Adaptive Optimal Control of AC/DC Systems

by

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(ABSTRACT)

The dissertation presents a new control strategy for two terminal HVDC systems embedded in an AC network. The control is based upon real-time measurements performed on the AC/DC system. Use is made of a technique for high speed accurate measurement of positive sequence voltages and currents, first developed in the field of computer relaying. The real-time measurements provides a term in the control law to compensate for inaccuracies following departure from the operating point. The control criterion is to damp out the electromechanical oscillations following a disturbance.

The main contribution of the dissertation is to describe a new optimal controller formulation which contains a measurement based component. Optimal controllers are commonly constructed using linearized equations of the system around the operating point. In DC systems this approach is of a very limited value because of a highly nonlinear nature of the system. With the controller developed in this dissertation, it becomes possible to describe the system as a nonlinear dy-
namic system. The approximation resulting from the usual linearization of the system equations is thus avoided, and leads to a better controller design.

The control technique is illustrated with a small AC/DC system. However, the equations formulated are sufficiently general, so that the technique can be applied to a larger system. Simulation results are included to represent the effectiveness of the developed controller.
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CHAPTER 1. INTRODUCTION

Advances in the field of control theory suggest better techniques for control of a multivariable dynamic system. Modern control theory incorporates the time domain differential equations for system representation, and eliminates the guess work involved in classical control theory for computation of the feedback gains in order to stabilize a dynamic system [1]-[3]. Optimal control theory is a branch of the modern control theory which provides the best control strategy for a selected performance criterion [4]-[9].

Control of HVDC converter, including the optimal control approach, for improvement of the system stability was the subject of previous research as documented in literature [10]-[24]. Also, extensive research was done in optimal control of synchronous generator excitation system to stabilize the machine following a disturbance [25]-[29]. However, the simultaneous modulation of both AC and DC controls for stability improvement was lacking in technical publications reviewed. In addition, almost all of the previous work used a simplified
system model, a parallel AC/DC transmission line connecting a generator to an infinite bus. In many studies Taylor series expansion was used to linearize the system model about the operating point, and higher order terms were ignored.

The objective of this research is to develop an adaptive optimal control strategy, as a hierarchical control scheme, for stability enhancement of the AC power system with an embedded DC link. The main goal is formulation of an algorithm which is sufficiently general to be applicable to any system with a two terminal DC transmission system. Proper system modeling in addition to selection of an adequate performance measure are the requirements for determination of the solution for the problem. Achievement of a closed loop control law which would not contradict the physical system constraints is required. Real-time measurements are used to provide a measurement based optimal control law.

The proposed scope of the research is to consider coordination between the controls of generator excitation system, HVDC rectifier, and HVDC inverter for formulation of the optimal control law. Also in a realistic system the structure of the AC system is more complex than a parallel AC/DC system. The system modeling should be capable of handling an embedded DC link in an AC network rather than a parallel AC/DC link. Furthermore, linearization of the system model alone is not desirable due to large excursions which can occur as a result of the changing operating point of the system. It is intended to separate the system mathematical model into two parts and investigate a solution technique.
The separation of the system model into two parts is an important aspect of the system modeling. The original form of the system equations are nonlinear. Taylor series expansion is used to obtain the linearized form of the equations. Thus, the second and higher order terms are ignored. To include the effect of the ignored terms, the linearized equations are subtracted from the original nonlinear form of equations. This is the residual which is a function of the system states and inputs. However, from the value of the states and inputs the value of the residual is computed, and its future value is extrapolated with utilization of an estimation algorithm.

The classical method of approach for determination of the optimal control law is a well developed concept. The variational or iterative approach, depending on the nature of the system, is used to formulate the optimal control law for a system such that the physical system constraints are satisfied and also the performance measure is minimized. In order to be able to achieve the optimal control law, information regarding the system model, the performance measure, and physical constraints are necessary. The mathematical model in general can be linear, nonlinear, time varying, or time invariant. The performance measure in general has a functional form (i.e. a function of a function) [32]. The control signal applied during the control interval causes the system to follow some state trajectory, and the performance measure assigns a real number to this trajectory. The control sequence which yields the optimum (minimum) value of the performance measure is referred to as the optimal control.

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The optimal control law is represented as a function of system states and time. In cases where the optimal control law is independent of the value of the initial state, the optimal control law implies a rule which determines the optimal control at any time. Hence, the optimal control law is a closed loop feedback type of control. However, in cases where the optimal control law has a functional relationship dependent on the value of the initial state, then the optimal control law is only valid for the system with that specific initial state. Therefore, the optimal control law is an open loop type of control.

In derivation of the necessary condition for optimal control, the Hamiltonian is formulated. The Hamiltonian is expressed by forming a relationship between the scalar function of performance measure and the system state equation in conjunction with utilization of Lagrange multipliers. Then the necessary conditions for an unconstrained optimal control problem are derived by taking the partial derivatives of Hamiltonian with respect to the states, Lagrange multipliers, and controls.

Application of optimal control theory to a power system begins with system modeling. The mathematical model of a power system is generally nonlinear and in many cases it is linearized about an operating point. A quadratic performance measure of linear regulator, or a linear tracking type is commonly used. For these cases the optimal control laws are closed loop. However, the control law is only valid for the specific operating point or small excursions in vicinity of the operating point. Therefore, the control law will not be optimal as the system operat-
ing point changes drastically or the excursions become large. The alternative approach uses the nonlinear mathematical model of the system for finding the optimal control law. Iterative techniques such as steepest descent, variation of extremals, quasilinearization, and gradient projection are available to solve the nonlinear two point boundary value problem. These techniques will determine an open loop control which is optimal only for specified set of initial conditions. Unfortunately an open loop control is not acceptable for control of power systems, due to possibility of error in measurements of the initial conditions. The consequence of measurement error under such a control law would be poor system performance.

From the above discussion it is concluded that the linearization of the state equation alone is not desirable for achievement of an optimal control, and the iterative approach for the nonlinear system of equations would not yield a closed loop optimal control law. Also, the parallel AC/DC system representation is not truly a valid representation of the power system with an embedded two terminal DC system.

Thus contributions of this dissertation are in regard to elimination of these two shortcomings. This dissertation concentrates on the following two goals:

1. Design of an adaptive optimal control strategy with utilization of power system nonlinear equations such that the limitations of small signal models are removed.
2. Coordination of AC and DC power modulation with consideration for large scale power systems with an embedded DC system.

The nonlinear equations are linearized by Taylor series expansion which forms the first part of the system model. The second part is obtained by subtracting the linear part from the original nonlinear equations. This residual which will be expressed as a function of time is a measurement based term, and an estimation algorithm is used for extrapolation of the future values of this term. The residual provides a term in the optimal control law, forming an adaptive optimal control strategy. The linear part is used for solution of the Riccati equation to obtain the feedback gains. The remaining residual is a forcing function of a vector differential equation which is solved for the purpose of modifying the optimal control law provided by the linear portion of the state equation. The network sensitivity approach (generation shift factors) is considered for modeling of the DC power modulation in a large scale AC power system and control of the generator accelerating power.

Chapter 2 of this dissertation contains literature review. Chapter 3 includes the mathematical development and derivation of optimal control law for a linear system with quadratic performance measure. It also illustrates the mathematical development of the proposed control algorithm. Chapter 4 is devoted to testing of the proposed control algorithm, using a fourth order aircraft model. The large scale power system modeling for the proposed control algorithm is explained in chapter 5. Chapter 6 describes the development of a computer program used for
simulation of the power system performance and computation of numerical values of the states and inputs for measurement based optimal controller. A small AC/DC system is used in this chapter as an example for explanation of the developed technique. The example system simulation results are documented in chapter 7. The conclusions and recommendations for future work are summarized in chapter 8.
CHAPTER 2. LITERATURE REVIEW

2.1: Introduction to HVDC Transmission and Control

Electric lighting and power transmission began with direct current. The first central power station, built by Thomas Edison in New York, was a DC system and began operation in 1882. The invention of transformers, polyphase machines and circuits in late nineteenth century favored generation and transmission of AC power. Although the advantages of AC power transmission were well accepted, some superior aspects of DC power transmission were not forgotten. Certainly replacement of the AC system by the DC system was of no interest, but a few economical and technical limitations of AC power transmission forced engineers to supplement the AC system by DC links. DC transmission required rectification of AC to DC at the sending end and inversion of DC to AC at the receiving end. For HVDC transmission the conversion process was performed
by three phase bridges, known as Graetz bridges. The Graetz circuit initially consisted of mercury arc valves, but the advancements of semiconductor technology and the invention of high power silicon controlled rectifiers favored the use of these solid state devices for circuit elements. The connection of asynchronous power systems and the cheaper cost of DC power transmission over long distances were examples of technical and economical advantages of HVDC transmission, respectively [33].

The fundamental control principle of converters for DC link has been developed for classical HVDC power transmission [33,34]. For steady state operation the rectifier operates to maintain a constant current (CC) or constant power, while the inverter operates to maintain a constant extinction angle (CEA). In other words, the rectifier controls the DC current, while the inverter controls the DC voltage. Additional control characteristics were provided due to rectifier ignition angle and inverter extinction angle limitations. Consequently, a constant ignition angle (CIA) control was provided for the rectifier, and it was activated when the ignition angle tended to fall below the minimum value. Also, when the inverter current dropped below the rectifier current setting by a value greater than the current margin, its constant current (CC) control was activated to take control. The control characteristic is shown in figure 1. The rectifier and inverter regulators used the difference between the reference settings and locally measured values to drive the control circuitry which determined the necessary firing angle of the converters.
Figure 1. HVDC Control Characteristic
2.2: Survey of the Previous Research

Previous research work in control of HVDC power transmission served two distinct objectives. The first objective was to improve the response of the DC current and voltages caused by setting adjustments or system disturbances, summarized in section 3 of this chapter. The second objective was to enhance the stability of AC system by rapid power modulation of DC link, explained in section 4. Concluding remarks on the literature review are included in section 5.

2.3: Control Strategies for Improvement of DC System

Response

Classical converter valve ignition control used individual valve firing circuits which derived each valve ignition signal from a comparison of a sinusoidal voltage with a common regulator signal as shown in figure 2 which is taken from [11]. The controller provided adequate performance for most operating conditions, but in case of large disturbances the DC voltage response was poor and DC current overshoot occurred.

D.P. Carroll et al., investigated the use of combined voltage and current feedback for converter control [10]. The technique did not use modern control theory, but
Figure 2. HVDC Integral Feedback Controller (From [11], courtesy of IEEE, © 1973)
it improved the system response. The main difficulty was the proper system modeling for large disturbances and selection of a voltage reference for the controller.

M.S. Sachdev et al., in 1973 for the first time applied the optimal control theory for improvement of HVDC link performance [11]. The link was equipped with conventional integral feedback controller and connected two AC buses. The parameters for the DC link were selected to match the performance of an available DC simulator. The modeling incorporated the nominal tee representation of the DC line, and the converters' linearized performance equations as well as that of the controllers. The state equation had the form:

\[ \dot{X}(t) = AX(t) + BU(t) \]  (2.3.1)

For optimal control of the system it was assumed that only the sending end converter valve ignition can be controlled. Therefore, a new control was fed to the summing junction of rectifier control circuitry. The optimal control vector \( \psi(t) \) was defined and state equation was modified to:

\[ \dot{X}(t) = AX(t) + E\psi(t) \]  (2.3.2)

It was desired to control the system such that a quadratic cost function, \( J(\psi) \), was minimized. State regulator and tracking techniques were considered. For state regulator problem the following functional was the performance measure:
The control vector which minimized $J(\psi)$ had the form,

$$\psi(t) = -R^{-1}E^TKX(t)$$

and $K$ was obtained from solution of the Riccati equation:

$$\dot{K} = -KA - A^TK + KER^{-1}E^TK - Q$$

For tracking problem it was desired to minimize the error in output, and the performance measure was as follows:

$$J(\psi) = \frac{1}{2}\int_0^\infty [e^T(t)Qe(t) + \psi^T(t)R\psi(t)]dt$$

And $e(t)$ was defined as:

$$e(t) = Z - Y(t)$$

where $Y(t)$ was the output and $Z$ was the desired output. The control law had the form,

$$\psi(t) = -R^{-1}E^TKX(t) - R^{-1}E^T((A - ER^{-1}E^TK)^T)^{-1}C^TQZ$$

and the Riccati equation was:

$$\dot{K} = -KA - A^TK + KER^{-1}E^TK - C^TQC$$
Initially a fourth order system was considered with rectifier and inverter current increments, controller input voltage increment ($\Delta V_{ce}$), and the increment of voltage at the mid-point of the DC transmission line ($\Delta V_e$) as the state variables. For the state regulator the optimal control was fed to the summing junction through a low pass filter ($\frac{1}{1 + Ts}$) and the filter output formed the fifth state, while the filter was not used for the tracking technique. Feedback gains were computed for this system. Step changes in value of current order at rectifier and voltage at inverter were simulated. The rectifier DC voltage and current trajectories for the original and the optimally controlled system were presented. A suboptimal controller was also designed based on locally measurable states at rectifier. The performance superiority of the optimally controlled system was obvious, but there were no significant differences between the trajectories of optimally and suboptimally controlled systems. Despite the authors statement that for the tracking problem the outputs reached the steady state faster than state regulator problem, no conclusion could have been drawn because of differences in weighting factors, system order, and ambiguity in selection of the desired output.

F. Nozari et al., also used the optimal control theory for control of HVDC link [12]. The assumption was that the link existed between strong AC systems. The converters' DC voltages were obtained from,

$$V_d = (3\frac{\sqrt{3}}{\pi}E_mn_b)\cos \theta - (3\frac{X_l}{\pi}n_b)I_d$$  \hspace{1cm} (2.3.10)$$

where:
\( E_m \): peak phase to ground AC voltage

\( n_b \): number of series bridges at each converter

\( X_t \): transformer commutating reactance

\( I_d \): direct current

\( \theta \): ignition angle for rectifier, extinction angle for inverter

The above equation was represented by,

\[
V_d = e - R_c I_d
\]  \hspace{1cm} (2.3.11)

where \( e \) was determined by the converter control. Transmission line was represented by multiple section nominal pi equivalent circuit as shown in figure 3 which is taken from [12]. The dynamic equations of the system were formulated for the multi-section line model, but for simplicity one-section line model is used here to represent the form of the state equation.

\[
\dot{e}_1 = -\frac{1}{T_1} e_1 + \frac{1}{T_1} u_1 \]  \hspace{1cm} (2.3.12)

\[
i_{d1} = \frac{1}{L_1} - \frac{R_1}{L_1} i_{d1} - \frac{1}{L_1} V_{t1} \]  \hspace{1cm} (2.3.13)

\[
i_1 = \frac{1}{l} V_{t1} - \frac{1}{l} V_{t2} + \frac{R}{l} i_1 \]  \hspace{1cm} (2.3.14)

\[
\dot{V}_{t2} = \frac{1}{C} i_1 - \frac{1}{C} i_{d2} \]  \hspace{1cm} (2.3.15)
Figure 3. HVDC System and Controller Model (From [12], courtesy of IEEE, © 1976)
\[
\dot{i}_2 = \frac{1}{L_2} V_{t2} - \frac{R_2}{L_2} i_{d2} - \frac{1}{L_2} e_2 \quad (2.3.16)
\]

\[
\dot{e}_2 = -\frac{1}{T_2} e_2 + \frac{1}{T_2} u_2 \quad (2.3.17)
\]

For power flowing from converter 1 to converter 2 the parameters were:

\[
R_1 = R_{C1} + R_{L1} \quad (2.3.18)
\]

\[
R_2 = -R_{C2} + R_{L2} \quad (2.3.19)
\]

where,

- \(R_{C1}, R_{C2}\) : equivalent resistances due to commutating reactances
- \(R_{L1}, R_{L2}\) : resistances of DC reactors
- \(L_1, L_2\) : reactances of DC reactors

The converter voltages behind the commutating reactances \((e_1, e_2)\) were bounded by their upper and lower limits, so some of the state variables were constrained.

The generalized state equation for a multi-section line had the following form:

\[
\dot{X} = AX + BU \quad X \in \mathbb{R}^n, \ U \in \mathbb{R}^m, \ m \leq n \quad (2.3.20)
\]

\[
X_i^{\text{min}} \leq X_i \leq X_i^{\text{max}}, \quad i = 1, \ldots, n \quad (2.3.21)
\]

The objective was to achieve controls \(U\) which would drive the above system from the initial point \(X_0\) to a desired point \(\bar{X}\) while minimizing the quadratic function:
\[ J(U) = \frac{1}{2} \int_{0}^{\infty} [(X - \hat{X})^T Q (X - \hat{X}) + (U - \hat{U})^T R (U - \hat{U})] dt \] (2.3.22)

where \( \hat{U} \) was the steady state control vector related to \( \hat{X} \) by:

\[ 0 = A\hat{X} + B\hat{U} \] (2.3.23)

Translations of the form,

\[ Y = X - \hat{X}, \quad V = U - \hat{U} \] (2.3.24)

were used to reduce the state equation to:

\[ \dot{Y} = AY + BV, \quad Y_0 = X_0 - \hat{X} \] (2.3.25)

Finally the quadratic cost function was reduced to:

\[ J(U) = \frac{1}{2} \int_{0}^{\infty} (Y^T Q Y + V^T R V) dt \] (2.3.26)

If the problem was solved as if there were no constraints and the optimal controls obtained were such that the trajectories were far from the boundary values, then the solution procedure would have been acceptable. However, if the unconstrained solution had yielded trajectories which were close to the boundary values or violated the constraints, then the solution technique would not have been acceptable. Since it was not possible to predict the result in advance, a more general technique for constrained optimal control was suggested. According to the
authors, the procedure was complicated and often could not have been expressed in a closed loop feedback form. Therefore this technique of solution was abandoned, and it was proposed to use limiters as shown in figure 3. The incorporation of limiters in the system achieved the approximation of an optimal control, and it was called 'linear feedback with implied constraints'. Design of a suboptimal controller was also presented based on local measurements, and the results were discussed for the suboptimal controller. The solution for optimal control was only used for comparison purposes, and no specific results of the optimal controller were given.

2.4: Control Strategies for Stabilization of AC System

Improvement of dynamic or transient stability of the AC system was achieved by proper control of HVDC system. System stability was assured if after a disturbance the machine rotor angle swings were damped out and reached a final equilibrium condition. However, if the angle between the machines increased continuously as a consequence of a large disturbance (such as faults on AC system) the system was transiently unstable. Dynamic instability was reached if the machine rotor angles oscillated around the equilibrium point and some machine(s) stepped out. The relatively slow response of the AC system to disturbances, especially when transmission lines were long, was the major contributor to stability problems. The time elapsed could have been as long as several
hundred milliseconds before a disturbance on AC system was followed by a response when the transmission line length was long. However, the control associated with a DC transmission system could have changed the power flow at very rapid rates. If the HVDC converters made a specific change in DC power flow at the proper time, it could have cancelled the negative effect of a severe system disturbance. Some of the AC system variables which changed following a disturbance were used as signals for design of a controller for the DC transmission system. The power flow in an AC line, the magnitude of power angle between buses, and the rate of change of frequency were generally the monitoring signals used to generate proper control signals for modulation of the DC power.

In 1966 H.A. Peterson et al., investigated the damping of power swings in a parallel AC/DC system [13]. The system model consisted of a machine connected to an infinite bus via a composite AC/DC line. For small changes in phase angle between the machine and infinite bus voltages the change in AC power flow through the AC line was:

$$\Delta P_{ac} = \frac{E_1 E_2}{X} \cos \delta_0 \frac{\Delta \omega}{s}$$  \hspace{1cm} (2.4.1)

where,

- $\delta_0$ : initial angle between the bus voltages
- $\Delta \omega$ : speed of the synchronous machine
- $s$ : Laplace operator

Small change in rotor speed was related to the change in accelerating power by:
\[ \Delta P_a = 2Hs\Delta \omega \] (2.4.2)

where H was the inertia constant of the machine. The variation of the DC power to damp out the swings were chosen to be a function of \( \Delta \omega \),

\[ \Delta P_{dc} = f(\Delta \omega) \] (2.4.3)

Therefore the small power change \( \Delta P \) was expressed as:

\[ \Delta P = \Delta P_a + \Delta P_{dc} + \Delta P_{ac} \] (2.4.4)

\[ \Delta P = 2Hs\Delta \omega + f(\Delta \omega) + \frac{E_1E_2}{X} \cos \delta_0 \frac{\Delta \omega}{s} \] (2.4.5)

Three type of functions considered for \( f \) were the following:

Type A: \( f(\Delta \omega) = K_0\Delta \omega \) (2.4.6)

Type B: \( f(\Delta \omega) = K_0\Delta \omega + \frac{K_1}{s}\Delta \omega \) (2.4.7)

Type C: \( f(\Delta \omega) = K_0\Delta \omega + \frac{K_1}{s}\Delta \omega + \frac{K_2}{s^2}\Delta \omega \) (2.4.8)

The transfer functions \( \frac{\Delta P_{dc}}{\Delta P} \) and \( \frac{\Delta P_{ac}}{\Delta P} \) were formulated to evaluate whether the power change \( \Delta P \) was carried by the AC or DC line after the steady state was reached. The results were,
TYPE A: $\Delta P_{dc} = 0$, $\Delta P_{ac} = \Delta P$ \hspace{1cm} (2.4.9)

TYPE B: $\Delta P_{dc} = a \Delta P$, $\Delta P_{ac} = \beta \Delta P$, $a + \beta = 1$ \hspace{1cm} (2.4.10)

TYPE C: $\Delta P_{dc} = \Delta P$, $\Delta P_{ac} = 0$ \hspace{1cm} (2.4.11)

Analog simulations of the system with a controller of type A, B, or C led to the conclusion that the AC power swings were damped out by proper selection of controller gain(s) which was a trial and error based procedure.

Later J.J. Dougherty et al., implemented a Type A controller on a different system model (Two machines connected by two parallel AC and a bipolar DC link) and obtained satisfactory results [14]. Also N.A. Vovos added an additional $s\Delta \omega$ term to the type C controller and proposed a technique for control of DC link in such a way that the reactive power was maintained constant at both terminals. However, at inverter terminal the reactive power varied due to losses of the DC line [15,16].

T. Machida in 'Improving Transient Stability of AC System by Joint Usage of DC System' used experimental results to develop a mathematical formulation [17]. The system model was a simplified parallel double AC/DC line which connected a generator to an infinite bus. Two major conclusions were drawn from experimentations on the artificial system. First, the more power transferred to the infinite bus by DC link during the steady state, the more stable the generator was for AC faults. Second, if the DC power increased after an AC fault, then
transient stability was greatly improved. The AC fault was placed on the mid-point of one of the AC lines, and after a short time the faulted line was taken out. Machida used an equal area criterion to justify how much the DC power must have been increased for stability improvement. Figure 4 which is taken from [17] shows this situation. If the DC power did not increase rapidly ($B_2 = 0$) the accelerating area $A_1 + A_2$ would been greater than decelerating area $B_1$ and generator step-out possibility existed. The above equation was represented by:

$$B_2 = (A_1 + A_2) - B_1$$ \hspace{1cm} (2.4.12)

$$\int_{\theta_c}^{\theta_m}(P'_d - P_d) d\theta = \int_{0}^{\theta_c}(P_l - P'_l) + (P_d - P'_d)] d\theta - \int_{\theta_c}^{\theta_m}(P'_l - P_l) d\theta$$ \hspace{1cm} (2.4.13)

where,

$P_d$ : steady state DC power

$P_l$ : steady state AC power

$P'$ : transient power

After rearrangement of the above equations and substitution of:

$$P'_g = P'_l + P'_d$$ \hspace{1cm} (2.4.14)

$$P_g = P_M$$ \hspace{1cm} (2.4.15)

where the mechanical power ($P_M$) was assumed to be constant for that period, the following result was obtained:
Figure 4. Artificial System and Equal Area Criterion (From [17], courtesy of IEEE, © 1966)
\[ P'_d = P_M \left( \frac{\theta_m - \theta_0}{\theta_m - \theta_c} - \frac{1}{\theta_m - \theta_c} \right) \left[ \int_{\theta_0}^{\theta_c} P_g \, d\theta + \int_{\theta_c}^{\theta_m} P_I \, d\theta \right] \]  \hspace{1cm} (2.4.16)

Determination of \( P'_d \) yielded the DC power increment from:

\[ \Delta P_d = P'_d - P_d \]  \hspace{1cm} (2.4.17)

It has been admitted that because of nonlinearities in the \( P'_s \) and \( P'_i \) they must be obtained experimentally. The controller sensed the AC voltage drop at the generator bus, and the DC power flow readjustment was done by regulation of inverter ignition angle \( \beta \). Upon a drop in AC voltage the inverter ignition angle \( \beta \) increased to prevent the drop of DC power flow (to avoid increase of the accelerating area \( A_2 \)). When the faulted line tripped, the inverter ignition angle \( \beta \) was brought back to its original value and rectifier ignition angle advanced (to increase the decelerating area \( B_2 \)). Consequently, the rectifier voltage increase caused an increase of DC power, and upon reclosure of the faulted line the angle \( \alpha \) was brought back to its original value. The results obtained from the artificial system showed transient stability enhancement of the system under consideration, but implementation of the controller on a system without exact prediction of the system performance would not have been possible.

Y. Yoshida et al., investigated the connection of two asynchronous AC power systems by a DC link [18]. Each system had its own automatic frequency control system. As a controller for the DC link the authors proposed an automatic frequency ratio control (AFRC) device. The input to the controller consisted of a
signal proportional to the difference of the normalized frequency deviation of the two system as follows:

$$\Delta F = k_a \frac{\Delta f_a}{f_a} - k_b \frac{\Delta f_b}{f_b}$$

(2.4.18)

where $k_a$ and $k_b$ were gain constants of the frequency detector. Their values were equal to the normalized inverse of the generating capacity of systems A and B. Figure 5 which is taken from [18] represents the system and the controllers. Three controllers investigated were as follows:

- **Type A**: proportional + deadband
- **Type B**: proportional + deadband with jumping factor
- **Type C**: proportional + differential + deadband with jumping factor

Type A controller was slow and the response characteristic of the DC link followed the frequency changes of the AC system. For type B controller when $\Delta F$ had reached the value of deadband, the controller caused a step change in DC power flow. In case of type C controller the frequency change was detected immediately and DC power modulation was much faster in order to help the system AFC. For purpose of implementation of type A or B controllers on the system the DC power modulation didn’t take place for very small frequency deviations, while for type C controller the DC power modulation was done continuously. The controller parameters determined were based on an exhaustive trial and error procedure to satisfy certain system performance. The simulation results were satisfactory subject to selection of proper values of the parameters and system operation policies.
CHAPTER 2. LITERATURE REVIEW

Figure 5. Representation of the System and Controllers (From [18], courtesy of IEEE, © 1968)
P.K. Dash et al., applied optimal control theory for transient stability improvement of an AC/DC power system [19]. The system model consisted of a machine connected to an infinite bus by parallel AC/DC lines. The machine was modeled using Park's equations, and the HVDC had an inverse cosine control with integral feedback. The AC line was represented by nominal pi equivalent circuit, and the DC line was represented by an equivalent LR circuit. The nonlinear system equations were expressed by:

$$ \dot{X} = F(X,U) \quad (2.4.19) $$

The linearized equation had the form:

$$ \Delta \dot{X} = A\Delta X + B\Delta U \quad (2.4.20) $$

$$ \Delta Y = C\Delta X \quad (2.4.21) $$

where,

$$ \Delta X = X - X_0 \quad (2.4.22) $$

$$ A = \frac{\partial F}{\partial X}, \quad B = \frac{\partial F}{\partial U} \quad (2.4.23) $$

$$ \Delta X = [\Delta \delta \ \Delta \omega \ \Delta I \ \Delta I_{dc} \ \Delta V_{cc} \ \Delta V_{dc} \ \Delta T_m]^T \quad (2.4.24) $$

$$ \Delta U = [\Delta I_{dc}(\text{ref.})] \quad (2.4.25) $$
Due to existence of disturbances in states of the system, a more general form of the state equation was presented,

\[ \Delta Y = \begin{bmatrix} \Delta \delta & \Delta \omega & \Delta V_j \end{bmatrix}^T \]  

(2.4.26)

\[ \Delta \dot{X} = A \Delta X + B \Delta U + D \Delta \omega_m \]  

(2.4.27)

\[ \Delta Y = C \Delta X \]  

(2.4.28)

where D was the input matrix for the disturbance vector. For nonoptimal control the first control strategy selected was:

\[ \Delta U = K_1 \Delta \delta + K_2 \Delta \omega \]  

(2.4.29)

but this type of control did not work well for the nonlinear system. Therefore the second control was chosen which had the following form:

\[ \Delta U = K_2 \Delta \omega + K_3 \Delta \dot{\omega} \]  

(2.4.30)

Or:

\[ \Delta U = G^* \Delta X + D^* \Delta \omega_m \]  

(2.4.31)

So:

\[ \Delta \dot{X} = [A + BG^*] \Delta X + [BD^*] \Delta \omega_m \]  

(2.4.32)
For application of the optimal control theory equation (2.4.27) was defined as:

\[ \dot{X} = AX + BU + DW \]  

(2.4.33)

Vectors \( Z \) and \( V \) were defined:

\[ Z^T = [X^T \ Y^T] , \quad V = \dot{U} \]  

(2.4.34)

And the transformed equation had the form:

\[ \dot{Z} = AZ + BV \]  

(2.4.35)

where,

\[ \hat{A} = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix} , \quad \hat{B} = \begin{bmatrix} B \\ D \end{bmatrix} \]  

(2.4.36)

The performance index to be minimized was:

\[ J(V) = \frac{1}{2} \int_0^\infty [Z^T U + V^T R V] dt \]  

(2.4.37)

The solution of the algebraic Riccati equation yielded an optimal closed loop control. Also a suboptimal controller was designed based on the measurable states. It was concluded that the nonoptimal, optimal, and suboptimal control techniques improved transient stability of the system, and when the nonoptimal control failed to achieve stability the optimal and suboptimal controls maintained the stability.
K. Kobayashi et al., introduced the idea of minimum time control for improve-
ment of transient stability [20]. The system model was a parallel double AC/DC
line which connected a synchronous machine to an infinite bus as shown in figure
6 which is taken from [20]. The generator swing equation was:

\[ \frac{M}{\omega_0} \ddot{\delta} + \frac{D}{\omega_0} \dot{\delta} = P_m - P_e \]  

(2.4.38)

The authors ignored the effects of governor and exciter, and assumed a constant
voltage behind the generator transient reactance. The rectifier controlled the DC
voltage, while the inverter controlled the DC current (opposite of classical HVDC
control scheme). The DC link was modeled as dynamic admittances connected
from rectifier and inverter AC buses to ground as shown in figure 6. Any change
in set-points of the converters were equivalent to numerical changes of some
driving-point admittances in the system admittance matrix. In formulation of the
state equations the rectifier voltage and the inverter current were selected as the
system inputs. The machine power angle and speed were chosen as the system
states. Hence,

\[ x_1 = \delta \quad , \quad x_2 = \dot{\delta} \]  

(2.4.39)

\[ u_1 = V_d \quad , \quad u_2 = I_d \]  

(2.4.40)

Kron formula was used to reduce the order of the system, while the generator and
the infinite buses identities were preserved. Since the original admittance matrix
of the system was dependent on set-point changes, the reduced order admittance
Figure 6. System Model and Single Line Diagram (From [20], courtesy of IEEE, © 1978)
matrix was also dependent on these changes. Hence, for the reduced order admittance matrix:

\[ Y_{12} = Y_{12}(u_1, u_2) \quad (2.4.41) \]

\[ \theta_{12} = \theta_{12}(u_1, u_2) \quad (2.4.42) \]

\[ P_e = G_{11}E_1^2 + Y_{12}E_1E_2 \cos(\delta - \theta_{12}) = P_e(x_1, u_1, u_2) \quad (2.4.43) \]

Finally, the state equations formulated were as follows:

\[ \dot{x}_1 = x_2 \quad (2.4.44) \]

\[ \dot{x}_2 = \left[ P_m - \frac{D}{\omega_0}x_2 - P_e(x_1, u_1, u_2) \right] \frac{\omega_0}{M} \quad (2.4.45) \]

To improve transient stability of the system, minimum time principle was applied for optimal switching of converters set-points. The performance index was:

\[ J(U) = \int_0^T J_d \, dt \quad (2.4.46) \]

The Hamiltonian was formulated as:

\[ H = p_1x_2 + p_2 \left[ P_m - \frac{D}{\omega_0}x_2 - P_e(x_1, u_1, u_2) \right] \frac{\omega_0}{M} - 1 \quad (2.4.47) \]

where \( p_1 \) and \( p_2 \) satisfied:
\[
\dot{p}_1 = -\frac{\partial H}{\partial x_1} \quad (2.4.48)
\]
\[
\dot{p}_2 = -\frac{\partial H}{\partial x_2} \quad (2.4.49)
\]

The optimal control was obtained from:

\[-p_2 P_e(x_1, u_1, u_2) > 0 \quad \rightarrow \quad P_e(x_1, u^*_1, u^*_2) = \bar{P}_e \quad (2.4.50)\]

\[-p_2 P_e(x_1, u_1, u_2) \leq 0 \quad \rightarrow \quad P_e(x_1, u^*_1, u^*_2) = P_e \quad (2.4.51)\]

where,

\[
\bar{P}_e = \max[ P_e(x_1, u_1, u_2) ] \quad , \quad u_1, u_2 \in U \quad (2.4.52)
\]

\[
P_e = \min[ P_e(x_1, u_1, u_2) ] \quad , \quad u_1, u_2 \in U \quad (2.4.53)
\]

constrained by upper and lower limits of \( u_1 \) and \( u_2 \). The digital and analog simulation results showed the optimal bang-bang control had a better damping influence on generator rotor angle oscillations than simple switching of the set points when full power was drawn from the machine during the acceleration period. However, since the optimal control was not realizable for on-line control, a suboptimal control was recommended.

C.E. Grund et al., have used the modern control theory and implementations of combined controller-observer to design a closed loop control law for dynamic
stability improvement of an AC system by control of a DC link [21]. To reduce the system order to a manageable size, coherency dynamic reduction technique based on commutation bus faults was used. The nonlinear reduced order system was linearized to identify observable and controllable states of the system. The observer was needed for reconstruction of necessary states of the system. The controllable system states were used to obtain state variable feedback gains for the controller. Separation principle made it possible to design observer and controller separately. A simplified model consisted of parallel AC/DC lines was used to evaluate the theory. The controller inputs were the change of rectifier bus frequency, and change of power in AC line which represented the difference in frequency deviations between terminal buses. The controller modulated the rectifier DC power by varying the rectifier DC current and the inverter DC voltage. It was concluded that controlability of HVDC link by power modulation with coordinated voltage modulation was more effective than power modulation alone.

2.5: Concluding Remarks on Literature Review

This technical literature review described the previous research work in control of HVDC link. The concentration was on optimal control of a DC link for DC system response improvement (section 3), in addition to nonoptimal and optimal control of DC link for stability improvement of an AC system (section 4). The
nonoptimal control of DC link was reviewed due to its importance for identification of AC system parameters used as monitoring signals. From section 3 the following conclusions can be drawn:

1. The DC line was modeled as nominal pi or nominal tee. Multi-section nominal pi equivalent circuit model did not have a great significance over one-section nominal pi equivalent circuit model.

2. The state equation formulation for optimal control of HVDC link in one case included the model of control circuitry, and the optimal control was a new input fed to the summing junction of the rectifier control circuitry. The problem was treated as an unconstrained optimal control case, and the equations were valid for small disturbances or minor set-point changes.

3. The state equation formulation for optimal control of HVDC link in another case did not include the control circuitry and assumes the voltages behind the commutating reactances as control variables. The problem was treated as a constrained optimal control case, but the solution was the same as the unconstrained problem and limiters were used for the restricted controls.

4. In both cases the AC system was assumed to be strong and the DC power modulation did not cause AC bus voltages to change, which is not true in many cases.
The following are the conclusions from section 4:

1. In almost all cases the system model was a machine connected to an infinite bus by parallel AC/DC lines.

2. For optimum system performance, the conventional controllers required gains and parameter adjustments by a trial and error procedure.

3. The optimal controllers improved system performance, but due to unavailability of all measurements at controller, the local measurements were used and the controllers were converted to suboptimal controllers.

4. The optimal control solution technique did not include the constraints directly.

5. The generators' governor and exciter system were not coordinated with DC link controller for optimal control of the system.

6. The state equations in most cases were the linearized version of nonlinear equations acceptable for dynamic stability studies; but most probably unsatisfactory for transient stability studies.

7. There was no consistent agreement in the AC system parameters used as monitoring signals.
8. In a majority of the cases, rectifier controlled the power modulation and joint rectifier-inverter control was rarely investigated.

9. The criterion of the performance measure for optimal control was either to minimize deviation of the states and the controls from the desired values, or minimize the transient time.
CHAPTER 3. DESIGN OF OPTIMALLY CONTROLLED SYSTEMS

3.1: Introduction to Optimal Control Theory

Application of optimal control theory to a physical system is a well developed concept. The optimal control theory determines the best control signal for a system such that the process satisfies the physical system constraints, and also minimizes a cost function which is referred to as the performance measure. In order to be able to formulate the optimal control law for a system, the following are required:

1. Mathematical model of the system

2. Physical constraints of the system
3. Performance measure criteria

The general mathematical model has the form,

$$\dot{X}(t) = a(X(t), U(t), t)$$  \hspace{1cm} (3.1.1)

where $a$ is a nonlinear time varying function of the states, the control, and time. After formulation of the mathematical model, the physical constraints on the states and controls should be specified. Generally the performance measure has the form,

$$J(U) = h(X(t), t_f) + \int_0^{t_f} g(X(t), U(t), t) dt$$  \hspace{1cm} (3.1.2)

where $h$ and $g$ are scalar functions and the final time $t_f$ may be fixed or free. The control signal applied between the initial and final times causes the system to follow some state trajectory, and the performance measure assigns a real number to this trajectory. The control $U^*(t)$ which yields the minimum value of the performance measure is referred to as the optimal control law.

The optimal control is represented as a function of the system states and time. In cases where the optimal control has the form,

$$U^*(t) = f(X(t), t)$$  \hspace{1cm} (3.1.3)

the function $f$ is the optimal control law. Since it is independent of the values of the states at any specific time, it has the form of a closed loop feedback type of
control law. However, in cases where the optimal control has a functional relationship such as,

$$ U^*(t) = f(X(t_0), t) $$

(3.1.4)

the function $f$ is the optimal control law only for the initial states $X(t_0)$. Therefore, the optimal control is an open loop type of control law.

In derivation of the necessary conditions for optimal control law the variational approach and the minimum principle of calculus of variation is utilized [7]. The elaborate and tedious intermediate mathematical steps are not of main interest, only the final results are of interest. The Hamiltonian is defined as:

$$ H(X(t), U(t), P(t), t) = g(X(t), U(t), t) + P^T(t)[a(X(t), U(t), t)] $$

(3.1.5)

where $P(t)$ is a column vector containing the Lagrange multipliers. The necessary conditions for an unconstrained optimal control law are:

$$ \dot{X}^*(t) = \frac{\partial H(X^*(t), U^*(t), P^*(t), t)}{\partial P(t)} $$

(3.1.6)

$$ \dot{P}^*(t) = -\frac{\partial H(X^*(t), U^*(t), P^*(t), t)}{\partial X(t)} $$

(3.1.7)

$$ 0 = \frac{\partial H(X^*(t), U^*(t), P^*(t), t)}{\partial U(t)} $$

(3.1.8)

The boundary condition is specified from the transversality condition:
\[
\frac{\partial h(X^*(t), t^* \partial t)}{\partial X(t)} - P^*(t^*)^T \delta X_f + [H(X^*(t), U^*(t), P^*(t), t^*) + \frac{\partial h(X^*(t), t^*)}{\partial t}] \delta t_f = 0
\] (3.1.9)

Application of equations (3.1.6) through (3.1.9) to the system state equation (3.1.1) and performance measure (3.1.2) yields the optimal control law.

### 3.2: Mathematical Formulation

The dynamics of a linear time varying system is described by:

\[
\dot{X}(t) = A(t)X(t) + B(t)U(t) \quad (3.2.1)
\]
\[
Y(t) = C(t)X(t) \quad (3.2.2)
\]

and the quadratic performance measure is:

\[
J(U) = \frac{1}{2} [e^T(t^*)H e(t^*)] + \frac{1}{2} \int_0^{t_f} [e^T(t)Q(t)e(t)] + [U^T(t)R(t)U(t)] dt \quad (3.2.3)
\]

where,

\[
e(t) = X(t) - X_d(t) \quad (3.2.4)
\]

for state tracking, or:
for output tracking. The variables in equations (3.2.1) through (3.2.5) are defined as follows:

\[ \begin{align*}
X(t) & : \text{state vector, } n \times 1 \\
U(t) & : \text{input vector, } r \times 1 \\
Y(t) & : \text{output vector, } m \times 1 \\
A(t) & : \text{state feedback matrix, } n \times n \\
B(t) & : \text{input matrix, } n \times r \\
C(t) & : \text{output matrix, } m \times n \\
J(U) & : \text{cost function, } 1 \times 1 \\
H & : \text{constant symmetric positive semi-definite matrix, } n \times n \\
Q(t) & : \text{symmetric positive semi-definite matrix, } n \times n \\
R(t) & : \text{symmetric positive definite matrix, } r \times r \\
e(t) & : \text{error vector, } n \times 1 \text{ or } m \times 1 \\
t_0 & : \text{initial time} \\
t_f & : \text{final time}
\end{align*} \]

The objective is to control the system such that the state vector (or output vector) stays near the desired vector, during the control period \((t_0, t_f)\). The performance measure is said to be of a regulator or of a tracking type, depending on whether the desired vector \(X_d\) is a zero vector or a non-zero vector. Following the com-
putation and implementation of the optimal control $U^*(t)$, the system is optimally controlled. The quadratic performance measure of the equation (3.2.3) has the advantage of being mathematically tractable, and producing a linear optimal feedback control law.

There are considerable difficulties in determination of the optimal control law, and further difficulties exist in implementation of the optimal controller. Even for a linear system it is not always possible to obtain a general analytical solution. The following conditions are needed for determination of an analytical solution:

1. States and controls are not bounded.

2. $R(t)$ is positive definite, and $Q(t) \& H$ are positive semi-definite.

3. System controllability and observability.

The first condition eliminates the physical limitations of the system, i.e. ignores saturation effects, by removing the constraints of upper and lower bounds on control. The second condition is purely mathematical, assuring that a meaningful optimization problem is being dealt with. The third condition is needed depending on the specification of the control period and the implementability of the optimal control law.
3.3: State Regulator Performance Measure

The system dynamics is represented by the state equation (3.2.1), and the performance measure is as shown in equation (3.2.3) with \( X_d \) equal to zero in equation (3.2.4). The Hamiltonian is formulated from equation (3.1.5),

\[
H = \frac{1}{2}[X^T(t)Q(t)X(t)] + \frac{1}{2}[U^T(t)R(t)U(t)] + P^T[A(t)X(t) + B(t)U(t)]
\]  

(3.3.1)

Applying equations (3.1.6) through (3.1.8) yield the following results:

\[
\dot{X}(t) = \frac{\partial H}{\partial P(t)} \quad \rightarrow \quad \dot{X}(t) = A(t)X(t) + B(t)U(t)
\]  

(3.3.2)

\[
\dot{P}(t) = -\frac{\partial H}{\partial X(t)} \quad \rightarrow \quad \dot{P}(t) = -Q(t)X(t) - A^T(t)P(t)
\]  

(3.3.3)

\[
0 = \frac{\partial H}{\partial U(t)} \quad \rightarrow \quad 0 = R(t)U(t) + B^T(t)P(t)
\]  

(3.3.4)

The term \( \frac{\partial H}{\partial U(t)} = 0 \) yields an extremum of Hamiltonian,

\[
U(t) = -R^{-1}(t)B^T(t)P(t)
\]  

(3.3.5)

which is the primitive form of the optimal control law. If existence of \( R^{-1}(t) \) and \( \frac{\partial^2 H}{\partial U^2(t)} > 0 \) are proven, the control law minimizes the performance measure. Since the second partial of Hamiltonian with respect to \( U(t) \) is \( R(t) \), and \( R(t) \) is a positive definite matrix then the inequality is satisfied and also the inverse of  

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R(t) exists. Substitution of the control law $U(t)$ into the state equation (3.3.2), and augmentation of the new state equation with costate equation (3.3.3) results in:

$$\dot{X}(t) = A(t)X(t) - B(t)R^{-1}(t)B^T(t)P(t)$$  \hspace{1cm} (3.3.6)

$$\begin{bmatrix} \dot{X}(t) \\ \dot{P}(t) \end{bmatrix} = \begin{bmatrix} A(t) & -B(t)R^{-1}(t)B^T(t) \\ -Q(t) & -A^T(t) \end{bmatrix} \begin{bmatrix} X(t) \\ P(t) \end{bmatrix}$$  \hspace{1cm} (3.3.7)

Equation (3.3.7) is a system of $2n$ time varying homogeneous differential equations. The solution of the augmented differential equation is unique, provided that the initial state $X(t_0)$ and $P(t_0)$ are known. However, the initial value of $P(t)$ is not known, but its final value is obtainable from the transversality condition of equation (3.1.9). In other words,

$$\delta\left\{ \frac{1}{2} [X^T(t_f)HX(t_f)] \right\}$$  \hspace{1cm} \frac{\partial}{\partial X(t)} - P(t_f) = 0$$  \hspace{1cm} (3.3.8)

Hence,

$$P(t_f) = HX(t_f)$$  \hspace{1cm} (3.3.9)

Therefore, the augmented differential equation can not be directly solved because of the split boundary conditions. However, there is no need to solve the augmented differential equation. For known initial condition, the solution has the form:
\[
[X(t) \quad P(t)]^T = \Phi(t, t_0) \quad [X(t_0) \quad P(t_0)]^T
\] (3.3.10)

Where \(\Phi(t, t_0)\) is the state transition matrix. The final value of states and costates are:

\[
\begin{bmatrix}
X(t_f) \\
P(t_f)
\end{bmatrix} =
\begin{bmatrix}
\varphi_{11}(t_f, t) & \varphi_{12}(t_f, t) \\
\varphi_{21}(t_f, t) & \varphi_{22}(t_f, t)
\end{bmatrix}
\begin{bmatrix}
X(t) \\
P(t)
\end{bmatrix}
\] (3.3.11)

Separation of the equation (3.3.11) into two equations leads to:

\[
X(t_f) = \varphi_{11}(t_f, t)X(t) + \varphi_{12}(t_f, t)P(t)
\] (3.3.12)

\[
P(t_f) = \varphi_{21}(t_f, t)X(t) + \varphi_{22}(t_f, t)P(t)
\] (3.3.13)

Now substitution of \(P(t_f)\) of equation (3.3.9) into equation (3.3.13) yields:

\[
HX(t_f) = \varphi_{21}(t_f, t)X(t) + \varphi_{22}(t_f, t)P(t)
\] (3.3.14)

Then from substitution of equation (3.3.12) into (3.3.14), \(P(t)\) is found. Thus,

\[
H[\varphi_{11}(t_f, t)X(t) + \varphi_{12}(t_f, t)P(t)] = \varphi_{21}(t_f, t)X(t) + \varphi_{22}(t_f, t)P(t)
\] (3.3.15)

\[
[H\varphi_{11}(t_f, t) - \varphi_{21}(t_f, t)]X(t) = [\varphi_{22}(t_f, t) - H\varphi_{12}(t_f, t)]P(t)
\] (3.3.16)

\[
P(t) = [\varphi_{22}(t_f, t) - H\varphi_{12}(t_f, t)]^{-1}[H\varphi_{11}(t_f, t) - \varphi_{21}(t_f, t)]X(t)
\] (3.3.17)

Kalman has shown that the above inverse matrix exists[57]. Equation (3.3.17) is put in a short form as follows,
\[ P(t) = K(t)X(t) \] (3.3.18)

In other words \( P(t) \) is a linear function of \( X(t) \). Substitution of \( P(t) \) of equation (3.3.18) into equation (3.3.5) yields the optimal control law in the desired form:

\[ U^*(t) = -R^{-1}(t)B^T(t)K(t)X(t) \] (3.3.19)

Or,

\[ U^*(t) = F(t)X(t) \] (3.3.20)

which is an indication of a linear, time varying, and closed loop optimal control law.

For time varying systems the analytical solution of the state transition matrix does not exist and iterative techniques using digital computers are utilized to obtain the solution. However, if the system is time invariant and matrices \( A, B, Q, \) and \( R \) are constants, then the inverse Laplace transform technique is one approach to solve for the state transition matrix. However, this does involve tedious work for large order systems. A remedy to this problem is to take advantage of equation (3.3.18) and use a direct approach by differentiating it. So,

\[ \dot{P}(t) = \dot{K}(t)X(t) + K(t)\dot{X}(t) \] (3.3.21)

Substitution of equations (3.3.2) and (3.3.3) into the above equation leads to:
\[-Q(t)X(t) - A^T(t)K(t)X(t) = \dot{K}(t)X(t) + K(t)[A(t)X(t) + B(t)U(t)] \quad (3.3.22)\]

Now substitution of the relationship (3.3.19) in the equation (3.3.22), and cancellation of X(t) results in following equation which has the desirable form.

\[\dot{K}(t) = -K(t)A(t) - A^T(t)K(t) + K(t)B(t)R^{-1}(t)B^T(t)K(t) - Q(t) \quad (3.3.23)\]

This equation is the well known Riccati equation, and for a given boundary condition it can be solved. Since the initial boundary condition is undetermined, then the transversality condition of equation (3.1.9) is used to find the final boundary value as follows.

\[
\frac{\delta}{\delta X(t)} \left[ \frac{1}{2} X^T(t_f)H X(t_f) \right] - P(t_f) = 0 \quad (3.3.24)
\]

\[P(t_f) = HX(t_f) \quad (3.3.25)\]

But from equation (3.3.18) at time \( t = t_f \) we have:

\[P(t_f) = K(t_f)X(t_f) \quad (3.3.26)\]

Then,

\[K(t_f) = H \quad (3.3.27)\]
which is the final boundary value for the Riccati equation. It can be shown that the solution of the Riccati equation with its final boundary value of equation (3.3.27) not only exists, but it is also unique[5].

To summarize the final result of this section, the essential equations are repeated here and the equation numbers are maintained. Given:

$$\dot{X}(t) = A(t)X(t) + B(t)U(t)$$  \hspace{1cm} (3.2.1)

and,

$$J(U) = \frac{1}{2}[X^T(t_f)HX(t_f)] + \frac{1}{2}\int_0^{t_f}[[X^T(t)Q(t)X(t)] + [U^T(t)R(t)U(t)]]dt$$  \hspace{1cm} (3.3.28)

The following optimal control law is unique and it exists.

$$U^*(t) = -R^{-1}(t)B^T(t)K(t)X(t)$$  \hspace{1cm} (3.3.19)

Where $K(t)$ is the solution of Riccati equation:

$$\dot{K}(t) = -K(t)A(t) - A^T(t)K(t) + K(t)B(t)R^{-1}(t)B^T(t)K(t) - Q(t)$$  \hspace{1cm} (3.3.23)

with final boundary value,

$$K(t_f) = H$$  \hspace{1cm} (3.3.27)

The optimal trajectories are the solution of the linear differential equation:
\[ \dot{X}(t) = [A(t) - B(t)R^{-1}(t)B^T(t)K(t)]X(t) \]  \hspace{1cm} (3.3.29)

\[ X(t_0) = X_0 \]  \hspace{1cm} (3.3.30)

The system of equation (3.2.1) does not require controllability as a requirement for the finite control interval \([t_0, t_1]\). This is due to the fact that the contribution of uncontrollable states to the cost function is always finite because of the finite control interval. The situation is quite different for an infinite control interval. The controllability is then a strict requirement to assure a finite value for \(J(U)\). An important result is that for a time invariant system with constant \(Q\), \(R\), and infinite control interval the matrix \(K(t)\) is constant. This result tremendously simplifies the implementation of the controller.

### 3.4: Output Regulator Performance Measure

If equation (3.2.5) is used instead of equation (3.2.4) in equation (3.2.3) with \(Y_d(t)\) equal to zero, then the performance measure is an output regulator type. Again the Hamiltonian is formed with,

\[ Y(t) = C(t)X(t) \]  \hspace{1cm} (3.4.1)

and steps of section 3 are followed to obtain the optimal control law. However, the following changes are needed:
Thus it is necessary to show that the new replacements are positive semi-definite. To show this property the system observability is necessary to avoid $C(t)$ being zero for all $t$. If $Q(t)$ is positive semi-definite, then $Y^T(t)Q(t)Y(t)$ is also positive semi-definite for all $Y(t)$. $Y(t) = C(t)X(t)$ results in $X^T(t)C^T(t)Q(t)C(t)X(t)$ being positive semi-definite for all $C(t)X(t)$. But, observability implies that each output $Y(t)$ is generated by a unique state $X(t)$. Therefore it can be concluded that the positive semi-definiteness of $X^T(t)C^T(t)Q(t)C(t)X(t)$ is for all $X(t)$ and that $C^T(t)Q(t)C(t)$ is positive semi-definite. Establishment of $C^T(t)HC(t)$ being positive semi-definite is identical to the above argument[4].

The optimal control law for output regulator is identical to control law for state regulator.

$$U^*(t) = -R^{-1}(t)B^T(t)K(t)X(t)$$

(3.4.2)

But, the Riccati equation of state regulator (equation (3.3.23)) is slightly modified as shown in the following.

$$\dot{K}(t) = -A^T(t)K(t) - K(t)A(t) + K(t)B(t)R^{-1}(t)B^T(t)K(t) - C^T(t)Q(t)C(t)$$

(3.4.3)

with the final boundary value,
\[ K(t_f) = C^T(t_f)HC(t_f) \]  

(3.4.4)

Naturally the solution of \( K(t) \) for output regulator is different from the state regulator. The optimal trajectories are obtained from the solution of the linear differential equation:

\[ \dot{X}(t) = [A(t) - B(t)R^{-1}(t)B^T(t)K(t)]X(t) \]  

(3.4.5)

\[ X(t_0) = X_0 \]  

(3.4.6)

The optimal control law is a function of system states rather than system outputs. This at first might be unacceptable, since the system outputs only might have been available. But the system observability guarantees the possibility of generation of the states from the outputs. The controllability is not a necessity for a finite control interval, but it is a requirement for the case of infinite control period. In cases of infinite control period for a controllable and time invariant system, again the feedback gains will be constant.

### 3.5: Output Tracking Performance Measure

This section focuses on the output tracking type of performance measure. The system dynamics is expressed by the equations (3.2.1) and (3.2.2). The perform-
The augmented state and costate equations with substitution of equation (3.5.6) into equation (3.5.3) results in,
\[
\begin{pmatrix}
X(t) \\
P(t)
\end{pmatrix} =
\begin{pmatrix}
A(t) & -B(t)R^{-1}(t)B^T(t) \\
C^T(t)Q(t)C(t) & -A^T(t)
\end{pmatrix}
\begin{pmatrix}
X(t) \\
P(t)
\end{pmatrix} +
\begin{pmatrix}
0 \\
C^T(t)Q(t)Y_d(t)
\end{pmatrix}
\tag{3.5.7}
\]

Equation (3.5.7) is a nonhomogeneous linear time varying differential equation of order \(2n\) with a forcing function dependent on the desired output. The solution of the differential equation for \(X(t)\) and \(P(t)\) has the form:

\[
[X(t) \quad P(t)]^T = \Phi(t_\text{f}, t) \ [X(t) \quad P(t)]^T + \int_t^{t_\text{f}} \Phi(t_\text{f}, \tau)[0 \quad C^T(\tau)Q(\tau)Y_d(\tau)]^T d\tau \tag{3.5.8}
\]

But from the transversality condition of equation (3.1.9),

\[
\frac{\delta}{\delta X(t_\text{f})} \left( \frac{1}{2} [C(t_\text{f})X(t_\text{f}) - Y_d(t_\text{f})]^T H [C(t_\text{f})X(t_\text{f}) - Y_d(t_\text{f})] \right) - P(t_\text{f}) = 0 \tag{3.5.9}
\]

or,

\[
P(t_\text{f}) = \frac{\delta}{\delta X(t_\text{f})} \left( \frac{1}{2} [X^T(t_\text{f})C^T(t_\text{f})HC(t_\text{f})X(t_\text{f}) - X^T(t_\text{f})C^T(t_\text{f})HY_d(t_\text{f}) - Y_d^T(t_\text{f})HC(t_\text{f})X(t_\text{f}) + Y_d^T(t_\text{f})HY_d(t_\text{f})] \right) / \delta X(t_\text{f})
\tag{3.5.10}
\]

which leads to:

\[
P(t_\text{f}) = C^T(t_\text{f})HC(t_\text{f})X(t_\text{f}) - C^T(t_\text{f})HY_d(t_\text{f}) \tag{3.5.11}
\]

Replacement of the integral term in equation (3.5.8) by two functions \(f_1(t)\) and \(f_2(t)\), and partitioning the state transition matrix results in:
\[
\begin{bmatrix}
X(t) \\
P(t)
\end{bmatrix} =
\begin{bmatrix}
\phi_{11}(t_f, t) & \phi_{12}(t_f, t) \\
\phi_{21}(t_f, t) & \phi_{22}(t_f, t)
\end{bmatrix}
\begin{bmatrix}
X(t) \\
P(t)
\end{bmatrix} +
\begin{bmatrix}
f_1(t) \\
f_2(t)
\end{bmatrix}
\tag{3.5.12}
\]

Or,
\[
X(t) = \phi_{11}(t_f, t)X(t) + \phi_{12}(t_f, t)P(t) + f_1(t)
\tag{3.5.13}
\]
\[
P(t) = \phi_{21}(t_f, t)X(t) + \phi_{22}(t_f, t)P(t) + f_2(t)
\tag{3.5.14}
\]

Substitution of \(P(t_f)\) of equation (3.5.11) into equation (5.5.14) results in a linear relationship between \(P(t)\) and \(X(t)\) as shown in the following:
\[
C^T(t_f)HC(t_f)X(t) - C^T(t_f)HY_d(t_f) =
\phi_{21}(t_f, t)X(t) + \phi_{22}(t_f, t)P(t) + f_2(t)
\tag{3.5.15}
\]

Now substitution of \(X(t_f)\) of equation (3.5.13) into equation (3.5.15) yields:
\[
C^T(t_f)HC(t_f)[\phi_{11}(t_f, t)X(t) + \phi_{12}(t_f, t)P(t) + f_1(t)] - C^T(t_f)HY_d(t_f) =
\phi_{21}(t_f, t)X(t) + \phi_{22}(t_f, t)P(t) + f_2(t)
\tag{3.5.16}
\]

Further simplifications are done in the following.
\[
[C^T(t_f)HC(t_f)\phi_{12}(t_f, t) - \phi_{22}(t_f, t)]P(t) = [\phi_{21}(t_f, t) - C^T(t_f)HC(t_f)\phi_{11}(t_f, t)]X(t)
\tag{3.5.17}
\]
\[+ [C^T(t_f)HY_d(t_f) + f_2(t) - C^T(t_f)HC(t_f)f_1(t)]
\]
The above equation is rewritten as:

\[ P(t) = K(t)X(t) - g(t) \]  

Equation (3.5.19) is the desirable relationship that is needed for a direct solution approach. Now substitution of equation (3.5.19) into equation (3.5.6) and further simplification yield the optimal control law.

\[ U^*(t) = -R^{-1}(t)B^T(t)[K(t)X(t) - g(t)] \]  

\[ U^*(t) = R^{-1}(t)B^T(t)[g(t) - K(t)X(t)] \]  

\[ U^*(t) = -R^{-1}(t)B^T(t)K(t)X(t) + R^{-1}(t)B^T(t)g(t) \]  

To solve for K(t) and g(t) equation (3.5.19) is differentiated:

\[ \dot{P}(t) = \dot{K}(t)X(t) + K(t)\dot{X}(t) - \dot{g}(t) \]  

Then equation (3.5.3) is substituted in the above equation and \( U(t) \) is replaced by the \( U^*(t) \) of equation (3.5.22):

\[ \dot{P}(t) = \dot{K}(t)X(t) + K(t)[A(t)X(t) + B(t)[ - R^{-1}(t)B^T(t)K(t)X(t) + R^{-1}(t)B^T(t)g(t)]] - \dot{g}(t) \]
The $\dot{P}(t)$ relation of equation (3.5.4) is substituted into equation (3.5.24) and $P(t)$ in equation (3.5.4) is replaced by equality of equation (3.5.19), then:

$$
-C^T(t)Q(t)C(t)X(t) + C^T(t)Q(t)Y_d(t) - A^T(t)K(t)X(t) + A^T(t)g(t) =
$$

$$
[K(t) + K(t)A(t) - K(t)B(t)R^{-1}(t)B^T(t)K(t)]X(t) + K(t)B(t)R^{-1}(t)B^T(t)g(t) - \dot{g}(t)
$$

(3.5.25)

$$
[K(t) + K(t)A(t) - K(t)B(t)R^{-1}(t)B^T(t)K(t) + C^T(t)Q(t)C(t) + A^T(t)K(t)]X(t) + [K(t)B(t)R^{-1}(t)B^T(t)g(t) - \dot{g}(t) - C^T(t)Q(t)Y_d(t) - A^T(t)g(t)] = 0
$$

(3.5.26)

The above equation is satisfied when the quantities inside each bracket are zero.

Thus,

$$
\dot{K}(t) = -A^T(t)K(t) - K(t)A(t) + K(t)B(t)R^{-1}(t)B^T(t)K(t) - C^T(t)Q(t)C(t)
$$

(3.5.27)

$$
\dot{g}(t) = -[A^T(t) - K(t)B(t)R^{-1}(t)B^T(t)]g(t) - C^T(t)Q(t)Y_d(t)
$$

(3.5.28)

From equation (3.5.19) at $t = t_f$,

$$
P(t_f) = K(t_f)X(t_f) - g(t_f)
$$

(3.5.29)

and the transversality condition of equation (3.5.11) the final boundary values are obtained as follows.

$$
K(t_f) = C^T(t_f)HC(t_f)
$$

(3.5.30)

$$
g(t_f) = C^T(t_f)HY_d(t_f)
$$

(3.5.31)
The differential equation of (3.5.27) is the Riccati equation, and it is identical to the one for the output regulator. Also, the boundary value of equation (3.5.30) is identical to the boundary value of the Riccati equation for the output regulator. Thus the Riccati equation and its boundary value are independent of the desired output \( Y_d(t) \). The solution of the Riccati equation only requires the knowledge of the matrices \( A(t), B(t), C(t), H, Q(t), R(t), \) and \( t_f \). It can be concluded that for a time invariant system, eigenvalues of the optimal closed loop tracking system are identical to the eigenvalues of the optimal closed loop output regulator system. The differential equation of (3.5.28) is adjoint to the state equation of the optimally controlled system. The adjoint relationship has the implication that the eigenvalues of equation (3.5.28) are the negative of the system eigenvalues. Therefore the closed loop system stability is independent of the desired output, but the desired output forms the forcing function of the system through the term \( g(t) \). In order to solve for \( g(t) \) the desired output should be known throughout the control interval \([t_0, t_f]\). In other words the present value of optimal control is dependent of the future values of the desired output.

To summarize the final results of this chapter the necessary equations needed to obtain the control law are repeated here. Given:

\[
\dot{X}(t) = A(t)X(t) + B(t)U(t) \quad (3.2.1)
\]

\[
Y(t) = C(t)X(t) \quad (3.2.2)
\]

and the performance measure of:
\[ J(U) = \frac{1}{2} [Y(t_f) - Y_d(t_f)]^T H [Y(t_f) - Y_d(t_f)] + \int_0^{t_f} \left[ [Y(t) - Y_d(t)]^T Q(t) [Y(t) - Y_d(t)] + [U^T(t) R(t) U(t)] \right] dt \] (3.5.32)

the optimal control law is:

\[ U^*(t) = R^{-1}(t) B^T(t) [g(t) - K(t) X(t)] \] (3.5.21)

Where \( K(t) \) is the solution of the Riccati equation:

\[ \dot{K}(t) = -A^T(t) K(t) - K(t) A(t) + K(t) B(t) R^{-1}(t) B^T(t) K(t) - C^T(t) Q(t) C(t) \] (3.5.27)

with final boundary value,

\[ K(t_f) = C^T(t_f) H C(t_f) \] (3.5.30)

And \( g(t) \) is the solution of the differential equation:

\[ \dot{g}(t) = - [A^T(t) - K(t) B(t) R^{-1}(t) B^T(t)] g(t) - C^T(t) Q(t) Y_d(t) \] (3.5.28)

with final boundary value,

\[ g(t_f) = C^T(t_f) H Y_d(t_f) \] (3.5.31)

The optimal trajectories are the solution of the linear differential equation:

\[ \dot{X}(t) = [A(t) - B(t) R^{-1}(t) B^T(t) K(t)] X(t) + B(t) R^{-1}(t) B^T(t) g(t) \] (3.5.33)
The same controllability condition of section 4 is required for the output tracking of this section. Also, the system observability is a necessity at all times.

3.6: Developed Control Algorithm

The material in this section describes the main contribution of this dissertation to the problem of nonlinear system control. For the first time a control algorithm is developed which uses the nonlinear representation of the system state equation and yields an adaptive optimal control law. In the following derivation it is shown that the optimal control is a closed loop feedback type of control law. This is a contribution in this field since all the existing algorithms which deal with the nonlinear form of the state equation produce an open loop type of optimal control law. Another disadvantage of the existing algorithms is the fact that the control law is computed iteratively for the given initial value of the states.

The developed control algorithm uses the nonlinear form of the state equation which is a function of system states and inputs. Taylor series expansion of the nonlinear state equation is first added then subtracted from the original nonlinear state equation. From the Taylor series expansion, the added terms contain the partial derivatives of the nonlinear function with respect to the states and inputs.
The partial derivatives form the state feedback and input matrices (A and B). This constitutes the first part of the state equation which is used to solve the Riccati equation. The original nonlinear function and the subtracted terms form the residual. The residual is computed in time as the state measurements become available. This approach makes it possible to express the residual as a function of time, but in reality the future values of the residual are unknown. However, the future values are predicted with utilization of an estimation algorithm. The estimation algorithm provides a time dependent functional form of the residual. The estimated residual forms the second part of the state equation. As more state measurements become available and time progresses, the accuracy in prediction of the future values of the residual improves significantly. The residual is the forcing function of a vector differential equation, and its solution becomes a part of the optimal control law.

The system dynamics is expressed by the state equation,

\[ \dot{X}(t) = A(t)X(t) + B(t)U(t) + f(t) \]  

and the output equation,

\[ Y(t) = C(t)X(t) \]

where \( f(t) \) is the additional function of time residual in the state equation which represents the residual in case of linearization. The performance measure is given by equation (3.2.3) with \( e(t) \) defined in equation (3.2.5). The initial mathematical
derivation of the optimal control law for the new form of the state equation is identical to the formulation of section 5. Thus the step by step procedure is not repeated, however the differences are summarized in the following.

1. Equations (3.5.1) and (3.5.2) will have the additional term $P^T(t)f(t)$.

2. Equation (3.5.3) will have the additional term $f(t)$.

3. The zero vector in equations (3.5.7) and (3.5.8) will be replaced by $f(t)$.

4. The $f_1(t)$ and $f_2(t)$ in the equations (3.5.12) through (3.5.18) will have different identities.

Substitution of equations (3.5.22) and (3.6.1) into equation (3.5.23) yield:

$$\dot{P}(t) = \dot{K}(t)X(t) + K(t)[(A(t)X(t) + B(t)[-R^{-1}(t)B^T(t)K(t)X(t) + R^{-1}(t)B^T(t)g(t)] + f(t)] - \dot{g}(t)$$

(3.6.3)

Using the equation (3.5.4) and substituting it in the equation (3.6.3) will result in:

$$- C^T(t)Q(t)C(t)X(t) + C^T(t)Q(t)Y(t) - A^T(t)K(t)X(t) + A^T(t)g(t) =$$

$$\left[\dot{K}(t) + K(t)A(t) - K(t)B(t)R^{-1}(t)K(t)\right]X(t) +$$

$$K(t)B(t)R^{-1}(t)B^T(t)g(t) + K(t)f(t) - \dot{g}(t)$$

(3.6.4)

Simplification of equation (3.6.4) leads to:
\[
[\dot{K}(t) + K(t)A(t) - K(t)B(t)R^{-1}(t)K(t) + A^T(t)K(t) + C^T(t)Q(t)C(t)]X(t) + \\
[K(t)B(t)R^{-1}(t)B^T(t)g(t) + K(t)f(t) - \dot{g}(t) - C^T(t)Q(t)Y_d(t)] - A^T(t)g(t) = 0
\] (3.6.5)

Again the quantity inside each bracket should be zero, consequently:

\[
\dot{K}(t) = -A^T(t)K(t) - K(t)A(t) + K(t)B(t)R^{-1}(t)B^T(t)K(t) - C^T(t)Q(t)C(t) \tag{3.6.6}
\]

\[
\dot{g}(t) = -[A^T(t) - K(t)B(t)R^{-1}B^T(t)]g(t) - C^T(t)Q(t)Y_d(t) + K(t)f(t) \tag{3.6.7}
\]

The final boundary values are identical to equations (3.5.30) and (3.5.31).

\[
K(t_f) = C^T(t_f)HC(t_f) \tag{3.6.8}
\]

\[
g(t_f) = C^T(t_f)HY_d(t_f) \tag{3.6.9}
\]

The summary of the final results are as follows. Given:

\[
\dot{X}(t) = A(t)X(t) + B(t)U(t) + f(t) \tag{3.6.1}
\]

\[
Y(t) = C(t)X(t) \tag{3.6.2}
\]

and the performance measure of:

\[
J(U) = \frac{1}{2}[Y(t_f) - Y_d(t_f)]^T H[Y(t_f) - Y_d(t_f)] + \\
\frac{1}{2}\int_0^{t_f} [Y(t) - Y_d(t)]^T Q(t)[Y(t) - Y_d(t)] + [U^T(t)R(t)U(t)]dt \tag{3.5.32}
\]
The optimal control law is:

\[ U^*(t) = R^{-1}(t)B^T(t)[g(t) - K(t)X(t)] \]  
(3.5.21)

Where \( K(t) \) is the solution of the Riccati equation:

\[ \dot{K}(t) = -A^T(t)K(t) - K(t)A(t) + K(t)B(t)R^{-1}(t)B^T(t)K(t) - C^T(t)Q(t)C(t) \]  
(3.6.6)

with final boundary value,

\[ K(t_f) = C^T(t_f)HC(t_f) \]  
(3.6.8)

And \( g(t) \) is the solution of the differential equation:

\[ \dot{g}(t) = -[A^T(t) - K(t)B(t)R^{-1}(t)B^T(t)]g(t) - C^T(t)Q(t)Y_d(t) + K(t)f(t) \]  
(3.6.7)

with final boundary value,

\[ g(t_f) = C^T(t_f)HY_d(t_f) \]  
(3.6.9)

The optimal trajectories are the solution of the linear differential equation:

\[ \dot{X}(t) = [A(t) - B(t)R^{-1}(t)B^T(t)K(t)]X(t) + B(t)R^{-1}(t)B^T(t)g(t) + f(t) \]  
(3.6.10)

\[ X(t_0) = X_0 \]  
(3.6.11)

All considerations regarding to controllability and observability discussed in section 5 are also true here.
As discussed in the beginning of this section, the residual $f(t)$ should be a known function of time for the entire control interval. This is necessary in order to be able to solve equation (3.6.7). In reality the residual is not known for the entire control interval and an estimation algorithm is used to predict its future values. As time progresses the estimation of the future values of the residual improves and it results in a better optimal control strategy.
CHAPTER 4. TESTING OF THE ALGORITHM

4.1: Introduction

The detail discussion of the optimal control theory with different types of quadratic performance measure is illustrated chapter 3. The developed control formulation intended for adaptive optimal control of HVDC terminals is documented in section 6 of chapter 3. A requirement for derivation of optimal control law in that section is the advance knowledge of the residual \( f(t) \) for the entire control interval.

For a nonlinear model of power system the residual \( f(t) \) is not known in advance and it has to be approximated. The approximation technique involves prediction of future values of function \( f(t) \) based on the previous measurements taken during the control interval. To investigate the validity of the inclusion of this approximation and its impact on the system performance, a small order test system is
selected for study purposes. The Linear Algebra and Systems (L-A-S) software package is used as a tool for this study.

4.2: L-A-S Language and Software Package

Linear Algebra and Systems (L-A-S) is a high level interactive conversational program language [35,36]. The L-A-S program is written in Fortran, but its use does not require the knowledge of Fortran programming. It is a useful tool for analysis and design of linear control systems. An L-A-S program consists of a sequence of operator statements. Each operator statement has the following structure:

\[
\text{label: input-field ( operator-field ) = output-field}
\]

The label is optional and is used in conjunction with program control operators for realization of recursive calculations. The input field contains the name(s) of array(s) used by the operator-field. The operator field contains mnemonic name of computational algorithm. The output-field contains the name(s) of array(s) where the result of the computation is stored. The five categories of operator statements are: input/output, linear algebra, systems, time response, and program control. A program written in L-A-S can be stored on disk-program-file (DPF). Similarly the computation result of an operator statement can be stored on the
disk-data-file (DDF). The operator statements are processed by L-A-S interpreter. The interpreter recognizes the algorithm which has to be executed, and performs the following:

1. It generates the operating system (OS) command for execution of the desired algorithm in a module.

2. It stores on common sharable area (CSA) a temporary file containing all numerical data necessary for execution of the algorithm.

3. It transfers the control to the generated operating system (OS) command.

The module commencing its execution by the generated OS command performs the following:

1. It halts the interpreter execution temporarily.

2. It reads the input data from CSA.

3. It executes the desired algorithm and calculates the output data.

4. It stores the output data on CSA.

5. It releases the L-A-S interpreter.
6. It stops the execution.

The original version of L-A-S language was developed in 1976 at the University of Campinas, Brazil by Professor Bingulac and his co-workers.

Two operators of L-A-S are used mainly for study of the test system, in addition to all other linear algebra and matrix manipulation operators. These two are identified by CRR and CE3 which solves the Riccati equation for a finite time interval and solves the state equation in forward time with discrete samples of input, respectively. In the following the two operators are explained.

I- CRR: Continuous Riccati Regulator

The operator statement that solves the Riccati equation has the following form:

\[ A, Q, S, H, DT, NT(CRR) = AK \]

where,

- **A**: A matrix, \( n \times n \)
- **Q**: Q matrix (weighting matrix), \( n \times n \)
- **S**: \( S = BR^{-1}B^T, n \times n \)
- **H**: H matrix (final boundary value), \( n \times n \)
- **DT**: \( \Delta t \) (time increment), \( 1 \times 1 \)
- **NT**: total time/DT, \( 1 \times 1 \)
- **AK**: solution of the Riccati equation, \( n \times (NT + 1) \)
II- CE3: Time Solution of the State Equation with Discrete Samples of Input

The operator statement that solves the state equation with discrete inputs has the following form:

\[ X_{0,NT,TT,A,B,U}(CE3) = X \]

where,

- \( X_{0} \): initial condition, \( n \times 1 \)
- \( NT \): number of time steps, \( 1 \times 1 \)
- \( TT \): total time, \( 1 \times 1 \)
- \( A \): \( A \) matrix, \( n \times n \)
- \( B \): \( B \) matrix, \( n \times r \)
- \( U \): input vector, \( NT \times r \)
- \( X \): solution of the state equation, \( NT \times n \)

4.3: Solution of Differential Equation with Given Final Value

Value

The CRR is used to solve the Riccati equation with its specified final boundary value. The CE3 is used for solving the \( g(t) \) equation to obtain the optimal control law and study the response of the optimally controlled system. However, the equation for \( g(t) \) cannot directly be solved by the operator CE3 due to availability of its final boundary value rather than the initial condition. Therefore,
some modifications are required to use a forward integration routine to integrate backward in time. Suppose,

\[ \dot{X}(t) = AX(t) + BU(t) \]  
\[ X(t_f) = X_f \]  

To solve the equation (4.3.1) with boundary condition of equation (4.3.2) by operator CE3, the necessary conversion is:

\[ \dot{M}(t) = [-A]M(t) + [-B]W(t) \]  
\[ M(t_0) = X_f \]  

where \( W(t) \) is the vector containing the discrete inputs in reverse order in time. Solution of equation (4.3.3) with initial condition of equation (4.3.4) is identical to solution of the original equation (equation (4.3.1) with final boundary value of equation (4.3.2)) but the solution is in reverse order in time. To obtain the proper order for solution of equation (4.3.1) from solution of equation (4.3.3) the following algebraic identity is used,
where $x_i(t)$ and $m_i(t)$ are the time responses of state $i$ of the vectors $X(t)$ and $M(t)$ stored in a column vector, respectively. Now suppose:

$$[B]U(t) = [B_1 \; B_2] [e^{-at} \; U_1(t)]^T \quad (4.3.6)$$

where $U_1(t)$ is the vector containing the discrete values of the inputs. Then the state equation is:

$$\dot{X}(t) = [A]X(t) + [B_1]e^{-at} + [B_2]U_1(t) \quad (4.3.7)$$

$$X(t_f) = X_f \quad (4.3.8)$$

Let us define:

$$N(t) = e^{-at} \quad (4.3.9)$$

then,

$$\dot{N}(t) = -ae^{-at} \quad (4.3.10)$$
Equation (4.3.7) can be written as:

\[
\begin{vmatrix}
\dot{X}(t)
\end{vmatrix} = \begin{vmatrix}
A & B_1 \\
0 & -\alpha
\end{vmatrix}
\begin{vmatrix}
X(t)
\end{vmatrix} + \begin{vmatrix}
B_2
\end{vmatrix} U_1(t)
\]  \hspace{1cm} (4.3.11)

\[
\begin{vmatrix}
X(t_f) \\
N(t_f)
\end{vmatrix} = \begin{vmatrix}
X_f \\
e^{-\alpha t_f}
\end{vmatrix} \hspace{1cm} (4.3.12)
\]

To solve the above equation by the operator CE3 the conversion method of equation (4.3.3) and (4.3.4) is used. In other words,

\[
\dot{M}(t) = \begin{vmatrix}
-A & -B_1 \\
0 & -\alpha
\end{vmatrix} M(t) + \begin{vmatrix}
-B_2
\end{vmatrix} W(t) \hspace{1cm} (4.3.13)
\]

\[
M(t_0) = \begin{vmatrix}
X_f \\
e^{-\alpha t_f}
\end{vmatrix} \hspace{1cm} (4.3.14)
\]

After solution of equation (4.3.13) with initial condition (4.3.14) the last state can be eliminated and the application of the identity of equation (4.3.5) will yield the solution of the original differential equation in proper order.
4.4: Definition of the Test Problem

The small order test system is an aircraft landing problem [9]. The aircraft landing problem is a fourth order system and it is only concerned with the final phase of landing, that is, the final 100 feet of aircraft descent in twenty seconds. It is intended to guide the aircraft in the longitudinal plane such that the aircraft altitude follows a desired exponential path. The longitudinal motion of the aircraft during the final phase of landing is obtained only by control of the elevator deflection angle. The assumption is that the velocity of aircraft is maintained constant during the landing. The general definition of aircraft coordinates and angles are shown in figure 7, and the transfer function relating the elevator deflection and pitch angle is as follows.

\[ \theta(s) = \frac{K_s(T_s s + 1)}{s\left(\frac{s^2}{\omega_s^2} + \frac{2\zeta_s}{\omega_s} + 1\right)} \delta(s) \quad (4.4.1) \]

where,

- \( \theta \): pitch angle
- \( \delta \): elevator deflection angle
- \( K_s \): short period gain
- \( T_s \): path time constant
- \( \omega_s \): short period resonant frequency
- \( \zeta \): short period damping ratio
Figure 7. Aircraft Coordinates and Angles
The numerical values of the related parameters are:

\[ K_i = -0.95 \text{ per second} \]
\[ T_i = 2.5 \text{ second} \]
\[ \omega_r = 1.0 \text{ rad. per second} \]
\[ \zeta = 0.5 \]

The altitude is related to the pitch angle by,

\[ h(s) = \frac{V}{s(T_s^2 + 1)} \theta(s) \]  \hspace{1cm} (4.4.2)

where the velocity \( V \) of the aircraft is 256 feet per second.

Equations (4.4.1) and (4.4.2) are used for formulation of the system state equation. The system constant matrices, states, and input are formulated and computed in the following.

\[ A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & 0 \\ 1 & 0 & 0 & 0 \\ 0 & a_{32} & a_{33} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \]  \hspace{1cm} (4.4.3)

\[ B = \begin{bmatrix} b_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \]  \hspace{1cm} (4.4.4)
\( C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \) (4.4.5)

\[ a_{11} = \frac{1}{T_s} - 2\zeta \omega_s \rightarrow a_{11} = -0.6 \] (4.4.6)

\[ a_{12} = \frac{2\zeta \omega_s}{T_s} - \omega_s^2 - \frac{1}{T_s^2} \rightarrow a_{12} = -0.76 \] (4.4.7)

\[ a_{13} = \frac{1}{VT_s^2} - \frac{2\zeta \omega_s}{V T_s} + \frac{\omega_s^2}{V} \rightarrow a_{13} = -0.00297 \] (4.4.8)

\[ a_{32} = \frac{V}{T_s} \rightarrow a_{32} = 102.4 \] (4.4.9)

\[ a_{33} = -\frac{1}{T_s} \rightarrow a_{33} = -0.4 \] (4.4.10)

\[ b_1 = \omega_s^2 K_s T_s \rightarrow b_1 = -2.375 \] (4.4.11)

\[ X(t) = [\dot{\theta}(t) \quad \theta(t) \quad \dot{h}(t) \quad h(t)]^T \] (4.4.12)

\[ U(t) = [\delta(t)] \] (4.4.13)

The desired pitch angle is zero, and the desired altitude is:
\[ h_d(t) = 100e^{-0.2t} \quad , \quad 0 \leq t \leq 20 \] (4.4.14)

The matrices H, Q, and R are defined in the following.

\[ H = [0] \] (4.4.15)

\[ Q: \quad q_{44} = 0.0001 \quad , \quad \text{all others are 0} \] (4.4.16)

\[ R = [1] \] (4.4.17)

And the desired vector is then,

\[ X_d(t) = [0 \quad 0 \quad -20e^{-0.2t} \quad 100e^{-0.2t}]^T \] (4.4.18)

The initial value of the state vector is:

\[ X_0(t) = [0 \quad -0.0781 \quad -20 \quad 100]^T \] (4.4.19)

### 4.5: Formulation of the Test Problem

The detail information about the test problem and its simulation is given in the last section, however its translation from a tracking problem to a regulator problem according to the developed control algorithm is illustrated in the following. The state equation is:
\[ \dot{X}(t) = AX(t) + BU(t) + f(t) \]  
(4.5.1)

where \( f(t) \) is a known vector. The performance measure is:

\[
J(U) = \frac{1}{2} [X(t_f) - X_d(t_f)]^T H [X(t_f) - X_d(t_f)] + \\
\frac{1}{2} \int_0^{t_f} \left( [X(t) - X_d(t)]^T Q [X(t) - X_d(t)] + [U^T(t) RU(t)] \right) dt
\]  
(4.5.2)

Using the translation,

\[ Z(t) = X(t) - X_d(t) \]  
(4.5.3)

then \( X(t) \) is:

\[ X(t) = Z(t) + X_d(t) \]  
(4.5.4)

Now substitution of the equation (4.5.4) into the equation (4.5.1) yields,

\[ \dot{Z}(t) + \dot{X}_d(t) = AZ(t) + AX_d(t) + BU(t) + f(t) \]  
(4.5.5)

In other words the state equation in terms of the new variable \( Z \) is:

\[ \dot{Z}(t) = AZ(t) + BU(t) + r(t) \]  
(4.5.6)

where,

\[ r(t) = AX_d(t) - \dot{X}_d(t) + f(t) \]  
(4.5.7)
And the performance measure is:

\[ J(U) = \frac{1}{2} Z^T(t) H Z(t) + \frac{1}{2} \int_{t_0}^{t_f} Z^T(t) Q Z(t) + U^T(t) R U(t) dt \] (4.5.8)

Now with the state equation of the equation (4.5.6) and the performance measure of the equation (4.5.8), the optimal control law can be obtained from the final results of section 6 of chapter 3. Thus, the optimal control law for the above equations is the equation (3.5.21) with \( X(t) \) replaced by \( Z(t) \). Also \( K(t) \) and \( g(t) \) are the solution of equations (3.6.6) and (3.6.7), with final boundary value of equations (3.6.8) and (3.6.9), and \( Y_d \) equal to zero.

### 4.6: Simulation of the Test System

Simulation of the aircraft landing problem serves as a test for verification of the validity of the developed control algorithm for systems with state equation of (4.5.1) which contain an additional function of time term. The vector \( f(t) \) when not known in advance will be extrapolated for the future control interval based on the history of its past values. A least square technique is used for prediction of the future values of the unknown vector. Verification of the validity of the developed control algorithm is shown by the following three simulation phases:

1. Achievement of the optimal control law for the aircraft landing problem defined by equation (4.4.1) through (4.4.19) with \( f(t) \) equal to zero.
Simulation of the optimally controlled system and storage of the trajectory of the first state, \( x_1(t) \).

2. Achievement of the optimal control law for the aircraft landing problem when \( a_{11} \) entry of the state feedback matrix, \( A \), is varied and \( f(t) \) is no longer zero, but its value is known. The function \( f(t) \) is adjusted such that the state equation is mathematically correct, pending the knowledge of \( x_1(t) \) which is furnished from phase 1. Simulation of the optimally controlled system with a known \( f(t) \) (excluding \( f(t) = 0 \)) and comparison of the results with the results of phase 1.

3. Prediction of \( x_1(t) \) during the multi-stage control interval from the previous values which were stored. The multi-stage control interval is formed by splitting the control interval into several equal time periods. Then for the first stage, \( f(t) \) which is linearly dependent on \( x_1(t) \) is assumed to maintain its constant value at initial time for the entire control interval. Based on this approximation the optimal control is computed, and the optimally controlled system is simulated up to the end of the first period. Then the values of \( x_1(t) \) is used to extrapolate the future values of \( f(t) \) for the reminder of the control interval and computation of the optimal control for the second period. This procedure is continued until the final control time is reached. Finally the simulation results of this phase are compared with the results of phase 1.

CHAPTER 4. TESTING OF THE ALGORITHM
The program written in L-A-S language for simulation of aircraft landing problem is illustrated in appendix A. The program consists of eight blocks which are executed in sequence as explained in the appendix.

4.7: Simulation Results of the Test System

The L-A-S program developed in appendix A is used for simulation of the three phases that are discussed in the previous section. The simulation results are illustrated in figures 8 through 17.

For phase 1 of the aircraft simulation, the 1,1 entry of A matrix is selected to be the original value which is -0.6. This selection causes the vector F in equation (A.1) to assume a zero value. Since F is zero, the value of \( x_1(t) \) needed according to equation (A.18) is of no importance and for simplicity it is chosen to be zero. The simulation results for phase 1 are shown in figures 8 and 9. The trajectories of the derivative of pitch angle and the pitch angle for the aircraft are illustrated in figure 8. The trajectories of the derivative of altitude and the altitude for the aircraft are shown in figure 9. These state trajectories are saved and used as the reference trajectories for the other two phases of the simulation.

For phase 2 of the aircraft simulation, three values of -0.59, -0.5, and 0.0 are selected for 1,1 entry of the A matrix. The \( x_1(t) \) from phase 1 is placed on a disk-data-file such that it is read by the program when needed. This is necessary since
Figure 8. Phase 1 Simulation of Aircraft: Derivative of pitch angle and pitch angle
Chapter 4. Testing of the Algorithm

Figure 9. Phase 1 Simulation of Aircraft: Derivative of altitude and altitude.
in this phase the function $f(t)$ is no longer zero. The change in the value of 1,1 entry of the A matrix results in a different solution of the Riccati equation. Hence, different sets of feedback gains are computed for each different selection of the 1,1 entry of A matrix. Since $x_i(t)$ is known and it is the correct trajectory of the first state from phase 1, the solution of the equation (A.20) is exact. The solution of this equation for phase 1 did not contain the influence of $f(t)$ since the vector $F$ of equation (A.1) was zero. For phase 2 the solution of equation (A.20) includes the influence of $f(t)$, and it makes a contribution to computation of the optimal input. Then the question that remains to be answered is whether the contribution is correct to produce the same trajectories as phase 1. In other words, proper compensation for computation of the system optimal input is made through the solution of equation (A.20) as a result of a change in some entry of A matrix which resulted in a different solution of the Riccati equation. If the trajectories of the phase 1 and 2 are identical, then the answer is yes and the proper compensation is made through the solution of equation (A.20).

Figure 10 and 11 show the comparison between the results of simulation for phase 1 and phase 2, when the 1,1 entry of A matrix is -0.59 for phase 2. Figure 10 illustrate the trajectories of the derivative of the pitch angle and the pitch angle, while figure 11 show the trajectories of the derivative of altitude and the altitude.

Figure 12 and 13 illustrate the comparison between the results of simulation for phase 1 and phase 2, when the 1,1 entry of A matrix is -0.50 for phase 2. Figure 12 show the trajectories of the derivative of the pitch angle and the pitch angle,
Figure 10. Phase 2 Simulation of Aircraft: Derivative of pitch angle and pitch angle, -0.60 & -0.59
Figure 11. Phase 2 Simulation of Aircraft: Derivative of altitude and altitude, -0.60 & -0.59
while figure 13 illustrate the trajectories of the derivative of altitude and the altitude.

Figure 14 and 15 show the comparison between the results of simulation for phase 1 and phase 2, when the 1,1 entry of A matrix is 0.0 for phase 2. Figure 14 illustrate the trajectories of the derivative of the pitch angle and the pitch angle, while figure 15 show the trajectories of the derivative of altitude and the altitude.

From the figures 10 through 15 it becomes evident that solution of the equation (A.20) makes proper compensation, independent of the three choices for 1,1 entry of A matrix, when the exact knowledge of the function $f(t)$ is available. This due to the fact that all the state trajectories almost coincide on the top of the reference trajectories. So, if some of the system dynamics are expressed as a known function of time, then according to the above simulation results, the $g(t)$ of the optimal control law, from the developed algorithm, makes the proper compensation for the changes in feedback gains through the computation of the equation (3.6.7) which contain $f(t)$ as the forcing function.

However, this is of limited practical benefit since the algorithm requires the exact knowledge of the states of the system which are generally unknown for future times. This is why in phase 3 it is attempted to estimate the future values of the function $f(t)$ from the past history of the function by utilization of an estimation algorithm.
Figure 12. Phase 2 Simulation of Aircraft: Derivative of pitch angle and pitch angle, -0.60 & -0.50
Figure 13. Phase 2 Simulation of Aircraft: Derivative of altitude and altitude, -0.60 & -0.50
Figure 14. Phase 2 Simulation of Aircraft: Derivative of pitch angle and pitch angle -0.60 & 0.0
Figure 15. Phase 2 Simulation of Aircraft: Derivative of altitude and altitude, -0.60 & 0.0
For phase 3 of the aircraft simulation, the zero value for the 1,1 entry of the A matrix is selected. Also, the \( x_1(t) \) is assumed not to be known which effectively results in \( f(t) \) to be unknown. Then the function \( x_1(t) \) is estimated by the use of a least square curve fitting algorithm, and dividing the 20 second control interval into five 4 second control periods. For the first 4 second of the control period \( x_1(t) \) assumed a constant value equal to its initial value. During this period, the derivative of the pitch angle which is the first state is sampled at every 0.4 seconds. The total of 11 samples are fed into a least square algorithm to find the coefficients of a polynomial expressing the dynamics of the first state as a function of time. The maximum order of this polynomial is limited to three, and the coefficients are as follows:

\[
\beta(0) = + 0.042636 \\
\beta(1) = - 0.016091
\]

where,

\[
x_1(t) = \beta(0) + \beta(1) t + \beta(2) t^2 + \beta(3) t^3 \tag{4.7.1}
\]

Now value of the first state is predicted for the remaining control interval from the equation (4.7.1) and the polynomial coefficients which are obtained from sampling of the first state in the first control period. The simulation of the next 4 seconds proceeded with this estimate of \( x_1(t) \). Then the next set of polynomial coefficients are obtained from the 10 new samples collected during the second control period plus the 11 old samples. The coefficients are:
\[ p(0) = + 0.036819 \]
\[ p(1) = - 0.014906 \]
\[ p(2) = + 0.001420 \]

The same process is continued for the third control period and the coefficients of the polynomial for the 31 samples are:

\[ p(0) = + 0.039880 \]
\[ p(1) = - 0.019841 \]
\[ p(2) = + 0.003020 \]
\[ p(3) = - 0.000140 \]

And the final set of the polynomial coefficients are obtained from the sampling of \( x_1(t) \) up to the end of the fourth period. The coefficients obtained for the simulation of the last 4 seconds are:

\[ p(0) = + 0.033757 \]
\[ p(1) = - 0.012511 \]
\[ p(2) = + 0.001387 \]
\[ p(3) = - 0.000047 \]

The simulation results for phase 3 which involves the estimation of the first state is illustrated in figure 16 and 17 which are compared with the reference trajectories of phase 1. From these state trajectories a few facts can be concluded. First,
since there are no penalties on the first three states, there are some oscillations in the trajectory of these states. Second, the least square estimation technique causes undesirable oscillations of the first three states during the last eight seconds of the landing. Third, inaccuracies involved in estimation of $x_1(t)$ has small impact on the fourth state, which is the one whose deviations being penalized, so its trajectory remains very close to the reference trajectory.

In order to eliminate the undesirable oscillations, a different estimation algorithm should be used to predict the future values of the first state. This can be simply accomplished, but was not done here. The simulation of aircraft landing is only for testing the feasibility of the developed control algorithm and is of no further interest. An alternative estimation algorithm will be considered for the power system.

The aircraft simulation results are very satisfactory and it has proven the validity of the developed optimal control algorithm, despite the poor estimator performance. With the confidence on the validity of the developed algorithm the work is pursued in the following chapters for implementation of this algorithm on a large scale AC/DC power system.
Figure 16. Phase 3 Simulation of Aircraft: Derivative of pitch angle and pitch angle, estimated $f(t)$
Figure 17. Phase 3 Simulation of Aircraft: Derivative of altitude and altitude, estimated f(t)
CHAPTER 5. POWER SYSTEM MODELING

5.1: Introduction

The optimal control theory and the developed control algorithm as discussed in chapter 3 requires the dynamic model of the system. A power system in general consist of generation systems, transmission systems, and loads. The generation system includes the turbine-generator sets, turbine speed control systems, and generator excitation control. The transmission system includes the power transformers, the AC transmission lines, and in some cases the DC transmission links. The DC transmission link includes converter stations, and their control system in addition to the DC transmission lines. The power system which is considered here is a large scale system and contains all of the above components. The power system modeling technique introduced here uses the positive sequence represen-
tation of the system, and the system parameters are obtained from data corresponding to the system one line diagram.

In the following sections the generator model, exciter model, HVDC model, and the network modeling method is illustrated. Finally the overall system representation which is compatible with the developed control algorithm is explained.

5.2: Generator Model

Proper generator modeling is an important factor for investigation of power system dynamics. A number of well accepted synchronous machine models are available which use the Park frame of reference [37]-[43]. The range of complexity of these models are in the following ascending order.

1. The model considers a constant voltage behind the transient reactance $X'_{d}$.

2. The model considers the d-axis transient effects and it requires one differential equation for $\dot{E}'_{q}$.

3. The model considers both the d and q-axis transient effects and requires two differential equations for $\dot{E}'_{q}$ and $\dot{E}'_{d}$.
4. The model in addition to the d-axis transient effects considers the d and q-axis subtransient effects, and it requires three differential equations for $\dot{E'}_q$, $\dot{E''}_q$, and $\dot{E''}_d$.

5. The model considers both the d and q-axis transient and subtransient effects, and it requires four differential equations for $\dot{E'}_q$, $\dot{E'}_d$, $\dot{E''}_q$, and $\dot{E''}_d$.

In addition to the number of differential equations required for the above models, there are two additional differential equations which represent the mechanical equations of motion (swings).

\[
\frac{2H}{\omega_0} \dot{\omega} + D(\omega - \omega_0) = P_m - P_e \tag{5.2.1}
\]

\[
\dot{\delta} = \omega - \omega_0 \tag{5.2.2}
\]

Where,

- $\omega$ : generator speed, rad/sec
- $\delta$ : generator rotor angle, rad
- $\omega_0$ : generator reference speed, rad/sec
- $H$ : inertia constant, per unit
- $D$ : damping coefficient, per unit/rad/sec
- $P_e$ : electrical power, per unit
- $P_m$ : mechanical shaft power, per unit
The optimal control of the power system that we are concerned with is a dynamic stability problem, and as documented in reference [44,45], model 2 suffices the requirement for this study. The phasor diagram of figure 18 represents the synchronous generator in transient state for model 2. From references [37,39,43], the following equations express the generator phasor relations and dynamics.

\[
E_q = E_t + r_d I_t + jx_q I_t \tag{5.2.3}
\]

\[
E_l = E_t + r_d I_t + jx_d I_d + jx_q I_q \tag{5.2.4}
\]

\[
E'_q = E_q - (x_q - x'q)I_d \tag{5.2.5}
\]

\[
E_l = E_q + (x_d - x_q)I_d \tag{5.2.6}
\]

\[
E_l = E'_q + (x_d - x'q)I_d \tag{5.2.7}
\]

\[
\dot{E'}_q = \frac{1}{T_{d0}}(E_{f_d} - E_l) \tag{5.2.8}
\]

where,

\(E_t\) : terminal voltage

\(E_q\) : voltage behind q-axis synchronous reactance

\(E'_q\) : voltage proportional to field flux linkages

\(E_l\) : voltage proportional to field current

\(E_{f_d}\) : field voltage acting along the q-axis

\(I_t\) : terminal current

CHAPTER 5. POWER SYSTEM MODELING
Figure 18. Synchronous Generator Phasor Diagram
$I_d$ : d-axis component of the terminal current

$r_o$ : armature resistance

$x_q$ : q-axis synchronous reactance

$x_d$ : d-axis synchronous reactance

$x_d'$ : d-axis transient reactance

$T_{\infty}$ : d-axis open circuit time constant

It should be noted that all voltages and currents in equations (5.2.3) and (5.2.4) are complex quantities, while in the remaining equations they are real valued quantities. The above synchronous generator model is also known as the third order generator model, since only three differential equations express the machine dynamics. They are equations (5.2.1), (5.2.2), and (5.2.8).

### 5.3: Exciter Model

A wide variety of standard exciter models are also available for control of the generator field voltage [39,40,46]. The IEEE Type I exciter model as shown in figure 19 is adapted for this study. The time domain differential equation representation of the exciter system is as follows.

$$
\dot{V}_y = \frac{1}{T_r}(E_t - V_y)
$$

(5.3.1)
Figure 19. IEEE Type 1 Exciter Model
\[ \dot{V}_r = \frac{1}{T_a} \left[ -V_r + \left( -V_y - V_z + V_{ref} + V_s \right) K_d \right] \] (5.3.2)

\[ \dot{V}_z = \frac{1}{T_f} \left\{ \frac{K_f}{T_e} \left[ V_r - (S_e + K_e) E_{fd} \right] - V_z \right\} \] (5.3.3)

\[ \dot{E}_{fd} = \frac{1}{T_e} \left[ V_r - (S_e + K_e) E_{fd} \right] \] (5.3.4)

where,

- $E_r$ : terminal voltage
- $E_{fd}$ : field voltage
- $V_y$ : output voltage of the filter
- $V_z$ : feedback voltage
- $V_r$ : output voltage of the regulator
- $V_{ref}$ : exciter reference voltage
- $V_s$ : auxiliary voltage for control purposes
- $K$ : block gain
- $T$ : block time constant

This is a fourth order exciter model. Thus, a generator and its excitation system forms a system of order seven. The auxiliary voltage $V_s$ is used as the exciter control for the developed control algorithm.
5.4: HVDC Model

The quasi steady state model of the HVDC is used for representation of the DC link [33,34]. This model expresses the DC voltage as a function of the primary converter AC voltage, converter transformer tap ratio, valve ignition angle, transformer commutating reactance, and the DC current. The DC link single line diagram is shown in figure 20. The DC voltages at rectifier and inverter are obtained from:

\[
V_{dr} = \frac{3\sqrt{2}}{\pi} q_r E_r \cos \alpha - \frac{3}{\pi} X_c r I_{dc} \tag{5.4.1}
\]

\[
V_{di} = \frac{3\sqrt{2}}{\pi} q_i E_i \cos \gamma - \frac{3}{\pi} X_c i I_{dc} \tag{5.4.2}
\]

\[
V_{dr} = R_d I_{dc} + V_{di} \tag{5.4.3}
\]

where,

\[ V_{dr} \] : rectifier DC voltage
\[ V_{di} \] : inverter DC voltage
\[ I_{dc} \] : DC current
\[ E_r \] : primary AC voltage at rectifier
\[ E_i \] : primary AC voltage at inverter
\[ q_r \] : transformer tap ratio at rectifier
\[ q_i \] : transformer tap ratio at inverter
\[ X_c \] : transformer short circuit reactance at rectifier
Figure 20. DC Link Single Line Diagram
\( X_d \): transformer short circuit reactance at inverter

\( R_d \): DC line resistance

\( \alpha \): rectifier ignition angle

\( \gamma \): inverter extinction angle

The relationship between the two terminal stations is expressed by equation (5.4.3) as an algebraic equation. Even though this is not an exact relationship due to the inductance of smoothing reactors and the DC line, the associated time constant is small and it can be neglected for dynamic stability studies [58]. The time constant of the control circuitry is also neglected for the same reason. Equations (5.4.1) through (5.4.3) contain the actual values of the parameters. The conventional base values from reference [47] are adapted for expression of the equations in per unit as as follows.

\[
V_{dr}^{pu} = a_r E_r^{pu} \cos \alpha - \frac{n}{6} X_{cr}^{pu} I_{dc}^{pu} \quad (5.4.4)
\]

\[
V_{di}^{pu} = a_i E_i^{pu} \cos \gamma - \frac{n}{6} X_{ci}^{pu} I_{dc}^{pu} \quad (5.4.5)
\]

The DC current in terms of AC terminal voltage, transformer tap ratio, transformer short circuit reactance, ignition angle, and the overlap angle is obtained from the integration of the AC voltage during the commutating process[43]. The equation for rectifier DC current in terms of the actual parameters is:

\[
I_{dc} = \frac{a_r E_r}{\sqrt{2} X_{cr}} [\cos \alpha - \cos(\alpha + \alpha)] \quad (5.4.6)
\]
where \( u \) is the overlap angle. The above equation in per unit is:

\[
I_{dc}^{pu} = \frac{3}{\pi} \frac{a_r E_r^{pu}}{X_{cr}^{pu}} \left[ \cos \alpha - \cos(\alpha + u) \right]
\]  

(5.4.7)

From the above equation the overlap angle is computed.

\[
u = \cos^{-1} \left[ \cos \alpha - \frac{X_{cr}^{pu} I_{dc}^{pu}}{a_r E_r^{pu} \frac{\pi}{3}} \right] - \alpha
\]  

(5.4.8)

The Fourier analysis of the AC current waveform at the primary side of the rectifier transformer results in a relationship between the AC primary current and the DC current in terms of actual values as follows.

\[
I_{ac} = a_r k_r \frac{\sqrt{6}}{\pi} I_{dc}
\]  

(5.4.9)

The equation (5.4.9) in per unit is,

\[
I_{ac}^{pu} = a_r k_r I_{dc}^{pu}
\]  

(5.4.10)

where \( k_r \) is a unit-less quantity obtained from the following equation, providing \( u \) does not exceed 60 degrees [43].

\[
k_r = \sqrt{\frac{\cos 2\alpha - \cos 2(\alpha + u)}{4[\cos \alpha - \cos(\alpha + u)]^2} + \frac{[2u + \sin 2\alpha - \sin 2(\alpha + u)]^2}{4[\cos \alpha - \cos(\alpha + u)]}}
\]  

(5.4.11)

The above equations are necessary to compute the real and reactive powers of the DC link at the rectifier station. Neglecting the converter losses,
\[ P_{ac}^{pu} = P_{dr}^{pu} = V_{dr}^{pu} I_{ac}^{pu} \]  \hspace{1cm} (5.4.12)

where \( P_d \) is the per unit DC power at rectifier. The per unit reactive power for support of the DC link is computed from:

\[ Q_{dr}^{pu} = E_r^{pu} I_{ac}^{pu} \sin \varphi \]  \hspace{1cm} (5.4.13)

where \( \varphi \) is the power factor angle and according to reference [43] it is computed from:

\[ \cos \varphi = \frac{1}{2k_r} [ \cos \alpha + \cos(\alpha + u) ] \]  \hspace{1cm} (5.4.14)

For an inverter \( \alpha \) is replaced by \( \gamma \) and the subscript \( r \) is replaced by \( i \) in equations (5.4.6) through (5.4.14). Therefore, the DC real and reactive powers are highly dependent on the converter ignition and extinction angles.

**5.5: Interconnected Generator-Network-HVDC Model**

The models of different components of a large scale power system are discussed in previous sections of this chapter. These components are connected through a complex transmission network. Figure 21 illustrates the interconnection of different components of the system. The dynamic elements which directly interact with the network are the generators and the HVDC link. The exciter uses the
terminal voltage of the generator to compute the error between this voltage and
the exciter reference voltage. This error drives the exciter control circuitry to ob-
tain the new value of the field voltage. This new field voltage in turn changes the
terminal voltage of the generator in order to reduce the error between the actual
voltage and reference voltage. The synchronous generator provides the real and
reactive power needs of the network. The HVDC link takes real and reactive
power from the network at the rectifier terminal, and it provides real power to the
network at the inverter terminal while taking reactive power from the network
at this terminal.

In the present study the loads are represented by constant shunt impedances in
the simulation program because of the simplicity of load modeling. This does not
impose a limitation on the loads which are not constant impedances. In the
admittance matrix reduction technique it is mandatory to represent the loads by
constant shunt impedances. However, the departure from constant impedances
are reflected in the state measurements obtained from the actual system and the
residual computation. For the same reason, the HVDC transmission system is
represented by constant impedances as follows:

\[ R_r = \frac{E_r^2}{P_{dr}} \quad , \quad X_r = \frac{E_r^2}{Q_{dr}} \]  \hspace{1cm} (5.5.1)

\[ R_i = -\frac{E_i^2}{P_{di}} \quad , \quad X_i = \frac{E_i^2}{Q_{di}} \]  \hspace{1cm} (5.5.2)
SYNCHRONOUS GENERATOR

AC NETWORK

EXCITER

HVDC LINK

Figure 21. Interconnected Generator-Network-HVDC Link Diagram
Then the overall transmission system is expressed by the network admittance matrix $Y_{bus}$. The dimension of the network admittance matrix is equal to the number of AC buses. To form the connection of the generators to the network, their q-axis synchronous reactances are used to form additional buses for the network. Hence, the dimension of the network admittance matrix is increased by the number of generators in the system. The network reduction technique is used to reduce the size of the admittance matrix by preserving the identity of the generator q-axis buses in addition to the reference bus and eliminating all the other system buses. Therefore,

$$
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix} =
\begin{bmatrix}
Y_1 & Y_2 \\
Y_3 & Y_4
\end{bmatrix}
\begin{bmatrix}
E_1 \\
E_2
\end{bmatrix}
$$

(5.5.3)

where,

- $I_1$: vector of current injections at non-generating buses
- $I_2$: vector of current injections at generating buses
- $E_1$: vector of voltages at non-generating buses
- $E_2$: vector of voltages at generating buses
- $Y$: admittance matrix

Since the vector $I_1$ is the injection vector for the non-generating buses, then it is identically zero. Thus,

$$
0 = Y_1E_1 + Y_2E_2
$$

(5.5.4)

$$
I_2 = Y_3E_1 + Y_4E_2
$$

(5.5.5)
Equation (5.5.4) is solved for $E_1$,

$$E_1 = -Y_1^{-1}Y_2E_2$$  \hspace{1cm} (5.5.6)

Substitution of equation (5.5.6) into equation (5.5.5) leads to:

$$I_2 = (Y_4 - Y_3Y_1^{-1}Y_2)E_2$$  \hspace{1cm} (5.5.7)

In other words,

$$I_2 = Y_{eq}E_2$$  \hspace{1cm} (5.5.8)

where,

$$Y_{eq} = Y_4 - Y_3Y_1^{-1}Y_2Y_4$$  \hspace{1cm} (5.5.9)

This equivalent admittance matrix represents the reduced order static model of the network.

In the following the generator dynamic equations are modified in order to include interaction of the generators through the network, and to form the desirable state variable form of the equations. The complex power of generator $i$ is:

$$S_i = E_{qi}I_{ti}^*$$  \hspace{1cm} (5.5.10)
where \( j \) is the number of generators in the system, \( E_{qi} \) is the fictitious bus voltage behind the q-axis synchronous reactance, and also \( I_n \) is obtained from equation (5.5.8) which is a component of \( I_z \) vector. Then,

\[
S_i = E_{qi} \left[ \sum_{k=1}^{j+1} Y_{ik} E_k \right]^* \quad i = 1, \ldots, j \quad (5.5.11)
\]

Now the real and reactive powers are:

\[
P_i = Re[S_i] \quad i = 1, \ldots, j \quad (5.5.12)
\]

\[
Q_i = Im[S_i] \quad i = 1, \ldots, j \quad (5.5.13)
\]

It also can be shown that the magnitude of the generator d-axis current can be computed from:

\[
I_{di} = \frac{Q_i}{E_{qi}} \quad i = 1, \ldots, j \quad (5.5.14)
\]

Now substitution of \( P_i \) relation of equation (5.5.12) into the equation (5.2.1) for each generator forms the primitive state variable form of the swing equation which will be modified later. Equation (5.2.2) remains unchanged, and the third dynamic equation which is the equation (5.2.8) is also modified by substitution of equation (5.2.6) into this equation. Then for a cylindrical rotor generator,

\[
\dot{E}_{q} = \frac{1}{T_{d0}} [E_{fd} - E_q] \quad (5.5.15)
\]

CHAPTER 5. POWER SYSTEM MODELING
where in the next chapter replacement of $\dot{E}'_q$ by $\dot{E}_q$ is illustrated with use of the example system.

The translation of the power system from the original representation to the reduced order model as discussed in this section is shown in figure 22. The loads and the HVDC are modeled by the constant shunt impedances. These impedances in addition to the q-axis synchronous reactance of generators are used for computation of the admittance matrix. Then the reduced order equivalent model of the interconnected network is obtained according to equation (5.5.9).

Before leaving this section it is important to notice that the network model completely ignores the effect of the DC power change. To include this factor, the network sensitivity technique is used to modify the swing equation (5.2.1). This method shows the approximate change in the line flows for changes in the generation on the network [48,49,50]. The generation shift factor is defined as the per unit increase in the loading of a line as a result of the generation increase at a bus by one per unit, and the consequent generation reduction by one per unit at the reference bus [49].

The rectifier and inverter AC buses are considered as generation buses for network sensitivity method, where the rectifier real power is a negative generation. However, the rectifier and inverter powers are related by the following derivation.
Figure 22. Translations in Power System Modeling
Thus, equation (5.5.20) shows the relationship between the rectifier and inverter powers as a function of inverter DC voltage and DC line resistance. Now the rate of change of the DC power at rectifier in terms of the change in inverter power is:

\[
\frac{dP_{dr}}{dP_{di}} = 1 + \frac{2R_d}{V_{di}^2}P_{di}
\]  

(5.5.21)

or,

\[
\Delta P_{dr} = [1 + \frac{2R_d}{V_{di}^2}P_{di}^0] \Delta P_{di}
\]  

(5.5.22)
From the network sensitivity method (generation shift factor) a change in rectifier power causes a change in the generator i power by the following expression:

\[ G_{i,r} = \frac{\Delta P_i}{\Delta P_{dr}} \]  
\[ \text{(5.5.23)} \]

where,

\[ G_{i,r} = -\frac{X_{r,ti}}{x_{qi}} \]  
\[ \text{(5.5.24)} \]

And,

\[ G_{i,i} = \frac{\Delta P_i}{\Delta P_{di}} \]  
\[ \text{(5.5.25)} \]

where,

\[ G_{i,i} = -\frac{X_{i,ti}}{x_{qi}} \]  
\[ \text{(5.5.26)} \]

The parameters of equations (5.5.23) through (5.5.26) are defined in the following.

- \( P_i \): electrical power of generator i
- \( P_{dr} \): rectifier DC power
- \( P_{di} \): inverter DC power
- \( G_{i,r} \): generation i shift factor for a change in rectifier power
- \( G_{i,i} \): generation i shift factor for a change in inverter power
$x_{qi}$ : q-axis synchronous reactance of generator i

$X_{i,n}, X_{i,n}$ : corresponding components of the reactance matrix

The inverter power is a positive generation while the rectifier power is a negative generation. In order to maintain consistency, the sign of the generation shift factor corresponding to the change in rectifier power is reversed and the rectifier power increment is assumed to be a positive quantity. Since the network sensitivity technique uses the linearized equations of the system, the superposition theorem applies and the total change in the generator i power as a result of a change in DC link power transfer is:

$$\Delta P_i = - G_{i,r} \Delta P_{dr} + G_{i,i} \Delta P_{di}$$ (5.5.27)

where $\Delta P_{dr}$ and $\Delta P_{di}$ are both positive quantities in equation (5.5.27). Now substitution of equation (5.5.22) into equation (5.5.27) results in:

$$\Delta P_i = - G_{i,r} [1 + \frac{2R_d P_d^0}{V_{di}^2}] \Delta P_{di} + G_{i,i} \Delta P_{di}$$ (5.5.28)

or,

$$\Delta P_i = \Omega_i \Delta P_{di}$$ (5.5.29)

where,

$$\Omega_i = - G_{i,r} [1 + \frac{2R_d P_d^0}{V_{di}^2}] + G_{i,i}$$ (5.5.30)
Now the swing equation of (5.2.1) is put in the proper state variable form as follows:

\[ \frac{2H}{\omega_0} \dot{\omega} + D(\omega - \omega_0) = P_m - Re[S] - \Omega \Delta P_{di} \]  

(5.5.31)

which includes the effect of DC power modulation.

This concludes the network modeling for a large scale AC/DC power system, and appropriate modification of the generator equations.

### 5.6: State Equation Representation of the Overall System

In previous sections of this chapter the modeling of individual components are discussed with exception of the turbine and its control system. It is assumed that the turbine shaft power \( P_m \) will remain constant since the time constant associated with the change in mechanical power is 4 to 5 times larger than the control interval chosen for this study. The validity of this assumption is well accepted throughout the literature.

The large scale network modeling modified the generator equations for representation of the overall power system. The reason for the modifications is to form the proper state variable form of the equations which expresses the dynamics of an interconnected system. The nonlinear state equation of the system is:
\[ \dot{X}(t) = F(X(t), U(t)) \] (5.6.1)

where,

\[ X(t) = [X_{G1} \quad X_{G2} \quad X_{G3} \quad \ldots \quad X_{Gj}]^T \] (5.6.2)

\[ U(t) = [U_{E1} \quad U_{E2} \quad U_{E3} \quad \ldots \quad U_{Ej} \quad U_{dc}]^T \] (5.6.3)

The \( X_{G1} \) through \( X_{Gj} \) are the state vectors for generators 1 to \( j \). The \( U_{E1} \) through \( U_{Ej} \) are the input vectors for excitation systems 1 to \( j \), and \( U_{dc} \) is the input for the HVDC link. The definition of each component of the state vector is:

\[ X_G = [\omega \quad \delta \quad E_q \quad V_y \quad V_z \quad V_r \quad E_{fd}]^T \] (5.6.4)

And the definition of each component of the input vector is:

\[ U_E = [V_{ref} + V_s] \] (5.6.5)

\[ U_{dc} = [\Delta P_{di}] \] (5.6.6)

The equation (5.6.1) is linearized, and the \( A \) and \( B \) matrices are obtained from:

\[ A = \frac{\partial F(X(t), U(t))}{\partial X(t)} \quad , \quad B = \frac{\partial F(X(t), U(t))}{\partial U(t)} \] (5.6.7)

which are evaluated at the initial value of the states and the inputs. Then the following translations are defined:
\[ Z(t) = X(t) - X(t_0) \] (5.6.8)

\[ V(t) = U(t) - U(t_0) \] (5.6.9)

The translated state equation with inclusion of the residual has the form:

\[ \dot{Z}(t) = AZ(t) + BV(t) + [F(X(t), U(t)) - F(X(t_0), U(t_0)) - AZ(t) - BV(t)] \] (5.6.10)

Now if the residual inside the bracket of equation (5.6.10) is measured in time and its future values are extrapolated, then it can be expressed as a function of time. Thus the final form of the state equation of the translated system is,

\[ \dot{Z}(t) = AZ(t) + BV(t) + f(t) \] (5.6.11)

where equation (5.6.11) is in the desired form and it is compatible with the state equation of the developed control algorithm.
CHAPTER 6. SIMULATION PROGRAM

6.1: Introduction

In order to be able to test the validity of the developed control algorithm for a power system, it is necessary to have the system measurements available. For achievement of the measurements it is required to develop a program which simulates the power system performance. This type of program is referred to as the transient stability program, and it is commonly used for study of power system dynamics and stability. The most common transient stability programs consider only the AC transmission system. The time constant associated with the network variables are extremely small and can be considered to be zero without significant loss of accuracy. Similarly the synchronous machine stator time constants may be taken as zero. Thus, relevant differential equations for these rapidly changing variables are transformed into algebraic equations. The most commonly accepted
method in transient stability program uses the Newton-Raphson iterative technique for solution of the network equations [37]. This technique has the superiority of fast convergence characteristic in comparison with the other available techniques. Also a fourth order Runge-Kutta integration method is widely used for integration of the differential equations.

Recently, inclusion of the HVDC model in a transient stability programs has become necessary due to the increase in number of the HVDC links in the AC transmission systems. There are many available methods to modify the existing transient stability programs to accommodate the HVDC model [43,55,56]. The unified or sequential techniques are two among many other methods which considers the HVDC model [43]. The unified technique modifies the Jacobian of Newton-Raphson method, and the sequential technique represents the DC link by real and reactive loads which are updated in each iteration of Newton-Raphson method as the converter AC bus voltages changes after each iteration.

In this chapter the development of a power flow and transient stability program for the example AC/DC system is illustrated. Then the state equation formulation and inclusion of the developed optimal controller in the transient stability program is explained.
6.2: AC Power Flow

The widely accepted method for AC power flow programs are Gauss, Gauss-Seidel, and Newton-Raphson techniques. The Newton-Raphson method in polar coordinate is adapted for the power flow program. The single line diagram of the example system is shown in the figure 23. The system data is shown in the figure 24, 25, 26, and 27. The 9 bus system has two generators, three transformers with two of them having the capability of off-nominal turns ratio, a two terminal HVDC link, a load bus, and an infinite bus. For the power flow the DC link terminal buses are represented by specified real and reactive load buses as explained in detail in section 3 of this chapter. The Newton-Raphson technique in the program uses the polar coordinate approach of the reference [37]. For the generator buses the real powers and the magnitude of the terminal voltages are specified. Also, the magnitude and angle of the voltage at reference bus is specified. The load is represented by a constant impedance, and it is included in formulation of the admittance matrix. The network equations are then coded, and the Jacobian is formulated. The mismatch between the specified and calculated real and reactive powers at each bus is computed and multiplied by the inverse of the Jacobian to obtain the increments of the bus voltages and angles. The increments are added to the previous values of the bus voltages and angles to obtain their updated values. From the new values of the bus voltages and angles the
Figure 23. Single Line Diagram of the Example System
### QUANTITIES IN PER UNIT ON 100 MVA BASE

<table>
<thead>
<tr>
<th>IDENTITY</th>
<th>FROM</th>
<th>TO</th>
<th>R</th>
<th>X</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0.00520</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>0.00164</td>
<td>0.02504</td>
<td>0.43087</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>0.00164</td>
<td>0.02504</td>
<td>0.43087</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>8</td>
<td>0.00820</td>
<td>0.12540</td>
<td>0.43087</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>8</td>
<td>0.00820</td>
<td>0.12540</td>
<td>0.43087</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>4</td>
<td>0</td>
<td>0.00613</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>5</td>
<td>0.00164</td>
<td>0.02504</td>
<td>0.43087</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>9</td>
<td>0</td>
<td>0.00613</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>6</td>
<td>0</td>
<td>0.01040</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
<td>9</td>
<td>0</td>
<td>0.01050</td>
<td>0</td>
</tr>
</tbody>
</table>

**IDENTITY:**

1: Line  
2: Transformer with nominal turns ratio  
3: Transformer with off-nominal turns ratio of 1 to 1.03

*Figure 24. Example System AC Line and Transformer Data*
QUANTITIES IN PER UNIT ON 100 MVA BASE

DC Line Resistance : 0.000975
Rectifier Transformer Reactance : 0.0069554
Inverter Transformer Reactance : 0.0066132
Specified Inverter DC Voltage : 0.8908114
Specified Rectifier DC Current : 16.0553020

DC Base Voltage : 557.476 KV
DC Base Current : 179.380 A
Rectifier Ignition Angle : 15 DEGREES
Inverter Extinction Angle : 17 DEGREES

Figure 25. Example System DC Link Data
<table>
<thead>
<tr>
<th>BUS NO.</th>
<th>VOLTAGE</th>
<th>GENERATION</th>
<th>LOAD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>26 KV</td>
<td>1640 MW</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>345 KV</td>
<td>590 MVAR</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>345 KV</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>500 KV</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>500 KV</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>26 KV</td>
<td>820 MW</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>500 KV</td>
<td>680 MVAR</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>345 KV</td>
<td>0</td>
<td>200/20</td>
</tr>
<tr>
<td>9</td>
<td>500 KV</td>
<td>INFINITE</td>
<td>-</td>
</tr>
</tbody>
</table>

Figure 26. Example System Bus Data
# GENERATORS DATA IN PER UNIT OF THE SYSTEM

**GENERATOR NUMBER 1:**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H$</td>
<td>57.600/sec.</td>
</tr>
<tr>
<td>$D$</td>
<td>0.500/rad./sec.</td>
</tr>
<tr>
<td>$r_a$</td>
<td>0</td>
</tr>
<tr>
<td>$x_d$</td>
<td>0.095</td>
</tr>
<tr>
<td>$x'_d$</td>
<td>0.01075</td>
</tr>
<tr>
<td>$T_{d0}'$</td>
<td>5.000 sec.</td>
</tr>
</tbody>
</table>

**GENERATOR NUMBER 2:**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H$</td>
<td>28.800/sec.</td>
</tr>
<tr>
<td>$D$</td>
<td>0.250/rad./sec.</td>
</tr>
<tr>
<td>$r_a$</td>
<td>0</td>
</tr>
<tr>
<td>$x_d$</td>
<td>0.190</td>
</tr>
<tr>
<td>$x'_d$</td>
<td>0.02150</td>
</tr>
<tr>
<td>$T_{d0}'$</td>
<td>5.000 sec.</td>
</tr>
</tbody>
</table>

**EXCITER DATA:**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_r$</td>
<td>0.050 sec.</td>
</tr>
<tr>
<td>$T_a$</td>
<td>0.050 sec.</td>
</tr>
<tr>
<td>$T_e$</td>
<td>0.500 sec.</td>
</tr>
<tr>
<td>$T_f$</td>
<td>1.000 sec.</td>
</tr>
<tr>
<td>$K_a$</td>
<td>40.000</td>
</tr>
<tr>
<td>$K_e$</td>
<td>-0.050</td>
</tr>
<tr>
<td>$K_f$</td>
<td>0.050</td>
</tr>
<tr>
<td>$S_e$</td>
<td>0.074</td>
</tr>
</tbody>
</table>

Figure 27. Example System Generators and Exciter Data
network equations are re-evaluated and the power mismatches are computed again. If the mismatches are less than the precision index of 0.001 the network solution is obtained, otherwise the Jacobian is re-computed and the iteration is continued until the convergence criteria is satisfied.

6.3: Inclusion of the DC Equations in AC Power Flow

A sequential approach is used for inclusion of the DC equations in the AC power flow [43]. In the sequential method the AC and DC equations are solved separately, thus the integration of the DC model is accomplished without significant restructuring of the AC power flow program. For an AC power flow iteration, each converter is simply modeled by specified real and reactive power injections at the AC terminal bus. The HVDC converter AC bus voltage is obtained from the AC power flow iteration, and it is used to solve the DC equations. The new value of the DC variables are used to compute the specified real and reactive injections for the HVDC converter AC bus of the next iteration. The iterative process continues until the convergence is reached.

Each converter station as shown in figure 28 is represented by four equations, one voltage equation, one current equation, and two control equations. Then a DC Jacobian is formulated. The formulation of the technique, using the equations in
Figure 28. HVDC Converter Single Line Representation
their per unit form as described in section 4 of chapter 5 is as follows, where superscript \( sp \) indicates the specified value of the variable. For rectifier:

\[
R(1) = V_{dr} - [a_r E_r \cos \alpha - \frac{\pi}{6} X_{cr} l_{dc}]
\]

(6.3.1)

\[
R(2) = V_{dr} - [R_d l_{dc} + V_{di}]
\]

(6.3.2)

\[
R(3) = \cos \alpha - \cos \alpha^{sp}
\]

(6.3.3)

\[
R(4) = I_{dc} - I_{dc}^{sp}
\]

(6.3.4)

For the inverter:

\[
RR(1) = V_{di} - [a_i E_i \cos \gamma - \frac{\pi}{6} X_{ci} l_{dc}]
\]

(6.3.5)

\[
RR(2) = \cos \gamma - \cos \gamma^{sp}
\]

(6.3.6)

\[
RR(3) = V_{di} - V_{di}^{sp}
\]

(6.3.7)

Even though it was previously stated that each converter is expressed by four equations, for a two terminal DC link the current (equation (6.3.2) ) is common to both rectifier and inverter terminals. Hence, repetition of this equation for the inverter is avoided. Therefore, the inverter is expressed by three equations.

The rectifier voltage equation is equation (6.3.1), the current equation is equation (6.3.2), and the two control equations are equations (6.3.3) and (6.3.4). The
rectifier control variables are the cosine of the ignition angle and the DC current. The inverter voltage equation is the equation (6.3.5), and the two control equations are equations (6.3.6) and (6.3.7). The inverter control variables are the cosine of the extinction angle and the DC voltage. The quantities inside the brackets are the specified values for each iteration, while the quantities outside the brackets are the calculated values. From these two sets of equations the two DC Jacobians are formulated.

\[
\begin{align*}
\Delta R(1) & = \begin{bmatrix} 1 & \frac{\pi}{6}X_{cr} & -E_r\cos\alpha & -a_rE_r \end{bmatrix} \begin{bmatrix} \Delta V_{dr} \\ \Delta I_{dc} \\ \Delta a_r \\ \Delta \cos\alpha \end{bmatrix} \\
\Delta R(2) & = \begin{bmatrix} 1 & -R_{d} & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta V_{dr} \\ \Delta I_{dc} \\ \Delta a_r \\ \Delta \cos\alpha \end{bmatrix} \\
\Delta R(3) & = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta V_{dr} \\ \Delta I_{dc} \\ \Delta a_r \\ \Delta \cos\alpha \end{bmatrix} \\
\Delta R(4) & = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta V_{dr} \\ \Delta I_{dc} \\ \Delta a_r \\ \Delta \cos\alpha \end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\Delta RR(1) & = \begin{bmatrix} 1 & -E_i\cos\gamma & -a_iE_i \end{bmatrix} \begin{bmatrix} \Delta V_{di} \\ \Delta a_i \\ \Delta \cos\gamma \end{bmatrix} \\
\Delta RR(2) & = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta V_{di} \\ \Delta a_i \\ \Delta \cos\gamma \end{bmatrix} \\
\Delta RR(3) & = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta V_{di} \\ \Delta a_i \\ \Delta \cos\gamma \end{bmatrix}
\end{align*}
\]

Where \(\Delta R\) and \(\Delta RR\) can be replaced by \(R\) and \(RR\), respectively. This is due to the fact that the mismatches, \(\Delta R\) and \(\Delta RR\) are defined as:

\[
\Delta R = R^{calculated} - R^{specified} \tag{6.3.10}
\]

\[
\Delta RR = RR^{calculated} - RR^{specified} \tag{6.3.11}
\]

with,
\[ R^{specified} = 0 \]  
\[ RR^{specified} = 0 \]

The DC Jacobians are inverted and multiplied by the residual vector to obtain the new increment vector of the variables. The new value of the variables are obtained from subtracting the increments from the previous value of the variables at the end of each iteration. It should be noted that the determinants of the DC Jacobians are non-zero, thus their inverse is guaranteed. Then the real and reactive power for the rectifier and inverter buses are computed from the new value of DC variables according to the equations (5.4.12) and (5.4.13) of chapter 5.

Based on the above formulation, after each iteration of the AC power flow and computation of the new value of the AC bus voltages and angles, the DC power flow is executed and the new value of DC variables are computed. The test for convergence is that the tolerance for AC real and reactive power in addition to tolerance for the DC residuals to be less than the 0.001 precision index.

Upon the convergence of the power flow, for a given value of DC voltage, DC current, rectifier ignition angle, and inverter extinction angle, the converter transformers tap ratios are obtained. Also any other information such as bus voltages, and line flows can be printed.
6.4: AC/DC Transient Stability

Transient stability program simulates the performance of the example system. Initially the power flow is solved to obtain the initial conditions for the differential equations which are to be integrated by the fixed time step Runge-Kutta algorithm. The dynamic equations consist of the generator and exciter differential equations which are integrated recursively for the simulation period. The inner loop for each numerical integration iteration is the solution of the network equations. After each integration step the values of $E'_q$ and $\delta_q$ are updated and kept constant for the solution of the network equations.

The generator terminals do not maintain the initial constant power injection following a system disturbance. The power balance is disturbed and change in flux linkages and speed are followed. The modeling technique of reference [37] was initially adapted for computation of generators real and reactive power for solution of the network equations. However after implementation of the method, it was realized that the convergence characteristic of the method was quite poor. Hence, the method of reference [51] was implemented. The results for the method of reference [51] showed an excellent convergence characteristic, and the computation time was roughly one tenth compared to the method of reference [37].

The method suggested in reference [51] express the real and reactive power at the generator terminal bus by,
\[
P = \frac{E_q E_t}{x'_d} \sin(\delta_q - \delta_t) + \frac{E_t^2}{2} \left( \frac{1}{x_q} - \frac{1}{x'_d} \right) \sin 2(\delta_q - \delta_t) \quad (6.4.1)
\]

\[
Q = \frac{E_q E_t}{x'_d} \cos(\delta_q - \delta_t) + E_t^2 \left( \frac{1}{x_q} - \frac{1}{x'_d} \right) \cos^2(\delta_q - \delta_t) - \frac{E_t^2}{x_q} \quad (6.4.2)
\]

where the armature resistance is neglected and the subscripts \( q \) and \( t \) represents the generator \( q \)-axis and terminal variables, respectively [52,53,54]. The above equations express the real and reactive power injections into the generator terminal bus. However, in power flow computation of the network real and reactive powers, the convention of powers injected out of each bus is adapted. Thus, the sign of the right hand terms in equations (6.4.1) and (6.4.2) are reversed in order to maintain the consistency, and then the real and reactive powers are added to the generator terminal bus powers. This modification is done by proper changes in power flow coding. Another advantage of this approach is that the size of the AC Jacobian is unchanged.

The disturbance simulated is a three phase fault on one of the parallel AC lines between buses 2 and 3, and the fault location is close to bus 3. The three phase fault is modeled as a low impedance connection between bus 3 and ground. The duration of the fault simulated is three cycles. Consequently one of the AC lines is removed to clear the fault. For transient stability program the first rectifier and inverter control equations are replaced by the following equations.

\[
R(3) = a_r - a_{r \text{ sp}} \quad (6.4.3)
\]

CHAPTER 6. SIMULATION PROGRAM
\[ RR(2) = a_i - a_i^{sp} \] (6.4.4)

Then accordingly the DC Jacobians for the rectifier and inverter are modified without any change of the variables. The change is necessary in order to maintain the DC voltage and current at their specified value by modulation of rectifier and inverter ignition and extinction angles, respectively. A special logic was built into the control of HVDC link. If during the disturbance the AC voltage of the converter that controls the voltage drops below 0.5 per unit, then the current setting of the terminal that controls the current adjusts itself to 0.01 percent of the original current setting. As soon as the disturbance is removed and the converter AC bus voltage exceeds the 0.5 per unit, then the classical control mode of a two terminal HVDC link is established.

### 6.5: State Equation Formulation for the Example System

In this section the necessary equations for state equation representation of the example system is explained in greater detail. According to the derivation, the equations for the components of the state feedback matrix, \( A \), and the input matrix, \( B \), are obtained. These two matrices are used by the optimal controller to obtain the optimal control law.

Applying equation (5.5.3) to the example system leads to:
where, $I_1$ through $I_8$ are the current injections at buses 1 through 8, with a zero value. Currents $I_9$ through $I_{11}$ are the current injections at the infinite bus, q-axis bus of generator 1, and q-axis bus of generator 2, respectively. Voltages $E_1$ through $E_8$ are the voltages for buses 1 through 8. Voltages $E_9$ through $E_{11}$ are the voltages of infinite bus, q-axis bus of generator 1, and q-axis bus of generator 2, respectively. The system data in addition to the q-axis synchronous reactance of the generators are used to formulate the admittance matrix. In admittance matrix formulation the HVDC link and the load are represented by constant impedances, according to equations (5.5.1) and (5.5.2). Then the equivalent admittance matrix is computed according to equation (5.5.9). Upon completion of the equivalent admittance matrix computation, the components of the voltage and current vectors are rearranged, and later numbered 1 through 3 for simplicity. Also the proper row and column rearrangements are made for the equivalent admittance matrix to maintain consistency. Hence, equation (5.5.8) is:
\[ I = Y_{eq} E \]  \hspace{1cm} (6.5.2)

where,

\[ I = [I_{10} \quad I_{11} \quad I_9]^T \]  \hspace{1cm} (6.5.3)

\[ E = [E_{10} \quad E_{11} \quad E_9]^T \]  \hspace{1cm} (6.5.4)

and the subscripts 10, 11, and 9 are changed to 1, 2, and 3, respectively. The rearranged equivalent admittance matrix is symmetric and its components in polar coordinate are:

\[
Y_{11} = 8.293 \hspace{0.5cm} , \hspace{0.5cm} \angle -86.323^\circ \\
Y_{21} = 0.200 \hspace{0.5cm} , \hspace{0.5cm} \angle +74.376^\circ \\
Y_{22} = 4.354 \hspace{0.5cm} , \hspace{0.5cm} \angle -89.521^\circ \\
Y_{31} = 7.505 \hspace{0.5cm} , \hspace{0.5cm} \angle +73.997^\circ \\
Y_{32} = 4.023 \hspace{0.5cm} , \hspace{0.5cm} \angle +86.791^\circ \\
Y_{33} = 14.370 \hspace{0.5cm} , \hspace{0.5cm} \angle +255.702^\circ
\]

The value of bus voltages are obtained from the power flow results and the current injections were computed from equation (6.5.2). The generators q-axis voltages and the infinite bus voltage in polar coordinate are:

\[
E_1 = 1.874 \hspace{0.5cm} , \hspace{0.5cm} \angle 62.120^\circ \\
E_2 = 1.911 \hspace{0.5cm} , \hspace{0.5cm} \angle 71.039^\circ \\
E_3 = 1.050 \hspace{0.5cm} , \hspace{0.5cm} \angle 0^\circ
\]
The value of current injections were compared with the value of the same currents from the power flow results and they were identical. This assured the correct computation of the equivalent admittance matrix.

Now from equations (5.5.12) and (5.5.13) and also the reduced order admittance matrix the real and reactive power of generators 1 and 2 are:

\[
P_1 = E_1^2 Y_{11} \cos(\theta_{11}) + E_1 E_2 Y_{12} \cos(\delta_1 - \delta_2 - \theta_{12}) + E_1 E_3 Y_{13} \cos(\delta_1 - \delta_3 - \theta_{13})
\]

\[
Q_1 = -E_1^2 Y_{11} \sin(\theta_{11}) + E_1 E_2 Y_{12} \sin(\delta_1 - \delta_2 - \theta_{12}) + E_1 E_3 Y_{13} \sin(\delta_1 - \delta_3 - \theta_{13})
\]

\[
P_2 = E_2 E_1 Y_{21} \cos(\delta_2 - \delta_1 - \theta_{21}) + E_2^2 Y_{22} \cos(\theta_{22}) + E_2 E_3 Y_{23} \cos(\delta_2 - \delta_3 - \theta_{23})
\]

\[
Q_2 = E_2 E_1 Y_{21} \sin(\delta_2 - \delta_1 - \theta_{21}) - E_2^2 Y_{22} \sin(\theta_{22}) + E_2 E_3 Y_{23} \sin(\delta_2 - \delta_3 - \theta_{23})
\]

where \(\delta\) is the angle of bus voltage and \(\theta\) is the angle of the admittances. Then equations (6.5.5) and (6.5.7) are substituted in the swing equations of each machine (equation (5.5.31)) to achieve the state equation form of these equations which would require additional information regarding to value of \(\Omega\).

The d-axis current of each generator is obtained from substitution of equations (6.5.6) and (6.5.8) into equation (5.5.14). Thus,
\[ I_{d1} = -E_1 Y_{11} \sin(\theta_{11}) + E_2 Y_{12} \sin(\delta_1 - \delta_2 - \theta_{12}) + E_3 Y_{13} \sin(\delta_1 - \delta_3 - \theta_{13}) \]  
\[ (6.5.9) \]

\[ I_{d2} = E_1 Y_{21} \sin(\delta_2 - \delta_1 - \theta_{21}) - E_2 Y_{22} \sin(\theta_{22}) + E_3 Y_{23} \sin(\delta_2 - \delta_3 - \theta_{23}) \]  
\[ (6.5.10) \]

Substitution of equations (6.5.9) and (6.5.10) into equation (5.2.5) of each machine eliminates the d-axis current component dependency. Thus,

\[ E'_1 = E_1 - (x_d - x'_d)[-E_1 Y_{11} \sin(\theta_{11}) + E_2 Y_{12} \sin(\delta_1 - \delta_2 - \theta_{12}) + E_3 Y_{13} \sin(\delta_1 - \delta_3 - \theta_{13})] \]  
\[ (6.5.11) \]

\[ E'_2 = E_2 - (x_d - x'_d)[E_1 Y_{12} \sin(\delta_2 - \delta_1 - \theta_{21}) - E_2 Y_{22} \sin(\theta_{22}) + E_3 Y_{23} \sin(\delta_2 - \delta_3 - \theta_{23})] \]  
\[ (6.5.12) \]

Equations (6.5.11) and (6.5.12) are differentiated with respect to time and equation (5.5.15) for each cylindrical rotor generators is written as,

\[ \dot{E'}_1 = \frac{1}{T'_{d01}}(E_{fd1} - E_1) \]  
\[ (6.5.13) \]

\[ \dot{E'}_2 = \frac{1}{T'_{d02}}(E_{fd2} - E_2) \]  
\[ (6.5.14) \]

Later, substitution of equations (6.5.13) and (6.5.14) into the differential equations yield,

\[ a\dot{E}_1 + b\dot{E}_2 + d\dot{\delta}_1 + c\dot{\delta}_2 = e \]  
\[ (6.5.15) \]
\[ g \dot{E}_1 + f \dot{E}_2 + h \dot{\delta}_1 + k \dot{\delta}_2 = l \]  

(6.5.16)

which are independent of the \( \dot{E}'_1 \) and \( \dot{E}'_2 \). Where,

\[ a = 1 + (x_d - x'_d)Y_{11} \sin \theta_{11} \]  

(6.5.17)

\[ b = - (x_d - x'_d)Y_{12} \sin(\delta_1 - \delta_2 - \theta_{12}) \]  

(6.5.18)

\[ c = (x_d - x'_d)E_{2}Y_{12} \cos(\delta_1 - \delta_2 - \theta_{12}) \]  

(6.5.19)

\[ d = -(x_d - x'_d)[E_{2}Y_{12} \cos(\delta_1 - \delta_2 - \theta_{12}) + E_{3}Y_{13} \cos(\delta_1 - \delta_3 - \theta_{13})] \]  

(6.5.20)

\[ e = \frac{1}{T_{d01}}(E_{fd1} - E_1) \]  

(6.5.21)

\[ f = 1 + (x_d - x'_d)Y_{22} \sin \theta_{22} \]  

(6.5.22)

\[ g = -(x_d - x'_d)Y_{21} \sin(\delta_2 - \delta_1 - \theta_{21}) \]  

(6.5.23)

\[ h = (x_d - x'_d)E_{1}Y_{21} \cos(\delta_2 - \delta_1 - \theta_{21}) \]  

(6.5.24)

\[ k = -(x_d - x'_d)[E_{1}Y_{21} \cos(\delta_2 - \delta_1 - \theta_{21}) + E_{3}Y_{23} \cos(\delta_2 - \delta_3 - \theta_{23})] \]  

(6.5.25)

\[ l = \frac{1}{T_{d02}}(E_{fd2} - E_2) \]  

(6.5.26)

Equations (6.5.15) and (6.5.16) forms a set of linear equation. Solving these equations for \( \dot{E}_1 \) and \( \dot{E}_2 \) results in:
\[ \dot{E}_1 = \frac{1}{af - bg}[(bh - df)\dot{\delta}_1 + (bk - cf)\dot{\delta}_2 + (ef - bh)] \]  
(6.5.27)

\[ \dot{E}_2 = \frac{1}{af - bg}[(dg - ah)\dot{\delta}_1 + (cg - ak)\dot{\delta}_2 + (al - eg)] \]  
(6.5.28)

where,

\[ \dot{\delta}_1 = \omega_1 - \omega_0 \]  
(6.5.29)

\[ \dot{\delta}_2 = \omega_2 - \omega_0 \]  
(6.5.30)

Now substitution of equations (6.5.29) and (6.5.30) into equations (6.5.27) and (6.5.28) results in two differential equations which shows the dynamics of the q-axis bus voltages of the generators 1 and 2. These two equations are the differential equation necessary as a part of system state equations.

The exciter differential equations for each generator is in the desirable form, except for the fact that in equation (5.3.1) the terminal voltage \( E_t \) should be expressed in terms of the state variables. To accomplish this, from reference [54] the generator d-axis current when the armature resistance is neglected is as follows.

\[ I_d = \frac{E_q - E_t \cos(\delta_q - \delta_i)}{x_d} \]  
(6.5.31)
Or,

\[ E_t = \frac{E_q - x_d I_d}{\cos(\delta_q - \delta_t)} \]  \hspace{1cm} (6.5.32)

Substitution of d-axis current of generators 1 and 2 from equations (6.5.9) and (6.5.10) separately into equations (6.5.31) yield:

\[ E_{t1} = \frac{1}{\cos(\delta_1 - \delta_{t1})} \{ E_1 - x_{d1} [ - E_1 Y_{11} \sin(\theta_{11}) + E_2 Y_{12} \sin(\delta_1 - \delta_2 - \theta_{12}) + E_3 Y_{13} \sin(\delta_1 - \delta_3 - \theta_{13})] \} \]  \hspace{1cm} (6.5.33)

\[ E_{t2} = \frac{1}{\cos(\delta_2 - \delta_{t2})} \{ E_2 - x_{d2} [ E_1 Y_{21} \sin(\delta_2 - \delta_1 - \theta_{21}) - E_2 Y_{22} \sin(\theta_{22}) + E_3 Y_{23} \sin(\delta_2 - \delta_3 - \theta_{23})] \} \]  \hspace{1cm} (6.5.34)

Then equations (6.5.33) and (6.5.34) are separately substituted in equation (5.3.1) for exciter of generators 1 and 2. It should be noted that the angle of the generator terminal bus voltage is not a state variable, and for substitution of its numerical values the initial value of the angle is used. The remainder of the exciter equations, equations (5.3.2) through (5.3.4), remain unchanged.

So far all the necessary steps are taken for deriving the state variable form of the system equations. If the numerical value of \( \Omega \) in equation (5.5.30) is known, then the work is complete. To obtain the numerical value of \( \Omega \) for generators 1 and 2 from equations (5.5.30), computation of the network reactance matrix is necessary. The network reactance matrix is computed by deleting the line

CHAPTER 6. SIMULATION PROGRAM

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conductances and the constant admittances used for representation of HVDC link and the load from the admittance matrix. The HVDC link and the load are represented by constant current sources. Even though at first this might seem as a contradiction in philosophy of modeling the HVDC and the load, it is necessary to mention that the network sensitivity approach is an approximation technique and no claim of exactness is made. Finally the newly formed susceptance matrix is inverted to obtain the network reactance matrix. The corresponding values of this matrix, according to the equations (5.5.24) and (5.5.26), are selected to compute the generation shift factors, $G_{ij}$. Then equation (5.5.30) is used to compute the value of the $\Omega$ for each generator. Therefore, the complete system dynamic equations are as follows.

For generator 1:

$$\dot{\omega}_1 = \frac{\omega_0}{2H_1} \{ - D_1(\omega_1 - \omega_0) + P_m 1 - E_1^2 Y_{11} \cos(\theta_{11}) - E_1E_2 Y_{12} \cos(\delta_1 - \delta_2 - \theta_{12}) - E_1E_3 Y_{13} \cos(\delta_1 - \delta_3 - \theta_{13}) - \Omega_1 \Delta P_{di} \}$$

(6.5.35)

$$\dot{\delta}_1 = \omega_1 - \omega_0$$

(6.5.36)

$$\dot{E}_1 = \frac{1}{af - bg}[(bh - df)(\omega_1 - \omega_0) + (bk - cf)(\omega_2 - \omega_0) + (ef - bh)]$$

(6.5.37)

$$\dot{V}_{y1} = \frac{1}{T_r} \{ \frac{1}{\cos(\delta_1 - \delta_{11})} \{ E_1 - x_d 1 [ - E_1 Y_{11} \sin(\theta_{11}) + E_2 Y_{12} \sin(\delta_1 - \delta_2 - \theta_{12}) + E_3 Y_{13} \sin(\delta_1 - \delta_3 - \theta_{13})] \} - V_{y1} \}$$

(6.5.38)
\[
\dot{V}_{r1} = \frac{1}{T_a} [ - V_{r1} + (- V_{y1} - V_{z1} + V_{ref1} + V_{s1}) K_d ] \quad (6.5.39)
\]

\[
\dot{V}_{z1} = \frac{1}{T_f} \left( \frac{K_f}{T_e} \{ V_{r1} - (S_e + K_e) E_{fd1} \} - V_{z1} \right) \quad (6.5.40)
\]

\[
\dot{E}_{fd1} = \frac{1}{T_e} [ V_{r1} - (S_e + K_e) E_{fd1} ] \quad (6.5.41)
\]

For generator 2:

\[
\dot{\omega}_2 = \frac{\omega_0}{2H_2} \{ - D_2(\omega_2 - \omega_0) + P_{m2} - E_2E_1Y_{12} \cos(\delta_2 - \delta_1 - \theta_{21}) - \\
E_2^2 Y_{22} \cos(\theta_{22}) - E_2E_3Y_{23} \cos(\delta_2 - \delta_3 - \theta_{23}) - \Omega_2 \Delta P_{di} \} \quad (6.5.42)
\]

\[
\dot{\delta}_2 = \omega_2 - \omega_0 \quad (6.5.43)
\]

\[
\dot{E}_2 = \frac{1}{af - bg} [(dg - ah)(\omega_1 - \omega_0) + (cg - ak)(\omega_2 - \omega_0) + (al - eg)] \quad (6.5.44)
\]

\[
\dot{V}_{y2} = \frac{1}{T_r} \left( \frac{1}{\cos(\delta_2 - \delta_{t2})} \{ E_2 - x_d \{ E_1Y_{21} \sin(\delta_2 - \delta_1 - \theta_{21}) - \\
E_2Y_{22} \sin(\theta_{22}) + E_3Y_{23} \sin(\delta_2 - \delta_3 - \theta_{23}) \} \} - V_{y2} \right) \quad (6.5.45)
\]

\[
\dot{V}_{r2} = \frac{1}{T_a} [ - V_{r2} + (- V_{y2} - V_{z2} + V_{ref2} + V_{s2}) K_d ] \quad (6.5.46)
\]

\[
\dot{V}_{z2} = \frac{1}{T_f} \left( \frac{K_f}{T_e} \{ V_{r2} - (S_e + K_e) E_{fd2} \} - V_{z2} \right) \quad (6.5.47)
\]

CHAPTER 6. SIMULATION PROGRAM
The nonlinear form of the system state equation, equation (5.6.1), is given by equations (6.5.35) through (6.5.48) for the example system. Now the equation (5.6.7) is applied to equations (6.5.35) through (6.5.48) to obtain the entries of the state feedback and input matrices. For simplicity the following are defined:

\[ M_1 = \frac{2H_1}{\omega_0} \]  
\[ M_2 = \frac{2H_2}{\omega_0} \]

Then, the non-zero components of A and B matrices are as follows.

\[ a_{1,1} = -\frac{D_1}{M_1} \]  
\[ a_{1,2} = \frac{1}{M_1} [E_1E_2Y_{12} \sin(\delta_1 - \delta_2 - \theta_{12}) + E_1E_3Y_{13} \sin(\delta_1 - \delta_2 - \theta_{13})] \]  
\[ a_{1,3} = -\frac{1}{M_1} [2E_1Y_{11} \cos(\theta_{11}) + E_2Y_{12} \cos(\delta_1 - \delta_2 - \theta_{12}) + E_3Y_{13} \cos(\delta_1 - \delta_2 - \theta_{13})] \]  
\[ a_{1,9} = -\frac{1}{M_1} [E_1E_2Y_{12} \sin(\delta_1 - \delta_2 - \theta_{12})] \]
\[ a_{1,10} = -\frac{1}{M_1} \left[ E_1 Y_{12} \cos(\delta_1 - \delta_2 - \theta_{12}) \right] \]  
\[ (6.5.55) \]

\[ a_{2,1} = 1 \]  
\[ (6.5.56) \]

\[ a_{3,1} = \frac{bh - df}{af - bg} \]  
\[ (6.5.57) \]

\[ a_{3,3} = -\frac{1}{af - bg} \frac{1}{T'd_{101}} \left[ 1 + (x_d - x'_d) Y_{22} \sin \theta_{22} \right] \]  
\[ (6.5.58) \]

\[ a_{3,7} = \frac{1}{af - bg} \frac{1}{T'd_{101}} \left[ 1 + (x_d - x'_d) Y_{22} \sin \theta_{22} \right] \]  
\[ (6.5.59) \]

\[ a_{3,8} = \frac{bk - cf}{af - bg} \]  
\[ (6.5.60) \]

\[ a_{3,10} = -\frac{1}{af - bg} \frac{1}{T'd_{102}} \left[ (x_d - x'_d) Y_{12} \sin(\delta_1 - \delta_2 - \theta_{12}) \right] \]  
\[ (6.5.61) \]

\[ a_{3,14} = \frac{1}{af - bg} \frac{1}{T'd_{102}} \left[ (x_d - x'_d) Y_{12} \sin(\delta_1 - \delta_2 - \theta_{12}) \right] \]  
\[ (6.5.62) \]

\[ a_{4,2} = \frac{1}{T_r} \left\{ \cos(\delta_1 - \delta_{t1}) [ - x_{d1} Y_{12} E_2 \cos(\delta_1 - \delta_2 - \theta_{12}) - \cos^2(\delta_1 - \delta_{t1}) - x_{d1} Y_{13} E_3 \cos(\delta_1 - \delta_3 - \theta_{13})] + \sin(\delta_1 - \delta_{t1}) [E_1 + x_{d1} E_1 Y_{11} \sin \theta_{11} - x_{d1} Y_{12} E_2 \sin(\delta_1 - \delta_2 - \theta_{12}) - x_{d1} Y_{13} E_3 \sin(\delta_1 - \delta_3 - \theta_{13})] \right\} \]  
\[ (6.5.63) \]
\( a_{4,3} = \frac{1}{T_r} \frac{1}{\cos(\delta_1 - \delta_{t1})} [1 + x_{d1} Y_{11} \sin \theta_{11}] \)  
\( a_{4,4} = - \frac{1}{T_r} \)  
\( a_{4,9} = \frac{1}{T_r} \frac{1}{\cos(\delta_1 - \delta_{t1})} [x_{d1} Y_{12}E_2 \cos(\delta_1 - \delta_2 - \theta_{12})] \)  
\( a_{4,10} = \frac{1}{T_r} \frac{1}{\cos(\delta_1 - \delta_{t1})} [-x_{d1} Y_{12} \sin(\delta_1 - \delta_2 - \theta_{12})] \)  
\( a_{5,4} = - \frac{K_a}{T_a} \)  
\( a_{5,5} = - \frac{1}{T_a} \)  
\( a_{5,6} = - \frac{K_a}{T_a} \)  
\( a_{6,5} = \frac{K_f}{T_f T_e} \)  
\( a_{6,6} = - \frac{1}{T_f} \)
\[ a_{6,7} = \frac{K_f}{T_f T_e} [ - (S_e + K_e) ] \]  
\[ (6.5.73) \]

\[ a_{7,5} = \frac{1}{T_e} \]  
\[ (6.5.74) \]

\[ a_{7,7} = \frac{- (S_e + K_e)}{T_e} \]  
\[ (6.5.75) \]

\[ a_{8,2} = - \frac{1}{M_2} [E_2 E_1 Y_{21} \sin(\delta_2 - \delta_1 - \theta_{21})] \]  
\[ (6.5.76) \]

\[ a_{8,3} = - \frac{1}{M_2} [E_2 Y_{21} \cos(\delta_2 - \delta_1 - \theta_{21})] \]  
\[ (6.5.77) \]

\[ a_{8,8} = - \frac{D_2}{M_2} \]  
\[ (6.5.78) \]

\[ a_{8,9} = \frac{1}{M_2} [E_2 E_1 Y_{21} \sin(\delta_2 - \delta_1 - \theta_{21}) + E_2 E_3 Y_{23} \sin(\delta_2 - \delta_3 - \theta_{23})] \]  
\[ (6.5.79) \]

\[ a_{8,10} = - \frac{1}{M_2} [2E_2 Y_{22} \cos(\theta_{22}) + E_1 Y_{21} \cos(\delta_2 - \delta_1 - \theta_{21}) + \\
E_3 Y_{23} \cos(\delta_2 - \delta_3 - \theta_{23})] \]  
\[ (6.5.80) \]

\[ a_{9,8} = 1 \]  
\[ (6.5.81) \]

\[ a_{10,1} = \frac{dg - ah}{af - bg} \]  
\[ (6.5.82) \]
\[ a_{10, 3} = - \frac{1}{af - bg} \frac{1}{T'_{do1}} [(x_d - x'_d)Y_{21} \sin(\delta_2 - \delta_1 - \theta_{12})] \] (6.5.83)

\[ a_{10, 7} = \frac{1}{af - bg} \frac{1}{T'_{do1}} [(x_d - x'_d)Y_{21} \sin(\delta_2 - \delta_1 - \theta_{12})] \] (6.5.84)

\[ a_{10, 8} = \frac{cg - ak}{af - bg} \] (6.5.85)

\[ a_{10, 10} = - \frac{1}{af - bg} \frac{1}{T'_{do2}} [1 + (x_d - x'_d)Y_{11} \sin \theta_{11}] \] (6.5.86)

\[ a_{10, 14} = \frac{1}{af - bg} \frac{1}{T'_{do2}} [1 + (x_d - x'_d)Y_{11} \sin \theta_{11}] \] (6.5.87)

\[ a_{11, 2} = \frac{1}{Tr} \frac{[x_dY_{21}E_1 \cos(\delta_2 - \delta_1 - \theta_{21})]}{\cos(\delta_2 - \delta_{t2})} \] (6.5.88)

\[ a_{11, 3} = \frac{1}{Tr} \frac{[- x_dY_{21} \sin(\delta_2 - \delta_1 - \theta_{21})]}{\cos(\delta_2 - \delta_{t2})} \] (6.5.89)

\[ a_{11, 9} = \frac{1}{Tr} \{ \frac{\cos(\delta_2 - \delta_{t2})}{\cos^2(\delta_2 - \delta_{t2})} \left[ - x_dY_{21}E_1 \cos(\delta_2 - \delta_1 - \theta_{21}) - x_dY_{23}E_3 \cos(\delta_2 - \delta_3 - \theta_{23}) \right] + \sin(\delta_2 - \delta_{t2}) (E_2 - x_dE_1Y_{21} \sin(\delta_2 - \delta_1 - \theta_{12}) + x_dY_{22}E_2 \sin \theta_{22} - x_dY_{23}E_3 \sin(\delta_2 - \delta_3 - \theta_{23})) \} \] (6.5.90)
\[ a_{11,10} = \frac{1}{T_r} \frac{\cos(\delta_2 - \delta_{12})}{[1 + x_{d2}Y_{22} \sin \theta_{22}]} \quad (6.5.91) \]

\[ a_{11,11} = -\frac{1}{T_r} \quad (6.5.92) \]

\[ a_{12,11} = -\frac{K_a}{T_a} \quad (6.5.93) \]

\[ a_{12,12} = -\frac{1}{T_a} \quad (6.5.94) \]

\[ a_{12,13} = -\frac{K_a}{T_a} \quad (6.5.95) \]

\[ a_{13,12} = \frac{K_f}{T_fT_e} \quad (6.5.96) \]

\[ a_{13,13} = -\frac{1}{T_f} \quad (6.5.97) \]

\[ a_{13,14} = \frac{K_f}{T_fT_e} \left[-(S_e + K_e)\right] \quad (6.5.98) \]

\[ a_{14,12} = \frac{1}{T_e} \quad (6.5.99) \]

\[ a_{14,14} = -\frac{(S_e + K_e)}{T_e} \quad (6.5.100) \]
Upon substitution of the initial values of the states and inputs from the base case power flow and the constant system parameters of figure 27 into equations (6.5.51) through (6.5.104), the numerical value of the non-zero components of A and B matrices are obtained. Figure 29 shows the numerical values of the A and B matrices.

\[
b_{5,1} = \frac{K_a}{T_a}
\]  

(6.5.101)

\[
b_{12,2} = \frac{K_a}{T_a}
\]  

(6.5.102)

\[
b_{1,3} = -\frac{\Omega_1}{M_1}
\]  

(6.5.103)

\[
b_{8,3} = -\frac{\Omega_2}{M_2}
\]  

(6.5.104)
### Non-Zero Components of the A Matrix

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<th>(A_{ij})</th>
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### Non-Zero Components of the B Matrix

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<th>j</th>
<th>(B_{ij})</th>
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</thead>
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</table>

Figure 29. Numerical Values of A and B Matrices for Example System
6.6: Inclusion of Optimal Controller in Transient Stability Program

The optimal controller block is added to the transient stability program, as shown in the flow chart of figure 30. The pre-disturbance state measurement vector initializes the $f(t)$ vector. After each five cycle, the new state measurements are obtained and used to compute and store $f(t)$. The collection of measurements and computation of $f(t)$ during the fault period are avoided, since they are grossly in error. As soon as the disturbance is isolated and the new measurements arrive, it is used to compute $f(t)$ and the estimation algorithm is executed to extrapolate the future values of $f(t)$ from the previously stored values of $f(t)$. Then the vector differential equation (3.6.7) is solved for $g(t)$ with the new estimate of $f(t)$ for computation of the optimal inputs. It is important to notice that according to the developed control algorithm, the solution of the Riccati equation is independent of the function $f(t)$. As long as the system A and B matrices remain unchanged (as is the case in our study), then based on equation (3.6.6) the time variable $K(t)$ matrix remains unchanged. Hence, the Riccati equation can be solved off-line. In other words the Riccati equation is solved before running the simulation program, and the $K(t)$ in forward and backward orders are saved in two separate data files. Then the simulation program accesses these data files to read in the value of $K(t)$ as the need arises.
Figure 30. Simulation Program Flow Chart
In summary the adaptive optimal controller coding executes the following steps in every five cycles of simulation time:

1. It collects and stores the measurements in an array.

2. It computes $f(t)$ and estimates the future values of this vector from the previous values, using an estimation algorithm.

3. It computes the new value of $g(t)$ from the new knowledge of $f(t)$.

4. If necessary it reads the new value of $K(t)$, and computes the optimal control law.

5. It maintains the computed value of the optimal inputs for the next five cycles.

The computer program for the solution of the Riccati equation is shown in Appendix B. Also, the simulation program is illustrated in Appendix C.
CHAPTER 7. SIMULATION RESULTS

7.1: Introduction

Design of the optimally controlled systems and formulation of the optimal control law for a linear system with regulator and tracking type of performance measure in addition to the developed control algorithm as applied to a nonlinear system is addressed in chapter 3. The power system modeling, including an embedded HVDC link, is treated in chapter 5. The example system and the simulation program written for the study of the developed control algorithm is illustrated in chapter 6. The detail derivation of the state equation, and formulation of the state feedback and the input matrices for the example system are also dealt with in chapter 6.

In this chapter results of the simulation runs for the example system with optimal controller for the linear and nonlinear system representations are illustrated and
discussed. The plotted variables for the optimally controlled system are compared to the variables of the system with classical control strategy. The word classical might cause some confusion. In this chapter classical control strategy means a control strategy which maintains a constant DC current at the rectifier and a constant DC voltage at the inverter. It certainly does not refer to any control strategy which involves the modulation of AC or DC controls with the use of some feedback signals.

In this chapter the rotor angles of generator 1 and 2 are plotted in degrees. Also, the exciter optimal inputs, DC current, rectifier AC voltage, and inverter AC voltage are plotted in per unit. The time scale is in seconds.

The developed optimal control algorithm for nonlinear representation of the system equations requires an estimation technique to predict the future values of the residual vector. Initially, the previous value of the residual vector \( j(t) \) which was computed from state measurements was used to obtain the mean value of the residual vector for estimation of its future values. Later, a moving average estimation technique with a window of 4 samples was used to improve the system performance. The conceptual illustration of figure 31 shows the differences between the two estimation techniques. The mean value estimation technique adds all the samples and the result is divided by the number of samples. While, the moving average estimation technique with a window of 4 samples adds the most recent four samples and the result is divided by four. In either case, the averaged
value of \( f(t) \) is used as the future value. As will be explained later, the moving average technique is by far the better technique to use.

The simulation involves the following cases:

1. Linearized power system state equation and a regulator type of performance measure with utilization of the steady state and time variable solution of the Riccati equation (Cases 1 and 2).

2. Linearized power system state equation and a tracking type of performance measure with utilization of the steady state and time variable solution of the Riccati equation (Cases 3 and 4).

3. Nonlinear representation of power system state equation and a regulator type of performance measure with utilization of a mean value estimator plus the steady state and time variable solution of the Riccati equation (Cases 5 and 6).

4. Nonlinear representation of power system state equation and a tracking type of performance measure with utilization of a mean value estimator plus the steady state and time variable solution of the Riccati equation (Cases 7 and 8). The alternative moving average technique with the above description (Cases 9 and 10).
Figure 31. Conceptual Illustration of the Estimators

CHAPTER 7. SIMULATION RESULTS
5. Nonlinear representation of power system state equation and a regulator type of performance measure with an approximate optimal control law and the two different estimators (Cases 11 and 12).

The simulation results of the above cases are summarized in figure 32. The detail discussion of each individual case is documented in the following sections of this chapter. The performance of the system with linearized equations is the best for case 4 when the tracking type of performance measure is used with the time variable solution of the Riccati equation. The performance of the system with nonlinear representation of the system equations is the best for case 10 when the time variable solution of the Riccati equation is used with a moving average estimation technique for prediction of the future values of the residual. However, the system performance of case 12 with approximate optimal control law is as good as the performance of case 10. Considering that case 12 is computationally about 50 times faster than case 10, despite the existence of small steady state errors in the inputs of case 12, it is preferred over case 10.

It is important to note that in figures 36, 41, 45, 49, 53, 57, 61, 65, 69, 73, 78, and 82 the voltage jumps in the rectifier and inverter AC voltage trajectories of the optimally controlled system appears to be worse with the optimal control. However, the objective of the optimization procedure was to minimize the rotor angle movement from the desired trajectory. This is achieved by the optimal controller at the expense of somewhat greater excursions in converter AC bus voltages. An optimal controller can be formulated which will again weights to voltage devi-
<table>
<thead>
<tr>
<th>CASE</th>
<th>TYPE</th>
<th>DESCRIPTION</th>
<th>PERFORMANCE</th>
<th>COMMENTS</th>
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<tr>
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<td>R,SR</td>
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<td>SMALL S.S. ERROR IN INPUTS</td>
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</table>

Type A: Linearized  
Type B: Nonlinear - Exact  
Type C: Nonlinear - Approximate  
R: Regulator  
T: Tracking  
S.S.: Steady State  
SR: Steady State Solution of Riccati Equation  
TR: Time Variable Solution of Riccati Equation  
AE: Mean Value (Average) Estimator  
ME: Moving Average (Window of 4) Estimator
ations from the desired values. The techniques developed in this dissertation are applicable for such a controller as well, although that case has not been studied explicitly here.
7.2: Linearized Equations and Regulator Performance Measure

A linearized state equation of the example system is used for representation of the dynamic system. Hence,

\[
\delta \dot{X}(t) = A \delta X(t) + B \delta U(t) \tag{7.2.1}
\]

where numerical values of the A and B matrices are given in figure 29. According to the translations (5.6.8) and (5.6.9), the translated system has the form:

\[
\dot{Z}(t) = AZ(t) + BV(t) \tag{7.2.2}
\]

The optimal control law from equation (3.3.19) for the linearized system of equation (7.2.2) with a state regulator type of performance measure is:

\[
V^*(t) = - R^{-1} B^T K(t) Z(t) \tag{7.2.3}
\]

The above equation requires the solution of the Riccati equation of (3.3.23). In solution of the Riccati equation, for the example system, the H matrix is chosen to be zero and the non-zero entries of the Q and R matrices are:

\[
Q(1,1) = 2.0 \\
Q(2,2) = 2.0
\]
Q(8,8) = 2.0
Q(9,9) = 2.0

R(1,1) = 100.0
R(2,2) = 100.0
R(3,3) = 0.100

In other words the only states which are penalized are the generators speed and rotor angle. A heavier weight is placed on the exciter inputs in comparison to the weight on the deviation of the DC power at the inverter, since the DC power modulation is the main control. The control interval is selected to be five seconds.

Translating equation (7.2.3) into the variables of the original system results in,

$$U^*(t) - U(t_0) = - R^{-1} B^T K(t)[X(t) - X(t_0)]$$  \hspace{1cm} (7.2.4)

or, from section 6 of chapter 5:

$$[V_{s1} \quad V_{s2} \quad \Delta P_{di}]^T = - R^{-1} B^T K(t)[X(t) - X(t_0)]$$  \hspace{1cm} (7.2.5)

where $V_{s1}$ and $V_{s2}$ are the optimal inputs for the excitation system and $\Delta P_{di}$ is the required change in the DC power at the inverter. Therefore the above equation is coded in the optimal controller block of the Fortran program which does the computation for obtaining the values of the optimal inputs. After computation of the optimal inputs, the exciter inputs are updated and the deviation of the DC power is added to the initial value of the inverter DC power. Then the total DC...
power is divided by the specified inverter DC voltage to compute the new value of the specified DC current at rectifier station.

CASE 1:

The simulation results for the linearized state equation and the state regulator type of performance measure with steady state solution of the Riccati equation are shown in figures 33 through 36. The reference variables for all of the simulation plots are the variables obtained from the simulation of the system with the classical control strategy, which maintains the DC current and the DC voltage constant by modulation of the rectifier ignition and inverter extinction angles. Also for the classical control strategy the exciter inputs are set to zero.

The rotor angle swings of generators 1 and 2 for the optimally controlled system are shown in figure 33. It is evident that the performance of the optimally controlled system is superior to the performance of the system with classical control strategy. The rotor swings are damped out very rapidly in case of the optimally controlled system, hence the dynamic stability of the system is improved. However, the selection of a state regulator type of performance measure is not quite adequate for the system under investigation due to the fact that after removal of the faulted line the final state of the system are no longer equal to the initial state. But the initial and final states are close enough to achieve good results. Selection of this type of performance measure and utilization of the steady state solution
of the Riccati equation results in 0.24 and 0.07 percent error in the final values of the rotor angles of generators 1 and 2, respectively.

The exciter optimal inputs for generators 1 and 2 are shown in figure 34. From this figure two facts are apparent. First, the optimal inputs are quite active, especially the optimal input for the exciter of generator 2 which remains effective longer than the optimal input for the exciter of generator 1. Second, a steady state error of 0.17 and 0.06 percent exist for the exciter optimal inputs of generators 1 and 2, respectively.

The rectifier DC current modulation is shown in figure 35. Also, a steady state error of 0.06 percent exists for the rectifier DC current. The steady state errors exist due to the fact that the steady state solution of the Riccati equation is used instead of the time variable solution of the Riccati equation.

The rectifier and inverter station AC voltages are shown in figure 36. It is evident that the increase and decrease in the DC current causes initial voltage jumps. However, as soon as the modulation activity of the DC current dies out the AC voltages are stabilized. Also, it is important to mention that the rectifier AC bus represents the characteristics of a weak system, and in the case with the classical control strategy the rotor oscillations of generator 1 causes AC voltage oscillations at rectifier station for quite some time. But, in the case with the optimal control strategy the rotor oscillations of generator 1 are damped out very rapidly and the AC voltage at rectifier station is regulated accordingly.
Figure 33. CASE 1: Rotor Angle Swings of Generators 1 and 2
Figure 34. CASE 1: Exciter Optimal Input of Generators 1 and 2
Figure 35. CASE 1: Rectifier DC Current Modulation
Figure 36. CASE 1: Converter AC Voltages
As mentioned previously the steady state errors are due to a particular choice of the steady state solution of the Riccati equation. If at final control time the value of vector $Z(t)$ (which contains the errors between the actual and initial or the desired values of the states) is not zero, it is multiplied by the steady state solution of the Riccati equation which is non-zero and produces a non-zero $KZ(t)$ term in optimal control law. This results in a non-zero optimal control vector $V(t)$ which expresses the errors between the actual and desired optimal inputs. In cases where the time variable solution of the Riccati equation is used, due to selection of a zero penalty on final value of the states in performance measure, the final boundary value of the Riccati equation becomes zero. Therefore, independent of the final value of the error vector $Z(t)$, the term $K(t)Z(t)$ is zero at final control time. This leads to a zero steady error in the inputs. The same argument can be used for cases in which the optimal control law contains $g(t)$, since the final boundary value of $g(t)$ is also zero.

To highlight the effect of the exciter optimal inputs on dynamic stability improvement of the system, the optimal inputs are computed but not fed into the exciters and the generators rotor swings of this run are shown in figure 37. Since the rectifier power is mostly provided by the generator 1 the modulation of the DC current alone will damp out the rotor swings of this generator, but it requires a greater time and the amplitude of the oscillations are much greater. However, the output power of generator 2 makes insignificant contribution to the rectifier power and the DC current modulation has very little effect in damping the rotor swings of this generator. This illustration proves the proper choice in thrust of
the research and verifies the fact that for optimal control of a power system with an embedded HVDC link both AC and DC powers should be optimally steered for assurance of dynamic stability of the system.
Figure 37. CASE 1: Rotor Angle Swings of Generators 1 and 2
CASE 2:

Figures 38 through 41 shows the simulation results when the time variable solution of the Riccati equation is used. The results are very similar to the simulation of the system with use of the steady state solution of the Riccati equation. Practically the generators rotor swings are almost identical. The error in final value of the rotor angles of generators 1 and 2 are 0.10 and 0.13 percent, respectively. The converter AC voltage profiles are also very similar. The difference becomes apparent when the optimal inputs are compared. The advantage of using the time variable solution of the Riccati equations is the fact that the steady state errors in optimal inputs are nonexistent.
Figure 38. CASE 2: Rotor Angle Swings of Generators 1 and 2
Figure 39. CASE 2: Exciter Optimal Input of Generators 1 and 2
Figure 40. CASE 2: Rectifier DC Current Modulation
Figure 41. CASE 2: Converter AC Voltages
7.3: Linearized Equations and Tracking Performance Measure

The linearized form of the state equation is identical to equation (7.2.2), and the performance measure of state tracking type is selected. The desired values of the states are obtained from two power flow runs. The base case power flow run provides the initial value of the states, while the power flow after removal of the faulted line provides the final value of the states. It is decided to guide the states of the system from the initial point to the final point along an exponential path with a time constant of one second. Hence, the desired path has the form:

\[ X_d(t) = Ae^{-t} + \Gamma \]  \hspace{1cm} (7.3.1)

where \( A \) and \( \Gamma \) are computed from the initial and final value of the states based on the results of the two power flow runs.

The optimal control law from equation (3.5.21) is:

\[ V^*(t) = R^{-1}B^T[g(t) - K(t)Z(t)] \]  \hspace{1cm} (7.3.2)

Solution of the Riccati equation is the same as the solution of this equation for the state regulator case, since \( C(t) \) in equation (3.5.27) is equal to the identity matrix. Again, taking advantage of \( C(t) \) being an identity matrix, \( g(t) \) is the solution of the equation (3.5.28) with \( Y_d(t) \) replaced by \( C(t)Z_d(t) \) and the final
boundary value of zero due to the fact that $H$ matrix is chosen to be a zero matrix. But, according to the translation of equation (5.6.8),

$$Z(t) = X(t) - X(t_0) \quad (7.3.3)$$

Therefore,

$$Z_d(t) = X_d(t) - X(t_0) \quad (7.3.4)$$

and $X_d(t)$ is available from equation (7.3.1). Given $Z_d(t)$ from equation (7.3.4), then equation (3.5.28) is solved and the value of vector $g(t)$ is obtained. Consequently the optimal inputs are computed from the following equation.

$$[V_{s_1} \ V_{s_2} \ \Delta P_d]^T = R^{-1}B^T(g(t) - K(t)[X(t) - X(t_0)]) \quad (7.3.5)$$

CASE 3:

The simulation results for the linearized state equation and the state tracking type of performance measure with steady state solution of the Riccati equation are shown in figures 42 through 45. The reference of the comparison are the variables obtained from the simulation of the system with classical control strategy.

The generator rotor angle swings are shown in figure 42. Superiority of the optimally controlled system in securing the dynamic stability of the system is evident from this figure. The selection of the performance measure is well justified,
and the error in the final value of the rotor angles of generators 1 and 2 is 0.59 and 0.08 percent, respectively.

The response of the exciter optimal inputs are shown in figure 43. It is clear that the inputs are quite active and their steady state error exists. The steady state error of the exciter optimal inputs for generators 1 and 2 are 0.16 and 0.06 percent, respectively.

The rectifier DC current modulation is shown in figure 44. It also contains a steady state error of 0.06 percent. The reason for the steady state errors is again selection of the steady state solution of the Riccati equation.

The rectifier and inverter AC voltages are shown in figure 45. The increase and decrease of the rectifier DC current causes voltage jumps. However, the AC voltage regulation for the rectifier bus, which is a weak bus, is far superior for the optimally controlled system when compared with the case of classically controlled system.
Figure 42. CASE 3: Rotor Angle Swings of Generators 1 and 2
Figure 43. CASE 3: Exciter Optimal Input of Generators 1 and 2
Figure 44. CASE 3: Rectifier DC Current Modulation
Figure 45. CASE 3: Converter AC Voltages
CASE 4:

Figures 46 through 49 shows the simulation results when the time variable solution of the Riccati equation is used. The simulation results are quite similar to the case 3. The major advantages are the smaller error in the final value of the generators rotor angle, and the optimal inputs. The error in the final value of the rotor angles of generators 1 and 2 are 0.46 and 0.03 percent, respectively. There is no error in the final value of the optimal inputs, since the final value of K(t) is equal to the H matrix which is identically zero.
Figure 46. CASE 4: Rotor Angle Swings of Generators 1 and 2
Figure 47. CASE 4: Exciter Optimal Input of Generators 1 and 2
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Figure 48. CASE 4: Rectifier DC Current Modulation
Figure 49. CASE 4: Converter AC Voltages
7.4: Nonlinear Equations and Regulator Performance

Measure

The formulation of the state equation in this section takes advantage of the developed control algorithm, but it is slightly different from the formulation of section 6 of chapter 5. Given:

\[ \dot{X}(t) = F(X(t), U(t)) \]  

(7.4.1)

then the Taylor series expansion results in,

\[ \dot{X}(t) = F(X_d(t), U_d(t)) + A[X(t) - X_d(t)] + B[U(t) - U_d(t)] \]  

(7.4.2)

where the A and B matrices are partials of the function F of equation (7.4.1) with respect to X(t) and U(t) evaluated at X_d(t) and U_d(t), respectively. This however results in time variable A and B matrices. At this point we introduce the assumption that the A and B matrices are to be evaluated at X(t_0) and U(t_0). Even though at first this assumption might be subject to objections, but later it will be shown that the error caused by this assumption is captured in the residual term of the state equation. Defining the translations,

\[ Z(t) = X(t) - X_d(t) \]  

(7.4.3)

\[ V(t) = U(t) - U_d(t) \]  

(7.4.4)
and substitution of equations (7.4.3) and (7.4.4) into equation (7.4.2) leads to:

\[ \dot{X}(t) = F(X_{d}(t), U_{d}(t)) + AZ(t) + BV(t) \]  \hspace{1cm} (7.4.5)

Adding and subtracting a few terms such that the equation (7.4.5) would have the mathematical form of equation (7.4.1) results in,

\[ \dot{X}(t) = F(X(t), U(t)) + AZ(t) + BV(t) + [F(X(t), U(t)) - F(X_{d}(t), U_{d}(t)) - AZ(t) - BV(t)] \]  \hspace{1cm} (7.4.6)

where the terms inside the bracket are the new terms. It is apparent that if the equal terms in equation (7.4.6) are cancelled, then this equation will express the mathematical form of equation (7.4.1). Now a term \( \dot{X}_{d}(t) \) is subtracted from both sides of the equation (7.4.6), hence:

\[ \dot{X}(t) - \dot{X}_{d}(t) = F(X_{d}(t), U_{d}(t)) + AZ(t) + BV(t) + [F(X(t), U(t)) - F(X_{d}(t), U_{d}(t)) - AZ(t) - BV(t) - \dot{X}_{d}(t)] \]  \hspace{1cm} (7.4.7)

Taking advantage of the equation (7.4.3) and cancellation of the first right hand side term preceding the equal sign with its counterpart inside the bracket results in:

\[ \dot{Z}(t) = AZ(t) + BV(t) + f(t) \]  \hspace{1cm} (7.4.8)

where,

\[ f(t) = F(X(t), U(t)) - AZ(t) - BV(t) - \dot{X}_{d}(t) \]  \hspace{1cm} (7.4.9)
and \( f(t) \) is the residual expressed by equation (7.4.9) which captures the inaccuracies resulting from the evaluation of A and B matrices at the initial value of the states and inputs rather than their desired values. The \( f(t) \) is computed during the control interval from the measured value of the state variables and its future value is extrapolated.

To minimize the error in the states and inputs of the original system from the desired values, the following regulator type of performance measure is utilized for the translated system.

\[
J(V) = \frac{1}{2} \int_{0}^{t_f} [Z^T(t)QZ(t) + V^T(t)RV(t)] \, dt \quad (7.4.10)
\]

From section 6 of chapter 3 the optimal control law for the translated system is,

\[
V^*(t) = R^{-1}B^T[g(t) - K(t)Z(t)] \quad (7.4.11)
\]

where \( K(t) \) is the solution of the Riccati equation (3.6.6) with \( C(t) \) equal to identity matrix and final boundary value of zero. The term \( g(t) \) is the solution of equation (3.6.7), again with \( C(t) \) equal to identity matrix and \( Y_f(t) \) equal to zero. Then,

\[
\dot{g}(t) = -[A^T - K(t)BR^{-1}B^T]g(t) + K(t)f(t) \quad , \quad g(t_f) = 0 \quad (7.4.12)
\]

Hence, measurement of the states and evaluation of the value of vector \( f(t) \) plus extrapolation of its future values yield \( g(t) \) from equation (7.4.12). But, as soon as the new set of measurements are obtained the value of \( f(t) \) is computed and its
extrapolation is reevaluated. Hence, equation (7.4.12) is repeatedly solved with the new forcing function \( f(t) \) during the control interval. Consequently, the optimal control law is:

\[
\begin{bmatrix} V_{s1} & V_{s2} & \Delta P_d \end{bmatrix}^T = R^{-1} B^T \{ g(t) - K(t)[X(t) - X_d(t)] \} \quad (7.4.13)
\]

CASE 5:

The simulation results for representation of the nonlinear form of the state equation and a regulator type of performance measure with steady state solution of the Riccati equation are shown in figures 50 through 53. The reference for comparison purposes are the system variables simulated with classical control strategy. The future value of the vector \( f(t) \) is predicted by taking the mean of the values evaluated from measurement of the state variables during the previous period and maintaining this value constant for the future time.

The rotor angle swings for generators 1 and 2 are shown in figure 50. From this figure it is evident that the optimal controls damp out the electromechanical oscillations. The error in the final value of the rotor angles for generator 1 and 2 are 3.57 and 0.18 percent, respectively.

The trajectories of the exciter optimal inputs are shown in figure 51. The optimal inputs are active and the error in final value of the input for exciter of generators 1 and 2 are 0.48 and 0.01 percent, respectively.
The rectifier current response is shown in figure 52. The trajectory shows a sudden initial increase of the DC current above the nominal rating of the HVDC link. Then a drop below the nominal rating of the link within a few steps which eventually shifts back toward the desired value of the DC current. The slow response of the DC current toward the desired value causes the slow response of the rotor angle of generator 1 toward the desired value. The above fact is because of the inaccuracy in prediction of the future values of \( f(t) \) and its burden on inaccurate computation of \( g(t) \) which is then included in the optimal control law. The error in the final value of the rectifier DC current is 0.24 percent.

The AC voltage profiles for the converter stations are shown in figure 53. The AC voltage response of the rectifier station, which is a weak bus, following the initial modulation of the DC current is ripple free.
Figure 50. CASE 5: Rotor Angle Swings of Generators 1 and 2
Figure 51. CASE 5: Exciter Optimal Input of Generators 1 and 2
Figure 52.  CASE 5:  Rectifier DC Current Modulation
Figure 53. CASE 5: Converter AC Voltages

CHAPTER 7. SIMULATION RESULTS
CASE 6:

Figures 54 through 57 show the simulation results when the time variable solution of the Riccati equation is utilized. The rotor angle swings of the generators 1 and 2 are shown in figure 54. Since the time variable solution of the Riccati equation is used then the rotor angles tend to approach the desired value at a faster rate. This is due to the fact that \( K(t) \) shifts toward zero as the final control time is approached. The estimated value of vector \( f(t) \) is multiplied by the matrix \( K(t) \) in differential equation (7.4.12), and as \( K(t) \) approaches zero the solution of this equation being \( g(t) \) also approaches zero. Therefore this phenomenon eventually eliminates the effect of inaccuracies of \( f(t) \) in the optimal control law. The error in the final value of the rotor angles of generators 1 and 2 are 1.42 and 0.13 percent, respectively.

The exciter optimal inputs are shown in figure 55. Again the effect of the time variable solution of the Riccati equation is apparent in moving the optimal inputs toward zero as the final control time is approached. The final value error in the exciter input of generators 1 and 2 are practically zero.

The rectifier DC current modulation is shown in figure 56. Use of the time variable solution of the Riccati equation also causes the final value of the DC current to approach the desired value. The error in the final value of the rectifier current is practically zero.
The AC voltages of the converter stations are shown in figure 57. The impact of the time variable solution of the Riccati equation in taking the AC voltages closer to their final value is quite clear.
Figure 54. CASE 6: Rotor Angle Swings of Generators 1 and 2
Figure 55. CASE 6: Exciter Optimal Input of Generators 1 and 2
Figure 56. CASE 6: Rectifier DC Current Modulation
Figure 57.  CASE 6: Converter AC Voltages
7.5: Nonlinear Equations and Tracking Performance

Measure

The formulation of state equation for the developed control algorithm is documented in section 6 of chapter 5. According to equation (5.6.11) the form of the state equation is,

\[ \dot{Z}(t) = A Z(t) + B V(t) + f(t) \]  

(7.5.1)

where the A, B, Z(t), V(t), and f(t) are defined in equations (5.6.7) through (5.6.10). \( Z_d(t) \) is defined as:

\[ Z_d(t) = X_d(t) - X(t_0) \]  

(7.5.2)

which is identical to equation (7.3.4). To minimize the errors in the states of the translated system of equation (7.5.1) from the desired values, the following performance measure is defined.

\[ J(V) = \frac{1}{2} \int_0^T \left[ (Z(t) - Z_d(t))^T Q (Z(t) - Z_d(t)) + V^T(t) R V(t) \right] dt \]  

(7.5.3)

From section 6 of chapter 3 the optimal control law for the above system is,

\[ V^*(t) = R^{-1} B^T [g(t) - K(t) Z(t)] \]  

(7.5.4)
where $K(t)$ is the solution of the Riccati equation (3.6.6) with $C(t)$ equal to identity matrix, and zero final boundary value. Also, the term $g(t)$ is solution of the equation (3.6.7) with $C(t)$ equal to identity, and $Y_d(t)$ is replaced by $Z_d(t)$. Hence,

$$
\dot{g}(t) = - [A^T - K(t)BR^{-1}B^T]g(t) - QZ_d(t) + K(t)f(t) , \quad g(t_f) = 0 \quad (7.5.5)
$$

As soon as a new set of measurements are obtained the value of $f(t)$ is computed and its extrapolation is reevaluated. Hence, equation (7.5.5) is repeatedly solved with the new forcing function $f(t)$ during the control interval. Consequently the values of the optimal inputs are computed from,

$$
[V_{s1} \; V_{s2} \; \Delta P_{di}]^T = R^{-1}B^T[g(t) - K(t)[X(t) - X(t_0)]] \quad (7.5.6)
$$

CASE 7:

The simulation results for representation of the nonlinear form of the state equation and tracking type of performance measure with the steady state solution of the Riccati equation are shown in figures 58 through 61. Again the reference of comparison is the system performance with classical control strategy. Also, the future value of the vector $f(t)$ is predicted by a straight line, obtained from the computation of the mean of previous values of $f(t)$ from the state measurements.

The rotor angle swings of generators 1 and 2 are shown in figure 58. The error in the final value of the rotor angles of generators 1 and 2 are 3.69 and 0.19 percent, respectively.
The optimal inputs for exciter of generators 1 and 2 are shown in figure 59. The error in the final value of the inputs for exciter of generators 1 and 2 are 0.75 and 0.08 percent, respectively.

The rectifier DC current modulation is shown in figure 60. The error in final value of the rectifier current is 0.25 percent.

The AC voltage profiles of the converter stations are shown in figure 61.
Figure 58. CASE 7: Rotor Angle Swings of Generators 1 and 2
Figure 59. CASE 7: Exciter Optimal Input of Generators 1 and 2

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Figure 60. CASE 7: Rectifier DC Current Modulation
Figure 61. CASE 7: Converter AC Voltages
CASE 8:

Figures 62 through 65 show the simulation results when the time variable solution of the Riccati equation is used. The rotor angle swings of generators 1 and 2 are shown in figure 62. The error in the final value of the rotor angles for generators 1 and 2 are 1.46 and 0.14 percent, respectively. The optimal inputs for exciter of generators 1 and 2 are shown in figure 63. The error in their final values are practically zero. The rectifier DC current modulation is shown in figure 64, and again the final error of this current is zero. The AC voltage responses of the rectifier and inverter stations are shown in figure 65.
Figure 62. CASE 8: Rotor Angle Swings of Generators 1 and 2
Figure 63. CASE 8: Exciter Optimal Input of Generators 1 and 2
Figure 64. CASE 8: Rectifier DC Current Modulation
Figure 65. CASE 8: Converter AC Voltages
The simulation results of cases 5, 6, 7, and 8 leads to the conclusion that utilization of the time variable solution of the Riccati equation assures a zero final value error for all of the inputs. Also, selection of the time variable solution of the Riccati equation causes a more rapid shift of the generators rotor angle toward their desired value. Even though the translated state equation formulation and performance measure of section 4 and 5 of this chapter are different, the simulation results show amazing similarities. The reason for similar simulation results are as follows.

1. The difference in computation of the optimal control law is evident from comparison of equations (7.4.13) and (7.5.6). The difference lies in the terms inside the bracket. The solution of the Riccati equation, K(t), in equation (7.4.13) is multiplied by the difference in the current value of the states and the desired value of the states, while in equation (7.5.6) it is multiplied by the difference in the current value of the states and the initial value of the states. However, from the power flow results the differences between the initial and final value of the states are minor. Therefore, the initial value of the states are almost equal to the desired value of the states.

2. The term g(t) in equations (7.4.13) and (7.5.6) is the solution of two different differential equations (7.4.12) and (7.5.5). The difference lies in the term \(-QZ_d(t)\) of equation (7.5.5). But \(Z_d(t)\) as expressed by equation (7.5.2) and explained above is the difference between the initial
and desired value of the states, and it is small. Consequently for the example system, the solution of the two equations (7.4.12) and (7.5.5) are almost identical.

From simulation results of section 4 and 5 of this chapter it is evident that the developed control algorithm damp out the rotor angle swings of generators 1 and 2 and improves the dynamic stability of the system. However, from the trajectory of the rotor angle of generator 1 it is clear that a second maxima exists, and following the second maxima the rotor angle moves toward the desired value at a very slow rate. As mentioned previously, comparison of the trajectories of the rotor angle of generator 1 for the cases involving the steady state and time variable solution of the Riccati equation makes clear that in case of the time variable solution of the Riccati equation the rotor angle approaches the desired value at a slightly faster rate. This also demonstrates that the type of solution of the Riccati equation alone is not responsible for the slow movement of the rotor angle toward the desired value. The problem involves the computed value of g(t) in the optimal control law which is the solution of the vector differential equation with f(t) as a forcing function. The inaccuracy in estimation of the future values of the f(t) results in the inaccurate computation of g(t). Thus, when the mean value estimation of f(t) is incorporated, the value of the first few residual vectors are much larger than the remainder, and this corrupts the estimation of the future values of f(t) as more measurements are obtained and f(t) is reevaluated for computation of g(t). Hence, a moving average estimation technique with a window of 4 samples is used to replace the mean value estimation technique.

CHAPTER 7. SIMULATION RESULTS
CASE 9:

The simulation results for representation of the nonlinear form of the state equation and tracking type of performance measure with the steady state solution of the Riccati equation are shown in figures 66 through 69. Again the reference of comparison is the system performance with classical control strategy. Also, the future values of the vector $f(t)$ is predicted by a straight line, obtained from computation of the moving average of the previous values of $f(t)$ from the state measurements.

The rotor angle swings of generators 1 and 2 are shown in figure 66. The error in the final value of the rotor angles of generators 1 and 2 are 0.85 and 0.02 percent, respectively.

The optimal inputs for exciter of generators 1 and 2 are shown in figure 67. The error in the final value of the inputs for exciter of generators 1 and 2 are 0.43 and 0.09 percent, respectively.

The rectifier DC current modulation is shown in figure 68. The error in final value of the rectifier current is 0.08 percent.

The AC voltage profiles of the converter stations are shown in figure 69.
Figure 66. CASE 9: Rotor Angle Swings of Generators 1 and 2

CHAPTER 7. SIMULATION RESULTS
Figure 67. CASE 9: Exciter Optimal Input of Generators 1 and 2
Figure 68. CASE 9: Rectifier DC Current Modulation
Figure 69. CASE 9: Converter AC Voltages
CASE 10:

Figures 70 through 73 show the simulation results when the time variable solution of the Riccati equation is used with the moving average estimation technique. The rotor angle swings of generators 1 and 2 are shown in figure 70. The error in the final value of the rotor angles for generators 1 and 2 are 0.05 and zero percent, respectively. The optimal inputs for exciter of generators 1 and 2 are shown in figure 71. The error in their final values are practically zero. The rectifier DC current modulation is shown in figure 72, and again the final error of this current is zero. The AC voltage responses of the rectifier and inverter stations are shown in figure 73.

The simulation result of cases 9 and 10 when compared with cases 7 and 8 show clearly that the rotor angle swing of generator 1 is improved as a result of incorporation of the moving average estimation technique. Following the second maxima, independent of the type of the solution of the Riccati equation, the rotor angle of generator 1 approaches the desired value at a very fast rate. This leads to the conclusion that the correct knowledge of the future values of the system states has an important effect in system performance. In this investigation the moving average estimation technique is superior over the ordinary mean value estimation technique, and it adequately predicts the future value of the residuals.
Figure 70.  CASE 10:  Rotor Angle Swings of Generators 1 and 2
Figure 71. CASE 10: Exciter Optimal Input of Generators 1 and 2
Figure 72. CASE 10: Rectifier DC Current Modulation
Figure 73. CASE 10: Converter AC Voltages
7.6: Nonlinear State Equation and Approximate Optimal Control Law

In this section the state equation formulation of the section 4 of chapter 7 is used. The translations are also identical to equations (7.4.3) and (7.4.4). The optimal control law for the regulator performance measure of equation (7.4.10) is given by equation (7.4.11). The optimal control law contains the term g(t) which is the solution of the equation (7.4.12):

$$\dot{g}(t) = -[A^T - K(t)BR^{-1}B^T]g(t) + K(t)f(t) \quad g(t_f) = 0 \quad (7.6.1)$$

The measurements arrive in discrete points in time during the total control interval. An example of this is shown in figure 74. Times $t_1$ and $t_2$ are the first two discrete points in time where the state measurements become available. At time $t_1$ the first state measurement vector is obtained. This vector is used to compute the residual $f(t_1)$. The value of $f(t_0)$ and $f(t_1)$ is used to predict the future value of the residual, which is a constant (shown as $f_1$) due to the nature of the estimator used in this dissertation. Since equation (7.6.1) is a differential equation with a known final boundary value, it is solved backward in time with $f_1$ as its forcing function. The solution for $g(t)$ contains a steady state and a transient period. In figure 74 the steady period is $[t_0, t_m]$ and the transient period is $[t_m, t_f]$. Since the time $t_1$ is the first point in the control interval at which control action is taken, it
Figure 74. Conceptual Illustration of the Solution for $g(t)$
falls in the interval during which the value of $g(t_i)$ from the solution for $g(t)$ is equal to the steady state solution of the differential equation. At time $t_2$ the second state measurement vector is obtained and $f(t_2)$ is computed. Based on the value of $f(t_2)$ and the previous values of the residual the estimator predicts the future value of the residual which is $f_2$. Since the time $t_2$ is still early in the total control interval, again the solution for $g(t)$ at this time is equal to its steady state value. The same argument can be used for subsequent points until time $t_m$ is reached. Therefore, the numerical integration approach for obtaining a solution for $g(t)$ at any of these discrete points which fall in the steady state period $[t_0, t_m]$ is time consuming and not justified. If the steady state solution of the Riccati equation is used, then the steady state solution of $g(t)$ is obtained from:

$$g(t) = [A^T - KBR^{-1}B^T]^{-1}Kf$$  \hspace{1cm} (7.6.2)$$

Thus from now on $g(t)$ is computed from equation (7.6.2) rather than the equation (7.6.1).

Let us assume at time $t_m$ the states have smoothly reached their desired values and also the estimator predicts a value of zero for the future values of the residual $f(t)$. Thus, from equation (7.6.2) the value of $g(t_m)$ is zero. If the assumption turns out to be the actual fact, then no approximation is introduced in the optimal control law of equation (7.4.13) and tremendous amount of computation time is saved. However, if the states do not reach their desired values at time $t_m$, then $f_m$ also would not be zero and the steady state solution of the differential equation is no longer equal to its exact solution for $g(t)$ from equation (7.6.1). Hence, use
of the steady state solution of \( g(t) \) according to equation (7.6.2) in optimal control law of equation (7.4.13) introduces an approximation. It is also important to note that \( t_m \) is not stationary, but moving along the time axis any time the differential equation is solved for \( g(t) \).

The simulation results with a mean value estimation technique and the approximate optimal control law are shown in figures 75 through 78.

CASE 11:

The rotor angle swings of generators 1 and 2 are shown in figure 75. It is evident from this figure that the error in final value of the rotor angle of generator 1 is quite significant. The error in final value of the rotor angles of generators 1 and 2 are 6.45 and 0.01 percent, respectively. The exciter optimal inputs are shown in figure 76. Again these inputs contain significant steady state errors, especially the optimal input for the exciter of generator 1. The errors of the inputs for exciter of generators 1 and 2 are 2.02 and 0.13 percent, respectively. The rectifier DC current modulation is shown in figure 77. The final value of the DC current also contain a steady state error of 1.52 percent. The AC voltage profiles at the rectifier and inverter stations are shown in figure 78. The trajectory of the voltages do not reach the final value specified by the power flow run.

As is evident from the results of the simulation with approximate value of \( g(t) \) in optimal control law, the system performance is not quite acceptable and the final states and inputs are not close to the desired values. The reason behind the poor
system performance is the fact that the steady state solution of the Riccati equation and also approximation of g(t) corrupts the values of the optimal inputs when the mean value estimation is used for prediction of $f$.
Figure 75. CASE 11: Rotor Angle Swings of Generators 1 and 2
Figure 76. CASE 11: Exciter Optimal Input of Generators 1 and 2
Figure 77. CASE II: Rectifier DC Current Modulation
Figure 78. CASE 11: Converter AC Voltages
CASE 12:

Figures 79 through 82 show the simulation results for the approximate optimal control law, when the moving average estimation technique with a window of 4 samples is used to compute $f$. The rotor angle swings of generators 1 and 2 are shown in figure 79. The error in final value of the rotor angles of generators 1 and 2 are 0.65 and 0.06 percent, respectively. The exciter optimal inputs are shown in figure 80. The errors of the inputs for exciter of generators 1 and 2 are 0.20 and 0.00 percent, respectively. The rectifier DC current modulation is shown in figure 81. The error in final value of the DC current is 0.13 percent. The AC voltage profiles at the rectifier and inverter stations are shown in figure 82.

Comparison of the rotor angle swings of generators 1 and 2, when the exact and approximate optimal control law is used with a moving average estimation technique in the optimal controller block of the simulation program, shows that there are no significant differences in the two trajectories (cases 10 and 12). Therefore, the approximate control law can be implemented rather than the exact optimal control law for the adaptive optimal control of the system. The approximate optimal control law is computationally about 50 times faster than the exact optimal control law. This is due to the fact that the numerical integration for solution of the differential equation involving the term $g(t)$ as required by the exact optimal control law is replaced by a simple matrix multiplication for the approximate optimal control law. Also, the need for storage of the time variable solution of the Riccati equation is eliminated. Consequently, implementation of the ap-
proximate control law removes the computational time limitations which can be of a serious problem for implementation of the exact optimal control law as an on-line optimal controller.
Figure 79. CASE 12: Rotor Angle Swings of Generators 1 and 2
Figure 80. CASE 12: Exciter Optimal Input of Generators 1 and 2
Figure 81. CASE 12: Rectifier DC Current Modulation
Figure 8. CASE 12: Converter AC Voltages

CHAPTER 7. SIMULATION RESULTS
8.1: Conclusions

An adaptive optimal control strategy is developed for enhancement of the dynamic stability of a power system with an embedded two terminal HVDC link. The optimal control algorithm requires the nonlinear mathematical model of the dynamic system. The Taylor series expansion provides the linear portion of the nonlinear differential equations which represents the power system. The state measurement technique is used to evaluate the value of the derivative of the translated state vector from the linearized state equation and the original nonlinear state equation. The difference of the two vectors, referred to as the residual, is computed repeatedly as time progresses and an estimation technique is utilized to predict the future value of the residuals. The residual which previously was
ignored throughout the literature, makes a contribution in computation of the optimal control law. Since the contribution of the residuals to the optimal control law requires the prediction of their future values, and any prediction algorithm introduces some inaccuracies, the optimal control law is theoretically considered to be suboptimal.

In large scale power system modeling only the dynamic equations of the generators and their corresponding exciters are used. The HVDC link and the load are represented by constant impedances. This by no means is a restriction since the system measurements reflect the exact characteristics of HVDC link and the load (for detail discussion see page 113). The interconnection of these dynamic elements (generators) through the AC network is represented by the admittance matrix, which later is reduced by the Kron formula and only the identity of the infinite bus in addition to the generators q-axis buses are preserved. In order to express the proper change of the generators electrical power as a result of DC power modulation, the network sensitivity approach (generation shift factor) is used and the swing equations are modified accordingly.

To enhance the dynamic stability of the system, the field voltage of the generators are modulated in addition to the specified DC current at the rectifier station. The actual inputs computed from the optimal control algorithm are the auxiliary excitation inputs and the deviation of the DC power from the original value at the inverter station. The auxiliary inputs are directly added to the exciter reference settings, therefore it actually can be thought that the reference voltage set-

CHAPTER 8. CONCLUSIONS AND RECOMMENDATIONS
tings are modulated. This causes the modulation of the field voltage which effectively results in change of the generator terminal voltage. The change in the generator terminal voltages causes the steering of the generator electrical power. However, this AC power steering is not very large due to physical limitations such as field saturation. The exciter control is quite effective in damping the low amplitude rotor swings for generators located remotely from the HVDC link. The simulation results showed when the exciter optimal inputs are blocked and not fed to the exciter, the rotor angle swing of generator 2 which is located remotely from the HVDC terminals does not fully damp out within the five second control interval. The deviation of the DC power is converted to deviation in rectifier DC current and consequently it is added to its specified value in order to modulate the DC power. The DC power modulation, which is considerably greater than the AC power modulation via the exciter optimal inputs, effectively steers the AC power at rectifier and inverter AC buses and is mostly responsible for securing the system dynamic stability. There is also a limitation on the maximum value of DC current. This is related to the limitation on the minimum value of the valve ignition angles. Thus, to avoid this problem, it is assumed that the converter transformer taps can be changed very quickly. Presently the change in tap setting of LTC require a time greater than the entire 5 second control interval. If in the future, fast tap changing can be accomplished, then the issue is resolved. However, an alternative approach is possible and is discussed in the recommendation section to remedy the problem.
The simulation results for the example system with a variety of different optimal control law formulations are documented in chapter 7. The system performance is considerably improved due to implementation of the optimal controllers. The closed loop optimal control law for the linearized system and the state regulator and tracking type of performance measure show improved system stability. The developed optimal control law, again a closed loop control law, and different formulations of the translated system show excellent system performance, depending upon the selection of proper estimation algorithm. The trajectories of the generator rotor angles show better damping when the moving average estimation technique with a window of 4 samples is used instead of the mean value estimation technique. An approximate optimal control law for the developed control algorithm is also investigated, and the simulation results show almost identical system performance and excellent damping of the rotor angle swings. The approximate optimal control law is fully implementable for on-line control purposes and it is computationally about 50 times faster than its exact counterpart. It also does not use the time variable solution of the Riccati equation and eliminates the need for very large memory.

The performance of the system with the developed optimal control strategy when compared with performance of the system with the optimal control strategy for the linearized equations reveals its superiority through the fact that the value the first minima of the rotor angles of generators 1 is reduced.
8.2: Recommendations for Future Work

During the course of this research, validity of the developed control algorithm was tested, and the simulation results proved the effectiveness of the algorithm. However, in the previous section it was mentioned that to avoid the upper or lower limitations on the valve ignition angle, the converter transformer tap is changed very quickly. The recommendation for future work address a solution to this problem and introduce a few additional thoughts for further investigations. The recommendations are as follows:

1. To maintain the converter transformers tap constant it is possible to place static var compensators on the primary side of the converter transformers, and introduce additional optimal controls for this devices to modulate the reactive power at these buses. The controls can optimally increase or decrease the supply of the reactive power at these buses when the DC current modulation becomes significantly greater or smaller than the HVDC link rating, respectively. If this is accomplished, then the restriction on transformer tap changing is removed through the control of SVC for raising or lowering the primary AC voltage, and a fixed transformer tap ratio can be maintained. This in addition will enhance the converter bus voltage regulation during the DC power modulation.
2. Inclusion of the load bus voltages in performance measure through the Jacobian (since it is not a state variable) to regulate the load bus voltages.

3. Inclusion of a salient pole synchronous generator model in power system modeling.

4. Application of the developed control algorithm to a power system with a multiterminal HVDC link.

5. Inclusion of the optimal controller as a subroutine in a general purpose simulation program, such as SIMPOW which is developed by ASEA, to further test the limitations of the developed control algorithm.
APPENDIX A. L-A-S AIRCRAFT LANDING SIMULATION PROGRAM

The aircraft landing simulation program is discussed in this appendix and the L-A-S program listing is shown following the discussion.

Block number 1 defines the input data which are matrices A, F, B, Q, R, and H (equations (4.4.3) through (4.4.17)). The F matrix is defined as,

\[
F = [(a_{11}^0 - a_{11}^s) \ 0 \ 0 \ 0]^T \quad (A.1)
\]

where the superscript o and s of \(a_{11}\) indicates the original and selected value of \(a_{11}\), respectively. Statements starting with semicolon are comment statements for the program. The operator DMA is used for definition of constant matrices. Commas of input field separate the entries of each row, and slashes separate each row. The output field contains the name of the array(s).
Block number 2 defines the necessary data for the operator CRR which solves the Riccati equation. Parameters DT and NT are the integration time increment and the number of discrete points in total control time, respectively. The matrix $S$ is computed from the equation:

$$
S = BR^{-1}B^T \quad (A.2)
$$

The output field AK is the array containing the time variable solution of the Riccati equation. The constant portion of this time variable solution is stored in array K, and it is used for the remainder of the program as the steady state solution of the Riccati equation.

Block number 3 computes the closed loop state matrix from the following relations with the use of equation (A.2).

Open Loop System: 

$$
\dot{X}(t) = AX(t) + BU(t) \quad (A.3)
$$

Optimal Control: 

$$
U^*(t) = R^{-1}B^T[g(t) - KZ(t)] \quad (A.4)
$$

Closed Loop System: 

$$
\dot{X}(t) = [A - SK]X(t) + ...... \quad (A.5)
$$

$$
[AC] = [A - SK] \quad (A.6)
$$

Block number 4 defines the matrices XDIS and XDDI which are the vectors containing the coefficients of the desired states, and the derivative of the desired states, respectively. Equation (4.5.7) is modified as follows,
\[ r(t) = AX_d(t) - \dot{X}_d(t) + f(t) \]  
\[ r(t) = ([A]X_{DIS} - [XDDI])e^{-0.2t} + f(t) \]  
\[ r(t) = [r_2]e^{-0.2t} + f(t) \]

where \( r_2 \) is a column vector. The above modification is valid, since the first two entries of \( X_{DIS} \) and \( XDDI \) are zero. The column vector \( [r_2] \) is then computed according to the quantity inside the bracket in equation (A.8).

Block number 5 prepares the necessary equations for solution of the differential equation \( g(t) \). The translation of equation (4.5.3) converted the problem from tracking type to a regulator type. Then \( Y_d(t) \) is zero and the terms containing \( Y_d(t) \) are excluded from equations (3.6.7) and (3.6.9). Thus,

\[ \dot{g}(t) = - [A^T - KS]g(t) + [K]r(t) \]  
\[ g(t_0) = [0] \]

And substitution of equation (A.9) into (A.10) leads to:

\[ \dot{g}(t) = - [A^T - KS]g(t) + [K][r_2]e^{-0.2t} + f(t) \]  

Using the definition of \([AC]\) from equation (A.6) in equation (A.12) results in,

\[ \dot{g}(t) = - [AC]^Tg(t) + [K][r_2]e^{-0.2t} + [K]f(t) \]  

APPENDIX A. L-A-S AIRCRAFT LANDING SIMULATION PROGRAM
Based on the derivation of equations (4.3.6) through (4.3.14) the term \([K][r_2]\) will increase by one the dimension of array \([ACT]\) which is the transpose of \([AC]\). The new variable is:

\[
x_5(t) = e^{-0.2t}
\]  
(A.14)

\[
x_5(t_f) = e^{-4}
\]  
(A.15)

Or,

\[
\dot{x}_5(t) = -0.2x_5
\]  
(A.16)

and,

\[
x_5(t_f) = 0.0183156
\]  
(A.17)

The first four entries of the fifth column are computed from \(-[K][r_2]\) and stored in array COL5. The first four entries of the fifth row (ROW5) are identically zero and the negative of the time constant of the exponential function is defined as NTAL forms the fifth entry of this row. Then column COL5 and ROW5 are tied to [ACT] to form a five by five array named AG. The function \(f(t)\) has the form:

\[
f(t) = Fx_1(t)
\]  
(A.18)

where for phase 1 the vector \(F\) is a zero column vector, and for phase 2 it is computed from equation (A.1), upon selection of a value for \(a_{11}\). Therefore, for
phase 1 \( x_1(t) \) is not necessary and it is defined as a zero column vector. However, after completion of simulation for phase 1, the first state trajectory is saved and its order is reversed by the relation of equation (4.3.5). The vector \( x_1(t) \) in reverse order is named \( XI/I \). Substitution of equation (A.18) into (A.13) and modifications according to equation (4.3.13) yield,

\[
\dot{g}(t) = \begin{bmatrix} ACT & -[K][r_2] \\ 0 & NTAL \end{bmatrix} g(t) + [MKF0][XI/I] \quad (A.19)
\]

or,

\[
\dot{g}(t) = [AG]g(t) + [MKF0]XI/I \quad (A.20)
\]

where,

\[
[MKF0] = - [K][F] \quad 0^T \quad (A.21)
\]

Also, from equations (4.3.14), (A.11), and (A.17) the initial condition \( G0 \) for equation (A.20) is formulated as,

\[
G0 = \begin{bmatrix} 0 \\ 0.0183156 \end{bmatrix}^T \quad (A.22)
\]

Parameters NT and TT are the number of points in total simulation time and the total simulation time, respectively. The definition of the above parameters and \( G0, AG, MKF0, \) and \( XI/I \) suffice the information required by the operator CE3 and the elaboration in use of this operator for solution of a differential equation with given final boundary value is discussed in section 3 of chapter 4. The sol-
olution of the differential equation (A.20) is stored in array G. Since the time solution of the fifth variable is of no interest, it is eliminated from array G, and the new array G1 contains the time solution of the first four variables. It is important to note that the time solution of the differential equation (A.19) is in reverse order.

Block number 6 computes the forcing function portion for simulation of the optimally controlled system. So far we have,

\[ U^*(t) = R^{-1}B^T[g(t) - KZ(t)] \]  \hspace{1cm} (A.23)

for the system of equation (4.5.6). Using the translation of equation (4.5.3),

\[ U^*(t) = R^{-1}B^T[g(t) - K[X(t) - X_d(t)]] \]  \hspace{1cm} (A.24)

Substitution of the equation (A.24) into equation (4.5.1) yields,

\[ \dot{X}(t) = AX(t) + BR^{-1}B^Tg(t) - BR^{-1}B^TKX(t) + BR^{-1}B^TKX_d(t) + f(t) \]  \hspace{1cm} (A.25)

\[ \dot{X}(t) = [A - BR^{-1}B^TK]X(t) + BR^{-1}B^Tg(t) + BR^{-1}B^TKX_d(t) + f(t) \]  \hspace{1cm} (A.26)

\[ \dot{X}(t) = [AC]X(t) + BV + BKHD + FX1T \]  \hspace{1cm} (A.27)

The second, third, and fourth terms of the above are the forcing function of the differential equation. The term BV is computed as follows:

\[ V = R^{-1}B^T[GT\|I] \]  \hspace{1cm} (A.28)
\[ BV = B \times V \quad (A.29) \]

Where GT is the transpose of G1 which is necessary for conformability, and \([II]\) is the square matrix in identity (4.3.5). The third and fourth terms, BKHD and FX1T of equation (A.27) are computed and added to the second term. Then,

\[ U1 = BV + FX1T + BKHD \quad (A.30) \]

Since the last three rows of U1 are all zero, they are eliminated and row 1 is stored in array U.

Block number 7 solves the state equation of the optimally controlled system, using operator CE3. The initial condition of the system X0 and the input matrix INPT are defined as follows.

\[ X0 = \begin{bmatrix} 0 & -0.0781 & -20 & 100 \end{bmatrix}^T \quad (A.31) \]

\[ INPT = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}^T \quad (A.32) \]

The input is necessary for the operator CE3 to obtain the time solution of the optimally controlled system. Also, UT is the transpose of U which is in proper form needed for operator CE3. Then arrays X0, NT, TT, AC, INPT, and UT are used as the input field for the operator CE3 to obtain the time solution of the system. State trajectories are stored in array X.
Block number 8 computes the eigenvalues of AC and stores it in array ACEV. The matrices A, F, K, ACEV are printed and the time solution of the optimally controlled system is plotted.

The L-A-S program listing is as follows:

```plaintext
;****BLOCK#1****
;PREPARE_THE_INPUT_DATA
;=*=*=*=*=*=*=*=*=*=*=*=*=*=*=*=*=*
;DEFINE_A_MATRIX
;=*=*=*=*=*=*=*=*=*=*=*=*=*=*=*=*=*
0(DMA)=A11
A11, -76, -003, 0/1, 102.4, -4/, 1(DMA) = A
;DEFINE_F_MATRIX
;=*=*=*=*=*=*=*=*=*=*=*=*=*=*=*=*=*
;WHERE_XDOT = AX + BU + F[X11]
-6(DMA) = F0
F0, A11(-) = F1
F1/0/0/0(DMA) = F
;DEFINE_B_MATRIX
;=*=*=*=*=*=*=*=*=*=*=*=*=*=*=*=*=*
-2.375/0/0/0(DMA) = B
;DEFINE_Q_MATRIX
;=*=*=*=*=*=*=*=*=*=*=*=*=*=*=*=*=*
///,...0001(DMA) = Q
;DEFINE_R_MATRIX
;=*=*=*=*=*=*=*=*=*=*=*=*=*=*=*=*=*
1(DMA) = R
;DEFINE_H_MATRIX
;=*=*=*=*=*=*=*=*=*=*=*=*=*=*=*=*=*
///,...0(DMA) = H
;****BLOCK#2****
;DEFINE_DT_AND_NT_FOR_SOLUTION_OF_RICCATI_EQUATION
;=*=*=*=*=*=*=*=*=*=*=*=*=*=*=*=*=*
;TOTAL_TIME = DT*NT____TOTAL_TIME = 20_SEC.
;WHERE_DT = TIME_INCREMENT__, __NT = #_OF_TIME_INCREMENT
50(DMA) = NT
.4(DMA) = DT
;COMPUTE_S = B*R(-1)*BT
;=*=*=*=*=*=*=*=*=*=*=*=*=*=*=*=*=*
;WHERE_R(-1) = INVERSE_OF_R_MATRIX__, __BT = TRANSPOSE_OF_B
B(T) = BT
R(-1) = RIN
B, RIN, BT(*) = S
;CALL_CRR_AND_SOLVE_RICCATI_EQUATION
;=*=*=*=*=*=*=*=*=*=*=*=*=*=*=*=*=*
;WHERE_AK_IS_THE_ARRAY_OF_TIME_VARIABLE_GAINS
A, Q, S, H, DT, NT(CRR) = AK
;COMPUTE_CONSTANT_GAINS_FROM_SOLUTION_OF_RICCATI_EQUATION
```

APPENDIX A. L-A-S AIRCRAFT LANDING SIMULATION PROGRAM 264
WHERE K IS THE CONSTANT_GAIN

AK,4(CTC) = K

***BLOCK#3*****

;COMPUTE_CLOSED_LOOP_A_MATRIX

WHERE AC = A-S*K IS THE_CLOSED_LOOP_A_MATRIX

S,K(*) = SK

A,SK(-) = AC

***BLOCK#4*****

;DEFINE_COEFF._OF_DESIRED_X_AND_DERIVATIVE_OF_DESIRED_X

XDIS = COEF. OF DESIRED X

XDDI = COEF. OF DERIVATIVE OF DESIRED X

//20/100(DMA) = XDIS

//4/-20(DMA) = XDDI

.COMPUTE_R2_OF_R

WHERE_R = [A*XDIS-XDDI] EXP + F = [R2] EXP + F

A,XDIS(*) = R1

R1,XDDI(-) = R2

***BLOCK#5*****

;PREPARE_FOR_SOLUTION_OF_DIFF_EQUATION_GDOT


OR_GDOT = [AC(T)]G - [K][R2]EXP - [K]F

DUE_TO_EXPONENTIAL_NATURE_OF_EXPR EVENTUALLY

[K][R2] FORMS THE FIFTH_COLUMN_OF_[AC(T)]_MATRIX

DEFINE_MONE = -1

-1(DMA) = MONE

INCREASE_DIMENSION_OF_GDOT_TO_5(COL5 & _ROWS)

.COMPUTE_COL5

WHERE_GDOT = [K][R2,MONE(*) = COL5

DEFINETIME_NEGATIVE_OF_XDIS_TIME_CONSTANT

2(DMA) = NTAL

DEFINE ROW 5

WHERE ACT IS THE TRANSPOSE_OF AC

AC(T) = ACT

FORM_AG_THE_5*5_A_MATRIX_OF_GDOT_EQUATION

ACT,COL5(CTI) = AG1

AG1,ROW5(RTI) = AG

FORM_THE_-K*F_PART_OF_GDOT_EQUATION

K,F,MONE(*) = MKF

DEFINE ZERO

APPENDIX A. L-A-S AIRCRAFT LANDING SIMULATION PROGRAM

265
WHERE_ZERO_IS_1*1_ZERO_MATRIX
0(DMA)=ZERO
CONSTRUCT_MKF0
WHERE_MKF0_IS_5*1_FORMED_FROM_MKF
MKF,ZERO(RTI)=MKF0
DEFINE_G0_THE_INITIAL_CONDITION_FOR_GDOT_EQUATION
\\.0183156(DMA)=G0
READ_DATA_FILES_I1_AND_X1
WHERE_I1_IS_THE_51*51_QQQ_MATRIX
AND_X1 IS_THE_KNOWN_51*1_VECTOR_OF_FIRST_STATE_VARIABLE
(RDF)= I1,[I1]
(RDF)=X1,[X1]
COMPUTE_XI1
WHERE_XI1 IS_X1_IN_REVERSE_TIME
I1,X1(*)=XI1
DEFINE_NT_AND_TT_FOR_SOLUTION_OF_DIFF_EQUATION_GDOT
WHERE_NT=#_OF_POINTS_IN_TT__TT=TOTAL_TIME
51(DMA)=NT
20(DMA)=TT
CALL_CE3_FOR_SOLUTION_OF_DIFF_EQUATION_GDOT
G,4(CTC)=G1
*****BLOCK#6*****
PREPARE_FOR_COMPUTATION_OF_OVERALL_SYSTEM_INPUT
COMPUTE_GT
WHERE_GT_IS_THE_4*51_TRANSPOSE_OF_G1
G1(T)=GT
COMPUTE_V
WHERE_V=R(-1)*BT*GT*I1 IS_FIRST_PORTION_OF_OPTIMAL_CONTROL
U=R(-1)*BT*[G-KZ]
RIN,BT,GT,I1(*)=V
B,V(*)=BV
DEFINE_XIT
WHERE_XIT IS_THE_TRANSPOSE_OF_X1
X1(T)=X1T
DEFINE_FX1T
WHERE_FX1T=F*X1T
F,X1T(*)=FX1T
READ_DATA_FILE_HD
WHERE_HD IS_THE_4*51_DESIRED_TRAJECTORY_ARRAY
(RDF)=HD,[HD]
B,RIN,BT,K,HD(*)=BKHD
BV,FXT,BKHD(+)=U1
U1,1(CTR)=U
;;;;;;BLOCK#7;;;;;;
;PREPARE_FOR_SYSTEM_SIMULATION
;=====================================
;DEFINE_X0
;=====================================
;WHERE_X0_IS_THE_INITIAL_CONDITION_VECTOR
0/-078/1/-20/100(DMA)=X0
;DEFINE_INPT
;=====================================
;WHERE_INPT_IS_THE_INPUT_MATRIX
1/0/0/0(DMA)=INPT
;DEFINE_UT
;=====================================
;WHERE_UT_IS_TRANSPOSE_OF_U_NEEDED_FOR_CE3_COMPATIBILITY
U(T)=UT
;CALL_CE3_FOR_SOLUTION_OF_DIFF.EQUATION_XDOT
;=====================================
;WHERE_X_CONTAINS_THE_VALUE_OF_TRAJECTORIES
X0,NT,TT,AC,INPT,UT(CE3)=X
;;;;;;BLOCK#8;;;;;;
;COMPUTE_EIGNVALUES_OF_AC
;=====================================
;WHERE_AC_IS_THE_CLOSED_LOOP_A_MATRIX
AC(EGV)=ACEV
;PRINT_A,F,K,ACEV(EIGNVALUES_OF_AC)_AND_PLOT_TRAJECTORIES
;=====================================
A,F,K,ACEV(OUT)=
X(PLT)=

APPENDIX A. L-A-S AIRCRAFT LANDING SIMULATION PROGRAM
APPENDIX B. FORTRAN PROGRAM

LISTING FOR SOLUTION OF THE RICCATI EQUATION

The computer program for time variable solution of the Riccati equation is documented in this appendix. The program listing is as follows:

```
//A240ROST JOB 34C84,NRUSHA,TIME = 2
/*PRIORITY  IDLE*/
/*JOBPARM  LINES = 2*/
//STEPI EXEC WATFIV
//SYSIN DD DATA
//WATFIV  NRUSHA,NLIST,PAGES = 100,TIME = (2,0)
REAL  MULT1(14,14),MULT2(14,14),MULT3(14,3),MULT4(14,3)
REAL  MULT5(14,14),MULT6(14,14)
DIMENSION GAIN(14,14),AMAT(14,14),AMATTR(14,14),BMAT(14,3)
DIMENSION BMATTR(3,14),RMA T(3,3),RINV(3,3),HMAT(14,14),QMAT(14,14)
DATA  AMAT/196*0./,BMAT/42*0./,RMA T/9*0./,RINV/9*0./
DATA  HMAT/196*0./,QMAT/196*0./
KCONT = 0
KPRINT = 1
DELTAT = -0.008
C  DEFINITION OF COMPONENTS OF A MATRIX
AMAT(1,1) = -1.6362461
AMAT(1,2) = -12.277406
AMAT(1,3) = -31.903317
AMAT(1,9) = 2.3332716
```
AMAT(1,10) = -.1435513
AMAT(2,1) = 1.0
AMAT(3,1) = 2.1778211
AMAT(3,3) = -.6648671
AMAT(3,7) = -1*AMAT(3,3)
AMAT(3,8) = -1.0
AMAT(3,10) = .0418853
AMAT(3,14) = -1*AMAT(3,10)
AMAT(4,2) = 3.2704796
AMAT(4,3) = 7.3823304
AMAT(4,4) = -20.
AMAT(4,9) = .146754
AMAT(4,10) = .653323
AMAT(5,4) = -800.0
AMAT(5,5) = -20.0
AMAT(5,6) = -800.0
AMAT(6,5) = 0.1
AMAT(6,6) = -1.0
AMAT(6,7) = -0.0024
AMAT(7,5) = 2.0
AMAT(7,7) = -0.048
AMAT(8,2) = 4.2741393
AMAT(8,3) = -1.0416643
AMAT(8,8) = -1.6362467
AMAT(8,9) = -18.615795
AMAT(8,10) = -28.542503
AMAT(9,8) = 1.0
AMAT(10,1) = -1.3500602
AMAT(10,3) = 0.0767264
AMAT(10,7) = -1*AMAT(10,3)
AMAT(10,8) = 2.6896578
AMAT(10,9) = -0.7557754
AMAT(10,10) = -1*AMAT(10,10)
AMAT(11,2) = 0.9870781
AMAT(11,3) = 1.1536667
AMAT(11,9) = 0.44158
AMAT(11,10) = 5.7495322
AMAT(12,11) = -20.0
AMAT(12,12) = -800.0
AMAT(12,13) = -20.0
AMAT(13,12) = 0.1
AMAT(13,13) = -1.0
AMAT(13,14) = -0.0024
AMAT(14,12) = 2.0
AMAT(14,14) = -0.048

C DEFINITION OF COMPONENTS OF B MATRIX
BMAT(5,1) = 800.
BMAT(12,2) = 800.
BMAT(1,3) = 0.8703102
BMAT(8,3) = -0.1325288

C DEFINITION OF COMPONENTS OF R MATRIX & COMPUTATION OF INVERSE
RMAT(1,1) = 100
RMAT(2,2) = 100
RMAT(3,3) = 0.1

APPENDIX B. FORTRAN PROGRAM LISTING FOR SOLUTION OF THE RICCATI EQUATION

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DO 10 I = 1, 3
10 RINV(I,I) = 1.0/RMAT(I,I)

C DEFINITION OF COMPONENT OF Q MATRIX
QMAT(1,1) = 2.
QMAT(2,2) = 2.
QMAT(8,8) = 2.
QMAT(9,9) = 2.

C COMPUTATION OF TRANSPOSE OF B MATRIX
DO 20 I = 1, 14
DO 20 J = 1, 3
20 BMAATTR(I,J) = BMAT(I,J)

C SPECIFICATION OF FINAL BOUNDARY VALUE
DO 30 I = 1, 14
DO 30 J = 1, 14
GAIN(I,J) = HMAT(I,J)
30 AMATTR(I,J) = AMAT(I,J)

100 CONTINUE

C COMPUTATION OF THE NEXT VALUE
CALL MULT(GAIN,AMAT,MULT1,14,14,14)
CALL MULT(AMATTR,GAIN,MULT2,14,14,14)
CALL MULT(GAIN,BMAT,MULT3,14,14,13)
CALL MULT(MULT3,RINV,MULT4,14,3,3)
CALL MULT(MULT4,BMATR,MULT5,14,3,14)
CALL MULT(MULT5,GAIN,MULT6,14,14,14)
DO 50 I = 1, 14
DO 50 J = 1, 14
50 GAIN(I,J) = GAIN(I,J) + (-MULT1(1,J)-MULT2(1,J))
                  + MULT6(I,J)-QMAI T(I,J))*DELTAT
KCONT = KCONT + 1
IF(KPRINT.NE.10) GO TO 200

C WRITE(6,*) KCONT
DO 60 I = 1, 14
WRITE(6,300) (GAIN(I,J),J = 1, 14)

300 FORMAT(1X,7E11.4)
60 CONTINUE
KPRINT = 0
200 KPRINT = KPRINT + 1
IF(KCONT.LT.610) GO TO 100
STOP
END

//DATA
//ENDWAT
/*

APPENDIX B. FORTRAN PROGRAM LISTING FOR SOLUTION OF THE RICCATI EQUATION
APPENDIX C. FORTRAN PROGRAM
LISTING FOR SIMULATION OF POWER SYSTEM

The computer program for simulation of the power system as discussed in chapter 6 is documented in this appendix. The program listing for case I0 is as follows:

```
//A240ROST JOB 34C84,NRUSHA,TIME = 80
/*PRIORITY IDLE
/*JOBPARM LINES = 9
//STEP1 EXEC WATFIV
//FT08F001 DD DSN = A34C84.GAINF6.FORWRD,
// DISP = SHR
//FT09F001 DD DSN = A34C84.GAINB6.BCKWRD,
// DISP = SHR
//SYSIN DD DATA
//WATFIV NRUSHA,NOLIST,PAGES = 500,TIME = (80,0)
REAL J1,JAC,ID,IDS,LTCR,LTCI,IDD,IQQ,KA,KE,KF,M1,M2,LTCRS,LTCIS
REAL LTCRO,LTCIO
LOGICAL L(8),LL(8)
DIMENSION P(8),Q(8),G(9,9),B(9,9),E(11),DELTA(11),PSCH(8)
DIMENSION DELP(8),DELQ(8),J1(16,16),WKAREA(999),BB(16,1),QSCH(8)
DIMENSION JAC(14,14),THETA(9,9),Z(9,9),LDC(4),LLDC(3)
DIMENSION R(4),RR(3),A1A(4,4),B1B(3,3),R1(4,1),RR1(3,1)
DIMENSION RA(6),XQ(6),XD(6),XDP(6)
DIMENSION ER(6),EIM(6),EQR(6),EIQ(6),EQMAG(6),EQANG(6),COMPA(6)
DIMENSION CURR(6),CURI(6),CURMAG(6),CURANG(6),COMPR(6),COMPI(6)
```
DIMENSION IQQ(6),IDD(6),EIMAG(6),EQPMAG(6),PELEC(6),TANG(6)
DIMENSION YY(14),DY(14),EE(11),IDDC(6),IQQC(6),EQPCON(6)
DIMENSION AMAT(14,14),AMATTR(14,14),BMAT(14,3),BMA ITR(3,14)
DIMENSION RMAT(3,3),RINV(3,3),RINVBT(3,14),BRINBT(14,14)
DIMENSION FMEAS(14),FMEASR(14,63),GAINF(14,14),FEST(14)
DIMENSION XTEMP(63),YTEMP(63),XDIS(14),XDDIS(14),GVEC(14)
DIMENSION DGVEC(14),GTEMP(14,7),FNLINR(14),AX(14),BU(14),U(3)
DIMENSION XZERO(14),QMAT(14,14)
COMPLEX Y(9,9)
COMPLEX EC(6),S(6),CUR(6),RAM(6),XQM(6),XDM(6),XDPM(6)
COMPLEX EQ(6),EI(6),CIDD(6),CIQQ(6)
COMPLEX EC1,EC6,EC10,EC11,TC10,TC11,CTC10D,CTC10Q,CTC11D,CTC11Q
COMMON /ELEC/H
COMMON /NIU/CGAMAM,K2,ISTAB
COMMON /TIM/TIME,KSWICH,IREMOV
COMMON /EXITER/VREF1,VREF2,VS1,VS2
COMMON /VOLT/EE,PELEC,EQPMAG,PMECH1,PMECH2
COMMON /OMEGA/SPEED
COMMON /EQUA/LLCNT,BRINBT,AMAT,BMAT,AMATTR,BETA,GAINF
COMMON /GGG/HH
COMMON /QMATRX/QMAT
DATA TR/.05/,VS1/0.1/,KA/40./,TA/-0.05/,FREQ/60.0/,
DATA SE/0.074/,TE/0.5/,KF/0.05/,TF/1./,PI/3.141593/,
DATA DI/5./,M1/3055775/,TDOP/.5./,D2/25./,M2/.1527887/,
DATA VS2/0.1/,TD/.0027778/,TLTCR/0.5/,TLTCI/0.5/,
DATA AMAT/196*0./,BMAT/42*0./,RMAT/9*0./,RINV/9*0./,
DATA GVEC/14*0./,GAINF/196*0./,OPTCON/3*0./,QMAT/196*0./

C DEFINITION OF INTEGRATION VARIABLES
H = .008
HH = 0.008

C DEFINITION OF CONSTANT PARAMETERS (PI AND EBS:PRECISION INDEX)
PI = 3.1415927
EBS = .001
ICONV = 0
KSWICH = 0

C DEFINITION OF HVDC CONSTANTS
XCR = .0139109/2.
XCI = .0132263/2.
RD = .000975

C DEFINITION OF HVDC DESIRED VALUES
IDS = 16.055302
VDIS = .8908114
ALFAS = 15.
GAMAS = 17.

C DEFINITION OF HVDC INITIAL VALUES
ALFA = 10.
GAMA = 10.
LTCR = 1.
LTCI = 1.
VDR = 1
VDI = 1
ID = 14

C DEFINITION OF AC CONSTANTS
TAP = 1.03
A1 = (1.0-TAP)/(TAP**2)
B1 = (TAP-1)/TAP
RA(1) = 0.0
XD(1) = .10
XQ(1) = .095
XDP(1) = .01075
RA(6) = 0.0
XD(6) = .199
XQ(6) = .19
XDP(6) = .0215

C DEFINITION OF SCHEDULED P, Q
PSCH(1) = 16.4
PSCH(3) = 0.0
QSCH(3) = 0.0
PSCH(4) = 0.0
QSCH(4) = 0.0
PSCH(5) = 0.0
QSCH(5) = 0.0
PSCH(6) = 8.2
PSCH(8) = 0.0
QSCH(8) = 0.0

C INITIALIZATION OF YBUS
DO 10 I = 1,9
DO 20 J = 1,9
Y(I,J) = CMPLX(0.0,0.0)
10 CONTINUE
20 CONTINUE

C INITIALIZATION OF JACOBIAN
DO 30 I = 1,16
DO 40 J = 1,16
J1(I,J) = 0.0
40 CONTINUE
30 CONTINUE

C INITIALIZATION OF MAGNITUDE AND ANGLE OF BUS VOLTAGES
DO 50 I = 1,11
E(I) = 1.0
DELTA(I) = 0.0
50 CONTINUE

C DEFINITION OF VOLTAGE MAGNITUDE FOR V. R. & SWING BUSES
E(1) = 1.02
E(6) = 1.02
E(9) = 1.05

C DEFINITION OF YBUS IN RECTANGULAR COORDINATES
Q1 = 192.30769
Q2 = 5.9
Q3 = .43087
RE = 5.2089024
Q4 = 79.531046
Q5 = 162.99919
Q6 = 96.153846
Q7 = 95.238095
Q8 = 6.8
Q9 = 7.9531046
Q10 = 0.215435
REE = 0.520890
Y(1,1) = CMPLX(0.0,-Q1/TAP-B1*Q1)
Y(1,2) = CMPLX(0.0,Q1/TAP)
Y(2,1) = Y(1,2)
Y(2,2) = CMPLX(RE + REE,-Q1/TAP + Q2 + Q3-Q4-A1*Q1-Q9 + Q10)
Y(2,3) = CMPLX(-RE,Q4)
Y(2,8) = CMPLX(-REE,Q9)
Y(3,2) = Y(2,3)
Y(3,3) = CMPLX(RE + REE,-Q4 + Q3-Q5-Q9 + Q10)
Y(3,4) = CMPLX(0.0,Q5)
Y(3,8) = CMPLX(-REE,Q9)
Y(4,3) = Y(3,4)
Y(4,4) = CMPLX(RE/2,-Q5-Q5-Q4/2 + Q3/2)
Y(4,5) = CMPLX(-RE/2,Q4/2)
Y(4,9) = CMPLX(0.0,Q5)
Y(5,4) = Y(4,5)
Y(5,5) = CMPLX(RE/2,-Q4/2-Q3/2-Q6/TAP-A1*Q6)
Y(5,6) = CMPLX(0.0,Q6/TAP)
Y(6,5) = Y(5,6)
Y(6,6) = CMPLX(0.0,-Q6/TAP-B1*Q6)
Y(7,7) = CMPLX(0.0,-Q7 + Q8)
Y(7,9) = CMPLX(0.0,Q7)
Y(9,4) = Y(4,9)
Y(9,7) = Y(7,9)
Y(9,9) = CMPLX(0.0,-Q7-Q5)
Y(8,3) = Y(3,8)
Y(8,8) = CMPLX(2*REE + 2,-2*Q9 + 2*Q10-0.2)
Y(8,2) = Y(4,5)

C DEFINITION OF REAL AND IMAGINARY PARTS OF YBUS
C AND CONVERSION OF YBUS FROM RECTANGULAR TO POLAR
DO 60 I = 1, 9
DO 70 J = 1, 9
G(I,J) = REAL(Y(I,J))
B(I,J) = AIMAG(Y(I,J))
Z(I,J) = SQRT((G(I,J))**2 + (B(I,J))**2)
IF(G(I,J),GT,0) THEN DO
THETA(I,J) = ATAN(B(I,J)/G(I,J))
ELSE DO
IF(G(I,J),NE,0) THEN DO
THETA(I,J) = ATAN(B(I,J)/G(I,J)) + PI
ELSE DO
IF(B(I,J),GT,0) THEN DO
THETA(I,J) = PI/2.
ELSE DO
THETA(I,J) = -PI/2.
END IF
END IF
END IF
70 CONTINUE
60 CONTINUE
K1 = 0
DO 75 I = 1, 9
DO 75 J = 1, 9
75 THETA(I,J) = -THETA(I,J)
C COMPUTATION OF COSINE OF ALF A & GAMA
CALFA = COS(ALFA*PI/180.)
CGAMA = COS(GAMA*PI/180.)
CALFAS = COS(ALFAS*PI/180.)
CGAMAS = COS(GAMAS*PI/180.)
CGAMAM = 0.978
1000 K1 = K1 + 1
C  COMPUTATION OF PSCH & QSCH AT BUSES 2 & 7 BASED ON DC VALUES
   NO = 2
   CALL KCOMP(NO,E,CALFA,ID,XCR,LTCR,RK,ALPU)
   ALPUR = ALPU
   CALL PQCOMP(NO,VDR,ID,E,RK,LTCR,PSCH,QSCH,CALFA,ALPUR)
   NO = 7
   CALL KCOMP(NO,E,CGAMA,ID,XCI,LTCI,VK,ALPU)
   ALPUI = ALPU
   CALL PQCOMP(NO,VDI,ID,E,VK,LTCl,PSCH,QSCH,CGAMA,ALPUI)
C  COMPUTATION OF P & Q BASED ON AC VALUES
DO 80 I = 1,8
   P(I) = 0.0
   Q(I) = 0.0
   DO 90 J = 1,9
      P(I) = (E(I)*E(J)*Z(I,J))*COS(THETA(I,J) + DELTA(I)-DELTA(J)) + P(I)
      Q(I) = (E(I)*E(J)*Z(I,J))*SIN(THETA(I,J) + DELTA(I)-DELTA(J)) + Q(I)
   90 CONTINUE
80 CONTINUE
C  COMPUTATION OF P & Q MISMATCHES
DO 100 I = 1,8
   DELP(I) = PSCH(I)-P(I)
   IF(I.NE.1.AND.I.NE.6) THEN DO
      DELQ(I) = QSCH(I)-Q(I)
   END IF
100 CONTINUE
C  TEST FOR CONVERGENCE
DO 110 I = 1,8
   IF(ABS(DELP(I)).LT.EBS) THEN DO
      L(I) = .TRUE.
   ELSE DO
      L(I) = .FALSE.
   END IF
   IF(I.NE.1.AND.I.NE.6) THEN DO
      IF(ABS(DELQ(I)).LT.EBS) THEN DO
         LL(I) = .TRUE.
      ELSE DO
         LL(I) = .FALSE.
      END IF
      END IF
110 CONTINUE
    .LL(8).AND.(ICONV.EQ.1)) THEN DO
      GO TO 5555
   ELSE DO
      END IF
   END IF
C  COMPUTATION OF J1 OF JACOBIAN
DO 120 I = 1,8
   DO 130 J = 1,8
      IF(I.NE.J) THEN DO
         J(I,J) = E(I)*E(J)*Z(I,J)*SIN(THETA(I,J) + DELTA(I)-DELTA(J))
      ELSE DO
         SUM = 0.
      END IF
   130 CONTINUE
120 CONTINUE

DO 140 K = 1,9
IF(K.NE.I) THEN DO
SUM = SUM-E(I)*E(K)*Z(I,K)*SIN(THETA(I,K) + DELTA(I)-DELTA(K))
END IF
140 CONTINUE
J1(I,J) = SUM
END IF
130 CONTINUE
120 CONTINUE
C COMPUTATION OF J2 OF JACOBIAN
DO 150 I = 1,8
DO 160 J = 1,8
IF(J.NE.1.AND.J.NE.6) THEN DO
IF(I.NE.J) THEN DO
J1(I+8,J) = E(I)*Z(I,J)*COS(THETA(I,J) + DELTA(I)-DELTA(J))
ELSE DO
SUM = 0.
DO 170 K = 1,9
IF(K.NE.I) THEN DO
SUM = SUM + E(K)*Z(I,K)*COS(THETA(I,K) + DELTA(I)-DELTA(K))
END IF
170 CONTINUE
J1(I+8,J) = 2.*E(I)*Z(I,J)*COS(THETA(I,I)) + SUM
END IF
END IF
160 CONTINUE
150 CONTINUE
C COMPUTATION OF J3 OF JACOBIAN
DO 180 I = 1,8
IF(I.NE.1.AND.I.NE.6) THEN DO
DO 190 J = 1,8
IF(I.NE.J) THEN DO
J1(I+8,J+8) = +E(I)*Z(I,J)*SIN(THETA(I,J) + DELTA(I)-DELTA(J))
ELSE DO
SUM = 0.
DO 200 K = 1,9
IF(K.NE.I) THEN DO
SUM = SUM + E(K)*Z(I,K)*SIN(THETA(I,K) + DELTA(I)-DELTA(K))
END IF
200 CONTINUE
J1(I+8,J+8) = SUM
END IF
END IF
190 CONTINUE
180 CONTINUE
C COMPUTATION OF J4 OF JACOBIAN
DO 210 I = 1,8
IF(I.NE.1.AND.I.NE.6) THEN DO
DO 220 J = 1,8
IF(I.NE.J) THEN DO
J1(I+8,J+8) = +E(I)*Z(I,J)*SIN(THETA(I,J) + DELTA(I)-DELTA(J))
ELSE DO
SUM = 0.
DO 230 K = 1,9
IF(K.NE.I) THEN DO
SUM = SUM + E(K)*Z(I,K)*SIN(THETA(I,K) + DELTA(I)-DELTA(K))
END IF
230 CONTINUE
APPENDIX C. FORTRAN PROGRAM LISTING FOR SIMULATION OF POWER SYSTEM
END IF
230 CONTINUE
   J1(I+8,J+8) = 2.*E(I)*Z(I,I)*SIN(THETA(I,I)) + SUM
END IF
220 CONTINUE
END IF
210 CONTINUE
C  REORDERING THE JACOBIAN
   DO 240 I = 9,12
      DO 250 J = 1,16
         J1(I,J) = J1(I+1,J)
      250 CONTINUE
   240 CONTINUE
   DO 260 J = 1,16
      J1(I3,J) = J1(I5,J)
      J1(I4,J) = J1(I6,J)
   260 CONTINUE
   DO 270 I = 1,14
      DO 280 J = 9,12
         J1(I,J) = J1(I,J+1)
      280 CONTINUE
   270 CONTINUE
   DO 290 I = 1,14
      J1(I,13) = J1(I,15)
      J1(I,14) = J1(I,16)
   290 CONTINUE
C  DEFINITION OF REDUCED ORDER JACOBIAN
   DO 300 I = 1,14
      DO 310 J = 1,14
         JAC(I,J) = J1(I,J)
      310 CONTINUE
   300 CONTINUE
C  DEFINITION OF BB
   DO 320 I = 1,8
      BB(I,1) = DELP(I)
   320 CONTINUE
   BB(9,1) = DELQ(2)
   BB(10,1) = DELQ(3)
   BB(11,1) = DELQ(4)
   BB(12,1) = DELQ(5)
   BB(13,1) = DELQ(7)
   BB(14,1) = DELQ(8)
C  DEFINITION OF ARGUEMENTS OF IMSL SUBROUTINE
   M = 1
   N = 14
   IA = 14
   IDGT = 0
   CALL LEQT2F(JAC,M,N,IA,BB,IDGT,WKAREA,IER)
   DO 330 I = 1,8
      DELTA(I) = DELTA(I) + BB(I,1)
   330 CONTINUE
   DO 340 I = 9,12
      E(I-7) = E(I-7) + BB(I,1)
   340 CONTINUE
   E(7) = E(7) + BB(13,1)
   E(8) = E(8) + BB(14,1)

APPENDIX C. FORTRAN PROGRAM LISTING FOR SIMULATION OF POWER SYSTEM  277
CALL HVDC(VDR,VDI,E,CALFA,CALFAS,VDRS,
+ CGAMA,CGAMAS,ID,IDS,VDIS,RK,VK,LTCR,LTCI,XCR,XCI,RD,P1,R,RR)
DO 345 I = 1,4
IF(ABS(R(I)).LT.EBS) THEN DO
 LDC(I) = 1
 ELSE DO
 LDC(I) = 0
 END IF
345 CONTINUE
DO 346 I = 1,3
IF(ABS(RR(I)).LT.EBS) THEN DO
 LLDC(I) = 1
 ELSE DO
 LLDC(I) = 0
 END IF
346 CONTINUE
ITES = LDC(1)*LDC(2)*LDC(3)*LDC(4)*LLDC(1)*LLDC(2)*LLDC(3)
IF(ITES.EQ.1) THEN DO
 ICONV = I
 END IF
IF(K1.GT.20) GO TO 5555
GO TO 1000
5555 LTCRS = LTCR
 LTCI = LTCI
 PDCS = VDIS*IDS
 LTCRO = LTCR
 LTCIO = LTCI
C START OF TRANSIENT STABILITY PROGRAM
ISTAB = 0
IPLACE = 1
IREMOV = 7
IMODE = 0
ICONV = 0
KGCALC = 620
MOVE = 4
NEQ = 14
TIME1 = 0.0
SPEED = 2*PI*FREQ
IFMEAS = 0
DO 370 I = 1,8
 L(I) = .FALSE.
 LL(I) = .FALSE.
370 CONTINUE
DO 375 I = 1,6,5
 ER(I) = E(I)*COS(DELT(I))
 EIM(I) = E(I)*SIN(DELT(I))
 EC(I) = CMPLX(ER(I),EIM(I))
 S(I) = CMPLX(P(I),-1*Q(I))
 CUR(I) = S(I)/CONJG(EC(I))
 RAM(I) = CMPLX(RA(I),0.)
 XQM(I) = CMPLX(0.,XQ(I))
 XDM(I) = CMPLX(0.,XD(I))
 XDPM(I) = CMPLX(0.,XDP(I))
 EQ(I) = EC(I) + RAM(I)*CUR(I) + XQM(I)*CUR(I)
EQR(I) = REAL(EQ(I))
EQI(I) = AIMAG(EQ(I))
EQMAG(I) = SQRT(EQR(I)**2 + EQI(I)**2)
CALL TANGEN(I,EQR,EQI,EQANG)
CURR(I) = REAL(CUR(I))
CURI(I) = AIMAG(CUR(I))
CURMAG(I) = SQRT(CURR(I)**2 + CURI(I)**2)
CALL TANGEN(I,CURR,CURI,CURANG)
COMPI(I) = XQ(I)*CURMAG(I)*COS(DELT(I)-CURANG(I))-RA(I)*CURMAG(I)
*SIN(DELT(I)-CURANG(I))
COMPR(I) = E(I) + RA(I)*CURMAG(I)*COS(DELT(I)-CURANG(I)) + XQ(I)
*CURMAG(I)*SIN(DELT(I)-CURANG(I))
CALL TANGEN(I,COMPR,COMPI,COMPA)
TANG(I) = DELT(I)-CURANG(I) + COMPA(I)
IQQ(I) = CURMAG(I)*COS(TANG(I))
IDD(I) = CURMAG(I)*SIN(TANG(I))
CIQQ(I) = CMPLX(IQQ(I)*COS(EQANG(I)),IQQ(I)*SIN(EQANG(I)))
CIDD(I) = CMPLX(IDD(I)*COS(EQANG(I)),IDD(I)*SIN(EQANG(I)))
EQPMAG(I) = EQMAG(I)-(XQ(I)-XDP(I))*IDD(I)
PELEC(I) = REAL(CUR(I)*CONJG(EQ(I)))

375 CONTINUE
DELTA(10) = EQANG(1)
DELTA(11) = EQANG(6)
PMECH1 = PELEC(1)
PMECH2 = PELEC(6)

C DEFINITION OF COMPONENTS OF A MATRIX

AMAT(1,1) = -1.6362461
AMAT(1,2) = -12.277406
AMAT(1,3) = -31.903317
AMAT(1,9) = 2.3332716
AMAT(1,10) = -.1435513
AMAT(2,1) = 1.0
AMAT(3,1) = 2.1778211
AMAT(3,3) = -.6648671
AMAT(3,7) = -1*AMAT(3,3)
AMAT(3,8) = -.1615107
AMAT(3,10) = .0418853
AMAT(3,14) = -1*AMAT(3,10)
AMAT(4,2) = 3.2704796
AMAT(4,3) = -1.0416643
AMAT(4,8) = -18.615795
AMAT(4,9) = -28.542503
AMAT(4,10) = 2.0
AMAT(4,4) = -20.
AMAT(4,9) = .146754
AMAT(4,10) = .653323
AMAT(5,4) = -800.0
AMAT(5,5) = -20.0
AMAT(5,6) = -800.0
AMAT(6,5) = 0.1
AMAT(6,6) = -1.0
AMAT(6,7) = -0.0024
AMAT(7,5) = 2.0
AMAT(7,7) = -0.048
AMAT(8,2) = 4.2741393
AMAT(8,3) = -1.0416643
AMAT(8,8) = -16.362467
AMAT(8,9) = -18.615795
AMAT(8,10) = -28.542503
AMAT(9,8)= 1.0
AMAT(10,1) = -0.3500602
AMAT(10,3) = 0.0767264
AMAT(10,7) = -1*AMAT(10,3)
AMAT(10,8) = 2.6896578
AMAT(10,10) = -0.7557754
AMAT(10,14) = -1*AMAT(10,10)
AMAT(11,2) = 0.9870781
AMAT(11,3) = 1.1536667
AMAT(11,9) = 0.44158
AMAT(11,10) = 5.7495322
AMAT(11,11) = -20.0
AMAT(12,11) = -800.0
AMAT(12,12) = -20.0
AMAT(12,13) = -800.0
AMAT(13,12) = 0.1
AMAT(13,13) = -1.0
AMAT(13,14) = -0.0024
AMAT(14,12) = 2.0
AMAT(14,14) = -0.048

C DEFINITION OF COMPONENTS OF B MATRIX
BMAT(5,1) = 800.
BMAT(12,2) = 800.
BMAT(1,3) = -0.8703102
BMAT(8,3) = -0.1325288

C DEFINITION OF COMPONENTS OF R MATRIX, AND INVERSE OF R
RMAT(1,1) = 100.
RMAT(2,2) = 100.
RMAT(3,3) = 0.1
DO 5010 I = 1,3
5010 RINV(I,I) = 1/RMAT(I,I)
QMAT(1,1) = 2
QMAT(2,2) = 2
QMAT(8,8) = 2
QMAT(9,9) = 2

C COMPUTATION OF TRANSPOSE OF B MATRIX
DO 5020 I = 1,14
DO 5020 J = 1,3
BMATR(I,J) = BMAT(I,J)
5020 CONTINUE

C COMPUTATION OF TRANSPOSE OF A MATRIX
DO 5030 I = 1,14
DO 5030 J = 1,14
5030 AMATTR(I,J) = AMAT(I,J)
CALL MULT(RINV,BMATTR,RINVBT,3,3,14)
CALL MULT(BMAT,RINVBT,BRINBT,14,3,14)

C DEFINITION OF EXCITER REFERENCE VALUES
VREF1 = (KE + SE)*EQMAG(1)/KA-VS1 + E(1)
VREF2 = (KE + SE)*EQMAG(6)/KA-VS2 + E(6)

C DEFINITION OF INTEGRATION VARIABLES
YY(1) = 2*PI*FREQ
YY(2) = DELTA(10)
YY(3) = EQPMAG(1)
YY(4) = E(1)
YY(5) = KA*(VREF1 + VS1-E(1))
YY(6) = 0.
YY(7) = EQMAG(1)
YY(8) = 2*PI*FREQ
YY(9) = DELTA(11)
YY(10) = EQPMAG(6)
YY(11) = E(6)
YY(12) = KA*(VREF2 + VS2-E(6))
YY(13) = 0.
YY(14) = EQMAG(6)

C MODIFICATION OF SCHEDULED P, Q
PSCH(1) = 0.0
QSCH(1) = 0.0
PSCH(6) = 0.0
QSCH(6) = 0.0

C DEFINITION OF NEW VOLTAGE MAGNITUDES & ANGLES
3000 CONTINUE
IF(ISTAB.EQ.50) THEN DO
LTCRS = LTCR0
LTCIS = LTCI0
WRITE(6,*) LTCRS,LTCIS
END IF
EQPCON(1) = YY(3)
EQPCON(6) = YY(10)
E(10) = EQMAG(1)
E(11) = EQMAG(6)
DELTA(10) = YY(2)
DELTA(11) = YY(9)
IF(ISTAB.GE.(IREMOV+1)) GO TO 2001
FAULT = 0.0
IF(ISTAB.GE.IPLACE) THEN DO
FAULT = 100000
END IF
IF(ISTAB.EQ.IREMOV) THEN DO
FAULT = 0.0
DO 671 I = 1, 8
E(I) = 2.0
END IF
Y(3,3) = CMPLX(RE + REE + FAULT,-Q4 + Q3-Q5-Q9 + Q10-FAULT)

C AND CONVERSION OF YBUS FROM RECTANGULAR TO POLAR
DO 400 I = 1, 9
DO 410 J = 1, 9
G(I,J) = REAL(Y(I,J))
B(I,J) = AIMAG(Y(I,J))
Z(I,J) = SQRT((G(I,J)**2 + (B(I,J))**2)
IF(G(I,J).GT.0) THEN DO
THETA(I,J) = ATAN(B(I,J)/G(I,J))
ELSE DO
IF(G(I,J).NE.0) THEN DO
THETA(I,J) = ATAN(B(I,J)/G(I,J)) + PI
ELSE DO
THETA(I,J) = PI/2.
ELSE DO
THETA(I,J) = -PI/2.
END IF
END IF
END IF
410 CONTINUE
400 CONTINUE
DO 420 I = 1,9
DO 430 J = 1,9
THETA(I,J) = -THETA(I,J)
430 CONTINUE
420 CONTINUE
2001 K2 = 0
2000 K2 = K2 + 1
C COMPUTATION OF PSCH & QSCH AT BUSES 2 & 7 BASED ON DC VALUES
NO = 2
CALL KCOMP(NO,E,CALFA,ID,XCR,LTCR,RK,ALPU)
ALPUR = ALPU
CALL PQCOMP(NO,VDR,ID,E,RK,LTCR,PSCH,QSCH,CALFA,ALPUR)
NO = 7
CALL KCOMP(NO,E,CGAMA,ID,XCI,LTCI,VK,ALPU)
ALPUI = ALPU
CALL PQCOMP(NO,VDI,ID,E,VK,LTCI,PSCH,QSCH,CGAMA,ALPUI)
C COMPUTATION OF P & Q BASED ON AC VALUES
DO 500 I = 1,8
P(I) = 0.0
Q(I) = 0.0
DO 510 J = 1,9
P(I) = (E(I)*E(J)*Z(I,J))*(COS(THETA(I,J) + DELTA(I)-DELTA(J))) + P(I)
Q(I) = (E(I)*E(J)*Z(I,J))*(SIN(THETA(I,J) + DELTA(I)-DELTA(J))) + Q(I)
510 CONTINUE
500 CONTINUE
GEN1P1 = -(E(1)*EQPCON(1)/XDP(1))*SIN(Delta(10)-Delta(1)) + ((E(1)**2)
/2)*((1/XDP(1))-1/XQ(1)))*SIN((2*(DELTA(10)-DELTA(1)))/P(1) = P(1) + GEN1P1
GEN2P2 = -(E(6)*EQPCON(6)/XDP(6))*SIN(Delta(11)-Delta(6)) + ((E(6)**2)
/2)*((1/XDP(6))-1/XQ(6)))*SIN((2*(DELTA(11)-DELTA(6)))/P(6) = P(6) + GEN2P2
GEN1Q1 = -(E(1)*EQPCON(1)/XDP(1))*COS(Delta(10)-Delta(1)) + ((E(1)**2)
/2)/((1/XDP(1))-1/XQ(1)))*((COS(DELTA(10)-DELTA(1)))**2) + (E(1)**2)/Q(I)
Q(I) = Q(I) + GEN1Q1
GEN2Q2 = -(E(6)*EQPCON(6)/XDP(6))*COS(Delta(11)-Delta(6)) + ((E(6)**2)
/2)/((1/XDP(6))-1/XQ(6)))*((COS(DELTA(11)-DELTA(6)))**2) + (E(6)**2)/Q(6)
Q(6) = Q(6) + GEN2Q2
C COMPUTATION OF P & Q MISMATCHES
DO 520 I = 1,8
DELP(I) = PSCH(I)-P(I)
520 CONTINUE
APPENDIX C. FORTRAN PROGRAM LISTING FOR SIMULATION OF POWER SYSTEM 282
DELQ(I) = QSCH(I) - Q(I)
520 CONTINUE
C TEST FOR CONVERGENCE
DO 530 I = 1, 8
IF(ABS(DELP(I)).LT.EBS) THEN DO
L(I) = .TRUE.
ELSE DO
L(I) = .FALSE.
ENDIF
END IF
IF(ABS(DELQ(I)).LT.EBS) THEN DO
LL(I) = .TRUE.
ELSE DO
LL(I) = .FALSE.
ENDIF
530 CONTINUE
GO TO 6666
ELSE DO
END IF
C COMPUTATION OF J1 OF JACOBIAN
DO 540 I = 1, 8
DO 550 J = 1, 8
IF(I.EQ.J) THEN DO
J1(I,J) = E(I)*E(J)*Z(I,J)*SIN(THETA(I,J) + DELTA(I)-DELTA(J))
ELSE DO
SUM = 0.
DO 560 K = 1, 9
IF(K.NE.I) THEN DO
SUM = SUM - E(I)*E(K)*Z(I,K)*SIN(THETA(I,K) + DELTA(I)-DELTA(K))
ENDIF
560 CONTINUE
J1(I,J) = SUM
ENDIF
550 CONTINUE
IF(I.EQ.1.AND.J.EQ.1) THEN DO
PARP = (EQPCON(1)*E(1)/XDP(1))*COS(Delta(10)-Delta(1))-(E(1)**2)*
((1/XDP(1))-((1/XQ(1))))*COS(2*(Delta(10)-Delta(1)))
J1(I,J) = J1(I,J) + PARP
ENDIF
IF(I.EQ.6.AND.J.EQ.6) THEN DO
PARP = (EQPCON(6)*E(6)/XDP(6))*COS(Delta(11)-Delta(6))-(E(6)**2)*
((1/XDP(6))-(1/XQ(6)))*COS(2*(Delta(11)-Delta(6)))
J1(I,J) = J1(I,J) + PARP
ENDIF
ENDIF
560 CONTINUE
C COMPUTATION OF J2 OF JACOBIAN
DO 570 I = 1, 8
DO 580 J = 1, 8
IF(I.EQ.J) THEN DO
J1(I,J + 8) = E(I)*Z(I,J)*COS(THETA(I,J) + DELTA(I)-DELTA(J))
ELSE DO
SUM = 0.
DO 590 K = 1, 9
IF(K.NE.I) THEN DO
APPENDIX C. FORTRAN PROGRAM LISTING FOR SIMULATION OF POWER SYSTEM

SUM = SUM + E(K) * Z(I,K) * COS(THETA(I,K) + DELTA(I) - DELTA(K))
END IF

590 CONTINUE
J1(I,J + 8) = 2. * E(I) * Z(I,I) * COS(THETA(I,I)) + SUM
IF (I.EQ.1.AND.J.EQ.1) THEN DO
  PARM = -(EOPCON(1)/XDP(1)) * SIN(DELTA(10) - DELTA(1)) + E(1) * ((1/XDP(1)) - (1/XQ(1))) * SIN(2 * (DELTA(10) - DELTA(1)))
  J1(I,J + 8) = J1(I,J + 8) + PARM
END IF
IF (I.EQ.6.AND.J.EQ.6) THEN DO
  PARM = -(EOPCON(6)/XDP(6)) * SIN(DELTA(11) - DELTA(6)) + E(6) * ((1/XDP(6)) - (1/XQ(6))) * SIN(2 * (DELTA(11) - DELTA(6)))
  J1(I,J + 8) = J1(I,J + 8) + PARM
END IF
END IF

580 CONTINUE

C COMPUTATION OF J3 OF JACOBIAN
DO 600 I = 1, 8
DO 610 J = 1, 8
IF (I.NE.J) THEN DO
  J1(I + 8,J) = -E(I) * E(J) * Z(I,J) * COS(THETA(I,J) + DELTA(I) - DELTA(J))
ELSE DO
  SUM = 0.
  DO 620 K = 1, 9
  IF (K.NE.I) THEN DO
    SUM = SUM + E(K) * Z(I,K) * COS(THETA(I,K) + DELTA(I) - DELTA(K))
  END IF
  620 CONTINUE
  J1(I + 8,J) = SUM
END IF
IF (I.EQ.1.AND.J.EQ.1) THEN DO
  PARM = -(E(I) * EOPCON(I)/XDP(I)) * SIN(DELTA(10) - DELTA(I)) + E(I) * (2) * ((1/XDP(I)) - (1/XQ(I))) * SIN(2 * (DELTA(10) - DELTA(I)))
  J1(I + 8,J) = J1(I + 8,J) + PARM
END IF
IF (I.EQ.6.AND.J.EQ.6) THEN DO
  PARM = -(E(6) * EOPCON(6)/XDP(6)) * SIN(DELTA(11) - DELTA(6)) + E(6) * (2) * ((1/XDP(6)) - (1/XQ(6))) * SIN(2 * (DELTA(11) - DELTA(6)))
  J1(I + 8,J) = J1(I + 8,J) + PARM
END IF
END IF
610 CONTINUE

C COMPUTATION OF J4 OF JACOBIAN
DO 630 I = 1, 8
DO 640 J = 1, 8
IF (I.NE.J) THEN DO
  J1(I + 8,J + 8) = + E(I) * Z(I,J) * SIN(THETA(I,J) + DELTA(I) - DELTA(J))
ELSE DO
  SUM = 0.
  DO 650 K = 1, 9
  IF (K.NE.I) THEN DO
    SUM = SUM + E(K) * Z(I,K) * SIN(THETA(I,K) + DELTA(I) - DELTA(K))
  END IF
  650 CONTINUE
  J1(I + 8,J + 8) = 2. * E(I) * Z(I,I) * SIN(THETA(I,I)) + SUM
END IF
IF(I.EQ.1.AND.J.EQ.1) THEN DO
  PARQ = -(EQPCON(1)/XDP(1)) * COS(DELTA(10) - DELTA(1)) + 2 * E(1) * ((1/XDP(1)) - (1/XQ(1))) * ((COS(DELTA(10) - DELTA(1)))**2) + 2 * E(1)/XQ(1)
  J1(I + 8, J + 8) = J1(I + 8, J + 8) + PARQ
END IF
IF(I.EQ.6.AND.J.EQ.6) THEN DO
  PARQ = -(EQPCON(6)/XDP(6)) * COS(DELTA(11) - DELTA(6)) + 2 * E(6) * ((1/XDP(6)) - (1/XQ(6))) * ((COS(DELTA(11) - DELTA(6)))**2) + 2 * E(6)/XQ(6)
  J1(I + 8, J + 8) = J1(I + 8, J + 8) + PARQ
END IF
END IF
640 CONTINUE
630 CONTINUE
C DEFINITION OF BB
DO 660 I = 1, 8
  BB(I, 1) = DELP(I)
  BB(I + 8, 1) = DELQ(I)
660 CONTINUE
C DEFINITION OF ARGUEMENTS OF IMSL SUBROUTINE
M = 1
N = 16
IA = 16
IDGT = 0
CALL LEQT2F(J1, M, N, IA, BB, IDGT, WKAREA, IER)
DO 670 I = 1, 8
  DELTA(I) = DELTA(I) + BB(I, I)
  E(I) = E(I) + BB(I + 8, I)
670 CONTINUE
CALL HVDCTR(VDR, VDI, E, CALFA, CALFAS, VDRS, LTCRS, LTCIS, + CGAMA, CGAMAS, ID, IDS, VDIS, RK, LTCR, LTCI, XCR, XCI, RD, PI, R, RR)
DO 680 I = 1, 4
  IF(ABS(R(I)).LT.EBS) THEN DO
    LDC(I) = 1
  ELSE DO
    LDC(I) = 0
  END IF
680 CONTINUE
DO 690 I = 1, 3
  IF(ABS(RR(I)).LT.EBS) THEN DO
    LLDC(I) = 1
  ELSE DO
    LLDC(I) = 0
  END IF
690 CONTINUE
ITES = LDC(1) * LDC(2) * LDC(3) * LDC(4) * LLDC(1) * LLDC(2) * LLDC(3)
IF(ITES.EQ.1) THEN DO
  ICONV = 1
  ELSE DO
    ICONV = 0
  END IF
IF(K2.GE.40) THEN DO
  GO TO 9000
  ELSE DO
    GO TO 2000
  END IF
6666 CONTINUE

APPENDIX C. FORTRAN PROGRAM LISTING FOR SIMULATION OF POWER SYSTEM
PARAMETERS
PTMPR(I) = -GEN1P1
PTMPR(6) = -GEN2P2
QTMPR(I) = -GEN1Q1
QTMPR(6) = -GEN2Q2
DO 1375 I = 1,6,5
ER(I) = E(I)*COS(Delta(I))
EIM(I) = E(I)*SIN(Delta(I))
EC(I) = CMPLX(ER(I),EIM(I))
S(I) = CMPLX(PTMPR(I),-1*QTMPR(I))
CUR(I) = S(I)/CONJG(EC(I))
EQ(I) = EC(I) + RAM(I)*CUR(I) + XQM(I)*CUR(I)
EQR(I) = REAL(EQ(I))
EQI(I) = AIMAG(EQ(I))
EQMAG(I) = SQRT(EQR(I)**2 + EQI(I)**2)
CALL TANGEN(I,EQR,EQI,EQANG)
CURR(I) = REAL(CUR(I))
CURI(I) = AIMAG(CUR(I))
CURMAG(I) = SQRT(CURR(I)**2 + CURI(I)**2)
CALL TANGEN(I,CURR,CURI,CURANG)
COMPI(I) = XQ(I)*CURMAG(I)*COS(Delta(I)-CURANG(I))-RA(I)*CURMAG(I)
* S(Delta(I)-CURANG(I))
COMPR(I) = E(I) + RA(I)*CURMAG(I)*COS(Delta(I)-CURANG(I)) + XQ(I)
* CURMAG(I)*SIN(Delta(I)-CURANG(I))
CALL TANGEN(I,COMPR,COMPI,COMPA)
TANG(I) = DELTA(I)-CURANG(I) + COMPA(I)
IQQ(I) = CURMAG(I)*COS(TANG(I))
IDD(I) = CURMAG(I)*SIN(TANG(I))
CIQQ(I) = CMPLX(IQQ(I)*COS(EQANG(I)),IQQ(I)*SIN(EQANG(I)))
CIIDD(I) = CMPLX(IDD(I)*COS(EQANG(I)),IDD(I)*SIN(EQANG(I)))
PELEC(I) = REAL(CUR(I)*CONJG(EQ(I)))
END
1375 CONTINUE
E(10) = EQMAG(1)
E(11) = EQMAG(6)
DO 973 I = 1,11
EE(I) = EE(I)
973 CONTINUE
IF(IMODE.EQ.1) GO TO 8700
C
C * BEGINNING OF OPTIMAL CONTROLLER BLOCK *
C
JSTAB = ISTAB/10
KSTAB = JSTAB*10
IF(I(ISTAB.EQ.KSTAB) THEN DO
IFMEAS = IFMEAS + 1
NFMEAS = IFMEAS-(MOVE-1)
CALL DERIV(NEQ,TIME,YY,FNLINR)
DO 5050 I = 1,14
XVEC(I) = YY(I)
5050 CONTINUE
C
APPENDIX C. FORTRAN PROGRAM LISTING FOR SIMULATION OF POWER SYSTEM
CALL MULT(BMAT,U,BU,14,3,1)
DO 5060 I = 1,14
FMEAS(I) = FNLINR(I)-AX(I)-BU(I)
5060 CONTINUE
DO 6000 I = 1,14
FMEASR(I,IFMEAS) = FMEAS(I)
IF(ISTAB.LT.610) THEN DO
READ(8,7000) (GAINF(I,J),J = 1,14)
ELSE DO
DO 6090 J = 1,14
GAINF(I,J) = 0.0
6090 CONTINUE
END IF
6000 CONTINUE
7000 FORMAT(1X,7E11.4)
IF(ISTAB.EQ.0) THEN DO
DO 6010 I = 1,14
BETA(I,1) = FMEASR(I,IFMEAS)
DO 6015 J = 2,4
6015 BETA(I,J) = 0.0
6010 CONTINUE
ELSE DO
DO 6020 I = 1,14
DO 6030 J = 1,IFMEAS
XTEMP(J) = (J-1)*H*10
YTEMP(J) = FMEASR(I,J)
6030 CONTINUE
IF(IFMEAS.LT.MOVE) THEN
DO 6937 J = 1,IFMEAS
BETA(I,1) = BETA(I,1) + YTEMP(J)
BETA(I,1) = BETA(I,1)/IFMEAS
BETA(I,2) = 0.0
BETA(I,3) = 0.0
BETA(I,4) = 0.0
ELSE DO
BETA(I,1) = 0.0
DO 6037 J = NFMEAS,IFMEAS
BETA(I,1) = BETA(I,1) + YTEMP(J)
BETA(I,1) = BETA(I,1)/MOVE
BETA(I,2) = 0.0
BETA(I,3) = 0.0
BETA(I,4) = 0.0
END IF
6020 CONTINUE
END IF
IF(ISTAB.NE.0) THEN DO
DO 6040 1CNT = 1,KGCALC
LCNT = 1CNT
CALL RKG111J(NEQ1,TIME1,GVEC,DGVEC)
6040 CONTINUE
DO 6050 1 = 1,14
GTEMP(1,1) = GVEC(I)
6050 CONTINUE
END IF
IF(ISTAB.NE.0) THEN DO

APPENDIX C. FORTRAN PROGRAM LISTING FOR SIMULATION OF POWER SYSTEM  287
CALL ZERO(XZERO)
DO 6060 I = 1,14
  XERROR(I) = XZERO(I) - XVEC(I)
6060 CONTINUE
CALL MULT(GAINF,XERROR,RESMUL,14,14,1)
DO 6070 I = 1,14
  RESULT(I) = RESMUL(I) + GTEMP(I,1)
6070 CONTINUE
CALL MULT(RINVBT,RESULT,OPTCON,3,14,1)
END IF
REWIND 9
TIME1 = 0
KGCALC = KGCALC - 10
DO 6061 I = 1,14
  GVEC(I) = 0.0
6061 CONTINUE
END IF
C *********************************************************************
C * END OF OPTIMAL CONTROLLER BLOCK *
C *********************************************************************
8700 CONTINUE
WRITE(6,1197) TIME, DELTA(10)*180/PI, DELTA(11)*180/PI, IDS, VS1, VS2,
, E(2), E(7), K2, ISTAB
1197 FORMAT(1X,F7.4, 1X,F7.4,1X,F7.4,1X,F7.4,1X,F7.4,1X,F7.4,1X,F7.4,
, 1X,F7.4,1X,F7.4,1X,12,1X,13)
IF(ISTAB.EQ.KSTAB) THEN
  VS1 = OPTCON(1)
  VS2 = OPTCON(2)
  PDCNEW = OPTCON(3) + PDCS
  IDS = PDCNEW/VDIS
END IF
CALL RKG1L(NEQ,TIME,YY,DYY)
IF(ABS(YY(5)).GT.1) THEN
  IF(YY(5).GT.0) THEN
    YY(5) = 1
  ELSE
    YY(5) = -1
  END IF
END IF
IF(YY(5).LT.0) THEN
  YY(5) = 1
END IF
END IF
IF(ABS(YY(12)).GT.1) THEN
  IF(YY(12).GT.0) THEN
    YY(12) = 1
  ELSE
    YY(12) = -1
  END IF
END IF
ISTAB = ISTAB + 1
ICONV = 0
KSWICH = 0
IF(ISTAB.LT.620) THEN
  GO TO 3000
END IF
9000 STOP
END
SUBROUTINE PQCOMP(NO,VD,ID,E,K,LTCTAP,PSCH,QSCH,CALFA,ALPU)
DIMENSION PSCH(7),QSCH(7),E(8)
REAL ID,IDS,LTCTAP,K
PD = VD*ID
ANG = ARCOS((CALFA + COS(ALPU))/(K*2))
SANGLE = SIN(ANG)
SD = E(NO)*K*LTCTAP*ID
PSCH(NO) = -PD
IF(NO.EQ.7) THEN
   DO
   PSCH(NO) = PD
END IF
QSCH(NO) = -SD*SANGLE
RETURN
END

SUBROUTINE HVDC(VDR,VDI,E,CALFA,CALFAS,VDRS,+
   + CGAMA,CGAMAS,ID,IDS,VDIS,RK,VK,LTCR,LTCI,XCR,XCI,RD,PI,R,RR)
DIMENSION R(4),RR(3),A1A(4,4),B1B(3,3),R1(4,1),RR1(3,1),WKAREA(99)
DIMENSION E(8)
REAL ID,IDS,LTCR,LTCI
R(I) = VDR-LTCR*E(2)*CALFA + (PI/6)*XCR*ID
R(2) = VDR-RD*ID-VDI
R(3) = CALFA-CALFAS
R(4) = ID-IDS
RR(1) = VDI-LTCI*E(7)*CGAMA + (PI/6)*XCJ*ID
RR(2) = CGAMA-CGAMAS
RR(3) = VDI-VDIS
DO 10 I = 1,4
DO 10 J = 1,4
A1A(I,J) = 0.0
10 CONTINUE
DO 15 I = 1,3
DO 15 J = 1,3
B1B(I,J) = 0.0
15 CONTINUE
A1A(1,1) = 1.
A1A(1,2) = PI*XCR/6
A1A(1,3) = -E(2)*CALFA
A1A(1,4) = -LTCR*E(2)
A1A(2,1) = 1.
A1A(2,2) = -RD
A1A(3,4) = 1.
A1A(4,2) = 1.
B1B(1,1) = 1.
B1B(1,2) = -E(7)*CGAMA
B1B(1,3) = -LTCI*E(7)
B1B(2,3) = 1.
B1B(3,1) = 1.
DO 20 I = 1,4
R1(I,1) = R(I)
20 CONTINUE
DO 25 I = 1,3
RR1(I,1) = RR(I)
M = 1
N = 4
IA = 4
IDGT = 0
CALL LEQT2F(A1A,M,N,IA,R1,IDGT,WKAREA,IER)
M = 1
N = 3
IA = 3
IDGT = 0
CALL LEQT2F(B1B,M,N,IA,RR1,IDGT,WKAREA,IER)
VDI = VDI-RR1(1,1)
LTC1 = LTC1-RR1(2,1)
CGAMA = CGAMA-RR1(3,1)
VD = VD-RR1(1,1)
ID = ID-RR1(2,1)
LTCR = LTCR-RR1(3,1)
CALFA = CALFA-RR1(4,1)
RETURN
END

SUBROUTINE KCOMP(NO,E,CALFA,ID,XC,LTCTAP,CONST,ALPU)
DIMENSION E(8)
REAL ID,LTCT
CONS = (XC*ID/(LTCTAP*E(NO)))*3.1415927/3.
CONS1 = CALFA-CONS
ALPU = ARCOS(CONS1)
ALFA = ARCOS(CALFA)
U = ALPU-ALFA
CONST = SQRT((COS(2*ALFA)-COS(2*ALPU))*2 + (2*U + SIN(2*ALFA)-SIN(2*ALPU))*2)/(4*(CALFA-COS(ALPU)))
RETURN
END

SUBROUTINE DERIV(NEQ,TIME,YY,DYY)
COMMON /ELEC/H
COMMON /VOLT/EE,PELEC,EQMAG,PMECH1,PMECH2
COMMON /EXITER/VREF1,VREF2,VS1,VS2
DIMENSION EE(l1),PELEC(6),EQMAG(6),YY(l4),DYY(l4)
REAL KA,KE,KF,M1,M2
DATA /0.05/ ,KA/40./ ,TA/0.05/ ,KE/-0.05/ ,SE/0.074/ ,TE/0.5/ ,KF I.05/,TF/1.0/ ,PI/3.141593/ ,FREQ/60./
DATA D1/.5/,M1/3055775/,TDOP/.25/,M2/1.527887/
DYY(1) = (-D1*(YY(1)-2*PI*FREQ)+ PMECH1-PELEC(1))/M1
DYY(2) = YY(1)-2*PI*FREQ
DYY(3) = (YY(7)-EQMAG(1))/TDOP
DYY(4) = (EE(1)-YY(4))/TR
DYY(5) = (-YY(5) + (-YY(4) + VS1 + VREF1-YY(6))*KA)/TA
DYY(6) = (-YY(6) + ((KE + SE)*YY(7) + YY(5))/TE)*KF)/TF
DYY(7) = (-KE + SE)*YY(7) + YY(5))/TE
DYY(8) = (-D2*(YY(8)-2*PI*FREQ) + PMECH2-PELEC(1))/M2
DYY(9) = YY(8)-2*PI*FREQ
DYY(10) = (YY(14)-EQMAG(6))/TDOP
DYY(11) = (EE(6)-YY(11))/TR
DYY(12) = (-YY(12) + (-YY(11) + VS2 + VREF2-YY(13))*KA)/TA
DYY(13) = (-YY(13) + ((KE + SE)*YY(14) + YY(12))/TE)*KF)/TF
DYY(14) = ((KE + SE)*YY(14) + YY(12))/TE
RETURN
END

SUBROUTINE RKGIL(NEQ,TIME,Y,DY)
C** RUNGE-KUTTA 4TH ORDER, FIXED STEP SIZE ROUTINE **

APPENDIX C. FORTRAN PROGRAM LISTING FOR SIMULATION OF POWER SYSTEM 290
C* PROVIDED BY R. G. LEONARD, VPI&SU ME DEPT
C*---------------------------------------------------
DIMENSION A(2),Q(14)
DATA Q/14*0./
COMMON /ELEC/H
REAL 1,Y(14),DY(14)
C*---------------------------------------------------
C* REAL*8 CONSTANTS .292893218813452475, 1.70710678118654752 *
C*---------------------------------------------------
A(1) = .2928932
A(2) = 1.707107
H2 = 0.5*H
CALL DERIV(NEQ,TIME,Y,DY)
DO 13 II= 1,NEQ
   B = H2*DY(II)-Q(II)
   Y(II) = Y(II) + B
13 Q(II) = Q(II) + 3.0*B-H2*DY(II)
TIME = TIME + H2
DO 20 J= 1,2
   CALL DERIV(NEQ,TIME,Y,DY)
   DO 20 II= 1,NEQ
      B = A(J)*H*DY(II)-Q(II))
      Y(II) = Y(II) + B
20 Q(II) = Q(II) + 3.0*B-A(J)*H*DY(II)
TIME = TIME + H2
CALL DERIV(NEQ,TIME,Y,DY)
DO 26 II= 1,NEQ
   B = (H*DY(II)-2.0*Q(II))/6.0
   Y(II) = Y(II) + B
26 Q(II) = Q(II) + 3.0*B-H2*DY(II)
RETURN
END
SUBROUTINE TANGAN(REA,AIM,ZAV)
PI = 3.1415927
IF(REA.GT.0) THEN DO
   ZAV = ATAN(AIM/REA)
END IF
IF(REA.LT.0) THEN DO
   ZAV = ATAN(AIM/REA) + PI
END IF
IF(REA.EQ.0.AND.AIM.LT.0) THEN DO
   ZAV = -PI/2.
END IF
IF(REA.EQ.0.AND.AIM.GT.0) THEN DO
   ZAV = PI/2.
END IF
IF(REA.EQ.0.AND.AIM.EQ.0) THEN DO
   ZAV = 0.0
END IF
RETURN
END
SUBROUTINE TANGEN(N,REA,AIM,ZAV)
DIMENSION REA(6),AIM(6),ZAV(6)
PI = 3.1415927
IF(REA(N).GT.0) THEN DO
   ZAV(N) = ATAN(AIM(N)/REA(N))
IF(REA(N).LT.O) THEN DO
ZAV(N) = ATAN(AIM(N)/REA(N)) + PI
END IF
IF(REA(N).EQ.O.AND.AIM(N).LT.O) THEN DO
ZAV(N) = -PI/2.
END IF
IF(REA(N).EQ.O.AND.AIM(N).GT.O) THEN DO
ZAV(N) = PI/2.
END IF
IF(REA(N).EQ.O.AND.AIM(N).EQ.O) THEN DO
ZAV(N) = 0.0
END IF
RETURN
END

SUBROUTINE HVDCTR(VDR,VDI,E,CALFA,CALFAS,VDRS,LTCRS,LTCIS,
+ CGAMA,CGAMAS,ID,IDS,VDIS,RK,VK,LTCR,LTCI,XCR,XCI,RD,PI,R,RR)

COMMON /NIU/CGAMAM,K2,ISTAB
COMMON /TIM/TIME,KSWICH,IREMOV
DIMENSION R(4),RR(3),A1A(4,4),B1B(3,3),R1(4,1),RR1(3,1),WKAREA(99)
DIMENSION E(8)

91 R(1) = VDR-LTCR*E(2)*CALFA + (PI/6.)*XCR*ID
R(2) = VDR-RD*ID-VDI
R(3) = LTCR-LTCRS
R(4) = ID-IDS
RR(1) = VDI-LTCI*E(7)*CGAMA + (PI/6.)*XCI*ID
RR(2) = LTCI-LTCIS
RR(3) = VDI-VDIS
DO 10 I = 1,4
  DO 10 J = 1,4
    A1A(I,J) = 0.0
  10 CONTINUE
DO 25 I = 1,3
  DO 25 J = 1,3
    B1B(I,J) = 0.0
  25 CONTINUE
A1A(1,1) = 1.
A1A(1,2) = PI*XCR/6.
A1A(1,3) = -E(2)*CALFA
A1A(1,4) = -LTCR*E(2)
A1A(2,1) = 1.
A1A(2,2) = -RD
A1A(3,3) = 1.
A1A(4,2) = 1.
B1B(1,1) = 1.
B1B(1,2) = -E(7)*CGAMA
B1B(1,3) = -LTCI*E(7)
B1B(2,2) = 1.
B1B(3,1) = 1.
DO 20 I = 1,4
  R1(I,1) = R(I)
  20 CONTINUE
DO 25 I = 1,3
  RR1(I,1) = RR(I)
  M = 1
N = 4
IA = 4
IDGT = 0
CALL LEQT2F(A1A,M,N,IA,R1,IDGT,WKAREA,IER)
M = 1
N = 3
IA = 3
IDGT = 0
CALL LEQT2F(B1B,M,N,IA,RR1,IDGT,WKAREA,IER)
TVDI = VDI-RR1(1,1)
TLTCI = LTCI-RR1(2,1)
TCGAMA = CGAMA-RR1(3,1)
TVDR = VDR-RR1(1,1)
TID = ID-RR1(2,1)
TLTCR = LTCR-RR1(3,1)
TCALFA = TALFA-(RI(4,1))/3
IF((TCALFA.GT.1.01 .OR.TCALFA.LT.0 .OR.TCGAMA.GT.CGAMAM).AND.(ISTAB.LT.IREMOV)) THEN DO
CALL HVDCSW(VDR,VDI,E,CALFA,CALFAS,VDRS,LTCRS,LTCIS,
    + CGAMA,CGAMAS,ID,IDS,VDIS,RK,VK,LTCR,LTCI,XCR,XCI,RD,PI,R,RR)
KSWICH = 1
ELSE DO
VDI = TVDI
LTCI = TLTCI
CGAMA = TCGAMA
VDR = TVDR
ID = TID
LTCR = TLTCR
CALFA = TALFA
IF(CALFA.GT.1 .AND.ISTAB.GT.7) THEN DO
LTCRS = LTCRS*CALFA*1.05
CALFA = .9999
WRITE(6,151) LTCRS
151 FORMAT(1X,'LTCRS IS CHANGED',F10.7)
GO TO 91
END IF
IF(CGAMA.GT.1 .AND.ISTAB.GT.7) THEN DO
LTCIS = LTCIS*CGAMA*1.05
CGAMA = .9999
WRITE(6,152) LTCIS
152 FORMAT(1X,'LTCIS IS CHANGED',F10.7)
GO TO 91
END IF
END IF
50 CONTINUE
RETURN
END

SUBROUTINE HVDCSW(VDR,VDI,E,CALFA,CALFAS,VDRS,LTCRS,LTCIS,
    + CGAMA,CGAMAS,ID,IDS,VDIS,RK,VK,LTCR,LTCI,XCR,XCI,RD,PI,R,RR)
DIMENSION R(4),RR(3),A1A(4,4),B1B(3,3),R1(4,1),RR1(3,1),WKAREA(99)
DIMENSION E(8)
REAL ID,IDS,LTCR,LTCI,LTCRS,LTCIS
IF(E(2).LT.0.5) THEN DO
    CIDS = 0.0001*IDS
ELSE DO
    CIDS = IDS*0.9
END IF

APPENDIX C. FORTRAN PROGRAM LISTING FOR SIMULATION OF POWER SYSTEM
END IF
VDRS = LTCRS*E(2)-(PI/6.)*XCR*CIDS
R(1) = VDI-LTCI*E(7)*CGAMA + (PI/6.)*XCI*ID
R(2) = VDI + RD*ID-VDR
R(3) = LTCI-LTCIS
R(4) = ID-CIDS
RR(1) = VDR-LTCR*E(2)*CALFA + (PI/6)+XCR +JD
RR(2) = LTCR-LTCRS
RR(3) = VDR-VDRS
DO 10 I = 1,4
   DO 10 J = 1,4
   A1A(I,J) = 0.0
10 CONTINUE
DO 15 I = 1,3
   DO 15 J = 1,3
   BIB(I,J) = 0.0
15 CONTINUE
A1A(1,1) = 1.
A1A(1,2) = PI*XCI/6
A1A(1,3) = -E(7)*CGAMA
A1A(1,4) = -LTCI*E(7)
A1A(2,1) = 1.
A1A(2,2) = RD
A1A(3,3) = 1.
A1A(4,2) = 1.
BIB(1,1) = 1.
BIB(1,2) = -E(2)*CALFA
BIB(1,3) = -LTCR*E(2)
BIB(2,2) = 1.
BIB(3,1) = 1.
DO 20 I = 1,4
   RL(I,1) = R(I)
20 CONTINUE
DO 25 I = 1,3
25 RR1(I,1) = RR(I)
M = 1
N = 4
IA = 4
IDGT = 0
CALL LEQT2F(A1A,M,N,IA,R1,IDGT,WKAREA,IER)
M = 1
N = 3
IA = 3
IDGT = 0
CALL LEQT2F(B1B,M,N,IA,RR1,IDGT,WKAREA,IER)
VDI = VDI-R1(1,1)
ID = ID-R1(2,1)
LTCI = LTCI-R1(3,1)
CGAMA = CGAMA-R1(4,1)
VDR = VDR-RR1(1,1)
LTCR = LTCR-RR1(2,1)
CALFA = CALFA-RR1(3,1)
RETURN
END
SUBROUTINE MULT(A,B,C,L,M,N)
DIMENSION A(L,M),B(M,N),C(L,N)
DO 10 I = 1, L
DO 10 J = 1, N
C(I,J) = 0.0
DO 10 K = 1, M
10 C(I,J) = C(I,J) + A(I,K) * B(K,J)
RETURN
END

SUBROUTINE DERIV1(NEQ1, TIME1, GVEC, DGVEC)
REAL KROFT(14,14), KBRIBT(14,14)
COMMON /GGG/ HH
COMMON /EQUA/ LLCNT, BRINBT, AMAT, BMAT, AMATTR, BETA, GAINF
COMMON /QMATRIX/QMAT
DIMENSION GVEC(14), DGVEC(14), GAINB(14,14), AMAT(14,14), BETA(14,4)
DIMENSION AMATTR(14,14), AXDIS(14), ROFT(14), FEST(14), XDIS(14)
DIMENSION XDDIS(14), BRINBT(14,14), BMAT(14,3)
DIMENSION ACLP(14,14), PART1(14), GAINF(14,14), UDES(3), BUDES(14)
DIMENSION XZERO(14), QMAT(14,14), ZDES(14), PART2(14)
DATA GAINB/196.0/
JIICNT = LLCNT/10
KIIICT = JIIICNT*10
IF(LLCNT.EQ.KIICNT) THEN DO
LLCNT = 1
DO 10 I = 1, 14
READ(9,100) (GAINB(I,J), J = 1, 14)
10 CONTINUE
100 FORMAT(1X,7E11.4)
END IF
TTIME = 4.96 - TIME1
CALL DESIRD(TTIME, XDIS, XDDIS)
CALL ZERO(XZERO)
CALL MULT(AMAT, XZERO, AXDIS, 14, 14, 1)
CALL INPUTD(UES)
CALL MULT(BMAT, UDES, BUDES, 14, 3, 1)
DO 20 I = 1, 14
FEST(I) = 0.0
DO 30 J = 1, 4
FEST(I) = BETA(I,J) * ((TTIME)**(J-1)) + FEST(I)
30 CONTINUE
ROFT(I) = AXDIS(I) + FEST(I) + BUDES(I)
20 CONTINUE
CALL MULT(GAINB, BRINBT, KBRIBT, 14, 14, 14)
CALL MULT(GAINB, ROFT, KROFT, 14, 14, 1)
DO 22 I = 1, 14
ZDES(I) = XDIS(I) - XZERO(I)
22 CONTINUE
CALL MULT(QMAT, ZDES, PART2, 14, 14, 1)
DO 21 I = 1, 14
KROFT(I) = KROFT(I) - PART2(I)
21 CONTINUE
DO 40 I = 1, 14
DO 40 J = 1, 14
ACLP(I,J) = AMATTR(I,J) - KBRIBT(I,J)
40 CONTINUE
CALL MULT(ACLP, GVEC, PART1, 14, 14, 1)
DO 50 I = 1, 14
DGVEC(I) = PART1(I) - KROFT(I)
50 CONTINUE
SUBROUTINE DESIRD(TIME,XDIS,XDDIS)

DIMENSION XDIS(14),XDDIS(14)
COMMON /OMEGA/SPEED

XDIS(1) = SPEED
XDIS(2) = 0.0044943*EXP(-1*TIME) + 1.0797057
XDIS(3) = -0.0164941*EXP(-1*TIME) + 1.890177
XDIS(4) = 1.02
XDIS(5) = XDIS(3)*(0.024)
XDIS(6) = 0.
XDIS(7) = XDIS(3)
XDIS(8) = 0.0044943*EXP(-1*TIME)
XDIS(9) = -0.0164941*EXP(-1*TIME)
XDIS(10) = 1.02
XDIS(11) = XDIS(3)*(0.024)
XDIS(12) = 0.
XDIS(13) = XDIS(3)
XDIS(14) = XDIS(3)

XDDIS(1) = 0.0
XDDIS(2) = -0.0044943*EXP(-1*TIME)
XDDIS(3) = 0.0164941*EXP(-1*TIME)
XDDIS(4) = 0.0
XDDIS(5) = XDDIS(3)*(0.024)
XDDIS(6) = 0.
XDDIS(7) = XDDIS(3)
XDDIS(8) = 0.0
XDDIS(9) = -0.0021203*EXP(-1*TIME)
XDDIS(10) = 0.0035399*EXP(-1*TIME)
XDDIS(11) = 0.0
XDDIS(12) = XDDIS(10)*(0.024)
XDDIS(13) = 0.
XDDIS(14) = XDDIS(10)
RETURN
END

SUBROUTINE RKG1L1(NEQ,TIME,Y,DY)

REAL l,Y(14),DY(14)

DO 13 II= 1,NEQ

B = ll2*DY(II)·Q(II)

Y(II) = Y(II) + B

Q(II) = Q(II) + 3.0·B-H2·DY(II)

APPENDIX C. FORTRAN PROGRAM LISTING FOR SIMULATION OF POWER SYSTEM
TIME = TIME + H2
DO 20 J = 1,2
CALL DERIV1(NEQ,TIME,Y,DY)
DO 20 II = 1,NEQ
B = A(J)*HH*DY(II)-Q(II))
Y(II) = Y(II) + B
20 Q(II) = Q(II) + 3.0*H2-A(J)*HH*DY(II)
TIME = TIME + H2
CALL DERIV1(NEQ,TIME,Y,DY)
DO 26 II = 1,NEQ
B = (HH*DY(II)-2.0*Q(II))/6.0
Y(II) = Y(II) + B
26 Q(II) = Q(II) + 3.0*B-H2*DY(II)
RETURN
END
SUBROUTINE LEAST(NUM,X,Y,ABC)
DIMENSION A(11,12),XPX(11,11),XPY(11),X(62),Y(62),B(11)
DIMENSION ABC(4)
DO 11 I = 1,14
ABC(I) = 0.0
CONTINUE
SUMY = 0.0
SUMX = 0.0
SUMXY = 0.0
SUMXS = 0.0
DO 4 I = 1,NUM
SUMY = SUMY + Y(I)
SUMX = SUMX + X(I)
SUMXY = SUMXY + X(I)*Y(I)
4 SUMXS = SUMXS + X(I)*X(I)
PMSSQ = 10.0E8
K = 1
N = K + 1
M = N + 1
XPX(1,1) = NUM
XPX(1,2) = SUMX
XPX(2,1) = SUMX
XPX(2,2) = SUMXS
XPY(I) = SUMY
XPY(2) = SUMXY
18 DO 12 I = 1,N
10 A(I,J) = XPX(I,J)
12 A(I,M) = XPY(I)
CALL GSJQR(N,M,A)
SUMSQ = 0.0
DO 16 I = 1,NUM
PROD = 0.0
16 DO 17 J = 1,K
17 PROD = PROD + A(J + 1,M)*X(I)**J
YHAT = A(1,M) + PROD
16 SUMSQ = SUMSQ + (Y(I)-YHAT)**2
IF(NUM-K-1.EQ.0) GO TO 42
CMSSQ = SUMSQ/(NUM-K-1)
IF(CMSSQ.GE.PMSSQ) GO TO 42
DO 15 I = 1,N

APPENDIX C. FORTRAN PROGRAM LISTING FOR SIMULATION OF POWER SYSTEM 297
IM = I - 1
ABC(I) = A(I, M)
15 CONTINUE
PMSSQ = CMSSQ
IF(K.EQ.10) GO TO 2000
K = K + 1
N = K + 1
M = N + 1
KPI = K + 1
KM1 = K - 1
DO 20 I = 1, KM1
XPX(KPI, I) = XPX(K, I + 1)
20 XPX(I, KPI) = XPX(KPI, I)
SUM1 = 0.0
SUM2 = 0.0
SUM3 = 0.0
DO 21 I = 1, NUM
XK = X(I)**K
SUM1 = SUM1 + XK*X(I)**(K - 1)
SUM2 = SUM2 + XK*XX
21 SUM3 = SUM3 + XK*Y(I)
XPX(KPI, K) = SUM1
XPX(KPI, KPI) = SUM2
XPX(K, KPI) = SUM1
XPY(KPI) = SUM3
DO 27 J = 1, K
27 B(J) = A(J, N)
IF(K.EQ.4) GO TO 42
GO TO 18
42 KM1 = K - 1
DO 25 I = 1, NUM
PROD = 0.0
25 CONTINUE
2000 CONTINUE
RETURN
END
SUBROUTINE GSJOR(N, M, A)
DIMENSION A(11, 12)
DO 16 K = 1, N
KPI = K + 1
DO 9 J = KPI, M
A(K, J) = A(K, J)/A(K, K)
9 CONTINUE
16 CONTINUE
RETURN
END
SUBROUTINE INPUTD(URES)
DIMENSION URES(3)
URES(1) = 1.0211242
URES(2) = 1.0211465
APPENDIX C. FORTRAN PROGRAM LISTING FOR SIMULATION OF POWER SYSTEM
UDES(3) = 0.0
RETURN
END
SUBROUTINE ZERO(XZERO)
DIMENSION XZERO(14)
COMMON /OMEGA/SPEED
XZERO(1) = SPEED
XZERO(2) = 1.0842
XZERO(3) = 1.8736830
XZERO(4) = 1.02
XZERO(5) = XZERO(3) * (0.024)
XZERO(6) = 0.
XZERO(7) = XZERO(3)
XZERO(8) = SPEED
XZERO(9) = 1.2398710
XZERO(10) = 1.9107770
XZERO(11) = 1.02
XZERO(12) = XZERO(10) * (0.024)
XZERO(13) = 0.0
XZERO(14) = XZERO(10)
RETURN
END
DATA
ENDWAT
//*
//
SELECTED REFERENCES


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