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Constraints on Topcolor Assisted Technicolor Models from Vertex Corrections

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Abstract

We use the LEP/SLD data to place constraints on Topcolor Assisted Technicolor Models. We find that due to a large negative shift in R_b induced by charged top-pion exchange, it is difficult to make the models compatible with experiment.

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1. Introduction

In top-color models of electroweak symmetry breaking, the top-color interaction becomes strong and broken at a scale Λ . This generates a top quark condensate which gives rise to a triplet of Goldstone bosons, the top-pions, which are absorbed into the W^\pm and the Z . In such models, the top-pion decay constant f_π , which determines the masses of the W^\pm and the Z , and the top mass m_t are related by [1]

$$f_\pi^2 = m_t^2 \left(\frac{N_c}{16\pi^2} \ln \frac{\Lambda^2}{\mu^2} \right). \quad (1)$$

Here, μ is a scale of the order of m_t . To obtain the correct masses for the W^\pm and the Z , one needs $f_\pi = v = 174$ GeV which implies $\Lambda \sim 10^{13-14}$ GeV. Because of this large hierarchy between m_t , f_π and Λ , top-color models typically require extreme fine tuning of the coupling constants to obtain the correct masses for the gauge bosons and the top.

In Ref. [2], Hill proposed to remedy this problem by lowering the top-color scale Λ to the order of a TeV. This lowers the value of f_π to about:

$$f_\pi \approx 50 \text{ GeV}.$$

In addition to the top-color interactions, Hill introduced technicolor [3] to generate a condensate of technifermions with a technipion decay constant F_π which satisfies

$$F_\pi^2 + f_\pi^2 = v^2 = (174 \text{ GeV})^2,$$

or

$$F_\pi^2 \approx (167 \text{ GeV})^2.$$

Thus, the majority of the W^\pm and Z masses come from the technifermion condensate, while the top quark condensate serves to make the top quark heavy. This type of model was dubbed “top-color assisted technicolor” and has been studied by many authors [4, 9, 10, 11, 14, 17, 19, 20].

However, it was pointed out by Burdman and Kominis [4] that the smallness of the top-pion decay constant f_π will have a dangerous effect on $R_b = \Gamma_{b\bar{b}}/\Gamma_{\text{had}}$. This is because the Yukawa coupling of the top quark and the left-handed bottom quark to the top-pions is given by

$$y_t = \frac{m_t}{f_\pi} \approx 3.5$$

which is very large. Since the top-pions¹ remain unabsorbed and physical in these models, there is a large radiative correction to the $Zb\bar{b}$ vertex coming from the charged-top-pion – top-quark loop.

The charged top-pion correction to the $Zb\bar{b}$ vertex is exactly the same as that of the charged Higgs correction in two Higgs doublet models with $v_1 = f_\pi$ and $v_2 = F_\pi$.

¹Actually, a linear combination of the top-pions and technipions are absorbed into the gauge bosons leaving the linear combination orthogonal to it physical. The absorbed Goldstone linear combination is mostly the technipion while the physical linear combination is mostly the top-pion.

As discussed by Grant in Ref. [5], the shift in the left handed coupling of the b to the Z due to this correction is given by²

$$\delta g_L^b = \frac{1}{2} \left(\frac{y_t^2}{16\pi^2} \right) \frac{v_2^2}{v^2} \left[-\frac{x}{(x-1)^2} \log x + \frac{x}{x-1} \right] \equiv \Delta(m_+), \quad (2)$$

where $x = m_t^2/m_+^2$. The $\frac{1}{2}$ in front is isospin, and the factor (v_2^2/v^2) is due to top-pion–technipion mixing. In the limit that the charged Higgs/top-pion mass m_+ goes to infinity, we find that δg_L^b goes to zero, i.e. the contribution decouples. However, because the Yukawa coupling y_t is so large, we find that δg_L is not small even for fairly large values of m_+ . For instance, if $m_+ = 1$ TeV, we find $\delta g_L = +0.003$. This amounts to a +0.7% shift in g_L , and a -1.3% shift in $\Gamma_{b\bar{b}}$. This would shift the theoretical value of $R_b = \Gamma_{b\bar{b}}/\Gamma_{\text{had}}$ by -1% from the Standard Model value of 0.2158 ($m_t = 174$ GeV, $m_H = 300$ GeV) down to about 0.2136. Given that the current experimental value of R_b is [6]

$$R_b = 0.21656 \pm 0.00074,$$

the difference would be at the 4σ level. For more realistic³ and smaller values of the top-pion mass m_+ , the shift in g_L^b , and thus the discrepancy between theory and experiment would be huge [4].

Since this is a 1-loop calculation for a Yukawa coupling which is large ($y_t \approx 3.5$), this result may not be particularly robust. However, the 1-loop result does serve as a guideline on how large the correction can be, and since the mass of the top-pion m_+ can be adjusted, we can use that freedom to hide our ignorance on the higher-order corrections. We will therefore refer to the value of m_+ used in Eq. 2 as the *effective* top-pion mass.

Of course, one cannot conclude that top-color assisted technicolor is ruled out on the basis of this observation alone. Indeed, it was pointed out by Hill and Zhang [7] that coloron dressing of the $Zb\bar{b}$ vertex actually shifts the left and right handed couplings of the b to the Z by

$$\frac{\delta g_L^b}{g_L^b} = \frac{\delta g_R^b}{g_R^b} = \frac{\kappa_3}{6\pi} C_2(R) \left[\frac{m_Z^2}{M_C^2} \ln \frac{M_C^2}{m_Z^2} \right]. \quad (3)$$

Here, κ_3 is the coloron coupling (to be defined in the next section) and M_C is the coloron mass. Again, we are using a 1-loop result for a large κ_3 , so it should be considered the *effective* coupling constant for our purpose. For $M_C \approx 1$ TeV, we find

$$\frac{\delta g_L^b}{g_L^b} = \frac{\delta g_R^b}{g_R^b} = 0.003 \kappa_3.$$

²We normalize the coupling so that at tree level, they are given by

$$g_L^b = -\frac{1}{2} + \frac{1}{3}s^2, \quad g_R^b = \frac{1}{3}s^2.$$

³The top-pion is a pseudo-Goldstone boson whose mass must be generated by ETC [8] interactions. Hill [2] estimates their masses to be around 200 GeV.

This leads to a positive shift in $\Gamma_{b\bar{b}}$ and R_b of

$$\frac{\delta\Gamma_{b\bar{b}}}{\Gamma_{b\bar{b}}} = 0.006 \kappa_3, \quad \frac{\delta R_b}{R_b} = 0.005 \kappa_3.$$

If $\kappa_3 \approx 2$, R_b would be shifted to the positive side by 1%. Furthermore, the Z' dressing of the $Zb\bar{b}$ vertex will also enhance R_b . In principle, therefore, it is possible to cancel the large negative top-pion contribution to R_b with an equally large but positive coloron and Z' contribution. The question is whether such a large correction is allowed by the other observables or not.

Clearly, one must consider all possible radiative corrections from all the particles involved and perform a global fit to the precision electroweak data. In this paper, we perform a systematic analysis of all relevant corrections to the $Zf\bar{f}$ vertices. The Z -pole data from LEP and SLD will be used to constrain the size of these vertex corrections and the effective top-color assisted technicolor parameters associated with them.

In section 2, we review top-color assisted technicolor and introduce the notation. The version we will be considering is the one with a strong $U(1)$ interaction “tilting” the vacuum. In section 3, we list all the relevant corrections we will be considering and discuss how they affect Z -pole observables. In section 4, we report the result of our fit to the latest LEP/SLD data. Section 5 concludes with a discussion on the interpretation of the result.

2. Top-color Assisted Technicolor

We concentrate our attention to the class of top-color assisted technicolor models which assume that the quarks and leptons transform under the gauge group

$$SU(3)_s \times SU(3)_w \times U(1)_s \times U(1)_w \times SU(2)_L$$

with coupling constants g_{3s} , g_{3w} , g_{1s} , g_{1w} , and g_2 . It is assumed that $g_{3s} \gg g_{3w}$ and $g_{1s} \gg g_{1w}$.⁴ The charge assignments of the three generation of ordinary fermions under these gauge groups are given in Table 1. Note that each generation must transform non-trivially under only one of the $SU(3)$'s and one of the $U(1)$'s, and that those charges are the same as that of the Standard Model color and hypercharge. This ensures anomaly cancellation.

Alternative charge choices to the one shown in Table 1 are possible. In the original model of Hill [2], the second generation was assigned $U(1)_s$ charges instead of those under $U(1)_w$ in order to distinguish it from the first generation. In the model recently proposed by Popovic and Simmons [10], both the first and second generations were given $SU(3)_s$ quantum numbers instead of those under $SU(3)_w$. Lane [11] discussed a more general form of $U(1)_{s,w}$ charge assignments that ensures

⁴Several authors have commented that having a strong $U(1)$ will cause the Landau pole to be situated not too far from the top-color scale [9, 10]. We will discuss this problem in a subsequent paper [12].

	$SU(3)_s$	$SU(3)_w$	$U(1)_s$	$U(1)_w$	$SU(2)_L$
$(t, b)_L$	3	1	$\frac{1}{3}$	0	2
$(t, b)_R$	3	1	$(\frac{4}{3}, -\frac{2}{3})$	0	1
$(\nu_\tau, \tau^-)_L$	1	1	-1	0	2
τ_R^-	1	1	-2	0	1
$(c, s)_L, (u, d)_L$	1	3	0	$\frac{1}{3}$	2
$(c, s)_R, (u, d)_R$	1	3	0	$(\frac{4}{3}, -\frac{2}{3})$	1
$(\nu_\mu, \mu^-)_L, (\nu_e, e^-)_L$	1	1	0	-1	2
μ_R^-, e_R^-	1	1	0	-2	1

Table 1: Charge assignments of the ordinary fermions.

anomaly cancellation. We will comment on the consequences of these alternative assignments at the end of section 5.

At scale $\Lambda \sim 1$ TeV, technicolor is assumed to become strong and generate a condensate (of something which we will leave unspecified) with charge $(3, \bar{3}, p, -p, 1)$ which breaks the two $SU(3)$'s and the two $U(1)$'s to their diagonal subgroups

$$SU(3)_s \times SU(3)_w \rightarrow SU(3)_c, \quad U(1)_s \times U(1)_w \rightarrow U(1)_Y,$$

which we identify with the usual Standard Model color and hypercharge groups.

The massless unbroken $SU(3)$ gauge bosons (the gluons G_μ^a) and the massive broken $SU(3)$ gauge bosons (the so called *colorons* C_μ^a) are related to the original $SU(3)_s \times SU(3)_w$ gauge fields $X_{s\mu}^a$ and $X_{w\mu}^a$ by

$$\begin{aligned} C_\mu &= X_{s\mu} \cos \theta_3 - X_{w\mu} \sin \theta_3 \\ G_\mu &= X_{s\mu} \sin \theta_3 + X_{w\mu} \cos \theta_3 \end{aligned}$$

where we have suppressed the color index, and

$$\tan \theta_3 = \frac{g_{3w}}{g_{3s}}.$$

The currents to which the gluons and colorons couple to are:

$$g_{3s} J_{3s}^\mu X_{s\mu} + g_{3w} J_{3w}^\mu X_{w\mu} = g_3 (\cot \theta_3 J_{3s}^\mu - \tan \theta_3 J_{3w}^\mu) C_\mu + g_3 (J_{3s}^\mu + J_{3w}^\mu) G_\mu,$$

where

$$\frac{1}{g_3^2} = \frac{1}{g_{3s}^2} + \frac{1}{g_{3w}^2}.$$

Since the quarks carry only one of the $SU(3)$ charges, we can identify

$$J_3^\mu = J_{3s}^\mu + J_{3w}^\mu$$

as the QCD color current, and g_3 as the QCD coupling constant.

Similarly, the massless unbroken $U(1)$ gauge boson B_μ and the massive broken $U(1)$ gauge boson Z'_μ are related to the original $U(1)_s \times U(1)_w$ gauge fields $Y_{s\mu}$ and $Y_{w\mu}$ by

$$\begin{aligned} Z'_\mu &= Y_{s\mu} \cos \theta_1 - Y_{w\mu} \sin \theta_1 \\ B_\mu &= Y_{s\mu} \sin \theta_1 + Y_{w\mu} \cos \theta_1 \end{aligned}$$

where

$$\tan \theta_1 = \frac{g_{1w}}{g_{1s}}.$$

The currents to which the B_μ and Z'_μ couple to are:

$$g_{1s} J_{1s}^\mu Y_{s\mu} + g_{1w} J_{1w}^\mu Y_{w\mu} = g_1 (\cot \theta_1 J_{1s}^\mu - \tan \theta_1 J_{1w}^\mu) Z'_\mu + g_1 (J_{1s}^\mu + J_{1w}^\mu) B_\mu,$$

where

$$\frac{1}{g_1^2} = \frac{1}{g_{1s}^2} + \frac{1}{g_{1w}^2}.$$

Again, since the fermions carry only one of the $U(1)$ charges, we can identify

$$J_1^\mu = J_{1s}^\mu + J_{1w}^\mu$$

as the Standard Model hypercharge current and g_1 as the hypercharge coupling constant.

The masses of the colorons and the Z' will be given by

$$\begin{aligned} M_C &= \mathcal{F} \sqrt{g_{3s}^2 + g_{3w}^2}, \\ M_{Z'} &= |p| \mathcal{F} \sqrt{g_{1s}^2 + g_{1w}^2}. \end{aligned}$$

where \mathcal{F} is the Goldstone boson decay constant associated with the breaking. Note that the mass of the Z' can be adjusted at will by adjusting the charge p of the condensate.

Below the symmetry breaking scale $\Lambda \sim 1$ TeV, the exchange of the massive colorons and the Z' give rise to effective four-fermion interactions of the form

$$\begin{aligned} \mathcal{L} &= -\frac{g_3^2}{2M_C^2} (\cot \theta_3 J_{3s}^\mu - \tan \theta_3 J_{3w}^\mu) (\cot \theta_3 J_{3s\mu} - \tan \theta_3 J_{3w\mu}) \\ &\quad -\frac{g_1^2}{2M_{Z'}^2} (\cot \theta_1 J_{1s}^\mu - \tan \theta_1 J_{1w}^\mu) (\cot \theta_1 J_{1s\mu} - \tan \theta_1 J_{1w\mu}). \end{aligned}$$

Since $\tan \theta_i \ll \cot \theta_i$ ($i = 1, 3$) by assumption, we neglect the J_{iw} terms and find

$$\mathcal{L} = -\frac{2\pi\kappa_3}{M_C^2} J_{3s}^\mu J_{3s\mu} - \frac{2\pi\kappa_1}{M_{Z'}^2} J_{1s}^\mu J_{1s\mu},$$

where we have defined

$$\kappa_i \equiv \frac{g_i^2}{4\pi} \cot^2 \theta_i, \quad (i = 1, 3).$$

Note that due to the hypercharge assignments, the Z' exchange interaction is attractive in the $\bar{t}t$ channel but repulsive in the $\bar{b}b$ channel while coloron exchange is attractive in both channels. Therefore, it is possible to arrange the coupling strengths κ_3 and κ_1 so that the combination of the coloron and Z' exchange interactions will condense the top, but not the bottom. (This is sometimes called *tilting* the vacuum.) Using the Nambu Jona-Lasinio approximation [13], we find that this requirement places the following constraint on the κ 's:

$$C_2(R)\kappa_3 + \frac{1}{9}\kappa_1 > \pi, \quad C_2(R)\kappa_3 - \frac{1}{18}\kappa_1 < \pi,$$

where $C_2(R) = \frac{N_c^2 - 1}{2N_c}$, $N_c = 3$. In the large N_c limit, $C_2(R) \approx \frac{N_c}{2} = \frac{3}{2}$, so the above constraint becomes

$$\kappa_3 + \frac{2}{27}\kappa_1 > \frac{2\pi}{3}, \quad \kappa_3 - \frac{1}{27}\kappa_1 < \frac{2\pi}{3}. \quad (4)$$

In addition, the requirement that the τ lepton does not condense leads to

$$\kappa_1 < 2\pi. \quad (5)$$

Under these conditions, the top quark condensate will form $\langle \bar{t}t \rangle \neq 0$ generating the top quark mass m_t and the top-pions with decay constant f_π which are related through Eq. 1. This breaks $SU(2)_L \times U(1)_Y$ down to $U(1)_{em}$, generating (smallish) masses for the W^\pm and the Z . The coupling of the top and bottom quarks to the top-pions is given by

$$y_t \left[\frac{1}{\sqrt{2}}(\bar{t}i\gamma_5 t)\pi_0 + \bar{t}_R b_L \pi^+ + \bar{b}_L t_R \pi^- \right]$$

where $y_t = m_t/f_\pi$.

The remainder of the masses of the W^\pm and the Z are assumed to come from a technifermion condensate in the usual fashion. The smaller fermion masses are generated through ETC interactions, including a small ETC mass for the top so that the top-pions will become massive.

3. Vertex Corrections in Top-color Assisted Technicolor

In previous attempts to constrain top-color assisted technicolor using precision electroweak measurements [14] attention had been focussed on the vacuum polarization corrections, namely the shift in the ρ parameter and Z - Z' mixing.

Focussing attention on vacuum polarization corrections has been the standard technique in analyzing precision electroweak data [15]. The main advantage in doing this is that vacuum polarization corrections modify the gauge boson propagators and are therefore universal: they correct *all* electroweak observables and therefore *all* the electroweak data can be used to constrain their sizes.

However, there are serious disadvantages also. First, each gauge boson couples to all particles that carry its charge so that the model under consideration must be specified *completely*. In top-color assisted technicolor models, this means that the charges and masses of the techni-sector must be specified which makes any limit highly model dependent. Second, in order to be able to use all electroweak data to constrain the vacuum polarization corrections, one often neglects the highly process-dependent vertex and box corrections which may not be negligible at all. In Ref. [14], the only corrections considered were vacuum polarization corrections coming from technifermions of specific models. Vacuum polarization and vertex corrections coming from ordinary fermion and top-pion loops were completely neglected.

A much better way to deal with top-color assisted technicolor and similar theories is to focus on *vertex corrections at the Z pole only*. This allows us to place severe constraints on the theory without specifying the technisector. All that is necessary is to specify the charges of the ordinary quarks and leptons. (A similar technique was used in Ref. [16] to constrain corrections to the $Zb\bar{b}$ vertex.)

Let us now list the vertex corrections that must be considered. They come in two classes, namely:

1. gauge boson mixing terms, and
2. proper vertex corrections.

Gauge boson mixing corrections to the $Zf\bar{f}$ vertices are due to the re-diagonalization of the gauge bosons from vacuum polarization corrections. At tree level, the Z couples to the current

$$J_Z^0 = J_{I_3} - s^2 J_Q,$$

where s^2 is shorthand for $\sin^2 \theta_w$. Z -photon mixing and Z - Z' mixing will modify this current to:

$$J_Z = J_{I_3} - (s^2 + \delta s^2) J_Q + \epsilon J_{1s},$$

where δs^2 and ϵ parametrize the size of the Z -photon and Z - Z' mixings, respectively. We neglect the small J_{1w} component of the $J_{Z'}$ current. We need not worry about the overall change in scale due to these corrections since the observables we will be looking at are all *ratios of coupling constants* from which such scale dependence vanishes.

Since we will be using only Z -pole observables in our analysis, δs^2 and ϵ will remain phenomenological parameters and will not yield any information on the vacuum polarization corrections which give rise to them. Vacuum polarizations are visible only when comparing processes at different energy scales, or processes involving different gauge bosons.⁵

The proper vertex corrections we must consider are the top-pion and coloron corrections discussed in the introduction and the Z' dressing corrections. We neglect all other corrections that vanish in the limit that all the fermion masses (except that

⁵ For instance, the S parameter is only visible when comparing neutral current processes at different energy scales and the T parameter is only visible when comparing neutral and charged current processes.

of the top) are taken to zero. We also make the simplifying assumption that the bottom-pions [17] are heavy enough so that their effects are negligible.

Since the couplings of the colorons and the Z' to the $SU(3)_w$ and $U(1)_w$ charges are highly suppressed, they can also be neglected. Then, with the charge assignment given in Table 1, the only vertices that receive coloron and Z' dressing corrections are $Zb\bar{b}$ and $Z\tau^+\tau^-$. The coloron correction was given in Eq. 3, and the Z' correction can be obtained by simply replacing κ_3 and M_C with κ_1 and $M_{Z'}$, respectively, and the color factor $C_2(R) = \frac{N^2-1}{2N} = \frac{4}{3}$ by the hypercharge squared:

$$\begin{aligned}\frac{\delta g_L(f)}{g_L(f)} &= \frac{\kappa_1}{6\pi} (Y_L^f)^2 \left[\frac{m_Z^2}{M_{Z'}^2} \ln \frac{M_{Z'}^2}{m_Z^2} \right], \\ \frac{\delta g_R(f)}{g_R(f)} &= \frac{\kappa_1}{6\pi} (Y_R^f)^2 \left[\frac{m_Z^2}{M_{Z'}^2} \ln \frac{M_{Z'}^2}{m_Z^2} \right].\end{aligned}$$

In the following, we will use $M_C = M_{Z'} = 1$ TeV.

Therefore, the couplings of the first and second generation fermions only receive corrections from photon- Z mixing:

$$\begin{aligned}\delta g_L(\nu_e) &= \delta g_L(\nu_\mu) = 0 \\ \delta g_L(e) &= \delta g_L(\mu) = \delta g_R(e) = \delta g_R(\mu) = \delta s^2 \\ \delta g_L(u) &= \delta g_L(c) = \delta g_R(u) = \delta g_R(c) = -\frac{2}{3}\delta s^2 \\ \delta g_L(d) &= \delta g_L(s) = \delta g_R(d) = \delta g_R(s) = \frac{1}{3}\delta s^2\end{aligned}$$

while the couplings of the third generation fermions receive all corrections:

$$\begin{aligned}\delta g_L(\nu_\tau) &= -\epsilon + 0.0021\kappa_1 g_L(\nu_\tau) \\ \delta g_L(\tau) &= \delta s^2 - \epsilon + 0.0021\kappa_1 g_L(\tau) \\ \delta g_R(\tau) &= \delta s^2 - 2\epsilon + 0.0085\kappa_1 g_R(\tau) \\ \delta g_L(b) &= \frac{1}{3}\delta s^2 + \frac{1}{3}\epsilon + (0.00023\kappa_1 + 0.0028\kappa_3)g_L(b) + \Delta(m_+) \\ \delta g_R(b) &= \frac{1}{3}\delta s^2 - \frac{2}{3}\epsilon + (0.00094\kappa_1 + 0.0028\kappa_3)g_R(b)\end{aligned}$$

Here, $\Delta(m_+)$ denotes the top-pion correction.

Given these expressions, we can now calculate how the Z -pole observables are shifted by non-zero values of δs^2 , ϵ , κ_1 and κ_3 , and fit the result to the experimental data. We will also let the QCD coupling constant $\alpha_s(m_Z)$ float in our fit so that the size of the QCD gluon dressing corrections can be adjusted. We define the parameter $\delta\alpha_s$ to be the shift of $\alpha_s(m_Z)$ away from its nominal value of 0.120:

$$\alpha_z(m_Z) = 0.120 + \delta\alpha_s.$$

Observable	Measured Value	ZFITTER Prediction
<u>Z lineshape variables</u>		
m_Z	91.1867 ± 0.0021 GeV	input
Γ_Z	2.4939 ± 0.0024 GeV	unused
σ_{had}^0	41.491 ± 0.058 nb	41.468 nb
R_e	20.783 ± 0.052	20.749
R_μ	20.789 ± 0.034	20.749
R_τ	20.764 ± 0.045	20.796
$A_{\text{FB}}^0(e)$	0.0153 ± 0.0025	0.0154
$A_{\text{FB}}^0(\mu)$	0.0164 ± 0.0013	0.0154
$A_{\text{FB}}^0(\tau)$	0.0183 ± 0.0017	0.0154
<u>τ polarization</u>		
A_e	0.1479 ± 0.0051	0.1433
A_τ	0.1431 ± 0.0045	0.1435
<u>SLD left-right asymmetry</u>		
A_e	0.1504 ± 0.0023	0.1433
<u>heavy flavor observables</u>		
R_b	0.21656 ± 0.00074	0.2158
R_c	0.1735 ± 0.0044	0.1723
$A_{\text{FB}}^0(b)$	0.0990 ± 0.0021	0.1004
$A_{\text{FB}}^0(c)$	0.0709 ± 0.0044	0.0716
A_b	0.867 ± 0.035	0.934
A_c	0.647 ± 0.040	0.666

Table 2: LEP/SLD observables [6] and their Standard Model predictions. The predictions were calculated using ZFITTER [18] with $m_t = 173.9$ GeV, $m_H = 300$ GeV, $\alpha_s(m_Z) = 0.120$, and $\alpha^{-1}(m_Z) = 128.9$.

4. Fit to LEP/SLD Data

In Table 2 we show the latest LEP/SLD data obtained from Ref. [6]. The correlation matrices for the errors in the Z -lineshape variables and the heavy flavor observables are shown in the appendix.

Of the 9 lineshape variables, the three R_ℓ ratios and the three forward-backward asymmetries are just ratios of coupling constants. Of the remaining three, the product

$$m_Z^2 \sigma_{\text{had}}^0 = 12\pi \frac{\Gamma_{e^+e^-} \Gamma_{\text{had}}}{\Gamma_Z^2}$$

is again just a ratio of coupling constants. All the other observables shown in Table 2 are ratios of coupling constants.

We therefore have 16 observables which we can use in our analysis. The shifts in these observables due to $\Delta(m_+)$ and non-zero values of δs^2 , ϵ , κ_1 , κ_3 , and $\delta\alpha_s$ are:

$$\frac{\delta\sigma_{\text{had}}^0}{\sigma_{\text{had}}^0} = 0.11 \delta s^2 + 0.93 \epsilon - 0.0013 \kappa_1 - 0.0005 \kappa_3 - 0.12 \delta\alpha_s + 0.40 \Delta$$

$$\begin{aligned}
\frac{\delta R_e}{R_e} &= \frac{\delta R_\mu}{R_\mu} = -0.86 \delta s^2 - 0.46 \epsilon + 0.0001 \kappa_1 + 0.0012 \kappa_3 + 0.31 \delta \alpha_s - 1.0 \Delta \\
&\frac{\delta R_\tau}{R_\tau} = -0.86 \delta s^2 + 2.7 \epsilon - 0.0096 \kappa_1 + 0.0012 \kappa_3 + 0.31 \delta \alpha_s - 1.0 \Delta \\
\frac{\delta A_{\text{FB}}^0(e)}{A_{\text{FB}}^0(e)} &= \frac{\delta A_{\text{FB}}^0(\mu)}{A_{\text{FB}}^0(\mu)} = -110 \delta s^2 \\
\frac{\delta A_{\text{FB}}^0(\tau)}{A_{\text{FB}}^0(\tau)} &= -110 \delta s^2 + 84 \epsilon - 0.043 \kappa_1 \\
\frac{\delta A_e}{A_e} &= -55 \delta s^2 \\
\frac{\delta A_\tau}{A_\tau} &= -55 \delta s^2 + 84 \epsilon - 0.043 \kappa_1 \\
\frac{\delta R_b}{R_b} &= 0.18 \delta s^2 - 1.6 \epsilon + 0.0004 \kappa_1 + 0.0044 \kappa_3 - 3.6 \Delta \\
\frac{\delta R_c}{R_c} &= -0.35 \delta s^2 + 0.46 \epsilon - 0.0001 \kappa_1 - 0.0012 \kappa_3 + 1.0 \Delta \\
\frac{\delta A_{\text{FB}}^0(b)}{A_{\text{FB}}^0(b)} &= -56 \delta s^2 + 1.1 \epsilon - 0.00009 \kappa_1 - 0.32 \Delta \\
\frac{\delta A_{\text{FB}}^0(c)}{A_{\text{FB}}^0(c)} &= -60 \delta s^2 \\
\frac{\delta A_b}{A_b} &= -0.68 \delta s^2 + 1.1 \epsilon - 0.00009 \kappa_1 - 0.32 \Delta \\
\frac{\delta A_c}{A_c} &= -5.2 \delta s^2
\end{aligned} \tag{6}$$

The top-pion correction $\Delta(m_+)$ in these expressions is fixed by choosing an effective top-pion mass m_+ . The remaining 5 parameters: δs^2 , ϵ , κ_1 , κ_3 , and $\delta \alpha_s$ are fit to the data given in Table 2, taking into account the correlations between the experimental errors given in the appendix.

We choose two reasonable values for the effective top-pion mass: $m_+ = 600$ and 1000 GeV. The value of Δ for these masses are

$$\begin{aligned}
\Delta(600 \text{ GeV}) &= 0.006 \\
\Delta(1000 \text{ GeV}) &= 0.003
\end{aligned}$$

The result of the fit for the $m_+ = 1000$ GeV case is:

$$\begin{aligned}
\delta s^2 &= -0.0004 \pm 0.0002 \\
\epsilon &= 0.0005 \pm 0.0005 \\
\kappa_1 &= 0.43 \pm 0.33 \\
\kappa_3 &= 2.9 \pm 0.8 \\
\delta \alpha_s &= -0.0008 \pm 0.0050
\end{aligned}$$

with the correlation matrix shown in Table 3. The quality of the fit was $\chi^2 = 12.6/(16 - 5)$. The strongest constraint on κ_3 comes from R_b , and the strongest constraint on κ_1 comes from R_τ . This is shown in Fig. 1.

As is evident from the figure, the region allowed by our fit overlaps with the region allowed by the vacuum tilting constraint: Eqs. 4 and 5. This is as expected

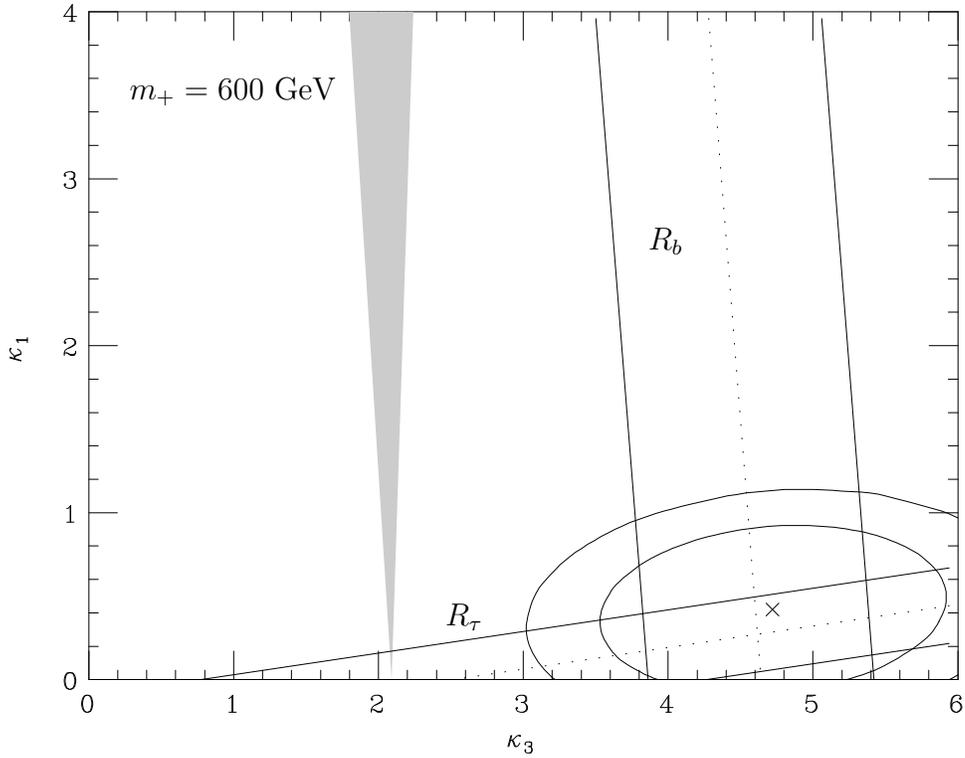
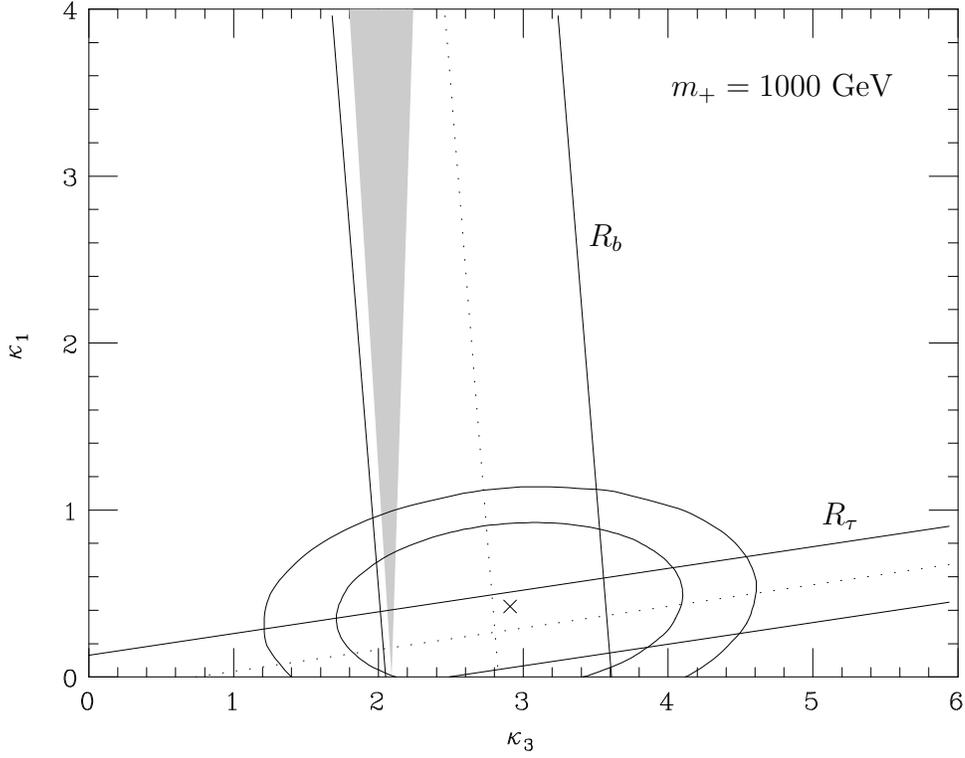


Figure 1: Limits on κ_1 and κ_3 for the $m_+ = 1000 \text{ GeV}$ and $m_+ = 600 \text{ GeV}$ cases. The contours show the 68% and 90% confidence limits. The shaded area is the region allowed by the requirement of vacuum tilting.

	δs^2	ϵ	κ_1	κ_3	$\delta\alpha_s$
δs^2	1.00	0.34	0.19	0.09	0.13
ϵ		1.00	0.74	0.23	0.17
κ_1			1.00	0.15	0.29
κ_3				1.00	-0.57
$\delta\alpha_s$					1.00

Table 3: The correlation matrix of the fit parameters.

from our discussion in the introduction: if the top-pion mass is large enough, then the top-pion correction is small enough to be cancelled by the coloron correction. Though the Z' correction can also be used to cancel the top-pion correction in R_b , it is suppressed by lepton universality.

For the $m_+ = 600$ GeV case, the fit result is identical to the $m_+ = 1000$ GeV case except for the limit on κ_3 which is

$$\kappa_3 = 4.7 \pm 0.8$$

This is also shown in Fig. 1. Obviously, to cancel the top-pion correction, one must move out of the region allowed by the vacuum tilting constraint.

5. Discussion and Conclusion

Our result demonstrates that the class of top-color assisted technicolor models we have considered is ruled out unless:

1. the effective top-pion mass is around a TeV. This means that either the higher-order corrections must suppress the 1-loop correction significantly, or that the top-pion is indeed as heavy as a TeV, or
2. 1-loop coloron corrections are enhanced significantly by higher-order corrections.

Hill [19] suggests that the top-pion contribution may be sufficiently suppressed by taking the top-pion decay constant $f_\pi \approx 100$ GeV. This will decrease the Yukawa coupling by a factor of 2 and suppress the top-pion correction by a factor of 4. However, this requires the top-color scale to be about $\Lambda \sim 1000$ TeV. This increase in the top-color scale will suppress enormously the coloron and Z' corrections, depriving them of any power to counteract the top-pion correction. Furthermore, an increase in top-color scale implies the necessity of fine tuning which is contrary to the original motivation of the theory.

It is interesting to note that the experimental values of $A_{\text{FB}}^0(b)$ and A_b actually *prefer* a large top-pion correction. In fact, these two observables contribute the most (6.8) to the overall χ^2 of the fit because the top-pion correction is not large enough

to make the agreement better.⁶ Therefore, finding a way to enhance the coloron correction may be the more phenomenologically viable path.

Since we have examined a model with a specific charge assignment, one can ask whether a different charge assignment may improve the situation. We have looked at several alternative scenarios and have found the following:

1. In the original formulation by Hill [2], the second generation was assigned $U(1)_s$ charges instead of $U(1)_w$ charges. This assignment would make κ_1 break lepton universality between the electron and the muon. As a result, the limits on κ_1 will be even tighter than when only the third generation carried the $U(1)_s$ charge.
2. In the model recently proposed by Popovic and Simmons [10], all three generations were assigned $SU(3)_s$ charges. This makes the coloron correction cancel exactly in the ratio $R_b = \Gamma_{b\bar{b}}/\Gamma_{\text{had}}$ so it cannot counteract the top-pion correction at all.
3. One can free κ_1 from the constraint of lepton universality if all three generations are assigned equal $U(1)_s$ charges and no $U(1)_w$ charge. However, that would make the Z' correction *decrease* the ratio $R_b = \Gamma_{b\bar{b}}/\Gamma_{\text{had}}$ since the denominator will grow faster than the numerator.⁷

There are of course other charge assignments that one can think of as was considered by Lane [11]. However, we feel that these examples more than aptly show that changing the charge assignments probably will not alleviate the problem.

To summarize: we have used the latest LEP/SLD data to place constraints on the size of relevant vertex corrections to Z -pole observables in top-color assisted technicolor models with a strong vacuum tilting $U(1)$. We find that it is difficult to make the models compatible with experiment unless the large top-pion correction to R_b can be suppressed, or the coloron correction enhanced.

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⁶ The bottom-pion correction, which we have neglected, may account for the deviation in $A_{\text{FB}}^0(b)$ and A_b .

⁷ One should also take into account the effect of direct Z' exchange [20] between the initial e^+e^- pair and the final $f\bar{f}$ pair in such models, but this was not done here.

Appendix: Correlations of LEP/SLD Data

	m_Z	Γ_Z	σ_{had}^0	R_e	R_μ	R_τ	$A_{\text{FB}}^0(e)$	$A_{\text{FB}}^0(\mu)$	$A_{\text{FB}}^0(\tau)$
m_Z	1.000	0.000	-0.040	0.002	-0.010	-0.006	0.016	0.045	0.038
Γ_Z		1.000	-0.184	-0.007	0.003	0.003	0.009	0.000	0.003
σ_{had}^0			1.000	0.058	0.094	0.070	0.006	0.002	0.005
R_e				1.000	0.098	0.073	-0.442	0.007	0.012
R_μ					1.000	0.105	0.001	0.010	-0.001
R_τ						1.000	0.002	0.000	0.020
$A_{\text{FB}}^0(e)$							1.000	-0.008	-0.006
$A_{\text{FB}}^0(\mu)$								1.000	0.029
$A_{\text{FB}}^0(\tau)$									1.000

Table 4: The correlation of the Z lineshape variables at LEP

	R_b	R_c	$A_{\text{FB}}^0(b)$	$A_{\text{FB}}^0(c)$	A_b	A_c
R_b	1.00	-0.17	-0.06	0.02	-0.02	0.02
R_c		1.00	0.05	-0.04	0.01	-0.04
$A_{\text{FB}}^0(b)$			1.00	0.13	0.03	0.02
$A_{\text{FB}}^0(c)$				1.00	-0.01	0.07
A_b					1.00	0.04
A_c						1.00

Table 5: The correlation of the heavy flavor observables at LEP/SLD.

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