DEVELOPMENT OF INTERACTIVE COMPUTER PROGRAMS FOR MECHANICAL ENGINEERING DESIGN: FATIGUE ANALYSIS, SECTION PROPERTIES, AND BEAM ANALYSIS

by

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DEDICATION

To my beloved parents, and , and my brothers and sister who have constantly encouraged me to pursue a higher education.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

NOMENCLATURE --- FATIGUE ANALYSIS

b = Kececioglu factor.

D = Dimension, in or m.

Ka = Surface finish factor.

Kb = Size and shape factor.

Kc = Reliability factor.

Kd = Temperature factor.

Ke = Fatigue Strength Reduction factor.

Kf = Miscellaneous factor.

Kt = Theoretical stress concentration factor.

L = Load line.

M = Moment, lb-in or N-m.

N = Number of cycles.

n = Safety factor, dimensionless.

P = Probability, %.

p = Exponent on dimensionless alternating stress term, see Eq. 1 and Table A-1.

Q = Notch sensitivity factor.

q = Exponent on dimensionless mean stress term, see Eq. 1 and Table A-1.

R_1 = A variable, see Eq. 1 and Table A-1.

R_2 = A variable, see Eq. 1 and Table A-1.

r = Notch radius, in or m.

S = Strength of a material, psi or Pa.

Se' = Endurance limit for a R. R. Moore rotating beam specimen.

Se'' = Significant endurance limit for infinite life, psi or Pa.

Se'''= Significant endurance limit for finite life, psi or Pa.

T = Torque, lb-in or N-m.

t = Temperature, °F or °C.

Z = Section modulus, in^3 or m^3.

Z_R = Standard deviation of the endurance limit.

τ = Shear stress, psi or Pa.

Subscripts: a = alternating.

e = equivalent except in Se', Se'', or Se''' where it indicates endurance.

m = mean.

u = ultimate.

y = yield.

Superscripts: ' = indicates a load including a measure of overloading, or a proportionality factor, usually n, safety factor.

NOMENCLATURE --- SECTION PROPERTIES

A = Area, in^2 or mm^2.

b = Base, in or mm.

h = Height, in or mm.

I = Area moment of inertia, in^4 or mm^4.

i = Index number of a point.

J = Polar area moment of inertia, in^4 or mm^4.

M = Static moment of an area, in^3 or mm^4.

n = Number of points.

R = Radius of gyration, in or mm.

r = Radius of a circle, in or mm.

x = Coordinate of x-axis.

y = Coordinate of y-axis.

θ = Angle between the original axis and arbitrary axis.

ϕ = Angle between the original axis and principal axis.

NOMENCLATURE --- BEAM ANALYSIS

E = Modulus of Elasticity, psi or Pa.

G = Structural damping constant, dimensionless.

H = A constant for rotating shaft or rotor in vibration, dimensionless.

I = Area moment of inertia, $in.^4$ or m^4.

i = Number of sections.

J = Mass moment of inertia, lb-sec or $Kg\text{-}m^2$.

L = Length, in or m.

M = Moment, lb-in or N-m.

m = Mass, $lb\text{-}sec/in^2$ or Kg.

P = Concentrated load, lb or N.

q = Magnitude of uniformly distributed load, lb/in or N/m.

q_1 = Magnitude of linearly varied distributed load, at the left end, lb/in or N/m.

q_2 = Magnitude of linearly varied distributed load, at the right end, lb/in or N/m.

S = Slope, radian.

V = Shear, lb or N.

W = Deflection, in or m.

w = Forced circular frequency, rad/sec.

x = Variable length, in or m.

$[P]$ = Point matrix.

$[F]$ = Field matrix.

$[U]$ = Product of matrices.

$[Z]$ = State vector.

Superscripts: L = Left.

R = Right.

CHAPTER I

1.0 INTRODUCTION

The computer is an essential tool that engineers have used to increase the effectiveness and efficiency of their work. New technologies have become feasible because of it. In order to get a quick, accurate, and optimized solution to a complicated design problem, there is little alternative but to use the computer. Therefore, the computer and the designer should work as a team; the computer does the analysis and the engineer supplies the creativity and decision-making components. This interactive process is called computer-aided design. But the engineer is not using the computer as frequently nor as fully as he should. Currently, major computer-aided design applications --- in order of use and market size --- are: electronics, drafting, cartography, architecture, and last and least, engineering.[1*] If the computer were more readily available to the engineer, and if the "canned" programs were tailored to the more general uses in engineering, the engineers would be more willingly to use the computer.

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*All numbers subscripted refer to references listed at the back of this thesis.

A large number of "canned" computer programs have been developed for engineering as listed in the Computer Program Abstracts[2] in batch form; i.e., using computer cards for input. In order to use these programs, one has to have some knowledge of programming and also has to know exactly what data have to be input card by card. Therefore, one has to read pages upon pages of instruction manual before one even attempts to use such programs. One small error in the input, such as wrong sequence of cards, data being put in the wrong location, or data in the wrong format will invalidate the entire run. The interactive computer system will eliminate this problem. Since all the instructions are given to the users step by step via the screen or printout of the terminal in a conversational form, all the user need to do is to follow the simple instructions. The user enters the data by answering the questions that are asked by the computer. Even one who has no previous programming knowledge will be in a position to produce quality work if the interactive program is written properly.

This thesis deals with three programs in mechanical engineering design: Fatigue Analysis, Section Properties, and Beam Analysis. The Fatigue Analysis program will size a mechanical component, circular, rectangular or any shape, to prevent fatigue failure. User can select one of the six most generally accepted fatigue failure lines: modified Goodman fracture line, modified Goodman yield line, Soderberg line, Gerber line, Quadratic line, and Kececioglu line. There is also a routine built in to find the significant endurance limit provided that the user can supply the value of the theoretical stress concentration factor along with other physical and environmental parameters. If the theoretical stress concentration factor is not known, the program will supply a list of references where the user can find the theoretical stress concentration factor. The user also has to supply a subroutine for the stress equations applicable to his problem where any equivalent stress theory can be used.

The cross sectional properties program is designed to find twenty different section properties, such as area, area moment of inertia, and radius of gyration about different axis of any shape plane cross section. Several computer and calculator programs exist for finding these properties, but they are complex. Data entry error is highly probable and cross section verification is not generally available for a specified shape. But the program developed here can find section properties of any shape that can possibly be thought of as well as graphically verifying its shape on an interactive basis.

The Beam Analysis program uses the transfer matrix method to find and also plot the graphs of deflection, slope, moment, and shear of any continuous beam with any kind of loadings, such as uniformly distributed load, concentrated load, concentrated moment, and so on. This can be done provided that the beam is deflected within the linear, elastic range. It will do static as well as forced, undamped, dynamic response analysis.

All three programs provide the option of using English or SI units. The micro-processor used was a Teketronix, model 4051, with 32 K memory, a CRT output and a hard copy facility. The programming language used was BASIC, an acronym for Beginners All-purpose Symbolic Instruction Code, and was developed at Dartmouth College, New Hampshire by Professors J. G. Kemeny and T. E. Kurtz under the terms of a grant from the National Science Foundation in 1965[3]. It is a high level computer language that is easy to use and is applicable to scientific and mathematical work.

The symbols used for the variables are different for each program. They are defined in the nomenclature of the individual program.

CHAPTER II

2.0 LITERATURE REVIEW

A large number of "canned" computer programs have been developed. Their number is increasing enormously. They are mainly designed for large computers, written in batch mode FORTRAN. Recently, some programs have been developed for the pocket size programmable calculators. These applications are getting increasing attention because of their advantages such as small size, low cost, and accessibility to almost anybody. Owing to their size limitation, they are used only for small program with small data storage. Sanderson[3] in 1973 recommended the use of BASIC in engineering design, but very seldom are engineering programs written in BASIC language.

Mischke[4, 5] used IOWA CADET (Computer Augmented Design Engineering Technique) program[6] as an illustration of computer application in mechanical design. This program had a number of fatigue analysis routines such as Marin's deterministic fatigue modification factors, significant strength with generalized Gerber failure diagram, and fatigue damage. All the routines were written in FORTRAN.

For fatigue analysis, Hewlett-Packard Company[7] in 1976 had developed one program that calculated any one of the seven variables (yield strength, significant endurance limit, cross sectional area, stress concentration factor, maximum load, minimum load, and safety factor) in the Soderberg's equation, provided that the numerical values of the other six variables were known. This program used only Soderberg's equation to size a mechanical component under fatigue loading. This program was designed for HP-67 or HP-97 series programmable calculators.

Shigley[8] in 1976 provided the equations and flow-charts for six short programs for programmable calculators. Only those that are related to this thesis are described here. The program included modified Goodman fracture line for prediction of a fatigue failure, the log S - log N diagram to determine the significant endurance limit for finite life when the life in number of loading cycles was given or vice versa, and the theoretical stress concentration factor for a rounded, shouldered shaft in bending. The program that computed the theoretical stress concentration factor used the curve-fitting technique which converted the immense amount of experimental data to a regression line form for the programmable calculator.

Pilkey and Jay[9] in 1974 had compiled a list of the existent section properties programs. Programs included SASA, developed by Structural Dynamics Research Corporation, Cincinnati, Ohio, which calculated both cross sectional properties and stresses using the finite element method proposed by Herrmann.[10,11] Such properties as moment of inertia, torsional constant, and warping constant could be found for a cross section of any shape.

TRW Systems Group, STRU-PAK, Redondo Beach, California[9], had developed three section properties programs: GENSECT1, GENSECT2, and STANSECT. GENSECT1 used an integration technique to compute the section properties of arbitrary plane cross section. The user had to input the x and y coordinates for points which defined the outline of the section.

Voids in a section were handled similarily. GENSECT2 only computed the section properties of a cross section that could readily be subdivded into rectangles, right triangles, and circles. Voids were handled in the same way and had to be composed of these same geometric figures. The user had to divide the cross section into rectangles, right triangles, and circles, specifying their dimensions and centroids. STANSECT calculated the section properties of eighteen types of standard cross section.

AREA$$, developed by General Electric, Bethesda, Maryland,[9] computed the section properties for any area bounded by straight lines and circular arc segments. This program used a problem-oriented language designed especially for geometric property calculations.

SECTION1, developed by United Computing Systems, Kansas City, Missouri,[9] calculated the area, centroid, moment of inertia, section modulus, extreme fiber distance, and radius of gyration of any section provided that it could be resolved into rectangles.

SECTION PROPERTIES, developed by North American Aviation, Canoga Park, California,[9] determined the section properties of any area defined by peripheral coordinates. Areas were restricted to those bounded by convex curves.

In 1975, Pilkey[12] developed BEAMSTRESS, a program which computed the section properties and stresses for an arbitrary cross section of a bar using the finite element method proposed by Hermann.[10,11] The cross section had to be modelled as an assemblage of quadrilateral finite elements of any size. The accuracy of the results would depend upon the fineness of these elements used. Properties computed included area, centroid, moment of inertia, radius of gyration, shear center, shear deformation coefficients, torsional constant, and warping constant. If the internal shear forces, bending moments, axial force, and axial torque were given, this program would compute the normal and shear stress distribution.

Wojciechowski[13] in 1976 developed a technique that replaced integration by summation of finite elements to find the section properties of a plane cross section. This technique applied only to areas bounded by straight lines, but because curves could be approximated by straight line segments, the method could be used on any shape cross section. The equations for computing area, static moment, and moment of inertia were derived. This method could be used by a programmable calculator.

Hewlett-Packard Company[14] in 1976 developed a section properties program using the same method developed by Wojciechowski[13] but had expanded it to include more properties, such as centroid, product of inertia, and moment of inertia about the centroid, the principal axis and also an arbitrary axis. The x and y coordinates of the vertices of the section were input sequentially for a complete, clockwise path. Holes in the section were input in the same manner but in a counter-clockwise path.

Using the transfer matrix method, Pilkey[15] in 1969 developed a generalized structural analysis program which could analyze static, stability, free dynamic, and forced dynamic motion. The user had to supply four subroutines: transfer matrix, initial parameter, time dependent loading, and time dependent displacements subroutines for the beam to be analyzed. Therefore, one had to be very familiar with the transfer matrix method in order to be able to write these subroutines. The output gave the deflection, slope, moment, and shear along the beam.

Paz and Cassaro[16] in 1974 presented the theory and described a computer program in Fortran IV for the analysis of continuous beams on discrete elastic supports using the five moment equations. This analysis permitted direct determination of the redundant moments at the supports of a continuous beam. This program accepted uniformly distributed loads and any number of concentrated loads in the cantilever or at any span of the beam. Output gave both bending moment and reactions of the supports only. Two sample problems were provided for demonstrating the capabilities of the program.

Pilkey and Jay[17] in 1974 compiled a list of beam analysis programs. SPIN, a program developed by Structural Dynamics Research Corporation, Cincinnati, Ohio,[17] calculated the critical speeds of rotating shafts and the natural frequencies in bending of multispan beams of arbitrary cross section. It also calculated the response due to sinusoidally applied forces. The deflections, bending moments, shear forces, and stresses created by static forces could also be found by forcing the shaft to zero speed. SPIN used a distributed mass method for dynamic analysis.

TRW Systems Group, STRU-PAK, Redondo Beach, California[17], developed two programs: MULTISPAN and STANBEAM. MULTISPAN used a static, Euler-Bernoulli analysis of multiple span beams with no dynamic motion. It allowed up to ten spans having uniform or piecewise variable cross sections. In-span supports were pinned; end supports could be fixed, pinned, or free. STANBEAM allowed only static bending analysis of single span beams with either uniform or piecewise variable cross sections. Internal forces and displacements were found using an integration procedure. Maximum shear and bending stresses were calculated by the usual VQ/IB and MC/I formulas.

COM/CODE Corporation, Alexandria, Virginia[17], developed LINKI, a beam analysis program which used a special purpose language called LINKI. It allowed static, stability, and free dynamic analysis of Euler-Bernoulli, Rayleigh, or Timoshenko beams and accepted inspan supports, variable cross sections, and foundations.

Dow Engineering Company, Houston, Texas[17] had a general two-dimensional beam analysis program called GENERAL ANALYSIS which could deal with static beams, beams on elastic foundations, and simple frames. The method used was finite differences and outputs were deflection, shear, and moment.

Southwest Research Institute[17] had two programs: DANAXXO and DANAXX4. DANAXXO computed the frequencies and eigenvectors of a beam with lumped mass using a stiffness matrix method of analysis. Response due to static loads could also be analyzed. DANAXX4 computed the time history of the response of a beam to applied force pulses and applied torque pulse which was represented by a lumped parameter system. The program allowed any combination of hinged, clamped, free, or guided flexural boundary conditions, but no damping was included. The response was determined by a step-by-step integration of the equations of motion using the linear acceleration method.

Pilkey[12] in 1975 developed another program called BEAMRESPONSE using transfer matrix method. Static, stability, and dynamic analysis could be performed for beams of uniform or variable cross section with any kind of loadings. Any number of in-span supports were acceptable, including extension springs, rotary springs, rigid supports, guides, shear releases, and moment releases. The program calculated the deflection, slope moment, and shear for static and steady state conditions, the critical load and mode shape for stability, and the natural frequencies and mode shapes for transverse vibrations.

Hewlett-Packard Company[18] in 1976 also developed four programs for the beam analysis used in their programmable calculators. All four programs were designed only for static response with the mass of the beam neglected. Each program dealt with only one type of beam. The four types of beam that could be applied were cantilever beam, simply supported beam, beam fixed at both ends and propped cantilever beam. Each program calculated the deflection, slope, moment, and shear at any specified point along the beam of uniform cross section. Distributed loads, point loads, applied moments, or combinations of all three might be applied. The equations used were based on elementary beam theory and applied only to simple beams.

A report[19] in 1975 described an interactive computer program, KINSYN III, for designing complex linkage. This program had both synthesis and analysis capabilities. It relied on continuous communication between the designer and the computer to arrive at a final solution. This program was developed by the kinematicians at Massachusetts Institute of Technology under the direction of Roger Kaufman, who is now at George Washington University. The designer created and altered a linkage on a "data tablet", a graphic device continually sensed by the computer. In turn the computer responded via a display screen. In this way, the designer experimented with the linkage design, while the computer guided and instructed him by performing the required calculations and by flashing pertinent design information on the display screen. Therefore, the designer would know immediately if the design was possible.

Shore, Wilson and Semsarzadeh[20] in 1975 developed STACRB, an interactive computer program with graphical output which analyzed horizontally curved and straight aligned bridge structures. This program used the finite element modelling method and allowed the user to: define the parameters that characterized the structure; modify an already discretized structure; and obtain graphical or tabular results on various structural components such as structure geometry, cross section geometry, and various results of analysis such as deflections, reactions, and stress resultants.

CHAPTER III

3.1 THEORY --- FATIGUE ANALYSIS

This program computes the dimension of a mechanical component whose cross section is either circular, rectangular, or expressable as a function of one variable by proportions. The dimension computed is the diameter, if the cross section is circular; otherwise, it is the dimension of the cross section which has all its other dimensions described in terms of this basis dimension.

3.1.1 FATIGUE FAILURE LINE

When one analyzes a fatigue failure, there are a number of fatigue failure lines available. This program provides the six most generally accepted fatigue failure lines: modified Goodman fracture line, modified Goodman yield line, Soderberg line, Gerber line, Quadratic line, and the Kececioglu line. Marin[21] has developed the general form of the fatigue failure lines. It is given in modified form as follows:

$$\left(\frac{S_a}{R_2\ Se''}\right)^p + \left(\frac{R_1\ S_m}{S_u}\right)^q = 1 \qquad (1)$$

Specifically, the exponents p and q control the fatigue line being used. Table A-1[22] gives the controlling parameters that select the fatigue failure line desired in the design.

For the Kececioglu factor, b, it depends on the type of material being used. From experimental data obtained by the application of the probabilistic "Design for Reliability" method, Kececioglu[23, 24] found the values of b for different materials. These are given in Table A-2.

3.1.2 PARAMETRIC METHOD

When using this Fatigue Analysis program, the user has to supply a subroutine program where any equivalent stress theory may be used. The stress equations in this subroutine have to be derived using the Parametric method developed by Mitchell and Zinskie[25].

In developing a fatigue design equation, it is necessary to solve the simultaneous equations of two lines: the general load line and the material fatigue safety failure line. This parametric method can apply to both straight and curved load lines. The parametric pair of equations which describes the general load line are of the form:

$$\left.\begin{aligned} S'_a &= f_a(L_a, n, D) \\ S'_m &= f_m(L_m, n, D) \end{aligned}\right\} \quad (2)$$

as a function of the parameter, n. Alternately, n is the safety factor as defined by Juvinall[26]. It is redefined here as:

$$n = \frac{\text{Maximum allowed external load}}{\text{Design external load}} \tag{3}$$

This safety factor, n, can also be considered as a proportionality constant that signifies overload. It will scale the externally applied loads if they are changed from the design load during the loading process. Therefore, any load that is changed directly proportional to the overload is a function of n and should be multiplied by the proportional factor, n. These are the parametric equations of Eq. 2. Substituting Eq. 2 into Eq. 1, one gets Eq. 4 which is the generalized fatigue equation for a mechanical component.

$$\left(\frac{f_a(L_a,\ n,\ D)}{R_2\ Se'''}\right)^p + \left(\frac{R_1\ f_m(L_m,\ n,\ D)}{S_u}\right)^q = 1 \tag{4}$$

To illustrate the concept of the parametric method, an example of a rotating shaft subjected to a transmitted torque, T, and an applied moment, M, is used. The shaft is assumed to be loaded such that, if more torque is demanded, the bending moments applied to the shaft will increase proportionally by the same factor, n. The design equation for this case has been found[22, 27], as

$$D = \frac{32\ n}{\pi} \left(\frac{M}{Se'''} + \frac{0.866\ T}{S_u} \right)^{1/3} \tag{5}$$

Using the maximum distortion energy theory, the equivalent stresses are:

$$\left.\begin{aligned} S_{em} &= \left(S_m^2 + 3\tau_m^2 \right)^{\frac{1}{2}} = \left(0^2 + 3(\tfrac{T}{2Z})^2 \right)^{\frac{1}{2}} = 0.866\ \frac{T}{Z} \\ S_{ea} &= \left(S_a^2 + 3\tau_a^2 \right)^{\frac{1}{2}} = \left((\tfrac{M}{Z})^2 + 0^2 \right)^{\frac{1}{2}} = \frac{M}{Z} \end{aligned}\right\} \tag{6}$$

But upon overload, both the torque, T, and the moment, M, go up proportionally by the same factor, n, that is:

$$T \longrightarrow n\ T \qquad \text{and} \qquad M \longrightarrow n\ M \tag{7}$$

thus

$$S'_{em} = \frac{0.866\ n\ T}{Z} \qquad \text{and} \qquad S'_{ea} = \frac{n\ M}{Z} \tag{8}$$

Equation 8 is the stress equation using the parametric method, as required for the subroutine.

Substituting Eq. 8 into Eq. 1 with the parameters R_1, R_2, p, and q set at 1, 1, 1, and 1 (see Table A-1), respectively for the modified Goodman fracture line, one gets:

$$\frac{n\ M}{Se'''\ Z} + \frac{0.866\ n\ T}{S_u\ Z} = 1 \tag{9}$$

Substituting the section modulus for a circular cross section,

$Z = \frac{\pi D^3}{32}$, and solving for shaft diameter, one gets:

$$D = \frac{32\ n}{\pi}\left(\frac{M}{Se''} + \frac{0.866\ T}{S_u}\right)^{1/3}$$

which is identical to Eq. 5.

3.1.3 SIGNIFICANT ENDURANCE LIMIT

This routine for computing the significant endurance limit uses a combination of data presented in: Section 6-13 to Section 6-22 of Shigley[28], Section 3-24 to Section 3-29 of Deutschman[29], and Chapter three of Sors[30].

The significant endurance limit is found by the equation,

$$Se'' = Se' \times Ka \times Kb \times Kc \times Kd \times Ke \times Kf \qquad (10)$$

3.1.3.1 SURFACE FACTOR, Ka

The data for the surface factor which is a function of the ultimate tensile strength were taken from Fig. 6-27 of Shigely[28].

Five types of surface finish are provided. For each type, an equation in terms of the ultimate tensile strength is obtained by polynomial regression. The equations used for each type of surface finish are given below.

$$\left.\begin{array}{l}
\text{For polished finish, } Ka = 1.0 \\
\text{For ground finish, } Ka = 0.89 \\
\text{For machined or cold drawn,} \\
Ka = -2.91 \times 10^{-17} S_u^3 + 2 \times 10^{-11} S_u^2 - 4.95 \times 10^{-6} S_u + 1.064 \\
\text{For hot rolled,} \\
Ka = -5.77 \times 10^{-17} S_u^3 + 3.41 \times 10^{-11} S_u^2 - 8 \times 10^{-6} S_u + 1.066 \\
\text{For as forged,} \\
Ka = -6.45 \times 10^{-17} S_u^3 + 3.63 \times 10^{-11} S_u^2 - 7.87 \times 10^{-6} S_u + 0.89
\end{array}\right\} \quad (11)$$

where the ultimate tensile strength, S_u, must be in psi. If SI units are used, the program automatically converts the tensile strength to English unit.

3.1.3.2 SIZE AND SHAPE FACTOR, Kb

The data for the size and shape factor, Kb, were taken from Fig. 42, Part II of Sors[30].

The factor, Kb, is determined by size, shape, and material. Since the size of the mechanical component is not known until the end of the computation, this factor, Kb, has to be treated separately while all the other factors: Ka, Kc, Kd, Ke, and Kf are calculated first. The dimension of the component is found by solving the simultaneous equations of the general load line and the selected fatigue failure line. An iteration process called the half interval search is used. For each iteration, a new size factor and significant endurance limit are computed. Using this value of the significant endurance limit, the simultaneous equations of the general load line and the fatigue failure line can be solved by iteration. If the root of these equations is found, then this set of size factor and significant endurance limit are correct for this design.

The cross section of the mechanical component must be either circular or rectangular. Moreover, it must be made of steel or light alloy, because only the experimental data of these two are available. Using a curve fitting technique, the data were converted to equations by regression[31]. These equations are given below where the dimension, D, must be in mm. If English units are used, the program automatically corrects the dimension to SI units.

For steel,

Circular cross section,

$$\left.\begin{array}{ll} \text{If } D<23, & Kb = 1 \\ \text{If } D>23 \text{ but} <130, & Kb = \dfrac{D}{-18.75 + 1,802\ D} \\ \text{If } D>130, & Kb = 0.59 \end{array}\right\} \quad (12)$$

Rectangular cross section,

$$\left.\begin{array}{ll} \text{If } D<19, & Kb = 0.88 \\ \text{If } D>19 \text{ but} <150, & Kb = 0.5061 + \dfrac{7.214}{D} \\ \text{If } D>150, & Kb = 0.55 \end{array}\right\} \quad (13)$$

For light alloy,

Circular cross section,

$$\left.\begin{array}{ll} \text{If } D<7, & Kb = 1.0 \\ \text{If } D>7 \text{ but } <41, & Kb = 0.515 + \frac{3.24}{D} \\ \text{If } D>41, & Kb = 0.59 \end{array}\right\} \quad (14)$$

Rectangular cross section,

$$\left.\begin{array}{ll} \text{If } D<7, & Kb = 0.88 \\ \text{If } D>7 \text{ but } <47, & Kb = 0.5061 + \frac{2.25}{D} \\ \text{If } D>47, & Kb = 0.55 \end{array}\right\} \quad (15)$$

3.1.3.3 RELIABILITY FACTOR, Kc

The reliability factor was computed using the method described in Shigley[28]. This uses the equation

$$Kc = 1 - 0.08\ Z_R \qquad (16)$$

where Z_R is the standard deviation of the endurance limit. The data of Table 6-2 in Shigley were fitted to a curve by regression. The relation between the reliability and the probability is given by

$$P = 100 - \text{reliability} \qquad (17)$$

A regression on data in terms of base ten logarithm was used to obtain the standard deviation, Z_R[31],

$$Z_R = 2.37 - 0.885(\log P) - 0.193(\log P)^2 - 0.0502(\log P)^3 - 0.00489(\log P)^4 \qquad (18)$$

The reliability factor, Kc, is then computed using Eq. 16.

3.1.3.4 TEMPERATURE FACTOR, Kd

The method used for the temperature factor, Kd, is described by Shigley[28]. From Shigley,

$$Kd = \frac{620}{460 + t} \qquad (19)$$

where t = operating temperature in °F.

The above equation is used only when t>160 °F (71 °C). The user should be warned that at operating temperatures above 570 °F (300 °C), creep and reduced yield strength may cause problems which are not accounted for by Eq. 19.[30]

3.1.3.5 FATIGUE STRENGTH REDUCTION FACTOR, Ke

The user must supply the theoretical stress concentration factor, Kt. If the user doesnot know the theoretical stress concentration factor, the references[32, 33, 34, 35] where tables or charts of Kt are given, will be helpful.

The user may supply the notch sensitivity factor, Q, or he can instruct the micro-processor to calculate it. The data for the notch sensitivity factor were taken from Fig. B-2 on page 892 in Deutschman[29]. Again, using regression, Eq. 20 is obtained. The notch radius, r, must be in inches. The program automatically corrects the radius data to English units if SI units are used.

$$
\left.
\begin{array}{l}
\text{Under bending or axial loading:} \\
\text{If } S_u < 50{,}000 \text{ psi (345 MPa)} \\
Q = -8828\, r^4 + 3345.3\, r^3 - 440.94\, r^2 + 24.62\, r + 0.18 \\
\text{If } S_u < 60{,}000 \text{ psi (414 MPa) but} \geq 50{,}000 \text{ psi (345 MPa)} \\
Q = -7031.25\, r^4 + 2671.9\, r^3 - 353.13\, r^2 + 20.2\, r + 0.28 \\
\text{If } S_u < 80{,}000 \text{ psi (552 MPa) but} \geq 60{,}000 \text{ psi (414 MPa)} \\
Q = -10{,}156.25\, r^4 + 3{,}825\, r^3 - 497.5\, r^2 + 27.05\, r + 0.23 \\
\text{If } S_u < 100{,}000 \text{ psi (690 MPa) but} \geq 80{,}000 \text{ psi (552 MPa)} \\
Q = -15{,}057.38\, r^4 + 5{,}165.4\, r^3 - 606\, r^2 + 29.23\, r + 0.3 \\
\text{If } S_u < 140{,}000 \text{ psi (965 MPa) but} \geq 100{,}000 \text{ psi (690 MPa)} \\
Q = 5{,}431{,}250\, r^5 - 1{,}236{,}125\, r^4 + 104{,}242.5\, r^3 - 4{,}010.7\, r^2 \\
\qquad + 71.06\, r + 0.33 \\
\text{If } S_u \geq 200{,}000 \text{ psi (1379 MPa)} \\
Q = -271{,}319\, r^4 + 37{,}276.5\, r^3 - 1{,}771\, r^2 + 35.03\, r + 0.67 \\
\\
\text{For torsional loading,} \\
\text{If } S_u < 60{,}000 \text{ psi (414 MPa)} \\
Q = -10{,}156.25\, r^4 + 3{,}825\, r^3 - 497.5\, r^2 + 27.05\, r + 0.23 \\
\text{If } S_u < 80{,}000 \text{ psi (552 MPa) but} \geq 60{,}000 \text{ psi (414 MPa)} \\
Q = -15{,}057.38\, r^4 + 5{,}165.4\, r^3 - 606\, r^2 + 29.23\, r + 0.3 \\
\text{If } S_u < 120{,}000 \text{ psi (827 MPa) but} \geq 80{,}000 \text{ psi (552 MPa)}
\end{array}
\right\} \quad (20)
$$

$$Q = 5{,}431{,}250\ r^5 - 1{,}236{,}125\ r^4 + 104{,}242.5\ r^3 - 4{,}010.7\ r^2 + 71.06\ r + 0.33$$

If $S_u < 180{,}000$ psi (1241 MPa) but $\geq 120{,}000$ psi (827 MPa)

$$Q = -271{,}319\ r^4 + 37{,}276.5\ r^3 - 1{,}771\ r^2 + 35.03\ r + 0.67$$

For aluminum alloy (based on 2024-T6 data)

$$Q = -8{,}815.2\ r^4 + 3{,}411.3\ r^3 - 462.64\ r^2 + 27.85\ r + 0.013$$

After the theoretical stress concentration factor and the notch sensitivity factor are known, the fatigue strength reduction factor is obtained by

$$Ke = \frac{1}{1 + Q(Kt - 1)} \tag{21}$$

If the stress concentration factor is accounted for on the stress side of the equation, Q should be set to zero to make Ke = 1.

3.1.3.6 MISCELLANEOUS FACTOR, Kf

This factor accounts for any miscellaneous effect that is not considered above. If there is no miscellaneous effect, Kf will be set equal to unity by the program.

3.1.3.7 ENDURANCE LIMIT FOR A R. R. MOORE ROTATING BEAM SPECIMEN, Se'

The calculation of the endurance limit for a R. R. Moore rotating beam specimen, Se', is described in Shigley[28].

$$\left.\begin{array}{l}\text{If } S_u < 200{,}000 \text{ psi } (1379 \text{ MPa}),\ Se' = 0.5\, S_u \\ \text{If } S_u \geq 200{,}000 \text{ psi } (1379 \text{ MPa}),\ Se' = 100{,}000 \text{ psi}\end{array}\right\} \quad (22)$$

3.1.4 SIGNIFICANT ENDURANCE LIMIT FOR FINITE LIFE, Se'''

For finite life, there are two methods to compute the significant endurance limit, Se'''. One is the log S - log N approach and the other is log S - linear N approach.

The equation for log S - log N approach is

$$\left.\begin{array}{l}Se''' = 10^{B} \\ \text{where } B = \left(\frac{\log N}{3} - 1\right)\left[\log Se'' - \log(0.9\ Kd\ Se')\right] + \log(0.9\ Kd\ Se')\end{array}\right\} \quad (23)$$

And the equation for log S - linear N approach is

$$Se''' = 0.9\ Kd\ Se' + \left(\frac{\log N}{3} - 1\right)(Se'' - 0.9\ Kd\ Se') \quad (24)$$

When a life of greater than one million cycles is desired, the infinite life significant endurance limit is used.

3.2 THEORY --- SECTION PROPERTIES

This method uses a technique that replaces integration by summation of finite elements to find the section properties of a plane cross section. Basically, the method divides a cross section into a series of trapezoids or rectangles, then adds or substracts the properties of the elemental areas to find the composite properties of the total area. It applies only to area bounded by straight lines, but because curves can be approximated by straight line segments, it can be used on any shape plane cross section.

To speed up the computation process, the program uses different methods for the circular cross section and holes. It requires the radius and the x and y coordinates of the center of the circular section or hole. It will give an exact result. The previous method requires a large number of data points in order to get a good approximation of the curve, which in turn requires many computations.

3.2.1 POLYGONAL CROSS SECTION

The method for computing the section properties of polygonal cross section uses difference equations which replace integration by summation of finite elements. To illustrate how this summation method works, an example of an arbitrary triangular cross section ABC shown in Fig. 1 is used. First the area of the trapezoid DFBC is found. Then the trapezoid FBAE and EACD are substracted. And the net

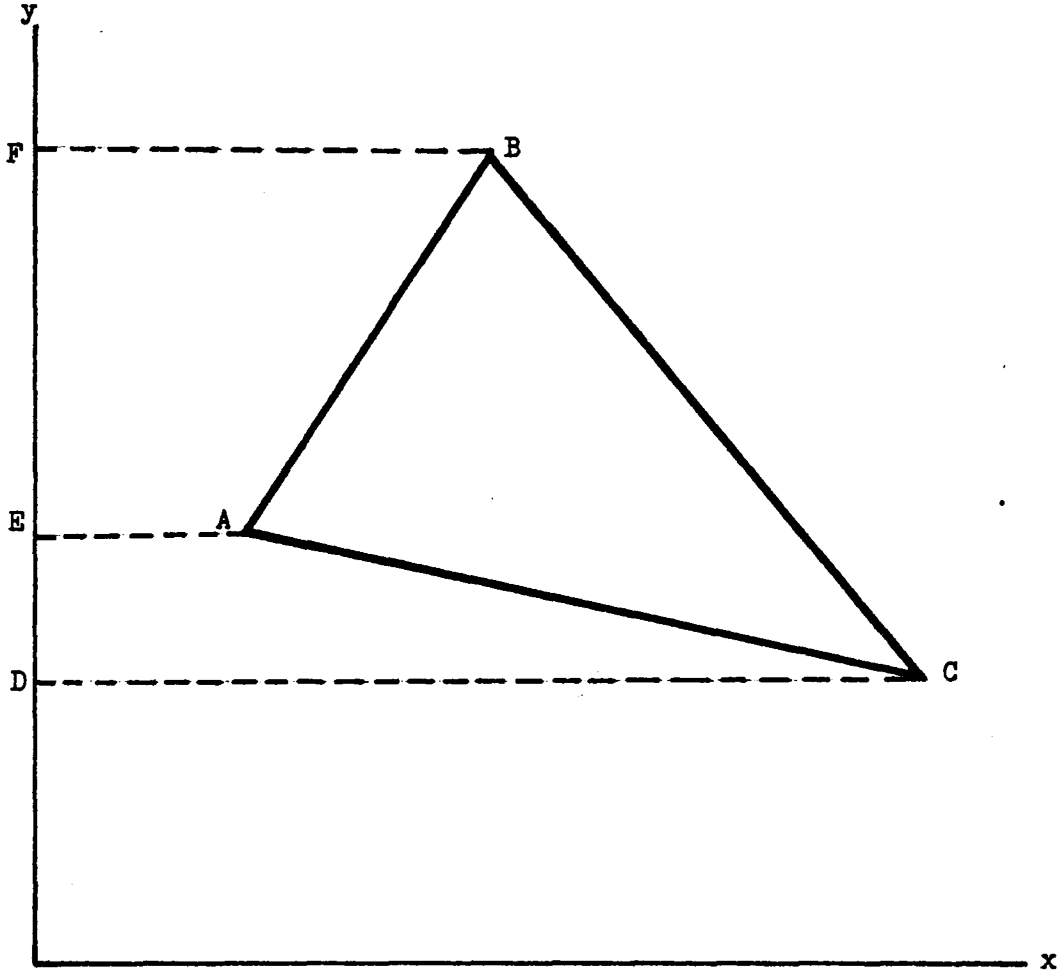


Figure 1. Area of Triangle ABC.

result will be the area of triangle ABC.

Since there are a few axes involved, they are clarified with the help of an irregular shape as shown in Fig. 2, where all the axes are labelled.

All plane cross sections are made up of connected straight lines (curved boundaries may be approximated by straight line segments). For each straight line, a trapezoid, a rectangle, or a triangle can always be formed with the axis, like the shaped area ABCD, as shown in Fig. 3. The area of such a trapezoid can be calculated from the basic rules of plain geometry as

$$\Delta A = -(y_{i+1} - y_i)(x_{i+1} + x_i)/2 \tag{25}$$

which is essentially the area of a trapezoid.

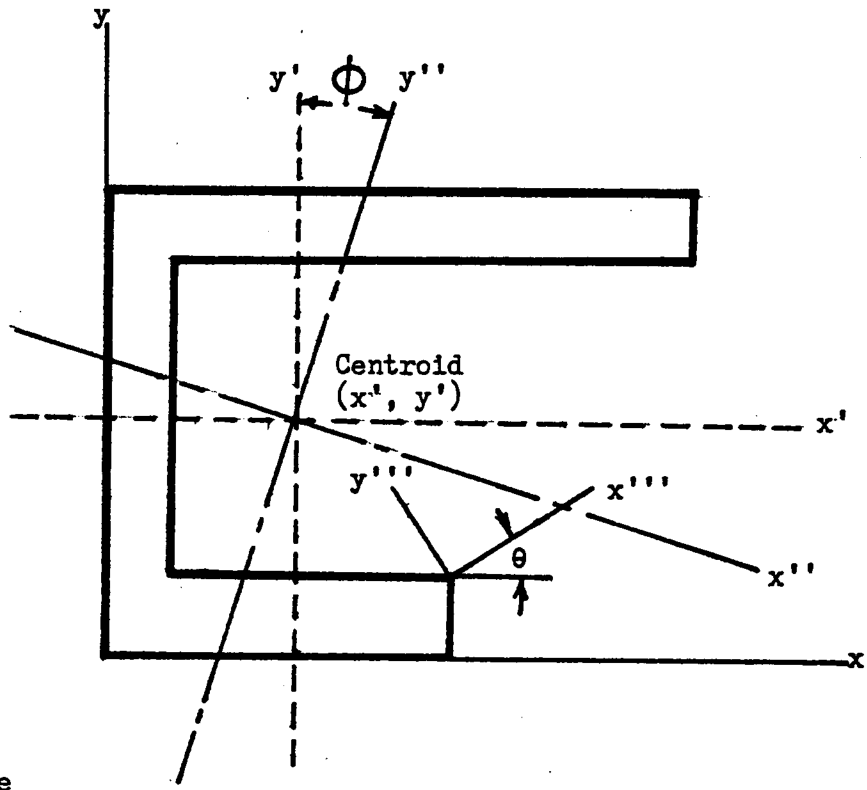
The general formula of the static moment of an area is defined as

$$M_y = \int x \, dA \tag{26}$$

where x is the distance between the centroid of the area and y-axis.

The static moment of ΔM_y of the trapezoid ABCD about the y-axis can be considered to consist of contributions from the rectangle AEGD and triangle EBF minus the contribution from triangle CFG. Using Eq. 26, the static moment ΔM_y can be found to be

$$\begin{aligned}\Delta M_y = {} & \left(\frac{x_i + x_{i+1}}{4}\right)(y_{i+1} - y_i)(x_i + x_{i+1})/2 + \tfrac{1}{2}\left(x_i - \tfrac{2}{3}\left(\frac{x_i - x_{i+1}}{2}\right)\right) \\ & \left(\frac{y_{i+1} - y_i}{2}\right)\left(\frac{x_i - x_{i+1}}{2}\right) - \tfrac{1}{2}\left(\frac{y_{i+1} - y_i}{2}\right)\left(\frac{x_i - x_{i+1}}{2}\right) \\ & \left(x_{i+1} + \tfrac{2}{3}\left(\frac{x_i - x_{i+1}}{2}\right)\right)\end{aligned} \tag{27}$$



where

x-y	Original axis.
x'-y'	Translated axis (axis translated to the centroid)
x''-y''	Rotated, principal axis.
x'''-y'''	Rotated, arbitrary axis (can be located at any point with any angle of rotation).
Φ	Angle between the translated axis and the principal axis.
θ	Angle between the arbitrary axis and the original axis.

Figure 2. Coordinate Definitions --- Section Properties Program.

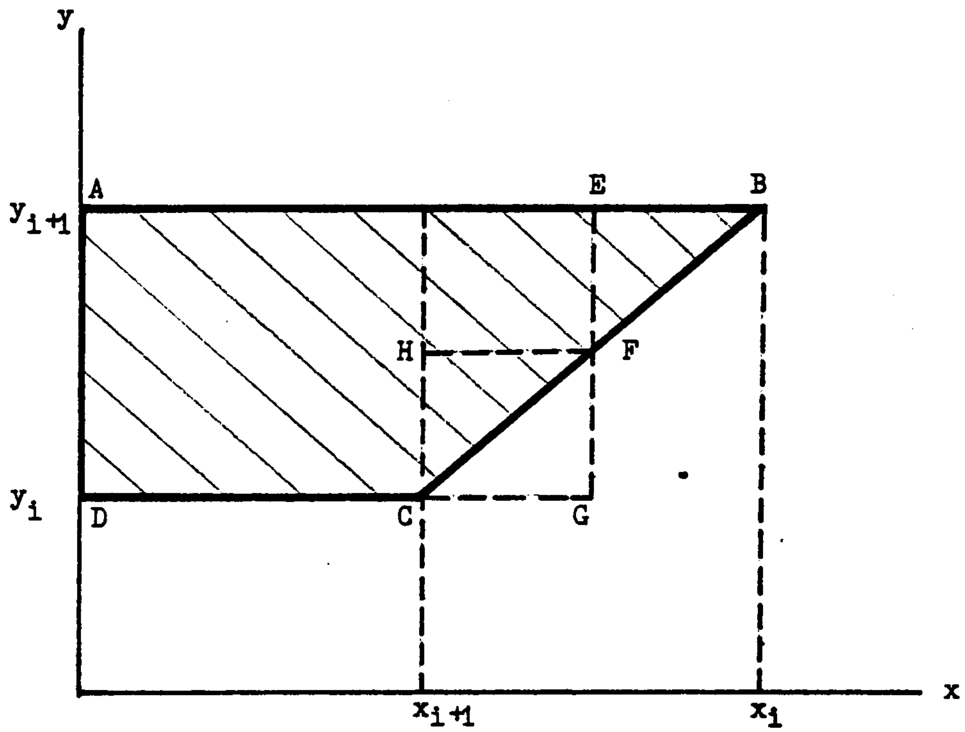


Figure 3. Trapezoid ABCD

which, after simplification and rearrangement, becomes

$$\Delta M_y = ((y_{i+1} - y_i)/8)((x_{i+1} + x_i)^2 + (x_{i+1} - x_i)^2/3) \tag{28}$$

The centroid of an area with respect to x-axis is defined as

$$x' = \frac{\int x \, dA}{\int dA} \tag{29}$$

Dividing Eq. 28 by the area gives

$$\Delta x' = -\frac{1}{A}((y_{i+1} - y_i)/8)((x_{i+1} + x_i)^2 + (x_{i+1} - x_i)^2/3) \tag{30}$$

The formula for calculating the area moment of inertia with respect to the y-axis is

$$I_y = \int x^2 \, dA \tag{31}$$

Similiar to the static moment, the area moment of inertia, I_y, of the trapezoid ABCD about y-axis consists of contributions from rectangle AEGD and triangle EBF minus contribution of triangle CFG. To transfer all the area moment of inertia to the y-axis, the parallel axis theorem is used.

$$I_y = I_{y'} + Ax^2 \tag{32}$$

And the area moment of inertia of a rectangular and a triangular sections with respect to the axis at their bases are given by

$$\left.\begin{aligned} I \text{ of rectangle} &= (b\,h^3)/3 \\ I \text{ of triangle} &= (b\,h^3)/36 \end{aligned}\right\} \quad (33)$$

Using Eqs. 31, 32 and 33, the area moment of inertia, ΔI_y, can be found as

$$\begin{aligned} \Delta I_y = &- \tfrac{1}{3}(y_{i+1} - y_i)\left(\frac{x_{i+1} + x_i}{2}\right)^3 + \frac{1}{36}\left(\frac{y_{i+1} - y_i}{2}\right)\left(\frac{x_{i+1} - x_i}{2}\right)^3 \\ &+ \tfrac{1}{2}\left(\frac{y_{i+1} - y_i}{2}\right)\left(\frac{x_{i+1} - x_i}{2}\right)\left(x_{i+1} - \tfrac{2}{3}\left(\frac{x_{i+1} - x_i}{2}\right)\right)^2 \\ &- \frac{1}{36}\left(\frac{y_{i+1} - y_i}{2}\right)\left(\frac{x_{i+1} - x_i}{2}\right)^3 - \tfrac{1}{2}\left(\frac{y_{i+1} - y_i}{2}\right)\left(\frac{x_{i+1} - x_i}{2}\right) \\ &\left(x_i + \tfrac{2}{3}\left(\frac{x_{i+1} - x_i}{2}\right)\right)^2 \end{aligned} \quad (34)$$

which, after simplification and rearrangement, becomes

$$\Delta I_y = - ((y_{i+1} - y_i)(x_{i+1} + x_i)/24)((x_{i+1} + x_i)^2 + (x_{i+1} - x_i)^2) \quad (35)$$

To find the total area, A, centroid, x', and area moment of inertia, I_y, of a plane cross section, the Eqs. 25,30, and 35 are first solved for each line segment and then the individual results are summed, that is

$$A = \sum \Delta A$$

or

$$A = -\sum_{i=0}^{n}(y_{i+1} - y_i)(x_{i+1} + x_i)/2 \tag{36}$$

$$x' = \sum \Delta x'$$

or

$$x' = -\frac{1}{A}\sum_{i=0}^{n}((y_{i+1} - y_i)/8)((x_{i+1} + x_i)^2 + (x_{i+1} - x_i)^2/3) \tag{37}$$

$$I_y = \sum \Delta I_y$$

or

$$I_y = -\sum_{i=0}^{n}((y_{i+1} - y_i)(x_{i+1} + x_i)/24)((x_{i+1} + x_i)^2 + (x_{i+1} - x_i)^2) \tag{38}$$

Also, the area moment of inertia about the axis through the centroid (x'-y') can be found by

$$I_{y'} = I_y - Ax'^2 \tag{39}$$

which is essentially the parallel axis theorem that transfers the area moment of inertia from the original axis (x-y) to the axis through centroid (x'-y').

The equations for static moment, M_x, centroid y', area moment of inertia with respect to x-axis, I_x, and area moment of inertia of the axis through centroid, $I_{x'}$, are found in the same manner. Their formulas are given as Eqs. 40 through 42.

Static moment, ΔM_x is given by

$$M_x = \int y \, dA$$

$$\Delta M_x = -((x_{i+1} - x_i)/8)((y_{i+1} + y_i)^2 + (y_{i+1} - y_i)^2/3) \tag{40}$$

Centroid, y', is given by

$$y' = \frac{\int y \, dA}{\int dA}$$

$$\Delta y' = \frac{1}{A}\left(\frac{x_{i+1} - x_i}{8}\right)((y_{i+1} + y_i)^2 + (y_{i+1} - y_i)^2/3)$$

$$y' = \sum \Delta y'$$

$$y' = \frac{1}{A}\sum_{i=0}^{n}((x_{i+1} - x_i)/8)((y_{i+1} + y_i)^2 + (y_{i+1} - y_i)^2/3) \qquad (41)$$

Area moment of inertia with respect to x-axis, I_x, is given by

$$I_x = \int y^2 \, dA$$

$$\Delta I_x = ((x_{i+1} - x_i)(y_{i+1} + y_i)/24)((y_{i+1} + y_i)^2 + (y_{i+1} - y_i)^2)$$

$$I_x = \sum \Delta I_x$$

$$I_x = \sum_{i=0}^{n}((x_{i+1} - x_i)(y_{i+1} + y_i)/24)((y_{i+1} + y_i)^2 + (y_{i+1} - y_i)^2) \qquad (42)$$

The area moment of inertia with respect to the axis through centroid, $I_{x'}$, is given by the parallel axis theorem.

$$I_{x'} = I_x - Ay'^2 \qquad (43)$$

The product area moment of inertia with respect to the axis x-y is given by

$$I_{xy} = \sum_{i=0}^{n}(1/(x_{i+1} - x_i))((y_{i+1} - y_i)^2(x_{i+1} + x_i)(x_{i+1}^2 + x_i^2)/8$$

$$+ (y_{i+1} - y_i)(x_{i+1}y_i - x_iy_{i+1})(x_{i+1}^2 + x_{i+1}x_i + x_i^2)/3$$

$$+ (x_{i+1}y_i - x_iy_{i+1})^2(x_{i+1} + x_i)/4) \qquad (44)$$

The product area moment of inertia about the axis translated to the centroid (x'-y') is given by

$$I_{x'y'} = I_{xy} - Ax'y' \tag{45}$$

The angle, Φ, between the translated axis (x'-y') and the principal axis (x''-y'') is given by

$$\Phi = \frac{1}{2} \tan^{-1} \left(- \frac{2 I_{x'y'}}{I_{x'} - I_{y'}}\right) \tag{46}$$

The area moments of inertia about the translated, rotated, principal axis (x''-y'') are given by

$$I_{x'\Phi} = I_{x'} \cos^2\Phi + I_{y'} \sin^2\Phi - I_{x'y'} \sin 2\Phi \tag{47}$$

$$I_{y'\Phi} = I_{y'} \cos^2\Phi + I_{x'} \sin^2\Phi + I_{x'y'} \sin 2\Phi \tag{48}$$

The arbitrary axis (x'''-y''') is the axis that can be fixed anywhere by specifying the x and y coordinates of the origin of the arbitrary axis. The angle between the original axis (x-y) and the arbitrary axis (x'''-y''') is θ. The area moments of inertia about the arbitrary axis (x'''-y''') are given by

$$I_{x'''a} = I_{x'} + A(y' - y''')^2 \tag{49}$$

$$I_{y'''a} = I_{y'} + A(x' - x''')^2 \tag{50}$$

$$I_{x'''y'''a} = I_{x'y'} + A(x' - x''')(y' - y''') \tag{51}$$

where x''' and y''' are distances from the arbitrary axis to the original axis (x-y).

Now, rotating to the angle θ, the area moments of inertia and the product area moment of inertia of the arbitrary axis become

$$I_{x'''} = I_{x'''a}\cos^2\theta + I_{y'''a}\sin^2\theta - I_{x'''y'''a}\sin 2\theta \tag{52}$$

$$I_{y'''} = I_{y'''a}\cos^2\theta + I_{x'''a}\sin^2\theta + I_{x'''y'''a}\sin 2\theta \tag{53}$$

$$I_{x'y'} = \tfrac{1}{2}(I_{x'''a} - I_{y'''a})\sin 2\theta + I_{x'''y'''a}\cos 2\theta \tag{54}$$

The polar moment, J, about the arbitrary axis (x'''-y''') is

$$J = I_{x'''} + I_{y'''} \tag{55}$$

3.2.2 CIRCULAR CROSS SECTION

For circular cross section, the formulas are commonly known. They are listed below.

Area of circle is

$$A = \pi R^2 \tag{56}$$

The centroid is always at the center.

The area moment of inertia and product area moment of inertia with respect to the axis translated to the centroid (x'-y') are

$$I_{x'} = \pi R^4/4 \tag{57}$$

$$I_{y'} = \pi R^4/4 \tag{58}$$

$$I_{x'y'} = 0 \tag{59}$$

Then the area moment of inertia and product area moment of inertia about any other axis are given by the parallel axis theorem. For the original axis (x-y), they are given by Eqs. 60, 61, and 62.

$$I_x = I_{x'} + Ay'^2 \tag{60}$$

$$I_y = I_{y'} + Ax'^2 \tag{61}$$

$$I_{xy} = I_{x'y'} + Ax'y' \tag{62}$$

Now, all the other properties, such as angle ϕ, the area moment of inertia about the translated, rotated principal axis (x''-y''), the area moment of inertia about the arbitrary axis (x'''-y'''), and the polar moment of inertia about the arbitrary axis can be calculated by the same equations as for the polygonal section, Eqs. 46-55.

3.2.3 RADIUS OF GYRATION

The radius of gyration of an area is the square root of the area moment of inertia of that area divided by the area itself. Therefore, radius of gyration with respect to the original axis is given as

$$\left.\begin{aligned} r_x &= (I_x/A)^{\frac{1}{2}} \\ r_y &= (I_y/A)^{\frac{1}{2}} \end{aligned}\right\} \tag{63}$$

And the radius of gyration about the axis through the centroid are given as

$$\left.\begin{aligned} r_{x'} &= (I_{x'}/A)^{\frac{1}{2}} \\ r_{y'} &= (I_{y'}/A)^{\frac{1}{2}} \end{aligned}\right\} \quad (64)$$

3.3 THEORY --- BEAM ANALYSIS

The basic philosophy of the transfer matrix method is based on the idea that a continuous and complicated system can be broken up into component parts with simple elastic and dynamic properties that can be expressed in matrix form. These component matrices are considered building blocks that, when fitted together and evaluated with the proper boundary conditions, will give the static and dynamic responses of the entire system. A continuous beam can be considered to have a number of elements linked together end to end in the form of a chain. Each element, represented by a transfer matrix, can be fitted together as a system by successive matrix multiplications. This method is so generalized that it can deal with any kind of continuous beam with any combinations of loadings. This method has few restrictions. However, the system must be loaded within the elastic range. This method is well documented in Chapter 3 of Pestel and Leckie[36].

This program, using transfer matrix method, computes and also plots the curves of deflection, slope, moment, and shear along the beam. Static and forced, undamped dynamic analysis can be performed for beams of uniform or variable cross section. Uniformly or linearly varied distributed loads, concentrated loads, concentrated applied moments, or combinations of all three may be applied. This program allows any combination of pinned, fixed, free, or guided flexural boundary conditions. A normally kinematically unstable condition can be handled if sufficient internal elastic supports are provided. In-span support can be elastic springs and/or elastic moment springs. Modelling for dynamic responses uses lumped mass. Rigid in-span indeterminants are not treated by this program.

For a given continuous beam with several sections, say i, each element or section is represented by the appropriate field and point transfer matrices. The state vectors from one end, $[Z]_0$, to the other end, $[Z]_i$, are related by the equation,

$$[Z]_i = [P]_i\ [F]_i\ [P]_{i-1}\ [F]_{i-1} \cdots\cdots [P]_j\ [F]_j \cdots\cdots [P]_1\ [F]_1\ [Z]_0$$

$$= [U]\ [Z]_0 \tag{65}$$

where $[F]_j$ = a field transfer matrix that describes the jth section of distributed stiffness with or without distributed loads.

$[P]_j$ = a point transfer matrix that describes the jth element at a point with no finite length.

The state vector, $[Z]$, has five components defined as:

$$[Z] = \begin{bmatrix} W \\ S \\ M \\ V \\ 1 \end{bmatrix} \tag{66}$$

The boundary conditions are as follows:

$$\begin{aligned} &\text{For pinned end, } W=0,\ M=0 \\ &\text{For fixed end, } W=0,\ S=0 \\ &\text{For free end, } M=0,\ V=0 \\ &\text{For guided end, } S=0,\ V=0 \end{aligned} \tag{67}$$

The field and point transfer matrices in Eq. 65 are known and the boundary conditions of both ends should be applied to Eq. 68.

$$\left[Z\right]_i = \left[U\right] \left[Z\right]_0 \tag{68}$$

Solution of Eq. 68 yields all the variables in the state vectors $\left[Z\right]_0$. Once $\left[Z\right]_0$ is known, the matrix multiplication process is repeated to yield the states at each desired point along the beam. In fact, matrix multiplications for many more stations can be determined the second time through in order to get a better resolution of the states within the beam.

3.3.1 DERIVATION OF TRANSFER MATRIX

Since the transfer matrices are the essential building blocks for the systems, some of them will be derived.

Field Matrix for a Massless Beam:

Figure 4 shows a massless straight beam with two displacements, the deflection W and the slope S, and the two corresponding forces, the shear force V and the bending moment M. Using the equilibrium condition requires that the sum of forces in vertical direction be zero and also the sum of the moments about point i-1 be zero, that is

$$\sum F_y = 0, \text{ gives} \quad V_i^L - V_{i-1}^R = 0$$

$$\sum M_{i-1} = 0, \text{ gives} \quad M_i^L - M_{i-1}^R - V_i^L L_i = 0$$

or

$$V_i^L = V_{i-1}^R \tag{69}$$

$$M_i^L = M_{i-1}^R + V_i^L L_i \tag{70}$$

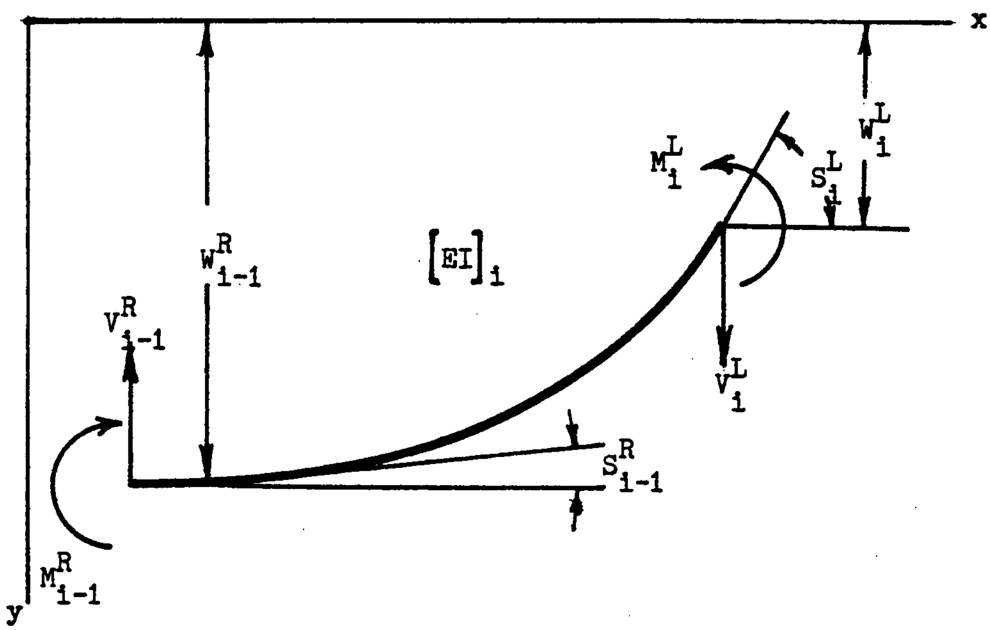


Figure 4. Massless straight beam.

From elementary beam theory, for a cantilever beam with concentrated moment at its free end, the end deflection and slope are given as

$$W = -\frac{ML^2}{2EI} \tag{71}$$

$$S = -\frac{ML}{EI} \tag{72}$$

And for a cantilever beam with a concentrated load at its free end, the end deflection and slope are

$$W = \frac{VL^3}{3EI} \tag{73}$$

$$S = \frac{VL^2}{2EI} \tag{74}$$

Using superposition and noting that the point i-1 has an initial deflection W_{i-1} and an initial slope S_{i-1}, the following equations can be obtained.

$$W_i^L = W_{i-1}^R - S_{i-1}^R L_i - M_i^L \frac{L_i^2}{2(EI)_i} + V_i^L \frac{L_i^3}{3(EI)_i} \tag{75}$$

$$S_i^L = S_{i-1}^R + M_i^L \frac{L_i}{(EI)_i} - V_i^L \frac{L_i^2}{2(EI)_i} \tag{76}$$

Substituting Eq. 70 into Eq. 75, gives

$$W_i^L = W_{i-1}^R - L_i S_{i-1}^R - \frac{L_i^2}{2(EI)_i} M_{i-1}^R - \frac{L_i^3}{6(EI)_i} V_{i-1}^R \tag{77}$$

Substituting Eq. 69 into Eq. 76, gives

$$S_i^L = S_{i-1}^R + \frac{L_i}{(EI)_i} M_{i-1}^R + \frac{L_i^2}{2(EI)_i} V_{i-1}^R \tag{78}$$

Equations 69,70,77 and 78 can be put in a matrix form. Adding an extension column, yields

$$\begin{bmatrix} W \\ S \\ M \\ V \\ 1 \end{bmatrix}_i^L = \begin{bmatrix} 1 & -L & -\frac{L^2}{2EI} & -\frac{L^3}{6EI} & 0 \\ 0 & 1 & \frac{L}{EI} & \frac{L^2}{2EI} & 0 \\ 0 & 0 & 1 & L & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} W \\ S \\ M \\ V \\ 1 \end{bmatrix}_{i-1}^R \tag{79}$$

Equation 79 is the field matrix for massless beam, see Fig. 5a.

The other field matrices are derived in a similiar way making use of elementary beam theory, other elastic and dynamic properties. Other matrices are listed below.

Field matrix for a massless beam with uniformly distributed load, q, see Fig. 5b.

$$\begin{bmatrix} 1 & -x & -\frac{x^2}{2EI} & -\frac{x^3}{6EI} & \frac{qx^4}{24\ EI} \\ 0 & 1 & \frac{x}{EI} & \frac{x^2}{2EI} & -\frac{qx^3}{6\ EI} \\ 0 & 0 & 1 & x & -\frac{qx^2}{2} \\ 0 & 0 & 0 & 1 & -\ qx \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{80}$$

Field matrix for a massless beam with linearly varied distributed load, q_1, q_2, see Fig. 5c, 5d.

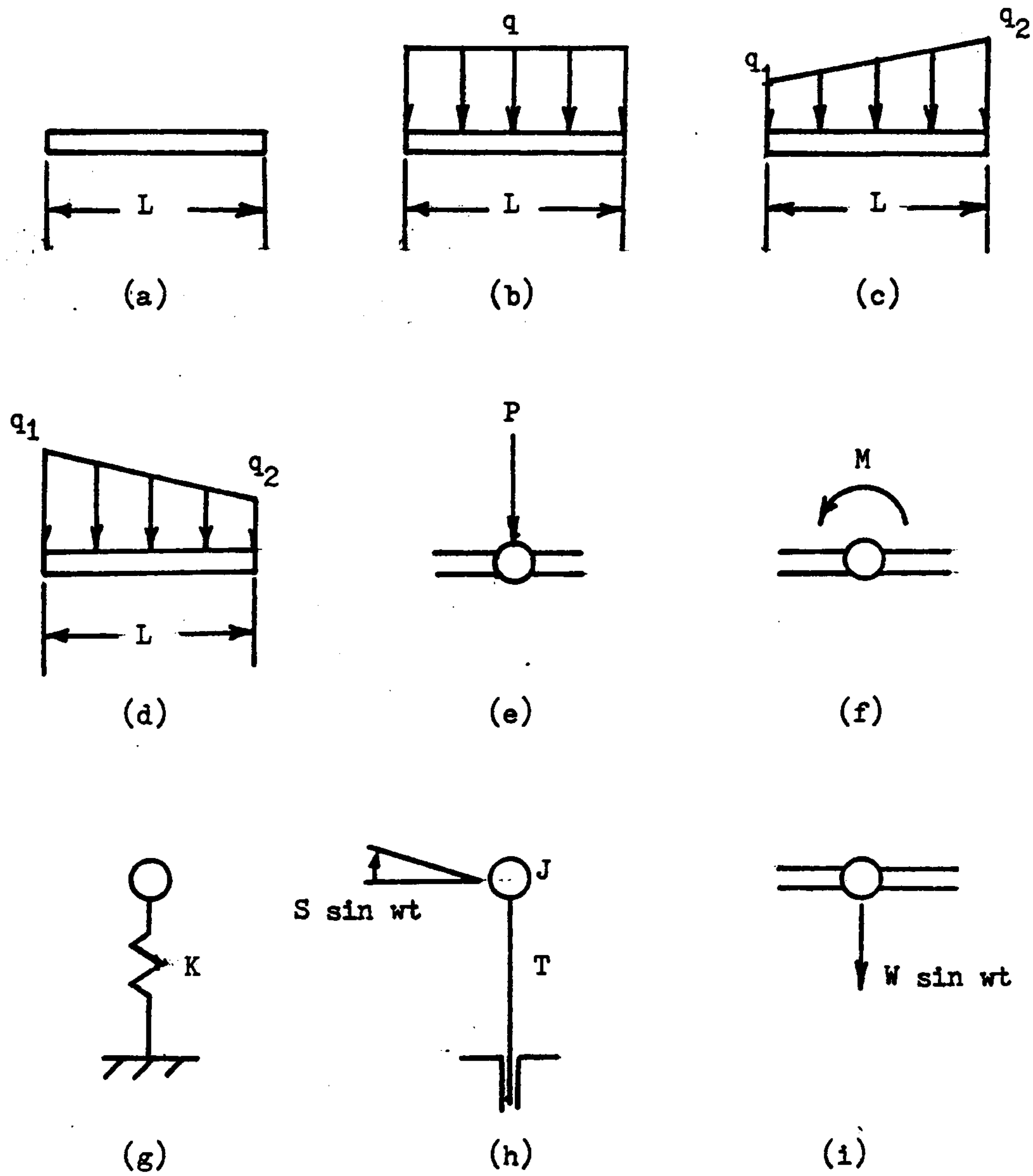


Figure 5. Diagram for Field and Point transfer matrics.

$$\begin{bmatrix} 1 & -x & -\frac{x^2}{2EI} & -\frac{x^3}{6EI} & \frac{q_1x^4}{24EI} - \frac{q_1x^5}{120LEI} + \frac{q_2x^5}{120LEI} \\ 0 & 1 & \frac{x}{EI} & \frac{x^2}{2EI} & \frac{-q_1x^3}{6EI} + \frac{q_1x^4}{24LEI} - \frac{q_2x^4}{24LEI} \\ 0 & 0 & 1 & x & \frac{-q_1x^2}{2} + \frac{q_1x^3}{6L} - \frac{q_2x^3}{6L} \\ 0 & 0 & 0 & 1 & -q_1x + \frac{q_1x^2}{2L} - \frac{q_2x^2}{2L}. \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{81}$$

The field matrix of Eq. 81, when $q_1 = q_2$, will become the same as the field matrix for a massless beam with uniformly distributed load.

Point matrix for a concentrated load, P, see Fig. 5e.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -P \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{82}$$

Point matrix for a concentrated moment, M, see Fig. 5f.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -M \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{83}$$

Point matrix for an elastic spring support with stiffness K, see Fig. 5g.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ K & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{84}$$

Point matrix for an elastic moment spring support with moment stiffness T, see Fig. 5h.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & T & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{85}$$

Lumped mass is used for modelling of dynamic response. Point matrix for a lumped mass, m, with frequency w, see Fig. 5i.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -mw^2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{86}$$

An overall point transfer matrix can be obtained by combining all the point matrices listed above and also adding in the moment term, (HJw^2).

This term[36] describes the gyroscopic or rotatory inertia effects of a beam or rotor in the dynamic case. The combined point transfer matrix becomes

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & HJw^2 + T & 1 & 0 & -M \\ K - mw^2 & 0 & 0 & 1 & -P \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad (87)$$

where H= -1 for bending vibration.

H= +1 for rotating shaft (equal angular direction of whirl and rotation).

H= -3 for rotating shaft (opposite angular direction of whirl and rotation).

If any element of the combined point transfer matrix is not needed, that particular element should be set to zero to give the required point transfer matrix.

An example of a simple cantilever beam with a concentrated load at its free end, shown in Fig. 6, is used to illustrate the transfer matrix method. This same example is done numerically by this program. It is shown together with more complex examples in the user's guide in Appendix A.3.

There is one field and one point transfer matrices in this problem. The field matrix is for the massless beam and the point matrix is for the concentrated load. The equation that describes this beam is

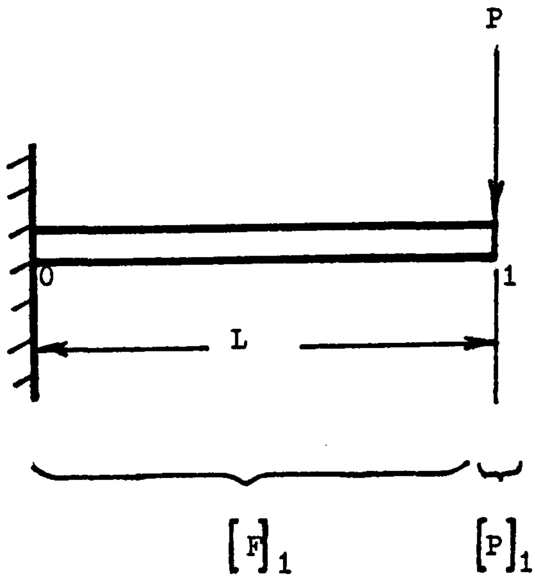


Figure 6. Cantilever beam with end load.

$$[Z]_1 = [P]_1 \; [F]_1 \; [Z]_0 \tag{88}$$

where $[P]_1$ is the point matrix shown in Eq. 82, and $[F]_1$ is the one shown in Eq. 79. Substitution yields

$$\begin{bmatrix} W \\ S \\ M \\ V \\ 1 \end{bmatrix}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -P \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -L & \frac{-L^2}{2EI} & \frac{-L^3}{6EI} & 0 \\ 0 & 1 & \frac{L}{EI} & \frac{L^2}{2EI} & 0 \\ 0 & 0 & 1 & L & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} W \\ S \\ M \\ V \\ 1 \end{bmatrix}_0$$

or

$$\begin{bmatrix} W \\ S \\ M \\ V \\ 1 \end{bmatrix}_1 = \begin{bmatrix} 1 & -L & \frac{-L^2}{2EI} & \frac{-L^3}{6EI} & 0 \\ 0 & 1 & \frac{L}{EI} & \frac{L^2}{2EI} & 0 \\ 0 & 0 & 1 & L & 0 \\ 0 & 0 & 0 & 1 & -P \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} W \\ S \\ M \\ V \\ 1 \end{bmatrix}_0 \tag{89}$$

The boundary conditions at the ends are

$$W_{x=0} = 0, \quad S_{x=0} = 0, \quad M_{x=L} = 0, \quad V_{x=L} = 0 \tag{90}$$

Applying the boundary conditions to Eq. 89 gives the moment and shear at point 0 as

$$\left.\begin{aligned} M_0 &= -PL \\ V_0 &= P \end{aligned}\right\} \tag{91}$$

Substituting the values of state vectors, W_0, S_0, M_0, and V_0 at point 0 into Eq. 89 gives the state vectors at point 1.

$$\left.\begin{aligned} W_1 &= \frac{PL^3}{2EI} - \frac{PL^3}{6EI} = \frac{PL^3}{3EI} \\ S_1 &= \frac{-PL^2}{EI} + \frac{PL^2}{2EI} = -\frac{PL^2}{EI} \\ M_1 &= 0 \\ V_1 &= 0 \end{aligned}\right\} \quad (92)$$

The intermediate results of deflection, slope, moment, and shear along the beam can now be computed using Eq. 89, where L is replaced by the variable length, x. Along with Eqs. 90 and 91, the results are

$$\left.\begin{aligned} W_x &= \frac{Px^2}{6EI}(3L - x) \\ S_x &= \frac{Px}{2EI}(x - 2L) \\ M_x &= P(x - L) \\ V_x &= P \end{aligned}\right\} \quad (93)$$

where $0 < x < L$.

Using Eq. 93, the curves of deflection, slope, moment, and shear can be plotted as a function of the length of the beam.

For the forced, undamped, dynamic case, a forcing circular frequency, w, has to be given. The same analysis procedure can be followed. It is similiar to the static case because static case is just a special dynamic case with the forcing circular frequency equal to zero.

CHAPTER IV

4.1 DISCUSSION OF RESULTS --- FATIGUE ANALYSIS

The application of this program is unlimited. It can be applied to shaft, bolt assembly, spring, and other problems that need a fatigue analysis. The component can be either circular, rectangular, or any shape, provided that the dimensions are all given in terms of the height of the cross section. Any equivalent stress theory can be used in the analysis. Any one of the six fatigue failure lines, which are the most generally accepted in the present, can be chosen. One can even use all six fatigue failure lines one by one and compare the results obtained from different theories.

The significant endurance limit is computed by using the known experimental data presented in the form of graphs. Since all the experimental data have been converted into equations by regression analysis, the computation of the significant endurance limit is easily accomplished.

4.2 DISCUSSION OF RESULTS --- SECTION PROPERTIES

Finding the sectional properties of an irregularly shaped cross section is a tedious task. This program provides a means of calculating twenty sectional properties of any shape cross section describable by straight lines. It is especially useful for extruded shapes which are usually not simple, but are frequently encountered in the design where volume and low cost is desired. An example of a hexagonal extruded bar is shown in the user's guide of this program in the Appendix A in Section A.2.

This program treats the polygonal section and circular section separately. For a polygonal section, the method of replacing integration by summation of finite elements is used to find the sectional properties. Although this method can apply to circular section by approximating the curves with straight line segments, it is time consuming and difficult to get an accurate result. Therefore, to speed up the computation, to make the input easier, and to obtain a more accurate result, the properties of circular cross section and circular holes are computed using the ordinary formulas of area and area moment of inertia about the centroid of a circle. The area moment of inertia with respect to other axis is computed using the parallel axis theorem. In this way, a circular section or a circular hole inside a cross section may be input by giving only the radius of the circle, and x and y coordinate of the center. This is done instead of approximating the curves by straight line segments and inputting the x and y coordinates of each line.

Another special feature of this program is that it provides a graphical verification of the input cross section in addition to the data list. This is especially useful in checking to see whether the holes are in the correct position and completely within the perimeters of the section. One good example of this error can be found in the Hewlett-Packard[14] page 02-10, example 4 in the Section Properties program. The original drawing of the L-shaped cross section with a circular hole is shown in Fig. B-7. If one examines the location of the hole carefully, one will find that the hole, with the given dimensions, is actually located partly outside the perimeter of the cross section instead of inside as shown in the figure. The error can be eliminated if the cross section is drawn to scale first. This obvious mistake can be observed and rectified. The corrected drawing together with the results are given in the user's guide in the Appendix A.2.

This program also computes the area moment of inertia about an arbitrary axis, that is, this axis can be located at any specified point with any angle of rotation. This is very useful in design.

4.3 DISCUSSION OF RESULTS --- BEAM ANALYSIS

This is a generalized beam analysis program which can analyze any kind of beam with any loadings. Even static indeterminate beam or beams that are normally kimatically unstable but have sufficient internal supports can be analyzed. The different types of beam that can be analyzed are unlimited. A few examples are given in the user's guide in Appendix A in Section A.3. This program has a list of sixteen different boundary conditions that cover all of the boundary cases. The output includes a drawing of the beam and its loadings together with a list of input data as a check. If any input error is present, corrections can be made before going into computation. The deflection, slope, moment, and shear along the beam are given in numerical values.

These are also given in the form of graphs. A general feeling of how the beam behaves can easily be obtained for this output. Completion of such problem by hand can be very time consuming and difficult because of so many loadings involved.

This program can also analyze a beam with different cross section and flexural stiffness in each section by inputting the different area moment of inertia and modulii of elasticity for each section.

CHAPTER V

5.0 CONCLUSION

The interactive system when properly programmed provides the engineer with information regarding a proposed design. This is done by providing results of an analysis in a graphic, comprehensible form and at a much faster rate than previously achieved through batch operation. Interactive computing is also easier to use especially by those who have no previous programming knowledge. Also, the use of graphical verification of the input data provides a easy check before the computation takes place, in this way, a correct result can be assured.

Another advantage is that the designer is able to exercise greater control of his computer-based system, and to execute a design or an analysis much faster. Apart from obvious savings in time, he is also able to consider alternative designs in greater depth and to achieve a greater degree of optimization. Thus, economies in all respects of the design should result.

CHAPTER VI

6.1 RECOMMENDATION --- FATIGUE ANALYSIS

The following points are recommended to be added to improve this program in the future:

1. The program now will only find the dimension of a mechanical component which will prevent fatigue failure, but, in some occasions, a safety factor of the component may be desired. Thus the future program should provide the option to find either the dimension or the safety factor of the component to prevent fatigue failure.
2. A tutorial section designed to help the user to write the subroutine of stress equations in Parametric form should be included. Step-by-step simple instructions and examples for illustration would be helpful. This way, the user who is not familiar with the Parametric method needs not to refer to the user's guide and its appendix. Thus, this program could stand alone. Thereby, the full assets of the interactive system could be utilized.
3. In the present program, the finite life region of the S-N curves is fixed between 0.9 S_u @ 1×10^3 cycles to Se'' @ 1×10^6 cycles. The future program should be able to have this region be varied, to suit some purposes.

4. Moreover, it is sometimes desired to determine the life of a given component at a specified safety factor. This analysis could be completed by fixing the safety factor and dimension, and sclving for the significant endurance strength of the component. This significant endurance strength can be used in the S-N equation to predict life.
5. A separate program should be considered to deal with the design under cumulative damage situations.
6. This program should be modified or a new program should be generated to replace the safety factor with a probability of failure. Statistical interference theory should be utilized.
7. A modification to the method should be considered so that different stress concentration factors may be used on different type stresses.
8. The theoretical stress concentration factor has to be supplied by the user in the present program, but with the help of graphics and more computing efforts, this factor can be computed with the user providing the necessary geometric information. This will increase the usefulness of the program.

6.2 RECOMMENDATION --- SECTION PROPERTIES

The twenty sectional properties obtained by this program are quite sufficient for general design, but another useful property --- the location of shear center should be added to the future program. In the case of some non-symmetrical cross sections, like a channel section which has only one axis of symmetry, the plane of the bending moment must pass through the shear center if twisting of the section is to be avoided. This problem usually arises when a thin open section is used and under these conditions, local failure or buckling may happen.

Another program should be considered which will compute the mass moments of inertia of three dimensional configurations. A generalized three dimensional version of this should be considered for such computations.

6.3 RECOMMENDATION --- BEAM ANALYSIS

For this program, the following points are recommended for improvement:

1. For the dynamic case, plots of amplitude and the phase angle of deflection, slope, moment, and shear as a function of exciting frequency should be included. In this way, the critical frequency can be recognized and avoided in the design.
2. In-span indeterminates in the form of fixed support should be allowed. These are not provided in the present program. This feature, although such in-span indeterminants exist only theoretically, will enlarge the application of this program. It can be approximated by assigning a large value for the stiffness of the elastic spring support; e.g., 1×10^{20} lb/in (1.13×10^{19} N/m). This will give a result that approaches the fixed support case.
3. The present program provides for the analysis of forced, undamped

dynamic response, but a damped dynamic case is desirable sometimes. The addition of this feature will require new complex transfer matrices for the beam and its loadings, which can be derived by replacing EI by EI(1 + jG), where j is the imaginary number and G is the structural damping constant. A new scheme of multiplying the complex matrices and also new boundary conditions are needed.

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APPENDIX A

USER'S GUIDES

A.1 USER'S GUIDE --- FATIGUE ANALYSIS

INTRODUCTION

Machine members are often found to have failed under the action of repeated or fluctuating stresses, and yet an analysis reveals that the actual maximum stresses were below the ultimate strength of the material and quite frequently even below the yield strength. These failures usually are under stresses repeated for large number of times. This failure is called a fatigue failure.

There are many theories that predict the fatigue failure. The six most generally accepted are: modified Goodman fracture line, modified Goodman yield line, Soderberg line, Gerber line, Quadratic line, and Kececioglu line. These six fatigue failure lines are available in the program to size a mechanical component, circular, rectangular, or any shape, to prevent fatigue failure. Any equivalent stress theories are allowed. It can also compute the significant endurance limit with the theoretical stress concentration factor and other physical and environmental parameters supplied by the user.

The program is written in BASIC and runs on a Teketronix microprocessor model 4051 with 32K memory.

THEORY

When one analyzes a fatigue failure, there are a number of fatigue failure lines. The general form of the fatigue failure line is

$$\left(\frac{S_a}{R_2\ Se'''}\right)^p + \left(\frac{R_1\ S_m}{S_u}\right)^q = 1 \qquad \text{(A-1)}$$

where S_a = alternating stress, psi or Pa.

S_m = mean stress, psi or Pa.

S_u = ultimate tensile strength, psi or Pa.

Se''' = significant endurance limit, psi or Pa.

R_1, R_2 = variables depend on the fatigue failure line being selected, see Table A-1.

p, q = exponents control the fatigue failure line being selected, see Table A-1.

Specifically, the exponents p and q control the fatigue failure line being used. Table A-1 lists the controlling parameters that select the desired fatigue failure line.

For the Kececioglu line, a Kececioglu factor, b, has to be supplied. This factor depends on the type of material being used. From experimental data obtained by the application of the probabilistic "Design for Reliability" method, Kececioglu found the values of b between 0.8914 and 1.0176, see Table A-2.

A subroutine program of stress equations where any equivalent stress theory may be used has to be supplied by the user. The stress equations must be written using the Parametric method described in the paper, "A Man-Machine Interactive Method for the Development of

Table A-1. Parameters for selection of the desired fatigue failure line.

Type Fatigue Failure Line	p	q	R_1	R_2
Modified Goodman fracture line	1	1	1	1
Modified Goodman yield line	1	1	$\frac{S_u}{S_y}$	$\frac{S_y}{Se'''}$
Soderberg line	1	1	$\frac{S_u}{S_y}$	1
Gerber line	1	2	1	1
Quadratic line	2	2	1	1
Kececioglu line	b*	2	1	1

*See Table A-2

Table A-2. Kececioglu factor.

Type of Steel	Diameter of Specimen	Brinell Hardness Number	Tensile Strength	Significant Endurance Limit	Kececioglu factor
AISI	inches	BHN	psi	psi	b
1018	0.375	130	60,400	26,200	0.8914
1038	0.375	164	68,600	31,000	0.9266
4130	0.375	207	106,100	39,700	1.0176
4340	0.375	233	116,400	48,700	0.9685
AISI	mm	BHN	MPa	MPa	b
1018	9.53	130	416.44	180.64	0.8914
1038	9.53	164	472.98	213.74	0.9266
4130	9.53	207	731.53	273.72	1.0176
4340	9.53	233	802.55	335.77	0.9685

Fatigue Design Equations", by L. D. Mitchell and J. H. Zinskie.[25]

The basic concept of the Parametric method is to introduce a proportionality factor, n, to any loads that will change during the process of overloading. This factor n can be considered as a proportionality constant that signifies overload, so any load that is affected directly and in the same proportion to the overload is a function of n and should be multiplied by n.

The fatigue failure equation and the stress equations are solved by Half Interval Search. This method converges slower but it avoids difficulties which other methods, like Newton's method or false position method, may encounter. It searches for sign changes of the equation. The interval between sign changes will be continuously halved until the desired accuracy or the tolerance of the root is reached.

The significant endurance limit is found by the equation,

$$Se'' = Se' \times Ka \times Kb \times Kc \times Kd \times Ke \times Kf \qquad \text{(A-2)}$$

where Se'' = significant endurance limit for infinite life, psi or Pa.

Se' = endurance limit of a R. R. Moore specimen, psi or Pa.

Ka = surface factor.

Kb = size and shape factor.

Kc = reliability factor.

Kd = temperature factor.

Ke = fatigue strength reduction factor.

Kf = miscellaneous effects factor.

The correction factors, Ka, Kb, Kc, Kd, and Ke are obtained from material presented in Section 6-13 to Section 6-22 of Mechanical Engineering Design, 2nd Ed., McGraw-Hill, N. Y., 1972, by J. E. Shigley, Chapter three of Fatigue Design of Machine Components, Pergamon Press, 1971, by L. Sors, and Section 3-23 to Section 3-29 of Machine Design, Macmillan Publishing Co. Inc., N. Y., 1975, by A. D. Deutschman, W. J. Michels, and C. E. Wilson. The experimental data of the correction factors are converted into regression lines, from which these factors are computed. After the correction factors are obtained, the significant endurance limit for infinite life, Se'', is computed using Eq. A-2. Then it is corrected for finite life to obtain the significant endurance limit for finite life, Se''', if required.

INPUT DATA REQUIRED

Units can be either in English or SI system. This depends on the option chosen by the user. The following data are needed.

1. Ultimate tensile strength of the material, psi or Pa.
2. Yield strength of the material, psi or Pa.
3. Significant endurance limit, psi or Pa.
4. Moment causing alternating stress, lb-in or N-m.

5. Moment causing a steady stress, lb-in or N-m.

6. Alternating axial force, lb or N.

7. Steady axial force, lb or N.

8. Alternating torque, lb-in or N-m.

9. Steady torque, lb-in or N-m.

10. Safety factor, dimensionless.

11. Lower limit of the dimension, in or m.

12. Upper limit of the dimension, in or m.

If the significant endurance limit is not known, the following physical and environmental parameters are required for computing the significant endurance limit.

13. Type of surface finish.

14. Reliability, %.

15. Operating temperature, °F or °C.

16. Theoretical stress concentration factor.

17. Notch sensitivity factor (optional).

18. Notch radius (optional), in or m.

19. Type of material.

20. Type of loading.

21. Miscellaneous effect factor (optional).

22. Shape of cross section.

23. Endurance limit for R. R. Moore rotating beam specimen (optional), psi or Pa.

24. Number of cycles.

EXAMPLE A-1

A rotating shaft is subjected to a transmitted, steady torque of 1,200 lb-in (135.58 N-m) and an applied stationary moment of 2,400 lb-in (271.16 N-m). Overload is caused by an increase of torque and moment. The safety factor is 1.8. The shaft is circular, machined, and made of steel with tensile strength of 200,000 psi (1.397×10^9 Pa) and yield strength of 150,000 psi (1.048×10^9 Pa). The shaft has a notch radius of 0.02 in (5.08×10^{-4} m) and a theoretical stress concentration factor of 2.5, and is operating at a temperature of 190 °F (87.8 °C). The reliability used is 99%. Determine the diameter of the shaft.

First, the stress equations using the Parametric method are derived. Using the maximum distortion energy theory, the equivalent mean and alternating stresses are:

$$S_{em} = (S_m^2 + 3\tau_m^2)^{\frac{1}{2}} = \left(0^2 + 3\left(\frac{T}{2Z}\right)^2\right)^{\frac{1}{2}} = 0.866\,\frac{T}{Z}$$

$$S_{ea} = (S_a^2 + 3\tau_a^2)^{\frac{1}{2}} = \left(\left(\frac{M}{Z}\right)^2 + 0^2\right)^{\frac{1}{2}} = \frac{M}{Z}$$

but upon overload, both the torque, T, and moment, M, go up proportionally by the same factor n. That is

$$T \longrightarrow nT, \quad \text{and} \quad M \longrightarrow nM$$

thus,

$$S'_{em} = \frac{0.866\,n\,T}{Z} \quad \text{and} \quad S'_{ea} = \frac{n\,M}{Z}$$

where the section modulus, $Z = \frac{\pi D^3}{32}$. Substituting in the section modulus, the stress equations become

$$S'_{em} = \frac{0.866\ n\ T}{\frac{\pi D^3}{32}} \quad \text{and} \quad S'_{ea} = \frac{n\ M}{\frac{\pi D^3}{32}}$$

These two equations are the required stress equations for the subroutine program.

The procedure of inputting the data is as follows:

English units are used.

Tensile strength = 200000 psi

Yield strength = 150000 psi

Instruct the micro-processor to find the significant endurance limit.

Type of surface finish, enter 3 for machined or cold drawn.

Reliability = 99%

Operating temperature = 190 °F

Theoretical stress concentration factor = 2.5

Notch radius = 0.02 in

Material is steel.

The shaft is under bending.

No miscellaneous effect factor is needed.

Method to be used for computing the finite life significant endurance limit, Se''', is log - log method. The code is 1.

The cross section of the shaft is circular.

No endurance limit for a R. R. Moore rotating beam specimen is known.

Finite life, number of cycle = 500000

The following is the subroutine program for stress equations, using the Parametric method, and written in BASIC. Using the variables defined by the program, the numerical values of these variables are entered before the stress equations. The variables should be numbered starting with line 6000 incrementing by 10's. The stress equations should start with line 6100 incrementing by 10's. After the stress equations are entered, a numbered return statement is typed. Then type RUN 600.

```
6000 T2 = 1200
6010 M1 = 2400
6020  N = 1.8
6100 A1 = N*M1/(PI*D↑3/32)
6110 A2 = 0.866*N*T2/(PI*D↑3/32)
6120 RETURN
RUN 600
```

Modified Goodman fracture line is selected for the design. Enter MGF, the code for this fatigue failure line.

Establish the limits on the half interval search for the solution.

Enter lower limit = 0.01 in

Enter upper limit = 10.00 in

The above procedure of entering the data is also shown in Figs. A-1 to A-6. The result together with the input data, the correction factors for computing the significant endurance limit, and the

```
*************************************************************
*                                                           *
*           FATIGUE ANALYSIS PROGRAM, ANALYTICAL            *
*                                                           *
*           By Yiu Wah Luk, VPI & SU, Spring 1978           *
*                                                           *
*************************************************************

You have entered a FATIGUE RESISTANT, INTERACTIVE DESIGN routine.
Components will be sized to prevent fatigue failure.

Do you want to use English Units? (Y or N)Y

We will use English Units throughout this routine.

Please enter the strength of the material to be used.

Tensile Strength, in psi, Su                     = 200000
Yield Strength,   in psi, Sy                     = 150000

Do you know the Significant Endurance Limit of the material? (Y or N)N
```

Figure A-1. Output for Example A-1.

```
This section of the program will calculate the Significant
Endurance Limit.

Enter the # for the types of surface finish used.
#1 for polished finish.
#2 for ground finish.
#3 for machined or cold drawn.
#4 for hot rolled.
#5 for as forged.
3

What is the reliability in %?99

What is the operating temperature in Degree F ?190

Do you know the Theoretical Stress Concentration Factor (Kt)?Y

Theoretical Stress Concentration Factor = 2.5

Do you know Notch Sensitivity (q)? (Y or N)N
What is the notch radius in inches?0.02

Is the material steel? (Y or N)Y
Is it under bending or axial loading? (Y or N)Y

Is there any miscellaneous-effect factor? (Y or N)N

The S-N curve is used for this determination.
A Log-Log or a Log-Linear S-N curve will be used.
The finite life region is 0.9*Su @1E3 cycles to Se'' @1E6 cycles.
What method do you want to use for computing the
Significant Endurance Limit (Se''')?
Enter #1 for Log-Log method.
Enter #2 for Log-Linear method.
1
```

Figure A-2. Output for Example A-1.

```
Is the cross section circular? (Y or N)Y

Do you know the Endurance Limit (Se') for rotating-beam
specimen? (Y or N)N

Is the design life infinite? (Y or N)N
Number of cycles = 500000
```

Figure A-3. Output for Example A-1.

```
You will now be requested to supply the PARAMETRIC DESIGN STRESS
EQUATIONS for your design problem.
You must write these equations in BASIC. Use the following instructions.
If you are unfamiliar with the development of such equations,
see user's guide and its appendix for guidance.
Enter component loads starting with line 6000 incrementing by 10's.
 Use M1=Moment causing alternating stress (lb-in).
 Use M2=Moment causing a steady stress (lb-in).
 Use F1=An alternating axial force (lbf).
 Use F2=A steady axial force (lbf).
 Use T1=An alternating torque (lb-in).
 Use T2=A steady torque (lb-in).
 Use  N=Safety factor.
Enter the numeric value for each of the variables used in
your stress equations.
Do this before you enter your stress equations.

Enter your stress equations starting with line 6100
incrementing by 10's.
 Use A1=Alternating stress.
 Use A2=Mean stress.
 Use PI=Pi.
 Use D=Basic dimension. N.B., All dimensions should be given in terms of
 D. In case of rectangular component, use proportions.
After your stress equations are entered, type a numbered return
statement. Then type run 600.
EXAMPLE...................................................
The following equations are in English Unit.
6000 T2=1200.
6010 M1=2400.
6020 N=1.8
6100 A1=N*M1/(PI*D↑3/32)
6110 A2=0.866*N*T2/(PI*D↑3/32)
6120 RETURN
RUN 600
```

Figure A-4. Output for Example A-1.

```
6000 T2=1200
6010 M1=2400
6020 N=1.8
6100 A1=N*M1/(PI*D↑3/32)
6110 A2=0.866*N*T2/(PI*D↑3/32)
6120 RETURN
RUN 600
```

Figure A-5. Output for Example A-1.

```
Select the fatigue failure line to be used in the design.

If Modified Goodman Fracture Line, enter MGF
If Modified Goodman Yield Line, enter MGY
If Soderberg Line, enter S
If Gerber Line, enter G
If Quadratic Line, enter Q
If Kececioglu Line, enter K
MGF

The Failure Line selected is Modified Goodman Fracture Line.
The failure equation is (Sa/(R2*Se''')↑m + (R1*Sm/Su)↑p = 1.
where M=1
where P=1
where R1=1
and where R2=1
Do you wish to change any parameters? (Y or N)N

The following entries will establish the limits on a Half
Interval Search for the solution to your problem.
What is the smallest basic dimension that you wish to try?
Give your answer in inches. Do not answer 0.0.  0.01
What is the largest dimension, in inches? 10
```

Figure A-6. Output for Example A-1.

```
**************************************************************************

Tensile Strength, Su.............................= 200,000 psi
Yield Strength, Sy...............................= 150,000 psi
Significant Endurance Limit, Se'''@500000 cyc.= 21,395 psi
Smallest dimension tried.........................= 0.01 inches
Largest dimension tried..........................= 10.00 inches
Safety Factor, N.................................= 1.80
Moment causing alternating stress, M1...........= 2,400 lb-in
Steady torque, T2................................= 1,200 lb-in

The following factors are used for computing Se''':
Surface factor (Ka)                      = 0.64
Size and shape factor (Kb)               = 0.81
Reliability factor (Kc)                  = 0.81
Temperature factor (Kd)                  = 0.95
Fatigue Strength Reduction factor (Ke)= 0.42
Miscellaneous factor (Kf)                = 1.00
Endurance Limit for Rotating-beam Specimen (Se')     = 100,000 psi
Significant Endurance Limit for infinite life (Se'')= 16,960 psi

The Failure Line selected is Modified Goodman Fracture Line.
The failure equation is (Sa/(R2*Se''')↑m + (R1*Sm/Su)↑p= 1.
where M=1
where P=1
where R1=1
and where R2=1

The design dimension is 1.2911 inches

**************************************************************************

Do you wish to convert the design dimension to SI unit? (Y or N)Y

The design dimension is 0.0328 m
```

Figure A-7. Output for Example A-1.

```
**************************************************************************

Tensile Strength, Su...........................= 200,000 psi
Yield Strength, Sy.............................= 150,000 psi
Significant Endurance Limit, Se'''@500000 cyc.= 25,779 psi
Smallest dimension tried.......................= 0.01 inches
Largest dimension tried........................= 10.00 inches
Safety Factor, N...............................= 1.80
Moment causing alternating stress, M1.........= 2,400 lb-in
Steady torque, T2..............................= 1,200 lb-in

The following factors are used for computing Se''':
Surface factor (Ka)                    = 0.64
Size and shape factor (Kb)             = 1.00
Reliability factor (Kc)                = 0.81
Temperature factor (Kd)                = 0.95
Fatigue Strength Reduction factor (Ke)= 0.42
Miscellaneous factor (Kf)              = 1.00
Endurance Limit for Rotating-beam Specimen (Se')     = 100,000 psi
Significant Endurance Limit for infinite life (Se'')= 20,865 psi

The Failure Line selected is Modified Goodman Yield Line.
The failure equation is (Sa/(R2*Se'''))↑m + (R1*Sm/Su)↑p= 1.
where M=1
where P=1
where R1=1.33333333333
and where R2=5.81872093284

The design dimension is 0.7491 inches

**************************************************************************

Do you wish to convert the design dimension to SI unit? (Y or N)Y

The design dimension is 0.0190 m
```

Figure A-8. Output for Example A-1.

```
*************************************************************************

Tensile Strength, Su...............................= 200,000 psi
Yield Strength, Sy.................................= 150,000 psi
Significant Endurance Limit, Se'''@500000 cyc.= 21,345 psi
Smallest dimension tried...........................= 0.01 inches
Largest dimension tried............................= 10.00 inches
Safety Factor, N...................................= 1.80
Moment causing alternating stress, M1..........= 2,400 lb-in
Steady torque, T2..................................= 1,200 lb-in

The following factors are used for computing Se''':
Surface factor (Ka)                        = 0.64
Size and shape factor (Kb)                 = 0.81
Reliability factor (Kc)                    = 0.81
Temperature factor (Kd)                    = 0.95
Fatigue Strength Reduction factor (Ke)= 0.42
Miscellaneous factor (Kf)                  = 1.00
Endurance Limit for Rotating-beam Specimen (Se')     = 100,000 psi
Significant Endurance Limit for infinite life (Se'')= 16,916 psi

The Failure Line selected is Soderberg Line.
The failure equation is (Sa/(R2*Se''')↑m + (R1*Sm/Su)↑p= 1.
where M=1
where P=1
where R1=1.33333333333
and where R2=1

The design dimension is 1.2983 inches

*************************************************************************

Do you wish to convert the design dimension to SI unit? (Y or N)Y

The design dimension is 0.0330 m
```

Figure A-9. Output for Example A-1.

```
*************************************************************************

Tensile Strength, Su...........................= 200,000 psi
Yield Strength, Sy.............................= 150,000 psi
Significant Endurance Limit, Se'''@500000 cyc.= 21,547 psi
Smallest dimension tried.......................= 0.01 inches
Largest dimension tried........................= 10.00 inches
Safety Factor, N...............................= 1.80
Moment causing alternating stress, M1..........= 2,400 lb-in
Steady torque, T2..............................= 1,200 lb-in

The following factors are used for computing Se''':
Surface factor (Ka)                     = 0.64
Size and shape factor (Kb)              = 0.82
Reliability factor (Kc)                 = 0.81
Temperature factor (Kd)                 = 0.95
Fatigue Strength Reduction factor (Ke)= 0.42
Miscellaneous factor (Kf)               = 1.00
Endurance Limit for Rotating-beam Specimen (Se')     = 100,000 psi
Significant Endurance Limit for infinite life (Se'')= 17,094 psi

The Failure Line selected is Gerber Line.
The failure equation is (Sa/(R2*Se''')↑m + (R1*Sm/Su)↑p= 1.
where M=1
where P=2
where R1=1
and where R2=1

The design dimension is 1.2696 inches

*************************************************************************

Do you wish to convert the design dimension to SI unit? (Y or N)Y

The design dimension is 0.0322 m
```

Figure A-10. Output for Example A-1.

```
******************************************************************************

Tensile Strength, Su..............................= 200,000 psi
Yield Strength, Sy................................= 150,000 psi
Significant Endurance Limit, Se'''@500000 cyc.= 21,551 psi
Smallest dimension tried..........................= 0.01 inches
Largest dimension tried...........................= 10.00 inches
Safety Factor, N..................................= 1.80
Moment causing alternating stress, M1.........= 2,400 lb-in
Steady torque, T2.................................= 1,200 lb-in

The following factors are used for computing Se''':
Surface factor (Ka)                        = 0.64
Size and shape factor (Kb)                 = 0.82
Reliability factor (Kc)                    = 0.81
Temperature factor (Kd)                    = 0.95
Fatigue Strength Reduction factor (Ke)= 0.42
Miscellaneous factor (Kf)                  = 1.00
Endurance Limit for Rotating-beam Specimen (Se')    = 100,000 psi
Significant Endurance Limit for infinite life (Se'')= 17,098 psi

The Failure Line selected is Quadratic Line.
The failure equation is (Sa/(R2*Se'''))↑m + (R1*Sm/Su)↑p= 1.
where M=2
where P=2
where R1=1
and where R2=1

The design dimension is 1.2691 inches

******************************************************************************

Do you wish to convert the design dimension to SI unit? (Y or N)Y

The design dimension is 0.0322 m
```

Figure A-11. Output for Example A-1.

```
**************************************************************************

Tensile Strength, Su...........................= 200,000 psi
Yield Strength, Sy.............................= 150,000 psi
Significant Endurance Limit, Se'''@500000 cyc.= 21,549 psi
Smallest dimension tried.......................= 0.01 inches
Largest dimension tried........................= 10.00 inches
Safety Factor, N...............................= 1.80
Moment causing alternating stress, M1.........= 2,400 lb-in
Steady torque, T2..............................= 1,200 lb-in

The following factors are used for computing Se''':
Surface factor (Ka)                    = 0.64
Size and shape factor (Kb)             = 0.82
Reliability factor (Kc)                = 0.81
Temperature factor (Kd)                = 0.95
Fatigue Strength Reduction factor (Ke)= 0.42
Miscellaneous factor (Kf)              = 1.00
Endurance Limit for Rotating-beam Specimen (Se')    = 100,000 psi
Significant Endurance Limit for infinite life (Se'')= 17,096 psi

The Failure Line selected is Kececioglu Line.
The failure equation is (Sa/(R2*Se''')↑m + (R1*Sm/Su)↑p= 1.
where M=1.5
where P=2
where R1=1
and where R2=1

The design dimension is 1.2693 inches

**************************************************************************

Do you wish to convert the design dimension to SI unit? (Y or N)Y

The design dimension is 0.0322 m
```

Figure A-12. Output for Example A-1.

Table A-3. Results of Example A-1 obtained by six fatigue failure lines.

Type of Failure Line used	Design Diameter	
	in	m
Modified Goodman fracture line*	1.2911	0.0328
Modified Goodman yield line*	0.7491	0.0190
Soderberg line	1.2983	0.0330
Gerber line	1.2696	0.0332
Quadratic line	1.2691	0.0322
Kececioglu line (with b=1.5)+	1.2693	0.0322

* If the user subscribes to the Modified Goodman theory, the largest dimension of the two Goodman solutions must be selected.

\+ This is an arbitrary choice of exponent used for this example only.

parameters of the fatigue failure line selected for this design are output and shown in Fig. A-7. The design diameter of the shaft is found to be 1.2911 in. (0.0328 m).

Using the same input data, the component is redesigned by the other five fatigue failure lines. The results using the other five fatigue failure lines are shown in Figs. A-8 to A-12. All six results are listed in Table A-3 for comparsion.

Normally, all these analyses are not carried out. Only the analysis that corresponds to the user's selected theory is computed. But the program provides the capability to redesign the component using other fatigue failure theories in order that the user can choose among alternatives.

From the results listed in Table A-3, the smallest dimension is obtained by modified Goodman yield line. This result is not correct for a fatigue design because the modified Goodman theory demands that one considers the yield line and the fracture line. Thus, the largest dimension of the two Goodman solutions must be selected. The most conservative solution is obtained using Soderberg line. This is not always true. It depends upon the location of the load line. In this example, the solutions from modified Goodman line, Soderberg line, Gerber line, Quadratic line, and Kececioglu line are quite close. This is true only for this particular example because of the load line. Other examples may give more significant differences between these theories.

EXAMPLE A-2. (Taken from problem 5-38a, Mechanical Engineering Design 3rd Ed., McGraw-Hill, New York, N. Y., 1977, by J. E. Shigley.)

Figure A-13 shows two views of a flat steel spring loaded in bending by the force F. The spring is assembled so as to produce a preload F_{min} = 900 N. The force then varies from this minimum value to a maximum of 3000 N. The spring is forged of AISI 1095 steel having the following properties:

S_u = 1,400 MPa

S_y = 950 MPa

H_B = 398

12 percent elongation in 50 mm.

Find the thickness t if Kt = 2.50 and a margin of safety of 90 percent is to be used.

let D be the thickness of the flat spring.

Since no reliability factor is mentioned, 50% is assumed.

Room temperature is assumed when operating, 24 °C.

Notch sensitivity factor is assumed to be 0.97.

Let the spring be designed for 100,000 cycles.

SI units are used.

The stress equations using the Parametric method are desired.

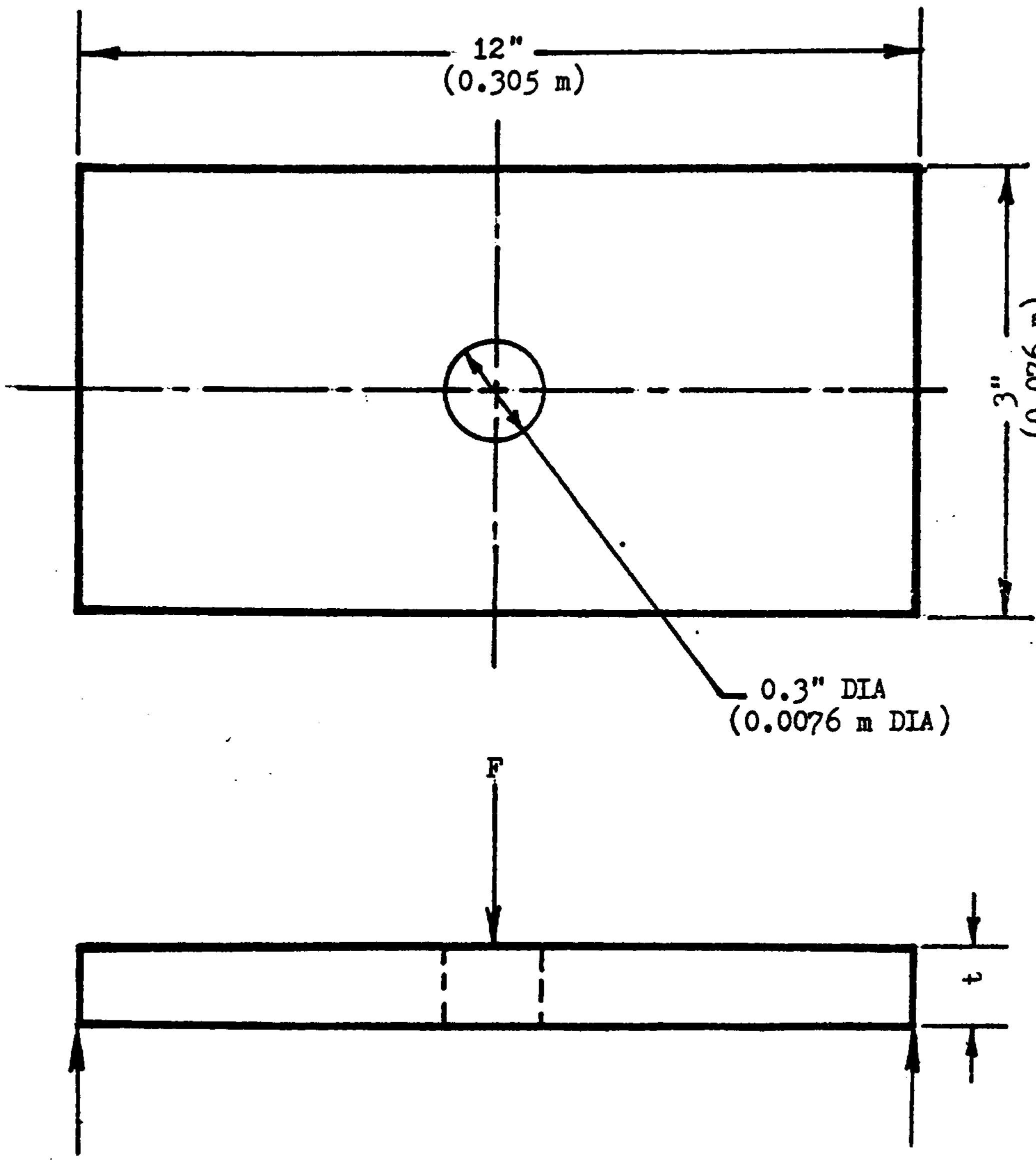


Figure A-13. Flat Steel Spring under Fatigue Loading.

Alternating force, $F_a = \frac{3000 - 900}{2}$

Mean force, $F_m = \frac{3000 + 900}{2}$

Using the maximum distortion energy theory, the equivalent mean and alternating stresses are obtained.

$$S_{em} = (S_m^2 + 3\tau_m^2)^{\frac{1}{2}} = \frac{M_m}{Z} = \frac{0.15\ (3000 - 900)}{2Z}$$

$$S_{ea} = (S_a^2 + 3\tau_a^2)^{\frac{1}{2}} = \frac{M_a}{Z} = \frac{0.15\ (3000 + 900)}{2Z}$$

But upon overload, maximum load will increase by factor n, so

$$F_{max} \longrightarrow n\ F_{max}$$

Substituting in the section modulus, $Z = \frac{0.075\ D^2}{6}$, yields

$$S'_{em} = \frac{6(3000\ n - 900)}{D^2}$$

$$S'_{ea} = \frac{6(3000\ n + 900)}{D^2}$$

These two equations are the required stress equations for the subroutine program.

The procedure of inputting the data is as follows:

SI units are used.

Tensile strength = 1400000000 Pa

Yield strength = 950000000 Pa

Instruct the micro-processor to find the significant endurance limit.

Type of surface finish, enter 5 for as forged.

Reliability = 50%

Operating temperature = 24 °C

Theoretical stress concentration factor = 2.5

Notch sensitivity factor = 0.97

Material is steel.

The spring is under bending.

No miscellaneous effect factor is needed.

Method used for computing the finite life significant endurance limit, Se''', is log - log method, the code is 1.

The cross section of the spring is rectangular.

No endurance limit for a R. R. Moore rotating beam specimen is known.

Finite life, number of cycles = 100000

The following is the subroutine program for stress equations, using the Parametric method and written in BASIC. Following the instructions provided by the program, the subroutine is

```
6000  N = 1.9
6100 A1 = 6*(3000*N - 900)/D↑2
6110 A2 = 6*(3000*N + 900)/D↑2
6120 RETURN
RUN 600
```

Soderberg line is selected for this design. Enter S, the code for Soderberg line.

Establish the limits on the half interval search for the solution.

Enter lower limit = 0.001 m

Enter upper limit = 0.10 m

The input data and result are shown in Figs. A-14 to A-15. The thickness of the flat spring is found to be 0.0146 m (0.5747 in).

```
******************************************************************************

Tensile Strength, Su............................= 1,400,000,000 Pa
Yield Strength, Sy..............................= 950,000,000 Pa
Significant Endurance Limit, Se'''@100000 cyc.= 167,994,384 Pa
Smallest dimension tried........................= 0.00 m
Largest dimension tried.........................= 0.10 m
Safety Factor, N................................= 1.90

The following factors are used for computing Se''':
Surface factor (Ka)                        = 0.25
Size and shape factor (Kb)                 = 0.88
Reliability factor (Kc)                    = 1.00
Temperature factor (Kd)                    = 1.00
Fatigue Strength Reduction factor (Ke)= 0.41
Miscellaneous factor (Kf)                  = 1.00
Endurance Limit for Rotating-beam Specimen (Se')    = 689,475,700 Pa
Significant Endurance Limit for infinite life (Se'')= 61,341,851 Pa

The Failure Line selected is Soderberg Line.
The failure equation is (Sa/(R2*Se''')↑m + (R1*Sm/Su)↑p= 1.
where M=1
where P=1
where R1=1.47368421053
and where R2=1

The design dimension is 0.0146 m

******************************************************************************

Do you wish to convert the design dimension to English unit? (Y or N)Y

The design dimension is 0.5747 inches

Do you wish to redesign the component using another
```

Figure A-14. Output for Example A-2.

```
failure line? Warning! A Modified Goodman approach requires
both a fracture and a yield analysis. (Y or N)N

Do you want to design a new component? (Y or N)N


        ---  END  ---
```

Figure A-15. Output for Example A-2.

A.2 USER'S GUIDE --- SECTION PROPERTIES

INTRODUCTION

The cross sectional properties of a machine member are usually needed in the design. To find these properties for an irregular shape cross section is time consuming and difficult. This program provides a means of computing twenty sectional properties of any shape cross section. The sectional properties computed are: area, location of centroid, area moment of inertia about different axis, and radius of gyration.

This program also provides a graphical verification of the input cross section together with the list of data. In this way, the cross section can be checked easily before the computation takes place.

The program is written in BASIC and used on a Teketronix microprocessor model 4051 with 32 K memory.

THEORY

Part of the theory is documented in the paper "Properties of plane cross sections", Machine Design, Jan., 22, 1976, pp. 105-107, written by F. Wojciechowski. The technique used is to replace integration by summation of finite elements to find the section properties of a plane cross section. This technique applies only to areas bounded by straight lines, but because curves could be approximated by straight line segments, the method can be used on any shape.

Basically, the method divides a cross section into series of trapezoids or rectangles, then the properties of each elemental area are added or substracted to give the composite properties of the desired cross section.

In order to speed up the input and computation, and also to obtain a more accurate result, the properties of circular cross section and circular holes are not computed using the above mentioned method. It uses the ordinary formulas of area and area moment of inertia about the centroid of a circle. Then the area moment of inertia with respect to other axis is computed using the parallel axis theorem. In this way, to input circular section or a circular hole inside a cross section, only the radius of the circle, and x and y coordinates of the center are needed, instead of approximating the curves by straight line segments and inputting the x and y coordinates of each line.

INPUT DATA NEEDED

Units can be either in English or SI system. This depends on the option chosen by the user. The following data are needed.

For circular section or circular hole:

Radius of circle, in or mm.

X and y coordinates of the center, in or mm.

For polygonal section or polygonal hole:

X and y coordinates of each vertices, in. or mm.

The following points should be noted when inputting a polygonal section or polygonal hole.

1. The polygonal section must be located entirely within the first quadrant, that is, both x and y coordinates must be positive.
2. The x and y coordinates of the vertices of the polygonal section or hole must be entered sequentially for a complete, clockwise path around the section or hole.
3. Be sure to end the path with the first vertice.

EXAMPLE B-1. (Taken from example 1, pp. 106, of the paper "Properties of plane cross sections", Machine Design, Jan., 22, 1976, written by F. Wojciechowski.)

A hollow hexagonal cross section is shown in Fig. B-1. Determine the area, centroid, area moment of inertia about the x-y axis and principal axis, and radius of gyration.

English units are used.

The outside perimeter is not a circular section.

Enter the x and y coordinates of the vertices of the hexagonal cross section sequentially for a complete, clockwise path around the section.

#1 x, y = 1, 0

#2 x, y = 0, 1.732

#3 x, y = 1, 3.464

#4 x, y = 3, 3.464

#5 x, y = 4, 1.732

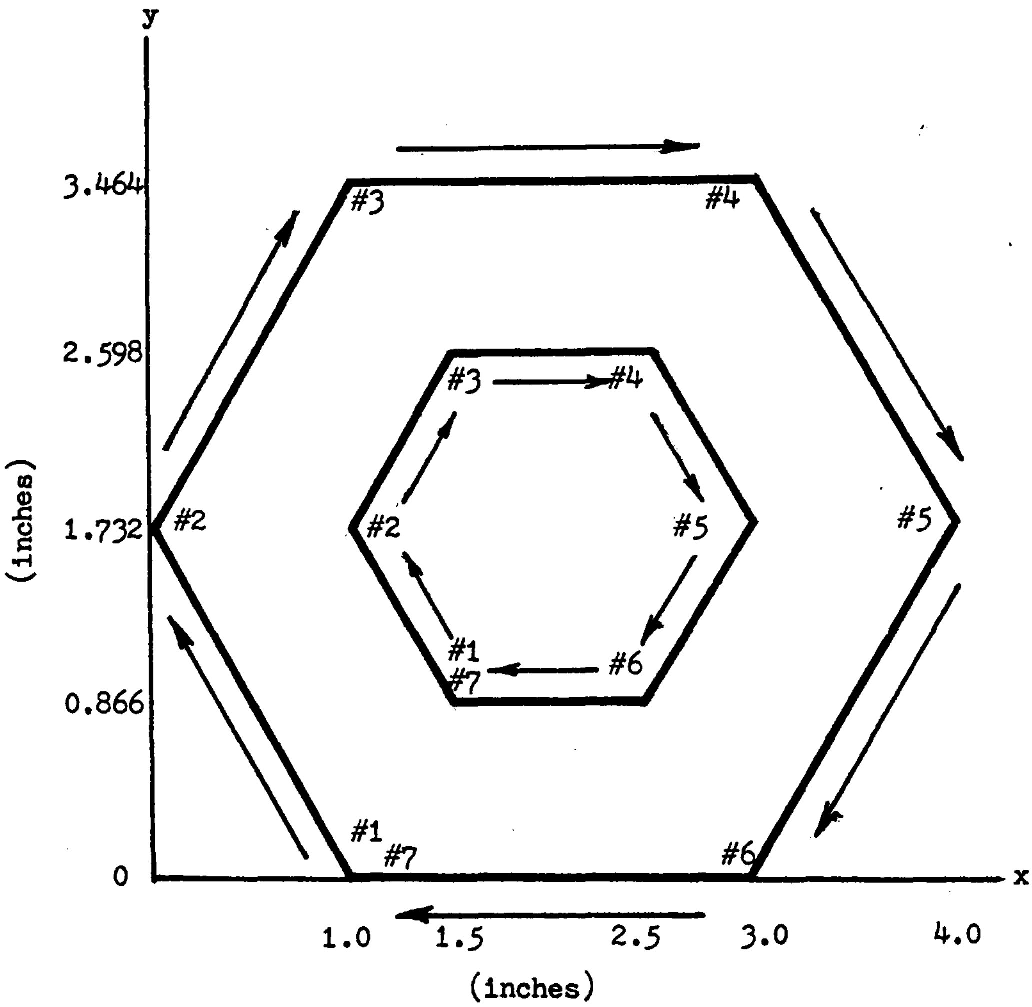


Figure B-1. A hollow hexagonal cross section.

#6 x, y = 3, 0

#7 x, y = 1, 0

For the hexagonal hole, enter x and y coordinates of the vertices sequentially for a complete clockwise path around the hole.

#1 x, y = 1.5, 0.866

#2 x, y = 1, 1.732

#3 x, y = 1.5, 2.598

#4 x, y = 2.5, 2.598

#5 x, y = 3, 1.732

#6 x, y = 2.5, 0.866

#7 x, y = 1.5, 0.866

Since the properties with respect to an arbitrary axis are not required, answer "N" when ask if the area moment of inertia about an arbitrary axis is desired.

The input data, the drawing of the cross section, and the results are shown in Figs. B-2 to B-6.

EXAMPLE B-2. (Taken from pp. 02-10, example 4, in the Section Properties program in <u>User's Guide of Mechanical Engineering Programs, Pac I, for HP-67 or HP-97 programmable calculator,</u> Hewlett-Packard Company, Oregon, 1976.)

For the part shown in Fig. B-7, compute the polar moment of inertia about point A. Point A denotes the center of a hole about which the part rotates. The area of the hole must be deleted from the cross section. (No unit was given, so English unit is assumed.)

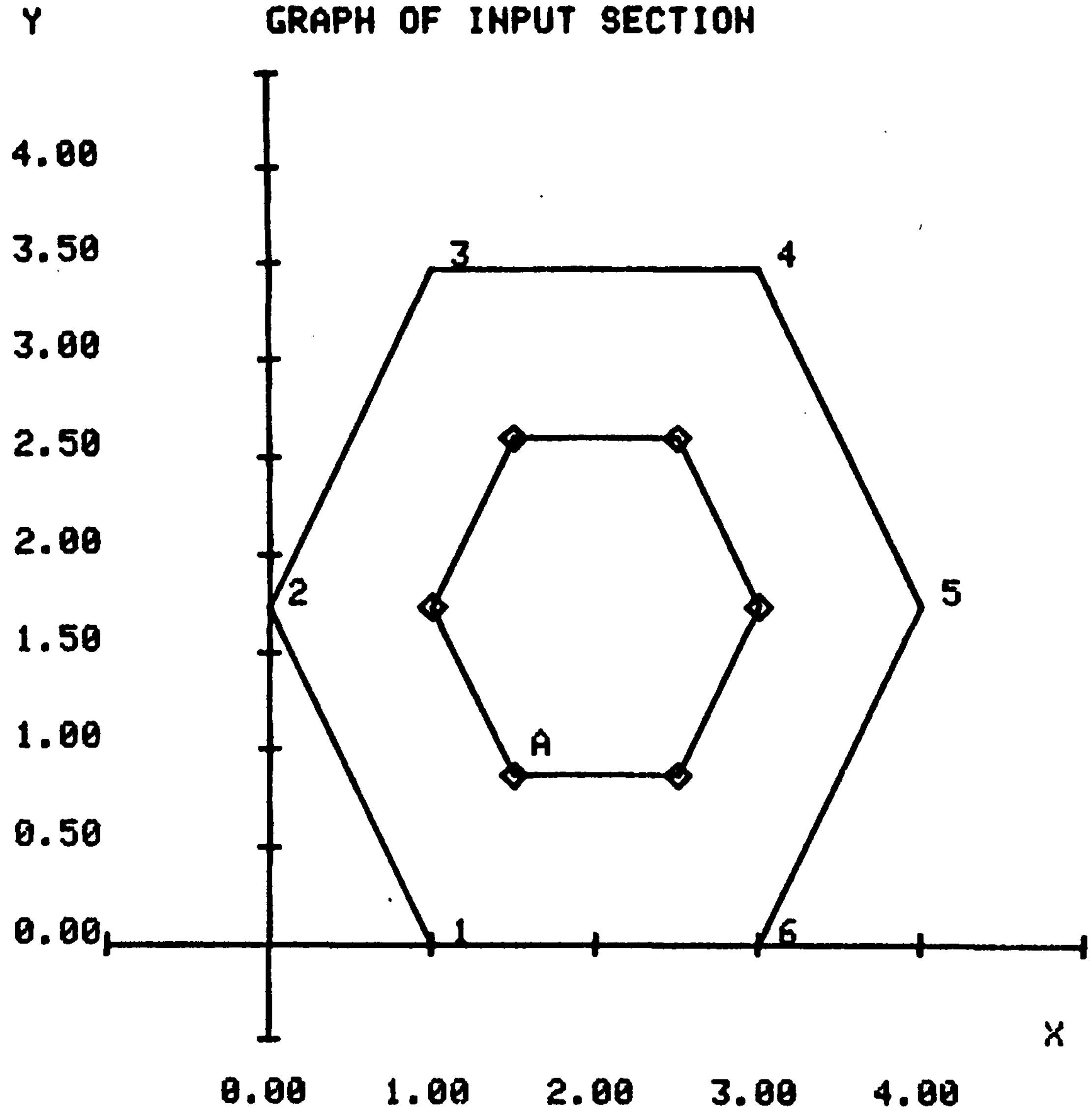
```
****************************************************************
*                                                              *
*          SECTION PROPERTIES OF POLYONAL SECTION              *
*                                                              *
*          By Yiu Wah Luk, VPI & SU, Spring 1978               *
*                                                              *
****************************************************************

Do you want to use SI units? (Y or N)N

We will use English Units throughout this program.

Is the outer perimeter a circular section? (Y or N)N


Please enter the X and Y coordinates of the vertices of the
polygon (which must be located entirely within the first
quadrant) sequentially for a complete, clockwise path around
the polygon. Units should be inches.
Be sure to end with the first point.
X(1), Y(1) = 1,0
X(2), Y(2) = 0,1.732
X(3), Y(3) = 1,3.464
X(4), Y(4) = 3,3.464
X(5), Y(5) = 4,1.732
X(6), Y(6) = 3,0
X(7), Y(7) = 1,0

Are there any holes in the section? (Y or N)Y
Are there any circular holes? (Y or N)N

How many polygon holes are there in the section?1

For polygon holes:
```

Figure B-2. Output for Example B-1.

```
Enter the X and Y coordinates of each vertices in a complete, clockwise
path. Units should be inches.
Be sure to end with the first point of each hole.

For polygonal hole #1:
X(1), Y(1) = 1.5,0.866
X(2), Y(2) = 1,1.732
X(3), Y(3) = 1.5,2.598
X(4), Y(4) = 2.5,2.598
X(5), Y(5) = 3,1.732
X(6), Y(6) = 2.5,0.866
X(7), Y(7) = 1.5,0.866
```

Figure B-3. Output for Example B-1.

```
All data are in inches.

Poly. Data (X,Y):
#1= 1, 0
#2= 0, 1.732
#3= 1, 3.464
#4= 3, 3.464
#5= 4, 1.732
#6= 3, 0

Poly.Hole'A'Data (X,Y):
#1= 1.5, 0.866
#2= 1, 1.732
#3= 1.5, 2.598
#4= 2.5, 2.598
#5= 3, 1.732
#6= 2.5, 0.866
```




```
Do you want to change any data? (Y or N)N
```

Figure B-4. Output for Example B-1.

```
*** SECTION PROPERTIES OF THE REQUIRED SECTION ***

Area...............................................= 7.79 in↑2

X coordinate of the centroid.......................= 2.00 inches

Y coordinate of the centroid.......................= 1.73 inches

Area moment of inertia about X-axis................= 31.50 in↑4

Area moment of inertia about Y-axis................= 39.29 in↑4

Area product of inertia............................= 27.00 in↑4

Area moment of inertia about X'-axis translated to
the centroid.......................................= 8.12 in↑4

Area moment of inertia about Y'-axis translated to
the centroid.......................................= 8.12 in↑4

Area product of inertia about translated axis......= 0.00 in↑4

Angle between translated axis and principal axis
(in degree), positive is counter-clockwise.........= 0.00

Area moment of inertia about the translated, rotated,
principal X''-axis.................................= 8.12 in↑4

Area moment of inertia about the translated,
rotated, principal Y''-axis........................= 8.12 in↑4


Do you want to get area moment of inertia about an
arbitrary axis? (Y or N)N
```

Figure B-5. Output for Example B-1.

```
Radius of gyration about X-axis........................= 2.01 inches

Radius of gyration about Y-axis........................= 2.25 inches

Radius of gyration about X'-axis translated to
the centroid...........................................= 1.02 inches

Radius of gyration about Y'-axis translated to
the centroid...........................................= 1.02 inches


Do you want to find the properties of another section?
(Y or N)N

--------------------     END     -------------------
```

Figure B-6. Output for Example B-1.

English units are used.

The outside perimeter is not a circular scetion.

For the L-shaped cross section, the x and y coordinates of the vertices are entered in a complete clockwise path.

#1 x, y = 0, 0

#2 x, y = 0, 2

#3 x, y = 5, 2

#4 x, y = 5, 1.4

#5 x, y = 0.8, 1.4

#6 x, y = 0.8, 0

#7 x, y = 0, 0

For the circular hole at A:

Radius = 0.25

x, y = 0.2, 0.6

To find the properties about the arbitrary axis at A with zero degree of rotation, enter:

x, y coordinates of the origin of the arbitrary axis = 0.2, 0.6

Angle of rotation of the arbitrary axis = 0 degree

The drawing of the cross section and the results are shown in Figs. B-8 to B-10. There is a difference between the original drawing, Fig. B-7, and the one from the output, Fig. B-8, because the hole was actually located partly outside the L-shaped cross section. The error can be eliminated if the cross section is first drawn to scale. This obvious mistake can be observed and rectified. However, it is not rectified for this example so that the solution obtained may be compared to the Hewlett Packard results.

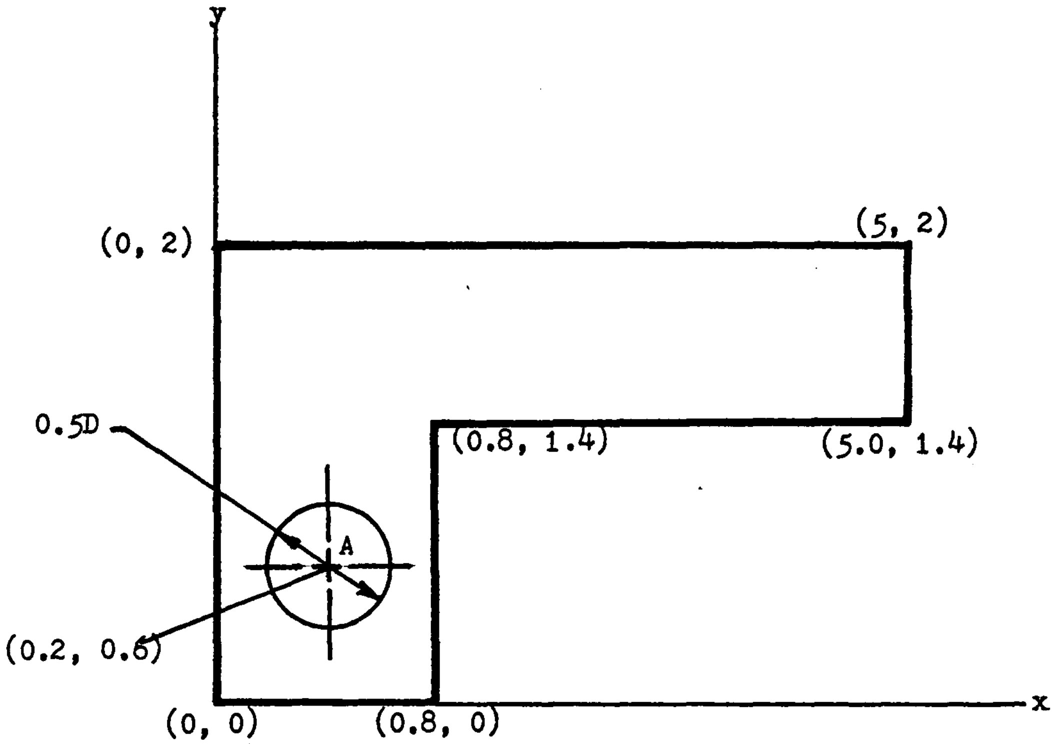
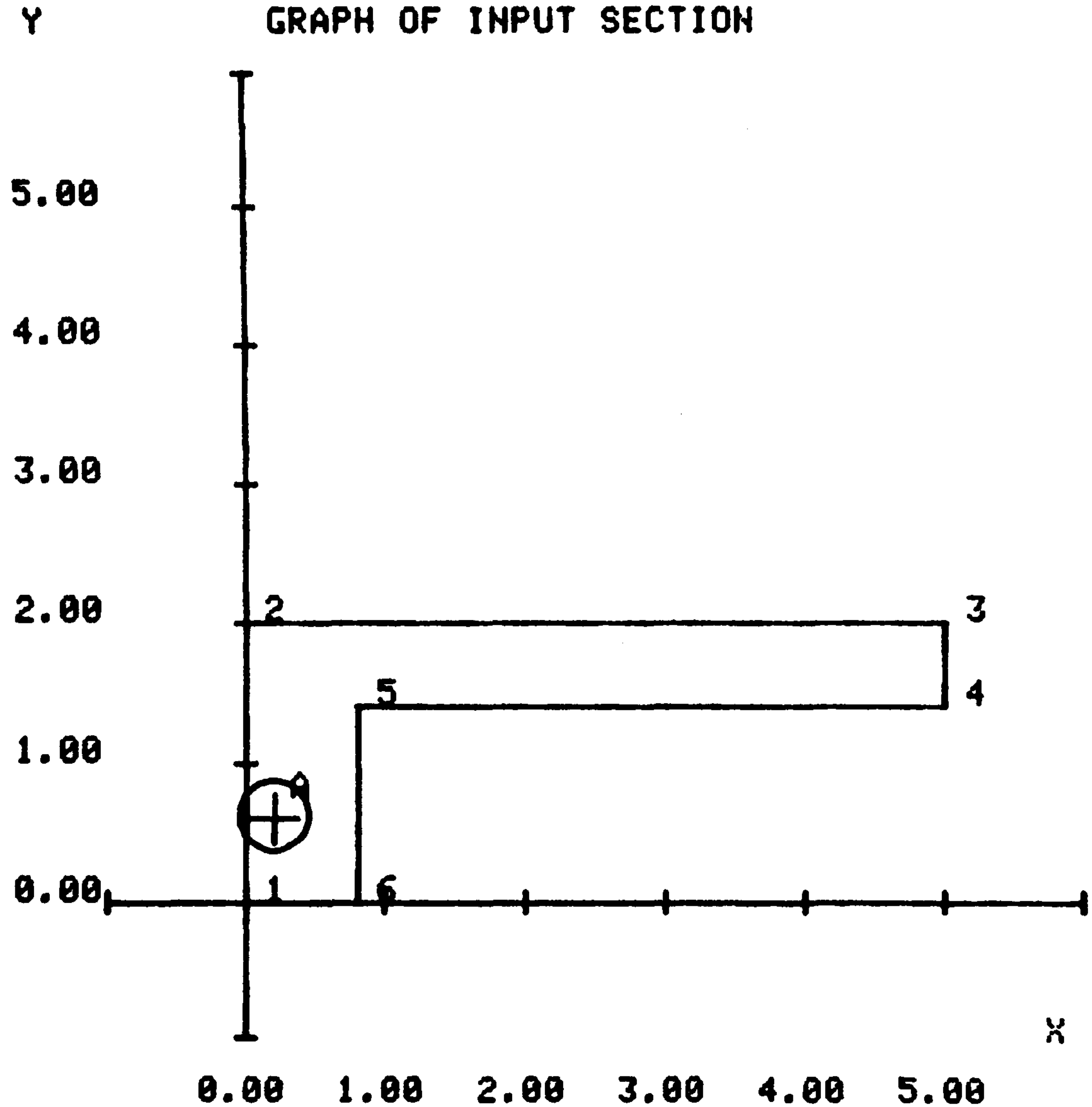


Figure B-7. L-shaped cross section with a circular hole.

```
All data are in inches.

Poly. Data (X,Y):
#1= 0, 0
#2= 0, 2
#3= 5, 2
#4= 5, 1.4
#5= 0.8, 1.4
#6= 0.8, 0

Cir. Hole 'A' Data:
Radius = 0.25
X, Y   = 0.2, 0.6
```




```
Do you want to change any data? (Y or N)N
```

Figure B-8. Output for Example B-2.

```
*** SECTION PROPERTIES OF THE REQUIRED SECTION ***

Area.............................................= 3.92 in↑2

X coordinate of the centroid.....................= 2.02 inches

Y coordinate of the centroid.....................= 1.47 inches

Area moment of inertia about X-axis..............= 9.42 in↑4

Area moment of inertia about Y-axis..............= 25.23 in↑4

Area product of inertia..........................= 13.04 in↑4

Area moment of inertia about X'-axis translated to
the centroid.....................................= 0.94 in↑4

Area moment of inertia about Y'-axis translated to
the centroid.....................................= 9.29 in↑4

Area product of inertia about translated axis......= 1.42 in↑4

Angle between translated axis and principal axis
(in degree), positive is counter-clockwise.........= 9.38

Area moment of inertia about the translated, rotated,
principal X''-axis...............................= 0.72 in↑4

Area moment of inertia about the translated,
rotated, principal Y''-axis......................= 9.52 in↑4


Do you want to get area moment of inertia about an
arbitrary axis? (Y or N)Y
```

Figure B-9. Output for Example B-2.

```
Please enter the angle between the original axis (X-Y) and
arbitrary axis (X'''-Y'''), in degree, positive is
counter-clockwise.....................................= 0
Enter X-axis coordinate of the arbitrary axis (inches)= .2
Enter Y-axis coordinate of the arbitrary axis (inches)= .6

Area moment of inertia about the arbitrary axis,X'''..= 3.91 in↑4

Area moment of inertia about the arbitrary axis,Y'''..= 22.22 in↑4

Polar area moment of inertia about the arbitrary
axis (X'''-Y''')......................................= 26.13 in↑4

Area product of inertia about the arbitrary
axis (X'''-Y''')......................................= 7.61 in↑4

Radius of gyration about X-axis.......................= 1.55 inches

Radius of gyration about Y-axis.......................= 2.54 inches

Radius of gyration about X'-axis translated to
the centroid..........................................= 0.49 inches

Radius of gyration about Y'-axis translated to
the centroid..........................................= 1.54 inches


Do you want to find the properties of another section?
(Y or N)N

--------------------    END    --------------------
```

Figure B-10. Output for Example B-2.

A.3 USER'S GUIDE ---BEAM ANALYSIS

INTRODUCTION

One of the most frequently encountered engineering designs is beam because it can be used for modelling many structures. This program, using transfer matrix method, computes and also plots the curves of deflection, slope, moment, and shear along the beam. Static and forced, undamped dynamic analysis can be performed for beams of uniform or variable cross section. Uniformly or linearly varied distributed loads, concentrated point loads, applied moments, or combinations of all three may be applied. This program allows any combination of pinned, fixed, free, or guided flexural boundary conditions, even normally kinematically unstable conditions can be handled if sufficient internal supports are provided. In-span support can be elastic springs and/or elastic moment spring. Modelling for dynamic response uses lumped mass. The programming language used is BASIC and the micro-processor used is a Teketronix model 4051 with 32 K memory.

THEORY

The theory, using transfer matrix method, is well documented in chapter 3 of the <u>Matrix Methods in Elastromechanics</u>, by Eduard C. Pestel and Frederick A. Leckie, McGraw-Hill, 1963. The method is based on the idea that a continuous beam can be broken up into component parts with simple elastic and dynamic properties that can be expressed in matrix form. These component matrices, when fitted together by successive matrix multiplication and evaluated with the proper boundary conditions will give the response of the entire beam.

For a given continuous beam with several sections, say i, each element or section is represented by the appropriate field and point transfer matrices. The state vectors from one end, $\left[Z\right]_0$, to the other end, $\left[Z\right]_i$, are related by the equation:

$$\begin{aligned}\left[Z\right]_i &= \left[P\right]_i \left[F\right]_i \left[P\right]_{i-1} \left[F\right]_{i-1} \cdots\cdots \left[P\right]_j \left[F\right]_j \cdots\cdots \left[P\right]_1 \left[F\right]_1 \left[Z\right]_0 \\ &= \left[U\right]\left[Z\right]_0 \end{aligned} \qquad \text{(C-1)}$$

where $\left[F\right]_j$ = a field transfer matrix that describes the j^{th} section of distributed stiffness with or without distributed loads

$\left[P\right]_j$ = a point transfer matrix that describes the j^{th} element at a point with no finite length.

i = number of sections.

$\left[Z\right]_0$ = state vector at beginning, usually the left end.

$\left[Z\right]_i$ = state vector at the termination end, usually the right end.

$\left[U\right]$ = product of all field and point transfer matrices.

The state vector, $\left[Z\right]$, has five components, defined as:

$$\left[Z\right] = \begin{bmatrix} W \\ S \\ M \\ V \\ 1 \end{bmatrix} \qquad \text{(C-2)}$$

where W = deflection

S = slope

M = moment

V = shear

The boundary conditions are as follows:

For pinned end, W=0, M=0

For fixed end, W=0, S=0

For free end, M=0, V=0 (C-3)

For guided end, S=0, V=0

where the symbols are defined same as Eq.C-2.

The field and point transfer matrices in Eq. C-1 are known and the boundary conditions of both ends should be applied to Eq. C-4.

$$[Z]_i = [U][Z]_0 \tag{C-4}$$

All the variables in the state vectors $[Z]_0$ and $[Z]_i$ can now be found. Once $[Z]_0$ is known, this matrix multiplication process is repeated to yield the states at each desired point along the beam.

INPUT DATA REQUIRED

The beam should first be divided into several sections, such that each section has the same distributed stiffness and a point element at the end if any.

Units can be either in English or the S. I. system. This depends on the option chosen by the user. For each section, the following data are needed:

1. Length of the section, inches or metres.
2. Modulus of elasticity, psi or Pa.
3. Area moment of inertia, in^4 or m^4.
4. Magnitude of uniformly distributed load, lb/in or N/m. (N.B.; Load is positive downward.)
5. Magnitude of linearly varied distributed load, on the left and on the right, lb/in or N/m. (N.B.; Load is positive downward.)
6. Magnitude of concentrated load, lb or N. (N.B.; Load is positive downward.)
7. Magnitude of moment, lb-in or N-m.(N.B.; Moment is positive in counter-clockwise direction.)
8. Stiffness of the support, lb/in or N/m.
9. Support moment stiffness, lb-in/rad or N-m/rad.
10. Magnitude of concentrated weight, lb or N.
11. Weight moment of inertia, lb-in^2 or N-m^2.
12. Forced circular frequency, rad/sec.
13. Enter the type of vibration of the beam or rotor.

    H, a factor for determining gyroscopic inertial effects in the dynamic case, given as:

$$H = \begin{cases} -1 & \text{for bending vibration} \\ +1 & \text{for rotating shaft (equal angular direction of whirl and rotation)} \\ -3 & \text{for rotating shaft (opposite angular direction of whirl and rotation)} \end{cases}$$

Enter 1 for H = -1

2 H = +1

3 H = -3

EXAMPLE C-1.

A cantilever beam, 8 inches (0.2032 m) long, has a concentrated load of 100 lb (444.82 N) at its free end, see Fig. C-1. The moment of inertia, 4.7 in^4 (1.956×10^{-6} m^4) is constant. The material used is steel and the modulus of elasticity is 30×10^6 psi (2.068×10^{11} Pa). Neglect the weight of the beam. Determine the deflection, slope, moment, and shear along the beam.

English units are used.

This is a static response.

<u>Section #1</u>

For field matrix, enter 1 for massless beam.

Length of this section = 8 inches.

Modulus of elasticity = 30×10^6 psi.

Area moment of inertia = 4.7 in^4.

For point matrix,

Concentrated load	= 100 lb.
Moment	= 0 lb-in.
Stiffness of support	= 0 lb/in.
Support moment stiffness	= 0 lb-in/rad.

<u>Section #2</u>

For field matrix, enter 0 for the last section.

Choose the boundary condition of fixed-free, enter 7.

Enter 10 increments for the section.

The micro-processor outputs the input data, the numerical values, and curves of deflection , slope, moment, and shear, see Figs. C-2 to C-9.

EXAMPLE 2 (Taken from page 87, Example 3-9, of Matrix Methods in Electromechanics, by Eduard C. Pestel and Frederick A. Lecke, McGraw-Hill, 1963, numerical values have been assumed for this example.)

A beam of uniform flexural stiffness EI is simply supported at one end, built-in at the other, and supported by a spring one third of the distance along its length, see Fig. C-10. It is subjected to a uniform distributed static load of 100 lb/in (1.75×10^4 N/m) between points 0 and 1 and a concentrated moment of -10000 lb-in (-1130 N-m) at point 1. Determine the deflection, slope, moment, and shear diagrams. Steel is the material used.

English units are used.

This is a static response.

Section #1

For field matrix, enter 2 for uniformly distributed load on massless beam.

Length of this section	= 10 inches
Modulus of elasticity	= 30×10^6 psi
Uniformly distributed load	= 100 lb/in

For point matrix,

Concentrated load	= 0 lb
Moment	= -10000 lb-in
Stiffness of support	= 282000 lb/in

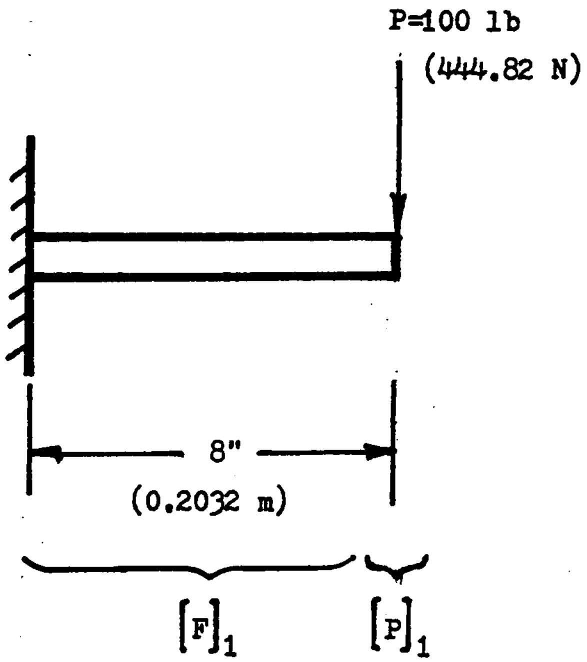


Figure C-1. Cantilever beam with end load.

```
******************************************************************
*                                                                *
*    THIS IS A BEAM ANALYSIS PROGRAM WHICH SOLVES STATIC OR      *
*    DYNAMIC (FORCED, UNDAMPED VIBRATION) RESPONSE.              *
*                                                                *
*    by Yiu Wah Luk, advised by Professor Larry D. Mitchell      *
*     Virginia Polytechnic Institute and State University       *
*             Department of Mechanical Engineering              *
*                        Spring 1978                            *
*                                                                *
******************************************************************

Do you want to use English units? (Y or N)Y


English units will be used throughout this program.

Is this a static analysis? (Y or N)Y

Please divide your beam into several field sections,
and enter the information required for each section.
N.B. Always start from the left end.


Press RETURN to proceed.
```

Figure C-2. Output for Example C-1.

```
For section #1:
The following Field Matrices are available.
0       the last section.
1       massless beam.
2       uniformly distributed load on massless beam.
3       linearly varied distributed load on massless beam.

Enter # for the required Field Matrix= 1

Enter length of this section (in)
(H.B. Do not enter zero.) =8
Enter Modulus of Elasticity (psi)=30E6
Enter Area Moment of Inertia (in↑4)=4.7


Accumulated Transfer Matrix including section #1 is as follows:

W  1.00E+000  -8.00E+000  -2.27E-007  -6.05E-007   0.00E+000
S  0.00E+000   1.00E+000   5.67E-008   2.27E-007   0.00E+000
M  0.00E+000   0.00E+000   1.00E+000   8.00E+000   0.00E+000
V  0.00E+000   0.00E+000   0.00E+000   1.00E+000   0.00E+000
1  0.00E+000   0.00E+000   0.00E+000   0.00E+000   1.00E+000


Press RETURN to proceed.
```

Figure C-3. Output for Example C-1.

```
For section #1:
Is there any concentrated load, moment, lumped mass (for dynamic
analysis only), or elastic support in this section? (Y or N)Y

Enter magnitude of the concentrated load, (lb)
(N.B. Load is positive downward) =100

Enter magnitude of the Moment (lb-in)
(N.B. Moment is positive in counter-clockwise direction.
Please put in the right sign.)=0

Enter stiffness of the support (lb/in)=0

Enter support moment stiffness (lb-in/rad)=0


Accumulated Transfer Matrix including section #1 is as follows:

W  1.00E+000  -8.00E+000  -2.27E-007  -6.05E-007   0.00E+000
S  0.00E+000   1.00E+000   5.67E-008   2.27E-007   0.00E+000
M  0.00E+000   0.00E+000   1.00E+000   8.00E+000   0.00E+000
V  0.00E+000   0.00E+000   0.00E+000   1.00E+000  -1.00E+002
1  0.00E+000   0.00E+000   0.00E+000   0.00E+000   1.00E+000


Press RETURN to proceed.
```

Figure C-4. Output for Example C-1.

```
For section #2:
The following Field Matrices are available.
0       the last section.
1       massless beam.
2       uniformly distributed load on massless beam.
3       linearly varied distributed load on massless beam.

Enter # for the required Field Matrix= 0
```

Figure C-5. Output for Example C-1.

*** BOUNDARY CONDITIONS ***

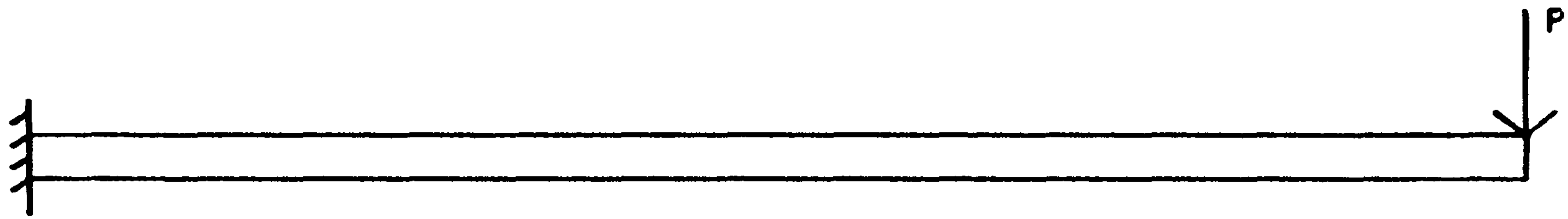
		RIGHT END			
		PINNED	FIXED	FREE	GUIDED
LEFT END	PINNED	1	2	3 (K.U.)	4
	FIXED	5	6	7	8
	FREE	9 (K.U.)	10	11 (K.U.)	12 (K.U.)
	GUIDED	13	14	15 (K.U.)	16 (K.U.)

where K. U. = Kinematically Unstable, unless internal supports exist.
Use these boundary conditions at your own risk.
Do not answer K. U. to the question below.

Enter # for the required Boundary Condition = 7

Figure C-6. Output for Example C-1.

```
Drawing of the Beam
```




```
DATA:
For section #1:
Length of this section......................= 8.00E+000 in
Modulus of Elasticity.......................= 3.00E+007 psi
Area Moment of Inertia......................= 4.70E+000 in↑4
Magnitude of Concentrated Load..............= 1.00E+002 lb

Do you want to change the data? (Y or N)N

How many increments would you like to have for each field section?10
```

Figure C-7. Output for Example C-1.

LENGTH (in)	DEFLECTION (in)	SLOPE (Radian)	MOMENT (lb-in)	SHEAR (lb)
0.00E+000	0.00E+000	0.00E+000	-8.00E+002	1.00E+002
8.00E-001	1.76E-006	-4.31E-006	-7.20E+002	1.00E+002
1.60E+000	6.78E-006	-8.17E-006	-6.40E+002	1.00E+002
2.40E+000	1.47E-005	-1.16E-005	-5.60E+002	1.00E+002
3.20E+000	2.52E-005	-1.45E-005	-4.80E+002	1.00E+002
4.00E+000	3.78E-005	-1.70E-005	-4.00E+002	1.00E+002
4.80E+000	5.23E-005	-1.91E-005	-3.20E+002	1.00E+002
5.60E+000	6.82E-005	-2.07E-005	-2.40E+002	1.00E+002
6.40E+000	8.52E-005	-2.18E-005	-1.60E+002	1.00E+002
7.20E+000	1.03E-004	-2.25E-005	-8.00E+001	1.00E+002
8.00E+000	1.21E-004	-2.27E-005	-1.09E-011	1.00E+002
8.00E+000	1.21E-004	-2.27E-005	-1.09E-011	0.00E+000

Do you wish to see the graphs for deflection, slope,
moment and shear? (Y or N)Y

Figure C-8. Output for Example C-1.

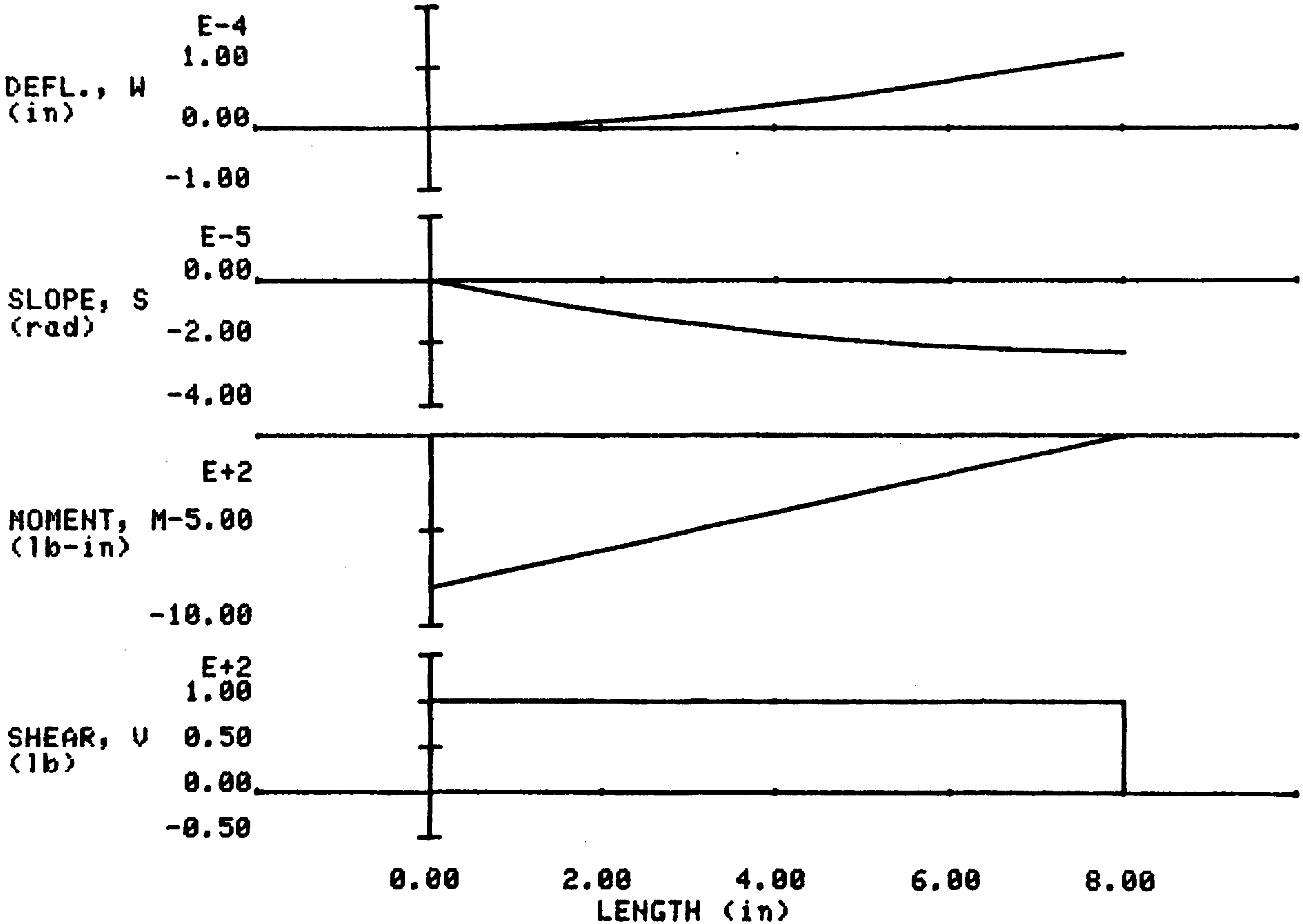


Figure C-9. Output for Example C-1.

Support moment stiffness = 0 lb-in/rad

Section #2

For field matrix, enter 1 for massless beam.

Length of this section = 20 inches

Modulus of elasticity = 30×10^{6} psi

Area moment of inertia = 4.7 in^{4}

No pcint matrix for this section.

Section #3

For field matrix, enter 0 for the last section.

Choose the boundary condition of pinned-fixed, enter 2.

Enter 10 increments for each section.

The micro-processor outputs the data and deflection, slope, moment, and shear diagrams, see Figs. C-11 to C-13.

EXAMPLE C-3.

The beam with 6 elastic supports of stiffness 1200 lb/in (2.10×10^{5} N/m) each is shown in Fig. C-14. The moment of inertia of the beam is constant, 144 in^{4} (6×10^{-5} m^{4}), and the modulus of elasticity is 20×10^{6} psi (1.38×10^{11} Pa). Determine the deflection, slope, mcment, and shear diagrams. Neglect the weight of the beam.

English units are used.

This is a static response.

Section #1

For field matrix, enter 1 for massless beam.

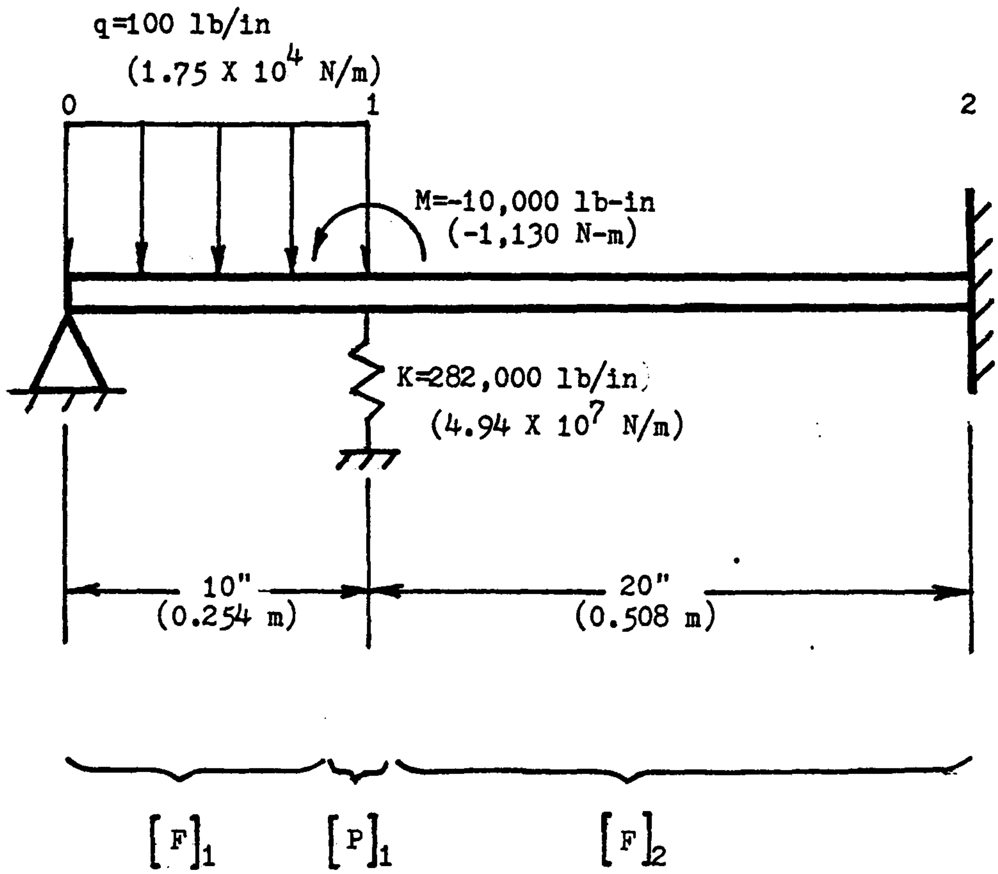
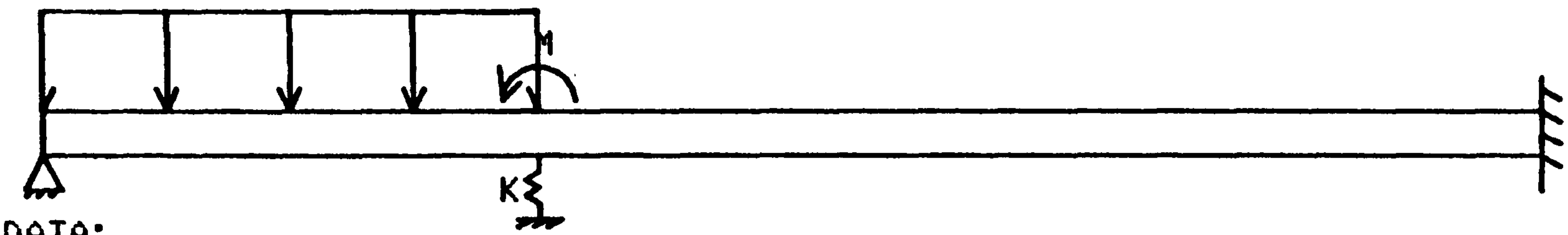


Figure C-10. Statically loaded, statically indeterminant beam.

Drawing of the Beam




```
DATA:
For section #1:
Length of this section......................= 1.00E+001  in
Modulus of Elasticity.......................= 3.00E+007  psi
Area Moment of Inertia......................= 4.70E+000  in↑4
Magnitude of uniformly distributed load....= 1.00E+002  lb/in
Magnitude of Moment.........................=-1.00E+004  lb-in
Stiffness of Support........................= 2.82E+005  lb/in

For section #2:
Length of this section......................= 2.00E+001  in
Modulus of Elasticity.......................= 3.00E+007  psi
Area Moment of Inertia......................= 4.70E+000  in↑4


Do you want to change the data? (Y or N)N


How many increments would you like to have for each field section?10
```

Figure C-11. Output for Example C-2.

LENGTH (in)	DEFLECTION (in)	SLOPE (Radian)	MOMENT (lb-in)	SHEAR (lb)
0.00E+000	0.00E+000	-9.32E-005	0.00E+000	1.58E+002
1.00E+000	9.30E-005	-9.27E-005	1.08E+002	5.81E+001
2.00E+000	1.85E-004	-9.19E-005	1.16E+002	-4.19E+001
3.00E+000	2.77E-004	-9.13E-005	2.42E+001	-1.42E+002
4.00E+000	3.68E-004	-9.18E-005	-1.68E+002	-2.42E+002
5.00E+000	4.61E-004	-9.39E-005	-4.60E+002	-3.42E+002
6.00E+000	5.57E-004	-9.85E-005	-8.52E+002	-4.42E+002
7.00E+000	6.59E-004	-1.06E-004	-1.34E+003	-5.42E+002
8.00E+000	7.71E-004	-1.18E-004	-1.94E+003	-6.42E+002
9.00E+000	8.96E-004	-1.34E-004	-2.63E+003	-7.42E+002
1.00E+001	1.04E-003	-1.55E-004	-3.42E+003	-8.42E+002
1.00E+001	1.04E-003	-1.55E-004	6.58E+003	-5.49E+002
1.20E+001	1.26E-003	-6.98E-005	5.48E+003	-5.49E+002
1.40E+001	1.33E-003	2.34E-007	4.39E+003	-5.49E+002
1.60E+001	1.27E-003	5.47E-005	3.29E+003	-5.49E+002
1.80E+001	1.12E-003	9.35E-005	2.19E+003	-5.49E+002
2.00E+001	9.09E-004	1.17E-004	1.10E+003	-5.49E+002
2.20E+001	6.64E-004	1.25E-004	-2.07E+000	-5.49E+002
2.40E+001	4.20E-004	1.17E-004	-1.10E+003	-5.49E+002
2.60E+001	2.08E-004	9.34E-005	-2.20E+003	-5.49E+002
2.80E+001	5.71E-005	5.45E-005	-3.29E+003	-5.49E+002
3.00E+001	0.00E+000	-3.47E-018	-4.39E+003	-5.49E+002

Do you wish to see the graphs for deflection, slope,
moment and shear? (Y or N)Y

Figure C-12. Output for Example C-2.

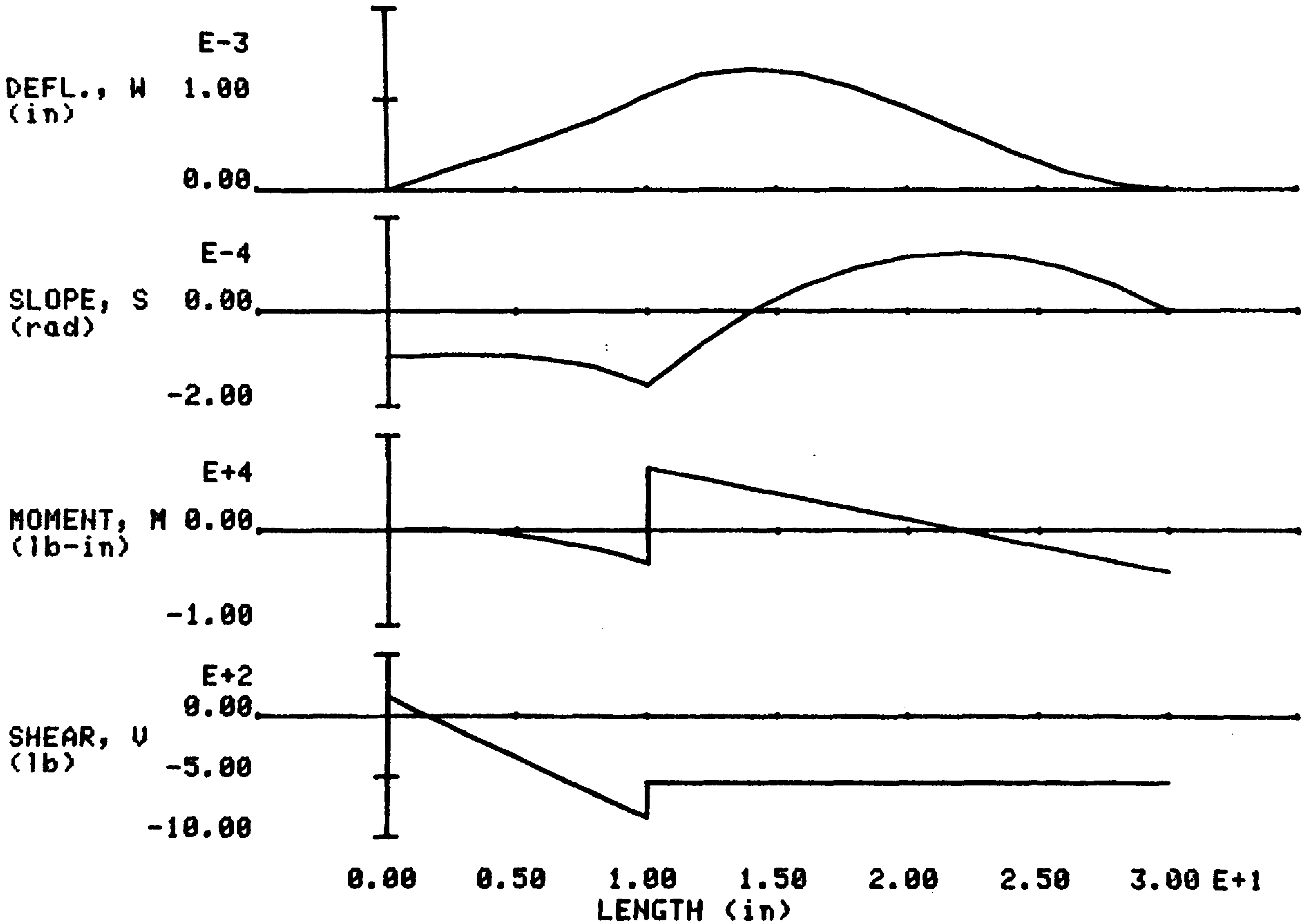


Figure C-13. Output for Example C-2.

Length of this section = 1 X 10^{-20} inches (Do not enter 0)

Modulus of elasticity = 20 X 10^6 psi

Area moment of inertia = 144 in^4

For point matrix,

Concentrated load = 0 lb

Moment = 0 lb-in

Stiffness of support = 1200 lb/in

Support moment stiffness = 0 lb-in/rad

The use of an almost zero length beam allows the immediate installation of a spring support.

Section #2

For field matrix, enter 3 for linearly varied distributed load on massless beam.

Length of this section = 480 inches

Modulus of elasticity = 20 X 10^6 psi

Area moment of inertia = 144 in^4

Linearly varied distributed load, on the left = 75 lb/in

Linearly varied distributed load, on the right = 100 lb/in

For the point matrix,

Concentrated load = 0 lb

Moment = 0 lb-in

Stiffness of support = 1200 lb/in

Support moment stiffness= 0 lb-in/rad

Section #3

For field matrix, enter 1 for massless beam.

Length of this section	= 480 inches
Modulus of elasticity	= 20×10^6 psi
Area moment of inertia	= 144 in^4

For point matrix,

Concentrated load	= 0 lb
Moment	= 0 lb-in
Stiffness of support	= 1200 lb/in
Support moment stiffness	= 0 lb-in/rad

Section #4

For field matrix, enter 2 for uniformly distributed load on massless beam.

Length of this section	= 480 inches
Modulus of elasticity	= 20×10^6 psi
Area moment of inertia	= 144 in^4
Uniformly distributed load	= 100 lb/in

For point matrix,

Concentrated load	= 0 lb
Moment	= 0 lb-in
Stiffness of support	= 1200 lb/in
Support moment stiffness	= 0 lb-in/rad

Section #5

For field matrix, enter 1 for massless beam.

Length of this section	= 480 inches

Modulus of elasticity = 20×10^6 psi

Area moment of inertia = 144 in^4

For point matrix,

Concentrated load = 0 lb

Moment = 0 lb-in

Stiffness of support = 1200 lb/in

Support moment stiffness = 0 lb-in/rad

Section #6

For field matrix, enter 3 for linearly varied distributed load on massless beam.

Length of this section = 480 inches

Modulus of elasticity = 20×10^6 psi

Area moment of inertia = 144 in^4

Linearly varied distributed load, on the left = 100 lb/in

Linearly varied distributed load, on the right = 75 lb/in

For point matrix,

Concentrated load = 0 lb

Moment = 0 lb-in

Stiffness of support = 1200 lb/in

Support moment stiffness = 0 lb-in/rad

Section #7

For field matrix, enter 0 for the last section.

Choose the boundary condition of free-free, enter 11.

Enter 10 increments for each section.

The micro-processor output of the data and deflection, slope, moment, and shear diagrams are shown in Figs. C-15 to C-19.

EXAMPLE C-4.

A shaft with two lumped mass, 800 lb (3559 N) and 1200 lb (5338 N) in weight, vibrates at a frequency of 10 rad/sec, see Fig. C-20. The area moment of inertia is constant, I=12 in^4 (5×10^{-6} m^4). It has a fixed end and the other end is an elastic support with stiffness, K=1×10^5 lb/in (1.75×10^7 N/m), and support moment stiffness, T=1×10^4 lb-in/rad (1130 N-m/rad). Find the deflection, slope, moment, and shear diagrams at this frequency. Steel is the material used. Weight moment of inertia is 100 lb-in^2 (0.287 N-m^2).

SI units are used.

This is a dynamic response problem forced by the concentrated load P sin wt where w=10 rad/sec. P is shown on Fig. C-20.

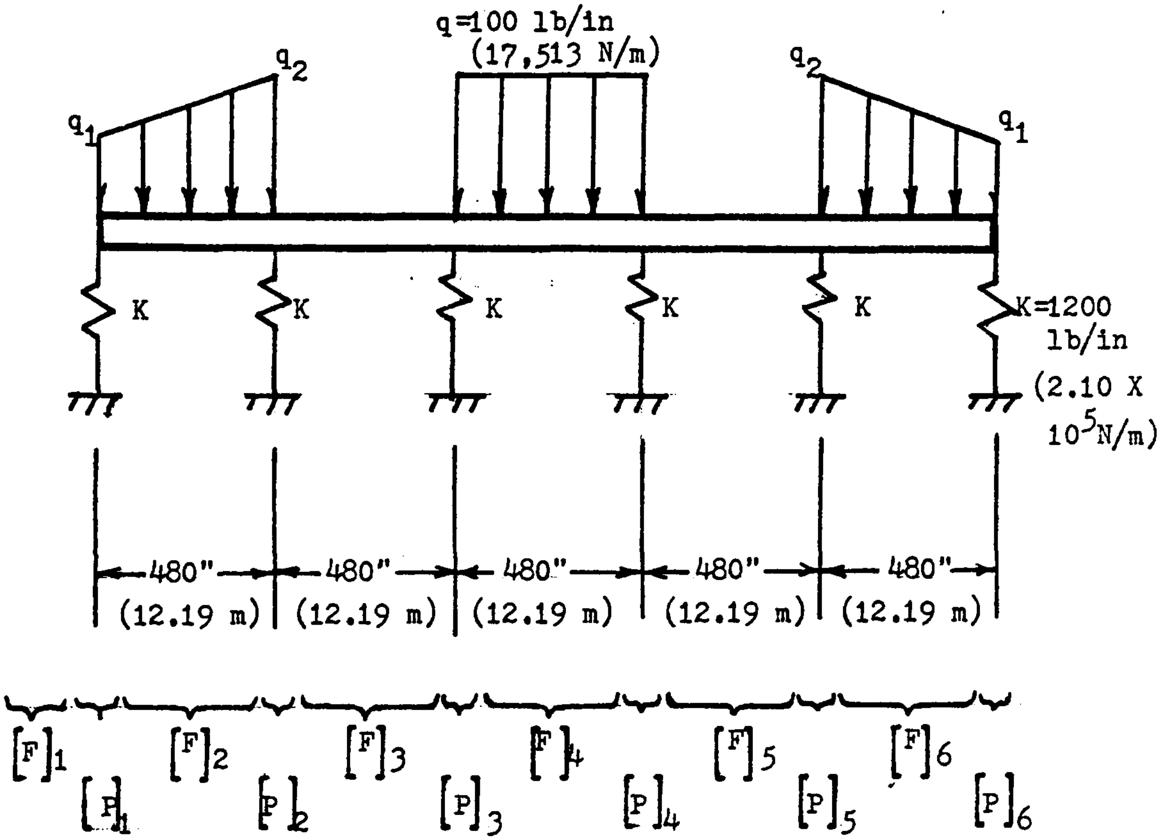
<u>Section #1</u>

For field matrix, enter 1 for massless beam.

Length of this section	= 0.254 m
Modulus of elasticity	= 2.068×10^{11} Pa
Area moment of inertia	= 5×10^{-6} m^4

For point matrix,

Concentrated load	= 0 N
Moment	= 0 N-m
Stiffness of support	= 0 N/m
Support moment stiffness	= 0 N-m/rad

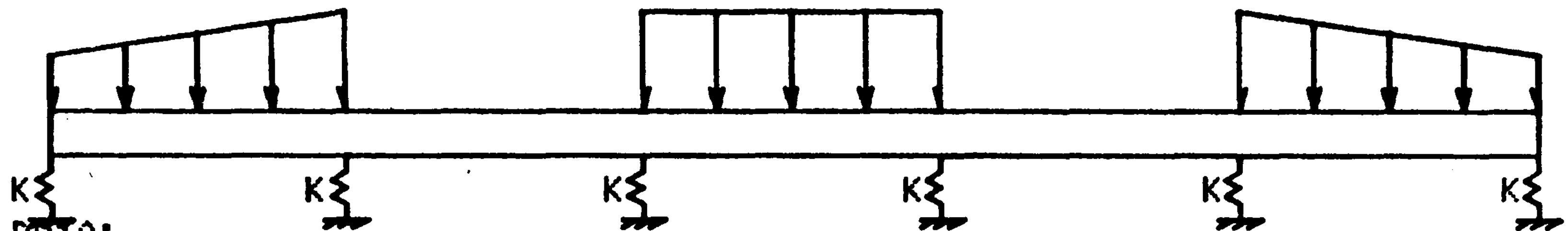


where q_1 = 75 lb/in (1.313×10^4 N/m)

q_2 =100 lb/in (17,513 N/m)

Figure C-14. Complex beam loading and supports.

Drawing of the Beam




```
DATA:
For section #1:
Length of this section......................= 1.00E-020 in
Modulus of Elasticity.......................= 2.00E+007 psi
Area Moment of Inertia......................= 1.44E+002 in↑4
Stiffness of Support........................= 1.20E+003 lb/in

For section #2:
Length of this section......................= 4.80E+002 in
Modulus of Elasticity.......................= 2.00E+007 psi
Area Moment of Inertia......................= 1.44E+002 in↑4
Left end of linearly-varied dist. load.....= 7.50E+001 lb/in
Right end of linearly-varied dist. load....= 1.00E+002 lb/in
Stiffness of Support........................= 1.20E+003 lb/in

For section #3:
Length of this section......................= 4.80E+002 in
Modulus of Elasticity.......................= 2.00E+007 psi
Area Moment of Inertia......................= 1.44E+002 in↑4
Stiffness of Support........................= 1.20E+003 lb/in

For section #4:
Length of this section......................= 4.80E+002 in
Modulus of Elasticity.......................= 2.00E+007 psi
Area Moment of Inertia......................= 1.44E+002 in↑4
Magnitude of uniformly distributed load....= 1.00E+002 lb/in
```

Figure C-15. Output for Example C-3.

```
Stiffness of Support........................= 1.20E+003 lb/in

For section #5:
Length of this section......................= 4.80E+002 in
Modulus of Elasticity.......................= 2.00E+007 psi
Area Moment of Inertia......................= 1.44E+002 in↑4
Stiffness of Support........................= 1.20E+003 lb/in

For section #6:
Length of this section......................= 4.80E+002 in
Modulus of Elasticity.......................= 2.00E+007 psi
Area Moment of Inertia......................= 1.44E+002 in↑4
Left end of linearly-varied dist. load.....= 1.00E+002 lb/in
Right end of linearly-varied dist. load....= 7.50E+001 lb/in
Stiffness of Support........................= 1.20E+003 lb/in

Do you want to change the data? (Y or N)N

How many increments would you like to have for each field section?10
```

Figure C-16. Output for Example C-3.

LENGTH (in)	DEFLECTION (in)	SLOPE (Radian)	MOMENT (lb-in)	SHEAR (lb)
0.00E+000	1.50E+001	-1.23E-001	0.00E+000	0.00E+000
1.00E-021	1.50E+001	-1.23E-001	0.00E+000	0.00E+000
2.00E-021	1.50E+001	-1.23E-001	0.00E+000	0.00E+000
3.00E-021	1.50E+001	-1.23E-001	0.00E+000	0.00E+000
4.00E-021	1.50E+001	-1.23E-001	0.00E+000	0.00E+000
5.00E-021	1.50E+001	-1.23E-001	0.00E+000	0.00E+000
6.00E-021	1.50E+001	-1.23E-001	0.00E+000	0.00E+000
7.00E-021	1.50E+001	-1.23E-001	0.00E+000	0.00E+000
8.00E-021	1.50E+001	-1.23E-001	0.00E+000	0.00E+000
9.00E-021	1.50E+001	-1.23E-001	0.00E+000	0.00E+000
1.00E-020	1.50E+001	-1.23E-001	0.00E+000	0.00E+000
1.00E-020	1.50E+001	-1.23E-001	0.00E+000	1.80E+004
4.80E+001	2.08E+001	-1.16E-001	7.78E+005	1.44E+004
9.60E+001	2.60E+001	-9.78E-002	1.38E+006	1.06E+004
1.44E+002	3.01E+001	-7.11E-002	1.79E+006	6.70E+003
1.92E+002	3.27E+001	-3.90E-002	2.02E+006	2.68E+003
2.40E+002	3.38E+001	-4.84E-003	2.05E+006	-1.46E+003
2.88E+002	3.32E+001	2.82E-002	1.88E+006	-5.72E+003
3.36E+002	3.11E+001	5.66E-002	1.50E+006	-1.01E+004
3.84E+002	2.79E+001	7.69E-002	9.06E+005	-1.46E+004
4.32E+002	2.39E+001	8.56E-002	9.44E+004	-1.92E+004
4.80E+002	1.99E+001	7.88E-002	-9.42E+005	-2.40E+004
4.80E+002	1.99E+001	7.88E-002	-9.42E+005	-4.29E+001
5.28E+002	1.65E+001	6.31E-002	-9.44E+005	-4.29E+001
5.76E+002	1.39E+001	4.74E-002	-9.46E+005	-4.29E+001
6.24E+002	1.20E+001	3.16E-002	-9.48E+005	-4.29E+001
6.72E+002	1.08E+001	1.58E-002	-9.50E+005	-4.29E+001
7.20E+002	1.05E+001	-7.28E-005	-9.52E+005	-4.29E+001
7.68E+002	1.09E+001	-1.60E-002	-9.54E+005	-4.29E+001
8.16E+002	1.20E+001	-3.19E-002	-9.56E+005	-4.29E+001
8.64E+002	1.39E+001	-4.78E-002	-9.58E+005	-4.29E+001
9.12E+002	1.66E+001	-6.38E-002	-9.60E+005	-4.29E+001

Figure C-17. Output for Example C-3.

9.60E+002	2.00E+001	-7.98E-002	-9.62E+005	-4.29E+001
9.60E+002	2.00E+001	-7.98E-002	-9.62E+005	2.40E+004
1.01E+003	2.41E+001	-8.69E-002	7.47E+004	1.92E+004
1.06E+003	2.81E+001	-7.86E-002	8.81E+005	1.44E+004
1.10E+003	3.15E+001	-5.88E-002	1.46E+006	9.60E+003
1.15E+003	3.37E+001	-3.13E-002	1.80E+006	4.80E+003
1.20E+003	3.44E+001	-1.07E-014	1.92E+006	2.33E-008
1.25E+003	3.37E+001	3.13E-002	1.80E+006	-4.80E+003
1.30E+003	3.15E+001	5.88E-002	1.46E+006	-9.60E+003
1.34E+003	2.81E+001	7.86E-002	8.81E+005	-1.44E+004
1.39E+003	2.41E+001	8.69E-002	7.47E+004	-1.92E+004
1.44E+003	2.00E+001	7.98E-002	-9.62E+005	-2.40E+004
1.44E+003	2.00E+001	7.98E-002	-9.62E+005	4.29E+001
1.49E+003	1.66E+001	6.38E-002	-9.60E+005	4.29E+001
1.54E+003	1.39E+001	4.78E-002	-9.58E+005	4.29E+001
1.58E+003	1.20E+001	3.19E-002	-9.56E+005	4.29E+001
1.63E+003	1.09E+001	1.60E-002	-9.54E+005	4.29E+001
1.68E+003	1.05E+001	7.28E-005	-9.52E+005	4.29E+001
1.73E+003	1.08E+001	-1.58E-002	-9.50E+005	4.29E+001
1.78E+003	1.20E+001	-3.16E-002	-9.48E+005	4.29E+001
1.82E+003	1.39E+001	-4.74E-002	-9.46E+005	4.29E+001
1.87E+003	1.65E+001	-6.31E-002	-9.44E+005	4.29E+001
1.92E+003	1.99E+001	-7.88E-002	-9.42E+005	4.29E+001
1.92E+003	1.99E+001	-7.88E-002	-9.42E+005	2.40E+004
1.97E+003	2.39E+001	-8.56E-002	9.44E+004	1.92E+004
2.02E+003	2.79E+001	-7.69E-002	9.06E+005	1.46E+004
2.06E+003	3.11E+001	-5.66E-002	1.50E+006	1.01E+004
2.11E+003	3.32E+001	-2.82E-002	1.88E+006	5.72E+003
2.16E+003	3.38E+001	4.84E-003	2.05E+006	1.46E+003
2.21E+003	3.27E+001	3.90E-002	2.02E+006	-2.68E+003
2.26E+003	3.01E+001	7.11E-002	1.79E+006	-6.70E+003
2.30E+003	2.60E+001	9.78E-002	1.38E+006	-1.06E+004
2.35E+003	2.08E+001	1.16E-001	7.78E+005	-1.44E+004
2.40E+003	1.50E+001	1.23E-001	-4.06E-004	-1.80E+004
2.40E+003	1.50E+001	1.23E-001	-4.06E-004	2.80E-006

Figure C-18. Output for Example C-3.

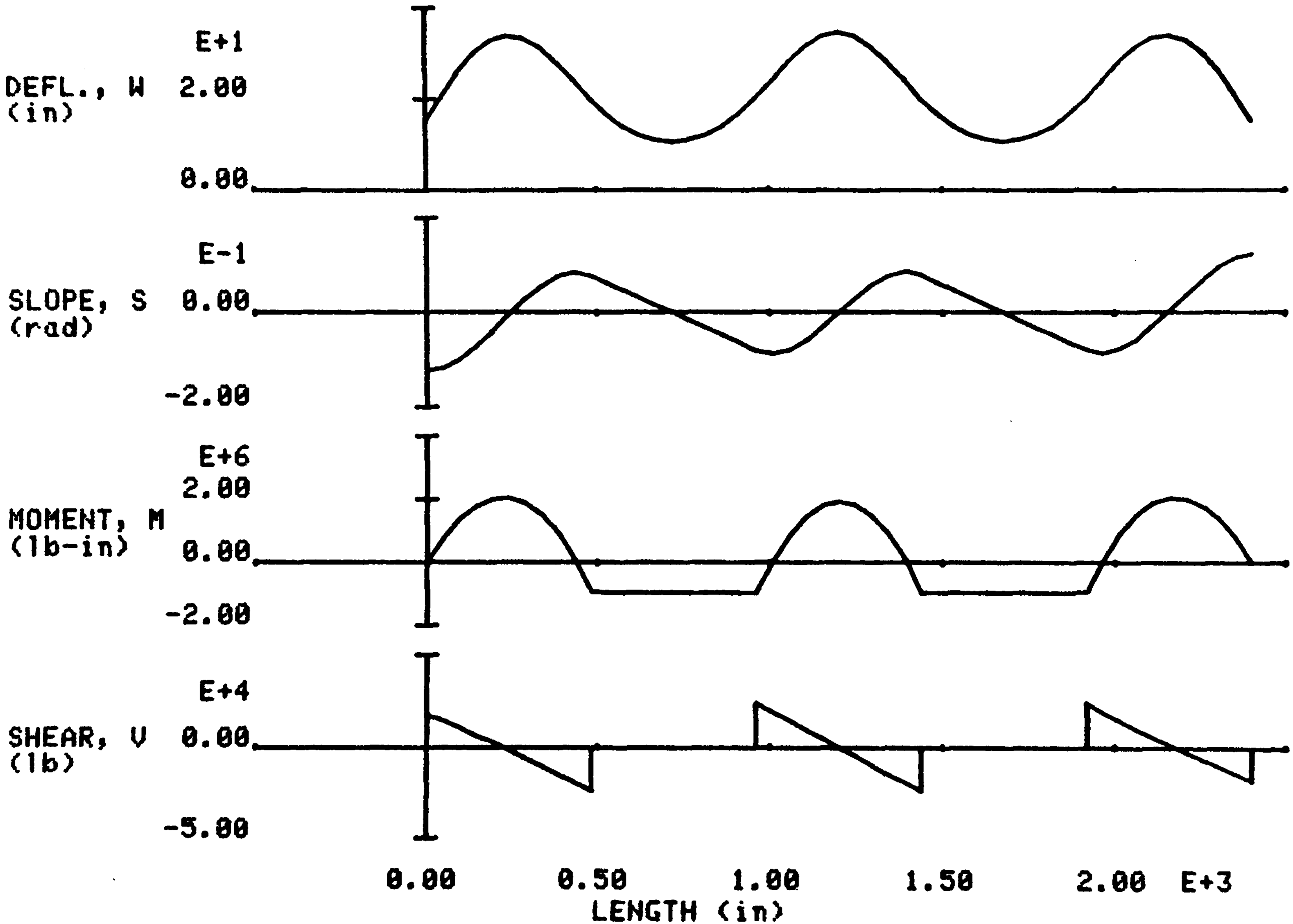


Figure C-19. Output for Example C-3.

Weight of concentrated load = 3559 N

Section #2

For field matrix, enter 1 for massless beam.

Length of this section = 0.508 m

Modulus of elasticity = 2.068×10^{11} Pa

Area moment of inertia = 5×10^{-6} m^4

For point matrix,

Concentrated load = 7500 N

Moment = 0 N-m

Stiffness of support = 1.75×10^7 N/m

Support moment stiffness = 1130 N-m/rad

Weight of concentrated load = 5338 N

Weight moment of inertia = 0.287 N-m^2

For type of vibration, enter 1 for bending vibration.

Section #3

For field matrix, enter 0 for the last section.

Choose the boundary condition of fixed-free, enter 7.

Enter 10 increments for each section.

The micro-processor outputs the data and the deflection, slope, moment, and shear diagrams, see Fig. C-21 to C-23.

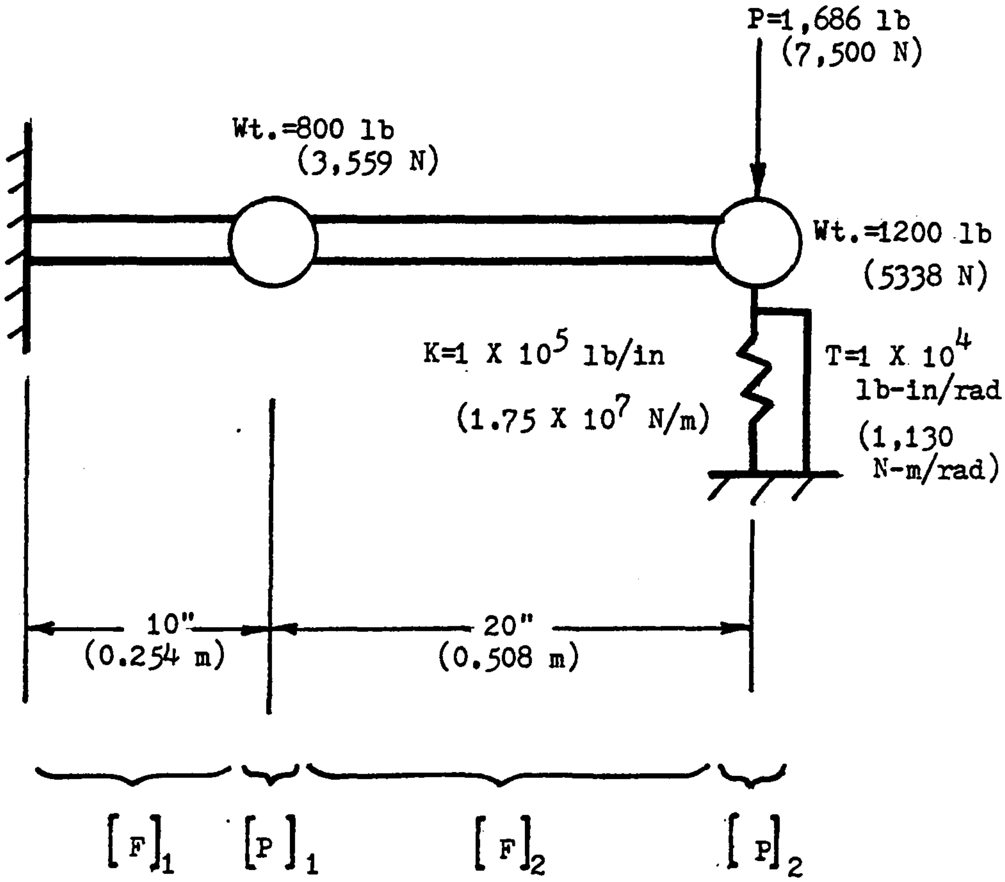
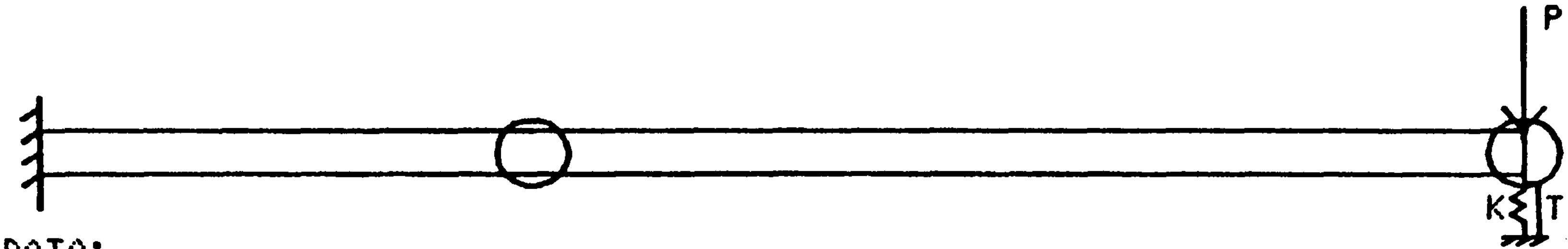


Figure C-20. Dynamically loaded shaft.

```
Drawing of the Beam
```




```
DATA:
Frequency..................................= 1.00E+001 rad/sec
For section #1:
Length of this section.....................= 2.54E-001 m
Modulus of Elasticity......................= 2.07E+011 Pa
Area Moment of Inertia.....................= 5.00E-006 m↑4
Magnitude of Concentrated Weight...........= 3.56E+003 N

For section #2:
Length of this section.....................= 5.08E-001 m
Modulus of Elasticity......................= 2.07E+011 Pa
Area Moment of Inertia.....................= 5.00E-006 m↑4
Magnitude of Concentrated Load.............= 7.50E+003 N
Stiffness of Support.......................= 1.75E+007 N/m
Support Moment Stiffness...................= 1.13E+003 N-m/rad
Magnitude of Concentrated Weight...........= 5.34E+003 N
Weight Moment of Inertia (WR↑2) about a
diameter of disc or element................= 2.87E-001 N-m↑2

Do you want to change the data? (Y or N)N

How many increments would you like to have for each field section?10
```

Figure C-21. Output for Example C-4.

LENGTH (m)	DEFLECTION (m)	SLOPE (Radian)	MOMENT (N-m)	SHEAR (N)
0.00E+000	0.00E+000	0.00E+000	-1.64E+003	2.15E+003
2.54E-002	5.05E-007	-3.96E-005	-1.59E+003	2.15E+003
5.08E-002	2.00E-006	-7.78E-005	-1.53E+003	2.15E+003
7.62E-002	4.45E-006	-1.15E-004	-1.48E+003	2.15E+003
1.02E-001	7.81E-006	-1.50E-004	-1.42E+003	2.15E+003
1.27E-001	1.21E-005	-1.84E-004	-1.37E+003	2.15E+003
1.52E-001	1.72E-005	-2.17E-004	-1.31E+003	2.15E+003
1.78E-001	2.31E-005	-2.49E-004	-1.26E+003	2.15E+003
2.03E-001	2.98E-005	-2.79E-004	-1.20E+003	2.15E+003
2.29E-001	3.73E-005	-3.08E-004	-1.15E+003	2.15E+003
2.54E-001	4.54E-005	-3.35E-004	-1.09E+003	2.15E+003
2.54E-001	4.54E-005	-3.35E-004	-1.09E+003	2.15E+003
3.05E-001	6.38E-005	-3.86E-004	-9.83E+002	2.15E+003
3.56E-001	8.46E-005	-4.32E-004	-8.74E+002	2.15E+003
4.06E-001	1.08E-004	-4.72E-004	-7.65E+002	2.15E+003
4.57E-001	1.32E-004	-5.07E-004	-6.55E+002	2.15E+003
5.08E-001	1.59E-004	-5.36E-004	-5.46E+002	2.15E+003
5.59E-001	1.87E-004	-5.60E-004	-4.37E+002	2.15E+003
6.10E-001	2.16E-004	-5.79E-004	-3.27E+002	2.15E+003
6.60E-001	2.46E-004	-5.93E-004	-2.18E+002	2.15E+003
7.11E-001	2.76E-004	-6.01E-004	-1.09E+002	2.15E+003
7.62E-001	3.07E-004	-6.03E-004	6.64E-001	2.15E+003
7.62E-001	3.07E-004	-6.03E-004	-1.92E-011	-1.46E-010

Do you wish to see the graphs for deflection, slope,
moment and shear? (Y or N)Y

Figure C-22. Output for Example C-4.

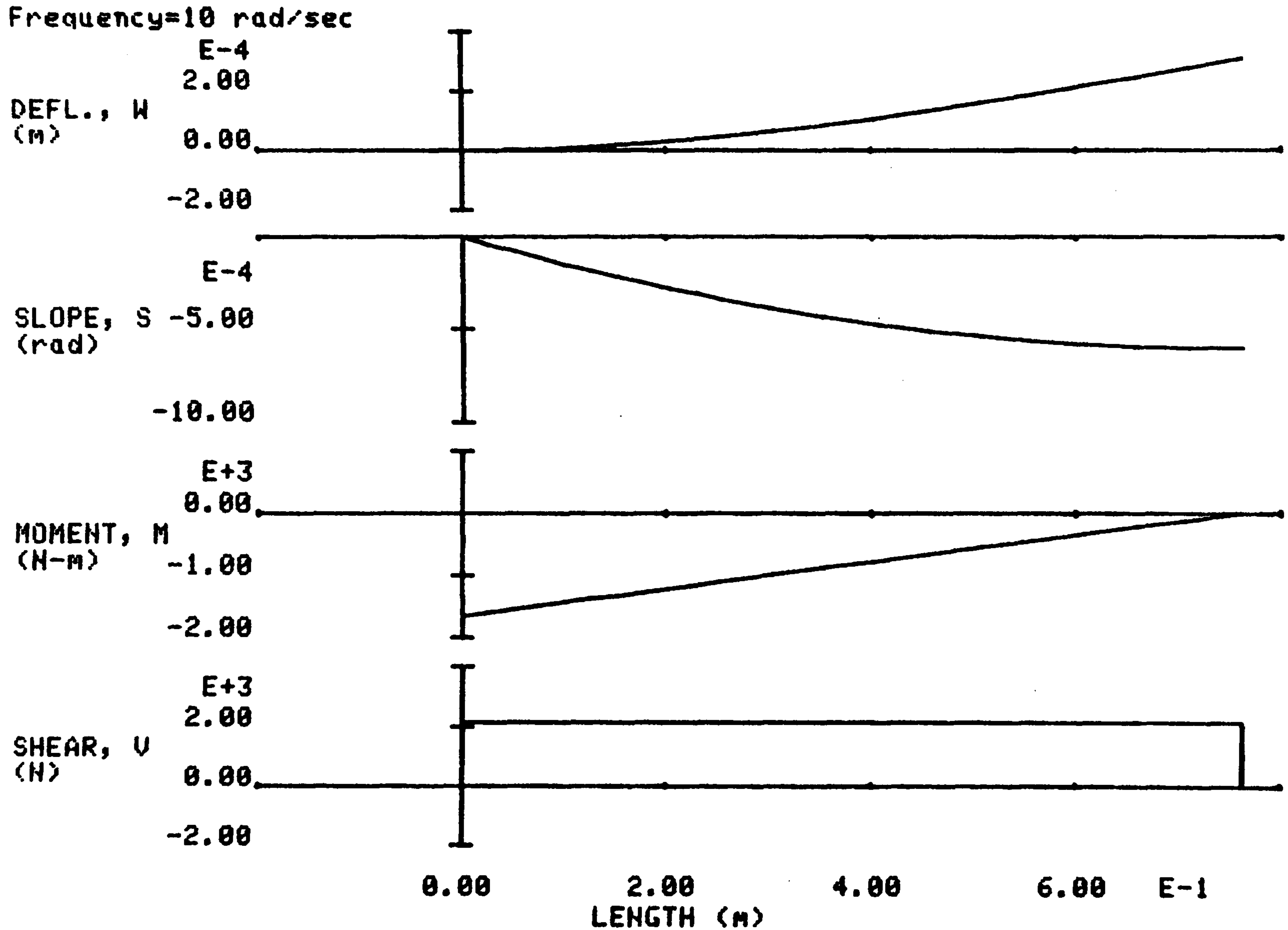


Figure C-23. Output for Example C-4.

APPENDIX B

APPENDIX B.1

PROGRAM LISTING --- FATIGUE ANALYSIS

```
150 INIT
160 PAGE
161 PRI "***************************************************************"
162 PRI "*                                                             *"
163 PRI "*            FATIGUE ANALYSIS PROGRAM, ANALYTICAL             *"
164 PRI "*                                                             *"
165 PRI "*            By Yiu Wah Luk, VPI & SU, Spring 1978            *"
166 PRI "*                                                             *"
167 PRI "***************************************************************"
170 I=0
180 M1=0
190 M2=0
200 F1=0
210 F2=0
220 T1=0
230 T2=0
240 J=0
250 Y=0
260 J5=0
265 N1=0
270 DIM A$(1),B$(6),C$(3),D$(5),E$(3),F$(31),G$(8)
280 PRINT "JJYou have entered a FATIGUE RESISTANT, INTERACTIVE DESIGN ";
290 PRI "routine._Components will be sized to prevent fatigue failure."
300 PRINT "JDo you want to use English Units? (Y or N)";
310 INPUT A$
320 IF A$="Y" THEN 420
330 IF A$<>"N" THEN 300
340 PRINT "JWe will use SI units throughout this routine."
350 J=3
360 B$="m"
370 C$="Pa"
380 D$="N-m"
390 E$="N"
400 G$="Degree C"
```

```
420 B$="inches"
430 C$="psi"
440 D$="lb-in"
450 E$="lbf"
460 G$="Degree F"
470 PRINT "JWe will use English Units throughout this routine."
480 PRINT "JPlease enter the strength of the material to be used.J"
490 PRINT "Tensile Strength, in ";C$;", Su                  = ";
500 INPUT S1
510 PRINT "Yield Strength,   in ";C$;", Sy                  = ";
520 INPUT S2
530 PRINT "JDo you know the Significant Endurance Limit of the ";
540 PRINT "material? (Y or N)";
550 INPUT A$
560 IF A$="Y" THEN 590
570 GOSUB 2730
580 GO TO 596
590 PRINT "Significant Endurance Limit, in ";C$;", Se'''   = ";
592 INPUT S3
594 J4=1
596 GOSUB 2460
600 PAGE
640 PRINT "Select the fatigue failure line to be used in the design.J"
650 PRINT "If Modified Goodman Fracture Line, enter MGF"
660 PRINT "If Modified Goodman Yield Line, enter MGY"
670 PRINT "If Soderberg Line, enter S"
680 PRINT "If Gerber Line, enter G"
690 PRINT "If Quadratic Line, enter Q"
700 PRINT "If Kececioglu Line, enter K"
710 INPUT R$
720 M=1
730 P=1
740 R1=1
750 R2=1
```

```
760 IF R$="MGF" THEN 840
770 IF R$="MGY" THEN 870
780 IF R$="S" THEN 940
790 IF R$="G" THEN 980
800 IF R$="Q" THEN 1020
810 IF R$="K" THEN 1070
820 PRINT "JYour response may have been in error; TRY AGAIN."
830 GO TO 640
840 F$="Modified Goodman Fracture Line."
850 Y=0
860 GO TO 1140
870 F$="Modified Goodman Yield Line."
880 IF J4=1 THEN 910
890 Y=1
900 GO TO 930
910 R1=S1/S2
920 R2=S2/S3
930 GO TO 1140
940 F$="Soderberg Line."
950 R1=S1/S2
960 Y=0
970 GO TO 1140
980 F$="Gerber Line."
990 P=2
1000 Y=0
1010 GO TO 1140
1020 F$="Quadratic Line."
1030 M=2
1040 P=2
1050 Y=0
1060 GO TO 1140
1070 F$="Kececioglu Line."
1080 PRINT "For Kececioglu Line, you have to supply Kececioglu's "
1090 PRINT "Constant which is depended on the material property."
1100 PRINT "Kececioglu's Constant, M = ";
```

```
1110 INPUT M
1120 Y=0
1130 P=2
1140 PRINT "JThe Failure Line selected is ";F$
1150 IF Y=1 THEN 1180
1160 PRI "The failure equation is (Sa/(R2*Se'''))↑m + (R1*Sm/Su)↑p = 1."
1170 PRI "where M=";M;"_where P=";P;"_where R1=";R1;"_and where R2=";R2
1180 PRINT "Do you wish to change any parameters? (Y or N)";
1190 INPUT A$
1200 IF A$="Y" THEN 640
1201 PRINT "JThe following entries will establish the limits on a Half "
1202 PRINT "Interval Search for the solution to your problem."
1210 PRINT "What is the smallest basic dimension that you wish to try?"
1220 PRINT "Give your answer in ";B$;". Do not answer 0.0.  ";
1230 J5=0
1240 INPUT D1
1250 PRINT "What is the largest dimension, in ";B$;"? ";
1260 INPUT D2
1270 D4=D1
1280 D3=D2
1290 D=D1
1300 GOSUB 2440
1310 IF J4=1 THEN 1360
1320 GOSUB 4890
1330 IF Y=0 THEN 1360
1340 R1=S1/S2
1350 R2=S2/S3
1360 X1=(A1/(R2*S3))↑M+(A2*R1/S1)↑P-1
1370 FOR T=1 TO 100
1380 D=(D4+D3)/2
1390 GOSUB 2440
1400 IF J4=1 THEN 1450
1410 GOSUB 4890
1420 IF Y=0 THEN 1450
1430 R1=S1/S2
```

```
1440 R2=S2/S3
1450 X2=(A1/(R2*S3))↑M+(A2*R1/S1)↑P-1
1460 IF ABS(X2)<=1.0E-5 THEN 1540
1470 IF X1*X2<=0 THEN 1510
1480 D4=D
1490 X1=X2
1500 GO TO 1520
1510 D3=D
1520 NEXT T
1530 J5=1
1540 PAGE
1550 PRINT USING "72""*""":
1560 IMAGE CFD,1X,S
1570 IMAGE CFD.2D,1X,S
1580 PRINT "JTensile Strength, Su.............................= ";
1590 PRINT USING 1560:S1
1600 PRINT C$
1610 PRINT "Yield Strength, Sy...............................= ";
1620 PRINT USING 1560:S2
1630 PRINT C$
1631 IF N1<=0 THEN 1640
1632 IF N1=>1000000 THEN 1640
1633 PRINT "Significant Endurance Limit, Se'''@";N1;" cyc.= ";
1634 GO TO 1650
1640 PRINT "Significant Endurance Limit, Se'' for infinite life= ";
1650 PRINT USING 1560:S3
1660 PRINT C$
1670 PRINT "Smallest dimension tried.........................= ";
1680 PRINT USING 1570:D1
1690 PRINT B$
1700 PRINT "Largest dimension tried..........................= ";
1710 PRINT USING 1570:D2
1720 PRINT B$
1730 PRINT "Safety Factor, N.................................= ";
1740 PRINT USING 1570:N
```

```
1750 PRINT
1760 IF M1=0 THEN 1800
1770 PRINT "Moment causing alternating stress, M1.........= ";
1780 PRINT USING 1560:M1
1790 PRINT D$
1800 IF M2=0 THEN 1840
1810 PRINT "Moment causing a steady stress, M2............= ";
1820 PRINT USING 1560:M2
1830 PRINT D$
1840 IF F1=0 THEN 1880
1850 PRINT "Alternating axial force, F1...................= ";
1860 PRINT USING 1560:F1
1870 PRINT E$
1880 IF F2=0 THEN 1920
1890 PRINT "Steady axial force, F2........................= ";
1900 PRINT USING 1560:F2
1910 PRINT E$
1920 IF T1=0 THEN 1960
1930 PRINT "Alternating torque, T1........................= ";
1940 PRINT USING 1560:T1
1950 PRINT D$
1960 IF T2=0 THEN 2000
1970 PRINT "Steady torque, T2.............................= ";
1980 PRINT USING 1560:T2
1990 PRINT D$
2000 IF J4=1 THEN 2210
2010 PRINT "JThe following factors are used for computing Se''':"
2020 IMAGE CFD.2D
2030 PRINT "Surface factor (Ka)                    = ";
2040 PRINT USING 2020:K1
2050 PRINT "Size and shape factor (Kb)             = ";
2060 PRINT USING 2020:K2
2070 PRINT "Reliability factor (Kc)                = ";
2080 PRINT USING 2020:K3
2090 PRINT "Temperature factor (Kd)                = ";
```

```
2100 PRINT USING 2020:K4
2110 PRINT "Fatigue Strength Reduction factor (Ke)= ";
2120 PRINT USING 2020:K5
2130 PRINT "Miscellaneous factor (Kf)                = ";
2140 PRINT USING 2020:K6
2150 PRINT "Endurance Limit for Rotating-beam Specimen (Se')    = ";
2160 PRINT USING 1560:S5
2170 PRINT C$
2180 PRINT "Significant Endurance Limit for infinite life (Se'')= ";
2190 PRINT USING 1560:S6
2200 PRINT C$
2210 PRINT "JThe Failure Line selected is ";F$
2220 PRINT "The failure equation is (Sa/(R2*Se''')↑m + (R1*Sm/Su)↑p= 1."
2230 PRI "where M=";M;"_where P=";P;"_where R1=";R1;"_and where R2=";R2
2240 IF J5=1 THEN 2260
2250 GO TO 2290
2260 PRINT "JA solution does not exist in your specified range of "
2270 PRINT "dimensions. Sorry about that, CHIEF!.... Please try again."
2280 GO TO 1210
2290 PRINT "JThe design dimension is ";
2300 PRINT USING "cfd.4d,1x,s":D
2310 PRINT B$
2320 PRINT
2330 PRINT USING "72""*""":
2331 IF J=3 THEN 2340
2332 PRINT "JDo you wish to convert the design dimension to SI ";
2333 PRINT "unit? (Y or N)";
2334 D=D*0.0254
2335 H$="m"
2336 GO TO 2345
2340 PRINT "JDo you wish to convert the design dimension to English ";
2341 PRINT "unit? (Y or N)";
2342 D=D*39.37007874
2343 H$="inches"
2345 INPUT A$
```

```
2346 IF A$="N" THEN 2364
2347 IF A$<>"Y" THEN 2331
2350 PRINT "JThe design dimension is ";
2351 PRINT USING "cfd.4d,1x,s":D
2352 PRINT H$
2364 PRINT "JDo you wish to redesign the component using another "
2366 PRINT "failure line? Warning! A Modified Goodman approach requires"
2368 PRINT "both a fracture and a yield analysis. (Y or N)";
2370 INPUT A$
2380 IF A$="Y" THEN 640
2390 PRINT "JDo you want to design a new component? (Y or N)";
2400 INPUT A$
2410 IF A$="Y" THEN 150
2420 PRINT "JJJ          ---  END  ---"
2430 GO TO 2720
2440 IF I=0 THEN 2460
2450 GO TO 6000
2460 PAGE
2461 PRINT "You will now be requested to supply the PARAMETRIC DESIGN ";
2462 PRINT "STRESS_EQUATIONS for your design problem."
2463 PRI "You must write these equations in BASIC. Use the following ";
2464 PRINT "instructions._If you are unfamiliar with the development ";
2465 PRINT "of such equations,_see user's guide and its appendix ";
2466 PRINT "for guidance."
2470 PRI "Enter component loads starting with line 6000 incrementing ";
2480 PRINT "by 10's."
2490 PRINT " Use M1=Moment causingHHHHHHH_______ alternating stress ";
2495 PRINT "(";D$;")."
2500 PRINT " Use M2=Moment causingHHHHHHH_______ a steady stress ";
2505 PRINT "(";D$;")."
2510 PRINT " Use F1=An alternating axial force (";E$;")."
2520 PRINT " Use F2=A steady axial force (";E$;")."
2530 PRINT " Use T1=An alternating torque (";D$;")."
2540 PRINT " Use T2=A steady torque (";D$;")."
2550 PRINT " Use  N=Safety factor."
```

```
2560 PRINT "Enter the numeric value for each of the variables used in "
2570 PRINT "your stress equations._Do this before you enter your ";
2580 PRINT "stress equations."
2590 PRINT "JEnter your stress equqtions starting with line 6100 "
2600 PRINT "incrementing by 10's."
2610 PRI " Use A1=Alternating stress. _ Use A2=Mean stress. _ Use PI=";
2620 PRINT "Pi. _ Use D=Basic dimension. N.B., All dimensions should ";
2630 PRINT "be given in terms of D. In case of rectangular component, ";
2640 PRI "use proportions. _After your stress equations are entered, ";
2650 PRINT "type a numbered return_statement. Then type run 600."
2660 PRINT "EXAMPLE..................................................."
2670 PRINT "The following equations are in English Unit."
2680 PRINT "6000 T2=1200. _6010 M1=2400. _6020 N=1.8 _6100 A1=N*M1/";
2690 PRINT "(PI*D↑3/32) _6110 A2=0.866*N*T2/(PI*D↑3/32) _6120 RETURN "
2700 PRINT "RUN 600J"
2710 I=I+1
2720 END
2730 REM *** SUBROUTINE FOR FINDING SE'''.
2740 REM *** THIS SECTION CALCULATES Ka (K1).
2750 J4=2
2760 PRI "LThis section of the program will calculate the Significant "
2770 PRINT "Endurance Limit."
2780 PRINT "JEnter the # for the types of surface finish used."
2790 PRINT "#1 for polished finish."
2800 PRINT "#2 for ground finish."
2810 PRINT "#3 for machined or cold drawn."
2820 PRINT "#4 for hot rolled."
2830 PRINT "#5 for as forged."
2840 INPUT I1
2850 IF J=3 THEN 2880
2860 S8=S1
2870 GO TO 2890
2880 S8=S1*1.450377397E-4
2890 GO TO I1 OF 2930,2950,2970,2990,3010
2900 PRINT "JError! Your input should be from 1 to 5, but you have "
```

```
2910 PRINT "entered ";I1;". Please try again."
2920 GO TO 2780
2930 K1=1
2940 GO TO 3020
2950 K1=0.89
2960 GO TO 3020
2970 K1=-2.91E-17*S8*S8*S8+2.0E-11*S8*S8-4.95E-6*S8+1.064
2980 GO TO 3020
2990 K1=-5.77E-17*S8*S8*S8+3.41E-11*S8*S8-8.0E-6*S8+1.066
3000 GO TO 3020
3010 K1=-6.45E-17*S8*S8*S8+3.63E-11*S8*S8-7.87E-6*S8+0.89
3020 IF K1<=1 THEN 3030
3022 K1=1
3026 REM *** THIS SECTION COMPUTES Kc (K3).
3030 PRINT "JWhat is the reliability in %?";
3040 INPUT I2
3050 P1=100-I2
3060 IF P1>0 THEN 3100
3070 PRINT "Error! The reliability should be in %, and cannot be "
3080 PRINT "greater than 100. Please try again."
3090 GO TO 3030
3100 P2=LGT(P1)
3110 Z1=2.37-0.885*P2-0.193*P2*P2-0.0502*P2↑3-0.00489*P2↑4
3120 K3=1-0.08*Z1
3130 REM *** THIS SECTION COMPUTES Kd (K4).
3140 PRINT "JWhat is the operating temperature in ";G$;" ?";
3150 INPUT I3
3160 IF J<>3 THEN 3180
3170 I3=9*I3/5+32
3180 IF I3<=160 THEN 3210
3190 K4=620/(460+I3)
3200 GO TO 3220
3210 K4=1
3220 REM *** THIS SECTION COMPUTES Ke (K5).
3230 PRINT "JDo you know the Theoretical Stress Concentration ";
```

```
3240 PRINT "Factor (Kt)?";
3250 INPUT A$
3260 IF A$="Y" THEN 3390
3270 IF A$<>"N" THEN 3230
3280 PRINT "JThe following references will help you to find the "
3290 PRINT "Theoretical Stress Concentration Factor:"
3300 PRINT "JR.E. Peterson, Stress Concentration FactorHHHHHHHHHHHHHH";
3310 PRINT "HHHHHHHHHHHH____________________________, John Wiley & Sons,"
3315 PRINT "N. Y., 1974."
3320 PRINT "JJ.E. Shigley, Mechanical Engineering DesignHHHHHHHHHHH";
3325 PRINT "HHHHHHHHHHHHHHHHHH______________________________, 3rd Ed.,"
3330 PRINT "McGraw-Hill, 1977, p. 663-670."
3340 PRI "JA.D. Deutschman, W.J. Michels, C.E. Wilson, Machine Design";
3345 PRINT "HHHHHHHHHHHHHH______________,"
3350 PRINT "Macmillan Publishing Co., Inc. N.Y., 1975, p. 894-901."
3360 PRINT "JL. Sors, Fatigue Design of Machine ComponentsHHHHHHHH";
3365 PRINT "HHHHHHHHHHHHHHHHHHHHHHHHHHHHH___________________________";
3366 PRINT "__________,"
3370 PRINT "Pergamon Press, 1971, p. 42-84 of part II. "
3372 PRINT "JJDo you wish to leave.the program to look up the Stress "
3373 PRINT "Concentration Factors? (Y or N)";
3374 INPUT A$
3375 IF A$="Y" THEN 2420
3376 IF A$<>"N" THEN 3372
3390 PRINT "JTheoretical Stress Concentration Factor = ";
3400 INPUT I4
3410 PRINT "JDo you know Notch Sensitivity (q)? (Y or N)";
3420 INPUT A$
3430 IF A$="N" THEN 3490
3440 IF A$<>"Y" THEN 3410
3450 PRINT "Notch Sensitivity (q) = ";
3460 J2=1
3470 INPUT Q
3480 GO TO 4230
3490 PRINT "What is the notch radius in ";B$;"?";
```

```
3500 INPUT R
3510 PRINT "JIs the material steel? (Y or N)";
3520 INPUT A$
3530 IF A$="N" THEN 4090
3540 IF A$<>"Y" THEN 3490
3550 J2=1
3560 IF J<>3 THEN 3580
3570 R=R*39.37007874
3580 PRINT "Is it under bending or axial loading? (Y or N)";
3590 INPUT A$
3600 IF A$="N" THEN 4020
3610 IF A$<>"Y" THEN 3580
3620 IF J=3 THEN 3650
3630 S4=S1
3640 GO TO 3660
3650 S4=S1*1.450377397E-4
3660 IF S4>50000 THEN 3720
3670 IF R<=0.16 THEN 3700
3680 Q=0.8
3690 GO TO 4230
3700 Q=-8828*R↑4+3345.3*R↑3-440.94*R*R+24.62*R+0.18
3710 GO TO 4230
3720 IF S4>60000 THEN 3780
3730 IF R<=0.16 THEN 3760
3740 Q=0.8
3750 GO TO 4230
3760 Q=-7031.25*R↑4+2671.9*R↑3-353.13*R↑2+20.2*R+0.28
3770 GO TO 4230
3780 IF S4>80000 THEN 3840
3790 IF R>0.16 THEN 3820
3800 Q=-10156.25*R↑4+3825*R↑3-497.5*R*R+27.05*R+0.23
3810 GO TO 4230
3820 Q=0.9
3830 GO TO 4230
3840 IF S4>100000 THEN 3900
```

```
3850 IF R>0.15 THEN 3880
3860 Q=-15057.38*R↑4+5165.4*R↑3-606*R*R+29.23*R+0.3
3870 GO TO 4230
3880 Q=0.92
3890 GO TO 4230
3900 IF S4>140000 THEN 3970
3910 IF R>0.08 THEN 3940
3920 Q=5431250*R↑5-1236125*R↑4+104242.5*R↑3-4010.7*R*R+71.06*R+0.33
3940 GO TO 4230
3950 Q=0.92
3960 GO TO 4230
3970 IF R>0.06 THEN 4000
3980 Q=-271319*R↑4+37276.5*R↑3-1771*R*R+35.03*R+0.67
3990 GO TO 4230
4000 Q=0.965
4010 GO TO 4230
4020 IF S4>60000 THEN 4040
4030 GO TO 3790
4040 IF S4>80000 THEN 4060
4050 GO TO 3850
4060 IF S4>120000 THEN 4080
4070 GO TO 3920
4080 GO TO 3970
4090 PRINT "JIs the material Aluminum alloy? (Y or N)";
4100 INPUT A$
4110 IF A$="N" THEN 4190
4120 IF A$<>"Y" THEN 4090
4130 J2=2
4140 IF R>0.16 THEN 4170
4150 Q=-8815.2*R↑4+3411.3*R↑3-462.64*R*R+27.85*R+0.013
4160 GO TO 4230
4170 Q=0.85
4180 GO TO 4230
4190 PRINT "JSorry! You have to suppy the Notch Sensitivity Factor."
4200 PRINT "Notch Sensitivity (q) = ";
```

```
4210 J2=0
4220 INPUT Q
4230 K5=1/(1+Q*(I4-1))
4240 PRINT "JIs there any miscellaneous-effect factor? (Y or N)";
4250 INPUT A$
4260 IF A$="Y" THEN 4300
4270 IF A$<>"N" THEN 4240
4280 K6=1
4290 GO TO 4320
4300 PRINT "Miscellaneous-effect Factor = ";
4310 INPUT K6
4320 PRINT "JThe S-N curve is used for this determination. "
4321 PRINT "A Log-Log or a Log-Linear S-N curve will be used. "
4322 PRINT "The finite life region is 0.9*Su @1E3 cycles to Se'' ";
4323 PRINT "@1E6 cycles."
4329 PRINT "What method do you want to use for computing the "
4330 PRINT "Significant Endurance Limit (Se''')?"
4340 PRINT "Enter #1 for Log-Log method."
4350 PRINT "Enter #2 for Log-Linear method."
4360 INPUT J1
4370 GO TO J1 OF 4420,4420
4380 PRINT "JError! Your input should be 1 or 2, but you have entered ";
4390 PRINT J1
4400 PRINT "Please try again. "
4410 GO TO 4320
4420 REM *** INFORMATION FOR COMPUTING Kb (SIZE FACTOR).
4430 IF J2=1 THEN 4560
4440 PRINT "JIs the material light alloy? (Y or N)";
4450 INPUT A$
4460 IF A$="Y" THEN 4550
4470 IF A$<>"N" THEN 4440
4490 PRINT "JSorry, I can only compute those for steel and light alloy."
4550 J2=2
4560 J3=1
4570 PRINT "JIs the cross section circular? (Y or N)";
```

```
4580 INPUT A$
4590 IF A$="Y" THEN 4620
4600 IF A$<>"N" THEN 4570
4610 J3=2
4620 PRINT "JDo you know the Endurance Limit (Se') for rotating-beam "
4630 PRINT "specimen? (Y or N)";
4640 INPUT A$
4650 IF A$="N" THEN 4700
4660 IF A$<>"Y" THEN 4620
4670 PRINT "Se' in ";C$;" = ";
4680 INPUT S5
4690 GO TO 4780
4700 IF J=3 THEN 4730
4710 S5=0.5*S1
4720 GO TO 4740
4730 S5=0.5*1.45038E-4*S1
4740 IF S5<100000 THEN 4760
4750 S5=100000
4760 IF J<>3 THEN 4780
4770 S5=S5*6894.757
4780 PRINT "JIs the design life infinite? (Y or N)";
4790 INPUT A$
4800 IF A$="N" THEN 4840
4810 IF A$<>"Y" THEN 4780
4820 N1=0
4830 GO TO 4880
4840 PRINT "Number of cycles = ";
4850 INPUT N1
4860 IF N1<1000000 THEN 4880
4870 N1=0
4880 RETURN
4890 REM *** SUBROUTINE TO COMPUTE Se'' WHICH DEPENDS ON D.
4900 REM *** THIS PART COMPUTES Kb (K2).
4910 IF J=3 THEN 4940
4920 N2=D*25.4
```

```
4930 GO TO 4950
4940 N2=D*1000
4950 GO TO J2 OF 4960,4970
4960 GO TO J3 OF 4980,5040
4970 GO TO J3 OF 5100,5060
4980 K2=N2/(-18.75+1.802*N2)
4990 IF N2>23 THEN 5010
5000 K2=1
5010 IF N2<130 THEN 5220
5020 K2=0.59
5030 GO TO 5220
5040 K2=0.5061+7.214/N2
5050 IF N2>19 THEN 5070
5060 K2=0.88
5070 IF N2<150 THEN 5220
5080 K2=0.55
5090 GO TO 5220
5100 K2=0.515+3.24/N2
5110 IF N2>7 THEN 5130
5120 K2=1
5130 IF N2<41 THEN 5220
5140 K2=0.59
5150 GO TO 5220
5160 K2=0.5061+2.25/N2
5170 IF N2>7 THEN 5220
5180 K2=0.88
5190 IF N2<47 THEN 5220
5200 K2=0.55
5210 REM *** THIS SECTION COMPUTES Se'''.
5220 S6=ABS(K1*K2*K3*K4*K5*K6*S5)
5230 S7=0.9*S1*K4
5240 IF N1=0 THEN 5300
5250 GO TO J1 OF 5260,5280
5260 S3=10↑((LGT(N1)/3-1)*(LGT(S6)-LGT(S7))+LGT(S7))
5270 GO TO 5310
```

```
5280 S3=S7+(LGT(N1)/3-1)*(S6-S7)
5290 GO TO 5310
5300 S3=S6
5310 RETURN
6000 T2=1200
6010 M1=2400
6020 N=1.8
6100 A1=N*M1/(PI*D*D*D/32)
6110 A2=0.866*N*T2/(PI*D*D*D/32)
6120 RETURN
6500 END
```

APPENDIX B.2

PROGRAM LISTING --- SECTION PROPERTIES

```
100 INIT
110 PAGE
120 PRI "**************************************************************"
130 PRI "*                                                            *"
140 PRI "*          SECTION PROPERTIES OF POLYONAL SECTION            *"
150 PRI "*                                                            *"
160 PRI "*          By Yiu Wah Luk, VPI & SU, Spring 1978             *"
170 PRI "*                                                            *"
180 PRI "**************************************************************"
190 SET DEGREES
200 DIM X(30),Y(30),X1(11,30),Y1(11,30),D1(21,3),T(5),I(9),A$(1),B$(1)
210 DIM J1(11)
220 T=1
230 T5=0
240 I=0
250 PRINT "JJDo you want to use SI units? (Y or N)";
260 INPUT A$
270 IF A$="Y" THEN 350
280 IF A$<>"N" THEN 250
290 PRINT "JWe will use English Units throughout this program."
300 T5=1
310 C$="inches"
320 D$="in↑2"
330 F$="in↑4"
340 GO TO 390
350 C$="mm"
360 D$="mm↑2"
370 F$="mm↑4"
380 PRINT "JWe will use SI units throughout this program."
390 REM *** ENTER DATA FOR BOUNDARY
400 PRINT "JIs the outer perimeter a circular section? (Y or N)";
410 INPUT A$
420 IF A$="N" THEN 480
430 IF A$<>"Y" THEN 390
```

```
440 T(1)=2
450 PRINT "JRadius of the circle (";C$;")  = ";
460 INPUT D1(1,1)
470 GO TO 590
480 PRI "JJPlease enter the X and Y coordinates of the vertices of the "
490 PRINT "polygon (which must be located entirely within the first "
500 PRINT "quadrant) sequentially for a complete, clockwise path around"
510 PRINT "the polygon. Units should be ";C$;"."
520 PRINT "Be sure to end with the first point."
530 GOSUB 4380
540 J1(1)=J
550 FOR M=1 TO J1(1)
560 X1(1,M)=X(M)
570 Y1(1,M)=Y(M)
580 NEXT M
590 PRINT "JAre there any holes in the section? (Y or N)";
600 INPUT A$
610 IF A$="N" THEN 1040
620 IF A$<>"Y" THEN 590
630 REM *** ENTER DATA FOR CIRCULAR HOLES
640 PRINT "Are there any circular holes? (Y or N)";
650 INPUT A$
660 IF A$="N" THEN 840
670 IF A$<>"Y" THEN 640
680 PRINT "How many circular holes are there inside the section?";
690 INPUT D2
700 T(2)=2
710 PRINT "JFor circular holes give:"
720 FOR T1=1 TO D2
730 PRINT "JRadius of the circular hole #";T1;" (";C$;") = ";
740 INPUT D1(T1+1,1)
750 PRINT "X and Y coordinates of the center of the hole #";T1;" (";C$;
760 PRINT ") = ";
770 INPUT D1(T1+1,2),D1(T1+1,3)
780 NEXT T1
```

```
790 PRINT "JAre there any polygon holes? (Y or N)";
800 INPUT A$
810 IF A$="N" THEN 1040
820 IF A$<>"Y" THEN 790
830 REM *** ENTER DATA FOR POLYGONAL HOLES
840 PRINT "JHow many polygon holes are there in the section?";
850 INPUT D3
860 IF T(2)=2 THEN 890
870 T(2)=3
880 GO TO 900
890 T(2)=4
900 PRINT "JFor polygon holes:"
910 PRINT "Enter the X and Y coordinates of each vertices in a ";
920 PRINT "complete, clockwise path. Units should be ";C$;"."
930 PRINT "Be sure to end with the first point of each hole.J"
940 FOR M=2 TO D3+1
950 PRINT "For polygonal hole #";M-1;":"
960 GOSUB 4380
970 J1(M)=J
980 FOR M1=1 TO J1(M)
990 X1(M,M1)=X(M1)
1000 Y1(M,M1)=Y(M1)
1010 NEXT M1
1020 PRINT
1030 NEXT M
1040 T(4)=1
1050 T(5)=1
1060 REM *** SET UP FOR CIR. OR POLY. BOUNDARY AND DRAW OUTER BOUNDARY
1070 GO TO T(1) OF 1100,1080
1080 GOSUB 4580
1090 GO TO 1110
1100 GOSUB 5590
1110 GO TO T(2) OF 1630,1130,1370,1130
1120 REM *** DRAW CIRCULAR HOLES
1130 WINDOW R(1),R(2),R(5),R(6)
```

```
1140 VIEWPORT V1,V2,V3,V4
1150 FOR T1=2 TO D2+1
1160 C=D1(T1,1)*PI/18
1170 C1=D1(T1,2)-D1(T1,1)
1180 C2=D1(T1,3)
1190 MOVE C1,C2
1200 FOR T2=360 TO 0 STEP -10
1210 ROTATE T2
1220 RDRAW 0,C
1230 NEXT T2
1240 MOVE D1(T1,2)+(R(2)-R(1))/50,D1(T1,3)+(R(2)-R(1))/50
1250 T(4)=T(4)+1
1260 B$=CHR(63+T(4))
1270 PRINT B$
1280 MOVE D1(T1,2),D1(T1,3)
1290 SCALE 1,1
1300 RMOVE -2,0
1310 RDRAW 4,0
1320 RMOVE -2,2
1330 RDRAW 0,-4
1340 WINDOW R(1),R(2),R(5),R(6)
1350 NEXT T1
1360 GO TO T(2) OF 1630,1630,1370,1370
1370 WINDOW R(1),R(2),R(5),R(6)
1380 VIEWPORT V1,V2,V3,V4
1390 REM *** DRAW POLYGONAL HOLES
1400 FOR T2=2 TO D3+1
1410 T(4)=T(4)+1
1420 MOVE X1(T2,1)+(R(2)-R(1))/50,Y1(T2,1)+(R(6)-R(5))/50
1430 B$=CHR(63+T(4))
1440 PRINT B$
1450 MOVE X1(T2,1),Y1(T2,1)
1460 FOR T3=2 TO J1(T2)
1470 DRAW X1(T2,T3),Y1(T2,T3)
1480 NEXT T3
```

```
1490 REM *** DRAW DIAMOND ON EACH POINT OF POLYGONAL HOLE
1500 MOVE X1(T2,1),Y1(T2,1)
1510 FOR T3=1 TO J1(T2)-1
1520 MOVE X1(T2,T3),Y1(T2,T3)
1530 SCALE 1,1
1540 RMOVE 1,0
1550 RDRAW -1,-1
1560 RDRAW -1,1
1570 RDRAW 1,1
1580 RDRAW 1,-1
1590 WINDOW R(1),R(2),R(5),R(6)
1600 NEXT T3
1610 NEXT T2
1620 REM *** PRINT DATA ON THE GRAPH
1630 WINDOW 0,130,0,100
1640 VIEWPORT 0,40,0,100
1650 MOVE 0,100
1660 PRINT "All data are in ";C$;".J"
1670 GO TO T(1) OF 1710,1680
1680 PRINT "Cir. Section Data:"
1690 PRINT "Radius = ";D1(1,1)
1700 GO TO 1750
1710 PRINT "Poly. Data (X,Y):"
1720 FOR T2=1 TO J1(1)-1
1730 PRINT "#";T2;"= ";X1(1,T2);", ";Y1(1,T2)
1740 NEXT T2
1750 GO TO T(2) OF 1950,1760,1850,1760
1760 PRINT
1770 FOR T4=2 TO D2+1
1780 T(5)=T(5)+1
1790 B$=CHR(63+T(5))
1800 PRINT "Cir. Hole '";B$;"' Data:"
1810 PRINT "Radius = ";D1(T4,1)
1820 PRINT "X, Y   = ";D1(T4,2);", ";D1(T4,3)
1830 NEXT T4
```

```
1840 GO TO T(2) OF 1950,1950,1850,1850
1850 PRINT
1860 FOR T2=2 TO D3+1
1870 T(5)=T(5)+1
1880 B$=CHR(63+T(5))
1890 PRINT "Poly.Hole'";B$;"'Data (X,Y):"
1900 FOR T3=1 TO J1(T2)-1
1910 PRINT "#";T3;"= ";X1(T2,T3);", ";Y1(T2,T3)
1920 NEXT T3
1930 NEXT T2
1940 REM *** CHANGE DATA
1950 WINDOW 0,130,0,100
1960 VIEWPORT 0,130,0,100
1970 MOVE 127,13
1980 PRINT "X"
1990 MOVE 0,4
2000 PRINT "Do you want to change any data? (Y or N)";
2010 INPUT A$
2020 IF A$="N" THEN 2520
2030 IF A$<>"Y" THEN 2000
2040 T(3)=1
2050 PRINT "LPlease enter 0 when you have finished changing data."
2060 PRINT "JDo you want to change the data for the outer boundary? "
2070 PRINT "(Y or N)";
2080 INPUT A$
2090 IF A$="N" THEN 2200
2100 IF A$<>"Y" THEN 2060
2110 GO TO T(1) OF 2170,2120
2120 PRINT "JFor Circular Boundary:"
2130 PRINT "Present Radius (";C$;")     = ";D1(1,1)
2140 PRINT "Enter new Radius (";C$;")   = ";
2150 INPUT D1(1,1)
2160 GO TO 2200
2170 PRINT "JFor Polygonal Boundary:"
2180 T1=1
```

```
2190 GOSUB 6000
2200 IF T(2)=1 THEN 2510
2210 PRINT "JDo you want to change the data for the holes? (Y or N)";
2220 INPUT A$
2230 IF A$="N" THEN 2510
2240 IF A$<>"Y" THEN 2210
2250 GO TO T(2) OF 2510,2260,2440,2260
2260 FOR T2=2 TO D2+1
2270 T(3)=T(3)+1
2280 B$=CHR(63+T(3))
2290 PRINT "JFor Circular Hole '";B$;"' :"
2300 PRINT "Present Radius (";C$;")      = ";D1(T2,1)
2310 PRINT "Enter new Radius (";C$;")    = ";
2320 INPUT H1
2330 IF H1=0 THEN 2350
2340 D1(T2,1)=H1
2350 PRINT "Present coordinate of centre (X,Y), in ";C$;"  = ";
2360 PRINT D1(T2,2);", ";D1(T2,3)
2370 PRINT "Enter new coordinate of centre (X,Y), in ";C$;"= ";
2380 INPUT H2,H3
2390 IF H2=0 OR H3=0 THEN 2420
2400 D1(T2,2)=H2
2410 D1(T2,3)=H3
2420 NEXT T2
2430 GO TO T(2) OF 2510,2510,2440,2440
2440 FOR T2=2 TO D3+1
2450 T(3)=T(3)+1
2460 B$=CHR(63+T(3))
2470 PRINT "JFor Polygonal Hole '";B$;"' :"
2480 T1=T2
2490 GOSUB 6000
2500 NEXT T2
2510 GO TO 1040
2520 REM *** CALCULATE SECTION PROPERTIES
2530 GO TO T(1) OF 2540,2960
```

```
2540 K1=J1(1)
2550 T1=1
2560 GOSUB 6180
2570 A1=A2
2580 X4=X5
2590 Y4=Y5
2600 I(1)=I1
2610 I(2)=I2
2620 I(3)=I3
2630 GO TO T(2) OF 2750,2750,2640,2640
2640 FOR T2=2 TO D3+1
2650 K1=J1(T2)
2660 T1=T2
2670 GOSUB 6180
2680 A1=A1-A2
2690 X4=X4-X5
2700 Y4=Y4-Y5
2710 I(1)=I(1)-I1
2720 I(2)=I(2)-I2
2730 I(3)=I(3)-I3
2740 NEXT T2
2750 X4=X4/A1
2760 Y4=Y4/A1
2770 GO TO T(2) OF 3270,2780,3270,2780
2780 J4=X4*A1
2790 J5=Y4*A1
2800 FOR T2=2 TO D2+1
2810 A3=PI*D1(T2,1)↑2
2820 J3=PI*D1(T2,1)↑4/4
2830 J6=J3+A3*D1(T2,3)↑2
2840 J7=J3+A3*D1(T2,2)↑2
2850 J8=A3*D1(T2,2)*D1(T2,3)
2860 J4=J4-D1(T2,2)*A3
2870 J5=J5-D1(T2,3)*A3
2880 A1=A1-A3
```

```
2890 I(1)=I(1)-J6
2900 I(2)=I(2)-J7
2910 I(3)=I(3)-J8
2920 NEXT T2
2930 X4=J4/A1
2940 Y4=J5/A1
2950 GO TO 3270
2960 A1=PI*D1(1,1)↑2
2970 X4=D1(1,1)
2980 Y4=D1(1,1)
2990 I(1)=5*PI*D1(1,1)↑4/4
3000 I(2)=I(1)
3010 I(3)=A1*X4*Y4
3020 GO TO T(2) OF 3270,2780,3030,3030
3030 A3=0
3040 J3=0
3050 J4=0
3060 J5=0
3070 J6=0
3080 J7=0
3090 FOR T2=2 TO D3+1
3100 K1=J1(T2)
3110 T1=T2
3120 GOSUB 6180
3130 J6=J6+X5
3140 J7=J7+Y5
3150 A3=A3+A2
3160 J3=J3+I1
3170 J4=J4+I2
3180 J5=J5+I3
3190 NEXT T2
3200 I(1)=I(1)-J3
3210 I(2)=I(2)-J4
3220 I(3)=I(3)-J5
3230 X4=(X4*A1-J6)/(A1-A3)
```

```
3240 Y4=(Y4*A1-J7)/(A1-A3)
3250 A1=A1-A3
3260 GO TO T(2) OF 3270,2780,3270,2780
3270 I(4)=I(1)-A1*Y4*Y4
3280 I(5)=I(2)-A1*X4*X4
3290 I(6)=I(3)-A1*X4*Y4
3300 R6=SQR(I(1)/A1)
3310 R7=SQR(I(2)/A1)
3320 R8=SQR(I(4)/A1)
3330 R9=SQR(I(5)/A1)
3340 IF I(4)-I(5)=0 THEN 3370
3350 G1=0.5*ATN(-2*I(6)/(I(4)-I(5)))
3360 GO TO 3380
3370 G1=0
3380 IMAGE CFD.2D,1X,S
3390 PRINT "L   *** SECTION PROPERTIES OF THE REQUIRED SECTION ***"
3400 PRINT "JArea.........................................= ";
3410 PRINT USING 3380:A1
3420 PRINT D$
3430 PRINT "JX coordinate of the centroid.................= ";
3440 PRINT USING 3380:X4
3450 PRINT C$
3460 PRINT "JY coordinate of the centroid.................= ";
3470 PRINT USING 3380:Y4
3480 PRINT C$
3490 PRINT "JArea moment of inertia about X-axis..........= ";
3500 PRINT USING 3380:I(1)
3510 PRINT F$
3520 PRINT "JArea moment of inertia about Y-axis..........= ";
3530 PRINT USING 3380:I(2)
3540 PRINT F$
3550 PRINT "JArea product of inertia......................= ";
3560 PRINT USING 3380:I(3)
3570 PRINT F$
3580 PRINT "JArea moment of inertia about X'-axis translated to "
```

```
3590 PRINT "the centroid......................................= ";
3600 PRINT USING 3380:I(4)
3610 PRINT F$
3620 PRINT "JArea moment of inertia about Y'-axis translated to "
3630 PRINT "the centroid......................................= ";
3640 PRINT USING 3380:I(5)
3650 PRINT F$
3660 PRINT "JArea product of inertia about translated axis......= ";
3670 PRINT USING 3380:I(6)
3680 PRINT F$
3690 PRINT "JAngle between translated axis and principal axis "
3700 PRINT "(in degree), positive is counter-clockwise.........= ";
3710 PRINT USING "FD.2D":G1
3720 K2=I(4)*COS(G1)+I(5)*SIN(G1)↑2-I(6)*SIN(2*G1)
3730 K3=I(5)*COS(G1)↑2+I(4)*SIN(G1)↑2+I(6)*SIN(2*G1)
3740 PRINT "JArea moment of inertia about the translated, rotated, "
3750 PRINT "principal X''-axis................................= ";
3760 PRINT USING 3380:K2
3770 PRINT F$
3780 PRINT "JArea moment of inertia about the translated, "
3790 PRINT "rotated, principal Y''-axis........................= ";
3800 PRINT USING 3380:K3
3810 PRINT F$
3820 PRINT "JJDo you want to get area moment of inertia about an "
3830 PRINT "arbitrary axis? (Y or N)";
3840 INPUT A$
3850 IF A$="N" THEN 4160
3860 IF A$<>"Y" THEN 3820
3870 PRINT "JPlease enter the angle between the original axis (X-Y) and"
3880 PRINT "arbitrary axis (X'''-Y'''), in degree, positive is "
3890 PRINT "counter-clockwise.....................................= ";
3900 INPUT G2
3910 PRINT "Enter X-axis coordinate of the arbitrary axis (";C$;")= ";
3920 INPUT X6
3930 PRINT "Enter Y-axis coordinate of the arbitrary axis (";C$;")= ";
```

```
3940 INPUT Y6
3950 K7=I(4)+A1*(Y4-Y6)↑2
3960 K8=I(5)+A1*(X4-X6)↑2
3970 K9=I(6)+A1*(X4-X6)*(Y4-Y6)
3980 I(7)=K7*COS(G2)↑2+K8*SIN(G2)↑2-K9*SIN(2*G2)
3990 I(8)=K8*COS(G2)↑2+K7*SIN(G2)↑2+K9*SIN(2*G2)
4000 K1=I(7)+I(8)
4010 I(9)=0.5*(K7-K8)*SIN(2*G2)+K9*COS(2*G2)
4020 PRINT "JArea moment of inertia about the arbitrary axis,X'''..= ";
4030 PRINT USING 3380:I(7)
4040 PRINT F$
4050 PRINT "JArea moment of inertia about the arbitrary axis,Y'''..= ";
4060 PRINT USING 3380:I(8)
4070 PRINT F$
4080 PRINT "JPolar area moment of inertia about the arbitrary "
4090 PRINT "axis (X'''-Y''')....................................= ";
4100 PRINT USING 3380:K1
4110 PRINT F$
4120 PRINT "JArea product of inertia about the arbitrary "
4130 PRINT "axis (X'''-Y''')....................................= ";
4140 PRINT USING 3380:I(9)
4150 PRINT F$
4160 PRINT "JRadius of gyration about X-axis.......................= ";
4170 PRINT USING 3380:R6
4180 PRINT C$
4190 PRINT "JRadius of gyration about Y-axis.......................= ";
4200 PRINT USING 3380:R7
4210 PRINT C$
4220 PRINT "JRadius of gyration about X'-axis translated to "
4230 PRINT "the centroid........................................= ";
4240 PRINT USING 3380:R8
4250 PRINT C$
4260 PRINT "JRadius of gyration about Y'-axis translated to "
4270 PRINT "the centroid........................................= ";
4280 PRINT USING 3380:R9
```

```
4290 PRINT C$
4300 PRINT "JJJDo you want to find the properties of another section? "
4310 PRINT "(Y or N)";
4320 INPUT A$
4330 IF A$="Y" THEN 100
4340 IF A$<>"N" THEN 4300
4350 PRINT "JJ---------------------   END   --------------------"
4360 END
4370 REM *** ENTER DATA FOR POLYGONAL SECTION
4380 PRINT "X(1), Y(1) = ";
4390 INPUT X(1),Y(1)
4400 IF X(1)=>0 AND Y(1)=>0 THEN 4450
4410 PRINT "JError! Your polygon should be located entirely within the "
4420 PRINT "first quadrant, that is, X and Y both have to be positive."
4430 PRINT "Please try again."
4440 GO TO 4380
4450 J=2
4460 PRINT "X(";J;"), Y(";J;") = ";
4470 INPUT X(J),Y(J)
4480 IF X(J)=>0 AND Y(J)=>0 THEN 4530
4490 PRINT "JError! Your polygon should be located entirely within the "
4500 PRINT "first quadrant, that is, X and Y both have to be positive."
4510 PRINT "Please try again."
4520 GO TO 4380
4530 IF X(J)=X(1) AND Y(J)=Y(1) THEN 4560
4540 J=J+1
4550 GO TO 4460
4560 RETURN
4570 REM *** SET UP WINDOW, VIEWPORT AND AXIS FOR CIRCULAR SECTION.
4580 W1=0
4590 W2=2*D1(1,1)
4600 W3=0
4610 W4=2*D1(1,1)
4620 V1=40
4630 V2=130
```

```
4640 V3=5
4650 V4=95
4660 GOSUB 4760
4670 W5=(R(2)-R(1))/2
4680 W6=(R(6)-R(5))/2
4690 REM *** DRAW CIRCULAR BOUNDARY
4700 WINDOW -W5,W5,-W5,W5
4710 VIEWPORT V1,V2,V3,V4
4720 E=D1(1,1)
4730 GOSUB 5540
4740 RETURN
4750 REM *** LABEL "NEAT" TIC
4760 PRINT "L                              Y          GRAPH OF INPUT SECTION"
4770 V1=V1+10.8
4780 V3=V3+8.4
4790 VIEWPORT V1,V2,V3,V4
4800 DIM R(8)
4810 R(1)=W1
4820 R(2)=W2
4830 R(3)=INT((V2-V1)/(8*1.8)) MAX 1
4840 R(5)=W3
4850 R(6)=W4
4860 R(7)=INT((V4-V3)/(3*2.8)) MAX 1
4870 REM *** CALCULATE NEW LIMITS AND INTERVALS
4880 R5=3
4890 GOSUB 5040
4900 R5=7
4910 GOSUB 5040
4920 WINDOW R(1),R(2),R(5),R(6)
4930 REM AXIS R(3),R(7),R(1)+R(3),R(5)+R(7)
4940 AXIS R(3),R(7)
4950 REM *** LABEL THEM
4960 R5=4
4970 E$="HHHJJ"
4980 GOSUB 5330
```

```
4990 R5=8
5000 E$="HHHHH"
5010 GOSUB 5330
5020 HOME
5030 RETURN
5040 REM *** R(R5) = MINIMUM NO. OF TICS
5050 R1=(R(R5-1)-R(R5-2))/R(R5)
5060 R2=10↑INT(LGT(R1))
5070 R1=R1/R2
5080 IF R1>2 THEN 5120
5090 IF R1=1 THEN 5160
5100 R2=2*R2
5110 GO TO 5160
5120 IF R1>5 THEN 5150
5130 R2=5*R2
5140 GO TO 5160
5150 R2=10*R2
5160 REM *** ADJUST DATA MIN
5170 R1=INT(R(R5-2)/R2)
5180 R3=R2*(R1+2)
5190 IF R3<R(R5-2) THEN 5220
5200 R3=R3-R2
5210 GO TO 5190
5220 R(R5-2)=R3
5230 REM *** ADJUST DATA MAX
5240 R1=INT(R(R5-1)/R2)
5250 R3=R2*(R1-2)
5260 IF R(R5-1)<R3 THEN 5290
5270 R3=R3+R2
5280 GO TO 5260
5290 R(R5-1)=R3
5300 REM *** R(R5) = ADJUST TIC INTERVAL
5310 R(R5)=R2
5320 RETURN
5330 REM LABEL AXIS
```

```
5340 R4=R(R5-1)
5350 R(4)=R(1)
5360 R(8)=R(5)
5370 R3=ABS(R(R5-3)+R4) MAX ABS(R(R5-2)-R4)
5380 R3=INT(LGT(R3)+1.0E-8)
5390 R2=10↑-R3
5400 R1=R(R5-2)-R4/2
5410 R(R5)=R(R5)+R4
5420 IF R(R5)>R1 THEN 5470
5430 MOVE R(4),R(8)
5440 PRINT E$;
5450 PRINT USING "-D.2D,S":R(R5)*R2
5460 GO TO 5410
5470 IF R3=0 THEN 5520
5480 R(R5)=R1
5490 MOVE R(4),R(8)
5500 PRINT E$;
5510 PRINT USING "3A,+FD,S":"  E";R3
5520 RETURN
5530 REM *** TO DRAW A CIRCLE
5540 MOVE E,0
5550 FOR T1=10 TO 360 STEP 10
5560 DRAW E*COS(T1),E*SIN(T1)
5570 NEXT T1
5580 RETURN
5590 REM *** SET UP FOR POLYGONAL SECTION
5600 GOSUB 5820
5610 M1=X8 MIN Y8
5620 M2=X9 MAX Y9
5630 W1=M1
5640 W2=M2
5650 W3=M1
5660 W4=M2
5670 V1=40
5680 V2=130
```

```
5690 V3=5
5700 V4=95
5710 GOSUB 4760
5720 MOVE X1(1,1),Y1(1,1)
5730 FOR T2=2 TO J1(1)
5740 DRAW X1(1,T2),Y1(1,T2)
5750 NEXT T2
5760 FOR T2=1 TO J1(1)-1
5770 MOVE X1(1,T2),Y1(1,T2)
5780 PRINT T2
5790 NEXT T2
5800 RETURN
5810 REM *** FIND THE MIN AND MAX OF X1(J1) AND Y1(J1)
5820 REM *** X8,Y8 ARE MIN; X9,Y9 ARE MAX
5830 X8=X1(1,1)
5840 Y8=Y1(1,1)
5850 FOR M=1 TO J1(1)-1
5860 IF X1(1,M)=>X8 THEN 5880
5870 X8=X1(1,M)
5880 IF Y1(1,M)=>Y8 THEN 5900
5890 Y8=Y1(1,M)
5900 NEXT M
5910 X9=X1(1,1)
5920 Y9=Y1(1,1)
5930 FOR M=1 TO J1(1)-1
5940 IF X1(1,M)<=X9 THEN 5960
5950 X9=X1(1,M)
5960 IF Y1(1,M)<=Y9 THEN 5980
5970 Y9=Y1(1,M)
5980 NEXT M
5990 RETURN
6000 REM *** SUBROUTINE FOR CHANGING DATA OF POLYGONAL SECTION
6010 PRINT "Enter point # = ";
6020 INPUT H1
6030 IF H1=0 THEN 6160
```

```
6040 IF H1<=J1(T1)-1 THEN 6090
6050 PRINT "JError! You have only ";J1(T1)-1;" data, your input ";H1;
6060 PRINT "is out of your data range."
6070 PRINT "Please try again."
6080 GO TO 6010
6090 PRINT "Present values (X,Y), in ";C$;"  =";X1(T1,H1);", ";Y1(T1,H1)
6100 PRINT "Enter new values (X,Y), in ";C$;"=";
6110 INPUT X1(T1,H1),Y1(T1,H1)
6120 IF H1<>1 THEN 6150
6130 X1(T1,J1(T1))=X1(T1,1)
6140 Y1(T1,J1(T1))=Y1(T1,1)
6150 GO TO 6010
6160 RETURN
6170 REM *** SUBROUTINE TO CALC. A,X,Y,Ix,Iy,Ixy FOR POLYGONAL SECTION
6180 A2=0
6190 X5=0
6200 Y5=0
6210 I1=0
6220 I2=0
6230 I3=0
6240 FOR K=1 TO K1-1
6250 A2=A2-(Y1(T1,K+1)-Y1(T1,K))*(X1(T1,K+1)+X1(T1,K))/2
6260 B1=(Y1(T1,K+1)-Y1(T1,K))/8
6270 C1=(X1(T1,K+1)+X1(T1,K))↑2+(X1(T1,K+1)-X1(T1,K))↑2/3
6280 X5=X5-B1*C1
6290 B2=(X1(T1,K+1)-X1(T1,K))/8
6300 C2=(Y1(T1,K+1)+Y1(T1,K))↑2+(Y1(T1,K+1)-Y1(T1,K))↑2/3
6310 Y5=Y5+B2*C2
6320 B3=(X1(T1,K+1)-X1(T1,K))*(Y1(T1,K+1)+Y1(T1,K))/24
6330 C3=(Y1(T1,K+1)+Y1(T1,K))↑2+(Y1(T1,K+1)-Y1(T1,K))↑2
6340 I1=I1+B3*C3
6350 B4=(Y1(T1,K+1)-Y1(T1,K))*(X1(T1,K+1)+X1(T1,K))/24
6360 C4=(X1(T1,K+1)+X1(T1,K))↑2+(X1(T1,K+1)-X1(T1,K))↑2
6370 I2=I2-B4*C4
6380 IF X1(T1,K+1)-X1(T1,K)=0 THEN 6460
```

```
6390 B5=(Y1(T1,K+1)-Y1(T1,K))↑2*(X1(T1,K+1)+X1(T1,K))
6400 B5=B5*(X1(T1,K+1)↑2+X1(T1,K)↑2)/8
6410 C5=(Y1(T1,K+1)-Y1(T1,K))*(X1(T1,K+1)*Y1(T1,K)-X1(T1,K)*Y1(T1,K+1))
6420 C5=C5*(X1(T1,K+1)↑2+X1(T1,K+1)*X1(T1,K)+X1(T1,K)↑2)/3
6430 C6=(X1(T1,K+1)*Y1(T1,K)-X1(T1,K)*Y1(T1,K+1))↑2
6440 C6=C6*(X1(T1,K+1)+X1(T1,K))/4
6450 I3=I3+(B5+C5+C6)/(X1(T1,K+1)-X1(T1,K))
6460 NEXT K
6470 RETURN
```

APPENDIX B.3.1

PROGRAM LISTING --- BEAM ANALYSIS (PART I)

```
100 INIT
110 PAGE
115 SET DEGREES
120 DIM T(5,5),T1(5,5),T2(5,5),T3(5),S(5),A$(1),F(30),L(30),E(30)
122 DIM I1(30),P(30),Q1(30),Q2(30),K1(30),K2(30),P1(30),M5(30),M(30)
124 DIM A(2,3),I2(30)
130 GOSUB 6000
131 T=T1
132 P1=0
133 M5=0
134 K1=0
135 K2=0
136 Q1=0
137 Q2=0
139 M=0
142 L=0
150 S=0
151 S(5)=1
152 J$="rad/sec"
200 PRI "**************************************************************"
201 PRI "*                                                            *"
210 PRI "*     THIS IS A BEAM ANALYSIS PROGRAM WHICH SOLVES STATIC OR *"
220 PRI "*     DYNAMIC (FORCED, UNDAMPED VIBRATION) RESPONSE.         *"
221 PRI "*                                                            *"
222 PRI "*     by Yiu Wah Luk, advised by Professor Larry D. Mitchell *"
223 PRI "*      Virginia Polytechnic Institute and State University   *"
224 PRI "*              Department of Mechanical Engineering          *"
225 PRI "*                       Spring 1978                          *"
226 PRI "*                                                            *"
227 PRI "**************************************************************"
230 PRINT "JJDo you want to use English units? (Y or N)";
240 INPUT A$
250 IF A$="Y" THEN 330
260 IF A$<>"N" THEN 210
```

```
270 PRINT "JJSI units will be used throughout this program."
280 U=1
285 G=9.806
290 B$="N/m"
300 C$="N"
310 D$="m"
311 E$="Pa"
312 F$="m↑4"
316 G$="N-m"
318 K$="N-m↑2"
319 L$="N-m/rad"
320 GO TO 380
330 U=2
335 G=386.088
340 B$="lb/in"
350 C$="lb"
360 D$="in"
362 E$="psi"
364 F$="in↑4"
366 G$="lb-in"
368 K$="lb-in↑2"
369 L$="lb-in/rad"
370 PRINT "JJEnglish units will be used throughout this program."
380 PRINT "JIs this a static analysis? (Y or N)";
390 INPUT A$
395 U1=1
396 W=0
400 IF A$="Y" THEN 460
410 IF A$<>"N" THEN 380
420 U1=2
430 PRINT "JSo this is a dynamic analysis."
440 PRINT "Enter frequency (";J$;") =";
442 INPUT W
460 PRINT "JPlease divide your beam into several field sections,"
470 PRINT "and enter the information required for each section."
```

```
475 PRINT "N.B. Always start from the left end."
476 PRINT "JJPress RETURN to proceed."
477 INPUT A$
478 IF A$<>" " THEN 480
480 N=1
490 PRINT "LFor section #";N;":"
500 PRINT "The following Field Matrices are available."
510 PRINT "0      the last section."
520 PRINT "1      massless beam."
530 PRINT "2      uniformly distributed load on massless beam."
540 PRINT "3      linearly varied distributed load on massless beam."
550 PRINT "JEnter # for the required Field Matrix= ";
560 INPUT F(N)
570 IF F(N)=0 THEN 2010
580 GO TO F(N) OF 620,620,620
590 PRINT "Error! There are only 3 Field Matrices, but you have entered"
600 PRINT F(N);". Please try again."
610 GO TO 550
620 PRINT "JEnter length of this section (";D$;")"
625 PRINT "(N.B. Do not enter zero.) =";
630 INPUT L(N)
640 PRINT "Enter Modulus of Elasticity (";E$;")=";
650 INPUT E(N)
660 PRINT "Enter Area Moment of Inertia (";F$;")=";
670 INPUT I1(N)
675 X=L(N)
677 E1=E(N)*I1(N)
680 GO TO F(N) OF 690,720,790
690 GOSUB 6110
710 GO TO 860
720 PRINT "Enter magnitude of the uniformly distributed load, (";B$;")"
722 PRINT "(N.B. Load is positive downward.)=";
730 INPUT Q1(N)
750 Q2(N)=Q1(N)
770 GOSUB 6200
```

```
780 GO TO 860
790 PRI "What is the magnitude of the linearly varied distributed load,"
800 PRINT "(N.B. Load is positive downward.) on the left (";B$;") =";
810 INPUT Q1(N)
820 PRINT "on the right (";B$;") =";
830 INPUT Q2(N)
840 GOSUB 6200
860 GOSUB 6600
870 GOSUB 6710
880 PRINT "LFor section #";N;":"
890 PRINT "Is there any concentrated load, moment, lumped mass (for ";
900 PRI "dynamic_analysis only), or elastic support in this section? ";
905 PRINT "(Y or N)";
910 INPUT A$
915 P(N)=0
917 H=0
918 I2(N)=0
920 IF A$="N" THEN 1960
930 IF A$<>"Y" THEN 890
935 P(N)=1
1020 PRINT "JEnter magnitude of the concentrated load, (";C$;")"
1022 PRINT "(N.B. Load is positive downward) =";
1030 INPUT P1(N)
1090 PRINT "JEnter magnitude of the Moment (";G$;") "
1095 PRINT "(N.B. Moment is positive in counter-clock";
1096 PRINT "wise direction._Please put in the right sign.)=";
1100 INPUT M5(N)
1150 PRINT "JEnter stiffness of the support (";B$;")=";
1160 INPUT K1(N)
1270 PRINT "JEnter support moment stiffness (";L$;")=";
1280 INPUT K2(N)
1290 IF U1=1 THEN 1480
1292 PRINT "Enter magnitude of concentrated weight (";C$;")=";
1293 INPUT M(N)
1295 IF ABS(K2(N))<1.0E-20 THEN 1480
```

```
1298 PRINT "Enter Weight Moment of Inertia, WR↑2 (";K$;")="
1299 INPUT I2(N)
1300 PRI "The following types of vibration of the support are available"
1310 PRINT "1  for bending vibration."
1320 PRI "2      rotating shaft (equal angular direction of whirl and ";
1330 PRINT "rotation."
1340 PRI "3      rotating shaft (opposite angular direction of whirl &";
1350 PRINT " rotation."
1360 PRINT "Enter the # for the required vibration =";
1370 INPUT D2
1380 GO TO D2 OF 1400,1420,1440
1390 PRI "Error! Your input should be 1, 2 or 3. But you have entered "
1395 PRINT D2;" . Please try again."
1398 GO TO 1300
1400 H=-1
1410 GO TO 1480
1420 H=1
1430 GO TO 1480
1440 H=-3
1480 GOSUB 6340
1900 GOSUB 6600
1910 GOSUB 6710
1960 N=N+1
2000 GO TO 490
2010 N1=N-1
2013 DIM I1(N1),I2(N1),P(N1),Q1(N1),Q2(N1),K1(N1),K2(N1),P1(N1),M5(N1)
2014 DIM M(N1),F(N1),L(N1),E(N1)
2015 DELETE 1,2014
2020 FIND 7
2025 APPEND 3815
2035 PRINT "L                        *** BOUNDARY CONDITIONS ***"
2036 VIEWPORT 0,130,0,100
2037 WINDOW 0,130,0,100
2038 MOVE 35,90
2039 DRAW 115,90
```

```
2040 MOVE 35,85
2050 DRAW 115,85
2060 MOVE 10,75
2070 DRAW 115,75
2080 MOVE 15,65
2090 DRAW 115,65
2100 MOVE 15,55
2110 DRAW 115,55
2120 MOVE 15,45
2130 DRAW 115,45
2140 MOVE 10,35
2150 DRAW 115,35
2160 MOVE 115,90
2165 DRAW 115,35
2170 MOVE 95,85
2180 DRAW 95,35
2190 MOVE 75,85
2200 DRAW 75,35
2210 MOVE 55,85
2220 DRAW 55,35
2230 MOVE 35,90
2240 DRAW 35,35
2250 MOVE 15,75
2260 DRAW 15,35
2270 MOVE 10,75
2280 DRAW 10,35
2285 DRAW 115,35
2290 MOVE 58,86
2300 PRINT "R I G H T     E N D"
2310 MOVE 40,78
2320 PRINT "PINNED      FIXED      FREE      GUIDED"
2330 MOVE 12,63
2340 PRINT "LJHEJHFJHTJJJHEJHNJHD"
2350 MOVE 17,68
2360 PRINT "  PINNED        1         2      3 (K.U.)       4"
```

```
2365 MOVE 17,59
2370 PRINT "  FIXED           5            6           7             8"
2375 MOVE 17,50
2380 PRINT "  FREE       9 (K.U.)      10      11 (K.U.)    12 (K.U.)"
2385 MOVE 17,40
2390 PRINT "  GUIDED        13            14      15 (K.U.)    16 (K.U.)"
2400 PRI "JJJJwhere K. U. = Kinematically Unstable, unless internal ";
2401 PRINT "supports exist. _Use these boundary conditions at your ";
2402 PRINT "own risk."
2405 PRINT "Do not answer K. U. to the question below."
2410 PRINT "JEnter # for the required Boundary Condition = ";
2420 INPUT D2
2430 IF D2=>9 THEN 2438
2435 GO TO D2 OF 2460,2490,2510,2530,2550,2570,2590,2610
2438 GO TO D2-8 OF 2630,2650,2670,2690,2710,2730,2750,2770
2440 PRINT "Error! The # for Boundary Condition is only from 1 to 16,"
2450 PRINT "but you have entered ";D2;". Please try again."
2455 GO TO 2410
2460 M1=1
2462 M2=3
2464 M3=2
2466 M4=4
2470 GOSUB 6490
2472 S(2)=A2
2474 S(4)=A3
2476 GO TO 3810
2490 M1=1
2492 M2=2
2494 M3=2
2496 M4=4
2498 GOSUB 6490
2500 S(2)=A2
2502 S(4)=A3
2504 GO TO 3810
2510 M1=3
```

```
   2512 M2=4
   2514 M3=2
3  2516 M4=4
   2518 GOSUB 6490
   2520 S(2)=A2
   2522 S(4)=A3
   2524 GO TO 3810
   2530 M1=2
   2532 M2=4
   2534 M3=2
   2536 M4=4
   2538 GOSUB 6490
4  2540 S(2)=A2
   2542 S(4)=A3
   2544 GO TO 3810
   2550 M1=1
   2552 M2=3
   2554 M3=3
   2556 M4=4
5  2558 GOSUB 6490
   2560 S(3)=A2
   2562 S(4)=A3
   2564 GO TO 3810
   2570 M1=1
   2572 M2=2
   2574 M3=3
6  2576 M4=4
   2578 GOSUB 6490
   2580 S(3)=A2
   2582 S(4)=A3
   2584 GO TO 3810
   2590 M1=3
   2592 M2=4
7  2594 M3=3
   2596 M4=4
```

```
   2598 GOSUB 6490
   2600 S(3)=A2
   2602 S(4)=A3
   2604 GO TO 3810
   2610 M1=2
   2612 M2=4
   2614 M3=3
   2616 M4=4
8  2618 GOSUB 6490
   2620 S(3)=A2
   2622 S(4)=A3
   2624 GO TO 3810
   2630 M1=1
   2632 M2=3
   2634 M3=1
   2636 M4=2
9  2638 GOSUB 6490
   2640 S(1)=A2
   2642 S(2)=A3
   2644 GO TO 3810
   2650 M1=1
   2652 M2=2
   2654 M3=1
   2656 M4=2
10 2658 GOSUB 6490
   2660 S(1)=A2
   2662 S(2)=A3
   2664 GO TO 3810
   2670 M1=3
   2672 M2=4
   2674 M3=1
11 2676 M4=2
   2678 GOSUB 6490
   2680 S(1)=A2
   2682 S(2)=A3
```

```
2684 GO TO 3810
2690 M1=2
2692 M2=4
2694 M3=1
2696 M4=2
12 2698 GOSUB 6490
2700 S(1)=A2
2702 S(2)=A3
2704 GO TO 3810
2710 M1=1
2712 M2=3
2714 M3=1
2716 M4=3
13 2718 GOSUB 6490
2720 S(1)=A2
2722 S(3)=A3
2724 GO TO 3810
2730 M1=1
2732 M2=2
2734 M3=1
2736 M4=3
14 2738 GOSUB 6490
2740 S(1)=A2
2742 S(3)=A3
2744 GO TO 3810
2750 M1=3
2752 M2=4
2754 M3=1
15 2756 M4=3
2758 GOSUB 6490
2760 S(1)=A2
2762 S(3)=A3
2764 GO TO 3810
2770 M1=2
2772 M2=4
```

16

```
2774 M3=1
2776 M4=3
2778 GOSUB 6490
2780 S(1)=A2
2782 S(3)=A3
3810 GOSUB 7930
3815 PRINT
3821 X=0
3822 X4=0
3823 PRI "JJHow many increments would you like to have for each field ";
3824 PRINT "section?";
3830 INPUT N3
3850 J2=0
3852 IMAGE 5(2E,3X)
3853 PRI "L LENGTH      DEFLECTION      SLOPE        MOMENT      SHEAR"
3855 PRINT "  (";D$;")            (";D$;")           (Radian)      (";G$;")";
3856 PRINT "     (";C$;")"
3857 PRINT
3860 IF P(N1)=0 THEN 3870
3862 N2=N3*N1+N1+1
3864 GO TO 3875
3870 N2=N3*N1+N1
3875 DELETE 2035,3822
3877 DELETE 7930,10680
3880 DELETE S1,S2,S3,S4,S5,X3
3882 DIM S1(N2),S2(N2),S3(N2),S4(N2),S5(N2),X3(N2),R(8)
3885 FOR J1=1 TO N1
3890 IF J1=1 THEN 3940
3900 N=J1-1
3905 IF P(N)=0 THEN 3940
3930 GOSUB 6340
3935 GOSUB 6800
3937 S=T3
3940 X1=L(J1)/N3
3942 N=J1
```

```
3944 E1=E(J1)*I1(J1)
3950 FOR J4=0 TO L(J1) STEP X1
3955 GO TO F(J1) OF 3960,3975,3975
3960 GOSUB 6110
3965 GOSUB 6800
3970 GO TO 3985
3975 GOSUB 6200
3980 GOSUB 6800
3985 J2=J2+1
3990 X3(J2)=X4
3995 S1(J2)=T3(1)
4000 S2(J2)=T3(2)
4005 S3(J2)=T3(3)
4010 S4(J2)=T3(4)
4012 PRINT USING 3852:X3(J2),S1(J2),S2(J2),S3(J2),S4(J2)
4013 X4=X4+X1
4014 X=X+X1
4015 NEXT J4
4016 X4=X4-X1
4017 X=0
4018 S=T3
4019 NEXT J1
4020 GOSUB 7800
4021 PRINT "JJJDo you wish to see the graphs for deflection, slope, "
4022 PRINT "moment and shear? (Y or N)";
4023 INPUT A$
4024 IF A$="Y" THEN 4028
4025 IF A$<>"N" THEN 4021
4026 GO TO 5999
4028 PAGE
4030 MOVE 60,0
4032 PRINT "LENGTH (";D$;")↑"
4033 S5=S4
4034 M6=9
4035 M7=29
```

```
4036 H$="SHEAR, V"
4037 I$=C$
4045 GOSUB 6880
4055 M6=32
4060 M7=52
4065 S5=S3
4066 H$="MOMENT, M"
4067 I$=G$
4070 GOSUB 6880
4080 H$="SLOPE, S"
4082 I$="rad"
4085 M6=55
4090 M7=75
4095 S5=S2
4100 GOSUB 6880
4110 H$="DEFL., W"
4112 I$=D$
4115 M6=78
4120 M7=98
4125 S5=S1
4130 GOSUB 6880
4132 IF U1=1 THEN 4140
4134 PRINT "Frequency=";W;" ";J$
4140 VIEWPORT 0,130,0,100
4150 WINDOW 0,130,0,100
4160 MOVE 0,0
4170 PRINT "JDo you want to analyze another beam? (Y or N)";
4180 INPUT A$
4190 IF A$="N" THEN 4240
4200 IF A$<>"Y" THEN 4170
4210 FIND 6
4220 OLD
4230 RUN
4240 PRINT "JJJJ                          ************   END   ************ "
5990 END
```

```
6000 REM *** SUBROUTINE TO PUT T1 AS AN IDENTITY MATRIX.
6010 FOR I=1 TO 5
6020 FOR J=1 TO 5
6030 IF I<>J THEN 6070
6040 T1(I,J)=1
6050 GO TO 6080
6070 T1(I,J)=0
6080 NEXT J
6090 NEXT I
6100 RETURN
6110 REM *** SUBROUTINE FOR MASSLESS BEAM TRANSFER MATRIX
6120 GOSUB 6000
6130 T1(1,2)=-X
6140 T1(1,3)=-X*X/(2*E1)
6150 T1(1,4)=-X↑3/(6*E1)
6160 T1(2,3)=X/E1
6170 T1(2,4)=X*X/(2*E1)
6180 T1(3,4)=X
6190 RETURN
6200 REM *** SUBROUTINE FOR UNIFORMLY OR LINEARLY VARIED DIST. LOAD.
6210 REM *** TRANSFER MATRIX.
6220 GOSUB 6000
6230 T1(1,2)=-X
6240 T1(1,3)=-X*X/(2*E1)
6250 T1(1,4)=-X↑3/(6*E1)
6260 T1(1,5)=Q1(N)*X↑4/(24*E1)-(Q1(N)-Q2(N))*X↑5/(120*L(N)*E1)
6270 T1(2,3)=X/E1
6280 T1(2,4)=X*X/(2*E1)
6290 T1(2,5)=-Q1(N)*X↑3/(6*E1)+(Q1(N)-Q2(N))*X↑4/(24*L(N)*E1)
6300 T1(3,4)=X
6310 T1(3,5)=-Q1(N)*X*X/2+(Q1(N)-Q2(N))*X↑3/(6*L(N))
6320 T1(4,5)=-Q1(N)*X+(Q1(N)-Q2(N))*X*X/(2*L(N))
6330 RETURN
6340 REM *** SUBROUTINE FOR CONC. LOAD, MOMENT, TORQUE AND ELASTIC
6350 REM *** SUPPORT, ELASTIC SUPPORT OR LUMPED MASS TRANSFER MATRIX.
```

```
6360 GOSUB 6000
6390 T1(3,2)=H*I2(N)*W↑2+K2(N)
6400 T1(3,5)=-M5(N)
6410 T1(4,1)=-M(N)*W↑2/G+K1(N)
6420 T1(4,5)=-P1(N)
6430 RETURN
6490 REM *** SUBROUTINE TO CALC. INITIAL PARAMETERS USING B. C.
6500 A(1,1)=T(M1,M3)
6510 A(1,2)=T(M1,M4)
6520 A(1,3)=-T(M1,5)
6530 A(2,1)=T(M2,M3)
6540 A(2,2)=T(M2,M4)
6550 A(2,3)=-T(M2,5)
6560 A1=A(1,1)*A(2,2)-A(2,1)*A(1,2)
6570 A2=(A(2,2)*A(1,3)-A(1,2)*A(2,3))/A1
6580 A3=(A(1,1)*A(2,3)-A(2,1)*A(1,3))/A1
6590 RETURN
6600 REM *** SUBROUTINE TO MULTIPLY 2 MATRICES, T & T1.(BOTH 5X5)
6610 FOR I=1 TO 5
6620 FOR J=1 TO 5
6630 T2(I,J)=0
6640 FOR K=1 TO 5
6650 T2(I,J)=T2(I,J)+T1(I,K)*T(K,J)
6660 NEXT K
6670 NEXT J
6680 NEXT I
6690 T=T2
6700 RETURN
6710 REM *** SUBROUTINE TO LIST OUT ACCUMULATED TRANSFER MATRIX.
6720 PRINT "JJAccumulatedHHHHHHHHHH___________ Transfer Matrix ";
6730 PRINT "including section #";N;" is as follows:J"
6740 IMAGE 5(2X,2E)
6750 FOR I=1 TO 5
6760 PRINT USING 6740:T(I,1),T(I,2),T(I,3),T(I,4),T(I,5)
6770 NEXT I
```

```
6780 PRINT "K1KHVKHMKHSKHWJJJJJJ"
6782 PRINT "Press RETURN to proceed.";
6784 INPUT A$
6786 IF A$<>" " THEN 6790
6790 RETURN
6800 REM *** SUBROUTINE TO MULTIPY A 5X5 AND A 1X5 MATRICES.
6810 FOR I=1 TO 5
6820 T3(I)=0
6830 FOR K=1 TO 5
6840 T3(I)=T3(I)+T1(I,K)*S(K)
6850 NEXT K
6860 NEXT I
```

DRAWCURVES

```
6870 RETURN
6880 REM *** DRAW THE CURVES OF W,S,M,V
6890 REM *** LABEL "NEAT" TIC
6900 B1=1.0E+300
6910 B2=-1.0E+300
6920 FOR I=1 TO N2
6930 B1=S5(I) MIN B1
6940 B2=S5(I) MAX B2
6950 NEXT I
6960 VIEWPORT 25,130,M6,M7
6970 R(1)=0
6980 R(2)=X3(N2)
6990 R(3)=INT(105/(8*1.8)) MAX 1
7000 R(5)=B1
7010 R(6)=B2
7020 R(7)=INT((M7-M6)/(3*2.8)) MAX 1
7030 REM *** CALCULATE NEW LIMITS AND INTERVALS
7040 R5=3
7050 GOSUB 7310
7060 R5=7
```

0 x   TOT LENGTH x

MIN Y   MAX

```
7070 GOSUB 7310
7080 WINDOW R(1),R(2),R(5),R(6)
7090 REM AXIS R(3),R(7),R(1)+R(3),R(5)+R(7)
```

```
7100 AXIS R(3),R(7)
7110 REM *** DRAW THE CURVE
7120 MOVE X3(1),S5(1)
7130 FOR J=1 TO N2
7140 DRAW X3(J),S5(J)
7150 NEXT J
7160 PRINT
7170 REM *** LABEL THEM
7180 IF M7>29 THEN 7220
7190 R5=4
7200 E$="HHHJJ"
7210 GOSUB 7600
7220 R5=8
7230 E$="HHHHH"
7240 GOSUB 7600
7250 VIEWPORT 0,130,M6,M7
7260 WINDOW 0,10,0,10
7270 MOVE 0,5
7280 PRINT H$;"_(";I$;")"
7290 HOME
7300 RETURN
7310 REM *** R(R5) = MINIMUM NO. OF TICS
7312 IF ABS(R(R5-1)-R(R5-2))=>1.0E-12 THEN 7320
7314 R1=R(R5-1)
7316 GO TO 7330
7320 R1=(R(R5-1)-R(R5-2))/R(R5)
7330 R2=10↑INT(LGT(R1))
7340 R1=R1/R2
7350 IF R1>2 THEN 7390
7360 IF R1=1 THEN 7430
7370 R2=2*R2
7380 GO TO 7430
7390 IF R1>5 THEN 7420
7400 R2=5*R2
7410 GO TO 7430
```

```
7420 R2=10*R2
7430 REM *** ADJUST DATA MIN
7440 R1=INT(R(R5-2)/R2)
7450 R3=R2*(R1+2)
7460 IF R3<R(R5-2) THEN 7490
7470 R3=R3-R2
7480 GO TO 7460
7490 R(R5-2)=R3
7500 REM *** ADJUST DATA MAX
7510 R1=INT(R(R5-1)/R2)
7520 R3=R2*(R1-2)
7530 IF R(R5-1)<R3 THEN 7560
7540 R3=R3+R2
7550 GO TO 7530
7560 R(R5-1)=R3
7570 REM *** R(R5) = ADJUST TIC INTERVAL
7580 R(R5)=R2
7590 RETURN
7600 REM LABEL AXIS
7610 R4=R(R5-1)
7620 R(4)=R(1)
7630 R(8)=R(5)
7640 R3=ABS(R(R5-3)+R4) MAX ABS(R(R5-2)-R4)
7642 IF R3>1.0E-7 THEN 7650
7644 R3=ABS(R(R5-3)) MAX ABS(R(R5-2))
7650 R3=INT(LGT(R3)+1.0E-8)
7660 R2=10↑-R3
7670 R1=R(R5-2)-R4/2
7672 IF R5=4 THEN 7680
7674 MOVE R(1),R(5)
7675 PRINT "HHHHHH";
7676 PRINT USING "-2D.2D,S":R(R5)*R2
7680 R(R5)=R(R5)+R4
7690 IF R(R5)>R1 THEN 7740
7700 MOVE R(4),R(8)
```

```
7710 PRINT E$;
7720 PRINT USING "-D.2D,S":R(R5)*R2
7730 GO TO 7680
7740 IF R3=0 THEN 7790
7750 R(R5)=R1
7760 MOVE R(4),R(8)
7770 PRINT E$;
7780 PRINT USING "3A,+FD,S":"  E";R3
7790 RETURN
7800 IF P(N1)=0 THEN 7920
7850 GOSUB 6340
7860 GOSUB 6800
7870 S1(N2)=T3(1)
7880 S2(N2)=T3(2)
7890 S3(N2)=T3(3)
7900 S4(N2)=T3(4)
7910 X3(N2)=X3(N2-1)
7915 PRINT USING 3852:X3(N2),S1(N2),S2(N2),S3(N2),S4(N2)
7920 RETURN
7930 REM *** SUBROUTINE TO DRAW THE BEAM.
7940 PAGE
7950 L2=0
7960 L3=0
7970 FOR J4=1 TO N1
7980 L2=L2+L(J4)
7990 NEXT J4
8000 VIEWPORT 3,127,75,95
8010 WINDOW 0,L2,0,20
8020 C1=3
8030 C2=80
8040 GOSUB 9460   :REM DRAW B.C.'S
8050 VIEWPORT 3,127,75,95
8060 WINDOW 0,L2,0,20
8070 L1=0
8080 FOR K=1 TO N1
```

TO 9400

```
8090 L3=L3+L(K)
8100 L4=(L3-L1)/50
8110 REM *** TO DRAW MASSLESS BEAM.
8120 MOVE L1,9
8130 DRAW L3,9
8140 MOVE L3,5
8150 DRAW L1,5
8160 MOVE L3,5
8170 GO TO F(K) OF 8710,8190,8340
8180 REM *** TO DRAW UNIFORMLY DISTRIBUTED LOAD.
8190 MOVE L1,9
8200 RDRAW L4,2
8210 RMOVE -L4,-2
8220 DRAW L1,18
8230 DRAW L3,18
8240 DRAW L3,9
8250 RDRAW -L4,2
8260 A2=9
8270 MOVE L1+L(K)/4,9
8280 GOSUB 10630
8290 MOVE L1+L(K)/2,9
8300 GOSUB 10630
8310 MOVE L1+3*L(K)/4,9
8320 GOSUB 10630
8330 GO TO 8710
8340 IF Q1(K)>Q2(K) THEN 8540
8350 REM *** TO DRAW LIN.-VARIED DISTRIBUTED LOAD WITH Q1<Q2.
8360 MOVE L1,9
8370 RDRAW L4,2
8380 RMOVE -L4,-2
8390 DRAW L1,14
8400 DRAW L3,18
8410 DRAW L3,9
8420 RDRAW -L4,2
8430 MOVE L1+L(K)/4,9
```

```
8440 A2=6
8450 GOSUB 10630
8460 MOVE L1+L(K)/2,9
8470 A2=7
8480 GOSUB 10630
8490 MOVE L1+3*L(K)/4,9
8500 A2=8
8510 GOSUB 10630
8520 GO TO 8710
8530 REM *** TO DRAW LIN.-VARIED DISTRIBUTED LOAD WITH Q1>Q2.
8540 MOVE L1,9
8550 RDRAW L4,2
8560 RMOVE -L4,-2
8570 DRAW L1,18
8580 DRAW L3,14
8590 DRAW L3,9
8600 RDRAW -L4,2
8610 MOVE L1+L(K)/4,9
8620 A2=8
8630 GOSUB 10630
8640 MOVE L1+L(K)/2,9
8650 A2=7
8660 GOSUB 10630
8670 MOVE L1+L(K)*3/4,9
8680 A2=6
8690 GOSUB 10630
8710 IF P1(K)=0 THEN 8820
8720 REM *** TO DRAW A POINT LOAD.
8730 MOVE L3,9
8740 A2=11
8750 VIEWPORT 3,130,75,95
8760 WINDOW 0,L2+L4,0,20
8770 GOSUB 10630
8780 VIEWPORT 3,127,75,95
8790 WINDOW 0,L2,0,20
```

```
8800 MOVE L3,18
8810 PRINT " P"
8820 IF M5(K)=0 THEN 9010
8830 REM *** TO DRAW A MOMENT.
8840 MOVE L3,9
8850 VIEWPORT 124*L3/L2,124*L3/L2+6,85,88
8860 WINDOW -1,1,0,1
8870 A1=1
8880 GOSUB 10580
8890 MOVE -1,0
8900 RDRAW 0.5,0.5
8910 RMOVE -0.5,-0.5
8920 MOVE 0,1.3
8930 PRINT "M"
8940 MOVE -1,0
8950 VIEWPORT 124*L3/L2-0.8,124*L3/L2+3,86.5,88
8960 WINDOW -2,1,0,1
8970 DRAW -1.5,1
8980 VIEWPORT 3,127,75,95
8990 WINDOW 0,L2,0,20
9000 REM *** DRAW ELASTIC AND TORQUE SUPPORT.
9010 IF K1(K)=0 AND K2(K)=0 THEN 9310
9020 MOVE L3,9
9030 VIEWPORT 124*L3/L2,124*L3/L2+6,73,80
9040 WINDOW 0,5,0,10
9050 MOVE 2.5,10
9060 DRAW 2.5,8
9070 RDRAW -1,-1
9080 RDRAW 1,-1
9090 RDRAW -1,-1
9100 RDRAW 1,-1
9110 RDRAW 0,-2
9120 RDRAW 1.5,0
9130 RDRAW -3,0
9140 RDRAW 1,0
```

```
9150 RDRAW -1,-1
9160 RMOVE 1,0
9170 RDRAW 1,1
9180 RMOVE 1,0
9190 RDRAW -1,-1
9200 MOVE 0,4
9210 PRINT "K"
9220 IF K2(K)=0 THEN 9310
9230 MOVE 2.5,9
9240 RDRAW 1,0
9250 RDRAW 0,-7
9260 MOVE 4,4
9270 PRINT "T"
9280 VIEWPORT 3,127,75,95
9290 WINDOW 0,L2,0,20
9300 MOVE L3,7
9310 IF M(K)=0 THEN 9370
9320 MOVE L3,7
9330 VIEWPORT 124*L3/L2,124*L3/L2+6,79,85
9340 WINDOW -1,1,-1,1
9350 A1=1
9360 GOSUB 10570
9370 VIEWPORT 3,127,75,95
9380 WINDOW 0,L2,0,20
9390 L1=L3
9400 NEXT K
9410 C1=127
9420 VIEWPORT 3,127,75,95
9430 WINDOW 0,L2,0,20
9440 GOSUB 9460
9450 RETURN
9460 REM *** SUBROUTINE FOR DRAWING BOUNDARY CONDITION.
9470 IF C1>100 THEN 9500
9480 IF D2<=4 THEN 9510
9482 IF D2<=8 THEN 9660
```

```
9484 IF D2<=12 THEN 10490
9486 IF D2<=16 THEN 9960
9490 GO TO 9510
9500 GO TO D2 OF 9510,9660,10490,9960,9510,9660,10490,9960
9502 GO TO D2-8 OF 9510,9660,10490,9960,9510,9660,10490,9960
9510 VIEWPORT C1-1.5,C1+1.5,C2-4,C2+4
9520 WINDOW 0,3,0,8
9530 MOVE 0,1
9540 DRAW 1.5,4
9550 DRAW 3,1
9560 DRAW 0,1
9570 MOVE 0.5,1
9580 DRAW 0,0.5
9590 MOVE 1.5,1
9600 DRAW 1,0.5
9610 MOVE 2.5,1
9620 DRAW 2,0.5
9630 MOVE 1.5,4
9640 DRAW 1.5,8
9650 GO TO 10550
9660 IF C1>100 THEN 9810
9670 VIEWPORT C1-1.5,C1,C2-3,C2+7
9680 WINDOW 0,1,0,10
9690 MOVE 1,10
9700 DRAW 1,0
9710 MOVE 1,9
9720 RDRAW -1,-1
9730 MOVE 1,7
9740 RDRAW -1,-1
9750 MOVE 1,5
9760 RDRAW -1,-1
9770 MOVE 1,3
9780 RDRAW -1,-1
9790 MOVE 1,3
9800 GO TO 10550
```

```
9810 VIEWPORT C1,C1+1.5,C2-3,C2+7
9820 WINDOW 0,1,0,10
9830 MOVE 0,10
9840 DRAW 0,0
9850 MOVE 0,9
9860 RDRAW 1,-1
9870 MOVE 0,7
9880 RDRAW 1,-1
9890 MOVE 0,5
9900 RDRAW 1,-1
9910 MOVE 0,3
9920 RDRAW 1,-1
9930 MOVE 0,3
9940 GO TO 10550
9950 REM *** TO DRAW GUIDED END
9960 IF C1>100 THEN 10230
9970 VIEWPORT C1-3,C1,C2-3,C2+7
9980 WINDOW 0,2,0,10
9990 MOVE 2,7
10000 DRAW 2,3
10010 MOVE 1,10
10020 DRAW 1,0
10030 MOVE 1,9
10040 RDRAW -1,-1
10050 MOVE 1,7
10060 RDRAW -1,-1
10070 MOVE 1,5
10080 RDRAW -1,-1
10090 MOVE 1,3
10100 RDRAW -1,-1
10110 A1=1
10120 MOVE 1.5,5.75
10130 VIEWPORT C1-1.5,C1,C2+2,C2+3.5
10140 WINDOW -1,1,-1,1
10150 GOSUB 10570
```

```
10160 VIEWPORT C1-3,C1,C2-3,C2+7
10170 WINDOW 0,2,0,10
10180 MOVE 1.5,4.25
10190 VIEWPORT C1-1.5,C1,C2+0.5,C2+2
10200 WINDOW -1,1,-1,1
10210 GOSUB 10570
10220 GO TO 10550
10230 VIEWPORT C1,C1+3,C2-3,C2+7
10240 WINDOW 0,2,0,10
10250 MOVE 0,7
10260 DRAW 0,3
10270 MOVE 1,10
10280 DRAW 1,0
10290 MOVE 1,9
10300 RDRAW 1,-1
10310 MOVE 1,7
10320 RDRAW 1,-1
10330 MOVE 1,5
10340 RDRAW 1,-1
10350 MOVE 1,3
10360 RDRAW 1,-1
10370 MOVE 0.5,5.75
10380 A1=1
10390 VIEWPORT C1,C1+1.5,C2+2,C2+3.5
10400 WINDOW -1,1,-1,1
10410 GOSUB 10570
10420 VIEWPORT C1,C1+3,C2-3,C2+7
10430 WINDOW 0,2,0,10
10440 MOVE 0.5,4.25
10450 VIEWPORT C1,C1+1.5,C2+0.5,C2+2
10460 WINDOW -1,1,-1,1
10470 GOSUB 10570
10480 GO TO 10550
10490 IF C1>100 THEN 10530
10500 MOVE 0,5
```

```
10510 DRAW 0,9
10520 GO TO 10550
10530 MOVE L2,5
10540 DRAW L2,9
10550 RETURN
10560 REM *** SUBROUTINE TO DRAW A CIRCLE
10570 MOVE A1,0
10580 FOR J4=0 TO 360 STEP 20
10590 DRAW A1*COS(J4),A1*SIN(J4)
10600 NEXT J4
10610 RETURN
10620 REM *** SUBROUTINE TO DRAW AN ARROW OF LENGTH A2
10630 RDRAW -L4,2
10640 RMOVE L4,-2
10650 RDRAW L4,2
10660 RMOVE -L4,-2
10670 RDRAW 0,A2
10680 RETURN
```

APPENDIX B.3.2

PROGRAM LISTING --- BEAM ANALYSIS (PART II)

```
3815 REM *** LIST DATA
3825 VIEWPORT 0,130,0,100
3835 WINDOW 0,130,0,100
3845 MOVE 0,97
3855 PRINT "Drawing of the Beam"
3865 MOVE 0,72
3875 PRINT "DATA:"
3885 IMAGE 2E,1X,FA
3900 IF U1=1 THEN 3905
3901 PRINT "Frequency..................................=";
3902 PRINT USING 3885:W,J$
3905 FOR K=1 TO N1
3915 PRINT "For section #";K;":"
3925 PRINT "Length of this section.....................=";
3935 PRINT USING 3885:L(K),D$
3945 PRINT "Modulus of Elasticity......................=";
3955 PRINT USING 3885:E(K),E$
3965 PRINT "Area Moment of Inertia.....................=";
3975 PRINT USING 3885:I1(K),F$
3985 GO TO F(K) OF 4155,3995,4035
3995 PRINT "Magnitude of uniformly distributed load....=";
4005 PRINT USING 3885:Q1(K),B$
4025 GO TO 4155
4035 PRINT "Left end of linearly-varied dist. load.....=";
4045 PRINT USING 3885:Q1(K),B$
4065 PRINT "Right end of linearly-varied dist. load....=";
4075 PRINT USING 3885:Q2(K),B$
4155 IF P1(K)=0 THEN 4185
4165 PRINT "Magnitude of Concentrated Load.............=";
4175 PRINT USING 3885:P1(K),C$
4185 IF M5(K)=0 THEN 4215
4195 PRINT "Magnitude of Moment........................=";
4205 PRINT USING 3885:M5(K),G$
4215 IF K1(K)=0 THEN 4245
```

```
4225 PRINT "Stiffness of Support.......................=";
4235 PRINT USING 3885:K1(K),B$
4245 IF K2(K)=0 THEN 4266
4255 PRINT "Support Moment Stiffness..................=";
4260 PRINT USING 3885:K2(K),L$
4266 IF U1=1 THEN 4275
4267 PRINT "Magnitude of Concentrated Weight...........=";
4268 PRINT USING 3885:M(K),C$
4270 IF ABS(K2(K))<1.0E-20 THEN 4275
4272 PRINT "Weight Moment of Inertia (WR↑2) about a "
4273 PRINT "diameter of disc or element...............=";
4274 PRINT USING 3885:I2(K),K$
4275 PRINT
4285 NEXT K
4295 REM *** CHANGE DATA
4305 PRINT "JDo you want to change the data? (Y or N)";
4315 INPUT A$
4325 IF A$="N" THEN 5100
4335 IF A$<>"Y" THEN 4305
4345 PAGE
4355 PRINT "JWhich section do you want to change?";
4365 INPUT N
4375 IF N=0 THEN 4378
4377 IF N<=N1 THEN 4385
4378 PRINT "Error! You only have ";N1;" sections, but you have entered "
4379 PRINT N;". Please try again."
4380 GO TO 4355
4385 IF U1=1 THEN 4392
4386 PRINT "Present frequency...............................=";
4387 PRINT USING 3885:W,J$
4388 PRINT "Enter new frequency=";
4389 INPUT W
4392 PRINT "For section #";N;":"
4395 PRINT "Present Length of this section .................=";
4405 PRINT USING 3885:L(N),D$
```

```
4415 PRINT "Enter new Length =";
4425 INPUT L(N)
4435 PRINT "Present Modulus of Elasticity....................=";
4445 PRINT USING 3885:E(N),E$
4455 PRINT "Enter new Modulus of Elasticity =";
4465 INPUT E(N)
4475 PRINT "Present Area Moment of Inertia...................=";
4485 PRINT USING 3885:I1(N),F$
4495 PRINT "Enter new Area Moment of Inertia =";
4505 INPUT I1(N)
4515 GO TO F(N) OF 4795,4525,4585
4525 PRINT "Present Uniformly Distributed Load...............=";
4535 PRINT USING 3885:Q1(N),B$
4545 PRINT "Enter new Uniformly Distributed Load =";
4555 INPUT Q1(N)
4565 Q2(N)=Q1(N)
4575 GO TO 4795
4585 PRINT "Present left side of lin-varied dist. load.......=";
4595 PRINT USING 3885:Q1(N),C$
4605 PRINT "Enter new left side of lin.-varied dist. load =";
4615 INPUT Q1(N)
4625 PRINT "Present right side of lin.-varied dist. load.....=";
4635 PRINT USING 3885:Q2(N),C$
4645 PRINT "Enter new right side of lin.-varied dist. load =";
4655 INPUT Q2(N)
4795 IF P1(N)=0 THEN 4845
4805 PRINT "Present Concentrated Load........................=";
4815 PRINT USING 3885:P1(N),C$
4825 PRINT "Enter new Concentrated Load =";
4835 INPUT P1(N)
4845 IF M5(N)=0 THEN 4895
4855 PRINT "Present Moment ..................................=";
4865 PRINT USING 3885:M5(N),G$
4875 PRINT "Enter new Moment =";
4885 INPUT M5(N)
```

```
4895 IF K1(N)=0 THEN 4945
4905 PRINT "Present Stiffness of Support......................=";
4915 PRINT USING 3885:K1(N),B$
4925 PRINT "Enter new Stiffness of Support =";
4935 INPUT K1(N)
4945 IF K2(N)=0 THEN 4975
4955 PRINT "Present Support Moment Stiffness................=";
4960 PRINT USING 3885:K2(N),L$
4966 PRINT "Enter new Support Moment Stiffness =";
4967 INPUT K2(N)
4975 IF U1=1 THEN 4990
4977 PRINT "Present magnitude of Concentrated Weight.........=";
4978 PRINT USING 3885:M(N),C$
4979 PRINT "Enter new magnitude of Concentrate Weight =";
4980 INPUT M(N)
4982 IF ABS(K2(N))<1.0E-20 THEN 4990
4984 PRINT "Present Weight Moment of Inertia (WR↑2)..........=";
4985 PRINT USING 3885:I2(N),K$
4986 PRINT "Enter new Weight Moment of Inertia (WR↑2) =";
4987 INPUT I2(N)
4990 PRI "JDo you want to change the data of another section?(Y or N)";
4992 INPUT A$
4995 IF A$="Y" THEN 4345
5005 IF A$<>"N" THEN 4990
5010 GOSUB 6090
5012 T=T1
5013 S=0
5014 S(5)=1
5015 FOR K4=1 TO N1
5025 E1=E(K4)*I1(K4)
5030 X=L(K4)
5032 H=K4
5035 GOSUB F(K4) OF 6180,6270,6270
5045 GOSUB 6600
5055 IF P(K4)=0 THEN 5085
```

```
5065 GOSUB 6410
5075 GOSUB 6600
5085 NEXT K4
5090 GO TO 2035
```

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DEVELOPMENT OF INTERACTIVE COMPUTER PROGRAMS FOR MECHANICAL ENGINEERING DESIGN: FATIGUE ANALYSIS, SECTION PROPERTIES, AND BEAM ANALYSIS

by

Yiu Wah Luk

(ABSTRACT)

This thesis presents the theory and describes three interactive computer graphics programs for mechanical engineering design: Fatigue Analysis, Section Properties, and Beam Analysis. The Fatigue Analysis program sizes up a mechanical component, circular, rectangular, or any shape, to prevent fatigue failure. Six most generally accepted fatigue failure lines are available and any equivalent stress theories are allowed. It can also calculate the significant endurance limit with the theoretical stress concentration factor supplied by the user.

The Section Properties program finds twenty section properties, such as area, area moment of inertia, and radius of gyration about different axis, of any shape plane cross section.

The Beam Analysis program, using transfer matrix method, computes and also plots the curves of deflection, slope, moment, and shear along the beam. Static and forced, undamped dynamic analysis can be performed for beams of uniform or variable cross section. Uniformly or linearly varied distributed loads, concentrated point loads, applied moments, or combinations of all three may be applied. This program

allows any combination of pinned, fixed, free, or guided flexural boundary conditions, even normally kinematically unstable condition can be handled if sufficient internal supports are provided. In-span support can be elastic springs and/or elastic moment spring. Modelling for dynamic response uses lumped mass.

All three programs provide the option of using either English or SI units. The programming language used is BASIC and the micro-processor used is a Teketronix model 4051 with 32 K memory.