Advanced instrumentation and measurement techniques for near surface flows

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(Abstract)

The development of aerodynamic boundary layers on wind turbine blades is an important consideration in their performance. It can be quite challenging to replicate full scale conditions in laboratory experiments, and advanced diagnostics become valuable in providing data not available from traditional means. A new variant of Doppler global velocimetry (DGV) known as cross-correlation DGV is developed to measure boundary layer profiles on a wind turbine blade airfoil in the large scale Virginia Tech Stability Wind Tunnel. The instrument provides mean velocity vectors with reduced sensitivity to external conditions, a velocity measurement range from $0 \text{ms}^{-1}$ to over $3000 \text{ms}^{-1}$, and an absolute uncertainty. Monte Carlo simulations with synthetic signals reveal that the processing routine approaches the Cramér-Rao lower bound in optimized conditions. A custom probe-beam technique is implanted to eliminate laser flare for measuring boundary layer profiles on a DU96-W-180 wind turbine airfoil model. Agreement is seen with laser Doppler velocimetry data within the uncertainty estimated for the DGV profile.

Lessons learned from the near-wall flow diagnostics development were applied to a novel benchmark model problem incorporating the relevant physical mechanisms of the high amplitude periodic turbulent flow experienced by turbine blades in the field. The model problem is developed for experimentally motivated computational model development. A circular cylinder generates a periodic turbulent wake, in which a NACA 63215b airfoil with a chord Reynolds number $Re_c = 170,000$ is embedded for a reduced frequency $k = \pi f c/V = 1.53$. Measurements are performed with particle image velocimetry on the airfoil suction side and in highly magnified planes within the boundary layer. Outside of the viscous region, the Reynolds stress profile is consistent with the prediction of Rapid Distortion Theory (RDT), confirming that the redistribution of normal stresses is an inviscid effect. The fluctuating component of the phase-averaged turbulent boundary layer profiles is described using the exact solution to laminar Stokes flow. A phase lag similar to that in laminar flow is observed with an additional constant phase layer in the buffer region. The phase lag is relevant for modeling the intermittent transition and separation expected at full scale.

(337 words)
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Attributions

During the course of my graduate studies, I have had the privilege to work with several faculty and students. The contributions of a few in particular warrant mention:

Dr. K. Todd Lowe is the primary adviser and committee chair for this research. He has advised all parts of the work presented, and has reviewed all included publications. The LDV measurements included in the journal article in Chapter III were performed by Dr. Lowe.

Dr. Eric Paterson is a committee member and Co-PI for the second half of this dissertation. He provided advice on the development of the model problem, as well as on the methods of analysis of the resulting data.

Di Zhang is a fellow doctoral candidate with whom I worked alongside on the wind turbine model problem research in the second half of this dissertation. The model problem was developed jointly, with Di performing all numerical modeling work. The raw numerical results used for analysis in Chapter VI were generated by Di as part of the collaborative research program, with scaling and analysis and performed by the author.
Chapter I.

Introduction

The continued development of advanced instrumentation has enabled new research and development in all fields of study. As the capacity increases to investigate more complex problems with higher degrees of accuracy, these techniques are then adapted to a wider variety of problems.

The recent interest in wind energy production over the past several decades has presented an opportunity for applying the already well developed ideas of fluid mechanics to the specific case of the high Reynolds number, high turbulence, low reduced frequency flow around thick, lifting bodies. The accompanying efforts to the overall field of wind turbine aerodynamics are many faceted: development of theory applicable in the regime of operation, application of instrumentation tailored to the specifics of the problem to provide detailed experimental results, and development of numerical models to enable high fidelity simulations of multi-scale flows. The last aspect is of particular interest, since validated models will directly lead to improved design and efficiency of wind turbine farms.

The present dissertation focuses on instrumentation development, with application to wind turbine aerodynamics. In the first portion, a new variant of Doppler global velocimetry is developed and characterized, for use in large wind tunnel facilities where wind turbine aerodynamic measurements often must be performed due to the high Reynolds number regime. In the second portion, a model problem is developed to address the specific conditions of high turbulence, unsteady interaction turbine wakes with additional downstream turbines, as is the case in wind farms. The model problem is motivated by the development of hybrid Reynolds-Averaged Navier Stokes – Large Eddy Simulation (RANS-LES) with an improved turbulent boundary layer transition model. The model problem was developed with insights both from literature and from preliminary simulations. The scope of the present dissertation focuses on the design of the model problem, high rate high resolution Particle Image Velocity (PIV) measurements on the model problem constructed in a mid-sized wind tunnel, and new insights into unsteady turbulent boundary layer observed over a model turbine blade.

1. Structure and contents

This dissertation is divided into seven chapters comprising two main research efforts, as well as several appendices:

Chapter 1 provides a general introduction and overview of the dissertation.
Chapter II contains background information and literature review of measurement techniques leading to the development of cross-correlation Doppler global velocimetry.

Chapter III is the first manuscript, “Cross-correlation Doppler global velocimetry (CC-DGV),” published in Optics and Lasers in Engineering (2015 71:51-61, DOI 10.1016/j.optlaseng.2015.03.012). This paper presents a background on DGV techniques and the development of the CC-DGV technique and measurements principle. The CC-DGV method provides mean velocity measurements with one camera per component, with a velocity measurement range from 0 ms\(^{-1}\) to over 3000 ms\(^{-1}\). Experimental results in two flows are presented: boundary layer profiles on a wind turbine blade model in the Virginia Tech Stability Wind Tunnel, and volumetric measurements and radial profiles in the Virginia Tech Hot Supersonic Jet at cold and hot conditions, respectively. Uncertainties are estimated for both flows, with the frequency shift uncertainty being an absolute uncertainty independent of velocity regime, and a geometry uncertainty dependent on the configuration of the system.

Chapter IV is the second manuscript, “Investigation of measurement sensitivities in cross-correlation Doppler global velocimetry” published in Optics and Lasers in Engineering (2016 86:44-52, DOI 10.1016/j.optlaseng.2016.05.003). This article presents an in-depth uncertainty analysis for CC-DGV. The Cramér-Rao lower bound (CRLB), which represents the theoretically lowest measurement variance for an unbiased data set, is computed from an analytical model for the DGV signals. Monte Carlo simulations are performed for synthetic, noisy signals at various simulated conditions processed using CC-DGV and compared to the CRLB. The signal estimator is found to achieve the best uncertainty in high velocity regimes, and for matched cell temperatures and small frequency step size of the incident laser frequency.

Chapter V contains the final manuscript to be submitted to Experiments in Fluids, titled “Unsteady boundary layer development on a wind turbine blade: a benchmark problem.” The motivation for a model problem to study the physics of high Reynolds number, high amplitude low reduced frequency periodic boundary layer flow over a thick airfoil is presented, and a model problem is developed for use in a joint experimental-numerical effort. Particle image velocimetry measurements are performed on a NACA 63215b airfoil suction side, including several high magnification planes in the low \(Re_q\) boundary layer with a high speed acquisition system, under steady inflow conditions and in the wake of a circular cylinder. In-plane components of the mean velocities and Reynolds stresses are presented, and the behavior of the normal stresses outside of the buffer region are shown to be dominated by inviscid effects via Rapid Distortion Theory. A triple decomposition is applied to the boundary layer profiles to separate the contributions of the ensemble mean, periodic, and stochastic turbulent contributions to the velocity. The periodic component agrees well with the laminar Stokes flow solution.
phase lag is observed in the boundary layer similar to that seen under laminar conditions, with an additional constant phase lag region in the buffer layer.

Chapter VI discusses the development of the model problem for the wind turbine aerodynamics effort, as well as extended details of the measurements configuration and processing routines. Additional supporting data in the wake of the cylinder is presented as a validation of the experimental design.

Chapter VII is a summary of the overall body of work, including conclusions for each research effort. An outlook on future work is included as well.

Formatting differs amongst the chapters due to the nature of the included manuscripts. Chapters III and IV follow the formatting guidelines of the Optics and Lasers in Engineering journal where they have both already been published. Chapter V is to be submitted to Experiments in Fluids, and follows formatting guidelines for that journal.

2. Achievements

Key results and conclusions of the present research include:

- Development and demonstration of a new Doppler global velocimetry variant, cross-correlation DGV, which provides mean velocity vectors with high spatial resolution and high velocity measurement range, and reduced sensitivity to external noise sources.

- Sensitivities and uncertainties in the CC-DGV technique have been characterized through estimation of the Cramér-Rao lower bound and several Monte Carlo simulations.

- A model problem was developed to study a flow of interest to the wind turbine community: the transitional boundary layer on a thick, cambered airfoil at high Reynolds number embedded in a high amplitude, low reduced frequency wake.

- Experimental characterization of the model problem. In particular, Rapid Distortion Theory effects was seen to dominate the flow outside of the boundary layer, while the periodic component of the turbulent boundary layer was similar to laminar Stokes flow with a constant phase region in the buffer layer.

References
Chapter II.

Instrumentation development background

This chapter provides an overview of velocimetry instrumentation techniques, with specific emphasis on laser based techniques.
1. Instrumentation for aerodynamic velocimetry

A diverse class of instruments exist for the measurement of fluid velocity. A brief overview of common techniques used in incompressible aerodynamics is presented with special emphasis on laser-based techniques.

1.1 Point-wise techniques

The most widespread and often simplest to implement velocity measurement techniques are those that give measurements at a single point or small volume in space. This category of measurement techniques is conducive to probe-based instruments, where a sensing element is placed directly in the flow at the point of interest. Pressure (Pitot) probes and hot wire anemometry point techniques have been a mainstay of aerodynamic measurements since the inception of the field, and continue to be ubiquitous due to their simplicity and robustness. Laser Doppler velocimetry (LDV) is a non-intrusive technique that uses the interference fringes of intersecting laser beams to determine the Doppler shift of scattered light within the flow in a very small measurement volume. In large-scale facilities, point measurements are often considered standard instrumentation. Point techniques are all compatible with time-resolved measurements.

1.1.1 Pressure probes

Pressure measurements are often synonymous with velocity measurements, by application of the Bernoulli equation in incompressible flow and the isentropic flow equations for compressible flow. The simplest type of pressure measurements are Pitot probes, which measure the total pressure in a flow when oriented into the flow direction. Pitot probes work on the principle of the Bernoulli equation, expressed in terms of the total \( p_0 \) and static pressures \( p_s \):

\[
p_0 = p_s + \frac{\rho u^2}{2}
\]  

The measurement principle is valid for inviscid flow and for tubes of sufficiently narrow diameter (Chue 1975). These probes often include static pressure ports, which allows for the direct measurement of the dynamic pressure.
via the difference of the total and static pressures $p_0 - p_s$; for incompressible flows this allows for direct computation of the velocity. The velocity measured is that along the streamline aligned with the probe total pressure port. Probes are typically aligned normal to the flow direction in order to measure the velocity in the streamwise direction. Specialized probes have been developed to measure account for the effects of non-parallel flow. Kiel probes (Kiel 1935) feature a shroud over the tip of the probe to deflect the flow to counteract the effects of mean or instantaneous yaw of the probe. Multi-hole probes, such as 5-hole (Dominy and Hodson 1992) and 7-hole (Sumner 2002) probes are capable of measuring the flow angle within some angle range based on the differential pressure from multiple ports arranged around the cone shaped tip of the probe. Fast response pressure probes capable of time resolved data have also been developed, see the review of (Sieverding et al. 2000). Linear probe arrays, known as “integrating wake rakes,” are also commonly used with control volume analysis to determine lift and drag on aerodynamic models in wind tunnels (Timmer and van Rooij 2001; Cerretelli et al. 2012; Joseph 2014).

### 1.1.2 Hot-wire anemometry

Hot-wire anemometry (HWA) describes a classification of instruments which are well suited for high rate measurements of fluctuating velocity and turbulence spectra. The measurement principle involves the use of a thin wire heated by an electrical current (King 1914). As air flows past the wire heat is convected away as a function of the instantaneous velocity. A common current configuration is that of the “constant temperature” anemometer, in which a bridge circuit is used as a feedback loop to maintain the temperature of the wire by adjusting the current; the fluctuating velocity is a function of the instantaneous electrical current. A review of HWA techniques is given in (Comte-Bellot 1976), and the method remains a staple of turbulence measurements to this day (Hutchins et al. 2015).

### 1.1.3 Laser Doppler velocimetry

Laser Doppler anemometry / velocimetry (LDA / LDV) is one of the earliest laser diagnostic methods (Yeh and Cummins 1964) and is in some ways analogous to hot-wire anemometry in that it takes point measurements at high repetition rates, and is thus well suited to acquire turbulence data at individual points. LDV works on the principle of the Doppler shift of light scattered by seed particles in a flow. The
most common presently used approach is known as the dual-beam configuration (Albrecht et al. 2003). Two or more laser beams at different optical frequencies are crossed to form a small measurements volume with an interference fringe pattern from the beam crossing. As a particle moves through the measurement volume, light is Mie scattered to a receiver and the interference fringe pattern is modulated by the Doppler shift frequency. The velocity is determined from the Doppler shift based on the geometry of the probe beams and receiver. One component of velocity can be determined from each beam crossing pair, i.e. two total beams can measure one component of velocity and three total beams can measure three components. Additional laser beams provide redundant measurements which can be used to improve the measurement accuracy. Advanced LDV techniques have been developed to provide data such as spatial resolution within the measurement volume (Czarske et al. 2002; Lowe and Simpson 2009), particle acceleration(Lehmann et al. 2002; Lowe and Simpson 2006), among others.

1.2 Planar techniques

Full-field measurements have become widespread over the past several decades due to the advancement of laser technology. Planar and volumetric measurements allow for evaluation of much larger regions of a flow field simultaneously, enabling insights into large-scale phenomena, however often with a trade-off of temporal resolution. Common laser-based planar velocimetry techniques include particle image velocimetry (PIV) and Doppler global velocimetry (DGV).

1.2.1 Particle image velocimetry

Particle image velocimetry (PIV) offers spatially resolved velocity measurements and is the most common technique currently used for planar flow-field measurements (Prasad 2000). PIV was first discussed in its present form by (ADRIAN 1984), and become popular with the incorporation of computer-based processing algorithms (Willert and Gharib 1991). In its simplest form, PIV involves the use of a digital camera to acquire two images of a seeded flow with a small delay time between frames. Illumination is typically provided by a double pulsed Nd:YAG laser. A two-dimensional spatial correlation is performed between corresponding small regions, or interrogation windows,
for the two frames. In each interrogation window, the peak correlation location corresponds to the statistical particle
displacement for the particles within the window. Single camera (planar) PIV provides two components of velocity
corresponding to the in-plane basis vectors of the image plane. Three-component measurements are achieved using a
stereoscopic configuration of two cameras imaging the same region with offset viewing angles (Willert 1997).

PIV has a high-pass spatial filter effect on the fluctuating velocity since the spatial resolution is set by the size of the
interrogation volume size. Uncertainty is often quoted as about 1% of the velocity magnitude, but is in fact very
complex and has been the focus of recent efforts to develop uncertainty analysis routines, an overview and
comparative analysis is given by (Sciacchitano et al. 2015), with particular success found in estimation of
uncertainty using the method of (Wieneke 2015). Several advancements in processing routines have been studied by
that same author, including self-calibration of stereoscopic image (Wieneke 2005) and scale-adaptive interrogation
windows sizes and shapes (Wieneke and Pfeiffer 2010).

PIV has been extended by many researchers for advanced applications. High rate time-resolved measurements have
been demonstrated by many researchers, including (Thurow et al. 2013). Volumetric measurements have also been
shown using holographic and tomographic techniques (Elsinga et al. 2006; Sheng et al. 2006). Methods for pressure
and acceleration measurements have been of interest over the past decade. Particle image velocimetry (PIV) and the
related particle tracking velocimetry (PTV) are both well-suited to this application, and indeed were used for
accelerometry before the extension of the method to include pressure estimation, for example (Jakobsen and
Dewhirst 1997). Multiple pulse systems, typically three or four pulses, are used to determine the Lagrangian
acceleration of a small particle grouping, such as the “pseudo-tracing” iterative scheme of (Liu and Katz 2006). For
acceleration from PTV, a true Lagrangian quantity is measured with reduced error in this quantity, but requires
interpolation to a regular grid for pressure estimation (Novaro and Scarano 2012). Two approaches can be used to
determine the pressure gradient field, via the momentum equation or the Poisson equation; a full review of both
approaches is given by (van Oudheusden 2013).
1.2.2 Doppler global velocimetry

Doppler global velocimetry (DGV) was developed over the past two decades, first introduced by (Komine et al. 1994) in conjunction with Northrop and NASA Langley (also known as planar Doppler velocimetry, (PDV) (Charrett et al. 2004)). Spatially resolved DGV offers a number of intrinsic benefits over other optical diagnostic techniques, related to the fundamental means of measuring Doppler shift via molecular absorption cells. As opposed to LDV and PIV, where individual particles or small groupings of particles generate the measurement signal, the frequency filtering used in DGV allows for multiple particles passing through a point in space to contribute to a data point. This then permits small particles and sparse seeding to be used, as well as the ability to integrate the signal in time to improve signal to noise ratio (SNR). The direct measurand is the Doppler shift frequency, which is a fundamental measure of velocity, thus simplifying post-processing and reducing errors introduced therein.

DGV measurements are based on the Mie scattering of light by seed particles, as with PIV and LDV. When light is scattered by particles in a flow, a Doppler shift in the incident laser frequency occurs, and thus the scattered light is

![Figure 1](image-url)  
**Fig. 1** Example transmission spectrum for iodine vapor at a cell temperature of 20°C and cell length of 5cm predicted from the code by (Forkey et al. 1997).
shifted by some amount relative to the incident light. This phenomenon is governed by the well-known Doppler equation:

$$\Delta \nu = \frac{(\hat{\sigma} - \hat{i}) \cdot \vec{V}}{\lambda_0}$$  \hspace{1cm} (2)

where $\Delta \nu$ is the measured frequency shift, $\lambda_0$ is the incident laser wavelength, $\hat{\sigma}$ and $\hat{i}$ are the spatial vectors along the scattered and incident light directions, respectively, and $\vec{V}$ is velocity of the particle.

Mie scattering is itself an elastic phenomenon, so any detected shift in frequency is due solely to Doppler effects. Since particles are compatible with the continuum flow assumption, then frequency shift is continuum flow velocity related within the limits of particle slip at very small time scales. In order to measure this frequency shift, vapor cells are used to selectively absorb light at known frequencies. Iodine vapor has many absorption lines in the tunable range of 532nm Nd:YAG and Nd:YVO$_4$ systems (Figure 1) (Forkey et al. 1997). By setting the incident laser wavelength to be situated on one of the absorption lines, the Doppler shifted light scattered by the particles in the flow will result in an intensity shift as seen through the vapor cell (Figure 2). Typically, light scattered from the flow is split into two paths and imaged; one path will include a vapor cell to attenuate the intensity as a function of frequency (signal detector), and the other would be unobstructed (reference detector) (Meyers 1995). Each signal detector in this design would measure a different component of velocity, as governed by the intensity shift.
Doppler equation (2). This method requires precise prior knowledge of the unshifted laser frequency to deduce position on the vapor cell absorption spectrum and thus to construct intensity ratios. An in depth study of the errors present in typical DGV systems is given by (Meyers et al. 2001).

In order to reduce the number of cameras needed by a factor of two, a technique known as 2ν-DGV (“two-frequency DGV”) was developed by (Charrett et al. 2004). Instead of using separate signal and reference detectors for each velocity component, a single detector is used with a vapor cell permanently attached in line, and the incident laser wavelength is modulated at a high rate between a location of nominally full transmission through the vapor cell and a position on an absorption line. When done at high enough rates (limited by the frame rate of the detector camera), time-resolved data can be acquired by this method. As in conventional DGV, raw data from 2ν-DGV consists of light intensity ratios.

A further technique requiring a single camera per component of velocity is known as frequency-modulation DGV (FM-DGV), first developed by (Müller et al. 1999) and further developed by (Fischer et al. 2007). FM-DGV allows for the removal of the reference camera by high frequency scanning of a portion of an absorption line. Uncertainties of 0.02 m/s have been achieved using this approach (Fischer et al. 2007). Temporal resolution can be achieved in practice to about 20 kHz for multiple-component measurements, and 100 kHz for single component measurements (Fischer et al. 2011). The ratios of spectral harmonics of the scanning frequency obtained from transmission signal through a caesium vapor cell are calculated to obtain frequency shift values. According to (Fischer et al. 2008), use of the full non-linear portion of the absorption line allows for measurement of Doppler shift frequencies up to ±430 MHz, which corresponds to ±260 m/s for the laser center wavelength and camera configuration used in that work. Various methods have been developed for FM-DGV in order to better control the laser frequency (Fischer et al. 2009a) and reduce noise (Fischer et al. 2009b). Some recent examples of the FM-DGV system in specialized applications include (Fischer et al. 2015; Schlüßler et al. 2015; Gürtler et al. 2016). A related method known as frequency shift key (FSK) DGV was developed by (Müller et al. 2007) uses the principle of FM-DGV with only four discrete scan points needed in the cycle.
2. Laser flare reduction techniques


Laser flare is the greatest inhibitor to near-surface measurements with optical techniques. Laser flare describes the saturation of camera/photodetector pixels which image the region encompassing and surrounding laser impingement on solid surfaces. This phenomenon arises from reflection of light from the solid surface, which is exacerbated by reflections inside of the imaging optics and electronic effects such as pixel blooming and leakage. In Particle Image Velocimetry (PIV), a high image contrast between tracer/seed particles is necessary for accurate cross-correlation results (Paterna et al. 2013). As such, even regions of the camera sensor that are not saturated can still fail to yield valid data if background levels are too high relative to Mie-scattered signal light (Chennaoui et al. 2008).

Near-wall measurements are of interest to the wind energy community so as to aid in prediction of wind turbine efficiency and power generation at a wide variety of conditions. Especially at high Reynolds numbers and high angles of attack, flow separation is a major contributor to unsteady drag and far field acoustic noise. The simplest means to reduce observed laser flare is to position the cameras at such an angle relative to the scattering surface as to minimize the solid angle through which scattered light can enter the camera lens. Several researchers employ this method, referred to in this paper as “grazing angles.” Examples of this approach include (Konrath et al. 2008) and (Pierce and Lu 2012), among others. To achieve Reynolds numbers approaching those experienced by wind turbines in the field, measurements are often performed in large-scale wind tunnel facilities where optical access is limited. In these cases, additional treatments become necessary.

For PIV applications, laser light is formed into a sheet to illuminate planes of seed particles in a flow (Willert and Gharib 1991). In order to illuminate regions near a solid surface, often the only available path for the laser sheet culminates with it directly illuminating or closely grazing by said surface. Photons impinging on any interface where the index of refraction changes (such as between the fluid and solid) will succumb to one of three possible sinks: absorption of light energy into the solid, specular or diffuse reflection off the surface, or transmission through the
material. In the case of a perfectly matched index of refraction interface, light will pass through cleanly, as in the study by (Uzol et al. 2002) using an acrylic turbo-pump with a solution of sodium iodide liquid. For each of these modes, mitigation strategies have been devised; a review of these techniques is given by (Paterna et al. 2013)

Previous attempts to combine multiple flare mitigation techniques have been limited. (Pierce and Lu 2012) used a polished transparent polycarbonate window panel modified in two ways. In the first configuration, a back-coated mirror was placed on top of the polycarbonate; this was noted to be the optimal performance, but not practicable in their application. In the second configuration, thin layers of flat black paint and fluorescent red dye (stated *prima facie* to have properties resembling rhodamine) are applied to the surface of the polycarbonate. It is herein suggested that in both configurations, the nature of the modification reduces the potential benefit of the transparent property of the polycarbonate.

(Konrath et al. 2008) have shown the most successful combination of methods, for an opaque delta wing model consisting of flat surfaces with leading edge rounding. The surface was polished to a mirror finish, and coated with flat black paint doped with rhodamine B fluorescent pigment. Using this method, measurements were achieved nominally down to the surface, but no exact value is reported for near-wall resolution. Similarly, (Paterna et al. 2013) found that flat black paint alone as applied to an opaque wooden surface showed worse behavior than several fluorescent and transparent configurations measured independently.

Mirrored surface treatments, such as those used by (Konrath et al. 2008) and (Pierce and Lu 2012) have convincingly been shown to be very effective at eliminating laser flare. However, for the flows of interest in the present study, a mirrored surface was ruled out due to the demands of additional concurrent measurements to be performed in future large-scale wind tunnel campaigns. Polishing of the airfoil (either in whole or part) would affect the surface roughness, which would in turn alter the boundary layer transition and drag.

**References**


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Chapter III.

Cross-correlation Doppler global velocimetry (CC-DGV)

This chapter presents the development of the new CC-DGV instrument, published as “Cross-correlation Doppler global velocimetry (CC-DGV)” in Optics and Lasers in Engineering (2015) (71 51:61, DOI 10.1016/j.optlaseng.2015.03.012). A additional application of the technique, an “Optical Wake Rake” for wake profiles in large wind tunnels, is presented in Appendix A.

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Cross-correlation Doppler global velocimetry (CC-DGV)

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1. Introduction

Doppler global velocimetry (DGV) is a laser diagnostic technique for fluid flows [1] and an area of continuous development. Laser techniques have been extensively used in fluid mechanics research owing to their potential for high rate and large spatial area measurements, and minimal disruption to the flow. The first laser-based flow velocimetry system, laser Doppler velocimetry (LDV), was introduced in 1964 by Yeh and Cummins [2]. LDV provides measurement rates in the kilohertz range in small volumes, either averaged or resolved within the measurement volume [3,4]. Particle image velocimetry (PIV) is one of the most prevalent contemporary techniques, and many advanced and specialty applications have been studied [5]. DGV is a more recent technique, which has the ability to directly sense the Doppler frequency shift of scattered light, while providing large-format spatially resolved measurements [6].

Descriptions of the operating principle for traditional DGV systems can be found in other works, e.g., Meyers [6] and Jenkins et al. [7], and discussions of common error sources are given by Ainsworth et al. [8] and Meyers et al. [9]. Several variations on DGV have been developed, with the goal of reducing measurement uncertainty by eliminating sources of error as well as system complexity, reviewed further to follow.

Two-frequency DGV (2ν-DGV) was developed by Charrett et al. [10] in order to reduce the number of cameras needed per measured component from two to one. This eliminates image registration issues as well as errors due to non-even splitting of scattered light. In addition to using only one camera per component, a technique known as frequency modulated DGV (FM-DGV) also removes dependence on absolute intensity value [11], by use of the harmonic quotient of the signal. A related technique, frequency shift key DGV (FSK-DGV), is a discretized version of FM-DGV using four frequency points per scan with a CCD camera sensor [12]. Interferometric DGV (I-DGV) systems have also been developed to eliminate the need to tune the laser frequency to specific absorption lines [13], and to eliminate the vapor absorption cell entirely [14,15]. These systems directly measure the Doppler frequency shift with greater sensitivity than standard DGV systems, however involve more complex opto-mechanical configurations and at present require temporal averaging.

The standard practice for sensing Doppler frequency in DGV is via the use of vapor cells. Typically, this demands strict regulation of the vapor cell side arm and body temperatures in order to accurately define the vapor cell transfer function between light frequency and proximity.
the transmission ratio of scattered light captured by the cell. Forkey et al. [16] found that for iodine vapor, the absorption spectrum was uniquely a function of cell temperature, internal pressure, and cell dimensions. Since cell dimensions are fixed for a given system, and vapor pressure can be determined from the temperature, dependence is reduced to temperature only. Forkey et al. [16] further found that the center frequency of each absorption line does not change with temperature; the depth and width of the lines both increase at increasing temperature. Special configurations known as “starved cells” and “saturated cells” aim to hold the cell transfer function fixed for a given system, and absolute light intensity independence of a Doppler-shifted spectrum from scattered light within a seeded flow, and a non-shifted spectrum of the incident laser beam. Cross-correlating these spectra yields the frequency shift between them.

A simple CC-DGV configuration for one-component measurements is shown in Fig. 1. As in traditional DGV systems, a flow of interest is seeded with tracer particles, and a region of interest is illuminated with a single-frequency laser. The laser frequency is swept over a range of several gigahertz. For comparison, the width of a typical absorption line is on the order of 1 GHz, as shown in Fig. 2 for the theoretical spectrum of a 5 cm molecular iodine cell at 30 °C body temperature. The range of frequencies chosen for a scan is such that one or more full absorption lines are captured.

A low power secondary beam path is established to monitor the incident laser frequency and act as a reference signal in the data processing. This secondary beam is split from the primary beam path immediately after the laser aperture, and passed through a vapor cell. Cross-correlation DGV (CC-DGV) is introduced herein to improve system robustness while matching the single camera per component configuration for one-component measurements. A simple CC-DGV configuration for one-component measurements is shown in Fig. 1. As in traditional DGV systems, a flow of interest is seeded with tracer particles, and a region of interest is illuminated with a single-frequency laser. The laser frequency is swept over a range of several gigahertz. For comparison, the width of a typical absorption line is on the order of 1 GHz, as shown in Fig. 2 for the theoretical spectrum of a 5 cm molecular iodine cell at 30 °C body temperature. The range of frequencies chosen for a scan is such that one or more full absorption lines are captured.

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A low power secondary beam path is established to monitor the incident laser frequency and act as a reference signal in the data processing. This secondary beam is split from the primary beam path immediately after the laser aperture, and passed through a vapor cell.

### 2. Cross-correlation Doppler global velocimetry

In this section, cross-correlation DGV is presented. A description of system configuration and features is presented in Section 2.1, followed by details of specific instrumentation in Section 2.2. Data processing routines are described in Section 2.3, with an uncertainty analysis in Section 2.4 and discussion in Section 2.5.

#### 2.1. CC-DGV system configuration

CC-DGV measures the Doppler shift of scattered light by comparison of two simultaneously acquired vapor absorption spectra: a
through a vapor cell before imaging directly onto a photodetector. As the laser frequency is swept, this reference signal will provide a non-Doppler-shifted absorption spectrum, which is also used to determine the laser frequency scan rate by comparison with the model spectrum.

The flow is imaged by a series of detector units, each consisting of a camera fitted with a laser-line band-pass filter and vapor absorption cell. The vapor cells attenuate scattered light based on the species' absorption spectrum. The Doppler shift frequency is a function of both the local velocity as well as the system geometry, according to the Doppler shift equation:

$$\Delta \nu = \frac{\left( \delta - \hat{i} \right) \cdot V}{\lambda}$$  \hfill (1)

where $\Delta \nu$ is the Doppler shift frequency, $\delta$ and $\hat{i}$ are the unit vectors defining the scattered light (camera orientation) and incident laser light directions, $V$ is the velocity vector of the fluid expressed in the global coordinate system, and $\lambda$ is the wavelength of the incident laser light. The vectors expressed in Eq. (1) are shown in Fig. 1.

### 2.2. Instrumentation

Frequency scanning is done on the continuous wave Coherent Inc. Verdi V6 diode-pumped solid state Nd:YVO$_4$ laser, with maximum output power of 6 W at 532 nm. The temperature of an internal etalon can be modulated to coarsely adjust the output frequency on the order of tens of gigahertz. For an absorption line of 1 GHz width, Doppler shifts up to 6 GHz can be detected, corresponding to a component of velocity measured, a distinct ($\delta - \hat{i}$) is needed. Methods for transformation of non-orthogonal and over-defined systems are given by Charrett et al. [20]. Image calibration is done by means of a grid calibration target. All detector units in the system image the target, and the resulting calibration images are used for both linear spatial mapping as well as image registration amongst the various detector units.

![Normalized Transmission vs Frequency](image)

**Fig. 2.** Theoretical vapor absorption spectrum for a 5 cm molecular iodine cell at 25°C. The center frequency marked as 0 frequency shift corresponds to 563265.061 GHz for the incident light optical frequency.

2.3. Processing routines

The CC-DGV data reduction routine is shown schematically in Fig. 3. The absorption spectrum recorded for each point within a flow will be Doppler-shifted relative to the non-shifted reference signal (Fig. 3, Panel A). The scattered light signal for each pixel is cross-correlated with the reference signal (Fig. 3, Panel B) according to

$$R(\nu_i) = \frac{1}{\sigma_{\nu_p}\sigma_{\nu_r}} \sum_{j=1}^{N} S_{\nu_p}(\nu_j) S_{\nu_r}(\eta_j - \nu_i)$$  \hfill (2)

where $R(\nu_i)$ is the value of the cross-correlation for a series of discrete phase shift values $\nu_i$, $S_{\nu_p}$ and $S_{\nu_r}$, also $\sigma_{\nu_p}$ and $\sigma_{\nu_r}$, are the mean-subtracted scattered light signals and their standard deviations, respectively, for the pixel and reference signals. $\eta_j$ is the incident light frequency at each of the $N$ points in the scan.

The shift associated with the peak value of the correlation coefficient data vector $R(\nu_i)$ corresponds to the Doppler shift frequency, to the accuracy of the discrete frequency step size of the laser frequency scan. To improve the accuracy to sub-scan interval levels, a least-squares parabolic fit is performed around the peak of the cross-correlation curve ($\pm 3$ scan intervals about the peak value, Fig. 3, Panel C) to determine the coefficients in the form

$$\hat{\nu}(\nu) = P_2 \nu^2 + P_1 \nu + P_0$$  \hfill (3)

where $\hat{\nu}(\nu)$ is the polynomial fit to the correlation coefficient data vector, and $P_i$ are the polynomial coefficients. The parabola maximum of Eq. (3) is then given by

$$\nu_{\text{max}} = -\frac{P_1}{2P_2}$$  \hfill (4)

such that this interpolated peak location $\nu_{\text{max}}$ is the Doppler shift frequency to be used on the left hand side of Eq. (1). This value represents the mean Doppler shift frequency over the duration of the laser frequency scan.

The component of velocity ultimately determined by Eq. (1) is that oriented along the ($\delta - \hat{i}$) vector direction. For each component of velocity measured, a distinct ($\delta - \hat{i}$) is needed. Methods for transformation of non-orthogonal and over-defined systems are given by Charrett et al. [20]. Image calibration is done by means of a grid calibration target. All detector units in the system image the target, and the resulting calibration images are used for both linear spatial mapping as well as image registration amongst the various detector units.

2.4. Uncertainty analysis

The following analysis considers uncertainty of the mean velocity at a single, finite-sized point, and is not indicative of turbulence levels in the flow. Each point in the post-processed results is constructed from corresponding points from each image. To reconcile the various cameras, each image is interpolated onto an evenly spaced grid shared by all cameras. As such, the spatial resolution of
the final measurement is limited by the individual camera with the poorest spatial resolution.

Measurement uncertainty is derived from several sources, each acting upon the terms in Eq. (1). However, only the Doppler frequency shift term is directly measured. The transformation of the velocity into a global, orthogonal system denoted $U, V, W$ is given by Charrett et al. [20] (in modified form for three cameras) as:

$$
\begin{bmatrix}
U_1 \\
U_2 \\
U_3
\end{bmatrix} =
\begin{bmatrix}
\hat{o}_x \
\hat{o}_y \
\hat{o}_z
\end{bmatrix} =
\begin{bmatrix}
\hat{i}_x \
\hat{i}_y \
\hat{i}_z
\end{bmatrix}_{1x} \begin{bmatrix}
\hat{i}_y \
\hat{i}_z \
\hat{i}_x
\end{bmatrix}_{1y} \begin{bmatrix}
\hat{i}_z \
\hat{i}_x \
\hat{i}_y
\end{bmatrix}_{1z}
$$

The subscripts of the elements in the rotation matrix refer to the camera index and global frame component of the resultant vector. For example, $\hat{o}_x$ denotes the component in the global frame $x$-direction of the vector difference of the camera 1 $\hat{o}$ vector with the incident laser $\hat{i}$ vector, as shown in Fig. 4. $U_i$ refers to the velocity component measured in the $\hat{i}$ vector direction of camera $i$. The velocity components of $[U_1, U_2, U_3]^T$ follow from the orthogonal velocity vector, $[U, V, W]^T$, weighted by the magnitude of the components of the matrix $\begin{bmatrix}
\hat{o}_x \
\hat{o}_y \
\hat{o}_z
\end{bmatrix}_{ij}, \{i = 1, 2, 3\}, \{j = x, y, z\}$.

In Eq. (4), the phase shift parameter $\nu_{\text{max}}$ can be broken down into the product of the voltage shift, $\Delta \psi$, corresponding to the peak correlation location, and the mapping coefficient between voltage and frequency, $\kappa$:

$$
\nu_{\text{max}} = (\Delta \psi) (\kappa)
$$

Aside from interferometric DGV, all DGV methods involving vapor absorption cells have an additional source of uncertainty relating to the way in which the transfer function between the experimentally observed and theoretical vapor absorption spectra is handled. For CC-DGV, either a mapping between the experimental and theoretical spectra can be used, or a calibration can be
performed as is done in FM-DGV. In this work, the mapping approach is used. Owing to the robustness of the cross-correlation function, the line center location of the experimental and theoretical spectra are sufficient information for determining the frequency mapping. The uncertainty with such a scheme is thus reduced to that of a single mapping coefficient relating the discrete scan step width to frequency.

A modified form of the left hand side of Eq. (5), accounting for this frequency mapping term, can now be defined as:

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \kappa \lambda \begin{bmatrix} \Delta \psi_1 \\ \Delta \psi_2 \\ \Delta \psi_3 \end{bmatrix}$$

(7)

where $\Delta \psi_i$ is the voltage shift determined from each camera. Substituting into Eq. (5) and solving for velocity gives the final form of the CC-DGV transformation equation:

$$\begin{bmatrix} U \\ V \\ W \end{bmatrix} = \begin{bmatrix} (\tilde{\omega} - \tilde{\omega})_{1x} & (\tilde{\omega} - \tilde{\omega})_{1y} & (\tilde{\omega} - \tilde{\omega})_{1z} \\ (\tilde{\omega} - \tilde{\omega})_{2x} & (\tilde{\omega} - \tilde{\omega})_{2y} & (\tilde{\omega} - \tilde{\omega})_{2z} \\ (\tilde{\omega} - \tilde{\omega})_{3x} & (\tilde{\omega} - \tilde{\omega})_{3y} & (\tilde{\omega} - \tilde{\omega})_{3z} \end{bmatrix}^{-1} \begin{bmatrix} \Delta \psi_1 \\ \Delta \psi_2 \\ \Delta \psi_3 \end{bmatrix}$$

(8)

Finally, the uncertainty in the orthogonal velocity components is found by application of the propagation of error equation (for quantity of interest $\alpha$ dependent on several parameters $\beta_i$):

$$\delta \alpha^2 = \sum \left( \frac{\partial \alpha}{\partial \beta_i} \delta \beta_i \right)^2$$

(9)

For simplicity, the transformation matrix in Eq. (8) is defined as $A$, following the notation of Charrett et al. [20]. Square powers in Eq. (10) imply element-by-element operations on the transformation matrix:

$$\begin{bmatrix} \delta U^2 \\ \delta V^2 \\ \delta W^2 \end{bmatrix} = (\delta \lambda^2) \begin{bmatrix} \Delta \psi_1^2 \\ \Delta \psi_2^2 \\ \Delta \psi_3^2 \end{bmatrix} + A^2 (\delta \lambda^2) \begin{bmatrix} \Delta \psi_1^2 \\ \Delta \psi_2^2 \\ \Delta \psi_3^2 \end{bmatrix} + A^2 (\delta \lambda^2) \begin{bmatrix} \Delta \psi_1^2 \\ \Delta \psi_2^2 \\ \Delta \psi_3^2 \end{bmatrix}$$

$$+ A^2 (\delta \lambda^2) \begin{bmatrix} \Delta \psi_1^2 \\ \Delta \psi_2^2 \\ \Delta \psi_3^2 \end{bmatrix}$$

(10)

The presence of system geometry vectors and Doppler shift magnitude in Eq. (10) imply that the final uncertainty values are highly dependent on the application. Configuration-specific uncertainty results are discussed in Section 3. The uncertainty contributions from the mapping coefficient and the laser frequency modulation are both neglected in the final analysis since the mapping coefficient error can be minimized through calibration, and the frequency modulation error is on the order of micrometers per second or lower.

For each frequency set point in the scan, the intensity value of a pixel is made up of the light scattered from several seed particles, all of which will pass through the same point in space during the camera integration time. When there is turbulence present, these particles will have a distribution of velocities. This in turn leads to different levels of light transmission through the vapor cells, and a final pixel intensity value summed from statistically random particle contributions. For a given measurement, this effect may alter the observed shape of non-linear portions of the absorption spectrum, causing the Doppler-shifted absorption lines to appear wider in frequency and with a higher minimum transmission relative to the turbulence-free case.

A model was created to assess the influence of turbulence on the cross-correlation processing routine. For each incident light frequency in a scan, a synthetic scattered light pixel signal was generated by creating a histogram of turbulent velocities, i.e., a histogram of the Doppler shifts, and summing their transmission through a model iodine transmission spectrum. The model spectrum was centered about the absorption line at 563244.039 GHz, with a cell length of 5 cm and body temperature of 20°C. This was then cross-correlated with a non-shifted reference signal to find the bias relative to the mean.

The biases for a range of mean velocity and turbulence intensity values are shown in Fig. 5. Turbulent fluctuations can either increase or decrease the intensity at every point in the spectrum, depending on the magnitude of the fluctuations and the transmission corresponding to such velocity fluctuations. In Fig. 5, the tendency for the biases to shift from positive to negative at higher mean velocities is due to the asymmetry of the spectrum around the absorption line. A low modulation depth feature on the lower frequency side of the line at 563244.039 GHz skews the relative line center of the received signal as the magnitude of turbulent fluctuations increases. This is also the cause of the jump in bias seen in the 500 ms1 and 1000 ms1 cases. Line choice will affect the magnitude and direction of the velocity bias depending on the symmetry of the line itself and that of the spectrum around the line; higher symmetry will reduce the bias. For all cases simulated, the bias error from the turbulence is below the measurement uncertainty for the validation cases presented in Section 3, and is thus neglected from the analysis.

2.5. Discussion

A limit of all velocimetry techniques is dynamic range. For a given physical configuration, the Doppler frequency shift varies linearly with velocity, i.e., the offset between scattered-light and reference signals increases linearly. With CC-DGV there is no theoretical limit on the maximum velocity measurable, provided a large enough scan range is possible. A range of several gigahertz corresponds to hypersonic velocities and is easily achievable using currently available laser systems. The query range of the cross-correlation determines the extent of frequency shifts at which to calculate the correlation between signals. Higher ranges allow for higher frequency shifts to be found, however increases the processing time. Zero-shift is always included, and the extent of the range is chosen based on the flow of interest, i.e., a value large enough to capture the highest expected velocity fluctuations around the mean. In practice, a velocity dynamic range from 0 m s−1 to over 600 m s−1 has been achieved.
Employing the cross-correlation algorithm for determination of Doppler frequency shift leads to a number of secondary considerations. The absolute intensities of scattered light does not factor into the determination of velocity. The algorithm looks only at the product of the Doppler-shifted scattered light signal and reference signal, and thus the absolute difference in intensity between these two signals does not factor into the result. As such, the scattered light intensity can be different in locations within the flow, such as may be caused by intensity gradients within a laser sheet. Further, different types of linear detectors may be used for each signal. In practice, the reference signal is measured using a single-point photodetector, while the scattered light from the flow is measured with a spatially-resolved camera sensor or detector array.

The robustness of the cross-correlation function leads to dropping the requirements for vapor cell temperature stability. As noted by Forkey et al. [16], the vapor cell transmission ratio is reduced to a unique function of temperature. Absorption spectra at various vapor temperatures are shown in Fig. 6 for an absorption line of molecular iodine, illustrating this point. Temperature mismatches between the cells within a system result in signals of different modulation depth and line width, where modulation depth is defined as the normalized difference between the full transmission intensity and the absorption line minimum transmission. The modulation depth of all signals must, however, be greater than the noise levels inherent in the signals. Should the signals achieve a 100% modulation depth, difficulties arise with the cross-correlation function; such a condition may be avoided by ensuring a low vapor pressure in all cells. Determination of the frequency interval per scan step is computed via comparison with the model spectrum; since absorption line center frequencies do not change with temperature, it is not required to precisely know the temperature of the cell, provided that it does not fluctuate during the scan. The use of the model spectrum to determine frequency intervals also implies that no calibration is required prior to each run. Several researchers, most recently Ecker et al. [21], have shown that the model spectrum adequately matches experimental data.

A challenge in the technique is that the mean seeding level must remain relatively constant during the duration of the laser frequency scan. Fluctuations in scattered light intensity with a timescale much shorter than the camera integration time will be averaged out in each image. Larger fluctuations in seeding levels with time scales on the same order as the camera integration time are seen as random noise, which does not correlate with the reference signal. The requirement for constant seeding is imposed in order to minimize the number of large-scale fluctuations, and to preclude slower drifts of the scattered light intensity, which would not be observed in the non-Doppler shifted reference. Use of a second camera and beamsplitter for each component (for example Meyers [6]) would eliminate the effects of temporally varying scattered light intensity, however would introduce an additional uncertainty source due to image registration between the two cameras. Finally, since the velocity measured from each scan represents a mean value over the duration of the scan, the time resolution of the results is limited by the total scan time.

3. Results

Measurements with the CC-DGV system were performed in two facilities. Boundary layer profiles on a wind turbine blade airfoil were taken in the subsonic Virginia Tech Stability Wind Tunnel and are presented in Section 3.1. In Section 3.2, volumetric velocity maps of a jet produced in the Virginia Tech Hot Supersonic Jet facility are presented.

3.1. Boundary layer profiles

The Virginia Tech Stability Wind Tunnel (Fig. 7) is a closed-loop subsonic wind tunnel with a test section of cross-section 1.83 m by 1.83 m and length of 7.3 m. Turbulence levels are below 0.05%, and the maximum Reynolds number is 5 million per meter chord [22]. Boundary layer profiles were acquired at a chord position of 44% on the pressure side of a DU96-W-180 wind turbine airfoil [23]. The model has a chord length of 0.457 m (18 in.) across the full spanwise length of the tunnel test section. The angle of attack was – 15°, corresponding to deep negative stall. An end-wall flow control system was also used to increase flow uniformity across the span [24].

Near-wall resolution was achieved using a modified form of the “laser fence” technique of Meyers et al. [25]. Single mode fiber optics were stripped and orthogonally cleaved, producing an expanding Gaussian beam profile from the fiber tip. The fibers were routed from outside of the tunnel, through a cavity in the airfoil model, and into surface pressure taps. The fiber tip was set to be flush with the airfoil surface (Fig. 7 inset). This method ensured that the illumination vector was in the direction of the local wall-normal. Scattered light

![Fig. 6. Temperature effect on the model iodine vapor absorption spectrum for a range of temperatures (5 cm cell length). The point labeled as 0 frequency shift corresponds to 563244.039 GHz for the incident light optical frequency. The position of the line center frequency does not change with temperature.](image)

![Fig. 7. Virginia Tech Stability Wind Tunnel in aerodynamic configuration looking downstream with 18° chord DU96-W-180 airfoil model installed (leading edge visible). The inset shows the location of the beam emitted normal to the airfoil surface.](image)
intensity diminishes as distance from the fiber tip increases, so laser power was set such that maximum practicable signal to noise ratio (SNR) was achieved in the boundary layer region of interest. Poly-alpha-olefin (PAO) seeding oil was supplied through a tube in the tunnel sidewall downstream of the airfoil model, and allowed to recirculate through the tunnel loop.

Cameras were placed outside of the test section, both above and below, in the spanwise direction. Scattered light observation directions and laser light illumination vector components are given in Table 1. A schematic of the system is shown in Fig. 8 along with raw images from each camera. SNR for each camera was approximately 40 dB in the beam core for the first centimeter. The cameras were aligned to be parallel to the local model surface direction at the measurement location, in order to minimize laser flare captured from within the pressure taps. Data from each detector unit was extracted along straight-line profiles from the center of the illumination cone and used in the transformation to tunnel axes components. The largest projected spatial dimension per pixel for the three cameras in the beam propagation direction was 180 μm.

In this configuration, the uncertainty in each of the velocity components is given in Table 2 for a representative free stream velocity of 50 ms⁻¹ in the streamwise direction. Contributions to the final uncertainty from each of the sources considered are also reported. The system geometry contribution refers to uncertainty in measuring the scattered light observation vectors, \( \delta \), and incident laser light vector, \( \vec{l} \). These vectors were determined by measuring the position of the cameras relative to the measurement plane. Uncertainties in this primary measurement were 1 cm or less; since stand-off distances were very large, the relative position uncertainty was therefore low.

The raw frequency shift uncertainty was estimated by studying the pixel-to-pixel variation within the camera images, and by comparison with validation results.

PAO seeding oil was atomized using an 8-Laskin nozzle seeder, with a mean particle diameter of approximately 1 μm. A camera integration time of 1000 ms was needed for maximum practicable SNR with the seeder operating at full capacity. 144 frequency scan points with a step size of 64.46 MHz were used. Including a 100 ms dwell time at each frequency, the total scan time for each case was on the order of 3 min.

Profiles were acquired at Reynolds numbers of 1.5 and 2 million, and results are shown in Fig. 9. A custom-built laser Doppler velocimeter (LDV) was used for validation data. The 2-component, long stand-off, fringe-type LDV was traversed in the wall-normal direction corresponding to locations within the DGV profile.

For the LDV results, a 1 W continuous wave laser operating at 532 nm was conditioned to produce three beams that cross approximately 1 m from the optics to form a measurement volume of nominally 200 μm diameter. For backscatter light collection, a 6 in. diameter, 750 mm focal length reflector was coupled to the objective lens of a 6 in. diameter, 1000 mm focal length refractor. Functionally, this optical system serves as a microscope or macro lens. Collecting light with a multi-mode fiber optic allows for cross-sections as small as 200 μm in diameter. The combination of the three beams crossing and the collection optics field of view resulted in a measurement volume of approximately 200 μm × 200 μm × 2 mm. Fringe spacings

### Table 1
Stability Wind Tunnel measurement geometry.

<table>
<thead>
<tr>
<th>Source</th>
<th>x</th>
<th>y</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laser; ( \vec{l} )</td>
<td>0.321</td>
<td>0.947</td>
<td>0.000</td>
</tr>
<tr>
<td>Camera 1: ( \delta_1 )</td>
<td>0.878</td>
<td>-0.224</td>
<td>0.423</td>
</tr>
<tr>
<td>Camera 2: ( \delta_2 )</td>
<td>-0.800</td>
<td>0.353</td>
<td>0.485</td>
</tr>
<tr>
<td>Camera 3: ( \delta_3 )</td>
<td>0.518</td>
<td>-0.095</td>
<td>-0.850</td>
</tr>
</tbody>
</table>

### Table 2
Uncertainties in Stability Wind Tunnel measurements based on 50 ms⁻¹ freestream flow.

<table>
<thead>
<tr>
<th>Uncertainty source contributions</th>
<th>Total uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>System geometry</td>
<td>( \sqrt{(\delta l)^2 + \delta \omega^2} )</td>
</tr>
<tr>
<td>Raw voltage shift</td>
<td>( \sqrt{(\delta l)^2 + (\delta \omega)^2} )</td>
</tr>
<tr>
<td>( U ) 0.26 ms⁻¹</td>
<td>1.28 ms⁻¹</td>
</tr>
<tr>
<td>( V ) 0.11 ms⁻¹</td>
<td>0.63 ms⁻¹</td>
</tr>
<tr>
<td>( W ) 0.44 ms⁻¹</td>
<td>1.02 ms⁻¹</td>
</tr>
</tbody>
</table>

Fig. 8. Camera configuration for boundary layer profile measurements in the Stability Wind Tunnel. Free-stream flow is aligned with the x-axis. Raw images from each camera are shown at right.
of the three fringe sets formed were 2.5 μm, 4.9 μm, and 7.6 μm. Since most sensitivity was in the parallel-to-surface direction (the 2.5 μm fringe set), only these results are presented here for comparison with CC-DGV. The entire optics assembly was mounted to a Velvex Bi-Slide motorized traverse with approximately 12 in. of travel. The light signal coupled into the collection optics fiber optic cable was transmitted to a photomultiplier tube and amplified with a low noise RF amplifier. Signals were digitized using an NI5154 high-speed digitizer. During data acquisition, the digitizer was triggered on single-burst signals, which were stored for subsequent processing. The root-mean-square (RMS) uncertainty of the LDV itself was ±0.4% for instantaneous readings, with a 1% error in the mean dominated by the sample set size and the high turbulence level of the flow.

A high degree of agreement is observed between CC-DGV and LDV measurements. Root-mean-square (RMS) deviations from the LDV profiles were computed for each case. For the $Re = 1.5 \times 10^6$ and $Re = 2 \times 10^6$ cases of Fig. 9, the RMS deviations were 2.77 ms$^{-1}$ and 1.34 ms$^{-1}$, respectively.

### 3.2. Supersonic jet velocity maps

The capability of using the CC-DGV system to reconstruct spatially-resolved velocity maps in three dimensions was leveraged in the Virginia Tech Hot Supersonic Jet facility [18] (Fig. 10). The jet uses a 38.1 mm (1.5 in.) exit diameter converging/diverging biconic nozzle, for a design Mach number of 1.65, with design nozzle pressure ratio $NPR = P_0/P_a$, where $P_0$ is the total pressure of the jet, and $P_a$ is the ambient pressure surrounding the jet) of 4.58. A 192 kW flanged in-line heater is located upstream of the nozzle to provide total temperature ratios (TTR ≡ $T_0/T_a$, where $T_0$ is the total temperature of the jet, and $T_a$ is the ambient temperature surrounding the jet) up to 3 at reduced flow rates.

Two detector units and two laser illumination directions were used (Fig. 11). The detector units were placed normal to the streamwise direction on opposite radial sides of the jet, each facing inward towards the flow. The largest projected spatial dimension per pixel for the two cameras was 57.2 μm and 54.0 μm in the image-frame horizontal and vertical planes, respectively. Laser sheets were formed and set to illuminate the same location within the flow. Both sheets were nominally contained in the $x$-$y$ plane of Fig. 11 ($± 1°$), launched from below the jet axis. One sheet was formed from a location just below the jet exit nozzle and launched downstream, while the other sheet was formed further downstream and launched upstream towards the jet nozzle. The scattered light observation and laser illumination vectors are shown in Table 3. A sample raw pixel signal is shown in Fig. 12 for the camera 1 laser 1 pair, along with the results for the cross-correlated Doppler shift for this pixel. All detector units and laser sending optics were mounted to a 2-axis traverse system to allow movement of the entire system in the streamwise and transverse radial directions ($x$- and $z$-directions of Fig. 11).

At every spatial measurement position, each laser sheet was independently imaged by both detector units. This provided four measured components of velocity; the three components with the consistently highest signal-to-noise ratios (SNR) were chosen for processing. Solid alumina seeding with a nominal particle size of 0.7 μm was supplied via a fluidized bed seeder through a port in the plenum. The camera integration times were set independently for each camera in order to maximize the SNR of the image; integration times ranging from 500 ms to 1500 ms were used depending on the observed scattered light intensity prior to each acquisition. 100
frequency scan points with a step size of 32.23 MHz were used. Including a 100 ms dwell time at each frequency, the total scan time for each plane ranged from 2–4 min, depending on acquisition time. Note that two scans, one with each laser illumination direction, were needed to determine the full three-component velocity vector.

Uncertainties for the velocity components are given in Table 4. The sensitivity of the cross-correlation algorithm and peak-finding subroutine are independent of velocity magnitude. Therefore, the uncertainty from this source, when compared with that of the subsonic boundary layer measurements, is only influenced by the terms of the $A$ matrix. System geometry uncertainties are larger for the supersonic jet measurements than for the subsonic boundary layer measurements due to the close proximity of the cameras to the measurement plane.

Volumetric data were acquired at cold conditions ($TTR = 1.0$) for a nozzle pressure ratio of $2.7 \pm 0.03$ and jet exit plane Mach number $M_j = 1.3$. Planar measurements normal to the nozzle exit plane at several radial positions were acquired at the same downstream station. These planar maps were then used to construct three-dimensional volumetric maps of the flow. Results are shown in Fig. 13 as iso-surfaces of constant velocity. For $U_j/U_f$ of 1.0, shock cells can be seen in the jet core as higher velocity regions, consistent with the over-expanded nature of the jet. The shear layer can be discerned in Fig. 13 as well; there is a lower radial spacing between the iso-surfaces at the lower normalized velocities, indicating a steeper spatial gradient.

Measurements were also taken at heated conditions ($TTR = 2.0$) at a nozzle pressure ratio (NPR) of $2.7 \pm 0.03$, with $M_j = 1.3$. In this case, the measurement plane bisected the tunnel centerline (as shown in Fig. 11), and was traversed in the streamwise, rather than transverse, direction. Radial profiles are presented in Fig. 14 for axial positions between $x_D$ of 0.7 and 2.6, by increments of $x_D=0.05$. A high degree of variability in the potential core as a result of the shock/expansion train of the biconic nozzle geometry is apparent, similar to a result of Powers et al. [27] (the same phenomenon is visible in Fig. 13 for $U_j/U_f = 1$ along the jet centerline).

The developing shear layer can be further analyzed by similarity scaling of the radial position, as shown in the right hand axes of Fig. 14. Here, the non-dimensional parameter $n_f$ is defined as

$$n_f = \frac{r - r_{0.5}}{x}$$

(11)

where $x$ is the streamwise position of the profile relative to the nozzle exit, $r$ is the radial distance from the jet centerline and $r_{0.5}$ is the radial position at which the mean velocity equals one half of the jet exit velocity, $U_f$. Lau et al. [27] showed that this scaling successfully collapses compressible shear layer data in round jets, while other researchers have corroborated this result [28, 29]. The results for this scaling indicate the expected trend that the outer shear layer scales well along increasing axial position. This fact provides a self-validation of the technique based upon the physics of the linearly-growing shear layer [27]. The inner portions of the shear layer, however, exhibit some interesting trends due to the compression/rarefaction of the imperfectly expanded jet. Notable is that the scaling holds for up to approximately 75% of the jet exit velocity. Above this value, the local influence of shock/expansion waves causes the scaling to fail since the scaling of Eq. (11) only accounts for free shear effects and not for rapid streamwise distortion.

Table 3. Uncertainties in supersonic jet measurements based on 500 ms $^{-1}$ freestream flow.

<table>
<thead>
<tr>
<th>Uncertainty source contributions</th>
<th>Total Uncertainty $\sqrt{\sum (\text{source})^2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>System geometry</td>
<td>$\sqrt{(6\Delta y^2)^2 + (\Delta \omega)^2}$</td>
</tr>
<tr>
<td>Raw voltage shift</td>
<td>$\sqrt{(\Delta x^2)^2 + (\Delta \omega)^2}$</td>
</tr>
<tr>
<td>$U$ 4.43 ms$^{-1}$</td>
<td>0.63 ms$^{-1}$</td>
</tr>
<tr>
<td>$V$ 16.91 ms$^{-1}$</td>
<td>0.84 ms$^{-1}$</td>
</tr>
<tr>
<td>$W$ 0.16 ms$^{-1}$</td>
<td>0.47 ms$^{-1}$</td>
</tr>
</tbody>
</table>

![Fig. 12](image-url) A single pixel signal for the $(b_1 - i_1)$ component is shown at left, compared with the non-Doppler shifted reference signal. The Doppler shift for this signal is shown graphically at right along with the result of the peak-finding algorithm.
4. Conclusions and outlook

Cross-correlation Doppler global velocimetry is a new DGV variant that improves robustness and dynamic range with a one-camera-per-component scheme that is independent from absolute scattered light intensity. Vapor absorption spectrum line center locations are independent of temperature and scattered light intensity, and thus the cross-correlation signal is not influenced by relative differences among the various sensors. CC-DGV has been demonstrated in two facilities at different flow regimes. Boundary layer profiles on an airfoil were acquired in a large-scale subsonic facility and validated with LDV measurements. Volumetric measurements were also taken in a supersonic jet, with measured mean velocities over 600 ms$^{-1}$, and validated with a well-accepted physics-based scaling from literature.

One aspect unique to CC-DGV is that there is no theoretical upper limit on measurable velocity. The large frequency domain of the scan allows for measurements to the hypersonic regime; with current equipment, a dynamic range from 0 ms$^{-1}$ to 3000 ms$^{-1}$ is notionally possible. All pixels in a data set are processed independently, and no a priori knowledge of velocity is needed. As such, high velocity gradients can be detected, such as would be expected in relevant shear flows.

CC-DGV has potential applicability to a wide variety of applications, specifically high-speed flows and large-scale facilities. In jets and high-speed separated flows, where there are expected to be disparate velocity regimes, the dynamic range of the technique allows for global context measurements. In large-scale facilities, the light collected by cameras is greatly reduced due to greater stand-off distances. In this case, increasing camera integration time at each frequency scan point represents an advantage, as the integration time can be optimized to provide sufficient SNR. CC-DGV is an expansion to the DGV/PDV implementation schemes, allowing for measurements in previously difficult applications. Related methods, such as 2ν-DGV, FSK-DGV, and FM-DGV, remain the optimal choice for smaller scale and time-resolved measurements.

Fig. 13. Iso-surfaces of constant velocity in the Hot Supersonic Jet at a total temperature ratio TTR = 1.0 and nozzle pressure ratio NPR = 2.7. The color scaling indicates radial distance from the jet centerline.

Fig. 14. Shear layer profiles for $0.7 \leq x/D \leq 2.6$ by 0.05 increments in the Hot Supersonic Jet at a total temperature ratio TTR = 2.0 and nozzle pressure ratio NPR = 2.7. Left: Radial scaling on nozzle exit diameter. Right: Radial similarity scaling based on linear shear layer growth [26].
Since the time resolution is limited to the acquisition time for a frequency scan, the system has thus far only been validated for mean measurements. Further work is needed to increase scattered light signal levels in order to reduce scan times. The most promising path forward is to increase incident laser power. In both configurations presented, the maximum available laser power was limited by the damage threshold of the fiber optics used for beam sending. Elimination of the fiber optics from the beam path can increase total laser power in the measurement location by an order of magnitude. Nevertheless, temporal resolution is at present a limit of the technique.

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References


Chapter IV.

Investigation of measurement sensitivities in cross-correlation Doppler global velocimetry

This chapter presents the characterization of the CC-DGV instrumented, published as “Investigation of measurement sensitivities in cross-correlation Doppler global velocimetry” in Optics and Lasers in Engineering (2016) (86 44:52, DOI 10.1016/j.optlaseng.2016.05.003)

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Investigation of measurement sensitivities in cross-correlation Doppler global velocimetry

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A B S T R A C T

Cross-correlation Doppler global velocimetry (CC-DGV) is a flow measurement technique based on the estimation of Doppler frequency shift of scattered light by means of cross-correlating two filtered intensity signals. The signal characteristics of CC-DGV result in fundamental limits for estimation variance as well as the possibility for estimator bias. The current study assesses these aspects theoretically and via Monte Carlo signal simulations. A signal model is developed using canonical numerical functions for the iodine absorption cell and incorporating Poisson and Gaussian signal noise models. Along with consideration of the analytical form of the Cramér–Rao lower bound, best practices for system settings are discussed. The CC-DGV signal processing routine is then assessed by a series of Monte Carlo simulations studying the effect of temperature mismatch between flow signal and reference detector cells, velocity magnitude, and discretization error in the frequency modulation. A measurement bias was observed; the magnitude of the bias is a weak function of the cell temperature mismatch, but it is independent of the flow velocity magnitude. The measurement variance was found to approach the Cramér–Rao lower bound for optimized conditions. A cyclical bias error resulting from the discrete nature of the laser frequency sweep is also observed with maximum errors of ±1.0% of the laser frequency scan step size, corresponding to peak errors of ±0.61 m s⁻¹ for typical settings. Overall, the signal estimator is found to perform best for matched cell temperatures, small frequency step size, and high velocity regimes, where the relative bias errors are collectively minimized.

1. Introduction

Doppler global velocimetry (DGV) is a measurement technique capable of providing non-intrusive, spatially resolved flow field measurements [1]. While advances in other measurement systems such as particle image velocimetry (PIV) have appropriately found widespread acclaim and opened new avenues of flow research, several aspects of DGV warrant continued interest. Specifically, the available dynamic range and comparatively reduced restrictions on particle resolvability enable its use in optically challenging facilities [2].

Since the introduction of DGV, several modifications and variants have been developed. A significant development came with the transition to high power continuous wave lasers, which offer narrower bandwidth and better frequency stability than pulsed laser systems [3]. With pulsed systems, velocity uncertainty of 2 m s⁻¹ was once considered the state of the art [4]. In 2004, a system designated the “two-frequency (2f) planar Doppler velocimetry” method was introduced, marking another phase in the evolution of the technique [5]. In this approach, the output frequency of a single laser is tuned to two frequencies in situ during data acquisition; a single detector acquires both the Doppler shifted signal image and the reference image in a full-transmission region of the vapor absorption spectrum. This single detector scheme was also employed in “frequency-modulated (FM) DGV” and the related technique “frequency-shift-key DGV,” the former of which can achieve minimum velocity standard deviations of 0.02 m s⁻¹ [6,7].

Cross-correlation DGV (CC-DGV) was recently introduced by the present authors as a complementary approach to related multiple frequency methods, particularly valuable in the high-speed flow regime [8]. As with two-frequency (2f), frequency-modulated (FM), and frequency-shift-key DGV, a single camera is required per component of velocity measured, thus eliminating errors due to camera registration and uneven beam-splitting effects.

All DGV techniques are based on the well-known Doppler shift equation

$$\Delta f = \frac{\Delta \nu}{c} \nu$$

(1)
where $\Delta \omega$ is the Doppler shift frequency of scattered light, $\hat{\omega} - \hat{i}$ is the directional sensitivity vector made up of the difference of the scattered light direction $\hat{\omega}$ and the incident laser light direction $\hat{i}$ unit vectors, $\hat{V}$ is the flow velocity vector for the scattering medium, and $\hat{i}$ is the incident laser wavelength. In its present form, the equation is valid with the assumptions that the local measured velocity is much less than the speed of light, and that the magnitude of any modulation of the laser frequency is much less than the laser frequency itself [9]. The former assumption is accurate on the order of parts per million in supersonic flows. The latter condition, which applies to several forms of DGV including two-frequency (2$v$) DGV, frequency-modulated (FM) DGV, and cross-correlation (CC) DGV, is similarly valid to parts per million or lower.

Absorption-based DGV techniques directly measure of Doppler frequency shift using a vapor absorption cell as a transfer function between light intensity and Doppler shift frequency [$\Delta \omega$ in Eq. (1)]. Systems employing 532 nm lasers and molecular iodine vapor cells have become common in DGV systems due to the tunability of the laser frequency and the presence of multiple, readily described features in the iodine absorption spectrum [10]. The process by which the Doppler shift frequencies are determined from raw intensity signals differ amongst the above-mentioned DGV techniques.

The focus of the present study is on sensitivities specific to cross-correlation DGV and the signal processing routines involved therein, such that CC-DGV is the only technique considered. As has been previously done for FM-DGV, the article investigates the effects of Doppler shift determination only, without consideration of the sensitivity vector [9]. For an analysis of the influence of the sensitivity vector, the reader is referred to Charrett et al. [11].

CC-DGV measures spatially resolved velocities averaged over the frequency scan time of the incident laser. Light from a continuous wave laser with narrow bandwidth is discretely modulated in frequency over a range on the order of several gigahertz. The components of the CC-DGV system are depicted in Fig. 1. Two beam paths of unequal power are established by means of a polarizing beamsplitter; the primary path is sent through a flow seeded with small particles, and the second is used for reference signals. The reference beam path is further split with a 50–50 beamsplitter. One reference path is directed onto a photodiode sensor to monitor stability of the beam power. The second reference beam is sent through a reference iodine vapor absorption cell and onto a second photodiode sensor. This signal serves as a non-Doppler shifted reference for the absorption spectrum and is used in subsequent signal processing. At each frequency point, Mie-scattered light from the primary beam in the flow is collected and filtered through an iodine vapor absorption cell before being imaged by a detector camera [8].

The cross-correlation DGV processing routine is shown schematically in Fig. 2. Doppler frequency is determined via cross-correlation of the non-Doppler shifted reference signal with the time-history signal from each pixel of the detector sensing the Mie-scattered, Doppler shifted light from the flow region. At each instant in time, the normalized transmission through the iodine vapor absorption cells in the Mie-scattered and reference beam paths will be separated as a function of the Doppler frequency shift. Typical camera integration times are on the order of several hundred milliseconds, and total scan times are on the order of about two minutes; therefore, the signal received must be considered a time mean. Several particles passing through the measurement volume of each pixel at potentially different velocities may contribute to each intensity reading [2]. This effect from unsteadiness and turbulence means that the intensity signal may be biased either higher or lower depending on the laser frequency relative to the absorption spectrum at that point, and thus cannot be mitigated by systematic calibration. Note, however, that even in the worst cases at high speeds and turbulence levels, bias errors from turbulence on the order of 0.1% of the mean are typical [8].

As noted by Forkey et al., the transmission ratio through pure iodine vapor in an absorption cell is dependent on cell length (Beer’s law), cell body temperature, and pressure within the cell [10]. For sealed cells of fixed size, the transmission becomes a function of temperature only, since vapor pressure itself is dependent on temperature. However, the line “center” location, where minimum transmission is achieved, is independent of temperature and remains constant. This characteristic enables CC-DGV to be performed without temperature stabilization of the vapor cells, as the Doppler shift can still be determined.

The aim of the present paper is to investigate and quantify...
measurement biases and ascertain where CC-DGV can be best applied. First, the signal model and Cramér–Rao lower bound are described in Section 2. The signal processing routine is then discussed in Section 2 along with the optimization of the signal and estimator. The results of several Monte Carlo simulations using synthetic signals are presented in Section 3, quantifying the bias errors arising from cell temperature mismatches and velocity-dependent discretization errors in the frequency sweep. A discussion of the implications on the CC-DGV method is finally offered in Section 4.

2. CC-DGV signals

2.1. Signal model

The CC-DGV signal is typically composed of scattered light intensity signals. The governing equation for the measurement technique is the Doppler shift equation Eq. (1), as described in Section 1. Laser light is scattered according to Mie theory, filtered through an iodine vapor cell with transmission function $r(f)$, where $f$ is the light frequency, and collected on a photodetector, typically a CMOS sensor. The photon arrival of $n_s$ scattered photons on the camera sensor is modeled as a Poisson process with the sensor quantum efficiency $\text{QE}$, yielding the signal $S$ sampled at each index $k$.

$$S(k) = \text{QE} n_s r(f_k)$$ (2)

2.2. Cramér–Rao lower bound

The theoretically lowest possible variance for an unbiased data set is determined by the Cramér–Rao lower bound (CRLB) [12–15]. A generalized derivation of the CRLB for several Doppler-based techniques including DGV is presented by Fischer et al. [16]. The CRLB is defined as the diagonal elements of the Fisher information matrix inverse, given by Fischer et al. as

![Image of cross-correlation and peak finding routines](image-url)
where \( p \) is the discrete joint probability function, \( y \) is the signal under consideration and \( \theta \) is a vector of unknown quantities in the measurement. For DGV in particular, two elements of \( \theta \) are used; the first represents the quantity of interest, either the velocity vector [16] or the Doppler shift frequency (neglecting contributions for geometric uncertainty) [9,17], and the second being a function of the scattered light power.

\[
\theta = \begin{bmatrix} \Delta f \\ f(P) \end{bmatrix}
\]

(4)

Two noise sources are considered by Fischer et al. [16]: “shot noise” with Poisson distribution, and electronic detector noise with Gaussian distribution. Since these sources are independent of one another, they can be considered separately and their final measurable variances added together after taking the inverse of Eq. (3). The CRLB can alternatively be presented in terms of signal-to-noise ratio (SNR) by summation of the signal strength compared to the noise bandwidth.

\[
\text{CRLB}(U)_{\text{Poisson}} = \frac{\sum_{k=0}^{N-1} r(f_k)^2}{\left( \sum_{k=0}^{N-1} r(f_k^2) \right)^2} \frac{\text{QE}}{2}\sigma_n^2
\]

\[
\text{CRLB}(U)_{\text{Gauss}} = \frac{\sum_{k=0}^{N-1} r(f_k)^2}{\left( \sum_{k=0}^{N-1} r(f_k^2) \right)^2} \frac{\text{QE}}{2}\sigma_n^2
\]

\[
\text{CRLB}(\Delta f) = \text{CRLB}(\Delta f)_{\text{Poisson}} + \text{CRLB}(\Delta f)_{\text{Gauss}}
\]

(5c)

\( r(f_k) \) is the normalized transmission of the vapor cell as a function of laser center frequency at signal index \( k \) for given cell conditions, QE is the quantum efficiency of the photodetector, and \( \sigma_n^2 \) is the variance of the electrical signal. It can be shown that a Gaussian distribution can serve as an approximate model for the Poisson photon shot noise, while also allowing for the incorporation of additional noise sources, collectively modeled with a constant variance [17,18]. Fischer et al. [17] describe the Gaussian noise source with a constant value of

\[
\sigma_n^2 = \left( \frac{\text{QE}}{h} \right)^2 (\text{NEP})^2 f
\]

(6)

where \( n_e \) is the number of electrons generated by the Poisson process in the detector element, \( T_e \) is the exposure duration, \( h \) is Planck’s constant, and \( \text{NEP} \) is the noise equivalent power of the sensor. The additional reference detector present in CC-DGV is ignored in this analysis. The secondary laser path directly illuminates the reference sensor, and as a result, the optical frequency differs between the reference signal and the Doppler shifted signal at the same instant in time. Whereas Eq. (5c) is derived for CRLB \( U \) of velocity (via the Doppler shift frequency), the frequency of the light measured by the reference detector is known within the resolution of the laser bandwidth and only the noise power in unknown. The reference signal is based on a greater number of photons compared to the corresponding Doppler shifted signal due to the direct illumination of the reference detector, and will inherently have a lower variance than the scattered light flow signal. The Doppler shifted signal will then always be the dominant source of uncertainty, in addition to uniquely containing information on the Doppler shift \( \Delta f \).

In order to benchmark the performance of the signal estimator in CC-DGV, the CRLB has been estimated for absorption line features at several side-arm temperatures. The vapor cell side-arm is a small extension of the gas-containing body of the cell that contains solid iodine and allows a location at which to apply temperature regulation. The vapor pressure is determined using the vapor pressure equation of Forkey et al. [10], for example as 26.627 Pa (0.19972 Torr) at 20 °C side arm temperature. Varying the vapor pressure in the cell effectively changes its optical thickness—higher pressure is thicker, and thus more absorptive, than lower pressure. Due to the complexity of the exact analytical forms of typical signals [10], the derivatives in Eqs. (5a) and (5b) were estimated numerically with the code developed by Forkey et al. for a 20 °C side-arm temperature.

The minimum variance theoretically possible for a given number of scanning samples can be computed. The factors influencing the measurement CRLB are ultimately signal-to-noise ratio, number of samples \( N \) in the signal, and the selection of the absorption line of interest and its features (contained within the function \( r(f_k) \) in Eqs. (5a) and (5b)). The optimization of these parameters is discussed later in Section 2.4, within the framework of the overall system optimization.

2.3. Signal processing routine

The cross-correlation function as used in CC-DGV is given as

\[
R_{(\nu)} = \frac{1}{\sqrt{\sigma_{\text{pix}}^2 \sigma_{\text{ref}}^2 N}} \sum_{j=1}^{N} S_{\text{pix}}(\eta_j)S_{\text{ref}}(\eta_j-\eta_i)
\]

(7)

where \( S_{\text{pix}} \) and \( S_{\text{ref}} \) are the Doppler shifted pixel signal and unshifted reference signal, respectively, and \( \sigma_{\text{pix}}^2 \) and \( \sigma_{\text{ref}}^2 \) are the respective signal variances. \( \eta \) is the incident (unshifted) light frequency at \( N \) points in the scan, and \( \nu_i \) and \( \nu_j \) are the correlation query points over which the correlation is summed to yield the correlation \( R_{(\nu)} \).

In order to determine the precise shift frequency, a second order polynomial \( R(\nu) \) with coefficients \( P \) is fit around the peak value and two points on either side, as described by

\[
\hat{R}(\nu) = P_2 \nu^2 + P_1 \nu + P_0
\]

(8)

The frequency shift corresponding to the peak of the polynomial function represents the time-averaged Doppler frequency shift in the measurement volume, as found from

\[
\Delta f = -\frac{P_2}{2P_1}
\]

(9)

The remainder of this paper focuses on assessing the variance and bias of this signal processing routine.

2.4. Optimization of algorithm

Based on the processing algorithm described above, the instrument parameters can be analyzed for identification of the optimal acquisition settings. Three choices in particular are considered here: peak-finding subroutine, number of samples \( N \), and scan bandwidth.

2.4.1. Peak-finding subroutine

The parabolic peak-finding subroutine was observed during initial development to have higher accuracy than a weighted
average approach. The choice of how many points around the peak value is free to be chosen during processing. Enough points must be used to properly capture the curvature of the peak region, while too many points extending into possible asymmetric regions of the correlation peak would yield false results. The number of samples directly relates to the choice of the parabola width; ultimately a parabola half-width of two sample points was found to be sufficient, as determined using the signal model and simulation approach developed below.

2.4.2. Number of samples

A key consideration for the number of samples arises from the Cramér–Rao lower bound, that for both Poisson and Gaussian noise the minimum uncertainty decreases with increased samples. Here, the number of samples refers to the number of discrete laser frequency values at which data is collected. Considering the signal processing routine, the correlation coefficient peak is better defined for a finer grid of acquisition points. Since CC-DGV yields mean velocities for typical camera integration times, there is no requirement for an acquisition to be below a given timescale of the flow. In practice, the maximum number of samples possible should be used. The primary restriction arises from data storage considerations, i.e. memory and disk space on the data acquisition system.

2.4.3. Scan bandwidth

The scan bandwidth is defined as the range over which the laser frequency is scanned during acquisition. The maximum possible bandwidth using present equipment is 10 GHz. A typical absorption line feature is on the order of 1 GHz, and thus instrumentation is not a limiting factor. The information of interest in the signal is contained in the rising and falling slopes of the absorption features, so the minimum bandwidth needed would be that defining one side of an absorption line. Consider Eq. (7) for a fixed spacing of frequency samples and constant signal noise. The bandwidth can be increased by adding samples to resolve additional absorption lines. This in turn increases the magnitude of the correlation coefficient. Thus the optimum would be the full width of an absorption feature, or an integer number of absorption features. This is similar to the result found by Fischer et al. for FM-DGV, wherein the optimal modulation amplitude is the full width at half maximum [6]. However, since for CC-DGV the full absorption feature needs to be observed both by the unshifted reference detector as well as the Doppler shifted signal cameras, the total scan bandwidth must be wider than the lines. Based on the expected velocity magnitude and direction, including turbulent fluctuations, the expected maximum Doppler shift can be estimated, and the bandwidth and center frequency set accordingly.

3. Monte Carlo simulations

A number of Monte Carlo simulations were performed to determine the sensitivities of the CC-DGV instrument to various test parameters and environmental conditions. The simulations are intended to study the effects of processing errors related to the vapor absorption cell characteristics, as well as measurement noise in the frequency shift term on the left hand side of Eq. (1). Velocity results are to be understood as the component of the velocity vector projected along the ideal \( \hat{\alpha} \) sensitivity vector direction of an optimally positioned detector (see Fig. 1).

3.1. Synthetic signal model

The first step in carrying out simulations was the creation of a model for synthetic data based on Eq. (2). The Poisson photon detection was modeled as a conditional probability that could be computationally implemented as a discrete random variable with uniform distribution \( x(k) \).

\[
P(k) = \frac{\varepsilon(f_k) \sum_{x(k) \leq 0} p(x(k))}{\sum_{x(k) \leq 0} p(x(k))} = \begin{cases} 1; & 0 \leq x(k) \leq 1 \\ 0; & \text{otherwise} \end{cases}
\]

The iodine vapor absorption process is highly non-linear, and described in detail by several researchers including [4,10,19]. The transmission function \( \varepsilon(f) \) for each signal was created using a program developed by Forkey et al. that generates iodine transmission spectra based on temperature, pressure, and length of the cell, along with a specified incident laser frequency range [10]. Both the non-Doppler shifted reference and the Doppler shifted signals are created synthetically with this program, the latter shifted in frequency according to Eq. (1). A maximum number of \( n = 10^{47} \) photons per case is simulated, roughly corresponding to the saturation level of the cameras used in previous CC-DGV work.

To create the Doppler shift, a large bandwidth of the absorption spectrum is generated and a velocity is prescribed, with the Doppler frequency shifted spectrum calculated a priori. The maximum and minimum of frequency range of the Doppler shifted signal are set relative to the non-shifted reference signal. In this way, there is no cycling or carry-over of intensity values at the ends of the frequency range of the signal, and large Doppler shifts (order of several gigahertz) can be simulated while maintaining dependence to the theoretical spectrum. A constant Gaussian detector noise is added to the signal with a constant variance equivalent to 100 photons. The quantum efficiency of the sCMOS cameras previously used with CC-DGV is QE = 0.57, and the analog-to-digital (A/D) conversion factor is 0.46 e/\text{count}.

The CRLB is computed using the flow signal only, and not accounting for the noise-free reference signal. The SNR is computed a priori to allow for ease of interpretation of the results. The SNR, in dB, is defined using the variance of the noise-free scattered light input signal \( S \), Poisson contribution \( P \), and Gaussian contribution \( G \) to the total signal as follows:

\[
\text{SNR} = 10 \log_{10} \left( \frac{\sigma^2_S}{\sigma^2_P + \sigma^2_G} \right)
\]

The input signal \( S \) is found by scaling the transmission function \( \varepsilon(f) \) with the number of scattered photons, the quantum efficiency, and the inverse of the A/D conversion factor. For the Poisson signal variance, the mean number of received photons for each data point is defined as the Poisson parameter \( \lambda_P \), and the signal variance equals the expected value \( E(P) = \lambda_P = 10^2 \) [20]. Note that the A/D conversion factor is included for each signal when calculating variance to allow comparison with the simulations.

For each study presented, \( N = 101 \) frequency scanning samples were used; the portion of the unshifted spectrum selected for the studies was centered at 18788.375 cm\(^{-1}\) (563.261 THz), with a total bandwidth of 0.380 cm\(^{-1}\) (11.392 GHz). An example synthetic signal at a 20°C side-arm temperature with an applied Doppler shift of 93.985 MHz (corresponding to 50 m s\(^{-1}\)) are shown in Fig. 3 for the noise-free case and for the full noise model with SNR = 35.7 dB. The noisy signal is normalized by the product of the number of scattered photons, the quantum efficiency, and the inverse of the A/D conversion factor.

The Poisson and Gaussian combined signal model was applied independently to each run to create a unique signal. The signal estimator routine used for all simulations includes both the cross-correlation and parabolic peak-finding steps discussed in Section 2.3. For statistical convergence, 10,000 independent iterations of each case were performed within each study in Sections 3.1.1, and 1000 independent iterations of each case for Section 3.1.2.
latter, the standard error of the mean was on the order of 0.01 m s\(^{-1}\) for 1000 iterations, sufficient for the objective of determining discretization-related mean bias.

### 3.1.1. CC-DGV sensitivity to instrumentation stability and noise

In large-scale facilities and real environments, precise regulation of vapor cell temperatures creates an additional level of system complexity, and may not be fully realizable. This effect of changing absorption due to changing cell conditions is muted in systems utilizing ‘starved’ cells [19]; however, such cells were not available in implementations for which past results have been reported using the technique. For the present simulations, the temperature of the reference detector cell is kept at 20 °C. Nine temperatures for the flow signal vapor cell (see Fig. 1, the cell in front of the camera) are simulated, ranging from 0–40 °C. The transmission spectra at the temperatures used are shown in Fig. 4. The length of both cells is kept constant at 5 cm, and the vapor pressure of iodine in the cell is determined as a function of the temperature. For each cell temperature, cases are run at three velocities, 5 m s\(^{-1}\), 50 m s\(^{-1}\), and 500 m s\(^{-1}\), representing three distinct velocity regimes. The velocity here represents the camera component velocity, i.e. the velocity measured along the $\hat{\delta} = \hat{i}$ sensitivity vector direction. CC-DGV has previously been demonstrated by the present authors in velocity regimes including low speed subsonic boundary layer flows and supersonic heated jet flows, where it was suggested that the sensitivity of the processing routine is independent of velocity regime [8].

![Fig. 3. Sample noise-free and SNR=35.74 dB synthetic signal at 20 °C side-arm temperature with 93.985 MHz applied Doppler shift.](image1)

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>SNR (dB)</th>
<th>$\mu_0$ (m (s^{-1}))</th>
<th>$\mu_n$ (m (s^{-1}))</th>
<th>$\mu_0$ (m (s^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<td>2.264</td>
<td>47.893</td>
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</tr>
<tr>
<td>5</td>
<td>27.3</td>
<td>2.720</td>
<td>48.429</td>
<td>497.602</td>
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<tr>
<td>10</td>
<td>30.6</td>
<td>3.242</td>
<td>49.009</td>
<td>498.292</td>
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<tr>
<td>15</td>
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<td>3.775</td>
<td>49.656</td>
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<td>20</td>
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<td>4.341</td>
<td>50.371</td>
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<tr>
<td>25</td>
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<td>4.954</td>
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<td>30</td>
<td>38.9</td>
<td>5.644</td>
<td>52.120</td>
<td>501.337</td>
</tr>
<tr>
<td>35</td>
<td>39.9</td>
<td>6.463</td>
<td>53.234</td>
<td>502.471</td>
</tr>
<tr>
<td>40</td>
<td>40.5</td>
<td>7.489</td>
<td>54.603</td>
<td>503.875</td>
</tr>
</tbody>
</table>

A logarithmic range of scattered photons from $n_0=10^2$ to $10^4$ was studied, ultimately corresponding to a peak SNR value of approximately 40.5 dB. Since the velocity is prescribed, the bias errors can be readily evaluated, and are presented in Table 1 as the mean value of 10,000 iterations for the $n_0=10^4$ scattered photon case. The bias error appears as a weak function of temperature, with higher temperatures producing stronger positive biases and vice versa. The temperature at which no bias exists is different for each of the three velocities, and can be explained by the discretization bias error discussed in Section 3.1.2, which represents a fixed bias error that is velocity dependent; the peak discretization bias error for the present simulation conditions was $\pm 0.61$ m s\(^{-1}\), see below. Given the magnitudes of the bias errors observed for mismatched cell temperatures, it is clear that a bias arises purely from the temperature effect. However, the magnitude of the total bias is essentially constant for the three velocity magnitudes for a given temperature. This provides the greatest benefit in the high-speed regime, where the relative bias is correspondingly lowest. This result, along with the allowance for high spatial gradients in scattered light intensity as discussed in [8], enables CC-DGV to be used in flows with both large ranges of velocities and large velocity gradients and confirms the consistent behavior across velocity regimes.

The simulation-specific CRLB results are plotted in Fig. 5, and are independent of Doppler shift magnitude. From Eq. (5c), there is a direct dependence on number of scattered photons, and the CRLB is seen to decrease with increasing SNR. The CRLB also decreases with increasing vapor cell body temperature. This is captured in the transmission function $r(f_k)$ and derivative term $r'(f_k)$, where at higher cell temperatures the transmission is lower at the line center, and the slope is greater at the line edges. Results for the measured velocity variance $\sigma_{\alpha}^2$ and the ratio of CRLB over velocity variance computed for each velocity case are shown in Fig. 6. Values for the variance and CRLB for the highest SNR case at each temperature and velocity magnitude are given in Table 2. The measurement variance decreases with increasing number of scattered photons and vapor cell temperature, the same trends as the CRLB.

Considering first the $U = 50$ m s\(^{-1}\) and $U = 500$ m s\(^{-1}\) cases in Fig. 6, the ratio of CRLB over measurement variance converges to unity, implying that the signal estimator is efficiently reaching the minimum possible uncertainty. Some results are shown to have variances a few percent higher than the CRLB; these minor discrepancies are believed to be the result of the biased estimator as well as neglecting the noise-free modeled reference signal in the CRLB. The variances of the two hottest cell temperature cases (35 °C and 40 °C) are both higher than the CRLB by up to about 15% for the peak SNR case. Referring to Fig. 4, the transmission curve approaches zero transmission at the center of one or both
absorption lines for these temperatures, causing an ambiguity in the CC-DGV processing algorithm and leading to elevated variance. To avoid this issue, minimum vapor cell transmission ratios of 5–10% should be used with CC-DGV. The case deviates from the results of the other two velocities, with variance ranging over 50% higher than the CRLB. This is due in part to the low magnitude of the Doppler frequency shift—the cross-correlation function in Eq. (7) for this velocity finds the strongest correlation at zero-shift, and the peak-finding subroutine is then solely responsible for detecting the shift. For all of the cases studied, the Poisson noise was the dominant contribution to the measurement variance. For the lowest SNR case, the Poisson variance was about 7 times the Gaussian variance for matched cell temperatures, and about 350 times the Gaussian variance at the highest SNR case.

3.1.2. Discretization errors

The laser frequency sweep performed during data acquisition is done in a discretized manner in order to allow sufficient time for light collection at each laser frequency sample by the receiving optics; this is the main motivation behind the parabolic peak-finding routine of Eqs. (9) and (10). A bias error dependent on the discrete frequency step size is expected as a result of the non-linearity of the vapor transmission spectrum about the line minima. The magnitude of this bias is assessed by simulating signals at a constant number of scattered photons, SNR=34.7 dB, and 20 °C side-arm temperature vapor cells for a range of camera component velocities at higher resolution than those corresponding to integer values of the discrete frequency scanning samples. As such, the role of the parabolic peak-finding routine is effectively isolated for study. Results for the frequency precision-dependent bias are shown in Fig. 7, as the normalized

<table>
<thead>
<tr>
<th>°C</th>
<th>SNR [dB]</th>
<th>( \sigma^2 ) [m^2s^-2]</th>
<th>( \sigma^2 ) [m^2s^-2]</th>
<th>( \sigma^2 ) [m^2s^-2]</th>
<th>CRLB [m^2s^-2]</th>
</tr>
</thead>
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<tr>
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<td>2.498</td>
<td>2.147</td>
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<td>2.231</td>
</tr>
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</tr>
<tr>
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<tr>
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</tr>
<tr>
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<td>0.041</td>
<td>0.030</td>
<td>0.029</td>
<td>0.026</td>
</tr>
</tbody>
</table>

Table 2: Velocity standard deviation values for 10,000 runs of the Monte Carlo simulations for each velocity and temperature case, at \( n_s=10^{47} \) simulated scattered photons, compared to the square root of CRLB.

Fig. 5. Cramér-Rao lower bound for each of the vapor cell temperature and number of scattered photons cases. The CRLB is independent of both the Doppler shift magnitude of the signal and the CC-DGV processing routine.

Fig. 6. Velocity measurement variance and CRLB divided by measurement variance for several flow signal path iodine cell side-arm temperatures with a fixed 20 °C reference cell and \( n_s=10^{47} \) simulated scattered photons. Column 1: \( U = 5 \text{ m/s}^-1 \); Column 2: \( U = 50 \text{ m/s}^-1 \); Column 3: \( U = 500 \text{ m/s}^-1 \). The colors correspond to those in Fig. 4. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)
bias error in velocity compared to the normalized distance between discrete scan samples. The \( \Delta \) operator represents the Doppler shift velocity increment corresponding to the discrete frequency step of the laser light. The location of zero on the abscissa corresponds to a Doppler shift magnitude exactly equal to the scan step size. A series of Doppler shift values corresponding to 1% of the frequency scan step size was investigated, for a total of 100 increments. As expected, a cyclical behavior across the sub-resolution scanning samples is observed, similar to the behavior observed by Shiptau et al. regarding fast Fourier transform-based frequency estimators in laser Doppler velocimetry [21].

The peak bias error resulting from the peak-finding step is within \( \pm 1.0\% \) of the step size, which for the case shown corresponds to maximum bias errors of \( \pm 0.61 \text{ m s}^{-1} \). Since this error scales with the frequency scanning step size, increasing the number of samples in the scan will reduce this bias error. There exists a slight net positive bias at the zero relative shift location for the present case. It is likely that this net bias is a result of the non-linearity of the vapor transmission spectrum about the minima of the line features, and thus dependent on choice of absorption line, line depth, and number of frequency scan samples used to construct the feature in the data. The reader is therefore cautioned not to generalize the precise results of this frequency resolution bias error, although the magnitude of the bias variation in the results is likely typical. For the 50 m s\(^{-1}\) case in Section 3.1.1, the discretization bias error found in the present study was 0.38 m s\(^{-1}\), consistent with the bias seen at matched temperatures. The cyclical behavior was found to be repetitive with a period of one frequency sample step size; and therefore, the bias error can be said to be a function of the frequency scan resolution rather than the absolute velocity magnitude. Further, the cyclical bias was found to be of the same absolute magnitude, independent of the Doppler shift frequency; at higher velocities, this bias is of lower relative impact and supports the assertion that CC-DGV is well-suited for high-speed flows.

4. Conclusions and outlook

The measurement uncertainties in cross-correlation Doppler global velocimetry (CC-DGV) were investigated using methods from information theory and numerical analysis. The operating principle of Doppler global velocimetry is based on the detection of the Doppler frequency shift of scattered light by means of selective filtering through a vapor absorption cell. Specific to CC-DGV, a laser frequency sweep of several gigahertz is performed with a cross-correlation between signals to determine the Doppler frequency shift. The present study addresses the sensitivities of the processing routine to several experimental parameters including signal-to-noise ratio, vapor cell side-arm temperature (optical thickness), and velocity magnitude.

The Cramér–Rao lower bound (CRLB) was calculated to establish a benchmark upon which the signal estimator can be compared. The CRLB is independent of the signal estimator and represents the lowest possible variance achievable for a given sampled signal. Based on the CRLB and the signal estimator, optimality conditions for acquisition parameters are discussed.

The robustness of the signal estimator in CC-DGV was studied by means of several Monte Carlo simulations. A signal model was developed using numerical models of the vapor absorption cells along with both Poisson and Gaussian signal noise models. Two or more vapor absorption cells are needed in CC-DGV, with one in the reference beam path and one in front of each camera measuring the Doppler shifted flow signal. Simulations were performed keeping the reference cell at fixed temperature and varying the flow signal vapor cell temperature. The signal estimator was found to be biased as a weak function of temperature, with positive bias errors at higher flow vapor cell temperatures, and negative bias errors at lower flow vapor cell temperatures relative to matched cell conditions. The ratio of the CRLB to the measurement variance approaches unity for cases of sufficient Doppler shift magnitude and transmission spectrum modulation depths below 100%, demonstrating the efficiency of the estimator.

In a previous study, the present authors presented results to demonstrate the dynamic range of CC-DGV across low and high velocity regimes [8]. At velocities representative of incompressible, moderate subsonic, and supersonic regimes, the signal estimator was found to yield approximately equal magnitude bias errors and variances for each temperature cell.

Finally, the bias error inherent in the parabolic peak-finding sub-routine employed due to the discrete nature of the laser frequency scan was investigated by simulating a range of Doppler frequency shifts that fall below the resolution of the laser frequency scan step size. Specifically, the effectiveness of the parabolic peak-finding routine is examined. The sub-resolution bias error was observed to be below \( \pm 1.0\% \) of the step size for the specific case studied, and was independent of the magnitude of the Doppler shift. The bias error is dependent on the normalized frequency difference between the true Doppler shift frequency value and the nearest discrete laser frequency shifts that fall below the resolution of the laser frequency scan step. As such, this behavior is not captured by the CRLB.

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References


Chapter V.

Unsteady boundary layer development on a wind turbine blade: a benchmark problem

This chapter describes the development of a model problem for investigation of an airfoil boundary layer in a periodic, turbulent wake. The content is to be submitted to Experiments in Fluids, published by Springer Publishing Company.
Unsteady boundary layer development on a wind turbine blade: a benchmark problem

Daniel R. Cadel, Di Zhang, K. Todd Lowe, and Eric G. Paterson

Keywords Design of experiments, boundary layer turbulence, computationally motivated experiments, periodic flow

Abstract

Wind turbines with thick blade profiles experience turbulent, periodic inflow at low reduced frequency and high Reynolds numbers, leading to unsteady blade loading and torque on the drive shaft. A benchmark, lab-scale problem is developed for joint consideration by experiment and computation; the experiments are the focus of this paper, as motivated by numerical model development. A cylinder at \( Re_d \sim 65,000 \) is placed 10.67 diameters upstream of a NACA 63215b airfoil at \( Re_c \sim 170,000 \) with reduced frequency of \( k \sim 1.5 \). Velocity contours on the airfoil suction side in the presence of the upstream cylinder indicate a redistribution of turbulent normal stresses from transverse to streamwise, consistent with existing rapid distortion theory. Particle image velocimetry is applied on the airfoil suction side, as well as in highly magnified boundary layer planes at several chordwise locations to yield velocity and turbulence profiles at several chord positions with and without the periodic inflow. A triple decomposition is applied to the boundary layer profiles to separate the ensemble mean, periodic, and stochastic turbulent contributions to the velocity. The phase-averaged boundary layer profiles are turbulent at all stations considered; however, the periodic \( \tilde{u} \) component is consistent with the exact laminar Stokes flow solution. A phase lag is observed between the edge velocity and that in the buffer region, with an inertial region between \( y^+ \sim 50 \) and \( y = \delta \) with a constant logarithmic slope. The phase lag compares well with that for a laminar flat plate, with an added constant lag region in the buffer layer.
1. Introduction

The aerodynamic boundary layer on an airfoil in an unsteady flow is important to the performance of the system. In most engineering applications, including the wind energy application of unsteady flow over turbine blades, turbulent flow is inherently present in the boundary layer flow over nearly all of the external surfaces. Turbulence changes the aerodynamics of the system, and if not properly controlled, can lead to higher drag and flow instabilities. Blade aerodynamics for wind turbines involves both unsteady inflow and high free stream atmospheric turbulence, each of which influence the boundary layer development, which in turn affects the operating efficiency of the turbine. Note that in the present work, unsteadiness refers to deterministic periodic flow, as opposed to stochastic turbulence. Wind turbines are commonly arranged in “farms” containing multiple systems, with some turbines operating in the wakes of others depending on the prevailing wind direction (Spalart et al. 2006; Cal et al. 2010; Gaumond et al. 2013). Note that a similar detached flow condition similarly arises in multi-element airfoils (Spalart et al. 2006). For the wind energy case, this typically occurs at very high Reynolds number and contains mean, periodic, and stochastic turbulent contributions to the total flow. The periodic flow within the turbine wakes is of low reduced frequency with large amplitude.

Power is generated by a wind turbine via the revolution of the blades, which turns a drive shaft to a generator. The fluctuating aerodynamic conditions on the blades leads to unsteady blade loading, which can be both time varying and asymmetrical amongst the individual blades on a turbine (Vijayakumar 2015). This causes fluctuating torque on the drive shaft, increased fatigue, and thus decreased lifespan, especially with the trend towards larger rotor diameters. Collectively, the aerodynamic factors enumerated by (Vijayakumar 2015) present a challenge for computational studies. In particular, accurate prediction of the reattachment and subsequent transition or bypass transition of a boundary layer on the blade immersed in the wake is important for assessing the aerodynamic performance.

Boundary layer transition is of particular interest as a result of high chord Reynolds numbers. A notable difference between laboratory scale and full scale, which corresponds to a Reynolds number difference of two orders of magnitude, involves the natural transition of the boundary layer. In reduced scale models without the presence of an unsteady inflow, the boundary layer is less likely to transition to turbulence, and transition devices such as trip strips
are often used to better match with the full-scale physics (Sagol et al. 2013). With a periodic inflow and moderate turbulence however, the boundary layer is likely to undergo bypass transition, wherein the boundary layer quickly becomes turbulent resulting in different growth.

Many efforts have been undertaken to study unsteadiness effects. (Reynolds and Hussain 1972) introduced a procedure for decomposing the flow field into time-mean, periodic, and stochastic turbulent contributions, which allows for evaluation of phase-averaged statistics. A reference signal, such as a surface pressure, is used to identify the instantaneous phase angle relative to the periodic disturbance. Many researchers have used this analysis method for circular cylinder wakes, including (Matsumura and Antonia 1993; Perrin et al. 2006a; Perrin et al. 2007; Perrin et al. 2008; Cao et al. 2014), among others. Significant analytical understanding came from application of Rapid Distortion Theory (RDT) (Goldstein and Atassi 1976). Subsequent improvements to the theory by (Atassi and Grzedzinski 1989) allow for the inclusion of airfoil angle of attack, camber, and thickness effects, and has been frequently used (Mish and Devenport 2006). The impact of periodic conditions on the boundary layer has been analytically studied for laminar boundary layers by many researchers beginning with (Stokes 1850), turbulent boundary layers (Gete and Evans 2003), and separated boundary layers (Simpson and Shivaprasad 1983), among others. A phase lag is present near the surface in all cases. A key difference between previous studies in periodic flows, such as those with turbomachinery applications, and the present problem of interest is the superposition of large amplitude periodic oscillations with high turbulence intensity. In the wind turbine case, periodic content comes from the rotating blades of upstream turbines, while large turbulent structures arise in both the atmospheric boundary layer as well as in the decaying turbine wake.

One of the remaining challenges in simulations of such viscous fluid flows is the modeling of anisotropic stresses and high Reynolds number boundary layer flows. Typically, resolution of the smallest scales requires use of direct numerical simulations (DNS), or highly resolved large eddy simulations (LES), for example Bentaleb and Leschziner (Bentaleb and Leschziner 2013). For problems of engineering interest, the computational cost of such approaches often cannot be justified, leading to the use of computationally less expensive RANS codes. These codes however introduce considerable modeling and simplification of terms in the Navier-Stokes equations via empirical closure coefficients.
Modeling of the farm-scale systems is important for siting as well as performance evaluation. A very large range of scales is present within the flow, up to tens of meters and seconds to minutes for turbulent structures in the atmospheric boundary layer (Vijayakumar 2015). Full resolution Direct Numerical Simulations (DNS) cannot be run for this problem due to the computational expense at full scale Reynolds number. Large Eddy Simulations (LES) is likewise limited by the grid requirements of capturing the small scales in the boundary layer and large scales in the wake. Hybrid LES-RANS models have been used to approach the problem, giving the benefit of resolved turbulence in the inflow and turbine wake with a Reynolds-averaged solution in the boundary layer (Spalart et al. 1997; Shur et al. 2008; Vijayakumar 2015). For the present work, a Delayed DES turbulence model (Spalart et al. 2006) is coincidently being developed incorporating the transition model based on that of (Langtry and Menter 2005) and (Menter et al. 2006). The present experiments are aimed at better understanding of the physics of boundary layer transition in low reduced frequency periodic flow in collaboration with model development and validation. A laboratory scale model problem is developed to focus on the relevant physics and studied experimentally with particle image velocimetry (PIV). This configuration is to be simulated numerically, and the ensuing experimental results are to be used for comparison and to motivate continued model development.

2. Design of experiment

The issue of modeling the flow over a wind turbine immersed in the wake of upstream turbines is complex. Simultaneously, there are periodic and stochastic turbulent contributions to the flow field, occurring at high Reynolds numbers. Simulation of all scales of interest in a fully resolved LES model is not computationally feasible for such a problem. The benefits of DES are apparent for this type of flow, however the problem is posed differently than in most DES simulations. For flow around a singular body with a low turbulence inflow the challenge is in switching between RANS and LES in a gray region along the boundary layer after which separation occurs. For the wind turbine operating in a wake, there is a second blade located downstream of the first, over which a boundary layer will develop; the LES solution in the wake must be imposed on a new gray region where a RANS region will be defined on the downstream blade. Resolved stresses spread across some wavenumber spectrum in the LES region must be reduced to be included in the RANS region. Modeled stress depletion then becomes a concern, where grid refinement can lead to artificially limited values of eddy viscosity and Reynolds stress (Spalart et al. 2006).
As a starting point, a benchmark configuration must be developed that encapsulates as best as possible the relevant physics while simplifying the flow to accommodate high fidelity simulations and experimental study in laboratory scale facilities. A model problem, consistent between the experiments and simulations, was developed for this purpose. Since the dynamics on a single blade was of interest, the complexity of the problem could be significantly reduced relative to the full wind turbine farm scale. Instead of modeling an entire wind turbine, a single blade can be considered. At full scale, chord Reynolds numbers are on the order of $10^6 - 10^7$. For the benchmark problem, the maximum feasible chord Reynolds number is on the order of $10^5$, significantly lower than the full scale. (Vijayakumar 2015) computationally studied a turbine blade in the atmospheric boundary layer and found the local flow angle fluctuations to be largest close to the rotor hub, with a fluctuation amplitude of 27% at 25% span, with a peak blade Reynolds number on the order of $10^7$.

The reduced Reynolds number presents the greatest deviation from the full scale physics with regard to the development of the boundary layer on the foil. At full scale, natural transition is expected. However, at lower Reynolds numbers, the influence of a periodic inflow with angle of attack fluctuations of $\pm 50^\circ$, bypass transition is instead expected. While different than the full-scale physics, this presents an opportunity for model development to correctly capture this phenomenon. A common model used to predict transition in CFD codes is the Menter-Langtry model (Langtry and Menter 2005; Langtry and Menter 2009), which adds correction terms to the production and dissipation of energy terms based on transport equations for the intermittency and transition-onset momentum thickness Reynolds number. These values are computed using local properties of the flow, and can therefore more accurately predict the effects of transitional boundary layers than models with only global parameters or purely empirical correlations. Incorporation of this model, or an improved version of the model from (Menter et al. 2015), is desired for the numerical model, and thus is also an objective for experimental study.

A relevant body of work exists in the rod-airfoil experiments of several researchers (Jacob et al. 2004; Henning et al. 2010; Violato et al. 2010; Greschner et al. 2004). In that effort, a benchmark problem was developed to study sound generation in turbomachinery applications. A circular cylinder “rod” is placed one chord length upstream of a NACA 0012 airfoil (chord to diameter ratio typically 10) to generate a periodic inflow to the airfoil. Chord Reynolds
numbers ranging from 35,000 (Violato et al. 2010) to 480,000 (Jacob et al. 2004) and reduced frequencies around 6 have been reported. The experimental data obtained for the benchmark problem was used to motivate numerical model development and provide a reference for comparison. For the present work related to wind energy, the reduced frequency of the rod-airfoil work was considered to be too large to yield applicable data, but the design was used as a starting point for the design of a new benchmark model problem.

### 2.1 Design methodology

The well-characterized properties of flow around cylinders allowed for a parametric design study for the present investigation. A cylinder was to be used to generate a periodic wake, in which an airfoil would be situated downstream. Key parameters to be selected were the freestream velocity and turbulence intensity; cylinder diameter; and airfoil profile, chord, angle of attack, and distance downstream of the cylinder. These physical parameters were further grouped via dimensional analysis into parameters including the cylinder and airfoil Reynolds numbers, airfoil reduced frequency, and normalized streamwise spacing. The reduced frequency, \( k \), is a non-dimensional parameter that describes the periodic nature of the flow, defined as

![PIV planes of interest](image)

**Fig. 1** Model problem (to scale) to study the effects of turbulent, periodic inflow on a lifting airfoil. Notional locations of physical, numerical, and experimental features are shown for reference. See Table 1 for relevant physical and non-dimensional parameters.
For wind energy applications, the reduced frequency is in the order of $10^0$ to $10^{-1}$ (Vijayakumar 2015).

\[ k = \frac{2\pi f c}{V} \]  

\[ (1) \]

Table 1 Model problem design parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cylinder diameter</td>
<td>$D$</td>
<td>1.5</td>
<td>in</td>
</tr>
<tr>
<td>Airfoil chord</td>
<td>$c$</td>
<td>4.0</td>
<td>in</td>
</tr>
<tr>
<td>Airfoil profile</td>
<td></td>
<td>NACA 63215b</td>
<td></td>
</tr>
<tr>
<td>Streamwise spacing</td>
<td>$L$</td>
<td>$L/D = 10.67$</td>
<td></td>
</tr>
<tr>
<td>Lateral offset</td>
<td>$h/c$</td>
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<td>Freestream velocity</td>
<td>$U_\infty$</td>
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<td>$m/s$</td>
</tr>
<tr>
<td>Aspect ratio</td>
<td>$AR$</td>
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</tr>
<tr>
<td>Cylinder Reynolds number</td>
<td>$Re_D$</td>
<td>63,500</td>
<td></td>
</tr>
<tr>
<td>Airfoil Reynolds number</td>
<td>$Re_c$</td>
<td>170,000</td>
<td></td>
</tr>
<tr>
<td>Reduced frequency</td>
<td>$k$</td>
<td>1.53</td>
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</tr>
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</table>

Key considerations in the final choice of dimensions included the Reynolds number-dependent shedding regime of the cylinder, the fluctuating angle of attack experienced by the airfoil, and the extent of the inviscid pressure field influence in the wake of the cylinder, all under the constraint of a tunnel facility at reduced scale. The final design is presented in Figure 1, showing physical features of the flow, and notional locations of the DES numerical RANS and Focus Regions (Spalart 2001) and experimental planes of interest. Tabulated values are given in Table 1. The freestream velocity of the wind tunnel facility is 26 m/s with turbulence intensity of 1%. The cylinder diameter is 1.5 inches, $Re_D = 63,500$, the airfoil is at zero degrees angle of attack with leading edge 16 inches downstream of the cylinder center with a 4 inch chord, $Re_c = 170,000$. The reduced frequency based on the expected shedding frequency of 125 Hz is $k = 1.53$. Additional information on the design of the experiment can be found in (Zhang et al.).

Wind turbine airfoils are generally thick; for example, the Delft family of profiles have thickness values in the range of $t/c = 15\% - 40\%$ (Timmer and van Rooij 2003), higher than typical for aviation applications. A profile thus needed to be selected for the benchmark problem that was similarly thick, but not so thick as to induce premature
separation. Twenty five airfoils from the NACA 6-series and Delft families were analyzed using an inviscid flow solver and implicit laminar and turbulent boundary layer solvers under steady inflow conditions, with transition and separation locations as metrics (Devenport and Schetz 1998). A NACA 63215b profile was finally selected, with a thickness of 15%, turbulent transition at \( x/c = 50\% \) and separation at \( x/c = 80\% \), based on the Michel method for flat plate transition with \( Re_{x,trans} = 250,000 \) (Schetz and Bowersox 2011). This mod “b” airfoil is a modified version of the NACA 6-series 63215 airfoil developed by (Hicks and Schairer 1979), modified on the suction side over the leading 30\% of the chord to produce higher lift and higher drag; the maximum thickness in 15\%. This results in a long region of close to zero pressure gradient, allowing the boundary layer at zero degree angle of attack and clean inflow to transition naturally. The unmodified NACA 63215 profile has previously been used in in the context of design analyses of wind plants (Tosun 2005) and turbine blade icing studies (Homola et al. 2010; Virk et al. 2010).

3. Facility

Measurements were performed in the Virginia Tech Open Jet Wind Tunnel. The facility is a blower-type wind tunnel powered by a 30 hp, 1180 RPM fan. Following the fan, the flow passes through a 4 meter long diffuser section, a honeycomb grid and two mesh screens, and a 5.5:1 contraction nozzle before entering the test section.

3.1 Wind tunnel and test section

Measurements were taken in the Virginia Tech Open Jet Wind Tunnel with a specially developed hard wall test section. The test section is 48” in length with a square cross section with side length 27 23/32 inches. The test section is made of four optically transparent acrylic panels with corner gussets to minimize vorticity at the wall junctions. The corner gussets were extended upstream into the contraction using foil tape, and varied slightly amongst subsequent measurement entries. On each of the four walls of the test section, the panel is bolted to an aluminum support frame at four streamwise locations. The cross-sectional geometry of the test section is shown in Figure 2, to scale. Flow is from left to right, and the dotted vertical lines indicate the interface with the upstream nozzle contraction and test section exit.
3.2 Cylinder and airfoil models

The circular cylinder and NACA 63215b airfoil were both mounted vertically within the test section and were solid aluminum. After machining, both models were nickel-chrome plated with a thickness of less than four ten-thousandths of an inch, and the airfoil further polished with a buffing wheel, all in order to achieve a near mirror finish. Since very near wall measurements in the boundary layer are of interest, the mitigation of laser flare was a priority; (Konrath et al. 2008; Pierce and Lu 2012) have shown high effectiveness for mirrored surfaces. Due to machining considerations, the airfoil trailing edge has a finite thickness of 0.21mm.

Holes were cut for the models in the acrylic floor and ceiling panels for the cylinder and airfoil. The airfoil was mounted to a rotation stage with the center of rotation at the quarter chord location. For each tunnel entry, an initial calibration was run to adjust the airfoil to zero degrees effective angle of attack based on velocity contours on the airfoil suction side. See Figure 2 for the relative placement of the cylinder and airfoil as installed in the test section.

Fig. 2 Cross-section of model problem geometry (to scale). Flow is from left to right. The solid horizontal lines denote the presence of solid acrylic walls, while the dotted vertical lines at $y = 0$ and $y = 48$ indicate the interface with the nozzle contraction and test section exit, respectively (note no physical barrier was present).
3.3 PIV instrumentation

Two particle image velocimetry systems were used to study the model problem, with LaVision DaVis version 8.3 software used for acquisition and initial processing of both systems. For the first system, the cameras were Photron Fastcam model SA1, with a 1024x1024 sensor, and the laser was a Photonics Industries International, Inc. diode-pumped Nd:YLF frequency doubled 527nm wavelength system with up to 70 mJ per pulse. This system had a data rate of 2920 Hz, and was used for all data presented unless otherwise noted. The second system used Imager pro X 4M double shutter CCD cameras with 2048 x 2048 pixel sensor operating at 4 Hz, and a Quantel EverGreen 200 Nd:YAG laser system with up to 200 mJ per pulse at frequency doubled 532 nm wavelength. Calibration was done using the method of Tsai (Tsai 1987) with an additional stereoscopic iterative self-calibration routine as described by (Wieneke 2005). Multi-pass processing with a final window size of 48x48 pixels was used for all data. Adaptive PIV, which allows for improved spatial resolution and accuracy by shaping and positioning the interrogation window as a function of local flow magnitude and gradient, was used for the boundary layer planes to improve near-wall resolution (Wieneke and Pfeiffer 2010). Post-processing and statistics were done in Matlab. High uncertainties were observed in the out-of-plane (spanwise) component in the boundary layer measurements due to low stereoscopic sensitivity from the camera placement, and therefore only in-plane velocities are reported in this paper.

Seed particles were introduced just downstream of the tunnel blower, which provided sufficient seed density and spreading by the time it reached the cylinder several meters downstream. DEHS oil with a mean particle size of 0.3 µm was used for the cylinder wake measurements, and smoke consisting of 50% pharmaceutical grade glycerine and 50% deionized water with a particle size of 0.2 - 0.3 µm was supplied from a Colt Turbo 4 smoke generator for all boundary layer planes. The smoke generator produced considerably higher spatial seed density, which proved necessary for the high magnification boundary layer planes.

3.4 Uncertainty analysis

The uncertainty in the presented PIV results is estimated using the standard propagation of error equation for a quantity $\alpha$ as a function of its dependencies $\beta_i$. 


The quantity of interest is the velocity vector, and the dependencies are the uncertainty in the velocity vector $\delta U$ determined from PIV and the uncertainty in the in-plane angle offset $\theta$ of the coordinate system global reference frame. For the full chord field of view measurements, the x-axis is aligned with the chordline, while for the boundary layer planes, the x-axis is aligned with the local surface angle. The velocity is defined in terms of $\delta U$ and $\theta$ for two components as

$$U = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} \delta U_1 \\ \delta U_2 \end{bmatrix}$$ (3)

Applying Eq. (2) to Eq. (3) gives a final uncertainty as a function of $\delta U$ and $\theta$

$$\delta U^2 = \left( \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} \delta U_1 \\ \delta U_2 \end{bmatrix} \right)^2 + \left( \begin{bmatrix} -\sin \theta & \cos \theta \\ -\cos \theta & -\sin \theta \end{bmatrix} \begin{bmatrix} \delta U_1 \\ \delta U_2 \end{bmatrix} \delta \theta \right)^2$$ (4)

The nominal flow angle after image correction is $0^\circ$, allowing for a simplification of Eq. (4) for the present case

$$\delta U^2 = \left( \begin{bmatrix} \delta U_1 \\ \delta U_2 \end{bmatrix} \right)^2 + \left( \begin{bmatrix} \delta U_2 \\ -\delta U_1 \end{bmatrix} \delta \theta \right)^2$$ (5)

The maximum uncertainty for the angle is estimated as $\pm 1^\circ$. This value is constant for a given image plane and can be considered a bias error, however curvature in the airfoil surface and corresponding changes in the wall-normal direction make a global correction impractical. The PIV velocity uncertainty is traditionally not as straightforward, however a statistical approach developed by (Wieneke 2015) and implemented in LaVision DaVis has been shown to be effective (Sciacchitano et al. 2015). Uncertainty estimates in the in-plane components varied with height off the wall. Representative values are given as $\delta = [0.52, 0.06]$ $m/s$ at the closest near-wall point and $\delta = [0.48, 0.38]$ $m/s$ for $y > \delta$ for the 1024x1024 pixel camera system in a boundary layer with the cylinder present upstream.
4. Results

Initial measurements were taken in the near-wake of the circular cylinder with the 1024x1024 pixel camera system to characterize the vortex shedding and confirm the experimental design parameters. The results are presented in (Cadel 2016), and were very similar to the flow field measured by several previous researchers, for example (Perrin et al. 2006b). Slightly lower magnitude of the Reynolds stresses were seen compared to their results, consistent with the difference in diameter Reynolds number. The wake was also measured with the same PIV system at a distance of $x/D = 9.43$ downstream of the cylinder center to characterize the inflow conditions at the location of the airfoil. Data was acquired in a plane normal to the streamwise flow direction with the cylinder installed but no airfoil model; when installed, the airfoil leading edge is located at $x/D = 10.67$. A discretized probability histogram of the instantaneous flow angle in the streamwise-transverse plane is presented in Figure 3. The distribution is double-peaked, due to the alternating positive and negative vorticity structures shed from the cylinder, and their associated vertical velocity. Fluctuations as far as $\pm 50^\circ$ are observed, with peaks at $\pm 23^\circ$. The wake half width was also obtained at this location, defined as the lateral location where the velocity deficit equals half of the maximum velocity deficit at the wake center; the half width was found to be $y_{1/2} = 0.0373 \text{ m} \left( y_{1/2}/D = 0.98 \right)$. This streamwise location is not yet in the region of self-similarity (Pope 2000).

The full flow field over the airfoil suction side was measured with the 2048x2048 pixel camera PIV system for global context. High magnification planes were then taken at several chordwise positions on the airfoil suction side with optimized camera grazing angles in order to extract profiles of velocity and turbulence statistics in the boundary layer both with and without the circular cylinder installed upstream.
4.1 Airfoil suction side

Full field measurements on the airfoil suction side were of interest to compare the large scale effects of the turbulent periodic inflow. Velocity fields were acquired with the 2048x2048 pixel camera PIV system, however in a planar (one camera) configuration. This approach allowed for measurements of in-plane velocities only, i.e. no spanwise w component. 2000 image pairs were acquired at a data rate of 4 Hz for each the cylinder-airfoil case and the airfoil only case, 1000 each with the camera focus optimized near the upstream and downstream halves of the chord. The image sets were then concatenated to provide data around the entire suction side. Note that compared to the 2920 Hz system, the data rate of the 4 Hz system is well above the timescale of the cylinder vortex shedding frequency, so each realization can be considered independent.

In the presence of the upstream cylinder, the incident flow angle is highly unsteady, since large vortices shed from the cylinder of the same order of magnitude as the airfoil chord. At any given instance in time, different portions of the airfoil may also be influenced by separate vortices, meaning the effective angle of attack is not uniform over the chord.

Mean velocity magnitude contours over the airfoil suction side are presented in Figure 4 for both the airfoil-only case and the cylinder-airfoil model problem case. Each case is normalized by the plenum pressure based bulk velocity $U_0$. Flow is left to right, with the coordinate origin located at the airfoil leading edge. Spatial coordinates

Fig. 4 Normalized mean velocity magnitude on the suction side of the NACA 63215b airfoil with steady inflow and with periodic inflow from the upstream cylinder. The black line indicates the path used for computing circulation, and $\bullet$ indicates the location of conditional averaging in the periodic inflow case.
are normalized on the airfoil chord length. For the steady inflow case, the flow behaves as expected for a lifting airfoil, with acceleration over the suction side until the slope surface reaches zero and a zero pressure gradient is expected. While the boundary layer is not resolved across the entire chord, a reduced velocity is observed near the surface towards the aft portion of the chord, suggesting a thickening of the boundary layer at this point.

The cylinder-airfoil case in Figure 4 shows a considerably lower mean velocity at all points in the observed flow field. A stratified inflow is present, due to the mean wake of the cylinder. Acceleration in the mean is still seen over the forward half of the airfoil chord, but with a reduced magnitude and spatial extent compared to the steady inflow case. The low pressure region on the suction side also appears to entrain higher momentum fluid from outside of the cylinder wake. The circulation was computed along the path indicated in Figure 4. For steady inflow the circulation was \( \Gamma_{\text{steady}} = 3.71 \, m^2/s \), and for the periodic inflow the circulation was \( \Gamma_{\text{per}} = 3.47 \, m^2/s \). A lower circulation is expected over the suction side with periodic inflow since the mean flow velocity is reduced in the cylinder wake. To account for the periodic nature of the flow, a conditional average was taken based on the transverse velocity at the point \( x/c = 0.20, y/c = 0.64 \). The data was divided into 8 bins between the smallest and largest observed fluctuations, with conditionally averaged circulation values found to be \( \Gamma = [3.56, 3.53, 3.49, 3.44, 3.44, 3.46, 3.42, 3.48] \, m^2/s \). Note that the wavelength of the vortex shedding is

**Fig. 5** Normalized Reynolds stresses on the suction side of the NACA 63215b airfoil.
smaller than the airfoil chord, implying that more than a single vortex is acting upon the airfoil at any instant. Higher circulations are seen during conditions of strong fluctuations, with lower circulations when the transverse fluctuation is small.

The periodic inflow in the cylinder-airfoil model problem has a significant impact on the Reynolds stresses. Compared with the steady inflow turbulence intensity of 1%, the velocity fluctuations in the cylinder wake are of much larger intensity, however the fluctuations have contributions from both the irrotational large scale structures, as well as the stochastic turbulence from the decay of said wake structures. The in-plane components of the Reynolds stress tensor are shown in Figure 5 for both cases. A median filter was applied to remove artifacts resulting from fixed optical distortions. For the clean inflow, both normal stresses $\overline{u'^2}$ and $\overline{v'^2}$ as well as the shear stress $\overline{u'v'}$ are nearly zero everywhere in the non-viscous region, as expected for steady flow. By comparison, the corresponding values for the cylinder-airfoil case exhibits higher values within the cylinder wake region. The shear stress $\overline{u'v'}$ shows a large increase specifically at the leading edge of the airfoil, due to the high distortion of turbulent structures at this point.

A notable feature of the normal stresses is a reduction of the $\overline{v'^2}$ component and corresponding increase in the $\overline{u'^2}$ component in the region above the airfoil surface. This phenomenon has been predicted analytically, originally by (Hunt and Graham 1978) for an instantaneously appearing flat plate in grid turbulence by applying a Rapid Distortion Theory approach. Two mathematical regions are defined, a viscous region corresponding to the boundary layer, and a source region where the turbulent stresses gradually vary from the viscous dominated values to the free stream values. Turbulence wavenumber spectra are used in the derivation; two model spectra for isotropic grid turbulence are applied. This approach has also been applied to cascades of compressor and turbine blades, for instance (La Riva et al. 2004).

From (Hunt and Graham 1978) it is possible to compare the theory to the presently measured airfoil stress profile. The normal and transverse shear averaged over 5% of the chord centered at a location of $x/c = 0.27$ are plotted in Figure 6, normalized by the inflow turbulence levels convected by the streamline curvature to map in height at $x/c = 0.27$. The solution of Hunt and Graham (Figure 5 in that work) has been rescaled in an effort to
approximately fit the present data. For isotropic grid turbulence, the Hunt and Graham solution scales in wall-normal position with the integral lengthscale for the given component, allowing comparison with the present results. The analytic results are scaled here with $L_{11} = 0.030m$ and $L_{22} = 0.060m$. The trend of the data is correct for both components, with the $u'^2$ curve closely following the analytic solution and the $v'^2$ component being of much higher magnitude. The physical mechanism involved is the redistribution of Reynolds stresses via the pressure rate of strain tensor in the velocity pressure gradient tensor term of the Reynolds stress transport equation (Pope 2000)

$$
\mathcal{R}_{ij} = \frac{p'}{\rho \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)}
$$

As fluctuations of $v/\partial y$ become smaller due to the reduced degree of freedom in the wall-normal direction, continuity necessitates that the turbulent energy be transferred to the streamwise and spanwise normal components. This result is observed for the case where the cylinder is present but not for the steady inflow case, since pressure fluctuations $p'$ in the form of shed vortices must be present for the mechanism to have an impact. The agreement

![Graph showing profiles of streamwise and wall-normal stresses](Fig. 6)

**Fig. 6** Profiles of the streamwise and wall-normal stresses at $x/c = 0.27$ compared to the results of (Hunt and Graham 1978) rescaled with $L_{11} = 0.030m$ and $L_{22} = 0.060m$. 

with the $\overline{v''}$ component follows the trend of (Hunt and Graham 1978) well, but the $\overline{u''}$ component is seen to overshoot near the wall and undershoot further away. Considering the original data of (Thomas and Hancock 1977), the collapse of the profile as streamwise distance increases does appear to trend upwards, as seen here. The outer region can be explained by the combination of two differences from the analytical solution case: high intensity anisotropic initial turbulence and a thick airfoil. Compared to the flat plate, the effect on the wall-normal component is qualitatively similar since the plate/airfoil presents an obstruction to fluctuations in that direction. The streamwise flow however is accelerated around the leading edge of the thick airfoil, preserving the large anisotropic nature of the streamwise turbulent stress. There is thus a contribution to the streamwise normal stress from the magnification of fluctuations by the acceleration, as well as from the redistribution from the wall-normal component.

The $L_{11}$ lengthscale was calculated analytically in the similarity region by (Glegg and Devenport 2017), and found to be $L_{11}/y_{1/2} \approx 0.8 - 0.9$ within the range $[x_{2}/y_{1/2}] < 1$. This corresponds in physical units to $L_{11} = 0.030m - 0.034m$ for the present downstream position, agreeing with the value needed of $L_{11} = 0.030$ to scale the results.

This result implies that the redistribution of normal stresses on the airfoil suction side is an effect of the deterministic flow field on the turbulence. With regards to the modeling effort, this effect is captured by the filtered LES equations, which apply the Navier Stokes equations to the large-scale turbulence.

### 4.2 Boundary layer profiles

Boundary layer mean velocity and turbulence profiles are of particular interest for the present study. The distribution of turbulent energy within the boundary layer and its development downstream is important for understanding the physical mechanisms, which in turn informs the numerical model development of the DDES and incorporated transition schemes. Boundary layer profiles were acquired with the 1024x1024 pixel camera system at 2920 Hz (5456 instances over two runs with the periodic cylinder inflow, and 2728 instances from a single run with steady inflow). Macro imaging with a typical magnification factor around 1.4 was set up in order to achieve a small field of view from a stand-off distance of about 18 inches.
For each profile, the exact position of the solid surface needed to be determined from the images themselves since uncertainty in physical position was too high to be compatible with boundary layer measurements. The mirror finish of the airfoil resulted in a reflection of particle images off the surface and into the camera frame. This then led to a nearly symmetrical phantom flow field through the solid surface when not masking out these regions in the PIV correlation processing. The center point of this symmetry could thus be taken as the wall position, to the accuracy of the interrogation window with overlap. Note a similar approach was used for boundary layer profiles using laser Doppler velocimetry by (Varano 2010).

For sub-pixel determination of the wall position, as well as for estimation of several boundary layer parameters, the data was fit to either the Blasius profile for laminar boundary layers or the law of the wall using the Clauser chart method. The Blasius profile for a laminar boundary layer is a function of the local edge velocity and position. The experimental data was shifted in wall height to overlap with the Blasius profile, thus giving the wall position. For turbulent profiles, the Clauser chart method is commonly used to estimate the friction velocity $u_f$ by finding the skin friction coefficient $C_f$ that satisfies the equation

$$\frac{U(y)}{U_{\infty}} = \left[ \frac{1}{\kappa} \sqrt{\frac{C_f}{2}} \ln \left( \frac{y U_{\infty}}{\nu} \right) + \frac{1}{\kappa} \sqrt{\frac{C_f}{2}} \ln \left( \frac{C_f}{2} \right) + B \sqrt{\frac{C_f}{2}} \right]$$

outside of $y^+ \sim 30$ (Clauser 1956). $\kappa$ and $B$ were taken as 0.41 and 5.0, respectively. For the present data, a least-squares fit of the profile in the region $30 < y^+ < 100$ was used to find the friction velocity. An additional degree of freedom was added to allow for shifting the wall height to obtain the best fit, thus determining the sub-pixel height for turbulent profiles. Wall positions for separated boundary layer profiles seen in the data were determined by plotting the phantom reflected portion of the profile along with the true flow field and estimating sub-pixel resolution wall position as the center of symmetry.


4.2.1 Steady inflow

Mean streamwise velocity profiles for chordwise positions of $x/c = [0.3, 0.4, 0.5, 0.6, 0.7, 0.8]$ are shown in Figure 7. Wall height is scaled on the boundary layer thickness, and velocities are scaled by the edge velocity. The local wall-parallel direction is used to define the coordinate system for each profile. The boundary layer stays laminar all the way until separation, which occurs between $x/c = 0.6$ to $x/c = 0.7$. Negative velocities are observed near the surface in the separated profiles, with a backflow ratio of $\gamma_{ph} = 0.46$ at a height of $y/\delta = 0.07$ at $x/c = 0.7$, indicating transitory detachment just before this location (Simpson and Chew 1981); this indicates the presence of a recirculation zone forming as the boundary layer lifts from the surface. Integral boundary layer parameters and statistics for the non-separated profiles are given in Table 2, and shown graphically in Figure 8. Uncertainty in Figure 8 is based on uncertainty in $U$ as a constant $0.5m/s$ as per Section 3.4. For error sources in $u_*$ the reader is referred to (Wei et al. 2005). The boundary layer thickness $\delta$ (defined as the location of 99% of the edge velocity), displacement thickness $\delta^*$, and momentum thickness $\theta$ all grow as the boundary layer develops downstream for each case. The momentum thickness Reynolds number is very low for all chord locations.

![Fig. 7 Mean profiles of $U$ velocity for the steady inflow case. Laminar flow up to $x/c = 0.6$, separated afterwards.](image)
Table 2 Boundary layer parameters for the no cylinder steady inflow and cylinder-airfoil cases.

<table>
<thead>
<tr>
<th>$x/c$</th>
<th>Inflow Case</th>
<th>BL State</th>
<th>$\delta$ (mm)</th>
<th>$\delta^*$ (mm)</th>
<th>$\theta$ (mm)</th>
<th>$Re_\theta$</th>
<th>$u_\tau$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>Steady</td>
<td>Laminar</td>
<td>0.884</td>
<td>0.265</td>
<td>0.040</td>
<td>71</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Periodic</td>
<td>Turbulent</td>
<td>2.033</td>
<td>0.323</td>
<td>0.096</td>
<td>130</td>
<td>1.287</td>
</tr>
<tr>
<td>0.4</td>
<td>Steady</td>
<td>Laminar</td>
<td>1.023</td>
<td>0.273</td>
<td>0.085</td>
<td>150</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Periodic</td>
<td>Turbulent</td>
<td>2.053</td>
<td>0.346</td>
<td>0.141</td>
<td>188</td>
<td>1.258</td>
</tr>
<tr>
<td>0.5</td>
<td>Steady</td>
<td>Laminar</td>
<td>0.997</td>
<td>0.303</td>
<td>0.110</td>
<td>191</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Periodic</td>
<td>Turbulent</td>
<td>2.248</td>
<td>0.338</td>
<td>0.161</td>
<td>216</td>
<td>1.243</td>
</tr>
<tr>
<td>0.6</td>
<td>Steady</td>
<td>Laminar</td>
<td>1.144</td>
<td>0.339</td>
<td>0.120</td>
<td>206</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Periodic</td>
<td>Turbulent</td>
<td>2.779</td>
<td>0.370</td>
<td>0.203</td>
<td>272</td>
<td>1.216</td>
</tr>
<tr>
<td>0.7</td>
<td>Steady</td>
<td>Separated</td>
<td>1.573</td>
<td>0.597</td>
<td>0.156</td>
<td>257</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Periodic</td>
<td>Turbulent</td>
<td>3.347</td>
<td>0.446</td>
<td>0.254</td>
<td>324</td>
<td>1.142</td>
</tr>
<tr>
<td>0.8</td>
<td>Steady</td>
<td>Separated</td>
<td>1.917</td>
<td>0.975</td>
<td>0.192</td>
<td>312</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Periodic</td>
<td>Turbulent</td>
<td>3.539</td>
<td>0.542</td>
<td>0.268</td>
<td>329</td>
<td>1.078</td>
</tr>
</tbody>
</table>

Fig. 8 Development of integral boundary layer parameters and turbulence parameters in the streamwise direction. Uncertainties based on $\delta U = 0.5 \text{m/s}^{-1}$. 
4.2.2 Periodic inflow

The presence of the upstream cylinder creates a periodic wake which results in a time-dependent boundary condition on the outer edge of the boundary layer. Profiles were acquired at the same chord location as in the steady inflow case, and the resulting mean statistics are shown in Table 2 and Figure 7. The boundary layer was found to be attached and turbulent at all of these locations, indicating that the unsteady inflow causes both early transition of the boundary layer as well as delayed separation. In the mean, the profiles all scale according the Clauser law of the wake; mean profiles are shown in Figure 9 along with the law of the wake and the one-equation model. Friction velocity was determined using the Clauser chart method, fit within the region $30 < y^+ < 100$.

![Fig. 9 Mean velocity profiles for the periodic inflow condition, scaled in wall units.](image1)

![Fig. 10 Probability density functions of boundary layer velocity components with periodic inflow. Mean profiles are blue circles, law of the wake is black squares, and Spalding one-equation model is black triangles.](image2)
model of Spalding (Spalding 1961). The closest near wall point for each profile falls in the buffer layer, consistent with the spatial resolution of the PIV instrumentation (Cadel 2016). At these low values of $Re_\theta$, (Purtell et al. 1981) found that the law of the wall is independent of Reynolds number, thus supporting its use in scaling of the present data. The friction velocity is seen to decrease for each case as the Reynolds number increases; the friction velocity is defined as $u_\tau = \sqrt{\tau_w/\rho}$, meaning that as the boundary layer develops the wall shear decreases until separation where the slope of the velocity profile at the surface is zero.

The periodic nature of the flow however means that the flow does not necessarily ever look like the mean, and is better described using a probability density at each location, see Figure 10. At each location, the extent of the spread of the profile is obvious. It is interesting to note that for the streamwise velocity, the width of the velocity band is relatively constant throughout the profile, while the wall-normal velocity increases significantly in width further from the wall. This result is consistent with the Rapid Distortion Theory results in Section 4.2, since fluctuations in this direction are diminished near the surface and redistributed to the streamwise direction.

### 4.2.3 Phase-averaging

The periodic nature of the flow in the wake of the cylinder lends itself well to phase-averaging of the boundary layer profiles. Time-signals outside of the boundary layer were observed to be periodic over the entire chord of the airfoil, and were thus suitable for analysis by this method. Phase-averaging was carried out with a Hilbert transform, using a similar approach to (Perrin et al. 2006b), however without the low pass and extrema filter used by those authors. In the present case, since no surface

![Phase-averaged velocity profiles](image)

**Fig. 11** Phase-averaged velocity profiles at $x/c = 0.60$. Friction velocity is determined using the law of the wake between $30 < y^+ < 100$. 

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pressure taps were available, a signal of streamwise velocity fluctuations $u'$ is used. A second order Butterworth bandpass filter with pass band from $100Hz - 140Hz$ is applied (since the Strouhal frequency is around $120Hz$). The Hilbert transform is applied to this signal, after which the envelope is divided out and phase determined as the inverse cosine of the ratio of the real part of the resulting signal. Sixteen phases were evaluated over a total range of $2\pi$. Individual time instances are assigned into bins based on the phase angle for that instance, with a bin width of $2\pi/16 = 22.5^\circ$ and approximately equal number of instances per bin. The resulting bins are phase-averaged following the approach of (Perrin et al. 2006b), following the original triple decomposition approach of (Reynolds and Hussain 1972) to separate large scale unsteadiness from turbulent fluctuations. The triple decomposition is represented as

$$U_p = \overline{U} + \bar{u} + u'$$

(8)

where the terms on the right side of Eq. 8 are the ensemble mean, periodic component, and stochastic fluctuations, respectively.

Mean velocity profiles for each phase are extracted and scaled using the Spalding profile in the region below $y^+ < 50$. Representative profiles at $x/c = 0.60$ are shown in Figure 11 along with the law of the wake and the Spalding model, as previously done by (Chesnakas and Simpson 1996). The scaling is done in this region due to the varying phase lag observed above $y^+ > 50$ to be discussed later. Friction velocities for all chordwise positions are shown in Figure 12.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig12.png}
\caption{Development of phase-averaged friction velocity $u_{\tau}$ for all chordwise positions, determined from the Spalding model below $y^+ < 50$.}
\end{figure}
The trend of decreasing $u_t$ as the boundary layer develops downstream holds true within each phase angle as well. This suggests that the boundary layer is able to react quickly to the local instantaneous boundary conditions. However, the deviation from the Spalding model at points close to the wall implies a phase lag between the free stream velocity and the buffer region.

An exact solution is known for the laminar boundary layer flow over an oscillating flat plate, also known as Stokes flow (Stokes 1850). While the airfoil suction side profiles measured in this work are turbulent, the boundary layer response in Stokes flow provides a useful reference for comparison and physical understanding. A flat plate is oscillated at a speed of $u = U_0 \cos(\omega t)$, where $\omega$ is an angular frequency. The boundary layer profile can then be represented as

$$u(y, t) = U_0 e^{-\eta} \cos(\omega t - \eta)$$

where $\eta = y \sqrt{\omega/2\nu}$ (Schlichting 1968). The profile is described by an inner and outer region with a relative phase lag. Using the triple decomposition of Eq. (8), the phase-averaged airfoil boundary layer profiles can be represented in a similar fashion to the Stokes flow via the periodic component profile $\tilde{a}$. The reference frame is changed to that of a oscillating solid surface by subtracting the value of the periodic component far from the surface for each phase angle. A comparison between the Stokes flow profiles with $\omega = 2.09 \text{ rad}$ and measured airfoil boundary layer profiles at $x/c = 0.70$ is given in Figure 13 for four instances in the cycle. The experimental data follows the same trend as the exact Stokes flow solution in each case, with deviations likely due to the turbulent nature of the airfoil boundary layer at this location. The agreement suggests that the oscillatory nature of the flow

![Fig. 13] Stokes flow representation of oscillating flow at four instances in the cycle. Experimental data •, Stokes flow solution --.
plays a significant role in the boundary layer profiles, and can be described in terms of the periodic cycle.

The phase lag in Stokes flow has been characterized both experimentally and theoretically, with a comparison given by (Tsahalis and Telionis 1974). The flow near the surface leads the flow further away by a maximum phase angle of $45^\circ$, with decreasing phase lag as distance increases. A phase overshoot is also observed in the outer region with the phase lagging behind the free stream. A phase lag has also been observed in a turbulent boundary layer with periodic flow. (Tardu et al. 1994) observed a phase lag primarily in the region below $y^+ < 12$ in a channel flow, where viscous forces dominate. A phase lag was also observed outside of the viscous region, however the slope and peak values of the phase lag varied with frequency. (Gete and Evans 2003) studied a flat plate turbulent boundary layer in a turbulent wake at $Re_x = 144,000$ at several reduced frequencies in the context of turbomachinery. A large phase difference near the wall on the order of hundreds of degrees was observed, with phase angle magnitude increasing with downstream position.

The phase lag through the boundary layer profile relative to the phase at the boundary layer edge location is computed here based on the streamwise velocity signal outside the boundary layer. Profiles of the phase lag within

![Profiles of velocity phase lag](image)

**Fig. 14** Profiles of velocity phase lag. The inner buffer region below approximately $y^+ < 50$ exhibits constant phase lag with a minimum near $-\pi/4$, an inertial region between $y^+ > 50$ to $y = \delta$ has a constant logarithmic slope, and the outer region beyond $y > \delta$ is dependent on the instantaneously passing fluctuation.
each phase angle are plotted for all chordwise locations in Figure 14 using wall unit scaling. Three distinct regions can be identified from the phase lag profiles. Beyond the boundary layer thickness $\delta$ in the outer region there is a slight increase in phase, similar to that seen in periodic laminar flow (Tsahalis and Telionis 1974). Nearest to the wall, a buffer region larger than that of (Tardu et al. 1994) is observed, corresponding to a region below approximately $y^+ < 50$. In this buffer region, the phase lag is roughly constant with wall height, with a magnitude converging towards $-\pi/4$, the same magnitude expected for laminar Stokes flow. The minimum value of the phase lag in the present data depends on the phase angle of the flow, indicating that the phase lag is dependent on the acceleration at the boundary layer edge. The dependency on acceleration of the flow implies that the phase lag is a pressure-driven effect. The height of the constant phase lag buffer region also appears to grow as the momentum thickness Reynolds number $Re_\theta$ increases, with a constant region observed beginning about at $x/c = 0.50$, corresponding to where an adverse pressure gradient is expected. During periods of large acceleration, i.e. around phases 5 and 13, the phase lag exhibits its largest magnitude and smallest magnitude phase lags in the buffer region. Conversely during periods of small acceleration, around phases 1 and 9, the minimum phase lag takes intermediate values. This suggests that the boundary edge condition is more easily imposed on the buffer region when the instantaneous acceleration is small, i.e. neutral pressure gradient, during which time the phase lag can vary between extremes.

Connecting the buffer and outer regions is an “inertial region,” which is subject to influence from both of the surrounding layers. The inertial region extends from approximately $y^+ > 50$ to $y = \delta$, consistent with the bounds of the wake region defined by (Wei et al. 2005). A similar trend of data with a constant logarithmic slope region was also observed by (Simpson et al. 1983) in separated periodic boundary layers. The phase lag varies in height with a constant mean logarithmic slope of $A \ln y^+$, with values of $A$ in the range of $13 – 26$ and independent of chord location and wall height. Despite the turbulent profile of the present data, the phase shift profile does not vary with streamwise distance as seen in turbulent (Gete and Evans 2003) and separated (Simpson et al. 1983) boundary layers; the phase angle is more consistent with laminar Stokes flow beyond the constant phase angle buffer region. Since the Stokes flow has an exact analytical solution, this suggests that the periodic flow in the boundary layer includes a deterministic component along with the mean turbulent flow.
5. Conclusions and outlook

A model problem was developed in order to replicate relevant physics of wind turbine blades operating in periodic wakes. The work is a joint effort between experiments and numerical modeling to motivate improvements to boundary layer transition modeling in hybrid RANS-LES turbulence schemes. The model problem consists of a 4 inch chord NACA 63215b airfoil located 10.67 diameters downstream of a 1.5 inch circular cylinder, at a chord Reynolds number of 170,000 and reduced frequency of 1.53. Compared to the steady inflow case, the high amplitude periodic wake was observed to cause early transition on the suction side of the airfoil, as well as delayed separation.

Outside of the viscous region, the flow is described by Rapid Distortion Theory principles. Within the boundary layer of the periodic inflow condition, a phase lag exists as a function of height off the surface. The boundary layer is turbulent, however the fluctuating response follows the exact solution for Stokes flow and the observed phase lag agrees with previous results for laminar flow conditions. Three regions were identified pertaining to the phase lag: an inner buffer region with a constant phase lag around $\pi/4$, an inertial region with constant logarithmic slope of the phase lag variation, and an outer region outside of the boundary layer. The existence of this phase lag is relevant to numerical wall modeling, since the flow beyond the boundary has a delayed effect on the conditions near the wall. The instantaneous profile directly influences the drag on a wind turbine blade, including both the skin friction drag in the case of turbulence and pressure drag in the case of separation, and is therefore of primary interest in accurate numerical models of wind turbine farms.

The benchmark model problem presents an opportunity for future adaptation to larger scale facilities. Scale-up of the problem for higher Reynolds number conditions will have significant impact on the boundary layer dynamics, bringing the model problem closer to the full-scale conditions. Turbulent transition of the boundary layer at higher Reynolds number is expected to be delayed relative to the present conditions, making the issue of transition modeling of even higher priority. The development of the model problem and results in a moderate scale facility provide a benchmark for initial development, and provide a framework for future measurements.
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Chapter VI.

Model problem background and measurement details

This chapter explores additional details of the research effort described in Chapter V. Decision points in the model problem design process are discussed, including an exploration of the design space. An in-depth discussion of the PIV instrumentation is presented including the rationale for various system settings and processing parameters. Measurements in the wake of the circular cylinder are presented to validate the technique and confirm the behavior of the model problem.
1. Model problem design methodology

The design of the model problem was a joint effort using the existing body of literature for relevant aspects of the problem, in conjunction with simulations of the problem with low fidelity models to study different iterations of the design and test the design space. Key results of the model problem design are discussed in Chapter V. Many additional details of the design are explored here to illustrate the decision process and interdependency of the parameters.

![Diagram of model problem design methodology](image)

**Fig. 1** Relation between experimental and numerical contributions to the research effort. Experimental contributions and shown in blue, numerical contributions in red, and shared milestones in black.
The design of the model problem and the research program overall were a collaborative effort between experimental and numerical approaches. A flow chart showing the contributions and interplay of the experimental and numerical portions is shown in Figure 1. Experimental contributions and shown in blue, numerical contributions in red, and shared milestones in black. First, the model problem was designed by synthesizing the findings of a review of experimental literature with low fidelity simulations to test proposed configurations. Once designed, the flow in the wake of the circular cylinder was measured experimentally and numerically computed with the newly developed turbulence model in order to validate both the behavior of the model problem and the numerical scheme. Numerical results for the airfoil then informed the locations at which experimental attention should be focused. Particle image velocimetry measurements in the boundary layer were then acquired to provide reference data for model development. Understanding of the model problem, such as the rapid distortion theory and Stokes flow analyses in Chapter V, also provide a framework for comparison with numerical results. Future work for the modeling work now involves tuning of the model and generalized application to wind farm aerodynamics and related problems of interest.

1.1. Circular cylinder flows

A consensus was reached early in the design process that the general scope of the model problem would be to place an airfoil in the wake of a blunt body, with all remaining parameters free to optimize. As a starting point, the benchmark problem of the flow around a circular cylinder was studied in depth. The periodic nature of the flow around turbine blades was of primary interest to the physics of the downstream foil, and a substantial database on the periodic nature of circular cylinders is available.

The flow around circular cylinders is a very well studied problem in the analytical, experimental, and computational fields and at a wide range of Reynolds numbers. Beginning with Rayleigh in 1896 (Rayleigh 1896), the concept of Reynolds number was applied to circular cylinders (Zdravkovich 1997). The alternating vortex shedding famously attributed to von Kármán has been further classified by the Reynolds number and turbulent transition regime
Laminar flow around circular cylinders also has been an area of interest due to the ability to transform a circle into an airfoil profile via conformal mapping (Milne-Thomson 1973). However since this implies inviscid flow, it does not apply to vortical shedding.

Experimental studies of circular cylinder flows have proliferated. Several works warrant mention due to their relevance to the present benchmark validation problem development. A review of vortex dynamics in circular cylinder wakes in the shear layer transition regime is provided by (Williamson 1996). The turbulence spectra within the far wake was studied by (Uberoi and Freymuth 1969). They studied a range of three orders of magnitude of the diameter Reynolds number and found a universal scaling for the isotropic turbulence at high wavenumbers. Considering the large scale shedding, seminal work by Cantwell and Coles used flying hot wire and phase-averaging to describe the wake structure and development at a diameter Reynolds number of 140,000 (Cantwell and Coles 1983). In particular, they noted the generality of the mechanism for turbulence production at saddle points in the velocity field in free shear flows. More recent work has expanded to the use of laser-based diagnostics; full-field and volumetric Particle Image Velocimetry yields results compatible with advanced statistical analysis methods such as Reynolds decomposition, phase-averaging, proper orthogonal decomposition (Braza et al. 2006; Perrin et al. 2006a), and velocity spectra from time-resolved PIV (Perrin et al. 2008).

Many computational studies have focused on cylinder wakes due to the wide applicability of the problem, such as DNS by (RAI 2010), LES and DES by (Ma and Karamanos 2000) at low Reynolds numbers, and LES by Breuer at higher Reynolds numbers (Breuer 2000), among countless others. Circular cylinder wake flows have also been used as a benchmark for initial development of turbulence models, such as the Scale Adaptive Simulation model of (Menter and Egorov 2005) and as a reference case for comparison of various modeling schemes (Spalart 2000).

1.2 Cylinder and airfoil sizing

The sizing of the cylinder and airfoil are mutually dependent, with both affecting the resulting values of the diameter and chord Reynolds numbers and the reduced frequency. The tunnel was to be always run at the maximum velocity
to provide the highest Reynolds numbers for the chosen configuration. The circular cylinder shedding, properties of the shedding frequency, wavelength, and vortex spacing are all determined from the Reynolds number. The relative size of the two objects has the biggest impact on the reduced frequency, since the cylinder diameter affects shedding frequency, and both frequency and airfoil chord length are used in the definition of reduced frequency, \( k = \frac{2\pi f_c}{L/V} \). A diagram of the design space is shown in Figure 2.

The final sizes were chosen through a cascading decision process starting with the cylinder diameter. An unrelated research program also involving a circular cylinder in the Open Jet facility was being developed, and a diameter of 1.5 inches was a compromise acceptable to both programs. This size corresponded to a Reynolds number of \( Re_D = 63,500 \) with a lateral vortex spacing of 58 mm and a shedding wavelength of 200 mm. The aspect ratio, based on the tunnel dimensions was 18, with a spanwise correlation length of \( A/D = 3 \), for a total of 6 periods.

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**Fig. 2** Design space of the cylinder-airfoil model problem. Similar experiments from the rod-airfoil family are indicated, along with the Reynolds number regimes expected for future scale-up and the full-scale wind farm system.
across the tunnel. Numerical simulations showed that the extent of the downstream pressure influence for this Reynolds number was $6D$, thereby setting a minimum distance at which the airfoil leading edge could be situated. The airfoil was ultimately set at the furthest downstream location practical of $10.67D$, in order to allow for wake growth. The airfoil chord length was then a tradeoff between minimizing reduced frequency while increasing $Re_c$. A value of 4 inches yielded a Reynolds number of $Re_c = 170,000$ and a reduced frequency of $k = 1.53$. The Reynolds number is one to two orders of magnitude smaller than expected in the full farm scale, but is high enough to expect transitional boundary layer flow. The reduced frequency is similarly larger than expected in the full scale, however the angle of attack fluctuations of up to $27^\circ$ are of the correct order of magnitude (Vijayakumar 2015). The choice of the NACA 63215b profile is detailed in Chapter V.

2. Particle Image Velocimetry (PIV)

Measurements were performed in several locations using Particle Image Velocimetry. Many results and key system information is discussed in Chapter V, with additional results and details of the configuration included here. Processing parameters used for each set were generally consistent amongst sets employing the same steps with individually refined parameters. Data acquisition, image pre-processing, and some of the velocity processing were performed in the LaVision Davis 8.2 and 8.3 environments.

Two PIV systems were used for the results presented in Chapter V. The high speed PIV system used for boundary layer measurements and for the cylinder wake measurements in the present chapter was developed by LaVision GmbH, and uses DaVis acquisition and processing software. A Photonics Industries International, Inc. diode-pumped Nd:YLF system with up to 70 mJ per pulse at frequency doubled 527 nm wavelength (model number DM30-527-DH) was used. The cameras were Photron FastCam SA1, capable of double frame imaging at frame rates of 2.92 kHz at a fully active sensor size of 1024 x 1024 pixels. The low speed PIV system used for the full-chord suction side measurements in Chapter V was also from LaVision GmbH using DaVis software. The laser was a Quantel EverGreen 200 Nd:YAG system with up to 200 mJ per pulse at frequency doubled 532 nm wavelength. The cameras were Imager pro X 4M double shutter CCD cameras with 2048 x 2048 pixel sensor operating at 4 Hz.
A key aspect of the LaVision DaVis software is the camera calibration routine. Camera calibration here refers to the spatial registration of the two stereo cameras relative to each other, and relative to a world frame. Multi-plane calibration plates with precisely machined geometry and features are imaged, and computer vision algorithms are employed to identify marks on the surfaces. As a general rule, the spatial calibration only holds where the calibration plate is in the image. Further, a sufficient number of features must be present within the image for the calibrations to be accurate. For early measurements with the cylinder alone, a LaVision Type 058-5 plate was sufficient to cover the field of view. For measurements of the airfoil boundary layer, this plate type was found to have too sparse dot features given the high magnification of the camera images. A custom calibration plate (Figure 3) was machined with a total size of 26mm on a side and mark spacing of 2mm in order to yield approximately 5 marks in a direction in the field of view, as per the recommendation of LaVision. This plate was named “Micro calibration plate” and programmed into the DAQ computers for both PIV systems. Note that the marks on the

![Mechanical drawing of the custom camera calibration plate](image)

**Fig. 3** Mechanical drawing of the custom camera calibration plate
calibration plate were bare aluminum, the result of an end mill removing the anodized surface; direct illumination with an LED light caused non-diffuse reflections from the marks, which biased the mark detection. Instead, only ambient scattered light from the overhead fluorescent lights was used, with long exposure times (up to 30 seconds) to provide adequate signal levels. The machining tolerance for the mark spacing was quoted as at most ±76 μm (0.003 inches), which was larger than the precision of the calibration routine itself.

Two camera calibration routines are possible once the plate marks have been identified. The first method employs a camera pinhole model to map both cameras (Tsai 1987). This method is based on a simplified model of the camera optics, and provides a physically based calibration. However, the method is intended for imaging through free-space, i.e. not through thick windows. The second method, intended for imaging with inherent optical distortions, fits a series of empirical 3rd order polynomial functions to define the image plane. A standard deviation of the fit is used as a metric to judge the accuracy of the calibration. For the boundary layer planes acquired with the high speed system and presented in Chapter V, the pinhole and 3rd order polynomial models had very similar standard deviation, with the pinhole model slightly worse. Nevertheless, the pinhole model was used for all boundary layer planes in this entry since the magnitude of the standard deviation was still small, and the model is more physical. The average of the pixel error standard deviations amongst boundary layer planes was 0.99 pixels and 1.03 pixels for cameras 1 and 2.

Following camera calibration, seed is added to the air and the cameras are refocused on the particles. A self-calibration routine is then run using images taken with the final camera focus. The self-calibration routine is an iterative correction to the camera calibration that accounts for misalignments in angle and position between the laser sheet and the initial placement of the camera calibration plate. For the present measurements, anywhere from one to fifteen runs with a 64x64 window size, 50% overlap, and 2 iterations per run were performed to achieve desired convergence.
For the boundary layer planes, a macro imaging was set up in order to achieve a small field of view from a stand-off distance of about 18 inches. In order to see close to the surface with minimal flare, grazing angles were used (Cadel et al. 2016). One camera was placed directly underneath the airfoil with the camera lens aligned with the airfoil span, towards the suction side of the airfoil (starboard side of wind tunnel). The second camera was upstream of the model below the tunnel approximately on the cylinder wake centerline, angled downstream. Band-pass filters at 527nm were added inside the macro extension tubes between the lenses and the camera sensor. The filters were removed while doing alignment and calibration of the cameras. Extension tubes were used on both cameras between the lens and the camera sensor (Figure 4). Scheimpflug adapters were not used, despite their common use in stereoscopic PIV; the internal springs in the Scheimpflug devices were not strong enough the support the weight of the lenses and extension tubes while also causing significant vibration during running. Since the field of view was small, the lack of Scheimpflugs had a relatively minimal impact on the image depth of field.

Seeding in the form of smoke (50% pharmaceutical grade glycerine, 50% deionized water) was supplied from a Colt Turbo 4 smoke generator for all boundary layer planes, and from a DEHS seeder for the cylinder and full chord airfoil measurements. The smoke generator provided higher seeding density, which was needed in the small field of view boundary layer planes.

For all data taken with the high speed PIV system, the first pre-processing step was subtraction of a Gaussian time filter, typically of length 9 images. For the low-speed system, a local minimum intensity subtraction was instead...
used (Wereley et al. 2002). The resultant images were finally used for stereoscopic PIV analysis. This includes a local sliding filter intensity correction as well as image correction to transform the camera images into the calibrated mutual world coordinate frame.

The spatial resolution possible for a set of particle images is a function of the camera sensor size, the field of view, the largest expected velocity fluctuations, and the time delay between successive images in a pair. Here, the spatial resolution differs from the camera pixel resolution as a result of the interrogation window size. PIV acts as a low pass filter, eliminating motions on a smaller scale than the net total motion of all the particles within an interrogation window over the measurement time. Additionally, displacements that are larger than the interrogation window size cannot be captured. As such, for a given PIV results, there is a minimum spatial scale (but no minimum velocity), and a maximum velocity (and maximum spatial scale governed by the field of view). The size of the interrogation window must be large enough to include enough particles as to give a strong correlation, and also large enough to capture the largest expected velocity fluctuations. Meanwhile, the minimum spatial scale defined by the interrogation window size limits the smallest wavenumbers that can be observed.

Standard interrogation window correlations act such that the maximum particle displacement detectable is half the window size, looking symmetrically about the center point in all directions to build the correlation function. In practice, the particle displacement should then be half of this already reduced size, for a net one quarter of the interrogation region size (LaVision 2006). An alternate approach known as Adaptive PIV allows for improved spatial resolution and accuracy by shaping and positioning the interrogation window as a function of local flow magnitude and gradient (Wieneke and Pfeiffer 2010). Using a multi-pass approach, successively smaller interrogation windows are used based on the correlation result from the previous iteration. The window is shifted based on the flow magnitude to ensure fluctuations are not lost beyond the scope of the window, and eliminating the symmetrical one half limitation in standard approaches. The aspect ratio and angle of the elliptically shaped and Gaussian weighted interrogation window are also adjusted on successive iterations based on the local gradient to improve the spatial resolution of the results.
A Matlab script was written to determine the smallest possible window size that would capture the largest expected fluctuations and the corresponding spatial resolution as a function of camera field of view for a given configuration. This assumes adequate seed density within the interrogation windows, and that scale-adaptive PIV is used allowing for motion from one edge of the frame to the other. An example case corresponding to the high-speed PIV system configuration with $dt = 6\mu s$ is shown in Figure 5. The global minimum spatial resolution depends on the $dt$ for the data set; this makes sense since the lengthscales and timescales of the flow are coupled.

All data was acquired using 2-camera stereoscopic systems, however for a few select cases in the high-speed PIV measurements, the cameras were noted to have been shifted between spatial calibration and image acquisition, and planar PIV was instead used. For all PIV planes, adaptive PIV was used with a minimum interrogation window size based on the results of the Matlab script (32x32 for the high speed system).

Uncertainties were found to be high for the third component of velocity, and as such only in-plane component are presented. These components are however taken from stereo-processed data for most plane locations, so a comparison study was performed between planar processing and stereo processing. Profiles of boundary layer velocity at $x/c = 0.75$ were extracted from the same raw data set processed with both stereo and planar correlation techniques, all other processing parameters being the same. Comparisons are shown in Figures 6 and 7 for the no cylinder steady inflow case and upstream cylinder case. Note that the profile from the entire image is shown without regard to the wall location. This is intentional, as the location of the wall is determined from the measured profile and not precisely known \textit{a priori}. In both cases, the $\overline{U}$ velocity profiles match closely, within about $0.1\ m/s$ for the
cylinder case, and with a peak difference of about 0.3 m/s for the no cylinder case. The $\bar{V}$ profiles show similar levels of agreement outside of the viscous region, but a higher difference within the boundary layer, particularly in the cylinder case with a difference of up to 1 m/s at the closest expected near-wall point. The Reynolds stress components are of larger magnitude for both normal stresses for both cases. The lower magnitudes measured with the stereo system are of the order of magnitude expected for a turbulent flat plate boundary layer. The shear stresses $\bar{u'}\bar{v'}$ for both cases are of very close for both planar and stereo cases, with higher discrepancy in the boundary layer but of the same order of magnitude.
Fig. 6 Comparison of stereo- and planar-processing at $x/c = 0.75$, without the upstream cylinder.

Fig. 7 Comparison of stereo- and planar-processing at $x/c = 0.75$, with the upstream cylinder.
2.1 Cylinder wake measurements

Stereoscopic PIV velocity measurements were performed in the near-wake at $x/D \sim 1$ downstream of the cylinder center, in the absence of the airfoil. The test section was characterized by Pitot-static probe. A cross section of streamwise velocity at a streamwise plane 7.44 inches downstream of the test section leading edge is shown in Figure 8. 10,658 realizations were acquired over four runs of the PIV system (based on camera memory limitations for a single run). The tunnel was turned off between each run, so velocities were normalized using the measured plenum total pressure $p_0$ from each run, with $U_0 = \sqrt{2p_0/\rho}$. The bulk mean velocity over the four runs was 26.48 m/s, leading to a Reynolds number $Re_D = 64,668$ with kinematic viscosity defined at $\nu = 1.56 \times 10^{-5}$. Mean velocity components are shown in Figure 9. The recirculation length was estimated from the mean streamwise contour as 1.288, consistent with previous researchers such as (Perrin et al. 2006a) (additional references within that article).

All six components of the Reynolds stress tensor were evaluated for the cylinder wake from the normalized, mean-subtracted velocity fluctuations. The results are plotted in Figure 10. The streamwise normal stress $\overline{u'\nu}$ exhibits peak values at about 1 diameter downstream of the cylinder center, and $\pm 0.5$ diameters in the $y$-direction. The vertical normal stress $\overline{v'^2}$ alternately exhibits its peak on the cylinder centerline about 1.5 diameters downstream of the cylinder center. This is just past the mean recirculation zone, and thus where the greatest interaction between the shed vortices is expected. The shear stress $\overline{u'v'}$ outside of the mean recirculation zone is negative on the upper half.
of the plane and positive on the lower half, since the $u'$ and $v'$ values at a specified location tend to always be of the same sign; for example in the upper half plane, when a vortex is shed from the upper surface the fluctuating velocity there will be positive $u'$ and negative $v'$, and when a vortex is shed from the lower half the fluctuating velocity will be negative $u'$ and positive $v'$. The spanwise normal stress $\overline{w'^2}$ is non-zero in the wake, indicating the presence of three-dimensionality in the wake. For the present Reynolds number, Norberg observed the spanwise correlation length to be on the order of $\Lambda/D = 3$ (Norberg 2001). The aspect ratio of the cylinder is $\Lambda/D = 18.5$, indicating that six periods should be present. The $\overline{u'w'}$ and $\overline{v'w'}$ shear stresses are both nearly zero, implying no correlation between these components.

![Fig. 9 Normalized velocity components in the wake of the circular cylinder.](image-url)
The wake shedding frequency and Strouhal number were computed in order to estimate the reduced frequency expected for the downstream airfoil. The Strouhal number is defined as

\[ St = \frac{fD}{U} \]  

where \( f \) is the shedding frequency. To find this value, time-frequency was studied at a point in the flow directly downstream of the cylinder, 0.4 diameters downstream of the trailing edge. The average transverse \( v \)-velocity value of 25 spatial points centered at \([x, y]/D = [1.26, –0.38]\) at every time step from the full ensemble was used to find the shedding frequency. A Welch power spectral density estimate with a Hanning window of length 256 and 50\% overlap was applied, and the peak frequency found to be 125.7 Hz, corresponding to a Strouhal number of 0.181.

For a cylinder at this Reynolds number, (Norberg 2001) compiled experimental data in the range of \( St = 0.186 – 0.196 \), just above the value measured here.

Phase-averaging within the cylinder wake was carried out using a Hilbert transform, using a similar approach to (Perrin et al. 2006b), however without the low pass and extrema filter used by those authors. In the present case, since no surface pressure taps were available, a signal of transverse velocity fluctuations \( v' \) located in the shear layer

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**Fig. 10** Normalized components of the Reynolds stress tensor in the wake of the circular cylinder.
was used. A second order Butterworth bandpass filter with pass band from $100 \text{Hz} - 140 \text{Hz}$ was applied (since the Strouhal frequency is around $120 \text{Hz}$). The Hilbert transform is applied to this signal, after which the envelope is divided out and phase determined as the inverse cosine of the ratio of the real part of the resulting signal. Sixteen phases were evaluated over a total range of $2\pi$. Individual time instances are assigned into bins based on the phase angle for that instance, with a bin width of $2\pi/16 = 22.5^\circ$ and approximately equal number of instances per bin. The resulting bins are phase-averaged following the approach of (Perrin et al. 2006b), following the original triple decomposition approach of (Reynolds and Hussain 1972) to separate large scale unsteadiness from turbulent fluctuations. The triple decomposition is represented as

$$U_p = \bar{U} + \bar{u} + u'$$

where the terms on the right side of Eq. 3 are the ensemble mean, periodic component, and stochastic fluctuations, respectively. Results for the phase-averaged streamwise $u$ velocity, spanwise $\sigma$ velocity, and the Reynolds shear stress $\bar{u}'\bar{v}'$ for each phase are presented in Figures 11-13, respectively. The results closely match those of (Perrin et al. 2006b), thus helping to validate the phase-averaging approach.

Fig. 11 Phase-averaged, normalized streamwise velocity $\bar{u}/U_0$ in the cylinder near-wake.
Fig. 12 Phase-averaged, normalized spanwise velocity $\bar{v}/U_0$ in the cylinder near-wake.

Fig. 13 Phase-averaged, normalized shear stress $\bar{u}\bar{v}'/U_0^2$ in the cylinder near-wake.
2.2 Verification of phase index cycle

The phase-averaged results presented throughout this dissertation are computed using a Hilbert transform as described above. To verify the correct behavior of the indexing, additional representations of the data are given here. A portion of the signal used for phase-averaging for the airfoil boundary layer profile at \( x/c = 0.60 \) is shown in Figure 14 as both the raw signal and after bandpass filtering from 100 Hz to 140 Hz; recall that the Strouhal shedding frequency from the cylinder is approximately 125 Hz. This demonstrates the periodic nature of the flow itself.

![Fig. 14 Raw and bandpass signal used for phase-averaging of the boundary layer profile at x/c = 0.70.](image)

The phase-averaged velocity at the boundary layer edge \( \gamma = \delta \) is a good metric for observing the cyclical nature of the phase-averaged results. As was seen from the Stokes flow solution, a phase offset exists within the boundary layer and beyond \( \gamma = \delta \), causing the peak magnitude of \( \langle \overline{U} + \bar{u} \rangle_e \) to sometimes occur at a value offset from 0°. The cycle of \( \langle U + \bar{u} \rangle_e \) at \( x/c = 0.70 \) is presented in Figure 15. 16 phases over 2\( \pi \) radians are computed in the phase-average, and the results are repeated for three periods for clarity. In this case, the peak value falls close to a phase angle of 2\( \pi \) radians.
3. Rapid distortion theory comparison

In Chapter V rapid distortion theory analysis is applied to the airfoil suction side and experimental results are compared with the analytical results of (Hunt and Graham 1978). A key result of the analysis was that the stress profiles were described by rapid distortion theory, and good agreement was seen between the analytical results and experiments. The redistribution of normal stresses is an inviscid effect on the turbulence in the cylinder wake. As such, the effect is captured in the filtered LES equations in the numerical model. To confirm this behavior, preliminary results from the delayed detached eddy simulation (DDES) performed by Di Zhang for the model problem under the same conditions were analyzed by the author in the same manner as the experimental results discussed in Chapter V.

Profiles from the numerical result at $x/c = 0.27$ normalized by the inflow profile from the simulation are plotted in Figure 16 along with the experimental and analytical results. The behavior from the DDES model follows the same trend as the experimental data; the agreement of $\overline{u'^2}/\overline{\nu'^2}$ between the analytical and numerical is closer than that of $\overline{u'^2}/\overline{\nu'_c^2}$, as with the experimental. An overshoot in $\overline{u'^2}/\overline{\nu'_c^2}$ is also observed near the wall for both numerical and experimental results, with a larger overshoot predicted from the DDES model. The numerical result also predicts an
undershoot in the $\overline{u'^2}/\overline{u'^2}_\infty$ profile compared to the analytical result at large $y/c$. This supports the claim in Chapter V that the undershoot is due to the difference in geometry between the (Hunt and Graham 1978) flat plate conditions and the thick airfoil with flow blockage in the present model problem. The collective agreement of the analytical, experimental, and numerical results demonstrates that the redistribution of Reynolds stresses is an inviscid effect on the turbulence, which is sufficiently captured in the filtered LES equations used in the DDES model and numerical scheme.

![Graph showing experimental and numerical profiles of the streamwise and wall-normal stresses](image)

**Fig. 16** Experimental and numerical profiles of the streamwise and wall-normal stresses at $x/c = 0.27$ compared to the results of (Hunt and Graham 1978) rescaled with $L_{11} = 0.030m$ and $L_{22} = 0.060m$.

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Chapter VII.

Conclusions and outlook

This dissertation has been comprised of two main research efforts, both focused on the development and application of advanced laser diagnostics techniques to aerodynamic problems. The first contribution is the development, characterization, and demonstration of a new variant of Doppler global velocimetry, “cross-correlation DGV.” The second contribution is the development of a benchmark model problem for joint experimental-numerical investigation of the turbulent, periodic flow over wind turbine blades, and the associated experimental measurements.

1. Conclusions

This collective body of work involves the development and application of advanced measurement techniques for near-wall turbulent flows. Viscous boundary layer flows are of particular importance to the aerospace field, and while a century of research has revealed a wealth of knowledge, advanced measurements in real-world conditions remain a challenge. For much of that time, laser-based diagnostic systems have enabled non-intrusive measurements very close to the solid surface, notably with laser Doppler velocimetry (LDV) techniques, in addition to the prolific use of hot wire anemometry. The benefits of high temporal data rates with these methods are balanced by their point-wise nature. In large scale facilities the problem is acute, since the expense of measurements is generally higher. Spatially resolved measurements are well-suited for these conditions but bring new experimental considerations such as the spatial and temporal resolution, and potential for contamination of data near solid surfaces from laser flare. The work in this dissertation aims to address these items by using two spatially resolving laser diagnostic techniques, Doppler global velocimetry (DGV) and particle image velocimetry (PIV), to measure boundary layer profiles in a large scale facility and in a model problem designed to highlight the flow physics of interest to the wind turbine aerodynamics community.

The first portion, comprised of Chapters II, III, IV, and Appendix A, describes the development of a novel variant of Doppler global velocimetry. DGV techniques are well known to be optimal for high speed flows, and have generally been limited to small scale facilities. Interest in measuring boundary layer profiles on large scale wind turbine blade models motivated the development of an instrument that performed well in low speed flows and in large scale facilities.

CC-DGV works on the principle of the Doppler shift equation to measure the Doppler shift of scattered light from a seeded flow. In all absorption-based DGV techniques, this is done by filtering the scattered light through a molecular
vapor cell to attenuate the light intensity as a function of optical frequency. The main contribution of CC-DGV is a mean velocity measurement technique with reduced dependency on the vapor cell physical conditions, upon which DGV measurements are typically very sensitive. The frequency of the incident laser light is slowly modulated and monitored to provide a reference signal of the frequency modulation. For each component of velocity to be measured, a photodetector and vapor cell are placed in the flow to collect scattered light. As per the sensitivity vector of the Doppler shift equation, the photodetectors are placed around the measurement volume to provide the best possible response to the velocity component of interest. With CC-DGV the conditions at the different camera locations and reference signal monitor need not be precisely maintained. This has a particular benefit in large and full scale facilities, where conditions are often not as easily controlled.

The instrument was characterized numerically via the Cramér-Rao lower bound (CRLB) and Monte Carlo simulations to better understand the sensitivities and error sources. The processing routine performs best at high velocities with small frequency step size and with matched cell temperatures, specifically when the modulation depth is below unity. A bias error is present in the routine, weakly dependent on the vapor cell temperature mismatch but independent of the magnitude of the velocity vector.

CC-DGV was demonstrated in two flows at disparate conditions. The first application was the measurement of boundary layer profiles on a DU96-W-180 wind turbine blade model in the Virginia Tech Stability Wind Tunnel at Reynolds numbers up to $Re_c = 2 \times 10^6$. An optical probe technique was developed to nearly eliminate the adverse effects of laser flare. Fiber optics were routed through the internal cavities of an 18 inch chord, 72 inch span model to project laser light outward through pressure taps in the model surface, thus with a propagation direction normal to the local surface angle. This probe technique is compatible only with DGV. Resulting profiles were found to agree with LDV validation data within the experimental uncertainty. The second application was in the Virginia Tech Hot Supersonic Jet, a 1.5 inch nozzle supersonic free shear flow facility, in order to demonstrate the range of uses of the method. Volumetric velocity was acquired at an exit plane Mach number $M_j = 1.3$ and at unheated and heated conditions of $TTR = 1.0$ and $TTR = 2.0$, and compared to literature.

The second portion of this dissertation explores the development of a benchmark model problem for studying the transitional boundary layer on a thick airfoil subjected to turbulent, periodic inflow, and the resulting physical insights. This material is presented in Chapters V and VI. This work is part of a joint effort between experiments and numerical modeling to better understand this category of flow commonly present in full-scale wind turbine farms. The resulting
physical insights from the experiments are to be taken into account for a combined delayed detached eddy simulation (DDES) turbulence model with incorporated boundary layer transition model by collaborators.

Wind turbines at full-scale conditions are often arranged in farms with tens or more of turbines positioned in each other’s wakes. The inflow conditions for downstream turbines are then periodic with high amplitude fluctuations, low reduced frequency, and large scale turbulence. The fluctuating aerodynamic conditions results in unsteady blade loading and drive shaft torques, which can reduce the operating lifespan of these components. Blade boundary layer transition plays a major role in the lift and drag of the turbine blade, and design of blade profiles and siting of turbines rely on numerical modeling of these phenomena. The complexity of the reattaching boundary layer on subsequent blades is an expensive problem for CFD and requires modeling. A benchmark model problem is developed and presented in this dissertation to approximate these conditions at a laboratory scale for use in both experiments and numerical simulation.

The benchmark model problem involves a circular cylinder serving to generate a high amplitude wake and a NACA 63215b airfoil directly downstream. Extensive reliance on existing literature was coupled with low fidelity modeling to reduce the design space to an optimal condition. Primary non-dimensional parameters of the model problem are a chord-based Reynolds number of $Re_c = 170,000$ and a reduced frequency of $k = 1.53$. The Reynolds number is 1-2 orders of magnitude below that of the farm scale, and therefore at present serves as a developmental problem with room for future scale-up.

Particle image velocimetry is used in several configurations to validate the design of the experiment and then investigate the boundary layer flow physics. The cylinder wake dynamics were studied with a 2920 Hz PIV system, and the high degree of agreement with literature was found. Time-independent instantaneous data was also acquired on the suction side of the airfoil under both steady inflow and periodic wake inflow conditions. The turbulence profile in the cylinder wake was verified to behave accordingly to the predictions of rapid distortion theory (RDT), indicating that the redistribution of Reynolds stresses in this system is an inviscid effect on the turbulence. Preliminary numerical results using a delayed detached eddy simulation also agree with the experimental and analytical profiles.

The boundary layer was measured with 2920 Hz PIV at several locations along the suction side of the airfoil chord. Momentum thickness Reynolds numbers are small, ranging from $Re_\theta = 130$ to $Re_\theta = 329$ over the chord for the periodic inflow condition. With the conditions of the present scale of the model problem, the steady inflow case has
transitory detachment around $x/c = 0.70$. When immersed in a periodic inflow however, the boundary layer is fully turbulent both in the mean and instantaneous profiles over the extent of reported results from $x/c = 0.30$ to $x/c = 0.80$, and scales well with the Spalding model. Phase-averaging and triple decomposition is applied to the boundary layer profiles based on the periodic streamwise velocity outside of the boundary layer. At all of the chordwise locations studied, the phase-averaged boundary layer profiles also scale with the law of the wake, with the friction velocity $u_\tau$ dependent on phase angle and streamwise location.

The periodic component of the phase-averaged profiles are compared with the exact solution to the laminar Stokes flow and found to agree well. The phase lag in the boundary layer is similarly found to agree with past results for laminar boundary layers with periodic oscillations, as opposed to results for turbulent and separated boundary layers under similar conditions. An additional constant phase lag region relative to the laminar case is observed for the present conditions in the buffer layer.

2. Outlook

The work presented in this dissertation motivates several avenues for continued development and investigation:

- Cross-correlation Doppler global velocimetry has been demonstrated in large scale facilities for wind turbine blade aerodynamics, and is compatible with scale-up to full-scale plants.

- Insights from the numerical analyses of the CC-DGV processing routine indicate that system uncertainty can be reduced further with further refined calibration techniques.

- Additional experimental characterization of the model problem can be performed to estimate parameters of interest in validation of new numerical models. Measurements on the airfoil pressure side are needed for completion of the circulation path and estimation of lift and drag. Near-wall resolution can be increased by using Particle Tracking Velocimetry (PTV) methods on the existing particle images for the Lagrangian velocity of individual particles in the sublayer. Spatially resolved data from PIV is desired over point techniques such as Laser Doppler Velocimetry or Hot Wire Anemometry in order to link the phenomena occurring simultaneously through the flow field, such as the observed phase lag.

- Investigation into the difference in the boundary layer thickness $\delta$ and displacement thickness $\delta^*$ between steady and periodic inflow conditions, and the abrupt change seen in $\delta$ and $\delta^*$ but not in momentum thickness $\theta$ at
separation in the steady inflow. Assessment of skin friction and drag from these integral parameters is also of interest.

- Additional analyses with the present data can be performed to study the phase-averaged Reynolds stress development in the boundary layer.

- The benchmark model problem should be scaled up to larger facilities, such as the Virginia Tech Stability Wind Tunnel to better approximate the full-scale conditions and to provide reference data for the numerical model developed based on the present measurements. Comparison of the periodic boundary layer component at full scale conditions with the Stokes flow exact solution should be pursued to determine range of applicability, including the extent of the constant phase lag region.
Appendix

A. Optical Wake Rake

This appendix presents the development of a non-intrusive technique for measuring wake profiles in a wind tunnel. The technique employs cross-correlation DGV, and was demonstrated in the Virginia Tech Stability Wind Tunnel. The information in this appendix was originally written as an unpublished technical report.
Non-Intrusive Wake Profile Measurements with a Laser Velocimeter (Optical Wake Rake)

Daniel R. Cadel

Abstract

A non-intrusive laser-based technique for airfoil wake profile measurements is presented using Doppler global velocimetry (DGV). The technique is applicable to large-scale facilities and test conditions where traditional instrumentation cannot be used, such as in cases of highly three-dimensional flow and conditions when flow blockage must be minimized. A laser probe beam is aligned through a flow along the desired profile direction, and seed particles are introduced to the flow. Doppler global velocimetry techniques are used to determine the Doppler shift from scattered light along the beam, providing spatially-resolved measurements of velocity. In the present proof-of-concept study, cross-correlation DGV was employed to obtain profiles over a reduced area at the center of a 1.83m by 1.83m cross-section subsonic wind tunnel. Areas for further development are identified and recommendations for best practices are presented.

Keywords: Doppler global velocimetry, wind tunnel testing, flow measurement technique

1. Introduction

The drag coefficient is one of the most fundamental measurements in airfoil characterization and testing [1]. This is commonly achieved with pressure measuring techniques and control volume analysis, with an appropriate correction scheme such as that of Maskell [2]. Very low uncertainties can be achieved with this approach, even below the level of discrepancy in results amongst similar tests in various facilities. So-called “wake rakes” are also a commonly used method for acquiring pressure profiles and cross-sections downstream of a test article. Examples in large-scale facilities include the Delft University Low-Speed Wind Tunnel [3], the University of Stuttgart Laminar Wind Tunnel [4], and the Virginia Tech Stability Wind Tunnel [5]. Wake rakes are limited in their operation such that the
system cannot be implemented for very high angles of attack. Particularly where the airfoil is past stall, buffeting from large size scale vortices can cause high loadings. At these high angles of attack, blockage effects are also significant. Moreover, since Pitot probes cannot capture vorticity, measurements beyond stall do not yield worthwhile results [6].

Direct evaluation of pressure fields using Particle image velocimetry (PIV) has become increasingly reliable and feasible over the past decade. A review of such methods is given by van Oudhesden [7]. A major application of such methods is aerodynamic force determination, including the steady [8-10] and unsteady [11] lift, drag, and pitching moment of airfoil sections using planar PIV. In all PIV-based pressure measurement techniques, several computationally intensive steps are required to obtain velocities, material acceleration, and integration for pressure gradient, ultimately yielding a large amount of flow information. Additionally, PIV acquisition settings must be carefully selected for the application, and may impose additional constraints [7].

2. Optical Wake Rake

An “optical wake rake” system concept is herein introduced to complement the above methods in cases of high blockage flows (Figure 1). This technique provides velocity measurements through use of optical diagnostics, thus providing data at spatial resolutions finer than standard systems, without any flow disruption, and at any realizable angle of attack. The motivation for such a system is not to provide data comparable to PIV pressure-techniques, but rather to extend the benefits of non-intrusive measurement to rapid production-run testing. Instrumentation requirements and data processing costs are lower than PIV-based techniques, and can be applied to a wide range of applications to which pressure rakes cannot.
Fig. 1 Optical Wake Rake probe beam in the Stability Wind Tunnel downstream of the 0.800 meter DU96-W-180, at the location where the pressure rake would sit. The inset shows a typical control volume for wind tunnel drag measurements. Nominal inflow and outflow profiles are shown for an airfoil at moderate angle of attack.

As in the case of the pressure rake, the conditions upstream are evaluated from wall pressure taps. Downstream, the velocity profile is measured directly, but the static pressure must be estimated from wall pressure taps. Static pressure along the downstream beam profile will differ from uniform as a result of separated flow and vortex dynamics. For the data processing, the static pressure will be assumed constant across the test section for a given spanwise, streamwise location; this static pressure estimation with the optical wake rake system thus contributes higher uncertainty.

A laser diagnostic system known as cross-correlation Doppler global velocimetry (CC-DGV) [12] was used for the proof-of-concept measurements. CC-DGV yields Reynolds-averaged Navier-Stokes (RANS) compatible results for velocity. A probe beam of collimated laser light is directed through the flow, nominally along the same axis defined
by linking the tips of the pressure probes of standard pressure rakes. Small “seed” particles are introduced in the flow to allow Mie-scattering of laser light. Photodetectors places around the flow collect Doppler-shifted light from the seed particles, governed by the Doppler shift equation

$$\Delta \nu = \frac{(\theta - \ell) \cdot \vec{V}}{\lambda_0}$$  \hspace{1cm} (1)

where $\Delta \nu$ is the measured Doppler frequency shift, $\lambda$ is the incident laser wavelength, and $(\theta - \ell)$ describes the vector difference between the Mie-scattered light direction (camera observation direction) and incident light direction, respectively [13]. A key practical benefit to using DGV instead of PIV is that individual seed particles need not be resolved for DGV. In fact, several particles passing through a point in space can all contribute to the Doppler-shifted scattered light received by the cameras. This allows for large fields of view, such as may be required in large scale facilities, and long integration times which in turn lowers the power requirements of the laser.

The velocity component measured by each camera is the component of the world-frame vector $\vec{V}$ in the $(\theta - \ell)$ direction; for each component of velocity, one linearly independent $(\theta - \ell)$ pair must be established. Since only a single laser beam direction can be used, cameras must be placed at appropriate relative positions. Uncertainties of 1.30 ms$^{-1}$ for the streamwise component have been achieved in similar configurations; see Cadel and Lowe [12,14] for a full accounting of instrument level uncertainty sources.

3. Implementation

Initial validation of the optical wake rake was performed in the Virginia Tech Stability Wind Tunnel. The Stability Wind Tunnel is a closed-loop subsonic wind tunnel with a test section of cross-section 1.83 m by 1.83 m and length of 7.3 m. A photograph of the probe beam installed in the facility can be seen in Figure 1. Turbulence levels at 57 ms$^{-1}$ are below 0.031% [15]. The airfoil model used for these measurements was an 800 mm chord length DU96-W-180 [3], with a span extending the width of the tunnel. A side-wall suction system was also used to increase flow uniformity [5]. Data was acquired at angles attack for which pressure rake measurements were also possible in order to provide validation data. A chord-based Reynolds number of $3 \times 10^6$ was used for all measurements.
A Verdi V6 diode-pumped, solid state, Nd:YVO₄, continuous wave, 532 nm laser operating at 6 Watts output power supplied the beam. Three pco.edge sCMOS cameras with 5.5 megapixel resolution were positioned around the tunnel. According to the momentum integral in Eq. 1, only the streamwise flow magnitude factors into the calculation of drag force. For the present measurement, only data from the camera with the highest sensitivity vector to the streamwise \((u)\) component was used, and is scaled accordingly assuming that the Doppler shift is entirely due to the \(u\)-velocity (i.e. spanwise and transverse components of velocity are zero). At non-stalled angles of attack this assumption is quite reasonable; at angles past stall, it may bias the results towards high velocity deficits in the wake since some of the Doppler shift contribution is from the spanwise and transverse components. Variation of the sensitivity vector with spatial position is not presently taken into account. A small field of view (approx. 600mm) was used due to constraints on optical access, and as such spatial variation of the sensitivity vector will not account for much error. A Vi-Count Compact mineral oil seeder was placed atop the tunnel and seed was supplied through a port hole in the ceiling downstream of the model and allowed to recirculate.

Camera registration and spatial mapping was achieved using Tsai’s camera pin-hole model as implemented in LaVision’s DaVis 8.2 software [16]. Using this approach, deviations on the order of a tenth of a millimeter for corresponding locations were achieved among the cameras (corresponding to sub-pixel accuracy). Mapped images are exported for CC-DGV processing with a suite of Matlab scripts.

For each point in the probe beam light propagation direction, the intensity of thirteen pixels in the beam-normal direction was averaged for each image in the scan to create the signal used for cross-correlation. A self-referencing technique was used to process data such that the magnitude of the Doppler shift was found with cross-correlation processing, and scaled in magnitude based on the validation data measured with the pressure rake. The pressure rake includes 112 total pressure probes (1.6mm diameter stainless steel tubes) and 7 static pressure probes (Dwyer model 167, 3mm diameter). In the central 528mm area, probes are laterally spaced by 7.6mm, and outside of this region probes are spaced by 25.4mm. Pressures are read using a series of four DTC Initium ESP-32HD 32-channel pressure scanners. Further details can be found in Joseph [5].

4. Results
Representative cases at a chord-based Reynolds number $Re = 3 \times 10^6$ and angles of attack $AoA = 0^\circ$ and $AoA = 10^\circ$ are plotted in Figure 2 along with pneumatic rake validation data for the central portion of the wake. The discrepancy in the velocity profile on opposite sides of the wake is due to a difference in static pressure; it is suspected that the bulk flow is turning at this location. Due to restrictions from concurrent measurements, only a small portion of the full tunnel width could be imaged, such that drag estimates are not possible with present data.

![Fig. 2 Optical Wake Rake and validation data at Reynolds number of $3 \times 10^6$ and $0^\circ$ (left) and $10^\circ$ (right) angle of attack](image)

**5. Discussion and Recommendations**

A new technique for acquiring wake profiles and drag force with a non-intrusive optical technique is explored. Initial evaluation of the system is performed in the full-scale Virginia Tech Stability Wind Tunnel, and the results presented show potential for use in cases, including high blockage flows, where standard instrumentation such as pressure rakes cannot be installed. The present data suggest that with proper extension to larger field of views and optimized camera positioning, the Doppler global velocimetry based method is capable of measuring wake profiles compatible with aerodynamic drag estimates.
Uncertainties for the DGV-based rake were found to be higher than with standard pressure rakes, and as such is not recommended for use when other methods are available. However, the uncertainties may be tolerable given the applicability to flows that cannot otherwise be studied by standard instrumentation. The absolute uncertainties in cross-correlation DGV make it well suited for high-speed flows, but correspondingly have a larger impact in the incompressible regime. Alternate Doppler global velocimetry methods, such as two-frequency (2ν) DGV [17] or frequency modulated (FM) DGV [18] offer lower uncertainties in the regime of interest.

Multiple cameras may be aligned to provide higher sensitivity along the streamwise direction, rather than the typically sought linear independence. Larger camera fields of view or multiple image planes must also be used to enable calculation of drag. Additional attention must also be paid to maintaining constant seeding density during the acquisition. This is standard in all scattering techniques, but for application in large facilities poses an additional challenge. To mitigate this effect, use of a second image to normalize each of the vapor-cell filtered images is recommended. This is standard in traditional DGV [19] as well as in 2ν-DGV [17], and a similar image normalization method is used in FM-DGV as well [18].

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