increases, the absorption peak moves to lower frequencies.

2.5.3. Changing the shape of the embedded masses

In this test different shapes of 36 periodic polypropylene masses embedded within a 4in sample of polyimide are tested. As shown in Figure 41, the basic geometrical shapes are spheres, diamonds, cylinders, and rectangles. Each shape has the same volume.

Figure 41: Changing the shape of the embedded masses
Figure 42: (a) Absorption coefficient and (b) transmission Loss of 4in polyimide with various shapes of 36 periodically spaced polypropylene masses – COMSOL
As shown in Figure 42a, the shape of the embedded masses has little effect on the absorption coefficient of the system. Since the volume and mass are held constant, the relative separation distance and surface area of each mass does not drastically change. A flat, round, or sharp tipped surface area also had little effect on increasing absorption.

As shown in Figure 42b, diamonds, cylinders, and rectangles achieve higher broadband TL than spheres. From 800-1600Hz a noticeable increase in TL is seen as the rectangular masses outperform the others in maximum TL. This shape has the largest projected surface area which leads to increased scattering effects upon normal acoustic incidence at high frequencies.

2.5.4. Changing the depth of a single layer of embedded masses

In this test the depth of one layer of 12 periodic polypropylene masses embedded within a 4in sample of polyimide is varied. As shown in Figure 43, the top layer is closest to the incident sound wave, and the bottom layer is closest to the base of the sample.

![Top layer](image1)
![Middle layer](image2)
![Bottom layer](image3)

**Figure 43:** Changing the depth of a single layer of embedded masses

As seen in Figure 44a, the low frequency absorption peak is increased as the layer of masses is moved towards the top boundary. Material close to the top free layer displaces at a higher magnitude than material close to the bottom fixed layer. The purple dotted curve shows that placing one layer of masses on the bottom layer is comparable to having no embedded masses in the foam. Similarly, the blue dotted curve shows that placing one layer of masses on the top layer is comparable to placing three layers of masses within the foam. As the masses are farther away from the base of the foam, the effective stiffness decreases (see Figure 8). The lowering of the effective stiffness moves the resonant frequency to lower frequencies, which is shown in Figure 44a as the masses on the top layer experience a higher low frequency absorption peak than the other arrangements. Therefore it is beneficial to place masses towards the free end.
in order to target low frequency absorption. As seen in Figure 44b, the addition of masses anywhere within the sample will increase broadband transmission loss when both boundaries are free because the stiffness is effectively the same in the axial direction of the sample.

Figure 44: (a) Absorption coefficient and (b) transmission loss of 4in polyimide with various depths of 12 periodically spaced polypropylene masses - COMSOL
2.5.5. Changing the radial distribution of the embedded masses

In this test the radial distribution of three periodic rows of 8 polypropylene masses are varied. As seen in Figure 45, the row of masses varies from an outer distribution along the perimeter of the sample to an inner distribution along the center of the sample.

![Diagram](https://via.placeholder.com/150)

Outer row  Middle row  Inner row

Figure 45: Changing the radial location of the embedded masses
Figure 46: (a) Absorption coefficient and (b) transmission loss of 4in polyimide of radially arranged polypropylene spherical masses with varying row location – COMSOL
As seen in Figure 46a, the peak absorption frequency is lowered as the masses are moved towards the center of the foam. While the outer perimeter of the sample is fixed to approximate friction in a duct, the maximum displacement occurs towards the center. The masses are therefore more effective as low frequency dynamic vibration absorbers when placed towards the center where displacement is the highest.

As seen in Figure 46b, placing the masses towards the center also results in a higher transmission loss at low frequencies. The degree of interaction among the masses increases as the separation distance decreases towards the center, resulting in higher impedance of the HG metamaterial.

2.5.6. Changing the random arrangement of the embedded masses

In this test 12, 24, and 36 polypropylene masses are randomly arranged within a 4in sample of polyimide and compared to a periodic arrangement. Figure 47 shows the random arrangement of 12, 24, and 36 masses, while Figure 48 shows periodic arrangements of 4, 8, and 12 masses in a sample of 4in foam. Three trials are run for each random study, and the average of the three trials is shown.

![Figure 47: Changing the random arrangement of embedded masses](image1)

![Figure 48: Changing the periodic arrangement of embedded masses](image2)
Figure 49: (a) Absorption Coefficient and (b) transmission loss of periodic vs. random spacing of polypropylene masses in 4in polyimide - COMSOL

As seen in Figure 49a, increasing the number of randomly arranged masses shifts the peak absorption frequency to lower frequencies. The increased mass and decreased separation distance as the number of elements is increased contributes to higher absorption and transmission loss at lower frequencies. It is also seen that the peak absorption coefficient is generally higher
with a periodic arrangement, indicating that the periodic arrangement is a more optimal design of a HG metamaterial.

In a periodic arrangement, all of the masses are tuned to the same resonant frequency, as dictated by their mass and equivalent stiffness determined by the distance embedded in foam and are therefore able to perform as an array of mass-spring-damper systems to increase the amount of acoustic energy dissipated at their resonant frequency. The randomly embedded masses, on the other hand, are embedded with various stiffness values determined by their randomly embedded distance in foam. Since each mass is tuned to a different resonant frequency, the dynamic vibration absorption effect does not occur at a single resonant frequency but is distributed over the frequency domain. Periodic arrangements are tunable by varying the positioning of the masses to attenuate target frequency bandwidths, whereas random arrangements are not tunable by their nature because there is no say in how the masses are positioned within the poro-elastic material.

As seen in Figure 49b, a higher number of randomly arranged masses result in a higher transmission loss across the full bandwidth. The choice of randomly and periodically embedding the masses within a poroelastic material both lead to an increase in transmission loss, while it does not appear that the random selection of embedded masses is an ideal configuration since it cannot be tunable. Both conditions lead to an increase in transmission loss due to diffraction effects of the incident wave impinging on the masses located randomly or periodically throughout the material.

2.6. Reverb room results for diffuse field absorption coefficient

Diffuse field absorption coefficient measurements are performed in NASA Langley’s reverberation chamber in the Structural Acoustics Loads and Transmission (SALT) Facility. A diffuse field takes into account all angles of sound incidence upon a sample. Empty room measurements are compared to a panel of melamine foam and a panel of melamine foam with periodic embedded masses. The melamine samples tested are 2ft x 4ft x 2in panels. As shown in Figure 50a, one layer of 7/16in 6gm diameter steel spheres are embedded periodically in a 19 x 9 grid within the panel. Three measurements are taken for both panels in three different locations in the room as shown in Figure 50b to obtain an average absorption coefficient measurement.
This study uses the ASTM International test method for the measurement of sound absorption in a reverberation room by measuring the decay rate [43]. A band of random noise is used as a test signal and turned on long enough for the sound pressure level to reach a steady state. When the signal is turned off, the sound pressure level decreases and the decay rate in each frequency band is measured. The absorption of the room and its contents is calculated, based on the assumptions that the incident sound field is diffuse before and during decay and that no additional energy enters the room during decay. The sound absorption area $A$ (m$^2$) is calculated from the Sabine formula shown below, where $V$ is the volume of the reverberation room (m$^3$), $c$ is the speed of sound (m/s), and $d$ is the decay rate (dB/s).

$$A = 0.9210 \frac{Vd}{c} \quad (35)$$

![Figure 50: (a) Cross-section of test specimen for reverb room and (b) specimen placed in reverb room](image)

Each sample is tested in three different random locations in the reverberation room and the average decay time and absorption coefficient is measured from 80-2000Hz in 20Hz increments. Figure 51 compares the T60 decay time of the melamine panel against noise floor measurements in an empty room. This plot shows that the decay time measurements of the melamine foam are noticeably above the noise floor measurements at all frequencies. It is noted that the sample size is smaller than suggested according to ASTM standard, but since the T60 times is noticeably different than empty room measurements, the results of a HG panel will be presented and compared to a baseline melamine panel.
Figure 51: T60 decay time for 2’x4’ melamine panel versus noise floor in reverb room

Figure 52 shows the percent increase in the absorption coefficient of embedding steel masses in a melamine panel, or the difference between an acoustic HG metamaterial and a poroelastic material. The percent increase in the absorption coefficient is calculated using the formula

\[
\text{Percent Increase in } \alpha = \left( \frac{\alpha_{\text{melamine steel}} - \alpha_{\text{melamine}}}{\alpha_{\text{melamine}}} \right) \times 100
\]  

(36)

The addition of the steel spheres within melamine shows an increase in absorption in a low frequency bandwidth of 200-600Hz. A 24.7% maximum increase in the absorption coefficient is found at 400Hz. At their low frequency resonance, an increase in displacement against the poro-elastic foam leads to increased damping losses. Thus the steel spheres perform as dynamic vibration absorbers in this frequency bandwidth. It is shown the results for normal incidence absorption coefficient in Figure 30a that melamine with embedded steel masses does not increase low frequency absorption in an impedance tube, whereas a melamine panel with embedded steel masses does increase low frequency absorption in a diffuse field environment. The absorption coefficient of melamine in the impedance tube is already high at low frequencies, which masked the effects of the embedded steel masses. In this environment, the effects of the masses are more pronounced as the thickness of the melamine foam does not perform all of the
contribution at low frequencies to increase absorption. These limited results show that HG metamaterial is effective at increasing low frequency absorption in a diffuse field environment.

![Figure 52: Percent increase in absorption coefficient when adding periodic masses to a melamine panel in reverb room](image)

2.7. Conclusions

Poro-elastic materials are effective at absorbing sound at high frequencies and are generally ineffective at low frequencies. Embedding masses within a poro-elastic material creates an array of resonant mass-spring-damper systems that operate at low frequencies. The displacement of the masses against the foam on resonance leads to an increase in damping losses. This results in increased absorption at low frequencies. The masses are more effective at attenuating low frequency sound with a larger impedance mismatch compared to the poro-elastic material. Furthermore, diffraction off of the masses causes absorption at higher frequencies.

Parametric studies are performed on acoustic HG metamaterial to study the factors that affect absorption and transmission loss. By changing the material of the embedded masses the frequency of absorption can be dynamically tuned according to a power relation. Heavier masses lead to higher displacements and a lowering of the natural frequency of the system. Similarly, by changing the size of the embedded masses the frequency of absorption can be dynamically tuned.
The increased size increases the mass and decreases the separation distance which both contribute to increasing low frequency absorption and transmission loss.

It is more effective to place the masses in a section of poro-elastic material where the displacement will be the highest. This tends to be near free boundaries that are not close to constrained walls. It is also found that a random versus periodic location of embedded masses both lead to increased absorption and transmission through the material due to resonant diffraction effects of the masses. Generally, as the periodic spacing between each element is decreased, the absorption and transmission increases because the elements interact more closely with each other. It is also shown that a periodic arrangement is more optimal for the embedded masses as compared to a random arrangement. In a periodic arrangement, the effective stiffness and thus resonant frequency of the masses on each layer are the same, thus resulting in vibration absorbers tuned to the same frequency to increase absorption at resonance.

At the peak absorption frequency, the majority of the acoustic energy is being sent through the material which is either dissipated as heat by the vibration of the material or passes through as a transmitted wave. HG metamaterial shows that poro-elastic material can be designed to achieve both high absorption and transmission loss at low frequencies. The absorption coefficient tests are backed by a steel plate which simulates the material bonded to a hard wall. TL tests, on the other hand, have no structural support and are simply used to measure the material characteristics of the HG metamaterial. Future work will take into account HG metamaterial bonded to a structure for TL measurements.

It is shown that low-frequency absorption and transmission loss is increased in the critical range from 200-250 Hz and below. HG metamaterials can be used for controlling low frequency sound radiation, improving low frequency transmission loss, and providing a more compact replacement to conventional materials. The result is a high performing system that increases both the absorption and the transmission loss at low frequencies.
3. MICROPERFORATED PANELS (MPP)

Microperforated panel (MPP) based acoustic metamaterials offer an alternative to heterogeneous metamaterials in increasing low frequency sound absorption. MPP consist of submillimeter size pores machined into a thin plate [44]. These pores distort the flow of air in a thin region as it passes through the panel. The modification of the flow results in viscous dissipation that increases as the velocity through the pores reaches a maximum [35]. Increased viscous dissipation increases the acoustic resistance, and therefore increases the absorption of the MPP.

In comparison to traditional sound absorbing material, MPP are cleanable, reclaimable, rugged, and lightweight. For example, these panels guard against dirt and prevent deterioration of acoustic foam when mounted on top. Furthermore, they are environmentally friendly due to their reusability. Applications have been made in room acoustics, environmental noise barriers, and silencers [45]. While MPP are usually more effective at lower frequencies than acoustic foams of the same thickness, they become ineffective in certain frequency bands. It is proposed that stacking multiple layers of MPP and/or combining with poro-elastic material will create an acoustic metamaterial with enhanced broadband sound absorption.

This chapter serves to develop analytical, numerical, and experimental tests to validate the acoustic properties of a microperforated panel. Numerical studies are performed to study the combination of a MPP with a poro-elastic material. These results will be used in the following chapter to develop a model of a MPP as a form of acoustic metamaterial.
3.1. Modeling of MPP

3.1.1. Analytical modeling of MPP

As seen in Figure 53, a MPP is spaced a distance $D$ from a hard surface to form a backing cavity. A normalized transfer impedance through the MPP is expressed as

$$Z_{tr} = \frac{p_1 - p_2}{\rho c v} \quad (37)$$

Where $p_1$ and $p_2$ are the upstream and downstream sound pressures, respectively, $v$ is the particle velocity in the pore, and $\rho c$ is the characteristic impedance of air. Because the thickness $t$ is small, the particle velocity is the same on both sides of the MPP and $v_1 = v_2 = v$.

In the following formulation, small holes known as apertures in a perforated panel are modeled as short tubes. When the apertures are very small the acoustic resistance becomes significant and must be taken into account. Maa [46] developed an approximate solution for apertures of sub-millimeter size. For normal incidence, the normalized specific acoustic impedance of the apertures is calculated by

$$z_L = \frac{R_L + j M_L}{\rho c} = r + j \omega m \quad (38)$$
where $R_L$ and $M_L$ are the specific acoustic resistance and reactance of the aperture, $\rho$ is the density of the air, $c$ is the sound velocity in air, and

$$r = \frac{g_1}{l^2 \rho} \left( \sqrt{1 + \frac{q^2}{32} + \frac{q \sqrt{2} l}{8 t}} \right)$$  \hspace{1cm} (39)$$

$$m = 0.294 (10^{-3}) \frac{t}{p} \left( 1 + \frac{1}{\sqrt{9 + \frac{q^2}{2}}} + 0.85 \frac{l}{t} \right)$$  \hspace{1cm} (40)$$

where $q = g_2 l \sqrt{f}$, $t$ is the panel thickness (mm), $l$ is the aperture diameter (mm), $p$ is the ratio of aperture area to panel in percentage, namely $p = 100\pi l^2 / 4b^2$, and $b$ is the distance between aperture centers (mm). For non-metallic material, $g_1 = 0.147$ and $g_2 = 0.316$.

A MPP mounted a distance $D$ from a rigid wall as seen in Figure 53 forms a resonant system. The normal specific acoustic impedance of the air behind the panel/membrane is

$$z_D = -j \cot \frac{\omega D}{c}$$  \hspace{1cm} (41)$$

where $\omega = 2\pi f$, $f$ is the frequency (Hz), and $c$ is the speed of sound of air. The normalized specific acoustic impedance of the entire system is thus

$$z = r + j \left( \omega m - \cot \frac{\omega D}{c} \right)$$  \hspace{1cm} (42)$$

The total acoustic impedance $z$ is therefore a combination of the MPP plus the cavity. MPP absorbers are most effective when the backing cavity is approximately one-quarter of the acoustic wavelength. In this case, the cotangent term is zero and the particle velocity in the pores is at a maximum.

The absorption coefficient can be calculated by the formula

$$\alpha = \frac{4Re(z)}{[1 + Re(z)]^2 + [Im(z)]^2} = \frac{4r}{(1 + r)^2 + \left( \omega m - \cot \frac{\omega D}{c} \right)^2}$$  \hspace{1cm} (43)$$

This method for calculating the absorption coefficient of a MPP backed by an air cavity is compared to experimental and numerical results. See Appendix B.4 for the MATLAB code.
3.1.2. Numerical modeling using COMSOL Multiphysics

Numerical models are built using COMSOL Multiphysics to calculate the absorption coefficient and transmission loss of a MPP system. In this study, two models are built and verified using experimental results. The first model is built to input the known impedance value of a MPP directly. The second model is built to input the known parameters of a MPP directly, such as the area porosity, thickness, and hole diameter. This model is advantageous when the reported impedance is not known, and allows the user to directly change MPP properties to suit the application. These two models are compared to each other and verified using experimental and analytical results.

3.1.2.1. MPP model using reported impedance values

Figure 54: COMSOL absorption coefficient model of MPP with air cavity using interior impedance boundary

This model is built in the poroelastic waves (ELW) module in COMSOL as shown in Figure 54. The geometry consists of a 3.786in diameter cylindrical tube modeled as air in the pressure acoustics domain. A sound hard wall boundary condition is applied to all outer walls. Plane wave radiation is applied to the top boundary with an incident pressure field of 1 Pa. The
fourth boundary adds an interior impedance boundary condition where the MPP exists. The known resistive and reactive components of a MPP backed by a 1in air cavity are uploaded to this boundary condition over a frequency range of 100-5,000 Hz in 50 Hz steps. Since the reactive component of the impedance consists of the MPP and cavity in series, the cavity impedance is subtracted from the total impedance as seen by equation 41, where $D = 1$in.

The second and third boundaries are spaced a distance $s = 3$in apart and the average pressure at each boundary is measured as $p_1$ and $p_2$, respectively. From these two pressure locations, the absorption coefficient is calculated using the transfer-function method [47]. Assuming plane wave propagation exists in the tube, there are two waves that propagate in opposite directions, namely the incident and reflected wave. Using the two pressure locations $p_1$ and $p_2$, the ratio of the incident and reflected wave can be calculated in COMSOL. The method to calculate the absorption coefficient using the two-microphone method is shown through equations 25 through 33 in section 2.3.1.

This model can be adjusted to include foam, multiple MPP layers, or position the MPP with a varying air gap. This study is performed with MPP with a backing air cavity varied from 1 inch to 4 inches in 1 inch increments. As seen in Figure 55, the absorption coefficient of a MPP with varying air cavities is calculated in COMSOL and compared to analytical results. The derivation of the analytical results was performed in section 3.1.1.1.

The results show that the resonant frequency corresponding to the peak absorption coefficient is determined by the backing cavity depth. The system has multiple resonances that occur where the cotangent term of the cavity reactance from equation 41 is zero and the absorption is near a maximum. This is also known as the quarter wavelength resonant frequencies. The absorption goes to zero when the cotangent term reaches infinity. By increasing the cavity depth, the lowest frequency absorption peak can be shifted to the left.
3.1.2.2. MPP model using interior perforated plate

The interior perforated plate model allows for the direct input of MPP parameters. The interior perforated plate boundary specifies the characteristic properties of a perforated plate in COMSOL Multiphysics. The transfer impedance through the boundary is calculated by the formula

$$\frac{Z}{\rho_c c_c} = \left( \frac{1}{\sigma} \sqrt{\frac{\theta k_{eq}}{\rho_c c_c}} \left( 1 + \frac{t_p}{d_h} \right) + \theta_f \right) + \frac{k_{eq}}{\sigma} \left( t_p + \delta_h \right)$$  \hspace{1cm} (44)

As shown by the impedance formula, the resistive component increases as the hole diameter is increased. Therefore MPP offer substantial resistance as the hole diameter is submillimeter in size. The properties of the MPP used in the study are seen in Table 5.
Table 5: Interior perforated plate COMSOL boundary specifications

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic Viscosity, $\mu$</td>
<td>$1.8 \times 10^{-5}$ (default, air)</td>
<td>Pa·s</td>
</tr>
<tr>
<td>Area Porosity, $\sigma$</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>Plate Thickness, $t_p$</td>
<td>0.381</td>
<td>mm</td>
</tr>
<tr>
<td>Hole diameter, $d_h$</td>
<td>0.21</td>
<td>mm</td>
</tr>
<tr>
<td>End correction, $\delta_h$</td>
<td>$0.25d_h$ (default)</td>
<td></td>
</tr>
<tr>
<td>Flow resistance, $\theta_f$</td>
<td>0.4</td>
<td></td>
</tr>
</tbody>
</table>

From the previous model as shown in Figure 54, the interior impedance boundary condition is replaced with the interior perforated plate boundary condition to replicate a MPP. The absorption coefficient of a MPP with varying air cavities are calculated in COMSOL and compared to analytical results as seen in Figure 56.

Figure 56: Absorption coefficient of MPP backed by an air cavity using interior perforated plate – COMSOL vs. analytical
The model for calculating the transmission loss of melamine foam with a MPP separated by a cavity depth is shown in Figure 57. This model is built using the poroelastic waves (ELW) module in COMSOL Multiphysics. The numerical results match up well to the analytical results. The model using the interior perforated plate is thus comparable to using reported impedance values for the MPP. By increasing the cavity depth of the interior perforated plate, the peak absorption can be moved to lower frequencies.

![Diagram](image)

Figure 57: COMSOL transmission loss model of MPP with air cavity backed by melamine foam using interior perforated plate

The properties used for melamine foam are referenced in Table 4. As shown in Figure 57, plane wave radiation is applied to the top and bottom boundaries, and an inlet pressure $p_0 = 1\text{Pa}$ is applied to the top boundary. Integration across the bottom boundary is used to calculate the outlet pressure, $p_2$. $p_0$ and $p_2$ are used to calculate the inlet and outlet powers, respectively. The transmission loss ($TL$) is calculated as the ratio of the inlet to outlet intensities, in dB.
A sound hard boundary is applied to all outer walls, and a frequency domain of 20-2000Hz is studies in 20Hz steps. The transfer impedance calculated in equation 44 is used to calculate the pressure drop across the MPP, which is then converted to decibels (dB) to calculate the transmission loss as shown in Figure 58. Results show that the transmission loss is increased at higher frequencies. The transmission loss is affected by the resistive and reactive component of the impedance of the MPP, and because the MPP is treated as a rigid plate there are no dips in TL due to resonant effects of a flexible plate. Experimental tests are performed in the following section to validate this model in an impedance tube.

![Figure 58: Transmission loss of MPP using transfer impedance equation](image)

3.2. Experimental setup using impedance tube

A MPP is tested for absorption and transmission loss experimentally in an impedance tube. To form a backing air cavity for the MPP, multiple 1 inch spacers are first designed using the CAD design tool Pro/ENGINEER and printed in plastic using a 3D printer. The outer diameter of the spacer is equal to the inner diameter of the impedance tube of 3.786in to ensure an airtight fit. The thickness of the spacer is 0.1in, and a lip of 0.2in thickness is extended on one side to attach the MPP. Figure 59 shows the combined printed spacer with a MPP stacked on top of a poro-elastic foam. The MPP is cut into a circle equal to the outer radius of the spacer, and the outer edges are glued to the lip of the spacer. Three additional spacers without a lip are
printed to extend the backing air cavity up to 4 inches. Absorption coefficient and transmission loss tests are performed following the procedure detailed in sections 2.3.

Figure 59: 1 inch 3D printed spacer with MPP (a,b) and 1 inch 3D printed spacer with MPP on top of (c) 4in polyimide and (d) 4in melamine

3.3. Results and validation

3.3.1. COMSOL test validation with MPP backed by a variable air cavity depth

Figure 60: MPP backed by a variable air cavity depth
As seen in Figure 60, the depth of the backing cavity separating a MPP from a rigid plate is varied from 1 to 4 inches in 1-inch increments. Experimental versus numerical COMSOL results are compared in Figure 61. For absorption coefficient tests the sample is backed by a rigid plate separated by an air cavity, and for transmission loss tests the sample is not backed by a rigid plate and a transmitted wave passes through the material.

The results in Figure 61a show that the peak absorption frequency is largely dictated by the air cavity depth backing the MPP. When the reactive component of the impedance of the cavity is zero, or the quarter wave resonance, the absorption is at a maximum. This is known as the resonance frequency and it occurs in harmonics that are integer multiples of the first resonance frequency. Furthermore, it is seen from equation 42 that the reactive component of the impedance is \( \cot \frac{\omega D}{c} \) where \( \omega \) is the frequency, \( D \) is the cavity depth, and \( c \) is the speed of sound in air. At the frequencies where this term reaches infinity the absorption goes to zero. Therefore, MPP by themselves in an air cavity are not effective broadband absorbers.

Figure 61b shows that the predicted transmission loss of a MPP mounted in a duct closely follows the experimental test. Experimental results show that there is a dip in TL near 200 Hz. This is expected to occur near the axial resonance mode of the panel when mounted in a duct. In this case, the MPP is flexible and the most efficient transfer of energy occurs near its flexible mode. The COMSOL model, on the other hand, treats the MPP as a transfer impedance shown by equation 44 and does not take into account flexible modes. Overall, the results show that the numerical models closely match experimental results. Now that the COMSOL model is validated it can be used for numerical studies.
Figure 61: (a) Absorption coefficient and (b) transmission loss of MPP – experimental vs. COMSOL
3.3.2. Melamine COMSOL test validation with MPP separated by a variable air cavity depth

![Diagram of Melamine with MPP separated by a variable air cavity depth](image)

Figure 62: Melamine with MPP separated by a variable air cavity depth

As seen in Figure 62, a single MPP is spaced a variable distance away from a sample of melamine foam. For absorption coefficient tests the foam is backed by a rigid plate, and for transmission loss tests the foam is not backed by a rigid plate so that a transmitted wave is allowed to pass through the material. The impedance of the MPP, the air cavity, and melamine foam act in series to increase sound absorption and transmission loss. Experimental versus numerical COMSOL results are compared in Figure 63.

Results show that the impedance of the MPP combines with the impedance of the melamine foam. As shown in Figure 61 with only the MPP, the absorption coefficient goes to zero when the reactive component of the impedance is zero. However, in Figure 63 when the MPP is backed by melamine foam, the absorption at these frequencies is non-zero. The melamine foam performs as an effective absorber at high frequencies where the wavelength of sound is smaller or comparable to the thickness of the foam. At high frequencies melamine dissipates the acoustic energy into heat through friction. This is shown where the absorption coefficient is increased at high frequencies. The results also show that the numerical models match experimental results. Now that the COMSOL model is validated it can be used for numerical studies.
Figure 63: (a) Absorption coefficient and (b) transmission loss of 2in melamine with MPP separated by a variable air cavity depth – experimental vs. COMSOL
3.3.3. Polyimide COMSOL test validation with MPP separated by a variable air cavity depth

As seen in Figure 64, a single MPP is spaced a variable distance away from a sample of polyimide foam. For absorption coefficient tests the foam is backed by a rigid plate, and for transmission loss tests the foam is not backed by a rigid plate so that a transmitted wave is allowed to pass through the material. As previous studies have shown, the impedance of the MPP, the air cavity, and polyimide foam act in series to increase sound absorption and transmission loss. Experimental versus numerical COMSOL results are compared in Figure 65.

Results in this section are similar to the previous study where melamine is used in place of polyimide foam. As shown in Figure 65 when the MPP is backed by polyimide foam, the impedance of the MPP is added to the impedance of polyimide to yield non-zero absorption across all frequencies. The foam is an effective absorber at high frequencies where the wavelength of sound is smaller or comparable to the thickness of the foam. This is shown at high frequencies where the absorption coefficient is increased. Results also show that the numerical models match experimental results. Now that the COMSOL model is validated it can be used for numerical studies.
Figure 65: (a) Absorption coefficient and (b) transmission loss of 2in polyimide with MPP separated by a variable air cavity depth – experimental vs. COMSOL
3.4. Numerical parametric studies of MPP

The goal of performing numerical parametric studies of a MPP is to study what parameters are most influential in increasing the absorption coefficient and transmission loss of a MPP system. In the following studies, the absorption coefficient and transmission loss of melamine and polyimide foam with various configurations of MPP are considered.

3.4.1. Studies of melamine with MPP

3.4.1.1. Melamine with MPP separated by a variable air cavity depth

Figure 66: 2in melamine with MPP separated by a variable air cavity depth

As seen in Figure 66, a single MPP is spaced a variable distance away from a sample of 2in melamine foam. Figure 67 shows the absorption coefficient and transmission loss of 2in melamine with a MPP separated by a variable air cavity depth.

The results show that the addition of a MPP with an increasing air cavity depth on top of melamine foam increases the absorption at low frequencies compared to a baseline melamine sample. The impedance of the MPP, the air cavity, and the melamine foam combine to alter the surface impedance of the sample and thus the low frequency absorption and transmission loss is increased. By removing melamine from this equation, the absorption coefficient will oscillate with frequency where zero point absorption occurs at one-half wavelength harmonics of the air cavity depth. The addition of the resistance and reactance provided by the melamine foam removes the case where the reactance of the MPP resonator approaches infinity and results in zero point absorption. Thus the MPP combined with melamine results in an effective broadband absorber. Melamine foam provides attenuation at high frequencies where the wavelength of
sound is smaller than the thickness of the foam. Furthermore, it is shown that adding a MPP with an air cavity to melamine foam increases the broadband transmission loss by about 6 dB.

The result is a system that absorbs low frequency sound and is also effective at high frequencies. It is shown especially as the air cavity increases that the absorption oscillates with frequency determined by the depth of the air cavity. Therefore it is beneficial to increase the depth separating the MPP from the melamine foam to achieve lower frequency absorption with the cost of the system being less compact.

Figure 67: (a) Absorption coefficient and (b) transmission loss of 2in melamine with MPP separated by a variable air cavity depth - COMSOL
3.4.2. Studies of polyimide with MPP

3.4.2.1. Polyimide with MPP separated by a variable air cavity depth

![Diagram of polyimide with MPP separated by variable air cavity depth]

Figure 68: 2in polyimide with MPP separated by a variable air cavity depth

As seen in Figure 68, a single MPP is spaced a variable distance away from a sample of 2in polyimide foam. Figure 69 shows the absorption coefficient and transmission loss of 2in melamine with a MPP separated by a variable air cavity depth.

The results shown in Figure 69 are nearly identical to the results using melamine foam with MPP separated by a variable air cavity depth. MPP combined with polyimide is found to result in an effective broadband absorber because polyimide foam provides attenuation at high frequencies where the wavelength of sound is smaller than the thickness of the foam, and the MPP shifts the absorption peak to lower frequencies. Furthermore, it is shown that adding a MPP with an air cavity to polyimide foam increases the broadband transmission loss by about 8dB at higher frequencies.
Figure 69: (a) Absorption coefficient and (b) transmission loss of 2in polyimide with MPP separated by a variable air cavity depth – COMSOL
3.5. Conclusions

Microperforated panels (MPP) offer an alternative to increasing low frequency sound absorption and transmission. The pores in MPP provide viscous dissipation as the velocity through the pores reaches a maximum. The addition of a MPP into a system can increase the low frequency absorption. Furthermore, as an air cavity backing the MPP increases, the peak absorption moves to lower frequencies.

This chapter serves to develop analytical, numerical, and experimental tests to validate the acoustic properties of a microperforated panel. Experimental and numerical tests are performed to verify a MPP backed by a variable air cavity depth, as well as a poro-elastic material with MPP separated by a variable air cavity depth. Numerical studies are then investigated to study the combination of a MPP with a poro-elastic material.

MPP with a backing air cavity is not effective as a broadband absorber, but it is found that the addition of a MPP with an air cavity backed by poro-elastic foam increases its broadband absorption capabilities. The addition of a poro-elastic material performs as an effective absorber at high frequencies where the wavelength of sound is smaller or comparable to the thickness of the foam where the acoustic energy is dissipated into heat through friction. Thus the MPP combined with poro-elastic material results in an effective broadband absorber. Furthermore, it is shown that adding a MPP with an air cavity to melamine foam increases the broadband transmission loss by about 6 dB.

The result is a system that absorbs low frequency sound and is also effective at high frequencies. It is shown especially as the air cavity increases that the absorption oscillates with frequency determined by the depth of the air cavity. Therefore it is beneficial to increase the depth separating the MPP from the melamine foam to achieve lower frequency absorption with the cost of the system being less compact. These results are used in the following chapter to investigate a MPP as a form of acoustic metamaterial.
4. MICROPERFORATED PANEL (MPP) BASED ACOUSTIC METAMATERIAL

This chapter investigates the use of a microperforated panel as a form of an acoustic metamaterial. Studies are performed with multiple MPP embedded periodically within an air cavity and embedded periodically within a poro-elastic material. A heterogeneous metamaterial is then combined with MPP and additional experimental and numerical parametric studies are performed.

4.1. Analytical modeling of MPP combined within an air cavity

This section presents an analytical model for predicting the absorption coefficient of multiple MPP backed by an air cavity. The analytical model will be compared against numerical results to verify the COMSOL model.

A MPP backed by an air cavity forms a resonance system consisting of a large number of micro-sized Helmholtz resonator holes in front of an acoustic cavity backed by a rigid material. The MPP can be placed in front of an air cavity or porous material. However, for the MPP it is not necessary to provide the extra acoustic resistance using porous materials.

The acoustic impedance at the front surface of a single MPP backed by an air cavity is

\[ Z_{1s} = R_{1p} + j\left[X_{1p} - \cot(kD_1)\right] \]  \hspace{1cm} (45)

where \( R_{1p} \) and \( X_{1p} \) is the resistance and reactance of a single MPP, respectively, and \( \cot(kD_1) \) is the reactance of the backing air cavity where \( k \) is the acoustic wave number in air. There are multiple resonant frequencies of a single MPP which can be calculated by setting the imaginary component to zero through the formula

\[ X_{1p} - \cot(kD_1) = 0 \]  \hspace{1cm} (46)

Between these multiple resonances there are zero points in absorption, which occur at frequencies where \( \cot(kD_1) \) is infinite.
To broaden the frequency range of absorption, it is effective to use multiple layers of MPP. As seen in Figure 70a, a double resonator structure is created by adding the two resonant air gaps in parallel. The equivalent circuit diagram is developed by Jung et al. [48] and is seen in Figure 70b.

\[ Z_{2p} = R_{2p} + jX_{2p} \]

\[ Z_{1p} = R_{1p} + jX_{1p} \]

\[ Z_{D2} = -j\cot(kD_2) \]

\[ Z_{D1} = -j\cot(kD_1) \]

Figure 70: Double MPP and backing cavity (a) schematic and (b) circuit diagram

For the double layer MPP system with a second MPP having an acoustic impedance of \( Z_{2p} = R_{2p} + jX_{2p} \) and an air thickness of \( D_2 \), the surface impedance in front of the second panel is

\[
Z_{2s} = R_{2p} + jX_{2p} - \frac{j \cot(kD_2) \left[ R_{1p} + j(X_{1p} - \cot(kD_1)) \right]}{R_{1p} + j[X_{1p} - \cot(kD_2) - \cot(kD_1)]}
\]

(47)
A similar procedure can be used for calculating the surface acoustic impedance of a multi-layer system. The absorption coefficient of the double MPP system is expressed as

$$\alpha = 1 - \left| \frac{Z_{2s} - 1}{Z_{2s} + 1} \right|^2$$ (48)

Figure 71 shows the absorption coefficient of a MPP backed by a 1in air cavity and the addition of a second MPP separated by a 1in air cavity using analytic vs. numerical COMSOL results. The results show agreement and the COMSOL numerical model will be used in the following parameter studies. See Appendix B.5 for the MATLAB code used to calculate the absorption coefficient of multiple MPP combined in series.

![Graph](image)

Figure 71: Absorption coefficient of multiple MPP with a 1in air cavity depth–COMSOL vs. analytical

By using an extra MPP the real component of the acoustic impedance is increased and the acoustic absorption coefficient is increased at lower frequencies. Furthermore, by extending the air cavity depth, the range of absorption can be extended to lower frequencies. At frequencies higher than the resonant frequency of a single MPP, the acoustic reactance of the two layers is approximately the same as a single layer alone [49]. Using an additional layer does not greatly increase the high frequency range of absorption.
4.2. Numerical parametric studies of MPP

The goal of performing numerical parametric studies of a MPP is to study what parameters are most influential in increasing the absorption coefficient and transmission loss of a MPP system. In the following studies, the absorption coefficient and transmission loss of melamine and polyimide foam with various configurations of MPP are considered.

4.2.1. Studies of MPP combined within an air cavity

In the following studies multiple MPP are combined within an air cavity with varying configurations. The advantages and disadvantages of embedding MPP with and without foam as the support material are also investigated.

4.2.1.1. Stacking multiple MPP layers with a periodic spacing within an air cavity

As seen in Figure 72, a MPP is spaced 1 inch away from a rigid backing cavity. Additional MPP layers are then stacked on top with a 1 inch air cavity separating each MPP. Up to 4 MPP layers are studied. This forms an acoustic metamaterial as the MPP is periodically spaced within an air cavity with a tunable depth. Figure 73 shows the absorption coefficient and transmission loss of multiple MPP stacked in 1 inch layers. For absorption coefficient tests the periodic MPP are backed by a rigid plate, and for transmission loss tests the periodic MPP are not backed by a rigid plate to allow a transmitted wave to pass through the material.

For a single MPP backed by an air cavity, the impedance of the MPP, which includes a real (resistive) and imaginary (reactive) component, is added in series with the reactive component of the backing air cavity. With each additional MPP added to the system, the
previous result is added in parallel with the reactance of the additional air cavity. Finally, this result is added in series with the additional MPP to obtain the surface impedance of the multiple MPP system.

By stacking additional layers of MPP the effective size of the air cavity is increased, thus lowering the resonant frequency of the system and lowering the peak absorption frequency. Results also show that an increase in the number of MPP layers leads to a broadband increase in transmission loss as more energy is dissipated across each layer due to the additional resistance provided by each MPP. The use of MPP within air cavity comes at the advantage of reducing the weight and thickness compared to using a poro-elastic material alone.

It is also seen in Figure 73 that the addition of multiple MPP prevents the absorption coefficient from dropping to zero which occurs when only one MPP is used. This is caused when the reactance of the single air cavity, \( \cot \frac{\omega D}{c} \), approaches infinity. However, when multiple MPP are used the reactance changes and does not allow the reactance to reach infinity and cause the absorption to drop to zero. The absorption coefficient still oscillates with frequency due to the reactance term, but the oscillations are not allowed to drop to zero as would occur with one MPP. Therefore, stacking multiple MPP in an air cavity is an effective solution for a broadband absorber.
Figure 73: (a) Absorption coefficient and (b) transmission loss of MPP stacked in 1 in layers – COMSOL
4.2.1.2. Varying the periodic spacing of MPP within an air cavity

Figure 74: Varying the periodic spacing of MPP within a 4in air cavity

As seen in Figure 74, a 4in air cavity is embedded with a varying number of MPP. Each MPP is evenly spaced within the sample, and as more MPP are added, the spacing between each decreases. Figure 75 shows the absorption coefficient and transmission loss of periodic MPP within a 4in air cavity. For absorption coefficient tests the periodic MPP are backed by a rigid plate, and for transmission loss tests the periodic MPP are not backed by a rigid plate to allow a transmitted wave to pass through the material.

In this study, each sample has the same effective air cavity but with a varied number of embedded MPP. As shown in Figure 75a, the absorption coefficient of one MPP oscillates with frequency with a maximum occurring at its quarter wave resonance near 600 Hz and a zero point absorption at its anti-resonance near 1700 Hz. The addition of multiple MPP within this 4-inch air cavity modifies the resistive and reactive components of the surface impedance to move the absorption peak to lower frequencies and also to remove the zero point absorption that occurs for a single resonator alone. The addition of multiple MPP results in a broadband absorber with higher absorption at low frequencies, but at the tradeoff of lowering the peak amplitude of the absorption coefficient that is only possible with a single resonator. Therefore, it is beneficial to add multiple MPP when broadband sound absorption is a concern. Furthermore, as seen in Figure 75b, adding multiple MPP within an air cavity increases the broadband transmission loss in an impedance tube.
Figure 75: (a) Absorption coefficient and (b) transmission loss of periodic MPP within a 4in air cavity – COMSOL
4.2.1.3. Air cavity with non-periodic embedded MPP

As seen in Figure 76, three layers of MPP are embedded in a 2in air cavity. The layers are spaced unevenly with three different air cavity depths of 1in, 3/4in and 1/4in. The six different combinations of these arrangements are shown above. Figure 77 shows the absorption coefficient and transmission loss of variously spaced MPP layers within a 2in air cavity. For absorption coefficient tests the periodic MPP are backed by a rigid plate, and for transmission loss tests the periodic MPP are not backed by a rigid plate to allow a transmitted wave to pass through the material.

As shown in Figure 77a, samples with a larger air cavity placed closest to the terminating end yield a higher absorption coefficient at low frequencies. This is because the resonant frequency is mainly determined by the air cavity closest to the backing wall where the most reflection occurs, and a larger cavity will yield higher absorption at low frequencies.

As shown in Figure 77b, the sample with the largest air cavity placed closer to the inlet yields a higher broadband transmission loss. This is because a larger component of the resistance is affected by the first two air cavities due to the higher amplitude of reflection at the inlet. Furthermore, samples with the same outlet air cavity depth exhibit the same transmission loss versus frequency, showing that the inlet and middle air cavities can be swapped freely without affecting the transmission loss.
Figure 77: (a) Absorption coefficient and (b) transmission loss of non-periodic MPP within a 2in air cavity – COMSOL
4.2.2. Studies of MPP embedded within melamine

4.2.2.1. Melamine with periodic embedded MPP

![Image of melamine foam with varying numbers of MPP](image)

Figure 78: 2in melamine with periodic embedded MPP

As seen in Figure 78, a 2in sample of melamine foam is embedded with a varying number of MPP. Each MPP is evenly spaced within the sample. Figure 79 shows the absorption coefficient and transmission loss of 2in melamine with a varying number of embedded MPP. For absorption coefficient tests the foam is backed by a rigid plate, and for transmission loss tests the foam is not backed by a rigid plate to allow a transmitted wave to pass through the foam.

The additional impedance provided by the MPP moves the absorption peak to lower frequencies. Instead of the MPP acting as multiple tiny resonators with an air cavity, the MPP is now acting within a porous medium that provides additional resistive and reactive impedance. The addition of multiple MPP embedded in melamine foam also causes the peak amplitude of the absorption coefficient to be higher than the case where the MPP is embedded within an air cavity. This is the result of the additional resistance and reactance of the melamine foam contributing to the surface impedance of the sample. By segmenting the thickness of the foam into many separate systems, higher frequencies are not as easily absorbed because the mechanism to absorb high frequency sound relies on the foam’s thickness being longer than the effective wavelength of the incident sound.

As more MPP are introduced to the system, the surface impedance is increased and the broadband transmission loss is increased as more sound energy is dissipated across each layer. The transmission loss increases over 5 dB in the low frequency range, and up to 20 dB in the high frequency range. Therefore, embedding multiple MPP in melamine is an effective solution for increasing broadband transmission loss and increasing low frequency absorption.
Figure 79: (a) Absorption coefficient and (b) transmission loss of 2in melamine with periodic embedded MPP – COMSOL
4.2.2.2. Melamine with non-periodic embedded MPP

Figure 80: Melamine with non-periodic embedded MPP

As seen in Figure 80, three layers of MPP are embedded in a 2in sample of melamine. The layers are spaced unevenly with three different layering depths of 1in, 3/4in and 1/4in. The six different combinations of these arrangements are shown above. Figure 81 shows the absorption coefficient and transmission loss of variously spaced MPP layers within a 2in sample of melamine. For absorption coefficient tests the foam is backed by a rigid plate, and for transmission loss tests the foam is not backed by a rigid plate to allow a transmitted wave to pass through the foam.

The random selection of embedded depths of the MPP within melamine does not have as significant of an effect as compared to a random selection of embedded depths of the MPP within an air cavity at low frequencies. Below 400 Hz, there is not a significant change in absorption or transmission loss due to the various depth configurations. There also does not appear to be a significant enhancement of using a random versus periodic spacing of MPP within melamine.

The addition of multiple MPP embedded in melamine is effective at increasing low frequency
absorption and transmission loss, but the various depths in which they are embedded does not largely affect the peak absorption frequency.

Figure 81: (a) Absorption coefficient and (b) transmission loss of 2in melamine with non-periodic embedded MPP – COMSOL
4.2.3. Studies of MPP embedded within polyimide

4.2.3.1. Polyimide with periodic embedded MPP

![2in polyimide with periodic embedded MPP](image)

Figure 82: 2in polyimide with periodic embedded MPP

As seen in Figure 82, a 2in sample of polyimide foam is embedded with a varying number of MPP. Each MPP is evenly spaced within the sample. Figure 83 shows the absorption coefficient and transmission loss of 2in polyimide with a varying number of embedded MPP. For absorption coefficient tests the foam is backed by a rigid plate, and for transmission loss tests the foam is not backed by a rigid plate to allow a transmitted wave to pass through the foam.

As shown in Figure 83, these findings are very similar to the melamine sample with periodic embedded MPP. As more MPP are embedded into polyimide, the low frequency absorption and transmission loss is increased. The transmission loss increases over 9 dB in the low frequency range, and up to 13 dB in the high frequency range compared to a baseline polyimide sample. Therefore, embedding multiple MPP in polyimide is an effective solution for increasing broadband transmission loss and increasing low frequency absorption.
Figure 83: (a) Absorption coefficient and (b) transmission loss of 2in polyimide with periodic embedded MPP – COMSOL
4.2.3.2. Polyimide with non-periodic embedded MPP

As seen in Figure 84, three layers of MPP are embedded in a 2in sample of polyimide. The layers are spaced unevenly with three different layering depths of 1in, 3/4in and 1/4in. The six different combinations of these arrangements are shown above. Figure 85 shows the absorption coefficient and transmission loss of variously spaced MPP layers within a 2in sample of polyimide. For absorption coefficient tests the foam is backed by a rigid plate, and for transmission loss tests the foam is not backed by a rigid plate to allow a transmitted wave to pass through the foam.

As shown in Figure 85, these results are very similar to melamine foam with non-periodic embedded MPP. Below 400 Hz, there is not a significant change in absorption or transmission due to the various depth configurations. The addition of the same number of MPP at different depths within polyimide foam does not show as large of an effect because the results are dominated by the impedance of the polyimide foam. The addition of multiple MPP embedded in polyimide foam is effective at increasing low frequency absorption and transmission loss, but the various depths in which they are embedded does not largely affect the peak absorption frequency.
Figure 85: (a) Absorption coefficient and (b) transmission loss of 2in polyimide with non-periodic embedded MPP – COMSOL
4.3. Study of MPP combined with HG metamaterial

This study investigates the effects of combining an acoustic HG metamaterial from Chapter 2 with a microperforated panel (MPP) on increasing the absorption and transmission loss compared to a baseline material. When combining a MPP with HG metamaterial, a new model is built using COMSOL Multiphysics and validated with experimental and analytical results. Numerical studies are performed with the validated model to test for the absorption coefficient and transmission loss.

4.3.1. Results and validation

The following results calculate the absorption coefficient and transmission loss of a HG metamaterial with MPP separated by various air gaps using experimental methods to verify the COMSOL numerical results. A HG metamaterial is combined with MPP and separated by a variable air cavity depth. In these studies, a 2in HG metamaterial refers to 2in polyimide with one center layer of 12 periodically spaced 1gm individual weight polypropylene masses, a 3in HG metamaterial refers to 3in polyimide with two layers of 12 periodically spaced 1gm polypropylene masses, and a 4in HG metamaterial refers to 4in polyimide with three layers of 12 periodically spaced 1gm polypropylene masses as shown in Figure 86.

Figure 86: Distribution of masses in varying layers of HG metamaterial

2in Foam HG
1 layer
12 masses

3in Foam HG
2 layers
24 masses

4in Foam HG
3 layers
36 masses
4.3.1.1. Melamine HG with MPP separated by a variable air cavity depth validation

As seen in Figure 87, a single MPP is spaced a variable distance away from a sample of 2in melamine HG metamaterial. Experimental versus numerical COMSOL results are compared in Figure 88. Results show that the numerical models match experimental results.

The impedance of the MPP, the air cavity, and melamine HG act in series to increase sound absorption and transmission loss. As the air cavity depth separating the MPP from melamine HG increases, the peak absorption coefficient moves to lower frequencies and the absorption coefficient oscillates as a function of frequency due to the increasing resonant cavity. Additional parameter studies using the numerical model are performed to study these effects.
Figure 88: (a) Absorption coefficient and (b) transmission loss of 2in melamine HG with MPP separated by a variable air cavity depth – experimental vs. COMSOL.
4.3.1.2. Polyimide HG with MPP separated by a variable cavity depth validation

Figure 89: 2in polyimide HG with MPP separated by a variable air cavity depth

As seen in Figure 89, a single MPP is spaced a variable distance away from a sample of 2in polyimide HG metamaterial. Experimental versus numerical COMSOL results are compared in Figure 90. Results show that the numerical models match experimental results.

The results of the added MPP are similar to results with melamine HG. As seen in Figure 90, the absorption coefficient is shown to sharply peak at lower frequencies due to the dynamic vibration effect of the embedded masses. This effect is more apparent in polyimide as compared to melamine foam because polyimide has a lower stiffness and porosity, allowing the masses to displace at larger amplitudes at resonance. Additional parameter studies using the numerical model are performed to study these effects.
Figure 90: (a) Absorption coefficient and (b) transmission loss of 2in polyimide HG with MPP separated by a variable air cavity depth – experimental vs. COMSOL.
4.3.2. Parametric studies of melamine HG with MPP

The following studies use the COMSOL numerical model to investigate the effects of combining a MPP with a melamine HG metamaterial on increasing the absorption coefficient and transmission loss.

4.3.2.1. Melamine vs. HG metamaterial with MPP separated by a variable air cavity depth

In the following tests, melamine with MPP separated by a variable air cavity depth is compared against melamine HG with MPP separated by a variable air cavity depth. The results are then compared against a baseline MPP of equal thickness. Figure 9 shows the numerical absorption coefficient and transmission loss of 2in melamine vs. melamine HG with MPP separated by a variable air cavity depth. For absorption coefficient tests the samples are backed by a rigid plate, and for transmission loss tests the samples are not backed by a rigid plate to allow a transmitted wave to pass through the samples.

The results show that the melamine HG metamaterial with MPP performs on the same magnitude as melamine with MPP in absorption and transmission loss. The low frequency absorption increase is primarily due to the additional impedance of the MPP plus resonant cavity at low frequencies. The dynamic vibration effects of the masses embedded within melamine foam is not significant at low frequencies. Due to the high stiffness and porosity of melamine foam, the magnitude of vibration is not as high to produce a significant increase in the absorption coefficient.
Figure 91: (a) Absorption coefficient and (b) transmission loss of 2in melamine vs. melamine HG with MPP separated by a variable air cavity depth – COMSOL
4.3.2.2. Melamine HG with MPP separated by a variable air cavity depth

In the following tests, a single MPP is spaced a variable distance away from a sample of melamine HG and compared against a baseline melamine sample and MPP of the same thickness. Figure 92 shows the numerical absorption coefficient and transmission loss of 2in melamine HG with MPP separated by a variable air cavity depth. For absorption coefficient tests the foam is backed by a rigid plate, and for transmission loss tests the foam is not backed by a rigid plate to allow a transmitted wave to pass through the foam.

The absorption peak is shifted to lower frequencies as a larger air cavity separating the MPP and melamine HG metamaterial is increased, showing that the resonant frequency of the system is decreased due to the cavity and MPP resonance. Additionally, the broadband TL increases as the air cavity separating the HG metamaterial and the MPP increases. The low frequency absorption as shown in Figure 92a is mainly due to the addition of the MPP plus air cavity, because the dynamic vibration effects of the masses embedded within melamine foam are not as significant at low frequencies. The peak displacement amplitudes are low due to the high stiffness and porosity of melamine foam.
Figure 92: (a) Absorption coefficient and (b) transmission loss of 2in melamine HG with MPP separated by a variable air cavity depth – COMSOL
4.3.3. Parametric studies of polyimide HG with MPP

The following studies use the COMSOL numerical model to investigate the effects of combining a MPP with a polyimide HG metamaterial on increasing the absorption coefficient and transmission loss.

4.3.3.1. Polyimide vs. HG metamaterial with MPP separated by a variable cavity depth

In the following tests, polyimide with MPP separated by a variable air cavity depth is compared against polyimide HG with MPP separated by a variable air cavity depth. The results are then compared against a baseline MPP of equal thickness. Figure 93 shows the numerical absorption coefficient and transmission loss of 2in polyimide vs. polyimide HG with MPP separated by a variable air cavity depth. For absorption coefficient tests the samples are backed by a rigid plate, and for transmission loss tests the samples are not backed by a rigid plate to allow a transmitted wave to pass through the samples.

The results show that the polyimide HG metamaterial with MPP outperforms the polyimide with MPP in absorption at low frequencies. Transmission loss results show that the polyimide HG metamaterial with MPP has a broadband increase in TL from about 300-800Hz. The addition of the polyimide foam allows for higher frequencies to be absorbed because the wavelengths are comparable or shorter than the thickness of the foam, allowing the sound waves to be dissipated into thermal energy by the foam and thus increase absorption and transmission loss. The low frequency absorption increase is due to both the dynamic vibration effects of the embedded masses in polyimide foam and the additional impedance of the MPP plus resonant cavity at low frequencies.
Figure 93: (a) Absorption coefficient and (b) transmission loss of 2in polyimide vs. polyimide HG with MPP separated by a variable air cavity depth – COMSOL
4.3.3.2. Polyimide HG metamaterial with MPP separated by a variable cavity depth

In the following tests, a single MPP is spaced a variable distance away from a sample of polyimide HG and compared against a baseline polyimide sample and MPP of the same thickness. Figure 94 shows the numerical absorption coefficient and transmission loss of 2in polyimide HG with MPP separated by a variable air cavity depth. For absorption coefficient tests the foam is backed by a rigid plate, and for transmission loss tests the foam is not backed by a rigid plate to allow a transmitted wave to pass through the foam.

When a MPP with an increasing air cavity is placed on top of the HG metamaterial, the peak absorption frequency stays the same, but the amplitude of the absorption coefficient increases due to additional resistance of the MPP and increasing cavity resonance at low frequencies. Additionally, the broadband TL increases as the air cavity separating the HG metamaterial and the MPP increases. The dynamic vibration effects of the masses embedded in polyimide foam are more noticeable than melamine foam, as shown in Figure 94a where the low frequency absorption peaks around 280 Hz. Therefore it is beneficial for low frequency absorption and transmission to combine a HG metamaterial with a MPP separated by an air cavity.
Figure 94: (a) Absorption coefficient and (b) transmission loss of 2in polyimide HG with MPP separated by a variable air cavity depth – COMSOL
4.4. Conclusions

Microperforated panels (MPP) offer an alternative to increasing low frequency sound absorption and transmission. This chapter investigated the use of MPP as an acoustic metamaterial by varying the periodicity of MPP within air and a poro-elastic layer, as well as combining a MPP with a poro-elastic heterogeneous metamaterial.

Studies are performed with multiple MPP embedded periodically within an air cavity and embedded periodically within a poro-elastic material. A heterogeneous metamaterial is then combined with MPP and additional experimental and numerical parametric studies are performed. Numerical studies include embedding multiple MPP layers within an air cavity, embedding multiple MPP layers within a poro-elastic material, and a poro-elastic heterogeneous metamaterial with MPP separated by a variable air cavity depth.

It is found that stacking multiple periodic MPP in an air cavity is an effective solution for a broadband absorber. As more MPP are introduced to the system, the surface impedance is increased and the broadband transmission loss is increased as more sound energy is dissipated across each layer. It is also found that embedding multiple periodic MPP within a poro-elastic material is an effective solution for increasing low frequency absorption and transmission loss. The transmission loss increases over 5 dB in the low frequency range, and up to 20 dB in the high frequency range. There did not appear to be a significant enhancement of using a random versus periodic spacing of MPP when embedding within a poro-elastic material.

Combining a HG metamaterial with a MPP yields a low frequency absorption peak with a magnitude that increases as the separation distance between the MPP and poro-elastic material increases. The introduction of the MPP also results in an increased broadband absorption and transmission loss of the system. The absorption coefficient is also shown to sharply peak at lower frequencies due to the dynamic vibration effect of the embedded masses. This effect is more apparent in polyimide as compared to melamine foam because polyimide has a lower stiffness and porosity, allowing the masses to displace at larger amplitudes at resonance. Overall the results have shown that by using MPP as a form of acoustic metamaterial the low-frequency absorption and transmission loss is increased in the critical range from 500 Hz and below.
5. ACTIVE HETEROGENEOUS (HG) ACOUSTIC METAMATERIAL

An active HG metamaterial combines active and passive elements of sound control to minimize acoustic transmission through a medium. The passive elements in this design utilize the high-frequency absorption capabilities of poro-elastic foam by dissipating acoustic energy into heat through friction and low-frequency absorption capabilities of the embedded masses in foam due to dynamic vibration absorption. The active elements refer to active masses embedded in foam that are driven by an active feedforward control system outlined in this section. By sending a control signal, the masses oscillate at a frequency to actively modify the surface impedance of the structure, resulting in destructive interference of the primary noise source at an error microphone located in the outlet of a duct and a total reduction of sound power.

The concept of an active acoustic HG metamaterial is to combine the effects of passive and active control into a compact system that can control low-frequency sound not possible in a purely passive system. The added benefit of active control allows the system to respond to changing signals in the noise source and adjust its behavior to attenuate different noise signals in the environment.

In this paper, two methods are used to reduce the sound power of a primary noise source. The first method sends a signal to the active embedded masses out of phase with a primary noise source to result in destructive interference and a total reduction in sound power. This method does not use a control algorithm and is only performed to evaluate the performance of the active HG metamaterial at a given frequency. The second method uses a Filtered-x LMS algorithm to actively cancel a primary noise source. This method does involve active noise control and allows the system to respond to changing frequencies in its environment. An active HG metamaterial is designed with the goal of attenuating sound below 400Hz.

5.1. Active HG metamaterial experiment overview

An active-passive HG metamaterial is designed for the following experiments. As seen in Figure 95, the samples consist of 3in thick polyimide foam with 12 embedded masses arranged periodically within the center cross-section. The two center masses with wires connecting to
them serve as the active masses, and the other ten masses are 7/16in diameter 1gm individual weight polypropylene spheres. This sample comprises a hybrid active-passive HG metamaterial design. Each active mass consists of a 7/16in diameter, 7/8in long plastic cylinder with copper wire wrapped numerous times around the perimeter. Two 3/8in diameter, 1/8in long neodymium magnets are stacked and placed in the center of the cylinder and capped off by two foam pieces. The total weight of each active masse is 6gm. By sending an oscillating electric signal through the wires, a magnetic field oscillates in the axial direction and causes the magnet to oscillate and thus excites the system at the frequency of excitation. When the actuators are turned off, the system will behave as a passive HG metamaterial and the system will attenuate sound at its tuned resonant frequency. By choosing active and passive masses of similar weight, the masses will act like embedded mass-spring-damper systems with similar tuned frequencies. Thus even when not activated, the system will have a passive effect which will be enhanced when the active inputs are applied.

For experimental tests, the active HG metamaterial is placed within an impedance tube in the transmission loss setup with an error microphone placed 38in downstream from the surface of the sample. The two active masses are actuated in phase to suppress downstream plane wave radiation in a duct.

Figure 95: (a) Cross-section of polyimide with 10 polypropylene masses and 2 active masses and (b) close-up of active mass
5.2. Out of phase active HG metamaterial test

The first method of active noise control uses destructive interference to cancel a primary noise source by sending a signal to the active masses that is out of phase with the primary noise source. The sample is placed 48 inches away from the noise source. The procedure for the out of phase tests is as follows. First, the active HG metamaterial is placed in the sample holder of an impedance tube with the levels measured by the error microphone placed downstream of the sample. A sine wave signal is sent to the two active masses in the HG metamaterial with a level adjusted to match an incoming sine wave signal sent to the upstream speaker of the same frequency. The phase of the active masses is adjusted to be 180 degrees out of phase with the primary noise source. The signals to the upstream speaker and the active masses are then activated and the reduction in SPL at the signal frequency is measured. The goal of this experiment is to reduce sound power at frequencies below 400Hz. To test this goal, three different frequencies, or tones, below 400Hz were used as the primary noise source and were determined in the section below, and the sound power reduction at each frequency is measured after sending an out of phase signal to the active embedded masses.

5.2.1. Determining the frequencies of excitation for the active masses

It is noted that the active masses are designed to attenuate a range of low frequency excitations, but since the tests are limited only three are chosen for this experiment. The methodology behind choosing these frequencies of excitation is outlined.

The first frequency of excitation is chosen where the passive HG metamaterial achieves the highest reduction in downstream pressure compared to a standard poro-elastic material. The idea is to further enhance the passive effects of HG metamaterial to reach the maximum attenuation at a tuned frequency. To find this frequency, a 3in sample of polyimide foam is placed in an impedance tube and a band limited white noise signal is sent to the upstream speaker. A downstream microphone measures the SPL in dB. Following this test, the 3in sample of polyimide is embedded with one layer of passive embedded masses and the downstream SPL is measured. Figure 96 compares the SPL between these two samples. The passive device demonstrates the highest drop in SPL in the downstream microphone near 130Hz. The first frequency of excitation will be chosen where the active masses can benefit from the dynamic
effects of the passive embedded masses. Therefore, the first frequency of excitation is chosen to be 130Hz.

Figure 96: Sound pressure level (dB) of 3in polyimide with 12 embedded masses in the middle layer (hybrid of 10 polypropylene spheres and 2 shakers)

The second frequency of excitation is chosen where the peak absorption frequency of the passive HG metamaterial occurs. To determine this frequency, the absorption coefficient of a 3in sample of polyimide embedded with one layer of 12 periodically spaced polypropylene 7/16in diameter spheres is tested using COMSOL Multiphysics. As seen in Figure 97, the peak absorption frequency is near 250Hz. Therefore, the second frequency of excitation is chosen to be 250Hz.

Figure 97: Absorption coefficient of 3in polyimide with 12 polypropylene masses in the middle layer - COMSOL
The third frequency of excitation is chosen where the frequency of peak transmission loss occurs. To determine this frequency, the transmission loss of a 3in sample of polyimide embedded with one layer of 12 periodically spaced polypropylene 7/16in diameter spheres is tested using COMSOL Multiphysics. As seen in Figure 98, the peak transmission loss is near 400Hz. Therefore, the third frequency of excitation is chosen to be 400Hz.

Figure 98: Transmission Loss of 3in polyimide with 12 polypropylene masses in the middle layer – COMSOL
### 5.2.2. Results of out of phase tests

Figure 99: (I) Time (s) vs. amplitude (Pa) of noise source (blue) vs. shakers (red) with source turned off, (II) noise cancellation (Pa) of shakers (green) with source turned on, and (III) noise cancellation (dB) of shakers at target frequency of (a) 130Hz, (b) 250Hz, and (c) 400Hz.

Figure 99 shows the results of the out of phase tests at the frequencies of 130Hz, 250Hz, and 400Hz. Row I shows how the signals from the speaker and the active masses are completely out of phase with each other with the same amplitude. Row II compares the amplitude reduction with and without the active masses turned on while the primary noise source is activated. Finally, Row III shows the noise reduction in dB at the downstream microphone at the selected frequencies.

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Reduction in SPL (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>130</td>
<td>17.250</td>
</tr>
<tr>
<td>250</td>
<td>18.504</td>
</tr>
<tr>
<td>400</td>
<td>16.580</td>
</tr>
</tbody>
</table>
Table 6 summarizes the reduction in SPL at the downstream microphone of the three tones. The reduction is calculated by subtracting the peak SPL at the target frequency with the active masses turned off and turned on while the primary noise source is activated. The results show a reduction in SPL from 16-18dB and demonstrate repeatability at different frequencies.

5.3. Theory of active noise control

In this section, the implementation of a controller using the filtered-x LMS algorithm [50] is presented. The basic theory uses a control algorithm to suppress plane wave radiation at an error sensor using a reference signal upstream of the active controller. The controller implementation in a duct is shown in Figure 100. The sound from the primary source is measured using a detection sensor, which is fed through the electronic controller and a secondary source signal is sent to the actuator. An error sensor is placed downstream of the secondary source, and the controller seeks to minimize the error using the filtered-x LMS algorithm.

![Diagram of noise control in a duct](image)

Figure 100: Control of noise in a duct using an external reference sensor
The digital controller is implemented as two separate FIR filters as shown in Figure 101.

![Figure 101: Digital implementation using FIR filters](image)

The model of the feedback path, \( \hat{F}(z) \), is estimated during a system identification phase prior to control. The control problem then becomes feedforward as shown in Figure 102.

![Figure 102: Equivalent digital feedforward block diagram](image)

The response of the analogue filters and data converters is included in \( C(z) \) and \( r(n) \) is a filtered reference signal. Assuming \( H(z) \) is an FIR filter with \( I \) coefficients then the error sequence can be written as

\[
e(n) = d(n) + \sum_{i=0}^{I-1} h_i r(n)
\]  

The mean square error of \( e(n) \) is a quadratic function of each of the filter coefficients \( h_i \), and these coefficients are adjusted to minimize the mean square error using the method of steepest descent as shown in equation 66.

\[
h_{i(new)} = h_{i(old)} - \mu \frac{\delta E[e^2(n)]}{\delta h_i}
\]
where $\mu$ is a convergence coefficient. An estimate of the true derivative can be obtained by taking the instantaneous value, which can now be used to update all the filter coefficients at every sample time as shown in equation 67.

$$\frac{\partial E[e^2(n)]}{\partial h_l} = 2E[e(n) \frac{\partial e(n)}{\partial h_l}] = 2E[e(n)r(n - i)]$$  \hspace{1cm} (51)

The filtered-x LMS algorithm is used to update the coefficients of an adaptive filter. It first calculates the output signal from the adaptive filter, then filters the input signal with the estimation of the plant and generates a filtered reference signal and updates the filter coefficients by using the following equation

$$h_l(n + 1) = h_l(n) - \alpha e(n)r(n - i)$$  \hspace{1cm} (52)

The filtered reference signal $r(n)$ is generated by filtering the reference signal with $\hat{C}(z)$, which models the true error path $C(z)$. This is computed in a system identification phase prior to control. The complete digital block diagram is shown in Figure 103. The theory behind this digital block diagram will be used for the active control tests in the following section by implementing the filtered-x LMS algorithm.

![Figure 103: Complete digital block diagram](image)

### 5.4. Active control tests

An adaptive feedforward signal processing scheme known as the filtered-x LMS algorithm [51; 52] is used to minimize the sound at the error signal. The error signal is represented by a $\frac{1}{2}$in B&K condenser microphone downstream of the sample in an impedance tube. Two studies are performed using the Filtered-x LMS algorithm based on the reference
signal used. The first study uses an ideal electronic reference signal sent to the primary noise source. The second study uses a microphone placed upstream of the sample as an external reference signal. This method allows for feedback from the secondary active noise control source.

A hybrid active-passive HG metamaterial is used for the active control tests as shown in Figure 95. A one input, one output study is performed where the input is the downstream error microphone, and the output is the signal to the two active embedded masses. The control algorithm minimizes the error signal in the downstream tube by sending a control signal to the active masses that are in phase within the HG metamaterial. The recorded sound pressure levels (SPL) in dB relative to 20µPa are presented to evaluate the efficiency of the active HG metamaterial.

Tonal tests were performed by sending the discrete tones of 130Hz and 250Hz to the speaker, and broadband tests were performed by sending a 400Hz white noise band limited signal to the speaker. Successful tonal and broadband control has been achieved.

5.4.1. Active control tests with an ideal electronic reference signal

![Schematic of active control test without external feedback](image)

Figure 104: Schematic of active control test without external feedback

The following tests use the Filtered-x LMS algorithm to cancel noise at an error microphone located downstream of the sample in an impedance tube with a noise source located...
in the upstream section. As shown by the schematic in Figure 104, this test uses a reference signal without feedback by using the electronic reference signal sent to the noise source.

The tests begin by focusing on the cancellation of tones. Two low frequency tones of 130Hz and 250Hz are chosen to compare to the out of phase tests. Then, a 400Hz white noise band limited signal is used to cancel broadband noise.

![Graphs showing active cancellation of a (a) 130Hz tone, (b) 250Hz tone, and (c) 400Hz white noise band limited signal without feedback](image)

Figure 105: Active cancellation of a (a) 130Hz tone, (b) 250Hz tone, and (c) 400Hz white noise band limited signal without feedback

Figure 105 shows the reduction in dB at the downstream error microphone using a primary noise source of 130Hz, 250Hz, and 400Hz white noise band limited. The noise reduction of the white noise signal does not significantly reduce the overall SPL across the bandwidth, but it does reduce sound in certain bandwidths. At 67.5 Hz for example the control reduced the sound by 8.6dB. The two tonal tests of 130Hz and 250Hz show a reduction in SPL of 25.0dB and 6.3dB, respectively.
5.4.2. Active control tests with external feedback

In comparison to the previous test, the following tests use a reference signal with feedback by placing a microphone in the upstream section of the impedance tube. The test schematic is shown in Figure 106. Two low frequency tones of 130Hz and 250Hz are used as the primary noise source to compare to the out of phase tests. Then, a 400Hz white noise band limited signal is used to cancel broadband noise.

Figure 106: Schematic of active control test with external feedback

Figure 107: Active cancellation of a (a) 130Hz tone, (b) 250Hz tone, and (c) 400Hz white noise band limited signal with feedback
Figure 107 shows the reduction in dB at the downstream error microphone using a primary noise source of 130Hz, 250Hz, and 400Hz white noise band limited. The noise reduction of the white noise signal does not significantly reduce the overall SPL across the bandwidth, but it does reduce sound in certain bandwidths. At 67.5Hz for example the control reduced the sound by 9.0dB. The two tonal tests of 130Hz and 250Hz show a reduction in SPL of 19.9dB and 26.9dB, respectively.

5.5. Results of active acoustic HG metamaterial tests

Table 7: Results of Active Acoustic HG Metamaterial Tests

<table>
<thead>
<tr>
<th>Test Control Method</th>
<th>dB Reduction at 130 Hz</th>
<th>dB Reduction at 250Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Out of Phase</td>
<td>17.2</td>
<td>18.5</td>
</tr>
<tr>
<td>Active Without Feedback</td>
<td>25.0</td>
<td>6.3</td>
</tr>
<tr>
<td>Active With Feedback</td>
<td>19.9</td>
<td>26.9</td>
</tr>
</tbody>
</table>

Table 8: Measured SPL of Primary Noise Source

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>SPL (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>130</td>
<td>97.3</td>
</tr>
<tr>
<td>250</td>
<td>94.3</td>
</tr>
<tr>
<td>400</td>
<td>106.8</td>
</tr>
<tr>
<td>400 white noise band limited</td>
<td>92.9 (OASPL)</td>
</tr>
</tbody>
</table>

Table 7 summarizes the results of the active acoustic HG metamaterial tests for the tones of 130Hz and 250Hz. Successful noise reduction has been demonstrated using an out of phase method as well as active control with and without feedback. Table 8 reports the incident sound pressure level produced by the primary noise source and indicates that the control methods are capable of attenuating this sound level at these certain frequencies.
5.6. Conclusions

In the low frequency range, poro-elastic foam is not effective at attenuating sound due to the longer wavelength of the incident wave relative to the foam thickness. To solve this problem, embedded masses are combined with foam to lower the resonant frequency of the system and increase absorption at lower frequencies due to a dynamic absorption effect as a portion of the wave is reflected back towards the source. This method is tunable by varying the size, spacing, or material of the embedded masses. An active control method has been introduced that adjusts the signal sent to an active embedded mass that vibrates at an excitation frequency to modify the surface impedance of the poro-elastic foam to destructively interfere with a primary noise source and attenuate sound at a downstream location. One criticism is that instead of performing as dynamic vibration absorbers, the active masses are performing as mini acoustic sources that destructively interfere with an incident sound field to minimize sound radiation. However, the SPL of the noise source at 400 Hz is measured to be 106.8 dB, indicating that the active masses would have to produce 106.8 dB of sound to produce a cancellation. Furthermore, when accelerometers are attached to the structure, it is clearly seen that the masses vibrate as opposed to functioning as a noise source. Lastly, other experiments have shown that active masses, when embedded in foam, can vibrate using a controller to attenuate vibration on a beam [28].

In this study, the benefits of a passive HG metamaterial are combined with active control methods to increase low frequency transmission loss through a poro-elastic material. Because the dynamics are controlled by an electric signal, the system can respond to changing frequencies of excitation. First, a test is performed where active masses are excited out-of-phase with a primary noise source to destructively interfere with an incoming noise source. Following these tests, a Filtered-x LMS algorithm is used to control the active masses based on an incoming signal.

While the active noise control tests were targeted to reduce the error at one downstream microphone, further studies can increase the number of inputs to reduce the sound pressure levels at various locations simultaneously. Furthermore, this system can be applied to more complicated acoustics beyond plane waves produced by an impedance tube. By increasing the number of outputs to multiple actuators, noise control at modes higher than plane waves could be achievable by independently changing the phase of each actuator. This study has demonstrated an effective active acoustic HG metamaterial for attenuating low frequency sound in a duct.
6. CONCLUSIONS AND FUTURE WORK

6.1. Conclusions

Throughout this research, advanced blanket concepts using acoustic metamaterials have been investigated to achieve increased absorption and transmission loss in the critical low frequency range below 500 Hz. Heterogeneous materials and microperforated panels have been implemented as forms of acoustic metamaterials that can be tuned to target low-frequency absorption and transmission loss by using passive and active control. Acoustic HG metamaterials and MPP metamaterials have been successfully designed, constructed, and tested using analytical, experimental, and numerical methods. Studies have been performed using normal incidence impedance tubes as well as a reverberant chamber at NASA Langley. Using the two key acoustic performance metrics of absorption and transmission loss, these materials have shown to have great control over the target absorption bandwidth frequencies. These absorbers show promise for the implementation in the payload fairing in expendable launch vehicles and aircraft applications.

Poro-elastic materials are effective at absorbing sound at high frequencies and are generally ineffective at low frequencies. Acoustic HG metamaterials are effective at low frequency absorption by acting as dynamic vibration absorbers at the resonance frequency of the embedded masses in poro-elastic foam. Embedding masses within a poro-elastic material creates an array of resonant mass-spring-damper systems that operate at low frequencies. The displacement of the masses against the foam on resonance leads to an increase in damping losses. This results in increased absorption at low frequencies. The masses are more effective at attenuating low frequency sound with a larger impedance mismatch compared to the poro-elastic material. Furthermore, diffraction off of the masses causes absorption at higher frequencies.

Parametric studies are performed on acoustic HG metamaterial to study the factors that affect absorption and transmission loss. By changing the material of the embedded masses the frequency of absorption can be dynamically tuned according to a power law. Heavier masses lead to higher displacements and a lowering of the natural frequency of the system. Similarly, by changing the size of the embedded masses the frequency of absorption can be dynamically tuned. The increased size of the masses increases the weight and also decreases the separation distance.
between each mass which both contribute to increasing low frequency absorption and transmission loss.

It is more effective to place the masses in a section of poro-elastic material where the displacement is the highest. This tends to be near free boundaries that are not close to constrained walls. It is also found that a random versus periodic location of embedded masses both lead to increased absorption and transmission through the material due to resonant diffraction effects of the masses. Generally, as the periodicity between each element is decreased, the absorption and transmission increases because the elements are closer together and are more influenced by their resonant behavior at low frequency. It is also seen that the peak absorption of periodic elements is higher than random elements, indicated that a periodic arrangement is more optimal for the embedded masses.

The resonance frequency of the HG metamaterial is dependent on the density of the embedded masses and the stiffness of the poro-elastic material. It is more desirable for achieving low-frequency attenuation to place the masses close to the free surfaces of the poro-elastic material to achieve a higher displacement at resonance leading to increased dynamic vibration absorption. For designing the material to target low-frequency absorption, the masses should be placed at a higher depth from the base surface. The mass law shows that heavier masses embedded in a poro-elastic material result in increased broadband transmission loss. By increasing the density, size, decreasing the spacing between masses, and placing the masses closer to free surfaces, the transmission loss can be increased. The attenuation capability of the HG metamaterial with a periodic distribution of masses performs better than the case in which the masses are placed randomly.

It has been shown that microperforated panels (MPP) as a form of acoustic metamaterial offer an additional opportunity for achieving low frequency absorption and transmission loss. The pores in MPP provide viscous dissipation as the velocity through the pores reaches a maximum. The addition of a MPP into a system increases the low frequency absorption. Furthermore, as an air cavity backing the MPP increases, the peak absorption moves to lower frequencies.
Analytical, numerical, and experimental tests are performed to validate the acoustic properties of a MPP backed by a variable air cavity depth, as well as a poro-elastic material with MPP separated by a variable air cavity depth. Numerical studies are then investigated to study the combination of a MPP with a poro-elastic material. The result is a system that absorbs low frequency sound and is also effective at high frequencies. It is shown especially as the air cavity increases that the absorption oscillates with frequency determined by the depth of the air cavity. Therefore it is beneficial to increase the depth separating the MPP from the melamine foam to achieve lower frequency absorption.

MPP is investigated as a form of acoustic metamaterial by periodically spacing multiple MPP layers within an air cavity as well as a poro-elastic material. Combining multiple MPP periodically results in an effective broadband absorber. As more MPP are introduced to the system, the surface impedance is increased and the broadband transmission loss is increased as more sound energy is dissipated across each layer. It is also found that embedding multiple periodic MPP within a poro-elastic material is an effective solution for increasing low frequency absorption and transmission loss. The transmission loss increases over 5 dB in the low frequency range, and up to 20 dB in the high frequency range. There does not appear to be a significant enhancement of using a random versus periodic spacing of MPP when embedding within a poro-elastic material.

Studies combining a HG metamaterial with a MPP have shown to yield an additional increase in low frequency absorption and transmission loss by combining the advantages of both noise control methods. The introduction of the MPP results in an increased broadband absorption and transmission loss of the system, and the absorption coefficient increases as the separation distance between the MPP and poro-elastic material increases. The absorption coefficient is also shown to sharply peak at lower frequencies due to the dynamic vibration effect of the embedded masses. This effect is more apparent in polyimide as compared to melamine foam because polyimide has a lower stiffness and porosity, allowing the masses to displace at larger amplitudes at resonance. Overall the results show that using MPP as a form of acoustic metamaterial increases the low-frequency absorption and transmission loss in the critical range from 500 Hz and below.
Active feedforward control is successfully implemented with acoustic HG metamaterial to achieve desired attenuation of tonal and broadband frequencies. An active control method is introduced that adjusts the signal sent to active embedded masses that vibrates at an excitation frequency to modify the surface impedance of the poro-elastic foam to destructively interfere with a primary noise source and attenuate sound at a downstream location. The benefits of a passive HG metamaterial have been combined with active control methods to increase low frequency transmission loss through a poro-elastic material. Because the dynamics are controlled by an electric signal, the system can respond to changing frequencies of excitation to cancel sound. A Filtered-x LMS algorithm is used to control the active masses based on an incoming signal. This study demonstrates an effective active acoustic HG metamaterial for attenuating low frequency sound in a duct. Results of active control tests show a reduction in 19.95 dB with a primary noise source of 130 Hz at 97.38 dB, and a reduction in 26.97 dB with a primary noise source of 250 Hz at 94.38 dB.

It is shown that by using poro-elastic acoustic metamaterials, low-frequency absorption and transmission loss are successfully increased in the critical frequency range from 500 Hz and below. These designs have the potential to be implemented in the Space Launch System (SLS) during lift-off to attenuate low-frequency noise. Furthermore, these designs can be implemented in the fuselage of next generation aircraft to reduce interior noise and noise transmitted to the environment. HG metamaterials and MPP can be used for controlling low-frequency sound radiation, improving low-frequency transmission loss, providing damping to the structure, and providing a lighter and thinner replacement to conventional materials. The result is a high performing system that increases both the absorption and the transmission loss at low frequencies.
6.2. Future work

Future work will focus on the real world applications of HG metamaterials and microperforated panels. These materials will be developed for the application of acoustic blankets bonded to a payload fairing and tested in an acoustic environment during a lift-off scenario. The commercial design and development of these materials will also be investigated.

In future work, the active control system developed in this study can be extended to include multiple inputs or multiple output actuators in a more complex system. This will allow the active HG metamaterial to control more complex modes by independently changing the phase of each actuator embedded within a poro-elastic material.

Future work will also develop an optimization algorithm where the optimal arrangement of the acoustic metamaterial and choice of material will be selected based on the desired attenuation frequency or frequency bandwidth. Future work will also investigate variable or random incidence absorption and transmission loss by extending the work performed in this study at the NASA reverberation chamber. A more thorough study on the applications of HG metamaterials to liners will also be investigated.
A. Experimental Test Procedures

A.1. Impedance Tube Requirements

The Impedance tube is essentially a tube with a test sample holder at one and a sound source at the other [47]. Two microphone ports are located along the wall of the tube. The tube is straight with uniform cross section and with rigid, smooth, non-porous walls without holes. The walls are heavy and thick enough to not be excited to vibrations in the working frequency range. The requirements for measuring the absorption coefficient include that the side wall thickness should be 10% of the cross dimension of the tube. The working frequency range is $f_1 < f < f_u$, where $f_1$ is the lower working frequency of the tube, which is limited by the accuracy of signal processing equipment, and $f_u$ is chosen to avoid occurrence of non plane wave mode propagation, $f_u s < 0.45 c_0$ (s is mic spacing, meters). The microphone spacing must be longer than 5% of the wavelength corresponding to lower frequency of interest. The tube should be long enough to cause plane wave development between the source and sample. The microphones should be side mounted with the diaphragm flush with interior surface of the tube. The signal processing equipment includes an amplifier, a 2 channel FFT analyzing system, and a generator capable of producing the required source signal. A membrane loudspeaker is located at the opposite end of the tube from test sample holder contained in insulating box. Elastic vibration insulation is applied between impedance tube and frame of loudspeaker. The signal generator emits a random stationary signal with a flat spectral density within frequency range of interest. The test specimen should fit snugly in holder. Sealing the crack about the edges of the sample with Vaseline to prevent edge effects is performed.

A.2. Absorption Coefficient Experimental Test Procedure

1. Place two ½” B&K Condenser Microphones flush into the holes of the impedance tube in the configuration shown, making sure to label the microphones as mic1 and mic2. Plug mic1 into the first slot of the microphone power supply, and plug mic2 into the second slot of the microphone power supply. Set the gains of each to 20dB.
2. Connect the NI DAQ to a power source and insert two 4 channel slots into the first and second chassis (you will only use the first chassis for this step). Connect a BNC cable from slot1 of the microphone power supply to Channel 0 of the NI Chassis, and connect a BNC cable from slot2 of the microphone power supply to Channel 1 of the NI Chassis.

3. Connect the Rane amplifier to a power source and connect the positive and negative terminals to the impedance tube speaker.

4. Connect the B&K DAQ to a power source and with a split BNC connector, connect channel 5 (or output 1) to ch1 of the rane amp and also to channel 0 of the second chassis on the NI DAQ.

5. Connect the B&K DAQ to a laptop loaded with PULSE Labshop and MATLAB using an Ethernet port and connect the NI DAQ to the same laptop using a USB port.

6. Open Pulse Labshop and under hardware setup, right click generator 1 and select properties. Under waveform click user defined, select a 1 Vpeak Signal, and load the file white_noise.wav. Click activate template (F2) and start generator (Shift+F8).

7. In Matlab open “VALDAQmc_GUI.m” and run.

8. In VALDAQ, click Detect Modules / Select Channels, and under chassis slot 1 place a check in Channel 0 and Channel 1, and under voltage enter ‘mic 1’ under Channel 0 and ‘mic2’ under Channel 1. Under chassis slot 2 place a check in channel 0 and under voltage enter ‘signal’.

9. Enter a sampling frequency of 8533 Hz, measurement Time of 60 s, a NFFT (# samples) of 8192, an overlap (# samples) of 2048, and select Reference Channel as mic1. Check ‘save time-domain data’.

10. Take the backing end of the impedance tube and place the steel backing plate flush against the back until it fits. Then slide the sample into the opening until it rests flat against the steel backing plate. Now load the backing end into the end of the impedance tube until the closure is sealed.

11. Turn on the mic power supply, and with the level set to 0 of ch1, turn on the Rane amplifier.

12. Bring up VALDAQ_Gui and click run, adjust the amplitude until the plot is closest to +/-5V. Abort the recording then click run again, and save the file as test_12.

13. Swap the position of mic1 with the position of mic2.
14. Bring up VALDAQ and click run, and save the file as test_21. Reduce the amplitude to 0 and stop the generator, and turn off the amp and mic power supplies.

15. Open ‘impedance_tube_absorption_NIA_NI.m’ and make sure to change line 41 sample_length to the correct dimensions in meters. Also change line 45 to short or long depending on the spacing between the microphones. Then click run and see the absorption coefficient and impedance plotted. The file will be written to test.mat, or the frequency vs absorption coefficient can be found as the saved variable ‘data’ with the first column as the frequency, and the second column as the absorption coefficient. The data is generally only good from 80-2000 Hz.

A.3. Transmission Loss Experimental Calculations and Test Procedure

The coefficient of interest is the first term of the transmission loss matrix, $\alpha(f)$. This term is the same sound TL coefficient as $\tau$. So by $\alpha(f) = \tau$, the STL is defined above. The four values A1,A2,B1,B2 can be used to solve for the STL coefficient. Defined in terms of sound pressure readings, $P_i$, and distances of the mics, $\delta x_i$.

\[
A_1(f) = \frac{-j P_1(f) - P_2(f)e^{-jk\delta x_1}}{2 \sin(k\delta x_1)}e^{-jk\delta x_2}
\]

\[
A_2(f) = \frac{j P_4(f) - P_3(f)e^{jk\delta x_4}}{2 \sin(k\delta x_4)}e^{jk\delta x_3}
\]

\[
B_1(f) = \frac{j P_1(f) - P_2(f)e^{jk\delta x_1}}{2 \sin(k\delta x_1)}e^{jk\delta x_2}
\]

\[
B_2(f) = \frac{-j P_4(f) - P_3(f)e^{-jk\delta x_4}}{2 \sin(k\delta x_4)}e^{-jk\delta x_3}
\]

The sound pressure at specific points in space and time are related to pressure measurements at other points in space and time through phase relationships. These relationships are typically gathered through the acquisition of complex ensemble average cross spectra, $\overline{\rho_{xy}}$. A measure of strict sound pressure magnitude at a point is acquired as an ensemble averaged autopower spectra, $\overline{\rho_{xx}}$. These two measurements can be utilized to modify the above eq.
By acquiring complex cross spectra from microphones 2, 3, 4, using mic 1 as reference, and the autopower spectra of microphone 1 with the impedance tube in two different end conditions, the STL coefficient can be solved. The two end conditions must be very different and are generally open ended, or anechoic, represented below with a subscripted O, and closed ended, or reverberant, represented below with a subscripted C.

\[
A_1(f)G_1^*(f) = \frac{-j \bar{G}_{11}(f) - \bar{G}_{13}(f)e^{-jk\delta X_1}}{2\sin(k\delta X_1)} e^{-jk\delta X_2}
\]

\[
A_2(f)G_1^*(f) = \frac{j \bar{G}_{14}(f) - \bar{G}_{12}(f)e^{jk\delta X_4}}{2\sin(k\delta X_4)} e^{jk\delta X_3}
\]

\[
B_1(f)G_1^*(f) = \frac{j \bar{G}_{11}(f) - \bar{G}_{12}(f)e^{jk\delta X_1}}{2\sin(k\delta X_1)} e^{jk\delta X_2}
\]

\[
B_2(f)G_1^*(f) = \frac{-j \bar{G}_{14}(f) - \bar{G}_{13}(f)e^{-jk\delta X_4}}{2\sin(k\delta X_4)} e^{-jk\delta X_3}
\]

\[
\alpha(\omega) = \frac{(A_{10}(\omega)G_{10}^*(f))(B_{2C}(\omega)G_{1C}^*(f)) - (A_{1C}(\omega)G_{1C}^*(f))(B_{20}(\omega)G_{1O}^*(f))}{(A_{20}(\omega)G_{1O}^*(f))(B_{2C}(\omega)G_{1C}^*(f)) - (A_{2C}(\omega)G_{1C}^*(f))(B_{20}(\omega)G_{1O}^*(f))}
\]

An open ended tube condition and a hard closed ended tube condition were used for both sized tubes. Two ¼” condenser microphones were used to measure the one autopower spectrum and the three cross spectra in each end condition.

1. Continuing from the configuration of the absorption coefficient tests, remove the backing plate and sample from the end of the tube. Now line up the second half of the impedance tube (not used in absorption tests) with the previous one. Using the same backing plate, insert the steel plate flush to the back as before, and insert the anechoic terminating foam into the plate. Now insert the plate into the aft end of the second tube and secure tightly with clamps.

2. Taking the middle tube (10”) place the sample in such that the top is flush with the incident inner lip, and carefully secure this tube to the aft end of the incident tube and the front end of the aft tube. Use clamps to secure the three tubes together, making sure that they are all parallel to each other.
3. Leaving the first two microphones where they are (according to the figure), place the third and fourth microphone flush into the holes of the aft tube in the configuration shown, making sure to label these mics as mic3 and mic4. Plug mic3 into the first slot of a second mic supply, and plug mic4 into the second slot of the mic supply. Set the gains of the second source to 30dB.

4. With a BNC connect the first slot of the second mic supply to ch2 of the first chassis of the NI DAQ and connect the second slot of the second mic supply to ch3 of the first chassis of the NI DAQ.

5. Turning on the two mic supplies and the amp (with zero level), open PULSE Labshop and load white_noise.wav into generator one, activate template and run generator.

6. Open VALDAQ and click detect modules. Place a check in channel 2 and 3 under slot 1, and enter mic3 and mic4 respectively. Click continue and keep the same numbers as previously.

7. In VALDAQ Click record and adjust the level of the amp until the time-domain values are within +_5V. click abort and then record for the full time, and save data as 1234_TL_ClosedTerm.

8. Swap microphones 1 and 2 in position and repeat step (7) but save to file 2134_TL_ClosedTerm.

9. Return mics to original position. Now swap mics 1 and 3 in position and change the gain of the first slot of the first mic supply to 30db and the first slot of the second mic supply to 20dB and repeat step (7) but save to file 3214_TL_ClosedTerm.

10. Return mics to original position. Now swap mics 1 and 4 in position and change the gain of the first slot of the second mic supply to 30dB and the second slot of the second mic supply to 20dB and repeat step (7) but save to file 4231_TL_ClosedTerm.

11. Return mics and gains to original position, and remove backing plate from the aft end of the tube. Repeat steps (7-10) with an open end condition but each file to OpenTerm and opposed to ClosedTerm.

12. In Matlab, open impedance_tube_TL_NIA.m and make sure the sample_length is the correct length of the sample in meters, and run. See the variable data for a plot of frequency versus transmission loss and test.mat for a file of the same data.
B. Matlab Codes

B.1. Absorption Coefficient Matlab Code

Absorption coefficient
% Calculate Absorption Coefficient with 2 microphones method
% 
% Microphone 1: closest to speaker
% Microphone 2: closest to sample
% s: distance between microphones in meters
% L: distance between microphone 1 and surface of sample in meters
% R: reflection coefficient at surface of sample
% R = \( [H12-\exp(-jks)] / [\exp(jks) - H12] * \exp(2jkL) \)
% k: wavenumber
% T: temperature in C
% c: wavespeed
% c = 20.05*sqrt(T+273)
% H12: transfer function between mic 1 and mic2
% H12 = p2 / p1
% alpha: absorption coefficient
% alpha = 1 - |R|^2
% Time data required:
% Ch1: time steps
% Ch2: mic #1
% Ch3: mic #2
% Two measurements are required: mics in regular position and mics switched

clc
clear all
close all

%----------------------------------------------- Variables -------------------------------%
% File name for regular configuration
fn1 = 'test_12.dat';
% File name for switched configuration
fn2 = 'test_21.dat';
% Number of spectral bins
nfft = 8192;
% Length of sample [m]
sample_length = 4*.0254;
% Temperature [C]
T = 25;
% Tube configuration, 'short' for high-freq and 'long' for low-freq config
tube_config = 'short';
% True if absorption coefficient and impedance need to be saved
save_data = true;
% File for saving data
save_filename = 'test.mat';

%-------------------------------- Load Data ---------------------------------
Nch=3;

% Data for measurement in regular configuration
fid = fopen(fn1,'r','l');
out1 = fread(fid,[Nch+1, inf],'float32');
cfclose(fid);
% Data for measurement in switched configuration
fid = fopen(fn2,'r','l');
out2 = fread(fid,[Nch+1, inf],'float32');
cfclose(fid);

%------------------- Compute Transfer Functions --------------------------

% Sampling frequency [Hz]
Fs = 8533;
% Uncalibrated transfer function in regular configuration
[H12a,freq] = tfestimate(out1(2,:),out1(3,:),hamming(nfft),nfft/4,nfft,Fs);
% Uncalibrated transfer function in switched configuration
[H12b,freq] = tfestimate(out2(3,:),out2(2,:),hamming(nfft),nfft/4,nfft,Fs);
% Magnitude of calibrated transfer function
tmp1 = abs(H12a .* H12b);
% Phase of calibrated transfer function
tmp2 = unwrap(angle(H12a .* H12b));
% Calibrated transfer function
H12_calib = sqrt(tmp1) .* exp(tmp2/2*1i);

%------------------ Compute Absorption Coefficient -------------------------
% Wave speed [m/s]
c = 343.2*sqrt((T+273.15)/293);
% Wavenumber
k = 2*pi*freq/c;

if strcmp(tube_config,'short')
    % High-frequency (short spacing)
    % Spacing between microphones [m]
    s = 0.07366;
    % Distance from microphone 1 to tube termination
    L1 = 0.605155;
elseif strcmp(tube_config,'long')
    % Low-frequency (long spacing)
    % Spacing between microphones [m]
s = 0.27686;
% Distance from microphone 1 to tube termination
L1 = 0.808355;
end

% Distance from microphone 1 to surface of sample
L = L1 - sample_length;
% Reflection Coefficient
R = (H12_calib-exp(-1i*k*s)) ./ (exp(1i*k*s)-H12_calib) .* exp(2i*k*L);
% Absorption coefficient
alpha = 1 - abs(R).^2;
% Normal impedance at surface of sample
impedance = (1 + R) ./ (1 - R);

%----------------------------------- Plot Results -----------------------------------%
figure(1)
plot(freq, alpha,'LineWidth',3)
ylim([0 1])
xlim([50 1950])
xlabel('Frequency [Hz]','FontSize',24)
ylabel('Absorption Coefficient','FontSize',24)
grid on
set(gca,'FontSize',24)

figure(3)
plot(freq, real(impedance), freq, imag(impedance), 'LineWidth',3)
xlim([50 1950])
xlabel('Frequency [Hz]','FontSize',24)
ylabel('Specific Acoustic Impedance Ratio','FontSize',24)
legend('Resistive','Reactive')
grid on
set(gca,'FontSize',24)

%----------------------------------- Save Data -----------------------------------%
if save_data
    save(save_filename,'freq','alpha','impedance')
end

data = [freq alpha];
B.2. Transmission Loss Matlab Code

% Calculate Transmission Loss with 4 microphones method
%
% Microphone 1: upstream tube, closest to speaker
% Microphone 2: upstream tube, closest to sample
% Microphone 3: downstream tube, closest to sample
% Microphone 4: downstream tube, closest to tube termination
% s: distance between microphones in meters
% x1: coordinate of microphone position #1 to reference sample surface in meters
% x2: coordinate of microphone position #2 to reference sample surface in meters
% x3: coordinate of microphone position #3 to reference sample surface in meters
% x4: Coordinate of microphone position #4 to reference sample surface in meters
% k: wavenumber
% T: temperature in C
% c: wavespeed
% c = 20.05*sqrt(T+273)
% H12: transfer function between mic 1 and mic 2
% H13: transfer function between mic 1 and mic 3
% H14: transfer function between mic 1 and mic 4
% Time data required:
% Ch1: time steps
% Ch2: mic #1
% Ch3: mic #2
% Ch4: mic #3
% Ch5: mic #4
% Eight measurements are required:
% - Open termination, mics in positions 1 2 3 4
% - Open termination, mics in positions 2 1 3 4
% - Open termination, mics in positions 3 2 1 4
% - Open termination, mics in positions 4 2 3 1
% - Closed termination, mics in positions 1 2 3 4
% - Closed termination, mics in positions 2 1 3 4
% - Closed termination, mics in positions 3 2 1 4
% - Closed termination, mics in positions 4 2 3 1
% Syntax for open termination filenames:
% 1234_filename1.dat
% 2134_filename1.dat
% 3214_filename1.dat
% 4231_filename1.dat
% Syntax for closed termination filenames:
% 1234_filename2.dat
% 2134_filename2.dat
% 3214_filename2.dat
% 4231_filename2.dat
clc
clear all
close all

%---------------------------- Variables -----------------------------%
% Common part of file names for open termination
filename1 = 'TL_OpenTerm';
% Common part of file names for closed termination
filename2 = 'TL_ClosedTerm';
% Gains in dB associated with each microphone, each row is a config, open termination
gains_OpenTerm = [20 20 30 30 ; 20 20 30 30 ; 30 20 20 30 ; 30 20 30 20]; % for Melamine (NASA)
% Gains in dB associated with each microphone, each row is a config, closed termination
gains_ClosedTerm = [20 20 30 30 ; 20 20 30 30 ; 30 20 20 30 ; 30 20 30 20]; % for Melamine (new)
nfft = 8192;
% Length of sample [m]
sample_length = 4*.0254; % 3.5 for Melamine, 3.75 for Polyimide
% Temperature [C]
T = 25;
% Atmospheric pressure [kPa]
ap = 101.8;
% Tube configuration, 'short' for high-freq and 'long' for low-freq config
tube_config = 'short';
% True if TL needs to be saved
save_data = true;
% File for saving data
save_filename = 'test.mat';

%----------------- Load Data and Compute Spectrum ------------------%
% Number of channels
Nch=5;
% Microphone number assigned to positions "1 2 3 4", each row is a config
mic_configs = {'1234_','2134_','3214_','4231_'};
mic_pos = [1 2 3 4 ; 2 1 3 4 ; 3 2 1 4 ; 4 2 3 1];
% Loop over number of configs
for ind = 1:4
    % Load data for open termination
    fn = strcat(mic_configs{ind},filename1, '.dat');
    fid = fopen(fn,'r','l');
    out = fread(fid,[Nch+1, inf],'float32');
    fclose(fid);
    % Apply gains to data
out = diag(1./10.^(gains_OpenTerm(ind,:)/20)) * out(2:5,:);
% Sampling frequency [Hz]
Fs = 8533;
% Compute transfer functions respective to microphone in position #1
[H1a(ind,:),freq] = tfestimate(out(ind,:),out(mic_pos(ind,1,:),hamming(nfft),nfft/4,nfft,Fs);
[H2a(ind,:),freq] = tfestimate(out(ind,:),out(mic_pos(ind,2,:),hamming(nfft),nfft/4,nfft,Fs);
[H3a(ind,:),freq] = tfestimate(out(ind,:),out(mic_pos(ind,3,:),hamming(nfft),nfft/4,nfft,Fs);
[H4a(ind,:),freq] = tfestimate(out(ind,:),out(mic_pos(ind,4,:),hamming(nfft),nfft/4,nfft,Fs);
% Compute coherence respective to microphone in position #1
[C1a(ind,:),freq] = mscohere(out(ind,:),out(mic_pos(ind,1,:),hamming(nfft),nfft/4,nfft,Fs);
[C2a(ind,:),freq] = mscohere(out(ind,:),out(mic_pos(ind,2,:),hamming(nfft),nfft/4,nfft,Fs);
[C3a(ind,:),freq] = mscohere(out(ind,:),out(mic_pos(ind,3,:),hamming(nfft),nfft/4,nfft,Fs);
[C4a(ind,:),freq] = mscohere(out(ind,:),out(mic_pos(ind,4,:),hamming(nfft),nfft/4,nfft,Fs);

% Load data for closed termination
fn = strcat(mic_configs{ind},filename2,'.dat');
 fid = fopen(fn,'r','l');
out = fread(fid,[Nch+1, inf],'float32');
fclose(fid);
% Apply gains to data
out = diag(1./10.^(gains_ClosedTerm(ind,:)/20)) * out(2:5,:);
% Compute transfer functions respective to microphone in position #1
[H1b(ind,:),freq] = tfestimate(out(ind,:),out(mic_pos(ind,1,:),hamming(nfft),nfft/4,nfft,Fs);
[H2b(ind,:),freq] = tfestimate(out(ind,:),out(mic_pos(ind,2,:),hamming(nfft),nfft/4,nfft,Fs);
[H3b(ind,:),freq] = tfestimate(out(ind,:),out(mic_pos(ind,3,:),hamming(nfft),nfft/4,nfft,Fs);
[H4b(ind,:),freq] = tfestimate(out(ind,:),out(mic_pos(ind,4,:),hamming(nfft),nfft/4,nfft,Fs);
% Compute coherence respective to microphone in position #1
[C1b(ind,:),freq] = mscohere(out(ind,:),out(mic_pos(ind,1,:),hamming(nfft),nfft/4,nfft,Fs);
[C2b(ind,:),freq] = mscohere(out(ind,:),out(mic_pos(ind,2,:),hamming(nfft),nfft/4,nfft,Fs);
[C3b(ind,:),freq] = mscohere(out(ind,:),out(mic_pos(ind,3,:),hamming(nfft),nfft/4,nfft,Fs);
[C4b(ind,:),freq] = mscohere(out(ind,:),out(mic_pos(ind,4,:),hamming(nfft),nfft/4,nfft,Fs);
end
% Clear data in memory
clear out
% Remove frequencies under cutoff frequency
cutoff_frequency = 40;
[tmp1,tmp2] = min(abs(freq-cutoff_frequency));
freq2 = freq(tmp2:end);

%------------------------ Calibrated Tranfer Functions ------------------------%
% Open termination
H11a = H1a(1,tmp2:end);
H21a = sqrt(abs(H2a(1,tmp2:end) .* H2a(2,tmp2:end))) .*
exp(1i*(unwrap(angle(H2a(1,tmp2:end)))-unwrap(angle(H2a(2,tmp2:end))))/2);
H31a = sqrt(abs(H3a(1,tmp2:end) .* H3a(3,tmp2:end))) .*...
\begin{verbatim}
exp(1i*(unwrap(angle(H3a(1,tmp2:end)))+unwrap(angle(H3a(3,tmp2:end))))/2);
H41a = sqrt(abs(H4a(1,tmp2:end) .* H4a(4,tmp2:end))).*
exp(1i*(unwrap(angle(H4a(1,tmp2:end)))+unwrap(angle(H4a(4,tmp2:end))))/2);
% Closed termination
H11b = H1b(1,tmp2:end);
H21b = sqrt(abs(H2b(1,tmp2:end) .* H2b(2,tmp2:end))).*...
exp(1i*(unwrap(angle(H2b(1,tmp2:end)))+unwrap(angle(H2b(2,tmp2:end))))/2);
H31b = sqrt(abs(H3b(1,tmp2:end) .* H3b(3,tmp2:end))).*...
exp(1i*(unwrap(angle(H3b(1,tmp2:end)))+unwrap(angle(H3b(3,tmp2:end))))/2);
H41b = sqrt(abs(H4b(1,tmp2:end) .* H4b(4,tmp2:end))).*...
exp(1i*(unwrap(angle(H4b(1,tmp2:end)))+unwrap(angle(H4b(4,tmp2:end))))/2);

%------------------------
% Transmission Loss
%------------------------
% Wave speed [m/s]
c = 343.2*sqrt((T+273.15)/293);
% Wavenumber
k = 2*pi*freq2'/c;
% Density of air at reference temperature [kg/m^3]
rho0  = 1.186;
% Density of air at actual measured temperature [kg/m^3]
rho = rho0 * (pa/101.325) * (293/(T+273.15));

if strcmp(tube_config,'short')
    % High-frequency (short spacing)
    % Spacing between microphones [m]
s = 0.07366;
    % Distance from microphone 1 to tube termination
    L1 = 0.605155;
elseif strcmp(tube_config,'long')
    % Low-frequency (long spacing)
    % Spacing between microphones [m]
s = 0.27686;
    % Distance from microphone 1 to tube termination
    L1 = 0.808355;
end

% Coordinate of microphone position #1 to reference sample surface
x1 = -0.45466;
% Coordinate of microphone position #2 to reference sample surface
x2 = x1 + s;
% Coordinate of microphone position #3 to reference sample surface
%x3 = 0.96774;
  x3 = 23*.0254;
% Coordinate of microphone position #4 to reference sample surface
x4 = x3 + s;
\end{verbatim}
% Wave coefficients for open termination
Aa = 1i * (H11a.*exp(1i*k*x2) - H21a.*exp(1i*k*x1)) ./ (2*sin(k*(x1 - x2)));
Ba = 1i * (H21a.*exp(-1i*k*x1) - H11a.*exp(-1i*k*x2)) ./ (2*sin(k*(x1 - x2)));
Ca = 1i * (H31a.*exp(1i*k*x4) - H41a.*exp(1i*k*x3)) ./ (2*sin(k*(x3 - x4)));
Da = 1i * (H41a.*exp(-1i*k*x3) - H31a.*exp(-1i*k*x4)) ./ (2*sin(k*(x3 - x4)));
P0a = Aa + Ba;
V0a = (Aa - Ba) / (rho*c);
Pda = Ca.*exp(-1i*k*sample_length) + Da.*exp(1i*k*sample_length);
Vda = (Ca.*exp(-1i*k*sample_length) - Da.*exp(1i*k*sample_length)) / (rho*c);

% Wave coefficients for closed termination
Ab = 1i * (H11b.*exp(1i*k*x2) - H21b.*exp(1i*k*x1)) ./ (2*sin(k*(x1 - x2)));
Bb = 1i * (H21b.*exp(-1i*k*x1) - H11b.*exp(-1i*k*x2)) ./ (2*sin(k*(x1 - x2)));
Cb = 1i * (H31b.*exp(1i*k*x4) - H41b.*exp(1i*k*x3)) ./ (2*sin(k*(x3 - x4)));
Db = 1i * (H41b.*exp(-1i*k*x3) - H31b.*exp(-1i*k*x4)) ./ (2*sin(k*(x3 - x4)));
P0b = Ab + Bb;
V0b = (Ab - Bb) / (rho*c);
Pdb = Cb.*exp(-1i*k*sample_length) + Db.*exp(1i*k*sample_length);
Vdb = (Cb.*exp(-1i*k*sample_length) - Db.*exp(1i*k*sample_length)) / (rho*c);

% Transfer matrix
T11 = (P0a.*Vdb - P0b.*Vda) ./ (Pda.*Vdb - Pdb.*Vda);
T12 = (-P0a.*Pdb + P0b.*Pda) ./ (Pda.*Vdb - Pdb.*Vda);
T21 = (V0a.*Vdb - V0b.*Vda) ./ (Pda.*Vdb - Pdb.*Vda);
T22 = (-Pdb.*V0a + Pda.*V0b) ./ (Pda.*Vdb - Pdb.*Vda);

% Transmission Loss
TL = 10*log10((1/4)*abs(T11 + T12/(rho*c) + rho*c*T21 + T22).^2);

%------------------------------- Plot Results ---------------------------------
figure(1)
plot(freq2,TL,'LineWidth',3);
xlim([50 1950])
grid on
xlabel('Frequency [Hz]' , 'FontSize',24)
ylabel('Transmission Loss [dB]' , 'FontSize',24)
legend('4" Polyimide - PP Balls' )
set(gca,'FontSize',24)

%------------------------------- Save Data -------------------------------------
if save_data
    save(save_filename,'freq2','TL')
end

clear all
close all
clc

f = [20:20:2000];
w = 2*pi*f;
rho0 = 1.225;
c0 = 343.2;
k0 = w./c0;
Z0 = rho0*c0;

d = 4*.0254; % thickness

sigma = 10000; % melamine normal flow resistivity N Pa*s/m^2

X = rho0*f/sigma;

%eq 2.28
Zc = rho0*c0*(1+0.057*X.^(-0.754)-i*0.087*X.^(-0.732));

%eq 2.29
k = (1/c0)*w.*(1+0.0978*X.^(-0.700)-i*0.189*X.^(-0.595));

%% Absorption Coefficient Calculation

%eq 2.17
Z1 = -i*Zc.*cot(k*d);

Z1real = real(Z1);
Z1imag = imag(Z1);
R1 = (Z1 - rho0*c0)./(Z1 + rho0*c0);
alpa = 1 - abs(R1).^2;

figure
plot(f,alpha)
ylim([0,1.2])
ylim([0,1.2])

alpha_data = [f; alpha]';
%% Transmission Loss Calculation

\[
\begin{align*}
T_{11} &= \cos(k*d); \\
T_{12} &= j*Z_c.*\sin(k*d); \\
T_{21} &= j*\sin(k*d)./Z_c; \\
T_{22} &= \cos(k*d); \\
TL &= -20*\log10(abs((2*exp(j*k0*d))./(T_{11}+T_{12}/Z_0+Z_0*T_{21}+T_{22}))); \\
\end{align*}
\]

figure
plot(f,TL)

TL_data = [f; TL]';

**B.4. Matlab Code for Calculating the Analytical Absorption Coefficient of MPP backed by an air cavity from a rigid wall**

clear all
close all
clc

\[
f = [0:20:2000];
\]

\[
w = 2*pi*f;
\]

\[
t = .4; \quad \% \text{thickness, mm}
\]

\[
l = .22; \quad \% \text{aperture diameter, mm}
\]

\[
b = 1/16*25.4; \quad \% \text{distance between aperture centers, mm}
\]

\[
D = 4*25.4; \quad \% \text{distance from a rigid wall mm}
\]

\[
p = 100*pi*l^2/(4*b^2); \quad \% \text{ratio of aperture area to panel area, }\%
\]

\[
g1 = 0.147; \quad \% \text{non-metallic constant } g1
\]

\[
g2 = 0.316; \quad \% \text{non-metallic constant } g2
\]

\[
q = g2*l*sqrt(f);
\]

\[
r = g1/l^2*t/p*(sqrt(1+q.^2/32)+q.*sqrt(2)/8*l/t);
\]

\[
m = 0.294*10^-3*(t/p)*(1+1./sqrt(9+q.^2/2)+0.85*l/t);
\]

\[
\rho = 1.275*10^-9; \quad \% \text{density of air kg/mm}^3
\]

\[
c = 343200; \quad \% \text{speed of sound in air mm/s}
\]

\[
z = r + j*(w.*m-cot(w*D/c));
\]

\[
\alpha = 4*r./((1+r.^2)+(w.*m-cot(w*D/c)).^2);
\]

figure
plot(f, alpha)
data = [f; alpha];

B.5. Matlab Code for calculating the Absorption Coefficient of double MPP

% Matlab Code for analytically determining the absorption
% coefficient of double MPP backed by an air gap from a rigid wall

clear all
close all
clc

f = [20:20:2000];
w = 2*pi*f;

% thickness, mm
%s aperture diameter, mm
% distance between aperture centers, mm
% distance from a rigid wall mm

p = 100*pi*l^2/(4*b^2);  % ratio of aperture area to panel area,
g1 = 0.147;  % non-metallic constant g1
% non-metallic constant g2
q = g2*l*sqrt(f);
r = g1*l^2*w/p*(sqrt(1+q.^2/32)+q.*sqrt(2)/8*l/t);
m = 0.294*10^-3*(t/p)*(1+1./sqrt(9+q.^2/2)+0.85*l/t);
rho = 1.275*10^-9;  % density of air kg/mm^3
c = 343200;  % speed of sound in air mm/s
k = w/c;  % wave number of air

alpha1 = 4*r./((1+r).^2+(w.*m-cot(w*D/c)).^2);  % absorption coefficient of 1 MPP

z = r + j*(w.*m);
gap = -j*cot(w*D/c);

Z2 = r + j*(w.*m) - (j*cot(k*D).*r+j*(m-cot(k*D)))./(r+j*(m-cot(k*D)-cot(k*D)));
R2 = (Z2-1)./(Z2+1);
alpha2 = 1-abs(R2).^2;  % absorption coefficient of 2 MPP

Z3 = r + j*(w.*m) - (j*cot(k*D).*Z2)./(r+j*(m-cot(k*D)-cot(k*D)));
R3 = (Z3-1)./(Z3+1);
alpha3 = 1-abs(R3).^2;  % absorption coefficient of 3 MPP
\[ Z_4 = r + j(w.m) - (j\cot(kD) \cdot (r + j(w.m) - (j\cot(kD) \cdot (Z_2)))/(r+j(m - \cot(kD) - \cot(kD))))/(r+j(m - \cot(kD) - \cot(kD))); \]

\[ R_4 = (Z_4-1)/(Z_4+1); \]

\[ \alpha_4 = 1 - \text{abs}(R_4)^2; \] % absorption coefficient of 4 MPP

figure
plot(f, \alpha_1, f, \alpha_2, f, \alpha_3, f, \alpha_4)
legend('1mpp', '2mpp', '3mpp', '4mpp')

data = [f; \alpha_1; \alpha_2; \alpha_3; \alpha_4]’;
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