Applications of Neutrino Physics

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(ABSTRACT)

Neutrino physics has entered a precision era in which understanding backgrounds and systematic uncertainties is particularly important. With a precise understanding of neutrino physics, we can better understand neutrino sources. In this work, we demonstrate dependency of single detector oscillation experiments on reactor neutrino flux model. We fit the largest reactor neutrino flux model error, weak magnetism, using data from experiments. We use reactor burn-up simulations in combination with a reactor neutrino flux model to demonstrate the capability of a neutrino detector to measure the power, burn-up, and plutonium content of a nuclear reactor. In particular, North Korean reactors are examined prior to the 1994 nuclear crisis and waste removal detection is examined at the Iranian reactor. The strength of a neutrino detector is that it can acquire data without the need to shut the reactor down. We also simulate tau neutrino interactions to determine backgrounds to muon neutrino and electron neutrino measurements in neutrino factory experiments.
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Chapter 1

Introduction

The neutrino was first postulated in 1930 by Wolfgang Pauli to account for a continuous electron spectrum in beta decay \[1\]. Pauli concluded that an additional particle that is spin 1/2, electrically neutral, and similar in mass to an electron would yield the measured results. It was not until 1956 that the neutrino\(^1\) was first detected by Cowan and Reines using neutrinos from nuclear reactors [2]. Neutrinos originating from the Sun and atmosphere were later detected with results that were unexpected, creating questions about the fundamentals of solar processes and particle physics.

Neutrino physics is a fast growing field which has entered, through iteration and innovation, into a precision era. Using the knowledge we have now, neutrinos can begin to aid in measuring quantities and solving problems in separate but related fields such as nuclear physics, in understanding the nuclear beta decays, and reactor monitoring. As we come to better understand the physics behind neutrinos we can apply this knowledge to other challenging and unique problems.

Exploring the basic fundamentals of neutrino oscillation, in chapter 2 we look at both the simplistic two neutrino case as well as the full three active neutrino case with extensions for possible sterile neutrinos. The chapter also presents the status of the field and how it has progressed beginning with the solar neutrino problem, where our understanding of neutrinos was questioned, to our current understanding with overwhelming evidence for flavor oscillation to what future experiments will be searching for.

In chapter 3 we focus on the recent reactor experiments which have measured the neutrino mixing

\(^1\)The particle detected is most accurately described as an electron anti-neutrino.
One of the largest theory errors in the reactor neutrino flux model is weak magnetism and it is examined in chapter 4. Historically, the weak magnetism correction was predicted and measured in the nuclear beta spectrum with the intent to verify a link between electromagnetism and the weak interaction. This is a very challenging quantity to measure and once the Z boson was detected, a lot of the motivation for measuring the weak magnetism correction was lost. Because weak magnetism influences the electron beta spectrum, it also effects the neutrino spectrum. It is particularly important for reactor neutrino experiments which measure deviations in the energy spectrum from what is expected. Typically, direct calculations of reactor neutrino flux assume that all beta decays have same form for the weak magnetism correction as predicted by the conserved vector current for Gamow-Teller decays. We do not know the exact correction for forbidden beta decays which account for a large portion of the decays that occur in nuclear reactors within the energy range relevant for reactor neutrino experiments. We used measured neutrino spectra to try to constrain a linear approximation of the weak magnetism correction. There are a variety of other experimental uncertainties that make this very challenging to measure.

In chapter 5, we describe an in-depth analysis for the use of neutrino detectors as a means to monitor nuclear reactors. The end goal is to be able to infer the power and plutonium content while the reactor is running. Completing a case study on the DPRK reactors, as well as the Iranian reactor, we discovered that under considerations of a surface detector within 20 meters of the reactor and generous assumptions of background reduction, the power can be well measured and it is possible to detect partial core replacement. The strength of a neutrino detector is that it can remotely acquire data without the need to shut the reactor down. As well, with the capability of measuring both power and plutonium content independently, we can actually get a degenerate measure of the plutonium content of the reactor through a burn-up analysis. We additionally simulated a measurement of long-lived isotopes with the goal of detecting hidden waste or removal of nearby waste. Combining a neutrino detector with traditional methods would unquestionably strengthen capabilities to detect diversion.

In chapter 6, we explore neutrino factories. A neutrino factory is a muon storage ring that allows the muons to decay in flight and has long straight sections where the muon is likely to decay. Further
down the path there is a detector which measures the oscillations of the muon decay products: a neutrino and an antineutrino. In these experiments, some neutrinos will oscillate into tau neutrinos which can interact with the detector and produce short-lived tau leptons. These leptons have a large probability to decay into either electrons or muons, which are miss-identified as a signal if the tau is not detected before it decays. It is very challenging to make a detector that can measure the tau directly so tau neutrino interactions are a background that must be considered. A Monte Carlo calculation was used to predict what the expected energy spectrum would be from tau decays.
Chapter 2

Neutrino oscillations

It has been well established that the flavor of a neutrino, identified by the charged lepton it produces in a weak charged current interaction, can change in flight from the neutrino source to a detector. It was not initially obvious which mechanism was responsible for the change but we know now that it is due to what has been labeled as neutrino oscillations. This chapter will cover the basics of neutrino oscillation following a traditional format. We begin first with two flavor neutrino oscillations in vacuum for a clean final result. The addition of matter interactions to the two flavor framework is briefly mentioned for completeness. The explanation is extended to the standard three neutrino flavors ($e, \mu, \tau$) with special attention given to sterile neutrino additions and motivations. Finally, the history and future of neutrino oscillations is discussed.

2.1 Two flavor vacuum oscillations

Neutrino oscillation was discussed in Ref. [3, 4]. In the absence of interactions, a neutrino in a stationary eigenstate of the Hamiltonian, $|\nu_i\rangle$, with energy $E_i$, will have the following time dependent form:

$$|\nu_i(t)\rangle = e^{-iE_it} |\nu_i\rangle.$$  \hspace{1cm} (2.1)

These eigenstates are also referred to as mass eigenstates because the free Hamiltonian can be written in terms of the diagonal matrix, diag($m_1^2, m_2^2$) where $m_1$ and $m_2$ are the neutrino masses for $|\nu_1\rangle$ and $|\nu_2\rangle$. However, weak interactions produce neutrinos in distinct flavor eigenstates, $|\nu_\alpha\rangle$, where $\alpha = e, \mu, \tau$. The flavor eigenstates can be written as a linear combination of mass eigenstates...
and are related through a unitary mixing matrix $U$ such that

$$|\nu_\alpha\rangle = U^*_{\alpha i} |\nu_i\rangle. \quad (2.2)$$

The probability to find a neutrino in flavor eigenstate $\beta$ at some time $t$ given that the neutrino was in state $\alpha$ at $t = 0$ is

$$P(\nu_\alpha \rightarrow \nu_\beta) = |\langle \nu_\beta | e^{-iE_\alpha t}|\nu_\alpha\rangle|^2$$

$$= |\langle \nu_j | U_{\beta j} e^{-iE_\alpha t} U^*_{\alpha i} |\nu_i\rangle|^2$$

$$= |e^{-iE_\alpha t} U_{\beta j} U^*_{\alpha i} \langle \nu_j | \nu_i\rangle|^2$$

$$= |e^{-iE_\alpha t} U_{\beta i} U^*_{\alpha i}|^2$$

$$= e^{-i(E_\alpha - E_\beta)t} U_{\beta i} U^*_{\alpha i} U_{\beta j} U^*_{\alpha j}$$

$$\approx e^{-i\frac{\Delta m^2_{ij} L}{4E}} U_{\beta i} U^*_{\alpha i} U_{\beta j} U^*_{\alpha j} \quad (2.3)$$

In the last step, under the limit $p_i \gg m_i$, $E_i$ is replaced by $E + \frac{m^2_i}{2E}$ where $E$ is the total neutrino energy and the time has been replaced with the distance traveled, $L$. $\Delta m^2_{ij}$ is called the mass splitting and is given by $m^2_i - m^2_j$. In the case of two neutrinos, $U$ is a 2×2 matrix that can be written in terms of a single mixing angle $\theta$ and one phase, $\phi$, that is irrelevant for neutrino oscillations.

$$U = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}$$

Then, with two neutrinos,

$$P(\nu_\alpha \rightarrow \nu_\alpha) = 1 - \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 L}{4E}\right) \quad (2.4)$$

and

$$P(\nu_\alpha \rightarrow \nu_\beta) = \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 L}{4E}\right) \quad (2.5)$$

Experiments that measure probabilities such as that in Eq. 2.4 are called disappearance experiments. Those that measure probabilities like that in Eq. 2.5 are called appearance experiments. It can be seen in Fig. 2.1 that a neutrino’s flavor identity will change with amplitude determined by the magnitude of the mixing angle $\theta$ and with frequency determined by the mass splitting, $\Delta m^2_{ij}$. Both parameters are fundamental to the neutrinos and experiments can probe oscillation phase space with a choice of neutrino energy to baseline distance ratio. The first oscillation maximum will occur at $L/E = 2\pi/\Delta m^2$. 
Figure 2.1: As a function of neutrino energy and fixed baseline, the neutrino survival probability is shown in the left panel while the appearance probability is shown in the right panel. The effects of the mixing angle and mass splitting are indicated by the corresponding arrows.

2.2 Matter effects

In the presence of matter, additional terms need to be added to the Hamiltonian to account for coherent forward scattering that happens in flight; this is similar to the effect that happens to photons as they travel through a medium. The weak interaction allows for two primary means of interaction with matter. The neutral current interaction, through the exchange of a virtual Z boson, is symmetric under the exchange of electron neutrino with muon or tau neutrinos. In the context of active neutrino oscillations, this amounts to an overall common phase shift but does not change the overall oscillation probabilities. The charged current interaction, through the exchange of a virtual W boson, allows the electron neutrino to interact with electrons in a way that the other flavors cannot causing a shift in the index of refraction for electron neutrinos. This adds a potential to the Hamiltonian that is specific to electron neutrinos

\[ V(t) = \pm \sqrt{2} G_F n_e(t), \]  

(2.6)

where \( G_F \) is the Fermi coupling constant, \( n_e(t) \) is the electron density, and \( E \) is the neutrino energy. The plus sign is taken for electron neutrinos while the negative sign is taken for electron anti-neutrinos. In terms of a two flavor neutrino oscillation, the Schrodinger equation can be written as

\[ i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \frac{1}{2E} \left[ \begin{array}{cc} m_1^2 & 0 \\ 0 & m_2^2 \end{array} \right] \begin{pmatrix} U \end{array} + 2E \begin{pmatrix} V(t) & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} U^\dagger \end{array} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}. \]  

(2.7)

In the case of a constant electron density, the oscillation probability can be written in terms of a
modified mixing angle and mass splitting,

\[ P(\nu_e \to \nu_\mu) = \sin^2 2\theta_{\text{matter}} \sin^2 \left( \frac{\Delta m^2_{\text{matter}} L}{4E} \right) \]  \hspace{0.5cm} (2.8)

where

\[ \sin^2 2\theta_{\text{matter}} = \frac{\sin^2 2\theta}{\sin^2 2\theta + \left( \cos 2\theta - \frac{2EV}{\Delta m^2} \right)^2} \]  \hspace{0.5cm} (2.9)

and

\[ \Delta m^2_{\text{matter}} = \Delta m^2 \sqrt{\sin^2 2\theta + \left( \cos 2\theta - \frac{2EV}{\Delta m^2} \right)^2}. \] \hspace{0.5cm} (2.10)

In the case that \( V=0 \), the oscillation probability returns to the original form with no matter effects. There is a resonant effect that occurs when \( \frac{2EV}{\Delta m^2} = \cos 2\theta \) and under this condition, \( \sin^2 2\theta_{\text{matter}} = 1 \) with maximal mixing. This also depends on the sign of the mass splitting which is degenerate in the vacuum, two-neutrino oscillation.

\section*{2.3 Three flavor}

It is now known that there are at least three neutrinos and of those, exactly three couple to the Z boson with masses that are less than half that of the Z \[5\]. To go from a two flavor oscillation framework to three flavor, a 3 \( \times \) 3 unitary mixing matrix is used. The Pontecorvo-Maki-Nakagawa-Sakata (PMNS) \[6\] parameterization is standard and follows similarly to the CKM matrix from the quark sector. With such,

\[ U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\phi_1} & 0 \\ 0 & 0 & e^{i\phi_2} \end{pmatrix}. \]

The same procedure done in Sec. 2.1 can be performed using this larger 3 \( \times \) 3 mixing matrix. The addition of the third neutrino flavor allows for CP violation through the \( \delta \) phase.

\section*{2.4 Sterile Neutrino}

There have been indications that the three neutrino oscillation framework is not sufficient to explain all of the results measured in neutrino experiments. Data from the Liquid Scintillator Neutrino Detector (LSND), in which muon antineutrinos oscillated into electron antineutrinos, could be fit
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by a mass splitting of about 1 eV$^2$\textsuperscript{[7]}. With only three neutrinos, there can only be two independent mass splittings and the mass splittings are orders of magnitude smaller. To incorporate this larger mass splitting into the neutrino oscillation picture, an additional neutrino would need to be added.

At the large electron-positron collider (LEP), $e^+e^-$ collisions were done at the Z resonance. The Z boson can decay into fermion anti-fermion pairs. This includes quarks, charged leptons, and neutrinos but excludes the top quark because it is too massive. Through a measurement of the invisible decay width, the experiment found the number of light active neutrino species to be $2.984 \pm 0.0082$\textsuperscript{[8]}. This implies that there are three neutrinos that have masses less than half the mass of the Z boson and also interact weakly. A fourth neutrino, with a mass well below the Z mass, would then not interact weakly and is therefore termed ”sterile”. The LEP experiment does not put a constraint on the number of sterile neutrinos.

The MiniBooNE experiment was designed to check the results of LSND by probing the same distance to energy ratio thus allowing sensitivity to the same mass splitting parameter space. In the antineutrino run for MiniBooNE, the collaboration found oscillations that are consistent with a LSND mass splitting\textsuperscript{[9, 10]}. The neutrino run observed excess events at low energies but it is not clear if it consistent with LSND\textsuperscript{[11]}. Reanalysis of the reactor antineutrino flux, which will be discussed in detail in chapter 3, has predicted an increase in antineutrino rates. Previous experiments that once were in agreement with the predictions now see a deficit. The deficit could be explained by a sterile neutrino oscillation with a mass splitting around that predicted by LSND or larger\textsuperscript{[12]}.

The expansion rate of the universe, during the radiation dominated era, is effected by the energy density of relativistic particles and primarily by photons and neutrinos. Measurements of the cosmic microwave background temperature and the expansion rate in the early universe can place constraints on the relativistic degrees of freedom, $N_{eff}$\textsuperscript{[13]}. The standard model, with 3 neutrino families, predicts $N_{eff}$ close to 3. Numbers higher than 3 could be indicative of additional neutrino flavors, in particular sterile neutrinos. Sterile neutrinos are not a unique solution as other light particles or changes in expansion rate can change the value of $N_{eff}$. Additionally, the expansion rate and neutrino oscillations can effect the ratio of protons to neutrons. Almost all of the neutrons become incorporated into $^4$He nuclei because of the large binding energy. Measuring the $^4$He mass fraction gives another method for measuring $N_{eff}$. Current estimates of $N_{eff}$ are close to 4\textsuperscript{[14]} but having a 1eV$^2$ sterile produces some tension with other cosmological models\textsuperscript{[15]}. For further
information on sterile neutrinos, see Ref. [13].

2.5 Current picture

2.5.1 Solar oscillations

An early indication for neutrino oscillation was through a measurement of neutrinos from the sun. One of the solar processes that yields the highest energy neutrinos is from boron-8 ($^8$B) beta decay. Predictions were made for the $^8$B flux with a variety of solar model choices [16] and the Homestake experiment detected these neutrinos from the sun using the interaction $^{37}$Cl ($\nu_e, e^-$) $^{37}$Ar [17]. After extracting and counting the argon, it was concluded that the number of neutrino interactions were low by a factor of a few. This raised concerns regarding the accuracy of the standard solar model and the discrepancy was labeled as the solar neutrino puzzle and later as the solar neutrino problem.

The Kamiokande-II experiment measured the $^8$B neutrinos through electron scattering, $\nu_e e^- \rightarrow \nu_e e^-$, in water [18]. The scattering imparts energy to the electron and can cause the electron to move faster than light in water. This process emits light in the form of Cherenkov radiation [19] that allows for a measurement of neutrino arrival time, direction, and energy spectrum. The experiment reported a ratio of measured events to predicted events of $0.46 \pm 0.13$ (stat.) $\pm 0.08$ (syst.). This was considered in agreement with the measurement done at Homestake.

The Soviet-American Gallium Experiment (SAGE) experiment measured neutrinos that were produced from the interaction $p + p \rightarrow d + e^+ + \nu_e + \gamma$ [20]. The predicted flux for these neutrinos is directly related to the observed solar luminosity and unaffected by changes to the solar model. More than 90% of solar neutrinos are produced from this process but the energies are too low for chlorine or water Cherenkov detectors. The SAGE experiment uses $^{71}$Ga in the process, $^{71}$Ga ($\nu_e, e^-$) $^{71}$Ge and can detect p-p neutrinos. The results found that, like the neutrinos from $^8$B, the p-p neutrino rate was lower than expected. The GALLEX experiment also used gallium atoms for the detection of solar neutrinos and found similar results [21, 22].

By this time, predictions were made that the missing effect is possibly due to neutrino oscillation with matter effects rather than issues with the solar model [23, 24, 25]. The Super-Kamiokande experiment, like Kamiokande-II uses neutrino-electron scattering in water [26, 27]; it too found a lower than expected detection rate.
Chapter 2. Neutrino oscillations

The Sudbury Neutrino Observatory (SNO) experiment was unique in that it detected $^8$B neutrinos through three reactions: charged current, neutral current, and electron scattering [28, 29]. The charged current reaction only occurs with electron flavor neutrinos, $\nu_e + d \rightarrow p + p + e^-$. The neutral current reaction can occur with all three neutrino flavors, $\nu + d \rightarrow p + n + \nu$. The elastic scattering reaction can also occur with all three neutrino types but the reaction is stronger with electron flavor neutrinos. Through a combination of the charged current and neutral current processes, one can determine that neutrinos have changed flavor from the electron flavor created in the sun independent of the solar model. It was found that a significant portion solar neutrinos were no longer electron neutrinos but were instead muon and tau neutrinos and that the total rate from the neutral current interaction was in agreement with the standard solar model prediction. The oscillation framework has gained additional support with the Borexino experiment which measured, in addition to the $^8$B and p-p neutrinos, neutrinos from the pep reaction [30, 31, 32, 33].

The Kamland [34, 35] experiment allowed for an orthogonal check to neutrino oscillations as a means to deal with the solar neutrino problem. Through a charged current interaction, Kamland detected anti-electron neutrinos that originated from distant nuclear reactors, on the order of 180 km. To good approximation the survival probability in this experiment is given by

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \approx \cos^4 \theta_{13} \left( 1 - \sin^2 2\theta_{12} \sin^2 \frac{\Delta m_{12}^2 L}{4E} \right). \quad (2.11)$$

In 2002, the Kamland experiment found $\cos^4 \theta_{13} \geq 0.92$, $0.86 < \sin^2 2\theta_{12} < 1$, and $\Delta m_{12}^2 = 6.9 \times 10^{-5}$. Combining Kamland with the solar experiments led to the conclusion that neutrino oscillation with large mixing angle matter effects was the best explanation for the solar neutrino problem.

2.5.2 Atmospheric oscillations

Cosmic rays can lead to the production of neutrinos and were of great importance to understanding additional neutrino oscillation properties. When cosmic rays strike particles in the atmosphere, they can produce mesons like pions and kaons. The mesons then will typically decay into muon and muon-neutrinos. Furthermore, the muons will decay. For a $\pi^+$ decay and skipping intermediate decay steps, the final products are $e^+ + \bar{\nu}_\mu + \nu_\mu + \nu_e$. This leads to an expectation of about two muon flavor neutrinos to every one electron flavor neutrino. The Kamiokande [36] experiment measured these atmospheric neutrinos but found a reduced number of muon-like events while maintaining
the expected electron like events. This was contrary to the Frejus experiment which found their muon to electron ratio to be in agreement with predictions [37] but similar to the Irvine-Michigan-Brookhaven (IMB) experiment [38].

The Super K [39] experiment gave tremendous support to oscillations as the explanation for the lack of muon neutrino events in atmospheric neutrino experiments. The experiment had sufficient statistics and angular resolution to bin events by energy, flavor, and zenith angle. By tracking the angle, one can identify if the neutrino had traveled only 15 km from directly overhead, 13,000 km from directly below, or somewhere between. The analysis was consistent with muon to tau neutrino oscillations with $5 \times 10^{-4} < \Delta m_{23}^2 < 6 \times 10^{-3} \text{eV}^2$ and $\sin^2 2\theta_{23} > 0.82$ at 90% confidence. These results were further supported by MACRO [40] and Soudan 2 [41]. The Main Injector Neutrino Oscillation Search (MINOS) experiment was much like the Super K experiment, measuring atmospheric neutrinos with a large detector, but it used a magnetic field in iron to distinguish between $\mu^+$ and $\mu^-$, allowing the distinction of neutrino and antineutrino and tests of CPT, a symmetry built into quantum field theory. Analysis of the data revealed similar oscillation parameters and no large distinction between muon neutrino oscillations and muon antineutrino oscillations [42, 43, 44].

The K2K experiment used an accelerator as a neutrino source to probe the same parameter space as the atmospheric oscillation findings [45]. The experiment used both a near detector to measure an oscillated neutrino spectrum and a far detector at 250 km to detect the oscillation. A ratio of the two is used to determine the oscillation probability. Analysis of the data indicated a good agreement with neutrino oscillations and the best fit had similar parameters to that from Super K.

Experiments went on to check that the disappearance of muon neutrinos was in fact from an oscillation to a tau neutrino and not some other particle or mechanism. There were some indirect verification such as checks through neutral currents that verify that oscillations were to active flavors and not to sterile [46] and some experiments had results that were inconsistent with other mechanisms like neutrino decay. The OPERA experiment was designed to detect tau neutrinos by measuring the tau leptons produced from a charged current interaction with the tau neutrino [47]. The tau leptons are very hard to detect and study because of their short lifetime (290 femtoseconds) and high interaction energy threshold (3.5 GeV). At this time, the experiment has observed three tau lepton candidates giving strong support for $\nu_\mu \rightarrow \nu_\tau$ appearance [48]. The Super K experiment performed analysis of their data to look for tau events as well [49, 50]. Within a water Cherenkov detector the tau decay creates a distribution of rings that make it hard to identify the initial particle.
For the analysis, the collaboration used a neural network to identify patterns that are characteristic of tau decay and not other backgrounds. At the time of that last publication, there was an estimate of about 180 tau events in the Super K detector. In addition, the IceCube collaboration has reported three tau neutrino candidates [51].

Additional tests of the atmospheric mass splitting $\Delta m_{32}^2$ has been done using electron antineutrinos from nuclear reactors. In addition, these experiments have a clean measurement of $\theta_{13}$ without strong impact from any of the other mixing angles. Three recent reactor experiments have placed strong constraints on the values for $\theta_{13}$ [52, 53, 54]. Chapter 3 will focus heavily on reactor neutrino experiments. $\theta_{13}$ has also been measured using accelerator experiments through detecting electron neutrinos in a muon neutrino beam. These sorts of experiments gave indications that the value for $\theta_{13}$ was not 0, in particular the T2K and MINOS experiments [55, 56], with data now favoring larger mixing angles [57].

### 2.5.3 Future experiments and considerations

At this time, constraints have been placed on all of the mixing angles and both of the mass splittings for the three neutrino framework [58].

\[
\begin{align*}
\sin^2 2\theta_{12} &= 0.857 \pm 0.024 \\
\sin^2 2\theta_{23} &> 0.95 \\
\sin^2 2\theta_{13} &= 0.095 \pm 0.010 \\
\Delta m_{31}^2 &= (7.5 \pm 0.20) \times 10^{-5} \text{ eV}^2 \\
|\Delta m_{32}^2| &= (2.32_{-0.08}^{+0.12}) \times 10^{-3} \text{ eV}^2
\end{align*}
\]

Besides measuring the oscillation parameters to better precision, there are still a few quantities that remain to be determined. The sign of the larger, atmospheric mass splitting, $\Delta m_{32}^2$ has yet to be measured. Experiments with large matter effects associated with the large mass splitting will be sensitive to the sign. If the sign is positive, which is the case if the third mass state is heavier than the second, then neutrinos are said to have a normal mass hierarchy. Alternatively, they are said to have an inverted mass hierarchy. Large atmospheric detectors with large matter effects from the Earth, such as PINGU [59], are good candidates to determine the hierarchy as well as long baseline
experiments, like NOνA [60] and LBNE [61]. The capability to determine the mass hierarchy is explored for a variety of experiments in Ref. [62]. Additionally, detection of a neutrino burst from a nearby supernovae could resolve it as well [63].

The value of the CP violating phase, $\delta$, has yet to be measured. Experiments that measure the difference between $P(\nu_\alpha \rightarrow \nu_\beta)$ and $P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta)$ are sensitive to the CP phase. Accelerator based neutrino experiments are commonly designed with the capability to switch between neutrinos and anti-neutrinos. Determination of the CP phase could be assisted when using a constrained value of $\theta_{13}$ from reactor experiments.

A experimental verification of Majorana neutrinos can be done through neutrino-less double beta decay. This decay process would emit two electrons that have the binding energy approximately split between them. Regular double beta decay would also emit two electron antineutrinos, carrying invisible energy away. Experiments look for a peak at the endpoint to indicated that a decay occurred that emitted no neutrinos. The Heidelberg-Moscow experiment claimed to measure neutrino-less double beta decay in 2001 [64] but no other experiment has found a positive result. There are several ongoing experiments searching for this decay [65, 66, 67, 68, 69] with a few to come [70, 71].
Chapter 3

Reactor Neutrinos

Reactor neutrinos originate from the beta decay of neutron rich isotopes which were produced through fission inside nuclear reactors. Within a typical nuclear reactor, there are four primary fissioning isotopes that yield over 99\% of the total energy produced through fissions: $^{235}\text{U}$, $^{238}\text{U}$, $^{239}\text{Pu}$, and $^{241}\text{Pu}$. When one of these isotopes fissions, it will usually split into two lighter nuclei which are neutron rich and will decay to stability. This process is known as beta-decay and, for an atom with atomic number $Z$ and atomic mass $A$, is of the form

$$
A_Z \rightarrow A_{Z+1} + e^- + \bar{\nu}_e. \quad (3.1)
$$

Thus, each beta-decay will yield an electron antineutrino\(^1\) and on average, one fission will lead to the production of approximately six neutrinos, of which about two are above 2 MeV. There are six neutrinos on average due to the fact that the neutron to proton ratio of stable nuclei is not linear. Once a fission occurs, there need to be about 6 total beta decays to convert the excess neutrons into protons. As a result, a 1 GW\(_{th}\) nuclear reactor will emit about $10^{20}$ neutrinos per second making nuclear reactors a great source for neutrinos. In fact, the first neutrino detection was done using reactor neutrinos by Cowan and Reines in 1956 from the Savannah River Plant [2]. They had originally considered using a nuclear explosion as the neutrino source but were later convinced to use neutrinos from a reactor.

By counting the detected neutrinos, the number of fissions per second can be inferred and related to the thermal power of the reactor. As would be expected, different fissioning isotopes will have a

\(^1\)For textual simplicity, electron antineutrinos will just be referred to as reactor neutrinos throughout the rest of this chapter.
different distribution of fission fragments, the daughter nuclei produced by fissions. The subsequent beta-decay from a fission fragment will have a characteristic neutrino energy spectrum specific to that beta-decay. Combining this together leads to the fact that the neutrino energy spectrum, integrated over all beta-decays spurred from one fissioning isotope, will be different than that from a different fissioning isotope. At any given time the neutrino signal from an active reactor will be some linear combination of neutrinos due to the four primary fissioning isotopes. For an accurate prediction of the measured event rate and spectrum, we require knowledge of what the neutrino energy spectrum is for each fissioning isotope.

There have been several experiments recently dedicated to studying neutrino oscillations of reactor neutrinos. These experiments are currently running in an attempt to measure the oscillation parameter $\theta_{13}$ [54, 53, 52]. $\theta_{13}$ is a mixing angle that lies within the unitary matrix that relates flavor and mass eigenstates. See chapter 2 for more details on the neutrino mixing matrix. Also, see results from the Daya Bay experiment for the current strongest constraint on $\theta_{13}$ [72] which is $\sin^2 2\theta_{13} = 0.089 \pm 0.008$.

Reactor neutrino experiments are sensitive to the effects of sterile neutrinos. In a disappearance experiment, like a reactor neutrino experiment, there is not a direct indication of what flavor the neutrino oscillated into, only that it oscillated into some other flavor. For this reason, a single neutrino oscillating into a sterile neutrino would look no differently than it oscillating into some other flavor. The primary focus in this chapter is a sterile neutrino with a mass splitting similar to that hinted at by the LSND experiment of about 1 eV$^2$. Having such a large mass splitting relative to the others causes a certain level of decoupling in the oscillation. Oscillations into sterile neutrinos happen over distances of a few tens of meters while oscillations driven by $\theta_{13}$ are over a distance of few kilometers. For a neutrino experiment with a baseline of a few kilometers, the sterile neutrino oscillation is very rapid over the energy range 1.8 MeV to 8 MeV, the relevant energies for reactor neutrinos. A disappearance experiment will measure an overall deficit in the number of neutrinos dictated by the average of the oscillation into the sterile mass splitting. This acts as an additional asymmetric normalization parameter that is not well constrained by current experiments.

The approximate reactor neutrino survival probability is given by:

$$P_{\bar{\nu}_e \bar{\nu}_e} = 1 - 4 s_{12}^2 c_{12}^2 c_{13}^4 c_{14}^4 \sin^2 \Delta_{21} - 4 c_{12}^2 s_{13}^2 c_{13}^2 c_{14}^4 \sin^2 \Delta_{31} - 4 s_{12}^2 s_{13}^2 c_{13}^2 c_{14}^4 \sin^2 \Delta_{42}$$

$$- 4 c_{12}^2 s_{13}^2 c_{13}^2 s_{14}^2 \sin^2 \Delta_{41} - 4 s_{12}^2 c_{13}^2 s_{14}^2 c_{14}^2 \sin^2 \Delta_{43} - 4 s_{13}^2 s_{14}^2 c_{14}^2 \sin^2 \Delta_{43}$$

$$\approx 1 - 4 s_{12}^2 c_{12}^2 c_{13}^4 c_{14}^4 \sin^2 \Delta_{21} - 4 s_{13}^2 c_{13}^2 c_{14}^4 \sin^2 \Delta_{31} - 4 s_{14}^2 c_{14}^4 \sin^2 \Delta_{41}$$

(3.2)

(3.3)
where $\Delta_{ij}$ is $\frac{1}{4E} \Delta m_{ij}^2$. It is assumed that $\Delta m_{21}^2 \ll |\Delta m_{31}^2| \ll |\Delta m_{41}^2|$. Due to the differences in these scales, $\Delta_{21}$ will effectively be a constant in a $\theta_{13}$ measurement with a baseline of a few kilometers. A detector, at a fixed distance from a reactor, with sufficiently good energy resolution can identify the energy where the maximum deficit occurs. This energy depends on the value $\Delta m_{31}^2$ and the size of the disappearance depends on $\theta_{13}$. A sterile neutrino when combined with other systematic parameters can mimic the signature of a non-zero $\theta_{13}$ oscillation. This is of particular concern when there is not a second detector at another distance that can isolate distance dependent parameters, such as oscillations, from the systematic parameters and model dependent predictions.

In a two detector setup, for example, in which systematic errors are correlated across the detectors, the ratio of event rates in the detectors will be approximately

$$\frac{R_{\text{far}}}{R_{\text{near}}} (E) \approx \frac{L_{\text{near}}^2}{L_{\text{far}}^2} \frac{P_{\bar{\nu}_e \nu_e} (E, L_{\text{far}})}{P_{\bar{\nu}_e \nu_e} (E, L_{\text{near}})},$$

(3.4)

Where $L_{\text{near}}$ and $L_{\text{far}}$ are the distances from the reactor to the near and far detector respectively. In the event that there is only one detector the expected rate needs to be predicted and the neutrino source and production needs to be well understood.

### 3.1 Flux models

Early predictions of neutrino energy spectra were made in early 1980s [73, 74, 75, 76, 77]. The standard was set by Schreckenbach et al. using beta decay measurements done at the Institut Laue-Langevin, ILL [78, 79, 80]. For these predictions, fissile targets were exposed to a thermal neutron flux and a magnetic spectrometer was used to measure the total integrated beta spectrum. This was done for $^{235}\text{U}$, $^{239}\text{Pu}$, and $^{241}\text{Pu}$ individually and was not done for $^{238}\text{U}$ until 2013 [81] due to its lack of a thermal fission cross section. Each integrated beta spectrum is then decomposed into contributing beta branches, inverted to give the neutrino spectrum contributions, and summed for the total neutrino spectrum.

These calculations have been redone recently by two independent groups. The new calculations will be referred to as flux MFL [82] and flux H [83] while the ILL calculation will be referred to as flux S. The calculations share common measurements taken at the ILL of the integrated electron spectrum but when the calculation was redone, current nuclear databases were used and additional corrections were considered. This has led to a change in the predictions since the 1980s calculation.
An allowed beta spectrum can be expressed by

\[ N_\beta (W) = K p^2 (W_0^i - W)^2 F (Z, W) L_0 (Z, W) C (Z, W) S (Z, W) G_\beta (Z, W) (1 + \delta_{WM} W) . \quad (3.5) \]

In Eq. 3.5, \( K \) is a normalization factor, \( p^2 (W_0^i - W)^2 \) is a phase space factor\(^2\) where \( W = E/(m_e c^2) + 1 \) and \( W_0 \) is \( W \) at the endpoint, \( F(Z, W) \) is the Fermi function, and the other factors are corrections. \( L_0 (Z, W) \) is due to a finite size correction to the electric charge distribution while \( C(Z, W) \) is due to a finite size correction of the hypercharge distribution. \( S(Z, W) \) corrects for a screening effect that bound electrons have in reducing the effective charge felt by outgoing electrons. \( G_\beta (Z, W) \) are radiative corrections from virtual and real photon emission. This term needs to be replaced by \( G_\nu (Z, W) \), radiative corrections to neutrinos, during the inverting procedure in order to calculate the neutrino spectrum. The final term is due to a weak magnetism correction that is, to first order, linear in beta energy. This weak magnetism term will be discussed in greater depth in chapter 4.

Flux H used a similar procedure to the 1980s calculation in which the total integrated spectra is decomposed into so-called virtual beta decays. The fit was done by taking a finite slice of the spectrum whose range ends at the highest energy data point and the slice was fit with a beta spectrum that has a free endpoint and amplitude following that from Eq. 3.5. After a fit was determined, the beta spectrum was subtracted from the total integrated spectrum and the process is repeated. Beta spectrum derived endpoints from the ENSDF database and thermal fission yields from the JEFF database were used to calculate the effective nuclear charge. Performing the decomposition yielded oscillations in the residuals that were smoothed out by taking an average over 250 keV bins. Once the decomposition was finished, the inversion process was done to map the virtual beta spectra into neutrino spectra and was re-summed.

These new calculations, flux MFL and flux H, have predicted an overall increase in the event rate from the previous calculation flux S. In using one of the new fluxes, neutrino experiments that had been in agreement with rates predicted by flux S now see a deficit. One possibility to explain the deficit is to introduce a sterile neutrino as to remove some of the neutrino flux via oscillation [12]. The ratio of observed to predicted rate was plotted for reactor experiments as a function of standoff distance. The data was then fit with a sterile neutrino with arbitrary mass and mixing and the best fit was found to be for neutrinos with mass splitting greater than about 1 eV\(^2\) and mixing of \( \sin^2 2\theta_{14} = 0.1 \) [12]. There are other possible sources for a lack of neutrinos and in particular the weak magnetism correction can easily remove a large portion of the flux.

\(^2\)Forbidden transitions will have additional corrections to the phase space factor.
Due to the lack of nuclear data, calculations of the effective average nuclear charge will have an associated error. Fissions lead to many fission fragments that have not been well studied and have not had their beta decay spectra measured. Instead most decays only have gamma spectroscopy measurements done. The problem this leads to is the pandemonium effect \[84, 85\] in which there can be a high energy gamma transition or a cascade of several low energy gamma transitions following a low energy beta decay. It can be hard to measure the low energy gamma causing errors in determining the beta endpoint and branching fractions. Simulations were done to find the effect of under- and over-estimating the pandemonium effect on the effective nuclear charge and the overall error was parameterized by a quadratic that is correlated across isotopes.

The inversion procedure itself has bias errors associated with approximating the data with a limited number of virtual beta decays. Many solutions exist to the inversion procedure and the choices made can preferentially pick out certain solutions. The error can be quantified through determining how the fit changes with different inversion procedure choices. There are also statistical errors that are propagated through from errors in the beta spectrum. The size of these errors can be determined through simulating the beta spectrum with randomized statistical fluctuations and inverting.

All of these errors need to be considered when using reactor neutrino flux model and can have sizable effects on the final measured rates. Using the calculations from one of these flux models and associated errors, we are able to predict the shape and rate of the neutrino flux entering a detector. Next we need to understand how these neutrinos interact with a detector and how we can distinguish signal from background.

### 3.2 Reactor neutrino detection

The most common method for detecting reactor neutrinos is to use the process of inverse beta-decay (IBD).

\[
p + \bar{\nu}_e \rightarrow n + e^+ \tag{3.6}
\]

Due to the mass difference between the proton and that of the neutron and positron, the interaction has a neutrino energy threshold of about 1.8 MeV given by \[\left( (m_n + m_e)^2 - m_p^2 \right) / 2m_p\]. Although the upper bound on reactor neutrino energy is not well known, the majority of the neutrino flux falls below 10 MeV. Figure 3.1 shows the neutrino spectra for the four primary fissioning isotopes per fission from flux MFL. It can be seen in the figure that a large portion, about two-thirds, of the
Figure 3.1: The left plot shows the average neutrino spectra per fission of $^{235}$U, $^{238}$U, $^{239}$Pu, and $^{241}$Pu. The right plot shows the average neutrino spectra measured through IBD from $^{235}$U, $^{238}$U, $^{239}$Pu, and $^{241}$Pu. Neutrino spectra from reference [82] are used in this figure.

The emitted reactor neutrino spectrum falls below 1.8 MeV, so this threshold does put a constraint on experiments. Alternative detection methods exist for measuring neutrinos below this threshold. Two of particular note are electron scattering and coherent scattering. Neither has a minimum threshold but electron scattering has the downside that the cross section is much lower than IBD and coherent scattering is a theoretical development at this stage. In the event that a detector was made for the purposes of measuring neutrinos through coherent scattering, the cross section would scale with the square of the neutron number and be much larger than IBD [86]. The downside is that the recoil energy decreases with the atomic mass, making measurements of low energy neutrinos very challenging.

Even with this threshold, IBD is beneficial in that it allows for a delayed coincidence measurement that significantly rejects backgrounds. IBD detectors are typically doped with an element that has a high neutron cross section and a discernible neutron capture signature. One such dopant is gadolinium (Gd) which has a high neutron capture cross section and leads to a 8 MeV gamma cascade. In such a detector, an IBD event produces a positron and neutron. The positron will quickly scatter and annihilate creating a prompt signal. At a short time later, the neutron will find an atom with a high neutron cross section and be absorbed. This is the delayed signal. Requiring an event to have both the prompt and delayed signal greatly reduces the number of backgrounds that can have the same signature as an event. Three of the predominate backgrounds that can mimic a signal are accidental, fast-neutron, and beta-delayed neutrons. These will be discussed in
Two ongoing experiments that use IBD detectors are the Double Chooz and Daya Bay experiments. The Double Chooz detector and Daya Bay detectors are similar in design. Within the innermost region is the liquid scintillator target doped with Gd surrounded by a second scintillator region for detecting escaped gammas. Outside is a third region of mineral oil for shielding external source gammas from the target region and is surrounded by PMTs that are used for detecting light from target events. Outside this are regions for identifying and excluding cosmic muons and some muon related events. Reactor experiments can reduce the cosmic muon flux and thus the background rate by placing the detectors deep underground.

The Double Chooz experiment uses neutrinos from two 4.25 GW$_{th}$ reactors. The target detector is a liquid scintillator doped with Gd that has about $6.75 \times 10^{29}$ protons on target. The detector is deployed 1.05 km from the reactors. There have been plans to add a near detector at a distance around 400 m, but no such detector is currently in use. The Daya Bay experiment has six 2.9 GW$_{th}$ reactors split into pairs over three separate locations. Original plans were made to begin taking data with two 20 tonne detectors at a near site with an effective distance of 0.51 km. Later a near site would install two 20 tonne detectors at effective distances of 0.56 km and a far site would install four 20 tonne detectors at an effective distance of 1.6 km. The actual deployment differed from the original plan in that six detectors were installed across all three sites before data collection. For the purpose of demonstrating how systematic uncertainties can effect a one detector setup, we examine the Double Chooz experiment [87] in its early stages as well as the Daya Bay experiment if it had started taking data with only one set of near detectors.

In an idealized situation where no systematic uncertainties are present and sufficient statistics exist, one neutrino detector at a distance of around 2 km$^3$ would be able to pin down both values of $|m_{31}^2|$ and $\theta_{13}$. In practice, however, multiple detectors are used. With two detectors, a ratio can be taken between the far and the near detector data, eliminating common factors leaving primarily oscillation parameters. In effect, the near detector is used to calibrate the expectation for the far detector.

---

The detected neutrino spectra, after integrating in the IBD cross section, from each primary fissioning isotope is shown in Fig. 3.1. The event peak occurs around 4 MeV for reactor neutrinos using an IBD detector. Then it follows that for $m_{31}^2$ eV$^2$ with the value $2.32 \times 10^{-3}$ and a neutrino energy of 4 MeV, an oscillation maximum occurs at a distance of about 2.1 km.
3.3 One detector flux dependence

The existence of multiple flux models can effect the fitting of $\theta_{13}$ in reactor experiments. We examine the early stages of the Double Chooz and Daya Bay experiments, considering what happens if nature follows one model and we choose to fit with another. In addition, we allow for the possibility of a sterile neutrino with a large mass. Due to the large mass, the sterile oscillations have a sufficiently short oscillation length that a detector sees as an averaging effect which causes a flat decrease in events across all relevant energies. By including this sterile neutrino, we are acknowledging the possible existence of sterile neutrinos and providing a means for the normalization to change. We always assume the new mixing angles associated with the sterile neutrino, as well as $\theta_{13}$, are zero in nature. In our fit, we vary $\theta_{13}$ and one of the sterile mixing angles, $\theta_{14}$, associated with a 1 eV$^2$ mass splitting. We examine three months of data for both experiments, simulated using GLoBES [88, 89], assume 121 energy bins$^4$, and use the $\chi^2$ function given in Eq. 3.7.

$$\chi^2 = \sum_{i,d} \left( \frac{(P_{i,d} - M_{i,d})^2}{M_{i,d}} \right) + \sum_{r,l} \left( \frac{\xi_{BU}}{\sigma_{BU}} \right)^2 + \sum_{d} \left( \frac{\xi_{DN}}{\sigma_{DN}} \right)^2 + \sum_{r} \left( \frac{\xi_{RN}}{\sigma_{RN}} \right)^2 + (\xi_{WM})^2 + (\xi_{Z})^2 + (\xi_{FN})^2$$  \hspace{1cm} (3.7)

$$P_{i,d} = \sum_{r,l} F_{i,d,r} \left( 1 + \xi_{WM} \sigma_{i,l}^{WM} + \xi_{Z} \sigma_{i,l}^{Z} + \xi_{FN} \sigma_{i,l}^{FN} \right) \left( 1 + \xi_{RN} + \xi_{d}^{DN} \right) \left( 1 + \xi_{r,l}^{BU} \right)$$  \hspace{1cm} (3.8)

Each of the $\xi$ in Eq. 3.7 and Eq. 3.8 are systematic parameters that are minimized over. Each $\sigma$ is an error associated with a $\xi$. $\sigma_{RN}$ is a reactor normalization error of 2%, $\sigma_{DN}$ is a detector normalization error of 0.15%, $\sigma_{BU}$ is an isotopic burn-up error of 2%, we use an energy scale error of 0.1%, and flux theory errors are given in the paper describing flux H [83]. The flux theory errors include weak magnetism errors $\sigma_{i,l}^{WM}$, effective nuclear charge errors $\sigma_{i,l}^{Z}$, flux normalization errors $\sigma_{i,l}^{FN}$, and stat and bias errors $\sigma_{i,l}^{SB}$. $M_{i,d}$ is the number of measured events in energy bin i at detector site d. $F_{i,d,r}$ is the number of predicted events in energy bin i at detector site d from reactor site r. $f_{l}$ is the isotopic composition for isotope l. We assume that each reactor has a constant isotopic composition of 40% $^{235}$U, 40% $^{239}$Pu, 10% $^{238}$U, and 10% $^{241}$Pu. We take the errors for $^{238}$U to be the same as the errors for $^{235}$U. We also use the $^{238}$U neutrino spectrum from flux MFL for flux H.

$^4$The number of bins is exceedingly high but a change to a larger bin width is not expected to change the result drastically.
\( \chi^2 \) are used as a goodness of fit test in order to quantify the level of disagreement between data sets. This particular \( \chi^2 \) uses a Gaussian approximation to what is more precisely a Poissonian process, that is counting events. In the limit of large statistics, the Poissonian \( \chi^2 \) is well described by a Gaussian. The first summation within the \( \chi^2 \) is where the predicted event rates are directly compared to the measured event rates. It is assumed that the data has 1\( \sigma \) statistical fluctuations given by the square of the binned event rate; this is a property of Poissonian statistics. In the simplest case with only one energy bin and no systematic parameters, a \( \chi^2 \) of 1 is given when the predicted number of events differs by 1\( \sigma \) from the measured number of events.

As seen in Eq. 3.8, there can be many systematic parameters that effect event rates. Each has an expected value and associated error that are determined prior to the experiment. In the case of our \( \chi^2 \), the parameters are all defined to be fluctuations about an expected value of 0. When this is not the case, the additional terms in the \( \chi^2 \) would be of the form \( (\xi - \mu)^2 / \sigma \) where \( \mu \) is the expected value and \( \sigma \) is the associated error. The systematic \( \xi \) parameters can fluctuate to create better agreement between the predicted and measured data but with the additional \( \chi^2 \) terms pull the systematic parameter towards the expected value \( \mu \). For this reason, they are called pull terms. When the systematic parameter deviates from the expected value by \( \sigma \) it contributes an additional unit to the \( \chi^2 \). Through minimizing the \( \chi^2 \), there is a balance between lowering the disagreement to the data and increasing the disagreement in the pull parameters.

The method becomes more intricate when the systematic errors have correlations. The weak magnetism, effective nuclear charge, and flux normalization errors are such systematic errors. If one of these errors fluctuates then there is a calculated response that is different for each energy bin and each isotope. For that reason, it can’t be written in the standard format and instead the 1\( \sigma \) errors are multiplied into the prediction. It then follows that if, for example, \( \xi^{WM} = 1 \), the pull term will increase the \( \chi^2 \) by 1 and the contribution to the predicted data from isotope \( l \) in energy bin \( i \) will be modified by a factor of \( \sigma^{WM}_{i,l} \). The final complication to the \( \chi^2 \) used in Eq. 3.7 is extra factor of \( 1 + M_{i,d} \sum_l f_l \left( \sigma^{SB}_{i,l} \right)^2 \) in the denominator of the first term. The stat and bias errors are not correlated across each energy bin or isotope and would need an additional pull term for each causing the addition of four times the number of bins in additional pull terms. Instead of adding those pull terms, the error is instead added in quadrature with the statistical error, reducing the computational complexity for minimizing the \( \chi^2 \).

A variety of minimization methods exist. In the limit where the systematic errors are small,
Models | Experiment | Label | Min $\chi^2$ value | $\sin^2 2\theta_{13}$ | $\sin^2 2\theta_{14}$
--- | --- | --- | --- | --- | ---
S/H | Double Chooz | A | 0.99 | 0.00 | 0.08
| | | B | 3.25 | 0.00 | 0.00
| Daya Bay | C | 5.23 | 0.00 | 0.035
| | D | 6.49 | 0.04 | 0.00
MFL/H | Double Chooz | E | 0.31 | 0.00 | 0.01
| | F | 0.34 | 0.005 | 0.00
| Daya Bay | G | 1.54 | 0.00 | 0.005
| | H | 1.39 | 0.025 | 0.00

Table 3.1: Values of minima and their locations for Fig. 3.2.

quadratic terms can be removed and the $\chi^2$ can be written in the form of $|Ax - b|^2$ where A is a m by n matrix, x is a n dimensional vector where each element holds the value of a systematic error and b is a m dimensional vector. The process of minimizing the $\chi^2$ then corresponds to finding the vector x that minimizes $|Ax - b|^2$. A fine attention to detail is required to find the appropriate values for each entry in the matrix A. The minimization of the $\chi^2$ in this linear form can be performed very quickly through the use of a singular value decomposition. Necessary non-linearities sometimes appear in data analysis and they can make it challenging to write in a meaningful linear format.

The minimum value of the $\chi^2$ can give an indication of how well the prediction agrees with the data. As a guideline for data with statistical fluctuations, a good fit is expected to get a unit of $\chi^2$ for each data bin. The actual value of the $\chi^2$ minimum should not be used when placing limits on things such as oscillation parameters. Instead, deviations from the minimum value should be used $\Delta \chi^2 = \chi^2 - \chi^2_{\text{min}}$ [90]. This mitigates the effect that additional terms have in artificially inflating the $\chi^2$. For example, if the original $\chi^2$ was for a detector at a 2km from a reactor, a second data set could be added to the $\chi^2$ for a very short baseline. Even though this data has not had any $\theta_{13}$ oscillation, it will increase the minimum value of the $\chi^2$. $\Delta \chi^2$ will remain unchanged and for that reason is a more appropriate method for quantifying sensitivities. With one degree of freedom, a $\Delta \chi^2$ of 1, varying only one parameter, such as $\theta_{13}$, will determine the range $\theta_{13}$ can take with 1σ agreement. Likewise, a $\Delta \chi^2$ value of 4, would correspond to a 2σ agreement and so on.

In Fig. 3.2 we plot contours of $\Delta \chi^2=4$, as defined in Eq. 3.7, in the physical region. A $\Delta \chi^2=4$
Figure 3.2: Contours of $\Delta \chi^2 = 4$ above the minimum value in the physical region are shown. Three months of simulated data is used. Minima are labeled by letters. Vertical lines represent two sigma for Daya Bay and Double Chooz with no sterile oscillation and the flux in the fit matching the true flux. Left plot compares flux S and flux H. The right plot compares flux MFL and flux H. Daya Bay sensitivities are shown in blue while Double Chooz sensitivities are shown in orange.
Figure 3.3: The ratio, for each bin, between flux H and the flux MFL, is given in blue. The inverse is given in orange. The survival probability is shown in green with $\sin^2 2\theta_{13} = 0.15$ and using a weighted average distance. The shaded region represents the statistical error. The data and oscillation probability is given for the Daya Bay near site.

was chosen to allow easy comparison to the $2\sigma$, one degree of freedom $\theta_{13}$ limits. On the left side of Fig. 3.2, we compare fluxes S and H and on the right side, we compare fluxes MFL and H. In Tab. 3.1, locations of the $\chi^2$ minima and their values are listed for each configuration. For Daya Bay, it can be seen in any of the contours, that fitting with a different flux than nature's, yields a very different limit for $\theta_{13}$ than the $2\sigma$ limit. Double Chooz, on the other hand, has much milder differences between limits. It may not seem surprising that there is a large disagreement between the contours when comparing flux S and flux H since there is greater than a $3\%$ average flux normalization difference between the two models. When comparing fluxes MFL and H, however, the overall normalization between the the models is comparable and it is not immediately obvious why there is still such a disparity between the two contours at Daya Bay.

In Fig. 3.3, the differences in the spectrum shape are examined to give insight into why the experiments react differently. For the distances and energies given, the survival probability has a positive slope for the Double Chooz experiment. If the Double Chooz detector was further from the reactor, the survival probability would have a minimum at a higher energy and a shape distinct from that of just a linear slope. We also see that the ratio of flux H to flux MFL has a positive slope. This matching of slopes allows a larger $\theta_{13}$ fit when we assume nature follows flux H and we fit using flux MFL. The key reason for the asymmetry between the experiments is the difference in baselines. Because the Daya Bay detector site chosen has a baseline that is shorter than that of Double Chooz,
it receives many more counts and thus has much narrower statistical error bars. For Double Chooz, the statistical errors are large enough to absorb the differences between flux H and flux MFL. As Double Chooz continues to take data, those error bars will shrink and eventually the experiment will run into the same issues as Daya Bay.

The Double Chooz collaboration mitigated some of the flux uncertainty through the use of the Bugey-4 experiment [91] for flux normalization. The Bugey-4 experiment used an IBD detector at a distance of 15 m from a 2800 MWth reactor and measured 300,000 events. The large neutrino sample set helps put stronger constraints on the flux normalization. After including the Bugey-4 normalization calibration, only the shape differences between the models are left. When we combine these two experiments in a fit where there is an additional free normalization parameter that is correlated across both experiments, we still find there to be a strong flux dependence. In Fig. 3.3, we show the $\chi^2$ given in Eq. 3.7 with an additional rate term that also compares the predicted events for the Bugey-4 experiment. An additional free normalization is included, correlated across both experiments and the true value of $\theta_{13}$ is chosen to be 0, true flux to be MFL, and fit with flux H. We find that even after the difference in predicted rates is removed from the flux models, the shape differences can still impact a $\theta_{13}$ measurement. The best fit in this case is for $\sin^2 2\theta_{13} = 0.05$.

It would helpful to place stronger constraints on the flux models through additional short baseline reactor experiments. In particular, experiments with reasonable energy resolution for detecting energy dependent deviations from the prediction. One of the largest theory errors, weak magnetism,
allows for rotation of the neutrino spectrum about a point close to 2.3 MeV. A short baseline reactor experiment could, in principle, put strong constraints on the value of the weak magnetism parameter, giving insight into a region of nuclear physics that is hard to measure by traditional means. These additional constraints would also allow for more precise measurements of neutrino properties in the future.
Chapter 4

Weak Magnetism

4.1 CVC and weak magnetism theoretical uncertainty

It was observed that the vector current coupling constant for muon decay was approximately the same as the vector current coupling constant for nuclear beta decay. It was then expected that there needs to be some symmetry, similar to that of electromagnetism, that prevented a renormalization by the strong force. This was the idea of a conserved vector current (CVC) and was applied to the weak interaction by Feynman and Gell-Mann [92]. With this expected symmetry, there exists a way to relate matrix elements of the weak vector current with corresponding electromagnetic amplitudes [93]. In fact, the weak vector current and the isovector part of the electromagnetic current form a single isotriplet vector within the standard model [94].

The fundamental weak current has the form  $\bar{q}\gamma_\mu (1 + \gamma_5) q$. Extending beyond pure V-A to account for finite size effects and only imposing Lorentz invariance, nucleon beta decay can have matrix elements of the form

$$\langle \beta | J^W_\mu | \alpha \rangle = \bar{u} (p_2) \left[ \gamma_\mu (g_V + g_A \gamma_5) - i \frac{\sigma_\mu \nu q^\nu}{m_1 + m_2} (g_M + g_T \gamma_5) + q_\mu \frac{m_1 + m_2}{m_1 + m_2} (g_S - g_P \gamma_5) \right] u (p_1)$$

(4.1)

where $q = p_1 - p_2$ is the momentum transfer and $\bar{u} (p_2)$ and $u (p_1)$ are free Dirac spinors [95]. Each of the $g_V, g_A, ..., g_P$ are form factors and have $q^2$ dependence because the interaction occurs between quarks confined to nuclei and it is not reasonable to treat the interaction the same way one would a point particle.
Decay rates can be parameterized by an $ft$ value where $t$ is the half-life and

$$f = \int_{m_e}^{E_{\text{max}}} dE \frac{F(\pm Z, E) pE (E_0 - E)^2}{(\pm Z, E) pE (E_0 - E)^2}$$  \hspace{1cm} (4.2)$$

accounts for the phase space. Just as in the flux calculations in chapter 3, additional corrections exist to the phase space that modify $f$. Fermi transitions where the change in total angular momentum $\Delta J = 0$ and $\Delta \pi = 0$, $J^P = 0^+$ to $0^+$, are called ”super-allowed” decays. Only $g_V$ and $g_S$ terms contribute and CVC predicts them to be 1 and 0 respectively (at $q^2 = 0$) yielding a constant $ft$ value for all such transitions. Measurements of decays with these transitions are found to be in strong agreement with this prediction.

Additional support for CVC would be found in the energy spectrum from Gamow-Teller transitions where $\Delta J = 0,1$ and $\Delta \pi = 0$. CVC predicts a shape correction factor, linear in energy, that depends on an interference between the the $g_M$ term in the interaction, also referred to as the weak magnetism term, and the $g_A$ axial vector term. In the impulse approximation, where the decaying nuclei is considered free, the size of the correction is proportional to the difference of the proton and neutron magnetic moments. The effect of the correction term is doubled when comparing $\beta^-$ to $\beta^+$ decays because of a sign change in the interaction when going from electrons to positrons. Gell-Mann suggested to look at the A=12 isotriplet [96] in which $^{12}$B can $\beta^-$ decay to $^{12}$C and $^{12}$N can $\beta^+$ to $^{12}$C. Using a triplet, such as this, highlights the isospin symmetry and helps reduce dependence on systematic uncertainties. The CVC theory predicts a slope correction to the energy spectrum for $^{12}$B to $^{12}$C of 0.43% MeV$^{-1}$ and a correction for $^{12}$N to $^{12}$C of -0.50% MeV$^{-1}$.

Experiments were done to measure the shape of these two transitions. Of particular note was that done by Lee, Mo, and Wu in 1963 using an iron-free magnetic spectrometer and then later reanalyzed in 1977 due to an erroneous Fermi function [97, 98]. After accounting for the theoretical allowed shape, including radiative and finite nuclear size corrections, shape correction factors were determined as shown in Fig. 4.1. The slope of the correction factor was fit to determine the weak magnetism correction and it was found to be in agreement with the theoretical CVC prediction. After averaging over two experimental setups, the best fit slope correction was found to be of 0.46% MeV$^{-1}$ for $^{12}$B to $^{12}$C and -0.50% MeV$^{-1}$ for $^{12}$N to $^{12}$C.

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Figure 4.1: Reprinted figure with permission from C. S. Wu, Y. K. Lee, and L. W. Mo, Phys. Rev. Lett. 39, 72 Published 11 July 1977. Copyright 1977 by the American Physical Society with accompanying text: "Shape correction factors for $^{12}$B and $^{12}$N. $S_{exp}/S = 1 + a \pm E$ measured with the narrow (3/16 in.) angular slits. The open circles for $^{12}$N are not used for fitting. The points are normalized to the value near the middle of each spectrum." [98]
Figure 4.2: In the left plot, the expected neutrino flux is shown in arbitrary units for allowed, first non-unique forbidden, and first unique forbidden decays as a function of neutrino energy for thermal fission of $^{235}$U. In the right plot, the fractional contribution from each decay type is shown. [99]

These experiments are found to be very challenging. "Several laboratories have, in the past, experimentally investigated the beta spectra of $^{12}$B and $^{12}$N. Although the ratio of the shape factors between these two spectra was found to be of the right order of magnitude, the deviation of each individual spectrum from the allowed shape was either several times larger than what was calculated, or the sign was opposite to what was predicted, or the individual spectrum was just not investigated." [97] The number of isotopes investigated has therefore been very limited. This is an issue when it comes to reactor neutrinos because fissions yield isotopes that have atomic mass around either 90 or 140, much higher than the isotopes investigated here.

Many of these have forbidden decays, different than the standard "allowed" Gamow-Teller and Fermi decays, with larger changes in angular momentum and possible changes in parity. Figure 4.1 shows the contribution to the $^{235}$U neutrino flux from allowed and first forbidden decays. Around half of the neutrino spectrum between 3 and 6 MeV is due to forbidden decays. This is a problem for neutrino flux predictions because unlike allowed transitions, the size of beta-decay shape corrections for forbidden decays are not well known. Of particular note is the weak magnetism correction which could be quite large. In Ref. [83], the size of the weak magnetism correction was calculated using gamma energy decay widths and the CVC hypothesis for 13 allowed Gamow-Teller decays. Excluding 3 decays that had log ft values higher than 7, the mean weak-magnetism slope parameter was found to be $0.67 \pm 0.26\%$ MeV$^{-1}$. Including the 3 large log ft decays, the size of the correction increases substantially to $4.78 \pm 10.5\%$ MeV$^{-1}$. Forbidden decays tend to have large log ft values and if the 3 sample decays examined are any indication, it is possible that the weak magnetism correction could be substantial.
Chapter 4. Weak Magnetism

In Ref. [100], the corrections to the neutrino spectrum were examined while using various assumptions about the operators involved in the forbidden transitions. Unique first forbidden decays have one unique operator, shape change, and weak magnetism correction. Non-unique, first forbidden decays have several operators each of which can individually give different predictions for the shape and strength of weak magnetism corrections. The combination of operators involved for each non-unique decay is not known. This causes large differences in the weak magnetism correction that depend on the assumptions made about the relative weights of the operators used in the flux calculations. A 4% uncertainty in the neutrino flux is expected due to the forbidden decay uncertainty alone. The analysis made some approximations that excluded some currents involved that could, in principle, also effect the weak magnetism correction. The author states, "Reducing the uncertainty within a purely theoretical framework would be difficult. An improvement will require either direct measurements of the antineutrino flux or a substantial improvement in our knowledge of the dominant forbidden beta transitions." To address this issue, we turn to actual data from ongoing reactor neutrino experiments with hopes of constraining the weak magnetism correction.

4.2 Experimental constraints

Reactor neutrino experiments measure the neutrino spectrum and therefore we can use past and ongoing experiments to constrain the value of the weak magnetism slope correction. To limit the influence of neutrino oscillation as well as give higher statistics, it is preferable to use the near detector sites for the constraint. For each of the experiments, we use the background subtracted rates for their detectors nearest to the nuclear reactors [54, 53, 52, 101]. The Daya Bay, Double Chooz, and RENO experiments are ongoing experiments designed with the goal to measure $\theta_{13}$. The Bugey-3 experiment is a past experiment with data published in 1994. Data was published for distances of 15 m, 40 m, and 95 m. With these distances, the experiment was probing neutrino oscillation in a region of parameter space different than the aforementioned experiments. The experiment provides a large number of events, nearly 100,000, at the 15 m standoff distance. The event shape is shown for each experiment in Fig. 4.2 where the event rates are normalized to yield the same number of events between 1.8 and 8 MeV. Visually, there is good agreement between Daya Bay, Double Chooz, and RENO but poor agreement with Bugey-3.

The best fit values for the weak magnetism slope correction and the corresponding $\Delta \chi^2 = 1$ uncertainties are shown for the various experiments in Fig. 4.4 using Eq. 4.3. This follows similarly to the
Figure 4.3: The neutrino event rates are shown as a function of the reconstructed neutrino energy for a Daya Bay near site, Double Chooz, Bugey-3 at 15 m, and the Reno near site. All rates are normalized to yield the same integrated count over the energies 1.8 MeV to 8.0 MeV. Prompt energies are converted into neutrino energies through an energy shift of 0.8 MeV, ignoring detector effects. The Bugey-3 data is shifted in energy by 1.8 MeV.
\( \chi^2 \) in Eq. 3.7 with a few key differences. We now use one free normalization parameter (no pulls) and combine the weak magnetism and effective nuclear charge theory errors into one linear slope correction “X”. We found that there was a strong correlation between the two errors and decided to treat them as indistinguishable. Although this parameter X is a general slope correction, it will be referred to as a weak magnetism correction. It has been found that the reactor composition has very little impact on measuring the weak magnetism correction. Even with 100% uncertainty in the composition, the weak magnetism errors only increase by about 10 to 20%. The average reactor composition is used for each experiment that provides it. If not provided, the composition is assumed to be 50% \(^{235}\text{U}\), 10% \(^{238}\text{U}\), 30% \(^{239}\text{Pu}\), and 10% \(^{241}\text{Pu}\).

\[
\chi^2 = \sum_i \frac{(P_i - M_i)^2}{M_i \left[ 1 + M_i \sum_l f_l (\sigma_{\text{Stat&Bias}}^{i,l})^2 \right]} + (\xi^{\text{FN}})^2 \tag{4.3}
\]

\[
P_i = \sum_l F_{i,l} \left( 1 + X (E_i - 2 \text{MeV}) + \xi^{\text{FN}} \sigma_{\text{FN}}^{i,l} \right) \left( 1 + \xi^{\text{Norm}} \right) \tag{4.4}
\]

The best fit values and \( \chi^2 \) are also listed in Tab. 4.1. It can be seen that for the three ongoing experiments, there is generally a very bad agreement between the predicted and measured spectrum. It is very challenging to reproduce the low energy spectrum and in the lowest energy bins, the disagreement is at the level of 50% as can be seen in Fig. 4.5. Presumably this is due to non-linearities in the detector response.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Best WM value [% MeV(^{-1})]</th>
<th>( \chi^2 )</th>
<th>Number of energy bins used</th>
<th>p-value (gof)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Daya Bay (near)</td>
<td>0.82</td>
<td>165.8</td>
<td>24</td>
<td>0</td>
</tr>
<tr>
<td>Daya Bay (near) with energy response</td>
<td>-0.89</td>
<td>24.4</td>
<td>24</td>
<td>0.44</td>
</tr>
<tr>
<td>Double Chooz</td>
<td>1.04</td>
<td>85.1</td>
<td>13</td>
<td>0</td>
</tr>
<tr>
<td>Bugey-3 (15 m)</td>
<td>-10.17</td>
<td>38.7</td>
<td>48</td>
<td>0.83</td>
</tr>
<tr>
<td>RENO (near)</td>
<td>7.38</td>
<td>1205.3</td>
<td>28</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4.1: The best fit values shown in Fig. 4.4 are listed with the corresponding \( \chi^2 \) values. The number of energy bins is also listed to show the level of agreement between the predicted spectrum and the measured.
Figure 4.4: Best fit value for the weak magnetism slope correction and $\Delta \chi^2 = 1$ error bars for a variety of reactor neutrino experimental data. The Daya Bay (near) with energy response uses the correction given in [54] to account for detector nonlinearities in the positron energy response. The shaded region shows the approximate theory prediction errors for Gamow-Teller weak magnetism slope.

Figure 4.5: In orange, the collaboration prediction for the no oscillation, background subtracted signal is shown with the prompt signal shifted by 0.78 MeV to represent a naive neutrino signal. The prediction event rates are taken from Ref. [53]. The data is compared directly to the predictions made assuming no detector response and using the same reactor flux model.
The only experiment of the four that has provided a non-linear positron detector response function is the Daya Bay experiment. "The scintillator nonlinearity for electrons is described by an empirical model \( f_{\text{scint}}(E_{\text{true}}) = \frac{E_{\text{vis}}}{E_{\text{true}}} = \frac{p_0 + p_3 \cdot E_{\text{true}}}{1 + p_1 \cdot e^{-p_2 \cdot E_{\text{true}}}} \)." [54] Best fit values for the empirical model were found to be \( p_0 = 1.0215 \), \( p_1 = 0.3224 \), \( p_2 = 1.0346 \), and \( p_3 = 0.0011 \) using the curve shown within the Daya Bay paper. By including this response, the prediction agrees much better with the data but the best fit value of weak magnetism shifts from 0.53% MeV\(^{-1} \) to -1.03% MeV\(^{-1} \).

It is expected that similar corrections are needed for the other experiments but they have not been explicitly quantified or parameterized in the literature. Monte Carlo calculations done within the collaborations seem to account for the spectrum discrepancies. The Double Chooz collaboration, for example, has good agreement between their own spectrum prediction and the measured spectrum when using their Monte Carlo. We expect that the best weak magnetism value will shift significantly once energy corrections are added for the remaining experiments, as it did with Daya Bay. So, not only are the experimental weak magnetism values predictions in disagreement with each other but there is still a lot of uncertainty in the value predicted by any particular experiment because of undeclared energy responses.

As it has been shown, there are linear slope differences between the flux models. This means that the choice of model will also change the weak magnetism best fit value. We redid the analysis using a different flux model and found that each experiment had a very similar shift to the best fit weak magnetism slope correction. Going from the flux model described in Ref. [83] to that described in Ref. [102], each best fit slope correction decreased between 1 and 2% MeV\(^{-1} \) for each experiment. This is illustrated in Fig. 4.6 where the \( \chi^2 \) is shown as a function of the weak magnetism slope correction for the Daya Bay near detector, using the non-linear energy correction function.

Additional energy corrections can effect the measured slope correction such as the energy calibration error. When we add in a 0.8% energy calibration error to the Daya Bay fit, the change to the \( \chi^2 \) and best fit for the weak magnetism correction is shown in Fig. 4.7. The 1\( \sigma \) errors go from approximately 0.37% MeV\(^{-1} \) to 2.98% MeV\(^{-1} \) with the 0.8% energy calibration error. This is a significant source of uncertainty in a slope correction measurement. For a precise measurement of the weak magnetism slope correction, the detector energy response needs to be very well understood. If the detector energy response is under control, then with Daya Bay statistics and systematics, the weak magnetism slope correction can be measured to the same level as the allowed Gamow-Teller
Figure 4.6: Using Eq. 4.3 for the Daya Bay experiment with the non-linear energy correction. The fit is shown in blue for flux H and shown in orange for flux MFL. The theory errors for the weak magnetism correction are shown in the shaded region where all forbidden decays are assumed to give allowed Gamow-Teller corrections.
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Figure 4.7: Using Eq. 4.3 for the Daya Bay experiment with the non-linear energy correction and flux H. The fit is shown in blue when not including any energy calibration error and in orange with a 0.8% energy calibration error.

4.3 Improvements on measurement

It would be preferable to have a stronger constraint on the slope correction parameter. 1 σ weak magnetism slope correction uncertainties are plotted as a function of runtime in Fig. 4.8. The energy calibration error was taken to be 2%\(^2\), composition error to be 5%. For the experiment simulation, a distance of 400m, reactor power of 5.8 GWth, detector mass of 40 tons, and a runtime of 1 year was used. The various curves demonstrate the impact of some important improvements on systematic errors. A statistics limit is shown as well for comparison. Knowing the energy scale error worse does not make the measurement much worse. It can be seen that knowing the energy scale error better would help significantly and that it is the dominating error for the majority of runtimes. Knowing the theory errors better would help, such as the statistic and bias theory error. As it stands, this error controls the asymptotic behavior of the WM error sensitivity. If we could eliminate other neutrino flux models and sterile neutrino dependence, normalization information could help as well.

\(^{2}\)This is understood to be a high reference point.
but it’s not that significant after a year of data.

Figure 4.8: 1σ WM sensitivities are shown as a function of runtime. Each curve demonstrates dependencies on particular systematic parameters. The solid blue curve gives a baseline for expected systematic parameters. The dashed red curve has no statistic and bias theory error. The solid orange curve has a normalization penalty. The dashed black curve has no energy calibration error.

It can be seen that even having no energy scale error, we are systematics limited and it is hard to get better sensitivity than around 0.5 % MeV$^{-1}$. Many competing errors become much more significant at the level. For example, the normalization theory error and reactor composition become important at that level even though they don’t have much impact while the other systematics are in play. It will be hard to improve past the 0.5 % MeV$^{-1}$ level.

4.4 Conclusion

The weak magnetism correction to the reactor neutrino spectrum, as well as its error, is hard to predict theoretically. There are large contributions of both unique and non-unique 1st forbidden decays. These decays could change the value of the weak magnetism correction dramatically. Additionally, detector related uncertainties have large effects on measured slope corrections. Without the ability to limit the weak magnetism correction sufficiently, reactor neutrino experiments require careful designed as to not depend on weak magnetism slope effects.
4.5 Note added in proof

After writing this chapter, ongoing reactor neutrino experiments have found a large spectral distortion (bump) that is not understood. There is the possibility that this is due to forbidden transitions in beta-decay [100] but it is unclear at this point. The origin of bump will need to be understood before any meaningful linear slope correction can be made.
Chapter 5

Reactor monitoring

5.1 Introduction

Short baseline reactor neutrino detectors can be effective at measuring eV^2 sterile neutrinos in addition to having high event rates for constraining reactor flux model uncertainties such as weak magnetism. Such detectors could additionally provide information regarding the state of the reactor for purposes of detecting the diversion of nuclear materials. Neutrinos carry useful information about the nuclear reactor and because they interact very weakly, they will always escape from the reactor and provide this information while the reactor is running without the need to shut the reactor down. This property is what allows neutrinos to be used for the measurement of nuclear reactor characteristics such as power, burn-up, and plutonium content. The idea of using neutrinos for reactor monitoring dates back to 1978 with Borovoi and Mikaelyan [103]. The capability to measure both the power [104] and the effect due to a changing plutonium content were experimentally verified by a group from the Kurchatov Institute at the Rovno power plant.

There have been many theoretical efforts since [105, 106, 107, 108, 109] with a variety of experimental assumptions. Some analyses assume that the reactor power is given from an external monitor and base the plutonium measuring capability on the fact that a $^{239}$Pu fission nets fewer neutrinos than a $^{235}$U fission. A decrease in neutrino rates at a known power would indicate plutonium generation and can be monitored over time. The target reactors considered are often large reactors in which the plutonium generation far exceeds the amount needed for a nuclear bomb which is deemed at 8 kg [110]; this is also termed a significant quantity. The most common type of reactor is a light water
reactor (LWR) which have fuel assemblies that are easy to keep track of. In addition, no plutonium nuclear weapons program started from LWR but instead started through graphite or heavy water (D$_2$O) moderated reactors. This analysis differs in that it assumes no external power information, examines much smaller reactors, and applies monitoring concepts to real-world scenarios.

The goal of this chapter is to explain why neutrinos have the capability to measure power and the plutonium content of a nuclear reactor independently, examine alternative neutrino capabilities, and explain how the neutrino data can then be converted into useful information for an organization such as the International Atomic Energy Agency (IAEA)$^1$. Among other things, one of the IAEA’s functions is "to establish and administer safeguards designed to ensure that special fissionable and other materials, services, equipment, facilities, and information made available by the Agency or at its request or under its supervision or control are not used in such a way as to further any military purpose; and to apply safeguards, at the request of the parties, to any bilateral or multilateral arrangement, or at the request of a State, to any of that State’s activities in the field of atomic energy" [111]. To this end, the IAEA verifies declared nuclear materials, in accordance to the non-proliferation treaty (NPT) using methods that are primarily based on material accountancy assisted by camera surveillance and tamper-proof seals. The "Additional Protocol", a legal document addition to the NPT, has also allowed the IAEA to perform tests that would check for undeclared nuclear material of which included environmental sampling [112].

A hypothetical measurement will be simulated for a historical situation in the Democratic People’s Republic of Korea (DPRK) and implications will be discussed for future applications in locations such as Iran. Much of the content of this chapter was originally done in Refs. [113, 114] and much of the structure from those references remains intact.

With feasibility and usefulness in mind, detectors on the order of about 5 tonnes with 100% efficiency will be considered. To compensate for a reduced efficiency, the detector will be made more massive. Table 5.1 lists a the detector mass needed to achieve the same level of significance for selected efficiencies. It is envisioned that the detector, shielding, and electronics will all fit within a standard 20' shipping container. Choosing a detector with a relatively small size comes with the trade-off that the distance between the detector and the reactor will have to be short, on the order of 20 m. This also means that the detector will require a surface deployment with large cosmic related backgrounds. A surface detector has

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$^1$An independent organization related to the United Nations system
Table 5.1: Actual detector mass in ton as a function of efficiency for a mineral oil based liquid scintillator (EJ-321L) with $8.6 \times 10^{22}$ protons per gram and a polyvinyltoluene based solid scintillator (EJ-200) with $5.1 \times 10^{22}$ protons per gram. Table and caption taken from Ref. [114]

<table>
<thead>
<tr>
<th>Efficiency [%]</th>
<th>25</th>
<th>40</th>
<th>60</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquid scintillator</td>
<td>20.1</td>
<td>12.5</td>
<td>8.4</td>
<td>6.3</td>
</tr>
<tr>
<td>Solid scintillator</td>
<td>34.0</td>
<td>21.3</td>
<td>14.2</td>
<td>10.6</td>
</tr>
</tbody>
</table>

5.2 Reactor physics

In the upcoming analysis, we need a way to convert the fission rates that dictate the neutrino spectrum into mass inventory, the mass of the isotopes within the reactor. The relationship between the two is controlled by the reactor physics of the core. Following Ref. [113], we introduce fission fractions, $\mathcal{F}_I$, which are defined by

$$\mathcal{F}_I = \frac{f_I}{\sum_I f_I} \quad \text{with} \quad \sum_I \mathcal{F}_I = 1,$$

(5.1)

where $f_I$ is the fission rate for isotope $I$. Fission fractions can then be expressed as a function of burn-up. Burn-up measures the number of fissions which have occurred per unit of fuel mass and has the units of MWd/t. Burn-up gives a measurement of the number of fissions that have occurred. Regardless of whether the reactor was run normally or with half the power and for twice as long, the reactor’s isotopic composition should be the same when neglecting radioactive decays. In principle, there may be sizable differences in isotopes with half-lives on the order of the difference in run-time. Isotopes with half-lives too short will equilibrate. Isotopes with half-lives too long will not decay much and with both reactor burn-rates producing the same number of isotopes, the number of atoms will be similar (and neutrino events are low). In Sec. 5.3.2, we examine the difference for the closest to ”just right” long lived isotopes. With an accurate reactor model, the fission fractions can be predicted when provided the burn-up. While the reactor is running, the power and fission fractions will determine the fission rate and spectrum of the neutrino events. At the same time, the fission rate is related to the mass by

$$f_I = \phi_n \sigma_I m_I,$$

(5.2)
where \( m_I \) is the mass of isotope \( I \), \( \sigma_I \) is the energy averaged fission cross section and \( \phi_n \) is the neutron flux. Unfortunately, the neutron flux and the fission cross section are changing in time along with the isotopic mass. Changes in the neutron absorption due to fission fragments will change the neutron flux and mass will change with the burn-up. We have performed evolution or burn-up calculations for several reactor types using the SCALE software suite [116]. We find that treating \( \phi_n \sigma_{\text{Pu}239} \) as a constant only introduces a 2% root mean square error when determining the plutonium mass for a graphite moderated reactor and comparable errors for other reactors studied. Using the evolution models, we can then measure the neutrino event spectrum, fit the fission fractions and convert the fission fractions into fission rates. With the fission rates, the model is used to find the mass of the isotope in question, \(^{239}\text{Pu}\) in our case. In the end, the neutrino spectrum predicts the burn-up and the burn-up predicts the mass.

An example of the time evolution of the \( \mathcal{F}_I \) for a graphite moderated, natural uranium fueled reactor is given in the left hand panel of Fig. 5.1, where the fission fractions are shown as a function of the burn-up. For this particular type of reactor, very little \(^{241}\text{Pu}\) is created and is not shown. \( \mathcal{F}_{\text{U}238} \) is approximately constant. Natural uranium is predominantly \(^{238}\text{U}\), with 99.274\% abundance, only a small percent of \(^{238}\text{U}\) fissions or captures neutrons, leaving the overall bulk unchanged. \(^{235}\text{U}\) has a high thermal fission cross section and \( \mathcal{F}_{\text{U}235} \) decreases steadily over time because there are no mechanisms to replace those fissioned atoms. \(^{239}\text{Pu}\) is generated after the capture of neutrons on \(^{238}\text{U}\) and similarly has a large thermal fission cross section. The overall effect seen is an anti-correlation between the fission fractions in \(^{235}\text{U}\) and \(^{239}\text{Pu}\). Any loss in \( \mathcal{F}_{\text{U}235} \) needs to be gained in \( \mathcal{F}_{\text{Pu}239} \) when the other two fission fractions are nearly constant. The anti-correlation is shown in the right hand panel of Fig. 5.1.

### 5.3 DPRK

The DPRK, in the time leading up to the 1994 nuclear crisis, provides a historically interesting scenario for study. It is a situation where conventional safeguards methods had difficulty while there is still sufficient public information to do a detailed study. Within the Yongbyon nuclear facility, in the DPRK, there are two nuclear reactors of concern: a 5 MW\(_{e}\), graphite moderated reactor and the Soviet supplied IRT that runs on Soviet supplied highly enriched uranium (HEU) drivers which we assume to be enriched to 80\% \(^{235}\text{U}\). In addition to the reactors, there is also a waste reprocessing facility used for the extraction of plutonium from the 5 MW\(_{e}\) reactor. In Fig.
Figure 5.1: The left hand panel shows the evolution of the fission fractions in a graphite moderated natural uranium fueled reactor as a function of burn-up. The right hand panel shows the anti-correlation of the fission fractions in $^{235}\text{U}$ and $^{239}\text{Pu}$. Figure and caption taken from Ref. [113].

5.2, the relative locations of the two reactors and waste facility are shown as well as contours of the expected measured neutrino event rates for a 5 tonne IBD detector over a year of data taking. We want to examine how well neutrino detectors could measure information about these two reactors in an attempt to understand the strengths and weaknesses a neutrino detector would have if used for future non-proliferation efforts.

The 5 MW$_e$ began running before IAEA safeguards were in place and during that time there was a 70 d shutdown in which the DPRK declared the removal of a few hundred damaged fuel elements and the separation of 90 g of plutonium [117, pp. 88]. Later environmental sampling done by the IAEA gave indications of three reprocessing campaigns which could imply that additional fuel was replaced during the 70 d shutdown and a larger amount reprocessed. At the time of the shutdown, we expect that there was about 8.8 kg of plutonium within the fuel, based on reactor simulations.

A later measurement of the fuel is challenging because the composition, to first order, is only a function of burn-up. A false declaration of power during the time between shutdown and safeguards enforcement could cause the reactor core to have the expected burn-up and composition regardless of whether or not there was a large fraction of the core replaced as seen in Fig. 5.3. A gamma analysis of the spent fuel at known locations could determine how much fuel was replaced. However, in 1994, the DPRK unloaded the spent fuel rapidly, eliminating any knowledge of the fuel positions. The amount of fuel replaced remains uncertain even 20 years later.
Chapter 5. Reactor monitoring

Figure 5.2: A map of relevant boundaries and geographies of the Yongbyon nuclear facility. Contours show expected inverse beta-decay event rates for a 5 tonne detector over the course of a year. X’s mark the location of various neutrino detectors used in the paper. The satellite image on which this map is based was taken on May 16 2013 by GeoEye-1. Figure and caption taken from Ref. [113].

Figure 5.3: Burn-up of the fuel in the 5 MW$_e$ reactor is shown as function of time measured in days since January 1, 1986. The blue curve is based on the values declared by the DPRK, i.e. no major refueling has taken place in 1989. The orange curve is derived assuming that the full core has been replaced with fresh fuel in 1989. Figure adapted from Ref. [117]. Figure and caption taken from Ref. [113].
Figure 5.4: In the left hand panel, 1 \sigma sensitivities to reactor power are shown for varying data collection periods using a 5 t detector at 20 m standoff from the 5 MW_e reactor. Fission fractions are free parameters in the fit. In the right hand panel, 1 \sigma sensitivities to burn-up, where power is a free parameter in the fit. The blue curve shows the history under the assumption of no diversion. The orange curve shows history for the case of a full core discharge in 1989. Figure and caption taken from Ref. [113].

The IRT reactor is of concern because it can run with or without additional natural uranium targets added. HEU fuel does not produce appreciable amounts of plutonium. In order to produce plutonium, \(^{238}\text{U}\) needs to capture neutrons but in HEU, when the density of \(^{235}\text{U}\) is high, most neutrons will induce fissions in \(^{235}\text{U}\) over capturing on \(^{238}\text{U}\). Through the addition of natural uranium targets, the reactor can produce about 0.5 kg of plutonium within the targets per 250 day run. If the targets are added and removed between IAEA visits this could be an additional source for plutonium used in nuclear weapons.

### 5.3.1 5 MW_e reactor

The following analysis of the 5 MW_e reactor was first presented in Ref.[113]. In the analysis, sensitivities to power, burn-up, and plutonium content are determined based on the declared power history. The declared history is displayed as blue curves in the various figures in this section. Comparisons are made to a hypothetical undeclared core swap to a fresh reactor core during the 70 day shutdown period, displayed as orange curves. The difficulty in determining the difference between the two curves lies in the fact that after 1992, power and burn-up are the same. As seen
in Fig. A.1, after the 1st inspection, all the fission rates from the four primary fissioning isotopes are identical with or without diversion. For the following analyses, a standard 5 t detector at 20 m standoff from the reactor is used, which for a data taking period of one year corresponds to about 95,000 events.

The simplest reactor property to measure is its power. For the analysis, we use a statistical $\chi^2$-function that has no additional pull terms in the sum:

$$\chi^2 = \sum_{i} \frac{1}{n^0_i} \left[ \left( N P_{th} \sum_{I} F_I S_{I,i} \right) - n^0_i \right]^2.$$  \hspace{1cm} (5.3)

In this equation, $F_I$ is the fission fraction for isotope $I$, $n^0_i$ is the measured number of neutrino events in energy bin $i$, and $S_{I,i}$ is the neutrino yield in energy bin $i$ for isotope $I$. $P_{th}$ is the thermal power and $N$ is a normalization constant. Moreover, the fission fractions $F_I$ are subject to a normalization constraint as given in Eq. 5.1 but are otherwise free and minimized over.

The resulting 1σ sensitivities are shown in the left hand panel of Fig. 5.4 where the reactor is assumed to follow the declared burn-up. The power curve for the diverted scenario is also shown for comparison. This analysis assumes precise knowledge of factors that effect the normalization of the measured neutrino rate such as the distance from the reactor to the detector. In addition, both the reactor and detector are treated as point sources. This can be corrected once the geometries are known. Any uncertainty in factors that effect the normalization will increase the 1σ sensitivities of $P_{th}$ correspondingly. Neglecting these potential sources of systematic uncertainty, a power accuracy of around 2% can be achieved.

A similar analysis can be done to determine the sensitivities for burn-up, $BU$, using Eq. 5.3. In this circumstance, $P_{th}$ is free in the fit and the fission fractions $F_I$ are now functions of burn-up, determined by a reactor core simulation done using the SCALE software suite. The results of this analysis are shown in the right hand panel of Fig. 5.4. Burn-up across the history of the reactor has a consistent error at the level of $\sim 100$ MWd/t.

Closely related to the burn-up is the amount of plutonium in the nuclear reactor. This analysis is done again using the same $\chi^2$ function found in Eq. 5.3. This time, $P_{th}$, $F_{U235}$, $F_{U238}$, and the relative contribution of the two plutonium fission rates are free parameters. The resulting sensitivities are shown as dashed black lines in Fig. 5.5 for both the number of plutonium fissions in the left panel and the fissile plutonium mass inventory in the right panel. Alternatively, one can use the burn-up sensitivity to constrain the plutonium content as well. This method has an overall
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Figure 5.5: $1\sigma$ sensitivities to plutonium are shown for varying data collection periods using a 5 t detector at 20 m standoff from the 5 MW$_e$ reactor. The blue curve shows the $^{239}$Pu history under the assumption of no diversion. The orange curve shows the $^{239}$Pu history if there had been diversion. Black dashed error bars show the $1\sigma$ sensitivity by measuring the plutonium fission rates with uranium fission rates and reactor power free in the fit. Solid black error bars show the $1\sigma$ sensitivity determined by constraining the burn-up using a reactor model. The left plot shows the errors on absolute plutonium fission rates and the right plot show the corresponding errors for plutonium mass with a shaded exclusion region from the assumption that all neutrons not needed for fission are available for the production of plutonium. Figure and caption taken from Ref. [113].

A stronger constraint on the values that the fission fragments can take and therefore it is expected to reduce the uncertainty in a plutonium measurement over the analysis with free parameters.

Akin to the burn-up analysis, we parameterize the fission fractions as a function of the burn-up and find the maximum burn-up that is allowed at $1\sigma$. We then use a reactor model to compute the difference in plutonium mass inventory between the reactor at the average reactor burn-up and the reactor at a burn-up $1\sigma$ higher, $\text{Pu(BU} + \delta\text{BU}) - \text{Pu(BU)}$. This gives a fissile plutonium sensitivity at $1\sigma$ and is shown as the solid black error bars in Fig. 5.5.

In the right hand panel, a very naive exclusion region is shown for comparison. It assumes that each of the 1.7 neutrons per fission not being used to sustain the chain reaction is instead available to produce more plutonium. This limit is shown as the shaded region in the right hand plot.

For a neutrino detector that starts at the first inspection in 1992, none of the observed quantities mentioned would be able to identify if a diversion took place during the 70 d shutdown as the fission rates of each fissile isotope match for either case. Constraints can be put on the any of those
quantities but with false declarations beforehand; the power, burn-up, and plutonium content could all be made identical between the cases. The one neutrino signature that could be different would be from long-lived isotopes within the reactor.

5.3.2 Long-lived isotope difference

Although, to good approximation, the fissile composition of a reactor is only a function of burn-up, there can be some differences in the byproducts of the fission. Nuclear fissions produce a wide array of isotopes, some of which have decay chains that have members with half-lives on the order of 100s of days and subsequent decays that produce neutrinos above the 1.8 MeV IBD threshold. The three isotopes that are the largest contributors to this group are $^{90}$Sr, $^{106}$Ru, and $^{144}$Ce with half-lives of 28.9 y, 371.8 d, and 284.9 d respectively. We call these long-lived isotopes (LLI).

If we consider the 5 MW$_e$ reactor, a fresh core irradiated since the 70 d shutdown will have produced the same number of LLI as one irradiated since the 1968 start if they both have the same total burn-up. The new core, however, will have had less time for these isotopes to decay away. We expect then that there will be some difference in the neutrino signature because of the difference in the LLI to help identify if the core had been replaced with a fresh core during the 70 d shutdown. The mass of the LLI in both circumstances are shown as a function of time in Fig. 5.6. It can be seen that at the time of the first inspection, indicated by the black vertical line, there are differences on the order of 20% for $^{106}$Ru and $^{144}$Ce and 5% for $^{90}$Sr. The difference becomes much less significant at the 1994 shutdown shown at the right edge of the plots.

We examined detection capabilities of diversion based solely on measuring the difference in the LLI contributions to the neutrino energy spectrum. There are two practical options for measurement periods. One is to measure immediately starting in 1992, when the safeguards entered into force, while the reactor is running and the other is to wait until shutdown and try to measure a difference in the afterglow. In one case, the difference in LLI is larger, but there is a very large reactor background from the ongoing production of short-lived isotopes. In the other, there is no reactor background, but the LLI difference is small.

We find that measuring for a year in 1992 with the reactor on, that the reactor background of 34,000 far exceeds the signal event rate of 69 in the range of 1.8 to 3.6 MeV. After the reactor is shutdown in 1994, a year of data collection would lead to the event rates shown in Fig. 5.7. Here the event
Figure 5.6: The masses of three LLI as a function of time since the Jan 1986 startup of the 5 MW$_e$ reactor. The solid curves show the LLI masses if the reactor follows the declared burn-up. The dashed curves show the LLI masses if there was a full core diversion during the 70 d shutdown. The black vertical line marks the first inspection in 1992.
Figure 5.7: The IBD event rates for one year of data and 20 m standoff are shown only from LLI contributions and no other sources of background. The rates are shown in blue for a core following the declared burn-up and are shown in orange if a core replacement took place during the 70 d shutdown. The left panel is for a 1994 shutdown measurement and the right panel is for a 1992 shutdown measurement.

rates are a factor of a few short of being significant. A longer data collection period would not help much because the source isotopes are decaying away and each subsequent period would yield fewer events. Additionally, the relative amount of background will increase with the signal to background ratio decreasing. An earlier shutdown, in the absence of cosmic backgrounds, could measure the difference in LLI contributions. The event rates are shown if the reactor was shutdown in 1992 also in Fig. 5.7.

From a purely statistical standpoint, the difference in LLI event rates yields a $\Delta \chi^2 = 16$ difference if the reactor was shutdown in 1992 with a year of data collection. If such a measurement took place, it would have to be compared to the expected LLI from a burn-up calculation. Uncertainties in the burn-up would add an additional pull parameter that will weaken the result. In this case, the burn-up would have to be known to within 20%.

If we remove the 1.8 MeV restriction, much more of the LLI spectrum becomes available to detect. To do this, we could use electron scattering or coherent neutrino nucleus scattering. A relative comparison of the cross sections are shown in Fig. 5.8 with all cross sections normalized to one ton of detector. Event rates are already low and for that reason electron scattering will not be a viable option so we instead turn to coherent scattering. The differential coherent scattering cross section
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Figure 5.8: The total cross sections are shown for IBD, electron scattering, and coherent scattering on two different elements as a function of neutrino energy. Each process is normalized to one ton of detector.

\[
\frac{d\sigma}{dT} = \frac{G_F^2}{4\pi} \left[ N - \left(1 - 4\sin^2\theta_W\right) Z \right]^2 M \left(1 - \frac{MT}{2E^2_\nu}\right) F\left(Q^2\right)^2. \tag{5.4}
\]

In this equation, \(\sigma\) is the cross section, \(G_F\) is the Fermi constant, \(N\) is the number of neutrons, \(Z\) is the number of protons, \(M\) is the mass, \(T\) is the recoil energy of the interaction, \(E_\nu\) is the incoming neutrino energy, and \(F(Q^2)\) is a form factor. There are two important facts to gather about this cross section. First, the strength increases with the square of the number of neutrons while the energy endpoint for nucleus recoil is inversely proportional to the mass. With a choice of detector isotope to use, we can go to a large atomic mass and have a much higher cross section than IBD, but we give up on recoil energy. Any threshold we place on the nuclear recoil energy will correspondingly restrict the minimum visible neutrino energy. We expect that a threshold of 0.2 KeV can be achieved using ionization detectors [118]. With this threshold, atomic mass above \(A=32\) will restrict measured neutrino energies to be above the IBD threshold of 1.8 MeV. As can be seen in Fig. 5.8, abundant isotopes of silicon have under an atomic mass of \(A=32\) and still have higher cross section than typical IBD processes, so there is the possibility to improve the detection of a LLI difference using this technology.

With a lower neutrino energy threshold, more LLI become important. To the list, we additionally consider the isotopes \(^{91}Y\), \(^{126}Sn\), \(^{137}Ce\), and \(^{154}Eu\) which have next to none of their neutrino spectrum
interact through IBD. Additional consequences of using coherent scattering exist. First, there is not the same one to one mapping between the measured energy signal and the energy of the incoming neutrino, as there is with IBD. At low energies, there are some defining characteristics of the neutrino spectrum for the fissioning isotopes that could really help distinguish between each. The fact that there is a redistribution of the events, however, will remove some of the prominent features and disperse them over a range of nuclear recoil energies. Second, in addition to more LLI events below 1.8 MeV, there will be many more events from the nuclear reactor. Only about 2 of 6 neutrinos are emitted above the IBD threshold from reactors so probing further will increase these background rates as well. Third, if the only signal is a nuclear recoil, there is not a delayed coincidence, like there is with IBD, to help with the background rejection from non-neutrino sources. Additionally, coherent scattering has not yet been demonstrated to detect neutrinos.

With these caveats, it is still worthwhile to make estimates of the expected rates and statistical capabilities of such a detector. Figure 5.9 shows the findings in which we plot the statistical $\chi^2$ as a function of atomic number. The average atomic mass, for each atomic number, was weighted by abundance. In addition, we plot the coherent scattering event rates for $^{20}$Ne, an isotope near the peak of what we found the statistical capability to be.

Overall, there is no strong improvement in the capabilities to measure a LLI difference when a coherent scattering detector is used over an IBD detector. There are still many uncertainties associated with the detector technology and errors in the normalization can completely negate any difference in LLI signal. At this time, IBD appears to be the better option for this purpose and it still would only be able to detect the LLI difference in select circumstances where the burn-up is well known and the difference in LLI is expected to be high. In addition, external backgrounds will have to be well under control as well. In the case of the DPRK, if neutrino safeguards were put in place starting at the first inspection, there would not be a substantial neutrino signature to distinguish between the declared core and the core after a full diversion during the 70 d shutdown with the same burn-up by 1992.

## 5.3.3 IRT reactor

Like in section 5.3.1 on the 5 MW$_e$ reactor, the analysis for the IRT reactor was first presented in Ref. [113]. The IRT is assumed to run for a 250 day period followed by a 100 day shutdown [117, pp. 148], and the fission rates are shown in Fig. A.2. The natural uranium targets that may be
Figure 5.9: Statistical $\chi^2$ as a function of atomic number are shown on the left and event rates for coherent scattering on one ton of neon over one year are shown on the right. The nuclear recoil energy threshold is assumed to be 0.2 KeV. The event rates are shown in blue if the reactor follows the declared burn-up and are shown in orange if there is a full core diversion during the 70 d shutdown. The first row has the reactor on background with events starting at the first inspection date in 1992. The second row is for the same time period had the reactor been shutdown. The third row is for data collected immediately after the 1994 shutdown.
Figure 5.10: 1σ sensitivities to reactor plutonium fissions are shown for 50 day collection periods using a 5 t detector, 20 m away from the IRT reactor. Black dashed error bars show the 1σ sensitivity resulting from measuring the plutonium fission rate with with uranium contributions and power free in the fit. The solid black error bars show the 1σ sensitivity determined using a burn-up model. The left plot shows driver only results and the right plot shows results for driver and targets combined. Figure and caption taken from Ref. [113].

Added provide much more $^{238}$U, changing the fission fractions substantially and allowing an order of magnitude increase in $^{239}$Pu production and fissions. As with the 5 MW$_e$ reactor, it is assumed that a 5 t neutrino detector is placed 20 m away from this reactor. A measurement of the power can be done using the $\chi^2$ from Eq. 5.3. Splitting the measurement into 50 d bins, a neutrino detector could determine the thermal power to within 0.6 MW in each data taking period. All other things the same, the addition of targets will increase the power output of the reactor. As long as the detector distance and mass were sufficiently well known, the errors would be small enough to clearly notice the power difference caused by the addition of breeding targets. Without much difficulty, operators could adjust the power to remain the same as the expected levels without targets. This would reduce plutonium production by about 25%.

The same techniques applied to the 5 MW$_e$ reactor can be used here to determine the 1σ errors on plutonium content. The constraints can be placed again either through the fission rates directly and using the $\chi^2$ from Eq. 5.3 and then converting these to a plutonium mass using Eq. 5.2 or by determining the errors on the burn-up first using a reactor model and then propagate the errors to the plutonium mass inventory. The 1σ errors are shown in Fig. 5.10 for the reactor both with and without the additional natural uranium targets. This circumstance, in particular, demonstrates how the reactor dynamics, which are quite different with and without the targets, control the capabilities
that a neutrino detector has to measure the plutonium content of the reactor. We found that both setups had similar error bars on raw plutonium fission rates, with and without the targets and as well as to the 5 MW$_e$ reactor results. The sensitivities to the actual mass content of the reactors is very different between all three. In the case with only drivers, a neutrino detector would be sensitive to tens of grams of plutonium. With both the drivers and targets, there is an order of magnitude increase in the errors into the hundreds of grams of plutonium. The 5 MW$_e$ reactor was found to have sensitivities on the order of a few kg. Neutrinos detectors can measure the fission rates and that is why a detector has similar sensitivities to the plutonium fissions for each of the different reactors and setups. The amount of the plutonium that fissions, on the other hand, is determined by the reactor physics. The neutron flux density in the fuel containing the plutonium is very different for the two configurations of the IRT with and without the natural uranium targets and so the reactor can house different amounts of plutonium and still have nearly the same number of plutonium atoms fission per second. The sensitivity to a variety of reactor types as a function of the thermal power is shown in Fig. 5.11.

The plutonium content sensitivity depends on whether or not there are additional natural uranium targets. This means that in the circumstance that we do not know if the targets are in place by other means, the neutrino detector either needs to be able to distinguish between the two cases or the error needs to include the difference in plutonium masses as well as use the larger error of the two. In hopes of being able to identify the existence of targets, we simulate the rates for the reactor with and without the targets assuming that the reactor power was controlled in such a way as to have the same total total fission rate in both cases. The rates are then compared through a simple statistical $\chi^2$ with no pull parameters and the disagreement was not found to be significant enough over the 250 d period to identify the presence of targets. We conclude that the difference in plutonium masses of 0.36 kg should be an additional error over the 250 d. Taking the IAEA estimation for the upper end of the range of plutonium produced in the IRT [117, p. 97] of 4 kg, we see that this requires about 8-10 reactor cycles. Since the errors from a neutrino measurement between each cycle are statistically independent we find the total error from a neutrino measurement taking 8 cycles to be $0.36 \, \text{kg} \sqrt{8} = 1.0 \, \text{kg}$. In the more realistic case of no plutonium production in the IRT this measurement translates into an upper bound of the same size from this source.
Figure 5.11: Absolute accuracy in the determination of the plutonium content based on the measurement of the neutrino spectrum as a function of the thermal power of the reactor. The different lines stand for different types of reactors as indicated by the labels: the first term indicates the type of moderator, whereas the second part denotes the fuel type, natural uranium (NU), low enriched uranium (LEU) and highly enriched uranium (HEU). This figure assumes a 5 t detector, a standoff of 15 m, and 90 days of data taking. The horizontal line labeled “IAEA goal” indicates the accuracy which corresponds to the detection of 8 kg of plutonium at 90% confidence level. Figure and caption taken from Ref. [113].
Figure 5.12: In the left hand panel events are shown for 200 days of data collection 20 m from the shut down IRT reactor and 1.2 km from the running 5 MW_e reactor. The IRT is assumed to only contribute to the detected neutrino spectrum through its long lived isotopes shown in black. The 5 MW_e reactor is assumed to be running either at the declared 8 MW_{th}, as shown in blue, or at 18 MW_{th}, as shown in orange. The right hand panel shows the 1σ sensitivities to reactor power resulting from this measurement. The blue curve shows the power history under the assumption of no diversion. The orange curve shows the power history if there had been diversion. Figure and caption taken from Ref. [113].
5.3.4 5 MW<sub>e</sub> reactor power measurement at IRT

In addition to direct monitoring of an adjacent reactor, there are some alternative capabilities that were examined in Ref. [113]. A neutrino detector at the IRT reactor will measure not only the neutrinos that originated within the IRT reactor, but also those that originated within the 5 MW<sub>e</sub> reactor. This is particularly useful during times when the IRT is shut down, which happens for approximately 100 days every year [117, pp. 148]. This will yield two measurement periods of 100 days each for the reactor power of the 5 MW<sub>e</sub> reactor during the crucial time, after the 70 d shutdown and before the first inspection, where the declared power was low, around 8 MW<sub>th</sub>, but the actual power would have been as high as 18 MW<sub>th</sub>, in order to bring the second core to the same final burn-up; see Fig. 5.3.

For this analysis, data collection is assumed to start shortly after an IRT shutdown at a point where all but the long-lived neutrino producing isotopes have decayed away. We assume that this leaves only the LLI: $^{90}\text{Sr}$, $^{106}\text{Ru}$, and $^{144}\text{Ce}$ to contribute strongly to the measured IBD spectrum; the shorter lived isotopes decay away significantly on the order of days. The number of atoms for each of the LLI was computed using SCALE and is shown in Tab. 5.2. Using the same detector setup as the previous IRT section, we use a 5 t detector at 20 m standoff from the IRT and 1.2 km from the 5 MW<sub>e</sub> reactor, see Fig. 5.2. Data is collected over two 100 day periods and the detected spectrum is shown in the left hand panel of Fig. 5.12. The signal event numbers are small and therefore we use the appropriate Poisson log-likelihood to define the $\chi^2$-function\(^2\)

\[
\chi^2 = 2 \sum_i \left[ n_i \log \frac{n_i}{n_i^0} - (n_i - n_i^0) \right] \quad \text{with} \quad n_i = N P_{th} \sum I S_{I,i} + LLI_i, \quad (5.5)
\]

where $LLI_i$ is the long lived isotope contribution in the bin $i$. Resulting sensitivities are shown in the right hand panel of Fig. 5.12. This corresponds to an uncertainty of about 3.8 MW<sub>th</sub> during the periods of interest. The difference in reactor power for a second core would be detected at 3.2 $\sigma$.

It is important to note that the event rates for this particular circumstance is very low and the

\(^2\)Gaussian $\chi^2$ are approximately correct for counting processes with large statistics. For low event rates, this is no longer a good approximation.
analysis does not include any backgrounds. Very significant background rejection would be needed to keep the signal from being absorbed into the background noise.

This result implies that a larger detector could be used to safeguard several reactors in a larger area. In particular, a detector that is sensitive to direction could identify the reactor that contributed the neutrino and get several power measurements simultaneously. Also, without the need to be close to a reactor, it could be placed underground allowing for greater background reduction [119].

### 5.3.5 Waste detection

In Ref. [113] examine how neutrino detectors can be used for detection of reprocessing nuclear waste. With sufficient insight of where waste might be disposed, a neutrino detector could be placed nearby and can see the signature of LLI, even after years of storage. Table 5.3 lists the number of atoms of each of the three primary LLI that would be expected in the waste at the point in time of the first inspection, roughly 3 years after the 70 d shutdown. In the following analysis, it is assumed that the complete core was removed during the 70 day shutdown and the resulting reprocessing wastes are stored together in one of three locations: the “suspected waste site”, building 500, or the Radiochemical Laboratory [117]. All three locations are shown in Fig. 5.2. For building 500, we assume that we cannot deploy inside the hatched area, since this facility was declared to be a military installation exempt from safeguards access [117, pp. 149]. The resulting standoff distances are shown in Tab. 5.4.

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Sr$^{90}$</th>
<th>Ru$^{106}$</th>
<th>Ce$^{144}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount (atoms)</td>
<td>$1.2 \times 10^{24}$</td>
<td>$1.4 \times 10^{22}$</td>
<td>$3.7 \times 10^{22}$</td>
</tr>
</tbody>
</table>

Table 5.3: Number of long-lived isotopes at day 2251 for a complete reactor core removed at day 1156 and stored for 3 years.

Due to the low event statistics, a Poisson log-likelihood is again used, as in Eq. 5.5, with the difference that the reactor events from the 5 MW$_e$ are now background and the signal are the $LLI_i$. Table 5.4 summarizes the results for each location. Figure 5.13 shows the event rate spectrum in the most promising of the setups considered, the case of the reprocessing plant. It is found that a detector around 25 m from the waste and 1.8 km from the 5 MW$_e$ reactor would have a 2 $\sigma$ signal after 55 days of data collection. The strongest contributor to detection capability is the distance
Chapter 5. Reactor monitoring

Figure 5.13: Total event rates are shown in purple for 1 year of integrated data collection starting in 1992 with a 5 t detector 25 m from spent fuel and 1.83 km from the 5 MW$_e$ reactor. The reactor contribution to total event rates are shown in red and long lived isotope contributions shown in blue.

from the source. Additionally, searching for the waste earlier would be more successful in terms of sensitivity. With half-lives on the order of a year, waiting three years before measuring the neutrino signal gives approximately 1/8th the signal.

Excluding any backgrounds other than that created by the nearby reactors, the background rates can be well constrained. This could be done either using a neutrino detector close to the reactor and getting a very strong power constraint or by using an spectral cut. The LLI do not have any strong neutrino signal above about 3.5 MeV and therefore event rates beyond this energy would strictly be background events originating from the reactors. Knowing the expected reactor neutrino shape, the normalization could be fit and subtracted off. In principle, this technique could be used for any background that has an understood shape with energies that go significantly beyond the signal threshold. As with the previous section, these event rates are very low and inclusion of additional backgrounds will overwhelm the signal.
<table>
<thead>
<tr>
<th>Location</th>
<th>Reactor Distance [m]</th>
<th>Fuel Distance [m]</th>
<th>Reactor Events</th>
<th>Fuel Events</th>
<th>$\chi^2$</th>
<th>$2\sigma$ Time [y]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Building 500</td>
<td>1980</td>
<td>80</td>
<td>10.1</td>
<td>0.9</td>
<td>0.34</td>
<td>$\geq$10</td>
</tr>
<tr>
<td>Suspected Waste Site</td>
<td>1060</td>
<td>25</td>
<td>35.3</td>
<td>8.9</td>
<td>8.22</td>
<td>0.33</td>
</tr>
<tr>
<td>Reprocessing Plant</td>
<td>1830</td>
<td>25</td>
<td>11.8</td>
<td>8.9</td>
<td>16.95</td>
<td>0.15</td>
</tr>
<tr>
<td>Reprocessing Plant</td>
<td>1800</td>
<td>100</td>
<td>12.2</td>
<td>0.6</td>
<td>0.12</td>
<td>$\geq$10</td>
</tr>
</tbody>
</table>

Table 5.4: Events are integrated over 1 year with a 5 t detector. The waste corresponds to a complete reactor core discharged in 1989 during the 70 day shutdown. Long lived isotopes are decayed 3 years before the measurement starts. The expected time to achieve a $2\sigma$ detection is given in the last column. Table and caption taken from Ref. [113].

### 5.3.6 Continuous neutrino observations

Under the circumstances that a neutrino detector could be used over a long period of core history, the events can be binned in time and used to track the overall progression of the core. As seen in Fig. 5.3, in order to match the declared burn-up with a fresh core that started during the 70 d shutdown, there will be periods where the power will have to be substantially different from the declared in order to make up for the extra time the previous core was running. Neutrino detectors are able to measure the reactor power very well as seen in Fig. 5.4. If the detector was present for the entire lifetime of the reactor, then the power history would be well known. Any deviation from the declared power would be identified. Integrating the measured power over the history of the reactor, the total burn-up can be well constrained as well. Together with measuring the power, the burn-up can simultaneously be measured, independently, through identifying the relative proportions of the fission fractions through a reactor model that can predict the fission fractions as a function of burn-up. Neutrino detectors then have two independent methods for tracking the burn-up. Disagreement between the two measurements would indicate that there have been alterations to the core. In the circumstance of the 5 MW$_e$ reactor, had a neutrino detector been present for its lifetime, the burn-up would be well known through an integrated power measurement. If the core was replaced with a fresh core during the 70 d shutdown, the burn-up, as determined through a direct measurement of the fission fractions, would disagree with that predicted by the integrated power measurement.
To determine sensitivity to such a situation, a modified version of Eq. 5.3 is used

$$\chi^2 = \sum_t \sum_i \frac{1}{n_{i,t}} \left[ (1 + \alpha_{\text{detector}}) P_{\text{th}}^t \sum_l F_l (BU^t) S_{l,i} - n_{i,t}^0 \right]^2 + \left( \frac{\alpha_{\text{detector}}}{\sigma_{\text{detector}}} \right)^2.$$  (5.6)

where $t$ is indexing the time interval for which a measurement is available. $\alpha_{\text{detector}}$ is a detector normalization parameter with uncertainty $\sigma_{\text{detector}}$. $P_{\text{th}}^t$ is the average reactor power in each time bin $t$. $F_l$ are the fission fractions which are a function of the burn-up in each time bin $t$, $BU^t$. The burn-up as a function of time is given by

$$BU^t = \left( \sum_{\tau=1}^{t-1} \frac{P_{\text{th}}^\tau \Delta \tau}{M_{\text{core}}} \right) + BU^0.$$  (5.7)

where $\Delta \tau$ is the width of the time bin, $BU^0$ the initial burn-up at the start of data taking and $M_{\text{core}}$ the mass of the reactor core in terms of fuel loading. If this initial burn-up $BU^0$ is well known, as it would be if data collection began at start-up, such an analysis greatly reduces the uncertainty in the total plutonium budget. In Tab. 5.5, the total error budget is given through the use of this method, labeled “method 2”, and is shown compared to the results if only the burn-up but not the power history is measured based on the results of the previous sections, labeled “method 1”. For method 2 we assumed that reactors start with a well known composition, that is $BU^0 = 0$ and a detector related uncertainty $\sigma_{\text{detector}} = 1\%$ is achievable and all the $P_{\text{th}}^t$ are free parameters in the fit. In the case of the 5 MW$_e$ reactor, for both analyses, the question is: What is the maximum change in $BU^x$ during the 70 day shutdown?

In Tab. 5.5, core 1 refers to the reactor core between the initial startup and the 70 d shutdown and core 2 refers to the time from the 70 d shutdown until the 1994 shutdown. In terms of plutonium generation, plutonium can be extracted during the 70 d shutdown or after the 1994 shutdown. The net plutonium uncertainty is then the sum of plutonium extracted from the core during the 70 d shutdown and the excess plutonium generated beyond that predicted in the final unloading. The parenthesis in the table list the amount of excess plutonium that can be generated in either core while only using data from that respective time period. The number beside it, not in parenthesis for Core 1, is the amount of plutonium that can be removed from the core before there is a detection at 1$\sigma$ while using data for the entire reactor lifetime. For core 2, this number is the maximum excess plutonium generated at the 1994 shutdown. Figure 5.14 displays the burn-up curves that represent the maximum removal of plutonium during the 70 d shutdown that would not be detected when using data over the whole reactor history. It can be seen from the figure that removal during the 70 d shutdown will also decrease the available plutonium at the 1994 shutdown causing an
Figure 5.14: The plots show the burn-up curve that allows for the maximum plutonium removal during the 70 d shutdown in orange. The blue declared burn-up curve is shown for comparison. The left-hand panel uses method 1 while the right-hand panel uses method 2.

After the 70 d shutdown, the core is assumed to be a weighted sum with a factor that can be adjusted between 0 and 1 that controls the amount of the core that is fresh and the amount that is unchanged. At value 1, the core is 100% fresh and at value 0, the core is 100% unchanged. The dynamics of both core types is assumed to behave the way it would in a reactor comprised entirely of the either a fresh or unchanged core. The value of $BU_x$ is translated into the resulting plutonium mass sensitivity by using the reactor model. The conversion process here converts the amount of burn-up that both types portions of the cores receive into the plutonium content individually and is summed. This amount of plutonium is slightly different than if the core is assumed to be homogeneous and completely described by the average burn-up of the two core sections.

It is clear that method 1 is less accurate but does not rely on continuity of knowledge whereas method 2 is much more accurate but requires continuity of knowledge. Method 2 still offers a significant advantage compared to conventional methods by providing its results in a timely fashion and not only at some later, unspecified time in the future.

For completeness we also list the plutonium mass sensitivities from the indirect method and the detection of reprocessing wastes in Tab. 5.6. Additionally, the burn-up curve is shown for the maximum amount of core that could be replaced while remaining within 1 $\sigma$ power deviation for an parasitic IRT measurement of the 5 MW$_e$ reactor.
<table>
<thead>
<tr>
<th>Reactor</th>
<th>Final Burn-up [MWd/t]</th>
<th>Final Pu [kg]</th>
<th>Method 1, 1σ Burn-up [MWd/t]</th>
<th>Method 1, 1σ Pu [kg]</th>
<th>Method 2, 1σ Burn-up [MWd/t]</th>
<th>Method 2, 1σ Pu [kg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>IRT/run</td>
<td>With targets</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 MW_e from 1st inspection</td>
<td>Core 1</td>
<td>178</td>
<td>8.83</td>
<td>178</td>
<td>9.5*</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>Core 2</td>
<td>648</td>
<td>27.7</td>
<td>95</td>
<td>3.29</td>
<td></td>
</tr>
<tr>
<td>5 MW_e from start-up</td>
<td>Core 1</td>
<td>178</td>
<td>8.83</td>
<td>138 (83)</td>
<td>6.68 (3.76†)</td>
<td>43 (1.9)</td>
</tr>
<tr>
<td></td>
<td>Core 2</td>
<td>648</td>
<td>27.7</td>
<td>52 (66)</td>
<td>1.81 (2.30†)</td>
<td>6.7 (6.9)</td>
</tr>
<tr>
<td>5 MW_e Core 3</td>
<td></td>
<td>307</td>
<td>14.6</td>
<td>51</td>
<td>2.17</td>
<td>3.2</td>
</tr>
<tr>
<td>5 MW_e Core 4</td>
<td></td>
<td>255</td>
<td>12.3</td>
<td>53</td>
<td>2.36</td>
<td>2.7</td>
</tr>
</tbody>
</table>

Table 5.5: Pu content and 1σ uncertainties are given for two analysis techniques for both the IRT and 5 MW_e reactors. Due to the inability to reliably detect the presence of targets in the IRT reactor, they are assumed to be in the reactor. The detection capability is given for each 250 day run of the IRT. The 5 MW_e reactor Pu error is a combination of removed Pu that may have occurred during the 70 day shutdown and the final Pu content in the reactor at the 1994 shutdown. The quantities are independent if data is only taken after the 1st inspection and correlated if taken from start-up. The flat burn-up analysis adds a fixed burn-up to each time bin and the final Pu error is the final Pu difference between the burn-up increased data and the expected data. The power constrained analysis assumes the starting fuel composition is known and the burn-up is given by the integration of the power with an assumed 1% detector normalization uncertainty. The Pu error is the maximum Pu difference attainable through power increases and fuel removal (in the case of the 5 MW_e reactor). Values are given for 1σ sensitivities for maximizing the Pu available for Core 1 or Core 2 respectively. Parenthesis are for uncertainties in cores using only data from the respective section. Core 3 and core 4 are additional fuel loads that are irradiated in the 5 MW_e reactor post-1994 [120] and are added for completeness. Table and caption taken from Ref. [113].
Table 5.6: 1\(\sigma\) uncertainties on the discharged plutonium for core 1 for the IRT parasitic measurement and for the detection of high-level reprocessing waste.

<table>
<thead>
<tr>
<th>Parasitic measurement</th>
<th>Core 1 burn-up [MWd/t]</th>
<th>Core 1 Pu [kg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parasitic measurement</td>
<td>51</td>
<td>2.55</td>
</tr>
<tr>
<td>Suspected Waste (25m)</td>
<td>56</td>
<td>2.76</td>
</tr>
<tr>
<td>Reprocessing Plant (25m)</td>
<td>34</td>
<td>1.67</td>
</tr>
</tbody>
</table>

Figure 5.15: The burn-up curve that allows for the maximum plutonium removal during the 70 d shutdown through a parasitic measurement of the 5 MW\(_e\) reactor from a neutrino detector at the IRT reactor is shown in orange. The declared burn-up is shown in blue for comparison.
5.4 Iran

The IR-40 reactor in Iran appears to be an ideal future candidate for neutrino reactor monitoring. The reactor is designed to run at 40 MW$_{th}$ with a heavy water (D$_2$O) moderator. A heavy water moderator, over light water, has a lower cross section for neutron capture allowing the reactor to run on natural uranium. The production of plutonium will then be higher in such a reactor and is of particular concern regarding the production of nuclear weapons. Reference [114] goes into the details for measuring the plutonium content analogous to what has been done in the previous section for the reactors in the DPRK.

Additionally, because the reactor design is a primary concern, there has been a suggestion to use low-enriched uranium (LEU) instead of natural uranium as the fuel. This would allow for a lower amount of fuel to be used and decreases the probability that $^{238}$U captures a neutron over $^{235}$U fissions. Overall this would decrease the amount of plutonium produced by the reactor. Reference [114] additionally looks at the capability to identify which core configuration is in use through a method of tracking the rate at which plutonium is produced. The last topic covered in that paper is the ability to detect if the reactor fuel is removed from the site. This section goes into further detail of this particular measurement.

After a reactor is shutdown, neutrinos continue to be emitted from the spent fuel. Within days, the majority of the short-lived isotopes will decay away leaving only the long-lived isotopes (LLI) as significant contributors to the measured neutrino events. The expected signal after a shutdown from the Iranian IR-40 nuclear core is examined in this chapter. The only isotopes considered to contribute to the measured neutrino spectrum are $^{90}$Sr, $^{106}$Ru, and $^{144}$Ce. Using SCALE to simulate the IR-40 reactor, the produced mass of each of these LLI can be predicted as a function of burn-up with the dependence shown in figure 5.16. Using these masses, the number of decays for each LLI that have occurred during a measurement can be determined though equation 5.8.

$$\Delta N_j = N_{0j}(BU) \left[1 - e^{-\frac{t}{\tau_j}}\right] e^{-\frac{t_0}{\tau_j}}$$ (5.8)

$N_{0j}(BU)$ is the number of atoms, at shutdown, for isotope j in the fuel if the reactor is shut down with burn-up, BU; $\tau_j$ is the lifetime of isotope j; and t is the measurement duration. For each LLI decay, there will be a number of neutrinos emitted with energies characteristic of the LLI and their subsequent fast daughter decays. The neutrino energy spectrum from each LLI and its daughter
Figure 5.16: Mass of the long-lived isotopes in Iranian IR-40 as a function of reactor burn-up while assuming constant 40MWth power and initial fuel load of 8.6 t uranium.

Products is shown in figure 5.17. These spectra can be used to find the expected inverse beta decay (IBD) event rates using equation 5.9.

\[
R_i = \frac{M}{4\pi r^2} \sum_{j=1}^{3} \Delta N_j \int_{E_i - \Delta E/2}^{E_i + \Delta E/2} S_j(E)\sigma(E)dE
\]  

(5.9)

In this equation, \(R_i\) is the total number of expected neutrino events in energy bin \(i\), with bin width \(\Delta E\), over a measurement time \(t\) and assuming the reactor shut down with burn-up \(BU\) and \(t_0\) time has passed before the measurement began. \(M\) is the number of target protons, which corresponds to \(4.3 \times 29\) protons in the case of a 5 tonne organic scintillator; \(r\) is the standoff distance, which is taken to be 17.5 m based on physical constraints of the reactor site; \(S_j(E)\) is the neutrino spectrum for isotope \(j\); and \(\sigma(E)\) is the IBD cross section. Event rates are shown in figure 5.18 for \(R_n\) integrated over energy with a 30 d data collection period. \(R_n\) represents our signal events. In order to simulate the entire neutrino spectrum expected, background events need to be added.

With a surface deployment, which is necessary for this application, muon related events are expected to be the largest contributor to the background. The two muon related event types of primary concern are fast neutrons and beta-delayed neutron events, both of which can mimic IBD. Fast neutrons can enter the detector and scatter off a proton, producing a signal similar to that expected from
a positron. The produced neutron thermalizes and eventually captures for the delayed coincidence signal. The energy spectrum is assumed to be flat for fast neutrons. Beta-delayed neutrons are when a cosmic ray produces a short-lived radioactive isotope that decays within the detector. In this analysis, every radioactive isotope produced in this way is assumed to be $^9$Li. The beta electron can be mistaken for a positron and the neutron later captures. Using Ref. [121], there are expected to be $1 \text{d}^{-1} \text{t}^{-1}$ fast neutron events and $43 \text{d}^{-1} \text{t}^{-1}$ beta-delayed neutron events in a surface detector. The background rate is found to be significantly higher than the signal rate.

There are a variety of inquiries that can be made about the fuel through a neutrino measurement. One such question is to ask how long it will take to detect the removal of spent fuel. For this sensitivity computation, equation 5.10 can be used if the measured events are split into 30 day time bins with the remainder in the final bin, between 0 and 30 days of data.

$$
\chi^2 = \sum_{m=0}^{t_{\text{bins}}} \sum_{n=0}^{n_{\text{bins}}} \frac{(R_{m,n} + B_{m,n} - B_{m,n})^2}{B_{m,n}}
$$

(5.10)

It is assumed that the spent fuel is removed from the detector and the only contribution to the measured data is from background events, $B_{m,n}$ while $B_{m,n} + R_{m,n}$ are expected. The additional m label is used to differentiate the event rates in each time bin. The background events, $B_{m,n}$, are simply given by multiplying the the daily background spectra by the time bin duration while $R_{m,n}$

Figure 5.17: Emitted neutrino spectra per LLI decay.
accounts for the exponential decay of the LLI. $t_{\text{bins}}$ is the number of time bins and $n_{\text{bins}}$ is the number of energy bins. Sample event rates for one time bin are shown in figure 5.19.

The fact that the LLI only produce neutrinos below 4 MeV means that events above 4 MeV can be used for better background control and estimation. Detecting the sensitivity to removal of spent fuel results from finding the measurement duration that corresponds to the desired $\chi^2$ for a given time the waste has been stationary before removal. Figure 5.20 shows this calculated time for detection at 90% CL for the removal of fuel from a reactor with runtime of 270d, both with and without reducing the background by a factor of 2. It was found that if the fuel was removed days after shutdown, a neutrino detector would be able to detect if the spent fuel was missing within 10’s of days at 90% confidence level. With a factor of 2 background suppression, waste removal could be detected within 90 days even if it was removed as late as a year after shutdown.

5.5 Conclusion

The capabilities of neutrino detectors have been explored for the historical scenario of the DPRK leading up to the 1994 crisis in which the outcome using traditional methods was less than desirable. Neutrino detectors were found to be capable of quantitatively measuring bulk properties of the
Figure 5.19: Sample event rates for a 30 day time bin, shortly after a 1000 day runtime shutdown. Statistical error bars are also shown.

Figure 5.20: Calculated time for the 90 % C.L. detection of spent fuel removal after 270 days of runtime.
reactor such as as power, burn-up, and plutonium content. Research reactors with thermal power below 100 MW\textsubscript{th} are ideal candidates for such neutrino detectors as the reactor dynamics and rates allow for sufficiently precise measurements of the plutonium content. These measurements are done in real time without the need to shutdown the reactor to acquire data. This property is particularly important in circumstances with intermittent access in which continuity of knowledge is hard to maintain. With access for the entire lifetime of the reactor, the neutrino detectors can simultaneously measure the burn-up through the power and fission fractions independently; disagreement between the two measurements would indicate a diversion. The same techniques have been applied to the IR-40 reactor in Iran and a neutrino detector was found to be a very capable safeguard. In all cases, a neutrino detector is a strong additional constraint to reactor foul-play especially if used in conjunction with existing technologies.
Chapter 6

Tau backgrounds

6.1 Neutrino factory

The accelerator based experiments that were discussed in Ch. 2 use meson decays as a neutrino source. The neutrinos in these experiments are nearly all muon flavored and these experiments are designed to limit the electron neutrinos. The contamination of other neutrino flavors can complicate oscillation analysis and the precision to which the $\nu_e$ and $\nu_\mu$ fluxes can be determined is important source of systematic uncertainty [122]. These issues can be circumvented by using muon decay from storage rings in experiments known as neutrino factories. The storage ring is designed to circulate muons at controlled energies and is shaped as to have long straight sections to promote a direction for the neutrinos. For negatively charged muons, the decay process is

$$\mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e.$$ (6.1)

With the fact that there are both a neutrino and anti-neutrino emitted, there are several oscillation channels that can be examined: $\nu_\mu \rightarrow \nu_\mu$ disappearance, $\nu_\mu \rightarrow \nu_e$ platinum channel, $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$ golden channel, and $\bar{\nu}_e$ disappearance. In addition, by swapping the charge of the muon, all the CP conjugate channels can also be observed. With a magnetic detector, the sign of the charged lepton produced through the charged current neutrino interaction can be distinguished and the oscillation channel determined. As an added benefit, the expected neutrino spectrum from muon decay is well understood. This allows for a calculation of the absolute neutrino flux since the stored muon current, momentum, and polarization are measured.
6.2 Tau misidentification

At the detector site, there is a non-zero probability that the arriving neutrinos from the muon ring will have oscillated into tau neutrinos. The tau neutrino can interact with the detector and assuming that the neutrino has sufficient energy, it can produce a charged tau lepton through the charged current interaction. The mass of the tau lepton is 1.776 GeV and thus has a threshold for interaction. It also has a short mean lifetime of $290 \times 10^{-15}$ s. Without the original intention of observing tau leptons directly, it can be very challenging to identify their presence and requires a great deal of spatial resolution. If the tau is not identified and the decay products are observed, then the decay products may act as a background to the process that is desired to be observed.

In the case of neutrino factories, the signal is the observation of a muon or electron that is presumably caused by the interaction of a muon neutrino or electron neutrino. Unfortunately, a little above 34% of the time, the charged tau will decay into either a muon or an electron. These will act as a background to oscillation experiments and will change the perceived oscillation probabilities. We either then need to be able to find a way to selectively remove the background while not removing much of the signal, possibly through momentum and energy cuts or pattern identification, or predict expected event rates and shape well enough that the background can be fit with minor impact to the desired physics. In either case the properties of the charged leptons decay products needs to be examined. Studies have been done to examine the size of the impact to various neutrino oscillation channels [123, 124, 125].

For Ref. [126], we needed to take into account the tau contamination. To this end, we used the GENIE [127] neutrino Monte Carlo generator to create a migration matrix that was used to convert a binned tau neutrino spectrum into a reconstructed muon or electron neutrino spectrum. We first generated differential cross sections for interactions between neutrinos and argon and iron nuclei. This was done using the shell command: `gmkspl -p 12,-12,14,-14,16,-16 -t 1000180400 -o xnuAr40.xml`, for $^{40}$Ar and with the specified output file "xnuAr40.xml". Then 10,000 events were simulated for tau neutrinos at each energy bin step with energies ranging between 2 GeV and 10 GeV with 0.125 GeV increments. To do this, environmental variables were set such that GEVGL=CC and GSPLOAD=xnuAr40.xml or other appropriate cross section files. The command: `gevgen -s -n 10000 -p 16 -t 1000180400 -e E\_\tau produces 10,000 events between an tau neutrino particle (-p 16) with energy $E\_\tau$ and an argon target (-t 1000180400). The events were written into a root file, in.root, using: `gntpc -i gntp.0.ghep.root -f gst -n 10000 -o in.root`. Subsequently, the root file was
Events that lead to the eventual production of an electron or muon were identified. We then subtracted the invisible energy carried off by the neutrinos, produced in the decay, from the original tau neutrino energy to determine the reconstructed muon or electron neutrino energy. Figure 6.1 shows a sample reconstructed $\nu_\mu$ energy spectrum from a mono-energetic tau neutrino source. At this step, the missing transverse momentum could be identified and binned but was not used in the analysis done in Ref. [126]. Reference [123] looked at angular cuts in detail and found that, "Any cuts that attempt to do so drastically reduce the direct muon events as well and hence worsen the sensitivity to the oscillation parameters."

The event rates were binned into probabilities by counting the number of occurrences that a tau neutrino with energy $E_\tau$ decays into a specific charged lepton with energy $E_l$ and divided that number by the total number of interactions that occurred with a tau with energy $E_\tau$; in this case, that number is 10,000. Each of these probabilities was placed as an entry in a migration matrix, $M_{i,j}^l$, for neutrino flavor $l$. This matrix need not be square and the dimensionality is dictated by the number of energy bins in the measured charged lepton spectrum and the tau neutrino spectrum. The background reconstructed neutrino spectrum for neutrino flavor $l$, expected from tau decays,
\[ S_i^l = \sum_j M_{i,j}^l \sigma(E_j) S_j^\tau \]  

where \( \sigma(E_j) \) is the tau neutrino charged current cross section at the energy, \( E_j \), and \( S_j^\tau \) is the binned tau neutrino energy spectrum. This converts the number of tau neutrinos expected through an oscillation calculation into the background measured neutrino spectrum. The matrices and cross sections are given in App. B.
Chapter 7

Conclusion

Over the last several years, there has been a strong focus on reactor experiments and through these experiments the mixing angle $\theta_{13}$ was found to be non-zero. Reactor experiments were particularly instrumental in this search because they have a simple dependence on $\theta_{13}$ without extra interference from still unknown parameters like CP violation and sign of the atmospheric mass splitting.

In light of two recent recalculations of the reactor neutrino flux model, we simulated reactor experiments and were able to show that the choice of flux model could cause a disagreement with data in a way that could be misunderstood as a neutrino oscillation. This is especially true when sterile neutrino oscillations are considered which allow for an effective free-normalization parameter. We found that experiments with only one neutrino detector need to be extra cautious without the capability to normalize the flux with another detector.

To help alleviate the flux uncertainty we can try to reduce the errors associated with the flux models. One way to accomplish this is to constrain the weak magnetism error, the largest theory error associated with these models. This error is particularly large because it is not well understood for forbidden decays and the models assume that weak magnetism correction is the same for these as it is for the allowed decays.

When considering forbidden decays, it is possible to have very large weak magnetism correction. One way of constraining the size of the correction would be to take a neutrino measurement directly and perform a fit. We looked at data from four experiments, Daya Bay, Double Chooz, RENO, and Bugey-3. We found that if we do not account for detector related systematics in the energy spectrum, then each experiment has a very different best fit for the weak magnetism value even
though they should all produce the same value under ideal circumstances. The Daya Bay experiment provided a detector response correction and when used, the best fit value shifted by $2\% \text{ MeV}^{-1}$. In addition, recent spectral features complicate this measurement further.

With a better understanding of neutrinos and the spectrum from nuclear reactors, the role of a neutrino detector could be reversed to monitor the properties of a nearby reactor. Using a rate and spectrum analysis, the total neutrino spectrum can be decomposed into contributions from each of the primary fissioning isotopes and the reactor power can be determined. We examined the capabilities of a 5 tonne neutrino detector within 20 meters of a neutrino reactor in the DPRK prior to the 1994 crisis and we found that we could measure the reactor power to within a few percent. As well, depending on the reactor type, the plutonium content could also be measured to well below one significant quantity (8 Kg) in one year.

The DPRK provided an interesting scenario for examination due to a 70 day shutdown in 1989 where it is possible that the entire reactor core was replaced with a fresh core. Even today, the details of this shutdown are unclear. Had there been a neutrino detector in such a scenario, it would have the capability to detect a diversion either through a deviation in power or a deviation in fissile composition.

In addition to monitoring the reactors directly, we examined the a neutrino detectors capability for indirect measurements. We looked at the ability to detect hidden reprocessing waste. In which case, we would be measuring the neutrinos from the long lived isotopes $^{90}\text{Sr}$, $^{106}\text{Ru}$, and $^{134}\text{Ce}$. If there were no other background and if close enough, we could achieve statistical significance. Unfortunately, it would be drowned out by the cosmic backgrounds with current detector technologies. We could also parasitically measure other nearby reactors. During shutdowns when the neutrino production from the nearby detector is low, other nearby reactors will contribute strongly to the measured neutrino spectrum. Like the reprocessed waste detection, the events are low for this. Additionally, we looked at how long it would take to detect if the on-site waste was removed.

Overall neutrino detectors are a strong addition to the reactor monitoring tools. This is particularly true for reactors that are of concern for proliferation such as low power (10s of MW) graphite and heavy water moderated reactors. As well, neutrino detectors can be extremely useful for situations where continuity of knowledge is an issue. With the ability to measure neutrinos for intermittent periods, such a detector can access information that would otherwise be inaccessible to traditional methods.
Appendix A

DPRK rates

The following two figures display the fission rates for the two nuclear reactors we examined in the DPRK in chapter 5. It can be easily seen in Fig. 5.3 that the fission rates are identical for the 5 MW\textsubscript{e} reactor after the 1st inspection regardless of diversion. Additionally, the substantial increase to the IRT plutonium fission rate is clearly shown in Fig. A.2.
Figure A.1: The fission rates of the four primary fissioning isotopes in the 5 MW$_e$ reactor are shown as a function of time measured in days since January 1, 1986. The solid lines use the declared power history while the dashed lines correspond to the evolutionary history of a completely new core starting after the 70 d shutdown. The solid and dashed distinction correspond to the two burn-up curves in Fig. 5.3.

Figure A.2: The fission rates of the four primary fissioning isotopes in the IRT are shown as a function of the reactor runtime. In the left panel, the rates are shown assuming an 80% $^{235}$U fuel enrichment without any natural uranium targets. The right panel shows the rates with the natural uranium targets added.
Appendix B

Tau contamination migration matrices and cross sections

In Ref. [126], the binning: 2.0,0.5,0.5,0.5,0.5,0.5,0.5,1.0,1.0,1.0,1.0 GeV was used for argon and iron migration matrices. The following matrices are used to convert a tau neutrino spectrum into a reconstructed muon or electron neutrino spectrum following Eq. 6.2. Additionally, a multiplicative factor of 0.17 needs to be added to account for the branching fraction in tau decays for both muon and electron neutrinos. Tau neutrino cross sections are listed at the end.

Muon reconstruction in argon:

\[
\begin{pmatrix}
0. & 0. & 0. & 0.796 & 0.727 & 0.621 & 0.53 & 0.448 & 0.401 & 0.257 & 0.201 & 0.149 & 0.11 \\
0. & 0. & 0. & 0.139 & 0.146 & 0.161 & 0.15 & 0.145 & 0.129 & 0.121 & 0.097 & 0.075 & 0.059 \\
0. & 0. & 0. & 0.06 & 0.095 & 0.117 & 0.131 & 0.135 & 0.124 & 0.129 & 0.102 & 0.085 & 0.063 \\
0. & 0. & 0. & 0.006 & 0.03 & 0.074 & 0.101 & 0.11 & 0.114 & 0.12 & 0.101 & 0.089 & 0.07 \\
0. & 0. & 0. & 0. & 0.002 & 0.025 & 0.065 & 0.089 & 0.097 & 0.111 & 0.102 & 0.088 & 0.079 \\
0. & 0. & 0. & 0. & 0. & 0.002 & 0.022 & 0.053 & 0.074 & 0.099 & 0.102 & 0.091 & 0.081 \\
0. & 0. & 0. & 0. & 0. & 0.001 & 0.018 & 0.044 & 0.079 & 0.085 & 0.093 & 0.08 \\
0. & 0. & 0. & 0. & 0. & 0. & 0.002 & 0.017 & 0.05 & 0.079 & 0.082 & 0.085 \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.002 & 0.025 & 0.063 & 0.076 & 0.078 \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.009 & 0.06 & 0.117 & 0.143 \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.007 & 0.049 & 0.098 \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.006 & 0.046 \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.006
\end{pmatrix}
\]
Electron reconstruction in argon:

\[
\begin{bmatrix}
0. & 0. & 0. & 0.788 & 0.723 & 0.633 & 0.533 & 0.464 & 0.407 & 0.269 & 0.205 & 0.156 & 0.12 \\
0. & 0. & 0. & 0.156 & 0.155 & 0.147 & 0.147 & 0.136 & 0.126 & 0.124 & 0.102 & 0.084 & 0.057 \\
0. & 0. & 0. & 0.05 & 0.096 & 0.125 & 0.137 & 0.131 & 0.12 & 0.122 & 0.104 & 0.085 & 0.071 \\
0. & 0. & 0. & 0.006 & 0.025 & 0.072 & 0.103 & 0.116 & 0.116 & 0.119 & 0.099 & 0.084 & 0.072 \\
0. & 0. & 0. & 0. & 0.002 & 0.022 & 0.062 & 0.089 & 0.101 & 0.114 & 0.103 & 0.085 & 0.075 \\
0. & 0. & 0. & 0. & 0. & 0.001 & 0.018 & 0.048 & 0.075 & 0.097 & 0.097 & 0.082 & 0.076 \\
0. & 0. & 0. & 0. & 0. & 0.001 & 0.015 & 0.041 & 0.076 & 0.094 & 0.091 & 0.076 \\
0. & 0. & 0. & 0. & 0. & 0. & 0.001 & 0.013 & 0.049 & 0.078 & 0.086 & 0.081 \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.002 & 0.023 & 0.055 & 0.08 & 0.08 \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.007 & 0.057 & 0.113 & 0.141 \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.007 & 0.048 & 0.099 \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.006 & 0.045 \\
\end{bmatrix}
\]

Muon reconstruction in iron:

\[
\begin{bmatrix}
0. & 0. & 0. & 0.789 & 0.73 & 0.625 & 0.53 & 0.453 & 0.397 & 0.254 & 0.201 & 0.151 & 0.111 \\
0. & 0. & 0. & 0.143 & 0.144 & 0.153 & 0.149 & 0.146 & 0.129 & 0.127 & 0.092 & 0.075 & 0.06 \\
0. & 0. & 0. & 0.064 & 0.096 & 0.12 & 0.131 & 0.129 & 0.124 & 0.128 & 0.103 & 0.083 & 0.064 \\
0. & 0. & 0. & 0.004 & 0.029 & 0.076 & 0.104 & 0.111 & 0.116 & 0.12 & 0.102 & 0.089 & 0.072 \\
0. & 0. & 0. & 0. & 0.002 & 0.024 & 0.063 & 0.088 & 0.097 & 0.113 & 0.106 & 0.092 & 0.079 \\
0. & 0. & 0. & 0. & 0.002 & 0.022 & 0.052 & 0.075 & 0.094 & 0.099 & 0.091 & 0.079 \\
0. & 0. & 0. & 0. & 0. & 0.002 & 0.018 & 0.044 & 0.079 & 0.092 & 0.086 & 0.081 \\
0. & 0. & 0. & 0. & 0. & 0. & 0.002 & 0.017 & 0.052 & 0.077 & 0.086 & 0.079 \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.002 & 0.025 & 0.063 & 0.076 & 0.079 \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.009 & 0.058 & 0.116 & 0.145 \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.006 & 0.048 & 0.102 \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.007 & 0.044 \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.006 \\
\end{bmatrix}
\]
Electron reconstruction in iron:

\[
\begin{pmatrix}
0.0 & 0.794 & 0.726 & 0.626 & 0.538 & 0.463 & 0.409 & 0.27 & 0.209 & 0.157 & 0.117 \\
0.0 & 0.0 & 0.148 & 0.154 & 0.152 & 0.144 & 0.136 & 0.124 & 0.123 & 0.096 & 0.077 & 0.063 \\
0.0 & 0.0 & 0.054 & 0.094 & 0.127 & 0.136 & 0.13 & 0.12 & 0.123 & 0.104 & 0.084 & 0.068 \\
0.0 & 0.0 & 0.005 & 0.025 & 0.073 & 0.103 & 0.116 & 0.117 & 0.115 & 0.1 & 0.09 & 0.071 \\
0.0 & 0.0 & 0.0 & 0.001 & 0.021 & 0.06 & 0.092 & 0.102 & 0.114 & 0.099 & 0.089 & 0.075 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.001 & 0.018 & 0.047 & 0.075 & 0.099 & 0.099 & 0.085 & 0.075 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.001 & 0.015 & 0.039 & 0.078 & 0.094 & 0.087 & 0.082 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.001 & 0.013 & 0.047 & 0.08 & 0.087 & 0.082 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.001 & 0.023 & 0.057 & 0.08 & 0.081 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.008 & 0.055 & 0.11 & 0.143 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.006 & 0.049 & 0.098 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.005 & 0.04 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.005
\end{pmatrix}
\]

Cross sections:

\[
10^{-38}\text{cm}^2
\begin{pmatrix}
40\text{Ar }\bar{\nu}_\tau & 40\text{Ar }\nu_\tau & ^{56}\text{Fe }\bar{\nu}_\tau & ^{56}\text{Fe }\nu_\tau \\
0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 \\
0.019751 & 0.089867 & 0.018516 & 0.087628 \\
0.059839 & 0.192761 & 0.058497 & 0.189357 \\
0.121939 & 0.295572 & 0.126104 & 0.291814 \\
0.198021 & 0.426289 & 0.198996 & 0.421425 \\
0.277844 & 0.563819 & 0.281078 & 0.557327 \\
0.440762 & 0.855117 & 0.445093 & 0.845066 \\
0.602016 & 1.15477 & 0.607987 & 1.14102 \\
0.767548 & 1.47846 & 0.775191 & 1.46077 \\
0.938613 & 1.82557 & 0.948205 & 1.80373
\end{pmatrix}
\]
Bibliography


[47] Search for $\nu_\mu \rightarrow \nu_\tau$ oscillation with the OPERA experiment in the CNGS beam. 2011.


[60] D.S. Ayres et al. NOvA: Proposal to build a 30 kiloton off-axis detector to study \( \nu(\mu) \rightarrow \nu(e) \) oscillations in the NuMI beamline. 2004.


[99] P. Huber. private communication.


