AN ANALYTICAL MODEL OF REINFORCED CONCRETE BEAMS
CONSIDERING STRAIN HARDENING AND CONFINEMENT EFFECTS

by

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Acknowledgment is made to and , who served on the author's graduate committee and offered encouragement along the way.

A special word of thanks is due my parents, , who encouraged me in my work at all times.

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I. INTRODUCTION

The purpose of this thesis is threefold: (1) to evaluate the current knowledge of flexural behavior in reinforced concrete, (2) to develop a mathematical model for predicting the moment-curvature and load-deflection curves, and (3), to study the effects of section and material properties on the rotation capacity of reinforced concrete beams.

In recent years the philosophy of limit design in reinforced concrete has evolved. The theoretical and experimental investigations in limit design have centered around the determination of the rotational capacities of hinging regions in reinforced concrete members. This is in contrast with the limit design theories applied to steel structures where the rotational capacity is considered to be sufficient. Because concrete was traditionally treated as a brittle material, its ductile properties have only recently been understood.

Essential to the development of limit design is a knowledge of the flexural behavior of reinforced concrete members when loaded to collapse. Load factors can then be applied which will insure against catastrophic structural failure. Based on the concept of concrete as a strain-softening material which will continue to carry load at ever-increasing curvatures, it seems probable that a true collapse, or limit method of design can be found which will economize design, especially design against dynamic structural failure. A description of the historical development and the current knowledge of the limit design concepts is presented as the first part of this thesis in the section entitled "Literature Review."

As a result of extensive testing by a number of researchers, it is felt that there now exists enough data to develop an analytical model to predict the flexural behavior of reinforced concrete. From these tests, it appears that the rotational capacity is strongly influenced by two parameters which have not previously received sufficient attention. These are: (1) the strain-hardening of the steel reinforcement and (2) the ultimate strain of the concrete. The mathematical model developed in this thesis includes these influences and the result is a better prediction of the moment-curvature and load-deflection curves. The relationships used to describe these parameters are discussed in the section entitled "Analytical Model."

Once the analytical model has been verified, it is possible to study the importance of additional parameters without performing extensive tests. The additional parameters studied in this thesis can be grouped under the two main parameters mentioned above. Those influencing the strain-hardening of the reinforcement are: (1) the steel yield stress and (2) the steel strain at the onset of strain hardening. Those affecting the ultimate concrete strain are: (1) stirrup spacing, (2) compression steel ratio, (3) slope of the moment diagram, and (4) the beam width. The relative importance of these parameters is the subject of the third part of the thesis in the section entitled "Discussion of Parameter Influence."

Finally, a summary of the results obtained from the investigation in this thesis is contained in the "Conclusions" section.

II. LITERATURE REVIEW

Richart, Brandtzaeg, and Brown[15] in 1928 did a series of tests to investigate the effect of triaxial loading on the strength of concrete. They applied varying hydraulic pressures to a series of conventional compression specimens. They concluded the following:

1. The strength of the concrete in triaxial compression was found to increase greatly with the magnitude of the smallest principle stress.
2. The high stresses resisted by the concrete in triaxial compression (the maximum principal stress reaching 24,600 psi in one instance) were always accompanied by very large strains. The axial deformation at maximum load ranged from .5% to more than 7%, depending on the magnitude of the stress and the quality of the concrete.
3. The tests of concrete of lean, medium, and rich mixtures in triaxial compression showed the rate of increase in the strength with increase in the smallest principle stress was largely independent of the proportions of the concrete mixture.
4. The triaxial compression tests showed that the presence of lateral pressures added to the strength of the specimen an amount approximately 4.1 times the magnitude of the lateral pressure. Thus, the magnitude of the maximum principle stress developed was roughly equal to the strength of the concrete in simple compression plus 4.1

times the lateral pressure.

5. Much of the deformation under triaxial loading was due to an inelastic reduction in volume or a compacting of the concrete under stress. The amount of compacting varied considerably with the richness of the mixture.

The results of this investigation can perhaps best be shown in the pictorial of Figure 1.

Chan[6] in 1955 began a series of investigations into the ultimate strength and deformation of plastic hinges in reinforced concrete. He tested a series of specimens subjected to axial loading of small eccentricity. The main variable was the ratio of lateral binding. He offered the following conclusions:

1. The ultimate strain of unbound concrete varied between 0.003 and 0.004 under the action of combined bending and axial loads.
2. Spalling of concrete cover and hence reduction in compressive area resulted in an abrupt drop in load, but lateral binding enabled the enclosed concrete to continue to resist load.
3. Lateral binding prevented brittle concrete failure after the spalling strain had been exceeded. Prior to the spalling stage, however, it had little or no influence. The confined concrete was held in equilibrium by arching across the gaps between adjacent binders, and failure occurred when the arches collapsed or when the binders dilated enough to preclude the binding effect.

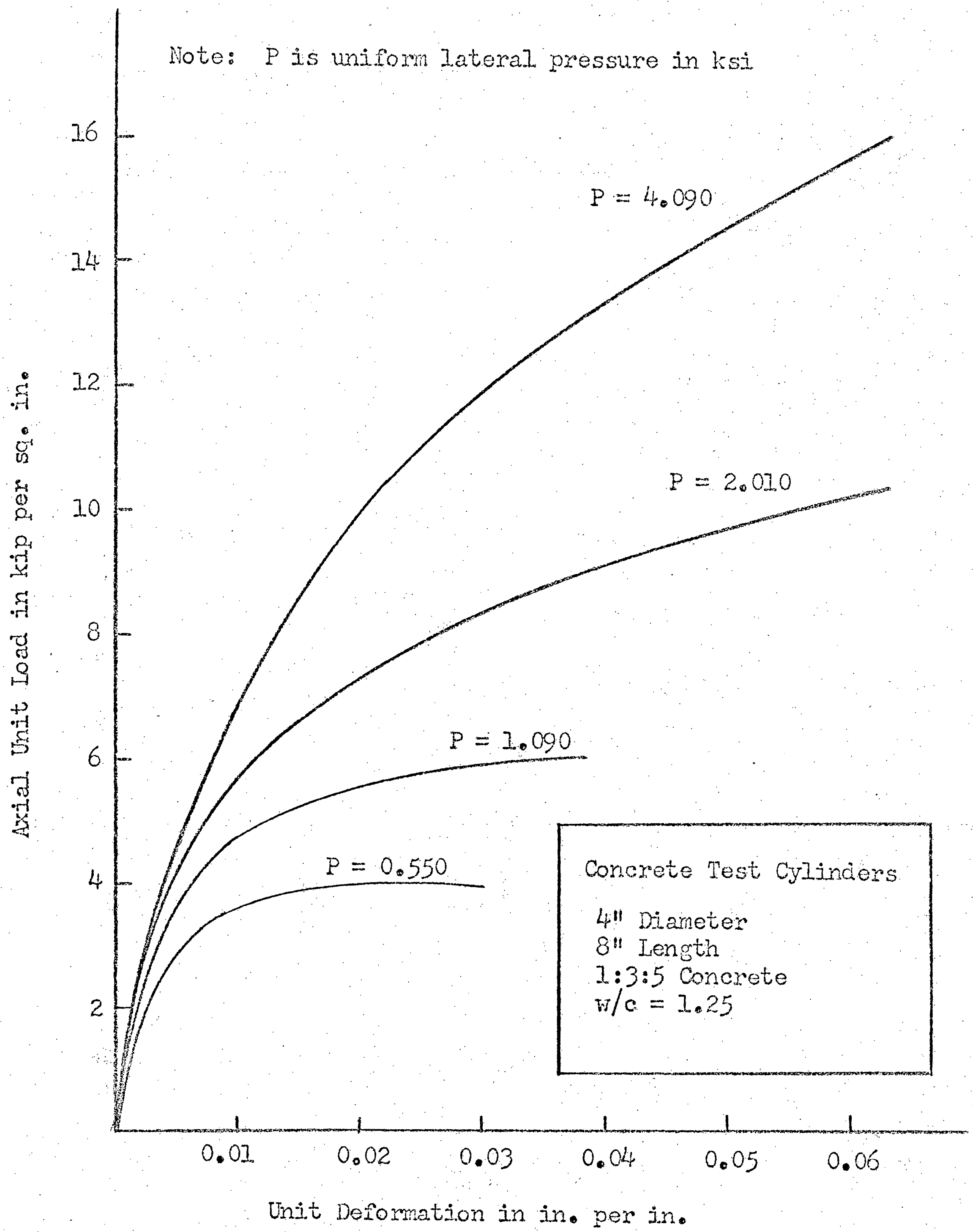


Figure 1. Effect of Triaxial Stress State on Concrete Stress-Strain Curve

4. Spiral binding was more efficient than rectangular stirrups in affording lateral support to the confined concrete.

Roy and Sozen[17] reported at the 1964 Flexural Mechanics Symposium their findings from a series of 60 axially-loaded, five by five by 25-inch prisms. Their object was to determine the effect of rectangular transverse reinforcement on the strength and ductility of the confined concrete. Their conclusions were as follows:

1. Rectangular ties, with or without longitudinal reinforcement, did not enhance the load-carrying capacity of the specimens. This was because the bending resistance of the ties was insufficient to restrain effectively the expansion of the concrete. Before the ties had deflected enough to develop membrane action, the concrete was destroyed.
2. The transverse ties effected a significant increase in the deformation capacity of the concrete. Many of the ultimate strains observed extended beyond five percent.

The first conclusion reached by Roy and Sozen disagrees with various other test results[1,17]. This disagreement is probably caused by the fact that Roy and Sozen's specimens were fabricated with the transverse steel immediately at the surface. Other similar tests have been conducted with a concrete cover of about one-fourth of an inch. Apparently the cover encloses both the steel and confined concrete long enough for the arching action of the concrete between the stirrups to take place. This hypothesis needs to be investigated to clarify the cause of the differences in test results.

In discussions of Roy and Sozen's paper, S. Stockl, V. V. Bertero, and C. Felippa challenge the statement that transverse rectangular ties do not increase load carrying capacity. Stockl reports on increase in strength of approximately 16 percent for a tie spacing of two inches. Bertero and Felippa report increases of 13 percent and 26 percent for spacings of $2\frac{1}{2}$ inches and $1\frac{1}{2}$ inches, respectively.

Sawyer(18) proposed at this same meeting an analysis based on a bilinear moment-curvature relationship which he presented as shown in Figure 2. He proposes that the ultimate moment, M_u, should be evaluated by the ultimate(4) strength design provisions of ACI 318-63, and that the ratio M_e/M_u be assigned the constant value 0.85. He also suggested the use of

$$\phi_u = 0.00274 \ / \ qd$$

to compute the ultimate curvature, where q and d are as defined in ACI 318-63.

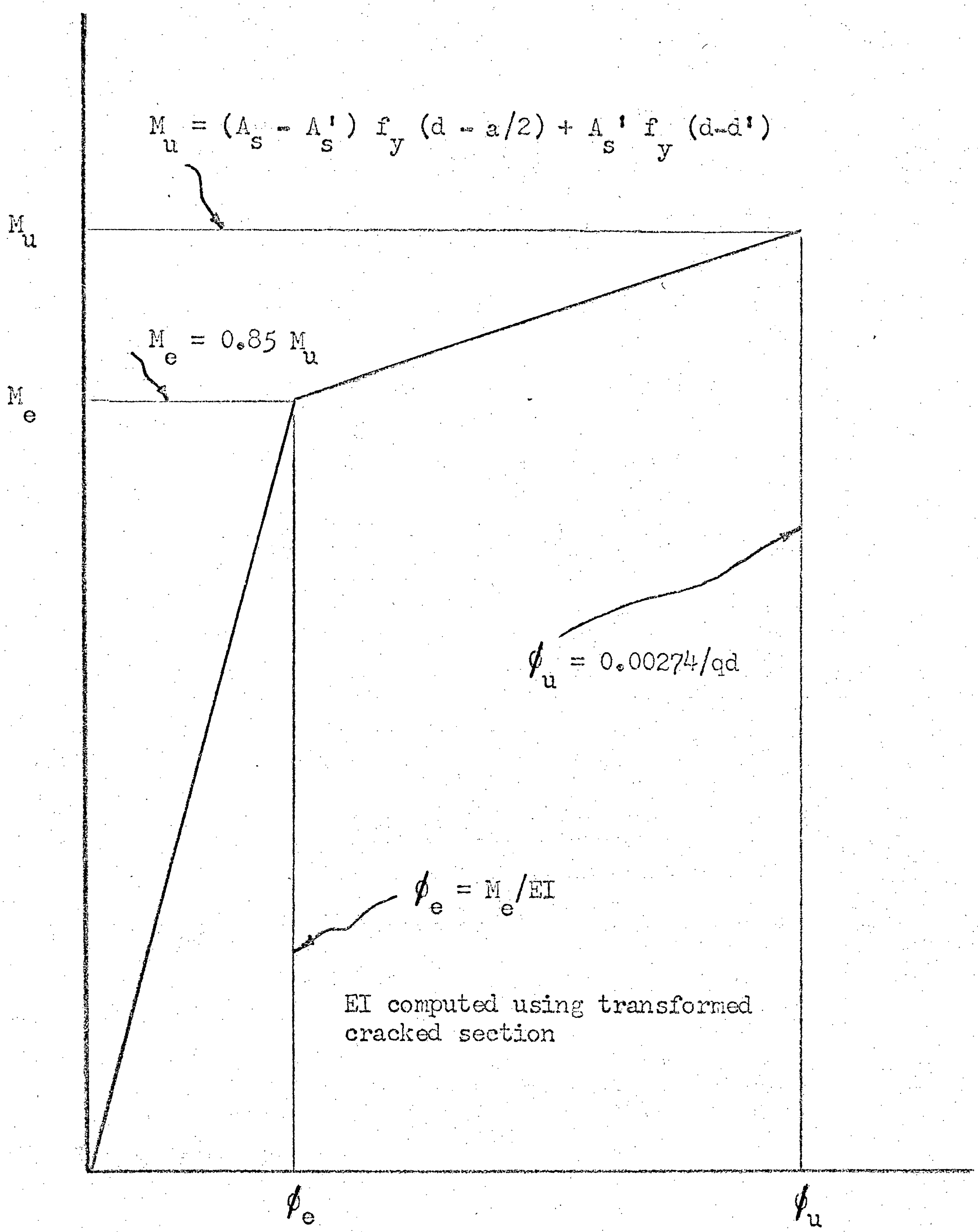


Figure 2. Sawyer's Bilinear Moment-Curvature Relationship

Barnard[3] at the Flexural Mechanics Symposium in 1964 presented a discussion of the flexural behavior of reinforced concrete beams when loaded to collapse. He discussed primarily the property of concrete as a strain-softening material, and the behavior of a beam section on the falling branch of the moment-curvature curve. He states that the finite discontinuity length under point loading can be explained as a property of the concrete failure mechanism -- crushing must occur over a finite length. Also observed is the phenomenon of sections outside the region of contamination decreasing in curvature as they lose strain energy along their respective unloading curves. Barnard warns that if the energy decrease outside the discontinuity exceeds the increase inside, sudden failure of a member in an indeterminate structure could occur prior to the collapse load being reached. He says this possibility of an energy release is a major obstacle to the development of limit analysis procedures, and it is essential to determine under what conditions this energy release is possible.

Rosenblueth and de Cossio[16] also presented a paper on the descending branch of the force-deformation curve at the symposium in 1964. They regard the problem as one of instability, but their arguments are essentially the same as Barnard's. They show that a moment-rotation approach to consideration of flexural behavior is capable of dealing with members whose moment-curvature curves possess a descending branch. In members of this type, when the section of maximum moment reaches the ultimate moment, it continues to increase in curvature, and thus the moment begins to fall off. Since adjacent moments are proportional to the moment at the critical section, these bending moments must also decrease. This seems to say that all curvatures in the area will decrease except at one section, with a corresponding decrease in deflection. This is contradicted by test results, which show increasing deflection for decreasing loads after ultimate moment has been reached at one section. Therefore, it is concluded that the traditional moment-curvature approach is inadequate for determining deflections after ultimate moment has been reached.

In 1964, Mattock[12] reported the results of tests on 37 beams. He demonstrated experimentally that the rotational capacity of a portion of a beam under a moment gradient is greater than that of the same beam under constant moment. In addition to moment gradient, Mattock investigated the effects of steel percentage, effective depth, concrete strength, and reinforcement steel yield stress. He concluded that equations derived from equilibrium of forces and compatibility of strains can be used to arrive at a close estimate of moments and curvatures as long as strain hardening of the reinforcement and variation in concrete compressive strain are considered.

Corley[7] in 1966 reported on a series of 40 beams tested as an extension of those reported by Mattock. Size of specimen and confinement of the concrete in compression were the primary variables investigated. He made the following conclusions:

1. The maximum concrete compressive strain can be much greater than 0.003. The strains can be reasonably

estimated by the following equation:

$$\epsilon_u = 0.003 + 0.02\ b/z + \left(\frac{p'' f_y}{20}\right)^2$$

where

ϵ_u = maximum concrete compressive strain at ultimate strength.

b = width of compression face.

z = distance from section of maximum moment to adjacent section of zero moment.

p'' = ratio of volume of binding steel (one stirrup plus compression steel between stirrups) to volume of bound concrete (product of area bound and stirrup spacing).

f_y = yield stress of reinforcing steel.

This equation may only be used when shear failure does not occur.

2. Member size does not have a significant direct influence on the maximum concrete compressive strain.

Nordell(13) in 1966 investigated a series of concrete beams with the objective of obtaining a better understanding of the hinge mechanism in beams subjected to static and dynamic loads. All of his beams were under-reinforced so that yielding of the tensile steel occurred before the concrete in compression failed. Among his conclusions were the following:

1. The size of the transverse reinforcement (closed stirrups) had a negligible effect on the rotational capacity of the beams. However, stirrup spacing was not varied.
2. Increasing the amount of confined concrete resulted in a greater ultimate rotation capacity.

Perhaps the most interesting aspect of Nordell's paper is his good correlation between experimental and computed data. This correlation was obtained by assuming the maximum stress of the confined concrete to be equal to that of the unconfined concrete, and the limiting strain was determined using experimental moment-rotation data. With the assumed stress-strain relationships for concrete, the neutral axis must move downward to preserve equilibrium. However, test observations indicate flexural cracks continue to move up throughout the loading sequence. This seems to suggest an inadequacy in the assumed stress-strain curve used by Nordell, and suggests the need for further study using higher maximum stresses for the confined concrete.

III. ANALYTICAL MODEL

Steel Stress-Strain Relationship

The stress-strain relationship for the tension and compression reinforcement is as shown in Figure 3. This relationship is the one used by Nordell[13]. An ultimate steel strain of 0.15 inch/inch, and a ratio of ultimate strength to yield strength of 1.63 are assumed in the equation of the strain hardening region. These values are the result of tests by Burns and Siess[5]; Gaston, Siess, and Newmark[8]; and Nordell[13].

Concrete Stress-Strain Relationships
Unconfined Concrete

The unconfined concrete stress-strain relationship used in the model is a bilinear one, illustrated in Figure 4. The yield stress is taken as 0.85 of the cylinder strength in accordance with ACI paragraph 1503[4]. The yield strain of 0.0012 is used in attempting to approximate the non-linear stress-strain curves proposed by Hognestad[10] and others. The value of ultimate strain depends on a number of variables. Corley[7] proposed the following equation which considers the binding ratio and the slope of the moment diagram:

$$\epsilon_u = 0.003 + 0.02\, b/z + (p'' f_y/20)^2$$

in which

b = beam width

z = distance from point of zero moment to point of maximum moment.

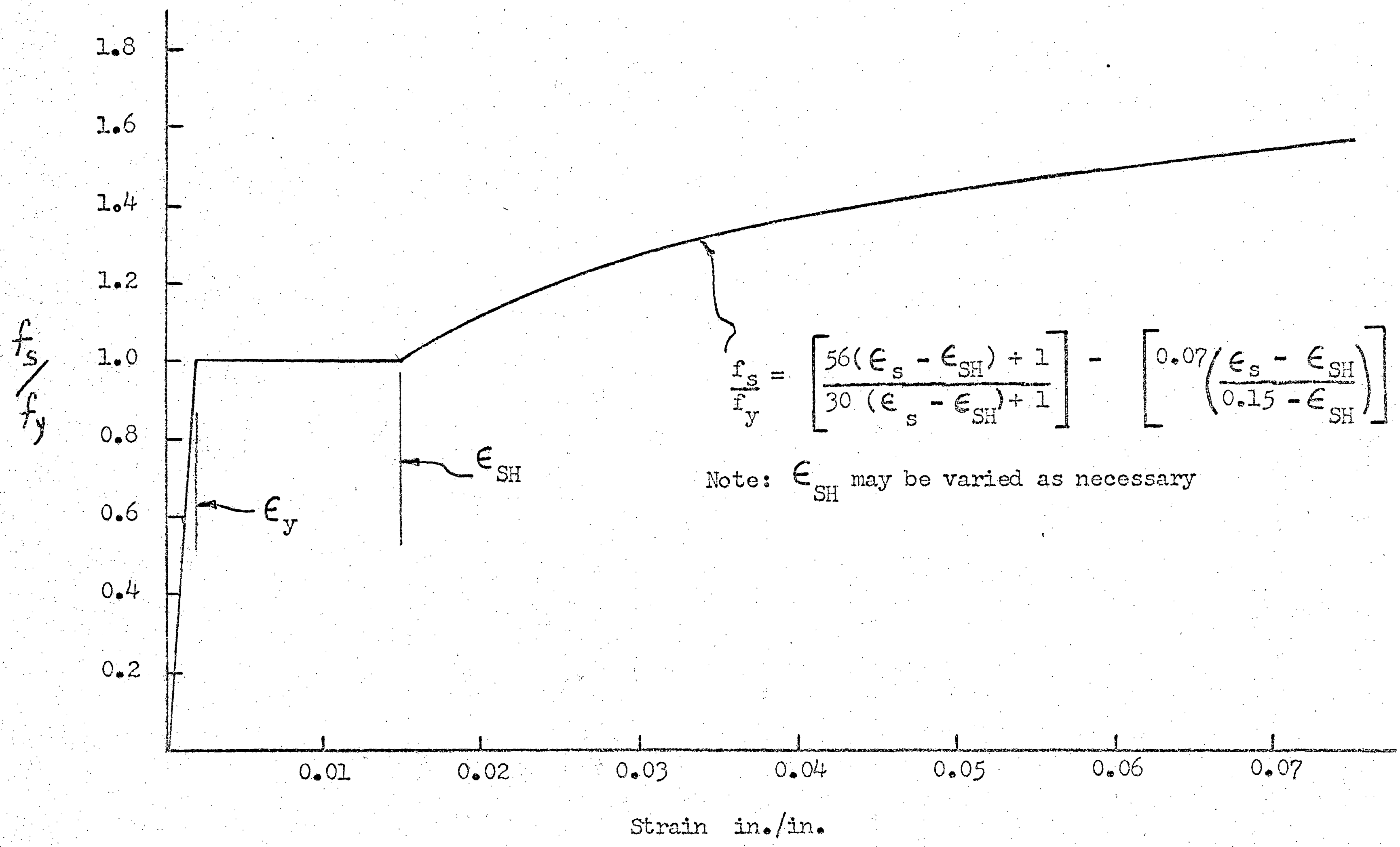


Figure 3. Steel Stress-Strain Curve

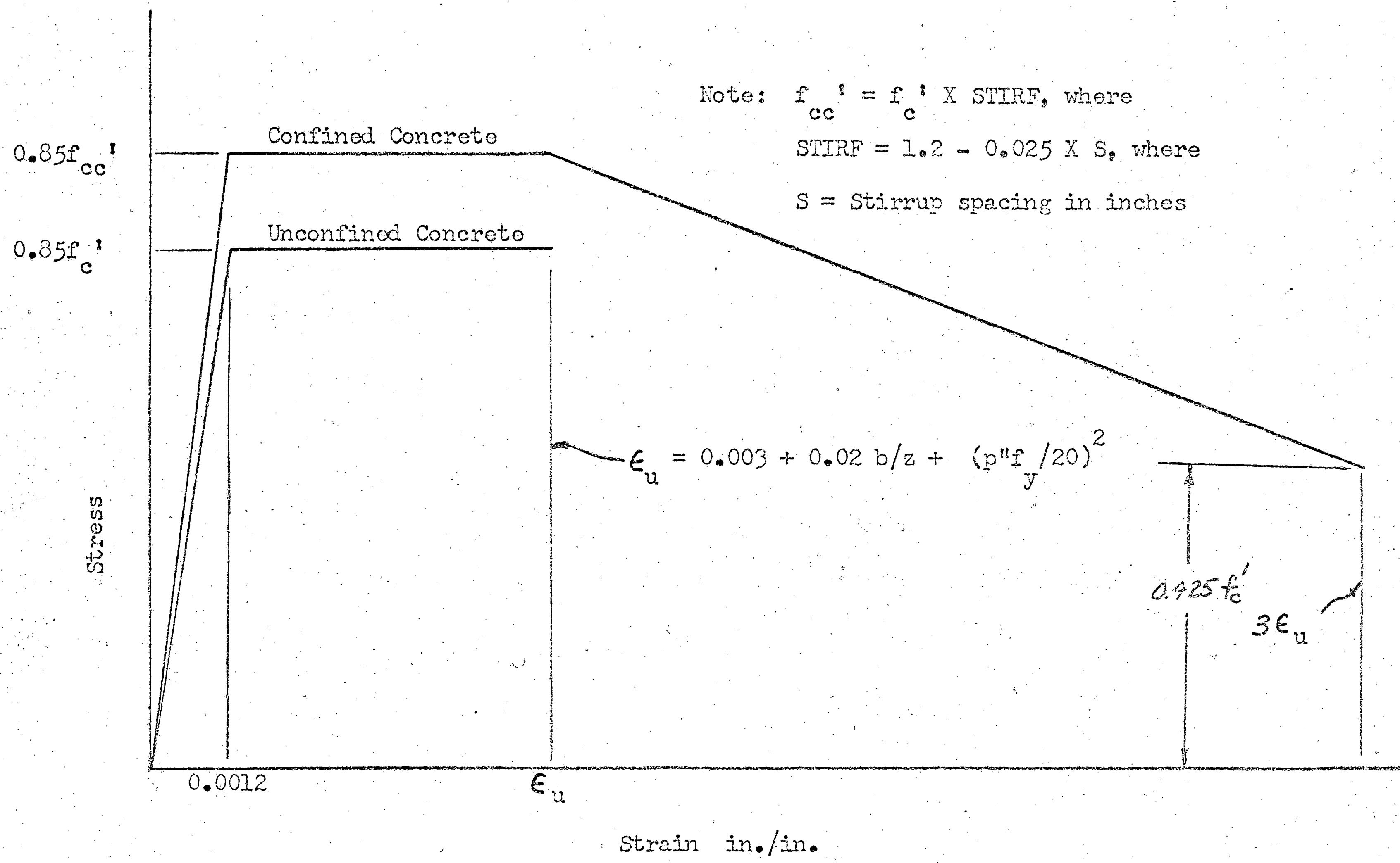


Figure 4. Concrete Stress-Strain Curves

p'' = ratio of volume of binding reinforcement (one stirrup plus compression steel) to volume of concrete enclosed (area enclosed by one stirrup multiplied by the stirrup spacing).

f_y = yield point of stirrup steel.

For small amounts of binding and constant moment, this expression reduces to the ACI value of 0.003. It should be emphasized that the values given by this equation are not strains in the generally accepted sense. Rather they are shortenings per unit gage length at the compression face measured over a length equal to the beam effective depth. Actually, there are many different values of strain along this face due to nonuniformity of material and the spread of plasticity, but for the purpose of this investigation, the values given by Corley's equation will be used as ultimate strains for the unconfined concrete.

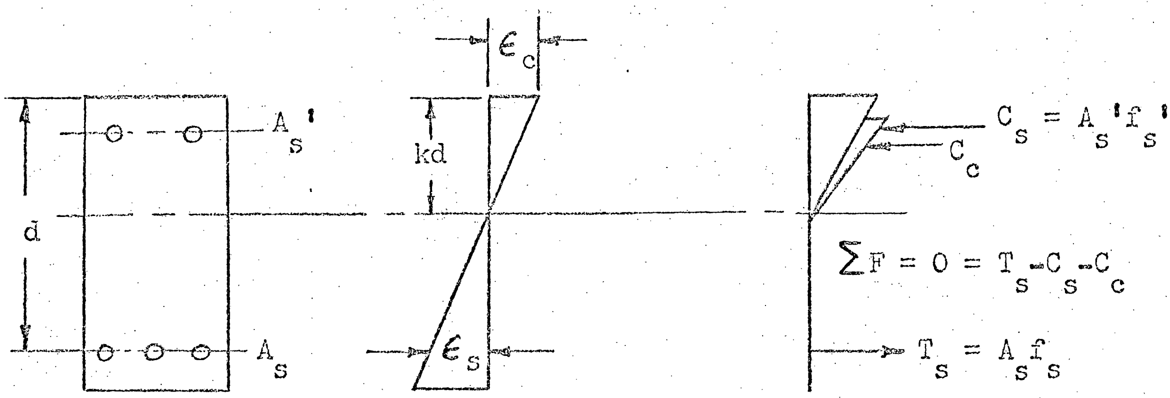
Confined Concrete

The confined concrete relationship used in the model is the tri-linear diagram shown in Figure 4. This takes into account the increase in compressive stress which researchers have witnessed in specimens subjected to triaxial loading. The increase in maximum stress allowed in the region of confinement is the result of investigations by S. Stockl, V. V. Bertero, and C. Felippa, as reported in their discussions of Roy and Sozen's[17] paper. This phenomenon of increasing stress capacity with increasing confinement has been commented on by other researchers, and the expression used here seems to be a quantitative median of the current information. As can be readily seen, stirrup spacing is the only parameter considered to affect the change in maximum stress. Stirrup spacing of eight inches or less is required for any increase, and the maximum increase allowed approaches 20 percent for very closely spaced stirrups.

Generation of Moment-Curvature Relationship

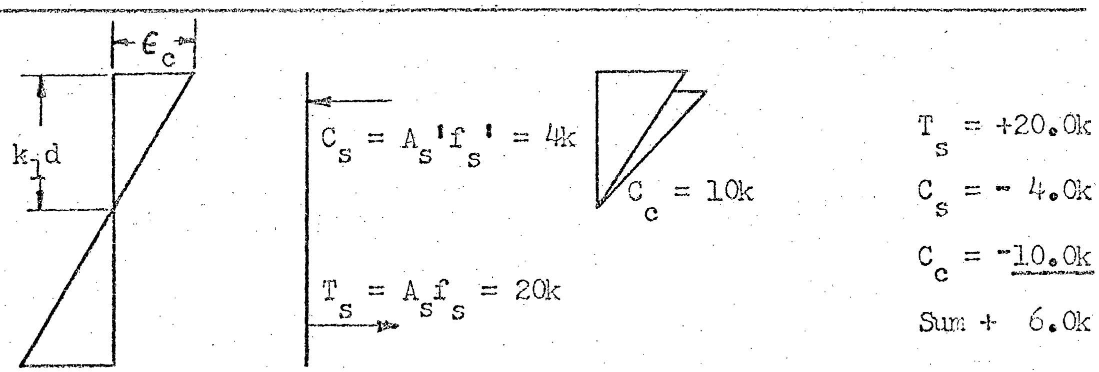
The primary function of the computer program included in this report is to develop the moment-curvature relationship for a reinforced concrete beam given the section and material parameters of that beam. The process of obtaining the moment-curvature relationship is iterative, starting with reasonable assumptions and proceeding until the statics and compatibility conditions are met. First, a value of concrete strain is set for the extreme compression fiber. Next, a trial compression zone depth is selected. With these two values assumed, the geometry of the section is fixed, as shown in Figure 5. The forces in the reinforcement and in the concrete are now computed, and a check is made for internal equilibrium. If the forces are not in equilibrium, the neutral axis is incremented up or down as required until the force equilibrium does check within the limits prescribed, plus or minus 0.1 kip in this program.

After equilibrium is reached, the resisting moment is calculated by taking moments of compressive forces about the line of action of the tensile force. The curvature is equal to the extreme fiber concrete



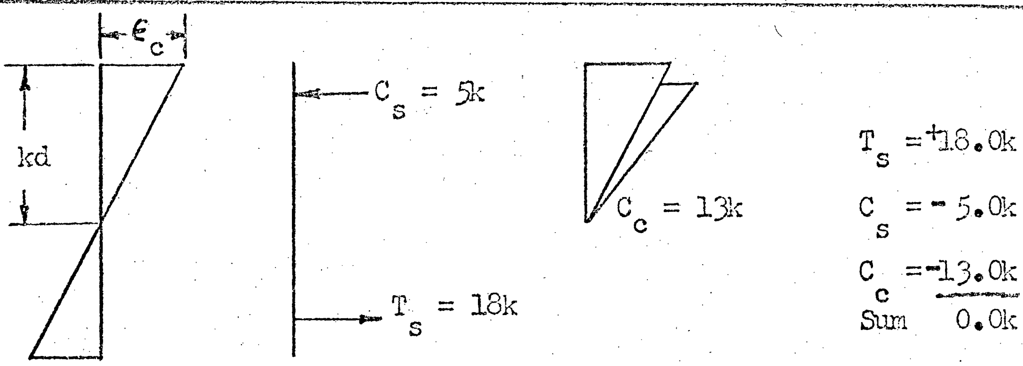
a. Section b. Strain c. Forces

Conditions in a General R - C Beam



a. Strain b. Steel Forces c. Concrete Force d. Summary

First Try to Balance Forces (k_1 trial value)



a. Strain b. Steel Forces c. Concrete Force d. Summary

Final Balance (k final value)

Figure 5. Basic Statics Involved in Finding One Point on the Moment Curvature-Curve

strain divided by the depth to the neutral axis.

When one point on the moment-curvature curve is found, the concrete strain is incremented and the procedure repeated. In the program included here, the concrete strain is incremented by 0.001 starting with 0.001 and continuing until either (1) the extreme fiber concrete strain reaches ϵ_u, (2) the tension steel strain reaches 0.15, or (3) the curvature reaches 0.02 radians/inch. As mentioned before, these limits can easily be extended, depending only on the computer storage available and the practical upper values of these parameters.

As an intermediate step, the program computes the value of yield moment and yield curvature, which are defined as those values of moment and curvature when the tension steel reaches first yield. To find these values, the concrete strain is incremented between its value just below reinforcement yield and its value just above reinforcement yield. The reinforcement strain is held constant in this process, establishing the geometry and allowing the equilibrium check to be carried out as before.

Computation of Deflections

The second moment-area theorem is the basis for the deflection calculations presented herein. With symmetry about midspan assumed, the midspan deflection is equal to the product of the area of the curvature diagram between midspan and either support multiplied by the distance from the support to the centroid of the area. Assuming a linear distribution between all discrete points on the moment-curvature curve, the equation for midspan deflection may be written

$$Y = 1/6 \sum_{1}^{N} \Big[\phi_i \,(2\,X_i^{\,2} - X_i\,X_{i-1} - X_{i-1}^{\,2}) + \phi_{i-1}\,(X_i^{\,2} + X_i\,X_{i-1} - 2\,X_{i-1}^{\,2}) \Big]$$

for a simple beam with one point loading as shown in Figure 6[13]. In this equation "N" is the number of increments along the beam from the support to midspan, and ϕ_i is the curvature at the point whose distance from the support is X_i.

The given model should predict deflections very well as long as all sections of the beam continue to increase in moment. When a descending moment-curvature branch is encountered, however, the traditional moment-curvature approach to computing deflections breaks down. Necessary to compute deflections after maximum moment has been reached is a knowledge of the spread of plasticity in the hinging region. Very little research has gone into studying the lengths of hinges, and for this reason any deflections found by the model beyond the point of maximum moment are to be ignored. The hinge length included in the program is merely a dummy, and is not intended to represent in any way the hinging behavior of the beams in question.

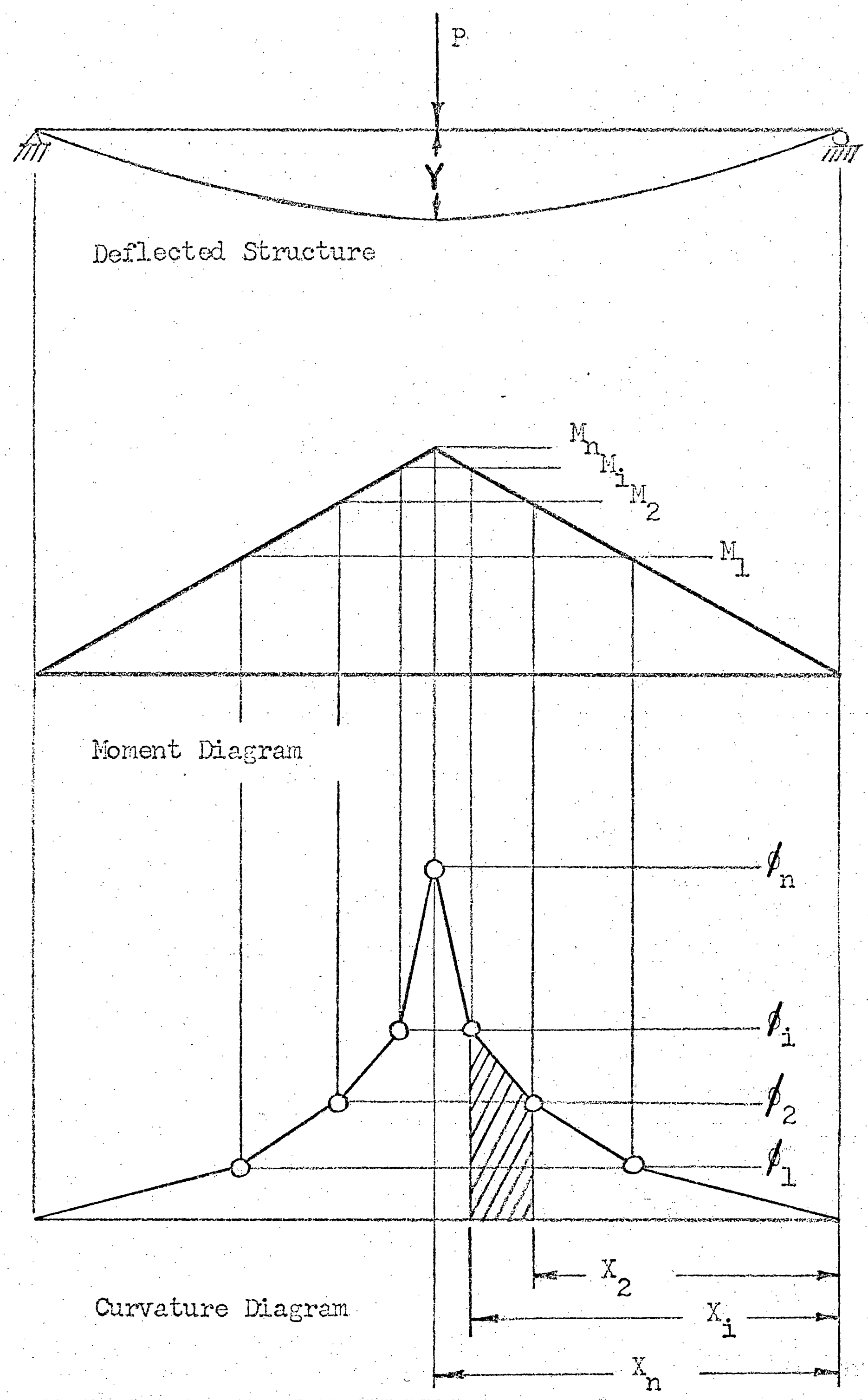


Figure 6. Deflected Structure, Moment Diagram, Curvature Diagram - Basis of Deflection Calculations

IV. MODEL VERIFICATION

A comparison between the experimental and analytical flexural behavior of 10 different beams is shown in Figures 7-18. For the first six beams, the analytical moment-curvature relationship is plotted only until the given value of ultimate strain at the extreme concrete fiber is reached. Experimental data is available for only the ascending or idealized bilinear portion of the moment-curvature curves of these six beams. The last four beams represent a very limited quantity of experimental data showing the descending branch of the moment-curvature curve. For these four beams, the analytical moment-curvature curve is plotted over the full range of curvature for which experimental data is available for the given beam. The section and material properties for the beams discussed here, as well as a description of their flexural behavior in tabular form, comprise Tables 1-20 in Appendix A.

Beam J6 is representative of the series of beams for which moment-curvature is plotted up to the point of ultimate extreme fiber concrete strain. The analytical curve for this beam closely describes the experimental result, as shown in Figure 7. The maximum rotational capacity predicted is a direct result of the assumed ultimate concrete strain. It is evident from Figure 7, as well as Figures 8-12, that some adjustment may be necessary in Corley's[7] equation for ultimate strain. For the wide range of member parameters included here, the maximum rotational capacity predicted varies between about 50% and 125% of the maximum experimental value. This is a much closer prediction than that by Sawyer[18] as shown in Figure 7, for the simple reason that Sawyer always considers the concrete ultimate strain to be Hognestad's[10] value of 0.0038.

Figure 13 describes the behavior of a singly-reinforced beam with no stirrups. The analytical curve closely describes the experimental data until the ACI ultimate strain of 0.003 is reached. After this, the analytical curve falls off while the experimental data continues to rise. This only means that the value of 0.003 is conservative, and a larger assumed ultimate strain value would give closer correlation with the experimental.

Figures 14-16 show the moment-curvature curves of three other beams for which experimental data in the range of large curvatures is available. The predicted moment at large curvatures is too large, and this indicates that the steel strain hardening influence has been overestimated.

Figures 17 and 18 are load-deflection curves, and the conservative estimate of deflections shown coincides with the conservative curvatures predicted for the given two beams. With reasonable adjustments in the ultimate strain expression, the actual available deflection could be more closely approximated.

Upon examination of the curves in this section which describe the flexural behavior of beams with a fairly broad range of member properties, it is concluded that the model does work with the following limitations: (1) the available rotational capacity is a function of the ultimate concrete strain assumed, and (2) the moment capacity is a function of the steel strain hardening assumed. Thus, the two parameters controlling an accurate prediction of the flexural behavior in reinforced concrete are ultimate concrete strain and reinforcement strain hardening.

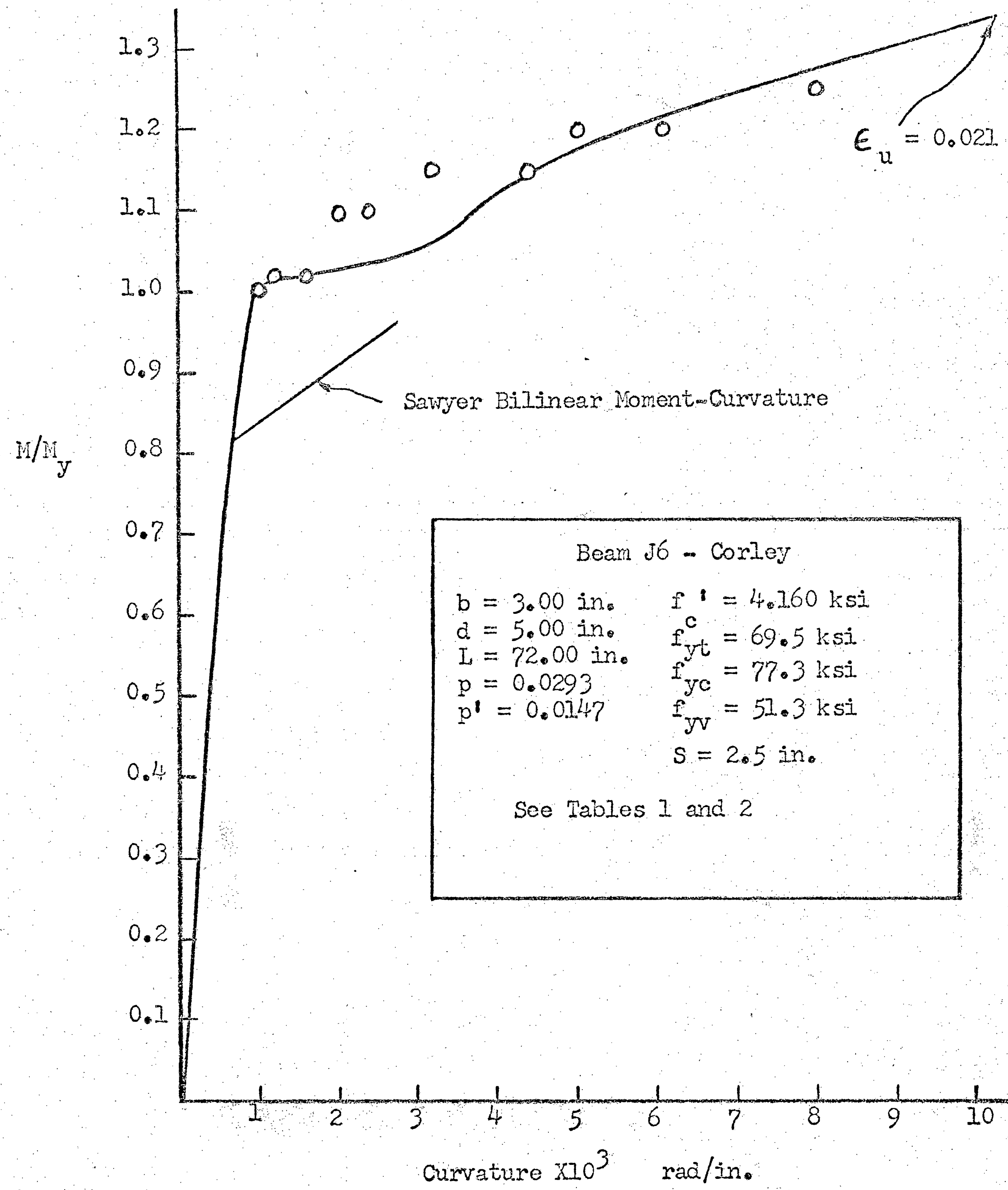


Figure 7. Analytical and Experimental Moment-Curvature Curves - Beam J6

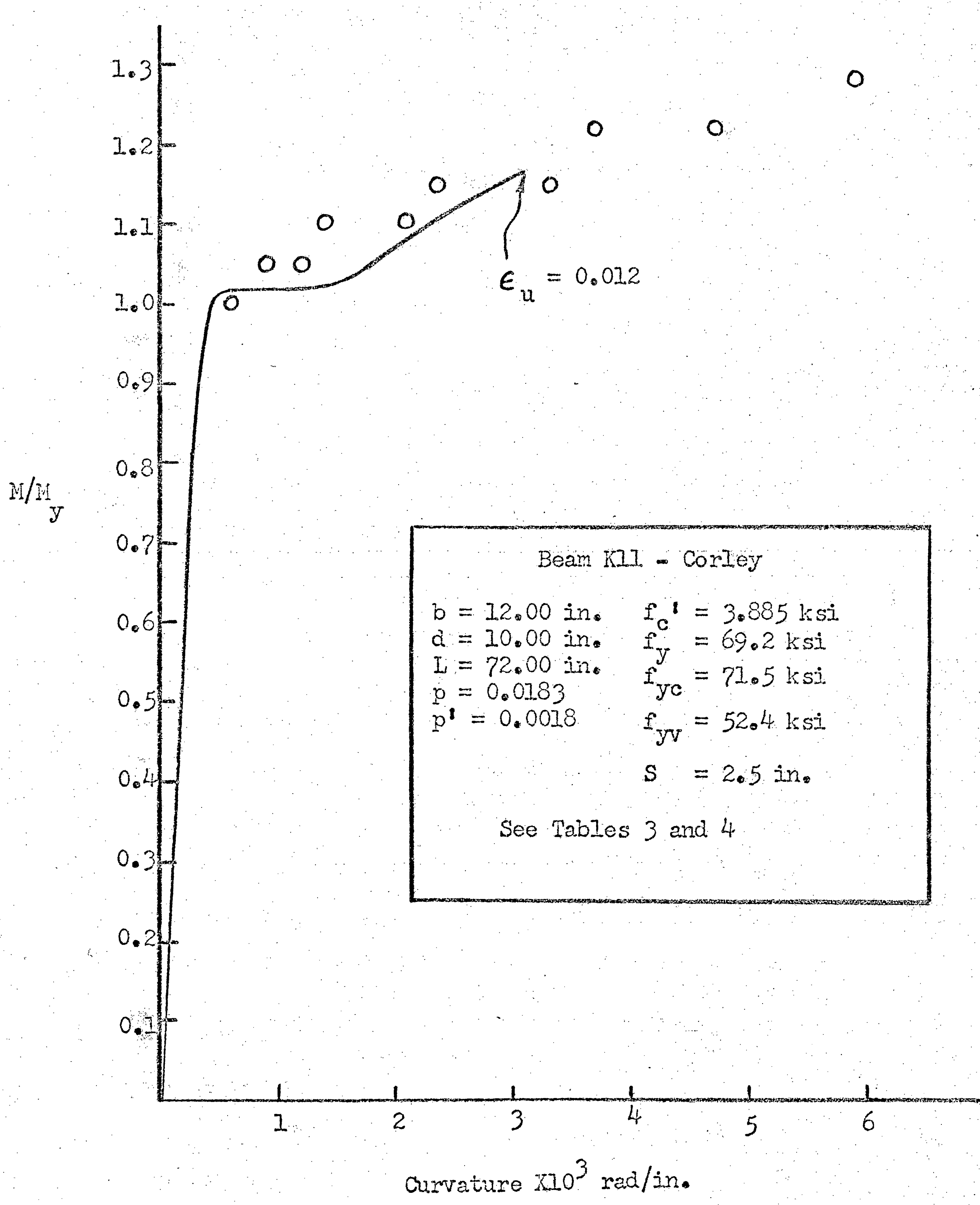


Figure 8. Analytical and Experimental Moment-Curvature Curves - Beam K11

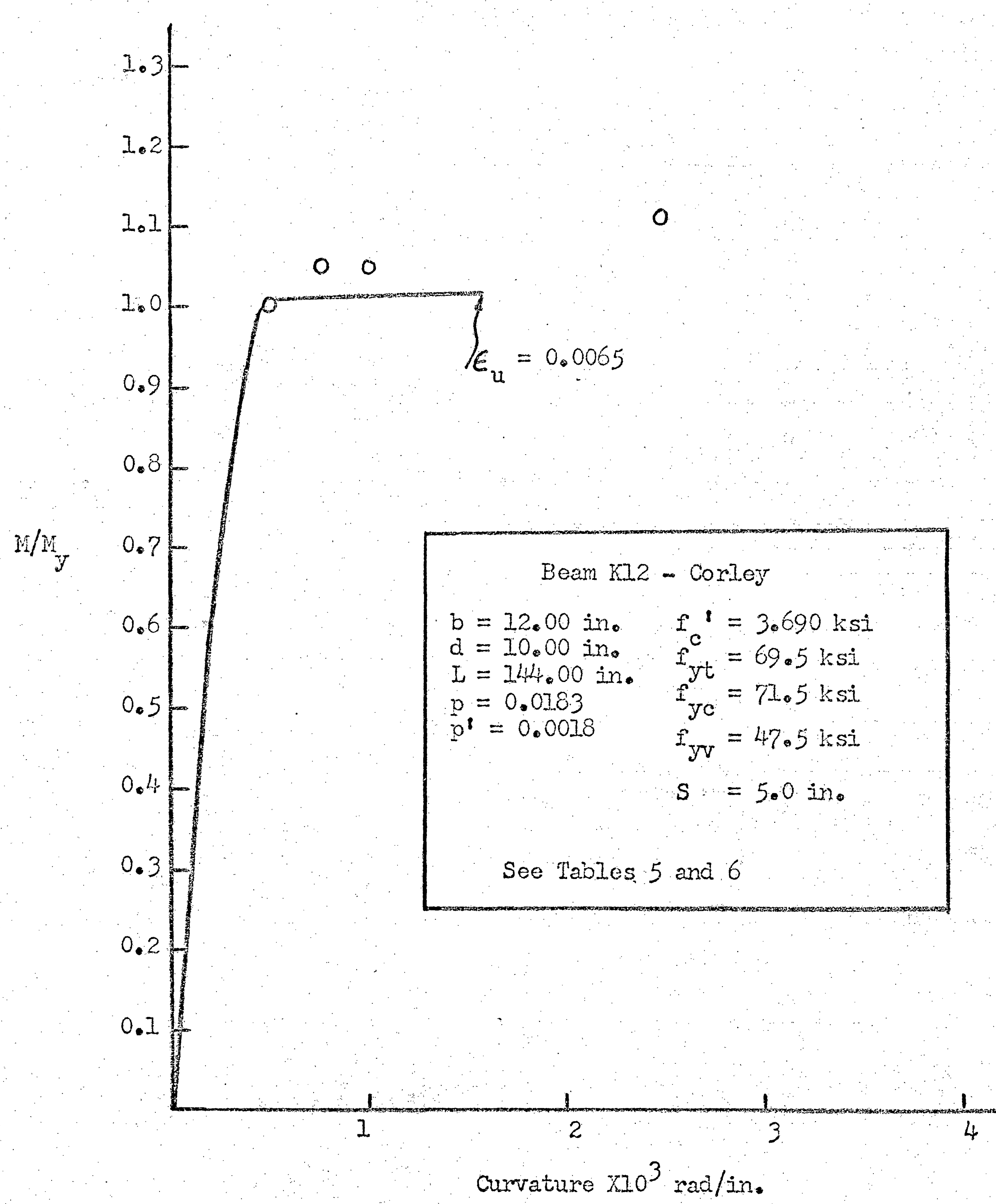


Figure 9. Analytical and Experimental Moment-Curvature Curves - Beam K12

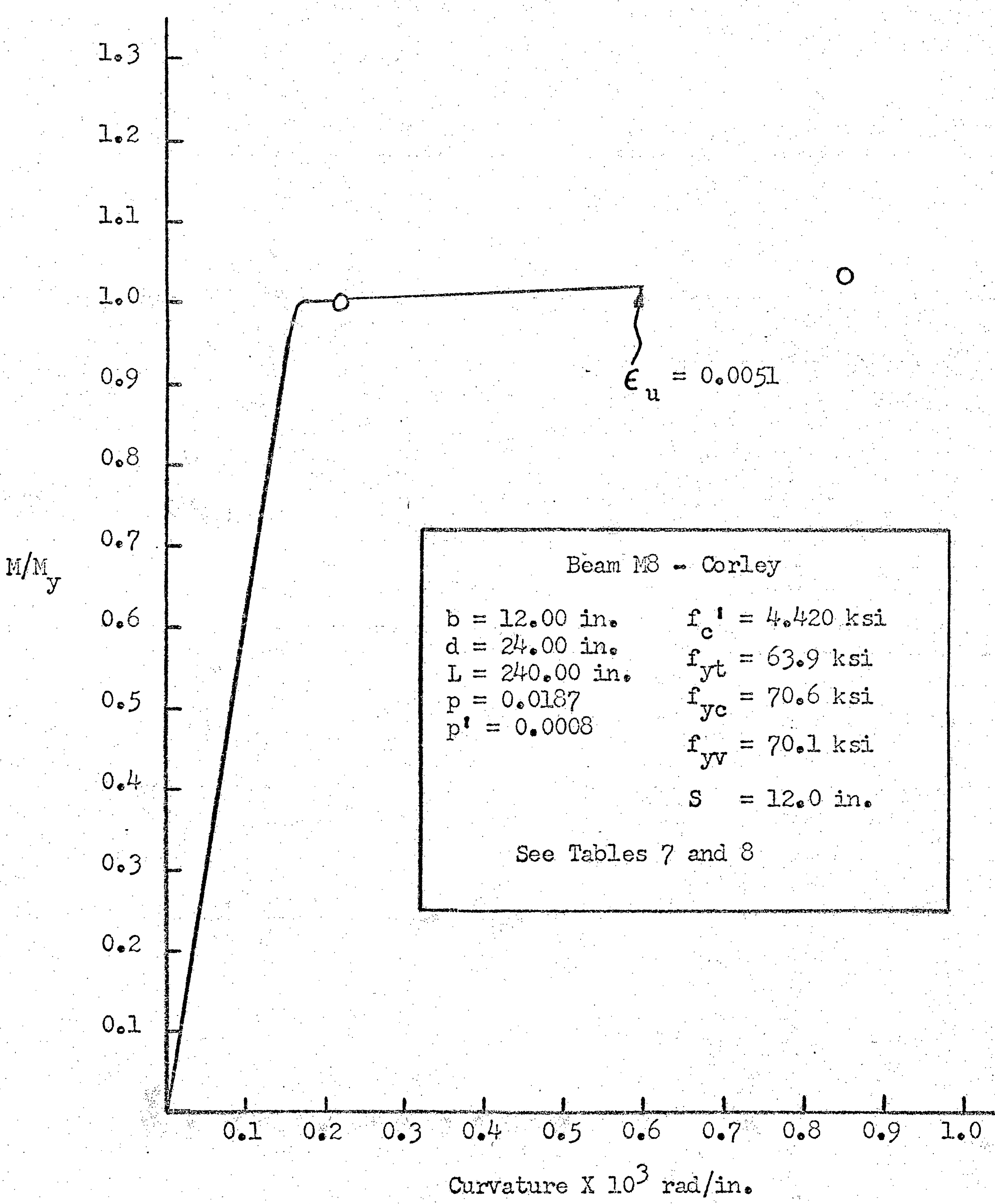


Figure 10. Analytical and Experimental Moment-Curvature Curves - Beam M8

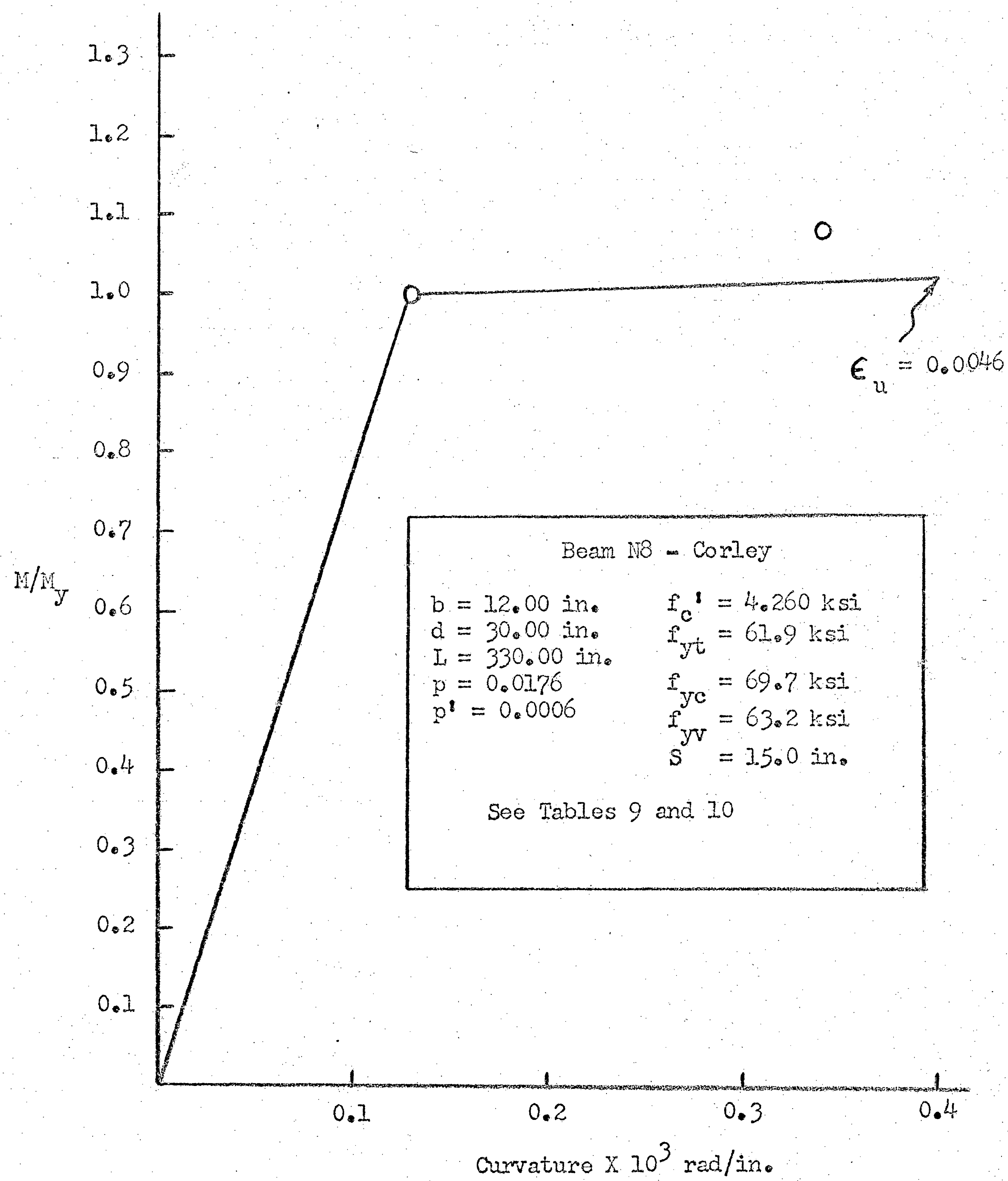


Figure 11. Analytical and Experimental Moment-Curvature Curves - Beam N8

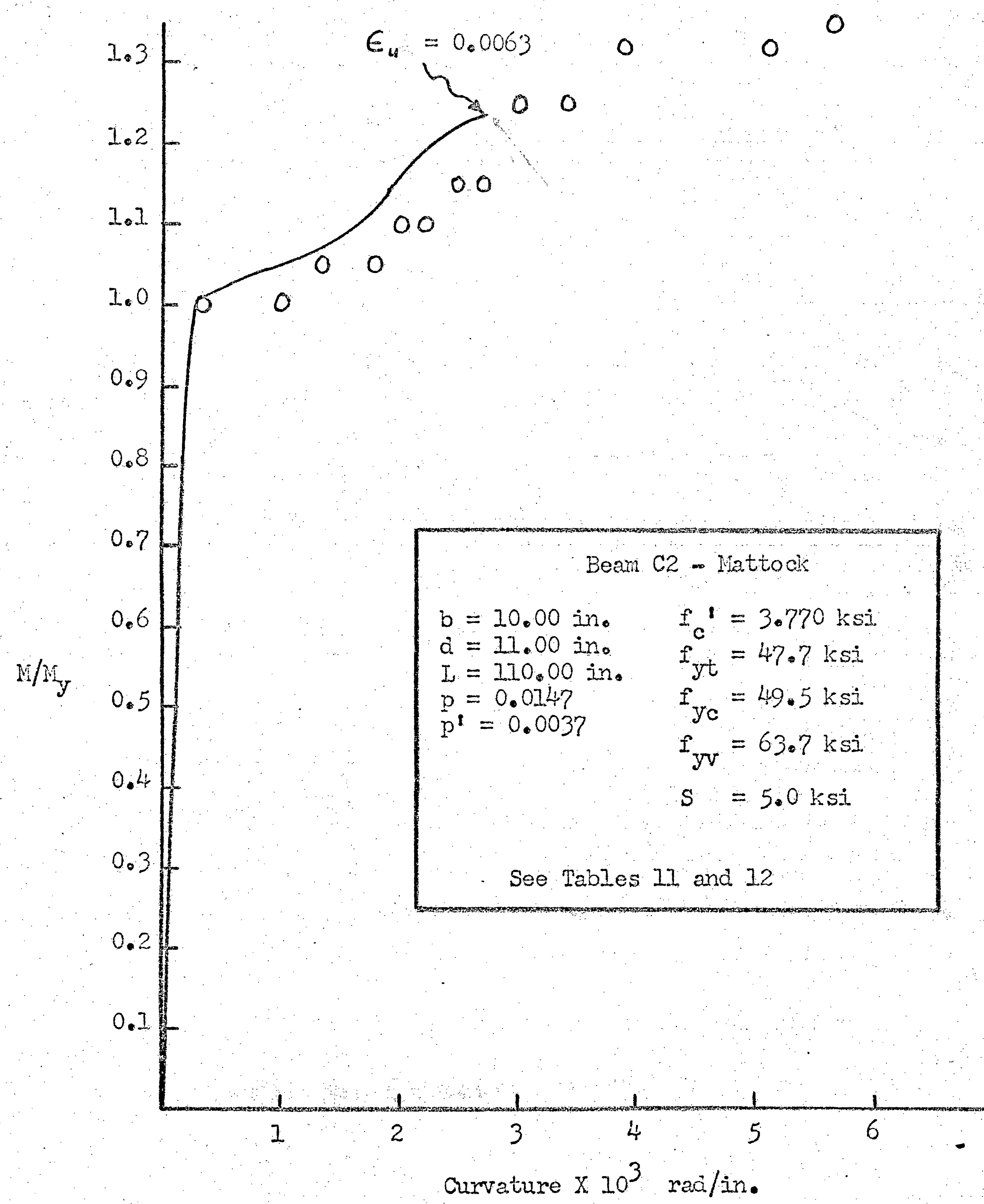


Figure 12. Analytical and Experimental Moment-Curvature Curves - Beam C2

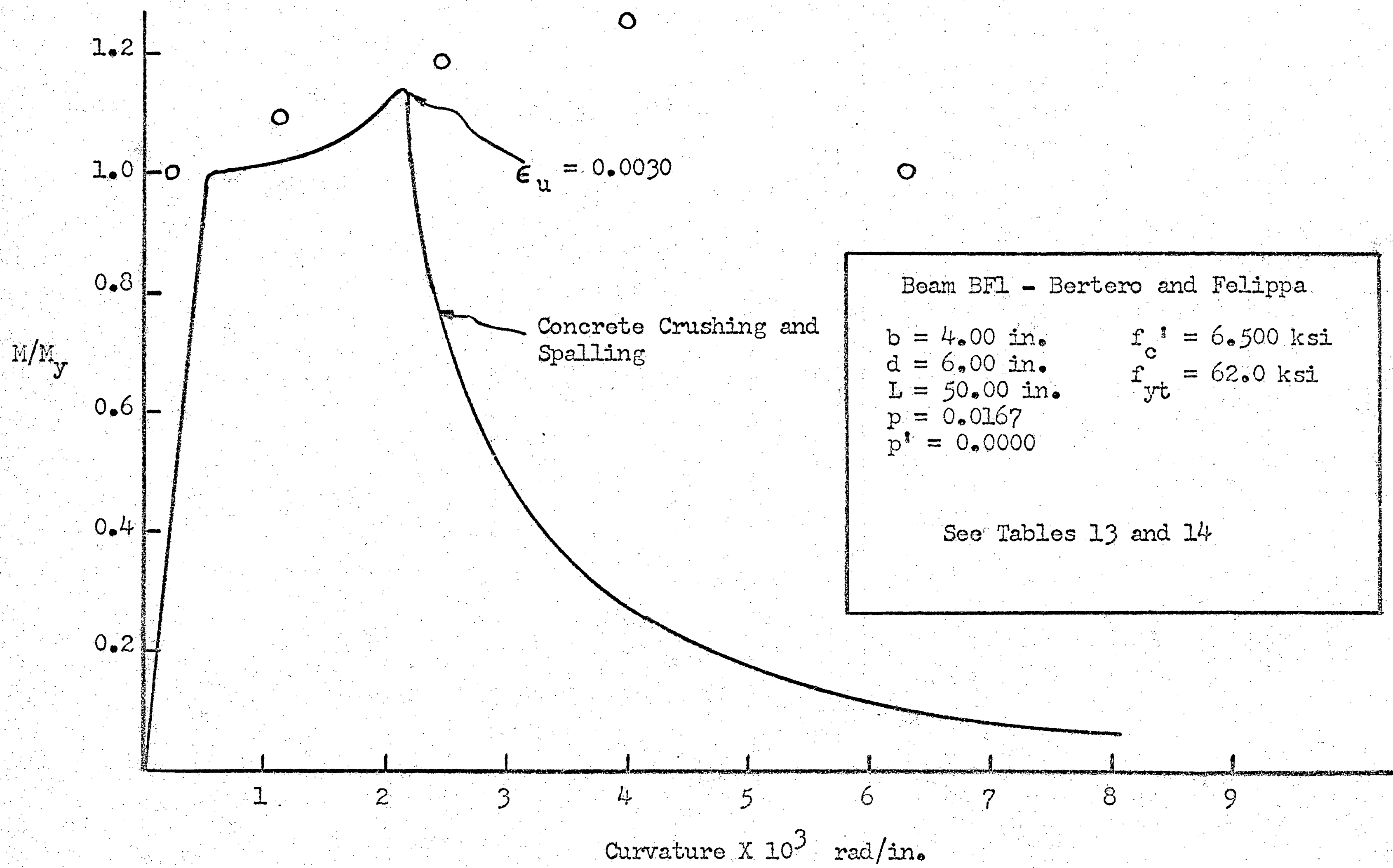


Figure 13. Analytical and Experimental Moment-Curvature Curves - Beam BF1

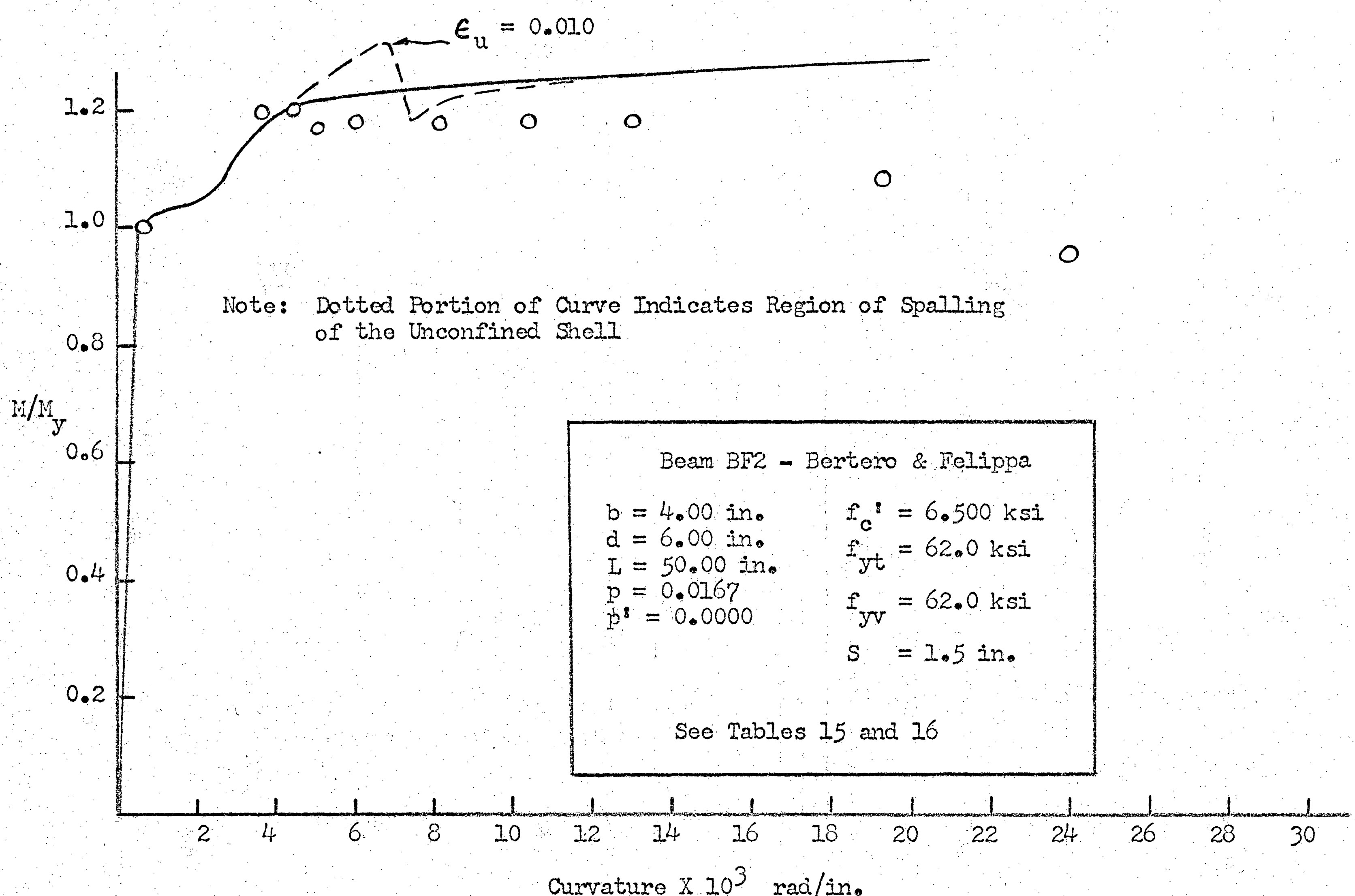


Figure 14. Analytical and Experimental Moment-Curvature Curves - Beam BF2

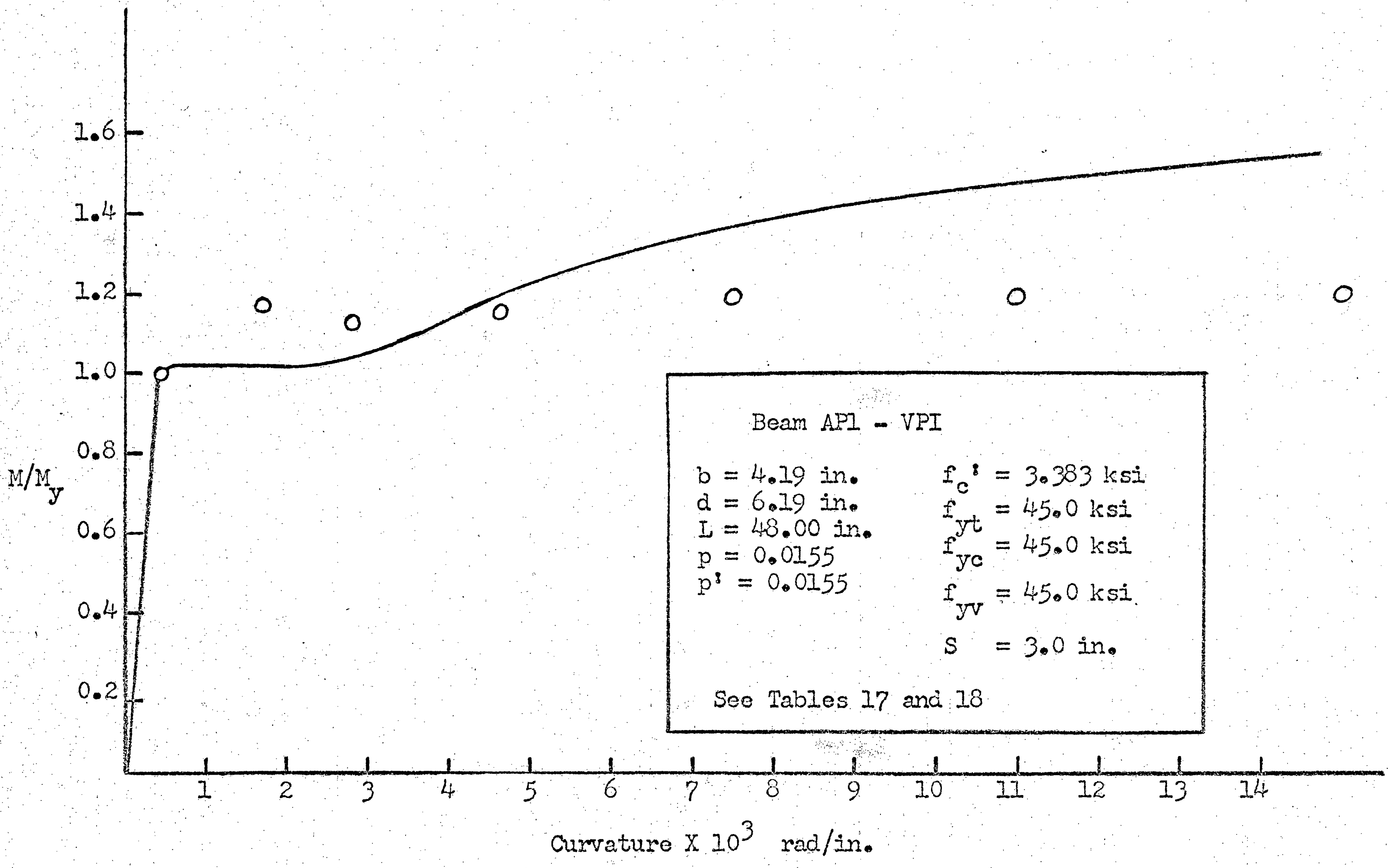


Figure 15. Analytical and Experimental Moment-Curvature Curves - Beam AP1

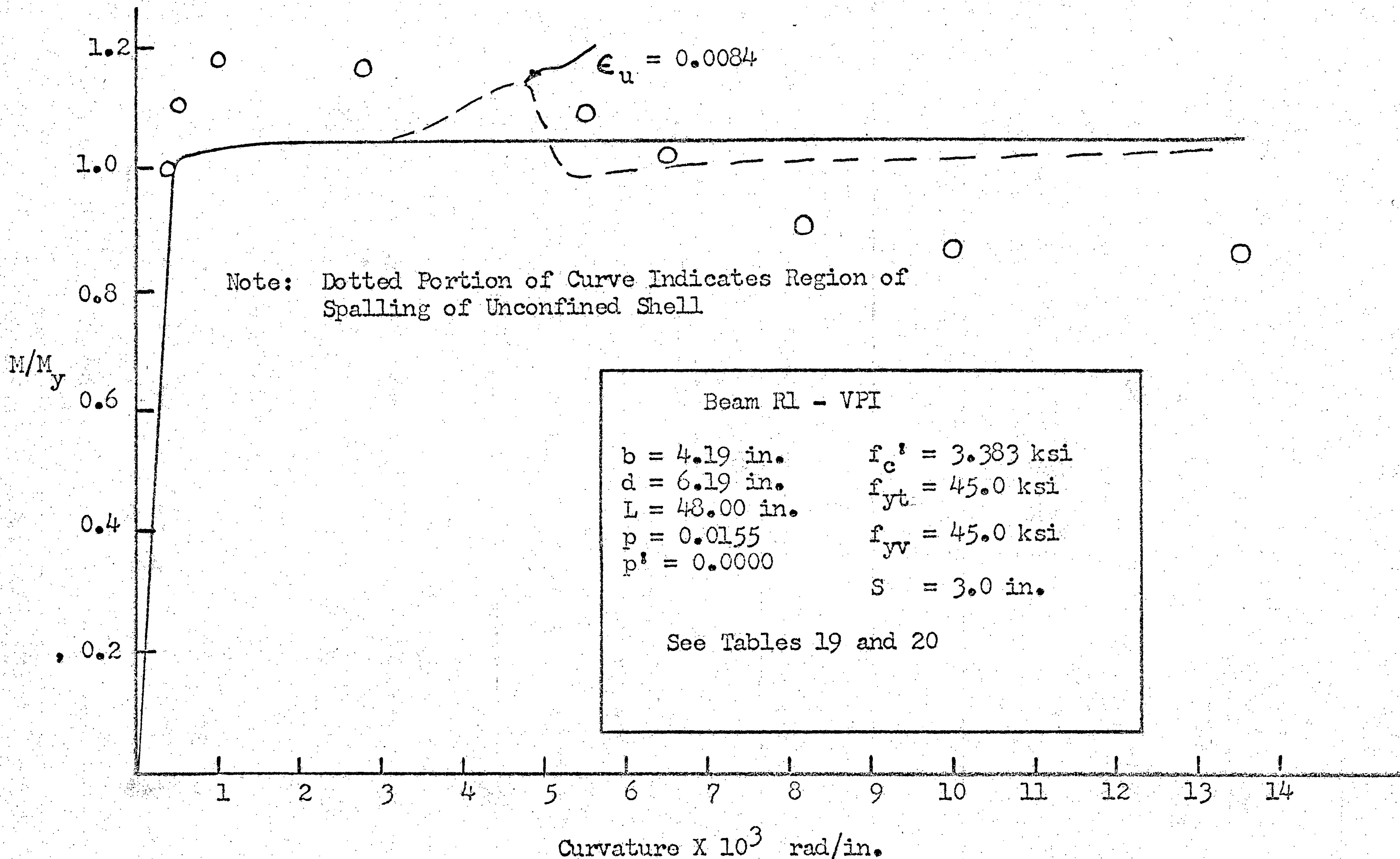


Figure 16. Analytical and Experimental Moment-Curvature Curves - Beam R1

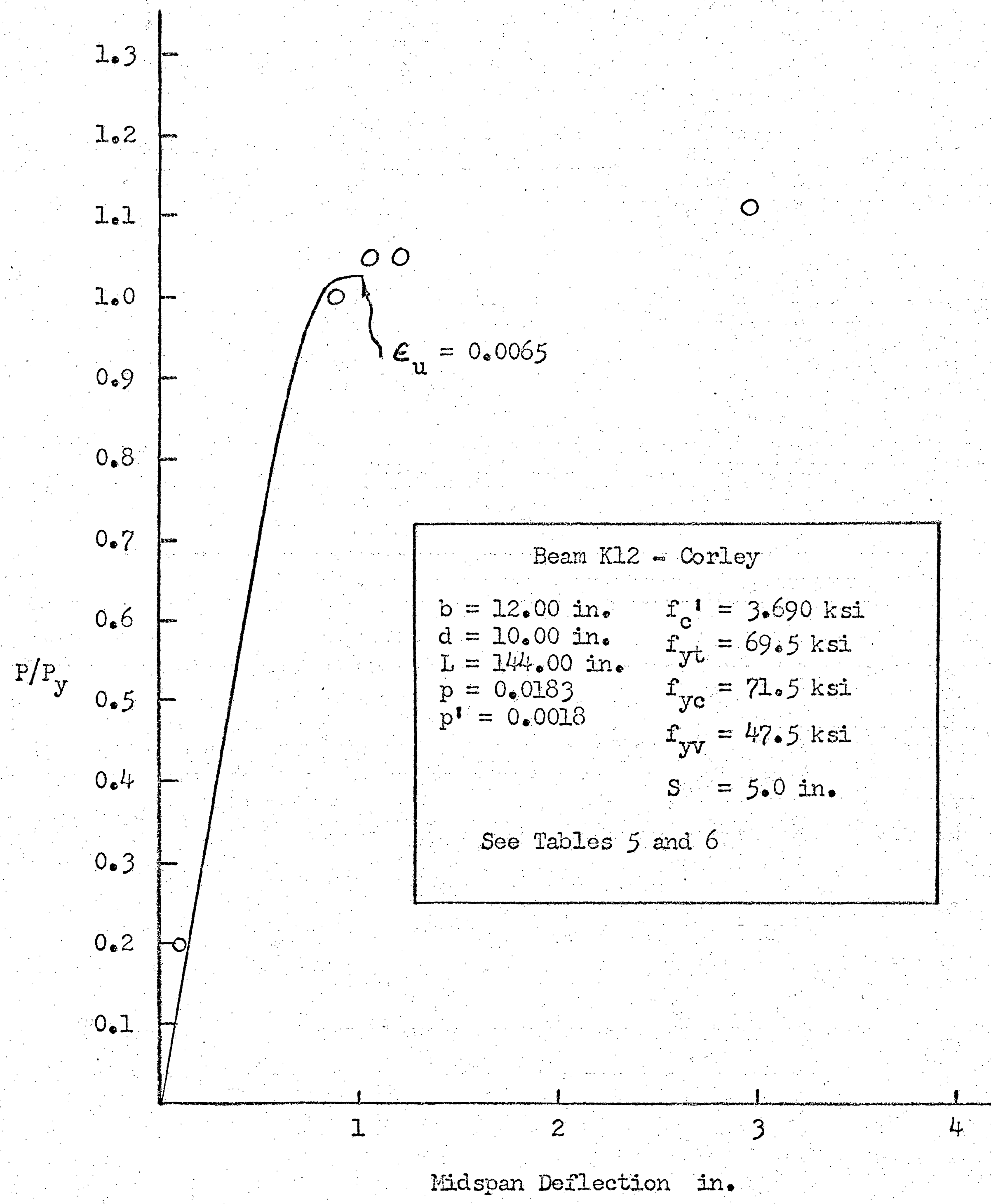


Figure 17. Analytical and Experimental Load-Deflection Curves - Beam K12

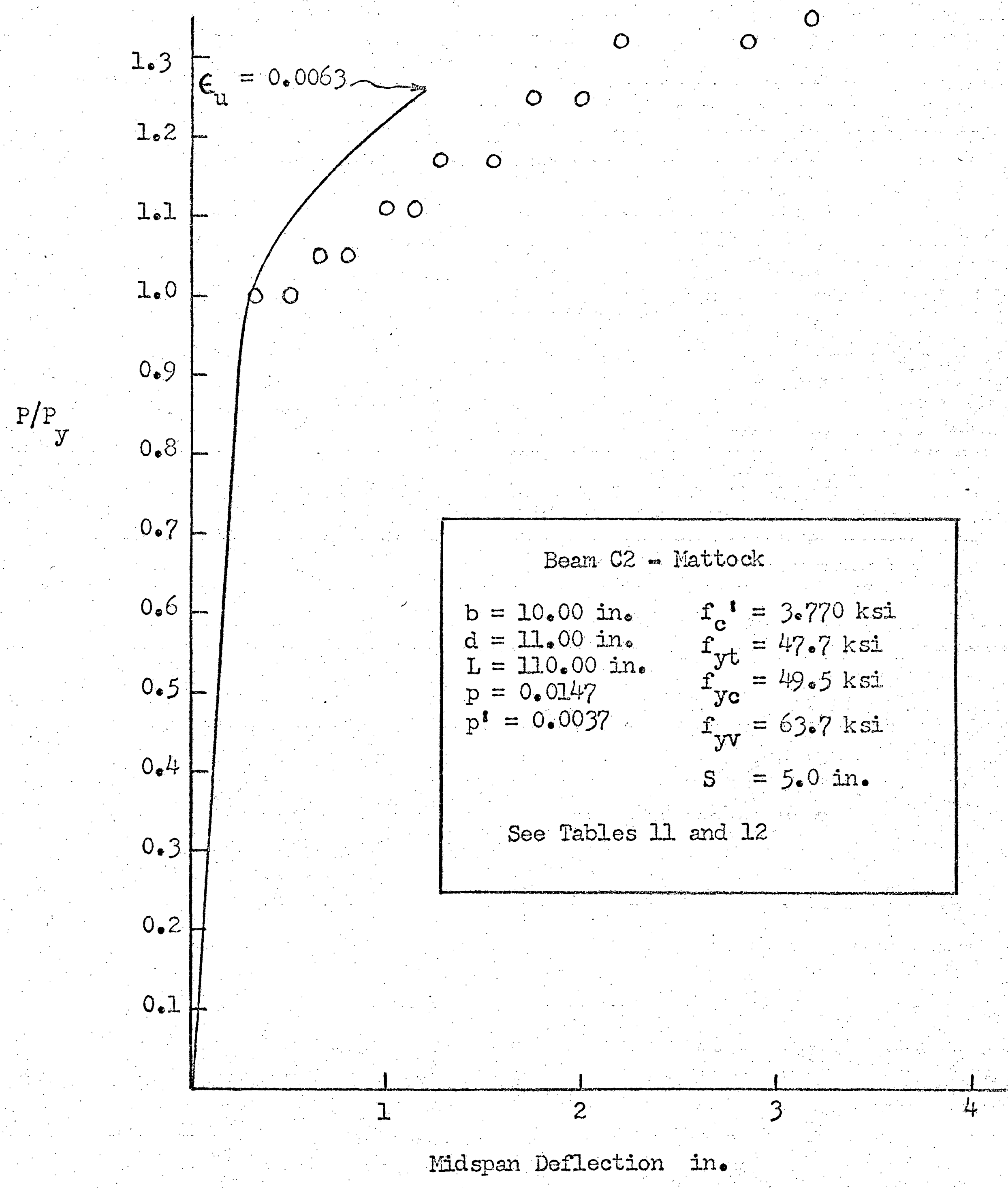


Figure 18. Analytical and Experimental Load-Deflection Curves - Beam C2

V. DISCUSSION OF PARAMETER INFLUENCE

Using the analytical model developed in this thesis, the effect of the most important beam parameters on moment capacity and rotational capacity is shown in Figures 19-25. These graphs illustrate the relative importance of the parameters involved, and they also give a quantitative estimate of the change in flexural behavior caused by varying each parameter individually.

Figure 19 shows the effect of varying tensile yield stress on the moment-curvature curve. The yield and ultimate moment capacities increase approximately directly with the increase in yield stress. Ultimate rotational capacity, however, decreases with increasing steel yield stress. This is caused by the increasing concrete area, and thus the increasing neutral axis depth, necessary to balance the tensile force.

The necessity for the inclusion of reinforcement strain hardening in the analytical model is shown in Figure 20. Ultimate curvature is not affected by this parameter, but the neglect of strain hardening will result in serious underestimation of moment capacity.

Figure 21 illustrates the effect stirrup spacing has on flexural behavior. The basic moment-curvature curve is determined by the other parameters involved, with stirrup spacing controlling the values of ultimate moment and ultimate curvature. Within the range of stirrup spacing considered here, approximately 50% greater rotational capacity is realized with each doubling of the number of stirrups.

Sufficient compression steel to develop fully the ductility of the tension steel was noted by Nordell[13] to be of primary importance in limit design. This conclusion is substantiated by Figure 22, which shows the effect of varying compression steel percentage. With no compression steel, the tension steel ductility is unused; it never enters the strain hardening region. The two curves including compression steel indicate how important a proper combination of steel percentages can be in developing sufficient rotational capacity to make limit design feasible. In the lower curve, there is half as much compression steel as tension steel; in the upper curve, the compression and tension areas are equal. The threefold increase in rotational capacity and the 30% increase in moment capacity indicates that compression steel percentage is perhaps the most important parameter in developing rotational capacity for limit design.

Span length is shown in Figure 23 to be a measure of the moment diagram slope. The effect of varying span length is seen here to be almost negligible for reasonable values of span length. For very short spans, the model presented here is inaccurate because shear is generally the mode of failure for such beams.

Figure 24 also shows beam width to have negligible influence on the moment-curvature curve. The three-inch wide beam actually has slightly higher ultimate curvature than the six or nine-inch wide beams. This is because the stirrups in a smaller beam (same stirrup size) are a slightly larger percentage of the concrete area bound. The conclusion reached here that effect of beam width is negligible is supported by Corley[7].

The effect of concrete compressive strength on the moment-curvature curve is also of interest. Figure 25 shows this effect, but its importance in limit design remains a moot question. There is a substantial improvement in flexural behavior as the concrete strength is increased from 2.0 ksi to 6.0 ksi, but most of this improvement is realized already at 4.0 ksi. Added strength beyond this point is useful, but probably not necessary to develop the rotational capacity needed.

The validity of the conclusions reached here rests for a large part on the values assumed for ultimate concrete strain. One suggestion for future investigation along these lines is that a statistical study be made of the variables affecting ultimate concrete strain, and that a new expression for ultimate concrete strain be formulated as a result of this study. Another area of investigation which needs attention before reinforced concrete flexural behavior can be fully understood is that phenomenon usually referred to as "hinge length." If a method can be formulated to accurately predict the length of hinge expected in a given member, then the computation of deflections will follow readily and accurately.

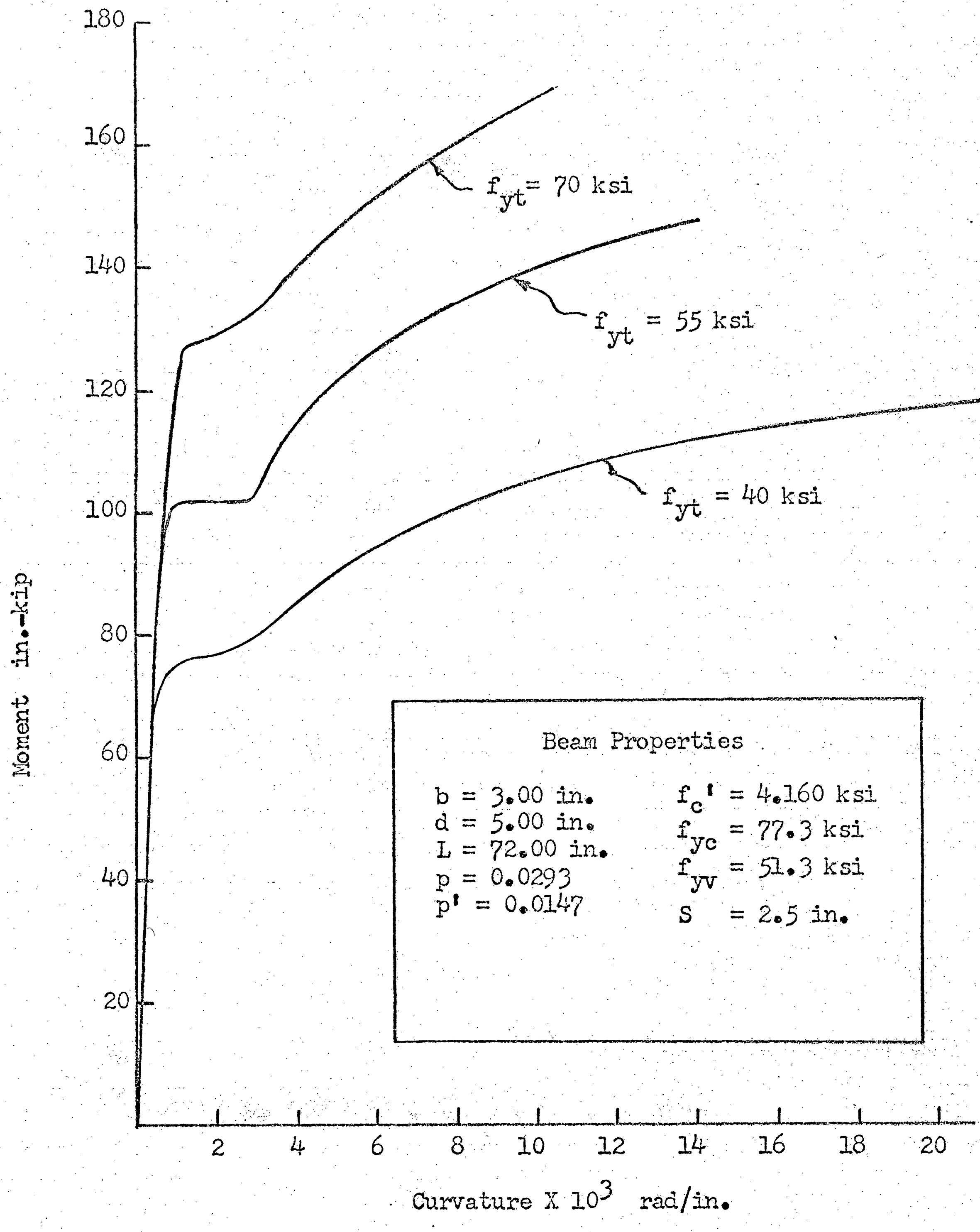


Figure 19. Effect of Change in Tension Steel Yield Stress on Moment-Curvature Curve

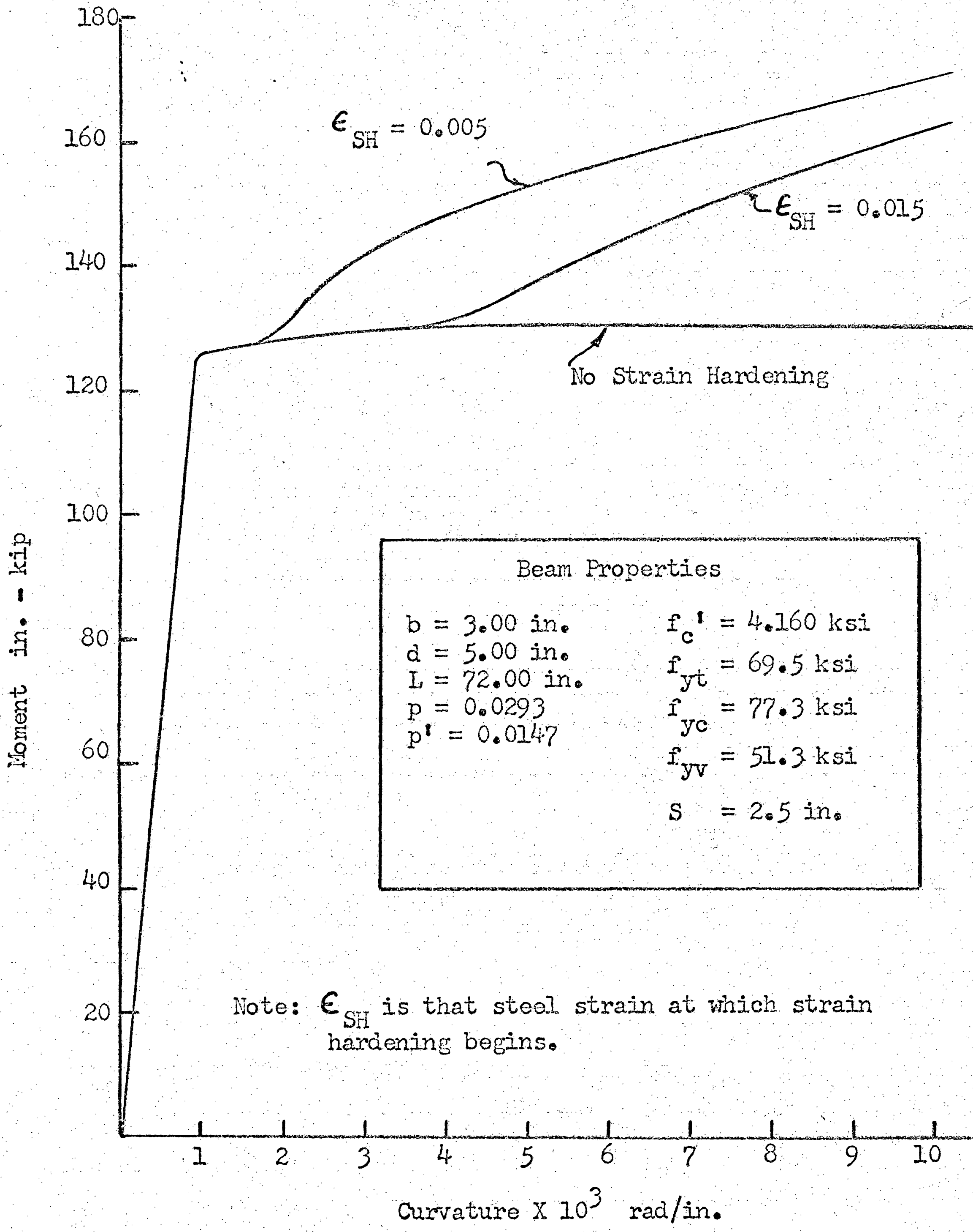


Figure 20. Effect of Reinforcement Strain Hardening on Moment-Curvature Curve

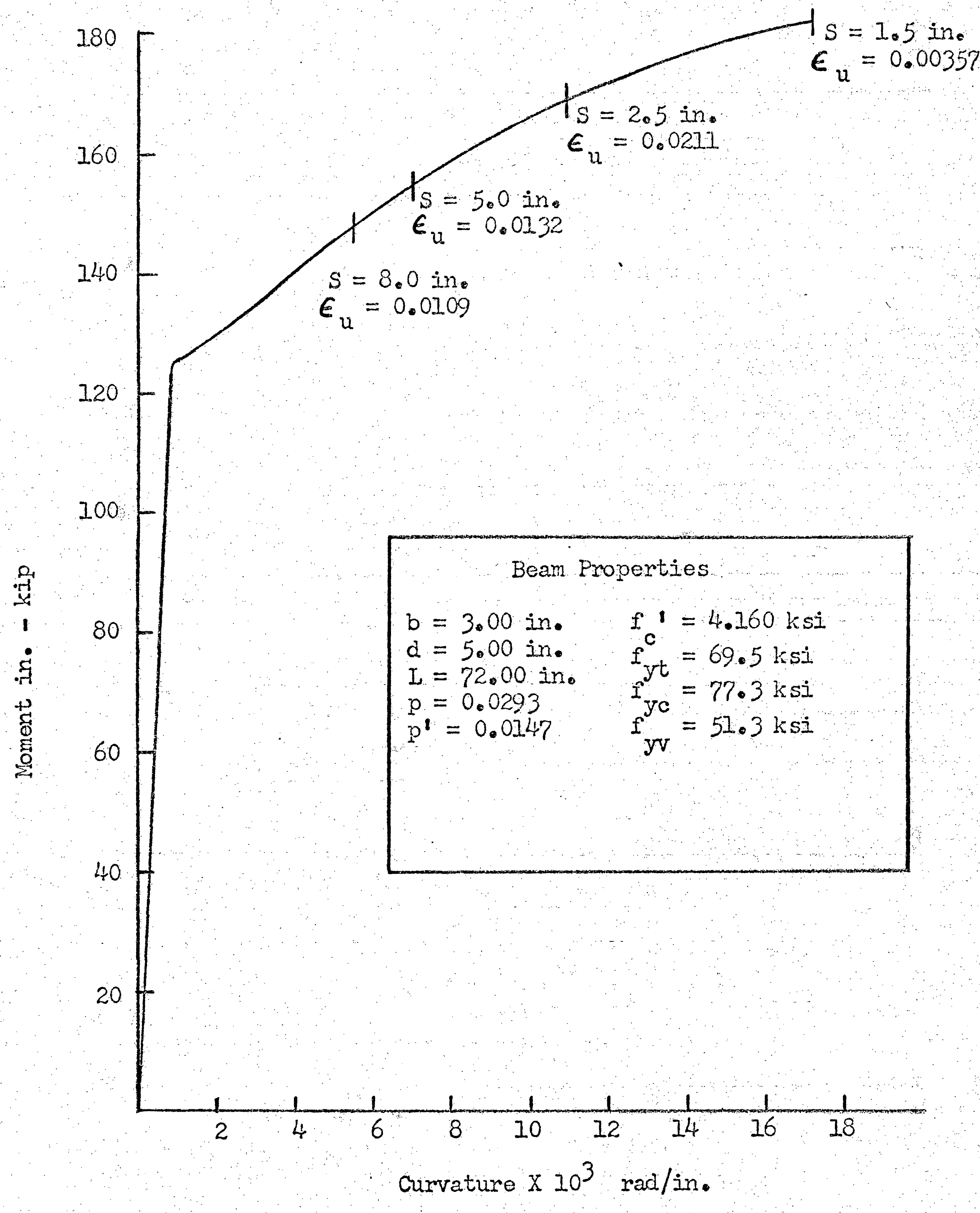


Figure 21. Effect of Stirrup Spacing on Moment-Curvature Curve

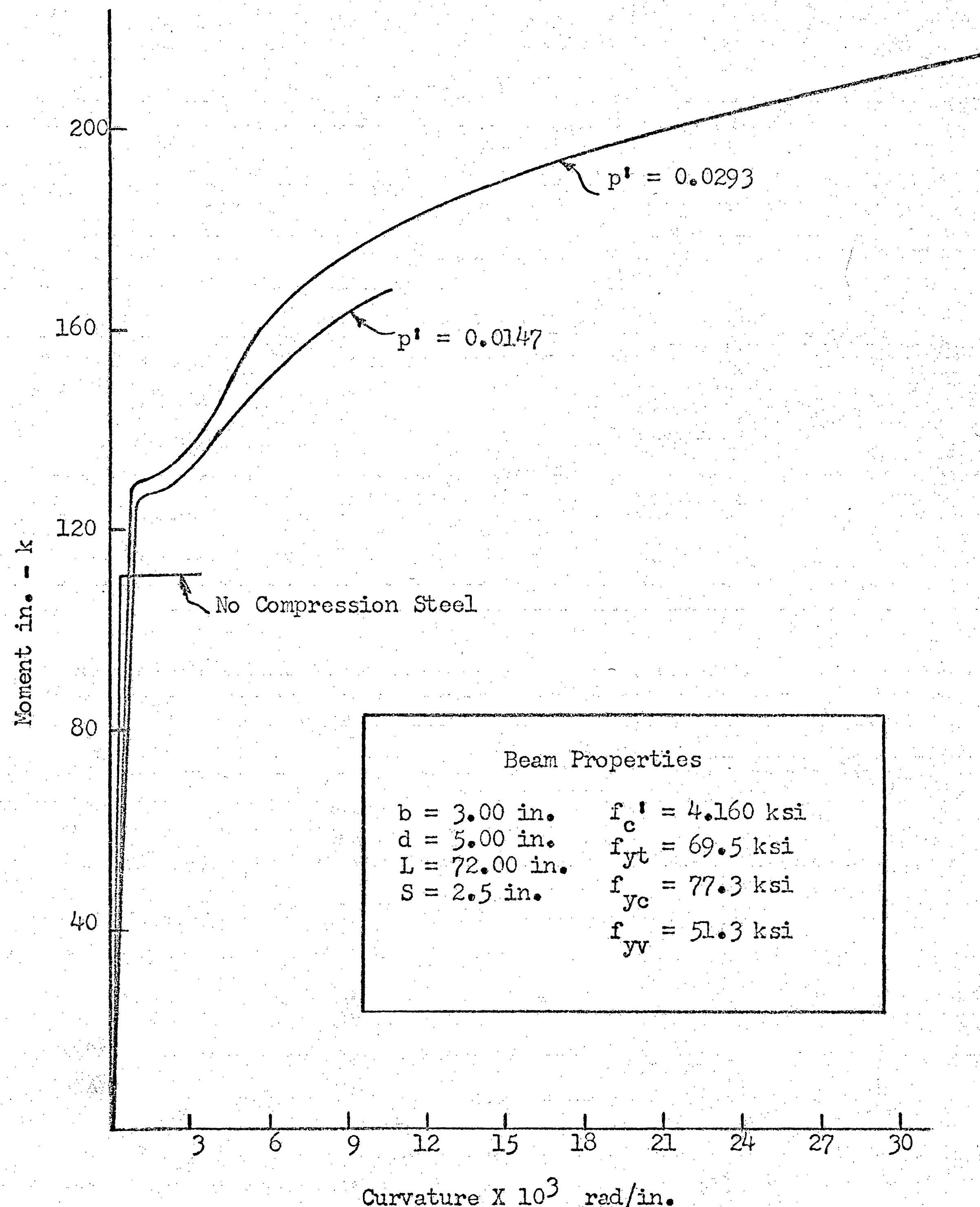


Figure 22. Effect of Compression Steel Percentage on Moment-Curvature Curve

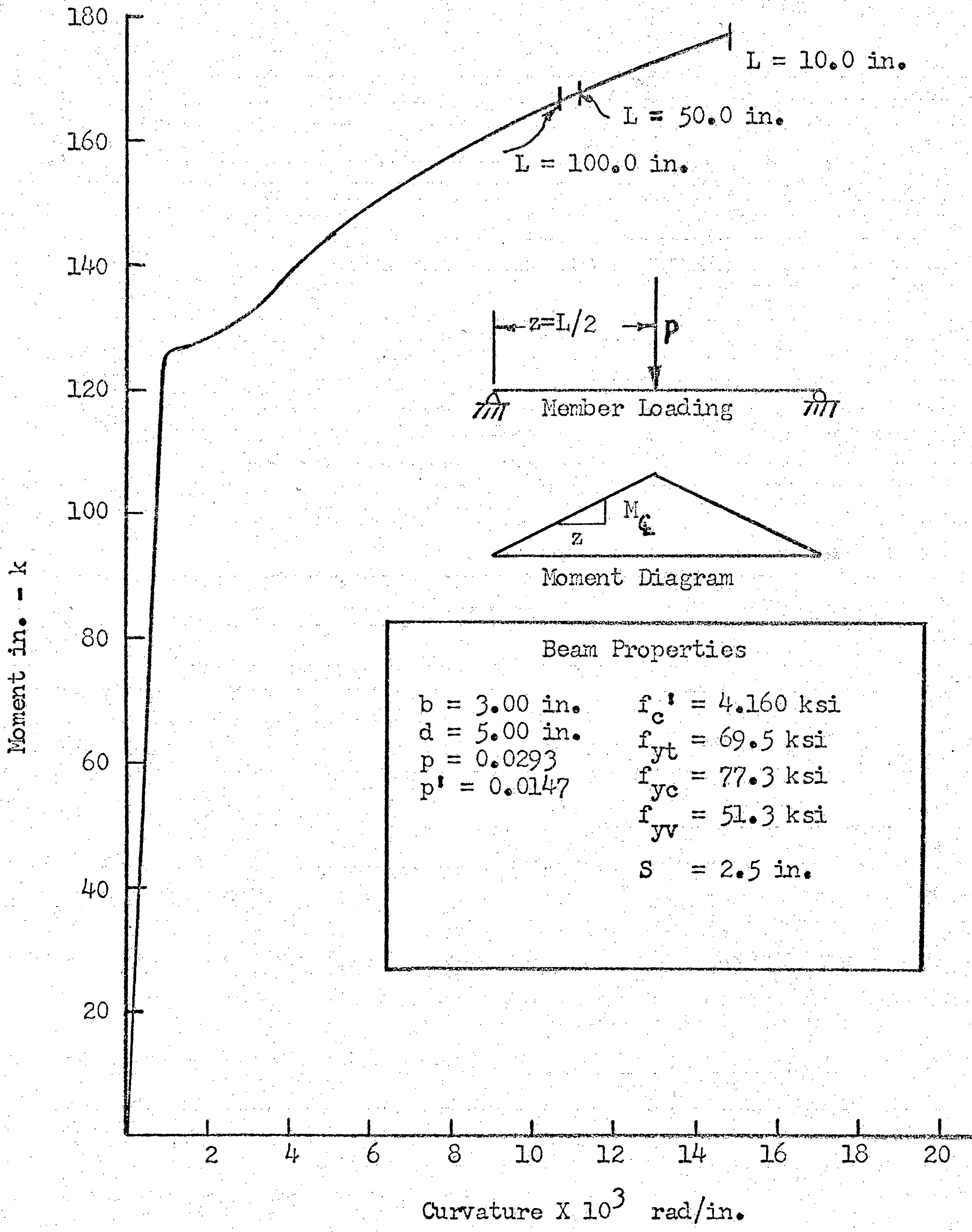


Figure 23. Effect of Span Length on Moment-Curvature Curve

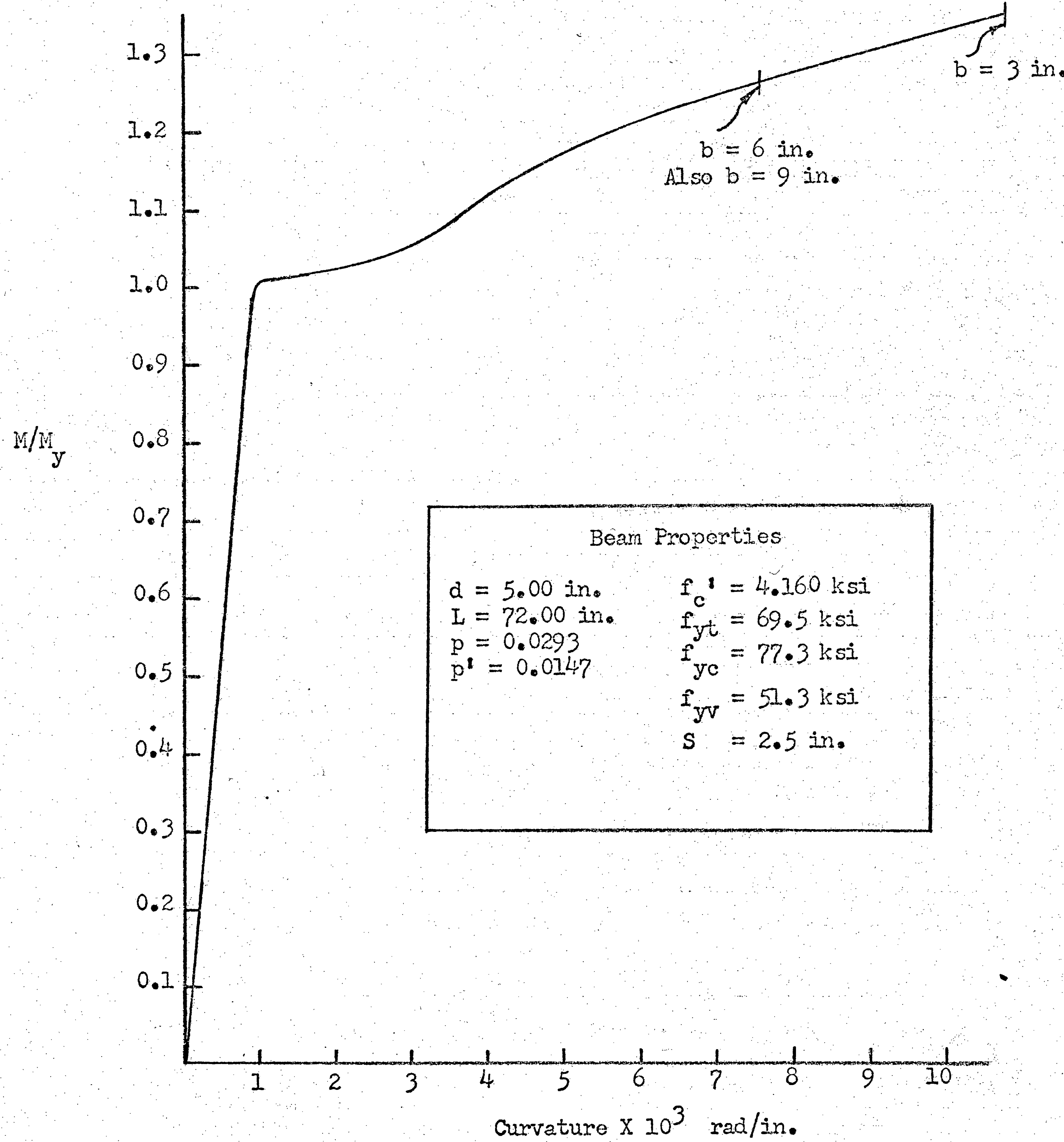


Figure 24. Effect of Beam Width on Moment-Curvature Curve

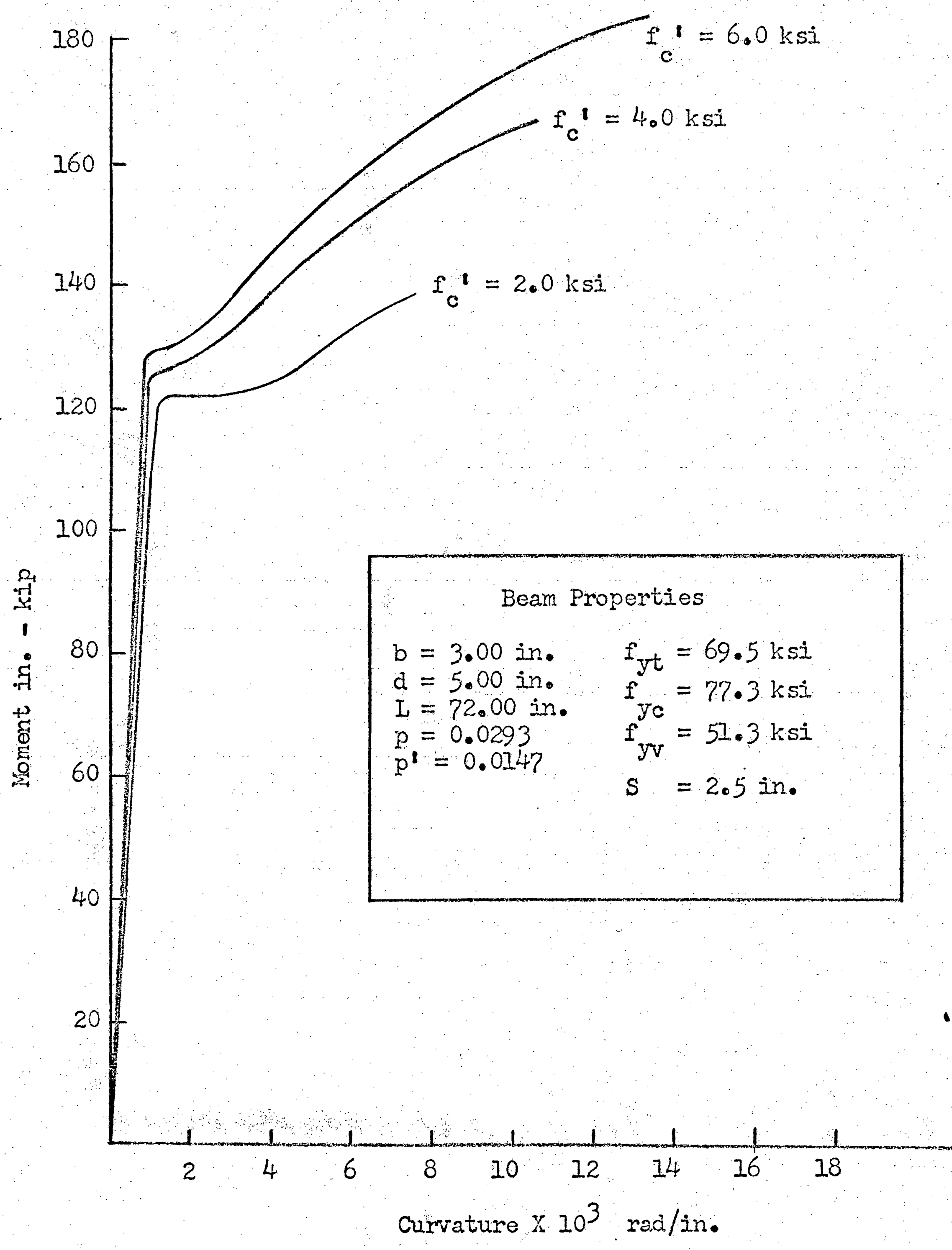


Figure 25. Effect of Concrete Compressive Strength on Moment-Curvature Curve

VI. CONCLUSIONS

1. The model presented here gives an accurate prediction of the flexural behavior of reinforced concrete if the concrete ultimate strain and the steel strain-hardening function are known.
2. As tension steel yield stress is increased, moment capacity is increased, but rotational capacity decreases.
3. Reinforcement strain hardening must be considered in order to predict flexural behavior accurately.
4. Closed rectangular stirrups significantly increase the rotational capacity of reinforced concrete beams.
5. Sufficient compression steel to develop fully the tension steel ductility is a necessity in limit design. Compression steel percentage may be the most important parameter in developing adequate rotational and moment capacity.
6. Span length has very little effect on the moment-curvature curve for a given section.
7. Beam width has negligible influence on flexural behavior.
8. Concrete compressive strengths of 4.0 ksi or greater are desirable for limit design.
9. Further research should include a statistical study of the parameters affecting ultimate concrete strain; also there should be a study of spread of plasticity and the formulation of a method of predicting hinge length in reinforced concrete beams.

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Appendix A

Tables Describing Flexural Behavior of Ten Beams

TABLE 1. BEAM J6

MOMENT-CURVATURE RELATIONSHIP FOR A R-C BEAM

BEAM DIMENSIONS	
BEAM WIDTH=	3.0000
CONFINED WIDTH=	2.0000
EFFECTIVE DEPTH=	5.0000
COMP STEEL DEPTH=	0.8125
DEPTH TO CONFINED CONCRETE=	0.5000
AREA OF TENSION STEEL=	0.4400
AREA OF COMPRESSION STEEL=	0.2200
SPAN LENGTH=	72.0000
TENSION STEEL PROPERTIES	
STEEL MODULUS=	29000.0000
YIELD STRESS=	69.5000
YIELD STRAIN=	0.0024
STRAIN HARDENING AT	0.0100
ULTIMATE STRAIN=	0.1500
COMPRESSION STEEL PROPERTIES	
STEEL MODULUS=	29000.0000
YIELD STRESS=	77.3000
YIELD STRAIN=	0.0027
STRAIN HARDENING AT	0.0050
ULTIMATE STRAIN=	0.1500
UNCONFINED CONCRETE PROPERTIES	
COMPRESSIVE STRENGTH=	3.5360
STRAIN AT BREAK POINT 1=	0.0012
ULT STRAIN UNCONFINED=	0.0211
CONFINED CONCRETE PROPERTIES	
CONFINED COMP STRENGTH=	4.0222
STRAIN AT BREAK POINT 1=	0.0012
STRAIN AT BREAK POINT 2=	0.0211
ULT STRAIN CONFINED=	0.0633

YIELD MOMENT= 125.12634

YIELD CURVATURE= 0.00094

TABLE 2. BEAM J6--FLEXURAL CALCULATIONS

CONC STRAIN IN./IN.	CURVATURE RAD/IN.	MOMENT IN-K	LOAD KIP	DEFLECTION IN.	TENSION STEEL STRESS KSI	COMPRESSION STEEL STRESS KSI	DEPTH TO NEUTRAL AXIS IN.
0.0010	0.00043	62.64	3.48	0.19	33.92	18.78	2.305
0.0020	0.00083	113.31	6.29	0.35	62.32	38.45	2.410
0.0030	0.00143	127.14	7.06	0.43	69.50	53.24	2.094
0.0040	0.00227	128.79	7.16	0.46	69.50	62.47	1.761
0.0050	0.00315	132.21	7.35	0.52	70.84	70.67	1.585
0.0060	0.00384	138.79	7.71	0.67	74.68	77.30	1.561
0.0070	0.00432	141.93	7.88	0.76	76.65	77.30	1.620
0.0080	0.00477	144.82	8.05	0.86	78.26	77.30	1.679
0.0090	0.00522	147.11	8.17	0.92	79.81	77.30	1.726
0.0100	0.00568	149.54	8.31	1.02	81.33	78.05	1.761
0.0110	0.00617	151.83	8.43	1.09	82.87	79.20	1.784
0.0120	0.00664	154.02	8.56	1.18	84.25	80.32	1.808
0.0130	0.00710	156.15	8.67	1.27	85.49	81.42	1.831
0.0140	0.00757	157.97	8.78	1.35	86.71	82.46	1.849
0.0150	0.00804	159.74	8.87	1.44	87.83	83.49	1.866
0.0160	0.00849	161.46	8.97	1.52	88.85	84.48	1.884
0.0170	0.00897	162.89	9.05	1.59	89.88	85.42	1.895
0.0180	0.00941	164.53	9.14	1.68	90.74	86.37	1.913
0.0190	0.00987	165.89	9.22	1.75	91.63	87.27	1.925
0.0200	0.01033	167.21	9.29	1.83	92.46	88.14	1.936
0.0210	0.01078	168.51	9.36	1.90	93.23	88.99	1.948
0.0220	0.01100	167.17	9.29	2.14	93.28	90.05	2.001
0.0230	0.01119	165.50	9.19	2.16	93.08	91.09	2.055
0.0240	0.01139	163.93	9.11	2.17	92.84	92.09	2.107
0.0250	0.01159	162.44	9.02	2.19	92.93	93.05	2.157
0.0260	0.01179	161.06	8.95	2.21	92.56	93.97	2.205
0.0270	0.01199	159.74	8.87	2.23	92.36	94.86	2.252

TABLE 3. BEAM K11

MOMENT-CURVATURE RELATIONSHIP FOR A R-C BEAM

BEAM DIMENSIONS	
BEAM WIDTH=	12.0000
CONFINED WIDTH=	11.1250
EFFECTIVE DEPTH=	10.0000
COMP STEEL DEPTH=	0.8125
DEPTH TO CONFINED CONCRETE=	0.4375
AREA OF TENSION STEEL=	2.2000
AREA OF COMPRESSION STEEL=	0.2200
SPAN LENGTH=	72.0000
TENSION STEEL PROPERTIES	
STEEL MODULUS=	29000.0000
YIELD STRESS=	69.2000
YIELD STRAIN=	0.0024
STRAIN HARDENING AT	0.0100
ULTIMATE STRAIN=	0.1500
COMPRESSION STEEL PROPERTIES	
STEEL MODULUS=	29000.0000
YIELD STRESS=	71.5000
YIELD STRAIN=	0.0025
STRAIN HARDENING AT	0.0050
ULTIMATE STRAIN=	0.1500
UNCONFINED CONCRETE PROPERTIES	
COMPRESSIVE STRENGTH=	3.3022
STRAIN AT BREAK POINT 1=	0.0012
ULT STRAIN UNCONFINED=	0.0120
CONFINED CONCRETE PROPERTIES	
CONFINED COMP STRENGTH=	3.7563
STRAIN AT BREAK POINT 1=	0.0012
STRAIN AT BREAK POINT 2=	0.0120
ULT STRAIN CONFINED=	0.0361
YIELD MOMENT=	1268.56999
YIELD CURVATURE=	0.00044

TABLE 4. BEAM K11--FLEXURAL CALCULATIONS

CONC STRAIN IN./IN.	CURVATURE RAD/IN.	MOMENT IN-K	LOAD KIP	DEFLECTION IN.	TENSION STEEL STRESS KSI	COMPRESSION STEEL STRESS KSI	DEPTH TO NEUTRAL AXIS IN.
0.0010	0.00023	718.93	39.94	0.10	38.12	23.55	4.320
0.0020	0.00044	1262.32	70.13	0.18	68.80	47.70	4.574
0.0030	0.00077	1289.18	71.62	0.19	69.20	68.87	3.898
0.0040	0.00110	1292.26	71.79	0.20	69.20	71.50	3.652
0.0050	0.00142	1293.02	71.83	0.21	69.20	71.50	3.527
0.0060	0.00170	1318.26	73.24	0.22	70.84	71.50	3.538
0.0070	0.00194	1353.62	75.20	0.27	73.17	72.26	3.605
0.0080	0.00218	1385.27	76.96	0.32	75.23	73.66	3.667
0.0090	0.00242	1412.02	78.45	0.38	77.09	75.00	3.722
0.0100	0.00265	1436.79	79.82	0.41	78.73	76.28	3.777
0.0110	0.00287	1459.00	81.06	0.46	80.23	77.51	3.828
0.0120	0.00310	1479.00	82.17	0.49	81.61	78.69	3.874
0.0130	0.00319	1419.32	78.85	0.56	80.43	79.96	4.071
0.0140	0.00330	1417.33	78.74	0.57	81.63	81.16	4.243
0.0150	0.00350	1430.56	79.48	0.55	82.69	82.21	4.286
0.0160	0.00370	1442.43	80.13	0.57	83.66	83.21	4.329
0.0170	0.00389	1453.12	80.73	0.61	84.55	84.18	4.372
0.0180	0.00408	1462.80	81.27	0.64	85.37	85.12	4.415
0.0190	0.00427	1470.63	81.70	0.67	86.16	86.02	4.454
0.0200	0.00445	1478.60	82.14	0.69	86.86	86.88	4.497
0.0210	0.00463	1484.92	82.50	0.72	87.53	87.72	4.536
0.0220	0.00481	1491.05	82.84	0.74	88.15	88.53	4.577
0.0230	0.00498	1496.12	83.12	0.76	88.74	89.30	4.616
0.0240	0.00516	1500.64	83.37	0.79	89.28	90.05	4.655
0.0250	0.00533	1504.64	83.59	0.82	89.79	90.78	4.694
0.0260	0.00549	1508.16	83.79	0.84	90.27	91.48	4.733
0.0270	0.00566	1511.23	83.96	0.86	90.71	92.15	4.772

TABLE 5. BEAM K12

MOMENT-CURVATURE RELATIONSHIP FOR A R-C BEAM

BEAM DIMENSIONS	
BEAM WIDTH=	12.0000
CONFINED WIDTH=	11.0000
EFFECTIVE DEPTH=	10.0000
COMP STEEL DEPTH=	0.8125
DEPTH TO CONFINED CONCRETE=	0.5000
AREA OF TENSION STEEL=	2.2000
AREA OF COMPRESSION STEEL=	0.2200
SPAN LENGTH=	144.0000
TENSION STEEL PROPERTIES	
STEEL MODULUS=	29000.0000
YIELD STRESS=	69.5000
YIELD STRAIN=	0.0024
STRAIN HARDENING AT	0.0100
ULTIMATE STRAIN=	0.1500
COMPRESSION STEEL PROPERTIES	
STEEL MODULUS=	29000.0000
YIELD STRESS=	71.5000
YIELD STRAIN=	0.0025
STRAIN HARDENING AT	0.0050
ULTIMATE STRAIN=	0.1500
UNCONFINED CONCRETE PROPERTIES	
COMPRESSIVE STRENGTH=	3.1365
STRAIN AT BREAK POINT 1=	0.0012
ULT STRAIN UNCONFINED=	0.0065
CONFINED CONCRETE PROPERTIES	
CONFINED COMP STRENGTH=	3.3717
STRAIN AT BREAK POINT 1=	0.0012
STRAIN AT BREAK POINT 2=	0.0065
ULT STRAIN CONFINED=	0.0196
YIELD MOMENT=	1256.66570
YIELD CURVATURE=	0.00046

TABLE 6. BEAM K12--FLEXURAL CALCULATIONS

CONC STRAIN IN./IN.	CURVATURE RAD/IN.	MOMENT IN-K	LOAD KIP	DEFLECTION IN.	TENSION STEEL STRESS KSI	COMPRESSION STEEL STRESS KSI	DEPTH TO NEUTRAL AXIS IN.
0.0010	0.00022	674.98	18.75	0.39	35.95	23.72	4.465
0.0020	0.00042	1181.72	32.83	0.71	64.71	48.03	4.727
0.0030	0.00070	1272.66	35.35	0.81	69.50	70.59	4.309
0.0040	0.00099	1275.89	35.44	0.83	69.50	71.50	4.050
0.0050	0.00128	1276.29	35.45	0.84	69.50	71.50	3.910
0.0060	0.00157	1277.01	35.47	0.85	69.50	71.50	3.824
0.0070	0.00172	1248.09	34.67	1.03	69.88	72.58	4.066
0.0080	0.00187	1232.71	34.24	1.06	70.74	74.08	4.275
0.0090	0.00208	1259.17	34.98	1.11	72.53	75.44	4.326
0.0100	0.00229	1281.35	35.59	1.17	74.15	76.73	4.376
0.0110	0.00248	1301.23	36.15	1.24	75.57	77.98	4.431
0.0120	0.00268	1317.77	36.60	1.30	76.88	79.17	4.482
0.0130	0.00287	1332.78	37.02	1.34	78.06	80.31	4.535
0.0140	0.00305	1345.74	37.38	1.40	79.14	81.41	4.585
0.0150	0.00324	1357.35	37.70	1.55	80.13	82.47	4.636
0.0160	0.00341	1367.79	37.99	1.61	81.04	83.48	4.687
0.0170	0.00359	1377.22	38.26	1.76	81.86	84.46	4.738
0.0180	0.00376	1384.96	38.47	1.81	82.66	85.39	4.785
0.0190	0.00393	1391.91	38.66	1.88	83.39	86.30	4.831
0.0200	0.00410	1398.15	38.84	2.01	84.06	87.16	4.878
0.0210	0.00426	1403.74	38.99	2.05	84.68	88.00	4.925
0.0220	0.00444	1412.42	39.23	2.14	85.37	88.80	4.958

TABLE 7. BEAM M8

MOMENT-CURVATURE RELATIONSHIP FOR A R-C BEAM

BEAM DIMENSIONS	
BEAM WIDTH=	12.0000
CONFINED WIDTH=	11.1250
EFFECTIVE DEPTH=	24.0000
COMP STEEL DEPTH=	0.8125
DEPTH TO CONFINED CONCRETE=	0.4375
AREA OF TENSION STEEL=	5.4000
AREA OF COMPRESSION STEEL=	0.2200
SPAN LENGTH=	240.0000
TENSION STEEL PROPERTIES	
STEEL MODULUS=	29000.0000
YIELD STRESS=	63.9000
YIELD STRAIN=	0.0022
STRAIN HARDENING AT	0.0100
ULTIMATE STRAIN=	0.1500
COMPRESSION STEEL PROPERTIES	
STEEL MODULUS=	29000.0000
YIELD STRESS=	70.6000
YIELD STRAIN=	0.0024
STRAIN HARDENING AT	0.0050
ULTIMATE STRAIN=	0.1500
UNCONFINED CONCRETE PROPERTIES	
COMPRESSIVE STRENGTH=	3.7570
STRAIN AT BREAK POINT 1=	0.0012
ULT STRAIN UNCONFINED=	0.0051
CONFINED CONCRETE PROPERTIES	
CONFINED COMP STRENGTH=	3.7570
STRAIN AT BREAK POINT 1=	0.0012
STRAIN AT BREAK POINT 2=	0.0051
ULT STRAIN CONFINED=	0.0154
YIELD MOMENT=	6940.15198
YIELD CURVATURE=	0.00017

TABLE 8. BEAM M8--FLEXURAL CALCULATIONS

CONC STRAIN IN./IN.	CURVATURE RAD/IN.	MOMENT IN-K	LOAD KIP	DEFLECTION IN.	TENSION STEEL STRESS KSI	COMPRESSION STEEL STRESS KSI	DEPTH TO NEUTRAL AXIS IN.
0.0010	0.00010	4169.44	69.49	0.46	37.49	26.75	10.467
0.0020	0.00019	6964.70	116.08	0.84	63.90	53.54	10.561
0.0030	0.00033	7040.55	117.34	0.86	63.90	70.60	9.138
0.0040	0.00047	7052.18	117.54	0.88	63.90	70.60	8.599
0.0050	0.00060	7055.79	117.60	0.89	63.90	70.60	8.304
0.0060	0.00069	7029.84	117.16	1.03	64.92	71.38	8.656
0.0070	0.00080	7267.75	121.13	1.13	67.32	72.93	8.731
0.0080	0.00091	7464.93	124.42	1.39	69.36	74.41	8.834
0.0090	0.00101	7627.24	127.12	1.63	71.15	75.82	8.944
0.0100	0.00110	7766.32	129.44	1.84	72.71	77.17	9.061
0.0110	0.00120	7884.59	131.41	2.02	74.11	78.45	9.178
0.0120	0.00129	7986.80	133.11	2.21	75.37	79.67	9.295
0.0130	0.00138	8077.82	134.63	2.38	76.49	80.83	9.415
0.0140	0.00147	8155.24	135.92	2.53	77.52	81.94	9.530
0.0150	0.00156	8223.95	137.07	2.68	78.46	83.01	9.645
0.0160	0.00164	8283.69	138.06	2.78	79.32	84.03	9.757
0.0170	0.00173	8359.54	139.33	2.95	80.21	85.00	9.837

TABLE 9. BEAM N8

MOMENT-CURVATURE RELATIONSHIP FOR A R-C BEAM

BEAM DIMENSIONS	
BEAM WIDTH=	12.0000
CONFINED WIDTH=	11.2500
EFFECTIVE DEPTH=	30.0000
COMP STEEL DEPTH=	0.8125
DEPTH TO CONFINED CONCRETE=	0.3750
AREA OF TENSION STEEL=	6.3200
AREA OF COMPRESSION STEEL=	0.2200
SPAN LENGTH=	330.0000
TENSION STEEL PROPERTIES	
STEEL MODULUS=	29000.0000
YIELD STRESS=	61.9000
YIELD STRAIN=	0.0021
STRAIN HARDENING AT	0.0100
ULTIMATE STRAIN=	0.1500
COMPRESSION STEEL PROPERTIES	
STEEL MODULUS=	29000.0000
YIELD STRESS=	69.7000
YIELD STRAIN=	0.0024
STRAIN HARDENING AT	0.0050
ULTIMATE STRAIN=	0.1500
UNCONFINED CONCRETE PROPERTIES	
COMPRESSIVE STRENGTH=	3.6210
STRAIN AT BREAK POINT 1=	0.0012
ULT STRAIN UNCONFINED=	0.0046
CONFINED CONCRETE PROPERTIES	
CONFINED COMP STRENGTH=	3.6210
STRAIN AT BREAK POINT 1=	0.0012
STRAIN AT BREAK POINT 2=	0.0046
ULT STRAIN CONFINED=	0.0138

YIELD MOMENT= 9900.86975

YIELD CURVATURE= 0.00013

TABLE 10. BEAM N8--FLEXURAL CALCULATIONS

CONC STRAIN IN./IN.	CURVATURE RAD/IN.	MOMENT IN-K	LOAD KIP	DEFLECTION IN.	TENSION STEEL STRESS KSI	COMPRESSION STEEL STRESS KSI	DEPTH TO NEUTRAL AXIS IN.
0.0010	0.00008	6202.75	75.18	0.70	38.09	27.18	12.967
0.0020	0.00016	9969.38	120.84	1.29	61.90	54.22	12.469
0.0030	0.00028	10062.31	121.97	1.32	61.90	69.70	10.810
0.0040	0.00039	10081.72	122.20	1.34	61.90	69.70	10.178
0.0050	0.00049	9962.90	120.76	1.46	61.90	69.70	10.201
0.0060	0.00059	10353.24	125.49	1.66	64.39	70.61	10.195
0.0070	0.00068	10699.92	129.70	2.10	66.75	72.17	10.298
0.0080	0.00077	10984.93	133.15	2.49	68.76	73.64	10.427
0.0090	0.00085	11221.46	136.02	2.84	70.52	75.04	10.564
0.0100	0.00093	11423.33	138.46	3.18	72.05	76.38	10.708
0.0110	0.00101	11595.66	140.55	3.47	73.42	77.65	10.851
0.0120	0.00109	11745.18	142.37	3.75	74.64	78.86	10.995
0.0130	0.00117	11874.04	143.93	4.00	75.75	80.01	11.136
0.0140	0.00124	11988.02	145.31	4.24	76.75	81.11	11.276
0.0150	0.00132	12119.09	146.90	4.51	77.76	82.16	11.376

TABLE 11. BEAM C2

MOMENT-CURVATURE RELATIONSHIP FOR A R-C BEAM

BEAM DIMENSIONS	
BEAM WIDTH=	6.0000
CONFINED WIDTH=	5.0000
EFFECTIVE DEPTH=	10.0000
COMP STEEL DEPTH=	0.8125
DEPTH TO CONFINED CONCRETE=	0.5000
AREA OF TENSION STEEL=	0.8800
AREA OF COMPRESSION STEEL=	0.2200
SPAN LENGTH=	110.0000
TENSION STEEL PROPERTIES	
STEEL MODULUS=	28300.0000
YIELD STRESS=	47.7000
YIELD STRAIN=	0.0017
STRAIN HARDENING AT	0.0105
ULTIMATE STRAIN=	0.1500
COMPRESSION STEEL PROPERTIES	
STEEL MODULUS=	28300.0000
YIELD STRESS=	49.5000
YIELD STRAIN=	0.0017
STRAIN HARDENING AT	0.0105
ULTIMATE STRAIN=	0.1500
UNCONFINED CONCRETE PROPERTIES	
COMPRESSIVE STRENGTH=	3.2045
STRAIN AT BREAK POINT 1=	0.0012
ULT STRAIN UNCONFINED=	0.0063
CONFINED CONCRETE PROPERTIES	
CONFINED COMP STRENGTH=	3.4448
STRAIN AT BREAK POINT 1=	0.0012
STRAIN AT BREAK POINT 2=	0.0063
ULT STRAIN CONFINED=	0.0188
YIELD MOMENT=	366.10812
YIELD CURVATURE=	0.00028

TABLE 12. BEAM C2--FLEXURAL CALCULATIONS

CONC STRAIN IN./IN.	CURVATURE RAD/IN.	MOMENT IN-K	LOAD KIP	DEFLECTION IN.	TENSION STEEL STRESS KSI	COMPRESSION STEEL STRESS KSI	DEPTH TO NEUTRAL AXIS IN.
0.0010	0.00025	336.19	12.23	0.25	43.78	23.09	3.984
0.0020	0.00084	383.04	13.93	0.39	47.70	38.29	2.391
0.0030	0.00148	397.37	14.45	0.48	49.26	49.50	2.022
0.0040	0.00193	422.65	15.37	0.73	52.75	49.50	2.077
0.0050	0.00234	442.34	16.09	0.95	55.41	49.50	2.139
0.0060	0.00273	457.57	16.64	1.13	57.59	49.50	2.194
0.0070	0.00287	450.74	16.39	1.20	57.85	49.50	2.436
0.0080	0.00300	441.88	16.07	1.22	58.00	49.50	2.670
0.0090	0.00330	450.37	16.38	1.21	59.31	49.50	2.725
0.0100	0.00361	457.16	16.62	1.32	60.49	49.50	2.772
0.0110	0.00390	463.47	16.85	1.43	61.52	49.50	2.819
0.0120	0.00419	469.36	17.07	1.53	62.43	49.50	2.866
0.0130	0.00448	473.88	17.23	1.62	63.29	49.50	2.905
0.0140	0.00476	478.11	17.39	1.71	64.05	49.50	2.944
0.0150	0.00503	483.15	17.57	1.81	64.74	50.02	2.983
0.0160	0.00532	486.75	17.70	1.92	65.45	50.93	3.006
0.0170	0.00560	491.16	17.86	2.02	66.06	51.83	3.038
0.0180	0.00588	494.31	17.97	2.15	66.66	52.67	3.061
0.0190	0.00614	498.28	18.12	2.25	67.16	53.50	3.092
0.0200	0.00642	501.05	18.22	2.33	67.67	54.28	3.116
0.0210	0.00669	503.66	18.31	2.46	68.14	55.03	3.139
0.0220	0.00696	506.10	18.40	2.54	68.58	55.76	3.163
0.0230	0.00724	509.19	18.52	2.63	69.02	56.44	3.178

TABLE 13. BEAM BF1

MOMENT-CURVATURE RELATIONSHIP FOR A R-C BEAM

BEAM DIMENSIONS	
BEAM WIDTH=	4.0000
CONFINED WIDTH=	0.0000
EFFECTIVE DEPTH=	6.0000
COMP STEEL DEPTH=	0.8125
DEPTH TO CONFINED CONCRETE=	0.5000
AREA OF TENSION STEEL=	0.4000
AREA OF COMPRESSION STEEL=	0.0000
SPAN LENGTH=	50.0000
TENSION STEEL PROPERTIES	
STEEL MODULUS=	29000.0000
YIELD STRESS=	62.0000
YIELD STRAIN=	0.0021
STRAIN HARDENING AT	0.0100
ULTIMATE STRAIN=	0.1500
COMPRESSION STEEL PROPERTIES	
STEEL MODULUS=	29000.0000
YIELD STRESS=	62.0000
YIELD STRAIN=	0.0021
STRAIN HARDENING AT	0.0100
ULTIMATE STRAIN=	0.1500
UNCONFINED CONCRETE PROPERTIES	
COMPRESSIVE STRENGTH=	5.5250
STRAIN AT BREAK POINT 1=	0.0012
ULT STRAIN UNCONFINED=	0.0030
CONFINED CONCRETE PROPERTIES	
CONFINED COMP STRENGTH=	5.5250
STRAIN AT BREAK POINT 1=	0.0012
STRAIN AT BREAK POINT 2=	0.0030
ULT STRAIN CONFINED=	0.0090
YIELD MOMENT=	130.77024
YIELD CURVATURE=	0.00056

TABLE 14. BEAM BF1--FLEXURAL CALCULATIONS

CONC STRAIN IN./IN.	CURVATURE RAD/IN.	MOMENT IN-K	LOAD KIP	DEFLECTION IN.	TENSION STEEL STRESS KSI	COMPRESSION STEEL STRESS KSI	DEPTH TO NEUTRAL AXIS IN.
0.0010	0.00046	106.24	8.50	0.10	50.49	18.24	2.189
0.0020	0.00125	133.69	10.69	0.16	62.00	28.52	1.598
0.0030	0.00214	148.97	11.92	0.23	62.00	36.66	1.404
0.0040	0.00228	116.86	9.35	0.26	58.07	62.00	1.754
0.0050	0.00243	102.08	8.17	0.26	54.62	62.00	2.060
0.0060	0.00258	89.76	7.18	0.26	51.56	62.00	2.329
0.0070	0.00273	79.45	6.36	0.27	48.59	62.00	2.568
0.0080	0.00288	70.71	5.66	0.27	46.20	62.00	2.779
0.0090	0.00303	63.28	5.06	0.28	43.78	62.00	2.969
0.0100	0.00319	56.91	4.55	0.29	41.55	62.00	3.139
0.0110	0.00334	51.42	4.11	0.29	39.66	62.00	3.292

TABLE 15. BEAM BF2

MOMENT-CURVATURE RELATIONSHIP FOR A R-C BEAM

BEAM DIMENSIONS	
BEAM WIDTH=	4.0000
CONFINED WIDTH=	3.0000
EFFECTIVE DEPTH=	6.0000
COMP STEEL DEPTH=	0.8125
DEPTH TO CONFINED CONCRETE=	0.5000
AREA OF TENSION STEEL=	0.4000
AREA OF COMPRESSION STEEL=	0.0000
SPAN LENGTH=	50.0000
TENSION STEEL PROPERTIES	
STEEL MODULUS=	29000.0000
YIELD STRESS=	62.0000
YIELD STRAIN=	0.0021
STRAIN HARDENING AT	0.0100
ULTIMATE STRAIN=	0.1500
COMPRESSION STEEL PROPERTIES	
STEEL MODULUS=	29000.0000
YIELD STRESS=	62.0000
YIELD STRAIN=	0.0021
STRAIN HARDENING AT	0.0100
ULTIMATE STRAIN=	0.1500
UNCONFINED CONCRETE PROPERTIES	
COMPRESSIVE STRENGTH=	5.5250
STRAIN AT BREAK POINT 1=	0.0012
ULT STRAIN UNCONFINED=	0.0101
CONFINED CONCRETE PROPERTIES	
CONFINED COMP STRENGTH=	6.4228
STRAIN AT BREAK POINT 1=	0.0012
STRAIN AT BREAK POINT 2=	0.0101
ULT STRAIN CONFINED=	0.0303
YIELD MOMENT=	130.77969
YIELD CURVATURE=	0.00055

TABLE 16. BEAM BF2--FLEXURAL CALCULATIONS

CONC STRAIN IN./IN.	CURVATURE RAD/IN.	MOMENT IN-K	LOAD KIP	DEFLECTION IN.	TENSION STEEL STRESS KSI	COMPRESSION STEEL STRESS KSI	DEPTH TO NEUTRAL AXIS IN.
0.0010	0.00047	110.83	8.87	0.10	52.58	17.95	2.133
0.0020	0.00133	134.55	10.76	0.16	62.00	26.73	1.507
0.0030	0.00225	136.40	10.91	0.18	62.80	33.93	1.332
0.0040	0.00297	146.06	11.68	0.25	67.42	45.98	1.346
0.0050	0.00366	153.17	12.25	0.32	71.05	58.82	1.367
0.0060	0.00432	158.68	12.69	0.38	74.03	62.00	1.388
0.0070	0.00497	163.29	13.06	0.45	76.51	62.00	1.409
0.0080	0.00559	167.33	13.39	0.50	78.62	62.00	1.430
0.0090	0.00620	170.99	13.68	0.57	80.42	62.00	1.451
0.0100	0.00682	173.63	13.89	0.61	82.08	62.00	1.466
0.0110	0.00699	169.47	13.56	0.74	82.08	62.00	1.574
0.0120	0.00715	163.67	13.09	0.75	80.96	62.00	1.678
0.0130	0.00731	158.23	12.66	0.75	80.02	62.00	1.778
0.0140	0.00748	154.95	12.40	0.76	80.06	62.00	1.872
0.0150	0.00778	158.36	12.67	0.70	82.50	62.00	1.928
0.0160	0.00818	159.25	12.74	0.73	83.26	62.00	1.956
0.0170	0.00857	160.20	12.82	0.76	83.95	62.06	1.985
0.0180	0.00897	160.58	12.85	0.79	84.67	63.10	2.006
0.0190	0.00934	161.59	12.93	0.83	85.23	64.14	2.034
0.0200	0.00973	162.03	12.96	0.86	85.84	65.11	2.055
0.0210	0.01012	162.49	13.00	0.89	86.40	66.05	2.076
0.0220	0.01049	162.96	13.04	0.92	86.91	66.97	2.097
0.0230	0.01086	163.43	13.07	0.95	87.39	67.85	2.118
0.0240	0.01122	163.91	13.11	0.98	87.83	68.72	2.139
0.0250	0.01161	163.84	13.11	1.00	88.32	69.52	2.153
0.0260	0.01196	164.31	13.14	1.03	88.70	70.33	2.174
0.0270	0.01230	164.77	13.18	1.06	89.05	71.12	2.195

TABLE 17. BEAM AP1

MOMENT-CURVATURE RELATIONSHIP FOR A R-C BEAM

BEAM DIMENSIONS	
BEAM WIDTH=	4.1875
CONFINED WIDTH=	2.4375
EFFECTIVE DEPTH=	6.1875
COMP STEEL DEPTH=	1.0000
DEPTH TO CONFINED CONCRETE=	0.6250
AREA OF TENSION STEEL=	0.4000
AREA OF COMPRESSION STEEL=	0.4000
SPAN LENGTH=	48.0000
TENSION STEEL PROPERTIES	
STEEL MODULUS=	29000.0000
YIELD STRESS=	45.0000
YIELD STRAIN=	0.0016
STRAIN HARDENING AT	0.0150
ULTIMATE STRAIN=	0.1500
COMPRESSION STEEL PROPERTIES	
STEEL MODULUS=	29000.0000
YIELD STRESS=	45.0000
YIELD STRAIN=	0.0016
STRAIN HARDENING AT	0.0150
ULTIMATE STRAIN=	0.1500
UNCONFINED CONCRETE PROPERTIES	
COMPRESSIVE STRENGTH=	2.8755
STRAIN AT BREAK POINT 1=	0.0012
ULT STRAIN UNCONFINED=	0.0176
CONFINED CONCRETE PROPERTIES	
CONFINED COMP STRENGTH=	3.2350
STRAIN AT BREAK POINT 1=	0.0012
STRAIN AT BREAK POINT 2=	0.0176
ULT STRAIN CONFINED=	0.0529

YIELD MOMENT= 96.02653

YIELD CURVATURE= 0.00040

TABLE 18. BEAM AP1--FLEXURAL CALCULATIONS

CONC STRAIN IN./IN.	CURVATURE RAD/IN.	MOMENT IN-K	LOAD KIP	DEFLECTION IN.	TENSION STEEL STRESS KSI	COMPRESSION STEEL STRESS KSI	DEPTH TO NEUTRAL AXIS IN.
0.0010	0.00045	96.17	8.01	0.09	45.00	15.99	2.228
0.0020	0.00146	99.46	8.29	0.10	45.00	15.61	1.368
0.0030	0.00248	99.09	8.26	0.23	45.00	15.03	1.209
0.0040	0.00342	104.53	8.71	0.29	47.34	16.76	1.169
0.0050	0.00432	112.54	9.38	0.30	51.38	19.78	1.158
0.0060	0.00522	119.30	9.94	0.41	54.62	22.56	1.149
0.0070	0.00614	124.74	10.39	0.52	57.27	24.91	1.140
0.0080	0.00706	129.79	10.82	0.63	59.46	27.17	1.133
0.0090	0.00801	133.04	11.09	0.72	61.33	28.70	1.124
0.0100	0.00896	136.23	11.35	0.78	62.90	30.22	1.116
0.0110	0.00990	139.66	11.64	0.87	64.23	31.84	1.111
0.0120	0.01087	141.28	11.77	0.94	65.40	32.68	1.104
0.0130	0.01184	143.35	11.95	1.00	66.40	33.71	1.098
0.0140	0.01280	145.51	12.13	1.07	67.27	34.77	1.094
0.0150	0.01377	147.15	12.26	1.13	68.03	35.60	1.089
0.0160	0.01474	149.07	12.42	1.21	68.70	36.55	1.085
0.0170	0.01573	149.73	12.48	1.26	69.30	36.93	1.081
0.0180	0.01668	150.97	12.58	1.30	69.81	38.29	1.079
0.0190	0.01761	151.39	12.62	1.33	70.24	40.41	1.079
0.0200	0.01855	151.26	12.61	1.79	70.64	42.09	1.078
0.0210	0.01949	151.40	12.62	1.85	71.00	43.72	1.077
0.0220	0.02015	151.92	12.66	1.90	71.21	45.00	1.092
0.0230	0.02039	151.00	12.58	1.92	71.24	45.00	1.128
0.0240	0.02055	150.31	12.53	1.93	70.96	45.00	1.168
0.0250	0.02071	149.64	12.47	1.94	70.87	45.00	1.207
0.0260	0.02087	148.98	12.41	1.95	71.00	45.00	1.246
0.0270	0.02103	148.34	12.36	1.96	70.72	45.00	1.284

TABLE 19. BEAM R1

MOMENT-CURVATURE RELATIONSHIP FOR A R-C BEAM

BEAM DIMENSIONS	
BEAM WIDTH=	4.1875
CONFINED WIDTH=	2.4375
EFFECTIVE DEPTH=	6.1875
COMP STEEL DEPTH=	1.0000
DEPTH TO CONFINED CONCRETE=	0.6250
AREA OF TENSION STEEL=	0.4000
AREA OF COMPRESSION STEEL=	0.0000
SPAN LENGTH=	48.0000
TENSION STEEL PROPERTIES	
STEEL MODULUS=	29000.0000
YIELD STRESS=	45.0000
YIELD STRAIN=	0.0016
STRAIN HARDENING AT	0.0150
ULTIMATE STRAIN=	0.1500
COMPRESSION STEEL PROPERTIES	
STEEL MODULUS=	29000.0000
YIELD STRESS=	45.0000
YIELD STRAIN=	0.0016
STRAIN HARDENING AT	0.0150
ULTIMATE STRAIN=	0.1500
UNCONFINED CONCRETE PROPERTIES	
COMPRESSIVE STRENGTH=	2.8755
STRAIN AT BREAK POINT 1=	0.0012
ULT STRAIN UNCONFINED=	0.0084
CONFINED CONCRETE PROPERTIES	
CONFINED COMP STRENGTH=	3.2350
STRAIN AT BREAK POINT 1=	0.0012
STRAIN AT BREAK POINT 2=	0.0084
ULT STRAIN CONFINED=	0.0251
YIELD MOMENT=	94.67977
YIELD CURVATURE=	0.00045

TABLE 20. BEAM R1--FLEXURAL CALCULATIONS

CONC STRAIN IN./IN.	CURVATURE RAD/IN.	MOMENT IN-K	LOAD KIP	DEFLECTION IN.	TENSION STEEL STRESS KSI	COMPRESSION STEEL STRESS KSI	DEPTH TO NEUTRAL AXIS IN.
0.0010	0.00036	75.65	6.30	0.07	36.24	18.46	2.751
0.0020	0.00097	97.55	8.13	0.12	45.00	29.77	2.054
0.0030	0.00166	98.37	8.20	0.13	45.00	38.76	1.803
0.0040	0.00237	98.00	8.17	0.23	45.00	45.00	1.687
0.0050	0.00307	98.03	8.17	0.27	45.00	45.00	1.629
0.0060	0.00363	101.24	8.44	0.32	46.63	45.00	1.651
0.0070	0.00413	105.07	8.76	0.36	48.68	45.00	1.695
0.0080	0.00460	108.56	9.05	0.40	50.38	45.00	1.738
0.0090	0.00485	106.21	8.85	0.41	50.84	45.00	1.854
0.0100	0.00501	100.44	8.37	0.42	49.63	45.00	1.997
0.0110	0.00516	95.20	7.93	0.43	48.27	45.00	2.131
0.0120	0.00532	92.71	7.73	0.44	48.21	45.00	2.254
0.0130	0.00550	94.72	7.89	0.43	50.43	45.00	2.364
0.0140	0.00574	95.29	7.94	0.45	51.25	45.00	2.437
0.0150	0.00601	95.44	7.95	0.47	51.76	45.00	2.495
0.0160	0.00627	95.71	7.98	0.49	52.20	45.00	2.553
0.0170	0.00655	95.56	7.96	0.50	52.73	45.00	2.597
0.0180	0.00682	95.51	7.96	0.52	53.21	45.00	2.640
0.0190	0.00708	95.52	7.96	0.54	53.64	45.00	2.684
0.0200	0.00733	95.59	7.97	0.56	54.02	45.00	2.727
0.0210	0.00758	95.69	7.97	0.57	54.36	45.00	2.771
0.0220	0.00782	95.82	7.99	0.59	54.66	45.00	2.814
0.0230	0.00805	95.97	8.00	0.61	54.92	45.00	2.858
0.0240	0.00831	95.73	7.98	0.62	55.31	45.77	2.887
0.0250	0.00853	95.91	7.99	0.64	55.52	46.61	2.930
0.0260	0.00879	95.70	7.98	0.66	55.85	47.38	2.959
0.0270	0.00899	95.89	7.99	0.67	56.01	48.16	3.003

Appendix B

Computer Program - Model for Predicting Flexural Behavior

```
$JOB    9           826151019 CIVIL ENGR. SERVICE
$IBJOB              MAP,NODECK
$ID                 G A AUSTIN
$IBFTC MAIN
C      SYMBOLS DEFINED
C      ALC          CENTROID OF CONCRETE COMP. FORCE SYSTEM.
C      AS           CROSS-SECTIONAL AREA OF TENSILE STEEL.
C      ASPR         CROSS-SECTIONAL AREA OF COMPRESSION STEEL.
C      AV           CROSS-SECTIONAL AREA OF STIRRUP.
C      B            OVERALL WIDTH OF SECTION.
C      BCC          WIDTH OF CONFINED CONCRETE.
C      CC           FORCE IN CONCRETE.
C      CS           FORCE IN COMPRESSION STEEL.
C      CURV         CURVATURE AT CENTER SECTION.
C      CURVY        CURVATURE AT YIELD.
C      D            DEPTH FROM EXT. COMP. FIBER TO TENS.STEEL CEN.
C      DBV          DEPTH TO CENTROID OF STIRRUP AT BOTTOM.
C      DEFL         DEFLECTION UNDER LOAD.
C      DNA          DEPTH FROM EXT. COMP.FIBER TO NEUTRAL AXIS.
C      DPR          DEPTH FROM EXT.COMP.FIBER TO COMP.STEEL CEN.
C      DV           DEPTH TO CENTROID OF STIRRUP.
C      E0           CONCRETE STRAIN AT MAXIMUM CONCRETE STRESS.
C      E1           ULTIMATE CONCRETE STRAIN.
C      EC           CONCRETE STRAIN.
C      ES           STEEL STRAIN.
C      ESC          STRAIN IN COMPRESSION STEEL.
C      ESH          STRAIN AT WHICH STEEL ENTERS ST. HAR. REGION.
C      EST          STRAIN IN TENSION STEEL.
C      EY           YIELD STRAIN OF STEEL.
C      FC           CONCRETE STRESS.
C      FCPR         COMPRESSIVE STRENGTH OF CONCRETE.
C      FCPRC        CONFINED COMP. STRENGTH OF CONCRETE.
C      FS           STEEL STRESS.
C      FSC          STRESS IN COMPRESSION STEEL.
C      FST          STRESS IN TENSION STEEL.
C      FY           YIELD STRESS IN STEEL.
C      FYC          YIELD STRESS IN COMPRESSION STEEL.
C      FYT          YIELD STRESS IN TENSION STEEL.
C      FYV          YIELD STRESS OF STIRRUPS.
C      L            LENGTH OF MEMBER.
C      LOAD         CONCENTRATED LOAD AT CENTER OF SPAN.
C      M            BENDING MOMENT AT CENTER SECTION.
C      MX           BENDING MOMENT AT X DISTANCE FROM LEFT END.
C      PDP          BINDING RATIO.
C      PHI          CURVATURE AT INTERMEDIATE POINT.
C      S            STIRRUP SPACING.
C      STIRF        CONFINED CONC.STRESS INCREASE FACTOR.
C      SUMF         SUM OF FORCES.
C      TS           FORCE IN TENSION STEEL.
C      ULTSS        ULTIMATE STEEL STRAIN.
```

```
C      VBR       VOLUME OF BINDING REINFORCEMENT.
C      VCB       VOLUME OF CONCRETE BOUND.
C      X         DISTANCE FROM LEFT END.
C      Y         INTERMEDIATE COMPONENT OF DEFLECTION.
C      YIELDM    YIELD MOMENT.
C      YS        MODULUS OF ELASTICITY OF STEEL.
C      YZ        NEUTRAL AXIS INCREMENT.
C      Z         DISTANCE FROM MAX TO ZERO MOMENT.
C      1. INCREMENT EC BY .001.
C      2. ASSUME NA.
C      3. CALCULATE EST AND ESC.
C      4. CALCULATE FST AND FSC.
C      5. CALCULATE TS AND CS.
C      6. CALCULATE CC.
C      7. CHECK FORCES.
C      8. INCREMENT NA UP OR DOWN.
C      9. REPEAT UNTIL FORCES CHECK.
       REAL L,M,LOAD,MX
       DIMENSION M(40),LOAD(40),DEFL(40),Y(165),X(165)
       DIMENSION MX(40,165),CURV(40),TOPS(40),PHI(40,165)
       DIMENSION FSC(40),FST(40),FSTA(40),FSCA(40),ESC(40)
       DIMENSION EST(40),DNA1(40)
       COMMON    ECC,DNA,DTS,E0,E1,E2,B,BCC,FCPR,FCPRC,FCP,
      1STIRF,D1,D2,D4,D5,D7,D8,D81,BOUT,D9,D10,D17,EXTS,
      2EXTS1,C1,C2,C3,C4,C5,C11,C17,C21,C7,CURVY
       EQUIVALENCE(DV,DTS)
     1 READ(5,104) AS,ASPR,AV,D,DPR,DV,B,BCC,L,FCPR
   104 FORMAT(F5.2,F5.2,F5.2,F8.4,F8.4,F8.4,F8.4,F8.4,F6.1
      1,F6.3)
       READ(5,116) FYT,FYC,FYV,S,E0,ESHT,ESHC,YS,ULTSS
   116 FORMAT(F5.1,F5.1,F5.1,F6.2,F7.5,F7.5,F7.5,F8.1,F5.3)
       READ(5,138) DBV
   138 FORMAT(F8.4)
       FCP=FCPR
       STIRF=1.2-.025*S
       IF(STIRF.LT.1.0) STIRF=1.0
       I=0
       EC=0.0
       DNA=.4*D
       FCPR=FCPR*.85
       FCPRC=FCPR*STIRF
       EYT=FYT/YS
       EYC=FYC/YS
       IF(ASPR.EQ.0.0.AND.AV.EQ.0.0) GO TO 140
       Z=L/2.0
       VBR=ASPR*S+(DBV-DV+BCC)*AV*2.0
       VCB=(DBV-DV)*BCC*S
       PDP=VBR/VCB
       E1=.003+.02*B/Z+(PDP*FYV/20.0)**2
       GO TO 139
```

```
140 E1=.003
139 E2=E1*3.0
    WRITE(7,111) B,BCC,D,DPR,DTS,AS,ASPR,L
111 FORMAT(1H1,39HMOMENT-CURVATURE RELATIONSHIP FOR A R-C
   1,5H BEAM///,2X,30HBEAM DIMENSIONS                    /,
   24X,28HBEAM WIDTH=                  ,F8.4/,
   34X,28HCONFINED WIDTH=              ,F8.4/,
   44X,28HEFFECTIVE DEPTH=             ,F8.4/,
   54X,28HCOMP STEEL DEPTH=            ,F8.4/,
   64X,28HDEPTH TO CONFINED CONCRETE=  ,F8.4/,
   74X,28HAREA OF TENSION STEEL=       ,F8.4/,
   84X,28HAREA OF COMPRESSION STEEL=   ,F8.4/,
   94X,26HSPAN LENGTH=               ,F10.4//)
    WRITE(7,112) YS,FYT,EYT,ESHT,ULTSS
112 FORMAT(2X,24HTENSION STEEL PROPERTIES/,
   14X,25HSTEEL MODULUS=           ,F11.4/,
   24X,25HYIELD STRESS=            ,F11.4/,
   34X,25HYIELD STRAIN=            ,F11.4/,
   44X,25HSTRAIN HARDENING AT      ,F11.4/,
   54X,25HULTIMATE STRAIN=         ,F11.4//)
    WRITE(7,113) YS,FYC,EYC,ESHC,ULTSS
113 FORMAT(2X,28HCOMPRESSION STEEL PROPERTIES/,
   14X,25HSTEEL MODULUS=           ,F11.4/,
   24X,25HYIELD STRESS=            ,F11.4/,
   34X,25HYIELD STRAIN=            ,F11.4/,
   44X,25HSTRAIN HARDENING AT      ,F11.4/,
   54X,25HULTIMATE STRAIN=         ,F11.4//)
    WRITE(7,114) FCPR,E0,E1
114 FORMAT(2X,30HUNCONFINED CONCRETE PROPERTIES/,
   14X,25HCOMPRESSIVE STRENGTH=    ,F11.4/,
   24X,25HSTRAIN AT BREAK POINT 1= ,F11.4/,
   34X,25HULT STRAIN UNCONFINED=   ,F11.4//)
    WRITE(7,115) FCPRC,E0,E1,E2
115 FORMAT(2X,28HCONFINED CONCRETE PROPERTIES/,
   14X,25HCONFINED COMP STRENGTH=  ,F11.4/,
   24X,25HSTRAIN AT BREAK POINT 1= ,F11.4/,
   34X,25HSTRAIN AT BREAK POINT 2= ,F11.4/,
   44X,25HULT STRAIN CONFINED=     ,F11.4//)
    EST(2)=0.0
    ESC(2)=0.0
    FST(2)=0.0
    FSC(2)=0.0
    LA=2
 30 EC=EC+.001
    LA=LA+1
    IF(D.LT.10.0) YZ=.3*D
    IF(D.GE.10.0) YZ=.4*D
    GO TO 31
 32 YZ=YZ/2.0
    DNA=DNA+YZ
```

```
      GO TO 31
   33 YZ=YZ/2.0
      DNA=DNA-YZ
   31 ESC(LA)=EC*(DNA-DPR)/DNA
      EST(LA)=EC*(D-DNA)/DNA
      FST(LA)=FS(EST(LA),FYT,ESHT)
      IF(EST(LA).LT.EST(LA-1)) FST(LA)=FST(LA-1)-YS*(EST(LA
     1-1)-EST(LA))
      IF(FST(LA-1).LT.FS(EST(LA-1),FYT,ESHT).AND.EST(LA).GT
     1.EST(LA-1)) GO TO 38
      GO TO 39
   38 FSTA(LA)=FST(LA-1)+YS*(EST(LA)-EST(LA-1))
      IF(FSTA(LA)-FS(EST(LA),FYT,ESHT)) 36,37,37
   36 FST(LA)=FSTA(LA)
      GO TO 39
   37 FST(LA)=FS(EST(LA),FYT,ESHT)
   39 FSC(LA)=FS(ESC(LA),FYC,ESHC)
      IF(ESC(LA).LT.ESC(LA-1)) FSC(LA)=FSC(LA-1)-YS*(ESC(LA
     1-1)-ESC(LA))
      IF(FSC(LA-1).LT.FS(ESC(LA-1),FYC,ESHC).AND.ESC(LA).GT
     1.ESC(LA-1)) GO TO 40
      GO TO 41
   40 FSCA(LA)=FSC(LA-1)+YS*(ESC(LA)-ESC(LA-1))
      IF(FSCA(LA)-FS(ESC(LA),FYC,ESHC)) 43,42,42
   43 FSC(LA)=FSCA(LA)
      GO TO 41
   42 FSC(LA)=FS(ESC(LA),FYC,ESHC)
   41 TS=AS*FST(LA)
      CS=ASPR*FSC(LA)
  148 ECC=EC*(DNA-DTS)/DNA
      C=CC(EC)
   34 SUMF=TS-CS-C
      IF(SUMF.GT.(-.10).AND.SUMF.LT.(+.10)) GO TO 50
      IF(SUMF.LE.(-.10)) GO TO 33
      IF(SUMF.GE.(+.10)) GO TO 32
   50 I=I+1
      DNA1(I)=DNA
      CURV(I)=EC/DNA
   73 M(I)=CC(EC)*(D-ALC(EC))+CS*(D-DPR)
      LOAD(I)=M(I)*4.0/L
      IF(FST(LA).EQ.FYT.AND.FST(LA).GT.FST(LA-1).AND.
     1EST(LA-1).LT.EYT) GO TO 117
      GO TO 61
  117 YIELDM=YM(EYT,EC,D,FYT,DPR,FYC,ESHC,AS,ASPR)
      GO TO 30
   61 IF(CURV(I).GT..03) GO TO 75
      IF(ECC.GT.E2) GO TO 75
      IF(EST(LA).LT..15.AND.EC.LT..026) GO TO 30
   75 WRITE(7,149) YIELDM,CURVY
  149 FORMAT(2X,27HYIELD MOMENT=                ,F12.5/,
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  12X,27HYIELD CURVATURE=            ,F12.5)
   WRITE(6,72)
72 FORMAT(1H1)
   WRITE(7,71)
71 FORMAT(6H  CONC,5X,9HCURVATURE,4X,6HMOMENT,4X,4HLOAD,
  12X,10HDEFLECTION,9H  TENSION,12H COMPRESSION,
  29H DEPTH TO/,7H STRAIN,46X,5HSTEEL,5X,5HSTEEL,4X,
  37HNEUTRAL/,53X,6HSTRESS,4X,6HSTRESS,5X,4HAXIS/,
  48H IN./IN.,4X,7HRAD/IN.,6X,4HIN-K,5X,3HKIP,6X,3HIN.,
  58X,3HKSI,7X,3HKSI,8X,3HIN.//)
   N=I
   DO 51 I=1,N
   DEFL(I) =0.0
   JA=L/2.0
   DO 51 J=1,JA
   X(J)=J
   MX(I,J)=LOAD(I)*X(J)/2.0
   KA=1
62 DO 53 K=KA,N
   IF(K-1) 20,21,22
20 STOP
21 IF(MX(I,J).LE.M(1).AND.MX(I,J).GE.0.0) GO TO 57
   GO TO 53
22 IF(MX(I,J).LE.M(K).AND.MX(I,J).GE.M(K-1)) GO TO 54
29 IF(MX(I,J).GE.M(K).AND.MX(I,J).LE.M(K-1)) GO TO 59
53 CONTINUE
52 DEFL(I)=DEFL(I)-Y(JA)-Y(JA-1)-Y(JA-2)
   GO TO 79
57 PHI(I,J)=CURV(K)-CURV(K)*(M(K)-MX(I,J))/M(K)
   IF(I-1) 23,94,95
95 IF(MX(I,J).LT.MX(I-1,J).AND.PHI(I,J).LT.PHI(I-1,J))
  1PHI(I,J)=PHI(I-1,J)-(CURV(1)/M(1))*(MX(I-1,J)-MX(I,J))
94 IF(J-1) 23,24,25
23 STOP
24 Y(J)=PHI(I,J)*(2.0*X(J)**2)/6.0
   GO TO 58
25 Y(J)=(PHI(I,J)*(2.0*X(J)**2-X(J)*X(J-1)-X(J-1)**2)+PH
  1I(I,J-1)*(X(J)**2+X(J)*X(J-1)-2.0*X(J-1)**2))/6.0
   GO TO 58
54 PHI(I,J)=CURV(K)-(CURV(K)-CURV(K-1))*(M(K)-MX(I,J))/(
  1M(K)-M(K-1))
   IF(MX(I,J).LT.MX(I-1,J).AND.PHI(I,J).LT.PHI(I-1,J))PH
  1I(I,J)=PHI(I-1,J)-(CURV(1)/M(1))*(MX(I-1,J)-MX(I,J))
   GO TO 60
59 PHI(I,J)=CURV(K)-(CURV(K)-CURV(K-1))*(MX(I,J)-M(K))/(
  1M(K-1)-M(K))
   GO TO 79
60 IF(J-1) 26,27,28
26 STOP
27 Y(J)=PHI(I,J)*(2.0*X(J)**2)/6.0
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      GO TO 58
   28 Y(J)=(PHI(I,J)*(2.0*X(J)**2-X(J)*X(J-1)-X(J-1)**2)+PH
     1I(I,J-1)*(X(J)**2+X(J)*X(J-1)-2.0*X(J-1)**2))/6.0
      GO TO 58
   79 HINGEL=6.0
      Y(JA)=CURV(I)*(L/2.0-HINGEL/4.0)*HINGEL/2.0
      DEFL(I)=DEFL(I)+Y(JA)
      GO TO 55
   58 DEFL(I)= DEFL(I)+Y(J)
      IF(J.EQ.JA) GO TO 77
      GO TO 51
   77 IF(I-1) 74,55,76
   74 STOP
   76 IF(DEFL(I).LE.DEFL(I-1)) GO TO 52
   55 SUBSC=I
      TOPS(I)=SUBSC*.001
      WRITE(7,56) TOPS(I),CURV(I),M(I),LOAD(I),DEFL(I),FST(
     1I+2),FSC(I+2),DNA1(I)
   56 FORMAT(F7.4,F12.5,F11.2,F8.2,F9.2,F11.2,F10.2,F11.3)
   51 CONTINUE
      GO TO 1
    8 STOP
      END
$IBFTC F1
      FUNCTION  FS(ES,FY,ESH)
      YS=29000.0
      EY=FY/YS
      IF(ES.LE.EY) FS=ES*YS
      IF(ES.GT.EY.AND.ES.LT.ESH) FS=FY
      IF(ES.GE.ESH) GO TO 6
      GO TO 7
    6 FS=FY*((56.*(ES-ESH)+1.)/(30.*(ES-ESH)+1.)-(.07*(ES-E
     1SH)/(.15-ESH)))
    7 RETURN
      END
$IBFTC F2
      FUNCTION FC(EC)
      COMMON    ECC,DNA,DTS,E0,E1,E2,B,BCC,FCPR,FCPRC,FCP,
     1STIRF,D1,D2,D4,D5,D7,D8,D81,BOUT,D9,D10,D17,EXTS,
     2EXTS1,C1,C2,C3,C4,C5,C11,C17,C21,C7,CURVY
      FCPR=FCP*.85
      IF(EC.LT.E0) FC=EC*FCPR/E0
      IF(EC.GE.E0.AND.EC.LE.E1) FC=FCPR
      IF(EC.GE.E1) FC=0.0
      RETURN
      END
$IBFTC F3
      FUNCTION FCC(EC)
      COMMON    ECC,DNA,DTS,E0,E1,E2,B,BCC,FCPR,FCPRC,FCP,
     1STIRF,D1,D2,D4,D5,D7,D8,D81,BOUT,D9,D10,D17,EXTS,
     2EXTS1,C1,C2,C3,C4,C5,C11,C17,C21,C7,CURVY
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      FCPR=FCP*.85
      IF(ECC.LT.E0) FCC=ECC*FCPR*STIRF/E0
      IF(ECC.GE.E0.AND.ECC.LE.E1) FCC=FCPR*STIRF
      IF(ECC.GT.E1.AND.ECC.LT.E2) FCC=FCPR*(1.445-ECC*.63/
     1.04)
      RETURN
      END
$IBFTC YIELDM
      FUNCTION YM(EYT,EC,D,FYT,DPR,FYC,ESHC,AS,ASPR)
      COMMON    ECC,DNA,DTS,E0,E1,E2,B,BCC,FCPR,FCPRC,FCP,
     1STIRF,D1,D2,D4,D5,D7,D8,D81,BOUT,D9,D10,D17,EXTS,
     2EXTS1,C1,C2,C3,C4,C5,C11,C17,C21,C7,CURVY
      SC=EC-.001
      SINC=.001
      GO TO 120
  118 SINC=SINC/2.0
      SC=SC+SINC
      GO TO 120
  119 SINC=SINC/2.0
      SC=SC-SINC
  120 EST1=EYT
      DNA=SC*D/(EST1+SC)
      ECC=SC*(DNA-DTS)/DNA
      FST1=FYT
      ESC1=(DNA-DPR)*SC/DNA
      FSC1=FS(ESC1,FYC,ESHC)
      TS=AS*FST1
      CS=ASPR*FSC1
      SUMF=TS-CS-CC(SC)
      IF(SUMF.GT.(-.10).AND.SUMF.LT.(+.10)) GO TO 150
      IF(SUMF.LE.(-.10)) GO TO 119
      IF(SUMF.GE.(+.10)) GO TO 118
  150 CURVY=SC/DNA
      YM=CC(SC)*(D-ALC(SC))+CS*(D-DPR)
      RETURN
      END
$IBFTC CC
      FUNCTION CC(EC)
      COMMON    ECC,DNA,DTS,E0,E1,E2,B,BCC,FCPR,FCPRC,FCP,
     1STIRF,D1,D2,D4,D5,D7,D8,D81,BOUT,D9,D10,D17,EXTS,
     2EXTS1,C1,C2,C3,C4,C5,C11,C17,C21,C7,CURVY
      D1=(E0/EC)*DNA
      D2=((E1-E0)/EC)*DNA
      D4=(1.0-E0/EC)*DNA
      D5=DNA-DTS
      D7=DNA*(1.0-E1/EC)
      D8=DTS-D7
      D81=D7-DTS
      BOUT=B-BCC
      D9=(E1/EC)*DNA
      D10=DNA-D9
      D17=(ECC-E0)*DNA/EC
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```
      EXTS=FCPRC-FCPR
      EXTS1=FCC(ECC)-FC(ECC)
      C1=FCPR*D1*B/2.0
      C2=FCPR*D2*B
      C3=FC(EC)*DNA*B/2.0
      C4=FCPR*D4*B
      C5=EXTS1*D5*BCC/2.0
      C11=EXTS*D1*BCC/2.0
      C17=EXTS*D17*BCC
      C21=EXTS*D2*BCC
      C7=(FCPRC+FCC(ECC))*BCC*D81/2.0
      IF(EC.LE.E0) GO TO 91
      IF(EC.GT.E0.AND.EC.LT.E1) GO TO 92
      IF(EC.GE.E1) GO TO 93
   91 CC=C3+C5
      GO TO 147
   92 IF(ECC.LE.E0) GO TO 9
      IF(ECC.GT.E0) GO TO 10
    9 CC=C1+C4+C5
      GO TO 147
   10 CC=C1+C4+C11+C17
      GO TO 147
   93 IF(ECC.LE.E0) GO TO 12
      IF(ECC.GT.E0.AND.ECC.LT.E1) GO TO 13
      IF(ECC.GE.E1) GO TO 14
   12 CC=C1+C2+C5
      GO TO 147
   13 CC=C1+C2+C11+C17
      GO TO 147
   14 CC=C1+C2+C11+C21+C7
  147 RETURN
      END
$IBFTC ALC
      FUNCTION ALC(EC)
      COMMON    ECC,DNA,DTS,E0,E1,E2,B,BCC,FCPR,FCPRC,FCP,
     1STIRF,D1,D2,D4,D5,D7,D8,D81,BOUT,D9,D10,D17,EXTS,
     2EXTS1,C1,C2,C3,C4,C5,C11,C17,C21,C7,CURVY
      AL1=C1*(D4+D1/3.0)
      AL2=C2*(D7+D2/2.0)
      AL21=C21*(D7+D2/2.0)
      AL3=C3*DNA/3.0
      AL4=C4*D4/2.0
      AL5=C5*(DTS+D5/3.0)
      AL7=FCC(ECC)*BCC*D81*(DTS+D81/2.0)
      AL71=(FCPRC-FCC(ECC))*BCC*D81*(DTS+D81*2.0/3.0)/2.0
      AL11=C11*(D4+D1/3.0)
      AL17=C17*(DTS+D17/2.0)
      IF(EC.LE.E0) GO TO 81
      IF(EC.GT.E0.AND.EC.LT.E1) GO TO 82
      IF(EC.GT.E1) GO TO 83
   81 ALC=(AL3+AL5)/CC(EC)
```

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      GO TO 146
   82 IF(ECC.LE.E0) GO TO 85
      IF(ECC.GT.E0) GO TO 110
   85 ALC=(AL1+AL4+AL5)/CC(EC)
      GO TO 146
  110 ALC=(AL1+AL4+AL11+AL17)/CC(EC)
      GO TO 146
   83 IF(ECC.LE.E0) GO TO 86
      IF(ECC.GT.E0.AND.ECC.LT.E1) GO TO 87
      IF(ECC.GE.E1) GO TO 88
   86 ALC=(AL1+AL2+AL5)/CC(EC)
      GO TO 146
   87 ALC=(AL1+AL2+AL11+AL17)/CC(EC)
      GO TO 146
   88 ALC=(AL1+AL2+AL11+AL21+AL7+AL71)/CC(EC)
  146 RETURN
      END
$ENTRY        MAIN

      DATA DECK

$IBSYS
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AN ANALYTICAL MODEL OF REINFORCED CONCRETE BEAMS
CONSIDERING STRAIN HARDENING AND CONFINEMENT EFFECTS

by

Glenn Alvin Austin, Jr.

Abstract

An analytical model is presented which predicts flexural behavior in reinforced concrete beams. The model considers reinforcement strain hardening and variation in ultimate concrete strain, two factors neglected in previous models. These parameters must be considered if an accurate prediction of the moment-curvature curve is desired.

The proposed model correlates well with experimental results obtained except for its prediction of ultimate rotational capacity. Since this is controlled largely by the predicted ultimate concrete strain, a better method of predicting this quantity will result in better ultimate curvature predictions.

Using the proposed model, the effect of various parameters on flexural behavior is discussed. The conclusions reached include:

1. Compression steel percentage may be the most important parameter in developing adequate rotational and moment capacity for limit design.
2. Closely spaced rectangular stirrups significantly increase the rotational capacity of reinforced

concrete beams.

3. Increasing tensile steel yield stress causes increasing moment capacity, but decreasing rotational capacity.

4. Concrete compressive strengths of four thousand pounds per square inch or greater are desirable for limit design.

5. Span length and beam width have little influence on flexural behavior.