FINITE ELEMENT ANALYSIS OF PIERCED SHEAR WALLS

by

John Walter Weaver

Thesis submitted to the Graduate Faculty of the

Virginia Polytechnic Institute

in partial fulfillment for the degree of

MASTER OF SCIENCE

in

Civil Engineering

APPROVED:

Dr. R. M. Barker, Chairman

Dr. G. A. Gray

Dr. G. W. Swift

Dr. R. D. Walker

JUNE 1970

Blacksburg, Virginia

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KLB 7-22-77

ACKNOWLEDGEMENT

Grateful appreciation is extended to , the author's major professor, for his teaching, patience, and guidance throughout this study.

Acknowledgement is expressed to each of the author's professors for their influence in his academic development.

The author wishes to thank the VPI Computing Center and the College of Engineering for financing this study. He also wishes to acknowledge NSF for its financial aid during his first year of graduate work.

Sincere gratitude is extended to and for typing this thesis.

For her untiring support and confidence, the author wishes to extend a special thanks to his beloved wife, .

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I. INTRODUCTION

The increased use of shear walls in multistory buildings as an economical method of providing lateral stability against the forces of wind, earthquake and possible explosion, has increased interest in developing an efficient method for their analysis. The conventional analysis of shear walls is complicated by the architectural and mechanical demands requiring them to contain openings for doorways, windows, as well as plumbing and ventilation ducts.

As illustrated in current literature, the finite element method lends itself nicely to the analysis of pierced shear walls, due to its ability to handle openings in any continuum in the normal course of analysis. A finite element analysis requires the use of a digital computer because the number of calculations necessary for obtaining a solution are too numerous to be done manually. However, in using the computer the analyst will also obtain displacements and corresponding stresses at almost any point he wishes.

In some instances it may be advantageous to carry the economical benefit of shear walls one step further by using "tapered" shear walls. The "tapering" of shear walls in tall buildings may institute a saving of material by varying the thickness of the wall through different sections of the building. The inherent ability of the finite element method to handle changes in section properties, arbitrary loadings as well as numerous boundary conditions for a structure seems to indicate that this method of analysis is idealy suited for use on general pierced shear wall configuration.

It is the intent of this study to illustrate the applicability of a refined finite element method of analysis to the problem of pierced shear walls. The finite element program developed incorporates a refined plane stress rectangular elements plus beam elements. Due to the generality of the pierced shear walls that may be analyzed using this refined finite element procedure, this method merits consideration as the preferred tool for the approximate solution of these problems.

There is little documented knowledge of the interaction of framed structures with their stiffening shear walls. The finite element program developed in this study, as mentioned earlier, makes use of beam elements, which may be used to represent beams, columns and/or diagonal bracing in framed buildings. This program is used to investigate the interaction between shear walls and their surrounding frames. This method of analysis may also be used to analyze the effect of infilled frames in a framed structure.

II. LITERATURE REVIEW

The use of shear walls in structures today may be separated into two broad functional classes:

a) Pierced shear walls with one or more bands of openings.

b) Shear walls with or without openings incorporated as a stiffening agent in multistory frames.

Both classes of shear walls are being used as a principle lateral load bearing entity in multistory buildings.

The prime structural unit in the class of pierced shear walls is the shear wall itself. This class of shear walls may be visualized as a series of vertical cantilever beams connected at discrete levels by lintel beams or floor slabs as illustrated in Fig. 12. Classically, these systems have been approximated as deep columns connected by an equivalent continuous medium to replace the discrete physical system. This method of analysis received popular recognition in the early 1960's as shown by the work of Rosman[13], as well as that of Coull and Choudhury[3]. Fundamental to this method of analysis are the assumptions that the walls deflect equally and that the connecting lintel beams possess a point of counterflexure at their midspan.

On reviewing the assumptions necessary for this method of analysis, it is apparent that the following major points have not been considered: 1) the stress concentrations at the corners of the opening; 2) the local distortions at the junction of the lintel beams and wall sections due to the forces and moments exerted by the lintel beams; 3) the distortion of the shear wall (the lengthening and shortening of the wall sections) and its influence on the lintel beams; 4) the influence of shear deformation in the wall section and; 5) the effect of axial forces on the lintel beams.

Coull and Puri[4] extended the afore mentioned analysis to include the effects of shear deformation in the walls. They investigated the validity of their method through a series of experimental model tests. With deflections from these tests on shear walls with one and two bands of opening, under uniform and point loads, they justified the need for considering shear deformation in shear wall analysis.

Sensmeier[15] formulated a finite difference solution for the governing differential equations of shear wall continuum to be used with a computer. He correlated the stresses from an analytical and photoelastic analysis of several pierced shear wall models loaded at the top only. The photoelastic studies reiterated the presence of high stresses at the corners of the opening but also indicated areas af almost neglectable stress in some of the continuous wall areas. Sensmeier investigated the influence of various length to depth ratios in his analytical and model studies. A portion of these studies will be presented later.

Late in 1969, Schwaighofer and Microys[14] presented a method of analysis of pierced shear walls which is simple to use and requires only methods familiar to most practicing engineers. This method reduces any shear wall configuration into an equivalent frame idealization. Although this method gave good correlations with a model study, its effectiveness is dependent on the proper conversion of the shear wall properties to those to be used in the equivalent frame.

Schwaighofer and Microys' method may also be applied to the second functional class of shear wall problems. In this class, the frame portion of the structure is the prime element and the shear wall is added as a stiffener for lateral loadings. This type of structure is shown in Fig. 19. Although this is a logical method of analysis, it is still dependent on the effectiveness of proportioning the equivalent frame.

A more classical method of analyzing frames with shear walls is to separate the frame and shear wall and analyze each separately. This type of method was used by Khan and Sbarounis[9]. Another method that has been presented is to model shear walls in framed structures as elastically restrained cantilevers such as that used by Gould[8]. Gould's method of analysis replaces the frame connections to the shear wall with linear and rotational springs. The constants for these springs are derived from the properties of the frame and shear wall configuration. A finite difference formulation is used to express the relationship between the frame and the shear wall in order to obtain the deflection and rotations for the shear wall. Then a separate computation is necessary to find the moments and shears for frame members connected to the shear walls. Gould's method does not include shear deformation in its analysis.

A common restriction on all of the above methods of analysis for either one or both types of shear wall configurations is that special effort must be put forth to analyze structures of varying material properties, different loading conditions, and varying cross section. These parameters commonly render a solution under one of the above methods impractical or increase its order of approximation.

With the development of large digital computers, the finite element method of structural analysis has been adopted by several individuals for the analysis of shear walls. The finite element method is more ideally suited for the solution of displacements and stress in any general shear wall configuration in that it can readily handle variations in thickness of the walls, irregularities and variations in loading, irregularities in the geometry and configuration of openings, and variations in material properties in the normal course of analysis. The finite element method of analysis is basically the idealization of a continuum as small discrete elements considered to be interconnected at their nodes. The finite elements possess the same material properties as does the continuum. When the assemblage of the elements is subjected to a given loading condition, displacements (linear and/or rotational) are induced at each node in the system. In the stiffness approach to finite element analysis, it is the structural stiffness in the form of a stiffness matrix which relates the loads to the nodal displacements through conventional matrix theory of structures. The total structural stiffness matrix is the sum effect of all the element stiffnesses.

Girijavallabhan[7] has investigated the use of plane stress triangular and rectangular elements in the finite element analysis of shear walls. The triangular element's advantage of being able to fit more general boundary conditions than the rectangular elements, is of little importance in the analysis of shear walls since most shear walls and their openings are orthogonal. Girijavallabhan found that it is more efficient to use rectangular elements in his analysis in that rectangular elements are more flexible than triangular ones, and are therefore more accurate because the displacement method of analysis converges from the stiff side.

The plane stress finite elements possess two linear orthogonal displacements at each node. Using plane stress elements it is necessary to represent lintel beams or beams connected to a shear wall in framed structures as bar elements (only two orthogonal displacements at each end) in order for them to be compatible with the rest of the shear wall idealization. If a beam is idealized as a bar, one loses the prime function of a beam - that of bending. Girijavallabhan developed a method whereby he idealized the lintel beams as a series of small finite elements in order to determine the displacements and rotations at the ends of these beams, then a slope deflection procedure was used to calculate the forces and moment on the end of the beams. He did not compare his finding with any type of model study.

MacLeod[10] has presented a rectangular finite element which, in a manner, possesses two orthogonal linear displacements as well as a rotation displacement at each node. He points out that this element may be beneficial in the analysis of shear walls in that it has the necessary degrees of freedom to be compatible with a standard plane frame beam element. He did not offer any comparisons with experimental data to verify the usefulness of his element. The increased interest of shear wall designers in the interaction of a beam and a shear wall questions the usefulness of MacLeod's element. It is the prime purpose of this work to investigate the use and the validity of MacLeod's element, as well as to verify the use of the rectangular plane stress element in the analysis of general shear wall structural systems.

III. THE GENERAL FINITE ELEMENT PROCEDURE

The finite element method (FEM) of structural analysis is equivalent to a Ritz procedure whereby the potential energy of a continuum, idealized as a system of discrete segments, is minimized in order to investigate a state of structural equilibrium. In the stiffness method of analysis, using the FEM procedure, the minimization of potential energy may be used to establish the relationship between loads applied to a structural system and their resulting displacements in that system. A review of the fundamental steps and specifications for the FEM will be presented herein, since the general theory of the FEM is covered in detail in the works of Clough[1] and Zienkiewicz[19].

The first step in the FEM is the idealization of the structural system into an imaginary arrangement of discrete elements. These elements are considered to be interconnected at a specific number of points lying on their boundaries.

The next step in this procedure is the selection of the element properties and the formulation of its displacement pattern or function. The displacement functions must uniquely define the state of strain within an element in terms of its unknown nodal displacements. Displacement functions are used to control the deformation of an element so that it will act as a continuum and in so doing, reduce discontinuites along its boundaries to a minimum.

A displacement function must include the following properties:

1) possess internal element continuity

2) be of a form that will provide for a state of constant strain

3) be of a form that will provide for rigid body displacements.

For the monotonic convergence of a solution by the FEM, it is necessary that the displacement function insure inter-element displacement compatability, thus preventing any areas of infinite strain. This last property is not essential in that valid solutions may be obtained in the limit as element sizes decrease when the first three properties are satisfied.

Let us now look at the FEM procedures in terms of its general matrix equations. In a finite element idealization, the element is defined by its nodes and boundaries. The internal displacements of the element $\{U\}$ may be expressed through the displacement function in matrix notation as

$$\{U\} = [M]\{\alpha\} \tag{1}$$

where $\{\alpha\}$ are undetermined constants. The nodal displacements $\{U_n\}$ may be expressed as

$$\{U_n\} = [C]\{\alpha\} \tag{2}$$

where $[C]$ is obtained from proper substitution of the nodal coordinates into matrix $[M]$. Strains may be expressed in matrix form as

$$\{\varepsilon\} = [Q]\{\alpha\} \tag{3}$$

where $[Q]$ is formed from the proper differentiation of the displacement function $[M]$. Using equation (2), the strains may be related to the nodal displacements as

$$\{\varepsilon\} = [Q]\,[C^{-1}]\,\{U_n\} = [B]\,\{U_n\} \tag{4}$$

The stresses and strains may be related through the relations of elasticity expressed by the elasticity matrix $\left[D\right]$ as

$$\left\{\sigma\right\} = \left[D\right] \left\{\varepsilon\right\} \tag{5}$$

For the plane stress element composed of isotropic homogenous material, the explicit elasticity matrix (3 x 3) is

$$\left[D\right] = \frac{E}{1 - \nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \tag{5a}$$

Using the principle of minimization of potential energy or the virtual work technique, as expressed in several works[1,19], the element stiffness matrix is found to be

$$\left[k\right] = t \left[C^{-1}\right]^T \left(\int_A \left[Q\right]^T \left[D\right] \left[Q\right] dxdy \right) \left[C^{-1}\right] \tag{6}$$

where t is the thickness of the element.

The forces acting on the element may be related to the nodal displacements as

$$\left\{F_e\right\} = \left[k\right] \left\{U_n\right\} \tag{7}$$

The standard methods of combining element stiffness matrices $\left[k\right]$ to obtain the total structural stiffness matrix $\left[K\right]$ may be used. Thus we have

$$\left\{F\right\} = \left[K\right] \left\{U_n\right\} \tag{8}$$

where $\left\{F\right\}$ is the matrix of equivalent nodal forces imposed on the structure.

Upon modification of the total stiffness matrix to include the effect of all prescribed displacement boundary conditions, this matrix becomes a positive definite, symmetric banded matrix. These properties of $[K]$ greatly facilitate the solution for the unknown displacements from equation (8) using any one of the several methods presented in the literature for use on the digital computer.

The solution of equation (8) to find the unknown displacements is made without any approximations in the mathematics after the initial idealization. The solutions obtained will be valid when the chosen displacement function allows the element to accurately approximate the continuum.

Once the nodal displacements are known, stresses anywhere in the element may be obtained by combining equations (4) and (5) as

$$\{\sigma\} = [D]\ [B]\ \{U_n\} \tag{9}$$

to express the stresses as a function of the nodal displacements.

In determining the stresses the analysis of the structure is completely outlined. Starting with the coordinates of the nodes, the elastic properties and physical dimensions of the elements, as well as our assumed displacement function, we have obtained the nodal displacements and element stresses for our structural system.

IV. DEVELOPMENT OF ELEMENT STIFFNESSES

In the course of this work it was deemed necessary to develop two separate FEM computer programs in order to fully investigate the usefulness and validity of MacLeod's element.[10]

The first of these programs uses a simple rectangular element with two inplane displacements at each node. The other incorporates MacLeod's element with three displacements at each node and plane frame beam elements. The development of each of these elements follows:

A. Rectangular 8 DOF Element

The 8 DOF element is so called because it allows two inplane displacements at each of its four corner nodes (see Fig. 1) - thus having a total of 8 degrees of freedom (DOF). This simple element has shown more rapid convergence characteristics than most of the general triangular plane-stress elements used to date.

The 8 DOF element used herein is essentially the same as presented by either Przemieniecki[12], Zienkiewicz[19], or Clough[1]. The displacement function used for this element is as follows.

$$\begin{aligned} u\,(x,y) &= \alpha_1 + \alpha_2\xi + \alpha_3\xi\eta + \alpha_4\eta \\ v\,(x,y) &= \alpha_5 + \alpha_6\xi + \alpha_7\xi\eta + \alpha_8\eta \end{aligned} \qquad (10)$$

where $\xi = x/a$ and $\eta = y/b$ are the dimensionless coordinates of any point in the element (see Fig. 1). This displacement function provides inter-element boundary compatability.

The square (8 x 8) element stiffness matrix may be formed

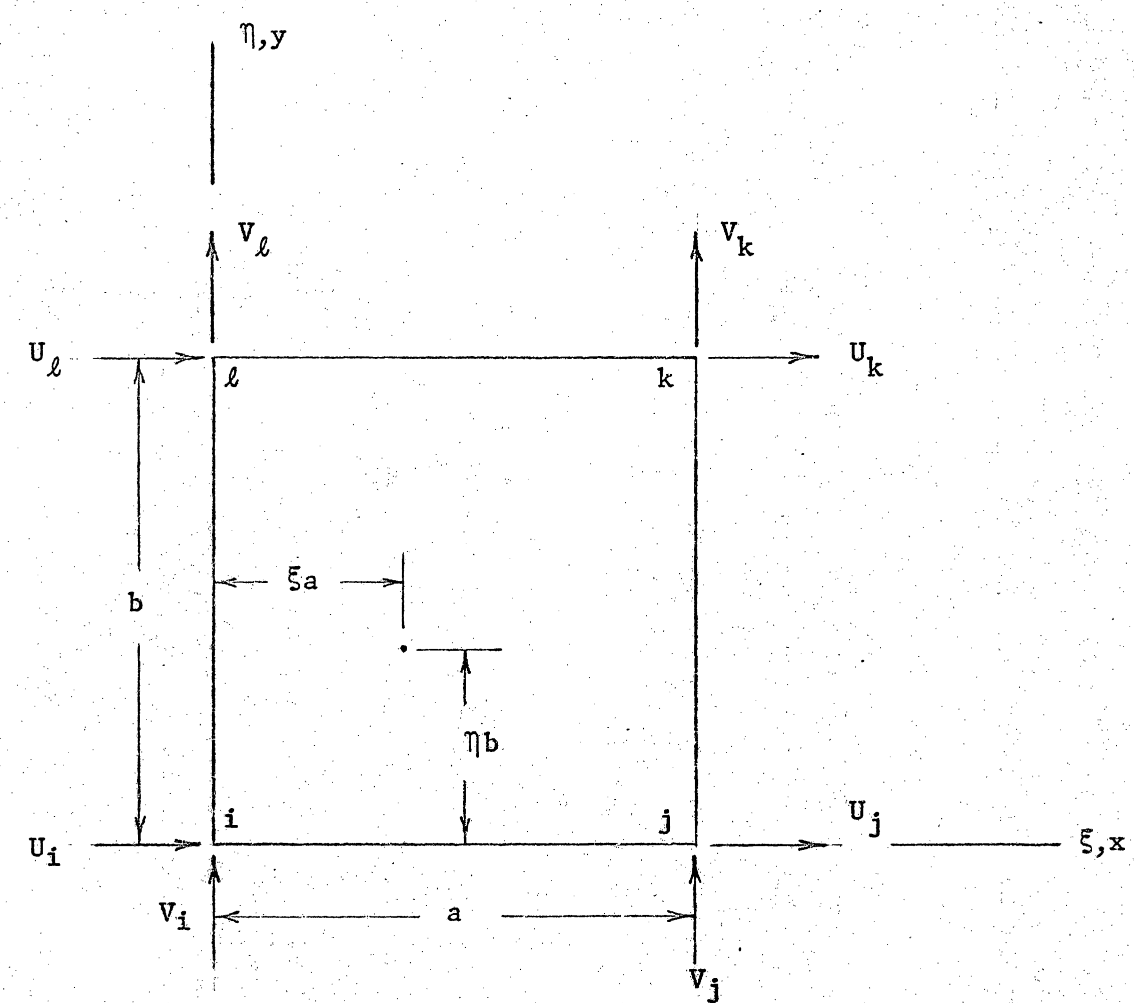


FIGURE 1. TYPICAL 8 DOF ELEMENT

explicitly, as given in expression 11, by using the FEM procedure previously reviewed.

The strains $\{\varepsilon\}$ are related to the coefficients $\{\alpha\}$ as

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \tau\, x\, y \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial \eta} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{Bmatrix} = \begin{bmatrix} 0 & \frac{1}{a} & \frac{\eta}{a} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\xi}{b} & \frac{1}{b} \\ 0 & 0 & \frac{\xi}{b} & \frac{1}{b} & 0 & \frac{1}{a} & \frac{\eta}{a} & 0 \end{bmatrix} \{\alpha\}$$

$$\{\varepsilon\} = [Q]\ \{\alpha\} \qquad \text{(3) repeated}$$

where a and b are the element dimensions.

The stresses are related to the strains by equation 5a presented earlier.

The element nodal forces and displacements in the coordinate directions may be related through the element stiffness matrix by equation (7) as

$$\{F_e\} = [k]\ \{U_n\} \qquad \text{(7) repeated}$$

where

$$\{F_e\}^T = (F_{xi}, F_{yi}, F_{xj}, F_{yj}, F_{xk}, F_{yk}, F_{xl}, F_{yl})$$

$$\{U_n\}^T = (U_i, V_i, U_j, V_j, U_k, V_k, U_l, V_l)$$

and matrix $[k]$ may be formed from the integration of expression (6) as the array given in expression (12).

$$[k] = \frac{Et}{12(1-\nu^2)} \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{matrix} \\ \begin{bmatrix} 4B+2AC & & & & & & & \\ -4B+AC & 4B+2AC & & & & & & \\ -2B-AC & 2B+2AC & 4B+2AC & & & \text{(Symmetric)} & & \\ 2B-2AC & -2B-AC & -4B+AC & 4B+2AC & & & & \\ D & E & -D & -E & 4A+2BC & & & \\ -E & -D & E & D & 2A-2BC & 4A+2BC & & \\ -D & -E & D & E & -2A-BC & -4A+BC & 4A+2BC & \\ E & D & -E & -D & -4A+BC & -2A-BC & 2A-2BC & 4A+2BC \end{bmatrix} \end{array} \quad (12)$$

Where: $A = a/b$

$B = b/a$

$C = 1-\nu$

$D = 3(1+\nu)/2$

$E = 3(1-3\nu)/2$

An explicit form of the matrix product of [D] and [B] , called matrix [DB], may be formulated to simplify the calculation of stresses from the displacements determined by equation (9).

Matrix DB would appear as:

$$\frac{E}{1-\nu^2}\begin{bmatrix} -A & A & B & -B & -CN & -DN & DN & -EN \\ -AN & AN & BN & -BN & -C & -D & D & C \\ -CF & CF & DF & CF & -AF & AF & BF & -BF \end{bmatrix} \qquad (13)$$

where $A = \dfrac{1-\eta}{a}$ $\qquad D = \xi/b$

$B = \eta/a$ $\qquad F = \dfrac{1-\nu}{2}$

$C = \dfrac{1-\xi}{b}$ $\qquad N = \nu$

B. Rectangular 12 DOF Element

The rectangular 12 DOF plane stress finite element as presented herein possesses three degrees of freedom at each of its corner nodes, thus allowing each node to undergo two orthogonal displacements in the coordinate directions plus an inplane "rotation". A twelve degree of freedom (DOF) element utilizing rotation at each node has been of great interest to the author for some time since such an element can be easily combined with plane frame beam elements. The combined use of a twelve DOF element and the plane frame beam element can be used to idealize most framed shear wall and pierced shear wall problems in that the beam element can model any member of a frame as well as any slender lintel beam. In view of these possibilities the development of such a twelve DOF element was attempted early in the course of this work. The most structurally desirous idealization of such an element would be one where the rotation used at each node would be that of classical particle rotation, namely,

$$\theta = \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \tag{14}$$

It was felt that the use of such a twelve DOF element in combination with beam elements in a finite element analysis would give a valid approximation of the actual structural system.

The development of the element stiffness matrix for this desired element was found to be mathematically impossible. Numerous symmetric and anti-symmetric displacement functions, as well as some derived from stress equilibrium conditions, were tried with no apparent success.

In March 1969, Macleod[10] introduced two rectangular elements possessing twelve DOF. Each of his elements (Type 1 and Type 2) possess a "rotation" of one of the intersecting sides at each corner node as shown in figures 2a and 2b. Upon combining these two elements in a "checkerboard" manner (so as to insure that no two elements with a common boundary are of the same type) there will be a unique "rotation" at each node as shown in figure 2c.

Macleod suggests that using this combination of his two element types gives a valid idealization of a structural system although there may be some loss of symmetry in truly symmetric problems as can be easily seen in figure 2c.

Felippa has recently shown that Macleod's ingenious formulation is a special member of the family of "isoparametric quadrilaterals" introduced by Ergatoudis, Irons, and Zienkiewicz.[5]

The use of Macleod's elements in a finite element analysis requires the development of two separate element stiffness matrices by the general rules stated earlier.

The displacement functions for both of Macleod's elements are

$$\begin{aligned} U(x,y) &= \alpha_4 + \alpha_7 x + \alpha_6 y + \alpha_{12} xy + \alpha_1 y^2 + \alpha_{10} xy^2 \\ V(x,y) &= \alpha_5 + \alpha_8 x + \alpha_2 y + \alpha_3 xy + \alpha_9 x^2 + \alpha_{11} x^2 y \end{aligned} \qquad (15)$$

where the unknown coefficients are arranged as shown in order to simplify the inversion of matrix [C] which relates the nodal coordinates

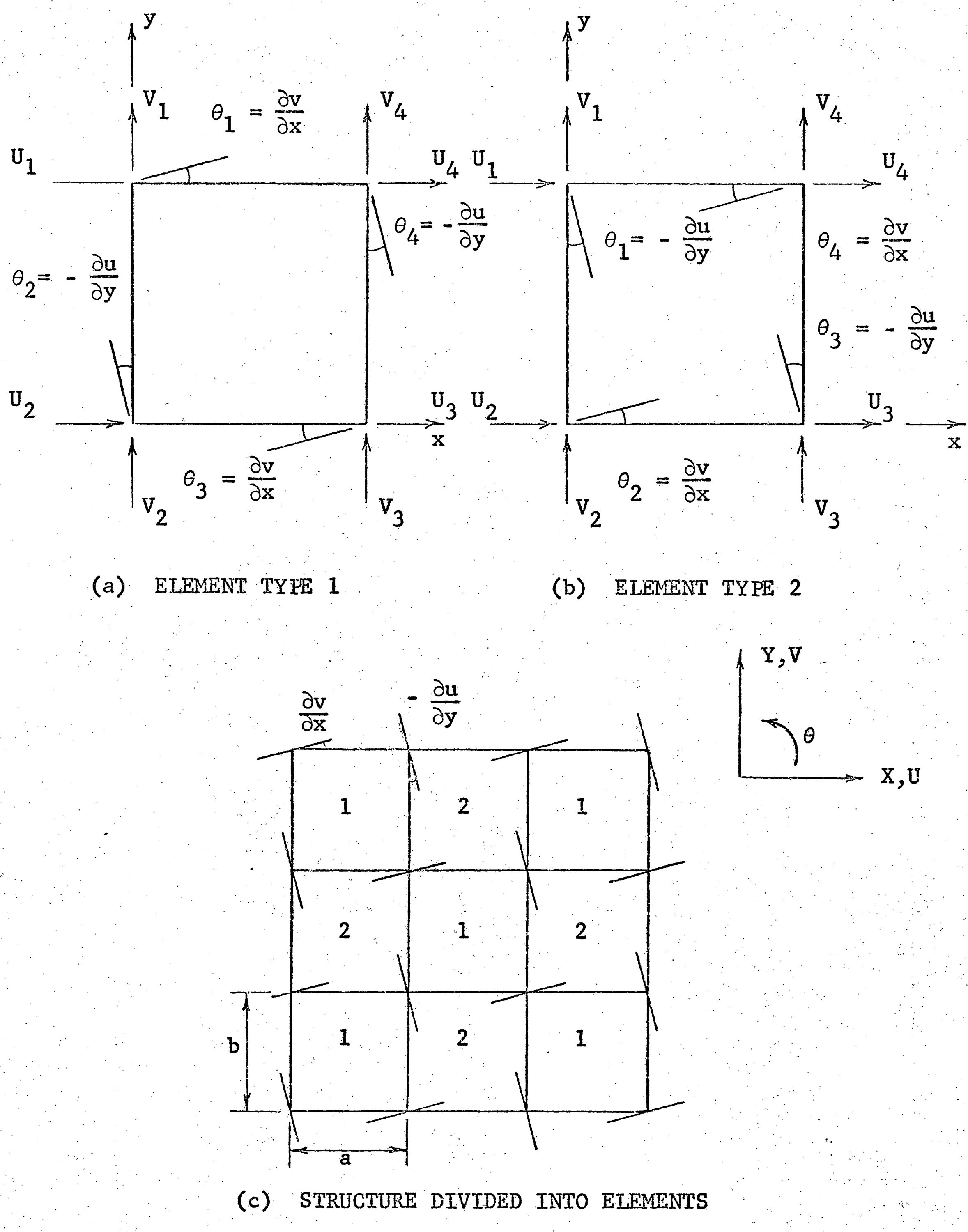


FIGURE 2. MACLEOD 12 DOF ELEMENTS

to the unknown coefficients. These displacement functions were chosen by trial and error to insure full displacement boundary compatibility between adjacent elements.

The stress strain relationship for these elements is the same as for the 8 DOF element discussed earlier; therefore, the $[D]$ matrix is the same as expressed in formulation (5a).

The explicit relationship of the strains to the unknown coefficients, matrix $[Q]$, is the same for both element types and may be expressed as

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} \equiv \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & y^2 & 0 & y \\ 0 & 1 & x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x^2 & 0 \\ 2y & 0 & y & 0 & 0 & 1 & 0 & 1 & 2x & 2xy & 2xy & x \end{bmatrix} \{\alpha\}$$

Since the matrices $[Q]$ and $[D]$ are not dependent upon element type, the fomulation of the element stiffness matrix is separated into two phases where:

$$[k'] = [Q]^T [D] [Q] \ dx \ dy \qquad (6)$$

so that:

$$[k] = t \ [C^{-1}]^T \ [k'] \ [C^{-1}] \qquad (7)$$

Since $[k']$ may be formed explicitly the element stiffness matrix $[k]$ may be formed in the computer by the proper multiplications of the element dependent $\left[C^{-1}\right]^T$ and $\left[C^{-1}\right]$ matrices. An explicit formulation of the $\left[k'\right]$ is presented in expression (18).

A similar procedure is used for the fomulation of the element stress matrix $\left[DQ\right]$ as the explicit product of the $\left[D\right]$ and $\left[Q\right]$ matrices. Using $\left[DQ\right]$ the stresses in the element may be found as

$$\left\{\sigma\right\} = \left[DQ\right] \left[C^{-1}\right] \left\{U_n\right\} \tag{9}$$

Expression (20) contains explicit form of matrix $\left[DQ\right]$, usable for both element types.

$$
[k'] = \frac{Et}{1-\nu^2}
\begin{array}{c}
\begin{array}{cccccccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12
\end{array} \\
\left[\begin{array}{cccccccccccc}
4DO & & & & & & & & & & & \\
\cdots & xy & & & & & & & & & & \\
2DO & B & F+DO & & & & & & & & & \\
\cdots & \cdots & \cdots & \cdots & & & & & \text{(Symmetric)} & & & \\
\cdots & \cdots & \cdots & \cdots & \cdots & & & & & & & \\
2CO & \cdots & CO & \cdots & \cdots & xyO & & & & & & \\
\cdots & xyR & BR & \cdots & \cdots & \cdots & xy & & & & & \\
2CO & \cdots & CO & \cdots & \cdots & xyO & \cdots & xyO & & & & \\
4GO & \cdots & 2GO & \cdots & \cdots & 2BO & \cdots & 2BO & 4FO & & & \\
4HO & DR & H & \cdots & \cdots & 2GO & D & 2GO & 4MO & S+4NO & & \\
4HO & F & T+2HO & \cdots & \cdots & 2GO & FR & 2GO & 4MO & N(R+4O) & W+4NO & \\
2GO & CR & GZ & \cdots & \cdots & BO & C & BO & 2FO & P+2MO & M & D+FO
\end{array}\right]
\end{array}
\qquad (18)
$$

Where:

$B = x^2y/2$	$H = x^2y^3/6$	$P = xy^4/4$
$C = xy^2/2$	$R = \nu$	$S = xy^5/5$
$D = xy^3/3$	$O = (1-\nu)/2$	$T = x^4y/4$
$F = x^3y/3$	$M = x^3y^2/6$	$W = x^5y/5$
$G = x^2y^2/4$	$N = x^3y^3/9$	$Z = (1+\nu)/2$

$$[DB] = \frac{Et}{1-\nu^2} \begin{array}{c} \begin{array}{cccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \end{array} \\ \left[\begin{array}{cccccccccccc} 0 & \nu & x\nu & 0 & 0 & 0 & 1 & 0 & 0 & y^2 & x^2\nu & y \\ 0 & 1 & x & 0 & 0 & 0 & \nu & 0 & 0 & y^2\nu & x^2 & y\nu \\ \gamma y & 0 & \frac{\gamma y}{2} & 0 & 0 & \frac{\gamma}{2} & 0 & \frac{\gamma}{2} & \gamma x & \gamma xy & \gamma xy & \frac{\gamma x}{2} \end{array}\right] \end{array}$$

(20)

Where: $\gamma = 1 - \nu$

x and y are the local coordinates of the element.

C. The Plane Frame Beam Element

This general beam element possesses six end-displacements. As shown in figure 3, these displacements consist of translations in the inplane coordinate directions (X_M, Y_M) as well as a rotation in the Z_M sense at both ends, respectively. The element stiffness matrix S_M may be formed for the member coordinate system through standard structural analysis procedures. The procedure used herein is the one presented by Gere and Weaver[6], whereby unit displacements are applied in each of the displacement directions, one at a time, and the resulting action forms the element stiffness matrix.

The member stiffness matrix may be transformed to the element stiffness matrix for the structure axes system by proper multiplication by an axes transformation matrix $\left[R_T\right]$ for the plane frame member where

$$\left[R_T\right] = \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix} \tag{21}$$

The $\left[R\right]$ matrix is composed of the direction cosines of the member in question, where γ is the angle between the structure axes and the member axes (see figure 3b). The 3x3 $\left[R\right]$ matrix is formed as

$$\left[R\right] = \begin{bmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C_x & C_y & 0 \\ -C_y & C_x & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{22}$$

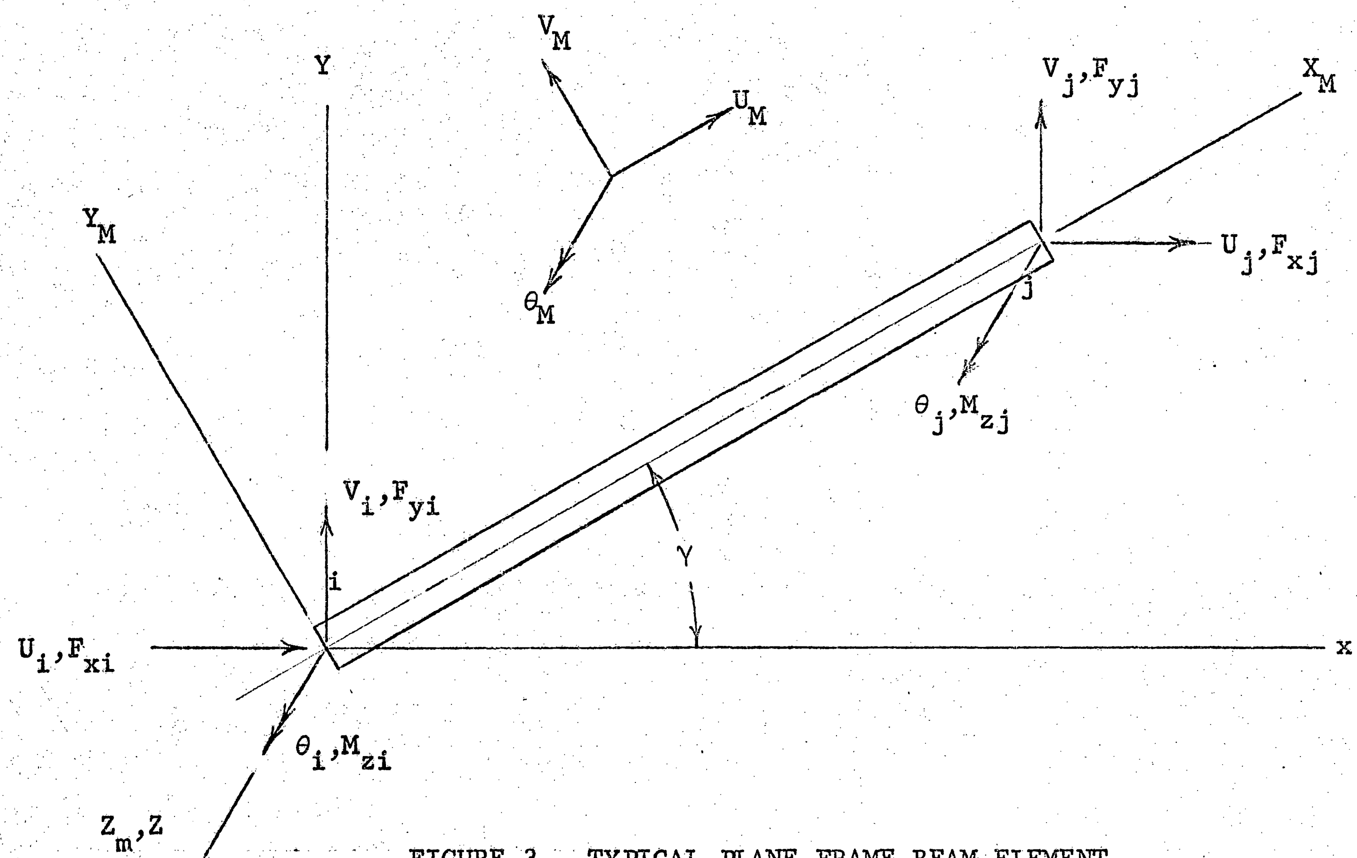


FIGURE 3. TYPICAL PLANE FRAME BEAM ELEMENT

The member stiffness matrix $[S]$ for the structure axes system may be formed as

$$[S] = [R_T]^T [S_M] [R_T] \qquad (23)$$

An explicit formulation for the stiffness matrix $[S]$ (as given by expression (24)) may be formed for a homogeneous beam of constant cross section, where E is Young's Modulus of elasticity, A is the area of the cross section perpendicular to the xy plane, L is the length of the member, and I is the moment of inertia of the cross section about the z axis.

Using the stiffness matrix $[S]$, the forces in the structure coordinate system may be related to the displacements in the same system for each member as

$$\{F_e\} = [S]\{U_n\} \qquad (25)$$

where

$$\{F_e\}^T = (F_{xi}, F_{yi}, M_{zi}, F_{xj}, F_{yj}, M_{zj})$$

$$\{U_n\}^T = (U_i, V_i, \theta_i, U_j, V_j, \theta_j)$$

The 6x6 $[S]$ matrix may be easily combined with the 12x12 element stiffness matrices of Macleod's elements in forming the total stiffness for the complete structure since each node of the elements and each end or node of the beams possess 3 DOF.

After the calculation of the displacements for the entire node network it is more beneficial to calculate the member end actions rather than the stresses as would be done for the rectangular elements.

$$
\frac{12\,EI}{L^3}
\begin{bmatrix}
C\,C_x^{\,2} + C_y^{\,2} & & & & & \text{(SYMMETRIC)} \\
(C - 1)\,C_x\,C_y & C\,C_y^{\,2} + C_x^{\,2} & & & & \\
-\frac{L}{2}\,C_y & \frac{L}{2}\,C_x & \frac{L^2}{3} & & & \\
-(C\,C_x^{\,2} + C_y^{\,2}) & -(C - 1)\,C_x\,C_y & \frac{L}{2}\,C_y & C\,C_x^{\,2} + C_y^{\,2} & & \\
-(C - 1)\,C_x\,C_y & -(C\,C_y^{\,2} + C_x^{\,2}) & -\frac{L}{2}\,C_x & (C-1)\,C_x C_y & C C_y^{\,2} + C_x^{\,2} & \\
-\frac{L}{2}\,C_y & \frac{L}{2}\,C_x & \frac{L^2}{6} & \frac{L}{2}\,C_y & -\frac{L}{2}\,C_x & \frac{L^2}{3}
\end{bmatrix}
\qquad (24)
$$

where

$$C = \frac{AL}{12\,I}$$

The beam end actions (figure 3) may be related to unit displacements in the structure coordinate system by classical beam theory and matrix analysis. Using these procedures a matrix of member end actions due to unit displacements $\left[A_{MD}\right]$ may be formed by multiplying the effects of the unit displacements by the magnitudes of the actual displacements $\left\{U_n\right\}$. The member end actions $\left[A_M\right]$ may be found as

$$\left[A_M\right] = \left[A_{MD}\right] \left\{U_n\right\} \qquad (26)$$

Refer to expression (27) for an explicit form of this relationship.

$$
\begin{bmatrix} AM_{i1} \\ AM_{i2} \\ AM_{i3} \\ AM_{j1} \\ AM_{j2} \\ AM_{j3} \end{bmatrix}
=
\begin{bmatrix}
\psi\, C_x & \psi\, C_y & 0 & -\psi\, C_x & -\psi\, C_y & 0 \\
-\varphi\, C_y & \varphi\, C_x & \beta & \varphi\, C_y & -\varphi\, C_x & \beta \\
-\beta\, C_y & \beta\, C_x & w & \beta\, C_y & -\beta\, C_x & \frac{w}{2} \\
-\psi\, C_x & -\psi\, C_y & 0 & \psi\, C_x & \psi\, C_y & 0 \\
\varphi\, C_y & -\varphi\, C_x & -\beta & -\varphi\, C_y & \varphi\, C_x & -\beta \\
-\beta\, C_y & \beta\, C_x & \frac{w}{2} & \beta\, C_y & -\beta\, C_x & w
\end{bmatrix}
\begin{bmatrix} U_i \\ V_i \\ \theta_i \\ U_j \\ V_j \\ \theta_j \end{bmatrix}
\tag{27}
$$

where

$$\psi = \frac{EA}{L} \; ; \quad \varphi = \frac{12\ EI}{L^3} \; ; \; \beta = \frac{6\ EI}{L^2} \; ; \quad w = \frac{4\ EI}{L}$$

$C_x = \cos\gamma$; $C_y = \sin\gamma$ (see figure 3)

V. COMPARISONS AND APPLICATIONS

Of initial interest is the comparison of solutions obtained from the finite element computer programs developed for the 8 DOF and 12 DOF elements with an exact elasticity solution. The validity and convergence of solutions from the two programs is investigated in the first of the following sections.

In the remaining sections, the usefullness and applicability of the 12 DOF program based on MacLeod's element is examined through comparisons with experimental data. These experimental tests were chosen from the literature in order to compare the finite element solutions with data from displacement studies, strain-gage work, other analytical studies, but most importantly with photoelastic studies. The scope of the models used (see Table 1) basically covers the whole range of general shear wall configurations, i.e., pierced models with slender or deep lintel beams and a model of a framed shear wall.

In each case studied, the results of the 12 DOF (also, 8 DOF where applicable) finite element solution are compared with the results of the model study used. In some instances these comparisons are complimented with the results of other analytical methods.

The instructions for using the programs are not covered in this section. The reader is referred to the appendices for the identification and solution of problems with these programs.

Table I. Model Properties

Model Identification	Modulus of Elasticity	Poissons Ratio	Thickness
Cantilever Subjected to end shear	30×10^{3} ksi	0.3	1.0 in.
Coull and Puri's Model #5	45.4×10^{4} psi	0.33	0.625 in.
Coull and Puri's Model #1	45.2×10^{4} psi	0.33	0.625 in.
Sensmeier's Model #3	46.0×10^{4} psi	0.39	0.259 in.
Sensmeier's Model #1	46.0×10^{4} psi	0.39	0.262 in.
Schwaighofer and Microys' Model	43.2×10^{4} psf	0.20	1.0 ft.

A. Comparison of the Finite Element Solutions With Elastic Theory

A cantilever beam subjected to end shear as presented by Connor and Will[2] will be used as the basis of this study. Connor and Will developed the elastic analysis of this cantilever as an example in a convergence study of triangular finite element solutions. They covered the development of the required elastic analysis in their work, therefore, only basic parts will be mentioned here. The elastic idealization of the beam is shown in Fig. 4, whereby 4a is statically equivalent to 4b. The force intensities, Px and Py, are constant over the thickness, t.

The finite element idealization of the cantilever is also adopted from the work of Connor and Will[2]. This idealization(Fig. 5) models only half of the beam in question. In so doing the neutral axis of the beam may be restrained from deformation as indicated by the presence of the roller supports along line $x = 0$ in Fig. 5. The node at $x = 0$, $y = 50$ is not allowed to displace in either the x or y directions, therefore its displacements u and v are set equal to zero. Also, to conform with the boundary conditions of the elastic solution, $v = 0$ for the node at $x = 25$, $y = 50$.

Figure 5 contains the dimensions of the beam while Table 1 must be consulted for the elastic properties and thickness.

The parabolic shear loading and the linear flexure loading, shown in Figures 4 and 5 are reduced to equivalent point loads at the discrete finite element nodes by the appropriate methods presented by Connor and Will.

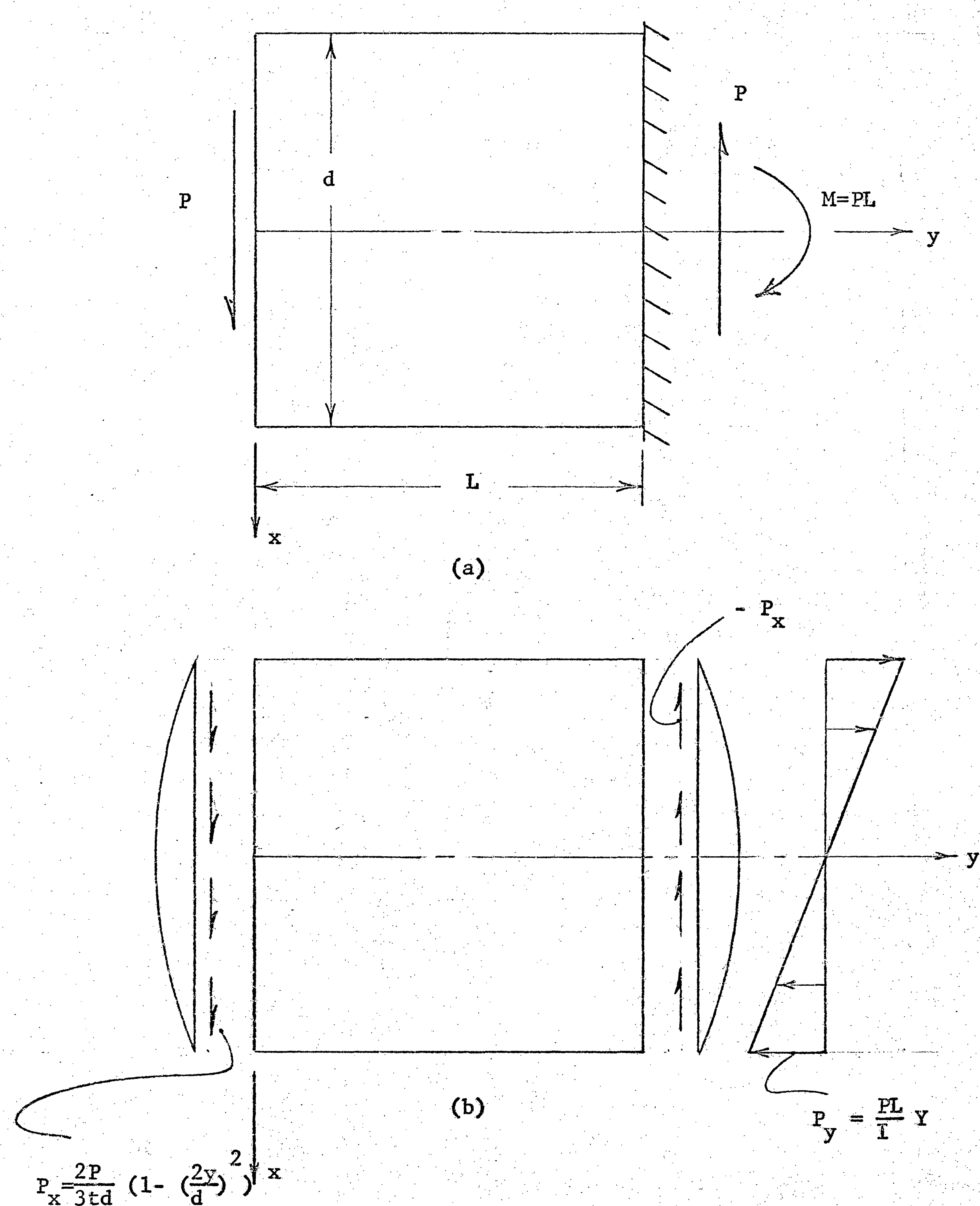


FIGURE 4. CANTILEVER SUBJECTED TO END SHEAR

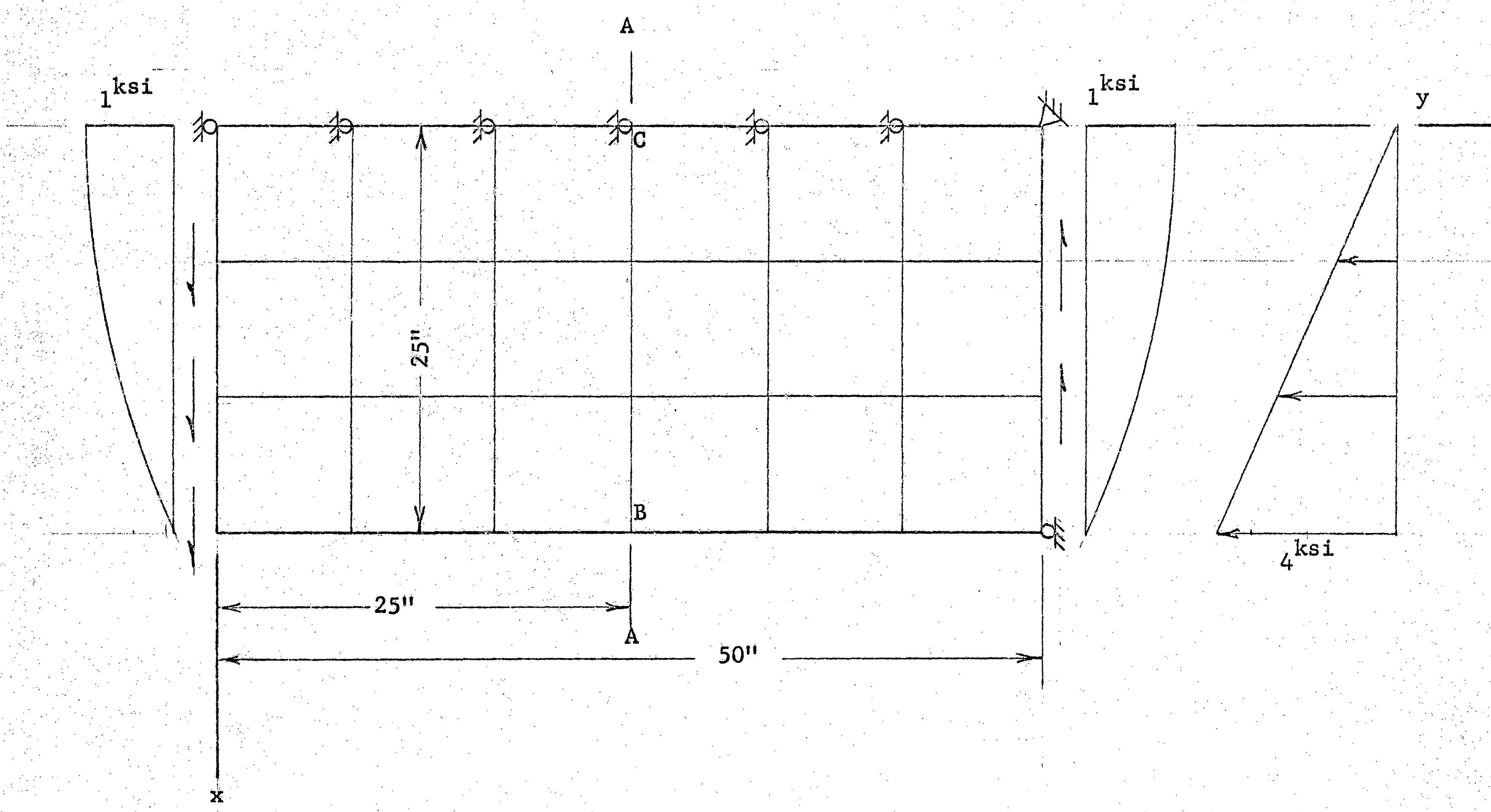


FIGURE 5. FINITE ELEMENT IDEALIZATION OF CANTILEVER SUBJECTED TO END SHEAR

In order to compare the convergence properties of each of the finite element programs, it was necessary to obtain several solutions of the beam utilizing idealizations of decreasing element dimensions and, therefore, increasing number of elements and total degrees of freedom. A direct stiffness finite element solution based on a compatible displacement function should converge to a lower bound of the strain energy as the number of elements increases. The different square element meshes used are recorded in Table II.

A comparison of the convergence to the actual value of the shear stress at the mid-point of the neutral axis (see Fig. 5, point C) is given in Fig. 6. It is apparent that the 12 DOF element gives a very good solution even for the coarse mesh, while the 8 DOF element requires a finer mesh to reach the same degree of accuracy. A monotonic convergence is evident in both cases, but from two different sides. This action of the 12 DOF element, has been recorded in the behavior of other higher order elements where rotations are considered. (See Reference 11)

Table II presents the convergence characteristics of solutions from the finite element programs as compared with the theoretical value of the bending stress (σy) at the mid-point of the lower edge (point B, Fig. 5). Here the lack of symmetrical rotations in MacLeod's element produces a noticeable effect on the average σy recorded in row 6. In reviewing rows 4 and 5 it is evident that the values recorded for each node (one value for each of two adjacent elements) indicate the variance from the theoretical of any one reading in similar mesh divisions is of the same order of magnitude for all readings.

Table II. Convergence of Bending Stress

1. DOF of Element	8	12	8	12	12	12	8	12
2. Total DOF of Structure	56	84	132	150	198	252	462	693
3. Element Mesh	3 x 6	3 x 6	5 x 10	4 x 8	5 x 10	6 x 12	10 x 20	10 x 20
4. σy from each adjacent element (ksi)*	-2.365 -1.640	-1.664 -2.418	-1.79 -2.228	-1.749 -2.300	-1.800 -2.243	-1.833 -2.200	-1.897 -2.117	-1.90 -2.12
5. Variance of each recorded σy from theoretical σy (ksi)*	.360 - .365	.336 - .418	.210 - .228	.251 - .300	.200 - .243	.167 - .200	.103 - .117	.100 - .120
6. σy average (ksi)*	-2.0025	-2.041	-2.009	-2.022	-2.021	-2.016	-2.007	-2.01

*NOTE: This table presents the values of σy at point B (Fig. 5) as calculated by the finite element programs for increasing mech sizes. The theoretical value of σy is -2.0 ksi.

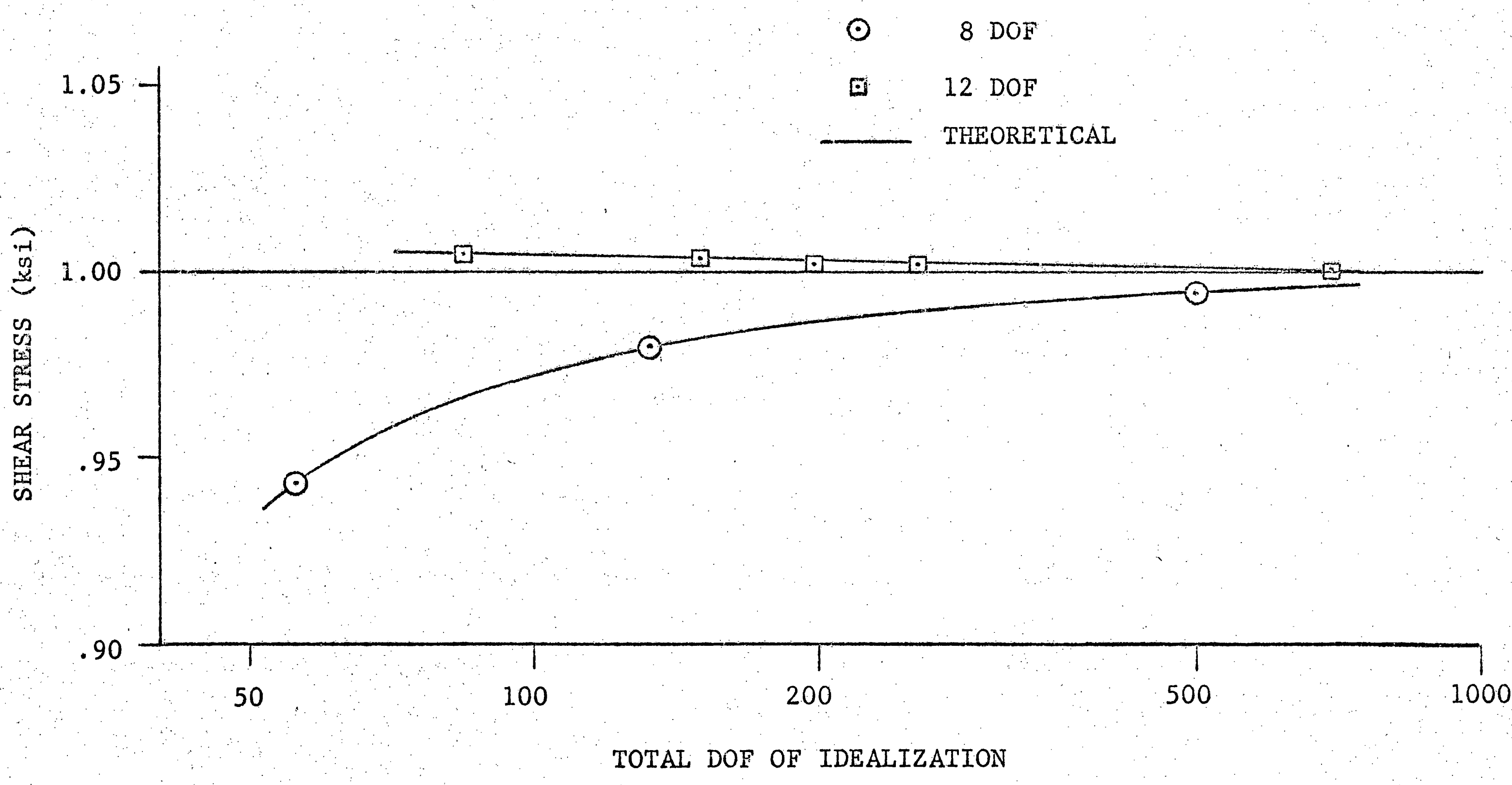


FIGURE 6. CONVERGENCE OF SHEAR STRESSES

The unsymmetric values obtained from the 12 DOF analysis for the coarse mesh sizes produce averages which are less accurate than those of the 8 DOF analysis. This lack of symmetry becomes negligible with finer mesh divisions. The effect of this non-symmetry in the coarser mesh divisions may possibly be eliminated by solving the problem again with interchanged element types and averaging the results from both solutions.

Figure 7 presents a comparison of the results of the finite element programs for a 5 x 10 mesh, with the theoretical bending stress and shear stress distributions through the depth of the beam along section A-A, Fig. 5. Both the 8 DOF and the 12 DOF solutions are quite good in comparison with these theoretical stresses.

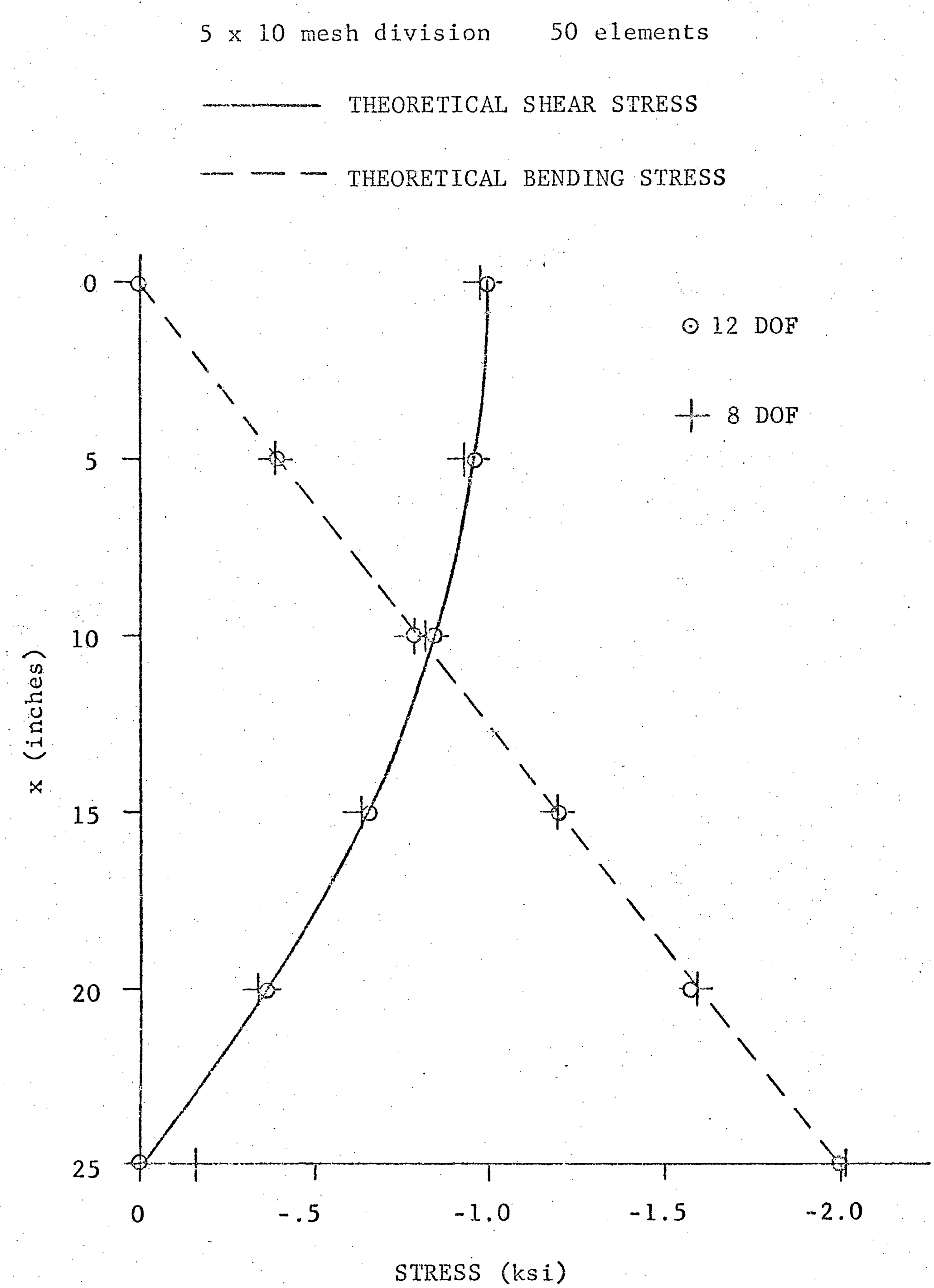


FIGURE 7. STRESSES ALONG SECTION A-A, FIG. 5

B. Application to Shear Wall Problems

The elastic properties and thickness of each of the shear wall models studied in this section are enumerated in Table I.

The shear-walls modeled here may be broken into three classes for discussion:

1) Pierced shear walls with deep lintel beams
2) Pierced shear walls with slender lintel beams
3) Shear walls in framed structures

Class 1 structures are idealized totally of rectangular elements while Class 2 and 3 structures are idealized by appropriate combinations of MacLeod's elements and general plane frame beam elements.

The finite element idealizations of the models presented in the figures of this section reflect the "checker board" element typing system required for the use of MacLeod's elements in the 12 DOF program. In these figures, a numeral one in the center of an element indicates that the element is of type 1, while elements without a numeral are understood to be of type 2.

The finite element idealizations of the models of class 1 structures for the 8 DOF program are essentially the same as those for the 12 DOF program except that the element typing is superfluous.

1. Coull and Puri Model #5

Model #5 of Coull and Puri[4] as illustrated in Fig. 9 is proportioned to represent a pierced shear wall in an eleven-story building. The deep lintel beams in this model have a length to depth ratio (l/d) of 1.2.

The finite element idealization used for the 12 DOF program is shown in Fig. 8. An 8 DOF solution was also obtained for this model.

In the experimental displacement analysis of this model, Coull and Puri clamped the base section below the first opening between roughened steel plates to simulate a fixed end condition. The finite element idealization is designed to approximate this clamped end condition.

The model is loaded with a single transverse load at the top of the model.

As presented in Fig. 9, the displacements calculated from the 12 DOF analysis agree more closely with the experimental results than do those of the 8 DOF analysis. Since the 12 DOF elements possess 4 more DOF than the 8 DOF elements, it follows that the 12 DOF displacement solution should be more flexible than the 8 DOF solution, as is shown in Fig. 9.

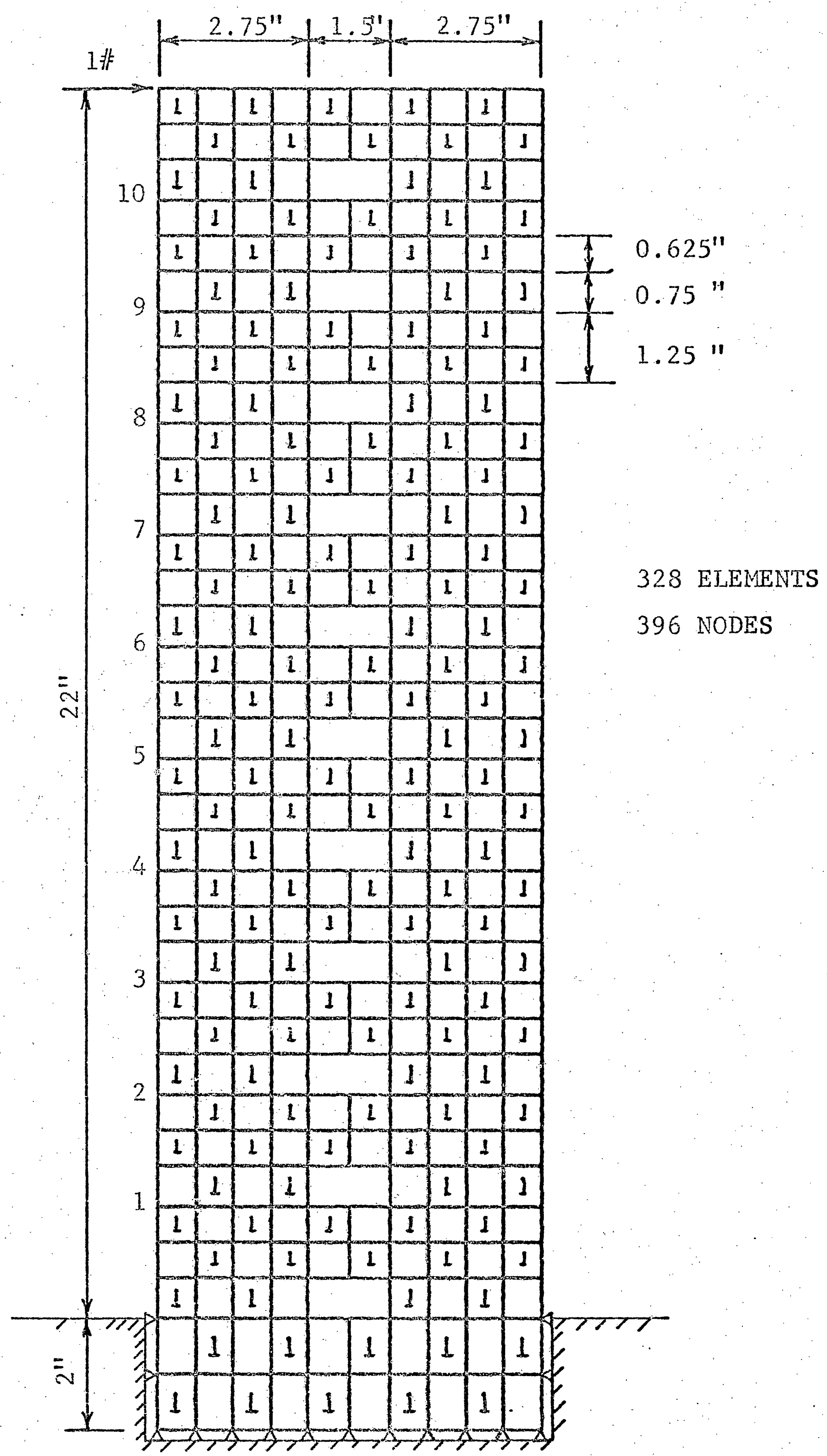


FIGURE 8. IDEALIZATION OF COULL AND PURI'S MODEL #5

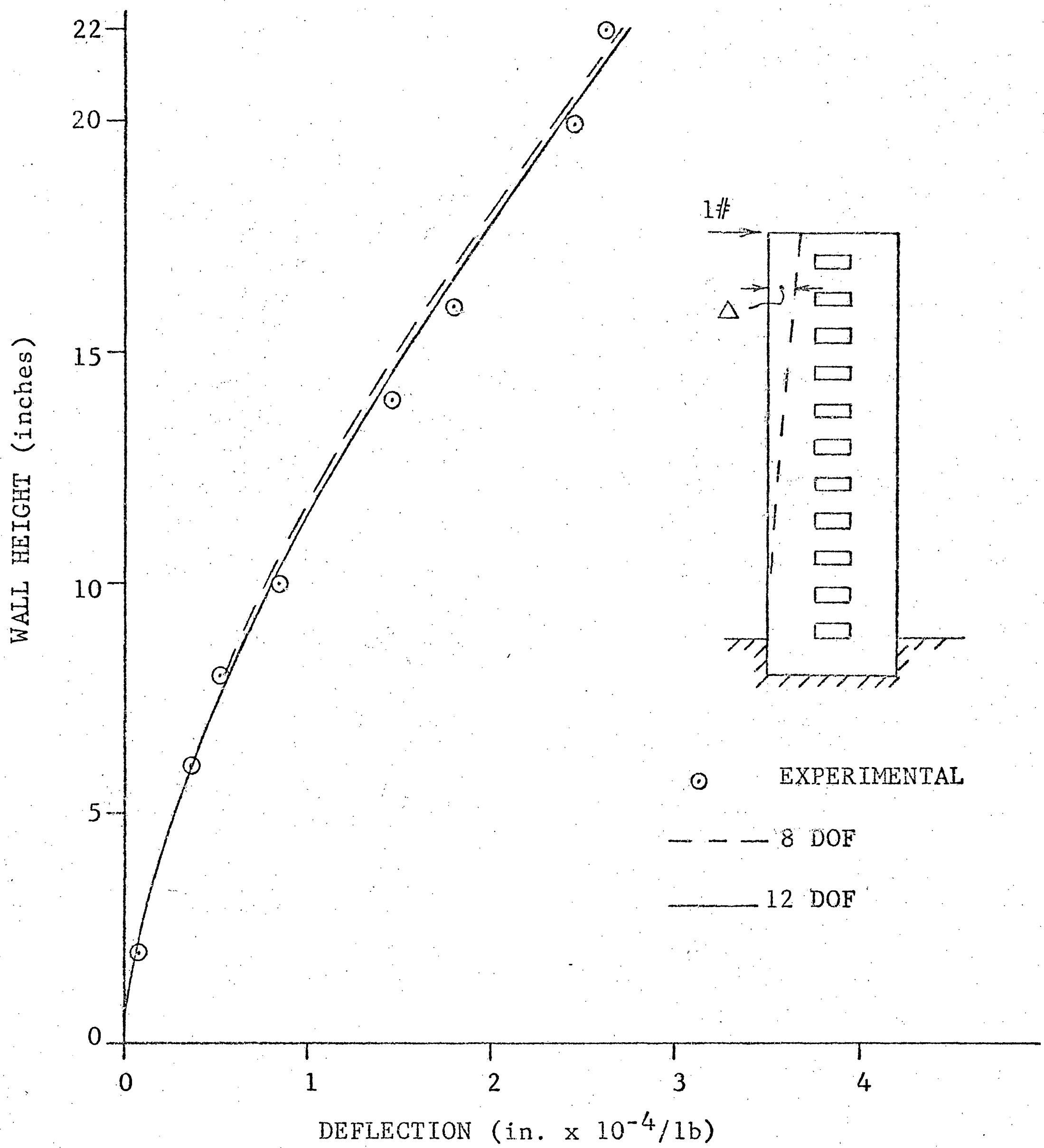
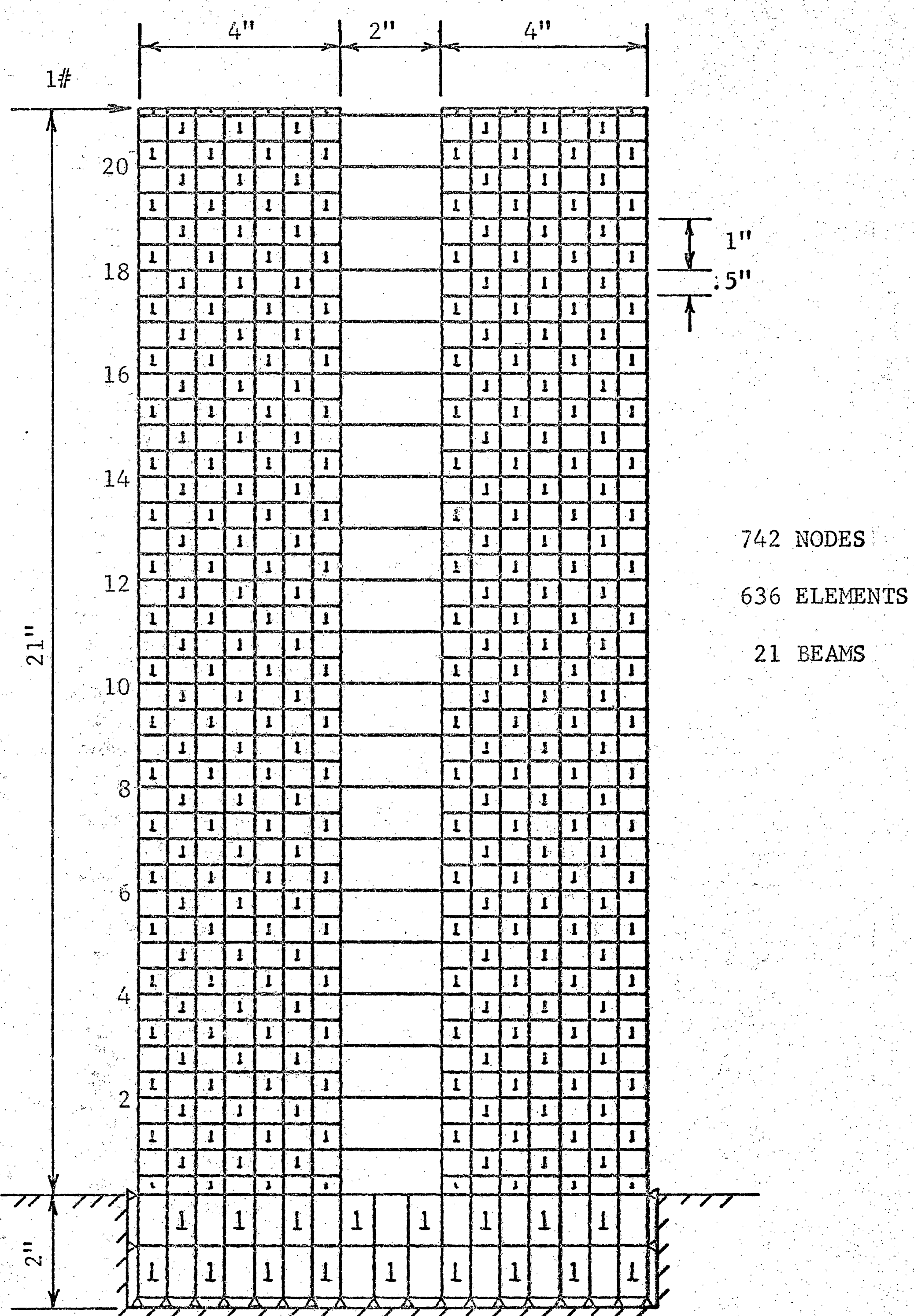


FIGURE 9. DEFLECTION OF COULL AND PURI'S MODEL #5

2. Coull and Puri Model #1

The second Coull and Puri model investigated is of a 21 story shear wall with slender lintel beams (1/d = 8) as illustrated in Fig. 11. The 12 DOF finite element idealization is given in Fig. 10, where the slender lintel beams are shown as beam elements and the wall sections are divided into rectangular elements. The idealization of this model is similar to that of Model #5 in that the base conditions and loading conditions are the same.

The 12 DOF displacement solution (using beam elements) provides a generally good correlation with the displacements recorded for the experimental analysis.



BEAM DEPTH = .25"

FIGURE 10. IDEALIZATION COULL AND PURI'S MODEL #1

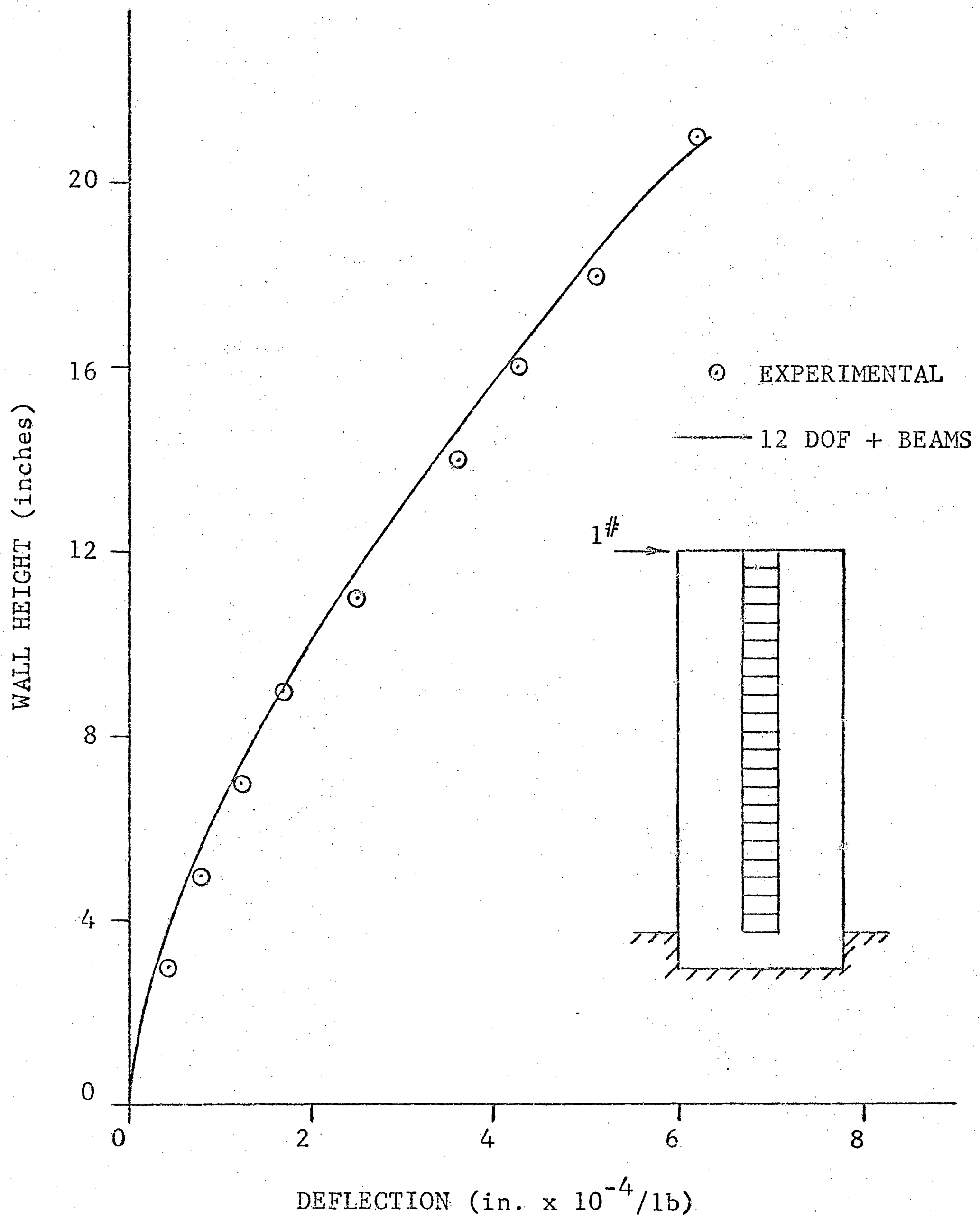


FIGURE 11. DEFLECTION OF COULL AND PURI'S MODEL #1

3. Sensmeier Model #3

Sensmeier presents the results of a detailed photoelastic study of several models of shear walls from five story buildings. His model #3 (Fig. 12) with deep lintel beams (l/d = 2) is idealized for the 12 DOF finite element analysis as illustrated in Fig. 13. This model was loaded with an approximate parabolic distributed shear load across its top in the experimental analysis. This shear loading was converted to equivalent concentrated loads applied at the nodes across the top of the wall in the finite element analysis. The total shear load was 149.3 pounds.

The fixed end condition of the model was set up in the finite element analysis by specifying each node along the base to be fixed (the x and y displacements and the rotation set equal to zero).

Figures 14 through 16 present comparisons of the stresses obtained by Sensmeier in his photoelastic analysis with those calculated by the 12 DOF finite element program.

The distributions of bending stress along the outside edge of the shear wall (Fig. 14) show the calculated values to be slightly larger than the experimental results.

A comparison of shear stresses across horizontal sections of the structure are presented in Figures 15 and 16. Figure 15 shows a comparison of the shear stresses through half the structure along the horizontal center line of the middle lintel beam. Figure 16 gives a similar comparison along a horizontal line half-way between the middle and next higher lintel beams.

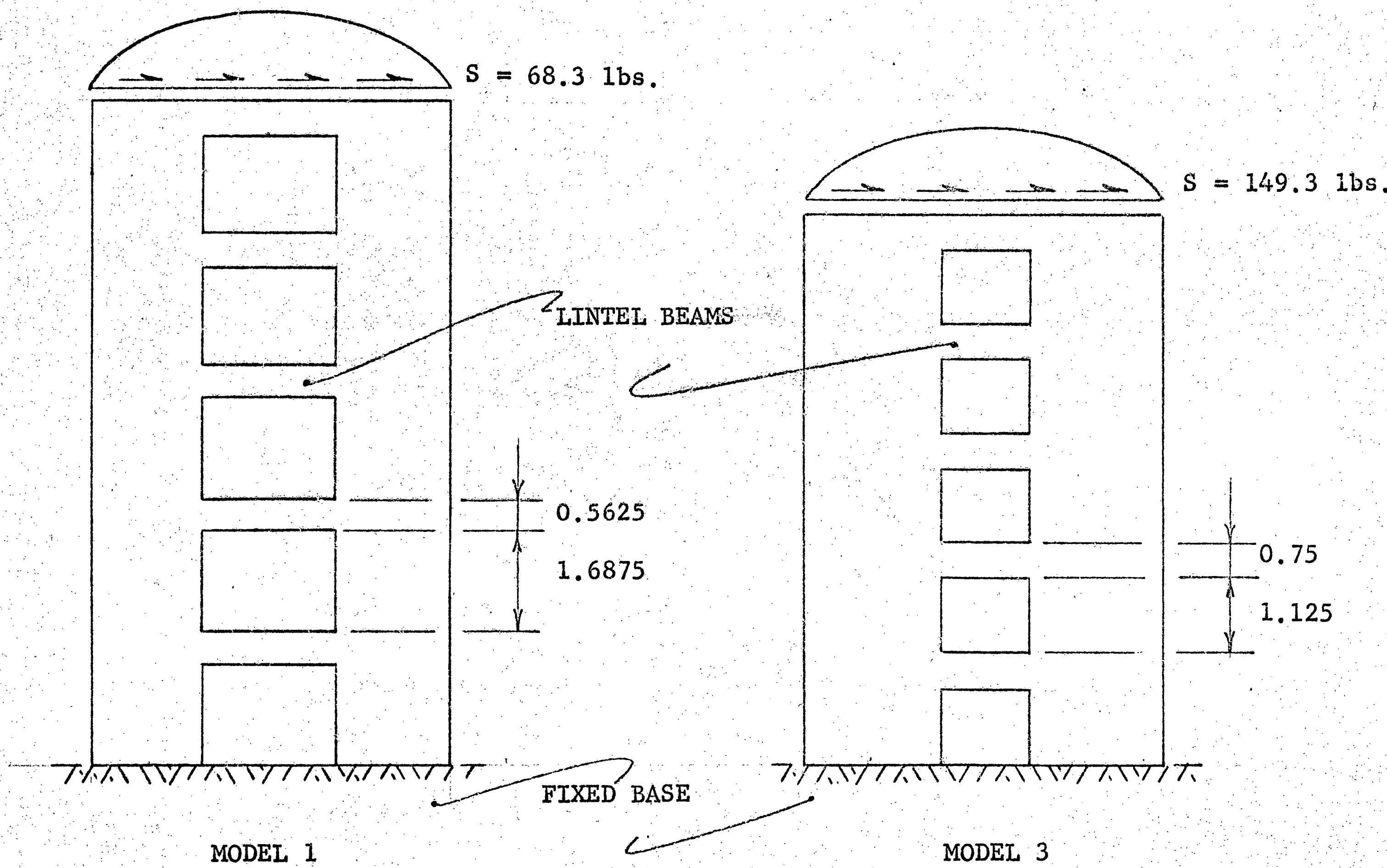


FIGURE 12. SENSMEIER'S MODELS

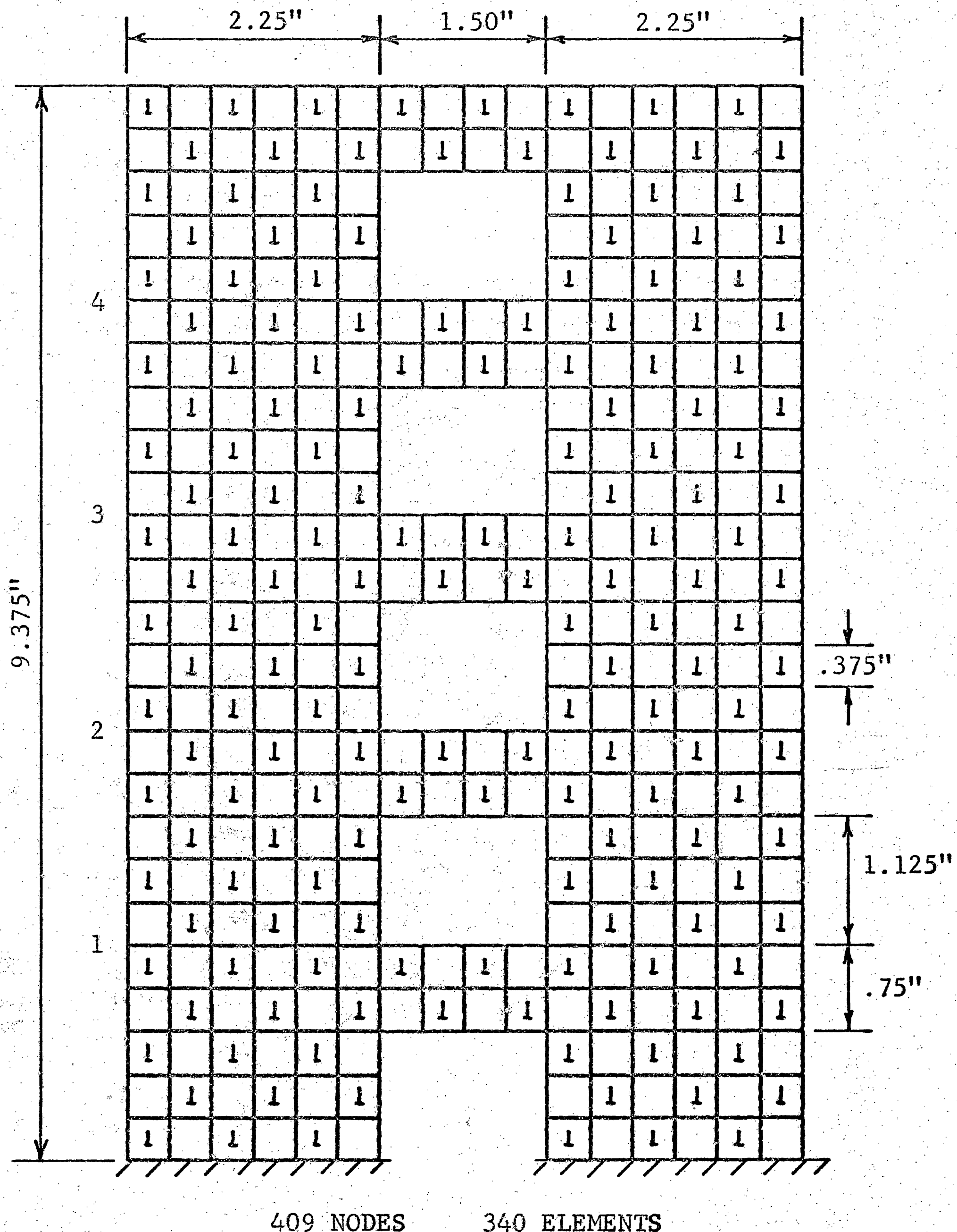


FIGURE 13. IDEALIZATION OF SENSMEIER'S MODEL #3

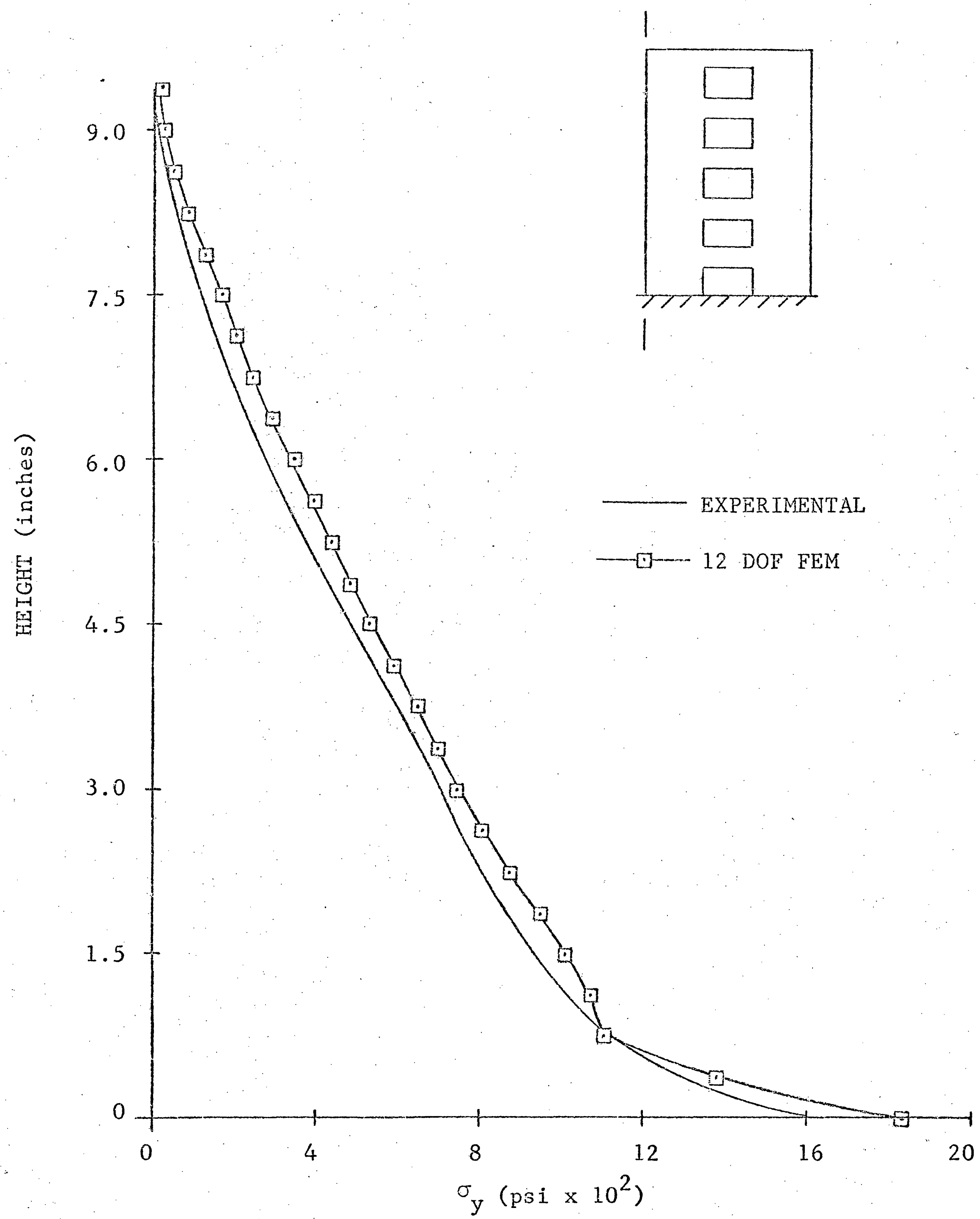


FIGURE 14. σ_y ALONG THE LEFT BOUNDARY OF SENSMEIER'S MODEL #3

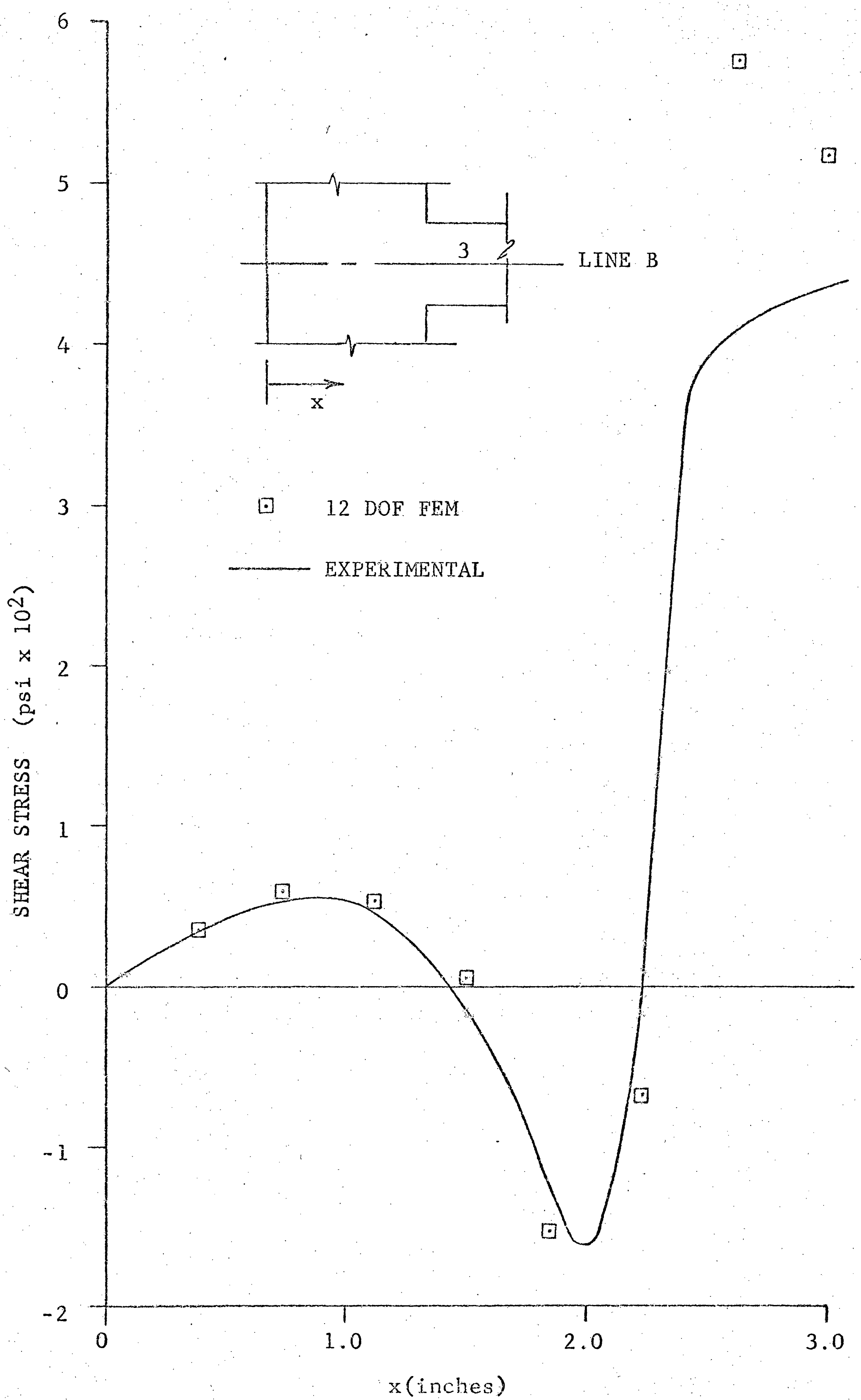


FIGURE 15. SHEAR STRESS ALONG LINE B
SENSMEIER'S MODEL #3

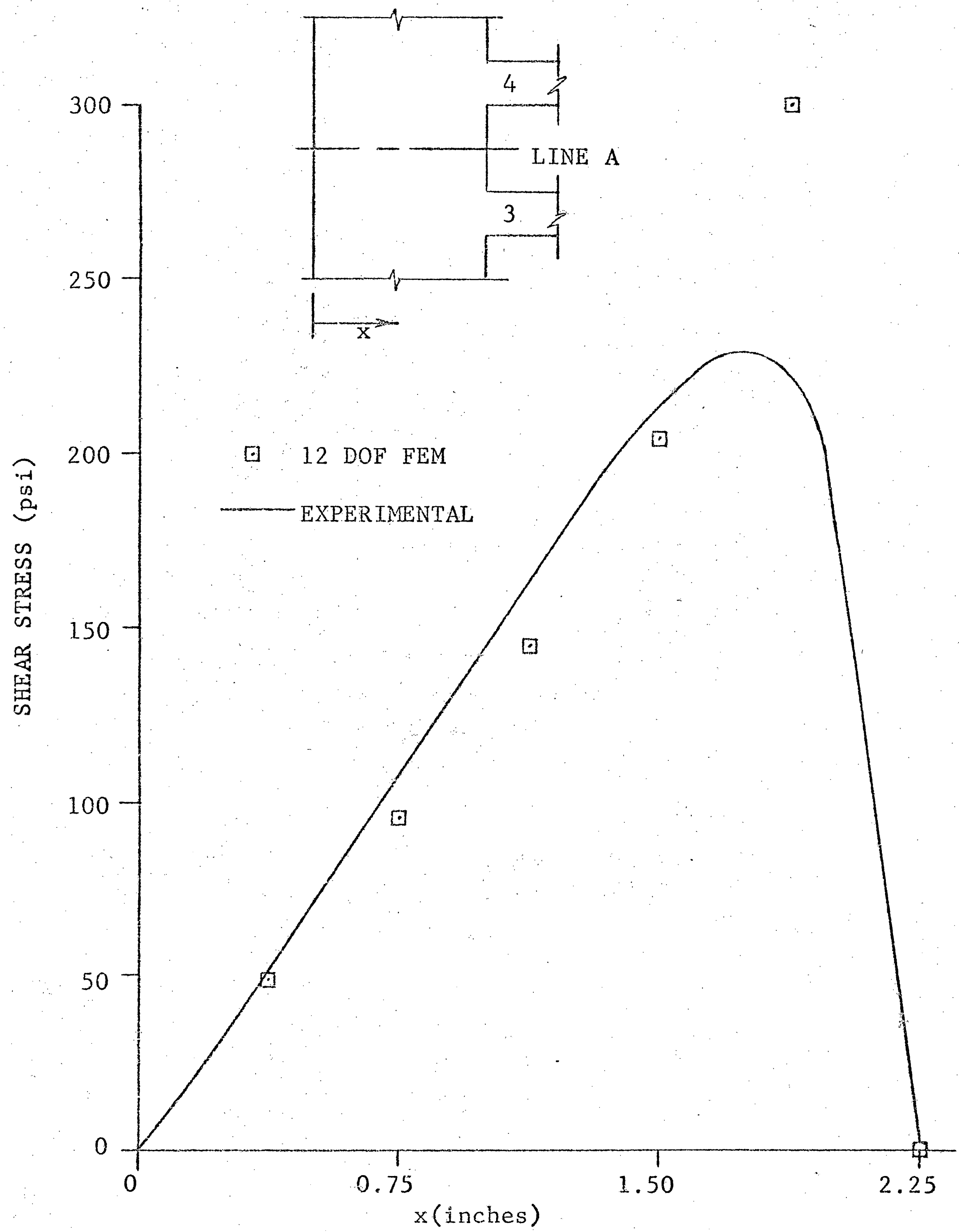


FIGURE 16. SHEAR STRESS ALONG LINE A

SENSMEIER'S MODEL #3

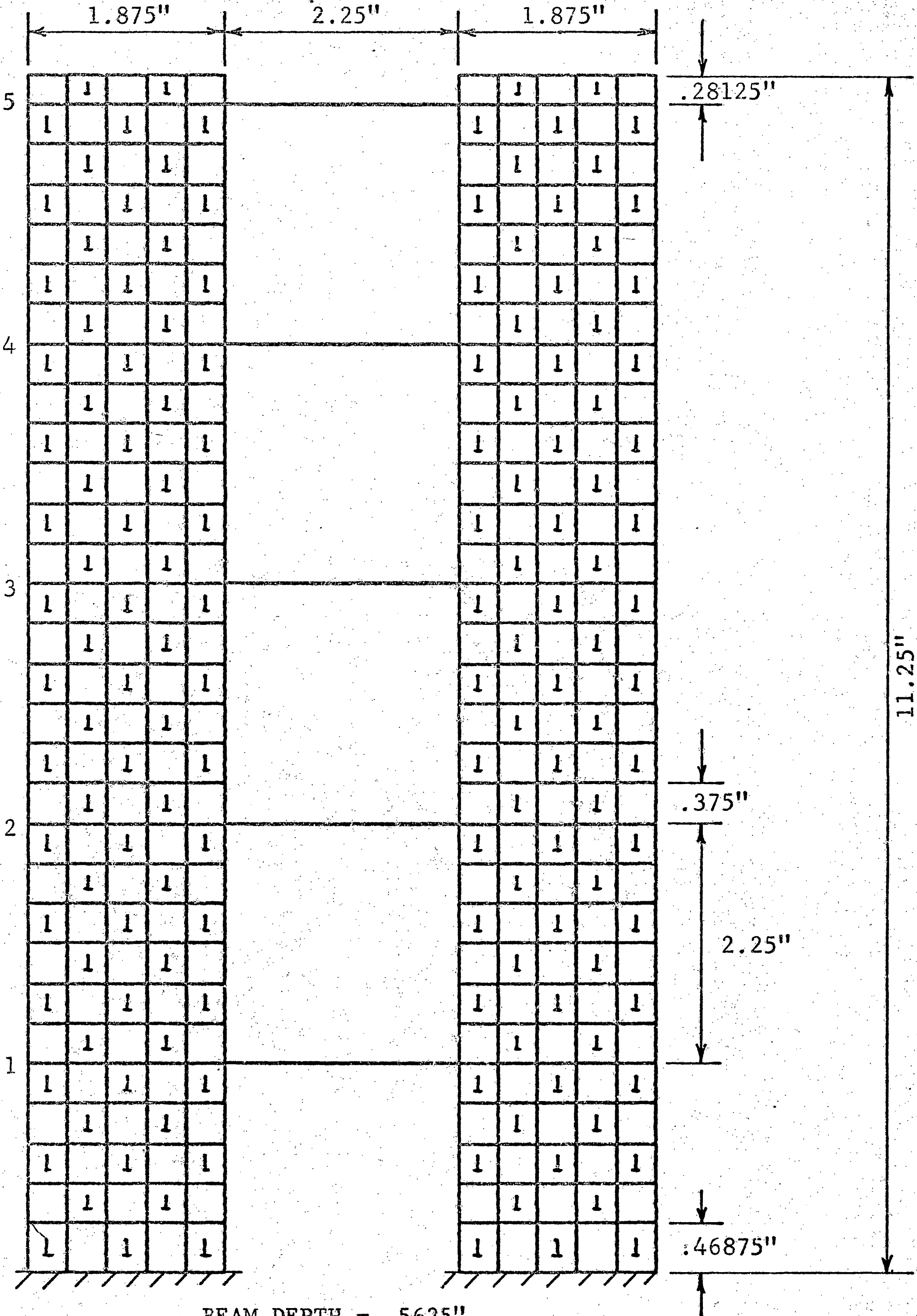
4. Sensmeier Model #1

Sensmeier's model #1 was chosen for analysis so that its lintel beams (1/d = 4) could be idealized as plane frame beam elements (Fig. 17). The actual shape and loading conditions of this five story model are presented in Fig. 12. The approximate experimental parabolic shear loading is applied to the finite element idealization in the same manner as discussed for model #3.

This model also has the same fixed base as model #3.

Upon completion of the finite element analysis of this model, several stress distributions were plotted for various sections of the structure. The comparison of bending stresses along the outside edge of the shear wall (Fig. 18) illustrates a definite difference in the theoretical and calculated results.

Other stress comparisons plotted for this model give even more erratic results and are not presented because it is apparent that the idealization of the lintel beams of this model (1/d = 4) as beam elements is not a valid assumption.



BEAM DEPTH = .5625"

372 NODES   300 ELEMENTS   5 BEAMS

FIGURE 17. IDEALIZATION OF SENSMEIER'S MODEL #1

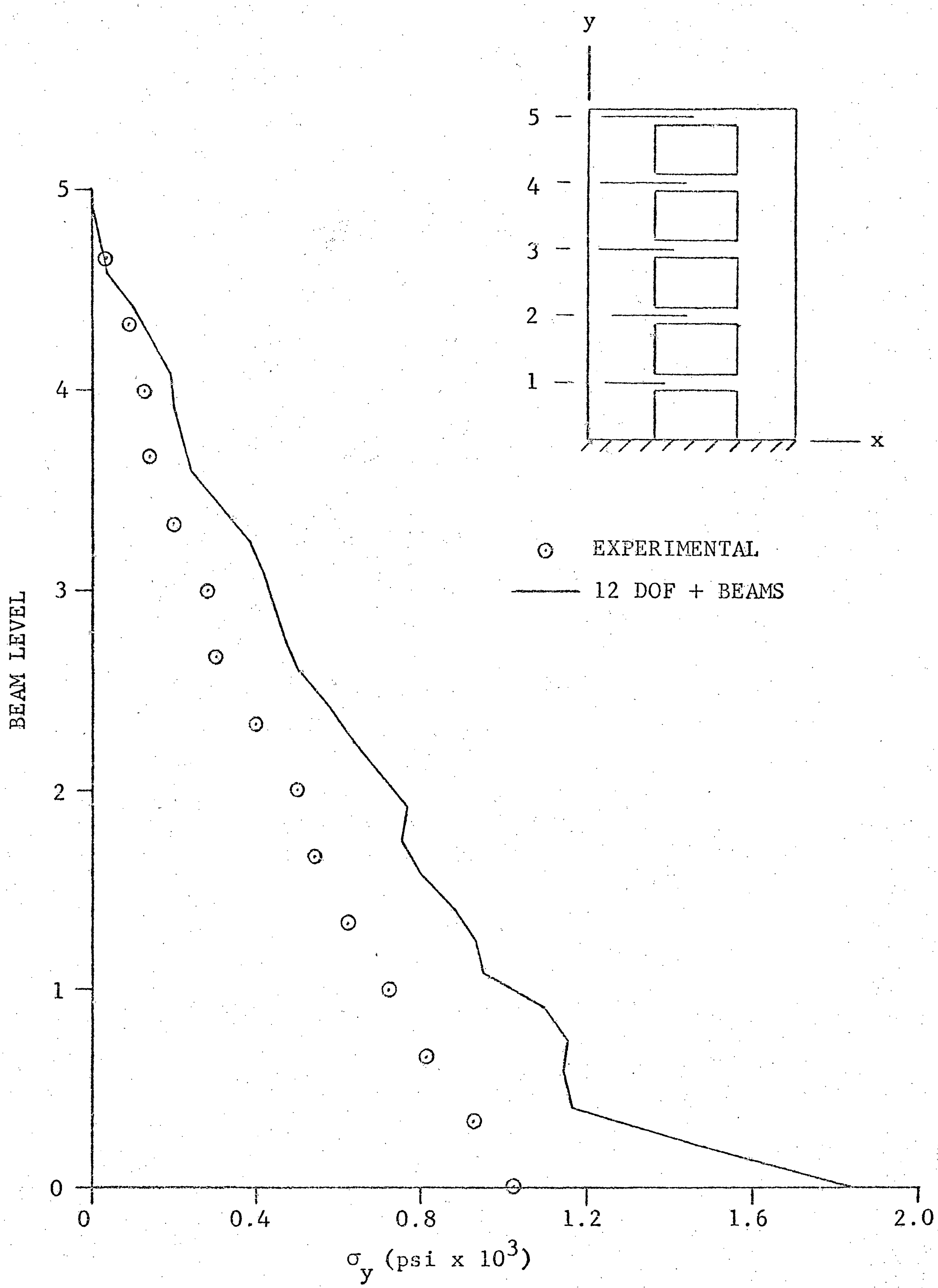


FIGURE 18. σ_y ALONG LEFT BOUNDARY OF SENSMEIER'S MODEL #1

5. Framed Shear Wall

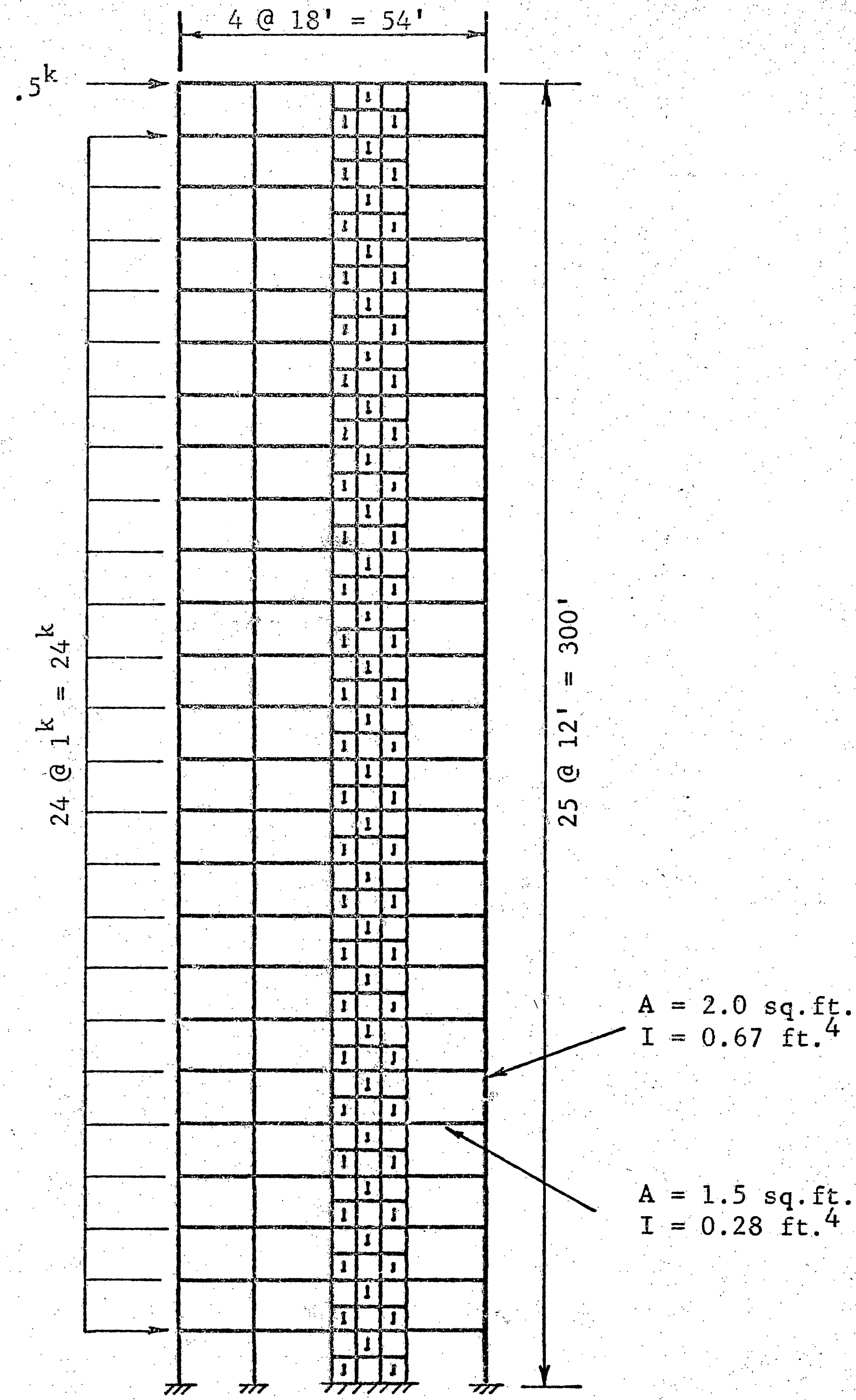
Schwaighofer and Microys suggest the use of an equivalent frame method of analysis for shear walls in framed structures. In their approach the shear wall is reduced to an equivalent frame which may be analyzed through the use of standard frame analysis computer programs. Verification of their procedure is achieved through comparison for analytical and experimental stresses for a twenty-five story model as illustrated in Fig. 19. The results of a strain gage analysis as well as a photoelastic analysis of this framed structure are presented in their study.

The 12 DOF finite element program is easily set up for the solution of this type of problem. The beams and columns are represented by the general plane frame beam elements used in the program and the shear wall is divided into a simple mesh of 12 DOF rectangular elements, as shown in Figure 19.

A uniformly distributed wind load was applied to the finite element idealization through a series of equivalent nodal forces applied at each node along that edge.

The fixed base condition was achieved by setting all the displacements of each node along the base of the structure equal to zero.

In this application, the 12 DOF solution produced stresses which show good correlation with the stresses along the left side of the inframed shear wall as recorded from both of the experimental methods used by Schwaighofer and Microys. The comparison of stresses from the experimental and finite element analyses is presented in Fig. 20.



282 NODES 150 ELEMENTS 150 BEAMS

FIGURE 19. SCHWAIGHOFER AND MICROYS' MODEL

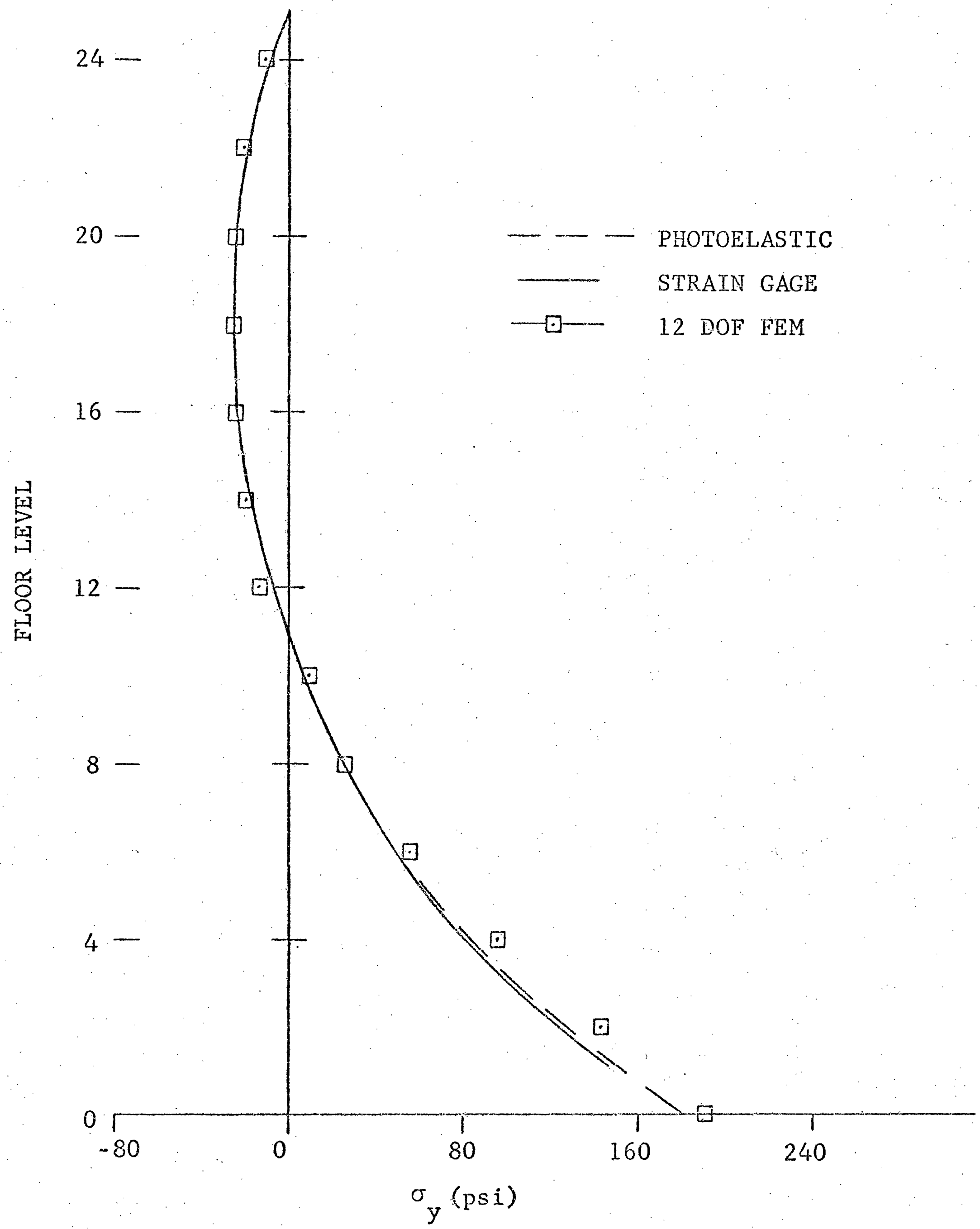


FIGURE 20. σ_y ALONG LEFT EDGE OF SHEAR WALL
SCHWAIGHOFER AND MICROYS' MODEL

VI. DISCUSSION

The finite element computer programs presented in this study may be used for the analysis of plane stress structures of orthogonal geometry. Both programs are quite similar. However, the 12 DOF programs uses beam elements whereas the 8 DOF program does not.

The programs accept data formulated from the finite element idealization of the structure and mathematically exact finite element solution is found. An analysis by the 8 DOF program produces two possible displacements in the coordinate directions at each node as well as three inplane stresses at the same nodes.

In addition to these quantities a solution by the 12 DOF method gives an inplane rotation at each node. If beams are used for a 12 DOF solution, three displacements, including rotation, and three member-end actions, including moment, are given for each end of the beam.

This discussion is directed toward the use of the rectangular elements in the analysis of shear wall configurations. The shear wall problems investigated in this study represent only a basic portion of the general configurations for which these finite element procedures may be used. The solution of pierced shear walls with several bands of openings of irregular arrangement is as easily obtained as that of one band of openings. The analysis of inframed shear walls is easily extended from the simple problem presented herein, to various combinations of frames and shear walls.
One such example may be a configuration where the shear wall does

not penetrate the full height of the frame. A natural extension of this procedure would be the analysis of infilled frames.

The addition of the rotational degrees of freedom to the basic 8 DOF rectangular element causes the resultant 12 DOF element to be more flexible than its basic counter part. The comparison of deflections for Coull and Puri's model #5 illustrates this behavior.

The rotations also enable the 12 DOF element to more accurately evaluate the shear stresses in problems where shear deformation is an important consideration as indicated in the analysis of the cantilever subjected to end shear.

The comparison of bending stresses as evaluated from the coarser mesh idealizations of this cantilever, reiterates the presence of non-symmetric displacement patterns in the fundamental definition of the 12 DOF elements. However, the effect of this non-symmetry may be eliminated by using finer mesh divisions.

An important aspect of this study is the use of the 12 DOF element in conjunction with the plane frame beam elements as a valid method of analysis of framed shear walls and pierced shear walls with slender lintel beams. For this evaluation, the presence of reliable experimental results are necessarily important.

The 12 DOF evaluation of displacements for Coull and Puri's model #1 showed the use of these rectangular elements combined with beam elements to give good correlations with the experimental displacements. The slenderness ratio (l/d) for the lintel beams of this model is 8.

In the evaluation of stresses for Sensmeier's model #1, possessing a lintel beam slenderness ratio of 4, the use of beam elements gives rather inadequate results. It is believed that these lintel beams are not slender enough to be truly idealized as beam elements.

The use of beam elements and 12 DOF elements in the finite element solution of shear wall and frame combinations has quite favorable possibilities as evidenced from the analysis of Schwaighofer and Microy's model.

As stated earlier the use of 12 DOF, in preference to the 8 DOF, rectangular elements produces solutions of increased flexibility for structures that are totally idealized by these rectangular elements.

The beam element may be used in connection with the 12 DOF elements in the analysis of shear wall and frame combinations where the beam element is used to represent true plane frame members. The applicability of the beam element as an appropriate idealization of slender lintel beams has not been fully realized in this study. However, the behavior of the beam elements in the study does warrant further study of this topic. This additional study would require experimental data for proper justification of analytical findings.

VII. CONCLUSIONS

Two rectangular finite element idealizations have been presented and compared for application to the analysis of general shear wall configurations.

Several conclusions may be drawn:

1) Use of the 12 DOF element produces valid results for displacements and stresses for moderate mesh divisions. Whereas, course mesh divisions show the influence of the non-symmetric displacement idealization.

2) The 12 DOF element is more flexible than the 8 DOF rectangular element.

3) The 12 DOF element gives more accurate results than the 8 DOF element where shear deformation is significant.

4) The 12 DOF element may be combined with the plane frame beam element in the analysis of inframed shear wall configurations.

5) Beam elements may be used to idealize "slender" lintel beams in pierced shear wall configurations for solutions using the 12 DOF element. Slender in this context means a length to depth ratio of the lintel beams of approximately 8 or more.

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APPENDIX 1

Description of 8 DOF and 12 DOF Programs

These finite element programs are named for the total number of degrees of freedom possessed by the basic rectangular element used. The 12 DOF program based on MacLeod's elements also contains plane frame beam elements.

All floating point mathematical operations in these programs are carried out and stored in double precision.

After the structure is divided into two numbering systems of nodes and elements, numbered in sequence from one in each case, the data is prepared for input into the programs on punched cards as indicated in Appendices 2 and 3.

Upon entry of the data into the program, the program lists this data in the output. After solution of the problem the nodal displacements are listed with each node number and its coordinates. Next the normal stresses, shear stresses, and principle stresses are listed for each node of each element. If beams are used in the 12 DOF program, the member end actions (forces in the x and y direction and inplane moment) are listed for each beam.

Both computer programs are dimensionless, therefore, any consistent set of units may be used. For example, the linear dimensions may be in inches, while the modulus of elasticity may be in psi, and loads in pounds.

In dividing the structure into elements and nodes for the finite element analysis, it must be kept in mind that the maximum allowable difference in any two node numbers of a single element is 24 and 19, for the 8 DOF and 12 DOF programs, respectively.

Any number of zero or non-zero boundary conditions may be specified for each structure. Also, any number of nodal loads may be specified for each structure.

Any number of problems may be batched for solution on any one program run.

Refer to the appendices on data preparation for the maximum number of nodes, elements, beams and different materials allowed in each program.

To fully utilize the capabilities of each program, it is necessary to use a computer with large storage capacity. The 8 DOF program requires 170,000 bytes of core storage and 1,100,000 bytes of auxiliary disk storage, whereas the 12 DOF program requires 220,000 bytes of core and an extra 1,350,000 bytes on disk.

A detailed listing of each program is given in Appendices 4 and 5.

APPENDIX 2

Preparation of Data for 12 DOF Program

This data input sequence, format, and explanation was adopted from the work of E. L. Wilson, see reference 17. In idealizing a problem for solution by this program the nodes, elements and beams are each numbered sequentially from one.

Card A: Identification card (18A4)

Columns 1 through 72 of this card contain information to be printed with the results.

Card B: Control card (5I5)

Columns		Maximum
1- 5	Number of nodal points	900
6-10	Number of elements	800
11-15	Number of beams	500
16-20	Number of different materials	20
21-25	Number of boundary pressure cards	100

Group C: Material property identification - one card for each different material or different beam type used. The program automatically interprets data on these cards as values of element material properties or beam properties.

For element properties each card contains (I5,4F10.0)

Columns	
1- 5	Material identification number
6-15	Modulus of elasticity for compression
16-25	Poisson's ratio
26-35	Modulus of elasticity for tension
36-45	Thickness of element

For beam properties each card contains (I5,3F10.0)

Columns	
1- 5	Material identification number
6-15	Modulus of elasticity
16-25	Cross sectional area parallel to Z-axis
26-35	Moment of inertia about Z-axis

Group D: Nodal point data - one card for each node (2I5,5F10.0)

Columns	1- 5	Nodal point number
	10	Node Code (see discussion)
	11-20	X-coordinate of node
	21-30	Y-coordinate of node
	31-40	UX
	41-50	UY
	51-60	UZ

Nodes must be given in numerical sequence. If cards are omitted, the omitted nodes are generated at equal intervals along a straight line between the nodes.

The Node Code parameter is used to control the type of boundary conditions input under the names of UX, UY, and UZ in columns 31-60.

If Node Code (the number in column 10) is:

0	UX	Specifies load in x-direction
	UY	Specifies load in y-direction
	UZ	Specifies moment about z-axis
1	UX	Specifies displacement in x-direction
	UY	Specifies load in y-direction
	UZ	z moment
2	UX	Specifies load in x-direction
	UY	Specifies displacement in y-direction
	UZ	z moment
3	UX	Specifies displacement in x-direction
	UY	Specifies displacement in y-direction
	UZ	z moment
4	UX	Specifies displacement in x-direction
	UY	Specifies displacement in y-direction
	UZ	Specifies rotation about z-axis

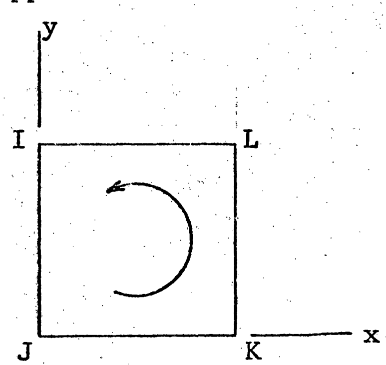
If a Node Code is not explicitly stated, it is set at 0.

Group E: Element cards - one card for each element (7I5)

Columns	1- 5	Element number
	6-10	Nodal point I
	11-15	Nodal point J
	16-20	Nodal point K
	21-25	Nodal point L
	26-30	Material Identification number
	31-35	Element type (1 or 2)

Element cards must be in element number sequence. If element cards are omitted, the program generates the omitted information by increasing each preceding value of I, J, K, and L by one. The generated material identification number is set equal to that of the last element. Each generated element type number (1 or 2) is set opposite to that of the last element. The last element card must always be given.

The nodal point specification sequence (I, J, K, L) must be followed explicitly. For the typical element these nodes appear as:



The maximum difference between any two nodal numbers on one element must be less than 20.

Group F: Beam cards - one card for each beam (4I5)

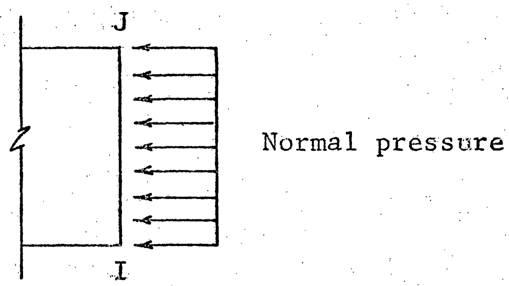
Columns	1- 5	Beam number
	6-10	Nodal point I
	11-15	Nodal point J
	16-20	Material Identification number

Beam cards must be in beam number sequence. If beam cards are omitted, the program generates the omitted data in the same manner as it does for elements.

Group G: Pressure Cards

One card for each element subjected to boundary pressure (2I5,F10.0)

Columns	1- 5	Nodal point I
	6-10	Nodal point J
	11-20	Normal pressure



As shown above, the element boundary must be on the left as one precedes from I to J for positive normal pressure.

APPENDIX 3

Preparation of Data for 8 DOF Program

This data input sequence, format and explanation was adopted from the work of E. L. Wilson, see reference 17.

Card A: Identification card (18A4)

Columns 1 through 72 of this card contain information to be printed with the results.

Card B: Control card (4I5)

Columns		Maximum
1- 5	Number of nodal points	1100
6-10	Number of elements	1000
11-15	Number of different materials	8
16-20	Number of boundary pressure cards	100

Group C: Material property identification - one card for each different material used. (I5,4F10.0)

Columns	
1- 5	Material identification number
6-15	Modulus of elasticity for compression
16-25	Poisson's ratio
26-35	Modulus of elasticity for tension
36-45	Thickness of element

Group D: Nodal point data - one card for each node (2I5,4F10.0)

Columns	
1- 5	Nodal point number
10	Node Code (see discussion)
11-20	x-coordinate of node
21-30	y-coordinate of node
31-40	UX
41-50	UY

Nodes must be given in numerical sequence. If cards are omitted, the omitted nodes are generated at equal intervals along a straight line between the nodes.

The Node Code parameter is used to control the type of boundary conditions input under the names of UX and UY in columns 31-50.

If Node Code (the number in column 10) is:

0	UX	Specifies load in x-direction
	UY	Specifies load in y-direction
1	UX	Specifies displacement in x-direction
	UY	Specifies load in y-direction
2	UX	Specifies load in x-direction
	UY	Specifies displacement in y-direction
3	UX	Specifies displacement in x-direction
	UY	Specifies displacement in y-direction

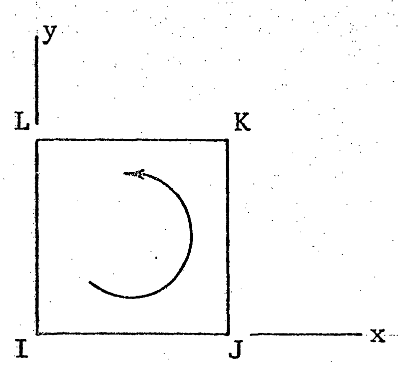
If a Node Code is not specified explicitly, it is set at 0.

Group E: Element Cards - one card for each element (6I5)

Columns	1- 5	Element number
	6-10	Nodal point I
	11-15	Nodal point J
	16-20	Nodal point K
	21-25	Nodal point L
	26-30	Material Identification number

Element cards must be in element number sequence. If element cards are omitted, the program generates the omitted information by increasing each preceding value of I, J, K, and L by one. The material identification number for each generated element is set equal to that of the last element. The last element card must always be given.

The nodal point specification sequence (I, J, K, L) must be followed explicitly. For the typical element these nodes appear as:



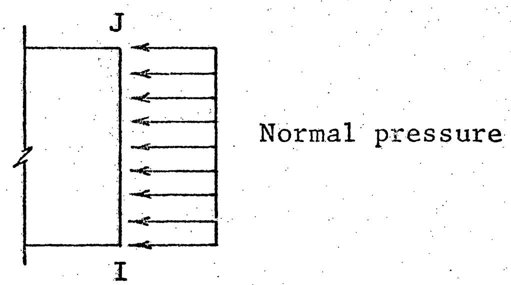
The maximum difference between any two nodal numbers must be less than 25.

Group F: Pressure Cards

One card for each element subjected to boundary pressure

(2I5,F10.0)

Columns	1- 5	Nodal point I
	6-10	Nodal point J
	11-20	Normal pressure



As shown above, the element boundary must be on the left as one precedes from I to J for positive normal pressure.

APPENDIX 4

12 DOF PROGRAM LISTING

```
C
C     FEM  12  DOF  PROGRAM
C
C
C     FINITE ELEMENT ANALYSIS USING TWO DIMENSIONAL PLANE STRESS
C     RECTANGULAR ELEMENTS WITH 3 DEGREES OF FREEDOM AT EACH NODE
C     AND PLANE FRAME BEAM ELEMENTS
C
C
C
      IMPLICIT REAL*8 (A-G,O-Z)
      REAL CONT
      INTEGER    CODE
      COMMON / HEAD/  ICRD,LIST,HED(18),IPAGE,LINE
      COMMON / DATUM / AS(800),BS(800),E(4,20),PR(100),
     2      X(900),Y(900),UX(900),UY(900),UZ(900),
     3      NUMNP,NUMEL,NUMMAT,NUMPC,MTYPE,CODE(900),
     4      IX(800,6),IBW,NE,IBC(100),JBC(100),
     5      IY(500,4),NUMBM
      COMMON / SOLN / A(120,60),B(120),MNUBLK,IJK,NB,ND
      DATA CONT/ 4HDATA/
      DEFINE FILE 1 (270,5000,L,IJK)
C
      ICRD = 5
      LIST= 6
C
C     CHECK FOR TERMINATION OF PROGRAM
C
   50 READ (ICRD,1000)  HED
 1000 FORMAT (18A4)
      IF (HED(2) .EQ. CONT )  STOP
C
   51 IPAGE = 1
C
C
C     DOCUMENT THE PROBLEM
C
      CALL DOCUMT
C
C     FORM STIFFNESS MATRIX
C
      CALL STIFF
C
C
C
C     SOLVE FOR AND PRINT OUT DISPLACEMENTS
C
      CALL  SYMSOL  (NE,IBW)
C
C
C
```

```
      CALL TITLE
      WRITE (LIST,2100)
      NB = 20
      ND = 60
      NBLK = 0
      N = 0
   60 NBLK = NBLK+1
      L = 0
   70 N = N+1
      L = L+1
      I = ND + (3*N-((NBLK-1)*ND))
      J = I-1
      K = I-2
      IF ( LINE .LT. 50 ) GO TO 65
      CALL TITLE
      WRITE (LIST,2100)
   65 WRITE (LIST,2200)  N,X(N),Y(N),A(K,NBLK),A(J,NBLK),A(I,NBLK)
      LINE = LINE +1
      IF ( N.EQ. NUMNP ) GO TO 80
      IF (L-NB) 70,60,60
   80 CONTINUE
 2100 FORMAT (1H0,9X,4HNODE,7X,1HX,11X,1HY,14X,7HX-DISPL,13X,7HY-DISPL,
     1 12X,9H"THETA-Z")
 2200 FORMAT ( 9X,I5,2F12.3,3D20.8)
C
C
C
C     CALCULATE AND PRINT OUT STRESSES
C
      CALL STRESS
C
C     START NEW PROBLEM
C
      GO TO 50
C
C
 5000 STOP
      END




      SUBROUTINE TITLE
C
C     TITLE SUBROUTINE
C
      COMMON / HEAD/  ICRD,LIST,HED(18),IPAGE,LINE
      WRITE (LIST,100) IPAGE
  100 FORMAT(1H1,3X,11HFEM  WEAVER,50X, 4HPAGE,I3)
      IF ( IPAGE .EQ. 1 ) WRITE (LIST,102)
  102 FORMAT(8X,75HFINITE ELEMENT ANALYSIS USING 12 DOF RECTANGULAR ELEM
```

```
     1ENT PLUS BEAM ELEMENTS)
      WRITE (LIST,101) HED
  101 FORMAT (1H0,10X,18A4)
      IPAGE= IPAGE +1
      LINE = 0
      RETURN
      END




      SUBROUTINE   DOCUMT
C
C     SUBROUTINE FOR DOCUMENTATION OF PROBLEM
C
      IMPLICIT REAL*8 (A-G,O-Z)
      INTEGER  CODE
      COMMON / HEAD/  ICRD,LIST,HED(18),IPAGE,LINE
      COMMON / DATUM / AS(800),BS(800),E(4,20),PR(100),
     2     X(900),Y(900),UX(900),UY(900),UZ(900),
     3     NUMNP,NUMEL,NUMMAT,NUMPC,MTYPE,CODE(900),
     4     IX(800,6),IBW,NE,IBC(100),JBC(100),
     5     IY(500,4),NUMBM
C
C     READ AND WRITE CONTROL INFORMATION AND MATERIAL PROPERTIES
C
      READ (ICRD,1003)  NUMNP,NUMEL,NUMBM,NUMMAT,NUMPC
      NE = 3*NUMNP
      CALL TITLE
      WRITE (LIST,2000)  NUMNP,NUMEL,NUMBM,NUMMAT,NUMPC
   56 DO 59  M=1,NUMMAT
      READ (ICRD,1001) MTYPE,(E(J,MTYPE),J=1,4)
      IF ( E(4,MTYPE)) 57,57,58
   57 WRITE (LIST,2011)  MTYPE,(E(K,MTYPE),K=1,3)
      GO TO 59
   58 WRITE (LIST,2010)  MTYPE, (E(I,MTYPE),I=1,4)
   59 CONTINUE
C
C     I/O  NODAL POINT DATA
C
      CALL TITLE
      WRITE (LIST,2004)
      L= 0
   60 READ (ICRD,1002) N,CODE(N),X(N),Y(N),UX(N),UY(N),UZ(N)
      IF ( N-1 ) 65,69,65
   65 CONTINUE
      ZX=N-L
      DX = (X(N)-X(L))/ZX
      DY = (Y(N)-Y(L))/ZX
   69 CONTINUE
      NL=L+1
```

```
   70 L=L+1
      IF ( N-L) 100,90,80
   80 CODE(L) = 0.0
      X(L) = X(L-1)+ DX
      Y(L) = Y(L-1)+ DY
      UX(L) = 0.0
      UY(L) = 0.0
      UZ(L) = 0.0
      GO TO 70
   90 DO 91 K=NL,N
      IF (LINE .LT. 50) GO TO 81
      CALL TITLE
      WRITE (LIST,2004)
   81 LINE = LINE + 1
   91 WRITE (LIST,2002)  K,CODE(K),X(K),Y(K),UX(K),UY(K),UZ(K)
      IF( NUMNP- N)  100,110,60
  100 WRITE (LIST,2009)  N
      STOP
  110 CONTINUE
C
C     I/O  ELEMENT PROPERTIES
      IF ( NUMEL ) 190,190,120
  120 CALL TITLE
C
      WRITE (LIST,2001)
      N=0
  130 READ (ICRD,1003) M,(IX(M,I),I=1,6)
      IF (M - NUMEL) 140,140,132
  132 WRITE (LIST,2019)  M
      STOP
  140 N=N+1
      IF(M-N)  170,170,150
  150 IX(N,1)=IX(N-1,1)+1
      IX(N,2)=IX(N-1,2)+1
      IX(N,3)=IX(N-1,3)+1
      IX(N,4)=IX(N-1,4)+1
      IX(N,5)=IX(N-1,5)
      IF ( IX(N-1,6) .EQ. 1 ) GO TO 160
      IX(N,6) = 1
      GO TO 170
  160 IX(N,6) =2
  170 CONTINUE
      I =IX(N,1)
      K =IX(N,3)
      BS(N) = DABS (Y(K) -Y(I))
      AS(N) = DABS (X(K)-X(I))
      IF ( AS(N)) 169,168,169
  168 WRITE (LIST,2018) N
      STOP
  169 CONTINUE
      AB = BS(N) / AS(N)
```

```
      IF (LINE .LT. 50 ) GO TO 171
      CALL TITLE
      WRITE (LIST,2001)
  171 LINE = LINE +1
      WRITE (LIST,2003) N,(IX(N,I),I=1,6),AB    ,AS(N),BS(N)
      IF(M-N) 180,180,140
  180 IF (NUMEL- N) 190,190,130
  190 CONTINUE
C
C     I/O BEAM PROPERTIES
C
      IF (NUMBM) 400,450,410
  400 WRITE (LIST,2040)
 2040 FORMAT (37HONO. BEAMS ENTERED AS NEGATIVE NUMBER)
      STOP
  410 CALL TITLE
      WRITE (LIST,2030)
 2030 FORMAT (1H0,9X,35HBEAM NO.     I      J    MATERIAL NO.)
      N=0
  420 READ (ICRD,1003) M,(IY(M,I),I=1,4)
      IF (M - NUMBM)  425,425,422
  422 WRITE (LIST,2041) M
 2041 FORMAT (14HOBEAM ERROR M=,I5)
      STOP
  425 N= N+1
      IF ( IY(M,4)) 426,430,426
  426 CONTINUE
      IF ( M-N ) 430,430,427
  427 IY(N,1)=IY(N-1,1)+1
      IY(N,2)=IY(N-1,2)+1
      IY(N,3)=IY(N-1,3)
      IY(N,4)= 6
  430 CONTINUE
      IF (LINE .LT. 50 ) GO TO 435
      CALL TITLE
      WRITE (LIST,2030)
  435 LINE= LINE + 1
      WRITE (LIST,2031) N,(IY(N,I),I=1,4)
 2031 FORMAT(10X,I6,I7,I6,I10,I9)
C
      IF ( M .GT. N) GO TO 425
      IF ( N .LT. NUMBM) GO TO 420
  450 CONTINUE
C
C     I/O  OF PRESSURE BOUNDARY CONDITIONS
C
      IF (NUMPC) 280,310,290
  280 WRITE (LIST,2020)
      STOP
  290 CALL TITLE
      WRITE (LIST,2005)
```

```
      DO 300  L=1,NUMPC
      READ (ICRD,1002) IBC(L),JBC(L),PR(L)
      IF(LINE .LT.50) GO TO 291
      CALL TITLE
      WRITE (LIST,2005)
  291 LINE= LINE + 1
  300 WRITE (LIST,2006)  IBC(L),JBC(L),PR(L)
  310 CONTINUE
C
C     DETERMINE BAND WIDTH
C
      J=0
      IF ( NUMEL .LT. 1 ) GO TO 350
      DO 340 N=1,NUMEL
      DO 340 I=1,3
      K=I+1
      DO 340 L=K,4
      KK =IABS (IX(N,I)-IX(N,L))
      IF (KK-J)  325,325,320
  320 J=KK
  325 CONTINUE
  340 CONTINUE
  350 CONTINUE
C     IBW = BAND-WIDTH FOR PROBLEM IN QUESTION
C
C     CHECK HALF BAND WIDTH WITH BEAMS
C
      IF ( NUMBM .LT. 1 ) GO TO 480
      K= 0
      DO 460 N=1,NUMBM
      JJ= IABS( IY(N,1)-IY(N,2))
      IF ( JJ .LT. K) GO TO 460
      K=JJ
  460 CONTINUE
C
      IF ( K .LT. J) GO TO 480
      J=K
  480 CONTINUE
      IBW = 3*(J+1)
      WRITE (LIST,2050) IBW
 2050 FORMAT (10X,34HHALF BAND WIDTH FOR THIS PROBLEM =,I3)
C
 5000 CONTINUE
C
C
C     FORMAT STATMENTS
C
 1001 FORMAT(I5,5F10.0)
 1002 FORMAT (2I5,5F10.0)
 1003 FORMAT  (7I5)
 2000 FORMAT (1H0   /
```

```
    110X,28HNUMBER OF NODAL POINTS......,I5//
    210X,28HNUMBER OF ELEMENTS..........,I5//
    X10X,28HNUMBER OF BEAMS.............,I5//
    310X,28HNUMBER OF DIFF. MATERIALS...,I5//
    410X,28HNUMBER OF PRESSURE CARDS....,I5/)
2001 FORMAT (1H0,5X,65HELEMENT NO.    I      J      K      L     MATERIAL
    1 EL.TYPE       B/A,11X,1HA,12X,1HB)
2002 FORMAT (7X,2I10,2F17.4,3D20.7)
2003 FORMAT (11X,I3,2X,4I6,I8,I11,3F13.3)
2004 FORMAT (1H0,9X,  18HNODAL POINT   TYPE,10X,6HX-ORD.,11X,6HY-ORD.,
    1  6X,14HX(LOAD/DISPL.), 6X,14HY(LOAD/DISPL.),6X,10H"Z-MOMENT")
2005 FORMAT (1H0,10X,28HPRESSURE BOUNDARY CONDITIONS/ 1H0,19X,1HI,6X,1
    1J,5X,8HPRESSURE)
2006 FORMAT (18X,I3,4X,I3,F13.4)
2009 FORMAT  (26HONODAL POINT CARD ERROR N=,I5)
2010 FORMAT (1H0,9X,  9HMAT'L NO.,18X,4HE(C),19X,2HNU,19X,4HE(T),16X,
    19HTHICKNESS,/  (14X,I3,7X,3D22.7,F22.6))
2011 FORMAT (1H0  /
    1 10X,31HPROPERTIES OF BEAM MATERIAL NO. ,I3//
    2 15X,26HMODULUS OF ELASTICITY.....,D15.8//
    3 15X,26HAREA OF CROSS-SECTION.....,F15.5//
    4 15X,26HI ABOUT Z-AXIS............,D15.8)
2018 FORMAT (37HONODAL COORD. ERROR - SIDE OF ELEMENT,I6,2X,8HEQUALS O
2019 FORMAT (18HOELEMENT ERROR M=,I5)
2020 FORMAT (33HONUMPC ENTERED AS NEGATIVE NUMBER)
     RETURN
     END




     SUBROUTINE STIFF
     IMPLICIT REAL*8 (A-G,O-Z)
     INTEGER  CODE
     COMMON / DATUM / AS(800),BS(800),E(4,20),PR(100),
    2      X(900),Y(900),UX(900),UY(900),UZ(900),
    3      NUMNP,NUMEL,NUMMAT,NUMPC,MTYPE,CODE(900),
    4      IX(800,6),IBW,NE,IBC(100),JBC(100),
    5      IY(500,4),NUMBM
     COMMON / SOLN / A(120,60),B(120),MNUBLK,IJK,NB,ND
     COMMON/ SMALK / R(12,12),C(12,12,2),D(12,12),S(12,12,2),DB(3,12)
     DIMENSION LM(4)
     DIMENSION SM(6,6)
C
C    FORM ELEMENT STIFFNESS MATRICES FOR BOTH TYPES OF ELEMENTS
C
C    D EQUALS QTDQ TIMES C
C    S EQUALS  CT TIMES QTDC*C
C    INITIALIZE MATRICES
C
     NB = 20
```

```
      ND = 3* NB
      ND2= 2*ND
      NBLK = 0
      IJK = 1
      DO 10 I=1,ND2
      B(I)= 0.0D0
      DO 10 J=1,ND
   10 A(I,J)= 0.0D0
C
C     CALCULATE MAX. NUMBER BLOCKS
      MNUBLK = (NUMNP * 3 / ND) +1
C
C
      N1 = 0
      N2=0
C
   60 NBLK = NBLK +1
      NH = NB*(NBLK+1)
      NM = NH-NB
      NL = NM- NB+1
      IBS= 3*NL-3
C     FORM ELEMENT STIFFNESS MATRIX
C
      IF ( NUMEL ) 220,220,20
   20 DO 210 N=1,NUMEL
      IF ( IX(N,5) .LT. 1 ) GO TO 210
C
C     DETERMINE IF THIS ELEMENT EFFECTS THIS BLOCK OF BIGK
C
      DO 80 I=1,4
      IF ( IX(N,I) .GE. NL .AND. IX(N,I) .LE.NM ) GO TO 90
   80 CONTINUE
      GO TO 210
   90 CONTINUE
C
      M =IX(N,6)
      IF( N.EQ.1) GO TO 100
      QA = DABS ( AS(N) / AR -1.0D0)
      QB = DABS ( BS(N) / BR -1.0D0 )
      IF ( QA .LT. 0.5D-5  .AND. QB .LT. 0.5D-5  .AND. IX(N,5) .EQ.
     1  MR ) GO TO 120
  100 CONTINUE
C
      CALL QTDQ (N)
      AR = AS(N)
      BR = BS(N)
      MR = IX(N,5)
  120 GO TO (125,135),M
  125 N1 = N1+1
      IF ( N1.EQ. 1) GO TO 130
      QA = DABS((2.0D0 * AS(N) /(A1+AR)) -1.0D0)
```

```
      QB = DABS((2.0D0 * BS(N) /(B1+BR)) -1.0D0)
      IF (QA .LT.0.5D-5 .AND.QB.LT.0.5D-5 )  GO TO 150
C
  130 CONTINUE
      A1 = AS(N)
      B1 = BS(N)
      MTYP1 = IX(N,5)
      CALL CINVER (N,M)
      GO TO 145
  135 N2= N2+1
      IF ( N2 .EQ. 1) GO TO 140
      QA = DABS((2.0D0 * AS(N) /(A2+AR)) -1.0D0)
      QB = DABS((2.0D0 * BS(N) /(B2+BR)) -1.0D0)
      IF ( QA .LT.0.5D-5  .AND. QB .LT.0.5D-5  ) GO TO 150
C
  140 CONTINUE
      A2= AS(N)
      B2= BS(N)
      MTYP2= IX(N,5)
      CALL CINVER (N,M)
  145 CONTINUE
C
C     MULTIPLY  (C* QTDQ *C)VOL
      DO 146 I=1,12
      DO 146 J=1,12
      D(I,J) = 0.0D0
      DO 146 K=1,12
      D(I,J)= D(I,J) + R(I,K) * C(K,J,M)
  146 CONTINUE
      DO 148 I=1,12
      DO 148 J=1,12
      S(I,J,M) = 0.0D0
      DO 148 K=1,12
      S(I,J,M) = S(I,J,M) + C(K,I,M) * D(K,J)
  148 CONTINUE
  150 CONTINUE
C
C
C     ADD ELEMENT STIFFNESS TO TOTAL STIFFNESS
C
  161 DO 166  I=1,4
  166 LM(I) = 3 * (IX(N,I) -1)
      IX(N,5) = -IX(N,5)
C
      DO 200 I=1,4
      DO 200 K=1,3
      II= LM(I)+ K -IBS
      KK = 3* (I-1) +K
      DO 205 J=1,4
      DO 205 L=1,3
      JJ= LM(J)+L-II+1- IBS
```

```
      LL = 3 *(J-1) + L
      IF (JJ) 185,185,170
  170 CONTINUE
      IF (IBW-JJ) 180,195,195
  180 WRITE (LIST,2004) N
 2004 FORMAT (10X,46HBAND WIDTH EXCEEDS THAT CALCULATED FOR PROBLEM)
      STOP
  185 CONTINUE
      GO TO 205
  195 CONTINUE
      A(II,JJ) = A(II,JJ) + S(KK,LL,M)
  205 CONTINUE
  200 CONTINUE
  210 CONTINUE
  220 CONTINUE
C
C     ADD BEAM STIFFNESSES TO TOTAL STIFFNESS MATRIX
C
      IF (NUMBM .EQ. 0) GO TO 490
      DO 480  N=1,NUMBM
      IF ( IY(N,3) .LT. 1 ) GO TO 480
C
C     DETERMINE IF THIS BEAM EFFECTS THIS BLOCK OF BIGK
C
      IF (IY(N,1).GE.NL.AND.IY(N,1).LE.NM) GO TO 332
      IF (IY(N,2).GE.NL.AND.IY(N,2).LE.NM) GO TO 332
      GO TO 480
  332 CONTINUE
C
      L = IY(N,1)
      K = IY(N,2)
      YY=      ( Y(K)-Y(L))
      XX=      ( X(K)-X(L))
      IF (N.EQ. 1) GO TO 450
C
      QA = 0.0D0
      QB = 0.0D0
      IF ( DABS(XX) .GT. 0.5D-7  ) QA=1.0D0
      IF ( DABS(XOLD) .GT. 0.5D-7  ) GO TO 331
      GO TO 335
  331 QA = DABS( XX/ XOLD -1.0D0)
  335 IF ( DABS(YY) .GT. 0.5D-7  ) QB=1.0D0
      IF ( DABS(YOLD) .GT. 0.5D-7  ) GO TO 340
      GO TO 345
  340 QB = DABS( YY/ YOLD -1.0D0)
  345 CONTINUE
      IF ( QA .LT. 0.5D-7  .AND. QB .LT. 0.5D-7  .AND. IY(N,3) .EQ.
     1 MTYPE ) GO TO 460
  450 CONTINUE
      CALL BMSTIF (N,L,K,SM)
      XOLD = XX
```

```
      YOLD = YY
      MTYPE = IY(N,3)
  460 CONTINUE
C
C     ADD BEAM STIFFNESS TO TOTAL STIFFNESS
C
      LM(1)= 3*(IY(N,1)-1)
      LM(2)= 3*(IY(N,2)-1)
      IY(N,3) = -IY(N,3)
C
      DO  470  I=1,2
      DO  470  K=1,3
      II = LM(I) + K - IBS
      KK = 3* (I-1) + K
      DO  465  J=1,2
      DO  465  L=1,3
      JJ = LM(J) + L- II+1 -IBS
      LL= 3* (J-1)+ L
      IF (JJ) 468,468,467
  467 CONTINUE
      IF (IBW -JJ)  469,471,471
  469 WRITE (LIST,2004) N
  468 CONTINUE
      GO TO 465
  471 CONTINUE
      A(II,JJ)=A(II,JJ)+ SM(KK,LL)
  465 CONTINUE
  470 CONTINUE
  480 CONTINUE
  490 CONTINUE
C
C
C
C     ADD CONCENTRATED FORCES  AT NODES
C
      DO 250 N=NL,NM
      IF ( N .GT. NUMNP ) GO TO 250
      ICODE = CODE(N)+ 1
      K = N *3 - IBS
      J = K -1
      I = K-2
      GO TO (251,252,253,254,250) ,ICODE
  251 B(J) = B(J) + UY(N)
  253 B(I) = B(I) + UX(N)
      GO TO 254
  252 B(J) = B(J) + UY(N)
  254 B(K) = B(K) + UZ(N)
  250 CONTINUE
C
C
C     BOUNDARY CONDITIONS
```

```
C
C     PRESSURE  B.C.
C
      IF (NUMPC) 260,310,260
  260 DO 300 L=1,NUMPC
      I=IBC(L)
      J=JBC(L)
      PP = PR(L)/2.0
C
      DX = (X(J)- X(I))* PP
      DY = (Y(I)- Y(J))* PP
C
      II = 3*I-IBS
      JJ= 3*J -IBS
      IF (II .LT. 2 .OR. II .GT. ND )  GO TO 270
  270 IF (JJ .LT. 2 .OR. JJ .GT. ND ) GO  TO 300
C
      B(II-1) = B(II-1)+ DY
      B(II) = B(II)+DX
      B(JJ-1) = B(JJ-1) + DY
      B(JJ) = B(JJ) +DX
C
  300 CONTINUE
  310 CONTINUE
C
C     DISPLACEMENT B.C.
C
      DO 400 M=NL,NH
      IF ( M .GT. NUMNP ) GO TO 400
C
      ICODE = CODE(M) + 1
      N =  3*M-2 -IBS
      U= UX(M)
C
      GO TO (400,315,320,330,330),ICODE
C
  315 CALL MODIFY (N,U,IBW,ND2)
      GO TO 400
  330 CALL MODIFY (N,U,IBW,ND2)
  320 N=N+1
      U= UY(M)
      CALL MODIFY (N,U,IBW,ND2)
      IF (ICODE.LT. 5 ) GO TO 400
      N = N+1
      U = 0.0D0
      CALL MODIFY (N,U,IBW,ND2)
  400 CONTINUE
C
C     WRITE BLOCK OF EQU'S ON DISK AND SHIFT UP LOWER BLOCK
C
      IJK =( NBLK* 6) -5
```

```
      WRITE (1'IJK) ((A(I,J),I=1,ND),J=1,IBW),(B(K),K=1,ND)
C
      DO 420 N=1,ND
      K=N+ND
      B(N)= B(K)
      B(K)= 0.0D0
      DO 420 M=1,ND
      A(N,M)= A(K,M)
  420 A(K,M) = 0.0D0
C
C     CHECK FOR LAST BLOCK
      IF ( NM .LE. NUMNP) GO TO 60
C
C      CHANGE ALL MATERIAL NUMBERS BACK TO + NUMBERS
C
      IF (NUMEL .LT. 1 ) GO TO 601
      DO 600 I=1,NUMEL
      J = IX(I,5)
      IX(I,5) =-IX(I,5)
  600 CONTINUE
  601 IF ( NUMBM .LT. 1 ) GO TO 651
      DO 650 I=1,NUMBM
      J = IY(I,3)
      IY(I,3) = -IY(I,3)
  650 CONTINUE
  651 CONTINUE
C
  500 RETURN
      END
```

```
      SUBROUTINE BMSTIF (N,L,K,SM)
      IMPLICIT REAL*8 (A-G,O-Z)
      INTEGER CODE
      COMMON / DATUM / AS(800),BS(800),E(4,20),PR(100),
     2      X(900),Y(900),UX(900),UY(900),UZ(900),
     3      NUMNP,NUMEL,NUMMAT,NUMPC,MTYPE,CODE(900),
     4      IX(800,6),IBW,NE,IBC(100),JBC(100),
     5      IY(500,4),NUMBM
      COMMON/ SMALK / R(12,12),C(12,12,2),D(12,12),S(12,12,2),DB(3,12)
      DIMENSION SM(6,6)
C
C
C     FORM BEAM STIFFNESS MATRIX
C
C
      XX=  X(K)- X(L)
      YY=  Y(K)- Y(L)
      XY=  DSQRT ( XX**2 + YY**2)
```

```
      CX=  XX/ XY
      CY=  YY/ XY
      M= IY(N,3)
      CON = 0.0D0
      CON=  E(2,M)* XY**2 /(12.0D0 * E(3,M))
C
C     E(1,M)= MODULUS,E(2,M)=XAREA , E(3,M)= MOMENT INERTIA Z
C
      DO 100 I=1,6
      DO 100 J=1,6
  100 SM(I,J)= 0.0D0
      DO 200 I=1,4,3
      SM(I,I)= CON * CX**2 + CY**2
      J=I+1
      SM(J,J)= CON * CY**2 + CX**2
      J=J+1
      SM(J,J)= XY**2 /3.0D0
  200 CONTINUE
      SM(2,1)= (CON-1)*CX*CY
      SM(5,4)= SM(2,1)
      SM(4,2)= -SM(2,1)
      SM(5,1)= -SM(2,1)
      SM(4,1)= -SM(1,1)
      SM(5,2)= -SM(2,2)
      SM(3,2)= CX*XY/2.0D0
      SM(6,2)= SM(3,2)
      SM(6,5)= -SM(3,2)
      SM(5,3)= -SM(3,2)
      SM(6,4)= CY* XY/2.0D0
      SM(6,1)= -SM(6,4)
      SM(3,1) = -SM(6,4)
      SM(4,3)= SM(6,4)
      SM(6,3)= SM(3,3)/ 2.0D0
      CON = 12.0D0 * E(1,M)* E(3,M) / XY**3
      DO 220  I=1,6
      DO 220  J=1,I
  220 SM(I,J) = SM(I,J)* CON
      DO 240  I=1,5
      J=I+1
      DO 240  K=J,6
      SM(I,K) = SM(K,I)
  240 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE CINVER (N,M)
      IMPLICIT REAL*8 (A-G,O-Z)
      INTEGER CODE
```

```
      COMMON / DATUM / AS(800),BS(800),E(4,20),PR(100),
     2      X(900),Y(900),UX(900),UY(900),UZ(900),
     3      NUMNP,NUMEL,NUMMAT,NUMPC,MTYPE,CODE(900),
     4      IX(800,6),IBW,NE,IBC(100),JBC(100),
     5      IY(500,4),NUMBM
      COMMON/ SMALK / R(12,12),C(12,12,2),D(12,12),S(12,12,2),DB(3,12)
      DIMENSION C1(12,12),C2(12,12)
      AA= 1.0D0 / AS(N)
      BB= 1.0D0 / BS(N)
      AB= AA*BB
      DD = AA**2
      EE = BB**2
      FF = DD* BB
      GG = EE* AA
      GO TO (100,200),M
C
C     C INVERSE FOR TYPE 1 ELEMENT
C
  100 DO 50 I=1,12
      DO 50 J=1,12
   50 C1(I,J)= 0.0D0
      C1(1,1) = EE
      C1(1,4) =-EE
      C1(1,6) = BB
      C1(2,2) = BB
      C1(2,5) =-BB
      C1(3,3) = BB
      C1(3,9) = BB
      C1(3,5) = AB*2.0D0
      C1(3,8)=-AB* 2.0D0
      C1(4,4) = 1.0D0
      C1(5,5) = 1.0D0
      C1(6,6)=-1.0D0
      C1(7,4)= -AA
      C1(7,7)=  AA
      C1(8,5)= -AA* 2.0D0
      C1(8,8)=  AA* 2.0D0
      C1(8,9)= -1.0D0
      C1(9,5)= DD
      C1(9,8)=-DD
      C1(9,9)= AA
      C1(10,1)=-GG
      C1(10,4)= GG
      C1(10,6)=-AB
      C1(10,7)= GG
      C1(10,10) = -GG
      C1(10,12) = -AB
      C1(11,2) = -FF
      C1(11,3) = -AB
      C1(11,5) = -FF
      C1(11,8)  = FF
```

```
      C1(11,9)   =-AB
      C1(11,11) = FF
      C1(12,6)   = AA
      C1(12,7)   =-AB* 2.0D0
      C1(12,10) =AB* 2.0D0
      C1(12,12) = AA
      DO 70 I=1,12
      DO 70 J=1,12
   70 C(I,J,1) = C1(I,J)
      GO TO 300
C
C
C     C INVERSE FOR TYPE 2 ELEMENT
C
  200 DO 60  I=1,12
      DO 60 J=1,12
   60 C2(I,J)= 0.0D0
      C2(1,1) = -EE
      C2(1,3) = -BB
      C2(1,4) =  EE
      C2(2,2) =  BB
      C2(2,5) = -BB
      C2(3,2) = -AB* 2.0D0
      C2(3,6) = -BB
      C2(3,11)=  AB* 2.0D0
      C2(3,12)= -BB
      C2(4,4) =  1.0D0
      C2(5,5) =  1.0D0
      C2(6,1) =  BB* 2.0D0
      C2(6,3) =  1.0D0
      C2(6,4) = -BB* 2.0D0
      C2(7,4) = -AA
      C2(7,7) =  AA
      C2(8,6) =  1.0D0
      C2(9,5) = -DD
      C2(9,6) = -AA
      C2(9,8) =  DD
      C2(10,1)=  GG
      C2(10,3)=  AB
      C2(10,4)= -GG
      C2(10,7)= -GG
      C2(10,9)=  AB
      C2(10,10)= GG
      C2(11,2) = FF
      C2(11,5) = FF
      C2(11,6) = AB
      C2(11,8) = -FF
      C2(11,11)= -FF
      C2(11,12)=  AB
      C2(12,1) = -AB* 2.0D0
      C2(12,3) = -AA
```

```
      C2(12,4) =  AB* 2.0D0
      C2(12,9) = -AA
      DO 80 I=1,12
      DO 80 J=1,12
   80 C(I,J,2) = C2(I,J)
  300 CONTINUE
      RETURN
      END




      SUBROUTINE SYMSOL (NN,IBW)
      IMPLICIT REAL*8(A-G,O-Z)
      COMMON / SOLN / A(120,60),B(120),MNUBLK,IJK,NB,ND
C
C     SOLVE BANDED SYMMETRIC SYSTEM OF SIMULTANEOUS EQUATIONS
C
      NN = 60
      NL = NN+1
      NH = NN+NN
      NBLK = 0
      IJK=1
      GO TO 150
C
C     REDUCE EQU'S BY BLOCKS
C
C     SHIFT BLOCK OF EQU'S
C
  100 NBLK =NBLK +1
      DO 125 N=1,NN
      NM = NN+N
      B(N)= B(NM)
      B(NM)= 0.0D0
      DO 125 M=1,IBW
      A(N,M) =A(NM,M)
  125 A(NM,M) = 0.0D0
C
C     READ IN NEXT BLOCK OF EQU'S
C
      IF (NBLK .EQ. MNUBLK ) GO TO 200
      IJK = (NBLK *6) +1
  150 READ (1'IJK) ((A(I,J),I=NL,NH),J=1,IBW),(B(K),K=NL,NH)
      IF (NBLK .EQ. 0 ) GO TO 100
C
C     REDUCE BLOCK OF EQU'S
C
  200 DO 300 N=1,NN
      IF ( A(N,1) .EQ. 0.0D0 ) GO TO 300
      B(N) =B(N)/ A(N,1)
      DO 275 L=2,IBW
```

```
      IF ( A(N,L) .EQ. 0.0D0 ) GO TO 275
      C =A(N,L) / A(N,1)
      I=N+L-1
      J=0
      DO 250 K=L,IBW
      J=J+1
      IF (A(I,J) .EQ. 1.0D0 .AND. B(I) .EQ. 0.0D0 ) GO TO 260
  250 A(I,J)= A(I,J) - C*A(N,K)
      B(I) =B(I)-A(N,L)* B(N)
  260 A(N,L) = C
  275 CONTINUE
  300 CONTINUE
C
C     WRITE BLOCK OF EQU'S BACK ON DISK
C
      IF( NBLK .EQ. MNUBLK) GO TO 400
      IJK = (NBLK *6) -5
      WRITE (1'IJK)   ((A(I,J),I=1,NN),J=2,IBW),( B(K),K=1,NN)
      GO TO 100
C
C     BACK SUBSTITUTION
C
  400 DO 450 M=1,NN
      N=NN+1-M
      DO 425 K=2,IBW
      L=N+K-1
  425 B(N) =B(N) - A(N,K)*B(L)
      NM =N + NN
      B(NM)= B(N)
  450 A(NM,NBLK) =B(N)
      NBLK = NBLK-1
      IF (NBLK .EQ. 0 ) GO TO  500
      IJK = (NBLK *6) -5
      READ (1'IJK) ((A(I,J),I=1,NN),J=2,IBW),(B(K),K=1,NN)
      GO TO 400
C
C     UNKNOWNS ARE STORED IN FIRST MNUBLK COLUMNS OF A IN 2ND BLOCK
C
  500 RETURN
      END




      SUBROUTINE  MODIFY  (N,U,IBW,NE)
C
C     MODIFY TOTAL STIFFNESS MATRIX FOR GIVEN DISPL. BOUNDARY CONDITIONS
C
      IMPLICIT REAL*8 (A,B,U)
      COMMON / SOLN / A(120,60),B(120),MNUBLK,IJK,NB,ND
C
```

```
      DO 250  M=2,IBW
      K= N- M +1
      IF (K) 235,235,230
  230 B(K) =B(K)- A(K,M)* U
      A(K,M) =0.0D0
  235 K=N+ M-1
      IF(NE -K)  250,240,240
  240 B(K)= B(K)- A(N,M)* U
      A(N,M) = 0.0D0
  250 CONTINUE
      A(N,1)= 1.0D0
      B(N)= U
      RETURN
      END




      SUBROUTINE STRESS
      IMPLICIT REAL*8 (A-G,O-Z)
      INTEGER CODE
      COMMON / HEAD/  ICRD,LIST,HED(18),IPAGE,LINE
      COMMON / DATUM / AS(800),BS(800),E(4,20),PR(100),
     2     X(900),Y(900),UX(900),UY(900),UZ(900),
     3     NUMNP,NUMEL,NUMMAT,NUMPC,MTYPE,CODE(900),
     4     IX(800,6),IBW,NE,IBC(100),JBC(100),
     5     IY(500,4),NUMBM
      COMMON / SOLN / A(120,60),B(120),MNUBLK,IJK,NB,ND
      COMMON/ SMALK / R(12,12),C(12,12,2),D(12,12),S(12,12,2),DB(3,12)
      DIMENSION U(12),SIG(6)
C
C
C     CALCULATE STRESSES AT EACH NODE OF THE ELEMENTS
C            AND END ACTIONS ON THE BEAMS.
C
C
      ND = 60
      IF ( NUMEL ) 150,150,99
   99 N1 = 0
      CALL TITLE
      WRITE (LIST,2000)
C
C     S WILL CONTAIN  4 SETS OF MATRIX DQ*C-ONE 3*12 FOR EACH NODE
C
      N2=0
      DO 100 N=1,NUMEL
      M= IX(N,6)
      GO TO (101,102), M
  101 N1= N1+1
      IF(N1.EQ. 1) GO TO 105
      QA = DABS( AS(N)/ A1 -1.0D0)
```

```
      QB = DABS( BS(N)/ B1 -1.0D0)
      IF ( QA .LT.0.5D-5  .AND. QB .LT.0.5D-5  .AND. IX(N,5) .EQ.
     1MTYP1 ) GO TO 106
  105 CONTINUE
C
      A1= AS(N)
      B1= BS(N)
      MTYP1 =IX(N,5)
      CALL CINVER (N,M)
C
  106 CONTINUE
      GO TO 109
  102 N2= N2 +1
      IF(N2.EQ. 1) GO TO 108
      QA = DABS( AS(N)/ A2 -1.0D0)
      QB = DABS( BS(N)/ B2 -1.0D0)
      IF (QA .LT.0.5D-5  .AND. QB .LT.0.5D-5  .AND. IX(N,5) .EQ.
     1MTYP2 ) GO TO 109
  108 CONTINUE
C
      A2 = AS(N)
      B2 = BS(N)
      MTYP2 = IX(N,5)
      CALL CINVER (N,M)
C
  109 CONTINUE
      IF ( N.EQ.1 .OR.N1.EQ. 1 .OR. N2.EQ.1 ) GO TO 110
      GO TO (111,112),M
  111 QA = DABS ((2.0D0*AS(N) /(A1+AR))-1.0D0)
      QB = DABS ((2.0D0*BS(N) /(B1+BR))-1.0D0)
      MR= 2*IX(N,5)- MTYP1 -MTYPR
      GO TO 113
  112 QA = DABS ((2.0D0*AS(N) /(A2+AR))-1.0D0)
      QB = DABS ((2.0D0*BS(N) /(B2+BR))-1.0D0)
      MR= 2*IX(N,5)- MTYP2 -MTYPR
  113 CONTINUE
      IF (QA.LT.0.5D-5  .AND.QB.LT.0.5D-5 .AND.MR.EQ.0)GO TO 120
  110 CONTINUE
      AR = AS(N)
      BR = BS(N)
      MTYPR = IX(N,5)
      CALL ELSTRS (N,M)
C
  120 CONTINUE
C     PLACE PROPER DISPL.  IN U FROM A
C
      DO 20 J=1,4
      DO 20 K=1,3
      I = 3*IX(N,J)-3 +K
      JJ = 3*J-3+K
      NBLK = (I-1) /ND +1
```

```
      II =    ND + I-(ND*(NBLK-1))
      U(JJ) = A(II,NBLK)
   20 CONTINUE
C
C     SOLVE FOR STRESSES
C
      DO 200 K=1,4
      DO 30 I=1,6
   30 SIG(I) = 0.0D0
      DO 35 I=1,3
      L=(3*K)+I-3
      DO 35 J=1,12
      SIG(I) = SIG(I)+ S(L,J,M)* U(J)
   35 CONTINUE
C
C     CALCULATE PRINCIPAL STRESSES
C
      AA =(SIG(1)+SIG(2))* 0.5D0
      BB =(SIG(1)-SIG(2))* 0.5D0
      CC = DSQRT( BB**2 + SIG(3)**2)
      SIG(4)=AA+CC
      SIG(5)=AA-CC
      IF ( BB .EQ. 0.0D0 ) GO TO 41
      SIG(6)=(DATAN(SIG(3)/ BB ))* 23.64788975D0
      GO TO 42
   41 SIG(6) = 90.0D0
   42 CONTINUE
      IF (LINE.LT.40) GO TO 40
      CALL TITLE
      WRITE (LIST,2000)
   40 CONTINUE
      IF ( K .EQ. 1 ) GO TO 45
      WRITE (LIST,2001) IX(N,K),(SIG(I),I=1,6)
      GO TO 46
   45 WRITE (LIST,2002)   N,IX(N,K),(SIG(I),I=1,6)
   46 LINE = LINE +1
  200 CONTINUE
  100 CONTINUE
  150 CONTINUE
 2000 FORMAT (1H0,9X,6HEL.NO.,6X,4HNODE,9X,8HX-STRESS,6X,8HY-STRESS,
     15X,9HXY-STRESS,5X,10HMAX-STRESS,4X,10HMIN-STRESS,4X,5HANGLE)
 2001 FORMAT (15X,I10,5X,5D14.4,F9.2)
 2002 FORMAT (1H0,9X,I5,I10,5X,5D14.4,F9.2)
C
C     SOLVE FOR BEAM END ACTIONS
C
      IF ( NUMBM .LT. 1 ) GO TO 500
      CALL TITLE
      WRITE(LIST,2100)
      DO 400 N=1,NUMBM
      L= IY(N,1)
```

```
      LL = IY(N,2)
      XX = X(LL) -X(L)
      YY = Y(LL) -Y(L)
      IF (N .EQ. 1) GO TO 320
      QA = 0.0D0
      QB = 0.0D0
      IF ( DABS(XX) .GT. 0.5D-7  ) QA=1.0D0
      IF ( DABS(XOLD) .GT. 0.5D-7  ) GO TO 300
      GO TO 305
  300 QA = DABS( XX/ XOLD -1.0D0)
  305 IF ( DABS(YY) .GT. 0.5D-7  ) QB=1.0D0
      IF ( DABS(YOLD) .GT. 0.5D-7  ) GO TO 310
      GO TO 315
  310 QB = DABS( YY/ YOLD -1.0D0)
  315 CONTINUE
      IF  ( QA .LT. 0.5D-7  .AND. IY(N,3) .EQ. MTYPE      .AND. QB .LT.
     1 0.5D-7 ) GO TO 330
  320 CONTINUE
      XOLD= XX
      YOLD= YY
      MTYPE = IY(N,3)
      CALL BMACT (N,XX,YY)
  330 CONTINUE
C
C     LOCATE PROPER DISPL. FOR THE BEAM,PLACE IN U
C
      DO 340 J=1,2
      DO 340 K=1,3
      I = 3*IY(N,J)-3 +K
      JJ= 3*J-3+K
      NBLK = (I-1) /ND +1
      II = ND + I -(ND*(NBLK-1))
      U(JJ) = A(II,NBLK)
  340 CONTINUE
C
C     SOLVE FOR END-ACTIONS
C
      DO 350  I=1,6
  350 SIG(I)=0.0D0
      DO 360  I=1,6
      DO 360  J=1,6
      SIG(I)= SIG(I)+ R(I,J)* U(J)
  360 CONTINUE
C
C     PRINT OUT BEAM END ACTIONS
C
      IF (LINE .LT. 45 ) GO TO 380
      CALL TITLE
      WRITE (LIST,2100)
  380 WRITE (LIST,2101)  N,L,(SIG(I),I=1,3),LL,(SIG(J),J=4,6)
      LINE= LINE+ 3
```

```
 2100 FORMAT (1H0,9X,14HBEAM NO.  NODE,5X,7HX-FORCE,9X,7HY-FORCE,9X,
     18HZ-MOMENT)
 2101 FORMAT (1H0,6X,2I8,3D16.5/ 15X,I8,3D16.5 )
  400 CONTINUE
  500 CONTINUE
      RETURN
      END




      SUBROUTINE  QTDQ (N)
      IMPLICIT REAL*8 (A-G,O-Z)
      INTEGER CODE
      COMMON / DATUM / AS(800),BS(800),E(4,20),PR(100),
     2      X(900),Y(900),UX(900),UY(900),UZ(900),
     3      NUMNP,NUMEL,NUMMAT,NUMPC,MTYPE,CODE(900),
     4      IX(800,6),IBW,NE,IBC(100),JBC(100),
     5      IY(500,4),NUMBM
      COMMON / SOLN / A(120,60),B(120),MNUBLK,IJK,NB,ND
      COMMON/ SMALK / R(12,12),C(12,12,2),D(12,12),S(12,12,2),DB(3,12)
C
C     FORM MATRIX  (QTDQ)AREA FOR 12 DOF ELEMENT
C
      MTYPE = IX(N,5)
      XNU = E(2,MTYPE)
      GAM=(1.0D0 - XNU)* 0.5D0
      CON = E(1,MTYPE) * E(4,MTYPE) / (1.0D0 - XNU**2)
      AA= AS(N)
      BB = BS(N)
      DO 100 I=1,12
      DO 100 J=1,12
  100 R(J,I)= 0.0D0
C     R(I,J)=(QTDQ)VOL
C
      R(2,2)= AA*BB
      R(7,7)= R(2,2)
      R(7,2)= R(2,2)* XNU
      R(6,6)= R(2,2)* GAM
      R(8,6)= R(6,6)
      R(8,8)= R(6,6)
      R(9,6)= R(6,6)* AA
      R(9,8)= R(9,6)
      R(12,6)= R(9,6)* 0.5D0
      R(12,8)= R(12,6)
      R(7,3) = R(7,2)* AA *0.5D0
      R(3,2) = R(2,2)* AA *0.5D0
      R(6,1) = R(6,6) * BB
      R(8,1) = R(6,1)
      R(6,3) = R(6,1)* 0.5D0
      R(8,3) = R(6,3)
```

```
      R(12,2) = R(7,2)*BB* 0.5D0
      R(10,7)=(AA*BB**3)/ 3.0D0
      R(12,7) = R(2,2) *BB *0.5D0
      R(12,12)= R(10,7)
      R(3,3)= R(10,7) * GAM
      R(1,1) = R(3,3) *4.0D0
      R(3,1)= R(3,3) * 2.0D0
      R(10,2)= R(10,7)* XNU
      R(11,2)= (BB*AA**3)/ 3.0D0
      R(12,9)= R(11,2)* GAM *2.0D0
      R(9,9) = R(12,9)* 2.0D0
      R(12,12) = R(12,12)+ ( R(12,9)* 0.5D0)
      R(11,7) = R(11,2) * XNU
      R(9,1) = R(9,6)* BB
      R(3,3) = R(3,3)+ R(11,2)
      R(12,3) = R(7,3)* BB * 0.5D0
      R(12,1) = R(9,1)* 0.5D0
      R(9,3)  = R(12,1)
      R(10,6) = R(12,1)
      R(11,6) = R(12,1)
      R(10,8) = R(12,1)
      R(11,8) = R(12,1)
      R(12,3) = R(12,3)+ R(12,1) *0.5D0
      R(11,3) = R(9,1) * BB / 3.0D0
      R(10,3) = R(10,2) * AA * 0.5D0 + R(11,3)
      R(10,1) = R(11,3) * 2.0D0
      R(11,1) = R(10,1)
      R(12,10)= R(9,1)* AA / 3.0D0
      R(12,11)= R(11,7)* BB* 0.5D0 + R(12,10)
      R(10,9) = R(12,10) * 2.0D0
      R(11,9) = R(10,9)
      R(11,10) =R(9,1)* R(2,2)* 4.0D0/ 9.0D0
      R(10,10) = R(11,10) + BB**5*AA / 5.0D0
      R(11,11) = R(11,10) + AA**5*BB / 5.0D0
      R(11,10) = R(11,10) + ( R(11,7)* BB**2 / 3.0D0)
      R(12,10) = R(12,10) + BB**4*AA / 4.0D0
      R(11,3) = R(11,3)+ AA**4* BB / 4.0D0
      DO 150 I=1,12
      DO 150 J=1,I
  150 R(I,J) = R(I,J)*CON
C
      DO 200 I=1,11
      J=I+1
      DO 200 K=J,12
      R(I,K) =R(K,I)
  200 CONTINUE
C
      RETURN
      END
```

```
      SUBROUTINE BMACT (N,XX,YY)
      IMPLICIT REAL*8 (A-G,O-Z)
      INTEGER CODE
      COMMON / DATUM / AS(800),BS(800),E(4,20),PR(100),
     2       X(900),Y(900),UX(900),UY(900),UZ(900),
     3       NUMNP,NUMEL,NUMMAT,NUMPC,MTYPE,CODE(900),
     4       IX(800,6),IBW,NE,IBC(100),JBC(100),
     5       IY(500,4),NUMBM
      COMMON/ SMALK / R(12,12),C(12,12,2),D(12,12),S(12,12,2),DB(3,12)
C
C     FORM BEAM END-ACTION MATRIX
C
      M= IY(N,3)
      XY= DSQRT ( XX**2 + YY**2)
      CX=  XX/ XY
      CY=  YY/ XY
      DO 100 I=1,6
      DO 100 J=1,6
  100 R(I,J)= 0.0D0
C
      GAM = E(1,M)* E(2,M) / XY
      R(1,1)= GAM * CX
      R(1,2)= GAM * CY
      GAM= 12.0D0 * E(1,M)* E(3,M) / XY**3
      R(2,1)= -GAM* CY
      R(2,2)=  GAM* CX
      GAM = GAM * XY/ 2.0D0
      R(3,1)=-GAM *CY
      R(3,2)= GAM *CX
      R(2,3)= GAM
      R(3,3)=4.0D0* E(1,M) * E(3,M) / XY
C
      R(1,4)= -R(1,1)
      R(1,5)= -R(1,2)
      R(2,4)= -R(2,1)
      R(2,5)= -R(2,2)
      R(2,6)=  R(2,3)
      R(3,4) =-R(3,1)
      R(3,5) =-R(3,2)
      R(6,6) = R(3,3)
      R(3,6) = R(3,3)/ 2.0D0
      R(6,3) = R(3,6)
      R(4,1) = -R(1,1)
      R(4,2) = -R(1,2)
      R(4,4) = R(1,1)
      R(4,5) = R(1,2)
      R(5,1) = -R(2,1)
      R(5,2) = -R(2,2)
      R(5,3) = -R(2,3)
      R(5,4) = R(2,1)
      R(5,5) = R(2,2)
```

```
      R(5,6) = R(5,3)
      R(6,1) = R(3,1)
      R(6,2) = R(3,2)
      R(6,4) = -R(3,1)
      R(6,5) = -R(3,2)
      RETURN
      END
```

```
      SUBROUTINE ELSTRS (N,M)
      IMPLICIT REAL *8 (A-G,O-Z)
      INTEGER CODE
      COMMON / DATUM / AS(800),BS(800),E(4,20),PR(100),
     2       X(900),Y(900),UX(900),UY(900),UZ(900),
     3       NUMNP,NUMEL,NUMMAT,NUMPC,MTYPE,CODE(900),
     4       IX(800,6),IBW,NE,IBC(100),JBC(100),
     5       IY(500,4),NUMBM
      COMMON/ SMALK / R(12,12),C(12,12,2),D(12,12),S(12,12,2),DB(3,12)
C
C     FORM ELEMENT STRESS MATRIX
C
C     BOTH C INVERSE ARE FORMED - FORM DB*C(M) FOR 4-NODES
C
      MTYPE = IX(N,5)
      GAM = E(1,MTYPE)                   /(1.0D0-E(2,MTYPE)**2)
      DO 200 K=1,4
      XX=0.0D0
      YY=0.0D0
      GO TO (25,22,23,24) ,K
   25 YY=BS(N)
      GO TO 22
   24 YY=BS(N)
   23 XX=AS(N)
   22 CONTINUE
      XNU=E(2,MTYPE)
C     FORM DB  (MEANING DQ/
C
      DO 210 I=1,3
      DO 210 J=1,12
  210 DB(I,J) = 0.0D0
      DB(1,2) = XNU
      DB(1,3) = XX* XNU
      DB(1,7) = 1.0D0
      DB(1,10) = YY**2
      DB(1,11) = XX**2*XNU
      DB(1,12) = YY
      DB(2,2) =1.0D0
      DB(2,3) = XX
      DB(2,7) = XNU
```

```
      DB(2,10) = YY**2 *XNU
      DB(2,11) = XX**2
      DB(2,12) = YY* XNU
      XNU= 1.0D0 -XNU
      DB(3,1) = YY* XNU
      DB(3,3) = DB(3,1)/ 2.0D0
      DB(3,6) = XNU / 2.0D0
      DB(3,8) = DB(3,6)
      DB(3,9) = XX * XNU
      DB(3,10) = XX* YY* XNU
      DB(3,11)= DB(3,10)
      DB(3,12)= DB(3,9) / 2.0D0
C     MULTIPLY DB*C(M) FOR EACH OF FOUR NODES
      DO 150 I=1,3
      L= 3*K+I-3
      DO 150 J=1,12
      S(L,J,M) = 0.0D0
      DO 150 II=1,12
      S(L,J,M) = S(L,J,M) + DB(I,II)*C(II,J,M)
  150 CONTINUE
  200 CONTINUE
C      S* GAM
      DO 250 I=1,12
      DO 250 J=1,12
      S(I,J,M)=S(I,J,M) * GAM
  250 CONTINUE
C                                                         THE END
      RETURN
      END
```

APPENDIX 5

8 DOF PROGRAM LISTING

```
C
C     FEM  8  DOF  PROGRAM
C
C
C     FINITE ELEMENT ANALYSIS USING TWO DIMENSIONAL PLANE STRESS
C     RECTANGULAR ELEMENTS WITH 2 DEGREES OF FREEDOM AT EACH NODE
C
C
C
      IMPLICIT REAL*8 (A-G,O-Z)
      REAL CONT
      INTEGER    CODE
      COMMON / HEAD/  ICRD,LIST,HED(18),IPAGE,LINE
      COMMON / DATUM / AB(1000),PR(100),E(4,8),
     1      X(1100),Y(1100),UX(1100),UY(1100),
     2      NUMNP,NUMEL,NUMMAT,NUMPC,MTYPE,CODE(1100),
     3      IX(1000,5),IBW,NE,IBC(100),JBC(100)
      COMMON / SOLN / A(100,50),B(100),MNUBLK,IJK
      DATA CONT/ 4HDATA/
      DEFINE FILE 1 (220,5000,L,IJK)
      ICRD = 5
      LIST= 6
C
C     CHECK FOR START OF NEW PROBLEM OR TERMINATION OF PROGRAM
C
   50 READ (ICRD,1000)  HED
 1000 FORMAT (18A4)
      IF (HED(2) .EQ. CONT )  STOP
   51 IPAGE = 1
C
C
      CALL DOCUMT
C
C     FORM STIFFNESS MATRIX
C
      CALL STIFF
C
      WRITE (6,7999)  IBW,MNUBLK
 7999 FORMAT (1H0,9X,17HHALF BAND WIDTH =,I5,2X,13H,NO. BLOCKS =,I5)
C
C
C     CALCULATE AND PRINT OUT DISPLACEMENTS
C
      CALL  SYMSOL  (NE,IBW)
C
C
      CALL TITLE
      WRITE (LIST,2100)
      NB = 25
      ND = 50
      NBLK = 0
```

```
      N = 0
   60 NBLK = NBLK+1
      L = 0
   70 N = N+1
      L = L+1
      I = ND + (2*N-((NBLK-1)*ND))
      J = I-1
      IF ( LINE .LT. 50 ) GO TO 65
      CALL TITLE
      WRITE (LIST,2100)
   65 WRITE (LIST,2200)  N,X(N),Y(N),A(J,NBLK),A(I,NBLK)
      LINE = LINE +1
      IF ( N.EQ. NUMNP ) GO TO 80
      IF (L-NB) 70,60,60
   80 CONTINUE
 2100 FORMAT (1H0,9X,4HNODE,7X,1HX,11X,1HY,14X,7HX-DISPL,13X,7HY-DISPL)
 2200 FORMAT (10X,I3,2F12.3,2D20.8)
C
C
C
C     CALCULATE AND PRINT OUT STRESSES
C
      CALL STRESS
C
C
C     START NEXT PROBLEM
C
      GO TO 50
C
C
 5000 STOP
      END




      SUBROUTINE   DOCUMT
C
C     SUBROUTINE FOR DOCUMENTATION OF PROBLEM
C        ( I/O ALL THE PROBLEM CHARACTERISTICS )
C
      IMPLICIT REAL*8 (A-G,O-Z)
      INTEGER  CODE
      COMMON / HEAD/  ICRD,LIST,HED(18),IPAGE,LINE
      COMMON / DATUM / AB(1000),PR(100),E(4,8),
     1     X(1100),Y(1100),UX(1100),UY(1100),
     2     NUMNP,NUMEL,NUMMAT,NUMPC,MTYPE,CODE(1100),
     3     IX(1000,5),IBW,NE,IBC(100),JBC(100)
C
C     READ AND WRITE CONTROL INFORMATION AND MATERIAL PROPERTIES
C
```

```
      READ (ICRD,1003) NUMNP,NUMEL,NUMMAT,NUMPC
      NE = 2*NUMNP
      CALL TITLE
      WRITE (LIST,2000)     NUMNP,NUMEL,NUMMAT,NUMPC
   56 DO 59  M=1,NUMMAT
      READ (ICRD,1001) MTYPE,(E(J,MTYPE),J=1,4)
      WRITE (LIST,2010)  MTYPE, (E(I,MTYPE),I=1,4)
   59 CONTINUE
C
C     I/O  NODAL POINT DATA
C
      CALL TITLE
      WRITE (LIST,2004)
      L= 0
   60 READ (ICRD,1002) N,CODE(N),X(N),Y(N),UX(N),UY(N)
      IF ( N-1 ) 65,69,65
   65 CONTINUE
      ZX=N-L
      DX = (X(N)-X(L))/ZX
      DY = (Y(N)-Y(L))/ZX
   69 CONTINUE
      NL=L+1
   70 L=L+1
      IF ( N-L) 100,90,80
   80 CODE(L) = 0.0
      X(L) = X(L-1)+ DX
      Y(L) = Y(L-1)+ DY
      UX(L) = 0.0
      UY(L) = 0.0
      GO TO 70
   90 DO 91 K=NL,N
      IF (LINE .LT. 50) GO TO 81
      CALL TITLE
      WRITE (LIST,2004)
   81 LINE = LINE + 1
   91 WRITE (LIST,2002)  K,CODE(K),X(K),Y(K),UX(K),UY(K)
      IF( NUMNP- N)  100,110,60
  100 WRITE (LIST,2009)  N
      STOP
  110 CONTINUE
C
C     I/O  ELEMENT PROPERTIES
C
      CALL TITLE
      WRITE (LIST,2001)
      N=0
  130 READ (ICRD,1003) M,(IX(M,I),I=1,5)
      IF (M - NUMEL) 140,140,132
  132 WRITE (LIST,2019)  M
      STOP
  140 N=N+1
```

```
      IF(M-N)  170,170,150
  150 IX(N,1)=IX(N-1,1)+1
      IX(N,2)=IX(N-1,2)+1
      IX(N,3)=IX(N-1,3)+1
      IX(N,4)=IX(N-1,4)+1
      IX(N,5)=IX(N-1,5)
  170 CONTINUE
      I =IX(N,1)
      K =IX(N,3)
      AS=DABS(X(K)-X(I))
      BS=DABS( Y(K)-Y(I))
      IF ( AS)  169,168,169
  168 WRITE (LIST,2018) N
      STOP
  169 CONTINUE
      AB(N)=    BS/AS
      IF (LINE .LT. 50 ) GO TO 171
      CALL TITLE
      WRITE (LIST,2001)
  171 LINE = LINE +1
      WRITE (LIST,2003) N,(IX(N,I),I=1,5),AB(N),AS,BS
      IF(M-N) 180,180,140
  180 IF (NUMEL- N) 190,190,130
  190 CONTINUE
C
C     I/O  OF PRESSURE BOUNDARY CONDITIONS
C
      IF (NUMPC) 280,310,290
  280 WRITE (LIST,2020)
      STOP
  290 CALL TITLE
      WRITE (LIST,2005)
      DO 300  L=1,NUMPC
      READ (ICRD,1002) IBC(L),JBC(L),PR(L)
      IF(LINE .LT.50) GO TO 291
      CALL TITLE
      WRITE (LIST,2005)
  291 LINE= LINE + 1
  300 WRITE (LIST,2006)  IBC(L),JBC(L),PR(L)
  310 CONTINUE
C
C     DETERMINE THE HALF-BAND-WIDTH FOR THIS PROBLEM  ( MAX. BEING 50)
C
      J=0
      DO 340 N=1,NUMEL
      DO 340 I=1,3
      K=I+1
      DO 340 L=K,4
      KK =IABS (IX(N,I)-IX(N,L))
      IF (KK-J)  325,325,320
  320 J=KK
```

```
  325 CONTINUE
  340 CONTINUE
C     IBW = HALF-BAND-WIDTH OF PROBLEM IN QUESTION
      IBW = 2*J +2
C
 5000 CONTINUE
C
C
C     FORMAT STATMENTS
C
 1001 FORMAT(I5,4F10.0)
 1002 FORMAT (2I5,4F10.0)
 1003 FORMAT  (6I5)
 2000 FORMAT (1H0    /
     110X,28HNUMBER OF NODAL POINTS......,I3/
     210X,28HNUMBER OF ELEMENTS..........,I3/
     310X,28HNUMBER OF DIFF. MATERIALS...,I3/
     410X,28HNUMBER OF PRESSURE CARDS....,I3)
 2001 FORMAT (1H0,5X,55HELEMENT NO.      I      J      K      L    MATERIAL
     1   B/A,11X,1HA,12X,1HB )
 2002 FORMAT (7X,2I10,2F17.4,2D20.7)
 2003 FORMAT (11X,I3,2X,4I6,I8,3F13.3)
 2004 FORMAT (1H0,9X,  18HNODAL POINT   TYPE,10X,6HX-ORD.,11X,6HY-ORD.,
     1  6X,14HX(LOAD/DISPL.), 6X,14HY(LOAD/DISPL.))
 2005 FORMAT (1H0,10X,28HPRESSURE BOUNDARY CONDITIONS/ 1H0,19X,1HI,6X,1H
     1J,5X,8HPRESSURE)
 2006 FORMAT (18X,I3,4X,I3,F13.4)
 2009 FORMAT  (26H0NODAL POINT CARD ERROR N=,I5)
 2010 FORMAT (1H0,9X,  9HMAT'L NO.,18X,4HE(C),19X,2HNU,19X,4HE(T),
     1 15X,9HTHICKNESS, / (14X,I3,7X,4D22.7 ))
 2018 FORMAT (37H0NODAL COORD. ERROR - SIDE OF ELEMENT,I6,2X,8HEQUALS 0)
 2019 FORMAT (18H0ELEMENT ERROR M=,I5)
 2020 FORMAT (33H0NUMPC ENTERED AS NEGATIVE NUMBER)
      RETURN
      END




C
      SUBROUTINE TITLE
C
C     TITLE SUBROUTINE
C
      COMMON / HEAD/  ICRD,LIST,HED(18),IPAGE,LINE
      WRITE (LIST,100) IPAGE
  100 FORMAT(1H1,3X,37HFEM-8 DOF RECTANGULAR ELEMENTS,WEAVER,
     1 25X,4HPAGE,I3 )
      WRITE (LIST,101) HED
  101 FORMAT (1H0,10X,18A4)
      IPAGE= IPAGE +1
```

```
      LINE = 0
      RETURN
      END




C
      SUBROUTINE STIFF
C
C
C     FORM THE TOTAL STIFFNESS MATRIX IN BLOCKS
C
C
      IMPLICIT REAL*8 (A-G,O-Z)
      INTEGER  CODE
      COMMON / DATUM / AB(1000),PR(100),E(4,8),
     1       X(1100),Y(1100),UX(1100),UY(1100),
     2       NUMNP,NUMEL,NUMMAT,NUMPC,MTYPE,CODE(1100),
     3       IX(1000,5),IBW,NE,IBC(100),JBC(100)
      COMMON / SOLN / A(100,50),B(100),MNUBLK,IJK
      DIMENSION LM(4),S(8,8)
C
      IB=IBW
C
C     CALCULATE  MAX. NO. OF BLOCKS (MNUBLK)
C
      MNUBLK =(NUMNP*2 / 50)+1
C
C
C     INITIALIZATION OF MATRICES
C
      NB = 25
      ND=2*NB
      ND2=2*ND
      NBLK=0
      IJK= 1
      DO 50 N=1,ND2
      B(N)= 0.0D0
      DO 50 M=1,ND
   50 A(N,M) = 0.0D0
C
C     FORM BIG K IN BLOCKS
C
   60 NBLK =NBLK+1
      NH=NB*(NBLK+1)
      NM= NH-NB
      NL=NM-NB+1
      IBS =2*NL-2
C
C     FORM ELEMENT STIFFNESS MATRIX
```

```
C
      DO 210 N=1,NUMEL
      IF ( IX(N,5) .LT. 1 ) GO TO 210
C
C     DETERMINE IF THIS ELEMENT AFFECTS THIS BLOCK OF BIGK
C
      DO 80 I=1,4
      IF ( IX(N,I) .GE. NL .AND. IX(N,I) .LE. NM ) GO TO 90
   80 CONTINUE
      GO TO 210
C
   90 IF (N .EQ. 1 ) GO TO 160
      ASB = DABS ((AB(N) /ABOLD) -1.0D0 )
      IF ( ASB .LT. 0.5D-14  .AND. IX(N,5)  .EQ. IX(N-1,5))  GO TO 161
  160 CONTINUE
      ABOLD = AB(N)
      CALL ELSTIF (S,N)
C
C     ADD ELEMENT STIFFNESS TO TOTAL STIFFNESS
C
  161 DO 166  I=1,4
  166 LM(I)=2*IX(N,I)-2
C
      IX(N,5) = -IX(N,5)
      DO 200 I=1,4
      DO 200 K=1,2
      II =LM(I)+K -IBS
      KK = 2*I-2+K
      DO 205 J=1,4
      DO 205 L=1,2
      JJ=LM(J)+L-II+1-IBS
      LL= 2*J-2+L
      IF (JJ) 185,185,170
  170 CONTINUE
      IF (IBW-JJ) 180,195,195
  180 WRITE (LIST,2004) N
 2004 FORMAT (10X,46HBAND WIDTH EXCEEDS THAT CALCULATED FOR PROBLEM)
      STOP
  185 CONTINUE
      GO TO 205
  195 A(II,JJ)= A(II,JJ) + S(KK,LL)
  205 CONTINUE
  200 CONTINUE
  210 CONTINUE
C
C
C     ADD CONCENTRATED FORCES  AT NODES
C
      DO 250 N=1,NUMNP
      ICODE = CODE(N)+ 1
      K= 2*N- IBS
```

```
      IF ( K .LT. 1 .OR.  K .GT. ND) GO TO 250
      GO TO (251,252,253,250),ICODE
  251 B(K)= B(K)+ UY(N)
  253 B(K-1)= B(K-1)+ UX(N)
      GO TO 250
  252 B(K) =B(K) +UY(N)
  250 CONTINUE
C
C
C     BOUNDARY CONDITIONS
C
C     PRESSURE  B.C.
C
      IF (NUMPC) 260,310,260
  260 DO 300 L=1,NUMPC
      I=IBC(L)
      J=JBC(L)
      PP = PR(L)/2.0
C
      DX = (X(J)- X(I))* PP
      DY = (Y(I)- Y(J))* PP
C
      II = 2*I-IBS
      JJ = 2*J-IBS
      IF ( II .LT. 2 .OR. II .GT. ND ) GO TO  270
C
      B(II-1) = B(II-1)+ DY
      B(II) = B(II)+DX
  270 IF ( JJ .LT. 2 .OR. JJ.GT. ND ) GO TO 300
      B(JJ-1) = B(JJ-1) + DY
      B(JJ) = B(JJ) +DX
C
  300 CONTINUE
  310 CONTINUE
C
C     DISPLACEMENT B.C.
C
      DO 400  M=1,NUMNP
C
      ICOD  = CODE(M) + 1
      N = 2*M-1-IBS
      IF ( N.LT. 1 .OR. N .GT. ND2 ) GO TO 400
      U= UX(M)
C
      GO TO (400,315,320,330),ICOD
C
  315 CALL MODIFY (N,U,IBW,ND2)
      GO TO 400
  330 CALL MODIFY (N,U,IBW,ND2)
  320 N=N+1
      U= UY(M)
```

```
      CALL MODIFY (N,U,IBW,ND2)
  400 CONTINUE
C
C
C     WRITE BLOCK OF EQU'S ON DISK AND SHIFT UP LOWER BLOCK
C
      IJK = (NBLK *5) -4
      WRITE (1'IJK) ((A(I,J),I=1,ND),J=1,IBW),(B(K),K=1,ND)
C
      DO 420 N=1,ND
      K=N+ND
      B(N)= B(K)
      B(K)= 0.0D0
      DO 420 M=1,ND
      A(N,M)= A(K,M)
  420 A(K,M) = 0.0D0
C
C     CHECK FOR LAST BLOCK
      IF ( NM .LE. NUMNP) GO TO 60
C
      DO 450 I=1,NUMEL
  450 IX(I,5) =-IX(I,5)
  500 RETURN
      END




      SUBROUTINE  ELSTIF (S,N)
C
C
C     FORM ELEMENT STIFFNESS MATRIX
C
C
      IMPLICIT REAL*8 (A-G,O-Z)
      INTEGER CODE
      COMMON / DATUM / AB(1000),PR(100),E(4,8),
     1      X(1100),Y(1100),UX(1100),UY(1100),
     2      NUMNP,NUMEL,NUMMAT,NUMPC,MTYPE,CODE(1100),
     3      IX(1000,5),IBW,NE,IBC(100),JBC(100)
      DIMENSION S(8,8)
C     INITIALIZE S
      DO 10 I=1,8
      DO 10 J=1,8
   10 S(I,J)= 0.0D0
C
      L = IX(N,5)
      XNU = E(2,L)
      EC = E(1,L)
      AR = AB(N)
      ARI= 1.0D0 / AR
```

```
      CON = EC *E(4,L) /(12.0D0 *(1.0D0-XNU**2))
      AA = CON * AR * 2.0D0
      BB = AA*2.0D0
      CC = CON * (1.0D0-XNU)*ARI
      DD = CC * 2.0D0
      EE = CON *1.5D0 *(1.0D0 +XNU)
      FF = CON *1.5D0 *(1.0D0 -3.0D0*XNU)
      OO = CON * ARI * 2.0D0
      PP = OO* 2.0D0
      QQ = CON * (1.0D0 -XNU)*AR
      RR = QQ * 2.0D0
C
      DO 20 I=1,7,2
   20 S(I,I)= BB+DD
      DO 30 I=2,8,2
   30 S(I,I)= PP+RR
C
      S(2,1) = EE
      S(7,1) = AA-DD
      S(7,2) = -FF
      S(8,1) = FF
      S(8,2) = -PP+QQ
      S(8,7) = -EE
      S(5,1) = -AA-CC
      S(5,2) = -EE
      S(7,5) = -BB+CC
      S(8,5) = -FF
      S(6,1) = -EE
      S(6,2) = -OO-QQ
      S(7,6) = FF
      S(8,6) = OO-RR
      S(6,5) = EE
      S(3,1) = S(7,5)
      S(3,2) = FF
      S(7,3) = S(5,1)
      S(8,3) = EE
      S(5,3) = S(7,1)
      S(6,3) = -FF
      S(4,1) = -FF
      S(4,2) = S(8,6)
      S(7,4) = EE
      S(5,4) = FF
      S(6,4) = S(8,2)
      S(4,3) = -EE
      S(8,4) = S(6,2)
C
      DO 40 I=1,8
      IF (I .EQ. 8) GO TO 40
      II =I+1
      DO 42 J=II,8
      S(I,J)= S(J,I)
```

```
   42 CONTINUE
   40 CONTINUE
C
      RETURN
      END




      SUBROUTINE  MODIFY  (N,U,IBW,NE)
C
C
C     ALTER THE TOTAL STIFFNESS MATRIX TO INCORPERATE ANY SPECIFIED
C      BOUNDARY CONDITIONS
C
C
      IMPLICIT REAL*8 (A,B,U)
      COMMON / SOLN / A(100,50),B(100),MNUBLK,IJK
C
      DO 250  M=2,IBW
      K= N- M +1
      IF (K) 235,235,230
  230 B(K) =B(K)- A(K,M)* U
      A(K,M) =0.0D0
  235 K=N+ M-1
      IF(NE -K)  250,240,240
  240 B(K)= B(K)- A(N,M)* U
      A(N,M) = 0.0D0
  250 CONTINUE
      A(N,1)= 1.0D0
      B(N)= U
      RETURN
      END



C
      SUBROUTINE SYMSOL (NN,IBW)
C
C
C     SOLVE THE SYSTEM OF BANDED,BLOCKED SIMULTANEOUS EQUATIONS
C     BY GUASSIAN ELIMINATION
C
C
      IMPLICIT REAL*8(A-G,O-Z)
      COMMON / SOLN / A(100,50),B(100),MNUBLK,IJK
C
      NN = 50
      NL = NN+1
      NH = NN+NN
```

```
      NBLK = 0
      IJK=1
      GO TO 150
C
C     REDUCE EQU'S BY BLOCKS
C
C     SHIFT BLOCK OF EQU'S
C
  100 NBLK =NBLK +1
      DO 125 N=1,NN
      NM = NN+N
      B(N)= B(NM)
      B(NM)= 0.0D0
      DO 125 M=1,IBW
      A(N,M) =A(NM,M)
  125 A(NM,M) = 0.0D0
C
C     READ IN NEXT BLOCK OF EQU'S
C
      IF (NBLK .EQ. MNUBLK ) GO TO 200
      IJK = (NBLK *5) +1
  150 READ (1'IJK) ((A(I,J),I=NL,NH),J=1,IBW),(B(K),K=NL,NH)
      IF (NBLK .EQ. 0 ) GO TO 100
C
C     REDUCE BLOCK OF EQU'S
C
  200 DO 300 N=1,NN
      IF ( A(N,1) .EQ. 0.0D0 ) GO TO 300
      B(N) =B(N)/ A(N,1)
      DO 275 L=2,IBW
      IF ( A(N,L) .EQ. 0.0D0 ) GO TO 275
      C =A(N,L) / A(N,1)
      I=N+L-1
      J=0
      DO 250 K=L,IBW
      J=J+1
      IF (A(I,J) .EQ. 1.0D0 .AND. B(I) .EQ. 0.0D0 ) GO TO 260
  250 A(I,J)= A(I,J) - C*A(N,K)
      B(I) =B(I)-A(N,L)* B(N)
  260 A(N,L) = C
  275 CONTINUE
  300 CONTINUE
C
C     WRITE BLOCK OF EQU'S BACK ON DISK
C
      IF( NBLK .EQ. MNUBLK) GO TO 400
      IJK = (NBLK *5) -4
      WRITE (1'IJK)   ((A(I,J),I=1,NN),J=2,IBW),( B(K),K=1,NN)
      GO TO 100
C
C     BACK SUBSTITUTION
```

```
C
  400 DO 450 M=1,NN
      N=NN+1-M
      DO 425 K=2,IBW
      L=N+K-1
  425 B(N) =B(N) - A(N,K)*B(L)
      NM =N + NN
      B(NM)= B(N)
  450 A(NM,NBLK) =B(N)
      NBLK = NBLK-1
      IF (NBLK .EQ. 0 ) GO TO  500
      IJK = (NBLK *5) -4
      READ (1'IJK) ((A(I,J),I=1,NN),J=2,IBW),(B(K),K=1,NN)
      GO TO 400
C
C     UNKNOWN DISPL. ARE STORED IN THE FIRST MNUBLK COLUMNS
C     OF THE 2ND BLOCK OF MATRIX A
C
  500 RETURN
      END




      SUBROUTINE STRESS
C
C     CALCULATE THE STRESSES AT THE NODES OF EACH ELEMENT
C
C
      IMPLICIT REAL*8 (A-G,O-Z)
      INTEGER CODE
      COMMON / HEAD/  ICRD,LIST,HED(18),IPAGE,LINE
      COMMON / DATUM / AB(1000),PR(100),E(4,8),
     1     X(1100),Y(1100),UX(1100),UY(1100),
     2     NUMNP,NUMEL,NUMMAT,NUMPC,MTYPE,CODE(1100),
     3     IX(1000,5),IBW,NE,IBC(100),JBC(100)
      COMMON / SOLN / A(100,50),B(100),MNUBLK,IJK
      DIMENSION DB(3,8),U(8),SIG(6)
      CALL TITLE
      WRITE (LIST,2000)
      ND = 50
      DO 100 N=1,NUMEL
C                     NODAL
C     PLACE PROPER DISPL.  IN U FROM A
C
      DO  20 J=1,4
      JJ= 2*J-1
      I = 2*IX(N,J)-1
      M = 0
      L = 0
   10 DO 11 K=1,MNUBLK
```

```
      IF (I .GT. L .AND. I .LE. L+ND ) GO TO 15
      L = L + ND
      GO TO 11
   15 NBLK = K
      GO TO 16
   11 CONTINUE
   16 M = M+1
      II =    ND + I-(ND*(NBLK-1))
      U(JJ) = A(II,NBLK)
      IF (M .GT. 1 ) GO TO 20
      JJ= JJ+1
      I= I+1
      L = 0
      GO TO 10
   20 CONTINUE
C
C     DETERMINE LOCAL COORDINATES
C
      K=IX(N,1)
      L=IX(N,3)
      AS= DABS(X(L)-X(K))
      BS= DABS(Y(L)-Y(K))
      DO 100 M=1,4
      XX = 0.0D0
      YY = 0.0D0
      GO TO (22,23,24,25) ,M
   25 YY = BS
      GO TO 22
   24 YY = BS
   23 XX = AS
   22 CONTINUE
      AT = AS
      BT = BS
      CALL ELSTRS (N,AT,BT,XX,YY,DB)
C     SOLVE FOR STRESSES
C
      DO 30 I=1,6
   30 SIG(I)= 0.0D0
      DO 35 I=1,3
      DO 35 J=1,8
      SIG(I)=SIG(I)+ DB(I,J)* U(J)
   35 CONTINUE
C
C     CALCULATE PRINCIPAL STRESSES
C
      AA =(SIG(1)+SIG(2))* 0.5D0
      BB =(SIG(1)-SIG(2))* 0.5D0
      CC = DSQRT( BB**2 + SIG(3)**2)
      SIG(4)=AA+CC
      SIG(5)=AA-CC
      IF ( BB) 36,37,36
```

```
   36 SIG(6)=(DATAN(SIG(3)/ BB ))* 23.6478897500
      GO TO 38
   37 SIG(6) = 90.0D0
   38 CONTINUE
      IF (LINE.LT.40) GO TO 40
      CALL TITLE
      WRITE (LIST,2000)
   40 CONTINUE
      IF (M.EQ.1 ) GO TO 45
      WRITE (LIST,2001) IX(N,M),(SIG(I),I=1,6)
      GO TO 46
   45 WRITE (LIST,2002)   N,IX(N,M),(SIG(I),I=1,6)
   46 CONTINUE
      LINE =LINE + 1
  100 CONTINUE
 2000 FORMAT (1H0,9X,6HEL.NO.,6X,4HNODE,9X,8HX-STRESS,6X,8HY-STRESS,
     15X,9HXY-STRESS,5X,10HMAX-STRESS,4X,10HMIN-STRESS,4X,5HANGLE)
 2001 FORMAT (15X,I10,5X,5D14.4,F9.2)
 2002 FORMAT (1H0,9X,I5,I10,5X,5D14.4,F9.2)
      RETURN
      END



      SUBROUTINE ELSTRS (N,AS,BS,XX,YY,DB)
C
C
C     FORM ELEMENT STRESS MATRIX
C
C
      IMPLICIT REAL*8 (A-G,O-Z)
      INTEGER CODE
      COMMON / DATUM / AB(1000),PR(100),E(4,8),
     1      X(1100),Y(1100),UX(1100),UY(1100),
     2      NUMNP,NUMEL,NUMMAT,NUMPC,MTYPE,CODE(1100),
     3      IX(1000,5),IBW,NE,IBC(100),JBC(100)
      DIMENSION DB(3,8)
C
      M= IX(N,5)
      ASB= AS*BS
      DB(1,5) = YY/ ASB
      DB(1,6) =(E(2,M)*XX)/ASB
      DB(1,7) = -DB(1,5)
      DB(1,4) = -DB(1,6)
      DB(2,5) = (E(2,M)*YY)/ASB
      DB(2,6) =  XX/ASB
      DB(2,7) = -DB(2,5)
      DB(2,4) = -DB(2,6)
      AS = 1.0D0 /AS
      BS = 1.0D0 /BS
```

```
      DB(1,1) = DB(1,5) -AS
      DB(1,3) = -DB(1,1)
      DB(1,2) = DB(1,6)- BS* E(2,M)
      DB(1,8) = -DB(1,2)
      DB(2,1) = DB(2,5) -AS* E(2,M)
      DB(2,3) = -DB(2,1)
      DB(2,2) = DB(2,6)-BS
      DB(2,8) = -DB(2,2)
      AS = (1.0D0 - E(2,M))/2.0D0
      BS = E(1,M) * E(4,M)  /(1.0D0 -E(2,M)**2)
      DB(3,1) = AS*DB(2,2)
      DB(3,2) = AS*DB(1,1)
      DB(3,7) = -DB(3,1)
      DB(3,8) = AS* DB(1,7)
      DB(3,5) = AS* DB(2,6)
      DB(3,6) =-DB(3,8)
      DB(3,3) = -DB(3,5)
      DB(3,4) = -DB(3,2)
      DO 50 J=1,8
      DO 50 I=1,3
      DB(I,J) = DB(I,J) *BS
   50 CONTINUE
C                                                          THE END
      RETURN
      END
```

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FINITE ELEMENT ANALYSIS OF PIERCED SHEAR WALLS

John Walter Weaver

Abstract

Previous discrete analysis techniques applied to shear walls with openings have utilized finite elements with linear displacement fields. These elements require relatively fine meshes in order to correctly describe the reverse bending behavior of the connecting beams and to avoid an idealization for the shear wall which is too stiff. This study presents the use of finite elements which have additional degrees of freedom and allow curved deformation patterns to exist. These refined elements are also combined with plane frame beam elements which are used to idealize slender connecting beams in pierced shear walls. This combination of elements is also applied to analysis of the interaction of shear walls with their surrounding frames in plane frame shear wall combinations.

A computer program is developed and presented which is capable of handling a wide range of general shear wall configurations. Only the geometry, loading conditions, and elastic properties are used as input to the program. The output consists of the input data, as a check, then the displacements and stresses at each node as well as member and actions for the beam elements if they are used.

Several refined finite element solutions are compared with experimental results for pierced shear wall problems of varying story heights and beam to shear wall stiffness ratios as well as

frame and shear wall configurations. These comparisons provide initial guidelines for the efficient use of this refined element in the analysis of general shear wall configurations.