

THE APPLICATION OF SEPARATION INDICES  
TO MULTISTAGE SYSTEMS

by

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## I. INTRODUCTION

Since the earliest theoretical studies of separation processes<sup>(1-5)</sup>, individuals have attempted to quantify how well separations could be performed. Numerical indices were often a convenient choice for this purpose. One of the most common of the "indices" used was the "separation factor"<sup>(4)</sup> which is usually defined for a binary system as,

$$\text{separation factor} = \frac{Y_{21}}{Y_{11}} \quad (1)$$

where  $Y_{ij}$ , the segregation fraction<sup>(1)</sup> is defined as,

$$Y_{ij} = \frac{n_{ij}}{2 \sum_{j=1} n_{ij}} \quad (2)$$

and  $n_{ij}$  is the number of moles of component  $i$  in region  $j$ . In chromatography, the "resolution"<sup>(2)</sup> is the commonly employed index that characterizes how well the separation has been performed,

$$\text{Resolution} = R_s = \frac{2 \times (\text{difference between retention times})}{\text{Sum of peak widths}} \quad (3)$$

However, many of the indices that have been defined are valid only for certain types of separation processes, e.g., the resolution, and do not apply to other separation processes.

Rony<sup>(1)</sup> has recently proposed a new separation index, the extent of separation, and has claimed that it is superior to any other index, including all of the various definitions proposed for the "separation

factor" as well as the "resolution". However, the types of systems that he treated were mainly as shown in Figure 1(a), i.e., systems which can be termed "single-feed double-output". In these systems, a single feedstream is separated into two product streams, and it is always possible to calculate the extent of separation knowing only the compositions of the product streams. However, most industrial separation processes are multistage in nature, i.e., a series of separating units or stages are arranged in the form of a cascade. Products from one stage become feeds to adjacent stages. A sketch of one such stage is shown in Figure 1(b). It is not immediately evident that the extent of separation can be applied to such systems.

The purpose of this work is to apply the extent of separation, and any other indices that appear useful, to the calculation of stage-by-stage separation in a multistage process. As was done by Rony,<sup>(1)</sup> attention will be restricted to binary systems, i.e., systems that contain only two components and only two regions or phases.

In section II, we will describe the multistage model and the McCabe-Thiele graphical technique that will be used as the basis of our calculations. The applicability of the extent of separation and the extent of purification<sup>(7)</sup> to this system will be discussed in Sections III and IV. In section V, a new separation index will be proposed and will be shown to be applicable to a binary multistage distillation system. In section VI, we will summarize other indices which were considered and found unsuitable for multistage systems. Section VII will cover the application of the new index to other common multistage processes.

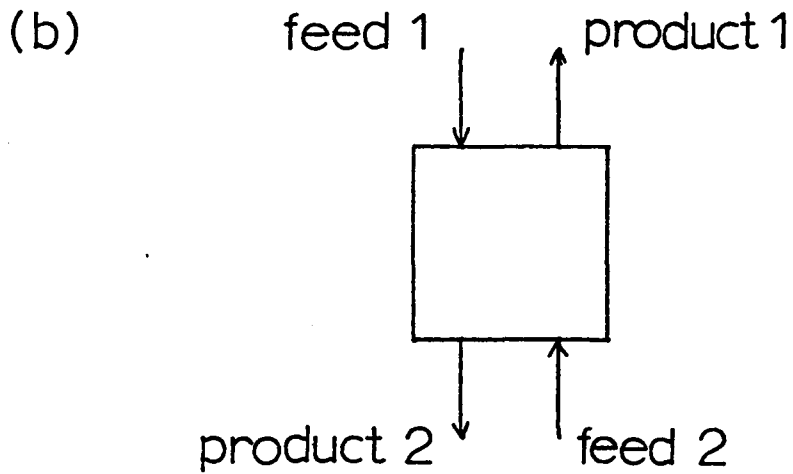
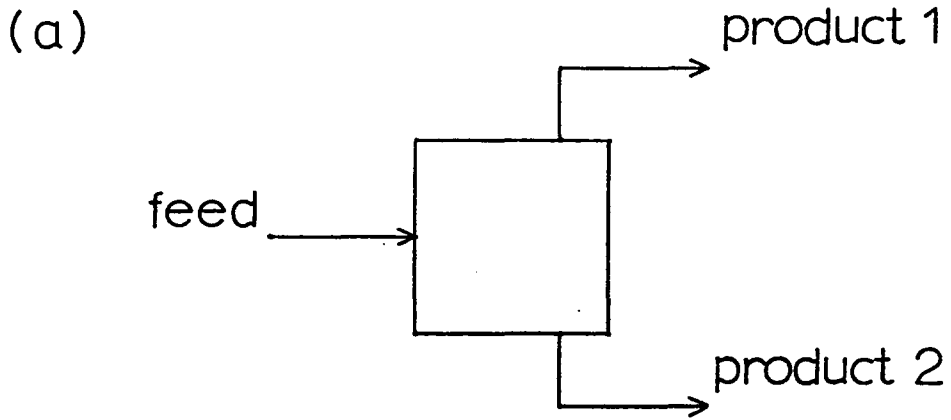


Figure 1. Schematic of a Separation Process

- (a) Single-feed Double-output
- (b) Double-feed Double-output

## II. MULTISTAGE MODEL

We shall use a simple continuous countercurrent distillation column as our model system for the calculation of different separation indices. Many standard texts on chemical engineering initiate discussions on multistage operations with distillation because of the generality and simplicity of the technique and its ease of modification to other types of processes.

Multistage Distillation. A typical continuous distillation column, one that contains both rectifying and stripping sections, is shown in Figure 2. The column is fed near its center with a feed of specified concentration. The plate on which the feed enters is called the feed plate. All plates above the feed plate constitute the rectifying section, and all plates below the feed, including the feed plate itself, constitute the stripping section. The vapor rising through the rectifying section is assumed to be completely condensed in the total condenser. Part of the condensate is collected as product D, while the remaining condensate is returned to the top plate of the tower as reflux L. At the bottom of the tower, the liquid stream,  $L'$ , is reheated by the reboiler, which vaporizes part of the liquid,  $V'$ , and returns it to the bottom plate. The rest of the liquid stream is withdrawn as bottom product B.

McCabe-Thiele Graphical Method. The McCabe-Thiele graphical technique is the easiest method for analyzing binary multistage separation processes. A typical McCabe-Thiele plot for a multistage

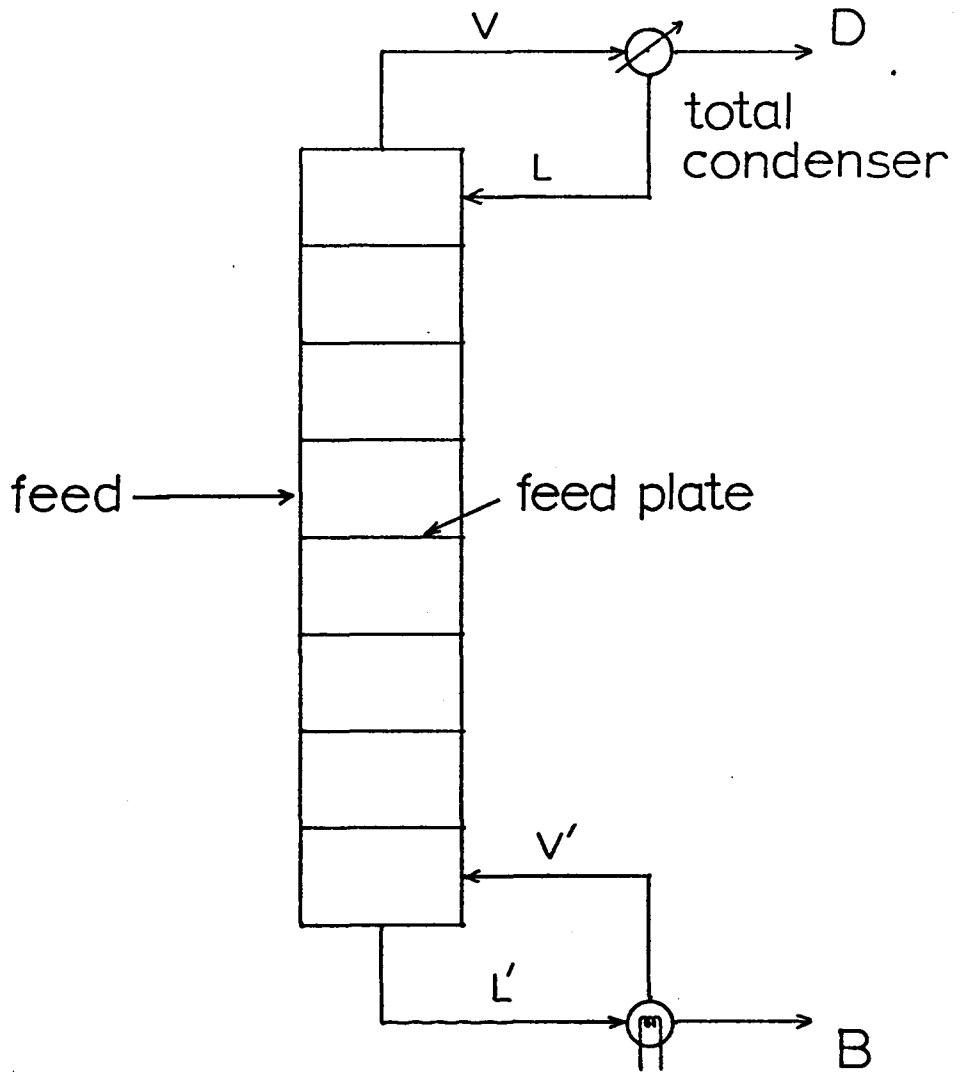


Figure 2. Schematic of a Continuous Distillation Column

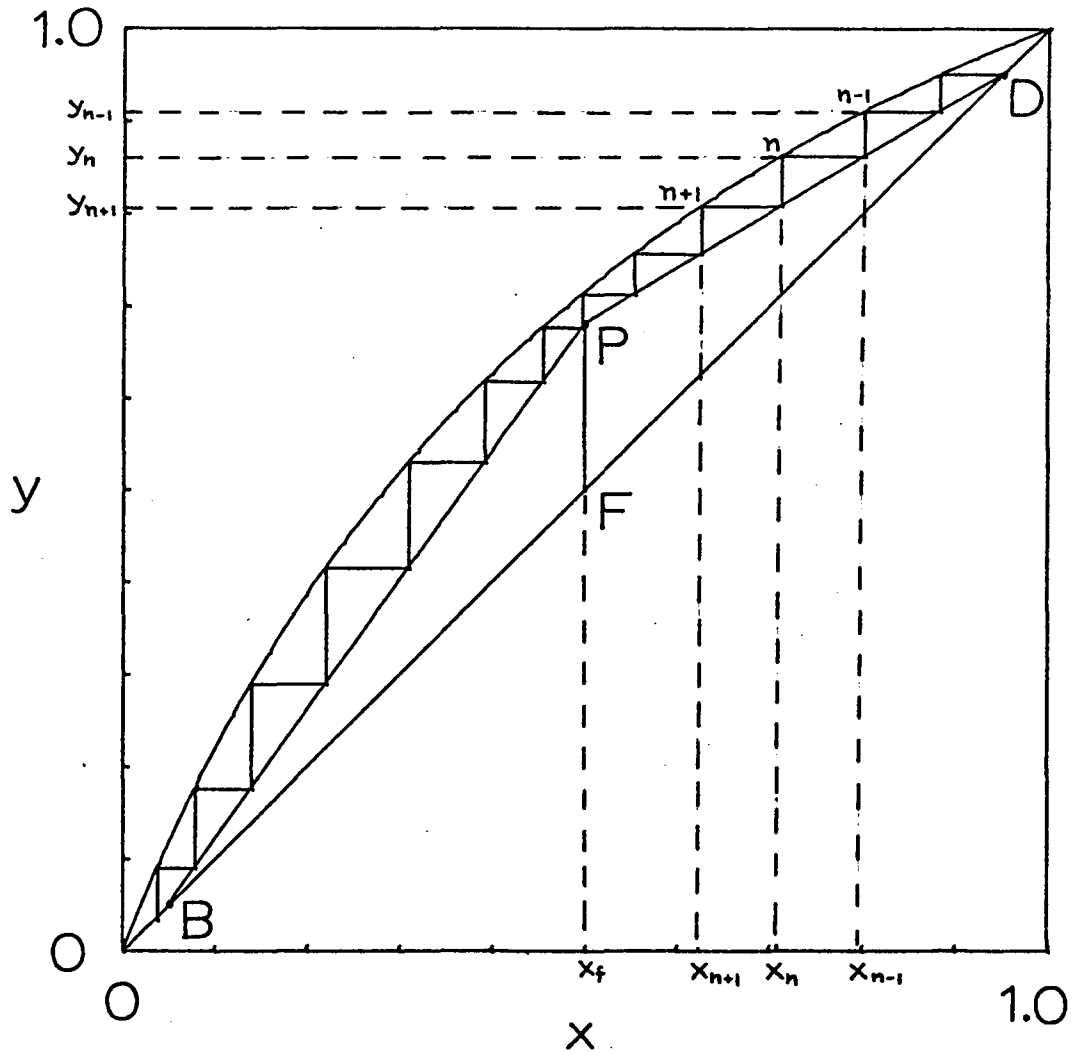


Figure 3. McCabe-Thiele Diagram for a Binary Multistage Distillation

distillation is shown in Figure 3. The details of this technique can be found in any standard text on chemical engineering unit operations<sup>(8)</sup>. Only a brief summary of the technique will be given here.

The method is based on a x-y equilibrium diagram, where x is the mole fraction of the more volatile component in the liquid and y is the mole fraction of the same component in the vapor. Points F, D and B, which represent the compositions of the feed, top and bottom products, respectively, are located on the diagonal line. PD and PB are the operating lines of the rectifying and stripping sections, respectively. The location of point P depends on the feed condition and the reflux ratios,  $L/V$  and  $L'/V'$ . The slopes of the lines PD and PB are  $L/V$  and  $L'/V'$  respectively. We step off a single equilibrium stage by drawing a horizontal line from D to meet the equilibrium curve and then a vertical line back to the "operating line". The compositions of the vapor and liquid streams through that stage are determined directly from the diagram. By repeating this operation until point B is reached or surpassed, the total number of stages required to separate the feed into top and bottom products, D and B, as well as the compositions in each stage, can be found.

The assumptions used in the McCabe-Thiele technique for distillation include<sup>(3)</sup>

1. Sensible heat changes throughout the column are negligible in comparison with latent heat of vaporization
2. The molar latent heats of vaporization of the components are equal

3. The heat of mixing of the components is negligible
4. Heat losses from the column can be neglected
5. Equilibrium is achieved in each stage.

The above assumptions are rather restrictive and seriously curtail the general utility of the method. Nevertheless, the technique represents a fairly good engineering approximation when sufficient thermodynamic data is lacking. In section VI, we will describe another graphical technique, the Ponchon-Savarit method, which can be used when assumptions 1 and 2 do not hold.



### III. THE EXTENT OF SEPARATION

The extent of separation,  $\xi$ , was recently proposed by Rony<sup>(1)</sup> and has been successfully applied to various separation processes<sup>(7, 9-13)</sup>. However, it has never been applied to multistage separation processes, such as a continuous distillation column, which are common in chemical process industries. The purpose of this section is to describe such an application. Definitions and important characteristics will be summarized in the following paragraph. We will also optimize a single stage separation process by a procedure that is based upon the extent of separation, which will later be used to design a multi-feed distillation system. Discussions on the theoretical results of the application of  $\xi$  and the difference in  $\xi$  to a multistage distillation process will be presented.

Description. Let us consider a binary system, a system that contains only two components ( $i = 1, 2$ ) and two regions ( $j = 1, 2$ ) (the term region is chosen in preference to phase, vessel, container, or fraction, all of which have more limited connotations). The quantity  $n_{ij}$ , is defined as the number of moles of component  $i$  in region  $j$ . Two simple dimensionless groups that can be formulated from a set of  $n_{ij}$  are the mole fraction,  $X_{ij}$

$$X_{ij} = \frac{n_{ij}}{\sum_{i=1}^2 n_{ij}} \quad (4)$$

and the segregation fraction,  $Y_{ij}$ ,

$$Y_{ij} = \frac{n_{ij}}{\sum_{j=1}^2 n_{ij}} \quad (5)$$

The distribution of either component 1 or 2 between the two regions is usually described in terms of a distribution coefficient,  $K_i$ ,

$$K_i = \frac{n_{i2}}{n_{i1}} \quad (6)$$

For a vapor-liquid system, the relative volatility,  $\alpha$ , is defined as

$$\alpha = \frac{X_{11}}{X_{21}} \cdot \frac{X_{22}}{X_{12}} \quad (7)$$

where regions  $j = 1$  and  $j = 2$  denote vapor and liquid phase, respectively.

In distillation,  $x$  and  $y$  are commonly used to represent the compositions of the more volatile component in the liquid and vapor streams respectively. Let us call component  $i = 1$  the more volatile component. Equation (7) then becomes,

$$\alpha \frac{x}{1-x} = \frac{y}{1-y} \quad (8)$$

Upon combining Eqs. (6) and (7), we obtain

$$\alpha = \frac{K_2}{K_1} \quad (9)$$

The extent of separation is defined as

$$\xi = \text{abs} \begin{vmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{vmatrix} \quad (10)$$

Since  $Y_{11} + Y_{12} = 1$  and  $Y_{21} + Y_{22} = 1$ , Eq. (10) can be simplified to,

$$\xi = \text{abs}(Y_{11} - Y_{21}) \quad (11)$$

or in terms of  $K_i$ ,

$$\xi = \text{abs} \left[ \frac{1}{1 + K_1} - \frac{1}{1 + K_2} \right] \quad (12)$$

According to Rony<sup>(1)</sup>, the extent of separation possesses all the desirable characteristics of a broadly applicable separation index. For example, it is

- a. Normalized,  $0 \leq \xi \leq 1$
- b. Invariant to a permutation of the component indices  $i$  or the region indices  $j$
- c. Independent of the units in which  $Y_{ij}$  is calculated
- d. Easily calculated
- e. Conceptually similar to the normalized De Donder extent of reaction,  $\xi_D$ <sup>(14)</sup>

and it applies to

- f. Any type of separation system
- g. Any initial level of component purity
- h. Any initial amount of components
- i. Any final level of component purification
- j. Any concentration profile (if one exists)
- k. Both separation and mixing processes
- l. Pairs of chemical components

m. Time normalization and minimum time analysis

n. Other forms of optimization.

As the result of the theoretical work presented in this thesis, we seriously question whether item f above is a correct statement.

Single Equilibrium Stage. Let us now consider a single equilibrium stage in which two components ( $i = 1,2$ ) each distribute between two regions ( $j = 1,2$ ). The stage is in thermodynamic equilibrium. The extent of separation can be calculated according to Eq. (12),

$$\xi = \text{abs} \left[ \frac{1}{1 + K_1} - \frac{1}{1 + K_2} \right] \quad (12)$$

If we combine Eqs. (9) and (12), we obtain

$$\xi = \text{abs} \left[ \frac{1}{1 + K_1} - \frac{1}{1 + \alpha K_1} \right] \quad (13)$$

For a given value of  $\alpha$ , Eq. (13) can be differentiated with respect to  $K_1$  and the result set equal to zero to yield the optimum value of  $K_1$ ,

$$K_1|_{\text{opt}} = \alpha^{-1/2} \quad (14)$$

and the optimum value of  $K_2$ ,

$$K_2|_{\text{opt}} = \alpha^{1/2} \quad (15)$$

It is important to note that there is only one optimum point and that this point is a maximum (second derivative is negative at

optimum values of  $K_1$  and  $K_2$ ). The corresponding maximum value of the extent of separation,  $\xi_{\max}$ , is

$$\xi_{\max} = \text{abs} \left[ \frac{\alpha^{1/2} - 1}{\alpha^{1/2} + 1} \right] \quad (16)$$

The region near the upper right-hand corner of a x-y equilibrium diagram of a binary system refers to the high temperature region. In this region, the component exists mostly in vapor form, i.e.,  $V > L$ . Therefore, according to the definition of  $K_i$  in Eq. (6),  $K_1$  and  $K_2$  will both be of very low value ( $\ll 1$ ) and become zero at  $y = 1$ , i.e., all vapor. Similarly, the region near the origin corresponds to the low temperature region and the component exists mostly in liquid form so that the values of  $K_1$  and  $K_2$  are very high ( $\gg 1$ ) and become infinite at the origin, i.e., all liquid.

According to Eq. (11),

$$\begin{aligned} \xi &= \text{abs}(Y_{11} - Y_{21}) \\ &= \text{abs} \left( \frac{n_{11}}{\sum_{j=1} n_{1j}} - \frac{n_{21}}{\sum_{j=1} n_{2j}} \right) \\ &= \text{abs}[\% \text{ of component 1 in vapor} \\ &\quad \text{phase} - \% \text{ of component 2 in} \\ &\quad \text{vapor phase}] \quad (17) \end{aligned}$$

According to the above equation,  $\xi$  can be viewed as a factor that indicates the equilibrium distribution of the components in a binary

Table 1

Variation of  $\xi$  as a Function of  $K_1$  and  $K_2^*$

Region	Chosen value of $K_1$	$K_2 = \alpha K_1$	$\xi$
mostly liquid	$\infty$	$\infty$	0
	10000	25000	0.00006
	1000	2500	0.00060
	100	250	0.00592
comparable amount of vapor and liquid	10	25	0.05245
	1	2.5	0.21429
	0.632	1.581	0.22515
mostly vapor	0.1	0.25	0.10909
	0.01	0.025	0.01449
	0.001	0.0025	0.00149
	0.0001	0.00025	0.00015
	0	0	0

---

\*  $\alpha = 2.5$

system. A higher value of  $\xi$  in a system indicates that the system is more readily separable than a system with a lower value of  $\xi$ . As an example, let us consider the case when the relative volatility of a system is 2.5. Table 1 gives values of  $K_2$  and  $\xi$  calculated from arbitrarily chosen values of  $K_1$ . It can be seen that the value of  $\xi$  is very small when the system exists mostly in the vapor or liquid form. It is, indeed, not desirable to have most of both components in either the vapor or the liquid phase, but rather at some optimum geometrical distribution between the two. This optimum point occurs at  $K_1 = 0.632$  and  $\xi_{\max} = 0.22515$ . These values can be calculated directly from Eqs. (14) - (16).

Application to a Multistage Distillation System. The liquid and vapor streams passing through a typical stage in each section of a distillation column as well as the feed plate are shown in Figure 4. The extent of separation can be calculated, according to Eq. (11) and Figure 4(a) as

$$\xi = \text{abs} \left[ \frac{y_n}{y_n + \frac{L}{V} x_n} - \frac{(1 - y_n)}{(1 - y_n) + \frac{L}{V}(1 - x_n)} \right] \quad (18)$$

in the rectifying section, and as

$$\xi = \text{abs} \left[ \frac{y_n}{y_n + \frac{L'}{V'} x_n} - \frac{(1 - y_n)}{(1 - y_n) + \frac{L'}{V'}(1 - x_n)} \right] \quad (19)$$

in the stripping section. As a further simplification, we shall treat only the case where the feed is saturated liquid. Then, according to

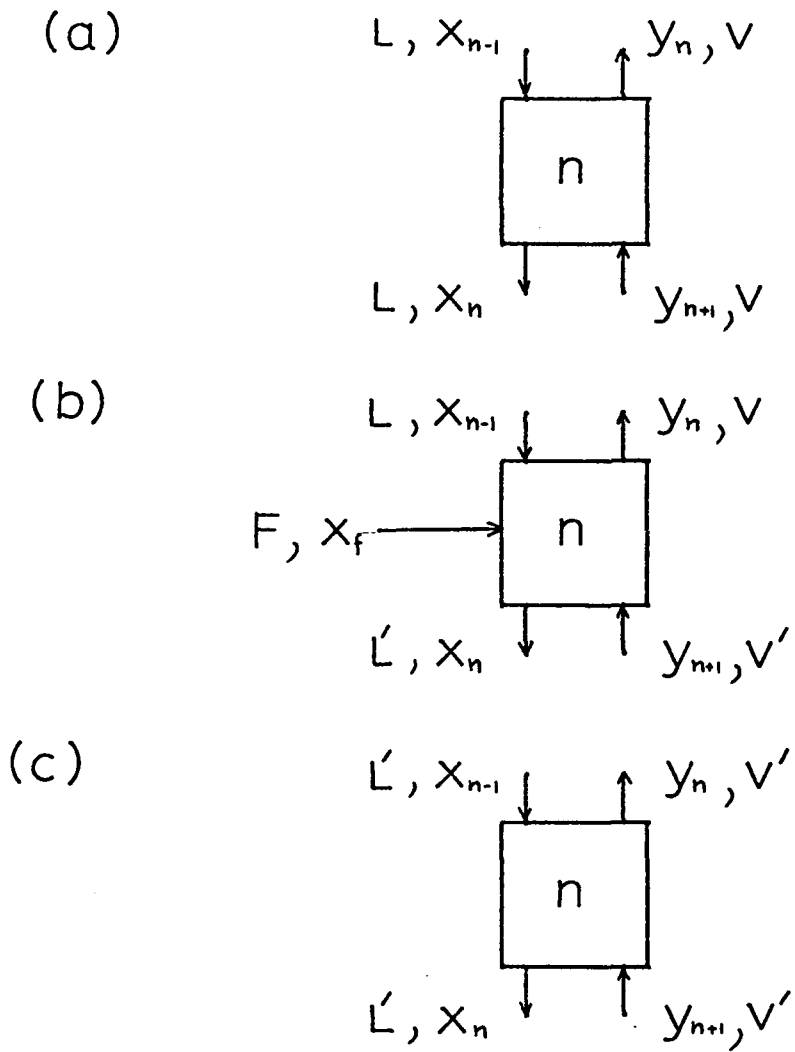


Figure 4. Schematic of Stage n in a Distillation Column

- (a) Rectifying section
- (b) Feed plate
- (c) Stripping section



Figure 4(b),  $V = V'$ ,  $L' = L + F$ , and

$$\xi = \text{abs} \left[ \frac{y_n}{y_n + \frac{L'}{V'} x_n} - \frac{(1 - y_n)}{(1 - y_n) + \frac{L'}{V'} (1 - x_n)} \right] \quad (20)$$

We will now apply Eqs. (18) through (20) to any simple distillation system in which the relative volatility,  $\alpha$ , can be assumed to be constant. Many binary mixtures, especially in the organic group, satisfy this assumption. For example, there is benzene-toluene ( $\alpha = 2.5$ ), n-heptane-n-octane ( $\alpha = 2.1$ ), ethanol-isopropanol ( $\alpha = 1.17$ ), and phenol-o-cresol ( $\alpha = 1.28$ ), to mention but a few binary systems that have approximately constant  $\alpha$ .

We will, for convenience, arbitrarily consider a binary system at the following typical operating conditions:

- a. Constant relative volatility  $\alpha = 2.5$
- b. Top product composition,  $x_D = 0.95$
- c. Bottom product composition,  $x_B \leq 0.05$
- d. Feed conditions,  $x_f = 0.50$ , saturated liquid.

Figure 5 gives McCabe-Thiele diagrams for the above system for the following typical and limiting reflux ratios.

- a.  $\frac{L}{V} = 1$ ,  $L'/V' = 1$  (total reflux)
- b.  $L/V = 0.60$ ,  $L'/V' = 1.40$
- c.  $L/V = 0.530$ ,  $L'/V' = 1.47$
- d.  $L/V = 0.523$ ,  $L'/V' = 1.477$  (minimum reflux ratio)

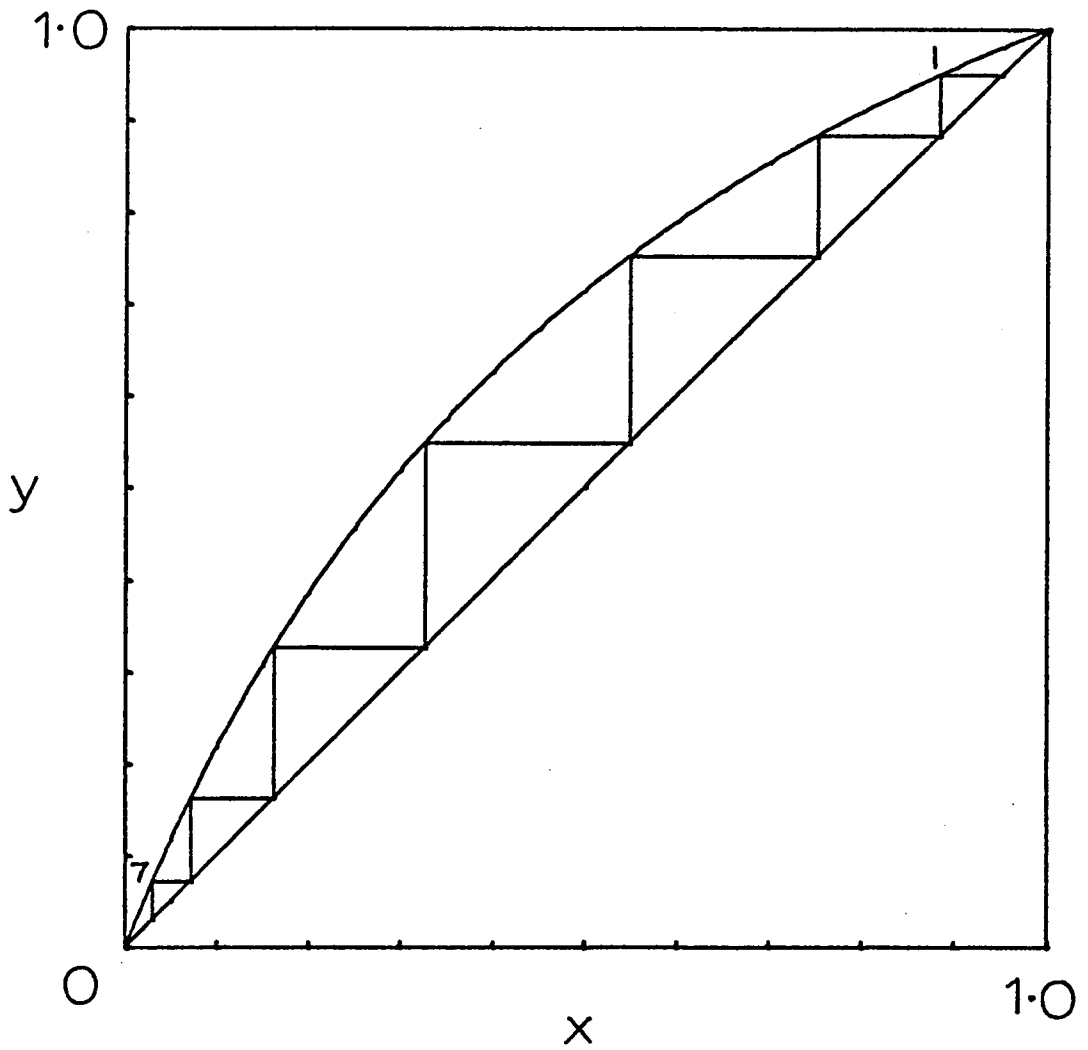


Figure 5a. McCabe-Thiele Diagram for a Distillation Column with Total Reflux ( $L/v = L'/v' = 1$ )

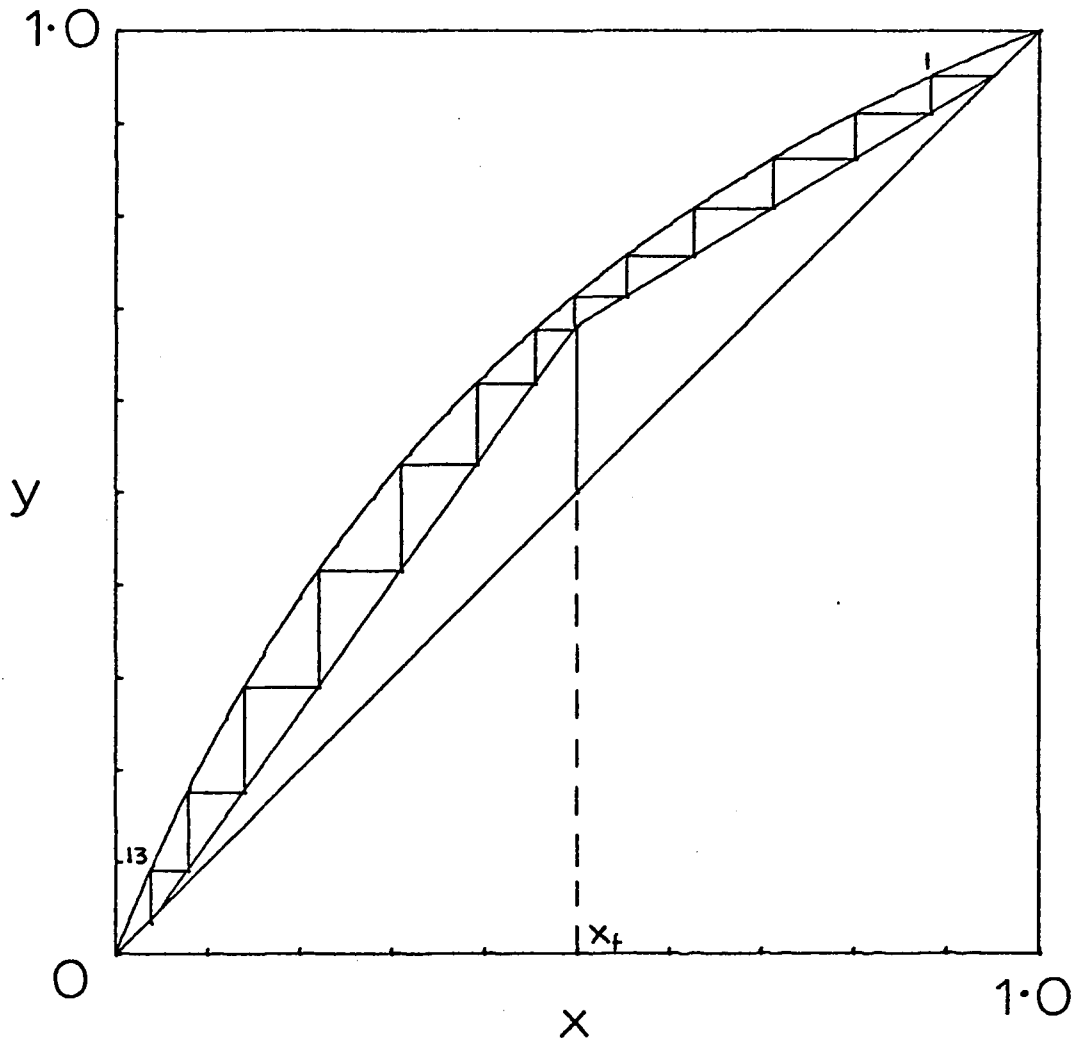


Figure 5b. McCabe-Thiele Diagram for a Distillation Column with  $L/v = 0.6$  and  $L'/v' = 1.40$

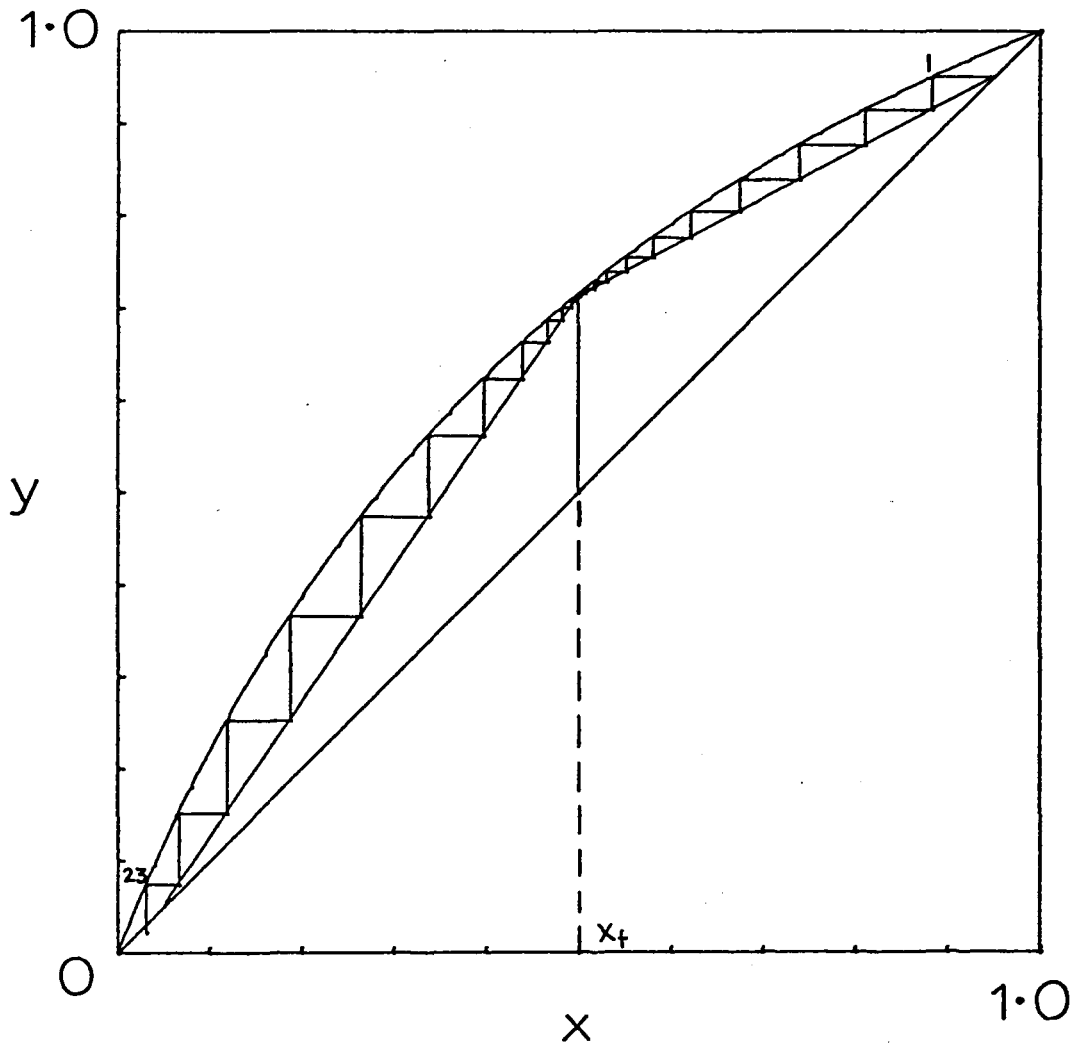


Figure 5c. McCabe-Thiele Diagram for a Distillation Column with  $L/v = 0.53$  and  $L'/v' = 1.47$

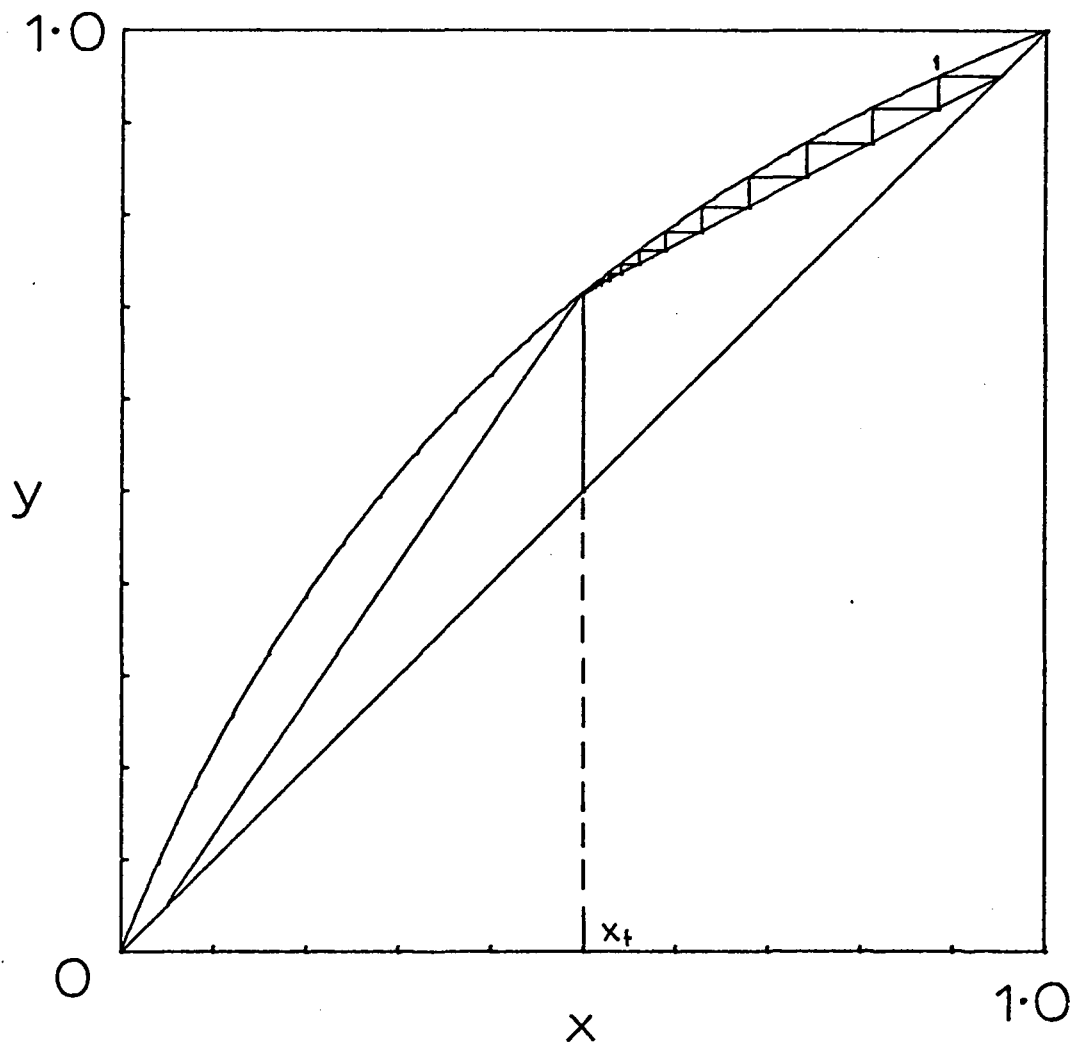


Figure 5d. McCabe-Thiele Diagram for a Distillation Column with  $L/v = 0.523$  and  $L'/v' = 1.477$  (minimum reflux ratio)

The system has a minimum reflux ratio of 0.523, at and below which even an infinite number of stages fails to give the required product compositions.

Discussion Concerning the Application of  $\xi$  to Binary Multistage Systems. Figure 6 gives plots of the extent of separation,  $\xi$ , versus the stage number,  $n$ , for the systems shown in Figure 5. Each stage is numbered in ascending order from the top plate of the column. In case (a), the total reflux case, the curve exhibits a maximum near the middle of the column. Cases (b) and (c) shows both maximum and minimum points and in case (d), the curve levels off after 20 stages. For a system of constant relative volatility,  $\xi_{\max}$  can be calculated in the manner described on page 12. This value, 0.22515, is shown in Figure 6 as a dotted line. We will discuss the reason why the curve show both maximum and minimum points in the following paragraph. It can be observed that in none of the four cases considered is the value of  $\xi$  close to zero. If  $\xi$  were a good separation index, we would expect it to approach zero when an individual stage is performing a poor separation. This is not the case in Figure 6(d) which corresponds to the minimum reflux ratio; even an infinite number of stages will not give the desired product. Despite this, the minimum value of  $\xi$  is 0.20995. In fact, it can be shown that  $\xi$  *will never be zero* except for the trivial situations when  $\alpha = 1$ , in which case the equilibrium curve becomes the diagonal line (see appendix B), and the cases when  $y_n = x_n = 1$  or 0.

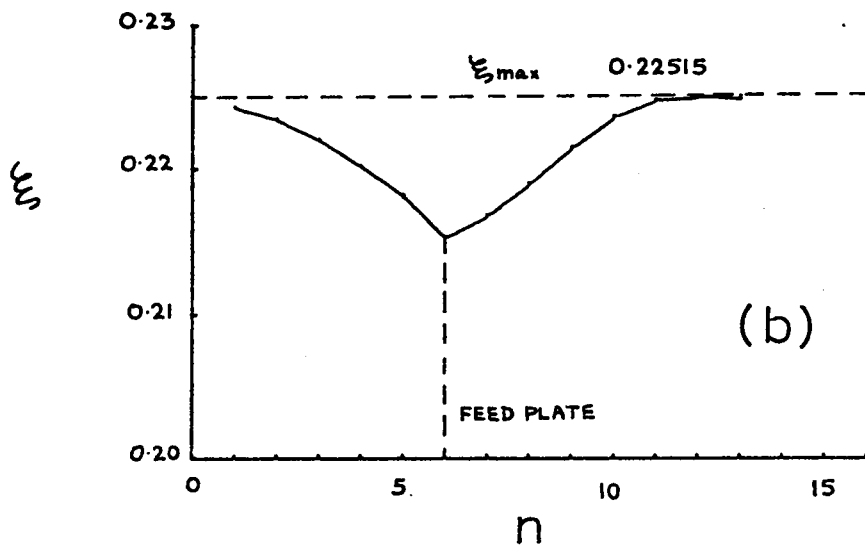
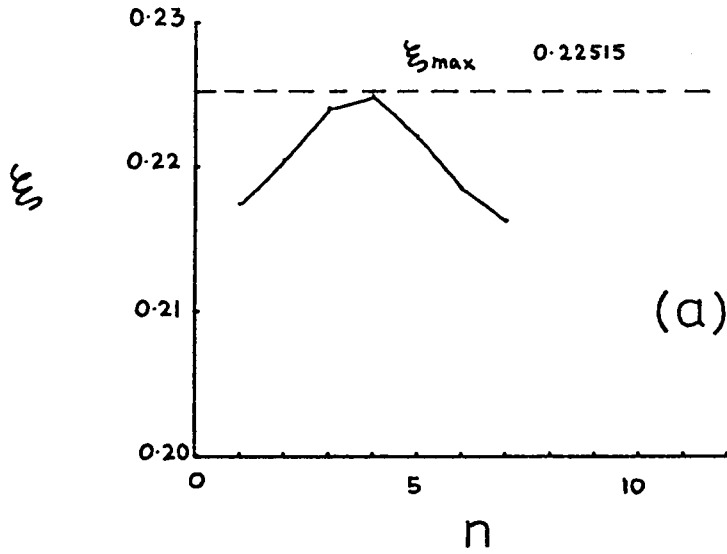


Figure 6.  $\xi$  versus  $n$

(a)  $L/v = L'/v' = 1$

(b)  $L/v = 0.6$  and  $L'/v' = 1.4$

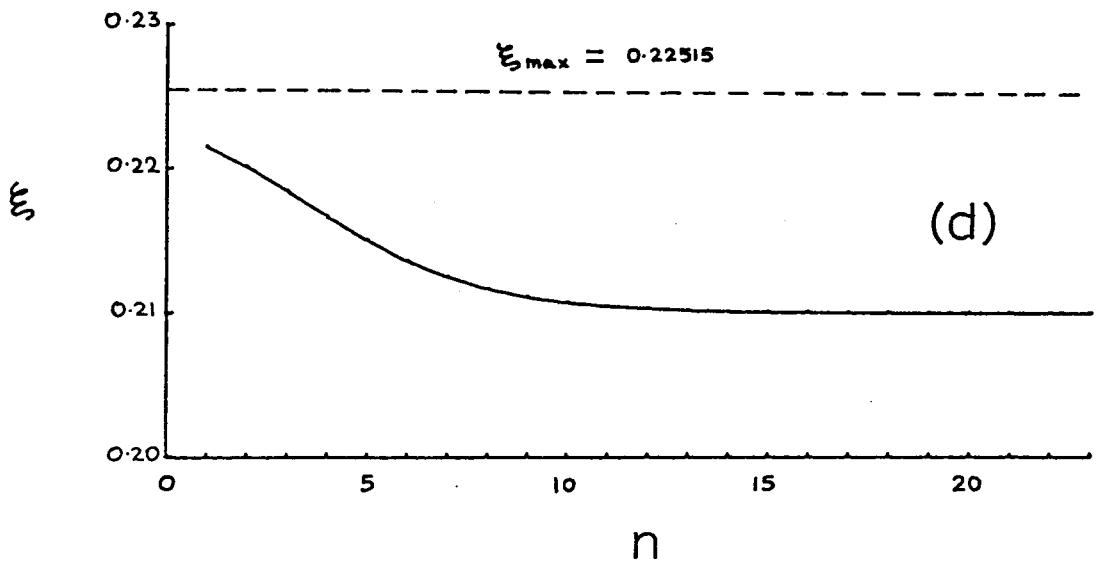
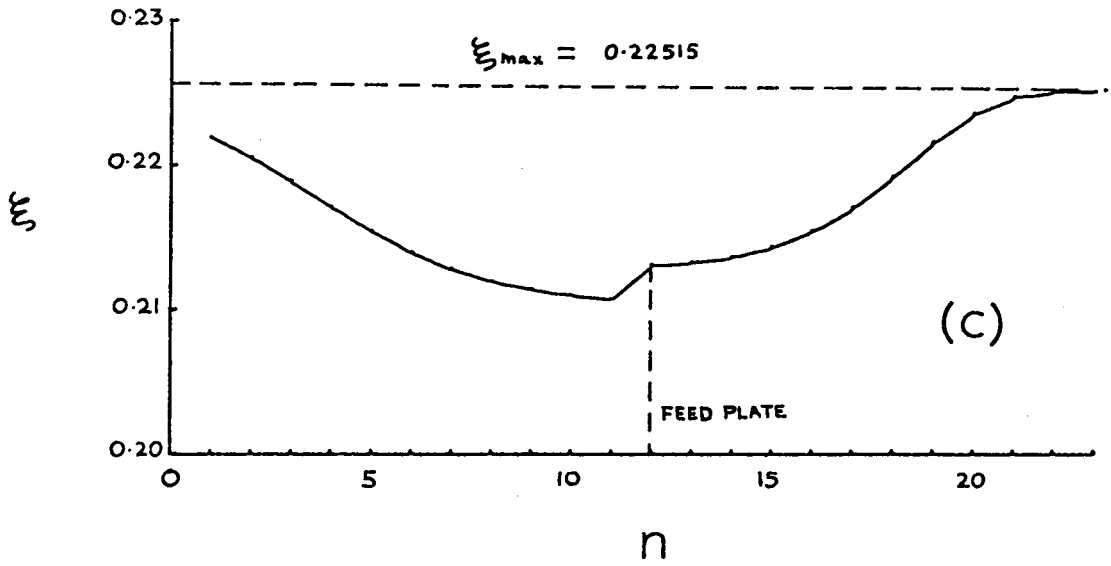


Figure 6.  $\xi_s$  versus  $n$

(c)  $L/v = 0.53$  and  $L'/v' = 1.47$

(d)  $L/v = 0.523$  and  $L'/v' = 1.477$



We have discussed in page 13 that the values of  $K_1$  and  $K_2$  may range from zero to infinity for a single stage system. However, these variations in  $K_1$  and  $K_2$  are somewhat limited in a multistage distillation system due to the introduction of refluxes. The vapor to liquid ratios are deliberately maintained constant in each section so that better distribution of components is obtained in each stage than what would have been without refluxes. As an example, let us consider the rectifying section. Without reflux, the system will exist mostly in vapor form, i.e., low  $K_1$  and  $K_2$ . By introducing liquid into this section, the equilibrium distribution of components is moved to a new and better value, i.e., higher  $K_1$  and  $K_2$ , that results in a better separation. The reverse is true for the stripping section. Each pair of  $K_1$  and  $K_2$ , which corresponds to a fixed point on an x-y equilibrium curve, is now dependent on the reflux ratio. However, the value of  $\xi_{\max}$  is independent on the reflux ratio because it is a function of relative volatility only, (Eq. (16)). But the location at which  $\xi_{\max}$  occurs in the x-y diagram depends on the reflux ratio. The y-coordinate at which  $\xi_{\max}$  occurs is given by (See appendix C).

$$y = \frac{\alpha - L/V \alpha^{1/2}}{(\alpha - 1)} \quad (21)$$

and the relations between  $K_1$ ,  $K_2$  and  $L/V$  are given by

$$1/K_1 = \frac{V}{L} \frac{y_n}{x_n} \quad (22)$$

$$\alpha/K_2 = \frac{y}{L} \frac{y_n}{x_n} \quad (23)$$

The value of  $y$  calculated from Eq. (21) can also be less or greater than one depending on the value of the reflux ratio for a system of constant  $\alpha$ . However, it is desirable to have this maximum point,  $\xi_{\max}$ , occur between  $y = 0$  and  $y = 1$ . The range of  $L/V$  for which this will occur is given by

$$\alpha^{-1/2} \leq L/V \leq \alpha^{1/2} \quad (24)$$

We have stated earlier that there is only one maximum value for  $\xi$  in a given system. This is true for a single stage system. For a multistage system, it is possible to have as many as  $\xi_{\max}$  as there are different reflux ratios in the system. This result is a consequence of the fact that each different reflux ratio changes the equilibrium distribution of the components between the vapor and liquid regions. Only when  $L/V$  is within the range of Eq. (24) will this maximum appear in the equilibrium diagram. As an example, let us consider the case,  $\alpha = 2.5$ . Equation (24) becomes,

$$0.6324 \leq L/V \leq 1.5811 \quad (25)$$

i.e., only when  $L/V \geq 0.6324$  and  $L/V \leq 1.5811$  is there a maximum in the  $x$ - $y$  equilibrium diagram. Now, let us consider the four cases shown in Figure 6.

Case (a)  $L/V = L'/V' = 1$ , the two maximum points merge into one at  $y = 0.6126$

Case (b)  $L/V = 0.6$ ,  $L'/V' = 1.40$ , only one  $\xi_{\max}$  at  $y = 0.1909$

Case (c)  $L/V = 0.53$ ,  $L'/V' = 1.47$ , only one  $\xi_{\max}$  at  $y = 0.1172$

Case (d)  $L/V = 0.523$ ,  $L'/V' = 1.477$ , only one  $\xi_{\max}$  at  $y = 0.1119$

In case (a),  $\xi_{\max}$  occurs within the range  $0 \leq y \leq 1$ . Since there is only one such point, the curve will show a maximum. However, the highest value of  $\xi$  in this system is 0.22496 (see table 9), not the maximum value of 0.22515. For this system,  $\xi_{\max}$  occurs only at  $y = 0.6126$ . During the procedure of stripping off stages, if the y-coordinate of any stage does not fall precisely on  $y = 0.6126$ ,  $\xi_{\max}$  for that system will be missed. For example, in Figure 5 (a), the y-coordinates of stage number 3 and 4 are 0.75248 and 0.54874 respectively (See table 5). The point  $y = 0.6126$  falls in between the two, so  $\xi_{\max}$  is missed. In cases (b) and (c), the reflux ratios of the rectifying section are outside the range of Eq. (25). For these cases, the maximum points are theoretically located beyond the point  $y = 1.0$ , so that  $\xi$  in this section is on a rising trend with  $y$ , i.e.,  $\xi$  decreases as  $n$  increases. In the stripping section,  $\xi_{\max}$  occurs at  $y = 0.1909$  and  $0.1172$  respectively. The curves in this section will exhibit a maximum similar to case (a). In case (d), as in (b) and (c), the maximum point is located at a point beyond  $y = 1.0$  so that  $\xi$  decreases as  $n$  increases. The curve levels off because in the pinch point region, there are very small changes in the values of  $x$  and  $y$  so that  $\xi$  remains essentially constant.

The Difference of  $\xi$  as a Separation Index. We have shown on page 13 that  $\xi$  gives some indication of the equilibrium distribution of the components in a system. The extent of separation depends solely on the coordinates of the point on the equilibrium curve and the reflux ratio. The compositions of the inlet streams to a stage have not yet been taken into account. These compositions are important factors because they partly determine the equilibrium compositions of that stage, which in turn, become feeds to adjacent stages. However,  $\xi$  is a very good index for a single stage system because good distribution of components corresponds directly to good separation. Because of its dependence on the equilibrium curve, perhaps the difference in the extent of separation between stages could be a better measure of separation in a binary multi-stage system. Large values of  $\Delta\xi$  would correspond to large changes in the compositions between stages. Figure 7 shows plots of  $\Delta\xi$  versus  $n$  for each of the cases considered in Figure 5. The irregularity in curve (c) is due to the feed stream and the change in reflux ratio. Curve (c) and (d) indicate that separation is best in stages located in the middle of each section. Curve (d) shows that in the region of the pinch point, separation is bad and  $\Delta\xi$  gradually decreases to zero. In case (a),  $\Delta\xi$  fails to represent the separation performance. For a total reflux system, the best stages are normally in the middle of the column, but the curve indicates that they are the worst. The reason for the dip in curve (a)

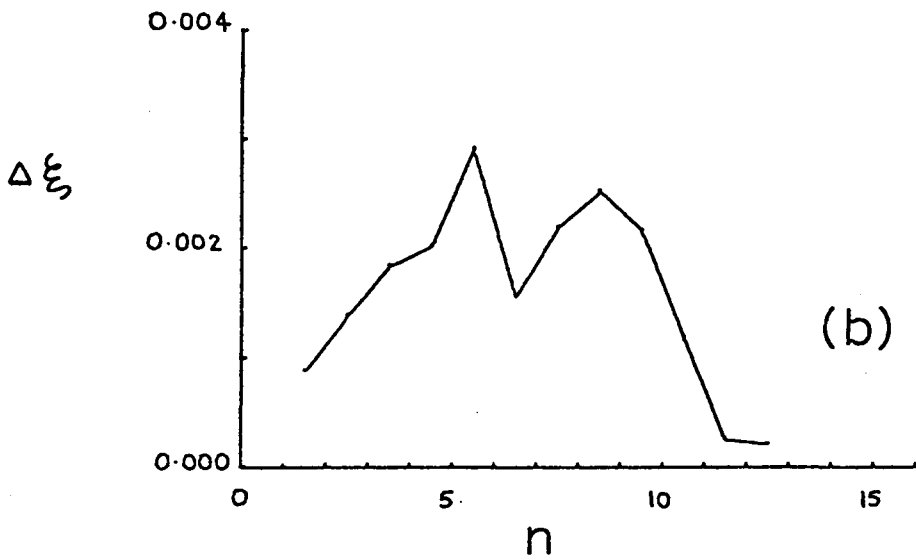
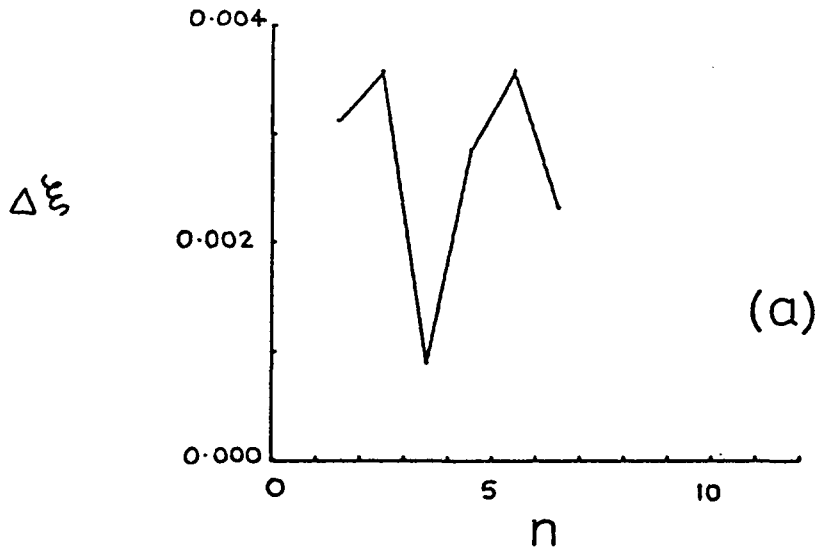


Figure 7.  $\Delta \xi$  versus  $n$

(a)  $L/v = L/v = 1$

(b)  $L/v = 0.6$  and  $L/v = 1.4$

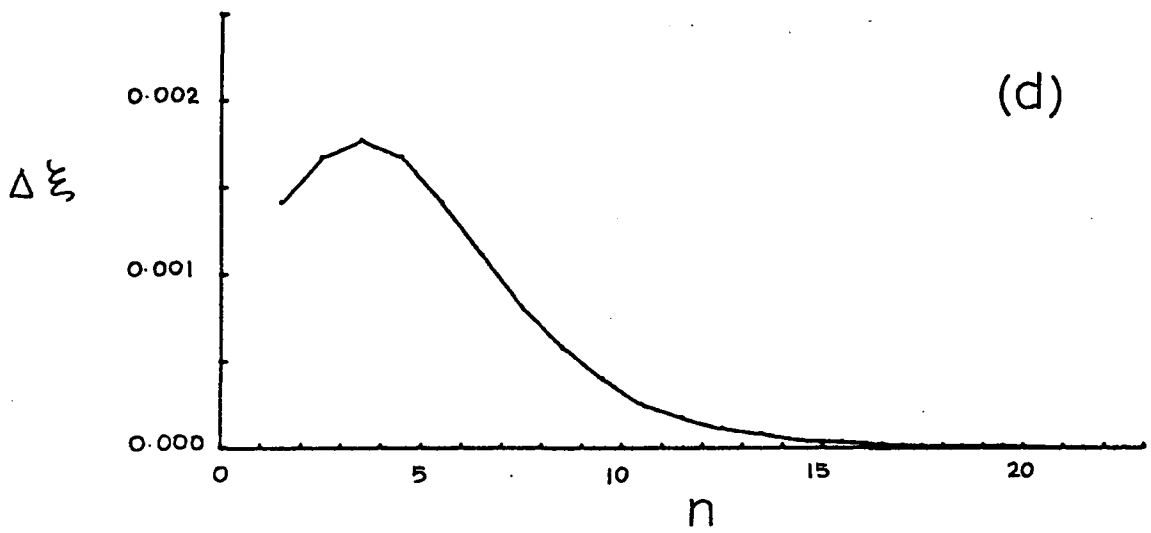
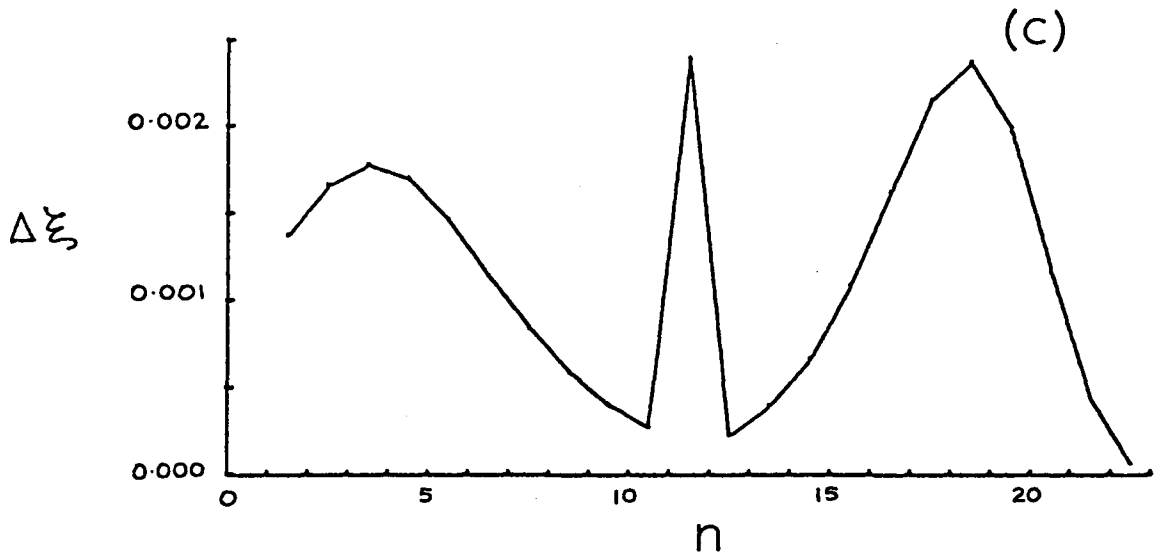


Figure 7.  $\Delta\xi$  versus  $n$

(c)  $L/v = 0.53$  and  $L'/v' = 1.47$

(d)  $L/v = 0.523$  and  $L'/v' = 1.477$

is that  $\xi_{\max}$  for this system occurs at  $y = 0.6126$  (see page 26) which is located between the compositions of stages number 3 and 4, i.e.,  $\xi$  for stages 3 and 4 are on the opposite sides of  $\xi_{\max}$  (See Figure 6(a)). The two values of  $\xi$  for stages 3 and 4 are comparable in magnitude, so that the difference between them is quite small. This phenomenon is not observed in the other three cases since  $\xi_{\max}$  occurs at low values of  $y$  for each case. The change in compositions from stage to stage at low values of  $y$  is not large and it is unlikely that two adjacent stages will fall on each side of the maximum point as in case (a).

On page 31 we will design a multi-feed distillation system in which  $\xi_{\max}$  is achieved in every stage. For such a system, it is obvious that  $\xi$  cannot be used as a separation index since it would mean that each stage performs equally well. It is also obvious that  $\Delta\xi$  cannot be used because for such a system;  $\Delta\xi$  will be zero for all stages in the column. Such results complement our observation in this section that  $\Delta\xi$  fails as a separation index, as is shown in case (a).

Multi-feed Distillation System. On page 25 we discussed how the location at which  $\xi_{\max}$  occurs is dependent upon the reflux ratio. By adjusting the reflux ratio, it is possible to locate  $\xi_{\max}$  at any value of  $y$ . We will make use of this fact to design a variable-reflux distillation system in which  $\xi_{\max}$  occurs at every stage in the column. Upon combining Eqs. (8) and (21), we obtain

$$\frac{L}{V} = \frac{y_n + \alpha(1 - y_n)}{\alpha^{1/2}} \quad (26)$$

or

$$\frac{L}{V} = \frac{\alpha^{1/2}}{1 + x_n(\alpha - 1)} \quad (27)$$

Starting with  $y_D$ , we can calculate the values of  $x$  in the first stage by using Eq. (8). The optimum reflux ratio for this stage can then be calculated from Eqs. (26) or (27). Once the reflux ratio is known, the equation for the operating line can be determined and hence, the value of  $y$  in the second stage. The calculations are repeated until the value  $x_B$  is surpassed (see appendix A).

Figure 8 is a schematic diagram of such a multi-feed distillation column. The simplest way to affect a change in the reflux ratio of a stage is to feed or withdraw the appropriate stream to or from that stage, respectively. For example, the reflux ratio of a stage,  $L/V$ , can be increased by adding liquid directly to the stage. The liquid composition must be the same as that of the equilibrium liquid composition of that stage. The condition of the actual feed stream is dictated by the column performance.

Figure 9 is a McCabe-Thiele diagram of the multi-feed distillation column of Figure 8. A product of 95% composition is withdrawn from the top, whereas the bottom product is less than 5%. The system has a constant relative volatility of 2.5 and the feed composition is 50%. For this system, 10 equilibrium stages and a slightly superheated vapor feed are required (See Table 11 and appendix A). This



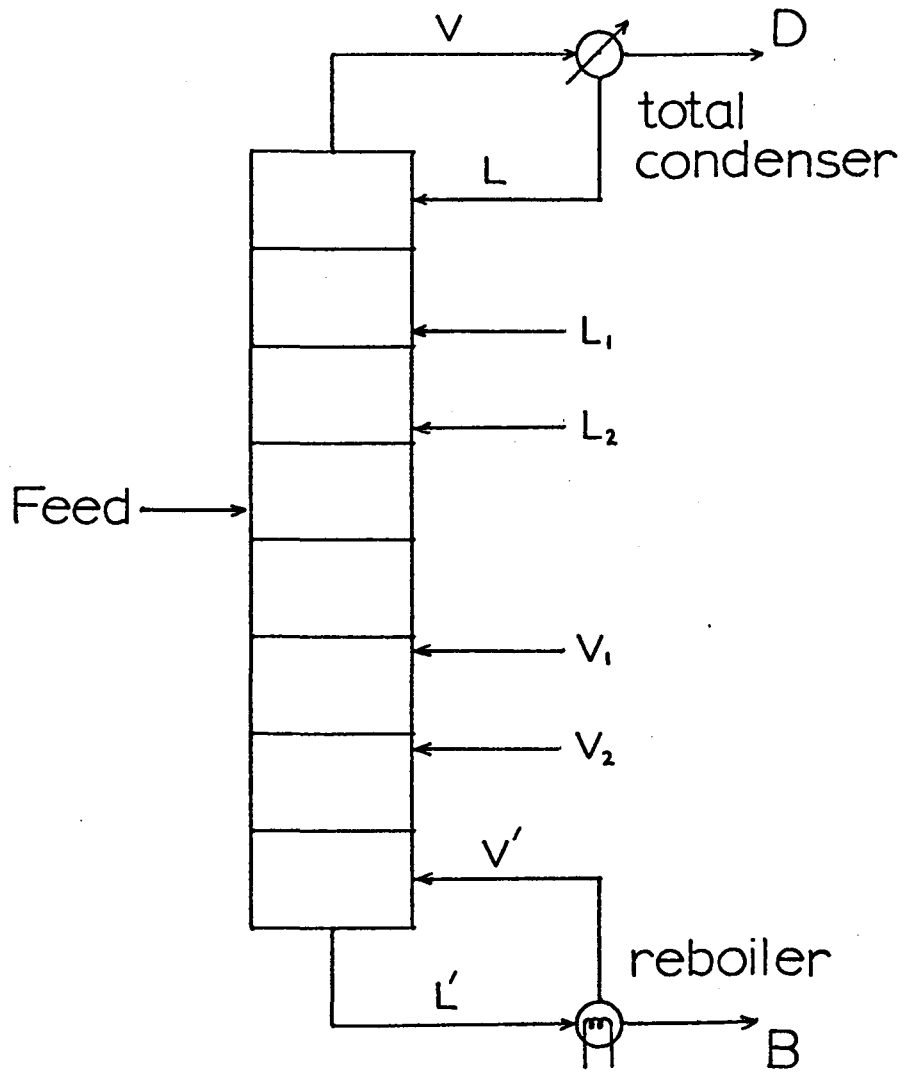


Figure 8. Schematic of a Multi-feed Distillation Column

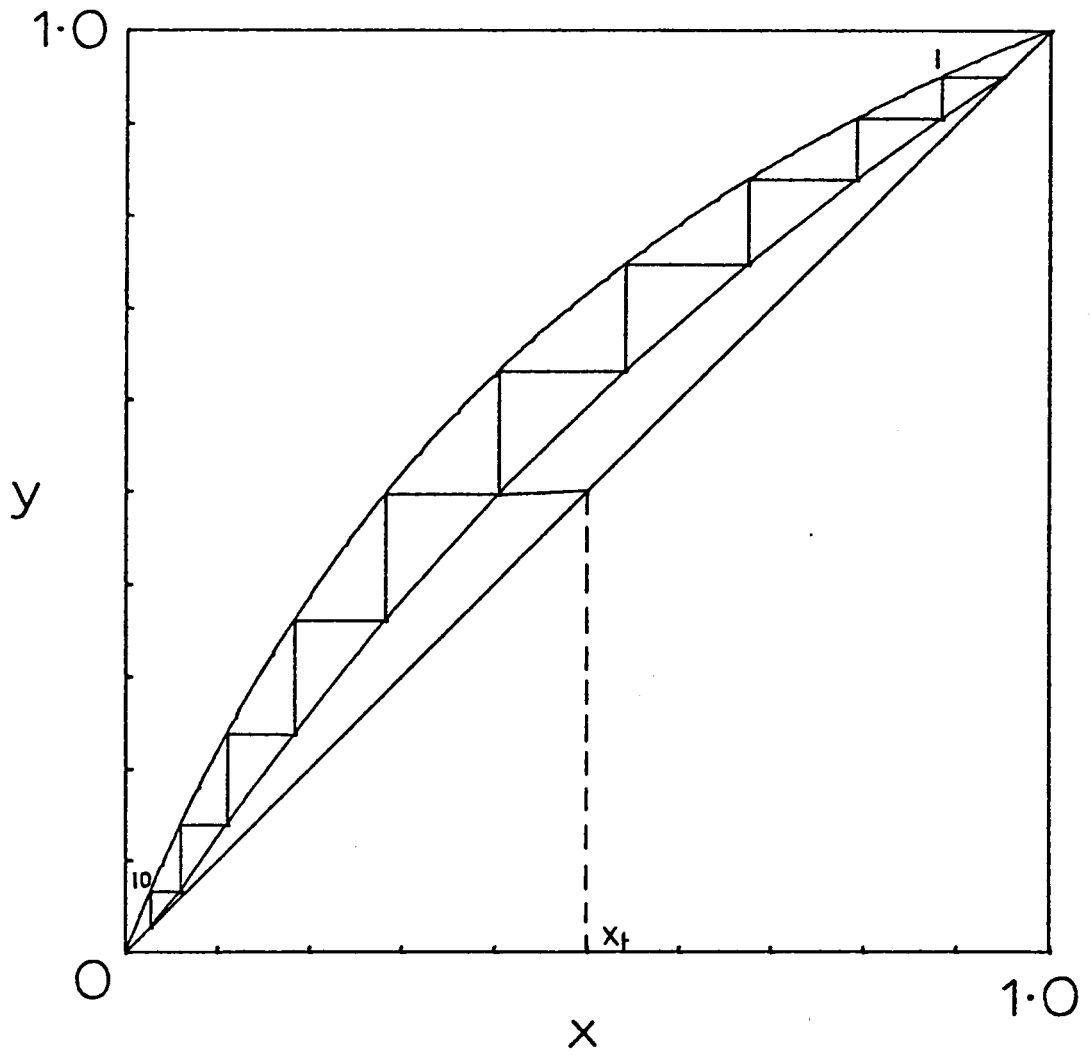


Figure 9. McCabe-Thiele Diagram for a Multi-feed Distillation Column

is a more efficient separation process than the single-feed system which, for the same top plate reflux ratio and feed composition, requires 11 equilibrium stages. A critical comparison between these methods is beyond the scope of this work. The production flowrates, refluxes and sidestreams must be taken into consideration. Further, the economic factors such as operating cost, capital cost must be considered.

#### IV. THE EXTENT OF PURIFICATION

In section III, we have shown that the extent of separation is not a suitable separation index for a multistage system. Rony<sup>(7)</sup>, in his discussion on the extent of separation, also proposed a new index, "the extent of purification", which was obtained by replacing the segregation fraction with the mole fraction in the definition of the extent of separation. However, he has not applied this index to any practical system. In this section, we will determine the applicability of the extent of purification to a multistage system.

Description. For a binary system, the extent of purification is mathematically defined as

$$\xi_p = \text{abs} \begin{vmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{vmatrix} \quad (28)$$

where  $X_{ij}$  is the mole fraction of component  $i$  in region  $j$ ,

$$X_{ij} = \frac{n_{ij}}{\sum_{j=1}^2 n_{ij}} \quad (29)$$

Since  $X_{11} + X_{12} = 1$ ,  $X_{21} + X_{22} = 1$ , Eq. (28) can be simplified to,

$$\xi_p = \text{abs}(X_{11} - X_{12}) \quad (30)$$

Application to Multistage Distillation System. The extent of purification can be calculated according to Eq. (30) and Figure 4,

$$\xi_p = \text{abs}(y_n - x_n) \quad (31)$$

Equation (31) applies to every stage in a distillation column.

Discussion. It can be seen from Eq. (31) that the extent of purification depends wholly on the coordinates,  $x$  and  $y$  of the equilibrium curve, and is not dependent on the slope of the operating line, which is an important factor in multistage calculation.  $\xi_p$  will become zero only when  $\alpha = 1$  (See appendix B) i.e., equilibrium curve becomes the diagonal. We conclude that the extent of purification is not suitable for a multistage system.

## V. A NEW SEPARATION INDEX FOR MULTISTAGE SYSTEMS

In section III and IV, we have demonstrated that the extent of separation and the extent of purification are not ideal indices for multistage processes. This observation is probably due to the fact that both indices do not include the compositions of the inlet streams to the stage under consideration. We cannot, knowing only the two outlet (equilibrium) stream compositions, determine what the compositions of the two inlet streams are, i.e., the two indices represent an underspecification of the single stage within a multistage column. We must know everything about the stage before we can make a useful calculation and any separation index must incorporate such information. With this in mind, we will propose a new separation index which will take these factors into consideration.

Description of the New Index,  $\xi_m$ . The new index,  $\xi_m$ , is defined as

$$\xi_m \equiv \text{abs} \begin{vmatrix} X_{11} & \bar{X}_{11} \\ X_{22} & \bar{X}_{22} \end{vmatrix} \quad (32)$$

where  $X_{ij}$  is the mole fraction of component  $i$  in region  $j$  leaving the stage

$\bar{X}_{ij}$  is the mole fraction of component  $i$  in region  $j$  entering the stage from an adjacent stage (see Figure 10).

This index was formulated after a series of trials and errors. Two of the characteristics of the index that can be immediately discerned are:

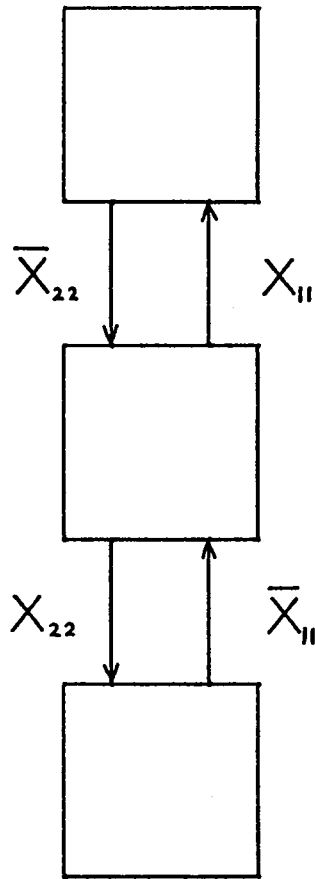


Figure 10. Nomenclature of a Stage in a Multistage Process

- a. It is normalized,  $0 \leq \xi_m \leq 1$
- b. It is *not* invariant to permutation of the component indices  $i$  or the region indices  $j$ .

Other properties of the index will be discussed along with its application in the following paragraphs.

Application to Multistage Distillation System. For any stage,  $n$ , in a distillation column, Eq. (32) can be written as

$$\xi_m = \text{abs}[y_n(1 - x_n) - y_{n+1}(1 - x_{n-1})] \quad (33)$$

The equation of the operating line in each section is

$$y = A + \frac{L}{V} x \quad (34)$$

where  $A$  is a constant.

Since, according to Figure 3,

$$y_{n+1} = A + \frac{L}{V} x_n \quad (35)$$

and

$$y_n = A + \frac{L}{V} x_{n-1} \quad (36)$$

substituting Eqs. (35) and (36) in Eq. (33), we obtain

$$\xi_m = \text{abs}\left[\left(1 + \frac{A}{L/V}\right)(y_n - y_{n+1})\right] \quad (37)$$

and from geometry

$$y_n - y_{n+1} = \frac{L}{V}(x_{n-1} - x_n) \quad (38)$$



Equation (37) can also be written in the form

$$\xi_m = \text{abs}[\left(\frac{L}{V} + A\right)(x_{n-1} - x_n)] \quad (39)$$

Equations (37) and (39) are identical.

From Eqs. (37) and (39), it can be seen that  $\xi_m$  represents the change in composition of either of the outlet streams multiplied by a correction factor. Figure 11 shows plots of  $\xi_m$  versus the stage number,  $n$ , for each of the cases considered in Figure 5. The change in composition of the liquid stream,  $\Delta x$ , and of the vapor stream,  $\Delta y$ , are also shown on the same graphs. In case (a),  $\xi_m$ ,  $\Delta x$  and  $\Delta y$  all fall on the same curve. In cases (b), (c) and (d), it can be clearly seen that, in the rectifying section,  $\Delta x$  curves follow  $\xi_m$  curves very closely where  $\Delta y$  curves fall far below  $\xi_m$ . However, in the stripping section, it is just the reverse, i.e.,  $\Delta y$  follows  $\xi_m$  more closely than  $\Delta x$ . This behavior is a consequence of the influence of the two factors,  $\left(\frac{L}{V} + A\right)$  and  $\left(1 + \frac{A}{L/V}\right)$ , which will be discussed in the next section.

Generally,  $\xi_m$  represents the performance of each stage in the distillation column quite well. In case (a), the best stages are in the middle of the column and are indicated by higher values of  $\xi_m$ . In cases (b) and (c), the stages in the middle of each section of the column are better than any other stages. In case (d), the minimum reflux ratio case, the index gradually decreases to zero as the separation in each stage becomes steadily poorer.

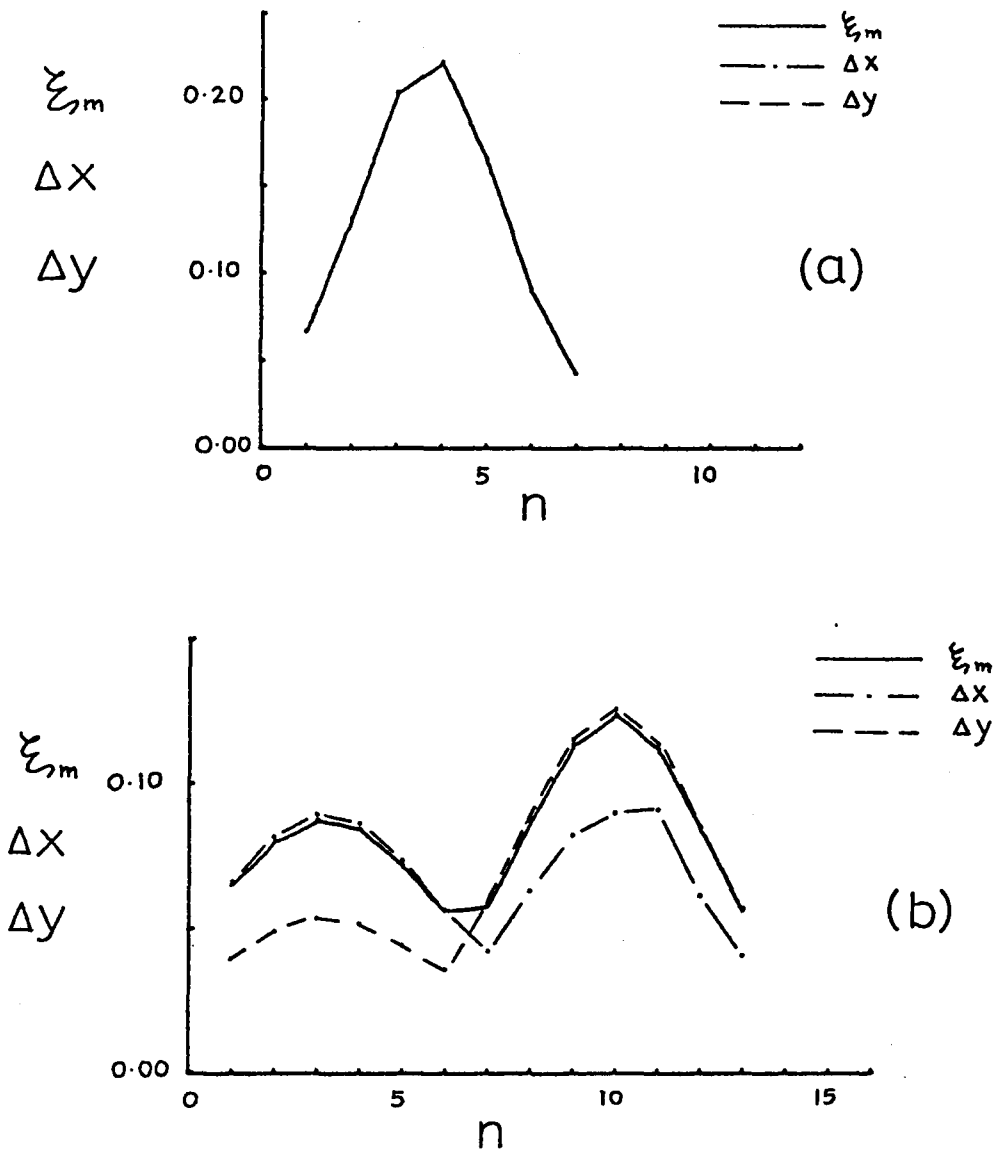


Figure 11.  $\xi_m, \Delta X, \Delta y$  versus  $n$

(a)  $L/v = L'/v' = 1$

(b)  $L/v = 0.6$  and  $L'/v' = 1.4$

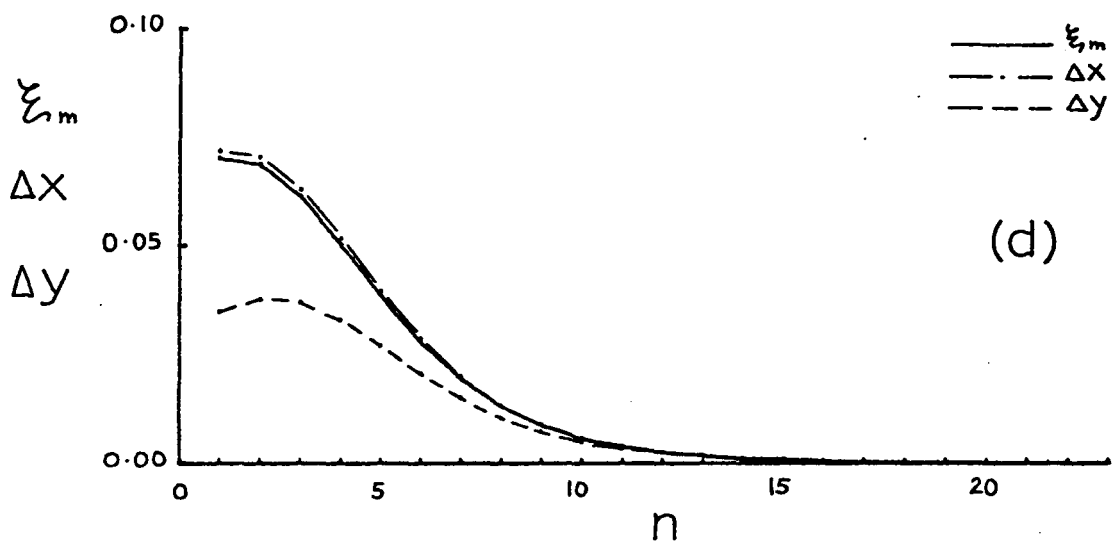
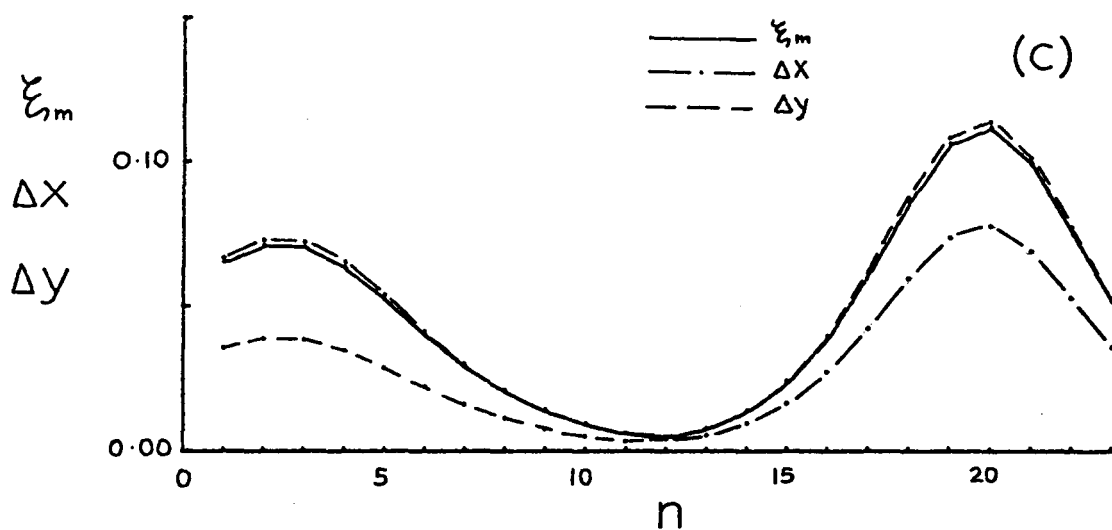


Figure 11.  $\xi_m$ ,  $\Delta X$ ,  $\Delta y$  versus  $n$

(c)  $L/v = 0.53$  and  $L'/v' = 1.47$

(d)  $L/v = 0.523$  and  $L'/v' = 1.477$

Factors  $\left(\frac{L}{V} + A\right)$  and  $\left(1 + \frac{A}{L/V}\right)$ . It was shown in the previous section that  $\xi_m$  can be written in the form  $\left[\frac{L}{V} + A\right]\Delta x$  or  $\left[\left(1 + \frac{A}{L/V}\right)\Delta y\right]$ . We will, in the following paragraphs, discuss the roles that these two factors play in a multistage distillation system.

Figure 12 shows the two operating lines in an x-y distillation equilibrium diagram. One end of each operating line always terminates at the diagonal. It can be clearly seen from Figure 12 that A, the value of y when x = 0, is always positive when  $L/V < 1$  and negative when  $L/V > 1$ . We will now discuss the two cases separately.

Case I.  $L/V \leq 1$ , i.e., rectifying section

Since the point  $(y_D, x_D)$  is always located on the operating line, according to Eq. (34), we can write

$$\left(\frac{L}{V} + A\right) = x_D + \frac{L}{V}(1 - x_D) \quad (40)$$

and

$$\left(1 + \frac{A}{L/V}\right) = (1 - x_D) - \frac{x_D}{L/V} \quad (41)$$

Since  $\frac{L}{V} \leq 1$  and  $0 \leq x_D \leq 1$ , therefore

- a.  $\left(\frac{L}{V} + A\right)$  is always equal to or less than one, and
- b.  $\left(1 + \frac{A}{L/V}\right)$  is always equal to or greater than one.

Case II.  $\frac{L}{V} \geq 1$ , i.e., stripping section

Since the point  $(y_B, x_B)$  is always located on the operating line, according to Eq. (34), we can write

$$\left(\frac{L}{V} + A\right) = x_B + \frac{L}{V}(1 - x_B) \quad (42)$$

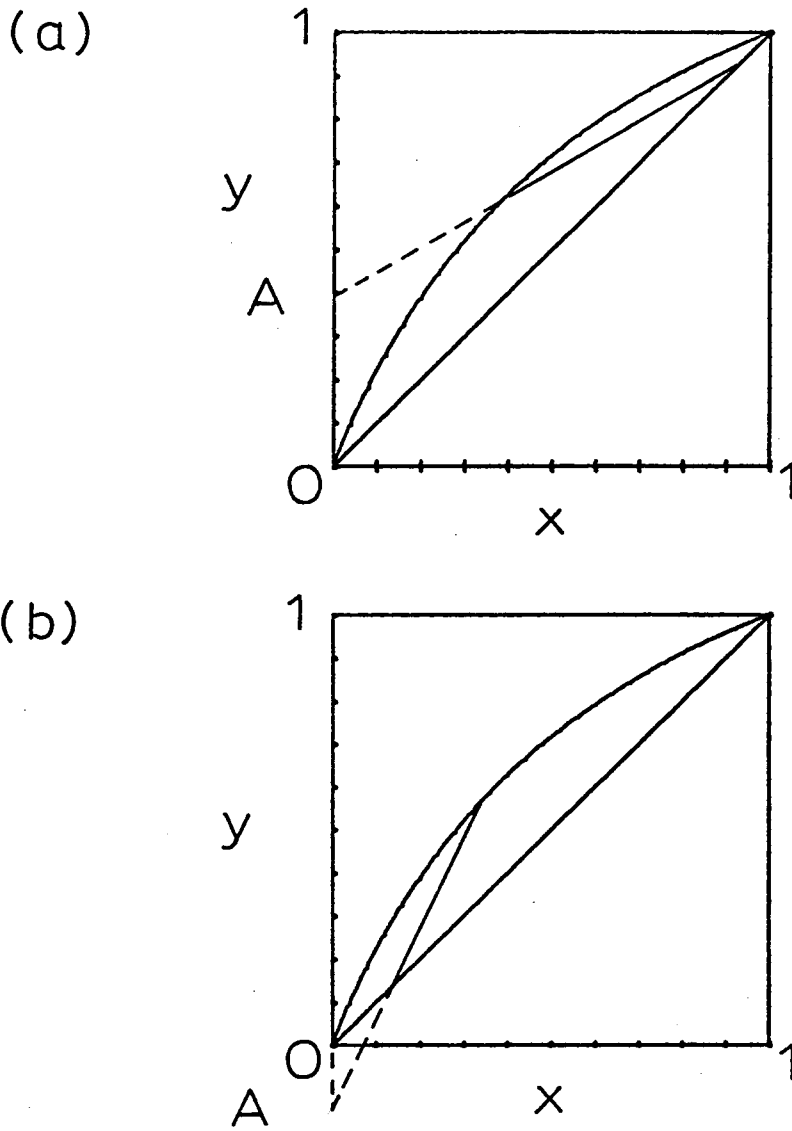


Figure 12. Operating Lines in a Distillation System

(a)  $L/v < 1$

(b)  $L/v > 1$

and 
$$\left(1 + \frac{A}{L/V}\right) = (1 - x_B) - \frac{x_B}{L/V} \quad (43)$$

Since  $L/V \geq 1$  and  $0 \leq x_B \leq 1$ , therefore

- a.  $\left(\frac{L}{V} + A\right)$  is always equal to or greater than one, and
- b.  $\left(1 + \frac{A}{L/V}\right)$  is always equal to or less than one.

Hence, in the rectifying section of a distillation column,

$$\begin{aligned} \xi_m &= \Delta y : (\text{a number which is } \geq 1) \\ &= \Delta x : (\text{a number which is } \leq 1) \end{aligned}$$

and in the stripping section,

$$\begin{aligned} \xi_m &= \Delta y : (\text{a number which is } \leq 1) \\ &\Delta x : (\text{a number which is } \geq 1) \end{aligned}$$

So,  $\xi_m$  can be interpreted as a number which is dependent both on the change in composition in a particular stage and the conditions at which the system is operating. Table 2 shows the values of the two factors which occur at limiting conditions for each section of a distillation column. We can see that, in the rectifying section,  $\left(\frac{L}{V} + A\right)$  varies between 0 to 1 while  $\left(1 + \frac{A}{L/V}\right)$  varies from 1 to  $\infty$ . The situation is just the reverse in the stripping section, where  $\left(1 + \frac{A}{L/V}\right)$  is restricted to values between 0 and 1 and  $\left(\frac{L}{V} + A\right)$  to values between 1 and  $\infty$ . It should be noted that  $\xi_m$  is still normalized even though the two factors become infinite at certain limiting conditions. As an example, let us consider the case  $L/V = 0$ , when  $\left(1 + \frac{A}{L/V}\right)$  becomes infinite. When  $L/V = 0$ , i.e., operating line is

Table 2

Values of  $(\frac{L}{V} + A)$  and  $(1 + \frac{A}{L/V})$  at Limiting Conditions

Section	$(\frac{L}{V} + A)$	$(1 + \frac{A}{L/V})$	Occurs when
Rectifying	1	1	$\frac{L}{V} = 1$
	$x_D$	$\infty$	$\frac{L}{V} = 0$
	1	$\frac{1}{L/V}$	$x_D = 1$
	$\frac{L}{V}$	1	$x_D = 0$
Stripping	1	1	$\frac{L}{V} = 1$
	$\infty$	$1 - x_B$	$\frac{L}{V} = \infty$
	1	$\frac{1}{L/V}$	$x_B = 1$
	$L/V$	1	$x_B = 0$

horizontal,  $\Delta y$  is also zero, and the value of  $\xi_m$ , calculated according to Eq. (37), is also zero. It can be concluded that, in the rectifying section, the factor  $(\frac{L}{V} + A)$  is normalized and is not as sensitive to changes in the operating conditions as the factor  $(1 + \frac{A}{L/V})$ , which is not normalized. Similarly, in the stripping section,  $(1 + \frac{A}{L/V})$  is normalized and is less sensitive to changes in the operating conditions than  $(\frac{L}{V} + A)$ , which is not normalized.

Discussions. For the system shown in Figure 11(a), both factors become unity at  $\frac{L}{V} = 1$  and  $\xi_m = \Delta x = \Delta y$ . For the system in Figure 11(b),  $L/V = 0.6$  and the values of the factors  $(\frac{L}{V} + A)$  and  $(1 + \frac{A}{L/V})$  for the rectifying section are 0.980 and 1.634 respectively. The value of  $(\frac{L}{V} + A)$  lies closer to unity than  $(1 + \frac{A}{L/V})$ , so that  $\xi$  is much closer to  $\Delta x$  than to  $\Delta y$ . In the stripping section,  $L/V = 1.40$ ,  $(\frac{L}{V} + A) = 1.40$  and  $(1 + \frac{A}{L/V}) = 0.986$ , values which indicate that  $\xi_m$  lies closer to  $\Delta y$  than to  $\Delta x$ . Similar arguments can be made with cases (c) and (d). We will now discuss the physical meaning of  $\xi_m$ .

The factors  $(\frac{L}{V} + A)$  and  $(1 + \frac{A}{L/V})$ , when used only in the rectifying and stripping sections, respectively, can be viewed as penalty factors since they are always less than unity except for two "perfect" conditions, where they become equal to unity. The two "perfect" conditions are:

1. Total reflux, i.e., no product is withdrawn from the system



2. Product, either at the top or bottom, is withdrawn in pure form, i.e.,  $x_D = 1$  or  $x_B = 0$ , which requires an infinite number of stages in the column.

If neither of these conditions is met, the two factors are less than one, which results in a lower value of  $\xi_m$ . If either of the two conditions is met, the factors become unity and there is no penalty (see table 2).

According to the above interpretation, the separation in the rectifying section is determined by the change in composition of the liquid stream,  $\Delta x$ , multiplied by the correction factor,  $(\frac{L}{V} + A)$ , while in the stripping section the separation is determined by the change in vapor stream composition,  $\Delta y$ , multiplied by another correction factor,  $(1 + \frac{A}{L/V})$ . It should be noted here that the two sections should be examined separately since they perform different functions. In the rectifying section, the liquid stream "enriches" the vapor stream, which is withdrawn as top product. In the stripping section, the vapor stream "strips" the liquid stream which is withdrawn as bottom product. It should also be noted from Eq. (38) that the index,  $\xi_m$ , actually is dependent upon the stream which is more sensitive to changes in composition in each section.

$$\begin{aligned} \text{i.e.,} \quad & \text{for } L/V < 1 & \Delta y < \Delta x \\ & \text{for } L/V > 1 & \Delta y > \Delta x \end{aligned}$$

$\xi_m$  has a value of unity only when products from both streams are withdrawn with 100% purity. This, indeed, means a perfect separation.

Since the values of  $(\frac{L}{V} + A)$  and  $(1 + \frac{A}{L/V})$  never become zero, except for the trivial cases when  $L/V = 0$ ,  $x_D = 0$  and  $L/V = \infty$ ,  $x_B = 1$ , respectively,  $\xi_m$  will become zero only when there is no separation, i.e.,  $\Delta x = \Delta y = 0$ .

The index can also be applied collectively to each section of a distillation column. Figure 13 shows how this can be done.  $x'_f$  and  $y'_f$  are the x and y coordinates of the point of intersection of the operating line (see Figure 15). Each section can be so considered because the factors  $(\frac{L}{V} + A)$  and  $(1 + \frac{A}{L/V})$  are constant in the section and the feed configuration of each section as a whole is the same as that at each individual stage, i.e., double-feed double-output. Equations (37) and (39) become

$$\text{Rectifying section} \quad \xi_m = \text{abs}[(\frac{L}{V} + A)(x_D - x'_f)] \quad (44)$$

$$\text{Stripping section} \quad \xi = \text{abs}[(1 + \frac{A}{L/V})(y'_f - y_B)] \quad (45)$$

The value of  $\xi_m$ , calculated from Eq. (44) actually represents the summation of  $\xi_m$  at every stage in the rectifying section and  $\xi_m$  in Eq. (45) represents the summation of  $\xi_m$  of every stage in the stripping section.

Let us now consider a single-feed double-output single-stage system (Figure 1(a)). If we denote the feed, product 1 and product 2 flow rates by  $F$ ,  $V$  and  $L$  respectively, and  $x_f$ ,  $y_{out}$  and  $x_{out}$  denote the composition of each stream, then the component mass balance of the system gives

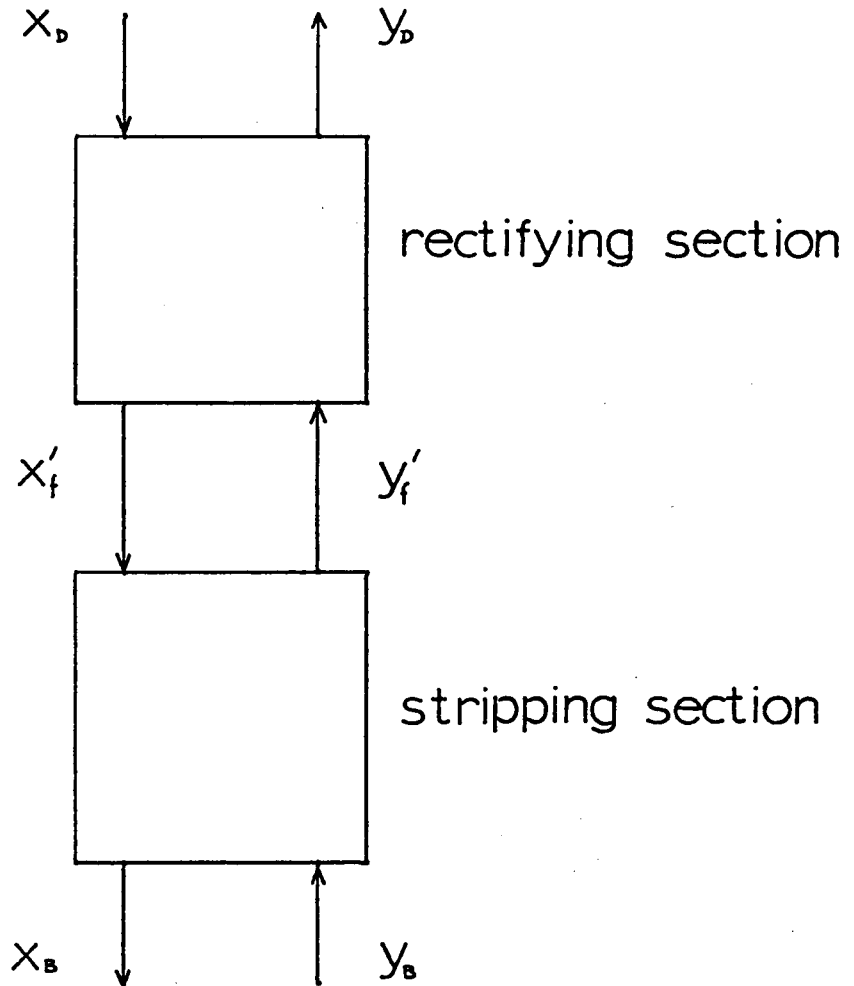


Figure 13 Schematic of a Distillation Column Arranged in Sections (feed stream has been omitted)

$$Fx_f = Vy_{out} + Lx_{out} \quad (46)$$

and overall mass balance gives

$$F = V + L \quad (47)$$

The system has six unknown variables and two equations, i.e., four variables can be set arbitrarily. We will fix the values of  $F$ ,  $V$ ,  $L$  and  $x_f$ .

Now let us fictitiously divide the feed stream into two streams so that we obtain a double-feed double-output configuration (see Figure 14(a)). Let us also specify that the flow rates of the two streams be  $V$  and  $L$ . Then, according to Eqs. (46) and (47), it is necessary that,

$$x_{in} = y_{out}$$

and

$$y_{in} = x_{out} \quad (48)$$

The new double-feed double-output system is, in fact, equivalent to one stage in a distillation column operated at total reflux. This is illustrated in Figure 14(b). If we now apply Eq. (33) to this system, we obtain

$$\begin{aligned} \xi_m &= \text{abs}[y_{out}(1 - x_{out}) - y_{in}(1 - x_{in})] \\ &= \text{abs}(y_{out} - x_{out}) \end{aligned} \quad (49)$$

$$= \text{abs}(y_{out} - y_{in}) \quad (50)$$

$$= \text{abs}(x_{in} - x_{out}) \quad (51)$$

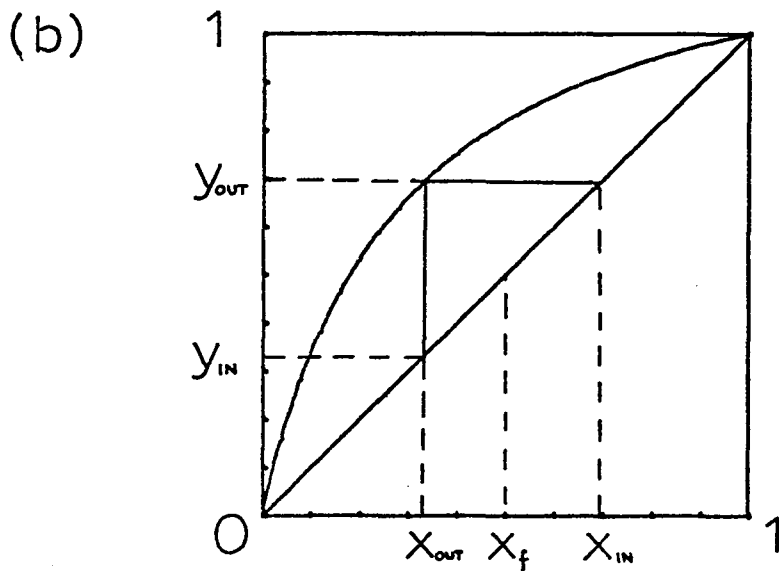
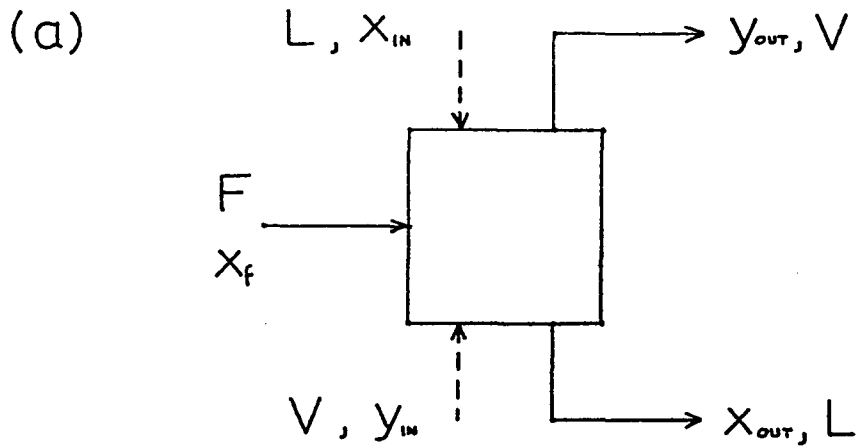


Figure 14. Equivalent Single Stage System

(a) Feed, product and fictitious streams

(b) Equivalent distillation system

Equation (49) is, indeed, the same as Eq. (31) which was derived for the extent of purification, i.e.,  $\xi_m$  reduces to  $\xi_p$  for a single stage system.

Figure 15 shows the comparison of a multistage distillation system with an "equivalent" single stage which produces the same top and bottom product. The feed arrangement of the equivalent system is single-feed double-output, as shown in Figure 1(a). The relative volatility of this equivalent system is approximately 360, and the relative volatility of the actual system is 2.5. If we calculate  $\xi_m$  for this system, we will not get the same value as the "overall"  $\xi_m$  which is the summation of the two  $\xi_m$  calculated from Eqs. (44) and (45). It should be noted here that the value of  $\xi_m$ , calculated from Eq. (44) cannot be added directly to the value of  $\xi_m$ , calculated from Eq. (45) because the two values represent  $\Delta x$  and  $\Delta y$  respectively. One value of  $\xi_m$  must be converted to the same basis as the other according to Eq. (38). The overall value of  $\xi_m$ , calculated this way, will be less than the value of  $\xi_m$ , calculated for the equivalent system because the former incorporates penalty factors,  $(\frac{L}{V} + A)$  and  $(1 + \frac{A}{L/V})$  which are less than unity. The fact that the values of  $\xi_m$  for each system are not the same, bring us to the conclusion that  $\xi_m$  differentiates a single stage system from a multistage system. It also points out the impossibility of representing a multistage system by an equivalent single stage.

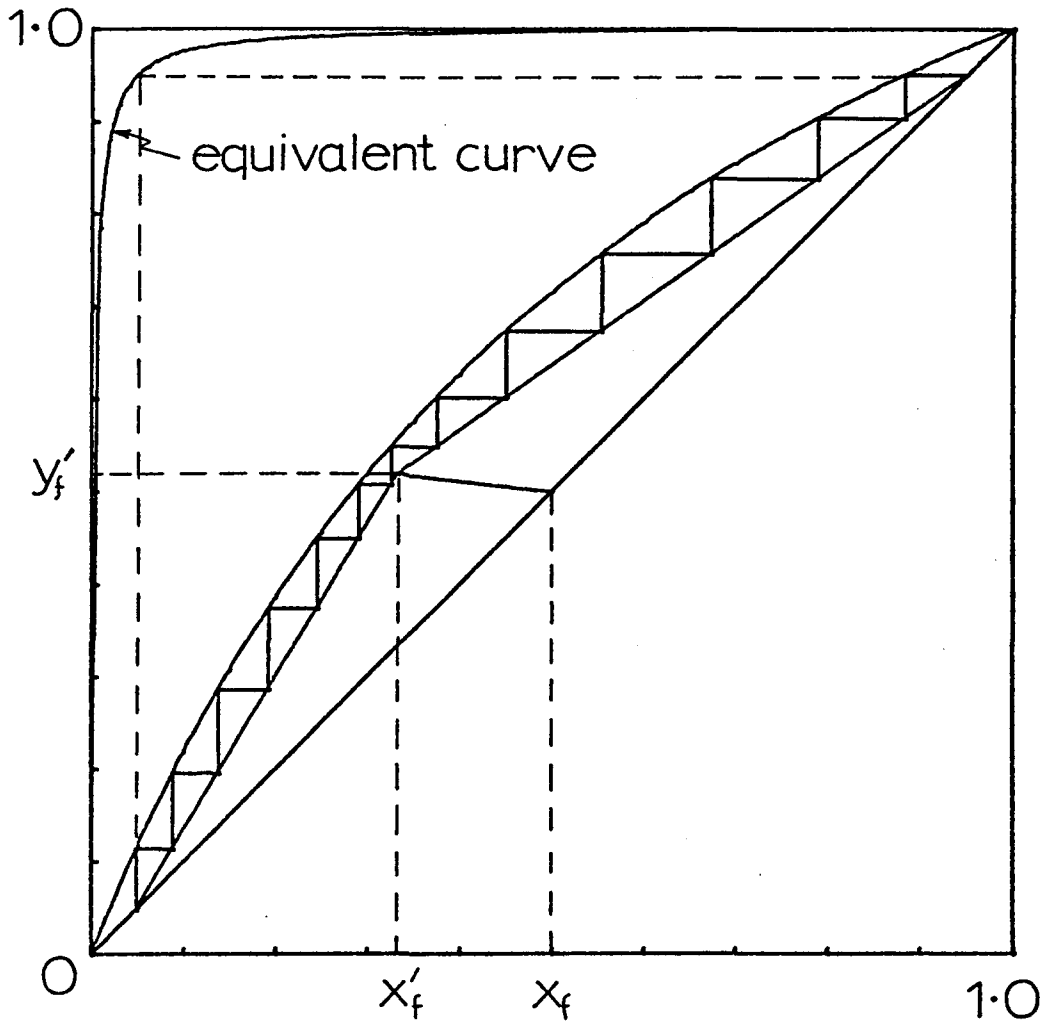


Figure 15. Comparison of Multistage and Equivalent Single Stage Systems

## VI. OTHER INDICES

While in the process of developing the new index,  $\xi_m$ , we tried a number of alternate indices which we will now briefly describe. They are:

$$\xi_1 = \text{abs}(x_{n-1} - x_n) \quad (52)$$

$$\xi_2 = \text{abs}(y_n - y_{n+1}) \quad (53)$$

$$\begin{aligned} \xi_3 &= [(x_{n-1} - x_n)^2 + (y_n - y_{n+1})^2]^{1/2} \\ &= [\xi_1^2 + \xi_2^2]^{1/2} \end{aligned} \quad (54)$$

$$\xi_4 = \text{abs} \begin{vmatrix} X_{21} & \bar{X}_{21} \\ \bar{X}_{12} & X_{12} \end{vmatrix} \quad (55)$$

$$= \text{abs}[(1 - A)(x_{n-1} - x_n)] \quad (56)$$

$$= \text{abs}\left[\frac{(1 - A)}{L/V} (y_n - y_{n+1})\right] \quad (57)$$

$$\xi_5 = \text{abs} \begin{vmatrix} X_{12} & \bar{X}_{12} \\ \bar{X}_{21} & X_{21} \end{vmatrix} \quad (58)$$

Equation (58) also reduces to Eqs. (56) and (57).

Discussions.  $\xi_1$  and  $\xi_2$  have straight forward physical interpretations. They respectively represent  $\Delta x$  and  $\Delta y$  between each stage in the system.  $\xi_3$  is the section of the operating line that lies within one stage in the McCabe-Thiele diagram (see Figure 3). These three indices do not differentiate a multistage system from an equivalent single stage system. Further,  $\xi_3$  is not normalized and normalization would involve a complicated mathematical expression.



Table 3

Values of  $(1 - A)$  and  $\frac{(1 - A)}{L/V}$  at Limiting Conditions

Section	$1 - A$	$\frac{1 - A}{L/V}$	Occur when
Rectifying	1	1	$L/V = 1$
	$1 - x_D$	$\infty$	$L/V = 0$
	$L/V$	1	$x_D = 1$
	1	$\frac{1}{L/V}$	$x_D = 0$
Stripping	1	1	$L/V = 1$
	$\infty$	$x_B$	$L/V = \infty$
	$L/V$	1	$x_B = 1$
	1	$\frac{1}{L/V}$	$x_B = 0$

$\xi_4$  and  $\xi_5$  are obtained from a permutation of the component and region indices in  $\xi_m$ . Table 3, similar to table 2 for  $\xi_m$ , shows the values of factors  $(1 - A)$  and  $\frac{(1 - A)}{L/V}$  of Eqs. (56) and (57), respectively, which occur at limiting conditions in each section of the distillation column. As before, we seek a factor which ranges from 0 to 1 in each operating section. It appears that  $(1 - A)$  and  $\frac{(1 - A)}{L/V}$  must be used in the rectifying and stripping sections, respectively. But for the typical operating conditions considered earlier (Figure 5), the values of  $(1 - A)$  and  $\frac{(1 - A)}{L/V}$  deviate so far from unity. As an example, let us consider case (b) in Figure 5, where  $L/V = 0.60$ ,  $x_D = 0.95$ ,  $x_B = 0.05$  and  $L'/V' = 1.40$ . The corresponding values for  $(1 - A)$  and  $\frac{(1 - A)}{L/V}$  are 0.62 and 0.729, respectively. On the other hand, if we use  $\frac{(1 - A)}{L/V}$  and  $(1 - A)$  for the top and bottom sections, respectively, the values are 1.034 and 1.02, i.e., the indices  $\xi_4$  and  $\xi_5$  follow the change in the product streams compositions. The two factors can then be called "merit factors" rather than "penalty factors".

## VII. APPLICATIONS TO OTHER MULTISTAGE PROCESSES

In section V, we have demonstrated the applicability of the new index,  $\xi_m$ , to a distillation process. We will now consider another graphical technique, the Ponchon-Savarit method, which involves less restrictive assumptions than the McCabe-Thiele method. Application of  $\xi_m$  to other common chemical engineering multistage processes such as gas absorption, leaching, and liquid-liquid extraction will also be considered. A slight modification of the index will be necessary for some processes due to the fact that the commonly used concentration units for such processes are in mass fraction units, not the mole fraction units with which we have used to define the index.

Ponchon-Savarit Graphical Technique. The limitations imposed by the assumption of constant molal overflow in the McCabe-Thiele method may be removed if the analysis of distillation equipment is based on enthalpy balances as well as material balances. The method usually referred to as the "Ponchon-Savarit method" is based on the graphical use of an enthalpy-concentration diagram<sup>(8)</sup>. Figure 16 is a typical Ponchon-Savarit construction for a distillation column. P and P' are the net flow of enthalpy points of the rectifying and stripping sections, respectively. Operating lines are drawn through these points. The enthalpy operating lines are used alternately with the lines to give a step-by-step determination



of the number of ideal stages required to accomplish a given separation. In comparing the Ponchon-Savarit method with the McCabe-Thiele method, the tie lines correspond to the equilibrium curve and the enthalpy operating lines on the enthalpy-concentration diagram to the material-balance operating line on the x-y diagram.

We have shown in appendix D that, by geometry,  $\xi_m$  can be written as

$$\begin{aligned} \xi_m = & (x_{n-1} - x_n) \left[ \frac{x_{n-1} - x_n}{y_n - y_{n+1}} + x_D \left( 1 - \frac{x_{n-1} - x_n}{y_n - y_{n+1}} \right) \right] \\ & + (x_D - x_{n-1})(y_D - y_{n+1}) \left[ 1 - \frac{L_{n-1}}{V_n} \frac{V_{n+1}}{L_n} \right] \end{aligned} \quad (59)$$

for the rectifying section and,

$$\begin{aligned} \xi_m = & (y_n - y_{n+1}) \left[ 1 + \frac{x_B \left( \frac{y_n - y_{n+1}}{x_{n-1} - x_n} - 1 \right)}{\frac{y_n - y_{n+1}}{x_{n-1} - x_n}} \right] \\ & + (y_{n+1} - y_B)(x_{n-1} - x_B) \left[ 1 - \frac{L_{n-1}}{V_n} \frac{V_{n+1}}{L_n} \right] \end{aligned} \quad (60)$$

for the stripping section.

If we assume the condition of constant reflux,

$$\text{i.e.} \quad \frac{L_{n-1}}{V_n} = \frac{V_{n+1}}{L_n} \quad (61)$$

the last portions of the expressions in Eqs. (59) and (60) become zero. Equations (59) and (60) can be further simplified, using Eqs. (34) and (38), to

$$\xi_m = \left(\frac{L}{V} + A\right)(x_{n-1} - x_n) \quad (62)$$

$$= \left(1 + \frac{A}{L/V}\right)(y_n - y_{n+1}) \quad (63)$$

which are the same as Eqs. (39) and (37), which have been derived for the McCabe-Thiele method.

The last terms of Eqs. (59) and (60) represent the fact that the reflux ratios in the system are not constant. The term can either be positive or negative depending on the sign of the factor

$$\left(1 - \frac{L_{n-1}}{V_n} \frac{V_{n+1}}{L_n}\right).$$

If  $\frac{L_{n-1}}{V_n} > \frac{L_n}{V_{n+1}}$ , the term is negative and  $\xi_m$  is smaller. This reflects from the fact that the reflux ratio becomes smaller as we proceed down from the top plate.

If  $\frac{L_{n-1}}{V_n} < \frac{L_n}{V_{n+1}}$ , the term is positive and  $\xi_m$  is larger, i.e., reflux ratio becomes larger as we proceed down the column.

It should be noted that, by definition,  $\xi_m$  is always normalized.

Application to Multistage Gas Absorption. Unlike distillation, the absorption and stripping sections in a gas absorption process are separated into two columns. Subject to certain simplifying assumptions, we can construct two McCabe-Thiele diagrams, one for each

section. These diagrams are shown in Figure 17.

Equation (V.11) of reference (3) expresses material balances in either section as

$$Y_{n+1} = \frac{L}{V} X_n + (Y_{out} - \frac{L}{V} X_{in}) \quad (64)$$

where  $Y = y/(1-y)$  (65)

$$X = x/(1-x) \quad (66)$$

Equation (64) is the equation of the operating in each section. If we plot this operating line equation on an x-y equilibrium diagram, the line will be slightly curved. However, at low solute concentrations, as usually is the case in gas absorption, the curvature can be neglected. It should be noted that the operating lines do not start or finish on the diagonal line,  $x = y$ . Equation (64) can then be written as

$$y_{n+1} = \frac{L}{V} x_n + A \quad (67)$$

where  $A = y_{out} - \frac{L}{V} x_{in} = \text{constant}$  (68)

The application of  $\xi_m$  leads to, as in the case of distillation,

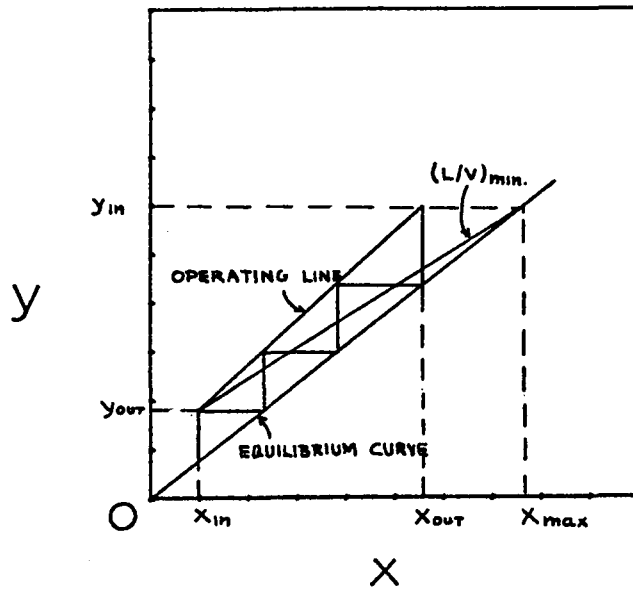
$$\xi_m = \text{abs}[\left(\frac{L}{V} + A\right)(x_{n-1} - x_n)] \quad (69)$$

$$= \text{abs}\left[\left(1 + \frac{A}{L/V}\right)(y_n - y_{n+1})\right] \quad (70)$$

In terms of X and Y, Eqs. (69) and (70) become

$$\xi_m = \text{abs}\left[\left(\frac{L}{V} + A\right)\left(\frac{X_{n-1}}{1+X_{n-1}} - \frac{X_n}{1+X_n}\right)\right] \quad (71)$$

(a)



(b)

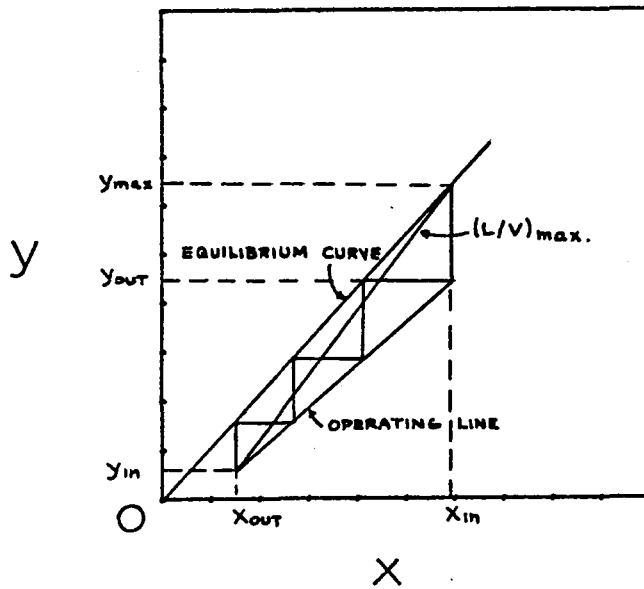


Figure 17. McCabe-Thiele Diagram for Gas Absorption

(a) Absorption column

(b) Stripping column



$$= \text{abs}\left[\left(1 + \frac{A}{L/V}\right)\left(\frac{Y_n}{1+Y_n} - \frac{Y_{n+1}}{1+Y_{n+1}}\right)\right] \quad (72)$$

Since the operating line in each section does not terminate on the diagonal, the reflux ratio in each section can now be equal to, greater or smaller than one. The factors  $\left(\frac{L}{V} + A\right)$  and  $\left(1 + \frac{A}{L/V}\right)$  must be interpreted differently from what has been done in distillation. As an example, let us consider the absorption section from which the composition of the flue gas must be less than 0.5%, i.e.,  $y_{\text{out}} \leq 0.005$ . If the feed is pure liquid, i.e.,  $x_{\text{in}} = 0$ , then  $A = 0.005$ . The factor  $\left(1 + \frac{A}{L/V}\right)$ , which is used when  $\frac{L}{V} \geq 1$ , becomes maximum at  $\frac{L}{V} = 1$  with a value of 1.005 and minimum of 1.0 at  $\frac{L}{V} = \infty$ . The factor  $\left(\frac{L}{V} + A\right)$ , which is used when  $\frac{L}{V} \leq 1$ , also is maximum at  $\frac{L}{V} = 1$  with a value of 1.005 and a minimum of 0.005 at  $\frac{L}{V} = 0$ . This means that, for this system,  $\frac{L}{V} = 1$  is the best reflux ratio above which separation is achieved at the expense of excessive liquid. Factor  $\left(\frac{L}{V} + A\right)$  becomes unity at  $\frac{L}{V} = 0.995$ , below this value,  $\left(\frac{L}{V} + A\right)$  is less than one and penalty is caused by either insufficient liquid or excessive gas. The optimum values of the two factors always occur at  $L/V = 1$ . Two extreme cases, where an infinite number of stages are required, are shown in Figure 17,  $\left(\frac{L}{V}\right)_{\text{min}}$  in the absorption column and  $\left(\frac{L}{V}\right)_{\text{max}}$  in the stripping column. These conditions correspond to the minimum and maximum reflux ratios in a distillation column, respectively.

Application to Leaching. Unlike distillation and gas absorption, concentration units for leaching are often expressed in mass fraction

rather than in mole fraction. A typical Ponchon-Savarit diagram for leaching is shown in Figure 18. Most standard texts on chemical engineering include this technique in their discussions<sup>(8)</sup>. Each stream is considered as a mixture of solid and solution. The solution, in turn, is a mixture of solute and solvent. The ratio of solid to solution is used in place of enthalpy in distillation and the abscissa denotes the solid free mass concentration of the solution. J is called a sum point and P, a difference point.

Because of the commonly used concentration units in leaching are mass fraction, we will modify the index,  $\xi_m$ , so that mass fractions can be used in place of mole fractions. Since the liquid leaving the stage with the solid and the solvent always have the same composition, Eq. (33) becomes

$$\xi_m = \text{abs}[X_n(1 - X_n) - X_{n+1}(1 - X_{n-1})] \quad (73)$$

where  $X_n$  is the mass fraction of solute in the liquid leaving stage n(solid free basis).

Since the value of X ranges from 0 to 1,  $\xi_m$  is normalized.

Application to Liquid-Liquid Extraction. There are a number of graphical methods which can be used in liquid-liquid extraction. The Ponchon-Savarit method is one of the simplest and does not involve many restrictions. A typical Ponchon-Savarit diagram for liquid-liquid extraction is shown in Figure 19. The graphical method based on a triangular equilibrium diagram<sup>(8)</sup> and the modified McCabe-

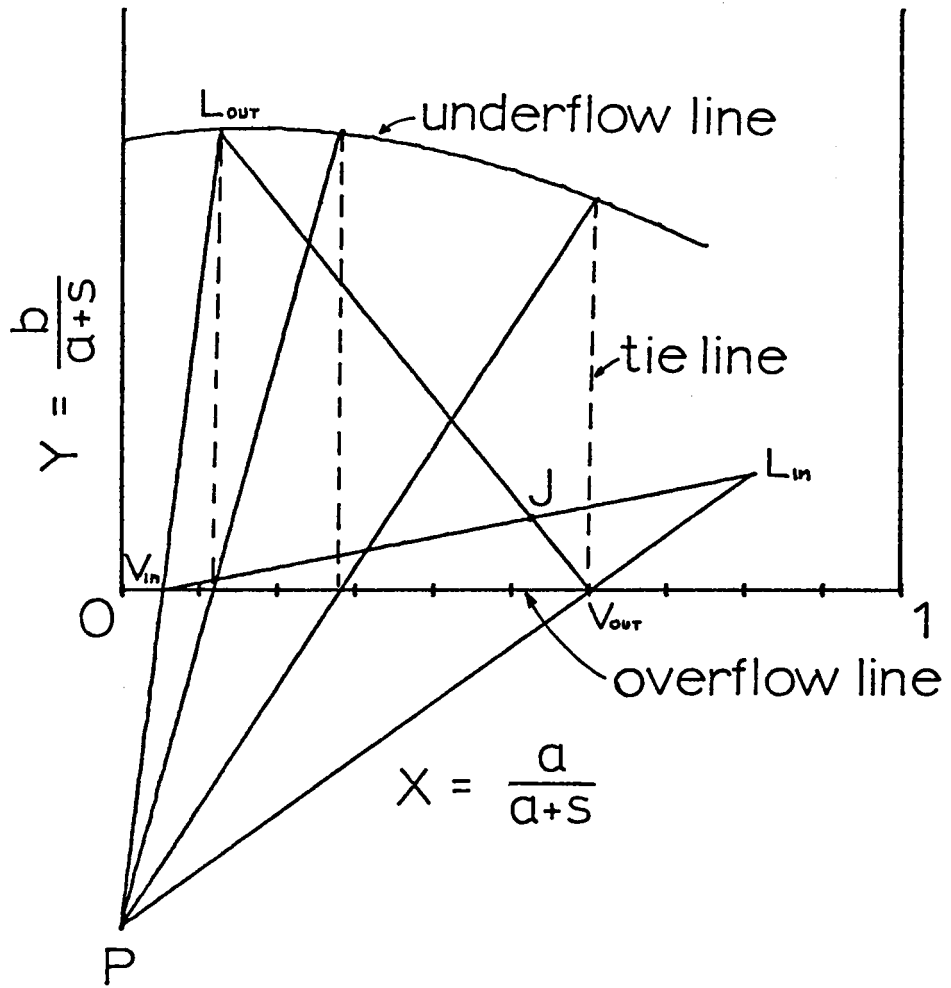


Figure 18. Ponchon-Savarit Diagram for Leaching.  $a$ , solute.  $b$ , solid.  $s$ , solvent.

Thiele method<sup>(8)</sup> will not be covered here. The application of the index to these methods are similar to methods previously discussed. Like leaching, mass fractions are preferred to mole fractions as concentration units. Each stream is considered as a mixture of solvent, the extracting agent, and solution. The solution, in turn, is a mixture of solute and diluent. The ratio of solvent to solution is used in place of enthalpy in distillation and the abscissa denotes the solvent free mass concentration of the solution.  $X_e$  and  $X_r$  denote the equilibrium concentrations of the live phases leaving a stage, the extract or solvent rich phase, and the raffinate or diluent rich phase.  $P_D$  and  $P_B$  are the difference points for extract enriching section and raffinate stripping section, respectively.

For liquid-liquid extraction, Eq. (33) becomes

$$\xi_m = \text{abs}[X_{e,n}(1 - X_{r,n}) - X_{e,n+1}(1 - X_{r,n-1})] \quad (74)$$

Since the values of  $X_e$  and  $X_r$  range from 0 to 1,  $\xi_m$  is normalized.



## VIII. RECOMMENDATIONS

1. The new separation index,  $\xi_m$ , is the best one that we have developed, but it is not necessarily the best possible index. The indices  $\xi_4$  and  $\xi_5$ , obtained from permutation of the component and region indices have similar characteristics as  $\xi_m$ .

2. A detailed comparison between countercurrent, crosscurrent and cocurrent continuous multistage systems might reveal some of the important characteristics of the index,  $\xi_m$ .

3. So far, we have considered only the plate-type or stage-wise processes. In processes such as gas absorption, ion exchange, etc., packed column are more commonly used. In such a column, separation is carried out by differential action rather than step-wise action. The concepts of Height of Transfer Unit (H.T.U.) and Height Equivalent to one Theoretical Plate (H.E.T.P.) are useful. Application of the new index,  $\xi_m$ , to such processes should be possible.

## IX. CONCLUSIONS

We have employed several different separation indices in a study of the stage-to-stage separation achieved in a multistage distillation column. We conclude that:

1. Both the extent of separation and the extent of purification are not useful separation indices in such a system.

2. The difference in the extent of separation from stage to stage is not a useful separation index or guide as to what is happening in the distillation column.

3. The extent of separation can be used to design a multi-feed distillation system.

4. A new separation index,  $\xi_m$ , gives a much more realistic comparison of the contributions of individual stages to the observed separation in a multistage distillation column.

5. The new separation index,  $\xi_m$ , can also be applied to a single stage separation system as well as to other types of multistage separation processes.

Table 4

Various Forms of  $\xi_m$  for Different Binary Multistage Processes

Process	$\xi_m$
Distillation	$\text{abs}[y_n(1-x_n) - y_{n+1}(1-x_{n-1})]$
Gas Absorption	$\text{abs}[y_n(1-x_n) - y_{n+1}(1-x_{n-1})]$
Leaching	$\text{abs}[X_n(1-X_n) - X_{n+1}(1-X_{n-1})]$
Liquid-liquid Extraction	$\text{abs}[X_{e,n}(1-X_{r,n}) - X_{e,n+1}(1-X_{r,n-1})]$



## X. SUMMARY

The applications of the extent of separation and the extent of purification to multistage separation processes have been studied in detail. Neither index is suitable for stage-to-stage calculations in such processes. This is due to the fact that, at any stage, both indices depend only on the outlet stream compositions. It is not possible to reconstruct the inlet stream compositions knowing only the equilibrium compositions of the outlet streams, i.e., both indices represent an underspecification of the single stage within a multistage system.

A new index,

$$\xi_m = \text{abs} \begin{vmatrix} X_{11} & X_{11} \\ X_{22} & X_{22} \end{vmatrix}$$

is proposed. The application of this new index to a multistage distillation process gives a much more realistic comparison of the contribution of individual stages to the observed separation. Applications to other types of multistage separation processes as well as a single stage system have also been studied. A summary of various forms of the new index  $\xi_m$  for different binary multistage processes are given in Table 4.

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## APPENDIX A

### SAMPLE CALCULATIONS

Calculation of the extent of separation. Let us consider, as an example, case (b) shown in Figure 5. (Table 6).

The equilibrium compositions of the vapor and liquid phases leaving the first stages are 0.95000 and 0.88372 respectively.

Then, according to Eq. (18),

$$\begin{aligned}\xi &= \text{abs}\left[\frac{0.95000}{0.95000 + 0.6 \times 0.88372} - \frac{0.0500}{0.0500 + 0.6 \times 0.11628}\right] \\ &= 0.22432\end{aligned}$$

Calculation of the new index,  $\xi_m$ . Let us consider the same example as above. The compositions of the vapor and liquid phases entering the first stages are 0.91023 and 0.95000 respectively.

Then, according to Eq. (33),

$$\begin{aligned}\xi_m &= \text{abs}[0.95000(1 - 0.88372) - 0.91023(1 - 0.95000)] \\ &= 0.06495\end{aligned}$$

Calculation for a Multi-Feed Distillation System. Consider a system with  $x_D = 0.95$ ,  $x_f = 0.50$ ,  $x_B \leq 0.05$  and  $\alpha = 2.5$ .

1st Stage  $y_D = 0.95$

The composition of the liquid leaving the stage is given by

$$x = \frac{y}{y + \alpha(1-y)}$$

$$\begin{aligned} &= \frac{0.95}{0.95 + 2.5 \times 0.05} \\ &= 0.88372 \end{aligned}$$

Therefore, the equilibrium compositions of the first stage is (0.95000, 0.88372).

The reflux ratio required to locate  $\xi_{\max}$  at  $y = 0.95$  is given by Eq. (26),

$$\begin{aligned} \frac{L}{V} &= \frac{0.95 + 2.5(1 - 0.95)}{1.581} \\ &= 0.67989 \end{aligned}$$

The equation of the operating line is

$$y = A + \frac{L}{V} x$$

Since the point (0.95, 0.95) is on the operating line, then,

$$\begin{aligned} A &= 0.95 - 0.67989 \times 0.95 \\ &= 0.30410 \end{aligned}$$

The point  $x = 0.883721$  is also on the operating line, therefore,

$$\begin{aligned} y &= 0.30410 + 0.67989 \times 0.88372 \\ &= 0.90494 \end{aligned}$$

The second point on the operating line is (0.90494, 0.88372).

The above calculation is repeated until  $x$  is equal to or less than 0.05 (see Table 11).

At the feed plate, the coordinates of the operating line is (0.49635, 0.40532), then,

$$\begin{aligned} f &= \frac{x_f - x}{y - x} \\ &= \frac{0.50 - 0.40532}{0.49635 - 0.40532} \\ &= 1.04007 \end{aligned}$$

i.e., the feed has to be slightly superheated.

APPENDIX B

For a multistage distillation system, the extent of separation is given by

$$\xi = \text{abs} \left[ \frac{y_n}{y_n + \frac{L}{V} x_n} - \frac{(1 - y_n)}{(1 - y_n) - \frac{L}{V}(1 - x_n)} \right]$$

If  $\xi = 0$ , then

$$\frac{y_n}{y_n + \frac{L}{V} x_n} = \frac{(1 - y_n)}{(1 - y_n) + \frac{L}{V}(1 - x_n)}$$

$$y_n(1 - y_n) + y_n(1 - x_n) \frac{L}{V} = (1 - y_n) y_n + \frac{L}{V} x_n(1 - y_n)$$

i.e.,  $y_n = x_n$

For a constant  $\alpha$  system, Eq. (8) can be written as

$$x_n = \frac{y_n}{y_n + \alpha(1 - y_n)}$$

then

$$y_n = \frac{y_n}{y_n + \alpha(1 - y_n)}$$

$$\alpha(1 - y_n) = (1 - y_n)$$

i.e.,  $\alpha = 1$

This implies that  $\xi$  will be zero only when  $\alpha = 1$  or when  $y_n = 1$ .

APPENDIX C

For a multistage distillation system, we can write

$$Y_{11} = \frac{1}{1 + K_1} = \frac{y_n}{y_n + \frac{L}{V} x_n}$$

According to Eq. (14),  $\xi_{\max}$  occurs when  $K_1 = \alpha^{-1/2}$  therefore,

$$\frac{1}{1 + \alpha^{-1/2}} = \frac{y_n}{y_n + \frac{L}{V} \left( \frac{y_n}{y_n + \alpha(1-y_n)} \right)}$$

$$y_n = \frac{\alpha - \frac{L}{V} \alpha^{1/2}}{(\alpha-1)} \quad (i)$$

and  $\frac{L}{V} = \frac{y_n - \alpha(1 - y_n)}{\alpha^{1/2}} \quad (ii)$

$$= \frac{\alpha^{1/2}}{1 + x_n(\alpha - 1)} \quad (iii)$$

Range of L/V in which  $\xi_{\max}$  occurs between  $y_n = 1$  and  $y_n = 0$ . For  $y_n = 1$ , Eq. (i) becomes

$$\alpha - 1 = \alpha - \frac{L}{V} \alpha^{1/2}$$

$$\frac{L}{V} = \alpha^{-1/2} = K_1$$

For  $y_n = 0$ , Eq. (i) becomes

$$0 = \alpha - \frac{L}{V} \alpha^{1/2}$$

$$\frac{L}{V} = \alpha^{1/2} = K_2$$

Therefore,  $\xi_{\max}$  occurs between  $y_n = 1$  and  $y_n = 0$  when

$$\alpha^{-1/2} \leq \frac{L}{V} \leq \alpha^{1/2}.$$



APPENDIX D

APPLICATION OF  $\xi$  TO PONCHON-SAVARIT METHOD

In the rectifying section, we can write, according to Eq. (33) and Figure 16,

$$\begin{aligned}\xi_m &= \text{abs}[y_n(1 - x_n) - y_{n+1}(1 - x_{n-1})] \\ &= ae \cdot bg - ac \cdot dg\end{aligned}$$

where  $ae$ ,  $bg$ ,  $ac$ , etc. represent line segments shown in Figure 16.

$$\begin{aligned}\xi_m &= (1 - eg)(bf + fg) - (1 - cg)(df + fg) \\ &= (1 - ef - fg)(bf + fg) - (1 - cf - fg)(df + fg) \\ &= -fg(bd - ce) + (bf - df) - ef \cdot bf + cf \cdot df \\ &= -fg(bd - ce) + bd + cf \cdot df \left(1 - \frac{ef}{df} \cdot \frac{bf}{cf}\right) \\ &= bd \left(1 - fg + fg \cdot \frac{ce}{bd}\right) + cf \cdot df \left(1 - \frac{ef}{df} \cdot \frac{bf}{cf}\right) \\ &= (x_{n-1} - x_n) \left[ x_D + (1 - x_D) \frac{x_{n-1} - x_n}{y_n - y_{n+1}} \right] \\ &\quad + (x_D - x_{n-1}) (y_D - y_{n+1}) \left[ 1 - \frac{L_{n-1}}{V_n} \cdot \frac{V_{n+1}}{L_n} \right] \\ &= (x_{n-1} - x_n) \left[ \frac{x_{n-1} - x_n}{y_n - y_{n+1}} + x_D \left(1 - \frac{x_{n-1} - x_n}{y_n - y_{n+1}}\right) \right] \\ &\quad + (x_D - x_{n-1}) (y_D - y_{n+1}) \left[ 1 - \frac{L_{n-1}}{V_n} \cdot \frac{V_{n+1}}{L_n} \right]\end{aligned}$$

Similarly, in the stripping section, we can show that

$$\xi_m = (y_n - y_{n+1}) \left[ 1 + \frac{x_B \left( \frac{y_n - y_{n+1}}{x_{n-1} - x_n} - 1 \right)}{\frac{y_n - y_{n+1}}{x_{n-1} - x_n}} \right]$$
$$+ (y_{n+1} - y_B)(x_{n-1} - x_B) \left[ 1 - \frac{L_{n-1}}{V_n} \cdot \frac{V_{n+1}}{L_n} \right]$$

APPENDIX E

This section contains a Hewlett-Packard calculator program, which was used to generate data in Tables 5 through 8. The program, when used in conjunction with a Hewlett-Packard plotter, draws the McCabe-Thiele diagram of the system being considered, step by step, along with the computation. Memory map and user's instruction are also given. Equipments needed are:

1. 9810 A Hewlett-Packard calculator Model 10
2. 9862 A Hewlett-Packard calculator plotter
3. 11215A Plotter Control Block (ROM).

Step

000	CLR		1		1
	STP		UP		UP
	XTO		.		
	0		1	050	FMT
	3		FMT		1
	8		1		UP
	XTO	030	5		0
	0		.		UP
	3		1		FMT
	7		FMT		1
010	CLR		1		DN
	1		6		CNT
	UP		CNT		CNT
	0		1	060	CNT
	FMT		UP		CNT
	1		FMT		CNT
	2	040	1		CNT
	FMT		DN		0
	1		0		.
	3		UP		0
020	CNT		1		5
	0		FMT		XTO
	UP		1		
	FMT		DN		

070	+		KEY		CNT
	0		DIY		CNT
	3		XFR	120	STP
	7		0		YTO
	XFR		3		0
	0		7		3
	3	100	FMT		7
	7		1		XTO
	UP		DN		0
	XFR		1		4
080	0		UP		1
	3		XFR		UP
	8		0	130	1
	x		3		KEY
	YTO		7		-
	a		X = Y		DN
	1	110	0		x
	+		1		YTO
	XFR		2		0
	0		0		4
090	3		GTO		0
	7		6		CLR
	-		2	140	STP
	a		CNT		XTO

	0		4	190	XFR
	4		0		0
	5		-		4
	UP		1		0
	FMT				+
		170	UP		
	1		a		CNT
	UP		-		CNT
	CLR		DIV		CNT
150	STP		XFR		CNT
	YTO		0		CNT
	b		4	200	CNT
	XTO		1		CNT
	a		+		CNT
	UP		DN		CNT
	0				XFR
		180	DIV		
	X = Y		YTO		0
	0		0		3
	2		3		8
160	2		6		DIV
	4		XFR		DN
	DN		0	210	+
	DIV		4		YTO
	XFR		1		0
	0		x		3

	5		XFR		XFR
	XFR		0		0
	0	240	4		3
	3		0		5
	6		+		UP
	GTO		YTO		XFR
220	0		0		0
	2		3		3
	5		5	270	6
	1		XFR		FMT
	b		0		1
	CNT		3		UP
	CNT	250	6		XFR
	CNT		FMT		0
	XTO		1		3
	0		DN		7
230	3		b		UP
	6		CNT		FMT
	UP		CNT	280	1
	XFR		CNT		DN
	0		UP		CNT
	4		FMT		CNT
	1	260	1		CNT
	x		DN		CNT

	CNT	310	3		5
	CNT		9		XFR
	CNT		CNT		0
	CNT		1		3
290	XFR		XEY		7
	0		-		XTO
	3		XFR	340	a
	5		0		UP
	UP		4		1
	XFR		5		XEY
	0	320	x		-
	4		YTO		XFR
	5		4		0
	-		2		3
300	UP		CNT		8
	XFR		0		x
	0		XTO	350	a
	3		3		+
	6		5		XEY
	XEY		CNT		DIV
	-	330	1		YTO
	DN		XTO		0
	DIV		+		3
	YTO		3		7



	a	4	XFR
	XEY	3	0
360	FMT	0	5
	1	XFR	410 UP
	DN	0	XFR
	XFR	4	0
	0	1	3
	3	x	7
	5	390 XFR	UP
	UP	0	XFR
	a	4	0
	UP	0	4
370	XFR	+	5
	0	XFR	
	3	0	420 X > Y
	7	3	STP
	STP	7	STP
	UP	YTO	STP
	XFR	400 0	STP
	0	3	GTO
	3	7	0
	6	FMT	3
380	X > Y	1	3
	0	DN	0

430 XFR

0

3

9

x

XFR

0

4

2

+

440 GTO

0

3

9

5

END

MEMORY MAP

<u>Register</u>	<u>Contents</u>
35	$y'_f$ and Stage Number
36	$x'_f$
37	$x_D$ and variables $x$ and $y$
38	$\alpha$
39	$L'/V'$
40	A
41	L /V
42	A'
43	-
44	-
45	$x_B$
46	-
47	-
48	-

Registers No. 0 through 34 are blank.

USER'S INSTRUCTION

Step	<u>Instruction</u>	Display		
		<u>x</u>	<u>y</u>	<u>z</u>
	Set plotter limits			
	Press: END			
	LOAD Program			
	Press: END			
	Press: CONTINUE	0	0	0
1	Enter: $\alpha$ (relative volatility)	$\alpha$		
	Press: CONTINUE			
	x-y equilibrium curve is drawn			
2	Enter: $x_D$ , L/V	L/V	$x_D$	
	Press: CONTINUE	0	0	0
3	Enter: $x_B$	$x_B$		
	plotter pen moves to coordinates ( $x_B$ , $x_B$ )			
4	Enter: $x_f$ , f	f	$x_f$	
5	Press: CONTINUE			
	operating lines and feed line are drawn and the equilibrium compositions of the first stage are displayed on screen	x	y	1

Repeat step 5 until all stages are drawn.

APPENDIX F

This section contains all the data generated by the program in Appendix E (Tables 5 through 8), and subsequent calculations (Tables 9, 10 and 12 through 15).

Table 5

Equilibrium Compositions in a Multistage Distillation with  $\frac{L}{V}=1$  and  $\frac{L'}{V'}=1$

n	y	x
1	.95000	.88372
2	.88372	.75248
3	.75248	.54874
4	.54874	.32723
5	.32723	.16287
6	.16287	.07220
7	.07220	.03019
	.03019	----

Data for Figure 5(a)

$$x_D = 0.95, x_B \leq 0.05$$

$$x_f = 0.50, \text{ saturated liquid feed}$$

$$\alpha = 2.5$$

Table 6

Equilibrium Compositions in a Multistage Distillation

with  $L/V = 0.6$  and  $L'/V' = 1.40$

n	y	x
1	.95000	.88372
2	.91023	.80221
3	.86133	.71302
4	.80781	.62704
5	.75623	.55374
6	.71225*	.49751
7	.67651	.45549
8	.61768	.39256
9	.52958	.31049
10	.41469	.22082
11	.28915	.13994
12	.17591	.07867
13	.09013	.03811
	.03336	-----

\*Feed plate

Data for Figure 5(b)

$x_D = 0.95$ ,  $x_B \leq 0.05$

$x_f = 0.50$ , saturated liquid feed

$\alpha = 2.5$

Table 7

Equilibrium Compositions in a Multistage Distillation

with  $L/V = 0.53$  and  $L'/V' = 1.47$

n	y	x
1	.95000	.88372
2	.91487	.81128
3	.87648	.73947
4	.83842	.67485
5	.80417	.62158
6	.77594	.58075
7	.75430	.55117
8	.73862	.53059
9	.72771	.51669
10	.72034	.50747
11	.71546	.50144
12	.71226*	.49753*
13	.70786	.49218
14	.70001	.48277
15	.68617	.46655
16	.66233	.43965
17	.62278	.39773



Table 7 (continued)

n	y	x
18	.56117	.33841
19	.47396	.26492
20	.36593	.18755
21	.25220	.11887
22	.15124	.06653
23	.07430	.03111
	.02223	----

\*Feed plate

Data for Figure 5(c)

$$x_D = 0.95, \quad x_B \leq 0.05$$

$x_f = 0.50$ , saturated liquid feed

$$\alpha = 2.5$$

Table 8

Equilibrium Compositions in a Multistage Distillation

with  $L/V = 0.523$  and  $L'/V' = 1.477$

n	y	x
1	.95000	.88372
2	.91534	.81219
3	.87793	.74205
4	.84124	.67944
5	.80850	.62808
6	.78164	.58878
7	.76108	.56029
8	.74618	.54043
9	.73579	.52695
10	.72875	.51799
11	.72406	.51210
12	.72098	.50825
13	.71897	.50576
14	.71766	.50415
15	.71682	.50312
16	.71628	.50245
17	.71593	.50202

Table 8 (continued)

n	y	x
18	.71571	.50174
19	.71556	.50156
20	.71547	.50145
21	.71541	.50138
22	.71537	.50133
23	.71535	.50130
	.71533	.50128

Data for Figure 5(d)

$$x_D = 0.95, x_B \leq 0.05$$

$x_f = 0.50$ , saturated liquid feed

$$\alpha = 2.5$$

Table 9

Calculated Values of  $\xi$  for Systems at Various Reflux Ratios

n	$\xi$ at L/V = 1	$\xi$ at L/V = 0.6	$\xi$ at L/V = 0.53	$\xi$ at L/V = 0.523
1	.21737	.22432	.22187	.22151
2	.22049	.22344	.22050	.22010
3	.22407	.22206	.21884	.21843
4	.22496	.22022	.21706	.21666
5	.22211	.21820	.21536	.21499
6	.21853	.21520*	.21390	.21358
7	.21622	.21684	.21276	.21247
8		.21903	.21191	.21166
9		.22155	.21132	.21108
10		.22371	.21092	.21068
11		.22489	.21065	.21043
12		.22514	.21304*	.21026
13		.22493	.21326	.21015
14			.21365	.21007
15			.21431	.21003
16			.21539	.21000
17			.21701	.20998
18			.21916	.20997

Table 9 (continued)

n	$\xi$ at L/V = 1	$\xi$ at L/V = 0.6	$\xi$ at L/V = 0.53	$\xi$ at L/V = 0.523
19			.22153	.20996
20			.22352	.20995
21			.22470	.20995
22			.22514	.20995
23			.22508	.20995

\*Feed plate

Data for Figure 6

$$x_D = 0.95, x_B \leq 0.05$$

$x_f = 0.50$ , saturated liquid feed

$$\alpha = 2.5$$

Table 10  
Difference in  $\xi$  from Stage to Stage  
For Systems at Various Reflux Ratios

n	$\frac{\Delta\xi}{L/V = 1}$	$\frac{\Delta\xi}{L/V = 0.6}$	$\frac{\Delta\xi}{L/V = 0.53}$	$\frac{\Delta\xi}{L/V = 0.523}$
1	.00312	.00088	.00137	.00141
2	.00358	.00138	.00166	.00167
3	.00089	.00184	.00178	.00177
4	.00285	.00202	.00170	.00167
5	.00358	.00291	.00146	.00141
6	.00231	.00155	.00114	.00111
7		.00219	.00085	.00081
8		.00252	.00059	.00058
9		.00216	.00040	.00040
10		.00118	.00027	.00025
11		.00025	.00239	.00017
12		.00021	.00022	.00011
13			.00039	.00008
14			.00066	.00004
15			.00108	.00003
16			.00162	.00002
17			.00215	.00001

Table 10 (continued)

Difference in  $\xi$  from Stage to Stage  
For Systems at Various Reflux Ratios

n	$\Delta\xi$ L/V = 1	$\Delta\xi$ L/V = 0.6	$\Delta\xi$ L/V = 0.53	$\Delta\xi$ L/V = 0.523
18			.00237	.00001
19			.00199	.00001
20			.00118	.00000
21			.00044	.00000
22			.00006	.00000
23				

Data for Figure 7

$$x_D = 0.95, x_B \leq 0.05$$

$x_f = 0.50$ , saturated liquid feed

$$\alpha = 2.5$$

Table 11

Multi-feed Distillation System

n	Coordinates on Equilibrium Curve		Coordinates on Operating Line		L/V
	y	x	y	x	
1	.95000	.88372	.90494	.88372	.67989
2	.90494	.79200	.83866	.79200	.72264
3	.83866	.67524	.74694	.67524	.78552
4	.74694	.54142	.63017	.54142	.87253
5	.63017	.40532	.49635	.40532	.98330
6	.49635	.28275	.36026	.28275	1.11026
7	.36026	.18384	.23768	.18384	1.23937
8	.23768	.11088	.13878	.11088	1.35566
9	.13878	.06055	.06582	.06055	1.44948
10	.06582	.02741	.02741	.02741	1.15895

Data for Figure 9

$$x_D = 0.95, x_B \leq 0.05$$

$$x_f = 0.50$$

$$\alpha = 2.5$$



Table 12

Calculated Values of  $\xi_m$ ,  $\Delta x$  and  $\Delta y$  for a  
Multistage Distillation with  $\frac{L}{y} = 1$  and  $\frac{L'}{y'} = 1$

n	$\xi_m$	$\Delta x$	$\Delta y$
1	.06628	.06628	.06628
2	.13120	.13120	.13120
3	.20390	.20390	.20390
4	.22151	.22151	.22151
5	.16501	.16501	.16501
6	.09067	.09067	.09067
7	.04201	.04201	.04201

Data for Figure 11(a)

$$x_D = 0.95, x_B \leq 0.05$$

$x_f = 0.50$ , saturated liquid feed

$$\alpha = 2.5$$

Table 13

Calculated Values of  $\xi_m$ ,  $\Delta x$  and  $\Delta y$  for a  
Multistage Distillation with  $\frac{L}{V} = 0.60$  and  $\frac{L'}{V'} = 1.40$

n	$\xi_m$	$\Delta x$	$\Delta y$
1	.06495	.06628	.03977
2	.07988	.08151	.04890
3	.08741	.08919	.05352
4	.08426	.08598	.05158
5	.07183	.07330	.04398
6	.05600	.05623	.03574
7	.05799*	.04202*	.05883*
8	.08684	.06293	.08810
9	.11326	.08207	.11489
10	.12375	.08967	.12554
11	.11162	.09088	.11324
12	.08455	.06127	.08478
13	.05596	.04056	.05677

\*Feed plate

Data for Figure 11(b)

$$x_D = 0.95, x_B \leq 0.05$$

$x_f = 0.50$ , saturated liquid feed

$$\alpha = 2.5$$

Table 14

Calculated Values of  $\xi_m$ ,  $\Delta x$  and  $\Delta y$  for a  
Multistage Distillation with  $\frac{L^*}{V} = 0.53$  and  $\frac{L'}{V'} = 1.47$

n	$\xi_m$	$\Delta x$	$\Delta y$
1	.06472	.06628	.03513
2	.07074	.07244	.03839
3	.07012	.07181	.03806
4	.06310	.06462	.03425
5	.05202	.05327	.02823
6	.03987	.04083	.02164
7	.02886	.02958	.01568
8	.02010	.02058	.01091
9	.01340	.01390	.00737
10	.00900	.00922	.00488
11	.00589	.00603	.00320
12	.00498*	.00391*	.00440*
13	.00773	.00535	.00785
14	.01362	.00941	.01384
15	.02346	.01622	.02384
16	.03891	.02690	.03955
17	.06063	.04193	.06161

Table 14 (continued)

n	$\xi_m$	$\Delta x$	$\Delta y$
18	.08581	.05932	.08721
19	.10630	.07349	.10803
20	.11191	.07737	.11373
21	.09935	.06868	.10096
22	.07571	.05234	.07694
23	.05124	.03542	.05207

\*Feed plate

Data for Figure 11(c)

$$x_D = 0.95, x_B \leq 0.05$$

$x_f = 0.50$ , saturated liquid feed

$$\alpha = 2.5$$

Table 15

Calculated Values of  $\xi_m$ ,  $\Delta x$  and  $\Delta y$  for a  
Multistage Distillation with  $\frac{L}{V} = 0.523$  and  $\frac{L'}{V'} = 1.477$

n	$\xi_m$	$\Delta x$	$\Delta y$
1	.06982	.07153	.03466
2	.06847	.07014	.03741
3	.06112	.06261	.03669
4	.05014	.05136	.03274
5	.03836	.03930	.02686
6	.02781	.02849	.02056
7	.01939	.01986	.01490
8	.01316	.01348	.01039
9	.00875	.00896	.00704
10	.00575	.00589	.00469
11	.00376	.00385	.00308
12	.00243	.00249	.00201
13	.00157	.00161	.00131
14	.00101	.00103	.00084
15	.00065	.00067	.00054
16	.00042	.00043	.00035
17	.00027	.00028	.00022

Table 15 (continued)

n	$\xi_m$	$\Delta x$	$\Delta y$
18	.00018	.00018	.00015
19	.00011	.00011	.00009
20	.00007	.00007	.00006
21	.00005	.00005	.00004
22	.00003	.00003	.00002
23	.00003	.00003	.00002

Data for Figure 11(d)

$$x_D = 0.95, x_B \leq 0.05$$

$x_f = 0.50$ , saturated liquid feed

$$\alpha = 2.5$$

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# THE APPLICATION OF SEPARATION INDICES TO MULTISTAGE SYSTEMS

by

Adisorn Chaipayungpan

## (ABSTRACT)

The applications of the extent of separation and the extent of purification to multistage separation processes have been studied in detail. Neither index is suitable for stage-to-stage calculations in such processes. This is due to the fact that, at any stage, both indices depend only on the outlet stream compositions. It is not possible to reconstruct the inlet stream compositions knowing only the equilibrium compositions of the outlet streams, i.e., both indices represent an underspecification of the single stage within a multistage system.

A new index,

$$\xi_m = \text{abs} \begin{vmatrix} X_{11} & \bar{X}_{11} \\ X_{22} & X_{22} \end{vmatrix}$$

is proposed. The application of this new index to a multistage distillation process gives a much more realistic comparison of the contribution of individual stages to the observed separation. Applications to other types of multistage separation processes as well as a single stage system have also been studied.