

# Constraints on R-parity violating couplings from lepton universality

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## Abstract

We analyze the one loop corrections to leptonic  $W$  and  $Z$  decays in an R-parity violating extension to the Minimal Supersymmetric Standard Model (MSSM). We find that lepton universality violation in the  $Z$  line-shape variables alone would strengthen the bounds on the magnitudes of the  $\lambda'$  couplings, but a global fit on all data leaves the bounds virtually unchanged at  $|\lambda'_{33k}| \leq 0.42$  and  $|\lambda'_{23k}| \leq 0.50$  at the  $2\sigma$  level. Bounds from  $W$  decays are less stringent:  $|\lambda'_{33k}| \leq 2.4$  at  $2\sigma$ , as a consequence of the weaker Fermilab experimental bounds on lepton universality violation in  $W$  decays. We also point out the potential of constraining R-parity violating couplings from the measurement of the  $\Upsilon$  invisible width.

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## I. INTRODUCTION

The assumption of R-parity conservation in supersymmetric model-building has long been an economical means of (1) avoiding certain phenomenological problems in SUSY models (*e.g.* proton decay), (2) ensuring that the lightest supersymmetric particle is available as a cure for the dark matter problem, and (3) reducing the SUSY model parameter space. (For recent reviews, see Ref. [1].) However, the recent discovery of neutrino mass at Super-Kamiokande [2] provides improved motivation for R-parity violating extensions to the Minimal Supersymmetric Standard Model (MSSM). Detailed analyses of the phenomenological constraints on such models are thus warranted to quantify the amount of R-parity violation permitted by current experimental data.

In this paper we consider the effects of R-parity violating extensions to the MSSM on *lepton universality* in  $W$  and  $Z$  decays. The R-conserving sector of the MSSM generates lepton universality violations proportional either to the lepton Yukawa couplings (due to Higgs interactions) or to the mass splittings of the sleptons (due to gauge interactions). Effects due to the Higgs sector will be considered in a future work [3] but will in general be negligible unless  $\tan\beta$  is quite large [4]. Effects due to gauge interactions are negligible if the slepton mass splittings are small. This is the case, for example, in supergravity (SUGRA) models with universal soft-breaking scalar masses at the SUGRA scale, in which the mass degeneracy is broken only by renormalization group running effects involving small Yukawa couplings. In R-parity violating models, however, R-parity violating interactions provide additional sources of lepton universality violation which may be significantly larger than these smaller effects.

The R-parity violating superpotential has the following form:<sup>1</sup>

$$W_{\mathcal{R}} = \frac{1}{2}\lambda_{ijk}\hat{L}_i\hat{L}_j\hat{E}_k + \lambda'_{ijk}\hat{L}_i\hat{Q}_j\hat{D}_k + \frac{1}{2}\lambda''_{ijk}\hat{U}_i\hat{D}_j\hat{D}_k, \quad (1.1)$$

where  $\hat{L}_i$ ,  $\hat{E}_i$ ,  $\hat{Q}_i$ ,  $\hat{U}_i$ , and  $\hat{D}_i$  are the MSSM superfields defined in the usual fashion [7], and the subscript  $i = 1, 2, 3$  is the generation index. Since *a priori* the interactions described by this superpotential have an arbitrary flavor structure, we generically expect that in the context of  $W$  and  $Z$  decays these will give rise to lepton universality violations. In this paper, we estimate the size of this violation and derive constraints on the R-parity violating couplings from LEP and Fermilab measurements of the leptonic observables in  $W$  and  $Z$  decays.

It is clear that the purely baryonic operator  $\hat{U}_i\hat{D}_j\hat{D}_k$  is irrelevant to our discussion. The other two operators may affect  $W$  and  $Z$  decays at one loop through vertex corrections to the  $W\ell_L\bar{\nu}$  and  $Z\ell_L\bar{\ell}_L$  vertices, with superparticles running in the loop. However, the couplings  $\lambda_{ijk}$  are already tightly constrained to be at most  $\mathcal{O}(10^{-2})$ , as the operator  $\hat{L}_i\hat{L}_j\hat{E}_k$  violates lepton universality in lepton decays at tree level [8]. The constraints on  $\lambda'_{ijk}$  are much less stringent. Previous limits cite upper bounds on  $\lambda'_{ijk}$  as large as the gauge couplings (*i.e.*

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<sup>1</sup>Because we are neglecting the soft breaking terms, we can rotate the bilinear terms away [5]. For possible effects of the soft breaking terms on  $W$  and  $Z$  decays see Ref. [6].

as large as 0.5) with the SUSY scale at 100 GeV [1]. One may thus expect significant radiative corrections induced by these couplings. Henceforth we will focus on the effects of the operator  $\hat{L}_i \hat{Q}_j \hat{D}_k$  only.

It is important to note that very strict constraints on the products of *different* R-violating couplings already exist from flavor-changing processes, *e.g.*  $\mu \rightarrow e\gamma$  constrains  $|\lambda'_{1ij}\lambda'_{2ij}| < 4.6 \times 10^{-4}$  [9]. However, these constraints can be easily satisfied by requiring only one of these couplings to be very small leaving the other coupling ill-constrained. The  $W$  and  $Z$  decay processes we consider here are flavor-conserving and involve the *same* R-violating coupling squared. Thus we can constrain the individual couplings rather than their products.

We emphasize that, in the absence of a complete calculation in the full theory, focussing attention on the violation of lepton universality provides a clear advantage over studying the effects of R-breaking couplings on the individual lepton-gauge boson couplings separately. This is because the R-conserving sector induces significant *universal corrections* to the lepton couplings which depend strongly on the choice of SUSY parameters. These corrections (along with the corrections to the hadronic partial widths) cancel when considering violations of lepton universality. Thus, the study of lepton universality violation lets us isolate the effects of R-breaking interactions without ad hoc assumptions about corrections from the R-conserving sector.

In the following calculations we neglect left-right squark mixing. Left-right squark mixing could be large only for the stop. However, since diagrams involving the stop contain down quarks with negligible mass, it will be seen that the contributions from these diagrams are numerically small (subleading in an expansion in  $m_W^2$  or  $m_Z^2$ ). Further, due to the chiral structure of the R-breaking interactions, two left-right mass insertions would be required in the diagram. Thus, such contributions would be further suppressed as long as the mixing parameter is perturbatively small.

Radiative corrections to individual  $Z \rightarrow \ell\bar{\ell}$  partial widths due to R-breaking interactions have previously been considered in Ref. [10] but lacked a consistent treatment of the R-conserving corrections. In this paper, we study the violation of lepton universality in  $W$  and  $Z$  decay to isolate the effects of R-breaking couplings and constrain their sizes. In determining the limits on R-breaking from  $Z$  decay, we perform a global fit to all the relevant LEP and SLD observables in which the corrections from both R-breaking and R-conserving interactions are parametrized and fit to the data. This provides a consistent accounting of R-conserving effects and allows us to improve the existing bounds on the R-breaking  $\lambda'$  couplings. A companion study of constraints on  $\lambda'$  and  $\lambda''$  couplings from LEP/SLD hadronic observables has been performed in Ref. [11].

## II. LEPTONIC W DECAYS

The relevant R-parity violating interactions expressed in terms of the component fields take the form

$$\begin{aligned} \Delta\mathcal{L}_R = \lambda'_{ijk} & \left[ \tilde{\nu}_{iL} \bar{d}_{kR} d_{jL} + \tilde{d}_{jL} \bar{d}_{kR} \nu_{iL} + \tilde{d}_{kR}^* \bar{\nu}_{iL}^c d_{jL} \right. \\ & \left. - (\tilde{e}_{iL} \bar{d}_{kR} u_{jL} + \tilde{u}_{jL} \bar{d}_{kR} e_{iL} + \tilde{d}_{kR}^* \bar{e}_{iL}^c u_{jL}) \right] + h.c. \end{aligned} \quad (2.1)$$

The one loop diagrams contributing to the decay  $W \rightarrow e_{iL}\bar{\nu}_{i'L}$  are shown in Figs. A 2 and A 2. At one loop, the neutrino flavor may differ from that of the tree level vertex ( $i \neq i'$ ) as a result of the R-parity violating interactions. Since neutrino flavor is indistinguishable in the detector, we should in principle sum over all three generations of antineutrino in the final state. However, since this is a one loop effect which does not interfere with the tree level flavor conserving decay, we will neglect it in our analysis and set  $i = i'$ .

The amplitude of each diagram in Figs. A 2 and A 2 are:

$$\begin{aligned}
& -N_C |\lambda'_{ijk}|^2 \left[ -i \frac{g}{\sqrt{2}} W^\mu(p+q) \bar{e}_{iL}(p) \gamma_\mu \nu_{iL}(q) \right] \times \\
(1a) & : 2 \hat{C}_{24}(0, 0, m_W^2; 0, m_{\bar{u}_{jL}}, m_{\bar{d}_{jL}}) \\
(1b) & : (d-2) \hat{C}_{24}(0, 0, m_W^2; m_{\bar{d}_{kR}}, m_{u_j}, 0) \\
& \quad - m_W^2 \hat{C}_{23}(0, 0, m_W^2; m_{\bar{d}_{kR}}, m_{u_j}, 0) \\
(2a) & : B_1(0; 0, m_{\bar{u}_{jL}}) \\
(2b) & : B_1(0; 0, m_{\bar{d}_{jL}}) \\
(2c) & : B_1(0; m_{u_j}, m_{\bar{d}_{kR}}) \\
(2d) & : B_1(0; 0, m_{\bar{d}_{kR}}) \tag{2.2}
\end{aligned}$$

The expression in the square brackets is the tree level amplitude. The definitions of the integrals  $B_1$ ,  $\hat{C}_{23}$ , and  $\hat{C}_{24}$  are presented in the Appendix. In the above expressions  $N_C = 3$  is the number of colors, and  $i, j, k$  are family indices; the final result must be summed over  $j$  and  $k$  to obtain the full correction for final state flavor  $i$ . We have set all the down type quark masses to zero. The up type quark masses  $m_{u_j}$  will also be set to zero except for the top quark ( $j = 3, m_{u_3} = m_t$ ).

Combining the expressions in Eq. (2.2), with appropriate factors of  $\frac{1}{2}$  for the wavefunction renormalizations, we obtain the one-loop shift of the  $W e_{iL}\bar{\nu}_{iL}$  coupling due to the  $\lambda'_{ijk}$  interaction:

$$\begin{aligned}
\delta g_{ijk} & = \delta g_{ijk}^{(u)} + \delta g_{ijk}^{(d)}, \\
\frac{\delta g_{ijk}^{(d)}}{g} & \equiv -N_C |\lambda'_{ijk}|^2 \left[ 2 \hat{C}_{24}(0, m_{\bar{u}_{jL}}, m_{\bar{d}_{jL}}) + \frac{1}{2} B_1(0, m_{\bar{u}_{jL}}) + \frac{1}{2} B_1(0, m_{\bar{d}_{jL}}) \right], \\
\frac{\delta g_{ijk}^{(u)}}{g} & \equiv -N_C |\lambda'_{ijk}|^2 \left[ (d-2) \hat{C}_{24}(m_{\bar{d}_{kR}}, m_{u_j}, 0) - m_W^2 \hat{C}_{23}(m_{\bar{d}_{kR}}, m_{u_j}, 0) \right. \\
& \quad \left. + \frac{1}{2} B_1(0, m_{\bar{d}_{kR}}) + \frac{1}{2} B_1(m_{u_j}, m_{\bar{d}_{kR}}) \right]. \tag{2.3}
\end{aligned}$$

We have suppressed the external momentum dependence of the  $B$  and  $C$  functions to simplify our expressions. The contributions of diagrams involving the down-type quark have been combined in  $\delta g_{ijk}^{(d)}$  and those involving the up-type quark in  $\delta g_{ijk}^{(u)}$ . The  $1/\epsilon$  poles of dimensional regularization cancel separately in each of these combinations so they are finite.

In the following, we evaluate the size of these shifts for a common squark mass of  $m_{\bar{q}} = 100$  GeV. To facilitate estimations for different squark masses, we provide approximate formulae.

- First, we evaluate  $\delta g_{i3k}^{(u)}$ , the contribution from diagrams involving the top quark  $u_3$ . For  $m_{u_3} = m_t = 175$  GeV,  $m_W = 81$  GeV, and  $m_{\bar{q}} = 100$  GeV, we find:

$$\frac{\delta g_{i3k}^{(u)}}{g} = -1.02\% |\lambda'_{i3k}|^2 \quad (2.4)$$

An approximate expression can be obtained by expanding the full expression of  $\delta g_{i3k}^{(u)}$  in powers of  $m_W^2$ :

$$\begin{aligned} \frac{\delta g_{i3k}^{(u)}}{g} &\approx -\frac{N_C}{(4\pi)^2} |\lambda'_{i3k}|^2 \\ &\times \left[ \frac{x}{4(1-x)^2} \{ x - 1 - (2-x) \ln x \} \right. \\ &\left. + \frac{m_W^2}{m_t^2} \frac{x}{3(1-x)^2} \{ 1 - x + \ln x \} \right] \end{aligned} \quad (2.5)$$

where  $x = m_t^2/m_{\bar{q}}^2$ . For  $m_{\bar{q}} = 100$  GeV, this expression is equal to  $(-1.11 + 0.09)\% |\lambda'_{i3k}|^2$ . Compared to the exact result above, we see that the leading order approximation is already fairly accurate.

- The contributions of the diagrams with massless quarks are numerically smaller and vanish as  $m_W \rightarrow 0$ . The correction with massless up-type quarks ( $j = 1, 2$ ) is:

$$\frac{\delta g_{ijk}^{(u)}}{g} = 0.22\% |\lambda'_{ijk}|^2 \quad (j = 1, 2) \quad (2.6)$$

The approximate form to leading order in  $m_W^2/m_{\bar{q}}^2$  is

$$\frac{\delta g_{ijk}^{(u)}}{g} \approx \frac{N_C}{(4\pi)^2} |\lambda'_{ijk}|^2 \frac{m_W^2}{9m_{\bar{q}}^2} \left( 1 - 3 \ln \frac{m_W^2}{m_{\bar{q}}^2} \right). \quad (2.7)$$

For  $m_{\bar{q}} = 100$  GeV, this gives  $0.31\% |\lambda'_{ijk}|^2$  which suffices for our purpose.

- The diagrams with down-type quarks contribute:

$$\frac{\delta g_{ijk}^{(d)}}{g} = 0.07\% |\lambda'_{ijk}|^2 \quad (2.8)$$

the approximate expression being

$$\frac{\delta g_{ijk}^{(d)}}{g} \approx \frac{N_C}{(4\pi)^2} |\lambda'_{ijk}|^2 \left( \frac{m_W^2}{18 m_{\bar{q}}^2} \right). \quad (2.9)$$

For  $m_{\bar{q}} = 100$  GeV, this gives  $0.07\% |\lambda'_{ijk}|^2$  to the accuracy shown.

Note that each contribution, Eqns. (2.5), (2.7), and (2.9) separately decouples in the limit  $m_{\bar{q}}^2 \rightarrow \infty$  as they should. Collecting everything together, the shift in the coupling of the  $i$ -th generation lepton to the  $W$  is given by:

$$\frac{\delta g_i}{g} = \sum_{j,k} \left[ \frac{\delta g_{ijk}^{(u)}}{g} + \frac{\delta g_{ijk}^{(d)}}{g} \right] = -0.95\% \sum_k |\lambda'_{i3k}|^2 + 0.29\% \sum_k |\lambda'_{i2k}|^2 + 0.29\% \sum_k |\lambda'_{i1k}|^2 \quad (2.10)$$

where we have summed over all possible generation indices  $j$  and  $k$ .

The current bound on lepton universality violation in leptonic  $W$  decays from  $D\theta$  is [13]

$$\frac{g_\tau}{g_e} = 1.004 \pm 0.019(stat.) \pm 0.026(syst.).$$

This places a constraint on

$$\delta \left( \frac{g_\tau}{g_e} \right) = \frac{\delta g_3}{g} - \frac{\delta g_1}{g}. \quad (2.11)$$

Note that R-conserving corrections do not contribute since they cancel in the ratio  $g_\tau/g_e$ . The constraint on the R-breaking couplings is thus

$$\begin{aligned} & -0.4 \pm 1.9(stat.) \pm 2.6(syst.) \\ & = \left\{ \sum_k |\lambda'_{33k}|^2 - 0.3 \sum_k |\lambda'_{32k}|^2 - 0.3 \sum_k |\lambda'_{31k}|^2 \right\} \\ & - \left\{ \sum_k |\lambda'_{13k}|^2 - 0.3 \sum_k |\lambda'_{12k}|^2 - 0.3 \sum_k |\lambda'_{11k}|^2 \right\}. \end{aligned} \quad (2.12)$$

Of the couplings  $\lambda'_{ijk}$  appearing in this expression, the  $i = 1$  couplings are already well constrained from neutrino-less double beta decay, neutrino masses, atomic parity violation, and low energy charged current universality. Using the  $2\sigma$  limits for the individual couplings charted in Ref. [14], we find for  $m_{\bar{q}} = 100$  GeV:

$$\begin{aligned} \sum_k |\lambda'_{11k}|^2 & \leq 0.00088, \\ \sum_k |\lambda'_{12k}|^2 & \leq 0.0055, \\ \sum_k |\lambda'_{13k}|^2 & \leq 0.079. \end{aligned} \quad (2.13)$$

The  $i = 3, j = 1$  couplings are also well constrained from  $R_{\tau\pi} = \Gamma(\tau \rightarrow \pi\nu_\tau)/\Gamma(\pi \rightarrow \mu\nu_\mu)$ . Again, using the  $2\sigma$  limits cited in Ref. [14] we find

$$\sum_k |\lambda'_{31k}|^2 \leq 0.036 \quad (2.14)$$

Therefore, we can neglect the  $\lambda'_{1jk}$  and  $\lambda'_{31k}$  terms in Eq. (2.12) and obtain

$$\sum_k |\lambda'_{33k}|^2 - 0.3 \sum_k |\lambda'_{32k}|^2 = -0.4 \pm 3.2 \quad (2.15)$$

where the systematic and statistical errors have been added in quadrature. If we neglect the  $32k$  term with a smaller numerical coefficient, this places a  $1\sigma$  ( $2\sigma$ ) upper bound on the  $33k$  term:

$$\sum_k |\lambda'_{33k}|^2 \leq 2.8 \quad (6.0), \quad (2.16)$$

which in turn translates into the limit

$$|\lambda'_{33k}| \leq 1.7 \quad (2.4). \quad (2.17)$$

Non-zero values of  $\lambda'_{32k}$  will weaken this bound.

As we will see later,  $Z$  decay data places a constraint on  $\sum_k |\lambda'_{33k}|^2$  at the  $\pm 0.1$  level. Therefore, for the  $W$  decay data to be competitive with the  $Z$  decay data, the error must be improved by more than an order of magnitude. While the Tevatron Run II may provide enough data to improve the statistical error considerably, improving the systematic error may prove a challenge [15].

### III. LEPTONIC Z DECAYS

We next consider the effect of R-parity violating interactions on flavor-conserving leptonic  $Z$  decays,  $Z \rightarrow e_{iL} \bar{e}_{iL}$ . Note that the  $\lambda'$  interaction in Eq. (1.1) involves only the left-handed lepton field. Therefore, at the one-loop level the right-handed coupling is unaffected. Neglecting all down-type quark masses, the amplitudes of the diagrams shown in Figs. A 2 and A 2 are

$$\begin{aligned}
& -N_C |\lambda'_{ijk}|^2 \left[ -i \frac{g}{\cos \theta_W} Z^\mu (p+q) \bar{e}_{iL}(p) \gamma_\mu e_{iL}(q) \right] \times \\
(3a) & : -2h_{uL} \hat{C}_{24} (0, 0, m_Z^2; 0, m_{\tilde{u}_{jL}}, m_{\tilde{u}_{jL}}) \\
(3b) & : +2h_{dR} \hat{C}_{24} (0, 0, m_Z^2; m_{u_j}, m_{\tilde{d}_{kR}}, m_{\tilde{d}_{kR}}) \\
(3c) & : -h_{uL} \left[ (d-2) \hat{C}_{24} (0, 0, m_Z^2; m_{\tilde{d}_{kR}}, m_{u_j}, m_{u_j}) \right. \\
& \quad \left. - m_Z^2 \hat{C}_{23} (0, 0, m_Z^2; m_{\tilde{d}_{kR}}, m_{u_j}, m_{u_j}) \right] \\
(3d) & : +h_{dR} \left[ (d-2) \hat{C}_{24} (0, 0, m_Z^2; m_{\tilde{u}_{jL}}, 0, 0) \right. \\
& \quad \left. - m_Z^2 \hat{C}_{23} (0, 0, m_Z^2; m_{\tilde{u}_{jL}}, 0, 0) \right] \\
(3e) & : h_{uR} m_{u_j}^2 \hat{C}_0 (0, 0, m_Z^2; m_{\tilde{d}_{kL}}, m_{u_j}, m_{u_j}) \\
(4a) + (4b) & : 2h_{eL} B_1 (0; 0, m_{\tilde{u}_{jL}}) \\
(4c) + (4d) & : 2h_{eL} B_1 (0; m_{u_j}, m_{\tilde{d}_{kR}}) \quad (3.1)
\end{aligned}$$

where

$$h_{fL} = I_3 - Q_f \sin^2 \theta_W, \quad h_{fR} = -Q_f \sin^2 \theta_W. \quad (3.2)$$

The tree level amplitude is  $h_{eL}$  times the expression in the square brackets. These corrections can be expressed as a shift in the coupling  $h_{eL}$ :

$$\delta h_{ijk} = \delta h_{ijk}^{(u)} + \delta h_{ijk}^{(d)}$$

$$\begin{aligned} \delta h_{ijk}^{(d)} &\equiv -N_C |\lambda'_{ijk}|^2 \left[ -2h_{uL} \hat{C}_{24} (0, m_{\tilde{u}_{jL}}, m_{\tilde{u}_{jL}}) \right. \\ &\quad \left. + h_{dR} \left\{ (d-2) \hat{C}_{24} (m_{\tilde{u}_{jL}}, 0, 0) - m_Z^2 \hat{C}_{23} (m_{\tilde{u}_{jL}}, 0, 0) \right\} \right. \\ &\quad \left. + h_{eL} B_1 (0, m_{\tilde{u}_{jL}}) \right] \\ \delta h_{ijk}^{(u)} &\equiv -N_C |\lambda'_{ijk}|^2 \left[ 2h_{dR} \hat{C}_{24} (m_{u_j}, m_{\tilde{d}_{kR}}, m_{\tilde{d}_{kR}}) \right. \\ &\quad \left. - h_{uL} \left\{ (d-2) \hat{C}_{24} (m_{\tilde{d}_{kR}}, m_{u_j}, m_{u_j}) - m_Z^2 \hat{C}_{23} (m_{\tilde{d}_{kR}}, m_{u_j}, m_{u_j}) \right\} \right. \\ &\quad \left. + h_{uR} m_{u_j}^2 \hat{C}_0 (m_{\tilde{d}_{kL}}, m_{u_j}, m_{u_j}) \right. \\ &\quad \left. + h_{eL} B_1 (m_{u_j}, m_{\tilde{d}_{kR}}) \right] \end{aligned} \quad (3.3)$$

Again, the dependence on the external momenta has been suppressed. The corrections which depend on the down-type quark have been grouped together in  $\delta h_{ijk}^{(d)}$  and those that depend on the up-type quark in  $\delta h_{ijk}^{(u)}$ . These combinations are separately finite. For simplicity, we again evaluate these shifts for a common squark mass of  $m_{\tilde{q}} = 100$  GeV.

- We begin with the top quark dependent contribution. For  $m_t = 175$  GeV,  $m_Z = 92$  GeV, and  $m_{\tilde{q}} = 100$  GeV, we find

$$\delta h_{i3k}^{(u)} = 0.63\% |\lambda'_{i3k}|^2. \quad (3.4)$$

This is well approximated by the leading  $m_Z = 0$  piece of the expansion in the  $Z$  mass:

$$\delta h_{ijk}^{(u)} \approx -\frac{N_C}{2(4\pi)^2} |\lambda'_{i3k}|^2 F(x) \quad (3.5)$$

where

$$F(x) = \frac{x}{1-x} \left( 1 + \frac{1}{1-x} \ln x \right) \quad (3.6)$$

and  $x = m_t^2/m_{\tilde{q}}^2$ . For  $m_{\tilde{q}} = 100$  GeV, this gives  $0.65\% |\lambda'_{i3k}|^2$ . The subleading terms from the individual diagrams contributing to  $\delta h_{i3k}^{(u)}$  are:

$$\begin{aligned} (3b) &: -\frac{N_C h_{dR}}{(4\pi)^2} |\lambda'_{i3k}|^2 \frac{m_Z^2}{2m_{\tilde{q}}^2} f\left(\frac{1}{x}\right) \\ (3c) &: \frac{N_C h_{uL}}{(4\pi)^2} |\lambda'_{i3k}|^2 \frac{m_Z^2}{m_t^2} f(x) \\ (3e) &: -\frac{N_C h_{uR}}{(4\pi)^2} |\lambda'_{i3k}|^2 \frac{m_Z^2}{m_t^2} g(x) \end{aligned} \quad (3.7)$$

where



$$f(x) \equiv -\frac{1}{18} \frac{1}{(1-x)^4} \left[ 2x^4 - 9x^3 + 18x^2 - 11x - 6x \ln x \right] \quad (3.8)$$

$$g(x) \equiv \frac{1}{12} \frac{1}{(1-x)^4} \left[ x^4 - 6x^3 + 3x^2 + 2x + 6x^2 \ln x \right] \quad (3.9)$$

The total subleading contribution for  $m_{\bar{q}} = 100$  GeV is  $-0.03\% |\lambda'_{ijk}|^2$ .

In Ref. [10], the leading and subleading contributions of diagrams (3b) and (3c) are shown<sup>2</sup> but the subleading contribution of diagram (3e) appears to have been omitted. We also disagree with the expression for (3b) in Ref. [10] by a factor of  $\frac{1}{2}$ . However, the numerical impact is negligible.

- The corrections involving massless quark loops vanish in the limit  $m_Z \rightarrow 0$  and give numerically small contributions. For the massless up-type quarks ( $u_1 = u$  and  $u_2 = c$ ) we find:

$$\delta h_{ijk}^{(u)} = -0.02\% |\lambda'_{ijk}|^2. \quad (3.10)$$

The leading order term in  $m_Z^2/m_{\bar{q}}^2$  is

$$\delta h_{ijk}^{(u)} \approx -\frac{N_C}{(4\pi)^2} |\lambda'_{ijk}|^2 \left[ h_{uL} \frac{m_Z^2}{9m_{\bar{q}}^2} \left( 1 - 3 \ln \frac{m_Z^2}{m_{\bar{q}}^2} \right) - h_{dR} \frac{m_Z^2}{18m_{\bar{q}}^2} \right]. \quad (3.11)$$

For  $m_{\bar{q}} = 100$  GeV, this gives  $-0.01\% |\lambda'_{ijk}|^2$ .

- The massless down-type quark dependent correction is:

$$\delta h_{ijk}^{(d)} = -0.06\% |\lambda'_{ijk}|^2 \quad (3.12)$$

The leading order term in  $m_Z^2/m_{\bar{q}}^2$  is

$$\delta h_{ijk}^{(d)} \approx -\frac{N_C}{(4\pi)^2} |\lambda'_{ijk}|^2 \left[ h_{uL} \frac{m_Z^2}{18m_{\bar{q}}^2} - h_{dR} \frac{m_Z^2}{9m_{\bar{q}}^2} \left( 1 - 3 \ln \frac{m_Z^2}{m_{\bar{q}}^2} \right) \right] \quad (3.13)$$

For  $m_{\bar{q}} = 100$  GeV, this gives  $-0.09\% |\lambda'_{ijk}|^2$ .

Combining everything together, and summing over the generation indices  $j$  and  $k$ , the shift of the  $i$ -th generation lepton coupling to the  $Z$  due to R-violating interactions is:

$$\begin{aligned} \delta h_i^R &= \sum_{j,k} \left[ \delta h_{ijk}^{(u)} + \delta h_{ijk}^{(d)} \right] \\ &= 0.61\% \sum_k |\lambda'_{i3k}|^2 - 0.08\% \sum_k |\lambda'_{i2k}|^2 - 0.08\% \sum_k |\lambda'_{i1k}|^2 \\ &\approx 0.61\% \sum_k |\lambda'_{i3k}|^2 \end{aligned} \quad (3.14)$$

where we drop the subleading terms. (This is equivalent to keeping only the diagrams involving the top and the stop.)

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<sup>2</sup>cf Eq. 9 of Ref. [10].

#### IV. FITS AND NUMERICAL ANALYSES

In order to place limits on the  $\lambda'_{ijk}$  couplings from  $Z$  decay, we need to know how the observables at LEP and SLD will be affected by the shifts  $\delta h_i^R$  in the left-handed coupling of  $e_i$  to the  $Z$ , as well as by other R-conserving vertex and oblique corrections.

The relevant observables are

$$R_\ell = \frac{\Gamma(Z \rightarrow \text{hadrons})}{\Gamma(Z \rightarrow \ell^+\ell^-)} = \frac{N_C \sum_{q=u,d,s,c,b} (h_{qL}^2 + h_{qR}^2)}{(h_{\ell L}^2 + h_{\ell R}^2)} \quad (4.1)$$

and

$$A_\ell = \frac{h_{\ell L}^2 - h_{\ell R}^2}{h_{\ell L}^2 + h_{\ell R}^2} \quad (4.2)$$

as well as

$$A_{\text{FB}}(\ell) = \frac{3}{4} A_e A_\ell \quad (4.3)$$

where  $\ell = e, \mu, \tau$ . The shift in  $R_\ell$  due to shifts in the coupling constants is

$$\begin{aligned} \frac{\delta R_\ell}{R_\ell} &= \frac{\delta \Gamma_{\text{had}}}{\Gamma_{\text{had}}} - \frac{2h_{\ell L} \delta h_{\ell L} + 2h_{\ell R} \delta h_{\ell R}}{h_{\ell L}^2 + h_{\ell R}^2} \\ &= \Delta_R - \frac{2h_{\ell L}}{h_{\ell L}^2 + h_{\ell R}^2} \delta h_\ell^R \\ &= \Delta_R + 4.3 \delta h_\ell^R \end{aligned} \quad (4.4)$$

where we have subsumed all the hadronic corrections and the lepton flavor independent oblique and vertex corrections into a single parameter  $\Delta_R$ , and the coefficient of  $\delta h_\ell^R$  is calculated for the value  $\sin^2 \theta_W = 0.2315$ .

We note that the  $\lambda'$  couplings we are trying to constrain also contribute to  $\Delta_R$  through  $\Gamma_{\text{had}}$  since they modify the  $Zq\bar{q}$  vertices<sup>3</sup> as well as the  $Z\ell\bar{\ell}$  vertices. However, since  $\Delta_R$  also subsumes highly *model dependent* corrections from the R-conserving sector, it can be considered an independent parameter from  $\delta h_\ell^R$  in our fit.

Similarly, the shift in  $A_\ell$  is given by

$$\begin{aligned} \frac{\delta A_\ell}{A_\ell} &= \frac{4h_{\ell L} h_{\ell R}^2 \delta h_{\ell L} - 4h_{\ell L}^2 h_{\ell R} \delta h_{\ell R}}{h_{\ell L}^4 - h_{\ell R}^4} \\ &= \Delta_A + \frac{4h_{\ell L} h_{\ell R}^2}{h_{\ell L}^4 - h_{\ell R}^4} \delta h_\ell^R \\ &= \Delta_A - 25 \delta h_\ell^R \end{aligned} \quad (4.5)$$

---

<sup>3</sup>The flavor dependence of R-violating corrections to the  $Zq\bar{q}$  vertices can be used to constrain  $\lambda'$  and  $\lambda''$  by looking at purely hadronic observables [11].

where we have subsumed all the lepton flavor independent oblique and vertex corrections into a single parameter  $\Delta_A$ , and the coefficient of  $\delta h_\ell^R$  is again calculated for the value  $\sin^2 \theta_W = 0.2315$ . We do not need to introduce another flavor independent parameter for  $A_{\text{FB}}(\ell)$  since

$$\frac{\delta A_{\text{FB}}(\ell)}{A_{\text{FB}}(\ell)} = \frac{\delta A_e}{A_e} + \frac{\delta A_\ell}{A_\ell} = 2\Delta_A - 25 \delta h_e^R - 25 \delta h_\ell^R. \quad (4.6)$$

Therefore, we can express all corrections from both R-breaking and R-conserving interactions in terms of just 5 parameters:  $\Delta_R$ ,  $\Delta_A$ , and  $\delta h_\ell^R$ ,  $\ell = e, \mu, \tau$ . A five parameter fit cannot be conducted, however, since a change in any one parameter can always be absorbed into the other four. We therefore define

$$\begin{aligned} \Delta_{R_e} &\equiv \Delta_R + 4.3 \delta h_e^R, \\ \Delta_{A_e} &\equiv \Delta_A - 25 \delta h_e^R, \\ \delta_{\mu e} &\equiv \delta h_\mu^R - \delta h_e^R, \\ \delta_{\tau e} &\equiv \delta h_\tau^R - \delta h_e^R. \end{aligned} \quad (4.7)$$

and perform a four parameter fit instead. Note that the parameters

$$\begin{aligned} \delta_{\mu e} &= 0.61\% \left\{ \sum_k |\lambda'_{23k}|^2 - \sum_k |\lambda'_{13k}|^2 \right\} \\ \delta_{\tau e} &= 0.61\% \left\{ \sum_k |\lambda'_{33k}|^2 - \sum_k |\lambda'_{13k}|^2 \right\} \end{aligned} \quad (4.8)$$

are measures of lepton universality violation. The dependence of all the observables we use in our fit to the four fit parameters is:

$$\begin{aligned} \frac{\delta R_e}{R_e} &= \Delta_{R_e} \\ \frac{\delta R_\mu}{R_\mu} &= \Delta_{R_e} + 4.3 \delta_{\mu e} \\ \frac{\delta R_\tau}{R_\tau} &= \Delta_{R_e} + 4.3 \delta_{\tau e} \\ \frac{\delta A_e}{A_e} &= \Delta_{A_e} \\ \frac{\delta A_\mu}{A_\mu} &= \Delta_{A_e} - 25 \delta_{\mu e} \\ \frac{\delta A_\tau}{A_\tau} &= \Delta_{A_e} - 25 \delta_{\tau e} \\ \frac{\delta A_{\text{FB}}(e)}{A_{\text{FB}}(e)} &= 2\Delta_{A_e} \\ \frac{\delta A_{\text{FB}}(\mu)}{A_{\text{FB}}(\mu)} &= 2\Delta_{A_e} - 25 \delta_{\mu e} \\ \frac{\delta A_{\text{FB}}(\tau)}{A_{\text{FB}}(\tau)} &= 2\Delta_{A_e} - 25 \delta_{\tau e} \end{aligned} \quad (4.9)$$

In table I, we show the most recent data of these observables from Refs. [16], [17], and [18]. The Standard Model predictions were calculated by ZFITTER v6.21 [19] with standard flag settings for the input values of  $m_t = 174.3$  GeV [20],  $m_H = 300$  GeV, and  $\alpha_s(m_Z) = 0.120$ . The limits on lepton universality violation is insensitive to the choice of the Higgs mass since Higgs couplings within the Standard Model do not violate lepton universality by any appreciable amount. The correlation matrix of the LEP  $Z$  lineshape data is shown in table II.

The result of the four parameter fit to all the data in table I is

$$\begin{aligned}\delta_{\mu e} &= 0.00038 \pm 0.00056 \\ \delta_{\tau e} &= -0.00013 \pm 0.00061 \\ \Delta_{A_e} &= 0.052 \pm 0.012 \\ \Delta_{R_e} &= 0.0007 \pm 0.0020\end{aligned}\tag{4.10}$$

with the correlation matrix shown in table III. The quality of the fit was  $\chi^2 = 8.3/(12 - 4)$ . In figures 5 and 6 we show the  $1\sigma$  constraints placed on  $\delta_{\mu e}$  and  $\delta_{\tau e}$  in the  $\Delta_{A_e} = \Delta_{R_e} = 0$  plane by each observable. It is seen that the strongest constraints come from  $R_\mu$ ,  $R_\tau$ , and  $A_\tau$  from the  $\tau$  polarization measurement at LEP. In figure 7, we show the 68% and 90% confidence contours on the  $\delta_{\mu e}$ - $\delta_{\tau e}$  plane.

The limits on  $\delta_{\mu e}$  and  $\delta_{\tau e}$  translate into limits on the R-breaking couplings:

$$\begin{aligned}\sum_k |\lambda'_{23k}|^2 - \sum_k |\lambda'_{13k}|^2 &= 0.062 \pm 0.093 \\ \sum_k |\lambda'_{33k}|^2 - \sum_k |\lambda'_{13k}|^2 &= -0.02 \pm 0.10 \\ \sum_k |\lambda'_{33k}|^2 - \sum_k |\lambda'_{23k}|^2 &= -0.083 \pm 0.093\end{aligned}\tag{4.11}$$

If we neglect the  $13k$  terms since they are already constrained to be small (recall that Eq. (2.13) shows the  $2\sigma$  upper bound), we obtain the following  $1\sigma$  ( $2\sigma$ ) upper bounds for the  $23k$  and  $33k$  terms:

$$\begin{aligned}\sum_k |\lambda'_{23k}|^2 &\leq 0.16 \quad (0.25) \\ \sum_k |\lambda'_{33k}|^2 &\leq 0.08 \quad (0.18)\end{aligned}\tag{4.12}$$

or

$$\begin{aligned}|\lambda'_{23k}| &\leq 0.40 \quad (0.50) \\ |\lambda'_{33k}| &\leq 0.28 \quad (0.42)\end{aligned}\tag{4.13}$$

The limit on  $\lambda'_{23k}$  should be interpreted as a limit on  $\lambda'_{232}$  since  $\lambda'_{231}$  and  $\lambda'_{233}$  are already fairly well constrained by other experiments [14]. If any of the  $13k$ -terms (in particular,  $\lambda'_{132}$  with a  $2\sigma$  upper bound of 0.28 [14]) are non-zero, these limits will be weakened.

It is interesting to note that since the measured value of  $R_\tau$  is smaller than the measured value of  $R_e$ , this pair prefers a negative value of  $\delta_{\tau e}$  (*cf.* Eq. 4.9). The same can be said of the pair  $A_{\text{FB}}(e)$  and  $A_{\text{FB}}(\tau)$ . On the other hand, the measured values of  $A_e$  (LEP and SLD)

and  $A_{LR}$  are all larger than the measured values of  $A_\tau$  (LEP and SLD) so these observables prefer a positive value of  $\delta_{\tau e}$ . (This is not apparent in figures 5 and 6 since they show the constraints in the  $\Delta_{A_e} = 0$  plane.) Due to this conflict, the central value of  $\delta_{\tau e}$  preferred by the global fit is virtually zero which satisfies neither the  $R$ 's nor the  $A$ 's. In fact,  $A_\tau$  from LEP, with its smaller fractional error, actually accounts for 2.8 out of 6.8 of the  $\chi^2$  of the fit.

If we perform our fit on the six  $Z$  line–shape parameters only, the result is

$$\begin{aligned}\delta_{\mu e} &= 0.00002 \pm 0.00061 \\ \delta_{\tau e} &= -0.00082 \pm 0.00070 \\ \Delta_{A_e} &= 0.055 \pm 0.033 \\ \Delta_{R_e} &= 0.0022 \pm 0.0022\end{aligned}\tag{4.14}$$

with  $\chi^2 = 1.9/(6 - 4)$ , and the correlation matrix is shown in table IV. The 90% confidence contour in the  $\delta_{\mu e}$ – $\delta_{\tau e}$  plane is shown in figures 7 and 8. This translates into

$$\begin{aligned}\sum_k |\lambda'_{23k}|^2 - \sum_k |\lambda'_{13k}|^2 &= 0.00 \pm 0.10 \\ \sum_k |\lambda'_{33k}|^2 - \sum_k |\lambda'_{13k}|^2 &= -0.13 \pm 0.11 \\ \sum_k |\lambda'_{33k}|^2 - \sum_k |\lambda'_{23k}|^2 &= -0.138 \pm 0.097\end{aligned}\tag{4.15}$$

Again, neglecting the  $13k$  term, we obtain the following  $1\sigma$  ( $2\sigma$ ) limits:

$$\begin{aligned}\sum_k |\lambda'_{23k}|^2 &\leq 0.10 \quad (0.20) \\ \sum_k |\lambda'_{33k}|^2 &\leq -0.02 \quad (0.09)\end{aligned}\tag{4.16}$$

or

$$\begin{aligned}|\lambda'_{23k}| &\leq 0.37 \quad (0.49) \\ |\lambda'_{33k}| &\leq \quad \quad (0.30)\end{aligned}\tag{4.17}$$

The negative central value for  $\delta_{\tau e}$  leads to a reduced upper bound for  $|\lambda'_{33k}|$ .

If we perform our fit on the two  $\tau$ –polarization observables and the four SLD observables only, the result is:

$$\begin{aligned}\delta_{\mu e} &= 0.0040 \pm 0.0046 \\ \delta_{\tau e} &= 0.0025 \pm 0.0013 \\ \Delta_{A_e} &= 0.062 \pm 0.013\end{aligned}\tag{4.18}$$

with  $\chi^2 = 0.84/(6 - 3)$ , and the correlation matrix is shown in table V. The 90% confidence contour in the  $\delta_{\mu e}$ – $\delta_{\tau e}$  plane for this case is also shown in figures 7 and 8. This translates into

$$\sum_k |\lambda'_{23k}|^2 - \sum_k |\lambda'_{13k}|^2 = 0.66 \pm 0.75$$

$$\begin{aligned}
\sum_k |\lambda'_{33k}|^2 - \sum_k |\lambda'_{13k}|^2 &= 0.41 \pm 0.22 \\
\sum_k |\lambda'_{33k}|^2 - \sum_k |\lambda'_{23k}|^2 &= -0.25 \pm 0.77
\end{aligned}
\tag{4.19}$$

Neglecting the  $13k$  term, we obtain the following  $1\sigma$  ( $2\sigma$ ) limits:

$$\begin{aligned}
\sum_k |\lambda'_{23k}|^2 &\leq 1.4 \quad (2.2) \\
\sum_k |\lambda'_{33k}|^2 &\leq 0.62 \quad (0.85)
\end{aligned}
\tag{4.20}$$

or

$$\begin{aligned}
|\lambda'_{23k}| &\leq 1.2 \quad (1.5) \\
|\lambda'_{33k}| &\leq 0.79 \quad (0.92)
\end{aligned}
\tag{4.21}$$

This time, the upper bounds are considerably larger.

This shows that had we used only the  $Z$  line-shape variables, which has been the case in previous analyses by other authors [10,12], or only the leptonic asymmetries, we would have reached drastically different conclusions concerning the limits on the R-breaking parameters. Only through a *global* analysis were we able to constrain the parameters in a consistent way.

## V. SUMMARY AND CONCLUSIONS

We find that flavor-*conserving* leptonic  $Z$  and  $W$  decays can be used to place significant constraints on the size of R-parity violating  $\lambda'$  couplings. Current bounds from lepton universality violation in leptonic  $Z$  decays from combined LEP/SLD data are

$$\begin{aligned}
|\lambda'_{23k}| &\leq 0.40 \quad (0.50) \\
|\lambda'_{33k}| &\leq 0.28 \quad (0.42)
\end{aligned}
\tag{5.1}$$

at the  $1\sigma$  ( $2\sigma$ ) level, assuming a common squark mass of  $m_{\tilde{q}} = 100$  GeV and the suppression of  $\lambda'_{13k}$  couplings. For larger (common) squark masses the above bounds should be interpreted as bounds on  $|\lambda'| \times \sqrt{F(x)/F(x_0)}$ , where  $F(x)$  is defined in Eq. 3.6 and  $x_0 = \frac{m_{\tilde{t}}^2}{(100\text{GeV})^2}$ .

Numerically, our numbers are not a significant improvement over those cited in Ref. [14]. However, the methods used to derive previous limits [10,12] were intrinsically flawed in that (1) R-conserving effects were not properly taken into account and (2) R-breaking effects on only the leptonic widths of the  $Z$  were considered. Indeed, had we also considered only the leptonic widths, our limits would have been those of Eq. (4.17). The analysis of this paper avoids these problems by focussing on *lepton universality violation* and performing a *global* fit on all LEP/SLD observables. The R-conserving effects are taken into account by parametrizing and fitting them to the data also. (Similar methods have been used in Ref. [21] to constrain flavor specific vertex corrections while taking into account the flavor universal oblique corrections.)

Current bounds on lepton universality in leptonic  $W$  decays provides the constraint

$$|\lambda'_{33k}| \leq 1.7 \quad (2.4) \tag{5.2}$$

at the  $1\sigma$  ( $2\sigma$ ) level. While not currently competitive with the  $Z$  decay bounds, the Fermilab results are complementary independent measurements, and they can be expected to improve dramatically at Tevatron Run II.

If the error on the LEP/SLD observables continue to shrink with the current central values, then eventually the region allowed by the line–shape variables and the asymmetries will fail to overlap in figure 8. In such a situation, not only the SM but the MSSM with R–parity violating couplings would be ruled out. In fact, no theory which introduces lepton universality violation in only the left–handed couplings would be viable.

Currently, the LEP and SLD observable provide the best limits on the  $\lambda'_{33k}$  couplings. However, one can potentially place a limit on the  $\lambda'$  couplings by looking at invisible decays of the  $\Upsilon$  and  $J/\Psi$  resonances at the  $B$  and  $\tau$ –charm factories [22]. The current bounds on  $\lambda'_{i33}$  imply that the correction to the invisible width of the  $\Upsilon$  resonance can be as large as 30%. A rough estimate shows that if the  $\Upsilon$  invisible width is found to agree with the Standard Model prediction with 5% accuracy, the  $\lambda'$  coupling would be constrained to be  $|\lambda'_{i33}| \leq 0.16$  at the  $2\sigma$  level. In addition, constraints on  $|\lambda'_{333}|$  will be available from forthcoming Tevatron studies of the decay  $t \rightarrow \tau b$  [23].

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## APPENDIX: FEYNMAN INTEGRALS

Here we make explicit our notation for the scalar and tensor integrals that appear in the calculation. The definitions of the integrals are similar to those of Ref. [24], but there are some small alterations made to take advantage of symmetries of the problem. The hat on the tensor integrals serves as a reminder of these differences.

### 1. Scalar Integrals

We define the functions  $B_0$  and  $\hat{C}_0$  by:

$$B_0 [p^2; m_1, m_2] \equiv i \mu^{4-d} \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 - m_1^2) [(k+p)^2 - m_2^2]} \quad (\text{A1})$$

$$\hat{C}_0 [p^2, q^2, (p-q)^2; m_1, m_2, m_3] \equiv i \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 - m_1^2) [(k+p)^2 - m_2^2] [(k+q)^2 - m_3^2]} \quad (\text{A2})$$

The general form of  $B_0$  is given by

$$B_0 [p^2; m_1, m_2] = \frac{-1}{(4\pi)^2} \left[ \Delta_\epsilon - \frac{m_1^2 \ln(m_1^2/\mu^2) - m_2^2 \ln(m_2^2/\mu^2)}{m_1^2 - m_2^2} + 1 + F(p^2; m_1, m_2) \right]$$

where  $\Delta_\epsilon = \frac{2}{4-d} - \gamma_E + \ln 4\pi$ , and [25]

$$F(p^2; m_1, m_2) = 1 + \frac{1}{2} \left( \frac{\Sigma}{\Delta} - \Delta \right) \ln \left( \frac{m_1^2}{m_2^2} \right) - \frac{1}{2} \sqrt{1 - 2\Sigma + \Delta^2} \ln \left( \frac{1 - \Sigma + \sqrt{1 - 2\Sigma + \Delta^2}}{1 - \Sigma - \sqrt{1 - 2\Sigma + \Delta^2}} \right) \quad (\text{A3})$$

with

$$\Sigma \equiv \frac{m_1^2 + m_2^2}{p^2}, \quad \Delta \equiv \frac{m_1^2 - m_2^2}{p^2}. \quad (\text{A4})$$

The function  $F(p^2; m_1, m_2)$  is well-behaved in the limit  $p^2 \rightarrow 0$ . For small  $p^2 \ll m_1^2, m_2^2$ , the behavior is:

$$F(p^2; m_1, m_2) = \frac{p^2}{2(m_1^2 - m_2^2)^2} \left[ m_1^2 + m_2^2 - \frac{2 m_1^2 m_2^2}{m_1^2 - m_2^2} \ln \left( \frac{m_1^2}{m_2^2} \right) \right] + \dots \quad (\text{A5})$$

$$F(p^2; m, m) = \frac{p^2}{6m^2} + \dots \quad (\text{A6})$$

$$F(p^2; 0, m) = \frac{p^2}{2m^2} + \dots \quad (\text{A7})$$

Useful special cases of the  $B_0$  function are:



$$B_0 [p^2; m, m] = \frac{-1}{(4\pi)^2} \left[ \Delta_\epsilon - \ln \frac{m^2}{\mu^2} + 2 - \sqrt{1 - \frac{4m^2}{p^2}} \ln \left( \frac{\sqrt{1 - \frac{4m^2}{p^2}} + 1}{\sqrt{1 - \frac{4m^2}{p^2}} - 1} \right) \right] \quad (\text{A8})$$

$$B_0 [p^2; 0, m] = \frac{-1}{(4\pi)^2} \left[ \Delta_\epsilon - \ln \frac{m^2}{\mu^2} + 2 - \left( 1 - \frac{m^2}{p^2} \right) \ln \left( 1 - \frac{p^2}{m^2} \right) \right] \quad (\text{A9})$$

The general form of the  $\hat{C}_0$  function is fairly complex and we refer the reader to Ref. [24]. It simplifies considerably for the following cases:

$$\hat{C}_0 [0, 0, p^2; m, 0, 0] = \frac{-1}{(4\pi)^2} \frac{1}{p^2} \left[ \ln \left( \frac{p^2}{m^2} \right) \ln \left( 1 + \frac{p^2}{m^2} \right) + \text{Li}_2 \left( -\frac{p^2}{m^2} \right) \right] \quad (\text{A10})$$

$$\hat{C}_0 [0, 0, p^2; 0, m_1, m_2] = \frac{-1}{(4\pi)^2} \frac{1}{4p^2} \left[ \ln^2 \left( \frac{1 - \Sigma + \sqrt{1 - 2\Sigma + \Delta^2}}{1 - \Sigma - \sqrt{1 - 2\Sigma + \Delta^2}} \right) - \ln^2 \left( \frac{m_1^2}{m_2^2} \right) \right] \quad (\text{A11})$$

where  $\Sigma$  and  $\Delta$  are defined as in Eq. (A4).

The following  $\hat{C}_0$  could be expressed in terms a sum of dilogarithms, but for our purposes it is simpler to reduce them to a Feynman parameter integral and either perform the integration numerically or, if an expansion is needed, to expand the integrand directly and then integrate.

$$\hat{C}_0 [0, 0, p^2; m, M, 0] = \frac{1}{(4\pi)^2} \int_0^1 dx \frac{1}{p^2(1-x) + m^2} \ln \left[ \frac{(m^2 - M^2)x + M^2}{(1-x)(M^2 - xp^2)} \right] \quad (\text{A12})$$

$$\hat{C}_0 [0, 0, p^2; m, M, M] = \frac{1}{(4\pi)^2} \int_0^1 dx \frac{1}{p^2(1-x) + (m^2 - M^2)} \ln \left[ \frac{(m^2 - M^2)x + M^2}{-p^2x(1-x) + M^2} \right] \quad (\text{A13})$$

## 2. Tensor Integrals

Definition and general form of  $B_1$ :

$$B_\mu [p; m_1, m_2] = i\mu^{4-d} \int \frac{d^d k}{(2\pi)^d} \frac{k_\mu}{(k^2 - m_1^2) [(k+p)^2 - m_2^2]} \equiv p_\mu B_1 [p^2; m_1, m_2] \quad (\text{A14})$$

$$B_1 [p^2; m_1, m_2] = -\frac{1}{2} B_0 [p^2; m_1, m_2] + \frac{1}{(4\pi)^2} \left( \frac{m_1^2 - m_2^2}{2p^2} \right) F(p^2; m_1, m_2) \quad (\text{A15})$$

From Eq. (A5), we find that in the limit  $p^2 \rightarrow 0$ :

$$B_1 [0; m_1, m_2] = -\frac{1}{2} B_0 [0; m_1, m_2] + \frac{1}{(4\pi)^2} \frac{1}{4(m_1^2 - m_2^2)} \left[ m_1^2 + m_2^2 - \frac{2 m_1^2 m_2^2}{m_1^2 - m_2^2} \ln \left( \frac{m_1^2}{m_2^2} \right) \right] \quad (\text{A16})$$

Other special cases:

$$B_1 [p^2; 0, m] = \frac{1}{(4\pi)^2} \frac{1}{2} \left[ \Delta_\epsilon - \ln \left( \frac{m^2}{\mu^2} \right) + 2 - \frac{m^2}{p^2} - \left( 1 - \frac{m^2}{p^2} \right)^2 \ln \left( 1 - \frac{p^2}{m^2} \right) \right] \quad (\text{A17})$$

$$\xrightarrow{p^2 \rightarrow 0} \frac{1}{(4\pi)^2} \frac{1}{2} \left[ \Delta_\epsilon - \ln \left( \frac{m^2}{\mu^2} \right) + \frac{1}{2} \right] \quad (\text{A18})$$

Useful relations among the  $B$ -functions:

$$0 = B_0 [p^2; m_1, m_2] + B_1 [p^2; m_1, m_2] + B_1 [p^2; m_2, m_1], \quad (\text{A19})$$

$$0 = (m_1^2 - m_2^2) B_0 [0; m_1, m_2] + (m_2^2 - m_3^2) B_0 [0; m_2, m_3] + (m_3^2 - m_1^2) B_0 [0; m_3, m_1]. \quad (\text{A20})$$

Definition of the  $C$ -functions: (Note the difference from the definitions in Ref. [24].)

$$\begin{aligned} C_\mu [p, q; m_1, m_2, m_3] &= i \int \frac{d^4 k}{(2\pi)^4} \frac{k_\mu}{(k^2 - m_1^2) [(k+p)^2 - m_2^2] [(k+q)^2 - m_3^2]} \\ &\equiv p_\mu \hat{C}_{11} + q_\mu \hat{C}_{12} \end{aligned} \quad (\text{A21})$$

$$\begin{aligned} C_{\mu\nu} [p, q; m_1, m_2, m_3] &= i \mu^{4-d} \int \frac{d^d k}{(2\pi)^d} \frac{k_\mu k_\nu}{(k^2 - m_1^2) [(k+p)^2 - m_2^2] [(k+q)^2 - m_3^2]} \\ &\equiv p_\mu p_\nu \hat{C}_{21} + q_\mu q_\nu \hat{C}_{22} + (p_\mu q_\nu + q_\mu p_\nu) \hat{C}_{23} + g_{\mu\nu} \hat{C}_{24} \end{aligned}$$

For the purpose of this paper, we will only need to evaluate these functions for  $p^2 = q^2 = 0$  (we neglect final state fermion masses).  $Q^2 = (p - q)^2 = -2p \cdot q$  will then be the invariant mass squared of the initial vector boson<sup>4</sup>. For this parameter choice, the  $C$ -functions can be expressed in terms of the  $B$ -functions and  $\hat{C}_0$  as:

$$\hat{C}_{11} = -\frac{1}{Q^2} \left\{ B_0 [0; m_1, m_2] - B_0 [Q^2; m_2, m_3] - (m_1^2 - m_3^2) \hat{C}_0 \right\} \quad (\text{A22})$$

$$\hat{C}_{12} = -\frac{1}{Q^2} \left\{ B_0 [0; m_1, m_3] - B_0 [Q^2; m_2, m_3] - (m_1^2 - m_2^2) \hat{C}_0 \right\} \quad (\text{A23})$$

$$(d-2) \hat{C}_{24} = -B_1 [Q^2; m_2, m_3] + (m_1^2 - m_2^2) \hat{C}_{11} + m_1^2 \hat{C}_0 \quad (\text{A24})$$

$$\begin{aligned} -Q^2 \hat{C}_{23} + 2 \hat{C}_{24} &= -B_1 [Q^2; m_2, m_3] - (m_1^2 - m_3^2) \hat{C}_{12} \\ &= -B_1 [Q^2; m_3, m_2] - (m_1^2 - m_2^2) \hat{C}_{11} \end{aligned} \quad (\text{A25})$$

We do not list expressions for  $\hat{C}_{21}$  nor  $\hat{C}_{22}$  since we do not use them in this paper.

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<sup>4</sup> We caution the reader that  $p$  and  $q$  defined here are different from those appearing in the figures.

TABLES

Observable	Measured Value	ZFITTER Prediction
<u>Z lineshape variables</u>		
$m_Z$	$91.1872 \pm 0.0021$ GeV	input
$\Gamma_Z$	$2.4944 \pm 0.0024$ GeV	unused
$\sigma_{\text{had}}^0$	$41.544 \pm 0.037$ nb	unused
$R_e$	$20.803 \pm 0.049$	20.739
$R_\mu$	$20.786 \pm 0.033$	20.739
$R_\tau$	$20.764 \pm 0.045$	20.786
$A_{\text{FB}}(e)$	$0.0145 \pm 0.0024$	0.0152
$A_{\text{FB}}(\mu)$	$0.0167 \pm 0.0013$	0.0152
$A_{\text{FB}}(\tau)$	$0.0188 \pm 0.0017$	0.0152
<u><math>\tau</math> polarization at LEP</u>		
$A_e$	$0.1483 \pm 0.0051$	0.1423
$A_\tau$	$0.1424 \pm 0.0044$	0.1424
<u>SLD left-right asymmetries</u>		
$A_{LR}$	$0.15108 \pm 0.00218$	0.1423
$A_e$	$0.1558 \pm 0.0064$	0.1423
$A_\mu$	$0.137 \pm 0.016$	0.1423
$A_\tau$	$0.142 \pm 0.016$	0.1424

TABLE I. LEP/SLD observables and their Standard Model predictions. The  $Z$  lineshape observables are from Ref. [16]. The rest of the data is from Ref. [17] and [18]. The Standard Model predictions were calculated using ZFITTER v.6.21 [19] with  $m_t = 174.3$  GeV [20],  $m_H = 300$  GeV, and  $\alpha_s(m_Z) = 0.120$  as input.

	$m_Z$	$\Gamma_Z$	$\sigma_{\text{had}}^0$	$R_e$	$R_\mu$	$R_\tau$	$A_{\text{FB}}(e)$	$A_{\text{FB}}(\mu)$	$A_{\text{FB}}(\tau)$
$m_Z$	1.000	-0.008	-0.050	0.073	0.001	0.002	-0.015	0.046	0.034
$\Gamma_Z$		1.000	-0.284	-0.006	0.008	0.000	-0.002	0.002	-0.003
$\sigma_{\text{had}}^0$			1.000	0.109	0.137	0.100	0.008	0.001	0.007
$R_e$				1.000	0.070	0.044	-0.356	0.023	0.016
$R_\mu$					1.000	0.072	0.005	0.006	0.004
$R_\tau$						1.000	0.003	-0.003	0.010
$A_{\text{FB}}(e)$							1.000	-0.026	-0.020
$A_{\text{FB}}(\mu)$								1.000	0.045
$A_{\text{FB}}(\tau)$									1.000

TABLE II. The correlation of the  $Z$  lineshape variables at LEP

	$\delta_{\mu e}$	$\delta_{\tau e}$	$\Delta_A$	$\Delta_R$
$\delta_{\mu e}$	1.00	0.53	0.22	-0.76
$\delta_{\tau e}$		1.00	0.28	-0.63
$\Delta_A$			1.00	-0.23
$\Delta_R$				1.00

TABLE III. The correlation matrix of the fit parameters using all data.

	$\delta_{\mu e}$	$\delta_{\tau e}$	$\Delta_A$	$\Delta_R$
$\delta_{\mu e}$	1.00	0.60	0.32	-0.79
$\delta_{\tau e}$		1.00	0.29	-0.69
$\Delta_A$			1.00	-0.33
$\Delta_R$				1.00

TABLE IV. The correlation matrix of the fit parameters using the  $Z$  line-shape data only.

	$\delta_{\mu e}$	$\delta_{\tau e}$	$\Delta_A$
$\delta_{\mu e}$	1.00	0.05	0.12
$\delta_{\tau e}$		1.00	0.41
$\Delta_A$			1.00

TABLE V. The correlation matrix of the fit parameters using the LEP  $\tau$ -polarization and SLD leptonic asymmetries only.

FIGURES

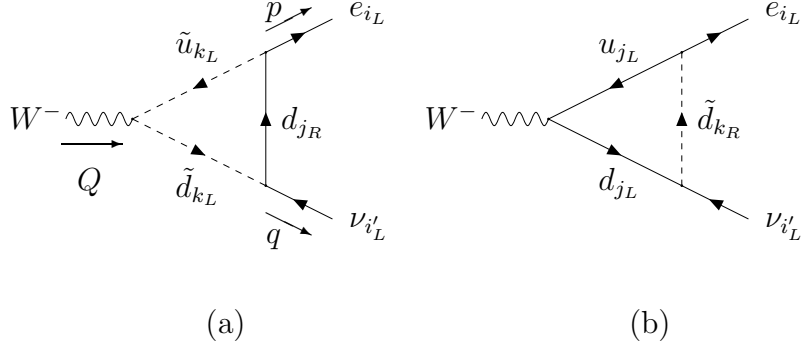


FIG. 1. Vertex corrections to  $W^- \rightarrow e_{iL} \bar{\nu}'_{iL}$  from R-parity violating interactions.

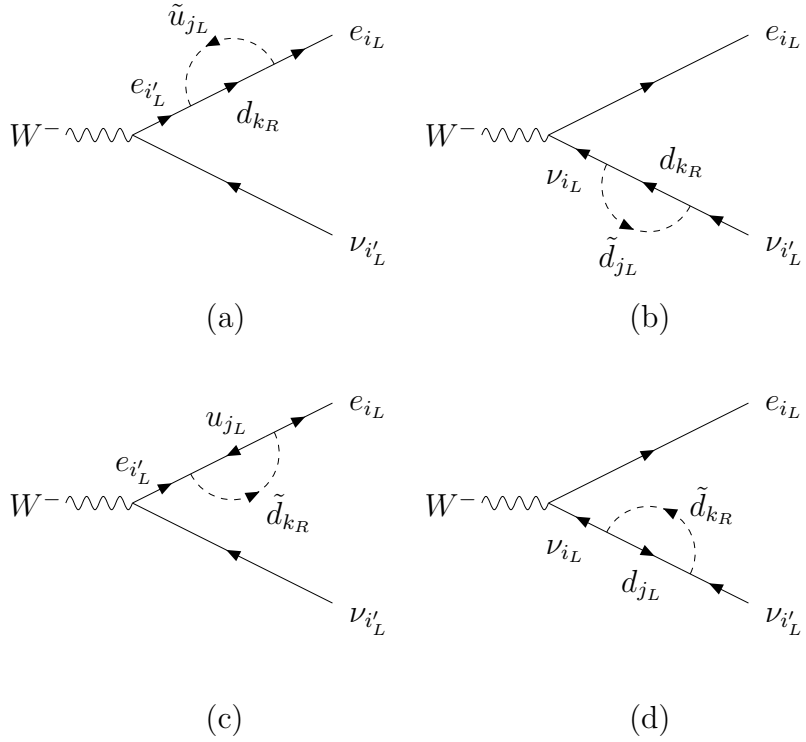


FIG. 2. Wavefunction renormalization corrections to  $W^- \rightarrow e_{iL} \bar{\nu}'_{iL}$  from R-parity violating interactions.

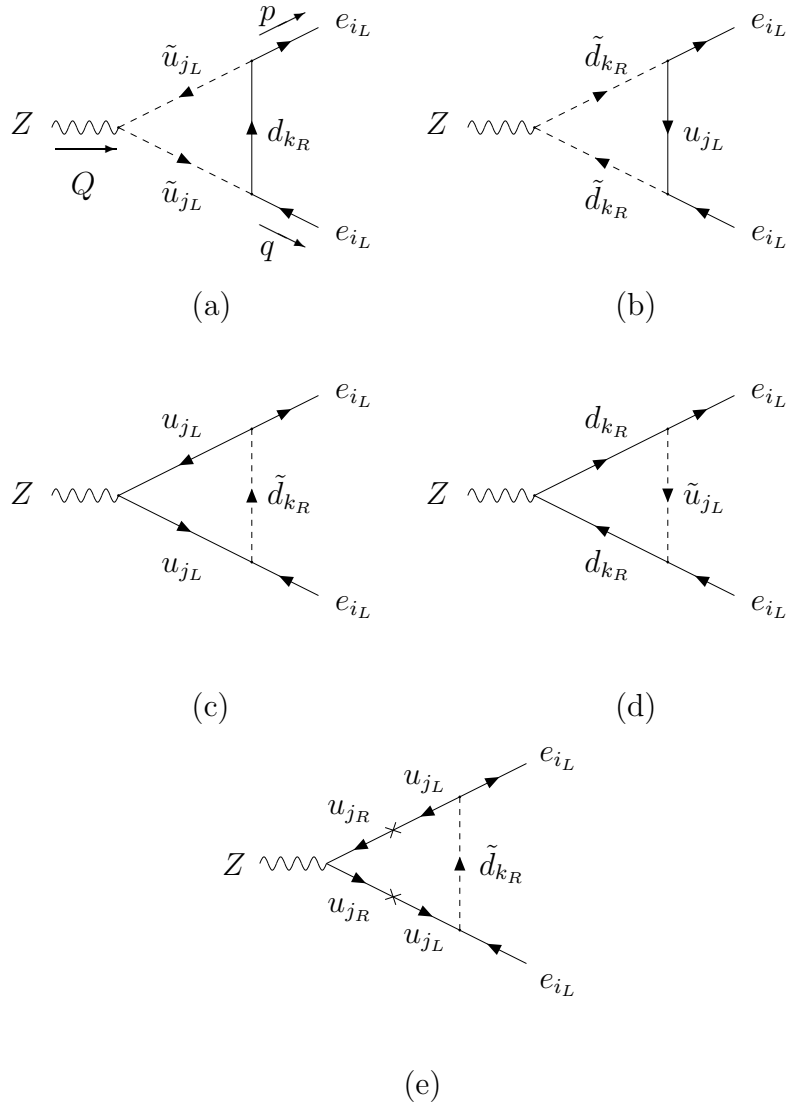


FIG. 3. Vertex corrections to  $Z \rightarrow e_{iL} \bar{e}_{iL}$  from R-parity violating interactions.



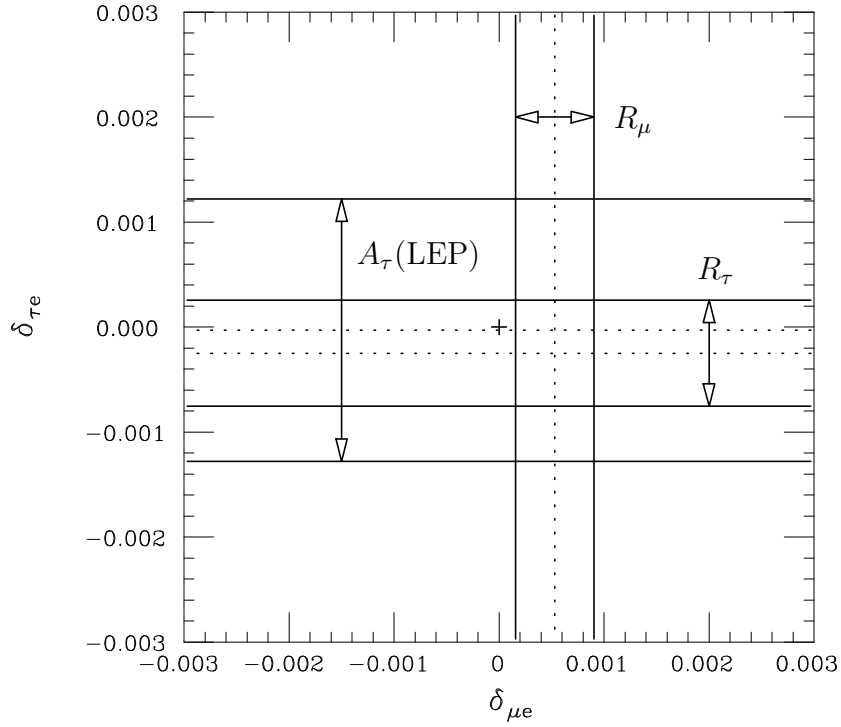


FIG. 5.  $1\sigma$  constraints on lepton universality violation.

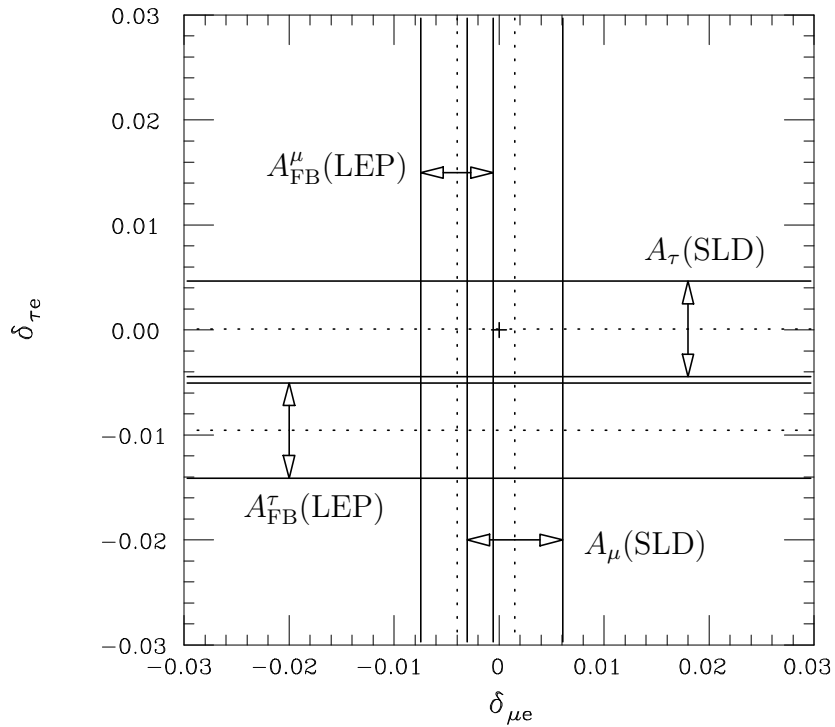


FIG. 6.  $1\sigma$  constraints on lepton universality violation. Note the larger scale with respect to the previous figure.



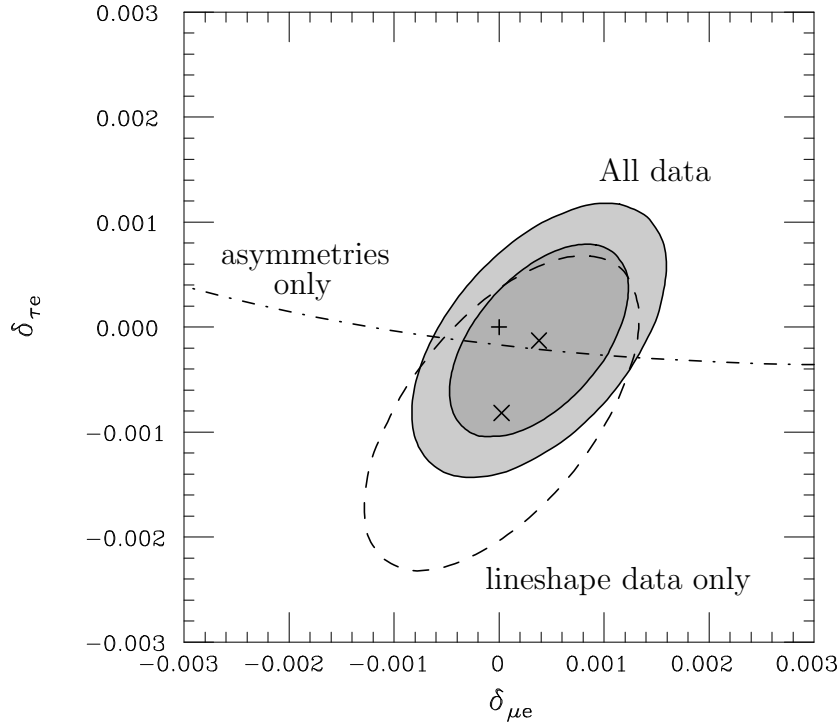


FIG. 7. Confidence contours from all data (68% and 90%, in gray), the  $Z$  line–shape data only (90%, dashed line), and the asymmetry data only (90%, dot-dashed line).

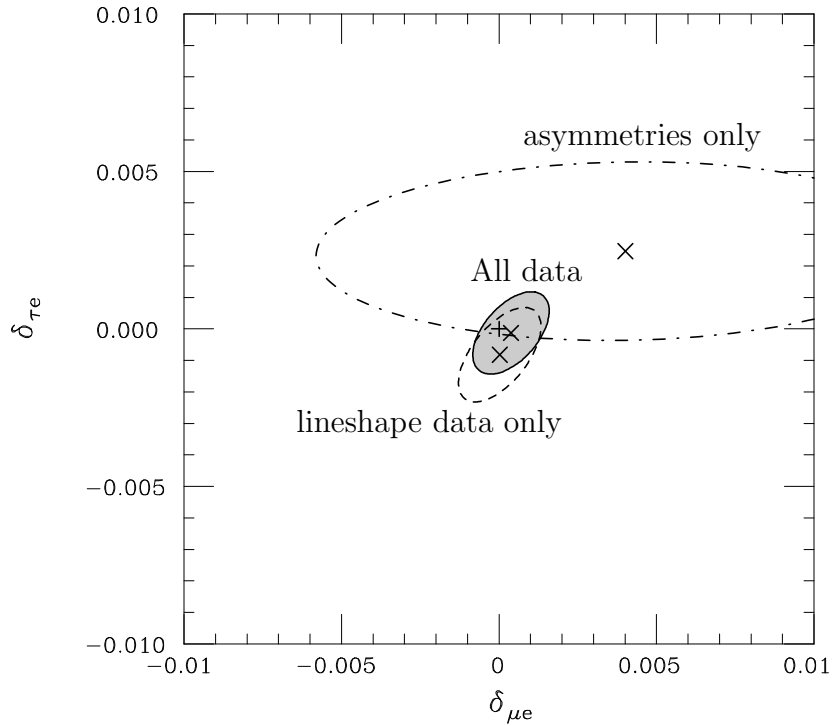


FIG. 8. 90% confidence contours from all data (gray),  $Z$  line–shape data only (dashed line), and asymmetry data only (dot-dashed line). Note the different scale with respect to the other figures.

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