Investigations on Solar Powered Direct Contact Membrane Distillation

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Abstract

Desalination is one of the proposed methods to meet the ever increasing water demands. It can be subdivided into two broad categories, thermal based desalination and electricity based desalination. Multi-effect Distillation (MED), Multi-Stage Flashing (MSF), Membrane Distillation (MD) fall under former and Reverse Osmosis (RO), Electro-Dialysis (ED) fall under later. MD offers an attractive solution for seawater as well as brackish water distillation. It shows highly pure yields, theoretically 100% pure. The overall construction of a MD unit is way simpler than any other desalination systems.

MD is a thermally driven diffusion process where desalination takes places in the form of water vapor transport across the membrane. It has low second law efficiency due to parasitic heat losses. The objective of the first part of the investigation is to thoroughly analyze a Direct Contact Membrane Distillation (DCMD) system from the view point of yield and exergy. The insights from exergy analysis are used in a design study, which is used for performance optimization. The first part concludes with a design procedure and design windows for large scale DCMD construction.

In the second part of the investigation, focus is moved to waveguide solar energy collector. The idea behind an ideal waveguide is to reduce the complexity of modeling solar energy collection. The mathematical model provided in this analysis can be extended to a family of non-imaging optics in solar energy and serves as a benchmarking analysis tool. A waveguide is suitable for low temperature operations due to limitations on maximum continuous temperature of operation. Thus, it becomes an ideal solution for DCMD applications. A levelized cost analysis is presented for a waveguide powered DCMD plant of a 30,000 capacity. A combination of waveguide and DCMD shows levelized cost of water at $1.80/m3, which is found to be lower than previously reported solar desalination water costs.
General Abstract

All parts of the globe are facing some kind of water related issues, some concerning quality, some concerning quantity where as some have to deal with both. Seawater Desalination is one of the proposed methods for water purification. However, in order to meet large demands, the process has to be efficient and sustainable.

In the first part of this thesis, a detailed investigation for Direct Contact Membrane Distillation shows insights into efficiency of the desalination. The outcomes from the study are used to build a design use case for a given capacity. In the later part of the thesis, a new solar collection method and mathematical model is proposed. The design developed in the first part is used as a reference design to build a solar collector powerblock to design a sustainable desalination technology.
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Chapter 1. Introduction

Water desalination is a technique of converting saline, impure water from sea or in-land reserve and converting it to potable water. Several desalination techniques exist today, such as, Multi Stage Flash (MSF), Multi Effect Distillation (MED), Vapor Compression Desalination (VC), Membrane Distillation (MD), Reverse Osmosis (RO), Forward Osmosis (FO), Electro-Dialysis (ED) etc. Each desalination technique has its advantages and disadvantages.

Membrane distillation is particularly attractive owing to simple construction, inexpensive operation and low maintenance. A MD unit has seawater and coolant separated by a hydrophobic membrane. The feed stream or saline water stream is heated above the temperature of coolant externally. Because of the temperature gradient, there exists a vapor pressure gradient and conjugately a vapor concentration gradient. The concentration gradient drives vapor from the saline channel to the coolant channel. It condenses on the coolant channel to form pure water. Hence, theoretically it is possible to produce permeate at 100% purity.

Based on the mode of operation and construction MD is classified into Direct Contact Membrane Distillation (DCMD), Air Gap Membrane Distillation (AGMD), Sweeping Gas Membrane Distillation (SGMD), Vacuum Membrane Distillation (VMD) etc. Goal of the present studies is to understand DCMD to a greater detail. In a DCMD module, permeate and saline streams are in direct contact with the hydrophobic membrane. DCMD shows simples construction among all MD techniques.

The heat and mass transfer problem in DCMD is solved in chapter 2. Heat transfer in DCMD can be characterized into two streams: 1) heat conduction, 2) heat transfer due to latent heat transfer. Latent heat transfer is a desirable phenomenon, where more latent heat transfer indicates more permeate flux. However, conduction heat transfer accounts to parasitic heat loss.
It can be said that the overall heat transfer in DCMD is a function of operational parameters such as stream inlet Reynolds numbers, inlet temperatures as well as geometrical parameters such as membrane stacking arrangement, membrane properties and construction. 2\textsuperscript{nd} chapter focuses on effects of these parameters on heat transfer in DCMD. The performance of DCMD is quantified into 2\textsuperscript{nd} Law Efficiency, also called as exergetic efficiency and recovery ratio. Overall the chapter provides valuable insights into design of a DCMD system using a more holistic approach.

In order to raise the stream temperatures on the feed side in a DCMD module, some form of heat source is required. It has been shown in the previous studies that solar energy can be used to raise the temperatures in DCMD modules since DCMD requires low stream temperatures. In chapter 3 a rigorous analysis is presented for waveguide as a power source for DCMD. A waveguide is a flat plate of optically transparent material which shows total internal reflection. Hence, the waveguide only suffers from optical absorption loss and losses to the surroundings. Chapter 3 provides insights into design procedure for a waveguide solar collector. Later part of the chapter shows cost calculations to obtain levelized cost of water from a waveguide DCMD plant.

The overall objective of the present work can be summarized as development of an efficient waveguide powered MD unit. A thorough literature review is presented in each chapter for more information on current state of art.
Chapter 2. Direct Contact Membrane Distillation – Modeling, Analysis and Design

Optimization

In light of recent events such as persistent drought in California, water has become a high value commodity, where demand exceeds supply by a large margin. Of the remedial technologies implemented, such as improved reclamation, efficient water delivery devices, and switching to non-traditional sources of water, sea-water distillation or desalination is one that has received wide attention recently. Among the various desalination techniques explored, thermal based direct contact membrane distillation (DCMD) is particularly advantageous because of its potential to provide high rejection rates, simple construction and low construction cost. Clean water recovery rate and thermal efficiency of DCMD are the two most important metrics for performance mapping which are strongly influenced by the temperature polarization effects and parasitic heat losses. The objective of the present study is to systematically perform a parametric numerical analysis leading to a careful characterization of exergy destruction inside the module. Information obtained from the numerical analysis is used to derive design windows on operational and geometric parameters to maximize recovery ratio with constraints on exergetic efficiency. For highest exergetic efficiency, the inlet stream Reynolds number for saline as well as permeate is found to be 300, and the inlet temperature of the saline stream is found to be 363K maintaining the permeate temperature at dead state temperature of 298K.

Keywords: Direct contact membrane distillation (DCMD), Exergy analysis, Exergy destruction, Fiber packing, and Drinking water.
1. Introduction

Climate change, and ever-increasing demand from the domestic and industrial entities have placed a tremendous strain on the water bodies. The stress has resulted in irregular and limited supply of potable water to the affected areas [1, 2]. Dependency on the precipitation makes it worse in areas such as California resulting into persistent drought like conditions [3]. Additionally, strain on the local water body leaves little to no room to address the fluctuations due to climatic conditions, precipitation, demand and natural calamities. In early 2016, Mekonnen and Hoekstra [2], presented that over 4 billion people in the world face scarcity of water at least few months in the year. The same study also shows that the severely affected areas face water scarcity all over the year and the numbers show that over 20% of the world population suffer from water shortage.

According to 2014 United Nations Water – Global Analysis and Assessment of Sanitation and Drinking-Water report, about 748 million people do not have access to clean drinking water, and of the people who have access; an estimated 1.8 billion people suffer from contaminated sources of water [1]. The reported numbers in [1] can be better visualized as, greater than 10% of the world population is deprived of an improved source of water, and more than 23% of the world population is in the danger of water borne diseases. Hence, as much as availability of clean water is important, purity of the available water is also an important issue in the global water supply systems.

Researchers have tried to address these issues by proposing technologies at all points in the water life cycle such as water reclamation, recycling, treatment and switching to non-traditional sources such as seawater [4]. Seawater purification techniques include Multi Stage Flash (MSF), Multi Effect Distillation (MED), Reverse Osmosis (RO), Forward Osmosis (FO), Electro-dialysis (ED), Nano-Filtration (NF) Membrane Distillation (MD), etc. [4, 5]. Membrane distillation
techniques are relatively newer and have received limited attention and commercialization efforts. MD is a thermally driven water purification process, which shows the potential to reduce the water production costs and produce pure water using waste-heat or solar energy. A MD system consists of a feed channel through which saline water flows and a coolant channel separated by a hydrophobic membrane, which is impermeable to liquid water but allows the passage of vapor. Since purification process can be described as mass transport of water vapor only, the yield from MD system has the highest purity of all other water purification techniques. Versatility of the process makes it useful with brackish as well as seawater as the feed sources[6, 7].

![Figure 1. Schematic and working of membrane distillation](image)

Based on the mode of operation, MD is subcategorized into Air Gap MD (AGMD), Sweeping Gas MD (SGMD), Vacuum MD (VMD) and Direct Contact MD (DCMD). The present study is focused on DCMD [6-10]. A DCMD system consists of saline water flow zone and pure water or permeate flow zone separated by a hydrophobic semi-permeable membrane as shown in
Fig.1. Pure water, which acts as both coolant and permeate collector is in direct contact with the membrane. The driving force for MD process is the vapor pressure difference across the membrane by virtue of the temperature difference between the hot saline flow and cold permeate flow. Since the hydrophobic porous membrane only allows transport of water vapor, the aforementioned partial pressure gradient drives the diffusion of vapor molecules from the saline water – membrane interface to the membrane – permeate interface where the vapor condenses into the cold permeate stream.

Several studies have focused on modeling the conjugate heat and mass transfer in DCMD [11-16]. Analytical resistance network models and numerical modeling approaches have been adopted to model several DCMD systems. The experimental studies and modeling approaches have studied the effects of operational and geometric parameters viz. inlet Reynolds number, temperature, salinity, fiber packing, etc. [13, 17, 18]. The two studied configurations are planar module and capillary membrane fiber arrangement. Capillary membrane fibers, also referred as hollow fiber membranes, have received wide attention due higher available surface area for heat and mass transfer [19-21]. The parameters can be broadly grouped into two categories, namely: (i) Geometrical parameters comprising packing arrangement, packing fraction, and packing angle; and (ii) Operational parameters consisting of temperature, and Reynolds number.

In previous works, the membrane packing is found to have a significant effect on the recovery [22, 23]. Membrane packing and arrangement directly influences flow characteristics, heat and mass transfer. Al-Obaidani et al. [23] presented a performance analysis of DCMD system with insights into energy and exergetic efficiency of the module for different packing fractions. The effect of local packing fraction of the randomly distributed fibers on the permeate flux yield and thermal efficiency was studied. An increase in permeate flux yield for increase in packing
fraction was observed. Similar analysis performed by Zhongwei et al. [22] also showed comparable results. The random distribution of fibers in both the studies was determined by Voronoi tessellation technique. Both studies show that recovery increases proportional to the increase in volume fraction. In an experimental study to evaluate fiber packing, Yang et al. [21] showed flux enhancement from 53% to 92% for several packing configurations. With additional components such as spacer knit fibers, they reported a 300% rise in the flux values in the laminar flow regime [21]. Overall, it is observed that hollow fiber packing has a significant role to play in designing more effective desalination systems. To the best of author’s knowledge, current literature lacks a systematic analysis to study the effect of hollow fiber packing.

Several researchers have studied the effect of feed and permeate temperatures on recovery and thermal efficiency. Al-Obaidani et al. [23] reported 38% increase in recovery and 5% increase in thermal efficiency for increasing feed temperature from 25 °C to 60°C. Song et al. [15] reported decrease in distillate flux with increase in permeate temperature while the flux increased with increase in saline feed inlet temperature. Qtaishat et al. [14] investigated effect of feed side temperature on local heat transfer coefficients both on permeate and feed side. A strong dependency between feed side temperature and feed side heat transfer coefficient was observed. Alklaibi [8, 24] showed that permeate flux increases 7 fold for change in feed temperature from 25 °C to 60°C.

Song et al. [17] discussed experimental and modeling results for a large scale DCMD system. They reported an increase in flux for increase in permeate inlet velocity due to heat transfer enhancement on permeate side with increase in the Reynolds number whereas in a different configuration Banat [25] has shown that the flux did not increase by an appreciable value for a five-fold increase in permeate mass flow rate. In another study, Ohta et al. [18] have established
that flux is strongly related to saline flow rate and weakly related to permeate flow rate. Aklaibi [24] pointed that the saline Reynolds number should be in the range 3000-5000 in order to seek a trade-off between heat transfer enhancement and pumping power.

Effect of feed salinity has been studied extensively, and it has been shown to have a very little effect on recovery as well as thermal efficiency [8, 12, 14, 17, 23]. Kurdian et al. [26] have shown an apparent error up to 11% for a salt solution other than just NaCl. Similarly, it has also been shown that NaCl solution cannot be assumed to be equivalent to the seawater in the previous studies [27, 28]. For the purpose of this study, the correlations for the properties of seawater are obtained from Refs.[27, 28].

Membrane physical properties also play an important role on the performance of DCMD. Mass transfer characteristics are affected by porosity, tortuosity and pore size; while heat transfer characteristics are affected by porosity, material of membrane. Additionally, both heat and mass transfer are also affected by overall contacting surface area of the membrane and thickness of membrane. Increasing the membrane thickness increases the heat transfer resistance. Increase in the heat transfer resistance reduces parasitic conduction heat losses and improves exergetic efficiency. On the other side, it also adds additional resistance to mass transfer, which negatively affects yield. Thus, there is a trade-off between exergetic efficiency and recovery ratio as a function of membrane properties. It has been shown that flux dropped by 70% for increase in membrane thickness from 0.25 to 1.55 mm. Increasing porosity was observed to have positive effects on vapor flux as well as thermal efficiency, since increase in porosity reduces mass transfer resistance [23].

Computational Fluid Dynamics (CFD) technique has been used to study the performance of DCMD in additional to one-dimensional (1-D) and two-dimensional (2-D) models developed by several researchers. CFD tool can be effectively used to understand the effects of fiber spacing,
packing arrangement, and use of turbulence promoters on heat and mass transfer. Yu et al. [19] studied single hollow fiber DCMD module using a 2-D CFD analysis. They investigated boundary layer build up, variation of Nusselt number, temperature polarization coefficient (TPC), heat and mass transfer coefficients as a function of various operational parameters. Yang et al. [21] studied the effects of membrane microstructure on heat and mass transfer, and the overall yield. A comprehensive review of the modeling approaches in MD points out that very few studies have used CFD approach to analyze the DCMD system [29]. Shirazi et al. [6, 10] also point out the need for more comprehensive CFD studies on MD systems.

Khayet [30] has presented a comprehensive review on energy efficiencies obtained by several researchers for MD systems and the corresponding specific energy consumption. Energy consumption in a MD system is characterized in the form of thermal efficiency, gained output ratio and exergetic or second law efficiency. Thermal efficiency ($\eta_{th}$) in context of MD is defined as the fraction of the heat contributing towards latent heat transfer. Gained output ratio (GOR) is defined as the ratio of enthalpy required to evaporate the distillate and the total heat consumption by the unit [30-32]. Zuo et al. [31] have shown that for increasing feed and permeate velocities, GOR only increases until a certain point after which it remains steady. As a result of the tradeoff between increase in pumping power and increase in GOR with increase in feed or permeate velocity, system needs to operate at an optimal value [31]. Although GOR and thermal efficiency talk about the energy consumption, they fail to connect work of separation in a system to the thermodynamic minimum work of separation in the desalination process. Thus, second law efficiency or exergetic efficiency gives a true metric to gauge performance of the MD module [33, 34]. It is defined as the ratio of least work of separation and actual work of separation. Mistry et al. [33] presented a detailed entropy analysis of various desalination techniques. The study also
points out that in a DCMD module, \(~34.5\%\) of the incoming exergy is destroyed in module, which is also the greatest source of exergy destruction in a DCMD system. The destruction is accountable to parasitic heat losses and transport resistance losses. Banat and Jwaied [25] estimated that 55.14\% of the input exergy is destroyed in the MD module. In another study, Speigler and El-Sayed have worked out exergy analysis for direct contact membrane distillation [34]. Taking the exergy analysis further, Criscuoli and Drioli presented energetic and exergetic analysis of an integrated solar MD system and showed that entropy generation in water separation process from seawater is always greater than zero, which points toward the inevitability of exergy destruction [35]. Since availability or exergy destruction is a function of heat and mass transfer in the system, to determine the cause-effect relationship between operational and geometrical parameters in DCMD with its exergetic performance, it is necessary to present a comprehensive parametric analysis.

This paper presents a new outlook towards DCMD systems with consideration towards tube arrangement, packing configuration in addition to operational parameters which have been extensively studied. The goal of the first part of the discussion is to establish the relationships between exergetic efficiency, \(\eta\) and recovery ratio \(\zeta\) with operational parameters such as inlet feed temperature, inlet Reynolds numbers of feed as well as permeate flow and geometrical parameters such as packing angle, volume fraction and packing arrangement. In the later part of the discussion, we have illustrated design directives for DCMD systems based on the discussions from the first part of the paper. Furthermore, a sample DCMD module design for a 10,000 L/day capacity unit is evaluated from the results of the parametric study.
The paper is organized into two sections: Section 2 discusses the mathematical modeling, and Section 3 discusses the results obtained from the numerical analysis and sample design windows are presented.

2. Mathematical Model:

![Representative computational geometries](image)

Figure 2. Representative computational geometries (a) rectangular packing (b) staggered packing (c) equivalent computational cell for rectangular packing (d) equivalent computational cell for staggered packing

Figure 2 shows two different configurations of DCMD system and the corresponding computational unit cell schematics considered in the present study. Figure 2(a) shows rectangular
arrangement and 2(b) shows staggered arrangement. The present study uses hollow fiber membranes, thus the DCMD system can be visualized as a collection of hollow fibers bunched in a shell as shown in Figs. 2a and b. Permeate flows inside the hollow fibers while saline feed flows in the shell in a countercurrent direction. The periodic configuration of the hollow tubular fibers in shell allows for identification of representative three-dimensional unit cell as indicated by the shaded areas in Figs. 2a and b. The three-dimensional unit cells as represented by the shaded areas in Figs. 2a and b are described in detail in Figs. 2c and d, respectively and form the computational domain for the present analysis. The unit cell in either packing scheme is described by a rectangular cell of width, $S_L$ and height, $S_R$ along $x$- and $y$- axis, respectively. The dimension along the tube axis ($z$- axis) is indicated by length, $L$. The inner and outer radii of the tubular fiber are represented by $r_i$ and $r_o$, respectively and the difference between $r_o$ and $r_i$ is the membrane thickness, $t_m$.

The packing angle of the ordered arrangement is calculated as: $\delta = \tan^{-1}\left(\frac{S_R}{S_L}\right)$ and the volume fraction for a constant module length is evaluated as: $\phi = \frac{A_p}{A_s}$, where $A_p$ and $A_s$ are permeate and saline flow areas, respectively. Hence, for given values of fiber radius ($r_i$), membrane thickness ($t_m$), packing angle ($\delta$), and volume fraction ($\phi$), the width ($S_L$), and height ($S_R$) of the unit cell can be evaluated using the aforementioned relations.

A steady state numerical model governing the conjugate heat and mass transfer in MD process is presented in this study. The conjugate heat and mass transfer is modeled as it has been adopted by several researchers in the literature [13, 15, 26, 29]. The model assumes the flow of both feed water and permeate to be incompressible. Membrane properties are assumed to be isotropic and constant over the temperature range under consideration. The membrane walls are assumed to form perfect no-slip boundaries. Membrane wetting is neglected and it is assumed that mass transfer in membrane is by molecular diffusion mechanism. The governing equations for the
computational domain comprising continuity, momentum and energy conservation in various zones are described in the following sections.

2.1. The saline/permeate zone

The temperature profile and flow characteristics in the saline and permeate region are obtained by solving continuity, momentum and energy equations, which are represented in non-dimensional form in Eqs. (1), (2) and (3), respectively.

\[ \frac{\partial u_i^*}{\partial x^*} + \frac{\partial v_i^*}{\partial y^*} + \frac{\partial w_i^*}{\partial z^*} = 0 \]  
\[ u_i \frac{\partial u_i^*}{\partial x^*} + v_i \frac{\partial u_i^*}{\partial y^*} + w_i \frac{\partial u_i^*}{\partial z^*} = - \frac{\partial p_i^*}{\partial x^*} + \frac{1}{Re_i} \left( \frac{\partial^2 u_i^*}{\partial x^{*2}} + \frac{\partial^2 u_i^*}{\partial y^{*2}} + \frac{\partial^2 u_i^*}{\partial z^{*2}} \right) \]  
\[ u_i \frac{\partial v_i^*}{\partial x^*} + v_i \frac{\partial v_i^*}{\partial y^*} + w_i \frac{\partial v_i^*}{\partial z^*} = - \frac{\partial p_i^*}{\partial y^*} + \frac{1}{Re_i} \left( \frac{\partial^2 v_i^*}{\partial x^{*2}} + \frac{\partial^2 v_i^*}{\partial y^{*2}} + \frac{\partial^2 v_i^*}{\partial z^{*2}} \right) \]  
\[ u_i \frac{\partial w_i^*}{\partial x^*} + v_i \frac{\partial w_i^*}{\partial y^*} + w_i \frac{\partial w_i^*}{\partial z^*} = - \frac{\partial p_i^*}{\partial z^*} + \frac{1}{Re_i} \left( \frac{\partial^2 w_i^*}{\partial x^{*2}} + \frac{\partial^2 w_i^*}{\partial y^{*2}} + \frac{\partial^2 w_i^*}{\partial z^{*2}} \right) \]  
\[ u_i \frac{\partial \theta_i^*}{\partial x^*} + v_i \frac{\partial \theta_i^*}{\partial y^*} + w_i \frac{\partial \theta_i^*}{\partial z^*} = \frac{1}{Re_i Pr_i} \left( \frac{\partial^2 \theta_i^*}{\partial x^{*2}} + \frac{\partial^2 \theta_i^*}{\partial y^{*2}} + \frac{\partial^2 \theta_i^*}{\partial z^{*2}} \right) \]  

The non-dimensional variables are defined as follows:

\[ x^* = \frac{x}{L}, \quad y^* = \frac{y}{L}, \quad z^* = \frac{z}{L}, \quad D^* = \frac{D}{L} \]  
\[ u_i^* = \frac{u_i}{w_i^{in}}, \quad v_i^* = \frac{v_i}{w_i^{in}}, \quad w_i^* = \frac{w_i}{w_i^{in}} \]  
\[ p_i^* = \frac{p_i}{\rho_i(w_i^{in})^2}, \quad \theta_i^* = \frac{i - T_p^{in}}{T_s^{in} - T_p^{in}} \]  
\[ Re_i = \frac{\rho_i w_i^{in} L}{\mu_i} = \frac{\rho_i w_i^{in} D}{\mu_i} \left( \frac{1}{D^*} \right), \quad Pr_i = \frac{\mu_i C_{p_i}}{K_i} \]
where the subscript “$i$” can be either $s$ to represent the saline flow or $p$ to represent the permeate flow. $u_i, v_i,$ and $w_i$ are the flow velocities in $x, y$ and $z$ directions respectively? $T_s$ and $T_p$ denote the local saline and permeate temperatures. The superscript “$in$” represents the boundary values at the inlet. The characteristic non-dimensional parameters that appear in Eq. (4) are the Reynolds number, $Re_i$ and Prandtl number, $Pr_i$.

2.2. The membrane zone

Membrane zone is the active zone in the physical system where the mass transport occurs. The properties of the membrane allow for water-vapor transport exclusively. The transport is modeled by molecular diffusion. Several researchers have compared the results from molecular diffusion with that from dusty gas model, Knudsen diffusion and a combination of Knudsen diffusion and molecular diffusion. For the pore scales in a commercial PTFE module, the transport phenomenon can be effectively modelled by molecular diffusion [36].

The membrane also provides path for heat conduction. The heat and mass transport thus forms a conjugate problem where heat transfer is a limiting factor towards the mass transport since higher heat transfer results in lower trans-membrane mass transport. Latent heat transfer across the membrane due to mass transport is modeled separately and it appears as interfacial source terms as discussed later in Section 2.3.

The mass transport is modeled as indicated in Eq. (5). The concentration gradients in the membrane along $x_s, y_s, z_s$ are considered while solving for mass transport. Similarly, the energy equation solves for temperature inside the membrane zone as indicated in Eq. (6).

$$\frac{\partial^2 C_{m_s}}{\partial x_m^2} + \frac{\partial^2 C_{m_s}}{\partial y_m^2} + \frac{\partial^2 C_{m_s}}{\partial z_m^2} = 0$$
\[
\frac{\partial^2 \theta_m^*}{\partial x_m^* 2} + \frac{\partial^2 \theta_m^*}{\partial y_m^* 2} + \frac{\partial^2 \theta_m^*}{\partial z_m^* 2} = 0
\] (6)

The local concentration of vapor on the membrane surface is obtained from: \( C_m = \frac{p_{sat}^{int}}{RT_{int}} \), where \( p_{sat}^{int} \) and \( T_{int} \) are the interface saturation pressures and temperatures corresponding to either saline zone or permeate zone. \( p_{sat}^{int} \) itself is a function of \( T_{int} \) and the relationship between the two is given by the Antoine’s Equation (Eq.(7)) \[37\]. Values for the constants \( A, B \) and \( C \) are obtained from the NIST webbook under properties of water \[38\].

\[
\log_{10}(p_{sat}^{int}) = A - \frac{B}{T_{int} + C}
\] (7)

2.3. Membrane – Fluid interfaces

The membrane fluid interfaces provide the active sites for evaporation of water at the free surfaces on the membrane. The coupling term on the energy equation and species transport thus appears at the saline-membrane and membrane-permeate interfaces. The non-dimensional form of the saline-membrane and permeate-membrane interface term on the energy equation is written as Eq. (8). The first term on the right hand side of the expression in Eq. (8) indicates the heat conduction to or from the fluid and the second term corresponds to the latent heat of evaporation. The sign on the second term determines whether the latent heat of evaporation is absorbed or evolved. It is positive for permeate zone and negative for saline zone.

\[
\frac{\partial \theta_i}{\partial x^*} + \frac{\partial \theta_i}{\partial y^*} = K^* \left( \frac{\partial \theta_m^*}{\partial x^*} + \frac{\partial \theta_m^*}{\partial y^*} \right) \pm \frac{Pr_s}{Sc \cdot Ja} \left( \frac{\partial C_m^*}{\partial x^*} + \frac{\partial C_m^*}{\partial y^*} \right)
\] (8)

In Eq. (8), \( Sc \) and \( Ja \) denote Schmidt and Jakob numbers, respectively and \( K^* \) denotes the ratio of membrane conductivity to the conductivity of the water.
\[ Sc_i = \frac{v_i}{D} \quad (9) \]
\[ Ja_i = \frac{C_{pi}(T_{i}^{in} - T_d)}{h_{fg}} \quad (10) \]

\( T_d \) denotes the global dead state temperature which is also used as the reference temperature for exergy calculations.

2.4. Boundary conditions

For the complete definition of the problem, following boundary conditions are imposed.

Saline Feed Channel Inlet –

\[ u_s^*(z^* = 0) = v_s^*(z^* = 0) = 0, \quad w_s^*(z^* = 0) = 1 \]
\[ \theta_s(z^* = 0) = 1 \]

Permeate Channel Outlet –

\[ \left. \frac{du_p^*}{dz^*} \right|_{(z^* = 0)} = 0, \quad \left. \frac{dv_p^*}{dz^*} \right|_{(z^* = 0)} = 0, \quad \left. \frac{d\theta_p^*}{dz^*} \right|_{(z^* = 0)} = 0 \]

Saline Feed Channel Outlet –

\[ \left. \frac{du_s^*}{dz^*} \right|_{(z^* = 1)} = 0, \quad \left. \frac{dv_s^*}{dz^*} \right|_{(z^* = 1)} = 0, \quad \left. \frac{d\theta_s^*}{dz^*} \right|_{(z^* = 1)} = 0 \]

Permeate Channel Inlet –

\[ u_p^*(z^* = 1) = v_p^*(z^* = 1) = 0, \quad w_p^*(z^* = 1) = 1 \]
\[ \theta_p(z^* = 1) = 0 \]

The membrane – fluid interfaces are treated as coupled walls with no-slip boundary condition and an additional source term as described in Eq. (8) for capturing the latent heat transport. Membrane end walls are treated as adiabatic walls and the unit cell lateral surfaces are treated with symmetry boundary condition.
Considering recovery ratios and overall mass flux to be significantly smaller than inlet mass flux, the mass transfer is only modeled through the membrane and there is no corresponding term affecting continuity equation. Thus, effect of vapor mass transfer through the membrane on the continuity equation is said to be negligible.

Input parameters of interest are selected based on the non-dimensional groups that appear in Eqs. (2), (3), (10) and (11) viz., \(Re_s, Re_p, Ja_s, D^*\) and \(K^*\). In addition to these, the effects of non-dimensional membrane thickness, \(\delta^*_m = \frac{\delta_m}{L}\), and membrane porosity, \(\epsilon\), are also evaluated. Variation in \(Ja_p\) is not considered, since elevating the input temperature of the permeate stream above the dead state temperature has associated exergy penalty. Thus, by maintaining \(Ja_p = 0\), unnecessary exergy destruction in the module is avoided. For parametric analysis except the variant, all other parameters are maintained at default values given as, \(Re_s = 375, Re_p = 375, D^* = 0.0125, \delta^*_m = 0.00125, K^* = 0.330, Ja = 0.029\) and \(\epsilon = 0.85\). These default values are computed at the central location of the selected range of dimensional parameters. The range for velocity on permeate and saline zones is studied in the range 0.12 m/s to 0.57 m/s. Jacob number is calculated for the range of saline inlet temperatures from 298 K to 363 K. Thermal conductivity of the membrane material is varied between 0.1 \(W/m - K\) to 0.4 \(W/m - K\). Membrane thickness is varied between 100 \(\mu m\) to 350 \(\mu m\). For the analysis the diffusion coefficient is iteratively obtained to match the validation results which is used in all later analysis, it is calculated to be 2.7e-07 kg m^{-2}s^{-1} Pa^{-1}.

2.5 Exergy analysis

Output from a DCMD system is measured as the mass of permeate produced for the mass of saline water circulated, the ratio of the two is defined as the recovery ratio, \(\zeta\). In a perfect DCMD
system, 100% recovery is desirable. However, under operational conditions, the mass transfer is severally affected by temperature polarization, parasitic heat conduction through the membrane, mass transfer resistance offered by the membrane, etc. Thus, effectiveness of a DCMD cannot be decided by $\zeta$ alone. The maximum available energy that can be used to produce the work of separation is referred to as exergy. The system can perform the work of separation until it reaches to the equilibrium state with the surroundings. The state variables namely temperature ($T_d$), pressure ($P_d$) and concentration ($w_d$) of the surrounding conditions are referred to as the dead state. The exergy analysis tool is used to characterize the energy used to produce useful work of pure water separation from the sea-water. The exergy analysis methodology developed in the previous studies [30, 31] is employed to characterize exergy destruction and determine exergetic efficiency as a function of geometrical and input parameters. The specific flow exergy of a mixture of $n$ constituents is given as,

$$ex = (h - h^{d*}) - T_d(s - s^{d*}) + \sum_{i=1}^{n} w_i(\mu_i^* - \mu_i^d)$$  \hspace{1cm} (12)$$

in which $h, s, w$ and $\mu$ are specific enthalpy, entropy, chemical potential and concentration of the constituent, respectively. Superscript ‘d’ represents the dead state, whereas, the following asterisk ‘*’ indicates that the properties are calculated at dead state temperature and pressure but at the feed concentration. Since for ideal liquid solutions, specific entropy is a function of temperature only and specific enthalpy is a function of temperature and pressure, a pressure correction is added in the specific enthalpy term. The rigorous formulation can be found in the previous studies by Sharqawy et al.[27, 28, 39]. The overall exergetic efficiency of the system is then evaluated as given in Eq. (13).
\[ \eta_{ex} = \frac{E_{x_{in}} - E_{x_{out}}}{E_{x_{in}}} \] (13)

3. Results and Discussion:

3.1. Model Validation

Figure 3. Verification of numerical scheme with Yu et al. (2011) for temperature polarization effects

The developed numerical scheme is compared with the numerical and experimental studies performed by Yu et al [19]. Table 1 shows the comparison of yields between the present numerical model and the experimental results obtained by Yu et al. [19] for four different saline and permeate inlet temperatures. It was observed that, there was less than 1% difference between previously reported experimental values and present simulation. In addition to yields, Fig. 3 shows verification of the present numerical scheme with Yu et al.’s prediction of temperature polarization coefficients [1]. The two membranes have different membrane properties and permeabilities and
hence, they exhibit different temperature polarization characteristics. The temperature polarization coefficient (TPC), defined as ratio of difference of local temperatures on either sides to difference of bulk temperatures on either sides, is higher towards the end which indicates that the temperature difference between bulk fluid and local at the surface is low at the boundaries in the respective domains and it increases towards the middle which results in lower TPC at the central region. The results show an overall effect of boundary layer formation. Higher value of TPC shows effective boundary layer heat transfer near the extremities of the channel which is due to counter-current operation. It can be concluded from the comparison that the temperature profiles as well as local fluxes computed from the present numerical simulation agree closely with the experimental results found in the literature.

Table 1. Comparison of absolute yields from experimental evaluation by Yu et al. (2011) with present simulations

<table>
<thead>
<tr>
<th>$T_{s, in}$ [K]</th>
<th>$T_{p, in}$ [K]</th>
<th>Experimental [kg/m²s]</th>
<th>Current Model [kg/m²s]</th>
</tr>
</thead>
<tbody>
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<td>327.2</td>
<td>294.0</td>
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<tr>
<td>333.9</td>
<td>294.4</td>
<td>0.00237</td>
<td>0.00235</td>
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<tr>
<td>334.8</td>
<td>312.8</td>
<td>0.00170</td>
<td>0.00175</td>
</tr>
<tr>
<td>337.6</td>
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<td>0.00272</td>
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<tr>
<td>337.6</td>
<td>304.0</td>
<td>0.00235</td>
<td>0.00245</td>
</tr>
</tbody>
</table>
3.2. Comparison of Staggered and Rectangular Grid for Default Parameters

Figure 4. Velocity and temperature profiles in rectangular packing, (a), (b), (c) show velocity profiles for entrance, mid and end regions respectively, (d), (e), (f) show temperature profiles for entrance, mid and end regions.

Figure 4a-c shows the local flow velocities and Fig. 4d-f shows the temperature profile for 3 sections along the length of the fiber for a rectangular packing arrangement. The figures correspond to the operating and design parameters of $Re_s = 575, Re_p = 300, Ja = 0.029, D^* = 0.0125, \delta_m^* = 0.00125, \gamma = 0.330$ and $\epsilon = 0.85$. The saline feed in the shell enters the domain at $z = 0$ and exits at $z = L$; while the permeate feed in the hollow fiber enters the domain at $z = L$ and exits at $z = 0$. The maximum velocity at the saline side increases from 0.2 m/s at its inlet ($z = 0$) in Fig. 4a to 0.35 m/s at its outlet ($z = L$) in Fig. 4c. It can be observed that for the mid-section and the end section, the velocity contours are almost identical, and thus it can be said that the flow is fully developed in less than half the length of the module. Similarly, permeate enters the module.
at $z = L$ as shown in Fig. 4c and moves to outlet at $z = 0$ as depicted in Fig. 4a. Boundary layer development is used to explain the trends in velocities moving from the start section to mid-section and then to the end section. It is further noted that flow on the permeate side also develops fully before $z = L/2$ as the velocity contours on the mid-section and the end section show a close agreement. In both saline and permeate side as observed from Figs. 4a-c, the flow velocity decreases from high value at the bulk to zero at the membrane interface due to boundary layer development.

As observed from Figs. 4d-f, the volumetric average temperature of saline feed flow (permeate flow) decreases (increases) from its inlet at $z = 0$ ($z = L$) to its corresponding outlet at $z = L$ ($z = 0$) due to the heat exchange by means of conduction through the membrane and latent heat exchange due to liquid-vapor phase change. It is also seen that the temperature profile changes considerably between the mid-section ($z = L/2$) in Fig. 4e and the two ends while the flow velocity does not change appreciably between the mid-section ($z = L/2$) in Fig. 4b and the corresponding saline ($z = L$) and permeate ($z = 0$) exits. The change in temperature profile increases the temperature polarization which leads to a decrease in permeate flux rate with increase in axial distance from permeate inlet as the partial pressure gradient that drives the desalination process also decreases.
Figure 5. Velocity and temperature profiles in staggered packing, (a), (b), (c) show velocity profiles for entrance, mid and end regions respectively, (d), (e), (f) show temperature profiles for entrance, mid and end regions.

The observations drawn for the rectangular packing from Fig. 4 are also valid for staggered packing as it can be seen from Fig. 5. Figure 5a-c illustrate local velocity profiles and Fig. 5d-f temperature profiles for the same three sections along the length of the fiber. Comparing Fig. 4d and Fig. 5d it can be seen that trans-membrane temperature difference is higher for rectangular packing than the staggered packing. As discussed before [2], arrangement has a direct effect on the shell side heat transfer, and it can be seen that, staggered packing shows enhanced heat transfer over rectangular packing. Moving towards mid-section and then towards the end-section of the module, it can be observed that the rectangular packing maintains a higher transmembrane temperature difference over staggered packing, which at some locations is as high as 5% which results in proportional higher mass flux.
3.3. Effect of Permeate Side Inlet Reynolds Number on Recovery Ratio and Exergetic Efficiency

Figure 6. Effect of permeate inlet Reynolds number, $Re_p$, variation on recovery ratio, $\zeta$, exergetic efficiency, $\eta$, (a) (b) (c) (d) and (e) (f) (g) respectively, for $Re_s = 375, D^* = 0.0125, \delta_m^* = 0.00125, K^* = 0.330, Ja = 0.029$ and $\epsilon = 0.85$ under rectangular packing

Figures 6a-d show recovery ratio $\zeta$, and Figs. 6e-h show exergetic efficiency $\eta$, both as functions of $Re_p$ for various packing angles $\delta = 15^\circ - 75^\circ, 30^\circ - 60^\circ, 45^\circ$ and volume fractions $\psi = 0.15, 0.3, 0.45, 0.6$. It is observed from Fig. 6 a-d that recovery ratio does not change appreciably for change in $Re_p$ from 375 to 1375. It shows that heat transfer enhancement on the permeate side has a little effect on the trans-membrane temperature difference. Correspondingly, in Fig. 6 e-h, we see a linear drop in exergetic efficiency. This can be accounted to increased viscous losses.

Analyzing Fig. 6a, it can be seen that with increase in $Re_p$ from 375 to 1375 for $\delta = 45^\circ$ and $\psi = 0.15$, $\zeta$ changes by less than 2%. However, from Fig. 6e it is seen that $\eta$ drops by 12.7% for the same set of parameters with increase in $Re_p$ from 375 to 1375. With increase in the $Re_p$, average temperature on permeate side drops, as well as the heat transfer is enhanced leading to higher local trans-membrane temperature differences and a concomitant increase in recovery ratio.
Nevertheless, the increase in trans-membrane temperature difference which results in higher conductive and latent heat transfer across the membrane, accompanied with increase in drag losses factor into the overall increase in exergy destruction with increase in \( Re_p \).

Further examining Figs. 6a and 6e, variation in \( Re_p \) is accompanied with a strong dependence on \( \delta \). Increase in \( \delta \) increases heat transfer on the shell side due to improved stream splitting and enhanced overall mixing in the cross-section. Thus, the temperature polarization on the saline side is reduced with increasing \( \delta \). It manifests as higher local trans-membrane temperature difference resulting into sharp rise in recovery with corresponding increase in exergy destruction due to conductive and latent heat transfer across the membrane. For instance, at \( Re_p = 1375 \), when \( \delta \) is varied from 15° to 45°, the recovery ratio increases by 400% and the exergetic efficiency drops by 18%.

Trends described earlier for variation in \( Re_p \) and \( \delta \) propagate for simultaneous change in volume fraction \( \phi \) as shown in Figs.6a-d and Figs. 6e-h. Interestingly, it is seen from Fig.6a-d, that increasing \( \psi \) increases the \( \zeta \) which also agrees with the previous studies [2]. A competing opposing trend is observed on \( \eta \) in Figs. 6e-h, where for a given \( Re_p \), \( \eta \) drops for increase in volume fraction when the packing angle is maintained constant. This can be attributed to two factors: (1) with increase in volume fraction of permeate, the overall saline mass flow rate at inlet decreases linearly with \( \psi \) varying from 0.15 to 0.6, thus the recovery ratio increases, (2) with increase in \( \psi \), heat transfer on shell side is enhanced, resulting into higher trans-membrane temperature differences as well as increased conduction and latent heat transfer losses. The two mechanisms overall reflect into higher recoveries with increase in \( \psi \) accompanied with increased fraction of exergy destruction. However, the increase in recovery ratio becomes less prominent for the change in \( \psi \).
from 0.45 to 0.6. Recovery doubles for increasing volume fraction from 0.15 to 0.3 and from 0.3 to 0.45, whereas it only increases by 33% for increase from 0.45 to 0.6.

Figure 7. Effect of permeate inlet Reynolds number, $Re_p$, variation on recovery ratio, $\zeta$, exergetic efficiency, $\eta$, (a) (b) (c) (d) and (d) (e) (f) (g) respectively, for $Re_s = 375, D^* = 0.0125, \delta_m^* = 0.00125, K^* = 0.330, Ja = 0.029$ and $\epsilon = 0.85$ under staggered packing

As explained for rectangular packing in Fig. 6, the trends observed for recovery ratio and exergetic efficiency with respect to $Re_p$, $\delta$ and $\psi$ follow the same behavior for staggered packing as shown in Fig. 7. Staggered packing shows overall better mixing characteristics than rectangular packing and thus shows lower trans-membrane temperature differences as discussed in Section 3.2. Staggered packing and rectangular packing show comparable performance for variation of operational parameter $Re_p$. Thus, it can be concluded that packing arrangement has negligible influence with tube-side heat transfer. Overall, it is seen that for the range of $Re_p$, $\delta$ and $\psi$ under consideration, there is less than 2% difference in the values for $\eta$ and $\zeta$ for the respective pairs of staggered and rectangular results. Overall, staggered arrangement shows higher $\eta$ and lower $\zeta$ compared to rectangular arrangement, which can be attributed to the better mixing characteristics of staggered packing. Due to better mixing and enhancement in heat transfer, staggered packing
shows higher conduction heat transfer resulting in lower recovery ratio. In comparison, rectangular packing shows higher mass flux and comparable heat conduction, resulting into higher overall heat transfer (conduction + latent heat transfer). This accounts for the ~2% lower value of $\eta$.

3.4 Effect of Saline Side Inlet Reynolds number on Exergetic Efficiency and Recovery Ratio

![Graphs showing the effect of permeate inlet Reynolds number, $Re_p$, variation on recovery ratio, $\zeta$, exergetic efficiency, $\eta$, (a) (b) (c) (d) and (e) (f) (g) respectively, for $Re_p = 375, D^* = 0.0125, \delta_m = 0.00125, K^* = 0.330, Ja = 0.029$ and $\epsilon = 0.85$ under rectangular packing.](image)

Figures 8a-d show effect of saline side inlet Reynolds number $Re_s$ on recovery ratio $\zeta$, and Figs. 8e-h show its effect on exergetic efficiency $\eta$, for a range of values of volume fraction, $\psi$ and packing angle, $\delta$. A non-linear trend is observed in Figs.8a-d for reduction in $\zeta$ as a function of $Re_s$. Change in $Re_s$ results into two most important phenomenon, (1) it changes the feed inlet mass flow rate, (2) it affects the shell-side heat transfer. It can be observed from Fig.8a that for increasing the $Re_s$ from 375 to 600, the behavior is non-linear and the non-linear heat transfer enhancement is a major influencing factor. Whereas for increase in $Re_s$ beyond 600, the trends become fairly linear and the recovery ratio is directly affected by the increase in saline inlet mass flow rates and not as much by the heat transfer enhancement. Beyond $Re_s = 600$, conduction dominates as the heat transfer mechanism, keeping the convection heat transfer almost constant.
The flow of saline on the exterior of the tube in the shell-side can be approximated to external flow on a plate. The heat transfer coefficient for saline flow over the tube in the laminar regime, thus increases in a square law fashion with increase in Reynolds number \( (h \propto Re_s^{1/2}) \) [40]. Since the thermal resistance to conduction heat transfer is inversely proportional to the heat transfer coefficient resulting in an inverse-square law relation to \( Re_s \), the recovery ratio as depicted in Fig. 8 also decreases in the same fashion with increase in \( Re_s \). For increase in \( Re_s \), increase in inlet feed mass flow rate amounts to drop in \( \zeta \). Consequently, proportion of inlet flow exergy increases without any appreciable increase in work done for separation in DCMD, which results into increase in \( \eta \) for increase in \( Re_s \).

Further, effect of packing angle \( \delta \) on \( \zeta \) and \( \eta \) for changes in \( Re_s \) is described from figs. 8a and e. As discussed in Section 3.3, \( \zeta \) increases for increase in \( \delta \). Typically, it is seen from Fig. 8a, for a given \( Re_s \), \( \zeta \) is \( \sim \)2.5 times higher for \( \delta = 30^\circ \) and \( \sim \)5 times higher for \( \delta = 45^\circ \) over \( \delta = 15^\circ \). Similarly, from Fig.8e, it is observed that \( \eta \) is \( \sim \)3 percent points lower for \( \delta = 30^\circ \) and \( \sim \)9 percent points lower for \( \delta = 30^\circ \), than for \( \delta = 15^\circ \). As discussed in Section 3.3, recovery ratio \( (\zeta) \), increases steadily for change in \( \psi \) from 0.15 to 0.45 (figs. 8a-d). Similar competing trend is observed for exergetic efficiency where, \( \eta \) increases steadily for change in volume fractions from 0.15 to 0.45 as illustrated in Figs. 8e-h. Recovery ratios are 4 to 8 times higher for \( \delta = 45^\circ \) against \( \delta = 15^\circ \) (Fig. 8a-d). The minimum exergetic efficiency for variation in saline Reynolds number for rectangular packing is observed for \( \delta = 45^\circ \), \( \psi = 0.6 \) and \( Re_s = 375 \) at 65%. Overall it is seen that \( \delta = 45^\circ \) has the highest recovery ratio among \( \delta = 15^\circ \), \( 30^\circ \) and \( 45^\circ \) for \( Re_s = 300 \) and \( \psi = 0.6 \) (Fig. 8d) although it shows the lowest exergetic efficiency (Fig. 8e) which comes out at 0.40 \%. 

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Figure 9. Effect of permeate inlet Reynolds number, $Re_s$, variation on recovery ratio, $\zeta$, exergetic efficiency, $\eta$, (a) (b) (c) (d) and (d) (e) (f) (g) respectively, for $Re_p = 375, D^* = 0.0125, \delta_m^* = 0.00125, K^* = 0.330, Ja = 0.029$ and $\epsilon = 0.85$ under staggered packing.

Similar trends are observed in the results obtained for staggered packing (fig 9). Heat transfer enhancement and mixing characteristics for staggered packing as described in Section 3.2 and 3.3, result into lower recovery ratios and higher exergetic efficiencies for change in $Re_s$ when compared to the rectangular packing. The range of $\eta$ obtained from staggered packing are 1-5% higher than the rectangular packing for the same set of parameters, whereas the $\zeta$ is about 1-8% lower. Maximum exergetic efficiency and maximum recovery ratio follow a similar trend in staggered packing where $\eta$ maxima is seen at the maximum saline Reynolds number considered, $Re_s = 1425$ and $\zeta$ maxima is seen at the minimum saline Reynolds number considered, $Re_s = 300$ for $\delta = 15^\circ, \psi = 0.15$ and $\delta = 45^\circ, \psi = 0.6$, respectively.

Comparing Figs.6-9, it can be said that enhancement in the overall heat transfer due to $Re_s$ is more prominent over $Re_p$ and has a more significant impact on the recovery ratio, $\zeta$ of DCMD.
3.5. Effect of Jakob Number on Exergetic Efficiency and Recovery Ratio

Figure 10. Effect of permeate saline inlet Jakob number, $Ja$, variation on recovery ratio, $\zeta$, exergetic efficiency, $\eta$, (a) (b) (c) (d) and (d) (e) (f) (g) respectively, for $Re_s = 575$, $Re_p = 300$, $D^* = 0.0125$, $\delta_m^* = 0.00125$, $K^* = 0.330$, $Ja = 0.029$ and $\epsilon = 0.85$ under rectangular packing

Figures 10 and 11 show the effect of saline inlet Jakob number on exergetic efficiency $\eta$ and recovery ratio $\zeta$ for rectangular and staggered packing, respectively. As mentioned earlier, the trends are reported only for variation in the saline Jakob number and not the permeate Jakob number. There is an obvious exergetic penalty associated with increasing the inlet stream temperature on the permeate stream, as well as it contributes to lower trans-membrane temperature differences. A counter argument can be made against this ideology which presents a thought that lowering the temperature on the permeate side might help in increasing the trans-membrane temperature differences thereby increasing the overall recoveries, however, to this, the amount of energy required to bring down the temperature of the inlet permeate stream below the dead state temperature would play a major role. Thus, for the purpose of this study the focus is narrowed down on only the saline Jakob number.
From Figs.10a and e, it can be observed that recovery ratio $\zeta$ and exergetic efficiency $\eta$ vary non-linearly with increasing Jakob number. The variation in $\zeta$ is attributed to the exponential behavior of vapor pressure as a function of local temperature given by the Antoine’s equation, Eq. (7). Local vapor pressure on the feed side increases with increase in saline inlet Jakob number. Thus, an overall higher trans-membrane vapor pressure difference is obtained which contributes to the increase in recovery ratio with increase in Jakob number. Contrary to the trends obtained for $\zeta$, it is observed that $\eta$ increases rapidly for increase in Jakob number from 0.01 to 0.05 after which is asymptotes to a constant value with further increase in $Ja_s$. This can be accounted to the efficient heat transfer taking place at higher temperatures as the Jakob number is increased. Thus, the entropy generation reduces relatively as we move towards higher Jakob numbers within the DCMD module. However, for increase in Jakob number beyond 0.05, increased overall conduction heat transfer through the membrane counteracts the enhanced heat transfer performance with elevation in the temperature of the interacting stream. Hence, overall exergetic efficiency of the DCMD module saturates for $Ja_s > 0.05$, as observed in figs. 10e-h.

![Figure 11](image)

Figure 11. Effect of permeate saline inlet Jakob number, $Ja$, variation on recovery ratio, $\zeta$, exergetic efficiency, $\eta$, (a) (b) (c) (d) and (d) (e) (f) (g) respectively, for $Re_s = 575$, $Re_p =$
300, \( D^* = 0.0125, \delta^*_m = 0.00125, K^* = 0.330, Ja = 0.029 \) and \( \epsilon = 0.85 \) under staggered packing.

From figs. 10,11a-e, for a given volume fraction \( \psi \), it is observed that values of \( \zeta \) are higher for higher packing angles and values of \( \eta \) are lower for higher packing angles as explained in Sections 3.2, 3.3 and 3.4. In addition, as the volume fraction is increased from 0.15 to 0.6, maximum \( \zeta \) progressively increases from 0.16 to 0.7 for rectangular packing and from 0.14 to 0.68 for staggered packing as shown in figs.10a-d and Figs.11a-d, respectively. For \( \delta = 15^\circ \), the peak exergetic efficiency drops from around 97% to 90% in rectangular packing (Figs. 10e-h) and drops from around 97% to 94% in staggered packing (figs. 11e-h) with increase in volume fraction from 0.15 to 0.6. For all volume fractions, the maximum \( \zeta \) is \( \sim 3 \) times higher for \( \delta = 45^\circ \) over \( \delta = 15^\circ \) and maximum \( \eta \) is about 5% lower.

Overall it is observed that for the highest recovery as well as efficiency, the DCMD module needs to be operated to as high temperature as possible. In the present study, the maximum absolute temperature of operation was maintained at 363K.
3.6. Effect of Membrane Properties on Exergetic Efficiency and Recovery Ratio

Figure 12. Effect of membrane property variation on recovery ratio, $\zeta$, exergetic efficiency, $\eta$, (a) (b) (c) (d) and (d) (e) (f) (g) respectively for $Re_s = 575, Re_p = 300, Ja = 0.029$ under staggered and rectangular packing.

Figure 12 shows the dependency of recovery ratio $\zeta$ and exergetic efficiency $\eta$ on non-dimensional membrane fiber diameter $d^*$, non-dimensional membrane thickness $\delta_m^*$, non-dimensional membrane thermal conductivity $K^*$, and membrane porosity $\epsilon$. Membrane properties play an important role in the behavior of the DCMD system since they directly influence the transport resistance, thermal resistance and effective surface area for the diffusion phenomenon. It is seen from Figs.12a-d that rectangular and staggered arrangements have lesser influence on $\zeta$ for different membrane characteristics. For non-dimensional fiber diameter, $d^*$, the $\eta$ maxima obtained for rectangular packing and staggered packing is different which can be attributed to the shell side heat transfer differences due to fiber packing. For variation in $K_m^*$, $\epsilon$ and $\delta_m^*$, the variation in $\eta$ seems to have no appreciable effect of arrangement.

Referring to Figs.12a and e, it is seen that $\zeta$ increases gradually for increase in the $\delta_m^*$ from 2e-04 to 9e-04. For this change in $\delta_m^*$, increase in thermal resistance reduces conductive heat loss, increases trans-membrane temperature difference. Thus, we see an increase in $\eta$ and $\zeta$. However,
further increase in $\delta_m$ beyond 9e-04 shows a non-linear drop in $\zeta$ and $\eta$ alike. It shows the effect membrane thickness has on the transport resistance. For higher membrane thicknesses, higher transport resistance becomes the limiting parameter for the performance.

$D^*$ plays an important role in the DCMD design process. It influences drag losses, conduction losses as well as the recoveries owing to the surface area changes it induces on the membrane interfaces. It is observed from figs 12b and f, that for smaller $D^*$, the recovery ratio is the highest. Increase in $D^*$ for any given $\psi$ increases overall package size. Thus, flow rates on saline as well as permeate side increase for constant $Re_s$ and $Re_p$. It is also accompanied by a corresponding rise in membrane-surface area and membrane volume. All of these factors contribute towards increased heat conduction losses and increased mass transfer losses. Thus it can be said that the value of $D^*$ should be as low as possible to facilitate higher recovery and better utilization of inlet flow exergies.

Higher thermal conductivity results in higher parasitic heat transfer losses. However, it is observed from figs. 12.c and g, that overall performance in the DCMD system is affected minimally due to changes in $K^*$. $\eta$ drops by <1% and $\zeta$ drops by <0.02% for increase in $K^*$ 4 times from 0.15 to 0.6. Overall, it can be seen that the recovery ratio and exergetic efficiency both drop with increase in thermal conductivity due to enhanced conduction heat transfer. Thus, lower thermal conductivity membranes are desired.

Membrane transport resistance is another limiting parameter for mass transfer. Highly porous membrane shows lower mass transfer resistances as well as lower heat transfer losses due to lower overall membrane thermal conductivity. The trends shown in figs.12d and h, are found to be in agreement with the general observation. The $\zeta$ and $\eta$ linearly increase for increase in $\epsilon$ for
rectangular packing whereas staggered packing shows non-linear variation. It can be concluded that for the best operation, a highly porous membrane with $\epsilon > 0.80$ should be selected.

3.7. Design Constraints and Example Calculations for a Rated System

The parametric studies yield some broad design constraints on the studied parameters. It has been observed that recovery ratio and exergetic efficiency show opposing trends except for $Ja, K^*$ and $\epsilon$. Thus, there lies a trade-off between the input parameters, geometric constraints and properties of the membrane as discussed in the previous sections. Thus, by imposing bounds on the $\eta$ as 50%, 65%, 80% and 95%, corresponding values of the studied parameters are obtained and are listed in Table 2. Overall it is seen that recovery ratio drops with increase in exergetic efficiency. Also it can be noted that the packing becomes less even for stricter constraints, the design has to move towards lower volume fractions and packing angles.

Table 2. Design Constraints for Select Exergetic Efficiency Bounds

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Bound on Exergetic Efficiency, $\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>50</td>
</tr>
<tr>
<td>Arrangement</td>
<td>Rectangular</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.6</td>
</tr>
<tr>
<td>$\delta$</td>
<td>45</td>
</tr>
<tr>
<td>$Re_p$</td>
<td>300</td>
</tr>
<tr>
<td>$Re_s$</td>
<td>575</td>
</tr>
<tr>
<td>$Ja$</td>
<td>0.065</td>
</tr>
<tr>
<td>$d^*$</td>
<td>2.20E-03</td>
</tr>
<tr>
<td>$\delta_m$</td>
<td>0.00125</td>
</tr>
<tr>
<td>$K^*$</td>
<td>0.437</td>
</tr>
</tbody>
</table>
To show the usability of the constraint windows, an example calculation of the plant design is shown in Table 3. A DCMD module for 10,000 L/day capacity is to be designed. Thus, selecting the overall best case scenario, we conclude that rectangular packing with $\psi = 0.6$ and $\delta = 45^\circ$ needs to be selected. Table 3 lists values of parameters corresponding to this case which yields a recovery ratio of 0.62%. Correspondingly, back calculating the saline water requirement at the feed, we require $2,000,000 L/day$ with a design factor of 1.25. Number of tubes required are calculated to be 1 billion. These calculations place the overall DCMD module size to be around, $3300 m^3$.

Table 3. Example calculation of a 10,000 L/day capacity desalination plant

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Parameter Value</th>
<th>Analogous Dimensional Form</th>
<th>Actual Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arrangement</td>
<td>Rectangular</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d^*$</td>
<td>1.25E-02</td>
<td>Fiber Diameter</td>
<td>2.50</td>
<td>mm</td>
</tr>
<tr>
<td>$\delta_m$</td>
<td>0.00125</td>
<td>Membrane Thickness</td>
<td>250</td>
<td>$\mu m$</td>
</tr>
<tr>
<td>$K^*$</td>
<td>0.437</td>
<td>Thermal Conductivity</td>
<td>0.2622</td>
<td>$W/mK$</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>0.85</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta$</td>
<td>45</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Re_p$</td>
<td>300</td>
<td>Permeate Inlet Velocity</td>
<td>1.20E-01</td>
<td>m/s</td>
</tr>
<tr>
<td>$Re_s$</td>
<td>575</td>
<td>Saline Inlet Velocity</td>
<td>2.30E-01</td>
<td>m/s</td>
</tr>
<tr>
<td>$Ja$</td>
<td>0.065</td>
<td>Saline Feed Temperature</td>
<td>333.0382135</td>
<td>$K$</td>
</tr>
</tbody>
</table>
4. Conclusion

The study highlights importance of geometrical constraints on the performance of DCMD system with hollow fiber membranes with saline water on the shell side and permeate on the inside of the fibers. It is observed from the studies that system performs better for higher volume fractions and evenly spaced fibers. Thus overall it is seen that rectangular packing with $\delta = 45^\circ$ and $\psi = 0.6$ performs the best with the maximum recovery ratio observed in the system $\sim 1.2\%$. Further studies are underway to perform constrained optimization of the DCMD system.

For operational parameter variations, the general conclusion visible from the present study suggests that the stream inlet Reynolds numbers on both, saline and permeate sides, should be lower and in order to maximize the recovery. To seek a balance between recovery and exergetic efficiency, the saline Reynolds number should be in the range $500 – 800$ which shows exergetic efficiency of at least $65\%$ with reasonable recoveries. Also, it can be concluded that $Re_s$ has a more significant impact on the overall performance than $Re_p$. Regardless of packing arrangement it was observed that higher saline inlet temperatures show better recoveries and exergetic efficiencies.

Selection of membrane is one of the important aspects of DCMD design process. The non-dimensional membrane thickness should be between $4 – 12e-04$. The fiber diameter is an important parameter since it heavily influences drag losses in the system, thus for better recoveries by maintaining acceptable recovery ratios, the non-dimensional fiber diameter should be in the range $4 – 8e-03$. Membrane conductivity was found to have a little effect on the exergetic efficiency for the practical range studied. The ideal value of thermal conductivity was observed to be close to the conductivity of standard PTFE membranes. It has also been found that recovery ratio and
exergetic efficiency max out for higher values of porosities. Thus, highly porous nature in the membrane is to be desired while selecting the membranes.

It is to be noted that, though the module efficiency falls in the range of 50-80%, it only accounts for fraction of the total exergy destruction in the overall system. Thus, more studies are underway to examine the overall exergetic performance of the MD system comprising of DCMD as well as other MD technologies to compare the performance. It can be further concluded that MD system in general are as much sensitive to their arrangement and overall design as they are to operational parameters. Thus, just by carefully arranging the membranes, the recovery ratios can be greatly improved.

**Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u )</td>
<td>Velocity in ( x ) direction</td>
<td>([m/s])</td>
</tr>
<tr>
<td>( v )</td>
<td>Velocity in ( y ) direction</td>
<td>([m/s])</td>
</tr>
<tr>
<td>( w )</td>
<td>Velocity in ( z ) direction</td>
<td>([m/s])</td>
</tr>
<tr>
<td>( L )</td>
<td>Length</td>
<td>([m])</td>
</tr>
<tr>
<td>( p )</td>
<td>Pressure</td>
<td>([Pa])</td>
</tr>
<tr>
<td>( T )</td>
<td>Temperature</td>
<td>([K])</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Density</td>
<td>([kg/m^3])</td>
</tr>
<tr>
<td>( \nu )</td>
<td>Kinematic Viscosity</td>
<td>([m^2/s])</td>
</tr>
<tr>
<td>( C_p )</td>
<td>Specific Heat Capacity</td>
<td>([J/kg - K])</td>
</tr>
<tr>
<td>( K_w )</td>
<td>Thermal Conductivity of Water</td>
<td>([W/m - K])</td>
</tr>
<tr>
<td>( \Delta H_v )</td>
<td>Latent Heat of Vaporization</td>
<td>([J/kg])</td>
</tr>
<tr>
<td>( D )</td>
<td>Diffusivity</td>
<td>([m^2/s])</td>
</tr>
<tr>
<td>( q_m )</td>
<td>Diffusive Mass Flux</td>
<td>([kg/m^2s])</td>
</tr>
</tbody>
</table>
\( C \)  Concentration \([\text{mol/L}]\)

\( W_m \)  Molecular Weight of Water \([\text{kg/kmol}]\)

\( Re \)  Reynolds Number

\( Pr \)  Prandtl Number

\( Sc \)  Schmidt Number

\( Ja \)  Jakob Number

\( \theta \)  Non-Dimensional Temperature

\( D^* \)  Non Dimensional Fiber Inner Diameter

\( K^* \)  Non Dimensional Thermal Conductivity

\( \delta_m^* \)  Non Dimensional membrane Thickness

\( \epsilon \)  Membrane Porosity

**Subscripts and superscripts**

\( p \)  Entity in the permeate domain

\( s \)  Entity in the saline domain

\( m \)  Entity in the membrane domain

\( \ast \)  Non Dimensional Quantity

\( i \)  Specie (Permeate or Saline)
References


Chapter 3. Analysis of Ideal Waveguide for Seawater Desalination Application

Persistent droughts and increasing demand for clean water have posed an un-surmountable challenge for existing water technologies. Desalination is proving to be an effective solution for water production, recycling and reclamation. However, all desalination techniques are energy intensive, hence making these environmentally sustainable is a challenge to the desalination research community. In our previous efforts, we emphasized on minimizing specific energy consumption in a Direct Contact Membrane Distillation by exergy analysis. This study tries to provide a solution to the energy supply side in the form of low cost waveguide for energy collection. The analysis presented in this paper provides analytical closed form solution to light and heat transport through a flat plate waveguide. Results obtained for light transport are then used to calculate levelized cost of water (LCOW) when a DCMD distillation plant is coupled to an ideal waveguide. With the current estimates, the LCOW is calculated at $1.80/m^3$. With further improvements in waveguide based collectors, this approach shows a positive way forward towards meeting the cost of existing systems such as large scale reverse osmosis units.

Keywords: Waveguide, Direct Contact Membrane Distillation, Concentration Ratio, LCOW, Sustainable Water, Solar Desalination
1. Introduction

Sustainable water is emerging as one of the active discussion topics. Many parts of the globe face unavailability of drinking water for several months in a year [2]. Water desalination has been effectively implemented to combat water shortage throughout in arid regions, in regions of limited availability such as middle eastern countries. It has emerged as one of the popular choices to solve the water crisis. As of 2015, there are more than 18,400 desalination plants in operation in over 150 countries which serve over 4% of the total world population [41]. Increasing awareness about desalination and water reclamation in addition to scarcity of fresh water from in-land resources have laid researchers to project the desalination capacity doubling in another decade. Of the total installed desalination capacity, about 88% of the desalinated water comes from Reverse Osmosis (RO) and Multi-stage Flash Distillation (MSF) [42]. Reverse osmosis requires electrical energy and MSF requires electrical and thermal energy in the form of high temperature steam. Majority of current electrical energy demands are met from non-conventional sources, where renewable energies account for 19.1% as of 2013 and fossil fuels taking away the major chunk at 78.3% [43]. Steam requirements for the MSF plants are evened out by fossil fuel based power plants. Hence, the current desalination landscape is a non-sustainable approach and the cost of water, in addition to the water production cost, has a hidden layer of cost in terms of environmental damage.

Energy requirements of a desalination plant can be broadly classified into two categories, thermal and electrical. Multi Effect Distillation (MED), Multi-stage Flash (MSF), Membrane Distillation (MD) require significant portion of the total energy as thermal energy, whereas Reverse Osmosis (RO), Electro-dialysis (ED) primarily require electricity. Both thermal and electrical energy requirements can be met by using one or more renewable energy sources such as
wind, solar, geothermal. Wind energy and geothermal energy installations suffer from limited availability, and requirement of complex conversion mechanisms. As the world embraces Decade of Sustainable Energy for All, SE4ALL, initiative by the United Nations General Assembly, of all the renewable energy sources, in a span of 4 years between 2009 and 2014, solar energy related technologies showed the highest growth rate, exceeding 45% [43]. As per the current projections by KPMG, solar assisted electricity is about to become 10% cheaper than that of coal counterpart in parts of the world [44]. Hence, use of solar energy makes a compelling case for supporting desalination projects.

In a direction aligning with the vision presented earlier, several researchers have directed their efforts towards developing sustainable solar desalination techniques [45-48]. Some of the efforts have involved combining traditional solar energy collection methods such as solar stills, parabolic concentrators, solar ponds whereas others have used a more direct approach by innovating into integrated solar-thermal desalination modules [45, 47]. In a comprehensive review on desalination driven by renewables, Garcia-Rodriguez[49] and Li et al.[50] have summarized several seawater and brackish water desalination technologies powered by solar pond, parabolic collectors, evacuated tubes and flat plate collectors. They also reported that many of the pilot scale plants used Multi Stage Flashing (MSF) or Multi Effect Distillation (MED), and it was found that MED is more suitable for indirect solar water desalination methods owing to its immunity to scaling and corrosion. In a comparison of competing solar technologies, Li et al. have also shown that it is possible to build cheaper photovoltaic (PV) installations which require lesser overall maintenance as opposed to solar ponds[50]. Concentrating collectors needs more land, more intensive maintenance schedules and overall complex design and construction.
In a study on sustainable desalination, Gude et al. [51] have used a combination of direct solar energy and PV to power desalination. Cipollina et al. [52] have proposed a solar collector, thermal energy storage integrated membrane desalination unit and shown possible use for various membrane distillation (MD) techniques such as DCMD, air gap membrane distillation (AGMD), vacuum membrane distillation (VMD) etc. Use of thermal energy storage provides additional allowance of thermal energy to provide for hours of weak or unavailability of sunlight. Banat et al. [53] presented a solar still coupled membrane distillation under indoor and outdoor conditions. Al – Kharabshesh [54] and Goswami [50] have demonstrated an evaporator condenser based desalination which uses solar energy in the evaporator. The incident solar radiation is used to heat the water in the evaporator using a condenser. It also uses recirculation brine for heat recovery, which reduces total energy requirement for heating saline feed.

Walton et al. have demonstrated use of membrane desalination which uses waste heat sources and solar energy to generate the hot streams. Bouguecha [55] demonstrated solar still powered MED. The study also concluded that multi effect solar still was the least expensive of the three technologies considered, the other two being RO and MD powered by PV. Renewable energy powered desalination passes the feasibility test if the required number of components and integration times are less. Since reverse osmosis and electro-dialysis only require electricity, it is easier to couple them to a PV array for power. Because of its simplicity, PV powered RO units have gained a lot of attention. Several researchers have developed small scale PV-RO units [56]. However, based on the design, rated capacity and implementation, it comes out that specific energy consumption varies between 1.5 $kWh/m^3$ to 8 $kWh/m^3$. Similarly, Rodriguez has also reported pilot studies for PV electro-dialysis units for capacities under 10 $kWh/m^3$. 

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Commercialization of solar desalination requires a feasibility and cost analysis to make a compelling case. In order to be acceptable desalinated water needs to meet the current water prices offered by utilities. In addition to the cost of plant design, materials, construction and assembly, they also include insurance, construction overheads, process equipment, auxiliaries, discharge locations, environmental compliance, operation and maintenance, interest rates etc. From the studies presented in the review article by Eltawil et al.[57], price tag on RE desalinated water comes anywhere between $2/m^3 to $20/m^3. Several researchers have put cost of RE powered desalination by using aforementioned methods anywhere between $1/m^3 to $18/m^3 where MSF is at the lower end and MD at the other extreme [58]. Costs associated with MD suffer due to their generally low recovery ratios, poor exergetic performance, high losses in terms of parasitic heat transfer within system. Additionally, in a techno-economic analysis of solar desalination, Fiorenza et al.[59] have pointed out that cost of water depends heavily on the total initial investment. They have also done analysis of levelized cost for multi-effect desalination powered by solar-thermal energy and PV-RO and project a cost of about $2/m^3 which is approximately 2.5 times more than the national average. In another similar study, Kalogirou has reported cost of water exceeding $2/m^3 for parabolic trough collection method [60]. It can be seen that solar desalinated water costs have a significant dispersion depending on the desalination technology.

As noted above choice of desalination is critical to minimize the overall costs. For the analysis presented in this paper, DCMD is selected. DCMD has the simplest overall construction of all desalination systems in existence. It consists of flow chamber consisting of membranes separating saline water and permeate. The saline water is heated above the permeate circulation temperature. The temperature difference results into local vapor pressure difference across the hydrophobic membrane. This vapor pressure difference drives water vapor across the membrane.
from saline water to permeate, where it condenses as pure water. Overall, MD has the purest yield as compared to other desalination techniques. It can be built at relatively low cost, since it does not require any pressurization to operate. However, it requires relatively large energy as thermal energy input. Required thermal energy requirement can be met from solar thermal sources or any waste heat energy sources. Thus, DCMD makes a compelling case for an overall inexpensive desalination construction which can help in bringing down LCOW.

It is understood that the LCOW depends on construction cost of desalination plant as well as the cost of RE installation. Cost of RE installation can be minimized by innovating inexpensive collection methods. The existing systems such as parabolic troughs have inherent limitations due to directional dependence. These limitations force them to use expensive auxiliaries for tracking. Other technologies such as solar ponds are dependent on heat transfer and lose major chunk of the total collected energy to atmosphere. Although recent trends show PV producing electricity at cheaper costs, PV modules have a bound on the quantum efficiency. Thus, a better method to generate thermal energy to support DCMD demand is required to make it sustainable.

![One dimensional waveguide based solar collector schematic](image)

**Figure 13. One dimensional waveguide based solar collector schematic**

Our focus in this article is to propose an ideal solar collection device as shown in Fig. 13. It illustrates construction of a waveguide solar collector. The ideal version of the waveguide assumes perfect total internal reflection and no escape cone losses. Objective of present discussion
is to propose an analytical closed form solution to a flat plate waveguide. The solution obtained from the mathematical formulation will help in calculating overall radiation reaching the receivers, also referred to as, direct radiation ‘\( I_r \)’. Direct radiation is the primary energy source for the heat transfer fluid (HTF) in the receiver. In this case water is selected as the HTF, as waveguide receiver assembly is directly coupled to a DCMD unit. Hence, the proposed system forms an ideal waveguide coupled desalination plant. The study provides useful insights into waveguide field size requirements for a given desalination capacity. It also provides directions for waveguide design based on recommended continuous maximum temperature of operation, \( T_{max} \) which is limited by the material of choice, and ambient conditions, \( T_{ao} \). The ideal model serves as a good use case to compare future non-imaging optics based solar collection methods. This study presents mathematical model in section 2, results obtained from the closed form solutions will be discussed in section 3. In the later part of the section 3, discussion will focus on the levelized cost of water (LCOW) and comparing it with the existing LCOWs.

Table 4. Properties of Optical Materials Used

<table>
<thead>
<tr>
<th>Material</th>
<th>Thermal Conductivity, ( k )</th>
<th>Absorptivity, ( \alpha )[m-1]</th>
<th>Continuous Maximum Temperature, ( T_{max} )[K]</th>
</tr>
</thead>
<tbody>
<tr>
<td>BK 7</td>
<td>1.11</td>
<td>0.21</td>
<td>422</td>
</tr>
<tr>
<td>Acrylic</td>
<td>0.2</td>
<td>0.5</td>
<td>360</td>
</tr>
</tbody>
</table>
2. Mathematical Model

Figure 13 shows the schematic of an ideal waveguide. Under one dimensional assumption, light absorption is a function of length only. Light propagation is assumed to be along ‘x’ direction only. Total length is denoted by ‘L’, waveguide thickness is denoted by ‘t’ and width by ‘w’, which is perpendicular to the plane of view. A uniform solar irradiation is considered for the analysis of value ‘i₀’. Variation in i₀ is considered explicitly to understand the effect of diurnal variation. Waveguide is assumed to have perfect total internal reflection (TIR) characteristics. Hence, once the radiation is trapped inside, it can either get absorbed by the material or travel towards the receivers by TIR.

The total energy transfer from waveguide to the receiver can be classified into two modes, thermal energy and direct radiation energy. Heat transfer is a function of surrounding conditions, material properties and geometrical parameters whereas direct radiation is a function of material properties and geometrical parameters only. The mathematical model is subdivided into two sub-parts, first part focuses on calculating direct radiation reaching to receivers on either sides, I_L and the second part provides a solution for the 1-D heat conduction problem for 2 boundary conditions. After successfully obtaining closed form solutions, further expression is derived for operation at T_{max} as a limiting case for conservative design windows. The limiting case is then used to obtain design parameters for LCOW estimation.

2.1. Concentration ratio

Under ideal circumstances, for perfectly transparent material showing complete lossless TIR, all of the incident radiation should reach the receivers. To quantify the direct radiation in more general terms, term concentration ratio, ‘CR’ is used. CR is calculated as the ratio of I_L to I₀. Further extending the discussion on lossless waveguide, the highest value CR can attain is the ratio
of waveguide length $L$ to thickness $t$. If TIR experiences absorption losses, $CR$ evaluated will be lower than the geometrical maxima of $CR$.

To evaluate direct radiation at either ends of the waveguide, consider an element of width ‘$dx$’ on surface. Hence, energy entering at any location ‘$x$’ can be written as,

$$e = I_0(W \cdot dx)$$

To cover the entire envelope of incident angles, the incoming energy is said to be uniformly distributed over angles $-\pi/2$ to $\pi/2$. Energy entering at any angle $d\theta$ can be written as,

$$e_{d\theta} = I_0(W \cdot dx) \left(\frac{d\theta}{\pi}\right)$$

Integrating $e_{d\theta}$ over the entire envelope should yield total radiation entering the waveguide.

Now focusing the analysis on the radiation inside the waveguide, as noted above, energy absorbed by the material is a function of path traveled by the incoming energy bundle. For energy transfer in positive $x$ direction, total path traveled, ‘$l_{+x}$’, before the energy bundle reaches the receivers can be calculated as,

$$l_{+x} = (L - x)/\sin\theta$$

Energy transfer in the waveguide material is modeled using exponential decay function of ‘$l_{+x}$’ and absorptivity of waveguide ‘$\alpha$’. Hence, direct radiation reaching either of the receivers can be calculated as,

$$I_L = \frac{I_0}{\pi t} \int_0^{\pi/2} \int_0^l I_0(W \cdot dx)e^{-\alpha(l-x)/\sin\theta}\left(\frac{d\theta}{\pi}\right)$$

Integrating over length, the expression of ‘$I_L$’ works out to,

$$I_L = \frac{I_0}{\pi t} \int_0^{\pi/2} \frac{\sin\theta}{\alpha} \left(1 - e^{-\alpha l/\sin\theta}\right) \cdot d\theta$$ (1)
2.2. Heat transfer in waveguide

Heat transfer in waveguide is conjugate to the direct radiation solved in section 2.1.

Establishing energy balance between $i_0$ and $i_L$ yields volumetric heat generation in the waveguide. Figure 14 shows energy flows in an ideal waveguide. It is assumed that heat generated due to absorption of radiation is uniformly generated in the waveguide volume. Volumetric generation term, $g'''$, is calculated as,

$$g''' = \frac{i_0}{t} - \frac{2i_0}{\pi Lt} \int_0^{\pi/2} \frac{\sin \theta}{\alpha} \left(1 - e^{-\alpha L/\sin \theta}\right) \cdot d\theta$$

1-D heat transfer equation in terms of temperature with volumetric generation can be written as,

$$k \frac{d^2T}{dx^2} - \frac{h}{t} (T - T_\infty) + g''' = 0$$

Let the non-dimensional temperature, $\bar{T}$, be $\frac{T - T_\infty}{T^*}$, where $T^* = \frac{g'''L}{k}$. Let the non-dimensional waveguide length be $\bar{x} = \frac{x}{L}$ and heat transfer constant be $m^2 = \frac{hL^2}{kt}$ which captures
combined effect of convection loss to atmosphere and conduction through material. Hence, Eq. (2) can be reiterated in terms of non-dimensional entities as,

\[
\frac{d^2 T}{d\bar{x}^2} - m^2 T + 1 = 0
\]  

(3)

Solution to Eq. (3) is obtained as a combination of general solution and particular solution, and it is expressed as,

\[
T = c_1 e^{m\bar{x}} + c_2 e^{-m\bar{x}} + \frac{1}{m^2}
\]  

(4)

Values of coefficients \(c_1\) and \(c_2\) in Eq. (4) are boundary condition specific, which in this case, depend on the receiver design.

Now consider a receiver where heat transfer fluid maintains constant temperature equal to the surrounding temperature \(T_\infty\). Hence the boundary condition in terms of non-dimensional temperature is written as, \(\bar{x} = 0, \bar{T} = 0\) and \(\bar{x} = 1, \bar{T} = 0\). Now solving Eq. (4) with the boundary condition to obtain \(c_1\) and \(c_2\), we get,

\[
T = \frac{1}{m^2} \left\{ 1 - \frac{\cosh m(\bar{x} - 1/2)}{\cosh(m/2)} \right\}
\]  

(5)

In another receiver design where heat transfer fluid takes out a constant heat flux, the boundary condition can be composed as, \(\bar{x} = 0, \frac{dT}{d\bar{x}} = -Nu[T_s - T_R]\) and \(\bar{x} = 1, \frac{dT}{d\bar{x}} = 0\). Where the local non-dimensional temperature at waveguide receiver ends is denoted by \(\bar{T}_s\), bulk temperature of HTF is denoted by \(\bar{T}_R\) and \(Nu\) denotes the Nusselt number. Solving Eq. (4) to obtain \(c_1\) and \(c_2\) for the pertinent boundary condition, we can write the solution for heat flux boundary condition as,

\[
T = \frac{Nu}{m} \left[ T_s - T_R \right] \left\{ \frac{\cosh \left( m(\bar{x} - \frac{1}{2}) \right)}{\sinh \left( \frac{m}{2} \right)} \right\} + \frac{1}{m^2}
\]  

(6)
2.3. Waveguide design

As discussed in sections 2.1 and 2.2, waveguide physical dimensions, material properties as well as surrounding conditions play an important role in both, heat conduction and direct radiation. Absorption of incident radiation leads to rise in waveguide temperature. Some of the absorbed energy is lost to atmosphere via natural convection and remaining part of energy travels towards the receiver. If the waveguide fails to reject heat to receiver or to the atmosphere and starts heating up towards the maximum operational temperature, it will start failing structurally. In addition to structural rigidity, optical properties are also severely affected with rise in temperature. In order to establish a design window on the waveguide dimensions, limiting case of $T_{max}$ provides a useful criteria.

From Eq. (5) and (6), it can be observed that, maximum temperature is attained at the midpoint of the waveguide. From the two cases represented by Eq. (5) and (6), the case with heat transfer to receiver will show lesser temperatures than the constant temperature boundary condition solution for receiver temperatures lower or equal to the ambient temperature. Therefore, Eq. (5) represents a more conservative estimation of waveguide absolute temperatures. In terms of $T_{max}$ and waveguide dimensions, Eq. (5) can be rewritten as,

$$\frac{h(T_{max} - T_\infty)}{I_0 \left[1 - \frac{2}{\pi} \int_0^{\pi/2} \frac{1 - e^{-L^2}}{L^2} \cdot d\theta\right]} = 1 - \operatorname{sech} \left(\frac{L}{\sqrt{2kt}}\right)$$

Solving Eq. (7) for a range of $h$ and material properties, design constraints for $t$ and $L$ can be obtained. To consider limiting case scenario from material safety standpoint, the highest temperature rise will be obtained for a case of lowest surface heat transfer coefficient. The analysis presented in section 3 is performed by keeping all other parameters constant except the independent variable and the holding variable. Default parameters is given under Table 5.
Table 5. Default Variable Values

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(L)</td>
<td>1 m</td>
</tr>
<tr>
<td>(t)</td>
<td>2 cm</td>
</tr>
<tr>
<td>(i_0)</td>
<td>1000 (W/m^2)</td>
</tr>
<tr>
<td>(h)</td>
<td>3 (W/m^2 - K)</td>
</tr>
<tr>
<td>(k)</td>
<td>1.1 (W/m - K)</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>0.2037 (m^{-1})</td>
</tr>
<tr>
<td>(T_\infty)</td>
<td>298 K</td>
</tr>
</tbody>
</table>

3. Results and Discussion

3.1. Effect of waveguide parameters on \(CR\)

Figure 15. Effect of waveguide parameters on waveguide temperatures for (a) material absorptivity, \(\alpha\) (b) surface heat transfer coefficient, \(h\) (c) material thermal conductivity, \(k\) (d) waveguide thickness, \(t\) (e) waveguide length, \(L\) (f) incident radiation intensity \(i_0\) for constant temperature boundary condition

Figure 15 shows effect of waveguide parameters on \(CR\) where sub-figures (a) shows dependency on \(L\), (b) shows dependency on \(t\), and (c) shows dependency on \(\alpha\). Overall it can be seen that \(CR\) increases for increase in \(L\), reduces for increase in \(t\) and reduces for increase in \(\alpha\). It
is seen from Fig. 15a that $CR$ increases non-linearly with increase in $L$ for constant $t$ and $\alpha$. This can be explained as a combined phenomenon of increased overall collection area and increase absorption within waveguide. The initial rapid increase in $CR$ for $L = 1 \text{ m}$ to $10 \text{ m}$, is due to the effect of increased collection surface area. Beyond $10 \text{ m}$ length it can be seen that increase in $CR$ becomes less prominent due to increased absorption owing to increased overall path length. Hence, for a reasonable performance, length of the waveguide needs to be maintained less than $10 \text{ m}$. In theory, shorter waveguides have better $CR$ for the given size than the bigger waveguides.

From Fig. 15b it can be seen that with increased $t$, $CR$ gradually drops. It can be accounted to increased cross-section area which spreads the traveling light flux over larger area for a given $L$ and $\alpha$. Increase in $t$ leads to increased overall mass of waveguide, material requirement and thereby cost. Hence, waveguides should have as low value of $t$ as possible, preferably smaller than $1.5 \text{ cm}$. Additionally, it is difficult to fabricate optically transparent panels with uniform optical properties for larger thicknesses, hence, it provides an additional benefit. Similarly, Fig. 15c shows dependency of $CR$ on material absorptivity, $\alpha$. Value of $\alpha$ in reality depends on coefficient of extinction and wavelength. However, for average wavelength and coefficient of extinction Fig.3c shows waveguide behavior over a spectrum of $\alpha$. Very high values of $\alpha$ can be thought of as scaled waveguide surfaces or yellowing of acrylic panels under aging. Hence, it captures the effect of surface level damage to optically transparent panels. It can be seen that $CR$ drops significantly for increase in $\alpha$. Absorption in the waveguide material increases exponentially for increase in $\alpha$, which implies, for ideal operations waveguide material needs to have as low value of $\alpha$ as possible, in addition proper cleaning routines and maintenance cycles must ensure consistent value of $\alpha$ in order to protect the waveguide performance from deteriorating.
3.2. Effect of waveguide parameters on heat transfer

Figure 16. Effect of waveguide parameters on waveguide temperatures for (a) material absorbptivity, $\alpha$ (b) surface heat transfer coefficient, $h$ (c) material thermal conductivity, $k$ (d) waveguide thickness, $t$ (e) waveguide length, $L$ (f) incident radiation intensity $i_0$ for heat flux boundary condition

Figure 16 and 17 show solutions to Eq. (5) and Eq. (6) for waveguide parameters. Subfigures in each indicate effect of absorptivity, $\alpha$ in subfigure a, effect of surface heat transfer coefficient, $h$ in b, material thermal conductivity, $k$ in c, effect of thickness, $t$ in d, effect of length, $L$ in e and effect of incident radiation, $i_0$ in subfigure f. Overall it can be seen that the maximum temperatures are achieved at the center of the waveguide. In general, maximum temperature increases with increase in $\alpha$, $L$, $i_0$ and increases with decrease in $h$, $k$, $t$. Figure 4a shows variation in waveguide temperatures with increasing $\alpha$ from $0.001 \text{ m}^{-1}$ to $100 \text{ m}^{-1}$. As described in section 3.1, absorption in the waveguide is a non-linear function of $\alpha$. Increase in $\alpha$ is accompanied by a non-linear absorption in the waveguide. It is followed by a non-linear rise in the volumetric heat generation, $g''''$. Hence, for increase in $\alpha$ from 0.1 to 1 m-1, the maximum temperature increases
by 64%, for increase in $\alpha$ from 1 to 10 m$^{-1}$, the increase is by 39% and the rise is limited to \( \sim 6\% \) for increase in $\alpha$ from 10 to 100. Low increase in temperature for raising $\alpha$ beyond 10 indicates that most part of the radiation was already absorbed in the waveguide, and increasing $\alpha$ beyond that seems to have little additional effect on maximum temperature. For the application where maximum energy needs to be transported across the waveguide in the form of direct radiation, $\alpha$ needs to be as low as possible.

Figure 16b shows the dependency of $T_{max}$ on heat transfer coefficient $h$. It shows the heat lost to atmosphere by the waveguide collection surface. As a rule of thumb, heat lost to atmosphere will be a waste of collected energy. Hence it needs to be as low as possible. However, if the receiver suffers from technical faults, and waveguide fails to lose the absorbed energy, it may lead to structural failures. Hence, analysis of $h$ proves to be useful for waveguide design. It can be seen from Fig. 16b that maximum temperature reduces for increasing $h$. Since $h$ appears in the governing equations under a square root, variation in $T_{max}$ is non-linear w.r.t. $h$. For increase in $h$ from 3 to 6 W/m$^2$ – $K$, maximum temperature drops by \( \sim 10\% \), similar for increase from 6 to 9 W/m$^2$ – $K$, it drops by 3%. Beyond 9 W/m$^2$ – $K$, effect of $h$ on the $T_{max}$ becomes less prominent. Under operational cases heat transfer coefficient beyond 9 represents a case of blowing winds.

Effect of material thermal conductivity is shown in Fig. 16c. Optically transparent materials usually also have low thermal conductivities. Low thermal conductivity results into lower heat conduction and under the case where the material has a generation term, the temperature of the material increases even more prominently as against for a material which can conduct the heat more efficiently. From Fig. 16c it can be seen that $T_{max}$ increases with increase in $k$. Since $k$ appears in the solution to the conduction equation under square root, the dependency of maximum
temperature on $k$ is non-linear. For increase in $k$ from 1 to 5 $W/m-K$, maximum temperature drops by $\sim 2\%$ and when increased from 10 to 15 $W/m-K$, it drops by 1.5% and the decrease becomes less prominent for further increase in material conductivity.

Thickness of waveguide contributes in the overall thermal inertia or area of cross section for heat conduction. Effect it has on the temperature profiles is illustrated in Fig. 16d. It can be seen that as $t$ increases, maximum temperature drops, but the effect is negligible for the range of parameters considered. For 6 times increase in $t$ from 0.005 to 0.03, maximum temperature only drops by $\sim 3\%$. As explained in section 3.1, $L$ affects the overall absorption as well as collection. Effect of $L$ on the temperature profiles is explained in Fig. 16e. It can be seen that $T_{max}$ increases non-linearly for increasing length. Generation term has exponential form with $L$ in the exponent. Hence, heat generation in the waveguide varies non-linearly for increasing the length. It can be seen from Fig. 16e that maximum temperature increases with increase in $L$ since waveguide collects more total energy. However, increase in maximum temperature reduces for $L$ increased over 10 $m$. For increase in length from 1 $m$ to 5 $m$, $T_{max}$ increases from 400 K to almost 600 K. It is important to note here that, to achieve higher overall collection one can go for more number of waveguides with smaller sized panels to ensure material safety instead of going for lesser number of panels of smaller size.

Overall energy collected by the waveguide is directly proportional to the solar irradiation. Figure 16 f shows the effect of solar irradiation. It can be seen that maximum temperature within waveguide increases linearly with increase in $i_0$. Value of $i_0$ varies hourly, daily, annually with latitude and longitude, hence this analysis helps in material selection as well.
Figure 17. Effect of waveguide parameters on waveguide temperatures for (a) material absorptivity, \( \alpha \) (b) surface heat transfer coefficient, \( h \) (c) material thermal conductivity, \( k \) (d) waveguide thickness, \( t \) (e) waveguide length, \( L \) (f) incident radiation intensity \( i_0 \) for heat flux boundary condition

Results for the Eq. (5) and Eq. (6) are extremely close to each other. Plots shown in Fig. 5 show the case for heat flux boundary condition on the receiver. It is to be noted that discussion for Fig.16 applies equally for Fig.17. However, the maximum temperatures are about 1-2 K lower in all cases for Eq. (5) over Eq. (6) which can be explained from the type of boundary condition. By coupling the waveguide to receiver where the receiver takes out heat flux continuously, it helps in keeping the waveguides cooler. However, it can be seen that effect of receiver parameters do not have a significant effect on the waveguide \( T_{max} \) as it can be seen from Fig.16 and Fig.17.

### 3.3. Effect of limiting \( T_{max} \) on design constraints

From the discussion on temperature profiles in section 3.2, it can be seen that the \( T_{max} \) in the waveguide is sensitive to \( h \), \( \alpha \), \( L \) and \( t \). In order to maintain the optical properties over a reasonable period of time as well as to maintain the structural rigidity, it is important to maintain
the maximum temperature in the safe operational range specified by the manufacturer for the given material. For the analysis, we have selected to evaluate the effect of BK7 glass and Acrylic on the waveguide size.

![Waveguide thickness as a function of length for maximum constant operational temperature](image)

Figure 18. Waveguide thickness as a function of length for maximum constant operational temperature of (a) BK-7 glass (b) Acrylic respectively

Figure 18 shows results for waveguide dimensions obtained using the Eq. (7). Figure 6 (a) shows the sizing calculations for BK7 glass, and Fig. 6b shows sizing calculations for Acrylic. It is observed from Fig. 18 that for any given \( L \), thickness requirements increase with decrease in \( h \). Which can be explained from the required area of cross-section for effective heat transfer from the waveguide. It can be observed that thickness required in Acrylic panels is much larger than that in BK7 mainly due to two reasons: (i) BK7 has lower \( \alpha \) against Acrylic, (ii) BK7 has higher thermal conductivity over Acrylic. The properties of BK7 and Acrylic are listed in Table 1. For calculation of overall power output from a waveguide, a conservative design calculation methodology is used. Hence the results corresponding to the lowest surface heat transfer coefficient are used for design windows which are listed in Table 6. It is observed from the analysis that \( L \) remains \(~0.3\ m\) for increase in thickness from 0.5 to 3 \( cm\) for Acrylic waveguides.
Table 6. Waveguide Size for Continuous Maximum Temperature Operation

<table>
<thead>
<tr>
<th>Thickness, t [m]</th>
<th>Acrylic</th>
<th>BK7</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.3032</td>
<td>0.3709</td>
</tr>
<tr>
<td>0.015</td>
<td>0.3055</td>
<td>0.3984</td>
</tr>
<tr>
<td>0.02</td>
<td>0.3078</td>
<td>0.4255</td>
</tr>
<tr>
<td>0.025</td>
<td>0.31</td>
<td>0.4523</td>
</tr>
<tr>
<td>0.03</td>
<td>0.3123</td>
<td>0.4787</td>
</tr>
</tbody>
</table>

3.4. LCOW from waveguide-desalination standalone unit

Based on the design windows calculated in section 3.3, the panel size is fixed. For the calculation of LCOW calculations, the waveguide is operated as direct radiation fed thermal power plant. The receiver coatings are assumed to be operating at 100%, they convert incident radiation completely to thermal energy used for heating water. The contribution of heat transfer component is completely neglected. The DCMD plant is built to produce 30,000 $m^3/day$. It has been reported before that, specific energy consumption in DCMD units is one of the highest among all desalination plants, and it is in the range 876 to 1356 $kWh/m^3$. For the purpose of analysis, an average value of 1000 $kWh/m^3$ is selected[61].

Table 7. LCOW Analysis of Solar-DCMD standalone unit

<table>
<thead>
<tr>
<th>MD-Solar</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plant Capacity (m3/day)</td>
</tr>
<tr>
<td>Number of Years</td>
</tr>
<tr>
<td>Discount Rate (%)</td>
</tr>
<tr>
<td>LCOE ($/kWh)</td>
</tr>
</tbody>
</table>
Carbon Tax ($/ton) 23
Emission factor for electricity (kgCO2-e/kWh) 1.22
Emission factor for natural gas (kgCO2-e/kWh) 0.184
Membrane Cost ($/m2) 12
Replacement Rate (1/y) 0.2

Annual Production Capacity (m3/yr.) 9855000
Ref Plant Capacity (m3/day) 24000
Ref Plant Unit Capital Cost ($/m3-day) 1131
Waveguide Cost ($) 201727941
Capital Cost ($) 234176981
Electricity to Transport Water (kWh) 1.2
Electricity for operation (kWh) 0
Thermal Energy for operation (kWh/m3) 1000
Normalized Capital Cost ($/m3) 1.55
Cost of Electricity ($/m3) 0.108
Brine Disposal Cost ($/m3) 0.0015
Membrane Replacement Cost ($/m3) 0.046
Pretreatment Cost ($/m3) 0.019
Labor Cost ($/m3) 0.03
Maintenance Cost ($/m3) 0.031
The overall LCOW analysis is presented in Table 7. It can be concluded from the analysis that, LCOW of solar-DCMD standalone unit comes at $1.80/m^3. From observation it is seen that the cost is competitive with previously reported desalination costs and lower than most of the studies. [50]

Figure 19. Box whisker plot showing cost comparison between waveguide-DCMD and prevalent solar desalination plants

Figure 19 shows dispersion in water production cost for several solar desalination techniques. It can be seen that LCOW from present study is lower than the median value previously reported for solar collector MSF (SC-MSF), solar collector MED (SC-MED) and PV-RO based existing plants.
4. Conclusion

The analysis presented a theoretical model of ideal waveguide. The mathematical model can be used to estimate the direct radiation, heat transfer losses and overall concentration ratio for a non-imaging optic element. The framework provides closed form analytical solutions for heat transfer and optical transmission problems. Hence it eliminates the need of numerical methods and computationally expensive techniques for energy analysis. It can be effectively used as an estimation tool for construction of non-imaging optics based power plants to support low temperature applications, PV installations and inexpensive Rankine cycle power plants.

It is observed from the initial part of the analysis that, for better optical performance, absorptivity of the material needs to be as low as possible. For increasing overall collection, increasing length yields non-linear increase in overall output which saturates at higher lengths. Length should be kept under 10 m for optimal gain through increased surface area accompanied by increased absorption. Material conductivity and thickness are found to have less significant impact on the heat transfer. Thickness should have value under 1.5 cm. However, surface heat transfer coefficient is found to have a significant role to play in the design procedure. The study presented a waveguide sizing analysis with fail-safe design considerations. The optimum waveguide size puts Acrylic waveguides of lengths ~0.3 m for thickness under 3 cm. Similar design analysis can be performed for any set of parameters at a future date using the mathematical framework provided in the present study.

From the levelized cost analysis, the waveguide – thermal based DCMD system shows promising results as the LCOW works out at $1.80/m^3. The LCOW number is comparable to that in the literature. Hence, for a carefully designed waveguide-thermal system, the LCOW can be further lowered. Future studies are underway to show better waveguide designs. For comparison
under more complex designs, in-house ray-trace codes have been implemented and more studies are underway.

_Nomenclature_

\( L \)  
Waveguide Length \([m]\)

\( w \)  
Waveguide Width \([m]\)

\( t \)  
Waveguide Thickness \([m]\)

\( \theta \)  
Angle of Incidence \([rad]\)

\( \alpha \)  
Material Absorptivity \([m^{-1}]\)

\( k \)  
Material Thermal Conductivity \([W/m-K]\)

\( h \)  
Heat Transfer Coefficient \([W/m^2-K]\)

\( I_0 \)  
Incident Radiation \([W/m^2]\)

\( I_L \)  
Direct Radiation \([W/m^2]\)

\( LCOW \)  
Levelized Cost of Water \([$/m^3]\)

\( CR \)  
Concentration Ratio

\( T \)  
Temperature \([K]\)

\( g'''' \)  
Volumetric Heat Generation \([W/m^3]\)

_Subscripts and Superscripts_

\( s \)  
Local Surface Value

\( r \)  
Receiver Bulk Value

\( \infty \)  
Atmospheric Value

\( max \)  
Maximum Value

\( o \)  
Operational Limit
References


Chapter 4. Conclusion and Future Work

The presented analysis showed elaborate exergy analysis for DCMD system. In the later part the analysis focused on formulating waveguide power source for DCMD and calculating the LCOW. Overall this thesis presents exergy based optimization of DCMD and waveguide solar collector for low temperature thermal applications such as DCMD.

4.1 Exergy Analysis of DCMD

In the first part of the analysis, a detailed exergy analysis was presented. The analysis evaluated effect of operational and geometrical parameters on exergetic efficiency and recovery ratio in DCMD. The highlights of the first part were root cause analysis of exergy destruction within DCMD module. Based on the elaborate analysis, later part of the second chapter focused on obtaining design windows for DCMD for exergetic efficiency constraints.

4.2 Waveguides for Desalination

Second part of the studies focused on analysis of ideal waveguide. The proposed mathematical framework showed flexibility and it can accommodate all geometrical parameters and effects of surroundings. Solutions were obtained for two receiver conditions and they were compared for effect of receiver design. The developed model was then used calculate waveguide size for safe continuous operation. Based on the sizing calculations for Acrylic, levelized cost of water was obtained at $1.80/m³. The levelized cost shows promising path for development of waveguide as a powersource for low temperature applications.
4.3 Future work

Future work for the first part of the studies includes development of experimental setup to test DCMD as well as AGMD and VMD systems. Based on the experimental setups and simple analytical model, a thorough system level analysis will help in choosing the most efficient MD technology. The most efficient MD technology will in effect yield lower levelized costs. The objective of this work is to bring MD costs as close to RO as possible to make a case for large scale implementation.

For the second part of the work, future work includes comparison of analytical results with numerical results from Monte Carlo Ray Trace packages. It will help in coining the analytical tool as a benchmark for non-imaging optics based solar collection methods. Further, an improved collector and waveguide is in the works, which will further enhance the overall collection and efficiency. Goal of this work is to lead towards more efficient non-imaging optic based solution for solar energy collection and design a standalone solar desalination MD technology.