We consider a search for the CP-violating angle $\delta_{\text{CP}}$ in long baseline neutrino oscillation experiments. We show that the subleading $\delta_{\text{CP}}$-dependent terms in the $\nu_\mu \rightarrow \nu_e$ oscillation probability can be easily obscured by the ambiguity of the leading term which depends on $|\Delta m^2_{31}|$. It is thus necessary to determine the value of $\Delta m^2_{31}$ with a sufficient accuracy. The $\nu_\mu$ survival events, which can be accumulated simultaneously with the $\nu_e$ appearance events, can serve for this purpose owing to its large statistics. Therefore, the combined analysis of $\nu_e$ appearance and $\nu_\mu$ survival events is crucial to provide a restrictive constraint on $\delta_{\text{CP}}$. Taking a test experimental setup, we demonstrate in the $\delta_{\text{CP}}$-$\Delta m^2_{31}$ plane that the analysis of $\nu_e$ appearance events leads to less restrictive constraints on the value of $\delta_{\text{CP}}$ due to the ambiguity of $\Delta m^2_{31}$ and that the combined analysis efficiently improves the constraints.

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I. INTRODUCTION

Neutrino oscillation has been strongly suggested by an accumulating number of experiments using a variety of neutrino sources [1, 2, 3], providing rich information on the flavor structure of the lepton sector. The mass- and the mixing-parameters of neutrinos are nonetheless still not completely known; following the definitions of Ref. [4], the unknown parameters include the value of the mixing angle $\theta_{13}$, the CP-violating angle $\delta_{\text{CP}}$, and the sign of $\Delta m^2_{31}$. The current upper bound on $\theta_{13}$ is expected to be improved down to $\sin^2 2\theta_{13} \sim 0.01$ by the next generation of nuclear reactor [5, 6, 7, 8, 9] and accelerator experiments [10, 11, 12], while $\delta_{\text{CP}}$ and the sign of $\Delta m^2_{31}$ are to be investigated by future long baseline neutrino oscillation experiments [10, 11, 12].

In this paper, we consider the search for the leptonic CP-violation angle by long baseline neutrino oscillation experiments using a conventional $\nu_\mu$ superbeam. CP violation can only be observed through the flavor-changing processes such as $\nu_\mu \rightarrow \nu_e$ [1]. It is challenging, however, to extract the value of $\delta_{\text{CP}}$ from this oscillation since the $\nu_e$ appearance probability is suppressed by the small value of $\sin^2 2\theta_{13}$. The search for $\delta_{\text{CP}}$ is made more difficult by backgrounds such as $\nu_e$-contamination of the incident beam, which is inseparable from the signal at the detector, and the neutral-current events that are misidentified as electron events. Further obstacles to extracting the value of $\delta_{\text{CP}}$ are the uncertainties in the other mixing parameters [12, 13, 14]. Among them, the uncertainty in the value of $\theta_{13}$, which is bounded only from above, has been widely studied [12, 13, 14]. The value of $\theta_{13}$ enters into the leading term of the $\nu_\mu \rightarrow \nu_e$ appearance oscillation probability, and the lack of knowledge on its value makes it difficult to search for the subleading effect of $\delta_{\text{CP}}$. The ambiguity of $|\Delta m^2_{31}|$ causes a similar problem: the value of $|\Delta m^2_{31}|$ also enters into the leading term of the appearance probability and its current experimental uncertainty is large enough to obscure the subleading $\delta_{\text{CP}}$-dependent effect. This uncertainty can also obscure the dependence on the sign of $\Delta m^2_{31}$ and make the $\delta_{\text{CP}}$ search more difficult. It is hence necessary to constrain the value of $|\Delta m^2_{31}|$ in searching for the CP-violating angle.

The value of $|\Delta m^2_{31}|$ can be precisely determined using $\nu_\mu$ survival events accumulated concurrently with the $\nu_e$ events [12, 13, 14]. This is due to the large number of $\nu_\mu$ events expected from the large $\nu_\mu$-flux available in $\nu_\mu$ superbeams. A combined analysis employing $\nu_\mu$ survival events as well as $\nu_e$ appearance events can thus place strong constraints on the values of $\delta_{\text{CP}}$ and $\Delta m^2_{31}$. We carry out an example analysis for a test setup fixing the parameters other than $\delta_{\text{CP}}$ and $\Delta m^2_{31}$. We demonstrate that the analysis of the $\nu_e$ appearance events alone does not sufficiently constrain the value of $\delta_{\text{CP}}$ in the presence of the ambiguity of $|\Delta m^2_{31}|$ and that the combined analysis efficiently improves the constraint.

This paper is organized as follows. In Sec. II we use an analytic expression of the $\nu_e$ appearance probability to show that the ambiguity in the leading term can obscure the dependence on $\delta_{\text{CP}}$ and on the sign of $\Delta m^2_{31}$. We point out that precise values of $|\Delta m^2_{31}|$ and $\sin^2 2\theta_{13}\sin^2 \theta_{23}$ are necessary to search for $\delta_{\text{CP}}$. We then use an analytic expression for the $\nu_\mu$ survival probability to show that it can constrain the value of $\delta_{\text{CP}}$. In Sec. III we consider a test setup and calculate allowed regions of parameters in the $\delta_{\text{CP}}$-$\Delta m^2_{31}$ plane fixing other parameters. We show that the combined analysis of $\nu_e$ appearance and $\nu_\mu$ survival events, in principle, gives a strong constraint on the value of $\delta_{\text{CP}}$ and on the sign of $\Delta m^2_{31}$. We conclude our
work and give discussions in Sec. [IV]

II. RELEVANCE OF THE AMBIGUITY OF $\delta m_3^2$ IN THE LEPTONIC CP VIOLATION SEARCH

Assuming that the number of neutrino generations is three, we define the mixing angles $\theta_{ij}$ ($\{i, j\} \subset \{1, 2, 3\}$), the CP-violating angle $\delta_{CP}$, and the quadratic mass differences $\delta m^2_{ij}$ as in Ref. [1]. We assume the parameter values to be $\delta m^2_{21} \simeq 8 \times 10^{-5}$ eV$^2$, $|\delta m^2_{31}| \simeq (2-3) \times 10^{-3}$ eV$^2$, $\sin^2 \theta_{12} \simeq 0.3$, $\sin^2 \theta_{23} \simeq 0.5$, and $\sin^2 2\theta_{13} \lesssim (0.1-0.2)$, which accommodate all neutrino oscillation experiments except for the LSND experiment [10].

We consider the case where the neutrino energy $E$ and the baseline length $L$ satisfies $(\delta m^2_{31}/L)/(2E) = O(1)$ so that the oscillation signal can be observed. We calculate the $\nu_{\mu} \rightarrow \nu_e$ oscillation probability in matter whose density $\rho$, and hence the electron number density $n_e$, is constant. Taking up to first order in $\delta m^2_{31}$ and in $a \equiv \sqrt{2G} \rho n_e E = (7.63 \times 10^{-5}$ eV$^2)(\rho/[g\,cm^{-3}])/(E/[GeV])$, we obtain [17]

\[
P(\nu_\mu \rightarrow \nu_e) = 4c_{13}^2 s_{13} s_{23}^2 \sin^2 \left[ \frac{\delta m^2_{21}}{4E} \left( 1 - 2s^2_{13} \right) \right] \times \sin^2 \left( \frac{\delta m^2_{21}}{4E} \left( 1 - 2s^2_{13} \right) \right) \left( \cos \delta_{CP} + \frac{2}{(2n + 1)\pi} \sin \delta_{CP} \right),
\]

where $s_{ij}$ and $c_{ij}$ denotes $\sin \theta_{ij}$ and $\cos \theta_{ij}$, respectively, and the top and the bottom of the double sign are taken when $\delta m^2_{21} > 0$ and $\delta m^2_{21} < 0$, respectively. The probability has an oscillatory dependence on the energy due to the sinusoidal factor, while the preceding factor in the first pair of brackets, depending weakly on the energy, gives the envelope of the oscillation. The leading terms of the argument and the envelope of the sinusoidal factor are given by $|\delta m^2_{21}|L/(4E)$ and $4c_{13}^2 s_{13} s_{23}^2$, respectively. The subleading terms of both the argument and the envelope depend on $\delta_{CP}$ and the sign of $\delta m^2_{31}$. One can obtain the knowledge on the value of $\delta_{CP}$ and the sign of $\delta m^2_{31}$ through subleading effects if the leading terms are known with sufficient accuracy; if not, the effects will be obscured by the ambiguity in the leading terms.

The argument of the sinusoidal factor is determined from the energy which gives the peak of the oscillation probability, or the peak energy in short. We thus evaluate the peak energy, including the corrections from energy dependence of the envelope, up to first order in $\delta m^2_{31}$ and in $a$ to obtain

\[
E_{\text{peak,n}} = \frac{L}{2(2n + 1)\pi} \left\{ \left| \delta m^2_{31} \right| \pm \left[ 1 - \frac{4}{(2n + 1)\pi^2} \right] \frac{|\delta m^2_{31}|}{(2n + 1)\pi} a' L(1 - 2s^2_{13}) + \delta m^2_{21} s_{12}^2 \right\} + \delta m^2_{21} \frac{c_{23} c_{12} s_{12}}{s_{13} s_{23}} \cos \delta_{CP} \pm \frac{2}{(2n + 1)\pi} \sin \delta_{CP},
\]

where $n = 0, 1, 2, \ldots$, $a' \equiv \sqrt{2G} \rho n_e = a/(2E) = (1.93 \times 10^{-4}$ km$^{-1})(\rho/[g\,cm^{-3}])$, and the top of the double sign is for $\delta m^2_{31} > 0$ and the bottom for $\delta m^2_{31} < 0$. Inverting the sign of $\delta m^2_{31}$ changes the signs of the subleading second and third terms in the braces of Eq. (2), and the dependence on $\delta_{CP}$ appears in the third term. The ambiguity of the leading term, which we denote by $\Delta(|\delta m^2_{31}|)$, must be smaller than these subleading terms to determine $\delta_{CP}$ and the sign of $\delta m^2_{31}$ from the observation of $E_{\text{peak,n}}$. The conditions on $\Delta(|\delta m^2_{31}|)$ are given, for typical values of the parameters and $n = 0$, by

\[
\Delta(|\delta m^2_{31}|) < \left( 1 - \frac{4}{\pi^2} \right) \frac{|\delta m^2_{31}|}{\pi} a' L(1 - 2s^2_{13}) + \delta m^2_{21} s_{12}^2 \simeq \left( 2 \times 10^{-4} \frac{L}{1000\,\text{km}} \frac{\rho}{2.6\,\text{g/cm}^3} + 2 \times 10^{-5} \right) \text{eV}^2,
\]

\[
\Delta(|\delta m^2_{31}|) < \delta m^2_{21} \frac{c_{23} c_{12} s_{12}}{s_{13} s_{23}} \simeq \left\{ \begin{array}{ll}
2 \times 10^{-4} \text{eV}^2 & \text{for } \sin^2 2\theta_{13} = 0.1 \\
8 \times 10^{-4} \text{eV}^2 & \text{for } \sin^2 2\theta_{13} = 0.01
\end{array} \right.
\]

The current experimental uncertainty in $|\delta m^2_{31}|$ is about $6 \times 10^{-4}$ eV$^2$ [2], which is larger or similar to the above critical values. It is therefore necessary to constrain the value of $\delta m^2_{31}$ within a smaller interval.

Similar analysis of the envelope leads to another set of conditions. To determine the sign of $\delta m^2_{31}$, we must impose the following condition on the ambiguity of $4c_{13}^2 s_{13} s_{23} = \sin^2 2\theta_{13} \sin^2 \theta_{23}$:

\[
\frac{\Delta(\sin^2 2\theta_{13} \sin^2 \theta_{23})}{\sin^2 2\theta_{13} \sin^2 \theta_{23}} < \frac{2a}{|\delta m^2_{31}|} \left( 1 - 2s^2_{13} \right) \simeq 0.3 \frac{L}{1000\,\text{km}} \frac{\rho}{2.6\,\text{g/cm}^3}.
\]
which is evaluated at \( E = E_{\text{peak}, \theta} \) using typical values of the parameters. To constrain the value of \( \delta_{\text{CP}} \), the condition is

\[
\frac{\Delta(\sin^2 2\theta_{13} \sin^2 \theta_{23})}{\sin^2 2\theta_{13} \sin^2 \theta_{23}} < \frac{\delta m_{31}^2}{|\delta m_{31}^2|} \frac{c_{23} s_{12} s_{13}}{s_{13} s_{23}} \approx \begin{cases} 0.3 & \text{for } \sin^2 2\theta_{13} = 0.1 \\ 0.9 & \text{for } \sin^2 2\theta_{13} = 0.01 \end{cases}.
\]

(6)

The current experimental bound on the value of \( \theta_{23} \) is \( \sin^2 2\theta_{23} > 0.9 \) \[ \text{[12, 13, 14]} \]. The value of \( \theta_{13} \), on the other hand, is bound only from above to date. Ignorance of the value of \( \theta_{13} \) is shown to be a major obstacle in determining the value of \( \delta_{\text{CP}} \) through long baseline experiments \[ \text{[12, 13, 14]} \]. A possible approach to this difficulty is to combine results from reactor neutrino experiments \[ \text{[15, 16, 17]} \]. Future experiments searching for the disappearance of \( \nu_e \) from reactors are expected to constrain the value of \( \theta_{13} \) independently of the values of other parameters such as \( \delta_{\text{CP}} \) and matter density. Reference \[ \text{[8]} \] suggests that future experiments can constrain the value of \( \sin^2 2\theta_{13} \) with \( \lesssim 10\% \) accuracy if the value of \( \sin^2 2\theta_{13} \) is as large as 0.1. Such reactor experiments can be performed in advance or concurrently with accelerator-based \( \delta_{\text{CP}} \) searches. In this prospect, we assume that the value of \( \sin^2 2\theta_{13} \sin^2 \theta_{23} \) will be known with a reasonable accuracy by the time of \( \delta_{\text{CP}} \) searches, and we keep the value of \( \sin^2 2\theta_{13} \sin^2 \theta_{23} \) fixed in this paper to explore the best-case scenario for the long baseline experiments.

The \( \nu_\mu \) survival events, which can be accumulated simultaneously with the \( \nu_e \) appearance events, can be used to constrain the value of \( \delta m_{31}^2 \) owing to the large statistics available. The energy dependence of the \( \nu_\mu \) survival probability is calculated, up to first order in \( \delta m_{31}^2 \) and in \( a \), as \[ \text{[15]} \]

\[
P(\nu_\mu \rightarrow \nu_\mu) = 1 - 4c_{13} s_{23}(1 - c_{13}^2 c_{23}^2) \left[ 1 + 2 \frac{a}{\delta m_{31}^2} \frac{s_{13}^2(1 - 2c_{13}^2 s_{23}^2)}{1 - c_{13}^2 s_{23}^2} \right] \times \sin^2 \left[ \frac{\delta m_{31}^2 L}{4E} \pm \frac{a L s_{13}^2 (1 - 2c_{13}^2 s_{23}^2)}{4E} \mp \frac{\delta m_{21}^2 L s_{13}^2 s_{23}^2 s_{12}^2 + c_{23}^2 c_{12}^2 - 2c_{23} s_{23} c_{12} s_{12} s_{13} \cos \delta_{\text{CP}}}{4E} \right] \frac{1}{1 - c_{13}^2 s_{23}^2},
\]

(7)

where the top and the bottom of the double sign in Eq. \[ \text{[11]} \] are taken when \( \delta m_{31}^2 > 0 \) and \( \delta m_{31}^2 < 0 \), respectively. Since the value of \( a s_{13}^2 (1 - 2c_{13}^2 s_{23}^2)/(1 - c_{13}^2 s_{23}^2) \) is negligibly small compared to the leading terms under the current experimental limits, the observation of the energy dependence of this mode gives the value of

\[
\left| \delta m_{31}^2 - \delta m_{21}^2 \frac{s_{13}^2 s_{23}^2 s_{12}^2 + c_{23}^2 c_{12}^2 - 2c_{23} s_{23} c_{12} s_{12} s_{13} \cos \delta_{\text{CP}}}{1 - c_{13}^2 s_{23}^2} \right|.
\]

(8)

We then obtain two possible values of \( \delta m_{31}^2 \), one being positive and the other being negative; their absolute values differ from each other by twice the \( \delta m_{31}^2 \) factor in Eq. \[ \text{[8]} \]. This constraint on \( \delta m_{31}^2 \) would contribute to determining the value of \( \delta_{\text{CP}} \) and the sign of \( \delta m_{31}^2 \). Consequently, a combined analysis of the \( \nu_e \) appearance and \( \nu_\mu \) survival events is crucial for measuring \( \delta_{\text{CP}} \). It gives the simultaneous constraint on \( \delta_{\text{CP}} \) and \( \delta m_{31}^2 \), and thus the result shall be presented in the \( \delta_{\text{CP}} \)-\( \delta m_{31}^2 \) plane.

III. NUMERICAL ANALYSIS OF AN EXAMPLE SETUP

In this section, we consider a test experiment setup and obtain constraints on \( \delta_{\text{CP}} \) and \( \delta m_{31}^2 \) from the oscillation event spectra, which are numerically calculated without employing any approximation formulae of the oscillation probabilities. We fix other parameters including \( \theta_{13} \) and \( \theta_{23} \) as mentioned in Sec. III. We show that the values of \( \delta_{\text{CP}} \) and \( \delta m_{31}^2 \) are significantly constrained by performing the combined analysis of \( \nu_e \) appearance and \( \nu_\mu \) survival events.

A. Example setup and the method of analysis

We consider the following example setup. A wide band beam of neutrinos is produced at the upgraded Alternating Gradient Synchrotron (AGS) at Brookhaven National Laboratory (BNL). We assume the flux of neutrinos given in Fig. [11] which is obtained by fitting the flux presented in Fig. 3 of Ref. \[ \text{[16]} \]. Neutrinos are detected by a water Čerenkov detector which has 500 kt of fiducial mass and is placed 770 km away from BNL in the Kimballton mine, Virginia \[ \text{[12]} \]. We exclusively consider quasi-elastic events \( \nu_e + n \rightarrow l^- + p \) as signals. We assume that its detection efficiency is unity, the data acquisition time is 5 \( \times \) 10^7 sec, and the matter density is 2.6 g/cm^3. The expected number of \( \nu_e \) events, within an energy bin \( E_i < E < E_{i+1} \) is evaluated as

\[
\langle N_i(\nu_\mu \rightarrow \nu_\mu) \rangle = TN \int_{E_i}^{E_{i+1}} dE \varepsilon(E) \frac{f(E)}{L^2} P(\nu_\mu \rightarrow \nu_\mu) \frac{d\sigma(E)}{dE},
\]

(9)

where \( T \) is the data acquisition time, \( N \) is the number of target nucleons in the fiducial volume of the detector, \( \varepsilon(E) \) is the detection efficiency, \( f(E) \) is the incident
The number of events in each bin is generated by the procedure. We generate the event number shown in Fig. 1 by a dotted line. The flux of electron neutrinos (dotted line) is assumed to be 0.7% of that of muon neutrinos with the same energy dependence.

\[ \nu_\mu \text{ flux, and } d\sigma(v_{\nu})/dE \text{ is the cross section of the detection reaction.} \]

We present in Fig. 2 the calculated event number spectra of \( \nu_e \) appearance events and those of \( \nu_\mu \) survival events. The parameters are taken as \( |\delta m_{31}| = 2.5 \times 10^{-3} \text{eV}^2, \delta m_{21}^2 = 8.2 \times 10^{-5} \text{eV}^2, \sin^2\theta_{12} = 0.84, \sin^2\theta_{23} = 1.00, \) and \( \sin^2\theta_{13} = 0.06 \). These figures show clear signals of \( \nu_e \) appearance and \( \nu_\mu \) disappearance between 1 GeV and 2 GeV.

It is necessary to take backgrounds into consideration. Significant sources of backgrounds in the \( \nu_e \) appearance signal are \( \nu_\mu \)-contamination in the incident \( \nu_\mu \) beam, and misidentification of neutral pions produced through neutral-current interaction. In the following, we take into account only the \( \nu_e \)-contamination which is not separable from signal at the detector. The number of the neutral-pion background is difficult to estimate theoretically since it depends on the model of the pion-production process, details of the detector design, and methods of the event selection; we leave the consideration of the neutral-pion background to a future work. The expected number of \( \nu_\mu \) events in an energy bin \( E_i < E < E_{i+1} \) is then given by \( \langle N_i^{(\nu_e)} \rangle = \langle N_i(\nu_\mu \rightarrow \nu_\mu) + \langle N_i(\nu_e \rightarrow \nu_\mu) \rangle \rangle \) defined as in Eq. (9) with the initial \( \nu_\mu \) replaced by \( \nu_e \). We assume in the following that the flux of \( \nu_\mu \)-contamination in the incident beam is 0.7% of the \( \nu_\mu \) flux with the same energy dependence, as shown in Fig. 1 by a dotted line.

We obtain the constraint on \( \delta_{CP} \) and \( \delta m_{31}^2 \) by the following procedure. We generate the event number spectrum \( N_i^{(\nu_e)}(\delta_{CP}, \delta m_{31}^2) \) for various values of \( \delta_{CP} \) and \( \delta m_{31}^2 \), while keeping the other parameters fixed. The number of events in each bin \( N_i^{(\nu_e)}(\delta_{CP}, \delta m_{31}^2) \) includes statistical errors and is distributed around \( \langle N_i^{(\nu_e)} \rangle \). We carry out hypothesis testing with the null hypothesis: "the parameter values are \( \delta_{CP}^{\text{true}} \) and \( \delta m_{31}^2(\text{true}) \)." We reject this null hypothesis if the deviation of \( N_i^{(\nu_e)}(\delta_{CP}, \delta m_{31}^2) \) from \( \langle N_i^{(\nu_e)} \rangle \) is of statistical significance, which we examine by the \( \chi^2 \) test using

\[ \chi^2 = \sum_{i=1}^{n_{\text{bin}}} \left[ \frac{N_i^{(\nu_e)}(\delta_{CP}, \delta m_{31}^2)}{N_i^{(\nu_e)}(\delta_{CP}^{\text{true}}, \delta m_{31}^2(\text{true}))} - 1 \right]^2, \]

where \( n_{\text{bin}} \) is the number of the energy bins. The allowed region is obtained as a region in which the null hypothesis cannot be rejected for a certain confidence level.

B. Constraints on the values of \( \delta_{CP} \) and \( \delta m_{31}^2 \)

We carry out the analysis illustrated in the previous subsection for three cases: using \( \nu_e \) appearance events...
only, using $\nu_\mu$ survival events only, and using both $\nu_e$ appearance and $\nu_\mu$ survival events. In the following, the values of the parameters except $\delta m^2_{31}$ and $\delta_{CP}$ are fixed to those given in Fig. 2, and we take $|\delta m^2_{31}| = 2.5 \times 10^{-3}$ eV$^2$ and $\delta_{CP}^{(true)} = -\pi/2, 0, \pi/2$, and $\pi$.

First, we show the analysis of the $\nu_e$ appearance spectrum only using $\chi^2(\nu_e)(\delta_{CP}, \delta m^2_{31})$. We present in Figs. 3 and 4 allowed regions for 68.3% and 95% confidence levels obtained from the $\nu_e$ appearance spectrum. We see that $\nu_e$ appearance events cannot always determine the sign of $\delta m^{2 \ (true)}_{31}$ and the value of $\delta_{CP}$. While there are indeed cases in which the wrong sign of $\delta m^{2 \ (true)}_{31}$ does not give any allowed region in the presented figures (e.g. Fig. 3(a)), there are cases where the wrong sign of $\delta m^{2 \ (true)}_{31}$ also gives allowed regions (e.g. Fig. 4(b)). The allowed region in such cases, however, is not a single extended region but a group of small isolated spots, which indicates that they are due to statistical errors and background. We also note that the allowed interval of $\delta_{CP}$ is enlarged by the ambiguity of $\delta m^{2 \ (true)}_{31}$ in some cases such as Fig. 3(a); in this case, the allowed intervals of the parameters are $\delta_{CP} \simeq (-0.5 \pm 0.25) \pi$ and $\delta m^{2 \ (true)}_{31} \simeq (2.5 \pm 0.2) \times 10^{-3}$ eV$^2$ for the 95% confidence level.

The allowed regions in Figs. 3 and 4 can be understood in terms of analytic expressions of Eqs. 11 and 12 as follows. We expect that the two oscillation spectra resemble each other and give a small value of $\chi^2(\nu_e)$ when their peak energies and peak oscillation probabilities are equal, i.e.

$$E_{peak} = E_{peak}^{(true)},$$

and

$$P(\nu_\mu \rightarrow \nu_e; \delta_{CP}, \delta m^{2 \ (true)}_{31}; E_{peak}) = P(\nu_\mu \rightarrow \nu_e; \delta_{CP}^{(true)}, \delta m^{2 \ (true)}_{31}; E_{peak}^{(true)}),$$

where $E_{peak}$ and $E_{peak}^{(true)}$ are given by Eq. 2 with $n = 0$. Solutions to these equations are shown in Figs. 3 and 4 by a dotted line for Eq. 11 and a broken line for Eq. 12. The allowed values of $\delta_{CP}$ and $\delta m^{2 \ (true)}_{31}$ are well constrained around the region where the dotted and the broken lines intersect or get close to each other, even for cases with the wrong sign of $\delta m^{2 \ (true)}_{31}$. It shows that it is difficult to distinguish two spectra satisfying the peak-matching conditions Eqs. 11 and 12. Difference of the two spectra away from the peak energy is not significant under the limited statistics and background.

Second, we present allowed regions obtained from $\nu_\mu$ survival spectra in Fig. 4. We show the result only for $\delta_{CP}^{(true)} = \pi/2$ since the results depend little on $\delta_{CP}^{(true)}$. The allowed region separates into two horizontal bands, one with $\delta m^{2 \ (true)}_{31} > 0$ and the other with $\delta m^{2 \ (true)}_{31} < 0$. The error of the two values of $\delta m^{2 \ (true)}_{31}$ shrinks down to about $\pm 3 \times 10^{-5}$ eV$^2$ at the 95% confidence level. This small error is due to the large statistics available. In terms of Eq. 7, the two bands correspond to the two possible values of $\delta m^{2 \ (true)}_{31}$ that give the same value of Eq. 5 and, consequently, the same energy dependence of the $\nu_\mu$ survival probability given in Eq. 7. These two values are indicated in Fig. 4 by dotted lines which lie inside the allowed regions.

Finally, we present the combined analysis by evaluating $\chi^2 = \chi^2(\nu_e) + \chi^2(\nu_\mu)$. The allowed regions from the combined analysis are presented in Figs. 5 and 6. It is shown that the isolated allowed regions are eliminated in these figures. The sign of $\delta m^{2 \ (true)}_{31}$ is hence well determined, and its absolute value is restricted to $(2.5 \pm 0.03) \times 10^{-3}$ eV$^2$, whose small error also reduces the error of $\delta_{CP}$ down to about $\pm 0.15 \pi$ or less around the true value. The synergy of $\nu_e$ appearance events and $\nu_\mu$ survival events excludes the fake allowed regions with the wrong sign of $\delta m^{2 \ (true)}_{31}$ and improves the precision of the parameters. The observation of both event types and the combined analysis are thus crucial.

IV. CONCLUSION AND DISCUSSIONS

We considered a search for the CP-violating angle $\delta_{CP}$ in long baseline neutrino oscillation experiments. We pointed out that it is necessary to take the ambiguities in $|\delta m^{2 \ (true)}_{31}|$, $\theta_{13}$, and $\theta_{23}$ into consideration in CP-violation searches. Their ambiguities can obscure the dependence of the $\nu_\mu \rightarrow \nu_e$ oscillation probability on $\delta_{CP}$ and on the sign of $\delta m^{2 \ (true)}_{31}$. We then showed that the $\nu_\mu$ survival events can be employed to precisely determine the value of $\delta m^{2 \ (true)}_{31}$, and be combined with $\nu_e$ appearance events to improve constraints on the value of $\delta_{CP}$ and to determine the sign of $\delta m^{2 \ (true)}_{31}$.

We numerically verified the significance of the combined analysis for an example setup. We assumed that the neutrino beam generated by the upgraded AGS beam at BNL is observed for $5 \times 10^7$ sec by a 500 kt water Čerenkov detector, placed 770 km away at Kinshukino mine. We took into account the ambiguity of $|\delta m^{2 \ (true)}_{31}|$ and fixed parameters other than $\delta m^{2 \ (true)}_{31}$ and $\delta_{CP}$. We considered only the $\nu_e$-contamination in the incident $\nu_\mu$ beam as a source of background. We obtained allowed regions in the $\delta_{CP}$-$\delta m^{2 \ (true)}_{31}$ plane through a $\chi^2$ analysis. The analysis of $\nu_e$ appearance events alone led to less restrictive constraints on the value of $\delta_{CP}$ and the sign of $\delta m^{2 \ (true)}_{31}$ due to the ambiguity of $|\delta m^{2 \ (true)}_{31}|$. In contrast, the combined analysis of $\nu_e$ appearance and $\nu_\mu$ survival events was capable of constraining the values of $\delta_{CP}$ and $|\delta m^{2 \ (true)}_{31}|$ with the errors of $\pm 0.15 \pi$ or less and $\pm 3 \times 10^{-5}$ eV$^2$, respectively, and also of determining the sign of $\delta m^{2 \ (true)}_{31}$.

The values of $\theta_{13}$ and $\theta_{23}$, which we have kept fixed in the present analysis, may not be known with sufficient precision by the time of $\delta_{CP}$ searches. The ambiguity in the value of $\sin^2 2\theta_{13} \sin^2 \theta_{23}$ can actually be a hurdle in searching for the value of $\delta_{CP}$ and the sign of $\delta m^{2 \ (true)}_{31}$, as discussed in Sec. 11 and in other studies [12,13,14]. This, however, does not change our major conclusions, i.e.: the ambiguity in the value of $|\delta m^{2 \ (true)}_{31}|$ is also a significant obstacle to the $\delta_{CP}$ search, and that ambiguity
FIG. 3: Allowed regions for 68.3% (gray solid line) and 95% (black solid line) confidence levels obtained from $\nu_e$ appearance events between 0.7 GeV and 3.1 GeV. The width of energy bins is taken as 0.2 GeV. Incident $\nu_e$-contamination and statistical errors are taken into account. The true parameters (cross) are taken to be $\delta m_{31}^{(\text{true})} = 2.5 \times 10^{-3}$ eV$^2$ and $\delta_{\text{CP}}^{(\text{true})} = -\pi/2, 0, \pi/2,$ and $\pi$ in (a), (b), (c), and (d), respectively, while $\delta m_{31}^2$ and $\sin^2 2\theta_{ij}$'s are fixed to those given in Fig. 2. The dotted line and the broken line show the solutions to Eqs. (11) and (12) in text, respectively.

FIG. 4: The same as Fig. 3 but $\delta m_{31}^{(\text{true})} = -2.5 \times 10^{-3}$ eV$^2$.

can be diminished by employing the $\nu_\mu$ survival events. First, note that the ambiguities of $\sin^2 2\theta_{13} \sin^2 \theta_{23}$ and of $|\delta m_{31}^2|$ affect two separate features of the $\nu_\mu \rightarrow \nu_e$ oscillation probability, namely the amplitude and the phase, as can be seen in Eq. (1). Since they have independent impacts on the $\delta_{\text{CP}}$-search in our approximation, the ambiguity of $|\delta m_{31}^2|$ will still have significant effects independently of $\theta_{13}$ and $\theta_{23}$. Second, the phase of the $\nu_\mu$ survival probability, Eq. (8), depends little on $\theta_{13}$ and $\theta_{23}$ for small $\theta_{13}$ below the current upper bound. Hence the value of $|\delta m_{31}^2|$ will be tightly constrained from the $\nu_\mu$ disappearance probability regardless of the ambiguity in $\theta_{13}$ and $\theta_{23}$. Our considerations are also supported by Ref. [14], although its authors analyzed a counting experiment using low-energy neutrinos and did not consider an experiment measuring the energy of neutrinos as we did.
of the oscillation. The improvement of the uncertainty in $\theta_{13}$ must await future experiments as discussed in Section II.

The combined analysis presented in this paper can be applied to more realistic case studies. For that purpose, it is necessary to take account of backgrounds other than $\nu_e$-contamination, especially that from single pion production events. A neutral pion becomes a source of background when the two photons from its decay are not separately detected due to the limited resolution of the detector and thereby misidentified as a $\nu_e$ appearance signal. The estimation of this background requires careful treatments. It is difficult to theoretically estimate the number of single-$\pi^0$-production events owing to its dependence on how strong-interaction processes are modeled. The number of misidentified events among them further depends on the details of the setup and method of experiments such as the design of the detector and the criteria of event selection. Nevertheless, there is a case study claiming that the number of background events from the neutral pions can be suppressed to be comparable to that from the incident $\nu_e$-contamination \cite{10}. We expect that the analysis presented in this paper shall remain effective when such excellent suppression of background is possible. We leave a realistic analysis with the consideration of $\pi^0$-background for a future work.

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[11] D. S. Ayres et al. [NOvA Collaboration], Fermilab-
FIG. 6: Allowed regions for 68.3% (gray solid line) and 95% (black solid line) confidence level. The values of the parameters are the same as in Fig. 3.

FIG. 7: The same as Fig. 6 but $\delta m_{31}^{2\text{(true)}} = -2.5 \times 10^{-3}\text{eV}^2$.


[16] A. Aguilar et al. [LSND Collaboration], Phys. Rev. D 64,