

Constraints on Gauged $B - 3L_\tau$ and Related Theories

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Abstract

We consider extensions of the Standard Model with an extra $U(1)$ gauge boson which couples to $B - (\alpha L_e + \beta L_\mu + \gamma L_\tau)$ with $\alpha + \beta + \gamma = 3$. We show that the extra gauge boson necessarily mixes with the Z , leading to potentially significant corrections to the $Zf\bar{f}$ vertex. The constraints on the size of this correction imposed by the Z -pole data from LEP and SLD are derived.

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I. INTRODUCTION

A persistent mystery in particle physics today is how nature distinguishes among the three generations of quarks and leptons and provides them with the observed mass hierarchy and mixings. A popular approach in constructing a model which can potentially explain this flavor problem is to extend the gauge symmetry of the Standard Model (SM) and permit the three generations to transform differently under the new symmetry. This idea is implemented, for instance, in topcolor [1] and topcolor assisted technicolor [2] models in which the third generation transforms differently from the first two.

In extending the SM gauge group and assigning charges to the matter fields, care is needed to ensure anomaly cancellation. However, extra care is necessary to further ensure *charge orthogonality* when the extended gauge group contains multiple abelian factor groups [3]. Without charge orthogonality the abelian charges will mix kinetically [4] under renormalization group running and the charge assignments lose scale-invariant meaning, rendering the model ill-defined.

In this paper, we examine these issues in the context of a series of models introduced in Ref. [5]. In these models, the SM gauge group is extended by an abelian factor to $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$, where the extra $U(1)_X$ gauge boson is coupled to some linear combination of baryon and lepton flavor numbers¹ which is anomaly free, *e.g.*

$$X = B - 3L_\tau, \quad B - 3L_e, \quad B - \frac{3}{2}(L_\mu + L_\tau), \quad \text{etc.}$$

These models were motivated by the desire to explain the masses and mixing in the neutrino sector. Unfortunately, the particle content of these models does not satisfy the charge orthogonality condition (COC). Consequently, the X and Y charges mix under renormalization group running and the model remains incomplete in the absence of a scale at which the charges are defined. While one can always assume that the charges are those at the scale at which the $U(1)_X$ symmetry breaks, thereby locking in the charges, at higher energy scales the two $U(1)$ gauge bosons will couple to some scale-dependent linear combination of X and Y charges. To avoid this one must go to the physical basis, as discussed in Ref. [3], in which the charges are orthogonal and scale invariant. However, the new scale invariant charges can no longer be associated with the SM weak hypercharge or any particular lepton flavor, and in general they will not even be rational. It is therefore necessary that the COC be imposed as an additional constraint if the initial charge assignment is to make any sense.

In the following, we show below that the COC cannot be satisfied in this class of models for *any* linear combination of B and $L_{e,\mu,\tau}$ which is anomaly-free without the addition of extra matter fields. Even when the COC is satisfied there can still be significant mixing between the Z and the X , and this can show up in Z -pole observables by breaking lepton universality. We use the LEP and SLD Z -pole data to place significant constraints on the size of this mixing. We find that, in the absence of additional sources of mixing, the mass of the X gauge boson is generically required to be at least a few hundred GeV.

¹A model in which only the baryon number is gauged was considered in Ref. [6].

II. ANOMALY CANCELLATION AND CHARGE ORTHOGONALITY

Consider a model with the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$ where $U(1)_Y$ is the putative weak hypercharge and $U(1)_X$ is an additional abelian factor group. We choose the charge assignments of the quarks and leptons to be

$$\begin{aligned} \begin{pmatrix} u_i \\ d_i \end{pmatrix}_L &\sim \left(3, 2, \frac{1}{6}; \frac{1}{3}\right), & u_{iR} &\sim \left(3, 1, \frac{2}{3}; \frac{1}{3}\right), & d_{iR} &\sim \left(3, 1, -\frac{1}{3}; \frac{1}{3}\right), \\ \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L &\sim \left(1, 2, -\frac{1}{2}; -\alpha\right), & e_R &\sim (1, 1, -1; -\alpha), & \nu_{eR} &\sim (1, 1, 0; -\alpha), \\ \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L &\sim \left(1, 2, -\frac{1}{2}; -\beta\right), & \mu_R &\sim (1, 1, -1; -\beta), & \nu_{\mu R} &\sim (1, 1, 0; -\beta), \\ \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L &\sim \left(1, 2, -\frac{1}{2}; -\gamma\right), & \tau_R &\sim (1, 1, -1; -\gamma), & \nu_{\tau R} &\sim (1, 1, 0; -\gamma), \end{aligned}$$

where $i = 1, 2, 3$ is the generation index, and α, β, γ are left arbitrary for the moment. In effect, the $U(1)_X$ gauge boson is chosen to couple to

$$X = B - (\alpha L_e + \beta L_\mu + \gamma L_\tau).$$

The addition of the right-handed neutrinos is necessary to make $U(1)_X$ a vectorial symmetry. Note that even though we list three right-handed neutrinos, any one whose $U(1)_X$ charge is chosen to be zero will effectively decouple completely from the theory. The minimal scalar sector necessary to break the gauge symmetry into the usual $SU(3)_C \times U(1)_{\text{em}}$ consists of the regular Higgs doublet

$$\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \sim \left(1, 2, \frac{1}{2}; 0\right)$$

and a neutral singlet

$$\chi^0 \sim (1, 1, 0, \delta), \quad \delta \neq 0,$$

to break $U(1)_X$. It is easy to show that anomaly cancellation leads to the condition

$$\alpha + \beta + \gamma = 3. \tag{1}$$

In non-GUT models with multiple gauged $U(1)$'s an additional constraint is necessary to ensure that these groups do not mix through radiative corrections. As has been discussed in Ref. [3], this requirement (at one loop) amounts to

$$\text{Tr}[Q_X Q_Y] = 0,$$

the charge orthogonality condition. In the model under consideration, the COC leads to the condition

$$\alpha + \beta + \gamma = -1.$$

Obviously, this and the anomaly cancellation condition Eq. (1) cannot be satisfied simultaneously. Therefore, the COC cannot be imposed regardless of the choice of α , β , and γ , and the model in its present form is ill-defined.

One way to rectify this problem is to change the charge assignments in the minimal scalar sector so that the kinetic mixing due to the scalars cancels that due to the fermions. However, this cannot be done so easily since the scalar charges are fixed by the requirement that they lead to the correct pattern of symmetry breaking, and also allow for the necessary Yukawa couplings to give masses to the fermions. Instead one may introduce new matter fields. The set of new matter fields that are necessary to impose the COC is not unique. For instance, the COC can be imposed upon the fermion sector by an introduction of a pair of fermions with charge assignments given by

$$\begin{aligned} N_L &\sim (1, 1, a, b), \\ N_R &\sim (1, 1, a, b), \end{aligned}$$

with $ab = -4$. If the scalar sector is extended so that neutrino mixing can be generated, the COC must be satisfied there also. Therefore, the complete phenomenology of the model cannot be worked out until all the extra fields have been specified. One can nevertheless place a constraint on these models, under a minimal set of assumptions, as we will discuss next.

III. VERTEX CORRECTIONS

Even with the COC initially imposed at high energies, the X and Y charges will mix once some of the particles decouple from renormalization group running. This mixing can be fairly large at the Z mass scale since it will be proportional to $\ln(\Lambda/m_Z)$, where Λ is the scale at which decoupling occurs. However, X - Y mixing will lead to the violation of lepton universality on the Z -pole which is well constrained by LEP and SLD data. In the following, we make the simplifying assumption that all the non-SM particles decouple at $M_X \gg m_Z$, the scale at which $U(1)_X$ breaks. Below M_X , only the SM particles survive decoupling and contribute to Z - X mixing. Comparison of the size of this mixing with the data will enable us to constrain M_X .

Consider possible corrections to the $Zf\bar{f}$ vertex at the Z -pole. There are two ways in which the X boson can correct the vertex. The first is by dressing the vertex as shown in Fig. 1(a). This correction leads to a shift in the $Zf\bar{f}$ couplings given by:

$$\frac{\delta h_{fL}}{h_{fL}} = \frac{\delta h_{fR}}{h_{fR}} = \frac{\alpha_X}{6\pi} (X_f^2) \left(\frac{m_Z^2}{M_X^2} \ln \frac{M_X^2}{m_Z^2} \right),$$

where X_f is the X -charge of fermion f , and $\alpha_X = g_X^2/4\pi$. The second is through Z - X mixing as shown in Fig. 1b. Since charge orthogonality is broken below M_X this correction does not vanish and is given by

$$\delta h_{f_L} = \delta h_{f_R} = -\frac{\alpha_X}{6\pi}(8s^2 X_f) \frac{m_Z^2}{M_X^2} \left[\ln \frac{M_X^2}{m_Z^2} + \frac{\left(\frac{1}{2} - \frac{4}{3}s^2\right)}{8s^2} \ln \frac{m_t^2}{m_Z^2} \right] \approx -\frac{\alpha_X}{6\pi}(8s^2 X_f) \left(\frac{m_Z^2}{M_X^2} \ln \frac{M_X^2}{m_Z^2} \right), \quad (2)$$

where s^2 is shorthand for $\sin^2 \theta_W$, and the tree level $Zf\bar{f}$ couplings are normalized as

$$h_{f_L} = I_{3f} - Q_f s^2, \quad h_{f_R} = -Q_f s^2.$$

The top mass dependent term in Eq. (2) is due to the decoupling of the top quark below m_t^2 , but we will neglect it since it is suppressed compared to the other term. Note that this correction is proportional to s^2 since it is only the $U(1)_Y$ part of the Z which mixes with the X .

Let us define

$$\xi \equiv \frac{\alpha_X}{6\pi} \left(\frac{m_Z^2}{M_X^2} \ln \frac{M_X^2}{m_Z^2} \right).$$

Then the sum of the dressing and mixing corrections can be written as

$$\begin{aligned} \delta h_{f_L} &= (h_{f_L} X_f^2 - 8s^2 X_f) \xi, \\ \delta h_{f_R} &= (h_{f_R} X_f^2 - 8s^2 X_f) \xi. \end{aligned}$$

To place a constraint on the size of ξ using the data from precision electroweak measurements, we follow the general procedure of Ref. [7]. We assume that the only significant non-SM vertex correction comes from ξ . Since we will only use LEP and SLD observables which are *ratios of coupling constants* in our analysis, oblique corrections will only manifest themselves through a shift in the the effective value of $\sin^2 \theta_W$ [8]. We introduce the parameter δs^2 to account for this deviation:

$$\sin^2 \theta_W = [\sin^2 \theta_W]_{\text{SM}} + \delta s^2.$$

We use δs^2 only as a fit parameter and extract no information from it so that our results are independent of the Higgs mass. The shifts in the left and right handed couplings are then

$$\begin{aligned} \delta h_{f_L} &= -Q_f \delta s^2 + (h_{f_L} X_f^2 - 8s^2 X_f) \xi, \\ \delta h_{f_R} &= -Q_f \delta s^2 + (h_{f_R} X_f^2 - 8s^2 X_f) \xi. \end{aligned}$$

IV. CONSTRAINTS FROM PRECISION ELECTROWEAK MEASUREMENTS

The shifts in the LEP/SLD observables due to the shifts in the coupling constants are easily calculable. For instance, the shift in the partial decay width $Z \rightarrow f\bar{f}$ is given by

$$\frac{\delta \Gamma_f}{\Gamma_f} = \frac{2h_{f_L} \delta h_{f_L} + 2h_{f_R} \delta h_{f_R}}{h_{f_L}^2 + h_{f_R}^2}$$

$$\begin{aligned}
&= -\frac{2(h_{f_L} + h_{f_R})}{h_{f_L}^2 + h_{f_R}^2} Q_f \delta s^2 \\
&\quad + \left[2X_f^2 - 8s^2 X_f \frac{2(h_{f_L} + h_{f_R})}{h_{f_L}^2 + h_{f_R}^2} \right] \xi.
\end{aligned}$$

Similarly, the shift in the parity violating asymmetry $A_f = (h_{f_L}^2 - h_{f_R}^2)/(h_{f_L}^2 + h_{f_R}^2)$ is given by

$$\begin{aligned}
\frac{\delta A_f}{A_f} &= \frac{4h_{f_L} h_{f_R}}{(h_{f_L}^4 - h_{f_R}^4)} (h_{f_R} \delta h_{f_L} - h_{f_L} \delta h_{f_R}) \\
&= \frac{4h_{f_L} h_{f_R}}{(h_{f_L}^2 + h_{f_R}^2)(h_{f_L} + h_{f_R})} [Q_f \delta s^2 + 8s^2 X_f \xi]
\end{aligned}$$

Note that the X boson dressing correction which is proportional to X_f^2 vanishes in $\delta A_f/A_f$.

The δs^2 and ξ dependence of the observables we use in our fit are as follows:

$$\begin{aligned}
\frac{\delta A_e}{A_e} &= -53.5 \delta s^2 - 99.0 \alpha \xi \\
\frac{\delta A_\mu}{A_\mu} &= -53.5 \delta s^2 - 99.0 \beta \xi \\
\frac{\delta A_\tau}{A_\tau} &= -53.5 \delta s^2 - 99.0 \gamma \xi \\
\frac{\delta A_{\text{FB}}(e)}{A_{\text{FB}}(e)} &= 2 \frac{\delta A_e}{A_e} = -107 \delta s^2 - 198 \alpha \xi \\
\frac{\delta A_{\text{FB}}(\mu)}{A_{\text{FB}}(\mu)} &= \frac{\delta A_e}{A_e} + \frac{\delta A_\mu}{A_\mu} = -107 \delta s^2 - 99.0 (\alpha + \beta) \xi \\
\frac{\delta A_{\text{FB}}(\tau)}{A_{\text{FB}}(\tau)} &= \frac{\delta A_e}{A_e} + \frac{\delta A_\tau}{A_\tau} = -107 \delta s^2 - 99.0 (\alpha + \gamma) \xi \\
\frac{\delta R_e}{R_e} &= -0.840 \delta s^2 + (1.18 + 1.09 \alpha - 2 \alpha^2) \xi + 0.307 \delta \alpha_s \\
\frac{\delta R_\mu}{R_\mu} &= -0.840 \delta s^2 + (1.18 + 1.09 \beta - 2 \beta^2) \xi + 0.307 \delta \alpha_s \\
\frac{\delta R_\tau}{R_\tau} &= -0.840 \delta s^2 + (1.18 + 1.09 \gamma - 2 \gamma^2) \xi + 0.307 \delta \alpha_s \\
\frac{\delta \sigma_{\text{had}}^0}{\sigma_{\text{had}}^0} &= 0.099 \delta s^2 \\
&\quad + \left[(1.599 \alpha^2 - 2.006 \alpha) - (0.401 \beta^2 + 0.916 \beta) - (0.401 \gamma^2 + 0.916 \gamma) - 0.471 \right] \xi \\
&\quad - 0.122 \delta \alpha_s \\
\frac{\delta R_b}{R_b} &= 0.182 \delta s^2 + 1.35 \xi \\
\frac{\delta R_c}{R_c} &= -0.351 \delta s^2 - 2.61 \xi \\
\frac{\delta A_{\text{FB}}(b)}{A_{\text{FB}}(b)} &= \frac{\delta A_b}{A_b} + \frac{\delta A_e}{A_e} = -54.1 \delta s^2 + (1.26 - 99.0 \alpha) \xi \\
\frac{\delta A_{\text{FB}}(c)}{A_{\text{FB}}(c)} &= \frac{\delta A_c}{A_c} + \frac{\delta A_e}{A_e} = -58.7 \delta s^2 - (4.80 + 99.0 \alpha) \xi
\end{aligned}$$

$$\begin{aligned}
\frac{\delta A_b}{A_b} &= -0.681 \delta s^2 + 1.26 \xi \\
\frac{\delta A_c}{A_c} &= -5.19 \delta s^2 - 4.80 \xi
\end{aligned}
\tag{3}$$

We have assumed that the right-handed neutrinos are heavy and only the left-handed ones contribute to the invisible width of the Z . The parameter $\delta\alpha_s$ gives the shift of the QCD coupling constant $\alpha_s(m_Z)$ from its nominal value of 0.120:

$$\alpha_s(m_Z) = 0.120 + \delta\alpha_s.$$

Note that the correction from Z - X mixing to the leptonic asymmetry parameters A_ℓ ($\ell = e, \mu, \tau$) appears with a large coefficient in Eq. (3). This means that lepton universality imposes a strong constraint on ξ .

Here, we list the results of fitting the expressions in Eq. (3) to the data listed in Table I [9] for two choices of α, β , and γ which were considered in Ref. [5]. In both cases $\alpha = 0$, so we do not need to consider interference between direct Z and X exchange. The correlations among the data used are shown in Tables II and III.

(i) $\alpha = \beta = 0, \gamma = 3$ case:

$$\begin{aligned}
\delta s^2 &= -0.00067 \pm 0.00019 \\
\xi &= 0.000015 \pm 0.000074 \\
\delta\alpha_s &= -0.0016 \pm 0.0032
\end{aligned}$$

The correlation among the fit variables are shown in Table IV while the constraints from various observables in the δs^2 - ξ plane are shown in Fig. 2. The quality of the fit was $\chi^2 = 25.6/(19 - 3)$ with the largest contributions coming from $A_{\text{FB}}(b)$ (5.3) and σ_{had}^0 (3.6). With such a large χ^2 , it is evident that including the X -corrections do not improve the agreement between theory and experiment.

To convert the limit on ξ into a limit on M_X , we must assume a value for g_X . For $g_X = g \approx 0.65$, the 1σ (2σ) upper bound on ξ translates into:

$$M_X \geq 860 \text{ (580) GeV.}$$

For $g_X = g' \approx 0.35$, the 1σ (2σ) bound is

$$M_X \geq 370 \text{ (220) GeV.}$$

Interestingly enough, these bounds agree with that derived in Ref. [5] which neglected both Z - X mixing and oblique corrections.

(ii) $\alpha = 0, \beta = \gamma = 1.5$ case:

$$\begin{aligned}
\delta s^2 &= -0.00063 \pm 0.00019 \\
\xi &= -0.00008 \pm 0.00014 \\
\delta\alpha_s &= -0.0007 \pm 0.0034
\end{aligned}$$

The correlation among the fit variables are shown in Table V while the constraints from various observables in the δs^2 - ξ plane are shown in Fig. 3. The quality of the fit was $\chi^2 = 25.3/(19 - 3)$ with the largest contributions coming from $A_{\text{FB}}(b)$ (4.7) and A_{LR} (3.3). For $g_X = g \approx 0.65$, the 1σ (2σ) upper bound on ξ translates into:

$$M_X \geq 1100 \text{ (500) GeV.}$$

For $g_X = g' \approx 0.35$, the 1σ bound is

$$M_X \geq 490 \text{ GeV.}$$

The 2σ limit on ξ does not lead to a constraint on M_X in this case since

$$\frac{m_Z^2}{M_X^2} \ln \frac{M_X^2}{m_Z^2} \leq \frac{1}{e},$$

with the maximum at

$$\frac{m_Z^2}{M_X^2} = \frac{1}{e}.$$

Since our analysis assumes $m_Z^2 \ll M_X^2$, the approximation breaks down in the environs of this scale anyway, invalidating any limits we may obtain.

The limits for other choices of α , β , and γ are similar. In Figs. 4 and 5, we plot the bounds on ξ and M_X for the $\alpha = 0$ models as functions of $\beta = 3 - \gamma$. As we can see, M_X is generally required to be of the order of a few hundred GeV.

V. DISCUSSION AND CONCLUSIONS

In this paper we have considered a class of models with an abelian factor group of type $B - (\alpha L_e + \beta L_\mu + \gamma L_\tau)$ and have explored some of their phenomenological consequences. In all cases considered, the quality of the fit to Z -pole electroweak observables is not improved over that of the Standard Model. The new physics parameter ξ violates lepton universality and is strongly constrained by the leptonic observables. The introduction of this parameter into the fit does not reconcile the experimental values of A_{LR} and $A_{\text{FB}}(b)$. Thus the model does not provide a solution to the A_b anomaly. We find that, under the assumption that the X boson is heavier than the Z , the Z -pole observables require that the mass of the extra gauge boson be of order a few hundred GeV.

In this analysis we have considered only kinetic mixing of the X and Z bosons due to SM particles between the scales M_X and m_Z , but more complicated versions of this model are possible. For variants in which non-SM particles charged under the $U(1)$'s decouple above M_X and cause the COC to be violated, the mixing will occur over a larger momentum range. If the scalar sector of the model includes fields which transform non-trivially under both $U(1)$'s, then their acquiring VEV's can lead to mass mixing between the X and the Z [5]. These additional sources of mixing may either dilute or sharpen the constraints obtained here, but must be considered on a model by model basis since they depend critically on the specifics of each model.

ACKNOWLEDGEMENTS

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TABLES

Observable	Measured Value	ZFITTER Prediction
<u>Z lineshape variables</u>		
m_Z	91.1872 ± 0.0021 GeV	input
Γ_Z	2.4944 ± 0.0024 GeV	unused
σ_{had}^0	41.544 ± 0.037 nb	41.474 nb
R_e	20.803 ± 0.049	20.739
R_μ	20.786 ± 0.033	20.739
R_τ	20.764 ± 0.045	20.786
$A_{\text{FB}}(e)$	0.0145 ± 0.0024	0.0152
$A_{\text{FB}}(\mu)$	0.0167 ± 0.0013	0.0152
$A_{\text{FB}}(\tau)$	0.0188 ± 0.0017	0.0152
<u>τ polarization at LEP</u>		
A_e	0.1483 ± 0.0051	0.1423
A_τ	0.1425 ± 0.0044	0.1424
<u>SLD left–right asymmetries</u>		
A_{LR}	0.15108 ± 0.00218	0.1423
A_e	0.1558 ± 0.0064	0.1423
A_μ	0.137 ± 0.016	0.1423
A_τ	0.142 ± 0.016	0.1424
<u>heavy quark flavor</u>		
R_b	0.21642 ± 0.00073	0.21583
R_c	0.1674 ± 0.0038	0.1722
$A_{\text{FB}}(b)$	0.0988 ± 0.0020	0.0997
$A_{\text{FB}}(c)$	0.0692 ± 0.0037	0.0711
A_b	0.911 ± 0.025	0.934
A_c	0.630 ± 0.026	0.666

TABLE I. LEP/SLD observables and their Standard Model predictions. All data is from Ref. [9]. The Standard Model predictions were calculated using ZFITTER v.6.21 [10] with default flag settings and $m_t = 174.3$ GeV [11], $m_H = 300$ GeV, and $\alpha_s(m_Z) = 0.120$ as input.

	m_Z	Γ_Z	σ_{had}^0	R_e	R_μ	R_τ	$A_{\text{FB}}(e)$	$A_{\text{FB}}(\mu)$	$A_{\text{FB}}(\tau)$
m_Z	1.000	-0.008	-0.050	0.073	0.001	0.002	-0.015	0.046	0.034
Γ_Z		1.000	-0.284	-0.006	0.008	0.000	-0.002	0.002	-0.003
σ_{had}^0			1.000	0.109	0.137	0.100	0.008	0.001	0.007
R_e				1.000	0.070	0.044	-0.356	0.023	0.016
R_μ					1.000	0.072	0.005	0.006	0.004
R_τ						1.000	0.003	-0.003	0.010
$A_{\text{FB}}(e)$							1.000	-0.026	-0.020
$A_{\text{FB}}(\mu)$								1.000	0.045
$A_{\text{FB}}(\tau)$									1.000

TABLE II. The correlation of the Z lineshape variables at LEP

	R_b	R_c	$A_{\text{FB}}(b)$	$A_{\text{FB}}(c)$	A_b	A_c
R_b	1.00	-0.14	-0.03	0.01	-0.03	0.02
R_c		1.00	0.05	-0.05	0.02	-0.02
$A_{\text{FB}}(b)$			1.00	0.09	0.02	0.00
$A_{\text{FB}}(c)$				1.00	-0.01	0.03
A_b					1.00	0.15
A_c						1.00

TABLE III. The correlation of the heavy flavor variables from LEP/SLD.

	δs^2	ξ	$\delta\alpha_s$
δs^2	1.00	-0.28	0.20
ξ		1.00	-0.23
$\delta\alpha_s$			1.00

TABLE IV. The correlations of the fit variables for the $\alpha = \beta = 0, \gamma = 3$ case.

	δs^2	ξ	$\delta\alpha_s$
δs^2	1.00	-0.33	0.26
ξ		1.00	-0.39
$\delta\alpha_s$			1.00

TABLE V. The correlations of the fit variables for the $\alpha = 0, \beta = \gamma = 1.5$ case.

FIGURES

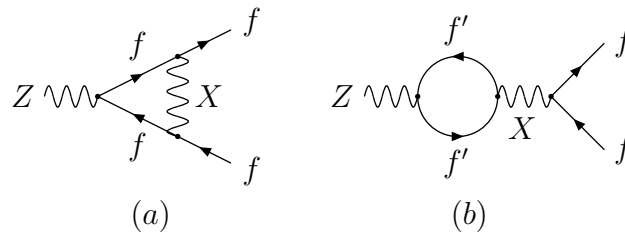


FIG. 1. One-loop vertex corrections to $Z \rightarrow f\bar{f}$. Wavefunction renormalization corrections are not shown.

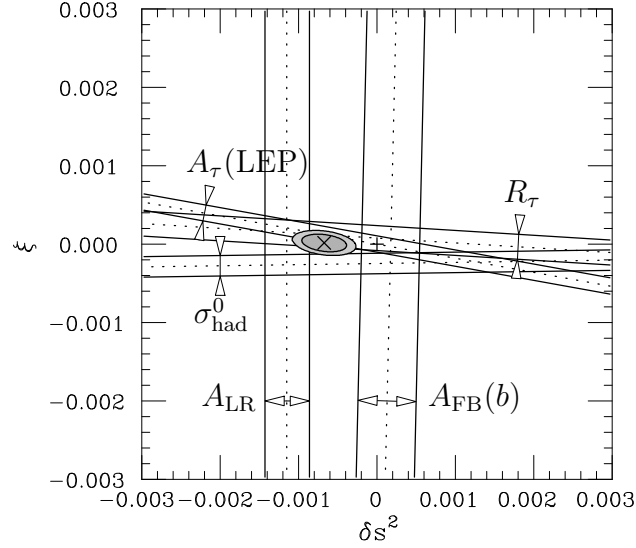


FIG. 2. The 68% and 90% confidence contours in the δs^2 - ξ plane for the $\alpha = \beta = 0$, $\gamma = 3$ case. The 1σ bounds from the observables leading to the strongest constraints are also shown.

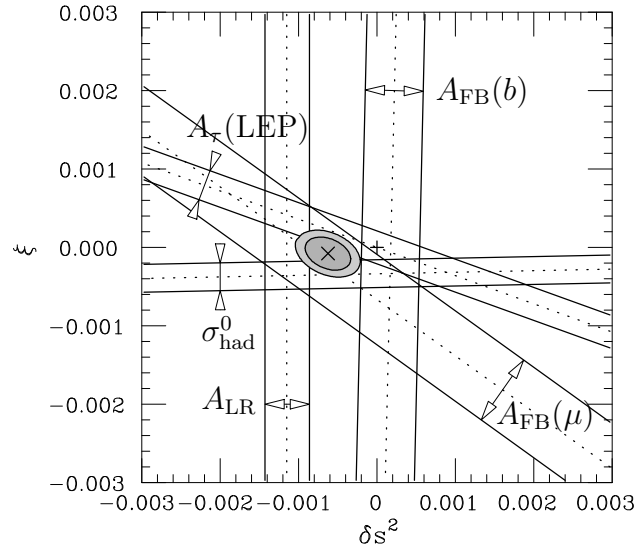


FIG. 3. The 68% and 90% confidence contours in the δs^2 - ξ plane for the $\alpha = 0$, $\beta = \gamma = 1.5$ case. The 1σ bounds from the observables leading to the strongest constraints are also shown.

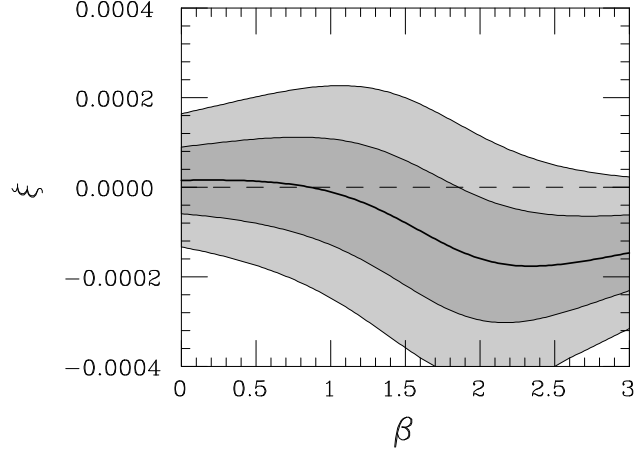


FIG. 4. The 1σ (dark gray) and 2σ (light gray) limits on ξ for $\alpha = 0$, $\beta + \gamma = 3$ models.

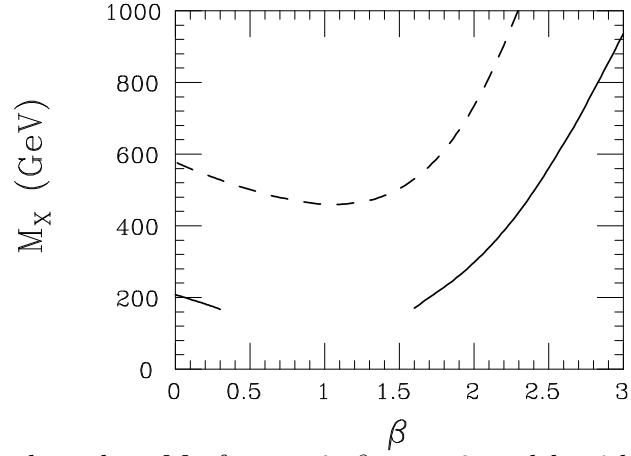


FIG. 5. The 2σ lower bound on M_X for $\alpha = 0$, $\beta + \gamma = 3$ models with $g_X = 0.65$ (dashed line) and $g_X = 0.35$ (solid line). No bound exists for the $g_X = 0.35$ case between $\beta \approx 0.4$ and $\beta \approx 1.5$.

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