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An analysis of Precision Electroweak Measurements: Summer 1998 Update.

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Abstract

We update our analysis of precision electroweak measurements using the latest data announced at Moriond, March 1998. Possible oblique corrections from new physics are parametrized using the STU formalism of Ref. [1], and non–oblique corrections to the $Zb\bar{b}$ vertex are parametrized using the ξ_b–ζ_b formalism of Ref. [2]. The implication of the analysis on minimal $SU(5)$ grand unification is discussed.

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1 Introduction

The analysis of precision electroweak measurements provides us with one of the few opportunities to constrain new physics beyond the Standard Model. The effectiveness of the approach is evident in the prediction of the top quark mass which was predicted to be around 180 GeV [3] well before its direct measurement some time later [4].

In the past few years, we have seen a few notable developments in the field of precision electroweak measurements. In addition to the ever increasing accuracy of the LEP and SLD measurements [5], a number of new or updated measurements have been announced:

- The University of Colorado Group announced a new measurement of the weak charge of Cesium 133 which improves the experimental error by a factor of 7 compared to their previous measurement [6, 7].
- The CCFR/NuTeV collaboration has announced a preliminary determination of $1 - M_W^2/M_Z^2$ from νN deep inelastic scattering, [8] which already improves on the previous result from CCFR [9] by a factor of 2.
- With the start of LEP2 and new analyses of data from CDF and D0, the error on the W mass has improved by more than a factor of 2 since the 1996 version of the Review of Particle Properties. [10, 11]
- Of the measurements done by LEP at the Z resonance, updated values of $A_{FB}^{0,b}$ from ALEPH [12] and $A_{\tau,e}$ from L3 [13] are noteworthy, as they shift the preferred value of $\sin^2\theta_{\rm eff}^{\rm lept}$ somewhat.

In light of these developments, it is worthwhile to revisit these data in hopes of assessing the status of the standard model and prospects for new physics.

In this letter, we present the constraints imposed on new physics from experimental data available as of June 1998. In section 2, we restrict our attention to oblique electroweak corrections and present the results in terms of the S, T, U parameters introduced in Ref. [1]. In section 3, we analyze the heavy flavor observables from LEP and SLD for possible non–oblique corrections to the $Zb\bar{b}$ vertex using the formalism of Ref. [2]. In this analysis, we let $\alpha_s(M_z)$ float and fit it to the data also. In section 4, we discuss the implications of our results for minimal $SU(5)$ grand unification. Section 5 concludes.

2 Constraints on Oblique Electroweak Corrections

The effects of new physics on electroweak observables can be quite difficult to quantify. Given the tremendous success of the standard model in accounting for the data, however, it is reasonable to restrict our attention by making some simplifying assumptions. This enables us to describe potential deviations from the standard model in terms of just a few parameters.

The simplest, but not necessarily comprehensive, assumptions are the following:

1. The electroweak gauge group is the standard $SU(2)_L \times U(1)_Y$. The only electroweak gauge bosons are the photon, the $W^\pm$, and the Z.

2. The couplings of new physics to light fermions are highly suppressed so that vertex and box corrections from new physics can be neglected (with the possible exception of processes involving the b quark). Only vacuum polarization (i.e. oblique) corrections need to be considered. Further justifications of this approximation have been discussed in Ref. [1]

3. The mass scale of new physics is large compared to the W and Z masses.

These assumptions let us express the virtual effects of new physics in terms of just three parameters defined as: [1]

$$\begin{aligned}
\alpha S &= 4s^2c^2\left[\Pi'_{\rm ZZ}(0) - \frac{c^2-s^2}{sc}\Pi'_{\rm Z\gamma}(0) - \Pi'_{\gamma\gamma}(0)\right], \\
\alpha T &= \frac{\Pi_{\rm WW}(0)}{M_W^2} - \frac{\Pi_{\rm ZZ}(0)}{M_Z^2}, \\
\alpha U &= 4s^2\left[\Pi'_{\rm WW}(0) - c^2\Pi'_{\rm ZZ}(0) - 2sc\Pi'_{\rm Z\gamma}(0) - s^2\Pi'_{\gamma\gamma}(0)\right].
\end{aligned} \tag{2.1}$$

Here, $\Pi_{\rm XY}(q^2)$ is the transverse part of the vacuum polarization function between gauge bosons X and Y and the prime represents a derivative with respect to q^2. α is the fine structure constant and s and c are shorthand notations for the sine and cosine of the weak mixing angle. Only the contribution of new physics to these functions are to be included. The parameters T and U are defined so that they vanish if new physics does not break the custodial $SU(2)$ symmetry. See Ref. [14] for a discussion on the symmetry properties of S.

The theoretical prediction for any observable will then consist of all the standard model corrections to its tree level prediction plus the possible corrections from new physics expressed in terms of S, T, and U. For instance, if the values of α, G_μ, and M_Z are used as input for the SM prediction, the shift in the ρ parameter, the effective value of $\sin^2\theta_w$ in leptonic asymmetries, and the W mass due to new physics will be given by

$$\begin{aligned}
\rho - [\rho]_{\rm SM} &= \alpha T \\
\sin^2\theta_{\rm eff}^{\rm lept} - [\sin^2\theta_{\rm eff}^{\rm lept}]_{\rm SM} &= \frac{\alpha}{c^2-s^2}\left[\frac{1}{4}S - s^2c^2T\right] \\
M_W/[M_W]_{\rm SM} &= 1 + \frac{\alpha}{2(c^2-s^2)}\left[-\frac{1}{2}S + c^2T + \frac{c^2-s^2}{4s^2}U\right],
\end{aligned} \tag{2.2}$$

where $[\mathcal{O}]_{\rm SM}$ denotes the Standard Model prediction of the observable $\mathcal{O}$.

All other electroweak observables we will be considering get their dependence on new physics corrections through ρ and $\sin^2\theta_{\rm eff}^{\rm lept}$. As a result, they only depend on S and T, while the W mass will be the sole observable which depends on U. By comparing standard model predictions with experimental measurements, we can determine the favored values of S, T, and U. The values of S, T and U obtained in this way give a quantitative measure of the potential size of radiative corrections from new physics. If the standard model predictions for particular values of $\alpha^{-1}(M_Z)$, $M_{\rm top}$ and $M_{\rm higgs}$ yielded $S = T = U = 0$, then this would mean perfect agreement between the Standard Model and experiment. On the other hand, non-zero values of S, T and U would imply either that the experiments prefer the existence of extra corrections from physics beyond the Standard Model, or that the values of $\alpha^{-1}(M_Z)$, $M_{\rm top}$ and $M_{\rm higgs}$ chosen in defining the "reference" Standard model were not optimal.

In table 1, we show the data we will be using to constrain S, T, and U. To the best of our knowledge, this is a comprehensive set of all precision electroweak measurements that are likely to have an impact on the analysis. We have excluded all the heavy flavor observables from the present analysis, since the impact of new physics on these quantities cannot be fully parametrized using S, T and U. We will return to the heavy flavor measurements in the next section. Some comments are in order:

- The $\nu_\mu e - \bar{\nu}_\mu e$ scattering parameters $g_V^{\nu e}$ and $g_A^{\nu e}$ are defined as

$$\begin{aligned} g_V^{\nu e} &\equiv 2 g_V^{\nu} g_V^{e}, \\ g_A^{\nu e} &\equiv 2 g_A^{\nu} g_A^{e}, \end{aligned}$$

  where g_V^f and g_A^f are the effective vector and axial–vector couplings of the fermion f to the Z. At tree level, we have

$$\begin{aligned} g_V^{\nu e} &= \rho \left(-\frac{1}{2} + 2s^2 \right), \\ g_A^{\nu e} &= \rho \left(-\frac{1}{2} \right). \end{aligned}$$

- The $\nu_\mu N - \bar{\nu}_\mu N$ deep inelastic scattering parameters g_L^2 and g_R^2 are defined as

$$\begin{aligned} g_L^2 &\equiv u_L^2 + d_L^2, \\ g_R^2 &\equiv u_R^2 + d_R^2, \end{aligned}$$

  where q_L and q_R are the effective left–handed and right–handed couplings of the quark q to the Z. At tree level, they are equal to

$$\begin{aligned} g_L^2 &= \rho \left(\frac{1}{2} - s^2 + \frac{5}{9} s^4 \right), \\ g_R^2 &= \rho \left(\frac{5}{9} s^4 \right). \end{aligned}$$

  The quantity measured by NuTeV is a certain linear combination of $u_{L/R}^2$ and $d_{L/R}^2$ which is roughly equal to the Paschos–Wolfenstein parameter: [18]

$$R^- = g_L^2 - g_R^2 = \rho \left(\frac{1}{2} - s^2 \right).$$

  See Ref. [8] for details.

- The weak charge of atomic nuclei measured in atomic parity violation experiments is defined as [19]

$$Q_W(Z,N) \equiv -2[C_{1u}(2Z+N) + C_{1d}(Z+2N)],$$

where C_{1q} $(q = u, d)$ are parameters in the parity violating low energy effective Lagrangian:

$$\mathcal{L}_{\rm PV} = \frac{G_\mu}{\sqrt{2}} \sum_{q=u,d} [C_{1q}(\bar{e}\gamma_\mu\gamma_5 e)(\bar{q}\gamma^\mu q) + C_{2q}(\bar{e}\gamma_\mu e)(\bar{q}\gamma^\mu\gamma_5 q)]\,.$$

At tree level, we have

$$Q_W(Z,N) = \rho[Z(1-4s^2) - N].$$

- The value of $\sin^2\theta_{\rm eff}^{\rm lept}$ from LEP is that derived from purely leptonic asymmetries only. We include both the LEP and SLD measurements in the fit with the quoted errors. Another approach has been taken in Ref. [20]

- The value of $\Gamma_{\ell^+\ell^-}$ is that derived from the LEP Z lineshape variables Γ_Z, $\sigma^0_{\rm had}$, and $R_\ell = \Gamma_{\rm had}/\Gamma_{\ell^+\ell^-}$. Using this one value in our analysis is equivalent to using all three with correlations taken into account.

- The value of the W mass is the average of direct determinations from LEP2 [10] and $p\bar{p}$ colliders [11].

To fix the reference Standard Model to which we compare the experimental data, we use the values [21, 22, 23]

$$\begin{aligned} M_{\rm top} &= 173.9\,\text{GeV}, \\ M_{\rm higgs} &= 300\,\text{GeV}, \\ \alpha^{-1}(M_Z) &= 128.9, \\ \alpha_s(M_Z) &= 0.12 \end{aligned} \tag{2.3}$$

Fitting to the data of table 1, we find

$$\begin{aligned} S &= -0.33 \pm 0.14 \\ T &= -0.14 \pm 0.15 \\ U &= 0.07 \pm 0.22 \end{aligned} \tag{2.4}$$

with the correlation matrix given by

$$\begin{bmatrix} 1 & 0.85 & -0.21 \\ 0.85 & 1 & -0.42 \\ -0.21 & -0.42 & 1 \end{bmatrix} \tag{2.5}$$

where rows and columns are labelled in the order S, T, U. The quality of the fit is $\chi^2 = 4.5/(11-3)$. Compared to the results of the 1996 [24] data, the major

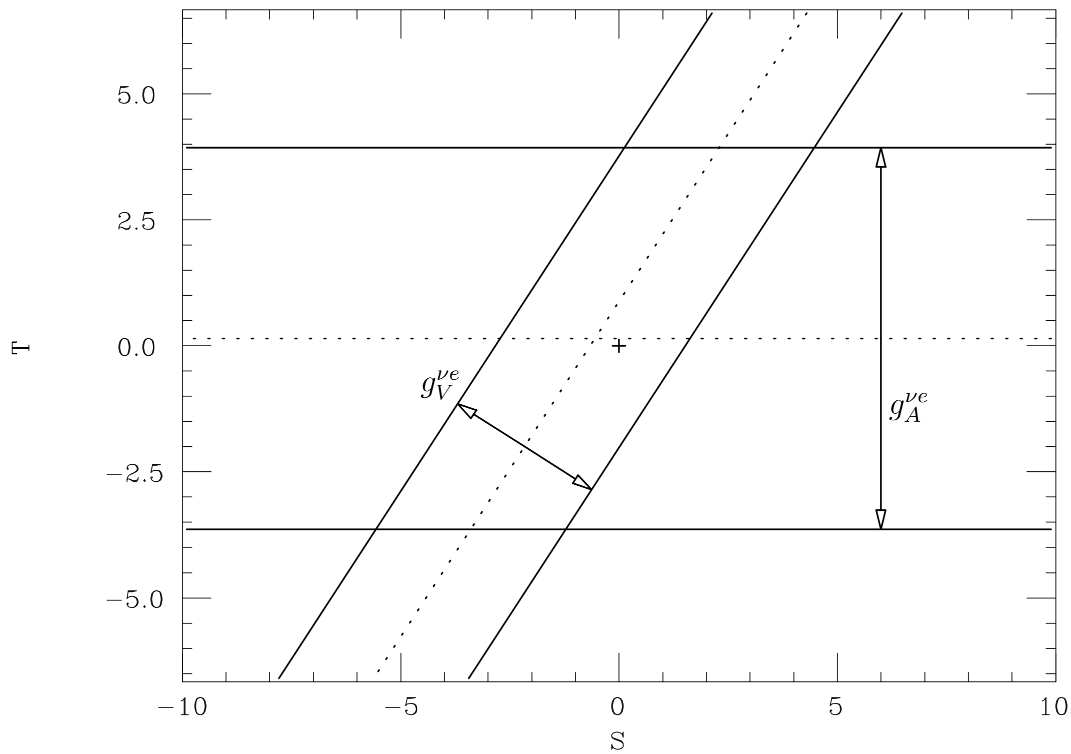
Observable	SM prediction	Measured Value	Reference
$\nu_\mu e$ and $\bar{\nu}_\mu e$ scattering			
$g_V^{\nu e}$	-0.0365	-0.041 ± 0.015	[15]
$g_A^{\nu e}$	-0.5065	-0.507 ± 0.014	[15]
Atomic Parity Violation			
$Q_W({}^{133}_{55}\mathrm{Cs})$	-73.19	-72.41 ± 0.84	[15]
$Q_W({}^{205}_{81}\mathrm{Tl})$	-116.8	-114.8 ± 3.6	[15]
$\nu_\mu N$ and $\bar{\nu}_\mu N$ DIS			
g_L^2	0.3031	0.3009 ± 0.0028	[15]
g_R^2	0.0304	0.0328 ± 0.0030	[15]
NuTeV	0.2289	0.2277 ± 0.0022	[8]
LEP/SLD			
$\Gamma_{\ell^+\ell^-}$	0.08392 GeV	0.08391 ± 0.00010 GeV	[5]
$\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$ (LEP)	0.23200	0.23157 ± 0.00041	[5]
$\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$ (SLD)	0.23200	0.23084 ± 0.00035	[5]
W mass			
M_W	80.315 GeV	80.375 ± 0.064 GeV	[10, 11]

Table 1: The data used for the oblique correction analysis. The value of $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$ for LEP is from leptonic asymmetries only. The W mass is the average of LEP2 [10] and $p\bar{p}$ [11] values. Definitions of $g_{V,A}^{\nu e}$ and $g_{L,R}^2$ can be found in the Review of Particle Properties [15]. The SM predictions for the W mass and the LEP/SLC observables were obtained using the program ZFITTER 4.9 [16], and the predictions for the other low energy observables were calculated from the formulae given in Ref. [17]. The parameter choice for the reference SM was $M_Z = 91.1867$ GeV [5], $M_{\mathrm{top}} = 173.9$ GeV [21], $M_{\mathrm{higgs}} = 300$ GeV, $\alpha^{-1}(M_Z) = 128.9$ [23], and $\alpha_s(M_Z) = 0.120$.

improvement is in the limits on U: the error has been reduced by more than a factor of 2. This can be directly traced to the improvement in the value of M_W.

In Figs. 1 to 4, we show the limits placed on S and T separately by each class of experiments. The bands in the upper figures represent the 1–σ limits placed on S and T by each observable. Note that there is an overall change in scale between Figs. 3 and 4. In Fig. 5, we compare the 90% confidence limits placed on S and T by the four classes of experiments we consider, while Fig. 6 shows the limits on S and T combining all experiments.

We can see from Fig. 6 that the current data favor either a small value of the Higgs mass or a larger value of $\alpha^{-1}(M_Z)$. The indications of a light Higgs would be consistent with low energy supersymmetry, which predicts a Higgs lighter than about 130 GeV [25], and typically gives small contributions to the oblique parameters [26].



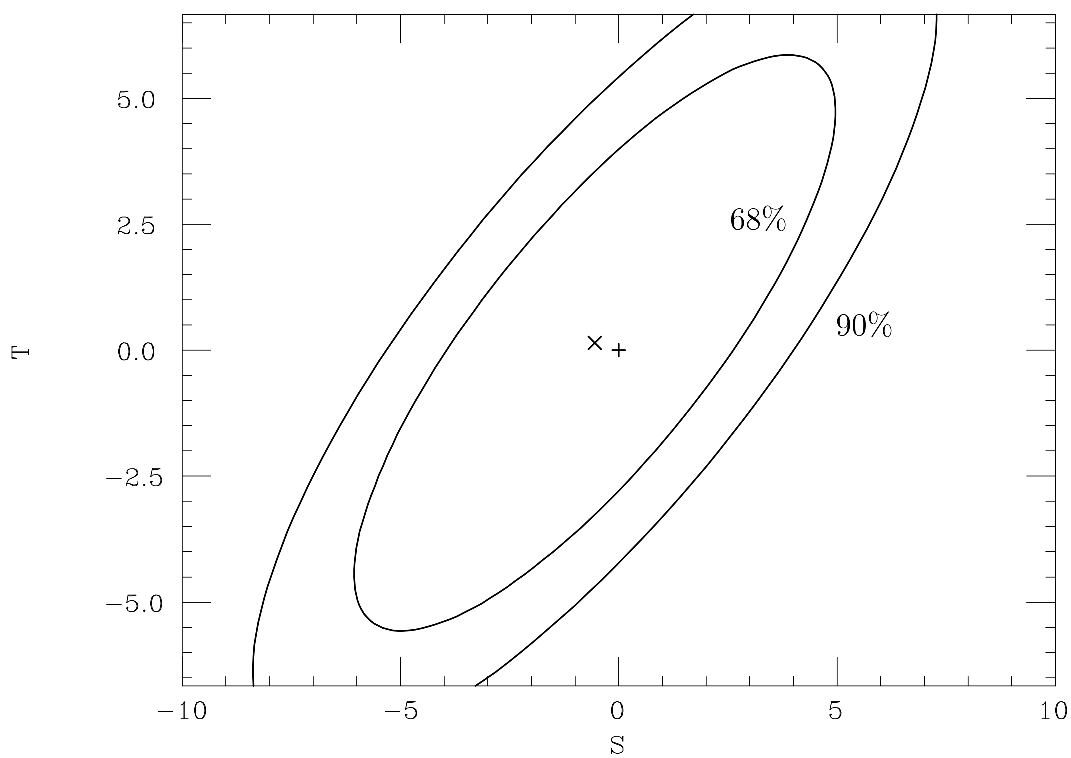


Figure 1: The limits on S and T from $\nu_\mu e$ and $\bar{\nu}_\mu e$ scattering experiments.

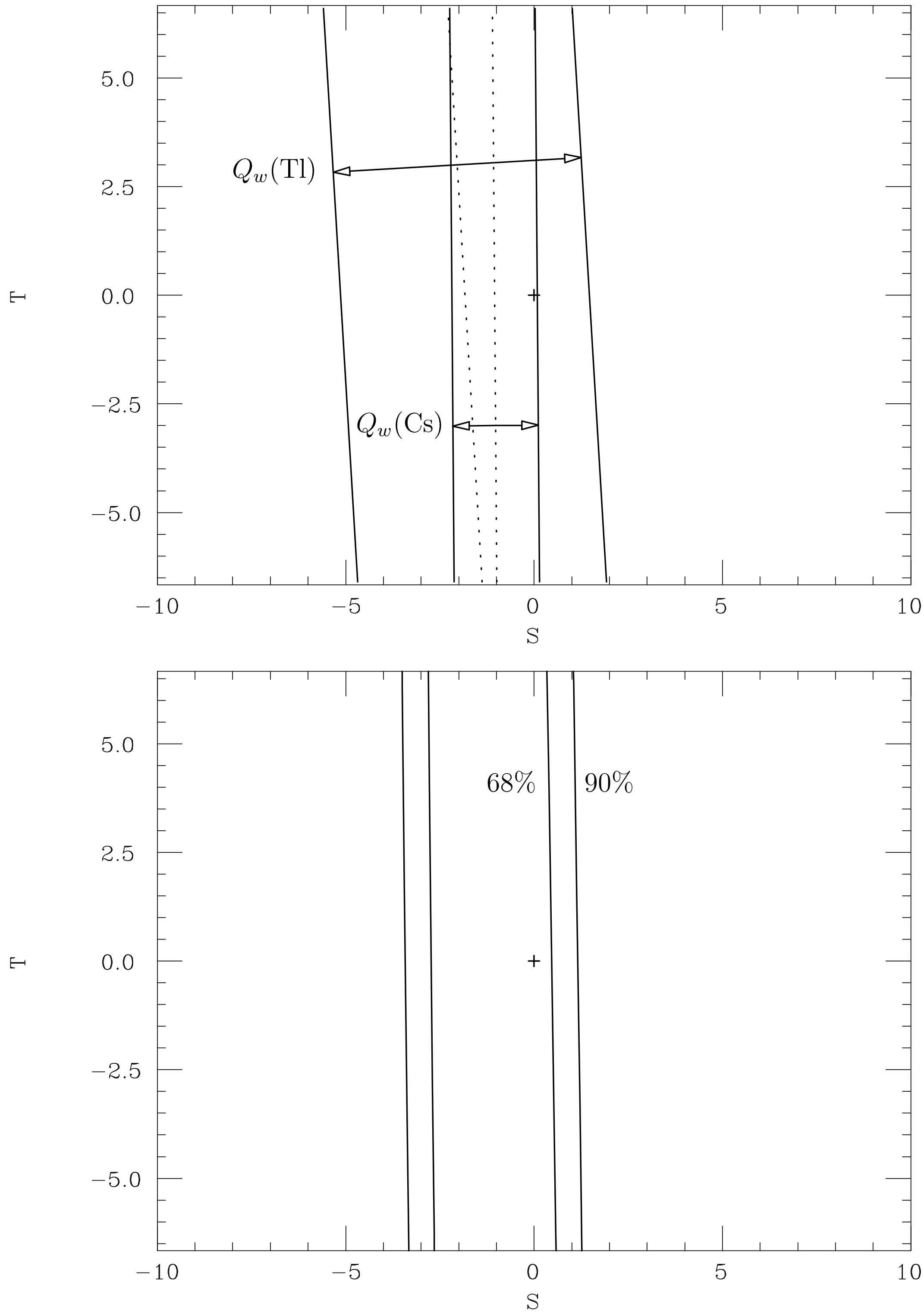


Figure 2: The limits on S and T from atomic parity violation experiments.

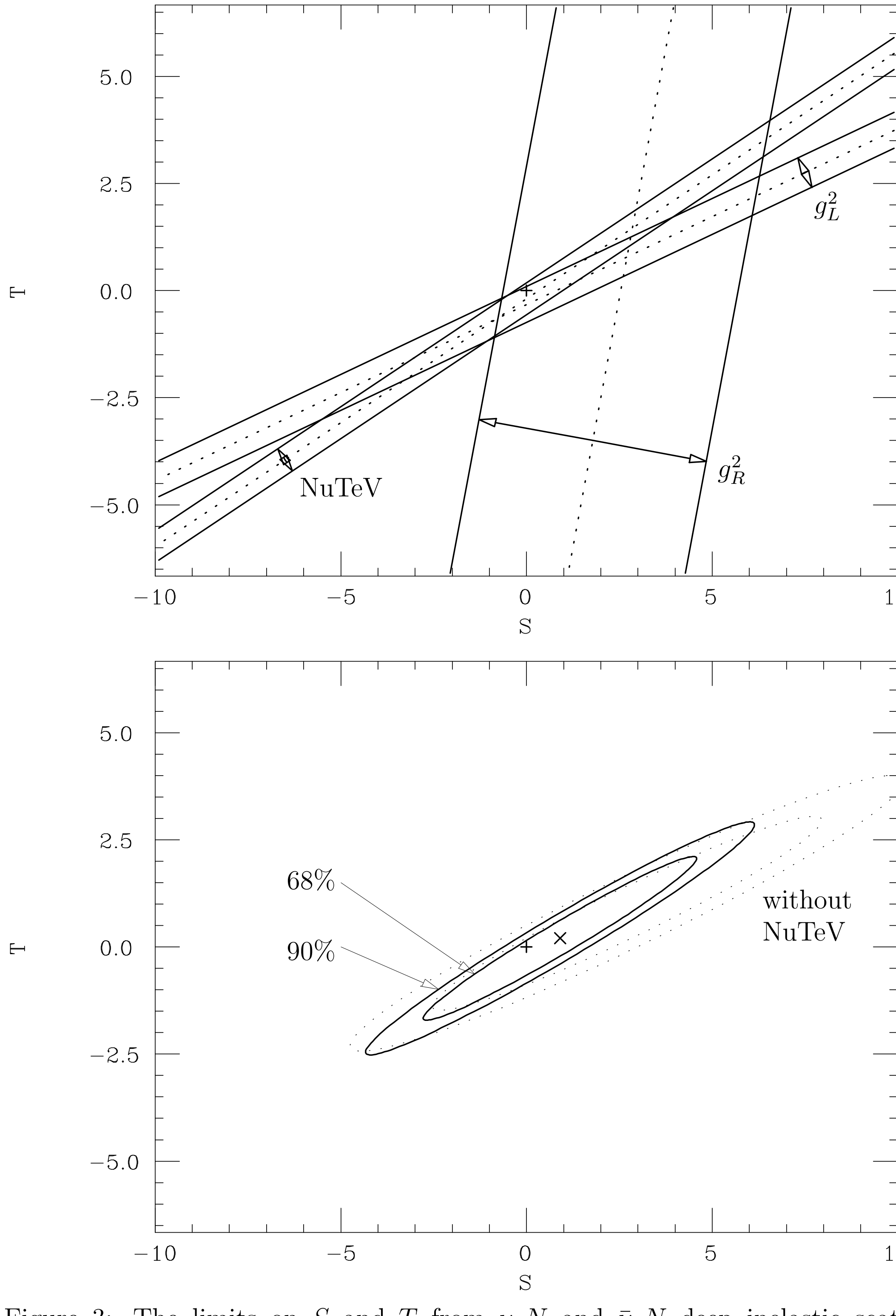


Figure 3: The limits on S and T from $\nu_\mu N$ and $\bar{\nu}_\mu N$ deep inelastic scattering experiments.

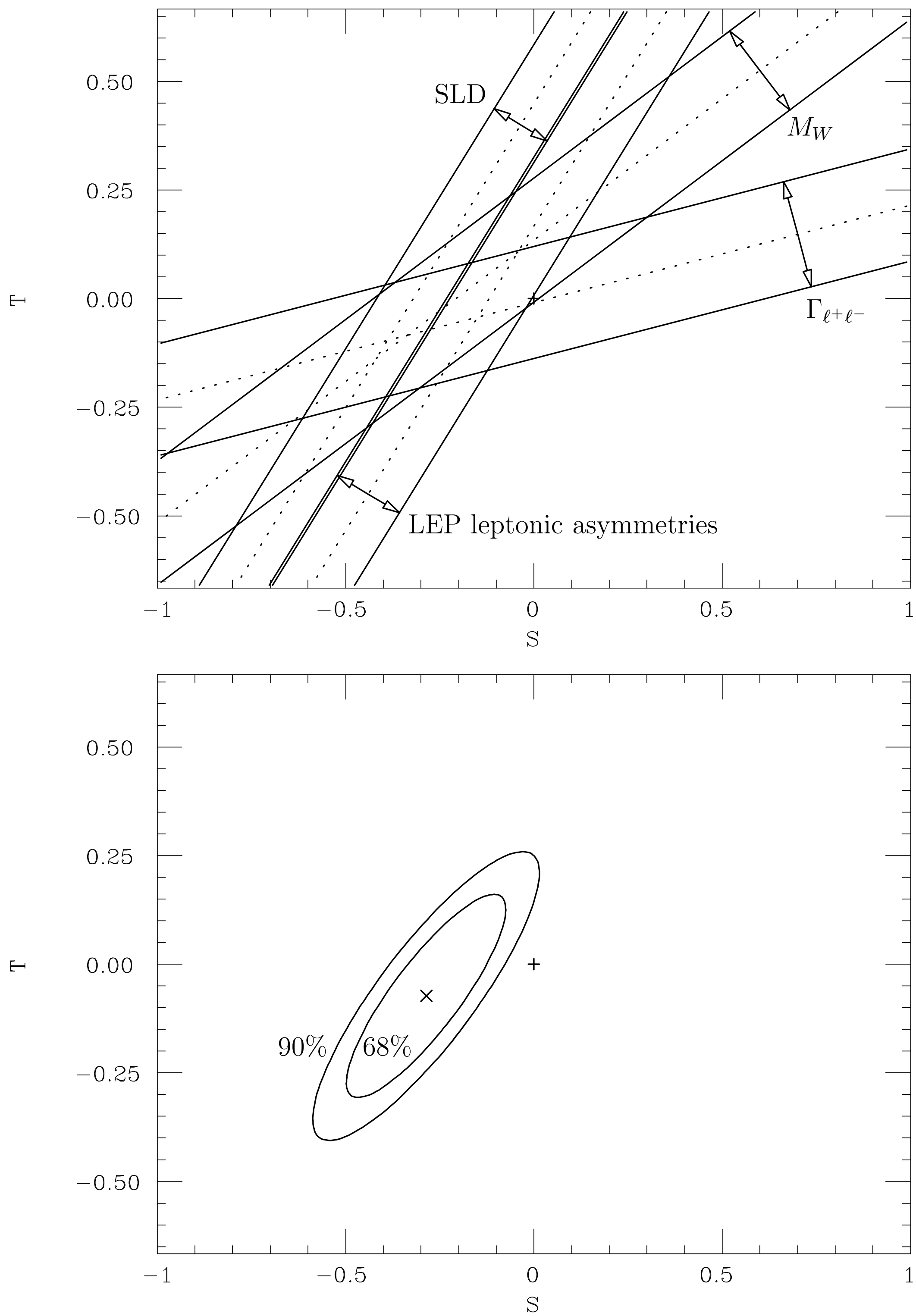


Figure 4: The limits on S and T from SLD, LEP, and M_W (LEP2 and $p\bar{p}$).

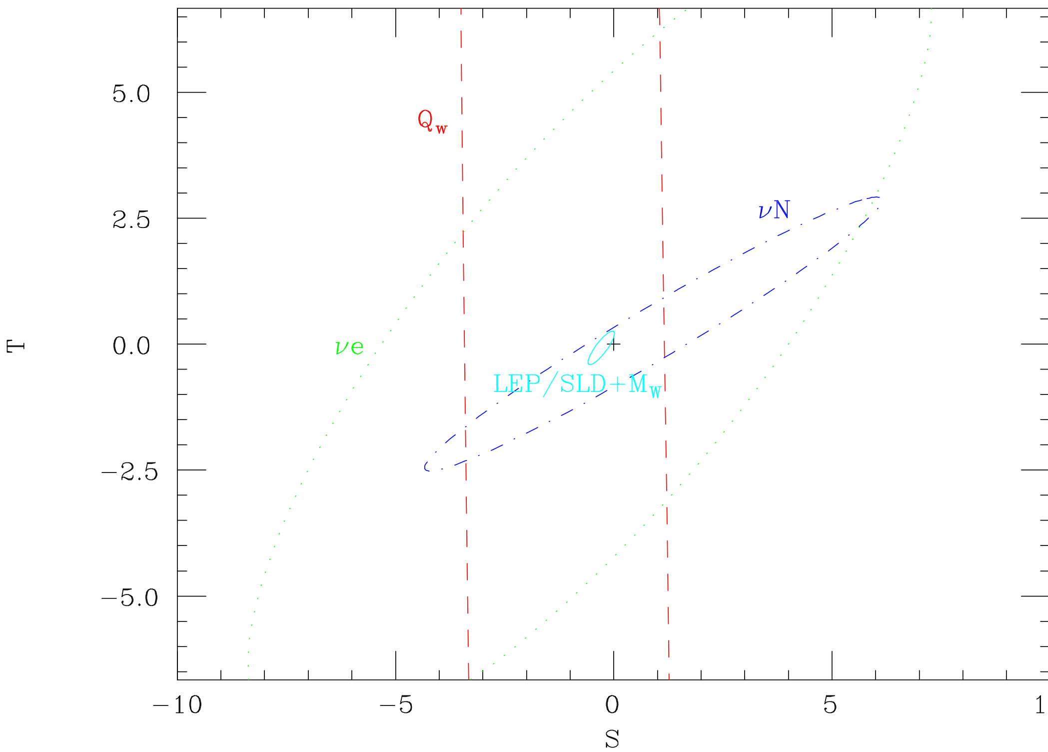


Figure 5: Comparison of the 90% likelihood contours from different experiments.

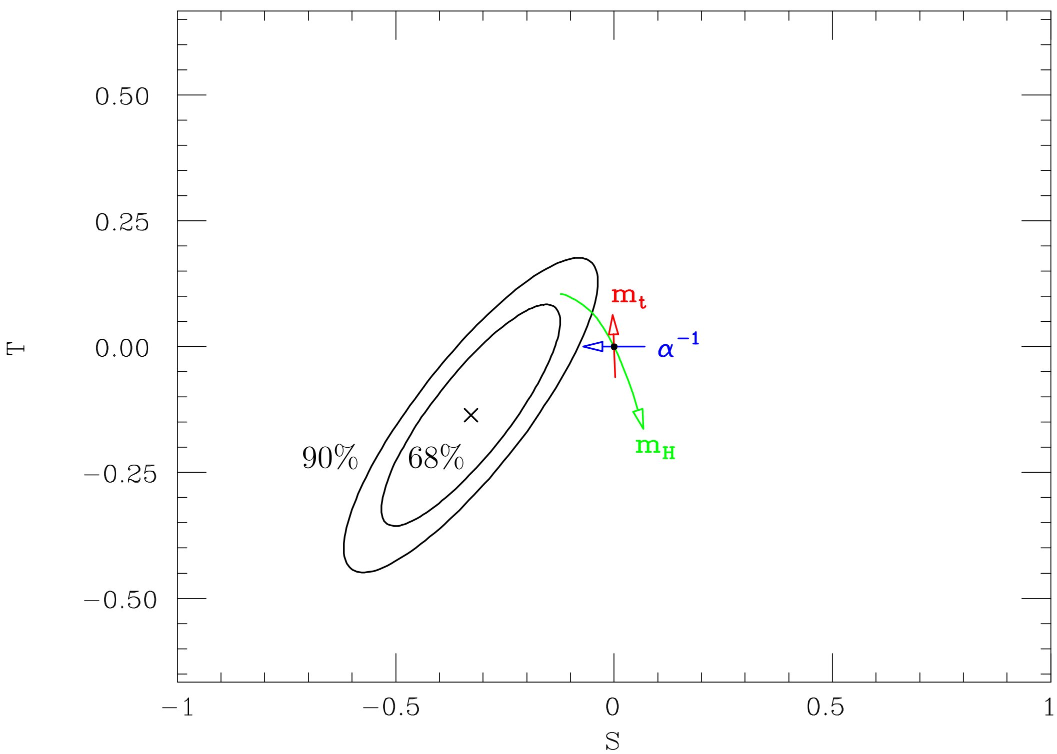


Figure 6: The limits on S and T : all experiments combined. The arrows show the range the SM point will move when M_{top}, M_{higgs}, and $\alpha^{-1}(M_Z)$ are varied. Red arrow: M_{top} varied from 168.7 to 179.1 GeV [21], Green arrow: M_{higgs} varied from 80 to 1000 GeV [22], Blue arrow: $\alpha^{-1}(M_Z)$ varied from 128.8 to 129.0 [23].

3 Constraints on Non-Oblique Corrections to the $Zb\bar{b}$ Vertex

In the previous section we excluded heavy flavor observables from our analysis, since in principle there could be corrections to these quantities that cannot be described solely in terms of S, T and U. In this section, we extend our analysis to include heavy flavor observables. This of course entails additional assumptions beyond those enumerated at the beginning of Sec. 2: in particular, we now assume:

1. The couplings of light (u, d, s, c) quarks to the Z are dictated solely by the standard model together with possible oblique corrections from new physics.

2. The couplings of the b to the Z may exhibit additional deviations from the standard model in the form of "direct" or "non-oblique" corrections; that is to say, the couplings of the b may receive appreciable corrections from vertex diagrams in addition to corrections from vacuum polarization diagrams.

These additional assumptions may appear on the face of it to be quite artificial, and indeed they do restrict considerably the class of models that are accurately described by our analysis. Just the same, however, these assumptions are valid for a large class of models. The reason is that the b is the isospin partner of the top, and hence its couplings can be modified by the mechanism responsible for generating the large top mass. Indeed, even in the standard model, the b receives "non-oblique" corrections that are absent for the first two generations. Appreciable non-oblique corrections to the b couplings would be expected generically in models with extended Higgs sectors and in models where the (t, b) doublet is involved directly in electroweak symmetry breaking.

Measurements of heavy flavor observables have shifted somewhat in recent years as experimental understanding has improved. [27] In particular, an apparent excess in the partial width of the Z to b quarks has decreased substantially, improving the comparison between the standard model and experiment. (cf. Figs. 7 and 8.) In this section we use the latest data [5] to determine how well the standard model describes the couplings of the b to the Z.

We begin by defining

$$\begin{aligned} \delta\rho &= \alpha T, \\ \delta s^2 &= \frac{\alpha}{c^2 - s^2}\left[\frac{1}{4}S - s^2 c^2 T\right]. \end{aligned} \tag{3.1}$$

$\delta\rho$ and δs^2 are just the shifts of the ρ parameter and $\sin^2\theta_{\rm eff}^{\rm lept}$:

$$\begin{aligned} \rho &= [\rho]_{\rm SM} + \delta\rho, \\ \sin^2\theta_{\rm eff}^{\rm lept} &= [\sin^2\theta_{\rm eff}^{\rm lept}]_{\rm SM} + \delta s^2. \end{aligned} \tag{3.2}$$

We write the left and right handed couplings of the b quark to the Z as

$$g_L^b = [g_L^b]_{\rm SM} + \frac{1}{3}\delta s^2 + \delta g_L^b,$$

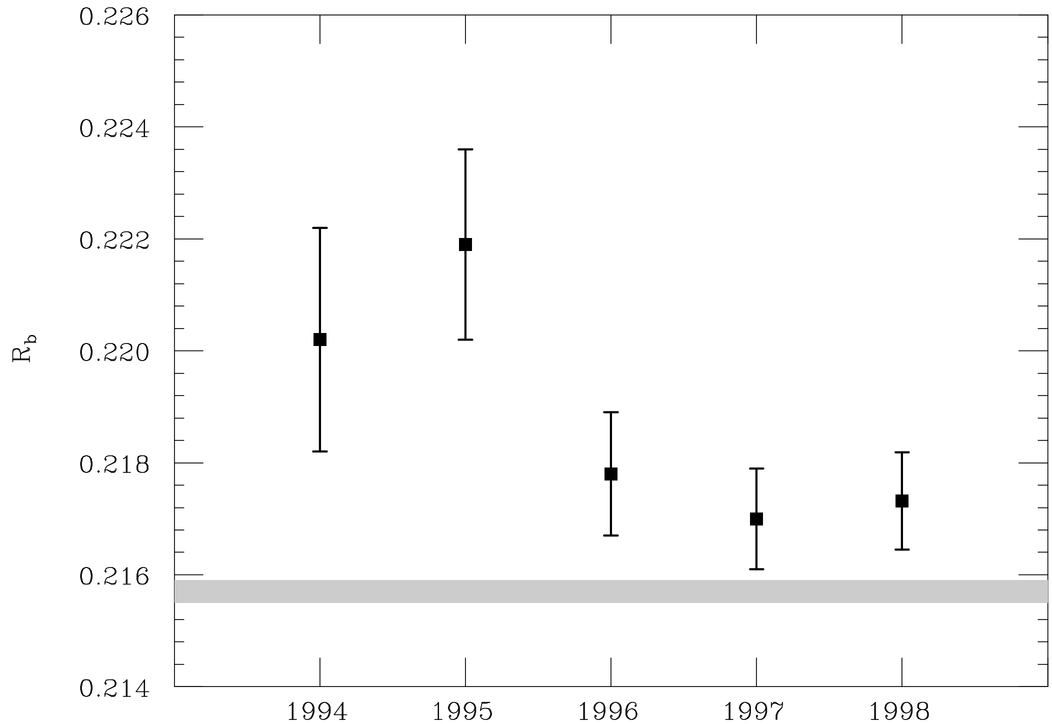


Figure 7: Change in the experimental value of R_b. The Standard Model prediction is shown by the shaded band.

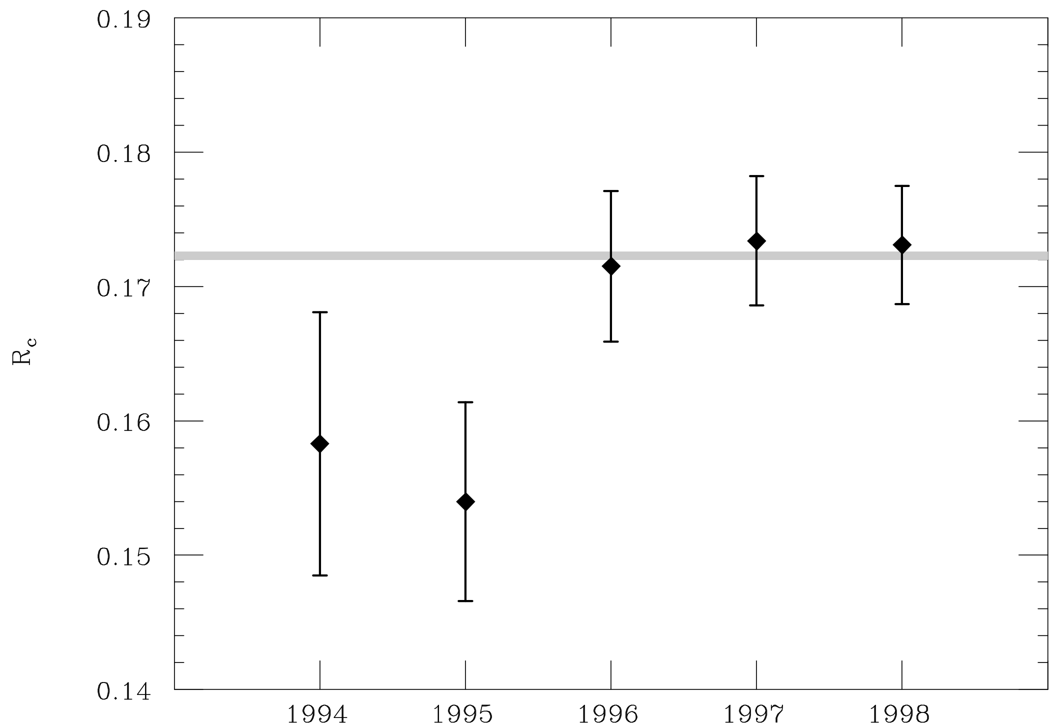


Figure 8: Change in the experimental value of R_c. The Standard Model prediction is shown by the shaded band.

$$g_R^b = [g_R^b]_{\rm SM} + \frac{1}{3}\,\delta s^2 + \delta g_R^b, \tag{3.3}$$

where we have included possible non–oblique corrections from new physics, δg_L^b and δg_R^b. Assuming that only the couplings of the b are significantly affected by non–oblique corrections, we can compute the dependence of electroweak observables on $\delta\rho$, δs^2, δg_L^b, and δg_R^b.

It is convenient to define the following linear combinations of δg_L^b and δg_R^b:

$$\begin{aligned} \xi_b &\equiv (\cos\phi_b)\delta g_L^b - (\sin\phi_b)\delta g_R^b, \\ \zeta_b &\equiv (\sin\phi_b)\delta g_L^b + (\cos\phi_b)\delta g_R^b, \end{aligned} \tag{3.4}$$

where

$$\phi_b \equiv \tan^{-1}|g_R^b/g_L^b| \approx 0.181. \tag{3.5}$$

By expanding $\Gamma_{b\bar{b}}$ about the point $\delta s^2 = \xi_b = \zeta_b = 0$, we find

$$\begin{aligned} \Gamma_{b\bar{b}} &= [\Gamma_{b\bar{b}}]_{\rm SM}\left\{1 + \delta\rho + \frac{2}{3}\left[\frac{g_L^b + g_R^b}{(g_L^b)^2 + (g_R^b)^2}\right]\delta s^2 \right. \\ &\qquad \left. + \left[\frac{2}{(g_L^b)^2 + (g_R^b)^2}\right]\Big(g_L^b\,\delta g_L^b + g_R^b\,\delta g_R^b\Big)\right\} \\ &= [\Gamma_{b\bar{b}}]_{\rm SM}\Big(1 + \delta\rho + 1.25\,\delta s^2 - 4.65\,\xi_b\Big) \end{aligned} \tag{3.6}$$

Similarly,

$$\begin{aligned} A_b &= \frac{(g_L^b)^2 - (g_R^b)^2}{(g_L^b)^2 + (g_R^b)^2} \\ &= [A_b]_{\rm SM}\left\{1 - \frac{4}{3}\left[\frac{g_L^b g_R^b(g_L^b - g_R^b)}{(g_L^b)^4 - (g_R^b)^4)}\right]\delta s^2 \right. \\ &\qquad \left. - \left[\frac{4 g_L^b g_R^b}{(g_L^b)^4 - (g_R^b)^4}\right]\Big(g_L^b\,\delta g_R^b - g_R^b\,\delta g_L^b\Big)\right\} \\ &= [A_b]_{\rm SM}\Big(1 - 0.68\,\delta s^2 - 1.76\,\zeta_b\Big). \end{aligned} \tag{3.7}$$

All the other observables get their dependence on δg_L^b and δg_R^b through either $\Gamma_{b\bar{b}}$ or A_b so they will depend on either ξ_b or ζ_b, but not both. The observables that depend on $\Gamma_{b\bar{b}}$ are:

$$\begin{aligned} \Gamma_Z &= [\Gamma_Z]_{\rm SM}\Big(1 + \delta\rho - 1.06\,\delta s^2 - 0.71\,\xi_b + 0.21\,\delta\alpha_s\Big), \\ \sigma^0_{\rm had} &= [\sigma^0_{\rm had}]_{\rm SM}\Big(1 + 0.11\,\delta s^2 + 0.41\,\xi_b - 0.12\,\delta\alpha_s\Big), \\ R_\ell \equiv \Gamma_{\rm had}/\Gamma_{\ell^+\ell^-} &= [R_Z]_{\rm SM}\Big(1 - 0.85\,\delta s^2 - 1.02\,\xi_b + 0.31\,\delta\alpha_s\Big), \\ R_b \equiv \Gamma_{b\bar{b}}/\Gamma_{\rm had} &= [R_b]_{\rm SM}\Big(1 + 0.18\,\delta s^2 - 3.63\,\xi_b\Big), \\ R_c \equiv \Gamma_{c\bar{c}}/\Gamma_{\rm had} &= [R_c]_{\rm SM}\Big(1 - 0.35\,\delta s^2 + 1.02\,\xi_b\Big). \end{aligned} \tag{3.8}$$

The parameter $\delta\alpha_s$ is a possible shift of $\alpha_s(M_Z)$ from our reference value of 0.120,

$$\alpha_s(M_Z) = 0.120 + \delta\alpha_s.$$

Note that only Γ_Z depends on $\delta\rho$. We will ignore Γ_Z in the following since including it will only place limits on $\delta\rho$ without affecting the other parameters. We will also omit all non–LEP/SLD observables since these are expected to have a negligible impact on the b couplings.

In an analogous way, we find

$$A_{\rm FB}^{b,0} = \frac{3}{4} A_e A_b = [A_{\rm FB}^b]_{\rm SM} \left(1 - 55.7\,\delta s^2 - 1.76\,\zeta_b\right). \tag{3.9}$$

The value of $A_{\rm FB}^{b,0}$ is the measured forward–backward asymmetry of the b with QCD corrections removed, so it naturally depends on the value of $\alpha_s(M_Z)$ used in the calculation. Since we let the value of $\alpha_s(M_Z)$ float in our fit, this should be taken into account. However, the dependence of the extracted value of $A_{\rm FB}^{b,0}$ on $\alpha_s(M_Z)$ is not straightforward since it depends on the details of each LEP detector. We estimated the sensitivity to $\alpha_s(M_Z)$ using the formulae in Ref. [28] and found it to be negligibly small as long as $|\delta\alpha_s| \lesssim 0.1$. The sensitivity to $\alpha_s(M_Z)$ is smaller than the systematic error ascribed to $A_{\rm FB}^{b,0}$. We will therefore ignore the $\alpha_s(M_Z)$ dependence of $A_{\rm FB}^{b,0}$ in our analysis, and similarly that of $A_{\rm FB}^{c,0}$.

The relationship between our parameters and others that have appeared in the literature is as follows. The parameter ϵ_b introduced in Ref. [29] was defined as

$$\begin{aligned} g_L^b - g_R^b &\equiv -\frac{1}{2}(1+\epsilon_b), \\ g_L^b + g_R^b &\equiv -\frac{1}{2}\left(1 - \frac{4}{3}\sin^2\theta_{\rm eff}^{\rm lept} + \epsilon_b\right). \end{aligned} \tag{3.10}$$

This definition assumes $\delta g_R^b = 0$, and the relation between ϵ_b and δg_L^b is given by

$$\epsilon_b = [\epsilon_b]_{\rm SM} - 2\delta g_L^b. \tag{3.11}$$

The parameters δ_{bV} and η_b introduced in Ref. [30] were defined as

$$\begin{aligned} \Gamma_{b\bar{b}} &\equiv \Gamma_{d\bar{d}}(1+\delta_{bV}), \\ A_b &\equiv A_s(1+\eta_b). \end{aligned} \tag{3.12}$$

They are related to ξ_b and ζ_b by

$$\begin{aligned} \delta_{bV} &= [\delta_{bV}]_{\rm SM} - 4.65\,\xi_b, \\ \eta_b &= [\eta_b]_{\rm SM} - 1.76\,\zeta_b. \end{aligned} \tag{3.13}$$

In table 2, we show the data used in our analysis. A fit to this data with δs^2, ξ_b, ζ_b, and $\delta\alpha_s$ as parameters, including the correlations between $\sigma_{\rm had}^0$ and R_ℓ, and among all heavy flavor observables yields:

$$\begin{aligned} \delta s^2 &= -0.00082 \pm 0.00025, \\ \xi_b &= -0.0021 \pm 0.0011, \\ \zeta_b &= 0.028 \pm 0.013, \\ \delta\alpha_s &= -0.006 \pm 0.005 \end{aligned} \tag{3.14}$$

Observable	ZFITTER prediction	Measured Value
$\sin^2\theta_{\rm eff}^{\rm lept}$ (LEP)	0.23200	0.23157 ± 0.00041
$\sin^2\theta_{\rm eff}^{\rm lept}$ (SLD)	0.23200	0.23084 ± 0.00035
$\sigma^0_{\rm had}$	41.468 nb	41.486 ± 0.053 nb
R_ℓ	20.749	20.775 ± 0.027
R_b	0.21575	0.21732 ± 0.00087
R_c	0.1723	0.1731 ± 0.0044
$A_{\rm FB}^{b,0}$	0.1004	0.0998 ± 0.0022
$A_{\rm FB}^{c,0}$	0.0716	0.0735 ± 0.0045
A_b	0.934	0.899 ± 0.049
A_c	0.666	0.660 ± 0.064

Table 2: The data used for the $Zb\bar{b}$ vertex correction analysis. All data are from Ref. [5]. The value of $\sin^2\theta_{\rm eff}^{\rm lept}$ for LEP is from leptonic asymmetries only. The parameter choice for the reference SM was $M_Z = 91.1867$ GeV [5], $M_{\rm top} = 173.9$ GeV [21], $M_{\rm higgs} = 300$ GeV, $\alpha^{-1}(M_Z) = 128.9$ [23], and $\alpha_s(M_Z) = 0.120$.

with the correlation matrix given by

$$\begin{bmatrix} 1 & 0.02 & -0.49 & 0.13 \\ 0.02 & 1 & -0.02 & 0.70 \\ -0.49 & -0.02 & 1 & -0.07 \\ 0.13 & 0.70 & -0.07 & 1 \end{bmatrix} \tag{3.15}$$

where the rows and columns are labelled in the order $(\delta s^2, \xi_b, \zeta_b, \delta\alpha_s)$. The quality of the fit was $\chi^2 = 2.5/(10-4)$.

The constraints imposed on δs^2, ξ_b, and ζ_b by the various observables are illustrated in Figs 9 through 12. The bands in Figs. 9 and 10 illustrate the $1-\sigma$ uncertainties on the various constraints. The 2–dimensional projections of the allowed regions onto the δs^2–ξ_b, δs^2–ζ_b planes are shown in Figs. 11 and 12.

In terms of δg_L^b and δg_R^b, the limits on ξ_b and ζ_b translate into

$$\begin{aligned} \delta g_L^b &= 0.0030 \pm 0.0026, \\ \delta g_R^b &= 0.028 \;\; \pm 0.013. \\ \mathrm{corr}(\delta g_L^b, \delta g_R^b) &= 0.9 \end{aligned} \tag{3.16}$$

Note the strong correlation between δg_L^b and δg_R^b (0.9) even though ξ_b and ζ_b were virtually uncorrelated (-0.02). This correlation stems from the fact that the error on ζ_b was so much larger than that on ξ_b, as is evident from Fig. 13. Therefore, some care is necessary when using these limits. For instance, if we assume that $\delta g_R^b = 0$, then the limits on δg_L^b will be given by

$$\delta g_L^b = -0.0020 \pm 0.0011. \tag{3.17}$$

and the central value of δg_L^b changes sign!

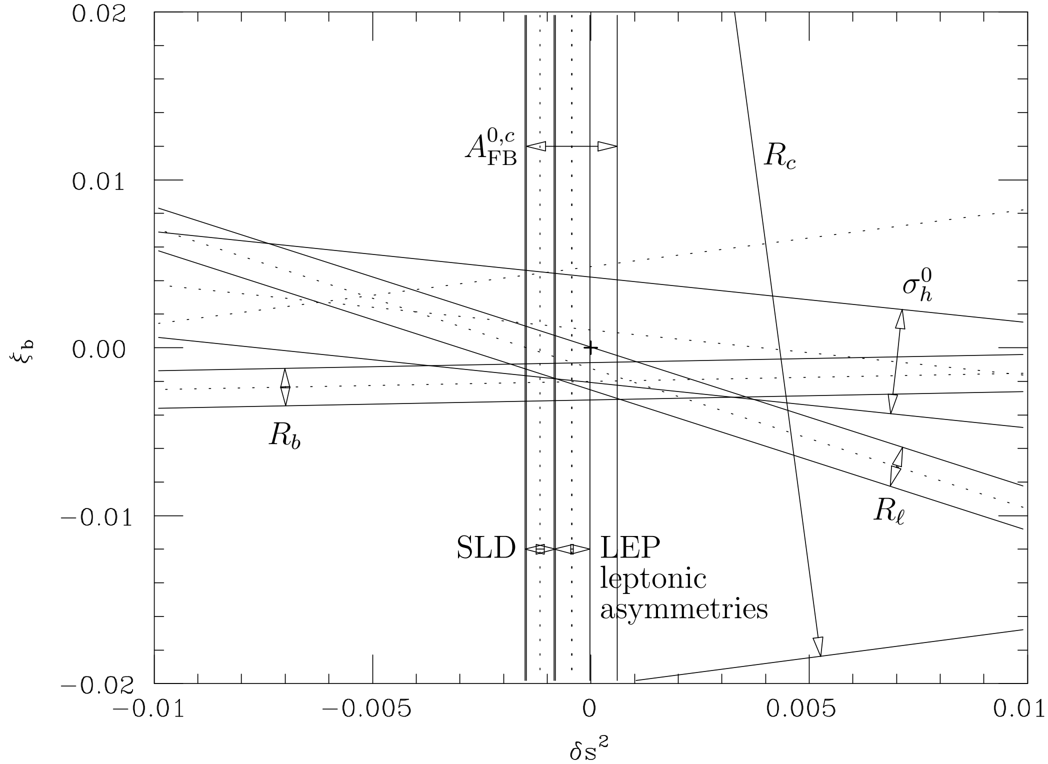


Figure 9: 1–σ limits on δs^2 and ξ_b.

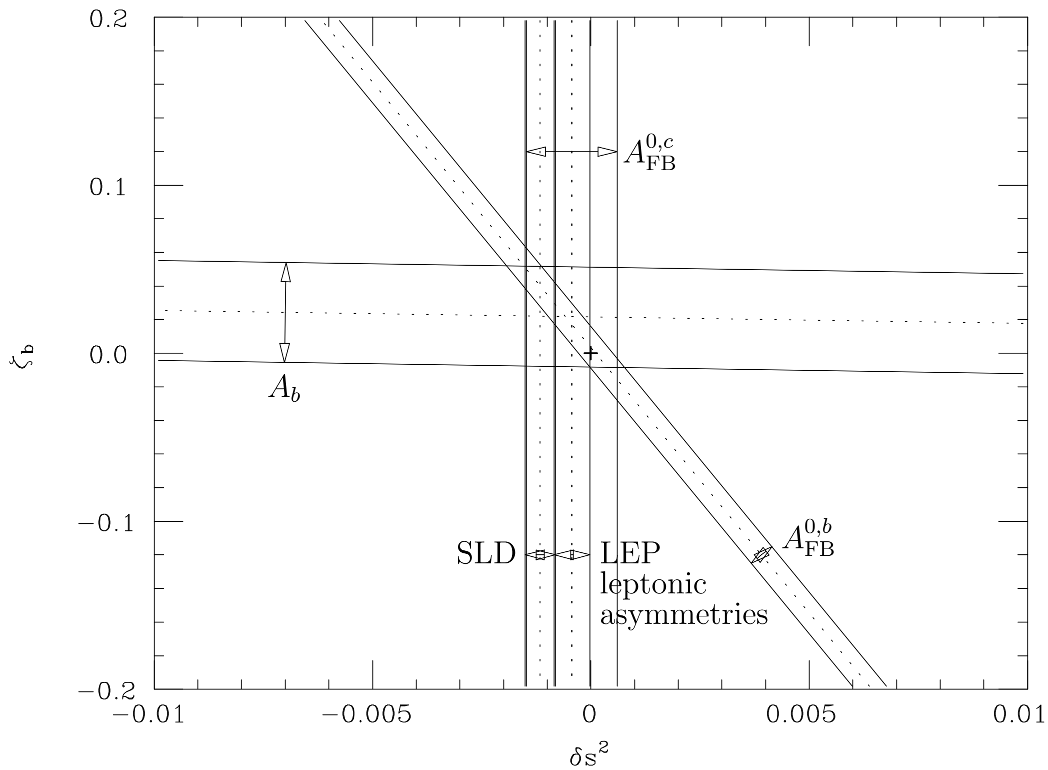


Figure 10: 1–σ limits on δs^2 and ζ_b.

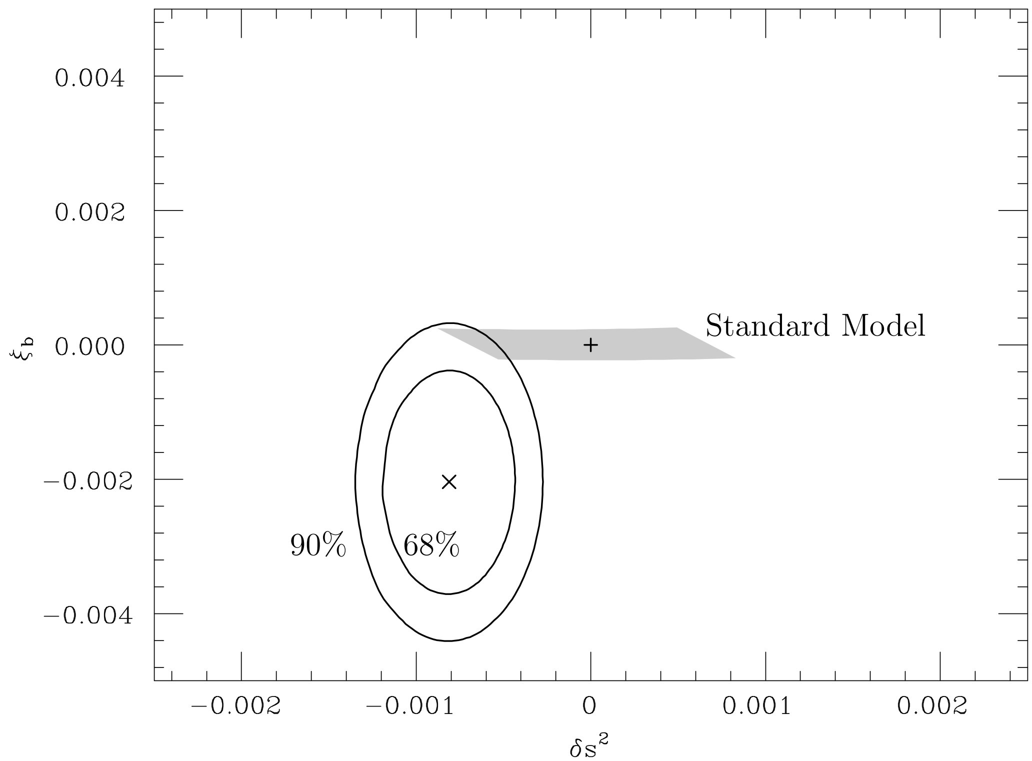


Figure 11: Limits on δs^2 and ξ_b. The shaded area represents the Standard Model points with $M_{\rm top} = 168.7 \sim 179.1$ GeV, and $M_{\rm higgs} = 80 \sim 1000$GeV.

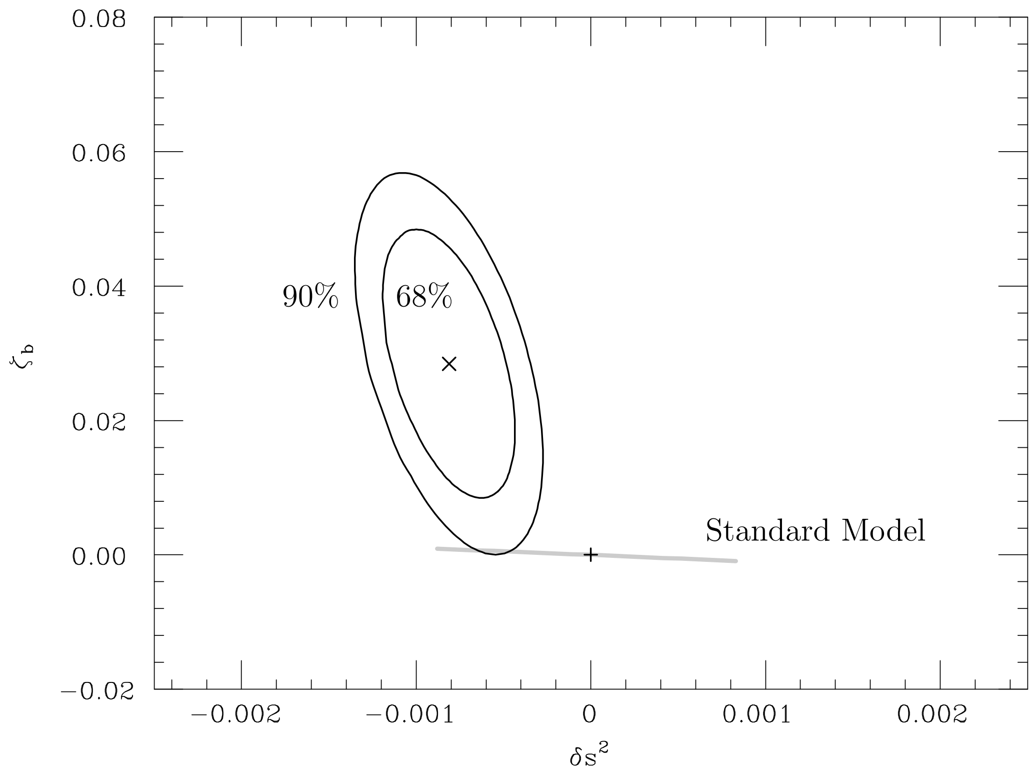


Figure 12: Limits on δs^2 and ζ_b. The shaded area represents the Standard Model points with $M_{\rm top} = 168.7 \sim 179.1$ GeV, and $M_{\rm higgs} = 80 \sim 1000$GeV.

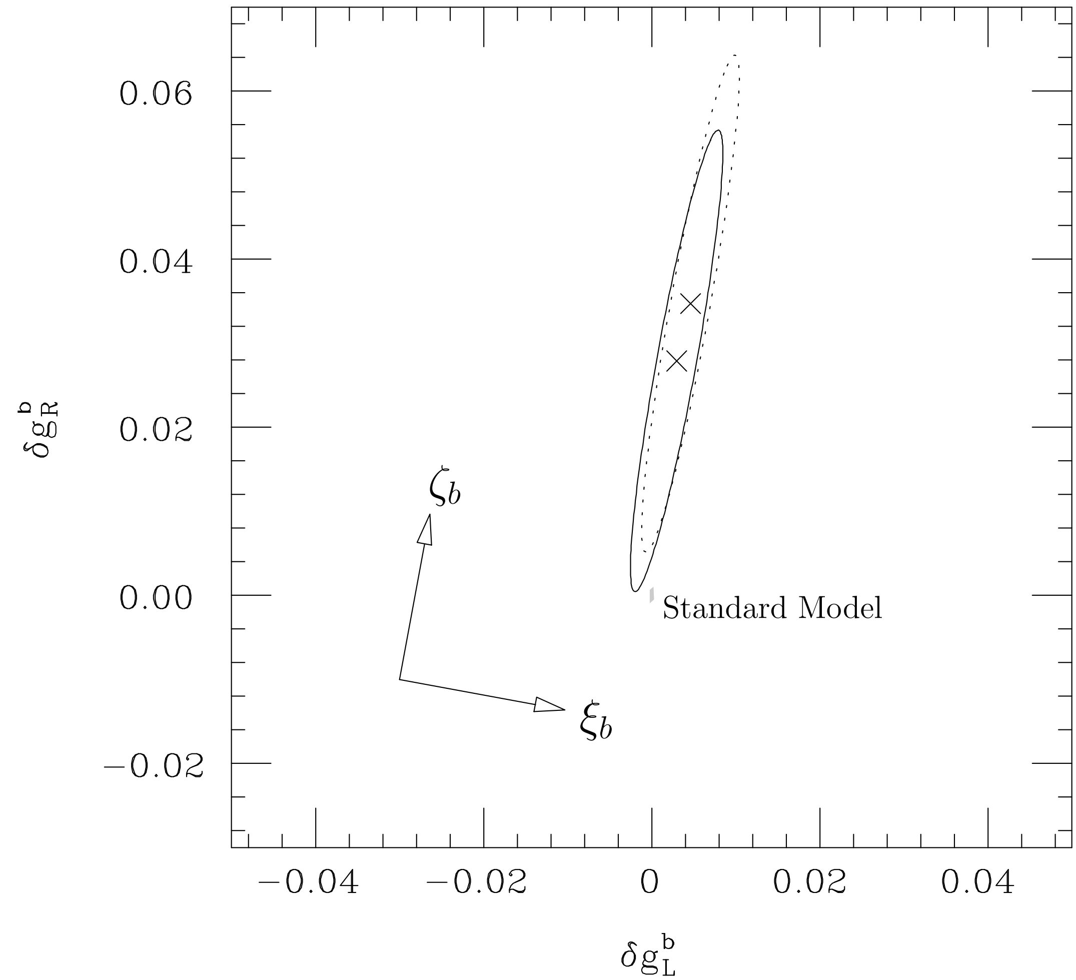


Figure 13: 90% confidence limits on g_L^b and g_R^b. Solid line – 98 data, dotted line – 97 data. The small shaded area around the origin represents the Standard Model points with $M_{\rm top} = 168.7 \sim 179.1$ GeV, and $M_{\rm higgs} = 80 \sim 1000$GeV.

It is clear from this analysis that g_R^b is one of the least well known of the precision electroweak observables. Given that the nominal standard model expectation is $g_R^b \sim 0.08$, the fractional error on g_R^b quoted in Eq. (3.16) amounts to roughly 15%. It would, of course, be of interest to find other measurements that could be used to reduce this error.

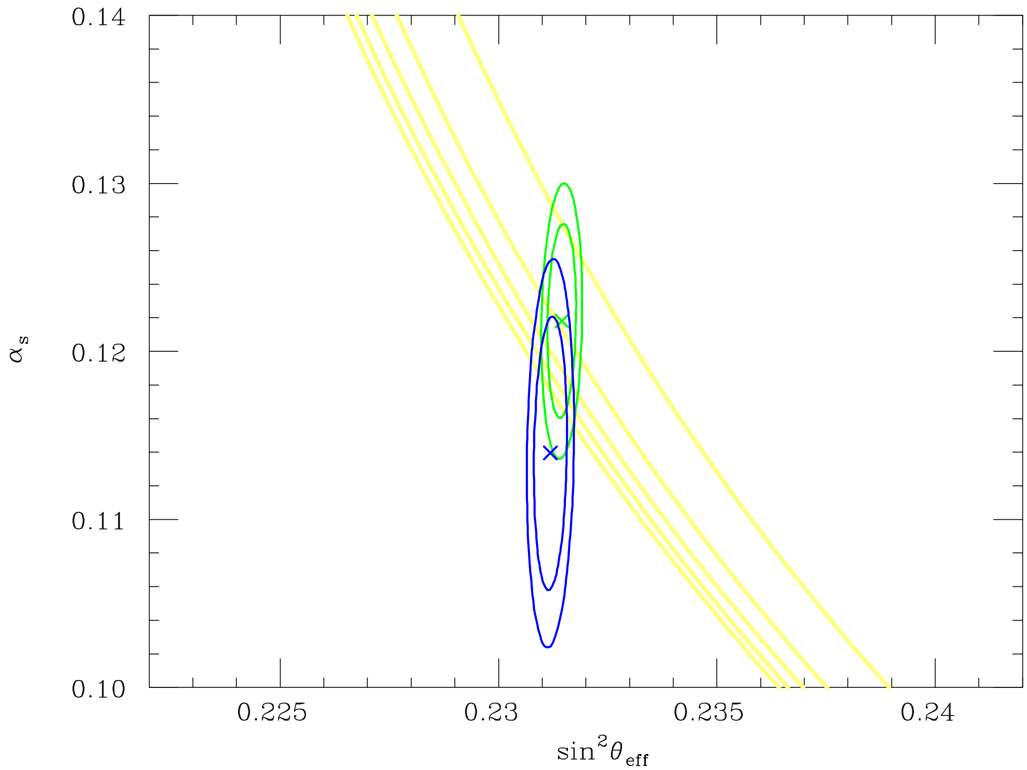


Figure 14: $\sin^2\theta_{\rm eff}^{\rm lept}$ and α_s. Blue contours: with non–zero ξ_b and ζ_b, Green contours: $\xi_b = \zeta_b = 0$. Yellow lines represent the 2–loop MSSM predictions with all SUSY particle masses set to a common value $M_{\rm SUSY}$. The five lines correspond respectively to $M_{\rm SUSY} = M_{\rm top}$, and 1, 2, 3, and 4 TeV from right to left. The thickness of the lines DOES NOT represent the theoretical error which is much larger.

4 Precision Electroweak Data and Supersymmetric $SU(5)$ Unification

We can also use the results of this analysis to assess the status of supersymmetric grand unification [31]. From the previous section, we can extract estimates for the values of $\sin^2\theta_{\rm eff}^{\rm lept}$ and $\alpha_s(M_Z)$:

$$\begin{aligned} \sin^2\theta_{\rm eff}^{\rm lept} &= 0.23118 \pm 0.00025 \\ \alpha_s(M_Z) &= 0.114 \pm 0.005 \end{aligned} \tag{4.1}$$

Note that allowing for non–zero ξ_b and ζ_b has lowered the preferred central values of both $\sin^2\theta_{\rm eff}^{\rm lept}$ and $\alpha_s(M_Z)$. It has been noted [32] that the prediction of $\alpha_s(M_Z)$ from supersymmetric grand unification is somewhat high relative to experiment. If, as our analysis suggests, the value of $\sin^2\theta_{\rm eff}^{\rm lept}$ is somewhat smaller than previously thought, the GUT prediction of $\alpha_s(M_Z)$ will increase still further: A smaller value of $\sin^2\theta_{\rm eff}^{\rm lept}$ implies a smaller ratio of g'/g at the Z mass scale, and this in turn increases the unification scale. Since $\alpha_s(M_Z)$ increases with the unification scale, a smaller value of $\sin^2\theta_{\rm eff}^{\rm lept}$ implies a larger value of $\sin^2\theta_{\rm eff}^{\rm lept}$.

In Fig. 14 we display the most naive prediction of $\alpha_s(M_Z)$ as a function of $\sin^2\theta_{\rm eff}^{\rm lept}$ for a few values of the SUSY mass scale, together with the 68 and 90 % confidence level error ellipses derived from our analysis. We can see that the GUT prediction of $\alpha_s(M_Z)$ is slightly high relative to experiment if $M_{\rm SUSY}\sim M_{\rm top}$ and the corrections from ξ_b and ζ_b are included. We also display the result for $\xi_b=\zeta_b=0$, for which the agreement is better. The GUT prediction of $\alpha_s(M_Z)$ can be lowered by threshold corrections to the standard model couplings at the GUT scale. [32]

More detailed analyses of the status of grand unification can be found in the literature [32]. Our point here is simply to note that the analysis of the previous section tends to shift $\alpha_s(M_Z)$ and $\sin^2\theta_{\rm eff}^{\rm lept}$ away from the SUSY $SU(5)$ prediction, if ξ_b and ζ_b turn out to be non-zero.

5 Conclusions

We have reviewed the status of precision electroweak data using the methods of Refs. [1, 2] to parametrize potential deviations from the standard model. Agreement between the standard model and experiment is quite good. Indeed, all of the parameters used in our analysis are found (for some choice of standard model parameters) to be consistent with zero at the 90% confidence level, indicating good agreement between the minimal standard model and experiment.

A few changes relative to previous analyses are apparent: First, the apparent excess in R_b reported in 1995 has decreased, and the contraints on U have improved substantially as knowledge of the W mass has improved. The overall quality of the fit is improved if either the Higgs mass is below our nominal value of 300 GeV, or if the inverse fine structure constant $\alpha^{-1}(M_Z)$ is somewhat larger than our nominal value of 128.9.

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