Essays on Housing Markets and Monetary Policy

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Dissertation submitted to the Faculty of the
Virginia Polytechnic Institute and State University
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy
in
Economics, Science

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April 22, 2015
Blacksburg, Virginia

Keywords: Housing Markets, Monetary Policy, DSGE Models

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(ABSTRACT)

This dissertation consists of three essays on housing markets and monetary policy. The first essay focuses on the impact of monetary policy on U.S. local housing markets and finds that monetary policy has uneven impacts on local housing markets, and that the magnitude of the impacts are correlated with housing supply regulations. The second essay studies the optimal interest rate rule in a DSGE model with housing market spillovers and finds that the optimal interest rate rule responds to house price inflation even when the stabilization of house price is not among the objectives of the policymaker.

The third essay is the core of this dissertation. I construct a dynamic stochastic general equilibrium (DSGE) model in this paper to study the fluctuations in the U.S. housing markets. The model features a market for newly built houses, a secondary market for old houses, and an endogenous term structure of nominal interest rates. Negative technological progress in the housing sector explains the upward trend in house prices over the past four decades. Housing preference and technology innovations explain about 80% of the volatility of housing investment, real price of new houses, and the old-to-new house price ratio. Monetary factors explain about 15% of the volatility of housing investment, but do not significantly contribute to the price fluctuations of either new or old houses. The preference innovation to old houses is the leading determinant of the run-up in the price of old houses relative to the price of new houses during the 10-year period before the Great Recession. The term structure is endogenous in this paper, and the intertemporal preference innovation makes a non-negligible contribution to the variations in nominal interest rates. Housing market conditions do not contribute much to the fluctuations of interest rates, but significantly affect the shape of the yield curve.

Acknowledgments

I am most grateful to my dissertation advisors, Richard Ashley and Kwok Ping (Byron) Tsang, for their guidance and support throughout the course of my PhD studies. The conversations with Byron on a daily basis over the past three years lifted me out of the pit of unimaginativeness and noncreativity. My gratitude also extends to other two members of my PhD committee, Suqin Ge and Wen You, and all other faculty, staff and my fellow graduate students in the Economics department.

I wish to thank Xue (Chris) Mei who motivated this five-year journey of mine toward an economist. Finally, I would like to thank my parents for their resigned patience and unconditional support. Without them, nothing would be meaningful to me.

Contents

List of Figures

List of Tables

1 Chapter 1

Introduction

The recent financial crisis has led to a re-examination of the role of housing in the macroeconomy. A rapidly growing literature investigates the linkage between monetary policy and housing market activities as well as the implications of house price fluctuations for monetary policy; see Del Negro and Otrok (2007), Goodhart and Hofmann (2008), and Calza, Monacelli and Stracca (2013) among many others.

In terms of methodology, vector autoregressive (VAR) models have been widely used in the empirical analysis of monetary policy issues since they were launched by Sims (1980). As suggested by Uhlig (2005), the key step in applying VAR methodology is in identifying the monetary policy shock. Usually this is done by appealing to certain informational orderings about the arrival of shocks or researchers' justification of "reasonable results."

In order to do serious policy analysis, we need a structural model with primitive interpretable shocks which are invariant to the class of policy interventions being considered. In practice, most macroeconomists now analyze policy using dynamic stochastic general equilibrium (DSGE) models. These models are promising for two reasons. First, a New Keynesian model represents a detailed economy that can generate the type of wedges we observe in the data from primitive interpretable shocks. Second, a New Keynesian model has enough microfoundations that both its shocks and its parameters are structural and invariant to policy interventions. The recent New Keynesian literature is typified by the work of Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007). Their models are widely considered to be the state-of-the-art New Keynesian model and are currently used to inform policymaking at the European Central Bank. Iacoviello and Neri (2010) investigate the housing market in an augmented version of the state-of-the-art model. They find that monetary factors are playing a bigger role in the housing cycle after 2000 and the spillovers from the housing market to the broad economy are non-negligible.

My dissertation joins this large literature of housing markets and monetary policy. It consists of three essays, where the first two are co-authored with Kwok Ping Tsang.

The first essay focuses on the impact of monetary policy on U.S. local housing markets. We start from the linearized present value model and decompose the log rent-price ratio into the expected present values of all future real interest rates, real housing premia and real rent growth for the housing markets in 23 U.S. metropolitan statistical areas. Based on the indirect inference bias-corrected VAR estimates, we find that monetary policy has uneven impacts on local housing markets, and that the magnitude of the impacts are correlated with housing supply regulations.

The second essay studies the optimal interest rate rule in a DSGE model with housing market spillovers developed by Iacoviello and Neri (2010). The policymaker is assumed to aim at minimizing overall economic fluctuations. We find that the optimal interest rate rule responds to house price inflation even when the stabilization of house price is not among the objectives of the policymaker.

The third essay is my job market paper. I develop a DSGE model that features a market for newly built houses, a secondary market for old houses, and an endogenous term structure of nominal interest rates. I then estimate the model using quarterly data from 1975 to 2013 and conduct various counterfactual analyses. Negative technological progress in the housing sector explains the upward trend in house prices over the past four decades. Housing preference and technology innovations explain about 80% of the volatility of housing investment, real price of new houses, and the old-to-new house price ratio. Monetary factors explain about 15% of the volatility of housing investment, but do not significantly contribute to the price fluctuations of either new or old houses. The preference innovation to old houses is the leading determinant of the run-up in the price of old houses relative to the price of new houses during the 10-year period before the Great Recession. The term structure is endogenously modeled and the intertemporal preference innovation makes a non-negligible contribution to the variations in nominal interest rates. Housing markets conditions do not contribute much to the fluctuations of interest rates, but significantly affect the shape of the yield curve.

Along this research agenda, a lot more work can be done in the future. Notice that standard DSGE models do have some shortcomings. First, those models usually have weak internal propagation mechanisms, i.e., they have to rely on external sources of dynamics to replicate the data. In particular, predetermined shocks are usually assumed to follow first-order autoregressive processes, which need to be highly persistent, or even close to unit roots, for a good match with the data. The same problem also applies to real business cycle (RBC) models; see Cogley and Nason (1995)'s discussion and the proposal of incorporating labor adjustment costs into the production function. How to strengthen the internal propagation mechanisms in DSGE models can be interesting.

Second, structural shocks are independently and identically distributed (the i.i.d assumption) in standard DSGE models. However, as the cross-equation restrictions embedded in DSGE models are often misspecified, some deviations from the i.i.d assumption are expected and usually accepted by researchers. For example, structural shocks implied by DSGE models often exhibit time-varying volatilities. Another issue is the possibility of structural changes in monetary policies. When we estimate a DSGE model using a dataset that spans a long period of time, say from 1975 to 2013, we should not believe that the monetary policy is always consistent. Instead, the monetary policy is supposed to be less aggressive during normal times and more aggressive during volatile times. In a Rational Expectations framework, economic agents' expectations on future changes in economic volatility and monetary policy affect their current decisions. In order to capture time-varying volatilities and possible structural changes in the monetary policy, we should consider incorporating regime switching of Hamilton (1989) into standard DSGE models. For example, we consider the following state-space representation of a dynamic linear model with switching in both measurement and transition equations:

$$y_t = H_{S_t} x_t + A e_t, \tag{1.1}$$

$$x_t = F_{S_t} x_{t-1} + G_{S_t} v_t, \tag{1.2}$$

$$\begin{pmatrix} e_t \\ v_t \end{pmatrix} \sim \mathbb{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} R_{S_t} & 0 \\ 0 & Q_{S_t} \end{pmatrix}\right), \tag{1.3}$$

where y_t is a $N \times 1$ vector of observed time series as a function of a $J \times 1$ vector of unobserved state variables; the unobserved state vector x_t follows a regime-switching vector autoregression; e_t is a $K \times 1$ vector of measurement errors and v_t is a $J \times 1$ vector of structural shocks. Coefficient matrices (H_{S_t}, F_{S_t}, and G_{S_t}) and volatilities (R_{S_t} and Q_{S_t}) all depend on the state S_t, which is an exogenous stochastic process following an M-regime Markov chain, where $S_t \in \{1, ..., M\}$ with transition matrix $P = [p_{ij}]$ defined as

$$p_{ij} = Pr[S_t = j | S_{t-1} = i] \text{ with } \textstyle\sum_{j=1}^{M} p_{ij} = 1. \tag{1.4}$$

Strengthening internal propagation mechanisms and modeling regime switching can improve standard DSGE models to a great extent. I regard both as promising areas of research in the future.

2 Chapter 2

The Impact of Monetary Policy on Local Housing Markets: Do Regulations Matter?

Abstract

This paper shows that monetary policy has uneven impacts on local housing markets, and that the magnitude of the impacts are correlated with housing supply regulations. We apply the linearized present value model, which allows the log rent-price ratio to be decomposed into the expected present values of all future real interests rates, real housing premia and real rent growth, to the housing markets in 23 U.S. metropolitan statistical areas. Based on the indirect inference bias-corrected VAR estimates, we find that MSAs that are more regulated have i) a higher variance in the log rent-price ratio, ii) a larger share of the variance explained by real interest rate, and iii) a stronger impulse response of house price to the real interest rate shock.

Keywords: Present value model, rent-price ratio, housing supply regulations.

JEL Classification: E31, G12, R31.

2.1 Introduction

Why does monetary policy have an uneven impact on different local housing markets? We show in this paper that the magnitude of the response in each market is highly correlated with its housing regulations. The more regulated the housing market, the larger is the response to a change in monetary policy.

The approach of this paper is straightforward. For each local housing market, we use the linearized present value framework (see Campbell and Shiller (1988)) to decompose the log rent-price ratio into its fundamental components. Then, we look at how the estimated model is related to two different indices of housing supply regulations. There are three major findings in this paper. First, MSAs that have more supply regulations also have a higher variance in the log rent-price ratio. Second, in the variance decomposition of the log rent-price ratio, a larger share of the variance is explained by real interest rate for more regulated MSAs. Third, house prices in more regulated MSAs have a stronger impulse response to the real interest rate shock.

Several studies have suggested that differences in the level and volatility of house price across metropolitan areas are due to differences in regulations (see Mayer and Somerville (2000), Glaeser, Gyourko and Saks (2005), and Paciorek (2013)). Furthermore, the monetary policy transmission to the housing markets also depends on regional heterogeneities, especially in the housing supply elasticity. To quantify the importance of regional heterogeneity in housing markets for the efficacy of monetary policy, Fratantoni and Schuh (2003) construct a heterogeneous-agent VAR in which the parameters and, therefore, impulse responses are time-varying. They show that regional housing markets exhibit significant variations in the responses to monetary shocks. By estimating a multi-region dynamic stochastic general equilibrium model, Leung and Teo (2011) find that differences in the price elasticity of housing supply can be related to the regional differences in the monetary propagation mechanism; a region with a higher adjustment cost in housing, i.e., low supply elasticity, responds more to monetary shocks. In this paper, we examine the relationship between housing supply regulations and the efficacy of a monetary policy in affecting the house price. In a work more directly related to this paper, Himmelberg, Mayer and Sinai (2005) argue that house price is typically more sensitive to changes in interest rates in cities where housing supply is relatively inelastic. We extend and strengthen their conclusion by providing empirical evidence from the present value framework.

The linearized present value model has been applied to the housing market. The log rent-price ratio is decomposed as an expected value of all future real interest rates, housing premia, and rent growth rates. Each expected component is then estimated through a vector autoregression (VAR) that consists of real interest rate, excess housing return, and rent growth, as well as a set of macroeconomic variables (see Campbell, Davis, Gallin and Martin (2009), Fairchild, Ma and Wu (2012), and Ambrose, Eichholtz and Lindenthal (2013)). Since interest rates are highly persistent, traditional maximum likelihood (ML) estimator of such models is likely to suffer from serious small sample bias so that the persistence in interest rates will be spuriously under-estimated, and the impulse responses and variance decompositions can be misleading; see Bauer, Rudebusch and Wu (2012) who employ the indirect inference estimator (A. A. Smith (1993) and Gourieroux, Monfort and Renault (1993)) for bias-corrected VAR estimates. By utilizing the indirect inference estimator, this paper revisits the application of the linearized present value model in 23 local housing markets and accounts for the volatility of the log rent-price ratio over the period 1978-2014.

2.2 The Linearized Present Value Model

We start with writing real house price at time t as P_t and real rent as R_t and defining the log of gross real return to housing over the period from t to $t+1$ as:

$$\phi_{t+1} \equiv \ln\left(\frac{P_{t+1} + R_{t+1}}{P_t}\right). \tag{2.1}$$

By using a first-order Taylor approximation as in Campbell and Shiller (1988), we are able to decompose the log of rent-price ratio at time t, $rp_t \equiv \ln(R_t/P_t) = \ln(R_t) - \ln(P_t) \equiv r_t - p_t$, into two separately identifiable components beyond a constant – the expected present value of all future real rates of housing return and the expected present value of all future real growth in housing rents (see Appendix A for details):

$$rp_t \simeq k + \mathbb{E}_t \left[\sum_{j=0}^{\infty} \rho^j \phi_{t+1+j} - \sum_{j=0}^{\infty} \rho^j \Delta r_{t+1+j} \right]. \tag{2.2}$$

Here, ϕ_t is the log of gross real return to housing, Δr_t is the growth rate of real rent. The parameter ρ is a discount factor that is defined as $(1 + e^{\overline{rp}})^{-1}$, where $\overline{rp}$ is the long-run log rent-price ratio. The constant k is equal to $(1-\rho)^{-1}[\ln(\rho) + (1-\rho)\ln(1/\rho - 1)]$.

By further defining real housing return, ϕ_t, as the sum of real risk-free interest rate, i_t, and real housing premium over that rate, π_t, we can rewrite the log of rent-price ratio as:

$$rp_t \simeq k + \mathbb{E}_t \sum_{j=0}^{\infty} \rho^j i_{t+1+j} + \mathbb{E}_t \sum_{j=0}^{\infty} \rho^j \pi_{t+1+j} - \mathbb{E}_t \sum_{j=0}^{\infty} \rho^j \Delta r_{t+1+j}, \tag{2.3}$$

or

$$rp_t \simeq k + \mathcal{I}_t + \Pi_t - \mathcal{G}_t, \tag{2.4}$$

where $\mathcal{I}_t$, Π_t, and $\mathcal{G}_t$ represent the expected present values of all future real interest rates, all future real housing premia, and all future real rent growth, respectively. It is worth noting that Equations (2.2)-(2.4) do not hold with exact equality, since the second- and higher-order moments have been ignored through a first-order Taylor approximation.

The expectation components, $\mathcal{I}_t$, Π_t, and $\mathcal{G}_t$, can be obtained from estimating a VAR model that consists of real interest rate i_t (a national variable), local real housing premium π_t, local real rent growth Δr_t, as well as national housing premium π_t^{US} and national real rent growth Δr_t^{US}:

$$\mathbf{Z}_t = \Sigma_0 + \Sigma_1 \mathbf{Z}_{t-1} + \Sigma_2 \mathbf{Z}_{t-2} + \epsilon_t, \tag{2.5}$$

where $\mathbf{Z}_t = (i_t, \pi_t, \Delta r_t, \pi_t^{US}, \Delta r_t^{US})'$ is a vector of $K = 5$ state variables. Σ_0 is a column vector of dimension K, each of Σ_1 and Σ_2 is a K-by-K matrix, ϵ_t is an error term.[1]

The VAR(2) model in Equation (2.5) can be rewritten in the form of a VAR(1):

$$\begin{pmatrix} \mathbf{Z}_t \\ \mathbf{Z}_{t-1} \end{pmatrix} = \begin{pmatrix} \Sigma_0 \\ \mathbf{0} \end{pmatrix} + \begin{pmatrix} \Sigma_1 & \Sigma_2 \\ \mathbf{I} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{Z}_{t-1} \\ \mathbf{Z}_{t-2} \end{pmatrix} + \begin{pmatrix} \epsilon_t \\ \mathbf{0} \end{pmatrix}, \tag{2.6}$$

or

$$\mathbb{Z}_t = \Gamma_0 + \Gamma \mathbb{Z}_{t-1} + \xi_t, \tag{2.7}$$

where $\mathbb{Z}_t = (\mathbf{Z}_t', \mathbf{Z}_{t-1}')'$, $\xi_t = (\epsilon_t', \mathbf{0}')'$. Given parameter estimates $\widehat{\Gamma}_0$ and $\widehat{\Gamma}$, the fitted values of $\mathcal{I}_t$,

[1] Engsted, Pedersen and Tanggaard (2012) argue that a properly specified VAR for return decomposition should include the log rent-price ratio, rp_t, as one of the state variables together with either real housing return or real rent growth, since log real house price p_t, hence rp_t, is in the time t information set. Given the approximate identity of Equation (2.2), a VAR that contains ϕ_t, rp_t, and a set of other state variables is equivalent to a VAR that contains Δr_t, rp_t, and the same set of other variables. As a result, one of the expectations at the right hand side of Equation (2.2) can be directly derived and the other expectation is backed out residually through the approximate identity. Moveover, as long as the VAR is properly specified, i.e., the log rent-price ratio is included as a state variable, the return variance decompositions are independent of which component is treated as a residual. However, if all three variables, rp_t, ϕ_t, and Δr_t, are included in the VAR system, the model becomes redundant and there would be a problem of multicollinearity, since knowing any two of the three equations for rp_{t+1}, ϕ_{t+1}, and Δr_{t+1}, one can infer the third, apart from the approximation error. Then, the VAR estimates would be meaningless. Here in the present work we exclude rp_t from the VAR but include both ϕ_t and Δr_t (of course, real housing return ϕ_t has been split into real interest rate i_t and real housing premium π_t). This exercise does not violate the argument of Engsted, Pedersen and Tanggaard (2012) that p_t is in the information set of time t and should be included in some form for the VAR to be legitimate, since the information in p_t has been included in ϕ_t. However, our method is fundamentally different from that of Engsted, Pedersen and Tanggaard (2012) in the sense that we are directly estimating the two expectations on the right-hand side of Equation (2.2) without arbitrarily assuming that the first-order Taylor apprriximation based linearized present value model holds exactly. Over our sample period, the U.S. housing markets experienced large fluctuations both in 1980s and in the recent financial crisis. Ignoring the pricing error that comes from omitting the second- and higher- order moments is problematic. Indeed, as we show in Sections 3 and 4, the pricing error is sizeable and it accounts for a large fraction of overall volatility of log rent-price ratio.

Π_t, and $\mathcal{G}_t$ are the first three elements of $(1-\rho)^{-1}(\mathbf{I}-\rho\widehat{\Gamma})^{-1}\widehat{\Gamma}_0 + (\mathbf{I}-\rho\widehat{\Gamma})^{-1}\widehat{\Gamma}\mathbb{Z}_t$, i.e.,

$$\widehat{\mathcal{I}}_t = e_1'[(1-\rho)^{-1}(\mathbf{I}-\rho\widehat{\Gamma})^{-1}\widehat{\Gamma}_0 + (\mathbf{I}-\rho\widehat{\Gamma})^{-1}\widehat{\Gamma}\mathbb{Z}_t], \tag{2.8}$$

$$\widehat{\Pi}_t = e_2'[(1-\rho)^{-1}(\mathbf{I}-\rho\widehat{\Gamma})^{-1}\widehat{\Gamma}_0 + (\mathbf{I}-\rho\widehat{\Gamma})^{-1}\widehat{\Gamma}\mathbb{Z}_t], \tag{2.9}$$

$$\widehat{\mathcal{G}}_t = e_3'[(1-\rho)^{-1}(\mathbf{I}-\rho\widehat{\Gamma})^{-1}\widehat{\Gamma}_0 + (\mathbf{I}-\rho\widehat{\Gamma})^{-1}\widehat{\Gamma}\mathbb{Z}_t], \tag{2.10}$$

where e_i, $i = 1, 2, 3$, is a column vector of dimension $2K$ with one as the i^{th} element, and zeros otherwise.

According to Equation (2.4), the fundamental log rent-price ratio follows

$$\widehat{rp_t} = k + \widehat{\mathcal{I}}_t + \widehat{\Pi}_t - \widehat{\mathcal{G}}_t. \tag{2.11}$$

The difference between actual log rent-price ratio and the fundamental value is a pricing error (or forecast discrepancy) $\widehat{e}_t$:

$$\widehat{e}_t = rp_t - \widehat{rp_t}. \tag{2.12}$$

In this paper, we treat the pricing error independently instead of combining it either with the expected future real rent growth as in Campbell, Davis, Gallin and Martin (2009) or with the expected future housing premia as in Fairchild, Ma and Wu (2012).[2]

[2]Campbell, Davis, Gallin and Martin (2009) follow the finance literature and treat the present value of future real rent growth as a residual. They attribute most of the variation in log rent-price ratio to changes in expected future real rent growth over the period 1997-2007. However, this phenomenon is mostly driven by the behavior of the forecast discrepancy. Fairchild, Ma and Wu (2012) treat the residual as part of the future real housing premia instead, and they find that the housing premia account for most variation in the rent-price ratio.

2.3 VAR Estimation

In this section, we estimate the VAR model in Equation (2.7) for each MSA. As Bauer, Rudebusch and Wu (2012) argue, due to the high persistence in interest rates, the conventional ML estimator of such a model likely suffers from serious small sample bias and tends to display less time-series persistence than does the true process. To improve on the performance of the ML estimator in small samples, they employ the indirect inference estimator that is proposed by A. A. Smith (1993) and Gourieroux, Monfort and Renault (1993) (see also Gourieroux, Renault and Touzi (2000)).

2.3.1 Indirect Inference Estimator

We begin with a brief discussion of the indirect inference estimator. The ML estimator of Γ can be obtained by applying OLS to each equation in the VAR system. Let $\widehat{\Gamma}$ denote the OLS estimator. The value of Γ which leads to a mean of the OLS estimator across a number of residual bootstrapping samples equal to $\widehat{\Gamma}$ is defined as the indirect inference estimator, $\widetilde{\Gamma}$, i.e.,

$$\widetilde{\Gamma} = \underset{\Gamma}{\operatorname{argmin}} \left\| \widehat{\Gamma} - \frac{1}{H} \sum_{h=1}^{H} \widehat{\Gamma}^h(\Gamma) \right\|, \tag{2.13}$$

where $\widehat{\Gamma}^h(\Gamma)$, $h = 1, \ldots, H$, is a set of OLS estimator if the data are generated under Γ.

Given the presence of high time-series persistence in real interest rate, we adopt the indirect inference estimator for the estimation of VAR models in this paper. In order to find a parameter matrix $\widetilde{\Gamma}$ that satisfies the condition in Equation (2.13), we iterate over the bootstrap algorithm provided in Bauer, Rudebusch and Wu (2012) for 5,000 times, discarding the first 500, using 100 bootstrap replications in each iteration and an adjustment parameter of 0.5.

One potential problem with the indirect inference bias correction procedure is that bias-corrected VAR estimates tend to exhibit explosive roots much more frequently than do OLS estimates. Since the VAR is assumed to be stationary, we need to ensure that all eigenvalues of $\widetilde{\Gamma}$ are less than one in modulus. Once the bias-corrected estimates have eigenvalues higher than one in modulus, we shrink the bias estimate toward zero until this restriction is satisfied.

2.3.2 Data Description

The data we use are at semi-annual frequency, ranging from the second half of 1978 (1978H2) to the second half of 2014 (2014H2). We use the house price index published by the Federal Housing Finance Agency (FHFA) as the measure of house prices. Housing rents come from the rent of primary residence published by the Bureau of Labor Statistics (BLS). The national CPI excluding shelter from BLS is used for obtaining real house price, P_t, and real housing rent, R_t. In order to obtain values of ρ and k in Equation (2.2) for each MSA, we use micro data from the 2000 Decennial Census of Housing (DCH) to benchmark the rent-price ratio in 2000; see Davis, Lehnert and Martin (2008) and Campbell, Davis, Gallin and Martin (2009) for a detailed procedure. Real interest rate, i_t, is the nominal 10-year Treasury yield less the median reading of 10-year inflation expectations from professional forecasters published by Blue Chip Economic Indicators and Livingston Survey. Real housing premium, π_t, is defined as the difference between real return to owner-occupied housing, $\phi_t \equiv \ln[(R_t + P_t)/P_{t-1}]$, and real interest rate, i_t.

Several measures of housing supply regulation have been constructed in the literature. One popular measure is the Wharton Residential Land Use Regulation Index (WRLURI) based on a 2005 survey; see Gyourko, Saiz and Summers (2008) and Saiz (2010). Another comprehensive measure is the index of housing supply regulation created by Saks (2008) for the late 1970s and 1980s. Both WRLURI and Saks-index are standardized to have a mean of zero and a standard deviation of one and are increasing in the degree of regulation. A community with an index value of positive one is one standard deviation above the national mean. Both regulation measures have data available for all the 23 MSAs in our sample, as shown in Table 2.1. These two measures have a positive correlation of around 0.6.

Table 2.1: Summary Statistics of Regulation Indices by MSA

MSA	WRLURI	Saks-index	MSA	WRLURI	Saks-index	MSA	WRLURI	Saks-index
Atlanta	0.04	-0.77	Honolulu	2.30	0.88	Philadelphia	1.03	0.47
Boston	1.54	0.86	Houston	-0.33	-0.52	Pittsburgh	0.07	0.26
Chicago	0.07	-1.01	Kansas City	-0.80	-0.95	Portland	0.30	0.94
Cincinnati	-0.55	0.16	Los Angeles	0.52	1.21	San Diego	0.51	1.60
Cleveland	-0.14	-0.25	Miami	0.78	0.47	San Francisco	0.87	2.10
Dallas	-0.33	-1.18	Milwaukee	0.28	0.19	Seattle	0.98	1.48
Denver	0.87	-0.68	Minneapolis	0.33	-0.16	St. Louis	-0.72	-0.66
Detroit	0.10	-0.69	New York	0.65	2.21			

The WRLURI for each MSA is calculated as the average across communities within that MSA, and therefore it is possible to be influenced by a few number of observations. For example, the unique observation within Honolulu (HI) makes it the most regulated MSA in our sample, whose regulation level is surprisingly higher than that of New York (NY). Another advantage of the Saks-index relative to WRLURI in our case is that it is based on the information in the first several years of our sample, whereas the later one from a survey in 2005 might be endogenously determined. Moreover, the Saks-index not only accounts for land use regulations. Instead, this combined index is constructed based on a lot of information beyond land use regulations, such as city-level environmental regulations, the importance of imposing controls on new construction as a method of limiting population growth, etc. We show results using both indices.

2.3.3 Estimation Results

We summarize the indirect inference estimation results in Table 2.2. The left panel shows the estimates of the first-order coefficient matrix and the right panel shows the second-order coefficient estimates. Rather than reporting the point estimates for each local market, we report the 25th percentile, the median, and the 75th percentile of each parameter estimate across the 23 metropolitan areas. The adjusted R^2 for each equation is listed in the last column.

Table 2.2: Summary of VAR Estimation Results

Dependent Variable		Coefficient on the 1st Lag of					Coefficient on the 2nd Lag of					$\overline{R}^2$
		i	π	Δr	π^{US}	Δr^{US}	i	π	Δr	π^{US}	Δr^{US}	
i	25th Percentile	1.411	-	-	0.071	-0.084	-0.412	-	-	-0.044	0.025	0.881
	Median	1.411	-	-	0.071	-0.084	-0.412	-	-	-0.044	0.025	0.881
	75th Percentile	1.411	-	-	0.071	-0.083	-0.412	-	-	-0.044	0.025	0.881
π	25th Percentile	-1.719	0.262	-0.225	-	-	0.055	0.208	-0.111	-	-	0.294
	Median	-0.742	0.379	0.202	-	-	0.950	0.317	0.042	-	-	0.395
	75th Percentile	-0.279	0.543	0.349	-	-	1.572	0.372	0.228	-	-	0.468
Δr	25th Percentile	-0.245	-0.040	0.292	-	-	0.007	-0.008	-0.021	-	-	0.094
	Median	-0.146	0.000	0.380	-	-	0.228	0.073	0.140	-	-	0.192
	75th Percentile	0.085	0.032	0.477	-	-	0.518	0.101	0.194	-	-	0.255
π^{US}	25th Percentile	-1.358	-	-	0.228	0.379	1.407	-	-	0.615	-0.433	0.325
	Median	-1.357	-	-	0.228	0.379	1.408	-	-	0.615	-0.433	0.325
	75th Percentile	-1.356	-	-	0.229	0.379	1.409	-	-	0.615	-0.433	0.325
Δr^{US}	25th Percentile	-0.414	-	-	-0.211	0.729	0.547	-	-	0.215	-0.261	0.183
	Median	-0.413	-	-	-0.211	0.729	0.547	-	-	0.215	-0.261	0.183
	75th Percentile	-0.413	-	-	-0.211	0.729	0.548	-	-	0.215	-0.261	0.183
Largest autoregressive root (median across 23 MSAs): 0.988												

Note: The dash symbol implies a constraint of a relevant coefficient of zero imposed on the VAR system.

Since national-level variables i_t, π_t^{US}, and Δr_t^{US} are specified to depend on their own lags only, their parameter estimates are identical across local markets, so that the 25th percentile, the median, and the 75th percentile are all equal (the slight discrepancy is a result of shrinking when the bias-corrected estimates exhibit explosive roots). The indirect inference parameter estimates are considerably different from the ML estimates in Table 2.7 (Appendix B). The former estimator yields much higher persistence in the VAR system than the later estimator. In particular, the largest autoregressive root has a median of 0.988 versus 0.832. In addition, the indirect inference estimator provides a higher $\overline{R}^2$ for each equation.[3]

[3] Campbell, Davis, Gallin and Martin (2009) include a set of macroeconomic conditions, including population growth, employment growth, and real personal income growth, in the VAR model. However, macroeconomic variables at MSA-level are only observed at annual frequency and they have to be converted into semi-annual frequency by assuming that their growth rates are constant throughout a given year. To avoid such an arbitrary assumption, we do not include macroeconomic conditions in this paper. In fact, in earlier attempts we find that macroeconomic conditions have little additional explanatory power to the housing variables, once the lags of the housing variables are included.

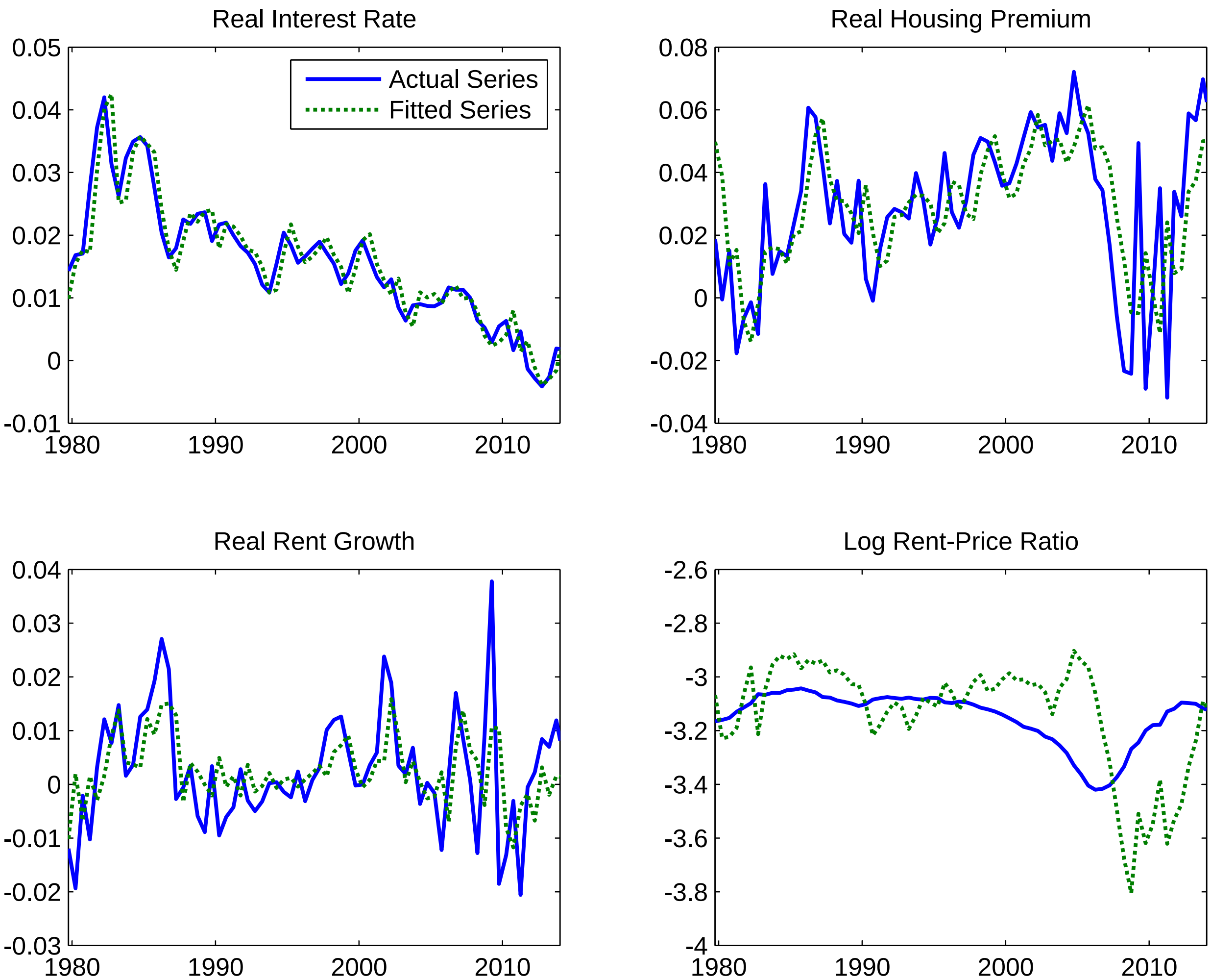


Figure 2.1: Actual vs. Fitted Series in the US National Market

In Figure 2.1, we plot both the actual and the fitted series of real interest rate, real housing premium, real rent growth, and log rent-price ratio in the U.S. national market. The VAR model with indirect inference bias correction fits the historical data of state variables very well. The fitted log rent-price ratio is slightly larger than the actual ratio on average, i.e., the pricing error is negative. The existence of this negative pricing error is consistent with the fact that the second- and higher-order terms in the Taylor approximation sum to a positive number; see Appendix A. A similar plot based on ML estimation results is shown in Figure 2.2. The ML estimates yield a much larger discrepancy (in magnitude) between actual log rent-price ratio and the fitted value.[4]

Table 2.3: Indirect Inference Estimator and Maximum Likelihood Estimator

	Standard Deviation	i_t	π_t	Δr_t	rp_t
USA	Actually Observed	0.998	2.495	1.076	9.974
	Implied by IIE	0.992	1.835	0.592	20.320
	Implied by MLE	0.920	1.466	0.545	4.230
Metro Median	Actually Observed	0.998	3.253	1.425	17.198
	Implied by IIE	0.992	2.452	0.832	22.853
	Implied by MLE	0.920	2.220	0.800	6.584

Note: All values reported in this table have been multiplied by 100.

While the log rent-price ratio implied by the ML estimates is much less volatile than the actual log rent-price ratio, the indirect inference estimator implies a much more volatile fitted log rent-price ratio. Notice that the VAR model is constructed to fit historical patterns of state variables in the system, rather than the VAR-computed log rent-price ratio. We compare the performance of indirect inference estimator versus ML estimator in terms of fitting the historical data of state variables and the log rent-price ratio (rp_t), in the upper panel of Table 2.3 for the U.S. national market and in the lower panel for the median metropolitan area. The first row of each panel displays the actually observed standard deviation. The standard deviations of fitted series implied by these two estimators are presented in the second and third row, respectively. The indirect inference estimator implies a standard deviation that is closer to the actual counterpart on all state variables both at the national level and the city level. This comparison suggests that the indirect inference estimator substantially outperforms the ML estimator in the sense that it captures a larger fraction of the variation in state variables and, therefore, yields much more reliable inferences on the impulse response functions in Section 5.

[4] In stock markets, Campbell and Shiller (1988) reject the null hypothesis that the fitted log dividend-price ratio statistically equals the actual counterpart. Instead, they observe substantial unexplained variation in the log dividend-price ratio.

2.4 Volatility Decomposition of Log Rent-Price Ratio

Given the bias-corrected estimates, $\widehat{\Gamma}_0$ and $\widehat{\Gamma}$, we compute the expected present value of all future real interest rates, $\widehat{\mathcal{I}}_t$, the expected present value of all future housing premia, $\widehat{\Pi}_t$, and the expected present value of all future real rent growth, $\widehat{\mathcal{G}}_t$, according to Equations (2.8)-(2.10). The pricing error $\widehat{e}_t$ is obtained through Equation (2.12). In this section, we examine how each of the four components determines the overall volatility of log rent-price ratio.

2.4.1 Volatility Decomposition and Regulations

The linearized present value model discussed in Section 3 decomposes the actual log rent-price ratio into four components (ignoring the constant term k henceforth):

$$rp_t = [\widehat{\mathcal{I}}_t \quad \widehat{\Pi}_t \quad -\widehat{\mathcal{G}}_t \quad \widehat{e}_t]\iota_4, \tag{2.14}$$

where ι_4 is a 4-by-1 column vector of ones. Notice that, since $\widehat{\mathcal{I}}_t$, $\widehat{\Pi}_t$, $\widehat{\mathcal{G}}_t$, and $\widehat{e}_t$ are correlated with one another, it is meaningless to attribute the volatility of log rent-price ratio to these four components by comparing variations in each single component with variations in the log rent-price ratio. Indeed, as shown in Campbell, Davis, Gallin and Martin (2009), the total variations in these four components are more than twice of the variations in the log rent-price ratio over the period 1975-1996, while the covariances among them dampen the fluctuations.

Let $\widehat{\Omega}$ denote the covariance matrix of the above four components. A simple Cholesky decomposition allows us to rewrite the log rent-price ratio as the sum of four orthogonal components:

$$\begin{aligned} rp_t &= [\widehat{\mathcal{I}}'_t \quad \widehat{\Pi}'_t \quad -\widehat{\mathcal{G}}'_t \quad \widehat{e}'_t]P\iota_4, \\ &= [\widehat{\mathcal{I}}'_t \quad \widehat{\Pi}'_t \quad -\widehat{\mathcal{G}}'_t \quad \widehat{e}'_t][P_1 \quad P_2 \quad P_3 \quad P_4]', \end{aligned} \tag{2.15}$$

where P is a 4-by-4 upper triangular matrix which satisfies $P'P = \widehat{\Omega}$, and P_i, $i = 1, ..., 4$, is the sum of all elements in row i of matrix P. Thus,

$$var(rp_t) = P_1^2 var(\widehat{\mathcal{I}}_t') + P_2^2 var(\widehat{\Pi}_t') + P_3^2 var(\widehat{\mathcal{G}}_t') + P_4^2 var(\widehat{e}_t'). \tag{2.16}$$

Notice that each of the orthogonalized terms, $\widehat{\mathcal{I}}_t'$, $\widehat{\Pi}_t'$, $-\widehat{\mathcal{G}}_t'$ and $\widehat{e}_t'$, has a standardized variance of one. Hence P_i^2, $i = 1, ..., 4$, stands for the variation of log rent-price ratio that is attributable to the i^{th} component.

The decomposition results are presented in the left panel of Table 2.4. We also conduct a similar decomposition for the fitted log rent-price ratio and present the results in the right panel. The bottom rows of the table show the correlations of variances and variance shares with both regulation indices.[5]

At the national level, the pricing error accounts for about 82% of the volatility of log rent-price ratio. Expected future real interest rates accounts for 16% and the other two expectation components do not contribute much. At the metropolitan level, the fraction of variation in log rent-price ratio that is attributable to the pricing error is between 35% and 93% across the 23 MSAs, with a median of 69% (see Appendix C for a discussion on pricing error). Expected future real interest rates is the second largest source and accounts for 23% of the overall volatility.

The variation in log rent-price ratio exhibits a significantly positive correlation with both regulation indices, which is in line with Mayer and Somerville (2000), Glaeser, Gyourko and Saks (2005), and Paciorek (2013). In more regulated markets, expected future real interest rates accounts for a significantly larger fraction of volatility of log rent-price ratio and the pricing error accounts for a smaller fraction. Apart from the pricing error, most of the variation in the *fitted* log rent-price ratio is accounted for by the expected future real interest rates, which contributes 70% at the national level and 85% at the metropolitan level. This fraction is much higher than what previous literature

[5] The result of Cholesky decomposition depends on the ordering of the variables. In our framework, $\widehat{\mathcal{I}}_t$ is the estimate of a national-level variable $\mathcal{I}_t$ which does not depend on any local conditions, and $\widehat{e}_t$ is a pricing error which is supposed to depend on all other three components. As a result, we fix $\widehat{\mathcal{I}}_t$ as the first variable and $\widehat{e}_t$ as the last one, and change the ordering of $\widehat{\Pi}_t$ and $\widehat{\mathcal{G}}_t$.

suggests (which does not correct for the bias in the VAR). Again, MSAs with more regulations have a more volatile fitted rent-price ratio, and real interest rate also contributes more to the variance.

There are two potential sources of error in $\widehat{e}_t$: (1) the VAR-computed $\widehat{\mathcal{I}}_t$, $\widehat{\Pi}_t$, and $\widehat{\mathcal{G}}_t$ might not be good estimates of $\mathcal{I}_t$, Π_t, and $\mathcal{G}_t$ under rational expectations and (2) the linearized present value model ignores the second- and higher-order moments. We have tried our best to deal with the former source of error by utilizing the indirect inference estimator and it turns out that the bias correction procedure considerably reduces the pricing error; see a comparison between Figures 2.1 and 2.2. In order to investigate the pricing error that comes from the second source, we conduct a Monte Carlo experiment in Appendix C. Our simulation exercise suggests that both the presence of and the time variation in log rent-price ratio create sizeable pricing error. A necessary condition for the Campbell-Shiller decomposition, which utilizes a first-order Taylor approximation around the steady-state log rent-price ratio, to perform well is that the log rent-price ratio series is relatively stable over time. In our empirical example, however, the log rent-price ratio experienced large fluctuations between 2000 and 2012, during which period the fitted log rent-price ratio largely deviates from the actual counterpart; see Figure 2.1.

Table 2.4: Volatility Decomposition of the Log Rent-Price Ratio

	Variance	Variance Shares (%)						Variance	Variance Shares (%)				
	rp_t	$\widehat{\mathcal{I}}_t$	$\widehat{\Pi}_t(1)$	$\widehat{\Pi}_t(2)$	$\widehat{\mathcal{G}}_t(1)$	$\widehat{\mathcal{G}}_t(2)$	$\widehat{e}_t$	$\widehat{rp}_t$	$\widehat{\mathcal{I}}_t$	$\widehat{\Pi}_t(1)$	$\widehat{\Pi}_t(2)$	$\widehat{\mathcal{G}}_t(1)$	$\widehat{\mathcal{G}}_t(2)$
USA	0.010	15.992	1.297	1.507	0.246	0.036	82.465	0.041	69.361	30.478	23.795	0.161	6.845
Atlanta	0.008	10.788	0.020	5.488	5.800	0.332	83.391	0.007	3.574	92.475	22.434	3.951	73.992
Boston	0.047	25.958	0.875	1.870	2.456	1.462	70.711	0.587	94.498	5.470	0.368	0.032	5.133
Chicago	0.016	11.430	1.769	6.635	5.256	0.390	81.545	0.111	85.502	14.460	1.603	0.038	12.895
Cincinnati	0.007	9.787	3.625	0.043	7.309	10.891	79.280	0.026	96.287	3.533	3.325	0.180	0.388
Cleveland	0.009	0.277	3.431	0.477	3.337	6.291	92.956	0.030	94.230	5.727	1.825	0.044	3.946
Dallas	0.006	4.125	3.458	1.791	0.925	2.592	91.492	0.011	74.747	24.436	3.776	0.817	21.477
Denver	0.019	33.807	1.826	1.204	0.540	1.162	63.826	0.005	0.008	85.217	40.596	14.775	59.396
Detroit	0.044	0.423	1.721	10.555	12.142	3.309	85.713	0.186	74.919	25.065	0.766	0.016	24.315
Honolulu	0.053	51.317	7.285	2.857	6.022	10.451	35.375	0.015	92.955	6.446	1.938	0.600	5.107
Houston	0.010	0.004	8.933	0.725	0.474	8.682	90.589	0.029	95.518	2.681	3.875	1.800	0.606
Kansas City	0.007	2.759	3.252	2.195	0.255	1.313	93.733	0.004	11.222	83.508	87.072	5.270	1.706
Los Angeles	0.046	23.059	7.598	0.067	0.663	8.195	68.679	0.103	34.433	65.010	13.416	0.557	52.150
Miami	0.053	19.118	2.271	13.938	24.944	13.277	53.668	0.031	51.104	48.096	15.976	0.800	32.920
Milwaukee	0.025	36.893	4.514	8.702	6.994	2.806	51.599	0.054	91.368	8.383	6.286	0.250	2.346
Minneapolis	0.032	26.241	0.071	0.007	6.539	6.603	67.149	0.058	78.431	20.889	19.495	0.680	2.074
New York	0.040	17.811	5.015	0.100	2.281	7.195	74.893	1.231	93.715	6.271	1.561	0.013	4.723
Philadelphia	0.030	55.182	1.588	2.635	1.225	0.178	42.006	0.147	92.278	7.607	3.341	0.115	4.381
Pittsburgh	0.012	54.381	4.877	6.463	5.425	3.838	35.318	0.036	95.888	3.673	3.207	0.439	0.905
Portland	0.065	47.671	2.285	6.589	4.348	0.044	45.696	0.113	73.764	26.117	8.680	0.119	17.557
San Diego	0.040	15.560	1.438	5.090	3.688	0.037	79.313	0.135	62.100	36.683	20.391	1.217	17.509
San Francisco	0.059	40.552	0.389	5.018	14.683	10.054	44.376	0.049	30.336	67.886	46.418	1.778	23.247
Seattle	0.059	46.763	9.261	2.053	0.737	7.945	43.240	0.071	87.742	11.739	8.138	0.519	4.120
St. Louis	0.016	41.399	0.831	0.993	3.661	3.499	54.109	0.083	95.453	4.393	4.458	0.153	0.088
Metro Median	0.030	23.059	2.285	2.195	3.688	3.499	68.679	0.054	85.502	14.460	4.458	0.519	5.107
Corr. w . Saks-index	0.747	0.443	0.224	-0.037	0.126	0.357	-0.490	0.476	0.131	-0.110	-0.093	-0.292	-0.113
	[0.000]	[0.017]	[0.152]	[0.434]	[0.283]	[0.047]	[0.009]	[0.011]	[0.276]	[0.309]	[0.337]	[0.088]	[0.304]
Corr. w . WRLURI	0.697	0.537	0.126	0.087	0.135	0.191	-0.569	0.236	0.049	-0.054	-0.180	0.016	0.101
	[0.000]	[0.004]	[0.283]	[0.347]	[0.270]	[0.192]	[0.002]	[0.139]	[0.413]	[0.404]	[0.205]	[0.471]	[0.323]

Note: (1) and (2) correspond to the two types of ordering. P-values in brackets.

2.5 House Price Volatility and the Monetary Shock

In this section, we examine the impact of a shock to real interest rate on housing markets and the local heterogeneity in the transmission of a monetary policy.

2.5.1 Orthogonal Impulse Response Functions

After obtaining the bias-corrected estimates of Σ_1 and Σ_2, labeled $\widehat{\Sigma}_1$ and $\widehat{\Sigma}_2$, we can derive the responses of $\widehat{\mathbf{Z}}_{t+\tau}$, $\tau \geq 0$, to a one-unit shock in $\mathbf{Z}_t$ as:

$$\widehat{IR}_{\mathbf{Z},\mathbf{Z},t+\tau} = \frac{\partial \widehat{\mathbf{Z}}_{t+\tau}}{\partial \widehat{\epsilon}_t'} = \begin{cases} \mathbf{I} & \text{if } \tau = 0, \\ \widehat{\Sigma}_1 & \text{if } \tau = 1, \\ \frac{\partial \widehat{\mathbf{Z}}_{t+\tau-1}}{\partial \widehat{\epsilon}_t'} \widehat{\Sigma}_1 + \frac{\partial \widehat{\mathbf{Z}}_{t+\tau-2}}{\partial \widehat{\epsilon}_t'} \widehat{\Sigma}_2 & \text{if } \tau \geq 2, \end{cases} \tag{2.17}$$

where the three subscripts of the $K \times K$ impulse response matrix ("$\widehat{IR}$") stand for response variables, impulse variables, and τ-period ahead prediction.

The Cholesky decomposition allows us to focus exclusively on the impact of a shock to one of the state variables, holding all others constant. The orthogonal impulse response functions are obtained by post-multiplying the impulse responses in Equation (2.17) with a lower triangular matrix $\widehat{P}$ satisfying $\widehat{\Sigma}_\epsilon = \widehat{P}\widehat{P}'$, where $\widehat{\Sigma}_\epsilon$ is the covariance matrix of the residual, $\widehat{\epsilon}$, i.e.,

$$\widehat{OIR}_{\mathbf{Z},\mathbf{Z},t+\tau} = \widehat{IR}_{\mathbf{Z},\mathbf{Z},t+\tau}\widehat{P}. \tag{2.18}$$

The j^{th}, $j = 1, ..., K$, column of the orthogonal impulse response matrix ("$\widehat{OIR}$") represents the responses of all K state variables to an orthogonal impulse in the j^{th} variable. The responses to an orthogonal impulse in real interest rate are represented by the first column of the orthogonal impulse response matrix and hence can be written as:

$$\widehat{OIR}_{\mathbf{Z},i,t+\tau} = \widehat{OIR}_{\mathbf{Z},\mathbf{Z},t+\tau} e_1, \tag{2.19}$$

the j^{th}, $j = 1, ..., K$, row of which represents the response of j^{th} state variable to an orthogonal impulse in real interest rate, e.g.,

$$\widehat{OIR}_{z_j,i,t+\tau} = e_j' \widehat{OIR}_{\mathbf{Z},i,t+\tau}, \tag{2.20}$$

with

$$z_1 = i, z_2 = \pi, z_3 = \Delta r.$$

Here, e_j, $j = 1, 2, 3$, is a column vector of dimension K with one as the j^{th} element, and zeros otherwise.

Based on the response functions of state variables to an orthogonal impulse in real interest rate, we are able to obtain the orthogonal impulse response functions of VAR-computed variables, including the expected future real interest rates $\widehat{\mathcal{I}}_t$, expected future real housing premia $\widehat{\Pi}_t$, expected future real rent growth $\widehat{\mathcal{G}}_t$, VAR-computed log rent-price ratio $\widehat{rp}_t$, log real housing rent $\widehat{r}_t$, and log real house price $\widehat{p}_t$; see Appendix D for a detailed derivation.

The standard error estimates of the orthogonal impulse responses are produced by bootstrapping from 1,000 simulated realizations. More specifically, we generate 1,000 replications $\widehat{OIR}^*_{\mathbf{Z},i,t+\tau}$ of the orthogonal impulse response estimates for state variables, conditional on $\widehat{\Gamma}_0$, $\widehat{\Gamma}$ and the covariance matrix of $\widehat{\epsilon}_t$, as though they were the population values. Each bootstrap sample of the residuals is drawn from a joint normal distribution. Replications $\widehat{OIR}^*_{\mathcal{I},i,t+\tau}$, $\widehat{OIR}^*_{\Pi,i,t+\tau}$, $\widehat{OIR}^*_{\mathcal{G},i,t+\tau}$, $\widehat{OIR}^*_{rp,i,t+\tau}$, $\widehat{OIR}^*_{r,i,t+\tau}$, and $\widehat{OIR}^*_{p,i,t+\tau}$ are derived based on Equations (2.28) through (2.33) in the appendix. It is worth noting that the bootstrapped standard error estimates for VAR-computed variables are usually large due to the presence of uncertainty, both in the coefficient matrices and in the state variables (see Campbell and Shiller (1987), Engsted and Tanggaard (2001), and Campbell and Vuolteenaho (2004)). As in Campbell and Vuolteenaho (2004), we bootstrap the standard error estimates by generating orthogonal impulse responses for the VAR-computed variables within each simulated realization conditional on the estimated coefficient matrix.[6]

2.5.2 Impulse Responses and Regulations

Since we use data at semi-annual frequency, each period stands for half a year. Table 2.5 shows the 0- to 8-period ahead orthogonal impulse responses of the fitted log real rent and the fitted log real price to a one-standard-deviation real interest rate shock for the U.S. national market and metropolitan areas (with the bootstrapped standard errors in parenthesis). In most markets, the fitted log real rent does not significantly respond to the real interest rate shock but the fitted log real price responds negatively. The response of log real price has a median of -0.043. Dallas (TX) and Houston (TX) are the only two exceptions where a raise in real interest rate induces a significant increase in house price.[7]

We present the correlation of the impulse responses of log real rent and log real price with housing regulations in Table 2.6. The impulse responses of log real price exhibit a negative correlation with both regulation indices, indicating that house price is more responsive to a change in real interest rate in more regulated markets. The result is robust when we discard the recent recession and focus particularly on the period 1978H2:2006H2. The intuition of this result is straightforward: housing regulations lower the the price elasticity of housing supply so that house price becomes more sensitive to an interest rate shock.

[6] Such standard error estimates do not incorporate full estimation of uncertainty of the impulse responses of VAR-computed variables. In order to capture full estimation uncertainty, one should estimate the VAR and the orthogonal impulse responses separately for each simulated realization.

[7] Dallas (TX) and Houston (TX) are the only two housing markets that experienced a large price fall in 1980s and have not fully recovered. It is reasonable that the linearized present value model fails to capture the sharp fluctuation in these markets, since the model depends on a first-order approximation. If the data for the 1980s are discarded, we are able to obtain more sensible results.

Table 2.5: Orthogonal Impulse Responses to a Real Interest Rate Shock

Response Variable	**Panel (1): Log Real Rent**								
Horizon (half a year)	0	1	2	3	4	5	6	7	8
USA	-0.003 (0.001)	-0.004 (0.002)	-0.004 (0.002)	-0.004 (0.003)	-0.003 (0.003)	-0.002 (0.003)	-0.001 (0.004)	0.000 (0.004)	0.001 (0.004)
Atlanta	-0.003 (0.002)	-0.005 (0.003)	-0.006 (0.004)	-0.005 (0.005)	-0.004 (0.005)	-0.002 (0.006)	0.000 (0.007)	0.002 (0.007)	0.005 (0.008)
Boston	-0.002 (0.002)	-0.001 (0.003)	-0.001 (0.003)	0.000 (0.004)	0.001 (0.005)	0.002 (0.005)	0.003 (0.006)	0.004 (0.006)	0.006 (0.007)
Chicago	-0.002 (0.001)	-0.003 (0.002)	-0.003 (0.003)	-0.002 (0.003)	-0.001 (0.003)	0.000 (0.004)	0.002 (0.004)	0.003 (0.005)	0.005 (0.005)
Cincinnati	-0.002 (0.002)	-0.005 (0.002)	-0.006 (0.003)	-0.006 (0.003)	-0.006 (0.003)	-0.005 (0.004)	-0.004 (0.004)	-0.003 (0.005)	-0.003 (0.005)
Cleveland	-0.004 (0.002)	-0.005 (0.003)	-0.005 (0.003)	-0.004 (0.004)	-0.003 (0.004)	-0.001 (0.005)	0.000 (0.005)	0.002 (0.006)	0.003 (0.006)
Dallas	-0.003 (0.002)	-0.004 (0.003)	-0.004 (0.003)	-0.004 (0.004)	-0.003 (0.004)	-0.003 (0.005)	-0.002 (0.005)	-0.002 (0.006)	-0.002 (0.007)
Denver	-0.003 (0.002)	-0.003 (0.003)	-0.003 (0.004)	-0.004 (0.005)	-0.004 (0.005)	-0.005 (0.006)	-0.006 (0.007)	-0.007 (0.007)	-0.008 (0.008)
Detroit	-0.002 (0.002)	-0.006 (0.003)	-0.006 (0.003)	-0.007 (0.004)	-0.007 (0.004)	-0.007 (0.005)	-0.006 (0.005)	-0.006 (0.006)	-0.005 (0.006)
Honolulu	-0.002 (0.002)	-0.002 (0.003)	-0.002 (0.004)	-0.001 (0.004)	-0.001 (0.005)	0.000 (0.006)	0.001 (0.006)	0.003 (0.007)	0.004 (0.008)
Houston	-0.004 (0.002)	-0.002 (0.003)	-0.001 (0.004)	-0.001 (0.005)	-0.001 (0.005)	-0.003 (0.006)	-0.004 (0.007)	-0.006 (0.007)	-0.008 (0.008)
Kansas City	-0.002 (0.002)	-0.001 (0.003)	0.000 (0.003)	0.002 (0.003)	0.003 (0.004)	0.005 (0.004)	0.007 (0.005)	0.008 (0.005)	0.009 (0.006)
Los Angeles	-0.002 (0.002)	-0.003 (0.003)	-0.003 (0.003)	-0.003 (0.004)	-0.002 (0.004)	-0.001 (0.005)	0.000 (0.005)	0.001 (0.006)	0.002 (0.006)
Miami	-0.002 (0.002)	-0.003 (0.003)	-0.006 (0.003)	-0.007 (0.004)	-0.009 (0.004)	-0.011 (0.005)	-0.013 (0.006)	-0.015 (0.006)	-0.016 (0.007)
Milwaukee	-0.002 (0.002)	-0.003 (0.002)	-0.002 (0.003)	0.000 (0.003)	0.002 (0.004)	0.004 (0.004)	0.006 (0.005)	0.007 (0.005)	0.009 (0.006)
Minneapolis	-0.003 (0.002)	-0.004 (0.003)	-0.004 (0.003)	-0.004 (0.004)	-0.002 (0.004)	-0.001 (0.005)	0.000 (0.005)	0.001 (0.006)	0.003 (0.006)
New York	-0.004 (0.002)	-0.006 (0.002)	-0.007 (0.003)	-0.008 (0.003)	-0.007 (0.004)	-0.007 (0.004)	-0.006 (0.005)	-0.005 (0.005)	-0.003 (0.006)
Philadelphia	-0.001 (0.002)	-0.001 (0.002)	0.000 (0.003)	0.001 (0.003)	0.003 (0.004)	0.005 (0.004)	0.008 (0.005)	0.010 (0.005)	0.012 (0.006)
Pittsburgh	-0.004 (0.002)	-0.007 (0.002)	-0.007 (0.003)	-0.007 (0.003)	-0.006 (0.004)	-0.006 (0.004)	-0.005 (0.005)	-0.004 (0.005)	-0.004 (0.006)
Portland	-0.003 (0.002)	-0.006 (0.003)	-0.009 (0.003)	-0.011 (0.003)	-0.012 (0.004)	-0.013 (0.005)	-0.013 (0.005)	-0.014 (0.006)	-0.014 (0.006)
San Diego	-0.003 (0.002)	-0.005 (0.003)	-0.006 (0.004)	-0.005 (0.004)	-0.004 (0.005)	-0.002 (0.006)	0.000 (0.006)	0.003 (0.007)	0.006 (0.007)
San Francisco	-0.002 (0.002)	-0.004 (0.003)	-0.005 (0.004)	-0.004 (0.005)	-0.003 (0.005)	-0.001 (0.006)	0.001 (0.007)	0.004 (0.007)	0.006 (0.008)
Seattle	-0.003 (0.002)	-0.005 (0.003)	-0.007 (0.003)	-0.010 (0.004)	-0.012 (0.005)	-0.014 (0.005)	-0.015 (0.006)	-0.017 (0.007)	-0.018 (0.007)
St. Louis	-0.001 (0.002)	-0.001 (0.002)	-0.001 (0.003)	0.000 (0.003)	0.001 (0.004)	0.002 (0.004)	0.003 (0.005)	0.004 (0.005)	0.005 (0.006)
Metro Median	-0.002 (0.002)	-0.004 (0.003)	-0.004 (0.003)	-0.004 (0.004)	-0.003 (0.004)	-0.002 (0.005)	0.000 (0.005)	0.001 (0.006)	0.003 (0.006)

Table 2.5 : Orthogonal Impulse Responses to a Real Interest Rate Shock, Continued

Response Variable					**Panel (2): Log Real Price**				
Horizon (half a year)	0	1	2	3	4	5	6	7	8
USA	-0.017 (0.014)	-0.021 (0.016)	-0.025 (0.012)	-0.028 (0.011)	-0.030 (0.010)	-0.032 (0.010)	-0.033 (0.010)	-0.034 (0.011)	-0.034 (0.011)
Atlanta	0.014 (0.008)	0.014 (0.008)	0.013 (0.007)	0.014 (0.007)	0.014 (0.007)	0.015 (0.007)	0.015 (0.007)	0.016 (0.007)	0.017 (0.008)
Boston	-0.191 (0.052)	-0.199 (0.057)	-0.205 (0.038)	-0.209 (0.034)	-0.211 (0.033)	-0.212 (0.035)	-0.212 (0.037)	-0.210 (0.040)	-0.208 (0.043)
Chicago	-0.061 (0.023)	-0.066 (0.026)	-0.071 (0.018)	-0.074 (0.016)	-0.077 (0.016)	-0.078 (0.016)	-0.079 (0.017)	-0.079 (0.018)	-0.079 (0.020)
Cincinnati	-0.043 (0.011)	-0.049 (0.012)	-0.053 (0.009)	-0.055 (0.008)	-0.056 (0.008)	-0.056 (0.009)	-0.055 (0.009)	-0.054 (0.010)	-0.052 (0.011)
Cleveland	-0.042 (0.010)	-0.050 (0.012)	-0.055 (0.009)	-0.059 (0.008)	-0.060 (0.008)	-0.060 (0.009)	-0.059 (0.010)	-0.057 (0.011)	-0.055 (0.012)
Dallas	0.040 (0.006)	0.040 (0.007)	0.038 (0.005)	0.038 (0.005)	0.037 (0.006)	0.036 (0.006)	0.035 (0.007)	0.034 (0.007)	0.033 (0.008)
Denver	0.010 (0.006)	0.011 (0.007)	0.011 (0.005)	0.011 (0.005)	0.011 (0.005)	0.010 (0.006)	0.009 (0.006)	0.008 (0.007)	0.007 (0.007)
Detroit	-0.074 (0.031)	-0.085 (0.037)	-0.090 (0.028)	-0.098 (0.025)	-0.102 (0.024)	-0.107 (0.025)	-0.109 (0.026)	-0.112 (0.028)	-0.113 (0.029)
Honolulu	-0.035 (0.008)	-0.036 (0.009)	-0.040 (0.007)	-0.039 (0.007)	-0.039 (0.007)	-0.037 (0.007)	-0.036 (0.008)	-0.034 (0.009)	-0.032 (0.010)
Houston	0.059 (0.011)	0.061 (0.013)	0.063 (0.009)	0.063 (0.008)	0.061 (0.009)	0.059 (0.010)	0.055 (0.011)	0.051 (0.012)	0.047 (0.013)
Kansas City	0.007 (0.005)	0.005 (0.007)	0.004 (0.005)	0.003 (0.004)	0.003 (0.004)	0.003 (0.005)	0.003 (0.005)	0.004 (0.005)	0.005 (0.005)
Los Angeles	-0.009 (0.034)	-0.010 (0.033)	-0.011 (0.018)	-0.012 (0.016)	-0.013 (0.015)	-0.014 (0.015)	-0.015 (0.015)	-0.016 (0.016)	-0.017 (0.016)
Miami	-0.025 (0.017)	-0.026 (0.019)	-0.030 (0.011)	-0.032 (0.011)	-0.034 (0.010)	-0.036 (0.011)	-0.038 (0.011)	-0.039 (0.011)	-0.040 (0.011)
Milwaukee	-0.049 (0.015)	-0.056 (0.017)	-0.062 (0.012)	-0.066 (0.011)	-0.068 (0.011)	-0.069 (0.011)	-0.069 (0.012)	-0.067 (0.013)	-0.065 (0.014)
Minneapolis	-0.043 (0.019)	-0.044 (0.022)	-0.048 (0.016)	-0.050 (0.013)	-0.051 (0.012)	-0.052 (0.013)	-0.053 (0.013)	-0.053 (0.014)	-0.052 (0.014)
New York	-0.272 (0.076)	-0.281 (0.086)	-0.289 (0.060)	-0.296 (0.052)	-0.300 (0.051)	-0.303 (0.052)	-0.305 (0.056)	-0.306 (0.060)	-0.306 (0.064)
Philadelphia	-0.082 (0.024)	-0.090 (0.027)	-0.097 (0.019)	-0.103 (0.017)	-0.106 (0.017)	-0.107 (0.018)	-0.107 (0.019)	-0.106 (0.021)	-0.105 (0.022)
Pittsburgh	-0.053 (0.013)	-0.060 (0.015)	-0.066 (0.010)	-0.069 (0.009)	-0.070 (0.009)	-0.069 (0.010)	-0.068 (0.011)	-0.067 (0.012)	-0.065 (0.013)
Portland	-0.059 (0.028)	-0.064 (0.032)	-0.069 (0.022)	-0.072 (0.019)	-0.075 (0.018)	-0.078 (0.018)	-0.080 (0.018)	-0.081 (0.019)	-0.082 (0.020)
San Diego	-0.051 (0.030)	-0.053 (0.032)	-0.056 (0.020)	-0.059 (0.018)	-0.062 (0.017)	-0.065 (0.017)	-0.067 (0.018)	-0.068 (0.019)	-0.069 (0.020)
San Francisco	0.003 (0.021)	0.000 (0.022)	-0.001 (0.013)	-0.002 (0.011)	-0.004 (0.010)	-0.007 (0.009)	-0.009 (0.009)	-0.011 (0.009)	-0.012 (0.009)
Seattle	-0.071 (0.021)	-0.075 (0.023)	-0.079 (0.015)	-0.082 (0.014)	-0.084 (0.014)	-0.085 (0.015)	-0.086 (0.016)	-0.086 (0.017)	-0.085 (0.019)
St. Louis	-0.074 (0.020)	-0.078 (0.022)	-0.083 (0.015)	-0.085 (0.013)	-0.087 (0.013)	-0.086 (0.014)	-0.086 (0.015)	-0.084 (0.016)	-0.083 (0.018)
Metro Median	-0.043 (0.019)	-0.050 (0.022)	-0.055 (0.013)	-0.059 (0.011)	-0.060 (0.011)	-0.060 (0.011)	-0.059 (0.012)	-0.057 (0.013)	-0.055 (0.014)

Previous literature finds that supply-side regulations play an important role in housing markets (see the discussion in the Introduction); observed regulations can explain a large fraction of differences in the volatility of house price between a highly regulated city and a relatively unregulated city, even when they face identical demand shocks. Using the real interest rate shock as the common demand shock to all local housing markets, our results are consistent with this argument.

Table 2.6: Correlation between Impulse Responses and Regulations

Sample Period	Response Variable	Regulation Index	Horizon (half a year) 0	1	2	3	4	5	6	7	8
78H2:14H2	Log Real Rent	Saks-index	-0.183	-0.252	-0.348	-0.356	-0.337	-0.285	-0.223	-0.162	-0.106
			[0.202]	0.123	[0.052]	0.048	[0.058]	0.094	[0.154]	0.230	[0.315]
		WRLURI	0.033	0.156	0.018	-0.033	-0.081	-0.083	-0.074	-0.054	-0.034
			[0.441]	0.239	[0.468]	0.441	[0.356]	0.354	[0.368]	0.403	[0.440]
	Log Real Price	Saks-index	-0.487	-0.480	-0.476	-0.472	-0.474	-0.476	-0.481	-0.485	-0.489
			[0.009]	0.010	[0.011]	0.011	[0.011]	0.011	[0.010]	0.010	[0.009]
		WRLURI	-0.318	-0.308	-0.307	-0.300	-0.299	-0.297	-0.297	-0.296	-0.296
			[0.070]	0.076	[0.077]	0.082	[0.083]	0.084	[0.084]	0.085	[0.085]
78H2:06H2	Log Real Rent	Saks-index	0.257	0.000	-0.180	-0.224	-0.209	-0.162	-0.105	-0.049	0.005
			[0.118]	0.500	[0.206]	0.152	[0.169]	0.229	[0.317]	0.413	[0.490]
		WRLURI	0.297	0.346	0.143	0.029	-0.047	-0.073	-0.077	-0.064	-0.047
			[0.085]	0.053	[0.257]	0.447	[0.415]	0.371	[0.364]	0.386	[0.415]
	Log Real Price	Saks-index	-0.246	-0.263	-0.289	-0.314	-0.358	-0.402	-0.447	-0.481	-0.496
			[0.129]	0.113	[0.091]	0.072	[0.047]	0.029	[0.016]	0.010	[0.008]
		WRLURI	-0.043	-0.072	-0.094	-0.111	-0.134	-0.157	-0.180	-0.196	-0.203
			[0.423]	0.371	[0.334]	0.307	[0.271]	0.237	[0.206]	0.185	[0.176]

Note: p-values in brackets.

2.6 Conclusion

We find that the uneven impacts of monetary policy are highly correlated with the supply regulations in the local housing markets. The more regulated the housing market, the larger its response to a real interest rate shock. While it is reasonable to assume that regulations are exogenously determined within our empirical framework, a strong correlation does not imply causality. We have not showed that housing supply elasticity is the only factor behind the uneven impacts of monetary policy. Also, within the linearized Campbell-Shiller framework, we have not considered how regulations can contribute to the second-order movements of the log rent-price ratio. We leave these issues for future research.

2.7 Appendix

2.7.1 Campbell-Shiller Decomposition

Starting from the realized log gross return to housing defined in Equation (2.1) and using lowercase letters for logs, we have

$$\begin{aligned}
\phi_{t+1} &= \ln(P_{t+1} + R_{t+1}) - \ln(P_t), \\
&= p_{t+1} + \ln\left(1 + \frac{R_{t+1}}{P_{t+1}}\right) - p_t, \\
&= p_{t+1} - p_t + \ln(1 + \exp(r_{t+1} - p_{t+1})).
\end{aligned} \tag{2.21}$$

Applying a Taylor approximation to the function $f(x) = \ln(1 + \exp(x))$ around $x = \overline{r - p}$ yields

$$\ln(1 + \exp(r_{t+1} - p_{t+1})) = \sum_{n=0}^{\infty} \frac{f^{(n)}(\overline{r-p})}{n!}(r_{t+1} - p_{t+1} - \overline{r-p})^n, \tag{2.22}$$

or equivalently,

$$\ln(1+\exp(r_{t+1}-p_{t+1})) = [-\ln(\rho) - (1-\rho)\ln(1/\rho - 1)] + (1-\rho)(r_{t+1}-p_{t+1}) + O((r_{t+1}-p_{t+1})^2), \tag{2.23}$$

where $\rho = (1 + \exp(\overline{r-p}))^{-1} \in (0, 1)$. The last term $O((r_{t+1} - p_{t+1})^2)$ is positive, since its sign is determined by the second-order derivative of $f(x)$,

$$f^{(2)}(x) = \frac{\exp(x)}{(1+\exp(x))^2} > 0,$$

Substituting Equation (2.23) into (2.21) yields:

$$\phi_{t+1} = p_{t+1} - p_t + [-\ln(\rho) - (1-\rho)\ln(1/\rho - 1)] + (1-\rho)(r_{t+1} - p_{t+1}) + O((r_{t+1} - p_{t+1})^2), \tag{2.24}$$

and

$$p_t = \rho p_{t+1} + [-\ln(\rho) - (1-\rho)\ln(1/\rho - 1)] + (1-\rho)r_{t+1} - \phi_{t+1} + O((r_{t+1} - p_{t+1})^2). \tag{2.25}$$

Iterating Equation(2.25) forward yields:

$$p_t = \frac{-\ln(\rho) - (1-\rho)\ln(1/\rho - 1)}{1-\rho} + \sum_{j=0}^{\infty} \rho^j((1-\rho)r_{t+1+j} - \phi_{t+1+j}) + \frac{O((r_{t+1} - p_{t+1})^2)}{1-\rho}, \tag{2.26}$$

and

$$r_t - p_t = \frac{\ln(\rho) + (1-\rho)\ln(1/\rho - 1)}{1-\rho} + \sum_{j=0}^{\infty} \rho^j(\phi_{t+1+j} - \Delta r_{t+1+j}) - \frac{O((r_{t+1} - p_{t+1})^2)}{1-\rho}, \tag{2.27}$$

which gives Equation (2.2) with a negative approximation bias under rational expectations.

2.7.2 Estimation without Bias Correction

Table 2.7: Summary of ML Estimation Results

Dependent Variable		Coefficient on the 1st Lag of					Coefficient on the 2nd Lag of					$\overline{R}^2$
		i	π	Δr	π^{US}	Δr^{US}	i	π	Δr	π^{US}	Δr^{US}	
i	25th Percentile	1.163	-	-	0.009	-0.025	-0.266	-	-	-0.002	-0.060	0.875
	Median	1.177	-	-	0.011	-0.006	-0.254	-	-	0.002	-0.049	0.876
	75th Percentile	1.187	-	-	0.020	0.015	-0.241	-	-	0.005	-0.031	0.877
π	25th Percentile	-1.449	0.269	-0.205	-	-	-0.487	0.150	-0.089	-	-	0.258
	Median	-0.719	0.399	-0.009	-	-	0.587	0.214	0.157	-	-	0.398
	75th Percentile	-0.147	0.476	0.265	-	-	1.455	0.304	0.354	-	-	0.461
Δr	25th Percentile	-0.374	0.006	0.208	-	-	0.124	-0.001	0.011	-	-	0.044
	Median	-0.115	0.031	0.306	-	-	0.373	0.056	0.128	-	-	0.176
	75th Percentile	0.051	0.045	0.350	-	-	0.663	0.077	0.207	-	-	0.236
π^{US}	25th Percentile	-0.779	-	-	0.372	-0.293	0.121	-	-	0.122	0.023	0.258
	Median	-0.509	-	-	0.472	-0.091	0.361	-	-	0.196	0.149	0.273
	75th Percentile	-0.386	-	-	0.541	0.048	0.565	-	-	0.269	0.279	0.298
Δr^{US}	25th Percentile	-0.346	-	-	-0.112	0.356	0.396	-	-	0.087	-0.131	0.106
	Median	-0.301	-	-	-0.082	0.455	0.519	-	-	0.108	-0.071	0.133
	75th Percentile	-0.202	-	-	-0.060	0.544	0.567	-	-	0.129	-0.029	0.145
Largest autoregressive root (median across 23 MSAs): 0.832												

Note: The dash symbol implies a constraint of a relevant coefficient of zero imposed on the VAR system.

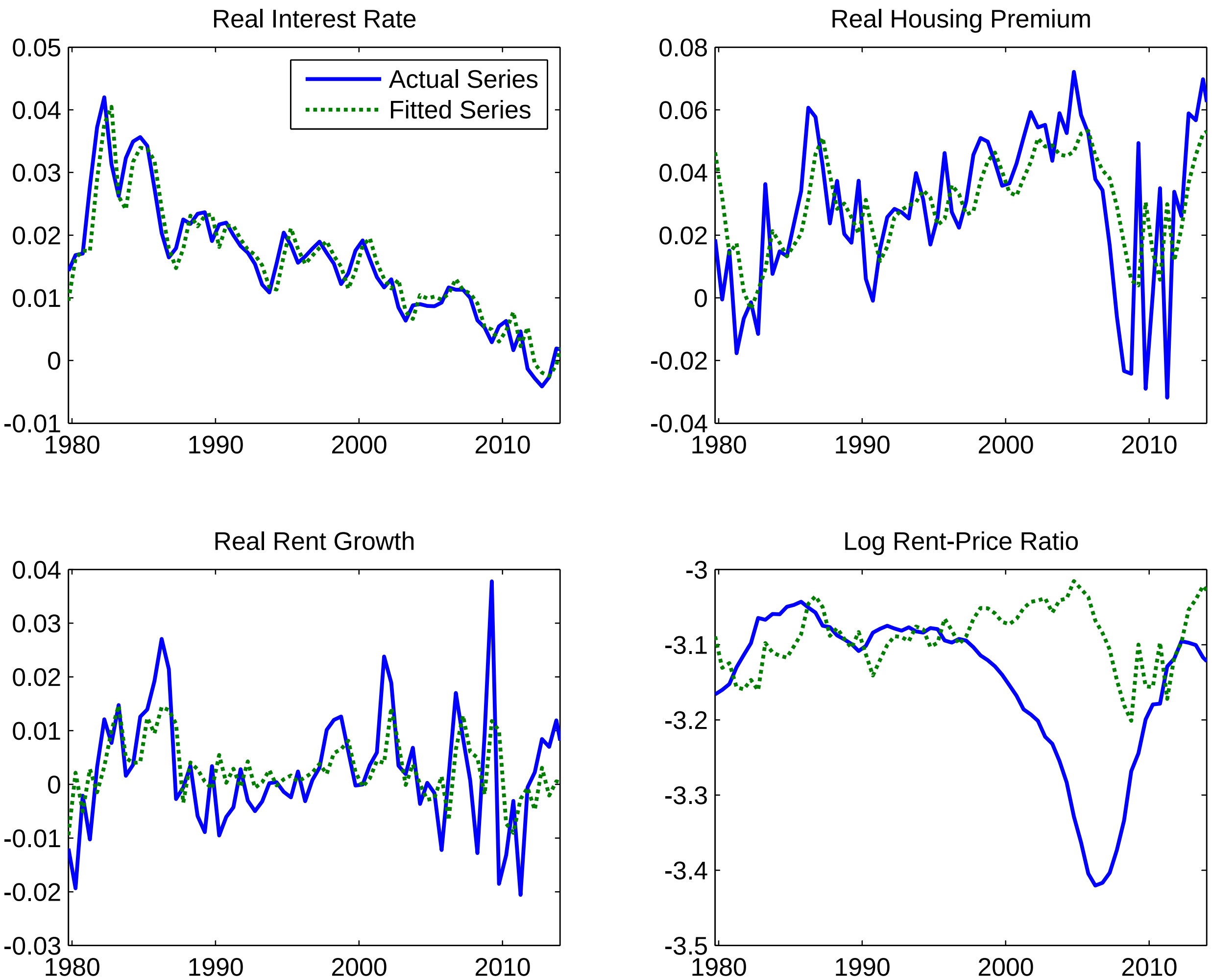


Figure 2.2: Actual vs. Predicted Series in the US National Market, ML Estimation Results

2.7.3 Sources of the Pricing Error

There are two potential sources of error in $\widehat{e}_t$: (1) the VAR-computed $\widehat{\mathcal{I}}_t$, $\widehat{\Pi}_t$, and $\widehat{\mathcal{G}}_t$ might not be good estimates of $\mathcal{I}_t$, Π_t, and $\mathcal{G}_t$ under rational expectations and (2) the linearized present value model ignores the second- and higher-order moments. We deal with the first source of error by utilizing the indirect inference estimator, and it turns out that the bias correction procedure considerably reduces the pricing error; see a comparison between Figures 2.1 and 2.2. In order to investigate the pricing error that comes from the second source, we conduct a Monte Carlo experiment for the U.S. national market by repeating the following steps 1000 times:

1. We keep i_t and Δr_t as in the original sample and construct a new series of log rent-price ratio with small fluctuations around its sample mean, $rp'_t = \overline{rp} + \nu_t$, where $\nu_t \sim \mathcal{N}(0, \sigma_\nu^2)$. Then, using rp'_t we back out the relevant housing premium π'_t.

2. We estimate the VAR with $\mathbf{Z}_t = (i_t, \pi'_t, \Delta r_t)$ and compute the pricing error, $\widehat{e'_t} = rp'_t - \widehat{rp'_t}$.

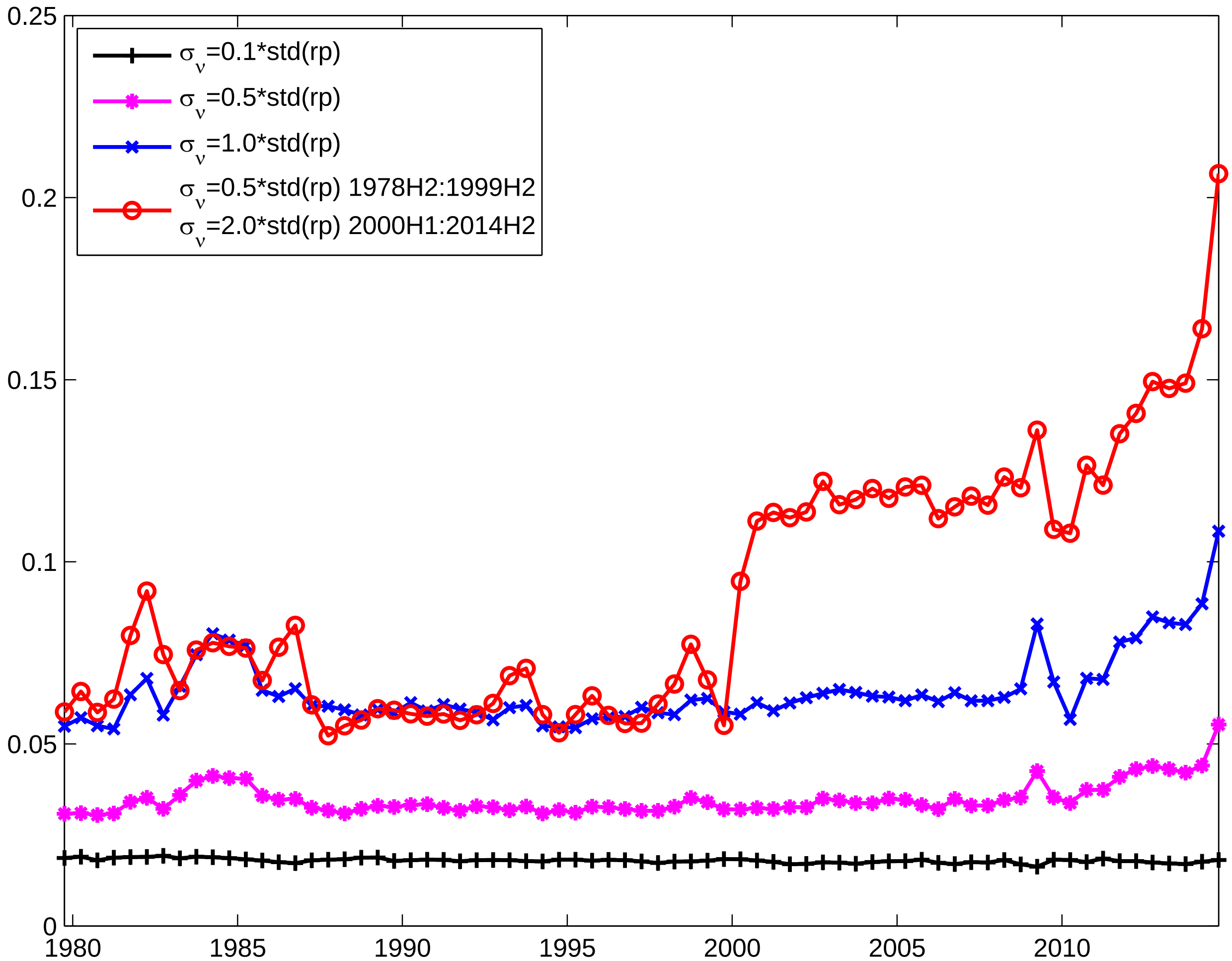


Figure 2.3: Simulated Pricing Errors

Figure 2.3 compares the average absolute value of the pricing error over 1000 repetitions when σ_ν takes on different values. The black line shows that, when σ_ν equals one tenth of the standard error of the actually observed log rent-price ratio, the pricing error is small and relatively stable. When we increase σ_v to one half of the observed standard error (the purple line), the pricing error becomes larger in magnitude and the series starts to vary over time. As the log rent-price ratio becomes as volatile as the observed series (the blue line), both the first and the second moments of the pricing error increase dramatically.

However, the constant standard error can hardly replicate the model-implied pricing error. Therefore, we assign different standard errors to the simulated log rent-price ratio for the earlier period 1978H2:1999H2 and the later period 2000H1:2014H2. As the red line shows, the pricing error shoots up once the simulated log rent-price ratio becomes more volatile.

This simulation exercise suggests that both the presence and the time variation of the second moment in the log rent-price ratio contribute to a sizeable pricing error. A necessary condition for the Campbell-Shiller decomposition, which utilizes a first-order Taylor approximation around the steady-state log rent-price ratio, to fit the data well is that the log rent-price ratio is relatively stable over time. In the data, however, the log rent-price ratio experienced large fluctuations between 2000 and 2012, during which period the fitted log rent-price ratio largely deviates from the actual counterpart; see Figure 2.1.

2.7.4 Orthogonal Impulse Response Functions

Given the response functions of state variables to an orthogonal impulse in real interest rate, those for VAR-computed $\widehat{\mathcal{I}}_t$, $\widehat{\Pi}_t$, and $\widehat{\mathcal{G}}_t$ are readily available via a linear combination. According to Equations (2.8) - (2.10), the responses of $\widehat{\mathcal{I}}_t$, $\widehat{\Pi}_t$, and $\widehat{\mathcal{G}}_t$ to an orthogonal impulse in real interest rate are:

$$\widehat{OIR}_{\mathfrak{z}_j,i,t+\tau} = \begin{cases} e_j'(\mathbf{I}-\rho\widehat{\Gamma})^{-1}\widehat{\Gamma}\left[\widehat{OIR}'_{\mathbf{Z},i,t+\tau} \quad \mathbf{0}\right]' & \text{if } \tau = 0, \\ e_j'(\mathbf{I}-\rho\widehat{\Gamma})^{-1}\widehat{\Gamma}\left[\widehat{OIR}'_{\mathbf{Z},i,t+\tau} \quad \widehat{OIR}'_{\mathbf{Z},i,t+\tau-1}\right]' & \text{if } \tau > 0, \end{cases} \tag{2.28}$$

with $\mathfrak{z}_1 = \mathcal{I}$, $\mathfrak{z}_2 = \Pi$, and $\mathfrak{z}_3 = \mathcal{G}$, which infer the response of the VAR-computed log rent-price ratio to an orthogonal impulse in real interest rate as,

$$\widehat{OIR}_{rp,i,t+\tau} = \widehat{OIR}_{\mathcal{I},i,t+\tau} + \widehat{OIR}_{\Pi,i,t+\tau} - \widehat{OIR}_{\mathcal{G},i,t+\tau}. \tag{2.29}$$

Given the log real rent at time $t-1$, r_{t-1}, the τ-period ahead prediction of log real rent with $\tau \geq 0$ can be expressed as:

$$\widehat{r}_{t+\tau} = r_{t-1} + \sum_{s=0}^{\tau} \widehat{\Delta r}_{t+s}, \tag{2.30}$$

and τ-period prediction of log real price takes the following form:

$$\widehat{p}_{t+\tau} = \widehat{r}_{t+\tau} - \widehat{rp}_{t+\tau}. \tag{2.31}$$

Then, we are able to obtain the τ-period ahead responses of the predicted log real rent and the predicted log real price to an orthogonal interest rate shock at time t,

$$\widehat{OIR}_{r,i,t+\tau} = \sum_{s=0}^{\tau} \widehat{OIR}_{\Delta r,i,t+s}, \tag{2.32}$$

$$\widehat{OIR}_{p,i,t+\tau} = \sum_{s=0}^{\tau} \widehat{OIR}_{\Delta r,i,t+s} - \widehat{OIR}_{rp,i,t+\tau}. \tag{2.33}$$

3 Chapter 3

Optimal Interest Rate Rule in a DSGE Model with Housing Market Spillovers

Abstract

This paper studies the optimal interest rate rule in a DSGE model with housing market spillovers (Iacoviello and Neri (2010)). We find that the optimal rule responds to house price inflation even when the stabilization of house price is not among the objectives of the policymaker, and that the strength of the response depends crucially on a few structural parameters.

Keywords: Optimal Interest Rate Rule, DSGE Model, House Price Inflation

JEL Classification: C68, E52, E58

3.1 Introduction

Since Taylor (1993), there is an expanding literature on monetary policy rule from either a positive (what the central bank does) or a normative (what the central bank should do) perspective. One normative question that arises is how the central bank, if at all, should respond to movements in house prices (or asset prices in general).

House prices are related to a number of macroeconomic variables, such as consumption and investment, over the business cycle. Although house price movements convey important information on economic activities, stabilizing house prices is not among the mandated objectives of central banks. In a large body of empirical work for models with a housing market, the monetary policy rule is usually specified to be responding only to money inflation and output gap (see Iacoviello (2005), Edge, Kiley and Laforte (2007) and Iacoviello and Neri (2010)).

Should the central bank react to house prices? The two opposing answers to this question are “leaning against asset-price bubbles" versus “cleaning up after the bubble bursts" (“lean" versus “clean"). Some argue that central banks should lean against surges in asset prices to unsustainable levels in order to avoid macroeconomic and financial instability (see Cecchetti, Genberg, Lipsky and Wadhwani (2000) and Borio and Lowe (2002)). Others argue that central banks should react to asset prices only to the extent that they contain information about future output growth and inflation. For example, Greenspan (2002) explains that, since it is very difficult to identify a bubble before its existence is confirmed by the bursting, the Federal Reserve does not directly react to financial imbalances. According to Bernanke and Gertler (1999), it is unnecessary for monetary policy to respond to changes in asset prices. Rules that directly target asset prices might have undesirable side effects of stifling the beneficial impact of the technology boom.

More recently, Kannan, Rabanal and Scott (2012) incorporate a financial sector into the model of Iacoviello and Neri (2010) and combine the macroprudential instrument with an augmented Taylor rule that also reacts to the growth rate of nominal credit. They find that strong monetary reactions to credit growth and house prices increase macroeconomic stability. However, whether a macroprudential instrument should be employed depends on the source of house price booms. In this paper, we assume that the policymaker aims at minimizing an ordinary loss function – a weighted variability of money inflation, wage inflation, output growth, and nominal interest rate – as suggested by Giannoni and Woodford (2003). Our goal is to examine whether the implementation of an interest rate rule that also responds to house price inflation reduces the policymaker's loss and how robust the optimal rule is to changes in a list of crucial structural parameters. Unlike Kannan, Rabanal and Scott (2012), we investigate the optimal policy rule without assuming the central bank knowing the source of house price booms. In particular, we do not intend to identify whether the observed fluctuations in house price are driven by fundamentals or not. Instead, given the housing market spillovers, we study an interest rate rule that responds not only to money inflation and output growth but also to house price inflation.

3.2 The Model

Iacoviello and Neri (2010) construct a DSGE model that allows for housing market spillovers to the broad economy. The model includes a consumption good sector and a housing sector. There are two types of households, patient and impatient, on the demand side that work, consume, and accumulate housing. The patient households own the capital of the economy and provide funds to firms and loans to the impatient households, who face the collateral constraints in equilibrium – their maximum borrowing is given by a fraction m (the loan-to-value ratio) of the expected present value of their home. On the supply side, the consumption sector combines capital and labor to produce consumption goods and business capital for both sectors. The housing sector combines business capital, labor, and land to produce new houses. Both capital and housing depreciate over time.

The model allows for sticky prices in the consumption sector and sticky wages in both sectors.

In each period, a fraction θ_π of retailers are able to set prices optimally, while another fraction $1-\theta_\pi$ only index prices to the previous period inflation rate with an elasticity of ι_π. Hence, the consumption sector Phillips curve takes the following form:

$$\ln \pi_t - \iota_\pi \ln \pi_{t-1} = \beta G_C (E_t \ln \pi_{t+1} - \iota_\pi \ln \pi_t) - \frac{(1-\theta_\pi)(1-\beta G_C \theta_\pi)}{\theta_\pi} \ln\left(\frac{X_t}{X}\right) + u_{p,t}, \tag{3.1}$$

where β is the discount factor of the patient households; G_C is the growth rate of consumption in the balanced growth path; π_t is the money inflation in the consumption sector; X_t is a markup over the marginal cost charged by retailers and X is the steady-state value; $u_{p,t}$ is an independently and identically distributed cost shock that affects inflation.

Similarly, the wage Phillips curves can be written as:

$$\ln \omega_{i,t} - \iota_{wi} \ln \pi_{t-1} = \beta G_C (E_t \ln \omega_{i,t+1} - \iota_{wi} \ln \pi_t) - \frac{(1-\theta_{wi})(1-\beta G_C \theta_{wi})}{\theta_{wi}} \ln\left(\frac{X_{wi,t}}{X_{wi}}\right), \tag{3.2}$$

$$\ln \omega'_{i,t} - \iota_{wi} \ln \pi_{t-1} = \beta' G_C (E_t \ln \omega'_{i,t+1} - \iota_{wi} \ln \pi_t) - \frac{(1-\theta_{wi})(1-\beta' G_C \theta_{wi})}{\theta_{wi}} \ln\left(\frac{X_{wi,t}}{X_{wi}}\right), \tag{3.3}$$

where $i = c, h$ (c and h denote the consumption sector and the housing sector, respectively). θ_{wi} characterizes the wage stickiness in sector i; $X_{wi,t}$ is the corresponding wage markup; $\omega_{i,t}$ is the nominal wage inflation in each sector, i.e. $\omega_{i,t} = \pi_t w_{i,t}/w_{i,t-1}$, where $w_{i,t}$ is the real wage. Parameters and variables with a prime refer to impatient households.

To close the model, the central bank sets the nominal interest rate, R_t, as a contemporaneous version of the Taylor rule that responds to the money inflation, π_t, and the GDP growth, $GDP_t/(G_C GDP_{t-1})$:

$$\text{Rule (I):} \quad R_t = R_{t-1}^{r_R} \pi_t^{(1-r_R)r_\pi} \left(\frac{GDP_t}{G_C GDP_{t-1}}\right)^{(1-r_R)r_Y} \left(\frac{q_t}{G_Q q_{t-1}}\right)^{(1-r_R)r_Q} \overline{rr}^{1-r_R} \frac{u_{R,t}}{s_t}, \tag{3.4}$$

i.e., the parameter r_Q that corresponds to the house price inflation, $q_t/(G_Q q_{t-1})$, is fixed at zero.[8] In

[8]Most related work specify a Taylor rule that responds to output gap, instead of output growth, beyond money

Equation (3.4), G_Q is the growth rate of house price in the balanced growth path; $\overline{rr}$ is the steady-state real interest rate; $u_{R,t}$ is an independently and identically distributed monetary shock; s_t is an AR(1) stochastic process that captures long-lasting deviations of inflation from its steady-state level, i.e. $\ln s_t = \rho_s \ln s_{t-1} + u_{s,t}$.

Besides $u_{p,t}$ in Equation (3.1) and $u_{R,t}$, $u_{s,t}$ in Equation (3.4), the model specifies a number of shocks, including intertemporal preference shock $u_{z,t}$, housing demand shock $u_{j,t}$, labor supply shock $u_{\tau,t}$, productivity shocks $u_{C,t}$ and $u_{H,t}$ in the two sectors, and the investment-specific technology shock $u_{K,t}$. Among them, the housing demand shock and the housing technology shock account for more than half of the volatilities of housing investment and house price (see Iacoviello and Neri (2010)).

Based on this model, we examine whether the optimal monetary policy reacts to house price inflation, under the assumption that the policymaker seeks to minimize an ordinary expected loss criterion as in Giannoni and Woodford (2003):

$$\begin{aligned}\mathfrak{L} = {} & \lambda_\pi var(\ln \pi_t - \iota_\pi \ln \pi_{t-1}) + \lambda_w var(\ln \pi_{wc,t} - \iota_{wc} \ln \pi_{t-1}) + \lambda_y var(\ln GDP_t - \ln(G_C GDP_{t-1})) \\ & + \lambda_r var(\ln R_t - \ln \overline{rr}), \end{aligned} \tag{3.5}$$

where $\pi_{wc,t} = \pi_t(w_{c,t} + w'_{c,t})/((w_{c,t-1} + w'_{c,t-1}))$ is the nominal wage inflation in the consumption sector.[9] Under the objective $\mathfrak{L}$, the policymaker minimizes the weighted variability of money inflation, wage inflation, output growth, and nominal interest rate, but does not put any weight on house price inflation.

inflation. Compared to output gap, output growth is first of all much easier to observe. Secondly, Sims (2013) suggests that responding to the growth rate of output is often welfare-improving.

[9]We don't consider the wage inflation in the housing sector, since both types of households contribute most of their labor to the production of consumption goods.

3.3 Optimal Monetary Policy

The baseline model is estimated for the period 1965:Q1-2006:Q4 by fixing r_Q at zero (see Iacoviello and Neri (2010) for details on the Bayesian estimation) and the loss function related parameters are taken from Giannoni and Woodford (2003) (see Table 4.1). We conduct a 4-dimensional optimization over $(r_R, r_\pi, r_Y, r_Q) \in [0,1] \times [0,15] \times [0,5]^2$ and find the combination that minimizes the loss function $\mathfrak{L}$ in Equation (3.5) as well as the region that satisfies the Blanchard-Kahn condition for determinacy.

Table 3.1: Parameter Values in a Calibrated Baseline Model

Structural Equations				Shock Processes		Loss Function	
Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value
β	0.9925	η	0.5325	ρ_{AC}	0.9576	λ_π	0.500
β'	0.9700	η'	0.4984	ρ_{AH}	0.9957	λ_w	0.500
j	0.1200	ξ	0.7186	ρ_{AK}	0.9236	λ_y	0.003
μ_c	0.3500	ξ'	0.9960	ρ_j	0.9563	λ_r	0.236
μ_h	0.1000	ϕ_{kc}	17.4921	ρ_z	0.9602		
μ_b	0.1000	ϕ_{kh}	10.7499	ρ_τ	0.8928		
μ_l	0.1000	θ_π	0.8201	ρ_s	0.9750		
X	1.1500	θ_{wc}	0.8165	σ_{AC}	0.0099		
X_{wc}	1.1500	θ_{wh}	0.9216	σ_{AH}	0.0198		
X_{wh}	1.1500	ι_π	0.7314	σ_{AK}	0.0136		
δ_h	0.0100	ι_{wc}	0.0799	σ_j	0.0431		
δ_{kc}	0.0250	ι_{wh}	0.4052	σ_R	0.0032		
δ_{kh}	0.0300	ζ_{kc}	0.8060	σ_z	0.0159		
m	0.8500	γ_{AC}	0.0032	σ_τ	0.0330		
ϵ	0.3417	γ_{AK}	0.0008	σ_p	0.0047		
ϵ'	0.5913	γ_{AH}	0.0027	σ_s	0.0339		
α	0.7936						

Note: Parameter λ_y is obtained as the value of 0.048 in Giannoni and Woodford (2003) divided by 16, since we are using quarterly data for each variable throughout this paper.

3.3.1 Determinacy and Uniqueness

According to Blanchard and Kahn (1980), the rational expectations equilibrium has a unique solution if and only if the number of unstable eigenvectors is exactly equal to the number of nonpredetermined variables. By searching over $(r_R, r_\pi, r_Y, r_Q) \in [0,1] \times [0,15] \times [0,5]^2$, we find that $r_\pi > 1$ is a sufficient condition for the equilibrium to be determinate. For $r_\pi \leq 1$, however, the equilibrium becomes indeterminate under any combination of (r_R, r_Y, r_Q). These results are consistent with the finding of Bullard and Mitra (2002) that determinacy is guaranteed by $r_\pi > 1$, except for a subtle difference. By considering a contemporaneous interest rate rule that reacts to inflation and output gap in a simple DSGE model with sticky prices, Bullard and Mitra (2002) find that a relatively low value of $r_\pi < 1$ can be "compensated" by choosing a sufficiently large value for r_Y for the equilibrium to be determinate. On the contrary, we do not find any (r_R, r_Y, r_Q) combination that supports the determinacy for $r_\pi \leq 1$ under the model considered in this paper.

3.3.2 Optimal Policy Rule

We examine the optimal interest rate rule under different scenarios and compare the optimal rule with its estimated counterpart. Let $V_\pi = var(\ln \pi_t - \iota_\pi \ln \pi_{t-1})$, $V_w = var(\ln \pi_{wc,t} - \iota_{wc} \ln \pi_{t-1})$, $V_y = var(\ln GDP_t - \ln(G_C GDP_{t-1}))$, and $V_R = var(\ln R_t - \ln \overline{rr})$ denote the four variance components in the loss function $\mathfrak{L}$. Similarly, we define the variability of house price inflation as $V_q = var(\ln q_t - \ln(G_Q q_{t-1}))$. We also define $r_\pi^* = (1 - r_R) r_\pi$, $r_Y^* = (1 - r_R) r_Y$, and $r_Q^* = (1 - r_R) r_Q$. The statistics and parameter values associated with both the estimated and the optimal interest rate rules are reported in Table 3.2.

Since the contemporaneous interest rate rule (I) is assumed to react to money inflation and output growth only in the baseline model, the parameter r_Q is fixed at zero. The persistence in the nominal interest rate, r_R, is estimated to be 0.61 and the estimates of r_π^* and r_Y^* are 0.55 and 0.19, respectively. When we release the zero restriction, the parameter estimates are $(r_R, r_\pi^*, r_Y^*, r_Q^*) = (0.61, 0.55, 0.18, 0.02)$, indicating that the estimated interest rate rule does not respond much to house price inflation.

The optimal interest rate rule parameters are $(r_R, r_\pi^*, r_Y^*, r_Q^*) = (0.71, 2.84, 1.08, 0.26)$ in our baseline model, which suggests that, in order to achieve macroeconomic stability, the interest rate rule should be more persistent and more responsive to both money inflation and output growth. More importantly, house price inflation is another variable that an interest rate rule should target due to the spillovers from the housing sector. Compared to the estimated interest rate rule, implementing the optimal rule reduces the policymaker's loss by more than 70%. Meanwhile, it yields positive side effects in terms of mitigating house price instability. In particular, the volatility of house price inflation under the optimal rule is about one-third less than what implied by the data.[10]

The central bank is assumed to set the nominal interest rate following a contemporaneous version of the Taylor rule in our baseline model. We also investigate another two specifications of the policy rule, one backward-looking and the other forward-looking:

$$\text{Rule (II):}\quad R_t = R_{t-1}^{r_R}\pi_{t-1}^{(1-r_R)r_\pi}\left(\frac{GDP_{t-1}}{G_C GDP_{t-2}}\right)^{(1-r_R)r_Y}\left(\frac{q_{t-1}}{G_Q q_{t-2}}\right)^{(1-r_R)r_Q}\overline{rr}^{1-r_R}\frac{u_{R,t}}{s_t}, \tag{3.6}$$

$$\text{Rule (III):}\quad R_t = R_{t-1}^{r_R}(E_t\pi_{t+1})^{(1-r_R)r_\pi}\left(\frac{E_t GDP_{t+1}}{G_C GDP_t}\right)^{(1-r_R)r_Y}\left(\frac{E_t q_{t+1}}{G_Q q_t}\right)^{(1-r_R)r_Q}\overline{rr}^{1-r_R}\frac{u_{R,t}}{s_t}. \tag{3.7}$$

As is shown in Table 3.2, neither the backward-looking nor the forward-looking policy rule responds much to house price inflation. However, in order to achieve macroeconomic stability, the policymaker should target house price inflation to a considerable extent under both rules.[11] Compared to the optimal contemporaneous rule (I), the optimal backward-looking rule (II) is more persistent and less responsive to price inflation, GDP growth, and house price inflation.

[10]Our objective loss function does not take into consideration the variability of house price inflation. Both intuition and preliminary quantitative attempts suggest that the optimal rule responds more to house price inflation when stabilizing house price (or/and real debt, which is closely related to the housing market) is also among the goals of the policymaker. Our basic argument in this paper is that the optimal interest rate rule responds to house price inflation even when the stabilization of house price is not among the objectives of the policymaker.

[11]Under the forward-looking policy rule (III), a 4-dimensional optimization over (r_R, r_π, r_Y, r_Q) yields strange results that the optimal rule is highly persistent and responding to price inflation only. In this case, a very high inertia parameter pushes down the parameters associated with both GDP growth and house price inflation. Instead, we fix the inertia parameter at its estimated value 0.59 and conduct an optimization over (r_π, r_Y, r_Q).

Table 3.2: Estimated and Optimal Interest Rate Rules

		Statistics						Parameter Values			
Policy Rule	$r_Q = 0$	V_π	V_w	V_y	V_R	V_q	$\mathfrak{L}$	r_R	r_π^*	r_Y^*	r_Q^*
Estimated (I)	YES	0.0221	0.0892	0.1933	0.0871	0.2675	0.0768	0.61	0.55	0.19	
Estimated (I)	NO	0.0229	0.0885	0.1827	0.0839	0.2510	0.0760	0.61	0.55	0.18	0.02
Optimal (I)		0.0143	0.0238	0.1038	0.0087	0.1857	0.0214	0.71	2.84	1.08	0.26
Estimated (II)	YES	0.0365	0.1091	0.2733	0.1040	0.3259	0.0982	0.68	0.43	0.10	
Estimated (II)	NO	0.0375	0.0906	0.2413	0.0714	0.3001	0.0816	0.68	0.42	0.11	0.01
Optimal (II)		0.0125	0.0202	0.1493	0.0192	0.2342	0.0213	0.78	1.54	0.40	0.09
Estimated (III)	YES	0.0396	0.1429	0.1948	0.1577	0.2916	0.1291	0.59	0.61	0.04	
Estimated (III)	NO	0.0435	0.1362	0.1884	0.1469	0.2902	0.1251	0.57	0.65	0.05	0.02
Optimal (III)		0.0163	0.0458	0.2998	0.0127	0.3890	0.0346	0.59	3.31	1.47	0.27

Note: Policy rules (I), (II), (III) refer to the contemporaneous rule in Equation (3.4), the backward-looking rule in Equation (3.6), and the forward-looking rule in Equation (3.7), respectively. All statistics are multiplied by 1000. The loss function $\mathfrak{L}$ is a weighted sum of V_π, V_w, V_y, and V_R.

Next, we wish to examine how robust the optimal interest rule in our baseline model is to changes in a list of structural parameters that are crucially important in Iacoviello and Neri (2010), including the wage share of patient households α, the steady-state value of the housing preference shock j, the loan-to-value ratio m, the price stickiness θ_π, the wage stickiness in the consumption sector θ_{wc}, and the wage stickiness in the housing sector θ_{wh}. While varying one of these parameters, we keep all other parameters at their baseline values. Table 3.3 reports the results.

The parameter α measures the labor income share of patient households in the economy. The optimal rule parameters r_π^* and r_Y^* are relatively robust. As α drops to 0.2 or when most of the households in the economy are impatient in accumulating housing, the optimal rule becomes less persistent but more responsive to house price inflation.

The parameter j is the steady-state value of the housing preference shock. The baseline value $j = 0.12$ is calibrated to match the ratio of business capital to annual GDP of about 2.1 and the ratio of housing wealth to GDP of about 1.35. As we increase the value of j or when institutional changes shift preference toward housing, the optimal rule responds more and more to house price inflation.

The parameter m denotes the loan-to-value ratio. The optimal rule parameters are insensitive to the value of m when $m < 1$. However, if impatient households are allowed to borrow more than the expected value of their housing, house price becomes more volatile. The optimal rule is hence considerably more sensitive to house price inflation.

The parameter θ_π denotes the price stickiness. As prices become non-sticky ($\theta_\pi = 0$), the optimal rule becomes more responsive to money inflation but less responsive to output growth and house price inflation, since money inflation is potentially volatile when more and more retailers could set prices optimally in each period.

The parameters θ_{wc} and θ_{wh} stand for the wage stickiness in the consumption sector and in the housing sector, respectively. When wages are not highly sticky in the consumption sector, the optimal rule is not persistent and not responsive to either output growth or house price inflation. Moreover, the response to money inflation is much smaller in magnitude compared to that in the baseline model. The loss function value is considerably higher than that in any other cases, suggesting that it is hard to achieve macroeconomic stability under flexible wages ($\theta_{wc} = 0$) and sticky prices ($\theta_\pi = 0.82$). This confirms the crucial role of sticky wages in Iacoviello and Neri (2010). Relatively, the optimal rule parameters are much less sensitive to the wage stickiness in the housing sector.

The analysis of Table 3.3 suggests that, in order to minimize the policymaker's loss, the interest rate rule should respond to house price inflation under a variety of scenarios except for the only case of low wage stickiness in the consumption sector. Another feature can be detected from Table 3.3 that, even the loss function $\mathfrak{L}$ does not account for house price volatility, a higher response to house price inflation r_Q always results in a lower value of V_q. This feature indicates an even higher response to house price inflation in the optimal rule when the policymaker cares about stabilizing house price.

Table 3.3: Sensitivity of the Optimal Interest Rate Rule

	Posited	Statistics						Optimal Values			
	Value	V_π	V_w	V_y	V_R	V_q	$\mathfrak{L}$	r_R	r_π^*	r_Y^*	r_Q^*
	0.20	0.0140	0.0303	0.1001	0.0103	0.1397	0.0249	0.48	3.27	1.13	0.50
	0.40	0.0141	0.0272	0.1024	0.0090	0.1560	0.0231	0.59	2.89	1.02	0.37
α	0.60	0.0142	0.0253	0.1037	0.0083	0.1706	0.0220	0.71	2.82	1.03	0.31
	0.79*	0.0143	0.0238	0.1038	0.0087	0.1857	0.0214	0.71	2.84	1.08	0.26
	1.00	0.0145	0.0227	0.1030	0.0091	0.1964	0.0210	0.70	3.06	1.20	0.25
	0.06	0.0144	0.0235	0.0952	0.0087	0.1922	0.0213	0.70	2.90	1.16	0.25
	0.12*	0.0143	0.0238	0.1038	0.0087	0.1857	0.0214	0.71	2.84	1.08	0.26
j	0.18	0.0143	0.0243	0.1156	0.0084	0.1719	0.0217	0.74	3.01	1.04	0.37
	0.24	0.0143	0.0246	0.1302	0.0090	0.1696	0.0219	0.72	2.83	0.90	0.38
	0.30	0.0142	0.0249	0.1467	0.0093	0.1667	0.0222	0.72	2.72	0.80	0.40
	0.65	0.0144	0.0234	0.1033	0.0091	0.1865	0.0214	0.70	2.82	1.06	0.30
	0.75	0.0144	0.0235	0.1028	0.0090	0.1923	0.0214	0.72	2.80	1.07	0.24
m	0.85*	0.0143	0.0238	0.1038	0.0087	0.1857	0.0214	0.71	2.84	1.08	0.26
	0.95	0.0142	0.0248	0.1048	0.0082	0.1640	0.0218	0.69	3.04	1.12	0.31
	1.05	0.0142	0.0272	0.1078	0.0069	0.0966	0.0227	0.72	3.62	1.22	0.68
	0.00	0.0073	0.0144	0.3146	0.0261	0.3556	0.0180	0.76	3.48	0.07	0.09
	0.20	0.0064	0.0144	0.2449	0.0162	0.2933	0.0150	0.78	3.06	0.21	0.08
θ_π	0.40	0.0067	0.0148	0.1946	0.0136	0.2513	0.0145	0.83	2.37	0.26	0.08
	0.60	0.0092	0.0161	0.1483	0.0135	0.2203	0.0163	0.85	1.96	0.37	0.09
	0.82*	0.0143	0.0238	0.1038	0.0087	0.1857	0.0214	0.71	2.84	1.08	0.26
	0.00	0.0140	0.4983	0.2886	0.0863	0.3323	0.2774	0.00	1.54	0.00	0.00
	0.20	0.0139	0.2672	0.2999	0.0830	0.3423	0.1610	0.00	1.62	0.00	0.00
θ_{wc}	0.40	0.0139	0.1313	0.3158	0.0828	0.3576	0.0931	0.00	1.74	0.00	0.00
	0.60	0.0136	0.0634	0.2742	0.0601	0.2936	0.0535	0.14	1.66	0.08	0.11
	0.82*	0.0143	0.0238	0.1038	0.0087	0.1857	0.0214	0.71	2.84	1.08	0.26
	0.00	0.0143	0.0250	0.1069	0.0103	0.1805	0.0224	0.78	2.22	0.73	0.35
	0.20	0.0143	0.0250	0.1065	0.0101	0.1806	0.0224	0.80	2.25	0.74	0.35
θ_{wh}	0.40	0.0143	0.0249	0.1063	0.0103	0.1796	0.0223	0.75	2.27	0.75	0.34
	0.60	0.0143	0.0248	0.1069	0.0100	0.1823	0.0222	0.75	2.30	0.78	0.32
	0.92*	0.0143	0.0238	0.1038	0.0087	0.1857	0.0214	0.71	2.84	1.08	0.26

Note: An asterisk * denotes the baseline model. All statistics are multiplied by 1000. The loss function $\mathfrak{L}$ is a weighted sum of V_π, V_w, V_y, and V_R.

3.4 Conclusion

Historical data imply that monetary policy rule reacts only to money inflation and GDP growth but not to house price inflation. In a DSGE framework with housing market spillovers, we examine whether the interest rate rule should respond to house price inflation in order to minimize the policymaker's loss. We find that, even when stabilizing house price is *not* one of the goals of the policymaker, it is optimal to implement an interest rate rule that responds to house price inflation.

4 Chapter 4

New and Old Housing Markets, Term Structure and the Macroeconomy

Abstract

I construct a dynamic stochastic general equilibrium (DSGE) model in this paper to study the fluctuations in the U.S. housing markets. The model features a market for newly built houses, a secondary market for old houses, and an endogenous term structure of nominal interest rates. Negative technological progress in the housing sector explains the upward trend in house prices over the past four decades. Housing preference and technology innovations explain about 80% of the volatility of housing investment, real price of new houses, and the old-to-new house price ratio. Monetary factors explain about 15% of the volatility of housing investment, but do not significantly contribute to the price fluctuations of either new or old houses. The preference innovation to old houses is the leading determinant of the run-up in the price of old houses relative to the price of new houses during the 10-year period before the Great Recession. The term structure is endogenous in this paper, and the intertemporal preference innovation makes a non-negligible contribution to the variations in nominal interest rates. Housing market conditions do not contribute much to the fluctuations of interest rates, but significantly affect the shape of the yield curve.

Keywords: DSGE Models, Housing Markets, Term Structure, Monetary Policy

JEL Classification: E23, E32, E44, E52, R31

4.1 Introduction

Over the past several decades, the U.S. housing markets have experienced large fluctuations that move together with business cycles, which has stimulated a growing literature on understanding the dynamic interactions between the housing sector and the macroeconomy. In the present work, I construct a dynamic stochastic general equilibrium (DSGE) model that characterizes both a market for newly built houses and a secondary market for old (or existing) houses and endogenously incorporates the term structure of nominal interest rates, motivated by two stylized facts in the U.S. housing markets.

Figure 4.1: U.S. National House Prices

First, as shown in Figure 4.1, the gap between old house price and new house price shrinks over time before the Great Recession, especially between 1997 and 2006. This paper aims at explaining the convergence between prices of old and new houses during the 10-year period prior to the recent recession by explicitly modeling both a new and an old housing market.[12]

Second, households usually borrow to purchase a house and carry the debt over a long period of time, usually 15 years or 30 years. Thus, the relevant borrowing rate is a long-term interest rate or mortgage rate, which ought to be distinguished from the short-term discount rate. In other words, the term structure of nominal interest rates plays a role in households' utility maximization problem and in the dynamics of housing markets. Thus, disturbances in the housing markets might in return affect the term structure. Incorporating the term structure into the DSGE framework enables us to learn more about its bidirectional relations with the housing markets.

Related Literature – Housing is one of the most important sectors over the business cycle. Before Maisel (1968), a substantial literature explores the effects of income and interest rates on the housing sector indicators, such as residential investment, taking the overall business cycle as given (see Ketchum (1954) and Alberts (1962)). However, development in the housing sector is not only a reflection of macroeconomic activities but also one of the driving forces of short-run economic fluctuations. In fact, the investment in the housing sector has been identified as an important channel through which monetary policy affects the economy (see Maisel (1968)) and the best precursor of the business cycle (see Leamer (2007)). More recently, Iacoviello and Neri (2010) apply a dynamic equilibrium model with nominal rigidities (see Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007)) to understanding sources and consequences of fluctuations in the U.S.

[12] There have been several indexes constructed for the U.S. house prices, among which the Quarterly Constant-Quality House Price Index by Census Bureau focuses on new single-family houses, and the Quarterly All-Transactions Index by the Federal Housing Finance Agency on existing single-family houses. Both of these two indexes are constructed after adjusting to the quality, including physical size, geographic location, housing type, climate, etc. To make these two indexes comparable, I fix the Quarterly Constant-Quality House Price Index for new houses and use the data on new and existing single-family house prices of 2012 published by Census Bureau and National Association of Realtors (NAR) to normalize the Quarterly Repeat-Transactions House Price Index for existing houses. The comparable indexes (in logs) from the first quarter of 1975 (1975:Q1) to the last quarter of 2013 (2013:Q4), deflated with the implicit price deflator for the nonfarm business sector, are plotted in the figure. The constant-quality property of both indexes ensures that the different patterns in these two house price indexes are not driven by quality-related factors. This paper tries to figure out what drives the changes in the price of old houses relative to the price of new houses between 1975:Q1 and 2013:Q4.

housing market. Their model features heterogeneity in households' discount factors, a multi-sector structure with consumption and housing goods, nominal rigidities, financing frictions tied to the households, and a rich set of structural shocks. The model attributes most of the volatility of housing investment and house price over the sample period 1965:Q1-2006:Q4 to the housing demand shock and the housing technology shock, and identifies the spillovers from the housing market to the broad economy as a channel of propagation. My contribution to this literature is that I make the term structure of interest rates endogenous in the DSGE model and investigate the bidirectional relations between housing markets and the term structure.

The literature on the term structure of interest rates extends back to Vasicek (1977) and Cox, Ingersoll and Ross (1985). In this literature, it is common to have latent factors driving the dynamics of the term structure of interest rates. These factors are usually interpreted as "level", "slope", and "curvature" factors. For example, Dai and Singleton (2000) construct a three-factor model of the term structure and provide a satisfactory fit of the data. However, the economic forces behind these latent factors are unclear before a number of recent studies focus on the relation between the term structure and macroeconomic dynamics. Ang and Piazzesi (2003) are among the first to incorporate macroeconomic factors in a term structure model. They show that a large fraction of the variations in bond yields can be explained by macro factors. Dewachter and Lyrio (2006) provide a macroeconomic interpretation for the latent factors: the level factor represents the long-run inflation expectation of agents, the slope factor captures business cycle conditions, and the curvature factor expresses an independent monetary policy factor. Hördahl, Tristani and Vestin (2006) and Rudebusch and Wu (2008) append the term structure to a set of standard macroeconomic aggregate relationships for output and inflation, and interpret the dynamics of yields and risk premia in terms of macroeconomic fundamentals. Bekaert, Cho and Moreno (2010) study the term structure in a New Keynesian model framework that features AS (Phillips Curve), IS (derived from utility maximization), and monetary policy equations. The effect of housing market dynamics on the term structure has never been considered, however. This paper contributes to the term structure literature by explicitly modeling housing markets. The transmission mechanism through which monetary policy changes lead to the housing market reactions in the presence of an endogenously adjusted term structure, and the question of whether and how changes in the housing markets affect the term structure are among the issues of interest here.

The second unique feature to the present work is that it explicitly models the secondary housing market; this has never been modeled in a dynamic equilibrium framework. In durable goods markets, however, there is a long-standing literature on the effects of secondary markets. For example, Anderson and Ginsburgh (1994), Hendel and Lizzeri (1999), and Johnson (2011) investigate the allocative effect of secondary markets. More recently, Chen, Esteban and Shum (2013) study different effects – substitution, allocative, and time consistency – of secondary markets in the U.S. automobile industry. They find that the existence of secondary markets is harmful to new car manufacturers and that transaction costs play a key role in secondary markets. This paper models a secondary housing market and examines the effects of transaction costs. Raising transaction costs reduces the role of the secondary market. In contrast, reducing transaction costs is able to make transactions in the secondary market frictionless. It is also found that a reduction in transaction costs in the secondary housing market not only reduces the overall macroeconomic fluctuations but also improves social welfare.

The objective of this paper is to understand the dynamics of housing markets, by studying a DSGE model that features nominal rigidities, the term structure, and the secondary housing market. The rest of this paper proceeds as follows. Section 2 describes the model economy. Section 3 estimates the model using Bayesian methods and performs a variety of counterfactual analyses. Section 4 examines the effects of transaction costs on the secondary housing market and finds the optimal level of transaction costs. Section 5 discusses the sources of housing market fluctuations. Section 6 gives a further discussion on the term structure. Section 7 concludes the paper.

4.2 The Model Economy

The model with housing spillovers builds on Iacoviello and Neri (2010), augmented with both new and old housing markets, as well as the term structure. On the demand side, there are two types of households: unconstrained (or lenders) and constrained (or borrowers), while constrained households face a borrowing constraint. On the supply side, the consumption sector combines capital and labor to produce consumption goods and business capital. The housing sector produces new houses combining business capital with labor and land. New houses enter the pool of old houses one period later and can be traded again in the secondary market. While the transaction of new houses is frictionless, matching buyers and sellers in the secondary market is costly.

4.2.1 Households

There is a continuum of measure 1 of agents in each of the unconstrained and constrained groups. The economic size of each group is measured by its wage share, which is determined by the parameter α in the production function. Within each group, a representative household maximizes the lifetime utility:

$$V = E_0 \sum_{t=0}^{\infty} (\beta G_C)^t z_t \left[\Gamma_C \ln(c_t - \epsilon c_{t-1}) + j_{o,t} \ln(h_{o,t}) + j_{n,t} \ln(h_{n,t}) - \frac{\tau_t}{1+\eta} \left(n_{c,t}^{1+\xi} + n_{h,t}^{1+\xi} \right)^{\frac{1+\eta}{1+\xi}} \right], \tag{4.1}$$

or

$$V' = E_0 \sum_{t=0}^{\infty} (\beta' G_C)^t z_t \left[\Gamma'_C \ln(c'_t - \epsilon' c'_{t-1}) + j_{o,t} \ln(h'_{o,t}) + j_{n,t} \ln(h'_{n,t}) - \frac{\tau_t}{1+\eta'} \left((n'_{c,t})^{1+\xi'} + (n'_{h,t})^{1+\xi'} \right)^{\frac{1+\eta'}{1+\xi'}} \right], \tag{4.2}$$

where variables without a prime refer to unconstrained households and those with a prime refer to constrained households. Consumption, working hours in the consumption sector, and working hours in the housing sector are denoted by c, n_c, and n_h, respectively. Owner-occupied old houses and new houses are denoted by h_o and h_n, the former of which is a stock variable whereas the later one is a flow variable.[13]

The utility function consists of three separable components: consumption, housing, and labor. First, in the consumption component, G_C is the growth rate of consumption along the balanced growth path; parameters ϵ and ϵ' measure the internal habits in consumption; scaling factors $\Gamma_C = (G_C - \epsilon)/(G_C - \beta\epsilon G_C)$ and $\Gamma'_C = (G_C - \epsilon')/(G_C - \beta'\epsilon' G_C)$ ensure that the marginal utilities of consumption are $1/c$ and $1/c'$ along the balanced growth path. Second, the housing component contains both old houses and new houses, which are perfect substitutes to each other. One can also think of it as a composite housing product of Cobb-Douglas form, $h_{o,t}^{j_{o,t}} h_{h,t}^{j_{n,t}}$, with a weight $j_{o,t}$ on old houses and $j_{n,t}$ on new houses. Preference shocks to two types of houses are measured by $j_{o,t}$ and $j_{n,t}$. Third, the specification of the disutility of labor allows for less than perfect labor mobility between sectors, characterized by parameters ξ, ξ', η, and η'. The term τ_t captures the shock to labor supply. Finally, z_t captures the shock to intertemporal preferences; the discount factors are β and β' which satisfy $\beta' < \beta$.

The shock processes z_t, τ_t, $j_{o,t}$, and $j_{n,t}$ follow:

$$\ln z_t = \rho_z \ln z_{t-1} + \varepsilon_{z,t}, \tag{4.3}$$

$$\ln \tau_t = \rho_\tau \ln \tau_{t-1} + \varepsilon_{\tau,t}, \tag{4.4}$$

$$\ln j_{o,t} = (1 - \rho_{j_o}) \ln j_o + \rho_{j_o} \ln j_{o,t-1} + \varepsilon_{j_o,t}, \tag{4.5}$$

$$\ln j_{n,t} = (1 - \rho_{j_n}) \ln j_n + \rho_{j_n} \ln j_{n,t-1} + \varepsilon_{j_n,t}. \tag{4.6}$$

where ρ_z, ρ_τ, ρ_{j_o}, and ρ_{j_n} are autoregressive parameters; j_o and j_n are the steady-state values of preference shocks; ε_z, ε_τ, ε_{j_o}, and ε_{j_n} are independently and identically distributed innovations with variances σ_z^2, σ_τ^2, $\sigma_{j_o}^2$, and $\sigma_{j_n}^2$.

Unconstrained households accumulate capital and houses, and make loans to constrained households. They rent capital to firms, choose the capital utilization rates, and sell the remaining unde-

[13]The rental market is not modeled in the present work. Both old and new houses are owner-occupied.

preciated capital. They maximize their lifetime utility, V, subject to the following budget constraint:

$$
\begin{aligned}
&c_t + \frac{k_{c,t}}{A_{k,t}} + k_{h,t} + k_{b,t} + q_t[f_t(h_{o,t} - (1-\delta_h)(h_{o,t-1} + h_{n,t-1})) + h_{n,t}] + p_{l,t} l_t + \frac{R_{L,t-1} b_{t-1}}{\pi_t} + TC_t \\
=&\frac{w_{c,t} n_{c,t}}{X_{wc,t}} + \frac{w_{h,t} n_{h,t}}{X_{wh,t}} + DIV_t + \left(R_{c,t} z_{c,t} + \frac{1-\delta_{kc}}{A_{k,t}} \right) k_{c,t-1} + \left(R_{h,t} z_{h,t} + 1 - \delta_{kh} \right) k_{h,t-1} \\
&+ p_{b,t} k_{b,t} + b_t + (p_{l,t} + R_{l,t}) l_{t-1} - \Phi_t - \frac{a(z_{c,t}) k_{c,t-1}}{A_{k,t}} - a(z_{h,t}) k_{h,t-1}.
\end{aligned} \tag{4.7}
$$

They choose consumption c_t, capital in the consumption sector $k_{c,t}$, capital $k_{h,t}$ and intermediate inputs $k_{b,t}$ in the housing sector, stock of old houses $h_{o,t}$ and flow of new houses $h_{n,t}$, land stock l_t, working hours $n_{c,t}$ and $n_{h,t}$, capital utilization rates $z_{c,t}$ and $z_{h,t}$, and borrowing b_t (or lending when it is negative) to maximize utility subject to the budget constraint in Equation (4.7). Intermediate inputs in the housing sector and land are priced at $p_{b,t}$ and $p_{l,t}$, respectively. The price of new houses is q_t. Old houses are priced at a fraction f_t of the new house price q_t. Loans are set in nominal terms and yield a riskless long-term nominal return of $R_{L,t}$. The term $A_{k,t}$ denotes the investment-specific technology shock. Real wages are denoted by $w_{c,t}$ and $w_{h,t}$, real rental rates by $R_{c,t}$ and $R_{h,t}$, and depreciation rates by δ_h, δ_{kc} and δ_{kh}. The terms $X_{wc,t}$ and $X_{wh,t}$ characterize the markup between the wage paid by the wholesale firm and the wage paid to the households in the consumption sector and the housing sector, respectively. The money inflation measures the change in the price of consumption goods, i.e., $\pi_t = P_t / P_{t-1}$, where P_t is the price of consumption goods at the retail level. Finally, DIV_t is a lump-sum profit from final good firms and from labor unions; TC_t stands for transaction costs in the secondary housing market; Φ_t is adjustment costs for capital; $z_{c,t}$ and $z_{h,t}$ are capital utilization rates in two production sectors; $a(z)$ is the convex cost of setting the capital utilization to z:

$$
DIV_t = \frac{X_t - 1}{X_t} Y_t + \frac{X_{wc,t} - 1}{X_{wc,t}} w_{c,t} n_{c,t} + \frac{X_{wh,t} - 1}{X_{wh,t}} w_{h,t} n_{h,t}, \tag{4.8}
$$

$$
TC_t = \frac{\phi}{2G_H} \left(\frac{h_{o,t}}{h_{o,t-1}} - G_H \right)^2 h_{o,t-1} q_t f_t, \tag{4.9}
$$

$$\Phi_t = \frac{\phi_{kc}}{2G_{KC}} \left(\frac{k_{c,t}}{k_{c,t-1}} - G_{KC} \right)^2 \frac{k_{c,t-1}}{G_{AK}^t} + \frac{\phi_{kh}}{2G_C} \left(\frac{k_{h,t}}{k_{h,t-1}} - G_C \right)^2 k_{h,t-1}, \tag{4.10}$$

$$a(z_{c,t}) = \widetilde{R}_C(\varpi z_{c,t}^2/2 + (1-\varpi)z_{c,t} + (\varpi/2 - 1)), \tag{4.11}$$

$$a(z_{h,t}) = R_h(\varpi z_{h,t}^2/2 + (1-\varpi)z_{h,t} + (\varpi/2 - 1)). \tag{4.12}$$

where G_H, G_Q, G_{KC}, and G_{AK} are growth rates associated with housing investment, house prices, capital in the consumption sector, and the investment-specific technology, respectively; $\widetilde{R}_c$ and R_h are the steady-state values of rental rates of the two types of capital (see more details in the appendix at the end of this paper); Y_t is the production of wholesale goods and X_t is the markup of final goods over wholesale goods. For later convenience, let $\zeta = \varpi/(1+\varpi)$.

Transaction Costs – It is worth a further discussion on the specification of transaction costs TC_t here. In the model, TC_t follows a quadratic form. An alternative specification would be a wedge that is proportional to the transaction of old houses, i.e.,

$$TC_t^w = \phi^w q_t f_t \left| h_{o,t} - (1-\delta_h)(h_{o,t-1} + h_{n,t-1}) \right|, \tag{4.13}$$

where ϕ^w is a parameter which measures the buying or selling cost as in Sommer, Sullivan and Verbrugge (2013). However, the sign of the transaction flow, $h_{o,t} - (1-\delta_h)(h_{o,t-1} + h_{n,t-1})$, is not determinate, since the discount factors β and β' are not different enough for the transaction in the secondary market to be unidirectional. This indeterminacy makes it impracticable to find the steady state in the case of wedge transaction costs.

Notice that the transaction flow can be detrended into $\widetilde{h}_{o,t} - ((1-\delta_h)/G_H)(\widetilde{h}_{o,t-1} + \widetilde{h}_{n,t-1})$, which is equal to $(1-(1-\delta_h)/G_H)\widetilde{h}_o - ((1-\delta_h)/G_H)\widetilde{h}_n$ in the steady state (more details are presented in the appendix). Because $\widetilde{h}_n$ is very small relative to $\widetilde{h}_o$, the transaction flow of old houses between two types of households is tiny around the steady state. The economy cannot get far away from the steady state without having two types of households trade old houses with each other through the secondary market. Off the steady-state path, households participate into the secondary market and incur quadratic transaction costs TC_t, which can be detrended into

$$\widetilde{TC}_t = \frac{\phi}{2}\left(\frac{\widetilde{h}_{o,t}}{\widetilde{h}_{o,t-1}} - 1\right)^2 \widetilde{h}_{o,t-1}\widetilde{q}_t f_t. \tag{4.14}$$

Next, I argue that the quadratic transaction costs are approximately equivalent to the progressive wedge transaction costs. As stated above, the economy stays around the steady state if households do not trade old houses via the secondary market. Suppose they choose to increase (decrease) their stock of old houses by $\Delta \times 100\%$ in period t, they need to buy (sell) about $\Delta \times 100\%$ of their previous period stock, $\widetilde{h}_{o,t-1}$, through the secondary market. The transaction costs in this case are:

$$\widetilde{TC}_t = \frac{\phi}{2}\left(\frac{\widetilde{h}_{o,t}}{\widetilde{h}_{o,t-1}} - 1\right)^2 \widetilde{h}_{o,t-1}\widetilde{q}_t f_t = \frac{\phi}{2}\Delta^2 \widetilde{h}_{o,t-1}\widetilde{q}_t f_t = \frac{\phi\Delta}{2}\left(\Delta\widetilde{h}_{o,t-1}\right)\widetilde{q}_t f_t, \tag{4.15}$$

where the term in parentheses, $\Delta\widetilde{h}_{o,t-1}$, stands for the transaction amount of old houses. As a result, the quadratic transaction costs in the model are equivalent to wedge transaction costs, where the cost rate $\phi\Delta/2$ is increasing in Δ, the magnitude of adjustment.

According to the model specification, both new and old houses depreciate at a constant rate of δ_h which is calibrated to be 0.01 later in Section 3. At this rate, new houses lose more than 75% of their value in 35 years and they can then be ignored or assumed to exit the market. Hence the pool of old houses contains properties at different ages, ranging from 1 quarter up to 35 years. In the steady state, the amount of old houses is equal across ages. It is implicitly assumed that households cannot identify the age of any particular old house. Instead, they purchase or sell an average of the pool, whose quality is constant over time around the steady state.

Constrained households do not accumulate capital and do not own finished good firms or land. Moreover, their maximum borrowing b'_t is given by the expected present value of their houses times the loan-to-value (LTV) ratio m:

$$
\begin{aligned}
& c'_t + q_t[f_t(h'_{o,t} - (1-\delta_h)(h'_{o,t-1} + h'_{n,t-1})) + h'_{n,t}] + \frac{R_{L,t-1}b'_{t-1}}{\pi_t} + TC'_t \\
= & \frac{w'_{c,t}n'_{c,t}}{X_{wc,t}} + \frac{w'_{h,t}n'_{h,t}}{X_{wh,t}} + DIV'_t + b'_t, \qquad (4.16) \\
& b'_t \leq mE_t\left(\frac{f_{t+1}q_{t+1}(1-\delta_h)(h'_{o,t} + h'_{n,t})\pi_{t+1}}{R_{S,t}}\right), \qquad (4.17)
\end{aligned}
$$

where $R_{S,t}$ is a short-term discount rate and

$$
DIV'_t = \frac{X'_{wc,t} - 1}{X'_{wc,t}} w'_{c,t}n'_{c,t} + \frac{X'_{wh,t} - 1}{X'_{wh,t}} w'_{h,t}n'_{h,t}, \qquad (4.18)
$$

$$
TC'_t = \frac{\phi'}{2G_H}\left(\frac{h'_{o,t}}{h'_{o,t-1}} - G_H\right)^2 h'_{o,t-1}q_t f_t. \qquad (4.19)
$$

Given the assumption $\beta' < \beta$, the borrowing constraint (4.17) holds with equality around the steady state for small shocks.

4.2.2 Production Technologies

Wholesale firms hire capital and labor, and purchase intermediate goods to produce wholesale goods Y_t and new houses IH_t. They seek to maximize their profit:

$$
\max \frac{Y_t}{X_t} + q_t IH_t - \left(\sum_{i=c,h} w_{i,t}n_{i,t} + \sum_{i=c,h} w'_{i,t}n'_{i,t} + R_{c,t}z_{c,t}k_{c,t-1} + R_{h,t}z_{h,t}k_{h,t-1} + p_{b,t}k_{b,t} + R_{l,t}l_{t-1}\right),
$$

where X_t is the markup of final goods over wholesale goods. The production technologies in two sectors are specified as:

$$
Y_t = [A_{c,t}(n^{\alpha}_{c,t}n'^{1-\alpha}_{c,t})]^{1-\mu_c}(z_{c,t}k_{c,t-1})^{\mu_c}, \qquad (4.20)
$$

$$
IH_t = [A_{h,t}(n^{\alpha}_{h,t}n'^{1-\alpha}_{h,t})]^{1-\mu_h-\mu_b-\mu_l}(z_{h,t}k_{h,t-1})^{\mu_h}k^{\mu_b}_{b,t}l^{\mu_l}_{t-1}. \qquad (4.21)
$$

In Equation (4.20), the consumption sector produces output with labor and capital. In Equation (4.21), new houses are produced with labor, capital, intermediate input, and land. The size of land is assumed to be fixed over time and normalized to one. The terms $A_{c,t}$ and $A_{h,t}$ measure productivity in the consumption sector and in the housing sector, respectively. The parameter α measures the labor income share of unconstrained households.

4.2.3 Price and Wage Rigidities

The model allows for sticky price in the consumption sector and sticky wages in both sectors. Price stickiness in the consumption sector is introduced by assuming monopolistic competition at the retail level, implicit costs of adjusting nominal prices following Calvo-style contracts (see Calvo (1983)), and partial indexation to lagged inflation of those prices that cannot be re-optimized. The resulting Phillips curve is:

$$\ln \pi_t - \iota_\pi \ln \pi_{t-1} = \beta G_C (E_t \ln \pi_{t+1} - \iota_\pi \ln \pi_t) - \frac{(1-\theta_\pi)(1-\beta G_C \theta_\pi)}{\theta_\pi} \ln (X_t / X) + \varepsilon_{p,t}, \tag{4.22}$$

where θ_π is the fraction of retailers that cannot set prices optimally but instead index prices to the previous period inflation.

Wage stickiness is modeled similarly. The Calvo-style pricing with partial indexation to inflation in the previous period yields the following wage Phillips curves:

$$\ln \omega_{i,t} - \iota_{wi} \ln \pi_{t-1} = \beta G_C (E_t \ln \omega_{i,t+1} - \iota_{wi} \ln \pi_t) - \frac{(1-\theta_{wi})(1-\beta G_C \theta_{wi})}{\theta_{wi}} \ln \left(\frac{X_{wi,t}}{X_{wi}} \right), \tag{4.23}$$

$$\ln \omega'_{i,t} - \iota_{wi} \ln \pi_{t-1} = \beta' G_C (E_t \ln \omega'_{i,t+1} - \iota_{wi} \ln \pi_t) - \frac{(1-\theta_{wi})(1-\beta' G_C \theta_{wi})}{\theta_{wi}} \ln \left(\frac{X_{wi,t}}{X_{wi}} \right), \tag{4.24}$$

where $i = c, h$, which denote the consumption sector and the housing sector, respectively. The parameter θ_{wi} characterizes the wage stickiness in sector i; $X_{wi,t}$ is the corresponding wage markup; and $\omega_{i,t}$ is the nominal wage inflation in each sector, i.e., $\omega_{i,t} = \pi_t w_{i,t} / w_{i,t-1}$, where $w_{i,t}$ is the real wage.

4.2.4 Monetary Policy

Following the term structure literature, I assume that yields of different maturities are driven by two latent factors L_t and S_t:

$$\ln R_{j,t} = \lambda_j + \mathbf{\Lambda}_j \mathbf{F}_t, \tag{4.25}$$

where $\ln R_{j,t}$ is the yield of maturity j periods, $j = 1, ..., J$; λ_j is a constant and $\mathbf{F}_t = (L_t, S_t)'$. The factor loadings on these two yield curve components, $\mathbf{\Lambda}_j$, are modeled as:

$$\mathbf{\Lambda}_j = \left(1, \quad \frac{1 - e^{-\delta j}}{\delta j}\right), \tag{4.26}$$

where δ denotes a decay parameter. The loadings on the first factor, L_t, are constant across the maturity spectrum. A positive shock to this factor induces an essentially parallel shift in the term structure that boosts the level of the whole yield curve, so the L_t factor is often called a “level" factor. The loadings on the second factor, S_t, decrease monotonically with the maturity. A positive shock to this factor increases short-term yields by much more than the long-term yields, thus S_t is usually called the “slope" factor. This setup is similar to the Nelson-Siegel term structure model (see Nelson and Siegel (1987)), without considering the “curvature" factor.[14]

[14] In the Nelson-Siegel model, the loadings on the “curvature" factor follow a hump-shaped pattern and are close to zero for short and long maturities but larger for intermediate maturities. I do not explicitly consider this factor but, later in the estimation, impose measurement errors to the yields of intermediate maturities. Imposing measurement errors partly takes care of this issue.

The dynamics of the these latent factors are specified as:

$$L_t = \gamma_L L_{t-1} + (1-\gamma_L)\ln \pi_t + \varepsilon_{L,t}, \tag{4.27}$$

$$S_t = \gamma_S S_{t-1} + (1-\gamma_S)\gamma_\pi \ln \pi_t + (1-\gamma_S)\gamma_Y \ln\left(\frac{GDP_t}{G_C GDP_{t-1}}\right) + \varepsilon_{S,t}, \tag{4.28}$$

where $\varepsilon_{L,t}$ and $\varepsilon_{S,t}$ are independently and identically distributed shocks to the level factor and to the slope factor respectively with variances σ_L^2 and σ_S^2. Here, GDP_t is defined as the sum of the value added of two sectors.

In Equation (4.27), the level factor L_t is interpreted as the underlying rate of inflation, as in Rudebusch and Wu (2008). This is actually a common interpretation in the recent macro-finance literature, such as Kozicki and Tinsley (2001), Dewachter and Lyrio (2006), and Hördahl, Tristani and Vestin (2006). Equation (4.28) is a classic Taylor rule, indicating that the slope factor S_t reacts to its own lag, inflation rate, and output growth.

4.2.5 Equilibrium

The consumption sector produces consumption goods, business investment, and intermediate inputs. The housing sector produces new houses. Old houses are traded in the secondary market directly between unconstrained and constrained households. The equilibrium conditions are:

$$C_t + IK_{c,t}/A_{k,t} + IK_{h,t} + k_{b,t} = Y_t - \Phi_t - (TC_t + TC'_t), \tag{4.29}$$

$$h_{n,t} + h'_{n,t} = IH_t, \tag{4.30}$$

$$(h_{o,t} - (1-\delta_h)(h_{o,t-1} + h_{n,t-1})) + \left(h'_{o,t} - (1-\delta_h)(h'_{o,t-1} + h'_{n,t-1})\right) = 0, \tag{4.31}$$

$$b_t + b'_t = 0, \tag{4.32}$$

where $C_t = c_t + c'_t$, $IK_{c,t} = k_{c,t} - (1-\delta_{kc})k_{c,t-1}$, and $IK_{h,t} = k_{h,t} - (1-\delta_{kh})k_{h,t-1}$. Equations (4.29) through (4.32) characterize the clearing conditions of the consumption good market, the new housing market, the old housing market, and the loan market (Walras' law).

4.2.6 Linear Deterministic Trends

Given the fact that historical data on consumption, residential investment, and house prices all exhibit linear trends, this model allows for linear deterministic trends in the technologies A_c, A_h, and A_k. Let the corresponding gross growth rates be respectively γ_{AC}, γ_{AH}, and γ_{AK}, i.e.,

$$\ln A_{c,t} = t\ln(1+\gamma_{AC}) + \ln Z_{c,t}, \text{ where } \ln Z_{c,t} = \rho_{AC}\ln Z_{c,t-1} + \varepsilon_{c,t}, \tag{4.33}$$

$$\ln A_{h,t} = t\ln(1+\gamma_{AH}) + \ln Z_{h,t}, \text{ where } \ln Z_{h,t} = \rho_{AH}\ln Z_{h,t-1} + \varepsilon_{h,t}, \tag{4.34}$$

$$\ln A_{k,t} = t\ln(1+\gamma_{AK}) + \ln Z_{k,t}, \text{ where } \ln Z_{k,t} = \rho_{AK}\ln Z_{k,t-1} + \varepsilon_{k,t}. \tag{4.35}$$

The terms $\varepsilon_{c,t}$, $\varepsilon_{h,t}$, and $\varepsilon_{k,t}$ are independently and identically innovations with zero mean and variances σ^2_{AC}, σ^2_{AH}, and σ^2_{AK}.

Because of the existence of these trends, the variables $Y_t, c_t, c'_t, k_{c,t}/A_{k,t}, k_{h,t}, q_t IH_t$ all grow at a common rate along the balanced growth path. The production function in the consumption sector infers that this common rate takes the following form:

$$\gamma_Y = \gamma_C = (1-\mu_c)\gamma_{AC} + \mu_c\gamma_{KC}. \tag{4.36}$$

Given $\gamma_Y = \gamma_{KC} - \gamma_{AK}$, it follows that

$$\gamma_Y = \gamma_{AC} + \frac{\mu_c}{1-\mu_c}\gamma_{AK}, \tag{4.37}$$

$$\gamma_{KC} = \gamma_{AC} + \frac{1}{1-\mu_c}\gamma_{AK}, \tag{4.38}$$

$$\gamma_{IH} = (1-\mu_h-\mu_b-\mu_l)\gamma_{AH} + (\mu_h+\mu_b)\left(\gamma_{AC} + \frac{\mu_c}{1-\mu_c}\gamma_{AK}\right), \tag{4.39}$$

$$\gamma_Q = (1-\mu_h-\mu_b)\left(\gamma_{AC} + \frac{\mu_c}{1-\mu_c}\gamma_{AK}\right) - (1-\mu_h-\mu_b-\mu_l)\gamma_{AH}. \tag{4.40}$$

4.3 Empirical Results

4.3.1 Data Description

The sample period is 1975:Q1 to 2013:Q4, and 17 observables are included in the analysis: real consumption, real business investment, real residential investment, real price of new houses, the old-to-new house price ratio, inflation rate, working hours and wage inflation in the consumption sector, working hours and wage inflation in the housing sector, nominal 3-month interest rate, and zero-coupon bond yields of maturities 4, 12, 24, 36, 48, and 60 quarters.

Real consumption, real business investment, and real residential investment are obtained from Bureau of Economic Analysis (BEA), expressed in per-capita term; inflation rate is the implicit price deflator for the nonfarm business sector; real price of new houses is the deflated Census Bureau House Price Index whereas the price of old houses is the Freddie Mac Quarterly Repeat-Transactions House Price Index; hours and wage inflations are obtained from Bureau of Labor Statistics (BLS); nominal short-term interest rate is the 3-month Treasury Bill Rate at secondary market from Board of Governors of the Federal Reserve System; zero-coupon bond yields of various maturities are obtained from the Federal Reserve Board (see Gurkaynak, Sack and Wright (2007)).

4.3.2 Calibration

The discount factors (β and β'), the production function parameters (μ_c, μ_h, μ_b, and μ_l), the depreciation rates (δ_h, δ_{kc}, and δ_{kh}), the LTV ratio (m), the steady state values of preference shocks (j_o and j_n), and the steady state gross price and wage markups (X, X_{wc}, and X_{wh}) are all calibrated. The parameter values are shown in Table 4.1.

Table 4.1: Calibrated Parameters

Parameter	Value	Parameter	Value	Parameter	Value
β	0.98285	δ_h	0.01	X	1.15
β'	0.90	δ_{kc}	0.025	X_{wc}	1.15
μ_c	0.35	δ_{kh}	0.03	X_{wh}	1.15
μ_h	0.10	m	0.85		
μ_b	0.10	j_o	0.20		
μ_l	0.10	j_n	0.04		

The model implies a zero inflation rate, i.e., $\pi = 1$, in the steady state, due to the assumption in Equations (4.27) and (4.28) that the central bank targets a zero inflation. The discount factor of unconstrained households is set at $\beta = 0.98285$, implying a steady-state quarterly long-term nominal interest rate of around 1.745 percent, as the sample average of the 60-quarter zero-coupon bond yield. The discount factor of constrained households β' is instead fixed at 0.90 to guarantee an impatience motive for constrained households large enough that they are arbitrarily close to the borrowing limit (see Iacoviello (2005)). The choices $\mu_c = 0.35$, $\mu_h = 0.10$, $\mu_b = 0.10$, and $\mu_l = 0.10$ are same as those in Iacoviello and Neri (2010). The usual markup on prices, 1.15, is chosen for X, X_{wc}, and X_{wh} (see Corsetti, Kuester, Meier and Muller (2013)). The depreciation rates are set at $\delta_h = 0.01$, $\delta_{kc} = 0.025$, and $\delta_{kh} = 0.03$. The LTV ratio is set at $m = 0.85$.[15] The steady-state values of preference shocks are chosen to be $j_o = 0.20$ and $j_n = 0.04$. These choices, together with other estimated parameters, imply a ratio of housing wealth to annual GDP of about 1.4 and an old-to-new house price ratio of around 0.7, as the mean over the sample period.

4.3.3 Prior and Posterior Distributions

The prior distributions of the structural parameters and shock processes are presented in the left panel of Tables 4.2 and 4.3. The labor income share of unconstrained households α has a prior

[15] The LTV ratio is actually varying over time. Data on the LTV ratio is indeed available at the Federal Housing Finance Agency (FHFA). However, this data does not distinguish between unconstrained and constrained households. While most households borrow through the financial markets in reality, the model assumes that a fraction α of the households do not borrow but lend to other households. Here, a sensible number of 0.85 is chosen for constrained households following Iacoviello and Neri (2010).

mean of 0.65 and a standard error of 0.05, as implied by Iacoviello (2005). The decay parameter δ has a prior mean of 0.06, as suggested by Diebold and Li (2006), and a relatively large standard error of 0.05. The consumption habit parameters (ϵ and ϵ'), the working disutility parameters (η and η'), the indexation parameters (ι_{π}, ι_{wc}, and ι_{wh}), the hour substitution parameters (ξ and ξ'), the utilization parameter (ζ), and the Calvo price and wage parameters (θ_{π}, θ_{wc}, and θ_{wh}) have the same prior distributions as in Iacoviello and Neri (2010). The transaction cost parameters (ϕ and ϕ') are assumed to have a Gamma-distributed prior with a mean of 5 and a relatively large standard error of 2.5. The adjustment cost parameters (ϕ_{kc} and ϕ_{kh}) are assumed to have a prior mean of 10 and a standard error of 2.5. The monetary policy parameters have similar priors to those in Christiano, Eichenbaum and Evans (2005). The technological growth rates (γ_{AC}, γ_{AH}, and γ_{AK}) all have a small prior mean of 0.005. All the autoregressive parameters (ρ_{AC}, ρ_{AH}, ρ_{AK}, ρ_{j_o}, ρ_{j_n}, ρ_{τ}, and ρ_z) have a Beta-distributed prior with mean 0.8 and standard error 0.1. All the structural shocks (σ_{AC}, σ_{AH}, σ_{AK}, σ_{j_o}, σ_{j_n}, σ_L, σ_S, σ_{τ}, σ_z, and σ_p) have an Inverse Gamma-distributed standard deviation with mean 0.1 and standard error 2, which corresponds to a rather loose prior. Measurement errors (σ_{n_h}, σ_{w_h}, σ_{R04}, σ_{R12}, σ_{R24}, σ_{R36}, and σ_{R48}) are also specified for the hours and the wage in the housing sector and the bond yields of intermediate maturities.

The right panels report the relevant posterior mean, median, and 90% probability intervals. The estimate of α, the labor income share of credit-unconstrained households, is 0.840, which implies a share of labor income accruing to the credit-constrained households of 16 percent. This value is close to the estimate obtained by Jappelli (1990), using the 1983 Survey of Consumer Finances. The estimate of the decay parameter $\delta = 0.013$ is less than the prior mean, most likely because the "curvature" factor of the term structure has been ignored in the estimation. Both unconstrained and constrained households exhibit a moderate degree of habit formation in consumption ($\epsilon = 0.452$ and $\epsilon' = 0.628$). The estimates of η and η', the labor supply elasticity, are close to their prior mean. The parameters ξ and ξ' are estimated to be 0.828 and 0.997, implying that hours in the two sectors are not perfect substitutes. The estimates of adjustment costs for capital are $\phi_{kc} = 23.131$ and

$\phi_{kh} = 11.073$. The estimate of ζ, the curvature of the capital utilization function, is 0.819, implying that $\varpi = 4.525$.

Table 4.2: Prior and Posterior Distribution of the Structural Parameters

	Prior Distribution			Posterior Distribution			
Parameter	Distribution	Mean	SD	Mean	5%	Median	95%
α	Beta	0.650	0.050	0.840	0.826	0.841	0.851
δ	Beta	0.060	0.050	0.013	0.011	0.013	0.016
ϵ	Beta	0.500	0.075	0.452	0.404	0.447	0.509
ϵ'	Beta	0.500	0.075	0.628	0.572	0.633	0.677
η	Gamma	0.500	0.100	0.641	0.444	0.606	0.825
η'	Gamma	0.500	0.100	0.512	0.399	0.505	0.613
ϕ	Gamma	5.000	2.500	0.104	0.022	0.092	0.188
ϕ'	Gamma	5.000	2.500	0.210	0.109	0.196	0.343
ϕ_{kc}	Gamma	10.000	2.500	23.131	20.820	22.875	27.503
ϕ_{kh}	Gamma	10.000	2.500	11.073	8.093	10.284	14.830
ι_{π}	Beta	0.500	0.200	0.838	0.747	0.833	0.918
ι_{wc}	Beta	0.500	0.200	0.065	0.024	0.065	0.105
ι_{wh}	Beta	0.500	0.200	0.509	0.293	0.491	0.776
ξ	Normal	1.000	0.100	0.828	0.744	0.827	0.929
ξ'	Normal	1.000	0.100	0.997	0.864	1.002	1.111
r_L	Beta	0.800	0.100	0.596	0.564	0.595	0.622
r_S	Beta	0.200	0.100	0.578	0.536	0.577	0.609
r_{π}	Normal	1.500	0.100	1.315	1.155	1.282	1.510
r_Y	Normal	0.200	0.100	0.800	0.703	0.806	0.901
θ_{π}	Beta	0.667	0.050	0.768	0.739	0.771	0.790
θ_{wc}	Beta	0.667	0.050	0.826	0.801	0.824	0.855
θ_{wh}	Beta	0.667	0.050	0.940	0.930	0.941	0.949
ζ	Beta	0.500	0.200	0.819	0.696	0.803	0.933
$100\gamma_{AC}$	Normal	0.500	1.000	0.312	0.295	0.312	0.326
$100\gamma_{AH}$	Normal	0.500	1.000	-0.042	-0.168	-0.050	0.058
$100\gamma_{AK}$	Normal	0.500	1.000	0.223	0.181	0.225	0.255

Note: The posterior distribution is obtained using the Metropolis-Hastings algorithm.

The transaction costs in the secondary market are characterized by $\phi = 0.104$ and $\phi' = 0.210$, indicating that unconstrained households incur a rate of 2.6% if they choose to change their stock of old houses by 50% while constrained households incur a higher rate of 5.25%. One explanation may be that since the later type of households are constrained, they are willing to adjust their housing stock at higher transaction costs.

Table 4.3: Prior and Posterior Distribution of the Shock Processes

	Prior Distribution			Posterior Distribution			
Parameter	Distribution	Mean	SD	Mean	5%	Median	95%
ρ_{AC}	Beta	0.800	0.100	0.895	0.867	0.894	0.922
ρ_{AH}	Beta	0.800	0.100	0.985	0.967	0.989	0.998
ρ_{AK}	Beta	0.800	0.100	0.970	0.957	0.970	0.989
ρ_{j_o}	Beta	0.800	0.100	0.971	0.956	0.972	0.982
ρ_{j_n}	Beta	0.800	0.100	0.967	0.956	0.967	0.975
ρ_{τ}	Beta	0.800	0.100	0.901	0.883	0.902	0.915
ρ_z	Beta	0.800	0.100	0.835	0.789	0.835	0.866
σ_{AC}	Inv.gamma	0.100	2.000	0.012	0.011	0.011	0.013
σ_{AH}	Inv.gamma	0.100	2.000	0.020	0.019	0.020	0.022
σ_{AK}	Inv.gamma	0.100	2.000	0.019	0.017	0.020	0.022
σ_{j_o}	Inv.gamma	0.100	2.000	0.039	0.030	0.038	0.049
σ_{j_n}	Inv.gamma	0.100	2.000	0.068	0.060	0.069	0.075
σ_L	Inv.gamma	0.100	2.000	0.009	0.008	0.009	0.009
σ_S	Inv.gamma	0.100	2.000	0.010	0.009	0.010	0.011
σ_{τ}	Inv.gamma	0.100	2.000	0.029	0.023	0.028	0.035
σ_z	Inv.gamma	0.100	2.000	0.016	0.014	0.016	0.018
σ_p	Inv.gamma	0.100	2.000	0.009	0.008	0.010	0.010
σ_{n_h}	Inv.gamma	0.100	2.000	0.240	0.218	0.241	0.255
σ_{w_h}	Inv.gamma	0.100	2.000	0.010	0.009	0.010	0.011
σ_{R04}	Inv.gamma	0.100	2.000	0.006	0.006	0.006	0.007
σ_{R12}	Inv.gamma	0.100	2.000	0.007	0.006	0.007	0.007
σ_{R24}	Inv.gamma	0.100	2.000	0.006	0.006	0.007	0.007
σ_{R36}	Inv.gamma	0.100	2.000	0.007	0.006	0.007	0.007
σ_{R48}	Inv.gamma	0.100	2.000	0.006	0.006	0.006	0.007

Note: The posterior distribution is obtained using the Metropolis-Hastings algorithm.

According to the estimation results, only 23.2 percent of the retailers are able to set prices optimally. A fraction $\theta_\pi = 0.768$ of retailers index prices to the previous period inflation rate with an elasticity equal to $\iota_\pi = 0.838$. As for wages, higher wage stickiness and larger wage indexation are found in the housing sector ($\theta_{wh} = 0.940$ versus $\theta_{wc} = 0.826$ and $\iota_{wh} = 0.509$ versus $\iota_{wc} = 0.065$). Estimates of the monetary policy rule are in line with previous evidence.

It is also found that the consumption sector exhibits the fastest rate of technological progress, followed by that in business investment and by that in the housing sector. The estimates of $\gamma_{AC} = 0.312\%$ and $\gamma_{AK} = 0.223\%$ are close to those in Iacoviello and Neri (2010). However, the estimate of γ_{AH} is negative. This estimate, $\gamma_{AH} = -0.042\%$, is considerably lower than the value obtained by Iacoviello and Neri (2010) but is consistent with the literature.[16] According to the estimation of Corrado, Lengermann, Bartelsman and Beaulieu (2006), the Total Factor Productivity (TFP) in the construction sector has been decreasing at an annual rate of 0.5% over the period 1987-2004; the contribution of labor and purchased inputs more than account for the real output growth of the construction sector; on the contrary, the TFP in the private nonfarm business sector has been increasing at an annual rate of 1.2%. One possible explanation for the negative technological progress is that restrictions and regulations on land use raise expensive barriers to building and lower the efficiency with which housing services are provided to occupants, as suggested by Albouy and Ehrlich (2012).

4.3.4 Model-Implied Land Price

Given the estimates of γ_{AC}, γ_{AK}, and γ_{AH}, the growth rate of consumption is $\gamma_C = 0.44\%$. The model implies that real land price, $p_{l,t}$, is also growing at this rate. One might argue that the convergence between old and new house prices before the Great Recession is simply a result of the run-up in residential land price, because land accounts for a larger fraction of the value of old houses. In fact, since land is a factor of the housing production, the causality goes the other way from the housing demand to land price.

Though it is an important aspect of the housing markets, land price has been ignored due to lack of data in the literature until recently. Two sets of data on land price index at quarterly frequency are available at the Lincoln Institute of Land Policy. In both datasets, the land price data are derived from data on housing values and estimates of structure costs using price indexes for housing and construction costs. The construction cost data are derived from publicly available data from the Bureau of Economic Analysis (BEA). The house price data are benchmarked to an estimate of the value of the stock of housing based on micro data from the 2000 Decennial Census of Housing and

[16]The negative technological progress in the housing sector confines to residential structures. In nonresidential structures such as highways, airports, bridges, and skyscrapers, an annual technological progress of around 1% is found by Gort, Greenwood and Rupert (1999).

2001 Residential Finance Survey, and are extrapolated forwards and backwards from the benchmark year using the Macromarkets LLC, formerly Case-Shiller-Weiss (CSW), repeat-sales index for the first data set, and the FHFA repeat-sales index for the second data set (see Davis and Heathcote (2007) for details). Both the CSW-based and the FHFA-based indexes suggest that real land price grows at a rate of 0.75% per quarter over 1975-2013. The model in the present work succeeds explaining about 60% of the land price growth.

Recently, Liu, Wang and Zha (2013) focus on modeling the comovements between land price and business investment. Based on a similar DSGE framework, they do not directly model the housing market but instead the land market, and interpret a shock to households' tastes for land as a housing demand shock. In addition, they assume that firms, instead of households, are credit constrained. Firms finance investment spending by using land as a collateral asset. Even they do not explicitly model the growth rate of land price, an implication of their specification is that land price is growing at the same rate as business investment, which is estimated to be $\gamma_{KC} = 0.68\%$ in the present work. In that case, the model would be able to explain more than 90% of the land price growth. This paper sticks with the current model specification for two reasons. First, land price is not among the primary concerns of the present work. Second, new and old houses cannot be distinguished from each other if households' demand for housing is simply modeled as their demand for land.

The deviation of land price from its trend growth implied by the model is plotted in Figure 4.2, together with the detrended FHFA-based and CSW-based land price indexes. Even when the estimation does not make use of the data on land price, the model replicates the fluctuations of land price over the sample period very well. This result reflects the fact that movements in land price are a result of house price fluctuations, which are in turn determined by demand and supply conditions. Learning the economic forces behind the fluctuations in the housing markets is among the objectives of this paper.

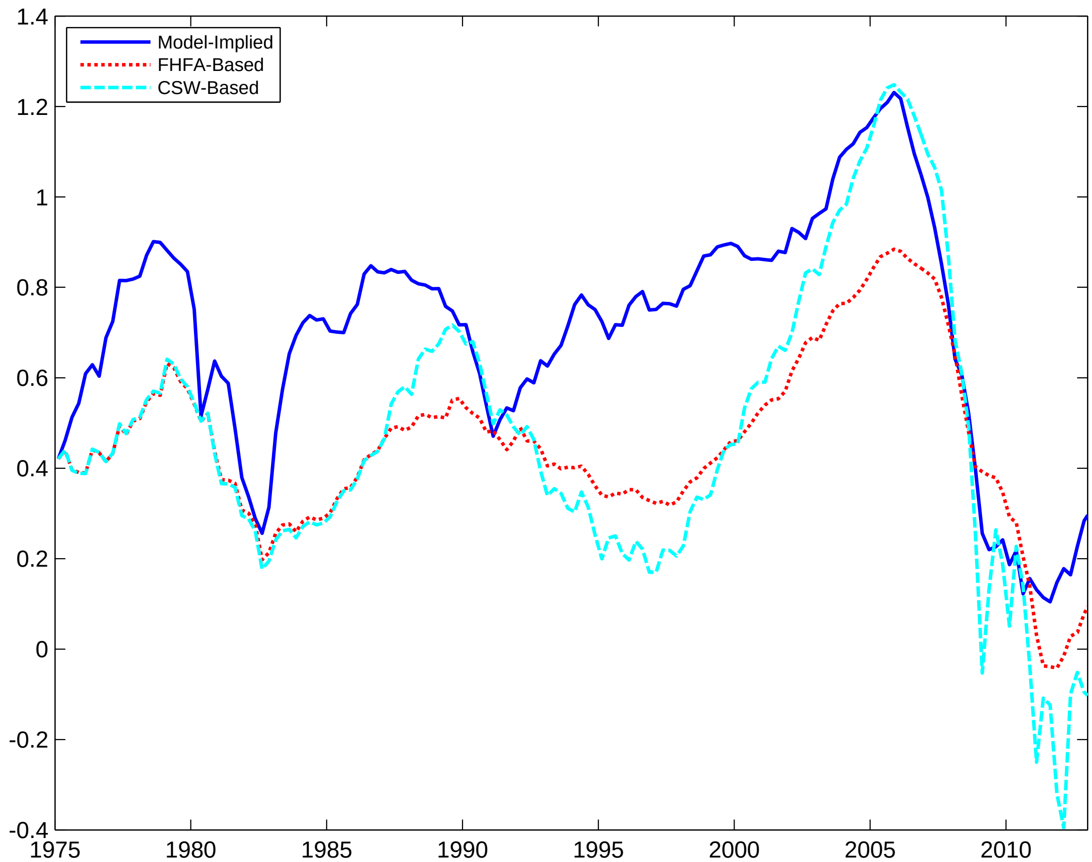
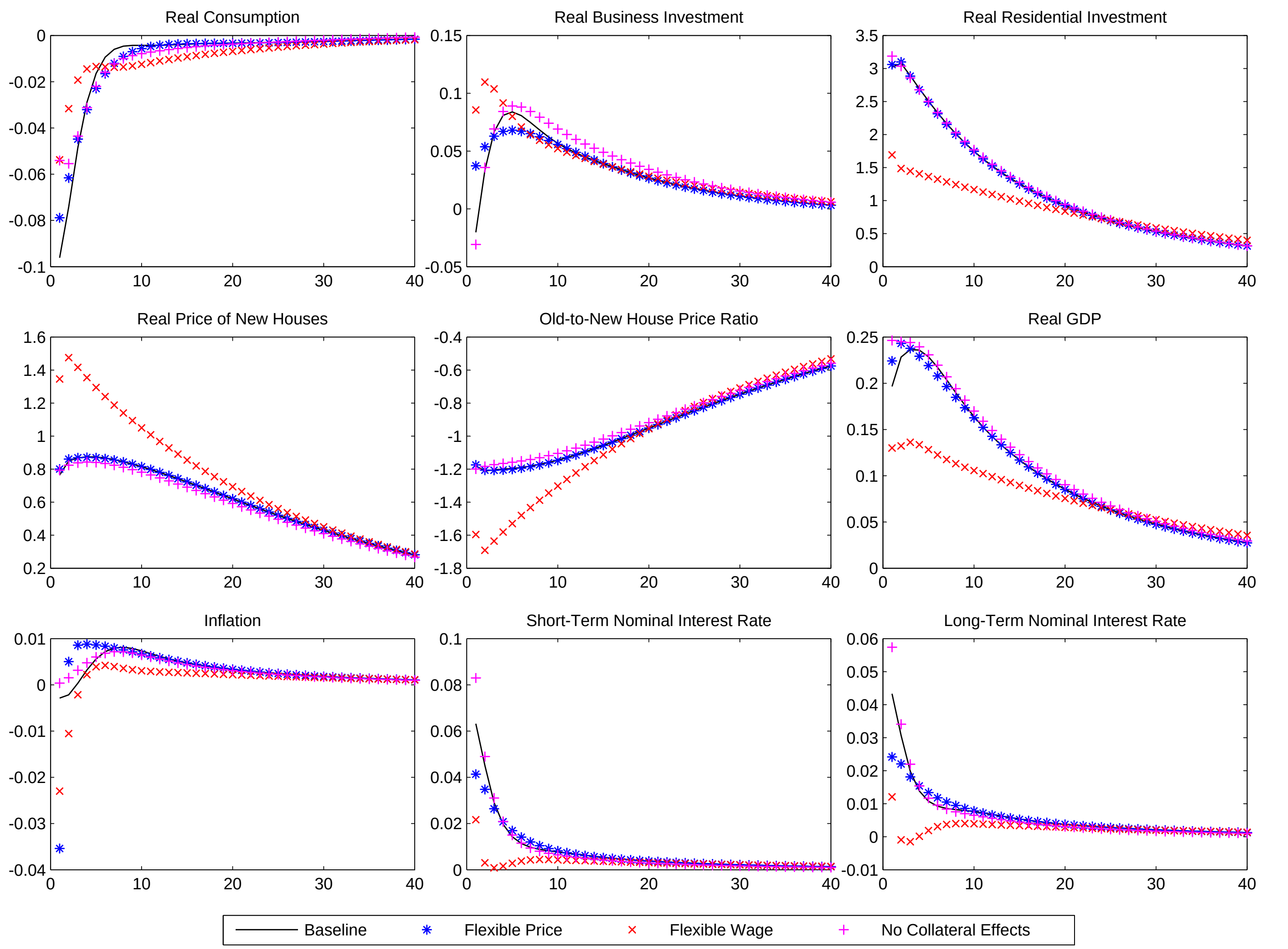


Figure 4.2: Land Price Fluctuations: A Comparison in Levels

4.3.5 Impulse Responses

The impulse responses of nine key variables – real consumption, real business investment, real residential investment, real price of new houses, the old-to-new house price ratio, real GDP, inflation, short-term nominal interest rate, and long term nominal interest rate – to preference (or demand) innovations, term-structure (or monetary) innovations, and the housing technology innovation are plotted in Figures 4.3 to 4.7.

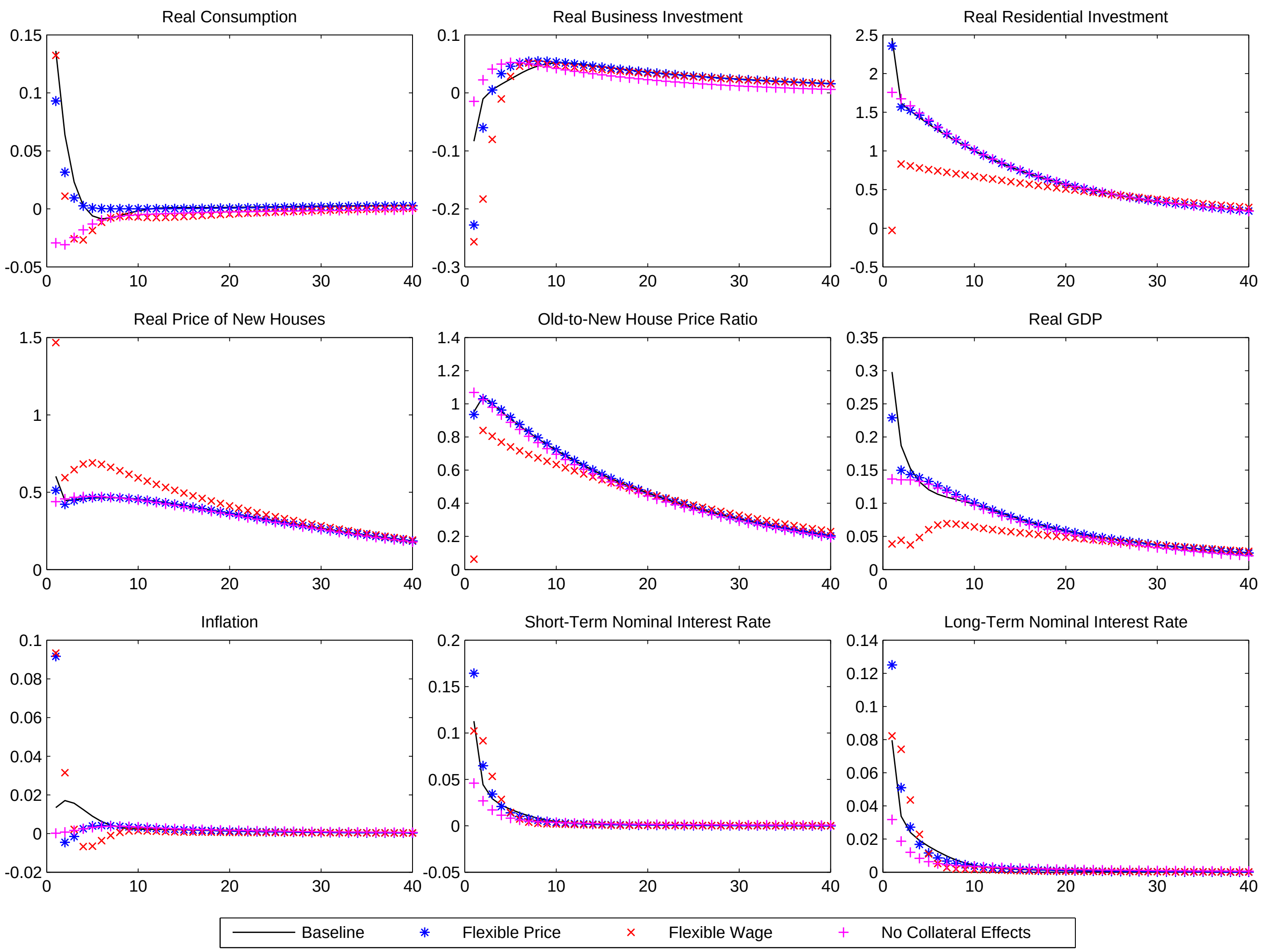
Preference Innovations Figure 4.3 plots impulse responses to the estimated preference innovation in new houses. A positive preference innovation in new houses discourages consumption and encourages business investment in the short run. Due to such a demand innovation, real investment in the housing sector increases by a large magnitude and stays above the steady-state level for a long time period. Real price of new houses also increases but old houses become relatively cheaper. The innovation in demand for new houses has a long-lasting positive impact on the real GDP and temporary impacts on inflation and nominal interest rates.



Note: The y-axis measures the percent deviation from the steady state.

Figure 4.3: Impulse Responses to a Preference Innovation in New Houses

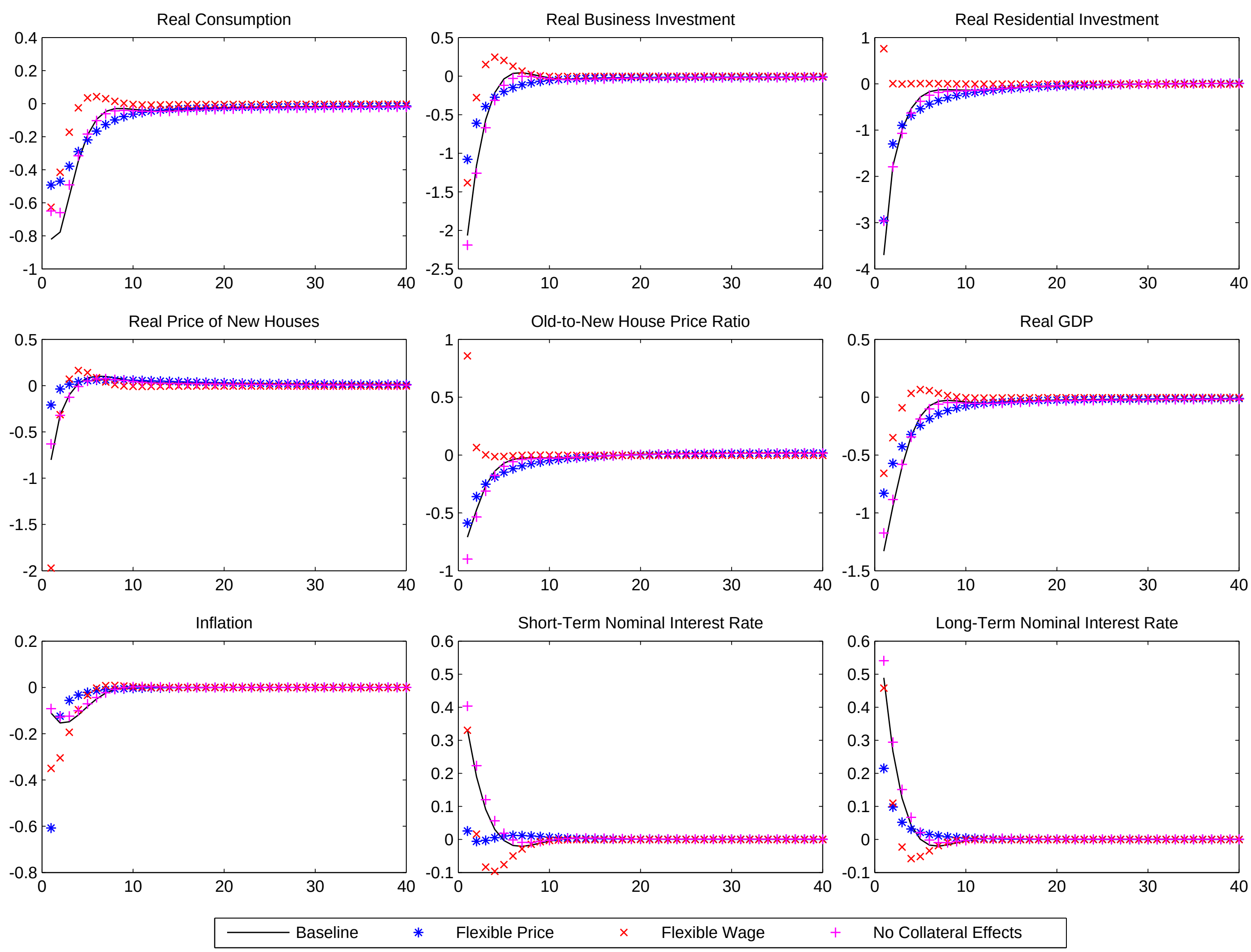
Figure 4.4 plots impulse responses to the estimated preference innovation in old houses. A positive preference innovation in old houses also encourages residential investment to a large extent but it does not have much impact on business investment. When households face such a disturbance, they switch from new houses to old houses, since new and old houses are perfect substitutes to each other. This switch increases their budget for consumption and stimulates real consumption in the short run. A positive innovation to the demand for old houses not only increases the real price of new houses but also shoots up the relative price of old houses. This innovation is another important driving force of the real GDP. It generates temporary and positive impacts on inflation and nominal interest rates.



Note: The y-axis measures the percent deviation from the steady state.

Figure 4.4: Impulse Responses to a Preference Innovation in Old Houses

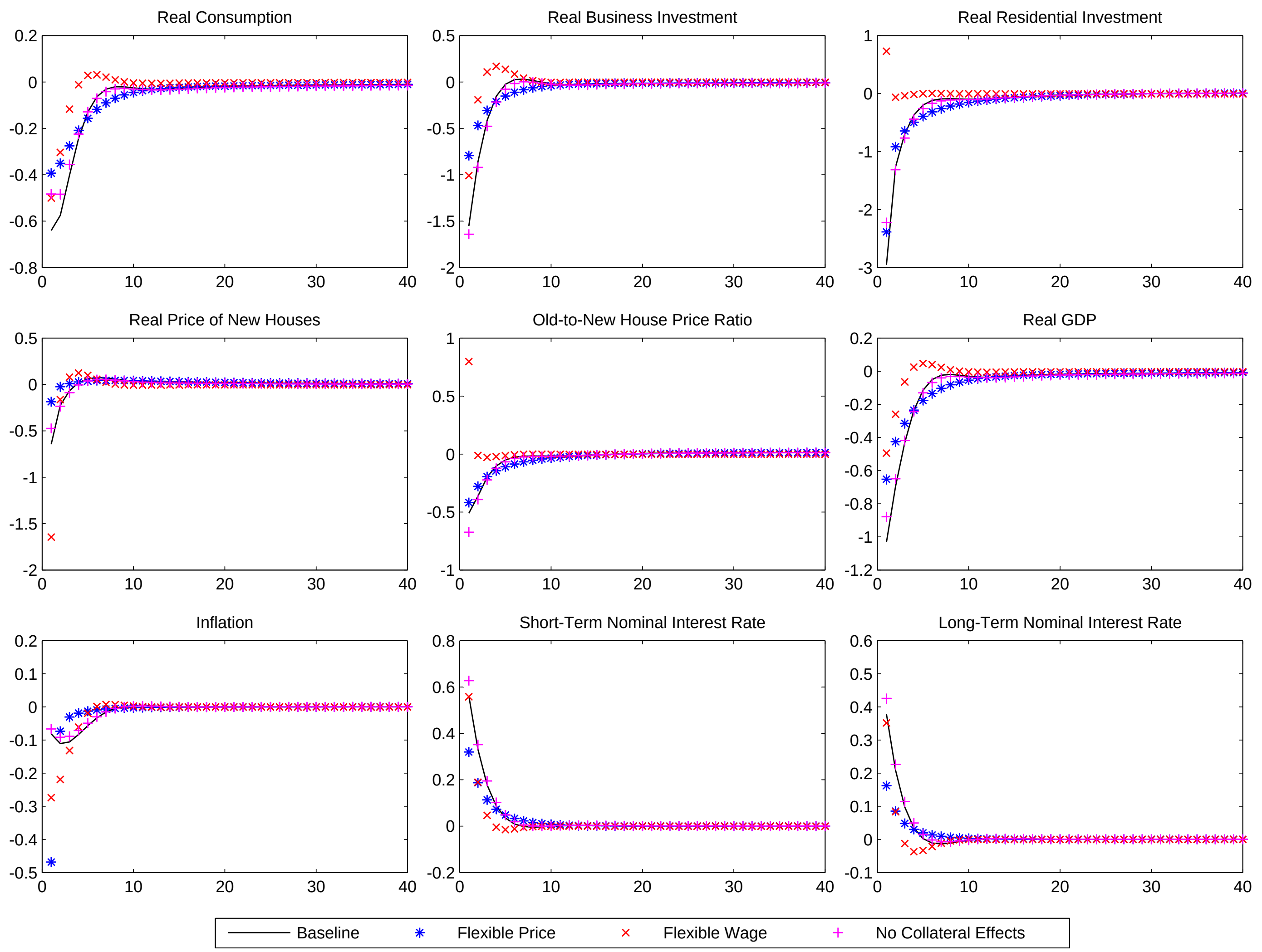
Term Structure Innovations Figures 4.5 and 4.6 plot impulse responses to the term structure level innovation and to the term structure slope innovation, respectively. Both level and slope innovations generate temporary (less than 8 quarters) impacts of similar pattern. All components of aggregate demand fall, with the largest drop in residential investment, followed by business investment, and by consumption. Real price of new houses drops and remains below the steady-state value for about 5 quarters. Old houses become cheaper relative to new houses. The slope innovation increases the short-term nominal interest rate by a larger extent, since the loadings on the slope factor decrease monotonically with the maturity.



Note: The y-axis measures the percent deviation from the steady state.

Figure 4.5: Impulse Responses to a Term Structure Level Innovation

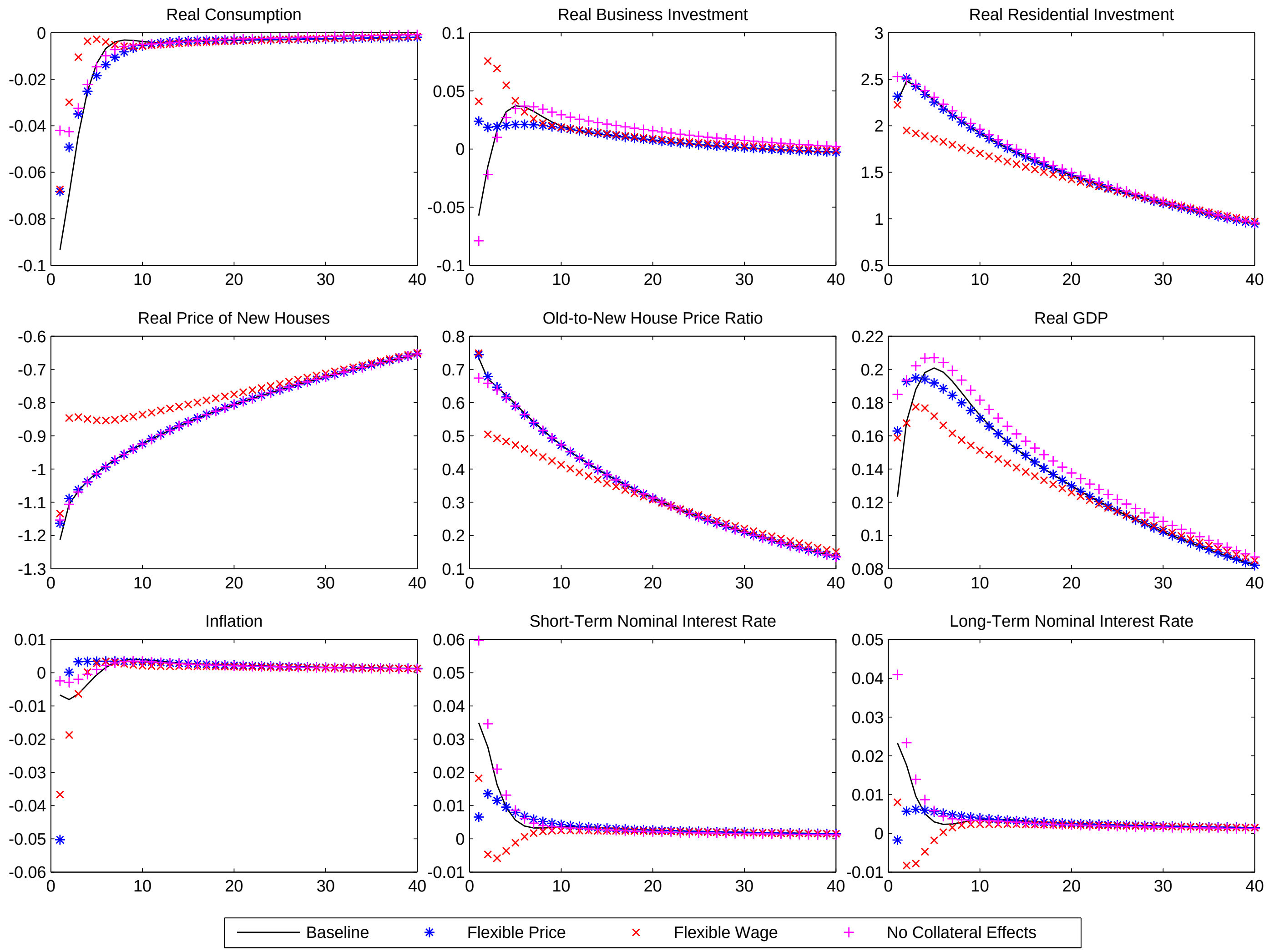
Compared to preference innovations, monetary factors generate larger changes in all components of aggregate demand. The large decline in residential investment is well-documented in the literature. Bernanke and Gertler (1995) find that the earliest and sharpest declines in final demand occur in residential investment in the case of a monetary policy shock.



Note: The y-axis measures the percent deviation from the steady state.

Figure 4.6: Impulse Responses to a Term Structure Slope Innovation

Housing Technology Innovation A positive technology innovation in the housing sector leads to a large rise in residential investment and a slight drop in business investment. Real consumption also drops in the short run. Due to a technological progress and hence a fall in construction costs, real price of new houses decreases. However, old houses become more expensive relative to new houses. This technology innovation stimulates the economy and has a hump-shaped impact on the real GDP. The innovation generates tiny impacts on inflation and nominal interest rates.



Note: The y-axis measures the percent deviation from the steady state.

Figure 4.7: Impulse Responses to a Housing Technology Innovation

4.3.6 Counterfactual Analyses

In this subsection, the scenarios with flexible price, flexible wages, and no collateral effects are examined. In particular, model simulations under $\theta_\pi = 0$ (flexible price), $\theta_{wc} = \theta_{wh} = 0$ (flexible wages), and $\alpha = 1$ (no collateral effects) are in turn conducted, holding all other parameters at their estimated values. The impulse responses for these three versions of the model are displayed in Figures 4.3 to 4.7. Three findings are worth mentioning.

First, price stickiness is important for the response of inflation. In the case of flexible price, all innovations generate larger impacts on the gross price inflation, simply because the price of consumption goods can flexibly vary at the retail level.

Second, wage stickiness has important implications for the housing markets. The combination of flexible house prices and sticky wages makes residential investment very sensitive to changes in preference conditions. Absent wage stickiness, the real price of new houses becomes more sensitive to preference innovations. Housing investment is sensitive to the term structure level and slope innovations only when wage stickiness is present. Facing positive term structure innovations, firms are discouraged to invest in the production of new houses in the case of sticky wages, because house price can always flexibly drop but wages remain sticky. The result is a lower level of investment in the housing sector. However, if wages are also flexible, the housing production costs do not change so that residential investment is insensitive to the term structure innovations. Wage stickiness also amplifies the responses of residential investment and new house price to the housing technology innovation.

Third, collateral effects are the key property for a positive response of consumption to a preference innovation in old houses. Without collateral effects, an increase in the demand for old houses would generate a fall in the real consumption. One explanation is that, facing a positive preference innovation in old houses, unconstrained households are willing to spend more on purchasing old houses.[17] In the case of no collateral effects or when α approaches 1, the supply of old houses falls down to zero and the price of old houses shoots up. As a result, a higher spending on old houses crowds out real consumption.

[17] According to the estimation results, there is a transaction flow of old houses from constrained to unconstrained households in the steady state.

4.3.7 Variance Decomposition

Table 4.4: Variance Decomposition of the Forecast Error

Variable	ε_c C Tech.	ε_h IH Tech.	ε_k IK Tech.	ε_{j_o} Old Pref.	ε_{j_n} New Pref.	ε_l Mon. L	ε_s Mon. S	ε_τ Lab. Sup.	ε_z Inter. Pref.	ε_p Cost Push
Real Consumption	22.90	0.13	4.45	0.19	0.15	14.07	7.87	29.43	7.12	13.69
Real Business Investment	5.98	0.01	66.40	0.04	0.06	6.68	3.74	6.68	3.73	6.68
Real Residential Investment	1.79	31.34	0.12	11.80	34.04	8.56	5.16	4.70	0.33	2.15
Real Price of New Houses	4.03	44.02	0.83	9.75	30.63	2.34	1.44	1.45	1.01	4.51
Growth of New House Price	1.97	29.74	0.21	7.75	11.86	18.93	12.50	0.50	0.35	16.20
Real Price of Old Houses	6.27	7.08	0.84	55.02	4.02	6.78	3.88	5.38	4.59	6.14
Growth of Old House Price	2.83	2.35	0.30	24.75	1.80	31.19	18.28	1.02	2.87	14.61
Old-to-New House Price Ratio	0.79	11.51	0.03	25.94	53.82	2.12	1.14	1.89	2.13	0.63
Real GDP	15.88	2.25	20.22	1.31	2.56	15.08	8.59	21.70	0.46	11.95
Inflation	5.39	0.04	0.23	0.16	0.08	12.59	6.39	1.29	6.18	67.65
Short-Term Interest Rate[1]	4.73	0.19	2.91	1.30	0.63	12.17	36.05	2.84	12.89	26.27
4-Quarter Interest Rate	3.55	0.14	2.15	0.97	0.46	9.88	26.43	2.11	9.68	19.85
12-Quarter Interest Rate	3.39	0.13	1.97	0.89	0.43	11.49	23.75	1.96	9.26	19.33
24-Quarter Interest Rate	3.33	0.12	1.83	0.84	0.39	14.61	21.25	1.85	9.11	19.55
36-Quarter Interest Rate	3.11	0.10	1.60	0.74	0.35	17.04	18.01	1.66	8.51	18.75
48-Quarter Interest Rate	3.13	0.09	1.51	0.71	0.33	20.79	16.39	1.60	8.55	19.33
Long-Term Interest Rate[2]	4.19	0.11	1.90	0.91	0.41	32.88	19.77	2.06	11.39	26.39
Level Factor	1.92	0.01	0.10	0.07	0.04	79.54	2.47	0.58	2.82	12.44
Slope Factor	2.84	0.27	2.84	1.09	0.64	27.53	51.48	2.67	4.44	6.20

Note: [1] 1-Quarter Interest Rate, [2] 60-Quarter Interest Rate. This table reports the 16-quarter conditional forecast error variance decomposition at the posterior mean parameter. All values are in percentage.

Table 4.4 presents results from the conditional variance decomposition of forecast error at the business cycle frequency.[18] Housing preference innovations (ε_{j_n} and ε_{j_o}) on the demand side and the housing technology innovation (ε_h) on the supply side explain about 80% of the volatility of residential investment, real price of new houses, and the old-to-new house price ratio. In particular, ε_{j_n} and ε_h each explains $30-40\%$ of the variations in residential investment and new house price. One half of the variation in the old-to-new house price ratio is explained by ε_{j_n}; another 25% by ε_{j_o}. The monetary components ε_l and ε_s explain $10-15\%$ of the variations in residential investment and the price of old houses, but do not contribute much to the variation in new house price. However, the

[18] The business cycle frequency is usually considered to be 3 to 5 years. Here, the 16-quarter conditional variance decomposition is reported.

level and slope innovations account for 30% of the variation in the growth rate of new house price and 50% of the variation in the growth rate of old house price. The housing demand and supply innovations do not have much explanatory power for real consumption and business investment.

As for nominal interest rates, the inflation innovation ε_p and the intertemporal preference innovation ε_z contribute about 20% and 10%, respectively, to the volatility. The term structure innovations ε_l and ε_s together explain $40-50\%$. The rest is attributable to the measurement error or the "curvature" factor which has been ignored in the estimation. Variations in both the level factor and the slope factor of the term structure are mostly explained by their own innovations, while $10-15\%$ are attributable to ε_z and ε_p. Housing demand and technology innovations both have limited effects on the term structure.

4.4 Application I: The Role of Transaction Costs

According to the specification of transaction costs in Equation (4.9) and (4.19), parameters ϕ and ϕ' do not have any impact on the steady state, where a tiny amount of old houses is transacted from constrained households to unconstrained households. However, these transaction cost parameters do have implications on the dynamics of the secondary housing market. Starting from the steady state, the transaction flows of old houses between two types of households under various combinations of (ϕ, ϕ') are simulated, given the set of estimated structural shocks. The results are presented in Figure 4.8. The vertical axis measures the transaction flow of old houses from constrained households to unconstrained households.

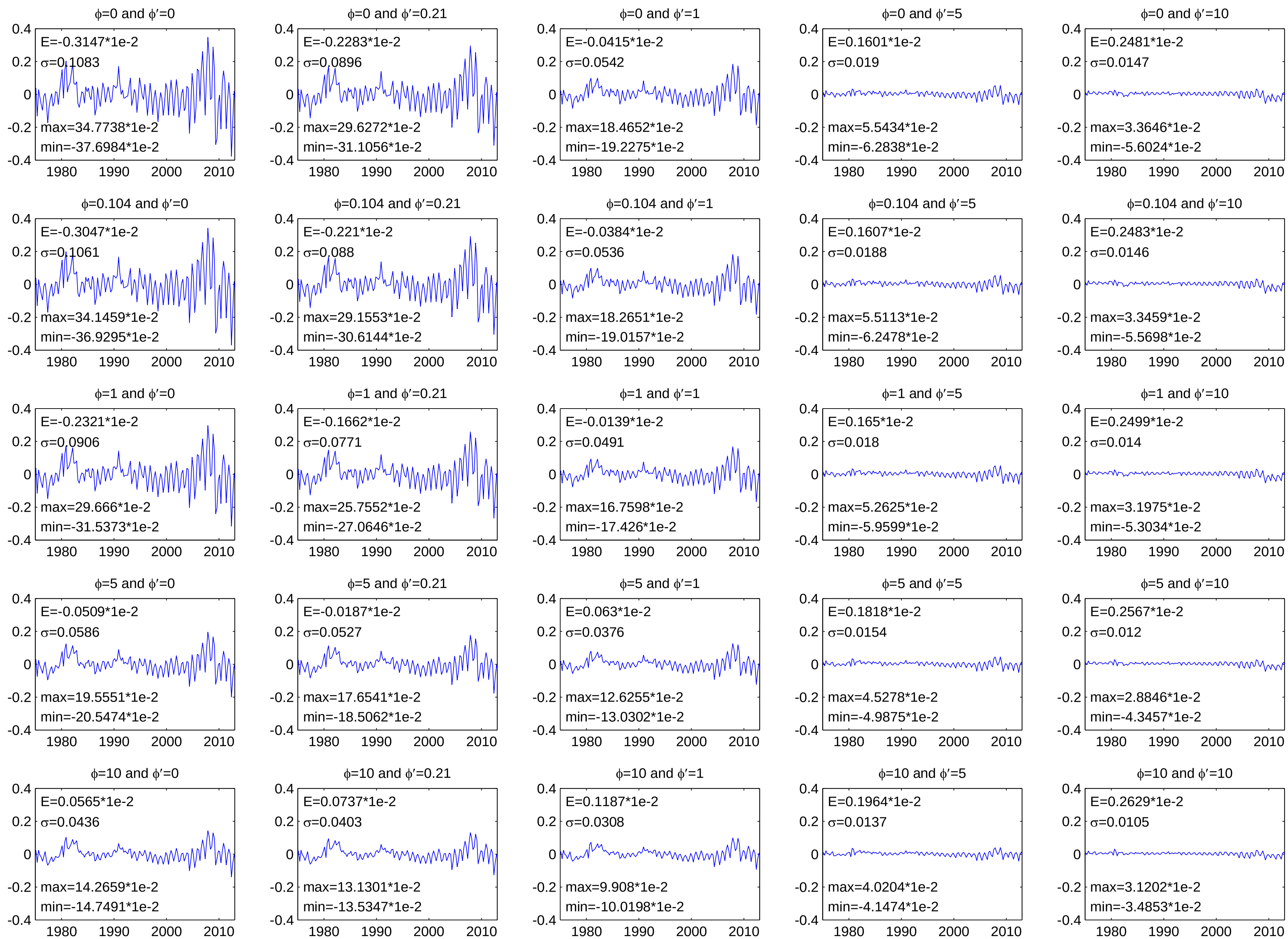


Figure 4.8: Transaction Costs and the Secondary Housing market

The graph located at the second row and the second column of the figure corresponds to the baseline model, where ϕ and ϕ' are estimated to be 0.104 and 0.210, respectively. In the baseline model, the transaction flow shows a maximum of 0.29 and a minimum of -0.31 with an average of -0.0022 and a standard error of 0.088. As either parameter increases (decreases), the transaction flow becomes less (more) volatile. In particular, the transaction flow is almost flat in the case of $(\phi, \phi') = (10, 10)$, whereas it shows the largest volatility when transaction costs are absent, i.e., $(\phi, \phi') = (0, 0)$. Further increasing the transaction cost parameters would with certainty shut the secondary market down.

Figure 4.8 suggests that, even though the transaction costs in the secondary housing market do not affect the steady state of the model, they generally discourage the two types of households from trading old houses with each other.

Knowing that the transaction costs stabilize the trade of old houses in the secondary market, it is straightforward to examine whether the existence of transaction costs is harmful to the overall stability of the macroeconomy. It is assumed that the policymaker seeks to minimize an ordinary expected loss criterion as in Giannoni and Woodford (2003):

$$\begin{aligned}\mathfrak{L} &= \lambda_\pi var(\ln \pi_t - \iota_\pi \ln \pi_{t-1}) + \lambda_w var(\ln \pi_{wc,t} - \iota_{wc} \ln \pi_{t-1}) \\ &\quad + \lambda_y var(\ln GDP_t - \ln(G_C GDP_{t-1})) + \lambda_r var(\ln R_{L,t} - \ln \overline{rr}), \end{aligned} \tag{4.41}$$

where $\overline{rr}$ is the steady-state value of long-run interest rate; $\pi_{wc,t} = \pi_t(w_{c,t} + w'_{c,t})/((w_{c,t-1} + w'_{c,t-1}))$ is the nominal wage inflation in the consumption sector.[19] Under the objective $\mathfrak{L}$, the policymaker minimizes a weighted variability of money inflation, wage inflation, output growth, and long-run nominal interest rate, with weights $\lambda_\pi = 0.5$, $\lambda_w = 0.5$, $\lambda_y = 0.048$, and $\lambda_r = 0.236$, as calibrated in Woodford (2003).

For later convenience, let $V_\pi = var(\ln \pi_t - \iota_\pi \ln \pi_{t-1})$, $V_w = var(\ln \pi_{wc,t} - \iota_{wc} \ln \pi_{t-1})$, $V_y =$

[19] The wage inflation in the housing sector is not considered, since both types of households contribute most of their labor to the production of consumption goods (see Sun and Tsang (2014)).

$var(\ln GDP_t - \ln(G_C GDP_{t-1}))$, and $V_r = var(\ln R_{L,t} - \ln \overline{rr})$ denote the four variance components in the loss function $\mathfrak{L}$. A 2-dimensional optimization over $(\phi, \phi') \in [0, +\infty)^2$ is conducted to identify the combination that minimizes the loss function $\mathfrak{L}$ in Equation (4.41). The results are presented in Table 4.5. Though the estimated values of transaction cost parameters are not large, the optimal values are zero for both types of households. Such a comparison suggests that reducing transaction costs improves overall macroeconomic stability.

Table 4.5: Estimated and Optimal Transaction Costs

	Statistics					Parameter Values		Social Welfare	
	V_π	V_w	V_y	V_r	$\mathfrak{L}$	ϕ	ϕ'	V_1^s	V_2^s
Estimated	0.0508	0.0378	0.5234	0.1011	0.0933	0.104	0.210	17.5197	-2.7162
Optimal	0.0511	0.0345	0.4255	0.1089	0.0889	0.000	0.000	17.5252	-2.7005

Note: All statistics are multiplied by 1000. The loss function $\mathfrak{L}$ is a weighted sum of V_π, V_w, V_y, and V_r. Social welfare V_1^s and V_2^s are computed under two different social welfare functions.

Another objective of the policymaker usually considered in the literature is social welfare (see Erceg, Henderson and Levin (2000), Faia and Monacelli (2007), and Schmitt-Grohé and Uribe (2007) among others). Social welfare is defined as the weighted average of two types of households' lifetime utilities, i.e.,

$$V_1^s = \alpha V + (1-\alpha) V', \tag{4.42}$$

$$\text{or } V_2^s = (1 - \beta G_C) V + (1 - \beta' G_C) V', \tag{4.43}$$

where V_1^s and V_2^s denote social welfare; the type-specific welfare V and V' are defined in Equations (4.1) and (4.2), respectively. The first social welfare function weights the welfare of the two types of households according to their economic sizes, α and $1-\alpha$. The second function is specified, following Rubio (2011) and Lambertini, Mendicino and Punzi (2013), such that both types of households achieve the same level of utility given the same constant consumption, housing, and working streams.[20]

[20]For the same constant consumption, housing, and working streams $(c, h_o, h_n, n_c, n_h) = (c', h'_o, h'_n, n'_c, n'_h)$, uncon-

The social welfare implied by different transaction costs conditional on the initial state's being the deterministic steady state are computed and presented in the last two columns of Table 4.5. While reducing overall economic fluctuations, a reduction in transaction costs also improves social welfare. This result is invariant to how these two types of households are weighted.

4.5 Application II: Sources of Housing Market Fluctuations

There are substantial fluctuations in the housing markets, associated with residential investment, real price of new houses, and the old-to-new house price ratio. Since around the year of 1997, there is an apparent convergence between the price of old houses and the price of new houses up until 2006, when the recent housing market bubble bursts. What factors are driving these changes?

4.5.1 What Moves the Housing Markets?

Figure 4.9 plots real residential investment (in log, with trend) and real price of new houses (in log, with trend) under different values of γ_{AH}, the housing technological trend. In the baseline model, $\gamma_{AH} = -0.042\%$, indicating that the TFP in the construction sector is *decreasing* at an annual rate of around 0.17%. The negative technological progress significantly explains the upward trend in house prices over the past four decades. Then, a variety of values are specifed for γ_{AH} and the model is simulated for each case. As the figure shows, an increase in γ_{AH} always promotes the investment in the housing sector and hence brings house price down.

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strained households' lifetime utility is $V = (1/(1-\beta G_C))U(c, h_o, h_n, n_c, n_h)$, whereas constrained households' lifetime utility is $V' = (1/(1-\beta' G_C))U(c', h'_o, h'_n, n'_c, n'_h)$. Weighting V by $1-\beta G_C$ and V' by $1-\beta' G_C$ ensures the same level of utility across the two types for given constant consumption, housing, and working streams. Lambertini, Mendicino and Punzi (2013) also consider an alternative social welfare function that weights the welfare of the two types of households equally and obtain consistent results.

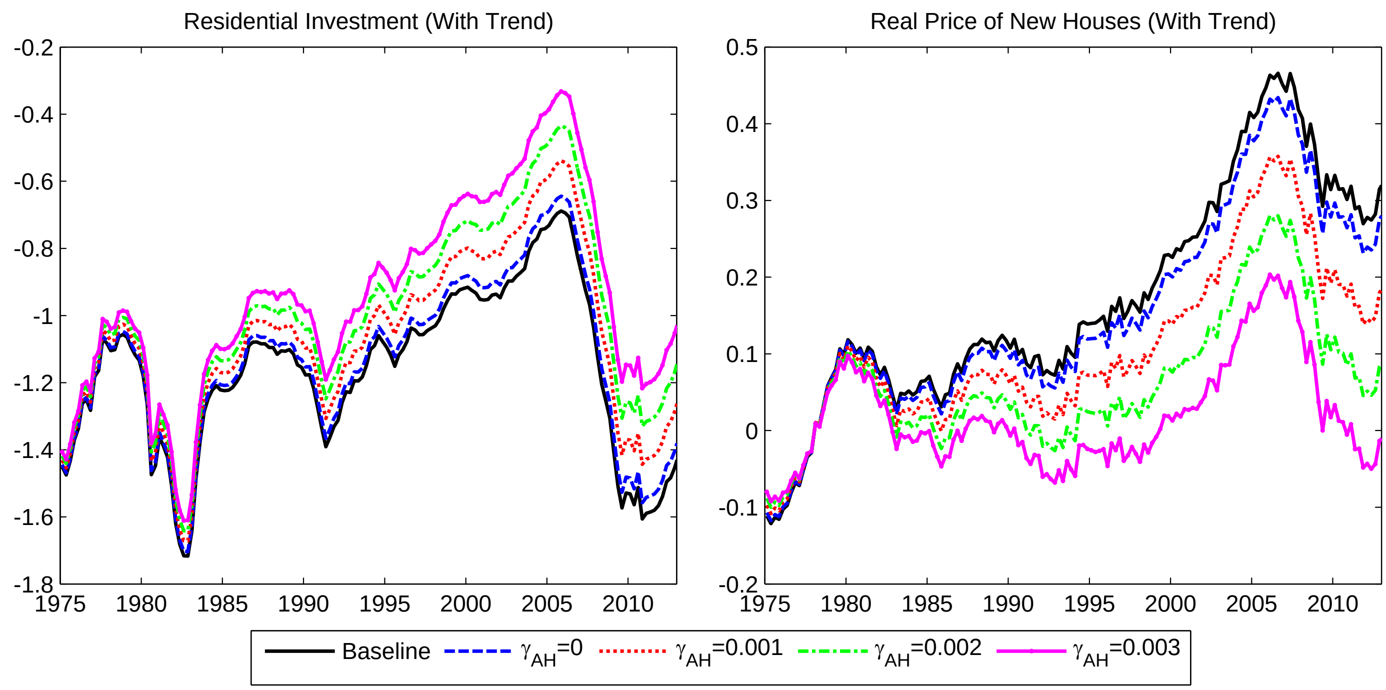

Figure 4.9: The Effects of the Housing Technological Trend

As of 2006, residential investment could be 50% higher and the price of new houses could be 25% lower, if the TFP in the housing sector is growing at the same rate as that in the consumption sector, i.e., $\gamma_{AH} = 0.003$.

Given that the upward trend in house prices is a result of the negative technological progress in the housing sector, I turn to exploring the dynamics of the detrended housing market variables. Each panel of Figure 4.10 plots real residential investment as well as two relevant counterfactual series, shutting down one of the innovations since 1975 and since 1997, respectively. Similar plots are presented in Figure 4.11 for the real price of new houses and in Figure 4.12 for the old-to-new house price ratio. Consistent with the variance decompositions, the housing technology innovation, ε_h, and the housing preference innovations, ε_{j_n} and ε_{j_o}, have large impacts on the dynamics of the housing markets.

Since the 1980s, real residential investment and real price of new houses are heavily influenced by the housing technology innovation ε_h, without which residential investment would have been 10%

to 20% lower between the mid-1980s and the mid-2000s and the new house price would have been 10% to 20% higher instead. The preference innovation in new houses ε_{j_n} has considerable impacts on residential investment and new house price before the 1990s. It also accelerates the drop in both series during the Great Recession. The preference innovation in old houses ε_{j_o} does not affect residential investment and new house price until 1997. After this time point, it contributes a lot to the run-up in the housing markets. The housing technology innovation ε_h does not have much impact on the ratio of old house price to new house price. Instead, this ratio is mainly affected by the housing preference innovations. Without the preference innovation in new houses ε_{j_n}, the old-to-new price ratio would have been around 10% higher before the 2006 financial crisis and 10% lower after the crisis. The preference innovation in old houses ε_{j_o} increases fluctuations of the house price ratio, especially after 1997. The fact that the old house price increases relative to the price of new houses between the mid-1990s and the mid-2000s is mainly explained by this innovation.

Among other innovations, the consumption sector technology innovation ε_c, the term structure innovations, ε_l and ε_s, and the labor supply innovation, ε_τ, have limited impacts on the housing markets.

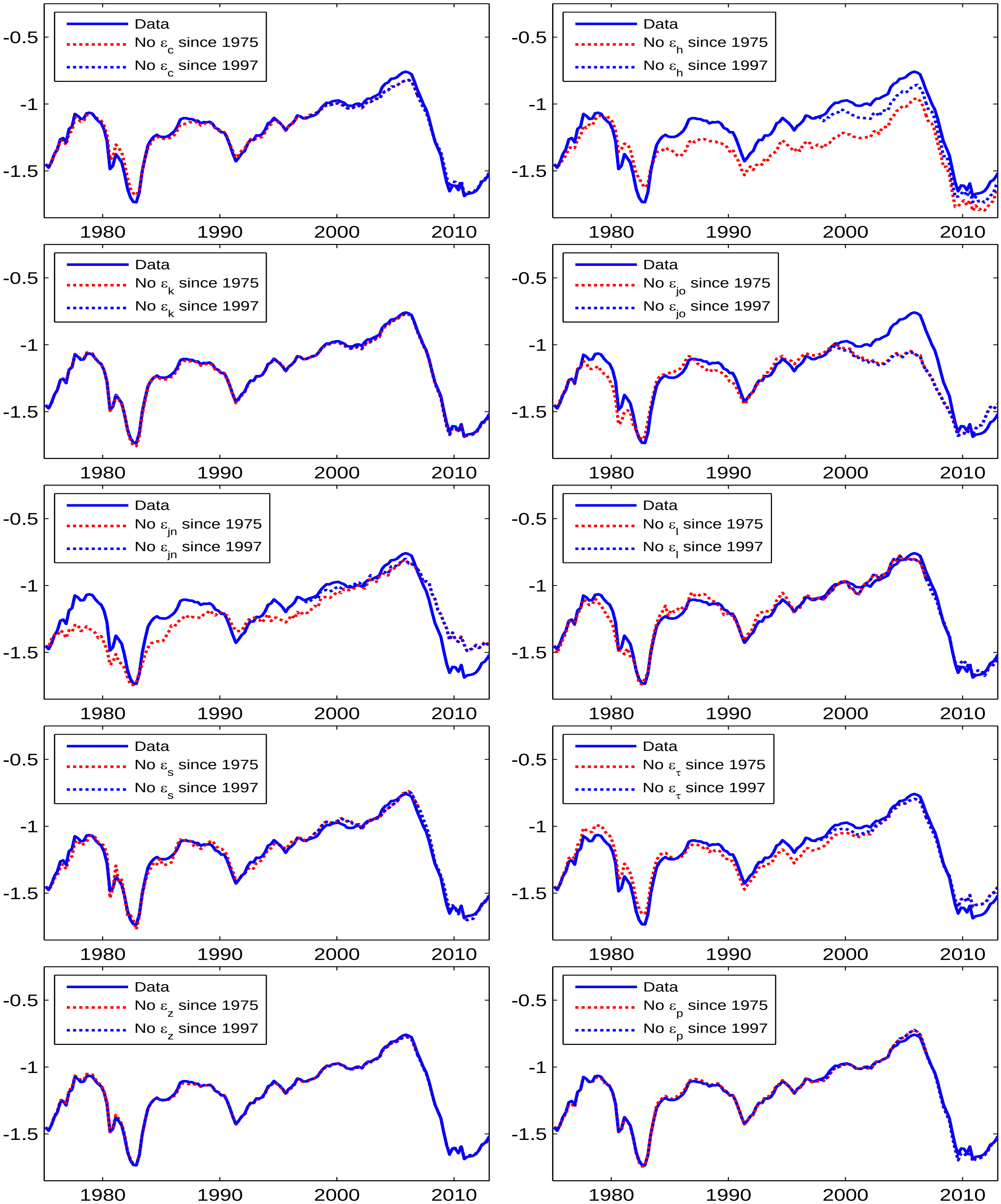

Figure 4.10: Counterfactuals on Residential Investment (in log)

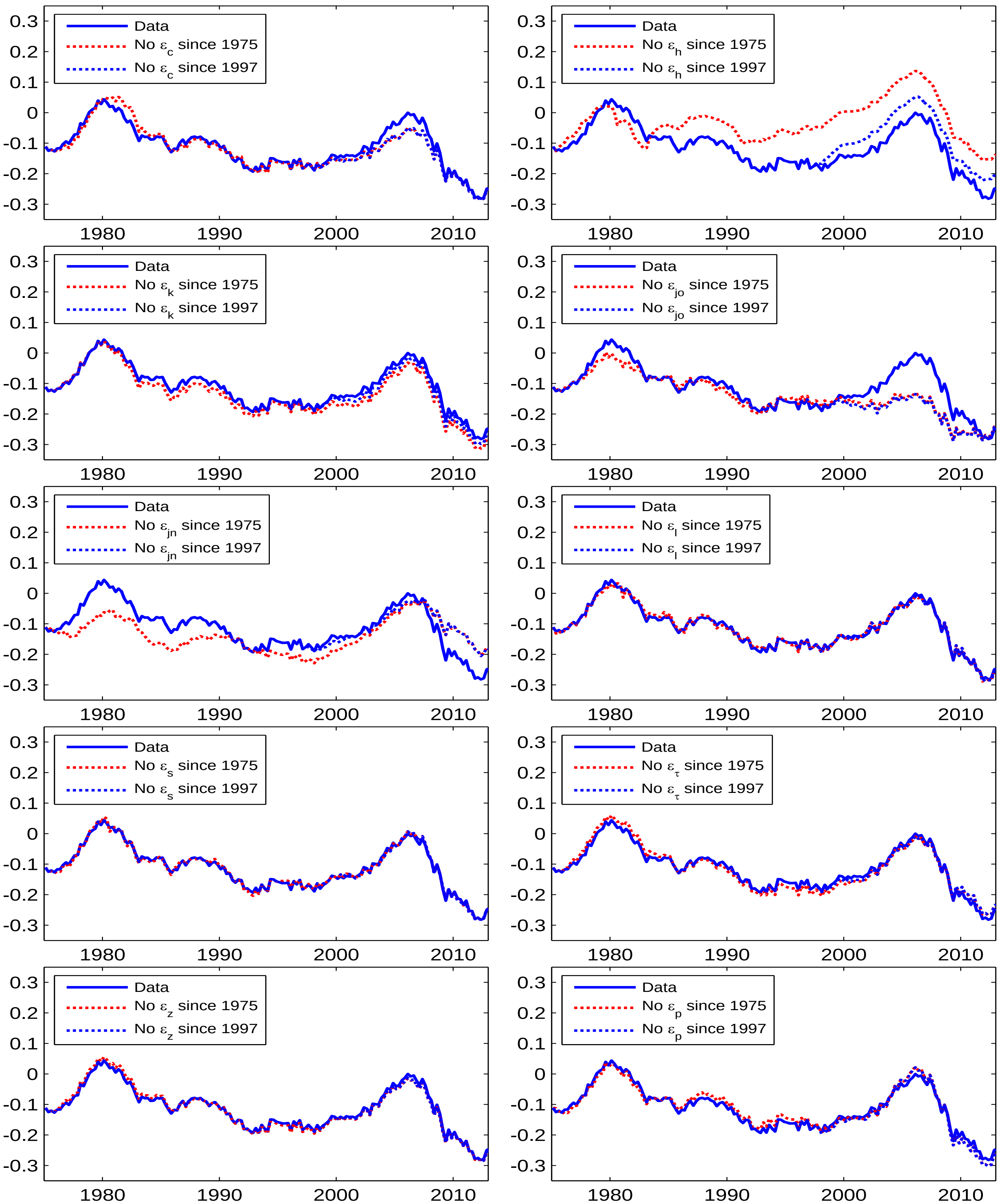

Figure 4.11: Counterfactuals on Real Price of New Houses (in log)

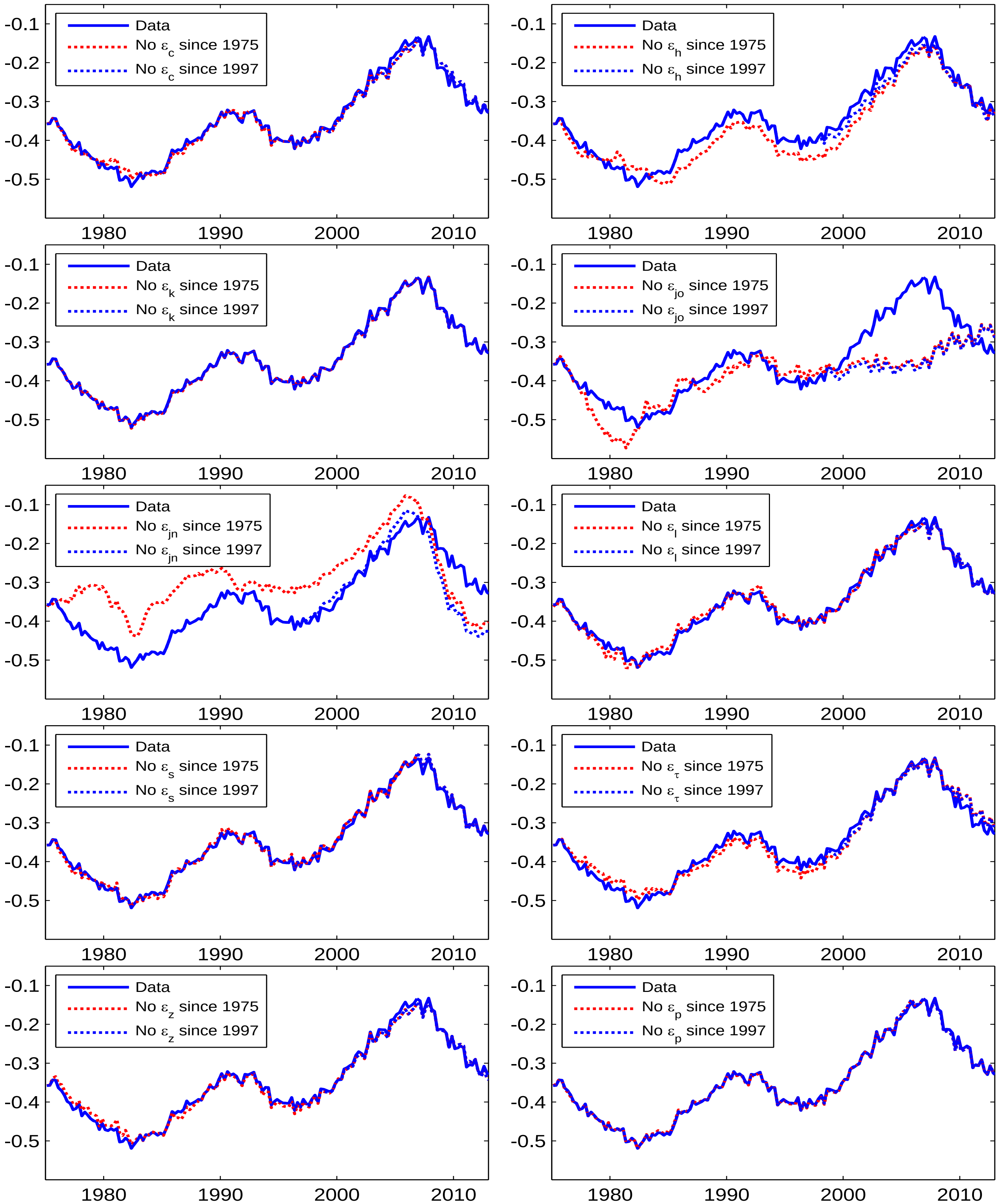

Figure 4.12: Counterfactuals on Old-to-New House Price Ratio (in log)

4.5.2 Understanding the Housing Preference Innovations

The previous subsection suggests that the housing preference innovations, ε_{j_n} and ε_{j_o}, account for a sizeable fraction of housing market fluctuations. In the DSGE model, these preference innovations are exogenously introduced, which represent genuine changes in consumer tastes given that the DSGE model "truly" characterizes the economy. However, the concern that these innovations could be nothing but the omitted factors that affect the demand for the two types of houses needs to be addressed. A standard multivariate time series analysis is then conducted for ε_{j_n} and ε_{j_o}, following Evans (1992),

$$\varepsilon_{j_n,t} = B_n \mathbf{x}_t + \nu_{n,t} \tag{4.44}$$

$$\varepsilon_{j_o,t} = B_o \mathbf{x}_t + \nu_{o,t}, \tag{4.45}$$

where B_n and B_o coefficient vectors; $\mathbf{x}$ is a vector of potentially relevant explanatory variables that affect housing preferences; ν_n and ν_o are independently and identically distributed error terms.[21]

The main determinants of the demand for housing are demographic. But other factors, such as income, price of housing, cost and availability of credit, consumer preferences, investor preferences, price of substitutes, and price of complements, all play a role. Some of these determinants have already been included in the model. Several variables that do not appear in the model are chosen as regressors, including the University of Michigan Index of Consumer Sentiment, the Economic

[21] Iacoviello and Neri (2010) conduct a similar analysis to examine if the preference innovation $u_{j,t}$ can be predicted, using the original framework of Evans (1992):

$$u_{j,t} = A(L)u_{j,t-1} + B(L)\mathbf{x}_{t-1} + \nu_t$$

where ν_t is a mean zero independently and identically distributed random variable, $A(L)$ and $B(L)$ are polynomials in the lag operator L, and $\mathbf{x}$ is a list of potential explanatory variables for housing demand. However, this setup contradicts their model assumption that $u_{j,t}$ is an independently and identically distributed random variable. The regressions (4.44) and (4.45) exclude the lags of each relevant dependent variable from the list of regressors, in order to be consistent with the assumptions in Section 2. By regressing the preference innovation on a variety of variables in their first lags, Iacoviello and Neri (2010) answer the question that whether the preference innovation can possibly be predicted. In the present work, a contemporaneous regression is conducted to answer a different question that how changes in the housing preference shocks can be explained by unmodeled factors.

Policy Uncertainty Index, the LTV ratio, the employment-to-population ratio, the population share between ages 16 and 19, the population share between ages 20 and 24, the population share above age 55, and the marriage rate.[22] Among these variables, marriage rate is available from 1979:Q1 to 2009:Q4 only. The trends in data, if exist, are all removed by subtracting a least-squares-fit straight line. Marriage rate is seasonally adjusted. Each variable is in the natural logarithm form. The results are presented in Table 4.6, in which the first four columns show the results with marriage rate excluded from the regressor list while the last four columns show the results with marriage rated included.

Columns (1) and (2) indicate that changes in the selected regressors explain only 10% of the variation in the preference innovation to new houses and 30% of the variation in the preference innovation to old houses. Consumer sentiment is statistically significant and has the expected sign in both regressions. As households become more optimistic about the economic prospects, they are willing to own more of both types of houses. The economic policy uncertainty does not significantly affect either of the preference innovations. A higher LTV ratio lowers the demand for old houses but does not affect the demand for the other type. One explanation is perhaps that an increase in LTV ratio is usually implemented during hard times when households attempt to save more for the future. The employment-to-population ratio affects the demand for new houses negatively and the demand for old houses positively. Age structure has important implications for the housing demand. As the share of population between ages 16 and 19 increases, the demand for old houses rises; the share of population above age 55 instead affects the demand for new houses negatively. This suggests that, firstly, households with more teenagers have higher demand for housing but they are likely to own more of old instead of new houses, and secondly, the elderly population tend not to own new houses. The backward-stepwise selection is then used to keep those variables that have significant impacts on the preference innovations. The main results presented in columns (3) and (4) stay the

[22] Data sources are as follows. Consumer Sentiment: Thomson Reuters/University of Michigan Surveys of Consumers; Uncertainty Index: Economic Policy Uncertainty; LTV Ratio: Federal Housing Finance Agency; the employment-to-population ratio and population shares: Federal Reserve Bank of St. Louis; marriage rate: Centers for Disease Control and Prevention.

same. When marriage rate is included in the regressor list, similar results are obtained in columns (5)-(8). An increase in marriage rate significantly promotes the households' demand for new houses. One explanation would be that as more people, usually young people, get married and leave their families, higher housing demand is created and newly married people prefer owning a new house. The results with backward-stepwise selection are similar.

Table 4.6: Understanding the Housing Preference Innovations

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	ε_{jn} NEW	ε_{jo} OLD	ε_{jn} NEW	ε_{jo} OLD	ε_{jn} NEW	ε_{jo} OLD	ε_{jn} NEW	ε_{jo} OLD
Consumer	0.162***	0.144***	0.146***	0.143***	0.149***	0.134***	0.135***	0.143***
Sentiment	(0.0436)	(0.0225)	(0.0381)	(0.0213)	(0.0466)	(0.0234)	(0.0414)	(0.0206)
Uncertainty	0.0219	-0.00867			-0.00187	-0.00363		
Index	(0.0341)	(0.0176)			(0.0389)	(0.0195)		
LTV	0.0155	-0.442***		-0.480***	-0.278	-0.677***		-0.695***
Ratio	(0.238)	(0.123)		(0.117)	(0.256)	(0.128)		(0.120)
Employment-to-	-1.156***	0.332**	-1.185***	0.210**	-1.466***	0.282	-1.474***	
Population	(0.286)	(0.148)	(0.281)	(0.0981)	(0.424)	(0.212)	(0.345)	
Population Share	0.0396	0.144***		0.166***	0.0250	0.115*		0.153***
b/w 16 and 19	(0.0959)	(0.0494)		(0.0438)	(0.123)	(0.0616)		(0.0489)
Population Share	0.130	0.104			-0.0199	0.127		
b/w 20 and 24	(0.187)	(0.0963)			(0.238)	(0.119)		
Population Share	-0.647**	0.0254	-0.484**		-0.774***	-0.238	-0.702***	-0.238**
above 55	(0.273)	(0.141)	(0.198)		(0.294)	(0.147)	(0.215)	(0.0921)
Marriage					0.0897**	-0.0108	0.0786**	
Rate					(0.0443)	(0.0222)	(0.0376)	
N	156	156	156	156	124	124	124	124
adj. R^2	0.108	0.305	0.126	0.308	0.153	0.362	0.172	0.372

Note: Standard errors in parentheses; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$; constant terms are not reported.

The results provide some evidence that part of the preference innovations can be captured by some omitted variables. However, the low goodness-of-fit in the multivariate regression analysis suggests that most of the movements in housing demand represent shifts in households' tastes for housing.

4.6 Application III: The Term Structure of Nominal Interest Rates

Modeling the term structure of nominal interest rates in a structural framework is a recent exercise. Before Rudebusch and Wu (2008), studies on the relationship between the term structure and macroeconomic factors focus particularly on statistical descriptions (see for example Ang and Piazzesi (2003)). Rudebusch and Wu (2008) develop a macro-finance model that combines the term structure with standard macroeconomic aggregate relationships for output and inflation. They find that up to 95% of the variations in interest rates are driven by the term structure level and factor shocks. However, their model specification is far from complete. In addition, the housing markets have never been considered among the determinants of the term structure. The present paper incorporates the term structure in a DSGE model that features a variety of structural innovations, including those related to the housing markets. As the variance decomposition in Table 4.4 shows, the term structure level and slope factors explain 40-50% of the variations in nominal interest rates. Among other innovations, the cost-push innovation and the intertemporal preference innovation also significantly contribute to the interest rate fluctuations.

How each of the structural innovations affects the shape of the yield curve? Each panel of Figure 4.13 shows the comparison between the data and the counterfactual scenario, with one of the innovations being shut down. Circles denote the average of the observed nominal interest rates over the whole sample period 1975:Q1-2013:Q4 at different maturities, whereas plus signs denote the average of the counterfactual nominal interest rates. A line of best fit using the method of least squares is added for each series. The shape of the yield curve is characterized by the intercept and the slope of the best-fit line, the later of which is also reported in each panel of the figure.

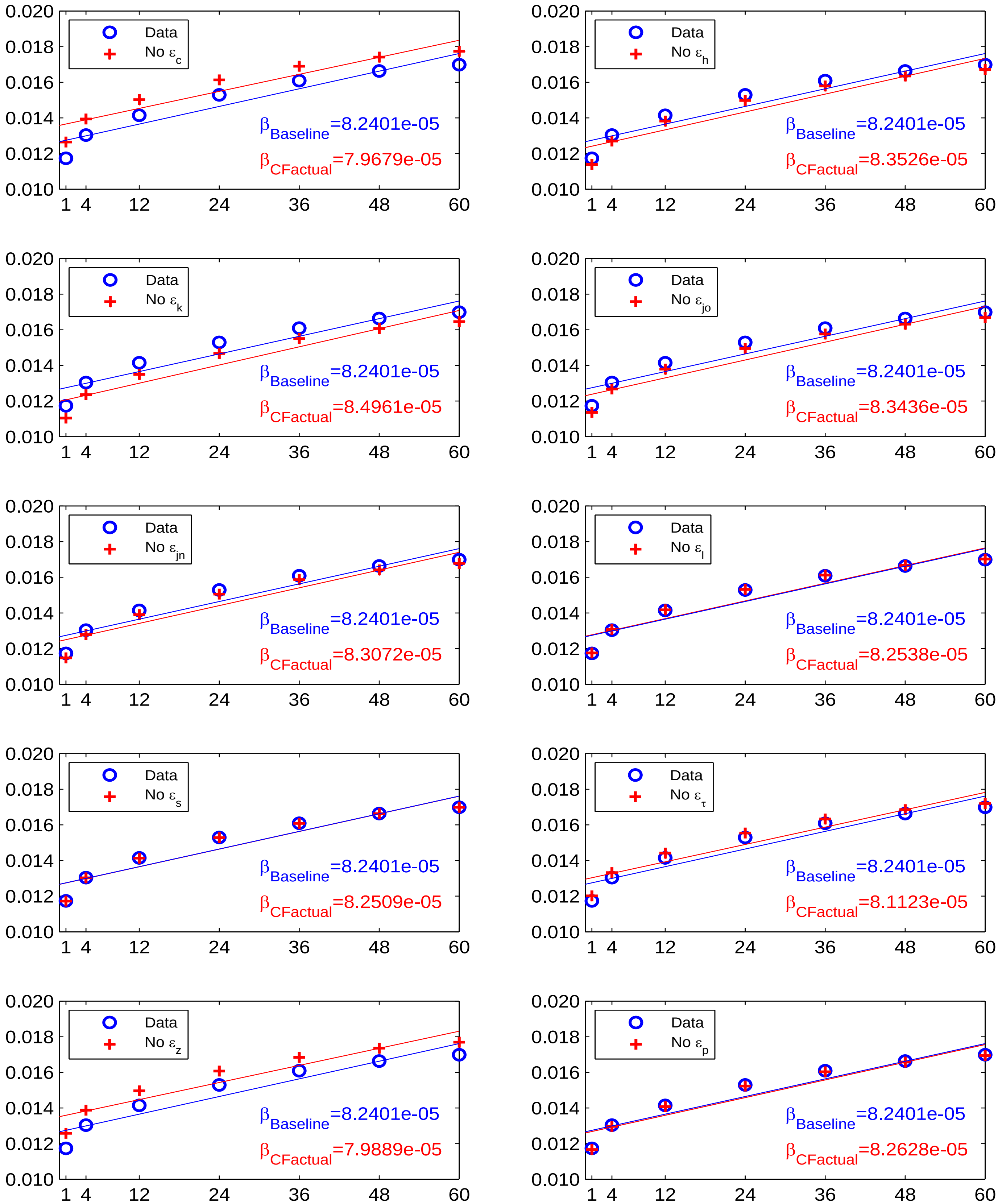

Figure 4.13: Structural Innovations and the Yield Curve

Over the sample period, even the term structure innovations, ε_l and ε_s, and the cost-push innovation ε_p significantly contribute to the variations in nominal interest rates, they do not change the average level of interest rates at any maturity, so that the shape of the yield curve is insensitive. Some of the structural innovations, such as the technology innovation in the consumption sector ε_c, the labor supply innovation ε_τ, and the intertemporal preference innovation ε_z, affect interest rates negatively. Others, including the housing productivity innovation ε_h, the investment-specific technology innovation ε_k, the housing preference innovations ε_{j_o} and ε_{j_n}, have positive impacts on interest rates. More importantly, the existence of these innovations significantly changes the shape of the yield curve. The shocks negatively affecting interest rates tend to make the yield curve steeper and those positively affecting interest rates make the yield curve flatter.

It is worth emphasizing that all of the housing market innovations, i.e., ε_h, ε_{j_o}, and ε_{j_n}, have been positively affecting nominal interest rates at different maturities and making the yield curve flatter over the entire sample period 1975:Q1-2013:Q4. The positive effect is even more evident between 1997 and 2006, during which housing market innovations together reduce the slope of the yield curve by 15%. To my knowledge, this is the first-ever finding on the relationship between the housing market conditions and the yield curve.

4.7 Conclusion

In this paper, I construct a DSGE model that features a market of newly built houses and a secondary market of old houses. I incorporate the term structure of nominal interest rates into the DSGE model in order to investigate the bidirectional relations between housing markets and the term structure. The negative TFP progress in the housing sector significantly explains the upward trend in house prices over the past four decades. The model attributes about 80% of the volatility of residential investment, real price of new houses, and the old-to-new house price ratio to housing preference and housing technology innovations. Monetary factors explain about 15% of the volatility of housing investment, but do not significantly contribute to the price fluctuations of either new or old houses. The preference innovation to old houses mainly explains the run-up in the old house price relative to the price of new houses between the mid-1990s and the mid-2000s. The intertemporal preference innovation has a non-negligible contribution to the dynamics of nominal interest rates. Housing market conditions do not contribute much to the fluctuations of interest rates, but significantly affect the shape of the yield curve.

The exogenously-specified shifts to housing preferences are related to several variables. Multivariate regressions suggest that consumer sentiment, the loan-to-value ratio, the employment-to-population ratio, and age structure, as well as marriage rate, are all potential variables that have explanatory power for the estimated housing preference innovations. Further research might improve such a DSGE model by taking these factors, such as age structure, into consideration. However, most of the movements in housing demand represent shifts in households' tastes.

4.8 Appendix: Derivations for the Model Economy

4.8.1 Unconstrained Households

The lifetime utility of unconstrained households is given by:

$$V_t = E_0 \sum_{t=0}^{\infty} (\beta G_C)^t z_t \left[\frac{G_C - \epsilon}{G_C - \beta\epsilon G_C} \ln(c_t - \epsilon c_{t-1}) + j_{o,t} \ln(h_{o,t}) + j_{n,t} \ln(h_{n,t}) - \frac{\tau_t}{1+\eta} \left(n_{c,t}^{1+\xi} + n_{h,t}^{1+\xi} \right)^{\frac{1+\eta}{1+\xi}} \right]. \tag{4.46}$$

The marginal utility of consumption is:

$$u_{c,t} = \left(\frac{G_C - \epsilon}{G_C - \beta\epsilon G_C} \right) \left(\frac{z_t}{c_t - \epsilon c_{t-1}} - \frac{\beta\epsilon G_C z_{t+1}}{c_{t+1} - \epsilon c_t} \right). \tag{4.47}$$

The marginal utility of old house is:

$$u_{h_o,t} = \frac{z_t j_{o,t}}{h_{o,t}}, \tag{4.48}$$

and the marginal utility of new house is:

$$u_{h_n,t} = \frac{z_t j_{n,t}}{h_{n,t}}. \tag{4.49}$$

The marginal disutilities of working in the goods sector and in the housing sector are:

$$u_{n_c,t} = z_t \tau_t \left(n_{c,t}^{1+\xi} + n_{h,t}^{1+\xi} \right)^{\frac{\eta-\xi}{1+\xi}} n_{c,t}^{\xi}, \tag{4.50}$$

$$u_{n_h,t} = z_t \tau_t \left(n_{c,t}^{1+\xi} + n_{h,t}^{1+\xi} \right)^{\frac{\eta-\xi}{1+\xi}} n_{h,t}^{\xi}. \tag{4.51}$$

Along the balanced growth path, consumption grows at the rate G_C every quarter, the marginal utility of consumption falls at this rate. Hence the transformed marginal utility $\widetilde{u}_{c,t} = u_{c,t} G_C^t$ is stationary around the steady state and is equal to:

$$\widetilde{u}_{c,t} = \left(\frac{G_C - \epsilon}{G_C - \beta\epsilon G_C} \right) \left(\frac{z_t}{\widetilde{c}_t - \frac{\epsilon}{G_C} \widetilde{c}_{t-1}} - \frac{\beta\epsilon z_{t+1}}{\widetilde{c}_{t+1} - \frac{\epsilon}{G_C} \widetilde{c}_t} \right). \tag{4.52}$$

Transformed consumption, $\widetilde{c}_t = c_t / G_C^t$, and the scaled marginal utility of consumption $\widetilde{u}_{c,t}$:

$$\widetilde{u}_c = z \left(\frac{G_C - \epsilon}{G_C - \beta\epsilon G_C} \right) \left(\frac{1}{\widetilde{c} - \frac{\epsilon}{G_C} \widetilde{c}} - \frac{\beta\epsilon}{\widetilde{c} - \frac{\epsilon}{G_C} \widetilde{c}} \right) = \frac{1}{\widetilde{c}}, \tag{4.53}$$

are both constant in the steady state, since z is equal to one.

The marginal utilities of housing $u_{h_o,t} = z_t j_{o,t} / h_{o,t}$ and $u_{h_n,t} = z_t j_{n,t} / h_{n,t}$ both decline at the rate G_H. Therefore the transformed marginal utilities $\widetilde{u}_{h_o,t} = u_{h_o,t} G_H^t$ and $\widetilde{u}_{h_n,t} = u_{h_n,t} G_H^t$ are stationary around the steady state and are equal to:

$$\widetilde{u}_{h_o,t} = \frac{z_t j_{o,t}}{\widetilde{h}_{o,t}}, \quad \widetilde{u}_{h_n,t} = \frac{z_t j_{n,t}}{\widetilde{h}_{n,t}}. \tag{4.54}$$

In the steady state, $\widetilde{u}_{h_o} = j_o/\widetilde{h}_o$ and $\widetilde{u}_{h_n} = j_n/\widetilde{h}_n$.

Notice that hours worked in the two sectors, $n_{c,t}$ and $n_{h,t}$, are stationary already in the level economy.

Unconstrained households' budget constraint is given by:

$$\begin{aligned}
&c_t + \frac{k_{c,t}}{A_{k,t}} + k_{h,t} + k_{b,t} + q_t[f_t(h_{o,t} - (1-\delta_h)h_{o,t-1} - (1-\delta_h)h_{n,t-1}) + h_{n,t}] + p_{l,t}l_t \\
&+ \frac{R_{L,t-1}b_{t-1}}{\pi_t} + TC_t \\
=&\frac{w_{c,t}n_{c,t}}{X_{wc,t}} + \frac{w_{h,t}n_{h,t}}{X_{wh,t}} + DIV_t + \left(R_{c,t}z_{c,t} + \frac{1-\delta_{kc}}{A_{k,t}}\right)k_{c,t-1} + (R_{h,t}z_{h,t} + 1 - \delta_{kh})\,k_{h,t-1} \\
&+ p_{b,t}k_{b,t} + b_t + (p_{l,t} + R_{l,t})l_{t-1} - \Phi_t - \frac{a(z_{c,t})k_{c,t-1}}{A_{k,t}} - a(z_{h,t})k_{h,t-1},
\end{aligned} \tag{4.55}$$

which can be transformed as follows:

$$\begin{aligned}
&\widetilde{c}_t + \frac{\widetilde{k}_{c,t}}{a_{k,t}} + \widetilde{k}_{h,t} + \widetilde{k}_{b,t} + \widetilde{q}_t\left[f_t\left(\widetilde{h}_{o,t} - \frac{(1-\delta_h)\widetilde{h}_{o,t-1}}{G_H} - \frac{(1-\delta_h)\widetilde{h}_{n,t-1}}{G_H}\right) + \widetilde{h}_{n,t}\right] + \widetilde{p}_{l,t}l_t \\
&+ \frac{R_{L,t-1}\widetilde{b}_{t-1}}{\pi_t G_C} + \widetilde{TC}_t \\
=&\widetilde{w}_{c,t}n_{c,t} + \widetilde{w}_{h,t}n_{h,t} + \left(1 - \frac{1}{X_t}\right)\widetilde{Y}_t + \left(\widetilde{R}_{c,t}z_{c,t} + \frac{1-\delta_{kc}}{a_{k,t}}\right)\frac{\widetilde{k}_{c,t-1}}{G_{KC}} + (R_{h,t}z_{h,t} + 1 - \delta_{kh})\,\frac{\widetilde{k}_{h,t-1}}{G_C} \\
&+ p_{b,t}\widetilde{k}_{b,t} + \widetilde{b}_t + (\widetilde{p}_{l,t} + \widetilde{R}_{l,t})l_{t-1} - \widetilde{\Phi}_t - \frac{a(z_{c,t})\widetilde{k}_{c,t-1}}{a_{k,t}G_{KC}} - a(z_{h,t})\frac{\widetilde{k}_{h,t-1}}{G_C},
\end{aligned} \tag{4.56}$$

where $\widetilde{c}_t = c_t/G_C^t$, $\widetilde{k}_{c,t} = k_{c,t}/G_{KC}^t$, $a_{k,t} = A_{k,t}/G_{AK}^t$, $\widetilde{k}_{h,t} = k_{h,t}/G_C^t$, $\widetilde{k}_{b,t} = k_{b,t}/G_C^t$, $\widetilde{q}_t = q_t/G_Q^t$, $\widetilde{h}_{o,t} = h_{o,t}/G_H^t$, $\widetilde{h}_{n,t} = h_{n,t}/G_H^t$, $\widetilde{p}_{l,t} = p_{l,t}/G_C^t$, $\widetilde{R}_{l,t} = R_{l,t}/G_C^t$, $\widetilde{b}_t = b_t/G_C^t$, $\widetilde{TC}_t = TC_t/G_C^t$, $\widetilde{w}_{c,t} = w_{c,t}/G_C^t$, $\widetilde{w}_{h,t} = w_{h,t}/G_C^t$, $\widetilde{R}_{c,t} = R_{c,t}G_{AK}^t$.

Note that

$$DIV_t = \frac{X_t - 1}{X_t} Y_t + \frac{X_{wc,t} - 1}{X_{wc,t}} w_{c,t} n_{c,t} + \frac{X_{wh,t} - 1}{X_{wh,t}} w_{h,t} n_{h,t}, \tag{4.57}$$

$$TC_t = \frac{\phi}{2G_H} \left(\frac{h_{o,t}}{h_{o,t-1}} - G_H \right)^2 h_{o,t-1} q_t f_t, \tag{4.58}$$

$$\Phi_t = \frac{\phi_{kc}}{2G_{KC}} \left(\frac{k_{c,t}}{k_{c,t-1}} - G_{KC} \right)^2 \frac{k_{c,t-1}}{G_{AK}^t} + \frac{\phi_{kh}}{2G_C} \left(\frac{k_{h,t}}{k_{h,t-1}} - G_C \right)^2 k_{h,t-1}, \tag{4.59}$$

$$a(z_{c,t}) = \widetilde{R}_c(\varpi z_{c,t}^2/2 + (1-\varpi) z_{c,t} + (\varpi/2 - 1)), \tag{4.60}$$

$$a(z_{h,t}) = R_h(\varpi z_{h,t}^2/2 + (1-\varpi) z_{h,t} + (\varpi/2 - 1)), \tag{4.61}$$

and the detrended counterparts of TC_t and Φ_t take the following form:

$$\widetilde{TC}_t = \frac{\phi}{2} \left(\frac{\widetilde{h}_{o,t}}{\widetilde{h}_{o,t-1}} - 1 \right)^2 \widetilde{h}_{o,t-1} \widetilde{q}_t f_t, \tag{4.62}$$

$$\widetilde{\Phi}_t = \frac{\phi_{kc}}{2} \left(\frac{\widetilde{k}_{c,t}}{\widetilde{k}_{c,t-1}} - 1 \right)^2 \widetilde{k}_{c,t-1} + \frac{\phi_{kh}}{2} \left(\frac{\widetilde{k}_{h,t}}{\widetilde{k}_{h,t-1}} - 1 \right)^2 \widetilde{k}_{h,t-1}. \tag{4.63}$$

The choice variables for unconstrained households are the following: $b_t, h_{o,t}, h_{n,t}, k_{c,t}, k_{h,t}, n_{c,t}, n_{h,t}, l_t$. The first-order conditions of unconstrained households' maximization problem are:

$$b_t: \quad u_{c,t} = \beta G_C E_t \left(u_{c,t+1} \frac{R_{L,t}}{\pi_{t+1}} \right), \tag{4.64}$$

$$\begin{aligned} h_{o,t}: \quad & u_{c,t} q_t f_t \left[1 + \frac{\phi}{G_H} \left(\frac{h_{o,t}}{h_{o,t-1}} - G_H \right) \right] \\ & = u_{h_o,t} + \beta G_C (1-\delta_h) E_t(u_{c,t+1} q_{t+1} f_{t+1}) + \beta G_C E_t \left[u_{c,t+1} q_{t+1} f_{t+1} \frac{\phi}{2G_H} \left(\frac{h_{o,t+1}^2}{h_{o,t}^2} - G_H^2 \right) \right], \end{aligned} \tag{4.65}$$

$$h_{n,t}: \quad u_{c,t} q_t = u_{h_n,t} + \beta G_C (1-\delta_h) E_t(u_{c,t+1} q_{t+1} f_{t+1}), \tag{4.66}$$

$$n_{c,t}: \quad u_{n_c,t} = u_{c,t} w_{c,t} / X_{wc,t}, \tag{4.67}$$

$$n_{h,t}: \quad u_{n_h,t} = u_{c,t} w_{h,t} / X_{wh,t}, \tag{4.68}$$

$$k_{c,t}: \quad u_{c,t}\left[\frac{1}{A_{k,t}}+\frac{\phi_{kc}}{G_{KC}}\left(\frac{k_{c,t}}{k_{c,t-1}}-G_{KC}\right)\frac{1}{G_{AK}^{t}}\right]$$

$$=\beta G_C E_t\left[u_{c,t+1}\left(R_{c,t+1}z_{c,t+1}+\frac{1-\delta_{kc}}{A_{k,t+1}}+\frac{\phi_{kc}}{2G_{KC}}\left(\frac{k_{c,t+1}^2}{k_{c,t}^2}-G_{KC}^2\right)\frac{1}{G_{AK}^{t+1}}-\frac{a(z_{c,t+1})}{A_{k,t+1}}\right)\right], \tag{4.69}$$

$$k_{h,t}: \quad u_{c,t}\left[1+\frac{\phi_{kh}}{G_C}\left(\frac{k_{h,t}}{k_{h,t-1}}-G_C\right)\right]$$

$$=\beta G_C E_t\left[u_{c,t+1}\left(R_{h,t+1}z_{h,t+1}+1-\delta_{kh}+\frac{\phi_{kh}}{2G_C}\left(\frac{k_{h,t+1}^2}{k_{h,t}^2}-G_C^2\right)-a(z_{h,t+1})\right)\right], \tag{4.70}$$

$$k_{b,t}: \quad u_{c,t}(p_{b,t}-1)=0, \tag{4.71}$$

$$l_t: \quad u_{c,t}p_{l,t}=\beta G_C E_t\left[u_{c,t+1}(p_{l,t+1}+R_{l,t+1})\right], \tag{4.72}$$

which can be transformed in the following way:

$$\widetilde{u}_{c,t}=\beta E_t\left(\widetilde{u}_{c,t+1}\frac{R_{L,t}}{\pi_{t+1}}\right), \tag{4.73}$$

$$\widetilde{u}_{c,t}\widetilde{q}_t f_t\left[1+\phi\left(\frac{\widetilde{h}_{o,t}}{\widetilde{h}_{o,t-1}}-1\right)\right]$$

$$=\widetilde{u}_{h_o,t}+\beta G_Q(1-\delta_h)E_t(\widetilde{u}_{c,t+1}\widetilde{q}_{t+1}f_{t+1})+\beta G_Q E_t\left[\widetilde{u}_{c,t+1}\widetilde{q}_{t+1}f_{t+1}\frac{\phi}{2}G_H\left(\frac{\widetilde{h}_{o,t+1}^2}{\widetilde{h}_{o,t}^2}-1\right)\right], \tag{4.74}$$

$$\widetilde{u}_{c,t}\widetilde{q}_t=\widetilde{u}_{h_n,t}+\beta G_Q(1-\delta_h)E_t(\widetilde{u}_{c,t+1}\widetilde{q}_{t+1}f_{t+1}), \tag{4.75}$$

$$u_{n_c,t}=\widetilde{u}_{c,t}\widetilde{w}_{c,t}/X_{wc,t}, \tag{4.76}$$

$$u_{n_h,t}=\widetilde{u}_{c,t}\widetilde{w}_{h,t}/X_{wh,t}, \tag{4.77}$$

$$\widetilde{u}_{c,t}\left[\frac{1}{a_{k,t}}+\phi_{kc}\left(\frac{\widetilde{k}_{c,t}}{\widetilde{k}_{c,t-1}}-1\right)\right]$$

$$=\frac{\beta}{G_{AK}}E_t\left[\widetilde{u}_{c,t+1}\left(\widetilde{R}_{c,t+1}z_{c,t+1}+\frac{1-\delta_{kc}}{a_{k,t+1}}+\frac{\phi_{kc}G_{KC}}{2}\left(\frac{\widetilde{k}_{c,t+1}^2}{\widetilde{k}_{c,t}^2}-1\right)-\frac{a(z_{c,t+1})}{a_{k,t+1}}\right)\right], \tag{4.78}$$

$$
\begin{aligned}
&\widetilde{u}_{c,t}\left[1+\phi_{kh}\left(\frac{\widetilde{k}_{h,t}}{\widetilde{k}_{h,t-1}}-1\right)\right] \\
=&\beta E_t\left[\widetilde{u}_{c,t+1}\left(R_{h,t+1}z_{h,t+1}+1-\delta_{kh}+\frac{\phi_{kh}G_C}{2}\left(\frac{\widetilde{k}^2_{h,t+1}}{\widetilde{k}^2_{h,t}}-1\right)-a(z_{h,t+1})\right)\right], \qquad (4.79)\\
&p_{b,t}=1, \qquad (4.80)\\
&\widetilde{u}_{c,t}\widetilde{p}_{l,t}=\beta G_C E_t\left[\widetilde{u}_{c,t+1}(\widetilde{p}_{l,t+1}+\widetilde{R}_{l,t+1})\right]. \qquad (4.81)
\end{aligned}
$$

The first-order conditions w.r.t. $z_{c,t}$ and $z_{h,t}$ are as follows:

$$
\begin{aligned}
R_{c,t} &= \frac{a'(z_{c,t})}{A_{k,t}},\\
R_{h,t} &= a'(z_{h,t}),
\end{aligned}
$$

which can be transformed as:

$$
\begin{aligned}
\widetilde{R}_{c,t} &= \frac{a'(z_{c,t})}{a_{k,t}} = \widetilde{R}_c(\varpi z_{c,t}+(1-\varpi)),\\
R_{h,t} &= a'(z_{h,t}) = R_h(\varpi z_{h,t}+(1-\varpi)).
\end{aligned}
$$

where $\widetilde{R}_c$ and R_h are the steady state values of the rental rates of the two types of capital.

4.8.2 Constrained Households

The lifetime utility of constrained households is given by:

$$
\begin{aligned}
V'_t =&E_0\sum_{t=0}^{\infty}(\beta' G_C)^t z_t\left[\frac{G_C-\epsilon'}{G_C-\beta'\epsilon' G_C}\ln(c'_t-\epsilon' c'_{t-1})+j_{o,t}\ln(h'_{o,t})+j_{n,t}\ln(h'_{n,t})\right.\\
&\left.-\frac{\tau_t}{1+\eta'}\left((n'_{c,t})^{1+\xi'}+(n'_{h,t})^{1+\xi'}\right)^{\frac{1+\eta'}{1+\xi'}}\right]. \qquad (4.82)
\end{aligned}
$$

The marginal utility of consumption is:

$$u'_{c,t} = \left(\frac{G_C - \epsilon'}{G_C - \beta'\epsilon' G_C}\right)\left(\frac{z_t}{c'_t - \epsilon' c'_{t-1}} - \frac{\beta'\epsilon' G_C z_{t+1}}{c'_{t+1} - \epsilon' c'_t}\right). \tag{4.83}$$

The marginal utility of old house is:

$$u'_{h_o,t} = \frac{z_t j_{o,t}}{h'_{o,t}}, \tag{4.84}$$

and the marginal utility of new house is:

$$u'_{h_n,t} = \frac{z_t j_{n,t}}{h'_{n,t}}. \tag{4.85}$$

The marginal disutilities of working in the goods and the housing sectors are:

$$u'_{n_c,t} = z_t \tau_t \left((n'_{c,t})^{1+\xi'} + (n'_{h,t})^{1+\xi'}\right)^{\frac{\eta'-\xi'}{1+\xi'}} (n'_{c,t})^{\xi'}, \tag{4.86}$$

$$u'_{n_h,t} = z_t \tau_t \left((n'_{c,t})^{1+\xi'} + (n'_{h,t})^{1+\xi'}\right)^{\frac{\eta'-\xi'}{1+\xi'}} (n'_{h,t})^{\xi'}. \tag{4.87}$$

The transformed marginal utility of consumption $\widetilde{u}'_{c,t} = u'_{c,t} G_C^t$ is stationary around the steady state and is equal to:

$$\widetilde{u}'_{c,t} = \left(\frac{G_C - \epsilon'}{G_C - \beta'\epsilon' G_C}\right)\left(\frac{z_t}{\widetilde{c}'_t - \frac{\epsilon'}{G_C}\widetilde{c}'_{t-1}} - \frac{\beta'\epsilon' z_{t+1}}{\widetilde{c}'_{t+1} - \frac{\epsilon'}{G_C}\widetilde{c}'_t}\right). \tag{4.88}$$

Transformed consumption, $\widetilde{c}'_t = c'_t / G_C^t$, and the scaled marginal utility of consumption $\widetilde{u}'_{c,t}$:

$$\widetilde{u}'_c = z\left(\frac{G_C - \epsilon'}{G_C - \beta'\epsilon' G_C}\right)\left(\frac{1}{1 - \frac{\epsilon'}{G_C}} - \frac{\beta'\epsilon'}{1 - \frac{\epsilon'}{G_C}}\right)\frac{1}{\widetilde{c}'} = \frac{1}{\widetilde{c}'}, \tag{4.89}$$

are both stationary in the steady state.

The marginal utilities of housing $u'_{h_o,t} = z_t j_{o,t}/h'_{o,t}$ and $u'_{h_n,t} = z_t j_{n,t}/h'_{n,t}$ both decline at the rate G_H. Therefore the transformed marginal utilities $\widetilde{u}'_{h_o,t} = u'_{h_o,t}G_H^t$ and $\widetilde{u}'_{h_n,t} = u'_{h_n,t}G_H^t$ are stationary around the steady state and are equal to:

$$\widetilde{u}'_{h_o,t} = \frac{z_t j_{o,t}}{\widetilde{h}_{o,t}}, \quad \widetilde{u}'_{h_n,t} = \frac{z_t j_{n,t}}{\widetilde{h}_{n,t}}. \tag{4.90}$$

In the steady state, $\widetilde{u}'_{h_o} = j_o/\widetilde{h}'_o$ and $\widetilde{u}'_{h_n} = j_n/\widetilde{h}'_n$.

Notice that hours worked in the two sectors, $n'_{c,t}$ and $n'_{h,t}$, are stationary already in the level economy.

Constrained households' budget constraint is given by:

$$\begin{aligned} & c'_t + q_t[f_t(h'_{o,t} - (1-\delta_h)h'_{o,t-1} - (1-\delta_h)h'_{n,t-1}) + h'_{n,t}] + \frac{R_{L,t-1}b'_{t-1}}{\pi_t} + TC'_t \\ =& \frac{w'_{c,t}n'_{c,t}}{X_{wc,t}} + \frac{w'_{h,t}n'_{h,t}}{X_{wh,t}} + DIV'_t + b'_t, \end{aligned} \tag{4.91}$$

which can be transformed as follows:

$$\widetilde{c}'_t + \widetilde{q}_t\left[f_t\left(\widetilde{h}'_{o,t} - \frac{(1-\delta_h)\widetilde{h}'_{o,t-1}}{G_H} - \frac{(1-\delta_h)\widetilde{h}'_{n,t-1}}{G_H}\right) + \widetilde{h}'_{n,t}\right] + \frac{R_{L,t-1}}{\pi_t}\frac{\widetilde{b}'_{t-1}}{G_C} + \widetilde{TC}'_t = \widetilde{w}'_{c,t}n'_{c,t} + \widetilde{w}'_{h,t}n'_{h,t} + \widetilde{b}'_t, \tag{4.92}$$

where

$$TC'_t = \frac{\phi'}{2G_H}\left(\frac{h'_{o,t}}{h'_{o,t-1}} - G_H\right)^2 h'_{o,t-1}q_t f_t, \tag{4.93}$$

$$DIV'_t = \frac{X_{wc,t}-1}{X_{wc,t}}w'_{c,t}n'_{c,t} + \frac{X_{wh,t}-1}{X_{wh,t}}w'_{h,t}n'_{h,t}. \tag{4.94}$$

The borrowing constraint is given by:

$$b'_t = mE_t\left(\frac{f_{t+1}q_{t+1}(1-\delta_h)(h'_{o,t} + h'_{n,t})\pi_{t+1}}{R_{S,t}}\right), \tag{4.95}$$

which can be transformed as:

$$\widetilde{b}'_t = mE_t\left(\frac{G_Q f_{t+1}\widetilde{q}_{t+1}(1-\delta_h)(\widetilde{h}'_{o,t}+\widetilde{h}'_{n,t})\pi_{t+1}}{R_{S,t}}\right). \tag{4.96}$$

The choice variables for constrained households are the following: $b'_t, h'_{o,t}, h'_{n,t}, n'_{c,t}, n'_{h,t}$. The first-order conditions of constrained households' maximization problem are:

$$b'_t: \quad u'_{c,t} = \beta' G_C E_t\left(\frac{u'_{c,t+1}R_{L,t}}{\pi_{t+1}}\right) + \lambda_t, \tag{4.97}$$

$$\begin{aligned} h'_{o,t}: \quad & u'_{c,t}q_t f_t\left[1+\frac{\phi'}{G_H}\left(\frac{h'_{o,t}}{h'_{o,t-1}} - G_H\right)\right] \\ &= u'_{h_o,t} + \beta' G_C(1-\delta_h)E_t(u'_{c,t+1}q_{t+1}f_{t+1}) + \beta' G_C E_t\left[u_{c,t+1}q_{t+1}f_{t+1}\frac{\phi'}{2G_H}\left(\frac{h'^2_{o,t-1}}{h'^2_{o,t}} - G_H^2\right)\right] \\ &\quad + E_t\left(\lambda_t\frac{m(1-\delta_h)q_{t+1}f_{t+1}\pi_{t+1}}{R_{S,t}}\right), \end{aligned} \tag{4.98}$$

$$h'_{n,t}: \quad u'_{c,t}q_t = u'_{h_n,t} + \beta' G_C(1-\delta_h)E_t(u'_{c,t+1}q_{t+1}f_{t+1}) + E_t\left(\lambda_t\frac{m(1-\delta_h)q_{t+1}f_{t+1}\pi_{t+1}}{R_{S,t}}\right), \tag{4.99}$$

$$n'_{c,t}: \quad u'_{n_c,t} = u'_{c,t}w'_{c,t}/X_{wc,t}, \tag{4.100}$$

$$n'_{h,t}: \quad u'_{n_h,t} = u'_{c,t}w'_{h,t}/X_{wh,t}. \tag{4.101}$$

which can be transformed in the following way:

$$\widetilde{u}'_{c,t} = \beta' E_t\left(\frac{\widetilde{u}'_{c,t+1}R_{L,t}}{\pi_{t+1}}\right) + \widetilde{\lambda}_t, \tag{4.102}$$

$$\begin{aligned} & \widetilde{u}'_{c,t}\widetilde{q}_t f_t\left[1+\phi'\left(\frac{\widetilde{h}'_{o,t}}{\widetilde{h}'_{o,t-1}} - 1\right)\right] \\ =& \widetilde{u}'_{h_o,t} + \beta'(1-\delta_h)E_t(\widetilde{u}'_{c,t+1}\widetilde{q}_{t+1}f_{t+1}G_Q) + \beta' G_Q E_t\left[\widetilde{u}'_{c,t+1}\widetilde{q}_{t+1}f_{t+1}\frac{\phi'}{2}G_H\left(\frac{\widetilde{h}'^2_{o,t+1}}{\widetilde{h}'^2_{o,t}} - 1\right)\right] \\ & + E_t\left(\widetilde{\lambda}_t\frac{m(1-\delta_h)\widetilde{q}_{t+1}f_{t+1}\pi_{t+1}G_Q}{R_{S,t}}\right), \end{aligned} \tag{4.103}$$

$$\widetilde{u}'_{c,t}\widetilde{q}_t = \widetilde{u}'_{h_n,t} + \beta'(1-\delta_h)E_t(\widetilde{u}'_{c,t+1}\widetilde{q}_{t+1}f_{t+1}G_Q) + E_t\left(\widetilde{\lambda}_t \frac{m(1-\delta_h)\widetilde{q}_{t+1}f_{t+1}\pi_{t+1}G_Q}{R_{S,t}}\right), \tag{4.104}$$

$$u'_{n_c,t} = \widetilde{u}'_{c,t}\widetilde{w}'_{c,t}/X_{wc,t}, \tag{4.105}$$

$$u'_{n_h,t} = \widetilde{u}'_{c,t}\widetilde{w}'_{h,t}/X_{wh,t}. \tag{4.106}$$

4.8.3 Firms

Wholesale firms solve the following maximization problem:

$$\max \frac{Y_t}{X_t} + q_t IH_t - \left(\sum_{i=c,h} w_{i,t}n_{i,t} + \sum_{i=c,h} w'_{i,t}n'_{i,t} + R_{c,t}z_{c,t}k_{c,t-1} + R_{h,t}z_{h,t}k_{h,t-1} + p_{b,t}k_{b,t} + R_{l,t}l_{t-1}\right).$$

The two production technologies are:

$$Y_t = [A_{c,t}(n^{\alpha}_{c,t}n'^{1-\alpha}_{c,t})]^{1-\mu_c}(z_{c,t}k_{c,t-1})^{\mu_c}, \tag{4.107}$$

$$IH_t = [A_{h,t}(n^{\alpha}_{h,t}n'^{1-\alpha}_{h,t})]^{1-\mu_h-\mu_b-\mu_l}(z_{h,t}k_{h,t-1})^{\mu_h}k^{\mu_b}_{b,t}l^{\mu_l}_{t-1}, \tag{4.108}$$

which can be transformed as:

$$\widetilde{Y}_t = [a_{c,t}(n^{\alpha}_{c,t}n'^{1-\alpha}_{c,t})]^{1-\mu_c}\left(\frac{z_{c,t}\widetilde{k}_{c,t-1}}{G_{KC}}\right)^{\mu_c}, \tag{4.109}$$

$$\widetilde{IH}_t = [a_{h,t}(n^{\alpha}_{h,t}n'^{1-\alpha}_{h,t})]^{1-\mu_h-\mu_b-\mu_l}\left(\frac{z_{h,t}k_{h,t-1}}{G_C}\right)^{\mu_h}\widetilde{k}^{\mu_b}_{b,t}. \tag{4.110}$$

The first-order conditions are:

$$n_{c,t}: \quad (1-\mu_c)\alpha\frac{Y_t}{X_t n_{c,t}} = w_{c,t}, \tag{4.111}$$

$$n'_{c,t}: \quad (1-\mu_c)(1-\alpha)\frac{Y_t}{X_t n'_{c,t}} = w'_{c,t}, \tag{4.112}$$

$$n_{h,t}: \quad (1-\mu_h-\mu_b-\mu_l)\alpha\frac{q_t IH_t}{n_{h,t}} = w_{h,t}, \tag{4.113}$$

$$n'_{h,t}: \quad (1-\mu_h-\mu_b-\mu_l)(1-\alpha)\frac{q_t IH_t}{n'_{h,t}} = w'_{h,t}, \tag{4.114}$$

$$k_{c,t-1}: \quad \mu_c \frac{Y_t}{X_t k_{c,t-1}} = R_{c,t} z_{c,t}, \tag{4.115}$$

$$k_{h,t-1}: \quad \mu_h \frac{q_t IH_t}{k_{h,t-1}} = R_{h,t} z_{h,t}, \tag{4.116}$$

$$k_{b,t}: \quad \mu_b q_t IH_t / k_{b,t} = p_{b,t} = 1, \tag{4.117}$$

$$l_{t-1}: \quad \mu_l q_t IH_t = R_{l,t}, \tag{4.118}$$

which can be transformed as:

$$(1-\mu_c)\alpha \frac{\widetilde{Y}_t}{X_t n_{c,t}} = \widetilde{w}_{c,t}, \tag{4.119}$$

$$(1-\mu_c)(1-\alpha)\frac{\widetilde{Y}_t}{X_t n'_{c,t}} = \widetilde{w}'_{c,t}, \tag{4.120}$$

$$(1-\mu_h-\mu_b-\mu_l)\alpha \frac{\widetilde{q}_t \widetilde{IH}_t}{n_{h,t}} = \widetilde{w}_{h,t}, \tag{4.121}$$

$$(1-\mu_h-\mu_b-\mu_l)(1-\alpha)\frac{\widetilde{q}_t \widetilde{IH}_t}{n'_{h,t}} = \widetilde{w}'_{h,t}, \tag{4.122}$$

$$\frac{\mu_c G_{KC}}{X_t}\frac{\widetilde{Y}_t}{\widetilde{k}_{c,t-1}} = \widetilde{R}_{c,t} z_{c,t}, \tag{4.123}$$

$$\mu_h G_C \frac{\widetilde{q}_t \widetilde{IH}_t}{\widetilde{k}_{h,t-1}} = R_{h,t} z_{h,t}, \tag{4.124}$$

$$\mu_b \widetilde{q}_t \widetilde{IH}_t / \widetilde{k}_{b,t} = 1, \tag{4.125}$$

$$\mu_l \widetilde{q}_t \widetilde{IH}_t = \widetilde{R}_{l,t}. \tag{4.126}$$

4.8.4 Wage Stickiness

As in Iacoviello and Neri (2010), two types of households supply their homogenous labor services to labor unions. There are two unions for each sector, each one acting in the interest of either type of households. The unions differentiate labor services, set nominal wages subject to a Calvo scheme and offer labor services to intermediate labor packers who assemble the differentiated labor services into the homogenous labor composites n_c, n'_c, n_h, n'_h and provide wholesale firms with labor services. Partial indexation of nominal wages to past inflation yields the following four wage Phillips curves:

$$\ln \omega_{c,t} - \iota_{wc} \ln \pi_{t-1} = \beta G_C (E_t \ln \omega_{c,t+1} - \iota_{wc} \ln \pi_t) - \epsilon_{wc} \ln(X_{wc,t}/X_{wc}), \tag{4.127}$$

$$\ln \omega'_{c,t} - \iota_{wc} \ln \pi_{t-1} = \beta' G_C (E_t \ln \omega'_{c,t+1} - \iota_{wc} \ln \pi_t) - \epsilon'_{wc} \ln(X'_{wc,t}/X_{wc}), \tag{4.128}$$

$$\ln \omega_{h,t} - \iota_{wh} \ln \pi_{t-1} = \beta G_C (E_t \ln \omega_{h,t+1} - \iota_{wh} \ln \pi_t) - \epsilon_{wh} \ln(X_{wh,t}/X_{wh}), \tag{4.129}$$

$$\ln \omega'_{h,t} - \iota_{wh} \ln \pi_{t-1} = \beta' G_C (E_t \ln \omega'_{h,t+1} - \iota_{wh} \ln \pi_t) - \epsilon'_{wh} \ln(X'_{wh,t}/X_{wh}), \tag{4.130}$$

with $\omega_{i,t}$ nominal wage inflation, that is $\omega_{i,t} = \pi_t w_{i,t}/w_{i,t-1}$ for each sector/household pair, and

$$\epsilon_{wc} = (1-\theta_{wc})(1-\beta G_C \theta_{wc})/\theta_{wc}, \tag{4.131}$$

$$\epsilon'_{wc} = (1-\theta_{wc})(1-\beta' G_C \theta_{wc})/\theta_{wc}, \tag{4.132}$$

$$\epsilon_{wh} = (1-\theta_{wh})(1-\beta G_C \theta_{wh})/\theta_{wh}, \tag{4.133}$$

$$\epsilon'_{wh} = (1-\theta_{wh})(1-\beta' G_C \theta_{wh})/\theta_{wh}. \tag{4.134}$$

4.8.5 Price Stickiness

Price stickiness in the consumption-business investment sector is introduced by assuming monopolistic competition at the retail level, implicit costs of adjusting nominal prices following Calvo-style contracts and partial indexation to lagged inflation of those prices that cannot be reoptimized. The resulting inflation equation is:

$$\ln \pi_t - \iota_\pi \ln \pi_{t-1} = \beta G_C (E_t \ln \pi_{t+1} - \iota_\pi \ln \pi_t) - \epsilon_\pi \ln(X_t/X) + u_{p,t}, \tag{4.135}$$

where $\epsilon_\pi = (1-\theta_\pi)(1-\beta G_C \theta_\pi)/\theta_\pi$.

4.8.6 Monetary Policy

It is assumed that yields of different maturities are driven by two zero-mean latent factors L_t and S_t:

$$\ln R_{j,t} = \lambda_j + \mathbf{\Lambda}_j \mathbf{F}_t, \tag{4.136}$$

where λ_j is a constant and $\mathbf{F}_t = (L_t, S_t)'$. The factor loadings on these two yield curve components, $\mathbf{\Lambda}_j$, are modeled as:

$$\mathbf{\Lambda}_j = \begin{pmatrix} 1, & \frac{1 - e^{-\delta j}}{\delta j} \end{pmatrix}, \tag{4.137}$$

where δ denotes a decay parameter and j maturity. The loadings on the first factor, L_t, are constant across the maturity spectrum. A positive shock to this factor induces an essentially parallel shift in the term structure that boosts the level of the whole yield curve, so the L_t factor is often called a “level" factor. The loadings on the second factor, S_t, decrease monotonically with the maturity. A positive shock to this factor increases short-term yields by much more than the long-term yields, thus S_t is termed the “slope" factor.

The dynamics of the these latent factors are specified as:

$$L_t = \gamma_L L_{t-1} + (1 - \gamma_L) \ln \pi_t + \varepsilon_{L,t}, \tag{4.138}$$

$$S_t = \gamma_S S_{t-1} + (1 - \gamma_S)\gamma_\pi \ln \pi_t + (1 - \gamma_S)\gamma_Y \ln \left(\frac{GDP_t}{G_C GDP_{t-1}} \right) + u_{S,t}, \tag{4.139}$$

where $u_{S,t} = \rho_S u_{S,t-1} + \varepsilon_{S,t}$; $\varepsilon_{L,t}$ and $\varepsilon_{S,t}$ are independently and identically distributed shocks to the level factor and to the slope factor respectively with variances σ_L^2 and σ_S^2. GDP_t is defined as the sum of the value added of two sector, i.e., $GDP_t = Y_t - k_{b,t} + \overline{q} IH_t$.

The interest rate data used in this study are the nominal 3-month interest rate and zero-coupon yields of maturities 4, 12, 24, 36, 48, and 60 quarters. In the setup, the short-term nominal rate $R_{S,t}$ refers to the 1-quarter rate $R_{1,t}$ and the long-term nominal rate $R_{L,t}$ refers to the 60-quarter rate $R_{60,t}$.

4.8.7 Market Clearing

The market clearing conditions are:

$$C_t + IK_{c,t}/A_{k,t} + IK_{h,t} + k_{b,t} = Y_t - \Phi_t, \tag{4.140}$$

$$h_{n,t} + h'_{n,t} = IH_t, \tag{4.141}$$

$$(h_{o,t} - (1-\delta_h)h_{o,t-1} - (1-\delta_h)h_{n,t-1}) + (h'_{o,t} - (1-\delta_h)h'_{o,t-1} - (1-\delta_h)h'_{n,t-1}) = 0, \tag{4.142}$$

$$b_t + b'_t = 0, \tag{4.143}$$

which can be transformed as:

$$\widetilde{C}_t + \widetilde{IK}_{c,t}/a_{k,t} + \widetilde{IK}_{h,t} + \widetilde{k}_{b,t} = \widetilde{Y}_t - \widetilde{\Phi}_t, \tag{4.144}$$

$$\widetilde{h}_{n,t} + \widetilde{h}'_{n,t} = \widetilde{IH}_t, \tag{4.145}$$

$$\left(\widetilde{h}_{o,t} - \frac{(1-\delta_h)\widetilde{h}_{o,t-1}}{G_H} - \frac{(1-\delta_h)\widetilde{h}_{n,t-1}}{G_H}\right) + \left(\widetilde{h}'_{o,t} - \frac{(1-\delta_h)\widetilde{h}'_{o,t-1}}{G_H} - \frac{(1-\delta_h)\widetilde{h}'_{n,t-1}}{G_H}\right) = 0, \tag{4.146}$$

$$\widetilde{b}_t + \widetilde{b}'_t = 0, \tag{4.147}$$

4.8.8 Linear Deterministic Trends

Suppose there are linear deterministic trends in the technologies A_c, A_h, and A_k. Let the corresponding gross growth rates be respectively:

$$\gamma_{AC}, \gamma_{AH}, \gamma_{AK}.$$

Because of these trends, the variables:

$$Y, c, c', \frac{k_c}{A_k}, k_h, qIH$$

all grow at a common rate along the balanced growth path.

The following is obtained from the production function,

$$\gamma_Y = \gamma_C = (1 - \mu_c)\gamma_{AC} + \mu_c \gamma_{KC}.$$

Since $\gamma_Y = \gamma_{KC} - \gamma_{AK}$, it follows that

$$\begin{aligned}
\gamma_Y &= \gamma_{AC} + \frac{\mu_c}{1-\mu_c}\gamma_{AK}, \\
\gamma_{KC} &= \gamma_{AC} + \frac{1}{1-\mu_c}\gamma_{AK}, \\
\gamma_{IH} &= (1 - \mu_h - \mu_b - \mu_l)\gamma_{AH} + (\mu_h + \mu_b)\gamma_Y, \\
&= (1 - \mu_h - \mu_b - \mu_l)\gamma_{AH} + (\mu_h + \mu_b)\left(\gamma_{AC} + \frac{\mu_c}{1-\mu_c}\gamma_{AK}\right), \\
\gamma_Q &= \gamma_Y - \gamma_{IH} = (1 - \mu_h - \mu_b)\left(\gamma_{AC} + \frac{\mu_c}{1-\mu_c}\gamma_{AK}\right) - (1 - \mu_h - \mu_b - \mu_l)\gamma_{AH}.
\end{aligned}$$

4.8.9 Steady State of the Model

Marginal utilities of consumption, old house, and new house are equal, respectively, to $1/\widetilde{c}$ (or $1/\widetilde{c}'$), $j_o/\widetilde{h}_o$ (or $j_o/\widetilde{h}'_o$), and $j_n/\widetilde{h}_n$ (or $j_n/\widetilde{h}'_n$). From the transformed consumption Euler equation, the steady state level of the real interest rate can be derived once $\overline{\pi} = 1$ has been imposed:

$$R_L = \frac{1}{\beta}. \tag{4.148}$$

From the Euler equations for the two capital stocks, the steady state values for the rental rates can be derived:

$$\widetilde{R}_c = \frac{G_{AK}}{\beta} - (1 - \delta_{kc}), \tag{4.149}$$

$$R_h = \frac{1}{\beta} - (1 - \delta_{kh}), \tag{4.150}$$

$$r \equiv \frac{R_L}{G_C} - 1. \tag{4.151}$$

Combining the Euler equation for k_c from the optimal demand for capital by firms in the good sector and the expression for $\widetilde{R}_c$, the following ratio is obtained:

$$\zeta_0 = \frac{\widetilde{k}_c}{\widetilde{Y}} = \frac{\mu_c G_{KC}}{\widetilde{R}_c X} = \frac{\beta \mu_c G_{KC}}{(G_{AK} - \beta(1 - \delta_{kc}))X}. \tag{4.152}$$

Combing the Euler equation for k_h from the optimal demand for capital by firms in the good sector and the expression for R_h the following ratio is obtained:

$$\zeta_1 = \frac{\widetilde{k}_h}{\widetilde{q}\widetilde{IH}} = \frac{\mu_h G_C}{R_h} = \frac{\beta \mu_h G_C}{1 - \beta(1 - \delta_{kh})}. \tag{4.153}$$

From the Euler equations for h_o and h_n:

$$\zeta_2 = \frac{f\widetilde{q}\widetilde{h}_o}{\widetilde{c}} = \frac{j_o}{1-\beta G_Q(1-\delta_h)}, \tag{4.154}$$

$$\zeta_3 = \frac{\widetilde{q}\widetilde{h}_n}{\widetilde{c}} = \frac{j_n}{1-f\beta G_Q(1-\delta_h)}. \tag{4.155}$$

From the Euler equations for h'_o, h'_n and b':

$$\zeta_4 = \frac{f\widetilde{q}\widetilde{h}'_o}{\widetilde{c}'} = \frac{j_o}{1-\beta' G_Q(1-\delta_h) - m(1-\beta'/\beta)G_Q(1-\delta_h)/R_S}, \tag{4.156}$$

$$\zeta_5 = \frac{\widetilde{q}\widetilde{h}'_n}{\widetilde{c}'} = \frac{j_n}{1-f\beta' G_Q(1-\delta_h) - fm(1-\beta'/\beta)G_Q(1-\delta_h)/R_S}, \tag{4.157}$$

$$\widetilde{\lambda} = \frac{1}{\widetilde{c}'}\left(1-\frac{\beta'}{\beta}\right). \tag{4.158}$$

Using the Walras' law $\widetilde{b}+\widetilde{b}'=0$ where $\widetilde{b}' = mG_Q(1-\delta_h)f\widetilde{q}(\widetilde{h}'_o+\widetilde{h}'_n)/R_S$, steady state repayment is $r\widetilde{b}'$:

$$r\widetilde{b}' = \frac{rmG_Q(1-\delta_h)}{R_S}f\widetilde{q}(\widetilde{h}'_o+\widetilde{h}'_n).$$

Define the adjusted depreciation rates:

$$\widetilde{\delta}_h = 1-\frac{1-\delta_h}{G_H}, \quad \widetilde{\delta}_{kc} = 1-\frac{1-\delta_{kc}}{G_{KC}}, \quad \widetilde{\delta}_{kh} = 1-\frac{1-\delta_{kh}}{G_{KH}}. \tag{4.159}$$

From above ratios and the budget constraints of the two types of households, the following is obtained:

$$\widetilde{k}_c = \zeta_0 \widetilde{Y}, \tag{4.160}$$

$$\widetilde{k}_h = \zeta_1 \widetilde{q}\widetilde{IH}, \tag{4.161}$$

$$f\widetilde{q}\widetilde{h}_o = \zeta_2 \widetilde{c}, \tag{4.162}$$

$$\widetilde{q}\widetilde{h}_n = \zeta_3 \widetilde{c}, \tag{4.163}$$

$$f\widetilde{q}\widetilde{h}'_o = \zeta_4 \widetilde{c}', \tag{4.164}$$

$$\widetilde{q}\widetilde{h}'_n = \zeta_5 \widetilde{c}', \tag{4.165}$$

$$\widetilde{h}_n + \widetilde{h}'_n = \widetilde{IH}, \tag{4.166}$$

$$\widetilde{\delta}_h \widetilde{h}_o - (1-\widetilde{\delta}_h)\widetilde{h}_n + \widetilde{\delta}_h \widetilde{h}'_o - (1-\widetilde{\delta}_h)\widetilde{h}'_n = 0, \tag{4.167}$$

$$\widetilde{c} + \widetilde{c}' + \widetilde{\delta}_{kc}\widetilde{k}_c + \widetilde{\delta}_{kh}\widetilde{k}_h + \widetilde{k}_b = \widetilde{Y}, \tag{4.168}$$

$$\begin{aligned} \widetilde{c} + \widetilde{q}\left[f\left(\widetilde{\delta}_h \widetilde{h}_o - (1-\widetilde{\delta}_h)\widetilde{h}_n\right) + \widetilde{h}_n\right] &= \frac{X-1}{X}\widetilde{Y} + r\widetilde{k}_c + r\widetilde{k}_h + \mu_l \widetilde{q}\widetilde{IH} + \widetilde{w}_c n_c + \widetilde{w}_h n_h \\ &+ \frac{rmG_Q(1-\delta_h)}{R_S} f\widetilde{q}(\widetilde{h}'_o + \widetilde{h}'_n), \end{aligned} \tag{4.169}$$

$$\widetilde{c}' + \widetilde{q}\left[f\left(\widetilde{\delta}_h \widetilde{h}'_o - (1-\widetilde{\delta}_h)\widetilde{h}'_n\right) + \widetilde{h}'_n\right] = \widetilde{w}'_c n'_c + \widetilde{w}'_h n'_h - \frac{rmG_Q(1-\delta_h)}{R_S} f\widetilde{q}(\widetilde{h}'_o + \widetilde{h}'_n). \tag{4.170}$$

The equations in the labor market satisfy from the demand side:

$$(1-\mu_c)\alpha \frac{\widetilde{Y}}{Xn_c} = \widetilde{w}_c, \tag{4.171}$$

$$(1-\mu_c)(1-\alpha) \frac{\widetilde{Y}}{Xn'_c} = \widetilde{w}'_c, \tag{4.172}$$

$$(1-\mu_h-\mu_b-\mu_l)\alpha \frac{\widetilde{q}\widetilde{IH}}{n_h} = \widetilde{w}_h, \tag{4.173}$$

$$(1-\mu_h-\mu_b-\mu_l)(1-\alpha) \frac{\widetilde{q}\widetilde{IH}}{n'_h} = \widetilde{w}'_h. \tag{4.174}$$

The total wage bill plus union dividends earned by each group follows:

$$\widetilde{w}_c n_c + \widetilde{w}_h n_h = \alpha \left((1-\mu_c)\frac{\widetilde{Y}}{X} + (1-\mu_h-\mu_b-\mu_l)\widetilde{q}\widetilde{IH} \right), \tag{4.175}$$

$$\widetilde{w}'_c n'_c + \widetilde{w}'_h n'_h = (1-\alpha) \left((1-\mu_c)\frac{\widetilde{Y}}{X} + (1-\mu_h-\mu_b-\mu_l)\widetilde{q}\widetilde{IH} \right). \tag{4.176}$$

Then,

$$\zeta_3\widetilde{c} + \zeta_5\widetilde{c}' = \widetilde{q}\widetilde{IH}, \tag{4.177}$$

$$\widetilde{c} + \widetilde{c}' + \widetilde{\delta}_{kc}\zeta_0\widetilde{Y} + \widetilde{\delta}_{kh}\zeta_1\widetilde{q}\widetilde{IH} + \mu_b\widetilde{q}\widetilde{IH} = \widetilde{Y}, \tag{4.178}$$

$$\begin{aligned} \widetilde{c} + \widetilde{\delta}_h\zeta_2\widetilde{c} + (1-f(1-\widetilde{\delta}_h))\zeta_3\widetilde{c} = {} & \frac{X-1}{X}\widetilde{Y} + r\zeta_o\widetilde{Y} + r\zeta_1\widetilde{q}\widetilde{IH} + \mu_l\widetilde{q}\widetilde{IH} \\ & + \alpha\left((1-\mu_c)\frac{\widetilde{Y}}{X} + (1-\mu_h-\mu_b-\mu_l)\widetilde{q}\widetilde{IH} \right) \\ & + \frac{rmG_Q(1-\delta_h)}{R_S}(\zeta_4 + f\zeta_5)\widetilde{c}', \end{aligned} \tag{4.179}$$

$$\begin{aligned} \widetilde{c}' + \widetilde{\delta}_h\zeta_4\widetilde{c}' + (1-f(1-\widetilde{\delta}_h))\zeta_5\widetilde{c}' = {} & (1-\alpha)\left((1-\mu_c)\frac{\widetilde{Y}}{X} + (1-\mu_h-\mu_b-\mu_l)\widetilde{q}\widetilde{IH} \right) \\ & - \frac{rmG_Q(1-\delta_h)}{R_S}(\zeta_4 + f\zeta_5)\widetilde{c}'. \end{aligned} \tag{4.180}$$

Using the formula for $\widetilde{q}\widetilde{IH}$:

$$\begin{aligned} & \left(1 + \widetilde{\delta}_h\zeta_2 + (1-f(1-\widetilde{\delta}_h))\zeta_3\right)\widetilde{c} \\ = & \left(\frac{X-1}{X} + r\zeta_0\right)\widetilde{Y} + (r\zeta_1 + \mu_l)(\zeta_3\widetilde{c} + \zeta_5\widetilde{c}') + \alpha\left((1-\mu_c)\frac{\widetilde{Y}}{X} + (1-\mu_h-\mu_b-\mu_l)(\zeta_3\widetilde{c} + \zeta_5\widetilde{c}') \right) \\ & + \frac{rmG_Q(1-\delta_h)}{R_S}(\zeta_4 + f\zeta_5)\widetilde{c}', \end{aligned} \tag{4.181}$$

$$\begin{aligned} & \left(1 + \widetilde{\delta}_h\zeta_4 + (1-f(1-\widetilde{\delta}_h))\zeta_5\right)\widetilde{c}' \\ = & (1-\alpha)\left((1-\mu_c)\frac{\widetilde{Y}}{X} + (1-\mu_h-\mu_b-\mu_l)(\zeta_3\widetilde{c} + \zeta_5\widetilde{c}') \right) - \frac{rmG_Q(1-\delta_h)}{R_S}(\zeta_4 + f\zeta_5)\widetilde{c}'. \end{aligned} \tag{4.182}$$

or equivalently,

$$\Big(1+\widetilde{\delta}_h\zeta_2+(1-f(1-\widetilde{\delta}_h))\zeta_3-(r\zeta_1+\mu_l)\zeta_3-\alpha(1-\mu_h-\mu_b-\mu_l)\zeta_3\Big)\,\widetilde{c}$$
$$-\left((r\zeta_1+\mu_l)\zeta_5+\alpha(1-\mu_h-\mu_b-\mu_l)\zeta_5+rmG_Q(1-\delta_h)(\zeta_4+f\zeta_5)/R_S\right)\widetilde{c}'$$
$$=\left(\frac{X-1}{X}+r\zeta_0+\frac{\alpha(1-\mu_c)}{X}\right)\widetilde{Y}, \tag{4.183}$$

and

$$\Big(1+\widetilde{\delta}_h\zeta_4+(1-f(1-\widetilde{\delta}_h))\zeta_5-(1-\alpha)(1-\mu_h-\mu_b-\mu_l)\zeta_5+rmG_Q(1-\delta_h)(\zeta_4+f\zeta_5)/R_S\Big)\,\widetilde{c}'$$
$$-\left((1-\alpha)(1-\mu_h-\mu_b-\mu_l)\zeta_3\right)\widetilde{c}=\frac{(1-\alpha)(1-\mu_c)}{X}\widetilde{Y}. \tag{4.184}$$

Redefining

$$\Xi_1\frac{\widetilde{c}}{\widetilde{Y}}-\Xi_2\frac{\widetilde{c}'}{\widetilde{Y}}=\Xi_3, \tag{4.185}$$
$$\Xi_4\frac{\widetilde{c}'}{\widetilde{Y}}-\Xi_5\frac{\widetilde{c}}{\widetilde{Y}}=\Xi_6, \tag{4.186}$$

delivers the following solution:

$$\frac{\widetilde{c}}{\widetilde{Y}}=\frac{\Xi_3\Xi_4+\Xi_2\Xi_6}{\Xi_1\Xi_4-\Xi_2\Xi_5}, \tag{4.187}$$
$$\frac{\widetilde{c}'}{\widetilde{Y}}=\frac{\Xi_1\Xi_6+\Xi_3\Xi_5}{\Xi_1\Xi_4-\Xi_2\Xi_5}, \tag{4.188}$$
$$\frac{\widetilde{q}\widetilde{IH}}{\widetilde{Y}}=\zeta_3\frac{\widetilde{c}}{\widetilde{Y}}+\zeta_5\frac{\widetilde{c}'}{\widetilde{Y}}. \tag{4.189}$$

Plug back to the rest equation,

$$(1+(\widetilde{\delta}_{kh}\zeta_1+\mu_b)\zeta_3)\frac{\Xi_3\Xi_4+\Xi_2\Xi_6}{\Xi_1\Xi_4-\Xi_2\Xi_5}+(1+(\widetilde{\delta}_{kh}\zeta_1+\mu_b)\zeta_5)\frac{\Xi_1\Xi_6+\Xi_3\Xi_5}{\Xi_1\Xi_4-\Xi_2\Xi_5}=1-\widetilde{\delta}_{kc}\zeta_0. \tag{4.190}$$

Rearrange the above equation and solve for f,

$$(1+(\widetilde{\delta}_{kh}\zeta_1+\mu_b)\zeta_3)(\Xi_3\Xi_4+\Xi_2\Xi_6)+(1+(\widetilde{\delta}_{kh}\zeta_1+\mu_b)\zeta_5)(\Xi_1\Xi_6+\Xi_3\Xi_5)=(1-\widetilde{\delta}_{kc}\zeta_0)(\Xi_1\Xi_4-\Xi_2\Xi_5). \tag{4.191}$$

In order to compute the levels of the variables in steady state, the value of hours worked is needed. The labor market equilibrium is of the kind:

$$(1-\mu_c)\alpha\frac{\widetilde{Y}}{X}=\widetilde{c}X_{wc}\left(n_c^{1+\xi}+n_h^{1+\xi}\right)^{\frac{\eta-\xi}{1+\xi}}n_c^{1+\xi}, \tag{4.192}$$

$$(1-\mu_h-\mu_b-\mu_l)\alpha\widetilde{q}\widetilde{IH}=\widetilde{c}X_{wh}\left(n_c^{1+\xi}+n_h^{1+\xi}\right)^{\frac{\eta-\xi}{1+\xi}}n_h^{1+\xi}, \tag{4.193}$$

so that the ratio of hours worked is:

$$\frac{n_h}{n_c}=\left(\frac{(1-\mu_h-\mu_b-\mu_l)\widetilde{q}\widetilde{IH}X}{(1-\mu_c)\widetilde{Y}}\right)^{\frac{1}{1+\xi}}, \tag{4.194}$$

where $X_{wc}=X_{wh}\equiv X_w$.

Plug back to get:

$$(1-\mu_c)\alpha\frac{\widetilde{Y}}{X\widetilde{c}}=X_w\left(1+\frac{(1-\mu_h-\mu_b-\mu_l)\widetilde{q}\widetilde{IH}X}{(1-\mu_c)\widetilde{Y}}\right)^{\frac{\eta-\xi}{1+\xi}}n_c^{1+\eta}, \tag{4.195}$$

knowing $\widetilde{c}/\widetilde{Y}$ and $\widetilde{q}\widetilde{IH}/\widetilde{Y}$, this can be solved for n_c,

$$n_c=\left(\frac{(1-\mu_c)\alpha\frac{\widetilde{Y}}{X\widetilde{c}}}{X_w\left(1+\frac{(1-\mu_h-\mu_b-\mu_l)\widetilde{q}\widetilde{IH}X}{(1-\mu_c)\widetilde{Y}}\right)^{\frac{\eta-\xi}{1+\xi}}}\right)^{\frac{1}{1+\eta}}, \tag{4.196}$$

$$n_h = \left(\frac{(1-\mu_c)\alpha \frac{\widetilde{Y}}{X\widetilde{c}}}{X_w \left(1 + \frac{(1-\mu_h-\mu_b-\mu_l)\widetilde{q}\widetilde{IHX}}{(1-\mu_c)\widetilde{Y}}\right)^{\frac{\eta-\xi}{1+\xi}}} \right)^{\frac{1}{1+\eta}} \left(\frac{(1-\mu_h-\mu_b-\mu_l)\widetilde{q}\widetilde{IHX}}{(1-\mu_c)\widetilde{Y}} \right)^{\frac{1}{1+\xi}}. \tag{4.197}$$

Similarly,

$$n'_c = \left(\frac{(1-\mu_c)(1-\alpha) \frac{\widetilde{Y}}{X\widetilde{c}'}}{X_w \left(1 + \frac{(1-\mu_h-\mu_b-\mu_l)\widetilde{q}\widetilde{IHX}}{(1-\mu_c)\widetilde{Y}}\right)^{\frac{\eta'-\xi'}{1+\xi'}}} \right)^{\frac{1}{1+\eta'}}, \tag{4.198}$$

$$n'_h = \left(\frac{(1-\mu_c)(1-\alpha) \frac{\widetilde{Y}}{X\widetilde{c}'}}{X_w \left(1 + \frac{(1-\mu_h-\mu_b-\mu_l)\widetilde{q}\widetilde{IHX}}{(1-\mu_c)\widetilde{Y}}\right)^{\frac{\eta'-\xi'}{1+\xi'}}} \right)^{\frac{1}{1+\eta'}} \left(\frac{(1-\mu_h-\mu_b-\mu_l)\widetilde{q}\widetilde{IHX}}{(1-\mu_c)\widetilde{Y}} \right)^{\frac{1}{1+\xi'}}. \tag{4.199}$$

Once the levels of hours worked by the two households in the two sectors are obtained, $\widetilde{Y}$, $\widetilde{k}_c$, and $\widetilde{k}_h$ can be computed,

$$\widetilde{Y} = \left[a_c \left(n_c^{\alpha} n_c'^{1-\alpha}\right)\right]^{1-\mu_c} \left(\frac{z_c \widetilde{k}_c}{G_{KC}}\right)^{\mu_c} = \left(n_c^{\alpha} n_c'^{1-\alpha}\right)^{1-\mu_c} \left(\frac{\widetilde{k}_c}{G_{KC}}\right)^{\mu_c}, \tag{4.200}$$

where $\widetilde{k}_c = \zeta_0 \widetilde{Y}$. As a result,

$$\widetilde{Y} = \left(n_c^{\alpha} n_c'^{1-\alpha}\right) \left(\frac{\zeta_0}{G_{KC}}\right)^{\frac{\mu_c}{1-\mu_c}}, \tag{4.201}$$

$$\widetilde{k}_c = \zeta_0 \left(n_c^{\alpha} n_c'^{1-\alpha}\right) \left(\frac{\zeta_0}{G_{KC}}\right)^{\frac{\mu_c}{1-\mu_c}}, \tag{4.202}$$

$$\widetilde{k}_h = \frac{\zeta_1 \widetilde{q}\widetilde{IH}}{\widetilde{Y}} \widetilde{Y} = \zeta_1 \left(\zeta_3 \frac{\Xi_3\Xi_4 + \Xi_2\Xi_6}{\Xi_1\Xi_4 - \Xi_2\Xi_5} + \zeta_5 \frac{\Xi_1\Xi_6 + \Xi_3\Xi_5}{\Xi_1\Xi_4 - \Xi_2\Xi_5} \right) \left(n_c^{\alpha} n_c'^{1-\alpha}\right) \left(\frac{\zeta_0}{G_{KC}}\right)^{\frac{\mu_c}{1-\mu_c}}, \tag{4.203}$$

$$\widetilde{k}_b = \mu_b \widetilde{q}\widetilde{IH} = \mu_b \left(\zeta_3 \frac{\Xi_3\Xi_4 + \Xi_2\Xi_6}{\Xi_1\Xi_4 - \Xi_2\Xi_5} + \zeta_5 \frac{\Xi_1\Xi_6 + \Xi_3\Xi_5}{\Xi_1\Xi_4 - \Xi_2\Xi_5} \right) \left(n_c^{\alpha} n_c'^{1-\alpha}\right) \left(\frac{\zeta_0}{G_{KC}}\right)^{\frac{\mu_c}{1-\mu_c}}. \tag{4.204}$$

$\widetilde{c}$ and $\widetilde{c}'$ can also be computed,

$$\widetilde{c} = \frac{\Xi_3\Xi_4 + \Xi_2\Xi_6}{\Xi_1\Xi_4 - \Xi_2\Xi_5}\widetilde{Y}, \tag{4.205}$$

$$\widetilde{c}' = \frac{\Xi_1\Xi_6 + \Xi_3\Xi_5}{\Xi_1\Xi_4 - \Xi_2\Xi_5}\widetilde{Y}, \tag{4.206}$$

$$\widetilde{q}\widetilde{IH} = \left(\zeta_3\frac{\Xi_3\Xi_4 + \Xi_2\Xi_6}{\Xi_1\Xi_4 - \Xi_2\Xi_5} + \zeta_5\frac{\Xi_1\Xi_6 + \Xi_3\Xi_5}{\Xi_1\Xi_4 - \Xi_2\Xi_5}\right)\widetilde{Y}. \tag{4.207}$$

Using the market clearing conditions for two types of houses,

$$0 = \widetilde{\delta}_h\widetilde{h}_o - (1-\widetilde{\delta}_h)\widetilde{h}_n + \widetilde{\delta}_h\widetilde{h}'_o - (1-\widetilde{\delta}_h)\widetilde{h}'_n, \tag{4.208}$$

$$\widetilde{IH} = \widetilde{h}_n + \widetilde{h}'_n, \tag{4.209}$$

$$\frac{\widetilde{h}_o}{\widetilde{h}_n} = \frac{\zeta_2}{\zeta_3}\frac{1}{f}, \tag{4.210}$$

$$\frac{\widetilde{h}'_o}{\widetilde{h}'_n} = \frac{\zeta_4}{\zeta_5}\frac{1}{f}, \tag{4.211}$$

the following can be obtained,

$$0 = \left(\widetilde{\delta}_h\frac{\zeta_2}{\zeta_3}\frac{1}{f} - (1-\widetilde{\delta}_h)\right)\widetilde{h}_n + \left(\widetilde{\delta}_h\frac{\zeta_4}{\zeta_5}\frac{1}{f} - (1-\widetilde{\delta}_h)\right)\widetilde{h}'_n, \tag{4.212}$$

$$\widetilde{IH} = \left(1 - \frac{\widetilde{\delta}_h\frac{\zeta_2}{\zeta_3}\frac{1}{f} - (1-\widetilde{\delta}_h)}{\widetilde{\delta}_h\frac{\zeta_4}{\zeta_5}\frac{1}{f} - (1-\widetilde{\delta}_h)}\right)\widetilde{h}_n, \tag{4.213}$$

$$\widetilde{h}_n = \frac{\widetilde{\delta}_h\frac{\zeta_4}{\zeta_5}\frac{1}{f} - (1-\widetilde{\delta}_h)}{\left(\widetilde{\delta}_h\frac{\zeta_4}{\zeta_5}\frac{1}{f} - (1-\widetilde{\delta}_h)\right) - \left(\widetilde{\delta}_h\frac{\zeta_2}{\zeta_3}\frac{1}{f} - (1-\widetilde{\delta}_h)\right)}\widetilde{IH}, \tag{4.214}$$

$$\widetilde{h}'_n = \widetilde{IH} - \widetilde{h}_n, \tag{4.215}$$

$$\widetilde{h}_o = \frac{\zeta_2}{\zeta_3}\frac{1}{f}\widetilde{h}_n, \tag{4.216}$$

$$\widetilde{h}'_o = \frac{\zeta_4}{\zeta_5}\frac{1}{f}\widetilde{h}'_n. \tag{4.217}$$

To find $\widetilde{q}$ and $\widetilde{IH}$ separately,

$$\widetilde{IH} = \left(n_h^{\alpha} n_h'^{1-\alpha}\right)^{1-\mu_h-\mu_b-\mu_l} \left(\frac{\widetilde{k}_h}{G_C}\right)^{\mu_h} \widetilde{k}_b^{\mu_b}, \tag{4.218}$$

and

$$\widetilde{q} = \widetilde{qIH} / \widetilde{IH}. \tag{4.219}$$

GDP_t is the sum of the value added of the two sectors, that is $GDP_t = Y_t - k_{b,t} + q_t IH_t = Y_t + (1 - \mu_b) q_t IH_t$. In the steady state, $\widetilde{GDP} = \widetilde{Y} + (1 - \mu_b)\widetilde{qIH}$.

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