A PROPOSED EXPERIMENT TO DETECT THE MAGNETIC

MONOPOLE OF THE RUDERMAN - ZWANZIGER MODEL

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Thesis submitted to the Graduate Faculty of the

Virginia Polytechnic Institute and State University

in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

in

Physics

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September 1970

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ACKNOWLEDGMENTS

The author is indebted to his advisor, Dr. Peter Trower, who has provided the inspiration, constructive criticism, and editorial assistance necessary for the completion of this manuscript. He would like to acknowledge the generous help he received from Dr. George Collins, Dr. John Ficenec, and Dr. Robert Brown regarding experimental procedure and magnetic monopole theory. The author wishes to thank all the members of his graduate committee, who provided useful comments regarding the final version of the paper. He is thankful for a loving wife,   , who not only provided understanding and encouragement but also performed the typing of the manuscript.

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INTRODUCTION

The formalism of quantum mechanics has been used successfully to predict the existence of new particles, codify their properties, and study their interaction with other particles. However we often obtain from this mathematical formalism results such as imaginary numbers and isolated singularities which do not have an obvious physical interpretation. It is very tempting for a physicist to disregard these extraneous mathematical features, reasoning that they result from a mathematical formulation which doesn't exactly fit the physical situation. However an antipathetical point of view -- that the mathematics is prediction of physics that is as yet unrealized -- is also a possible approach. When P.A.M. Dirac in 1930 used the mathematical formalism of quantum mechanics and special relativity to describe a free electron, a difficulty arose. He found not only positive, but also negative kinetic energy values were predicted for the electron. Dirac chose not to dismiss these negative kinetic energy states, but instead interpreted them as the result of a new particle, the electron's antiparticle, the positron. A year later, the positron was discovered by Anderson.

In 1931 Dirac, concerned with the reason for quantization of electric charge, studied the motion of an electron in an electromagnetic field. He interpreted the resulting wave equation as that of an electron in the field of an isolated magnetic pole. This result did not give a value for the electronic charge, but rather related the quantization of electric charge to that of magnetic charge! This implied the existence of a carrier of the elementary magnetic charge, forthwith known as the magnetic monopole.

Dirac's success in predicting the positron motivated a search which is still unsatisfied. Cosmic rays have been examined for signs of free monopoles while protons from accelerators at Berkeley, Brookhaven, Geneva, and Serpukhov have been used to bombard nuclei in an attempt to produce free monopoles. All experiments to date have had negative results. Therefore it would be easy to conclude that monopoles do not exist. However attempts to find a conservation law which would prohibit the existence of the monopole have also been fruitless.

If we heed Gell-Mann's totalitarian principle (Everything which is not forbidden is compulsory.), we must attempt to find non-free monopoles. Ruderman and Zwanziger have put forward an explanation for the negative experimental results. They argue that monopoles are likely to be produced in pairs, which are tightly bound and never become free particles. Such a pair has an analogue in positronium with the electron and positron. There the system, described classically in terms of Keplerian orbits, has a lifetime against annihilation depending on the separation of the particles or, equivalently, on the strength of the interaction. Annihilation of two such monopoles, which we will call poleonium, Ruderman and Zwanziger point out could cause certain anomalous cosmic ray events.

We propose here an experiment to investigate the cosmic ray events of the type which could result from formation and annihilation of monopole pairs in the Ruderman-Zwanziger model. We however first review Dirac's arguments, calculate the monopole's properties, and investigate the incidence of evokation of monopoles in current particle physics theory. We critically review previous monopole experiments, and summarize their results. Finally, using the model proposed by Ruderman and Zwanziger to give identifying properties of bound monopole pairs, we propose an experiment designed to detect the materialization of the magnetic monopole.

THE THEORY OF DIRAC MONOPOLES

In this section we review Dirac's magnetic monopole theory, theories using monopoles, and properties of magnetic monopoles.

Dirac Monopole Theory

Magnetic monopoles resulted from Dirac's investigation of an electron's motion in an electromagnetic field.[1,2] For a free electron, its wave function is

$$\psi = \psi_1(\vec{x}, t)\, e^{i\beta} \tag{1}$$

where ψ_1 is a wave function defined at each point in four-space and whose modulus is everywhere equal to the modulus of ψ. The wave function ψ is undetermined to within an arbitrary phase factor which we represent by β in $e^{i\beta}$. Although the value of β at a point has no physical meaning, the difference between the values of β at two different points, the relative phase, is measurable.

Since the wave function must be continuous and single valued at any point on a closed curve, the change in phase, $\Delta\beta$, around this curve must be an integral multiple of 2π. This change in phase may be different for different wave functions by arbitrary multiples of 2π, but if we consider a very small closed curve, the continuity condition requires that $\Delta\beta$ be very small for all the wave functions. Thus, the change in phase is independent of which wave function representing different states of the electron we examine.

If we operate on the wave function in eq.(1) with the four momentum operators

$$P_4 = iW = -i\hbar \frac{\partial}{\partial(it)} \quad ; \quad P_j = -i\hbar \frac{\partial}{\partial x_j} \; , \; j=1,2,3 \tag{2}$$

we obtain

$$iW\psi = e^{i\beta}\left(-i\hbar \frac{\partial}{\partial(it)} + \hbar K_4\right)\psi_1 \tag{3a}$$

and

$$P_j \psi = e^{i\beta}\left(-i\hbar \frac{\partial}{\partial x_j} + \hbar K_j\right)\psi_1 \tag{3b}$$

where

$$K_4 = \frac{\partial\beta}{\partial(it)} \quad ; \quad K_j = \frac{\partial\beta}{\partial x_j} \; , \; j = 1,2,3 \tag{4}$$

It now follows that if ψ satisfies a wave equation involving the momentum-energy operators (P_j, iW), then ψ_1 will satisfy the corresponding equation with these replaced by $(P_j + \hbar K_j, iW + \hbar K_4)$.

Let

$$\frac{e}{c} A_j = \hbar K_j \quad , \quad j=1,2,3 \tag{5a}$$

$$ie A_4 = \hbar K_4 \tag{5b}$$

Then ψ_1 will satisfy the wave equation with the operators replaced by

$$(P_j + (e/c)A_j, iW + ieA_4) \qquad j = 1,2,3$$

Associating $\vec{A}$ and A_4 with the electromagnetic four potential, we see that if ψ satisfies the wave equation for an electron in the absence of any field, then ψ_1 will satisfy the equation of an electron in an electromagnetic field where

$$\vec{\nabla} \times \vec{A} = \vec{H} \tag{6a}$$

$$-\vec{\nabla} A_4 - \frac{1}{c}\frac{\partial \vec{A}}{\partial t} = \vec{E} \tag{6b}$$

We can now relate the A's to the change in phase of a wave function around a closed curve. Using Stokes' Theorem and defining the field strength tensor

$$F_{\mu\nu} = \frac{\partial A_\nu}{\partial x_\mu} - \frac{\partial A_\mu}{\partial x_\nu} \quad , \mu, \nu = 1, 2, 3, 4 \tag{6c}$$

the change in phase around a closed curve in four-space can be written as

$$\oint A_\mu \, d\ell_\mu = \int_S F_{\mu\nu} \, dx_\mu \, dx_\nu$$

$$= \int_S \frac{e}{\hbar c} \vec{H} \cdot d\vec{S} + \frac{ie}{\hbar} \int dt \int \vec{E} \cdot d\vec{\ell} \, . \tag{7a}$$

However

$$\int \vec{E} \cdot d\vec{\ell} = 0 \tag{7b}$$

since it represents the change in potential energy over a closed loop. We can now interpret the change in phase around a closed curve in terms of the flux of the vector $\vec{H}$ through the curve.

What happens to the change in phase of a wave function around a small closed curve when the wave function vanishes on the curve? The vanishing of the wave function requires two conditions since it is complex so that the points at which it vanishes are constrained to lie along a line, traditionally called "nodel line". If the "nodel line" intersects our small closed curve, we can no longer say that the change in phase must be small. The change in phase can only be determined to within $2\pi n$, where n is an integer. Therefore

$$\Delta\beta = 2\pi \sum n + \left(\frac{e}{\hbar c}\right) \int \vec{H} \cdot d\vec{S} \tag{8}$$

where the summation is over all "nodel lines" that pass through our small closed curve. If the summation does not vanish, some "nodel lines" must terminate inside the surface bounded by the curve. The end points of the "nodel lines" are interpreted to be singularities of the electromagnetic field.

Because the change in phase for all wave functions around a vanishing small closed curve is vanishing small, then in the integration of (8), the total change in phase vanishes. The magnetic flux crossing the surface bounded by the curve is just

$$4\pi g = 2\pi n \hbar c/e \tag{9}$$

Thus, the interpretation of the end points as singularities in the electromagnetic field led Dirac to a value of magnetic flux which is quantized in terms of electric charge. The objects which contained unit quantized magnetic charge Dirac called magnetic monopoles.

Theories Using Monopoles

Dirac magnetic monopoles have been used to explain some unresolved problems in particle physics.

Porter[3] and later Goto[4] have evoked magnetic monopoles in an attempt to explain the origin of extensive large cosmic showers[5] which contain large numbers of particles and have very high total energies. They argue that if Dirac monopoles were present in primary cosmic rays, they would be accelerated by galactic magnetic fields and could obtain energies of 10^{20} eV. If these monopoles then fell on the earth's atmosphere, they would bremsstrahlung, producing copious showers of particles. In order to account for the frequency of observed extensive air showers, only 10^{-14} of the primary cosmic rays would have to be Dirac monopoles.

Schiff has used the monopole to permit the fractionally charged quark to exist within hadrons, and at the same time to prohibit the quark from appearing as free particle.[6] Schiff modifies Dirac's quantization to include monopoles which have a finite size characterized by a radius R, the radius of a hadron. This modification requires the quantization of electric charge within a range R of each other. Using his selection rule[7], which requires that quark clusters which lie within range, R, of each other have integer baryon number, he shows that quarks can exist within clusters of range R. Isolated quarks can not be seen because they have fractional baryon number, which is prohibited by the selection principle, and they have fractional electric charge, which is prohibited by quantization of charge through monopoles.

Schwinger has developed a particle theory based on fundamental particles, called "dyons", which carry both electric and magnetic charge.[8] Using "dyons", Schwinger shows that all particle interactions are electromagnetic in nature, the "strong" interaction being accounted for by the interaction of the magnetic charge of "dyons".

Electrodynamic Consequences of Monopoles

We describe here the anticipated properties of the magnetic monopole.

If magnetic monopoles exist, Maxwell's equations would become

$$\vec{\nabla}\cdot\vec{D} = 4\pi\rho_e \tag{10a}$$

$$\vec{\nabla}\cdot\vec{B} = 4\pi\rho_m \tag{10b}$$

$$\vec{\nabla}\times\vec{H} - 1/c\,\frac{\partial\vec{D}}{\partial t} = \frac{4\pi}{c}\vec{J}_e \tag{10c}$$

$$\vec{\nabla}\times\vec{E} + 1/c\,\frac{\partial\vec{B}}{\partial t} = -\frac{4\pi}{c}\vec{J}_m \tag{10d}$$

where

$$\vec{D} = \epsilon\vec{E} \tag{11}$$

$$\vec{B} = \mu\vec{H} \tag{12}$$

The sign of the magnetic current density $\vec{J}_m$ would be required from magnetic charge conservation,

$$\vec{\nabla}\cdot\vec{J}_m = -\frac{d\rho_m}{dt} \tag{13}$$

The equations are symmetric with respect to the interchange of electric and magnetic fields: $\vec{B}$ for $\vec{D}$, $\vec{H}$ for $\vec{E}$, and ρ_m for ρ_e ; μ for ϵ and $\vec{J}_m$ for $\vec{J}_e$.

Amaldi[9] points out that, even in a system with monopoles present, if $\vec{E}$ and $\vec{H}$ should retain their spatial symmetry properties, (i.e., the first being a polar vector -- odd under the parity operation -- while the second an axial vector -- even under the parity operation), the magnetic pole strength

$$g = \int \rho_m \, dV \tag{14}$$

should be a psuedoscaler and the magnetic density current

$$\vec{J}_m = \rho_m \vec{v} \tag{15}$$

should behave as an axial vector with respect to space reflections.

Ramsey[10] has pointed out that in a theory which includes magnetic poles, the CPT theorem would be replaced by a CPTM theorem where M is an operation which carries each pole into a pole of equal, but opposite sign (i.e. anti-pole).

We now consider the effect of monopoles on the stress-energy tensor,

$$T_{\mu\nu} = \left|\begin{array}{ccc:c} & T_{ij} & & -icS_1 \\ & & & -icS_2 \\ & & & -icS_3 \\ \hdashline -icS_1 & -icS_2 & -icS_3 & u \end{array}\right| \quad \mu,\nu = 1,2,3,4 \tag{16}$$

where T_{ij} is the Maxwell stress tensor

$$T_{ij} = \frac{1}{4\pi}\left\{ E_i D_j + H_i B_j - \tfrac{1}{2}\delta_{ij}(\vec{E}\cdot\vec{D} + \vec{H}\cdot\vec{B})\right\} \quad i,j = 1,2,3 \tag{17}$$

the Poynting vector

$$\vec{S} = \frac{c}{4\pi}\,\vec{E}\times\vec{H} \tag{18}$$

and the energy density

$$u = \frac{1}{8\pi}\left(\vec{E}\cdot\vec{D} + \vec{H}\cdot\vec{B}\right) \tag{19}$$

$T_{\mu\nu}$ is invariant under the transformation

$$\begin{aligned} \vec{E} &= \vec{E}'\cos\phi + \vec{H}'\sin\phi & \qquad \vec{H} &= -\vec{E}'\sin\phi + \vec{H}'\cos\phi \\ \vec{D} &= \vec{D}'\cos\phi + \vec{B}'\sin\phi & \qquad \vec{B} &= -\vec{D}'\sin\phi + \vec{B}'\cos\phi \end{aligned} \tag{20}$$

where ϕ is an arbitrary mathematical parameter. By applying the same transformation to the "sources"

$$\rho_e = \rho_e' \cos\phi + \rho_m' \sin\phi \qquad \rho_m = -\rho_e' \sin\phi + \rho_m' \cos\phi$$
$$\vec{J}_e = \vec{J}_e' \cos\phi + \vec{J}_m' \sin\phi \qquad \vec{J}_m = -\vec{J}_e' \sin\phi + \vec{J}_m' \cos\phi \tag{21}$$

not only the Poynting vector (18) and energy density (19), but also Maxwell's equations (10) remain invariant.

Physical Properties of Monopoles

Charge

Dirac specified the charge of the monopole from the quantization condition to be

$$g = \frac{n\hbar c}{2e} \qquad , n \text{ is integer.} \tag{22}$$

Mass

At the time of his prediction Dirac speculated that the mass of the monopole should be very large because no particles of low mass (Only the proton, neutron, electron, and positron were known.) had been found which resembled monopoles.

It is customary to define a mass for the monopole from a classical radius of the monopole equal to the classical radius of the electron,

$$r_g = \frac{n^2 g^2}{m_g c^2} = \frac{e^2}{m_e c^2} = r_e \tag{23}$$

Using this definition we obtain

$$m_g = n^2 \left(\frac{g}{e}\right)^2 m_e$$
$$= 2.4 \ \mathrm{GeV}/c^2 \qquad \text{for } n=1 \tag{24}$$
$$= 9.6 \ \mathrm{GeV}/c^2 \qquad \text{for } n=2 . \tag{25}$$

Electric Dipole Moment

Amaldi et al.[11] pointed out that since Dirac treated monopoles as Fermions of spin ½, they would necessarily have an electric dipole moment

$$\mu_g = \frac{\hbar g}{2 m_g c} = e\, r_g \tag{26}$$

the direction of which would be, for positive poles, opposite in direction to that of the spin and for negative poles, the same as that of the spin.

Ionization

Bauer[12] and Cole[13] have estimated the rate of ionization energy loss for monopoles by the semi-classical impact parameter method. The assumptions are that the electrons in the medium are free, and at rest, and the force between the monopole and the electron is

$$|\vec{F}| = \left| \frac{e}{c} [\vec{v} \times \vec{H}] \right| \approx \frac{egv}{cr^2} \tag{27}$$

where r is the separation distance of the monopole and electron. The impulse given to the electron by the passing monopole is

$$\Delta p_\perp = \int_{-\infty}^{+\infty} F_\perp \, dt = \int_{-\infty}^{+\infty} F_\perp \frac{dx}{v} = \frac{eg}{c} \int_{-\infty}^{+\infty} \frac{\cos\theta}{r^2} dx \tag{28}$$

where $F_\perp$ is the component of the force field at the position of the electron normal to the trajectory of the monopole, θ is the angle describing this component, and v is the velocity of the monopole. The integral is evaluated easily, and energy transferred to the electron at a distance b is

$$\Delta E(b) = \frac{(\Delta p)^2}{2 m_e} = \frac{2}{m_e} \left(\frac{ge}{bc} \right)^2 \tag{29}$$

Since there are $2\pi NZ\,b\,db\,dx$ electrons per path length dx that are between a distance b and b + db from the monopole, the energy loss per path length dx is

$$-\frac{dE}{dx} = 2\pi NZ \int b\,db\,\frac{(\Delta p)^2}{2m_e}$$

$$= 4\pi NZ\,\frac{g^2e^2}{m_ec^2}\int_{b_{min}}^{b_{max}}\frac{db}{b}$$

$$= 4\pi NZ\,\frac{g^2e^2}{m_ec^2}\,\ln\left(b_{max}/b_{min}\right) \tag{30}$$

In order to obtain a value of b_{max} the adiabatic condition is used. This states that one cannot induce transitions from one quantum state to another by a perturbation if the variation of the perturbation is small during the period of the system. In our case it can be assumed that the duration of the perturbation is the time b/v during which the monopole is near the electron. In order to produce transitions, the condition

$$\frac{b}{v} < \frac{1}{\langle\nu\rangle}$$

where $\langle\nu\rangle$ is an appropriate average of the frequencies of the atom, must be fulfilled. Treating this relativistically, we obtain for b_{max}

$$b_{max} = \frac{v}{\langle\nu\rangle\left(1-v^2/c^2\right)^{1/2}} \tag{31}$$

Quantum mechanics gives a limit for b_{min} since the electron can only be localized with respect to the monopole to the accuracy of its de Broglie wavelength

$$b_{min} = \frac{\hbar}{p} = \frac{\hbar\left(1-\frac{v^2}{c^2}\right)^{1/2}}{m_e v} \tag{32}$$

Equation (30) can now be written as

$$-\frac{dE}{dx} = 4\pi N Z \frac{g^2 e^2}{m_e c^2} \ln\left[\frac{m_e v^2}{\hbar\langle r\rangle(1 - v^2/c^2)}\right] . \tag{33}$$

Since the ionization for relativistic particle of charge z = 1 is given by[14]

$$-\frac{dE}{dx} = 4\pi N Z \frac{e^4}{m_e v^2} \ln\left[\frac{m_e v^2}{\hbar\langle r\rangle(1 - v^2/c^2)}\right] \tag{34}$$

the energy loss for a monopole in terms of the energy loss for the charged particle with the same velocity is

$$\left(-\frac{dE}{dx}\right)_{pole} = \left(-\frac{dE}{dx}\right)_{\substack{charged\\particle}} \times \left(\frac{g}{e}\right)^2 \left(\frac{v}{c}\right)^2 \tag{35}$$

For a charged particle and monopole, moving with velocity $v \to c$, the energy loss for the monopole is greater than the energy loss for the charged particle by a factor

$$(g/e)^2 \simeq 4700 \tag{36}$$

Bremsstrahlung

Bauer[12] has also calculated the energy loss due to bremsstrahlung for monopoles using a semi-classical approach which does not give the familiar logarithmic dependence of the energy loss. We have calculated the energy loss, but have taken as a model the derivation given by Rossi[15] for charged particles. The assumption here is that the energy radiated per unit time for a charged particle undergoing acceleration is given by classical electrodynamics to be

$$B = \frac{2g^2}{3c^3} a^2 \simeq \frac{g^2}{c^3} a^2 \tag{37}$$

where a is the acceleration of the particle.

The monopole in a frame of reference (primed frame) in which it is initially at rest and the nucleus, charge Ze, moves with a velocity $-\mathrm{v}$, exerts a force on the nucleus given by

$$|\vec{F}| = \left| \frac{Ze}{c} [\vec{v} \times \vec{B}] \right| = \frac{Ze}{c} \frac{vg}{r^2} \tag{38}$$

Relativistically, the maximum value of the acceleration of the monopole is given by

$$a'_{max} = \frac{Zegv}{m_g c b^2} \frac{1}{(1 - v^2/c^2)^{1/2}} \tag{39}$$

where b is the impact parameter. The acceleration has the magnitude of the order of a'_{max} for a time of the order of

$$\tau' = \frac{b}{c} \left(1 - \frac{v^2}{c^2} \right)^{1/2} \tag{40}$$

Therefore, the energy radiated during the collision has, in the primed reference system, the approximate value

$$\begin{aligned} P' &\approx \frac{g^2}{c^3} (a'_{max})^2 \tau' \\ &= \frac{g^2}{c^3} \frac{Z^2 e^2 g^2 v^2}{m_g^2 c^2 b^4} \frac{1}{(1 - v^2/c^2)} \frac{b}{c} \left(1 - \frac{v^2}{c^2} \right)^{1/2} \\ &= \frac{Z^2 e^2 g^4 v^2}{m_g^2 c^6 b^3} \frac{1}{(1 - v^2/c^2)^{1/2}} \end{aligned} \tag{41}$$

In order to find the energy, P, radiated during the collision in the reference frame in which the nucleus is at rest, we assume that the intensity of the radiation in the primed system is symmetrical so that the total momentum of the radiation field vanishes in the primed reference frame. Then the energy radiated in the unprimed frame is

$$P = \frac{P'}{(1 - v^2/c^2)^{1/2}} = \frac{Z^2 e^2 g^4 v^2}{m_g^2 c^6 b^3} \frac{1}{(1 - v^2/c^2)} \tag{42}$$

The frequency distribution in the primed system is related to the distribution in the unprimed system by means of the relativistic

Doppler effect:

$$\nu = \frac{1 + \frac{v}{c}\cos\varphi'}{(1 - v^2/c^2)^{1/2}}\,\nu' \tag{43}$$

where φ' is the angle of emission of the electromagnetic wave in the primed system. The frequency spectrum in the primed system will rapidly drop to zero for frequencies larger than

$$\nu_1' = \frac{1}{\tau'} = \frac{c}{b(1 - v^2/c^2)^{1/2}} \quad . \tag{44}$$

In the unprimed system this frequency is given by

$$\nu_1 \approx \frac{\nu_1'}{(1 - v^2/c^2)^{1/2}} = \frac{c}{b(1 - v^2/c^2)} \quad . \tag{45}$$

Since we have assumed the frequency spectrum to be constant, the energy per unit frequency interval is the ratio of the total energy emitted to the maximum frequency

$$\frac{dP}{d\nu} = \frac{P}{\nu_1} = \frac{z^2 e^2 g^4 v^2}{m_g^2 c^7 b^2} \quad . \tag{46}$$

Equation (46) can not hold for frequencies larger than

$$\nu_2 = E/\hbar \tag{47}$$

since a particle cannot radiate a photon of energy larger than its own kinetic energy.

In order to obtain the energy radiated into photons by a monopole traversing a thickness dx of absorber, we integrate (46) over the frequency range. The number of collisions with impact parameters between b and b + db in a thickness dx is given by

$$2\pi N b\, db\, dx$$

so the energy loss is given by

$$dE = \int_{b_{min}}^{b_{max}} \int_0^{\nu_2} \left(\frac{dP}{d\nu}\right) d\nu \, 2\pi N b \, db \, dx . \tag{48}$$

Substituting equation (46) in our above relation, we have

$$\frac{dE}{dx} = \int_{b_{min}}^{b_{max}} \int_0^{E/\hbar} \left(\frac{z^2 e^2 g^4 v^2}{m_g^2 c^7 b^2} \right) d\nu \, 2\pi N b \, db$$

$$= \frac{2\pi N z^2 e^2 g^4 v^2}{m_g^2 c^7 \hbar} \, E \, \ln \frac{b_{max}}{b_{min}}$$

$$= \left(\frac{e^2}{\hbar c}\right)\left(\frac{g^4}{m_g^2 c^4}\right)\left(\frac{v^2}{c^2}\right) 2\pi N z^2 E \ln \frac{b_{max}}{b_{min}} . \tag{49}$$

Rossi[15] has derived the energy loss due to bremsstrahlung for electrons and obtains

$$\frac{dE}{dx} = \left(\frac{e^2}{\hbar c}\right)\left(\frac{e^4}{m_e^2 c^4}\right) 2\pi N Z^2 E \ln \frac{b_{max}}{b_{min}} . \tag{50}$$

Assuming that the logarithmic factor, which depends only on the screening of the atomic electrons, is of the same order for electrons and monopoles, we can write the bremsstrahlung energy loss of monopoles in terms of that from electrons

$$\left(\frac{dE}{dx}\right)_{poles} = \left(\frac{dE}{dx}\right)_{electrons} \times \left(\frac{g^4}{e^4}\right)\left(\frac{m_e^2}{m_g^2}\right)\left(\frac{v^2}{c^2}\right), \tag{51}$$

which in the extreme relativistic limit ($v \to c$) becomes

$$\left(\frac{dE}{dx}\right)_{poles} \approx \left(1 \times 10^7\right)\left(\frac{m_e}{m_g}\right)^2 \left(\frac{dE}{dx}\right)_{electrons} . \tag{52}$$

<u>Čerenkov Radiation</u>

Using the symmetry of Maxwell's equations, Tompkins[16,17] has generalized to include monopoles the calculations of Fermi[18] for Čerenkov emission of charged particles moving through a medium. Tompkins found that the Čerenkov emission for monopoles was

$$\left(\frac{dE}{dx}\right)_{\check{c}} = g^2/\mu \; M \tag{53}$$

compard to Fermi's result for charged particles,

$$\left(\frac{dE}{dx}\right)_{\check{c}} = \frac{e^2}{\epsilon} M \tag{54}$$

where M is a constant depending on the magnetic permeability, μ , and dielectric constant, ϵ , of the medium.

We have attempted, unsuccessfully, to calculate the Čerenkov emission for monopoles following the procedure used by Jackson for charged particles.[19] Our attempt has failed because we can not derive the energy loss due to the $\vec{B}$ field of a moving monopole interacting with the magnetic moments of the electrons of the medium. Further investigation of this problem will be made after the author gains a better understanding of the macroscopic effect of a moving monopole on a medium of permeability μ .

Atomic Binding of Monopoles

Malkus[20] has tried to determine if the presence of a magnetic monopole in an atom could modify the electronic structure and lead to bound states. He sketched various approaches for treating this problem. One of these, which is qualitative in nature, leads to the conclusion that a pole can be bound to an atom or a molecule, but with an energy of the order of electron volts. We now reproduce this argument.

If we place a pole at a distance r from the center-of-mass of an atom or molecule, the magnetic field produced by the pole

$$H = g/r^2 \tag{55}$$

will polarize the molecule with a resulting change in energy given by

$$\Delta E = E_p - E_d \quad . \tag{56}$$

E_p is caused by attractive long ranged paramagnetic effects while E_d is caused by repulsive short ranged diamagnetic effects. At large distances, we expect that there is a place where ΔE has a negative minimum. Quantitatively these two terms can be approximated by

$$E_p = -\frac{e\hbar}{2m_e c} H \sum_{i=1}^{Z} (m_z^i + 2S_z^i) \tag{57a}$$

and

$$E_d = \frac{e^2}{m_e c^2} H^2 \sum_{i=1}^{Z} \rho_i^2 \tag{57b}$$

where m_z^i, s_z^i are the eigenvalues of the operators representing the z components of the orbital momentum and spin and ρ_i^2 is the average of the square of the perpendicular distance from the z-axis of the i-th electron. (The magnetic field is taken as the axis of quantization.)

The condition for binding is given by

$$\frac{d}{dr} \Delta E = 0 \tag{58a}$$

or

$$\left(\frac{e\hbar g}{m_e c r^3}\right) \sum_{i=1}^{Z} (m_z^i + 2S_z^i) - \left(\frac{4e^2 g^2}{m_e c^2 r^5}\right) \sum_{i=1}^{Z} \rho_i^2 = 0 \quad . \tag{58b}$$

The value for r where ΔE is a minimum is given by

$$r = \left[\frac{4eg}{\hbar c} \frac{\sum_{i=1}^{Z} \rho_i^2}{\sum_{i=1}^{Z} (m_z^i + 2S_z^i)}\right]^{1/2} \quad . \tag{59}$$

For the case of the hydrogen atom

$$Z = 1 \quad , \quad m_z = 0 \quad , \quad S_z = 1/2 \quad , \quad \rho^2 = \left(\frac{2\hbar^2}{m_e^2 c^2}\right)^2$$

we obtain

$$r = 1.06\,\text{Å} \quad , \quad \Delta E = -0.85\,\text{eV} \quad . \tag{60}$$

Thus the poles can be bound to a paramagnetic atom or molecule only with energies comparable to the chemical bond.

Amaldi et al.[11] have pointed out that if the Dirac monopoles were Fermious of spin ½, they would have an electric dipole moment. This moment would give rise to an attractive force when placed in the electric field of an electron or of a nucleus which would result in a deepening of the pole-nucleus potential well. To estimate the order of magnitude of this effect consider the monopole placed at the surface of a nucleus with its spin in the most favorable direction. The corresponding energy is given by

$$\begin{aligned} E &= \frac{-\mu_g Z e}{r_n^2} \\ &= -\frac{Z e^2 r_g}{r_o^2 A^{2/3}} \\ &= -3.2 \frac{Z}{A^{2/3}} \text{ MeV} \end{aligned} \tag{61}$$

if we assume

$$r_o = 1.2 \times 10^{-13} \text{ cm} \tag{62a}$$

and

$$r_g = r_e = 2.8 \times 10^{-13} \text{ cm} \quad . \tag{62b}$$

The binding energy in elements such as copper and aluminium is found to be about -5MeV.

EXPERIMENTAL ATTEMPTS TO DETECT FREE MONOPOLES

We are motivated to believe in the existence of a magnetic monopole because it would make our mathematical description of nature more symmetric. We are encouraged in this belief because there is no known prohibition against the monopole's existence. Thus cosmic ray and accelerator experiments have been performed to search for the monopole. These experiments are reviewed here and their results summarized.

Accelerator Experiments

In accelerator experiments, an attempt is made to produce monopoles through the interaction of high energy protons with target nuclei. These experiments have the advantage of a large flux of incident protons, but the disadvantage of being limited by the energies of existing accelerators, thus restricting the possible mass of the monopole. Monopole search experiments with accelerators are of three types on the basis of the different particular properties used to identify Dirac monopoles.

In Type I experiments a paramagnetic (or ferromagnetic) target irradiated in the beam of protons is sufficiently thick (a few g/cm^2) that an appreciable fraction of any monopoles produced will lose energy in it by ionization and stop inside the target itself. After irradiation the target is placed in a pulsed magnetic field in an attempt to drag the monopoles out. In Type II experiments, a magnetic field of several hundred gauss is applied to the target during the exposure of the proton beam in the hopes of extracting the poles when they are produced. Such poles would be accelerated in a vacuum pipe by solenoids and brought directly to a detector. In Type III experiments poles produced in a thin target, which they would escape by virtue of their initial velocity, would be deflected and accelerated by a magnetic field and then brought to a detector.

Bradner and Isbell[21] were the first to perform an accelerator experiment to search for monopoles. They attempted all three types of experiments! Using 6.2 GeV protons from the Bevatron at Lawarence Radiation Laboratory, they tried to produce monopoles of mass up to that of one proton. In the first part of the experiment, the accelerated protons interacted with a thin aluminium target which was in a magnetic field. If monopoles had been produced, they would have been driven by the magnetic field into an emulsion stack where they could be identified by their large ionization loss. In the second part of the experiment, a thick copper target was bombarded by the protons. The monopoles produced in this target would diffuse through the target and be accelerated by a magnetic field into an emulsion stack. Finally they placed a polyethylene target which had been bombarded by the protons for a long period of time in the magnetic field, hoping to extract monopoles. In all three cases no monopoles were seen.

In the period 1961-63 three sets of experiments were made along similiar lines of the Bevatron experiment: two at Geneva using the internal proton beam (28 GeV) of the CERN-PS and one at Brookhaven with the internal proton beam (30 GeV) of the AGS. In these experiments monopoles of up to about three proton masses could have been detected.

Fidecaro et al.[22] experiments were Type I and II, using copper and aluminium targets bombarded by protons. Any monopoles produced would have been accelerated by a magnetic field into a scintillation counter array. They also looked at a polyethylene target which was being irradiated by protons to detect radiation which might have originated in monopole-antimonopole annihilation. None of these experiments produced any evidence of magnetic monopoles.

Amaldi et al.[11] have also performed all three types of experiments at CERN, using the 28 GeV protons. The experiments of the first two types were similiar to those performed by Fidecaro et al..[22] In the third type of experiment a beryllium target was irradiated by the protons in a magnetic field. If a monopole had been produced, it would have been directed into emulsion stacks for identification. In all three types of experiments, no monopoles were seen.

Purcell et al.[23] at the Brookhaven AGS looked for monopoles produced either in nucleon-nucleon collisions or by secondary γ rays. The experiment, of type III, was based on the idea that any monopoles, produced by the protons interacting with the target (Be, C, CH_2, Al), would escape and be stopped in a liquid downstream. The liquid also served as a target for possible electromagnetic production of monopole pairs by energetic photons. Any monopoles stopped in the liquid would be drawn by a solenoid into a counter or emulsion stack. No events attributable to monopoles were seen.

The most recent accelerator search for monopoles has been carried out at the I.P.H.E. synchrotron at Serpukhov.[24] This 70 GeV proton accelerator is capable of producing a monopole with mass up to 5 proton masses using a hydrogen target, and up to 7 proton masses using a lead target. The experiment was of Type I and monopoles which were produced would be trapped in tungsten plates that surrounded the target. After exposure these plates were placed in a magnetic field which would extract any monopoles trapped in them, and direct the monopoles to a nuclear emulsion stack for detection. Again no monopole was detected.

Cosmic Ray Experiments

Cosmic ray experiments, unlike accelerator experiments, are not limited by the mass of the monopole that can be produced. Cosmic ray experiments performed so far have also been of three types. Type CI attempts to directly measure the instantaneous flux of monopoles produced in the upper atmosphere by primary cosmic rays. Type CII relies on extracting monopoles, using pulsed magnetic fields, trapped in "potential collectors". These materials have been exposed for geological times to the flux of primary cosmic rays. In experiments of Type CIII, one tries to observe the electric field generated (see eq.(10c)) from a magnetic current caused by the motion of samples of "potential collectors".

An experiment of Type CI was first performed by Malkus[20] who assumed that monopoles produced by cosmic rays in the upper atmosphere would travel in the general direction of the Earth's magnetic field.

Using a solenoid to concentrate geomagnetic lines and to accelerate the poles, he directed the poles into a nuclear emulsion, but failed to see any tracks attributable to monopoles. Recently other groups have performed Type CI experiments using the method of Malkus, but they also obtained similiar results.[25,26] The Type CI experiment performed by Green[27] attempted to detect the flux of monopoles in primary cosmic radiation by using the difference in Čerenkov emission for charged particles and monopoles. He found no monopoles.

Petukhov and Yakimenko[28] have searched for monopoles in a Type CII experiment using as a "potential collector" a piece of the Likhote-Alin meteorite. The sample was heated to a gaseous state while in a magnetic field. Monopoles trapped in the sample would be forced by the magnetic field into a counter where they could be detected. The total time for the treatment of the sample was 112 minutes and only one count was recorded.

Goto, Kolm, and Ford[29] used a portable 170 kG pulsed magnet in a Type CII experiment to extract monopoles trapped up to a depth of several centimeters in a magnetic outcrop of the Adirondack Mountains. No tracks attributable to monopoles were observed in the emulsion used for detection. In another CII experiment Kolm, Filz, and Yagoda[30] attempted to extract monopoles from deep-sea sediment and to detect them with nuclear emulsions. Several tracks, geometrically compatible with those of south monopoles were seen; however; although they exhibited constant ionization, it was not sufficiently dense to be the tracks of Dirac monopoles. They then performed a second experiment using scintillation detectors[31] which produced negative results.

The Type CII experiment by Fleischer <u>et al</u>.[32] utilized a 265 kG pulsed magnetic field to extract and solid-state detectors to observe magnetic monopoles from maganese nodulas formed on the floor of the Southern Ocean and from maganese pavement at the bottom of the North Atlantic Ocean.[33] Finally, they attempted to observe monopole tracks in samples of mica and of obsidian.[34] No tracks were found which could be attributed to monopoles.

In relation to Type CIII experiments, Vant-Hull[35] has built a superconducting quantum interferometer to detect magnetic charge. The passage of a monopole through the device generates a voltage which shifts the quantum phase by an amount proportional to the pole strength. The interferometer then detects this change in phase. In samples free of magnetic monopoles Vant-Hull was able to determine div $\vec{B}$ to be less than 1.2×10^{-14} Wb/Kg.

Alvarez[36] has invented an "electromagnetic monopole detector" to be used in Type CIII experiments. The device permits the analysis of a sample suspected of containing monopoles without changing any of the sample's properties. The sample under study is circulated through a coil which forms a closed superconducting circuit. By detecting a change in the current caused by the induced electromagnetic force associated with the change in flux in the electric circuit, the presence of a monopole would be known. The flux generated by a single Dirac monopole is

$$4\pi g \approx 4 \times 10^{-7} \text{ e.m.u.} \quad .$$

The generalized Faraday-Newmann law states that

$$e.m.f. = \oint \vec{E} \cdot d\vec{l} = -i_m - \frac{d}{dt} \Phi(\vec{B}) \tag{63a}$$

which reduces, in the absence of a variable flux of $\vec{B}$, to

$$e.m.f. = \oint \vec{E} \cdot d\vec{l} = -i_m \quad . \tag{63b}$$

If a "potential collector" containing an excess, N_g, of monopoles of one charge is carried by a wheel rotating at a frequency f through a coil of N turns, the magnetic current generated is

$$i_m = N_g \cdot g \cdot f. \tag{64}$$

If f is 100 Hz and N is 2×10^6, the observed d.c. voltage should be about 10^{-6} eV, which can be measured without major difficulties.

Alvarez has used this detector with a sample of moon rock brought back by Apollo 11.[37] In searching the 8.37 kilograms of lunar surface materials, he found no monopoles.

An experiment by Berkowitz and Kronenberg[38] is a combination of all three types of cosmic ray experiments. The authors exploited the property of the earth's geomagnetic field to collect and concentrate magnetic poles into a small area near the North Pole. There the monopoles were trapped by the snow from which they were extracted using a strong magnetic field and collected on sheets of aluminium and other materials. These sheets were then analysed by means of the electromagnetic detector of the Alvarez group. No monopoles were found.

Summary of Experimental Results

No monopoles have been detected experimentally to date. However this effort has succeeded in placing limits on free monopole production cross sections and the monopole mass. In Table I we summarize these limits which result from nucleon-nucleon collisions in accelerator experiments. These negative results may be explained if the monopole exists only if its mass is greater than the lower limit and/or its atomic binding is stronger than expected.

In Table II we list the results of experiments to detect monopoles in "potential collectors". The product of the area over which the poles were collected and the time of collection is given as a measure of the abundance. All negative results except those obtained by use of the Alvarez detector can be explained if the binding of monopoles to matter is much stronger than predicted.

Table III lists the results of experiments to detect the flux of monopoles in cosmic radiation. The only way that these negative results may be explained is if there are no free monopoles or if their properties are completely different from those predicted by theory.

Table I. Results of accelerator experiments to detect the magnetic monopole

Accelerator (protons)	Year	σ (cm^2)	mass limit (in proton masses)	other features	Reference
Bevatron Berkeley (6.3 GeV)	1959	10^{-40}	1.0	binding energy in polyethylene between 3 and 20 eV	21
CERN-PS Geneva (27.5 GeV)	1961	10^{-39}	≈ 2.9	insensitive if large binding energies exists	22
CERN-PS Geneva (28 GeV)	1961-63	10^{-40}	≈ 2.9	binding energy in matter between .6 and 60 eV	11
AGS Brookhaven (30 GeV)	1963	10^{-40}	3.0	--	23
AGS Brookhaven (30 GeV)	1963	10^{-37}	≈ 5.9	cross section for photo production (coherent)	23
AGS Brookhaven (30 GeV)	1963	10^{-34}	≈ 2.4	cross section for photo production (incoherent)	23
I.P.H.E. Serpukhov (70 GeV)	1970	10^{-41}	5.0	assumed monopoles could be extracted by 220 KG magnet	24

Table II. Results of experiments to extract free monopoles from matter.

Method (year)	Area x Time ($cm^2 \cdot sec$)	Mass Limit (in proton masses)	number/ nucleon	Reference
Extraction from magnetic outcrop (1963)	3×10^{12}	4,000	8×10^{-28}	29
Extraction from deep sea sediment (1966)	4×10^{16}	40,000	3×10^{-31}	30
Extraction from Mn nodule (1969)	2.8×10^{14}	13,000	1.3×10^{-25}	32
Stored tracks in minerals (1969)	2.3×10^{18}	200,000	--	34
Stored monopoles in Apollo 11 soil (1970)	4.3×10^{17}	300	4.8×10^{-28}	37
Monopoles in Artic Ice (1966)	10^{13}	--	--	38
Monopoles in commercial copper (1968)	--	--	6×10^{-25}	35
Extraction from Meteorite* (1963)	--	--	--	28

* Cross section for monopole production by cosmic rays $\leq 10^{-40}$ cm^2.

Table III. Results of monopole searches in primary cosmic radiation.

Method (year)	Monopole Intensity on Surface of Earth (cm^{-1} sec^{-1} sr^{-1})	Other Features	Reference
Used ionization properties as means of identification (1951)	$< 10^{-10}$	Assumed mass of monopoles $\leq 10^3$ mass of proton	20
Used Solenoid to extract monopoles from cosmic rays (1966)	$< 10^{-13}$	Assumed mass of monopole $\leq 7 \times 10^3$ mass of proton	25
Used Calorimeters to search for monopoles (1969)	$< 10^{-12}$	Energy of monopole $\geq 10^{13}$ eV	26
Used Čerenkov emission to detect monopoles (1967)	$< 4 \times 10^{-6}$	--	27

RUDERMAN-ZWANZIGER MODEL

Ruderman and Zwanziger[39] have put forward an explanation for the negative results of experimental searches for free monopoles in keeping with the existence of monopoles. We review here their model, its predictions, and the experimental evidence which they use to support it.

Limits on Mass

Ruderman and Zwanziger point that at large separations and low velocities, the interaction of a pole-antipole pair can be described by a magnetic Coulomb attractive potential

$$V(r) = -g^2/r \quad . \tag{65}$$

However, since g is at least 137 times greater than e, the force between the monopole pair is 10^4 greater than the force between an electron-positron pair. This magnetic potential only dominates at large distances, since at separations smaller than one pion Compton wavelength strong interaction forces may contribute through the exchange of a meson. The magnetic potential should be valid down to a distance

$$r_o = \frac{\hbar}{m_\pi c} \approx 10^{-13} \text{ cm} \quad , \tag{66}$$

the Compton wavelength of the pion. In order for the vacuum to be stable against spontaneous production of pole pairs at separations down to r_o, the mass of the pole pair must be greater than the Coulomb potential for all values for r down to r_o. The mass of the monopole, m_g, must satisfy the equation

$$2 m_g c^2 - g^2/r_0 > 0 \tag{67a}$$

or

$$2 m_g c^2 - g^2 m_\pi c/\hbar > 0 \quad , \tag{67b}$$

therefore,

$$m_g c^2 > \frac{g^2 m_\pi c^2}{2\hbar c} \approx 10 \text{ GeV} \quad . \tag{68}$$

It is interesting to note that if we replace the mass and charge of the monopole with that of the electron, we obtain

$$m_e c^2 > \frac{e^2 m_\pi c^2}{2\hbar c} \approx .5 \text{ MeV} . \tag{69}$$

Soft Photon Emission

Because the monopole is coupled so strongly to the electromagnetic field, (The coupling constant is $g^2/hc \approx 137.$) the creation and acceleration of a magnetic monopole pair will always be accompanied by the emission of many soft photons. Ruderman and Zwanziger[39] state that the probability that a monopole pair is suddenly emitted with velocity $v \approx c$ without an energy loss exceeding ΔE to any dipole photon is approximately

$$\exp\left[-\left(8g^2/3\pi\hbar c\right) \ln\left(m_g c^2/\Delta E\right)\right] .$$

For magnetic pair emission at an extreme relativistic relative velocity, the energy emitted into "soft" photons would be

$$\left(g^2 m_\pi/m_g\right)\left[1 - \frac{v^2}{c^2}\right]^{-2} m_g c^2 \quad .$$

Thus the kinetic energy with which the monopoles leave the region, $r < r_o$, should be much smaller than that of light -- the accompanying soft photons.

Bound State Prediction

The above consideration of the soft photons suggests that the energy of the monopole pair in their center of mass may have to exceed the rest mass energy, $2m_g c^2$, enormously in order for the poles to have enough kinetic energy to escape their mutual attraction. Thus instead of free monopoles being produced, it is overwhelmingly more likely that the poles, when produced, will assume Keplerian orbits. The lifetime of the poleonium against subsequent annihilation will depend on the time it takes to radiate away enough energy to bring them within an annihilation distance r_o.

The lifetime of poleonium can be estimated from the uncertainty principle

$$\Delta E \, \Delta t \approx \hbar \tag{70}$$

where

$$\Delta E = g^2/r \tag{71}$$

For a radial excursion of $r = 10^{-12}$ cm, the lifetime of the bound state is of order of 10^{-23} sec.

Number of Photons Emitted

The production of poleonium results in photons from two sources; bremsstrahlung from the mutual attraction and annihilation radiation. The multiplicity of these photons, which depends of course on the dynamical details, is roughly of the order[39]

$$g^2/\hbar c \approx 10^2 \qquad (72)$$

The energy spectrum of these photons depends on the pair separation and the lifetime of the bound system. For a radial excursion of 2×10^{-12} cm, the soft photon emission goes roughly like $g^2\ dw/w$ and cuts off sharply below 1 MeV.

Evidence to Support Theory

In support of their model, Ruderman and Zwanziger point to the observation of a number of unexplained, energetic, narrow, pure photon showers.[40-42] These showers each contain in a narrow cone some tens of photons and have no incident charged particles. The events were observed in nuclear emulsion stacks flown in balloons at high altitudes. Ionization and scattering measurements were used to identify the observed particles as electron pairs. The energy of the photon causing each electron pair was determined by measuring the opening angle of the pair. This is possible since a photon of a given energy in the Coulomb field of a nucleus originates an electron pair whose opening angle depends both on the energy of the photon and the momentum transferred to the nucleus. The most probable opening angle, θ_p, of a pair created by a photon of energy E_γ, given by Borsellino[43], is

$$\theta_p = \frac{4\, m_e c^2}{E_\gamma} \qquad (73)$$

Thus the values of the photon energy is only a rough estimate because of the nonuniqueness of the opening angle.

Table IV lists the energy of the photon associated with each pair, estimated from the measured opening angle, and the radial and longitudinal distance of the electron pair from the origin of the first pair.

Figure 1 shows a two dimensional projection of the pairs and their opening angles for two events. A radiation length in emulsion is 2.9 cm, so we see that the incident neutral particle had to travel more than a radiation length before any electron pairs were seen. It is difficult to explain the shower in terms of π° decay. Since the pairs have a most probable opening angle of annihilation the order of 10^{-3} rad., the maximum angle between the two photons from π° decay should be of the same order. If equipartition of energy between the two photons is assumed, the energy of the π° and the angle θ_p between its two decay products are related by the formula

$$\theta_p \approx \frac{2\, m_\pi c^2}{E_\pi} = 2/\gamma \tag{74}$$

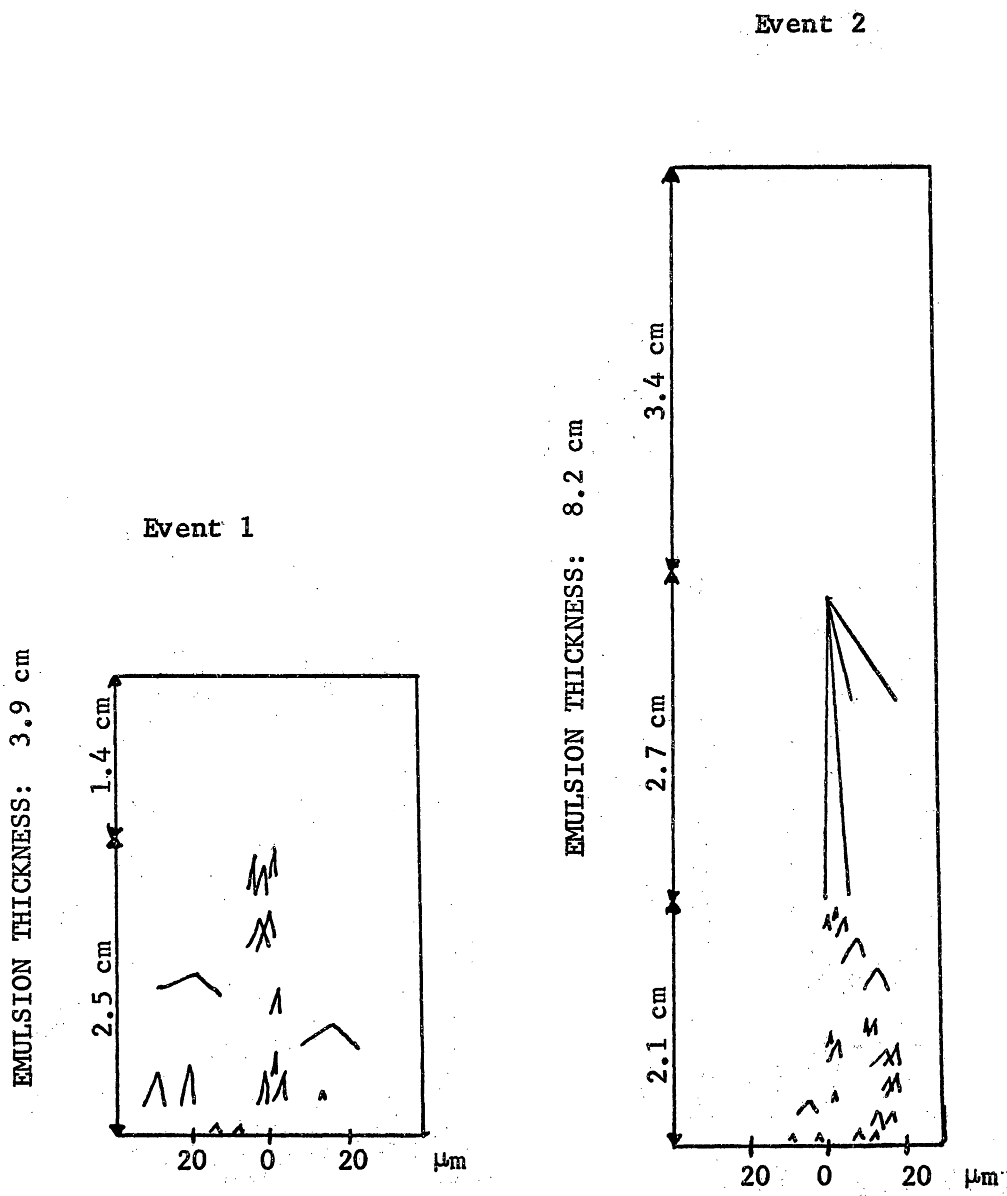
which gives an energy of 270 GeV for the π° . The sum total energy of the pairs given in Table IV is a factor of five lower than this number. Assuming for the moment that the π° hypothesis is correct and that all the energies are grossly underestimated, the origin of the π° must be found. A π° with γ = 2,000 will travel only 2 or 3 mm before decaying to photons. But from Figure 1, we see that there was no charged particle which could cause a nuclear interaction that could create a π° of this energy in five to twenty π° lifetimes.

From the above, an interpretation in terms of a purely electromagnetic process becomes more appealing. Perhaps these showers can

Table IV. Data from two narrow shower events.[41]

Event	Pair no.	Angle (mrad.)	Energy (GeV)	Radial distance (μm)	Distance from first pair (mm)
1	1	0.2	10.0	--	--
	2	0.2	10.0	6.0	.76
	3	1.0	2.0	2.0	2.20
	4	1.0	2.0	< 1.0	5.03
	5	1.0	2.0	2.5	6.80
	6	10.0	0.2	3.5	11.82
	7	0.6	3.3	1.5	12.16
	8	10.0	0.2	28.0	16.56
	9	0.6	3.3	30.0	18.67
	10	1.0	2.0	3.0	19.50
	11	2.0	1.0	35.0	19.85
	12	1.0	2.0	2.0	20.00
	13	< 1.0	> 2.0	3.5	21.00
	14	8.0	< 0.3	22.0	24.50
			$\Sigma E_\gamma > 40$	($\theta_\rho \approx 1$ mrad, $\theta^{max} = 10$ mrad)	
2	1	1.0	2.00	0.5	--
	2	0.2	10.00	0.0	.55
	3	1.0	2.00	1.5	27.24
	4	0.6	3.30	0.0	27.44
	5	2.0	1.00	4.5	27.69
	6	10.0	0.20	7.0	30.51
	7	20.0	0.10	12.0	32.99
	8	25.0	0.08	63.0	35.66
	9	1.0	2.00	11.0	36.42
	10	--	--	11.0	36.42
	11	0.4	5.00	0.0	37.24
	12	1.0	2.00	0.0	37.29
	13	0.1	20.00	17.0	39.16
	14	25.0	0.08	24.5	39.56
	15	1.0	2.00	15.0	41.21
	16	8.0	0.25	18.0	41.61
	17	0.3	6.60	10.0	42.46
	18	0.8	2.50	15.5	43.01
	19	50.0	0.04	11.5	44.21
	20	3.0	0.66	4.5	45.46
	21	2.0	1.00	21.0	45.91
	22	50.0	0.04	27.0	46.46
	23	< 2.0	> 1.0	9.0	46.76
	24	< 2.0	> 1.0	3.5	46.81
			$\Sigma E_\gamma > 62$	($\theta_\rho \approx 1.2$ mrad, θ^{max}=50mrad)	

Figure 1. Projected points of origin and opening angles of the pairs.



be explained by an electro-photonic cascade initiated by a single high energy photon. This process, however, fails to explain the energy distribution of pairs in the development of the showers. In event 2, for example, the first pair has an opening angle ten times greater than the opening angle of the second pair; moreover, its two electrons do not travel in the same direction as the shower. It is difficult to see how the second pair could be a result of the first, which has less energy. Further, in the one photon hypothesis, the pairs 2 and 3 of event 1 must be the materialization of photons radiated by the electrons of pair number 1. The photons associated with pair 2 and 3 would make an angle of at least 5 and 1 milliradians, respectively, with the direction of pair 1. But the distribution of angle between the electron and the photon in the bremsstrahlung process is the same as for the photon and positron in pair production.[44] Thus it is improbable that an electron of a pair whose opening angle is 0.1 millirad. should radiate a photon with an angle ten times larger.

An interpretation of the event as the materialization and successive multiplication of even three simultaneous photons has two difficulties: some of the pairs have too high energy and there are still too many pairs. The experimental data are best explained if 10-12 pairs are the materialization of the same number of incident photons whose energies vary between 2 to 10 GeV.

Ruderman and Zwanziger[39] speculate that these 10-12 photons result from the formation, decay, and annihilation of bound monopole pairs in the emulsion. The fact that almost a radiation length of material is traversed before any photon conversion suggests the possibility that a high-energy photon, interacting in the first radiation length of material with a nucleus, created a monopole pair whose interaction and annihilation explain the number and energy of the photons. The mechanism for the production of poleonium could be analogous to that for electron-positron pairs. For a photon incident on a nucleus with proton number Z, and atomic number A, the probability for electronic pair production is a thickness dx in g/cm^2 is given by[45]

$$\Phi_{pair} = 4\alpha N \frac{Z^2}{A} r_e^2 \left[\frac{7}{9} \ln(183 Z^{-1/3}) - \frac{1}{54} \right] \tag{75}$$

For a monopole of charge $g = 137e$ and mass ≈ 10 GeV/c^2, its classical radius, r_g, is equal to the classical radius of the electron, r_e. Thus at large energy it appears that the probability for monopole pair production is about equal to the probability for electron-positron pair production.

The γ of the center-of-mass of the observed photon showers can also be explained in terms of bound monopoles. In order that the momentum transfer, $\vec{q}$, to the participating nucleus be small compared to the nuclear radius, we must have

$$|q^2| = \left(\frac{8 m_g^2}{2E_\gamma} \right)^2 \gg \frac{1}{R^2} \approx \left(\frac{m_\pi c^2}{A^{1/3}} \right)^2 \tag{76}$$

Therefore

$$E_\gamma \gtrsim \frac{4A^{1/3} m_g^2 c^2}{m_\pi} \tag{77}$$

Then for a bound pair of approximate mass $\leq 2m_g/c^2$, the center-of-mass of the photon shower has, for a silver nucleus which is the main component of nuclear emulsions,

$$\gamma = \frac{E_\gamma}{2 m_g c^2} \approx \frac{2 A^{1/3} m_g}{m_\pi}$$

$$= A^{1/3} (g^2/\hbar c) \approx 10^3 \qquad (78)$$

This γ is consistent with the γ calculated for the events observed by Debenedetti.[41] The γ given above also gives an estimate of the energy of the incident photon, from equation (77). For $\gamma = 10^3$, the energy of the photon is approximately 10^{13} eV.

PROPOSED EXPERIMENT

The anomalous events pointed out by Ruderman and Zwanziger seem to fit their theory of bound monopole pairs, yet they are too few in number to give definite evidence for the existence of monopoles and to rule out the possibility that they are not statistical fluctuations. We propose here an experiment to examine a large number of photon showers in order to test the Ruderman-Zwanziger model for monopole pair production.

Production Process

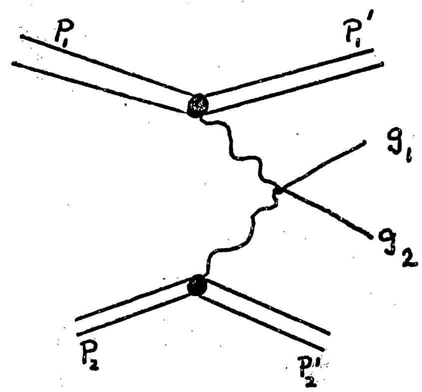
A possible mechanism for the production of monopoles would be through strong interactions, such as proton-proton collisions where we could have

$$p + p \rightarrow p + p + g^{+} + g^{-} \tag{79}$$

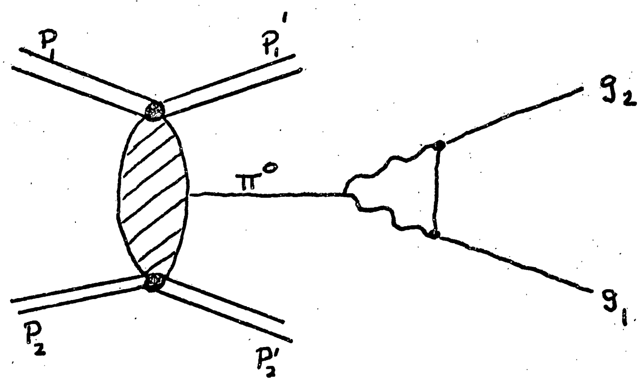
where the g's represent the monopole pair. Amaldi _et al._[11] have considered the production processes shown in Figure 2. They have made a rough estimate of total cross section in each case and obtain a value of the order of 10^{-35} cm^2 for monopoles with mass equal to 3 proton masses. Amaldi _et al._[11] state that the cross section calculations, which are based on perturbation theory, may be invalid because of the large coupling (eq.(72)) of the monopole to the electromagnetic field. Since the production of monopoles of mass 10 GeV/c^2 at rest in the center-of-mass system requires at least 20 GeV center-of-mass energy, the incident proton energy would have to be greater than 200 GeV. This fact plus the very low cross section forces us to look for other mechanisms for monopole production.

Figure 2. Possible processes for monopole production.

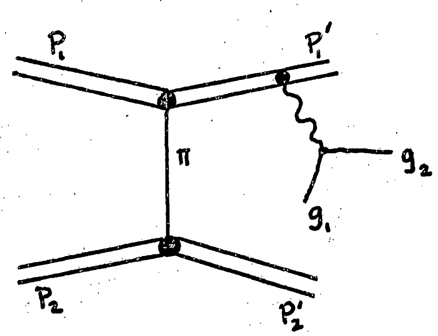
(a) Production of a pair of poles involving only the interaction of the Coulomb fields of the two colliding protons.



(b) Production of a pair of poles via the production of a virtual pion and its decay into two gamma rays.



(c) Production of a pair of poles by a virtual gamma ray, following the exchange of a virtual pion between the proton and the nucleon.



The monopole pair appears to be an "overgrown" electron-positron pair which carry magnetic charge instead of electric charge. We have seen that the probability for monopole production above threshold is the same as that for electron-positron production (eq.(75)). Therefore we will look for monopole pairs in the process of pair production in the field of a nucleus by a high energy photon. The analogy to electron-positron production is seen in our equation

$$\gamma + N \rightarrow N + g^{+} + g^{-} \tag{80}$$

As was seen in the last section, the incident photon energy should be of order 10^{13} eV in order to produce monopole pairs. Photons of this energy exist in cosmic radiation at high altitudes, but the flux is very small. For instance, at an altitude of 13,000 feet, the flux of photons with energy greater than 1 TeV is given by[46]

$$I \approx 10^{-8} \text{photons/cm}^2 \cdot \text{sec} \cdot \text{ster.} \tag{81}$$

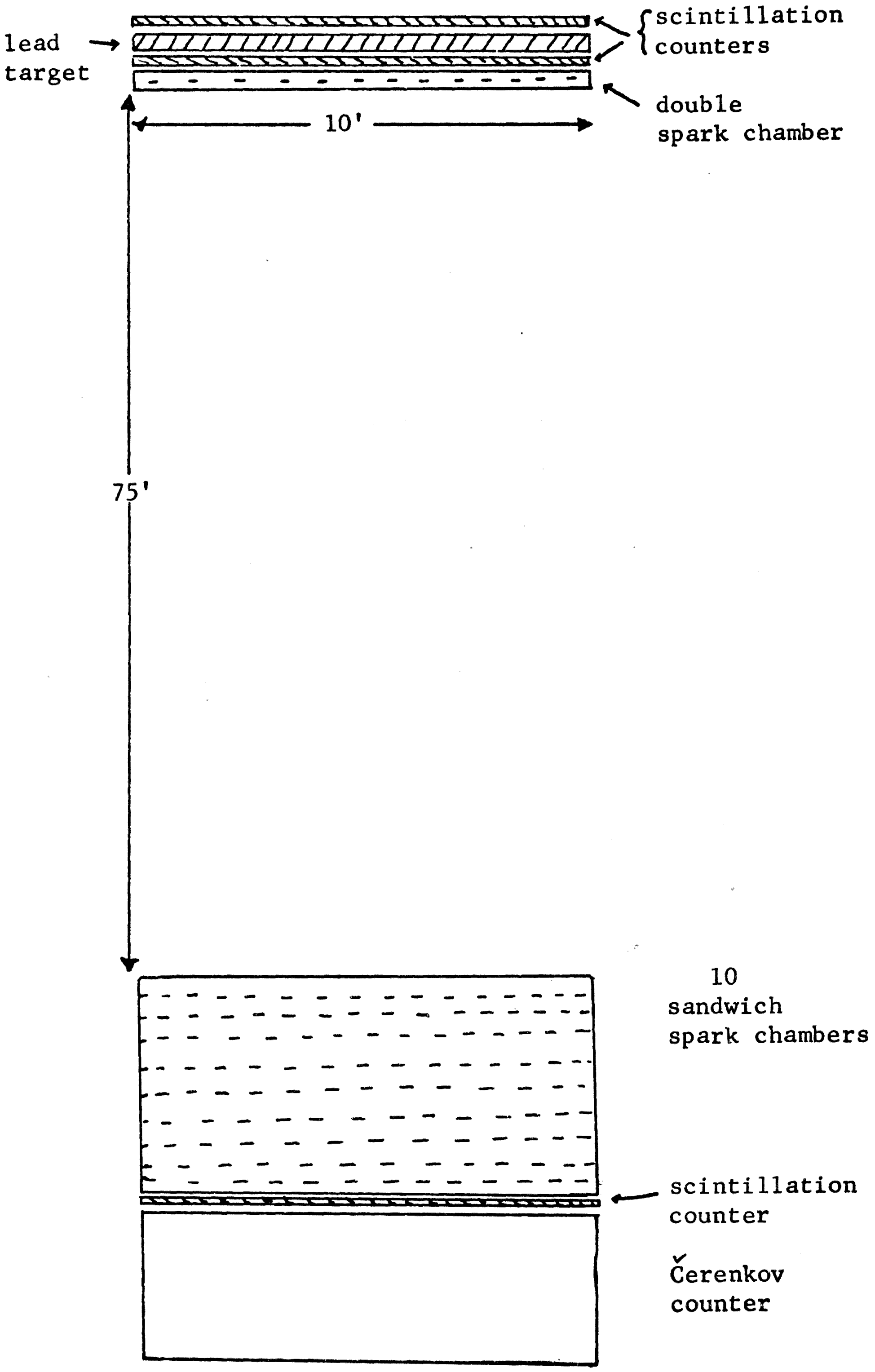
Experimental Equipment

A schematic of the experimental set-up is shown in Figure 3.

The probability for production of monopole pairs by high energy photons is proportional to the square of the proton number of the target (Z^2). We will use a high Z material, lead, of one radiation length thickness (.51 centimeters) as a target.

Scintillation counters, used to detect charged particles, will be placed above and below the target. We want to investigate only photon produced monopole pairs, so the counters will be used in anti-coincidence. That is, they will be used as part of a trigger system

Figure 3. Schematic of experimental set-up.



to tell us when we have a neutral particle incident on the target.

Directly below the target will be two proportional wire spark chambers, one x-y, one u-v. These chambers, which have high multiparticle detection efficiency, will tell us the coordinates of any charged particles leaving the target. If a photon incident on the target produces an electron-positron pair, the spark chamber will allow us to detect these particles and obtain an estimate of the x-y coordinates of the interaction vertex. If bound monopole pairs are produced, some of their radiated photons may convert to electrons-positron pairs before leaving the target. If this does happen, then we can also determine this interaction vertex.

Seventy-five feet below the target will be an array of 10 similiar double spark chambers (x-y and u-v) with .2 radiation lengths of lead in between each double chamber. The large distance from the target is needed in order to resolve the separate photons produced in the photon showers. The photons from monopole pair production are expected to be in a very small cone with an average separation of photons on the order of a milliradian. The .2 radiation lengths of lead above each chamber will allow the gradual conversion of the photons to electron-positron pairs, which can be detected by spark chambers. In order to resolve two different pairs in the chamber, they must be separated by a least .1 inches. We can not hope to identify each electron in the pair because their separation is much smaller than the resolution of the chamber. The array of counters will allow us to study the development of the electron-photon shower as a function of shower depth. Each spark chamber will give us the number of electron-positron pairs and the x-y coordinates.

Underneath the spark chamber array will be another scintillation counter and a total absorption Čerenkov counter. The scintillation counter, in anticoincidence with the counter above the target, will be used as part of the triggering system. The Čerenkov counter will consist of a large tank of water which is viewed by photomultiplier tubes. Using the tank, we can obtain a rough estimate (within 50%) of the total energy of the shower.

A Typical Event

The three scintillation counters; S1 above the target, S2 below the target, and S3 above the Čerenkov counter will be used to trigger the system. We will require that we have a signal (indicating at least one charged particle) from S3, in anticoincidence (no signal) with S1, indicating no charged particle incident on the target. We will use S2 in anticoincidence with S3 when we want to look at the case of no charged particles leaving the target. If our trigger system indicates a neutral particle incident on the target, we will then test the signal from the Čerenkov counter to see if the pulse height indicates a shower energy compatable with a neutral incident particle energy of 10^{12} eV. If we do have sufficient incident energy, we will then pulse the spark chambers and read out the coordinate points on each of the planes.

Our data will consist of x-y or u-v points from each spark chamber, a pulse height from the Čerenkov counter, which will tell us the energy of the shower, and indicator that tells us if counter S2 had a charged particle passing through it. All the data will be written on magnetic tape for computer analysis.

Analysis of Data

The data from the spark chamber will allow us to determine the number and coordinates of the charged particles, but we must first eliminate the spurious data points. These points occur whenever there is more than one particle passing through the chamber at the same time. If we have two particles passing through a x-y chamber, our data will consist of an (x_1,x_2) and (y_1,y_2). There are now four possible points which can be formed from these coordinates. Two points, (x_1,y_1) and (x_2,y_2), represent the points the charged particles passed through. The points (x_1,y_2) and (x_2,y_1) represent spurious points which have no physical significance and must be eliminated.

For each .2 radiation length of lead in the chamber array we will use double chambers (one x-y, one u-v) to aid in eliminating spurious points. For each pair of chambers, we will take the x-y coordinates from the "above" chamber and make all possible point combinations. For this point we will calculate the corresponding point on the "below" chamber (u-v chamber), and ask that there be a data point within a small radius of this point. If we do locate a point on the "below" chamber, then we have located two points on the track of a charged particle passing through the array. By examining all points on all pairs of chambers in this way, we can determine the number and positions of the charge particles on each plane.

Once we know two points for the particle we can calculate its line trajectory and determine where it was produced simply by checking above each layer of lead to see if there is a point lying on the trajectory. We may also in this way obtain a rough estimate of the location of the interaction in the target by projecting all of the particle trajectories in the array back to the target.

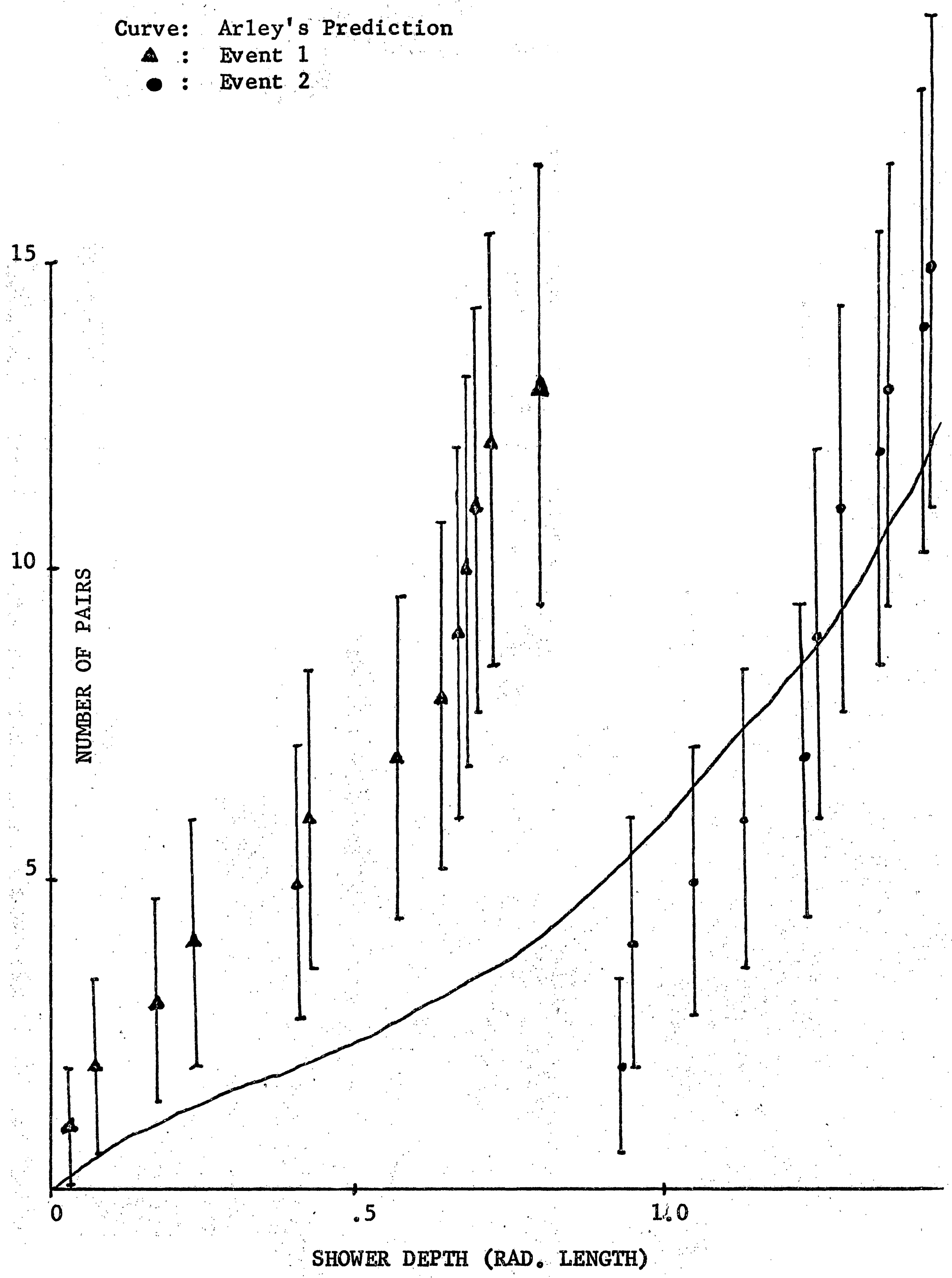
Experimental Requirements

In monopole pair production, photons are produced through the acceleration and deacceleration of the poles, and through their mutual attractive Coulomb interaction. Ruderman and Zwanziger[39] state that the multiplicity is essentially constant and of the order of 100. The multiplicity of showers produced by bremsstrahlung and pair production depends on the energy of the incident particle, its charge and the medium surrounding the particle.

If we have a high energy photon incident on our lead target, the photon may produce an electron-positron pair or a bound monopole pair. By observing the interaction in the sandwich spark chamber array below the target, we can distinguish the two types of events. In the case of electron-positron production, a cascade shower will be initiated by these two particles. In the case of monopole production, since they radiate about 100 photons in less than 10^{-23} sec, we would also have a cascade shower, but one produced by about 100 "incident" particles!

Arley[47] derived a theory of electromagnetic cascades which appear to agree with observed photon-electron cascade showers for incident energies of the order 10^{12} eV. In Figure 4 we have plotted Arley's

Figure 4. Number of pairs versus shower depth.



prediction for the number of electron-positron pairs as a function of shower depth in radiation lengths, and the data from the two anomalous events observed by Debenedetti.[41] Since no errors were quoted on this data, we have taken the error to be $\pm\sqrt{N}$, where N is the number of pairs. With a little imagination, we can state that the two events do agree with the theory, but we must point out that the lowest energy pair that Debenedetti recorded is 100 MeV. This is encouraging in that the cut-off energy of the photons in bound monopole production is a few MeV and many low energy pairs could have been undetected in the events of Debenedetti. We must therefore determine absolutely the number of pairs as a function of shower depth in order to distinguish between electron-positron production and bound monopole pair production.

Using Arley's theory[47] as a base, we will perform a Monte-Carlo calculation on the development of photon showers produced by one incident photon. From this calculation we can obtain more information about the possible fluctuations in shower development and be able to estimate what can be expected for electron-positron pair production.

Specious Events

Our triggering system permits us to trigger on all neutral particles incident on the target, so we will have to eliminate events caused by neutrons or π^0's.

The neutrons will interact strongly with the target and the secondary particles produced will mainly be hadrons. We can eliminate these events from our data by observing the secondary particles and their interaction with our chamber array. A π^o incident on the target presents a more serious problem. The π^o will decay into photons which will cause a cascade in our chamber array. We will first do a Monte-Carlo calculation of the resulting shower to determine the possible fluctuations in shower development due to photons from π^o decay. We believe that these fluctuations will not be so large that the π^o decay events can not be separated from the bound monopole production events.

Location of Experiment

A reasonable flux of cosmic rays with energy greater than 10^{12} eV exists at mountain altitudes. We propose to perform our experiment atop Mt. Evans which is located in Colorado at an altitude of 13,000 feet, where the incident flux of photons with energy greater than 10^{12} eV[46] is given by equation (81).

Our target and detection array will have a cross-sectional area of approximately 10 feet squared, so our maximum solid angle will be

$$d\Omega \cong \frac{(\pi)5^2}{75^2} \approx .02 \text{ ster.}$$

The probability for monopole production, assumed to be equal to the probability of electron-positron production, is .8 in one radiation length of lead. The number of expected monopole events in a time T, is given by

N = (flux) x (probability) x (time) x (area of target) x (solid angle).

From this we calculate the number of monopole events per day as

$$N = (10^{-8}\text{photons/cm}^2\cdot\text{sec}\cdot\text{ster})\times(.8\ \text{monopoles/photon})\times(8.6\times10^4\ \text{sec/day})\times(9\ \text{meters}^2)\times(.02\ \text{ster})$$

$$N \approx 2\ \text{monopole events/day}$$

The dead time of our system, the time during which the system is insensitive to events, is determined by the time it takes to read out all of the data from spark chambers and place it on magnetic tape. This time is typically less than a second so we will not be limited by this factor.

We will perform our experiment over a 3 month period in order to obtain enough data to make conclusive statements about the Ruderman-Zwanziger model for bound monopole pairs.

REFERENCES

1. P. A. M. Dirac, Proc. Roy. Soc. (London) Ser. A 133, 60 (1931).
2. P. A. M. Dirac, Phys. Rev. 74, 817 (1948).
3. N. A. Porter, Nuovo Cimento 16, 958 (1960).
4. E. Goto, Progr. Theor. Phys. 30, 700 (1963).
5. T. E. Cranshaw and W. Gilbraith, Phil. Mag. 2, 804 (1957).
6. L. I. Schiff, Phys. Rev. 160, 1257 (1967).
7. L. I. Schiff, Phys. Rev. Letters 17, 612 (1966).
8. J. Schwinger, Science 165, 757 (1969).
9. E. Amaldi, Old and New Problems in Elementary Particles, (Academic Press, New York, 1968) p. 1.
10. N. F. Ramsey, Phys. Rev. 109, 225 (1958).
11. E. Amaldi, et al., CERN Report 63-13 (1963) (unpublished).
12. E. Bauer, Proc. Cambridge Phil. Soc. 47, 777 (1951).
13. H. J. D. Cole, Proc. Cambridge Phil. Soc. 47, 196 (1951).
14. E. Segre, Nuclei and Particles, (W. A. Benjamin, Inc., New York, 1965) p. 28.
15. B. Rossi, High-Energy Particles, (Prentice-Hall, Inc., New York, 1952) pp. 57-59.
16. D. R. Tompkins, Phys. Rev. 138, B248 (1965).
17. D. R. Tompkins, Phys. Rev. 140, B443 (1965).
18. E. Fermi, Phys. Rev. 57, 485 (1940).
19. J. A. Jackson, Classical Electrodynamics, (John Wiley and Sons, Inc., New York, 1962) pp. 444-446.
20. W. V. R. Malkus, Phys. Rev. 83, 899 (1951).

21. H. Bradner and W. Isbel, Phys. Rev. 114, 603 (1959).

22. M. Fidecaro, et al., Nuovo Cimento 22, 657 (1961).

23. E. Purcell, et al., Phys. Rev. 129, 2326 (1963).

24. I. Gurevich, et al., Phys. Letters 31B, 394 (1970).

25. W. Carithers, et al., Phys. Rev. 149, 1070 (1966).

26. A. Erlykin and V. Yakovlev, JETP 29, 992 (1969).

27. P. J. Green, II., Louisiana State University and Agricultural and Mechanical College, 1967, (Ph.D. Thesis) (unpublished).

28. V. Petukhov and M. Yakimenko, Nucl. Phys. 49, 87 (1963).

29. E. Goto, et al., Phys. Rev. 132, 387 (1963).

30. H. H. Kolm, Phys. Today 20, 69 (1967).

31. H. H. Kolm, Application of Modern Physics to the Earth and Planetary Interiors, (John Wiley and Sons, London, 1968) p. 661.

32. R. Fleischer, et al., Phys. Rev. 177, 2029 (1969).

33. R. Fleischer, et al., Phys. Rev. 184, 1393 (1969).

34. R. Fleischer, et al., Phys. Rev. 184, 1398 (1969).

35. L. Vant-Hull, Phys. Rev. 173, 1412 (1968).

36. L. Alvarez, et al., University of California Radiation Laboratory Report, UCRL-19756 (1970) (unpublished).

37. L. Alvarez, et al., Science 167, 701 (1970).

38. H. Berkowitz and S. Kronenberg, U. S. Army Electronics Command, Fort Monmouth, N. J., Technical Report, ECON-2710 (1966) (unpublished).

39. M. Ruderman and D. Zwanziger, Phys. Rev. Letters 22, 146 (1969).

40. M. Schien, et al., Phys Rev. 95, 855 (1954) and 99, 643 (1955).

41. A. Debenedetti, et al., Nuovo Cimento 12, 954 (1954).

42. M. Koshiba and M. Kaplon, Phys. Rev. 100, 327 (1955).

43. A. Borsellino, Phys. Rev. 89, 1023 (1953).

44. H. Bethe and W. Heither, Proc. Roy. Soc. A146, 83 (1934).

45. B. Rossi, ob. cit., p. 81.

46. L.W. Jones, Proceedings of the Tenth International Cosmic Ray Conference (International Union of Pure and Applied Physics, Calgary, Canada, 1967) p. 480.

47. N. Arley, Proc. Roy. Soc. A168, 519 (1938).

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A PROPOSED EXPERIMENT TO DETECT THE MAGNETIC MONOPOLE OF THE RUDERMAN-ZWANZIGER MODEL

Donald M. Stevens

Abstract

In 1931 P.A.M. Dirac postulated the existence of a quantum of magnetic charge, the magnetic monopole. Since its prediction, cosmic ray and accelerator experiments have been performed to detect the monopole, but all have failed. M. Ruderman and D. Zwanziger have put forward an explanation for the negative experimental results. They argue that monopoles are likely to be produced in pairs, which are tightly bound and never become free. Ruderman and Zwanziger point to certain anomalous cosmic ray events as evidence of production of bound monopole pairs. In this paper we review Dirac's arguments, calculate the monopole's properties, and critically review previous monopole experiments. Using the model proposed by Ruderman and Zwanziger to give identifying properties, we propose an experiment designed to detect the materialization of the magnetic monopole.