A SCHEME FOR IDENTIFICATION OF A SET OF NONLINEAR EQUATIONS

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SYMBOLS

Term	
x	n-dimension vector of outputs (states)
u	m-dimension vector of inputs (controls)
A	n x n matrix of coefficients of the outputs
B	n x m matrix of coefficients of the inputs
y	n-dimension vector of identifier outputs
C	n x n matrix of coefficients of the identifier error
G	n x n matrix of coefficients of the identifier outputs
H	n x m matrix of coefficients of the identifier inputs
g_{ij}	an element of G -- row i, column j
h_{ij}	an element of H -- row i, column j
e	error -- y - x
$V(e;t)$	Lyapunov function
P	n x n coefficient matrix of the error
φ	n x n coefficient matrix -- G - A
ψ	n x m coefficient matrix -- H - B
φ_i	n x 1 column matrix of φ
ψ_i	n x 1 column matrix of ψ
Δx_i	difference between the state and the equilibrium value
Δu_k	difference between the k^{th} input and the corresponding equilibrium value
Γ	n x n matrix of parameters of $\dot{G}$
Λ	n x m matrix of parameters of $\dot{H}$
V	velocity (ft/sec)
γ	flight path angle (rad)

SYMBOLS (CONT.)

q	pitch rate (rad/sec)
θ	attitude angle (rad)
C_T	nondimensional thrust factor
δ_e	elevator angle (deg)
δ_f	flap angle (deg)
α	angle of attack (deg)
ρ	density (slugs/ft^3)
g	acceleration of gravity (ft/sec^2)
S	wing surface area (ft^2)
D	drag (lbs)
L	lift (lbs)
M	pitch moment (ft-lbs)
$\bar{c}$	mean aerodynamic chord (ft)
I_y	moment of inertia about the y-axis (slugs/ft^2)
QC_D	3 x 3 x 3 coefficient matrix of the coefficient of drag
QC_L	3 x 3 x 3 coefficient matrix of the coefficient of lift
QC_M	3 x 3 x 3 coefficient matrix of the coefficient of pitching moment
$C_{D_{\delta_e}}$	partial derivative of the coefficient of drag with respect to elevator angle
$C_{M_{\delta_e}}$	partial derivative of the coefficient of pitching moment with respect to elevator angle
C_{M_q}	partial derivative of the coefficient of pitching moment with respect to nondimensional pitch rate
$\dot{}$	time derivative
$\vert_o$	evaluated at equilibrium

I. INTRODUCTION

The approach to automatic control of an aircraft can be divided into three parts. The first is to choose a reference condition, usually one in which the aircraft is in equilibrium. Second, the equations of motion are linearized about the reference condition. And third, a controller is designed to provide satisfactory performance of the aircraft in the neighborhood of the reference condition. In most cases, the coefficients of the linearized equations can be assumed to be constants if the motion remains within the specified range of the reference conditions, and thus, a controller with constant feedback elements can be developed for the linearized equations of motion associated with any particular reference condition.

Automatic control by this method was first used in the development of autopilots. Given a reference condition, in the case of autopilots, an equilibrium condition, the controller is designed to maintain the aircraft at the reference. When a deviation from the reference is sensed, the controller causes the aircraft to return to the reference.

In order for the aircraft to have certain handling qualities, the automatic controller was refined so that it augmented the pilot's input. The reference condition, and thus the linearized equations of motion and the feedback control law, is updated whenever the error in the linearized equations of motion caused by a significant deviation from the reference is greater than the allowable error. For conventional aircraft, this type of feedback control can work satisfactorily over a wide range of flight conditions because the variation in the coefficients of the linearized equations of motion is small and thus the number of reference conditions needed is also small.

If the equations of motion are linearized for vertical and/or short take-off and landing (V/STOL) aircraft, the coefficients of the equations of motion can no longer be assumed to be constant, and in fact, because of the large variation in velocity and dependence on the thrust levels, some of the coefficients may vary significantly. The use of a fixed gain controller is no longer adequate and a control program that is updated at frequent intervals becomes a necessity. In order to develop an efficient controller for V/STOL type aircraft, information regarding the variation of the aircraft parameters based only on inputs and states would facilitate construction of a feedback controller.

The method of system identification may be useful in supplying the information needed by a controller. Once the equations of motion have been linearized, information regarding the variation of the coefficients of these equations can be determined using only the inputs and states. A feedback controller can then be designed from the information, and as the coefficients of the linearized equations vary, the feedback gains can be varied in order that specific aircraft dynamics, or handling qualities, are retained.

A model reference approach to the problem of system identification has been developed (4). In this approach, a model is used to identify an unknown system. By applying an input to the model and the unknown system, the parameters of the model are adjusted so that the error between the model and the unknown system is reduced. With proper model parameter adjustment, the error will go to zero asymptotically and the model will become a representation of the system.

Lyapunov's direct method was used by Narendra, Tripathi, Kudva, and Lüders (1) -- (4), to design a model reference system to identify an unknown system. Lyapunov's direct method has an advantage over the methods that use the gradient approach because it is globally stable, that is, convergence is guaranteed regardless of the size of the errors.

Narendra first used the Lyapunov's direct method in the identification of a system of equations with constant coefficients--the linear time-invariant problem (1). The identification scheme was developed in order to insure global stability based on measurements of the states and inputs only. Because the coefficients are constants, convergence is guaranteed.

Next, Narendra adapted the linear time-invariant results to a linear problem with slowly time-varying elements. The identification scheme was used in conjunction with a feedback control algorithm for a VTOL aircraft (2). For this problem, convergence is guaranteed to within a bounded error which can be made smaller by a correct choice of certain gains. The results obtained from the identification and control of this problem were satisfactory.

The previous methods of identification were based on the assumption that the equations of motion are linear and that the parameters are either constants or slowly varying functions of time. The equations of motion for an aircraft are nonlinear, and in order to develop a controller for the aircraft similar to those used previously, the equations would have to be linearized about a number of equilibrium conditions and the constants for each would have to be stored in an on-board computer. When the aircraft is away from this equilibrium condition, a new reference condition and the linearized equations of motion associated with this reference are used in the identification procedure. Thus, the identification procedure is begun again about a new set of equilibrium conditions. As the aircraft is flown, the reference is updated at frequent intervals. Because of the number of reference conditions needed to insure reliable identification, a computer with a large storage capacity is necessary.

If a system represented by a set of nonlinear equations could be identified without the need of frequent updating then the computer storage and updating problems could be minimized. The identification of such a system, specifically the set of nonlinear equations that represent a VTOL aircraft, is what this paper proposes. First linearizing the set of nonlinear equations about a reference point, and then letting certain parameters in the linear equations vary, the work of Narendra, et al, is applied. This set of linear equations which have the same input/output characteristics as the nonlinear equations, has a wider range of validity than the linear equations with constant parameters. This method of identification is still globally stable, but because of the varying parameters has only a bounded convergence.

II. LINEAR IDENTIFICATION SCHEME

The linear identification scheme developed by Narendra has produced satisfactory results and is the basis for the scheme presented in this paper (2). In order to maintain continuity in this paper, the scheme will be presented here.

The general set of linear equations can be represented by the matrix differential equation

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

where x and u are small perturbations, A is an n x n matrix and B is an n x m matrix, and A and B may or may not have time varying elements, but they are unknown. The model equation to be used to identify the elements of A and B is

$$\dot{y} = C(y-x) + G(t)x + H(t)u \quad . \tag{2}$$

Subtracting equation 1 from equation 2 gives the error equation

$$\dot{e} = Ce + (G-A)x + (H-B)u \quad . \tag{3}$$

The coefficient matrices of the model equation are varied in order that

$$\lim_{t \to \infty} e \to 0 \tag{4}$$

$$\lim_{t \to \infty} G \to A \tag{5}$$

$$\lim_{t \to \infty} H \to B \tag{6}$$

Equation 3 is stable if the scaler function V(e;t), also known as the Lyapunov function, satisfies certain requirements (5):

1. V(e;t) has continuous first partial derivatives;
2. V(e;t) > 0 for all x $\neq$ 0 and for all t, (7)
   V(0;t) = 0 for all t;

$$3. \quad \dot{V}(e;t) \leq 0 \text{ for all } x \neq 0 \text{ and for all } t, \qquad \dot{V}(0;t) = 0 \text{ for all } t. \tag{8}$$

The choice of the Lyapunov function is (1):

$$V = e^T Pe + \sum_i^n \varphi_i^T \varphi_i + \sum_i^m \psi_i^T \psi_i \tag{9}$$

where

$$\varphi_i = \begin{bmatrix} g_{1i} - a_{1i} \\ g_{2i} - a_{2i} \\ \cdot \\ \cdot \\ \cdot \\ g_{ni} - a_{ni} \end{bmatrix} \quad \text{and} \quad \psi_i = \begin{bmatrix} h_{1i} - b_{1i} \\ h_{2i} - b_{2i} \\ \cdot \\ \cdot \\ \cdot \\ h_{mi} - b_{mi} \end{bmatrix} \tag{10}$$

and i denotes the i^{th} column of

$$\varphi = G - A \quad \text{and} \quad \psi = H - B \ , \tag{11}$$

respectively. The first and second requirements for stability are satisfied with this function if P is chosen to be positive definite. Then, taking the time derivative of equation 9:

$$\dot{V} = \dot{e}^T Pe + e^T P\dot{e} + \sum_i^n (\dot{\varphi}_i^T \varphi_i + \varphi_i^T \dot{\varphi}_i) + \sum_i^m (\dot{\psi}_i^T \psi_i + \psi_i^T \dot{\psi}_i) \tag{12}$$

And, substituting equation 3 for e and rearranging terms:

$$\begin{aligned} V = {} & (e^T C^T + x^T \varphi^T + u^T \psi^T)Pe + e^T P(Ce + \varphi x + \psi u) \\ & + \sum_i^n (\dot{\varphi}_i^T \varphi_i + \varphi_i^T \dot{\varphi}_i) + \sum_i^m (\dot{\psi}_i^T \psi_i + \psi_i^T \dot{\psi}_i) \\ = {} & e^T C^T Pe + e^T PCe + x^T \varphi^T Pe + e^T P\varphi x + u^T \psi^T Pe + e^T P\psi u \\ & + 2 \sum_i^n \varphi_i^T \dot{\varphi}_i + 2 \sum_i^m \psi_i^T \dot{\psi}_i \end{aligned}$$

$$\dot{V} = e^T(C^TP + PC)e + \sum_i^n (x_i \varphi_i^T Pe + e^T P \varphi_i x_i + 2\varphi_i^T \dot{\varphi}_i)$$

$$+ \sum_i^m (u_i \psi_i^T Pe + e^T P \psi_i u_i + 2\psi_i^T \dot{\psi}_i) \quad . \tag{13}$$

Letting

$$- Q = C^TP + PC \tag{14}$$

Then

$$\dot{V} = - e^TQe + 2 \sum_i^n (x_i \varphi_i^T Pe + \varphi_i^T \dot{\varphi}_i) + 2 \sum_i^m (u_i \psi_i^T Pe + \psi_i^T \dot{\psi}_i)$$

$$= - e^TQe + 2 \sum_i^n \varphi_i^T (Pex_i + \dot{\varphi}_i) + 2 \sum_i^m \psi_i^T (Peu_i + \dot{\psi}_i) \tag{15}$$

To insure that requirement 3 (equation 8) is satisfied, Q must be positive definite, and since it was previously shown that P is positive definite, it follows from equation 14 that C must be negative definite. Since the sign of the other two elements of equation 15 is unknown, let

$$\dot{\varphi}_i = - Pex_i \quad \text{and} \quad \dot{\psi}_i = - Peu_i \quad , \tag{16}$$

then equation 3 will be globally stable regardless of the value of φ_i and ψ_i. In general matrix form:

$$\dot{\varphi} = - Pex^T \quad \text{and} \quad \dot{\psi} = - peu^T \tag{17}$$

If A and B are constants, then $\dot{\varphi} = \dot{G}$ and $\dot{\psi} = \dot{H}$ or

$$\dot{G} = - Pex^T \quad \text{and} \quad \dot{H} = - Peu^T \tag{18}$$

and equation 4 is satisfied. If, however, A and B contain slowly varying elements, the $\dot{\varphi}$ and $\dot{\psi}$ are approximated by $\dot{G}$ and $\dot{H}$. In this case, the error will not asymptotically vanish. When the error is large, the dominant term in the equation for the time derivative of the Lyapunov function (equation 15) is $- e^TQe$ and the error equation (equation 3) is still globally stable. But, when the error is small, the other two terms in equation 15 begin to dominate and zero error cannot be insured. Therefore, for slowly varying elements, the error equation is globally stable but the error is bounded in the neighborhood of $e = 0$. The magnitude of this neighborhood is determined by the choice of certain parameters to be discussed later.

When both e and $\dot{e}$ are identically zero, then equation 3 becomes

$$0 = (G-A)x + (H-B)u \quad , \tag{19}$$

and for identification the solution to this equation is trivial,

$$G = A \text{ and } H = B \quad . \tag{20}$$

For certain inputs, it is possible that the solution be nontrivial, but for complete controllability and a large input domain, the only possible solution will be the trivial one.

III. IDENTIFICATION OF A NONLINEAR SPRING

In general, the equations of motion can be written in the form

$$\dot{x} = f(x,u;t) \tag{21}$$

where x represents the n-dimensional vector of states, u represents the n-dimensional vector of inputs, and f is an n-dimensional vector of nonlinear functions. The usual procedure for solving the set of equations is to linearize them around some reference condition. Expanding the i^{th} nonlinear function in a Taylor's series about the reference (x_o,u_o):

$$f_i(x,u;t) = f_i(x_o,u_o;t) + \sum_i^n \left(\frac{\partial f_i}{\partial x_j}\right)\Bigg|_o \Delta x_j + \sum_k^n \left(\frac{\partial f_i}{\partial u_k}\right)\Bigg|_o \Delta u_k + \text{hot.} \tag{22}$$

The hot represents higher order terms which are neglected assuming the perturbations, Δx_j and Δu_k, to be small. Then rearranging terms

$$\Delta\dot{x}_i = f_i(x,u;t) - f_i(x_o,u_o;t) = \sum_i^n \left(\frac{\partial f_i}{\partial x_j}\right)\Bigg|_o \Delta x_j + \sum_k^n \left(\frac{\partial f_i}{\partial u_k}\right)\Bigg|_o \Delta u_k \tag{23}$$

In a more general form, letting:

$$a_{ij} = \left(\frac{\partial f_i}{\partial x_j}\right)\Bigg|_o \quad \text{and} \quad b_{ik} = \left(\frac{\partial f_i}{\partial u_k}\right)\Bigg|_o \tag{24}$$

the linearized equations can be written in the matrix form given by

$$\dot{x} = Ax(t) + Bu(t) \tag{25}$$

where Δx is replaced by $x(t)$ and Δu is replaced by $u(t)$. The elements of the coefficient matrices, A and B, are constant for constant reference conditions and vary for nonconstant reference conditions. This is the form studied by Narendra and the nonlinear equations can be identified as long as the states and inputs remain within relatively small bounds.

In order to increase the range of validity of the linearized equations it appears A and B could be allowed to vary, not only according to the reference condition, but also according to the state, and indirectly the input. Because the manner in which A and B vary is unknown, the identification scheme described in the previous section may be used to determine the dependence of A and B on the state.

Following the procedure for identification shown in the previous section, the identifying equation is

$$\dot{y} = C(y-x) + G(t)x + H(t)u \tag{26}$$

So that the error equation

$$\dot{e} = Ce + (G-A)x + (H-B)u \tag{27}$$

is stable, G and H are varied according to the laws given by

$$\dot{G} = -\,Pex^T \tag{28}$$

$$\dot{H} = -\,Peu^T \tag{29}$$

Making a minor modification so each element in $\dot{G}$ and $\dot{H}$ can be adjusted separately, equations 28 and 29 become (2):

$$\dot{G} = -\Gamma \otimes Pex^T \tag{30}$$

$$\dot{H} = -\Lambda \otimes Peu^T \tag{31}$$

where $\otimes$ represents element by element multiplication and the gain matrices, Γ and Λ, have positive elements. The error in identification will be bounded because the elements of A and B vary.

A. Equations of Motion

In order to determine the effectiveness of identifying a nonlinear system with a linear system that has varying coefficients, the two degree of freedom nonlinear spring problem will be used. The spring's motion can be described by the nonlinear equation:

$$m\ddot{x} + g_1(x)\dot{x} + g_2(x)x = g_3(\dot{x})u \quad . \qquad (32)$$

For the spring problems studied:

$m = 1$ (mass)

$g_1(x) = 5(1 + \epsilon_1 x^2)$ (damping nonlinearity)

$g_2(x) = (1 + x^2)$ (spring stiffness nonlinearity)

$g_3(\dot{x}) = (1 + \epsilon_2 \dot{x}^2)$ (input nonlinearity)

$0 \leq \epsilon_1, \epsilon_2 < 1$

Equation 32 can be put in the form of equation 21 by letting:

$$x = x_1 \qquad (34)$$

$$\dot{x} = x_2 \qquad (35)$$

Thus, the equations of motion become:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -5(1 + \epsilon_1 x_1^2)x_2 - (1 + x_1^2)x_1 + (1 + \epsilon_2 x_2^2)u \end{aligned} \qquad (36)$$

The equilibrium point

$$x_1 = x_2 = 0 \qquad (37)$$

is used as a reference and the nonlinear equations of motion are linearized:

$$\begin{aligned} \Delta\dot{x}_1 &= \left(\frac{\partial f_1}{\partial x_1}\right)\bigg|_o \Delta x_1 + \left(\frac{\partial f_1}{\partial x_2}\right)\bigg|_o \Delta x_2 + \left(\frac{\partial f_1}{\partial u}\right)\bigg|_o \Delta u + \text{hot} \\ \Delta\dot{x}_2 &= \left(\frac{\partial f_2}{\partial x_1}\right)\bigg|_o \Delta x_1 + \left(\frac{\partial f_2}{\partial x_2}\right)\bigg|_o \Delta x_2 + \left(\frac{\partial f_2}{\partial u}\right)\bigg|_o \Delta u + \text{hot} \end{aligned} \qquad (38)$$

Then, assuming small perturbations, solving for the partial derivatives, and substituting Δx with x and Δu with u, the equations in matrix form are given by:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \tag{39}$$

Comparing the nonlinear equations with the linear equations, 37 and 39, respectively, it is apparent that the elements of the second row of the matrix differential equation 39 have to be allowed to vary depending on the values of ϵ_1 and ϵ_2. The identifying model equations become:

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \begin{bmatrix} y_1 - x_1 \\ y_2 - x_2 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ h_{21} \end{bmatrix} u \tag{40}$$

where g_{21}, g_{22}, and h_{21} are allowed to vary depending on the source of the nonlinearity in order that equation 40, the identifier, has the same input/output relationship as equation 36.

B. Computer Solution

The equations of motion of the nonlinear spring were identified in a continuous time mode on an IBM 370 computer. A Runge-Kutta integration scheme was used to integrate the equations. The time step was .02 of a second over a time period of 50 seconds.

Two different control programs were used in the study of the nonlinear spring. One control was a step input of .4. The second control consisted of a ramp of 0 to .4 over the first ten seconds and a constant value of .4 for the remaining forty seconds, Figure 1.

The controls were applied to the equations of motion for three cases of nonlinearity in the spring. The first consisted of a nonlinearity in the spring stiffness, $\epsilon_1 = \epsilon_2 = 0$. For the second case a nonlinearity in the damping was added, $\epsilon_1 = .2$ and $\epsilon_2 = 0$. And, the third case consisted of the stiffness nonlinearity and a nonlinearity in the forcing function, $\epsilon_1 = 0$ and $\epsilon_2 = .3$.

In each of the cases of nonlinearity described on the previous page, the initial conditions of the elements of the coefficient matrices of the identifying equations were the linearized values of the corresponding coefficients of the linearized equations, equation 38. With the elements of the G matrix, g_{21} and g_{22}, having zero initial conditions, two more tests were run. The first consisted of the original nonlinearity, and in the second, the nonlinearity was added to the damping factor, $\epsilon_1 = .2$ and $\epsilon_2 = 0$.

C. Choice of Parameters

Before a system of nonlinear equations can be identified, certain parameters must be chosen. These parameters, the matrices C, P, Γ , and Λ (see equations 26, 30 and 31), determine the speed of identification and the magnitude of the error. To make it easier to determine the other parameters, C and P were chosen to be diagonal matrices. This uncouples the error equations and, to some extent, the elements in the G and H matrices.

For stability, as was shown, C must be negative definite. Since it is diagonal, each element of C should be negative. The effect of C is felt mainly in the initial stage of identification, or when the error is large, and has little effect after this stage. The identification did not appear to be greatly affected by different choices in the elements of C, so a value of -20 was chosen for each element on the diagonal.

The choice of the elements for the P matrix directly affected the choice in the elements of Γ and Λ, as can be seen in equations 30 and 31. Because of this, the elements of P were used to obtain a rough identification and the elements of Γ and Λ were used to improve the identification. For stability, P is positive definite, and for the examples solved in this section, the elements of P consist of 100 on the diagonal and 0 elsewhere.

The parameters that had the major effect on identification were the elements of Γ and Λ. The equations in which they appear are equations 30 and 31:

$$\dot{G} = -\Gamma \otimes Pex^T \tag{41}$$

$$\dot{H} = -\Lambda \otimes Peu^T \tag{42}$$

These equations show that each element in Γ and Λ correspond to a specific element in $\dot{G}$ and $\dot{H}$. Through experimentation, it was found that large values of $\dot{G}$ and $\dot{H}$ caused too great a change in G and H and rapid identification was not possible. Also, if $\dot{G}$ and $\dot{H}$ were too small, the identification was possible only after a long time and, even then, the bounded error was too large. If the elements of $\dot{G}$ and $\dot{H}$ were on the same order as the corresponding elements in G and H, then in most cases, identification was good.

D. Results

The trajectories of the states were not affected noticeably by the addition of the nonlinearities. The trajectories of the states for the step input are given in Figure 2 and for the ramp input in Figure 3. In all the identification problems studied in this section, the error in identification of the states is less than .1%, Figure 4, except when the initial condition of the identifying coefficients, g_{21} and g_{22}, are zero, Figure 5. The error in identifying x_2 for this case reached a maximum of about 3% within three seconds of the beginning of the identification, but it quickly decreased to less than .1% for the remainder of the identification.

The first problem studied was the identification of the nonlinear spring equation with a nonlinearity only in the spring stiffness, $(1 + x^2)$ and $\epsilon_1 = \epsilon_2 = 0$. Figure 6a shows the actual variation of the spring stiffness for a step input. For an initial value corresponding to the linear value of equation 39, the element allowed to vary in order to identify the spring stiffness nonlinearity is g_{21}. The variation of g_{21} is the same as the variation of the spring stiffness, Figure 6b. When the initial condition of g_{21} is zero, the difference between the spring stiffness and the identifying element is small after approximately three seconds of identification, Figure 7.

To determine the effect of control on the identification of the nonlinearity in the spring stiffness, the previous problem was studied with a second control scheme. This control scheme consisted of a ramp from 0 to .4 for the first ten seconds and a constant value of .4 for the remainder of the identification, Figure 1b. The difference in the variation of the identifier, g_{21}, and the variation of the spring stiffness is negligible, Figure 8.

A second nonlinearity, a nonlinearity in the damping, was added to the spring equation, $\epsilon_1 = .2$. For identification of this nonlinearity, a second element, g_{22} of the identifier, had to be allowed to vary, equation 40. Both the spring stiffness and the damping nonlinearities were identified successfully for each of the control programs. For the step input, Figure 9 shows the variations in the stiffness nonlinearity and the identifier element g_{21}, and Figure 10 shows the variation in the damping and the identifier element g_{22}. Figures 11 and 12 show the variation of the nonlinearities and their respective identifying elements for the case of the ramp input.

Setting the initial conditions of g_{21} and g_{22} to zero, the step input was applied to the identifier. Identification of the nonlinearities was successful shortly after the step input was applied. The variations of g_{21} and g_{22} are given in Figures 13 and 14, respectively.

Including nonlinearities in the step and ramp inputs, the inputs were applied to the equation of motion that had a stiffness nonlinearity only. The nonlinearities in the stiffness and inputs were not successfully identified, Figures 15 to 18. The problem of nontriviality discussed earlier may have occurred. The error in the identifier of x_2 can be written as:

$$\dot{e}_2 = c_{22}e_2 + (g_{21} - a_{21})x_1 + (g_{22} - a_{22})x_2 + (h_{21} - b_{21})u \ . \qquad (43)$$

At equilibrium for a step input and no error in the identification of the states, the error equation becomes:

$$0 = (g_{21} - a_{21})x_1 + (h_{21} - b_{21})u \ . \qquad (44)$$

Any number of sets of values for g_{21} and h_{21} exist that will satisfy equation 44. Thus, equation 44 has a solution other than the trivial solution, $g_{21} = a_{21}$ and $h_{21} = b_{21}$.

For the problem when the nonlinearity was in the damping and the stiffness, equation 43 becomes:

$$0 = (g_{21} - a_{21})x_1 \qquad (45)$$

when equilibrium is reached. In this case as in the last discussed,

$x_2 = 0$ and the element g_{22} does not enter into the identification equation. Only one value of g_{21} will satisfy equation 45, $g_{21} = a_{21}$. This may cause some problem in the identification of the damping nonlinearity, but this does not appear to be the case.

For the identification problems studied, the choice of inputs did not affect the identification of the nonlinearities. This fact is apparent in the last case studied, the identification of nonlinearities in the spring stiffness and the input. Though the nonlinearities were not identified, the equilibrium value of g_{21} for the step input, Figure 15b, was the same as the equilibrium of g_{21} for the ramp input, Figure 17b. The equilibrium values of h_{21} for the two inputs were also the same. Figure 16b shows the variation of h_{21} for a step input, and Figure 18b shows the variation of h_{21} for a ramp input.

IV. IDENTIFICATION OF EBF/STOL

The identification scheme developed in the previous section was adapted to the problem of identifying the nonlinear equations of motion of an externally blown flap short take-off and landing aircraft. The equations of motion are developed in Appendix A. The identification scheme was developed for continuous time identification, therefore, the equations of motion and the identification equations were integrated using Hamming's fourth order predictor-corrector integration method on an IBM 370 computer. This method uses the values of the four preceding time steps to compute the next set of values. The time step was .01 of a second and the equations were integrated for 100 seconds.

From Appendix A, the states used in the identification are the velocity, the flight path angle, the pitch rate, and the attitude angle. The controls used as inputs are nondimensional thrust factor, elevator deflection, and flap deflection. As was stated in the development of the identification scheme, a set of equilibrium values is needed as a reference for the identification equations. This reference was arbitrarily chosen as:

states:

$$V = 180 \text{ fps.}$$
$$\gamma = 0 \qquad (46)$$
$$q = 0$$
$$\theta = -.09848^\circ$$

controls:

$$C_T = .5399$$
$$\delta_e = -3.976^\circ \qquad (47)$$
$$\delta_f = 49.97^\circ$$

and is the reference for all problems solved in this section regardless of the values of the states.

To study the problem, two control programs were used. The first consisted of keeping the elevator and flap angles at the equilibrium values while applying a sinusoidal variation to the non-dimensional thrust factor about the equilibrium condition. A comparison was made of the outputs from the nonlinear equations, the identification equations, and a linear set of equations based on the equilibrium condition.

For the second control program, a linear variation in controls was used so that the states would begin at the equilibrium values and change until another equilibrium condition was attained when the velocity was approximately 120 feet per second. The controls varied linearly for the first half of the computation and then were held constant for the remainder of the time. Using this control scheme, the effect on the error of identification of the number of elements in the G and H matrices calculated using the identifying equations, 30 and 31, was studied, as was the effect of different initial guesses of certain elements in the G matrix.

A. Choice of Parameters

The manner in which the parameters were chosen in this section is the same as that used in the nonlinear spring problem. The matrices C and P were again chosen to be diagonal to uncouple the error equations. C consisted of -50 for each element on the diagonal and each element of the diagonal of P was 1000. The elements of Γ and Λ were chosen so that the elements $\dot{G}$ and $\dot{H}$ were on the same order as the respective elements in G and H. These values are given in Table 2.

The speed of identification is affected in another manner similar to the one cited in the previous section. Studying equations 30 and 31, if the differences in the actual values of the states and inputs and their respective reference values, the perturbation values x and u, change by a factor of 10 or more, then the magnitudes of $\dot{G}$ and $\dot{H}$ change for constant errors. This change in $\dot{G}$ and $\dot{H}$ has the same effect as improper choices of Γ and Λ, thus Γ and Λ must be changed accordingly to maintain small errors in identification.

The variation of the elements of Γ and Λ for the EBF/STOL problems studied was determined experimentally. The control programs were run and, whenever the elements of $\dot{G}$ and $\dot{H}$ were too small or too large, the corresponding elements of Γ and Λ were changed by a factor of 10 in order to offset the magnitude of the perturbation values. The values of Γ and Λ are given in Table 2. Whenever an element in A or B was known to be constant, then the value for the corresponding element of Γ and Λ was zero. This was the case in the fourth row of A and B which will be discussed later.

B. First Control Program: Sinusoidal variation of one control

In this program, the flap and elevator angles were held at the equilibrium values, equation 44, while the nondimensional thrust varied sinusoidally about the equilibrium value, Figure 19:

$$C_T = .5399 + .1 \sin(\pi t/10). \qquad (48)$$

This control scheme was applied to the three sets of equations -- the nonlinear equations of motion from Appendix A, the linear equations of motion based on the equilibrium values of equations 43 and

44, and the nonlinear identifying equations. The variations in the states produced by the nonlinear equations of motion are given in Figures 20 and 21. A relative per cent error given by

$$\varepsilon_i = \text{per cent error} = \frac{e_i}{x_i} \times 100 \ , \qquad (49)$$

where x_i is the i^{th} perturbation state of the nonlinear equations of motion and e_i is the i^{th} difference between the nonlinear perturbation state and the state calculated by the identifying equations of the linear equations of motion, was used to compare the identifier and linear equations of motion with the nonlinear equations of motion.

The initial values of the elements of G and H were those given in Table 1. Using the identifier equations, 2, 45 and 46, three elements in G, g_{11}, g_{22}, and g_{32}, were allowed to vary while the other elements in G and all the elements in H were held constant. The variation of g_{11}, g_{22} and g_{32}, as calculated by the identifier, are given in Figure 22.

The errors in identification of all four states were less than the errors of the linear equations of motion, Figures 23-26. For the linear equations of motion, the relative error was smallest in the velocity equation. It was on the order of 3% or less. The identifier was able to reduce the error by a factor of 10^{-2}, and the error in identification was on the order of .01% or less, Figure 23. The reduction in error of the flight path angle was not as great, although the identifying error was one-tenth that of the linear error, Figure 24. The per cent error of the linear equation for the flight path angle was 10% or higher, while the error of the identifier was less than 1% at all times.

The most disappointing result for this problem was in identification of the pitch rate. The errors for both the linear and identifier equations were at most times less than 10%, and, on the average, the identifier had errors less than the linear equations, but there is not a significant reduction in error by the identifier, Figure 25.

As was cited earlier, there is no need to identify the nonlinear equation describing the attitude angle because it is always given by $\dot{\theta} = q$ which is a linear equation. Because it is linear and the fact that the identification is based on the actual states, in this case, pitch rate, q, there is no error in the identification of the attitude angle. Although the linearized equation and nonlinear equation for attitude angle are the same, the linearized equation is based on results obtained from the linearized equations of motion, specifically the linear value for pitch rate. Since the linear value for pitch rate is in error the linearized equation describing attitude angle produces an error. These errors are found in Figure 26.

Studying the variation in the errors shows peaks at certain intervals. It appears that these peaks occur when the corresponding states intersect the equilibrium values. Because of the choice of equation for error analysis, equation 48, it is obvious that the per cent error will be undetermined when the perturbation, x_i, is zero. For this reason, when the perturbation nears zero the error increases greatly in magnitude.

C. Second Control Program: Linear variation three controls.

For the first fifty seconds, each of the controls, nondimensional thrust factor, elevator angle, and flap angle, were varied linearly from the equilibrium values used as the reference to another set of equilibrium values, then were held constant for the remaining fifty seconds, Figure 27. This control scheme was applied to the nonlinear equations of motion and the identifier, and allows the velocity to oscillate about another equilibrium at 120 feet per second. The other states oscillate around a new set of equilibrium values, Figures 28 and 29.

The first problem studied was to determine the effect on identification error caused by the number of elements in G and H allowed to vary. Using the identifying equations, the elements in the top row of the G and H matrix were varied, since the errors are uncoupled, the information obtained about the error in velocity should be sufficient, Figure 30. With the initial values equal to the corresponding linear values, Table 1, the elements of the first row in G, g_{11}, g_{12}, g_{14}, and of the first row in H, h_{11}, h_{12}, h_{13}, were varied, Figures 31 and 32. (The element g_{13} is always zero and therefore identification of this element is not needed.) The per cent error in velocity was on the order of 10^{-2} or less. This error is on the same order as the error in the first control program yet the velocity changed by more than sixty feet per second in this control program, Figure 30a.

Two of the elements, g_{12} and h_{11}, were held constant at the corresponding equilibrium values and the same control routine was applied. The variation in the other four elements was very close to that obtained when six elements varied, Figures 33 and 34. The error obtained when only four elements varied was of the same order when six elements varied, but close comparison shows the error produced when four elements varied to be slightly less, Figure 30b.

When only two elements were allowed to vary, g_{11} and h_{12}, the error lessened slightly, Figure 30c. Comparing the variations of g_{11}, for the three cases (six elements varying, Figure 31a, four elements varying, Figure 33a, and two elements varying, Figure 35a) only a slight difference in the values of g_{11} is apparent. This is also true for the variations of h_{12} (Figures 32b, 34a, and 36a.) Noting the fact that at least one element in a row must vary for identification of that row, it appears that the less elements in a given row allowed to vary, the smaller the error. Since only one element in a given row must be allowed to vary for identification, another benefit is only a few parameters need be adjusted.

The control scheme was then used in the identification of three elements in G, g_{11}, g_{22} and g_{32}, and three elements in H, h_{12}, h_{22} and h_{33}, with initial values equal to the linear values given in Table 1. The variation of these elements is given in Figures 35 and 36. The errors in identification of velocity and flight path angle, Figures 39a and 40a, are small, but the error in pitch rate often reaches 10%, Figure 41a. As in the first control scheme, this error is the hardest to decrease. Most of the peaks again occur when the pitch rate nears the equilibrium value, and thus, the percentage error increases tremendously. Except for these peaks the error is on the order of two or three per cent.

In order to determine the effect of the initial conditions of the elements of G and H on identification, the second control program was applied to the identifier with the initial values of g_{11}, g_{22}, and g_{32} set to zero. These three elements of G and the three elements of H, h_{12}, h_{22}, and h_{33}, were allowed to vary and the variations of these elements were compared to the variations of the same elements when all initial values were the linear values of Table 1. The results were mixed. The error in velocity decreased, Figure 39, the error in flight path angle remained the same, Figure 40, and the error in pitch rate increased, Figure 41. And of the six elements allowed to vary, only the variations of g_{22} and h_{22}, Figures 19a and 10b, are the same as when the initial value of g_{22} was the linearized value, Figures 35b and 36b.

As discussed earlier, there may be a problem associated with nontriviality. It appears that this problem has arisen here. The elements in the first and third rows varied in different manners but the errors were always small. As far as state identification is concerned there is no problem. But, if the variations of the elements of A and B are needed a problem may arise because of the different ways in which the same elements of G and H varied.

CONCLUSIONS

A method to identify a system of nonlinear equations using only inputs and states has been presented. So the work of Narendra could be applied, the nonlinear equations were linearized about a reference condition. The coefficients of the linear equations were allowed to vary so that the linear equations had the same input/output relation as the nonlinear equations.

Of the problems associated with identification, the main drawback is the choice of parameters. Although it would seem necessary to vary all the elements, it was found that varying only one element in a given equation produced reasonable identification. Though only a few parameters were needed, guesswork played a large role in their selection. Further work in this area would aid in all identification problems.

In most cases, state identification was satisfactory. The number of coefficients allowed to vary, the initial values of these coefficients, and different inputs seemed to have little effect on the errors in the states. However, some of the variations of the coefficients differed for two of these cases. The choice of input had no effect on the identification of the coefficients.

The greatest effect on the identification of the coefficients in the nonlinear spring problem occurred when elements in the H matrix were allowed to vary. It may be possible to improve the identification with another choice of the parameters or with a nonconstant input.

For different initial conditions on the identifier elements in the EBF/STOL problem, the equilibrium values of the elements differed.

Although the choice of initial conditions did not effect the final values when only one coefficient of a given equation was allowed to vary, when two coefficients were allowed to vary, different initial conditions produced different final values in the coefficients.

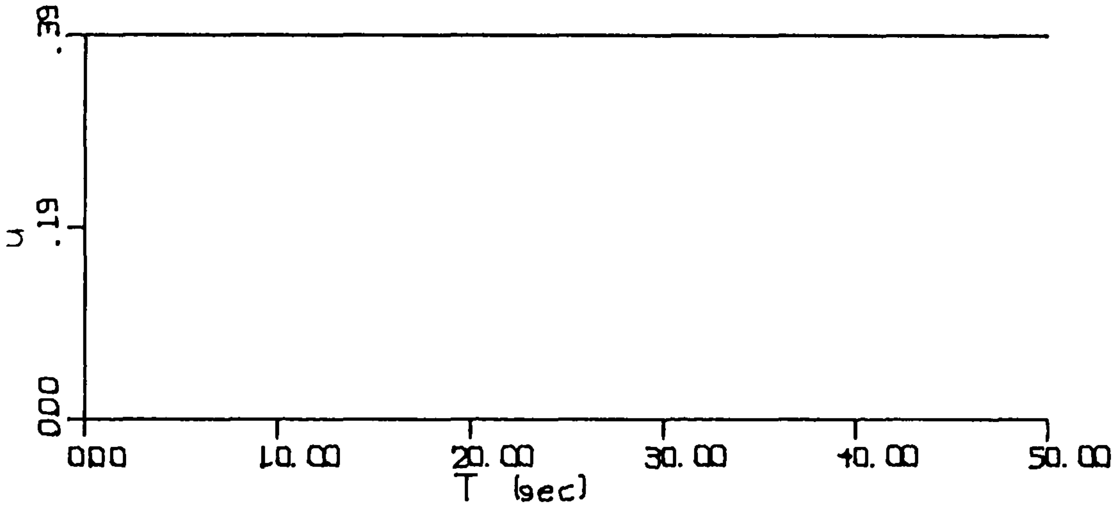
The number of coefficients allowed to vary in the identifier for the EBF/STOL problem seemed to have little effect on the variation of these elements. Whenever a coefficient was held constant, the other varying coefficients seemed to adjust slightly.

The problems encountered above are similar to the nontriviality problems cited earlier. To avoid these problems and to keep the number of parameters to be considered at a minimum, it appears practical to allow only one coefficient in a given equation to vary. And so that the coefficient is dependent on that state rather than control, a coefficient of the state should be chosen.

For further study along this line of nonlinear system identification, a large number of distinct control programs may be applied to determine to a greater extent the dependence of the identification on control. Also, studying the effect of which coefficients are allowed to vary should prove beneficial in developing a useful identification algorithm.

The identification procedure presented may be used in feedback control problems. Since the use of a feedback controller necessitates knowledge of the coefficients of the equation of motion, the identification of the trajectories of the coefficients should help in adjusting the feedback parameters in order to maintain the desired aircraft handling qualities.

a. Step Input



b. Ramp Input

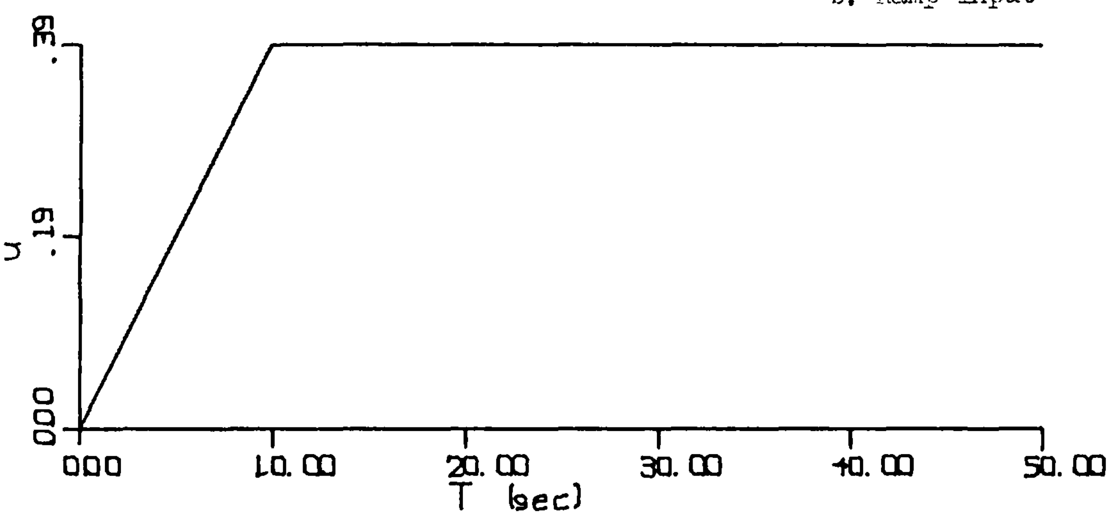
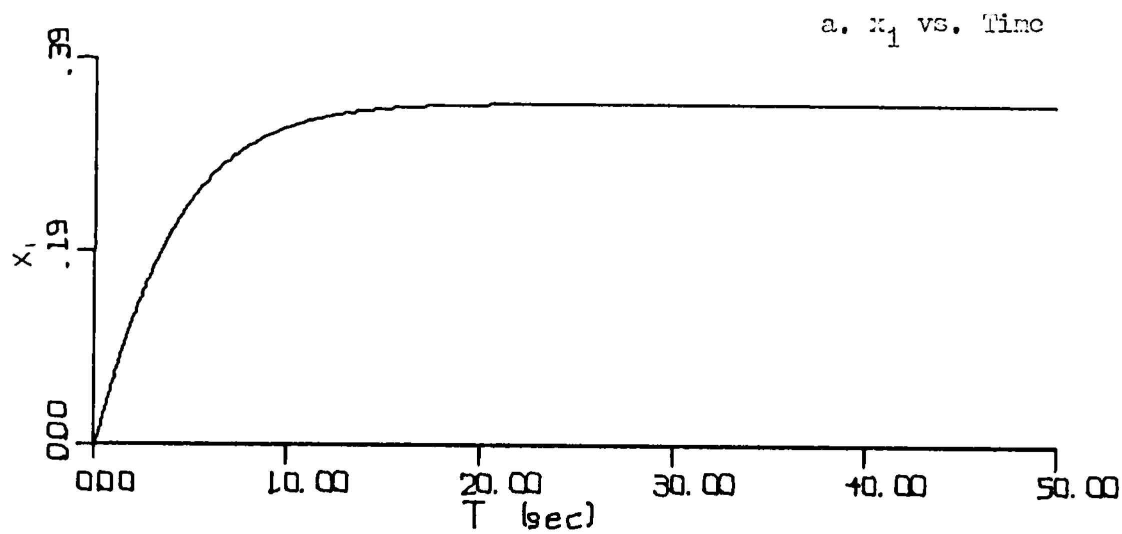


Figure 1: Controls vs. Time



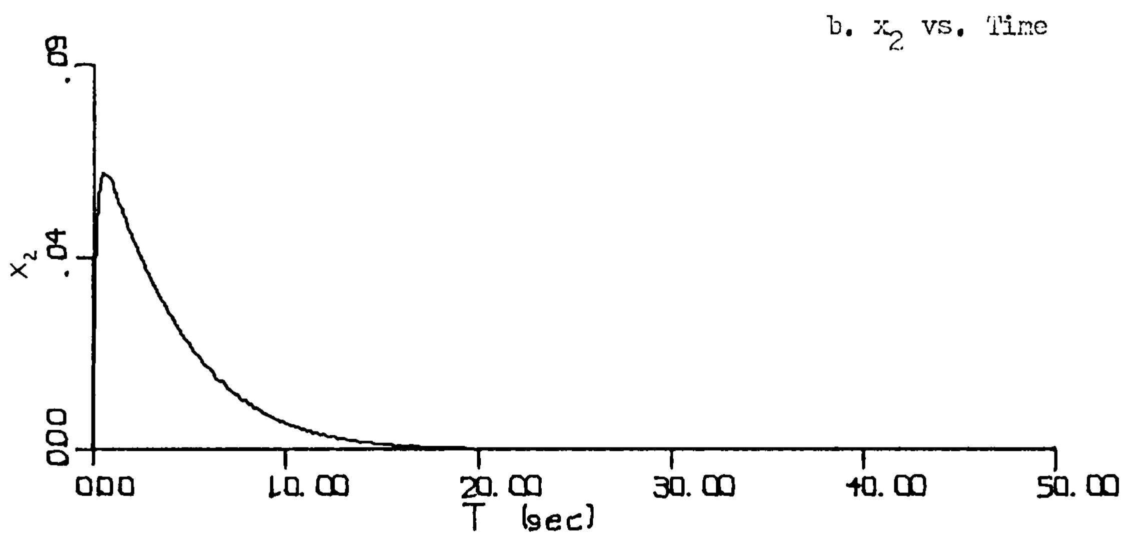


Figure 2: States vs. Time for Step Input

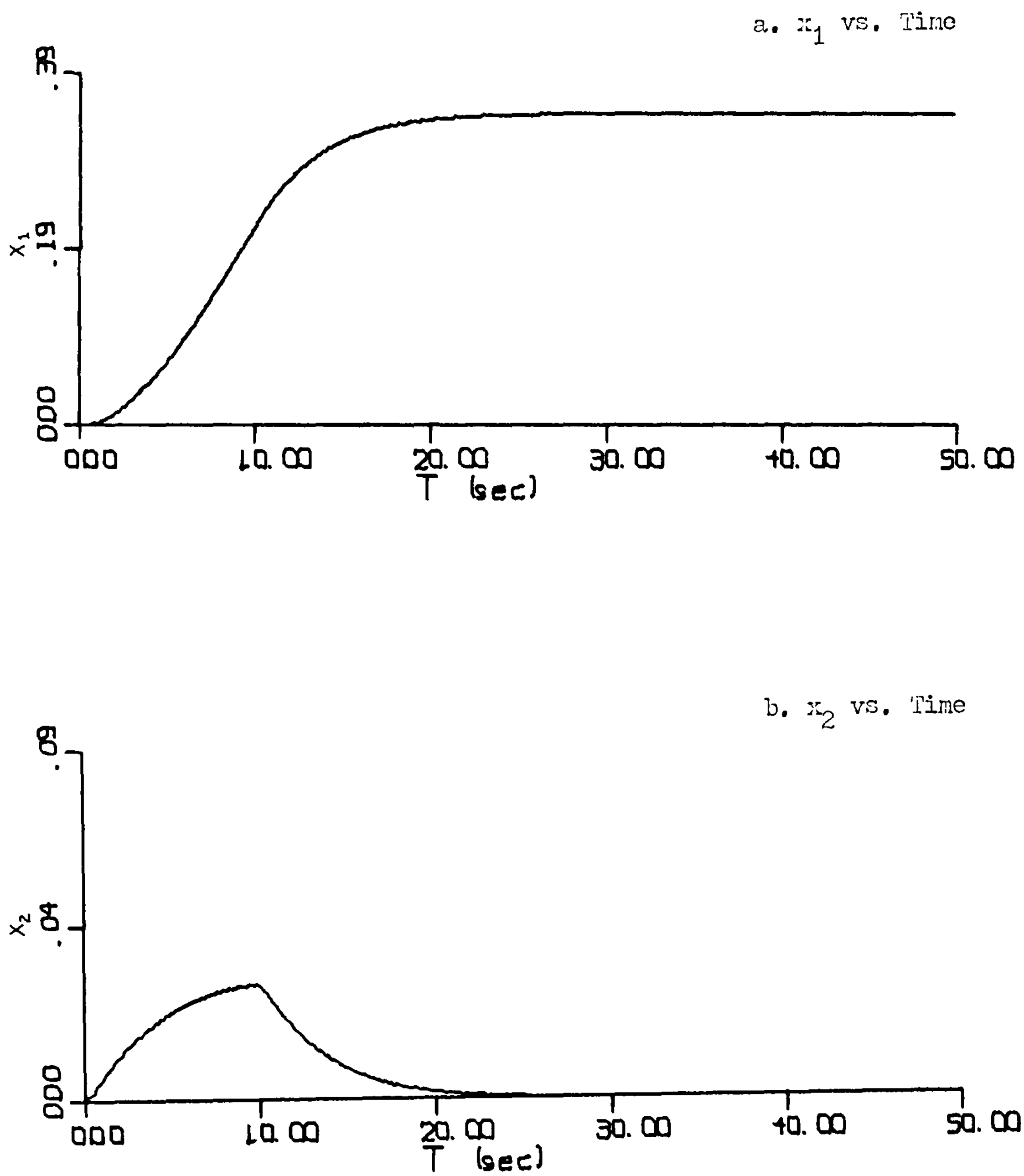
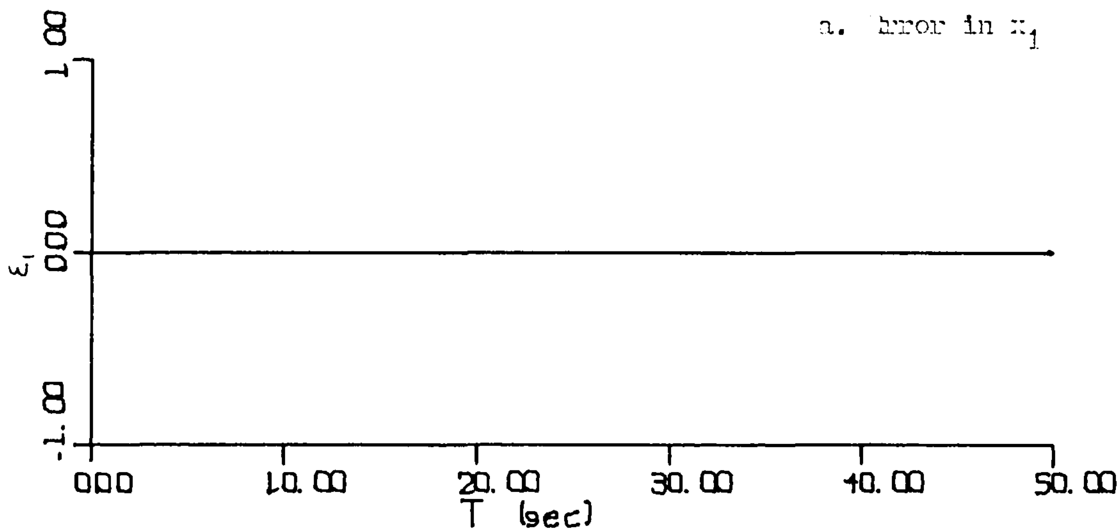


Figure 3: States vs. Time for Ramp Input



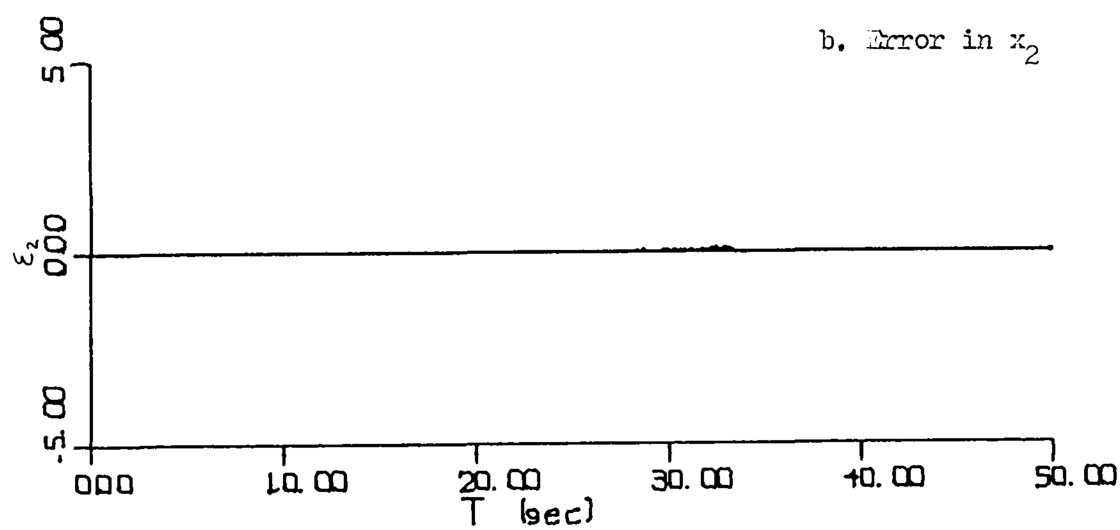


Figure 4: Errors in State Identification--Initial values of G are linear values

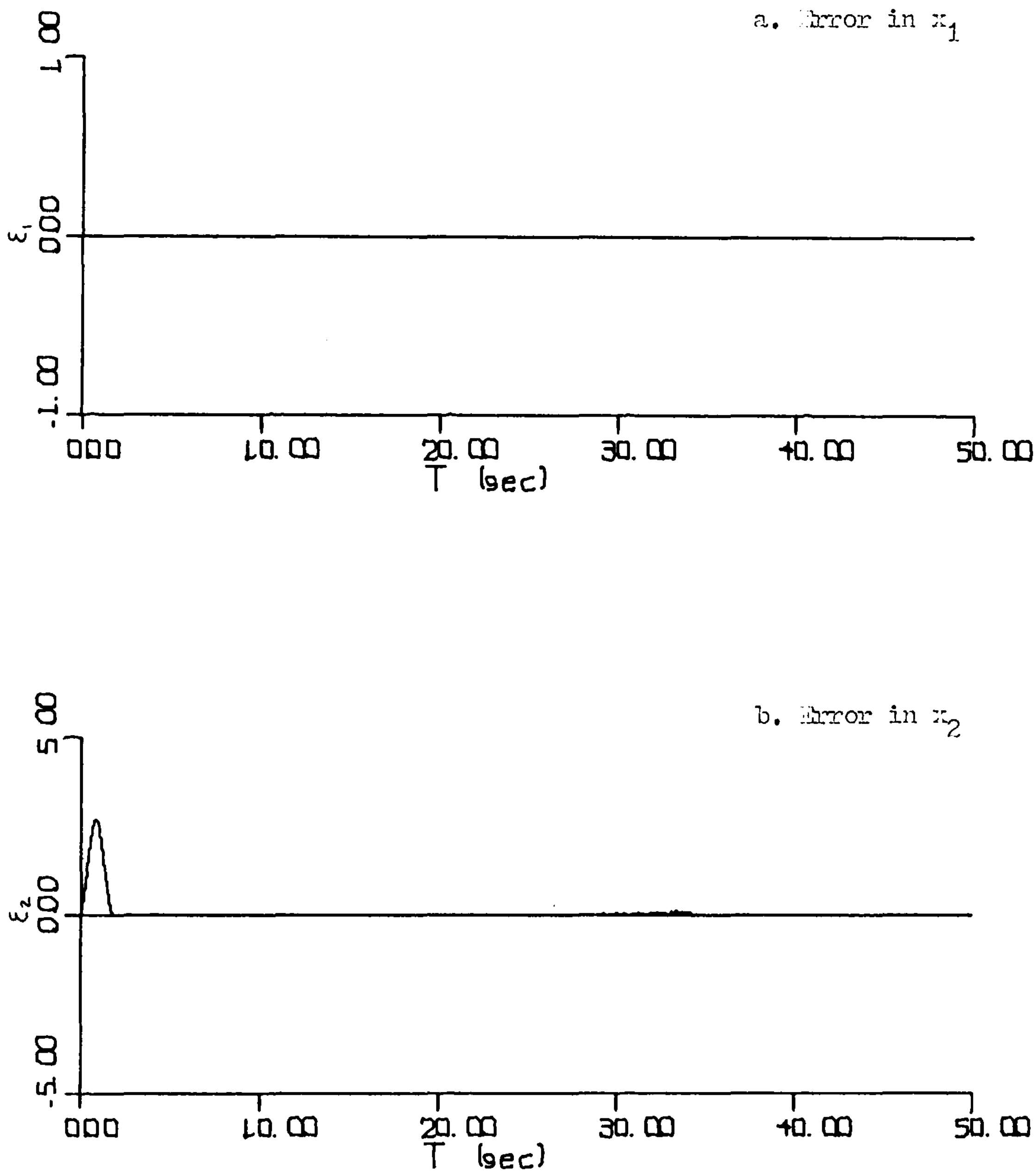


Figure 5: Errors in State Identification--Initial values of G are zero

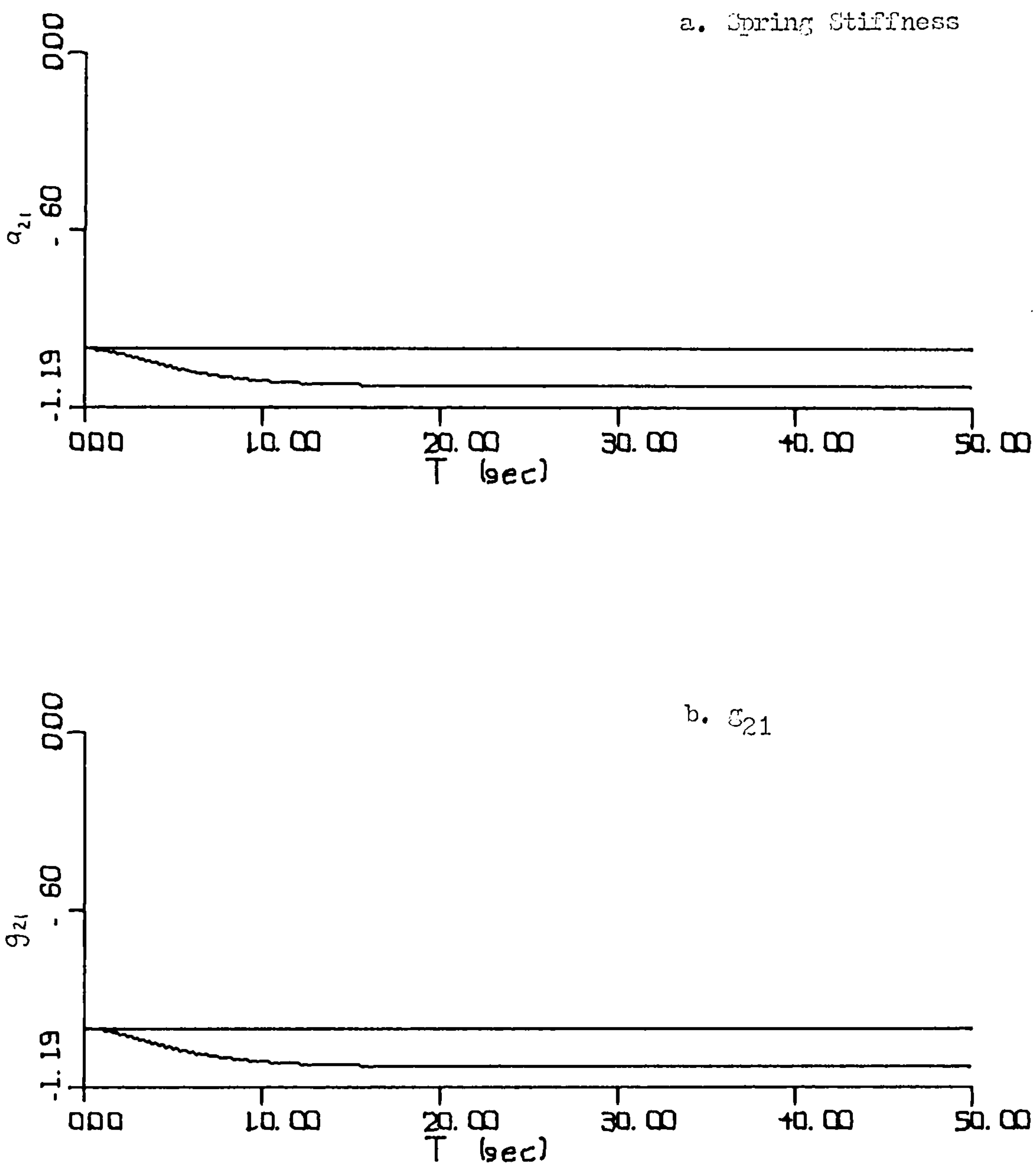


Figure 6: Variation of Spring Stiffness (Step Input)

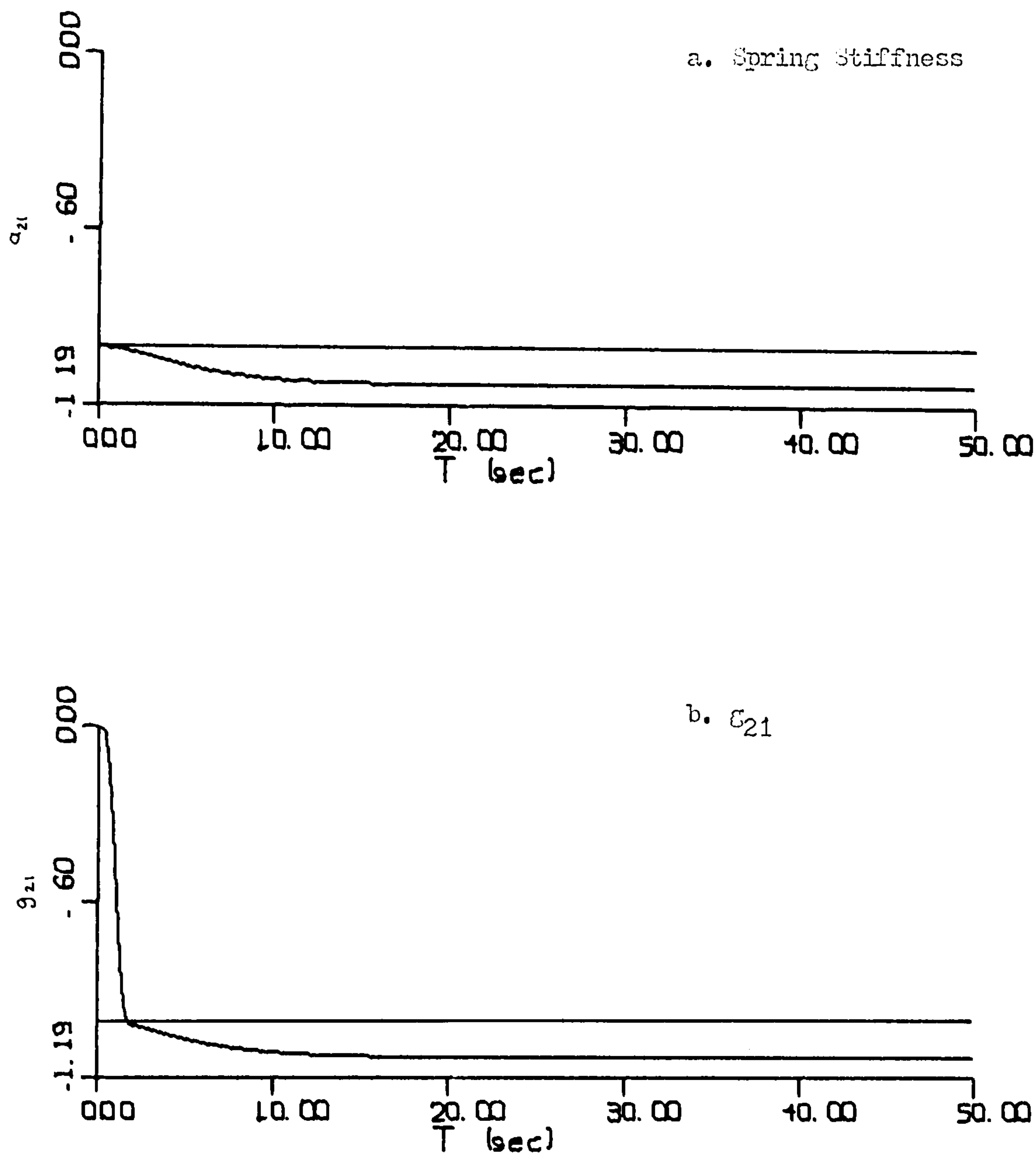
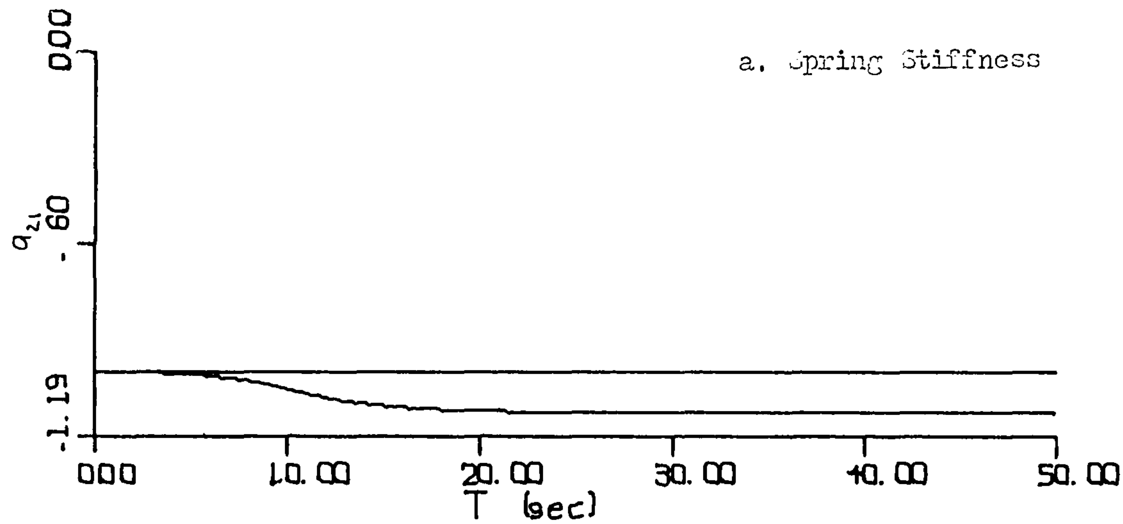


Figure 7: Variation of Spring Stiffness--Initial value of g_{21} is zero (Step Input)



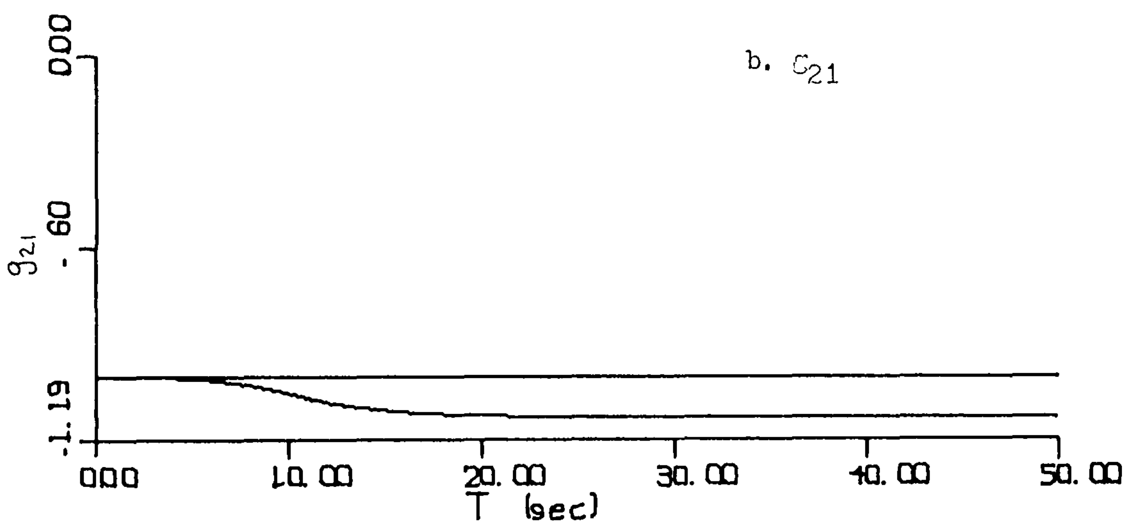


Figure 8: Variation of Spring Stiffness (Ramp Input)

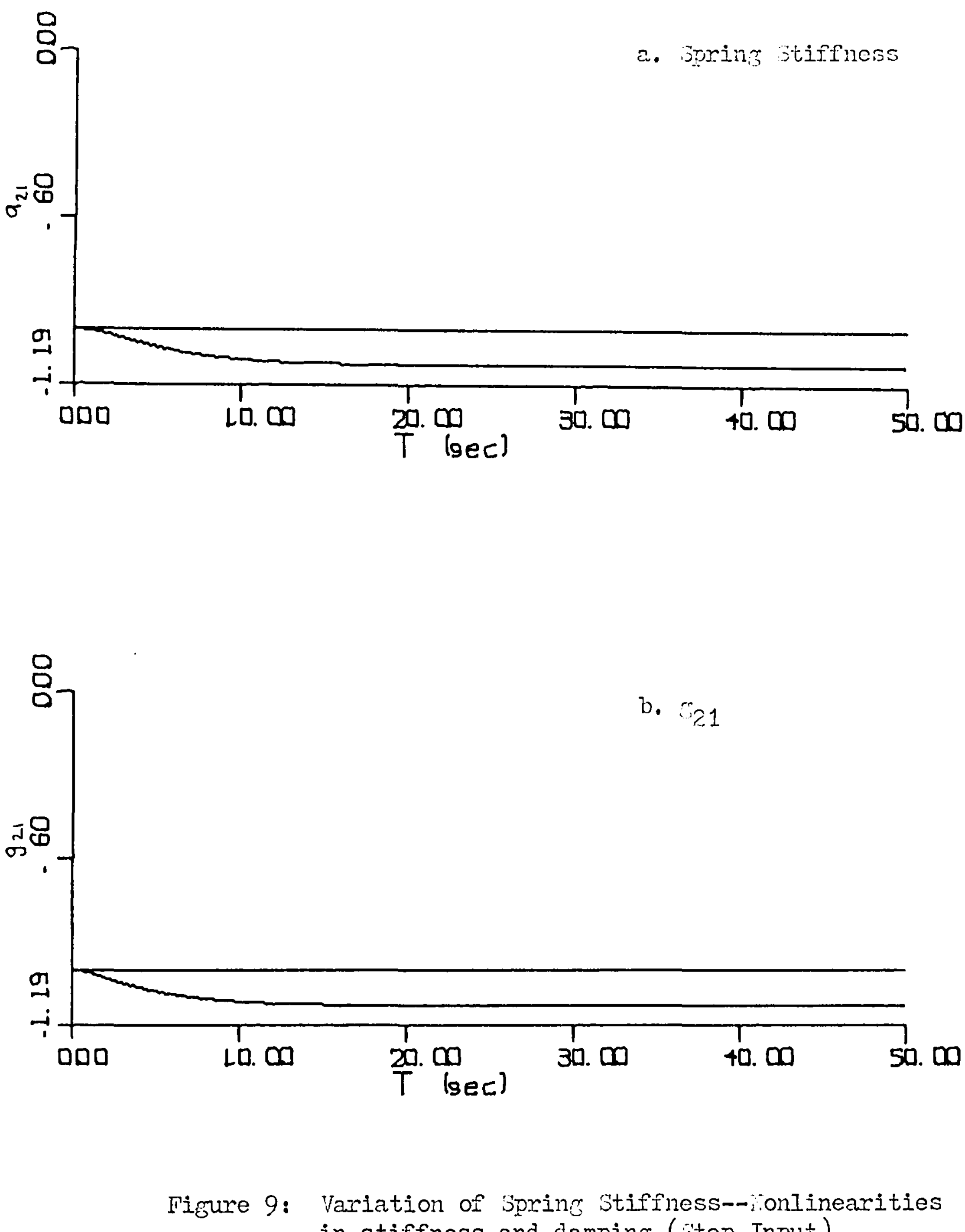


Figure 9: Variation of Spring Stiffness--Nonlinearities in stiffness and damping (Step Input)

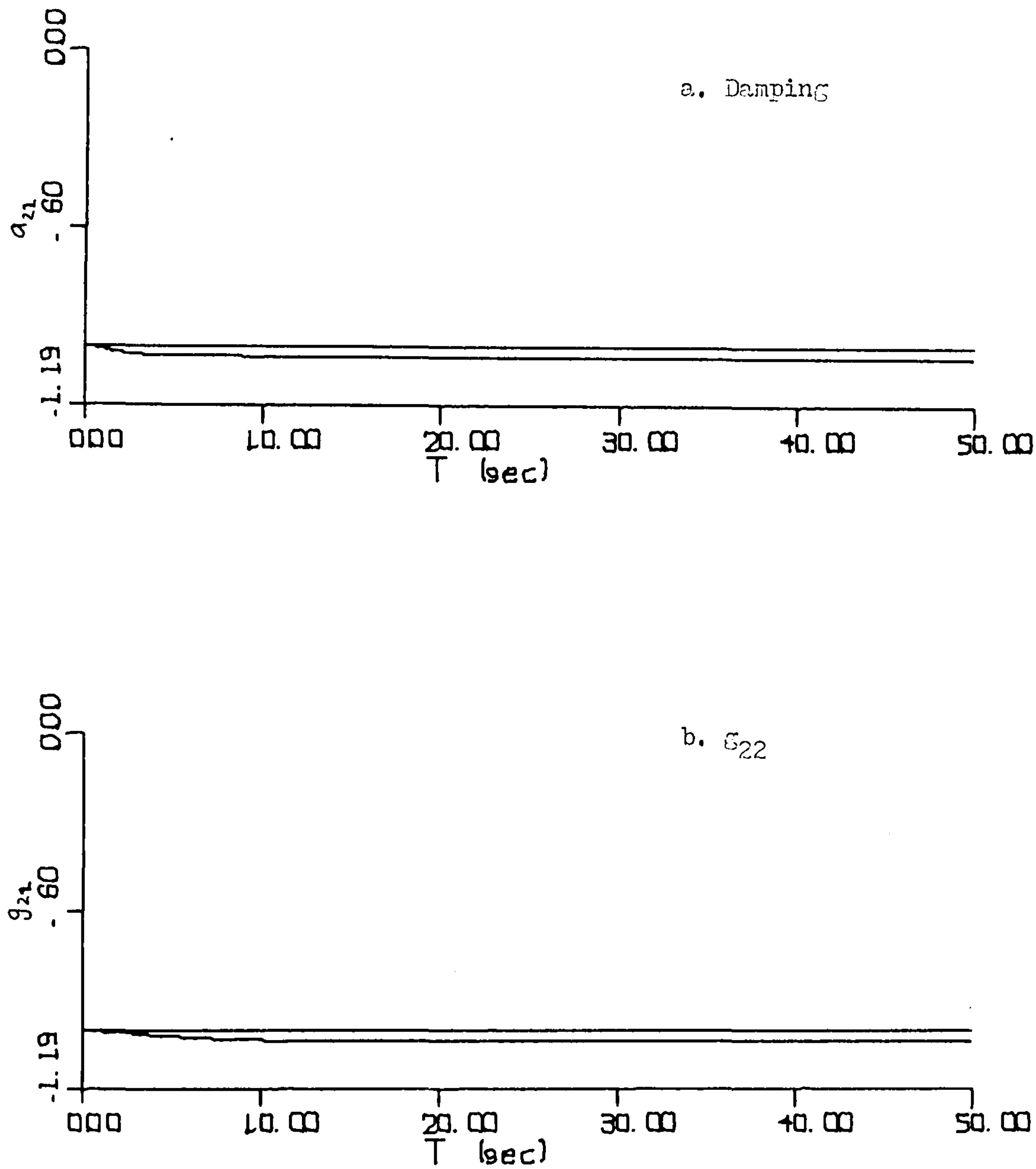


Figure 10: Variation of Damping--Nonlinearities in stiffness and damping (Step Input)

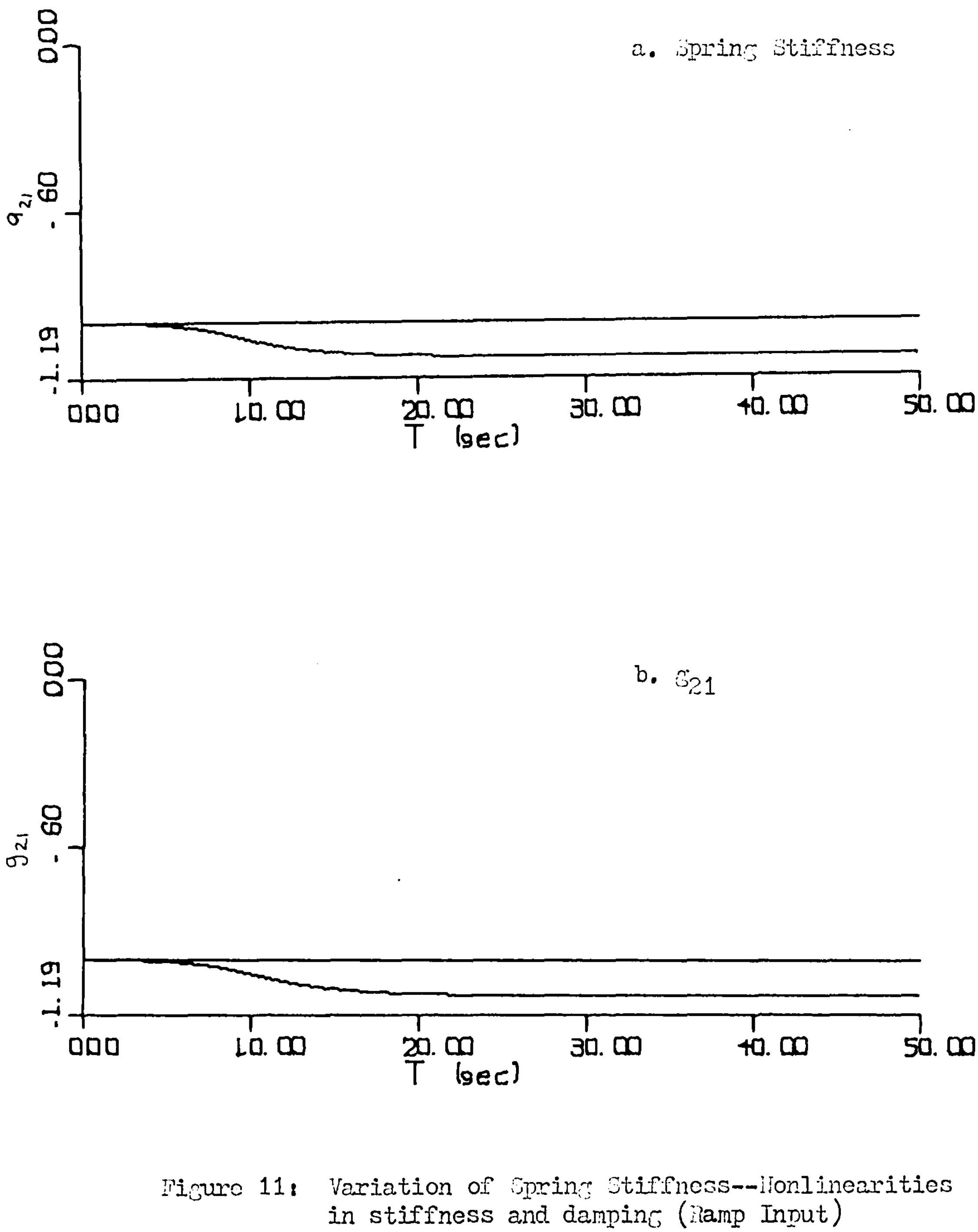


Figure 11: Variation of Spring Stiffness--Nonlinearities in stiffness and damping (Ramp Input)

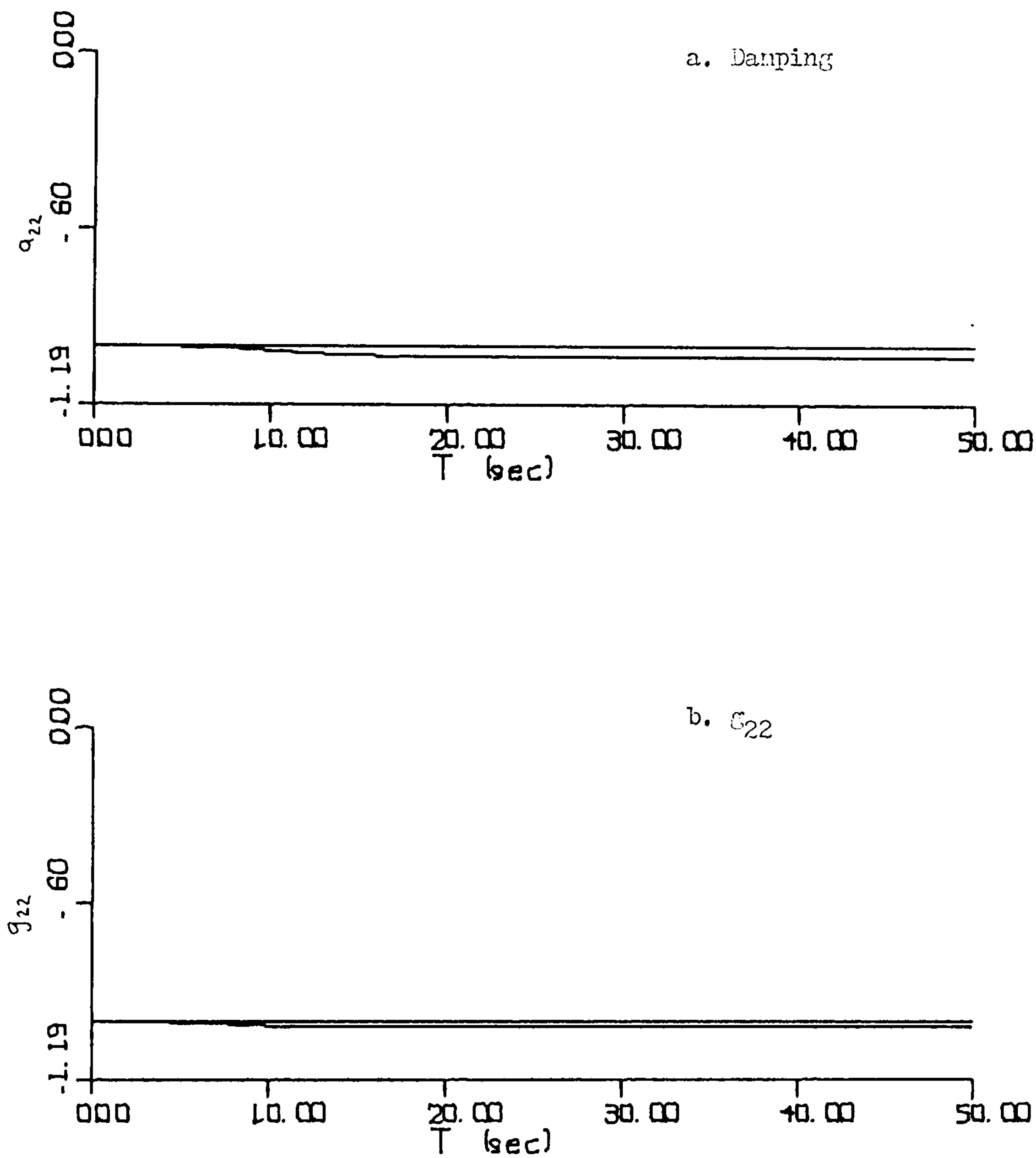


Figure 12: Variation of Damping--Nonlinearities in stiffness and damping (Ramp Input)

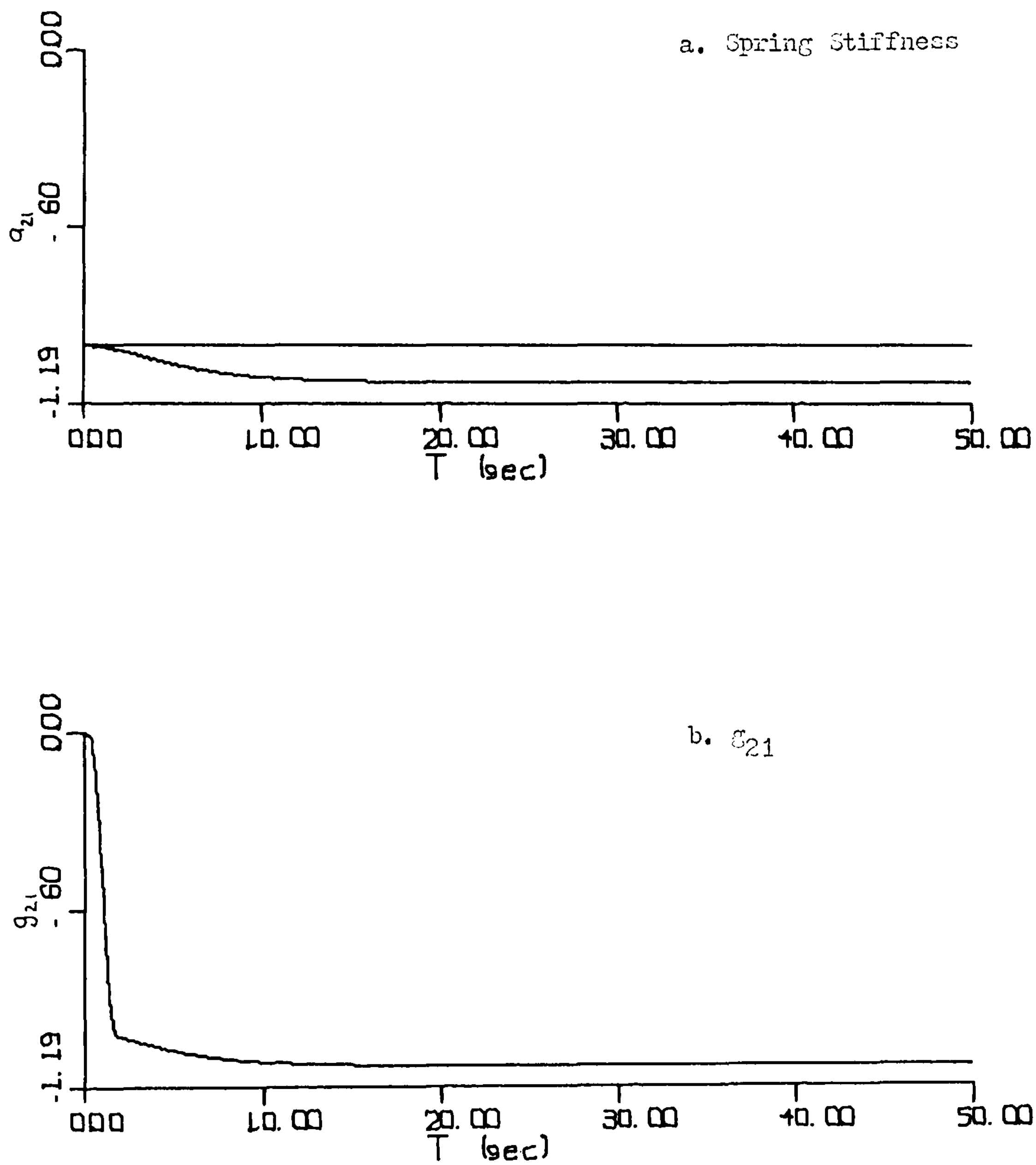
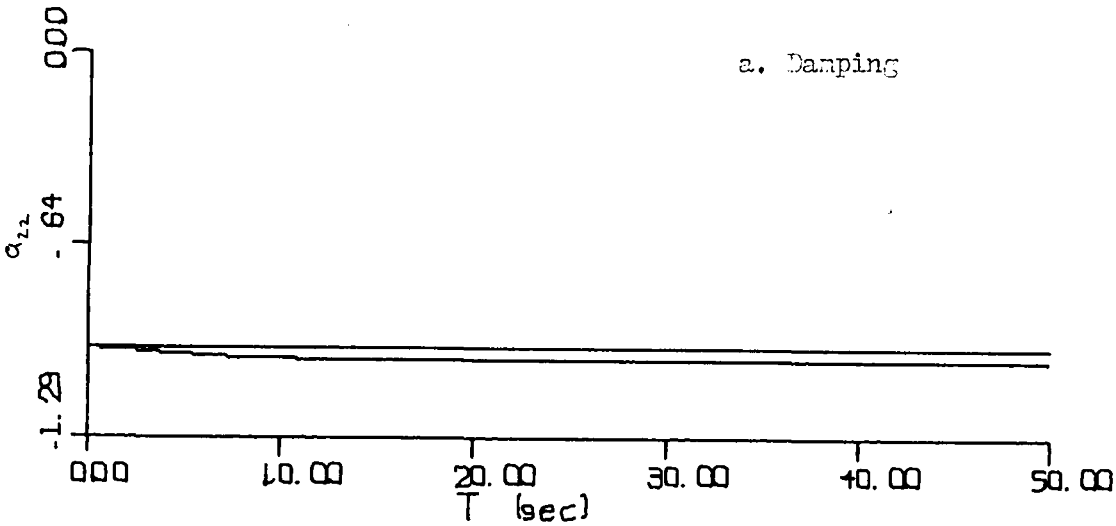


Figure 13: Variation of Spring Stiffness--Nonlinearities in stiffness and damping and initial value of g_{21} is zero (Step Input)



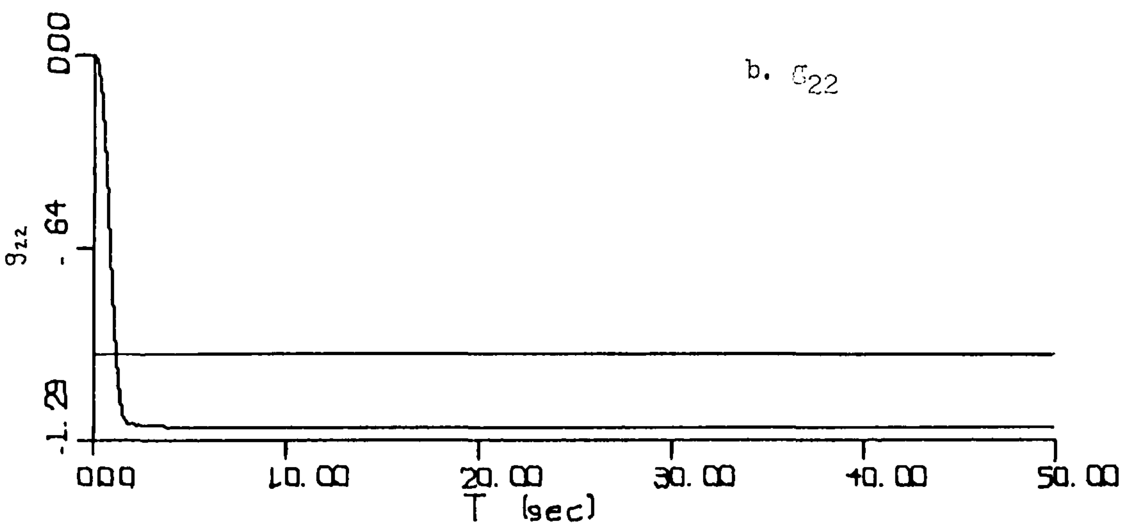


Figure 14: Variation of Damping--Nonlinearities in stiffness and damping and initial value of g_{22} is zero (Step Input)

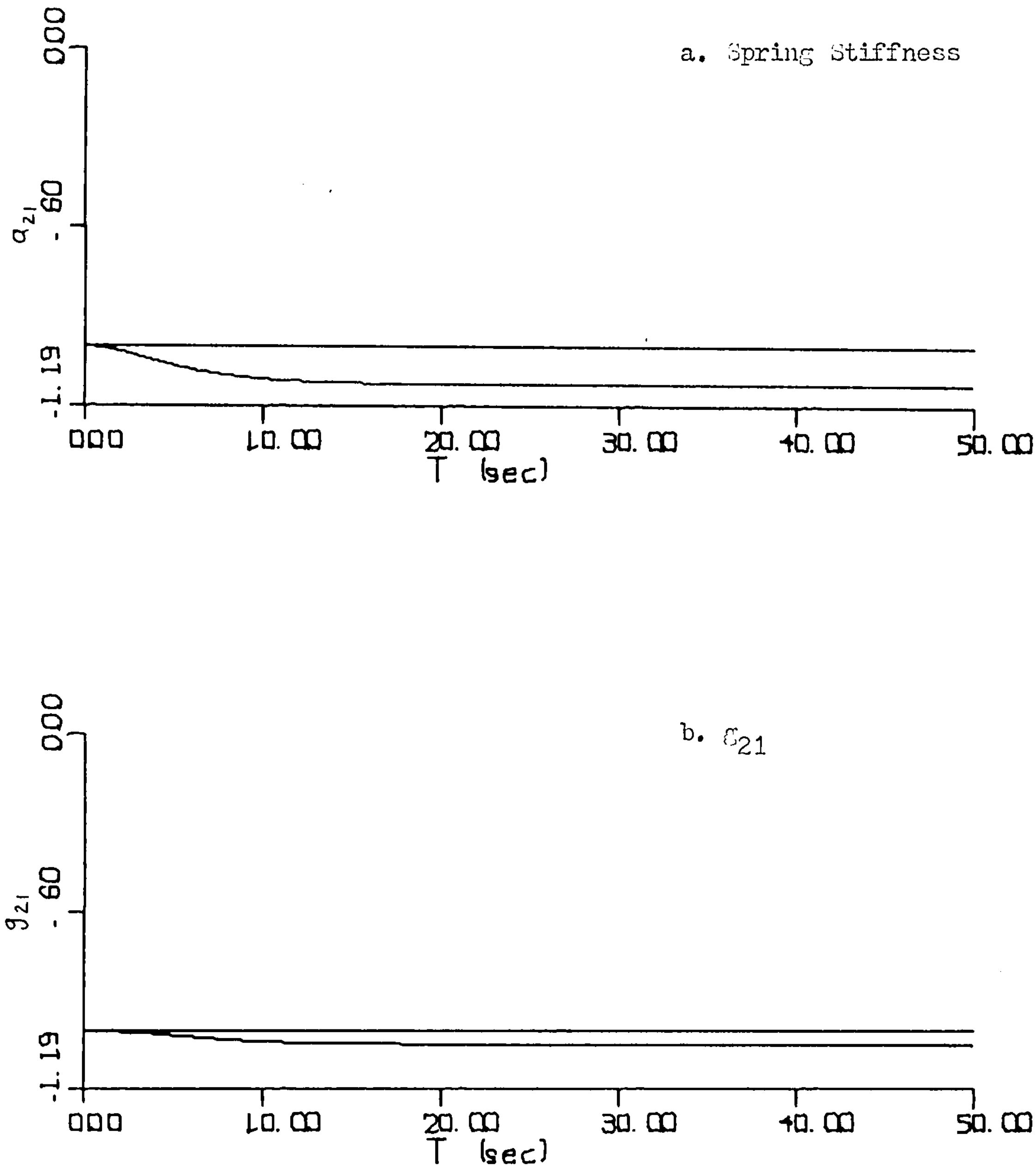
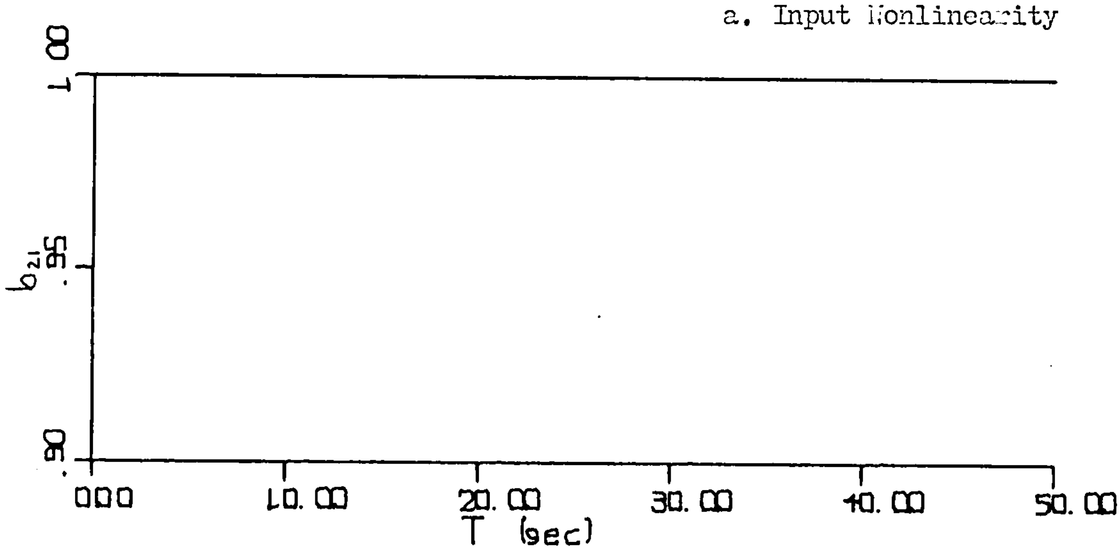


Figure 15: Variation of Spring Stiffness--Nonlinearities in stiffness and input (Step Input)

a. Input Nonlinearity



b. h_{21}

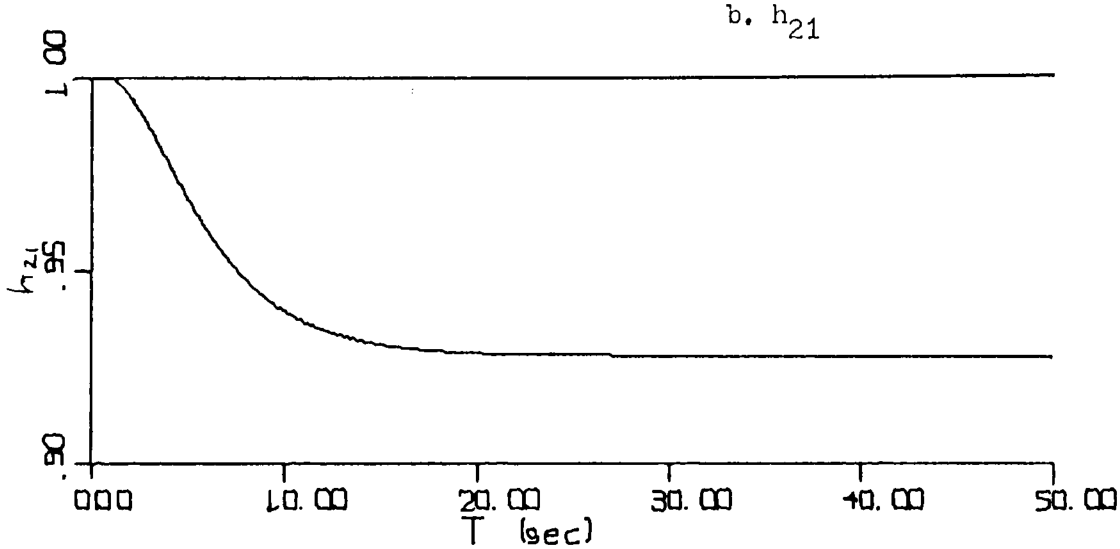
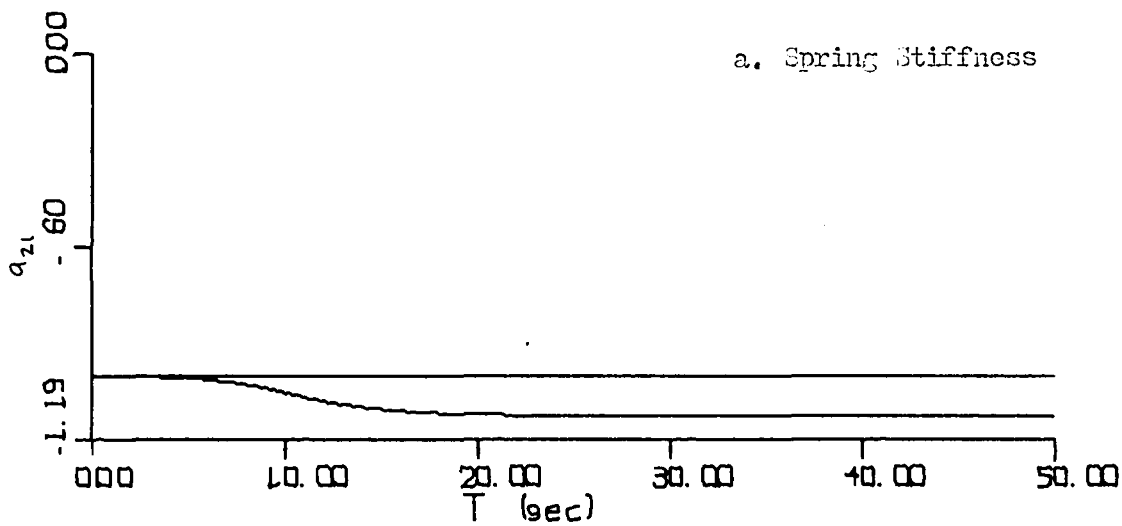


Figure 16: Variation of Input Nonlinearity--Nonlinearities in stiffness and input (Step Input)



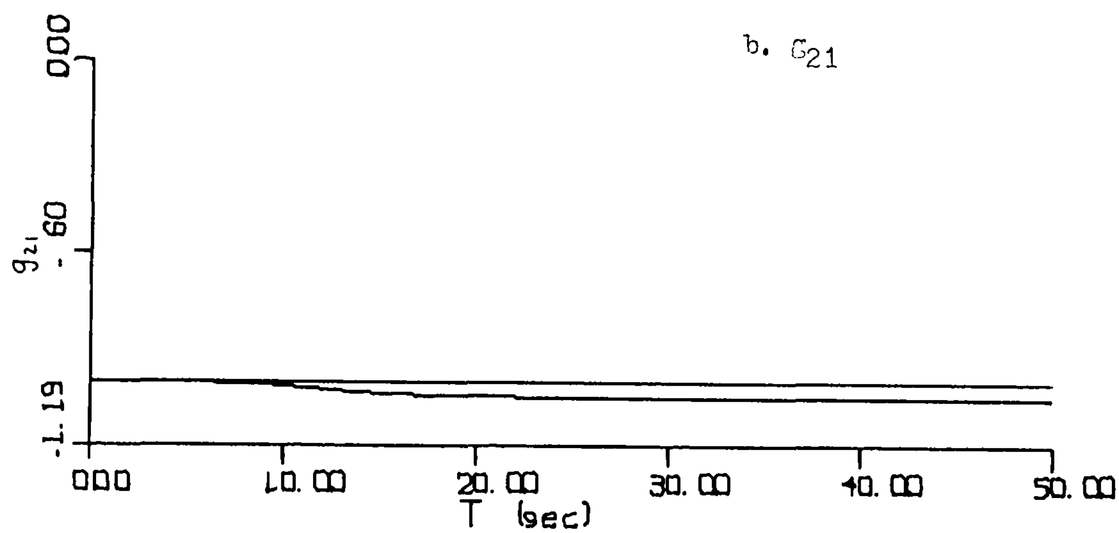
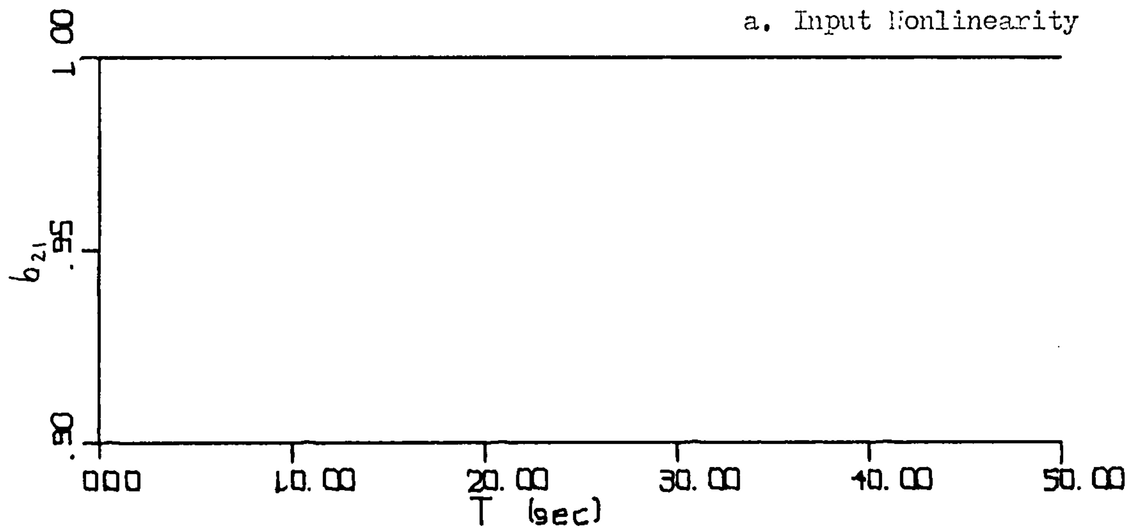


Figure 17: Variation of Spring Stiffness--Nonlinearities in stiffness and input (Ramp Input)



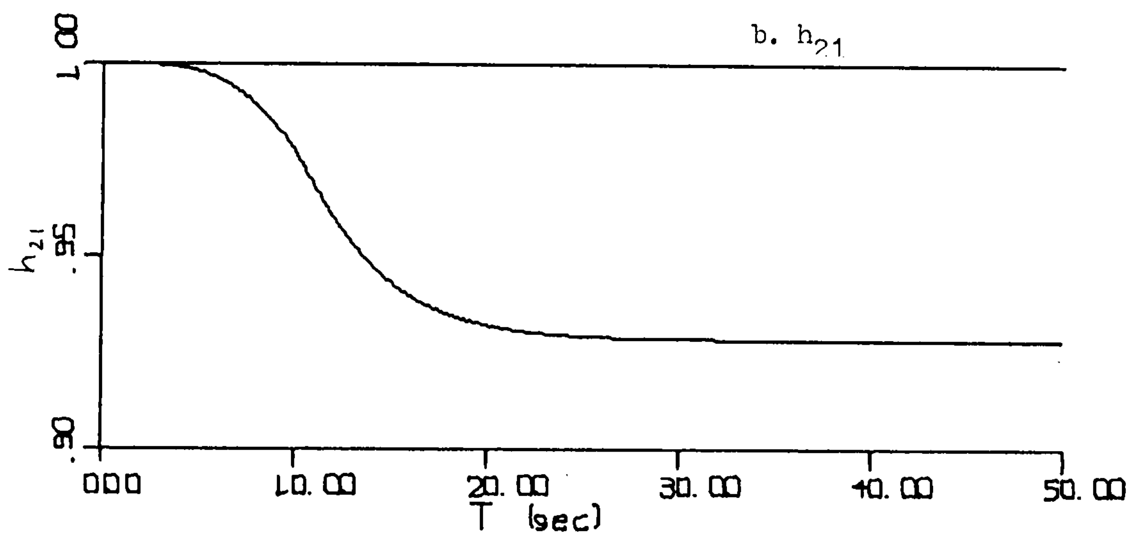


Figure 18: Variation of Input Nonlinearity--Nonlinearities in stiffness and input (Ramp Input)

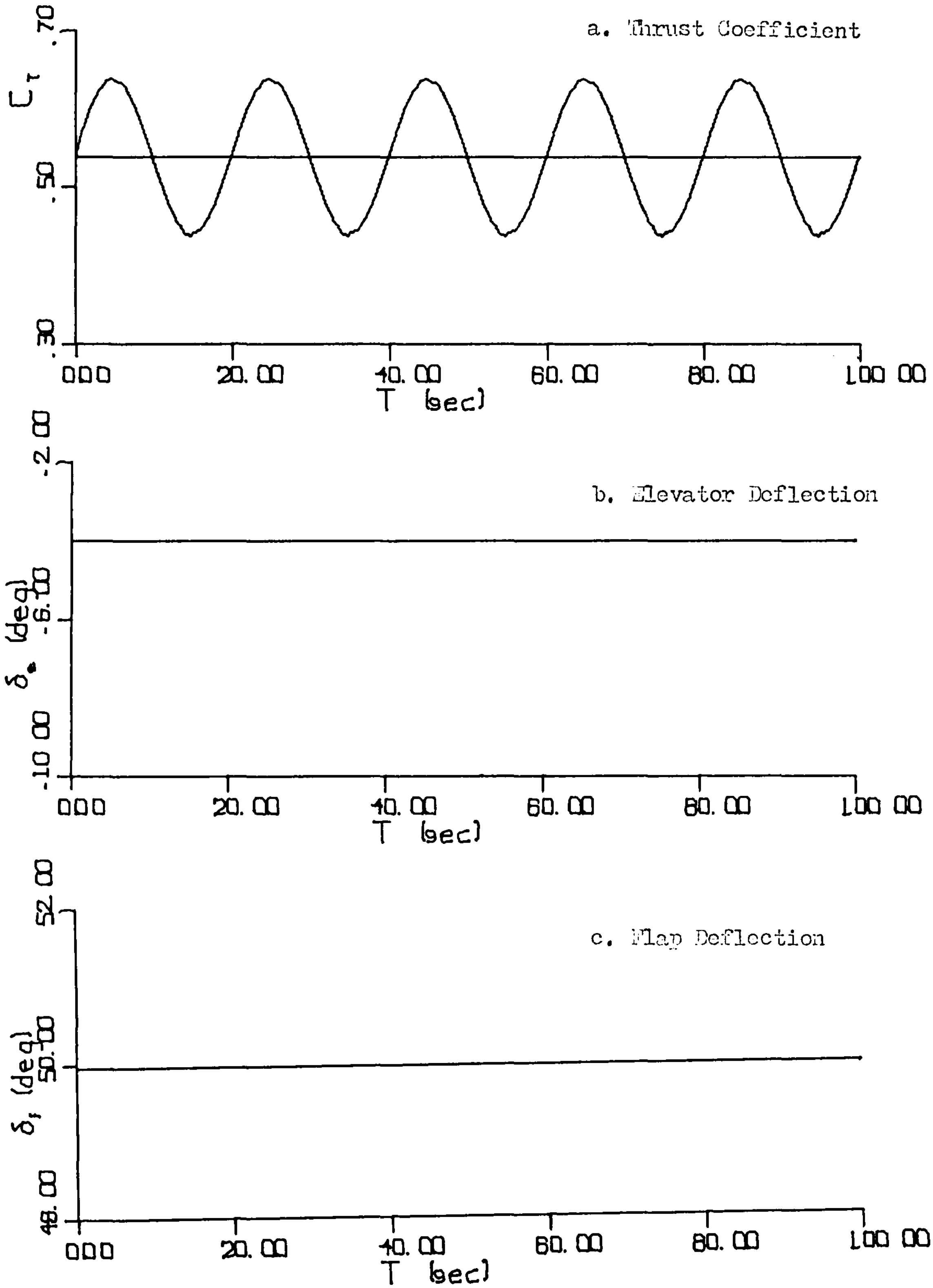


Figure 19: Controls vs. Time

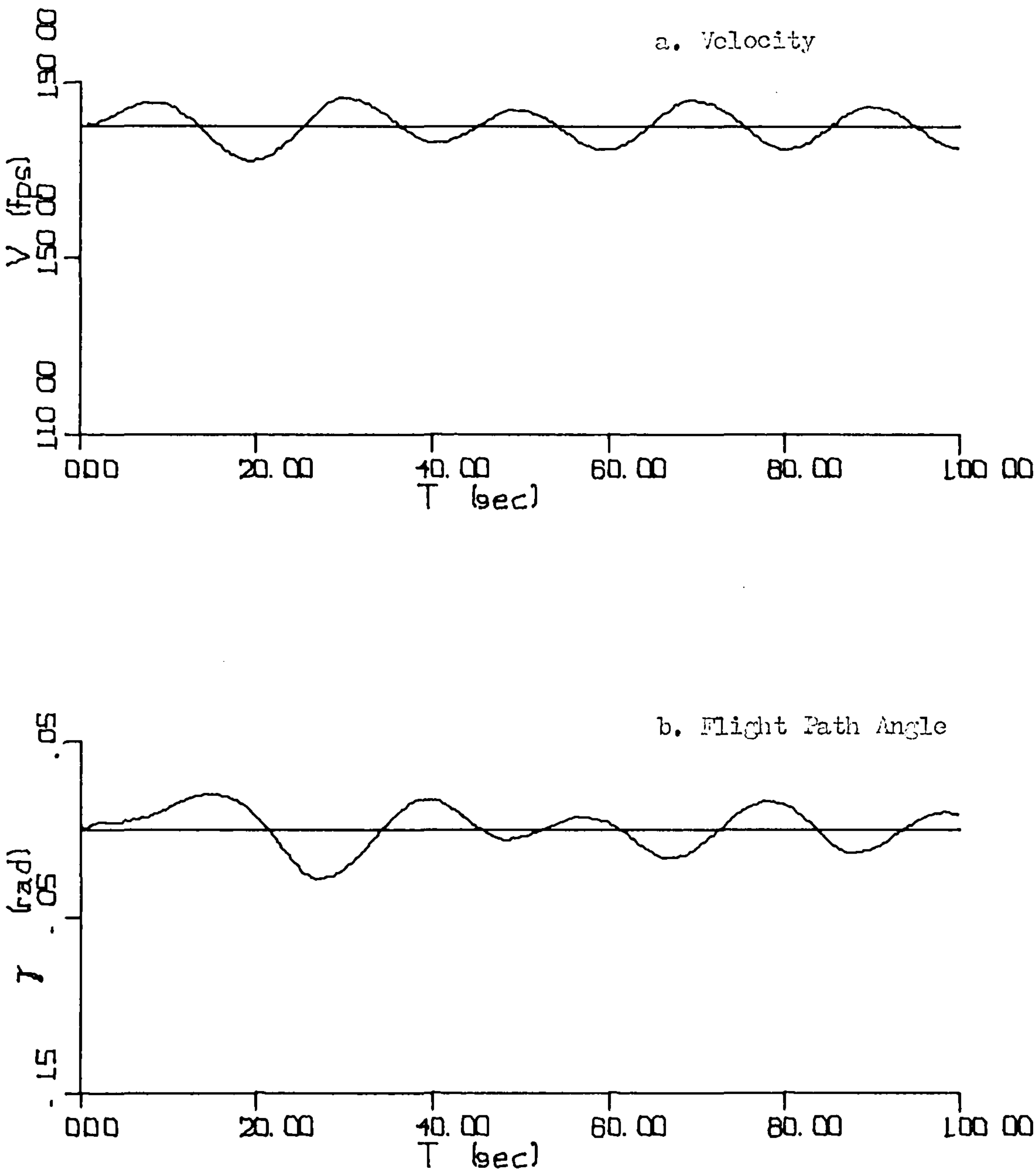
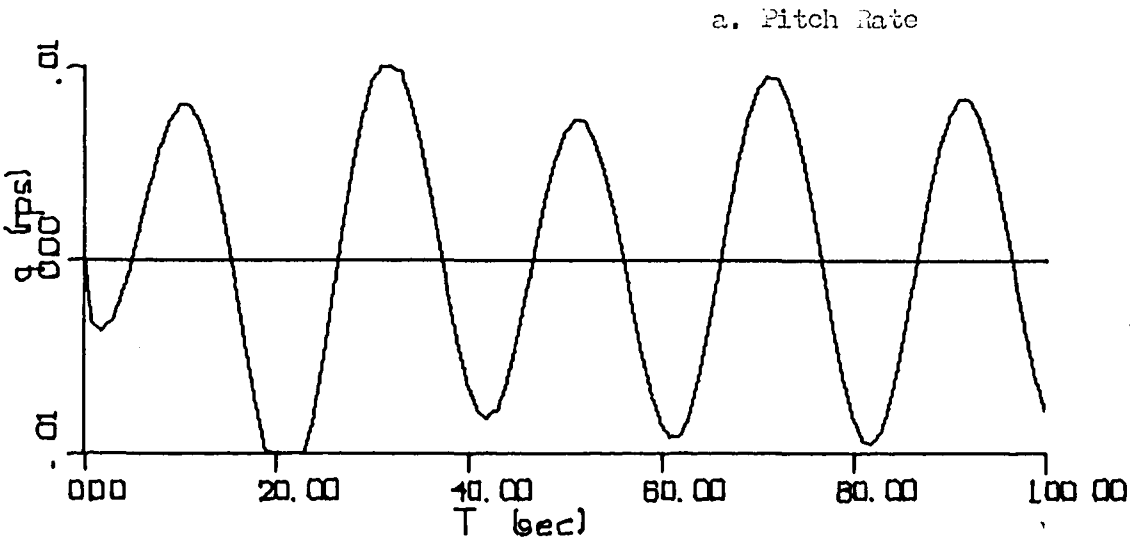


Figure 20: States vs. Time



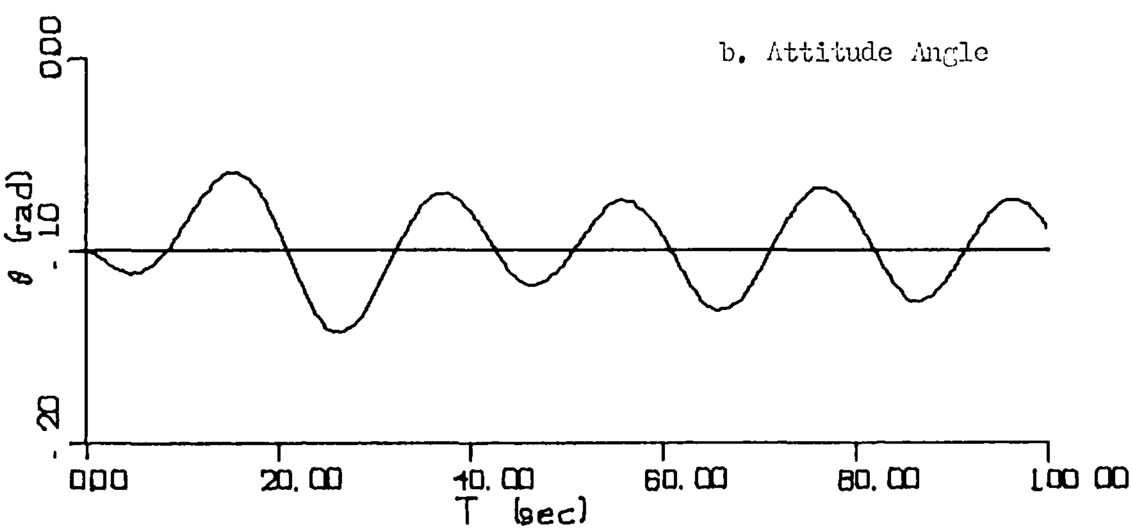


Figure 21: States vs. Time

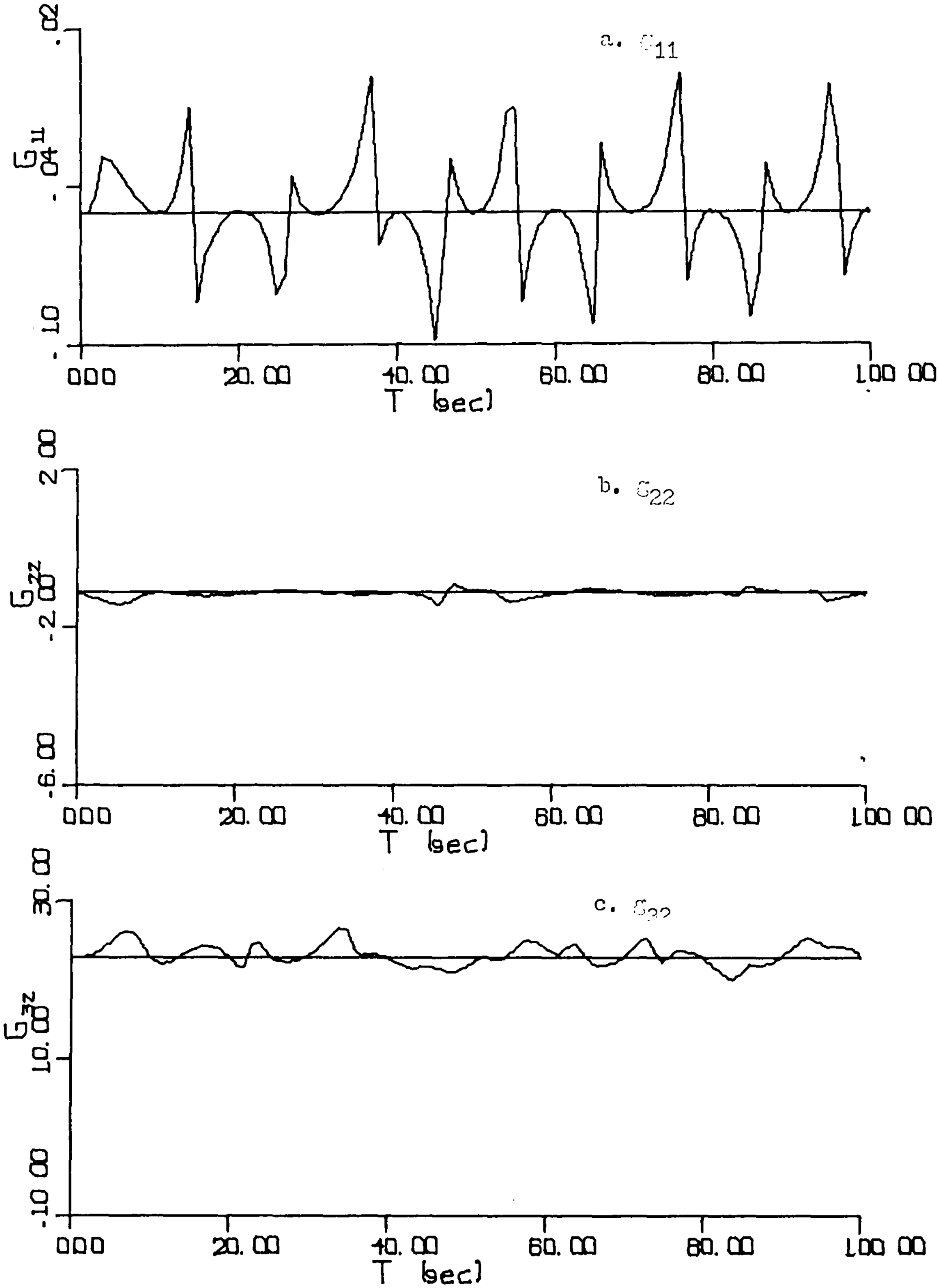


Figure 22: Variation of elements in G matrix

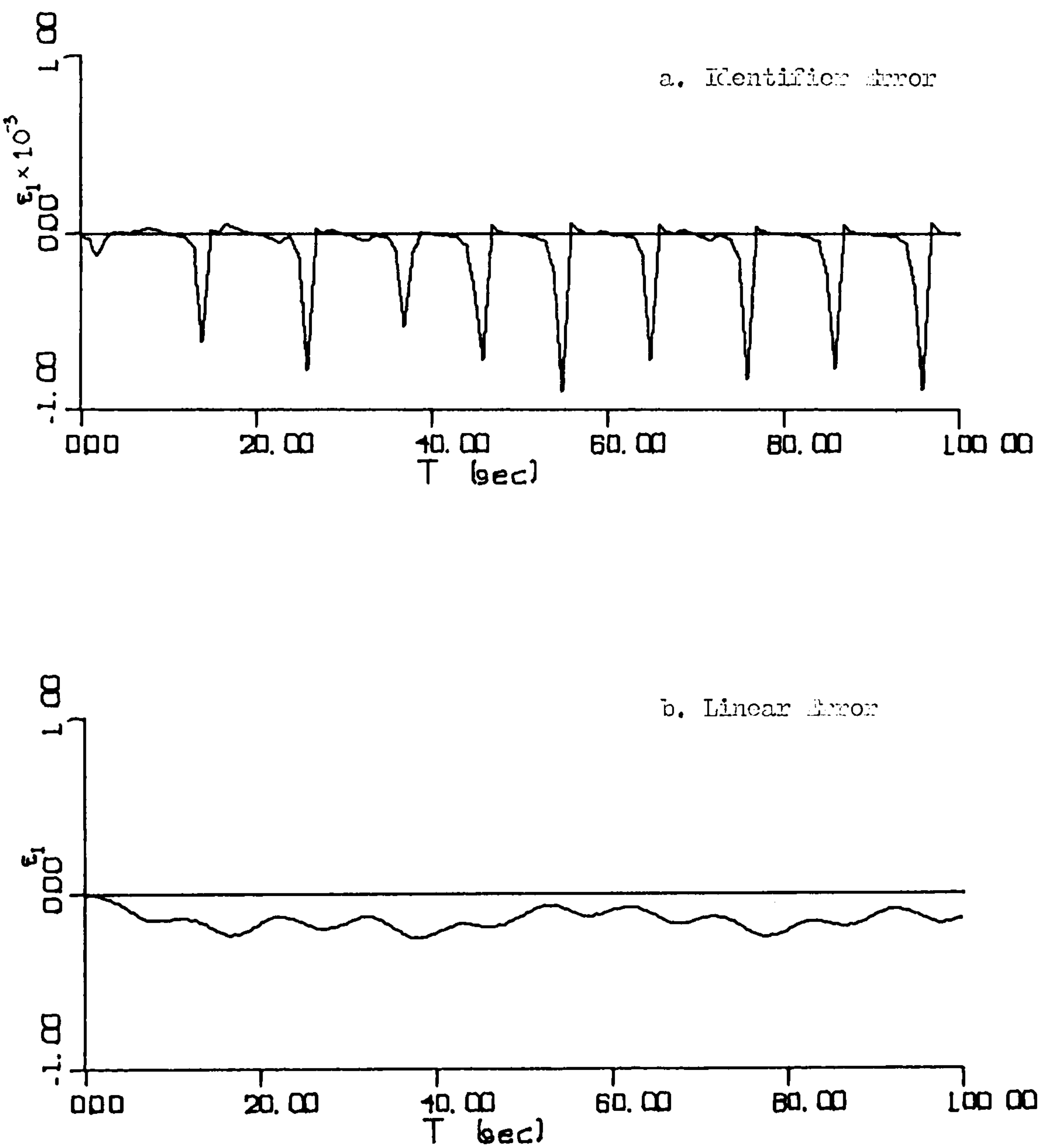
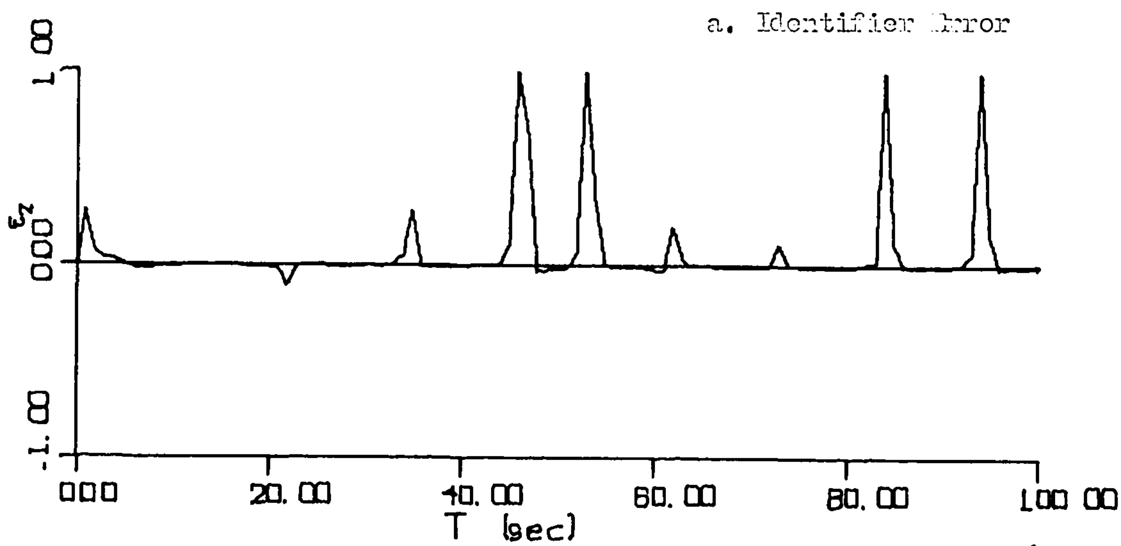


Figure 23: Errors in Velocity



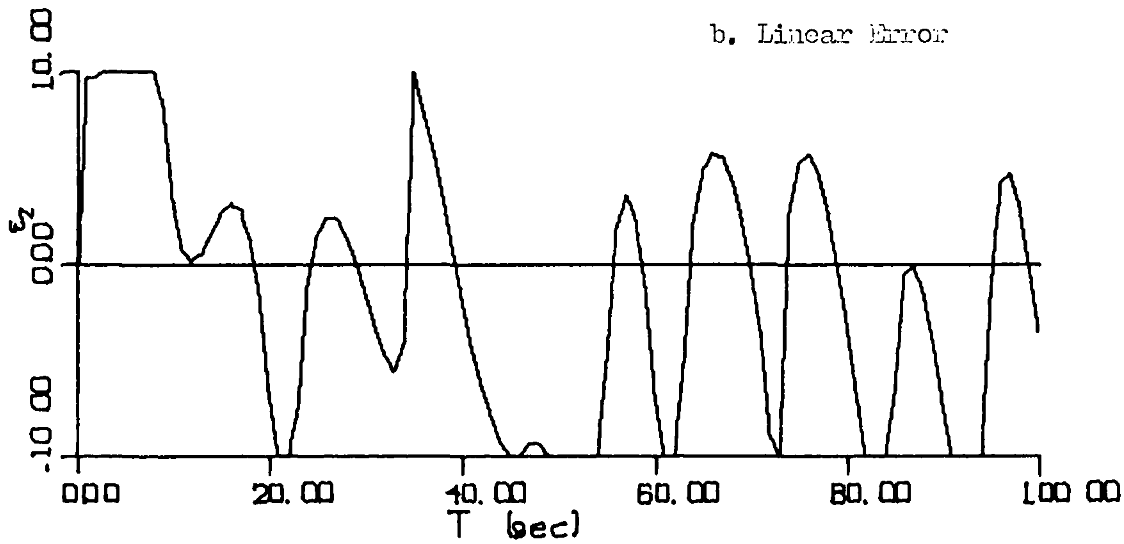
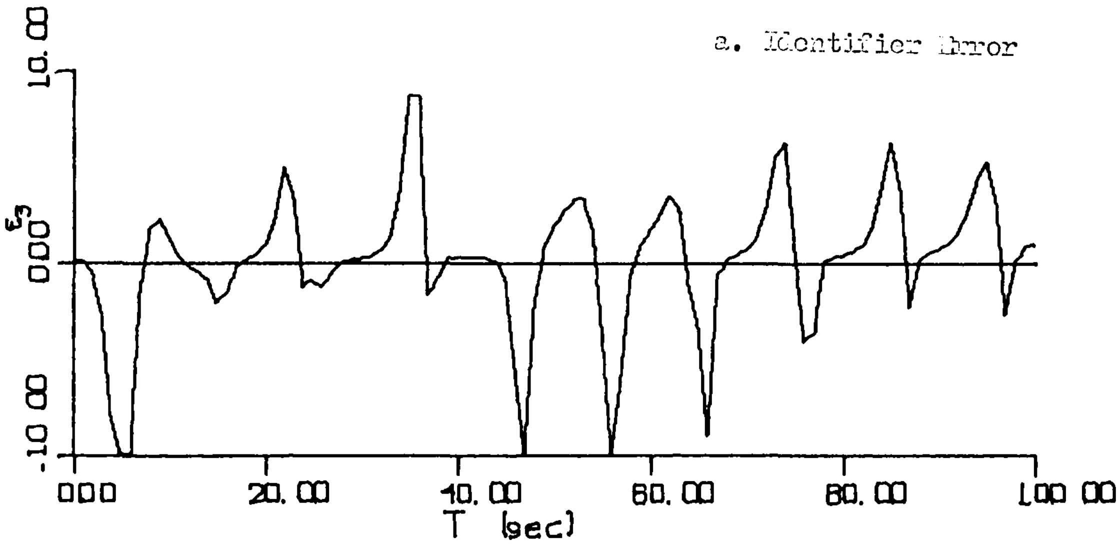


Figure 24: Errors in Flight Path Angle



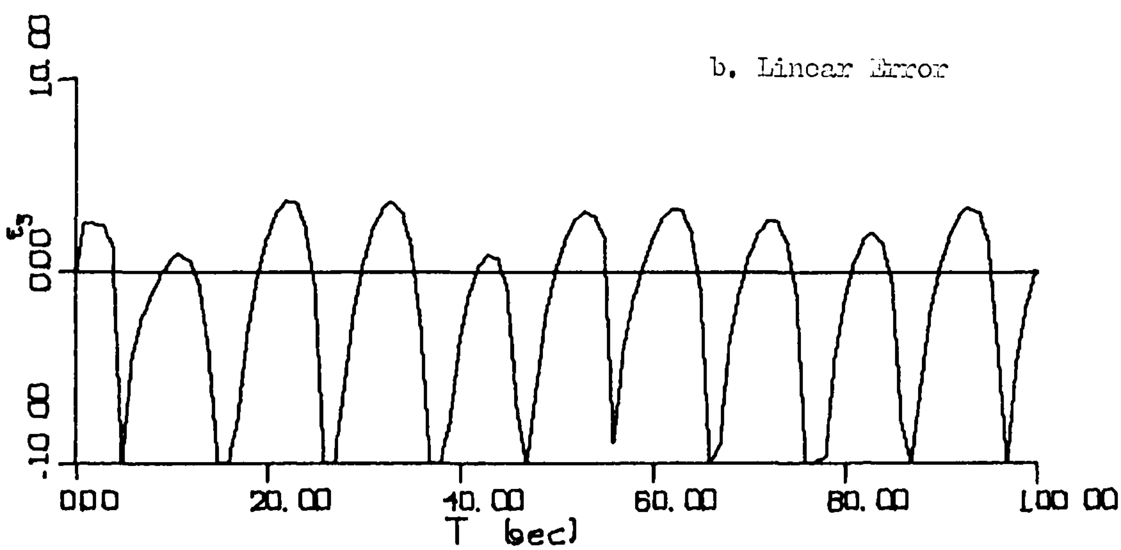


Figure 25: Errors in Pitch Rate

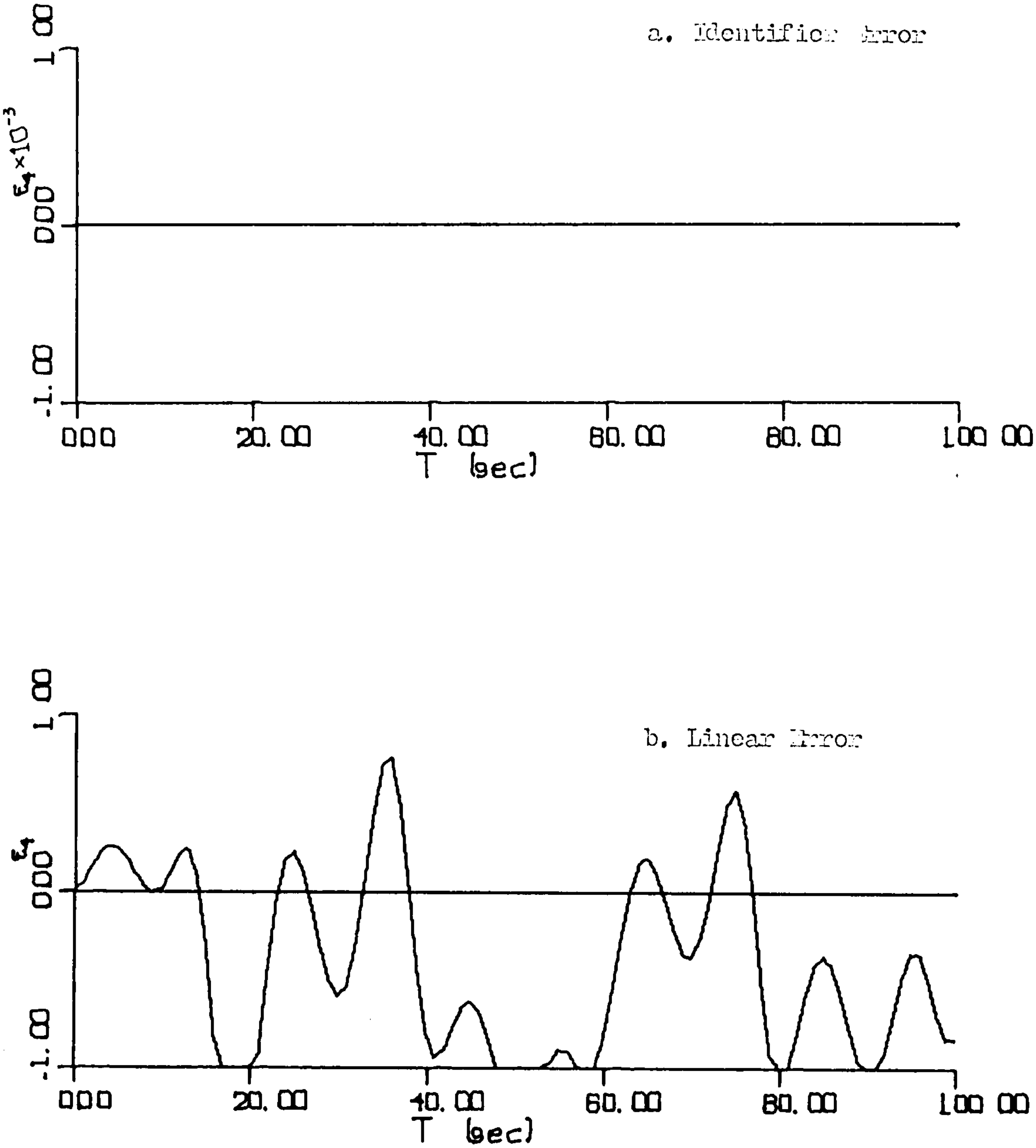


Figure 26: Errors in Attitude Angle

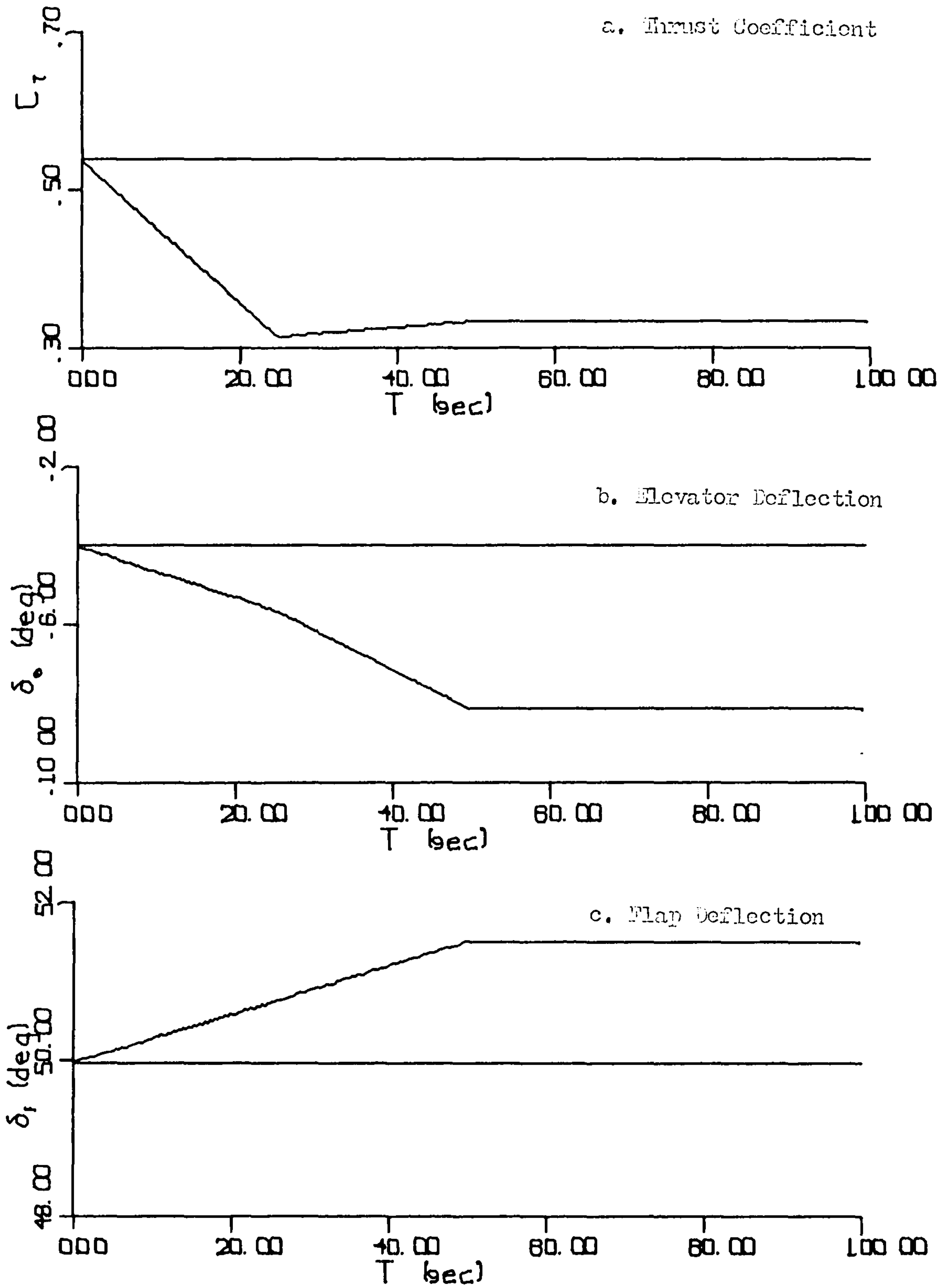


Figure 27: Controls vs. Time

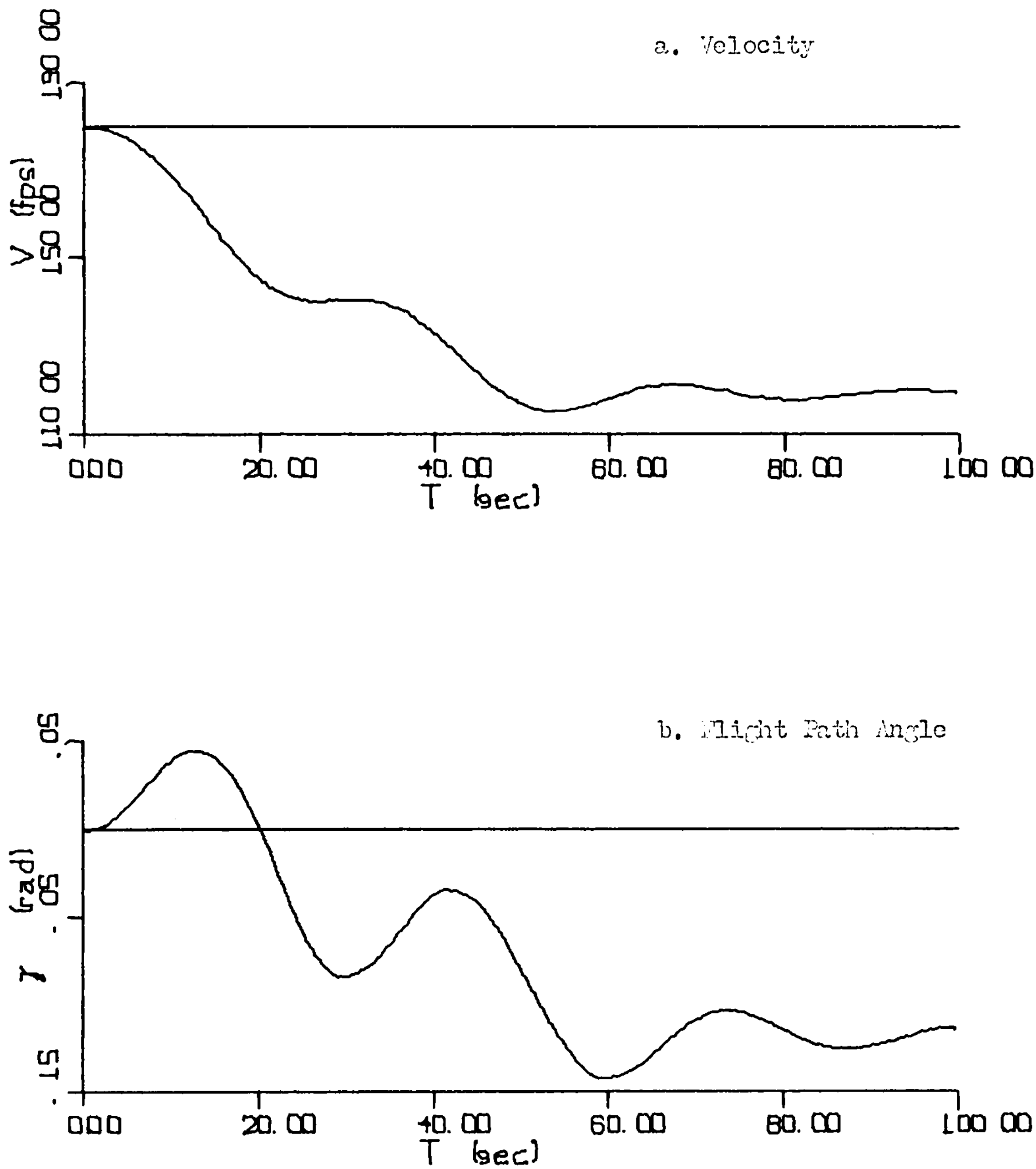
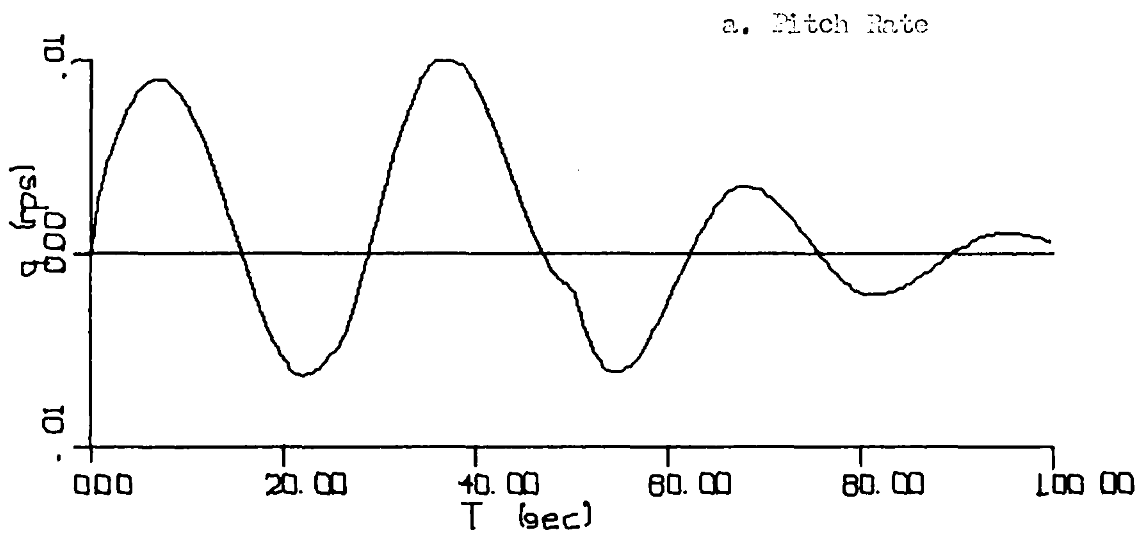


Figure 28: States vs. Time



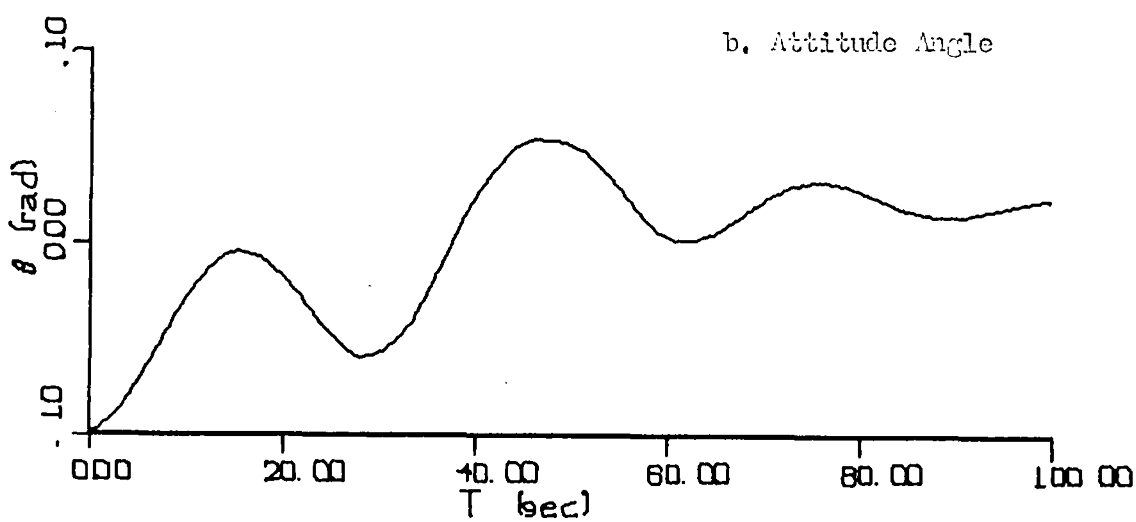


Figure 29: States vs. Time

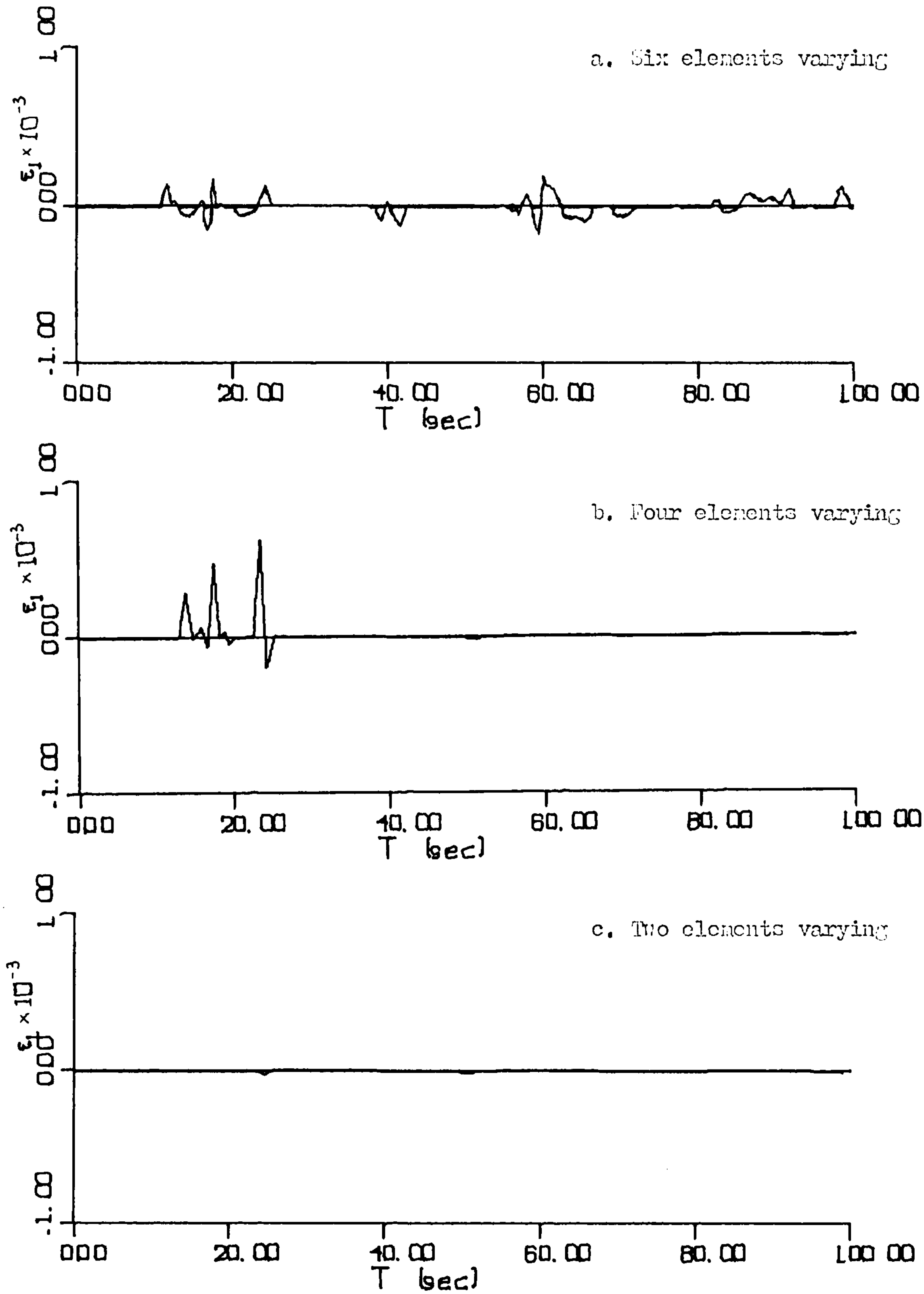


Figure 30: Identifier error in velocity

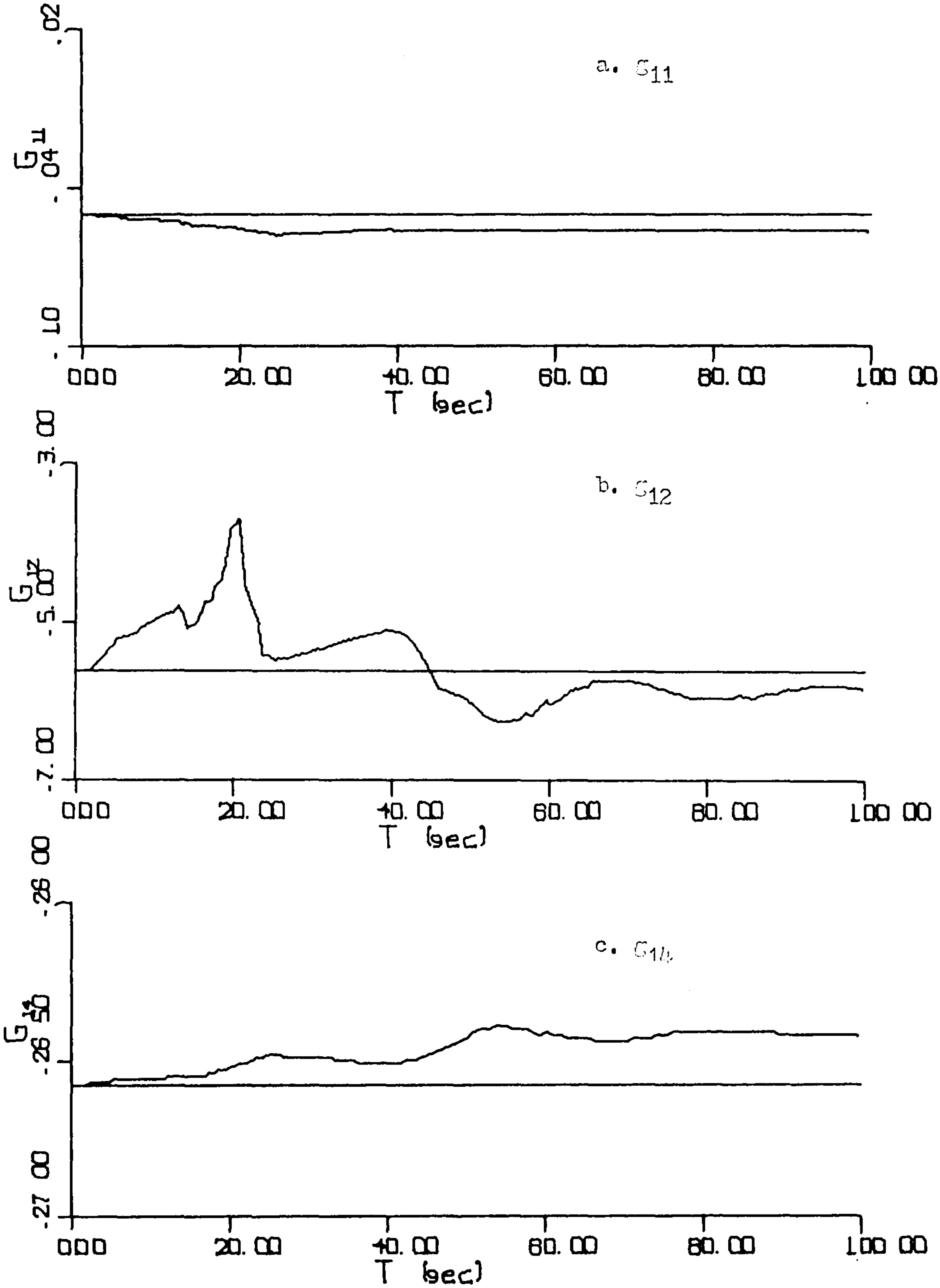


Figure 31: Variation of three elements in first row of G

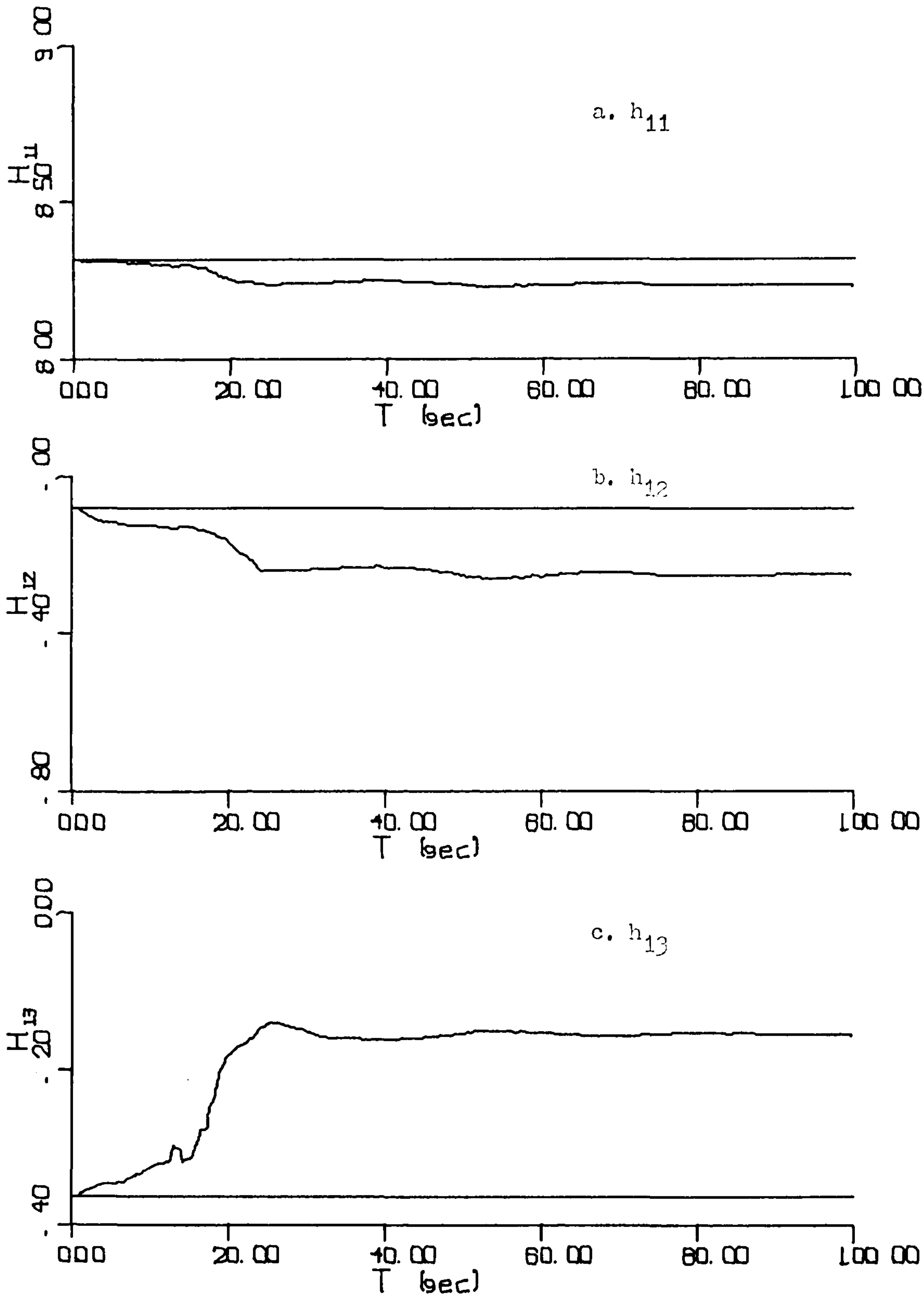


Figure 32: Variation of three elements in first row of H

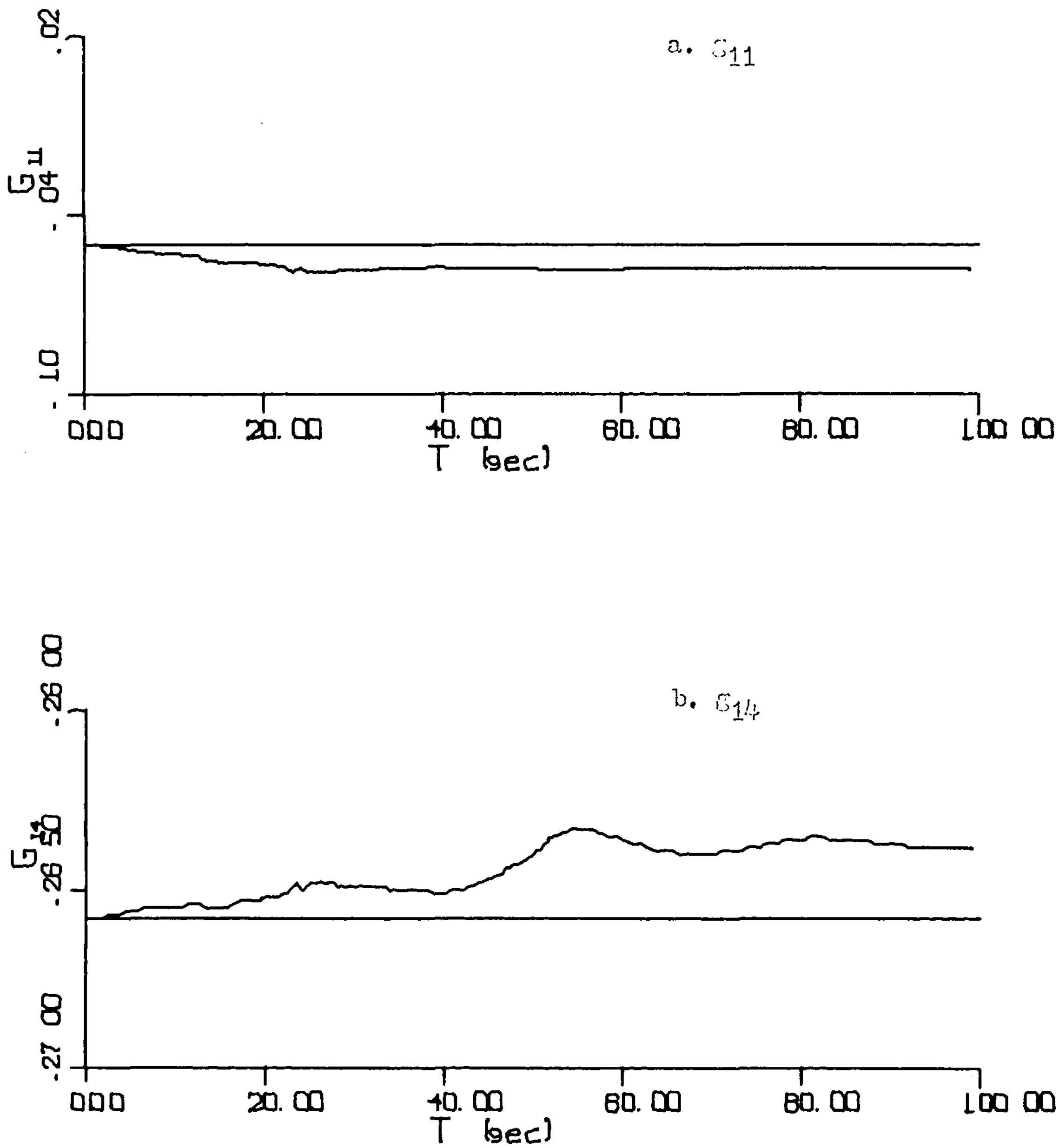
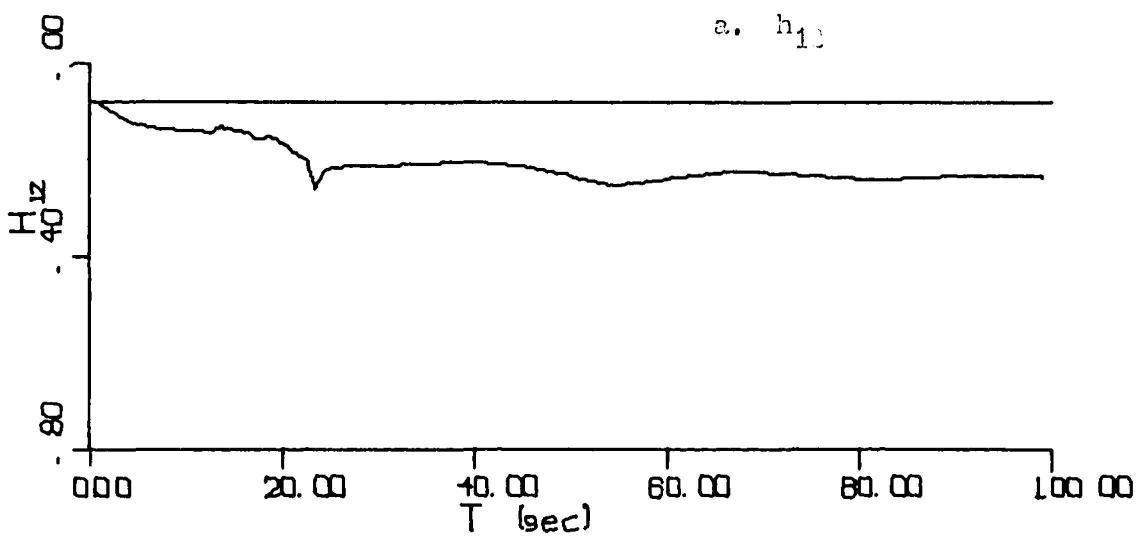


Figure 33: Variation of only two elements in first row of G



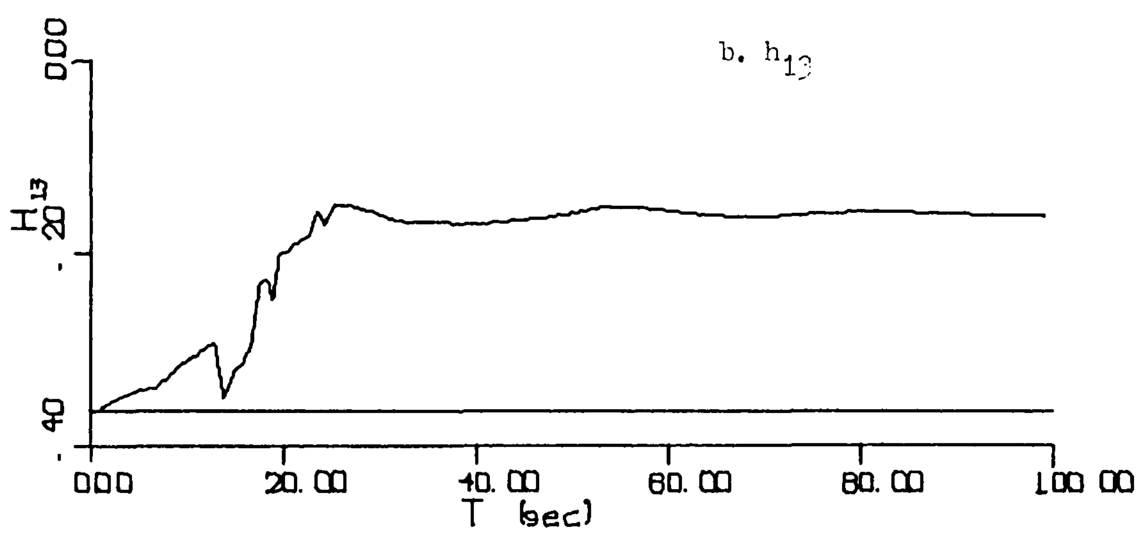


Figure 34: Variation of only two elements in first row of H

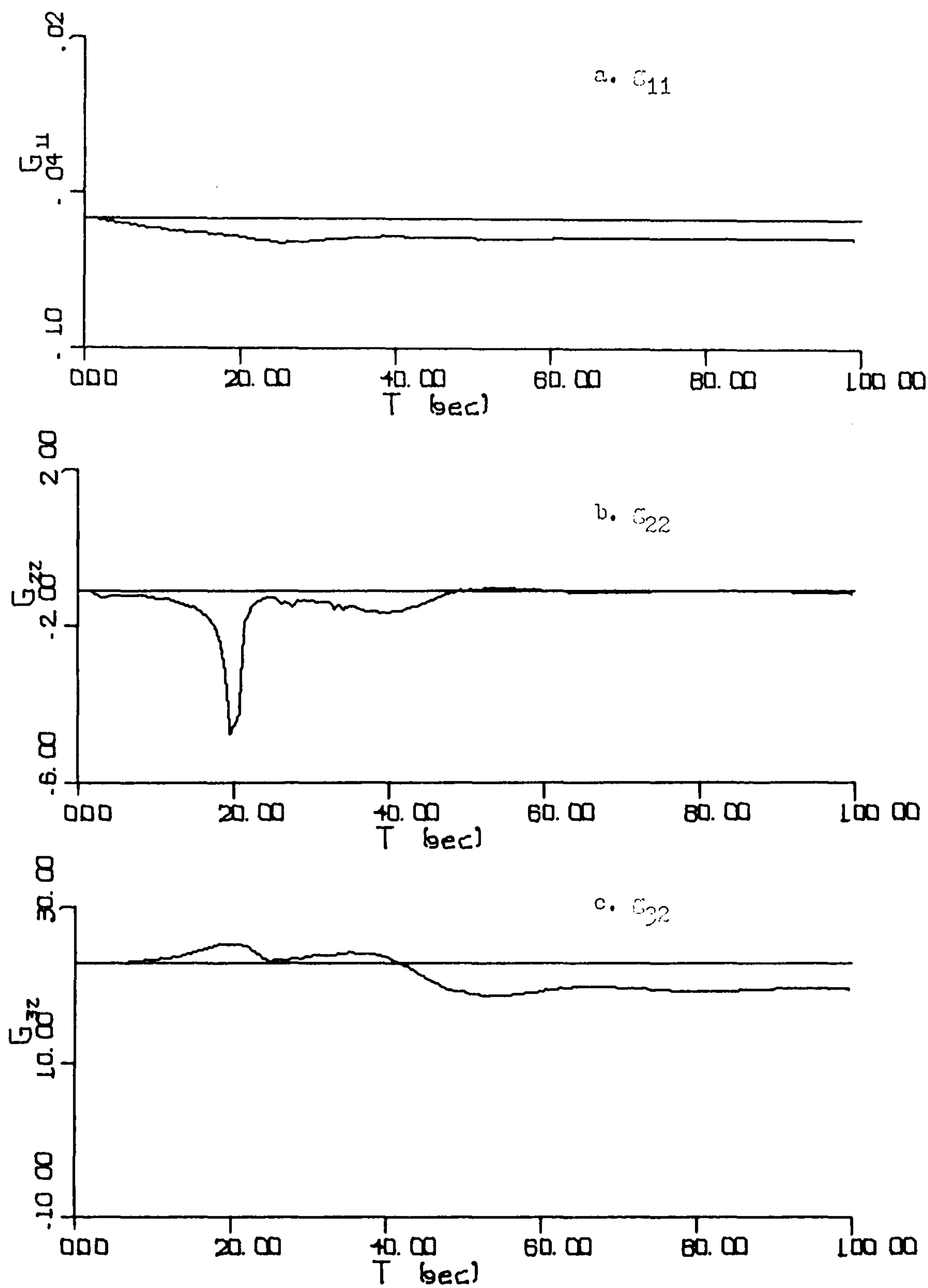
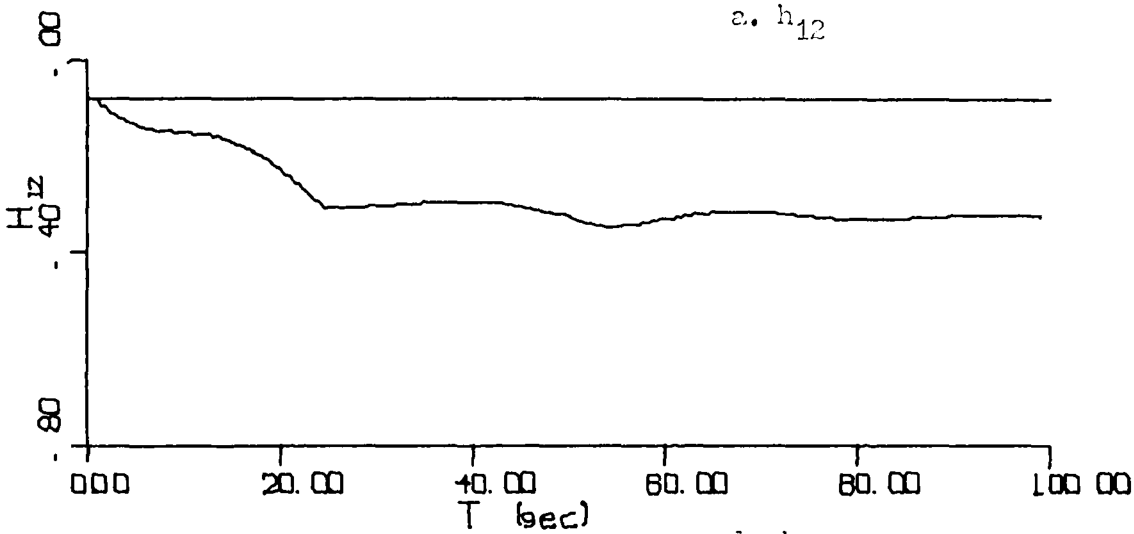
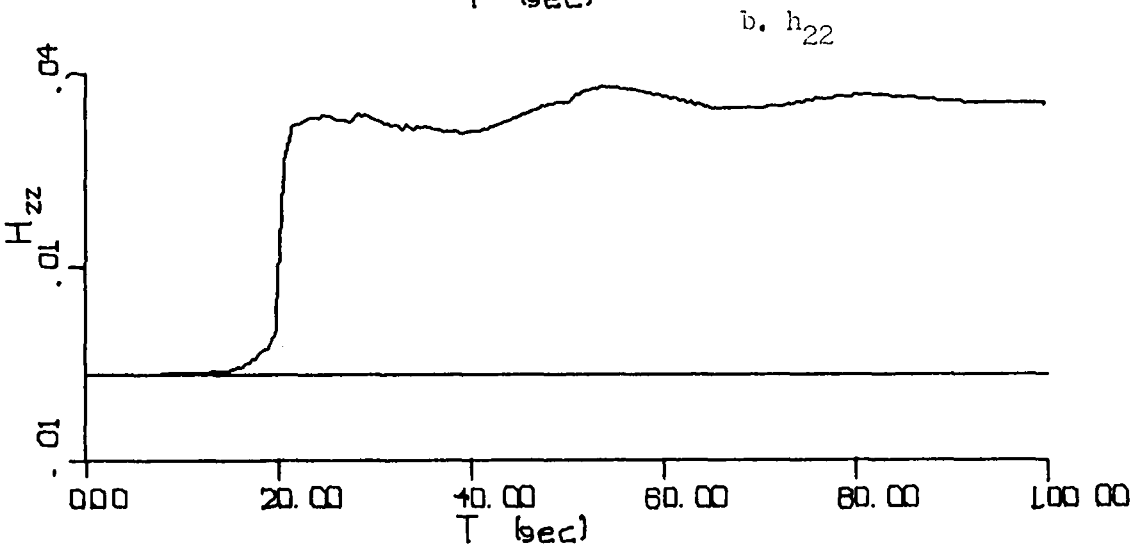


Figure 35: Variation of one element in each row of G





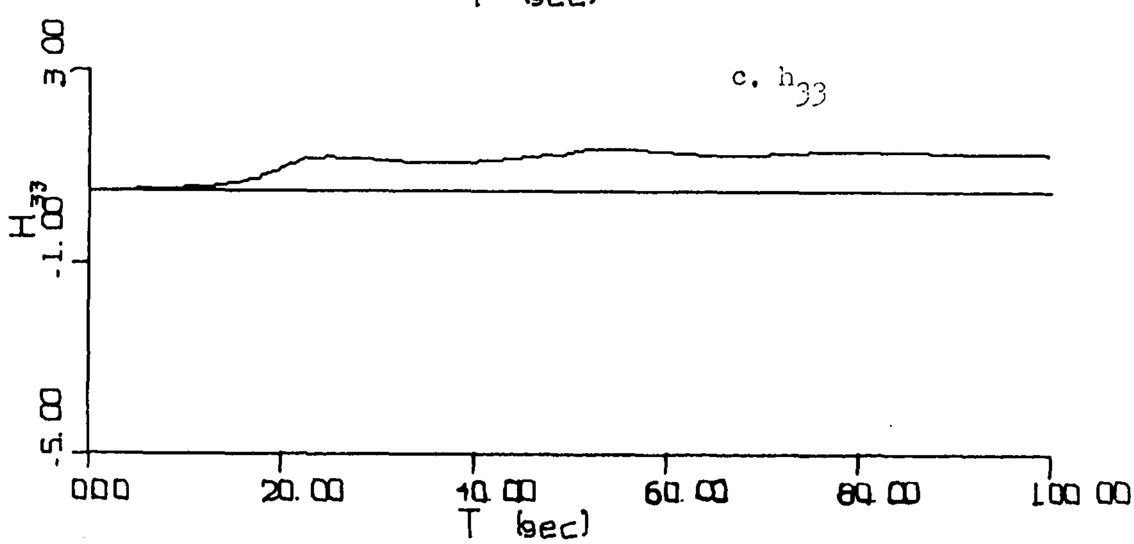


Figure 36: Variation of one element in each row of H

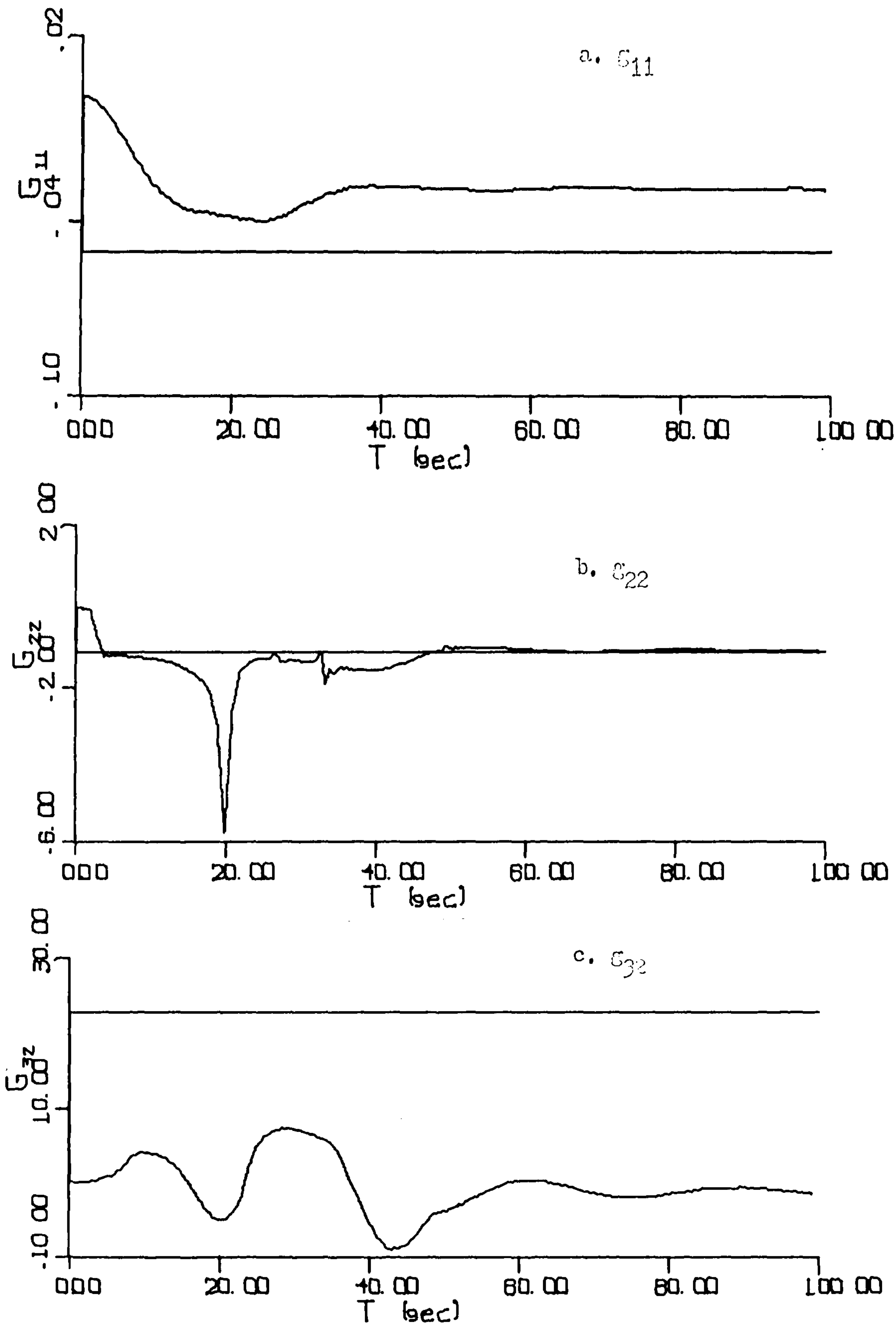


Figure 37: Variation of elements in G (Nonconstant elements have initial value zero)

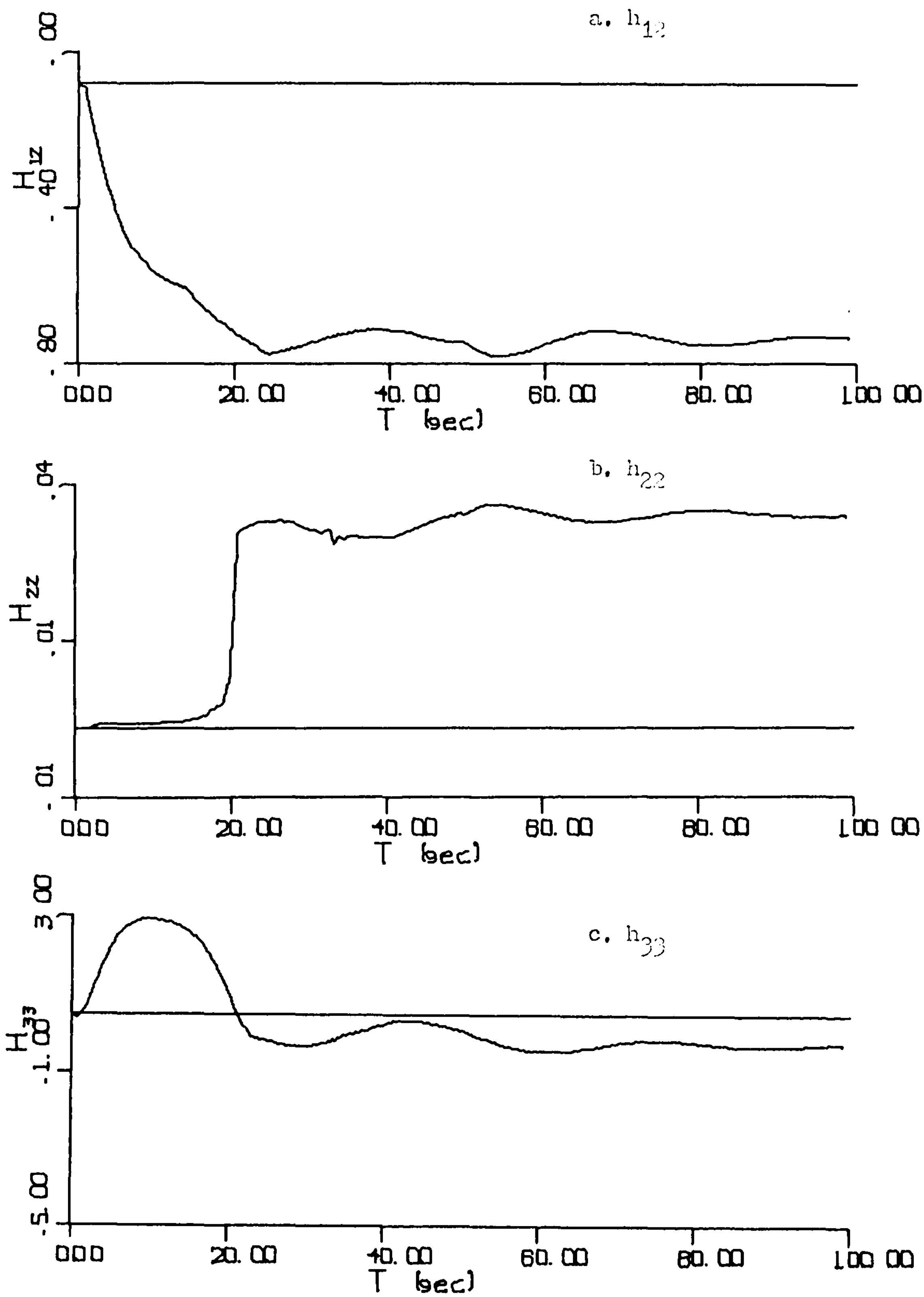


Figure 38: Variation of elements in H (Nonconstant elements of G have initial values of zero)

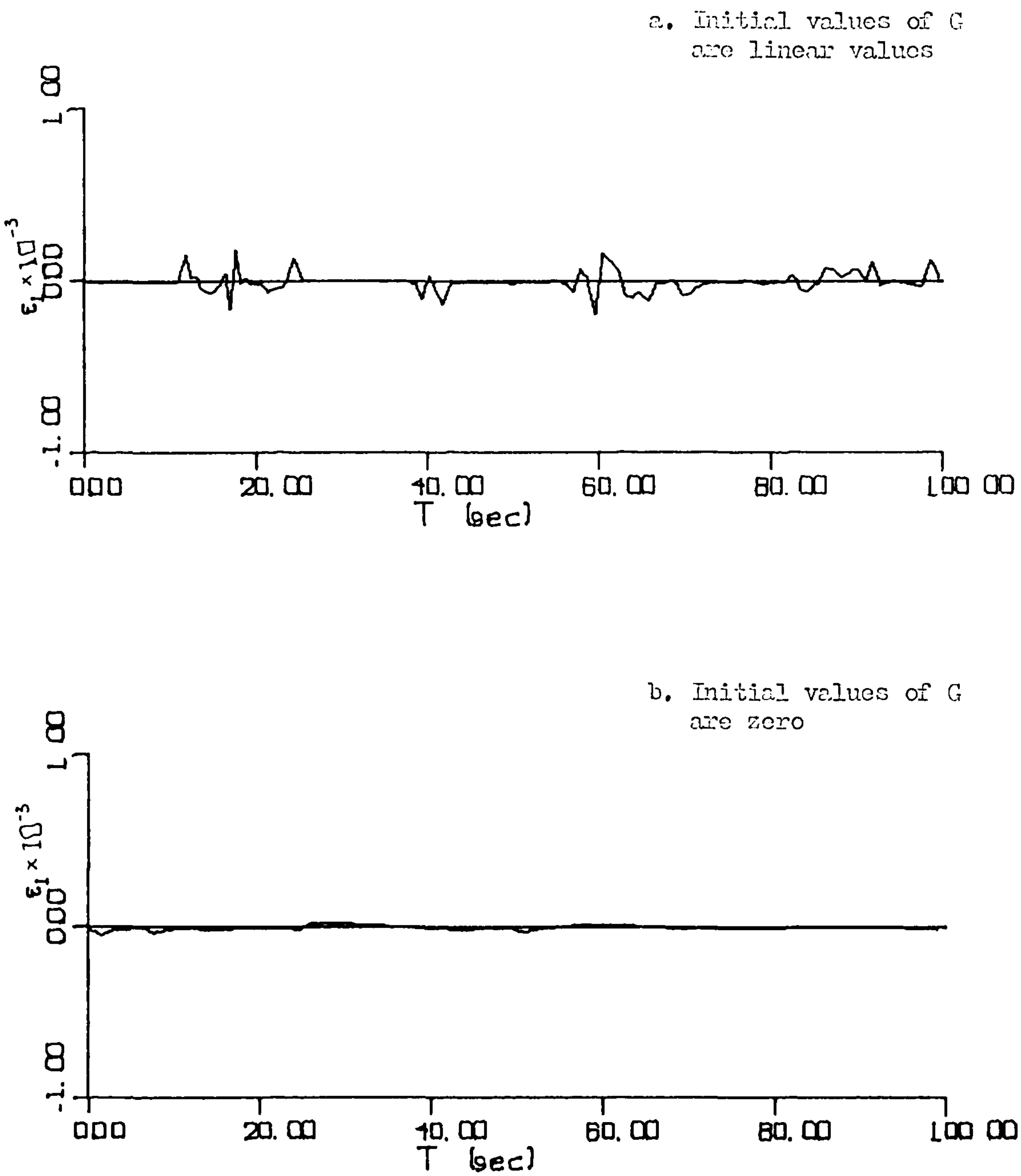
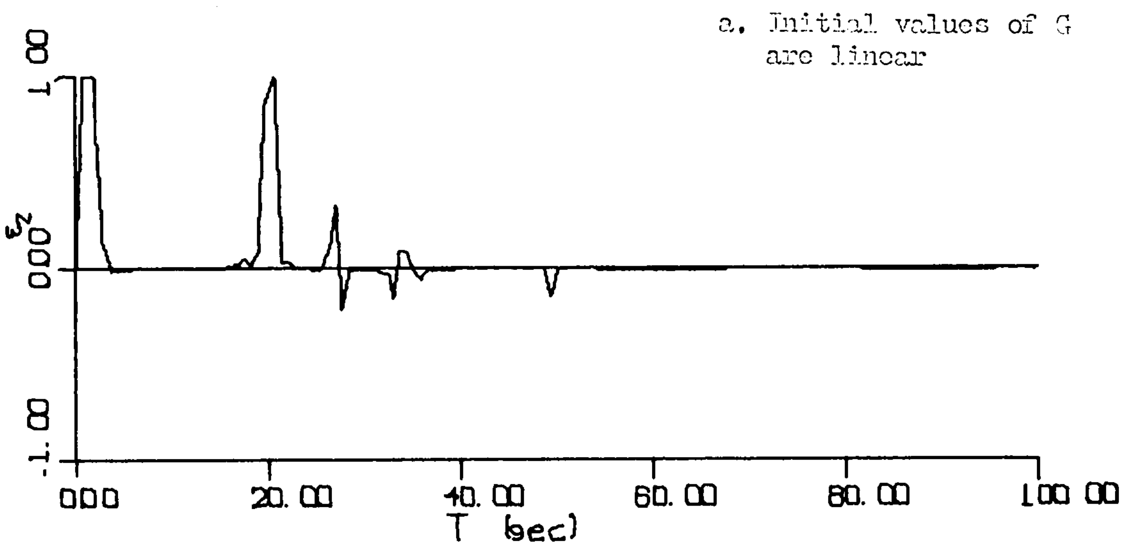


Figure 39: Error in identifier velocity



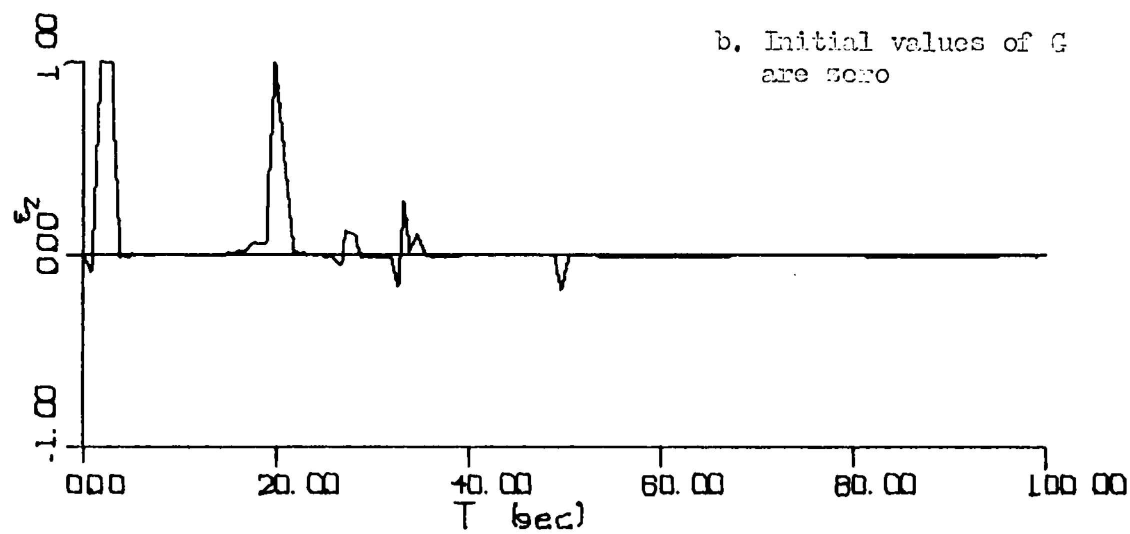


Figure 40: Error in identifier flight path angle

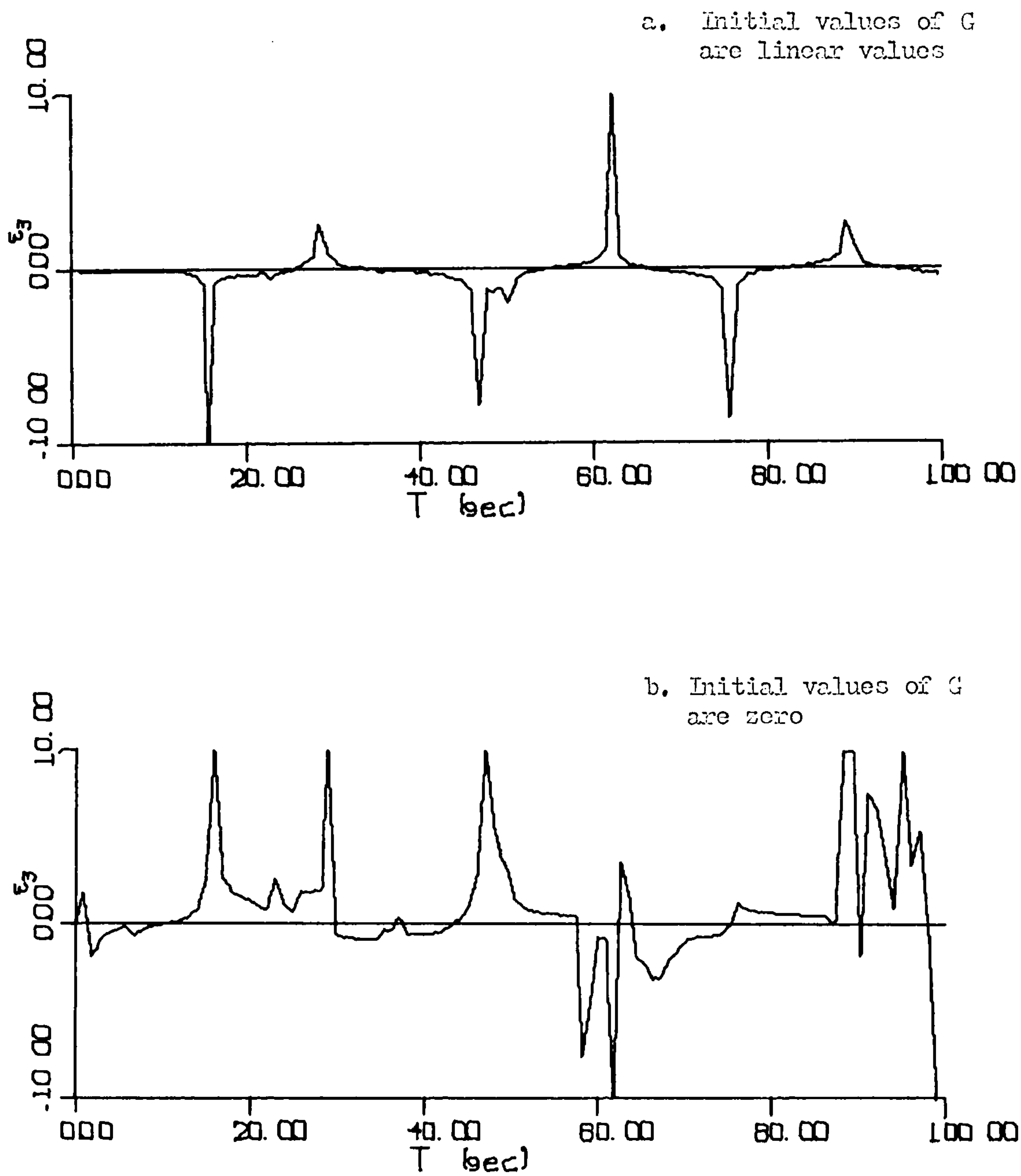


Figure 41: Error in identifier pitch rate

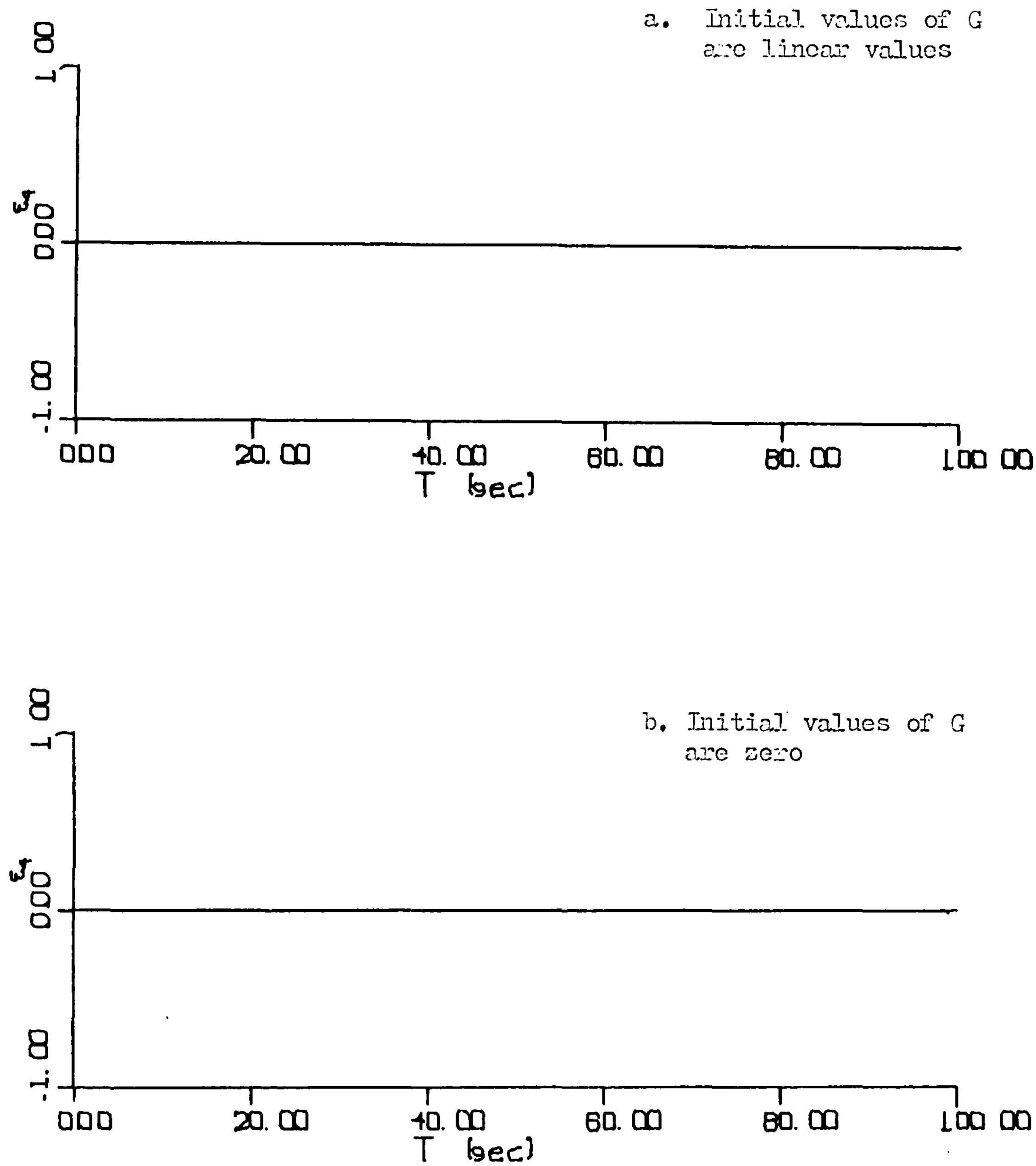


Figure 42: Error in identifier attitude angle

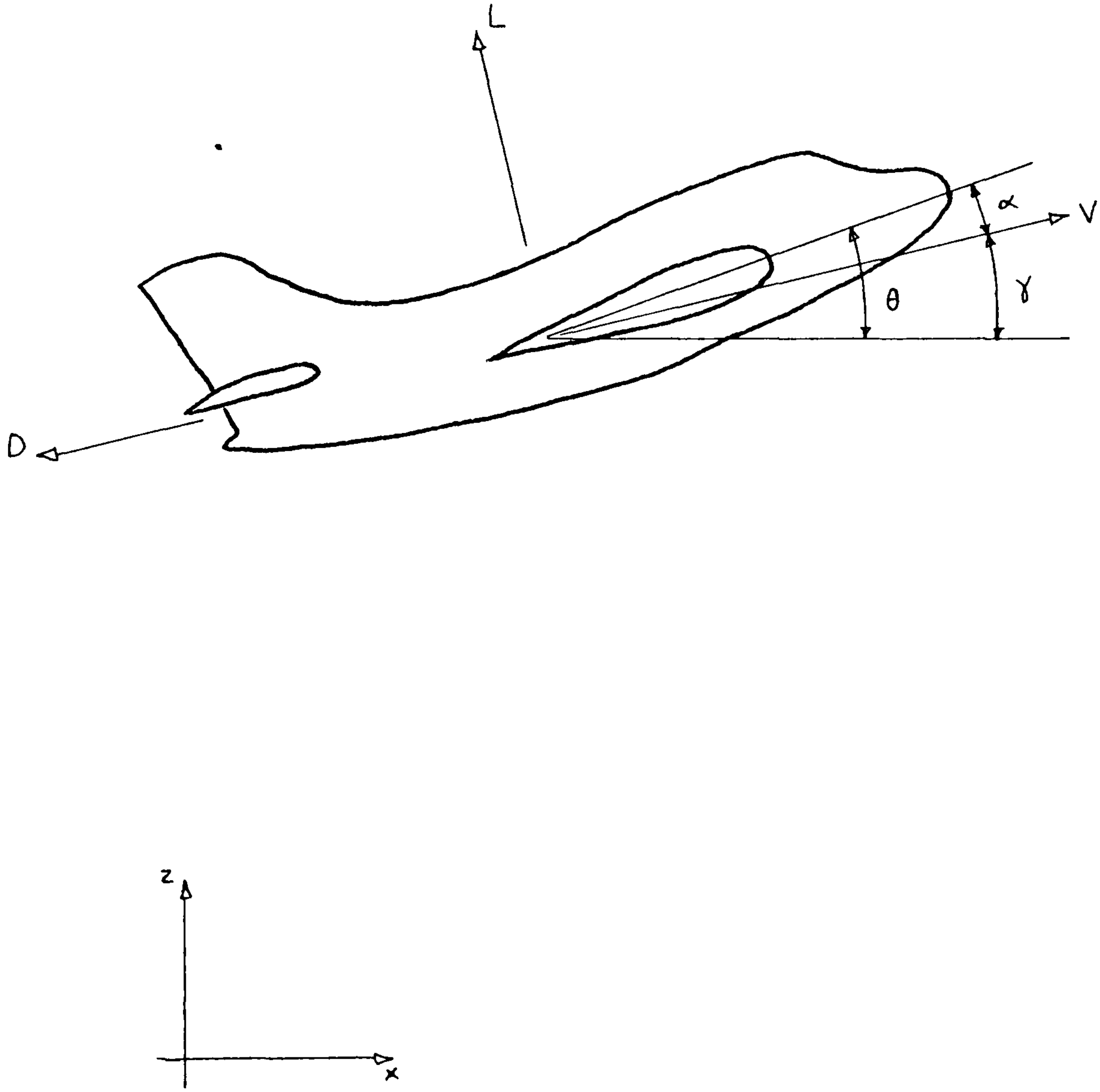


Figure 43: Aircraft Forces

Coefficient matrices:

$$A = \begin{bmatrix} -0.04990 & -5.61796 & 0.0 & -26.53200 \\ 0.00089 & -1.11252 & 0.0 & 1.11252 \\ 0.03169 & 22.92350 & -26.06740 & 22.92350 \\ 0.0 & 0.0 & 1.0 & 0.0 \end{bmatrix}$$

$$B = \begin{bmatrix} 8.31729 & -0.07941 & -0.36385 \\ 0.18315 & -0.00173 & -0.00220 \\ -5.28127 & -0.93662 & 0.51330 \\ 0.0 & 0.0 & 0.0 \end{bmatrix}$$

Equilibrium states:

$$V = 180 \text{ ft/sec}$$

$$= 0 \text{ rad}$$

$$q = 0 \text{ rad/sec}$$

$$= -.09848 \text{ rad}$$

Equilibrium controls:

$$C_T = .5399$$

$${}_e = -3.976^{\circ}$$

$${}_f = 49.97^{\circ}$$

Table 1: Values for the A and B coefficient matrices when the EBF/STOL equations of motion are linearized about the equilibrium values for V, , q, and , and controls C_T, ${}_e$, and ${}_f$.

Gain matrices:

$$\Gamma = \begin{bmatrix} .1 & 10000 & 0 & 100 \\ 0 & 10000 & 0 & 0 \\ 0 & 1000 & 0 & 0 \\ 0 & 0. & 0 & 0 \end{bmatrix} \qquad \Lambda = \begin{bmatrix} 10 & 10 & 10 \\ 0 & .1 & 0 \\ 0 & 0 & 100 \\ 0 & 0 & 0 \end{bmatrix}$$

Variation of the gains:

$$\begin{aligned} \gamma_{11} &= .1 && \text{for } 0 \le x_1 < 5 \\ &= .01 && 5 \le x_1 < 20 \\ &= .001 && 20 \le x_1 < 50 \\ &= .0001 && 50 \le x_1 \end{aligned} \quad , \quad \begin{aligned} \gamma_{12} &= 10000 && \text{for } 0 \le x_2 < .015 \\ &= 1000 && .015 \le x_2 < .045 \\ &= 100 && .045 \le x_2 \end{aligned}$$

$$\begin{aligned} \gamma_{14} &= 100 && \text{for } 0 \le x_4 < .04 \\ &= 10 && .04 \le x_4 \end{aligned} \quad , \quad \begin{aligned} \gamma_{22} &= 10000 && \text{for } 0 \le x_2 < .022 \\ &= 1000 && .022 \le x_2 < .045 \\ &= 100 && .045 \le x_2 \end{aligned}$$

$$\begin{aligned} \gamma_{32} &= 1000 && \text{for } 0 \le x_2 < .065 \\ &= 100 && .065 \le x_2 \end{aligned} \quad , \quad \begin{aligned} \lambda_{11} &= 10 && \text{for } 0 \le u_1 < .8 \\ &= 1 && .8 \le u_1 \end{aligned}$$

$$\begin{aligned} \lambda_{12} &= 10 && \text{for } 0 \le u_2 < .45 \\ &= 1 && .45 \le u_2 < 1.6 \\ &= .1 && 1.6 \le u_2 \end{aligned} \quad , \quad \begin{aligned} \lambda_{13} &= 10 && \text{for } 0 \le u_3 < .6 \\ &= 1 && .6 \le u_3 < 1 \\ &= .1 && 1 \le u_3 \end{aligned}$$

$$\begin{aligned} \lambda_{33} &= 100 && \text{for } 0 \le u_3 < .2 \\ &= 10 && .2 \le u_3 < .7 \\ &= 1 && .7 \le u_3 \end{aligned}$$

Table 2: Values and the variations of Γ and Λ for the EBF/STOL problem

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APPENDIX

The Equations of Motion of EBF/STOL

The model used as an example for the identification procedure was the set of equations of motion of a short take-off and landing (STOL) aircraft. Specifically, a type of STOL that utilizes the deflection of the exhaust gases from jet engines to augment the lift of the aircraft during take-off and landing. This is known as an externally blown flap short take-off and landing aircraft, or simply EBF/STOL.

The equations of motion were derived by summing the forces and moments about the axis system shown in Figure 43. The thrust forces do not appear explicitly because they are incorporated in the lift and drag forces. The longitudinal equations were used to describe the motion of the aircraft since the motion is essentially uncoupled from the lateral motion (6).

Summing the forces along each axis in the x-z plane:

$$F_x = - D - mg \sin \quad = m\dot{V}$$

$$F_y = L - mg \cos \quad = mV\dot{\gamma} \;\; ;$$

and, summing the moments about the y-axis:

$$M_y = I_y\ddot{\theta} = M \quad .$$

The angular velocity about the y-axis is:

$$\dot{\theta} = q$$

And, the angular acceleration is given by:

$$\ddot{\theta} = \dot{q} \quad .$$

The four equations of motion are:

$$\dot{V} = - \frac{D}{m} - \frac{G}{m} \sin\gamma$$

$$\dot{\gamma} = \frac{L}{mV} - \frac{g}{V}\cos\gamma$$

$$\dot{\theta} = q$$

$$\dot{q} = \frac{M}{I_y}$$

where

$$D = \tfrac{1}{2}\rho V^2 S C_D$$

$$L = \tfrac{1}{2}\rho V^2 S C_L$$

$$M = \tfrac{1}{2}\rho V^2 S C_M \bar{c} \quad .$$

For this problem, it is assumed that the aerodynamic coefficients, C_D, C_L, and C_M, are functions of the variables:

$$C_D = f(\alpha,\ C_T,\ \delta_f,\ \delta_e)$$

$$C_L = g(\alpha,\ C_T,\ \delta_f,\ \delta_e)$$

$$C_M = h(\alpha,\ C_T,\ \delta_f,\ \delta_e,\ q) \quad .$$

They can be expanded in a least squares data fit of second order of three of the variables:

$$C_D(\alpha,\ C_T,\ \delta_f) = \sum_i^3 \sum_j^3 \sum_k^3 QC_{D_{ijk}} \alpha^{i-1} C_T^{\,j-1} \delta_f^{\,k-1}$$

$$C_L(\alpha,\ C_T,\ \delta_f) = \sum_i^3 \sum_j^3 \sum_k^3 QC_{L_{ijk}} \alpha^{i-1} C_T^{\,j-1} \delta_f^{\,k-1}$$

$$C_M(\alpha,\ C_T,\ \delta_f) = \sum_i^3 \sum_j^3 \sum_k^3 QC_{M_{ijk}} \alpha^{i-1} C_T^{\,j-1} \delta_f^{\,k-1}$$

where QC_D, QC_L, and QC_M are the coefficients of the equations formed by the data fit. Then including the dependence on δ_e and q, the aerodynamic coefficients are:

$$C_D(\alpha,\ C_T,\ \delta_f,\ \delta_e) = \sum_i^3 \sum_j^3 \sum_k^3 QC_{D_{ijk}} \alpha^{i-1} C_T^{\,j-1} \delta_f^{\,k-1} + C_{D_{\delta_e}}\delta_e$$

$$C_L(\alpha,\ C_T,\ \delta_f,\ \delta_e) = \sum_i^3 \sum_j^3 \sum_k^3 XC_{L_{ijk}} \alpha^{i-1} C_T^{\,j-1} \delta_f^{\,k-1} + C_{L_{\delta_e}} \delta_e$$

$$C_M(\alpha,\ C_T,\ \delta_f,\ \delta_e,\ q) = \sum_i^3 \sum_j^3 \sum_k^3 XC_{M_{ijk}} \alpha^{i-1} C_T^{\,j-1} \delta_f^{\,k-1} + C_{M_{\delta_e}} \delta_e + C_{M_q} q$$

where $C_{D_{\delta_e}}$, $C_{L_{\delta_e}}$, $C_{M_{\delta_e}}$, and C_{M_q} are assumed to be constants.

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A SCHEME FOR IDENTIFICATION OF A SET OF NONLINEAR EQUATIONS

by

Terrence Joseph Hertz

(ABSTRACT)

A method for the identification of a nonlinear set of equations is presented. The method is based on the work done by Narendra, Tripathi, Kudva, and Luders for linear identification. The nonlinear equations are linearized and the coefficients of the linear equations are allowed to vary so the input/output relationship of the two sets of equations are the same. The variation of the coefficients of the linear equations are then identified using a form of Narendra's, et al, identification scheme.

The scheme was used to identify a set of nonlinear equations that describe the motion of a STOL vehicle. Two control schemes were employed. The first control program maintained the states close to an equilibrium position and the second caused the states to vary from one equilibrium to another.