THE CONSTITUTIONAL APPROACH TO THE FISCAL PROCESS:
AN INQUIRY INTO SOME LOGICAL FOUNDATIONS,
by
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Dissertation submitted to the Graduate Faculty of the Virginia Polytechnic Institute and State University in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY in Economics

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June 1978

Blacksburg, Virginia
To my wife,
PREFACE

A question of recurring intrigue to social scientists is the logic of individual and social commitment to rules, when full discretion would be available and would appear to be a dominating alternative. The logical counterpart of this broad question on the domain of fiscal theory is centered around the raison d'être of the constitutional approach to the fiscal process. The present study reflects the interest of the author in this theme and represents an attempt to look into the nature of considerations which may rationalize such an approach.

My intellectual debt to who has inspired much of the substance of my work is extremely weighty. Particular thanks are also due to

and for their many useful comments and suggestions. I also owe thanks to from whom I have received great help and encouragement especially during the initial stages of my work.

I finally would like to express my deep gratitude to my parents for their long-standing generosity.

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Chapter One

INTRODUCTION

1.1 Two Basic Paradigms of the Fiscal Decision Process

Generally it is agreed that classical writers in the English tradition of public finance theory like Ricardo, Mill, Edgeworth and Pigou had essentially a "taxation-only" view of fiscal theory. Public expenditures were, in this perspective, considered to be fundamentally unproductive; and, in consequence, the main task assigned to fiscal theory was essentially that of ascertaining criteria for equalizing the sacrifice or burden involved in taxation and/or to minimize its total magnitude.

At the same time, the Italian tradition in public finance theory had, as early as the 1880s, made rudimentary efforts through the work of Mazzola, Pantaleoni and De Viti De Marco, to examine the public economy within an exchange framework, thus attempting to obtain an integrated picture of the two sides of the fiscal account. However, it was not perhaps until Wicksell's contribution in 1896 that the need was recognized for considering simultaneously both the expenditure and the tax-side of the public sector budget if meaningful inferences about the fiscal structure were to be obtained.

The work subsequently developed by Lindahl, Musgrave, Bowen and, more recently, Samuelson, reinforced the view that the "taxation-only"
perspective of the fiscal process bypassed the central problem of how to allocate resources for the provision of public goods. An integrated analysis thus seemed to be indicated on theoretical grounds.

Observation of particular real-world processes of budget determination suggests, however, that in certain circumstances the basic implication of modern fiscal theory has been empirically refuted. In fact, fiscal processes are in some cases arranged so that the choice of the tax structure is apparently made quite independently from the choices on the expenditure side of the budget. While the latter choices are made recurrently in each budgetary period, major tax changes are, instead, undertaken only from time to time, and often remain in force for several years. Only relatively minor departures from the basic tax structure seem to be allowed in in-period budgetary decision-making. That is, the choice of the amount and composition of public expenditure programs is undertaken with a pre-selected tax rule in the background.

From the standpoint of economic theory and policy, an inescapable dilemma seems thus to emerge. Either the world of practical realities in budget determination processes has not yet fully recognized and explored the potential gains that theory suggests will exist under the "simultaneous-decisions" paradigm, or reworking of the theory is required to take account of some as-yet-ignored factors that might explain the adoption of a "separation-of-decisions" model.
Professor James Buchanan\textsuperscript{1} has examined this question and has advanced a potential explanation for the dichotomization of tax and expenditure decisions. According to his argument, the conventional model of simultaneous determination of public expenditures and tax-shares does not take into account some costs of budget determination which might be sufficiently important to render the dichotomization of decisions a preferable solution on efficiency grounds.

The core of the present study focuses on such an alternative theoretical model of fiscal decision. More specifically, it is devoted to the analysis of some mechanisms that are expected to be at work when a "separation-of-decisions" institutional framework is adopted and that might, at least conceptually, explain the savings in some of those cost items that the conventional "simultaneous-decisions" model neglects.\textsuperscript{2}

1.2 Optimization of the Fiscal Decision Process

Before concentrating exclusively on the specific kind of cost


\textsuperscript{2} Professor Buchanan's argument seems to refer to the part of the budget devoted to the provision and financing of public goods. It is clear that other functions usually performed by the fiscal structure, for example to correct for externalities, may provide another rationale for decisions on the tax-side to be taken quite independently from decisions on expenditures. This investigation focuses on the first kind of argument.
that will represent the central subject of subsequent analysis it would be methodologically convenient to start out by drawing a general picture of the basic cost components that seem to be most relevant to the choice of an optimal fiscal decision-making structure.

Perhaps the most fundamental cost item that is absent in the conventional budget determination analysis involves the absorption of resources that will take place as a result of the process of collective decision-making itself. As Professors Buchanan and Tullock have pointed out, the process of reaching collective agreement usually involves real resource absorption either in the form of direct consumption or indirectly, and perhaps more importantly, in terms of the product foregone as a consequence of the time allocated to the decision-making process. It seems advantageous to distinguish between two components of the costs of decision-making. First, consider what may be termed the variable part of these costs. This item will depend, in general, on an array of factors among which one of the most important is the degree of uncertainty associated with the pay-off that will eventually accrue to the choosing individual as a result of

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3 In the present study the terms costs of decision-making and bargaining or negotiation costs will be used interchangeably.

4 The terminology here will depart from that made familiar by F. Knight, Risk, Uncertainty and Profit (New York: Houghton Mifflin and Co., 1921). In fact, as it will soon become apparent, the discussion will be developed in an essentially probabilistic framework. To that extent, the term risk rather than uncertainty would conform better with that author's taxonomic suggestion.
the collective decision undertaken. In the specific context of choice to be examined here, uncertainty refers basically to the inability of each individual to predict his own economic position in the social setting to which the tax structure will ultimately apply. Since the magnitude of an individual's tax payment is presumed to be associated with his economic status, uncertainty in the above sense will therefore mean the impossibility of an exact prediction of an individual's tax payment.5

The basic idea relating uncertainty to decision-making costs is that increased uncertainty will tend to reduce disagreement among individuals in conflicting positions and therefore to reduce negotiation costs.6 As noted before, the specific mechanisms through which this effect is produced will be the subject of detailed investigation in the following chapters. For the moment it will suffice to note that uncertainty may be generated in the tax-decision context through two somewhat different processes. One consists of extending the duration of the tax-rule so that it applies to a whole sequence of

5It should by this time be clear the the sense in which the term uncertainty is used here differs from the one that corresponds to the inability of each individual to predict the outcome of the collective decision process itself. The latter is the uncertainty about the choice of a tax rule while the former is the uncertainty of the effects of a tax rule once it has been chosen.

budgetary periods rather than to a single one. It is presumed that as one moves further away from the point in time where the tax institution is determined, uncertainty will increase since, as time elapses, the greater the chances of potential change in initial economic position.

The alternative way of generating uncertainty is to introduce a time lag between the time when the choice of the tax rule is made and the time when it starts being effective. This period is defined here by the term *vaccatio legis*.

The two modes of generating uncertainty are, obviously, distinct processes since the existence of a *vaccatio legis* can conceivably be combined with, say, a single-period duration of the tax law; and, conversely, one can certainly have a multi-period validity of the tax rule associated with a *vaccatio legis* of zero duration. At the same time, while both processes essentially imply a separation of tax from expenditure decisions, they will nevertheless have different cost implications, as will soon become apparent. To that extent, the choice of an appropriate mix of the two uncertainty-generating mechanisms might be in order if a optimal solution to the fiscal decision process is sought.

The second component of the costs of decision-making may now be introduced. This has an intuitive explanation and basically corresponds to the fixed cost of negotiating an agreement on the division of the tax burden. It is therefore presumed that the process of
negotiation both in general and in the particular domain of fiscal reform involves some overhead-like costs, i.e., costs which do not significantly vary with the number of time units (budgetary periods) to which the decision applies. In other words, these set-up costs reflect the fact that the fiscal decision-making process like other productive processes involves economies of scale with respect to some of its cost components and over some range of "activity."  

7 Although not explicitly singled out like other cost items in his model, it seems that this corresponds to what Professor Buchanan had in mind when he asserted:

... The possible need to create institutions that will "make choices in advance" arises only when it is predicted that similar choice situations will be confronted recurrently either over a sequence of time periods or over separated events. (J. M. Buchanan, The Demand and Supply of Public Goods, 1968, p. 152)

[italics added]

The conventional model of choice contains no recognition, explicit or implicit, of the predicted repetition or recurrence of fiscal choices, roughly similar one to the other, over a whole series of time periods. (J. M. Buchanan, The Demand and Supply of Public Goods, p. 153)

In fact it seems that the "similarity" that can be found with respect to choice situations is the basic factor which may account for the asymptotic properties typical of scale economies. Another possible interpretation of the concept of "similarity" in this context would refer to the "isomorphic" perspective that tends to emerge in a constitutional choice setting. See R. Strotz, "How Income Ought to be Distributed: A Paradox in Distributive Ethics," Journal of Political Economy 6 (June 1958). Although in a slightly different context, A. C. Michalos, "The Costs of Decision Making," Public Choice 9 (1970) pointed out the existence of overhead costs of decision-making. More recently Jo Anna Gray, "On Indexation and Contract Length," Journal of Political Economy 86 (1978) introduced this cost item explicitly in a model of wage contracting.
Another basic item of the cost structure to be considered might be termed information costs. Take the case of the "simultaneous-decisions" model such as developed, say, in Samuelson's familiar formulation. The difficulties associated with the attainment of an "efficient" solution in this paradigm are well-known. As a rule, enormous information problems emerge in order to have a true revelation of preferences for public goods on the part of individuals due to "free-riding," and the imperfect information consequently obtained will tend, of course, to generate inefficiency costs.

According to one interpretation, these costs emerge whenever the tax-sharing arrangements are such that they do not generate a Pareto efficient solution in the provision of public goods and services. Individual equilibrium as represented by the equalization of marginal tax-price and marginal evaluation of the public good is not here required, provided that aggregate conditions are satisfied. In an alternative and more strict interpretation, inefficiency costs will be zero only in the case of a Lindahl type tax-sharing arrangement where individual equilibrium is also attained. Implicit in this interpretation is the view that deviations from individual equilibria, even if an optimal quantity of the public good and service is provided, will tend to generate tensions in the social system that can only be accommodated at a cost.  

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8 Using A. Breton's terminology, the existence of such deviations would essentially imply that "coercion" is exerted by the tax structure. See A. Breton, The Economic Theory of Representative Government (Chicago: Aldine Publishing Company, 1974).
For present purposes both interpretations will do. The relevant question is how these types of costs vary as the structure is shifted from the "simultaneous-decisions" paradigm towards the "separation-of-decisions" alternative. Will they increase or decrease? Two distinct aspects seem to be important in this connection. On the one hand, as soon as the moment when the tax structure is chosen deviates from the moment of benefit provision, the distributional conflict implicit in the revelation of preferences difficulties may be expected to be attenuated and, to that extent, one would predict a tendency to a more correct revelation of preferences on the part of individuals. On the other hand, however, the introduction of a time lag between decision and implementation creates the possibility of forecast errors as represented, in particular, by the emergence of potential deviations between predicted and realized patterns of demand for public goods and services. These errors and the kind of information costs that they imply tend of course to be aggravated as the time lag increases.

Two countervailing forces thus exist with respect to the behavior of information costs; and, in general, it seems impossible to know a priori which one will be dominant. Therefore, it will be convenient
to treat them in the following discussion as two separate sub-components of the information costs.\footnote{9}

Finally, another kind of cost which should be considered in any integrated view of the problem of designing an optimal fiscal decision process is adaptation costs. These refer first to the fact that changes in the tax structure are usually followed by a period of time when the private sector attempts to adjust to the new fiscal conditions, this adaptation being in general a costly process. If a time lag is introduced between the moment when the alterations in the fiscal system occur and the moment when its implementation commences, the costs involved in this adaptation may be reduced. Also, inefficient precautionary behavior, which tends to occur when a given tax change induces anticipation of future possible changes, might similarly be substantially attenuated with delayed implementation of the fiscal law.\footnote{10}

\footnote{9}It may be asked at this point whether the information costs due to preference revelation difficulties should not be more appropriately included in the negotiation costs considered previously since the latter essentially accounted for the consequences of the distributinal conflict implicit in the choice of tax structures. This apparently artifical separation can be, however, defended on the grounds that even if the information problem of knowing true demand patterns for public goods could somehow be solved (say, by the introduction of a new demand revealing device) the basic distributional problem of how to allocate the gains-from-trade among individuals would still remain.

\footnote{10}This effect of tax changes on anticipations has been examined by M. Feldstein, "On the Theory of Tax Reform," \textit{Journal of Public Economics} 6 (1976).
The important aspect to be noted is that the duration of a vaccatio legis has implications not only on the degree of uncertainty that will prevail at the constitutional stage but also on the magnitude of the adaptation costs. It seems more difficult to predict what will be the effects of variations in the duration of the tax institution on adaptation costs. In fact, as the duration increases, adaptation possibilities will certainly increase in the more distant periods. However, a problem may arise if the "magnitude" of the change implicit in the tax institution may itself vary with an extension of the duration of the institution.

Having described the basic cost components that seem most relevant to the choice of an optimal fiscal decision process, it might be useful to consider a simple illustration where these separate elements are brought together into an integrated picture of institutional optimization.

It has been noted earlier that the decision-making costs of negotiating fiscal arrangements tend to be inversely related to the degree of uncertainty prevailing at the moment of choice of the fiscal structure. It was also observed that this degree of uncertainty tends, in turn, to be positively associated with the time lag between decision and implementation and that such lags may vary either by changing the extent of the vaccatio legis or, alternatively, the extent of the budget period sequence over which the tax institution is intended to remain effective. In the following discussion these
are taken as the two basic constitutional control variables and it is assumed that total expected decision-making costs associated with a tax institution of duration $u$ are given by\textsuperscript{11}

$$D(u,s) = a + \int_{0}^{u} e^{-\sigma(t+s)} \, dt,$$  \hspace{1cm} (1.1)

where $a$ denotes the fixed cost of negotiation, $s$ indicates the time duration of the \textit{vaccatio legis} and $\sigma > 0$ represents the absolute value of the expected rate of decline (assumed constant) of the marginal cost of decision-making as uncertainty increases. Thus, $t+s$ is taken here as a proxy for the degree of uncertainty associated with the choice situation. It may also be noted, by observing the variable component of (1.1) that it decreases as $s$ increases and that it increases at a decreasing rate as $u$ expands, i.e., it is assumed here (and will later be proved) that an absolute decrease in decision-making costs tends to occur when, by augmenting the \textit{vaccatio legis} duration, uncertainty increases; at the same time, when the duration of the tax institution is extended by one unit, the marginal cost of negotiation declines but remains positive since infra-marginal units are unaffected by uncertainty.

\textsuperscript{11}For analytical convenience we develop the following illustration for the continuous, rather than the discrete, case although the latter would perhaps fit better into real-world practical possibilities. Discounting due to time preference is, for simplicity, ignored.
The information costs involved in tax constitution design may now be considered. As noted before, two cost components may be distinguished here. Consider first the information costs incurred to obtain an approximation to a correct revelation of preferences for public goods. It was indicated earlier that there is a presumption of a decline in this kind of cost as uncertainty is increased. Assume, therefore, that this cost component is described by

$$I(u,s) = \beta + \int_{0}^{u} e^{-\delta(t+s)}dt,$$

where $\delta > 0$ is the absolute value of the rate of decline of the marginal cost of information as uncertainty increases and $\beta$ denotes the (instantaneous) information cost that is incurred when the duration of tax institutions approaches zero.

The second component of information costs corresponds to the forecast error about future preferences for public goods. The magnitude of this kind of cost will obviously tend to increase both with the duration of the tax institution and the extent of the *vacatio legis*. Suppose, therefore, that this component is given by

$$F(u,s) = \int_{0}^{u} [e^{\mu(t+s)} - 1] \, dt,$$

where the bracketed expression stands for the expected variance (at time distance $t+s$ from the choice moment) of the predicted demand patterns for public goods about their actual values. Therefore, $\mu > 0$ represents the rate of increase of the expected variance as uncertainty increases.
Finally, the costs involved in the adaptation of the private sector to tax changes must be considered. It is assumed that these costs will tend to decrease in absolute value as the *vaccaatio legis* increases since more time is allowed for taxpayer adjustment. As with respect to variations on the duration of the tax institution it is assumed that as this duration increases, adaptation costs will increase but at a declining rate. This reflects both the presumption that the "magnitude" of tax changes implicit in the tax institutions becomes more significant as its duration increases and the anticipation of an increased scope for adaptation in the more distant periods. Analytically, the expression for these costs may therefore be represented in a way similar to the one corresponding to the variable portion of the negotiation costs, namely by

\[ A(u,s) = \int_0^u e^{-\gamma(t+s)} \, dt , \]  

(1.4)

where \( \gamma > 0 \) denotes the absolute value of the rate of decline of the marginal adaptation cost as uncertainty is increased.

Now, adding together the cost components indicated in (1.1) through (1.4) one obtains the total cost of the fiscal decision process, \( K \) where

\[ K(u,s) = D(u,s) + I(u,s) + F(u,s) + A(u,s) . \]  

(1.5)

Therefore, the problem of selecting an optimal decision process would amount, in the present context, to the choice of the values of the
constitutional control variables $u$ and $s$ so as to minimize the average cost per time unit, $\bar{K}$, where

$$\bar{K}(u,s) = \frac{K(u,s)}{u},$$

that is,

$$\text{Min} \quad u > 0, \ s > 0 \quad \left\{ \bar{K}(u,s) = \frac{1}{u} \left( \alpha + \beta + \int_0^u [e^{-\sigma(t+s)} + e^{-\delta(t+s)} + \\ + e^{\mu(t+s)} - e^{-\gamma(t+s) - 1}] \, dt \right) \right\}. \quad (1.7)$$

The necessary conditions for a minimum are given by

$$\bar{K}_u = \frac{1}{u} \left[ e^{-\sigma(u+s)} - e^{-\delta(u+s)} + e^{\mu(u+s)} - 1 - \bar{K} \right] = 0 \quad (1.8)$$

$$\bar{K}_s = \frac{1}{u} \int_0^u [-e^{-\sigma(t+s)} - \sigma e^{-\delta(t+s)} + \mu e^{\mu(t+s)} - \gamma e^{-\gamma(t+s)}] \, dt > 0 \quad (1.9)$$

$$s\bar{K}_s = 0 \quad (1.10)$$

$$m > 0, \ s > 0. \quad (1.11)$$

Figures 1.1a and 1.1b illustrate diagrammatically these conditions for the case of an interior solution in both constitutional control variables. Figure 1.1a is drawn for $s$ set at its optimal value $s^*$; similarly, Figure 1.1b is traced for $u$ set at its optimal value $u^*$. Curve $bb$ depicts the marginal cost with respect to changes in $u$, $K_u$.

\[\text{It may be noted that a corner solution } u = 0 \text{ is not considered since } \bar{K} \text{ is not defined for such value.}\]
Figure 1.1a
Optimal Duration of Tax Institution

Figure 1.1b
Optimal Vaccatio legis of Tax Institution
and a the corresponding average cost $\overline{K}$.\(^{13}\) Curve dd in Figure 1.1b represents the marginal cost with respect to changes in s, due to forecast errors while ee refers to the absolute value of the marginal cost due to adaptation, decision-making and preference revelation.\(^{14}\)

It can be shown\(^{15}\) that a reduction in the information cost parameter $\beta$, through, say, the implementation of a demand revealing process,\(^{16}\) will reduce the optimal duration of tax institutions and will increase the optimal length of the _vaccatio legis_. The first result is due to the fact that $\beta$ represents a fixed cost of information and therefore a reduction in its value requires a shorter duration of the tax institution $u$ in order to obtain appropriate dilution. The second result is basically due to a substitutability of sorts that exists with respect to the cost effects of the variables $u$ and $s$.\(^{17}\) This experiment is illustrated in Figures 1.1a and 1.1b where the new cost curves are represented by dashed lines, and $u^{**}$ and $s^{**}$ indicate the new optimal values of the control variables. Similarly, it can be...

\(^{13}\)In (1.8) these costs are actually deflated by $u$.

\(^{14}\)In (1.9) these costs are deflated by $u$.

\(^{15}\)See Appendix to Chapter 1.


\(^{17}\)This substitutability is expressed by the fact that $\bar{K}_{su} = \bar{K}_{su} > 0$, as shown in the Appendix.
shown that an increase in the fixed cost of negotiation, $\alpha$, will in-
crease the optimal length of tax institutions and reduce the *vaccatio*
*legis* period. On the other hand, it can also be shown⁴¹⁸ that even
in the simple formulation considered above, comparative static analysis
with respect to changes in the parameters $\sigma$, $\delta$, $\mu$ and $\gamma$ do not in
general generate unambiguous effects on the optimal values of the
constitutional control variables.

1.3 Outline of the Study

The main purpose of the above discussion was simply to give an
illustration of what might represent an integrated perspective of
fiscal decision process optimization. It attempted to bring together
into the same picture the cost components presumed most relevant to
the choice of some optimal characteristics of tax institutions. The
scope of the subsequent investigation will now be limited.

It has been assumed in the above analysis that the costs of
negotiation would vary inversely with the degree of uncertainty
implicit in a constitutional choice situation. In the next chapter
an attempt is made, by developing the basic constitutional model of
this study, to investigate the mechanisms through which savings in
that kind of cost may actually emerge as a result of increased
uncertainty. Risk aversion is assumed to characterize individual

⁴¹⁸ See Appendix to Chapter 1.
behavior at the constitutional stage, and the magnitude of the after
tax income to be divided among individuals, groups or parties is
assumed to be known with certainty. A unique "constitutional" utility
of income function is assumed as the relevant operator for making
choices. Three basic propositions will be advanced which describe the
mechanisms that are expected to work at the constitutional stage.
At the same time, a fourth proposition will attempt to characterize
the general nature of the tax structures that tend to emerge optimally
at the constitutional stage under an Harsanyi-type equiprobability
situation. In Chapter 3 the analysis is extended to the case of risk
preference and the major implications for the working of the basic
constitutional mechanisms are examined. In Chapter 4 the assumption
of a net income opportunity set known with certainty is relaxed. In
Chapter 5 an attempt is made to extend the discussion of one of the
propositions derived in Chapter 2 to a context where a simple majority
voting rule is assumed to prevail in the post-constitutional stage
with respect to the amount of public goods provided.

Chapter 6 contains the summary of the results.
Appendix to Chapter One

First reproduce the necessary conditions (1.7) and (1.8) for an interior solution

\[
\overline{K}_u = \frac{1}{u} \left[ e^{-\sigma(s+u)} + e^{-\delta(s+u)} + e^\mu(s+u) + e^{-\gamma(s+u)} - 1 - \overline{K} \right] = 0 \tag{1.8}
\]

\[
\overline{K}_s = \frac{1}{u} \int_0^u \left[ - \sigma e^{-\sigma(t+s)} - \delta e^{-\delta(t+s)} + \mu e^\mu(t+s) - \gamma e^{-\gamma(t+s)} \right] dt = 0. \tag{1.9}
\]

Consider now the corresponding Hessian

\[
H = \begin{bmatrix}
\overline{K}_{uu} & \overline{K}_{us} \\
\overline{K}_{su} & \overline{K}_{ss}
\end{bmatrix}
\]

where, by using equalities (1.8) and (1.9).

\[
\overline{K}_{uu} = \frac{1}{u} \left[ - \sigma e^{-\sigma(s+u)} - \delta e^{-\delta(s+u)} + \mu e^\mu(s+u) - \gamma e^{-\gamma(s+u)} \right] \tag{1.13}
\]

\[
\overline{K}_{us} = \frac{1}{u} \left[ - \sigma e^{-\sigma(s+u)} - \delta e^{-\delta(s+u)} + \mu e^\mu(s+u) - \gamma e^{-\gamma(s+u)} \right] = \overline{K}_{uu} \tag{1.14}
\]

\[
\overline{K}_{su} = \frac{1}{u} \left[ - \sigma e^{-\sigma(s+u)} - \delta e^{-\delta(s+u)} + \mu e^\mu(s+u) - \gamma e^{-\gamma(s+u)} \right] = \overline{K}_{us} = \overline{K}_{uu} \tag{1.15}
\]

\[
\overline{K}_{ss} = \frac{1}{u} \int_0^u \left[ \sigma^2 e^{-\sigma(t+s)} + \delta^2 e^{-\delta(t+s)} + \mu^2 e^\mu(t+s) + \gamma^2 e^{-\gamma(t+s)} \right] dt \tag{1.16}
\]
Assuming that the second order conditions are satisfied at the optimal values $u^*, s^*$ one will have

$$
\overline{K}_{uu} > 0 \quad \overline{K}_{uu} \overline{K}_{ss} - (\overline{K}_{su})^2 > 0,
$$

which imply

$$
\overline{K}_{ss} > \overline{K}_{uu} = \overline{K}_{su} = \overline{K}_{us}.
$$

To find the effects $du^*/d\beta$ and $ds^*/d\beta$ one uses $H$ to get

$$
\begin{bmatrix}
\overline{K}_{uu} & \overline{K}_{us} \\
\overline{K}_{su} & \overline{K}_{ss}
\end{bmatrix}
\begin{bmatrix}
du^* \\
ds^*
\end{bmatrix}
= 
\begin{bmatrix}
1/(u^*)^2 \\
0
\end{bmatrix}
$$

(1.19)

where the last column vector indicate the partial derivatives $-\overline{K}_{u\beta}$ and $-\overline{K}_{s\beta}$.

Therefore,

$$
du^*/d\beta = \overline{K}_{ss}/(u^*)^2 |H| > 0 \quad (1.20)
$$

$$
ds^*/d\beta = -\overline{K}_{su}/(u^*)^2 |H| < 0 . \quad (1.21)
$$

The effects $du^*/d\alpha$ and $ds^*/d\alpha$ are obtained from the above expressions simply by replacing $\beta$ with $\alpha$. The effects of changes in $\sigma, \delta$ and $\gamma$ on $u^*$ and $s^*$ may be obtained in the following way (the calculations are developed only with respect to changes in $\sigma$ since the other effects are obtained similarly).
\[
\begin{align*}
\begin{bmatrix}
\tilde{K}_{uu} & \tilde{K}_{us} \\
\tilde{K}_{su} & \tilde{K}_{ss}
\end{bmatrix}
\begin{bmatrix}
\frac{du^*/d\sigma}{ds^*/d\sigma}
\end{bmatrix}
&=
\begin{bmatrix}
\frac{1}{u^*} [(s^*+u^*)e^{-\sigma(s^*+u^*)} - \frac{1}{u^*} \int_0^{u^*} e^{-\sigma(t+s)} dt]
- \frac{1}{u^*} \int_0^{u^*} e^{-\sigma(t+s)} [1 - \sigma(t+s)] dt
\end{bmatrix}
\begin{bmatrix}
\frac{1}{u^*} [e^{-\sigma(s^*+u^*)} - \frac{1}{u^*} \int_0^{u^*} (t+s)e^{-\sigma(t+s)} dt]
- \frac{1}{u^*} \int_0^{u^*} e^{-\sigma(t+s)} [1 - \sigma(t+s)] dt
\end{bmatrix}
\end{align*}
\]

\[
\frac{du^*}{d\sigma} = \frac{1}{|H|} \left\{ \tilde{K}_{ss} \left[ \frac{1}{u^*} [(s^*+u^*)e^{-\sigma(s^*+u^*)} - \frac{1}{u^*} \int_0^{u^*} (t+s)e^{-\sigma(t+s)} dt] \right] - \tilde{K}_{su} \frac{1}{u^*} \int_0^{u^*} e^{-\sigma(t+s)} [1 - \sigma(t+s)] dt \right\}.
\]

The sign of (1.22) cannot be determined on a priori grounds.

Similarly, one may obtain

\[
\frac{ds^*}{d\sigma} = \frac{1}{|H|} \left\{ \tilde{K}_{uu} \frac{1}{u^*} \int_0^{u^*} e^{-\sigma(t+s)} [1 - \sigma(t+s)] dt - \tilde{K}_{su} \frac{1}{u^*} [(s^*+u^*)e^{-\sigma(s^*+u^*)} - \frac{1}{u^*} \int_0^{u^*} (t+s)e^{-\sigma(t+s)} dt] \right\}
\]

and again the direction of the effect is indeterminate. Using (1.18), one may note however by inspecting (1.22) and (1.23) that if

\[- \tilde{K}_{uu} > | - \tilde{K}_{ss} |\]

then \(du^*/d\sigma\) and \(ds^*/d\sigma\) will have opposite signs.

The effects \(du^*/du\) and \(ds^*/du\) are in turn given by

\[
\begin{align*}
\begin{bmatrix}
\tilde{K}_{uu} & \tilde{K}_{us} \\
\tilde{K}_{su} & \tilde{K}_{ss}
\end{bmatrix}
\begin{bmatrix}
\frac{du^*/du}{ds^*/du}
\end{bmatrix}
&=
\begin{bmatrix}
\frac{1}{u^*} [- (s^*+u^*)e^\mu(s^*+u^*) + \frac{1}{u^*} \int_0^{u^*} (t+s)e^\mu(t+s) dt]
\end{bmatrix}
\begin{bmatrix}
\frac{1}{u^*} \int_0^{u^*} e^\mu(t+s) [1 - (t+s) \mu - 1] dt
\end{bmatrix}
\end{align*}
\]

from which it may be obtained
\[
\frac{du^*}{d\mu} = \frac{1}{|H|} \left\{ \bar{K}_{ss} \frac{1}{u^*} \left[ -(s^*+u^*)e^{\mu(s^*+u^*)} + \frac{1}{u^*} \int_0^{u^*} (t+s)e^{\mu(t+s)} dt \right] - \right.
\]
\[
\bar{K}_{us} \frac{1}{u^*} \int_0^{u^*} e^{\mu(t+s)} \left[ -(t+s)\mu - 1 \right] dt \right\} \quad (1.24)
\]
\[
\frac{ds^*}{d\mu} = \frac{1}{|H|} \left\{ \bar{K}_{uu} \frac{1}{u^*} \int_0^{u^*} e^{\mu(t+s)} \left[ -(t+s)\mu - 1 \right] dt - \right.
\]
\[
\bar{K}_{su} \frac{1}{u^*} \left[ -(s^*+u^*)e^{\mu(s^*+u^*)} + \frac{1}{u^*} \int_0^{u^*} (t+s)e^{\mu(t+s)} dt \right] \right\} \quad (1.25)
\]

The signs of these expressions are again indeterminate but it may be similarly noted that if \( |-\bar{K}_{uu}| > |\bar{K}_{su}| \) then \( du^*/d\mu \) and \( ds^*/\mu \) will have opposite signs.
Chapter Two

NEGOTIATION COSTS UNDER UNCERTAINTY

2.1 Introduction

The present chapter is devoted to the discussion of the effects of uncertainty on one of the basic cost components of the fiscal decision process, namely the negotiation or decision-making costs of selecting tax-sharing arrangements. Specifically, the chapter analyzes the mechanisms through which savings of this kind of costs tend to occur in consequence of the uncertainty that is normally associated with a constitutional choice setting. First, a negotiation costs function in a very general form will be described. Then, a choice model will be developed from which four basic propositions about fiscal constitutional decision-making are derived. Finally, the implications of two of these propositions for the amount of resources devoted to bargaining activities by the parties in conflict will be discussed. This will be accomplished by resorting to a more specific formulation of the negotiation cost function discussed earlier.

2.2 A Negotiation Costs Function

The typical situation to be analyzed is one where a set of N individuals attempts to reach an agreement on how to divide among themselves a given tax cost. For the moment, the benefits that
correspondingly accrue on the expenditure side of the budget are assumed to be fixed. The main objective is to investigate the effect of uncertainty on the costs of negotiating such an agreement; for this purpose, it is assumed initially that the aggregate amount of real resources devoted to strategic bargaining in such situations is described by the differentiable function

\[ D = D(N^*/N, N, H, S). \]  

The argument \( N^*/N \) is the minimum proportion of individuals required to agree in order to have a fiscal collective decision taken. As Professors Buchanan and Tullock have pointed out, one would in general expect \( D_1 > 0 \), i.e., the larger the fraction of the voting body required to agree, other things equal, the greater the costs of reaching agreement. When a unanimity rule prevails (\( N^*/N = 1 \)) these costs presumably reach a maximum.

The second argument is, as defined earlier, the total number of individuals involved in the decision-making process. The presumption is that \( D_2 > 0 \) since, other things equal, the costs of organizing decisive coalitions tend to increase as the total size of the voting body expands.

The third argument, \( H \), is some index of the degree of homogeneity of the choosing group as expressed, say, by the variance of individual


\[ \text{2Ibid., p. 112.} \]
most-preferred outcomes. Although this will be the object of more careful consideration in a later part of this chapter, the plausible hypothesis may be advanced to the effect that the aggregate amount of resources devoted to the reconciliation of divergent individual choices will tend to be smaller the greater the degree of homogeneity of the voting group, i.e., $D_3 > 0$. At the same time, one would expect that similarity of external characteristics such as economic positions held and tastes are factors which tend to increase the degree of homogeneity of the group.\footnote{Ibid., p. 115.}

Finally, $S$ is some measure of the "amount at stake" at the negotiations table. It represents some aggregate indicator of the magnitude of the potential gains from trade that might emerge from the bargain. Although this aspect will also be discussed more carefully later in this chapter, the proposition that the larger the stake at hand the greater will be the magnitude of the resources devoted to strategic activities seems plausible. Therefore, $D_4 > 0$.\footnote{Ibid., p. 98 and ff.}

A simplification of (2.1) allows a concentration on the aspects most relevant to the argument. Assume that the decision-making group is composed of two persons only, A and B, who are required to reach an agreement on a tax-sharing scheme.\footnote{Generalization of the most important propositions to the case of N individuals is undertaken in the Appendix 1 to this chapter.} With this simplification (2.1)
can be reduced to

\[ D = D(H, S), \quad (2.2) \]

and attention will now focus on the potential effects of uncertainty on the arguments \( H \) and \( S \).

### 2.3 A Model of Fiscal Constitutional Choice

In the previous chapter it was suggested that there are two basic ways of introducing uncertainty in a fiscal decision-making context, namely by increasing the duration of tax institutions or by creating a *vaccatio legis*. In the present formulation, and for expositional convenience, the latter procedure only will be treated but the argument could be translated into the multiperiod setting implicit in the first alternative. Therefore, consider a single representative post-constitutional budgetary period in which it is provisionally assumed that a public spending program has been exogenously fixed.\(^6\) The total tax cost associated with this program is presumed to be known with certainty and equal to \( T \). Assume further that there are two economic positions in the community, say 1 and 2, one for each individual. The tax-sharing arrangement to be chosen is supposed to allocate the given total cost on the basis of external criteria associated with the two economic positions, i.e., the tax structure will divide \( T \) between the two economic positions so that

---

\[ T = T_1 + T_2 \]  \hspace{1cm} (2.3)

\[ T_i \geq 0 \quad i = 1, 2 \]  \hspace{1cm} (2.4)

where \( T_i \) indicate the tax associated with position \( i \). Provisionally assume further that for every possible allocation of the tax cost between the economic positions, and taking into account the potential detrimental effects of the tax arrangement on the productive side of the economy, the corresponding after-tax incomes or surpluses are known with certainty and are given by the monotonic decreasing functions

\[ m_i(T_i) = Y_i(T_i) - T_i \quad i = 1, 2 \]  \hspace{1cm} (2.5)

where \( Y_i \) is the before-tax income accruing to position \( i \) and it incorporates as a part the benefits derived from the expenditure side of the budget. \(^7\) Since \( m_1(T_1) \) and \( m_2(T_2) \) are monotonic in \( T_1 \) and \( T_2 \), respectively, the former will, for notational convenience, be hereafter used as the relevant choice variables of the representative individual. Suppose that the set of after-tax income opportunities that will emerge from alternative tax-sharing arrangements is described by

\[ m_2 = v(m_1) - g(T) , \]  \hspace{1cm} (2.6)

where \( g(T) \) is a scale factor which reflects the effect of the overall level of the budget, \( T \), on the net-income opportunity set. For the

\(^7\)There is no need to assume that these benefits are equally distributed between the two economic positions.
time being $g(T)$ is constant since $T$ is assumed fixed. The function $v$ is assumed to be at least twice differentiable and the following conditions are supposed to hold

$$v' < 0 \quad (2.7a)$$

$$v'' < 0 \quad (2.7b)$$

$$0 < \hat{m}_1 = v'^{-1}(-1) < \hat{m}_2 = v(\hat{m}_1) - g(T). \quad (2.7c)$$

The first condition does not need much explanation; it simply states that the increase in the after-tax income in one position can be obtained only by reducing the after-tax income of the other position.

The second condition indicates that the net income possibility frontier is concave, i.e., further equal increments in one of the after-tax incomes will require increasing reductions in the other. The implications of the abandonment of this assumption are considered in the next chapter. The third condition states that the maximum aggregate surplus income for the community which results from a tax-sharing arrangement that allocates net income $m_1$ to position 1,\(^8\) corresponds to an after-tax income distribution that is assumed to lie somewhere between the fully egalitarian distribution, $m_1^e = m_2^e$, and the one which allocates the whole

---

\(^8\)The converse situation with respect to the two positions could have, of course, been arbitrarily assumed.
aggregate surplus to position 2. In fact, (2.7c) implies that at maximum aggregate surplus the net income allocated to position 2, \( \hat{m}_2 = v(\bar{m}_1) - g(T) \), exceeds the corresponding net-income allocated to position 1, \( \hat{m}_1 \), and that the latter is positive. This assumption reflects the following presumptions:

1. The completely egalitarian distribution is relatively inefficient in terms of the magnitude of total aggregate net income due to incentive type considerations.

2. A completely unequal distribution is also relatively inefficient in that it may generate social tensions that can only be accommodated at considerable cost.

3. Position 2 is more productive than position 1 in terms of income-generating ability.

Figure 2.1 illustrates diagrammatically, for given \( T \), the net income opportunity frontier described by (2.6) and conditions (2.7). Point

\[ \frac{d}{dm_1} [m_1 + v(m_1) - g(T)] = 0 \]

from which it may be obtained

\[ v^{-1}(-1) = \hat{m}_1 \]

where \( v^{-1} \) is the inverse function of \( v' \). At the same time it is assumed, in order to simplify the subsequent argument and without loss of generality, that over the relevant range of budgetary variations the allocation of the full tax cost \( T \) to any one of the two positions generates a negative surplus in that position. This implicitly places some restrictions on the range of possible tax distributions.
Figure 2.1

Net Income Opportunity Locus
E' corresponds to the egalitarian distribution and M' indicates the distribution where aggregate net income is maximized. It should be noted that if incentive type considerations were not allowed, the locus would be reduced to a straight line with slope -1 since in that case the distribution of the surplus would be irrelevant to the magnitude of aggregate social income.

Now, assume that individuals A and B currently occupy positions 2 and 1, respectively, and consider the problem faced by one of these individuals, say A, who attempts to select his optimal tax-sharing arrangement. It is clear that, in the absence of altruistic motivation, and since A is able to identify his present position, his most-preferred tax-scheme will be the one that shifts the whole tax burden to position 1. Similarly, for B the optimal fiscal structure will be the one that places the full tax cost on position 2. In terms of the earlier discussion, this setting where full identification of economic positions is possible, will correspond to the one that characterizes the "simultaneous-decisions" fiscal paradigm. Assume now that the probabilities of occupying positions depend on the extent of the *vaccatio legis*. Specifically, suppose that $p_1^A(t)$ and $p_2^A(t) = 1 - p_1^A(t)$ indicate the subjective probabilities A assigns to the occupation by him of position 1 and 2, respectively, where t indicates the duration of the *vaccatio legis*. It seems reasonable to suppose that

$$A \quad p_2^A(0) = 1 \quad \text{and} \quad dp_2^A(t)/dt < 0 \quad (2.8)$$

i.e., starting with full identification of the economic position
currently hold, as the uncertainty generated by the vaccatio legis increases, A will reduce the prospect that he will occupy in the future this position. Also let

$$t^{**} = \text{glb} \{ t : \lim_{t \to t^+} p_2^A(t) = \frac{1}{2} \}.$$ (2.9)

That is to say, in the limit, as \( t \) tends to some \( t^{**} \) a situation is approached where A will consider himself equally likely to occupy either one of the two possible positions.\(^{10}\) This limit case will, therefore, essentially correspond to Harsanyi's equiprobability situation\(^ {11}\) and one would certainly expect the time length required to reach

\(^{10}\) The convergence to the equiprobability situation may be illustrated, for \( t \) discrete, by a Markov process. Suppose \((1 - \rho)\) and \(\rho < 1\) denote the probability of switching and not switching social positions, respectively, when \( t \) is increased by one unit. Then,

$$p_2^A(t) = p_2^A(t - 1)\rho + [1 - p_2^A(t - 1)] (1 - \rho)$$

$$= p_2^A(t - 1)(2\rho - 1) + (1 - \rho)$$

The solution of the above difference equation can be seen to be

$$p_2^A(t) = \frac{1}{2} (2\rho - 1)^t + \frac{1}{2}$$

Clearly, as \( t \to \infty \), and since \( |2\rho - 1| < 1 \), \( p_2^A(t) \) tends to \( 1/2 \). If, as it seems reasonable to assume, \( \rho > 0.5 \) the convergence is monotonic.

this limit probability to depend on the specific characteristics of
the social context to which the situation refers. In particular, the
degree of social mobility and the frequency of social contingencies
are factors that presumably will affect the magnitude of $t^{**}$.

Assuming, on the other hand, that $B$ has the same information as
$A$ and makes similar probability estimates one should have of course $^{12}$
\[ P_B(t) = P_A(t) \] (2.10)

Suppose now that a positive vaccatio legis period, $t > 0$, is given and
consider the problem faced by individual $A$ who attempts to determine
his optimal tax-sharing arrangement. Assume that a unique von Neumann-
Morgenstern utility function $U(m_1)$ exists which evaluates the surplus
accruing to each economic position. $^{13}$ Assume that $U' > 0$ and suppose
also, for the time being, that $U'' < 0$ so that risk aversion prevails
at the constitutional stage.$^{14}$ Consider, therefore, the decision
calculus of $A$ who, for given $t$, attempts to select his optimal distri-
bution of surpluses between the two economic positions, $(m_1^*, m_2^*)$, so as

---

$^{12}$ This extreme uniform knowledge assumption is actually not re-
quired for the results below but it facilitates exposition.

$^{13}$ The uniqueness property of constitutional utility functions has
been justified, at least with respect to the equiprobability situation,
by W. Vickrey, "Utility, Strategy and Social Decision Rules," Quarterly
Journal of Economics 74 (November 1960); and D. Mueller, "Constitutional
(February 1973).

$^{14}$ The implications for the argument of admitting risk preference
are discussed in Chapter 2.
to maximize his expected utility, that is,

$$\text{Max}_{m_1, m_2 \geq 0} \{S(m_1, m_2, t) = p_1(t)U(m_1) + p_2(t)U(m_2)\}$$

s.t. $m_2 = v(m_1) - g(T)$ \hspace{1cm} (2.11)

where the superscripts on the probabilities have been dropped for notational simplicity. For an interior solution $(m_1^*, m_2^*)$, the first-order condition is

$$\frac{p_1(t)U'(m_1^*)}{p_2(t)U'(m_2^*)} = -v'(m_1^*)$$ \hspace{1cm} (2.12)

i.e., A will allocate net incomes so as to equalize the ratio of (weighted) marginal utilities with the marginal rate of transformation.

---

15 The problem analyzed here bears some similarity with the question discussed on the optimal income taxation literature, namely in that it refers to the optimal allocation of a given fixed tax cost among positions. Important differences exist, nevertheless, between the two approaches. As an instance, it may be noted that in view of the constitutional perspective taken here the (constitutional) utility function used in the present formulation is a concept with some logical independency from the net-income opportunity set concept. In particular it may reflect some typical constitutional attitude towards risk. This contrasts with the optimal income taxation literature where the social welfare function to be maximized is, in general, based on a utility function which is simultaneously used to derive net incomes. Also, the probability calculus plays in the present formulation a role that has no direct analogue in the optimal income taxation literature. See, for example, J. A. Mirlees, "An Exploration in the Theory of Optimum Income Taxation," Review of Economic Studies 38 (1971); and A. Atkinson, "How Progressive Should the Income Tax Be?," in Longman's Essays on Modern Economics, London, 1973. For a survey of recent contributions and references to the literature, see D. Bradford and H. Rosen, "The Optimal Taxation of Commodities and Income," American Economic Review 66 (May 1976).
of net incomes. At the same time, by inspecting (2.11) it may be noted that since $U$ is strictly concave so is $S$ and therefore the solution $(m_1^*, m_2^*)$ is a global maximum.

2.4 The Consensus Effect

Let, for given $t$, $m_i^A(t)$ and $m_i^B(t)$ ($i = 1, 2$) indicate the optimal net incomes allocated to position $i$, as viewed by individual A and B, respectively. Also, since the analysis deals with a two-person model, take as a measure of dispersion of individual most-preferred choices, the following

$$ W_i(t) = [m_i^A(t) - m_i^B(t)]^2 \quad (i = 1, 2) \quad (2.13) $$

A first proposition is now examined:

PROPOSITION 1 (Consensus Effect)—As the vaccatio legis increases the dispersion of individual most-preferred choices is reduced. That is, $W_i(t)$ is monotone decreasing in $t$.

The proof of this proposition\textsuperscript{16} will be arbitrarily developed with respect to the income allocated to position 1, but it can be immediately applied to the income allocated to position 2. From (2.13), differentiating $W_i(t)$ with respect to $t$, one gets

\textsuperscript{16}The generalization of this proposition to the n-person case ($n \geq 2$) is discussed in Appendix 1 to this chapter.
\[ W_1'(t) = 2 \left[ m_{1A}^*(t) - m_{1B}^*(t) \right] \frac{dm_{1A}(t)/dt}{dm_{1B}(t)/dt} \]  

(2.14)

In order to obtain the sign of the second bracketed expression differentiate totally equilibrium condition (2.12) to get

\[ p_1'(t)U'(m_1^*) dt + p_2'(t)U'(m_2^*) v' dt + p_1(t)U''(m_1^*) dm_1^* + \]

\[ + p_2(t) \left[ U''(m_2^*) (v')^2 + U'(m_2^*) v'' \right] dm_1^* = 0 \]  

(2.15)

which after rearrangement gives

\[ \frac{dm_1^*}{dt} = - \frac{p_1'(t)U'(m_1^*) + p_2'(t)U'(m_2^*) v'}{p_1(t)U''(m_1^*) + p_2(t)\left[ U''(m_2^*) (v')^2 + U'(m_2^*) v'' \right]} \]  

(2.16)

Now it can be easily shown that the denominator of (2.16) is

\[ d^2S/dm_1^2 \]  

which, since \( S \) has been seen to be strictly concave, is negative. At the same time, for individual A, \( p_1'(t) > 0 \) and \( p_2'(t) < 0 \) according to (2.8) so that the numerator is positive and therefore \( dm_{1A}^*/dt > 0 \). Similarly for individual B using (2.10) \( dm_{1B}^*/dt < 0 \) and therefore the second bracketed expression on (2.14) is positive. That is to say, as uncertainty is increased, each individual will allocate a smaller after-tax income to the position which initially he perceives as having a greater probability of occupying. This in turn clearly implies that the first bracketed expression in (2.14) is negative since we have assumed A to have originally a higher probability of occupying position 2. This produces

\[ W_1'(t) < 0 \]  

(2.17)
which describes the consensus effect. Therefore, up to the equi-probability limiting situation it is observed that an increase in the duration of the *vaccatio legis* will reduce the dispersion of separate individual optimal choices and, to that extent, effectively raise the homogeneity of the group involved in decision-making. Consider now:

**PROPOSITION 2 (Asymptotic Full Consensus)**—Full consensus is approached as the equiprobability situation is approached. That is

\[ \lim_{t \to t^*} W_i(t) = 0. \]

Since \( W_i(t) \) is monotone decreasing in \( t \) and is bounded from below it should have a limit. Clearly this limit has to be zero since at the equiprobability situation both individuals have the same net-income opportunity set, possess an identical utility function and face similar probability prospects so that their choices have to be identical. That is, if individuals A and B most-preferred choices for \( t > t^* \) are denoted by \( m_{iA}^{**} \) and \( m_{iB}^{**} \), respectively,\(^{17}\)

\[ m_{iA}^{**} = m_{iB}^{**} = m_{i}^{**} \]  

(2.18)

\(^{17}\)Although not being the subject of present concern, it may be noted in passing that this distribution of net income which obtains full consensus under equiprobability is regarded by some authors as possessing strongly appealing ethical properties since, it is argued, it is the one derived under the most impersonal set of circumstances. See, for example, J. Harsanyi, "Cardinal Utility in Welfare Economics and in The Theory of Risk-Taking," *Journal of Political Economy* 61 (1953), and R. Strotz, "How Income Ought to be Distributed: A Paradox in Distributive Ethics," *Journal of Political Economy* 66 (June, 1958).
which, of course, imply, for \( t > t^{**} \)

\[
W_1(t) = 0. \tag{2.19}
\]

Let now \((\hat{m}_1, \hat{m}_2)\) indicate the vector of individual net incomes that maximize aggregate income (corresponding to point \( M' \) in Figure 2.1), and let \((m_1^e, m_2^e)\) denote the full egalitarian distribution, i.e.,

\[ m_1^e = m_2^e \]

(point \( E' \) in Fig. 2.1). Consider now:

**PROPOSITION 3 (Efficiency-Equity Trade-off)** -- The optimal tax structure at the equiprobability limiting situation will generate an after-tax income distribution which, in general, lies somewhere between the full egalitarianism and the distribution corresponding to maximum aggregate net income. That is

\[
\hat{m}_1 < m_1^{**} < m_1^e
\]

or, equivalently,

\[
\hat{m}_2 > m_2^{**} > m_2^e.
\]

In view of assumption (2.7c) which states that \( \hat{m}_2 > \hat{m}_1 \) one should have \( U'(\hat{m}_2) < U'(\hat{m}_1) \). At the same time it was seen that at maximum aggregate income \( v'(\hat{m}_1) = -1 \).\(^{18}\) Therefore,

\[
\frac{U'(\hat{m}_1)}{U'(\hat{m}_2)} > 1 = -v'(\hat{m}_1) \tag{2.20}
\]

so that, by recalling that at the equiprobability situation \( p_1(t) = p_2(t) \), it is seen that condition (2.12) is not satisfied at the distribution.

\(^{18}\)See Footnote 9 this chapter.
that generates maximum aggregate net income. On the other hand, at the full egalitarian distribution $U'(m_1^e) = U'(m_2^e)$ and in view of assumptions (2.7b) and 2.7c) the marginal rate of transformation $-v'(m_1^e)$ should be greater than one. Therefore

$$\frac{U'(m_1^e)}{U'(m_2^e)} = 1 < -v'(m_1^e). \quad (2.21)$$

Expressions (2.20) and (2.21) together imply that between $(\hat{m}_1, \hat{m}_2)$ and $(m_1^e, m_2^e)$ there should be some intermediate distribution $(m_1^{**}, m_2^{**})$ where optimum condition (2.12) is satisfied. Figure 2.2 illustrates this point. On the abcissa are represented the alternative distributions in terms of the $m_1$ component, where $\hat{m}_1$ denotes the maximum net income allocable to position 1. The upward sloping curve indicates for each $m_1$ the marginal rate of transformation of net incomes, $-v'$. This schedule should intersect the horizontal line through 1 at the value of $m_1$ which maximizes aggregate net income, i.e., $\hat{m}_1$. The downward sloping curve depicts in turn, for any $m_1$, the ratio of marginal utilities (equally weighted due to the equiprobability assumption) and clearly it should cross the -1 horizontal line at $m_1 = m_1^e$ since at that point $U'(m_1^e) = U'(m_2^e)$. Equilibrium condition (2.12) is satisfied at the value of $m_1$ for which those two curves intersect and it is seen that such value, $m_1^{**}$, lies between $\hat{m}_1$ and $m_1^e$ implying therefore a trade-off between efficiency and equity. It can also be seen that, should the $-v'$ schedule have intersected the horizontal line through 1 to the right of $m_1^e$, one would have instead $m_1^{e} < m_1^{**} < \hat{m}_1$. 
Figure 2.2

The Efficiency-Equity Trade-off
In the particular case where \( v'(m^e_1) = 1 \) no trade-off will of course arise between efficiency and equity, both criteria being simultaneously met.

More specific implications of Proposition 3 with respect to optimal fiscal arrangements at the constitutional stage when majority voting is the decision rule which prevails at the post-constitutional stage, are explored in Chapter 5. Consider now in Figure 2.3 a diagrammatical illustration of the three propositions so far discussed. The non-negative quadrant is a simple replication of Fig. 2.1 which indicates the set of possible net-income distributions among which individuals A and B will choose. Each pair of values \((m_1, m_2)\) on this locus is translated into utility terms by resorting to the constitutional utility function \(U\) depicted in the northwest and southeast quadrants. The curve indicated in the southwest quadrant thus indicates the corresponding utility possibility frontier. It can easily be seen that the slope of this curve equals \(U'(m_1)/U'(m_2)v'\).

For a given \( t < t^{**}\), individual A assigns probabilities \(p_2(t) > p_1(t)\) to positions 2 and 1, respectively. Line aa with slope \(\theta(t) = -p_2(t)/p_1(t)\) reflects these probability weights attached to social positions. Expected utility maximization requires A to choose the point in the southwest quadrant locus where the slope of the curve equals the slope of the probability line aa since it is here that

---

19 Strotz, "How Income Ought to be Distributed:..." (1958), used a similar diagram although it did not include variable conditions of uncertainty.
Figure 2.3
Optimal Distribution Determination
condition (2.12) is met. L describes such a point. From this point and working back to the first quadrant the corresponding pair of optimal income values \((m_1^*, m_2^*)\) is found. Now, as \(t\) increases, the probability weights attached to each position change and so will change the slope of the line aa. As this occurs it can be seen that A's optimum choice slides downwards along the curve from L, i.e., further and further away from the distribution more favourable to the position that initially A finds it more probable to occupy. Point P describes the equilibrium that may eventually be reached if the equiprobability limiting situation is attained. In this case \((t > t^{**})\), line aa reaches a final position depicted by a ** a ** with slope -1. The optimum pair of income values that corresponds to point P is \((m_1^{**}, m_2^{**})\) and, as noted earlier this point lies somewhere between the maximum aggregate income distribution \((\hat{m}_1, \hat{m}_2)\) and the completely egalitarian distribution \((m_1^e, m_2^e)\).

Now it may be readily seen that the point on the utility possibility locus which represents B's optimal choice will, similarly, approach P but this time moving from R in a northwest direction. This will be so since for \(t < t^{**}\) B has a higher probability of occupying position L.

The implications of this convergence of choices or consensus effect on the magnitude of negotiation costs are analyzed later in this chapter. They do not represent, however, the only channel through which uncertainty bears on those costs. A second mechanism is normally expected to be at work under the "separation-of-decisions" paradigm and
this will be the object of subsequent attention. Before that, however, a specific implication of the present formulation for the consensus mechanism may be discussed. This refers to the potential influence of the magnitude of the budget on the minimum extent of the *vaccatio legis* required before the consensus effect will operate. This point may be more easily seen by inspecting again Figure 2.3. According to previous reasoning, the optimal utility-of-income distribution for, say, individual A under the "simultaneous-decisions" paradigm \((t = 0)\) is indicated by point Q, i.e., the more "egoistic" allocation. \(^{20}\) Now, as one departs from this polar situation and allows \(t\) to be positive, the probability line with slope \(\theta(t)\) ceases to be vertical. At the same time, provided \(v'(0) \neq 0\) and \(U'(0)\) is finite, the slope of the utility possibility frontier at \(Q\) (i.e., at \(m_1 = 0\)) which is given by

\[
\frac{U'(0)}{U'[v(0) - g(\bar{T})v'(0)]}
\]

is finite. Therefore, it seems clear that, if the above conditions hold, one will keep observing the initial corner distribution \(Q\) as A's optimum choice until \(\theta(t)\) is equal to \((2.22)\). This implies that some minimum extent of the *vaccatio legis* may be required before the consensus mechanism starts operating. This minimum period is given by

\[^{20}\text{Obviousl}y the term egoistic can be used here only in a rather qualified sense since the other optimal distributions obtained for different values of \(t\) are no less than \(Q\) derived from the maximization of self-interest.\]
\( \hat{t} = \theta^{-1} \left[ \frac{U'(0)}{U'[v(0) - g(\bar{T})]v'(0)} \right] . \) \hspace{1cm} (2.23)

The point to be noted with respect to this expression is the dependence of \( \hat{t} \) on \( g(\bar{T}) \), i.e., the scale variable of the net income opportunity locus. This implies that the size of the budget, \( \bar{T} \), may potentially affect the minimum time lag required to have any effect of the consensus mechanism.

Differentiating (2.23) with respect to \( \bar{T} \) gives

\[
\frac{d\hat{t}}{d\bar{T}} = (\theta^{-1})' \cdot \left\{ \frac{U'(0)U''[v(0) - g(\bar{T})]g'v'}{U'[v(0) - g(\bar{T})]v'} \right\} \text{where } (\theta^{-1})' > 0. \hspace{1cm} (2.24)
\]

The sign of (2.24) depends on the sign of \( g' \). Suppose that for some reason, the budget is initially oversized. This would imply \( g'(\bar{T}) > 0 \), i.e., raising the magnitude of the budget will further contract the net-income opportunity locus. The analysis suggests that in this case \( \hat{t} \) would increase with an increase in the budget size. Conversely, if the budget is initially inefficient in the downward direction \( (g'(\bar{T}) < 0) \), an increase in the budget will expand outwardly the opportunity locus of income distributions and one would anticipate a reduction in the minimum vaccatio legis required to observe the consensus effect operating. Finally, if the budget is efficiently sized to begin with \( (g' = 0) \) no effect at the margin will, of course, be exerted on \( \hat{t} \).

An intuitive explanation for the above results may be obtained by noting that the initial operation of the consensus mechanism depends crucially on the individual valuation placed on the marginal
units of the stake at negotiation. If this stake is large, implying a comparatively lower valuation in these units, a slight change in the assignment of probabilities to positions as a result of a small increase in the degree of uncertainty, will be sufficient to induce the individual to shift these units to the position where they are more highly valuated. Conversely, a smaller stake would require a greater initial change in probabilities in order to induce the representative individual to shift the marginal income units to the position of smaller probability of occupation.

2.5 The Aspirations-Level Effect

This section discusses the second mechanism that is expected to operate under the "separation-of-decisions" fiscal paradigm. Let

\[ S_j = S_j(m_1^{*j}, m_2^{*j}, t) \]

indicate, for given \( t \), the expected utility of individual \( j \)'s most-preferred income distribution \( (m_1^{*j}, m_2^{*j}) \), \( j = A, B \),

\[ S_j(m_1^{*j}, m_2^{*j}, t) = p_1^j(t)U(m_1^{*j}) + p_2^j(t)U(m_2^{*j}). \] (2.25)

Let also \( t^o_j \) denote the vaccatio legis which, for individual \( j \), minimizes \( |m_1^{*j} - m_2^{*j}| \), i.e., which induces \( j \) to select among his set of optimal distributions for different \( t \), the most egalitarian one. In view of assumption (2.7c) and Proposition 3 it can be readily seen that \( t^o_A = t^{**} \), i.e., the most egalitarian distribution chosen by \( A \) is the one selected under equiprobability. For \( B \), on the other hand, \( t^o_B \) is the vaccatio legis which induces him to choose the fully egalitarian distribution \((m_1^e, m_2^e)\) where \( t^o_B < t^{**} \). This can be immediately seen
from Fig. 2.3 where it was already noted that, as $t$ increases to $t^{**}$, A's optimal choice moves from point $Q$ to point $P$, so that the latter represents his most egalitarian optimal distribution, while B's optimum choice moves from $R$ to $P$ so that $E$ denotes his most egalitarian optimal choice.

**PROPOSITION 4 (Aspiration-level Effect)**—As the vaccatio legis increases, and up to his most egalitarian optimal distribution, the expected utility associated with the individual's most-preferred after-tax distribution declines. That is,

$$\frac{dS_j(t)}{dt} < 0 \text{ for } t < t_j^0 \text{ and } S_j(t) \text{ reaches a global minimum at } t = t_j^0.$$ 

In order to analyze the behavior of $S_j$ as $t$ varies, differentiate $S_j$ with respect to $t$ to obtain

$$
\frac{dS_j(t)}{dt} = \frac{dp_j^1(t)}{dt} U(m_j^{*1}) + p_j^1(t)U'(m_j^{*1}) \frac{dm_j^{*1}}{dt} + \\
\frac{dp_j^2(t)}{dt} U(m_j^{*2}) + p_j^2(t)U'(m_j^{*2})v' \frac{dm_j^{*2}}{dt}.
$$

(2.26)

Note that the second and fourth terms cancel out since, if an interior solution for $(m_1^{*j}, m_2^{*j})$ is assumed, equilibrium condition (2.12) will be satisfied. At the same time, $dp_j^1(t)/dt = -dp_j^2(t)/dt$ and, therefore, (2.26) is reduced to

Note that the second and fourth terms cancel out since, if an interior solution for $(m_1^{*j}, m_2^{*j})$ is assumed, equilibrium condition (2.12) will be satisfied. At the same time, $dp_j^1(t)/dt = -dp_j^2(t)/dt$ and, therefore, (2.26) is reduced to

21See Appendix 1 to this chapter for a generalization to the n-person case.
The second derivative of $S_j(t)$ is, in turn,

$$
\frac{d^2 S_j(t)}{dt^2} = \frac{d^2 p_j^1(t)}{dt^2} [U(m_{\text{1}}^*) - U(m_{\text{2}}^*)] + 
[U'(m_{\text{1}}^*) - U'(m_{\text{2}}^*)] \frac{dp_j^1(t)}{dt} \frac{dm_{\text{1}}^*}{dt}.
$$

Now, consider first, the case of individual B. For him it was noted before that, for some $t = t_B^O < t^{**}$, B will select the fully egalitarian distribution $(m_1^e, m_2^e)$ as his most-preferred choice. Therefore, for $t = t_B^O$, (2.27) will vanish, implying that $S_B$ reaches a stationary point at the fully egalitarian distribution. Inspection of (2.28) indicates that it is a minimum. In fact, the first bracketed expression vanishes at $m_1^e = m_2^e$ and, since for $B, \frac{dm_{\text{1}}^*}{dt} < 0$ due to the consensus effect and also $dp_{\text{1}}^B/dt < 0$, the second expression is positive. Hence, (2.28) is positive implying a minimum. At the same time, since the sign of (2.27) at the end point $t = 0$ (where $U(m_{\text{2}}^*) = 0$) is negative this minimum is a global one.

For individual A, it was observed that $t_A^O = t^{**}$. Inspection of (2.27) suggests that, since for any $t < t_A^O = t^{**}$, $dp_{\text{1}}^A/dt > 0$ and $U(m_{\text{2}}^*) > U(m_{\text{1}}^*)$, $dS_A(t)/dt$ is negative for $t < t_A^O = t^{**}$ which implies in turn that $S_A$ will reach a global minimum at $t = t_A^O = t^{**}$. Referring again to Figure 2.3, the above reasoning suggests that the minimum of $S_A$ will occur at $P$ when the equiprobability situation is attained and
(m_1, m_2) represents the chosen distribution. At the same time, S_B reaches a minimum at point E and then increases up to point P. For individual B, E represents therefore a saddle point—minimum with respect to changes in t, maximum with respect to changes in m_B^—while for A, point P is a saddle point.

The main point to be emphasized in the above analysis is that, except for possible ranges like the arc EP just discussed, one would expect an increase in the degree of uncertainty implicit in the "separation-of-decisions" model to reduce the expected gains associated with the individuals most-preferred tax-sharing arrangements. It would not be surprising if this decline in the individuals maximum aspiration levels had some bearing on the amount of resources individuals are willing to invest in strategic activities, i.e., in the magnitude of the negotiation costs. The following discussion is an attempt to analyze the potential impact of the mechanisms described under Propositions 1 and 4, on the magnitude of such costs.

2.6 Uncertainty and Strategic Investment

It seems reasonable to expect that the amount of resources a representative individual, say A, is willing to invest in strategic activities, depends on the expected rewards and costs associated with such an investment. Under the "separation-of-decisions" fiscal paradigm, a basic hypothesis which seems to underlie this approach is that, for a representative individual say A, the net expected revenue from
an investment in strategy of amount $X_A$ might be described by a function such as

$$\pi^A(t) = R[S^A(t), W^A(t), X_A(t), \overline{X}_B(t)] - C^A[X_A(t)] \quad (2.29)$$

where $C^A[X_A(t)]$ and $R[\cdot]$ indicate the cost and expected gross return from $X_A(t)$, respectively. The latter is assumed to depend also on:

- $S^A(t)$, a measure of the magnitude of the stake at negotiation as viewed by individual A (the maximum potential gains obtainable from the bargaining situation);
- $W^A(t)$, the expected variance of individual optimal distributions; and
- $\overline{X}_B(t)$, the expected amounts of resources B will devote to strategic investment.

All the arguments of the function $\pi^A$ have been made parametric on $t$ since under the "separation-of-decision" fiscal paradigm, and as it will soon become apparent, all of them are presumed to adjust when the degree of uncertainty changes.

To illustrate how these factors may interact under the "separation-of-decisions" hypothesis consider the following ad hoc specification of (2.29):

$$\pi^A(t) = S^A(t) \frac{W(t)}{\hat{W}} f^A[X_A(t), \overline{X}_B(t)] - C^A[X_A(t)]. \quad (2.30)$$

Here, expected gross revenue is the product of three factors. One is the magnitude of the stake, $S^A(t)$, which may be taken as the expected utility of A's most-preferred net income distribution as defined in (2.25). Second, a measure of the degree of consensus with respect to net income distribution, $W(t)/\hat{W}$, which indicates the ratio of the divergence of individual optimal choices $W(t)$, as defined say in (2.13),
to the maximum possible value of $W(t)$, $\hat{W}$. If full consensus would exist, $W = 0$ and there would be no point in investing in strategy since no positive return would be expected from such investment. As one departs, however, from full consensus, potential returns from strategic investment will emerge and they will be at a maximum when full disagreement prevails, i.e., $W = \hat{W}$. Finally, $f^A[X_A(t), \overline{X}_B(t)]$ is a "production function" with a negative input, $\overline{X}_B(t)$, indicating A's expected effectiveness in bargaining. Besides reflecting A's own skills and the potential influence of other exogenous factors, $f^A$ is assumed to depend on the amount of resources A and B devote to strategic activities.

Suppose that diminishing returns prevail and that an increase in B's investment will raise the marginal productivity of A's own investment, so that $f^A_1 > 0$, $f^A_2 < 0$, $f^A_{11} < 0$, $f^A_{22} > 0$, $\frac{f^A_1}{f^A_{22}} = \frac{f^A_{21}}{f^A_{22}} > 0$ and, with proper normalization, $0 < f^A < 1$. Assume marginal cost $c^A$ is constant. Mutatis mutandi it may be supposed B faces a net return function $\pi^B(t)$ similar to (2.30).

Now, for a given level of uncertainty, A's optimum amount of investment in strategy will, for an interior solution, be given by

$$\frac{2\pi^A}{2X_A} = S^A(t) \frac{W(t)}{\hat{W}} \frac{f^A_1}{f^A_1 - c^A} = 0. \quad (2.31)$$

---

22 It should perhaps be noted in this connection the distinction between the potential returns from strategic investment and the potential returns from the bargain. When, say, $W = 0$ the former is zero while the latter is certainly positive and equal to $S^A$. 
A non-cooperative Nash solution to investment in strategy will be given by the pair of values \( X_A^* (t) \) and \( X_B^* (t) \) that satisfy

\[
\begin{align*}
\frac{\partial \pi^A}{\partial X_A} &= 0, \\
\frac{\partial \pi^B}{\partial X_B} &= 0.
\end{align*}
\]

The effect of increased uncertainty on \( X_A^* \) can be readily obtained by differentiating (2.31) with respect to \( t \). This gives

\[
\frac{dX_A(t)}{dt} = - \frac{f_{11}^A}{S^A} \frac{dS_A(t)}{dt} - \frac{f_1^A}{W(t)f_1^{A1}} \frac{dW(t)}{dt} - \frac{f_{12}^A}{f_{11}^A} \frac{d\overline{X}_B(t)}{dt}.
\]  

(2.32)

From the previous discussion of (2.27) it is known that, up to the egalitarian distribution, \( dS_A(t)/dt < 0 \), so that the first term on the right-hand side is negative. Also, from (2.17) it is known that the second term should be negative. The third term, however, is of ambiguous sign since it is not known a priori whether \( d\overline{X}_B(t)/dt \) is positive or negative. By solving the system composed of (2.32) together with a similar equation for \( B \), for \( dX_A/dt \) and \( dX_B/dt \) it can be shown\(^\text{23}\) that these derivatives will both be negative or positive according to whether

\[
- \frac{f_{11}^A}{f_{12}^A} > - \frac{f_{12}^B}{f_{11}^B}.
\]  

(2.33)

\(^{23}\)See Appendix 2 to Chapter 2.
It can be similarly shown\(^{24}\) that a stable solution would require the left-hand side of (2.33) to exceed the right-hand side, so that

\[
J(t) = -\frac{dX_A(t)}{dt} - \frac{dX_B(t)}{dt} > 0, \tag{2.34}
\]

which is a measure of the savings in negotiation costs that will result from a marginal increase in the *vaccatio legis*.

Finally it should be noted that expression (2.32) breaks down, for individual A, the three basic channels through which uncertainty bears on the magnitude of his strategic investment. The first term corresponds to the savings due to the operation of the aspirations-level effect. Similarly, the second term refers to the savings generated by the consensus effect. Finally, the third term corresponds to the resources A saves by anticipating a decline in B's own strategic investment in consequence of increased uncertainty. This effect, of course, is conditional on the operation of the other two effects.

Figure 2.4 illustrates the effect of an increase in the *vaccatio legis* on the resources devoted to strategy by A and B. \(R_A\) and \(R_B\) are, for given \(t\), reaction curves of individual A and B as described by equation (2.31) and the corresponding one for B. \(R_A\) and \(R_B\) represent, in turn, similar lines after \(t\) has increased. \(TU\) subsumes, therefore, for individual A, the operation of the consensus and aspirations-level effect while, at the same time, \(VT\) represents the induced effect based on the anticipation of B's adjustment.

\(^{24}\)See Appendix 2 to Chapter 2.
Figure 2.4

Strategic Investment
Appendix 1 to Chapter Two

The following discussion refers to a model of fiscal constitutional choice similar in most respects to the one developed in the text but allowing now for a n-person setting.

Consider therefore the case of n individuals an n social positions where \( n \geq 2 \). Assume that individuals are indexed according to the position that they occupy when \( t = 0 \), so that if \( p_{ij}(t) \) indicates, for given \( t \), the probability of individual \( i \) occupying position \( j \) \((i,j = 1, \ldots, n)\), then \( p_{ii}(0) = 1 \) and \( p_{ij}(0) = 0 \) \((j \neq i)\). Assume further that for any individual \( i \) and for any \( t < t^{**} \), where as before

\[
t^{**} = \text{glb} \{ t : \lim_{t \to t} p_{ii}(t) = \frac{1}{n} \},
\]

the following relationships hold

\[
d_{pij}/dt < 0 \quad (2.35)
\]

\[
d_{pij}/dt = dp_{ik}/dt > 0 \quad (j,k \neq i) \quad (2.36)
\]

The first of these assumptions says that, up to the equiprobability situation, as uncertainty increases individual \( i \) will consider less likely to occupy the social position which for him has initially the higher probability of occupation, i.e., position \( i \). The second assumption states that as uncertainty increases the probability of occupying positions different from \( i \) increases "uniformly" across these positions.
Since this is assumed to hold for any \( t < t^* \) and since \( p_{ij}(0) = 0 \) \((j \neq i)\) it in turn implies that for any \( t < t^* \) individual \( i \) considers as equally likely to occupy any of the positions \( j \neq i \), i.e.,
\[
p_{ij}(t) = p_{ih}(t) \quad (j,h \neq i).
\]
To simplify the argument, incentive effects are neglected here and therefore it is assumed that a fixed total social net income \( Z \) is to be distributed among the \( n \) social positions. Let \( m_{ij} \) indicate the income share allocated by individual \( i \) to position \( j \) and let \( U(m_{ij}) \) denote, as in the text, the strictly concave constitutioinal utility function. For given \( t \), the problem faced by individual \( i \) is therefore
\[
\text{Max} \quad \{L(m_{i1}, \ldots , m_{in}, t) = \sum_{k=1}^{n} p_{ik} U(m_{ik}) + \lambda(Z - \sum_{k=1}^{n} m_{ik})\} \quad (2.37)
\]
\[
m_{i1}, \ldots , m_{in} \geq 0
\]
where \( \lambda \) is a Lagrangean multiplier. The first-order necessary conditions for an interior solution are
\[
L_k = p_{ik} U'(m_{ik}^*) - \lambda = 0 \quad (k = 1, \ldots , n) \quad (2.38)
\]
\[
L_\lambda = Z - \sum_{k=1}^{n} m_{ik}^* = 0. \quad (2.39)
\]
Inspection of (2.38) immediately suggests that, since \( p_{ij}(t) = p_{ih}(t) \) for \( j,h \neq i \), and since \( U' \) is monotone, then \( m_{ij}^* = m_{ih}^* \) for \( j,h \neq i \), i.e., income will be equalized across the positions of lower probability of occupation. At the same time the second-order conditions for a global maximum are satisfied since it can be immediately seen from (2.37) that \( L \) is strictly concave.
Consensus Effect

Now to obtain $d m_{ii}^* / d t$ differentiate totally (2.38) and (2.39) to obtain

$$\frac{d m_{ii}^*(t)}{dt} = \frac{|v^{ii}|}{|D|}$$

(2.40)

where $|D|$ is the determinant of the bordered Hessian of $L$. At the same time, $|v^{ii}|$ is the determinant

$$|v^{ii}| =
\begin{vmatrix}
 p_{11} U''(m_{11}^*) & 0 & 0 & \ldots & -p_{11} U'(m_{11}^*) & \ldots & 0 & -1 \\
 0 & p_{12} U''(m_{12}^*) & 0 & \ldots & -p_{12} U'(m_{12}^*) & \ldots & 0 & -1 \\
 0 & 0 & p_{13} U''(m_{13}^*) & \ldots & -p_{13} U'(m_{13}^*) & \ldots & 0 & -1 \\
 \vdots & \vdots & 0 & \ddots & \ddots & \ddots & \ddots & \ddots \\
 0 & 0 & \ldots & \ddots & \ddots & \ddots & \ddots & \ddots \\
 0 & 0 & \ldots & \ldots & \ddots & \ddots & \ddots & \ddots \\
 0 & 0 & \ldots & \ldots & \ldots & \ddots & \ddots & \ddots \\
 -1 & -1 & -1 & 0 & \ldots & \ldots & \ldots & 0
\end{vmatrix}

$$

By expanding $|v^{ii}|$ in terms of the elements of the $i^{th}$ row and their co-factors it may be obtained

$$|v^{ii}| = -p_{ii} U'(m_{ii}^*) |v^{ii}| + (-1)^{n+2+i} |v^{ii}|_{i n+1}$$

(2.41)

where $|v^{ii}|_{i n+1}$ and $(-1)^{n+1+i} |v^{ii}|_{i n+1}$ are the co-factors of the $i^{th}$ and the $(n+1)^{th}$ elements of the $i^{th}$ row, respectively. Let, for notational simplification,

$$p_{ik} U'(m_{ik}^*) = p_{ih} U'(m_{ih}^*) = c > 0 \quad (k, h \neq i)$$

(2.42)
and

\[ p_{lk} U^{m_{lk}}(m^*) = p_{ih} U^{m_{ih}}(m^*) = b < 0. \quad (k, h \neq i) \quad (2.43) \]

It can be immediately seen that

\[ |v_{ii}^{\text{ii}}| = |B_{n-1}| \quad (2.44) \]

where \( B_{n-1} \) is the \((nxn)\) symmetric matrix

\[
B_{n-1} = \begin{bmatrix}
  b & 0 & 0 & \cdots & 0 & -1 \\
  0 & b & 0 & \cdots & 0 & -1 \\
  0 & 0 & b & \cdots & 0 & -1 \\
  \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
  \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
  0 & 0 & 0 & \cdots & b & -1 \\
-1 & -1 & -1 & \cdots & -1 & 0
\end{bmatrix} \quad (2.45)
\]

Similarly,

\[
v_{i \text{ n+1}}^{\text{ii}} = \begin{bmatrix}
  b & 0 & 0 & \cdots & -c & \cdots & 0 \\
  0 & b & 0 & \cdots & -c & \cdots & 0 \\
  0 & 0 & b & \cdots & -c & \cdots & 0 \\
  \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
  \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
  0 & 0 & 0 & \cdots & -c & \cdots & b \\
-1 & -1 & -1 & \cdots & 0 & \cdots & -1
\end{bmatrix} \quad (2.46)
\]

Dividing the elements of the \(i^{th}\) column of \(v_{i \text{ n+1}}^{\text{ii}}\) by \(c\) and transferring this column to the right-hand side of \(v_{i \text{ n+1}}^{\text{ii}}\) it can be seen that

\[ |v_{i \text{ n+1}}^{\text{ii}}| = (-1)^{n-i}c |B_{n-1}| \quad (2.47) \]
and using (2.44) and (2.47)

\[ |v_{ii}| = -p_{ii}' U'(m_{ii}^*) |B_{n-1}| + (-1)^{2n+2} c |B_{n-1}| = \]

\[ = [c - p_{ii}' U'(m_{ii}^*)] |B_{n-1}| \]  \hspace{1cm} (2.48)

Now it can be seen from (2.45) that

\[ |B_{n-1}| = b |B_{n-2}| - b^{n-2}. \]  \hspace{1cm} (2.49)

This is a difference equation with solution

\[ |B_{n-1}| = -(n-1)b^{n-2}. \]  \hspace{1cm} (2.50)

Using (2.50) in (2.48) one therefore gets

\[ |v_{ii}| = [p_{ii}' U'(m_{ii}^*) - c] (n-1)b^{n-2} \]  \hspace{1cm} (2.51)

and using (2.42) and (2.43)

\[ \frac{dm_{ii}^*}{dt} = \frac{[p_{ii}' U'(m_{ii}^*) - p_{ik}' U'(m_{ik}^*)] (n-1) [p_{ik}'' U''(m_{ik}^*)]^{n-2} |D|}{|D|}, \]  \hspace{1cm} (2.52)

The first bracketed expression in the numerator is negative since

\[ p_{ii}' < 0 \text{ and } p_{ik}' > 0. \]  \hspace{1cm} \text{The second bracketed expression will, for different } n, \text{ alternate in sign starting with a positive sign for } n = 2. \]  \hspace{1cm} \text{At the same time, since } L \text{ as defined in (2.37) is strictly concave, the bordered Hessian } |D| \text{ will, for different } n, \text{ also alternate in sign starting with a plus sign for } n = 2. \]  \hspace{1cm} \text{Therefore, for any } n \geq 2, \text{ one should have}

\[ \frac{dm_{ii}^*}{dt} < 0. \]  \hspace{1cm} (2.53)
This implies that as uncertainty increases, individual i will reduce the income allocated to the position which has initially for him a higher probability of occupation, i.e., position i. The proof that

$$\frac{dm^*_ij}{dt} > 0 \quad (i \neq j)$$

may be obtained simply by observing that, as noted previously, income is equalized across the positions $j \neq i$. Therefore, since total income $Z$ will be exhausted, one should have for $i \neq j$

$$\frac{dm^*_ij}{dt} = -\frac{1}{n-1} \frac{dm^*_ii}{dt} > 0.$$  \hfill (2.54)

Now consider the effect of an increase in $t$ on the dispersion of optimal individual choices with respect say to income allocated to position $j$ as measured by

$$W_j(t) = \frac{1}{n} \sum_{h=1}^{n} \left[ m^*_hj - \left( \frac{1}{n} \sum_{h=1}^{n} m^*_hj \right) \right]^2.$$  \hfill (2.55)

Let $m^*_{hj}$ denote the income allocated optimally by individual $h$ to position $j$ when the equiprobability situation is attained. Since each and every individual has the same net income opportunity set, possesses an identical constitutional utility function and faces similar probability prospects in such situation, $m^*_{hj}$ should be identical for all $h$. Define therefore $m^*_{j} = m^*_{hj}$. Adding and subtracting $m^*_{j}$ in the brackets of (2.55) we have

$$W_j(t) = \frac{1}{n} \sum_{h=1}^{n} \left[ m^*_{hj} - m^*_{j} + m^*_{j} - \left( \frac{1}{n} \sum_{h=1}^{n} m^*_{hj} \right) \right]^2.$$  \hfill (2.56)
\[ \frac{1}{n} \sum_{h=1}^{n} (m^*_h - m^{**}_h)^2 + 2(m^*_h - m^{**}_h)[m^{**}_h - (\sum_{h} m^*_h)/n] + [m^{**}_h - (\sum_{h} m^*_h)/n]^2]. \] (2.57)

After calculations and simplification this reduces to

\[ W_j(t) = \frac{1}{n} \left\{ \sum_{h=1}^{n} (m^*_h - m^{**}_h)^2 - \left[ \sum_{h=1}^{n} (m^{**}_h - m^*_h)/n \right]^2 \right\}. \] (2.58)

Since due to (2.53) and (2.54) \( m^{**}_{hj} \) approaches monotonically \( m^{**}_j \) for any \( h \), it may be readily seen from (2.58) that

\[ W_j'(t) < 0 \] (2.59)

which implies the consensus effect.

**Aspirations-level Effect**

Let the expected utility of individual \( i \)'s most-preferred income allocation be given by

\[ S(t) = \sum_{k=1}^{n} p_{ik}U(m^*_{ik}) \] (2.60)

Differentiating \( S \) with respect to \( t \) gives

\[ S'(t) = \sum_{k=1}^{n} p'_{ik}U(m^*_{ik}) + \sum_{k=1}^{n} p_{ik}U'(m^*_{ik}) \frac{dm^*_{ik}}{dt} \] (2.61)

But, from (2.38), \( p_{ik}U'(m^*_{ik}) = \lambda \) for any \( k \). Therefore
\[ S'(t) = \sum_{k=1}^{n} p'_{ik} U(m^*_{ik}) + \lambda \sum_{k=1}^{n} \frac{d m^*_{ik}}{dt} \]  
(2.62)

and since \( \sum_{k=1}^{n} \frac{d m^*_{ik}}{dt} = 0 \) one finally gets

\[ S'(t) = \sum_{k=1}^{n} p'_{ik} U(m^*_{ik}) \]  
(2.63)

and this expression can be easily seen to be negative up to the equiprobability situation (or equivalently in this case, up to the egalitarian distribution). In fact, it has been assumed that \( p'_{ih} = p'_{ij} \) for \( h,j \neq i \) and it has been noted before that income will be equalized across the positions other than \( i \). Therefore, let for any \( j \neq i \)

\[ p'_{ij} = d > 0 \]  

\[ U(m^*_{ij}) = e > 0 \]  

Using (2.35) and (2.36) and since \( \sum_{k=1}^{n} p'_{ik} = 0 \) this implies that

\[ p'_{ii} = -(n - 1)d \]  
(2.64)

At the same time, since up to the equiprobability situation \( p_{ii} > p_{ij} \)

(2.38) implies that \( U'(m^*_{ii}) < U'(m^*_{ij}) \) and therefore that

\[ U(m^*_{ii}) > U(m^*_{ij}) = e \]  
(2.65)

Now, \( S'(t) \) may be expressed as

\[ S'(t) = p'_{ii} U(m^*_{ii}) + (n - 1)de \]  
(2.66)
and using (2.64) and (2.65) it may be seen that, up to the equi-probability situation,

\[ S'(t) < 0 \]  \hspace{1cm} (2.67)

which implies the aspirations-level effect.
Appendix 2 to Chapter Two

Proof that stability implies $dX_A(t)/dt, dX_B(t)/dt < 0$.

From (2.30), the first-order condition for an interior solution is

$$\frac{\partial \pi}{\partial X_A} = S^A(t) \frac{W(t)}{\hat{W}} f_1^A - c^A = 0. \tag{2.68}$$

Totally differentiating (2.68) and solving for $dX_A(t)/dt$ gives

$$dX_A(t)/dt = (1/c)[a - bdX_B(t)/dt] \tag{2.69}$$

where

$$a = - \frac{[dS^A(t)/dt]}{W(t)} \frac{W(t)}{\hat{W}} f_1^A - S^A \frac{1}{\hat{W}} [dW(t)/dt] f_1^A > 0$$

$$b = [S^A W(t)/\hat{W}] f_{12}^A > 0$$

$$c = [S^A W(t)/\hat{W}] f_{11}^A < 0.$$ 

Since for B one should have similarly

$$dX_B(t)/dt = (1/h)[d - edX_A(t)/dt] \tag{2.70}$$

where $d,e > 0$ and $h < 0$, substituting (2.70) into (2.69) gives

$$dX_A(t)/dt = \frac{a - db/h}{c - eb/h}$$

where the numerator is positive and the denominator of ambiguous sign. Clearly,
\[
\frac{dX_A(t)}{dt} < 0 \Rightarrow c - eb/h < 0 \Rightarrow -c/b > -e/h . \tag{2.71}
\]

Now, a stable non-cooperative Nash equilibrium would require the slope of the reaction curve of A in Figure 2.4 to exceed the slope of the reaction curve of B, i.e., that

\[
\frac{dX_B}{dX_A} \bigg|_{R_A} = -\frac{f^A_{11}}{f^A_{12}} > \frac{dX_B}{dX_A} \bigg|_{R_B} = -\frac{f^B_{12}}{f^B_{11}} . \tag{2.72}
\]

It is easily seen that this corresponds to condition (2.71). Therefore, stability implies

\[
dX_A(t)/dt < 0 \quad \text{and} \quad dX_B(t)/dt < 0 . \quad \text{q.e.d.}
\]
3.1 Introduction

In the previous chapter the analysis has proceeded under the assumption that risk aversion prevails at the constitutional level of choice. This assumption has been perhaps the one most frequently adopted in the literature with respect to the characterization of individual risk behavior at that stage of choice.\(^1\) It may be recalled that an extreme version of this assumption has been strongly defended by Rawls\(^2\) as the typical attitude in such a choice setting. In fact, Rawls, drawing on a previous argument by W. Fellner,\(^3\) has asserted that the basic choice criterion that tends to emerge in the "original position" is the "difference principle" or maximinimization, i.e., one where the concern of the constitutional framer is focused on the interests of the worst-off individual. This extreme position


has been severely criticized essentially on the following two grounds: 4 First, it does not allow for the possible trade-off between a very large gain (no matter how large, and no matter how high the chances of its occurrence) to the non-worst-off individuals and a very small loss (no matter how small, and how small the chances of its occurrence) to the worst-off individual. Second, it does not take into account other presumably relevant aspects of the distribution across the non-worst-off individuals.

These two objections raise, therefore, serious doubts about the adequacy of the maximin rule as a reasonable choice criterion for the constitutional level of decision-making. In general it would seem more plausible to anticipate risk-aversion at such a choice level but not in the extreme form suggested by Rawls. This was implicitly the viewpoint in the analysis developed earlier. In the present chapter, however, an attempt is made to extend that analysis to the case of risk-preference. The basic reason for doing so is that part of the defense of the existence of risk aversion at the constitutional stage, with the latter defined in some equivalent sense to the concept of the "original position," may not in some circumstances apply as well to the particular and more specific domain of fiscal choice.

Such defense has been essentially grounded on the idea that the nature of the goods to be distributed may be so fundamentally important to the representative individual that, in face of the possibility of losing some of these goods, he will tend to severely discount the prospect of successive further advantages. "Grave" risks are, so to speak, involved in the choice situation.

It seems clear that the relevance of the above argument is crucially dependent on the qualitative and quantitative structure of the goods included in the "basket" to be constitutionally divided. Hence, as we move away from Rawls "original position" paradigm where the "basket" encompasses such goods as civil rights, powers, opportunities, income and wealth, to the consideration of much more restricted sets of goods, the above argument may lose part of its force. With respect to the specific area of constitutional choice in which we are presently interested, namely the choice of fiscal institutions, the "basket" of goods is dramatically circumscribed not only because a more restricted set of dimensions is taken into account, such as for example income and wealth, but also, and perhaps more importantly, because in some cases possibly only a small fraction of these dimensions may be potentially affected by the constitutional fiscal decisions. Conceptually at least, one may envisage circumstances

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5Rawls, A Theory of Justice, p. 62.
where it would be difficult to consider the problem of the constitutional division of a given tax burden as one involving "grave" risks. This may happen, for example if the predicted overall level of taxation is relatively small as compared with the magnitude of the before-tax income, the fiscal constitutional convention occurs with considerable frequency and other non-fiscal constitutional provisions assure *per se* a substantial degree of redistribution. In such circumstances, the choice of optimal tax-sharing arrangements at the constitutional level might not be adequately characterized as a "serious wealth oriented" activity,\(^6\) as a potentially important means of providing "insurance." That is to say, it does not seem legitimate in these circumstances to exclude on *a priori* grounds the possibility of the representative individual exhibiting, at least over some ranges, some degree of risk-preference with respect to the choice of fiscal institutions.\(^7\)

As noted before, the present chapter attempts, first, to discuss the basic consequences of admitting risk-preference at the constitutional level of fiscal choice. What is its effect on the consensus and aspirations-level mechanisms described earlier? Does it impair

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\(^6\)This terminology was used by J. Hirschleifer, "Investment Decision Under Uncertainty: Applications of the State-Preference Approach," *Quarterly Journal of Economics* 80 (May 1966). He contrasts the case of "serious wealth oriented" activities where risk aversion is assumed to prevail, to the case of small bets, taken more or less frequently, where risk-preference may occur.

\(^7\)It is interesting to note that nowhere in his original formulation of the separation-of-decisions paradigm has Professor Buchanan used the risk-aversion assumption.
the savings in negotiation costs that, according to our previous analysis, seems to be such an important ingredient to the internal logic of the "separation-of-decisions" paradigm? What is, on the other hand, the impact of this new assumption on the optimal division of the net-of-tax income? These are the questions to be addressed in the following discussion. Secondly, another feature of the constitutional model of fiscal choice is illuminated in the analysis, namely the substitutability that may potentially exist between conditions on the productive and utility sides of the economy. In particular it will be seen that suitable restrictions placed on the productive opportunities will force the constitutional model with risk-preference to work and generate results "as if" risk-aversion were actually prevailing at the constitutional setting. This result will enable us, in turn, to comment on Samuelson's model of social income distribution under "equal ignorance." Finally, some previous assumptions concerning the shape of the productive opportunities are reconsidered in the light of the above results.

3.2 A Fiscal Constitutional Model with Risk-Preference

As compared with the model developed in Chapter 2, the basic modification to be introduced in the present analysis involves the "constitutional" utility function $U(m_i)$ which is assumed now to exhibit risk-preference. Therefore, suppose in the following
discussion that $U' > 0$ and $U'' > 0$. The other assumptions of the model are retained for the time being.

Referring again to our previous Figure 2.3 it may be readily recognized that, once a convex utility function replaces the previous risk-averse one, the concave net-income opportunity frontier of the first quadrant need not translate into a concave utility possibility frontier as depicted in the southwest quadrant. In fact, the convexity of the utility function now countervails the concavity of the income possibility locus so that the final effect on the shape of the utility possibility curve is indeterminate. Analytically, this point may be noted by representing the utility possibility locus as

$$U(m_1) = s \left[ U(m_2) \right], \quad (3.1)$$

from which after twice differentiation we may obtain

$$s'' = \left( \frac{U''(m_1)}{U'(m_2)v'} - \frac{U'(m_1)v''}{U'(m_2)(v')^2} - \frac{U''(m_1)U''(m_2)}{[U'(m_2)]^2} \right) \frac{1}{v'U'(m_2)}. \quad (3.2)$$

And since now $U'' > 0$ the sign of this expression is clearly ambiguous.

For the purpose of discussing the impact of risk-preference on the working of the two constitutional mechanisms described earlier in Propositions 1 and 4, it is most convenient to consider separately the two following polar situations:

1. **Mild risk-preference and strong incentive effects of taxation.**

First assume that, over the whole range of possible distributions we have
Using (3.2) it is possible to obtain, after simplification and rearrangement, the following relationship

\[
- \frac{v''}{v'} < - \frac{U''(m_1)}{U'(m_1)} + v' \frac{U''(m_2)}{U'(m_2)}.
\]  

(3.4)

As suggested before, this expression relates in a certain way the degree of curvature on the utility and productive sides of the economy. Indeed, \(- \frac{U''(m_1)}{U'(m_1)}\) may be viewed as a measure of the curvature of the utility schedule,\(^8\) evaluated at \(m_1\). It is, in fact, the curvature measure proposed by Arrow and Pratt to represent the local degree of risk-aversion (preference).\(^9\) The right-hand side of (3.4) may thus be seen to vary inversely with the degree of risk-preference. Similarly, \(\frac{v''}{v'}\) may be interpreted as a corresponding curvature measure for the net-income possibility frontier, \(v-g\), and it may be noted that a very accentuated curvature of this locus implies essentially the existence of strong incentive effects of taxation, i.e., rapidly increasing reductions in aggregate net income as one departs from the taxation scheme which generates maximum social income.

\(^8\)More precisely, a measure of the algebraic value of the curvature.

Inequality (3.4) corresponds therefore to the case where strong incentive effects on the productive side of the economy dominate the (opposing) effect of mild risk-preference on the utility side so that the final result is a utility possibility frontier which still exhibits concavity. This case is illustrated diagrammatically in Figure 3.1.

In order to investigate the working of the consensus effect in the present case one may start by noting that the second-order condition for a maximum, is again satisfied here. In fact, from $d^2S/dm_1^2 < 0$, one obtains

$$- v'' < - \frac{p_1 U''(m_1)}{p_2 U'(m_2)} - \frac{U''(m_2)}{U'(m_2)} (v')^2,$$

and using the first-order equilibrium condition (2.12) and rearranging,

$$- \frac{v''}{v'} < v', \frac{U''(m_1)}{U'(m_1)} - \frac{U''(m_2)}{U'(m_2)} (v')^2$$

which corresponds to inequality (3.4), which is assumed to be satisfied. Now, in the previous chapter the consensus mechanism was essentially described by the expression

$$\frac{dm^*_1}{dt} = - \frac{p_1' U'(m_1^*) + p_2 U'(m_2^*) v'}{p_1 U''(m_1^*) + p_2 [ U''(m_2^*) (v')^2 + U'(m_2^*) v'']}$$

(2.16)

which, for individual A, who was assumed to have a higher probability of occupying position 2, was seen to be positive. That is, as uncertainty increases A tends to shift his optimum distribution in
Figure 3.1

Mild Risk Preference and Strong Incentive Effects
favor of position 1. Under the present circumstances, despite the fact that \( U'' > 0 \), we may note that (2.17) is still positive since the denominator corresponds to the expression of the second-order condition that is satisfied. The consensus effect is therefore expected to work in the present circumstances. What about the aspirations-level effect? By observing the earlier expression (2.21) and (2.22) it may be readily recognized that nothing has to be changed regarding the former conclusions, the reason being that neither of these expressions depends on the existence or absence of risk-aversion, i.e., on the sign of \( U'' \). The egalitarian distribution \( m^e_1 = m^e_2 \) thus still corresponds to the minimum of expected utility \( S \) and, up to this distribution, the aspirations-level effect will therefore be expected to operate. Turn now to the second polar situation.

2. **Strong risk-preference and mild incentive effects of taxation.**

In this case, the concavity of the net-income possibility locus is assumed not to be sufficiently accentuated, over the whole range of distributions, to compensate for the opposing effect of the convexity of \( U''(m_1) \). The latter influence prevails and, therefore, we have

\[
s'' > 0. \tag{3.5}
\]

This situation is graphically depicted in Figure 3.2. Inspection of this diagram suggests that the maximum of expected utility may systematically correspond to a corner distribution rather than to an
Figure 3.2

Strong Risk Preference and Mild Incentive Effects
interior point. In fact, taking again the case of individual A, it can be immediately seen that, for a wide range of values of \( t \), the slope at point Q of the probability line \( aa \), \( \Theta(t) = -\frac{p_2}{p_1} \), exceeds in absolute value the slope of the utility possibility locus, i.e.,

\[
\Theta(t) < \frac{U'(0)}{U'[v(0) - g]} v'(0)
\]

and, comparing with (2.12), this is nothing but the corresponding condition for a local corner maximum at point Q. Because of his intense risk-preference, A chooses therefore an extreme income distribution. Now, as the vaccatio legis increases, the slope of \( aa \) diminishes in absolute value while this line rotates around Q. Clearly, this point keeps being A's most-preferred distribution up to the moment when, possibly, line \( aa \) touches simultaneously points Q and N. This may occur:

(a) Only in the equiprobability limiting situation, \( t > t^{**} \), if the intercepts with the axis of the net-income possibility locus have the same value, i.e., if \( v(0) - g(T) = v^{-1}[g(T)] \). In this case, both individuals will be indifferent, in the limit, between any of the two extreme distributions represented by points Q and N.

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19 See Chapter 2, p.45 for the discussion of corner solutions under risk-aversion.
(b) Before that limiting situation, if \( v(0) - g(\bar{T}) < v^{-1}[g(\bar{T})] \). In this case A's optimum choice experiences a sudden jump from Q to N as soon as the *vaccatio legis* exceeds some value \( \bar{t} < t^{**} \). This discontinuous behavior will not occur, on the other hand, with respect to B for whom N represents the most-preferred distribution whatever the degree of uncertainty prevailing at the constitutional stage.

(c) Never, if \( v(0) - g(\bar{T}) > v^{-1}[g(\bar{T})] \). This is the converse situation of (b), but now point Q corresponds to the most efficient of the two extreme distributions so that A's optimum choice will become independent of the degree of uncertainty.

The above considerations clearly suggest some contrasting features between the present model and the one discussed in Chapter 2. First, there is a possible discontinuity of individual choices as uncertainty varies, a characteristic of the present formulation that is typically absent in the case of risk-aversion. Observed jumps in optimal taxation schemes as uncertainty varies at the constitutional stage may therefore be an indicator of the existence of ranges of risk-preference on the part of individuals.

Second, the relative efficiency of the extreme net-income distributions plays a special role in explaining the way optimal
individual choices evolve with uncertainty variation. In the risk-aversion case, in contrast, no such special role is assigned to particular distributions, i.e., there are no focal distributions on which relative efficiency comparisons may be exclusively concentrated.

Finally, and perhaps most importantly, over substantial ranges of uncertainty variation the consensus effect will not be working in the present formulation. Circumstances may exist (case (a) above) where only in the equiprobability situation will one observe a potential reduction in the dispersion of individual most-preferred distributions. Since the cost savings stemming from the operation of such a mechanism will thus not occur, this raises the question of whether the "separation-of-decisions" fiscal paradigm may have its potential applicability limited to the domain of risk-aversion.

Clearly, an answer to this question, even if tentative, requires as a preliminary step, an investigation of the possible operation of the other cost-saving mechanism, the aspirations-level effect. This aspect may now be considered and for convenience the following analysis refers to the discussion of cases (a), (b) and (c) above.

Take first, the situation dealt with in (a), where it is assumed that \( v(0) - g(\bar{T}) = v^{-1}[g(\bar{T})] \) and consider again individual A. It is easily seen that in the present circumstances A's expected utility from his most-preferred net-income distribution is reduced to
\[ S = p_2 \ U[m_2 = v(0) - g(T)] \]  
(3.6)

since, without loss of generality, it may be assumed that \( U(m_1 = 0) = 0 \). As \( t \) increases and \( p_2 \) decreases while approaching 1/2, \( S \) declines since, as noted before, A's optimal income vector, \( I_0, v(0) - g(T) \), remains the same. Eventually, \( S \) will reach the minimum

\[ S = \frac{1}{2} \ U[m_2 = v(0) - g(T)] \]  
(3.7)

when the equiprobability situation is attained. Hence, it may be concluded that the aspirations-level mechanism is working in the present circumstances.

Consider now the case where there is an asymmetry in the productivity of the extreme distributions. Take first, the situation described in (b) where \( v(0) - g < v^{-1}(g) \), i.e., where A presently occupies the position which generates the less productive extreme distribution. It is easy to see that in this case A's expected utility of his most-preferred tax distribution will decline as the vaccatio legis increases over the range \( 0 < t < \bar{t} \). For \( t > \bar{t} \), however, there will be an increase in A's expected utility. That is to say, there is a range of uncertainty variation over which the aspirations-level effect will work in a perverse direction. Clearly, if a difference in productivity of the extreme distributions represented by points Q and N in Fig. 3.2 exists but is not significant enough, the potential practical implications of this perverse range of uncertainty variation will lose much of their relevance. This
will be so since in such a case $t$ will probably be too large to have any reasonable chance of being exceeded in most real-world circumstances of constitutional choice. With respect to individual B, on the other hand, it easily seen that, since his most-preferred distribution is insensitive to $t$, the aspirations-level effect will be at work throughout the whole range of uncertainty variation.

Finally, it may be noted that case (c) is similar to (b) with the roles of A and B interchanged, so that no further discussion is required.

To sum up the conclusions of the discussion so far, it may be noted that with the exception of possible ranges of high degree of constitutional uncertainty the assumption of risk-preference does not eliminate the savings in negotiation costs that were expected to emerge under the "separation-of-decisions" fiscal paradigm. At worst, if risk-preference is very strong and/or taxation incentive effects are very mild, one of the constitutional mechanisms, namely the aspirations-level effect, will still be at work. On the other hand, if risk-preference is mild and/or taxation incentive effects are severe, the model works and generates results "as if" risk aversion has been assumed to begin with.

3.3 Samuelson's Theorem on the Distribution of Social Income

Having analyzed the two polar situations of risk-preference, and before attempting to generalize the conclusions obtained, it seems
appropriate to use the results already derived to comment on a model of constitutional choice developed by Samuelson. 11 Samuelson's formulation emerges as an attempted improvement over Lerner's model of optimal distribution of social income with known but unassigned utility schedules. 12 Although it does not refer explicitly to the question of taxation, it can be readily put, however, in a comparable basis. Specifically, Samuelson states:

I shall show that, if the crucial equal-ignorance assumption were really acceptable to every reasonable observer, then each and every person (subject to the postulated concave utility function that renders him a risk averter) would vote for a regime of equal distribution of income and this constitutional feature would be instituted by unanimous vote" (italics in the original). (p. 175).

And he goes on to say:

It will suffice to state the following obvious theorem on (each) concave function:

Let total social income be divided up into \( n \) non-negative magnitudes \((y_1, y_2, \ldots, y_n)\) with \( \sum y_j = Y \) and let each \( y_j \) be written on a separate card. Now let the deck of cards be shuffled and be dealt out at random to each of us \( n \) individuals, so that every permutation is equally likely and each of us can expect with equal probability to receive each \( y_j \). Then you (and I) will prefer that all the \( y \)'s be equal to \( Y/n = \sum y_j/n \) rather than anything else (including the historical distribution of incomes under mixed capitalism). Thus even if it makes no sense to sum different \( F_i(y_i) \), the fact that each \( F_i^" < 0 \) implies

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11P. Samuelson, "A. P. Lerner at Sixty."

Each \( \frac{1}{n} \sum_{j=1}^{n} F_i(y_i) \) is at a maximum subject to

\[
\sum_{j=1}^{n} y_j = Y \text{ for } y_1 = y_2 = \ldots = y_n = Y/n \text{ where } 1 \leq j \leq n
\]

\[
F_i(y_1) = \ldots = F_i(y_n).
\]

Clearly, Samuelson's formulation differs in some respects from the one previously developed here. It may be noticed, for example, that in his analysis each individual possesses a different utility function at the constitutional stage which contrasts with our assumption of a unique "constitutional" utility schedule. However this difference is not indeed relevant for the point to be made. For present purposes it is sufficient to note that, in contrast with the formulation in this study, Samuelson's theorem refers to the case of a fixed amount of social income, i.e., no incentive-type considerations are taken into account. Samuelson considers, however, in a footnote, the potential implications for his own theorem of relaxing such an assumption. He states, in fact,

Since redistribution in real life can never be of the distortionless type, egalitarianism will not (and should not) be carried all the way. But, since it is a theorem that dead-weight distortion grows only with the squares of the amounts redistributed, the optimal compromise between "equity" and "efficiency" will always involve some sacrifice of the latter to the former (and vice versa). (p. 176, f.1)

Samuelson recognized, therefore, that the introduction of incentive-type considerations will tend to generate a compromise between equity and efficiency, an aspect that was also emphasized in Proposition 3 of
the previous chapter. What he apparently failed to see, however, is that allowance for the possible variability of total social income has another potential implication for his own model namely that it may permit under certain circumstances the abandonment of the postulated concavity of the constitutional utility function. The analysis of case 1 above suggested in fact that if incentive considerations are strong enough, risk-aversion turns out not to be a necessary requirement for observing both the consensual and egalitarian proclivities of the constitutional mechanisms.

3.4 Extension of the Results

Before concluding, it should be pointed out that there are two ways in which the results derived may be, somewhat, generalized. The first, is easily obtained by abandoning the polar situations discussed above. That is, rather than assuming that the "constitutional" utility function exhibits either risk-aversion or risk-preference throughout the whole relevant range, we may admit instead that such a function combines both attitudes toward risk. Some portions of the schedule may be concave while others may exhibit convexity. As a diagrammatical example, Figure 3.3 depicts a utility possibility locus that may emerge if the constitutional utility function resembles the shape suggested by Friedman and Savage\textsuperscript{13} for choices involving risk.

Figure 3.3

A Friedman-Savage Utility Frontier
Although no particular representativeness as a typical constitutional contour seems attributable on a priori grounds to such a utility function, it may nevertheless serve for illustrative purposes. Inspection of this figure suggests that as uncertainty increases, A's most-preferred net-income distribution would slide downwards starting from point Q to point T. Since the probability line will be then supposedly tangent to T and U, these two distributions will thus be indifferent. For greater values of $t$ the optimal distribution would move from U to V and it will reach the later point in the equiprobability limiting situation. It is easily seen that with this particular curvature both mechanisms will be operating provided point V is above the 45° line.

The second avenue for possible generalization of the previous results may be readily recognized by referring again to the discussion of case 1 above. The analysis suggested the existence of some potential substitutability between conditions on the productive and on the utility side of the economy but the argument so far developed focused only on the possible implications of alternative shapes of the utility function. Clearly, substitutability runs, however, in both directions which means that the shape of the net-income opportunity locus may, under certain circumstances (namely the existence of strong risk-aversion at the constitutional stage), also have a wide range of potential variation without affecting the logic of the
"separation-of-decisions" model. In other words, the domain of applicability of the two constitutional mechanisms described in Propositions 1 and 4 is potentially much wider than that suggested by the set of restrictions (2.7) imposed on the income opportunity locus.
Chapter Four
UNCERTAINTY ON NET-INCOME OPPORTUNITIES

4.1 Introduction

So far the constitutional model of fiscal choice has been developed under the assumption that the after-tax income possibility frontier is known with certainty by the representative choosing individual. In the present chapter this assumption is relaxed in order to explore the potential implications of such modification for the working of the consensus and aspirations-level mechanisms discussed before. Allowance for uncertainty with respect to the opportunity set essentially represents an attempt to introduce a further dimension of realism into the previous constitutional model, by eliminating an asymmetry of sorts that exists in such a formulation. In fact, it has been assumed that individuals are unable, at the constitutional stage, to predict with exactitude the social positions that they will occupy in the future post-constitutional context, while at the same time supposing their ability to fully anticipate the magnitude of the after-tax residual to be divided constitutionally. This informational asymmetry is removed as soon as it is admitted that uncertainty exists with respect not only to who eventually gets which slice of the pie but also to the size of the pie itself. In diagrammatical terms, and referring again to Figure 2.3, this means that the after-tax income opportunity locus of the first-quadrant is no longer known with certainty at the constitutional
stage. At the same time, since the latter represents a net-income schedule—where net income may be defined as

\[ m = B + y - T, \] (4.1)

B, y, and T representing respectively benefits from public expenditures, gross private income and taxes—it seems clear that the variability of such a locus may originate in uncertainty with respect to either one or more of the above components B, y or T. For present purposes, however, the discrimination among these possible distinct sources of uncertainty is of no interest, the important aspect being only the overall variability exerted on m.

Two further points should be noted in connection with the introduction of uncertainty about the net-income locus. First, it may be observed that under the present circumstances specific assumptions have to be made with respect to the instruments available at the constitutional stage. In other words, since variability extends now to the object of choice itself, it seems natural to ask what are the control variables available to the representative individual at the constitutional convention. Here, a whole range of different specifications seems possible. As an illustrative example one may envisage the situation where constitutional powers exist such that say the amount of net income accruing to one of the two positions could be made certain in advance while the other bears the full burden of variability, by having its income residually determined. In the formulation to be developed below, however, the specific assumption to be made in this
connection is that the representative individual constitutionally selects among tax institutions that are able to generate preselected relative income distributions. In other words, it is supposed that by using appropriate tax institutions post-constitutional relative income distributions can be chosen constitutionally with certainty although the absolute level of income accruing in the future to each position is, nevertheless, random. This assumption will be further clarified below.

Second, taking into consideration the logic of the constitutional model developed so far, where the degree of uncertainty with respect to the occupation of social positions was assumed to be positively related to the extent of the vaccatio legis, it seems only natural to make the degree of variability of the net income locus also positively associated with the length of such period, t.

4.2 The Model

Initially, therefore, consider the problem faced by individual A who attempts to select his optimal post-constitutional after-tax relative income ratio \( \frac{\bar{m}_1}{\bar{m}_2} \), knowing as before that for a given t he has probabilities \( p_1(t) \) and \( p_2(t) \) of occupying positions 1 and 2, respectively. The variables \( \bar{m}_1 \) and \( \bar{m}_2 \) are now the random net-incomes accruing to positions 1 and 2 and it is assumed that they are given by

\[
\bar{m}_1(m_1, t) = m_1 [1 + h(t)\bar{z}]
\]  

(4.2)
where $\bar{z}$ is a symmetric random variable\(^1\) with expected value equal to zero, $E(\bar{z}) = 0$, and finite variance, $E(\bar{z}^2) = \sigma^2 < \infty$. The function $h(t)$ is monotone increasing in $t$, the *vaccatio legis*, where the latter works here as a shift parameter of the distribution of $\bar{z}$. Furthermore, in order to rule out negative net incomes, $-1/a < h(t) < 1/a$ for any $t$, where $[-a,a]$ is the domain of $\bar{z}$. At the same time, $m_1$ stands for the choice variable available to the representative individual. It can be immediately seen, in fact, that by choosing $m_1$ the individual automatically selects the post-constitutional relative income distribution, $\bar{m_1}/\bar{m_2} = m_1/v(m_1)$. The function $v(m_1)$ describes a locus similar to the net-income opportunity curve discussed earlier and it is supposed that the previous set of conditions (2.7) holds here again.

Using (4.2) and (4.3) one may obtain

\[
E(\bar{m}_1) = m_1, \quad E(\bar{m}_2) = v(m_1) \quad (4.4)
\]

\[
\text{Var}(\bar{m}_1) = m_1^2(1 + h^2\sigma^2), \quad \text{Var}(\bar{m}_2) = v^2(1 + h^2\sigma^2). \quad (4.5)
\]

It can be seen from (4.5) that an increase in the *vaccatio legis* period increases the variability of the net-income opportunity frontier. This is as it should be according to previous observations. At the same time, (4.4) suggests that the parametric shifts on the distribution of

\(^1\)Symmetry is a stronger assumption than required by the argument. It is used here to facilitate the exposition.
are of the mean-preserving type so that they correspond to "pure" uncertainty effects.

For given \( t \), the problem faced by, say, individual A is therefore to select \( m_1 \) so as to

\[
\text{Max } R = p_1 E[U(\tilde{m}_1)] + p_2 E[U(\tilde{m}_2)]
\]

subject to (4.2) and (4.3). Diagrammatically this choice problem is illustrated in Figure 4.1. For given \( t \), A is called upon to select a ray along the origin (parametric on \( m_1 \)) when he is uncertain not only with respect to the position he will occupy in the future but also about the specific point along the way which will be ultimately observed. The contour in the diagram denotes, for any chosen ray, the expected value of both net incomes.

The first-order necessary conditions for an interior maximum are given by

\[
p_1 E[U'(\tilde{m}_1)(1 + h\tilde{z})] + p_2 E[U'(\tilde{m}_2)(1 + h\tilde{z})v'] = 0.
\]

Since \((1 + h\tilde{z})\) and \((1 + h\tilde{z})v'\) are the (random) value increments to positions 1 and 2, respectively, of one additional unit of \( m_1 \) allocated to that position, \( U'(1 + h\tilde{z}) \) and \( U'(1 + h\tilde{z})v'\) are the corresponding utility increments in each of the two positions. The optimal individual tax-distribution is therefore pushed to the point where the expected increments of utility, weighted by position occupation probabilities, are equalized.
Figure 4.1

Variable Net Income Locus
The second-order condition for a maximum is, in turn, satisfied giving

\[ p_1 E[U''(1 + h\bar{z})^2] + p_2 E[U''(v')^2(1 + h\bar{z})^2 + \]
\[ + v''U'(1 + h\bar{z})] < 0. \quad (4.8) \]

4.3 The Consensus Effect

As noted before, the basic question under consideration refers to the potential implications of the new kind of uncertainty presently introduced to the operation of the two constitutional mechanisms discussed earlier. Consider, first, the consensus mechanism. Differentiating totally equilibrium condition (4.7) and rearranging one obtains

\[ \frac{dm_1^*}{dt} = -\frac{\phi(t, m_1^*) + \psi(t, m_1^*)}{p_1 E[U''(1+h\bar{z})^2] + p_2 E[U''(v')^2(1+h\bar{z})^2 + v''U'(1+h\bar{z})]} \quad (4.9) \]

where

\[ \phi(t, m_1^*) = p_1 E[U'(1+h\bar{z})] + p_2 E[U'(1+h\bar{z})v'] \quad (4.10) \]

\[ \psi(t, m_1^*) = p_1 E[U''m_1^*(1+h\bar{z})h'\bar{z}] + p_1 E[U'h'\bar{z}] \]
\[ + p_2 E[U''v(1+h\bar{z})h'\bar{z}v'] + p_2 E[U'h'\bar{z}v'] \quad (4.11) \]

Now the denominator of (4.9) is certainly negative since it corresponds to the expression of the second-order condition stated in (4.8). The
numerator, in turn, is composed of two terms. The first, \( \phi(t, m_1^*) \) it can be immediately seen to be positive. The second term, \( \psi(t, m_1^*) \), is however of indeterminate sign. In fact, it can be shown that

\[ p_1 E[U'zh'] < 0 \]

while, at the same time, the first and third terms of \( \psi \) are of indeterminate sign. This introduces the major point to be presently noted, namely that, in contrast with what happened in the discussion of the basic model of Chapter 2, in the present formulation a possibility arises, theoretically at least, of having \( dm_1^*/dt < 0 \), over some ranges, i.e., the consensus mechanism working in a perverse direction. This will happen if \( \psi \) is sufficiently negative to make the

\[ 2 \text{To prove that } p_1 E[U'zh'] < 0 \text{ it must be shown that } E[U'z] < 0. \]

With decreasing marginal utility, \( U' \), for \( z > 0 \)

\[ U'(m_1^* + m_1^*h z) < U'(m_1^*). \]

Multiplying both sides by \( z \),

\[ U'(m_1^* + m_1^*h z)z < U'(m_1^*)z. \]

Similarly, for \( z < 0 \),

\[ U'(m_1^* + m_1^*h z) < U'(m_1^*) \]

and

\[ U'(m_1^* + m_1^*h z)z < U'(m_1^*)z. \]

Therefore, for any \( z \),

\[ U'(m_1^* + m_1^*h z)z < U'(m_1^*)z. \]

Taking expected values on both sides, finally

\[ E[U'(m_1^* + m_1^*h z)z] < E[U'(m_1^*)z] = 0 \quad \text{q.e.d.} \]
numerator of (4.9), $\phi + \psi$, also negative. What economic interpretation may be given to this result? It seems that an intuitive explanation for such outcome may be obtained in the following way. An increase in the *vaccatio legis* by one unit has, under the present formulation, two distinct effects on the individual equilibrating adjustment. On the one hand, it twists the probabilities attached to social positions in favour of the one presently considered as of less likely occupation. This effect, which was also found in the basic model of Chapter 2, is described by the function $\phi$, and unambiguously calls for an "altruistic" shift in the optimal distribution, i.e., a shift in favor of the position with lower probability of occupation. On the other hand, however, the increase in uncertainty under the present formulation entails also an increase in the variability of post-constitutional net incomes. If risk aversion is not uniform along the income scale, it is possible that the loss in expected utility due to such increased variability is greater at a lower point in the income scale (as associated with position 1) where a greater sensitivity to risk may exist, than at a higher point (as associated with position 2) where the opposite may hold. If this happens, the individual may not have on this particular account an incentive to shift the distribution in an "altruistic" direction (i.e., in favour of position 1). On the contrary he may even take an

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"egoistic" move since by doing so he may avoid, to a greater extent, the detrimental impact of this kind of uncertainty. This second effect of uncertainty which did not occur in our previous model of Chapter 2 is described by the function $\psi$. It may be seen that, other things equal, the probability of a perverse consensus effect is higher, the greater the degree of (absolute) risk aversion at $m_1^*$ and the lower the risk-aversion at $m_2^*$. In fact, we may note from the differentiation of the equilibrium condition (4.7) that the function $\psi$ corresponds to

$$\psi = p_1 \frac{\partial}{\partial t} E[U'(1+h\tilde{z})] + p_2 \frac{\partial}{\partial t} E[U'(1+h\tilde{z})v'].$$  \hfill (4.12)

Expanding $U'(m_1)$ around $m_1^*$,

$$U'(m_1^* + m_1^* h\tilde{z}) = U'(m_1^*) + m_1^* h\tilde{z} U''(m_1^*) +$$

$$+ (m_1^* h\tilde{z})^2 U'''(m_1^*)/2 + O[(m_1^* h\tilde{z})^3]. \hfill (4.13)$$

Multiplying both sides by $(1+h\tilde{z})$, taking the expected value and differentiating with respect to $t$ one may obtain as an approximation

$$\frac{\partial}{\partial t} E[U'(1+h\tilde{z})] = 2m_1^* h\tilde{z} U''(m_1^*) + m_1^* h\tilde{z} U'''(m_1^*).$$

Performing similar calculations with respect to the second term of (4.12) we may finally obtain

---

*Note that the third absolute central moment vanishes due to our assumption of symmetry of $\tilde{z}$.\"
\[ \psi = 2p_1 m_1^* hh' \sigma^2 U'' (m_1^*) + p_1 m_1^* h'h' \sigma^2 U''' (m_1^*) + \]

\[ + 2p_2 v'h'h' \sigma^2 U'' (v) + p_2 v^2 h'h' \sigma^2 U''' (v). \]  

(4.14)

As noted before, the sign of \( \psi \) is indeterminate but one may see through (4.14) that, as also previously observed, the probability of a perverse consensus effect is higher the higher, other things equal, the absolute risk aversion at \( m_1^* \), as approximated by \(-U''(m_1^*)\), and the lower the absolute risk aversion at \( m_2^* = v \), as approximated by \(-U''(v)\).

One final point should be noted before concluding the analysis of the consensus mechanism. Proposition 2 of Chapter 2 suggested that in the "long-run," i.e., as the equiprobability situation is approached, the implicit isomorphism of the model there developed assured per se final convergence of separate individual choices to unique values. Again, the nature of the present model is such that under full uncertainty both individuals face the same probability prospects, a similar "constitutional" utility function and an identical net income opportunity set. Hence, their choices by "insufficient reason" cannot but be the same. However, and this is the aspect that was emphasized in the above analysis, this asymptotic property of the model does not preclude the possibility of having, over some intermediate ranges of uncertainty variation, diverging individual choices about optimal-tax distributions. That is to say, a possible local property of the model, has been identified.

At the same time, the potential relevance of such conceptual possibility for the determination of optimal fiscal decision processes
in practice has, of course, to be appraised in terms of the plausible
range of values of the variables and functional relationships in the
model, particularly the ones included in the function \( \psi \). To illus-
trate, it can be seen from (4.14) that if:

a) Risk sensitivity as approximated by \(-U''\) does not signifi-
cantly vary along the income scale (which implies \( U''' = 0 \)),
b) Information costs rise rapidly as the vaccatio legis increases
so that only a limited initial range of variation in \( t \) is
admissible\(^5\) or, equivalently, \( p_2 \) is substantially greater
than \( p_1 \),
c) Incentive effects are negligible over the relevant range so
that \( \nu = -1 \).

then the perverse case of the consensus effect is very unlikely to
occur.

4.4 The Aspirations-level Effect

Turn now to the aspirations-level effect. Under the present assump-
tions, the effect of a marginal increase in the vaccatio legis on the
level of A's expected utility \( R \) as described in (4.6) is given by

\[
\frac{dR(m^*_1, t)}{dt} = p_1E[U(m_1^*]) + p_1E[U'(m_1^*) [m_1^*h'z + \frac{dm^*_1}{dt} (1+hz)] +
\]

\[
p_2E[U(m_2^*]) + p_2E[U'(m_2^*) [vh'z + \frac{dm^*_2}{dt} (1+hz)v']].
\]

\(^5\) See Chapter 1.
Rearranging and noticing that two terms cancel out since equilibrium condition (4.7) is assumed to be continuously satisfied,

\[ \frac{dR(m_1^*, t)}{dt} = p_1'E[U(m_1^*)] + p_2'E[U(m_2')] + p_1E[U'(\bar{m}_1)m_1^*h'z] + p_2E[U'(\bar{m}_2)vh'z]. \] (4.16)

The first and second terms of (4.16) are analogous to those that were found in the model of Chapter 2 and, as before, their sum is negative up to the egalitarian distribution. The last two terms on the other hand have no counterpart in our earlier formulation. They essentially reflect the loss in expected utility with respect to both positions due to the increased variability of the size of the stake as \( t \) expands and both of them can be seen to be negative.\(^6\) This additional reduction in \( R \), as \( t \) increases, is a factor which clearly reinforces the aspirations-level effect. At the same time, since the last two terms do not vanish at the egalitarian distribution one can infer that an interior minimum of \( R \), i.e., in the range \( 0 < t < t^{**} \), if it exists, can only occur at \( t > t_e \) where \( t_e \) denotes the extent of the \textit{vaccatio legis} which induces the individual to select the egalitarian distribution as his optimal choice.

To conclude, the discussion of the case of uncertain net-income may be summarized by noting the following main implications of such assumption for the model: First, with respect to the consensus

\(^6\)See, for a similar proof, footnote 2 this chapter.
mechanism it was observed that a possibility arises, theoretically at least, of having the effect working in the wrong direction, i.e., tending to aggravate the divergence among separate individual optimal choices about tax-sharing arrangements. This possibility, however, may occur as a local phenomenon at most, i.e., only over limited ranges of uncertainty variation. Asymptotically, the consensual properties of the model are not ipso facto affected. Second, with respect to the working of the aspirations-level effect it was observed that this constitutional mechanism is comparatively reinforced due to the fact that the variability of the stake at negotiation essentially introduces an additional reason for discounting the expected utility associated with the most-preferred taxation schemes. It can be shown, however, that the way uncertainty affects constitutional expectations is important in this respect.\footnote{The reinforcement of the aspirations-level effect is dependent on the assumption of a "pure" uncertainty effect. The Appendix discusses the case of a non-mean preserving shift in \( \tilde{z} \), more specifically the case where there is overestimation of the post-constitutional opportunity set. As one would expect, it can be seen that the direction of the aspirations-level effect becomes ambiguous in this case.}
Appendix to Chapter Four

This Appendix discusses the case of a non-mean preserving shift in \( \tilde{z} \), where there is overestimation of the post-constitutional opportunity set. Consider, therefore, the case where

\[
\tilde{m}_1 = m_1 [1 + h(t)] 
\]

\[
\tilde{m}_2 = v(m_1) [1 + h(t)]
\]

(4.2')

(4.3')

where \( \tilde{u} \) is a non-negative random variable with expected value equal to unity and finite variance \( \sigma^2_u \) and \( h(t) \) is a non-negative function such that \( h' > 0 \). Thus,

\[
E(\tilde{m}_1) = m_1 (1 + h) \quad E(\tilde{m}_2) = v(m_1)(1 + h)
\]

(4.4')

\[
\text{Var}(\tilde{m}_1) = m_1^2 (1 + h)^2 \sigma^2_u \quad \text{Var}(\tilde{m}_2) = v^2 (1 + h)^2 \sigma^2_u
\]

(4.5')

i.e., a non mean preserving shift in the distribution of \( u \). Equilibrium condition (4.6) is, in the present case, given by

\[
p_1 E[U' \tilde{u}] + p_2 E[U' \tilde{uv}] = 0.
\]

(4.6')

Differentiating totally,

\[
\frac{dm_1^*}{dt} = \frac{\phi(t, m_1^*) + \psi(t, m_1^*)}{p_1 E[U'' (1 + h) \tilde{u}^2] + p_2 E[U'' (v')^2 (1 + h) \tilde{u}^2 + U' \tilde{uv}']}
\]

(4.7')
where

\[ \phi (t, m_1^*) = p_1 E[U'(\tilde{u})] + p_2 E[U'(\tilde{v})] > 0 \] (4.8')

\[ \psi(t, m_1^*) = p_1 E[U'' m_1^* h' u^2] + p_2 E[U'' v'h' u^2 v'] < 0 . \] (4.9')

The denominator of (4.7') is clearly negative. In the numerator \( \phi \) is positive and \( \psi \) is, as in the case in the text, of ambiguous sign.

The role of the degree of risk aversion at \( \tilde{m}_1 \) and \( \tilde{m}_2 \) (as approximated by \( -U'' \)) in influencing the sign of \( \psi \) is, however, made more clear in the present formulation. With respect to the aspirations-level effect one has in this case, and after simplifications,

\[ \frac{dR(m_1^*, t)}{dt} = p_1 E[U'(\tilde{m}_1)] + p_2 E[U'(\tilde{m}_2)] + p_1 E[U' m_1^* h' \tilde{u}] + \]

\[ + p_2 E[U' v'h' \tilde{u}] . \] (4.10')

The first two terms can be seen to be negative up to the egalitarian distribution, as happened in the case in the text. The two remaining terms, however, are in the present case and in contrast to the formulation in the text, both positive since they are (weighted) expected values of positive random variables. Thus, due to the systematic overestimation of the post-constitutional net-income opportunity set as implied by (4.4') the direction of the aspirations-level effect becomes ambiguous. This is as one would expect since such biased expectations compensate to some extent, and may possibly dominate, the discounting of expected utility that on the other hand occurs in consequence of the uncertainty with respect to the occupation of social positions.
5.1 Introduction

The investigation has so far concentrated on the three basic constitutional mechanisms described under Propositions 1, 2 and 4 of Chapter 2. In this chapter, however, another typical feature of the constitutional fiscal model has also been discussed at a general level. This is the inherent tendency to generate, under the stated assumptions, solutions which reflect a compromise between efficiency and equity considerations. This was the substance of Proposition 3. In the present chapter an attempt is made to extend this discussion. In order to obtain more specific implications of such proposition for the fiscal institutions that emerge constitutionally, a somewhat different specification of the model previously developed will be used.

Since at the present moment the interest is on the asymptotic properties of the formulation, it is useful to focus on the equiprobability situation although the potential relevance of the argument seems not to be confined to such a limiting situation. At the same time, since an attempt is made to bring explicitly into the analytic framework a post-constitutional voting rule, it is convenient to develop an n-person model rather than a simple two-person formulation. A Harsanyi type equiprobability situation may be considered where each of n individuals at a constitutional stage possesses full information.
about the set of demand curves for a given public good that will prevail at a representative post-constitutional period and admits as equally likely his own attachment to any one of the possible schedules. The marginal cost schedule of the public good is also known, and for convenience, it is assumed to be constant over quantities of the public good. It is further assumed that simple majority voting is the decision rule that will prevail at the post-constitutional stage with respect to the amount of the public good to be provided.

Assume also that individual tax-prices per unit of the public good must be uniform over quantities. No complementary redistributional measures are allowed, i.e., the only distributional activity is the one embedded in the vector of tax-prices to be chosen constitutionally.

The central problem faced by the representative individual at the constitutional stage is, therefore, to assign optimally the vector of \( n \) tax-prices to the set of distinct demand schedules. What tax structure will tend to emerge under the above circumstances? What is the preferred structure of tax-prices for the representative individual?

The above description of the problem at hand immediately suggests some basic similarity with the question addressed in the recent work developed by Professors Brennan and Buchanan. Their approach like that of this paper is, in fact, essentially concerned with the derivation

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of characteristics of the fiscal structure that tend to emerge opti-
mally when the distinction between the constitutional and post-
constitutional stages of choice is brought into consideration. The
major difference between the two formulations lies perhaps in the
characterization of the post-constitutional setting. While Brennan
and Buchanan's model considers a post-constitutional stage where bud-
getary choices are dominated by a Leviathan-type bureaucrat, the
formulation here assumes such choices to be obtained under simple
majority voting. In a sense, therefore, and as it will soon become
apparent, the median voter of the model described here represents the
analytical analogue of Brennan and Buchanan's bureaucrat. Also, it
may be noted that the model more closely resembles a game against
nature where the equiprobability calculus assumes an explicit role.
while the Brennan-Buchanan's formulation on the other hand suggests
a game against a well-defined set of interests of the opposing player,
namely the preferences of the bureaucrat.

The basic result of the analysis essentially represents an appli-
cation of the trade-off between efficiency and equity described earlier
in Proposition 3. More specifically it suggests that, under the
assumption of risk aversion the preferred tax-sharing arrangement will
reflect some compromise between two separate normative principles,
one of which calls for the maximization of aggregate taxpayers surplus
and the other of which fosters equalization of separate individual
surpluses. An immediate implication of the analysis is that the ful-
fillment of the Samuelson's well-known efficiency conditions will
emerge constitutionally only in the special case of risk-neutrality.

Finally, the possible existence of some constitutional concern for individual post-constitutional disequilibrium will be considered. The main implications of this possibility for the optimal tax structure are examined.

5.2 The Model

Consider a set of n individuals who face the constitutional choice of a tax-sharing arrangement. Assume that in the post-constitutional stage there are n distinct downward sloping demand curves for a given public good, to be allocated among the n individuals (one to each). We assume these individuals to have identical initial private (i.e., before introducing the public sector) incomes. Let \( D_i(G), i = 1, \ldots, n \) describe the demand curve in inverse form of the \( i^{\text{th}} \) individual, where \( G\) indicates the amount of the public good. Define \( r = (r_1, \ldots, r_n) \) as a vector of tax-prices per unit of the public good where \( r_i \) is the tax-price assigned to the demand curve \( D_i \). For simplicity, income effects are neglected throughout the analysis. Simple majority voting will be used at the post-constitutional stage to select the quantity of the public good. Let

\[
G(t) = G [D_1^{-1}(r_1), \ldots, D_n^{-1}(r_n)] \tag{5.1}
\]

indicate the amount selected under this rule for a given vector \( r \) of tax-prices. \( \bar{G} \) will represent, therefore, the choice of the median voter. Define
\[ S_i(t) = \int_0^G D_i(G) \, dG - r_i G \quad i = 1, \ldots, n \]  \hspace{1cm} (5.2)

as the surplus of the \(i^{th}\) taxpayer at the post-constitutional stage if \(r\) is the tax structure. Also let

\[ X_i(t) = - [D_i(G) - r_i]^2 \quad i = 1, \ldots, n \]  \hspace{1cm} (5.3)

indicate, for individual \(i\), a measure of the deviation at the margin between demand-price and tax-price, given \(r\).

Finally, denote by \(\alpha\) the marginal cost of providing the public good which is assumed constant.

The problem faced by the representative individual at the constitutional stage, who considers himself equally likely to possess each of the \(n\) demand curves for the public good, will therefore be to select a tax-vector \(r\) so as to maximize

\[ U(r) = \sum_{i=1}^{n} U(S_i, X_i). \]  \hspace{1cm} (5.4)

subject to a constraint of budget balance. Expression (5.4) indicates that the individual objective function is an equally weighted sum of equiprobable events. \(U\) is a "constitutional" evaluator, common to all individuals, which depends on two basic factors: The first is the surplus accruing to a given position and the justification for its inclusion seems intuitive. The other reflects the existence of some constitutional concern for the extent of deviations between individual demand-prices and tax-prices. The reason is that the magnitude of
individual disequilibria implied by such discrepancies essentially affect the extent of disagreement on the quantity of the public good to be provided at the post-constitutional stage. The avoidance of the social tensions potentially created by such disagreement might emerge, therefore, in some circumstances as a plausible constitutional objective. It is thus assumed that $U_2 > 0$ and also that $U_{22} < 0$. Of course $U_1 > 0$, and it is provisionally supposed that $U_{11} < 0$, so that risk aversion with respect to the distribution of the taxpayer's surpluses is assumed. It is further assumed that $U$ is separable, so that $U_{12} = U_{21} = 0$.

In Lagrangean form, the individual optimization problem may therefore be expressed as

$$\text{Max} \{ L(t) = U(t) + \lambda [\alpha - \sum_{i=1}^{n} r_i] \}. \quad (5.5)$$

For an interior solution, the first-order necessary conditions may be expressed, after calculations, in the following way:

$$L_j = \frac{\partial C}{\partial r_j} \left[ \sum_{i=1}^{n} U_1 S_i (D_i - r_i) - 2 \sum_{i=1}^{n} U_2 X_i \frac{dD_i}{dG} (D_i - r_i) \right] - U_1 S_j \bar{G} + 2U_2 X_j (D_j - r_j) - \lambda = 0 \quad j = 1, \ldots, n \quad (5.6)$$


3 The second-order conditions for a maximum are satisfied since $U(t)$ is concave.
\[ L_\lambda = \alpha - \sum_{i=1}^{n} r_i = 0. \quad (5.7) \]

The interpretation of this expression will be clarified as soon as some particular cases of it which are of most interest are examined.

Consider, first, the case where there is no constitutional concern for individual post-constitutional disequilibrium. In this case \( U_2 = 0 \) and (5.6) reduces to

\[ L_j = \frac{\partial G}{\partial r_j} \sum_{i=1}^{n} U_i (D_i - r_i) - U_j \left[ G - \lambda \right] = 0, \quad j = 1, \ldots, n. \quad (5.8) \]

The interpretation of (5.8) may be readily obtained in the following way. Take first the case of those individuals who, at the optimal \( r \), do not exert marginal influence on the level of \( G \), i.e., the non-median voters. Since we are assuming away corner solutions, for these individuals (5.8) will reduce to

\[ -U_j \left|_{S_k} \right. \bar{G} = \lambda. \quad (5.9) \]

Since \( U_j \) is monotonic, this implies that full equalization of surpluses will occur among the non-median voters at the optimal \( r \). Take now the case of an individual, say \( m \), who at the optimal \( r \) is able at the margin to influence the level of \( G \), i.e., the median voter. Here, a change in \( r_m \) will not only alter \( m \)'s surplus for a given \( G \) but will also change the surpluses of all other individuals via the change in \( G \) induced by \( \partial r_m \). This effect is given by the term
in (5.8) which represents the marginal change in aggregate weighted surplus. At equilibrium, this "external" effect exerted by m has therefore to be balanced against the "private" effect indicated in (5.9). One may thus note, by inspecting (5.8) that if the marginal weighted aggregate surplus \( \sum_{i=1}^{n} u_{i} |s_{i}| (D_{i} - r_{i}) \) is, say, negative, the median voter will have, at the optimal taxation scheme, a smaller surplus than the ones of the non-median voters. This implies, of course, that the tax-price paid by the median voter will be higher than the one that would assure full equalization of surpluses across all the voters. That is, the median voter suffers in this case a private loss, in relative terms, in order to be induced to choose a "socially" more convenient level of output of the public good. The converse result will emerge, of course, if marginal aggregate surplus is positive at equilibrium and if it is zero full equalization of surpluses across individuals will occur.

Consider now the case where risk neutrality prevails at the constitutional stage. \( U_2 \) is still assumed to be zero but now

\[ U_1 = \mu = \text{constant}. \]  

Therefore, (5.8) reduces to

\[ \frac{\partial G}{\partial r_m} = \sum_{i=1}^{n} u_i |s_i| (D_i - r_i) \]  

(5.10)

---

4Both terms "external" and "private" are used here in a very qualified sense since even if \( \frac{\partial G}{\partial r_i} = 0 \) a change in \( r_i \) will affect other individuals through the budget constraint (5.7).
For the non-median voter, thus

\[ -\mu \bar{G} = \lambda \]  

(5.13)

which now does not require, of course, equalization of their surpluses.

Similarly, using (5.13) in (5.12), for the median voter

\[ \frac{\partial \bar{G}}{\partial r_j} \mu \sum_{i=1}^{n} (D_i - r_i) - \mu \bar{G} - \lambda = 0, \quad j = 1, \ldots, n. \]  

(5.12)

\[ \frac{\partial \bar{G}}{\partial r_m} \mu \sum_{i=1}^{n} (D_i - r_i) = 0 \]  

(5.14)

which implies \( \Sigma (D_i - r_i) = 0 \). Thus, the familiar Samuelsonian efficiency conditions will emerge as a constitutional result only in the special case where risk neutrality is the typical attitude prevailing at that stage of choice. Only in that case, where individual taxpayer's surpluses are fully substitutable at the margin, will the optimal choice focus on the total aggregate surplus and not at all on its division among separate taxpayers. In other words, only in this special case does only "efficiency" matter.

The above points may be clarified with a diagrammatical illustration. In Figure 5.1 there are three individuals and three equiprobable

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Figure 5.1

The Constitutional Choice of Tax Structure
demand curves for public good $G$, $D_A$, $D_B$, and $D_C$. Let $r_1 = (r_{A1}, r_{B1}, r_{C1})$ indicate the vector of tax-prices that will generate, under majority voting, a level of output $G_1$ such that taxpayers surpluses are equalized, i.e., $(Pr_{A1}RP-RUT) = Mr_{B1}LM = Fr_{C1}IHF$. Denote by $\hat{G}$ the output of the public good for which Samuelson’s efficiency conditions hold ($ED_1 = MC$). Assume arbitrarily that $\hat{G} < G_1$. Now, it is clear that at $G_1$ the marginal aggregate weighted surplus $\sum_{i}^{\infty} U_i | S_i$ $(D_i - r_1)$ $(i = A, B, C)$ is negative. This means, by (5.8), that the surplus of the median, $S_B$ has to be reduced ($r_{B1}$ increased). As this occurs and the quantity of $G$ is reduced, the (equalized) surpluses of the non-median voters increase. The tax vector $r_2 = (r_{A2}, r_{B2}, r_{C2})$ describes the optimal scheme in this example. The quantity of the public good chosen under $r_2$ will be $G^*$ and the corresponding surpluses will be related in the following way $Mr_{B2}NM < Pr_{A2}QVP = Fr_{C2}JKF$. At the same time, it may be noted that $G^* > \hat{G}$, i.e., the quantity of the public good chosen at the post-constitutional stage under the optimal tax-arrangement will be located somewhere between the level where full equalization of surpluses occur, $G_1$, and the level at

7 Note that $U_i | S_i$ is equal to all individuals at $G_1$ due to full equalization of surpluses.
which unweighted aggregate surplus is maximized, $\hat{c}$. This suggests, therefore, an application of the trade-off between equity and efficiency that was previously noted in Proposition 3.

Consider now the case where there exists some constitutional concern for individual post-constitutional disequilibrium, i.e., $U_2 > 0$. Assume first the case of risk neutrality. In this case (5.6) reduces to

$$L_j = \frac{\partial \bar{G}}{\partial r_j} \mu \sum_{i=1}^{n} (D_i - r_i) - 2 \sum_{i=1}^{n} U_2 \frac{\partial \bar{G}}{\partial G} (D_i - r_i) + 2U_2 \frac{\partial \bar{G}}{\partial G} \sum_{j=1}^{G} (D_j - r_j) - \mu \bar{G} - \lambda = 0, j = 1, \ldots, n \tag{5.15}$$

For the non-median voters we thus have

$$2U_2 \frac{\partial \bar{G}}{\partial G} \sum_{k=1}^{G} (D_k - r_k) - \mu \bar{G} - \lambda = 0 \tag{5.16}$$

---

8This may be proved in the following way: according to (5.9) at equilibrium, for the non-median voters

$$-\beta \bar{G} - \lambda = 0$$

where constant $\beta$ represents the common value of $U_1|_{S_k}$ for all non-median voters. At the same time, from the above discussion the surplus of the median is smaller than the ones of the non-mediants which implies that for the former

$$-U_1|_{S_m} \bar{G} - \lambda < 0.$$ 

Now, since the median should be in individual equilibrium at the margin $U_1|_{S_m} (D_m - r_m) = 0$ and therefore

$$\sum_{i=1}^{n} U_1|_{S_i} (D_i - r_i) = \beta \sum_{i \neq m} (D_i - r_i).$$

Thus if $\bar{G} = \hat{G}$ one should have $\beta \sum_{i \neq m} (D_i - r_i) = 0$ which implies that (5.8) will not be satisfied for the median voter. The above reasoning holds a fortiori for the case of $\bar{G} < \hat{G}$ (note that $\partial \bar{G}/\partial r_m < 0$).
which implies\(^9\)

\[ D_k - r_k = \text{constant.} \quad (5.17) \]

At the same time, since the median voter has to be in individual equilibrium at the margin \((D_m = r_m)\), in his case,\(^{10}\) from \((5.15)\)

\[ \frac{\partial G}{\partial r_m} [\cdot] - \mu G - \lambda = 0. \quad (5.18) \]

It may be shown\(^{11}\) that the bracketed expression has to be zero. By inspecting \((5.15)\) one may readily note that this in turn implies

\[ D_i - r_i = 0 \quad i = 1, \ldots, n \]

i.e., each taxpayer will be in full private equilibrium at the margin. A Lindahl type fiscal arrangement will, thus, tend to emerge in this case as the optimal constitutional choice.

\(^9\)Note that \( U_2 |_{X_k} (D_k - r_k) \) is monotonic increasing on \((D_k - r_k)\).

In fact, \( \frac{\partial [U_2 |_{X_k} (D_k - r_k)]}{\partial (D_k - r_k)} = -2(D_k - r_k)^2 U_{22} |_{X_k} > 0. \)

\(^{10}\)We may observe that the median voter exerts now a second kind of "external" effect in that by altering \( G \) he modifies the marginal deviations of all taxpayers. The valuation of this effect is given by the second term in brackets.

\(^{11}\)See Appendix.
If one reintroduces now the assumption of risk aversion the main modifications with respect to the Lindahl scheme are the following: First, tax-prices among the non-median voters are relatively raised for those individuals with higher surpluses and lowered for those with smaller surpluses. In fact, (5.16) is replaced by

\[ 2U_2 \left| \frac{D}{S_2} - r_k \right| - U_1 \left| \frac{G}{S_k} - \lambda \right| = 0 \]  

(5.19)

which indicates that higher surpluses at the Lindahl solution will imply now a greater (algebraic) value of the second term thus requiring a higher \( r_k \) for equilibrium. The converse applies, of course, to the case of lower surpluses. Here taxpayers will experience a relative reduction in tax-prices to compensate them, so to speak, for the lower level of their surpluses. Hence, a noticeable difference arises between the present taxation scheme and the one that results when there is no constitutional concern for individual disequilibrium \( (U_2 = 0) \), in that the latter requires equalization of surpluses across non-median voters while the former does not.

Second, the "external" effect exerted by the median voter on the taxpayer's surpluses accounts now for distributional considerations. This entails that the tax-price of the median voter will be selected so that his choice with respect to \( G \) will depart now from the "efficient" level implied by the Lindahl scheme towards a level which generates more equity.
5.3 Two Assumptions Reconsidered

Two of the assumptions of the above model may be noted, namely the prohibition of post-constitutional lump-sum transfers and the requirement of uniform tax-prices. These two assumptions seem to be crucial for some of the results obtained. In fact, if any one of them is relaxed the conflict between equity and efficiency might completely disappear. A marginal tax-price vector may be chosen such that the "efficient" output of the public good will be selected by the median voter and appropriate choice of the infra-marginal tax schedules, combined or not with lump-sum transfers, may then produce the equalization of surpluses that is required, in the limit, by the constitutional perspective.

The prohibition of supplementary lump-sum transfers at the post-constitutional stage may be rationalized by the existence of some constitutional concern about the political feasibility of implementing the constitution in the future. In fact, direct redistributional activities undertaken at the post-constitutional stage may involve greater political resistance than the ones embedded in a scheme of uniform tax-prices and, to that extent, "indirect" redistribution might be the only feasible solution available at the constitutional stage.


At the same time, the requirement of uniform tax-pricing may also be rationalized, to some extent, without the need to invoke the existence of predicted political costs of constitutional implementation. Such rationalization becomes indeed possible as soon as the previous assumption that the shape and location of the (equiprobable) demand curves for the public good are known with certainty at the constitutional stage is relaxed. To see this point assume that \( G^* \), the quantity of the public good chosen by the median voter, is now, due to predicted fluctuations of the demand curves (more specifically, the one of the median voter), a random variable. For heuristic convenience take the hypothetical case where full qualization of surpluses may be attained simultaneously with Samuelson's efficiency conditions at the expected level of \( G^* \) under a uniform tax-pricing scheme. Consider now in Figure 5.2 the situation of two non-median voters with identical demand curves (not depicted) for whom two alternative tax-pricing schedules are open to choice. One is the uniform schedule represented by the horizontal line \( rr \). Alternatively, lines \( r'r' \) and \( r''r'' \) indicate (at the margin) two non-uniform tax-schedules which vertically add to \( rr \) and, if allocated one to each taxpayer, will generate the same expected cost for both. Now, would the representative constitutional framer be indifferent between these two alternative tax schemes? The answer is no if risk aversion is the typical constitutional attitude. In this case the representative individual will favor the adoption of a uniform tax structure since this is the scheme that minimizes the variance of cost-shares around
Figure 5.2

Choice of Alternative Tax-Price Schedules
the optimal value. In the figure, if \( G^* \) randomly varies, say, over the range \( G - \varepsilon, G + \varepsilon \) it may be easily shown that, other things equal, the uniform tax-pricing rule \( \tau_r \) corresponds to the optimal tax-sharing arrangement.\(^{14}\)

The above considerations reflect, of course, a partial equilibrium view. Since there is no conflict, to begin with, between efficiency and equity nothing can in fact be gained by relaxing the constraint of uniform tax-pricing and, something is indeed lost. A more general analysis in which this constraint is active should also take into account the potential effect of moving to a non-uniform tax-pricing arrangement both on the level of \( G^* \) and on the distribution of surpluses across taxpayers.

The above illustration, however, was able to suggest the major point to be noted, namely that a further margin of adjustment with respect to risk may exist when the quantity of the public good post-constitutionally provided is a random variable. To that extent there is a built-in tendency of the tax structure to approximate the

\[^{14}\text{The representative individual will attempt to select the tax-cost functions } C_1(G^*) \text{ and } C_2(G^*) \text{ so as to maximize}
\[
\int_{G-\varepsilon}^{G+\varepsilon} f(G^*) \{ U [-C_1(G^*)] + U [-C_2(G^*)] \} dG^* + \lambda [\Theta - C_1(G^*) - C_2(G^*)]
\]

where \( f \) is the density function of \( G^* \), \( \Theta \) is the fraction of total marginal cost \( \alpha \) to be divided between the two taxpayers and \( U \) is a risk averse utility function. From the first-order conditions we easily get

\[
\int -fU' \mid _{-C_1} dG^* = \int -fU' \mid _{-C_2} dG^*
\]

which obviously requires \( C_1 = C_2 \), i.e., the uniform schedule.
constant-share scheme and, therefore, one certainly will expect that some "efficiency" will be optimally sacrificed at the margin even when the constraint of uniform tax-pricing is abandoned.
Appendix to Chapter Five

Proof that

\[ \mu \sum_{i=1}^{n} (D_i - r_i) - 2 \sum_{i=1}^{n} U_{x_i} \frac{\partial D_i}{\partial r_i} (D_i - r_i) = 0. \]

From (5.16), for the non-median voter

\[ 2U_2 |\frac{D_k - r_k}{x_k} - \mu \bar{G} - \lambda = 0 \quad (5.16) \]

which, as we have seen, implies

\[ D_k - r_k = c \text{ (constant)}. \quad (5.17) \]

Suppose arbitrarily that \( c < 0 \). This implies

\[ \mu \bar{G} + \lambda < 0 \quad (5.18a) \]

since \( U_2 > 0 \). Now, using (5.18a) in (5.18),

\[ \frac{\partial \bar{G}}{\partial r_m} [\cdot] = \mu \bar{G} + \lambda < 0. \quad (5.18b) \]

But, since \( D_m - r_m = 0 \) and \( c < 0 \) the bracketed expression has to be negative (recall that \( \mu > 0 \) and \( \frac{\partial D_i}{\partial \bar{G}} < 0 \)) which implies

\[ \frac{\partial \bar{G}}{\partial r_m} [\cdot] > 0 \]
thus contradicting (18b). The same contradiction would arise if one had assumed $c > 0$. Therefore, the bracketed expression has to be equal to zero in these cases as well as, of course, in the case where $c = 0$. 
Chapter Six

SUMMARY

The central theme of the analysis developed in the present study has been the identification of some logical properties that are expected to characterize the "separation-of-decisions" paradigm of the fiscal decision process. The latter refers to an institutional framework where collective choices of tax-sharing arrangements are assumed to be made in advance with respect to public expenditure decisions. A model was developed where the effects of this time lag introduced between the two sets of decisions—the vaccatio legis—were analyzed.

Two broad sets of results were derived. The first refers to the non-asymptotic properties of the model, that is, the ones that are expected to hold when the vaccatio legis increases up to the limiting situation where equiprobability is attained. Two basic mechanisms which potentially operate when the vaccatio legis increases were identified. One, the Consensus Effect, tends to make separate individual most-preferred choices with respect to tax structures more similar to each other. The other, the Aspirations-level Effect, reduces the expected utility associated with each individual's most-preferred choice. It was seen that these mechanisms, when they are simultaneously at work, reinforce the effect of each other in reducing the amount of resources devoted to strategic activities and therefore in saving negotiation costs. Situations were examined, however, where either one or
both of the mechanisms may not operate or may even operate in the "wrong" direction. In particular, the consensus effect is not expected to work for some initial range of variation in the *vaccatio legis*. The extent of such range was seen to be possibly dependent on the size of the budget. Also, when risk preference exists at the constitutional stage, this mechanism may again not operate. Finally, uncertainty with respect to net income opportunities may provide circumstances where the consensus effect works "perversely," i.e., where individual choices diverge rather than converge when the *vaccatio legis* increases.

There is no *a priori* reason, however, for the aspirations-level mechanism not to work under the above circumstances so that the logic of the "separation-of-decisions" model is not necessarily affected by such possibilities. On the other hand, the converse situation may occur and cases were examined where the consensus effect may work nicely while the aspirations-level mechanism operates in the "wrong" direction. This may happen, for example, if there is overestimation at the constitutional stage of the post-constitutional net income opportunities. If nothing else, what the above results seem to suggest is that the identification and logical separation of the two basic constitutional mechanisms is not a tautological conceptual exercise.

The second set of propositions analyzed deals with asymptotic properties of the "separation-of-decisions" formulation. It was pointed out the basic isomorphism of the model, that is, the fact that, since under Laplace's "insufficient reason" individuals possess essentially
the same decision ingredients (similar probability prospects, unique utility evaluator, identical net income opportunities) their choices cannot but be the same. The importance of this proposition comes perhaps in an indirect way. It shows that possible violations of some non-asymptotic properties, namely the consensus mechanism, can be at most a local phenomenon. In the "long-run," full consensus is inescapable.

A second asymptotic property discussed refers to the trade-off between equity and efficiency that is expected to be reflected in tax schemes selected under such limiting circumstances. It was shown that if tax-prices are uniform over quantities, if no complementary redistributitional measures are allowed and if risk aversion prevails then there is no presumption that constitutionally preferred tax arrangements will fulfill Samuelson's well-known efficiency conditions. The analysis suggested that, in general, such arrangements will reflect a compromise between two separate normative principles, one of which calls for the maximization of aggregate taxpayers surplus and the other of which promotes equalization of separate individual surpluses.
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THE CONSTITUTIONAL APPROACH TO THE FISCAL PROCESS:

AN INQUIRY INTO SOME LOGICAL FOUNDATIONS

by

A. S, Pinto Barbosa

(ABSTRACT)

The central theme of the analysis developed in the study is the identification of some logical properties that are expected to characterize the "separation-of-decisions" paradigm of the fiscal decision process. The latter refers to an institutional framework where collective choices of tax-sharing arrangements are assumed to be made in advance with respect to public expenditure decisions. A model is developed where the effects of this time lag introduced between the two sets of decision—the vaccatio legis—are analyzed.

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