STRESS CHANGES IN REINFORCED
CONCRETE MEMBERS DUE TO CREEP

by

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I. INTRODUCTION

The subject of creep in concrete has been studied by numerous investigators both analytically and experimentally over a long period of time dating as far back as 1916.

A recent study of the "Redistribution of Bending Moment in Continuous Structures of Reinforced Concrete" by Ko-Chi Hsu (6) led this writer to pursue the topic further. Hsu developed generalized moment-curvature relations for several reinforced concrete members and studied the redistribution of bending moments in detail in a fixed-end beam loaded uniformly and subjected to creep. In his particular illustration, the redistribution did not result in significantly large changes of moment, but the basic study suggested that other forms of indeterminate beams and frames might yield larger and more significant moment changes.

A more detailed perusal of the basic creep equations led this writer to study the phenomenon of stress changes as well as moment changes. It was noted that creep caused stress changes which were independent of moment changes. Further investigation led to the realization that stress changes could be significantly large because of creep, even in those cases where the bending moment remained constant, as in a loaded cantilever beam.

A detailed analytical study was then outlined to develop the relation between creep and stress changes for a variety of beams of different geometry, material properties and creep coefficients.

Basic formulas were derived, ranges of parameters established and data developed using the IBM 1620 Digital Computer. The data was used to produce several forms of graphical curves. The use of the curves is illustrated in typical design problems.

II. SYMBOLS AND NOTATIONS

A_c Area of concrete

A_s Area of tension steel

A'_s Area of compression steel

b Width of beam

c Creep coefficient

d Distance from extreme compressive fiber to centroid of tensile reinforcement

d' Distance from extreme compressive fiber to centroid of compressive reinforcement

ϵ_c Strain of the concrete

ϵ_s Strain of the tension steel

ϵ'_s Strain of the compression steel

E_c Modulus of elasticity of the concrete

E_s Modulus of elasticity of the steel

f Stress

f_c Stress in the concrete

f_s Stress in the tension steel

f'_s Stress in the compression steel

I Moment of inertia

jd Internal moment arm

k Proportion of depth from the compression face to the neutral axis

kd Distance from extreme compressive fiber to neutral axis of the section

L Length of structural member

M	Bending moment
n	Ratio of E_s to E_c
n.a.	Neutral axis
p	Proportion of tension steel - A_s/bd
p'	Proportion of compression steel - A_s'/bd
P_c	Internal compressive force on the concrete of the beam
P_s	Internal tensile force on the tension reinforcement of the beam
P_s'	Internal compressive force on the compression reinforcement of the beam
w	Uniformly distributed load

III. GENERAL DISCUSSION OF THE CREEP PHENOMENOM

A. Definition of Creep and Brief Discussion of Creep Theories.

Under sustained loads plain concrete and concrete members experience a phenomenon known as creep. Creep in concrete is the time-dependent portion of the total strain resulting from a given load. The stress-strain curve in Figure 1 (7) depicts the creep property in concrete.

There are several theories explaining this phenomenon. Creep may be partly due to viscous flow of the cement-water paste, closure of internal voids, and/or crystalline flow in aggregates, but it is believed that the major portion is caused by seepage of adsorbed water from the gel that is formed by hydration of the cement. (10)

B. The Nature of Creep.

There are many factors relating to the quality of concrete that affect the magnitude and rate of creep. Some of these factors are the water-cement ratio, the type of aggregate, the conditions of curing, the temperature changes, size of the concrete mass, age at the time of loading, and the intensity and duration of loading.

1. Effect of Water-Cement Ratio.

The magnitude and rate of creep is greater in concretes of high water-cement ratio than it is in concretes of low water-cement ratio. This may be due to the fact that higher water-cement ratios cause increases in the size of the pores in the paste structure. Thus water may more readily escape the concrete mass, larger voids will result in the concrete and the water of adsorption may be expelled more readily from the gel when the concrete is subjected to compressive stresses.

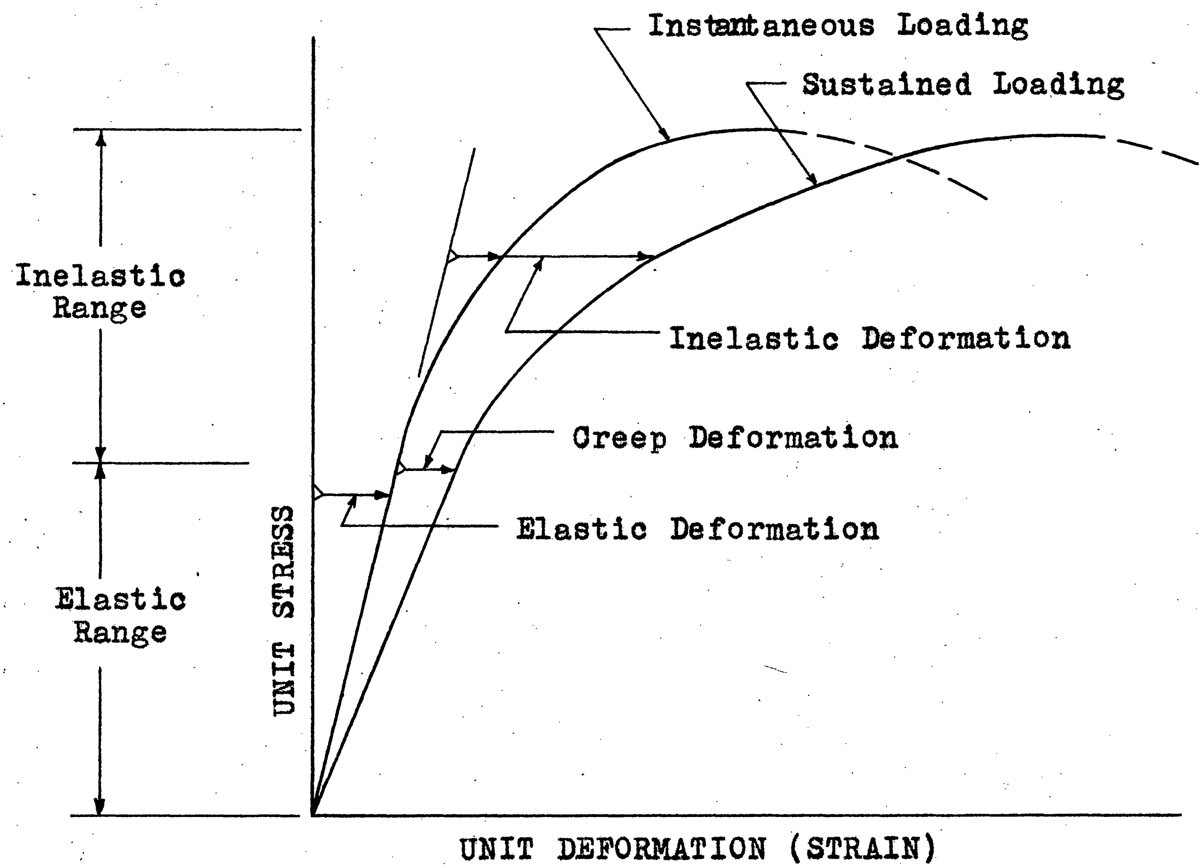


Figure 1. Stress-Strain Curves

Since the water-cement ratio is a measure of the strength of the concrete, it can be stated that low strength concretes experience a faster rate and greater magnitude of creep than do high strength concretes. (10)

2. Effect of Kind of Aggregate.

Davis, Davis and Hamilton (3) ran several tests on concrete specimens to study the effects of different kinds of aggregates on creep. In their tests, all aggregates were uniformly graded from fine to coarse and were batched in a saturated, surface-dry condition; their absorption factors were generally low. Therefore, any variations in creep were not due to the moisture conditions of the aggregates. Identical pastes were used in all mixes so that creep variations in the specimens were not due to seepage from the gel. Since the concrete specimens varied only as to the kind of aggregate, it may be stated that crystalline flow in the aggregates caused the different values of creep. It is possible that variations in particle shape, surface texture, and pore structure of the aggregates may have had some influence. From the test data, it was concluded that the creep variations between the specimens were so small that the influence of different kinds of aggregates on creep may be considered negligible in comparison to the other factors affecting creep.

3. Effect of the Humidity During Curing Stage.

The moisture conditions during the curing stage of plain concrete and concrete members has a marked effect on the amount of creep in the concrete. Concrete cured under humid conditions will creep less and at a lower rate than will that cured under dry conditions. Tests conducted by Davis and Troxell (10) on concrete cylinders of equal size and under equal sustained loads, showed that cylinders cured in 70 percent relative humidity experienced about two and one-half times the creep deformation of those cured in water. An increase in the humidity of the atmosphere reduces the loss of moisture to the surrounding atmosphere; this slows down the flow of moisture to the outer surface of the concrete, and thus reduces the loss of the adsorbed water from the gel. A factor affecting compressive creep in concrete members is that of drying shrinkage at or near the surface of the member. This drying shrinkage results in a reduction of the cross-sectional area remaining in compression and therefore causes higher stresses (and elastic strain) on the central core of the concrete. The higher the relative humidity during curing, the less drying shrinkage there will be.

4. Effect of Size of Mass.

Tests have shown that the size of a concrete mass has considerable effect on the creep of the concrete. From the results of creep tests conducted by Davis, Davis and Hamilton (3), it was shown that under equal values of sustained stresses for a given period of time, the creep deformation in a 10-inch diameter cylinder is only about one-half of that in a 6-inch diameter cylinder; both cylinders were the same length. The fact that under equal sustained stresses creep is less in larger masses of concrete is probably due to the reduced seepage. In large concrete masses the paths travelled by the expelled water will be longer with a resulting increase in the frictional resistance to the flow of water from the interior of the mass. For curing of similar shaped specimens in dry air, the smaller specimen has more area per unit volume than does the larger specimen, and is therefore more likely to be affected by surface drying. Thus, the effect of size on creep may become more pronounced when the concrete masses are placed in a dry atmosphere.

5. Effect of Age When First Loaded.

The magnitude of creep in a concrete member depends to a large extent on the age of the member when it was first loaded. The longer the concrete is allowed to age before it is loaded, the less the amount of total creep. An explanation for this is that the expulsion of moisture from the gel becomes more difficult as the porosity is decreased through hydration. The rate of creep is greatest at the time of load application and decreases rapidly as the magnitude of the creep increases for about the first six months. After the first six months the rate of creep will continue to decrease and the magnitude of creep will continue to increase slightly throughout the life of the loaded structure. Figure 2 is a typical creep-time curve. (8)

Figure 3 shows the percentage of long time creep versus duration of load (7) for cured-in-air portland cement concretes at whatever age first loaded. It can be seen from the graph that 10.5 percent of the long time creep took place within the first day the concrete was loaded. In about four years 98 percent of the long time creep will have occurred.

6. Summary of the Creep Phenomenon.

From the above information explaining the nature of creep, it can be stated that creep is dependent to a large

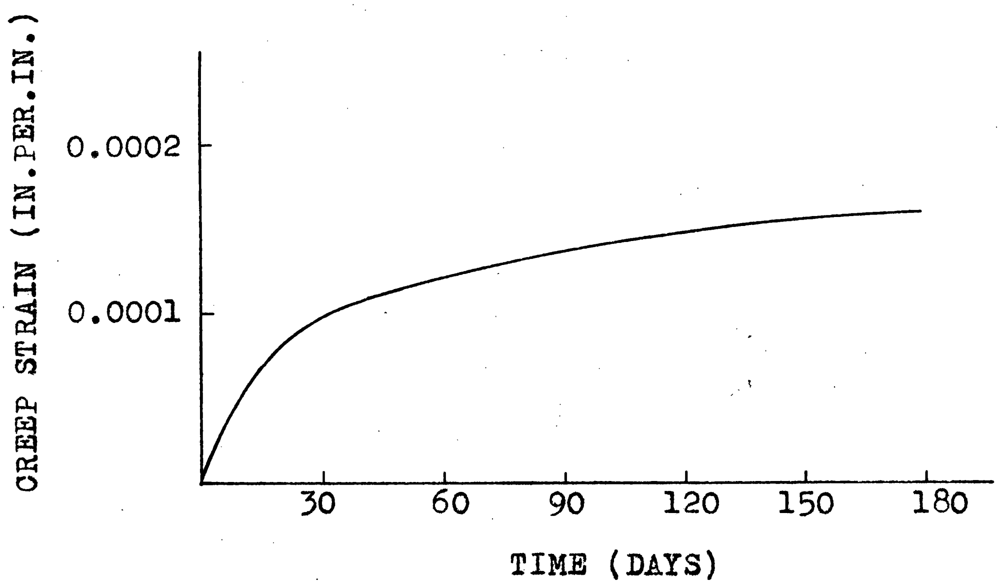


Figure 2. Creep-Time Curve

extent on the age of concrete when it is first loaded, the water-cement ratio or strength of the concrete, and on the environment in which curing takes place. To a lesser extent creep is affected by the size of the mass and the kind of aggregate used. It follows that the magnitude and rate of creep is greater for small members of high water-cement ratio cured in a dry atmosphere and loaded at an early age. The rate is greatest immediately after loading, but decreases

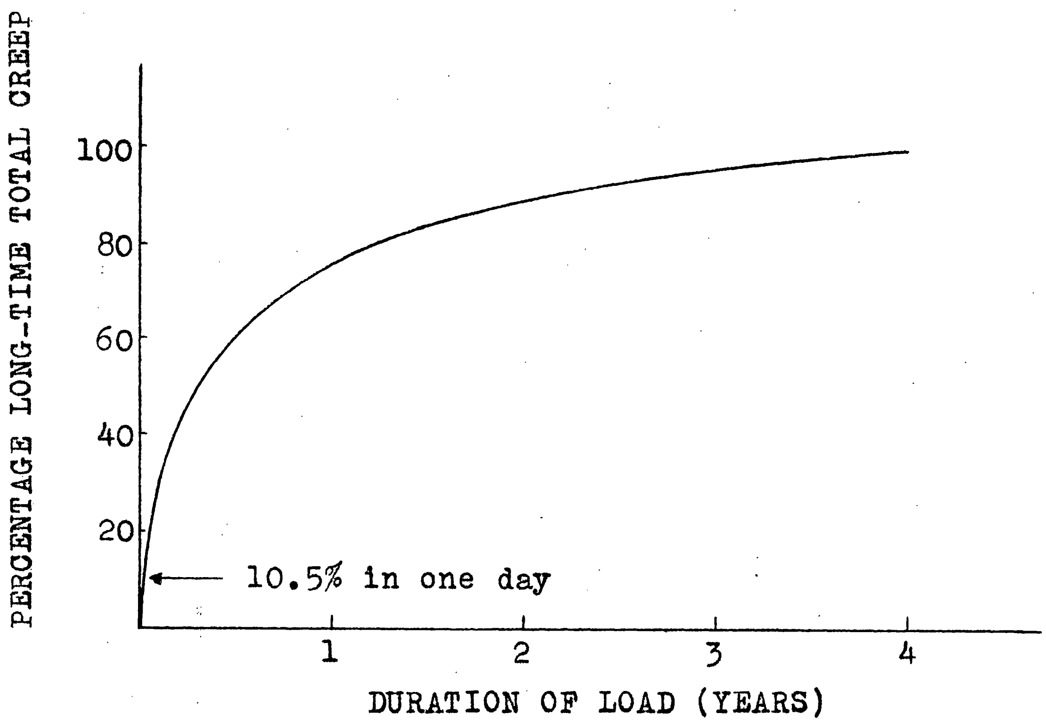


Figure 3. Percentage Creep - Duration of Load

quite fast. The magnitude continues to increase indefinitely but after a time becomes essentially negligible.

C. Deflections In Reinforced Concrete Beams.

1. Effect of Compressive Reinforcement on Beam Deflections.

The creep phenomenon has a marked effect on deflections in concrete structures. In a series of long-time deflection tests conducted by Washa and Fluck (12), additional deflections due to creep in reinforced concrete beams subjected to a sustained load for two and one-half years were observed to be one to two times as much as the original elastic deflections. The tests showed that the inclusion of arbitrary amounts of compression steel in simply supported reinforced concrete beams was very effective in reducing total deflection (including creep) by preventing excessive creep deformation. In several members of the same width and depth, the inclusion of compression steel equal in amount to the tension steel reduced the average total deflection by approximately one-half. The inclusion of half as much compression steel as tension steel caused a reduction of the average total deflection by approximately one-third.

Large creep deflections in beams are due to the continued straining of the concrete on the compression side. When compression steel is used in the member, the compression steel takes a large amount of the compressive force which results in a reduction in concrete compressive stresses and an effective decrease in creep deformation.

2. Method of Predicting Long Time Total Deflections.

Care must be taken in predicting long time deflections in reinforced concrete members. With a knowledge of the properties and age of the member rough values for long time total deflection may be calculated by modifying the formulas for elastic deflections.

Creep in concrete may be defined as the increase in strain due to a sustained stress. The modulus of elasticity of a material is the slope of the stress-strain curve; therefore, as creep takes place the slope (modulus of elasticity) becomes smaller. The smaller modulus of elasticity will result in a smaller calculated value of the moment of inertia of the assumed cracked transformed section. With the new values of modulus of elasticity and moment of inertia used in the formula for elastic deflection, a rough value for long-time total deflection may be calculated.

3. Elastic and Creep Recovery.

Since concrete develops both elastic and creep deformation while under load, it will show both elastic and creep recovery after removal of the load. The effect of duration of the load must be considered in discussing creep recovery just as it was considered in discussing creep deformation. The rate of creep recovery is much slower at late load-removal ages.

Figure 4 is a graphical illustration of the deformation and recoveries in a concrete specimen.

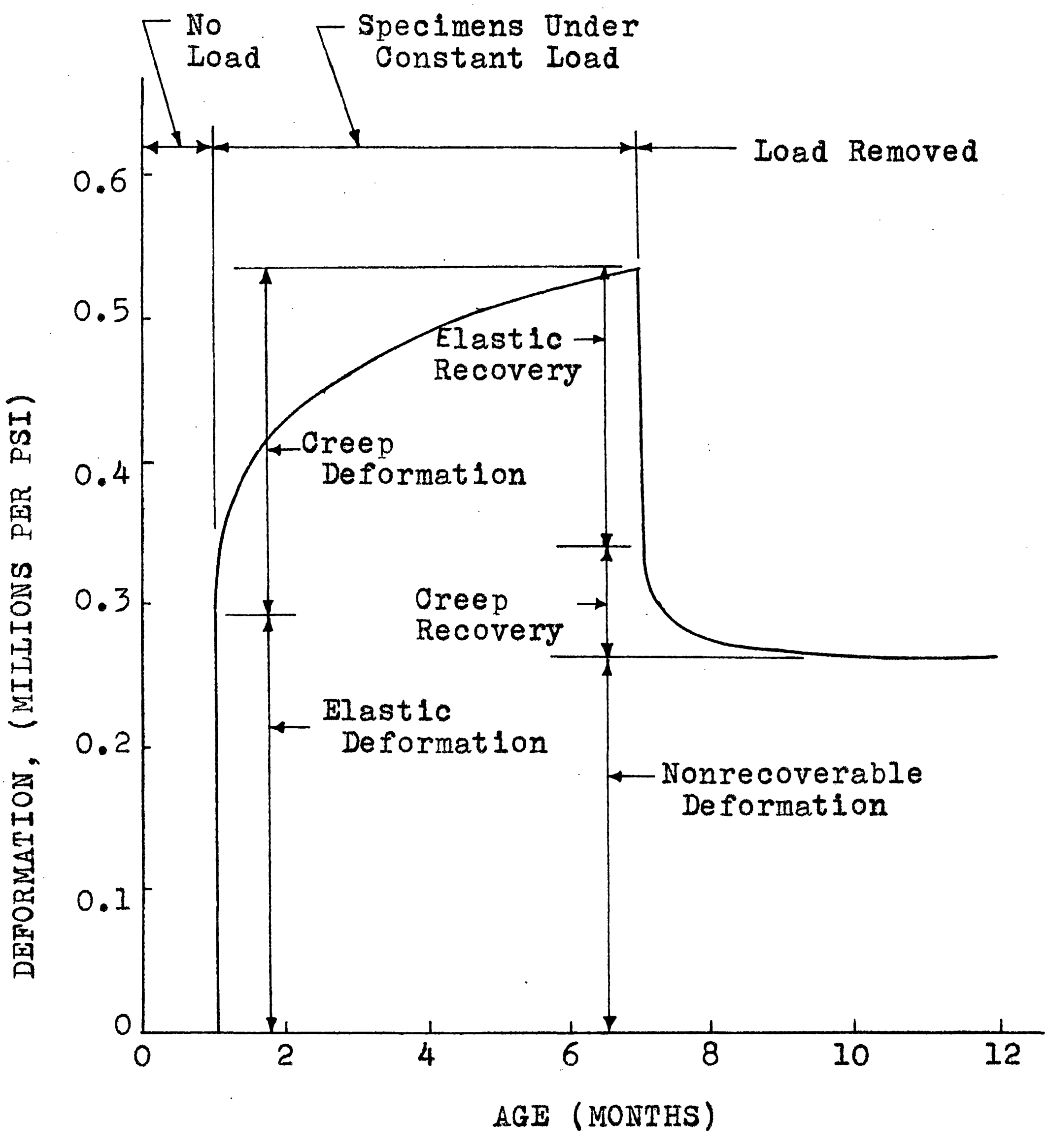


Figure 4. Elastic and Creep Deformation

D. The Nature of Stress Changes in Reinforced Concrete Members.

There are three physical phenomena that may result in stress changes in reinforced concrete members. These are the shrinkage phenomenon, the creep phenomenon, and the phenomenon of the redistribution of bending moments. The redistribution of bending moments may be caused by shrinkage or creep. The effects of shrinkage and creep can be separated analytically; however, in the physical action of the structure it is extremely difficult, and in most instances impossible, to measure these two effects separately.

1. <u>Shrinkage Effect on Stress Changes</u>.

Shrinkage takes place in concrete because of a natural loss in moisture independent of stress. Most of the shrinkage takes place during the curing period of the first few months. Thus, the most active shrinkage period may coincide with the period of most active creep.

The effect of temperature variation upon concrete is to produce expansion when the temperature rises and to produce contraction when the temperature falls. An accepted average value for the thermal coefficient of expansion of concrete is 0.000006 inches/inch/degree Fahrenheit, although this may vary from 0.000004 to 0.000007. (10)

The contractions in concrete due to temperature variation and shrinkage have the same mathematical relationships, and in design work it is customary to assume a combined coefficient for shrinkage plus temperature. Temperature effects in concrete are also usually considered to produce the same physical action as shrinkage. Therefore, this discussion of the effects of shrinkage will be assumed to include the effects of temperature change. Shrinkage stresses in concrete are caused by external and/or internal restraints to the free contraction of the concrete. Figure 5 depicts the strains developed in a restrained reinforced concrete element due to shrinkage. (2) If the element were unreinforced and unrestrained, it would tend to shorten from section A-A to section B-B due to shrinkage. However, the reinforcing steel at the bottom of the member restrains the free shrinkage, causing movement from section A-A to section C-C. This will result in the steel being stressed in compression from A' to C' while the concrete surrounding the steel will be stressed in tension from B' to C' if it does not crack. The concrete near the top of the element may be stressed in compression.

Induced stresses in the steel and concrete due to shrinkage may be calculated by multiplying the unit shrinkage at the point in question by the modulus of elasticity of the respective material. Ultimate values of shrinkage deformation of ordinary mixes and relatively thin members of plain concrete may be obtained from Scherer's formula (7) for unit shrinkage:

$$\delta = 0.00125\ (0.90 - h)$$

wherein δ is unit shrinkage and h is the relative humidity of the surrounding atmosphere. For a fully saturated curing condition in fog or water, h will equal one and the concrete element will experience expansion.

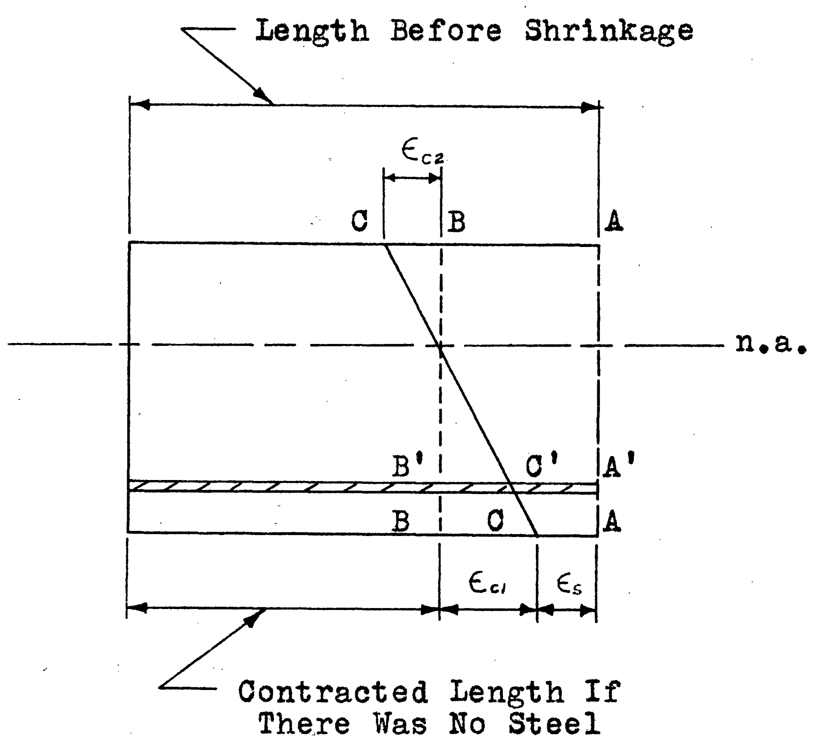


Figure 5. Shrinkage In A Concrete Element

2. Creep Effect on Stress Changes.

Creep strain occurs in a reinforced concrete member when the member is subjected to an applied stress. This applied stress may be due to the external loads on the member, to external restraints against shrinkage, to rotation of the ends of the member, to displacement of the ends of the member, or to any combination of these things.

Tensile and compressive reinforcement comprise internal restraints to creep in the concrete. Therefore, if any creep strain occurs in a reinforced concrete member, it will cause changes in the steel and concrete stresses in that member. This thesis is a detailed study of the stress changes caused by creep in the concrete. Section IV will explain how these stresses may be calculated.

Figure 6 is an illustration of the elastic and creep strain distribution developed in a reinforced concrete element subjected to an applied stress.

3. The Redistribution of Bending Moments.

The stiffness of a structural member may be defined as the moment necessary to rotate one simply supported end through a unit angle when the other end is fixed, partially fixed or simply supported. Stiffness takes the form $\alpha \frac{EI}{L}$, where α is a constant dependent upon the end conditions of the member. As creep occurs in a reinforced concrete member, the stiffness of the member decreases. This is due to the fact that as creep occurs in the concrete, the concrete modulus of elasticity (E) is less, and therefore the moment of inertia (I) is less.

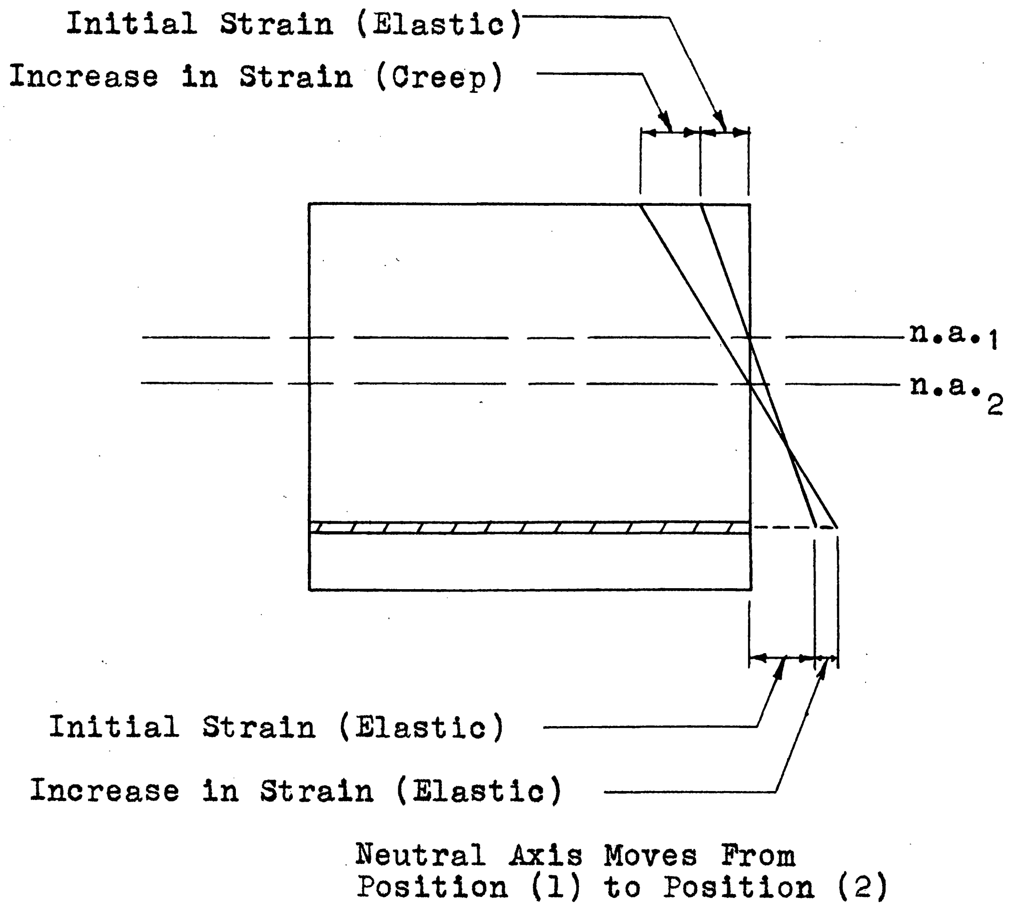


Figure 6. Elastic and Creep Strain Distribution In A Concrete Element

In an indeterminate reinforced concrete structure, the stiffness is not normally uniform throughout the structure because of the variation in reinforcement. When the stiffness of the structure decreases due to creep of the concrete, the less stiff sections of the structure experience a larger percentage change in stiffness than the stiffer sections of the structure. This non-uniform stiffness will produce a redistribution of bending moments in which the stiffer sections of the structure will experience an increase in bending moment and the less stiff sections will experience a decrease in bending moment. The change of bending moment at any section will result in a corresponding change of stress at that section.

Shrinkage action in a reinforced concrete structure will quite often cause translations and/or rotations in the individual members of the structure. These movements may induce additional bending effects and thereby produce a redistribution of bending moment throughout the entire structure. For every change of bending moment there will be a corresponding change of stresses.

4. <u>Beneficial and Detrimental Effects of Stress Changes on the Structure</u>.

Shrinkage and creep and the redistribution of bending moments in a reinforced concrete structure all cause changes of stress throughout the structure. Shrinkage action in concrete structures usually has a detrimental effect because of the large bending moments that are often induced by it. Excessive strains and stresses due to shrinkage may be large enough to cause cracks which if continuously exposed to weathering could eventually cause deterioration and become a serious problem. In one respect shrinkage in concrete is beneficial; if it does not produce cracking in the member, it will cause the concrete to grip the steel tightly, thus decreasing the possibility of bar slippage. Occasionally excessive strains and stresses due to creep may induce large deflections which also become a problem. However, both shrinkage and creep strains may be effectively decreased by the addition of compressive reinforcement.

Creep in concrete is usually advantageous because it produces a redistribution of stresses along the longitudinal axis of the members throughout the concrete structure. The stresses in adjacent members, or in a different section of the same member may be increased at one point and decreased at another, but the over-all effect usually results in a beneficial smoothing out of the stress pattern throughout the structure. Thus, creep has a beneficial influence upon continuous concrete structures, in the same way that the ductility of structural steel facilitates beneficial stress adjustment in continuous steel structures.

IV. INVESTIGATION OF STRESS CHANGES IN REINFORCED CONCRETE MEMBERS

A. Assumptions Used in Derivations.

To investigate the changes of stresses caused by creep in reinforced concrete members, several assumptions will be made as follows:

1. Stresses remain within the allowable working range specified by the 1956 A.C.I. code.
2. Beam cross-sections are plane and remain plane after bending of the beam.
3. Concrete below the neutral axis on the tension side of the member takes no tensile stress. The concrete is assumed to be cracked.
4. The compressive stress distribution on the cross-section is linear.
5. The reinforcing steel experiences no creep.
6. Creep is assumed to take place free of shrinkage.
7. Creep strain is directly proportional to elastic strain and within the allowable working range both are proportional to the compressive stress on the concrete.

As previously stated in section C-3, reinforced concrete members subjected to a constant bending moment experience a redistribution of stresses due to creep. Therefore by starting with an applied moment on the cross-section of a beam, equations of stress per unit moment in terms of the geometry of the cross-section and properties of the materials will be derived.

B. Derivation of Equations for Finding "k".

Before the stress/moment equations are derived it is necessary to derive the equation for k (the proportion of depth from the compression face to the neutral axis) for tensile reinforced and double reinforced beams taking into consideration the creep in the concrete.

Since the modulus of elasticity of concrete becomes smaller as creep takes place, the value of the concrete modulus will be considered to be $\frac{1}{c}E_c$. When creep is zero, $c = 1$ and E_c is the elastic modulus. $\frac{E_s}{\frac{1}{c}E_c} = cn$ and cn becomes the modular ratio.

1. Tensile Reinforced Section.

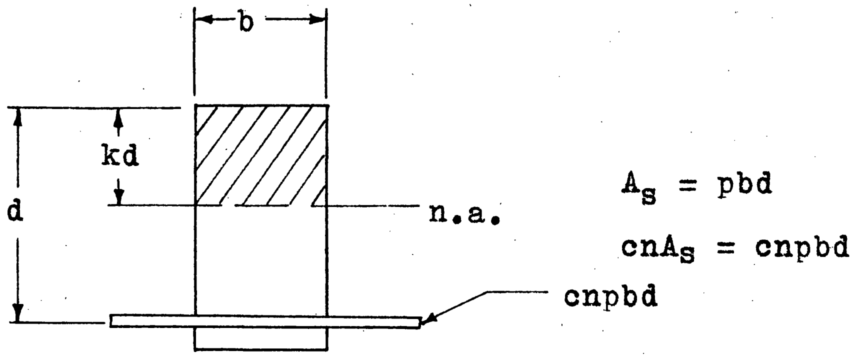


Figure 7. Tensile Reinforced Section

Taking the moment of areas about the neutral axis and solving for k

$$bkd \frac{kd}{2} = cnpbd \left[d - kd \right]$$

$$k^2 + (2\ cnp)k - 2\ cnp = 0$$

$$k = \sqrt{2\ cnp + (cnp)^2} - cnp$$

2. <u>Double Reinforced Section</u>.

$$A_s' = p'bd$$

$$(cn - 1)A_s' = (cn - 1)p'bd$$

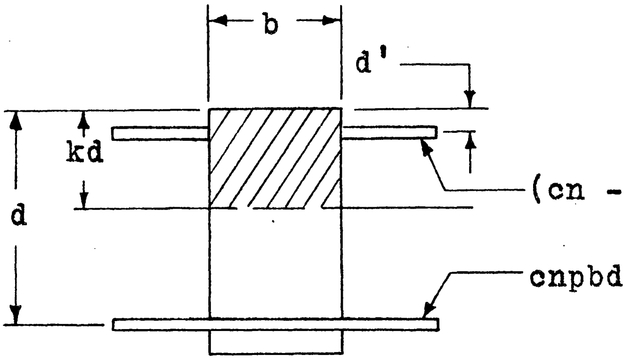


Figure 8. Double Reinforced Section

Taking the moment of areas about the neutral axis and solving for k

$$bkd\left(\frac{kd}{2}\right) + (cn - 1)p'bd(kd - d') - cnpbd\ (d - kd) = 0$$

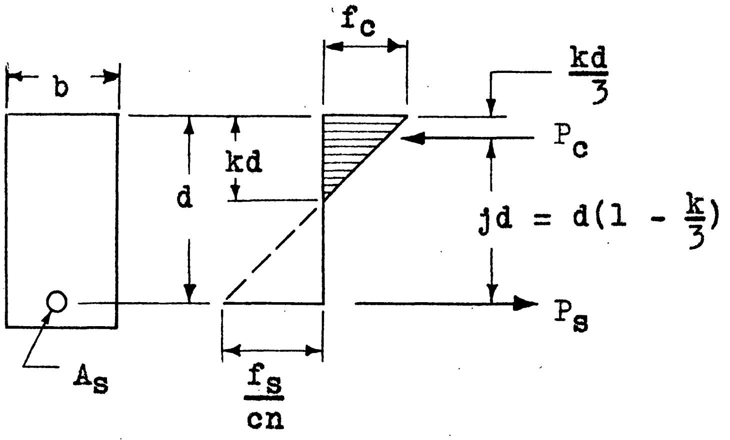
$$k^2 + 2(cnp + cnp' - p')k - 2\left(cnp + cnp'\frac{d'}{d} - p'\frac{d'}{d}\right) = 0$$

$$k = \sqrt{2\left(cnp + cnp'\frac{d'}{d} - p'\frac{d'}{d}\right) + (cnp + cnp' - p')^2}$$

$$- (cnp + cnp' - p')$$

C. Derivation of Stress/Moment Equations.

1. Tensile Reinforced Section.



(a) (b)

Figure 9. Tensile Reinforced Section and Stress Diagram

a. Concrete Stress/Moment Equation.

$$M = P_c \, jd$$

$$M = \tfrac{1}{2} f_c \, b \, kd \left[(1 - \frac{k}{3}) d \right]$$

$$\frac{f_c}{M} = \frac{2}{bd^2 \, k(1 - \frac{k}{3})}$$

b. Steel Stress/Moment Equation.

$$M = P_s \, jd$$

$$M = f_s pbd \quad d(1 - \frac{k}{3})$$

where $P_s = A_s f_s = pbd \, f_s$; $\frac{f_s}{M} = \frac{1}{bd^2 \, p(1 - \frac{k}{3})}$

From Section B-1

$$k = \sqrt{2 \, cnp + (cnp)^2} - cnp$$

2. Double Reinforced Section.

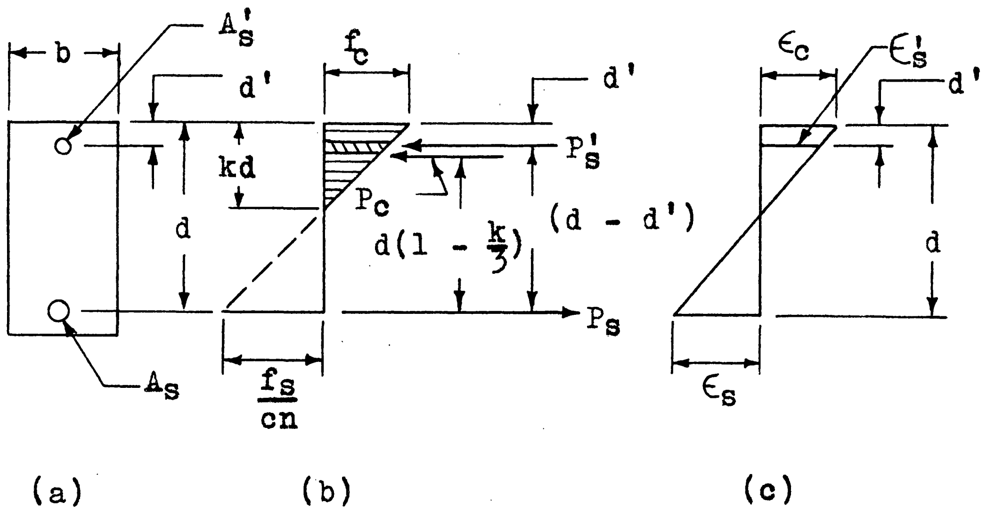


Figure 10. Double Reinforced Section, Stress Diagram, and Strain Diagram.

a. Concrete Stress/Moment Equation.

Summing moments about A_s (Fig. 10 (b))

$$M = P_s' \left[d - d'\right] + P_c \left[d(1 - \frac{k}{3})\right]$$

$$M = f_s' A_s' (d - d') + \tfrac{1}{2} f_c b kd^2 (1 - \frac{k}{3})$$

From the strain diagram (Fig. 10 (c))

$$\frac{\epsilon_s'}{\epsilon_c} = \frac{kd - d'}{kd}$$

$$\text{so } \epsilon_s' = \left(\frac{kd - d'}{kd}\right) \epsilon_c$$

$$\text{then } \frac{f_s'}{E_s} = \left(\frac{kd - d'}{kd}\right) \frac{f_c}{E_c \frac{1}{c}}$$

$$\text{and } f_s' = \left(\frac{kd - d'}{kd}\right)\left(\frac{E_s}{E_c}\right) f_c c = \left(1 - \frac{d'}{kd}\right) cnf_c$$

Now, $M = (1 - \frac{d'}{kd})cnf_c \quad p'bd \quad (d - d')$

$$+ \tfrac{1}{2} f_c\ b\ kd^2(1 - \tfrac{k}{3})$$

When $A_s' = p'bd$

Simplifying, $M = f_c bd^2 \left[(1 - \frac{d'}{kd})\ cnp'(1 - \frac{d'}{d}) + \tfrac{1}{2} k\ (1 - \tfrac{k}{3}) \right]$

and $$\frac{f_c}{M} = \frac{1}{bd^2 \left[(1 - \frac{d'}{kd})cnp'(1 - \frac{d'}{d}) + \tfrac{1}{2}k(1 - \tfrac{k}{3}) \right]}$$

b. Compressive Steel Stress/Moment Equation.

Summing moments about A_s (Fig. 10 (b))

$$M = P_s' \left[d - d' \right] + P_c \left[d(1 - \tfrac{k}{3}) \right]$$

$$M = f_s'\ A_s' \left[d - d' \right] + \tfrac{1}{2}f_c\ b\ kd \left[d(1 - \tfrac{k}{3}) \right]$$

From the strain diagram (Fig. 10 (a))

$$\frac{\epsilon_c}{\epsilon_s'} = \frac{kd}{kd - d'}$$

$$\text{so } \epsilon_c = (\frac{kd}{kd - d'})\epsilon_s'$$

$$\text{then } \frac{f_c}{E_c \frac{1}{C}} = (\frac{kd}{kd - d'})\ \frac{f_s'}{E_s}$$

$$\text{and } f_c = (\frac{kd}{kd - d'})\frac{E_c}{C E_s}\ f_s' = (\frac{kd}{kd - d'})\ \frac{f_s'}{cn}$$

Now, $M = f'_s\, p'bd^2(1 - \frac{d'}{d})$

$$+ \tfrac{1}{2} f'_s\, bd^2 \frac{k}{cn}(\frac{kd}{kd - d'})(1 - \frac{k}{3})$$

when $A'_s = p'bd$

Simplifying, $M = f'_s\, bd^2 \left[p'(1 - \frac{d'}{d}) + \frac{k}{2cn}(\frac{kd}{kd - d'})(1 - \frac{k}{3})\right]$

and $$\frac{f'_s}{M} = \frac{1}{bd^2\left[p'(1 - \frac{d'}{d}) + \frac{k}{2cn}(\frac{kd}{kd - d'})(1 - \frac{k}{3})\right]}$$

c. Tensile Steel Stress/Moment Equation.

Summing moments about A_s (Fig. 10 (b))

$$M = P'_s\left[d - d'\right] + P_c\left[d(1 - \frac{k}{3})\right]$$

$$M = f'_s\, A'_s\left[d - d'\right] + \tfrac{1}{2} f_c\, b\, kd\left[d(1 - \frac{k}{3})\right]$$

From the strain diagram (Fig. 10 (a))

$$\frac{\epsilon_c}{\epsilon_s} = \frac{kd}{d - kd}$$

so $\epsilon_c = (\frac{k}{1 - k})\epsilon_s$

then $\frac{f_c}{E_c \frac{1}{c}} = (\frac{k}{1 - k})\frac{f_s}{E_s}$

and $f_c = (\frac{k}{1 - k})\frac{E_c}{E_s c} f_s = (\frac{k}{1 - k})\frac{f_s}{cn}$

$$\frac{f'_s}{kd - d'} = \frac{f_s}{d - kd}$$

$$f_s' = \left(\frac{kd - d'}{d - kd}\right) f_s$$

Now, $$M = \left(\frac{kd - d'}{d - kd}\right) f_s\, p'\, bd\, (d - d')$$

$$+ \tfrac{1}{2} f_s \left(\frac{b\,kd}{cn}\right)\left(\frac{k}{1 - k}\right)\left(d - \frac{kd}{3}\right)$$

when $A_s' = p'\, bd$

Simplifying,

$$M = f_s\, bd^2 \left[\left(\frac{k - \frac{d'}{d}}{1 - k}\right) p'\left(1 - \frac{d'}{d}\right) + \frac{k^2}{2cn\,(1 - k)} \left(1 - \frac{k}{3}\right) \right]$$

and

$$\frac{f_s}{M} = \frac{1}{bd^2 \left[p' \left(\frac{k - \frac{d'}{d}}{1 - k}\right)\left(1 - \frac{d'}{d}\right) + \frac{k^2}{2cn(1 - k)}\left(1 - \frac{k}{3}\right) \right]}$$

From Section B-2,

$$k = \sqrt{2\left(cnp + cnp'\,\frac{d'}{d}\right) + (cnp + cnp' - p')^2} - (cnp + cnp' - p')$$

D. Tables of Stress/Moment.

1. Use of Digital Computer to Generate Tables.

To get an idea of the magnitude and the rate of change of f /M, variations in the geometry of the cross-section and the properties of the concrete were studied. A wide range and choice of parameters was decided upon and the equations were programmed on the IBM 1620 Digital Computer.

2. Dimensions and Parameters of Beams Used In Tables.

All values of parameters were chosen to cover the widest possible range of practical values. Enough intermediate points were evaluated to provide continuity to the plotted curves. Limitations on the values of the parameters are as follows:

$$d = 10.0'',\ 10\sqrt[3]{2}\,'',\ 20.0''$$

$$\frac{b}{d} = 0.3,\ 0.5,\ 1.0,\ 1.5$$

$$p = 0.002,\ 0.005,\ 0.010,\ 0.020,\ 0.030$$

$$\frac{p'}{p} = 0.0,\ 0.2,\ 0.4,\ 0.6$$

$$N = 6,\ 8,\ 10,\ 12,\ 15$$

$$C = 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7$$

For the cases of tensile reinforcement only, it will be noted that f/M varies inversely as the cube of depth d, so that the chosen values of depth give f/M values in the ratio of 8:4:1. Creep coefficients were chosen for the pure elastic case (C = 1), the more usual working range (C = 2 to C=4) and for a range beyond ($C = 5$ to $C = 7$) in order to better evaluate effects of excessive creep. Values of f/M were calculated for all combinations of the above parameters and are tabulated in a separate supplement to this thesis. (9)

E. Graphs of Percent Change In Stress at Constant Bending Moment vs. Creep Coefficient.

1. Explanation and Discussion of Graphs.

The graphs (see Figures 11 through 25) show that for a beam subjected to constant moment creep causes a reduction in compressive concrete stresses, an increase in tensile steel stresses, and when present, an increase in compressive steel stresses. The percentage loss in concrete stress is always about four or five times greater than the percentage gain in the tensile steel stress (see Figures 11 through 25) and the percentage gain in compressive steel stress is always about three to five times greater than the percentage loss in the concrete stress. (See Figures 14 through 22.)

The graphs for the double reinforced members (Figures 14 through 22) show several hundred percent gain in calculated stresses in the compression steel as creep takes place. This increase may not be physically possible if the compression steel yields. If the design stresses were low, the curves could be used through most or all of their range. Quite often the size of a member is dictated by factors other than the design load. In these cases of lower working stresses the range of curves may be quite useful.

The compression steel curves can be said to have a stress cut off point at the yield limit. Strain parallels stress up to this cut off point, after which the curves lose their value.

2. Use of Graphs for Design Problems.

The fact that these graphs are non-dimensional makes them useful for design purposes. For example, if an engineer designs a beam and would like to predict the long time stress changes, he can easily do so by the following method:

1. Establish a value of c for the concrete under working conditions. (c = 4 is a good value for long time predictions of stress changes.)
2. For the design value of n, choose the proper graph.
3. Then at the given value of c, proceed vertically on the graph to the curve with the known percent tension steel. Intermediate values may be found by interpolation.
4. Next, move horizontally and read the corresponding value of the percent change in stress. Multiplying this value by the elastic stress value will give the amount of stress change due to creep.

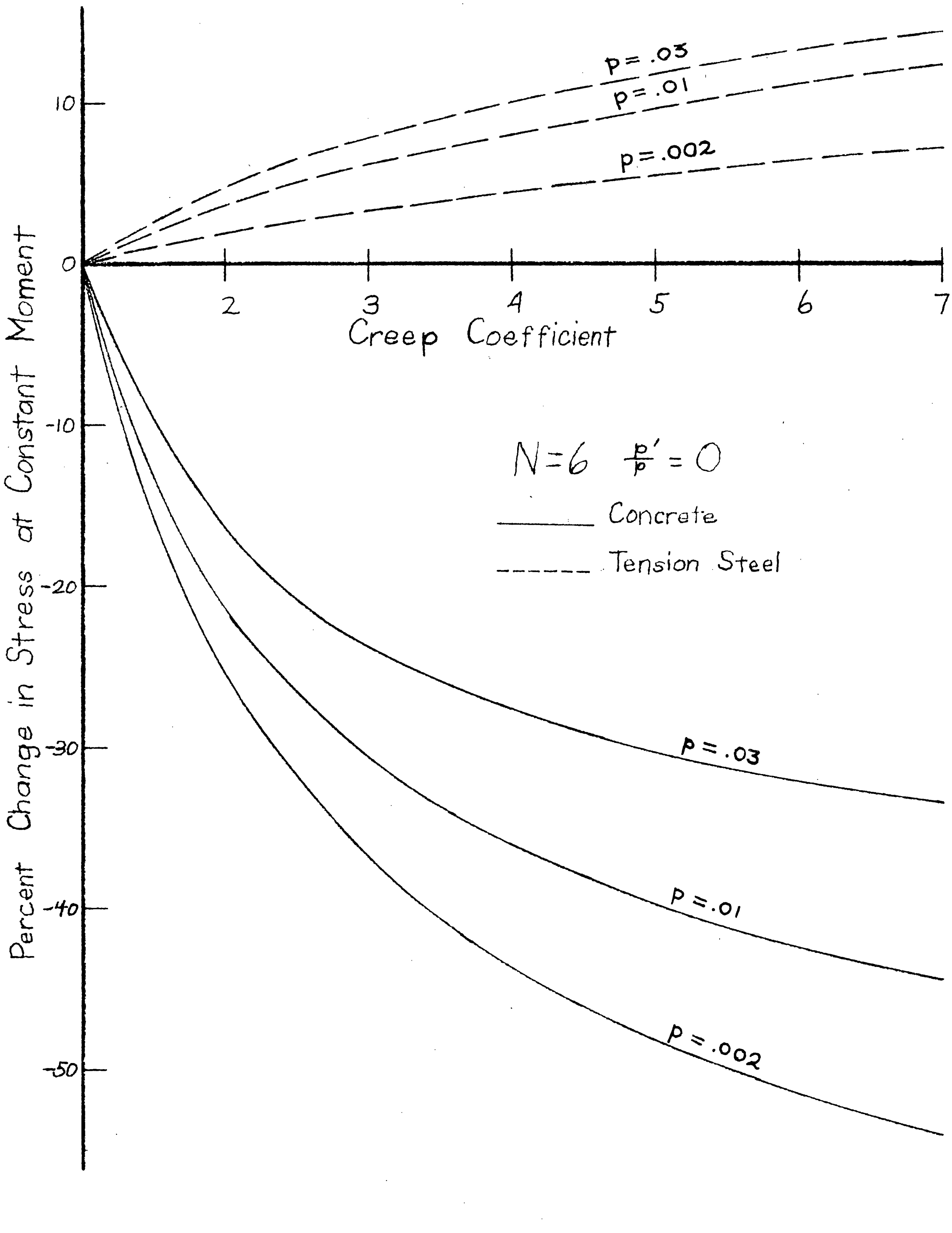


FIG. 11 CHANGE IN STRESS – CREEP COEFFICIENT

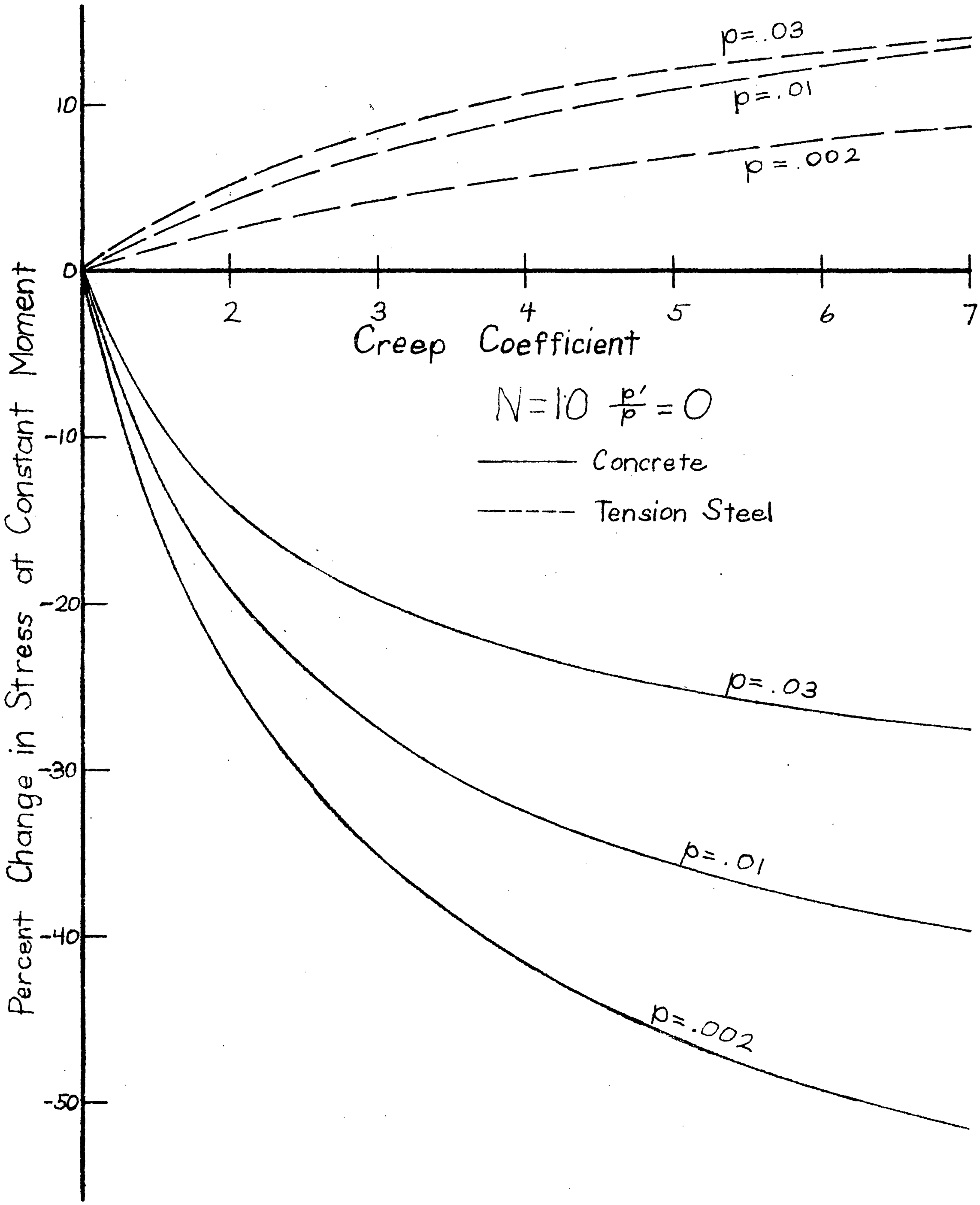


FIG. 12 CHANGE IN STRESS — CREEP COEFFICIENT

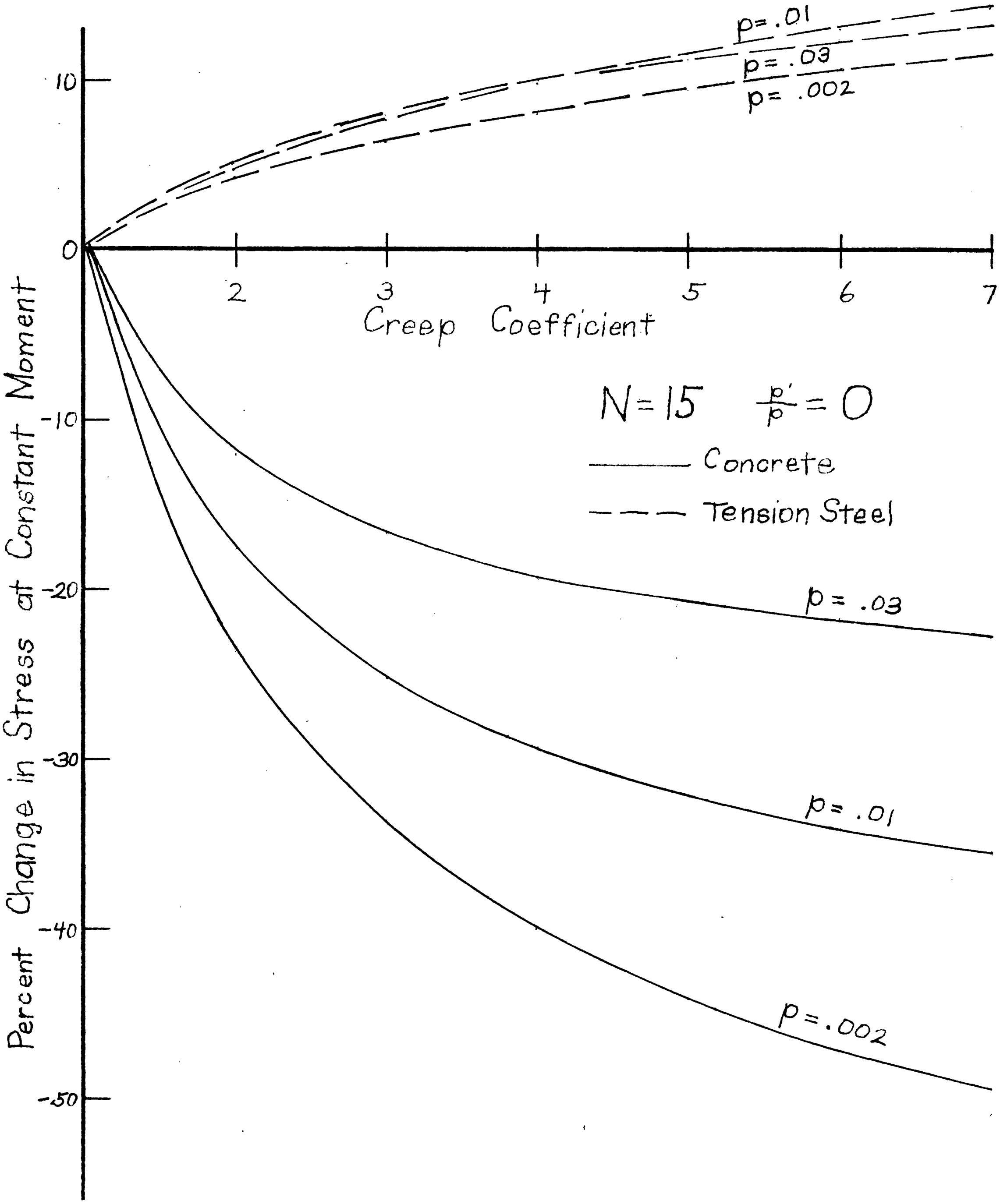


FIG. 13 CHANGE IN STRESS - CREEP COEFFICIENT

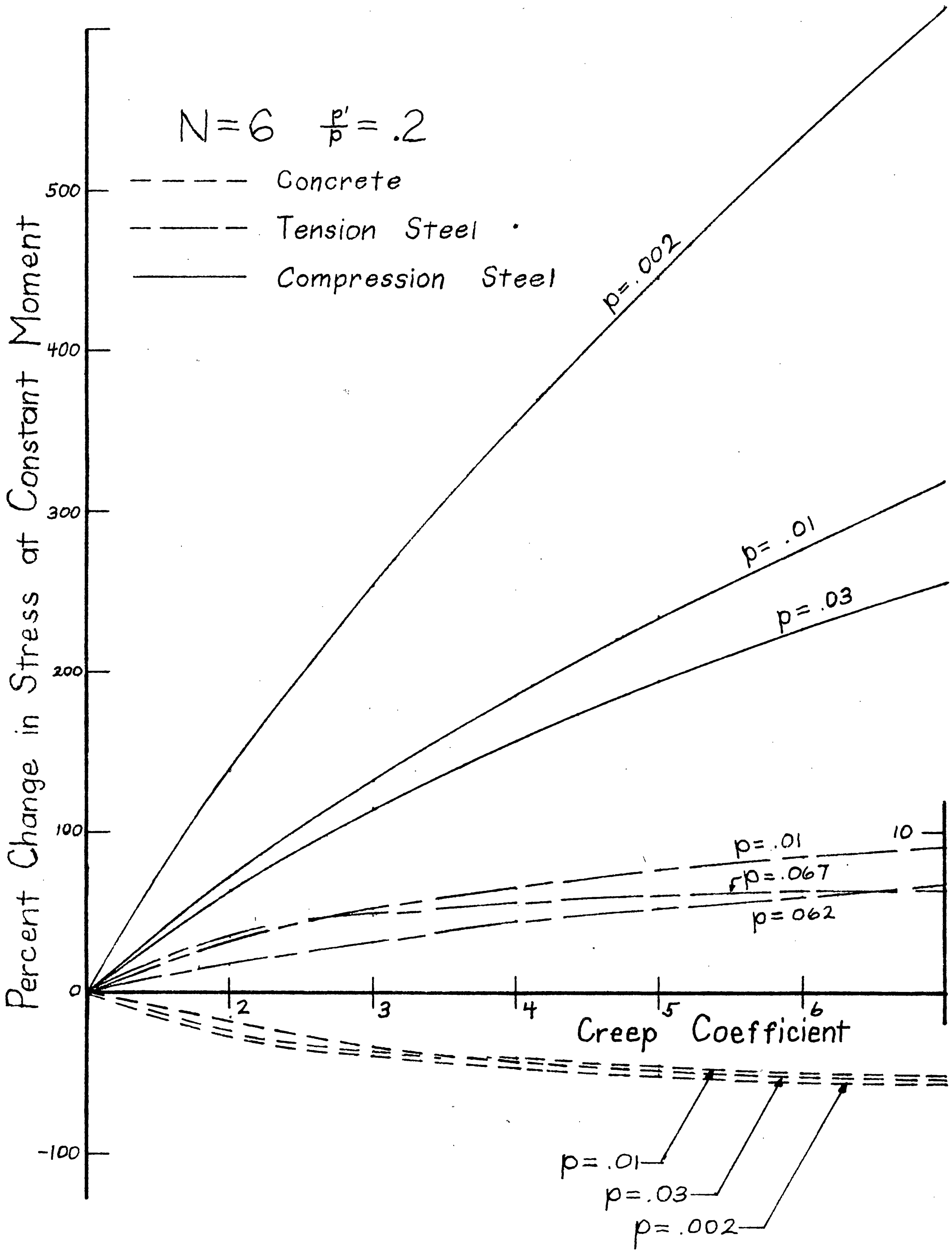


FIG. 14 CHANGE IN STRESS - CREEP COEFFICIENT

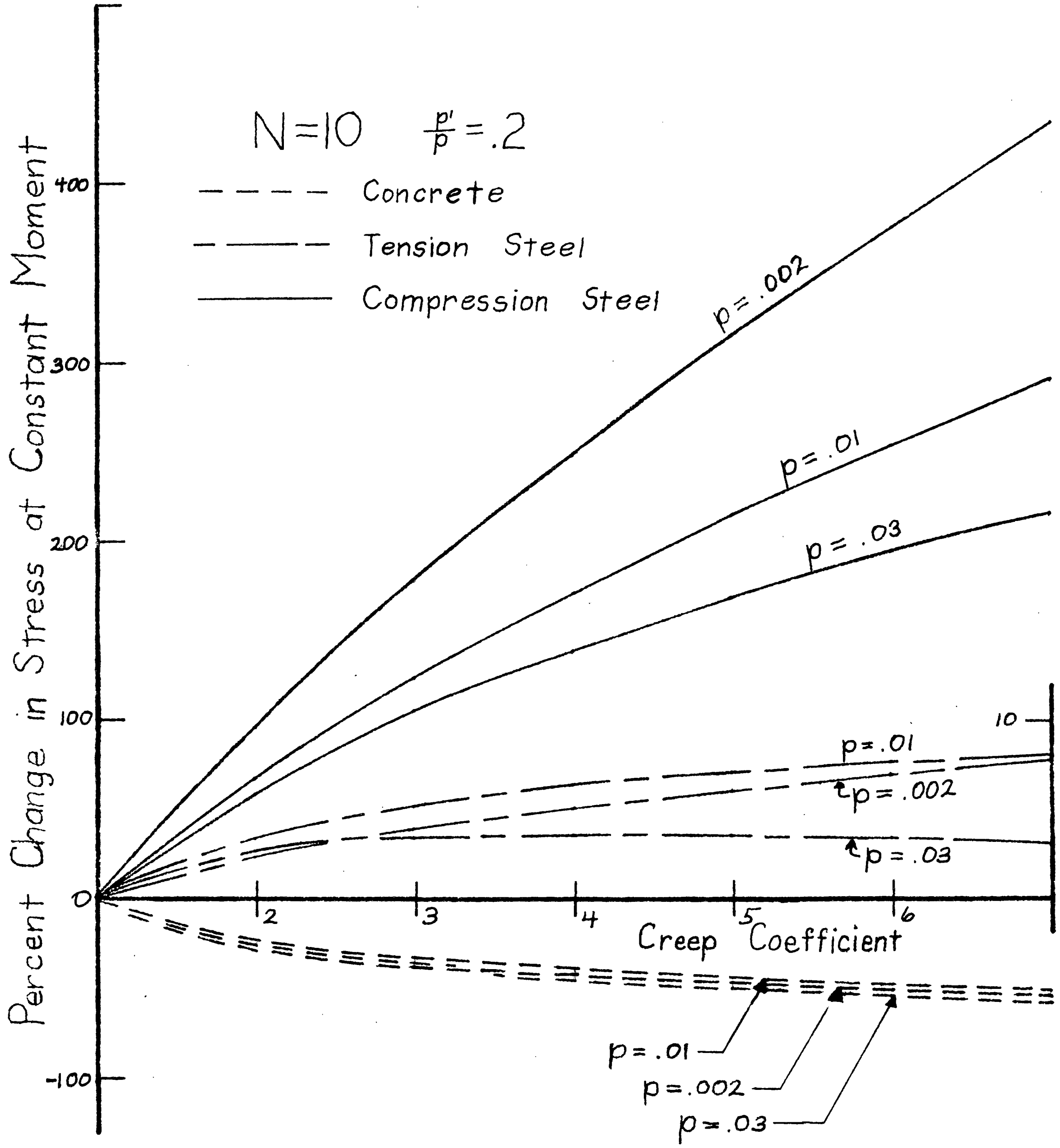


FIG 15 CHANGE IN STRESS - CREEP COEFFICIENT

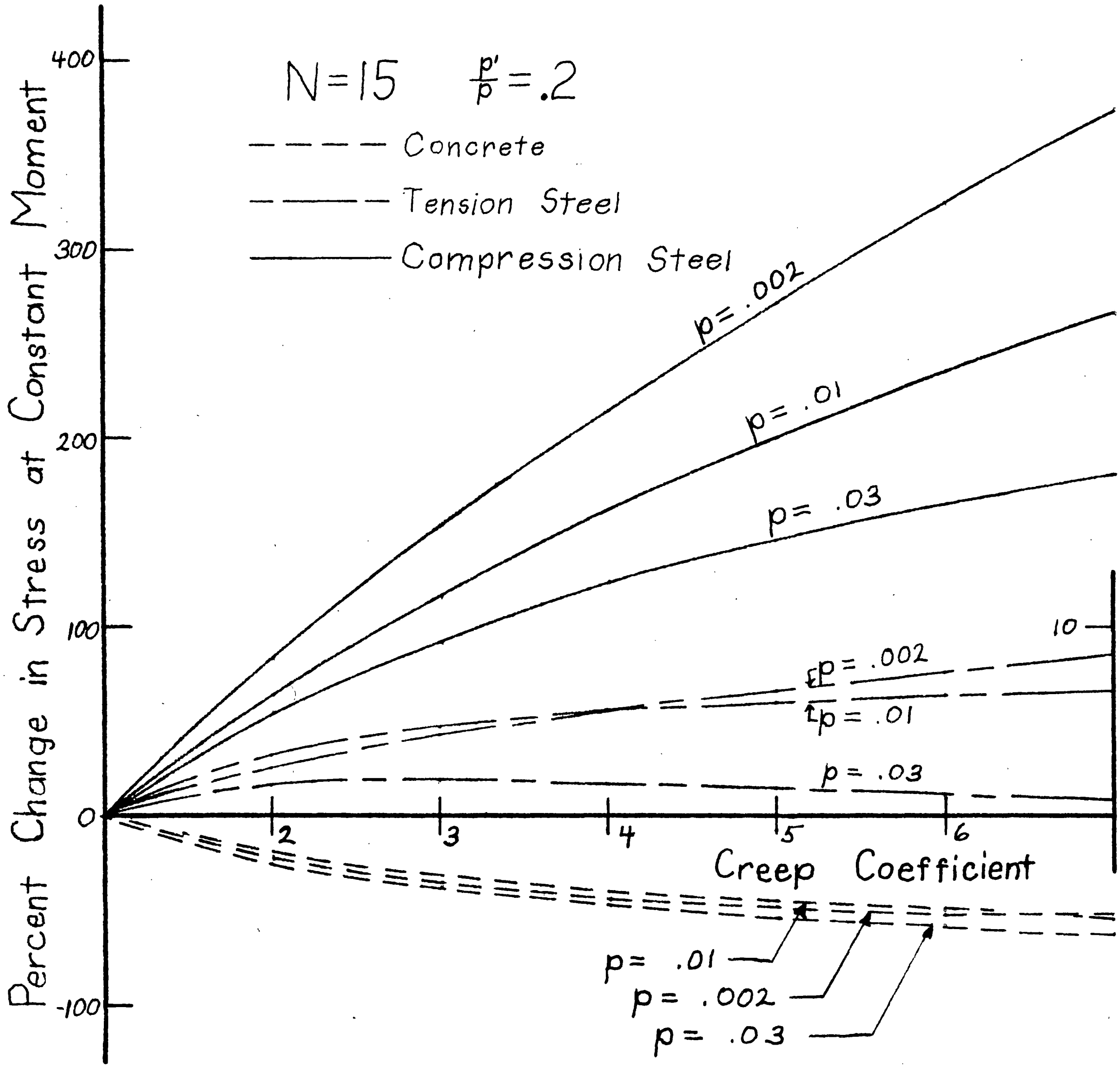


FIG. 16 CHANGE IN STRESS - CREEP COEFFICIENT

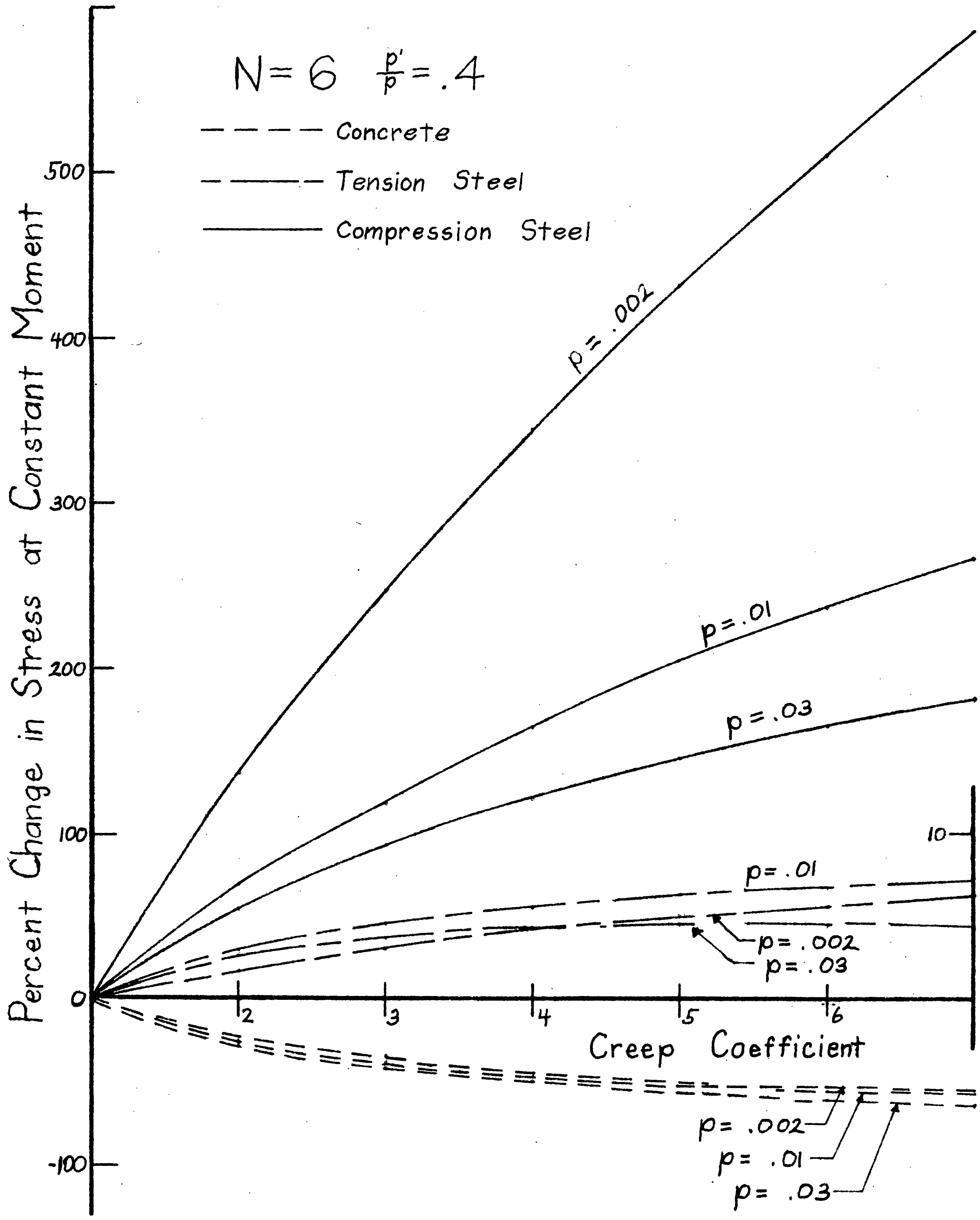


FIG. 17 CHANGE IN STRESS - CREEP COEFFICIENT

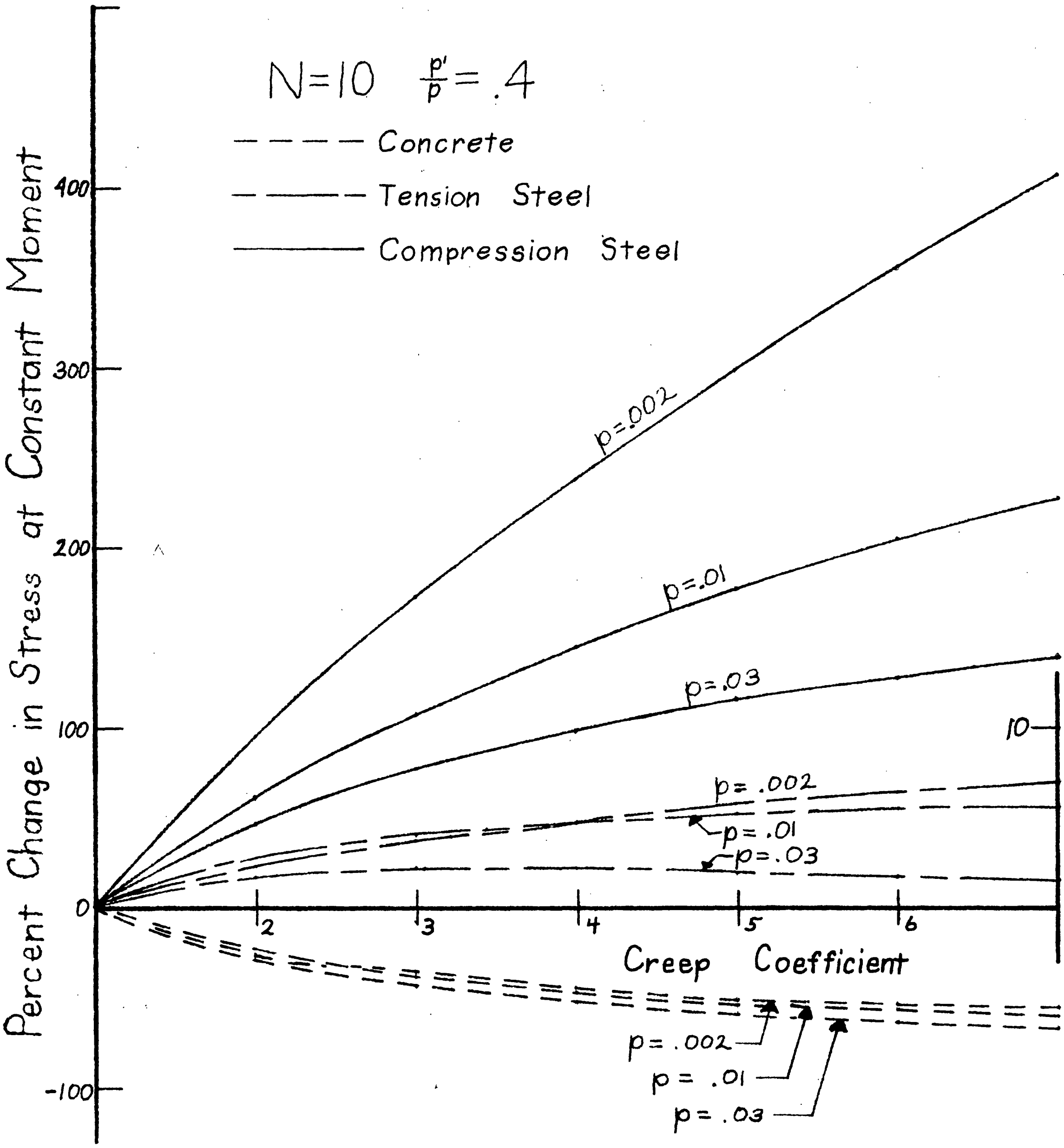


FIG. 18 CHANGE IN STRESS – CREEP COEFFICIENT

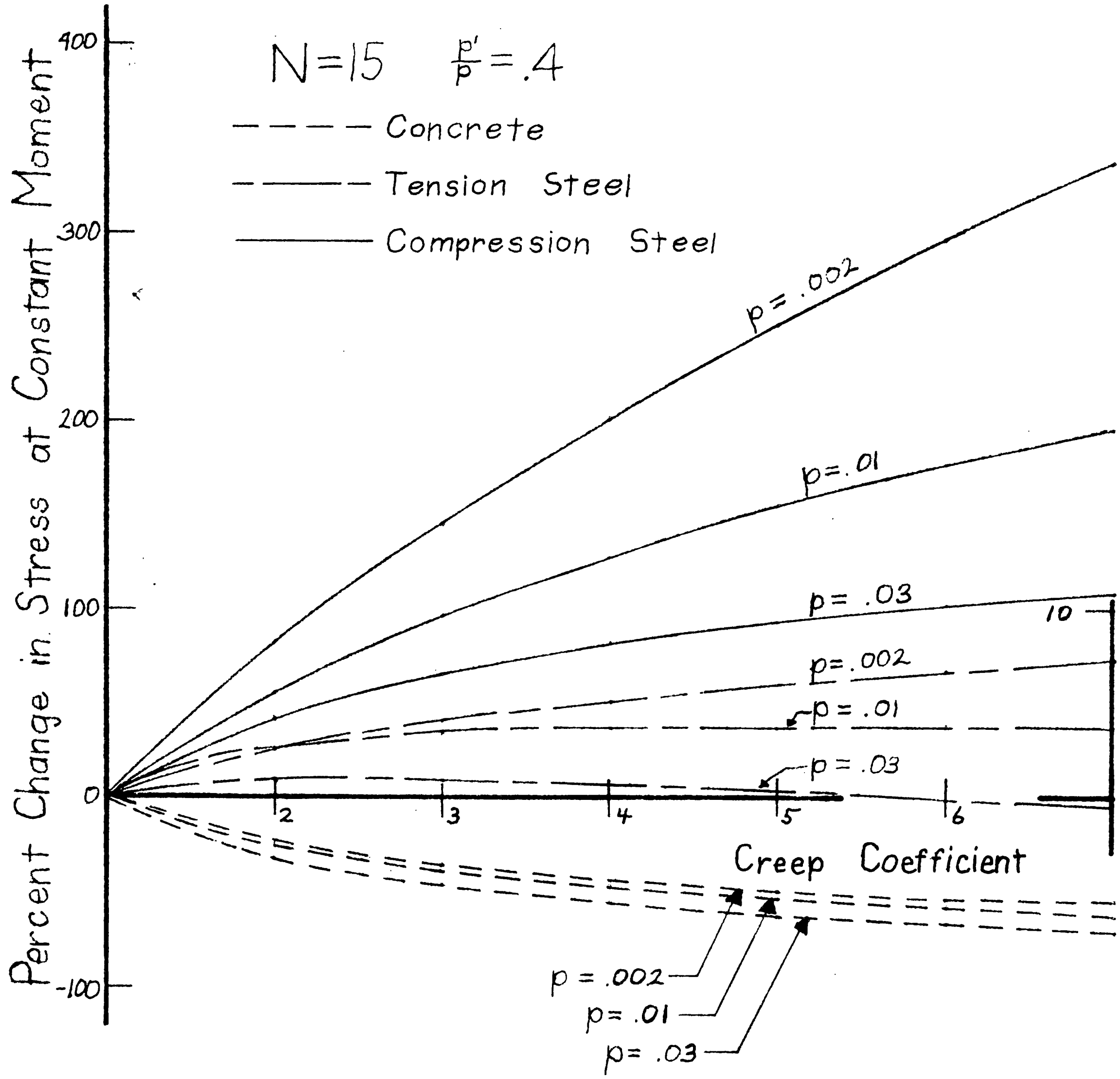


FIG. 19 CHANGE IN STRESS — CREEP COEFFICIENT

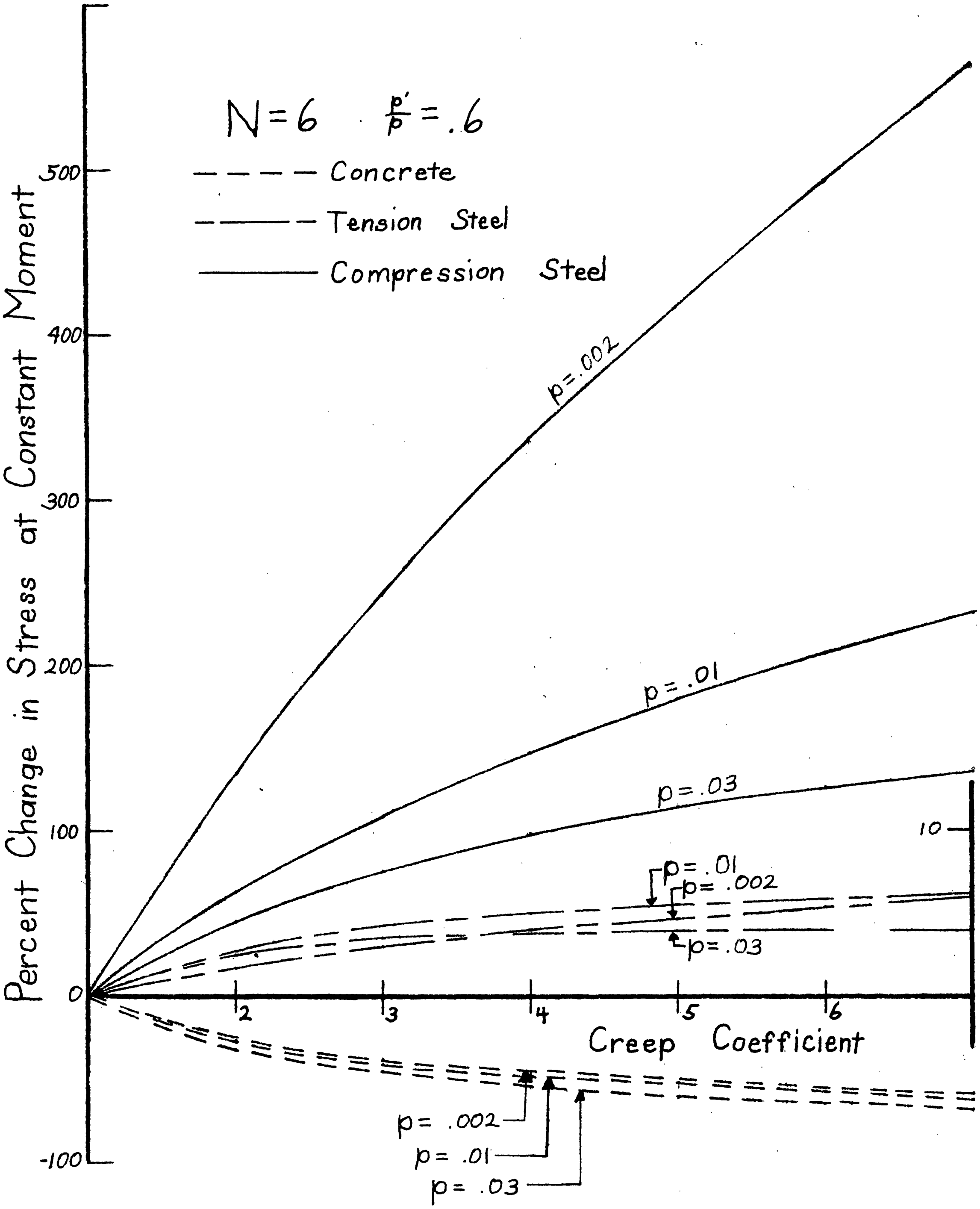


FIG. 20 CHANGE IN STRESS – CREEP COEFFICIENT

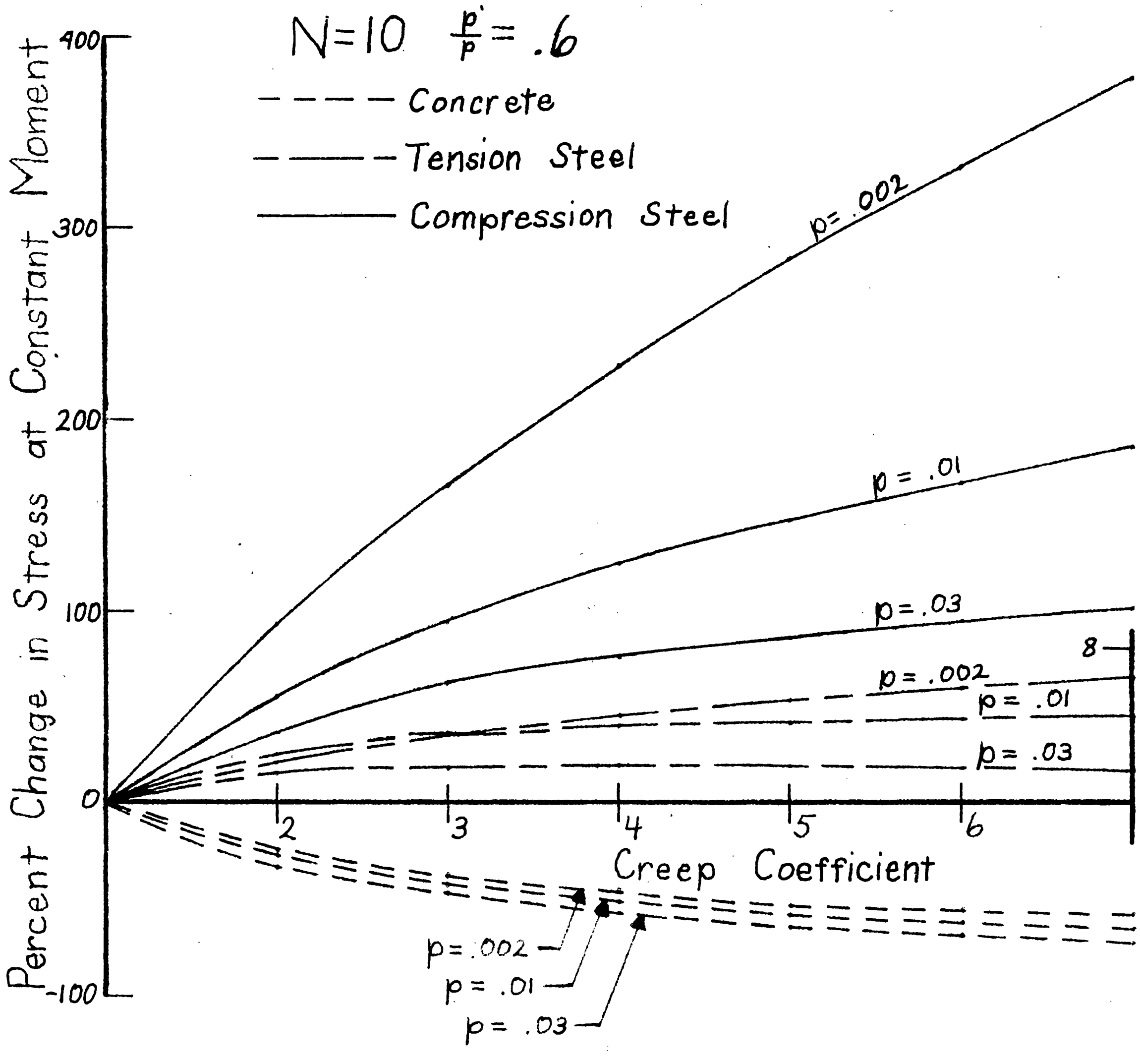


FIG. 21 CHANGE IN STRESS – CREEP COEFFICIENT

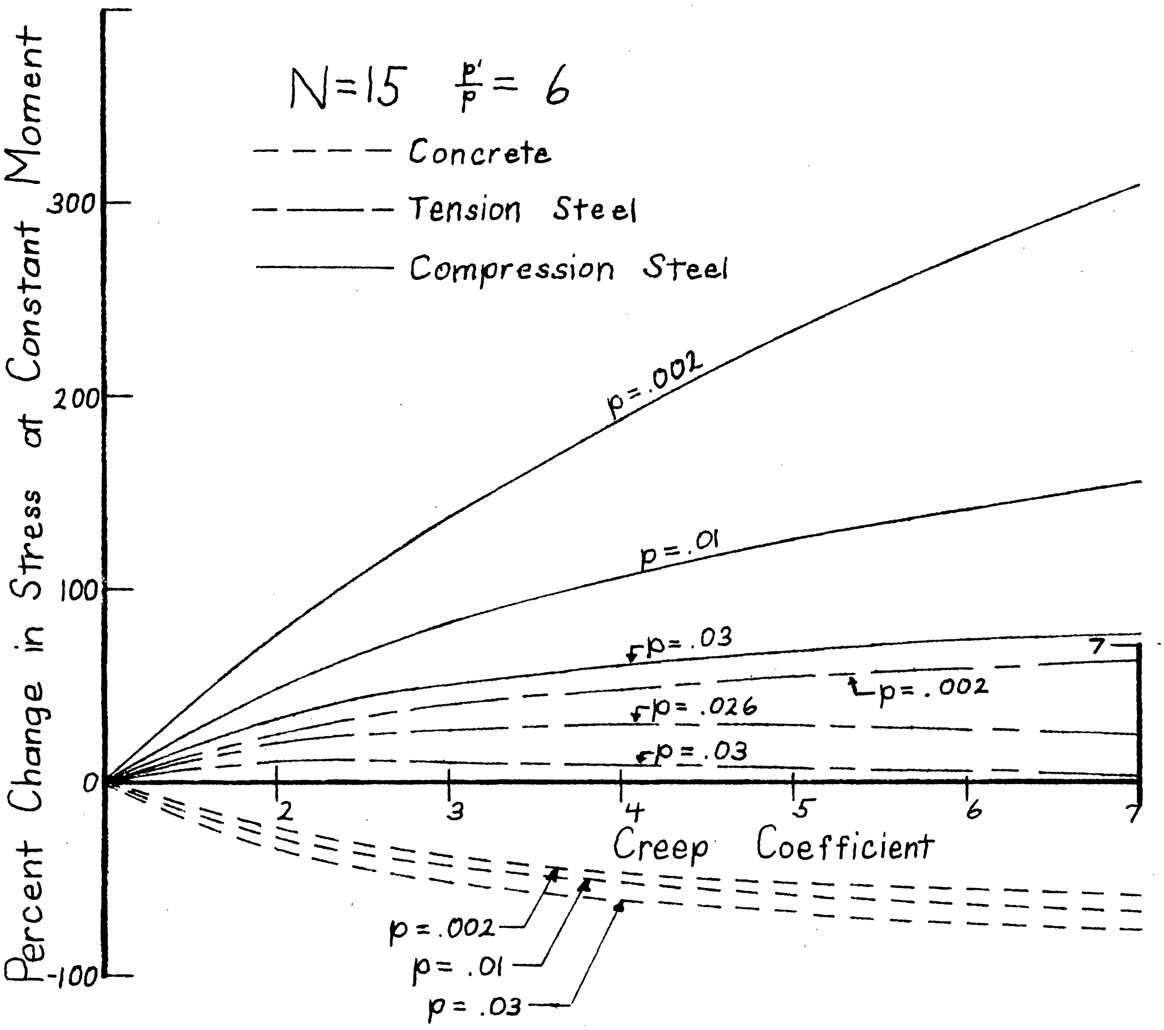


FIG. 22 CHANGE IN STRESS - CREEP COEFFICIENT

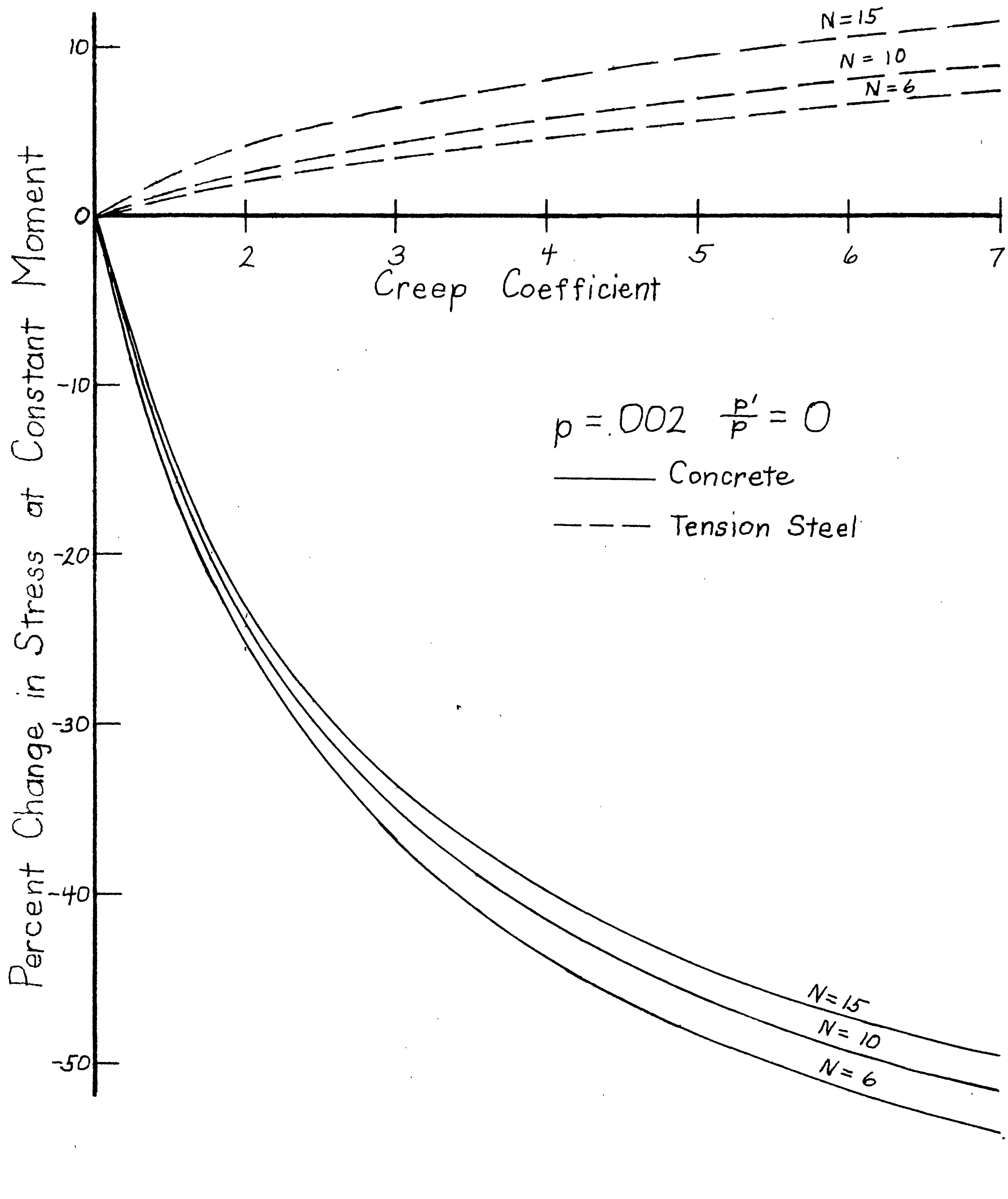


FIG. 23 CHANGE IN STRESS - CREEP COEFFICIENT

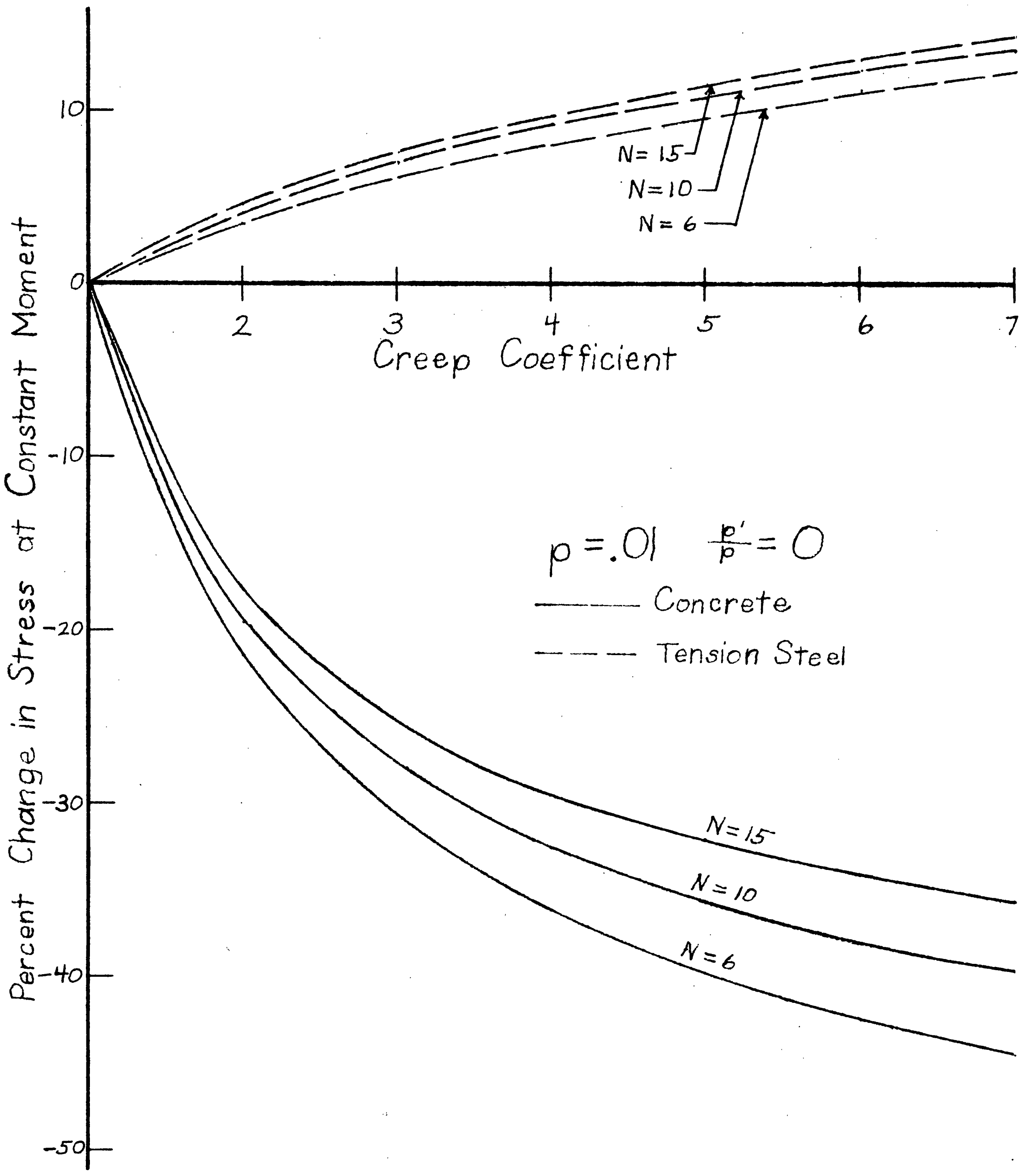


FIG. 24 CHANGE IN STRESS - CREEP COEFFICIENT

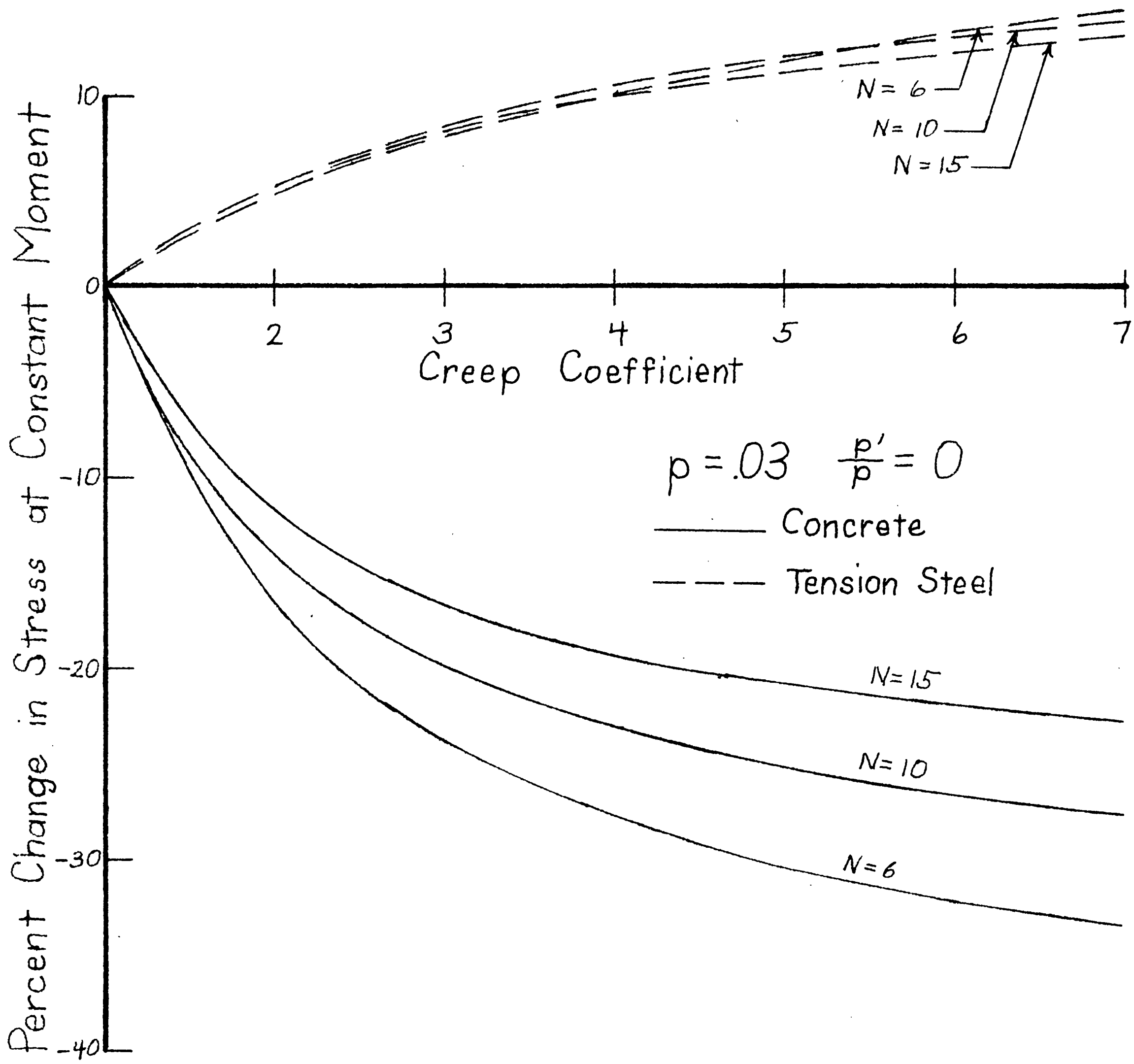


FIG. 25 CHANGE IN STRESS - CREEP COEFFICIENT

F. Design Problems.

Some example problems have been worked to illustrate the use of the developed curves and to show the magnitude of stress changes that occur in reinforced concrete beams. The initial elastic stresses are calculated and the stresses are then calculated with the effects of creep included.

Problem 1(a) is a cantilever beam with tension steel only. Problem 1(b) is the same beam except for the inclusion of compression steel. Problem 2(a) is a balanced design cantilever beam with tensile steel only, and Problem 2(b) is the same as Problem 2(a) except for the inclusion of compression steel. Problems 3 and 4 with different steel quantities are fixed-ended beams in which both the redistribution of bending moments and the redistribution of stresses will be calculated.

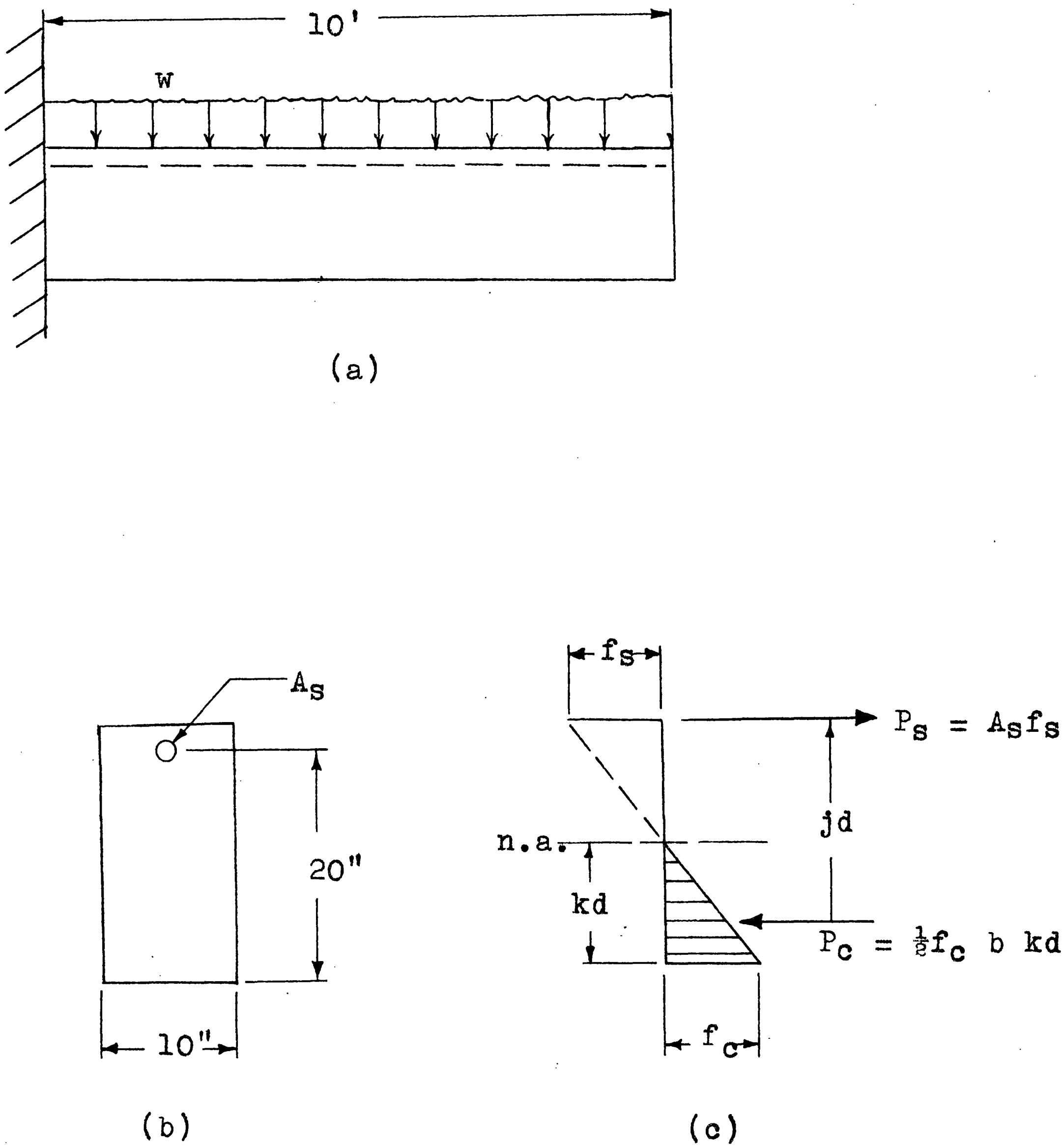


Figure 26. Tensile Reinforced Cantilever Beam

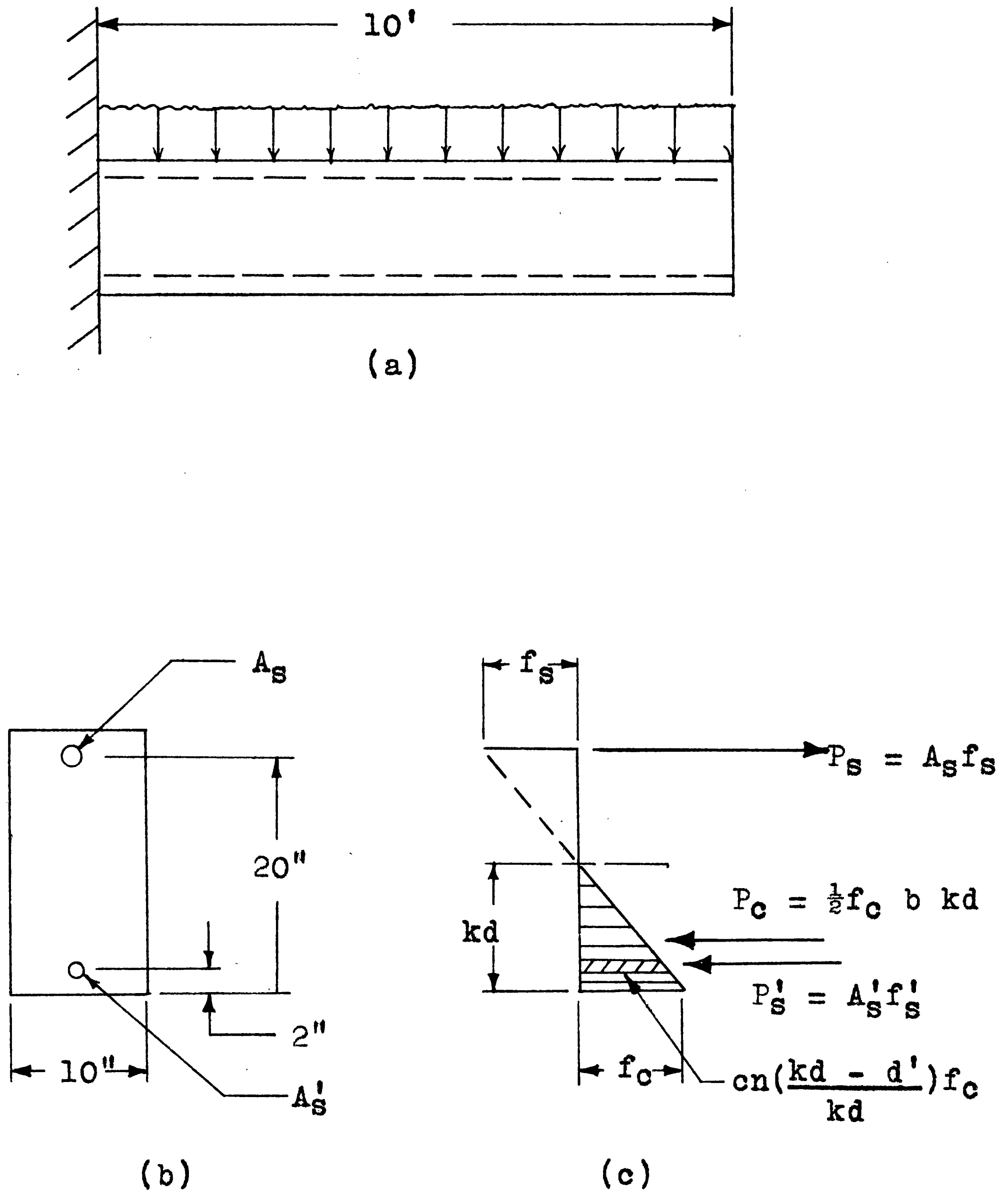


Figure 27. Double Reinforced Cantilever Beam

1. <u>Cantilever Beam - Overreinforced Design.</u>

a. Tensile Reinforced Section.

With reference to Figure 26,

3000 psi concrete

$$n = 10, \frac{p'}{p} = 0$$

$$p = 0.02$$

$$W_{DL} + W_{LL} = 1760 \text{ lb/ft.}$$

$$A_s = 4.0 \text{ sq.in.}$$

$$M = \frac{w\,l^2}{2} = \frac{(1760)(10)^2}{2} = 88{,}000 \text{ lb.ft.}$$

Elastic case (c = 1)

$$k = \sqrt{2cnp + (cnp)^2} - cnp$$

$$k = \sqrt{2(1)(10)(.02) + \left[1(10)(.02)\right]^2}$$

$$- (1)(10)(.02) = .463$$

$$fc = \frac{2M}{(jd)b\ (kd)} = \frac{2(88{,}000)(12)}{(16.91)(10)(9.26)}$$

$$fc = 1348 \text{ psi}$$

$$fs = \frac{M}{(jd)\ A_s} = \frac{(88{,}000)(12)}{(16.91)(4)}$$

$$fs = 15{,}611 \text{ psi}$$

For $n = 10$ and $p'/p = 0$, Figure 12 is the proper graph to use. Since there is no curve for $p = .02$, values of percent change in stress are interpolated.

For three different cases of creep,

c	Percent Change In Concrete Stress	Percent Change In Steel Stress
2	- 16.2	+ 4.8
3	- 23.0	+ 7.9
4	- 26.8	+ 10.1

Stresses in Beam after Creep

c	Concrete Stress	Steel Stress
2	1129	16362
3	1038	16844
4	987	17192

b. Double Reinforced Section.

With reference to Figure 27,

3000 psi concrete

$n = 10,\ \frac{p'}{p} = 0.4$

$p = 0.02,\ 0.008$

$W_{DL} + W_{LL} = 1760$ lb./ft.

$A_s = 4$ sq.in., $A_s' = 1.6$ sq. in.

$M = 88,000$ lb.ft.

$$k = \sqrt{2(cnp + cnp'\frac{d'}{d} - p'\frac{d'}{d}) + (cnp + cnp' - p')^2}$$

$$- (cnp + cnp' - p')$$

$k = .444$

Summing moments about A_s to find jd

Material	Stress Volume	Moment Arm	Moment
Concrete	$\frac{1}{2}fc(10)(8.88) = 44.4\ fc$	17.04	757 fc
Comp. steel	$10(\frac{6.88}{8.88})fc(1.6) = \underline{12.4fc}$	18	$\underline{223\ fc}$
	56.8fc		980 fc

$$jd = \frac{980\ fc}{56.8\ fc} = 17.28$$

$$fc = \frac{M}{(jd)(56.8)} = \frac{(88,000)(12)}{(17.28)(56.8)}$$

$fc = 1216$ psi

$$f's = cn\ (\frac{kd - 2}{kd})fc = (1)(10)(\frac{6.88}{8.88})(1216)$$

$f's = 9421$ psi

$$fs = \frac{M}{(jd)(A_s)} = \frac{(88,000)(12)}{(17.28)(4)}$$

$fs = 15389$ psi

For $n = 10$ and $p'/p = 0.4$, Figure 18 is the proper graph to use. Since there is no curve for $p = .02$, values of percent change in stress are interpolated.

For three different cases of creep,

C	Concrete Stress	Percent Change In Tension Steel Stress	Compression Steel Stress
2	-22.9	2.8	62.4
3	-34.3	4.0	112.1
4	-41.8	4.6	153.5

C	Stresses in Beam After Creep Concrete	Tension Steel	Compression Steel
2	937	15823	15304
3	799	16008	19980
4	708	16088	23881

2. Cantilever Beam - Balanced Design.

a. Tensile Reinforced Section.

With reference to Figure 26,

3000 psi concrete

$n = 10, \frac{p'}{p} = 0$

$p = 0.0117, \ p' = 0$

$w_{DL} + w_{LL} = 1493$ lb./ft.

$A_s = 2.34$ sq.in., $A'_s = 0$

As in Problem 1(a),

$M = 74{,}667$ lb.ft.

$k = 0.381$

$fc = 1348$ psi

$fs = 21928$ psi

For $n = 10$ and $p'/p = 0$, Figure 12 is the proper graph to use. Since there is no curve for $p = 0.0117$, values of percent change in stress will be interpolated.

C	Percent Change In Concrete Stress	Steel Stress
2	-18.8	4.3
3	-26.8	7.2
4	-31.3	9.4

C	Stresses in Beam After Creep Concrete Stress	Steel Stress
2	1095	22874
3	987	23514
4	926	23999

b. Double Reinforced Section.

With reference to Figure 27,

3000 psi concrete

$n = 10, \frac{p'}{p} = 0.4$

$p = 0.0117, \; p' = 0.00468$

$w_{DL} + w_{LL} = 1493$ lb./ft.

$A_s = 2.34$ sq.in., $A'_s = 0.94$ sq. in.

As in Problem 2(b),

$M = 76,667$ lb.ft.

$k = 0.358$

$fc = 1191$ psi

$f'_s = 8584$ psi

$fs = 21666$ psi

For $n = 10$ and $p'/p = 0.4$, Figure 18 is the proper graph to use. Since there is no curve for $p = 0.0117$, values of percent change in stress will be interpolated.

	Percent Change In		
C	Concrete Stress	Tension Steel Stress	Compression Steel Stress
2	-29.2	2.3	58.8
3	-38.9	3.4	103.0
4	-47.0	3.8	138.5

	Stresses in Beam After Creep		
C	Concrete	Tension Steel	Compression Steel
2	879	22172	13628
3	727	22393	17424
4	931	22496	20473

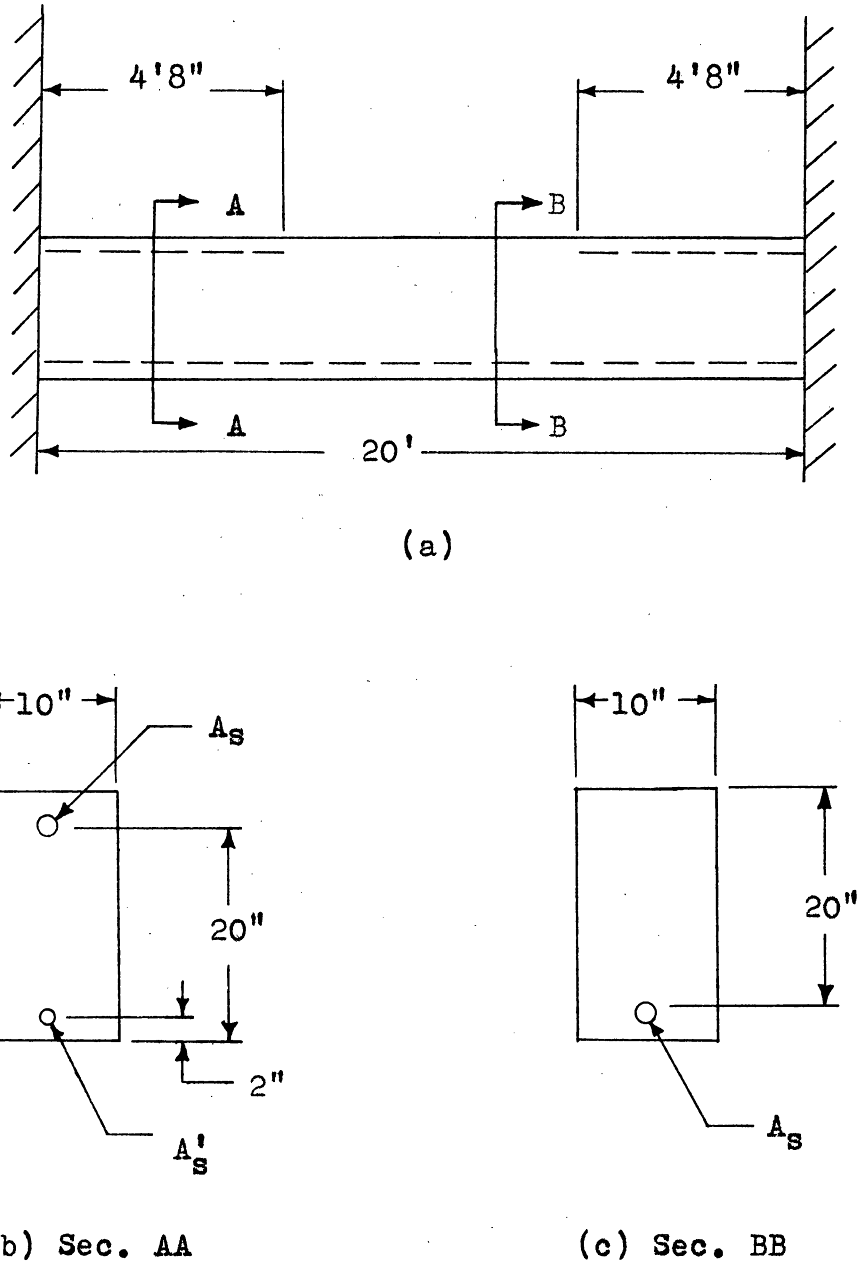


Figure 28. Fixed-Ended Beam

3. <u>Fixed-Ended Beam - Overreinforced Design.</u>

With reference to Figure 28,

3000 psi concrete

$n = 10$, $w_{DL} + w_{LL} = 2133$ lb./ft.

Section AA

$p = 0.01$, $p' = 0.006$, $\frac{p'}{p} = 0.6$

$A_s = 2.00$ sq. in., $A'_s = 1.20$ sq. in.

Section BB

$p = 0.006$, $p' = 0$, $\frac{p'}{p} = 0$

$A_s = 1.20$ sq. in., $A'_s = 0$

Moments and stresses in Sections AA and BB based on a uniform moment of inertia.

$$M_{AA} = \frac{wl^2}{12}$$

$$M_{AA} = \frac{(2133)(12)(240)^2}{12} = 853{,}333 \text{ lb. in.}$$

$$M_{BB} = \frac{wl^2}{24}$$

$$M_{BB} = \frac{(2133)(12)(240)^2}{24} = 426{,}667 \text{ lb. in.}$$

Section AA

As in Problem 1 (b),

$f_c = 1156$ psi

$f_s = 23918$ psi

$f'_s = 8057$ psi

Section BB

As in Problem 1 (a),

$f_c = 810$ psi

$f_s = 19692$ psi

Considering the non-uniform moment of inertia of Sections AA and BB,

$$I_{AA} = \frac{b(kd)^3}{3c} + nA_s\ (d-kd)^2 + nA'_s(kd - d')^2$$

$$I_{AA} = \frac{10(6.60)^2}{3(1)} + (10)(2)(20-6.6)^2$$

$$+ (10)(1.2)(6.60 - 2)^2 = 4803 \text{ in.}^3$$

$$I_{BB} = \frac{b(kd)^3}{3c} + nA_s\ (d-kd)^2$$

$$I_{BB} = \frac{(10)(5.83)^3}{3(1)} + (10)(1.2)(20 - 5.83)^2$$

$$= 3070 \text{ in.}^3$$

By the conjugate beam technique and considering the non-uniform moment of inertia, the resulting bending moments* are found to be:

$$M_{AA} = 881{,}423 \text{ lb. in.}, \quad M_{BB} = 398{,}577 \text{ lb.in.}$$

Stresses in Section AA	Stresses in Section BB
As in Problem 1 (b)	As in Problem 1 (a)
$f_c = 1194$ psi	$f_c = 757$ psi
$f_s = 24705$ psi	$f_s = 18395$ psi
$f'_s = 8322$ psi	

* For a more detailed explanation of a similar technique see reference 6.

For Section AA, $n = 10$ and $p'/p = 0.6$, Figure 21 is the proper graph to use. For Section BB, $n = 10$ and $p'/p = 0$, Figure 10 is the proper graph to use.

Section AA

C	Concrete Stress	Percent Change In Tension Steel Stress	Compression Steel Stress
2	-28.0	2.0	55.2
3	-41.3	2.7	94.8
4	-49.5	3.1	125.6

Section BB

C	Concrete Stress	Percent Change In Tension Steel Stress
2	-22.3	3.5
3	-32.6	6.0
4	-39.0	7.9

Each case of creep causes a change in the moments of inertia of the two sections and thereby causes a redistribution of bending moments. The changes in stress due to the redistribution of bending moments are included in the stress calculations below.

Moments of inertia and bending moments due to creep, and the percent change in bending moments.

Section AA

C	I	M	Percent Change In Bending Moments
2	4165	883,363	0.22
3	3818	885,915	0.51
4	3594	888,598	0.81

Section BB

C	I	M	Percent Change In Bending Moments
2	2576	396,637	-0.49
3	2262	394,085	-1.13
4	2033	391,403	-1.80

Total Percent Change in Stresses

Section AA

C	Concrete	Tension Steel	Compression Steel
2	-28.2	2.2	55.4
3	-41.8	3.2	95.3
4	-50.3	3.9	126.4

Section BB

C	Concrete	Tension Steel
2	-21.8	3.0
3	-31.5	4.9
4	-37.3	6.1

Stresses in Beam After Creep

Section AA

C	Concrete Stress	Tension Steel Stress	Compression Steel Stress
2	857	25237	12933
3	695	25502	16253
4	594	25659	18840

Section BB

C	Concrete Stress	Tension Steel Stress
2	592	18955
3	519	19291
4	475	19517

4. Fixed-Ended Beam, Balanced Design.

With reference to Figure 28,

3000 psi concrete

$n = 10$, $w_{DL} + w_{LL} = 2133$ lb./ft.

Section AA:

$p = 0.01185$, $p' = 0.006$, $p'/p = 0.506$

$A_s = 2.37$ sq.in., $A'_s = 1.20$ sq. in.

Section BB:

$p = 0.006$, $p' = 0$, $p'/p = 0$

$A_s = 1.20$ sq. in., $A'_s = 0$

Moments and stresses in Section AA and BB based on a uniform moment of inertia.

As in Problem 3,

M_{AA} = 1006,667 lb.in.

M_{BB} = 503,333 lb.in.

Section AA

As in Problem 1 (b),

fc = 1291

fs = 23986

f'_s = 9264

Section BB

As in Problem 1 (a),

fc = 956

fs = 23230

Considering the non-uniform moment of inertia of Sections AA and BB,

As in Problem 3,

I_{AA} = 5449

I_{BB} = 3070

By the conjugate beam technique and considering the non-uniform moment of inertia, the resulting bending moments* are found to be:

M_{AA} = 1048,533 lb.in.

M_{BB} = 461,467 lb.in.

Stresses in Section AA

As in Problem 1 (b)

fc = 1344 psi

fs = 24983 psi

f'_s = 9649 psi

Stresses in Section BB

As in Problem 1 (a)

fc = 877 psi

fs = 21298 psi

* For a more detailed explanation of a similar technique see reference 6.

For Section AA, n = 10 and p'/p = 0.506, values of percent change in stress are interpolated from the graphs of Figures 18 and 21. For Section BB, n = 10 and p'/p = 0, Figure 12 is the proper graph to use.

Section AA

C	Concrete Stress	Percent Change In Tension Steel Stress	Compression Steel Stress
2	-27.5	2.0	55.4
3	-40.5	2.7	95.4
4	-48.8	3.1	126.8

Section BB

C	Concrete Stress	Percent Change In Tension Steel Stress
2	-22.0	3.6
3	-32.1	6.0
4	-38.4	8.0

As in Problem 3, each case of creep causes a change in the moment of inertia of the two sections and thereby causes a redistribution of bending moments. The changes in stress due to the redistribution of bending moments are included in the stress calculations below.

Moments of inertia and bending moments due to creep, and the percent change in bending moments.

Section AA

C	I	M	Percent Change In Bending Moments
2	4663	1049,849	0.13
3	4240	1052,202	0.35
4	3967	1054,871	0.60

Section BB

C	I	M	Percent Change In Bending Moments
2	2576	460,151	-0.29
3	2262	457,798	-0.80
4	2033	455,129	-1.37

Total Percent Change in Stresses

Section AA

C	Concrete	Tension Steel	Compression Steel
2	-27.6	2.1	55.5
3	-40.9	3.1	95.8
4	-49.4	3.7	127.4

Section BB

C	Concrete	Tension Steel
2	-21.7	3.3
3	-31.3	5.2
4	-37.0	6.6

Stresses in Beam After Creep

Section AA

C	Concrete	Tension Steel	Compression Steel
2	973	25515	15003
3	794	25766	18894
4	681	25906	21943

Section BB

C	Concrete	Tension Steel
2	687	21991
3	602	22410
4	552	22695

V. DISCUSSION OF RESULTS

An evaluation of the graphs is given in Section E-1, page 36.

A study of the illustrated problems shows how the variations of stress versus creep are further affected by variations in the geometry and composition of the beams.

The beam sizes and shapes and the steel quantities were chosen to represent typical practical values.

In Problems 1(a) and 2(a) the values of tensile stress in the steel increased approximately 10 percent due to creep for $c = 4$ while the values of compressive stress in the concrete decreased approximately 28 percent. The inclusion of compression steel in these members affected these changes markedly. When compression steel was used (Problems 1(b) and 2(b)) the tensile steel stress increased only approximately four percent while the concrete stress decreased approximately 44 percent, a much larger decrease. The stresses in the compression steel increased more than 135 percent in both problems. Even though the compression steel experienced extremely large increases in stresses, it is interesting to note that these stresses remained within the allowable working range as specified by the ACI Code. (1) It is also noted that compression steel stresses approximately doubled their initial values substantiating the recommendation of the Code which permits using twice the computed stress capacity based on calculated elastic strain.

For the beams in Problems 3 and 4, the differences in the moments of inertia between the end-sections and the mid-section of each beam were taken into consideration in computing the initial elastic moments throughout the beam. In comparison with computations based on an assumed constant moment of inertia, this more exact moment analysis resulted in slightly higher computed moments in the stiffer sections of the beam and slightly lower computed moments in the less stiff sections of the beam. In Problems 3 and 4, the tensile steel stresses in the end sections (24705 to 25659 and 24983 to 25906) exceeded the allowable stress (24,000 psi) for the elastic case as well as for the cases of c = 2, 3, and 4 when the variation in moment of inertia was taken into account. For an assumed constant moment of inertia, the calculated elastic tensile steel stresses were 23918 psi (Problem 3) and 23986 psi (Problem 4).

It is also noted that stress changes due to creep shown in examples 1(a), 1(b), 2(a) and 2(b) are entirely independent of moment. In these loaded cantilever beams the moments remain constant throughout the creep period.

In Problems 3 and 4 the stress changes included the effects of moment changes due to moment redistribution as well as the stress changes due to creep. For a proper evaluation these effects should be separated and noted. In Problem 4, for example, where $c = 4$ the decrease in the concrete stress of 49.36 percent in the end section includes an increase of 0.60 percent due to moment change. The decrease in concrete stress due to creep alone would be 49.96 percent (49.36 + 0.60) if there had been no change in moment. Similarly, for the mid-section of the beam when $c = 4$, the decrease in concrete stress of 37.01 percent includes 1.37 percent decrease due to moment change. The decrease in concrete stress due to creep alone is 35.64 percent (37.01 - 1.37).

VI. CONCLUSIONS

From a review of the literature, a thorough study was made on the nature of creep and the factors affecting creep. The equations developed and the graphs presented in this thesis provide a convenient source of data that can be used in a practical way to determine the effects of the creep phenomenon on the stresses in reinforced concrete members. Illustrated problems show how this data can be used.

The graphs provide a reasonable working range of parameters. For the researcher interested in more precise values than can be obtained from the graphs, the detailed data has been collected and published separately. (9) The data was developed on an IBM 1620 Digital Computer and the program is included with the data.

Development of the data and its application to the problems has brought out certain cause and effect relations. These are discussed in detail in Section E-1 with reference to the graphs and in Section V with reference to the illustrated problems.

In summary the main points may be listed as follows:

1. For a beam subjected to constant bending moment, creep always causes a reduction in compressive concrete stresses, an increase in tensile steel stresses, and when present, an increase in compressive steel stresses.

2. All other factors being the same, beams of high strength concrete will experience larger decreases in concrete stresses than beams of low strength concrete when creep occurs.
3. Due to creep, a double reinforced concrete beam subjected to an applied moment will experience less than half as much increase in tensile stresses in the tension steel and nearly two-thirds more decrease in compressive stresses in the concrete than a similar beam reinforced with equivalent tension steel only.
4. As creep occurs, the computed stiffness of a reinforced concrete beam is reduced. Sections of the beam with lesser stiffness experience a larger percent decrease in stiffness than sections of the beam with greater stiffness.
5. In indeterminate structures of reinforced concrete, the effect of creep is to cause a redistribution of bending moments throughout the structure. The resulting changes in moments cause corresponding changes in stresses in the concrete and the steel which are in addition to the stress changes due to the creep phenomenon alone.

Although this thesis does not deal extensively with the subject of the redistribution of moments in indeterminate structures due to creep, the phenomenon is discussed and is included as a part of illustrated problems 3 and 4. A further study of this subject may reveal cases where the stress changes due to moment redistribution are considerably larger than those in the two illustrated problems. A natural sequel to this thesis would be such a study involving the effects of geometry and loading.

This thesis also does not deal with the subject of stress changes in reinforced concrete members due to shrinkage. In many cases the shrinkage effects on stresses may be more significant than the creep effects. Although shrinkage effects and creep effects can not be separated in the physical structure, it is possible to separate these two effects analytically, and thereby study them independently. An extensive study of shrinkage and shrinkage effects would be a significant additional topic.

Although the work of this thesis has been developed for reinforced concrete structures, it should be borne in mind that the work could be profitably extended to cover prestressed concrete structures.

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Abstract

STRESS CHANGES IN REINFORCED CONCRETE MEMBERS DUE TO CREEP

Under sustained loads plain concrete and concrete members experience the phenomenon known as creep. Creep varies in rate and magnitude depending upon the nature and magnitude of load and upon the physical characteristics of the concrete. From a review of the literature, creep coefficients are studied as to how the values are affected by mixes, loadings, time, curing, and quality of aggregates and materials.

Within the allowable working stress range of concrete, creep strain can be considered proportional to the induced stress. Assuming known stress - creep strain - time relations, a series of equations have been derived analytically to calculate the stress changes in both the steel and concrete in reinforced concrete beams of rectangular cross-section due to creep relaxation. The data has been tabulated and graphed. Extensive use of the digital computer has been made to thoroughly investigate these stress changes within a wide range of parameters including magnitude of load, strength of concrete, geometric proportion of the beams, and the amount and type of reinforcing steel.