# THE USE OF FLYING-SQUADS IN THE OPERATION OF MUNICIPAL FIRE SUPPRESSION ACTIVITIES

by

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It is my privilege to acknowledge the encouragement and guidance received during the course of this research from my advisor, He has made a significant contribution to the "learning experience" in my graduate education. I am also indebted to and for their critique of the research and their helpful comments.

I dedicate this work to my parents, and for their many sacrifices during my learning period.

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# CHAPTER I

# INTRODUCTION

Fire protection is one of the primary community services provided by most municipal governments. In larger cities, fire protection is provided by a full-time, professional fire department.

With limited budgets and increasing manpower costs, fire service management is faced with problems of developing innovative concepts for resource utilization. There are some indications that the fire service function is being appropriated an increasingly larger portion of the overall city budgets [53]. Also, the largest single cost in the operation of a municipal fire department is manpower. On the average, over 95% of the fire department budget is allocated to manpower [46]. This study investigates an alternative method of manning a fire company, which is the basic fire fighting unit and is usually manned by 4 to 6 fire fighters, in order to reduce the total manning requirements of a fire department.

## Problem Description

The research presented in this thesis is based on the concept of a flying squad as a manpower transport vehicle. As presently conceptualized, a flying squad is to be dispatched to the origin of an alarm where it will join the engine and ladder companies to form a complete response force. Each company will be manned with less than the usual

4 to 6 fire fighters, and the additional manpower will be supplied by a flying squad.

The decision problem involves determining whether it would be advantageous to introduce a flying squad into municipal fire departments, and if it is used, to determine (a) the number of flying squads required, (b) the locations/allocations of flying squads, (c) the size of each squad and the job skill levels of each member of a flying squad, and (d) the deployment strategy of a flying squad (i.e., fixed base versus patrolling base).

The objective function minimizes an estimate of a cost to society which includes the estimated fire losses and the cost of operating flying squads. The constraints that will be placed on the use of a flying squad will include (a) the allowable response time, (b) the potential demand for fire suppression services, (c) the workload of a flying squad, and (d) the number of flying squads to have.

## Purpose of Study

The purpose of this research is to define and structure the planning problems encountered in the use of a flying squad in municipal fire suppression operations. In addition, the formulation of a mathematical model of these problems and the development of a solution procedure are also part of the goals of this research. The research is intended to provide decision-makers, such as fire chiefs and city managers, with an aid to use in determining the applicability of a flying squad in their municipal fire departments.

## Scope of Research

All of the four planning problems mentioned earlier will be discussed in this research. In the problems of determining the location/allocation and the deployment strategy of a flying squad, only a fixed base situation will be considered.

The problems as it actually exists in the Roanoke Virginia Fire Department is used as a case study to apply the solution procedure developed for this research. The actual data was collected from the Roanoke Fire Department and the Roanoke City Planning Department.

## Solution Approach

In developing a solution procedure for the problem under consideration, the first step was to clearly define the problem. The second step was to take the problem definition and develop a mathematical model of it. In formulating the model, the opinions of professionals in the field as gathered through a written survey were also utilized to develop the appropriate constraints and objective function [6,7,15,17,23,49].

The four decision problems under investigation are interrelated in that the solution to one affects the solution of the others. The approach used in this research is to treat the problems of "location/allocation" and "how many" flying squads simultaneously, and the problems of the "deployment strategy" and "size and type" independently. The mathematical model developed is a mixed integer programming problem having a similar structure to the Capacitated Warehouse Location Problem (CWLP).

After a mathematical model of the problem had been developed, the third step in the solution approach was to formulate the solution procedure. Several different solution algorithms of CWLP were considered as a possible basis for the solution of the model in this research. Of these algorithms, the heuristic developed by Akinc and Khumawala [1] was chosen because of its computational efficiency in terms of both computer time and storage requirements.

In the present research, the algorithm developed by Akinc and Khumawala was modified so that the restrictions on the number of flying squads to use can be met. In addition, other minor changes were made to accomodate the specific mathematical model being solved.

## Thesis Organization

The first section of the body of this report (i.e., Chapter II) concerns the background information necessary for understanding the problems under consideration. The associated planning problems of fire suppression services are described, and relevant literature is also mentioned.

The third chapter of this study is concerned with the development of the decision model. The discussion in this chapter includes the constraints and criteria involved with each problem. In Chapter IV, the parameters used in the model along with their function in the model itself are discussed. The mathematical model is then presented and a discussion is given concerning its various components and their purpose.

The solution procedure is presented in Chapter V. A description of several algorithms is given along with a detailed presentation of the Akinc and Khumawala procedure. This is followed by a discussion of the modifications that were made to the Akinc and Khumawala procedure in order to make it compatible with the decision problem being solved in this study.

The application of the solution procedure developed for this research to the Roanoke Virginia Fire Department is discussed in Chapter VI. The associated data and the solution results are presented.

The conclusions of the research are given in Chapter VII, along with a description of potential extensions to the current research. The appendices that are included in this thesis contain information concerning (a) a questionnaire survey sent to fire departments, (b) the Akinc and Khumawala procedure, and (c) the computer program used in the solution procedure.

# CHAPTER II

## BACKGROUND INFORMATION

As mentioned earlier, part of the purpose of this research is to define and structure the planning problems encountered in the use of flying squads in municipal fire suppression operations, and to develop a solution procedure for these problems.

This chapter is concerned with a description of literature relevant to the analysis of the problem. The literature surveyed include such topics as (a) fire department operations, (b) planning fire department operations, (c) relation of response time and distance, (d) fire hazard and focal points, and (e) special squads and new fire apparatus. A discussion of literature surveyed on the solution methodology is presented in Chapter V.

### Fire Department Operations

Fire protection is one of the primary community services provided by most municipal governments. The types of public protection organizations are widely varied. One of the most familiar type in most larger cities is the public fire department. It is a department of the municipal government in which the head of the department is directly responsible for all fire protection activities in his municipality. A less common type is a fire protection agency in a department of public safety. In such organization, the public safety department performs several important functions including police service [46].

In organized fire fighting, the job is divided into functions carried out by teams called "companies" [46]. In large cities, companies may be grouped into districts, divisions, or battalions for command and administrative purposes. Basically, fire companies are of two general types: (a) engine company and (b) ladder company. An engine company is a specially modified truck which carries hose in several sizes, a pump capable of drafting water and increasing or regulating the pressure, and a water tank with a typical capacity of 500-1,000 gallons. In addition, an engine company is usually equipped with an assortment of fire fighting tools and appliances, including self-contained masks, hand-held extinguishers, and salvage covers and equipment. The ladder company commonly carries a hydraulically operated aerial ladder of 65 to 100 feet in length plus an assortment of several ladders for manual use of 10 to 30 feet in length. The ladder company carries a greater assortment of ladders, tools, and rescue devices than can be accommodated by an engine company. On the fire scene, the engine company undertakes the actual fire extinguishment by applying extinguishing agents such as water to the fire. The ladder company conducts forcible entry, rescue, ventilation, and salvage.

Studies by the National Fire Protection Agency suggests that five to six fire fighters for each company are needed for effective operation [46]. Although in smaller cities, companies are sometimes equipped with fewer fire fighters per company, but not less than four men are considered as efficient unit. In modern fire service practice, fire fighters are trained for basic tasks, and there is great flexibility in company operations as circumstances demand [46].

Fire alarms are received via public telephones, call boxes, police units, and other means. When an alarm is received, some combination of suppression resources is dispatched. The types and number of resources dispatched as a response force depends on the nature of the alarm and the location of the alarm. Regardness of how companies are organized, a response force would include at least one chief officer. A chief officer is responsible for all operations on the fire scene including placing "help calls" as he determines additional equipment and manpower are needed.

Normally, a minimum response force to alarms originating from "low value" structures (such as single family dwellings, small merchantiles or shops) would be two engines, one ladder, and a chief officer. To "high value" structures (such as large merchantiles, manufacturing or storage establishments, hotels, schools, and hospitals) four engines, two ladders, and at least one chief officer would be dispatched [46].

## Planning Fire Department Operations

A large portion of research conducted in recent years concentrates on fire suppression resource allocation and concerns the positioning and repositioning of vehicles, and determination of response districts. Literature relevant to these planning problems is presented below.

The problem of positioning is the problem of deciding where to locate fire stations and/or fire companies. This problem has received most of the attention of recent researchers. A decision on a positioning policy is a fairly permanent one. Thus, fire stations will be positioned over a long period of time at assigned locations. The most widely researched case is where the region to be protected is homogeneous, i.e., the rate and nature of fires are equal throughout the region. It is probably unrealistic to believe that any region behaves exactly this way. However, this case has been considered worthy of attention because (a) it is easy to deal with mathematically, and (b) empirical evidence has indicated that more realistic situations are often effectively approximated by results from the homogeneous case [12]. Another case is where the region to be protected by a positioning policy has heterogeneous demand, i.e., the variations in the rate and nature of fires in different parts of the region. In the problem of positioning, two possible situations can be assumed. In one case, the fire stations can be placed anywhere. In the second case, the number of possible locations is limited to a fixed set of candidates because of economic limitations, availability of lands, political considerations, etc. The former case is the one in which the location space is continuous, and the latter case the space is discrete.

Historically, the first systematic approach to this problem was developed by Valinsky [47] in the early 1950's. His analysis proceeded in four general steps. First, a set of positions was developed which satisfied the maximum response distance criteria of the Insurance Service Office (ISO). Second, an analysis of the burnable material in each part of the city was made in order to increase the coverage in areas with high risk. The last two steps involved a further refinment of the positioning policy on the basis of its ability to handle short term emergencies. Recently, more mathematical positioning approaches have been developed. Several of these approaches are mentioned below. Using historical information about the rate and nature of emergencies in the district, Raouf [35] calculated the value of a position as the weighted sum of the rectilinear distances from the location to the points where alarms originated. Santone and Berlin's [40] study also adopted this approach. However, in contrast to Raouf's analysis of rectilinear disdance, response time was calculated by determining the shortest time path from the street netwrok. In Jane Hogg's [18] study for Glasgow, Scotland, travel times through the street pattern of Glasgow were estimated and the best combination of "t" station sites from a set of "n" possible sites which yield the minimum total travel time to fires were selected. In her master's thesis [19], Hogg formulated a model to minimize the costs to society by estimating the fire loss to be anticipated with a given response pattern and the cost of providing the fire stations. She indicated that the results obtained from this additional criteria function were superior to those obtained by merely minimizing average response time. Stacey [43] modeled the positioning problem as one of allocating a given number of fire stations over possible locations in a fashion which would maximize the service to a set of demand points.

A set-covering approach has also been performed by several authors. ReVelle, Marks, and Liebman [36], Toregas, Swain, ReVelle, and Bergman

[45], and Colmer and Gilsinn [4] have reported efforts on such approaches. They indicated that when any set-covering procedure is to be used, both the accuracy of the assumptions and the efficiency of the technique must be considered. The exact solution of a set-covering problem is generally difficult, and normally large formulations can not be solved exactly by any known means due to excessive computer time and storage requirements. Most authors adopted heuristic procedures, i.e., schemes which produce good approximate solutions, but may not yield a truly optimal solution. A variety of heuristic set-covering procedures which are known to give nearly optimal solutions have been developed. Several satisfactory schemes are surveyed by Colmer and Gilsinn [4].

A closely related planning problem to the positioning problem is the repositioning problem. This problem deals with emergency situations such as when a substantial amount of fire suppression resources are involved with a working fire, it may be necessary to "move up" some available units from neighboring fire stations to avoid leaving that particular area unprotected. Though repositioning is a common practice in fire protection jurisdictions, the only published researches on this problem known to the author are by Swersey [44], Walker and Shinnar [50], Chaiken [3], and Kolesar and Walker [25]. In Swersey's report, the positioning was formulated to minimize a combination of city-wide average response time and the cost of company movements. His model was based on a warehouse location problem format. Walker and Shinnar investigated techniques for determining both the exact optimal solution and heuristic solutions to a warehouse location problem. Their results indicated that the heuristic approach provides solutions which are nearly as good as the exact technique solution, and the computer time required by the exact scheme is too large to be efficiently utilized in an operational system for repositioning. Chaiken [3] suggested that a repositioning procedure should permit neighborhoods to maintain some minimum level of coverage after repositioning. Chaiken mentioned that in practice the objective of repositioning is usually to minimize the number of neighborhoods without minimum coverage, rather than minimize city-wide average response time. Kolesar and Walker's report corrected this deficiency. The repositioning problem is addressed sequentially in four steps: (a) identifying the need for relocation, (b) selecting the empty locations which must be covered by a repositioning, (c) determining units which should be moved, and (d) refining the repositioning plan by considering the cost of relocating units.

Another planning problems which has received recent attention is the determination of response districts. That is, the determination of the area of the city to which a given company is to respond on a first alarm call. Second and higher alarm districts must also be set for each company. The approach normally adopted is to construct these areas so as to minimize average city-wide response time. Carter, Chaiken, and Ignall [2] studied the question of the best boundary between the response of two cooperating emergency units. Their results were that the best dispatch policy depend on the workload of the two units. When units are not very busy, the best policy is always to send the closest unit. When one of the units becomes much busier than the other, it may be a better policy to dispatch the less busy unit even if it is not closer to the alarm. Larson and Stevenson [30] viewed the problem as the design of response districts which minimizes the average response time. Under the assumptions of a homogeneous region, they investigated the variations in average response time resulting from non-optimal positionings of response units. They concluded that due to insensitivities it may not be necessary to quantize geographical data finely and to try laboriously to find the "optimal" solutions to redistricting and facility location problems. They also claimed that redistricting and facility location based on rather crude assumptions may yield mean travel times very near the minimum possible.

## Relation of Response Time and Distance

It seems generally agreed among fire researchers that the ideal measure of the effects of a suppression allocation would be the reduction in loss of life and property resulting from the plan. However, it also seems generally agreed that estimation of such losses is difficult to obtain, at least prior to implementation of the plan [12]. For this reason, many researchers have turned to surrogate measures which are more easily quantified and closely related to fire losses.

The most widely used of these measures is response time, i.e., the time duration between the report of a fire and the arrival of fire suppression forces. The popularity of this criterion has derived from the following assumptions:

(a) Fire losses increase as the time from the ignition of fire to arrival of suppression forces increases.

(b) Response time is more closely measured and studied than fire losses.

Though it is a well conceived fact that the fire spread will tend to increase with pre-burn time, the pattern of this loss is not well defined. For example, if fire tends to reach a dangerous size at about the same time it is reported, then response time would become a very important element. If, on the other hand, fires tend to remain in a minor state for a fairly long time, the time would be relatively unimportant to fire loss.

Hogg's [19] report analyzed fires in the United Kingdom during 1963 and 1967 to identify the extent of fires with respect to the response time. In the analysis, fires were divided into three stages of growth. Stage 1 was the period when the fire was contained to its room of origin; Stage 2 was the period during which the fire had spread beyond its room of origin; and Stage 3 was the period after the fire had spread to a different floor of the building. The proportion of fires which had advanced to each of the stages was recorded with the associated response time of suppression forces. Table 1 shows the results of this analysis for dwellings. The results indicate a tendency for fires to be found in more advanced stages when response times are longer.

The development of a fire within a structure and the effects of fire fighting operations were investigated by Labes [28]. As shown in Figure 1, the first significant event following the start of a fire is

Table 1

Dwelling Fires Observed by Fire Stage Reached and Response Time [19]

| Response Time in minutes | Stage 1 Observed | Stage 2 Observed | Stage 3 Observed | Observed Frequency |
|---|---|---|---|---|
| 2 | .842 | .088 | .070 | 57 |
| 3 | .801 | .079 | .120 | 316 |
| 4 | .793 | .089 | .118 | 1212 |
| 5 | .780 | .096 | .124 | 2092 |
| 6 | .768 | .093 | .139 | 2209 |
| 7 | .771 | .094 | .136 | 1665 |
| 8 | .783 | .078 | .139 | 1186 |
| 9 | .736 | .083 | .182 | 749 |
| 10 | .733 | .102 | .165 | 570 |
| 11 | .744 | .091 | .165 | 328 |
| 12 | .730 | .073 | .197 | 274 |
| 13 | .739 | .085 | .176 | 176 |
| 14 | .702 | .083 | .215 | 121 |
| 15 | .638 | .079 | .283 | 127 |
| 16 | .570 | .038 | .392 | 79 |
| 17 | .684 | .053 | .263 | 57 |
| 18 | .533 | .117 | .350 | 60 |
| 19 | .636 | .000 | .364 | 33 |
| 20 | .507 | .090 | .403 | 67 |
| 21 | .515 | .056 | .429 | 231 |

All fires with response time of less than two minutes or greater than twenty-one minutes were treated as if the time was two minutes or twenty-one minutes, respectively.

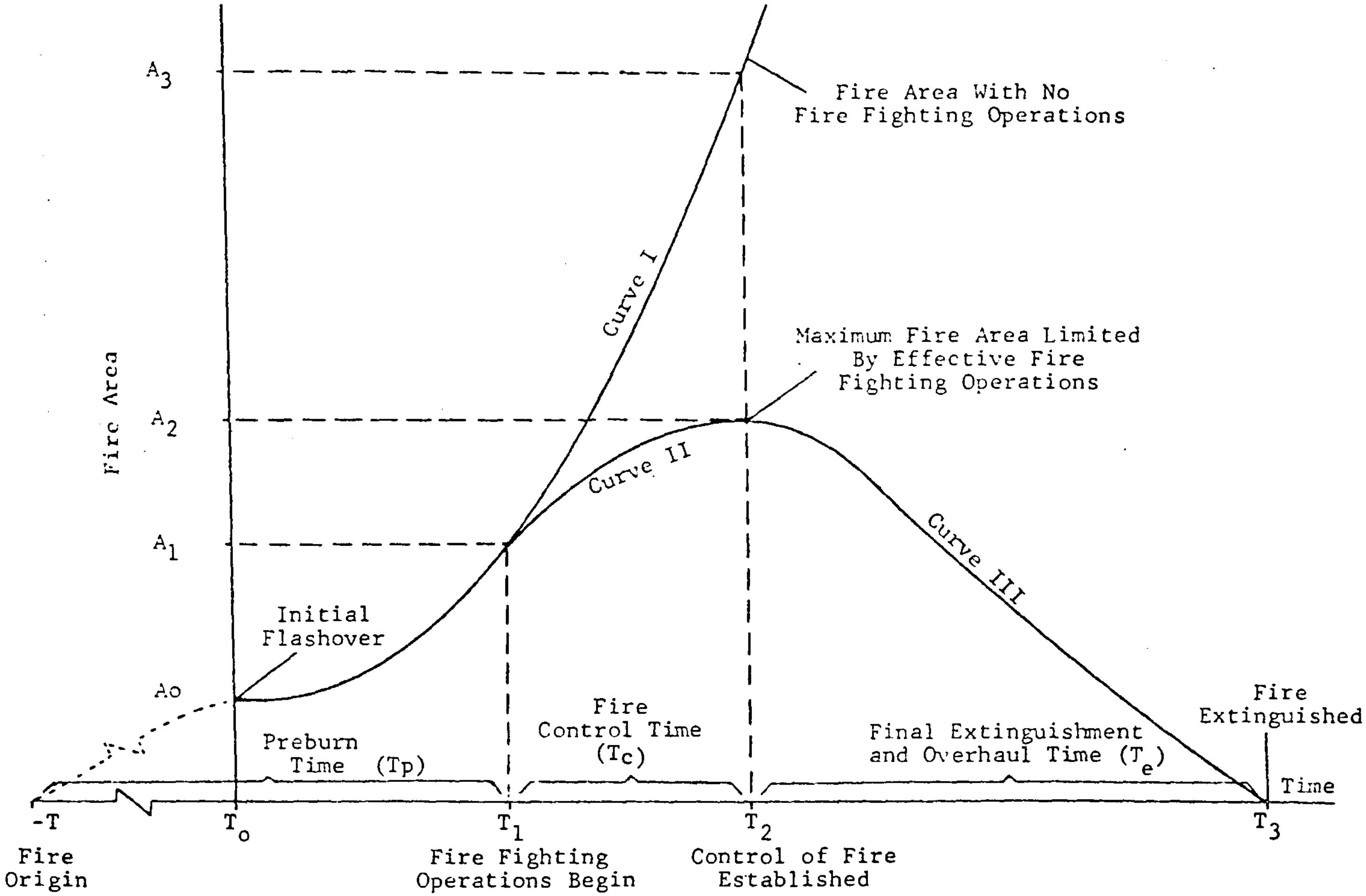


Figure 1. Fire Time-History Curves [28]

flashover. His experimental evidence indicates that in the absence of fire fighting operations, the fire spread would proceed at an exponential rate, as shown by Curve I in Figure 1. The activity of the fire fighting decreases this rate of development (Curve II), and the fire area after reaching some maximum value ($A_2$) begins to decrease (Curve III).

Several studies on the estimation of response time have been reported. In most cases, response time is estimated from distance obtained by consideration of the geographic locations of fires and the responding companies. The various studies employed several different measures of distance for this purpose. More often, one of two measures was employed. The first of these is Euclidean distance, i.e., the straight line path from the fire station to the fire scene. The second is rectilinear distance, i.e., the path from the fire station to the fire scene through a rectangular grid street pattern. Each of these approximate distances can be readily determined if the locations of a fire station and fire scene are known.

Mitchell's [32] analysis of travel time and distance relation was based on 922 incidents recorded in Fullerton, California during 1968. Applications of linear regression to the data produced the following relationships:

$$\text{Response Time} = \begin{cases} 2.377 + 1.587 \times (\text{Euclidean distance}) \\ \\ 2.328 + 1.302 \times (\text{rectilinear distance}) \end{cases}$$

The response time and distance are measured in minutes and miles, respectively.

In the study by Kolesar and Walker [26], data on response time and actual response distance, D, were recorded on 1722 incidents in New York City. Through the statistical regression procedure employed, they concluded that the square root relationship provided a better fit when response distances were short, while the linear relationship was better for larger runs. Direct fitting of such a relationship to the data produced the following expression:

$$\text{Response Time in Minutes} = \begin{cases} 3.05\sqrt{D} & \text{if } D \leq 1.2 \text{ miles,} \\ 1.67 + 1.39 \times (D) & \text{if } D > 1.2 \text{ miles.} \end{cases}$$

## Fire Hazard and Focal Points

It will also be necessary to develop measures for specifying the spacial distribution of demand for service within a given region. Santone and Berlin [40] measured this variation in fire hazard by a "weighting function". The weighting function is defined as a heuristic function which quantifies a judgment of the professional authorities. This judgment concerns the relative significance of each of the structural and human factors as related to the hazard of a potential fire on a given property. In their study, the following were defined as the critical variables.

(a) the number of persons endangered,

(b) the number of stories in each building,

(c) construction type,

(d) age of the structure, and

(e) square area of each floor.

The weighting function was devised by systematically correlating structural and demographic characteristics of all land use types with the statistical results of an analysis of the fire history data. The explicit physical characteristics of each structure, as well as implicit characteristics according to land use classification, were quantified subjectively as they related to the need for fire protection. The formulation of the weighting function is expressed as:

Structure = population + height in stories + construction type + age + area in square feet,

Weighting Function = Structure x Occupancy type.

Stacey [43] considered two other measures of the fire risk in a neighborhood. He argued that the fire risk should be proportional to the size of the things protected. Thus, he suggested using the population of a neighborhood and the value of the property in a given region as surrogate measures of the fire risk. Both measures were calculated for each of 44 areas in Dayton, Ohio, and a weighted sum of the two were used as a criterion for positioning suppression companies.

A much more detailed approach was employed by Valinsky [48] in New York City. A fire proneness index was calculated by dividing the average number of buildings with a particular occupancy which reported fires by the number of such buildings. This index represented the average number of fires per thousand structures having a particular occupancy type. Values obtained for different types are shown in Table 2.

In Colmer and Gilsinn's [4] report, two concepts of fire demand zone

Table 2

New York City Fire-Proneness Index [48]

| Type | Ratio of Fires to 1,000 Structures |
|---|---|
| Dwellings | 0.3 |
| Tenements | 8.2 |
| Warehouses | 9.3 |
| Factories | 6.9 |
| Garages | 1.4 |
| Hotels | 36.1 |
| Theaters | 9.0 |
| Store Buildings | 1.4 |
| Grocery Stores | 1.8 |
| Restaurants | 4.2 |
| Loft Buildings | 10.6 |

and focal points were mentioned. A fire demand zone was defined as a section of a city with relatively homogeneous land use. The demand for fire service for the zone was assumed to occur at one point called the focal point. They indicated that these concepts can be made operational in the following manner:

(a) The ultimate size of the fire demand zone should be related to a non-critical travel time. For example, if the city considers that 30 seconds is a critical response time, then the fire demand zone should not be larger than 30 seconds travelling time.

(b) A fire demand zone can be a single complex of buildings, i.e., a factory or a hospital, or an area of relatively homogeneous structures.

(c) The focal points are chosen to be points within the fire demand zone representing the principal hazard, or the centroid of the zone computed by weighting all hazards in the zone.

## Special Squads and New Fire Apparatus

Recently several reports on the use of "special squads" and newly introduced fire apparatus to reduce the total manning requirements have been made in a few cities. Some of these are mentioned below.

Gerhard [14] recommends the use of a "Task force" in San Jose, California. The essence of the plan is that manpower and equipment will be measured against the job to be accomplished. The term task force came to public attention during World War II in the course of naval operations in the Pacific. This method of utilizing men and equipment is considered to be adoptable to the fire service, and experience for brush fires in

Los Angeles, California starting in 1952 has proved this to be a fact. Under the plan, the task force will consist of a two-piece engine company and a two-piece ladder company, both apparatus of the engine company will be a triple combination pumper (i.e., tanker-pumper-hose wagon). The single-engine companies under this plan are called "satellites" and consist of a single triple combination pumper. The task force will be manned by 10 personnel that always respond from a single station. The satellite engine companies responding from single-company stations will be generally manned by 3 men. When the first-arriving is a satellite company, it is expected to perform only one basic operation. The engine company with a 500 gallon tank of water and manned with 3 men can perform one of the following but not two or more simultaneously, in the following order of priority:

(a) rescue,

(b) protect exposures from fire,

(c) confine the fire,

(d) extinguish, using the water and equipment on the apparatus, or,

(e) when the fire is determined to be greater than the satellite can handle, lay two 1.5 inch lines from the hydrant to the fire.

The task force commander is the fireground officer in charge until the arrival of the battalion chief. Individual company officers are responsible to this commander and will perform those functions which he determines are necessary at the fire scene. This allows units to respond as a single group without staggering their arrival and fragmenting fire ground operations.

Paddock [34] reported the use of a "flying squad" which went into service in 1969 in the city of Orange, California. This unit consisted of a specially equipped 3/4-ton pick-up truck and two men per shift who were centrally located. The special equipment includes a skid-mounted 200-gallon water tank, a portable pump, hose, tools, and map compartments. Men assigned to the flying squad have cross-trained so that they could handle any situation. Further, they have received extensive courses in Fire Science, Rescue Technique, and Advanced First Aid. Due to speed and versatility, the unit was mainly used for fire patrol and as a backup unit to larger equipment. The squad was also used for off-the-road situations such as brush fires, where large apparatus are too large and heavy to be efficiently utilized.

Cookingham [5] reported the use of a "Hot-Shot Squad",which is similar to but more heavily manned than a flying squad, currently being used in Fort Worth, Texas. Twenty-three men are assigned to the squad. Cookingham claims that this provides approximately the same level of fire fighting strength as 46 men assigned in the conventional manner to fire stations. The squad is made up of selected men who receive specialized in-service training in all phases of fire fighting. The men have the aptitudes and skills for handling electric, hydraulic, and mechanical tools and equipment. They are also experts in rescue and salvage work. The squad uses a specially designed truck which is equipped with salvage, rescue, and forcible entry tools. Its most important activity is to provide additional, highly skilled manpower to fight critical fires. It also handles many other emergency assignments in all areas of the city. Also, the Chicago Fire Department describes a special "Snorkel Squad No. 1" which served principally as a rescue unit [55].

Among the newly introduced fire apparatus are the Quick Response Vehicle (QRV) and the Quint [54]. The QRV is a small pumper unit on a four wheel drive truck chassis of 11,000 pounds gross vehicular weight capacity. This unit carries 250 gallons of water, approximately 1,500 feet of fire hose, and a 400 gallon per minute fire pump. First aid equipment, breathing apparatus, and a few hand tools are also equipped. The QRV's are manned with two fire fighters but will be able to carry up to 6 men when necessary to act as a squad or manpower transport unit, The unit is expected to meet several needs. First, many non-fire emergencies can be efficiently handled by two men and a small vehicle, Second, compared to heavy pumpers, fuel consumption and maintenance will be reduced. Third, this small vehicle is very maneuverable and has quick acceleration, thus response times will be decreased in many areas. Fourth, the QRV allows heavy equipment to be kept available for serious fires.

The Quint is so named because it contains five basic fire fighting components: a pump, a water tank, a load of hose, a number of portable ladders, and a powered elevating device. The Quint is manned with four fire fighters. The unit is able to perform as ladder companies presently operate. Second, the Quint can act as an attack unit within a task force since it can provide elevated streams of water supplied by other pumpers.

## Flying Squad

The term flying is indebted to police patrol operations. In police circles, the phenomenon of police forces crossing sector boundaries to respond to calls in nearby sectors has been labelled "flying" [31].

To better understand the concept of a flying squad being used in the current research, the traditional method of dispatching is briefly mentioned below. As mentioned earlier, the two basic fire fighting apparatus in most fire departments are the engine and ladder company which are usually manned by 5 to 6 fire fighters for each company. Every fire station houses at least one engine company, and ladder companies are distributed among some of these stations. The basic response force to a structure fire usually consists of two engine companies and one ladder company, and therefore, requires response from at least two stations. Figure 2a and 2b describe the situations where an engine company is the nearest unit and the second arriving engine and ladder companies respond from the same location and where the first arriving is an engine and ladder companies from the same location and the second arriving is an engine company, respectively. Figure 2c varies in that the second arriving is nearer than the ladder company and all units are dispatched from three seperate locations.

The basic concept of a flying squad (i.e., manpower transport vehicle) is to dispatch this unit to the origin of an alarm where it will join the engine and ladder companies to form a complete response force. Each company will be manned by less than the usual five to six fire fighters, and the additional manpower will be supplied by a flying squad. By introducing a flying squad, it is intended to develop an alternative method of

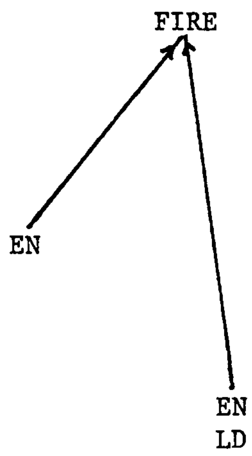


Fig. 2a Situation A

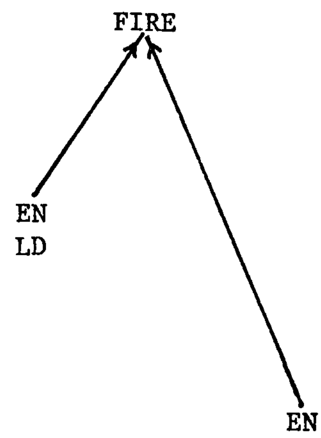


Fig. 2b Situation B

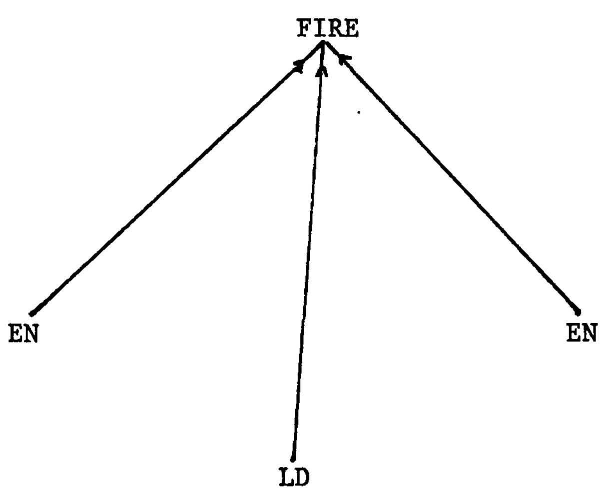


Fig. 2c Situation C

Figure 2. Traditional Dispatching Method

Key: EN and LD denote the engine company and ladder company, respectively.

manning the companies. It is anticipated that a flying squad will reduce the total manpower requirements. For example, if the fire protection provided by 14 conventionally manned companies (e.g., 5 men each) can be effectively substituted for 14 companies manned by 3 men each and 2 stratigically located flying squads with 6 men each, then a reduction of 16 fire fighters or 23% of manpower can be realized. Figure 3 illustrates the deployment of the complete basic response force if a flying squad is housed in an existing fire station. Figures 3a, 3b, and 3c are the same as Figures 2a, 2b, and 2c except for that a flying squad is housed with an engine company, engine and ladder companies, and a ladder company, respectively. Similarly, the situations where a flying squad is housed other than the existing fire stations can be illustrated.

There are four basic problems in planning the operations of a flying squad. These are determination of (a) the number of flying squads to have, (b) the locations/allocations of flying squads, (c) the size of a squad and the job skill levels of each member of the flying squad, and (d) the deployment strategy of a flying squad. To the author's knowledge no research has been reported on any of these problems.

The current research focuses on these planning problems for the operations of a flying squad. Detailed discussion of the problems is presented in Chapter III.

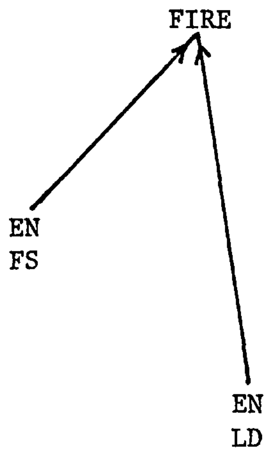


Fig. 3a Situation A

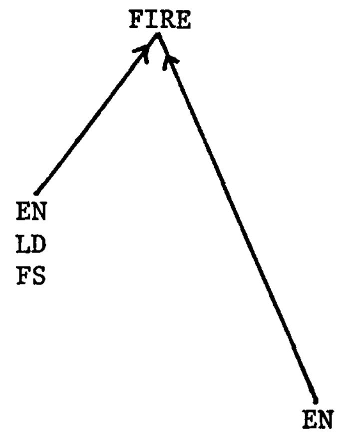


Fig. 3b Situation B

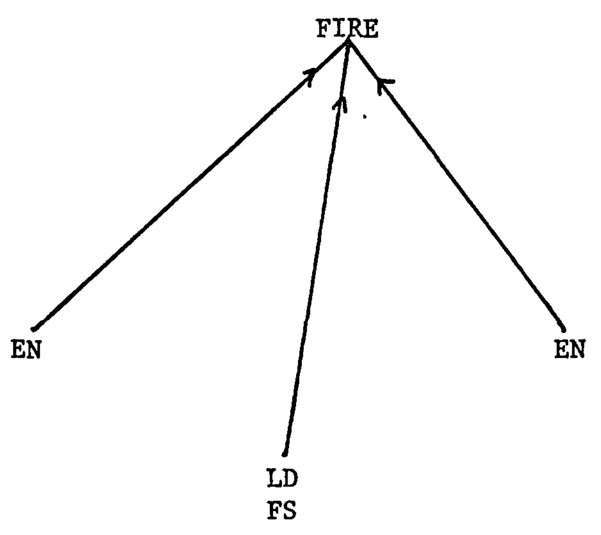


Fig. 3c Situation C

Figure 3. Proposed Dispatching Method

Note: FS is used to denote a flying squad.

## CHAPTER III

## THE DECISION PROBLEM

The current research is concerned with these following problems: determination of (a) the number of flying squads to have, (b) the location/allocation of a flying squad, (c) the deployment strategy of a flying squad, and (d) the size and the job skill levels of each member of a flying squad.

As a part of the research, a survey was sent to major fire department chief officers (see Appendix A for a copy of this survey) in order to ascertain what factors should be considered in the use of flying squads. The opinions of chief officers were utilized to develop the appropriate constraints and objective function. The results of the responses received are shown in Table 3. In the remainder of this chapter, the above mentioned problems and the objective function for the decision problem are discussed.

### The Number of Flying Squads

Decisions are to be made by each individual city as to the level of service they wish to maintain. For example, if a city wishes to maintain a high level of service, a large number of squads is likely needed. If, on the other hand, a city considers that a high level of service is not necessary (e.g., due to its physical composition), a smaller number of squads would be sufficient. Thus, the figure to be considered will vary

Table 3

Factors to be Considered in the Use of a Flying Squad

A. The Number of Squads to Have

(a) response time element

(b) level of service to maintain

(c) area of the city

(d) degree of manpower reduction necessary

(e) number of alarms requiring their service

(f) fire problem (i.e., types of fire responses)

(g) number of companies presently in service

B. Location/Allocation of a Squad

(a) response time and distance

(b) physical characteristics (i.e., availability of land, accessibility to main throughfare, etc.)

(c) number of present fire stations and capacity of a squad

(d) within a region, locate as centrally as possible

(e) locate in heavy fire incident area

C. Deployment Strategy

(a) whether the squads to respond on an initial alarm or on the report of an actual fire

(b) as needed, unit or fragment

(c) travel distances should be short enough to give the back-up men when vitally needed

(d) problem of other company awareness of their location

Table 3 (continued)

D. Size and Job Skill Levels

(a) size of squad to be pre-determined by the size of initial response assignments

(b) teamwork training

(c) the squad unit to be cross-trained including skills in rescue work

(d) the size depends on manpower reduction on a company

(e) the squad to have its own leadership (company officer rank)

(f) should only be used for manpower not to replace company in each city such as an industrial city versus a residential community. This figure may also be affected by several other factors such as economics, political situations, areas to be covered, etc.

The group of decision-makers who will be concerned with the number of squads to have will include the fire chief and city manager. It will be their decision that will determine the maximum number of squads to have when a budgetary limitation has been considered. The decision problem is to determine the optimum number of squads which optimizes the objective function to be discussed later in this chapter and satisfies a given set of constraints. In the research, a sensitivity analysis will be employed. The effect of additional squads on the system after obtaining a feasible solution will be observed.

After analyzing the survey responses received from fire chiefs and related journal literature, it was determined that the dominant considerations by a decision-maker in deciding on the number of squads to use appear to be (a) the level of service, (b) the fire service demand, and (c) the manpower consideration. In a mathematical model to be developed in Chapter IV, all three factors will be included as constraints. A discussion of each of these factors is presented below.

Level of Service

Response time has been considered as a major indicator of the level of service because it is widely believed that the faster the response, the less the loss to life and property involved in the fire assuming all other factors remains the same [41]. It is also one of the main ways in which a citizen may evaluate the effectiveness of his fire protection service. Response time is defined as the time from the report of a fire to the arrival of suppression forces at the fire scene. Ideally, the time from the ignition to the report of a fire should also be considered, but the fire department generally has less control over the "time to report" than it does of the "time from report to arrival". Response time depends on fire department communications, the number and positioning of fire stations, and the ability of the drivers to identify the optimum route and to drive to the scene of fire. But response time also depends on non-controllable factors, such as weather conditions, the road network of a city, traffic conditions, etc.

There are several response time measures that are being used among fire operation researchers. Among these are (a) the percent of calls responded to in less than "t" minutes, and (b) the average response time. More discussion of this response time is presented in the following section.

Fire Service Demand

The fire service demand can be measured by the number and severity of alarms received. Areas with high densities of serious alarms have high fire service demand, while areas with infrequent and inconsequential alarms have low demand. The most elementary measure of fire service demand is the number of alarms received and the type of alarms, i.e., residential, commercial, industrial, false alarm, etc. The level of such demand provides a criterion which is even more obvious than response time. Thus, relatively heavier suppression forces should be allocated to areas with high alarm rates and serious fire risks.

The geographical distribution of alarms was analyzed by Nilsson and Swartz [33] on data from Alexandria, Virginia. The land area of the city of Alexandria was divided into 1500 square feet areas, and the number of alarms in each area tabulated. Results of this analysis showed that the number of alarms per area ranged from 0 to 62. Thus, a substantial geographical distribution of alarms in certain areas was indicated. Similar studies done in Fullerton, California, Wichita Falls, Texas, and New York, New York, also reached such a conclusion [12].

### Manpower Restriction

At a time when a reduction of manpower level within a fire department organization is increasingly pressured by budget cuts and inflationary costs, the manpower restriction plays a very important role [14]. With its limited budget, a fire department must assume the responsibilities of developing ways of providing the best possible service and offering equitable service to all citizens and areas. As shown in Table 4, by far the largest portion of fire department expenditures is allocated to manpower. As mentioned earlier, on the average over 95% of the fire department budget is allocated to manpower [46]. Since flying squads are manpower transport vehicles, this constraint affects the number of squads to have and the crew size of both squads and companies.

## Location/Allocation of a Flying Squad

Another important issue in the use of a flying squad is the determi-

Table 4

Fire Department Budgets[53]

(1970)

| City | 1970 F.D. Budget % of General Revenue | 1970 F.D. Expenditures (in thousand dollars) | 1970 Salaries % of F.D. Budget |
|---|---|---|---|
| Chicago, Ill. | 12 | $ 68,478 | 98 |
| Los Angeles, Cal. | 12 | 62,348 | 98 |
| New York City, N.Y. | 4 | 233,296 | 99 |
| Atlanta, Ga. | 10 | 8,544 | 89 |
| Pittsburgh, Pa. | 13 | 11,181 | 98 |
| St. Louis, Mo. | 9 | 12,211 | 98 |
| Washington, D. C. | 4 | 22,683 | 94 |
| Cincinnati, Ohio | 5 | 11,215 | 96 |
| Long Beach, Cal. | 10 | 8,137 | 96 |
| Miami, Fla. | 19 | 7,619 | 99 |
| Portland, Ore. | 14 | 8,857 | 95 |
| San Jose, Cal. | 12 | 6,855 | 87 |
| Wichita, Kan. | 12 | 4,445 | 91 |
| Baton Rouge, La. | 10 | 3,246 | 89 |
| Columbia, S. C. | 18 | 1,391 | 94 |
| Lansing, Mich. | 13 | 3,325 | 91 |
| Montgomery, Ala. | 19 | 2,503 | 96 |
| Richmond, Va. | 7 | 5,620 | 77 |
| Wichita Falls, Tex. | 8 | 962 | 92 |

nation of the locations of squads and the design of their response districts in the face of spatially distributed demand patterns. When a limited number of squads is strategically located, it is possible to better service the community than if it was randomly located.

When locating flying squads, two possible situations are considered. In one case, the squads can be positioned anywhere (i.e., a continuous space problem). In the second case, the number of potential locations is limited to a fixed set of candidates (i.e., a discrete space problem). The opinions of professionals gathered through the survey mentioned earlier indicate that the latter case is more appropriate to the positioning of fire suppression forces. The set of candidates may include already existing fire stations as well as any other potential sites. The potential sites should possess the physical characteristics such as accessibility to main throughfare, availability of land, etc.

The design of response districts involves determining what portion of a city to assign to each squad. Districts are formed by combining seperate areas or regions of the city into mutually exclusive sets or groups. As the basic areas, fire station first alarm territories (e.g., there are 30 such areas in the city of Atlanta, Georgia), census tracks (e.g., there are 109 tracks in Atlanta), and others can be used. However, the use of the smaller basic areas is more desirable because this allows more refined and flexible response district definitions. The major constraints for this planning problem are response time and workload feasibility. They are discussed below.

Response Time

As in the previous planning problem, a dominant concern in the location of squads is response time. Ideally, minimizing its average response time to fires while providing equal average response time is desired. However, it is very difficult, if not impossible, to attain both at the same time. If the squads are positioned to minimize the average response time in the city, many more squads will be assigned to the high incidence region than to the low incidence region, and the resulting response times within two regions will not be equal. If this criteria is used, the residents of the low incidence region will possibly be concerned that they are being penalized for having fewer fires. If, on the other hand, the response time is made equal in both regions, the response time to many fires in the high incidence region will be increased in order to reduce those in the low incidence region, and the average response time throughout the city will be larger than its feasible minimum. Under this policy, the squads in the high incidence region would have a larger workload than the squads in the low incidence region. An illustration of this situation for two regions of a city having widely varying rates of fire incidence is given in Figure 4.

Because of this situation, some compromise is usually made between the above mentioned two objectives in order to attain a reasonable and useful allocation of squads. The criteria used in this research establishes the maximum limits in terms of response time to different physical compositions (e.g., 3 minutes to commercial regions and 5 minutes to residential regions). To utilize this criteria, an adequate representa-

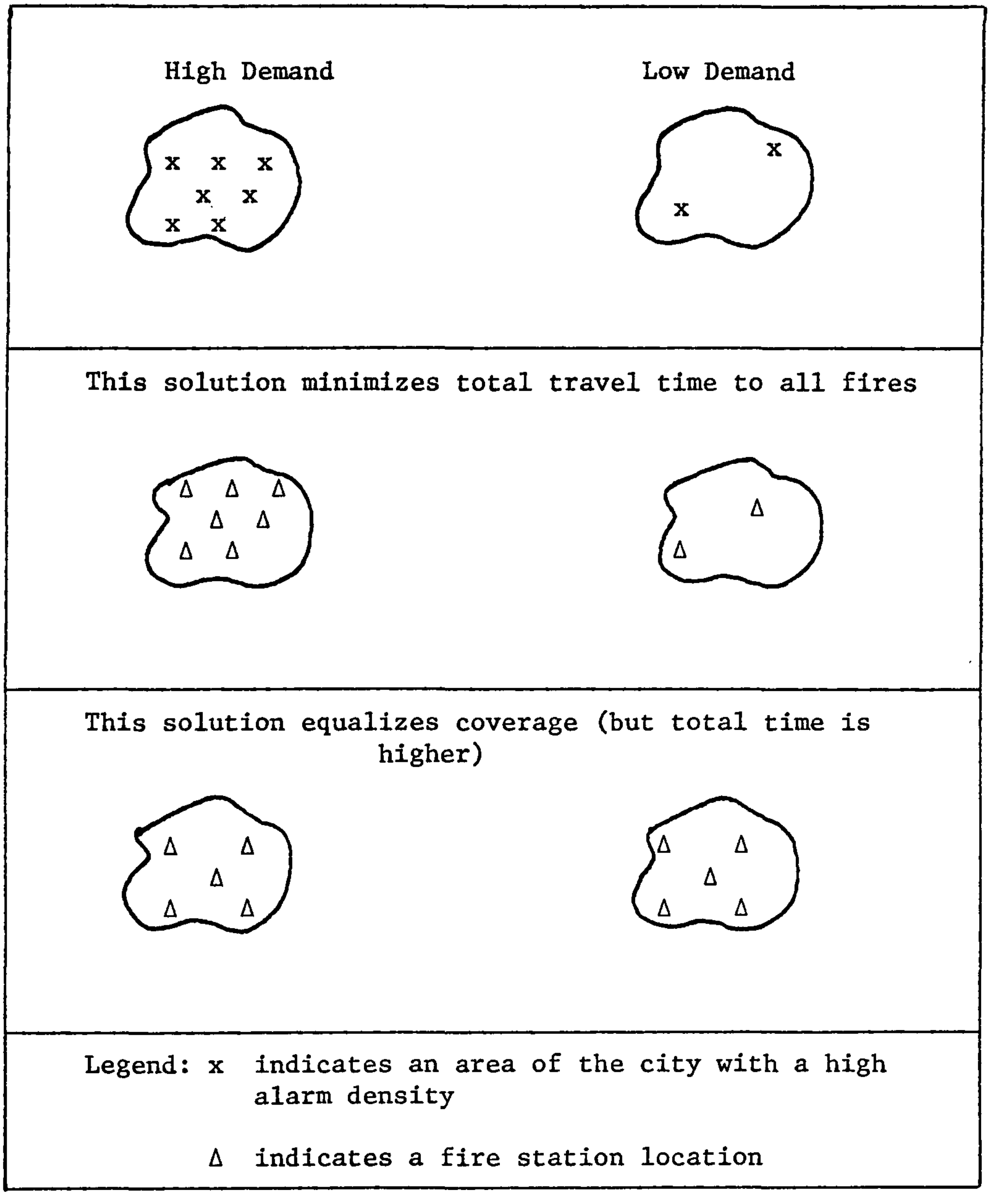


Figure 4. Two Allocation Strategies for Fire Companies

tion of the street network is necessary. Fortunately, in many cities, the highway department has coded the street network for present and future years [40]. Such a network is represented by a uniquely numbered series of nodes and links. The data listed includes distance between intersections (i.e., links between nodes). With this street network, the shortest response time from one node, representing a potential squad site, to all other nodes can be approximated by using the distance-to-response time relationships mentioned in Chapter II.

## Workload Feasibility

Another concern in the location/allocation of squads is workload feasibility. Besides the duty of answering alarms, fire fighters are involved with training and housekeeping required by the "live-in" system. Even though fire fighters devote only a small portion of their time answering alarms, preventing any unit from being overtaxed is very important because this may affect the morale and productivity of fire fighters [12].

As mentioned in the previous section, one of the appropriate measures of defining the workload of a squad is by the number and type of alarms responded. Most fire departments keep records of the number of calls they received and the types of services they provided. Obviously, the squad responding to a structural fire may consume more time and effort than if it were dispatched to a trash bin fire. The workload associated with a region depends on the frequency and type of alarms received. In the research, the estimated mean value of the workload is used to compute the level of workload in a region. The workload requirement of a region is defined as the product of structural fire occurence probability and the number of structures assuming that the region consists of homogeneous structures.

The available squad-hours limits the number of fires a squad can respond. The workload of a squad will be limited to a pre-determined number of structural fires, such as 120 fires.

## Deployment Strategy of a Flying Squad

The problem associated with a deployment strategy is how to dispatch a flying squad. Flying squads can be dispatched from fixed-base stations or from patrolling units.

Fixed-base stations are those which respond to an alarm from fixed locations. Included in the fixed-base category are nearly all fire companies and most ambulances. The best known and most widely researched class of problems in the location of fire stations are of this type. Such problems are generally static in that the locations selected are maintained for a relatively long period of time, usually several years. Patrolling situations are those which respond to an alarm from constantly moving vehicles. The police and security patrols are among the best examples of this.

In order to fully describe a deployment strategy, two questions are to be answered: (a) how many squads should be dispatched, and (b) which units should be dispatched? The procedure for dispatching appears to be similar in most fire departments. When an alarm is received and the location of an alarm is determined, dispatchers consult a "running card" to see which units should be dispatched. This card also contains information concerning how many units to send. In major cities, computerized dispatching system is being implemented. Study [14] shows that this new system can better response time by about a minute which is a considerable contribution to emergency situations.

The first element of a policy for dispatching flying squads to an alarm is a plan of designating how many squads to dispatch to various types of alarms. If too many units are dispatched, suppression forces will be left in a poor position to respond to subsequent alarms. If too few are dispatched, additional injuries and property losses may be incurred before adequate forces can be brought to the fire scene. The majority of the modern fire jurisdictions deal with this problem by establishing fixed responses for various types of alarms. A complete suppression force usually requires responses from several fire stations. For example, two engine and one ladder companies manned by a total of 15 fire fighters dispatched from stations A and B might always be assigned to a reported dwelling house fire in region C. Additional units are dispatched when the chief officer at the fire scene determines that the initially dispatched forces will be inadequate. In the research, this question is answered by locating at least one flying squad within a response time of $S_j$ from the focal point with respect to fire service demand of a region, where j is the region index.

The question of which units to dispatch is also to be answered. In the overwhelming number of modern fire protection jurisdictions, this decision is made on the basis of the closest units [12]. If two flying squads are required, the two closest units are to be chosen for dispatch.

## The Size and the Job Skill Levels of Each Member of a Flying Squad

In addition to the previously mentioned three problems, it will also be necessary to determine the size and the job skill levels of each member of a squad. For example, a decision must be made as to how many men are to be assigned to each squad, and what are the job responsibilities for each member of a squad. The manpower needed with flying squads is to be borrowed from the existing companies by reducing the crew size of present companies. The results of a survey indicate that a flying squad is to be manned by sufficient personnel so that upon response to an alarm the same number of personnel be present as presently experienced. Also, the size of a squad is to be pre-determined by the size of initial response assignments.

For example, if two engine and one ladder companies manned by five men on each apparatus are usually dispatched to a given type of alarm, and if the crew size is reduced to three men each, then the appropriate size of a flying squad would be six men. Spillman [42] reported a study done by the Dallas Fire Department which consisted of a series of actual tests in an effort to determine comparative effectiveness of various manpower levels in performing fire suppression operations. A brief outline of what an engine company is capable of performing in the delivery of water on various manning levels is shown in Table 5.

As to the job skill levels, men on the squad unit would have to be cross-trained so that they could operate any piece of equipment. In modern fire service practice, most fire fighters are trained for basic

Table 5

Comparative Effectiveness of Fire Crews[42]

<u>A. Three-man Engine Company Can</u>:

(a) provide back-up water supply from a hydrant and then man one 2.5" hose lines, or

(b) lay two 2.5" hose lines and man one 2.5" hose line using the water in the engine tank, or

(c) lay and man two 1.5" hose lines, for a total water carried in the engine tank.

<u>B. Four-man Engine Company Can</u>:

provide back-up water supply from a hydrant and lay and man:

(a) one 2.5" hose line, or

(b) two 1.5" hose lines.

<u>C. Five-man Engine Company Can</u>:

provide back-up water supply from a hydrant and lay and man one 2.5" and one 1.5" hose lines.

tasks and there is great flexibility in company operations as circumstances demand [46]. Therefore, no difficulties seem to arise in achieving this goal. It may be desirable to divide the squad members into personnel assigned to assist engine and/or ladder companies. This simplifies the integration of the forces at the fire scene. The squad must be capable of operating independently with its own leadership since they might respond to small fires or car accidents with no other equipment being dispatched. This gives the ability to work as a unit with no supervision.

## Objective Function for the Decision Problem

The criterion used in this research concerns the expected cost to society. This societal cost may include capital costs for constructing facilities, variable costs for operating the facilities, and the expected fire losses. Though, this cost to society has been considered as the ultimate measure of fire protection effectiveness [12], only a limited amount of related research using a societal cost criteria has been reported. These reports include those by Guild and Rollin [16] and Hogg [19, 20]. They are briefly discussed below.

The model of Guild and Rollin estimates the average societal cost of assigning a given number of companies to the area to be protected. Included in the societal cost are costs of maintaining the companies and costs due to anticipated fire losses. However, the anticipated fire losses are assumed to be simple multiple of response time which is not a true reflection of the actual situation. Hogg's model involves a nonlinear functional relationship between loss and response time. Hogg reports that results derived from the cost based resource allocation were superior to the ones based entirely on response time.

In the research, a non-linear functional relationship between fire loss and response time for different types of structures in different regions will be incorporated (see pp. 14 for a discussion of this function). Fires in different types of structures (e.g., a residential versus an industrial) and different region of a city (e.g., a dwelling house in high-income versus low-income family residential areas) may result in varying degree of fire related damages for a given response time of suppression forces. This non-linear scheme enables the squads to be located closer to the high-value areas. The expected fire losses at each region can then be determined by applying the fire loss to response time relationships to the expected number of fires and the response time of the closest unit. A detailed discussion of this relationship is given in Chapter IV.

Each time an additional squad is located, there are incurred costs associated with manpower, facilities, etc. It is abvious that a higher facility construction cost will be incurred if a facility is to be built in down town areas than if it is in rural areas, mainly because of higher lot cost in the down town area. Also, some cost savings can be realized if a squad is housed in an already existing station instead of in a new building to be built. Thus, a squad will have a variable construction cost depending on the alternative site to be located.

The sum of the expected fire losses in a city and the manpower cost associated with squads and companies will be considered as a cost to society. The criterion for the decision problem minimizes such a cost.

CHAPTER IV

# DECISION MODEL

The purpose of this chapter is to describe the development of the model of the decision problem involved in using flying squads. This discussion of the decision model includes (a) the description of the decision variables and objective function that are to be used, (b) the constraints involved in the problem, (c) a concise problem statement, and (d) the mathematical model itself. The notation that is used in this chapter is also used in the discussion of the solution procedure in Chapter V.

## Description of the Decision Variables and Objective Function

For a particular period of time (usually one year) under consideration, each region of a city has potential fire service demand. The level of service provided by a flying squad must satisfy its potential demand. The measure of potential fire service demand for region j used in this research is expressed in terms of structural fires as follows: when it is assumed that a city is divided into regions of homogeneous type k structures, then

Potential Fire Service Demand

= (the probability of fire occurence per type k structure, $P_{kj}$) x (the number of type k structures, $W_{kj}$).

Then, the sum of these potential fire service demands for all regions represents total fire service demand of a city.

The product of $P_{kj}$ and $W_{kj}$ represents the expected number of type k structural fires in region j. Unfortunately, little is known about the relationship between the fire loss and response time of a suppression force. However, this expression can be obtained by either of the following ways: (a) by conducting a series of actual fires, or (b) by quantifying the subjective opinions of the professionals in the field (see pages 14 and 42 for a discussion of this approach). Assume that the fire loss (e.g., property damage) for a single type k structural fire as a function of response time, $t_{ij}$, of a flying squad located at site i for a given company crew size, $m_1$, can be expressed as:

$$g_k(t_{ij}|m_1) = (a_{kj}t_{ij}^2 + b_{kj})\ F_{kj} \tag{1}$$

where $a_{kj}$ and $b_{kj}$ are constants which may assume alternative values for different values of $m_1$, and $F_{kj}$ represents the median value of a structure in region j which consists of homogeneous type k structures. Then, the expected fire losses from region j is:

$$\sum_i \left[P_{kj}\ W_{kj}\ F_{kj}(a_{kj}t_{ij}^2 + b_{kj})\right]\ x_{ij} \tag{2}$$

where the decision variable $x_{ij}$ represents the fraction of demand at region j which is supplied by a flying squad located at site i. The value of $x_{ij}$ will be greater than 0 but less than or equal to 1 if region j is protected by a flying squad located at site i, and will be 0, otherwise. Another decision variable used in the model concerns where to locate a flying squad. Specifically, let $y_i$ assume the value of 1, if a flying squad is located at potential site i, and 0, otherwise.

As mentioned earlier in Chapter III, the objective function consists of manpower costs for operating flying squads and companies, and the expected fire losses. The manpower cost associated with a new system can be expressed as follows:

$$\begin{aligned} \text{Manpower Cost} &= (\text{Manpower Cost for Flying Squads}) \\ &\quad + (\text{Manpower Cost for Companies}) \\ &= C\, m_3\, \Sigma\, y_i + C\, n\, m_1 \\ &= C(m_3\, \Sigma\, y_i + n\, m_1) \end{aligned} \tag{3}$$

where C, $m_3$, and n denote the average salary of a fire fighter, the crew size of a flying squad, and the number of companies presently in use in a city, respectively.

Then, the combined cost of (2) for all regions and (3) represents a cost to society when a flying squad is introduced as an alternative method of manning the companies. The objective function is to minimize this cost. This objective function, F, can be expressed mathematically as follows:

$$\begin{aligned} F = \sum_i \sum_j \, [P_{kj}\, W_{kj}\, F_{kj}(a_{kj} t_{ij}^2 + b_{kj})]\, x_{ij} \\ + C(m_3 \sum_i y_i + n\, m_1). \end{aligned} \tag{4}$$

The last decision variable under consideration is the crew size of a flying squad and/or a company(assumed homogeneous throughout the region). As mentioned earlier in Chapter III, the former is to be found as a function of the latter, and vice versa. For example, if "r" companies manned by $m_2$ men on each apparatus are usually dispatched as an initial response force and the proposed company crew size is $m_1$, where $m_2 > m_1$, then the crew size of a flying squad can be found as:

$$m_3 = r(m_2 - m_1) \tag{5}$$

when it is assumed that the complementary manpower is supplied by one flying squad. Therefore, the combined manpower of a flying squad and "r" companies manned by $m_1$ men each constitutes the manpower level of initial response force presently being experienced. The value of this decision variable will be found by solving the same problem for all crew size under consideration with appropriate expressions of (2) and (3). Therefore, the crew size which yields the minimum objective function value will be selected.

The purpose of this study can then be stated as trying to find values of decision variables $x_{ij}$, $y_i$, and $m_1$ or $m_3$ that will minimize F. This objective function will be subject to constraints that will be discussed in the following section.

## Problem Constraints

In formulating the planning problems of "how many squads to have" and "locations/allocations of flying squads", several constraints are placed upon the fire protection system. Some of these constraints were mentioned in Chapter III. These constraints can be summarized as either (a) response time constraints, (b) fire service demand constraints, (c) manpower constraints, or (d) workload feasibility constraints. Each of these restrictions will be discussed below.

Response Time

An acceptable response time may vary from region to region as a decision-maker determines the level of service to be provided. Regions with high fire hazard in terms of both frequency and value of fire loss should be better protected than those with low fire hazard. Since most cities are composed of several physical compositions such as residential, commercial, industrial, etc. the variations in the level of service to be provided must be considered in the model. For example, a typical constraint may be to locate at least one flying squad within 5 minutes of response time for residential area and 3 minutes for commercial area.

Fire Service Demand

This constraint is concerned with restrictions that each region of a city is properly protected by a squad. As mentioned earlier in the previous section, the measure for fire service demand used in this research is the number of expected structural fires occuring in a region. The variations of demand for a different type of region are also to be reflected in the model.

Manpower Constraint

The decision-maker would be responsible for supplying this constraint. When the amount of capital available for manpower is transformed into the number of men, this figure along with the company crew size limits the number of flying squads to operate.

Workload Feasibility

This constraint is placed so that none of the flying squads is physically overtaxed. Even though most of the fire fighter's duty time is said to be unproductive (in most cases over 80% of the time is "idle") [12], fire fighters must always be in good physical condition to respond to an alarm. The measure for this constraint used in this research is the number of structural fires to respond (e.g., 90 fires). This constraint is also to be supplied by a decision-maker.

Once these constraints have been formulated, they can be combined with the objective function discussed in the previous section to form a model of the decision model. The following sections will present a concise statement of the problem being considered and its mathematical model.

## Concise Problem Statement

The problem that will be considered in this research will be to determine (a) the number of flying squads to have, (b) the locations/ allocations of flying squads, and (c) the crew size of flying squads and companies. The problem will be subject to the following constraints: (a) the response time should not exceed the predetermined threshold, e.g., 5 minutes and 3 minutes for residential and commercial areas, respectively, (b) the potential fire service demand for each region must be satisfied, (c) the total manpower level should not exceed specified manpower restrictions, and (d) the expected workload of a flying squad should not exceed its workload feasibility.

Assumptions

The formulation of the problem is based on the following assumptions:

(a) All the squads are identical and indivisible.

(b) Each region consists of homogeneous physical composition, i.e., residential, commercial, industrial, etc.

(c) The area is divided into regions so that the response time within a region does not exceed travel time threshold.

(d) The demand for fire service for a region is assumed to occur at the focal point.

(e) All the companies are already strategically located, therefore, only the locations of flying squads are to be considered.

(f) The function expressing fire loss in terms of response time for a given company crew size is known.

## Mathematical Model

The following is a mathematical representation of the problem described in the previous section. Consider the following notation:

$i$ represents the $i^{th}$ alternative site for a flying squad,

$j$ represents the $j^{th}$ demand region,

$k$ represents an index of the $k^{th}$ type physical composition of demand region $j$ (e.g., if $k = 1 \Rightarrow$ residnetial, if $k = 2 \Rightarrow$ commercial, etc.),

$I$ represents the maximum value of $i$,

$J$ represents the maximum value of $j$,

$K$ represents the maximum value of $k$,

$P_{kj}$ represents the fire occurence probability for type k structures in demand region j,

$W_{kj}$ represents the number of type k structures in demand region j,

$F_{kj}$ represents the median value of a type k structure in demand region j,

n represents the number of companies presently in use,

$m_1$ represents the proposed company crew size,

$m_2$ represents the present company crew size (used in page 49),

$m_3$ represents the flying squad crew size,

$t_{ij}$ represents the shortest response time path from potential site i to demand region j,

$g_k(t_{ij}|m_1)$ represents the function expressing the fire loss for a type k structure in terms of response time of a flying squad, $t_{ij}$, for a given $m_1$ level,

$a_{kj}$ and $b_{kj}$ represent constants associated with the fire loss estimating function, $g_k(t_{ij}|m_1)$,

$S_j$ represents the response time threshold for demand region j,

$N_j$ represents the set of potential sites for flying squads which are within the response time threshold, $S_j$, from demand region j (i.e., $N_j = \{i|t_{ij} \leq S_j\}$),

$x_{ij}$ represents the fraction of fire service demand at region j which is supplied by a flying squad located at potential site i ($0 \leq x_{ij} \leq 1$),

$y_i$ represents a zero-one decision variable, i.e.,

$$y_i = \begin{cases} 1 \text{ if a flying squad is located at potential site i,} \\ 0 \text{ otherwise} \end{cases}$$

c represents the average annual salary of a fire fighter,

$M_1$ represents the lower bound on the number of flying squads to have which is to be computed from a set-covering problem (model (M2)),

$M_2$ represents the upper bound on the number of flying squads to have, set by the decision-maker,

$a_{ij}$ and $z_i$ represent a coefficient and decision variable associated with a set-covering problem, i.e.,

$$a_{ij} = \begin{cases} 1 \text{ if demand region j is covered by potential site i} \\ 0 \text{ otherwise, and} \end{cases}$$

$$z_i = \begin{cases} 1 \text{ if a squad is located at potential site i} \\ 0 \text{ otherwise,} \end{cases}$$

$\alpha_i$ represents the maximum number of structural fires flying squad i can respond to, set by a decision-maker.

The decision problem described previously can be expressed mathematically as follows:

Find $x_{ij}$, $y_i$ for $\forall$ i and j, and $m_1$ and $m_3$

to minimize

(M1)

$$\sum_{j=1}^{J} \sum_{i \varepsilon N_j} [P_{kj} W_{kj} F_{kj} (a_{kj} t_{ij}^2 + b_{kj})] x_{ij} + C(m_3 \sum_{i=1}^{I} y_i + n\, m_1) \quad (1)$$

Subject To:

(A) Fire service demand and response time threshold for each demand region must be satisfied.

$$\sum_{i \varepsilon N_j} x_{ij} = 1, \quad \forall\ j \quad (2)$$

(B) Manpower restrictions must be satisfied, i.e., the number of located flying squads must not exceed the maximum number of squads to have, $M_2$, and be equal to or greater than the number of flying squads so that the problem is to be feasible, $M_1$.

$$M_1 \leq \sum_{i=1}^{I} y_i \leq M_2 \quad (3)$$

where,

(M2)

$$M_1 = \underset{\underline{z}}{\text{minimum}} \sum_{i=1}^{I} z_i \quad (4)$$

S. T.

$$\sum_{i=1}^{I} a_{ij} z_i \geq 1, \quad \forall\ j$$

and $z_i = 0, 1 \ \forall\ i$

(C) The workload feasibility must be satisfied.

$$\sum_{j=1}^{J} P_{kj} W_{kj} x_{ij} \leq \alpha_i, \quad \forall \ i \tag{5}$$

(D) The demand region can be served only if it is assigned to the flying squad located.

$$y_i \geq x_{ij}, \quad \forall \ i \text{ and } j \tag{6}$$

(E) The following structure requirements must be satisfied.

$$y_i = 0, 1 \quad \forall \ i \tag{7}$$

$$\text{and} \quad x_{ij} \in [0,1], \quad \forall \ i \text{ and } j \tag{8}$$

Model (M1) is a mixed integer programming program having a similar structure to the Capacitated Warehouse Location Problem (see [24] for an example of this). The solution procedure developed for this problem will be discussed in Chapter V.

# CHAPTER V

# SOLUTION PROCEDURE

This chapter is concerned with the solution of the decision problem that was developed in the previous chapter. The mathematical model (M1) presented in Chapter IV is a mixed integer programming problem having a similar structure to the Capacitated Warehouse Location Problem (CWLP).[1]

Recently many different procedures have been discussed in the literature that have dealt with the solution of CWLP models by mathematical programming methods. Several of these methods will be mentioned in the next section, and then the solution procedure developed for the problem in the current research will be presented. This procedure is a modification of an efficient branch and bound algorithm by Akinc and Khumawala. The required modifications for the solution of the problem in this study are discussed in this chapter. A small sample problem is solved to further illustrate the entire solution procedure.

## Background

The CWLP is a generic name given to the following problem. A number of potential sites are selected where warehouses may be positioned, each

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[1] Although the problem is called the warehouse location problem, it occurs in many other operational situations: vendor selection, missile site selection, facility location-allocation, drilling site selection, lock-box problems, etc. [9].

site having a fixed cost to be incurred if the warehouse is opened there. In addition to the fixed cost, there are transportation costs from the warehouses to the customers and the capacity limitations of the open warehouses. The CWLP is then to select the subset of the potential sites to open which (a) satisfy customer demands (i.e., product sales), and (b) minimizes the sum of the fixed and transportation costs.

The flying squad problem and the CWLP are analogous in the following manner. A number of potential squad sites (i.e., "warehouse" location sites) are known to be available where squads can be housed. Associated with each potential site, there are incurred operating costs associated with manpower, facilities, etc., if a squad is housed there. In addition to the operating cost, there are expected fire losses from the squad locations to the fire scene. The flying squad problem is then to select the subset of potential sites to locate flying squads which (a) satisfy the fire service demand in terms of structural fires (i.e., "product sales") of each region of a city, and (b) minimizes the sum of the flying squad operating costs and expected fire loss.

In its simplest mathematical form, the capacitated warehouse location problem can be expressed as a mixed integer program as follows (with m customers and n potential warehouse sites) [24]:

(P1) $$\text{minimize } Z = \sum_{i=1}^{m} \sum_{j=1}^{m} c_{ij} x_{ij} + \sum_{i=1}^{n} F_i y_i \qquad (1)$$

$$\text{subject to: } \sum_{i=1}^{n} x_{ij} \geq D_j \quad \forall\, j \qquad (2)$$

$$\sum_{j=1}^{m} x_{ij} \leq S_i y_i \quad \forall\ i \tag{3}$$

$$x_{ij} \geq 0 \quad \forall\ i, j \tag{4}$$

and $$y_i = 0 \text{ or } 1 \quad \forall\ i \tag{5}$$

where $c_{ij}$ = the per unit cost of shipping goods from warehouse i to customer j,

$x_{ij}$ = the decision variable denoting the amount of goods shipped from warehouse i to customer j,

$F_i$ = the fixed cost associated with warehouse i,

$y_i$ = the decision variable indicating whether warehouse is opened ($y_i$=1) or closed ($y_i$=0),

$D_j$ = the demand at customer j,

and $S_i$ = the capacity of warehouse i.

The majority of the literature on the CWLP can be classified under two broad catagories: (a) research directed towards developing exact methods to obtain an optimum solution to the problem, and (b) heuristic algorithm for finding "good" solutions to problems with large number of potential warehouses and customers. The exact methods are generally the outgrowth of application of the implicit enumeration method which was first introduced by Land and Doig [29]. The literature on exact algorithms for the CWLP includes the branch and Bound applications by Sá [38] and Davis and Ray [8]; and a modified B&B approach by Ellwein [9].

The heuristic algorithms are computationally efficient methods for finding "good" solution to large sized problems. However, their major drawback is the fact that they yield solutions which can not be guaranteed to be optimum. Heuristic algorithms developed for the CWLP include those of Jandy [22], Sá [39], Khumawala [24], and Akinc and Khumawala [1] (which represents the basis for this study). Each of these approaches will be briefly discussed before presenting the algorithm developed for the problem in the current research.

Sá (1968)

Sá applies a branch and bound technique to solve the CWLP. His algorithm applies several dominance rules at each of the B&B tree nodes, in addition to determining a lower and an upper bound. The lower bound at each node is obtained in the usual manner, by ignoring the zero-one restrictions on free $y_i$'s and solving the resultant linear programming problem. The upper bound at each node is found by considering all free $y_i$ equal to one. Finding the upper bound in this manner facilitates one of his dominance rules that will be discussed later. The minimum of the upper bounds found to date in the tree search constitutes the current upper bound.

In an effort to reduce the size of the B&B tree, Sá uses two dominance rules which identify those nodes that can not contain optimum solution. These rules are as follows:

(a) Maximum Fixed Cost Rule. An upper bound, FMAX, is found on the total fixed cost that a solution can occur. Any solution which has fixed costs in excess of this upper bound is discarded. This bound is calculated as follows:

$$FMAX = g^* - VMIN \tag{6}$$

where $g^*$ is the current lower bound and VMIN is a lower bound on the variable cost that any solution can have. The VMIN is obtained from the solution of the transportation problem which uses all the warehouses. Clearly, any node at which the fixed cost of the open warehouses is greater than FMAX can not have any solution better than $g^*$. Thus, such node can be discarded.

(b) Upper Bound Rule. Sá's use of this rule can be explained as follows. Consider branching by constraining $y_k$, where k is a free warehouse at some non-terminal node. If the upper bound of the closed branch (along $y_k = 0$) exceeds that the open branch (along $y_k = 1$), the node resulting from the closed branch need not be considered any further.

Sá suggests that the success of his algorithm depends partly on the manner in which the next warehouse is chosen to be constrained. He considered two branching decision rules, which are based on the $y_i$ obtained when solving the relaxed problem for the lower bound. Sá's computational experience has shown that the maximum $y_i$ rule makes better use of the upper bound rule, i.e., many branches are pruned. The minimum $y_i$ rule, on the other hand, leads to a good solution (current upper bound) relatively soon, but takes longer to verify optimality due to the ineffectiveness of the upper bound rule.

Sá tested his method on several problems. A majority of these problems were derived from the original data of Kuehn and Hamburger [27]. His B&B method was capable of solving some 16 x 50 and 12 x 12 problems in reasonable time on an IBM 360/65. However, it failed to converge on 25 x 50 problems within 15 minutes of computer time.

Davis and Ray (1969)

Davis and Ray formulated the CWLP in a slightly different manner than given in (P1). Their formulation is "tightier" in the sense that they change constraint set (3) in (P1) into:

$$x_{ij} - \text{Minimum } (D_j, S_i)\, y_i \leq 0 \qquad (7)$$

The presence of constraint (7) forces $y_i$ to take the value of one if any customer's demand is supplied entirely by warehouse i, or if the full capacity of warehouse i is used to supply a single customer.

The B&B algorithm is then applied to the problem in the usual manner. When lower bounds are computed at the nodes, the values of the free $y_i$ usually turn out to be integers (one or zero) due to constraint (7). The faster convergence of nodes to terminal nodes necessitates only a small number of branches to be made. Finding the lower bound by solving this tightier version of the problem is more difficult. The presence of constraint (7) increases the number of constraints to be considered, and wipes out the efficiencies that can be derived from the use of transportation codes. Therefore, the computational efficiency of this algorithm is very much a function of how rapidly the lower bounds are obtained for each node.

Davis and Ray report that using this technique they have solved problems with from 20 to 50 potential warehouses in less than 25 minutes on a Burroughs 5500. Since information on the types of problems solved is not available, it is difficult to comment on the computational efficiency of this algorithm.

Ellwein (1970)

Ellwein developed and used several optimality and feasibility tests that reduce the number of nodes which need to be constrained. These tests are summarized below.

(a) FMAX. The y-vectors having the fixed cost component in excess of FMAX are discarded.

(b) CMIN. For any solution vector y to be feasible, it must have enough capacity to accommodate all the customers' demand. If CMIN = $\Sigma D_j$, then any solution vector encompassing a total capacity level (sum of the capacities of open warehouses) which is less than CMIN is dropped from consideration.

(c) VMIN. When all the warehouses are considered open, the solution (VMIN) to the resultant transportation problem is a lower bound on the transportation cost of any other solution. When warehouse i is deleted from the transportation problem resulting in VMIN, the optimum cost will clearly increase; say to $VMIN_i$, i.e.,

$$VMIN_i - VMIN = \Delta V_i \geq 0 \tag{8}$$

$\Delta V_i$ can be termed the lower bound on the reduction in transportation costs due to having warehouse i in the solution as opposed to not having it. If, for any warehouse k, this bounds exceeds its fixed cost, then warehouse k must be open in the optimum solution, thus all the y-vectors with $y_k = 0$ can be dropped from consideration.

Ellwein applies this test only at the initial node when all the warehouses are free because applying this test at every nodes can be computationally very expensive. Note that the application of the VMIN rule would require the solution of $S[K_2^{(t)}] + 1$ transportation problems.[2]

(d) Duality Constraints. Several constraints are obtained from the duality properties of the transportation problems to further reduce the number of y-vectors to be evaluated explicitly. These constraints are found and used as follows: Given a particular solution vector $\bar{y}$, the dual of (P1), DL($\bar{y}$) can be written as

$$\text{Max } g = \sum_{i=1}^{n} F_i y_i + \sum_{j=1}^{m} v_j D_j - \sum_{i=1}^{n} u_i S_i y_i \qquad (9)$$

$$\text{DL}(\bar{y}) \quad \text{subject to:} \quad v_j - u_i \leq c_{ij} \quad \forall\ i \text{ and } j \qquad (10)$$

$$\text{and} \quad u_i,\ v_j \geq 0 \quad \forall\ i \text{ and } j \qquad (11)$$

Let ($y^*$, $g^*$) be the incumbent solution and $u^*$ and $v^*$ be the corresponding vectors of dual activities. For any other vector y to have a solution value less than $g^*$, it must satisfy the following "duality constraints":

$$\sum_{i=1}^{n} F_i \hat{y}_i + \sum_{j=1}^{m} v_j^* D_j - \sum_{i=1}^{n} u_i^* S_i \hat{y}_i < g^* \qquad (12)$$

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[2] $S[K_2^{(t)}]$ is used to denote the number of free warehouses at the $t^{th}$ node.

therefore, any $\hat{y}$ not satisfying constraint (12) can not have $g < g^*$, and it can be dropped from further consideration.

Ellwein reports that his algorithm was able to solve the 16 site problems of Sá [38] in less than 3 minutes on IBM 360/67.

Jandy (1967)

Jandy obtains an approximation solution to the CWLP by solving a sequence of transportation problems. To describe the procedure, let $[c_{ij}^{(v)}]$ be the per unit transportation cost used for the solution of the $v^{th}$ transportation problem, and $[x_{ij}^{(v)}]$ be the corresponding solution. $c_{ij}^{(v)}$ is given by:

$$c_{ij}^{(v)} = \begin{cases} c_{ij} + F_i/S_i, \text{ if } v=1 \\ \\ c_{ij} + F_i/\sum_j x_{ij}^{(v-1)}, \text{ if } v>1 \end{cases} \tag{13}$$

These transportation problems are iteratively solved until $x_{ij}^{(v-1)} = x_{ij}^{(v)}$, for all i and j, i.e., the solution converges to $x_{ij}^{(v)}$. The corresponding design, $\dot{y}$ is obtained from:

$$\dot{y} = \begin{cases} 1 \text{ if } \sum_j x_{ij} > 0 \\ \\ 0 \quad \text{otherwise.} \end{cases}$$

At this point the algorithm considers pairwise interchanges between open and closed warehouses. The algorithm terminates when no more pairwise exchanges can profitably be made.

Jandy does not report any computational results except for four small sample problems that can be solved by hand. Therefore, the quantity of the procedure for large problems is not known.

Sá (1969)

As mentioned earlier, Sá's B&B procedure was incapable of solving large sized problems with a reasonable computational effort. This led him to devise a heuristic method which consists of two phases. Phase I is similar to the "add" heuristic of Kuehn and Hamburger [27] and the "drop" heuristic of Feldman, Lehrer and Ray [11]. Phase II, on the other hand, considers profitable pairwise exchange of closed and open warehouses in an attempt to improve the solution found at the end of Phase I.

The algorithm starts with all the warehouses open and closes one warehouse at a time whenever doing this reduces the total cost. This "drop" process is carried out until no more drops can be made without increasing the total cost. At this point, the algorithm attempts to "add" previously closed warehouses. Whenever a warehouse is profitably added, i.e., with a resultant decrease in the total cost, the algorithm returns to the "drop" process to see if any other warehouses can now be dropped. When no "drop" or "add" is possible Phase I ends with a solution which represents a total optimum.

The above solution indicates that the solution at hand can only be improved by interchanging elements taken from the sets of open and closed warehouses. In Phase II, Sá proposes to consider not all, but only simple exchanges, i.e., replacing an open warehouse with a closed one. Whenever a single exchange is found profitable, the algorithm leaves Phase II after making the interchange, and enters Phase I to investigate the possibility of further "drops" and "adds". When no single exchanges are possible in Phase II the procedure terminates with the heuristic solution.

Sá reports favorable computational results on the modified Kuehn and Hamburger [27] problems.

Khumawala (1973)

Khumawala's method aims at tracing a few paths (from the initial node to a terminal node) of the B&B tree to reach candidate solutions. The basis of the algorithm is the computation of lower and upper bounds ($\Delta_i$ and $\Omega_i$, respectively) on the savings in variable costs that accrue from having a warehouse i in the solution as opposed to not having it. They are computed as follows:

(a) Lower Bound ($\Delta_i$). When the warehouse i is added to the problem, the solution improves for each customer's unit demand by

$$\nabla_{ij} = \min_{k \epsilon K_1 \cup K_2 - \{i\}} [\ \max(c_{kj} - c_{ij}, 0)\ ] \text{ for } i \ \epsilon\ K_2 \tag{14}$$

where $K_1$ and $K_2$ are used to denote sets containing the indices of opened and free warehouses, respectively. Then, $\Delta_i$ is the solution to the following simple knapsack problem:

$$\text{(P2)} \qquad \text{Max} \ \sum_j \nabla_{ij} D_j t_j \tag{15}$$

$$\text{subject to:} \quad \sum_j D_j t_j \le S_i \quad \forall\, i \tag{16}$$

$$\text{and} \quad 0 \le t_j \le 1 \quad \forall\, j \tag{17}$$

(b) Upper Bound ($\Omega_i$). Let $u_r$, $r \varepsilon K_1$ and $V_j$ ($j=1,2,\ldots,n,n+1$) be the optimum dual variables associated with the solution $Z_r(K_1)$ where n+1 is the dummy customer and $Z_r(K_1)$ is the optimal solution to the transportation problem with only the warehouses contained in the set $K_1$. Let,

$$w_{ij} = \text{Min } (c_{ij} + V_{n-1} - V_j, 0) \quad j=1,2,\ldots, n \tag{18}$$

Note that $w_{ij} \ge 0$ and measures the per unit savings in the variable cost of customer j's demand if supplied from warehouse i. Then, the upper bound is the solution to the following simple knapsack problem:

$$\text{(P3)} \qquad \text{Max} \quad \sum_j w_{ij} D_j h_j \tag{19}$$

$$\text{s.t.} \quad \sum_j D_j h_j \le S_i \quad \forall\, i \tag{20}$$

$$0 \le h_j \le 1 \quad \forall\, j. \tag{21}$$

The algorithm starts by tracing a "good" path with all the warehouses free and computes $\Delta_i$ for all the warehouses. If $\Delta_i > F_i$ for any i, then warehouse i is fixed open. Given the set of fixed open warehouses, $\Omega_i$'s are computed as the upper bound on the savings in variable costs if warehouse i's were opened. If $\Omega_i < F_i$ for any i, then warehouse i is fixed closed. Note that whenever a warehouse is fixed open (closed), the $\Omega_i(\Delta_i)$ of the remaining free warehouses change. This allows for repetitive attempts to fix the warehouses open and closed. This cycle is carried out until no more warehouses can be fixed open or fixed closed. At this time, the algorithm selects a free warehouse to be constrained (open or closed). The algorithm alternates between fixing warehouses open or closed whenever possible, and constraining until a terminal solution is reached. This terminal solution is an upper bound to the problem. The heuristic decision rules that guide the selection of warehouses to be constrained are based on $\Delta_i$, $\Omega_i$, and $S_i$ of the free warehouses.

Depending on the six particular branch selection rules used, the procedure generates several candidate solutions, all of which may not necessarily be different. Since each solution is very efficiently obtained, Khumawala suggests generating all of these solutions and then naming the one with the least cost as the heuristic solution to the problem.

Khumawala reports quiet encouraging computational results, in terms of both the quality of the solution and the computational time and storage requirements. The computational tests were made on a large class of problems including all the test problems used by Sá [38] and Ellwein [9].

## The Akinc and Khumawala Procedure

The procedure that will be used to solve the problem of the current decision model is based on an efficient branch and bound algorithm that was developed by Akinc and Khumawala in 1977. One of the major reasons that this procedure was selected over those mentioned in the previous section is that this procedure appears to be computationally the most efficient. Its computational efficiency enables it to obtain rapidly "good" solutions to large problems. Akinc and Khumawala claim that their procedure obtained optimal solutions to the majority of relatively large problems at reasonable computational time and storage, whereas Sa' and Ellwein either had not attempted these problems, or had to terminate computation prematurely due to excessive computer time and/or storage [1].

The basic steps in the Akinc and Khumawala procedure are determining (a) the criterion for fixing free[3] squads to open, (b) the criterion for fixing free squads closed, (c) the lower bound, (d) the upper bound, (e) node selection rules, and (f) branch selection rules. The procedure ends when no further improvement can be made, i.e., lower bound is as large as the best solution obtained so far. A detailed discussion of the various parts of this procedure is given in Appendix B.

## Modifications to the Akinc and Khumawala Procedure

Several modifications were required in order to use the basic procedure developed by Akinc and Khumawala for the solution of the problem in this study. In essence, the Akinc and Khumawala procedure was developed to obtain a "good" solution without the limitation on the number of warehouses to open. Their procedure for solving a CWLP, like all B&B procedures, generates a sequence of partial assignments and analyzes the

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[3]The terms "open", "closed", and "free" squads will respectively mean a potential squad assigned to be use, a potential squad assigned not to be used, and a potential squad assigned not yet assigned either open or closed.

completions of each in search of successively better feasible solutions. Their procedure opens and closes as many warehouses at a time as they can be profitably (when a decrease in a total cost is realized) opened and closed, and terminates when no better feasible solution can be obtained.

The mathematical model developed in Chapter IV contains a constraint which limits the number of flying squads to open. Also, the model developed for the current research has an imbedded set-covering problem which allows the determination of the lower bound of the number of squads to open so that the problem is feasible. Further, the transportation costs from warehouse to customer used in the Akinc and Khumawala procedure is expressed as a simple multiple of the distance between two points, whereas the flying squad problem deals with a non-linear fire loss function.

The modifications required to adjust the above mentioned structural differences between the models of a CWLP and a flying squad problem will be described in detail below.

The first modification required was to open the squads one at a time. This approach is similar to the "add" heuristic of Kuehn and Hamburger [27]. By doing so, the solution can be obtained without exceeding the maximum allowable number of squads to open, i.e., the procedure terminates when the number of open squads reaches the maximun allowable number of squads. Also, a sensitivity analysis on the system such as cost/benefit ratio as an additional squad is added to the system can be done with already available information, thereby avoiding additional computations. This modification is accomplished by opening squad "t" from the set of free squads with the largest non-negative difference between the lower bound on the savings in fire losses due to having squad t open, and the operating cost associated with flying squad t.

In a mathematical form, this modification is expressed as follows: when squad i is added to the problem, the solution improves for each region's unit demand by [8],

$$\nabla_{ij} = \min_{k\varepsilon K_1 \cup K_2 - \{i\}} \{\max(c_{kj} - c_{ij}, 0)\} \tag{22}$$

The lower bound on the savings in fire losses, $\Delta_i$, is the solution to

(P4)

$$\max_{\underline{x}} \Delta_i = \sum_j \nabla_{ij} x_{ij} \tag{23}$$

$$\text{s.t.} \quad \sum_j x_{ij} \leq S_i \quad \forall\ i \tag{24}$$

$$\text{and } 0 \leq x_{ij} \leq D_j \tag{25}$$

where, $K_1$ and $K_2$ represent the sets containing the indices of squads to be opened and unconstrained (free), respectively,

$c_{ij}$ represents the per unit fire loss from squad i to region j,

$D_j$ represents the fire service demand of region j,

$x_{ij}$ represents the amount of fire service demand supplied from squad i to region j,

$S_i$ represents an upper limit on the workload of squad i,

$F_i$ represents the operating cost associated with squad i.

Then, select squad "t" such that

$$\Delta_t - F_t = \max_{i\varepsilon K_2} (\Delta_i - F_i) \geq 0 \tag{26}$$

and open the squad t.

The second modification of the Akinc and Khumawala procedure concerns an imbedded set-covering problem. The solution to this set-covering problem constitutes the lower bound on the number of squads to have so that the solution is feasible. Since the solution is found only with regard to response time constraints, it is the lower bound to the problem. When the workload of the squad is large compared to the fire service demand, this lower bound would also be one of the feasible solutions to the problem. This second modification allows terminating the procedure when the lower bound computed from the set-covering problem exceeds the maximum allowable number of squads to open, thereby avoiding further computations. When this is the case, there is no solution to the problem.

The computation of this lower bound is based on a set-covering problem as follows:

(P5)

$$\text{minimum } M_1 = \sum_i^I z_i \tag{27}$$

$$\text{s.t. } \sum_i^I a_{ij} z_i \geq 1, \ \forall\, j \tag{28}$$

$$\text{and } \quad z_i = 0, 1 \ \forall\, i \tag{29}$$

where, $M_1$ represents the lower bound on the number of squads so that the problem is feasible,

$$z_i = \begin{cases} 1, & \text{if squad i is open,} \\ 0, & \text{otherwise} \end{cases}$$

$$a_{ij} = \begin{cases} 1, & \text{if region } j \text{ is covered (i.e., region } j \text{ is located within} \\ & \text{the response time threshold from squad } i\text{) by squad } i \\ 0, & \text{otherwise} \end{cases}$$

$I$ represents the number of potential sites for squads,

$J$ represents the number of regions in a city.

To solve the set-covering problem, exact and heuristic algorithms may be applied. When the number of decision variables is small, the exact B&B method of Land & Doig [29] would be appropriate.[4] However, when the number of decision variables becomes large, a heuristic method should be used. Ignizio [21] reports that his heuristic solution to set-covering problem is able to handle problems upto 800 x 800 with reasonable computer time and storage requirements. The size of flying squad problem may vary from city to city depending on its population and area. If the census track is used as a basic area (region) and if each census track is to have a potential site for a squad, then the size of the problem will be m by m, where m is the number of census tracks in a city. In Table 6, the number of census tracks for many cities are listed.

The last modification in the Akinc and Khumawala procedure concerns with the variable cost matrix from squads to regions. Akinc and Khumawala tested their procedure against problems of which transportation cost is expressed as a linear function of distance. As mentioned earlier, the

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[4]Using the Toregas, et. al [45] test problem of $I = 30$ and $J = 30$, the optimum solution to model (P5) was obtained using the Land & Doig algorithm with a CPU time of 4 seconds on an IBM 370/165.

Table 6. Census Track [52]

(1960)

| City, State | No. of Census Tracks | Population in 1,000 | No. of Housing Units in 1,000 |
|---|---|---|---|
| Albuquerque, N.M. | 43 | 201 | 60 |
| Atlanta, GA. | 109 | 487 | 154 |
| Chicage, IL. | 939 | 3,550 | 1,124 |
| Columbia, S.C. | 28 | 97 | 28 |
| Denver, CO. | 98 | 493 | 174 |
| Fort Worth, TX. | 80 | 456 | 125 |
| Indianapolis, ID. | 145 | 476 | 158 |
| Las Vegas, NV. | 14 | 64 | 22 |
| Lincoln, NB. | 34 | 128 | 43 |
| Long Beach, CA. | 71 | 344 | 133 |
| Manhattan, N.Y. | 323 | 1,698 | 727 |
| Memphis, TN. | 103 | 497 | 151 |
| Miami, FA. | 59 | 291 | 120 |
| Nashville, TN. | 43 | 170 | 53 |
| New Orleans, LA. | 162 | 627 | 202 |
| Philadelphia, PA. | 378 | 2,002 | 649 |
| Pittsburg, PA. | 189 | 604 | 196 |
| Richmond, VA. | 61 | 219 | 69 |
| San Francisco, CA. | 135 | 740 | 310 |
| San Jose, CA. | 67 | 204 | 68 |
| St. Louis, MO. | 126 | 750 | 262 |
| Salt Lake, City, UT. | 49 | 189 | 64 |
| Seattle, WA. | 120 | 557 | 215 |
| Washington, D.C. | 96 | 763 | 262 |

fire loss function used in the model developed in Chapter IV is to be expressed as a non-linear relationship with response time. This last modification can be easily accomplished by assigning the values of non-linear fire loss function from sites to each region in a city into a "fire loss" matrix. For example, the fire loss matrix can be constructed from a "response time" matrix, T, as follows:

Response Time Matrix (T)

| Potential sites | Demand Regions 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| A | 1 | 2 | 3 | 4 | M* | 4 |
| B | 3 | 2 | 1 | 3 | M | 2 |
| C | 4 | 3 | M | 2 | 1 | 1 |
| D | 3 | 4 | 1 | M | 2 | 3 |

M* is used to denote that the route is not permitted.

If the fire loss function is expressed as, say, $t_{ij}^{2} + 5$, substitute each element in a "T" matrix into the fire loss function, and construct a fire loss matrix, F, as follows:

Fire Loss Matrix (F)

| Potential sites | Demand Regions 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| A | 6 | 9 | 14 | 21 | $\infty$ | 21 |
| B | 14 | 9 | 6 | 14 | $\infty$ | 9 |
| C | 21 | 14 | $\infty$ | 9 | 6 | 6 |
| D | 14 | 21 | 6 | $\infty$ | 9 | 14 |

With these modifications in mind, in the next section the solution procedure that has been developed for the problem being considered in this study, (M1), will be presented.

## Solution Procedure for "Flying-squad" Problem

A detailed step by step statement of the solution procedure for model (M1) is presented in this section. A flow chart is given in Figure 5 to facilitate its exposition. A small sample problem is also solved to further illustrate the entire solution procedure. In addition to the notation presented in Chapter IV, the following terminology will be used to present the solution procedure.

### Terminology

$K_0^{(t)}$ represents the closed station set at node t,

$K_1^{(t)}$ represents the open station set at node t,

$K_2^{(t)}$ represents the free station set at node t,

NS represents the number of open station,

z* represents the current upper bound,

$U^{(t)}$ represents the upper bound for node t,

$L^{(t)}$ represents the lower bound for node t,

TD represents total fire service demand,

$P_1^{(t)}$ represents the total capacity (i.e., workload) of the open stations at node t. This can be obtained by adding the capacity of the stations that are opened,

$P_2^{(t)}$ represents the maximum capacity of any complete design of the current node t. This is given by

$$\sum_{i \varepsilon K_1 U K_2} \alpha_i,$$

$\Delta_i$ represents a lower bound on the savings in fire losses due to having station i open,

$\Omega_i$ represents an upper bound on the savings in fire losses due to opening station i.

The computations of $U^{(t)}$, $L^{(t)}$, $\Delta_i$, and $\Omega_i$ are presented in Appendix B.

Step 1: Data Input. Obtain and input necessary data concerning the problem under consideration.

Step 2: Problem Formulation. Formulate the decision problem model as presented in model (M1).

Step 3: Set-Covering Problem (A1-A3).[5] Solve the set-covering problem (model M(2)) and find the lower bound, $M_1$, on the number of stations to open. If this lower bound exceeds the maximum allowable number of stations to open, $M_2$, the problem is infeasible and stop. Otherwise, continue.

Step 4: Initialization (A4). Start with all the potential stations as free stations. At node 1, therefore, the free stations, i.e., set ($K_2^{(1)}$) consists of the indices of all the potential stations; the open and the closed stations sets, $K_1^{(1)}$, and $K_0^{(1)}$ are empty. The current upper bound, z*, and initial lower bound, $L^{(1)}$, are set equal to $\infty$ and 0 by default, respectively. The number of open stations, NS, is zero at the initial node. The total capacity of the open stations at the initial node ($P_1^{(1)}$) is set to zero. $P_2^{(1)}$ is computed as the sum of the capacities of all stations. This quantity is computed at every node in order to avoid branching to infeasible nodes, i.e., nodes from which no feasible designs can emanate.

Step 5: Fixing Free Station Open (A5-A11). The $\Delta_i$'s are computed for

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[5] Figure in the parenthesis refer to the steps in flow chart in Figure 5.

all free stations to see if a station can be fixed open. If there are some stations with positive $\Delta_i$, the corresponding station with the largest $\Delta_i$ is fixed open by transferring its index from the free set $K_2^{(t)}$ to the open set $K_1^{(t)}$ and adding its capacity to $P_1^{(t)}$. Also, increase the number of open stations, NS, by one. If the number of open stations thus far reaches the maximum allowable number of stations, $M_2$, the procedure stops, and the incumbent is the best solution. If the $\Delta_i$'s computed for the free stations are all negative, go to Step 7. If the free station set $K_2^{(t)}$ is empty, go to Step 7. Otherwise, go to Step 6.

Step 6: Fixing Free Stations Closed (B1-B6). As can be seen from the flow chart in Figure 5, the $\Delta_i$ and $\Omega_i$ are applied cyclically, i.e., if the $\Delta_i$ test is applied and if it is successful in fixing open a station, then the $\Omega_i$ test is applied to see if any stations can be fixed closed. The cycle stops when no more stations can be fixed open or fixed closed. The procedure now computes the $\Omega_i^{(t)}$ for all free stations. If the open set $K_1^{(t)}$ is such that $P_1^{(t)}$ is less than the total fire service demand, TD, no stations can be fixed closed. If there is sufficient capacity obtained in the set $K_1^{(t)}$, the procedure investigates the possibility of fixing stations closed on the basis of their negative $\Omega_i^{(t)}$. If there are free stations with negative $\Omega_i^{(t)}$, they are fixed closed by updating the appropriate sets and quantities. If the free station set $K_1^{(t)}$ is empty, go to Step 7, otherwise go to Step 5.

Step 7: Node Evaluation (C1-C9). When the cycle ends, the node evaluation procedure starts. First the upper bound for the current node, $U^{(t)}$, is computed. A check is made to see if $U^{(t)}$ is better (smaller) than the current upper bound, z*. If so, the current upper bound is updated, i.e., $z^* = U^{(t)}$, and all the nodes whose lower bound, $L^{(t)}$, is greater than the current upper bound, z*, are discarded (C4). Whenever the current node becomes terminal, i.e., $K_2^{(t)} = \emptyset$, it is discarded since no more branches can be made from it. If the current node is not terminal, the corresponding lower bound is computed, and the node is added to the list of active nodes if its lower bound does not exceed the current upper bound, z*.

If the upper bound is not any better than the current upper bound, i.e., $U^{(t)} \geq z^*$, a check is made to see if the current node is terminal. If so, it is discarded from the list of active nodes. If the current node is not terminal, i.e., $K_2^{(t)} \neq \emptyset$, the lower bound is computed. If the lower bound exceeds the value of the incumbent solution, the current node is again discarded. Otherwise, the node is added to the list of the active nodes, and the procedure goes to Step 8.

Step 8: Node Selection (D5-D7). A node is selected for further branching only if the previous branching operation is complete (see Step 9), i.e., the nodes corresponding to both open and the closed branches have been evaluated. A check is made to insure that the list of the active nodes is not empty. If the list is empty, the procedure terminates and the incumbent is the best solution. If some active nodes still exist, a node with the least lower bound is selected to be branched from.

Step 9: Branch Selection (D1-D4, D8-D9). A free station is selected according to some Branch Selection Rule, i.e., the largest $\Omega$ rule (see

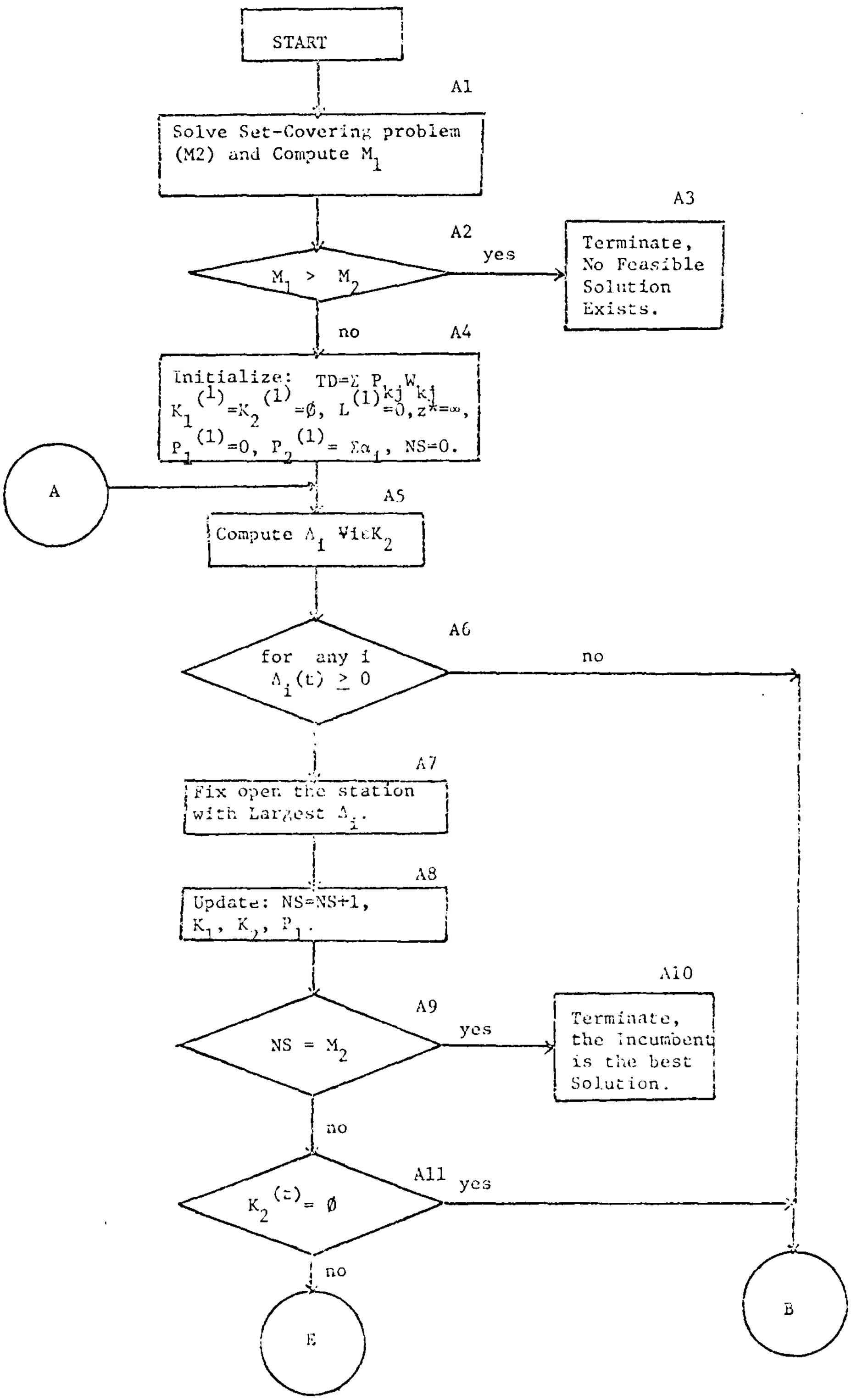


Figure 5. Flow Chart

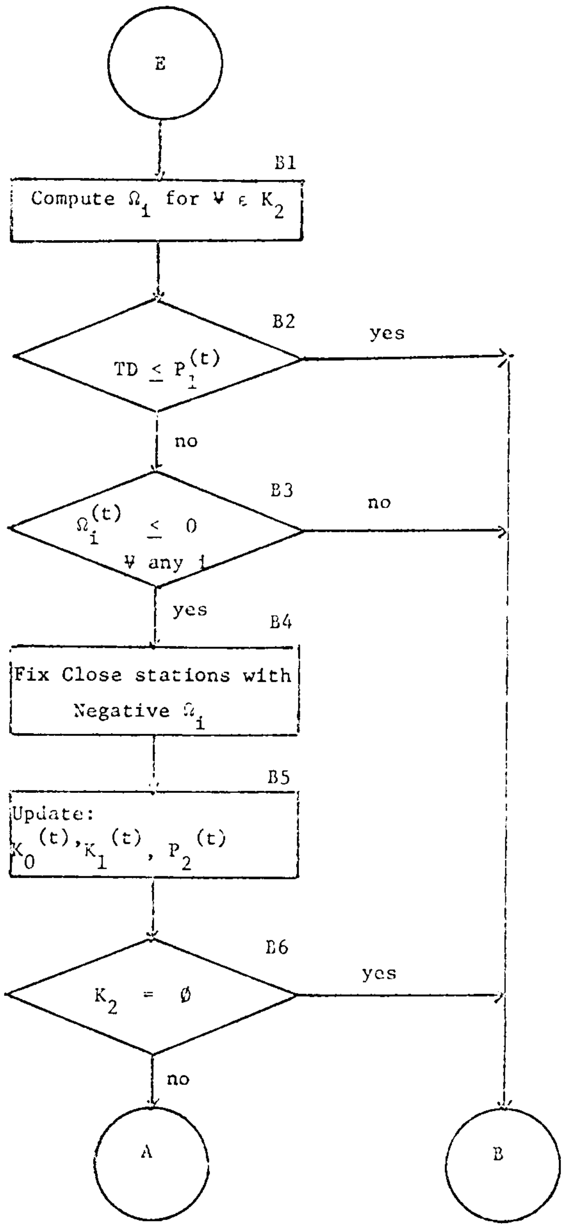


Figure 5 (cont.) Flow Chart

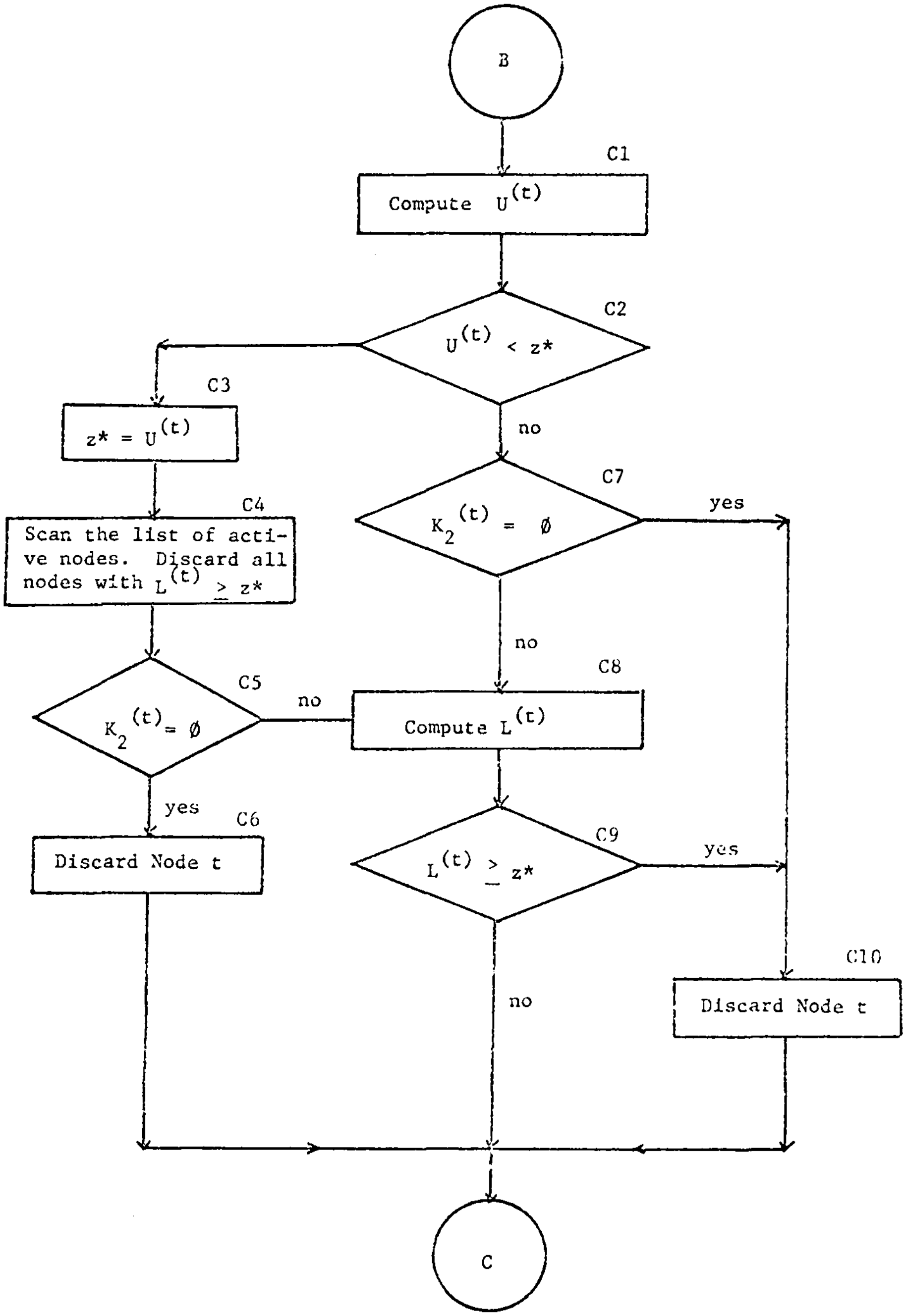


Figure 5 (cont.) Flow Chart

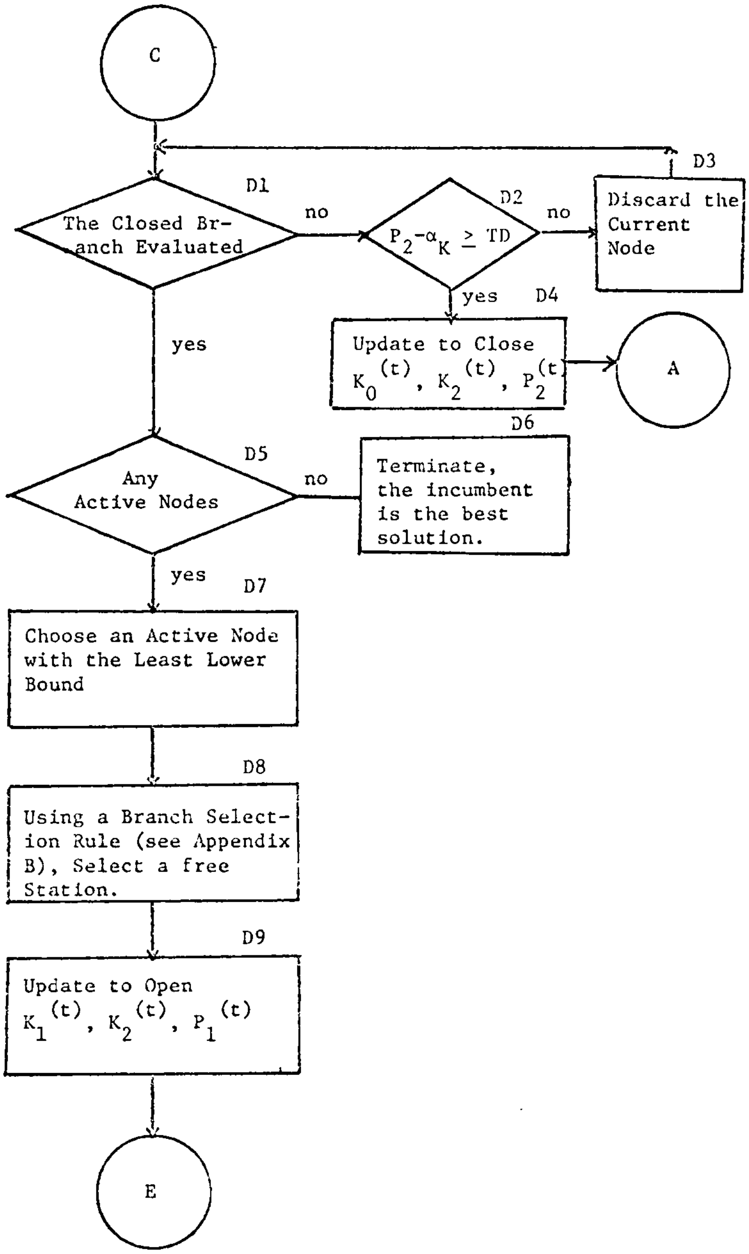


Figure 5 (cont.) Flow Chart

Appendix B for a discussion of this). The selected free station k is constrained open by updating sets $K_1^{(t)}$, $K_2^{(t)}$, and the quantities NS and $P_1^{(t)}$. The procedure goes to Step 6 to see if any free stations can be fixed closed. When the open branch operation is complete, the closed branch is to be evaluated. A test is made to insure that closing the selected station does not result in an infeasible node. If infeasibility does not arise, the procedure closes the selected station by updating the sets $K_0^{(t)}$, $K_2^{(t)}$, and the quantity $P_2^{(t)}$. The procedure now enters Step 5 to see if a station can be fixed open for the remaining free stations.

## Example Problem

A small example problem is now solved in order to better understand the solution procedure for Steps 3 through 9. The data for this problem is given is Table 7. A complete B&B tree corresponding to this problem is presented in Figure 6. The maximum allowable number of stations to open, $M_2$, is set to equal to 4 in this problem.

Step 3: Set-Covering Problem. A set-covering problem is solved using the set-covering coefficient matrix given below. The coefficient was assigned the value of 1 if the route is permitted, and 0, otherwise. The lower bound, $M_1$, is found to be 3 which is less than the maximum allowable number of stations to open, $M_2$. Therefore, there exists a feasible solution, and the procedure proceeds to Step 4.

Step 4: Initialization. Initialize the sets:

$$K_0^{(1)} = K_1^{(1)} = \emptyset,\ K_2^{(1)} = \{A,B,C,D\},$$

Table 7. Problem Data for the Sample Problem

| Operating Cost | Potential Sites (i) | Fire Losses $[g(t_{ij} \mid m_1)]$ Demand Regions (j) | | | | | | | Workload ($\alpha_i$) |
|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | |
| 100 | A | 6 | 5 | 16 | M* | M | 17 | 11 | 35 |
| 70 | B | M | 16 | 5 | 9 | 21 | M | M | 30 |
| 60 | C | 16 | 9 | M | 16 | M | 9 | 11 | 35 |
| 110 | D | M | 14 | 15 | 7 | 9 | 13 | M | 50 |
| Fire Service Demand ($P_{kj}$ $W_{kj}$) | | 15 | 10 | 10 | 15 | 5 | 15 | 20 | |

M* indicates that the route is not permitted.

and the quantities:

$$NS = 0,\ z^* = \infty,\ L^{(1)} = 0,\ P_1^{(1)} = 0,$$
$$P_2^{(1)} = 150, \text{ and } TD = 90.$$

Set-Covering Coefficient Matrix

| Potential Sites | Demand Regions | | | | | | |
|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| A | 1 | 1 | 1 | 0 | 0 | 1 | 1 |
| B | 0 | 1 | 1 | 1 | 1 | 0 | 0 |
| C | 1 | 1 | 0 | 1 | 0 | 1 | 1 |
| D | 0 | 1 | 1 | 1 | 1 | 1 | 0 |

Step 5: Fixing Free Station Open. Applying the $\Delta_i$ procedure on the free stations A, B, C, and D the values are computed as:

$$\Delta_A = 90,\ \Delta_B = 30,\ \Delta_C = 0,\ \Delta_D = -20.$$

Since station A has the largest positive $\Delta_i$, station A is fixed open. The sets and quantities are updated as:

$$K_1^{(1)} = K_1^{(1)} \cup \{A\} = \{A\}\ ,\ K_2^{(1)} = K_2^{(1)} - \{A\} = \{B,C,D\}$$
$$NS = NS + 1, \text{ and } P_1^{(1)} = 0 + 35 = 35.$$

The number of open stations so far is less than $M_2$, and the free station set $K_2^{(1)}$ is not empty. Therefore, the procedure goes to Step 6.

Step 6: Fixing Free Stations Closed. $\Omega_i^{(1)}$'s for all free stations are computed which yield:

$$\Omega_B = 40,\ \Omega_C = -60, \text{ and } \Omega_D = -100.$$

Since $TD = 90 > 35 = P_1^{(1)}$, no free stations can be fixed closed. The procedure goes to Step 7.

Step 7: Node Evaluation. Since $P_1^{(1)}$ is less than TD, the upper bound remains $\infty$. The test at (C7) indicates that the node is not terminal.

Thus $L^{(1)}$ is computed which gives 840. This initial node is added to the list of active nodes, and the procedure goes to Step 8.

Step 8: Node Selection. The initial node which is the only active node so far is selected to be branched from (D7).

Step 9: Branch Selection. Since this is the first time to branch, the procedure starts from step D8. Using the largest $\Omega$ selection rule, station B is selected. Therefore, for the open branch node, the sets $K_1^{(2)}$ and $K_2^{(2)}$ are updated as:

$$K_1^{(2)} = K_1^{(1)} \cup \{B\} = \{A,B\},\ K_2^{(2)} = K_2^{(1)} - \{B\} = \{C,D\}$$

$$\text{and } P_1^{(2)} = P_1^{(1)} + \alpha_B = 35 + 30 = 65.$$

The procedure now goes to Step 6 to see if any free stations can be fixed closed.

Step 6: Fixing Free Stations Closed. $\Omega_i^{(2)}$ for free stations C and D are computed as:

$$\Omega_C = 60, \text{ and } \Omega_D = 40.$$

Since TD = 90 > 65 = $P_1^{(2)}$, no free stations can be fixed closed (B2). The procedure goes to Step 7 to evaluate $L^{(2)}$ and $U^{(2)}$.

Step 7: Node Evaluation. $P_1^{(2)} <$ TD implies that $U^{(2)} = \infty$. The tests at step C2 and C7 lead the procedure to compute the lower bound for the current node 2. The lower bound, $L^{(2)}$, is found as 900. This current node 2 is added to the list of active nodes. The last branching operation is not complete, hence the closed branch (corresponding to station B) is now evaluated. The test at step D2 reveals that station B can be constra-

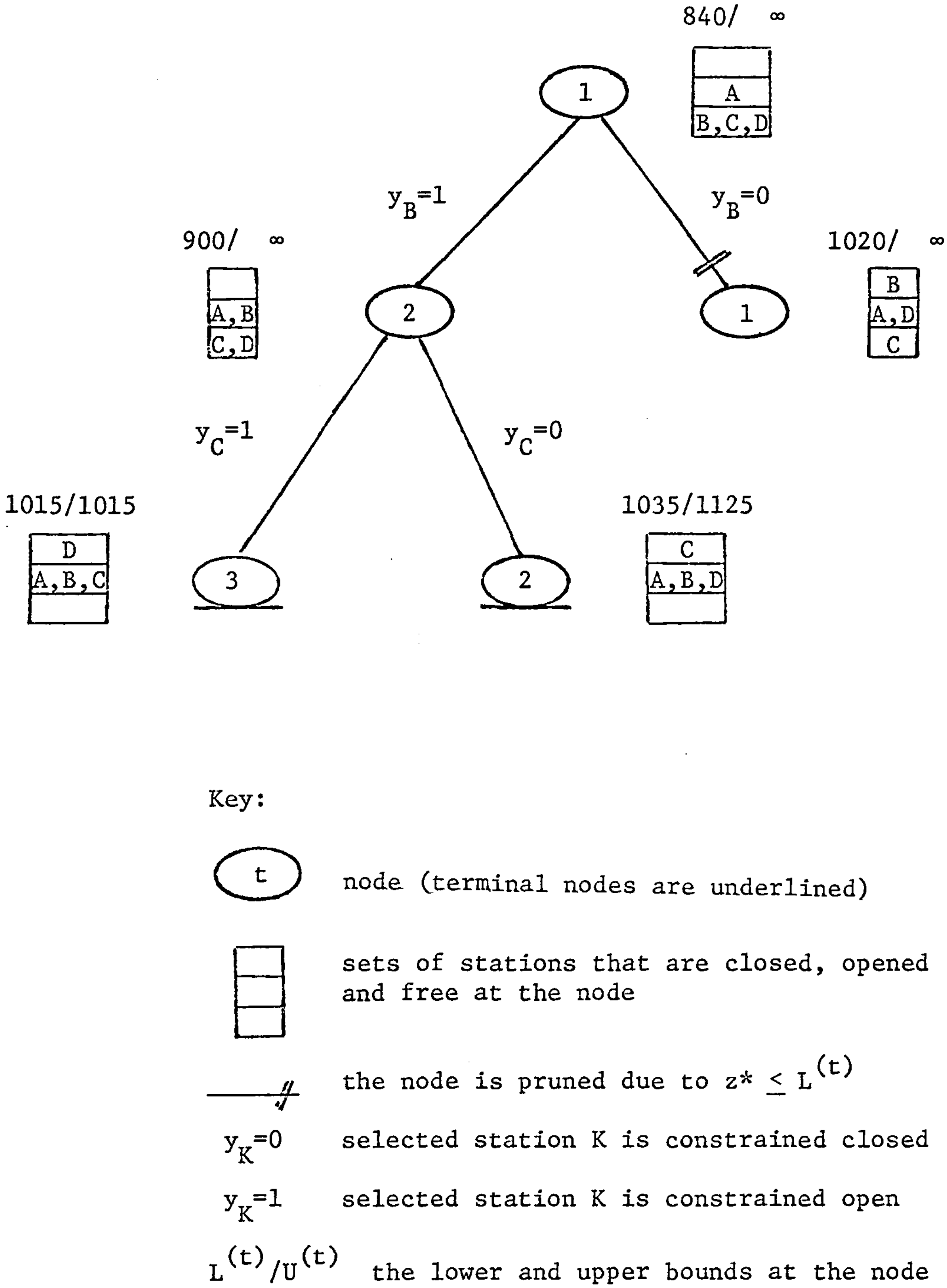


Figure 6. The B&B tree for the Sample Problem

ined closed since $P_2^{(1)} = 150 - 30 = 120 > TD = 90$. The sets are updated as:

$$K_0^{(1)} = K_0^{(1)} \cup \{B\} = \{B\}$$

$$K_2^{(1)} = K_2^{(1)} - \{B\} = \{C,D\}.$$

Now, the procedure goes to Step 5 to see if a free station can be fixed open.

Step 5: Fixing Free Station Open. $\Delta_i$ for free stations C and D are computed as:

$$\Delta_C = 0, \text{ and } \Delta_D = 4985.$$

Since station D has the largest positive $\Delta_i$, station D is fixed open. The sets and quantities are updated as:

$$K_1^{(1)} = K_1^{(1)} \cup \{D\} = \{A,D\}\ ,\ K_2^{(1)} = K_2^{(1)} - \{D\} = \{C\}$$

$$P_1^{(1)} = P_1^{(1)} + \alpha_D = 35 + 50 = 85,\ NS = NS + 1 = 2.$$

The number of open stations is still less than four. Therefore the procedure goes to Step 6 to see if free station c can be fixed closed.

Step 6: Fixing Free Station Closed. $\Omega_i^{(1)}$ for free station C is found as 190. Since $TD > P_1^{(1)}$, free station C can not be fixed closed(B2). The procedure goes to Step 7 to evaluate $L^{(1)}$ and $U^{(1)}$.

Step 7: Node Evaluation. Again, $P_1^{(1)} < TD$ implies that $U^{(1)} = \infty$. The lower bound is calculated as 1,020.

Up to this point one full branching operation is completed. Presently there are two (2) active nodes with the information given in Table 8. Having a branch operation completed, the procedure now goes to Step 8 to

Table 8

Information on the Active Nodes

| | Node 1 Closed Branch | Node 2 Open Branch |
|---|---|---|
| $K_0^{(t)}$ | B | $\emptyset$ |
| $K_1^{(t)}$ | A, D | A, B |
| $K_2^{(t)}$ | C | C, D |
| $U^{(t)}$ | $\infty$ | $\infty$ |
| $L^{(t)}$ | 1,020 | 900 |

select a node from the list of active nodes for further branching. The active node 2 which has the smallest lower bound is selected.

Step 9: Branch Selection. Among two free stations C and D, station C is selected according to the largest $\Omega$ selection rule. For the open branch (node 3), the following sets and quantities are updated:

$$K_1^{(3)} = K_1^{(2)} \cup \{C\} = \{A,B,C\}$$

$$K_2^{(3)} = K_2^{(2)} - \{C\} = \{D\}, \; K_0^{(3)} = \emptyset$$

$$NS = NS + 1 = 3, \text{ and } P_1^{(3)} = P_1^{(2)} + \alpha_C = 100.$$

Now, the procedure goes to Step 6 to see if free station D can be fixed closed.

Step 6: Fixing Free Station Closed. $\Omega_i^{(3)}$ for free station D is found as -20. Since TD< $P_1^{(3)}$ and $\Omega_D^{(3)} \leq 0$, station D is fixed closed. The sets are updated as:

$$K_0^{(3)} = K_0^{(2)} \cup \{D\} = \{D\}$$

$$K_1^{(3)} = \{A,B,C\}$$

$$\text{and } K_2^{(3)} = K_2^{(2)} - \{D\} = \emptyset.$$

As there are no more free stations left, Step 5 is bypassed, and the procedure goes to Step 7 to evaluate $U^{(3)}$ and $L^{(3)}$.

Step 7: Node Evaluation. For the current node 3, the upper and lower bounds are computed as 1,015 and 1,015, respectively. Now $U^{(3)} <$ z*. Thus, z* is updated, i.e., $z^* = U^{(3)}$. Since it is terminal, i.e., $K_2 = \emptyset$, the current node is discarded. The last branching is not complete; hence, the closed branch (corresponding to station C) is now evaluated.

The test at step D2 reveals that station C can be constrained closed since $P_2^{(2)} > TD$. The sets are updated as:

$$K_0^{(2)} = K_0^{(2)} \cup \{C\} = \{C\}, \text{ and}$$

$$K_2^{(2)} = K_2^{(2)} - \{C\} = \{C\}.$$

Now, the procedure goes to Step 5 to see if free station D can be fixed open.

Step 5: Fixing Free Station Open. $\Delta_i$ for free station D is computed as 40. Since station D has the positive $\Delta_i$, station D is fixed open. The sets and quantities are updated as:

$$K_1^{(2)} = K_1^{(2)} \cup \{D\} = \{A,B,D\}$$

$$K_2^{(2)} = K_2^{(2)} - \{D\} = \emptyset$$

$$\text{and } P_1^{(2)} = P_1^{(2)} + \alpha_D = 115.$$

As there are no more free stations left, Step 6 is bypassed and the procedure goes to Step 7.

Step 7: Node Evaluation. For node 2, the upper and lower bounds are computed as 1,125 and 1,035, respectively. Nodes 1 and 2 are discarded because $U^{(1)}$ and $U^{(2)}$ are greater than z*, and the node is terminal. When the procedure attempts to select a new node to branch further, it will not find any active nodes. Thus, the current solution is the best solution to the problem with stations A, B, and C are opened. The corresponding allocations to the demand regions are given in Table 9.

Table 9

Best Allocations

| Stations | Demand Regions | | | | | | |
|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| A | 15 | 10 | 0 | 0 | 0 | 0 | 0 |
| B | 0 | 0 | 10 | 15 | 5 | 0 | 0 |
| C | 0 | 0 | 0 | 0 | 0 | 15 | 20 |

## Algorithm Convergence

Convergence, as it is used for this algorithm, implies that the algorithm will reach a solution that the decision-maker finds acceptable. This can be broken down into two different convergence considerations. The first consideration is the attainment of a solution in a finite number of steps. The second consideration is the achievement of a satisfactory solution.

The first consideration of convergence, that of the attainment of a solution in a finite number of steps, is dependent upon the types of data characteristics. In other words, as is the case with most mixed integer programming algorithms, the procedure is usually more efficient for certain types of data than for others. The performance of the algorithm is affected not only by the size of the problems but also by the level of the operating costs of a flying squad. The number of possible solutions to the problems increases at an exponential rate with an increase in the number of potential stations [1]. However, the computation time does not parallel this growth. The rate of increase of computing time is claimed to be considerably less than the rate of increase in the number of solutions. The effect of the number of regions on the computing time is relatively minor [1].

Relatively large operating costs affect the power of the lower bounds. Thus, more frequent prunings are made due to fixing more stations closed. The rate of convergence will mainly depend on the number of potential stations and the level of operating costs of a flying squad. Akinc and Khumawala [1] claim that the basic algorithm terminates in a finite number of steps even for large practical problems.

The second convergence consideration is that of the achievement of an optimum solution. Many heuristic algorithms are capable of handling large problems, but they have the inherent disadvantage that their solutions can not be guaranteed to be optimal. Besides proper testing of the heuristic algorithms for their "goodness" depends in many cases on the availability of efficient exact algorithms, so that the heuristic solutions can be compared with the exact solutions. Unfortunately, exact algorithms capable of solving large problems are not known to the author. Akinc and Khumawala [1] report that they obtained optimal solutions to the majority of the 25 x 50 Kuehn and Hamburger [27] problems with reasonable computing time.

In order to consider a more complete discussion of the convergence of this algorithm, readers are recommended to consult the work of Akinc and Khumawala [1].

## Computer Program

The computer program coded in FORTRAN IV that was developed to affect the solution procedure described in this chapter is listed in Appendix C. The computer program was validated by comparison to a total enumeration result for a small sample problem.

The size of problem that can be solved using this computer program is limited by the dimension size of the program. The computer program as it is set up in Appendix C is arranged to work problems with 25 potential stations and 40 demand regions. Storage for this program was done using computer core only because the size of the example problems worked presented no storage problem. The next chapter will consider applications of the solution procedure to the Roanoke Virginia Fire Department as a case study.

CHAPTER VI

# APPLICATION OF SOLUTION PROCEDURE

This chapter illustrates the application of the solution procedure presented in Chapter V. To illustrate the use of the solution procedure, a problem as it actually exists in the Roanoke Virginia Fire Department is modeled and solved.

## Estimation of Parameters

The City of Roanoke is an urban area covering 28 square miles with a population of approximately 92,000 people (1970 census)[51]. There are areas within the city with high value commercial and industrial construction and others with medium value residential construction. The high value areas contain most of the high rise structures in the city.

The Roanoke Fire Department maintains 10 engine and 4 ladder companies manned by an average of 5 fire fighters per apparatus. Those 14 companies are housed in 9 fire stations located throughout the city.

The development of an estimate of each parameter will now be described. In this example, unless otherwise specified, the same value of the parameters will be used for all regions under consideration.

Potential Flying Squad Sites: i. The existing 9 fire stations in the city are chosen as potential sites for a flying squad, primary because those already existing fire stations meet the physical characteristics requirements needed to house a flying squad. The locations of present fire stations are shown in Figure 7.

Demand Regions: j. A census track is chosen as a basic area because available data such as the number of structures from a census track report can be directly used. There are 20 such areas in the city of Roanoke. The census tracks along with the locations of fire stations are also shown in Figure 7.

Response Time Threshold for Region j: $S_j$. For this example problem, the response time threshold is set-up in such a fashion that a flying squad is to arrive within 2 minutes after the first due company. In a mathematical form, this can be expressed as follows:

$$S_j = (ct_{ij} + 2) \text{ minutes}$$

where, $ct_{ij}$ denotes the response time of the first due company located at fire station i for demand region j. The expected arrival times of the first due company for 20 census tracks under consideration are shown in Table 12.

The Number of Structures and the Median Value of a Structure in Region j: $W_{kj}$ and $F_{kj}$. The census track report includes information on the number of structures and the median value of the structures (in dollars) in a given track. The 1970 census report of Roanoke, which is the most updated one available, will be used for this example problem. The information on the number of structures and the median value of the structures for 20 tracks are shown in Table 10.

CENSUS TRACTS IN THE ROANOKE, VA. SMSA

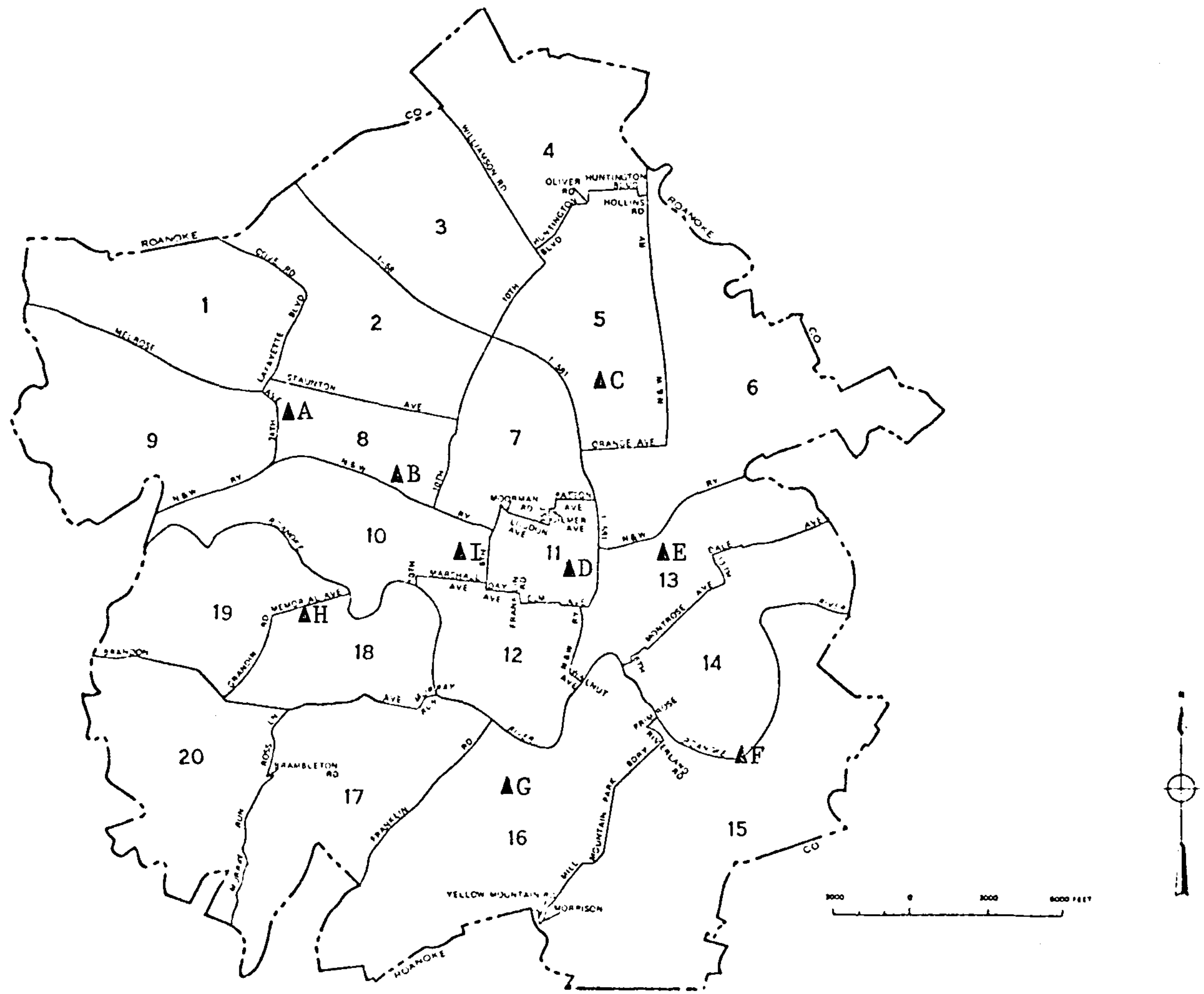


Figure 7. Census Tracks and Fire Station Locations (marked by ▲)

Table 10

Data of the City of Roanoke, VA.

(1970)

| Census Track | Number of Structures | Median Value of a Structure in $ | Potential Fire Service Demand |
|---|---|---|---|
| 1 | 1,480 | $ 13,000 | 12 |
| 2 | 1,407 | 11,900 | 11 |
| 3 | 1,944 | 15,000 | 16 |
| 4 | 1,933 | 16,400 | 16 |
| 5 | 2,168 | 12,400 | 18 |
| 6 | 855 | 8,700 | 7 |
| 7 | 2,184 | 7,600 | 18 |
| 8 | 1,281 | 8,300 | 10 |
| 9 | 605 | 13,300 | 5 |
| 10 | 1,628 | 12,300 | 13 |
| 11 | 496 | 8,300 | 4 |
| 12 | 2,697 | 10,100 | 22 |
| 13 | 2,174 | 6,600 | 18 |
| 14 | 1,378 | 8,400 | 11 |
| 15 | 1,044 | 10,300 | 8 |
| 16 | 2,244 | 29,900 | 18 |
| 17 | 914 | 23,900 | 7 |
| 18 | 1,982 | 14,700 | 16 |
| 19 | 2,367 | 13,700 | 19 |
| 20 | 2,001 | 18,800 | 16 |

Fire Occurence Probability: $P_{kj}$. Following the approach of Valinsky [47], the probability of fire occurence is estimated by dividing the number of structures which reported fires by the number of structures. In 1972, the Roanoke Fire Department experienced 267 structural fires while there were approximately 33,000 structures. The fire service demands for each of 20 census tracks, which were defined in Chapter IV as the product of fire occurence probability and the number of structures in a given region, are included in Table 10.

Fire Loss Function: $g(t_{ij}|m_1)$. As mentioned earlier in Chapter IV, the fire loss function is to be expressed as a function of response time of a flying squad. By using the relationship developed by Mitchell [32], response times from the focal points of each census tracks to the potential flying squad sites are estimated from the corresponding Euclidean distances measured. The response time-distance relationship is expressed as:

$$\text{Response Time} = 2.377 + 1.587 \times (\text{Euclidean distance})$$

where response time and distance are measured in minutes and miles, respectively. The response times computed are shown in Table 11.

The fire loss as a function of response time, $t_{ij}$, is estimated from the graph shown in Figure 1 [28] such as:

$$\text{Fire Loss} = (0.0628\, t_{ij}^2 + 0.148)\, F_{kj}$$

where $F_{kj}$ is the median value of a structure in census track j.

Company and Flying Squad Crew Size: $m_1$ and $m_3$. A company crew size of three is used in this example problem. A figure of three is

Table 11

Expected Response Time
Fire Station i to Census Track j

| j \ i | A | B | C | D | E | F | G | H | I |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 3.31 | 4.55 | 5.53 | 6.03 | 7.01 | 8.25 | 7.01 | 4.79 | 5.29 |
| 2 | 3.31 | 3.56 | 4.30 | 4.79 | 5.53 | 7.26 | 6.27 | 4.55 | 4.30 |
| 3 | 4.55 | 4.55 | 4.05 | 5.29 | 5.78 | 7.51 | 7.26 | 5.78 | 5.04 |
| 4 | 5.53 | 5.53 | 4.55 | 6.03 | 6.27 | 8.25 | 8.25 | 6.77 | 6.03 |
| 5 | 5.29 | 4.55 | 3.07 | 4.55 | 4.79 | 6.52 | 6.77 | 6.03 | 4.79 |
| 6 | 6.03 | 4.79 | 3.56 | 3.81 | 3.56 | 5.04 | 5.78 | 6.03 | 4.55 |
| 7 | 4.55 | 3.31 | 3.31 | 3.31 | 4.05 | 5.53 | 5.29 | 4.55 | 3.31 |
| 8 | 3.31 | 2.82 | 4.30 | 4.30 | 5.29 | 6.52 | 5.53 | 3.81 | 3.56 |
| 9 | 3.31 | 4.55 | 6.03 | 6.03 | 7.01 | 8.25 | 6.77 | 4.30 | 5.04 |
| 10 | 4.05 | 2.82 | 4.55 | 3.81 | 4.79 | 6.03 | 4.79 | 3.31 | 2.82 |
| 11 | 5.04 | 3.81 | 4.05 | 2.57 | 3.56 | 4.79 | 4.55 | 4.55 | 3.07 |
| 12 | 5.29 | 4.05 | 4.79 | 3.31 | 4.05 | 4.79 | 3.56 | 4.05 | 3.31 |
| 13 | 6.03 | 4.79 | 4.30 | 3.07 | 2.57 | 4.05 | 4.55 | 5.53 | 4.05 |
| 14 | 7.01 | 5.53 | 5.29 | 4.05 | 3.31 | 3.07 | 4.79 | 6.27 | 5.04 |
| 15 | 7.75 | 6.52 | 6.52 | 5.04 | 4.55 | 2.82 | 4.55 | 6.77 | 5.78 |
| 16 | 6.27 | 5.29 | 6.03 | 4.30 | 4.55 | 4.55 | 2.57 | 4.79 | 4.30 |
| 17 | 5.29 | 4.79 | 6.27 | 4.79 | 5.53 | 6.03 | 3.81 | 3.81 | 4.30 |
| 18 | 4.79 | 4.05 | 5.53 | 4.05 | 5.04 | 5.78 | 4.05 | 3.07 | 3.56 |
| 19 | 4.05 | 4.05 | 6.03 | 5.29 | 6.27 | 7.01 | 5.29 | 2.82 | 4.30 |
| 20 | 5.29 | 5.29 | 7.01 | 6.03 | 6.77 | 7.26 | 5.04 | 3.81 | 5.29 |

chosen because a minimum of three men are needed to ride an apparatus to a fire scene [6,7,23], and because most structural fires can be effectively put under control with a three man crew [14]. Therefore, the crew size of a flying squad will be 6 men according to the expression defined in Chapter IV.

Maximum Number of Flying Squads to Have: $M_2$. Presently, the Roanoke Fire Department maintains approximately 70 fire fighters per shift (14 apparatus x 5 fire fighters per apparatus per shift). When the company crew size is reduced to three, as mentioned earlier, 28 men are available for flying squads. This figure consitutes over 4 but less than 5 flying squads manned by 6 fire fighters per squad. Since the purpose of introducing a flying squad to the municipal fire department is to reduce the manpower level, the maximum number of flying squads to have is limited to 4 in this example problem.

Workload of a Flying Squad: $\alpha_i$. The workload of a flying squad is limited to 180 structural fires per period (1 year). This number represents the average number of alarms currently responded to by a company.

Average Salary of a Fire Fighter: C. The annual salary of $14,000 is used in this example problem.

## Solution

The previously discussed parameters were inputed to a computer program that was developed to solve the decision problem. The computer program is included in Appendix C.

The solution requires two flying squads at stations A and E. The response district of each flying squad is shown in Figure 8. The total manpower requirement for the Roanoke Fire Department if flying squads are introduced is 54 men per shift (i.e., 2 flying squads manned by 6 men each plus 14 apparatus manned by 3 men per company). This manpower level represents 16 men or 23% less than the present manpower level the City maintains (i.e., 14 apparatus manned by an average of 5 fire fighters minus 54 men).

An analysis of the results of the solution reveals that for the fires in 25% of the census tracks a flying squad arrives at the fire scene at the same time as the first due company. Also, for 50% of the census tracks a flying squad arrives before the arrival of a complete response force (i.e., 2 engine and 1 ladder companies). The largest response time obtained for a flying squad is 5.53 minutes. However, this represents less than 1 minute of elapsed response time from the closest fire station for that particular demand region (see census track 4 in Table 12). The expected arrival times of the first and second due engine companies, a ladder company, and a flying squad are shown in Table 12.

The results obtained from this study demonstrate that flying squads have a potential use as an alternative method of manning the fire companies in municipal fire departments.

CENSUS TRACTS IN THE ROANOKE, VA. SMSA

Response District of
Flying Squad No. 1
Located At Station A.

Response District of
Flying Squad No. 2
Located At Station E.

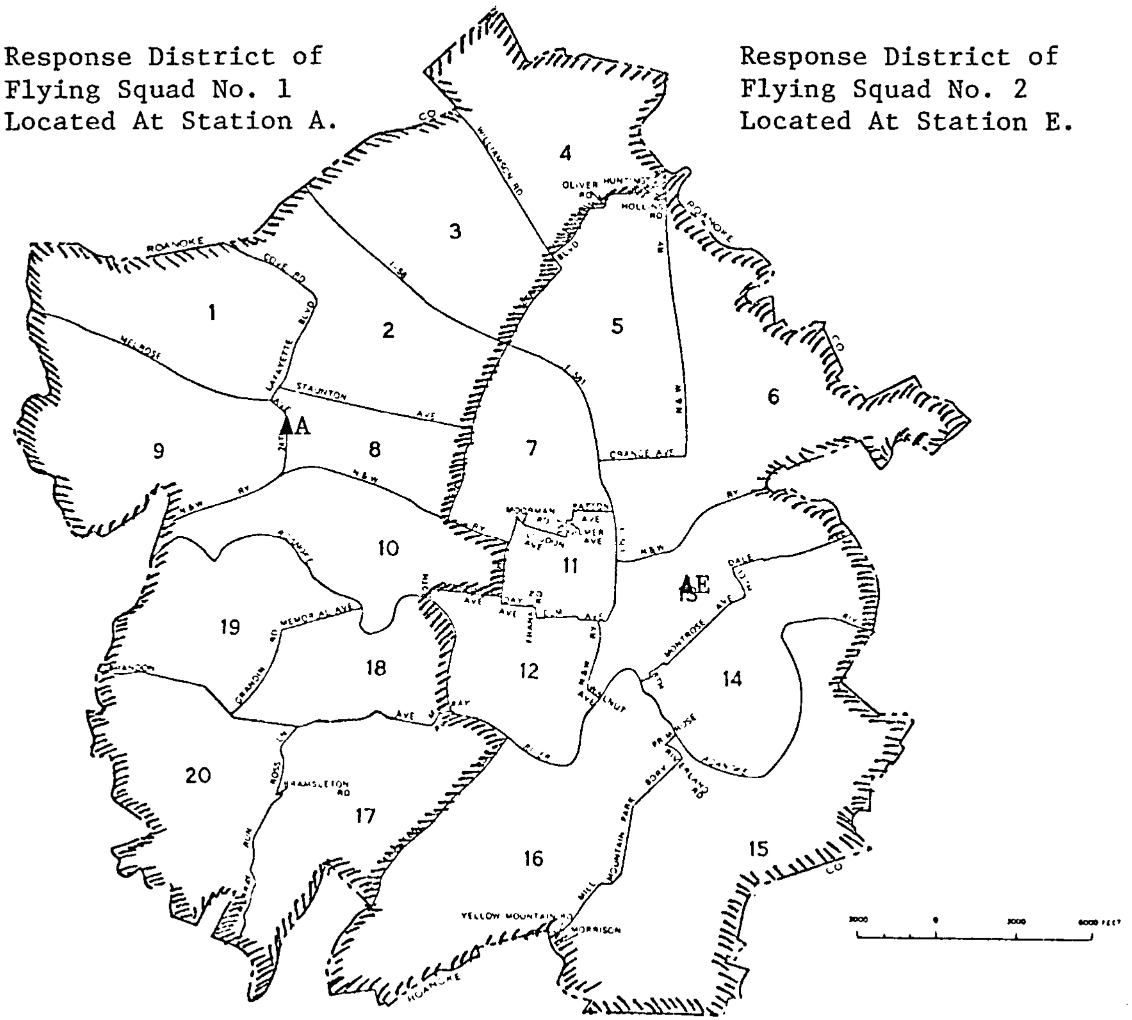


Figure 8. Response Districts of Two Flying Squads

Table 12

Expected Arrival Time of Units

| Census Track | First Due Engine Co. | Second Due Engine Co. | Ladder Co. | Flying Squad |
|---|---|---|---|---|
| 1 | 3.31 | 4.55 | 4.79 | 3.31 |
| 2 | 3.31 | 3.56 | 4.30 | 3.31 |
| 3 | 4.05 | 4.05 | 4.05 | 4.55 |
| 4 | 4.55 | 4.55 | 4.55 | 5.53 |
| 5 | 3.07 | 3.07 | 3.07 | 4.79 |
| 6 | 3.56 | 3.56 | 3.56 | 3.56 |
| 7 | 3.31 | 3.31 | 3.31 | 4.05 |
| 8 | 2.82 | 3.31 | 3.56 | 3.31 |
| 9 | 3.31 | 4.30 | 4.30 | 3.31 |
| 10 | 2.82 | 2.82 | 2.82 | 4.05 |
| 11 | 2.57 | 2.57 | 3.07 | 3.56 |
| 12 | 3.31 | 3.31 | 3.31 | 4.05 |
| 13 | 2.57 | 3.07 | 3.07 | 2.57 |
| 14 | 3.07 | 3.31 | 4.05 | 3.31 |
| 15 | 2.82 | 4.55 | 5.04 | 4.55 |
| 16 | 2.57 | 4.30 | 4.30 | 4.55 |
| 17 | 3.81 | 3.81 | 3.81 | 5.29 |
| 18 | 3.07 | 3.56 | 3.07 | 4.79 |
| 19 | 2.82 | 4.05 | 2.82 | 4.05 |
| 20 | 3.81 | 5.29 | 3.81 | 5.29 |

## CHAPTER VII

## RESULTS, SUMMARY, AND RECOMMENDATION

There were three primary objectives of this study. The first was to define and structure the planning problems encountered in the use of a flying squad in municipal fire suppression operations. The responses of a written questionnaire (shown in Appendix A) sent to the professionals in the field were used along with related journal literature to meet this purpose.

The second was to formulate a mathematical model which accurately describes the decision problem. The relevant criteria and constraints were described and modelled. This purpose was completed as shown in Chapter IV. The model itself could be expanded to include other cost factors, such as personal injuries and insurance premiums.

The third purpose, that of developing a solution procedure of a decision model, was completed as shown in Chapter V. A solution procedure for the decision model was developed and tested on a case problem. Although the procedure was only applied to a relatively small city, Roanoke, Virginia, it did work efficiently.

### Extensions

Extensions of this research could be made in two areas; (a) the use of a flying squad, and (b) additional research areas. The extensions in the use of a flying squad would include:

(a) Determining the acceptability of a flying squad concept by Insurance Service Office (ISO) which conducts municipal fire protection surveys and establishes fire insurance rates.

(b) Determining the impact of a flying squad used in municipal fire departments, such as the reaction of fire fighters union.

The extension of additional research area would be:

(a) Determining the location/allocation and a deployment strategy based on a patrolling situation using a similar concept to a police patrol.

(b) Combining the flying squads and companies as a whole in a decision model.

(c) Extending the present decision model to include additional criteria and constraints, such as insurance premiums.

(d) Combining the present procedure with a computer simulation which would generate data to be used as input in the solution procedure.

In concluding this report, it might be useful to emphasize the significance of this research. A contribution to the existing "body of knowledge" resulted from four research activities. They are (a) defining and structuring a potentially useful problem which had not been previously researched, (b) developing a mathematical model for this problem, (c) developing a solution procedure for the problem, and (d) applying the model and solution procedure to an existing fire department.

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APPENDICES

# APPENDIX A

## Written Survey

This appendix includes a survey sent to the Fire Departments listed below:

| | |
|---|---|
| *Atlanta, Georgia | *New York, New York |
| *Fort Worth, Texas | *Orange, Calif. |
| Garden Grove, Calif. | *St. Louis, Missouri |
| Los Angeles, Calif. | *San Jose, Calif. |
| Miami, Florida | |

The survey itself consists of an accompanying explanatory letter (Figure A1) and a two page questionnaire form (Figure A2).

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* These fire departments responded to the survey.

Dear Fire Chief:

We are currently involved in research concerning "flying squad" (i.e., manpower transport vehicle), and the application of flying squad to municipal fire departments. We are seeking information concerning the factors that must be considered before a decision to use a flying squad can be made.

Fire department budgets have shown a marked increase in the past few years. It is also well known that the largest single cost factor in the operation of a municipal fire department is manpower. By introducing a flying squad we intend to develop an alternative method of manning the companies so as to improve the operations of a fire department. It is anticipated that a flying squad will reduce the total manpower requirements of a fire department.

Under our conceptualization, a flying squad is to be dispatched to the origin of an alarm where it will join the engine and ladder companies to form a complete response force. Each truck will be manned with less than the usual 4-6 fire fighters, and the additional manpower will be supplied by a flying squad.

As examples of the kind of information we are seeking, consider the following. Suppose you have been given the assignment of determining whether or not to use a flying squad concept in the place of the conventional company manning method. What factors must be considered in making your decision? If you decide to use a flying squad, what factors must be considered in determining (a) the number of flying squads to use, (b) the locations/allocations of flying squads, (c) the size of each squad and the job skill levels of each member of a flying squad, and (d) the deployment strategy of a flying squad (i.e., fixed base versus patrolling base).

In addition to the decision problems we have thus far mentioned, many more will have to be looked at. Would you please include other problems and decision factors that you believe will present themselves when considering the use of a flying squad for municipal fire department operations?

The following pages have been included for your convenience. They contain a form that can be filled out quite easily with information we are seeking. We hope you will take the time necessary to answer this letter as the results of our research may be of great value to the management of municipal fire department operations.

Thank you very much for your cooperation.

Figure A1. Accompanying Letter

## SURVEY SHEET

A. The number of flying squds to use.
Factors to be considered:

a.

b.

c.

d.

Suggestions and recommendations:

B. The locations/allocations of flying squads.
Factors to be considered:

a.

b.

c.

d.

Suggestions and recommendations:

C. The size of a squad and the job skill levels of each member of the flying squad. Factors to be considered:

a.

b.

c.

d.

Suggestions and recommendations:

Figure A2. Information Form

D. The deployment strategy of a flying squad.
Factors to be considered:

a.

b.

c.

d.

Suggestions and recommendations:

E. The response force to a residential and high density district should be at least ( / ) men, ( / ) engine truck(s), and ( / ) ladder truck(s), respectively.

F. The minimum number of men that should accompany the engine or ladder truck to start the fire suppression operation assuming that a flying squad is due within two minutes. ( ) men.

G. For an effective operation, the time delay between the arrivals of the first truck and a flying squad should not exceed ( ) minute(s).

H. Would you want a flying squad as
( ) a manpower transport vehicle only
( ) a manpower transport vehicle and mini pumper
( ) other (please specify: )

Please include other problems and factors that you believe will present themselves in considering the use of a flying squad in the operation of municipal fire suppression activities:

Date: ____________________ Signature: ____________________

Figure A2(cont.). Information Form

# APPENDIX B

## The Akinc and Khumawala Procedure

The purpose of this appendix is to describe the Branch and Bound algorithm for the capacitated warehouse location problem developed by Akinc and Khumawala. Before the procedure is discussed, the definition of the following notation is made.

Let, $c_{ij}$ = the per unit transportation cost from warehouse i to customer j,

$D_j$ = the demand for customer j,

$S_i$ = an upper limit on the capacity of warehouse i,

$F_i$ = the fixed cost associated with warehouse i,

$x_{ij}$ = amount supplied from warehouse i to customer j,

$$y_i = \begin{cases} 1, & \text{if warehouse } i \text{ is open} \\ 0, & \text{otherwise,} \end{cases}$$

m = the number of customers,

n = the number of potential warehouses,

$K_0$ = the sets containing the indices of $y_i$'s which are constrained to be 0 (closed),

$K_1$ = the sets containing the indices of $y_i$'s which are constrained to be 1 (open),

$K_2$ = the sets containing the indices of $y_i$'s which are unconstrained (free).

$|K_2|$ = the number of elements in the set $K_i$, where i = 0, 1, or 2.

In its simplest form, the capacitated warehouse location problem can be formulated as a mixed integer program as follows:

$$\text{minimize} \quad z = \sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij} x_{ij} + \sum_{i=1}^{n} F_i y_i$$

$$\text{(P1)} \qquad \begin{aligned} \text{subject to:} \quad & \sum_{j=1}^{m} x_{ij} \leq S_i y_i, \quad \forall\, i \\ & \sum_{i=1}^{n} x_{ij} \geq D_j, \quad \forall\, j \\ & x_{ij} \geq 0, \quad \forall\, i \text{ and } j \\ & y_i = 0 \text{ or } 1, \ \forall\, i. \end{aligned}$$

The procedure begins by defining $K_0 = K_1 = \emptyset$ and $K_2 = \{1, 2, \ldots, n\}$. If at any node it can be shown that a particular free warehouse i is open (or closed) in the best completion reached from it, then the branch with $y_i = 0$ (or 1) can be pruned at this node. This procedure will be referred to as fixing open (closed) a warehouse, to distinguish from constraining open (closed) due to branching. The particular pruning devices in the algorithm are based on the following result due to Erlenkotter [10].

LEMMA. If $B \subseteq A$, then for $r \in B$, $Z[A-\{r\}]-Z[A] \leq Z[B-\{r\}] - Z[B]$, where Z[A] is the optimum solution value of the transportation problem having the sources belonging to the set A; this transportation problem will be referred to as T[A].

Criterion for Fixing Free Warehouses Open

At the initial node Ellwein [9] computes $\Delta V_i = Z[K_1 \cup K_2 - \{i\}] - Z[K_1 \cup K_2]$, where $K_1 = \emptyset$ and $K_2 = \{1, 2, \ldots, n\}$. $\Delta V_i$ is a lower bound on the savings in the variable costs due to having warehouse i open in any solution reached from the initial node. Therefore, $\Delta V_i \geq F_i \rightarrow y_i = 1$, i.e., if the lower bound on the variable cost savings due to warehouse i at the initial node is sufficient enough to cover its fixed costs, then warehouse i must be open in the optimum solution. Both Ellwein [9] and Akinc and Khumawala limit the use of this test to the initial node since its greatest potential benefit is realized at this stage. This test requires the solutions to $|K_2|+1$ transportation problems at any non-terminal nodes of the B&B tree which would be computationally very expensive. Apparently for this reason Akinc and Khumawala use a substitute test, called "delta" test at all other non-terminal nodes. The delta test computes a lower bound on $\Delta V_i$ which will be denoted by $\Delta_i$. Thus $\Delta_i \geq F_i \rightarrow y_i = 1$, $i \in K_2$.

To compute $\Delta_i$ it is not necessary to solve any transportation problems. When the source i is added to this problem, the solution improves for each customer's unit demand by

$$\nabla_{ij} = \min_{k \in K_1 \cup K_2 - \{i\}} [\max(c_{kj} - c_{ij}, 0)].$$

Therefore, $\Delta_i$ is the solution to

$$\max_{0 \leq x_{ij} \leq D_j} \left[ \sum_j \nabla_{ij} x_{ij} \right]$$

$$\text{s.t.} \quad \sum_j x_{ij} \leq S_i, \quad \forall \ i.$$

Criterion for Fixing Free Warehouses Closed

A simple criterion for fixing free warehouses closed at any non-terminal node is to compute $\Omega V_i = Z[K_1] - Z[K_1 U\{i\}]$, $i\varepsilon K_2$. The $\Omega V_i$ is an upper bound on the savings in variable costs due to opening warehouse i. Then $\Omega V_i \leq F_i \rightarrow y_i = 0$.

However, this test requires solutions to a large number of $(|K_2| + 1)$ of transportation problems. Alternatively, a weaker bound on these savings which will be denoted by $\Omega_i$ is computed. Thus, if $\Omega_i \leq F_i \rightarrow y_i = 0$.

The $\Omega_i$ is computed as follows: We have

$$Z[K_1 U\{i\}] = \min_{x_{ij}\geq 0} \sum_{r\varepsilon K_1\{i\}} \sum_j c_{rj} x_{rj}$$

$$\text{s.t.} \quad \sum_j x_{ij} \leq S_i \tag{1}$$

$$\sum x_{rj} \leq S_r, \quad r\varepsilon K_1 \tag{2}$$

$$\sum_{r\varepsilon K_1 U\{i\}} x_{rj} \leq D_j, \quad \forall\, j \tag{3}$$

Let $u_r$ and $v_j$ be the optimal dual multipliers of $T[K_1]$. Assign them to constraints (2) and (3) respectively and form the Lagrangian: $L[K_1 U\{i\}] =$

$$\min_{0\leq x_{ij}\leq D_j} \Big[ \sum_{r\varepsilon K_1 U\{i\}} \sum_j c_{rj} x_{ij} + \sum_j v_j (D_j - \sum_{r\varepsilon K_1 U\{i\}} x_{rj})$$

$$+ \sum_{r\varepsilon K_1} u_r(-S_r + \sum_j x_{rj})\Big]$$

$$\text{s.t.} \quad \sum_j x_{ij} \leq S_i.$$

Substituting $Z[K_1]$ for $\sum_j v_j D_j - \sum_{r\varepsilon K_1} u_r S_r$, we get

$$L[K_1 U\{i\}] = Z[K_1] + \min_{0 \le x_{rj} \le D_j} \left[ \sum_{r\varepsilon K_1} \sum_j (c_{rj} - v_j + u_r) x_{rj} - \sum_j (v_j - c_{ij}) x_{ij} \right]$$

$$\text{s.t.} \qquad \sum_j x_{ij} \le S_i.$$

The optimality of $u_r$ and $v_j$ implies $c_{rj} - v_j + u_r \ge 0$; thus $x_{rj} = 0$ for $r\varepsilon K_1$, $\forall$ j in the above. Hence,

$$L[K_1 U\{i\}] = Z[K_1] - \max_{0 \le x_{ij} \le D_j} \left[ \sum_j w_{ij} x_{ij} \,\middle|\, \sum_j x_{ij} \le S_i \right],$$

where, $w_{ij} = v_j - c_{ij}$.

This leads to

$$\Omega_i = Z[K_1] - L[K_1 U\{i\}] = \max_{0 \le x_{ij} \le D_j} \left[ \sum_j w_{ij} x_{ij} \,\middle|\, \sum_j x_{ij} \le S_i \right].$$

Note that the $\Delta_i$ and $\Omega_i$ at a node depend on the sets $K_0$ and $K_1$. Therefore, these tests are applied in a cyclic manner. Whenever a free warehouse is fixed open (or closed), the possibility of fixing closed (open) another free warehouse is investigated. The cycle stops when fixing another remaining free warehouse open or closed is not possible.

## The Lower Bound (LB)

The usual method for obtaining the LB is to solve the original integer problem without the integrality restrictions. Accordingly, the LB at any node for CWLP is obtained by solving the following transportation problem:

$$LB = \min_{x_{ij} \geq 0} \sum_{i \epsilon K_1 U K_2} \sum_j (c_{ij} + g_i/S_i)x_{ij} + \sum_{i \epsilon K_1} F_i$$

$$\text{s.t.} \quad \sum_{i \epsilon K_1 U K_2} x_{ij} = D_j, \quad \forall\, j$$

$$\sum_j x_{ij} \leq S_i, \quad i \epsilon\, K_1 U K_2 \qquad \text{(P2)}$$

where $g_i = \begin{cases} F_i, & \text{if } i \epsilon K_2 \\ 0, & \text{otherwise.} \end{cases}$

<u>The Upper Bound (UB)</u>

In the algorithm the UB at the node is simply $Z[K_1] + \sum_{i \epsilon K_1} F_i$. The LB at a node is an upper bound to (P1) if the solution to (P2) at that node results in all $y_i = 0$ or 1.

<u>The Node Selection Rules (NSR)</u>

The authors use two NSR's which are called MIN $K_2$ and LLB rules. These rules are parallel to the LIFO and priority type rules discussed in Geoffrion and Marsten [13]. The MIN $K_2$ rule selects the node with the least number of free variables. The LLB rule, on the other hand, selects the node with the least lower bound.

Because of this conflict they combined two rules into a hybrid (HR). Two parameters $\alpha$ and $\beta$ are specified. HR employs the LLB rule until the number of non-terminal nodes reaches the level $\beta$. At this time it switches to MIN $K_2$ rule. When the number of non-terminal nodes is reduced to $\alpha$, the LLB is again used. By selecting the values of the parameters $\alpha$ and $\beta$, the authors claim that the computer storage requirements can be effectively controlled.

Branch Selection Rules (BSR)

Akinc and Khumawala designed BSRs which would (a) use available information, thereby avoiding additional computations, and (b) offer greatest possible discrimination between the two emanating nodes. The following eight BSRs were developed:

1. the largest omega rule - - - - $\max_{i \varepsilon K_2} [\Omega_i - F_i]$,
2. the smallest omega rule - - - $\min_{i \varepsilon K_2} [\Omega_i - F_i]$,
3. the largest delta rule - - - - $\max_{i \varepsilon K_2} [\Delta_i - F_i]$,
4. the smallest delta rule - - - $\min_{i \varepsilon K_2} [\Delta_i - F_i]$,
5. the maximum y rule - - - - - - $\max_{i \varepsilon K_2} [y_i | 0 < y_i < 1]$,
6. the minimum y rule - - - - - - $\min_{i \varepsilon K_2} [y_i | 0 < y_i < 1]$,
7. the largest capacity ryle - - $\max_{i \varepsilon K_2} [S_i]$,
8. the smallest capacity rule - - $\min_{i \varepsilon K_2} [S_i]$.

# APPENDIX C

## Computer Program Listing

This appendix supplies the listing of the computer program used to solve the example problem in Chapter VI. The usage requirements of the program are listed in the program.

```
C*******************************************************************************************C
C                                                                                           C
C                                                                                           C
C      THIS PROGRAM SOLVES THE FLYING SQUAD PROBLEM DISCUSSED IN THIS                       C
C      THESIS.  IT FINDS THE BEST SET OF FLYING SQUAD LOCATIONS AMONG                       C
C      POTENTIAL SITES WHICH MINIMIZES THE SUM OF EXPECTED FIRE LOSSES                      C
C      AND OPERATING COSTS OF FLYING SQUADS AND FIRE COMPANIES.                             C
C                                                                                           C
C                                                                                           C
C      THE PROGRAM AS IT IS SET-UP NOW CAN SOLVE THE PROBLEMS WITH                          C
C      UPTO 30 POTENTIAL FLYING SQUAD SITES AND 40 DEMAND REGIONS.                          C
C                                                                                           C
C      THE PROGRAM ITSELF IS BROKEN DOWN INTO FOUR MAJOR SECTIONS:                          C
C                                                                                           C
C      (1) MAIN PROGRAM - THIS MAIN PROGRAM IS DESIGNED TO (A) READ DATA,                   C
C      (B) INITIALIZE THE PROGRAM, (C) COMPUTE THE UPPER BOUND, (D)                         C
C      COMPUTE THE LOWER BOUND, (E) EVALUATE A NODE, (F) PRINT PERTI-                       C
C      NENT INFORMATION, AND (G) CALL SUBROUTINES IN SECTIONS (2), (3),                     C
C      AND (4).                                                                             C
C                                                                                           C
C      (2) SUBROUTINE DELTA - THIS SUBROUTINE COMPUTES THE LOWER BOUND                      C
C      IN THE SAVINGS OF EXPECTED FIRE LOSSES IF A POTENTIAL FLYING                         C
C      SQUAD SITE K IS LOCATED.  THE FLYING SQUAD SITE WHICH WOULD                          C
C      YIELD A MAXIMUM NON-NEGATIVE DIFFERENCE BETWEEN THE SAVINGS AND                      C
C      THE COST OF OPERATING FLYING SQUAD IS TO BE OPENED.                                  C
C                                                                                           C
C      (3) SUBROUTINE OMEGA - THIS SUBROUTINE COMPUTES THE UPPER BOUND                      C
C      IN THE SAVINGS OF EXPECTED FIRE LOSSES IF A POTENTIAL FLYING                         C
C      SQUAD SITE J IS LOCATED.  THE POTENTIAL FLYING SQUAD SITE WHOSE                      C
C      OPERATING COST EXCEEDS ITS EXPECTED SAVINGS IN FIRE LOSSES IS                        C
C      FIXED CLOSED.                                                                        C
C                                                                                           C
```

```
C       (4) SUBROUTINE TRALG - THIS SUBROUTINE SOLVES A TRANSPORTATION          C
C       PROBLEM ONLY WITH ALREADY OPENED FLYING SQUAD SITES AS SOURCES.         C
C                                                                               C
C                                                                               C
C       THE USERS ARE EXPECTED TO INPUT THE FOLLOWING DATA REQUIREMENTS:        C
C       NW = THE NUMBER OF POTENTIAL FLYING SQUAD SITES,                        C
C       NC = THE NUMBER OF DEMAND REGIONS,                                      C
C       STAT(I) = THE DISTANCE IN MILES FROM THE FOCAL POINT OF DEMAND          C
C         REGION I TO THE CLOSEST FIRE STATION,                                 C
C       DMND(J) = THE POTENTIAL FIRE SERVICE DEMAND OF REGION J,                C
C       FCOST(I) = THE OPERATING COST OF FLYING SQUAD IF LOCATED AT             C
C         THE POTENTIAL SITE I,                                                 C
C       CPCTY(I) = THE MAXIMUM WORKLOAD OF FLYING SQUAD I,                      C
C       VALUE(J) = THE MEDIAN VALUE OF A STRUCTURE IN REGION J,                 C
C       TDIS(I,J) = THE SHORTEST PATH FROM POTENTIAL FLYING SQUAD SITE I        C
C         TO THE FOCAL POINT OF DEMAND REGION J, AND                            C
C       BDR = ANY INTEGER VALUE FROM 1 TO 8 OR ALL OF THEM.                     C
C                                                                               C
C                                                                               C
C*******************************************************************************C
C
C
      IMPLICIT INTEGER (A-Z)
      REAL TTIME,TSTAT
      REAL SEN
      REAL MINX,MINY,ZIM
      REAL A,B,C
      INTEGER*2 STATUS,STND,KZERO,KONE,KTWO,KK,SOLTN
      COMMON /ALI/ DMND(41)
      COMMON /BANU/ TRPC(30,41),FCOST(30),CPCTY(30),NODE,NW,NC,LB(100),
     1            UB(100),KZERO(100),KONE(100),KTWO(100),STATUS(100,30)
      COMMON /KEREM/ DEL(100,30),DDEL(41),KK(100,41)
```

```
      COMMON /SERYA/ MEGA(100,30),TTCAP,TDMND,TCTY1(100),
     1              NTR,FCPC(100,30)
      COMMON /HELEN/ M,N,LS(30),MR(30,41),NTC,NT(30,41),MB(30,41)
      DIMENSION TCTY2(100),SOLTN(30),STND(100),TVC(100),ASSAGN(100,30)
      DIMENSION VALUE(40),TDIS(30,41),STAT(40),TSTAT(40),TTIME(30,41)
C
      NUM=9999999
C
C    $$$$$$ READ DATA $$$$$$
      READ(5,1001) NW, NC
      M=NC+1
      READ(5,1005)(STAT(I),I=1,NC)
      READ(5,1001)(DMND(J),J=1,NC)
      READ(5,1001)(FCOST(I),I=1,NW)
      READ(5,1001)(CPCTY(I),I=1,NW)
      READ(5,1001)(VALUE(J),J=1,NC)
      DO 31 I=1,NC
   31 TSTAT(I)=2.328+1.302*STAT(I)/5.28
      WRITE(6,1006)(TSTAT(I),I=1,NC)
      DO 33 I=1,NW
      READ(5,1005)(TDIS(I,J),J=1,NC)
   33 CONTINUE
      DO 32 I=1,NC
      TSTAT(I)=TSTAT(I)+2.0
   32 CONTINUE
      DO 34 I=1,NW
      DO 35 J=1,NC
      TTIME(I,J)=2.328+1.302*TDIS(I,J)/5.28
      IF(TTIME(I,J).GT.TSTAT(J)) GO TO 38
      TRPC(I,J)=VALUE(J)*(0.00828*TTIME(I,J)*TTIME(I,J)+0.148)
      GO TO 35
   38 TRPC(I,J)=NUM
```

```
   35 CONTINUE
   34 CONTINUE
      DO 51 I=1,NW
      WRITE(6,1007)I,(TTIME(I,J),J=1,NC)
   51 CONTINUE
      DO 36 I=1,NW
   36 TRPC(I,M)=0
    1 CONTINUE
      READ(5,1003,END=99911) BDR
      DO 37 I=1,NW
      WRITE(6,1000) I,FCOST(I),(TRPC(I,J),J=1,NC),CPCTY(I)
   37 CONTINUE
      WRITE(6,1002)(DMND(J),J=1,NC)
 1000 FORMAT(I3,I7,20I6,I3)
 1001 FORMAT(7I10)
 1002 FORMAT(T9,20I6)
 1003 FORMAT(16I5)
 1005 FORMAT(20I4)
 1006 FORMAT(///,T9,20F5.2)
 1007 FORMAT(I5,T9,20F5.2)
C     INITIALIZE....................
      NTR=0
      ITER=0
      CLNG=100
      NODE=1
      TDMND=0
      DO 10 J=1,NC
      KK(1,J)=1
      DO 17 I=2,NW
      IF (TRPC(I,J).GT.TRPC(KK(NODE,J),J)) GO TO 17
      KK(NODE,J)=I
   17 CONTINUE
```

```
   10 TDMND=TDMND+DMND(J)
C     WRITE(6,1001) TDMND
      TTCAP=0
      DO 20 I=1,NW
   20 TTCAP=TTCAP+CPCTY(I)
C     WRITE(6,1001) TTCAP
      DO 1618 I=1,NW
      C=FCOST(I)
      A=C/CPCTY(I)
      FCPC(1,I)=A
C     WRITE(6,1001) FCPC(1,I)
 1618 CONTINUE
      IF (TTCAP-TDMND) 30,40,50
   30 WRITE (6,299) TTCAP,TDMND
  299 FORMAT ('0','PROBLEM INFEASIBLE',2X,'TOT CAP=',5X,I10,5X,'TOT DEM
     1=',5X,I10)
      GO TO 9911
   40 WRITE (6,399)
  399 FORMAT('0','ALL STATIONS ARE OPEN')
      GO TO 9911
   50 DO 16 I=1,CLNG
   16 STND(I)=0
      FLAG=0
      STND(1)=1
      NONODE=1
      LBND=0
      UBND=999999999
      DO 100 I=1,NW
  100 STATUS(NODE,I)=2
      KZERO(NODE)=0
      KONE(NODE)=0
      KTWO(NODE)=NW
```

```
      TCTY1(NODE)=0
      TCTY2(NODE)=TTCAP
      TVC(NODE)=0
      FIRST=1
  391 CALL DELTA
      TRANSD=0
      LDEL=-NUM
      DO 110 I=1,NW
      IF(STATUS(NODE,I).NE.2) GO TO 110
      IF(DEL(NODE,I).LT.0) GO TO 110
      IF(DEL(NODE,I).LE.LDEL) GO TO 110
      K=I
      LDEL=DEL(NODE,I)
      TRANSD=1
  110 CONTINUE
      IF(TRANSD.EQ.0) GO TO 200
      WRITE(6,2220) K,DEL(NODE,K)
      STATUS(NODE,K)=1
      KONE(NODE)=KONE(NODE)+1
      KTWO(NODE)=KTWO(NODE)-1
      TCTY1(NODE)=TCTY1(NODE)+CPCTY(K)
C
 1172 CALL OMEGA
      TVC(NODE)=NTC
      FIRST=0
      IF (TCTY1(NODE).LE.TOMND) GO TO 200
      TRANSD=0
      DO 140 I=1,NW
      IF(STATUS(NODE,I).NE.2) GO TO 140
      IF (MEGA(NODE,I).GT.0) GO TO 140
      KZERO(NODE)=KZERO(NODE)+1
      KTWO(NODE)=KTWO(NODE)-1
```

```
      TCTY2(NODE)=TCTY2(NODE)-CPCTY(I)
      STATUS(NODE,I)=0
      DO 18 J=1,NC
      IF (KK(NODE,J).NE.I) GO TO 18
      MIN1=NUM
      DO 19 L=1,NW
      IF(STATUS(NODE,L).EQ.0.OR.TRPC(L,J).GE.MIN1) GO TO 19
      KK(NODE,J)=L
      MIN1=TRPC(L,J)
   19 CONTINUE
   18 CONTINUE
      TRANSO=1
      WRITE(6,2221) I,MEGA(NODE,I)
  140 CONTINUE
      IF (TRANSO.EQ.1) GO TO 391
  200 WRITE(6,2222) (STATUS(NODE,I),I=1,NW)
C
C     UPPER BOUND ...................
      ITER=ITER+1
      UB(NODE)=NUM
      IF (TCTY1(NODE).LT.TDMND) GO TO 39
      UB(NODE)=TVC(NODE)
      DO 435 I=1,NW
      ASSAGN(NODE,I)=0
      IF (STATUS(NODE,I).NE.1) GO TO 435
      UB(NODE)=UB(NODE)+FCOST(I)
  435 CONTINUE
   39 CONTINUE
C
C     LOWER BOUND ...................
      LB(NODE)=0
      DO 48 J=1,NC
```

```
      SMLL=NUM
      DO 43 I=1,NW
      ASSAGN(NODE,I)=0
      IF (STATUS(NODE,I)-1) 43,42,41
   42 IF (SMLL.LE.TRPC(I,J)) GO TO 43
      SMLL=TRPC(I,J)
      K=I
      GO TO 43
   41 IF (SMLL.LE.(TRPC(I,J)+FCPC(NODE,I))) GO TO 43
      SMLL=TRPC(I,J)+FCPC(NODE,I)
      K=I
   43 CONTINUE
      LB(NODE)=LB(NODE)+SMLL*DMND(J)
      ASSAGN(NODE,K)=ASSAGN(NODE,K)+DMND(J)
   48 CONTINUE
      DO 481 I=1,NW
      IF (STATUS(NODE,I).NE.1) GO TO 481
      LB(NODE)=LB(NODE)+FCOST(I)
  481 CONTINUE
      WRITE(6,2226) NODE,UB(NODE),LB(NODE),SWHS,FLAG
 2226 FORMAT ('0',5I25)
C
C     NODE EVALUATION      .....................
      UBALT=0
      IF (KTWO(NODE).NE.0) GO TO 251
      IF (UB(NODE).GE.UBND) GO TO 250
      UBND=UB(NODE)
      WRITE (6,5012) NONODE
 5012 FORMAT (' ','NEW SOLUTION',I15,'   NODES REMAIN')
      UBALT=1
      DO 249 I=1,NW
  249 SOLTN(I)=STATUS(NODE,I)
```

```
      STND(NODE)=0
      NONODE=NONODE-1
      GO TO 253
  251 IF (LB(NODE).LT.UBND) GO TO 255
  250 STND(NODE)=0
      NONODE=NONODE-1
      WRITE (6,5013)
 5013 FORMAT ('0','NODE TERMINATED')
      GO TO 253
  255 IF (UB(NODE).GE.UBND) GO TO 253
      UBND=UB(NODE)
      WRITE (6,5012) NONODE
      UBALT=1
      DO 254 I=1,NW
      IF (STATUS(NODE,I).NE.1) GO TO 256
      SOLTN(I)=1
      GO TO 254
  256 SOLTN(I)=0
  254 CONTINUE
      WRITE (6,5015) (SOLTN(I),I=1,NW)
 5015 FORMAT(16I5)
  253 IF (UBALT.NE.1) GO TO 329
      TERM=0
      DO 327 K=1,CLNG
      IF(STND(K).EQ.0) GO TO 327
      TERM=1
      IF(LB(K).LE.UBND) GO TO 327
      STND(K)=0
      NONODE=NONODE-1
  327 CONTINUE
      IF (TERM.EQ.0) GO TO 10001
C
```

```
C       NSR
   329 IF (FLAG.EQ.1) GO TO 4444
       IF (NONODE.GE.75) SWITCH=2
       IF(NONODE.LE.40) SWITCH=1
       WITCH=SWITCH
       IF (WITCH.EQ.SWITCH) GO TO 333
       WRITE (6,2224) SWITCH
  2224 FORMAT ('0',' SWITCHED TO',I5)
       DO 5966 I=1,CLNG
       IF (STND(I).EQ.0) GO TO 5966
       WRITE(6,5967) I,(STATUS(I,J),J=1,NW),KTWO(I)
  5967 FORMAT (' ',I5,'***',25I4,I10)
  5966 CONTINUE
   333 IF (SWITCH.EQ.2) GO TO 2951
       NODE=0
       LBND=NUM
       DO 252 I=1,CLNG
       IF (STND(I).EQ.0) GO TO 252
       IF (LB(I).GE.LBND) GO TO 252
       LBND=LB(I)
       NODE=I
   252 CONTINUE
       GO TO 7983
  2951 NODE=0
       K2MIN=NUM
       DO 7967 I=1,CLNG
       IF (STND(I).EQ.0) GO TO 7967
       IF (KTWO(I).GT.K2MIN) GO TO 7967
       IF (KTWO(I).EQ.K2MIN.AND.LB(NODE).LE.LB(I)) GO TO 7967
       MODE=I
       K2MIN=KTWO(I)
  7967 CONTINUE
```

```
 7983 IF (NODE.EQ.0) GO TO 10001
      WRITE (6,5016) NODE
 5016 FORMAT('0','NODE CHOOSEN TO BE BRANCHED FROM IS:',I5)
C      1 LARGEST OMEGA
C      2 SMALLEST OMEGA
C      3 LARGEST DELTA
C      4 SMALLEST DELTA
C      5 LARGEST CAPACITY
C      6 SMALLEST CAPACITY
C      7 LARGEST Y
C      8 SMALLEST Y
      GO TO (7936,7946,7956,7966,7976,7986,7996,8006),BDR
 7936 LOMEG=-NUM
      DO 643 I=1,NW
      IF (STATUS(NODE,I).NE.2) GO TO 643
      IF (MEGA(NODE,I).LE.LOMEG) GO TO 643
      LOMEG=MEGA(NODE,I)
      SWHS=I
  643 CONTINUE
      GO TO 7979
 7946 SMOMEG=NUM
      DO 645 I=1,NW
      IF (STATUS(NODE,I).NE.2) GO TO 645
      IF (MEGA(NODE,I).GE.SMOMEG) GO TO 645
      SWHS=I
      SMOMEG=MEGA(NODE,I)
  645 CONTINUE
      GO TO 7979
 7956 LGDEL=-NUM
      DO 655 I=1,NW
      IF (STATUS(NODE,I).NE.2) GO TO 655
      IF (DEL(NODE,I).LE. LGDEL) GO TO 655
```

```
      SWHS=I
      LGDEL=DEL(NODE,I)
  655 CONTINUE
      GO TO 7979
 7966 SMDEL=NUM
      DO 665 I=1,NW
      IF(STATUS(NODE,I).NE.2) GO TO 665
      IF(DEL(NODE,I).GE.SMDEL) GO TO 665
      SWHS=I
      SMDEL=DEL(NODE,I)
  665 CONTINUE
      GO TO 7979
 7976 LARCAP=NUM
      DO 675 I=1,NW
      IF (STATUS(NODE,I).NE.2) GO TO 675
      IF (FCPC(NODE,I).GE.LARCAP) GO TO 675
      SWHS=I
      LARCAP=FCPC(NODE,I)
  675 CONTINUE
      GO TO 7979
 7986 SMCAP=-NUM
      DO 685 I=1,NW
      IF (STATUS(NODE,I).NE.2) GO TO 685
      IF (FCPC(NODE,I).LE.SMCAP) GO TO 685
      SWHS=I
      SMCAP=FCPC(NODE,I)
  685 CONTINUE
      GO TO 7979
 7996 LGVAY=-NUM
      DO 695 I=1,NW
      IF (STATUS(NODE,I).NE.2) GO TO 695
      IF (ASSAGN(NODE,I).LE.LGVAY) GO TO 695
```

```
      SWHS=I
      LGVAY=ASSAGN(NODE,I)
  695 CONTINUE
      GO TO 7979
 8006 SMVAY=NUM
      DO 705 I=1,NW
      IF (STATUS(NODE,I).NE.2) GO TO 705
      IF (ASSAGN(NODE,I).GE.SMVAY) GO TO 705
      SWHS=I
      SMVAY=ASSAGN(NODE,I)
  705 CONTINUE
 7979 CONTINUE
      WRITE(6,5017) SWHS
 5017 FORMAT('0','STATION CHOSEN TO BE FORCED IS:',I5)
C    BRANCHING
C
C     OPEN
      ONODE=NODE
      DO 4884 I=1,CLNG
      IF(STND(I).EQ.0) GO TO 4774
 4884 CONTINUE
      GO TO 10002
 4774 NODE=I
      WRITE(6,9921) NODE,SWHS
 9921 FORMAT ('0','FOR NEW NODE:',I5,I10,' IS BEING OPENED')
      DO 392 I=1,NW
      DEL(NODE,I)=DEL(ONODE,I)
  392 STATUS(NODE,I)=STATUS(ONODE,I)
      STATUS(NODE,SWHS)=1
      TCTY1(NODE)=TCTY1(ONODE)+CPCTY(SWHS)
      TCTY2(NODE)=TCTY2(ONODE)
      KZERO(NODE)=KZERO(ONODE)
```

```
      KONE(NODE)=KONE(ONODE)+1
      KTWO(NODE)=KTWO(ONODE)-1
      DO 393 J=1,NC
  393 KK(NODE,J)=KK(ONODE,J)
C     DO 393 I=1,NW DELTAS ARE NOT UPDATED
      STND(NODE)=1
      NONODE=NONODE+1
      FLAG=1
      GO TO 1172
C
C     CLOSE
 4444 NODE=ONODE
      TCTY2(NODE)=TCTY2(NODE)-CPCTY(SWHS)
      IF (TOMND.GT.TCTY2(NODE)) GO TO 1175
      WRITE (6,9923) NODE,SWHS
 9923 FORMAT ('0','OTHER BRANCH OF',I5,I5,' IS BEING CLOSED')
      STATUS(NODE,SWHS)=0
      DO 118 J=1,NC
      IF (KK(NODE,J).NE.SWHS) GO TO 118
      MIN1=NUM
      DO 119 I=1,NW
      IF (STATUS(NODE,I).EQ.0.OR.TRPC(I,J).GE.MIN1) GO TO 119
      KK(NODE,J)=I
      MIN1=TRPC(I,J)
  119 CONTINUE
  118 CONTINUE
      KTWO(NODE)=KTWO(NODE)-1
      KZERO(NODE)=KZERO(NODE)+1
      FLAG=0
      GO TO 391
 1175 WRITE (6,1176) SWHS
 1176 FORMAT ('0','CLOSING',I5,' LEAVES INSUFFICIENT CAPACITY')
```

```
      STND(NODE)=0
      NONODE=NONODE-1
      FLAG=0
      GO TO 329
10001 WRITE(6,532) (SOLTN(I),I=1,NW)
  532 FORMAT ('0','OPTIMUM SOLUTION',20I5)
      WRITE(6,279) UBND,NTR,ITER
  279 FORMAT ('0','OPTIMUM COST=',I10,' # TRANS=',I5,' # ITER=',I5)
      GO TO 9911
10002 WRITE (6,5817) (SOLTN(I),I=1,NW)
 5817 FORMAT ('0','BEST SOLUTION FOUND:',20I5)
      WRITE (3,4792) UBND,NTR,ITER
 4792 FORMAT('0','COST=',I10,' #TRANS=',I5,' #ITER=',I5)
 9911 GO TO 1
99911 STOP
 2222 FORMAT ('0',25I5)
 2220 FORMAT('0','STATION',I10,5X,'OPENED',I15)
 2221 FORMAT('0','STATION',I10,5X,'CLOSED',I15)
      END
C
C
      SUBROUTINE DELTA
      IMPLICIT INTEGER (A-Z)
      INTEGER*2 STATUS,KZERO,KONE,KTWO,KK
      COMMON /ALI/ DMND(41)
      COMMON /BANU/ TRPC(30,41),FCOST(30),CPCTY(30),NODE,NW,NC,LB(100),
     1        UB(100),KZERO(100),KONE(100),KTWO(100),STATUS(100,30)
      COMMON /KEREM/ DEL(100,30),DDEL(41),KK(100,41)
C
      NUM=9999999
      DO 11 J=1,NC
      MIN1=NUM
```

```
      DO 12 I=1,NW
      IF (I.EQ.KK(NODE,J).OR.STATUS(NODE,I).EQ.J.OR.TRPC(I,J).GE.
     1MIN1) GO TO 12
      MIN1=TRPC(I,J)
   12 CONTINUE
      DDEL(J)=MIN1-TRPC(KK(NODE,J),J)
C     WRITE(6,2000) KK(NODE,J), J, DDEL(J)
C2000 FORMAT(T15,'DELTA (',I2,',',I2,') = ',I3)
   11 CONTINUE
      DO 15 I=1,NW
      DEL(NODE,I)=0
      CTY=CPCTY(I)
   19 MAX=0
      K=0
      TR=0
      DO 20 J=1,NC
      IF (KK(NODE,J).NE.I.OR.DDEL(J).LE.MAX) GO TO 20
      TR=1
      K=J
      MAX=DDEL(J)
   20 CONTINUE
      IF(TR.NE.1) GO TO 25
      IF(DMND(K).GE.CTY) GO TO 18
      DEL(NODE,I)=DEL(NODE,I)+MAX*DMND(K)
      CTY=CTY-DMND(K)
      DDEL(K)=0
      GO TO 19
   18 DEL(NODE,I)=DEL(NODE,I)+MAX*CTY
   25 DEL(NODE,I)=DEL(NODE,I)-FCOST(I)
   15 CONTINUE
      DO 35 I=1,NW
      IF(STATUS(NODE,I).EQ.1) DEL(NODE,I)=0
```

```
   35 CONTINUE
C        WRITE (6,8534)
C8534 FORMAT('0','DELTAS')
C        WRITE(6,2222) (DEL(NODE,I),I=1,NW)
C2222 FORMAT (16I8)
         RETURN
         END
C
C
         SUBROUTINE OMEGA
         IMPLICIT INTEGER (A-Z)
         INTEGER*2 STATUS,KZERO,KONE,KTWO,KK
         REAL USBCAP,C
         COMMON /ALI/ DMND(41)
         COMMON /BANU/ TRPC(30,41),FCOST(30),CPCTY(30),NODE,NW,NC,LB(100),
        1            UB(100),KZERO(100),KONE(100),KTWO(100),STATUS(100,30)
         COMMON /KEREM/ DEL(100,30),OMEG(41),KK(100,41)
         COMMON /SERYA/ MEGA(100,30),TTCAP,TDMND,TCTY1(100),
        1            NTR,FCPC(100,30)
         COMMON /HELEN/ M,N,LS(30),MR(30,41),NTC,NT(30,41),MB(30,41)
         DIMENSION U(30),V(41)
C
C
         M=NC+1
         N=KONE(NODE)
         LK=1
         I=1
         DMND(M)=0
         IF (TCTY1(NODE).GE.TDMND) DMND(M)=TCTY1(NODE)-TDMND
         WRITE (6,97821) TCTY1(NODE),TDMND,DMND(M)
97821 FORMAT ('0','K1 CAP',I10,' TDMND=',I10,' SLACK=',I10)
  222 DO 1102 K=LK,NW
```

```
      IF (STATUS(NODE,K).EQ.1) GO TO 1103
 1102 CONTINUE
      GO TO 177
C
C
 1103 LK=K+1
      DO 1104 J=1,NC
 1104 MR(I,J)=TRPC(K,J)
      MR(I,M)=0
      LS(I)=CPCTY(K)
      IF (I.GE.N) GO TO 177
      I=I+1
      GO TO 222
  177 CONTINUE
      NTR=NTR+1
C
      CALL TRALG
      DO 1 I=1,N
      U(I)=-MB(I,M)
C     WRITE(6,1001) I,U(I)
    1 CONTINUE
      DO 2 J=1,M
      DO 3 I=1,N
      IF (NT(I,J).EQ.0) GO TO 3
      V(J)=U(I)+MR(I,J)
C     WRITE(6,1000) J,V(J)
      GO TO 2
    3 CONTINUE
      V(J)=0
    2 CONTINUE
C
      IF (KTWO(NODE).EQ.0) RETURN
```

```
C
C
      DO 2101 I=1,NW
      IF (STATUS(NODE,I).NE.2) GO TO 2101
      U(I)=V(M)
      USBCAP=0
      IF (TCTY1(NODE).LT.TDMND) USBCAP=TDMND-TCTY1(NODE)
      CTY=CPCTY(I)
      MEGA(NODE,I)=0
      DO 2102 J=1,NC
 2102 OMEG(J)=V(J)-U(I)-TRPC(I,J)
  444 MAX=0
      K=0
      DO 2103 J=1,NC
      IF (OMEG(J).LE.MAX) GO TO 2103
      MAX=OMEG(J)
      K=J
 2103 CONTINUE
      IF (K.EQ.0) GO TO 2107
      ZAR=DMND(K)
      IF (ZAR.GE.CTY) GO TO 128
      USBCAP=USBCAP+ZAR
      MEGA(NODE,I)=MEGA(NODE,I)+MAX*ZAR
      OMEG(K)=0
      CTY=CTY-ZAR
      GO TO 444
  128 MEGA(NODE,I)=MEGA(NODE,I)+MAX*CTY
      USBCAP=USBCAP+CTY
 2107 IF (USBCAP.EQ.0) GO TO 2101
      MINX=FCOST(I)/USBCAP
      FCPC(NODE,I)=MINX
 2101 CONTINUE
```

```
      DO 2114 I=1,NW
      IF(STATUS(NODE,I).NE.2) GO TO 2114
      MEGA(NODE,I)=MEGA(NODE,I)-FCOST(I)
C     WRITE(6,8535) I,MEGA(NODE,I)
 2114 CONTINUE
C1000 FORMAT('0','V(',I2,')=',I5)
C1001 FORMAT('0','U(',I2,')=',I5)
C8535 FORMAT('0','OMEGA (',I2,')=',I5)
C
C     TEMPORARY WRITE UP PF OMEGAS
C4121 FORMAT ('0','OMEGAS')
C4122 FORMAT (//10I13)
C     TEMPORARY WRITE UPS OF OMEGAS
C
C
      RETURN
      END
      SUBROUTINE TRALG
      IMPLICIT INTEGER (A-Z)
      COMMON /ALI/ LD(41)
      COMMON /HELEN/ M,N,LS(30),MR(30,41),NTC,NT(30,41),MB(30,41)
      DIMENSION MA(30,41),ACR(30),ACC(41),LRS(30),LCD(41)
      DIMENSION RKI(30),RKJ(41)
      DIMENSION NZR(30), NZC(41),MZ(30,41), NXR(30), NXC(41)
C
C
      LGNN=99999999
      LGNSS=LGNN+1
      ITN=0
      DO 1 I=1,N
      LRS(I)=LGNSS
      DO 1 J=1,M
```

```
      IF(MR(I,J).LT.LRS(I))LRS(I)=MR(I,J)
    1 CONTINUE
      DO 3 I=1,N
      DO 3 J=1,M
      MB(I,J)=MR(I,J)
      IF(MB(I,J).EQ.LGNN)GO TO 3
      MB(I,J)=MB(I,J)-LRS(I)
    3 CONTINUE
      DO 4 J=1,M
      LCD(J)=LGNSS
      DO 4 I=1,N
      IF(MB(I,J).LT.LCD(J))LCD(J)=MB(I,J)
    4 CONTINUE
      DO 55 J=1,M
      IF(LCD(J).EQ.0) GO TO 55
      DO 5 I=1,N
      IF(MB(I,J).EQ.LGNN)GO TO 5
      MB(I,J)=MB(I,J)-LCD(J)
    5 CONTINUE
   55 CONTINUE
  900 DO 200 I=1,N
  200 LRS(I)=LS(I)
      DO 201 J=1,M
  201 LCD(J)=LD(J)
      DO 6 I=1,N
    6 NZR(I)=0
      DO 7 J=1,M
    7 NZC(J)=0
      DO 2 I=1,N
      DO 2 J=1,M
    2 NT(I,J)=0
      DO 8 I=1,N
```

```
      DO 8 J=1,M
      MZ(I,J)=1
      IF(MB(I,J).NE.0)GO TO 8
      MZ(I,J)=0
      NZR(I)=NZR(I)+1
      NZC(J)=NZC(J)+1
    8 CONTINUE
      DO 1500 I=1,N
      DO 1500 J=1,M
      MA(I,J)=0
      IF(MZ(I,J).NE.0)GO TO 1500
      MA(I,J)=LRS(I)
      IF(LCD(J).LT.LRS(I))MA(I,J)=LCD(J)
 1500 CONTINUE
      DO 1501 I=1,N
      ACR(I)=0
      DO 1502 J=1,M
 1502 ACR(I)=ACR(I)+MA(I,J)
      ACR(I)=ACR(I)-LRS(I)
 1501 CONTINUE
      DO 1503 J=1,M
      ACC(J)=0
      DO 1504 I=1,N
 1504 ACC(J)=ACC(J)+MA(I,J)
      ACC(J)=ACC(J)-LCD(J)
 1503 CONTINUE
 1515 RKEY=0
      CKEY=0
      RED=0
      DO 1510 I=1,N
      IF(NZR(I).EQ.0) GO TO 1510
      RED=1
```

```
      IF(ACR(I).GT.0) GO TO 1510
      DO 1511 J=1,M
      IF(MZ(I,J).NE.0)GO TO 1511
      NT(I,J)=MA(I,J)
      MA(I,J)=0
      MZ(I,J)=1
      NZR(I)=NZR(I)-1
      NZC(J)=NZC(J)-1
      LRS(I)=LRS(I)-NT(I,J)
      LCD(J)=LCD(J)-NT(I,J)
      DO 1512 K=1,N
      IF(MZ(K,J).NE.0)GO TO 1512
      IF(LCD(J).GE.LRS(K))GO TO 1512
      ACR(K)=ACR(K)-MA(K,J)+LCD(J)
      ACC(J)=ACC(J)-MA(K,J)+LCD(J)
      MA(K,J)=LCD(J)
      IF(NZR(K).EQ.0) GO TO 1516
      IF(ACR(K).GT.0) GO TO 1516
      RKEY=1
 1516 IF(NZC(J).EQ.0) GO TO 1512
      IF(ACC(J).GT.0) GO TO 1512
      CKEY=1
 1512 CONTINUE
      IF(LCD(J).NE.0)GO TO 1540
      NZC(J)=0
      DO 1541 L=1,N
      IF(MZ(L,J).NE.0)GO TO 1541
      MZ(L,J)=1
      NZR(L)=NZR(L)-1
 1541 CONTINUE
 1540 CONTINUE
      IF(NZR(I).EQ.0)GO TO 1510
```

```
1511 CONTINUE
1510 CONTINUE
     IF(RKEY.EQ.1)GO TO 1515
     IF(RED.EQ.0)GO TO 100
1525 CKEY=0
     RKEY=0
     RED=0
     DO 1520 J=1,M
     IF(NZC(J).EQ.0)GO TO 1520
     RED=1
     IF(ACC(J).GT.0)GO TO 1520
     DO 1521 I=1,N
     IF(MZ(I,J).NE.0)GO TO 1521
     NT(I,J)=MA(I,J)
     MA(I,J)=0
     MZ(I,J)=1
     NZC(J)=NZC(J)-1
     LRS(I)=LRS(I)-NT(I,J)
     LCD(J)=LCD(J)-NT(I,J)
     NZR(I)=NZR(I)-1
     DO 1522 K=1,M
     IF(MZ(I,K).NE.0)GO TO 1522
     IF(LRS(I).GE.LCD(K))GO TO 1522
     ACC(K)=ACC(K)-MA(I,K)+LRS(I)
     ACR(I)=ACR(I)-MA(I,K)+LRS(I)
     MA(I,K)=LRS(I)
     IF(NZC(K).EQ.0)GO TO 1526
     IF(ACC(K).GT.0)GO TO 1526
     CKEY=1
1526 IF(NZR(I).EQ.0)GO TO 1522
     IF(ACR(I).GT.0)GO TO 1522
     RKEY=1
```

```
1522 CONTINUE
     IF(LRS(I).NE.0)GO TO 1550
     NZR(I)=0
     DO 1551 L=1,M
     IF(MZ(I,L).NE.0)GO TO 1551
     MZ(I,L)=1
     NZC(L)=NZC(L)-1
1551 CONTINUE
1550 CONTINUE
     IF(NZC(J).EQ.0)GO TO 1520
1521 CONTINUE
1520 CONTINUE
     IF(RED.EQ.0)GO TO 100
     IF(CKEY.EQ.1)GO TO 1525
     IF(RKEY.EQ.1)GO TO 1515
     SUM=0
     DO 1707 I=1,N
1707 SUM=SUM+NZR(I)
     IF(SUM.EQ.0)GO TO 100
     MAXS=0
     MINZ=LGNN
     DO 1528 I=1,N
     DO 1528 J=1,M
     IF(MZ(I,J).NE.0)GO TO 1528
     EX=ACR(I)+ACC(J)
     IF(MINZ.LT.EX)GO TO 1528
     SX=LRS(I)+LCD(J)
     IF(MINZ.EQ.EX.AND.MAXS.GE.SX)GO TO 1528
     KI=I
     KJ=J
     MINZ=EX
     MAXS=SX
```

```
1528 CONTINUE
     NT(KI,KJ)=MA(KI,KJ)
     MA(KI,KJ)=0
     MZ(KI,KJ)=1
     NZR(KI)=NZR(KI)-1
     NZC(KJ)=NZC(KJ)-1
     LRS(KI)=LRS(KI)-NT(KI,KJ)
     LCD(KJ)=LCD(KJ)-NT(KI,KJ)
     DO 1530 I=1,N
     IF(MZ(I,KJ).NE.0)GO TO 1530
     IF(LCD(KJ).GT.LRS(I))GO TO 1530
     ACR(I)=ACR(I)-MA(I,KJ)+LCD(KJ)
     ACC(KJ)=ACC(KJ)-MA(I,KJ)+LCD(KJ)
     MA(I,KJ)=LCD(KJ)
1530 CONTINUE
     IF(LCD(KJ).NE.0)GO TO 1570
     NZC(KJ)=0
     DO 1571 L=1,N
     IF(MZ(L,KJ).NE.0)GO TO 1571
     MZ(L,KJ)=1
     NZR(L)=NZR(L)-1
1571 CONTINUE
1570 CONTINUE
     DO 1531 J=1,M
     IF(MZ(KI,J).NE.0)GO TO 1531
     IF(LRS(KI).GT.LCD(J))GO TO 1531
     ACC(J)=ACC(J)-MA(KI,J)+LRS(KI)
     ACR(KI)=ACR(KI)-MA(KI,J)+LRS(KI)
     MA(KI,J)=LRS(KI)
1531 CONTINUE
     IF(LRS(KI).NE.0)GO TO 1580
     NZR(KI)=0
```

```
      DO 1581 L=1,M
      IF(MZ(KI,L).NE.0)GO TO 1581
      MZ(KI,L)=1
      NZC(L)=NZC(L)-1
 1581 CONTINUE
 1580 CONTINUE
      GO TO 1515
  100 DO 101  I=1,N
  101 NXR(I)=0
      DO 102 J=1,M
  102 NXC(J)=0
      NOPT=0
      DO 103 I=1,N
      IF(LRS(I).EQ.0)GO TO 103
      NXR(I)=1
      NOPT=NOPT+1
  103 CONTINUE
      IF(NOPT.EQ.0)GO TO 700
  115 MOPT=0
      DO 110 I=1,N
      IF(NXR(I).EQ.0)GO TO 110
      DO 111 J=1,M
      IF(NXC(J).EQ.1)GO TO 111
      IF(MB(I,J).NE.0)GO TO 111
      NXC(J)=1
      MOPT=MOPT+1
  111 CONTINUE
  110 CONTINUE
      IF(MOPT.EQ.0)GO TO 800
      MORE=0
      DO 112 J=1,M
      IF(NXC(J).EQ.0)GO TO 112
```

```
      DO 113 I=1,N
      IF(NXR(I).EQ.1)GO TO 113
      IF(MB(I,J).NE.0)GO TO 113
      IF(NT(I,J).EQ.0)GO TO 113
      NXR(I)=1
      MORE=1
  113 CONTINUE
  112 CONTINUE
      IF(MORE.NE.0)GO TO 115
  800 MINX=LGNSS
      ITN=ITN+1
      NTC=0
      DO 163 I=1,N
      DO 163 J=1,M
  163 NTC=NTC+NT(I,J)*MR(I,J)
      DO 150 I=1,N
      IF(NXR(I).EQ.0)GO TO 150
      DO 151 J=1,M
      IF(NXC(J).EQ.1)GO TO 151
      IF(MB(I,J).LT.MINX)MINX=MB(I,J)
  151 CONTINUE
  150 CONTINUE
      IF(MINX.EQ.LGNSS)GO TO 993
      IF(MINX.EQ.LGNN)GO TO 997
      DO 160 I=1,N
      DO 160 J=1,M
      IF(MB(I,J).EQ.LGNN)GO TO 160
      IF(NXR(I).EQ.0.AND.NXC(J).EQ.1)MB(I,J)=MB(I,J)+MINX
      IF(NXR(I).EQ.1.AND.NXC(J).EQ.0)MB(I,J)=MB(I,J)-MINX
  160 CONTINUE
      GO TO 900
  700 ITN=ITN+1
```

```
      NTC=0
      DO 710 I=1,N
      DO 710 J=1,M
      NTC=NTC+NT(I,J)*MR(I,J)
  710 CONTINUE
      DO 10 J=1,M
      WRITE(6,20) (NT(I,J), I=1,N)
      WRITE(6,20) (MR(I,J),I=1,N)
   20 FORMAT(T10,10I8)
   10 CONTINUE
  998 CONTINUE
      DO 2205 I=1,N
      DO 2205 L=1,M
      MB(I,L)=MR(I,L)-MB(I,L)
 2205 CONTINUE
      GO TO 9999
  997 WRITE(6,4075)
 4075 FORMAT('0','MINX EQUALS 9999')
      GO TO 9999
  993 CONTINUE
      DO 2100 I=1,N
 2100 NXR(I)=0
      DO 2102 J=1,M
 2102 NXC(J)=0
      LKI=0
      DO 2103 I=1,N
      IF(LRS(I).EQ.0)GO TO 2103
      NXR(I)=1
      LKI=LKI+1
      RKI(LKI)=I
 2103 CONTINUE
      LKJ=0
```

```
      DO 2110 J=1,M
      IF(LCD(J).EQ.0)GO TO 2110
      LKJ=LKJ+1
      RKJ(LKJ)=J
 2110 CONTINUE
 2115 DO 2114 I=1,N
      IF(NXR(I).EQ.0)GO TO 2114
      DO 2111 J=1,M
      IF(NXC(J).EQ.1)GO TO 2111
      IF(MB(I,J).NE.0)GO TO 2111
      NXC(J)=1
      NZC(J)=I
      DO 2116 L=1,LKJ
      PL=L
      IF(J.EQ.RKJ(L))GO TO 2200
 2116 CONTINUE
 2111 CONTINUE
 2114 CONTINUE
      DO 2112 J=1,M
      IF(NXC(J).EQ.0)GO TO 2112
      DO 2113 I=1,N
      IF(NXR(I).EQ.1)GO TO 2113
      IF(MB(I,J).NE.0)GO TO 2113
      IF(NT(I,J).EQ.0)GO TO 2113
      NXR(I)=1
      NZR(I)=J
 2113 CONTINUE
 2112 CONTINUE
      GO TO 2115
 2200 CONTINUE
      KJJ=RKJ(PL)
      MAX=LCD(KJJ)
```

```
2230 KII=NZC(KJJ)
     KJJ=NZR(KII)
     DO 2232 L=1,LKI
     QL=L
     IF(KII.EQ.RKI(L))GO TO 2231
2232 CONTINUE
     IF(MAX.GT.NT(KII,KJJ))MAX=NT(KII,KJJ)
     GO TO 2230
2231 IF(MAX.GT.LRS(RKI(QL)))MAX=LRS(RKI(QL))
     KJJ=RKJ(PL)
2241 KII=NZC(KJJ)
     NT(KII,KJJ)=NT(KII,KJJ)+MAX
     IF(KII.EQ.RKI(QL))GO TO 2240
     KJJ=NZR(KII)
     NT(KII,KJJ)=NT(KII,KJJ)-MAX
     GO TO 2241
2240 LCD(RKJ(PL))=LCD(RKJ(PL))-MAX
     LRS(RKI(QL))=LRS(RKI(QL))-MAX
     GO TO 100
9999 RETURN
     END
```

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THE USE OF FLYING SQUADS IN THE OPERATION OF MUNICIPAL FIRE SUPPRESSION ACTIVITIES

by

Chang H. Park

(ABSTRACT)

This study is concerned with the use of "Flying squads" in municipal fire departments. Specifically, the research effort involves development of a mathematical model and solution procedure for the optimum use of flying squads as manpower transport vehicles.

The exact problem considered in this thesis is that of determining (a) the number of flying squads to use, (b) the locations/allocations of each flying squad, (c) the deployment strategy of each squad, and (d) the size of each squad and the job skill levels of each member of the squads. In formulating the model, the opinions of professionals in the field, as gathered through a written survey, were utilized to develop the appropriate constraints and objective function.

The objective function minimizes an estimate of the cost to society which includes the estimated fire losses and the cost of operating flying squads. The model involves constraints associated with (a) the maximum response time, (b) the potential demand for fire services, (c) the workload of a flying squad, and (d) the maximum number of squads to have.

The mathematical model developed is a mixed integer programming problem having a similar structure to the Capacitated Warehouse Location Problem. A Branch and Bound type algorithm based on previous work by

Akinc and Khumawala is developed to solve the problem. An example of this problem as it actually exists in the Roanoke Virginia Fire Department is modeled and solved.

The results obtained from this study demonstrates that flying squads have a potential use as an alternative method of manning the companies in municipal fire departments.